

Arthur CHARPENTIER

bijgevoegde stelling

Temporal dependencies for natural events

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Modeling daily windspeed and temperature

Consider some stationary time series $(X_t)_{t \in \mathbb{Z}}$, and define $\gamma_X(h) = \text{cov}(X_t, X_{t-h})$ and its Fourier transform

$$f_X(\omega) = \frac{1}{2\pi} \sum_{h \in \mathbb{Z}} \gamma_X(h) \cos(\omega h).$$

Define autocorrelations as $\rho_X(h) = \gamma_X(h)/\gamma_X(0)$.

Long rang dependence: $\sum_{h \in \mathbb{Z}} |\rho_X(h)| = \infty$.

HASLETT & RASFERY (1989), ARFIMA processes for windspeed

$$(1 - L)^d \Phi(L) X_t = \Theta(L) \varepsilon_t, \text{ where } d \in (0, 1).$$

GARMA processes, HOSKING (1981, 1984),

$$(1 - 2uL + L^2)^{-\lambda} \Phi(L) X_t = \Theta(L) \varepsilon_t, \text{ where } \lambda \in (0, 1/2), |u| < 1.$$

$\omega = \cos^{-1}(u)$ is related to the seasonality of the series (e.g. $\omega = 2\pi/365$).

For daily windspeed $\hat{\lambda} = 0.126$ and $\hat{\sigma}^2 = 4.47^2$.

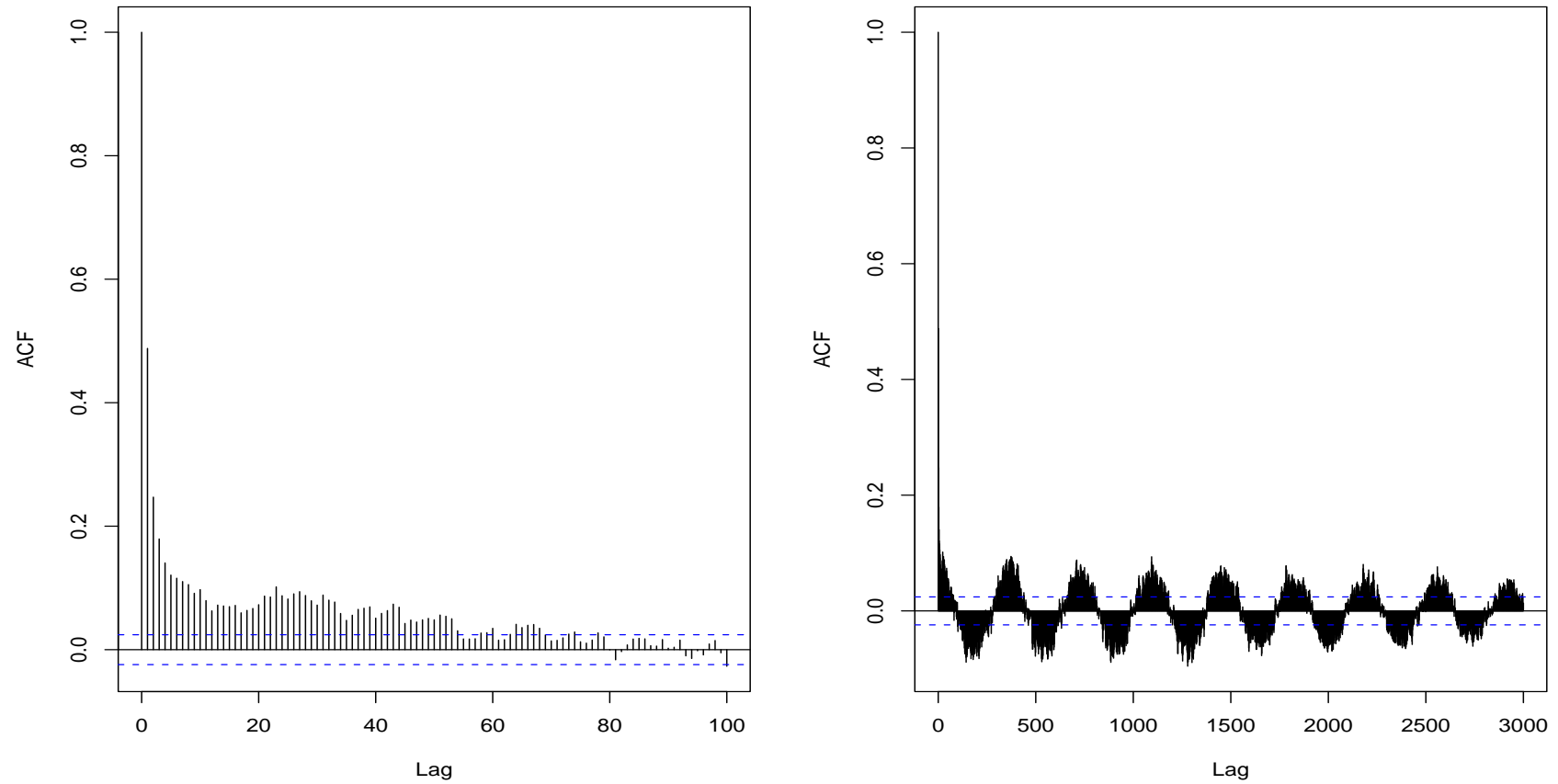


Figure 1: Autocorrelations, daily windspeed in Ireland.

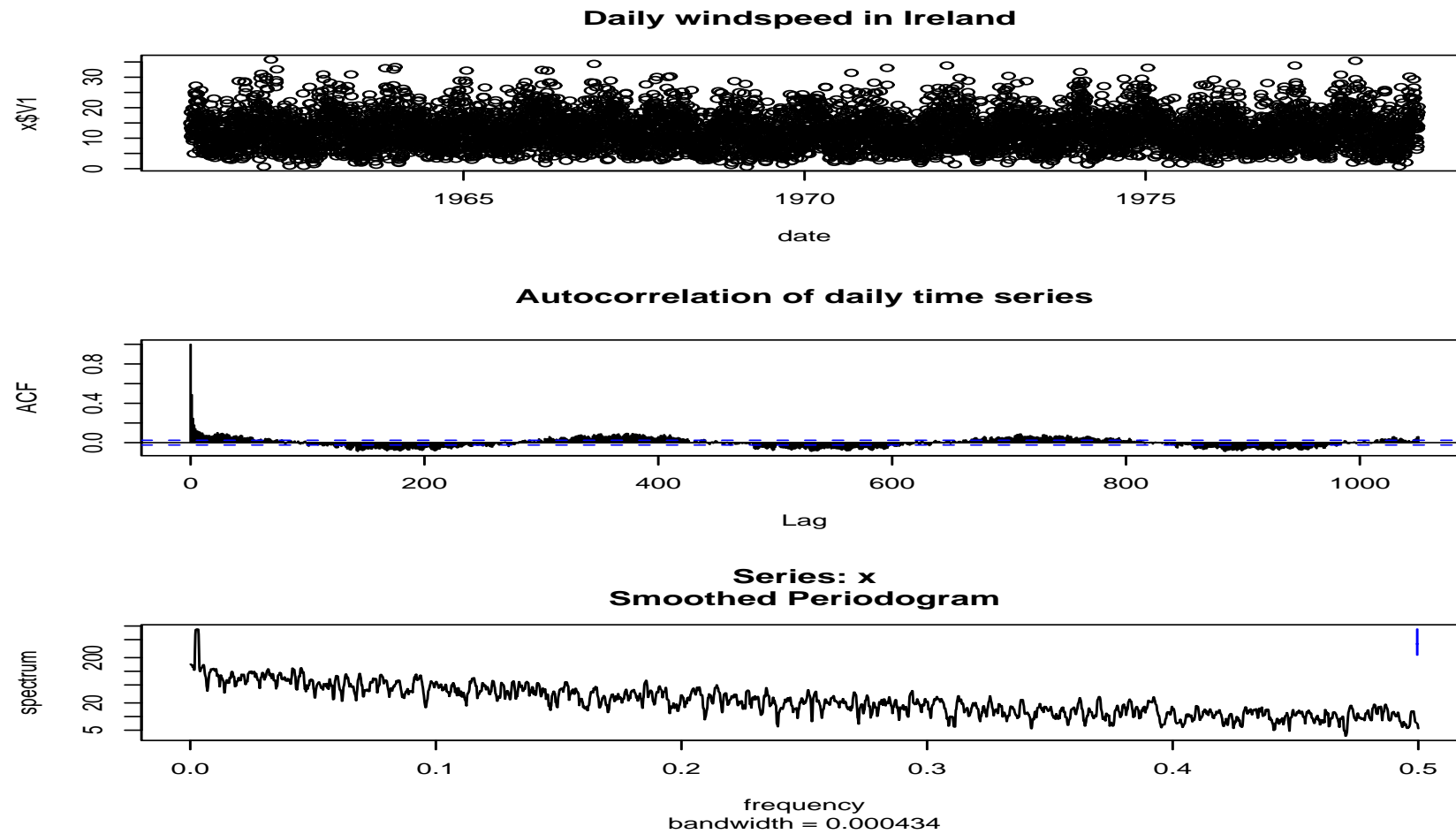


Figure 2: Daily windspeed in Ireland.

		15 knots	20 knots	25 knots
2 days	<i>GARMA</i>	47.7%	8.5%	0.3%
	seasonal <i>ARMA</i>	36.9%	3.6%	0.0%
	ratio	0.775	0.421	0.117
3 days	<i>GARMA</i>	46.3%	8.0%	0.3%
	seasonal <i>ARMA</i>	27.7%	1.4%	0.0%
	ratio	0.597	0.176	0.010
4 days	<i>GARMA</i>	45.3%	7.6%	0.3%
	seasonal <i>ARMA</i>	20.6%	0.5%	0.0%
	ratio	0.454	0.062	0.001

Table 1: Probabilities to have strong wind during consecutive days.

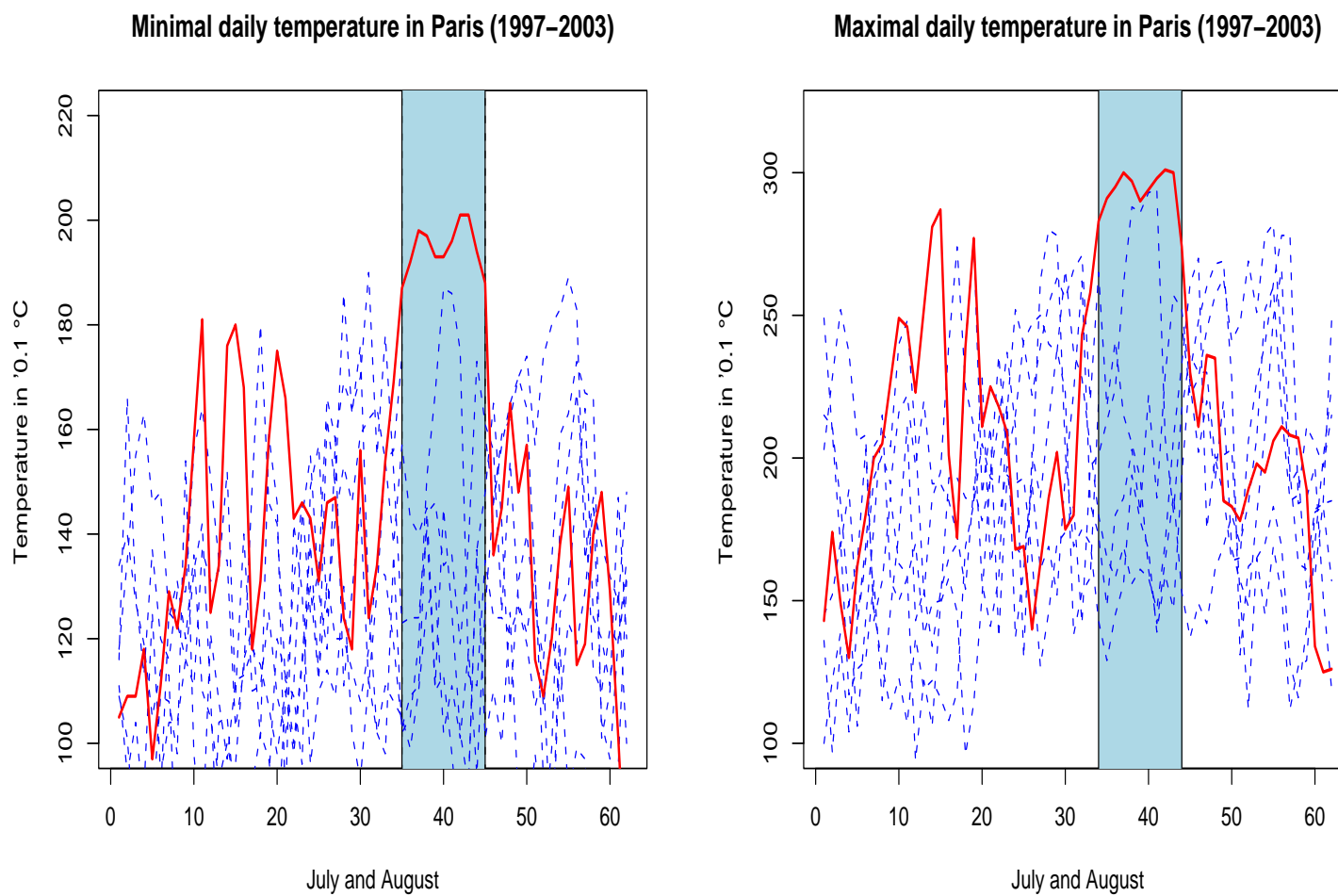


Figure 3: Daily temperature in Paris, years 1997 to 2003.

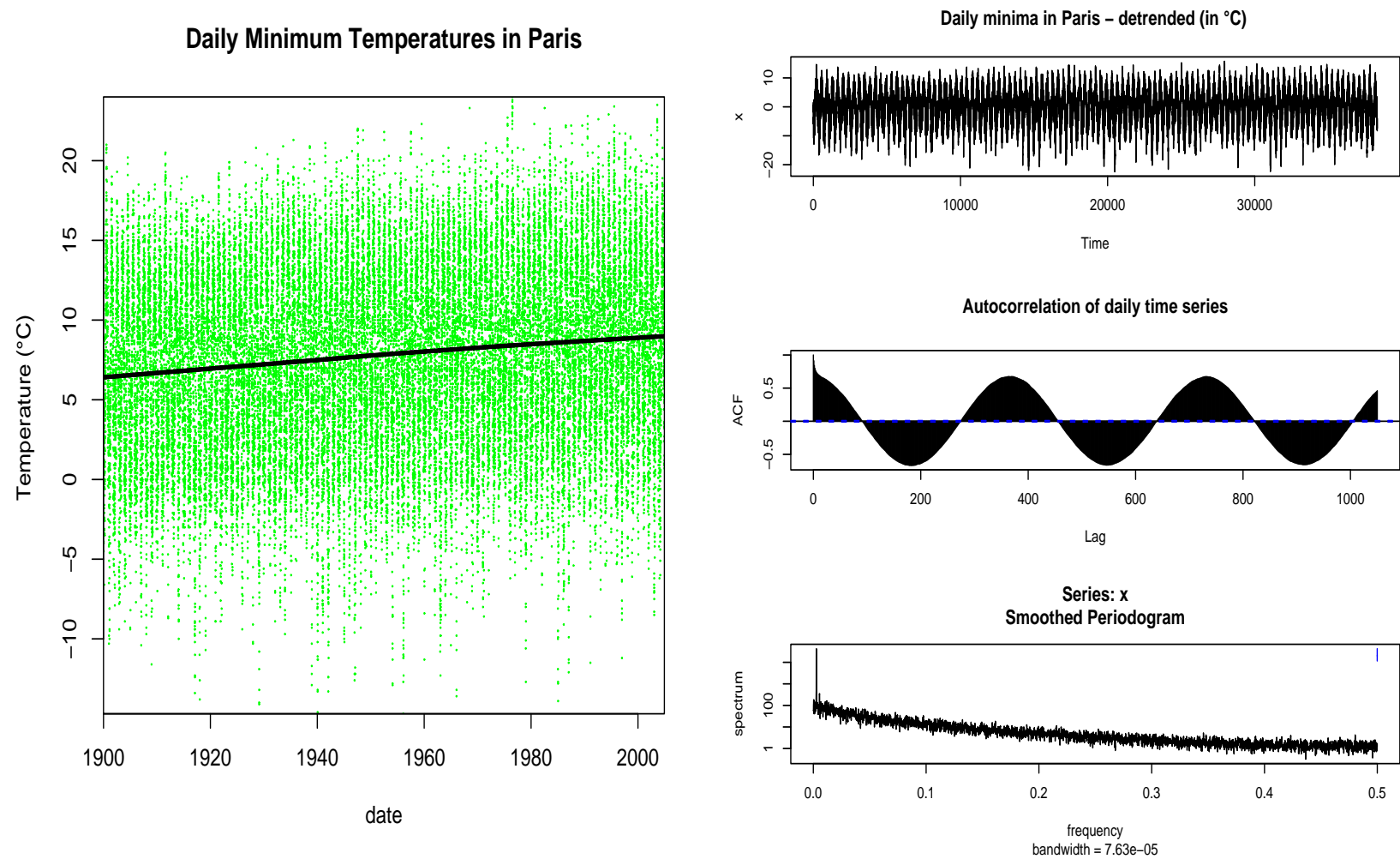


Figure 4: Trend of the series, and analysis of the series of residuals.

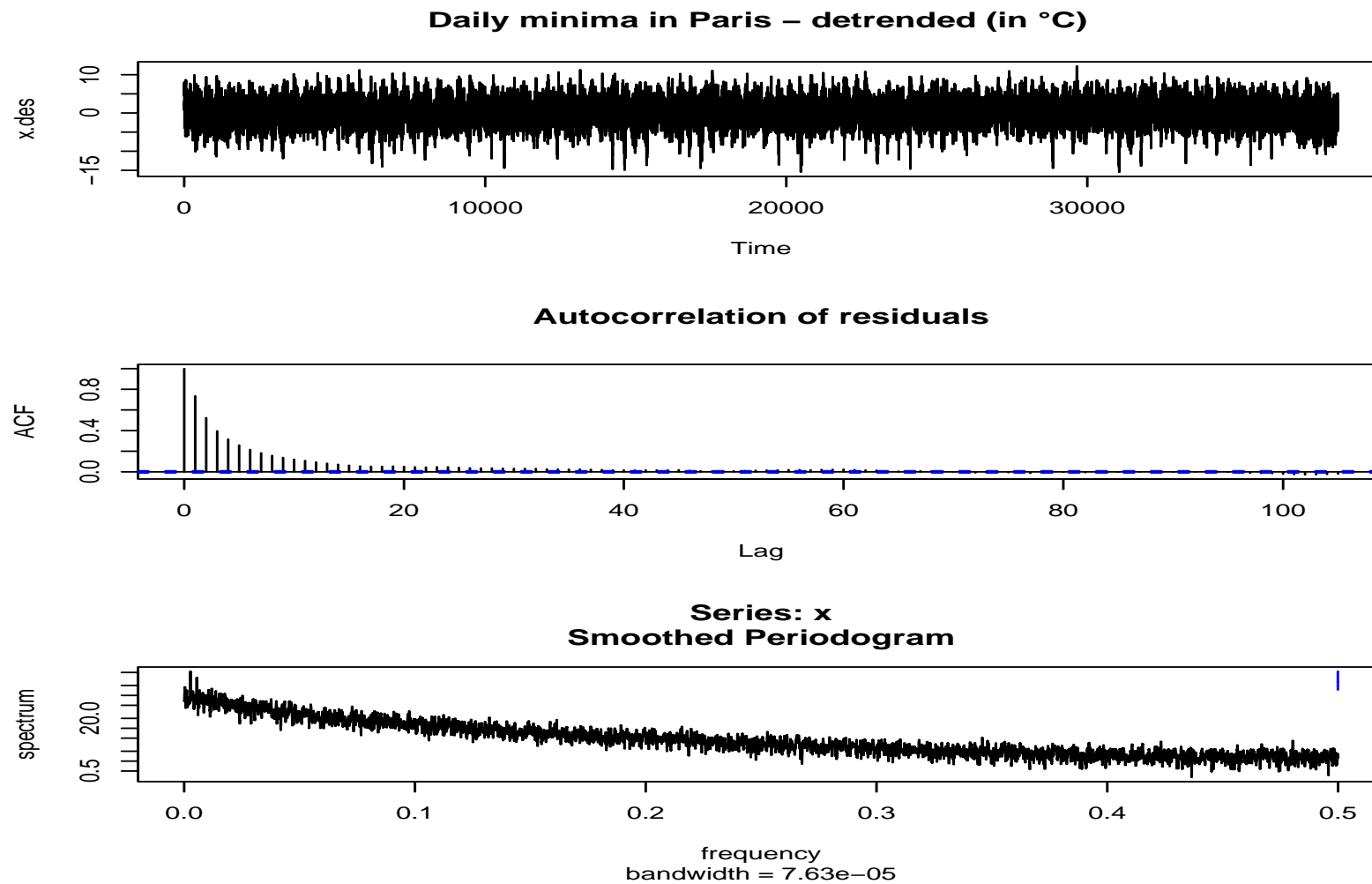


Figure 5: Residuals $(\varepsilon_t)_{t \in \mathbb{Z}}$.

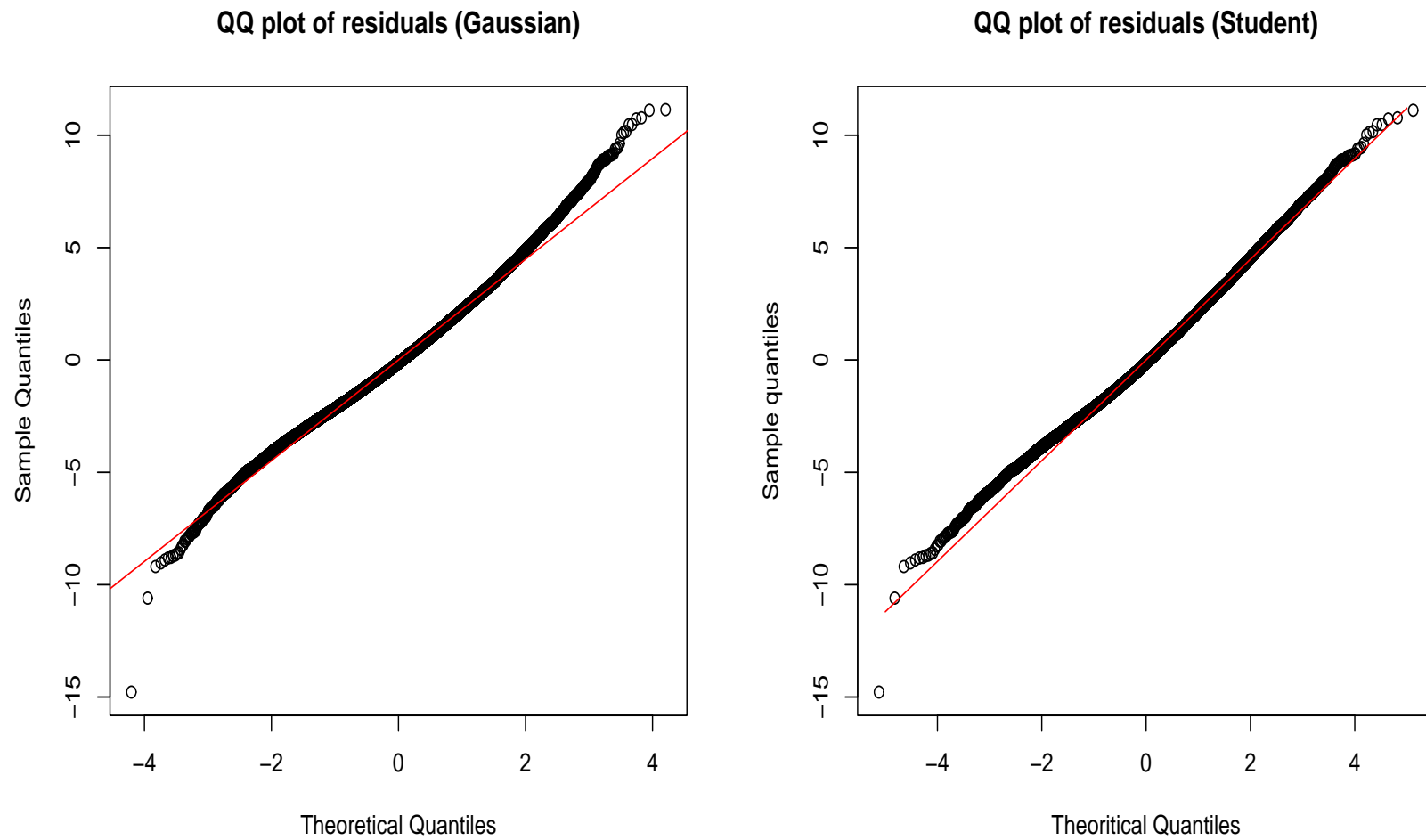


Figure 6: Residuals $(\varepsilon_t)_{t \in \mathbb{Z}}$, versus Gaussian and t -distribution.

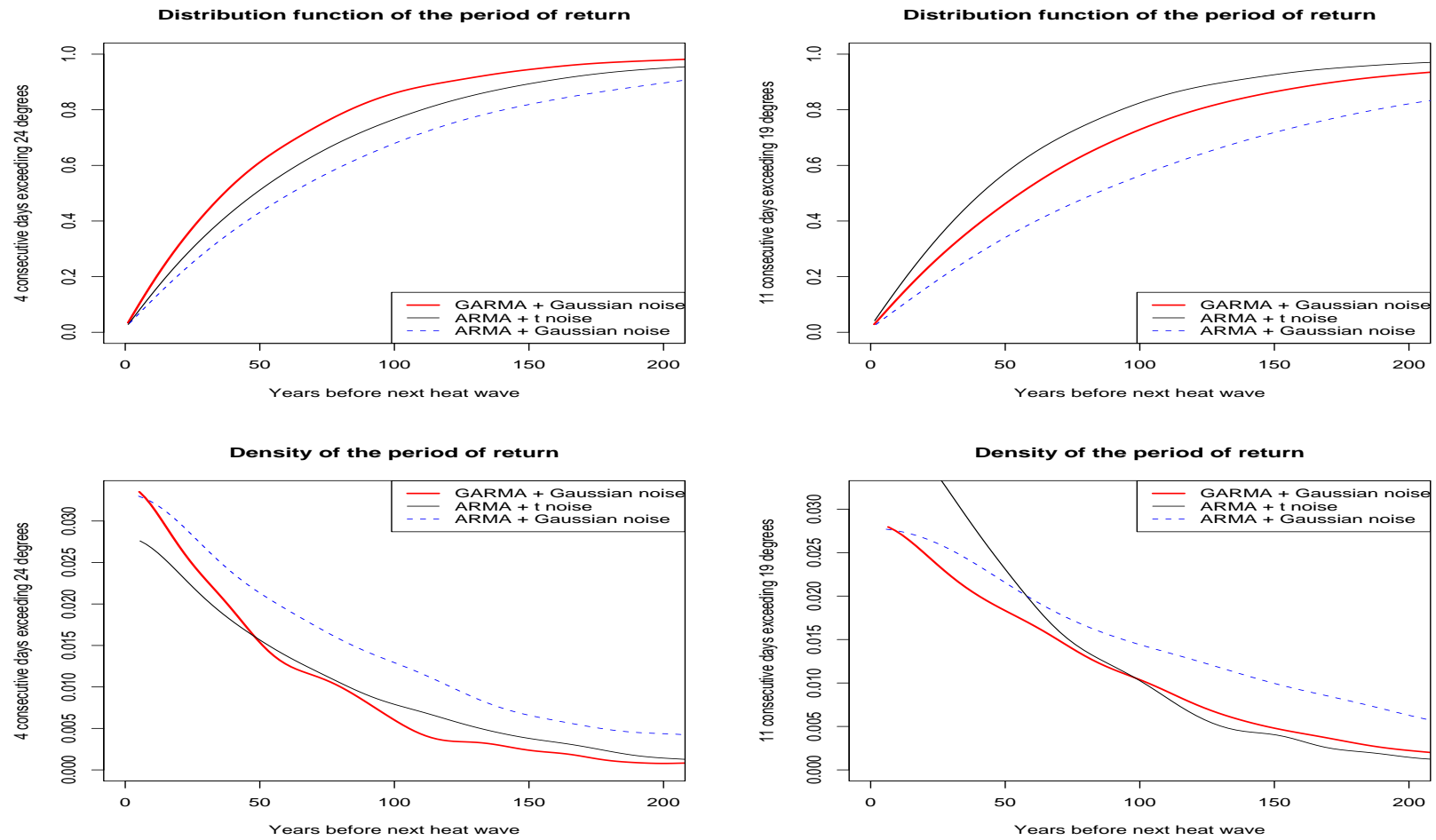


Figure 7: Survival distributions and densities of time before the next heat wave event (optimistic scenario).

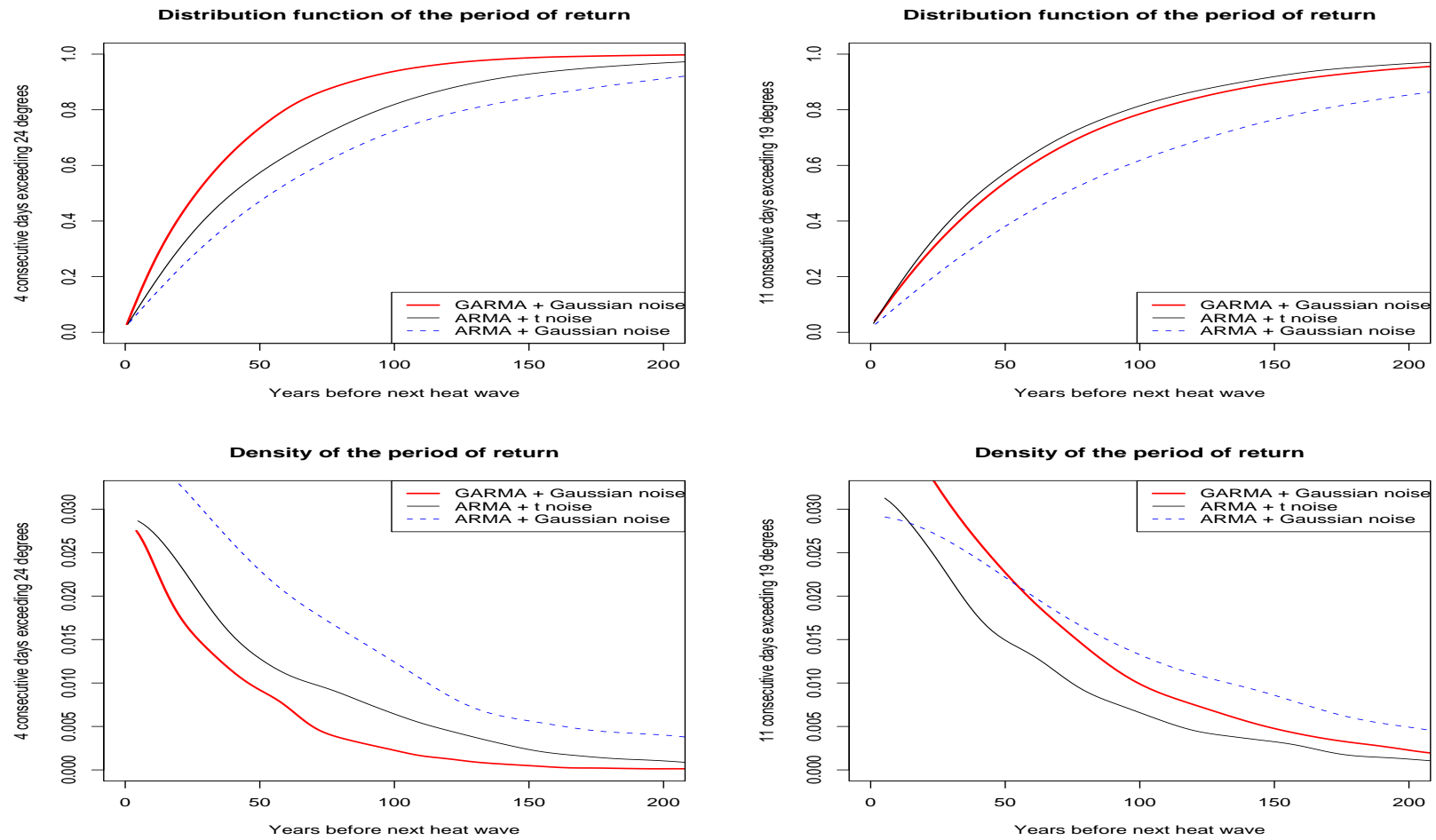


Figure 8: Survival distributions and densities of time before the next heat wave event (pessimistic scenario).

- heat wave, type (A): 11 consecutive days with temperature exceeding 19° C,

	short memory short tail noise	short memory heavy tail noise	long memory short tail noise
optimistic	88 years	69 years	53 years
pessimistic	79 years	54 years	37 years

Table 2: Periods of return (expected value, in years) before the next heat wave similar with August 2003 (type (A)).

- heat wave, type (B): 3 consecutive days with temperature exceeding 24° C,

	short memory short tail noise	short memory heavy tail noise	long memory short tail noise
optimistic	115 years	59 years	76 years
pessimistic	102 years	51 years	64 years

Table 3: Periods of return (expected value, in years) before the next heat wave similar with August 2003 (type (B)).

Modeling flood events

Two classical approaches, based on annualized maxima,

- HURST (1951), long memory Gaussian process
- GUMBEL (1958), i.i.d. observations Gumbel distributed

Based on 100 years of observations, not enough to assess if observations are, or not, independent: impact on return periods.

Another idea, TODOROVIC & ZELENHASIC (1970) and TODOROVIC & ROUSSELLE: consider flood event as a marked Poisson process.

Financial ACD (**Autoregressive Conditional Duration**) approach of ENGLE & RUSSELL (1998).

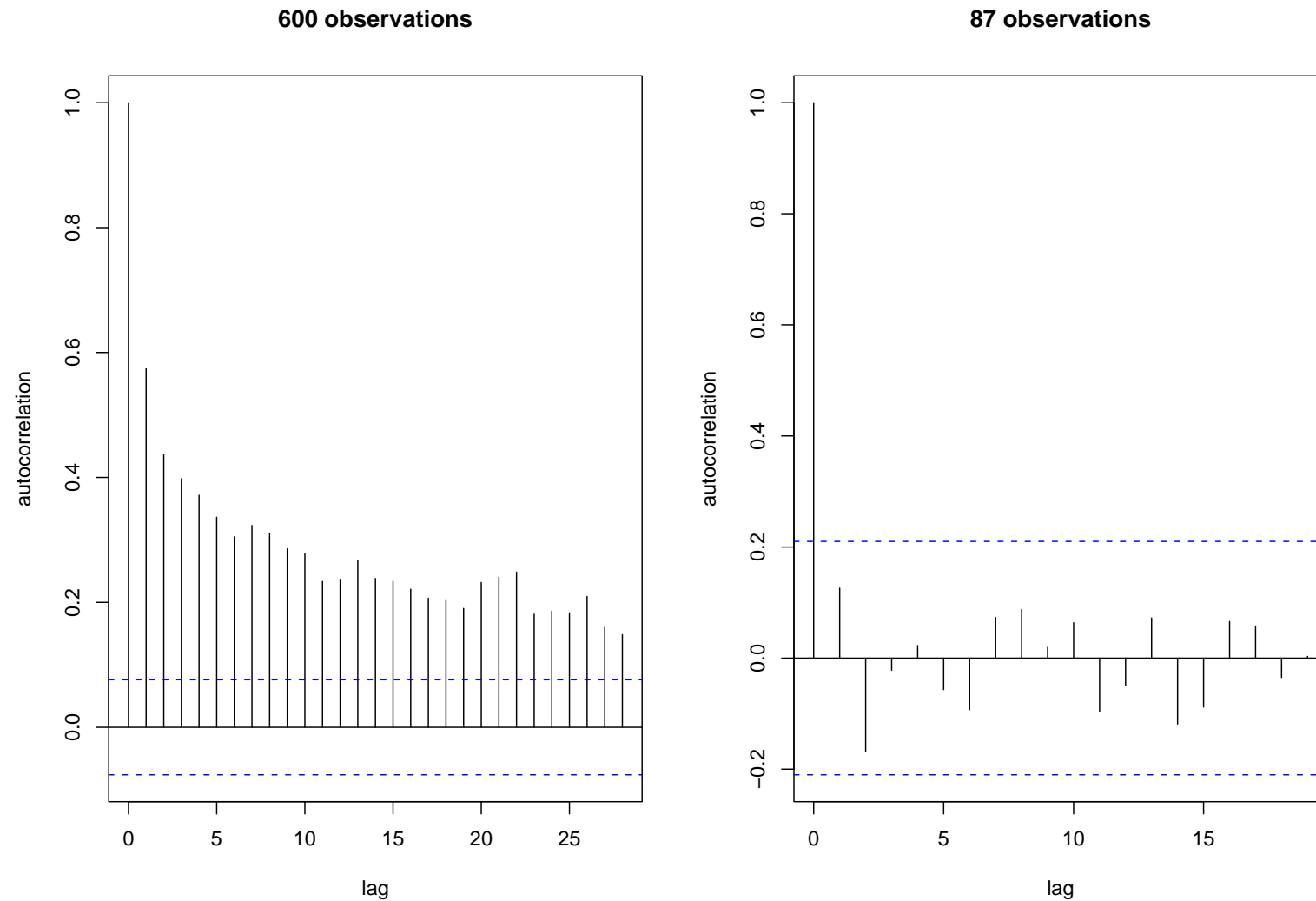


Figure 9: Autocorrelation function of the Nile annual maxima series.

Let X_i denote observed durations ($X_i = T_i - T_{i-1}$), and define $\mathcal{H}_i = \sigma(X_1, \dots, X_{i-1})$. Assume that

$$\left\{ \begin{array}{l} X_i = \Psi_i \cdot \varepsilon_i, \text{ where } (\varepsilon_i) \text{ i.i.d. (Exponential or Weibull)} \\ \mathbb{E}(X_i | \mathcal{H}_i) = \Psi_i \\ \Psi_i = \omega + \sum_{k=1}^p \alpha_k X_{i-k} + \sum_{k=1}^q \beta_k \Psi_{i-k}, \end{array} \right.$$

i.e.

$$X_i = \omega + \sum_{k=1}^{\max\{p,q\}} (\alpha_k + \beta_k) \eta_{i-k} + \eta_i,$$

where $\eta_i = X_i - \Psi_i = X_i - \mathbb{E}(X_i | \mathcal{H}_{i-1})$.

Two duration model in ENGLE & LUNDE (2003): time between two flood events, and duration of the flood. Marked process, with peak of the flood and volume.

Distribution of residuals (ε_i): exponential in finance, mixed Weibull in hydrology.

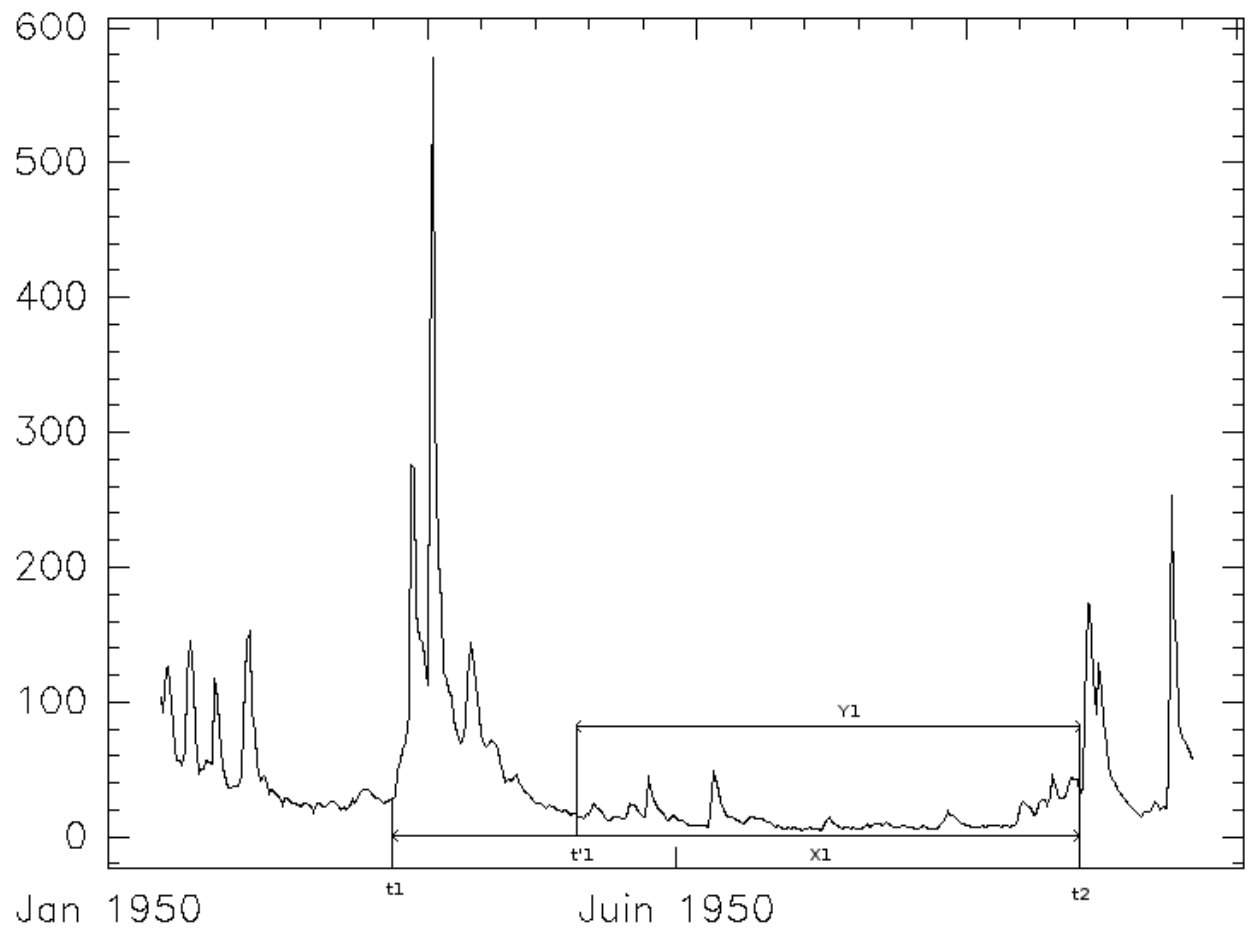


Figure 10: Two durations model

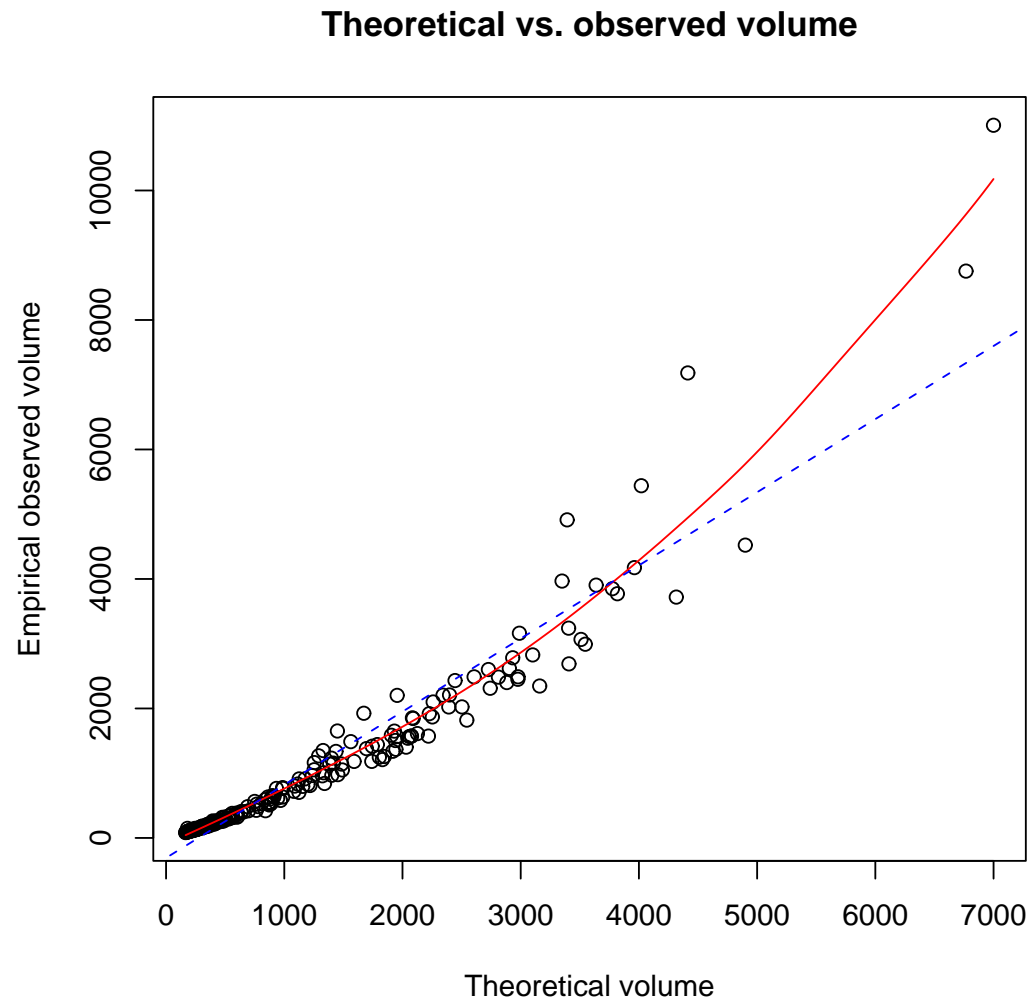


Figure 11: Relationship between peak, volume and duration of a flood.

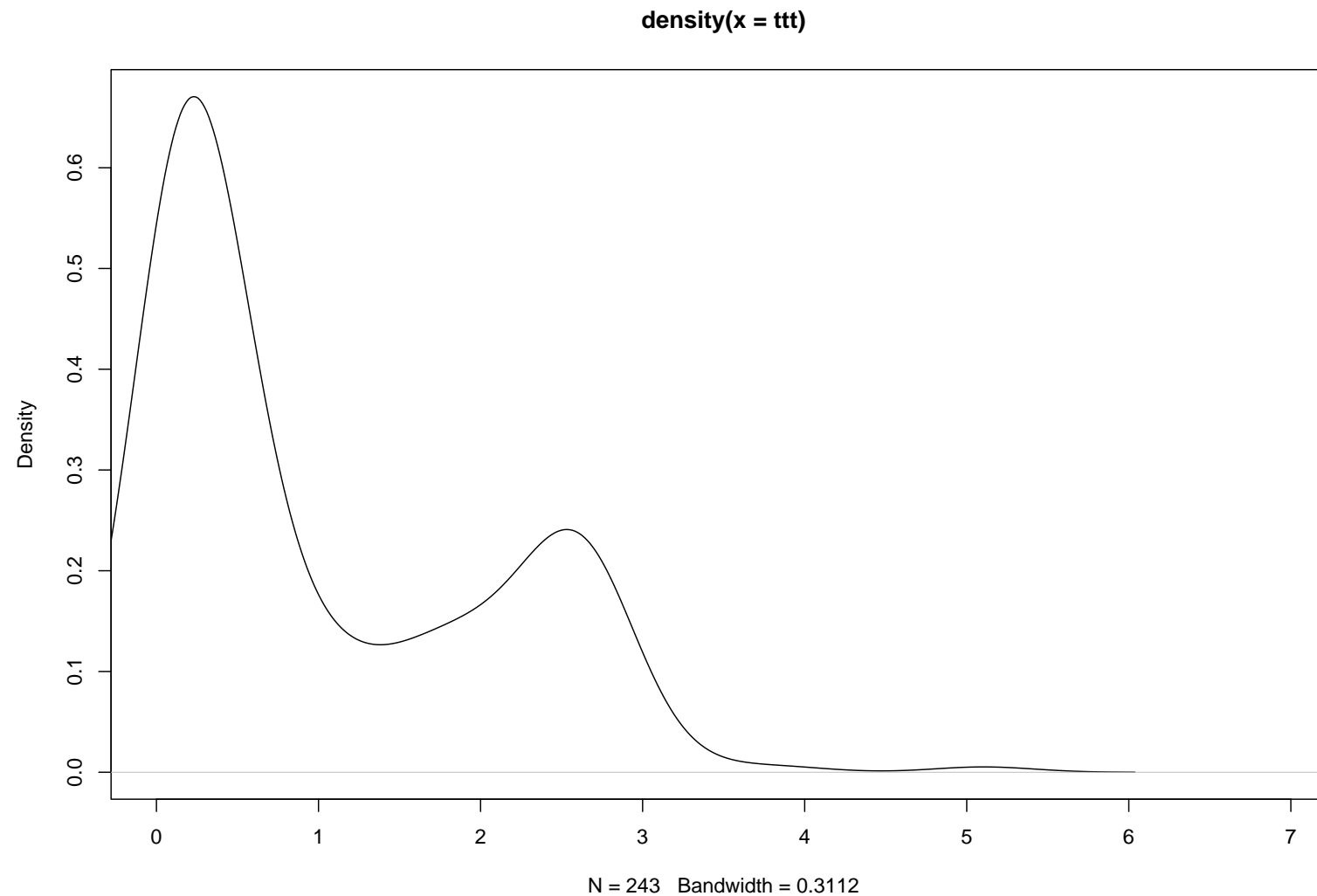


Figure 12: Kernel density estimation of residuals

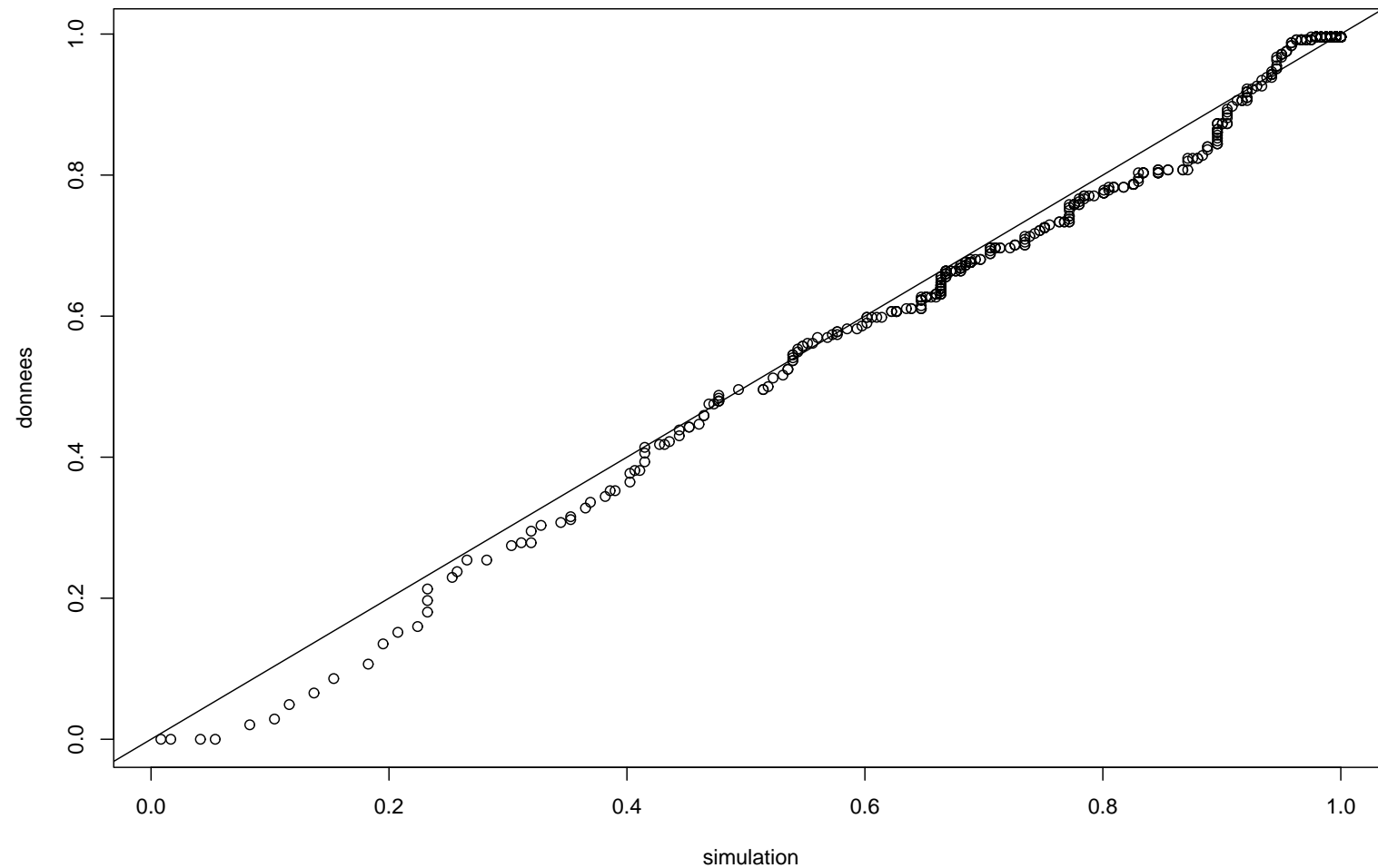


Figure 13: Empirical distribution function of the data vs. Weibull mixture.

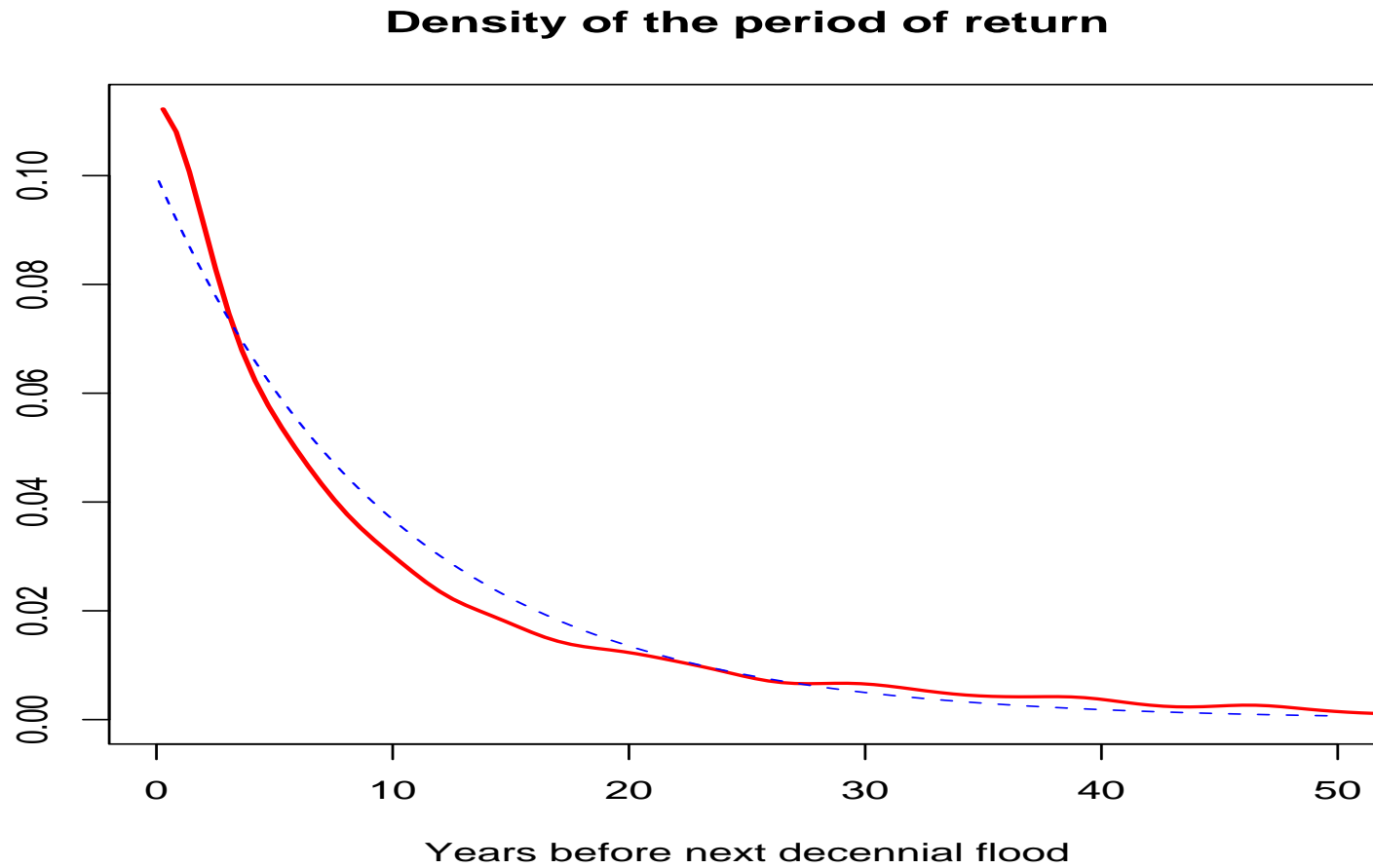


Figure 14: Flood duration distribution.

Structure of the bijgevoegde stelling

The bijgevoegde stelling is based on several papers,

BOUËTTE, J.C., CHASSAGNEUX, J.F., SIBAÏ, D., TERRON, R. & CHARPENTIER, A. (2006). *Windspeed in Ireland: long memory or seasonal effect ?* *Stochastic Environmental Research and Risk Assessment*. **20**, 141 - 151.

CHARPENTIER, A. (2006). *2003 heat wave and its return period*. submitted to *Weather*.

CHARPENTIER, A. AND SIBAÏ, D. (2006). *Dynamic flood modelling: Combining Hurst and Gumbel's approach*. submitted to *Journal of Hydrology*.