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Mesure et interpretation du cisaillement cosmologique

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THÈSE DE DOCTORAT UNIVERSITÉ PARIS VI

Pierre et Marie Curie

École doctorale d'astronomie et astrophysique

présentée par:

Elisabetta Semboloni

pour obtenir le grade de

DOCTEUR de l'UNIVERSITÉ PARIS 6

**Mesure et interpretation
du
cisaillement cosmologique**

soutenue le

14 11 2006

devant le jury composé de:

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Dr. Yannick Mellier	Directeur de Thèse
Dr. Alain Blanchard	Rapporteur
Dr. Francis Bernardeau	Rapporteur
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Résumé de thèse

L'analyse du cisaillement cosmologique est aujourd'hui un puissant moyen d'étude des propriétés statistiques du spectre de la matière noire, quelle que soit sa nature. On a montré récemment que l'approche tomographique permet de contraindre l'équation d'état de l'énergie sombre. Aujourd'hui, des relevés de galaxies de plus en plus vastes et profonds permettent d'analyser le cisaillement et de contraindre les paramètres cosmologiques avec une précision encore jamais atteinte. Parallèlement, il est nécessaire d'affiner les méthodes d'extraction du signal tout en contrôlant le biais qui pourrait conduire à une interprétation cosmologique erronée.

Le sujet de cette thèse s'inscrit dans ce contexte scientifique.

On y décrit la méthode d'extraction du signal, qui est ensuite appliquée aux données des relevés CFHTLS-Wide et CFHTLS-Deep. Les résultats de cette analyse démontrent le potentiel prometteur de cette dernière génération de relevés. D'autres aspects liés à l'interprétation du signal mesuré et aux différents types de biais susceptibles de l'affecter sont abordés.

Title

Analysis and interpretation of the cosmic shear signal

Thesis Summary

Cosmic shear analysis has proven to be a powerful tool for the investigation of the statistical properties of the power spectrum of dark matter fluctuations without making assumption on his nature. Recently it has been shown how tomography can give constraints on the evolution of the equation of state of dark energy. Deep and wide surveys, like the CFHTLS, LSST SNAP and DUNE are being performed today or the will start very soon. Their aim is to use the cosmic shear signal to get cosmological constraints with a precision never reached before. In fact, due to the sensitivity to the evolution of the density matter fluctuations, cosmic shear constraints are complementary to those derived by the cosmic microwave background analysis. On the other hand, we need to refine the method used to estimate the cosmic shear signal and to limit the bias that could affect the cosmological interpretation. This thesis is made with that purpose. I describe the aspects of cosmic shear measurement, and present the results of the cosmic shear analysis of the first release of the CFHTLS Deep and Wide survey. Also the results of a first tomographic study are presented showing the potential of this new generation of cosmic shear surveys. Throughout this work different sources of bias are explored, especially the bias related to the method used to estimate the shear signal. I also talk about some signal interpretation aspects, such as the evaluation of the cosmic variance and the effect of masking on the expected signal. Finally, I give conclusions and explicate future prospects.

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Chapitre 1

Introduction

La compréhension de la formation des structures à grande échelle dans l'Univers représente une des questions centrales de la cosmologie des dernières années. Grâce à des observations de plus en plus nombreuses et précises, il a été possible d'accumuler les évidences convergeant vers une description exhaustive et cohérente des principales propriétés de l'Univers. L'**homogénéité de l'Univers à grande échelle** est indiscutablement une hypothèse solide, en accord avec les observations des grands relevés de quasars (QSOs) et de galaxies. De même, l'observation du rayonnement cosmologique fossile (CMB) et de la distribution de ses fluctuations de température confirme son **isotropie**. Des fluctuations quantiques primordiales dont l'origine reste aujourd'hui mal comprise, seraient à l'origine de ces fluctuations du CMB et de la formation des structures. L'amplitude de ces inhomogénéités aurait augmenté progressivement sous l'effet de la gravité pour finalement produire des systèmes denses autogravitants par effondrement gravitationnel. La dynamique des systèmes gravitants (galaxies et amas de galaxies), les effets de lentille gravitationnelle et les propriétés statistiques de la distribution des galaxies à grande échelle suggèrent cependant que ce modèle doit inclure un ingrédient supplémentaire, une composante dominante de **matière noire froide dont la nature est inconnue, mais qui doit être non-baryonique**.

L'échelle angulaire à laquelle différentes zones du CMB sont en équilibre thermique au moment de la recombinaison suggère par ailleurs que l'Univers ait connu une phase d'expansion très rapide (quasi-exponentielle) avant l'époque de la recombinaison. L'hypothèse de l'existence d'une *phase d'inflation* permet de résoudre un certain nombre de paradoxes des modèles d'Univers de Friedmann-Robertson-Walker (platitude, horizon, monopoles). Elle conduit à des prédictions remarquables sur les propriétés du spectre des fluctuations initiales (loi de puissance, invariance d'échelle, champ gaussien) qui peuvent être aujourd'hui soumises à l'épreuve des tests observationnels.

Mais la découverte la plus inattendue et fascinante de cette décennie est sans conteste l'accélération de l'expansion de l'Univers. Les mesures des courbes de luminosité des supernovae de type Ia (SNIa) lointaines montrent une progression avec le redshift qui ne peut pas être expliquée si l'Univers ne subit pas une accélération. Il en résulte que la distance de luminosité doit contenir un terme supplémentaire à la composante matérielle ou à la courbure qui agit comme une pression négative : **l'énergie sombre**. Tout comme la matière noire, cette composante est de nature totalement in-

connue. Elle représente certainement une contribution de l'énergie du vide. Mais sa valeur étant plus de 120 fois plus faible que celle prédite pendant l'inflation, les physiciens se trouvent confrontés à une nouvelle énigme: comment l'énergie du vide a-t-elle pu s'atténuer d'un facteur supérieur à 100?

L'exploration des propriétés de l'énergie sombre est donc une des belles aventures des prochaines décennies et il est donc important de trouver les outils et les méthodes d'investigation poussée. L'analyse du CMB, qui est capable de fournir les meilleures contraintes sur beaucoup de paramètres cosmologiques, ne peut pas révéler des informations très riches sur l'énergie sombre. En effet, le CMB est au premier ordre un instantané de l'Univers à $z \approx 1000$, il ne peut pas donner une réponse univoque sur la nature de l'énergie sombre car son empreinte est portée principalement par les événements de l'histoire récente de l'Univers. Ce n'est pas le cas pour d'autres approches, comme les observations des supernovae, les effets de distorsion gravitationnelle, les amas de galaxies, les oscillations acoustiques des baryons, les observations de la forêt Lyman-alpha, ou toute autre observation de l'Univers proche (relativement au CMB, c'est à dire pour des redshifts de l'ordre de 1). C'est la raison pour laquelle les expériences futures avec SNAP, DUNE et LSST, dont les objectifs sont d'exploiter de façon systématique les mesures de distance des SNIa et du cisaillement cosmologique, ouvrent tant de perspectives. Ce sont en effet deux voies indépendantes et complémentaires pour étudier le comportement de l'énergie sombre.

L'analyse du cisaillement cosmologique est parmi les techniques les plus efficaces pour connaître les propriétés de la distribution de matière dans l'Univers. Un peu paradoxalement, c'est à ce titre qu'elle apparaît comme un outil d'investigation de l'énergie sombre.

L'analyse du cisaillement gravitationnel cosmologique et son interprétation cosmologique est l'objet de mon rapport. Dans une première partie, j'y rappelle brièvement les notions cosmologiques qui seront utiles à la compréhension de l'ensemble de mon travail. Les définitions concernant la déflexion de la lumière y sont rappelées, en soulignant comment il est possible, à travers la mesure du cisaillement cosmologique, de connaître les propriétés statistiques de la distribution de matière dans l'Univers. L'analyse du cisaillement cosmologique dans les données du CFHTLS représente la partie principale de mon doctorat et sera donc l'élément central de cette thèse (chap. 5). D'autres points strictement corrélés tels que l'évaluation des effets non gaussiens dans la mesure du cisaillement cosmologique (chap. 6) ou celui du *masking* dans la reconstruction du spectre (chap. 7) seront abordés dans une deuxième partie. Ces questions sur lesquelles j'ai focalisé mon attention pendant cette dernière année d'étude sont parmi les points clefs pour pousser l'analyse du cisaillement cosmologique au-delà des limites actuelles. A ce sujet, le chapitre 8 présente une étude sur la façon de réduire le biais dans l'estimation du cisaillement à partir des images astronomiques. La conclusion de cette thèse porte sur une analyse conjointe des données VIRMOS-Desart, CFHTLS et RCS. Cette comparaison, que l'on aimerait étendre à tous les relevés de cisaillement gravitationnel cosmologique publiés ces dernières années, vise à établir quantitativement l'accord entre les différentes données et mesurer ainsi la fiabilité des résultats du cisaillement cosmologique.

Chapitre 2

L'Univers à grande échelle

Cette section introduit les grandeurs caractéristiques qui décrivent l'évolution des fluctuations de densité de matière, depuis leur origine pendant les premières phases d'expansion de l'Univers, à travers toutes ses différentes époques et jusqu'à nos jours.

2.1 Introduction

2.1.1 Le spectre primordial

La formation des structures dans l'Univers peut être naturellement décrite par l'évolution du champ de fluctuations de densité de matière. Il est défini de la façon suivante:

$$\delta(\mathbf{x}, t) = \frac{\rho(\mathbf{x}, t) - \bar{\rho}(t)}{\bar{\rho}(t)} \quad (2.1)$$

où $\rho(\mathbf{x}, t)$ est la densité de matière à la position désignée par la coordonnée comobile \mathbf{x} et par la coordonnée temporelle t , et $\bar{\rho}(t)$ est la densité moyenne au temps t . Le champ $\delta(\mathbf{x}, t)$ est supposé être homogène et isotrope et peut généralement s'exprimer par ses composantes de Fourier:

$$\delta(\mathbf{x}, t) = \int d^3 \mathbf{k} \tilde{\delta}(\mathbf{k}, t) \exp(i\mathbf{k}\mathbf{x}) \quad (2.2)$$

où \mathbf{k} représente le vecteur d'onde. On peut décrire les propriétés du champ de fluctuations $\delta(\mathbf{x}, t)$ en mesurant ses moments. Les théories inflationnaires prédisent que la distribution des fluctuations primordiales serait décorrelée et que leur distribution en amplitude est Gaussienne. Dans ce cas, les moments d'ordre deux décrivent complètement le champ des fluctuations.

La fonction de corrélation entre deux points \mathbf{x} et $\mathbf{y} = \mathbf{x} + \mathbf{r}$ (où on a sous entendu la dépendance temporelle) d'un champ statistiquement isotrope et homogène dépend uniquement du module de la distance r entre les deux points:

$$\xi_\delta(r) = \langle \delta(\mathbf{x})\delta(\mathbf{x} + \mathbf{r}) \rangle \quad (2.3)$$

La transformée de Fourier de la fonction de corrélation est le spectre de puissance des fluctuations $P(k)$:

$$\langle \tilde{\delta}(\mathbf{k})\tilde{\delta}^*(\mathbf{k}') \rangle \equiv (2\pi)^3 \delta_{D(\mathbf{k}=\mathbf{k}')} P(k) \quad (2.4)$$

qui décrit complètement les propriétés statistiques de la distribution de matière après l'inflation. On indique avec f^* le complexe conjugué d'une quantité complexe f et δ_D dénote la fonction de Dirac. Cependant, à cause de l'effondrement gravitationnel des structures, le spectre s'écarte progressivement de sa distribution initiale et des caractéristiques non-gaussiennes apparaissent. Le spectre primitif $P_i(k)$ (*i.e.* après l'inflation) est généralement supposé invariant d'échelle (spectre de Harrison Zeld'ovich) et s'écrit donc $P_i(k) = A k$, où A est une constante de proportionnalité dont aujourd'hui on ne peut pas encore prédire l'amplitude. On décrira ensuite comment cette valeur peut être déterminée par des observations.

La description de l'évolution du spectre est alors contenue dans la fonction de transfert, $T(k)$:

$$P_{rec}(k) = T^2(k)P_i(k) , \quad (2.5)$$

mais n'est valide que dans l'approximation du régime linéaire où les modes restent découplés. Les propriétés de cette fonction de transfert sont abordées dans la section suivante. J'y discute comment les fluctuations de matière évoluent dans le régime linéaire à partir du spectre initial, jusqu'à l'époque du découplage matière-radiation, époque après laquelle les fluctuations de densité évoluent sous l'action de la gravité et de l'expansion de l'Univers.

2.1.2 Dynamique de l'Univers

L'évolution du spectre de fluctuations primordial dépend de la dynamique de l'Univers. Elle est décrite par l'évolution de sa métrique via les équations de Einstein. Sous l'hypothèse d'homogénéité et d'isotropie, la géométrie de l'espace-temps peut être décrite par la métrique Friedmann-Robertson-Walker (FRW):

$$ds^2 = c^2 dt^2 - a(t)^2 [dw^2 + f_K(w)(d\theta^2 + \sin^2(\theta)d\phi^2)] \quad (2.6)$$

où c est la vitesse de la lumière, t la coordonnée temporelle, w , θ , ϕ sont les coordonnées spatiales sphériques. Le paramètre $a(t)$ tient compte de l'évolution temporelle de la partie spatiale de la métrique. La métrique FRW décrit à la fois une sphère ($K > 0$), une parabole ($K < 0$) ou une géométrie Euclidienne ($K = 0$), selon l'expression de $f_K(w)$:

$$f_K(w) = \begin{cases} K^{-1/2} \sin(K^{1/2}w) & \text{si } K > 0 \\ w & \text{si } K = 0 \\ (-K)^{-1/2} \sinh((-K)^{1/2}w) & \text{si } K < 0 \end{cases}$$

où K est le paramètre de courbure.

Le *redshift* est le décalage spectral qui affecte les rayons lumineux, dû au fait que la partie spatiale de la métrique change avec le temps. On peut démontrer cet effet de la façon suivante. On considère une source S et un observateur O . La source émet un photon au temps t_s qui sera reçu par l'observateur à l'instant t_o . On définit la distance comobile à l'instant t comme la quantité $|ds|_t/a(t)$. Si la source et l'observateur n'ont pas du mouvement propre la distance comobile w_{SO}

est donc constante. Pour un rayon de lumière $ds^2 = 0$ et donc:

$$\int_O^S dw = \int_{t_o(t_s)}^{t_s} \frac{cdt}{a(t)} = \text{Const}$$

c'est à dire que l'intégrale ne change pas avec le changement du temps d'émission t_s et donc:

$$\frac{dt_0}{dt_s} = \frac{a(t_o)}{a(t_s)}$$

Deux photons qui sont émis avec un intervalle de temps dt_s sont reçus par l'observateur avec un intervalle de temps $dt_0 = \frac{a(t_o)}{a(t_s)} dt_s$. Une série d'impulsions lumineuses séparées par un intervalle de temps dt définit un signal de fréquence $\nu = 1/dt$ et aussi de longueur d'onde $\lambda = dt/c$:

$$\frac{\nu_s}{\nu_o} = \frac{\lambda_o}{\lambda_s} = \frac{a(t_o)}{a(t_s)}$$

On définit le redshift $z \equiv \frac{a(t_o)}{a(t_s)} - 1$ qui exprime directement le changement de longueur d'onde de la lumière pendant sa propagation. En général on suppose le paramètre de $a_0 = 1$ à l'époque actuelle et donc $a = (z + 1)^{-1}$. Vu sa définition, le redshift représente un paramètre naturel avec lequel décrire la "distance" des sources cosmologiques.

Pour une cosmologie FRW les équations d' Einstein s'écrivent:

$$\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\rho(t) - \frac{Kc^2}{a^2} \\ \left(\frac{\ddot{a}}{a}\right) &= -\frac{4\pi G}{3}\left(\rho(t) + \frac{3P(t)}{c^2}\right) \end{aligned} \quad (2.7)$$

où l'on a supposé que le tenseur énergie-impulsion $T_{\mu\nu}$ du fluide cosmique est celui d'un fluide parfait, caractérisé par une densité ρ et une pression P dont les seuls éléments non nuls sont: $T_{00} = \rho c^2$ et $T_{ii} = P$ pour $i = 1,2,3$ et dont l'équation d'état est $P = w\rho c^2$.

On définit le paramètre de Hubble $H(t) \equiv \frac{\dot{a}}{a}(t)$. Ces équations sont vraies à tous les temps et aussi à l'époque actuelle caractérisée par le paramètre temporel t_0 . On a défini le *paramètre de densité* $\Omega_0 \equiv \rho_0/\rho_{0,crit}$ et le *paramètre de courbure* $\Omega_{K,0} \equiv -Kc^2/(H_0^2 a_0)$ où $\rho_{0,crit} = \frac{3H_0^2}{8\pi G}$ représente la *densité critique* à l'époque actuelle (*i.e.* la densité pour laquelle $K = 0$). La conservation de l'énergie-impulsion peut être obtenue en imposant $T_{\mu\nu}^{;\mu} = 0$, ce qui conduit à:

$$\frac{d}{dt}[a^3 c^2 \rho] = -3P \frac{da^3}{dt} \quad (2.8)$$

ce qui donne:

$$d[\rho a^{3(1+w)}] = 0 \quad (2.9)$$

Pour ce qui concerne l'équation d'état des photons et de la matière on a respectivement $w_0 = 1/3$ et $w_0 = 0$, et donc $\rho_r \propto a^{-4}$ et $\rho_m \propto a^{-3}$ ce qui comporte une différente perte d'énergie de la matière baryonique et des photons pendant l'expansion de l'Univers donnée par l'équation:

$$H(z) = H_0 \sqrt{(\Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + (1+z)^2 \Omega_{K,0})} \quad (2.10)$$

où on a défini $\Omega_{m,0}$ $\Omega_{r,0}$ les paramètres de densité de la matière et de la radiation l'époque actuelle et puisque à l'époque actuelle la matière domine la radiation $\Omega_{m,0} = \Omega_0$ (dans la suite on écrira simplement Ω_m à la place de $\Omega_{m,0}$ sous-entendant que ce paramètre est mesuré à l'époque actuelle). Encore, l'équation 2.10 implique $\Omega_{K,0} = 1 - \Omega_0$ ce qui montre la signification de la densité critique. Si $\rho_0 > \rho_{0,crit}$ à un certain instant l'Univers commence à se contracter grâce à l'interaction gravitationnelle, dans le cas contraire il continuera à s'étendre pour toujours. La densité critique est la densité pour laquelle l'Univers continue son expansion avec une vitesse nulle dans la limite $t \rightarrow \infty$. L'équation 2.10 a aussi deux implications fondamentales. La dynamique de l'Univers, qui est donnée par le paramètre de Hubble, change au cours de son histoire: dans une première époque, sa dynamique est dominée par le rayonnement, puis la matière et le paramètre de courbure vont progressivement déterminer son devenir. On verra dans le paragraphe 2.6, que l'introduction d'une nouvelle composante, l'énergie sombre, modifie la dynamique de l'équation 2.10. Enfin, par le fait que l'Univers est en phase d'expansion, en arrière dans le temps (*i.e.* $t \rightarrow 0$) le paramètre $a(t) \rightarrow 0$ tandis que la densité devient infinie. On appelle cette singularité à $t = 0$ *Big Bang*.

2.2 Évolution des fluctuations: phénoménologie

On considère une sur-densité ρ_b homogène et sphérique de rayon b dans un champ de densité moyenne ρ_a . On peut considérer cette sur-densité, comme un univers isolé du reste de l'Univers défini par un paramètre de densité locale $\rho_b/\rho_a = \Omega_b > 1$. On définit de la même façon les paramètres de Hubble H_a et H_b tels que

$$H_b^2 = \frac{8\pi G}{3}\rho_b - \frac{c^2}{b^2} \quad \text{et} \quad H_a^2 = \frac{8\pi G}{3}\rho_a \quad (2.11)$$

Le contraste de densité $\delta = (\rho_b - \rho_a)/\rho_a$ est alors donné par:

$$\delta \simeq \frac{3c^2}{8\pi G b^2 \rho_a} \quad (2.12)$$

où on a utilisé le fait que dans le régime linéaire $H_b^2 - H_a^2 \simeq 0$. De plus, dans le régime linéaire $b \propto a$ et donc pendant l'époque dominée par le rayonnement $\delta \propto a^2$ (car $\rho_a \propto a^{-4}$), tandis que pendant l'époque dominée par la matière $\delta \propto a$ (car $\rho_a \propto a^{-3}$). L'effondrement de la structure engendre une onde de pression dans le fluide qui s'oppose à la croissance de la fluctuation. Pour suivre son évolution, il faut alors comparer deux temps caractéristiques: le temps d'effondrement gravitationnel et celui que l'onde de pression emploie pour effacer la sur-densité. Le premier est donné par $t_{grav} \sim \frac{1}{\sqrt{G\rho_b}}$ ¹. Le second est $t_{press} = \frac{b}{c_s}$ où c_s est la vitesse caractéristique de l'onde de pression. Donc, l'effondrement peut se produire lorsque $t_{grav} < t_{press}$. On définit la longueur de Jeans,

$$T_J = \frac{c_s}{\sqrt{G\rho_b}} \simeq \frac{c_s}{\sqrt{G\rho_a}}, \quad (2.13)$$

1. Le potentiel gravitationnel associé à la masse M_b de densité constante ρ_b est $\Phi = GM_b/b$. La force gravitationnelle ressentie par une masse m à la position $c < b$ est donnée par $f = -m\vec{\nabla}\Phi(c) = m\frac{d^2c}{dt^2}$. C'est formellement l'équation d'un oscillateur harmonique $\frac{d^2c}{dt^2} - \omega^2 c$ avec $\omega = \sqrt{\frac{4\pi G\rho_b}{3}}$, dont la période $T = \frac{2\pi}{\omega} \sim \frac{1}{\sqrt{G\rho_b}}$

comme la taille maximale au-delà de laquelle les fluctuations ne peuvent plus croître. Dans un Univers dominé par le rayonnement $c_s^2 \simeq 1/3c^2$ la condition d'effondrement devient:

$$b > \frac{c}{\sqrt{3G\rho_a}} . \quad (2.14)$$

Cette taille caractéristique correspond approximativement à celle du rayon de Hubble $d_H = cH^{-1}$ et représente celle de l'horizon (*i.e.* la distance maximale que les photons ont pu parcourir depuis le Big bang). Ainsi, puisque la taille de l'horizon croît avec le temps, les fluctuations de différentes tailles entrent progressivement dans l'horizon et stoppent donc leur croissance. En revanche, pendant l'époque dominée par la matière, $c_s = 0$, et donc toutes les fluctuations peuvent s'effondrer indépendamment de leur taille.

Pendant l'époque où la taille des fluctuations est plus petite que l'horizon et jusqu'à la recombinaison, d'autres effets sont susceptibles de modifier leur aspect. Le destin des baryons et celui des particules de matière noire non collisionnelle est néanmoins différent. En ce qui concerne les fluctuations de matière baryonique, leur destin est déterminé par l'effet de *free streaming* ou amortissement non collisionnel lié à la présence de matière noire et par l'amortissement de Silk (*Silk damping*) résultant des interactions entre les baryons et les photons. Dans les premières phases de l'Univers ces interactions qui sont plus précisément des diffusions Thompson sont si fréquentes que l'on peut considérer l'ensemble photons-électrons comme un seul fluide. Les photons sont libres de diffuser dans des régions moins denses et donc à cause des interactions avec les électrons transportent la matière baryonique hors des sur-densités. Les sur-densités dont la taille est du même ordre de grandeur ou plus petite que la taille maximale parcourue par les photons dans leur marche au hasard jusqu'à l'époque de la recombinaison sont alors gommées. Quand l'époque de la recombinaison approche, le libre parcours moyen des photons augmente et ces interactions deviennent plus rares puis s'estompent. L'effet de free-streaming est aussi capable d'effacer les fluctuations dans la composante baryonique; en effet la matière noire n'interagissant pas avec les photons, ceux-ci peuvent diffuser dans des régions sous-denses. Toutes les fluctuations dont la taille est comparable au parcours effectué par les particules de matière noire peuvent ainsi être gommées².

L'échelle maximale des fluctuations qui vont s'amortir puis disparaître dépend de la nature de la matière noire . Pour des modèles de matière noire chaude (HDM) le libre parcours moyen est plus élevé, ce qui efface toutes les structures dont la taille est plus petite que ≈ 40 Mpc. C'est typiquement la taille des super-amas. Dans le cas de modèles de matière noire froide (CDM), cet effet est négligeable et aucun obstacle ne s'oppose à une formation de structures de petite taille. Cependant, l'expansion intervient aussi dans le processus de croissance des structures. Si elle est trop rapide, l'effondrement (collapse) gravitationnel peut être stoppé. Lorsque le temps caractéristique d'expansion $t_{exp} = \frac{1}{H} = 1/\sqrt{G\rho_r}$ pendant l'époque de rayonnement est inférieur à celui de l'effondrement d'une fluctuation de matière noire, $t_{grav} = \frac{1}{\sqrt{G\rho_{DM}}}$, l'effondrement n'est pas possible. On peut traduire ces résultats trouvés dans la description de l'évolution du spectre de matière noire:

$$\left(\frac{\delta\rho}{\rho}\right)^2 = \int |\tilde{\delta}_k|^2 d^3\mathbf{k} \propto |\tilde{\delta}_k|^2 k^3 . \quad (2.15)$$

2. En réalité la situation est plus complexe car la matière noire peut condenser librement et par conséquent entraîner préférentiellement les baryons dans les régions où la matière noire est sur-dense.

Dans la phase où l'Univers est dominé par le rayonnement, pour un mode de taille caractéristique inférieure à l'horizon on a $k \ll k_{equiv}$:

$$|\delta_k|^2 k^3 \simeq Cost \rightarrow |\delta_k|^2 \propto k^{-3} . \quad (2.16)$$

Si la taille caractéristique reste toujours plus grande que l'horizon alors:

$$|\delta_k|^2 k^3 \propto |\delta_{k_0}|^2 k^3 \rightarrow \delta_k \propto k . \quad (2.17)$$

La fonction de transfert dans ces deux cas limites est donc respectivement $T(k) \propto k^{-2}$, pour les modes avec $k \ll k_{equiv}$, et $T(k) = 1$, pour les modes avec $k \gg k_{equiv}$.

2.3 Limites de l'approche phénoménologique

La phénoménologie de la formation des structure ne décrit que dans ses grandes lignes l'évolution des fluctuations de densité. La physique de la croissance des structures cosmologiques est plus rigoureusement décrite par la résolution des équations de conservation (équation de conservation et d'Euler) couplées entre les différentes espèces et les équations d'Einstein dans une métrique perturbée, où des perturbations stochastiques de la métrique d'origine quantique sont la source des fluctuations initiales. Dans le cas le plus simple on suppose des perturbations stochastiques scalaires (*i.e.* la métrique est perturbée au premier ordre en utilisant deux champs scalaires stochastiques). L'évolution des fluctuations dépend alors de leur nature.

Dans le cadre des modèles inflationnaires, les fluctuations quantiques engendrées sont de type adiabatique (*i.e.* préservent leur entropie dans l'espace). La résolution des perturbations dans le fluide couplé comporte l'écriture d'un système hiérarchique pour le plasma baryons-photons pour chaque mode k . La résolution de ce système donne des fluctuations de température des photons. Bien qu'il s'agisse d'un système couplé de longueur infinie, il est tout de même possible d'effectuer des approximations qui permettent de couper la hiérarchie. La figure 2.1 montre l'évolution d'un mode k obtenu avec cette approche. On voit qu'après l'entrée dans l'horizon la perturbation de densité de la matière baryonique reste couplée à celle du rayonnement, et cela jusqu'à la recombinaison. Puis les fluctuations de matière baryoniques peuvent croître de nouveau. Le plasma est sujet à des oscillations acoustiques tandis que les fluctuations de matière noire peuvent continuer de croître avec un taux qui dépend du facteur d'échelle comme $\propto \ln(a)$. Après la recombinaison les fluctuations de matière noire ont alors une amplitude beaucoup plus élevée que celle des baryons. Ceux-ci tombent alors dans le potentiel créé par les particules de matière noire pour former les structures.

La présence de la matière noire permet donc d'expliquer comment les structures parviennent à se former relativement rapidement en dépit de la pression et de l'expansion qui affectent fortement les baryons. En fait, le contraste de température $\Delta T/T \simeq 10^{-5}$ à l'échelle comobile des structures mesurée dans le CMB, qui peut être mis en relation avec $\delta\rho/\rho$ baryonique, ne pourrait pas expliquer des contrastes de densité $\delta\rho/\rho \gg 1$ observés à l'époque actuelle si l'on excluait la matière non collisionnelle. La figure 2.1 montre comment dans la réalité la situation est plus compliquée, mais le comportement général reste le même que celui donné par la description phénoménologique précédente. Pour les modes dont la taille reste toujours plus grande que celle du rayon de Hubble pendant l'ère radiative, $P(k) \propto k$ après la recombinaison. Pour les modes avec $k \ll k_{eq}$,

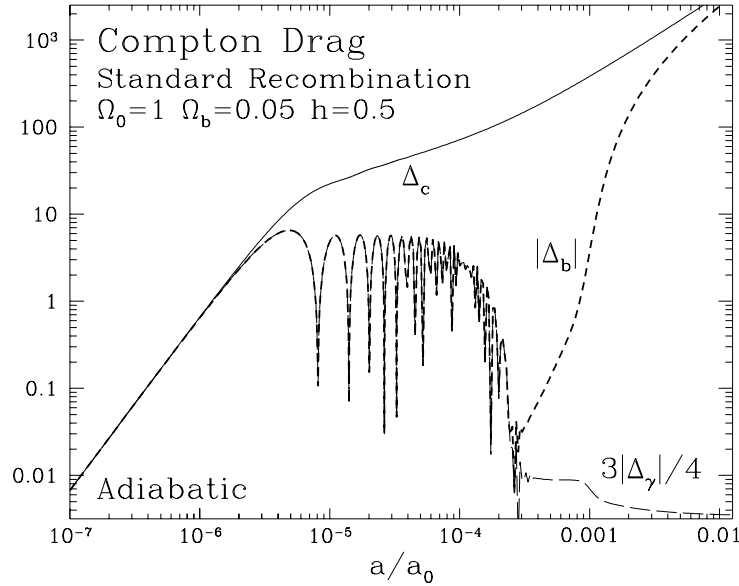


FIG. 2.1 – *Passage d'une fluctuation primordiale à travers l'horizon (Hu 1995). On voit l'évolution des fluctuations pour les différents composants.*

$P(k) \propto k^{-3}$. Pour rendre compte de cette histoire, il est possible de trouver des ajustements de la fonction de transfert $T(k)$ et de la caractériser par le paramètre $\Gamma = \Omega_m h^2$ qui détermine la forme du spectre. Pour les limites $k \rightarrow 0$ et $k \rightarrow \infty$ on retrouve respectivement $P(k) \propto k$ et $P(k) \propto k^{-3}$.

2.4 Normalisation du spectre

Outre la forme du spectre de puissance, il faut aussi connaître son amplitude et donc déterminer sa normalisation, A . L'inflation ne prédit malheureusement pas sa valeur et il est donc indispensable d'avoir recours aux observations astronomiques pour la mesurer. Plusieurs approches sont possibles. La première est une mesure de la normalisation du spectre donnée par l'amplitude des fluctuations du champ de température du CMB à très grande échelle, notamment grâce à COBE. Cette normalisation a l'avantage de se fonder sur les échelles correspondant au régime linéaire des perturbations et peut donc être directement utilisée comme une référence de A . La deuxième est basée sur les comptages de galaxies. On mesure les fluctuations de densité du nombre de galaxies δn à l'époque actuelle sur une sphère de rayon $R = 8h^{-1} Mpc$. En supposant que la mesure ne soit pas biaisée, c'est à dire que les fluctuations du nombre de galaxies suivent les fluctuations de densité de matière, on obtient:

$$\left\langle \frac{\delta n}{n} \right\rangle_R = \int d^3k P(k,0) W(kR) \approx 1, \quad (2.18)$$

et l'on appelle usuellement cette quantité σ_8 . La masse contenue dans cette sphère correspond à celle d'un amas de galaxies $\simeq 10^{15} M_\odot$. Du point de vue dynamique cette échelle indique que les amas sont les structures qui aujourd'hui établissent la transition vers le régime non-linéaire. La

constante σ_8 ne peut donc pas à priori être reliée à la constante de normalisation A sans un modèle d'évolution non-linéaire qui établit le lien entre ces deux paramètres. Dans la pratique, on voit que la relation entre A et σ_8 ne dépend pas trop de l'évolution non-linéaire du spectre, cela parce que l'écart au régime perturbatif est encore faible pour $\left\langle \frac{\delta\rho}{\rho} \right\rangle_R \simeq 1$. Pour cette raison, aborder la normalisation du spectre en terme de σ_8 a effectivement un sens.

2.5 Évolution des fluctuations après la recombinaison

Pour décrire l'évolution du spectre après la recombinaison, on part des équations d'Euler et de Poisson pour suivre l'évolution du fluide de matière en fonction du champ de gravité et de l'expansion de l'Univers [Bernardeau et al. 2002]. Les équations d'Euler et de continuité du fluide peuvent être obtenues en exprimant, via les moments de l'équation de Vlasov, la conservation de densité du fluide $f(\mathbf{x}, \mathbf{p}, t)$ dans l'espace des phases (\mathbf{x}, \mathbf{p}) . Dans un espace en expansion il est pratique d'utiliser des coordonnées comobiles $\mathbf{x}(t)$ et un champ de vitesse comobile $\dot{\mathbf{x}}(t)$:

$$\mathbf{r}(t) = a(t)\mathbf{x}(t) \quad \text{et} \quad \mathbf{v}(t) = H(t)\mathbf{r}(t) + \mathbf{u}(t) , \quad (2.19)$$

où $a(t)$ est le facteur d'expansion de l'Univers et $\mathbf{u}(\mathbf{x}, t) = a(t)\dot{\mathbf{x}}$ est la vitesse particulière du fluide par rapport au mouvement d'expansion d'ensemble. On définit aussi le potentiel Φ tel que $\frac{1}{m} \frac{d\mathbf{p}}{dt} = \frac{d\mathbf{u}}{dt} = -a\nabla\Phi$, relatif aux sur-densités locales:

$$\begin{aligned} \frac{df}{dt} &= \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{x}} \dot{\mathbf{x}} + \frac{\partial f}{\partial \mathbf{p}} \dot{\mathbf{p}} \\ &= \frac{\partial f}{\partial t} + \frac{\mathbf{p}}{a^2 m} \frac{\partial f}{\partial \mathbf{x}} - m \frac{\partial f}{\partial \mathbf{p}} \nabla\Phi(\mathbf{x}) = 0 , \end{aligned} \quad (2.20)$$

où l'on a utilisé la relation reliant la vitesse et l'impulsion $\mathbf{u} = \mathbf{p}/m$, où m est la masse des particules du fluide.

Les moments de la distribution $f(\mathbf{x}, \mathbf{p}, t)$ sont définis par:

$$\begin{aligned} \int d^3\mathbf{p} f(\mathbf{x}, \mathbf{p}, t) &\equiv \rho(\mathbf{x}, t) \\ \int d^3\mathbf{p} \frac{\mathbf{p}}{am} f(\mathbf{x}, \mathbf{p}, t) &\equiv \rho(\mathbf{x}, t) \mathbf{u}(\mathbf{x}, t) \\ \int d^3\mathbf{p} \frac{\mathbf{p}_i \mathbf{p}_j}{a^2 m^2} f(\mathbf{x}, \mathbf{p}, t) &\equiv \rho(\mathbf{x}, t) [\mathbf{u}_i(\mathbf{x}, t) \mathbf{u}_j(\mathbf{x}, t) + \sigma_{ij}(\mathbf{x}, t)] \end{aligned} \quad (2.21)$$

où σ_{ij} est le tenseur des contraintes du fluide. Dans la suite on ignorera ce terme qui concerne la déviation par rapport à un seul flot dans l'espace des phases en posant $\sigma_{ij} = 0$. Cette approximation est vraie dans le cas d'un fluide parfait sans pression. Dans le cas de chocs ou de vortex cette approximation n'est plus valable. En ce qui nous concerne, il s'agit de phénomènes qui se situent bien au-delà des régimes linéaire et perturbatif non linéaire auxquels on s'intéresse ici et nous ne les discuterons pas.

Les moments d'ordre zéro et du premier ordre de l'équation 2.20 donnent le système:

$$\frac{\partial \delta}{\partial t} = \frac{1}{a} \nabla [1 + \delta(\mathbf{x}, t)] \mathbf{u}(\mathbf{x}, t) \quad (2.22)$$

$$\frac{\partial \mathbf{u}(\mathbf{x}, t)}{\partial t} + \frac{1}{a} (\mathbf{u}(\mathbf{x}, t) \nabla) \mathbf{u}(\mathbf{x}, t) = -H(t) \mathbf{u} - \frac{1}{a} \nabla \Phi(\mathbf{x}, t) \quad (2.23)$$

$$\Delta \Phi = 4\pi G a(t)^2 \bar{\rho}(t) \delta(\mathbf{x}, t) , \quad (2.24)$$

où l'équation 2.24 représente l'équation de Poisson exprimée en coordonnées comobiles. La résolution du système 2.22, 2.23 et 2.24 est compliquée. Toutefois, on peut utiliser l'approximation linéaire et perturbative non linéaire pour trouver des solutions analytiques.

2.5.1 Le régime linéaire

Dans le cas où le contraste de densité est beaucoup plus petit que l'unité et que le gradient de la vitesse particulière du fluide est petit par rapport à la constante de Hubble H , on a:

$$|\delta| \ll 1 \quad \text{et} \quad \frac{1}{a} \frac{\partial \mathbf{u}_i}{\partial \mathbf{x}_j} \ll H$$

On peut donc écrire au premier ordre:

$$\begin{aligned} \frac{\partial \delta}{\partial t} + \frac{1}{a} \nabla \mathbf{u} &= 0 \\ \frac{\partial^2 \delta}{\partial t^2} + 2 \frac{\dot{a}}{a} \frac{\partial \delta}{\partial t} &= 4\pi G \bar{\rho} \delta . \end{aligned} \quad (2.25)$$

Les solutions du système 2.25 sont de la forme:

$$\delta(\mathbf{x}, t) = D_+(t) \delta_+(\mathbf{x}) + D_-(t) \delta_-(\mathbf{x}) \quad (2.26)$$

et donc les modes du spectre de puissance $P(k, t)$ évoluent de façon indépendante et peuvent être décrits selon la loi suivante:

$$P(k, t) = [D_+(t)]^2 P(k) , \quad (2.27)$$

où $P(k)$ est le spectre post-recombinaison. L'expression de $D_+(t)$ dépend du modèle d'Univers considéré.

Dans un Univers Einstein de Sitter ($\Omega_m = 1, \Omega_\Lambda = 0$) on a $D_+(t) = a(t)$ qui concorde avec le résultat trouvé avec l'approche phénoménologique pour un Univers dominé par la matière. Dans un cas plus général on peut l'exprimer de la façon suivante:

$$D_+(a) = \frac{5}{2} H(a) \Omega_m \int_0^a \frac{da'}{a'^3 H(a')} . \quad (2.28)$$

2.5.2 Le régime perturbatif non linéaire

Le régime linéaire est de moins en moins valable au fur et à mesure que l'on se rapproche de l'époque actuelle ou que l'on s'intéresse aux petites échelles. On peut néanmoins trouver des

solutions perturbatives du contraste de densité. Pour cela il est plus pratique de travailler dans l'espace de Fourier. Si on prend la divergence de l'équation 2.23 on trouve:

$$\frac{\partial}{\partial t}(\nabla \mathbf{u}(\mathbf{x}, t)) + H(t) \nabla \mathbf{u}(\mathbf{x}, t) + \frac{1}{a} \mathbf{u}(\mathbf{x}, t) \nabla (\nabla \mathbf{u})(\mathbf{x}, t) = -\frac{1}{a} \Delta \Phi \quad (2.29)$$

On peut donc remplacer directement l'équation de Poisson dans l'équation 2.29. La transformée de Fourier de l'équation 2.29 et 2.22 donne alors:

$$\begin{aligned} \frac{\partial \tilde{\delta}(\mathbf{k}, t)}{\partial t} + \frac{1}{a} \tilde{\theta}(\mathbf{k}, t) &= - \int d^3 \mathbf{k}_1 d^3 \mathbf{k}_2 \delta_{(\mathbf{k}=\mathbf{k}_1+\mathbf{k}_2)} \tilde{\theta}(\mathbf{k}_1, t) \tilde{\delta}(\mathbf{k}_2, t) \alpha(\mathbf{k}_1, \mathbf{k}_2) \quad (2.30) \\ \frac{\partial \tilde{\theta}(\mathbf{k}, t)}{\partial t} + H(t) \tilde{\theta}(\mathbf{k}, t) + 4\pi G a \bar{\rho} \tilde{\delta}(\mathbf{k}, t) &= -\frac{1}{a} \int d^3 \mathbf{k}_1 d^3 \mathbf{k}_2 \delta_{(\mathbf{k}=\mathbf{k}_1+\mathbf{k}_2)} \tilde{\theta}(\mathbf{k}_1, t) \tilde{\theta}(\mathbf{k}_2, t) \beta(\mathbf{k}_1, \mathbf{k}_2) \end{aligned}$$

où on a défini:

$$\alpha(\mathbf{k}_1, \mathbf{k}_2) = \frac{(\mathbf{k}_1 + \mathbf{k}_2) \mathbf{k}_1}{k_1^2} \quad \text{et} \quad \beta(\mathbf{k}_1, \mathbf{k}_2) = \frac{|\mathbf{k}_1 + \mathbf{k}_2|^2 (\mathbf{k}_1 \mathbf{k}_2)}{2k_1^2 k_2^2} \quad (2.31)$$

et $\theta(\mathbf{x}, t) = \nabla \mathbf{u}(\mathbf{x}, t)$ On décrit alors le contraste de densité comme une série des puissances du contraste de densité en régime linéaire:

$$\delta(\mathbf{x}, t) = \sum_{n=1}^{\infty} \delta^{(n)}(\mathbf{x}, t) \quad (2.32)$$

où $\delta^{(n)}(\mathbf{x}, t) \propto (\delta^{(1)}(\mathbf{x}, t))^n$ et $\delta^{(1)}(\mathbf{x}, t)$ est le contraste de densité en régime linéaire.

De la même façon on développe $\theta(\mathbf{x}, t)$ comme:

$$\theta(\mathbf{x}, t) = \sum_{n=1}^{\infty} \theta^{(n)}(\mathbf{x}, t) \quad (2.33)$$

où $\theta^{(1)} = -a \frac{\partial \delta^{(1)}}{\partial t}$ (première équation du système 2.25).

Dans le cas Einstein de Sitter, dans l'espace de Fourier les équations 2.32 et 2.33 deviennent:

$$\tilde{\delta}(\mathbf{k}, t) = \sum_{n=1}^{\infty} a^n(t) \tilde{\delta}^{(n)}(\mathbf{k}) \quad \text{et} \quad \tilde{\theta}(\mathbf{k}, t) = -H(t) a(t) \sum_{n=1}^{\infty} a^n(t) \tilde{\theta}^{(n)}(\mathbf{k}) \quad (2.34)$$

où $\delta^{(n)}(\mathbf{k})$ et $\theta^{(n)}(\mathbf{k})$ sont donnés respectivement par:

$$\begin{aligned} \tilde{\delta}^{(n)}(\mathbf{k}) &= \int d^3 \mathbf{q}_1 \frac{1}{(2\pi)^3} \dots \int d^3 \mathbf{q}_n \frac{1}{(2\pi)^3} \delta_{D(\mathbf{k}=(\mathbf{q}_1+\dots+\mathbf{q}_n))} F(\mathbf{q}_1, \dots, \mathbf{q}_n) \tilde{\delta}^{(1)}(\mathbf{q}_1) \dots \tilde{\delta}^{(1)}(\mathbf{q}_n) \quad (2.35) \\ \tilde{\theta}^{(n)}(\mathbf{k}) &= \int d^3 \mathbf{q}_1 \frac{1}{(2\pi)^3} \dots \int d^3 \mathbf{q}_n \frac{1}{(2\pi)^3} \delta_{D(\mathbf{k}=(\mathbf{q}_1+\dots+\mathbf{q}_n))} G(\mathbf{q}_1, \dots, \mathbf{q}_n) \tilde{\delta}^{(1)}(\mathbf{q}_1) \dots \tilde{\delta}^{(1)}(\mathbf{q}_n) \end{aligned}$$

$F(\mathbf{q}_1, \dots, \mathbf{q}_n)$ et $G(\mathbf{q}_1, \dots, \mathbf{q}_n)$ expriment le couplage entre le mode \mathbf{k} et tous les autres. Ces fonctions peuvent être évaluées pour n'importe quel ordre grâce à des lois de récursivité (voir aussi

[Bouchet et al. 1992] pour une dérivation).

$$\begin{aligned}
F_1 &= 1 \\
F_2 &= \frac{5}{7} + \frac{1}{2} \frac{\mathbf{q}_1 \mathbf{q}_2}{(q_1 q_2)} \left(\frac{q_1}{q_2} + \frac{q_2}{q_1} \right) + \frac{2}{7} \frac{(\mathbf{q}_1 \mathbf{q}_2)^2}{(q_1^2 q_2^2)} \\
G_1 &= 1 \\
G_2 &= \frac{3}{7} + \frac{1}{2} \frac{\mathbf{q}_1 \mathbf{q}_2}{(q_1 q_2)} \left(\frac{q_1}{q_2} + \frac{q_2}{q_1} \right) + \frac{4}{7} \frac{(\mathbf{q}_1 \mathbf{q}_2)^2}{(q_1^2 q_2^2)}
\end{aligned} \tag{2.36}$$

Dans le cas plus général on remplace $a(t)^n$ avec $D_+(t)^n$ dans l'équation 2.32 et on utilise la relation 2.25 pour trouver l'expression de $\theta(\mathbf{x}, t)$, $\delta(\mathbf{x}, t)$. Elle est alors injectée dans 2.30 pour retrouver la nouvelle expression de $F(\mathbf{q}_1, \dots, \mathbf{q}_n)$ et $G(\mathbf{q}_1, \dots, \mathbf{q}_n)$. Cependant, Bouchet et al. (1992) ont montré que la dépendance de ces fonctions avec le modèle cosmologique choisi est petite.

2.5.3 Evolution non linéaire et non-gaussianité du champ

En régime linéaire les modes restent découplés, la gaussianité du champ primordial reste alors conservée. Toutefois, d'après l'équation 2.35, on constate que dès le régime perturbatif on s'écarte des propriétés gaussiennes du champ δ . C'est une conséquence de l'apparition du couplage entre les modes. Ce processus est facilement expliqué en termes de l'évolution du collapse gravitationnel. En effet, au fur et à mesure que les structures s'effondrent le champ δ devient asymétrique (la valeur des pics négatifs (sous-densités) est fixée par le fait que $\rho > 0$. La valeur des pics positifs (sur-densités), en revanche, augmente de plus en plus. En plus de la variance, il faut alors connaître les autres moments pour déterminer les propriétés du champ. Le premier ordre de non gaussianité du champ δ est donné par les statistiques à trois points. Si on considère le développement perturbatif illustré plus bas on trouve:

$$\langle \delta(\mathbf{x}_1) \delta(\mathbf{x}_2) \delta(\mathbf{x}_3) \rangle = \langle (\delta^{(1)}(\mathbf{x}_1) \delta^{(1)}(\mathbf{x}_2) \delta^{(2)}(\mathbf{x}_3)) \rangle + 2 \text{ termes obtenus par permutation} \tag{2.37}$$

Les termes portent tous la même contribution, donc il suffit de multiplier par 3 le résultat concernant une des composantes.

Dans le cas d'un Univers Einstein de Sitter, un changement de variable dans l'expression $F_2(\mathbf{k}_1, \mathbf{k}_2)$ donné par 2.36, donne:

$$\tilde{\delta}^{(2)}(\mathbf{k}) = \int d^3 \mathbf{k}' \left[\frac{5}{7} + \frac{1}{2} \left(\frac{1}{|\mathbf{k}'|^2} + \frac{1}{|\mathbf{k} - \mathbf{k}'|^2} \right) \mathbf{k} \cdot (\mathbf{k} - \mathbf{k}') + \frac{2}{7} \frac{(\mathbf{k} \cdot (\mathbf{k} - \mathbf{k}'))^2}{|\mathbf{k}'|^2 |\mathbf{k} - \mathbf{k}'|^2} \right] \tilde{\delta}(\mathbf{k}') \tilde{\delta}(\mathbf{k} - \mathbf{k}') \tag{2.38}$$

On obtient donc:

$$\begin{aligned}
\langle \delta(\mathbf{x}) \delta(\mathbf{x}_2) \delta(\mathbf{x}_3) \rangle &\sim 3 \int \frac{d^3 \mathbf{k}}{(2\pi)^3} a(t) \int \frac{d^3 \mathbf{k}'}{(2\pi)^3} a(t) \int \frac{d^3 \mathbf{k}_1}{(2\pi)^3} a(t) \int \frac{d^3 \mathbf{k}_2}{(2\pi)^{3/2}} a(t) \\
&\langle \tilde{\delta}(\mathbf{k}) \tilde{\delta}(\mathbf{k} - \mathbf{k}') \tilde{\delta}(\mathbf{k}_2) \tilde{\delta}(\mathbf{k}_3) \rangle F_2(\mathbf{k} - \mathbf{k}', \mathbf{k}) \exp(i(\mathbf{k}_1 \mathbf{x}_1 + \mathbf{k}_2 \mathbf{x}_2 + \mathbf{k} \mathbf{x}))
\end{aligned}$$

En utilisant le fait que $\delta^{(1)}$ est un champ Gaussien:

$$\begin{aligned}
\langle \tilde{\delta}^{(1)}(\mathbf{k}_1) \tilde{\delta}^{(1)}(\mathbf{k}_2) \tilde{\delta}^{(1)}(\mathbf{k}_3) \tilde{\delta}^{(1)}(\mathbf{k}_4) \rangle &= \langle \tilde{\delta}^{(1)}(\mathbf{k}_1) \tilde{\delta}^{(1)}(\mathbf{k}_2) \rangle \langle \tilde{\delta}^{(1)}(\mathbf{k}_3) \tilde{\delta}^{(1)}(\mathbf{k}_4) \rangle \\
&+ 2 \text{ autres termes obtenus par permutation des indices}
\end{aligned} \tag{2.39}$$

La composante qui associe \mathbf{k} et \mathbf{k}' est nulle à cause de la fonction $F_2(\mathbf{k}, \mathbf{k}')$; on n'a donc que deux termes qui demeurent:

$$\langle \tilde{\delta}(\mathbf{k})\tilde{\delta}(\mathbf{k}_2)\tilde{\delta}(\mathbf{k}_3) \rangle \sim 6(2\pi)^3 a(t)^4 P(k_1)P(k_2)F_2(\mathbf{k}_1, \mathbf{k}_2)\delta_{(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3=0)}$$

2.5.4 Le régime non linéaire

La résolution analytique du système 2.22, 2.23, 2.24, dans le régime non linéaire n'est pas possible. Cependant, il est possible de trouver des expressions empiriques du spectre non linéaire $P_{nl}(k)$ en fonction du spectre $P(k)$ en régime linéaire. Elles permettent d'ajuster le champ des fluctuations δ mesuré dans des simulations à N -corps. Ce type d'approche suggéré pour la première fois par Hamilton et al. (1991) est basé sur une procédure de calibration entre le régime initial linéaire et un régime final fortement non linéaire. L'hypothèse à la base de ce type de calibration est que le collapse gravitationnel engendre un changement d'échelle. On considère une sphère isolée; la relation entre la taille initiale r_i et finale r_f de la sphère est donnée par: $r_f = (\rho_f/\rho_{bg})^{1/3}r_i$, où ρ_f représente la densité finale de la sphère et ρ_{bg} représente la densité moyenne du fond. Cela suggère que la fonction de corrélation en régime non linéaire ξ_{nl} à l'échelle r_{nl} soit liée à la taille r_l avant collapse gravitationnel de la façon suivante:

$$r_l = [1 + \bar{\xi}_{nl}(r_{nl})]^{1/3}r_{nl} \quad (2.40)$$

où $\bar{\xi}(r)$ représente la valeur de la fonction de corrélation moyennée dans le rayon comobile r . Avec cette calibration on peut trouver une fonction f_{nl} telle que:

$$\bar{\xi}_{nl}(r_{nl}) = f_{nl}[\bar{\xi}_l(r_l)] \quad (2.41)$$

Les équations 2.40 et 2.41 permettent de trouver une expression empirique de la fonction $f_{nl}(x)$ en utilisant des simulations à N -corps. Pendant longtemps le modèle de "clustering stable" (Davis & Peebles 1997) a été considéré comme celui décrivant le mieux les structures après effondrement gravitationnel. Il correspond à un régime dans lequel l'amas est à l'équilibre et isolé par rapport à son environnement. Le comportement de la fonction $f_{nl}(x)$ dans les deux cas limites est: $f_{nl}(x) \propto x$ quand $x \ll 1$ (régime linéaire) et $f_{nl}(x) \propto x^{3/2}$ pour $x \gg 1$ (clustering stable)³. Cette méthode permet une description du spectre dans le régime de transition entre les deux cas limites linéaire et fortement linéaire, les seuls cas qui peuvent être traités avant l'introduction de cette calibration. Cependant, il avait été validé par Hamilton et al. (1991) que sur des simulations N -corps avec $\Omega_m = 1$. Peacock and Dodds (1996) généralisent ce type de procédure de calibration pour des autres cosmologies. La calibration proposée par Peacock et Dodds 1996 a été ultérieurement remise en cause dans son fondement car l'état de "clustering stable" ne semble pas compatible avec une formation hiérarchique des structures où les structures semblent accroître leur masse grâce à des fusions de halos continus. Smith et al. (2003) proposent une alternative basée sur l'ajustement du spectre en régime fortement non linéaire à celui des modèles de halo. Cette approche devrait donner

3. Dans le régime de clustering stable le contraste de densité doit se maintenir constant et donc $\xi_{nl} \propto a^3$. Dans le régime linéaire l'évolution du contraste de densité est décrite par 2.28. Dans le cas Einstein de Sitter qui correspond au seul cas traité par Hamilton et al (1991), $\xi_l \propto a^2$. Successivement, Carrol, Press et Turner (1992) donnent une expression très précise pour l'équation 2.28 en fonction de la cosmologie obtenue par ajustement.

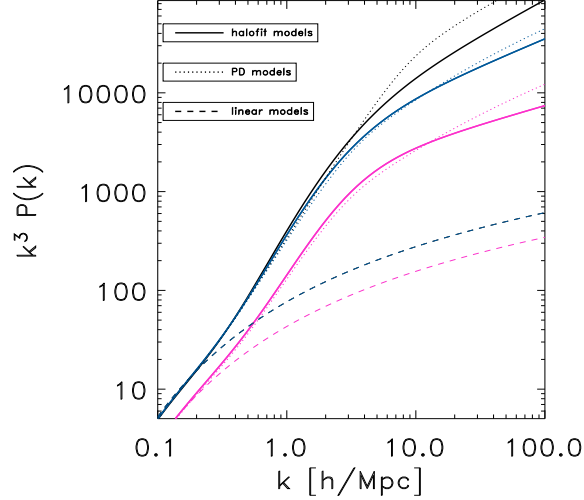


FIG. 2.2 – Différence entre le spectre de fluctuations en approximation linéaire, celui proposé par Peacock & Dodds 1996 et par Smith et al. 2003 pour trois différentes cosmologies. Du haut en bas: Λ CDM (noir) ($\Omega_m = 0.3$, $\sigma_8 = 0.8$, $h = 0.7$), Λ CDM (bleu) ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $\sigma_8 = 0.8$, $h = 0.7$) et TCDM (magenta) ($\Omega_m = 1.$, $\sigma_8 = 0.6$, $h = 0.5$).

des prédictions plus fiables à petite échelle. Dans tous les cas, les incertitudes de la représentation du régime non-linéaire impliquent à ce jour une imprécision de 10 – 15% entre les prédictions et les données. Le plot 2.2 montre la différence entre l’approximation linéaire, la méthode PD et la méthode halofit pour trois cosmologies différentes: Λ CDM, TCDM et OCDM.

2.6 Dynamique de l’énergie sombre

Comme nous l’avons anticipé dans l’introduction, la dynamique de l’Univers actuel suggère la présence d’une forme d’énergie dont la contribution est aujourd’hui dominante et qui impose une expansion accélérée de l’Univers. Le modèle le plus simple que l’on peut imaginer est celui d’une constante cosmologique. Elle peut être insérée directement dans les équations d’Einstein. Les équations d’Einstein pour une cosmologie FRW et un fluide cosmologique parfait 2.7 avec constante cosmologique s’écrivent alors:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) - \frac{Kc^2}{a} + \frac{\Lambda(t)}{3} \quad (2.42)$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G}{3}\left(\rho(t) + \frac{3P(t)}{c^2}\right) + \frac{\Lambda(t)}{3} \quad (2.43)$$

On définit naturellement $\Omega_\Lambda = \frac{\Lambda(t)c^2}{3H^2}$. Cette équation donne $P = -\rho c^2$ analogue à l’expression des photons et des baryons. Dans le cas d’un Univers plat, hypothèse largement confortée par les

observations des anisotropies du CMB, on trouve:

$$H(z) = H_0 \sqrt{(\Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + \Omega_{\Lambda,0})} \quad (2.44)$$

A partir d'un certain redshift dépendant de la valeur de Λ , l'énergie sombre commence à dominer. La valeur la plus vraisemblable selon l'ensemble des observations actuelles est $\Omega_{\Lambda,0} \simeq 0.7$, ce qui implique que la matière sombre commence à dominer vers $z < 2$.

Si d'un côté les observations cosmologiques concordent avec cette valeur, il est incroyablement difficile de la justifier sur le plan théorique et notamment de la réconcilier avec l'énergie associée à l'état fondamental du champ électromagnétique, ce que l'on appelle densité d'énergie associée au vide. Des théories qui représentent des extensions de l'action avec laquelle on dérive les équations d'Einstein, associent cette énergie à un champ scalaire ϕ évoluant dans un potentiel. Le formalisme de ces théories s'inspire de celui adopté pour construire des modèles d'inflation. Parmi celles-ci, on trouve les théories tenseur-scalaires, dont les modèles de quintessence représentent une sous-classe très populaire. Établir analytiquement des équations à partir de l'action de Friedmann modifiées sort du cadre de cette thèse. Une discussion détaillée ainsi qu'une liste exhaustive de références est présentée dans les papiers Tegmark (2002) et Esposito-Farèse (2001) qui représentent une introduction générale aux théories tenseur-scalaire de la gravitation. En ce qui nous concerne ici, ce que l'on retiendra, c'est que la définition de ces équations permet de définir, dans le cas des théories de quintessence, une pression P_x et une densité d'énergie ρ_x associée au champ ϕ :

$$\rho_x = \frac{c^2}{8\pi G} \left(\frac{1}{2} \dot{\phi}^2 + U(\phi) \right) \quad (2.45)$$

$$P_x = \frac{1}{8\pi G} \left(-\frac{1}{2} \dot{\phi}^2 + U(\phi) \right) \quad (2.46)$$

où $U(\phi)$ est le potentiel cosmologique dans lequel le champ ϕ évolue. En choisissant un potentiel $U(\phi)$ approprié on peut alors, déduire l'expression de $H(z)$ en fonction du redshift. Du point de vue observationnel, on peut donc trouver des contraintes sur l'équation d'état de l'énergie sombre, $w = P_x/(c^2\rho_x)$ en fonction du redshift. Selon la fonction choisie pour décrire l'évolution de l'équation d'état la dynamique de l'Univers change; en particulier, il change le moment auquel l'énergie sombre commence à dominer l'histoire de l'Univers. La mesure de courbes de luminosité des SNIa dont la luminosité à un redshift donné dépend de la métrique de l'Univers, peut directement tester l'évolution de l'équation d'état comme développement en série autour du redshift $z = 0$. D'un autre côté, la présence de l'énergie sombre, en accélérant l'expansion de l'Univers, ralentit la croissance des fluctuations de densité de matière et donc le moment du passage du régime linéaire au régime non linéaire. L'évolution du contraste de densité est plus ou moins accentué selon le type d'équation d'état caractérisant l'énergie sombre. Comme il sera plus clairement expliqué dans la suite, le cisaillement cosmologique est proportionnel à l'amplitude des fluctuations de densité que la lumière traverse et donc le signal mesuré dépend de l'évolution de l'équation d'état de l'énergie sombre.

Chapitre 3

Le cisaillement gravitationnel cosmologique

Dans ce chapitre on décrit le formalisme utilisé dans l'étude du cisaillement gravitationnel cosmologique, c'est à dire l'analyse statistique de la déflexion de la lumière sur des échelles cosmologiques par les grandes structures de l'Univers. J'y présente plus particulièrement les quantités et les notations propres à ce type d'analyse (voir aussi les revues Bartelmann & Schneider 2001, van Waerbeke & Mellier 2003, van Waerbeke & Mellier 2005). L'origine physique du cisaillement cosmologique peut être résumée en quelques lignes. En effet, il est connu depuis la formulation de la Relativité Générale par Albert Einstein 1915, que la distribution de la matière affecte la trajectoire des rayons lumineux. Dans le cas d'une source étendue telle qu'une galaxie, l'image observée est affectée par une distorsion et une amplification dues à la distribution de la matière entre la source et l'observateur. Dans certains cas ces effets peuvent atteindre de fortes amplitudes, lorsque le champ de gravité subi par les photons est très intense. Il engendre alors des cas spectaculaires de lentilles gravitationnelles comme les images multiples et les arcs. De tels cas existent dans la nature et sont systématiquement répertoriés par les grands relevés d'imagerie astronomiques. Mais le cas le plus fréquent correspond à des effets de faible amplitude qu'il n'est pas possible de détecter à l'oeil. Cependant, si on pouvait connaître les propriétés intrinsèques des sources on pourrait les comparer avec celles des images observées et déterminer ensuite les propriétés de la densité de matière projetée le long de la ligne de visée. Dans la réalité, on ne connaît pas les propriétés intrinsèques de chaque source, mais on peut faire quelques hypothèses concernant les propriétés statistiques sur leur distribution en ellipticité, les comparer à celles des images, puis en déduire des propriétés du champ de matière projetée. Les caractéristiques de ce champ dépendent de la géométrie de l'Univers, de la nature de ses composantes (matière, énergie, courbure) et du champ de fluctuations primordial $\delta(\mathbf{x})$.

3.1 L'équation des lentilles

On s'intéresse à la propagation de la lumière dans la métrique FRW. La séparation ξ entre deux rayons proches appartenant au même faisceau dans un espace-temps quelconque peut être écrite comme (voir annexe A):

$$\frac{d^2 \xi}{d\lambda^2} = \mathcal{T}\xi(\lambda) \quad (3.1)$$

où λ est appelé paramètre affine et paramétrise les géodésiques. La matrice \mathcal{T} est appelée matrice optique de distorsion et son expression dépend des dérivées de la métrique. On peut expliciter l'équation 3.1 dans un Univers avec une métrique 2.6:

$$\frac{d^2 \mathbf{x}}{d\omega^2} - K\mathbf{x} = 0, \quad (3.2)$$

où on a défini la distance comobile $\mathbf{x} = a^{-1}\xi$. On voit donc que la courbure de la métrique K affecte la géométrie du faisceau et que dans le cas $K = 0$ elle reste effectivement inchangée, comme on s'attendait dans le cas d'une géométrie Euclidienne.

Toutefois, la métrique FRW ne tient pas compte des inhomogénéités de matière locales qui sont à l'origine des lentilles gravitationnelles. L'équation qui exprime la déformation du faisceau

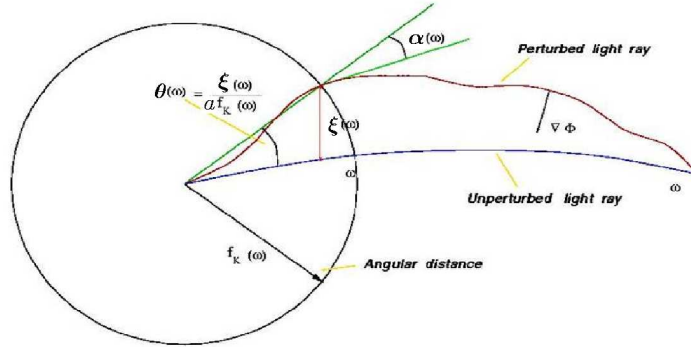


FIG. 3.1 – Illustration de la déflexion d'un rayon de lumière causée par les inhomogénéités de matière entre les sources (à droite) et l'observateur (à gauche). α représente l'angle de déflexion, i.e. la différence entre l'angle observé et celui attendu pour un Univers homogène. Image fournie par Yannick Mellier.

due aux inhomogénéités locales peut être déduite de la façon suivante. En supposant que la taille caractéristique des fluctuations est beaucoup plus petite que le rayon de Hubble, que le potentiel gravitationnel Φ correspondant aux fluctuations de densité soit tel que $\Phi \ll c^2$, que la vitesse propre v de déplacement des fluctuations soit telle que $v \ll c$ alors on peut définir une métrique de Minkowski perturbée au premier ordre post-Newtonien donnée par:

$$ds^2 = \left(1 + \frac{2\Phi}{c^2}\right) dt^2 - \left(1 - \frac{2\Phi}{c^2}\right) d\mathbf{x} \quad (3.3)$$

On utilise le principe de Fermat pour en déduire une relation entre la déviation du rayon de sa trajectoire non perturbée et le champ gravitationnel déflecteur (voir annexe B) :

$$\frac{d^2 \mathbf{x}}{dw^2} = -\frac{2}{c^2} \nabla_{\perp} \Phi \quad (3.4)$$

où $\nabla_{\perp} \Phi(\mathbf{x}(\boldsymbol{\theta}, w), w)$ indique la différence de la partie du potentiel Φ perpendiculaire à la ligne de visée.

La séparation angulaire comobile \mathbf{x} entre deux rayons séparés d'un angle $\boldsymbol{\theta}$ à la coordonnée radiale w est alors donnée par:

$$\frac{d^2 \mathbf{x}}{dt^2} = \frac{2}{c^2} \Delta[\nabla_{\perp} \Phi(\mathbf{x}(\boldsymbol{\theta}, w), w)] \quad (3.5)$$

où Δ indique la différence de la partie perpendiculaire du potentiel gravitationnel entre les deux rayons. Autrement dit, la séparation entre les rayons change selon la composante des forces de marée perpendiculaire à la direction de propagation du faisceau.

Les deux contributions, celle due aux inhomogénéités localisées et celle due à la courbure de l'Univers peuvent être sommées:

$$\frac{d^2 \mathbf{x}}{dt^2} - K \mathbf{x} = \frac{2}{c^2} \Delta[\nabla_{\perp} \Phi(\mathbf{x}(\boldsymbol{\theta}, w), w)] . \quad (3.6)$$

En considérant un observateur à $w = 0$ on peut résoudre cette équation différentielle en utilisant les conditions aux limites:

$$\mathbf{x} \Big|_{w=0} = 0 ; \quad \frac{d\mathbf{x}}{dw} \Big|_{w=0} = \boldsymbol{\theta} \quad (3.7)$$

sans pourtant en obtenir une solution sous forme explicite:

$$\mathbf{x}(\boldsymbol{\theta}, w) = f_K(w) \boldsymbol{\theta} - \frac{2}{c^2} \int_0^w dw' f_K(w - w') \Delta[\nabla_{\perp} \Phi(\mathbf{x}(\boldsymbol{\theta}, w'), w')] . \quad (3.8)$$

Cependant, dans le cas de faibles déflexions on peut toujours utiliser l'approximation de Born. Plus précisément dans l'hypothèse où le changement de distance comobile entre les rayons dus aux perturbations est beaucoup plus petit que la distance comobile sans perturbation par les inhomogénéités on peut écrire:

$$\frac{|\mathbf{x}(\boldsymbol{\theta}, w') - f_K(w') \boldsymbol{\theta}|}{f_K(w') \boldsymbol{\theta}} \ll 1 \quad (3.9)$$

où $f_K(w') \boldsymbol{\theta}$ est la distance comobile entre les rayons dans un Univers homogène. L'approximation de Born consiste à évaluer $\Phi(\mathbf{x}(\boldsymbol{\theta}, w), w)$ en fonction du potentiel $\Phi(f_K(w) \boldsymbol{\theta}, w)$ mesuré le long du rayon non-perturbé et à remplacer la différence $\Delta[\nabla_{\perp} \Phi]$ par le gradient de la différence des potentiels $\nabla_{\perp}[\Delta\Phi]$. A ce stade on peut aussi supposer que $\Delta\Phi = \Phi$ car la séparation entre les rayons dépend de la variation du potentiel et donc elle reste inchangée si on pose $\Phi = 0$ le long du rayon de référence. On peut donc définir l'angle de déflexion $\boldsymbol{\alpha}$ engendré par les inhomogénéités de la façon suivante (la signification géométrique des variables utilisées est représentée dans la figure 4.1):

$$\boldsymbol{\alpha}(\boldsymbol{\theta}, w) = \frac{f_K(w) \boldsymbol{\theta} - \mathbf{x}(\boldsymbol{\theta}, w)}{f_K(w)} = \frac{2}{c^2} \int_0^w dw' \frac{f_K(w - w')}{f_K(w)} [\nabla_{\perp} \Phi(f_K(w') \boldsymbol{\theta}, w')] \quad (3.10)$$

3.2 Amplification et cisaillement gravitationnel

On peut définir le Jacobien qui relie l'angle observé entre les deux rayons et l'angle que l'on aurait observé dans le cas d'un Univers homogène. L'équation 3.10 donne donc:

$$\mathcal{A}_{ij}(\boldsymbol{\theta}, w) = \mathcal{I}_{ij} - \frac{\partial \alpha_i(\boldsymbol{\theta}, w)}{\partial \theta_j} = \delta_{ij} - \frac{2}{c^2} \int_0^{w'} \frac{f_K(w-w')}{f_K(w)} f_K(w) \Phi_{ij}[f_K(w')\boldsymbol{\theta}, w'] \quad (3.11)$$

Dans l'approximation de Born la matrice \mathcal{A} est symétrique et on peut définir deux quantités, le **cisaillement** $\gamma = \gamma_1 + i\gamma_2$ et la **convergence** κ :

$$\mathcal{A} = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix} \quad (3.12)$$

On peut aussi définir un potentiel projeté, $\phi(\boldsymbol{\theta})$ tel que:

$$\phi(\boldsymbol{\theta}, w) = \frac{2}{c^2} \int_0^w dw' \frac{f_K(w-w')}{f_K(w)} f_K(w') \Phi[f_K(w')\boldsymbol{\theta}, w'] , \quad (3.13)$$

d'où l'on déduit:

$$\kappa = \frac{1}{2}(\phi_{,11} + \phi_{,22}), \quad \gamma = \frac{1}{2}(\phi_{,11} - \phi_{,22}) + i\phi_{,12} \quad (3.14)$$

Par ailleurs, l'équation de Poisson en coordonnées comobiles 2.24 s'écrit:

$$\nabla_x^2 \Phi = \frac{3H_0^2}{2a} \Omega_m \delta \quad (3.15)$$

Cette relation est utile pour exprimer la convergence κ en fonction des fluctuations de densité, δ , et faire ressortir explicitement une partie de la forte dépendance de κ en fonction du paramètre cosmologique Ω_m .

$$\kappa(\boldsymbol{\theta}, w) = \frac{3H_0^2 \Omega_m}{2c^2} \int_0^w dw' \frac{f_K(w') f_K(w-w')}{f_K(w)} \frac{\delta[f_K(w')\boldsymbol{\theta}, w']}{a(w')} \quad (3.16)$$

Mais cette expression restrictive ne donne la convergence que pour une source placée à une distance w . En pratique, on mesure la convergence κ en moyennant sur un ensemble de sources placées à différents redshifts et il convient de prendre en compte les propriétés de la distribution des sources $G(w)dw$:

$$\kappa(\boldsymbol{\theta}) = \frac{3H_0^2 \Omega_m}{2c^2} \int_0^{w_H} dw \bar{W}(w) f_K(w) \frac{\delta[f_K(w)\boldsymbol{\theta}, w]}{a(w)} \quad (3.17)$$

où la fonction $\bar{W}(w)$ est donnée par:

$$\bar{W}(w) \equiv \int_0^{w_H} dw' G(w') \frac{f_K(w-w')}{f_K(w')} . \quad (3.18)$$

w_H est la distance de l'horizon, définie comme la distance comobile pour laquelle on atteint un redshift infini.

D'après l'équation 3.17, les propriétés du champ de matière peuvent donc être reliées à celles du champ bi-dimensionnel κ . En effet, soient le spectre bi-dimensionnel $P_\kappa(l)$ de la convergence

κ , et $\langle \kappa(\mathbf{l})\kappa(\mathbf{l}') \rangle = (2\pi)^2 \delta_{D(l=l')} P_\kappa(l)$ la transformée de Fourier de la fonction de corrélation de $\xi(r) \equiv \langle \kappa(\boldsymbol{\theta})\kappa(\boldsymbol{\theta} + \mathbf{r}) \rangle$. Comme on pouvait s'y attendre il existe alors une relation entre le spectre des fluctuations tri-dimensionnel $P_\delta(k)$ et le spectre à deux dimensions de la convergence $P_\kappa(l)$ (voir annexe C):

$$P_\kappa(l) = \frac{9H_0^4 \Omega_m^2}{4c^4} \int_0^{w_H} dw \frac{\bar{W}^2(w)}{a^2(w)} P\left(\frac{l}{f_k(w)}, w\right). \quad (3.19)$$

Il faut souligner que les quantités κ et γ sont des fonctions du même champ scalaire ϕ , elles ne sont donc pas indépendantes. La relation entre ces deux champs est particulièrement simple si on écrit 3.14 dans l'espace de Fourier:

$$\tilde{\kappa}(l) = \frac{1}{2}(l_1^2 + l_2^2)\tilde{\phi}(\mathbf{l}) \quad \text{et} \quad \tilde{\gamma}(l) = \left[\frac{1}{2}(l_1^2 - l_2^2) + il_1 l_2\right]\tilde{\phi}(\mathbf{l}) \quad (3.20)$$

où l_1 et l_2 sont les composantes de \mathbf{l} . On déduit alors immédiatement:

$$\tilde{\gamma}(\mathbf{l}) = \tilde{\mathcal{D}}(\mathbf{l})\tilde{\kappa}(\mathbf{l}) \quad (3.21)$$

avec:

$$\tilde{\mathcal{D}}(l) = \frac{l_1^2 - l_2^2 + 2il_1 l_2}{|l|^2} \quad (3.22)$$

en multipliant par $\tilde{\mathcal{D}}^*(\mathbf{l})$ (on remarque que $\tilde{\mathcal{D}}\tilde{\mathcal{D}}^* = 1$) la relation 3.21, on trouve aussi $\tilde{\kappa}(\mathbf{l}) = \tilde{\mathcal{D}}(\mathbf{l})\tilde{\gamma}(\mathbf{l})$. La traduction de 3.21 dans l'espace réel est donnée par la convolution:

$$\kappa(\boldsymbol{\theta}) = \int d^2\boldsymbol{\theta}' \mathcal{D}^*(\boldsymbol{\theta} - \boldsymbol{\theta}')\gamma(\boldsymbol{\theta}') \quad (3.23)$$

où $\mathcal{D}^*(\boldsymbol{\theta})$ est la transformée de Fourier inverse de $\tilde{\mathcal{D}}^*(\mathbf{l})$:

$$\mathcal{D}(\boldsymbol{\theta}) = \frac{\theta_2^2 - \theta_1^2 - 2i\theta_1\theta_2}{|\boldsymbol{\theta}|^4} \quad (3.24)$$

La quantité $\kappa(\boldsymbol{\theta})$ est réelle et donc on doit avoir $\mathcal{I}m[D(\boldsymbol{\theta}) * \gamma(\boldsymbol{\theta})] = 0$. Ce qui est vrai, selon la définition de γ (eq. 3.14). Enfin la relation 3.21 permet de démontrer que le spectre de puissance de la masse projetée $P_\kappa(l)$ et celui du cisaillement $P_\gamma(l)$ sont identiques.

3.3 Les statistiques du deuxième ordre

La mesure la plus directe du spectre P_κ est celle de la fonction de corrélation:

$$\xi_\kappa(\phi) = \xi_\gamma(\phi) = \langle \gamma(\boldsymbol{\theta})\gamma(\boldsymbol{\theta} + \phi) \rangle = \int_0^\infty l \frac{dl}{2\pi} P_\kappa(l) J_0(l\phi) \quad (3.25)$$

où la moyenne $\langle \rangle$ est une moyenne d'ensemble.

Il est aussi utile de mesurer le champ de convergence ou bien le champ de cisaillement dans une ouverture circulaire de rayon ϑ avec l'estimateur défini de la façon suivante:

$$\bar{\kappa}(\vartheta) = \int_{\theta < \vartheta} d\theta U(\theta)\kappa(\theta), \quad \bar{\gamma}(\vartheta) = \int_{\theta < \vartheta} d\theta U(\theta)\gamma(\theta), \quad (3.26)$$

D'une façon générale on peut toujours relier ces quantités au spectre de puissance :

$$\langle \bar{\kappa}(\vartheta)^2 \rangle = \frac{1}{2\pi} \int_0^\infty l dl P_\kappa(l) \left(\int_0^\vartheta d\phi J_0(l\phi) U(\phi) \right)^2 \equiv \frac{1}{2\pi} \int_0^\infty l dl P_\kappa(l) [\mathcal{W}(l\vartheta)]^2 \quad (3.27)$$

où $[\mathcal{W}(l\vartheta)]^2$ représente le filtre que l'on applique au spectre projeté. Grâce à la relation 3.23 on peut réécrire le même genre d'expression aussi pour les moments du deuxième ordre du champ $\bar{\gamma}(\vartheta)$. Dans l'étude du cisaillement cosmologique on utilise couramment deux types de filtres: le filtre top-hat et le filtre compensé. Dans le cas du filtre top-hat on a:

$$\bar{\gamma}(\vartheta) = \frac{1}{\pi\vartheta} \int_{\theta < \vartheta} d^2\theta \gamma(\theta) \quad (3.28)$$

La variance est alors donnée par (*top-hat variance*):

$$\langle \gamma^2(\vartheta) \rangle = \frac{1}{2\pi} \int_0^\infty l dl P_\kappa(l) [\mathcal{W}_{TH}(l\vartheta)]^2 \quad \text{avec} \quad \mathcal{W}_{TH}(l\vartheta) = \left[2 \frac{J_1(l\vartheta)}{l\vartheta} \right]. \quad (3.29)$$

Une alternative particulièrement intéressante est la variance de la masse dans une ouverture (*aperture mass variance*), M_{ap}^2 , qui mesure la convergence dans un filtre compensé, c'est à dire tel que:

$$\int_{\theta < \vartheta} \theta U(\theta) d^2\theta = 0 \quad (3.30)$$

Une famille de filtres de ce type est donnée dans Schneider et al. 1998. Ce sont les filtres U tels que $U(\theta) = u(\theta/\vartheta)/\vartheta^2$ avec $u(x)$ donnée par:

$$u(x) = \frac{(s+2)^2}{\pi} (1-x^2)^s \left(\frac{1}{s+2} - x^2 \right) \quad (3.31)$$

où l est un paramètre arbitraire. Dans les mesures de M_{ap} présentées dans cette thèse on choisit un filtre de cette famille avec $s = 1$. Dans ce cas, M_{ap}^2 est donnée par:

$$\langle M_{ap}^2(\vartheta) \rangle = \frac{1}{2\pi} \int_0^\infty l dl P_\kappa(l) [\mathcal{W}_{map}(l\vartheta)]^2 \quad \text{avec} \quad \mathcal{W}_{map}(l\vartheta) = \left[\frac{24J_4(l\vartheta)}{l\vartheta} \right] \quad (3.32)$$

Le filtre compensé est un filtre passe-bande piqué au mode $l \approx 5/\vartheta$. Il peut être utilisé pour reconstruire le spectre de convergence dans l'espace réel et s'avère plus approprié que le filtre top-hat qui intègre tous les modes dont $l \lesssim 1/\vartheta$. Cette différence est claire lorsqu'on compare leurs comportements dans l'espace de Fourier. La contrepartie est qu'étant une intégrale sur un nombre limité de modes, le rapport signal sur bruit est plus faible que pour le top-hat. C'est pour cette raison que nous sélectionnons systématiquement différentes statistiques du deuxième ordre. Chacune d'elles peut être plus ou moins indiquée selon le type d'information que l'on cherche à extraire des données. Il est important de noter que $\langle M_{ap}^2(\vartheta) \rangle$ calculée pour une échelle donnée explore le spectre à des échelles environ 5 fois plus petites que la mesure de $\langle \gamma^2(\vartheta) \rangle$ à la même échelle angulaire.

On peut démontrer que M_{ap} peut aussi être exprimée en fonction du cisaillement. Pour cela, on définit les composantes tangentielle et radiale du cisaillement:

$$\gamma_t = -\mathcal{R}e[\gamma e^{-(2i\theta)}] \quad \gamma_r = -\mathcal{I}m[\gamma e^{-(2i\theta)}] \quad (3.33)$$

où θ est l'angle entre la ligne qui connecte le centre de la boîte et le point où on mesure le cisaillement,

$$M_{ap}(\vartheta) = \int_{\theta < \vartheta} d^2\theta U(\theta) \kappa(\theta) = \int_{\theta < \vartheta} d^2\theta Q(\theta) \gamma_t(\theta) \quad (3.34)$$

où la fonction $Q(\theta)$ est définie comme:

$$Q(\theta) = \frac{2}{\theta^2} \int_0^\theta d\theta' \theta' U(\theta') - U(\theta) \quad (3.35)$$

Les statistiques de M_{ap} ont une grande importance dans l'étude du cisaillement. On en verra dans la suite la raison, quand on abordera dans le prochain chapitre les problèmes concernant les biais systématiques dans l'estimation du signal de cisaillement mesuré à partir de l'ellipticité des galaxies.

3.4 Que mesure t-on et pourquoi?

L'équation 3.19 montre que les statistiques du deuxième ordre sont surtout sensibles à la densité de matière Ω_m , à la normalisation du spectre σ_8 , au paramètre de Hubble h et à la distribution des sources. Dans le cas d'un Univers EdS, dans l'approximation linéaire et pour un spectre de puissance du type $P(k) \propto k^n$ on obtient par exemple une expression particulièrement utile [Bernardeau et al. 1997]

$$\langle k(\vartheta)^2 \rangle \simeq 10^{-4} \sigma_8^2 \Omega_m^{1.6} \left(\frac{\vartheta}{1 \text{deg}} \right)^{-(n+2)} z_s^{1.4} \quad (3.36)$$

Le signal mesuré dépend donc de plusieurs paramètres et est de ce point de vue une quantité dégénérée. Ce que l'on mesure est effectivement un signal dont la forme et l'amplitude dépendent de la géométrie de l'Univers et du spectre de puissance. La dépendance avec la géométrie est contenue dans les termes de distance. Dans le cas plus général on s'attend à ce que les dépendances soient un peu différentes de celles montrées dans l'équation 3.36. En plus, si on veut tenir compte de l'évolution de spectre pendant l'époque radiative, le signal $\langle k(\vartheta)^2 \rangle$ dépendra aussi du paramètre Γ qui caractérise la fonction de transfert $T(k)$ (voir chapitre 2). La dépendance avec le redshift moyen des sources est à remarquer. Cette dépendance est due au terme de distances contenu en 3.19. Cela veut dire que pour une cosmologie donnée le signal $\langle k(\vartheta)^2 \rangle$ varie avec le redshift. La mesure du cisaillement dans des relevés de différentes profondeurs peut être utilisée pour valider l'origine cosmologique du signal. D'autre part, le type de cosmologie caractérise l'évolution du spectre de puissance et donc la comparaison entre le signal mesuré à différentes profondeurs donne aussi des informations concernant l'évolution du spectre des fluctuations dans les époques les plus récentes de l'histoire de l'Univers. Comme on a déjà anticipé dans le chapitre 2, c'est par ce moyen que l'on peut donc apporter des contraintes sur l'évolution de l'équation d'état de l'énergie sombre [Benabed & van Waerbeke 2004].

Chapitre 4

La mesure du cisaillement

Dans le chapitre précédent on a introduit l'équation des lentilles et les relations qui permettent de relier les champs κ et γ au champ de fluctuations de matière δ . Dans ce chapitre j'établis comment on peut estimer le champ du cisaillement γ à partir de l'ellipticité observée des galaxies.

4.1 Grandeurs caractéristiques des galaxies

On considère la distribution d'intensité d'une galaxie $\mathcal{I}(\boldsymbol{\theta})$. En première approximation, la distribution $\mathcal{I}(\boldsymbol{\theta})$ représente une ellipse et elle est donc complètement définie par ses premier et deuxième moments. Le premier moment donne le barycentre optique de l'ellipse $\boldsymbol{\theta}_c$:

$$\theta_{c,i} = \frac{\int d^2\boldsymbol{\theta} \theta_i W(\mathcal{I}(\boldsymbol{\theta}))}{\int d^2\boldsymbol{\theta} W(\mathcal{I}(\boldsymbol{\theta}))} \text{ avec } i,j = 1,2 \quad (4.1)$$

Le filtre $W(\mathcal{I})$ coupe l'intégrale quand l'intensité devient plus faible qu'une valeur limite. La présence de ce filtre permet de limiter les effets du bruit.

Les moments d'ordre deux de la distribution d'intensité représentent la matrice de forme de la galaxie.

$$\mathcal{I}_{ij} = \frac{\int d^2\boldsymbol{\theta} (\theta_i - \theta_{c,i})(\theta_j - \theta_{c,j}) W(\mathcal{I}(\boldsymbol{\theta}))}{\int d^2\boldsymbol{\theta} W(\mathcal{I}(\boldsymbol{\theta}))} \quad (4.2)$$

La matrice de forme est symétrique et peut donc être interprétée comme celle d'une ellipse de grand axe a et de petit axe b dont l'orientation est définie par un angle α . On définit l'ellipticité complexe $\chi = \chi_1 + i\chi_2$:

$$\chi_1 = \frac{\mathcal{I}_{11} - \mathcal{I}_{22}}{Tr\mathcal{I}} = \frac{a^2 - b^2}{a^2 + b^2} \cos(2\alpha) \quad (4.3)$$

$$\chi_2 = \frac{2\mathcal{I}_{12}}{Tr\mathcal{I}} = \frac{a^2 - b^2}{a^2 + b^2} \sin(2\alpha) \quad (4.4)$$

On peut aussi définir la brillance de surface de l'objet, telle que:

$$S = (\mathcal{I}_{11}\mathcal{I}_{22} - 2\mathcal{I}_{12}) = Det(\mathcal{I}) \quad (4.5)$$

Les mêmes définitions peuvent être utilisées pour définir les paramètres de forme des objets dans le plan de la source $I^0(\theta)$, χ_0 et S^0 , respectivement la distribution d'intensité, l'ellipticité complexe et la brillance de surface de la source. Selon le théorème de Etherington, et dans le cas où il n'y a pas d'absorption ni d'émission, la brillance de surface est conservée par effet de lentille gravitationnelle. On a donc :

$$I(\theta) = I^0(\alpha(\theta)) \quad (4.6)$$

où α est la position angulaire d'un élément de la source dans le plan source. Notons par la suite

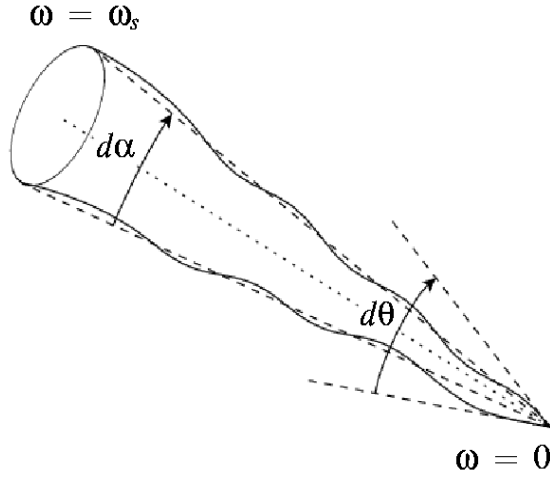


FIG. 4.1 – Schéma représentant la déformation du faisceau émis par une source caractérisée par la coordonnée radiale w_s et une taille angulaire $d\alpha$. La taille angulaire $d\theta$ observée est le résultat des déflexions le long de toute la ligne de visée.

$\alpha_0 = \alpha(\theta_0)$ Si on fait l'hypothèse que la source est assez petite alors on peut linéariser

$$\alpha(\theta) = \alpha_0 + \left. \frac{\partial \alpha(\theta)}{\partial \theta} \right|_{(\theta_0)} (\theta - \theta_0) \quad (4.7)$$

ce qui est équivalent à supposer que la matrice d'amplification est constante sur une échelle de la taille de la source. En utilisant l'équation 4.6 et la définition de la matrice de déflexion \mathcal{A} dans l'expression 4.2 on trouve la relation entre \mathcal{I}^0 et \mathcal{I} :

$$\mathcal{I}_{ij} = \mathcal{A}_{ik} \mathcal{I}_{kl}^0 \mathcal{A}_{lj}^t \quad (4.8)$$

où la matrice \mathcal{A} est donnée par l'expression 3.12 et est évaluée au centre optique de l'image θ_0 . De même on trouve l'amplification de la luminosité totale de la source :

$$\mu \equiv \frac{S^{obs}}{S_0} = \frac{1}{\text{Det}(\mathcal{A}(\theta_0))} = \frac{1}{(1 - \kappa)^2 + \gamma} \quad (4.9)$$

et son ellipticité:

$$\chi_0 = \frac{\chi_{obs} - 2g + g^2 \chi_{obs}^*}{1 + |g|^2 - 2\text{Re}(g\chi_{obs}^*)} \quad (4.10)$$

où on a défini le cisaillement réduit par:

$$g = \frac{\gamma}{1 - \kappa} \quad (4.11)$$

Dans le cas des effets de lentille gravitationnelle faible, on a $\kappa \ll 1$ $|\gamma| \ll 1$. Par conséquent l'amplification devient $\mu = 1 + 2\kappa(\theta, w)$, mais surtout la relation entre la forme de la source et son image se simplifie:

$$\chi_{obs} = \chi_0 + 2g = \chi_0 + 2\gamma . \quad (4.12)$$

Cette expression remarquable signifie que les vecteurs ellipticité se composent d'une façon linéaire dans le cas des effets de lentille gravitationnelle faible.

En pratique, on utilise plus fréquemment une définition un peu différente de l'ellipticité donnée par:

$$e = \frac{\mathcal{I}_{11} - \mathcal{I}_{22} + 2i\mathcal{I}_{12}}{\mathcal{I}_{11} + \mathcal{I}_{22} + 2Tr(\mathcal{I})^{(1/2)}} \quad (4.13)$$

La relation entre χ et e est $\chi = 2e/(1 + |e|^2)$ et en régime faible on a:

$$e_{obs} = e_0 + \gamma \quad (4.14)$$

L'équation 4.9 donne une méthode directe pour estimer le champ κ à partir de la mesure de l'amplification de la source. Cette option serait possible si nous connaissions parfaitement la luminosité absolue des sources ou si nous étions en mesure de faire des hypothèses solides sur ces luminosités. Cette condition limite la possibilité d'estimer κ à des cas où il est possible de faire des hypothèses supplémentaires sur la luminosité intrinsèque des sources. L'effet d'amplification peut être quand même mesuré sur des classes particulières d'objets telles que les SNIa ou les quasars (QSOs)¹.

L'équation 4.14 donne la possibilité de cartographier le champ γ à partir de la mesure d'ellipticité des galaxies. On peut raisonnablement faire l'hypothèse que l'orientation des galaxies est distribuée de façon aléatoire et isotrope. Dans ce cas, $\langle \chi_0 \rangle = 0$, où $\langle \rangle$ est une moyenne d'ensemble sur plusieurs galaxies dans la même région du ciel utilisée et l'équation 4.14 offre un'estimation du champ de cisaillement $\langle \gamma \rangle$ dans cette région.

4.2 Modes gravitationnels et non-gravitationnels

Comme on le verra par la suite, la mesure du cisaillement γ à partir des ellipticités des galaxies est un processus compliqué où de nombreux facteurs interviennent, certains étant particulièrement délicats à contrôler. Outre les problèmes techniques, ces difficultés ont aussi un impact néfaste sur les objectifs même puisqu'ils conduisent à des effets systématiques qui engendrent des biais dans l'estimation des contraintes cosmologiques.

Au cours de ces dernières années, différentes méthodes visant à quantifier la contamination du champ d'ellipticité des galaxies par des facteurs extérieurs au cisaillement ont été développées. Parmi ces tests, la division du champ d'ellipticité mesuré en modes du type gravitationnels (ou type E) et non gravitationnels (ou type B) a marqué une étape déterminante pour valider et

1. Voir par exemple Ménard & Dalal 2005 et Scranton et al. 2005.

crédibiliser les mesures de cisaillement gravitationnel. Cette méthode s'appuie sur le fait que le champ gravitationnel, et donc le champ de convergence κ , est un scalaire.

On considère en effet le champ gravitationnel projeté ϕ et on a $\nabla^2\phi = 2\kappa$ et aussi $\nabla \times \nabla\kappa = 0$. En mesurant γ directement sur des images et en utilisant 3.23 pour trouver le champ de convergence κ on peut trouver $\nabla^4\phi = \nabla \times \nabla\kappa \neq 0$. En d'autres termes, le champ κ mesuré n'est pas en général un champ scalaire. Nous pouvons donc décomposer le champ en modes E et B .

On écrit l'équation 3.23 dans le cas d'un champ $\kappa = \kappa_E + i\kappa_B$ complexe.

$$\kappa_E + i\kappa_B = \mathcal{D}^*(\boldsymbol{\theta}) * \gamma(\boldsymbol{\theta}) \quad (4.15)$$

et on définit un potentiel $\phi_c = \phi_E + i\phi_B$ tel que:

$$\nabla^2\phi^{E,B} = 2\kappa^{E,B}. \quad (4.16)$$

Le champ de cisaillement est alors obtenu en appliquant l'opérateur:

$$T \equiv (\partial_{11} - \partial_{22})/2 + i\partial_{12} \quad (4.17)$$

au champ ϕ_c et on obtient:

$$\gamma \equiv T\phi_c = \left[\frac{1}{2}(\phi_{,11}^E - \phi_{,22}^E) - \phi_{,12}^B \right] + i \left[\phi_{,12}^E + \frac{1}{2}(\phi_{,11}^B - \phi_{,22}^B) \right]. \quad (4.18)$$

Soient les vecteurs γ_E et γ_B tels que:

$$\gamma_E \equiv \mathcal{D}\kappa_E \quad \text{et} \quad \gamma_B \equiv \mathcal{D}\kappa_B \quad (4.19)$$

La partie réelle et imaginaire de l'équation 4.15 et la définition 4.19 impliquent alors:

$$\begin{aligned} \mathcal{R}e(\mathcal{D}^* * \gamma) &= \mathcal{D}^* * \gamma_E \\ \mathcal{I}m(\mathcal{D}^* * \gamma) &= \mathcal{D}^* * \gamma_B \end{aligned} \quad (4.20)$$

Et donc:

$$\begin{aligned} \gamma_E &= \mathcal{D} * \mathcal{R}e(\mathcal{D}^* * \gamma) \\ \gamma_B &= \mathcal{D} * \mathcal{I}m(\mathcal{D}^* * \gamma) \end{aligned} \quad (4.21)$$

Cette relation peut être écrite dans l'espace de Fourier:

$$\begin{aligned} \tilde{\gamma}_E &= [\cos(2\varphi) + i\sin(2\varphi)] [\cos(2\varphi)\gamma_1 + \sin(2\varphi)\gamma_2] \\ \tilde{\gamma}_B &= [\cos(2\varphi) + i\sin(2\varphi)] [\cos(2\varphi)\gamma_2 - \sin(2\varphi)\gamma_1] \end{aligned} \quad (4.22)$$

où φ est l'angle entre le vecteur \mathbf{l} et l'axe des abscisses. La division en modes E et B s'écrit donc :

$$\begin{aligned} \tilde{\gamma}_e &\equiv |\tilde{\gamma}_E| = \tilde{\gamma}_1 \cos(2\varphi) + \tilde{\gamma}_2 \sin(2\varphi) \\ \tilde{\gamma}_b &\equiv |\tilde{\gamma}_B| = \tilde{\gamma}_2 \cos(2\varphi) - \tilde{\gamma}_1 \sin(2\varphi) \end{aligned} \quad (4.23)$$

Si le champ d'ellipticité est un estimateur du champ du cisaillement gravitationnel, alors $\phi_B = 0$ et donc le champ γ_B est nul et $\nabla^2\kappa = 0$. Chaque fois que l'on mesure l'ellipticité des galaxies

$\gamma = \gamma_1 + i\gamma_2$ on peut donc effectuer une décomposition du champ en modes gravitationnels (ou modes E) et modes non-gravitationnels (ou modes B). Cette séparation peut être utilisée pour quantifier l'effet des erreurs systématiques dans la mesure du cisaillement. Ces modes B peuvent être soit des résidus de mauvaise correction d'effets instrumentaux, soit des modes réels, de source astrophysique, engendrés par des effets de nature différente des lentilles gravitationnelles.

Pour l'estimateur statistique M_{ap} , la décomposition est faite de façon naturelle, car la mesure du champ κ avec un filtre compensé ne prend en compte que des modes gravitationnels. La variance M_{ap}^2 définie dans l'équation 3.34 est en effet construite en utilisant γ_t , l'ellipticité tangentielle des galaxies qui ne porte que des modes de type E [Crittenden et al. 2002]. Il en résulte que, de la même façon, on peut définir $M_{\perp}(\vartheta)$

$$M_{\perp} = \int_{\theta < \vartheta} d^2\theta Q(\theta) \gamma_r(\theta) \quad (4.24)$$

en permutant γ_t avec γ_r . Les moments de M_{\perp} ne contiennent donc que des modes B.

Si le champ des galaxies suit le champ du cisaillement on doit donc trouver $\langle M_{\perp} \rangle = 0$. De même pour les statistiques d'ordre supérieur. L'image 4.2 permet de visualiser la différence des lignes de champ type E par rapport à celles d'un champ de type B . Dans un référentiel local un champ de type E ou irrotationnel ne montre que des configurations où les orientations du champ sont soit tangentielles soit radiales. Les champs de type B ou rotationnel n'ont en revanche que des configurations orientées selon les deux directions orientées à 45 degrés des composantes E . Par conséquent, une rotation des galaxies de 45 deg par rapport à leur barycentre transforme un champ de type E dans un champ de type B .

4.3 La mesure des statistiques à deux points dans la pratique

On peut construire des estimateurs de γ à partir de l'ellipticité observée de façon assez simple, soit pour le filtre compensé 3.30, soit pour le filtre top-hat 3.28, mesurés dans une boîte de taille ϑ centrée en θ_c :

$$\hat{\gamma} = \frac{\sum_i w_i e_i^{obs}}{\sum_i w_i} \quad \text{et} \quad \hat{M} = \frac{\sum_i w_i U(|\theta_i - \theta_c|) e_{t,i}^{obs}}{\sum_i w_i} \quad (4.25)$$

la sommation étant limitée aux galaxies dont la distance au centre de la boîte est inférieure à la taille de la boîte. w_i représente le poids associé à chaque galaxie.

Les estimateurs des statistiques du deuxième ordre associées à ces filtres sont donnés par:

$$\hat{\gamma}^2 = \frac{\sum_i w_i \sum_j w_j w_{j \neq i} e_i^{obs} e_j^{obs}}{\sum_i \sum_{j \neq 1} w_i w_j} \quad ; \quad \hat{M}^2 = \frac{\sum_i w_i \sum_{j \neq i} w_j U(|\theta_j - \theta_c|) U(|\theta_i - \theta_c|) e_{t,i}^{obs} e_{t,j}^{obs}}{\sum_i \sum_{j \neq i} w_i w_j} \quad (4.26)$$

où on prend soin d'éliminer l'auto-corrélation entre les galaxies (termes $i = j$). En effet, même si $\langle e^0 \rangle = 0$ on a $\langle e^0 e^0 \rangle = \sigma_e^2 \neq 0$ et donc en incluant cette composante dans l'estimateur sa valeur serait alors largement biaisée (le terme σ_e varie entre 0.33 pour les images spatiales et 0.44 pour les images au sol. Dans tous les cas il domine largement le signal!!).

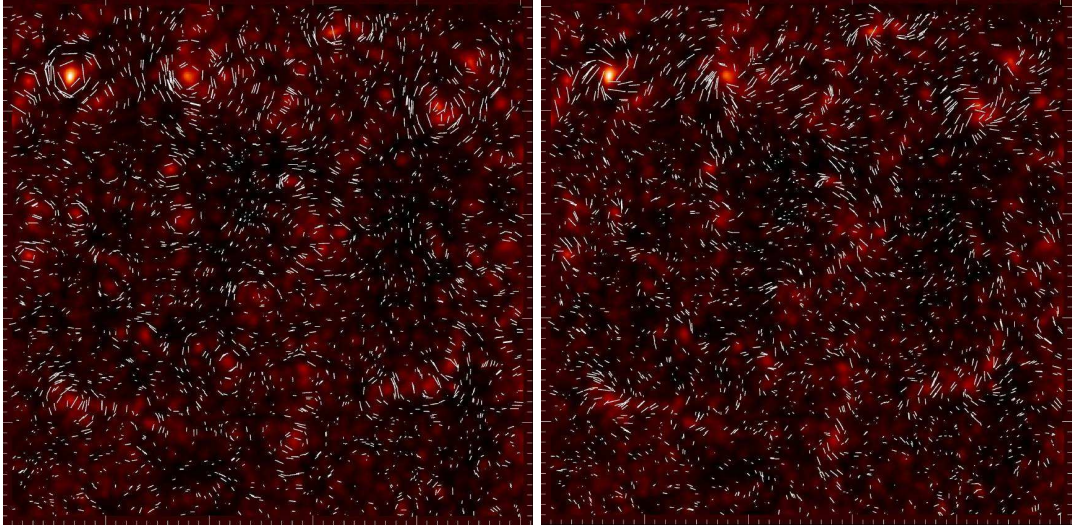


FIG. 4.2 – La figure de gauche montre le champ des ellipticités des galaxies observé dans le cas d'un champ irrotationnel, i.e. champ de type E . La figure de droite montre la même image mais dans le cas d'un champ à divergence nulle, i.e. champs de type B . Les différents degrés de rouge indiquent le champ de densité projetée. Les traits en blanc représentent l'orientation des galaxies du champ. Image fournie par Yannick Mellier.

Dans un relevé de galaxies, la moyenne d'ensemble pour une taille donnée correspond à la moyenne sur plusieurs boîtes, éventuellement pondérées:

$$\langle \hat{\gamma}^2 \rangle = \frac{\sum_{i=1}^N w_i \hat{\gamma}_i^2}{\sum_{i=1}^N w_i} \quad \text{et} \quad \langle \hat{M}^2 \rangle = \frac{\sum_{i=1}^N w_i \hat{M}_i^2}{\sum_{i=1}^N w_i} \quad (4.27)$$

où N est le nombre de boîtes et w_i le poids relatif de la boîte i . Enfin, avec la transformation $\gamma_t \rightarrow \gamma_r$ on déduit automatiquement l'estimateur de M_{\perp} . Il a été toutefois démontré [Crittenden et al. 2002, Schneider et al. 2002a, Pen, van Waerbeke & Mellier 2002] qu'utiliser la fonction de corrélation des ellipticités pour ensuite en dériver les autres statistiques présente deux avantages. Le premier, c'est de pouvoir mesurer la variance de M_{ap} sans être affecté par la présence des zones masquées. Dans les secteurs de champs masqués le filtre n'est plus compensé et il perd les propriétés présentées précédemment. Exprimer les statistiques à deux points avec la fonction de corrélation ξ permet en revanche de continuer à utiliser M_{ap} et de pouvoir (partiellement) décomposer en mode E et B le cisaillement cosmologique calculé par la variance top-hat et par la fonction de corrélation du cisaillement ξ . Pour cela, on décompose les fonctions de corrélation sous la forme de deux nouvelles composantes:

$$\xi_+ = \xi_t + \xi_r, \quad \xi_- = \xi_t - \xi_r \quad (4.28)$$

De façon analogue à 4.26 on définit les estimateurs $\xi_+(\theta)$ et $\xi_-(\theta)$ à partir de l'ellipticité observée

des galaxies:

$$\hat{\xi}_{\pm}(\theta) = \frac{\sum_i w_i \sum_j w_j (e_{t,i}^{obs} e_{t,j}^{obs} \pm e_{r,i}^{obs} e_{r,j}^{obs}) \Delta(|\theta_i - \theta_j|)}{\sum_i w_i w_j \Delta(|\theta_i - \theta_j|)} \quad (4.29)$$

où $\Delta(|\theta_i - \theta_j|)$ est une fonction dont la valeur est l'unité pour toutes les paires dont la séparation est comprise dans l'intervalle $\theta - \delta\theta$ et $\theta + \delta\theta$, où $\delta\theta$ représente la taille du bin utilisé, et nulle ailleurs. Si on définit la quantité:

$$\xi'(\theta) = \xi_-(\theta) + 4 \int_{\theta}^{\infty} \frac{d\theta'}{\theta'} \xi_-(\theta) + 12\theta^2 \int_{\theta}^{\infty} \frac{d\theta'}{\theta'^3} \xi_-(\theta') \quad (4.30)$$

on peut démontrer que:

$$\xi_E(\theta) = \frac{\xi_+(\theta) + \xi'(\theta)}{2}, \quad \xi_B(\theta) = \frac{\xi_+(\theta) - \xi'(\theta)}{2}, \quad (4.31)$$

On voit d'après l'intégrale 4.30 que pour diviser la fonction de corrélation du cisaillement en parties E et B à une échelle donnée r on a besoin d'intégrer ξ_- entre θ et l'infini. En pratique cela n'est évidemment pas possible car les relevés de galaxies ont une taille finie. Si on appelle R la taille maximale à laquelle on peut calculer la fonction de corrélation, on peut diviser les intégrales en deux termes intégrés respectivement dans les intervalles $[\theta, R]$ et $[R, \infty]$. Pour chaque échelle angulaire $\theta < R$ les modes E et B sont déterminés à une constante inconnue près, provenant de l'intégration du deuxième et troisième termes de 4.30, entre R et l'infini. Les signes avec lesquels cette constante apparaît dans ξ_E et ξ_B sont opposés. La somme de ξ_E et ξ_B reste donc constante, quelle que soit la valeur de cette constante.

De la même façon on peut écrire $\langle M_{ap}^2 \rangle$ et la composante E de la variance top-hat en fonction de ξ_+ et ξ_- :

$$\langle M_{ap}^2(\vartheta) \rangle = \frac{1}{2} \int_0^{\infty} dl l \int d\theta \theta [\xi_+(\theta l) J_0(l\theta) + \xi_-(\theta) J_4(l\theta)] [\mathcal{W}_{map}(l\vartheta)]^2 \quad (4.32)$$

$$\langle |\gamma|_E^2(\vartheta) \rangle = \frac{1}{2} \int_0^{\infty} dl l \int d\theta \theta [\xi_+(\theta l) J_0(l\theta) + \xi_-(\theta) J_4(l\theta)] [\mathcal{W}_{TH}(l\vartheta)]^2 \quad (4.33)$$

où \mathcal{W}_{TH} et \mathcal{W}_{map} représentent les filtres définis par 3.29 et 3.32. En utilisant la définition de ces filtres on trouve:

$$\langle M_{ap}^2(\vartheta) \rangle = \frac{1}{2} \int d\theta \frac{\theta}{\vartheta^2} [\xi_+ T_+(\frac{\theta}{\vartheta}) + \xi_- T_-(\frac{\theta}{\vartheta})] \quad (4.34)$$

$$\langle |\gamma|_E^2(\vartheta) \rangle = \frac{1}{2} \int d\theta \frac{\theta}{\vartheta^2} [\xi_+ S_+(\frac{\theta}{\vartheta}) + \xi_- S_-(\frac{\theta}{\vartheta})] \quad (4.35)$$

avec:

$$T_+(x) = 576 \int_0^{\infty} \frac{ds}{s^3} J_0(xs) (J_4(s))^2, \quad T_-(x) = 576 \int_0^{\infty} \frac{ds}{s^3} J_4(xs) (J_4(s))^2 \quad (4.36)$$

$$S_+(x) = 4 \int_0^{\infty} \frac{ds}{s} J_1(xs) (J_4(s))^2, \quad S_-(x) = 4 \int_0^{\infty} \frac{ds}{s} J_1(xs) (J_4(s))^2$$

En utilisant les propriétés des fonctions de Bessel, on trouve, après un peu de calculs et d'efforts [Schneider et al. 2002a]:

$$T_+(x) = \begin{cases} \frac{6(2-15x)^2}{5} \left[1 - \frac{2}{\pi} \arcsin\left(\frac{x}{2}\right) \right] \frac{x\sqrt{4-x^2}}{100\pi} (120 + 2320x^2 - 75x^4 + 132x^6 - 9x^8) & \text{pour } x < 2 \\ 0 & \text{pour } x > 2 \end{cases}$$

$$T_-(x) = \begin{cases} \frac{192}{35\pi} x^3 \left(1 - \frac{x^2}{4} \right)^{7/2} & \text{pour } x < 2 \\ 0 & \text{pour } x > 2 \end{cases}$$

$$S_+(x) = \begin{cases} \frac{1}{\pi} \left[4 \arccos\left(\frac{x}{2}\right) - x\sqrt{4-x^2} \right] & \text{pour } x < 2 \\ 0 & \text{pour } x > 2 \end{cases}$$

$$S_-(x) = \begin{cases} \frac{x\sqrt{4-x^2}(6-x^2) - 8(3-x^2)\arcsin(x/2)}{4\pi} & \text{pour } x < 2 \\ \frac{4(x^2-3)}{x^4} & \text{pour } x > 2 \end{cases}$$

Les expressions pour les composantes de type B , $\langle |\gamma|_B^2(\vartheta) \rangle$ et $\langle M_\perp(\vartheta) \rangle$ sont équivalentes à celles 4.34 et 4.35 en changeant les signes:

$$\langle M_\perp^2(\vartheta) \rangle = \frac{1}{2} \int d\theta \frac{\theta}{\vartheta^2} [\xi_+ T_+(\frac{\theta}{\vartheta}) - \xi_- T_-(\frac{\theta}{\vartheta})] \quad (4.37)$$

$$\langle |\gamma|_B^2(\vartheta) \rangle = \frac{1}{2} \int d\theta \frac{\theta}{\vartheta^2} [\xi_+ S_+(\frac{\theta}{\vartheta}) - \xi_- S_-(\frac{\theta}{\vartheta})] \quad (4.38)$$

Les expressions pour $T_+(x)$, $T_-(x)$ sont limitées $0 < x < 2$. Cela veut dire que l'équation 4.34 permet d'évaluer $\langle M_{ap}(\vartheta) \rangle$ sans aucune ambiguïté si l'on connaît les fonctions de corrélation du cisaillement dans l'intervalle $[0, 2\vartheta]$. Pour un relevé de taille R on peut mesurer $\langle M_{ap}^2 \rangle$ jusqu'à une échelle angulaire $R/2$. La fonction $S_-(x)$ en revanche n'est pas à support borné et donc la composante E , $\langle |\gamma|_E^2(\vartheta) \rangle$ est déterminée à une constante près, comme dans le cas de $\xi_E(\vartheta)$ et de $\xi_B(\vartheta)$.

4.4 Ellipticité intrinsèque et biais systématiques

Puisque le cisaillement cosmologique ne peut donner que des modes de corrélation de type E , l'éventuelle présence de modes B pose un problème sérieux d'interprétation des données car ils pourraient être artificiels ou réels. En cas d'absence de mode B , l'interprétation du signal est univoque, sinon, cette contamination révèle une source de bruit qui peut aussi affecter les modes E . L'interprétation des modes B dans l'estimation des paramètres cosmologique est un problème auquel pour le moment on n'a pas de réponse définitive. Si l'on veut parvenir à exploiter le cisaillement cosmologique de façon optimale dans les années qui viennent, il est pourtant important de faire un inventaire aussi exhaustif et détaillé que possible de ces contaminations.

L'origine des modes B peut provenir d'une mauvaise évaluation des paramètres de forme des objets. Mais comme on le verra, plusieurs tests permettent d'avoir une bonne estimation du niveau de contamination du signal cosmologique par des erreurs systématiques. Supposons donc, pour

le moment, que nous savons nous affranchir des erreurs liées à l'extraction du signal d'ellipticité dans les relevés des galaxies (nous y reviendrons plus tard, chapitre 8). En reprenant l'expression $e^{obs} = e^0 + \gamma$, la corrélation à deux points est alors donnée par:

$$\langle e_i^{obs} e_j^{obs} \rangle = \langle e_i^0 e_j^0 \rangle + \langle e_i^0 \gamma_j \rangle + \langle \gamma_i e_j^0 \rangle + \langle \gamma_i \gamma_j \rangle \quad (4.39)$$

Le premier terme représente le terme de corrélation intrinsèque (EE), le deuxième et le troisième représentent l'intégration cisaillement-ellipticité intrinsèque (EC) et le dernier est le terme dû au cisaillement pur, celui que l'on cherche à mesurer. Ainsi, lorsqu'on calcule la fonction de corrélation du cisaillement à partir de $\langle e_i^{obs} e_j^{obs} \rangle$ on mesure en réalité la somme de ces trois différents termes. À la différence du terme de cisaillement, les termes EE et EC peuvent avoir, a priori, une composante de type B non nulle. La première conséquence est donc que le signal B mesuré dans les relevés de galaxies n'est pas nul. La deuxième est que ces termes peuvent aussi affecter l'amplitude mesurée de la composante E du signal. Il peut en résulter un biais dans la mesure des paramètres cosmologiques.

Le problème de l'estimation du biais provoqué par la présence de contributions EE et EC a été abordé de façon indépendante par différents auteurs [Crittenden et al. 2001, King & Schneider 2002, Hirata & Seljak 2003, Heymans & Heavens 2003, Hirata & Seljak 2004, King 2005, Heymans 2006b]. En particulier, Heymans et al. 2006b démontre, à partir de simulations N-corps, que les termes de corrélation intrinsèque et d'interférence cisaillement-ellipticité intrinsèque ne sont pas négligeables, dépendent de la morphologie des galaxies, et donnent une contribution différente aux modes E et B . Pour les galaxies elliptiques la partie E de la fonction de corrélation intrinsèque est comparable avec la fonction de corrélation du cisaillement. Pour les galaxies spirales elle est consistante avec zéro. Cette différence liée à la morphologie est due au fait que les galaxies spirales interagissent différemment avec le champ de marée des amas de galaxies par rapport aux galaxies elliptiques. Le tracé 4.3 montre la partie E et B de la fonction de corrélation EE pour les galaxies spirales et elliptiques. Cet effet peut être éliminé grâce à l'estimation précise des redshifts des sources car la corrélation physique entre les galaxies diminue avec la distance pour devenir quasi-nulle à des échelles plus grandes de celle des amas de galaxies. Tant Heymans et al. 2006b que King & Schneider 2002 confirment que le terme EE disparaît si on élimine les paires dont la distance est plus petite que $10h^{-1}Mpc$. La corrélation EC est un effet plus subtil qui peut être expliqué de la façon suivante. Prenons un couple de galaxies A et B situées à des distances respectives $D_A > D_B$ de l'observateur. On notera leur distance relative D_{AB} . La galaxie B est alignée selon le champ de marée de son halo. Ce halo défléchit aussi la lumière émise par A. La corrélation entre les images peut donc être différente de zéro puisque la déflexion induite par la lentille est proportionnelle à l'efficacité de la lentille $\equiv D_B D_{AB} / D_A$. Le graphe 4.4 montre ce phénomène pour un plan source $z_s \sim 1$ corrélé avec toutes les galaxies d'avant plan, soit elliptiques, soit spirales. Ce phénomène pourrait en principe être observable car il prédit une contrepartie de modes B négative et une amplitude qui décroît progressivement lorsque la profondeur du relevé augmente. Ces prédictions suggèrent que la mesure de σ_8 due aux statistiques de deuxième ordre du cisaillement cosmologique devrait être biaisée. Il est difficile de prédire l'amplitude du biais totale dû aux termes EE et EC . En particulier, la contribution du terme EC dépend de la distribution en redshift et elle est opposée à celle due aux corrélations EE . Enfin, dans la mesure des paramètres cosmologiques par le cisaillement cosmologique autres facteurs d'incertitude interviennent tel que celui sur la

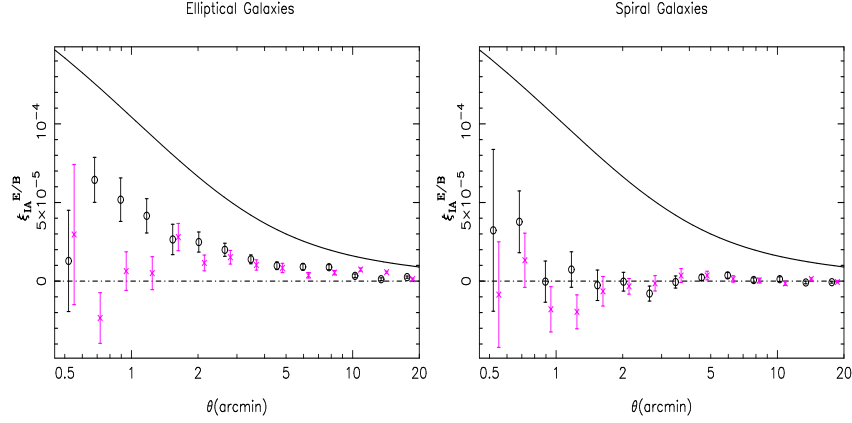


FIG. 4.3 – Modes E (points noirs) et B (croix magenta) dus à la corrélation intrinsèque des sources pour une distribution dont le redshift médian $z_m \sim 0.7$. La corrélation EE est comparée avec la fonction de corrélation $\xi_E(\vartheta)$ que l'on attend pour la même distribution en redshift. L'effet total sur un relevé réaliste contenant des galaxies spirales et elliptiques devrait être nul [Heymans et al. 2006b]

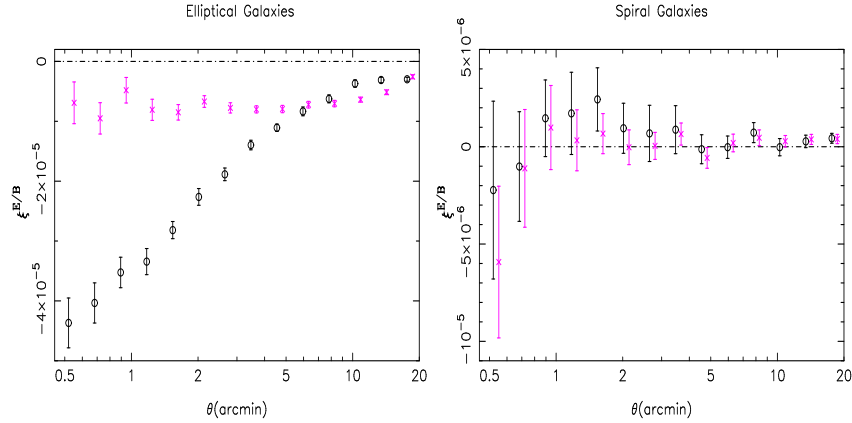


FIG. 4.4 – Graphe montrant les modes E (points noirs) et B (croix roses) dus à la corrélation EC pour un plan sources $z_s \sim 1$ avec les galaxies d'avant plan $0 < z < 1$.

distribution en redshift dont l'impact peut être aussi importante, notamment dans l'estimation de σ_8 [van Waerbeke et al. 2006].

Dans les années qui viennent, établir le niveau de fiabilité de la mesure des paramètres cosmologiques par cisaillement gravitationnel est un des objectifs scientifiques principaux à l'intérieur de la communauté; ce besoin est encore plus justifié vis à vis des nouveaux résultats *WMAP* – 3 publiés par Spergel et al. 2006, lesquels suggèrent une valeur de σ_8 en général plus faible que celle qui avait été mesurée sur les relevés récents. Le biais apporté par la présence des termes EE et

EC n'est que l'un des aspects sur lesquels il faudra travailler dans les années qui viennent pour exploiter la mesure du cisaillement cosmique et avoir des contraintes fiables. King & Schneider 2003, Heymans et Heavens 2003 et King 2005 proposent des éléments qui suggèrent comment minimiser les effets des corrélations EE et EC . Il faut pour cela mesurer les fonctions de corrélation à deux points dans plusieurs intervalles de redshift. Cette opération permet de distinguer les différentes contributions dues aux termes EE , EC grâce aux différents comportements de ces termes en fonction du redshift des sources et des lentilles. Elle peut s'appliquer, en principe au moins, aux relevés de galaxies multi-couleur tels que le CFHT Legacy Survey, pour lesquels on peut déterminer les redshifts de chaque source.

4.5 La correction de la PSF et la méthode KSB

La relation 4.10 permet de relier l'ellipticité des sources à celle observée et de construire un estimateur de γ . La situation réelle est malheureusement plus compliquée. On mesure l'ellipticité des galaxies sur des images dont les paramètres de forme sont dégradés par toute une série d'effets. L'altération globale est décrite par la Point Spread Function (PSF). On appelle PSF $P(\boldsymbol{\theta})$ la réponse impulsionnelle de l'instrument (*i.e.* la réponse à une distribution de Dirac). La relation entre la distribution d'intensité de la galaxie avant le passage à travers l'atmosphère $I(\boldsymbol{\theta})$ et l'intensité de la galaxie mesurée $I^{obs}(\boldsymbol{\theta})$, s'exprime de la façon suivante:

$$I^{obs}(\boldsymbol{\theta}) = \int d^2\vartheta I(\boldsymbol{\vartheta})P(\boldsymbol{\theta} - \boldsymbol{\vartheta}) \quad (4.40)$$

La PSF prend en compte des effets de l'atmosphère mais aussi tous les effets instrumentaux pouvant altérer $I(\boldsymbol{\theta})$, comme par exemple les défauts d'asservissement du guidage du télescope, les aberrations optiques, les dégradations produites par le détecteur: transfert de charges, dilution par les captures des électrons dans les profondeurs du substrat, les défauts des images (saturation des étoiles, gradients locaux du fond de ciel par les halos des étoiles brillantes ou par les résidus de remise à plat des champs) ou encore les effets de traitement et co-addition des images combinés aux effets d'échantillonnage spatial.

La PSF est donc en général une fonction anisotrope. La correction de la PSF reste aujourd'hui une des étapes les plus délicates dans l'analyse du cisaillement cosmologique. Les effets de distorsion ont des amplitudes de quelques pour-cents et donc un biais d'amplitude comparable dans la mesure de l'ellipticité est une crainte parfaitement fondée.

On dispose de différentes techniques permettant d'obtenir un estimateur de γ à partir de l'ellipticité observée en corrigeant l'effet de la PSF. La méthode KSB développée par Kaiser et al. (1995) est la plus couramment utilisée. Récemment d'autres techniques, telles que la décomposition en *shapelets* ([Kuijken 2006, Massey & Refregier 2005]) ou l'isotropisation de la PSF [Bernstein & Jarvis 2002] ont été développées. Une description de l'ensemble des approches ainsi qu'une première inter-comparaison de leurs performances sont présentées par Heymans & al. 2006a. Ce travail est le premier d'une série dans le cadre du Shear Testing Program (STEP) (pour plus de détails voir la page web <http://www.physics.ubc.ca/heyman/step.html>). Le but est d'évaluer ces méthodes à partir d'un jeu de simulations partagées par toutes les équipes proposant des méthodes de mesure

du cisaillement cosmologique, en vue d'améliorer la capacité à estimer le cisaillement à partir des mesures d'ellipticité des galaxies (voir aussi 8).

La méthode que j'utiliserai pour l'analyse des données dans cette thèse est la méthode KSB. J'en explique ici les principales caractéristiques en suivant la notation proposée par Kaiser et al. 1995 (voir aussi [Erben et al. 2001]). Reprenons la définition 4.2 des moments d'ordre deux de l'intensité en y introduisant un filtre W , d'échelle typique σ , centré sur le centre optique de la galaxie:

$$\mathcal{I}_{ij} = \int d^2\theta (\theta_i - \theta_{c,i})(\theta_j - \theta_{c,j})(\mathcal{I}(\boldsymbol{\theta}))W(|\boldsymbol{\theta} - \boldsymbol{\theta}_c|^2/\sigma^2) \quad (4.41)$$

Ce filtre n'affecte pas la définition de l'ellipticité complexe 4.2. L'hypothèse à la base de la méthode KSB consiste à supposer que la partie anisotrope de la PSF est plus petite que sa partie isotrope. On décompose donc la PSF en un terme anisotrope $q(\boldsymbol{\theta})$ et un terme isotrope $P(\boldsymbol{\theta})$:

$$P(\boldsymbol{\theta}) = \int d^2\vartheta P^{iso}(\boldsymbol{\theta} - \boldsymbol{\vartheta})q(\boldsymbol{\vartheta}) \quad (4.42)$$

et on ne retient que le premier ordre en q dans les calculs [Bartelmann & Schneider 2001]. Soit $\hat{\chi}_0$ l'ellipticité intrinsèque de la source convoluée par la partie isotrope de la PSF. La relation entre l'ellipticité $\hat{\chi}_0$ et celle observée χ_{obs} est donnée par:

$$\chi_{obs} = \hat{\chi}_0 + P^g g - P^{sm} q \quad \text{avec} \quad P^g = P^{sh} - P^{sm}(P^{*,sm})^{-1}P^{*,sh} \quad (4.43)$$

P^{sh} et P^{sm} sont appelés *shear tensor* (tenseur de cisaillement) et *smear tensor* (tenseur d'étalement). Ils sont calculés directement sur l'image de la galaxie à partir de la distribution d'intensité pondérée par le filtre $W(\theta)$. On remarque que P^{sh} est la réponse de la source au cisaillement dans le cas où on n'a aucun effet dû à la PSF. Il est donc corrigé par le terme P^g compte tenu de la PSF. En effet, P^g dépend aussi de $P^{*,sm}$ et $P^{*,sh}$ qui sont calculés sur les étoiles dont la forme est effectivement celle de la PSF². Le vecteur q exprime la partie anisotrope de la PSF et est estimé en appliquant (4.43) aux étoiles. En effet, pour les étoiles $\chi_{iso} = 0$ et $g = 0$ et donc:

$$q = (P^{sm*})^{-1}\chi^* \quad (4.44)$$

Pour obtenir un estimateur de g on corrige d'abord l'anisotropie des galaxies. Soit χ^{ani} , l'ellipticité des galaxies corrigées de l'effet anisotrope. On a:

$$\chi^{ani} = \chi^{obs} + P^{sm} q \quad (4.45)$$

et enfin on obtient:

$$g = (P^g)^{-1}(\chi^{ani} - \hat{\chi}_0) \quad (4.46)$$

On peut alors utiliser cette équation pour exprimer g en moyennant la mesure d'ellipticité dans des régions où il peut être considéré constant. Il faut simplement supposer que $\langle \hat{\chi}_0 \rangle = 0$, c'est à dire que les galaxies n'ont pas une direction privilégiée en l'absence du cisaillement. On obtient donc:

$$\langle g \rangle = \langle P^g \rangle^{-1} \langle \chi^{ani} \rangle \quad (4.47)$$

2. Les étoiles sont des source ponctuelles et donc la réponse instrumentale est quasi identique à la réponse impulsionnelle. Pour cette raison, la distribution de lumière observée d'une étoile permet de connaître la PSF à sa position.

Alternativement, on peut considérer que $\langle (P^g)^{-1} \hat{\chi}_0 \rangle = 0$ et définir:

$$\langle g \rangle = \langle (P^g)^{-1} \chi^{ani} \rangle \quad (4.48)$$

Les deux estimateurs étant corrects.

Chapitre 5

Mesure du cisaillement cosmologique avec le CFHTLS

Dans ce chapitre je décris les procédures qui nous ont conduit des images à l'interprétation cosmologique des premières mesures du cisaillement cosmologique dans le CFHT Legacy Survey [Semboloni et al. 2006a, Hoekstra et al. 2006] .

5.1 Le CFHT Legacy Survey

Le Canada-France-Hawaii Telescope Legacy Survey (CFHT) est un relevé photométrique du ciel dans le domaine visible mis en place par les agences de recherche française et canadienne. Il comporte trois relevés indépendants, le CFHTLS-Deep (ou Deep), le CFHTLS-Wide (Wide) et le CFHTLS-Very Wide (Very Wide), qui partagent 500 nuits d'observation. La distribution des nuits à chaque période de prise de données est organisée par un groupe scientifique de suivi, le CFHTLS Steering Group. L'acquisition des données du relevé devrait s'étaler entre 2003 et 2008.

Le relevé CFHTLS utilise la caméra panoramique Megacam [Boulade et al., 2002] construite par le CEA et qui est installée au foyer primaire du télescope CFHT. Cette caméra est composée de 36 CCD assemblés en mosaïque de 9×4 détecteurs selon 4 bandes horizontales. Chaque CCD est composé de 2048×4612 pixels couvrant un angle de 0.2 arcsec. Megacam couvre donc un champ de 1×1 deg² ce qui fait d'elle encore à ce jour la plus grande caméra CCD astronomique au monde. La caméra est fixée sur le nouvel instrument focal MegaPrime dont la construction fut conduite sous la responsabilité du CFHT. L'ensemble instrumental est une rénovation complète de l'environnement focal du foyer primaire, comprenant un nouveau correcteur grand champ, un système de guidage et d'autofocus, un environnement électronique d'acquisition des images Megacam, un module gérant les filtres et un obturateur (construits par le CEA). Le CFHT a la charge de l'opération de l'ensemble instrumental MegaPrime/Megacam ainsi que de la prise des données scientifiques et de calibration. Il assure aussi toutes les étapes pour construire les fichiers de pré-calibration (masterfiles) et le processus de pré-calibration des images. Le traitement de plus haut niveau, qui inclut notamment les calibrations, les co-additions des images, la génération des images

cartes de pondération, tous les catalogues et les meta-données de contrôle qualité, est exécuté par le centre Terapix, à l’IAP. Terapix assure aussi la production au cours du temps de plusieurs jeux de données. L’archivage et la distribution sont assurés par le CADC, à Victoria.

Pour ses deux composantes Deep et Wide, le CFHTLS est considéré comme le premier d’une nouvelle génération de relevés spécialement conçus et conduits pour l’analyse du cisaillement cosmologique, l’observation des *SNIa* et la production de grands catalogues profonds panchromatiques de galaxies.

Les CFHTLS-Wide et le CFHTLS-Deep, dont les objectifs primaires en termes de contraintes cosmologiques sont différents, ont des caractéristiques et des modes d’opération différents. En ce qui concerne leurs spécifications pour le cisaillement gravitationnel cosmologique, les deux relevés étant observés dans cinq différentes bandes (u^* , g' , r' , i' , z'), ils fournissent pour la première fois la possibilité de déterminer le redshift photométrique de l’ensemble des sources. Cette information supplémentaire est déterminante pour interpréter avec précision le signal gravitationnel. Elle permet d’éviter de calibrer la distribution en redshift des sources à partir de données externes. Mais elle s’avère encore plus importante pour exploiter l’évolution du signal du cisaillement avec le redshift, contrôler certains effets systématiques et quantifier les effets de l’énergie sombre sur le taux de croissance des fluctuations et les propriétés du spectre de puissance. L’effet devrait être plus marqué pour les données du CFHTLS Deep, qui est le premier relevé aussi vaste (quatre champs non-corrélés de 1deg^2 chacun) à atteindre une magnitude limite finale de 28.7 dans la bande i' . Cela correspond à une densité de galaxies d’environ $100/\text{arcmin}^2$, un échantillon sans précédent pour l’étude de l’Univers lointain, la formation des structures et l’analyse morphologique des galaxies. En ce qui nous concerne, il permettra de construire un sondage de type “tomographique” de la distribution des condensations de matière dans l’Univers.

Le CFHTLS-Wide est un relevé beaucoup plus vaste que le Deep. Il couvre une aire totale de 170deg^2 , répartie en 3 champs non-corrélés. Sa magnitude limite dans la bande i' est de 24.5. Ses objectifs principaux sont l’étude du cisaillement galaxie-galaxie, qui permet de sonder les propriétés des halos des galaxies [Hoekstra et al. 2004, Hoekstra et al. 2005] et l’étude du cisaillement cosmologique à grande échelle avec un très bon rapport signal sur bruit. Les plus grandes échelles angulaires qu’explore le CFHTLS-Wide seront de 5 degrés. À ces échelles, on s’affranchit des problèmes concernant les incertitudes sur le régime non-linéaire des fluctuations, ce qui permet d’interpréter sans équivoque les contraintes sur les paramètres cosmologiques. À titre de comparaison, à l’issue du projet le CFHTLS-Wide aura une surface environ 20 fois plus grande que VIRMOS-Descart, dont les contraintes sur σ_8 étaient précises au niveau de quelques pourcents [Van Waerbeke, Mellier & Hoekstra 2005]. Leur profondeur étant par ailleurs identique, on s’attend à un gain sur le rapport signal sur bruit d’un facteur cinq. Les positions des champs Deep et Wide sur le ciel sont montrées sur la carte 5.1. Elles ont été choisies de façon à bénéficier de recouvrements avec d’autres relevés. Par exemple, le champ de COSMOS est contenu dans le D2, un des champ du VVDS [Le Fèvre et al 2004] est dans D1, un des champs de DEEP2 [Madgwick et al. 2003] couvre le champ D3 et le champ W1 contient les champs du VVDS et du relevé XMM-LSS [Pierre et al. 2006]. Le recouvrement des champs du CFHTLS avec des champs VVDS et DEEP2 est un point particulièrement important car il permet de construire une calibration précise des redshifts photométriques grâce aux redshifts spectroscopiques mesurés par ces relevés.

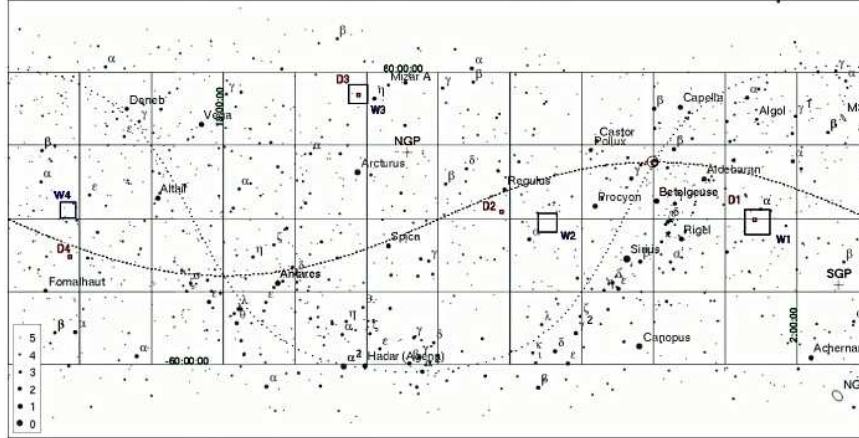


FIG. 5.1 – Carte montrant les positions des champs CFHTLS-Deep et CFHTLS-Wide sur le ciel (<http://www.cfht.hawaii.edu/Science/CFHLS/cfhtlsdeepwidefields.html>)

5.2 L’analyse des données T0001

Pendant ma première année de thèse j’ai analysé les données du CFHTLS-Deep publiées dans la première livraison de données (*release*) officielle (T0001) produite par l’équipe de Terapix¹. Elle contient l’ensemble des données acquises entre le 1er juin 2003 et le 22 juillet 2004, soit 25 deg² du relevé Wide (mais incomplet en couleur) et les 4 champs du Deep (mais loin de la profondeur finale). Les caractéristiques des données que j’ai analysées et utilisées pour obtenir des contraintes cosmologiques sont données dans la table 5.1. Etablir la qualité de cette première *release* et son potentiel pour l’analyse du cisaillement gravitationnel était une étape critique et décisive pour la suite à donner à l’ensemble du projet. Les problèmes techniques du correcteur grand champ auxquels le CFHT a dû faire face étaient parmi les inquiétudes les plus sérieuses. La dégradation des images hors de l’axe optique était telle qu’elle pouvait en effet remettre en cause la faisabilité technique de certains objectifs scientifiques du CFHTLS, et plus singulièrement sa composante de cisaillement gravitationnel.

La mesure du cisaillement cosmologique demande en effet une qualité d’image très élevée car après correction PSF on doit être capable de mesurer des effets de l’ordre de quelques pour-cents. Le seeing (*i.e.* la taille d’une étoile convoluée par la PSF) des images co-additionnées doit toujours être inférieure à 1'', et on demande que la distorsion du champ soit petite mais aussi stable dans le temps, ceci sur la totalité du champ couvert par la caméra. La PSF est échantillonnée par les étoiles, puis interpolée pour la modéliser aux positions des galaxies. Une PSF instable échantillonnerait mal les propriétés de chaque image conservée puis additionnée pour l’analyse du cisaillement gravitationnel si elle varie trop brutalement. Le premier pas dans mon analyse a donc été de valider la qualité de l’instrument MegaPrime/Megacam.

1. Pour plus d’informations consulter la page web <http://terapix.iap.fr/>

TAB. 5.1 – *Table synthétique du jeu des données T0001 pour les champs D1, D3 et D4 et que j’ai utilisées pour analyser le cisaillement gravitationnel dans les champs Deep. Les magnitudes sont données dans le système AB. Les détails sur la définition de la magnitude, sur celles de l’ouverture, du seeing et de la complétude peuvent être trouvés à l’adresse web http://terapix.iap.fr/article.php?id_article=383*

	D1	D3	D4
RA (J2000)	02:25:59	14:19:27	22:15:31
DEC (J2000)	-04:29:40	+52:40:56	-17:43:56
Effective FOV (deg ²)	0.80	0.77	0.77
Exp. time u^* (s)	10560	4620	16680
Median seeing u^* (arc-sec.)	1.15	0.88	1.05
Completeness u^* 50% (mag.)	26.4	26.0	26.2
Exp. time g'	7515	8010	11250
Median seeing g' (arc-sec.)	0.98	0.95	0.99
Completeness g' 50% (mag.)	26.4	26.5	26.2
Exp. time r'	17280	20820	26400
Median seeing r' (arc-sec.)	0.87	0.93	0.85
Completeness r' 50% (mag.)	26.1	26.4	25.9
Exp. time i'	52000	59640	58800
Median seeing i' (arc-sec.)	0.88	0.92	0.88
Completeness i' 50% (mag.)	26.1	26.2	25.8
Exp. time z'	12240	15120	-
Median seeing z' (arc-sec.)	0.86	0.85	-
Completeness z' 50% (mag.)	24.5	24.6	-

5.3 Création des masques

Les images du relevé sont examinées à l'oeil pour construire les masques. Ces masques visent à effacer des objets ou des secteurs d'image dont la présence peut biaiser le signal ou en diminuer le rapport signal sur bruit. Les traces rectilignes d'astéroïdes ou d'avions, les images fantômes ou les halos de diffusion des étoiles font partie des objets indésirables. Ce sont en général des événements relativement peu nombreux et facilement identifiables. Les fausses détections de pics discutées ci-dessous demandent plus d'attention et un peu d'explications.

La détection des objets et la mesure de leurs paramètres de forme sont déterminés avec deux logiciels différents: IMCAT² et SExtractor³. Les paramètres de fonctionnement sont réglés pour avoir une mesure optimale sur des sources de petite taille. Pour cette raison la mesure des paramètres de forme des objets étendus n'est pas optimisée⁴. Ce sont en effet les galaxies avec le plus bas rapport signal sur bruit individuel qui contribuent le plus au signal du cisaillement sur des échelles cosmologiques. La méthode de détection de IMCAT a le grand avantage de fournir les moments d'intensité pondérés par un filtre $W(|\theta|^2/\sigma^2)$ ⁵. IMCAT fournit aussi une correction de PSF immédiate, grâce à l'implémentation de la méthode KSB. Il est en revanche très sérieusement affecté par le problème de segmentation. En effet, la détection des objets est faite en passant le filtre $W(|\theta|^2/\sigma^2)$ sur toute l'image. La taille σ varie entre des limites fixées par l'utilisateur. Une maximisation du rapport signal sur bruit en fonction de la taille du filtre donne la taille de l'objet [Kaiser et al. 1995]. Si on recherche les petits objets, la taille du filtre ne peuvent plus recouvrir celles des grands objets de l'image et on détecte alors des faux objets correspondant à des pics locaux situés à l'intérieur des objets étendus. Ce phénomène affecte particulièrement les étoiles saturées, où IMCAT a tendance à produire systématiquement des fausses détections en identifiant des pics de densité le long de la figure de diffraction produite par les supports du miroir secondaire ainsi que le long des lignes de transfert de charge (voir figure 5.2).

Ces détections peuvent introduire un biais dans le signal car elles peuvent s'accompagner d'une orientation privilégiée des objets incorrectement détectés similaire avec celle d'un cisaillement gravitationnel (modes E). Les galaxies trop étendues sont donc aussi masquées. La méthode de détection de SExtractor, qui considère que des pixels connexes dont la luminosité est au-dessus d'un certain seuil par rapport à la luminosité du ciel font parties de la même structure, garantit une meilleure stabilité vis à vis de ce problème de segmentation. Cependant le problème n'est pas totalement résolu. Les masques ont aussi pour but d'éliminer des zones dont le signal sur bruit est inférieur à 80% par rapport à la moyenne de l'image. En effet, chaque pause individuelle du CFHTLS est prise avec un petit décalage angulaire par rapport à la précédente de façon à réduire les fluctuations dues au changement de sensibilité entre les pixels sur l'image additionnée. Ce processus conduit à une élimination systématique des zones entre les CCD et entre les bords de la caméra.

Il faut souligner que l'opération de *masking* ne devrait pas engendrer de biais dans la mesure

2. Logiciel développé par N. Kaiser pour l'analyse du cisaillement gravitationnel. Pour plus d'informations consulter la page web: <http://www.ifa.hawaii.edu/kaiser/imcat/>

3. Logiciel développé par E. Bertin. et S. Arnouts [Bertin & Arnouts 1996]. Pour plus d'informations voir la page web: http://terapix.iap.fr/rubrique.php?id_rubrique=91/

4. L'angle sous-tendu par une galaxie spirale de taille typique de 50 Kpc à un redshift $\simeq 1$ est quelques secondes d'arc.

5. Les versions plus récentes de SExtractor calculent désormais ces moments pondérés

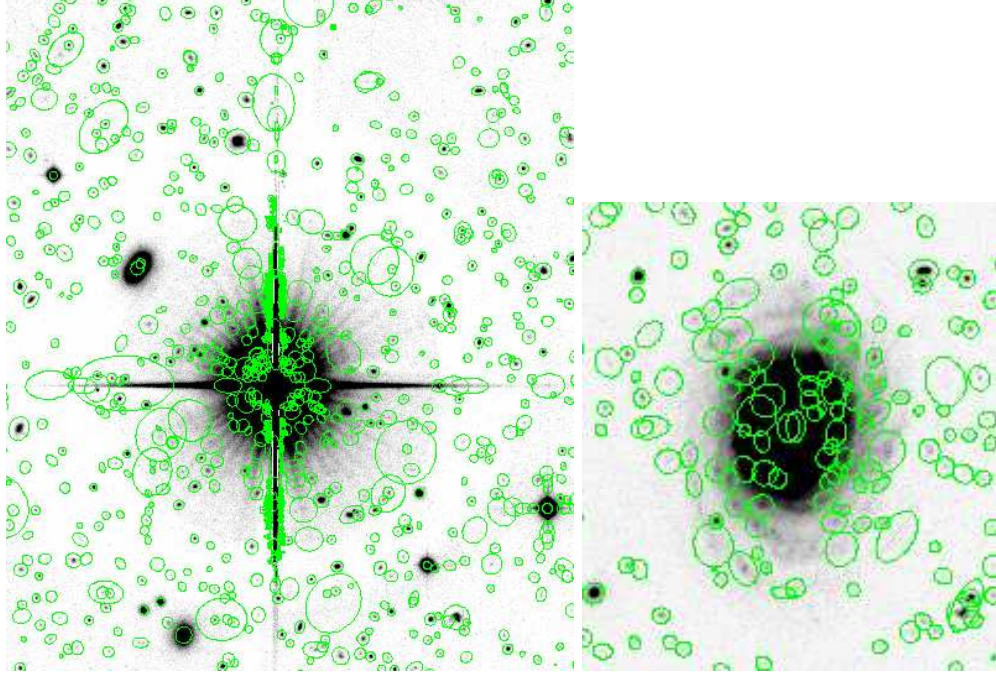


FIG. 5.2 – Exemple de fausses détections autour d’une étoile saturée (panneau de gauche) et d’une galaxie étendue (panneau de droite) dans le champ D1. Le logiciel IMCAT détermine des pics locaux qu’il identifie comme des objets.

du cisaillement car les critères avec lesquels les masques sont construites sont indépendants de la mesure du cisaillement ou de la forme des objets. À l’issue de cette étape, les masques couvrent une surface d’environ 30% du champ total.

5.4 Création des catalogues

Les champs utilisés pour mon analyse ne concernent que D1, D3 et D4. Le champ D2 a été exclu car il n’était pas assez profond pour composer un ensemble raisonnablement homogène avec les trois autres. Les catalogues ont été produits pour la bande i' et pour la bande r' , indépendamment. Etant donnée la nature achromatique du cisaillement, la comparaison du signal entre les deux bandes permet de consolider l’hypothèse de la nature gravitationnelle du signal mesuré. En fait, c’est cette comparaison du signal mesuré pour le champ D4 dans les deux bandes qui m’a conduite à choisir de travailler dans la bande r' car le même champ dans la bande i' présentait des systématiques. L’équipe Terapix avait eu des problèmes avec la solution astrométrique de ce champ, ce qui expliquerait la mauvaise qualité des résultats.

Le catalogue final contient des galaxies de magnitude $21.5 < r' < 25.5$. Des coupures ont été effectuées pour éviter la contamination des étoiles et pour ne pas franchir la limite de complétude

inférieure à 80%. Le catalogue a aussi été limité en taille; seuls les objets dont le rayon effectif dépasse celui des étoiles et il est inférieur à 10 pixels sont retenus. Puis on rejette un objet de chaque paire dont la distance de séparation est inférieure à 5 seconde d'arc (Van Waerbeke et al 2000). Cette sélection évite les mesures de paramètres de forme biaisés par la corrélation entre les deux objets d'une paire. Finalement, un poids $w = \frac{1}{\sigma_n^2 + \sigma_\epsilon^2}$ est affecté à chaque objet du catalogue. La quantité σ_n est la dispersion d'ellipticité dans une boîte contenant les 20 objets les plus proches dans un plan (taille, magnitude); le facteur σ_ϵ évite de donner un poids trop élevé aux galaxies dont l'ellipticité est mesurée avec un grand rapport signal/bruit. Le signal mesuré pour ces objets est en effet largement dominé par la dispersion due à l'ellipticité intrinsèque des galaxies qui est 0.44 dans le cas du Deep Survey. La densité finale des galaxies utilisées est d'environ 20/arcmin² et la taille effective du relevé (avec les masques) est de 2.1 deg².

5.5 Analyse des systématiques

La première étape pour déterminer la qualité des images Megacam et évaluer l'instrument pour le cisaillement cosmologique est d'établir la qualité de la correction de la PSF sur les étoiles que l'on utilise pour modéliser ensuite la PSF à la position des galaxies. La figure 5.3 montre l'allure de la PSF dans les 3 champs D1, D3 et D4. On constate qu'elle varie considérablement sur l'ensemble du champ Megacam. Il est donc indispensable de corriger la PSF sur chaque CCD de façon indépendante.

La correction de PSF utilisée est la méthode KSB introduite dans le chapitre 4. Les tenseurs $P^{*,sm}$, $P^{*,sh}$ et q dont on a besoin pour corriger l'ellipticité des galaxies sont interpolés en utilisant une fonction rationnelle dont la forme est:

$$p^\alpha = a_0 + a_1x + a_2y + a_3x^2 + a_4xy + a_5y^2 + c(xy) \quad (5.1)$$

avec

$$c(x,y) = \frac{b_0 + b_1x + b_2y + b_3x^2 + b_4xy + a_6y^2 + b_6y^3 + b_7y^4}{1 + b_8x + b_9y} \quad (5.2)$$

Hoekstra 2004 démontre en effet, que la partie anisotrope de la PSF est bien reproduite par un modèle du type 5.2. La partie polynomiale tient compte des variations de PSF à basse fréquence engendrées par exemple par un mauvais suivi du télescope. Cependant, ce modèle peut être insuffisant quand des autres facteurs dus à l'optique du télescope introduisent une composante anisotrope de la PSF qui varie avec une fréquence plus élevée. L'introduction d'une fonction rationnelle permet de reproduire ce type d'anisotropie sans devoir utiliser un polynôme d'ordre plus élevé. La présence de termes d'ordre plus élevé selon l'axe vertical y est due à l'asymétrie du CCD dont la dimension verticale est double de celle horizontale.

La dégradation de la qualité de l'image vers les bords de la caméra est bien confirmée sur le panneau en bas de l'image 5.4. Cette figure montre la moyenne des composantes de l'ellipticité (e_t, e_r) calculée dans des anneaux concentriques autour du centre de la caméra. Les deux panneaux du haut montrent cependant que la PSF est non seulement efficacement corrigée (l'ellipticité moyenne est réduite d'un facteur 10) mais aussi que le champ de distorsion résiduel est homogène sur toute la surface de la mosaïque. La corrélation résiduelle entre les étoiles (voir figure 5.5) montre que

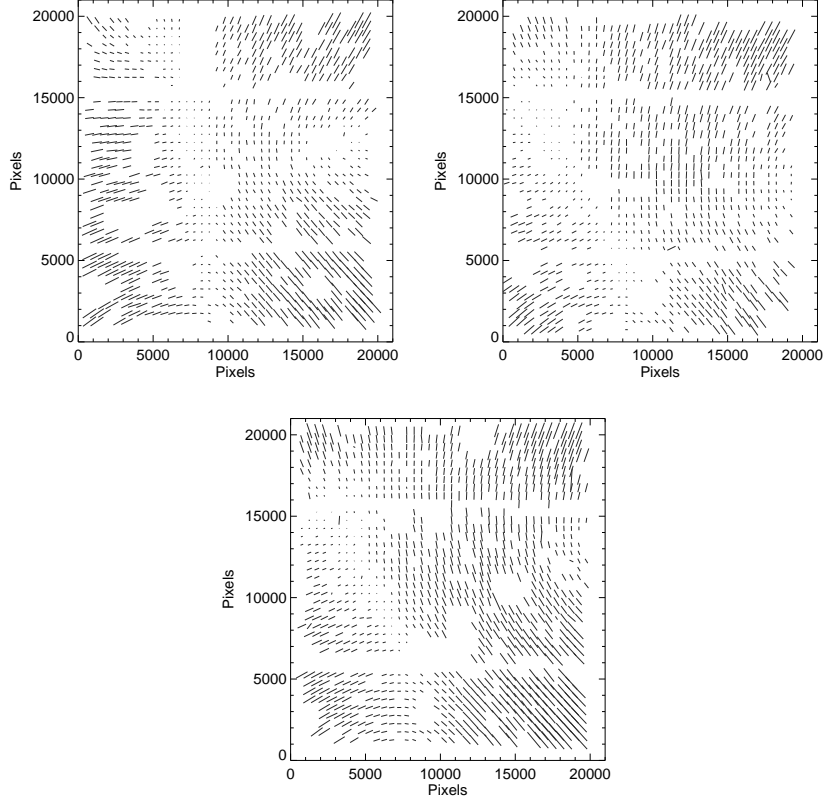


FIG. 5.3 – L’ellipticité des étoiles sur les champs de Megacam qui témoigne de l’anisotropie de la PSF pour les champs D1 D3 et D4. Pour tous les trois champs, $\langle e \rangle$ est seulement de quelques pour-cents au centre de la caméra, mais elle atteint environ 10% vers les bords (voir aussi fig.5.4). Il faut noter que les distorsions du correcteur grand champ qui étaient mesurées pendant les premiers mois précédant la mise en service de l’instrument ont atteint des valeurs jusqu’à deux fois plus élevée et elles affectaient sur plus de 50% du champ.

la distorsion résiduelle après correction de la PSF a une amplitude d’environ 1% de celle attendue de la corrélation entre les galaxies induite par le cisaillement cosmologique. Pour identifier plus précisément le signal et les systématiques, les données sont divisées en composante E et B . L’efficacité de la correction de la PSF sur les étoiles ne garantit pas en effet qu’elle l’est aussi à la position des galaxies. Nous avons déjà signalé que les étoiles ne sont pas assez nombreuses pour échantillonner une PSF fortement variable spatialement. La PSF peut être bien corrigée à la position des étoiles mais insuffisante entre les étoiles. Pour tester la qualité de la correction de la PSF entre les étoiles on utilise un estimateur ξ_{sys} [Bacon et al. 2003, Heymans 2003], qui mesure

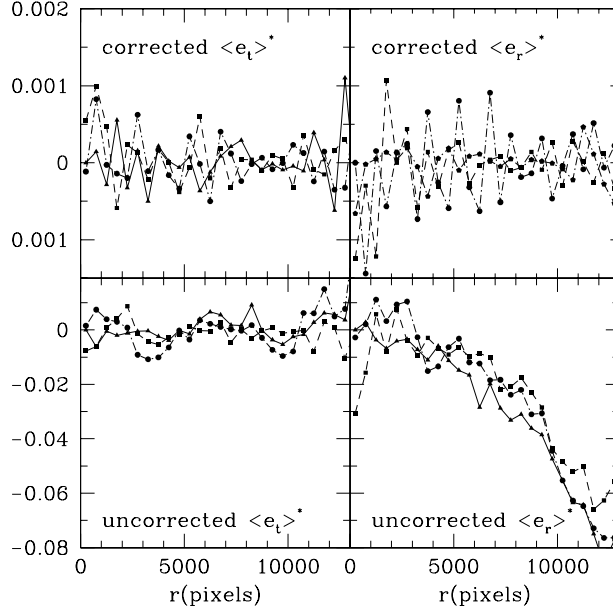


FIG. 5.4 – Comportement des composantes e_t , e_r de l'ellipticité des étoiles du centre au bord de la caméra. La moyenne est faite dans des anneaux concentriques autour du centre de la caméra; r représente la distance moyenne des étoiles de l'anneau. Le bruit statistique plus élevé à petit rayon résulte uniquement du fait que les anneaux internes ne sont pas construits à nombre d'objets constant mais ont de moins en moins d'objets lorsque le rayon décroît.

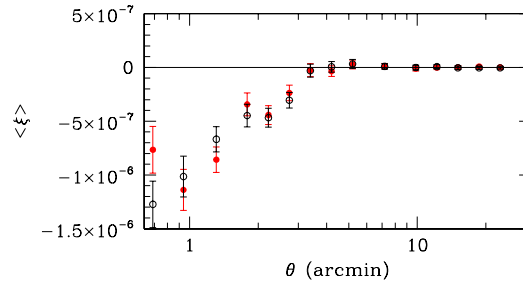


FIG. 5.5 – Composantes E (en rouge) et B (en noir) de la fonction de corrélation à deux points entre les étoiles après correction de la PSF. Les barres d'erreur représentent l'erreur statistique.

la corrélation entre les étoiles, avant correction de la PSF, e^* , et les galaxies après correction, e_{gal} ,

$$\xi_{sys} = \frac{\langle e^* e_{gal} \rangle}{\langle e^* e^* \rangle / 2}. \quad (5.3)$$

Dans le cas d'une bonne correction $\xi_{sys} = 0$, sinon il y a une partie de la PSF résiduelle, *i.e.* $\gamma_{mes} = \gamma_{true} + \alpha e^*$. La courbe de gauche de la figure 5.6 montre les statistiques du deuxième ordre mesurées sur les galaxies comparée avec ξ_{sys} et confirme encore une fois la bonne qualité de la correction de la PSF. Celle de droite montre les mêmes statistiques à deux points, comparées avec la contrepartie *B* du signal. La variance $\langle M_{ap}^2 \rangle$ et la fonction de corrélation ξ présentent des modes *B* à petite échelle. Une inspection plus détaillée montre que les objets dans l'intervalle de magnitude 25. $\langle r' \rangle < 25.5$ sont à l'origine de ces modes *B*. Ceux dans l'intervalle $21.5 < r' < 25$. ne montrent aucun signal de type *B* pour toutes les échelles et toutes les trois statistiques. L'origine des modes *B* pourrait vraisemblablement être due à une distorsion résiduelle engendrée par une mauvaise correction de la PSF sur les objets faibles. L'amplitude mesurée des corrélations de type *B* est aussi en accord avec les prédictions faites par Heymans et al. 2006b et donc on peut faire l'hypothèse que la corrélation intrinsèque galaxie-galaxie (EE) et la corrélation galaxie-lentille (EC) définies dans le chapitre précédent, soient à l'origine des modes *B*. Cependant il est difficile de prédire l'effet intégré le long de la ligne de visée, faute d'une estimation précise du redshift.

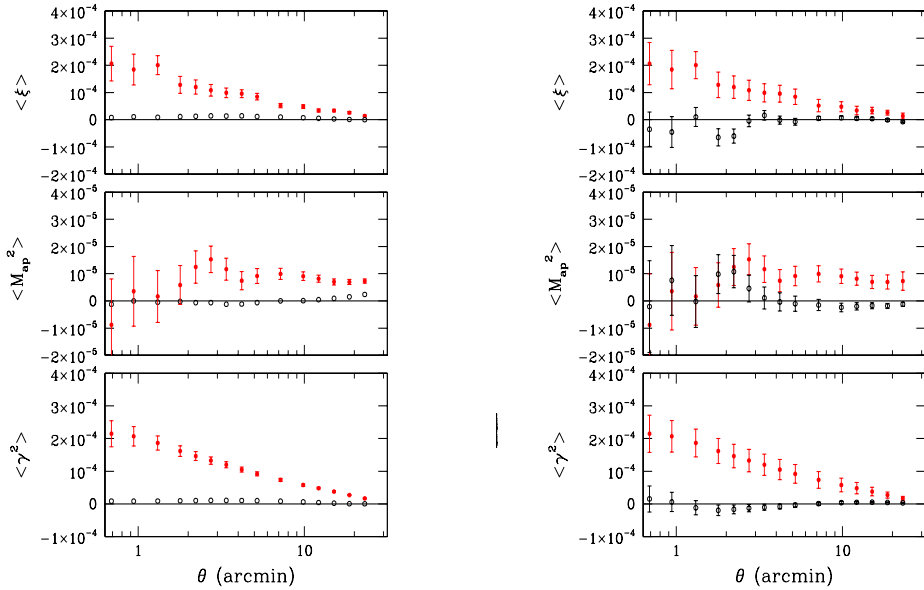


FIG. 5.6 – Mesure des statistiques du deuxième ordre: la fonction de corrélation ξ , la variance $\langle M_{ap}^2 \rangle$ et la variance $\langle |\gamma|^2 \rangle$. À gauche: les modes *E* (en rouge) concernant les trois statistiques sont comparées avec ξ_{sys} (en noir). Les barres d'erreur sur les modes *E* incluent seulement la partie statistique du bruit. À droite: les modes *E* (en rouge) sont comparés avec les modes *B* (en noir). Les barres d'erreur des modes *E* représentent l'erreur totale (erreur statistique + variance cosmique) calculée selon Schneider et al. 2002.

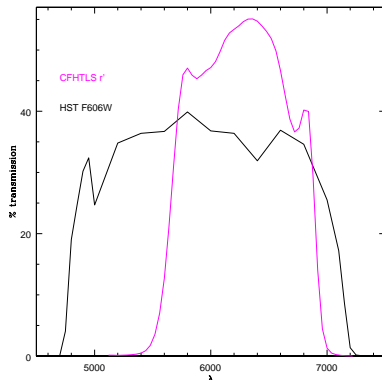


FIG. 5.7 – Transmission des filtres F606 de la caméra ACS de Hubble Space Telescope comparée à celle du filtre r' de Megacam.

5.6 Interprétation du signal

Après avoir déterminé la qualité du signal je suis passée à son interprétation cosmologique. Comme il a déjà été souligné dans le chapitre 4, le cisaillement cosmologique est avant tout sensible aux paramètres Γ , σ_8 , Ω_m et à la distribution en redshift des sources. Le CFHTLS-Deep dispose de plusieurs bandes donc en principe on peut déterminer le redshift de chaque source. Mais comme on le verra dans la section 5.9 la mesure des redshifts photométriques dans le champs de Deep était encore assez incertaine au moment de cette analyse, principalement par manque de données spectroscopiques, et seule une analyse qualitative des résultats a été possible. Pour une analyse détaillée, il a donc fallu calibrer la distribution en redshift en utilisant les redshifts photométriques du Hubble Deep Field North et South [Fernández-Soto et al. 1999]. Le *HDF* constitue un jeu de deux champs panchromatiques du ciel profond (magnitude limite en bande $F814 = 28$) obtenus à partir d'images prises par la caméra WFPC2 montée sur le Hubble Space Telescope. La surface totale du champ est de 5.3 arcmin^2 .

La bande F606 de ce relevé est assez similaire à celle r' du CFHTLS Deep (voir figure 5.7). À partir d'une comparaison croisée effectuée ultérieurement entre les données du champ D2, de celles de COSMOS et des catalogues HDF dans les filtres F814 et F606 nous avons pu confirmer que la relation entre ces deux filtres était particulièrement simple: $I_{F606W} = 1.05r' - 1.00$. Pour l'intervalle de magnitude du relevé CFHTLS-Deep ($21.5 < r' < 25.5$), $\Delta mag < 0.2$

La distribution en redshift est paramétrée de la façon suivante [Van Waerbeke et al. 2002]:

$$n(z) = \frac{\beta}{z_s \Gamma\left(\frac{1+\alpha}{\beta}\right)} \left(\frac{z}{z_s}\right)^\alpha \exp\left[-\left(\frac{z}{z_s}\right)^\beta\right], \quad (5.4)$$

La fonction de distribution est obtenue en coupant les catalogues *HDF* dans la même tranche de magnitude et en associant à chaque galaxie une magnitude mag_i pondérée par le même poids w_i utilisé pour calculer les statistiques à deux points. L'image 5.8 montre la distribution pour le

meilleur ajustement. Il correspond aux paramètres $\alpha = 1.98$, $\beta = 0.66$ $z_s = 0.098$. L'estimation

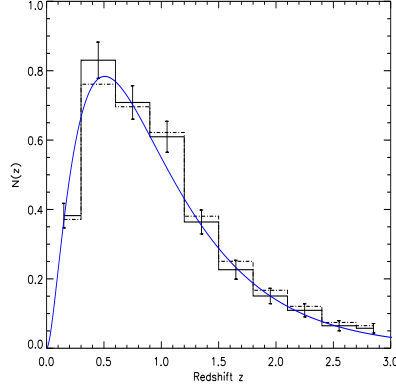


FIG. 5.8 – Courbe de la distribution en redshift déterminée avec le meilleur ajustement des paramètres pour l'échantillon HDF $21.5 < r' < 25.5$. L'histogramme (ligne continue) tient compte de la pondération des galaxies et il est comparé avec celui (ligne discontinue) où les galaxies ne sont pas pondérées. Les erreurs sont les erreurs statistiques

des paramètres cosmologiques est ensuite faite dans un plan à 4 paramètres $(\sigma_8, \Omega_m, h, z_s)$, tous les autres étant fixés avec les a priori donnés par les résultats de l'analyse du CMB-WMAP première année. Les paramètres α et β sont également fixés. On construit la fonction de vraisemblance \mathcal{L} en utilisant les mesures de la variance top-hat $\langle \gamma^2(\theta) \rangle$:

$$\mathcal{L} = \frac{1}{(2\pi)^{n/2}} |\mathbf{S}|^{1/2} \exp \left[-\frac{1}{2} (\mathbf{d} - \mathbf{s})^T \mathbf{S}^{-1} (\mathbf{d} - \mathbf{s}) \right] \quad (5.5)$$

où \mathbf{d} représente le vecteur des points mesurés, \mathbf{s} le vecteur les points construits à partir du modèle et \mathbf{S} est la matrice de variance-covariance. A la matrice de variance-covariance participent le bruit statistique et la variance cosmique. Cette matrice est calculée en suivant Schneider et al. 2002b pour une surface de 2.1 deg^2 et une densité de galaxies de $20/\text{arcmin}^2$ (voir aussi le chapitre 6 pour une explication plus détaillée concernant le calcul de la variance cosmique). La partie concernant le bruit statistique est remplacée par le bruit mesuré sur les données. Pour construire \mathbf{S} on a besoin d'un modèle de référence. Dans notre cas, j'ai choisi celui correspondant au modèle le plus vraisemblable selon l'analyse de WMAP première année ($\Gamma = 0.21$, $\sigma_8 = 0.88$, $\Omega_\Lambda = 0.7$) [Spergel et al. 2003]. Aux échelles que l'on considère il faut tenir compte de l'évolution non-linéaire du spectre. On choisit d'utiliser soit le schéma PD, soit le schéma de halofit.

Les contraintes sur Ω_m et σ_8 sont obtenues en marginalisant sur h dans l'intervalle $[0.6, 0.8]$ et sur z_s dans l'intervalle $z_s = 0.098^{+0.0129+0.0209}_{-0.014-0.0161}$ représentant les limites $\pm 1\sigma$ et $\pm 2\sigma$. L'Univers est par ailleurs supposé plat. Pour $\Omega_m = 0.3$ on trouve $\sigma_8 = 0.94 \pm 0.15$ dans le schéma PD et $\sigma_8 = 0.90 \pm 0.14$ dans le schéma halofit. Ces deux résultats sont assez similaires parce que, comme on l'a vu dans le chapitre 3, les deux schémas ne divergent significativement qu'à très petites

échelles⁶. A cause du petit nombre des points mesurés à ces échelles et du bruit statistique qui les affecte, ces points ont une importance mineure sur la valeur des contraintes cosmologiques. La figure 5.9 montre les contraintes dans le plan $[\Omega_m, \sigma_8]$. On étudie aussi les contraintes sur l'équation d'état de l'énergie sombre supposée telle que $P = w_0\rho$. La figure 5.10 montre ces contraintes dans le plan $[w_0, \Omega_m]$ dans le schéma PD marginalisant sur $\sigma_8 = [0.7, 1.]$, $h = [0.6, 0.8]$ et le même intervalle que précédemment pour z_s .

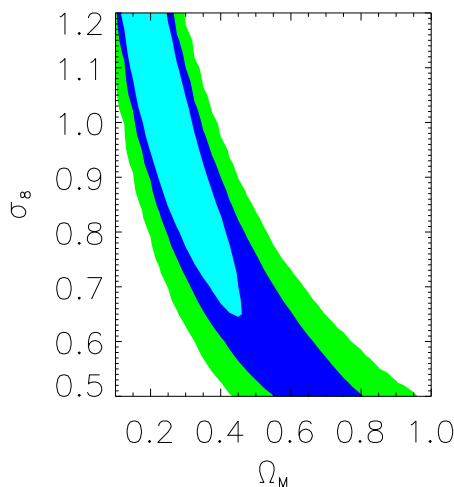


FIG. 5.9 – Contraintes dans le plan $[\Omega_m, \sigma_8]$ en utilisant le schéma PD. Les différentes couleurs montrent les régions pour les niveaux de confiance 66%, 95% et 99%.

5.7 Le CFHTLS Wide

Les données du CFHTLS-Wide disponibles à l'époque de la première *release* ont été co-additionnées par Henk Hoekstra. Une comparaison à l'analyse des mêmes champs faite à l'IAP a permis de valider la robustesse des techniques d'analyse et des mesures. Le champ du Wide contient 13 deg² du champ W1 et 9 deg² du champ W3 dans la bande i' . La méthode utilisée est différente de celle utilisée par Terapix. La méthode d'analyse en revanche est similaire à celle du CFHTLS Deep, bien qu'il s'agisse d'installation de la méthode KSB modifiée par Henk Hoekstra. La figure 5.11 montre la comparaison entre la fonction de corrélation mesurée sur les données du relevé CFHTLS-Wide et le signal ξ_{sys} défini par l'équation 5.3. La figure 5.12 montre les statistiques à deux points $\xi(\vartheta)$, $\langle M_{ap}^2(\vartheta) \rangle$ et $\langle |\gamma|^2(\vartheta) \rangle$ divisées en composantes E et B. Le signal mesuré est compatible avec

6. Par exemple à une échelle angulaire de 1 arcmin pour un redshift de 1 la taille physique correspondante pour un modèle EdS est inférieure à 1 Mpc/h et donc selon le plot 2.2 le changement entre la méthode Peacock & Dodds 1996 et Smith et al. 2003 n'est pas significatif.

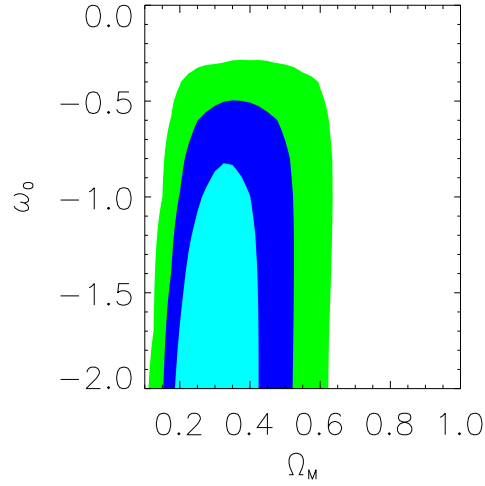


FIG. 5.10 – *Contraintes dans le plan $[\Omega_m, w_0]$ dans le même schéma pour un Univers plat et w constant. Les différentes couleurs montrent les régions pour les niveaux de confiance 66%, 95% et 99%. Les contraintes sur l'énergie sombre sont assez larges mais elles suggèrent clairement une valeur de $w_0 < -0.8$.*

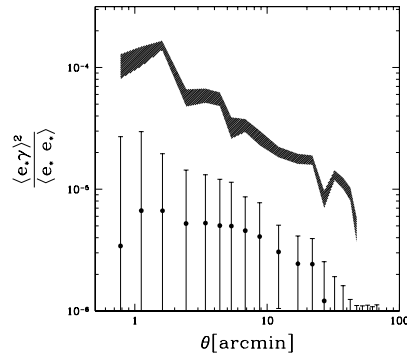


FIG. 5.11 – *Comparaison de la mesure de la fonction de corrélation à deux points sur les galaxies (modes E) avec l'estimateur ξ_{sys} défini dans la section 5.5. D'après Hoekstra et al 2006*

celui du CFHTLS-Deep si on tient compte de la différence du redshift moyen ($\langle z_s \rangle \simeq 1.0$ pour le Deep et $\langle z_s \rangle \simeq 0.8$ pour le Wide) due à la différence de profondeur des deux relevés.

Les paramètres cosmologiques ont été obtenus de façon analogue à celle utilisée pour le CFHTLS

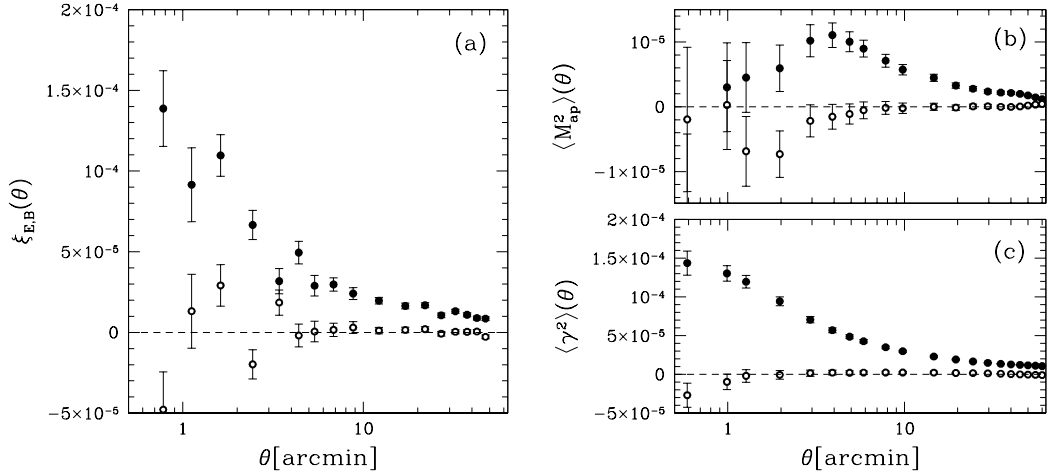


FIG. 5.12 – Mêmes courbes que 5.6 pour le CFHTLS Wide. La composante E (en noir) est comparée avec la contrepartie B (en blanc) du signal. Les erreurs ne prennent en compte que le bruit statistique. D’après Hoekstra et al. 2006.

Deep. La distribution en redshift a aussi été calibrée grâce aux catalogues *HDF* North et South. Pour les calibrer, on a cette fois utilisé la similitude entre la bande F814 et la bande i' de Megacam dans la tranche de magnitude utilisée pour nos mesures ($21.5 < i' < 24.5$). Un contrôle croisé entre les différents relevés CFHTLS-Wide, COSMOS et HDF donne $F_{814} = 1.02i' - 0.43$, soit une différence de magnitude $\Delta mag = 0.1$

Le schéma PD donne $\sigma_8 = 0.85 \pm 0.06$ pour $\Omega_m = 0.3$ (voir figure 5.13), compatible avec ce que l’on trouve pour le CFHTLS Deep. Dans le cas d’un Univers plat avec w constant on trouve aussi un résultat similaire à celui du CFHTLS Deep (voir figure 5.14): $w_0 < -0.4$.

5.8 Contraintes cosmologiques CFHTLS Deep+ Wide

La disponibilité des données provenant du CFHTLS Deep et du CFHTLS Wide qui nous ont permis de vérifier la cohérence des contraintes peuvent évidemment servir à une analyse conjointe. En particulier, la confrontation de deux relevés de profondeur différente donne des informations plus précises sur l’équation d’état de l’énergie sombre [Benabed & van Waerbeke 2004] car dans l’intervalle de redshift $0 < z < 3$ (qui contient donc les deux relevés) l’évolution du spectre des fluctuations de matière est très sensible aux propriétés de l’énergie sombre.

Dans la figure 5.15, je montre les mêmes contraintes cosmologiques précédemment montrées pour le CFHTLS-Deep (5.9,5.10) et pour le CFHTLS-Wide (5.13,5.14), obtenues par l’union des deux jeux de données. Les intervalles dans lesquels on fait varier les paramètres sont les mêmes que pour chaque relevé individuel. Le meilleur ajustement donne $\sigma_8 = 0.86 \pm 0.05$ dans le schéma

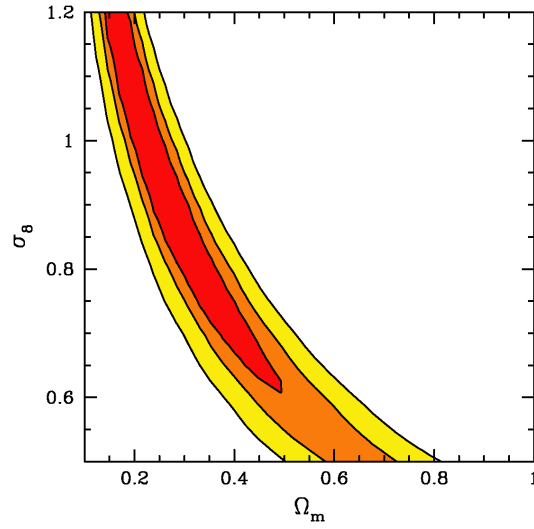


FIG. 5.13 – *Mêmes résultats que dans 5.9. D'après Hoekstra et al. 2006*

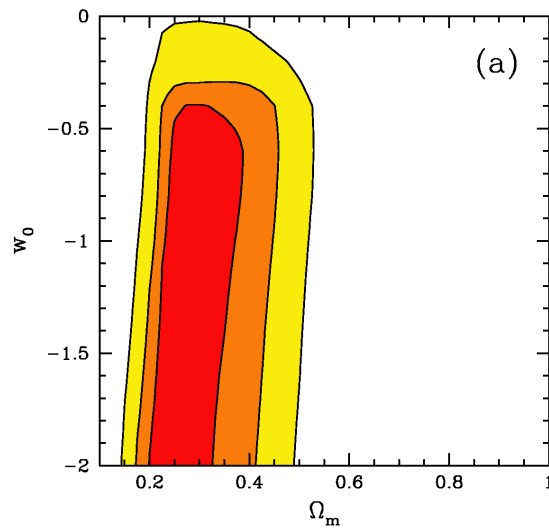


FIG. 5.14 – *Mêmes résultats que dans 5.10. D'après Hoekstra et al. 2006*

halofit et 0.89 ± 0.06 dans le schéma PD pour $\Omega_m = 0.3$.

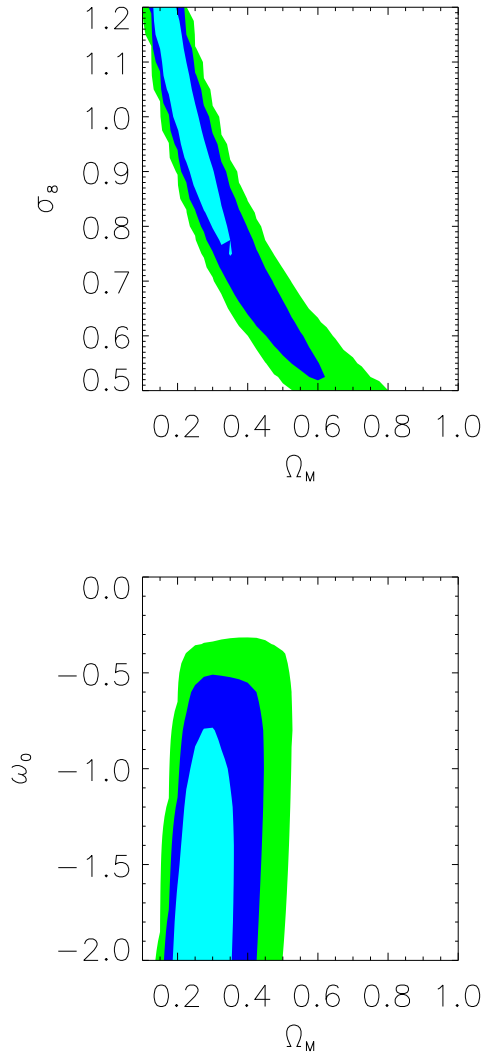


FIG. 5.15 – *Contraintes pour le CFHTLS Wide+Deep dans le schéma PD.*

5.9 Evolution du signal avec le redshift

L'observation d'une source dans plusieurs filtres permet de déterminer son redshift par photométrie, c'est à dire en comparant des flux relatifs mesurés dans différents filtres. Plusieurs logiciels de mesure de redshifts photométriques sont aujourd'hui disponibles; par exemple Le-Phare⁷ ou bien

⁷ Le Phare a été développé par S. Arnouts & O. Ilbert. Plus d'informations concernant ce code et son emploi peuvent être trouvées à la page web: www.lam.oamp.fr//arnouts/LEPHARE.html

hyper-z⁸. Pour l'évaluation des redshifts du CFHTLS Deep, j'ai utilisé ce dernier.

Le principe est de comparer les flux relatifs des différentes bandes avec ceux construits à partir de modèle-types qui reproduisent la distribution spectrale d'énergie (SED) des types spectraux de galaxies les plus courants. Le spectre d'une source étant décalé en fonction de son redshift, les rapports entre les différents flux sont modifiés selon une loi complexe mais qui peut être quantifiée. Donc, si on minimise la différence entre le rapport des flux observés entre chaque bande disponible et celui d'une série de modèles spectraux en fonction du redshift on peut estimer le redshift le plus probable de la source. Naturellement, le type spectral de la galaxie observée est inconnu, donc cette minimisation est faite dans l'espace de tous les modèles de distribution spectrale. Il faut noter que la plupart des modèles spectraux de la bibliothèque Hyper-z caractérise un type morphologique. Par la suite, on fera donc un usage sans distinction entre ces deux définitions de type.

En pratique, on minimise donc l'équation:

$$\chi^2(z) = \sum_{i=1}^{N_{filtres}} \left[\frac{F_{obs,i} - b \times F_{temp}(z)}{\sigma_i} \right]^2 \quad (5.6)$$

où F_{obs} et F_{temp} représentent respectivement le flux observé dans chaque filtre et le flux calculé pour un certain modèle spectral. σ_i est l'incertitude liée à la mesure dans chaque filtre et b un certain facteur de proportionnalité. Le facteur b dépend de la transmission des filtres.

Les modèles spectraux sont construits en considérant le taux de formation d'étoiles, la quantité de poussière, le taux de métallicité et le taux de rougissement dû aux poussières présentes dans la zone de formation d'étoiles. La présence de nuages d'hydrogène neutre extragalactique est aussi prise en compte. Elle est importante pour les objets à haut redshift qui voient leurs photons aux fréquences $\lambda_{Ly\alpha} = 1216\text{\AA}$ et $\lambda_{Ly\beta} = 1026\text{\AA}$ absorbés par ces nuages.

L'ajustement étant fait dans l'espace de modèles, il est recommandé de disposer de filtres pour lesquels les différents modèles montrent les différences les plus marquées. En pratique, puisque les filtres sont larges on ne peut pas identifier les raies d'émission associées à certaines fréquences caractéristiques; la détermination des redshifts est donc articulée autour de caractéristiques spectrales qui affectent le continuum, comme les cassures à 912\AA ou à 4000\AA dont l'amplitude change significativement selon le type spectral.

Le manque de couverture des filtres autour de ces caractéristiques spectrales peut causer des dégénérescences. Hyper-z peut alors trouver un minimum pour une morphologie différente de la morphologie réelle de l'objet qui peut donc être accompagné d'un redshift inexact. Si ce phénomène devient trop fréquent la distribution en redshift estimée par les redshifts photométriques n'est plus fiable et certainement biaisée. Pour le CFHTLS, le jeu de filtres couvre les bandes centrées sur 3500\AA (u^*), 5000\AA (g'), 6000\AA (r') 8000\AA (i'), 9000\AA (z'). La figure 5.16 illustre l'effet du décalage spectral sur le spectre d'une galaxie en fonction de son redshift. La détermination des redshifts photométriques dans le CFHTLS est dominée par la cassure à 4000\AA , et donc, pour des redshifts trop élevés la localisation de cette cassure n'est plus observable avec ce système de filtres. En pratique le modèle spectral d'une galaxie avec redshift $z \gtrsim 1.2$ sera confondu avec celui d'une galaxie d'autre modèle spectral à bas redshift, en général $z \leq 0.3$ (voir figure 5.17). La méthode

8. Hyper-z a été développé par Bolzonella et al. [Bolzonella, Miralles & Pelló 2000]. Plus d'informations concernant ce code et son emploi peuvent être trouvées à la page web: <http://webast.ast.obs-mip.fr/hyperz/>

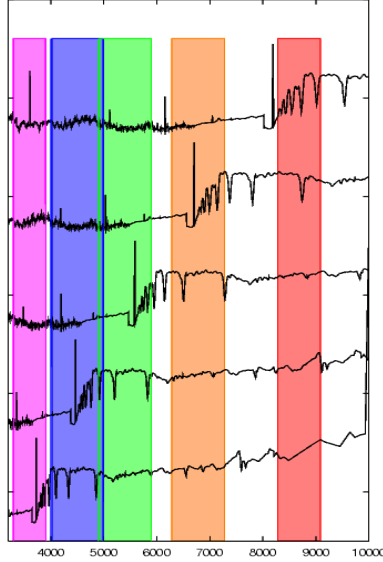


FIG. 5.16 – La figure ci-dessus montre l'évolution du spectre observé dans les bandes u^* , g' , r' , i' , z' de Megacam en fonction du redshift de la source. Les redshifts sont 0, 0.3, 0.5, 0.8, 1.2. La cassure à 4000 Å pour un redshift $z > 1.2$ ne sera plus observée par ce jeu de filtres. Cette image a été construite grâce à la génération d'un spectre synthétique par S. Charlot. Illustration communiquée par Y. Mellier.

que j'ai utilisée pour déterminer la magnitude de chaque source dans les 5 filtres est la suivante. Chaque objet a été détecté dans la bande r' d'où on a calculé ses paramètres de forme en utilisant le logiciel SExtractor. La magnitude dans les autres filtres est mesurée en conservant les paramètres déterminés dans la bande r' . De cette façon, l'ouverture utilisée est la même. On vérifie la cohérence de nos résultats en les comparant les magnitudes avec celles obtenues en utilisant la bande i' ou l'image multi-couleur (image chi2) pour la détection des objets. La dispersion moyenne de la magnitude des objets est entre 0.1 et 0.3 dans les différentes bandes. Cette erreur sur la magnitude est bien plus élevée que celle suggérée par SExtractor sur chaque objet en chaque bande mais elle reflète plus fidèlement l'incertitude associée à la façon avec laquelle on calcule les couleurs. Ce sont ces couleurs et leurs incertitudes qui ensuite sont données comme paramètres d'entrée du code. La distribution en redshift obtenue par cette procédure est montrée sur la figure 5.18. Elle ne semble pas montrer d'anomalies grossières. Malheureusement, seule une analyse qualitative du résultat a été possible à l'époque car les données spectroscopiques dans le champ VVDS n'étaient pas publiques, raison pour laquelle les contraintes cosmologiques ont été estimées en calibrant avec le HDF Nord et Sud. Malgré ces limitations, j'ai décidé d'étudier l'évolution du signal en coupant les données en deux sous-catalogues. Chacun contenait la moitié des objets pour avoir un bruit statistique comparable. La figure 5.19 montre que l'évolution du signal $\langle \gamma^2(\theta) \rangle$ suit les prédictions cosmologiques. Les mesures dans les deux échantillons concordent avec les modèles à "bas" et "haut" redshift. Ces modèles sont obtenus en prenant les valeurs du meilleur ajustement de

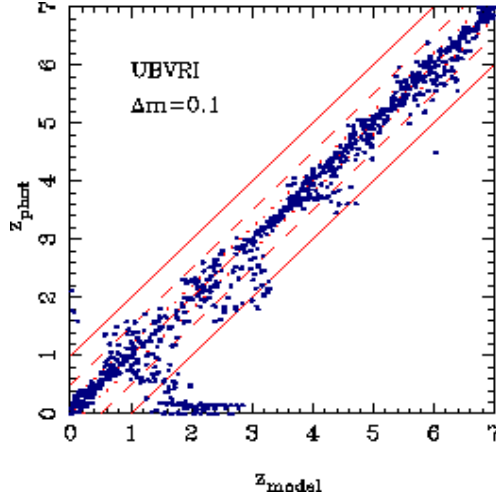


FIG. 5.17 – Exemple de la dégénérescence résultant du manque d'identification de la cassure à 4000 \AA . Le jeu des filtres CFHTLS est similaire à celui montré dans ce plot, l'erreur en magnitude est en moyenne plus élevée dans certaines bandes, la dispersion (et aussi le biais) sur l'estimation de redshift photométrique montré ici est donc comparable avec celui qu'on attend pour les données du CFHTLS-Deep. La disponibilité de la bande z' pour les champs D1 et D3 devrait permettre de sonder des redshifts légèrement plus élevés. Image extraite de Bolzonella et al. 2000.

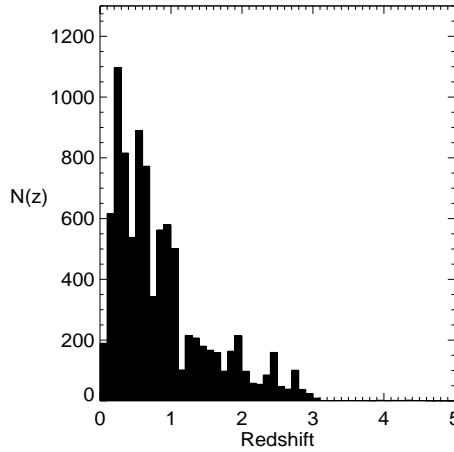


FIG. 5.18 – Distribution en redshift des sources pour le champ D1 déterminée avec Hyper-z.

la distribution $n(z)$ HDF, α , β , z_s en coupant cette distribution dans les intervalles $0.3 < z < 1.0$ (bas redshift) et $z > 0.8$ (haut redshift). La coupure tient compte du fait que nos redshifts ont

une incertitude au mieux de 0.2 (voir figure 5.18). Le signal à “bas” redshift apparaît un peu

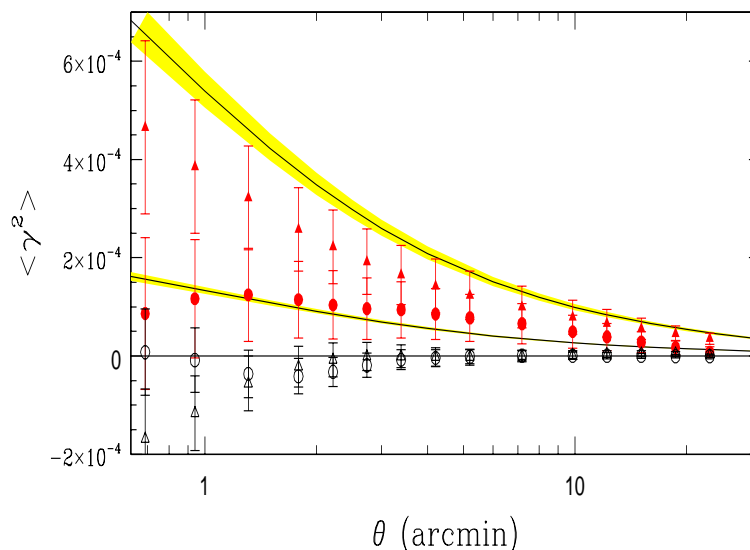


FIG. 5.19 – Composante E du signal mesuré sur les échantillons à “bas” (points rouges) et “haut” redshift (triangles rouges). Les barres d’erreur incluent la variance statistique et la variance cosmique relatives à chaque sous échantillon. Les respectives composantes B du signal sont aussi montrées. L’erreur statistique est montrée sur ces dernières. Les lignes noires continues représentent le modèle cosmologique (Λ CDM avec $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$, $\sigma_8 = 0.8$ et $\Gamma = 0.21$) caractérisée par $z_s = 0.098$. Pour l’échantillon à “bas” redshift la distribution des sources appartient à l’intervalle $0.3 < z < 1.0$, pour celui à “haut” redshift elle comprend sources avec $z > 0.8$. Les zones ombragées en jaune délimitent les régions correspondant à l’erreur $\pm 1\sigma$ sur z_s .

plus élevé que les prédictions, l’inverse est vrai pour le signal à “haut” redshift. Cela n’est pas étonnant, vu le faible niveau de précision de ces estimations de redshift. Il est réaliste de penser que des objets à haut redshift soient dans l’échantillon à bas redshift et inversement, en dépit de nos précautions. Ce résultat, dont la limitation principale est l’absence de calibration par un échantillon spectroscopique, montre qu’il est déjà possible d’envisager l’étude du cisaillement dans le CFHTLS avec une approche tomographique. Un problème sérieux est cependant à considérer pour les objets très faibles (le CFHTLS Deep atteindra une magnitude limite $i' = 28.7$) car il est indispensable de disposer de calibrations spectroscopiques fiables.

Chapitre 6

Variance cosmique

On appelle variance cosmique, l'incertitude associée aux propriétés du spectre des fluctuations de densité de matière $P(\kappa)$ à une échelle κ provenant du fait que la partie d'Univers observée dans un relevé, quelle que soit sa taille, ne représente qu'une des réalisations possibles du champ des fluctuations $\delta(\mathbf{x}, t)$. La variance cosmique se traduit par une source d'erreur supplémentaire dans la mesure des paramètres cosmologiques, mais qui dépend de l'échelle considérée du modèle d'univers. La variance cosmique associée aux statistiques du deuxième ordre est généralement calculée en faisant l'hypothèse que le champ est Gaussien. Pour un modèle cosmologique donné, elle peut être évaluée en produisant un nombre N de réalisations du champ de convergence κ (*i.e. de densité*). La variance cosmique est, par construction, la variance entre les N réalisations à une échelle donnée avec la condition que N soit suffisamment grand. Il est possible de construire des réalisations stochastiques d'un champ Gaussien (*Gaussian stochastic processes*) tel que κ , par convolution d'un bruit blanc avec la fonction de corrélation du champ κ [Salmon 1996]. Une alternative à cette méthode, est d'utiliser l'approche analytique suggérée par Schneider et al. 2002b. Cette dernière méthode a été utilisée pour obtenir les contraintes cosmologiques dans le relevé CFHTLS. Cependant, on sait que l'approximation de champ Gaussien, légitime à grande échelle, devient de moins en moins correcte aux petites échelles où le champ de fluctuations entre en régime non linéaire.

Dans ce chapitre j'étudie, à l'aide des simulations de tracé de rayons, l'impact de l'approximation du champ Gaussien sur le calcul de la variance cosmique. J'aborde ensuite les conséquences des effets non Gaussiens sur la possibilité de contraindre l'équation de l'énergie sombre avec l'étude tomographique [Benabed & van Waerbeke 2004].

6.1 Définition de la variance cosmique en régime Gaussien

Comme je l'ai déjà souligné dans la section 4.3, les quantités $\hat{\xi}_{\pm}$ définies par l'expression 4.29 sont des estimateurs non biaisés des fonctions ξ_{\pm} . L'erreur liée à cet estimateur est donnée par la partie diagonale de la matrice de covariance $\text{Cov}(\hat{\xi}_{\pm}(\theta_1); \hat{\xi}_{\pm}(\theta_2))$ définie par:

$$\text{Cov}(\hat{\xi}_{\pm}(\theta_1); \hat{\xi}_{\pm}(\theta_2)) = \langle (\hat{\xi}_{\pm}(\theta_1) - \xi_{\pm}(\theta_1))(\hat{\xi}_{\pm}(\theta_2) - \xi_{\pm}(\theta_2)) \rangle \quad (6.1)$$

Comme il sera plus clair dans la suite, la valeur de la variance cosmique d'un champ quelconque dépend de sa valeur même. Lorsqu'elle est mesurée directement sur les données cette valeur est très bruitée; pour cette raison on utilise un modèle cosmologique de référence pour calculer 6.1, soit de façon analytique, soit à l'aide de réalisations gaussiennes du champ [Salmon 1996]. Les interprétations cosmologiques du signal de cisaillement mesuré dans les relevés de première génération, comme Virmos-Descart ou RCS, ou dans de plus récents comme le CFHTLS, ont été conduites en choisissant comme modèle de référence celui qui semble le plus vraisemblable selon les données de l'expérience WMAP. Dans ce cas, on peut ne se concentrer que sur les termes de covariance de ξ_+ car $\xi_+ = \xi_E$ et toutes les statistiques à deux points peuvent être écrites en termes de ξ_+ sans devoir diviser le champ en modes E et B ¹.

Le développement de l'expression 6.1, comporte une séparation en trois termes [Schneider et al. 2002b]:

$$\text{Cov}(\hat{\xi}_+(\theta_1); \hat{\xi}_+(\theta_2)) = D\delta_k(\theta_1 - \theta_2) + q_{+++} + \langle \text{corrélations du } 4^{\text{eme}} \text{ ordre} \rangle \quad (6.2)$$

où δ_k représente un delta de Kroneker dont la valeur est non nulle seulement si $|\theta_1 - \theta_2| < \Delta\theta$, la taille du bin choisie pour calculer la fonction de corrélation. En d'autres termes, le premier terme, qui représente le bruit dû à l'ellipticité intrinsèque des sources, est un terme diagonal. Le deuxième terme, est un terme de couplage entre le signal et le bruit. Dans le troisième terme interviennent les fonctions de corrélation du 4^{eme} ordre. Schneider et al. 2002b donnent une expression analytique pour le premier et pour le deuxième terme:

$$D = 3.979 \times 10^{-9} \left(\frac{\sigma_e}{0.3}\right)^4 \left(\frac{A}{1 \text{ deg}^2}\right)^{-1} \left(\frac{n}{30 \text{ arcmin}^{-2}}\right)^{-2} \left(\frac{\theta}{1 \text{ arcmin}}\right) \left(\frac{\Delta\theta/\theta}{0.1}\right)^{-1} \quad (6.3)$$

$$q_{+++} = \frac{2\sigma_e^2}{\pi An} \int_0^\pi d\varphi \xi_+ \left(\sqrt{\vartheta_1^2 + \vartheta_2^2 - 2\vartheta_1\vartheta_2 \cos \varphi}\right) \quad (6.4)$$

où σ_e est la dispersion de l'ellipticité intrinsèque, n la densité numérique des galaxies et A l'aire du relevé. Ces expressions sont obtenues en moyennant sur l'ensemble des positions des galaxies dans un relevé de surface totale A , dans le cas où les effets de bord sont négligeables, (*i.e.* $\theta_1, \theta_2 \ll A$) ainsi que ceux des masques. En pratique, on peut mesurer le premier terme directement à partir des données pour s'affranchir de toute approximation. Le deuxième terme, tout comme le troisième, ne peut pas être estimé à partir des données car son amplitude dépend de la vraie valeur de la fonction de corrélation ξ_+ . Le troisième terme présente une complication supplémentaire car il dépend de la vraie valeur de la fonctions de corrélation à quatre points. Celle-ci ne peut être déterminée que si l'on connaît les moments d'ordre quatre du champ des fluctuations. Cependant, dans le cas de statistique gaussienne il peut être ré-écrit comme la somme des produits des moments d'ordre deux:

$$\langle \text{corrélations du } 4^{\text{eme}} \text{ ordre} \rangle = r_{+0} + r_{+1} \quad (6.5)$$

1. Calculer la matrice de variance-covariance de la fonction de corrélation avec un modèle cosmologique de référence, alors que c'est précisément la valeur de cette fonction qui est l'objet de la mesure (et donc on ne connaît pas sa valeur!!) peut sembler incohérent. C'est pourtant correct, si la variance cosmique ainsi calculée ne varie pas trop en fonction des paramètres cosmologiques. Néanmoins, dans le futur avec l'augmentation progressive du rapport signal sur bruit des données des très grands relevés, il est envisageable d'exploiter directement le signal mesuré comme modèle. Dans ce cas le calcul de la matrice de variance-covariance doit tenir compte de la division du signal de cisaillement en modes E et B .

dont l'expression analytique est donnée par Schneider et al. 2002:

$$\begin{aligned}
r_{+0} &= \frac{2}{\pi A} \int_0^\infty d\phi \phi \int_0^\pi d\vartheta_1 \xi_+(|\psi_a|) \int_0^\pi d\vartheta_2 \xi_+(|\psi_b|), \\
r_{+1} &= \frac{1}{(2\pi)A} \int_0^\infty d\phi \phi \int_0^{2\pi} d\varphi_1 \xi_-(|\psi_a|) \\
&\quad \times \int_0^{2\pi} d\varphi_2 \xi_-(|\psi_b|) [\cos 4\varphi_a \cos 4\varphi_b + \sin 4\varphi_a \sin 4\varphi_b],
\end{aligned} \tag{6.6}$$

avec ψ_a, ψ_b définis en accord avec Schneider et al. 2002.

Les intégrales 6.6 peuvent être évaluées seulement si on connaît la valeur de ξ_-, ξ_+ dans tout l'espace².

On remarque, que les intégrales q_{++}, r_{+0} et r_{+1} dépendent toutes de l'inverse de la surface du relevé et donc on peut recalibrer immédiatement la covariance pour un relevé de taille quelconque pourvu que les échelles angulaires soient beaucoup plus petites que la taille du relevé.

L'évaluation des quantités q_{++}, r_{+0} et r_{+1} pour un modèle cosmologique donné permet de connaître la variance cosmique correspondante pour les autres statistiques du deuxième ordre, telles que $\langle M_{ap}^2 \rangle$ et $\langle |\gamma|^2 \rangle$, en utilisant des filtres appropriés [Schneider et al. 2002b].

6.2 Variance cosmique d'un champ non-Gaussien

L'approximation du champ Gaussien est de moins en moins vraie aux petites échelles. On peut étudier à l'aide des simulations de tracés de rayons, quel est l'impact de l'approximation 6.5 sur l'évaluation de la variance cosmique [Semboloni et al. 2006b].

En utilisant un jeu de simulations à N-corps Λ CDM, on construit 64 lignes de visée, chacune avec une surface projetée de $\mathcal{S} = 7 \times 7 \text{ deg}^2$ selon la méthode décrite dans l'appendice A de Hamana et al. 2002. Le jeu de simulations est composé de 7 simulations de côté 200 Mpc, 4 de côté 400 Mpc, 3 de côté 600 Mpc, et 2 de côté 800 Mpc. Pour construire le plus grand nombre possible de lignes de visée, des permutations et des rotations de ces cubes sont aussi utilisées. Le modèle cosmologique est en accord avec celui proposé par Spergel et al. 2006, sauf pour la normalisation du spectre qui vaut ici $\sigma_8 = 1$. Elle est sensiblement plus grande que dans Spergel et al. 2006 ($\sigma_8 \approx 0.75$) pour des raisons historiques, les simulations ayant été produites à une époque où les données observationnelles suggéraient une valeur voisine de $\sigma_8 \approx 1$.

Pour chaque ligne de visée on construit 64 cartes de masse projetée κ en choisissant une distribution des sources $f(z) = \delta_{D(z=z_s)}$ avec $0.2 < z_s < 3$ avec résolution $\gtrsim 0.4$ arcmin. Van Waerbeke et al. 2002 montrent que les effets de la non Gaussianité du champ devraient commencer à être importants autour de 10 arcmin pour un redshift des sources voisin de l'unité. On s'attend à ce que cette échelle varie légèrement en fonction du redshift moyen des sources z_s , d'une part parce que la taille physique correspondante à une échelle angulaire donnée change, et d'autre part parce que la formation des structures est plus avancée à bas redshift. Pour évaluer l'effet de la non linéarité

2. Dans la pratique si on s'intéresse à la mesure des fonctions de corrélations à des échelles plus petites que $\approx 1 \text{ deg}$ on peut couper les intégrales à des échelles supérieures, par exemple $\approx 3 \text{ deg}$, pour calculer la variance cosmique de la façon analytique suggérée par Schneider et al. 2002b.

sur la matrice de variance-covariance, on compare $\text{Cov}_{\text{Gauss}}\xi_+(\vartheta_1, \vartheta_2) \equiv r_{+0} + r_{+1}$ donnée par l'expression 6.6 au terme $\text{Cov}\xi_+(\vartheta_1, \vartheta_2)$ mesuré directement comme étant la covariance entre les N simulations:

$$\text{Cov}\xi_+(\vartheta_1, \vartheta_2) = \frac{\sum_{i=1}^N (\xi_{+,i}(\vartheta_1) - \bar{\xi}_+(\vartheta_1))(\xi_{+,i}(\vartheta_2) - \bar{\xi}_+(\vartheta_2))}{N} \quad (6.7)$$

où $\bar{\xi}_+, \bar{\xi}_-$ sont les fonctions de corrélation moyennes sur les N simulations. Ce terme est directement comparable avec $\text{Cov}_{\text{Gauss}}\xi_+(\vartheta_1, \vartheta_2)$ car on utilise un nombre très grand d'objets et un bruit nul³.

Soit $\mathcal{F}(\vartheta_1, \vartheta_2)$ le rapport entre $\text{Cov}\xi_+(\vartheta_1, \vartheta_2)$ et $\text{Cov}_{\text{Gauss}}\xi_+(\vartheta_1, \vartheta_2)$. L'objectif est de recalibrer la matrice de covariance en régime Gaussien en fonction de θ_1, θ_2 et de corriger les prédictions Gaussiennes en y incluant des effets non Gaussiens. Avant tout, on remarque que si la dépendance $\propto 1/A$ dans l'équation 6.6 est vraie, la valeur $\mathcal{F}(\vartheta_1, \vartheta_2)$ mesurée ne devrait pas dépendre de la taille \mathcal{A} utilisée pour calculer 6.7. En particulier, on devrait toujours avoir un rapport $\mathcal{F}(\vartheta_1, \vartheta_2) = 1$ à grande échelle. La figure 6.2 montre la partie diagonale de la matrice $\mathcal{F}(\vartheta_1, \vartheta_2)$ pour différentes tailles \mathcal{A} utilisée pour calculer la covariance (6.7). Elles correspondent à $1/4, 1/9$, et $1/16$ de la surface totale de chaque image. Dans chaque cas, la covariance 6.7 est calculée en supposant que chaque sous-image est indépendante. La valeur N correspond au nombre total de sous-images utilisées. La valeur de l'expression analytique $\text{Cov}_{\text{Gauss}}\xi_+(\vartheta_1, \vartheta_2)$ est calculée en utilisant comme modèle de référence les fonctions $\bar{\xi}_+, \bar{\xi}_-$ moyennes des 64 réalisations. On constate deux faits importants:

- la valeur de $F(\vartheta)$ pour tous les relevés dont la taille $\mathcal{A} \ll \mathcal{S}$ est assez similaire, tandis que dans le cas $\mathcal{A} = \mathcal{S}$ elle diffère significativement des autres.
- la limite $F(\vartheta_1, \vartheta_2) = 1$ n'est pas atteinte asymptotiquement.

Pour comprendre l'origine de ces phénomènes et choisir une taille \mathcal{A} optimale pour la calibration, il faut tenir compte de plusieurs effets qui pourraient modifier l'amplitude de la variance cosmique mesurée sur les simulations. Il s'agit de la variance intrinsèque des simulations et du biais introduit par la mesure de la fonction de corrélation dans une boîte de taille finie (Peebles 1974).

Tout d'abord on remarque que la mesure de $\mathcal{F}(\vartheta_1, \vartheta_2)$ effectuée avec une taille $\mathcal{A} = \mathcal{S}$ est biaisée. En effet, les lignes de visée correspondant à cette taille angulaire représentent une dimension physique équivalente à celle des simulations. La variance que l'on mesure entre ces réalisations sous-estime l'amplitude réelle parce que les conditions de périodicité dans chaque cube doivent être respectées (par exemple le nombre de particules est le même). Si l'on divise ces cônes et qu'on traite chaque sous-image comme des ensembles indépendants on minimise l'effet, et il faut donc choisir une taille $\mathcal{A} < \mathcal{S}$. Cependant l'expression 6.6 n'est vraie que dans le cas où les effets de bord sont négligeables. La convergence entre les mesures dans le cas $\mathcal{A} < \mathcal{S}$, qui est visible sur la figure 6.2, montre que l'on peut sélectionner une taille quelconque parmi celles indiquées. On mesure en effet, la valeur du rapport $\mathcal{F}(\vartheta_1, \vartheta_2)$ pour les tailles $\mathcal{A} = 3.06, 5.44$, et 12.25 deg^2 et plusieurs plan sources sans trouver une dépendance de la taille. On peut donc conclure que la différence en amplitude du ratio $\mathcal{F}(\vartheta_1, \vartheta_2)$ pour différents choix de la taille \mathcal{A} est liée à la taille finie du jeu de simulations. Par ailleurs, on montrera dans la suite qu'elle est en accord avec la variance associée à chaque courbe par la méthode de bootstrap. Le fait que la limite asymptotique diffère de la valeur unité aux grandes échelles s'explique par le comportement de ξ_+ moyen, qui

3. Si on n'utilise pas un nombre suffisant de mesures, la valeur de la variance augmente, pour un simple problème de sous-échantillonnage.

diminue lorsque $\mathcal{A} < \mathcal{S}$, comme le montre la figure 6.1. Il s'agit d'un biais qui provient de la mesure d'une fonction de corrélation sur un support fini (Peebles 1974). L'effet sur ξ_- est négligeable. Cette

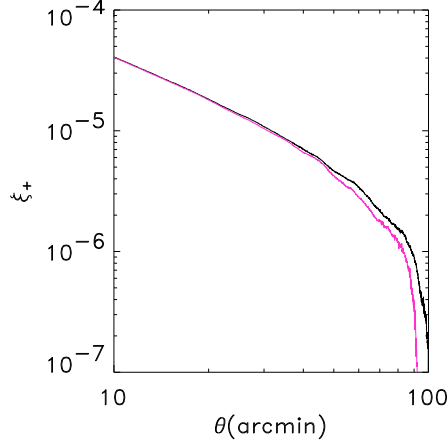


FIG. 6.1 – Comparaison entre la fonction de corrélation moyenne ξ_+ mesurée à partir de 64 simulations de taille originale (en noir) et celle mesurée en découpant les images en 9 sous-images (en magenta).

interprétation est confortée par le fait que le comportement est identique pour tous les plans sources utilisés et que dans le cas $\mathcal{A} = \mathcal{S}$ le rapport $\mathcal{F}(\vartheta_1, \vartheta_2) \gtrsim 0.8$ pour tous les plans sources. L'écart résiduel enregistré dans le cas $\mathcal{A} = \mathcal{S}$ peut être attribué au fait que les simulations ne sont pas complètement indépendantes (on utilise des permutations du même jeu de simulations à N-corps), et à la perte des modes dont on a parlé plus haut. Finalement, j'ai vérifié qu'en utilisant un autre jeu de simulations construit selon une technique différente et dont je parlerai dans la suite donne le même type de comportement (la fonction $\mathcal{F}(\vartheta_1, \vartheta_2)$ cette fois converge vers l'unité comme on le voit dans les plots 6.6).

Choisir $\bar{\xi}_+$ et $\bar{\xi}_-$ mesurés sur les simulations avec $\mathcal{A} < \mathcal{S}$ permettrait d'éviter cet effet à grande échelle. Cependant, le calcul numérique de 6.6 demande aussi la connaissance de ξ_+ et ξ_- grandes échelles. Enfin, on remarque que ce problème qui concerne majoritairement les grandes échelles est marginale si on recalibre la matrice $\mathcal{F}(\vartheta_1, \vartheta_2)$ que aux petites échelles.

On peut donc conclure que la taille $\mathcal{A} = 5.44 \text{ deg}^2$ est optimale. De plus, pour éviter une contamination par le biais à grande échelle il est raisonnable de ne recalibrer la matrice variance-covariance que pour des échelles $\vartheta < 10$ arcmin. La limite de précision de la calibration de la matrice non Gaussienne est donc donnée par la variance intrinsèque due au fait que le nombre de simulations est limité. On peut avoir un idée de son amplitude en mesurant la variance de la variance entre les différents jeux de sous-images. Plus précisément, puisque chaque image a été divisée en 9 sous-images, on dispose de 9 jeux composés chacun de 64 images. Sur tous ces jeux, on calcule la variance donnée par 6.7 puis on compare leur différence. La figure 6.3 montre l'erreur sur la variance estimée à partir des 9 jeux en utilisant 6.7. Cette erreur d'environ 30%, et donc la

limite de précision qu'on peut atteindre pour notre calibration est de l'ordre de 10%.

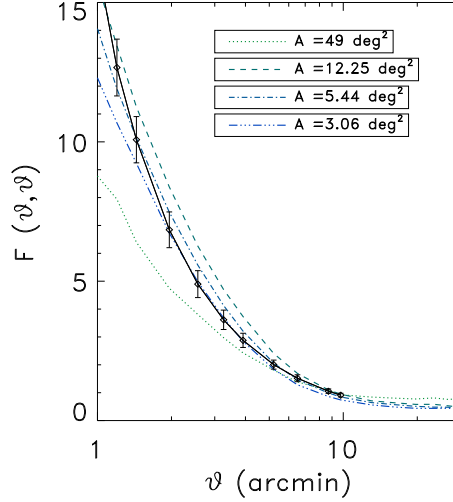


FIG. 6.2 – *Partie diagonale du rapport $\mathcal{F}(\vartheta_1, \vartheta_2)$ pour le jeu de simulations correspondant à $z_s = 1$. Les différentes courbes montrent que ce rapport varie en fonction de la taille \mathcal{A} des sous-images utilisées pour mesurer la variance de la fonction de corrélation ξ_+ . La courbe en noir est l'ajustement obtenu avec une taille $\mathcal{A} = 5.44$. Les barres d'erreur sont calculées par bootstrap sur 1000 réalisations et concordent avec l'erreur de 10% estimée en analysant plusieurs jeux de sous-images (voir texte).*

La figure 6.4 montre la partie diagonale de la fonction $\mathcal{F}(\vartheta_1, \vartheta_2)$ pour quelques plans source. Elle suggère de paramétrer la fonction $\mathcal{F}(\vartheta_1, \vartheta_2)$ avec une expression de la forme:

$$\mathcal{F}(\vartheta_1, \vartheta_2) = \frac{\alpha(z)}{[\vartheta_1 \vartheta_2]^{\beta(z)}} \quad (6.8)$$

où les paramètres α et β sont fixés pour chaque redshift par l'ajustement de la fonction 6.8 à la valeur $\mathcal{F}(\vartheta_1, \vartheta_2)$ mesurée pour des angles $1 < \vartheta_1, \vartheta_2 < 10$ arcmin. La limite inférieure est fixée pour éviter tout problème dû à la pixelisation. Avec cette procédure, on calcule α et β pour 9 plans source avec les redshifts $z_s = [0.4, 0.5, 0.8, 1.2, 1.4, 1.5, 2.1, 3.0]$.

On remarque que la valeur de $\mathcal{F}(\vartheta_1, \vartheta_2)$ augmente pour les petits redshifts comme l'on pouvait s'y attendre. L'échelle de transition ϑ_c pour laquelle $\mathcal{F}(\vartheta_1, \vartheta_2) > 1$, change aussi. L'ajustement effectué en utilisant l'équation 6.8 s'avère en bon accord avec les points mesurés. La figure 6.2 montre le résultat de ce type d'ajustement pour le cas particulier où $z_s = 1$. En se déplaçant hors de la diagonale le résultat se dégrade légèrement mais l'écart ne dépasse jamais 20%. Cette limitation n'est pas gênante à ce niveau car au fur et à mesure que ϑ_1 et ϑ_2 divergent les coefficients de corrélation deviennent rapidement négligeables comparés à ceux des régions où $\vartheta_1 \simeq \vartheta_2$. Une

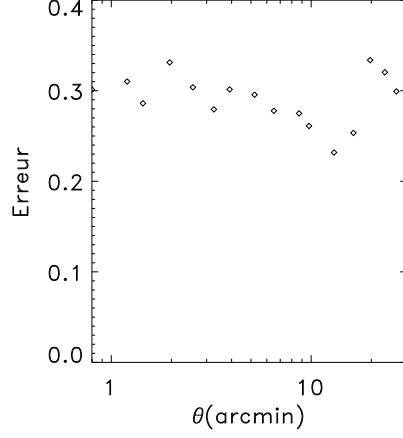


FIG. 6.3 – Incertitude mesurée sur les 9 jeux construits à partir des sous-images (voir texte).

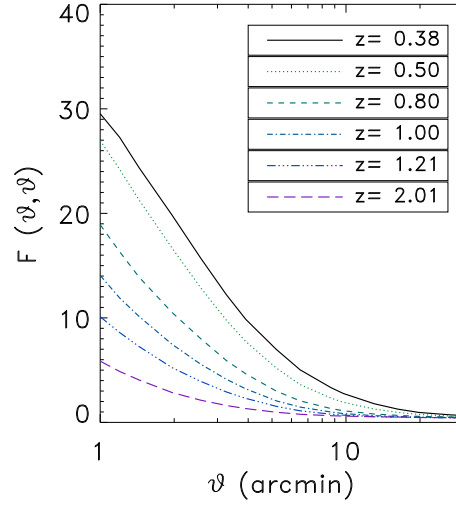


FIG. 6.4 – Partie diagonale de la matrice $\mathcal{F}(\vartheta_1, \vartheta_2)$ pour différents redshifts des sources.

fois déterminés α et β pour les 9 plans source, j'ai construit une relation empirique qui permet de calibrer $\text{Cov}_{\text{Gauss}}\xi_+(\vartheta_1, \vartheta_2)$ à petite échelle en fonction du redshift des sources.

$$\begin{aligned}\alpha(z) &= \frac{a_1}{z^{a_2}} + a_3 \\ \beta(z) &= b_1 z^{b_2} \exp(-z^{b_3}) + b_4.\end{aligned}\tag{6.9}$$

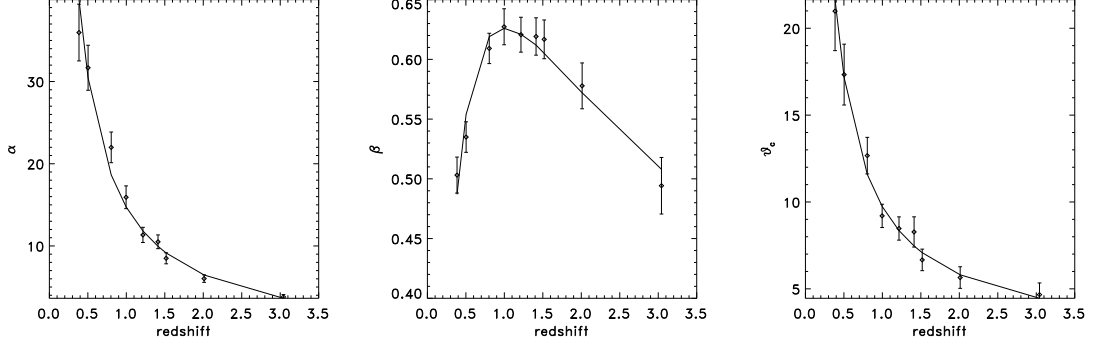


FIG. 6.5 – *Mesure des coefficients α , β et θ_c pour différents plans source. Les barres d’erreur sont calculées par bootstrap sur 1000 réalisations. La courbe représente l’ajustement effectué en utilisant les fonctions décrites dans le texte.*

Avec cette procédure, j’ai pu mesurer l’évolution de l’échelle angulaire de transition ϑ_c en fonction du redshift des sources et l’ajuster avec une expression analytique de la forme $\vartheta_c = \frac{t_1}{z^{t_2}} + t_3$. La figure 6.5 montre l’ajustement correspondant aux valeurs des paramètres suivantes: $(a_1, a_2, a_3) = (16.90, 0.95, -2.19)$, pour α , $(b_1, b_2, b_3, b_4) = (1.62, -0.68, -0.68, -0.03)$ pour β et $(t_1, t_2, t_3) = (8.07, 0.95, 1.65)$ pour θ_c .

6.3 Coherence de la méthode et dépendance de σ_8

Pour vérifier la consistance de la méthode j’utilise deux jeux de simulations de tracés de rayons, caractérisés par $z_s \sim 0.5$ et $z_s \sim 1.0$, réalisées avec le code TreePM (White 2002). Une description détaillée peut être trouvée dans Heymans et al. 2006b. Ces deux jeux sont de taille plus petite que celle de Hamana. Elles sont chacune composée de 12 images de taille $\mathcal{S} = 25 \text{ deg}^2$. Le modèle cosmologique est encore un ΛCDM en accord avec Spergel et al. 2006 avec une valeur de $\sigma_8 = 0.8$.

L’analyse de ces simulations doit permettre de vérifier les résultats de la section précédente. Les courbes de la figure 6.6 montrent que pour les deux jeux la valeur de $\mathcal{F}(\vartheta_1, \vartheta_2)$ converge vers l’unité pour des grandes échelles angulaires. L’effet dû à l’absence de modes qu’on a évoqué auparavant est ici négligeable car la taille angulaire des lignes de visée est déjà, à l’origine, plus petite que celle des cubes des simulations à N-corps. C’est effectivement démontré en utilisant des sous-images de taille $\mathcal{A} = 5.44 \text{ deg}^2$ car le rapport $\mathcal{F}(\vartheta_1, \vartheta_2)$ ne change pas à petite échelle. En revanche, on retrouve qu’à cause de la mesure biaisée de la fonction de corrélation, aux grandes échelles le rapport devient plus petit que l’unité lorsque $\mathcal{F}(\vartheta_1, \vartheta_2)$ est calculé sur les images de taille $\mathcal{A} < \mathcal{S}$.

Ces deux jeux peuvent aussi être utilisés pour étudier qualitativement le changement de comportement du rapport $\mathcal{F}(\theta_1, \theta_2)$ avec la normalisation σ_8 . La courbe de la figure 6.7 montre comment $\mathcal{F}(\theta_1, \theta_2)$ varie avec ce paramètre.

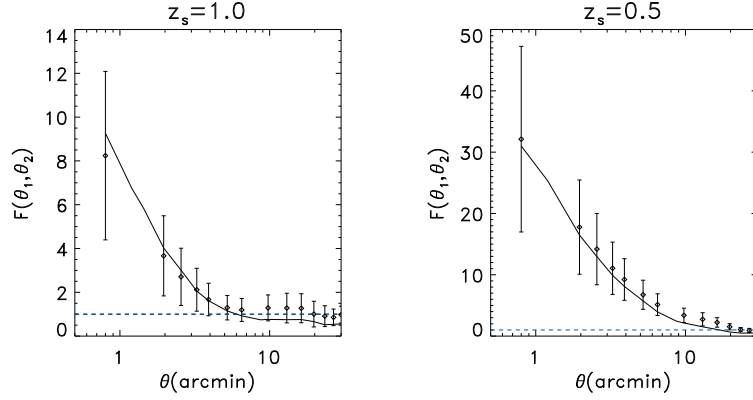


FIG. 6.6 – Le panneau de gauche montre les valeurs du rapport $\mathcal{F}(\vartheta_1, \vartheta_2)$ mesuré le long de la diagonale pour le jeu de simulations avec $z_s = 1$. Les valeurs obtenues en utilisant une taille $\mathcal{A} = \mathcal{S}$ (losanges) sont comparées à celles obtenues en choisissant $\mathcal{A} = 5.44 \text{ deg}^2$. Les barres d'erreur sont calculées par bootstrap sur 1000 réalisations. Le panneau de droite montre les mêmes résultats pour le jeu caractérisé par $z_s = 0.5$.

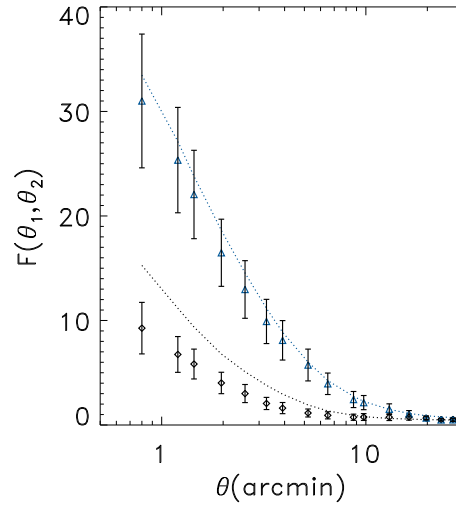


FIG. 6.7 – Les valeurs du rapport $\mathcal{F}(\theta_1, \theta_2)$ pour $z_s = 1$ et $z_s = 0.5$ trouvées avec le jeu de simulations avec $\sigma_8 = 1$. (lignes pointillées en bleu et noir) sont comparées avec celles obtenues avec les simulations avec $\sigma_8 = 0.8$ (losanges en noir et triangles en bleu). Les erreurs ne sont montrées que sur les jeux ayant $\sigma_8 = 0.8$ pour plus de clarté.

6.4 Effet de la non linéarité du champ sur les contraintes cosmologiques

À la lumière de ces résultats il faut explorer les implications de la calibration sur la précision avec laquelle on peut contraindre les paramètres cosmologiques par cisaillement gravitationnel. La figure 6.8 montre le changement de la variance cosmique attendu par l'introduction de la correction non Gaussienne pour une collection de relevés présents et futurs. Les caractéristiques de ces relevés sont résumées dans le tableau 6.1.

TAB. 6.1 – *Caractéristiques principales des relevés présentés sur la figure 6.8.*

Nom	Δ (deg ²)	n_{gal}/arcmin^2	$\langle z_s \rangle$
GEMS	0.25	65	1
COSMOS	1.6	80	1.2
CFHTLS Wide	50	15	0.8
SNAP deep	15	300	1.4
SNAP wide	260	120	1.2

La figure 6.8 montre la différence entre l'erreur totale en approximation Gaussienne et celle avec la correction non Gaussienne dans l'évaluation de la variance cosmique associée à la fonction de corrélation $\xi(\theta)$. Au fur et à mesure que l'erreur statistique diminue, l'importance de cette correction 6.8 devient de plus en plus importante. Cependant, la plus grande partie des contraintes cosmologiques fournies par le cisaillement proviendra des relevés couvrant de vastes zones du ciel. Cela sera encore plus vrai pour les relevés du futur tel que DES, DUNE, KIDS, LSST ou SNAP. Dans ce cas, le terme correctif de la variance pour des échelles angulaires $\vartheta < 10$ arcmin, même s'il s'avère important, ne devrait pas beaucoup affecter les résultats.

En revanche, cette correction devrait rendre plus compliquée la mesure de l'équation d'état de l'énergie sombre par cisaillement cosmologique. En effet, les contraintes sur l'équation d'état de l'énergie sombre proviennent de la comparaison du signal du cisaillement à différents redshifts, mais à petite échelle [Benabed & van Waerbeke 2004]. Une variance entre 5 et 10 fois plus grande à des échelles autour d'une minute d'arc suggérée par la figure 6.7 semble changer sensiblement les résultats montrés par Benabed & van Waerbeke 2004. Cependant, seule une étude plus approfondie en utilisant des simulations avec une valeur de σ_8 plus proche de la valeur actuellement acceptée peut fournir une réponse quantitative.

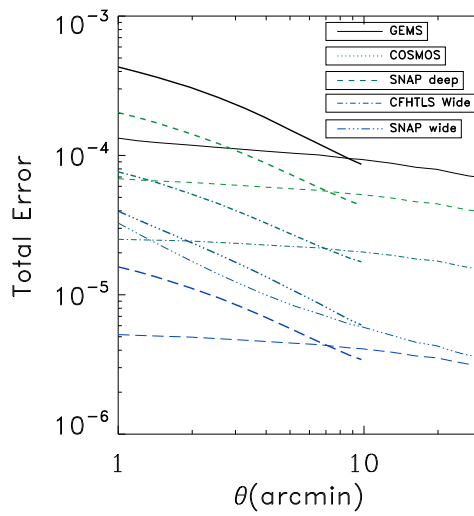


FIG. 6.8 – La figure montre l'erreur totale en régime gaussien, i.e. le résultat des contribution 6.3,6.4 et 6.6, sur une famille représentative de relevés. La densité et la taille utilisées pour évaluer 6.3,6.4 et 6.6 pour chaque relevé sont indiquées dans la table 6.1. La valeur σ_ϵ de la dispersion de l'ellipticité intrinsèque est prise 0.3 pour les relevés spatiaux (SNAP, COSMOS et GEMS) et 0.44 pour les relevés au sol (CFHTLS Wide). Pour chaque relevé, cette erreur est comparée avec celle que l'on trouve après la recalibration des termes 6.6 qui est effectuée en utilisant les redshifts indiqués dans la table 6.1.

Chapitre 7

Effet du “masking” sur la reconstruction du spectre

Avec la disponibilité de grands relevés et d’algorithmes de plus en plus performants dans la reconstruction du champ κ à partir de l’ellipticité des galaxies, comme par exemple ceux dérivés des relations 3.21 ou 3.23, il devient envisageable de mesurer directement le spectre de puissance de la masse projetée par transformée de Fourier des cartes de masse projetée. Cependant, la présence de masques devra être prise en compte car, comme on le discute dans ce chapitre, elle affecte le spectre. La méthode que j’adopte permet d’obtenir des prédictions sur le spectre affecté par la présence des masques qui peuvent être directement comparées au spectre observé.

7.1 Effet de la présence des masques: génération de modes E, B et ambigus

En utilisant l’équation 4.23 on peut écrire $\tilde{\gamma}_1(\mathbf{l})$ et $\tilde{\gamma}_2(\mathbf{l})$ en fonction de $\tilde{\gamma}_e(\mathbf{l})$ et $\tilde{\gamma}_b(\mathbf{l})$:

$$\begin{aligned}\tilde{\gamma}_1(\mathbf{l}) &= \tilde{\gamma}_e(\mathbf{l}) \cos(2\phi) - \tilde{\gamma}_b(\mathbf{l}) \sin(2\phi) \\ \tilde{\gamma}_2(\mathbf{l}) &= \tilde{\gamma}_e(\mathbf{l}) \sin(2\phi) - \tilde{\gamma}_b(\mathbf{l}) \cos(2\phi)\end{aligned}\tag{7.1}$$

La présence des masques sur un relevé peut être formalisée de la façon suivante:

$$\gamma_{mes}(\boldsymbol{\theta}) = \gamma(\boldsymbol{\theta})\mathcal{F}(\boldsymbol{\theta})\tag{7.2}$$

où $\gamma(\boldsymbol{\theta})$ est la valeur du champ de cisaillement à une certaine position angulaire $\boldsymbol{\theta}$, $\mathcal{F}(\boldsymbol{\theta})$ est la fonction fenêtre dont la valeur est l’unité dans le cas d’absence de masques et nulle autrement, $\gamma_{mes}(\boldsymbol{\theta})$ est le signal mesuré. En utilisant la relation 7.1 et la transformée de Fourier de l’équation 7.2 on trouve:

$$\tilde{\gamma}_{mes}(\mathbf{l}) = \int \frac{d^2\mathbf{q}}{(2\pi)^2} \left[\tilde{\gamma}_e(\mathbf{q}) \begin{pmatrix} \cos(2\phi_q) \\ \sin(2\phi_q) \end{pmatrix} + \tilde{\gamma}_b(\mathbf{q}) \begin{pmatrix} -\sin(2\phi_q) \\ \cos(2\phi_q) \end{pmatrix} \right] \tilde{\mathcal{F}}(\mathbf{l} - \mathbf{q})\tag{7.3}$$

où ϕ_q représente l'angle entre \mathbf{q} et l'axe des abscisses. Dans le cas d'un champ de cisaillement on peut poser $\gamma_B(\mathbf{l}) = 0$. Selon la définition 7.1 on peut définir deux quantités $\gamma_{E,mes}(\mathbf{l})$ et $\gamma_{B,mes}(\mathbf{l})$ qui sont des combinaisons linéaires des composantes de $\gamma_{mes}(\mathbf{l})$, par analogie à ce qui a été fait dans le chapitre 4.

$$\tilde{\gamma}_{E,mes}(\mathbf{l}) = \int \frac{d^2\mathbf{q}}{(2\pi)^2} \tilde{\gamma}_E(\mathbf{l}) \tilde{\mathcal{F}}(\mathbf{l} - \mathbf{q}) \cos(2\phi_{lq}) \quad (7.4)$$

$$\tilde{\gamma}_{B,mes}(\mathbf{l}) = \int \frac{d^2\mathbf{q}}{(2\pi)^2} \tilde{\gamma}_E(\mathbf{l}) \tilde{\mathcal{F}}(\mathbf{l} - \mathbf{q}) \sin(2\phi_{lq}) \quad (7.5)$$

où ϕ_{lq} est l'angle entre \mathbf{l} et \mathbf{q} . Enfin, on peut écrire les moments du deuxième ordre mesurés:

$$\langle |\tilde{\gamma}_{mes} \tilde{\gamma}_{mes}|^2 \rangle = \langle |\tilde{\gamma}_{E,mes} \tilde{\gamma}_{E,mes}|^2 \rangle + \langle |\tilde{\gamma}_{B,mes} \tilde{\gamma}_{B,mes}|^2 \rangle + \langle |\tilde{\gamma}_{E,mes} \tilde{\gamma}_{B,mes}|^2 \rangle \quad (7.6)$$

L'équation 7.6 montre qu'à cause de l'introduction de la fonction \mathcal{F} , même si le champ de cisaillement est de type E , le spectre de puissance mesuré contient à priori trois différentes composantes: une de type E , l'autre de type B et une troisième qui provient du couplage entre les deux, que l'on appelle une composante "ambiguë". On remarque que la définition de la fonction \mathcal{F} est générale et qu'à chaque fois qu'une fonction de transport $\mathcal{F} \neq 1$ est appliquée au champ γ original il n'y a aucune raison que le spectre mesuré soit le vrai spectre du cisaillement. Les différentes composantes de l'équation 7.6 sont données par:

$$\begin{aligned} \langle |\tilde{\gamma}_{E,mes}(\mathbf{l}) \tilde{\gamma}_{E,mes}(\mathbf{l})|^2 \rangle &= \int \frac{d^2\mathbf{q}}{(2\pi)^2} P_\kappa(\mathbf{q}) |\tilde{\mathcal{F}}(\mathbf{l} - \mathbf{q})|^2 \cos(2\phi_{ql})^2 \\ \langle |\tilde{\gamma}_{B,mes}(\mathbf{l}) \tilde{\gamma}_{B,mes}(\mathbf{l})|^2 \rangle &= \int \frac{d^2\mathbf{q}}{(2\pi)^2} P_\kappa(\mathbf{q}) |\tilde{\mathcal{F}}(\mathbf{l} - \mathbf{q})|^2 \sin(2\phi_{ql})^2 \\ \langle |\tilde{\gamma}_{E,mes}(\mathbf{l}) \tilde{\gamma}_{B,mes}(\mathbf{l})|^2 \rangle &= \int \frac{d^2\mathbf{q}}{(2\pi)^2} P_\kappa(\mathbf{q}) |\tilde{\mathcal{F}}(\mathbf{l} - \mathbf{q})|^2 \cos(2\phi_{ql}) \sin(2\phi_{ql}) \end{aligned} \quad (7.7)$$

En suivant la définition générale du spectre de puissance on définit alors:

$$\begin{aligned} (2\pi)^2 P_{E,mes}(\mathbf{l}) &= \langle |\tilde{\gamma}_{E,mes}(\mathbf{l}) \tilde{\gamma}_{E,mes}(\mathbf{l})|^2 \rangle \\ (2\pi)^2 P_{B,mes}(\mathbf{l}) &= \langle |\tilde{\gamma}_{B,mes}(\mathbf{l}) \tilde{\gamma}_{B,mes}(\mathbf{l})|^2 \rangle \\ (2\pi)^2 P_{A,mes}(\mathbf{l}) &= \langle |\tilde{\gamma}_{E,mes}(\mathbf{l}) \tilde{\gamma}_{B,mes}(\mathbf{l})|^2 \rangle \end{aligned}$$

où on a utilisé le symbole A pour la composante ambiguë ou couplée. L'image 7.1 montre deux exemples réalistes de masques que j'ai utilisés pour quantifier les effets sur la mesure du spectre de puissance. Le premier exemple est un masque appliquée à un des champs de VIRMOS-Dcart. Sa dimension est de $(2048 \times 24) \times (4096 \times 8)$ pixels. Chaque pixel couvre une taille angulaire de 0.206 arcsec ce qui donne une couverture totale de 2.8 deg \times 1.9 deg. Le deuxième exemple est celui du champ D1 du CFHTLS dont j'ai donné les caractéristiques dans le chapitre 5. Les courbes 7.2 montrent comment le spectre mesuré est altéré par les effets des masques décrits selon les équations

1. Du point de vue mathématique l'existence des modes B et ambiguës s'explique par un manque d'information complète. Plus précisément, la séparation en modes E et B pour un champ quelconque n'est bien définie que lorsque le champ peut être mesuré dans un domaine fermé et connecté. Cette condition n'est pas rigoureusement vérifiée dans les relevés réels.

7.7 pour ces deux cas. Le spectre théorique utilisé pour ces graphiques est en accord avec le modèle de WMAP-3 [Spergel et al. 2006] avec une constante de normalisation $\sigma_8 = 0.8$. Les résultats dans le cas du champ VIRMOS-Descart et du champ D1 sont assez similaires. Dans les deux cas l'isotropie du spectre est perdue à cause de l'asymétrie introduite par le masque, dont les structures apparaissent pour certains modes caractéristiques préférentiels. Cependant, l'amplitude des modes B et A n'excède jamais 10% de celle des modes E .

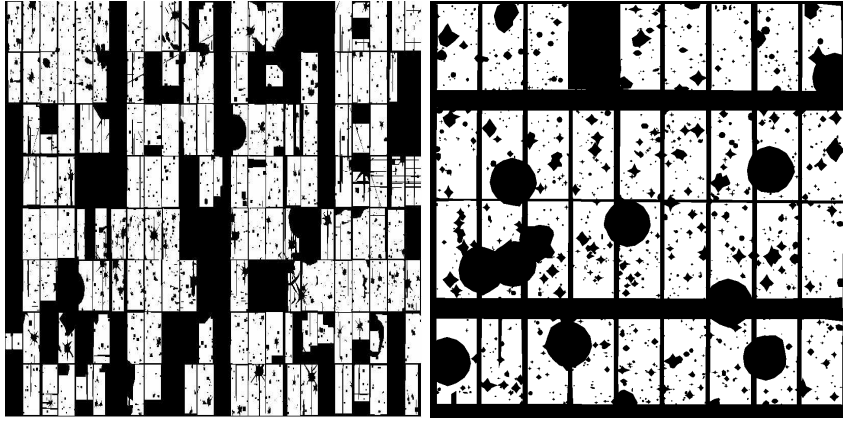


FIG. 7.1 – Exemple réaliste de masques: à gauche le champ F14 (taille $2.8 \times 1.9 \text{ deg}^2$) du relevé VIRMOS-Descart et à droite le champ D1 (taille 1 deg^2) du relevé CFHTLS.

7.2 Effet du couplage bruit-masking

On peut maintenant généraliser 7.2 et estimer l'amplitude des modes E, B et A dans le cas où le bruit dû à l'ellipticité intrinsèque de la population source n'est pas nul. L'équation

$$\gamma_{mes}(\boldsymbol{\theta}) = (\gamma(\boldsymbol{\theta}) + \epsilon(\boldsymbol{\theta}))\mathcal{F}(\boldsymbol{\theta}) \quad (7.8)$$

est équivalente à l'équation 7.2, mais avec la présence d'un terme de bruit $\epsilon(\boldsymbol{\theta}) = \epsilon_1(\boldsymbol{\theta}) + i\epsilon_2(\boldsymbol{\theta})$. Les composantes mesurées deviennent alors:

$$\begin{aligned} \langle |\tilde{\gamma}_{E,mes}(\mathbf{l})\tilde{\gamma}_{E,mes}(\mathbf{l})|^2 \rangle &= (2\pi)^2 P_{E,mes}(\mathbf{l}) + \int \frac{d^2\mathbf{q}}{(2\pi)^2} \int \frac{d^2\mathbf{q}'}{(2\pi)^2} \cos(2\phi_{lq}) \cos(2\phi_{lq'}) \tilde{\mathcal{F}}(\mathbf{l}-\mathbf{q}) \tilde{\mathcal{F}}^*(\mathbf{l}-\mathbf{q}') \\ &\quad [2 \langle \gamma_E(\mathbf{q})\epsilon(\mathbf{q}') \rangle + \langle \epsilon(\mathbf{q})\epsilon(\mathbf{q}') \rangle] \\ \langle |\tilde{\gamma}_{B,mes}(\mathbf{l})\tilde{\gamma}_{B,mes}(\mathbf{l})|^2 \rangle &= (2\pi)^2 P_{B,mes}(\mathbf{l}) + \int \frac{d^2\mathbf{q}}{(2\pi)^2} \int \frac{d^2\mathbf{q}'}{(2\pi)^2} \sin(2\phi_{lq}) \sin(2\phi_{lq'}) \tilde{\mathcal{F}}(\mathbf{l}-\mathbf{q}) \tilde{\mathcal{F}}^*(\mathbf{l}-\mathbf{q}') \\ &\quad [2 \langle \gamma_E(\mathbf{q})\epsilon(\mathbf{q}') \rangle + \langle \epsilon(\mathbf{q})\epsilon(\mathbf{q}') \rangle] \\ \langle |\tilde{\gamma}_{E,mes}(\mathbf{l})\tilde{\gamma}_{B,mes}(\mathbf{l})|^2 \rangle &= (2\pi)^2 P_{A,mes}(\mathbf{l}) + \int \frac{d^2\mathbf{q}}{(2\pi)^2} \int \frac{d^2\mathbf{q}'}{(2\pi)^2} \sin(2\phi_{lq}) \cos(2\phi_{lq'}) \tilde{\mathcal{F}}(\mathbf{l}-\mathbf{q}) \tilde{\mathcal{F}}^*(\mathbf{l}-\mathbf{q}') \\ &\quad [2 \langle \gamma_E(\mathbf{q})\epsilon(\mathbf{q}') \rangle + \langle \epsilon(\mathbf{q})\epsilon(\mathbf{q}') \rangle] \end{aligned}$$

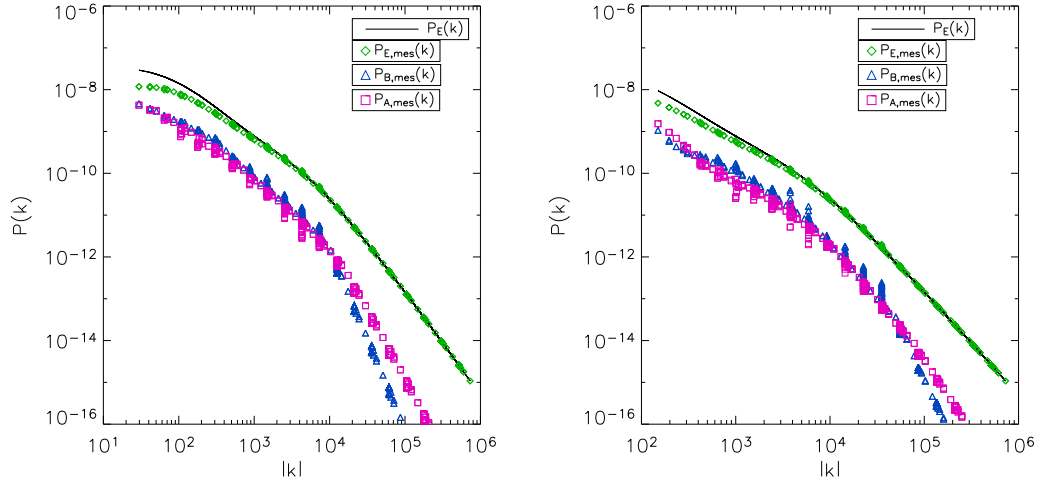


FIG. 7.2 – À gauche: Composantes E (losanges verts) B (triangles bleus) et A (carrés magenta) du spectre mesuré dans le cas des masques du relevé VIRMOS-Descart. À droite: les mêmes composantes pour le champ D1 du CFHTLS.

où $P_{E,mes}(\mathbf{l})$, $P_{B,mes}(\mathbf{l})$ et $P_{A,mes}(\mathbf{l})$ représentent les composantes du spectre mesurées dans le cas du bruit nul et en présence des masques 7.7. On choisit ici un modèle de bruit simple donné par l'ellipticité intrinsèque des galaxies et caractérisé par sa dispersion σ_ϵ :

$$\langle \epsilon(\boldsymbol{\theta})\epsilon(\boldsymbol{\theta}') \rangle = \sigma_\epsilon^2 \delta_{(\boldsymbol{\theta}=\boldsymbol{\theta}')} \quad (7.9)$$

non corrélée avec la valeur du cisaillement. En utilisant la transformée de Fourier de l'équation 7.9 on trouve alors:

$$\begin{aligned} P_{E,mes}^{noise}(\mathbf{l}) &= P_{E,mes}(\mathbf{l}) + \sigma_\epsilon \int \frac{d\mathbf{q}}{(2\pi)^2} |\tilde{\mathcal{F}}(\mathbf{l}-\mathbf{q})|^2 \cos(\phi_{lq})^2 \\ P_{B,mes}^{noise}(\mathbf{l}) &= P_{B,mes}(\mathbf{l}) + \sigma_\epsilon \int \frac{d\mathbf{q}}{(2\pi)^2} |\tilde{\mathcal{F}}(\mathbf{l}-\mathbf{q})|^2 \sin(\phi_{lq})^2 \\ P_{A,mes}^{noise}(\mathbf{l}) &= P_{A,mes}(\mathbf{l}) + \sigma_\epsilon \int \frac{d\mathbf{q}}{(2\pi)^2} |\tilde{\mathcal{F}}(\mathbf{l}-\mathbf{q})|^2 \cos(\phi_{lq}) \sin(\phi_{lq}) \end{aligned} \quad (7.10)$$

Donc, dans le but de mesurer la bonne valeur du spectre dans le cas d'absence de masques, la première équation doit être corrigée et elle devient:

$$P_{E,mes}^{noise}(\mathbf{l}) = P_{E,mes}(\mathbf{l}) + \sigma_\epsilon \int \frac{dq^2}{(2\pi)^2} |\tilde{\mathcal{F}}(\mathbf{l}-\mathbf{q})|^2 (\cos(\phi_{lq})^2 - 1) \quad (7.11)$$

ce qui permet d'éliminer le biais apporté par l'ellipticité intrinsèque dans la mesure du spectre. On remarque que dans le cas où les masques ne sont pas présents, le bruit n'a aucun effet sur la mesure, *i.e.* on peut mesurer le spectre directement à partir des données sans avoir de biais. En revanche,

dans le cas de la présence des masques le bruit intervient dans un terme de couplage avec les masques. La figure 7.3 montre les prédictions pour chaque composante, pour un bruit d'ellipticité intrinsèque caractérisé par $\sigma_\epsilon = 0.3$ dans le cas des champs VIRMOS-Descart et D1 utilisés plus haut. L'effet du bruit lié à l'ellipticité intrinsèque des galaxies est négligeable à petits $|k|$, tandis

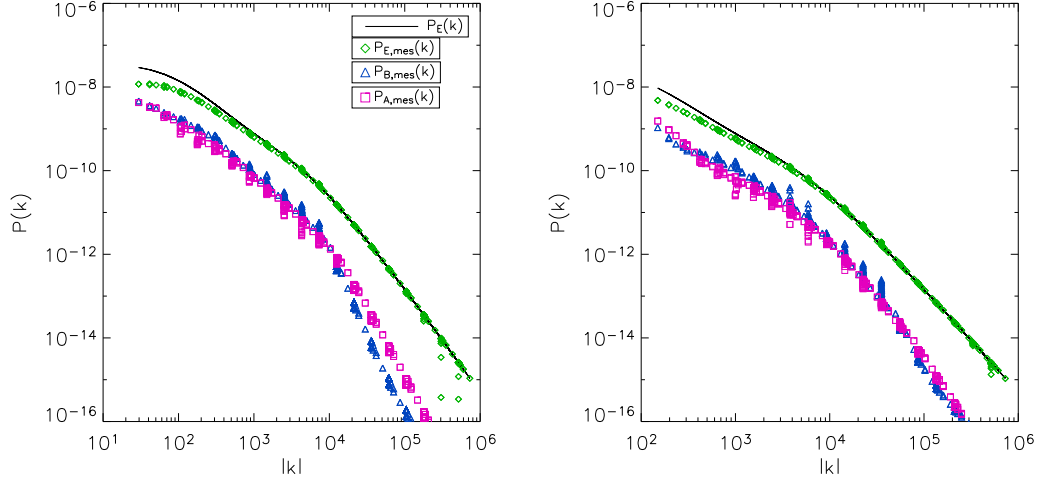


FIG. 7.3 – *A gauche: composantes E (losanges verts) B (triangles bleus) et A (carrés magenta) du spectre mesuré résultant de l'utilisation de masques et en présence de bruit du à l'ellipticité intrinsèque des galaxies dans le relevé VIRMOS-Descart. A droite: mêmes analyses pour le champ D1 du CFHTLS.*

qu'à grand $|k|$ (*i.e.* petites échelles angulaires dans l'espace réel) il apporte des modifications aux composantes E et B du spectre mesuré. Ceci est en accord avec le fait que le bruit statistique affecte plus les petites échelles angulaires que les grandes, comme on le voit aussi clairement d'après l'expression 6.3.

La méthode présentée dans ce chapitre permet à priori de prédire le spectre mesuré dans un relevé par inversion dans l'espace de Fourier des cartes de masse projetée quel que soit le bruit et les masques utilisés, sans besoin d'un modèle analytique. Elle peut être utilisée pour contraindre les paramètres cosmologiques en utilisant la méthode des moindres carrés présentée dans le chapitre 5. Il faut pour cela minimiser la différence entre le spectre mesuré et le spectre théorique affecté par le couplage bruit-masques, $(P_{mes}(k) - P_{mes}^{theo}(k))^2$, pour chaque composante E , B et A .

Chapitre 8

Biais de calibration dans la mesure du cisaillement cosmologique

On a déjà souligné à plusieurs occasions dans cette thèse, l'importance de réduire l'effet du biais dans la mesure du cisaillement cosmologique à partir de l'ellipticité mesurée dans un relevé de galaxies. Dans le chapitre 4 j'ai abordé le problème du biais causé par la corrélation intrinsèque des galaxies à l'intérieur des amas et par la corrélation entre ellipticité intrinsèque et lentilles d'avant plan qui peuvent affecter le signal. Les conséquences de la sous-estimation de la variance cosmique ont aussi été abordées tandis que celles d'une évaluation incorrecte de la distribution en redshift des sources, dont je parlerai dans le chapitre suivant, ont été discutées par van Waerbeke et al. 2006.

Dans ce chapitre je discute du possible biais lié à la méthode utilisée pour mesurer l'estimateur de cisaillement γ défini par 4.46. J'ai évoqué dans le chapitre 4 qu'aujourd'hui on dispose de plusieurs techniques qui permettent de dériver un estimateur du cisaillement à partir d'un relevé de galaxies. Ces techniques ont été implémentées par différents groupes et éventuellement testées de façon indépendante. Pour obtenir des contraintes fiables sur le spectre primordial à partir des mesures du cisaillement gravitationnel, il est nécessaire de valider et améliorer chaque méthode sur un ensemble de simulations réalistes partagées par les équipes développant ces outils. Le programme STEP (Shear Testing Program) est une collaboration internationale dont le but est d'évaluer, comparer, améliorer et valider toutes les méthodes d'analyse du cisaillement existantes et de rendre publiques leurs caractéristiques et performances.

La mesure de la quantité 4.46 à partir d'un relevé de galaxies comporte une grande quantité d'opérations. Le résultat dépend donc d'un grand nombre de facteurs et l'impact de chacun d'eux sur les relevés sol ou espace ne peut pas être exploré sans une analyse très longue et approfondie. Pour cette raison il est prévu que le programme STEP se déroule en plusieurs années. Progressivement, depuis environ deux ans, des simulations indépendantes construites par plusieurs groupes et utilisant différentes méthodes¹ sont mises à la disposition de la communauté. Les simulations

1. Chaque méthode doit être validée pour l'ensemble des conditions d'observation qui peuvent être raisonnablement envisagées (*i.e.* PSF, seeing, distribution spatiale et/ou densité d'étoiles, bruit, etc...). À terme, elles doivent permettre de les tester sur un jeu d'images simulées reflétant tous les cas de façon la plus réaliste possible. Le choix

contiennent un signal de cisaillement dont l'amplitude est inconnue. Elle sont ensuite analysées par chaque participant, et les catalogues du cisaillement estimé sont envoyés sur un site commun pour leur comparaison. Le biais entre signal mesuré et signal injecté est ensuite communiqué à tous les participants au programme. Plusieurs aspects de la mesure du cisaillement qui peuvent être à l'origine de biais sont aussi explorés. Comme par exemple, l'imprécision sur la position du centroïde, la sensibilité des mesures à la luminosité, à la taille caractéristique ou au rapport signal sur bruit des objets.

La première analyse STEP [Heymans et al. 2006a], STEP1, a permis de voir que les diverses implémentations de la méthode KSB souffrent de biais très différents. STEP1 fut aussi le premier test des méthodes d'analyse du cisaillement par shapelets. En ce qui me concerne, je ne m'intéresse ici qu'à la méthode KSB et à ses différentes versions car c'est celle que j'ai personnellement utilisée. Dans la suite je décris comment il a été possible, grâce aux simulations STEP1 et STEP2, d'identifier les origines du biais qui affectait ma propre chaîne d'analyse et comment j'ai pu le corriger.

8.1 Validation de la méthode KSB

8.1.1 Les simulations STEP1

Les deux jeux de simulations avec lesquels ont été effectués les premiers tests dans le cadre du programme STEP ont été réalisés d'une part en utilisant le logiciel Skymaker (voir la page web <http://terapix.iap.fr/cplt/oldSite/soft/skymaker/>) et d'autre part en utilisant un code de shapelets [Massey et al. 2004]. La façon dont les galaxies sont générées par les deux logiciels est différente. Dans Skymaker les galaxies sont composées d'une partie centrale (bulbe) caractérisée par un profil de type de Vaucouleurs, et d'un disque dont le profil est une loi exponentielle. Le rapport entre bulbe et disque peut varier selon la galaxie. La génération des galaxies par shapelets permet de couvrir un ensemble de types morphologiques très vaste. Les coefficients de chaque type ont été déterminés à partir des galaxies observées dans le champ HDF [Massey et al. 2004].

Un signal de cisaillement, constant sur tout le champ, est ensuite injecté dans les catalogues de galaxies simulées, finalement chaque objet est convolué par une PSF caractéristique reproduisant celle de relevés au sol.

Les simulations construites avec Skymaker qui m'intéressent pour la suite, sont composées de plusieurs jeux d'images. Chacun est constitué de 64 images de 4096×4096 pixels de 0.206 arcsec avec une densité de galaxies d'environ 15 arcmin^{-1} (c'est donc typiquement une image correspondant aux données du CFHTLS-Wide). À chaque set est appliqué un signal de cisaillement $\gamma_1 = (0.0, 0.005, 0.01, 0.05, 0.1)$; la composante γ_2 du signal est toujours nulle. Les six modèles de PSF terrestres avec lesquels on a convolué chaque set sont illustrés dans la figure 8.1.

8.1.2 Différentes implémentations de la méthode KSB

Dans cette section, je présente les résultats obtenus en utilisant trois méthodes différentes d'analyse, toutes basées sur la correction KSB.

d'utiliser différents logiciels pour construire les jeux de simulations permet de minimiser les risques que les résultats dépendent des simulations.

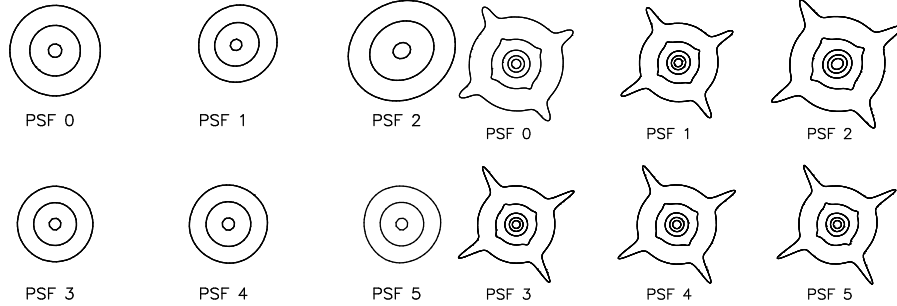


FIG. 8.1 – Parties internes (panneau de gauche) et externe (panneau de droite) des modèles de PSF utilisées pour convoluer les catalogues de galaxies de Skymaker. Pour le panneau de gauche les contours correspondent à 3, 25 et 90 pour-cents de la luminosité totale. Pour le panneau de droite ils correspondent à 0.003 0.03 et 25 pour-cents de la luminosité totale. D’après Heymans et al. 2006.

Méthode I

La première méthode est celle utilisée pour l’analyse du CFHTLS- Deep, que j’ai décrite dans le chapitre 5.

Méthode II

La deuxième méthode est inspirée par celle de Douglas Clowe telle qu’elle est présentée dans Heymans et al. 2006a, où elle porte la référence C1. Cette méthode a la particularité de mesurer les moments de l’intensité de lumière de chaque galaxie en utilisant une taille r_{fix} pour le filtre gaussien égal à la taille r_g^2 caractéristique des étoiles. Cette taille est déterminée par la valeur moyenne r_g^* de la taille r_g des étoiles sélectionnées dans le diagramme (mag, r_g) après une réjection par sigma-clipping.³ Les autres méthodes présentées dans le papier STEP1 utilisent un filtre, dont la taille est proportionnelle à celle de la galaxie, et un filtre dont la taille est proportionnelle à celle des étoiles, pour évaluer les vecteurs q et e^* et les tenseurs P^{sm*} et P^{sh*} calculés sur les étoiles. Alternativement, d’autres méthodes calculent ces dernières quantités sur les étoiles en utilisant un filtre dont la taille est adaptée à celle de la galaxies en question.

Méthode III

La méthode KSB implémentée par Henk Hoekstra est sans doute une des plus performantes. Le changement le plus important par rapport à la méthode I et II consiste à remplacer le lissage du scalaire $Tr(P^g)$ dans l’espace des paramètres (r_g, mag) par une interpolation avec un ajuste-

2. Le logiciel IMCAT fournit deux paramètres qui peuvent être utilisés pour identifier la taille des objets. Le paramètre r_g représente la taille optimale du filtre gaussien défini dans la section 4.5. Le paramètre r_h représente la taille correspondant à la moitié du flux total de l’objet (équivalent au paramètre FLUX_RADIUS fourni par SExtractor).

3. Le sigma-clipping a ici pour objectif d’éliminer de fausses détections parmi les étoiles sélectionnées dans le plan taille-luminosité. À la suite de l’opération de sélection dans le diagramme taille-luminosité, on effectue un ajustement polynomial des quantités vectorielles q et e^* et tensorielles $P^{*,sm}$ et $P^{*,sh}$ définies dans la section 4.5. Ces quantités sont utilisées ensuite pour corriger la PSF à la position des galaxies. Tous les objets pour lesquels au moins une des valeurs parmi ces quatre quantités s’écarte de plus de 3σ de sa valeur moyenne sont rejetés de l’échantillon.

ment polynomial dans ce même espace⁴. Le poids attribué à chaque galaxie est recalculé selon la pondération proposée par Hoekstra et al. 2000. Les critères de détection et de sélection restent les mêmes par ailleurs.

Les caractéristiques principales concernant chaque méthode sont résumées dans la table 8.1 et la comparaison est illustrée par la figure 8.2 pour la PSF0 et $\gamma_1 = 0.1$. L'amélioration apportée par la méthode III par rapport à la méthode I est évidente. Le biais qui affecte la méthode I est d'environ 20%, en accord avec celui trouvé dans l'analyse des simulations STEP2 [Massey et al. 2006] et celui trouvé dans l'analyse de CFHTLS-Deep (voir section suivante).

Cependant, l'origine du biais dans la méthode I est difficile à établir compte tenu de la multitude d'opérations de correction. Le choix de fixer la taille du filtre gaussien égale à celle de la PSF (méthode II) semble réduire partiellement le biais engendré par la procédure de correction implémentée dans la méthode I. La figure 8.2 montre que le biais obtenu est similaire à celui produit par la méthode C1 de Douglas Clowe [Heymans et al. 2006a]. Le plot 8.3 montre que le biais dans la méthode II reste constant tant que $r_{fix} \gtrsim 0.8r_g^*$. La méthode III est donc celle qui montre le biais le plus petit. La différence entre les trois méthodes est en accord avec la dispersion en termes de biais trouvée dans STEP1 pour les différentes implémentations de KSB. Cette dispersion et la dépendance du biais avec des paramètres tels que la magnitude et la taille des galaxies, démontrent que la méthode KSB est instable par rapport aux critères de sélection. Il n'est donc pas surprenant que ces méthodes très similaires aient parfois besoin d'une correction par une constante multiplicative permettant de rectifier leurs écarts d'amplitude [Heymans et al. 2006a].

D'après la série des panneaux de la figure 8.4, la méthode III donne des résultats qui limitent le biais autour de 5-6 pour-cents pour tous les types de PSF. La dépendance du biais de la taille rg est reportée dans 8.5 pour le jeu d'images PSF5 et $\gamma_1 = 0.1$.

4. Dans Heymans et al. 2006a, Henk Hoekstra propose un ajustement polynomial dans l'espace r_g , Ludovic van Waerbeke propose de son côté un ajustement dans l'espace $r_{h,mag}$. Dans la méthode III utilisé dans ce chapitre, l'ajustement est fait en fonction du paramètre r_h , ce qui donne le meilleur résultat.

TAB. 8.1 – Tableau récapitulatif des principales caractéristiques des méthodes I,II et III.

Méthode	I	II	III
Méthode de détection :	hfindpeaks (getshapes $m = 0.8 * r_g$)	hfindpeaks (getshapes $m = 0.8 * r_g^*$)	hfindpeaks (getshapes $m = 0.8 * r_g$)
correction PSF: (pol. 2 ^{eme} ordre)	$e^*, (P^{*,sm})^{-1} P^{*,sh}, q = (P^{*,sm})^{-1} e^*$ P^{sm} tenseur	$e^*, (P^{*,sm})^{-1} P^{*,sh}, q = (P^{*,sm})^{-1} e^*$ P^{sm} tenseur	$e^*, (P^{*,sm})^{-1} P^{*,sh}, q = (P^{*,sm})^{-1} e^*$ P^{sm} tenseur
correction P^g	$\frac{1}{2}Tr(P^g)$ lissé 20 objets (rg,mag)	$\frac{1}{2}Tr(P^g)$ lissé 20 objets (rg,mag)	4 ^{eme} ordre fit $\frac{1}{2}Tr(P^g)$ (rh)
Poids	Erben et al. 2000 avec 20 objets ^a	Erben et al. 2000 avec 20 objets	Hoekstra et al. 2000 ^b
Critères de sélection:			
-limite de détection	$\nu > 12$	$\nu > 12$	$\nu > 12$
-ellipticité observée	-	-	$ e_{obs} < 0.5$
- P^g :	$P_{smooth}^g > 0.01$	$P_{smooth}^g > 0.01$	-
-taille:	$r_g > r_{g,max}^*$ ^c	$r_g > r_{g,max}^*$	$r_g > r_{g,max}^*$
-distance entre objets:	$d > 10$ pixels	$d > 10$ pixels	$d > 10$ pixels

^a Le bruit qui affecte la mesure de l'ellipticité d'une galaxie est fonction de ses caractéristiques (*i.e.* taille, luminosité, rapport signal sur bruit). A partir de cette idée, Erben et al. 2000 suggère d'associer à chaque galaxie une incertitude σ_γ sur le cisaillement égale à la dispersion d'ellipticité corrigée dans une boîte glissante contenant le nombre n de plus proches voisins à la galaxie, dans l'espace des paramètres considéré. L'espace des paramètres peut être choisi de façon différente. Dans ce cas précis, j'ai choisi l'espace (r_g, mag) . Le poids associé à chaque galaxie est $1/(\sigma_\gamma^2 + \sigma_\epsilon^2)$ où le terme σ_ϵ est la dispersion de la distribution en ellipticité intrinsèque des galaxies

^b Hoekstra et al. 2000 propose de calculer l'imprécision Δ_χ pour chaque composante de l'ellipticité mesurée telle qu'elle est dérivée directement de la définition de l'ellipticité [Hoekstra et al. 2000]. L'erreur sur la quantité $\gamma = \chi_{ani}/P^g$ (voir la définition dans la section 4), où χ_{ani} est l'ellipticité corrigée de la partie anisotrope de la PSF. Elle est donnée par Δ_χ/P^g . A cette erreur de mesure s'ajoute celle due à l'ellipticité intrinsèque des sources. Le poids optimal est donné par $1/(\sigma_{mes}^2 + \sigma_\epsilon^2) = P^g^2/(\sigma_\epsilon^2 P^{\gamma^2} + \langle \Delta_\chi \rangle^2)$.

^c $r_{g,max}^*$ représente la taille maximale des étoiles sélectionnées

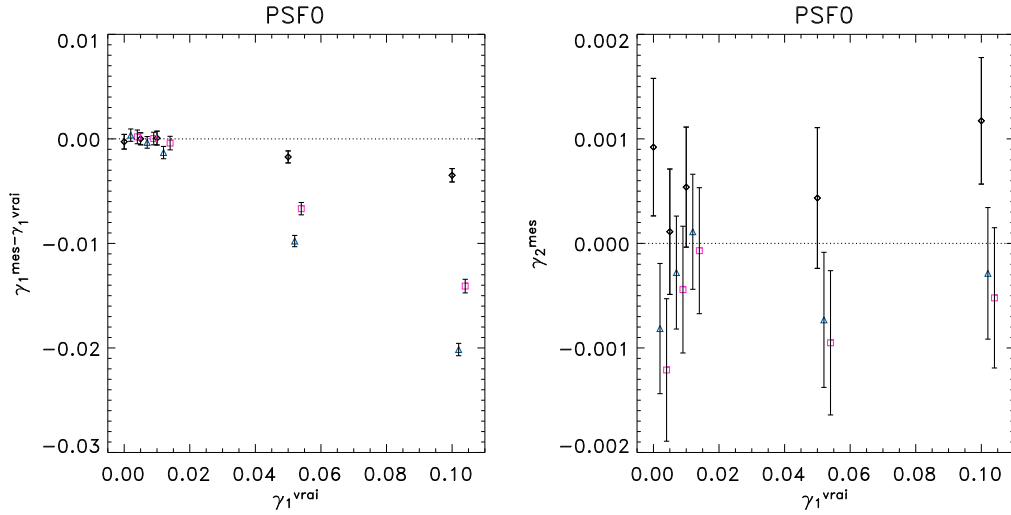


FIG. 8.2 – *Biais des composantes γ_1 et γ_2 en fonction de la vraie valeur de γ_1 . La méthode I représentée par les triangles bleus a un biais d'environ 20%. La méthode II représentée par les carrés magenta montre un biais d'environ 15%. La méthode III (losanges noirs) a un biais autour de 3%.*

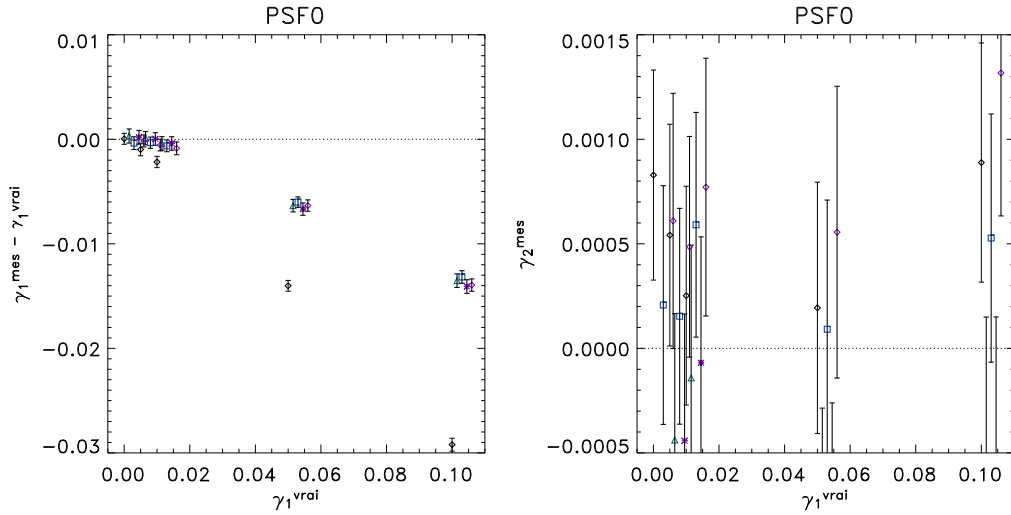
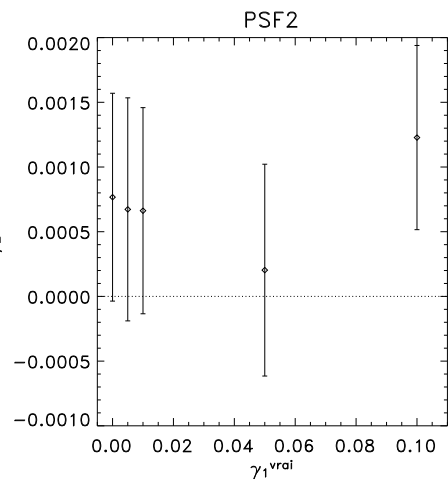
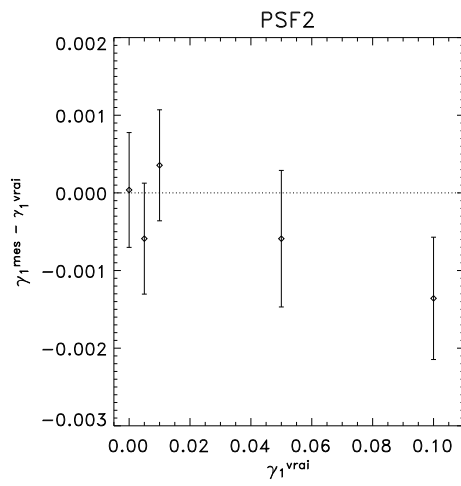
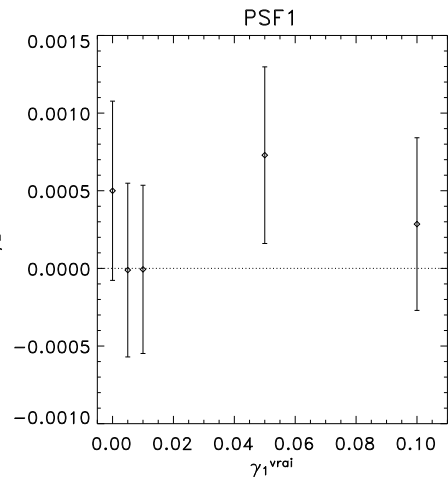
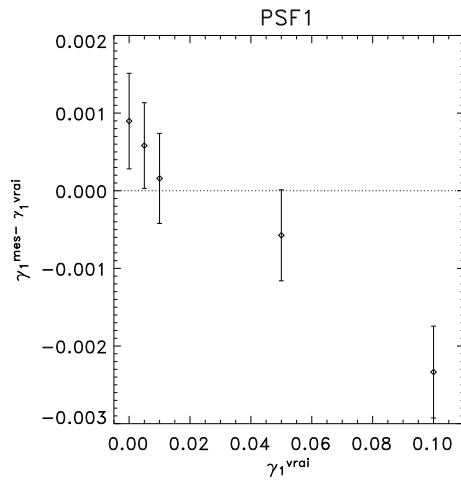
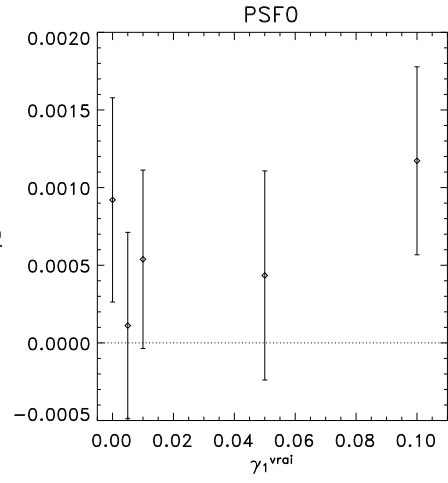
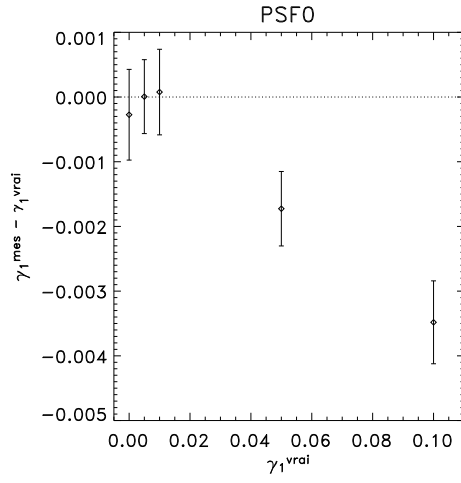


FIG. 8.3 – *Les courbes montrent les biais obtenus avec la méthode II pour la PSF0 en utilisant des filtres gaussiens de taille $r_{fix} = m * r_g^*$ où m vaut respectivement 0.6, 0.8, 0.9, 1.0, 1.2. Excepté le cas $m = 0.6$, où le filtre résultant est trop petit par rapport à la taille des objets, les autres choix comportent un biais équivalent. Le biais minimal est obtenu pour une taille $r_{fix} > 0.8r_g^*$.*



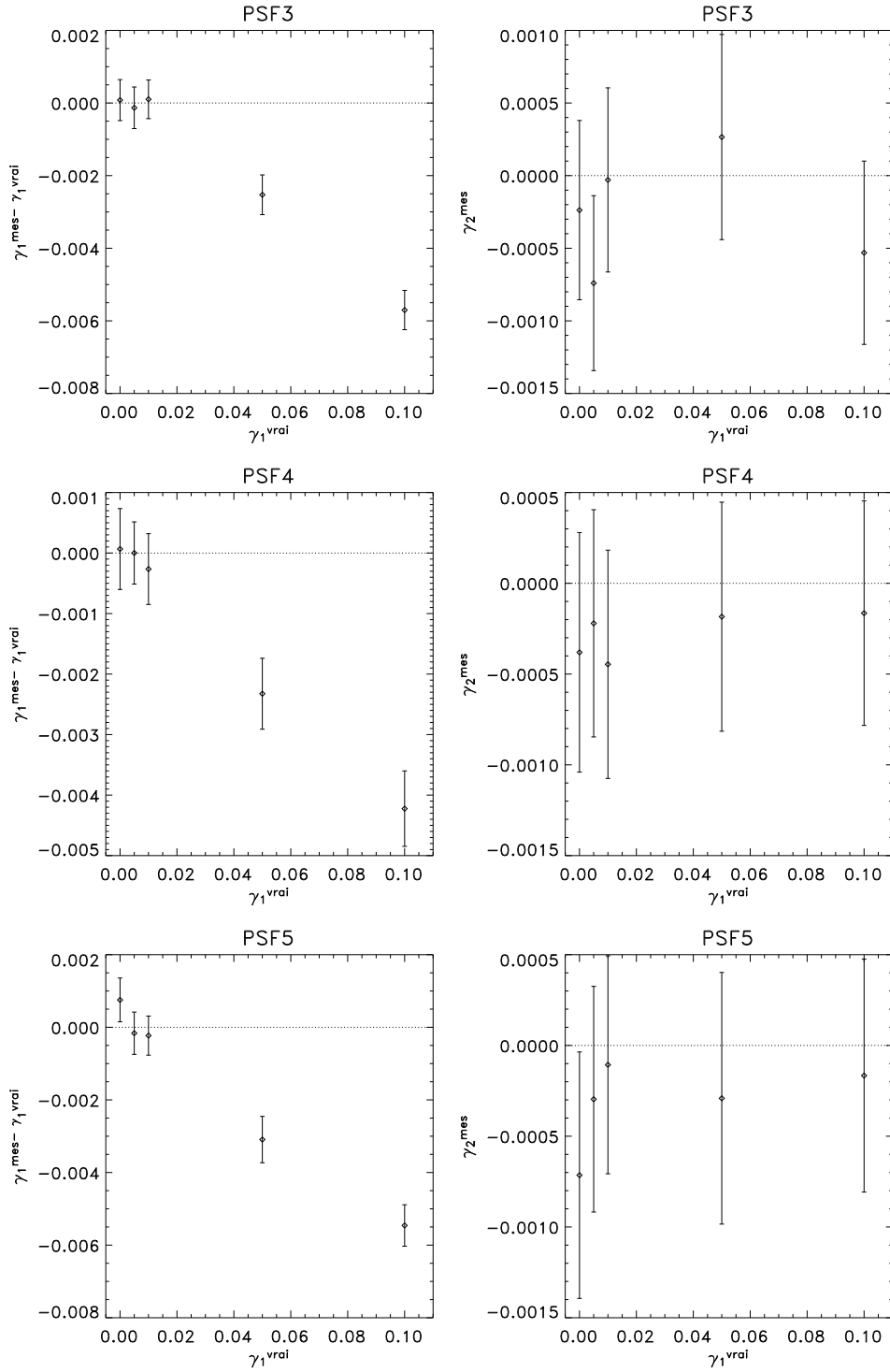


FIG. 8.4 – *Biais affectant les composantes γ_1 et γ_2 mesurées avec la méthode III pour chaque PSF des simulations STEP1*

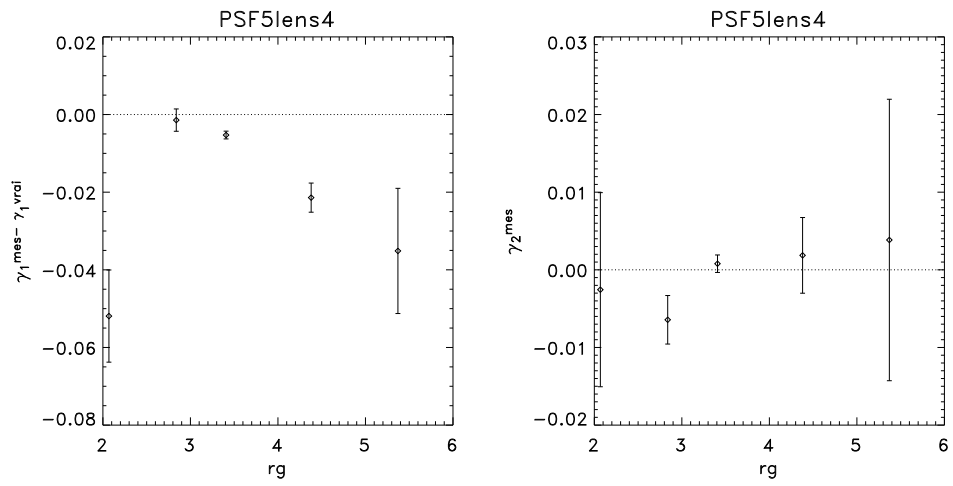


FIG. 8.5 – Dépendance du biais de la taille de l'objet r_g . Comme dans la plupart des méthodes KSB, le cisaillement a tendance à être sous-estimé pour des objets de taille beaucoup plus grande que celle des étoiles. La mesure du cisaillement pour le premier point est aussi sous-estimée. Cela montre que l'inclusion d'objets dont la taille est comparable à celle des étoiles peut aussi biaiser le signal. Ces objets sont exclus dans les courbes de la figure 8.4.

8.2 Biais de calibration dans le CFHTLS-Deep

La méthode I est celle que j'ai utilisée pour analyser les données CFHTLS-Deep. Comme on vient de le constater, elle comporte un biais qui est de l'ordre de 20%. Ce résultat est effectivement confirmé par la figure 8.6 qui compare l'ellipticité mesurée par Henk Hoekstra et moi sur le même échantillon de galaxies. À priori un biais de ce genre devrait donner un biais pouvant atteindre 40% dans l'estimation de la fonction de corrélation à deux points du cisaillement⁵, pourtant les résultats concernant la fonction de corrélation $\xi(\theta)$ (voir figure 8.7) révèlent un désaccord bien plus faible. La variance top-hat (voir figure 8.8) semble être sous-estimée à grande échelle, mais l'effet est tout de même plus faible que ce que on attendrait d'après le biais représenté sur la figure 8.6.

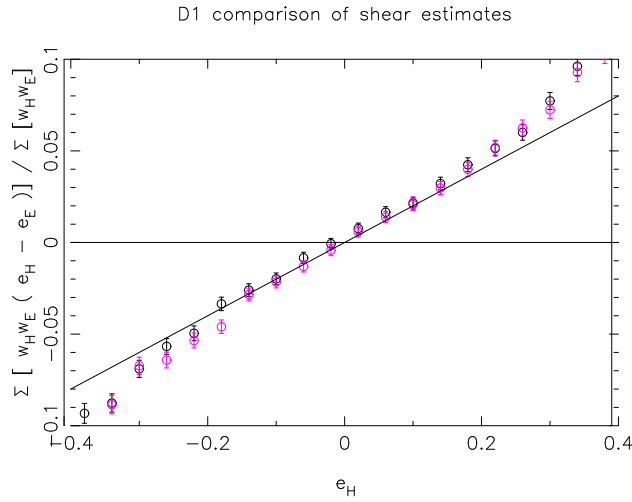


FIG. 8.6 – *Comparaison entre les composantes de l'ellipticité mesurées par Henk Hoekstra e_H et moi e_E dans le champ D1. Le plot montre la différence $\sum[w_H w_E (e_H - e_E)] / \sum[w_H w_E]$ pour chaque composante de l'ellipticité mesurée, en fonction de l'ellipticité mesurée par Henk Hoekstra. Les points noirs sont relatifs à la composante e_1 , ceux en magenta à la composante e_2 . Les poids respectifs w_H et w_E sont pris en compte. Figure fournie par Catherine Heymans.*

Le faible désaccord entre les statistiques à deux points visibles dans les figures 8.7 et 8.8 valide les contraintes cosmologiques provenant de l'analyse du jeu de données T0001 du CFHTLS-Deep. En effet, la variance cosmique qui affecte les paramètres cosmologiques est encore trop élevée pour que l'effet du biais montré à grande échelle sur la figure 8.8 soit significatif. Malgré cela, plusieurs points devraient être éclaircis. L'analyse des simulations STEP2 a révélé un biais significatif dans ma chaîne d'analyse, biais confirmé par l'analyse des simulations STEP1 et pour la comparaison des catalogues de cisaillement fournis par Henk Hoekstra par le CFHTLS-Deep. Mais heureusement, l'analyse des simulations STEP1 a permis une amélioration de ma procédure et le biais

5. Une approche du genre $\gamma_{mes} = \gamma_{vrai} + \alpha\gamma_{vrai}$ donne une fonction de corrélation à deux points mesurée au premier ordre ($\alpha = 20\% \ll 1$) par $\xi_{mes} = \xi_{vrai} + 2\alpha\xi_{vrai}$. C'est à dire sous-estimée d'une quantité égale au double du biais mesuré sur l'ellipticité.

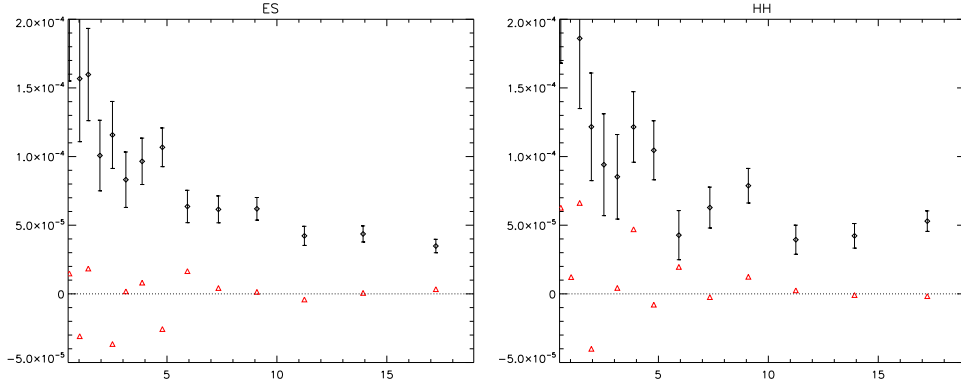


FIG. 8.7 – Fonction de corrélation du cisaillement pour les champs $D1$, $D3$ et $D4$ mesurée respectivement par moi (panneau de gauche) et par Henk Hoekstra (panneau de droite). Les modes E sont représentés en noir, les modes B en rouge. Pour plus de clarté les barres d'erreur sur les modes B n'ont pas été ajoutées. Figures fournies par Ludovic van Waerbeke et Henk Hoekstra

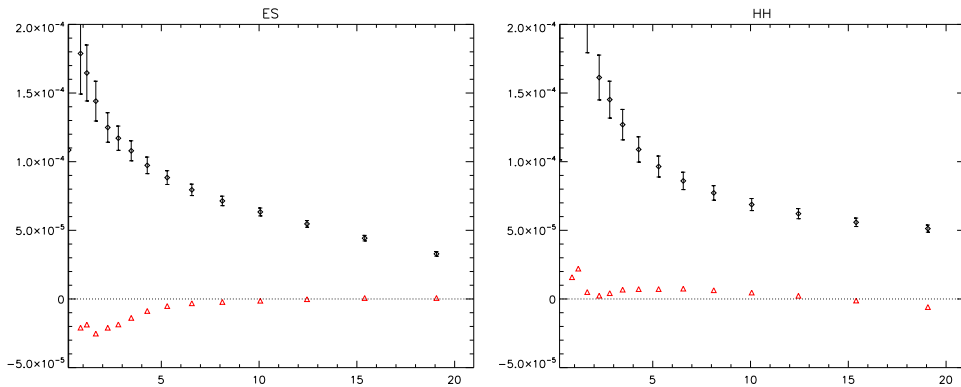


FIG. 8.8 – Même chose que 8.7 pour la variance top-hat. Figures fournies par Ludovic van Waerbeke et Henk Hoekstra.

est maintenant réduit à quelques pour-cents. Des validations ultérieures, fondées sur l'analyse des images fournies par STEP2 devraient confirmer la tendance et éventuellement suggérer d'autres améliorations. Le désaccord entre les statistiques à deux points demande aussi d'être ultérieurement étudié, car il suggère que le biais soit non pas une constante multiplicative, mais plutôt un champ, ce qui peut expliquer la variation de ce biais en fonction de l'échelle angulaire. Par ailleurs, l'analyse comparative des données de la première release CFHTLS Wide avait montré que les résultats sur les statistiques à deux points obtenus avec la chaîne d'analyse utilisée à Paris étaient en très bon accord avec ceux obtenus par Henk Hoekstra [Hoekstra et al. 2006]. Cela laisse penser que le

biais soit une fonction non banale des paramètres des galaxies tels que la taille, le rapport signal sur bruit ou encore la luminosité. Des ultérieures vérifications sont en cours.

Chapitre 9

Conclusions et perspectives

Le travail décrit dans cette thèse montre comment exploiter les mesures de cisaillement gravitationnel effectuées à partir de grands relevés de galaxies pour contraindre les paramètres cosmologiques. On y discute spécifiquement l'analyse des statistiques à deux points qui, pour le moment, sont les seules qui disposent d'un rapport signal-sur-bruit suffisant pour permettre une interprétation cosmologique des données (voir tableau 9.1). Les aspects concernant la mesure et la nature du signal, les sources de bruit et les biais systématiques sont abordés.

TAB. 9.1 – *Contraintes récentes sur la valeur de σ_8 pour $\Omega_m = 0.3$, obtenues par différents relevés. Entre parenthèses j'indique les auteurs respectifs des travaux.*

Nom du relevé	valeur σ_8
GEMS (Heymans et al. 2005)	0.68 ± 0.12
RCS (Hoekstra et al. 2002b)	$0.86^{+0.19}_{-0.13}$
VIRMOS-Descart (van Waerbeke et al. 2005)	0.83 ± 0.07
CFHTLS-Deep (Semboloni et al. 2006a)	0.94 ± 0.15
CFHTLS-Wide (Hoekstra et al. 2006)	0.85 ± 0.06

Sur le plan de l'interprétation cosmologique des données, je me suis concentrée sur l'exploitation scientifique du relevé CFHTLS Deep et Wide. L'ensemble des résultats concernant $\Omega_m - \sigma_8$ et le paramètre w de l'équation d'état de l'énergie sombre, dans le contexte d'un univers plat dont l'indice du spectre initial des fluctuations de la matière noire, fournissent des contraintes nouvelles. Pour w elles sont même parmi les toutes premières contraintes sur les propriétés de l'énergie sombre par le cisaillement gravitationnel cosmologique. Ces résultats concordent avec ceux provenant des autres approches, notamment ceux de WMAP.

La publication des nouveaux résultats de WMAP-3 a montré qu'il existait un décalage de 1.5σ entre les prédictions du CFHTLS Deep + Wide (et aussi avec les données de Virmos-Descart et le RCS) et WMAP3+LSS. Cet écart est plutôt une différence qu'il est difficile de déclarer significative à ce stade. Cependant, nous avons étudié quelles pourraient être les sources de désaccord, en supposant que le problème provienne du cisaillement cosmologique. Au cours de ce travail, j'ai

largement souligné que c'est seulement en perfectionnant les méthodes d'extraction du signal et en contrôlant les possibles sources de biais qu'il sera possible d'exploiter au mieux le potentiel des données des relevés de nouvelle génération.

Il est certain que parmi les points critiques figure la mesure du cisaillement lui-même et les problèmes de correction de la PSF. Mais les effets tels que la corrélation entre les ellipticités intrinsèques des galaxies et dans la corrélation croisée des ellipticités des galaxies d'arrière plan avec celles des lentilles d'avant plan doivent aussi être pris en compte dans l'erreur et dans la matrice de covariance pour l'analyse cosmologique des statistiques d'ordre deux. L'étude des effets non gaussiens sur l'estimation de la variance cosmique, montre indiscutablement que les contraintes provenant des relevés couvrant des petites surfaces du ciel, tels que GEMS et COSMOS, peuvent être altérées. Ne pas prendre en compte ces effets non-gaussiens conduit aussi à une sous-estimation des erreurs. Enfin, van Waerbeke et al. 2006 ont récemment montré que les effets de sous-échantillonnage de la distribution en redshift des galaxies peuvent conduire à une sous-estimation de la variance sur la distribution des sources et à une imprécision significative sur leur redshift moyen. L'indécision sur la détermination de la distribution en redshift des sources limite la précision avec laquelle on peut contraindre les paramètres cosmologiques. La distribution des sources est donc aussi affectée par la variance cosmique et son impact dépend de la dimension angulaire et de la profondeur du relevé ainsi que de la façon dont est échantillonné le redshift des sources jusqu'à la limite de complétude des données. Dans le cas de relevés comme les CFHTLS, Virmos-Descart ou RCS, la calibration sur des relevés spectroscopiques (ou multibande) de petite taille engendre donc une source d'erreur supplémentaire. L'erreur attendue est de l'ordre de 20% pour un relevé comme le CFHTLS qui utilise le HDF North+South comme calibrateur de sa distribution en redshift. Cette imprécision a pu être réduite à un 2% environ en utilisant les redshifts photométriques mesurés sur le relevé CFHTLS-Deep et calibrés sur les redshifts spectroscopiques du VVDS (Ilbert et al. 2006). Dans le but d'obtenir des contraintes plus fiables il est donc logique de reprendre les données provenant des relevés de première génération (Virmos-Descart, RCS par exemple) et de les ré-analyser conjointement avec le CFHTLS.

Une ré-analyse conjointe de plusieurs relevés (Benjamin et al. en prep.), avec les corrections dues à la non gaussianité de la matrice variance-covariance et l'utilisation des nouveaux redshifts CFHTLS-Deep, montre que les désaccords entre les résultats fournis par WMAP et les contraintes provenant du cisaillement cosmologique s'estompent. Pour le moment, nous n'avons que des résultats partiels et préliminaires car nous travaillons en ce moment même sur cette analyse. Ils montrent cependant clairement que l'analyse conjointe des relevés VIRMOS-Descart, CFHTLS-Wide et CFHTLS-Deep donne des contraintes dans le plan $[\sigma_8, \Omega_m]$ compatibles avec celles de WMAP-3 (figure 9.3). Les contraintes sur σ_8 pour chaque relevé sont montrés dans la table 9.2. Les valeurs mesurées pour les relevés Virmos-Descart, CFHTLS-Wide et CFHTLS-Deep sont légèrement différentes à cause de la différente distribution en redshift des sources utilisée pour la calibration. Cela est particulièrement visible dans le cas du relevé CFHTLS-Wide, dont le redshift moyen estimé en utilisant les champs HDF North et South est $\bar{z} = 0.76$, tandis que celui obtenu en utilisant les champs CFHTLS Deep est $\bar{z} = 1.00$. La figure 9.2 montre cette différence.

La variance relative à chaque relevé est finalement plus petite que celle trouvée auparavant. Cela est dû au fait que ces résultats sont obtenus avec une valeur fixée de la pente du spectre $\Gamma = 0.21$.

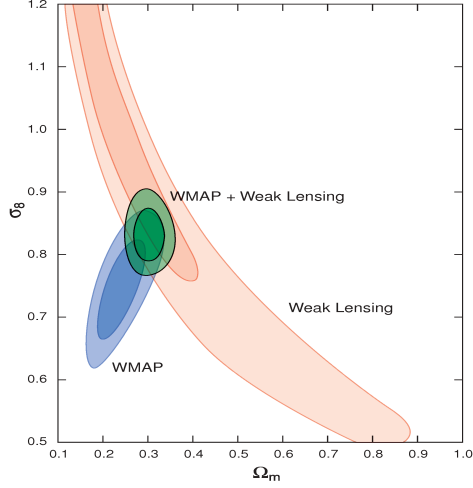


FIG. 9.1 – Comparaison entre les contraintes dans le plan $[\Omega_m, \sigma_8]$ obtenues par analyse des données WMAP (différents niveaux de bleu) et celles données par l’analyse du cisaillement gravitationnel dans le relevé CFHTLS Deep+Wide (différents niveau d’orange). Les intervalles de confiance pour les données conjointes CFHTLS+WMAP sont aussi montrés (différents niveaux de vert). Les contours délimitent les régions à 68% et 95% de confiance. D’après Spergel et al. 2006.

TAB. 9.2 – Contraintes sur la valeur de σ_8 pour $\Omega_m = 0.3$, obtenues sur les relevés de galaxies utilisés dans le figure 9.3. La valeur σ_8^p précédemment obtenue est comparée avec σ_8^n obtenue avec la nouvelle analyse qui utilise les redshifts photométriques publiés par Ilbert et al. 2006 pour calibrer de la distribution des sources. La valeur σ_8^{NG} est celle déterminée en ajoutant la correction non-gaussienne de la covariance à petite échelle.

Nom du relevé	σ_8^p	σ_8^n	σ_8^{NG}
VIRMOS-Descart (van Waerbeke et al. 2005)	0.83 ± 0.07	0.89 ± 0.05	0.86 ± 0.06
CFHTLS-Deep (Semboloni et al. 2006a) ^a	0.94 ± 0.15	0.96 ± 0.07	1.04 ± 0.09
CFHTLS-Wide (Hoekstra et al. 2006)	0.85 ± 0.06	0.75 ± 0.04	0.73 ± 0.07

^a Les données photométriques du catalogue publié par Ilbert et al. 2006 s’arrêtent à une magnitude égale a 25 dans la bande i' . Par conséquent les valeurs σ_8^n et σ_8^{NG} sont calculées dans un intervalle de magnitude légèrement différent de celui utilisé par Semboloni et al. 2006a

D’autres relevés devraient être ajoutés à cette analyse conjointe, comme le RCS [Hoekstra et al. 2002b] et GaBoDS [Hetttersheid et al. 2006], et la variance cosmique sur la distribution en redshift, qui devrait être de l’ordre de 2% pour un relevé de la taille du CFHTLS-Deep [van Waerbeke et al 2006], devra être incluse dans la matrice de covariance.

Pour conclure, aujourd’hui on a une vision beaucoup plus claire des obstacles qui devraient être surmontés pour établir des contraintes de haute précision avec le cisaillement cosmologique. Au cours des dernières années on a commencé à étudier tous ces biais sous plusieurs aspects,

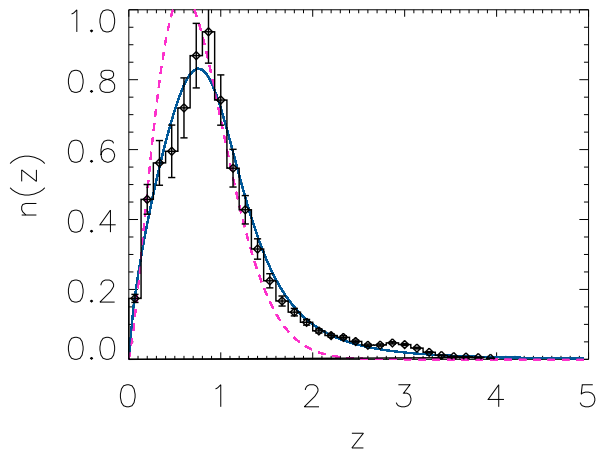


FIG. 9.2 – Comparaison entre la distribution en redshift du CFHTLS-Wide obtenue en utilisant les redshift photométriques du CFHTLS-Deep publiés par Ilbert et al. 2006 (courbe continue en bleu) et celle précédemment utilisée par Hoekstra et al. 2005 (courbe pointillée en magenta). L’histogramme montre la distribution en redshift mesurée dans le CFHTLS-Deep pour des objets avec $21.5 < i' < 24.5$. Les barres d’erreur comprennent l’erreur statistique et l’erreur du 2% due à la variance cosmique estimée selon van Waerbeke et al. 2006.

le but étant de comprendre leurs origines, de les corriger ou les réduire en partie et d’en tenir compte rigoureusement dans le calcul de l’erreur et de la matrice de covariance. Dans le futur il est prioritaire de continuer à intégrer ces méthodes de correction des biais qui ont déjà été suggérés dans l’analyse du cisaillement gravitationnel.

Pendant que les techniques de mesure du cisaillement s’affinent, des échantillons de plus en plus riches s’accumulent et des nouvelles voies d’exploration et d’analyse s’ouvrent, avec l’implication de nouvelles problématiques. La tomographie, dont j’ai montré en cette thèse la faisabilité, représente sans doute un outil parfaitement adapté pour reconstruire l’histoire du spectre de puissance des fluctuations. Mais elle ne pourra être exploitée dans toutes ses possibilités que si l’indétermination sur les propriétés de la distribution en redshift est réduite à moins de 10%. C’est faisable techniquement, mais c’est un objectif qui n’est pas encore atteint pour des galaxies faibles auxquelles on s’intéresse majoritairement dans l’analyse du cisaillement cosmologique dans les relevés les plus profonds.

Les relevés des nouvelle génération ouvrent aussi la possibilité de mesurer les statistiques d’ordre supérieur, telles que la dissymétrie (skewness) ou les fonctions de corrélation à trois points. Il a été démontré [Bernardeau et al. 2003] que la mesure de la skewness ne dépend que de la valeur de Ω_m et pas de σ_8 . Sa mesure permettrait donc de briser la dégénérescence typique des mesures des statistiques à deux points. Une tentative sur les données de Virgos-Descart [Pen et al 2003], et

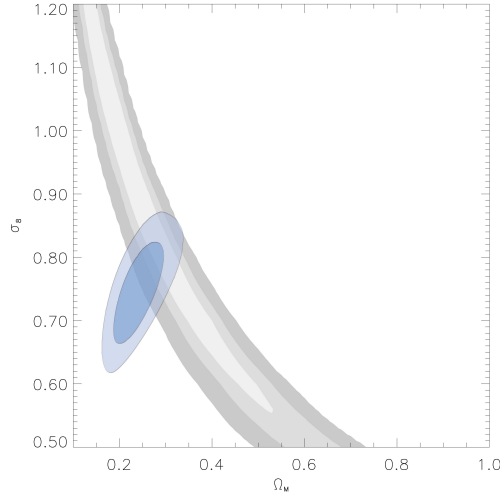


FIG. 9.3 – *Comparaison entre les contraintes dans le plan $[\Omega_m, \sigma_8]$ obtenues par l'analyse des données WMAP (différents niveaux de bleu) et celles données par l'analyse du cisaillement gravitationnel des relevés CFHTLS Deep, CFHTLS Wide et Virmos-Descart (différents niveaux de gris). Les contours délimitent les régions avec 68%, 95% et 99% de confiance. Figure fournie par Jonathan Benjamin.*

une mesure de $\langle M_{ap}^3 \rangle$ [Jarvis et al. 2004] sur les données du relevé CTIO a montré que ce type de mesure est aujourd'hui faisable. Cependant, c'est encore aujourd'hui une mesure techniquement difficile et incertaine. D'une part, il est nécessaire de construire un estimateur des statistiques à trois points par un algorithme rapide et qui soit capable de gérer des données bruitées, en présence des masques. D'autre part, il est nécessaire d'étudier à l'aide des simulations le bruit statistique et celui dû à la variance cosmique qui affectent cette mesure. Bien que prometteur, c'est un champ d'analyse qui est encore dans une phase de défrichage et nous sommes donc encore loin de pouvoir exploiter cosmologiquement ce signal.

Annexe A

Propagation des rayons

L'équation de propagation des rayons dans un espace-temps avec une géométrie non Euclidienne 3.1 a été formulée par Sachs en 1961. Dans cette annexe je résume les grandes lignes de ce calcul en adoptant la notation utilisée par Schneider et al. 1992 et Seitz et al. 1994. Le calcul est fait dans le cadre de la théorie de la relativité générale. On adopte une signature métrique $(+, -, -, -)$ et une notation tensorielle en accord avec Weinberg 1972. Le développement du formalisme mathématique nécessaire qui est utilisé ici peut être trouvé dans Weinberg 1972, Misner, Thorne et Wheeler 1970 et Landau & Lifshitz 1980.

A.1 Définition des rayons de lumière

Les équations des Maxwell pour un champ électromagnétique qui se propage dans le vide sont données par:

$$\begin{aligned}\vec{\nabla} \times \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t} \\ \vec{\nabla} \times \mathbf{H} &= \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}\end{aligned}\tag{A.1}$$

et

$$\begin{aligned}\vec{\nabla} \cdot \mathbf{E} &= 0 \\ \vec{\nabla} \cdot \mathbf{H} &= 0\end{aligned}\tag{A.2}$$

On définit un potentiel quadri-dimensionnel $A^i = (\phi, \mathbf{A})$ tel que:

$$\begin{aligned}\mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} - \vec{\nabla} \phi \\ \mathbf{H} &= \vec{\nabla} \times \mathbf{A}\end{aligned}\tag{A.3}$$

On peut aussi définir un tenseur F_{ij} tel que:

$$F_{\alpha\beta} = \frac{\partial A_\alpha}{\partial x^\beta} - \frac{\partial A_\beta}{\partial x^\alpha} .\tag{A.4}$$

Le premier couple d'équations de Maxwell A.1 donne:

$$\frac{\partial F_{\alpha\beta}}{\partial x^\gamma} + \frac{\partial F_{\beta\gamma}}{\partial x^\alpha} + \frac{\partial F_{\gamma\alpha}}{\partial x^\beta} = 0 \quad (\text{A.5})$$

et le second couple A.2:

$$\frac{\partial F^{\alpha\beta}}{\partial x^\alpha} = 0 \quad (\text{A.6})$$

La solution des équations A.1 et A.2 est alors:

$$A^\alpha = \mathcal{R}e(A_0^\alpha \exp(iS)) \quad (\text{A.7})$$

A_0^α représente l'amplitude complexe tandis que $S = -k^\alpha x_\alpha$ représente la phase réelle. Le 4-vecteur k^α est nul, *i.e.* $k^\alpha k_\alpha = 0$.

Dans un espace-temps quelconque les équations de Maxwell A.1 et A.2 doivent être écrites dans leur forme covariante:

$$F_{[\alpha\beta;\gamma]} = 0 \quad \text{et} \quad F_{;\alpha}^\alpha = 0 \quad (\text{A.8})$$

où ; exprime la dérivée covariante¹. Dans le cas d'un espace-temps générique les équations de Maxwell n'ont pas une solution d'ondes planes A.7. Malgré cela, dans la plupart des cas, et notamment dans le cas du cisaillement cosmologique, on peut utiliser l'approximation d'ondes courtes (WBS) qui permet de justifier l'hypothèse d'ondes localement planes, dans le cas où la longueur d'onde $\lambda \equiv \frac{1}{k^0} = \frac{c}{w_0}$ est petite par rapport au rayon de courbure L de l'espace-temps. La solution des équations de Maxwell A.8 peut être ré-écrite comme un développement en λ/L :

$$A^\alpha = \mathcal{R}e[(a^\alpha + \epsilon b^\alpha + \epsilon^2 c^\alpha \dots) \exp(iS/\epsilon)] \quad (\text{A.9})$$

De cette façon on peut avoir une onde d'amplitude presque constante et de phase variable. Le facteur $\epsilon \propto \lambda/L$ tient compte de l'ordre d'approximation. Il peut être, après développement, compté directement dans l'amplitude et la phase.

On appelle *rayons de lumière* les lignes caractérisées par le vecteur $k_\alpha \equiv S_{;\alpha}$:

$$\frac{dx^\alpha}{d\lambda} = k^\alpha \quad (\text{A.10})$$

où λ est un paramètre affine qui paramétrise la courbe. Chaque paramètre $\lambda' = a\lambda + b$ avec a, b constants le long de la courbe peut être utilisé pour définir les rayons de lumière. En utilisant l'auto orthogonalité de k^α on obtient $k_{;\beta}^\alpha k^\beta = 0$ et enfin l'équation des géodésiques des rayons de lumière:

$$\frac{d^2 x^\alpha}{d\lambda^2} + \Gamma_{\beta\gamma}^\alpha \frac{dx^\beta}{d\lambda} \frac{dx^\gamma}{d\lambda} = 0 \quad (\text{A.11})$$

1. On rappelle la définition de la dérivée covariante pour un 4-vecteur f^α : $f_{;\beta}^\alpha = \frac{\partial f^\alpha}{\partial x^\beta} + \Gamma_{\beta\gamma}^\alpha f^\gamma$ où Γ représente le symbole de Christoffel.

A.2 Déformation d'un faisceau optique

On s'intéresse maintenant à la propagation d'un ensemble de géodésiques de lumière, caractérisé par une fonction de phase définie. On caractérise les géodésiques de cet ensemble par $x^\alpha = f^\alpha(\lambda, y^i)$ où λ est le paramètre affine et y^i est une série de paramètres qui identifie les propriétés de chaque rayon. Selon la définition des géodésiques de type lumière, le vecteur d'onde k^α associé aux géodésiques est:

$$k^\alpha = \frac{\partial x^\alpha}{\partial \lambda} = \frac{\partial f^\alpha}{\partial \lambda} \quad (\text{A.12})$$

On considère maintenant un rayon de référence γ . Les vecteurs

$$\delta x^\alpha = \frac{\partial f^\alpha}{\partial y^i} \delta y^i \quad (\text{A.13})$$

connectent le rayon de référence γ aux rayons infiniment proches identifiés par un changement de paramètres $y^i + \delta y^i$. Pour un changement de paramètre affine $\lambda \rightarrow \lambda' = \lambda + h(y^i)$:

$$\begin{aligned} y^i &= g^i(y'^i) \\ x^\alpha &= f^\alpha(\lambda + h(y^i), g^i(y'^i)) = f'^\alpha(\lambda', y'^i) \end{aligned} \quad (\text{A.14})$$

et grâce à l'équation A.12 on trouve:

$$\delta x'^\alpha = \frac{\partial f'^\alpha}{\partial y'^i} \frac{\partial g^i}{\partial y'^j} \delta y'^j + \frac{\partial f'^\alpha}{\partial \lambda} \frac{\partial h}{\partial y'^j} \delta y'^j = \frac{\partial f'^\alpha}{\partial y'^i} \delta y'^i + k'^\alpha \delta h = \delta x^\alpha + k^\alpha \delta h \quad (\text{A.15})$$

On veut connaître comment les vecteurs de connexion δx^α changent le long du rayon γ (*i.e.* en fonction de λ). Par définition cette variation est donnée par la dérivée covariante des vecteurs δx^α par rapport au vecteur k^α tangent à la courbe

$$\frac{D\delta x^\alpha}{\partial \lambda} \equiv \delta \dot{x}^\alpha \equiv \delta x^\alpha_{;\beta} k^\beta \quad (\text{A.16})$$

On peut démontrer que:

$$\delta x^\alpha_{;\beta} k^\beta = k^\alpha_{;\beta} \delta x^\beta \quad (\text{A.17})$$

i.e. la dérivée de Lie du vecteur δx^α est nulle le long de γ . En utilisant l'équation A.13 on peut ré-écrire A.17:

$$\begin{aligned} \delta x^\alpha_{;\beta} k^\beta &= \left(\frac{\partial \delta x^\alpha}{\partial x^\beta} + \Gamma^\alpha_{\beta\gamma} \delta x^\gamma \right) k^\beta = \frac{\partial^2 f^\alpha}{\partial x^\beta \partial y^i} \delta y^i \frac{\partial f^\beta}{\partial \lambda} + \Gamma^\alpha_{\beta\gamma} \delta x^\gamma k^\beta \\ &= \frac{\partial^2 f^\alpha}{\partial \lambda \partial x^\alpha} \frac{\partial f^\beta}{\partial y^i} \delta y^i + \Gamma^\alpha_{\beta\gamma} k^\gamma \delta x^\beta = \left(\frac{\partial k^\alpha}{\partial x^\beta} + \Gamma^\alpha_{\beta\gamma} k^\gamma \right) \delta x^\beta = k^\alpha_{;\beta} \delta x^\beta \end{aligned} \quad (\text{A.18})$$

Mis à part des termes contenant les symboles de Christoffel qui sont identiques dans l'expression de droite et dans celle de gauche, il reste à démontrer

$$\delta y^i \left[\frac{\partial f^\beta}{\partial y^i} \frac{\partial}{\partial x^\beta} \left(\frac{\partial f^\alpha}{\partial \lambda} \right) \right] = \delta y^i \left[\frac{\partial f^\beta}{\partial \lambda} \frac{\partial}{\partial x^\beta} \left(\frac{\partial f^\alpha}{\partial y^i} \right) \right] \quad (\text{A.19})$$

pour prouver l'égalité A.17.

Pour cela, on peut utiliser les équivalences:

$$\frac{\partial f^\alpha}{\partial \lambda} \frac{\partial}{\partial x^\beta} \equiv \frac{\partial}{\partial \lambda} \quad \text{et} \quad \frac{\partial f^\alpha}{\partial y^i} \frac{\partial}{\partial x^\beta} \equiv \frac{\partial}{\partial y^i} \quad (\text{A.20})$$

qui démontrent l'égalité A.19 et donc l'égalité A.17.

Les dérivées par rapport à λ et y^i sont les dérivées directionnelles le long des courbes coordonnées et elles sont donc définies comme le produit entre le vecteur tangent à la courbe et le gradient de la fonction. Enfin, l'équation de transport pour δx^α le long de γ est:

$$\delta \dot{x}^\alpha = k_{;\beta}^\alpha \delta x^\beta \quad (\text{A.21})$$

On peut identifier parmi les rayons proches ceux qui ont la même phase S lesquels doivent satisfaire $S_{;i} \delta x^i = 0$. À partir de la relation A.15 on trouve que pour ces rayons

$$k_\alpha \delta x^\alpha = 0 \quad (\text{A.22})$$

quelle que soit la paramétrisation. De plus, pour la famille de géodésiques γ , γ_1 et γ_2 avec $S = \text{const}$, on trouve, grâce aux équations A.15 et A.22, que le produit $\delta x_1^\alpha \delta x_{2\alpha}$ est invariant par changement de paramétrisation. La collection des rayons de lumière caractérisée par une phase S constante est appelée *faisceau*.

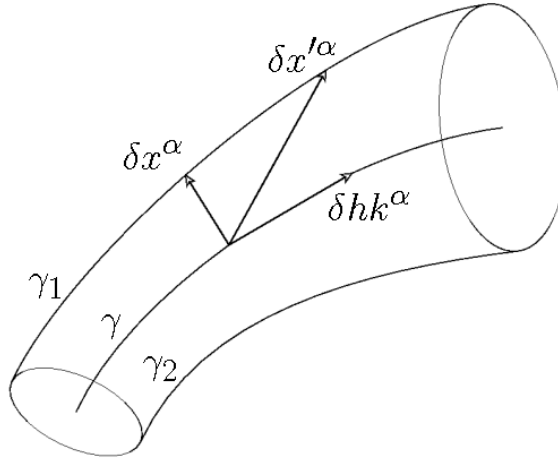


FIG. A.1 – Représentation graphique de la signification des vecteurs de connexion δx^α . Un changement de paramétrisation comporte le changement du vecteur de connexion.

On peut donner une interprétation physique à l'équation A.21 ou bien à A.22. Pour cela, on se place dans le système de référence d'un observateur qui croise le faisceau à un événement P^α . Soient τ le temps propre de l'observateur et $\frac{\partial dx^\alpha}{d\tau} = u^\alpha$ sa vitesse. En accord avec l'équation A.15 on peut toujours paramétriser les géodésiques des rayons de façon à avoir $u^\alpha \delta x_\alpha = 0$ en P^α . Ainsi, dans le référentiel de l'observateur, δx_α est un vecteur tridimensionnel dont la composante temporelle est nulle et donc il connecte tous les rayons qui croisent l'observateur à un instant donné. Pour les

rayons du faisceau (*i.e.* $S = \text{const}$) l'équation A.22 devient alors $\delta \mathbf{x} \mathbf{k} = 0$. Les rayons d'un faisceau croisent l'observateur à un même instant et leur séparation représente une surface bidimensionnelle perpendiculaire au vecteur d'onde \mathbf{k} .

Les produits $\delta \mathbf{x}_1 \delta \mathbf{x}_2$ caractérisent donc la forme et la taille du faisceau vu par l'observateur. On peut trouver leur modification pendant la propagation du faisceau grâce à A.21. On décrit cette surface bidimensionnelle par deux quadri-vecteurs (E_1^α, E_2^α) qui définissent une base orthonormale dans l'espace perpendiculaire à k^α relatif au rayon de référence γ . On demande aussi que les vecteurs (E_1^α, E_2^α) soient parallèlement transportés le long de γ (*i.e.* $E_{i;\beta}^\alpha k^\beta = 0$). On peut donc décomposer le vecteur δx^α :

$$\delta x^\alpha = -\xi_1 E_1^\alpha - \xi_2 E_2^\alpha + \xi_0 k^\alpha \quad (\text{A.23})$$

où les composantes ξ_1, ξ_2 représentent la composante transverse. Les relations trouvées plus haut, ne dépendant pas de sa vitesse u^α de l'observateur, on choisit donc par simplicité un observateur dont la vitesse u^α est parallèle au vecteur γ . Pour chaque événement P le long de γ on peut calculer ξ_1 et ξ_2 sur le plan bidimensionnel paramétrisé par (E_1^α, E_2^α) . En particulier l'équation de transport A.21 dans ce référentiel devient:

$$\dot{\xi}_i = k^i_{;j} E_i^j \xi_j = \mathcal{S}_{ij} \xi_j \quad (\text{A.24})$$

Dans le dernier paragraphe on a ainsi défini la matrice de déformation optique qui représente la variation de la séparation entre les rayons le long du paramètre λ projeté sur le plan

$$\mathcal{S}_{ij} = k^{\alpha}_{;\beta} E_i^\alpha E_j^\beta \quad (\text{A.25})$$

A.3 Matrice de déformation optique

L'expression de la matrice \mathcal{S} peut être explicitée grâce à la définition des scalaires optiques [Sachs 1961]. Ces sont des quantités qui définissent les propriétés physiques observables du faisceau et dont on peut étudier le transport pendant la propagation du rayon.

On définit le vecteur unitaire complexe $\epsilon^\alpha \equiv E_1^\alpha + i E_2^\alpha$ appartenant au sous-espace bi-dimensionnel décrit par l'écran (E_1^α, E_2^α) . A chaque événement le long de γ le tenseur

$$P_\beta^\alpha \equiv \delta_\beta^\alpha + k^\alpha k_\beta - k^\alpha u_\beta - u^\alpha k_\beta = \frac{1}{2}(\epsilon^\alpha \epsilon^{*\beta} + \epsilon^{*\alpha} \epsilon_\beta) \quad (\text{A.26})$$

représente le projecteur dans le plan (E_1^α, E_2^α) . En utilisant la définition du projecteur P_β^α , le transport de k^α peut être ré-écrit comme:

$$k_{\alpha;\beta} = \sigma_{\alpha\beta} + \theta P_{\alpha,\beta} + P_{(\alpha} k_{\beta)} \quad (\text{A.27})$$

avec:

$$\theta = \frac{1}{2} k^\alpha_{;\alpha} \quad \sigma = \frac{1}{2} k_{\alpha;\beta} \epsilon^{*\alpha} \epsilon^{*\beta} \quad \sigma_{\alpha\beta} = \mathcal{R}e(\sigma \epsilon_\alpha \epsilon_\beta) \quad (\text{A.28})$$

où θ et σ sont les scalaires optiques introduits par Sachs en 1961. La matrice \mathcal{S}_{ij} (eq.A.25) devient:

$$\mathcal{S} = \begin{pmatrix} \theta - \mathcal{R}e(\sigma) & \mathcal{I}m(\sigma) \\ \mathcal{I}m(\sigma) & \theta + \mathcal{R}e(\sigma) \end{pmatrix} \quad (\text{A.29})$$

Si on différentie l'équation A.24 on obtient l'équation 3.1 du chapitre 3:

$$\frac{d^2 \boldsymbol{\xi}}{d\lambda^2} = \mathcal{T} \boldsymbol{\xi}(\lambda) \quad (\text{A.30})$$

avec:

$$\mathcal{T} = \frac{d\mathcal{S}}{d\lambda} + \mathcal{S}^2 = \begin{pmatrix} \mathcal{R} - \mathcal{R}e\mathcal{F} & \mathcal{I}m\mathcal{F} \\ \mathcal{I}m\mathcal{F} & \mathcal{R} + \mathcal{R}e\mathcal{F} \end{pmatrix} \quad (\text{A.31})$$

Les expressions de \mathcal{F} et \mathcal{R} dépendent de la métrique. En effet on peut démontrer que:

$$\mathcal{R} = \frac{d\theta}{d\lambda} + \theta^2 + |\sigma|^2 = -\frac{1}{2} R_{\alpha\beta} k^\alpha k^\beta \quad (\text{A.32})$$

$$\mathcal{F} = \frac{d\sigma}{d\lambda} + 2\sigma\theta = -\frac{1}{2} C_{\alpha\beta\gamma\delta} \epsilon^{\alpha*} k^\beta \epsilon^{\gamma*} k^\delta \quad (\text{A.33})$$

où $R_{\alpha\beta}$ et $C_{\alpha\beta\gamma\delta}$ représentent le tenseur de Ricci et le tenseur de Weyl. L'introduction des scalaires optiques facilite l'interprétation physique de l'équation de déformation du rayon le long de sa trajectoire. En effet, la variation de la distance $l \equiv \delta x^\alpha \delta x_\beta$ entre deux rayons proches est:

$$\frac{dl}{d\lambda} = l(\theta + \sigma_{\alpha\beta} \epsilon^\alpha \epsilon^\beta) \quad (\text{A.34})$$

où le scalaire θ représente alors une expansion isotrope du faisceau, tandis que $\sigma_{\alpha\beta}$ représente une distorsion.

A.4 Déflexion des rayons dans la métrique FRW

On explicite maintenant la relation A.30 dans le cas d'un espace-temps avec une métrique FRW. On peut ré-écrire la métrique FRW 2.6 sous la forme:

$$ds^2 = c^2 dt^2 + a(t)^2 \left[\frac{1}{\sqrt{1 - Kr^2}} dr^2 + r^2 (d\theta^2 + \sin^2(\theta) d\phi^2) \right] \quad (\text{A.35})$$

Tout d'abord, on remarque qu'à cause de l'isotropie \mathcal{F} est nul. On n'a donc qu'à calculer le scalaire \mathcal{R} . Pour cela on considère un observateur de vitesse u_o^α à l'événement O où le faisceau croise l'observateur. λ augmente en remontant le temps et à l'événement O on a $\tilde{k}^\alpha u_\alpha|_O = -1$. \tilde{k}^α est le vecteur d'onde observé normalisé par la fréquence angulaire observée w/c (i.e. $k^\alpha = -(w/c)\tilde{k}^\alpha$). Soit w_0 la fréquence de la lumière à son émission, alors $w = (1+z)w_0$ à cause du décalage introduit par l'expansion de l'Univers. Donc finalement $\tilde{k}^0 = -(1+z)$. Le quadri-vecteur \tilde{k}^α doit être de type lumière et donc $\tilde{k}^\alpha \tilde{k}_\alpha = 0$. Cette condition avec $\tilde{k}^0(\lambda) = -(1+z)$ et l'orthogonalité par rapport à l'écran $E_1^\alpha E_2^\alpha$ donne:

$$\tilde{k}^\alpha = (1+z)(-1, 1/\sqrt{-g_{11}}, 0, 0) \quad (\text{A.36})$$

où g_{11} représente le coefficient de la métrique associé à la variable r et donc $g_{11} = -a(t)^2 \sqrt{1 - Kr^2}$. Les composantes du tenseur de Ricci dans la métrique FRW dans la forme A.35 sont données par:

$$R_{00} = 3 \frac{\ddot{a}}{a} \quad \text{composante temporelle} \quad (\text{A.37})$$

$$R_{ii} = -\frac{1}{c^2} \left[\frac{\ddot{a}}{a} + 2 \frac{\dot{a}^2}{a} + 2 \frac{k^2}{c} \right] \quad \text{avec } i = 1, 2, 3 \quad (\text{A.38})$$

Le scalaire $\mathcal{R}(z)$ est:

$$\mathcal{R}(z) = -\frac{1}{c^2}8\pi G\rho(1+z)^2 = -\frac{1}{a^2c^2}8\pi G\rho = -\left[\frac{\ddot{a}}{2a^4} + \frac{kc^2}{2a^4}\right] \quad (\text{A.39})$$

où on a utilisé la seconde équation de Friedmann pour obtenir la dernière égalité (deuxième équation du système 2.7 du chapitre 2 dans le cas d'une constante cosmologique nulle).

On peut maintenant utiliser l'expression de $\mathcal{R}(z)$ pour ré-écrire l'équation de déformation du faisceau A.30 en fonction du redshift. C'est une représentation avec une variable naturelle dans l'étude de la propagation des faisceaux à travers l'Univers. Mais de façon équivalente, dans cette thèse, on a préféré paramétriser les équations de propagation des faisceaux en utilisant le paramètre w qui est la composante radiale de la métrique FRW dans la forme 2.6. Pour ré-écrire l'équation A.30 on a donc besoin de relier la variable w à λ . Il est aussi utile de trouver une expression en fonction des distances comobiles $\mathbf{x} = a^{-1}\boldsymbol{\xi}$. L'équation de propagation du faisceau en coordonnées comobiles est:

$$\frac{d^2}{d\lambda^2}(a^{-1}\mathbf{x}(\lambda)) = \mathcal{R}(\lambda)\mathcal{I}a^{-1}\mathbf{x}(\lambda) \quad (\text{A.40})$$

où \mathcal{I} représente la matrice identité.

Le changement de variables est possible en remarquant que $dx^\alpha = \widetilde{k}^\alpha d\lambda$ et donc $da = -\frac{\dot{a}}{ca}d\lambda$. De plus on s'intéresse aux géodésiques de type lumière $cdt = -adw$ et donc enfin $d\lambda = \frac{a}{a^2}dw$. Grâce à ces relations, après un peu d'algèbre on peut écrire:

$$\frac{d^2}{dw}\mathbf{x}(w) = -K\mathbf{x}(w) \quad (\text{A.41})$$

ce qui correspond à la relation 3.2 du chapitre 3.

Annexe B

Angle de déflexion en champ faible

L'équation des géodésiques A.11 peut être obtenue à partir du principe d'action; le long d'une géodésique le Lagrangien

$$\mathcal{L}(x^\alpha, \dot{x}^\alpha) \equiv \frac{1}{2} g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta \quad (\text{B.1})$$

est un extremum, c'est à dire que la variation:

$$\delta \int_\gamma \mathcal{L}(x^\alpha, \dot{x}^\alpha) d\lambda = 0 \quad (\text{B.2})$$

par rapport aux paramètres $(x^\alpha, \dot{x}^\alpha)$ est nulle. $\dot{x}^\alpha = \frac{dx^\alpha}{d\lambda}$ est la dérivée par rapport au paramètre affine λ

Pour une géodésique de type lumière $\mathcal{L} = 0$. Le principe de Fermat implique que le temps t qu'un rayon émis par une source S emploie pour rejoindre un observateur O est stationnaire, *i.e.* les géodésiques de type lumière sont caractérisées par $\delta t = 0$.

On considère maintenant une métrique du type 3.3:

$$ds^2 = \left(1 + \frac{2\Phi}{c^2}\right) dt^2 - \left(1 - \frac{2\Phi}{c^2}\right) d\mathbf{x} \quad (\text{B.3})$$

Pour un rayon de lumière $ds^2 = 0$ et donc:

$$dt \sim \pm \frac{1}{c} \left(1 - \frac{2\Phi}{c^2}\right) dl \quad (\text{B.4})$$

où $dl = \sqrt{d\mathbf{x} d\mathbf{x}}$. Le signe change selon le choix, si le cône de lumière est orienté vers le passé ou le futur. Ici, on peut choisir le signe positif sans perdre en généralité, et donc aussi:

$$ct \sim \int_\gamma \left(1 - \frac{2\Phi}{c^2}\right) dl \quad (\text{B.5})$$

On peut choisir la coordonnée z comme celle qui connecte l'observateur et la source. Si on applique le principe de Fermat on obtient donc:

$$c\delta t = 0 = \int \left(\frac{\partial n}{\partial x} \delta x + \frac{\partial n}{\partial y} \delta y \right) \left[\left(\frac{dx}{dz} \right)^2 + \left(\frac{dy}{dz} \right)^2 + 1 \right]^{1/2} dz \quad (\text{B.6})$$

$$+ \int n \left[\left(\frac{dx}{dz} \right)^2 + \left(\frac{dy}{dz} \right)^2 + 1 \right]^{-1/2} \left(\frac{dx}{dz} \frac{d\delta x}{dz} + \frac{dy}{dz} \frac{d\delta y}{dz} \right) dz \quad (\text{B.7})$$

où on a défini $n \equiv 1 - \frac{2\Phi}{c^2}$. Le deuxième terme de B.6 peut être intégré par parties pour chaque composante x, y . En utilisant le fait que $\delta x = \delta y = 0$ soit en S soit en O on trouve enfin que le principe de Fermat est respecté si et seulement si:

$$\frac{\partial n}{\partial x} \left[\left(\frac{dx}{dz} \right)^2 + \left(\frac{dy}{dz} \right)^2 + 1 \right]^{1/2} = \frac{d}{dz} \left[n \frac{dx}{dz} \left[\left(\frac{dx}{dz} \right)^2 + \left(\frac{dy}{dz} \right)^2 + 1 \right]^{-1/2} \right] \quad (\text{B.8})$$

pour ce qui concerne la variable x . Pour la variable y on obtient la même expression. On peut donc continuer le calcul en ne considérant que la variable x et appliquer ensuite le résultat à la variable y . Enfin, on peut ré-écrire l'équation B.8 comme:

$$\frac{\partial n}{\partial x} = \frac{d \left[n \frac{dx}{dl} \right]}{dl} \quad (\text{B.9})$$

et de même pour la variable y . Si on met ensemble les deux résultats on trouve l'équation 3.4.

Annexe C

Équation de Limber

La relation entre le spectre bi-dimensionnel $P_\kappa(l)$ et le spectre tri-dimensionnel $P_\delta(k)$ 3.19 est obtenue grâce à l'équation de Limber. Dans cet annexe je décris les principales étapes qui permettent de déduire cette relation.

Reprenons la définition de $\kappa(\boldsymbol{\theta})$ en fonction du champ de fluctuations $\delta[f_K(w)\boldsymbol{\theta},w]$

$$\kappa(\boldsymbol{\theta}) = \frac{3H_0^2\Omega_m}{2c^2} \int_0^{w_H} dw \bar{W}(w) f_K(w) \frac{\delta[f_K(w)\boldsymbol{\theta},w]}{a(w)} \quad (\text{C.1})$$

la fonction de corrélation à deux points du champ de masse projeté le long de la ligne de visée est donc donnée par:

$$\langle \kappa(\boldsymbol{\theta})\kappa(\boldsymbol{\theta}') \rangle = \int_0^{w_H} dw \mathcal{F}(w) \int_0^{w_H} dw' \mathcal{F}(w') \langle \delta[f_K(w)\boldsymbol{\theta},w] \delta[f_K(w')\boldsymbol{\theta}',w'] \rangle \quad (\text{C.2})$$

où on a défini:

$$\mathcal{F}(w) \equiv \frac{3H_0^2\Omega_m}{2c^2} \bar{W}(w) \frac{f_K(w)}{a(w)} \quad (\text{C.3})$$

On peut supposer qu'au-delà d'une certaine longueur $L_{max} \ll w_H$ il n'y a plus de cohérence entre les fluctuations, c'est à dire que l'intégrale est limité aux valeurs de w' telles que $|w' - w| < L_{max}$. Cette approximation est en accord avec le fait que $P(k) \propto k$ pour $k \rightarrow 0$. Cela est équivalent à dire que dans l'espace réel, la fonction de corrélation du champs de fluctuations $\xi_\delta(r) \propto 1/r$ (voir def. 2.3) pour $r \rightarrow \infty$. Pour des distances plus petites que L_{max} on fait l'hypothèse que $f_k(w') \approx f_k(w)$; de même $\mathcal{F}(w') \approx \mathcal{F}(w)$. En faisant le changement de variable $\Delta w = w' - w$ l'équation C.2 devient alors:

$$\langle \kappa(\boldsymbol{\theta})\kappa(\boldsymbol{\theta}') \rangle = \int_0^{w_H} dw \mathcal{F}^2(w) \int d(\Delta w) \langle \delta[f_K(w)\boldsymbol{\theta},w] \delta[f_K(w)\boldsymbol{\theta}',w + \Delta w] \rangle \quad (\text{C.4})$$

Pour pouvoir remplacer le spectre de fluctuations dans l'équation C.4 on a besoin de formuler une autre hypothèse. On sait en effet que le champ de fluctuations δ dépend du temps (voir def. 2). Cette dépendance est cachée ici dans la dépendance de la variable angulaire w . En effet, pour un rayon de lumière dans la métrique FRW on a $c|dt| = adw$ ce qui veut dire que a priori les valeurs

$\delta[f_K(w)\boldsymbol{\theta},w]$ et $\delta[f_K(w)\boldsymbol{\theta}',w + \Delta w]$ ne sont pas calculées pour la même époque cosmologique. Cependant, on peut supposer que le champ de fluctuations ne change pas pendant le temps de propagation du faisceau à travers la longueur maximale de cohérence L_{max} et donc on a:

$$\langle \kappa(\boldsymbol{\theta})\kappa(\boldsymbol{\theta}') \rangle = \int_0^{w_H} dw \mathcal{F}^2(w) \int d(\Delta w) \xi_\delta(\sqrt{(f_K(w)^2(|\boldsymbol{\theta} - \boldsymbol{\theta}'|)^2 + \Delta w^2)}, w)$$

Si on appelle $\Delta\theta = |\boldsymbol{\theta} - \boldsymbol{\theta}'|$, on peut ré-écrire cette équation comme la fonction de corrélation à deux points du champ κ , $\xi_\kappa(\Delta\theta)$:

$$\xi_\kappa(\Delta\theta) = \int_0^{w_H} dw \mathcal{F}^2(w) \int d(\Delta w) \xi_\delta(\sqrt{(f_K(w)^2(\Delta\theta)^2 + \Delta w^2)}, w) \quad (\text{C.5})$$

On peut utiliser cette équation pour trouver la relation entre les spectres de puissance P_δ et celui de P_κ . On écrit pour cela l'équation C.2 dans l'espace de Fourier:

$$\begin{aligned} \langle \kappa(\boldsymbol{\theta})\kappa(\boldsymbol{\theta}') \rangle &= \int_0^{w_H} dw \mathcal{F}^2(w) \int dw' \int \frac{d^3 k}{(2\pi)^3} \int \frac{d^3 k'}{(2\pi)^3} \\ &\langle \tilde{\delta}[\mathbf{k},w] \tilde{\delta}^*[\mathbf{k}',w'] \rangle > e^{-i(f_K(w)\mathbf{k}_\perp \boldsymbol{\theta} + k_3 w)} e^{i(f_K(w')\mathbf{k}'_\perp \boldsymbol{\theta}' + k'_3 w')} \end{aligned} \quad (\text{C.6})$$

où \mathbf{k}_\perp représente la partie de \mathbf{k} orthogonale à la ligne de visée, *i.e.* appartenant au plan qui contient $\boldsymbol{\theta}$. Avec les mêmes approximations utilisées pour passer de l'équation C.2 à l'équation C.5:

$$\begin{aligned} \langle \kappa(\boldsymbol{\theta})\kappa(\boldsymbol{\theta}') \rangle &= \int_0^{w_H} dw \mathcal{F}^2(w) \int dw' \int \frac{d^3 k}{(2\pi)^3} P_\delta(k,w) \\ &e^{-i(f_K(w)\mathbf{k}_\perp(\boldsymbol{\theta} - \boldsymbol{\theta}'))} e^{-ik_3 w} e^{ik_3 w'} \\ &= \int_0^{w_H} dw \mathcal{F}^2(w) \int \frac{d^2 k}{(2\pi)^2} P_\delta(\mathbf{k}_\perp, w) e^{-i(f_K(w)\mathbf{k}_\perp(\boldsymbol{\theta} - \boldsymbol{\theta}'))} \end{aligned} \quad (\text{C.7})$$

où on a utilisé la définition du spectre de puissance $\langle \delta(\mathbf{k})\delta^*(\mathbf{k}') \rangle = (2\pi)^3 P_\delta(k)\delta_{(k=k')}$ entre la première et la seconde ligne ci-dessus et:

$$\int dw e^{i w k_3} = 2\pi \delta_{D(k_3=0)} \quad (\text{C.8})$$

ce qui impose $k_3 = 0$. Donc, seule la composante des vecteur d'onde dans le plan orthogonal à la ligne de visée contribue à l'intégrale, soit $P_\delta(k) = P_\delta(k_\perp)$.

Selon la définition de la transformée de Fourier et du spectre de puissance on a donc :

$$P_\kappa(l) = \int_0^{w_H} dw \frac{1}{f_K(w)^2} \mathcal{F}^2(w) \int d^2 l P_\delta\left(\frac{l}{f_K(w)}, w\right) \quad (\text{C.9})$$

où on a introduit le vecteur bi-dimensionnel $\mathbf{l} = \mathbf{k}_\perp f_K(w)$.

Bibliographie

- [Bacon, Réfrégier & Ellis 2000] Bacon, D., Réfrégier, A., Ellis, R., 2000, MNRAS 318, 625
- [Bacon et al. 2003] Bacon, D., Massey, R. J., Réfrégier, A., Ellis, R., 2003, MNRAS 344, 673
- [Bartelmann & Schneider 2001] Bartelmann, M., Schneider, P., 2001, Phys. Rep. 340, 294.
- [Benabed & Bernardeau 2001] Benabed, K., Bernardeau, F., 2001, Phys. Rev. D 64, 083501
- [Benabed & van Waerbeke 2004] Benabed, K., van Waerbeke, L., 2004, Phys. Rev. D 70, 123515
- [Bernardeau et al. 1997] Bernardeau, F., Van Waerbeke, L., Mellier, Y., 1997, A&A 322, 1
- [Bernardeau et al. 2002] Bernardeau, F., Colombi S., Gaztañaga, E., Scoccimarro R., 2002, PhR,367, 1 L., Mellier, Y., 1997, A&A 322, 1
- [Bernardeau et al. 2003] Bernardeau, F., Van Waerbeke, L., Mellier, Y., 2003, A&A 397, 405
- [Bernstein & Jarvis 2002] Bernstein, G. M., Jarvis, M., 2002, AJ,123,583
- [Bertin & Arnouts 1996] Bertin, E., Arnouts, S., 1996, A&A, 117, 393
- [Bernstein & Jarvis 2002] Bernstein, G., Jarvis, M., 2002, AJ, 123, 583
- [Bolzonella, Miralles & Pelló 2000] Bolzonella, M., Miralles, J.-M., Pelló, R., 2000, A&A 363, 476-492
- [Boulade et al. 2003] Boulade, O., Charlot, X., Abbon, P., Aune, S., Borgeaud, P., Carton, P.-H., Carty, M., Da Costa, J. Deschamps, H., Desforge, D., Eppele, D., Gallais, P., Gosset, L., Granelli, R., Gros, M., de kat, J., Loiseau, D., Ritou, J.L., Strazynski, P., Vignal, N., Vigroux, L., 2003, SPIE 4841, 72
- [Bouchet et al. 1992] Bouchet, F., Juskiwics, R., Colombi, S., Pellat, R., 1992, ApJ, 394, L5
- [Brodwin et al. 2006] Brodwin, M., Lilly, S.J., Porciani, C., McCracken, H.J., Le Fèvre, O., Foucaud, S., Crampton, D., Mellier, Y., 2006, ApJS, 162, 20
- [Bruzual & Charlot 1993] Bruzual, G., Charlot, S. 1993. ApJ 405, 538
- [Carrol, Press & Turner 1992] Carrol, S. M., Press, W., H., Turner, E. L., 1992 ARA&A, 30, 499
- [Cooray & Huterer 1999] Cooray, A. R., Huterer, D. 1999, ApJ 513, 95
- [Cooray & Hu] ,Cooray A., Hu W, 2001 ApJ, 554, 66
- [Crittenden et al. 2001] Crittenden, R., Natarajan, P., Pen, U., Theuns, T., 2001, ApJ 559, 552
- [Crittenden et al. 2002] Crittenden, R., Natarajan, P., Pen, U., Theuns, T., 2002, ApJ 568, 20
- [Erben et al. 2001] Erben, T., Van Waerbeke, L., Bertin, E., Mellier, Y., Schneider, P., 2001, MNRAS 366, 717
- [Esposito-Farèse 2001] , Esposito-Farèse G., Polarski D., 2001, Phys. Rev. D 63, 063504

- [Fahlman et al. 1994] Fahlman, G., Kaiser, N., Squires, G., Woods, D., 1994, ApJ 437, 56 q
- [Fernández-Soto et al. 1999] Fernández-Soto, A., Lanzetta, K. M., Yahil, A., 1999, ApJ, 513, 34
- [Hamana et al. 2002] Hamana, T., Colombi, S., Thyon, A., Devriendt, J. E. G. T., Mellier, Y., Bernardeau, F., 2002, MNRAS, 330, 365
- [Hamilton et al. 1991] Hamilton, A. J. S., Kumar, P., Lu, E., Matthews, A., 1991, ApJ, 374, L1-L4
- [Heymans 2003] Ph. D. Thesis
- [Heymans & Heavens 2003] Heymans, C., Heavens, A., 2003, MNRAS 339, 711
- [Heymans et al. 2005] Heymans, C., Brown, M. L., Barden, M., Caldwell, J. A. R., Jahnke, K., Rix, H.-W. et al., 2005, MNRAS 361, 160.
- [Heymans et al. 2006a] Heymans, C., Van Waerbeke, L., Bacon, D., Bergé, J., Bernstein, G., Bertin, E., et al., 2006, MNRAS, 368, 1323
- [Heymans et al. 2006b] Heymans, C., White, M., Heavens, A., Vale, C., van Waerbeke, L. 2006, MNRAS, 371, 750
- [Hirata & Seljak 2003] Hirata, C., Seljak, U., 2003, MNRAS 343, 459
- [Hirata & Seljak 2004] Hirata, C., Seljak, U., 2004, PhRvD 70, 6
- [Hettterscheidt et al. 2006] Hettterscheidt, M., Simon, P., Schirmer, M., Hildebrandt H., Schrabback, T., Erben, T., Schneider, P., 2006, pre-print astro-ph/0606571
- [Hoekstra et al. 2000] Hoekstra, H., Franx, M., Kuijken, K., 2000, ApJ 532, 88
- [Hoekstra et al. 2002a] Hoekstra, H., Yee, H.K.C., Gladders, M.D., Barrientos, L.F., Hall, P.B., Infante, L., 2002, ApJ 572, 55
- [Hoekstra et al. 2002b] Hoekstra, H., Yee, H.K.C., Gladders, 2002, ApJ, 577, 595
- [Hoekstra 2004] Hoekstra, H., 2004, MNRAS 347, 1337.
- [Hoekstra et al. 2004] Hoekstra, H., Yee, H. K. C., Gladders M.D., 2004, ApJ, 606, 67
- [Hoekstra et al. 2005] Hoekstra, H., Hsieh B. C., Yee, H. K. C., Lin, H., Gladders M.D., 2005, ApJ, 635, 73
- [Hoekstra et al. 2006] Hoekstra H., Mellier Y., van Waerbeke L., Semboloni E., Fu L., Hudson M.J., Parker L.C., Tereno I., Benabed K., 2006, ApJ, 647, 116
- [Ilbert et al. 2006] Ilbert O. et al., 2006, A& A, 457, 841
- [Jain & Seljak 1997] Jain, B., Seljak, U., 1997, ApJ 484, 560
- [Jarvis et al. 2004] Jarvis, M., Jain, B. Bernstein, G., 2004, 352, 338
- [Jarvis et al. 2005] Jarvis, M., Jain, B. Bernstein, G., Dolney, D., 2005, preprint astro-ph/0502243
- [Kaiser et al. 1994] Kaiser, N., Squires, G., Fahlman, G., Woods, D., 1994, in Durret, F., Mazure, A., Tran Thanh Van, J., Eds., Clusters of Galaxies. Editions Frontières. Gif-sur-Yvette, p. 269.
- [Kaiser et al. 1995] Kaiser, N., Squires, G., Broadhurst, T., 1995, ApJ 449, 460
- [King & Schneider 2002] King, L., Schneider, P., 2002, A&A, 398, 23
- [King 2005] King, L., 2005, A&A, 441, 47
- [Kuijken 2006] Kuijken, C., 2006, A&A, 456, 827
- [Landau & Lifshitz 1980] Landau, L. D., Lifshitz, E. M., 1980, The classical theory of fields, Butterworth-Heinemann Editions.

- [Lahav & Suto 2004] Lahav, O., Suto, Y. 2004. *Living Reviews in Relativity*. Published by the Max Planck Institute for Gravitational Physics.
- [Le Fèvre et al. 2004] e Fèvre et al., 2004, A&A, 428, 1043L
- [Le Fèvre et al. 2005] Le Fèvre, O., Vettolani, P., Garilli, B., Tresse, L., Bottini, B., et al. 2005, A&A 439, 845
- [Linder & Jenkins 2003] Linder, E. V., Jenkins, A. 2003, MNRAS 346, 583
- [Luppino & Kaiser 1997] Luppino, G., Kaiser, N., 1997, ApJ 475, 20
- [Madgwick et al. 2003] Madgwick et al, 2003, ApJ, 599, 997
- [Massey et al. 2004] Massey, R., Refregier, A., Conselice, C. J., David J., Bacon, D., 2004, MNRAS, 348, 214
- [Massey & Refregier 2005] Massey, R., Refregier, A., 2005, MNRAS, 363, 197
- [Massey et al. 2006] Massey, R. et al., 2006, submitted MNRAS, preprint: astro-ph/0608643
- [McCracken et al. 2003] McCracken, H., Radovich, M., Bertin, E., Mellier, Y., Dantel-Fort, M., Le Fèvre, O., Cuillandre, J.-C., Gwyn, S., Foucaud, S., Zamorani, G., 2003, A&A 410, 17
- [Mellier 1999] Mellier, Y., 1999. ARAA 37, 127.
- [Misner Thorne et Wheeler] Misner, C. W., Thorne, K. S., Wheeler J. A., 1970, Gravitation, Freeman Editions
- [Ménard & Dalal 2005] Ménard B., Dalal N., 2005, MNRAS, 358, 101
- [Munshi & Coles 2003] Munshi, D. , Coles, P., 2003, MNRAS, 338, 846
- [Munshi & Valageas 2005] Munshi, D. , Valageas, P., 2005, MNRAS, 360, 1401
- [Peacock & Dodds 1996] Peacock, J.A., Dodds, S.J., 1996, MNRAS 280, L9
- [Peebles 1974] Peebles, P.J.E., 1974, A&A, 32, 197
- [Pen, van Waerbeke & Mellier 2002] Pen, U.L., Van Waerbeke, L., Mellier, Y., 2002, ApJ 567, 31
- [Pen et al 2003] Pen, U.L., Zhang, T., van Waerbeke, L., Mellier, Y., Zhang, P., Dubinski, J., 2003, Apj, 592, 664P
- [Perlmutter et al. 1999] Perlmutter, S., Aldering, G., Goldhaber, G., Knop, R. A., Nugent, P., Castro, P. G., Deustua, S., Fabbro, S., Goobar, A., Groom, D. E, et al., 1999, ApJ 517, 565
- [Pierre et al. 2006] ierre et al., 2006, pre print: astro-ph/0607170
- [Réfrégier 2003] Réfrégier, A., ARAA 41, 643
- [Riess et al. 1998] Riess, Adam G., Filippenko, Alexei V., Challis, Peter, Clocchiatti, Alejandro, Diercks, Alan, Garnavich, Peter M., Gilliland, Ron L., Hogan, Craig J., Jha, Saurabh, Kirshner, Robert P., et al., 1998, ApJ 116, 1009
- [Riess et al. 2004] Riess, Adam G., Strolger L. , Tonry J. , Casertano, S., Ferguson H.C. , Mobasher B., Challis P., Filippenko A.V. , Jha S. , Li W. , Chornock R. , Kirshner R.P. , Leibundgut B. , Dickinson M., Livio M., Giavalisco M., Steidel C. C. , Benítez T. , Tsvetanov, Z., 2004 ApJ 607, 687
- [Sachs 1961] Sachs, R. K. ,1961, Proc. R. Soc. A 264, 309
- [Salmon 1996] Salmon, J., 1996, ApJ, 460, 598
- [Schneider et al. 1992] Schneider, P., Ehlers J., Falco ,E. E., 1992, Gravitational Lenses, Springer
- [Schneider 1996] Schneider, P., 1996, MNRAS 283, 853

- [Schneider et al. 1998] Schneider, P., Van Waerbeke, L., Jain, B., Kruse, G., 1998, MNRAS 296, 873
- [Schneider et al. 2002a] Schneider, P., van Waerbeke, L., M., Mellier, Y., 2002, A& A 389, 729
- [Schneider et al. 2002b] Schneider, P., van Waerbeke, L., Kilbinger, M., Mellier, Y., 2002, A& A 396, 1
- [Scoccimarro et al. 1999] Scoccimarro R, Zaldarriaga M, Hui L., 1999, ApJ, 527, 1S
- [Scranton et al. 2005] Scranton R.,Brice M.,Richards, G. T.,Nichol R. T. ,Myers A. D., Jain B. ,Gray A.,Bartelmann M., Brunner R. J., Connolly A.J. , Gunn J. E.,Sheth R.K, Bahcall, A.,Brinkman J.,Loveday J., Schneider D. P. Thakar A. ,York D.G.,2005,ApJ 633,589S
- [Seitz et al. 1994] Seitz, S., Schneider, P., Ehlers, J., 1994 , Class. Quantum Grav., 11, 2345
- [Semboloni et al. 2006a] Semboloni, E., Mellier, Y. and van Waerbeke, L., Hoekstra, H., Tereno, I., Benabed, K., Gwyn , S. D. J., Fu, L. Hudson, M. J. Maoli, R., Parker, L. C.
- [Semboloni et al. 2006b] Semboloni, E., van Waerbeke L., Heymans, C. , Hamana, T. , Colombi, S., White M., Mellier, Y.
- [Smith, et al. 2003] Smith, R. E., Peacock, J. A., Jenkins, A., White, S. D. M., Frenk, C. S., Pearce, F. R., Thomas, P. A., Efstathiou, G., Couchman, H. M. P., 2003, MNRAS 341, 1311
- [Spergel et al. 2003] Spergel, D. N., Verde, L., Peiris, H. V., Komatsu, E., Nolta, M. R., Bennett, C. L., Halpern, M., Hinshaw, G., Jarosik, N., Kogut, A., Limon, M., Meyer, S. S., Page, L., Tucker, G. S., Weiland, J. L., Wollack, E., Wright, E. L., 2003, ApJS 148, 175 n, H. M. P., 2003, MNRAS, 341, 1311
- [Spergel et al. 2006] Spergel, D. N., R. Bean, O. Doré, M. R. Nolta, C. L. Bennett, G. Hinshaw, N. Jarosik, E. Komatsu, L. Page, H. V. Peiris, L. Verde, C. Barnes, M. Halpern, R. S. Hill, A. Kogut, M. Limon, S. S. Meyer, N. Odegard, G. S. Tucker, J. L. Weiland, E. Wollack, E. L. Wright 2006, astro-ph/0603449
- [Tegmark 2002] Tegmark, M., 2002, Phys. Rev. D, 66, 10
- [Van Waerbeke et al. 2000] Van Waerbeke, L., Mellier, Y., Erben, T., Cuillandre, J.-C., Bernardeau, F., Maoli, R., Bertin, E., McCracken, H., Le Fèvre, O., Fort, B., Dantel-Fort, M., Jain, B., Schneider, P., 2000, A&A 358, 30
- [Van Waerbeke et al. 2001] Van Waerbeke, L., Mellier, Y., Radovich, M., Bertin, E., Dantel-Fort, M., McCracken, H. J., Le Fèvre, O., Foucaud, S., Cuillandre, J.-C., Erben, T., Jain, B., Schneider, P., Bernardeau, F., Fort, B., 2001, A& A 374, 757
- [Van Waerbeke et al. 2002] Van Waerbeke, L., Mellier, Y., Pelló, R., Pen, U.-L., McCracken, H. J., Jain, B., 2002, A& A 393, 369
- [Van Waerbeke & Mellier 2003] Van Waerbeke, L., Mellier, Y., astro-ph/0305089, Lecture given at the Aussois winter school, France, January 2003
- [Van Waerbeke & Mellier 2005] Van Waerbeke, L., Mellier, Y., Proceedings of the IAU Symposium 225 "Cosmological Physics with Gravitational Lensing", Y. Mellier & G. Meylan eds. Cambridge University Press 2005
- [Van Waerbeke, Mellier & Hoekstra 2005] Van Waerbeke, L., Mellier, Y., Hoekstra, H., 2005, A&A 429, 75
- [van Waerbeke et al. 2006] , van Waerbeke, L., White, M., Hoekstra, H., Heymans, C., 2006, Astroparticle Physics, 26, 91

- [White 2002] White, M., 2002, Apjs,143, 241
- [White & Hu] White M., Hu W., 2000, ApJ, 537, 1
- [Weinberg 1972] einberg, S., 1972, Gravitation and Cosmology: principals and Applications of the General Theory of Relativity, Wiley Editions
- [Wittman et al. 2000] Wittman, D. M., Tyson, A. J., Kirkman, D., Dell'Antonio, I., Bernstein, G., 2000, Nature 405, 143

Cosmic Shear Analysis with CFHTLS Deep data[★]

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Abstract. We present the first cosmic shear measurements obtained from the T0001 release of the Canada-France-Hawaii Telescope Legacy Survey. The data set covers three uncorrelated patches (D1, D3 and D4) of one square degree each, observed in u^* , g' , r' , i' and z' bands, to a depth of $i' = 25.5$. The deep, multi-colour observations in these fields allow for several data-quality controls. The lensing signal is detected in both r' and i' bands and shows similar amplitude and slope in both filters. B -modes are found to be statistically zero at all scales. Using multi-colour information, we derived a photometric redshift for each galaxy and use this to separate the background source sample into low- z and high- z subsamples. A stronger shear signal is detected from the high- z subsample than from the low- z subsample, as expected from weak lensing tomography. While further work is needed to model the effects of errors in the photometric redshifts, this result suggests that it will be possible to obtain constraints on the growth of dark matter fluctuations with lensing wide field surveys. The combined Deep and Wide surveys give $\sigma_8 = 0.89 \pm 0.06$ assuming the Peacock & Dodds non-linear scheme (P&D), and $\sigma_8 = 0.86 \pm 0.05$ for the halo model and $\Omega_m = 0.3$. We assumed a Cold Dark Matter model with flat geometry and have marginalized over the systematics, the Hubble constant and redshift uncertainties. Using data from the Deep survey, the 1σ upper bound for w_0 , the constant equation of state parameter is $w_0 < -0.8$.

1. Introduction

Cosmological weak lensing, also called cosmic shear, can be used to probe the dark matter distribution in the universe. Weak lensing observations complement other probes such as CMB anisotropies (Spergel et al. 2003), type Ia supernovae (Riess et al. 2004, Riess et al. 1998, Perlmutter et al. 1999), and redshift surveys

(Lahav & Suto 2004). Weak lensing also has the advantage of being free of any assumption regarding the light versus matter distributions (Mellier 1999, Bartelmann & Schneider 2001, Van Waerbeke & Mellier 2003, Réfrégier 2003).

It has only recently been shown that cosmic shear measurement is technically feasible (Bacon, Réfrégier & Ellis 2000, Kaiser et al. 2000, Van Waerbeke et al. 2000, Wittman et al. 2000). Unfortunately, the deepest weak lensing survey has a sky coverage limited to less than one deg² and the widest to ~ 10 deg². Moreover, most surveys were performed in one colour only, and even rough redshift information was not available. These limitations restricted the use of weak lensing as a cosmological probe to a very small number of parameters. Early weak lensing surveys were primarily focused on the measurements of the normalization of the dark matter power spectrum, σ_8 , and the mass density parameter, Ω_m . The most recent cosmic

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[★] Based on observations obtained with MEGAPRIME/MEGACAM, a joint project of CFHT and CEA/DAPNIA, at the Canada-France-Hawaii Telescope (CFHT) which is operated by the National Research Council (NRC) of Canada, the Institut National des Sciences de l'Univers of the Centre National de la Recherche Scientifique (CNRS) of France, and the University of Hawaii. This work is based in part on data products produced at TERAPIX and the Canadian Astronomy Data Centre as part of the Canada-France-Hawaii Telescope Legacy Survey, a collaborative project of NRC and CNRS.

shear surveys reach a relative accuracy of about 10% on $\Omega_m \sigma_8^{0.5}$ (Mellier 1999, Bartelmann & Schneider 2001, Van Waerbeke & Mellier 2003, Réfrégier 2003), but the uncertainty on other parameters is still fairly large.

Second generation cosmic shear surveys are now under way and will provide the community with multi-colour data of excellent image quality, over a wide field of view. The Canada-France-Hawaii Telescope Legacy Survey (CFHTLS)¹, using the recently built MEGAPRIME/MEGACAM wide field camera, belongs to this generation. The *CFHTLS-Wide* survey (the core of the CFHTLS cosmic shear survey) will provide a large sky coverage of 170 deg.², and the deep four deg.² *CFHTLS-Deep* will provide shear information on smaller scales and as a function of lookback time, out to higher redshift than the *CFHTLS-Wide*.

Both surveys will ultimately consist of complete and homogeneous panchromatic data in u^*, g', r', i', z' . The data were taken between June 1st 2003 and July 22, 2004 as part of a preliminary survey to provide detailed quality assessments and propose technical or operational improvements, when necessary. The CFHTLS-Wide has the same depth as VIRMOS-Desart, but so far, the available data is only in one colour and covers ≈ 20 deg². A cosmic shear analysis with the wide data is performed in Hoekstra et al. 2005. The CFHTLS-Deep has been observed in all five filters, therefore photometric redshift are available, and will be used in this work. Moreover, the CFHTLS D1 Deep field is located in the VIRMOS-VLT Deep Survey (VVDS) F02 field which has several thousand of galaxy redshifts (Le Fèvre et al. 2005) and near infrared data (on a tiny area). A combination of large and small scales from the Wide and the Deep data will ultimately provide an excellent data set to probe the nature of dark energy in the universe (Cooray & Huterer 1999, Benabed & Bernardeau 2001, Linder & Jenkins 2003, Benabed & van Waerbeke 2004, Jarvis et al. 2005). In this work, we describe the first CFHTLS cosmic shear studies based on Deep data, and then combine the Wide and Deep data analysis to derive constraints on Ω_m and σ_8 .

The organization of the paper is as follows. In section 2 we introduce the notation and define the statistics we use. The data set is described in Section 3. In section 4 and 5 we present results and residual systematics and we discuss them. Conclusions and perspectives are outlined in Section 6.

2. Theoretical background

The theory of weak lensing has been previously been discussed in detail in the literature, including the physical motivations of various approximations (e.g. Bartelmann & Schneider 2001). Following Hoekstra 2004, Van Waerbeke et al. 2002,

Van Waerbeke, Mellier & Hoekstra 2005 we follow the notation of Schneider et al. 1998.

We introduce the power spectrum of the convergence κ as :

$$P_\kappa(k) = \frac{9}{4} \Omega_0^2 \int_0^{\chi_H} \frac{d\chi}{a^2(\chi)} P_{3D} \left(\frac{k}{f_K(\chi)}; \chi \right) \times \left[\int_\chi^{\chi_H} d\chi' n(\chi') \frac{f_K(\chi' - \chi)}{f_K(\chi')} \right]^2, \quad (1)$$

where $f_K(\chi)$ is the comoving angular diameter distance out to radial distance $\chi(z)$, and $n(\chi)$ is the redshift distribution of the sources. $P_{3D} \left(\frac{k}{f_K(\chi)}, \chi \right)$ is the 3-dimensional mass power spectrum, and κ is a 2-dimensional wave vector perpendicular to the line-of-sight.

Cosmic shear can be studied using three different 2-point statistics, which differ only by their filtering schemes. These various statistics have different wavelength sensitivities to the power spectrum and therefore the effect of systematics on each is different. This enables the comparison of multiple cross-checked solutions. Two-point statistics are measured as a function of scale θ_c , which could either be a galaxy pair separation or smoothing window radius. The relation between each two-point statistics and the power spectrum of the gravitational convergence (*i.e.* the projected dark matter power spectrum) can be expressed as follows:

– Top-hat variance:

$$\langle \gamma^2 \rangle_{\theta_c} = \frac{2}{\pi \theta_c^2} \int_0^\infty \frac{dk}{k} P_\kappa(k) [J_1(k\theta_c)]^2. \quad (2)$$

– Shear correlation function:

$$\langle \xi \rangle_{\theta_c} = \frac{1}{2\pi} \int_0^\infty dk k P_\kappa(k) J_0(k\theta_c). \quad (3)$$

– Aperture mass variance:

$$\langle M_{\text{ap}}^2 \rangle_{\theta_c} = \frac{288}{\pi \theta_c^4} \int_0^\infty \frac{dk}{k^3} P_\kappa(k) [J_4(k\theta_c)]^2, \quad (4)$$

with the aperture mass variance defined as:

$$M_{\text{ap}}(\theta_c) = \int_{\theta < \theta_c} d^2\theta \kappa(\theta) U(\theta), \quad (5)$$

where $U(\theta)$ is a compensated filter such as:

$$U(\theta) = \frac{9}{\pi \theta_c^2} \left(1 - \frac{\theta^2}{\theta_c^2} \right) \left(\frac{1}{3} - \frac{\theta^2}{\theta_c^2} \right). \quad (6)$$

M_{ap} can be expressed in terms of the tangential shear component inside a circle as follows (Kaiser et al. 1994, Schneider 1996) :

$$M_{\text{ap}}(\theta_c) = \int_{\theta < \theta_c} d^2\theta \gamma_t(\theta) Q(\theta), \quad (7)$$

where the tangential shear component $\gamma_t(\theta)$ at the position θ is given by:

$$\gamma_t(\theta) = -\mathcal{R}e \left(\gamma(\theta) \right) e^{-2i\phi} \quad (8)$$

¹ <http://www.cfht.hawaii.edu/Science/CFHLS/>

and the function $Q(\theta)$ is defined as :

$$Q(\theta) = \frac{2}{\theta^2} \int_0^\theta d\theta' \theta' U(\theta') - U(\theta). \quad (9)$$

The aperture mass statistic as a tool for the cosmic shear analysis has been discussed in many papers (Schneider et al. 1998, Pen, van Waerbeke & Mellier 2002, Munshi & Coles 2003, Munshi & Valageas 2005). This statistic is sensitive to curl-free correlations (E-modes) generated by the (scalar) gravitational potential. Curl correlations (B-modes) are then easily derived using the same statistics, after rotating each galaxy by 45 *deg*. If the only signal present is due to lensing, then the B-modes should be zero at all scales. This simple procedure is therefore a powerful diagnostic tool to assess systematic residuals in cosmic shear signal.

Unfortunately, the M_{ap} statistic is sensitive to the smallest accessible angular scales, where cosmic shear signal depends on the poorly-known non-linear evolution of the dark matter power spectrum. This shortcoming forces us to compute E- and B-modes on larger angular scales in a different way. For this we use the top-hat shear variance and the shear correlation functions. These functions are usually derived from the ξ_+ and ξ_- shear correlation functions:

$$\begin{aligned} \xi_+(r) &= \langle \gamma_t(\theta) \gamma_t(\theta + r) \rangle + \langle \gamma_r(\theta) \gamma_r(\theta + r) \rangle, \\ \xi_-(r) &= \langle \gamma_t(\theta) \gamma_t(\theta + r) \rangle - \langle \gamma_r(\theta) \gamma_r(\theta + r) \rangle, \end{aligned} \quad (10)$$

where γ_t and γ_r are the tangential and radial projections of the shear onto the local frame joining two galaxies separated by a distance r . Following Crittenden et al. 2001a, we define

$$\xi'(r) = \xi_-(r) + 4 \int_r^\infty \frac{dr'}{r'} \xi_-(r') - 12r^2 \int_r^\infty \frac{dr'}{r'^3} \xi_-(r'). \quad (11)$$

The E and B shear correlation functions are given by

$$\xi^E(r) = \frac{\xi_+(r) + \xi'(r)}{2} \quad \xi^B(r) = \frac{\xi_+(r) - \xi'(r)}{2}. \quad (12)$$

A similar relation can be found for the aperture mass and the top-hat statistics as showed in Crittenden et al. 2001b. Crittenden et al. 2001b also pointed out ξ^E and ξ^B can only be derived up to an integration constant which depends on the extrapolated signal outside the measurement range.

Finally, the amplitude of the lensing signal depends on the galaxy redshift distribution $n(z)$ (see Eq.(1)). As in previous works (see Van Waerbeke et al. 2002, Van Waerbeke, Mellier & Hoekstra 2005), we use the following redshift distribution:

$$n(z) = \frac{\beta}{z_s \Gamma\left(\frac{1+\alpha}{\beta}\right)} \left(\frac{z}{z_s}\right)^\alpha \exp\left[-\left(\frac{z}{z_s}\right)^\beta\right], \quad (13)$$

where α , β and z_s parameters are derived from deep photometric redshift catalogues. The lensing signal can be predicted for any redshift range using Eq.(1) and Eq.(13).

3. The Deep CFHTLS T0001 data set

The Deep CFHTLS data used in this work consists of u^* , g' , r' , i' and z' stacked MEGACAM images that form the first CFHTLS release (hereafter T0001). The release is composed of stacked images, catalogues and relevant meta-data produced from observations in four uncorrelated fields that were carried out at CFHT with the MEGAPRIME instrument between June 1st 2003 and July 22, 2004. Details regarding each field are listed on the CFHTLS web pages².

Each MEGACAM image consists of an array of 9×4 EEV CCDs of 2048×4612 pixels each (Boulade et al. 2003). The pixel scale is 0.186" and the camera covers a total field of 1 degree \times 1 degree. There are two large gaps of 82 arc-second between rows of CCDs. In order to produce complete fields, the gaps have been filled by organizing observations in a series of exposure sequences with large offsets. This results in an heterogeneous pixel illumination at the borders of each CCD. This spatial flux variation induces a varying pixel signal-to-noise ratio that is taken into account by using pixel weight maps together with hand-made masks (see Section 4) to discard noisy areas of each field.

The stacks include only MEGACAM images with seeing better than 1.0" ³ and airmass below 1.4 have been selected. However, because there were fewer u^* -band images than for the other filters, we relaxed the selection criteria for this filter and kept all u^* images with seeing below 1.4". Only three of the four Deep fields have been selected for cosmic shear studies. The D2 Deep field has been dropped from our sample because it is significantly shallower than the other three fields.

Data were calibrated and processed at CFHT and the TERAPIX data center. The full T0001 release is archived at CADC⁴ and available to any CFHTLS registered user. A description of the data processing pipeline used to produce the deep T0001 stacks is beyond the scope of the paper, but the details can be found on the TERAPIX web pages⁵. Photometric and astrometric methods and quality assessments done on these data are explained in a short explanatory supplement⁶. The processing (astrometric and

² <http://www.cfht.hawaii.edu/Science/CFHLS/cfhtlsdeepwidefields.html>

³ We use the seeing definition of TERAPIX as twice the median flux radius of a selection of point sources on each CCD. FLUX_RADIUS as measured by SExtractor, is the radius of the disk that contains 50% of the total flux. For a Gaussian profile the SExtractor seeing is almost equal to the Full Width at Half Maximum (FWHM). For a typical MEGACAM PSF, it is slightly larger (10%) than the true PSF FWHM.

⁴ <http://cadwww.dao.nrc.ca/cfht/cfhtls/>

⁵ http://terapix.iap.fr/article.php?id_article=382

⁶ http://terapix.iap.fr/article.php?id_article=383

photometric calibrations, pixel re sampling, image warping and stacking, catalogue production) uses the current first generation TERAPIX software tools and closely follows the one used for the VIRAMOS-Descart survey that is described in McCracken et al. 2003. We refer to this paper, and to the TERAPIX and CFHT⁷ web pages for further details.

The accuracy of the photometric calibrations can be estimated from the stellar colour-colour plots and the galaxy counts in all bands given on the TERAPIX T0001 pages and is also discussed in the more detailed stellar analysis done by Schulteiss et al. (in preparation). In all bands, the cumulative internal and systematic photometric errors are 0.05 mag up to AB=22.5, and never larger than 0.1 to the 80% completeness limit (\approx AB=25.5). This uncertainty is sufficient for the cosmic shear studies on this paper.

Table 1 summarizes the T0001 stacks used in this work. The completeness limits have been computed by adding randomly simulated stars (Moffat profiles) inside a 2000×2000 area of each Deep field and by running the detection and photometry again, using the MAG_AUTO magnitude of SExtractor⁸ software (Bertin & Arnouts 1996). The completeness was also checked using galaxy counts⁹.

The D4- z' data have not been released by TERAPIX because the astrometric internal accuracy was below the scientific requirements. The large rms error found in the D4- z' data is not been fully understood, but it results in a large number of galaxy mismatches during the u^*, g', r', i', z' catalogue cross-identification. This hampers reliable panchromatic studies for many galaxies detected in this field. A further investigation reveals that the D4- i' astrometric solution is also slightly off, while data quality in other filters are excellent. Although it has no impact on the D4- u^*, g', r', i' photometric studies, a quick weak lensing analysis of the D4- i' field shows it has more systematic residuals than D1- i' and D3- i' . In contrast, the three Deep r' band data have similar quality and do not show systematics residual differences. We therefore used the r' band as the reference data set for all comparison between the fields, and only use the deep D1/D3 i' band data for colour comparisons, when needed.

4. Detection of the shear signal

4.1. Galaxy shape parameters

Catalogues and shape measurements of galaxies are produced using the IMCAT software (Kaiser et al. 1995, hereafter KSB). For each object the centroid position and the half-light radius r_h are measured. These parameters

⁷ <http://www.cfht.hawaii.edu/Science/CFHTLS-DATA/dataprocessing.html>

⁸ http://terapix.iap.fr/rubrique.php?id_rubrique=91

⁹ see http://clix.iap.fr/T0001/Plots/CFHTLS_D_i_galcount_T0001.png and http://clix.iap.fr/T0001/Plots/CFHTLS_D_r_galcount_T0001.png

are then used to derive orientations and raw ellipticities of galaxies from the weighted second moments I_{ij} of the galaxy light distribution. In order to minimize the noise contribution each moment is filtered using a Gaussian filter $W(\theta)$ of size r_h :

$$I_{ij} = \int d^2\theta W(\theta) \theta_i \theta_j(\theta) f(\theta), \quad (14)$$

where $f(\theta)$ is the surface brightness.

The raw ellipticity is given by:

$$\mathbf{e} = \left(\frac{I_{11} - I_{22}}{Tr(I)}; \frac{2I_{12}}{Tr(I)} \right). \quad (15)$$

where $Tr(I)$ represents the trace of the matrix I . We use the KSB method to get an unbiased estimator of the shear γ . This method has been tested by several teams and it has been demonstrated that it provides robust and reliable shear measurements from ground based data (see the comprehensive critical investigation of KSB and other techniques by Heymans et al. 2005b, and also references therein).

4.1.1. PSF correction: the principle

Let us assume the shear-free intrinsic ellipticity of a galaxy is \mathbf{e}^0 . On the detector, its shape is eventually modified by the distortions produced by gravitational lensing effects and systematics that increase the smearing and the anisotropic component of the PSF (atmosphere, optical aberrations). Assuming these distortions are small, Kaiser et al. 1995 demonstrated the observed ellipticity, \mathbf{e}^{obs} , can be written:

$$\mathbf{e}_\alpha^{obs} = \mathbf{e}_\alpha^0 + P_{\alpha\beta}^{sh}\gamma + P_{\alpha\beta}^{sm}\mathbf{q}, \quad (16)$$

where \mathbf{q} is the anisotropic component of the PSF and γ is the gravitational shear. P^{sh} and P^{sm} are called the shear and the smear polarisability. Their values depend on the galaxy surface brightness and on the filter properties $W(\theta)$. \mathbf{q} can be derived directly from the data, by measuring the ellipticity of stars in each field, \mathbf{e}_* , such as:

$$\mathbf{q}_\alpha = \frac{\mathbf{e}_\alpha^*}{P_{\beta\beta}^{sm}}. \quad (17)$$

The shear polarisability is however altered by the isotropic smearing component of the PSF. It results in a modification of the shear polarisability

$$P^\gamma = P^{sh} - \frac{P_{*}^{sh}}{P_{*}^{sm}} P^{sm}, \quad (18)$$

where P^γ is called pre-seeing shear polarisability and $P_{*}^{sh/sm}$ refers to stars (Luppino & Kaiser 1997). Provided the assumption $\langle e_0 \rangle = 0$ is valid, an unbiased estimator of the shear γ is given by:

$$\gamma = \langle P_\gamma^{-1}(\mathbf{e}^{obs} - P^{sm}\mathbf{q}) \rangle. \quad (19)$$

Table 1. Summary table of T0001 D1, D3 and D4 deep stacks used in this work. Magnitudes are instrumental AB. Details on magnitude, aperture, seeing and completeness definitions are given in the explanatory page http://terapix.iap.fr/article.php?id_article=383

	D1	D3	D4
RA (J2000)	02:25:59	14:19:27	22:15:31
DEC (J2000)	-04:29:40	+52:40:56	-17:43:56
Effective FOV (deg ²)	0.80	0.77	0.77
Exp. time u^* (s)	10560	4620	16680
Median seeing u^* (arc-sec.)	1.15	0.88	1.05
Completeness u^* 50% (mag.)	26.4	26.0	26.2
Exp. time g'	7515	8010	11250
Median seeing g' (arc-sec.)	0.98	0.95	0.99
Completeness g' 50% (mag.)	26.4	26.5	26.2
Exp. time r'	17280	20820	26400
Median seeing r' (arc-sec.)	0.87	0.93	0.85
Completeness r' 50% (mag.)	26.1	26.4	25.9
Exp. time i'	52000	59640	58800
Median seeing i' (arc-sec.)	0.88	0.92	0.88
Completeness i' 50% (mag.)	26.1	26.2	25.8
Exp. time z'	12240	15120	-
Median seeing z' (arc-sec.)	0.86	0.85	-
Completeness z' 50% (mag.)	24.5	24.6	-

4.1.2. Object selection

Prior to cosmic shear analysis, all CFHTLS images are checked by eye and masks are drawn by hand. These masks are designed to avoid elongated defects, like saturated stars, as well as large foreground galaxies with extended bright halo that may contaminate the shape of underlying faint galaxies (see Van Waerbeke et al. 2001 for details). We should emphasize that masks are *only* drawn using criteria ($z = 0$ galaxies, bright stars, CCD defects) that are not correlated with the lensing signal. In addition, we used the weight map images produced by TERAPIX for each stack to reject all pixels with a relative weight amplitude less than of 80%. This rejection step reduces significant spatial variation of the detection threshold and keeps the averaged redshift distribution of lensed galaxies stable over the field. The rejection scheme removes the CCD boundaries from all of the fields, and is essentially equivalent to singling out each CCD region, as was done earlier in VIRMOS-Descart survey. The gain in homogeneity is however preserved at the expense of the sky coverage. About 30% of the initial area is lost after the masking process.

Stars needed for the PSF correction are selected along the stellar locus of the magnitude/size diagram (Fahlman et al. 1994), from the region where stars are about one magnitude fainter than the saturation level and where they cannot be confused with faint galaxies. The P_{\star}^{sm} and P_{\star}^{sh} values are derived at all MEGACAM image positions from a PSF mapping that samples the PSF smearing and PSF anisotropy at the position of each star, and by interpolating their values between the stars. This operation is done on each CCD separately, as suggested by Hoekstra 2004. The PSF is mapped using a composite model of a second order polynomial and a rational func-

tion, $p^{\alpha}(x, y)$:

$$p^{\alpha}(x, y) = a_0 + a_1x + a_2y + a_3x^2 + a_4xy + a_5y^2 + c(x, y) \quad (20)$$

where $c(x, y)$ is the rational function chosen as:

$$c(x, y) = \frac{b_0 + b_1x + b_2y + b_3x^2 + b_4xy + b_5y^2 + b_6y^3 + b_7y^4}{1 + b_8x + b_9y} \quad (21)$$

The second order polynomial terms models the smooth low frequency PSF component, while the rational function provides a model for the high frequency PSF terms (Hoekstra 2004).

The correction is made in two steps. First, the coefficients of the rational function are determined. Since the CFHTLS Deep fields are much deeper than the RCS (Hoekstra et al. 2002) and the VIRMOS-Descart (Van Waerbeke et al. 2000, Van Waerbeke et al. 2001) surveys, the density of selected stars is higher and we do not need to map the PSF using external stellar fields. Each field has about 100 stars per CCD, so the high frequency PSF terms can be reasonably well sampled down to 0.5 arc-minute, and all coefficients of the rational function can be constrained with sufficient accuracy. In a second step, the polynomial terms are determined.

We also compared the rational function solution against the second order polynomial interpolation. We found the results are not very different from our composite model, although the rational function improves the quality and stability of the PSF mapping.

Once ellipticity is corrected we keep in the sample all objects with angular size larger than the seeing disk and smaller than two arc-seconds. Following

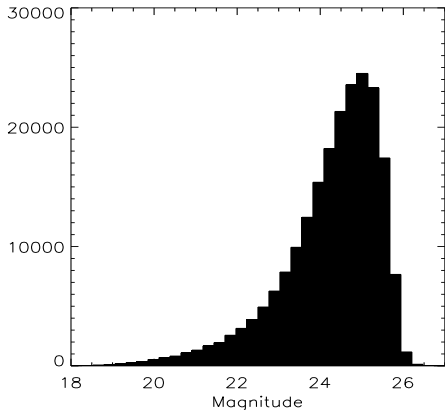


Fig. 1. Magnitude distribution of galaxies in the three fields in the r band. More detailed galaxy count plots, for each filter and for each Deep field, are available on the web at http://terapix.iap.fr/article.php?id_article=382

Van Waerbeke et al. 2000, we automatically reject one galaxy in every close pairs with angular separation less than 12 arcsec in order to avoid contamination of ellipticity measurements by overlapping isophotes of neighboring galaxies.

The magnitude distribution of the final object catalogue is shown in Fig.1. The limiting magnitude corresponding to a 80% completeness limit is $r'_{AB} = 25.5$. Bright objects with magnitude smaller than 21.5 and faint objects with magnitude larger than 25.5 are also removed from the galaxy sample. The final galaxy number density of the cosmic shear catalogue is about $20/\text{arcmin}^2$.

As proposed by Erben et al. 2001 we assign an ellipticity dispersion σ_g to each object corresponding to the ellipticity dispersion in a box containing its 20 nearest neighbors in the (*magnitude, size*) space. Weighted 2-point statistics are computed assigning to each galaxy a weight given by $1/(\sigma_g^2 + \sigma_e^2)$ where σ_e is the ellipticity dispersion of the unlensed galaxies. A different noise estimation (Hoekstra et al. 2000) gives similar results.

5. Residual systematics

5.1. Quality of the PSF correction

A visual inspection of the MEGAPRIME PSF (Fig.2) shows that the PSF anisotropy has significant variation over the field and may also be very large at the boundaries¹⁰. The PSF correction is therefore a critical step and its reliability demands careful verifications. In addition to the usual B-mode analysis shown in the next section, in this section we carry out several analyses of the systematics.

¹⁰ This strong PSF anisotropy has been considerably reduced by the CFHT staff, after the T0001 release. It should no longer be a critical issue for next releases.

The quality of the PSF correction and its homogeneity over the MEGACAM field camera can be assessed by comparing the mean star ellipticity before and after PSF correction (Fig.3). The average stellar ellipticities $\langle e_t \rangle$ and $\langle e_r \rangle$ are plotted as a function of the radial distance from the center of the field, r . It is interesting to note that the radial ellipticity component degrades much more and much faster than the tangential ellipticity. However, the PSF correction done by the PSF mapping is very good, for each Deep field. After correction, the dispersion of star ellipticities is about 2×10^{-3} at any point of the camera. There is no significant change in the residual error as function of position. The small increase in the fluctuation of star shapes at very small distances is due to higher Poisson noise: each radial bin has the same width, so the innermost circle encompasses the smallest area and contains fewer stars than the others.

Finally, we checked the residual amplitude of the shape correlation function between corrected stars. We found it to be two order of magnitude smaller than the expected lensing signal at all scales probed by this work (Fig. 4).

The tests discussed above only guarantee that the PSF correction is excellent in the neighborhood of selected stars or on angular scales larger than, or close to, the mean angular distance between stars. In regions where no stars were selected or on small scales, the local PSF correction residuals may be larger than the average. A useful test of systematic residuals on small scales has been proposed by Bacon et al. 2003 and Heymans et al. 2005b. Assuming the PSF model derived from stars and applied to galaxies is unable to remove all systematic contributions, the star-galaxy cross correlation will be non-zero and may vary as function of angular scale. If the residual is small, Bacon et al. 2003 showed the systematic residual can be expressed as follows:

$$\xi_{sys} = \frac{\langle e_* e_{gal} \rangle^2}{\langle e_* e_* \rangle}; \quad (22)$$

where e_{gal} is the *corrected* galaxy ellipticity and e_* is the *uncorrected* star ellipticity. We use the ξ_{sys} to compute the contribution of systematics for both top-hat and compensated filter. Fig. 5 shows they are consistent with zero at all scales between 0.5 arc-minute to 30 arc-minutes. This confirms that residual systematics are negligible in the MEGACAM Deep fields.

5.2. Independent analysis of r' and i' data

The robustness of cosmic shear signal can also be assessed by comparing results obtained using different filters for the same galaxy sample. Because gravitational lensing is achromatic, we expect the shape and amplitude of cosmic shear to be identical for data taken in different filters. Any significant difference between two bands provides a diagnostic of the PSF corrections. A first attempt at comparing shear measurements in different filters was made by Kaiser et al. 2000 using the CFHT12K camera. The I and V bands showed significantly different signals that

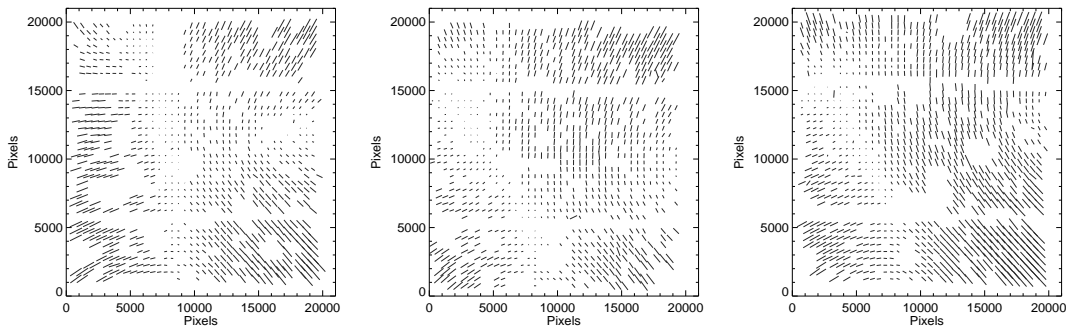


Fig. 2. The mean ellipticity of the stars shows the behavior of the PSF anisotropy for the three fields D1, D3 and D4. For all of the three fields, $\langle e \rangle$ is few percent in the central part and it becomes about 10% in the corners (see also fig.3).

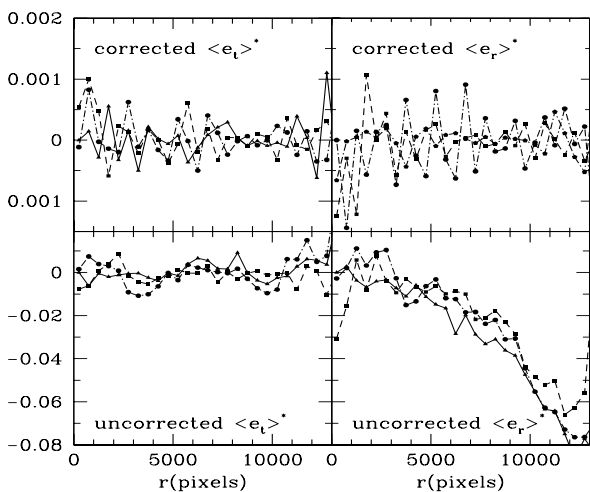


Fig. 3. Bottom panels show the mean tangential (left panel) and radial (right panel) uncorrected stellar ellipticity as a function of the distance to the center of the camera for D1rT001 (filled triangles), D3rT001 (filled squares) and D4rT001 (filled circles) fields. Top panels show the same quantities after PSF correction.

were inconsistent with the change in redshift distribution between the two filters.

The Deep photometry provides a sample of the same galaxies detected in different filters, so the signal is expected to be the same. However, these filters have different depths, and the shallowest colours do not have enough galaxies to allow a comparison of the signal between all colours using the same galaxies. This limitation affects mainly the u^* and g' bands. Furthermore, these bands are more sensitive to atmospheric dispersion than other filters. We expect their PSF anisotropy to be larger than for r' , i' and z' bands and its correction may also depend more on

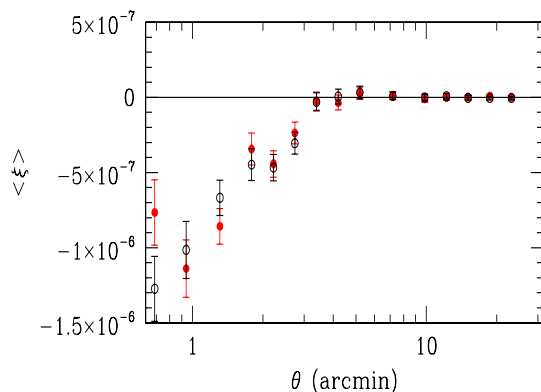


Fig. 4. E-modes (red filled circles) and B-modes (black open circles) top-hat two point statistics of corrected stars show the smallness of residual PSF systematics.

the relative differences between the averaged spectral energy distributions of stars used for the PSF calibration and of galaxies. Hence, u^* and g' are not well suited for weak lensing analysis. The comparison between the r' and the z' bands doesn't give many informations because of the limited size of the matched sample. We therefore decided only to focus on the comparison between r' and i' bands.

We computed the two-point statistics using the same objects in i' -band and r' -band in the D1 and D3 fields only. As reported before, D4 was discarded from this study because it shows higher systematic residuals in i' bands than the two other fields. It is worth noting that both r' and i' band images have been processed (flat fielding, astrometric and photometric calibrations, image selection, image stacking) in a totally independent way. The only correlations between the two samples are the software tools and the pipeline scheme used at TERAPIX.

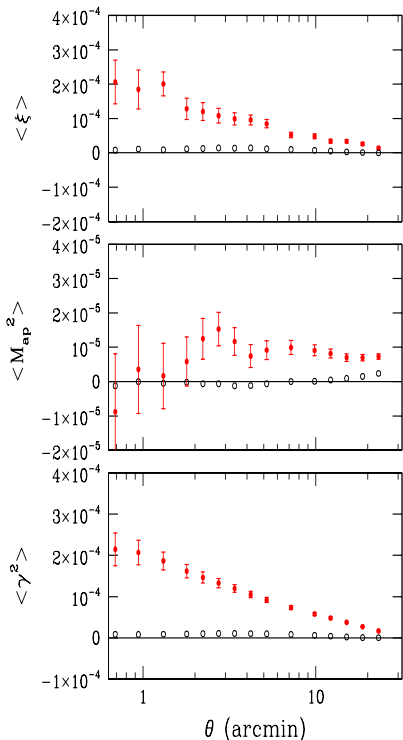


Fig. 5. Two-point signal statistics (red filled circles): top-hat (bottom) and M_{ap} (middle) correlation function (top), compared with residual systematics (black open circles). Signal error bars are statistical ones.

The r' and i' ellipticity catalogues have been computed and PSF-corrected independently, starting from the r' and i' T0001 stacked images. The galaxy cross-identification is done at the very end of the processing to compare the results. Fig. 6 shows the comparison of the E - and B -modes of the top-hat shear variance for both the i' and r' data sets. The error bars are estimated as the quadrature sum of the statistical and the systematic error ξ_{sys} defined by Eq.(22). The amplitude of the latter is bigger in the r' band as shown by the residual B -modes in this filter. The r' and i' bands results are remarkably similar, both in shape and amplitude, they agree to within 1σ at all scales.

6. Characterization of the shear signal

6.1. Two point statistics

The ellipticity correlation functions $\xi_+(r)$ and $\xi_-(r)$ are measured from the weighted mean of all pairs with angular separation r . The correlation function is computed using equations (11) and (12). The M_{ap} and the top-hat statistics are also computed as a function of the correlation

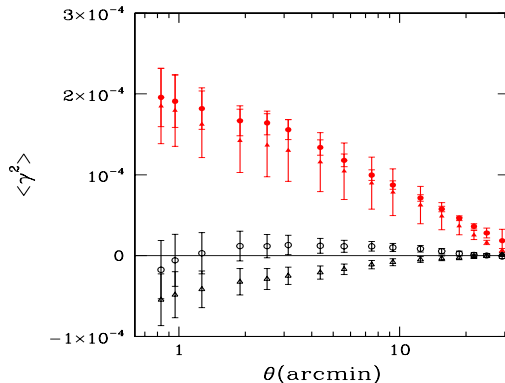


Fig. 6. Top-hat variance of the E -modes in the i' -band (red filled circles) and in the r' -band (red filled triangles) for the same data set. Top-hat variance B -modes in the i' -band (black open circles) and in r' -band (black open triangles).

functions $\xi_+(r)$ and $\xi_-(r)$ following Schneider et al. 2002 and Crittenden et al. 2001b. Fig.7 shows the two-point statistics for the three deep fields D1, D3 and D4. Error bars including statistical noise and cosmic variance are computed from the $\xi_+(r)$ and $\xi_-(r)$ as described in Schneider et al. 2002.

The cosmic variance contribution is computed using the CFHTLS T0001 Deep survey properties: an effective density (after masking) of 20 gal/arcmin^2 , an effective area of 2.1 deg^2 , and an ellipticity dispersion per ellipticity component of 0.3 (the latter was measured from the corrected ellipticity). However, the error calculation described in Schneider et al. 2002 is only valid for a single connected field with a number density of n equally-sized galaxies. We therefore replace the statistical error component by the Poisson noise measured from the data, using the weights (computed as described above) and positions of each galaxy. For the top-hat variance and the correlation function, the free integration constant is chosen so that the B -modes on scales between 15 and 25 arc-minutes vanish. Although its amplitude is meaningless, one can see that the B -mode is flat and stable over that range of angular scales.

In contrast, the M_{ap} statistic does not have an undetermined integration constant (as explained previously), so the B -mode amplitude is a physical property. Fig.7 shows the presence of B -modes. Note that the M_{ap} filter for a given size θ is mostly sensitive to scales around $\simeq \theta/5$. This explains why the other two-point statistics do not show B -modes at the same scales. The B -mode at such small scales may result from intrinsic alignment of galaxies (King & Schneider 2002, Heymans & Heavens 2003) or from the correlation between intrinsic ellipticity and shear (Hirata & Seljak 2004). If these systematics are real, we expect to correct them in future work by using the pho-

tometric redshifts. A further investigation confirms that the B-modes come from weak objects (i.e. $25.0 < r'_{AB} < 25.5$), and that a magnitude cut that rejects objects with magnitude fainter than 25.0 gives zero B-modes at all the scales, even for the M_{ap} statistic. However, we keep these objects in our catalogues because a deep sample will be necessary to study the evolution of signal with redshift. In addition, the presence of B-modes at small scales will be taken into account when we estimate cosmological parameters.

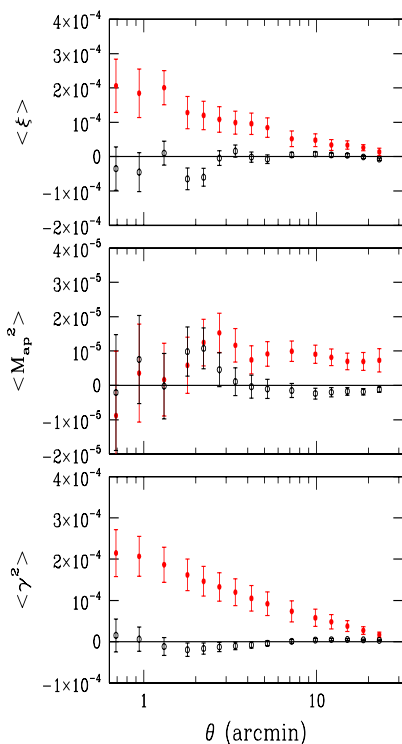


Fig. 7. Two-point statistics for all the three fields combined. Red filled circles show E-modes, black open circles show B-modes. E-mode error bars include the statistical error and the cosmic variance contribution, while B-modes are affected only by statistical error.

6.2. Evolution of signal with redshift

The cosmological nature of the two-point statistical signal can be established by comparing its amplitude as function of source redshifts with theoretical expectations of the gravitational instability paradigm and the gravitational lensing theory (Bernardeau et al. 1997, Jain & Seljak 1997). To first order, the signal should increase as $z_s^{1.5}$

Table 2. Mean photometric redshift in magnitude bins.

magnitude bin	mean redshift
$18.5 \leq i_{AB} \leq 24$	$\langle z \rangle = 0.850$
$19.0 \leq i_{AB} \leq 24$	$\langle z \rangle = 0.853$
$19.5 \leq i_{AB} \leq 24$	$\langle z \rangle = 0.858$
$20.0 \leq i_{AB} \leq 24$	$\langle z \rangle = 0.865$
$20.5 \leq i_{AB} \leq 24$	$\langle z \rangle = 0.876$
$21.0 \leq i_{AB} \leq 24$	$\langle z \rangle = 0.892$
$21.5 \leq i_{AB} \leq 24$	$\langle z \rangle = 0.913$
$22.0 \leq i_{AB} \leq 24$	$\langle z \rangle = 0.942$
$22.5 \leq i_{AB} \leq 24$	$\langle z \rangle = 0.981$
$23.0 \leq i_{AB} \leq 24$	$\langle z \rangle = 1.035$
$23.5 \leq i_{AB} \leq 24$	$\langle z \rangle = 1.100$

(Bernardeau et al. 1997, Jain & Seljak 1997, so even a rough separation of galaxies into low- and high-redshift populations should split the cosmological lensing signal accordingly.

The CFHTLS T0001 data sets are well suited for this analysis. The observations can be used to sample the high redshift universe up to $z \simeq 1$. There are enough of galaxies to divide into two subsets based on their estimated photometric redshifts.

Photometric redshifts were measured using the `hyper-z` public software¹¹ (Bolzonella, Miralles & Pelló 2000). `hyper-z` uses the multi-band photometric data of a galaxy to derive its most likely redshift and spectral energy distribution (SED) based on the Bruzual & Charlot evolution models (Bruzual & Charlot 1993).

We used the D1 and D3 u^* , g' , r' , i' and z' images and the D4 u^* , g' , r' , i' images (the D4- z' stacked image is missing in T0001). Photometric catalogues were produced by the `SExtractor` software. All galaxies were first detected in the r' band reference image. Magnitude and colours of galaxies are then computed using the r' -center positions and inside an aperture scaled according to the size of each galaxy in r' -band. The χ^2 minimization was performed assuming magnitude errors derived from `SExtractor`, which range between $\Delta mag = 0.03$ and $\Delta mag = 0.1$ in all bands.

Fig. 8 shows the photometric redshift distribution of the galaxies in D1 field down to $i' = 24.0$. This subsample can be compared with the VVDS spectroscopic redshift distribution obtained from 11000 spectra in the same region (Le Fèvre et al. 2005). There are no apparent discrepancies that would make the separation into photometric low- and high-redshift galaxies unreliable. Beyond $i' = 24.0$, large spectroscopic redshift samples are not yet available, but we don't have any reason to believe that our photometric redshift accuracies will degrade significantly for the $i' < 24.0$ sample.

The cosmic shear catalogue can therefore be split into two samples with equal numbers of galaxies at high and low redshifts with reasonable confidence and can be

¹¹ <http://webast.ast.obs-mip.fr/hyperz/>

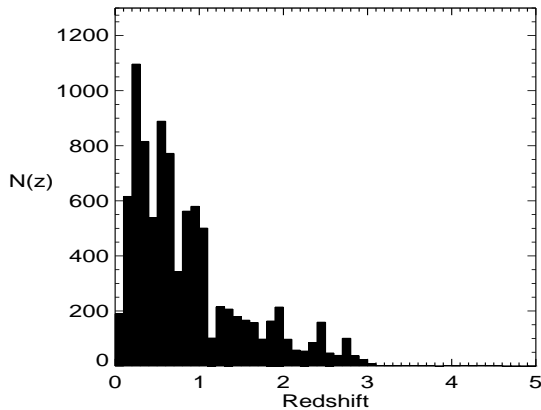


Fig. 8. Density of galaxies of D1 field as a function of photometric redshift.

compared with cosmological predictions. Poisson noise is therefore similar in the two subsamples, but photometric redshift errors are expected to be larger in the high-redshift tail.

Fig. 9 shows the top-hat shear variance measured for the two populations. The low- z sample ranges in $0.3 \lesssim z \lesssim 1.$, while the high- z galaxies have $z \gtrsim 1.0$. Error bars include Poisson noise and cosmic variance (see Sect. 4.2). The difference between the two samples demonstrates the cosmological nature of the signal. An indicative comparison of signals with theoretical predictions is also plotted.

The relative lensing amplitude for the two source galaxy populations is less sensitive to cosmic variance fluctuations, and agrees with the predictions.

Contamination by galaxies with incorrect photometric redshifts is likely important, in particular for the faintest galaxies and the high- z tail (further information about degeneracy of photometric redshifts in the case of missing infrared bands can be found on `hyper-z` user’s guide).

In spite of potential contamination by incorrect photometric redshifts, the cosmological imprint of large-scale structure detected in the Deep CFHTLS data shows that MEGAPRIME is suitable for cosmic shear studies. Fig. 9 also demonstrates that the CFHTLS Deep survey has promising potential for tomographic studies that explore the evolution of the dark matter power spectrum with look-back time.

7. Parameter estimation

7.1. Derivation of the likelihood function

In this Section, we describe the estimation of cosmological parameters. In a subsequent paper, we will perform a complete parameter estimate, combining weak lensing with other cosmological probes. Therefore, here we limit the

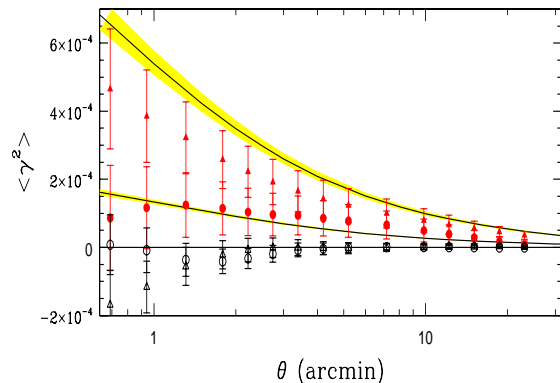


Fig. 9. Top-hat variance for “high- z ” subsample (red filled triangles) and “low- z ” subsample” (red filled circles). B-modes for the two subsamples are also shown. Error bars include statistical noise and cosmic variance. The data are compared with theoretical fiducial model ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $\sigma_8 = 0.88$ and $h = 0.7$) and source distribution $n(z)$ modeled by Eq.13, with $\alpha = 1.98$, $\beta = 0.66$ and $z_s = 0.0981$. The low- z source selection is simulated using $n(z)$ between $0.3 < z < 1.2$ and zero otherwise (bottom line). Likewise, the red filled triangles and the black open triangles are the E- and B-modes of the “high- z ” sample. The data are compared with the same theoretical model with a high- z source selection simulated using $n(z)$ between $z > 0.8$ and zero otherwise. Shaded areas show models within $z_s = 0.0981^{+0.013}_{-0.011}$ that represent the 1σ error region on z_s as derived from the likelihood parameter estimation.

analysis to the normalization of the mass power spectrum (σ_8) and matter density (Ω_m) measurements. The shape parameter Γ is given by the Cold Dark Matter paradigm $\Gamma = \Omega_m h$, where h is the reduced Hubble constant. We allow the characteristic redshift of the source distribution to vary around the best fit that will be described in the next sub-section.

To measure cosmological parameters, we adopt a maximum-likelihood method. Let d_i be the input data vector (i.e. the top-hat shear variance as a function of scale θ_i), and $m_i(\Omega_m, \sigma_8, n(z))$ the prediction, function of the parameters to be estimated. The likelihood function of the data is then:

$$\mathcal{L} = \frac{1}{(2\pi)^n |\mathbf{C}|^{1/2}} \exp \left[-\frac{1}{2} (d_i - m_i) \mathbf{C}^{-1} (d_i - m_i)^T \right], \quad (23)$$

where $n = 16$ is the number of angular scale bins and \mathbf{C} is the 16×16 covariance matrix of the top-hat shear,

$$C_{ij} = \langle (d_i - m_i)^T (d_j - m_j) \rangle, \quad (24)$$

and \mathbf{C} can be decomposed as $\mathbf{C} = \mathbf{C}_n + \mathbf{C}_s$, where \mathbf{C}_n is the statistical noise and \mathbf{C}_s the cosmic variance covariance matrix.

As discussed above, the matrix \mathbf{C}_s is computed according to Schneider et al. 2002, assuming an effective survey

area of the CFHTLS Deep fields: 2.1 square degrees, a number density of galaxies $n_{gal} = 20/arcmin^2$, and an intrinsic ellipticity dispersion of $\sigma_e = 0.3$ per component.

The cosmic variance is computed assuming Gaussian statistics. While this assumption becomes inappropriate on small angular scales, errors on such scales are dominated by the statistical noise contribution, so the Gaussian approximation remains an excellent one (Van Waerbeke et al. 2002). The covariance matrix components are derived for a fiducial cosmological model corresponding to the best fit of WMAP data proposed by Spergel et al. 2003: $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, $\sigma_8 = 0.88$, $\Gamma = 0.21$ (the reduced Hubble constant is $h=0.7$). The B -mode is calibrated by marginalizing around $B = 0$ within the 1σ interval.

7.2. Parameter estimation

The source redshift distribution is calibrated using the Hubble Deep Field (HDF) catalogues (Fernández-Soto et al. 1999), which provide a more accurate estimate of redshift in absence of infrared data in CFHTLS fields. It turns out that the F606 filter of WFPC2 is a good match to the MEGACAM r' filter within our 1σ magnitude error. We select all galaxies with $21.5 < r' < 25.5$. The Hubble Deep Fields provide a sample at high redshifts that overlaps with the redshift range expected for the CFHTLS Deep fields.

We use the source redshift distribution model of Eq.12 and perform a χ^2 fit, allowing the parameter z_s to vary. We then identify the $\pm 1\sigma$ and $\pm 2\sigma$ uncertainties, which we marginalized over in the cosmological parameter estimation. We find $\alpha = 1.9833$, $\beta = 0.6651$, $z_s = 0.0981^{+0.013+0.021}_{-0.011-0.016}$. Figure 10 shows the unnormalized weight in magnitude slices in the Deep catalogues. The effect of down-weighting faint galaxies is taken into account in the source redshift estimation. Figure 11 shows the best fit model and the underlying photometric redshifts from the Hubble Deep Fields (solid line). Error bars are Poisson errors. The dashed-dotted line on Figure 11 shows the redshift distribution one would have if we ignore the weighting. The best fit redshift distribution model has a mean source redshift of ≈ 1.01 , nearly 0.2 higher in z than the Wide survey (Hoekstra et al. 2005).

The constraints on Ω_m and σ_8 are obtained after marginalization of the reduced Hubble constant $h \in [0.6, 0.8]$ and over the $\pm 2\sigma$ limits of the source redshift parameter z_s . The resulting constraints in the Ω_m - σ_8 plane are given in Figure 12. This figure shows that the CFHTLS Deep field gives constraints as good as previous lensing measurements, despite its small field of view. This is the consequence of the larger fraction of high redshift galaxies, which are more strongly lensed. Using the Peacock & Dodds (1996) non-linear scheme, we obtain $\sigma_8 = 0.94 \pm 0.15 \pm 0.20$ ($\pm 1\sigma \pm 2\sigma$) for $\Omega_m = 0.3$. Error bars are the one and two σ errors respectively. The Smith et al. (2003) halo model gives $\sigma_8 = 0.90 \pm 0.14 \pm 0.20$, which

agrees with previous normalization measurements. The similarity between the result obtained using the Peacock & Dodds and that using the halo fitted model is not surprising. Indeed, on scales $\gtrsim 1'$, which dominate our signal, the difference between the two models of power spectrum is $\lesssim 5\%$. On smaller scales, we would expect an increasing discrepancy between these different ways to estimate σ_8 .

We then measure σ_8 by combining these constraints with those obtained on the CFHTLS Wide survey (see Hoekstra et al. 2005 for the details). The result of this joint analysis is shown in Figure 13, and remarkably, the Ω_m - σ_8 degeneracy is partially broken. This is the consequence of measuring the large and small scales simultaneously, as shown in Jain & Seljak 1997. For $\Omega_m = 0.3$, we get $\sigma_8 = 0.89 \pm 0.06 \pm 0.12$ using Peacock & Dodds (1996) for the non-linear scheme and $\sigma_8 = 0.86 \pm 0.05 \pm 0.11$ for the halo model (Smith et al. 2003).

The power spectrum normalization is in very good agreement with results from medium-redshift and low-source-redshift weak lensing surveys (Hoekstra et al. 2002, Van Waerbeke & Mellier 2005). It is remarkable that the parameters of the redshift distribution, which have been estimated from a different survey, are such that the normalization σ_8 lies within the errors of previous measurements. This is strong evidence that deep, medium and shallow lensing surveys are in cosmological agreement, hence reinforcing the ability of cosmic shear to probe the mass distribution at different redshifts and different scales.

Weak lensing can also be used to constrain dark energy. Figure 14 show the upper limit on w_0 , the constant equation of state parameter derived from the Deep data only. Here we used only the Peacock & Dodds non-linear prescription (a detailed discussion on non-linear power spectrum correction in the context of Dark Energy can be found in Hoekstra et al. 2005, which also includes a joint analysis of the Wide and Deep data). We obtain $w_0 < -0.8$ at 1σ , and the contours show that this result is independent of Ω_M . This is particularly interesting because lensing combined with either cosmic microwave background (Jarvis et al. 2005) or supernovae will provide a strong constraint on the dark energy equation of state.

8. Summary and conclusion

This paper describes the first cosmic shear studies of CFHTLS Deep data using the T0001 CFHTLS release. It uses data collected in u^* , g' , r' , i' and z' with MEGAPRIME/MEGACAM over the first year of the survey. Only between 1% and 15% of the Deep data are therefore in hand depending on the field and on the filter, and so the survey is still 2-3 magnitude below the final goal.

The T0001 data have been used to assess the capabilities of MEGAPRIME/MEGACAM and to clarify the potential and the science drivers of the CFHTLS Deep survey for weak lensing studies.

The correction for PSF anisotropy works very well, showing that residual systematics are almost zero at

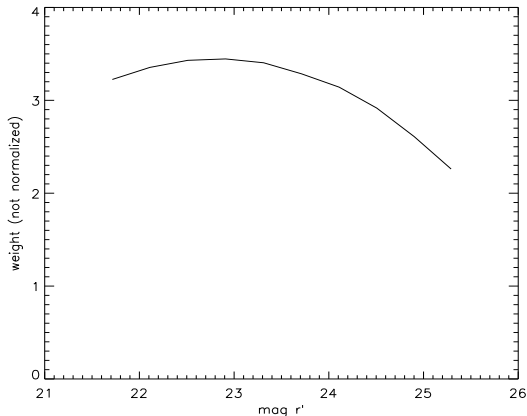


Fig. 10. Plot of mean weight per galaxy as function of magnitude $21.5 < r'_{AB} < 25.5$.

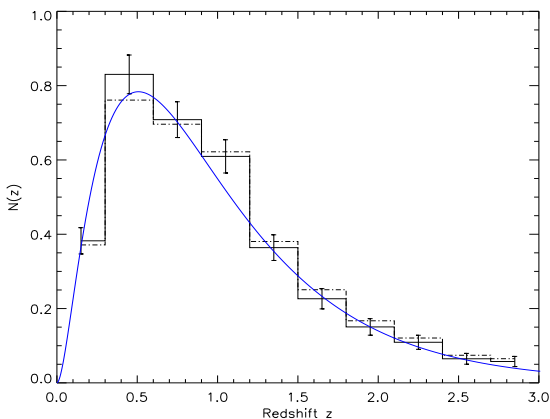


Fig. 11. The histogram shows the photometric redshift distribution of $21.5 < r'_{AB} < 25.5$ galaxies of Hubble Deep Field North and South used in this work. The central solid line is the best fit model. The solid line histogram is that magnitude weighted redshift distribution. The dashed-dot histogram shows the redshift distribution if the galaxies were not magnitude weighted.

all scales probed by a MEGACAM field. This is confirmed by the star-galaxy cross-correlation analysis. This also demonstrates that the CFHT-ELIXIR-TERAPIX calibration/reduction pipelines can deliver co-added images which have the required lensing quality. However, the presence of B-modes by weak objects at small scales should be further investigated.

The cosmic shear signal has been detected in the r' -band. Its consistency and achromaticity has been checked by independent r' - and i' -analysis of the same data sets.

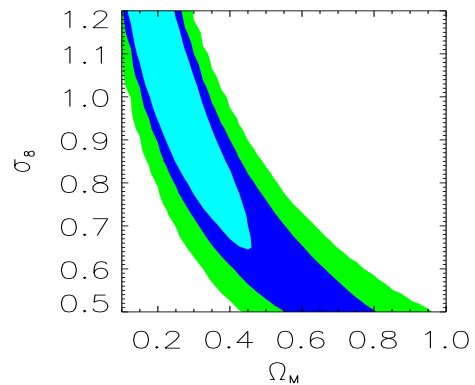


Fig. 12. Ω_m and σ_8 constraints with the Deep data only. The contours show 0.68, 0.95 and 0.999 confidence regions. Errors include statistical, covariance and residual systematic contributions. The models are pure Cold Dark Matter fit to the data, marginalized over the redshift distribution (see Section 7.2 for the details).

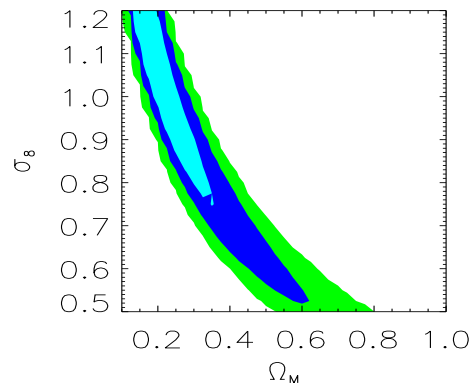


Fig. 13. Same as Figure 12, combined with the CFHTLS Wide data (Hoekstra et al. 2005). For $\Omega_m = 0.3$ we have $\sigma_8 = 0.86 \pm 0.05$ at 1σ (see Section 7.2 for details of the error calculation).

We have presented results for three standard two-point shear statistics.

Thanks to the depth of the CFHTLS Deep sample, and using the photometric redshifts derived from the u^* , g' , r' , i' and z' images, the galaxy sample was split into low- and high-redshift sources, and the cosmic shear signal was measured on the two subsamples separately. Both subsamples show zero B-modes and the shear amplitude of the high- z sample is clearly higher than the low- z one, with a ratio in agreement with the cosmic shear predictions. The amplitude of the signals from the two subsamples are

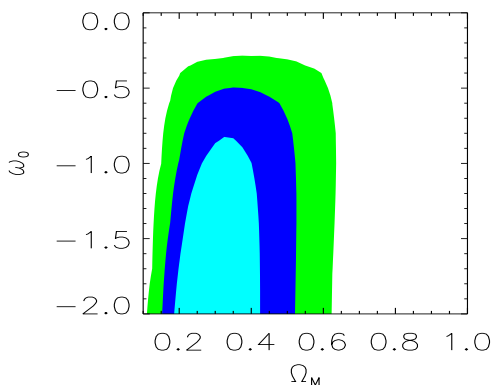


Fig. 14. Dark energy constraints from the Deep data only. Hidden parameters are marginalized using a flat prior over $\sigma_8 \in [0.7, 1.0]$, $h \in [0.6, 0.8]$ and within the $\pm 2\sigma$ boundaries of the redshift parameter z_s (see Section 7.2).

different from each other at all scales with a significance level higher than 5-sigma and their shapes follow theoretical expectations of Λ -CDM dominated universe. This strong evidence for the cosmological nature of the signal shows that the CFHTLS Deep data will allow us to explore the growth rate of cosmic shear signal with redshift, and hence the evolution of the dark matter power spectrum as function of lookback time.

Using only Deep data, and marginalizing over h and the redshift of sources, we have derived constraints on σ_8 and Ω_m . We show that the degeneracy between these two parameters is partially broken when the analysis is combined with data from Wide survey. Assuming $\Omega_m = 0.3$, we found that $\sigma_8 = 0.89 \pm 0.06$ for P&D and $\sigma_8 = 0.86 \pm 0.05$ with the halo model, in excellent agreement with Van Waerbeke, Mellier & Hoekstra 2005 ($\sigma_8 = 0.83 \pm 0.07$) and Hoekstra et al. 2002 ($\sigma_8 = 0.86 \pm 0.05$). Likewise, we derive $w_0 < -0.8$ using Deep data alone (see Hoekstra et al. 2005 for a deep+wide analysis).

Our results show that everything is in place to make a full scientific use the CFHTLS lensing data, and that soon with deeper Deep survey data and wider Wide survey data, we will be able to provide the best cosmological constraints from weak lensing to date. In particular, we expect to explore the growth rate of structure from a tomographic cosmic shear measurement, and to better constrain cosmological models from the non-Gaussian features derived from a joint analysis of two-point and three-point statistics. The analysis of three-point statistics in CFHTLS data goes beyond the scope of this paper. Indeed, the Deep data used for this paper, is not wide enough for such a measurement. However, three-point statistics will be investigated using future samples both for the Deep and Wide survey.

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References

- Bacon, D., Réfrégier, A., Ellis, R., 2000, MNRAS 318, 625
 Bacon, D., Massey, R. J., Réfrégier, A., Ellis, R., 2003, MNRAS 344, 673
 Bartelmann, M., Schneider, P., 2001, Phys. Rep. 340, 294.
 Benabed, K., Bernardeau, F., 2001, Phys. Rev. D 64, 083501
 Benabed, K., van Waerbeke, L., 2004, Phys. Rev. D 70, 123515
 Bernardeau, F., Van Waerbeke, L., Mellier, Y., 1997, A&A 322, 1
 Bertin, E., Arnouts, S., 1996, A&A, 117, 393
 Bolzonella, M., Miralles, J.-M., Pelló, R., 2000, A&A 363, 476-492
 Boulade, O., Charlot, X., Abbon, P., Aune, S., Borgeaud, P., Carton, P.-H., Carty, M., Da Costa, J. Deschamps, H., Desforge, D., Eppele, D., Gallais, P., Gosset, L., Granelli, R., Gros, M., de kat, J., Loiseau, D., Ritou, J.L., Strazynski, P., Vignal, N., Vigroux, L., 2003, SPIE 4841, 72
 Brodwin, M., Lilly, S.J., Porciani, C., McCracken, H.J., Le Fèvre, O., Foucaud, S., Crampton, D., Mellier, Y., 2006, ApJS, 162, 20
 Bruzual, G., Charlot, S. 1993. ApJ 405, 538
 Cooray, A. R., Huterer, D. 1999, ApJ 513, 95
 Crittenden, R., Natarajan, P., Pen, U., Theuns, T., 2001a, ApJ 559, 552
 Crittenden, R., Natarajan, P., Pen, U., Theuns, T., 2001b, ApJ 568, 20
 Erben, T., Van Waerbeke, L., Bertin, E., Mellier, Y., Schneider, P., 2001, MNRAS 366, 717
 Fahlman, G., Kaiser, N., Squires, G., Woods, D., 1994, ApJ 437, 56
 Fernández-Soto, A., Lanzetta, K. M., Yahil, A., 1999, ApJ, 513, 34
 Heymans, C., Heavens, A., 2003, MNRAS 339, 711
 Heymans, C., Brown, M. L., Barden, M., Caldwell, J. A. R., Jahnke, K., Rix, H.-W. et al., 2005b, MNRAS 361, 160.
 Heymans, C., Van Waerbeke, L., Bacon, D., Bergé, J., Bernstein, G., Bertin, E., et al., 2005a, preprint astro-ph/0506112
 Hirata, C., Seljak, U., 2003, MNRAS 343, 459
 Hirata, C., Seljak, U., 2004, PhRvD 70, 6
 Hoekstra, H., Franx, M., Kuijken, K., 2000, ApJ 532, 88

- Hoekstra, H., Yee, H.K.C., Gladders, M.D., Barrientos, L.F., Hall, P.B., Infante, L., 2002, *ApJ* 572, 55
- Hoekstra, H., 2003, IAU Symposium 216, "Maps of the Cosmos", ASP Conference Series, eds. M. Colless & L. Staveley-Smith, Sydney, July 2003, preprint astro-ph/0310908
- Hoekstra, H., 2004, *MNRAS* 347, 1337.
- Hoekstra H., Mellier Y., van Waerbeke L., Semboloni E., Fu L., Hudson M.J., Parker L.C., Tereno I., Benabed K., 2005, preprint astro-ph/0511089
- Jain, B., Seljak, U., 1997, *ApJ* 484, 560
- Jarvis, M., Jain, B. Bernstein, G., Dolney, D., 2005, preprint astro-ph/0502243
- Kaiser, N., Squires, G., Fahlman, G., Woods, D., 1994, in Durret, F., Mazure, A., Tran Thanh Van, J., Eds., *Clusters of Galaxies*. Editions Frontières. Gif-sur-Yvette, p. 269.
- Kaiser, N., Squires, G., Broadhurst, T., 1995, *ApJ* 449, 460
- Kaiser, N., Wilson, G., Luppino, G., 2000, preprint astro-ph/0003338
- King, L., Schneieder, P., 2002, *A&A*, 398, 23
- Lahav, O., Suto, Y. 2004. *Living Reviews in Relativity*. Published by the Max Planck Institute for Gravitational Physics.
- Le Fèvre, O., Vettolani, P., Garilli, B., Tresse, L., Bottini, B., et al. 2005, *A&A* 439, 845
- Linder, E. V., Jenkins, A. 2003, *MNRAS* 346, 583
- Luppino, G., Kaiser, N., 1997, *ApJ* 475, 20
- Mellier, Y., 1999. *ARAA* 37, 127.
- McCracken, H., Radovich, M., Bertin, E., Mellier, Y., Dantel-Fort, M., Le Fèvre, O., Cuillandre, J.-C., Gwyn, S., Foucaud, S., Zamorani, G., 2003, *A&A* 410, 17
- Munshi, D., Coles, P., 2003, *MNRAS*, 338, 846
- Munshi, D., Valageas, P., 2005, *MNRAS*, 360, 1401
- Peacock, J.A., Dodds, S.J., 1996, *MNRAS* 280, L9
- Pen, U.L., Van Waerbeke, L., Mellier, Y., 2002, *ApJ* 567, 31
- Perlmutter, S., Aldering, G., Goldhaber, G., Knop, R. A., Nugent, P., Castro, P. G., Deustua, S., Fabbro, S., Goobar, A., Groom, D. E, et al., 1999, *ApJ* 517, 565
- Réfrégier, A., *ARAA* 41, 643
- Riess, Adam G., Filippenko, Alexei V., Challis, Peter, Clocchiatti, Alejandro, Diercks, Alan, Garnavich, Peter M., Gilliland, Ron L., Hogan, Craig J., Jha, Saurabh, Kirshner, Robert P., et al., 1998, *ApJ* 116, 1009
- Riess, Adam G., Strolger L., Tonry J., Casertano, S., Ferguson H.C., Mobasher B., Challis P., Filippenko A.V., Jha S., Li W., Chornock R., Kirshner R.P., Leibundgut B., Dickinson M., Livio M., Giavalisco M., Steidel C. C., Benítez T., Tsvetanov, Z., 2004 *ApJ* 607, 687
- Schneider, P., 1996, *MNRAS* 283, 853
- Schneider, P., Van Waerbeke, L., Jain, B., Kruse, G., 1998, *MNRAS* 296, 873
- Schneider, P., van Waerbeke, L., Kilbinger, M., Mellier, Y., 2002, *A&A* 396, 1
- Smith, R. E., Peacock, J. A., Jenkins, A., White, S. D. M., Frenk, C. S., Pearce, F. R., Thomas, P. A., Efstathiou, G., Couchman, H. M. P., 2003, *MNRAS* 341, 1311
- Spergel, D. N., Verde, L., Peiris, H. V., Komatsu, E., Nolta, M. R., Bennett, C. L., Halpern, M., Hinshaw, G., Jarosik, N., Kogut, A., Limon, M., Meyer, S. S., Page, L., Tucker, G. S., Weiland, J. L., Wollack, E., Wright, E. L., 2003, *ApJS* 148, 175
- Van Waerbeke, L., Mellier, Y., Erben, T., Cuillandre, J.-C., Bernardeau, F., Maoli, R., Bertin, E., McCracken, H., Le Fèvre, O., Fort, B., Dantel-Fort, M., Jain, B., Schneider, P., 2000, *A&A* 358, 30
- Van Waerbeke, L., Mellier, Y., Radovich, M., Bertin, E., Dantel-Fort, M., McCracken, H. J., Le Fèvre, O., Foucaud, S., Cuillandre, J.-C., Erben, T., Jain, B., Schneider, P., Bernardeau, F., Fort, B., 2001, *A&A* 374, 757
- Van Waerbeke, L., Mellier, Y., Pelló, R., Pen, U.-L., McCracken, H. J., Jain, B., 2002, *A&A* 393, 369
- Van Waerbeke, L., Mellier, Y., astro-ph/0305089, Lecture given at the Aussois winter school, France, January 2003
- Van Waerbeke, L., Mellier, Y., Proceedings of the IAU Symposium 225 "Cosmological Physics with Gravitational Lensing", Y. Mellier & G. Meylan eds. Cambridge University Press 2005
- Van Waerbeke, L., Mellier, Y., Hoekstra, H., 2005, *A&A* 429, 75
- Wittman, D. M., Tyson, A. J., Kirkman, D., Dell'Antonio, I., Bernstein, G., 2000, *Nature* 405, 143

FIRST COSMIC SHEAR RESULTS FROM THE CANADA-FRANCE-HAWAII TELESCOPE WIDE SYNOPTIC LEGACY SURVEY¹

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ABSTRACT

We present the first measurements of the weak gravitational lensing signal induced by the large-scale mass distribution in the universe from data obtained as part of the ongoing Canada-France-Hawaii Telescope Legacy Survey (CFHTLS). The data used in this analysis are from the Wide Synoptic Survey, which aims to image ~ 170 deg² in five filters. We have analyzed an effective area of ~ 22 deg² (31 pointings) of i' data spread over two of the three survey fields. These data are of excellent quality, and the results bode well for the remainder of the survey: we do not detect a significant “ B ” mode, suggesting that residual systematics are negligible at the current level of accuracy. Assuming a cold dark matter model and marginalizing over the Hubble parameter $h \in [0.6, 0.8]$, the source redshift distribution, and systematics, we constrain σ_8 , the amplitude of the matter power spectrum. At a fiducial matter density $\Omega_m = 0.3$ we find $\sigma_8 = 0.85 \pm 0.06$. This estimate is in excellent agreement with previous studies. A combination of our results with those from the Deep component of the CFHTLS enables us to place a constraint on a constant equation of state for the dark energy, based on cosmic shear data alone. We find that $w_0 < -0.8$ at 68% confidence.

Subject headings: cosmology: observations — dark matter — gravitational lensing

Online material: color figures

1. INTRODUCTION

Weak gravitational lensing of distant galaxies by intervening massive structures provides us with a unique and unbiased way to study the distribution of matter in the universe. Although weak lensing has many applications in astronomy, from the study of galaxy dark matter halos (e.g., Brainerd et al. 1996; Hudson et al. 1998; McKay et al. 2001; Hoekstra et al. 2004) to galaxy clusters (e.g., Clowe et al. 1998; Dahle et al. 2002; Hoekstra et al. 2002a; Cypriano et al. 2004), much recent work has been devoted to the measurement of the statistical signal induced by large-scale structure (aka cosmic shear).

The reason for the recent popularity of cosmic shear is the fact that the signal is a direct measure of the projected matter power spectrum over a redshift range determined by the lensed sources and over scales ranging from the linear to nonlinear regime. This straightforward interpretation of the signal is rather unique in the growing set of tools available for cosmology and is an important

feature if we are to determine the values of cosmological parameters with high precision. We note, however, that the relation between the lensing signal and the cosmological parameters depends critically on the redshift distribution of the source galaxies. Provided that the latter information is available, weak lensing studies enable us to measure important parameters such as the matter density Ω_m and the amplitude of the power spectrum σ_8 . These observations also have the potential to constrain quintessence models (Benabed & van Waerbeke 2004). The combination of cosmic shear with other well-understood probes, such as the cosmic microwave background, is particularly powerful (e.g., Contaldi et al. 2003; Tereno et al. 2005).

The measurement of the signal, however, is not without challenges, which explains why cosmic shear has only recently appeared as a useful tool in cosmology. First of all, the induced change in the shapes of distant galaxies is small (less than 1%), much smaller than the intrinsic shapes of the sources themselves. As a result, large areas of the sky need to be surveyed in order to reduce the statistical errors. The first detections, reported only a few years ago (Bacon et al. 2000; Kaiser et al. 2000; van Waerbeke et al. 2000; Wittman et al. 2000), were based on relatively small areas (a few deg² at most). Recent developments in the construction of wide field imaging cameras on 4 m+ class telescopes have made it possible to image much larger portions on the sky to warrant accurate measurements of the lensing signal. For instance, van Waerbeke et al. (2005) presented results based on ~ 12 deg² of deep imaging data from the VIRMOS-Descart Survey. Other competitive results are based on shallower data, which cover a larger area, such as the Red-Sequence Cluster Survey (RCS; Hoekstra et al. 2002b) based on 53 deg² and the 75 deg² Cerro Tololo Inter-American Observatory (CTIO) Lensing Survey (e.g., Jarvis et al. 2003, 2006).

The second challenge is the careful removal of observational distortions, introduced by the telescope optics and atmosphere. Much work has been devoted to deal with the point-spread function (PSF), and several correction schemes have been developed

¹ Based on observations obtained with MegaPrime equipped with MegaCam, a joint project of CFHT and CEA/DAPNIA, at the Canada-France-Hawaii Telescope (CFHT), which is operated by the National Research Council (NRC) of Canada, the Institut National des Science de l’Univers of the Centre National de la Recherche Scientifique (CNRS) of France, and the University of Hawaii. This work is based in part on data products produced at TERAPIX and the Canadian Astronomy Data Centre as part of the Canada-France-Hawaii Telescope Legacy Survey, a collaborative project of NRC and CNRS.

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(e.g., Kaiser et al. 1995; Bernstein & Jarvis 2002; Refregier 2003) and tested (e.g., see Heymans et al. [2005] for the most up to date discussion). Fortunately, the separation of the signal into gradient (“*E*”-mode) and curl (“*B*”-mode) components provides a nontrivial test of the level of residual systematics, including the presence of intrinsic alignments of galaxies or observational distortions in the images. In addition, several other tests can be performed to test the accuracy of the corrections (e.g., Hoekstra et al. 2002c; Bacon et al. 2003; Heymans 2003, 2005; van Waerbeke et al. 2005). The recent results presented in van Waerbeke et al. (2005) and Jarvis & Jain (2005) have demonstrated that these systematics can be accurately corrected for, resulting in measurements free of *B* modes.

The next step is to survey much larger areas on the sky to sufficient depth in multiple filters, which enables us to probe the evolution of the matter power spectrum. The Canada-France-Hawaii-Telescope Legacy Survey (CFHTLS) aims to image $\sim 170 \text{ deg}^2$ in the *i'*-filter down to $i'_{AB} = 24.5$. This is comparable to the depth reached in the VIRMOS-Descart survey but an order of magnitude larger in survey area, thus resulting in a significant reduction in statistical errors. In addition, the fields are observed in four additional filters to ensure photometric redshift information for the sources. This is an important part of the survey, as it enables tests for intrinsic alignments and the study of the evolution of the matter power spectrum. The latter significantly improves the constraints on cosmological parameters (e.g., Hu 1999; Simon et al. 2004). Of particular interest is the improvement in the constraints on the equation of state of the dark energy (e.g., Benabed & van Waerbeke 2004).

In this paper we present the first results from the CFHTLS cosmic shear program, based on 31 pointings, resulting in an effective area of $\sim 22 \text{ deg}^2$ of *i'* data. These data comprise observations done in the first three semesters of the survey and already provide a significant improvement over previous work. The structure of the paper is as follows. In § 2 we discuss the CFHTLS and the data used in this paper, as well as the data reduction process. The weak lensing analysis is described in § 3. The resulting cosmic shear signal is presented in § 4, and the implications for cosmological parameters are discussed in § 5.

2. DATA

The CFHTLS⁸ is a joint Canadian-French program to make efficient use of Megaprime, the CFHT wide field imager, and to address a number of fundamental problems in astronomy. Megaprime, equipped with MegaCam, a 36 CCD mosaic camera with a field of view of $\sim 1 \text{ deg}^2$, enables us to obtain deep images of large areas of the sky. The survey itself consists of three independent parts, each with their own primary science driver. The survey has been allocated more than 450 nights over a ~ 5 year period.

The results presented in this paper deal with data collected as part of the wide synoptic survey, or “Wide Survey” for short. Once completed, it will cover $\sim 170 \text{ deg}^2$ in three patches of $49\text{--}72 \text{ deg}^2$ each through the whole filter set (u^*, g', r', i', z') down to $i' = 24.5$. The survey allows the study of the large-scale structure and matter distribution in the universe through weak lensing and the galaxy distribution. The former application is the focus of this paper. We also use cosmic shear results based on multi-color data from the Deep component of the survey, which are described in detail in Semboloni et al. (2005).

For the analysis presented here we use 19 pointings in the *i'*-band from the W1 field and 12 pointings from the W3 field,

resulting in an effective survey area of 22 deg^2 after masking. The layout of the current data with respect to the final survey is presented in Figure 1. These data were obtained during the 2003B, 2004A, and 2004B observing semesters. In addition, 4 deg^2 of the W2 field have been observed, but the area is not contiguous and we omit these data from the current analysis. Although more data have been taken since, we have chosen to limit the current data set to these three semesters as it represents a significant improvement over previously published results and provides a reference for future analyses.

2.1. Data Reduction

Detrended data (debiased and flatfielded) are provided by CFHT to the community through the Canadian Astronomical Data Centre. The detrending is done using the Elixir pipeline developed at CFHT. The pipeline also provides photometric zero points and in most cases a reasonable first-order astrometric solution.

The photometric calibrations are based on observations of standard stars during the observing run. These zero points are only valid under photometric conditions. We therefore examine the magnitudes of a large number of objects in the images to check the stability of the photometric zero point and scale the images to the brightest image. Most data were taken under photometric conditions, and the corrections are found to be small.

The astrometric solutions provided by the Elixir pipeline are more reliable for the more recent data, whereas a small fraction of chips of the earliest observations require a significant revision of the initial astrometry. The provided astrometric solution, however, is not accurate enough for the purpose of our study and needs to be improved. This process is too time-consuming to be done manually and is done using an additional pipeline. The USNO-A2 (Monet et al. 1998) catalog could be used to refine the astrometry, but the number density of sources is often too low to warrant stable results. Instead, we retrieved a red image from the second-generation Digital Sky Survey (POSS II; Reid et al. 1991) for each pointing. These observations have small geometric distortions. The astrometry of the POSS II image is calibrated using the USNO-A2 catalog. SExtractor (Bertin & Arnouts 1996) is used to generate a catalog of sources with accurate astrometry, with a number density significantly higher than the USNO-A2 catalog. In addition, the POSS II images have been taken more recently, thus reducing the effects of proper motions of the stars.

This new astrometric catalog is matched to each of the MegaCam images. The exposures have been taken with different offsets, in order to fill the gaps between the chips. We combine the matched catalogs for each exposure into a master catalog, which contains the average positions of the matched objects. This master catalog is used to derive the final second-order astrometric solution for each chip. This procedure ensures that in the overlapping area, the objects in each exposure are accurately matched to the same position, which is crucial when stacking the images for a weak lensing analysis: errors in the astrometry lead to additional anisotropies in the images.

Tests show that the resulting astrometric solution for each pointing is sufficiently accurate to stack all data into a large image. However, in this case different chips can contribute to the image at a given position. If the PSF properties “jump” between chips, this gives rise to a complicated PSF anisotropy pattern. Instead, we use only those regions of the sky that in the final stacks were all observed by the same chip. This avoids complicated behavior of the PSF, at the expense of losing approximately 20% of the survey area. In the future we plan to investigate in

⁸ See <http://www.cfht.hawaii.edu/Science/CFHTLS>.

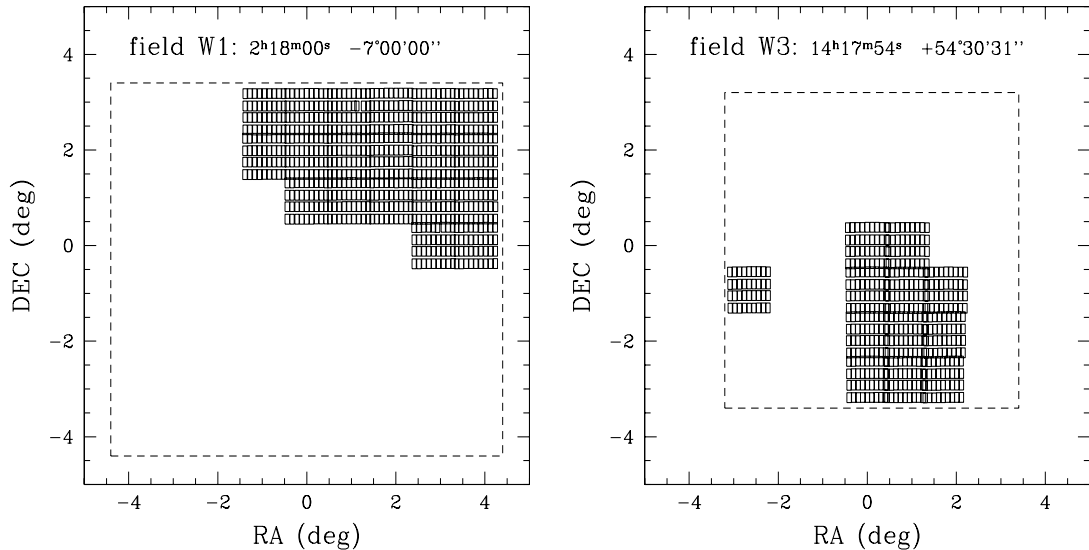


FIG. 1.—Layout of the observations of the W1 and W3 field. Each rectangle corresponds to one of the MegaCam chips. The dashed box indicates the planned size of each of the two fields, 72 deg^2 for W1 and 49 deg^2 for W3. Currently, 19 deg^2 of W1 and 12 deg^2 of W3 have been observed.

more detail to what extent we can deal with the PSF on full mosaic images.

Before stacking the images, we identify stars on chip 22 (which has the best image quality; see Fig. 2) and measure the FWHM of the PSF and PSF anisotropy. If the seeing of some of the exposures is significantly worse than the others, or if the images show extreme PSF anisotropy, those images are discarded before stacking the images. We stack each chip separately, using the SWarp routine. The resulting images are cropped such that the overlapping regions remain. These images are used in the weak lensing analysis presented here. Typically, the stacked images

consist of seven images, each with an integration time of 620 s, but because of reasons described above, in a few cases we end up with stacks of six images. The improvement in the PSF behavior warrants the minor decrease in depth.

Figure 2 shows the seeing distribution (or PSF size) as measured from the stacked images. Figure 2a shows the distribution for stars selected on chip 22, which typically has the best image quality. For this particular chip we find a median seeing of $0''.71$. Toward the edge of the field of view the image quality degrades and a more accurate representation of the PSF size distribution is presented in Figure 2b, which shows a histogram of the distribution

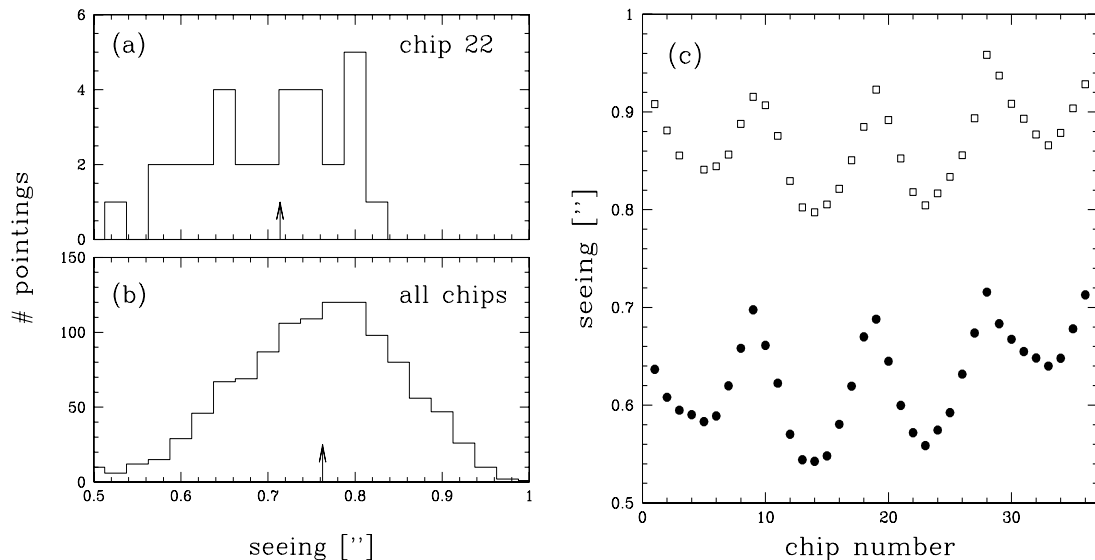


FIG. 2.—(a) FWHM of stars measured from the stacked images of chip 22 for all pointings. This chip typically has the best image quality (see panel c). (b) FWHM measured for all chips, providing a more accurate representation of the PSF size distribution. The arrows indicate the median seeing for both distributions. (c) The PSF size as a function of chip number for the best seeing image (*filled circles*) and the worst seeing image (*open squares*). The camera consists of four rows of nine chips each. The image quality is best in the center of the camera and degrades toward the edge, as indicated by these measurements.

based on all chips. In this case the median FWHM is $0''.76$. We note that recent changes to Megaprime have led to a significant improvement in image quality and a large reduction in PSF anisotropy, thus reducing the level of systematics in future cosmic shear measurements.

Nevertheless, the image quality in these early data is better than $1''$ for all data used here. Figure 2c shows the PSF size as a function of chip number for the best seeing image (*filled circles*) and worst seeing image (*open squares*). The camera consists of four rows of nine chips, which results in the “periodic” changes in the seeing. In both cases the PSF size is best in the middle of the camera and increases toward the edges.

3. WEAK LENSING ANALYSIS

The weak lensing analysis presented here is based on the method proposed by Kaiser et al. (1995) and Luppino & Kaiser (1997) with a number of modifications that are described in Hoekstra et al. (1998, 2000). This particular method has been tested in great detail (e.g., Hoekstra et al. 1998, 2002c; Bacon et al. 2001; Erben et al. 2001; Heymans et al. 2005) and is widely used for cosmic shear studies. Several promising alternative methods have been developed recently (e.g., Bernstein & Jarvis 2002; Refregier 2003), and tests are underway to ensure accurate measurements of the lensing signal.

Although the Kaiser et al. (1995) method is not the “ultimate” technique to extract the lensing signal from the images, it has proven to be one of the most accurate techniques currently available when tested on simulated data as part of the Shear TEsting Program (STEP; Heymans et al. 2005). In this experiment, various weak lensing pipelines were used to measure the shear in simulated images. The simulated data consisted of a series with five different shears (constant across the images) and five different (fairly realistic) PSFs, each introducing different systematics. Hence, STEP provides a test of how one can correct for both PSF anisotropy and the size of the PSF. However, the simulations do not capture all details of real PSFs or imperfections introduced by the stacking process. In this experiment, the Hoekstra et al. (1998, 2000) implementation of the Kaiser et al. (1995) approach was able to recover the lensing signal with an accuracy better than 2%. This accuracy is sufficient for published cosmic shear results and is acceptable for the results presented here, but we note that improvements are likely needed when analyzing the complete CFHTLS data set.

As mentioned earlier, we analyze the stacked images for each chip individually. The first step in the analysis is the detection of objects, for which we used `hfindpeaks`, the hierarchical peak-finding algorithm from Kaiser et al. (1995). We select objects that are detected with a significance greater than 5σ over the local sky. In addition to the positions of the objects, the peak finder also provides fair estimates of the object’s size. We use this information to remove all objects smaller than the PSF. Inspection shows that these are either extremely faint objects or, more relevant, spurious detections of diffraction spikes, etc. The remaining objects are analyzed in more detail, which yields estimates for the size, apparent magnitude, and shape parameters (polarization and polarizabilities). The apparent magnitudes are corrected for galactic extinction using the results from Schlegel et al. (1998).

The images were inspected by eye to mask out areas where the shape measurements could be compromised. Potential sources can be cosmetic, such as bleeding stars, halos, diffraction spikes, but also astronomical, such as H II regions or spiral structure in resolved galaxies. We separate the stars and galaxies on the ba-

sis of their half-light radii. The galaxy shapes are corrected for observational distortions as described below. The final, corrected catalogs for each chip are combined into a large catalog for each field.

For the weak lensing analysis we use only galaxies brighter than $i'_{AB} = 24.5$, which leaves a sample of 9.7×10^5 galaxies in the W1 field and 6.5×10^5 galaxies in the W3 field. The surveyed area is $\sim 22 \text{ deg}^2$, resulting in an average galaxy number density of $\sim 20 \text{ galaxies arcmin}^{-2}$. However, the shape measurement errors are larger for faint, small galaxies. Therefore, these galaxies should be given less weight for an optimal estimate of the lensing signal (Hoekstra et al. 2000).

Instead it is more convenient to define an effective number density of galaxies n_{eff} , which is related directly to the measurement error in the shear for an area of 1 arcmin^2 , which is the relevant quantity. *Hubble Space Telescope* observations indicate that for well-resolved galaxies the intrinsic shapes of galaxies results in a dispersion $\langle \gamma^2 \rangle^{1/2} \approx 0.3$ (Hoekstra et al. 2000). If all galaxies were measured “perfectly,” the error in the shear measurement would be given by $\sigma_\gamma = 0.3/(n_{\text{eff}})^{1/2}$. For the CFHTLS data this yields an effective number density of $\sim 12 \text{ galaxies arcmin}^{-2}$.

3.1. Correction for PSF Effects

The observed shapes of the galaxies cannot be used to measure the lensing signal, because observational distortions have significantly altered their shapes in a systematic fashion: PSF anisotropy introduces coherent alignments in the galaxy shapes and the seeing circularizes the images. The key to an accurate measurement of the weak lensing signal lies in the adequate correction for these systematic effects. As mentioned above, our pipeline has been tested extensively (e.g., Hoekstra et al. 2002c; Heymans et al. 2005) and has been shown to be able to recover the weak lensing shear with an accuracy of $\sim 2\%$.

The first step in this procedure is the identification of a sample of moderately bright stars that can be used to quantify the properties of the PSF (anisotropy and size). The pattern of PSF anisotropy changes from observation to observation, although we found that the Megacam PSF is relatively stable, in particular when compared to the previous CFH 12k camera (e.g., Hoekstra 2004). The PSF anisotropy also varies across the field of view, and this spatial variation is captured by fitting a second-order polynomial to the shape parameters of the stars for each chip.

We found that a second-order polynomial provided an excellent fit to the data. Note that the choice of model depends on the properties of the camera. For instance, van Waerbeke et al. (2005) used rational functions (as suggested by Hoekstra 2004) to describe the pattern of the CFH 12k camera. We intend to improve our characterization following the procedure developed by Jarvis & Jain (2005), which combines information from a series of exposures, effectively resulting in a denser sampling of the PSF variation.

A typical example of the PSF anisotropy as a function of position across the Megacam mosaic is presented in Figure 3. To show the spatial variation in more detail, we have subtracted the mean anisotropy across the field. The resulting pattern is coherent across the field, even though the fits were obtained from individual chips.

Having quantified the PSF anisotropy and its spatial variation, we can undo its effect following Kaiser et al. (1995) and Hoekstra et al. (1998). The same stars used to study the PSF anisotropy are also used to correct for the diluting effect of seeing,

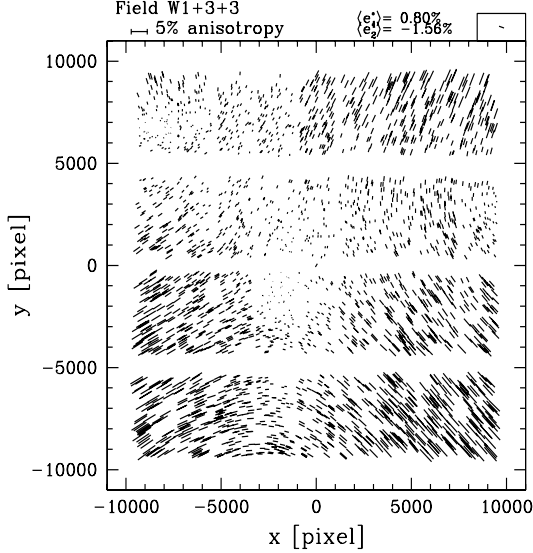


FIG. 3.—Typical example of the PSF anisotropy as a function of position for a stacked Megacam image. The sticks indicate the direction of the major axis of the PSF, and the length is proportional to the observed ellipticity of the PSF. In order to show the higher order spatial dependence of the anisotropy, we have subtracted the average ellipticity. The direction of the average PSF anisotropy is indicated in the top right box, and the amplitude is indicated as well. Although the PSF anisotropy is determined for individual chips, the figure clearly shows a large-scale coherent pattern.

as described in Luppino & Kaiser (1997) and Hoekstra et al. (1998).

4. MEASUREMENTS

To quantify the lensing signal, we measure the ellipticity (or shear) correlation functions from the galaxy shape catalogs. These correlation functions, in turn, can be related to the various two-point statistics that are commonly used in the literature. The use of the ellipticity correlation functions also allow for the separation of the signal into *E*-mode (gradient) and *B*-mode (curl) components. Gravitational lensing arises from a gravitational potential, and it is therefore expected that the lensing signal is curl-free. We note that Schneider et al. (2002b) showed that source clustering gives rise to small-amplitude *B* modes. The level of the latter *B* modes is much smaller than the *B* modes expected from systematics in the current analysis. Therefore, the amplitude of the observed *B* mode provides a measure of residual systematics.

In § 5 we briefly discuss how one of these observable two-point statistics relates to the matter power spectrum and cosmology. For more detailed discussions we refer to Schneider et al. (1998) and Bartelmann & Schneider (2001).

The two ellipticity correlation functions that are measured are

$$\xi_{tt}(\theta) = \frac{\sum_{i,j}^{N_i} w_i w_j \gamma_{t,i}(\mathbf{x}_i) \cdot \gamma_{t,j}(\mathbf{x}_j)}{\sum_{i,j}^{N_i} w_i w_j} \quad (1)$$

and

$$\xi_{rr}(\theta) = \frac{\sum_{i,j}^{N_i} w_i w_j \gamma_{r,i}(\mathbf{x}_i) \cdot \gamma_{r,j}(\mathbf{x}_j)}{\sum_{i,j}^{N_i} w_i w_j}, \quad (2)$$

where $\theta = |\mathbf{x}_i - \mathbf{x}_j|$; γ_t and γ_r are the tangential and 45° rotated shear in the frame defined by the line connecting the pair of galaxies. The weights w_i are proportional to the inverse square of the uncertainty in the shear (Hoekstra et al. 2000). For the following, it is more useful to consider

$$\xi_{\pm}(\theta) = \xi_{tt}(\theta) \pm \xi_{rr}(\theta), \quad (3)$$

i.e., the sum and the difference of the two observed correlation functions. As shown by Crittenden et al. (2002), one can derive *E*- and *B*-mode correlation functions by integrating $\xi_{+}(\theta)$ and $\xi_{-}(\theta)$ with an appropriate window function

$$\xi^E(\theta) = \frac{\xi_{+}(\theta) + \xi'_{-}(\theta)}{2} \quad \text{and} \quad \xi^B(\theta) = \frac{\xi_{+}(\theta) - \xi'_{-}(\theta)}{2}, \quad (4)$$

where

$$\xi'_{-}(\theta) = \xi_{-}(\theta) + 4 \int_{\theta}^{\infty} \frac{d\vartheta}{\vartheta} \xi_{-}(\vartheta) - 12\theta^2 \int_{\theta}^{\infty} \frac{d\vartheta}{\vartheta^3} \xi_{-}(\vartheta). \quad (5)$$

Evaluating the latter equation requires knowledge of the ellipticity correlation function ξ_{-} out to infinity. However, as shown in Schneider et al. (2002b), the expressions for ξ^E and ξ^B can also be expressed as

$$\xi^E(\theta) = \frac{\xi_{-}(\theta) + \xi'_{+}(\theta)}{2} \quad \text{and} \quad \xi^B(\theta) = -\frac{\xi_{-}(\theta) - \xi'_{+}(\theta)}{2}, \quad (6)$$

where

$$\xi'_{+}(\theta) = \xi_{+}(\theta) + 4 \int_0^{\theta} \frac{d\vartheta}{\vartheta} \xi_{+}(\vartheta) - 12\theta^2 \int_0^{\theta} \frac{d\vartheta}{\vartheta^3} \xi_{+}(\vartheta). \quad (7)$$

In this case, we need to know ξ_{+} to arbitrarily small radii. In practice, we know ξ_{+} only to some minimum radius θ_{\min} . Provided θ_{\min} is small, we can approximate ξ_{+} to be constant for $\theta < \theta_{\min}$. As a result, the correction to the correlation functions is also a constant. As discussed below, the measurement of the *B* mode of the aperture mass statistic allows us to identify the range of scales where the observed *B* mode vanishes. We use this range to estimate the correction to the correlation functions such that $\langle \xi_B \rangle$ vanishes.

We also present results for other frequently used two-point statistics, namely, the aperture mass variance $\langle M_{\text{ap}}^2 \rangle(\theta)$ and the top-hat smoothed variance $\langle \gamma^2 \rangle(\theta)$. Of these statistics, the aperture mass is of some particular interest, because only for this case are the *E* and *B* modes uniquely defined, whereas the decompositions of the shear correlation function and the top-hat variance in *E* and *B* modes are defined up to a constant (Crittenden et al. 2002; Pen et al. 2002). The aperture mass is defined as

$$M_{\text{ap}}(\theta) = \int_0^{\infty} d^2\vartheta U(\vartheta) \kappa(\vartheta), \quad (8)$$

where κ is the dimensionless surface density or convergence. Provided $U(\vartheta)$ is a compensated filter (i.e., a filter such that a constant surface density within the aperture yields $\langle M_{\text{ap}} \rangle = 0$), the aperture mass can be expressed in terms of the observable

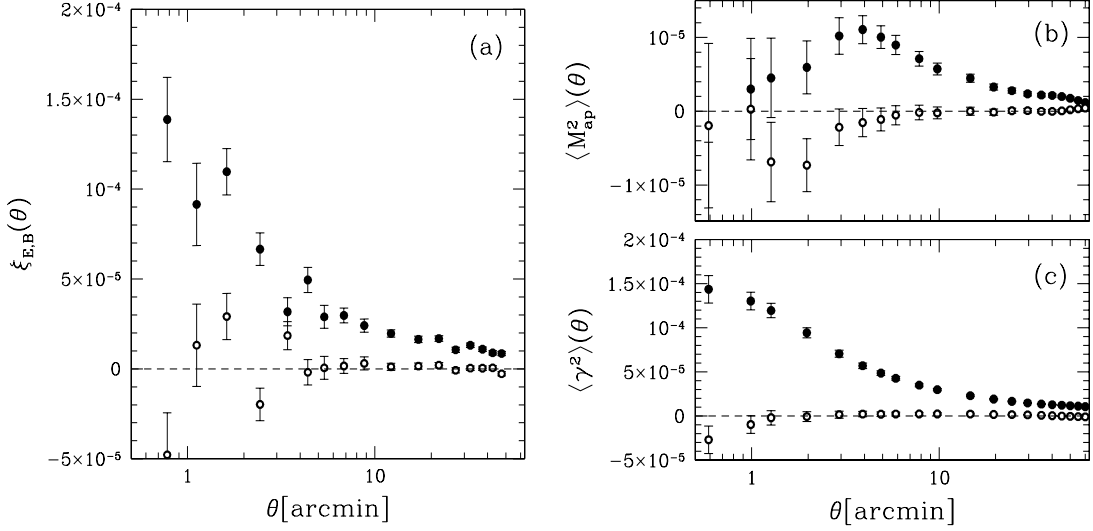


FIG. 4.—(a) E - and B -mode shear correlation functions (filled and open points, respectively) measured from ~ 22 deg² of i' data from the CFHT Legacy Survey. The error bars only indicate the statistical errors determined for the combined signal from the data from W1 and W3 fields. (b) E - and B -mode measurements for the aperture mass statistic. Note that the B mode is consistent with zero on all scales. (c) Same as above, but now for the top-hat variance.

tangential shear γ_t using a different filter function $Q(\vartheta)$ [which is a function of $U(\vartheta)$],

$$M_{ap}(\theta) = \int_0^\theta d^2\phi Q(\vartheta) \gamma_t(\vartheta). \quad (9)$$

We use the filter function suggested by Schneider et al. (1998):

$$U(\theta) = \frac{9}{\pi\theta^2} \left(1 - \frac{\vartheta^2}{\theta^2}\right) \left(\frac{1}{3} - \frac{\vartheta^2}{\theta^2}\right) \quad (10)$$

for $\theta \leq \vartheta$, and 0 elsewhere. The corresponding $Q(\theta)$ is given by

$$Q(\theta) = \frac{6}{\pi\theta^2} \left(\frac{\vartheta^2}{\theta^2}\right) \left(1 - \frac{\vartheta^2}{\theta^2}\right) \quad (11)$$

for $\theta \leq \vartheta$, and 0 elsewhere. The E - and B -mode aperture masses are computed from the ellipticity correlation functions using

$$\langle M_{ap}^2 \rangle(\theta) = \frac{1}{2} \int \frac{d\vartheta d\vartheta'}{\theta^2} \left[\xi_+(\vartheta) T_+ \left(\frac{\vartheta}{\theta}\right) + \xi_-(\vartheta) T_- \left(\frac{\vartheta}{\theta}\right) \right], \quad (12)$$

and

$$\langle M_{\perp}^2 \rangle(\theta) = \frac{1}{2} \int \frac{d\vartheta d\vartheta'}{\theta^2} \left[\xi_+(\vartheta) T_+ \left(\frac{\vartheta}{\theta}\right) + \xi_-(\vartheta) T_- \left(\frac{\vartheta}{\theta}\right) \right]. \quad (13)$$

The $T_{\pm}(x)$ vanish for $x > 2$, so that $\langle M_{ap}^2 \rangle$ can be obtained directly from the observable ellipticity correlation functions over a finite interval. Expressions for $T_{\pm}(x)$ can be found in Crittenden et al. (2002) and Schneider et al. (2002b). Similarly, the top-hat variance $\langle \gamma^2 \rangle$ can be expressed in terms of the ellipticity correlation functions, but with different weight functions:

$$\langle \gamma^2 \rangle_{E,B}(\theta) = \int \frac{d\vartheta d\vartheta'}{2\theta^2} \left[\xi_+(\vartheta) S_+ \left(\frac{\vartheta}{\theta}\right) \pm \xi_-(\vartheta) S_- \left(\frac{\vartheta}{\theta}\right) \right], \quad (14)$$

where $S_{\pm}(x)$ vanish for $x > 2$ and the expressions for $x \leq 2$ can be found in Schneider et al. (2002a).

4.1. Lensing Signal

The results for the various two-point statistics are presented in Figure 4. The filled points in Figure 4a indicate the observed E -mode shear correlation function as a function of angular scale. The error bars indicate only the statistical uncertainty in the measured signal and ignore any contribution from cosmic variance that is present in the E -mode signal. However, when estimating cosmological parameters, as discussed in § 5, we include estimates for cosmic variance. The open points correspond to the B mode. The signal presented in Figure 4 is based on the 22 deg² of data for the W1 and W3 fields.

As mentioned previously, in order to separate the signal into E and B modes, we have to define a zero-point for the B mode for the shear correlation function and top-hat variance. On the basis of the absence of B modes in the observed aperture mass variance, shown in Figure 4b, the correlation function in Figure 4a has been adjusted such that the mean B mode is zero on scales larger than $10'$.

Figure 4c shows the top-hat variance $\langle \gamma^2 \rangle$ as a function of scale. All three statistics have been used in the past to estimate cosmological parameters, and each has its own distinct advantage and disadvantage. For instance, the aperture mass provides an absolute calibration of the B mode but is most sensitive to the matter power spectrum on small scales. To probe the largest scales available in the surveyed area, we chose to use the top-hat variance for our estimates of cosmological parameters, presented in § 5.

The results presented in Figure 4 combine the measurements of the W1 and W3 fields. However, it is useful to compare the signals obtained from the individual fields as well, to check for consistency. The aperture mass variances for the two fields are shown in the upper panel of Figure 5. The solid points correspond to W1, whereas the open points are for W3. For reference we also show the results from van Waerbeke et al. (2005), indicated by the shaded region. The latter uses a similar range in

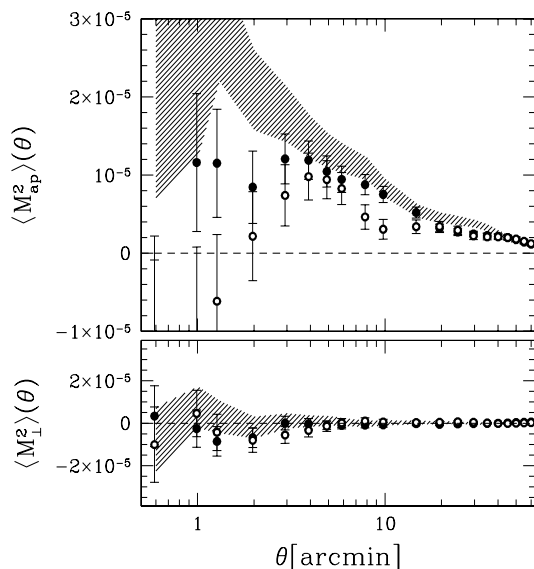


FIG. 5.—*Top*: Aperture mass variance determined from W1 (filled circles) and W3 (open circles) separately. For reference, the shaded region indicates the 1σ area around the measurement from the VIRMOS-Descart survey (van Waerbeke et al. 2005), which is of similar depth. The W1 measurements agree well with the latter, but the W3 signal is somewhat lower. On large scales (beyond $10'$) the agreement between all three measurements is excellent. Note, however, that the error bars do not include cosmic variance. *Bottom*: Aperture mass B mode for all three measurements, which are all consistent with zero on all scales.

apparent magnitude, although the filter is different (I_C vs. i' used here). Despite this difference, the amplitude should be comparable to our measurements. On scales beyond $10'$, all three measurements are in good agreement, but on small scales, the results from W3 appear lower than expected. The origin of this discrepancy is not clear, and increasing the surveyed area for this field might provide a better understanding. The lower panel of Figure 5 shows the corresponding B modes, which are all consistent with zero on all scales.

This paper presents the first CFHTLS results using one of two reduction and analysis pipelines. An independent analysis was performed by the Paris group, and for reference we also show a comparison with their preliminary results in Figure 6. A detailed discussion of this alternative analysis will be presented in L. Fu et al. (2006, in preparation). The top panels in Figure 6 show the aperture mass variance for the W1 and W3 fields. The points correspond to the results from the analysis presented in this paper. The shaded regions indicate the 1σ area around the measurements from the Paris pipeline (L. Fu et al. 2006, in preparation). The lower panels show the corresponding measurements of the B -mode signals. The results from the two pipelines agree very well. We note, however, that the Paris results are based on conservatively masked images, resulting in a smaller effective survey area than the one presented in this paper.

4.2. Tests for Residual Systematics

The absence of a significant B mode is very encouraging. However, by itself it cannot guarantee that the results are free of systematics (the presence of a B mode provides strong evidence for residual systematics, but the converse is not true). Fortunately, several other tests can be performed as well. These tests

essentially provide additional checks on the correction for PSF anisotropy, and we present results in this section.

If the correction for the size of the PSF (i.e., seeing) is imperfect, this can result in calibration biases (e.g., Hirata & Seljak 2003). Unfortunately, there is no good test of this correction, based on the data alone. Instead, one needs to rely on a comparison with simulated data sets, such as STEP (Heymans et al. 2005). As mentioned above, the results of the latter experiment suggest that we can recover the shear to better than 2%. We assume a systematic error of this magnitude when estimating cosmological parameters.

Depending on the set of cosmological parameters one is interested in, one can marginalize over the uncertainty in this correction, with little loss in accuracy (Ishak et al. 2004). This approach, however, may fail when studying the effect of massive neutrinos (e.g., Abazajian & Dodelson 2003). However, such an approach does not take into account the variable bias that might arise from pointing to pointing seeing variations. Fortunately, as shown in Vale et al. (2004), the signal is not affected significantly by small variations in calibration errors, thus reducing the complexity of the problem for current data sets.

As was shown in Hoekstra (2004) and van Waerbeke et al. (2005) the PSF anisotropy leads to a systematic signal that can have different E and B modes. In these studies, the systematic B mode was ~ 3 times lower than the systematic E mode. Furthermore, the source of an imperfect correction for PSF anisotropy can be twofold. First of all, the correction method itself might introduce errors. This has been studied in great detail (e.g., Hoekstra et al. 1998; Heymans et al. 2005), demonstrating that the correction scheme used here is sufficiently accurate. Second, as pointed out in Hoekstra (2004), an incorrect model for the spatial variation of the PSF anisotropy can lead to considerable errors.

Recently, Jarvis & Jain (2005) have presented an interesting way to improve the PSF model using the large number of exposures taken in a weak lensing survey. They analyze the PSF variation using a principal component analysis to identify the dominant PSF anisotropy patterns. Under the assumption that only a few parameters are needed to describe most of the variation, this approach allows for a much more detailed modeling of the PSF. We plan to implement this method for future analyses of the CFHTLS data.

We can test both of these potential points of failure. The accuracy of the model describing the spatial variation of the PSF anisotropy can be examined by correcting the measured shapes of the stars and computing the aperture mass variance of the residuals. In this case, any residual signal is caused by imperfections of the model (Hoekstra 2004). Figure 7 shows the results of this test. For reference, the upper panel shows the aperture mass variance of the stars before correction for PSF anisotropy. The E (filled circle) and B modes (stars) are very similar on scales $< 10'$, but on larger scales the E mode is significantly larger. Given that this signal is larger than the cosmic shear signal on scales larger than $10'$, this figure clearly demonstrates the importance of a careful correction for PSF anisotropy.

The bottom panel of Figure 7 shows the aperture mass variance using the residual shapes of stars after they have been corrected for PSF anisotropy. The E and B modes are indistinguishable on all scales. The results indicate that the adopted PSF model is inaccurate on scales between $4'$ and $20'$. Note, however, that the level of the residuals is about a factor of 30 smaller than the cosmic shear signal. Although there is clearly room for improvement regarding the modeling of the spatial variation of the PSF, the results presented in Figure 7 suggest that this is not a dominant source of error in our current analysis.

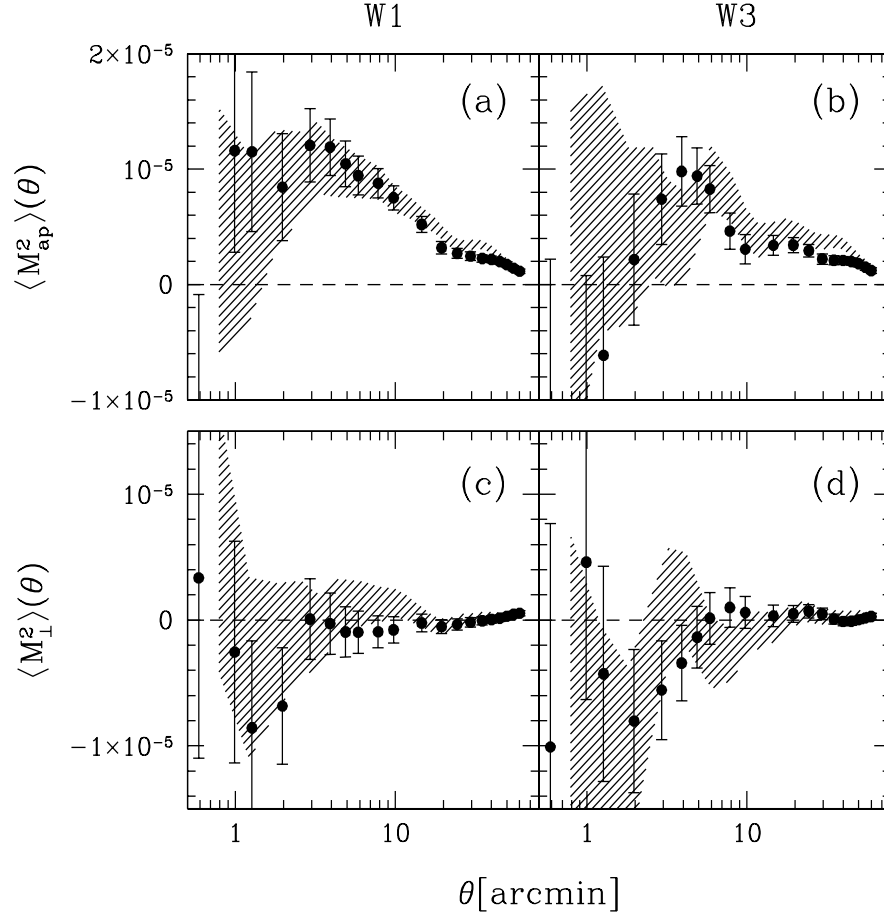


FIG. 6.—(a, b) Aperture mass variance from the W1 and W3 fields, respectively. The points correspond to the results presented here. The shaded regions indicate the 1σ area around the measurements from the Paris pipeline (L. Fu et al. 2006, in preparation). (c, d) The aperture mass B -mode signal for the W1 and W3 fields. The results from the two pipelines agree very well. Note that the Paris results are based on a smaller survey area than the one presented in this paper.

The next step is to examine how well the shapes of the background galaxies have been corrected. The amount of residual systematics left in the weak lensing signal due to imperfect PSF correction can be estimated from the correlation between the uncorrected stars and the corrected galaxies. Bacon et al. (2003) and Heymans (2003) defined a useful estimator

$$\xi_{\text{sys}} = \frac{\langle e^* \gamma \rangle^2}{\langle e^* e^* \rangle}, \quad (15)$$

where e^* is the ellipticity of the stars before PSF correction and γ is the shear estimate of the galaxies. The estimator is conveniently normalized by the star ellipticity autocorrelation function, which allows for a direct comparison to the lensing signal $\langle \gamma(r) \gamma(\theta + r) \rangle$. Note that this estimator is sensitive to imperfections in the model for PSF anisotropy and imperfections in the correction scheme itself.

The points in Figure 8 correspond to the resulting value for ξ_{sys} as a function of scale. The indicated error bars are those of the shear correlation function and are displayed to show the level of systematics with respect to the 1σ statistical error in the lensing signal. The shaded region corresponds to the 1σ region around the observed shear correlation itself, which is an order of

magnitude larger than ξ_{sys} . These results demonstrate that the signal presented in Figure 4 is not significantly affected by systematics and can be used to provide reliable constraints on cosmological parameters. Note, however, that as the CFHTLS progresses, the level of residual systematics needs to be reduced.

5. COSMOLOGICAL PARAMETERS

The observed two-point statistics can be related to the matter power spectrum $P_\delta(k)$, which depends on a range of cosmological parameters. For the study presented here, the most relevant parameters are the matter density Ω_m and the normalization σ_8 . In the case of cosmic shear, some of the dependence on cosmology also enters through the angular diameter distances to the sources.

5.1. Method

As discussed above, in order to probe the largest scales available in the surveyed area, we choose to use the top-hat variance $\langle \gamma^2 \rangle$, which can be expressed in terms of the power spectrum through

$$\langle \gamma^2 \rangle(\theta) = 2\pi \int_0^\infty dl l P_\delta(l) \left[\frac{J_1(l\theta)}{\pi l \theta} \right]^2, \quad (16)$$

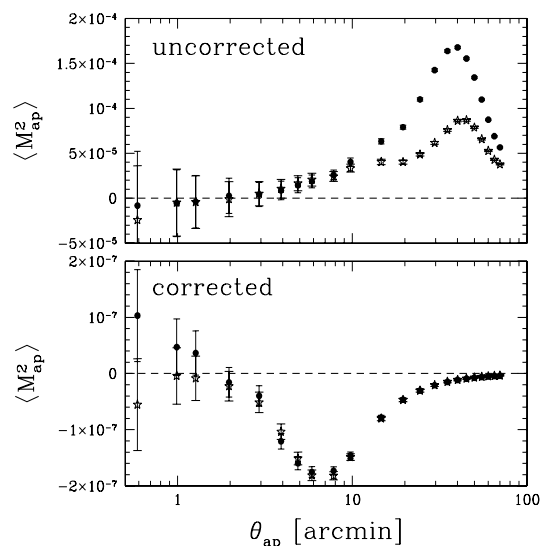


FIG. 7.—*Top*: Aperture mass variance computed from the uncorrected shapes of the stars. The *E* (filled circles) and *B* modes (stars) are very similar on scales $< 10'$, but on larger scales the *E* mode is larger. *Bottom*: The aperture mass using the residual shapes of stars after correcting for PSF anisotropy. The *E* and *B* modes are indistinguishable on all scales. The results suggest that the adopted PSF model is inadequate on scales between $4'$ and $20'$. Note, however, that the lensing signal on these scales is ~ 30 times larger than the residuals presented here.

where θ is the radius of the aperture used to compute the variance, and J_1 is the first Bessel function of the first kind. The function $P_\kappa(l)$ is not the power spectrum itself, but the convergence power spectrum, defined as

$$P_\kappa(l) = \frac{9H_0^4 \Omega_m^2}{4e^4} \int_0^{w_H} dw \left[\frac{\bar{W}(w)}{a(w)} \right]^2 P_\delta \left(\frac{l}{f_K(w)}; w \right), \quad (17)$$

where w is the radial (comoving) coordinate, w_H corresponds to the horizon, $a(w)$ is the cosmic scale factor (normalized to $a = 1$ today), and $f_K(w)$ is the comoving angular diameter distance. Note that $f_K(w) = w$ for the adopted flat geometry; $\bar{W}(w)$ is the source-averaged ratio of angular diameter distances D_{ls}/D_s for a redshift distribution of sources $n(w)$:

$$\bar{W}(w) = \int_w^{w_H} dw' n(w') \frac{f_K(w' - w)}{f_K(w')}. \quad (18)$$

Hence, it is important to know the redshift distribution of the sources, in order to relate the observed lensing signal to $P_\kappa(l)$ and consequently, cosmological parameters.

For our selection of background galaxies ($21.5 < i' < 24.5$), the redshift distribution should be very similar to the one used by van Waerbeke et al. (2005) in their analysis of the VIRMOS-Desart data. Currently, only i' images have been processed, but as the survey progresses photometric redshifts for the sources will be determined, which greatly enhances our ability to constrain the cosmology, and to minimize the uncertainty in the source redshift distribution.

Unfortunately, the current observational constraints on the redshift distribution are still limited, although a number of redshift surveys are targeting the faint galaxies and redshift range used for weak lensing (e.g., DEEP2: Davis et al. 2003; VVDS:

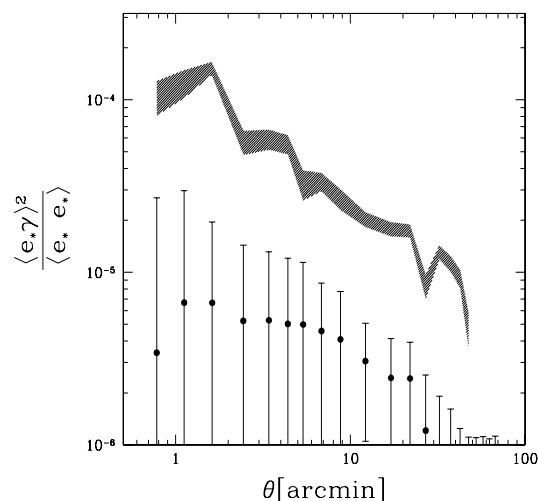


FIG. 8.—Points with error bars show the residual systematic correlation function ζ_{sys} as defined in the text. The indicated error bars are those of the shear correlation function and are displayed to show the level of systematics with respect to the 1σ statistical error. The shaded region corresponds to the 1σ region around the observed shear correlation itself, which is an order of magnitude larger than ζ_{sys} .

Le Fèvre et al. 2004). For the analysis here, the best information on the source redshifts comes from photometric redshift studies of the Hubble Deep Fields (HDFs; Fernández-Soto et al. 1999). It is convenient to parameterize the redshift distribution using (e.g., Efstathiou et al. 1991; Brainerd et al. 1996)

$$n(z) = \frac{\beta}{z_s \Gamma[(1 + \alpha)/\beta]} \left(\frac{z}{z_s} \right)^\alpha \exp \left[- \left(\frac{z}{z_s} \right)^\beta \right]. \quad (19)$$

The best-fit parameters are obtained from a least-squares fit to the photometric redshift distribution, taking into account the adopted weights for the shape measurements of the sources (see the discussion in § 4). The weight is predominantly a function of apparent magnitude, and the average value is shown in Figure 9. As fainter galaxies are, on average, at higher redshift, such a weighting scheme will modify the true source redshift distribution into an “effective” one. To compute the best-fit parameters, we use the Poisson errors in the counts as a function of redshift, thus ignoring field-to-field variation in the HDF redshift distributions.

The solid histogram in Figure 10 shows the effective source redshift distribution. Comparison with the dashed histogram (which corresponds to the unweighted case) shows that the weighting scheme slightly lowers the mean source redshift. The smooth curve corresponds to the best-fit redshift distribution, which has parameters $\alpha = 1.35$, $\beta = 1.654$, and $z_s = 0.668$. This corresponds to a mean source redshift of $\langle z \rangle = 0.76$. To quantify the uncertainties in the source redshift distribution, we keep α and β fixed but vary z_s to identify the 68% and 95% confidence intervals. This yields $z_s \in [0.632, 0.703]$ with 68% confidence and $z_s \in [0.613, 0.721]$ with 95% confidence. We marginalize over the latter interval when estimating the cosmological parameters.

The scales probed by the measurements are affected significantly by the nonlinear growth of structure. As shown by Jain & Seljak (1997) and Schneider et al. (1998), we cannot use the linear power spectrum, but it is necessary to use the nonlinear

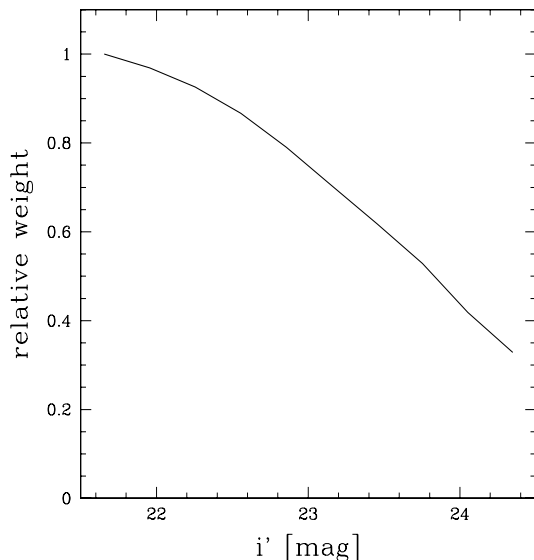


FIG. 9.—Average weight per galaxy as a function of apparent i' -magnitude. Note that the weights are not normalized.

power spectrum. Two different approaches to calculate the nonlinear power spectrum have been proposed, and we present results for both. The first is based on the scaling formula suggested by Hamilton et al. (1991), which was extended to a wider range of cosmologies by Peacock & Dodds (1996). Peacock & Dodds (1996) provide a prescription to compute the power spectrum, using a fitting formula that is calibrated using numerical simulations. More recently, Smith et al. (2003) suggested an approach based on the halo model to better capture the breakdown of the stable clustering assumption in the Peacock & Dodds (1996) prescription. We note that both prescriptions are based on relatively small numbers of numerical simulations and that their accuracy is limited. Comparison with recent numerical simulations suggest that they are accurate to $\sim 5\%$; this number depends on the region of parameter space that is probed and larger errors can occur (M. White 2005, private communication).

5.2. Constraints on Ω_m and σ_8

Cosmological parameters are estimated by comparing the predicted signal m_i to the observed top-hat variance d_i as a function of scale θ_i . We consider cold dark matter (CDM) models with a flat geometry (i.e., $\Omega_m + \Omega_\Lambda = 1$). We vary the parameters of the model, focusing on constraining the matter density Ω_m and the normalization σ_8 . For these parameters we limit the calculations to $\Omega_m \in [0, 1]$ and $\sigma_8 \in [0.5, 1.2]$.

As discussed above, we vary the source redshift distribution through $z_s \in [0.613, 0.721]$, assuming a flat prior. Furthermore, the signal depends somewhat on the value for the Hubble parameter, for which we use $h \in [0.6, 0.8]$ as motivated by the findings of the *Hubble Space Telescope* Key Project (Freedman et al. 2001). In § 5.1 we also consider w_0 , the dark energy equation of state. The maximum likelihood function is given by

$$\mathcal{L} = \frac{1}{(2\pi)^n |\mathbf{C}|^{1/2}} \exp \left[-\frac{1}{2} (d_i - m_i) \mathbf{C}^{-1} (d_i - m_i)^T \right]. \quad (20)$$

Here \mathbf{C}^{-1} is the covariance matrix. \mathbf{C} can be decomposed as $\mathbf{C} = \mathbf{C}_n + \mathbf{C}_s$, where \mathbf{C}_n is the statistical noise and \mathbf{C}_s the cos-

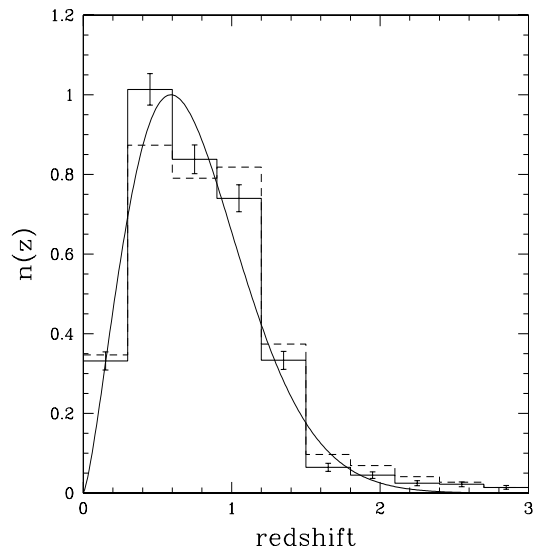


FIG. 10.—Effective redshift distribution of galaxies with $21.5 < I_{AB} < 24.5$ determined from the HDF-N and HDF-S. The error bars are the Poisson errors from the finite number of galaxies in each bin (note that these do not include cosmic variance). The smooth solid curve represents the best-fit model with a mean source redshift of $\langle z \rangle = 0.81$. The dashed histogram corresponds to the redshift distribution when the adopted weighting scheme is not taken into account.

mic variance covariance matrix. The matrix \mathbf{C}_s is computed according to Schneider et al. (2002a) for our fiducial Λ CDM cosmology, assuming an effective survey area of 13.5 deg^2 for the CFHTLS W1 and 8.5 deg^2 for W3, a number density of galaxies $n_{\text{gal}} = 12 \text{ arcmin}^{-2}$, and an intrinsic ellipticity dispersion of $\sigma_e = 0.3$ per component. At large scales our cosmic variance estimate is very robust (as we are in or approaching the linear regime). The estimate is less accurate on the smallest scales (it really requires numerical simulations to improve the estimate). The top-hat variance, however, probes a large range of scales, and as a result the measurements are highly correlated. Consequently, the limitations of our approach are not as severe compared to other statistics. To test this, we found that removing the smallest scale measurements does not change our estimates of the cosmological parameters, but it does degrade the breaking of the degeneracy between Ω_m and σ_8 .

Figure 11 shows the joint constraints on Ω_m , σ_8 using the Smith et al. (2003) model for the nonlinear power spectrum. Formally, the best-fit values are $\Omega_m = 0.225$ and $\sigma_8 = 1.0$, but since these two parameters are highly degenerate, the best-fit value has little meaning. Instead, for reference with other cosmic shear studies, we estimate the value of σ_8 for a fiducial matter density of $\Omega_m = 0.3$. For the Peacock & Dodds (1996) model, we obtain a value of $\sigma_8 = 0.88 \pm 0.06$ (68% confidence). The error includes the statistical errors, cosmic variance, and calibration errors. For the Smith et al. (2003) model we obtain a slightly lower value of $\sigma_8 = 0.85 \pm 0.06$. For $\Omega_m < 0.4$, the degeneracy between the two parameters is well described by $\sigma_8 \propto \Omega_m^{-0.6}$. The estimates for σ_8 based on the W1 and W3 separately also agree well. For $\Omega_m = 0.3$, we find $\sigma_8 = 0.87 \pm 0.07$ for W1 and $\sigma_8 = 0.75 \pm 0.12$ for W3.

These estimates are in excellent agreement with published cosmic shear results from other large surveys. Van Waerbeke et al. (2005) list a value of $\sigma_8 = 0.83 \pm 0.07$ based on the VIRMOS-DESCART survey. Hoekstra et al. (2002b) obtained $\sigma_8 = 0.86^{+0.04}_{-0.05}$

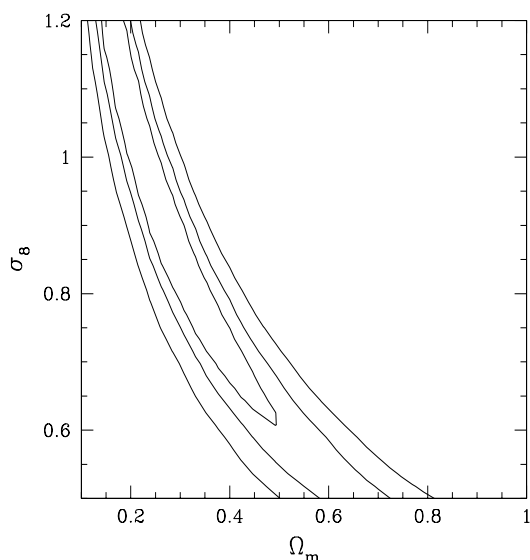


FIG. 11.—Joint constraints on Ω_m and σ_8 from the CFHTLS Wide data using the Smith et al. (2003) model for the nonlinear power spectrum. The contours indicate the 68.3%, 95.4%, and 99.7% confidence limits on two parameters jointly. We marginalized over the Hubble parameter and source redshift distribution as described in the text. [See the electronic edition of the Journal for a color version of this figure.]

from the RCS. Jarvis et al. (2006) found $\sigma_8 = 0.81^{+0.15}_{-0.10}$ (95% confidence) from the CTIO lensing survey. The constraints obtained here also agree well with those derived from the CFHTLS Deep survey, and a detailed comparison, including combined constraints with our measurements, are presented in Semboloni et al. (2005).

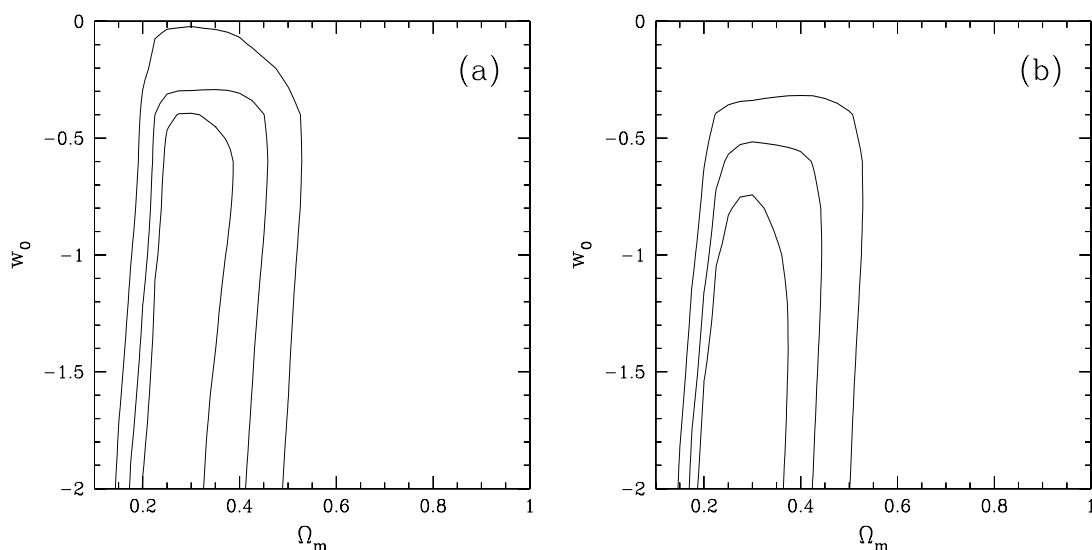


FIG. 12.—(a) Dark energy constraints using the measurements from the W1 and W3 fields. The contours indicate the 68.3%, 95.4%, and 99.7% confidence limits on two parameters jointly. We marginalized over $\sigma_8 \in [0.7, 1.0]$, $h \in [0.6, 0.8]$, and the source redshift distribution as described in the text. (b) Results when the measurements from the Deep component (Semboloni et al. 2005) are included. We used the Peacock & Dodds (1996) prescription for the nonlinear power spectrum. [See the electronic edition of the Journal for a color version of this figure.]

5.3. Constraints on Dark Energy

One of the major goals of the CFHTLS is the measurement of the equation of state of dark energy. The current lack of (photometric) redshift information seriously limits the accuracy of such a measurement. Given the current limitations of the data, we choose a simple model with a constant equation of state:

$$p = w_0 \rho. \quad (21)$$

The left panel in Figure 12 shows the joint constraints on Ω_m and w_0 based on the measurements of the lensing signal from the W1 and W3 fields. We marginalized over $\sigma_8 \in [0.7, 1.0]$, $h \in [0.6, 0.8]$ and the source redshift distribution as described above and considered the range of $-2 < w_0 < 0$. To obtain these results we used the Peacock & Dodds (1996) fitting formula to obtain the nonlinear power spectrum. We emphasize that there is not yet a reliable analytical fit to the nonlinear dark matter power spectrum available for nontrivial dark energy models. In particular, the halo model proposed in Smith et al. (2004) cannot provide an accurate description of the $w_0 \neq -1$ models because the power spectrum does not depend on w_0 at $z = 0$. We know that a change in w_0 should affect the background and therefore the change in structure clustering, which Smith et al. (2004) do not take into account. McDonald et al. (2005) have recently extended the halo model to smaller scales for $w_0 \neq -1$ dark energy models. We will present a detailed dark energy measurement in a subsequent work, taking into account new model fitting to numerical simulations, and the CFHTLS Type Ia supernova constraints (L. van Waerbeke et al. 2006, in preparation).

Although we currently lack photometric redshift information, the measurements of the Deep fields presented by Semboloni et al. (2005) probe the matter power spectrum at a slightly higher mean redshift, compared to the measurements presented here. Hence, we can improve our constraints on w_0 by combining our results to those obtained from the Deep survey (Semboloni et al.

2005). We stress that this should be considered a consistency check rather than a complete dark energy analysis. The result of this analysis is presented in the right panel of Figure 12, which gives $w_0 < -0.8$ with 68% confidence.

6. CONCLUSIONS

We have presented the first cosmic shear analysis based on $\sim 22 \text{ deg}^2$ (31 pointings) of deep i' imaging data from the CFHT Legacy Survey. These observations cover parts of two of three survey fields and already provide a significant increase in area compared to previous work. These early data show a strong variation of the PSF over the field of view. We note that recent changes to Megaprime have led to a significant reduction in PSF anisotropy. Nevertheless, our early results are very encouraging as we do not detect a significant B mode, suggesting that the derived lensing signal is free of systematics at the current level of accuracy. Comparison with an independent pipeline (L. Fu et al. 2006, in preparation) shows good agreement and demonstrates that the signal can be recovered robustly.

We assume a Λ CDM model with a flat geometry and derive joint constraints on the matter density Ω_m and the normalization of the matter power spectrum σ_8 , while marginalizing over the Hubble parameter and the source redshift distribution. We consider two models to calculate the nonlinear power spectrum. For a fiducial matter density of $\Omega_m = 0.3$ we find $\sigma_8 = 0.88 \pm 0.06$ for the Peacock & Dodds (1996) model. Similarly, we obtain a value of $\sigma_8 = 0.85 \pm 0.06$ using the Smith et al. (2003) approach. These estimates are in excellent agreement with previous studies

(Hoekstra et al. 2002b; Jarvis et al. 2006; van Waerbeke et al. 2005).

In the coming years we expect to image a total $\sim 140 \text{ deg}^2$ in five filters, allowing us to include photometric redshift information for the source galaxies. This will greatly enhance our ability to constrain cosmological parameters, most notably the equation of state of the dark energy (see, e.g., forecasts in Tereno et al. 2005). The measurements from Semboloni et al. (2005), based on the Deep component of the CFHTLS, probe a higher mean source redshift, thus providing crude redshift leverage. When combining our results with the measurements from Semboloni et al. (2005), we find $w_0 < -0.8$ (68% confidence) based on cosmic shear measurements alone.

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REFERENCES

- Abazajian, A., & Dodelson, S. 2003, *Phys. Rev. Lett.*, 91, 1301
 Bacon, D. J., Massey, R. J., Refregier, A. R., & Ellis, R. S. 2003, *MNRAS*, 344, 673
 Bacon, D. J., Refregier, A., Clowe, D., & Ellis, R. S. 2001, *MNRAS*, 325, 1065
 Bacon, D. J., Refregier, A., & Ellis, R. S. 2000, *MNRAS*, 318, 625
 Bartelmann, M., & Schneider, P. 2001, *Phys. Rep.*, 340, 291
 Benabed, K., & van Waerbeke, L. 2004, *Phys. Rev. D*, 70, 13515
 Bernstein, G. M., & Jarvis, M. 2002, *AJ*, 123, 583
 Bertin, E., & Arnouts, S. 1996, *A&AS*, 117, 393
 Brainerd, T. G., Blandford, R. D., & Smail, I. 1996, *ApJ*, 466, 623
 Clowe, D., Luppino, G. A., Kaiser, N., Henry, J. P., & Gioia, I. M. 1998, *ApJ*, 497, L61
 Contaldi, C. R., Hoekstra, H., & Lewis, A. 2003, *Phys. Rev. Lett.*, 90, 1303
 Crittenden, R. G., Natarajan, P., Pen, U.-L., & Theuns, T. 2002, *ApJ*, 568, 20
 Cypriano, E. S., Sodr , L., Kneib, J.-P., & Campusano, L. E. 2004, *ApJ*, 613, 95
 Dahle, H., Kaiser, N., Irgens, R. J., Lilje, P. B., & Maddox, S. J. 2002, *ApJS*, 139, 313
 Davis, M., et al. 2003, *Proc. SPIE*, 4834, 161
 Efsthathiou, G., Bernstein, G., Katz, N., Tyson, J. A., & Guhathakurta, P. 1991, *ApJ*, 380, L47
 Erben, T., Van Waerbeke, L., Bertin, E., Mellier, Y., & Schneider, P. 2001, *A&A*, 366, 717
 Fern andez-Soto, A., Lanzetta, K. M., & Yahil, A. 1999, *ApJ*, 513, 34
 Freedman, W. L., et al. 2001, *ApJ*, 553, 47
 Hamilton, A. J. S., Kumar, P., Lu, E., & Matthews, A. 1991, *ApJ*, 374, L1
 Heymans, C. 2003, Ph.D. thesis, Oxford Univ.
 Heymans, C., et al. 2006, *MNRAS*, in press (astro-ph/0506112)
 Hirata, C., & Seljak, U. 2003, *MNRAS*, 343, 459
 Hoekstra, H. 2004, *MNRAS*, 347, 1337
 Hoekstra, H., Franx, M., & Kuijken, K. 2000, *ApJ*, 532, 88
 Hoekstra, H., Franx, M., Kuijken, K., & Squires, G. 1998, *ApJ*, 504, 636
 Hoekstra, H., Franx, M., Kuijken, K., & van Dokkum, P. G. 2002a, *MNRAS*, 333, 911
 Hoekstra, H., Yee, H. K. C., & Gladders, M. D. 2002b, *ApJ*, 577, 595
 ———. 2004, *ApJ*, 606, 67
 Hoekstra, H., Yee, H. K. C., Gladders, M. D., Barrientos, L. F., Hall, P. B., & Infante, L. 2002c, *ApJ*, 572, 55
 Hu, W. 1999, *ApJ*, 522, L21
 Hudson, M. J., Gwyn, S. D. J., Dahle, H., & Kaiser, N. 1998, *ApJ*, 503, 531
 Ishak, M., Hirata, C., McDonald, P., & Seljak, U. 2004, *Phys. Rev. D*, 69, 83514
 Jain, B., & Seljak, U. 1997, *ApJ*, 484, 560
 Jarvis, M., Bernstein, G. M., Fischer, P., Smith, D., Jain, B., Tyson, J. A., & Wittman, D. 2003, *AJ*, 125, 1014
 Jarvis, M., & Jain, B. 2005, *ApJ*, submitted (astro-ph/0412234)
 Jarvis, M., Jain, B., Bernstein, G. M., & Dolney, D. 2006, *ApJ*, 644, 71
 Kaiser, N., Squires, G., & Broadhurst, T. 1995, *ApJ*, 449, 460
 Kaiser, N., Wilson, G., & Luppino, G. A. 2000, *ApJL*, submitted (astro-ph/0003338)
 Le F vre, O., et al. 2004, *A&A*, 428, 1043
 Luppino, G. A., & Kaiser, N. 1997, *ApJ*, 475, 20
 McDonald, P., Trac, H., & Contaldi, C. 2006, *MNRAS*, 366, 547
 McKay, T. A., et al. 2001, *ApJ*, submitted (astro-ph/0108013)
 Monet, D., et al. 1998, *The USNO-A2.0 Catalogue* (Washington, DC: USNO)
 Peacock, J. A., & Dodds, S. J. 1996, *MNRAS*, 280, L19
 Pen, U.-L., Van Waerbeke, L., & Mellier, Y. 2002, *ApJ*, 567, 31
 Refregier, A. 2003, *MNRAS*, 338, 35
 Reid, I. N., et al. 1991, *PASP*, 103, 661
 Schlegel, D. J., Finkbeiner, D. P., & Davis, M. 1998, *ApJ*, 500, 525
 Schneider, P., van Waerbeke, L., Jain, B., & Kruse, G. 1998, *MNRAS*, 296, 873
 Schneider, P., van Waerbeke, L., Kilbinger, M., & Mellier, Y. 2002a, *A&A*, 396, 1
 Schneider, P., van Waerbeke, L., & Mellier, Y. 2002b, *A&A*, 389, 729
 Semboloni, E., et al. 2005, *A&A*, submitted (astro-ph/0511090)
 Simon, P., King, L. J., & Schneider, P. 2004, *A&A*, 417, 873
 Smith, R. E., et al. 2003, *MNRAS*, 341, 1311
 Tereno, I., Dor , O., van Waerbeke, L., & Mellier, Y. 2005, *A&A*, 429, 383
 Vale, C., Hoekstra, H., van Waerbeke, L., & White, M. 2004, *ApJ*, 613, L1
 van Waerbeke, L., Mellier, Y., & Hoekstra, H. 2005, *A&A*, 429, 75
 van Waerbeke, L., et al. 2000, *A&A*, 358, 30
 Wittman, D. M., Tyson, J. A., Kirkman, D., Dell'Antonio, L., & Bernstein, G. 2000, *Nature*, 405, 143

Tracking quintessence by cosmic shear

Constraints from VIRMOS-DescaRT and CFHTLS* and future prospects

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ABSTRACT

Context. Dark energy can be investigated in two complementary ways, by considering either general parameterizations or physically well-defined models. This article follows the second route and explores the observational constraints on quintessence models where the acceleration of our universe is driven by a slow-rolling scalar field. The analysis focuses on cosmic shear, combined with type Ia supernovae data and cosmic microwave background observations.

Aims. This article examines how weak lensing surveys can constrain dark energy, how they complement supernovae data to lift some degeneracies and addresses some issues regarding the limitations due to the lack of knowledge concerning the non-linear regime.

Methods. Using a Boltzmann code that includes quintessence models and the computation of weak lensing observables, we determine the shear power spectrum and several two-point statistics. The non-linear regime is described by two different mappings. The likelihood analysis is based on a grid method. The data include the “gold set” of supernovae Ia, the WMAP-1 year data and the VIRMOS-DescaRT and CFHTLS-deep and -wide data for weak lensing. This is the first analysis of high-energy motivated dark energy models that uses weak lensing data. We explore larger angular scales, using a synthetic realization of the complete CFHTLS-wide survey as well as next space-based missions surveys.

Results. Two classes of cosmological parameters are discussed: *i*) those accounting for quintessence affect mainly geometrical factors; *ii*) cosmological parameters specifying the primordial universe strongly depend on the description of the non-linear regime. This dependence is addressed using wide surveys, by discarding the smaller angular scales to reduce the dependence on the non-linear regime. Special care is paid to the comparison of these physical models with parameterizations of the equation of state. For a flat universe and a quintessence inverse power law potential with slope α , we obtain $\alpha < 1$ and $\Omega_{Q0} = 0.75^{+0.03}_{-0.04}$ at 95% confidence level, whereas $\alpha = 2^{+18}_{-2}$, $\Omega_{Q0} = 0.74^{+0.03}_{-0.05}$ when including supergravity corrections.

Key words. Gravitational lensing. Cosmology: theory – cosmological parameters. Methods – data analysis.

1. Introduction

Cosmological observations provide increasing compelling evidences that the expansion of the universe is accelerating and that the cosmic history of the universe seems today dominated by another component than its matter and radiation content (see e.g. Peebles & Ratra 2003; Carroll 2001; Padmanabhan 2003; Peter & Uzan 2005, chap. 12 for reviews and references therein). If so, one of the most challenging issue of fundamental physics is to understand the cause of this acceleration, a question often referred to as the nature of the *dark energy*. Various solutions, from the introduction of a new type of matter to a modification of general relativity to describe the

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gravitation interaction, have been considered. A classification of these models with some relevant observational tests that can help to distinguish between each class from their underlying new physics is discussed in Uzan (2004) and Uzan, Aghanim & Mellier (2004).

Dark energy appears in the Friedmann equations through its *effective* density and pressure. Data are usually interpreted assuming the validity of the Copernican principle (so that the dynamics of spacetime is completely described by a single function, the scale factor a) and the validity of Einstein equations (and thus the standard Friedmann equations), so that the density and pressure of the dark energy component are defined by $\rho_{\text{de}} = (3/8\pi G)(H^2 + K/a^2) - \rho_{\text{m}} - \rho_{\text{r}}$ and $P_{\text{de}} = (-1/8\pi G)(\ddot{a}/a + H^2 + K/a^2)$, where $H = \dot{a}/a$ is the Hubble parameter and a dot refers to a derivative with respect to the cosmic time, while ρ_{m} and ρ_{r} are the density of pressureless matter and radiation, respectively. It follows that the equation of state of the dark energy corresponds to

$$3\Omega_{\text{de}}w = -1 + \Omega_K + 2q, \quad (1)$$

where $q = -a\ddot{a}/\dot{a}^2$ is the deceleration parameter and $\Omega_K = -K/a^2H^2$. From this point of view, w characterizes the dynamics of the cosmic expansion. More precisely, it parameterizes the deviation, $H(z) - \bar{H}(z)$, between the Hubble function of the observed Universe, $H(z)$, and that predicted for a universe filled only with pressureless matter and radiation, $\bar{H}(z)$. It is therefore equivalent to specify $w(z)$ or $H(z) - \bar{H}(z)$. However, when general relativity is assumed to describe gravity, w reduces to $P_{\text{de}}/\rho_{\text{de}}$ so that, in addition to the deviation from $\bar{H}(z)$, it also gives some insight on the properties of dark energy (see e.g. Martin, Schmid & Uzan (2005) for a case in which w does not reduce to the equation of state of a matter component).

Although effective equation of state derived from observations is a key empirical information on the rough nature of dark energy, a detailed description of its properties demands more thoughtful data interpretation. For example, all geometrical observables rely on the integration of the Hubble parameter, hence on a double integration of the equation of state w , that eventually dilutes or totally washes out its possible redshift dependence. If w is close to -1 , as observations tend to indicate, then it is in general difficult to demonstrate by geometrical tests that $w \neq -1$ or that $dw/dz \neq 0$; both would exclude a pure cosmological constant. Exploring early properties of dark energy models would be even more challenging since for $w \simeq -1$ the ratio between the matter and dark energy densities scales approximately as $(1+z)^3$ so that dark energy is dynamically negligible at redshift $z \gtrsim 2$. It leaves little freedom to determine the scaling of the dark energy density and to demonstrate that it is not properly described by a power law $[(1+z)^n]$, as would be the case for a constant w (see e.g. Kujat et al. 2002).

From the theoretical point of view two routes can be followed. One can either exhibit a general “model-independent” parameterization of the equation of state of the dark energy, as discussed in the previous paragraph, or rely on a completely specified theoretical models. A useful parameterization has to be realistic, in the sense that it should reproduce predictions of a large class of models, it has to minimize the number of free parameters and to be simply related to the underlying

physics (see e.g. Linder & Huterer 2005). Because the result of the analysis will necessarily have some amount of parameterization dependence (Basset et al. 2002), choosing the specified physical model strategy seems preferable to break degeneracies. In particular, it enables to compute without any ambiguity their signature both in low and high redshift surveys, such as the cosmic microwave background (CMB). The increasingly flourishing number of models hampers to provide a comprehensive set of unambiguous predictions to constrain physical models one by one with present-day observations, but there are still several benefits in exploring dark energy this way, in particular when weak lensing surveys are used together with CMB observations. This is deeply related to the evolution of dark energy properties and the growth rate of structure with look-back time, as discussed below.

At low redshift, w suffices to get observable that are all functions of $H(z)$ (see for example Peebles 1993; Peter & Uzan 2005). This is the case of all background quantities (e.g. luminosity distance, angular distance, look-back time, etc.) as well as of the linear growth factor of density perturbations. It follows that the equation of state encodes all relevant information, provided the amplitude of the power spectrum is calibrated by adding a new parameter, σ_8 , the variance of the density perturbation on a scale of $8h^{-1}$ Mpc (see e.g. Benabed & Bernardeau 2001; Benabed & van Waerbeke 2003; Doran et al. 2000). However, as far as weak lensing is concerned, it was shown (Benabed & van Waerbeke 2003; Benabed & Bernardeau 2001) that, for a fixed redshift of the sources, the modification of the growth factor in the linear regime was degenerate with the normalization factor. Hence normalizing on the CMB avoids this problem, and at the same time it is important to describe the non-linear regime. The use of CMB together with weak lensing data is therefore a logical way to constrain specified theoretical models, beyond the description by an empirical equation of state.

At higher redshift, and in particular to relate the amplitude of the matter density power spectrum to the one of the primordial power spectrum, one would need to include a description of the evolution of the perturbations of dark energy. In particular, this effect becomes increasingly important as w approaches zero (Benabed & Bernardeau 2001). Note that dark energy perturbations have a non-adiabatic component that also requires a detailed model to be described. This depends on the physical model of dark energy and cannot be incorporated in a simple model-independent way.

While the ability of lensing data to constrain the equation of state of dark energy has been widely studied (Benabed & van Waerbeke 2003; Hu & Jain 2004; Jain & Taylor 2003), there have been very few analysis with real data. Hoekstra et al. (2005) and Semboloni et al. (2005) used the CFHTLS wide and deep data to constrain a constant equation of state. Jarvis et al. (2005) analyzed the 75 square degrees CTIO lensing survey, combined with type Ia supernovae (Sn Ia) data and CMB, assuming a constant equation of state and a parameterization of the form proposed by Chevallier & Polarski (2001) and then by Linder (2003). In this article, we consider a class of completely defined

quintessence models, realized by a self-interacting scalar field. Hence all observational signatures (Sn Ia, lensing, CMB) can be explicitly computed, with no ambiguity in the way to deal with the perturbations of dark energy. This theoretical extension of the standard Λ CDM model involves only one additional parameter, needed to characterize the self-interacting potential of the quintessence field. It follows that, as discussed above, we will be able to normalize our initial power spectrum on the CMB angular power spectrum and, as a consequence, σ_8 and any possible dependence on the shape of the analytical fit of the transfer function will disappear from our discussion; the value of σ_8 will be an output of each model. The problem of the pivot redshift (see § 2.2) that appears when combining different datasets also disappears in that approach. As a conclusion, this approach is very efficient in terms of the number of extra-parameters and of the interpretation of the data. Let us emphasize that, even though we also consider Sn Ia and CMB data, we will focus on weak lensing - cosmic shear data. This article, being the first analysis of the CFHTLS data for dark energy studies, illustrates the power and the problems of lensing survey in studying dark energy.

The article is organized as follows. In Section 2, we define the quintessence models we are considering and we recall their main properties. We also compare them to various parameterizations proposed in the literature. Section 3 focuses on cosmic shear. After a reminder on theoretical issues, we describe the weak lensing data used for our analysis and the computational pipeline, finally we outline the likelihood analysis on real (§ 3.5) and synthetic (§ 3.6) datasets. In Section 4 we combine weak lensing data with type Ia supernovae and CMB temperature anisotropies; Figure 10 and Table 1 summarize the constraints on quintessence parameters. We finish, in Section 5, by an estimation of the proficiency of two possible space-based wide field imagers to unveil the nature of dark energy.

2. Modelling dark energy

2.1. Quintessence models

In this work, we consider the simplest class of quintessence models (Ratra & Peebles 1988; Wetterich 1988) in which a scalar field Q is slow-rolling in a runaway potential. Numerous forms of potentials have been proposed but we restrict to two classes of potentials. Let us briefly summarize the properties of these models.

The first class of potentials is an inverse power law (Ratra & Peebles 1988; Wetterich 1988)

$$V(Q) = M^4(Q/M_p)^{-\alpha}, \quad (2)$$

often quoted as Ratra-Peebles (RP) potential. $M_p \equiv (8\pi G)^{-1/2}$ is the reduced Planck mass. The potential depends on two free parameters: α is a positive index while M is a mass scale that has to be adjusted to fit Ω_Q today, once α is given. In particular, if Ω_Q dominates today then M and α are related by

$$\log\left(\frac{M}{1 \text{ GeV}}\right) \sim \frac{19\alpha - 47}{4(\alpha + 1)}. \quad (3)$$

The second class of potential is an extension of the previous that takes supergravity (SUGRA) corrections into account when $Q \sim M_p$ (Brax & Martin 1999),

$$V(Q) = M^4(Q/M_p)^{-\alpha} \exp(Q^2/2M_p^2). \quad (4)$$

Both potentials have a similar dynamics as long as $Q \ll M_p$ but differs at low redshift, in particular concerning their equation of state. In the SUGRA case, it is pushed toward -1 and one expects $w_0 \sim -0.82$ (Brax & Martin 1999).

With such well-defined models, the dynamics of the background is completely characterized by the Klein-Gordon equation for the scalar field,

$$\ddot{Q} + 3H\dot{Q} + \frac{dV}{dQ} = 0, \quad (5)$$

in addition to the Friedmann equation

$$\left(H^2 + \frac{K}{a^2}\right) = \frac{8\pi G}{3} \left[\rho_m + \rho_r + \frac{\dot{Q}^2}{2} + V(Q)\right] \quad (6)$$

allowing for this new matter contribution. These equations characterize background and low redshift observations and in particular the linear growth factor. Let us stress that, at this level of description, one can describe the quintessence component as a fluid with a time-dependent equation of state. This is due to the fact that the speed of sound, c_s^2 , is given by

$$c_s^2 = 1 + \frac{4}{3} \frac{1}{H\dot{Q}} \frac{dV}{dQ} \quad (7)$$

and that the equation of state evolves as

$$\dot{w} = -3H(1+w)(c_s^2 - w). \quad (8)$$

These models share the interesting property to possess scaling solutions which are attractor of the dynamical evolution. In general, but depending on the initial conditions, the dynamics starts with an early kinetic phase ($\dot{Q}^2 \gg V$) in which $w \sim 1$ so that $\rho_Q \propto (1+z)^6$. The field behaves as $Q = Q_i - A(1+z)$ and it freezes to a constant value. Since the kinetic energy decreases while the potential remains constant, this regime cannot last

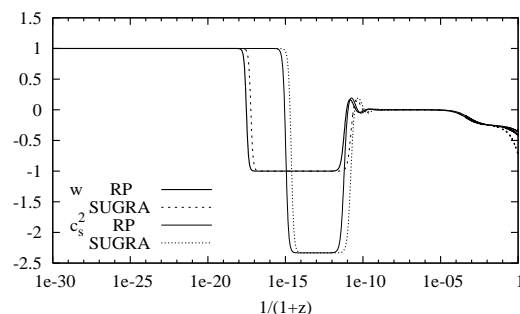


Fig. 1. Evolution of the equation of state w and of the sound speed c_s with the redshift for an inverse power law quintessence model with $\alpha = 6$, including or not the supergravity correction. We recover, from high to low redshift, the kinetic, slow-rolling and tracking phases described in the text.

forever. When the potential starts dominating, the equation of state shifts suddenly to $w \sim -1$. During this transition regime, $\rho_Q \propto (1+z)^0$. Then, there is a potential regime that lasts until the tracking regime during which

$$w = c_s^2 = \frac{\alpha w_B - 2}{\alpha + 2} \quad (9)$$

where w_B stands for the equation of state of the fluid dominating the background. At that stage, the scalar field is slow-rolling so that $w < 0$. Fig. 1 depicts the evolution of w and c_s^2 during these various regimes.

When cosmic microwave background anisotropies and large scale structures are considered, one needs to include the description of the evolution of the perturbations, and in particular include those of the scalar field described by

$$\delta\ddot{Q} + 3H\delta\dot{Q} + \left(\frac{k^2}{a^2} + \frac{d^2V}{dQ^2}\right)\delta Q + S = 0 \quad (10)$$

where S encodes the perturbations of the metric of the space-time and k is the comoving wavenumber of the perturbation. It was shown (Brax et al. 2000; Riazuelo & Uzan 2002) that there exists an attraction mechanism for super-Hubble wavelength so that the spectrum is insensitive to the initial conditions for the scalar field.

2.2. Models and parameterizations

Most data, and in particular supernovae data, are being analyzed using a general parameterization of the equation of state. These parameterizations are useful to extract model-independent information from the observations but the interpretation of these parameters is not always straightforward. In this paragraph, we remind the properties of some interesting parameterizations and compare them to the quintessence models we are considering.

Let us first recall that general parameterizations of the equation of state as

$$w(a) = w(a_0) + [w(a_m) - w(a_0)] \Gamma(a, a_t, \Delta) \quad (11)$$

were shown to allow an adequate treatment of a large class of quintessence models (Corasaniti & Copeland 2003; Basset et al. 2002). Such a parameterization involves four parameters $\{w(a_0), w(a_m), a_t, \Delta\}$ and a free function Γ varying smoothly between one at high redshift to zero today. Even though it reproduces the equation of state of most quintessence models, it is not economical in terms of number of parameters since most quintessence potentials involve one or two free parameters. If one assumes that the parameterization is supposed to describe the dynamics of a minimally coupled scalar field, the knowledge of w is sufficient but in a more general case one would need more information: The background dynamics depends on the potential and its first derivative, which can be related to w and \dot{w} . Accounting for perturbations, one needs to know the second derivative of the potential [see Eq. (10)] which can be inferred from \ddot{w} (Dave et al. 2002).

Since we expect dark energy to have observable consequences on the dynamics only at late time, one can consider

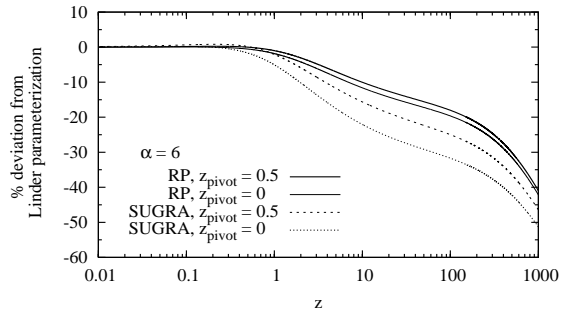


Fig. 2. Deviation of quintessence equation of state for Ratra-Peebles (solid) and SUGRA (dotted) models with $\alpha = 6$ from the generalized parameterization, Eq. (11), setting $z_{\text{pivot}} = 0$ (thick) or $z_{\text{pivot}} = 0.5$ (thin). Fitting the previous one up to $z \lesssim 0.3$, a deviation larger than 2% occurs at $z \approx 1$ for Ratra-Peebles models while at $z \approx 0.5$ for SUGRA models.

an equation of state obtained as a Taylor expansion around a pivot point,

$$w(a) = w_{\text{pivot}} + w_a(a_{\text{pivot}} - a). \quad (12)$$

This form depends on only two parameters and is a generalization of the parameterization proposed by Chevallier & Polarski (2001) and then Linder (2003) where $a_{\text{pivot}} = 1$. Two considerations are in order when using such a parameterization. First, the redshift band on which this is a good approximation of the equation of state is unknown. Clearly, compared with the form (11), it is unlikely to describe dark energy up to recombination time; see Fig. 2. Secondly, when combining observables at different redshift such as weak lensing, Sn Ia and CMB, one should choose the value of a_{pivot} in such a way that the errors in w_{pivot} and w_a are uncorrelated (Hu & Jain 2004). It follows that the pivot redshift is the redshift at which w is best constrained. In particular it was argued that it is important to choose $a_{\text{pivot}} \neq 1$ for distance-based measurements. The problem lies in the fact that the pivot redshift is specific to the observable. In this respect, dark energy models defined by a Lagrangian are more suitable, yielding to a definite equation of state as a function of redshift, hence more general than a Taylor expansion around a pivot point. Eventually, one can read out the values of w_{pivot} and w_a at whatever redshift. Fig. 3 depicts the value of w_{pivot} and w_a for the quintessence models we use, Eqs. (2) and (4), for the pivot redshifts $z_{\text{pivot}} = 0$ and $z_{\text{pivot}} = 0.5$. This complication, arising when one wants to combine datasets with different z_{pivot} , will also make it more difficult to infer constraints on the physical models from the constraints on the parameterization.

There is an alternative way to get a first hint on the nature of dark energy. It may be useful to consider the plane (w, w') where $w' \equiv dw/d \ln a$ is the derivative of w with respect to the number of e -folds. It was recently shown by Caldwell & Linder (2005) and Scherrer (2005) that quintessence models occupy a narrow part of this plane. This can be understood from Eq. (8) which implies that $w' + 3(1 - w^2) = 3(1 + w)(1 - c_s^2)$. For

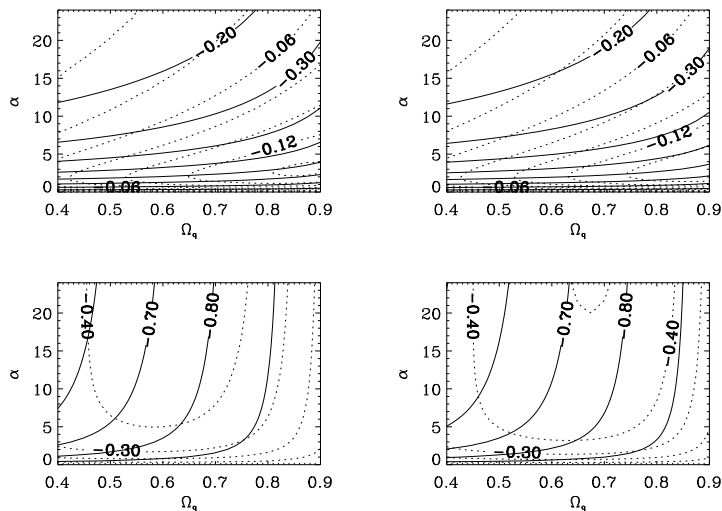


Fig. 3. Contour plots of the quintessence equation of state. We compare the equation of state of two quintessence models with the parameterization (12) for two values of the pivot redshift: $z_{\text{pivot}} = 0$ (left) and $z_{\text{pivot}} = 0.5$ (right). Solid lines correspond to level contours for w_{pivot} while dotted lines correspond to level contours of w_a . We have chosen the spacing of all the contour lines to be $\Delta w = 0.1$, except for the plots in the upper line, where $\Delta w_a = 0.02$. The upper line corresponds to Ratra-Peebles models, Eq. (2), while the lower line corresponds to SUGRA models, Eq. (4). Due to the exponential correction, w_0 is always smaller for SUGRA models because the potential is flatter and the field is rolling slower. Also, the value of w_{pivot} and w_a are more sensitive to the choice of z_{pivot} for SUGRA models than for Ratra-Peebles models.

quintessence models, $1 + w > 0$, and Eq. (7) implies that $c_s^2 < 1$ (because $Q > 0$ and $V' < 0$) so that

$$w' > -3(1 - w^2), \quad (13)$$

without any assumptions on the dynamics of the scalar field. In Caldwell & Linder (2005), two classes of quintessence models were exhibited, namely “thawing” models, in which $w \sim -1$ initially and increases as Q rolls down the potential, and “freezing” models, in which $w > -1$ initially and tends toward -1 as Q rolls down the potential. “Freezing” models contain tracking models and in particular the Ratra-Peebles models, Eq. (2), and SUGRA models, Eq. (4), considered in this work. Using a combination of numerical simulations and physical arguments, they concluded that

$$3w(1 + w) < w' < 0.2w(1 + w) \quad (14)$$

for “freezing” models. From an observational point of view, the analysis of supernovae data (Riess et al. 2004) showed that if the universe is flat then $w_0 = -1.31^{+0.22}_{-0.28}$ and $w'_0 = -1.48^{+0.90}_{-0.81}$ after marginalizing on Ω_{m0} . If one further imposes that $w_0 > -1$ then $w_0 < -0.76$ and $w'_0 = -0.6 \pm 0.5$. In the case where w is assumed constant then $w = -1.02^{+0.13}_{-0.19}$ and $w < -0.72$ at 68% and 99% confidence level respectively.

This phase-space analysis creates a link between the various parameterizations and the physical models. It can be shown (Scherrer 2005) that different classes of models (e.g. k-essence, Chaplygin gaz, quintessence, etc.) lies in different parts, hence offering a way to distinguish between these models without measuring $w(z)$. In particular, Fig. 4 depicts the dynamics of some Ratra-Peebles and SUGRA models in the (w, w')

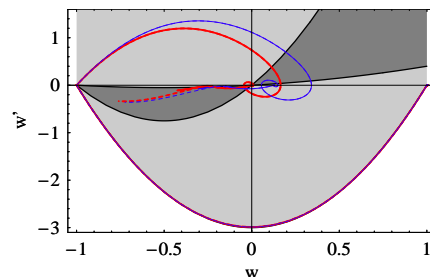


Fig. 4. Dynamics of the two quintessence models in the plane (w, w') . The shaded regions correspond to the constraints (13) in light gray and (14) in dark grey. We have considered a Ratra-Peebles (solid) and SUGRA (dash) models with $\alpha = 6$ (thick/red) and $\alpha = 11$ (thin/blue). Only in the tracking regime the models are compatible with Eq. (14).

plane, superposed to the regions where the inequalities (13) and (14) hold. Notice that the trajectories for the SUGRA potential are essentially the same of the Ratra-Peebles one, deviating just at low redshift towards the cosmological constant solution.

2.3. Summary

Quintessence models require only *two* parameters to describe the whole dynamics (with no redshift limitation). Compared to a pure cosmological constant, described by only one number, this gives us one extra parameter. In terms of extra-parameter

with respect to a standard Λ CDM, this is equivalent as considering a constant equation of state for dark energy.

On the other hand, a parameterization of the equation of state of the dark energy is sufficient to describe the low redshift universe. But, the parameterizations which describe accurately quintessence models involve at least *four* extra-parameters. It is thus more economical to work directly with the physical model. In that case, the evolution of perturbations can be inferred from w but this is not the case in more general situations.

Parameterizations with fewer parameters have a smaller dynamical range and are likely to be bad descriptions of dark energy at high redshift, notably for CMB computation. Besides, when combining dataset, the choice of the pivot redshift may induce some difficulties in interpreting the constraints on the parameters in terms of constraints on the physical models.

So considering directly a well-defined physical model instead of a parameterization is economical concerning the number of extra-parameter and avoids the problem of the pivot redshift. It allows us to compute the prediction of the models at all redshift (Sn Ia, weak lensing, and CMB). On the other hand, it concerns only a small class of models.

This discussion shows that both routes are complementary. In particular, it would be worth to evaluate to which accuracy constraints of order 1% on a given parameterization constrain physical models.

3. Cosmic shear

Gravitational lensing by large scale structures of the universe produce weak distortion fields and collectively modify the shape of background galaxies (see e.g. Bartelmann & Schneider 2001; Mellier 1999; Réfrégier 2003). Though this is a very weak signal and a challenging task, it has been detected almost simultaneously by van Waerbeke et al. (2001), Wittman et al. (2000), Kaiser et al. (2000), Bacon et al. (2000) and it now routinely observed by many groups around the world (see van Waerbeke & Mellier 2003; Hoekstra 2003 for recent reviews on observations). Over the past years the huge efforts carried out by these groups to deal with critical systematics considerably improved the reliability of the lensing signal and strengthened the ability of cosmic shear to constrain cosmological models from the statistical analyses of galaxy ellipticities.

3.1. Generalities

The gravitational lensing effect depends on the second order derivatives of the gravitational potential. The convergence, κ , and the shear, $\gamma = (\gamma_1, \gamma_2)$, describe the distortion of background images by the matter along the line of sight. These components are related by

$$\Delta\kappa = (\partial_1^2 - \partial_2^2)\gamma_1 + 2\partial_1\partial_2\gamma_2. \quad (15)$$

The evolution of the convergence is dictated by the Sachs equation (Sachs 1962; see Uzan & Bernardeau 2000 for a more modern description). The convergence in the direction θ can be

related to the matter distribution integrated along the line of sight

$$\kappa(\theta, \chi) = \frac{3}{2} \frac{\Omega_{m0} H_0^2}{c^2} \int_0^\chi \frac{S_K(\chi - \chi') S_K(\chi')}{S_K(\chi)} \times \frac{\delta_m[S_K(\chi')\theta, \chi']}{a(\chi')} d\chi' \quad (16)$$

for sources located at a radial distance χ defined by

$$\chi(z) = \int_0^z \frac{dz'}{H(z')} \quad (17)$$

and S_K is the angular diameter distance given by

$$S_K(\chi) = \begin{cases} \sin(\sqrt{K}\chi)/\sqrt{K} & K > 0 \\ \chi & K = 0 \\ \sinh(\sqrt{-K}\chi)/\sqrt{-K} & K < 0 \end{cases}. \quad (18)$$

If the sources have a distribution given by $n_\chi(\chi)d\chi = n(z)dz$ then the effective convergence takes the form

$$\kappa(\theta) = \int_0^{\chi_H} n_\chi(\chi) \kappa(\theta, \chi) d\chi \quad (19)$$

where χ_H is the comoving radial distance of the horizon. Decomposing the convergence in 2-dimensional Fourier modes,

$$\kappa(\theta) = \int \frac{d^2\ell}{2\pi} \hat{\kappa}(\ell) e^{i\ell\theta}, \quad (20)$$

the shear power spectrum, defined by $\langle \hat{\kappa}(\ell) \hat{\kappa}(\ell') \rangle = P_\kappa(\ell) \delta^{(2)}(\ell + \ell')$, can be related to the 3-dimensional power spectrum of matter density perturbations P_m by

$$P_\kappa(\ell) = \frac{9}{4} \frac{H_0^4 \Omega_{m0}^2}{c^4} \int_0^{\chi_H} \left[\frac{g(\chi)}{a(\chi)} \right]^2 P_m \left[\frac{S_K(\chi)}{\ell}, \chi \right] d\chi \quad (21)$$

in the small angle approximation (see e.g. Bartelmann & Schneider 2001, Peter & Uzan 2005 chap. 7). The function g is given by

$$g(\chi) = \int_\chi^{\chi_H} n_\chi(\chi') \frac{S_K(\chi' - \chi)}{S_K(\chi')} d\chi'. \quad (22)$$

Note that the window function $W(z) \equiv [g(\chi)/a(\chi)]^2$ is peaked around $z \approx z^*/2$ for a distribution of sources $n(z)$ approximately peaked at redshift z^* . This will be useful in the choice of the pivot redshift. Let us stress that expressions (16) and (21) assume the validity of the Poisson equation and thus of general relativity. These expressions may be slightly different in more general contexts and even be used to test general relativity (see Uzan & Bernardeau 2001; Schmid et al. 2005).

Neither κ nor P_κ are directly observable, but only filtered quantities can be obtained. Cosmic shear can be measured by various types of 2-point statistics which differ only by the chosen filtering scheme. This implies that their sensitivity to the power spectrum, and also to systematics, are different. In this work we consider two of these statistics, namely the aperture map variance, defined by

$$\langle M_{\text{ap}}^2 \rangle(\theta_c) = \frac{288}{\pi} \int \ell P_\kappa(\ell) \left[\frac{J_4(\ell\theta_c)}{\ell^2 \theta_c^2} \right]^2 d\ell, \quad (23)$$

which is a bandpass estimate of the convergence power spectrum, and the top-hat shear variance

$$\langle \gamma^2 \rangle(\theta_c) = \frac{8}{\pi} \int \ell P_\kappa(\ell) \left[\frac{J_1(\ell\theta_c)}{\ell\theta_c} \right]^2 d\ell, \quad (24)$$

which is a lowpass estimate of P_κ . Here J_n are the Bessel functions of the first kind. Both statistics can be deduced from two linear combinations of the radial and tangential components of the shear variance, $\xi_\pm \equiv \langle \gamma_r^2 \rangle \pm \langle \gamma_t^2 \rangle$ (see e.g. Bartelmann & Schneider 2001), which are directly estimated from the shapes of background galaxies.

3.2. Matter power spectrum and non-linear regime

In the previous expressions, and in particular in Eqs. (16) and (21), P_m refers to the full 3-dimensional power spectrum of pressureless matter, including cold dark matter and baryons.

In the linear regime, the growth factor D_+ is the growing solution of

$$\ddot{D}_+ + 2H\dot{D}_+ - \frac{3}{2}H^2\Omega_m D_+ = 0. \quad (25)$$

The second term that describes the damping due to the cosmological expansion contains all the effect of dark energy on D_+ . In this regime, the total effect on lensing, Eq. (21), is similar to the one obtained from a single redshift plane (see Benabed & van Waerbeke 2003; Schmid et al. 2005) so that the integrated growth effect is degenerated with the normalization of the spectrum.

Dealing with low-redshift sources, the lensing predictions and analysis involve the non-linear power spectrum. This regime cannot be described analytically from a perturbation approach (see however the proposal by Crocce & Scoccimarro 2005) and one would need to rely on N -body simulations.

N -body simulations including quintessence were recently performed using the GADGET code (Dolag et al. 2003) or an adaptive refinement tree code (Klypin et al. 2003). It was argued by Dolag et al. (2003) that the halo concentration distribution around the mean value does not depend on the cosmology, while the concentration parameter depends on the dark energy equation of state at the cluster formation redshift through the linear growth factor. However a systematic study confirming this claim is lacking. Klipin et al. (2003) and Mainini et al. (2003) show that dark energy changes the virial density contrast, Δ_c , which induces a change in the power spectrum at small scales ($k \gtrsim 1h \text{ Mpc}^{-1}$) but, for constant w , the error is smaller than the error in the expected non-linear model (Jarvis et al. 2005).

Instead of specific N -body simulations accomodating quintessence, in order to compute the non-linear matter power spectrum one can deal with linear-to-non-linear mappings, for instance based on the stable clustering *ansatz* (Hamilton et al. 1991; Peacock & Dodds 1996) or to a halo model (e.g. Ma & Fry 2000; Seljak 2000; Smith et al. 2003). These mappings have been tested for several cosmologies including Λ CDM but not for dynamical dark energy models.

Given their robustness and the precision level we can actually reach, we can hope that they remain valid for the class of models we are considering. Indeed this is a very strong assumption that can be justified by the fact that we do not expect the scalar field to cluster on small scales so that it is unlikely to affect the small scale behaviour of matter but by its influence on the expansion rate. Arguing that the clustering scale of the quintessence field is given by its Compton wavelength, Ma et al. (1999) propose an analytic approximation for the Peacock & Dodds (1996) formula to include quintessence with constant equation of state, claiming a 10% level accuracy. McDonald et al. (2005) propose a recipe to extend the aforementioned mappings to $w \neq -1$ cases, attaining a better accuracy for $k \lesssim 10 \text{ Mpc}^{-1}$ by systematically exploring a wider parameter space. Nevertheless, upcoming weak lensing measurements require an improved description of the smaller scales physics (Huterer & Takada 2005), eventually including hydrodynamics (Zhan & Knox 2004; Jing et al. 2005).

In conclusion, it seems early to decide how dark energy, and quintessence in particular, modify the mapping calibrated on Λ CDM. For that reason, in this work we will consider two linear-to-non-linear mappings, by Peacock & Dodds (1996) and Smith et al. (2003), and try to identify the parameters that are not sensitive to this choice. We will also try to quantify how the other parameters are affected so that we can estimate how our ignorance of the non-linear regime limits the use of weak lensing.

Hopefully, as we will also show, weak lensing data in the linear regime shall be able to be used. In that case, we can get an interesting constraint on the growth factor without messing with non-linear physics.

3.3. Lensing data

We use three sets of data for weak lensing, the VIRMOS-Descart (van Waerbeke et al. 2005), the deep field of the CFHTLS survey (Semboloni et al. 2005) and the wide field of CFHTLS (Hoekstra et al. 2005).

The details regarding the survey properties, image and catalogue processing of VIRMOS-Descart data are described in van Waerbeke et al. 2001 and McCracken et al. 2003. The shear measurement and error analysis are described in van Waerbeke et al. 2005 and summarized in Table 2 of that paper. This survey covers an effective (the unmasked area) area of 8.5 deg^2 spread in four fields. It probes lensed galaxies down to the limiting magnitude $I_{AB} = 24.5$ has an effective galaxy number density (after all selection processes) of 15 gal/arcmin^2 , and explores angular scales up to 50 arc-minutes.

For the CFHTLS deep and wide, all relevant details are given in Semboloni et al. 2005 and Hoekstra et al. 2005, respectively. The deep covers an effective area of 2.2 deg^2 in three fields, down to $I_{AB} = 26$, but only samples angular scales up to 30 arc-minutes. The effective galaxy number density is 22 gal/arcmin^2 . In contrast, the wide has a much larger effective area than the deep (22 deg^2) but only spread in two fields. It explores angular scales up to one degree at about the same depth as VIRMOS-Descart. The effective galaxy number den-

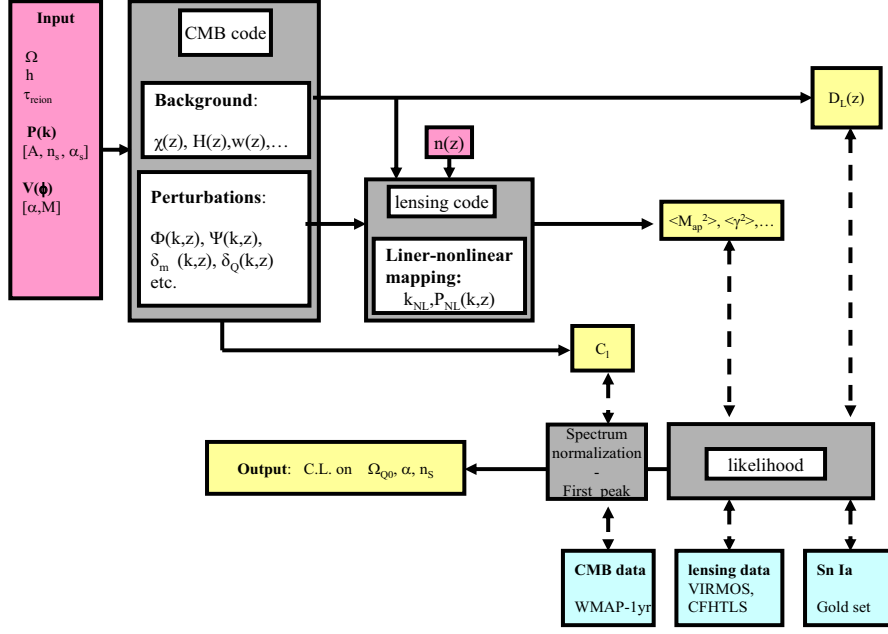


Fig. 5. Pipeline implemented for this work. Presently, we restrict to three free cosmological parameters, $\{\Omega_{Q0}, \alpha, n_s\}$, keeping fixed the others. The lensing code manipulates both background's and perturbations quantities computed by the CMB code, using a sources distribution $n(z)$ depending on the used dataset. In particular, the source redshift parameter z_s is left to vary and marginalized over afterwards. Finally, the likelihood is computed using (either real or synthetic) cosmic shear and Sn Ia data, both separately and jointly. The temperature CMB data are used to fix the amplitude A of the power spectrum at decoupling, and to put (conservative) constraints on the (Ω_{Q0}, α) parameters sub-space using the location of the first peak. In the CMB section of the pipeline, we indicate by δ_m and δ_Q energy density fluctuations in matter and quintessence components, respectively, while $\Phi(k, z)$ and $\Psi(k, z)$ are the scalar perturbations of the metric (Bardeen potentials) in Fourier space. In the lensing section, we denote by k_{NL} the scale at which the power spectrum becomes non-linear, $P(k_{NL}, z) \sim 1$. See § 3 for details.

sity is 13 gal/arcmin^2 (the final selection produced a catalogue somewhat less deep than VIRMOS-Descart). Note that once completed the wide survey will be composed of three compact areas that will sample angular scales up to 5 degrees in three independent fields so that linear scales will be explored with much more accuracy than present-day CFHTLS wide data.

Each survey used complementary photometric or spectroscopic galaxy samples to derive the redshift distribution of the lensed galaxy samples. As discussed in the VIRMOS-Descart and CFHTLS cosmic shear papers, it is convenient to describe the redshift distribution by a three-parameter function

$$n(z) = \frac{\beta}{z_s \Gamma\left(\frac{1+\alpha}{\beta}\right)} \left(\frac{z}{z_s}\right)^\alpha \exp\left[-\left(\frac{z}{z_s}\right)^\beta\right]. \quad (26)$$

α and β are obtained from a fit of photometric redshift distributions derived from external redshift calibration surveys. The CFHTLS and VIRMOS are different lensing data sets, but they were obtained with the same telescope (although with different instruments) and with the same exposure time. Therefore the two lensing surveys have to be calibrated using the same external redshift data set in order to preserve the homogeneity of the analysis. In van Waerbeke et al. (2005), VIRMOS was calibrated using the Hubble Deep Fields and MS1008 (see

van Waerbeke et al. 2001 for the details), while the CFHTLS lensing data (Hoekstra et al. 2006; Semboloni et al. 2006) were calibrated using the HDF only. The HDF and MS1008 data in visible and near infrared bands produced accurate photometric redshifts (Yahata et al. 2000; Athreya et al. 2002). We decided to use the HDF only for both VIRMOS and CFHTLS and abandon the MS1008 field. We have checked that the results discussed in this paper do not depend whether or not we include the MS1008 calibration field, since the sample variance due to the use of combined redshift calibration sets is absorbed in the redshift error (van Waerbeke et al. 2006).

In the following, we will use for the CFHTLS-deep survey (Semboloni et al. 2005)

$$\alpha = 1.9833, \quad \beta = 0.6651, \quad z_s = 0.0981^{+0.0129+0.0209}_{-0.0114-0.0161} \quad (27)$$

giving a mean redshift $\langle z \rangle = 1.01$, while for the CFHTLS-wide and the VIRMOS-Descart surveys (Hoekstra et al. 2005)

$$\alpha = 1.35, \quad \beta = 1.654, \quad z_s = 0.668^{+0.035+0.053}_{-0.036-0.055} \quad (28)$$

giving a mean redshift $\langle z \rangle = 0.76$. We quote the 1σ and 2σ errors. We use the same parameters for the VIRMOS-Descart and CFHTLS-wide redshift distributions since both surveys have a similar depth and the same effective galaxy number density.

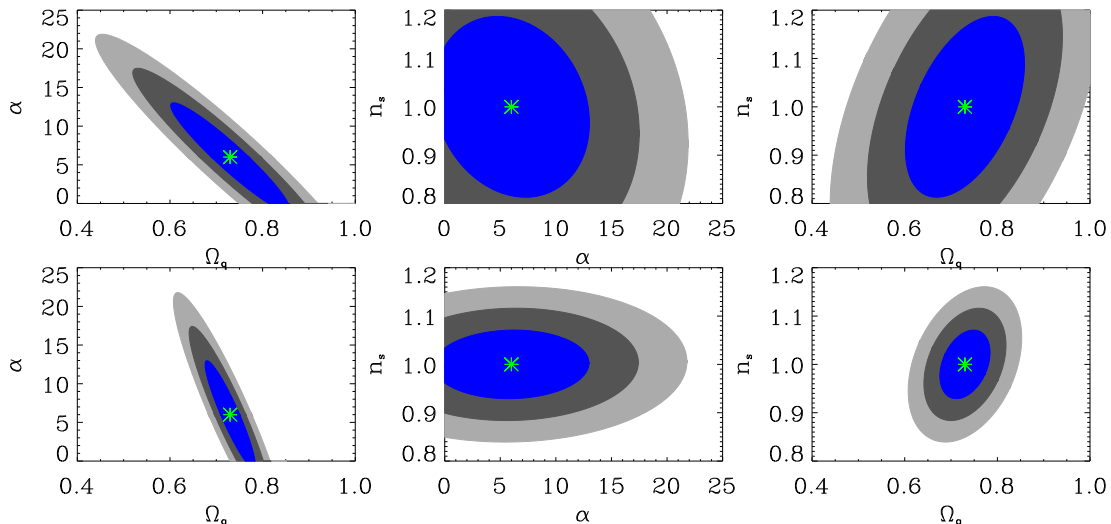


Fig. 6. Fisher analysis of the top-hat shear variance of the cosmic shear on cosmological parameters $(\alpha, \Omega_{\text{q}}, n_s)$, for Ratra-Peebles (top line) and SUGRA (bottom line) models – contours at 68 and 95% C.L. We employ a synthetic realization of a CFHTLS-wide like survey, covering 170 deg^2 , with 20 gal/arcmin^2 and intrinsic ellipticity $\sigma_e = 0.4$. The fiducial model is defined by $(\alpha, \Omega_{\text{q}}, n_s) = (6, 0.73, 1)$ while $(h, \tau_{\text{reion}}, \Omega_{\text{b}0} h^2) = (0.72, 0.17, 0.024)$ are kept fixed. The goal of this analysis is to help in choosing the range of the grid for the likelihood analysis. See § 3.4 for discussion.

3.4. Description of the pipeline and choice of the free parameters

The pipeline we developed is summarized in Fig. 5. We compute the evolution of background and perturbations power spectra in linear regime by means of a Boltzmann code allowing for photons, neutrinos, baryons, cold dark matter and quintessence scalar field. Notice that this code deals with several gauge choices and can also account for scalar-tensor theories of gravity (Riazuelo & Uzan 2002), hence allowing to study deviations from general relativity in this framework as well as extended quintessence scenarios (Uzan 1999; Amendola 2000; Chiba 1999).

Using this code we compute the CMB temperature (TT) angular power spectrum (C_ℓ) in order to fix the amplitude of the initial matter power spectrum at the redshift of the last scattering. We do it by matching the computed C_ℓ with WMAP-1yr data (Hinshaw et al. 2003) at a high multipole, to be preferred when studying quintessence since at low multipoles the integrated Sachs-Wolfe effect is dominant so that temperature anisotropies are not directly related to the density perturbations. Definitely, we use the multipole $\ell \simeq 110$ of WMAP-1yr data, where the total relative error of the TT spectrum is smaller than 3%. This normalization procedure holds until the correlation between multipoles is weak and does not take into account the measurement errors on the amplitude of the TT spectrum. However, we expect that the final results would not be strongly affected by a more accurate normalization. Notice that, as mentioned earlier on, σ_8 data is not used to normalize the spectrum.

Hence, its value may be evaluated from the matter power spectrum and compared with the observed values.

Once the linear matter power spectrum is known at every redshift, the weak lensing add-on code (Schimd et al. 2005) computes the non-linear power spectrum using two mappings, by Peacock & Dodds (1996) and Smith et al. (2003). We consider wavevectors ranging up to $10 h \text{ Mpc}^{-1}$ (corresponding to 1 arcmin at $z \sim 1$ for cosmologies close to ΛCDM). By Limber projection, we deduce the shear power spectrum allowing for a source redshift distribution of the form (26). To finish, several two-point statistics in real space are computed, namely the top-hat shear variance $\langle \gamma^2 \rangle$, the aperture mass variance $\langle M_{\text{ap}}^2 \rangle$, as well as the two-point correlations ξ_{\pm} . Let us stress that, like the Boltzmann code, the lensing add-on code works with scalar-tensor theories of gravity as well.

Cosmological parameters are estimated by comparing the predicted signal m_i to the data d_i as a function of scale X_i (which reduces to an angular scale θ_i , a redshift z_i or a multipole ℓ_i respectively for lensing, supernovae and CMB data). We vary the parameters of the model, disposing them on a regularly spaced grid and evaluate, at each grid point, the likelihood function,

$$\mathcal{L} = \frac{1}{(2\pi)^n |C|^{1/2}} \exp \left[-\frac{1}{2} (d_i - m_i)^T C^{-1} (d_i - m_i) \right]. \quad (29)$$

Here C^{-1} is the data covariance matrix, including Poisson shot noise and cosmic variance.

We focus on constraining the dark energy density and the parameter α of the quintessence potentials and restrict to a low-dimensional parameter space. Ideally, one would include

at least nine cosmological parameters: the spatial curvature, the Hubble constant, the parameter α of the potential, the dark energy and matter (both dark matter and baryonic) density parameters, the reionization optical depth, the amplitude and spectral index of the initial power spectrum. In addition, one should include the three parameters accounting for the redshift distribution of sources, Eq. (26).

Given the result of the analysis of CMB data (Spergel et al. 2003) we have assumed a spatially flat universe so that $\Omega_{m0} = 1 - \Omega_{Q0}$. The amplitude of the initial power spectrum is fixed by the normalization on the CMB. We have also assumed that the reduced Hubble constant, the reionization optical depth, and the baryon energy density today are fixed to $h = 0.72$, $\tau_{\text{reion}} = 0.17$, and $\Omega_{b0}h^2 = 0.024$, respectively.

We preliminarily performed a Fisher matrix analysis on the parameters space $(\alpha, \Omega_{Q0}, n_s)$ in order to estimate approximately the extent of the 1σ region and decide the sampling steps of the grid for the computation of the likelihood. Figure 6 depicts the 68%, 95%, and 99% confidence levels (C.L.) after for cosmic shear (top-hat variance) corresponding to a synthetic CFHTLS-wide like survey covering 170 deg^2 , with $20 \text{ galaxies/arcmin}^2$ and an intrinsic ellipticity of 0.4. The top line refers to Ratra-Peebles models and the bottom line to SUGRA models. Interestingly, SUGRA models appear to be more constrained. Notice that the Fisher analysis holds only locally, around a fiducial model (marked by a cross) which is fixed at $\alpha = 6$, $\Omega_{Q0} = 0.73$, and $n_s = 1.0$.

In the final likelihood analysis we allow to vary the cosmological parameters in the following ranges

$$\Omega_{Q0} \in [0.4, 0.9], \quad \alpha \in [0, 25], \quad n_s \in [0.9, 1.1]. \quad (30)$$

For the cosmic shear data, we also allow one of the source redshift parameters to vary. We choose z_s , always marginalized in the final analysis, over its 2σ interval; see Eq. (27) and (28).

Indeed, one can criticize these assumptions but these are sufficient *i)* to give us an idea of the parameter space available for dark energy realized by quintessence and *ii)* to discuss to which extent weak lensing can improve the constraints on dark energy models. From a pragmatic point of view, we were limited by computational capacities and a more complete analysis will follow. Let us stress that such an analysis requires an investigation of the potential degeneracies of the dark energy parameters with the standard cosmological parameters and in particular, one would need to quantify how allowing for dark energy changes the allowed range of variation of the other cosmological parameters. This is left for further studies.

3.5. Likelihood analysis: Joint cosmic shear data

Using both real and synthetic cosmic shear data, we perform the likelihood analysis aiming to investigate to which extent the constraints on cosmological parameters, and in particular the quintessence ones, depend on the linear-to-non-linear mapping and on the selection effects of the two-points statistics.

In this section, we focus on the first issue by combining top-hat variance data of VIRMOS-Descart, CFHTLS-deep and

CFHTLS-wide (22 deg^2 sub-sample) surveys; see Table 1 for numerical results of individual parameters constraints. Figure 7 depicts the results quoting the 68%, 95% and 99% confidence level contours. In particular, we compute the non-linear spectra by both the Peacock & Dodds (1996) and Smith et al. (2003) procedures, for Ratra-Peebles and SUGRA models as well, hence allowing for two kinds of comparisons.

Firstly, we can compare Ratra-Peebles and SUGRA likelihood contours, disregarding the non-linear mappings. There are two striking differences. One is the strong constraint on Ω_{Q0} found in the SUGRA case. The reason for this is that, since the amplitude of the power spectrum is kept fixed, the (σ_8, Ω_{m0}) degeneracy implies a strong constraint on Ω_{m0} and consequently on Ω_{Q0} through the flat universe prior. This effect is stronger in SUGRA models than in Ratra-Peebles ones since the former approach a Λ CDM at low redshift. The other difference is the well defined degeneracy found in the (Ω_{Q0}, α) plane for the Ratra-Peebles case. This feature will allow to put a stronger constraint on Ratra-Peebles' α than in SUGRA's α when combining with other data.

Secondly and perhaps more interestingly, by comparing the non-linear mappings, for both Ratra-Peebles and SUGRA models two classes of cosmological parameters come out: Those concerning the quintessence, α and Ω_{Q0} , are found to be essentially independent of the non-linear mapping used. This supports the claim of Simpson & Bridle (2005) that cosmic shear is sensitive to dark energy mostly through the background dynamics. The second class involves the cosmological parameters accounting for the primordial universe, here n_s only. In this case the corresponding likelihood contours strongly depend on the chosen mapping, regardless of the quintessence potential we used. Hence it seems not possible to constrain the primordial spectral index, at least jointly with quintessence parameters, until a stable formulation of the non-linear regime of structure formation will be available.

3.6. Likelihood analysis: Synthetic data

Using wide surveys, we can eventually investigate the effect of the non-linear regime of structures formation. For this purpose, we perform a likelihood analysis using a synthetic realization of the full CFHTLS-wide survey. This consists on synthetic data vectors of top-hat and aperture mass variance and on a synthetic covariance matrix. The former are computed at a fiducial model, which we take to be a Λ CDM with the current CFHTLS-wide redshift distribution. The covariance matrix is computed using the analytical approximation derived in Schneider et al. (2002). It depends on three main features of the survey: the effective area A , the effective galaxy number density n_{gal} and the dispersion of the distribution of ellipticities σ_e . For these parameters we used the values that are expected at the end of the CFHTLS-wide campaign, respectively: $A = 170 \text{ deg}^2$, $n_{\text{gal}} = 20 \text{ gal/arcmin}^2$, and $\sigma_e = 0.4$. Notice we assume a larger ellipticity dispersion and a higher density of galaxies than those obtained in Hoekstra et al. (2005). They correspond to a different galaxy weighting scheme than the one used with current data.

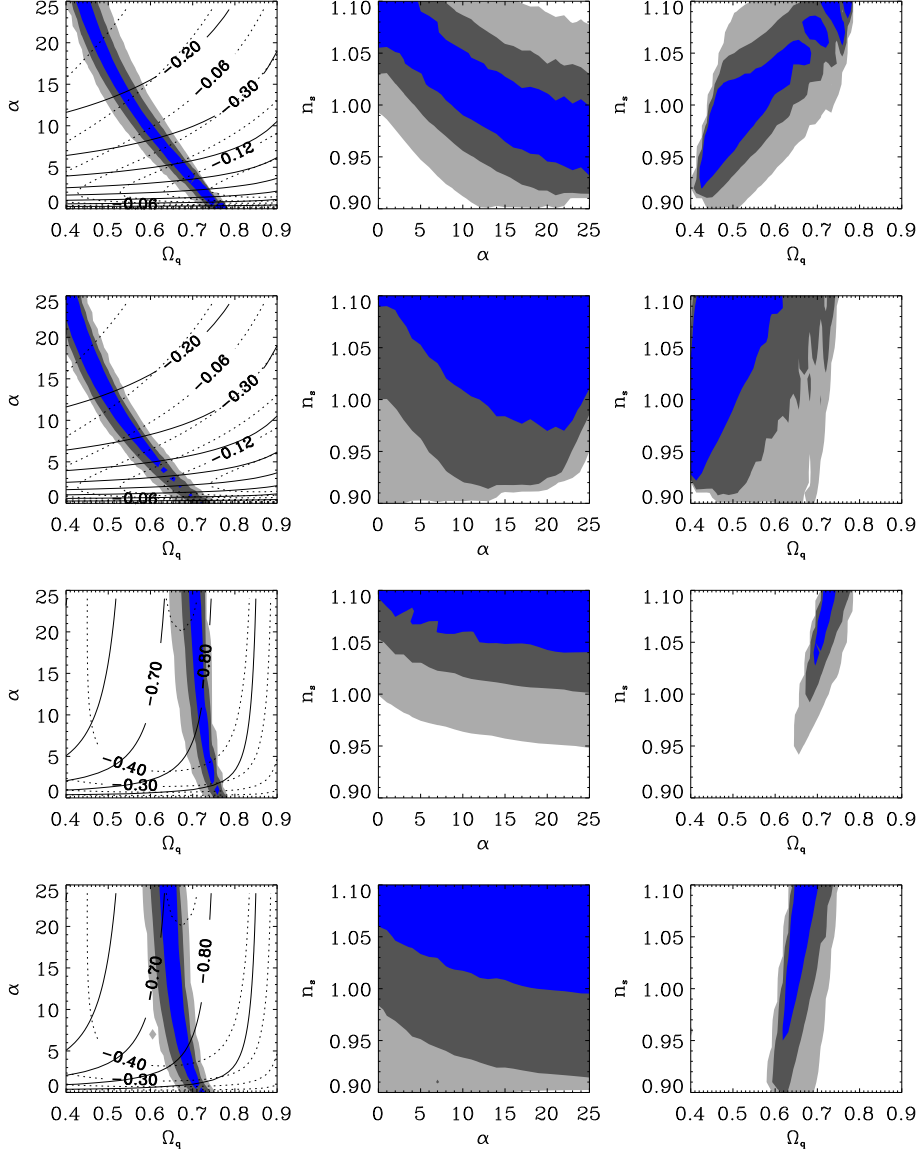


Fig. 7. Joint likelihood analysis of VIRMOS-Descart, CFHTLS-deep, and CFHTLS-wide/22 deg² top-hat variance data – contours at 68%, 95%, and 99% C.L. for the variables (Ω_{Q0}, α) , (α, n_s) and (Ω_{Q0}, n_s) . On the quintessence parameters sub-space, we have added the contour lines of (w_{pivot}, w_a) discussed in Fig. 3, assuming $z_{\text{pivot}} = 0.5$. The two upper lines are dedicated to Ratra-Peebles models [Eq. (2)] and the two lower to SUGRA models [Eq. (4)]. For each class of models, the non-linear spectrum has been computed using both the Peacock & Dodds (1996) procedure, first and third lines, and the halo model approach by Smith et al. (2003), second and fourth lines. See § 3.5 for discussion.

Figures 8 and 9 outline the likelihood analysis of Ratra-Peebles and SUGRA models, respectively – contours at 68%, 95% and 99% confidence level. We show only the results achieved using the mapping by Peacock & Dodds (1996), those achieved using the mapping by Smith et al. (2003) being in agreement at a 10% level.

Firstly, by comparing the results achieved using the top-hat variance data of the 22 deg² sub-sample (first line) with those of the synthetic 170 deg² field (second line), it is evident the gain achievable by the full survey. In particular, it is worth noticing that the pure quintessence parameters sub-space is less dependent on the survey area, while the constraints on the primordial

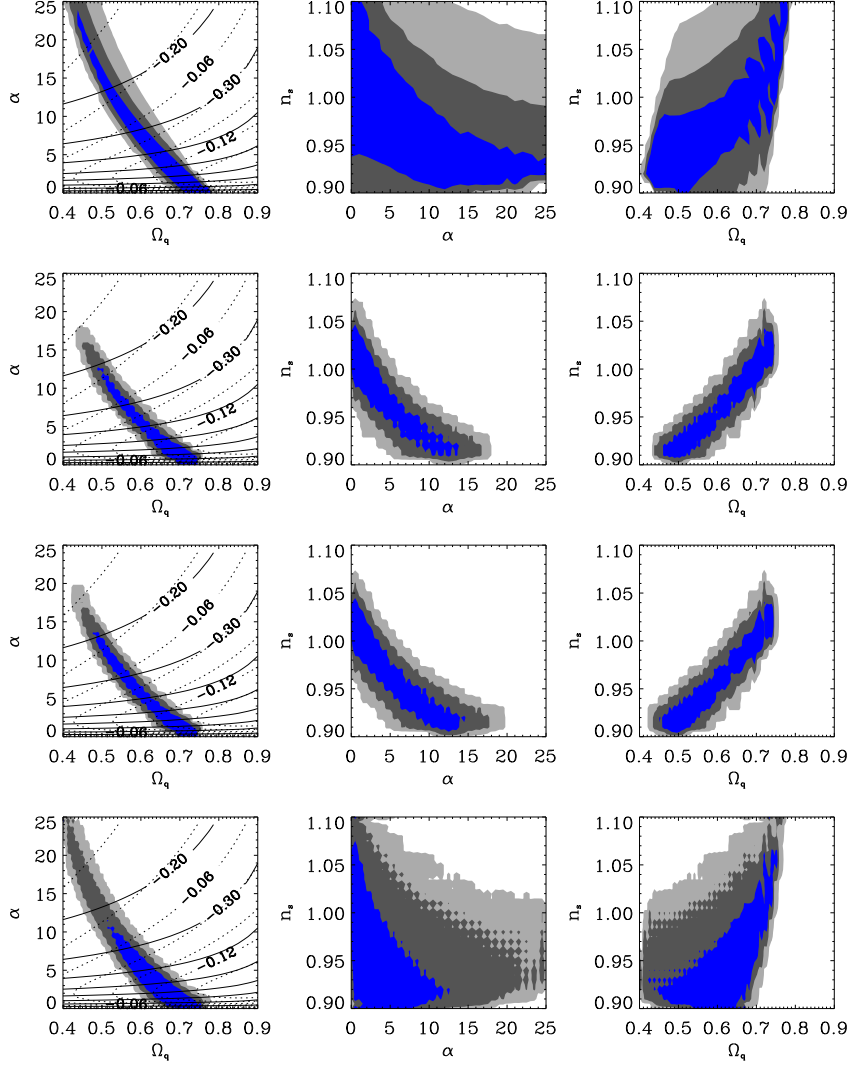


Fig. 8. CFHTLS-wide constraints on Ratra-Peebles models. From left to right, we present the likelihood analysis (68, 95, and 99% C.L.) for the variables (Ω_{Q0}, α) , (Ω_{Q0}, n_s) and (Ω_{Q0}, α) and we have added the contour lines discussed in Fig. 3, assuming $z_{\text{pivot}} = 0.5$. The first line depicts the analysis of the actual dataset based on 22 deg^2 (W1+W3; see § 3.6) while the three other lines are based on the synthetic data for a field of 170 deg^2 . The second and third line show the top-hat shear variance and aperture mass variance, respectively. The fourth line describes the analysis of the same simulated data for the top-hat shear variance but using only angular scales larger than 20 arcmin to cut out the non-linear part of the matter power spectrum. In particular, we conclude from the left column that the parameters describing the quintessence sector (α and Ω_{Q0}) are not affected by the choice of the statistics and are well estimated by the linear part of the power spectrum. Here we use the Peacock & Dodds (1996) mapping for the non-linear power spectrum. See § 3.6 for discussion.

spectral index strongly depend on it. Hence, the distinction of cosmological parameters in two classes introduced in the previous section seems confirmed.

The second and third lines show the likelihood contours for the top-hat and aperture mass variances, respectively. They are consistent, depicting compatible confidence level regions, for both Ratra-Peebles and SUGRA models. There are two main differences in the properties of aperture mass and top-hat

variances. Measurements of aperture mass variance at different scales are less correlated than top-hat variance ones, since the former is a narrow filtered version of the shear power spectrum; and their effective range extend to only about 1/5 of the top-hat variance one. Hence they follow more accurately the shape of the power spectrum at their measured range. For a power spectrum featureless at most scales but these, the independent measurements render this statistic the most convenient

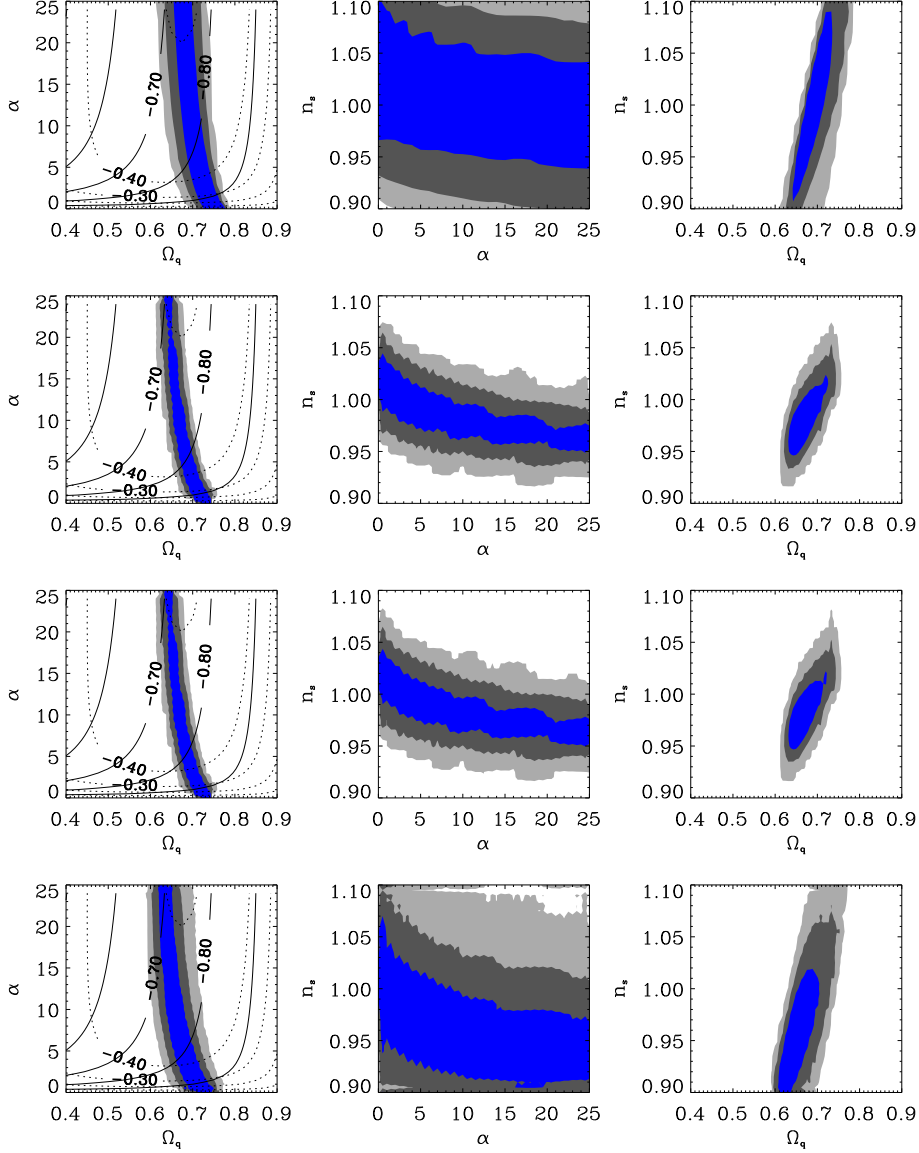


Fig. 9. CFHTLS-wide constraints on SUGRA models. This figure is analogous to Fig. 8. We reach the same conclusion as for Fig. 8. We recover that the contours are almost independent on the value of α . See § 3.6 for discussion.

to use. In this case, contours from aperture mass variance are expected to be smaller than top-hat variance ones. But in general, the loss of information regarding the behaviour of models at other scales diminishes the capability of distinguishing between models. Consequently, the aperture mass variance contours are, in general, expected to be larger than top-hat variance ones. The difference is larger when analysing cosmic shear most important parameters. Since we are studying parameters to which cosmic shear is moderately sensitive to, the difference is not noticeable in our results of Fig. 8 and 9, but it is striking

when using VIRMOS-Descart data to constrain the (σ_8, Ω_m) plane (van Waerbeke et al. 2001). Furthermore, for our data with measurements up to less than 1 deg, the largest power spectrum scale probed by the aperture mass variance is the one probed by the top-hat variance at around 10 arcmin and its full range effectively lies on non-linear scales, rendering its inferred parameters' constraints less reliable, due to non-linear modelling uncertainties. For these two reasons, we will choose to use top-hat variance only, when producing the final results.

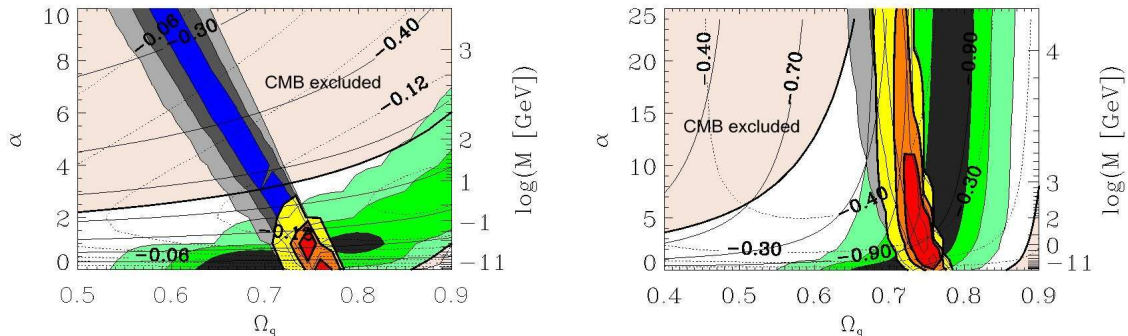


Fig. 10. Joint analysis of the quintessence parameters (Ω_{Q0}, α) from CMB, Sn Ia and cosmic shear (top-hat variance), for Ratra-Peebles models (left panel) and SUGRA models (right panel). The likelihood analysis uses the “gold” set for Sn Ia (green contours), VIRMOS-Descart + CFHTLS-deep + CFHTLS-wide (22 deg²) top-hat variance data for the cosmic shear (blue contours), and both combined (red contours). Contours correspond to 68, 95, and 99% C.L. According to WMAP-1yr measurements, the location of the first acoustic peak of the TT power spectrum of CMB, allowing for binning between neighboring multipoles, excludes regions of the parameter space (shaded) nearly degenerate with Sn Ia constraints; see § 4.2 for details. On the right axes, we quote an indicative mass scale of the quintessence potential, weakly dependent on Ω_{Q0} but Ω_{Q0} is small; see § 2 for details. Finally, contours of w_{pivot} (solid) and w_a (dotted) are superposed, setting $z_{\text{pivot}} = 0$. As for Ratra-Peebles models, supernovae data put strong constraints on α which are tightened by cosmic shear, while for SUGRA models all observables are fairly insensitive to the value of this parameter. The corresponding constraints are given on Table 1.

By using the full wide survey, we can try to disentangle the effects of the non-linear regime of structures formation by cutting off the small angular scales from the final analysis. In such a way, we can better investigate the distinction of cosmological parameters in the two classes discussed above, probing if dark energy primarily hangs on the background dynamics. The plots on the bottom line of Fig. 8 and 9 depict the analysis of the top-hat variance when taking into account only angular scales larger than 20 arcmin, corresponding to wavevectors $k \lesssim 1h \text{ Mpc}^{-1}$ at $z \lesssim \langle z \rangle / 2 \sim 0.5$, where therefore the effects of the non-linear regime are sub-dominant. The spread in the likelihood contours is more relevant for plots involving the primordial spectral index n_s , while the quintessence parameters (α, Ω_{Q0}) are not so much affected. Hence, one can study dark energy by cosmic shear using this technique even if not properly knowing how to deal with the non-linear regime. Obviously this conclusion has to be confirmed by a more complete study involving a larger parameter space, to account for other degeneracies.

The first line of Fig. 8 and 9 show the current results of CFHTLS-wide, using W1 and W3 data from an effective sky coverage of 22 deg². The corresponding marginalized constraints on α and Ω_{Q0} are a factor of 2 larger than the ones found with the synthetic data, depicted on the second line of the same figures. This is consistent with the reduction of the data error bars, which are proportional to $\sigma_e^2 n_{\text{gal}}^{-1} A^{-1/2}$, if cosmic variance is not taken into account. We must caution that the gain in the parameters space cannot be estimated with precision by this simple argument, namely by assuming

$$\text{gain} \sim \frac{r_{\sigma_e}^2}{r_{n_{\text{gal}}} \sqrt{r_A}}, \quad (31)$$

where the several factors r are the ratios between the features of the two surveys. In fact, the gain in the data error bars does

not translate linearly into a gain in the parameters space confidence levels. That happens only in the Fisher matrix approximation, and even there, only in the case of statistical uncorrelated parameters. Furthermore, this reasoning does not take into account the extra constraining power coming from measurements at larger scales, as discussed earlier on, or simply coming from the fact of disposing of more degrees of freedom for the likelihood calculations.

It is also worthwhile to notice that, in all cases, (from Fig. 7 to Fig. 9), a Λ CDM model with a Harrison-Zel’dovich spectrum [$(\alpha, n_s) = (0, 1)$] is compatible with the data for $\Omega_{Q0} \sim 0.7$ at 99% confidence level.

4. Combining with other observables

4.1. Sn Ia

We combine the cosmic shear data by VIRMOS-Descart and CFHTLS-deep and -wide (22 deg² sub-sample) surveys with the type Ia supernovae “gold” set by Tonry et al. (2003). In particular, we evaluate confidence intervals considering cosmic shear and Sn Ia both separately and jointly. Indeed, since the distance modulus depends only on the background dynamics, we can restrict to the quintessence cosmological parameters α and Ω_{Q0} .

Figure 10 depicts both the independent and combined analysis for Ratra-Peebles and SUGRA models, using the top-hat variance data for the cosmic shear and computing the non-linear spectrum by the Peacock & Dodds (1996) mapping. The corresponding results are summarized in Table 1; these marginalized results for each parameter do not assume any prior knowledge of the other, apart from the flat priors implied by the range of the grid. Let us emphasize that the constraints obtained on (w_0, w_a) inferred from those on the parameters of

Table 1. Results of separate and joint analysis of cosmic-shear and Sn Ia data on quintessence parameters (α, Ω_{Q0}) at 68% (95%) confidence level. For Ratra-Peebles models, we quote only upper limits for α , the best-fit being always at $\alpha = 0$. Only for indicative purposes, we quote 68% confidence level limits on (w_{pivot}, w_a) parameters computed at $z_{\text{pivot}} = 0$, setting *n.c.* when not constrained. Remind that in this class of models $w_0 \geq -1$. See § 4.1 for discussion.

	Ratra-Peebles				SUGRA			
	α	Ω_{Q0}	w_0	w_a	α	Ω_{Q0}	w_0	w_a
$\langle \gamma^2 \rangle$	< 14 (25)	$0.63^{+0.14(+0.15)}_{-0.17(-0.20)}$	n.c.	n.c.	$13^{+12(+12)}_{-5(-13)}$	$0.72^{+0.01(+0.03)}_{-0.02(-0.05)}$	$\lesssim -0.79$	$\gtrsim -0.45$
Sn Ia	< 1 (3)	$0.74^{+0.09(+0.16)}_{-0.10(-0.11)}$	$\lesssim -0.75$	$\gtrsim -0.13$	$12^{+5(+11)}_{-12(-12)}$	$0.77^{+0.04(+0.07)}_{-0.04(-0.08)}$	$\lesssim -0.84$	$\gtrsim -0.43$
$\langle \gamma^2 \rangle + \text{Sn Ia}$	< 1 (1)	$0.75^{+0.02(+0.03)}_{-0.04(-0.04)}$	$\lesssim -0.7$	$\gtrsim -0.13$	$2^{+7(+18)}_{-1(-2)}$	$0.74^{+0.03(+0.03)}_{-0.04(-0.05)}$	$\lesssim -0.84$	$\gtrsim -0.38$

the potential differ when we change the form of the potential. This confirms that constraints on the equation of state derived from a general parameterization have to be interpreted with care.

Concerning Ratra-Peebles models, the weak lensing and Sn Ia contours are closer to mutual orthogonality, so narrow joint constraints are expected. Data strongly favor a quintessence component close to a cosmological constant, the best-fit lying always at $\alpha = 0$, hence in Table 1 we prefer to present the constraints as an upper limit. As it is clear from Fig. 10, Sn Ia are much more constraining for this parameter than weak lensing. However, even though Sn Ia alone reject $\alpha \geq 1$ at the 68% level, as was well known (Podariu & Ratra 2000), the information of weak lensing further narrows the interval.

As far as SUGRA models are concerned, for a wide interval of α (approximately $\alpha > 5$) both the luminosity distance and the shear two-point correlations are (almost) independent of this parameter, as it is clear from Fig. 10. In fact, this conclusion was already reached studying the CMB temperature anisotropies (Brax et al. 2000), arguing that the equation of state does not strongly depend on the slope of the potential leaving both distances and linear growth factor almost unchanged. In terms of statistical significance, this means that the likelihood with respect to Sn Ia or weak lensing data alone is almost flat and so the best fit value has little meaning (see also Caresia, Matarrese & Moscardini 2004). In Table 1 we list the results in the form of a parameter value, which we take to be the likelihood weighted average of α , plus or minus the necessary deltas to form the confidence intervals obtained. However, for a substantial interval of α , weak lensing and Sn Ia lead to different, almost non-intersecting, ranges of Ω_{Q0} : Sn Ia favoring a higher value of Ω_{Q0} while weak lensing a lower one. Thus, even though both observables have limited sensitivity to constrain the parameter α , this fact allows to obtain a reasonable constraint from the joint likelihood, as that interval of α is rejected by the data. The joint confidence interval are substantially reduced, with a distinctive maximum of likelihood at $\alpha = 2$. Notice, however, that this is an unstable situation. In fact, if cosmic shear or supernovae contours slightly change their orientation and size, due for example to a larger uncertainty on the redshift of sources, the joint contour will easily degrade. On the contrary, the extreme situation, with both contours vertical and parallel for all α , would imply the abandon of the subjacent quintessence model. It has to be stressed that every systematic effect on weak-lensing or supernovae data as well, relying for

instance on the data analysis procedure, strongly affects the final result; therefore special care is necessary when combining several datasets.

Provided that it is not evident which pivot redshift should be used when parameterizing these quintessence models by an equation of state of the form of Eq. (12), we superpose to the (Ω_{Q0}, α) plane, in Fig. (10), the contours for w_{pivot} and w_a corresponding to $z_{\text{pivot}} = 0$. Let us stress that the final results in terms of the equation of state parameters are only indicative. Interestingly, notice from Fig. 3 that for Ratra-Peebles models the estimation of w_{pivot} and w_a should not change if using $z_{\text{pivot}} = 0.5$ instead of $z_{\text{pivot}} = 0$. In fact, for this class of models, noticeably different values on these parameters only appear when considering a high pivot redshift. On the contrary, for SUGRA models the choice of the pivot redshift would be relevant already at low redshift. See Table 1 for specific constraints on w_0 and w_a .

Recently, weak lensing data (Semboloni et al. 2005; Hoekstra et al. 2005) were used to put constraints on w under the assumption it is a constant parameter. In that particular case, one has two characteristic redshifts, z_a and z_{de} , defining the beginning of the acceleration phase [$\ddot{a}(z_a) = 0$], and of the domination of the dark energy [$\Omega_m(z_{\text{de}}) = \Omega_{\text{de}}(z_{\text{de}})$]. They are given by

$$(1 + z_a)^{3w} = -\frac{1}{1 + 3w} \frac{\Omega_{\text{de}0}}{\Omega_{\text{m}0}}, \quad (1 + z_{\text{de}})^{3w} = \frac{\Omega_{\text{de}0}}{\Omega_{\text{m}0}} \quad (32)$$

so that $z_a > z_{\text{de}}$. When w becomes negative and large in absolute value, z_a and z_{de} tend to zero so that dark energy just starts almost today to dominate the universe and is redshifted in a way that it does not affect even low redshift observables. It follows that we expect the data to be insensitive to the value of w in that regime so that one can get only an upper limit on its value, as found by Semboloni et al. (2005) and Hoekstra et al. (2005). Indeed, such a situation cannot be achieved with a physical model as considered here because by construction it imposes that $w > -1$. It follows that our approach is a physically motivated way of imposing a prior on w . Note also that Fig. 1 shows that $w = \text{constant}$ is not a good approximation of these two classes of models. At best the constraint on a constant w can be related to some redshift average of the equation of state. For these reasons, it is difficult to deduce a constraint on the physical models from a constraint on a constant w , even though the results of Table 1 are compatible with them.

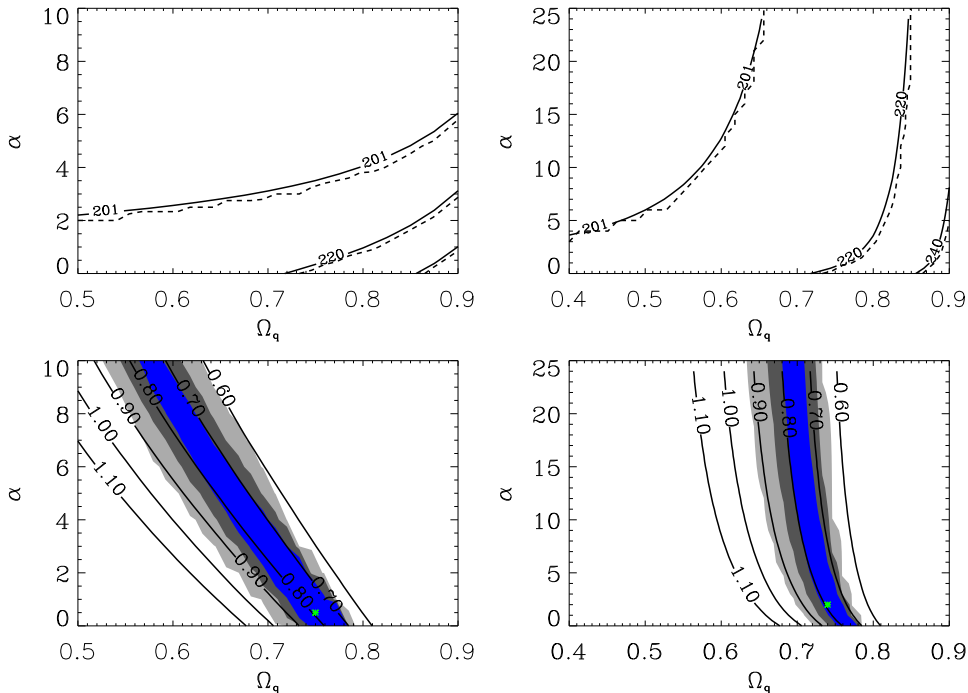


Fig. 11. Top line: Location of the first acoustic peak of the temperature CMB power spectrum, for Ratra-Peebles (left panel) and SUGRA (right panel) models. It is estimated using an analytical approximation lying on the solution of the background equations only (solid line), or by the full computation of C_ℓ s (dashed line). The central set of lines corresponds to the location of the first peak according to the best-fit of WMAP-1yr data, $\ell = 220.1 \pm 0.8$ (Page et al. 2003), while the left and right ones correspond to the multipoles $\ell = 201$ and $\ell = 240$, defining the smallest and the largest multipoles contributing to the three points of binned data defining the first peak (Hinshaw et al. 2003). Bottom line: Contour levels of σ_8 for Ratra-Peebles (left panel) and SUGRA (right panel) models with $n_s = 1$, superposed to the cosmic shear data contours (VIRMOS-Descart + CFHTLS-deep + CFHTLS-wide/22deg²). For indicative purposes, the star point marks the best-fit from the joint cosmic shear and SN Ia analysis, roughly sitting at $\sigma_8 = 0.8$. Remind that the spectrum was normalized on the CMB so this plots show the consistency with a normalization at $z = 0$. See § 4.2 for details.

4.2. CMB

CMB temperature anisotropies have been extensively used (see e.g. Baccigalupi et al. 2002; Jassal et al. 2004; Corasaniti et al. 2004) but several degeneracies amongst the cosmological parameters prevent to accurately constrain the cosmological parameters using CMB data only. Hence, concerning weak lensing, several studies already attempt to combine CMB and cosmic shear in order to constrain the cosmological parameters (see e.g. Contaldi, Hoekstra & Lewis 2003; Ishak et al. 2004; Tereno et al. 2004).

As for dark energy, it affects the CMB anisotropies angular power spectrum at least in two ways (Brax et al. 2000). Firstly, the angular diameter distance is modified so that the peak structure is shifted. In particular the location of the first acoustic peak, depending on the geometry of the universe, provides an estimate of the angular diameter distance to the last scattering surface. However, also pre-recombination effects can shift the peaks from their true geometrical locations (Doran et al. 2000; Kamionkowski & Buchalter 2000). Secondly, the time evolution of the dark energy strongly affects the integrated Sachs-

Wolfe effect. This effect is more relevant at low multipoles, modifying the amplitude of the spectrum, but it also leads to an additional shift of the Doppler peaks.

Beside using CMB data to normalize the power spectra (see § 3.4), we use them in still another way, by noticing that even without a statistical analysis of CMB data, we can strongly constrain cosmological parameters, and notably quintessence ones, simply by using the location of the first Doppler peak. Moreover, even without solving the perturbations equations and computing the TT spectrum, one can compute the acoustic scale just by solving the equation for the background evolution. The location of acoustic peaks is then estimated allowing for the shifts induced by the dark energy by means of a fitting formulae (Doran & Lilley 2002). In conclusion, the location of the first acoustic peak will be a function of the quintessence parameters (Ω_{Q0}, α), with a negligible dependence on the primordial spectral index n_s due to the shift correction. Using this analytic approximation, we individuate, in Fig. 10, the region of the (Ω_{Q0}, α) plane compatible with the location of the first acoustic peak of WMAP-1yr data (Hinshaw et al. 2003; Page et al. 2003) including the bins' contributions to the three

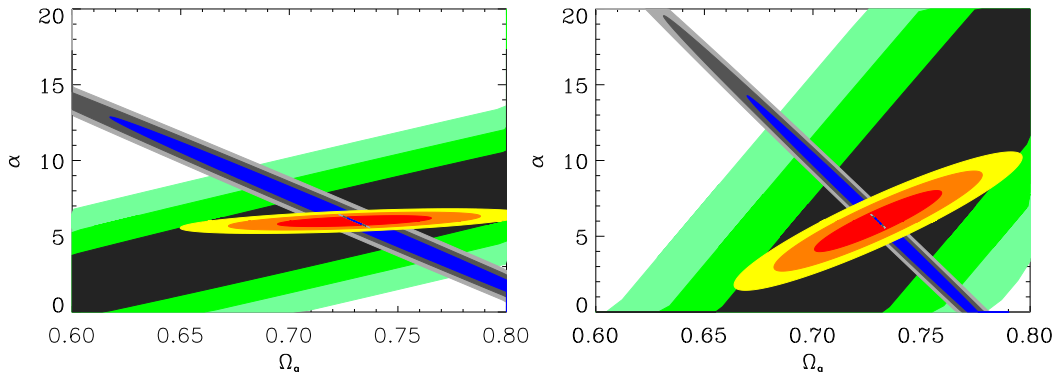


Fig. 12. Fisher analysis for Ratra-Peebles (left) and SUGRA (right) models, for cosmic shear top-hat variance (blue ellipses), Sn Ia “gold” set (green) and temperature CMB angular power spectrum by WMAP-1yr data (red) – contours at 68%, 95%, and 99% C.L. For the CMB, the noise matrix was computed using the public code by Verde et al. (2003). For cosmic shear, we consider both CFHTL-wide survey ($A = 170 \text{ deg}^2$, $n_{\text{gal}} = 20 \text{ gal/arcmin}^2$, $\sigma_e = 0.4$) and a space based survey ($A = 20,000 \text{ deg}^2$, $n_{\text{gal}} = 35 \text{ gal/arcmin}^2$, $\sigma_e = 0.3$), quoted as type II in the text. We consider only the quintessence parameters while the other parameters are kept *fixed*. See § 5 for discussion.

points defining the peak, namely $201 < \ell < 240$. Consistently with the normalization procedure we used, this result is in excellent agreement with that deduced by the complete computation of the TT spectrum, as shown in Fig. 11 (top panels). Let us notice that the region of the parameter space (Ω_{Q0}, α) compatible with the position of the first acoustic peak is degenerate with the Sn Ia constraints; the reason being that, since the pre-recombination effects of quintessence on the definition of the acoustic horizon at last scattering are negligible, the acoustic scale eventually depends only on the Hubble parameter like the luminosity distance, hence both are ultimately affected by quintessence approximatively in the same way.

As mentioned before, we may compute, a posteriori, the σ_8 value for each CMB normalized model. The obtained σ_8 iso-contours in the (Ω_{Q0}, α) plane are shown in Fig. 11 (bottom panels), setting $n_s = 1$. The σ_8 range that corresponds to the confidence levels found in this work, is in agreement with current cosmic shear σ_8 constraints; see Hoekstra et al. (2005) for a recent result and van Waerbeke & Mellier (2003) for a compilation of results. This shows the normalizations on the CMB and at $z = 0$ are compatible. It is interesting to note that the directions of the $\Omega_{Q0} - \alpha$ degeneracy for lensing data are lines of constant σ_8 . Being the curvature of the universe kept fixed, a strong constraint in σ_8 implies a strong constraint in Ω_{Q0} , through the well-known $\sigma_8 - \Omega_{m0}$ degeneracy, but only if Ω_{Q0} and α are not much correlated. This is what we observe in the SUGRA case. In general, a contour in the (σ_8, Ω_m) plane will move as a function of α and a strong constraint in σ_8 does not necessarily imply a strong constraint in Ω_{Q0} . This is what is obtained for the Ratra-Peebles case.

The computational tools we have developed (see Riazuelo & Uzan 2000; Schmid et al. 2005) allow us to compute, in the same framework, distance modulus, CMB anisotropies and weak lensing - cosmic shear effects. Hence, in principle there is no problem to combine Sn Ia and lensing data with CMB data. As already noticed, we have restricted here to a small parameter space. Indeed, a joint analysis with

CMB data require a wider parameter space, possibly using the so-called “normal parameters” (see e.g. Sandvik et al. 2004), an option left for a future study. In such a case, we would integrate our pipeline with a Markov chain Monte Carlo code (Tereno et al. 2005) developed for the likelihood analysis of cosmic shear. Furthermore, we will be able to include also the analysis of nucleosynthesis constraints, by means of a suitable code for quintessence models (Coc et al. 2006) which is able to deal with ordinary and extended quintessence models like the Boltzmann code and the lensing code used for this study.

5. Weak lensing: prospects on future data

Beyond CFHTLS, several next generation cosmic shear surveys are proposed in order to pin down w and the dark energy properties with exquisite details. It is interesting to explore the capabilities of such surveys to constrain the models discussed in this paper.

A comprehensive study of all projects and observing strategies is however beyond the scope of this work, so we deliberately focus on rather simple concepts where both sky coverage and depth are increased, assuming systematics related to shape measurement can be discarded. In particular, we consider two possible layouts achievable by space-based missions. We will indicate by

- *type I*, a deep survey that would cover about $1,000 \text{ deg}^2$, one magnitude deeper than the CFHTLS-wide providing an effective galaxy number density around 50 gal/arcmin^2 ;
- *type II*, a wider but shallow survey, covering $20,000 \text{ deg}^2$ and yielding 35 gal/arcmin^2 .

Furthermore, we assume an intrinsic ellipticity distribution of galaxies similar to the one observed with current cosmic shear surveys with HST.

A Fisher analysis of Ratra-Peebles and SUGRA models, restricted to the parameters (Ω_{Q0}, α) keeping fixed the others, was performed around a fiducial model defined by

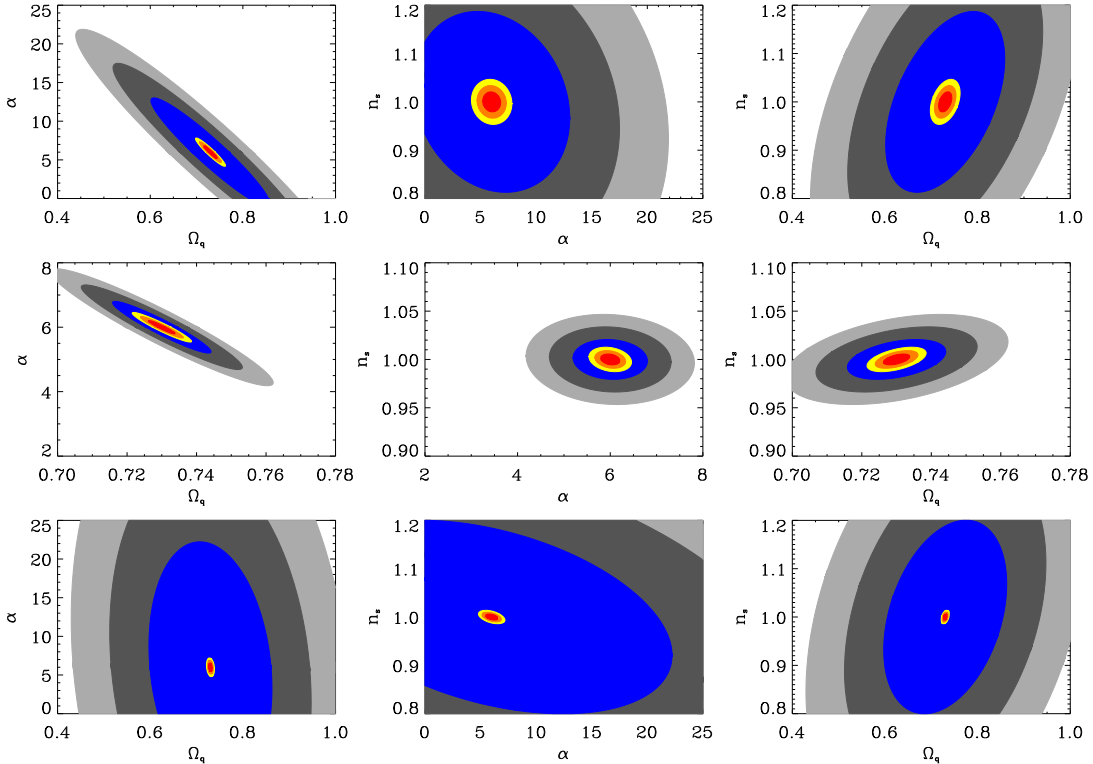


Fig. 13. Fisher analysis of weak lensing (top-hat variance) for Ratra-Peebles models, considering a CFHTLS-wide like survey and two space-based surveys layout (68, 95%, and 99% C.L.). Upper panels, analogous to the the likelihood analysis of Sec. 3 which also assumes all the other parameters fixed, compare a CFHTLS-wide like survey (wider ellipses) with a deep space-based survey with $A = 1,000 \text{ deg}^2$, $n_{\text{gal}} = 50 \text{ gal/arcmin}^2$, $\sigma_e = 0.3$. Middle panels compare the deep space-based survey with a wider but shallow space-based survey with $A = 20,000 \text{ deg}^2$, $n_{\text{gal}} = 35 \text{ gal/arcmin}^2$, $\sigma_e = 0.3$. Bottom panels compare the CFHTLS-wide like survey with the type I space-based survey, but marginalizing over τ_{reion} , left to vary. Compared with the first line, the last one shows the effect of a wider, more realistic parameters space. See § 5 for discussion.

$(\alpha, \Omega_{Q0}, n_s, h, \tau_{\text{reion}}, \Omega_{\text{baryon}} h^2) = (6, 0.73, 1, 0.72, 0.17, 0.024)$. The results are presented in Fig. 12. Weak lensing surveys (blue ellipses) together with Sn Ia “gold” set (green ellipses) and CMB WMAP-1yr data (red ellipses) show that both quintessence parameters can in principle be determined with a 10% accuracy. The degeneracy with respect to Sn Ia and CMB would be almost totally broken. It is worth noticing that, since all cosmological parameters but the quintessence ones are kept fixed, we have to take with care the astonishing gain with respect to a CFHTLS-wide like survey (larger ellipses) achievable by a space mission of type II (smaller ellipses).

On scales larger than 10 degrees, the flat sky approximation used in § 3.1 does not hold anymore. Indeed, an angular distance of 15 degrees on a sphere has a 1% deviation from the same distance on a plane, and for larger scales spherical harmonics must be considered (Stebbins 1996). These effects are not taken into account in the results of Fig. 12, where the gain observed between the two weak lensing ellipses essentially corresponds to Eq. (29), together with contributions from the fact that measurements from two surveys observing at dif-

ferent scale ranges have different cosmic variances and different degrees of freedom when fitting models to the data. In particular, a strategy of dividing the covered surveys areas in patches of 100 deg^2 is assumed. Hence, given the large ratio between both sky coverages, this is the dominant factor in the gain.

For indicative purposes, and allowing for the effect of systematic discussed in § 4.1, we restrict the Fisher analysis of weak lensing (top-hat variance) to Ratra-Peebles models, considering a CFHTLS-wide like survey and both the space-based surveys of type I and type II; see Fig. 13. As in § 4.1, we evaluate the parameters space $(\alpha, \Omega_{Q0}, n_s)$. In the upper line, analogous to the likelihood analysis of Sec. 3, which also assumes all the other cosmological parameters fixed, we compare the CFHTLS-wide like survey (wider ellipses) with the space-based surveys of type I (smaller ellipses). The middle line is analogue, but comparing space-based surveys of type I (wider ellipses) with the space-based surveys of type II (smaller ellipses); notice that the apparent rotation of the ellipses is simply due to a rescaling of the axes. As for quintessence parameters,

a space-based survey of type II gives 99% C.L. contours approximately 3 times smaller than those achievable by a type I survey. Indeed more cosmological parameters have to be taken into account. As an example, allowing the reionization optical depth τ_{reion} to vary, the likelihood contours get strongly modified; see the third line of Fig. 13, where we compare a CFHTLS-wide like and type II space-based surveys, marginalizing over τ_{reion} . They get larger and the degeneracies directions are changed. The strong impact of the reionization optical depth in cosmic shear results comes from the degeneracy with between τ_{reion} and the normalization of the spectrum.

The great predictive power of space-based surveys will allow to simultaneously constrain a large number of parameters with a good precision, even if not so high as the one showed in the analysis of a small number of parameters. However, it is important to notice that we have not taken into account the possible use of tomography based on a decomposition of lensing data into several lensed/source planes, nor any use of higher order statistics than the top-hat shear variance. So, even if several issues have been neglected in the present study, our conclusions are likely not over-optimistic.

6. Conclusions

In this article, we have investigated the constraints set by weak lensing, supernovae and CMB data on two families of quintessence models.

In such a situation where a physical model is fully specified, we can treat both the background and perturbation evolution without any ambiguity, in particular when dealing with high redshift data. Such an approach is thus complementary, as discussed in Section 2, to those based on a parameterization of the dark energy sector, in particular when trying to infer constraints on a physical model from those on the parameters of the equation of state. This also enables to get rid of the pivot redshift problem when combining different data sets. From a more theoretical point of view, such models cannot lead to an equation of state $w < -1$, contrary to an arbitrary parameterization. It is thus interesting to determine whether there is a tension in the data when such a physical constraint is imposed, as would be concluded from various studies indicating that $w < -1$ is favored.

To achieve this task, we have used a set of numerical tools that allow to compute background, CMB and lensing signatures of a large class of cosmological models including quintessence and some extensions such as scalar-tensor theories. We have focused our analysis on three cosmological parameters, the index of the primordial power spectrum and two parameters describing the quintessence models, along with an extra-parameter for the sources distribution (see § 3.4). Although one can criticize such a small parameter space, it is sufficient to give an idea of the parameter space available for quintessence models and to discuss how weak lensing data can improve the constraint on dark energy. This choice was also driven by numerical limitations but our analysis will be extended to a larger set of parameters in a near future.

We have normalized the initial power spectrum to the CMB so that σ_8 is now a prediction of the models and is not used for

normalization. Note also that we do not require to specify an analytical form for the transfer function. Weak lensing predictions are also sensitive to the linear to non-linear mapping and we have discussed the effect of such mapping on the constraints with care. In particular, we have shown that, while the parameters of the primordial spectrum are sensitive, those of the dark energy sector remain robust. We have also tested the possibility to cut the weak lensing data sets (such as the CFHTLS-wide) in order to reduce the influence of the non-linear regime.

This analysis is the first one using CFHTLS data to study the dark energy and illustrates the complementarity of these observations with other data sets. To finish, we have also forecast how space-based wide field imagers will improve our knowledge of dark energy. In particular, we have considered two possible strategies, the first deeper and the second wider but shallower. The latter turns out to be more suited to track dark energy as far as cosmic shear is concerned. The constraints on the two classes of quintessence models considered in this article are shown on Fig. 10 and Table 1. They can be summarized as follows: For a flat universe and a quintessence inverse power law potential with slope α , we get $\alpha < 1$ and $\Omega_{Q0} = 0.75^{+0.03}_{-0.04}$ at 95% confidence level, whereas $\alpha = 2^{+18}_{-2}$, $\Omega_{Q0} = 0.74^{+0.03}_{-0.05}$ when including supergravity corrections.

In the future, we plan to improve this analysis by first comparing it to a similar analysis based on a parameterization of the equation of state, by enlarging the parameter space, by addressing more carefully the problem of the redshift distribution of galaxies and by shifting from a grid method to an MCMC method.

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References

- Amendola, L. 2000, Phys. Rev. D, 62, 043511.
- Athreya, R., et al. 2002, A&A, 384, 743.
- Baccigalupi, C., et al. 2002, Phys. Rev. D, 65, 063520.
- Bacon, D., Réfrégier, A., & Ellis, R. 2000, MNRAS, 318, 625.
- Bartelmann, M., & Schneider, P. 2001, Phys. Rep., 340, 291.
- Benabed, K., & Bernardeau, F. 2001, Phys. Rev. D, 64, 083501.
- Benabed, K., & van Waerbeke, L. 2003, Phys. Rev. D, 67, 123515.
- Basset, B. A., et al. 2002, MNRAS, 336, 1217.
- Basset, B. A., Corasaniti, P. S., & Kunz, M. 2004, Phys. Rev. D, 69, 083517.
- Bennett, C. L., et al. 2003, ApJS, 148, 1.
- Brax, P., & Martin, J. 1999, Phys. Lett. B, 468, 40.
- Brax, P., Martin, J., & Riazuelo, A. 2000, Phys. Rev. D, 62, 103505.

- Caldwell, R. R., & Linder, E. V. 2005, *Phys. Rev. Lett.*, 95, 141301.
- Caresia, P., Matarrese, S., & Moscardini, L. 2004, *ApJ*, 605, 21.
- Carroll, S. M. 2001, *Living Rev. Rel.*, 4, 1.
- Chevallier, M., & Polarski, D. 2001, *Int. J. Mod. Phys. D*, 10, 213.
- Chiba, T. 1999, *Phys. Rev. D*, 60, 083508.
- Coc, A., et al. 2006 [astro-ph/0601299].
- Contaldi, C., Hoekstra, H., & Lewis, A. 2003, *Phys. Rev. Lett.*, 90, 221.
- Corasaniti, P. S., & Copeland, E. J. 2003, *Phys. Rev. D*, 67, 063521.
- Corasaniti, P. S., et al. 2004, *Phys. Rev. D*, 70, 083006.
- Crocco, M., & Scoccimarro, R. 2005 [astro-ph/0509418 and astro-ph/0509419].
- Dave, R., Caldwell, R. R., & Steinhardt, P. J. 2002, *Phys. Rev. D*, 66, 023516.
- Dolag, K., et al. 2003, *A&A*, 416, 853.
- Doran, M., et al. 2001, *ApJ*, 559, 501.
- Doran, M., et al. 2001, *Phys. Rev. D*, 64, 123520.
- Doran, M., & Lilley, M. 2002, *MNRAS*, 330, 965.
- Hamilton, A. J., et al. 1991, *ApJ*, 374, L1.
- Hinshaw, G., et al. 2003, *ApJS*, 148, 135.
- Hoekstra, H. 2003, *IAU Symposium 216, Maps of the Cosmos*, ASP Conference Series, eds. M. Colless & L. Staveley-Smith, Sydney, July 2003 [astro-ph/0310908].
- Hoekstra, H., et al. 2005, accepted by *ApJ* [astro-ph/0511089].
- Hu, W., & Jain, B. 2004, *Phys. Rev. D*, 70, 043009.
- Hu, W., & White, M. 1997, *Phys. Rev. D*, 56, 596.
- Huterer, D., & Takada, M. 2005, *Astropart. Phys.*, 23, 369.
- Ishak, M., Hirata, C., McDonald, P., & Seljak, U. 2004, *Phys. Rev. D*, 69, 083514.
- Jarvis, M., Jain, B., Bernstein, G., & Dolney, D. 2005, accepted by *ApJ* [astro-ph/0502243].
- Jassal, H. K., Bagla, J. S., & Padmanabhan, T. 2004, *MNRAS*, 356, L11.
- Jain, B., & Taylor, A. 2003, *Phys. Rev. Lett.*, 91, 141302.
- Jing, Y. P., et al. 2005 [astro-ph/0512426].
- Kaiser, N., Wilson, G., & Luppino, G. 2000 [astro-ph/0003338].
- Kamionkowski, M., & Buchalter, A. 2000 [astro-ph/0001045].
- Klypin, A., et al. 2003, *ApJ*, 599, 31.
- Kujat, J., et al. 2002, *ApJ*, 572, 1.
- Linder, E. V., 2003, *Phys. Rev. Lett.*, 90, 091301.
- Linder, E. V., & Huterer, D., 2005, *Phys. Rev. D*, 72, 043509.
- Ma, C. P., & Fry, J. N. 2000, *ApJ*, 543, 503.
- Ma, C. P., et al. 1999, *ApJ*, 521, L1.
- Mainini, A., et al. . 2003, *ApJ*, 599, 24.
- Martin, J., Schmid, C., & Uzan, J.-P., 2006, *Phys. Rev. Lett.*, 96, 061303.
- McCracken, H. J., et al. 2003, *A&A*, 410, 17.
- McDonald, P., Trac, H., & Contaldi, C. 2006, *MNRAS*, 366, 547.
- Mellier, Y. 1999, *ARA&A*, 37, 127.
- Padmanabhan, T., 2003, *Phys. Rep.*, 380, 235.
- Page, L., et al. 2003, *ApJS*, 148, 233
- Peacock J. A., & Dodds, S. J. 1996, *MNRAS*, 280, L19.
- Peebles, P. J. E. 1993, *Principles of physical cosmology* (Princeton University Press).
- Peebles P. J. E., & Ratra, B. 2003, *Rev. Mod. Phys.* 75, 559.
- Peter, P., & Uzan, J.-P. 2005, *Cosmologie primordiale* (Belin).
- Podariu, S., & Ratra, B. 2000, *ApJ*, 532, 109.
- Ratra B., & Peebles, P. J. E. 1988, *Phys. Rev. D*, 37, 3406.
- Refregier, A. 2003, *ARA&A*, 41, 645.
- Riazuelo, A., & Uzan, J.-P. 2000, *Phys. Rev. D*, 62, 083506.
- Riazuelo, A., & Uzan, J.-P. 2002, *Phys. Rev. D*, 66, 023525.
- Riess, A. G., et al. , 2004, *ApJ*, 607, 665.
- Sachs, R. K. 1962, *Proc. Roy. Soc. A*, 270, 103.
- Sandvik, H. B., et al. 2004, *Phys. Rev. D*, 69, 063005.
- Scherrer, R.J. 2005 [astro-ph/0509890].
- Schimd, C., Uzan, J.-P., & Riazuelo, A. 2005, *Phys. Rev. D*, 71, 083512.
- Schneider, P., Ehlers, J., & Falco, E. E. 1992, *Gravitational lenses* (Springer.)
- Schneider, P., van Waerbeke, L., Kilbinger, M., & Mellier, Y. 2002, *A&A*, 396, 1
- Seljak, U. 2000, *MNRAS*, 318, 203.
- Semboloni, E., et al. 2005 [astro-ph/0511090].
- Simpson, F., & Bridle, S. 2005, *Phys. Rev. D*, 71, 083501.
- Smith, R. E., et al. 2003, *MNRAS*, 341, 1311.
- Spergel, D. N., et al. 2003, *ApJS*, 594, 1.
- Stebbins, A., 1996, [astro-ph/9609149].
- Terano, I., et al. 2005, *A&A*, 429, 383.
- Uzan, J.-P. 1999, *Phys. Rev. D*, 59, 123510.
- Uzan, J.-P., 2004, *AIP Conference Proceedings 736, Phi in the Sky: The Quest of Cosmological Scalar Fields* [astro-ph/0409424].
- Uzan, J.-P., & Bernardeau, F. 2000, *Phys. Rev. D*, 63, 023004.
- Uzan, J.-P., & Bernardeau, F. 2001, *Phys. Rev. D*, 64, 083004.
- Uzan, J.-P., Aghanim, N., & Mellier, Y. 2004, *Phys. Rev. D*, 70, 083533.
- van Waerbeke, L., & Mellier, Y. 2003, Lecture given at the Aussois winter school, France, January 2003 [astro-ph/0305089].
- van Waerbeke, L., Mellier, Y., Erben, T., et al. 2000, *A&A*, 358, 30.
- van Waerbeke, L., Mellier, Y., Radovich, M., et al. 2001, *A&A*, 374, 757.
- van Waerbeke, L., Mellier, Y., & Hoekstra, H. 2005, *A&A*, 429, 75.
- van Waerbeke, L., et al. 2006, in preparation.
- Verde, L., et al. 2003, *ApJS*, 148, 195.
- Wetterich, C. 1988, *Nucl. Phys. B*, 302, 668.
- Wittman, D. M., et al. 2000, *Nature*, 405, 143.
- Yahata, N., et al. 2000, *ApJ*, 538, 493.
- Zhan, H., & Knox, L. 2004, *ApJ*, 616, L75.

The Shear TEsting Programme 2: Factors affecting high precision weak lensing analyses

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ABSTRACT

The Shear TEsting Programme (STEP) is a collaborative project to improve the accuracy and reliability of weak lensing measurement, in preparation for the next generation of wide-field surveys. We review sixteen current and emerging shear measurement methods in a common language, and assess their performance by running them (blindly) on simulated images that contain a known shear signal. We determine the common features of algorithms that most successfully recover the input parameters. A desirable goal would be the combination of their best elements into one ultimate shear measurement method. In this analysis, we achieve previously unattained discriminatory precision via a combination of more extensive simulations and pairs of galaxy images that have been rotated with respect to each other. That removes the otherwise overwhelming noise from their intrinsic ellipticities. Finally, the robustness of our simulation approach is confirmed by testing the relative calibration of methods on real data.

Weak lensing measurement has improved since the first STEP paper. Several methods now consistently achieve better than 2% precision, and are still being developed. However, we can now distinguish all methods from *perfect* performance. Our main concern continues to be the potential for a multiplicative shear calibration bias: not least because this can not be internally calibrated with real data. We determine which galaxy populations are responsible and, by adjusting the simulated observing conditions, we also investigate the effects of instrumental and atmospheric parameters. We have isolated several previously unrecognised aspects of galaxy shape measurement, in which focussed development could provide further progress towards the sub-percent level of precision desired for future surveys. These areas include the suitable treatment of image pixellisation and galaxy morphology evolution. Ignoring the former effect affects the measurement of shear in different directions, leading to an overall underestimation of shear and hence the amplitude of the matter power spectrum. Ignoring the second effect could affect the calibration of shear estimators as a function of galaxy redshift, and the evolution of the lensing signal, which will be vital to measure parameters including the dark energy equation of state.

1 INTRODUCTION

The observed shapes of distant galaxies become slightly distorted by the (differential) gravitational deflection of a light bundle as it passes near foreground mass structures. Such “cosmic shear” happens regardless of the nature and state of the foreground mass. It is therefore a uniquely powerful probe of the cosmic mass distribution, dominated by dark matter. Observations of gravitational lensing are directly and simply linked to theories of structure formation that are otherwise ill-equipped to predict the distribution of light (for reviews, see Bartelmann & Schneider 2001; Wittman 2002; Refregier 2003). Measurements are *not* limited by astrophysical bias (e.g. Dekel & Lahav 1999; Gray *et al.* 2002; Hoekstra *et al.* 2002b; Smith *et al.* 2003; Weinberg *et al.* 2000), which affects optical surveys, nor by unknown physics of distant supernovae (e.g. Hillebrandt & Niemeyer 2000; James *et al.* 2006; Sullivan *et al.* 2006; Travaglio, Hillebrandt, & Reinecke 2006), nor by the uncertain relations between the mass of galaxy clusters and their observable X-ray luminosity or temperature (e.g. Huterer & White 2003; Pierpaoli, Scott & White 2001; Viana, Nichol & Liddle 2002). Gravitational lensing is a purely geometric effect, requiring knowledge of only deflection angles and distances. By directly observing the growth of the mass structures over cosmic time, and by investigating the large-scale geometry of the universe, it is also an effective probe of dark energy (Semboloni *et al.* 2006a; Hoekstra *et al.* 2006; Jarvis *et al.* 2006; Schimd *et al.* 2006) and can test alternative theories of gravity that move beyond general relativity (White & Kochanek 2001).

The practical use of weak lensing in cosmology effectively began with the simultaneous detection of a coherent cosmic shear signal by four independent groups (Bacon, Refregier & Ellis 2000; Kaiser, Wilson & Luppino 2000; Van Waerbeke *et al.* 2000; Wittman *et al.* 2000). Since then, the field of weak lensing has advanced dramatically. Large, dedicated surveys with ground- and space-based telescopes have recently measured the projected 2D power spectrum of the large-scale mass distribution and drawn competitive constraints on the matter density parameter Ω_m and the amplitude of the matter power spectrum σ_8 (Maoli *et al.* 2001; Rhodes *et al.* 2001; Van Waerbeke *et al.* 2001; Hoekstra *et al.* 2002; Bacon *et al.* 2003; Refregier *et al.* 2002; Jarvis *et al.* 2003; Brown *et al.* 2003; Hamana *et al.* 2003; Massey *et al.* 2005; Rhodes *et al.* 2004; Van Waerbeke *et al.* 2005; Heymans *et al.* 2005; Jarvis *et al.* 2005; Hoekstra *et al.* 2006; Semboloni *et al.* 2006a; Hettercheidt *et al.* 2006; Schrabback *et al.* 2006; Dahle 2006). The results from these efforts are found to be in broad agreement and are rapidly becoming more credible, with the most recent publications presenting several different diagnostic tests to determine the levels of systematic error. Ambitious plans are being laid for dedicated telescopes both on the ground (e.g. VST-KIDS, DES, VISTA darkCAM, Pan-STARRS, LSST) and in space (e.g. DUNE, SNAP, JDEM). Indeed, future weak lensing surveys were recently identified as the most promising route to understanding the nature of dark energy by the joint NSF-NASA-DOE Astronomy and Astrophysics Advisory Committee (AAAC) and NSF-DOE High Energy Physics Advisory Panel (HEPAP) Dark Energy Task Force¹. The importance of weak lensing in future cosmological and astrophysical contexts seems assured.

However, the detection and measurement of weak gravitational lensing presents a technical challenge. The $\sim 1\%$ distortion

induced in the observed shapes of galaxies is an order of magnitude smaller than their typical intrinsic ellipticities, and a similar factor smaller than the spurious shape distortions created by convolution with the telescope’s point spread function (PSF). Correction for these effects is crucial and complex. To test the reliability of weak lensing measurements, it has therefore been necessary since the first detections to manufacture simulated images that closely resemble real data but contain a known shear signal. Bacon *et al.* (2001), Erben *et al.* (2001) and Hoekstra *et al.* (2002) ran their shear measurement methods on such images. By comparing the input and mean measured shears, they determined the calibration error inherent to each technique, and in some cases discovered (and hence corrected) a multiplicative calibration bias. This is most important because it cannot be self-calibrated from a survey itself. Other systematics can be checked for in real data via correlation of the galaxies and the PSF, or via an E - B decomposition (Schneider *et al.* 2002; Crittenden *et al.* 2002; Schneider & Kilbinger 2006). These early tests determined that the first successful shear measurement methods were accurate to $\leq 10\%$ of the signal.

To maximise progress in this technical field, and to foster the exchange of data and theoretical knowledge within the weak lensing community, we launched the Shear TESting Programme (STEP). In the first STEP paper, (Heymans *et al.* 2005, STEP1), we parametrized the performance of methods in terms of their multiplicative shear calibration bias m , an additive residual shear offset c and, in some cases, a nonlinear responsivity to shear q . That analysis confirmed that the main difficulty in weak lensing lies in the calibration of the shear signal, but encouragingly showed that all of the methods used on existing weak lensing surveys achieve better than $\sim 7\%$ accuracy. Shear measurement error is therefore not currently a dominant source of error.

Unfortunately, this accuracy will not be sufficient to realise the potential of the ambitious and much larger future surveys. STEP1 found that the most accurate shear measurement methods were successfully calibrated to within a few percent, but the limited size and precision of the first STEP simulations forbade any finer analysis than this. The morphologies of galaxies in the first simulated images were also overly simplistic, in a way that did not fully test the assumptions of some shear measurement methods that galaxies lack substructure and complex shapes.

In this second STEP paper, we include complex galaxy morphologies and conduct a more precise test of current and developing shear measurement algorithms to the $\leq 0.5\%$ level. We achieve this precision through the combination of a more extensive set of simulated images and an ingenious use of galaxy pairs rotated with respect to each other (Nakajima & Bernstein 2006). This removes the otherwise dominant noise from galaxies’ intrinsic ellipticities. The new set of simulated images has also been designed to span a wide range of realistic observing conditions and isolate several potentially challenging aspects of shear calibration in which the accuracy of shear recovery may begin to deteriorate. The data set is sufficiently large for it to be divided into different simulated observing conditions and for independent tests to be carried out within each. We thereby test the effects of the following parameters on shear measurement precision:

¹ <http://www.nsf.gov/mps/ast/detf.jsp>

- Complex galaxy morphology
- Galaxy size
- Galaxy magnitude
- Selection effects related to galaxy ellipticity
- Direction of the shear signal relative to the pixel grid
- PSF size
- PSF ellipticity

Sixteen different shear measurement codes have been run on the simulated images. These can be categorised into four distinct categories. We provide a brief description of each algorithm, and outline the relative successes of each method. The STEP programme has dramatically sped the development of new shear measurement methods (e.g. Refregier & Bacon 2003; Bernstein & Jarvis 2002; Massey & Refregier 2005; Kuijken 2006; Nakajima & Bernstein 2006, Bridle *et al.* in preparation), and we particularly focus on these. However, these methods necessarily remain experimental, and development continues. The results from such methods should therefore be taken as an indication of progress rather than a judgement on their ultimate potential.

This paper is organised as follows. In §2, we describe the simulated images. In §3, we review the different shear measurement methods used by each author, translating them into a common language for ease of comparison, and categorising them into four distinct groups. In §4, we compare each author’s measured shear with the input signal, and split the simulations in various ways to isolate areas of potential difficulty in shear measurement. Because of the number of different methods used, this is a rather daunting process. In §5, we provide some perspective on the results, assessing the relative performance of the different methods, and the categories of methods. In §6, we derive some general conclusions and outline suggestions for future development.

2 SIMULATED IMAGES

We have used the Massey *et al.* (2004a) simulation package to manufacture artificial images that closely resemble deep r -band data taken in good conditions with the *Suprime-Cam* camera on the *Subaru* telescope. We specifically mimic the weak lensing survey data of Miyazaki *et al.* (2002b). The *Subaru* telescope was built with careful consideration of weak lensing requirements, and has reliably obtained the highest quality weak lensing data to date (Miyazaki *et al.* 2002a; Wittman 2005, Kasliwal *et al.* in preparation). It therefore represents the current state-of-the-art, and will most closely match future dedicated survey instruments. The simulated images are publicly available for download from the STEP website².

To aid the interpretation of our results, the simulated images incorporate several “unrealistic” simplifications: neither the noise level, the input shear signal nor the PSF vary as a function of position. This does not adversely affect the validity of the results, as any combination of PSF size, PSF ellipticity, and shear signal can usually be found in one of the images. However, it does let us simply average the measured shear for the large number of galaxies in each image, without explicitly keeping track of either the shear or PSF applied to each object. As in STEP1, the main figure of merit throughout our analysis will be the mean shear measured within each image, $\langle \bar{\gamma} \rangle$, and deviations of that from the known input shear γ^{input} . If the mean shear can be determined without bias for any

Image set	PSF description	Galaxy type
A	Typical Subaru PSF ($\sim 0.6''$)	shapelets
B	Typical Subaru PSF ($\sim 0.6''$)	pure exponential
C	Enlarged Subaru PSF ($\sim 0.8''$)	shapelets
D	Elliptical PSF aligned along x -axis	shapelets
E	Elliptical PSF aligned at 45°	shapelets
F	Circularly symmetric Subaru PSF	shapelets

Table 1. The six different sets of images used in the STEP2 analysis are carefully chosen to isolate and test particular aspects of weak shear measurement. Either the PSF shape, or the form of galaxies’ intrinsic morphology varies in a prescribed way between sets.

input shear (and for any PSF), all of the commonly-used statistics typical in cosmic shear analysis should also be unbiased (but the distribution of the shear estimates will affect their noise level).

To address the specific topics outlined in the introduction, we manufactured six sets of simulated images. These span a range of realistic observing conditions, in a carefully orchestrated way that will isolate various effects. The differences between the images are described in table 1. Each set contains $128 \ 7' \times 7'$ images, with a pixel scale of $0.2''$. In the first simulated image of each set, the galaxies are not sheared. For the next 63 images, which all feature the same patch of sky in order to maximise sensitivity to shear calibration, the galaxies are sheared by a random amount. This amount is chosen with a flat PDF within $|\gamma^{\text{input}}| < 6\%$. To concentrate on cosmic shear measurement rather than cluster mass reconstruction, this limit is smaller than the maximum shears used in STEP1. However, the shears are now crucially chosen from a continuous distribution and are allowed to be in any direction relative to the pixel grid. Note that we are really attempting to measure “reduced shear” (Seitz & Schneider 1997) throughout this analysis, although there is explicitly zero convergence in the simulations. The input signals were not disclosed to any of the groups analysing the data.

We can predict the signal to noise ratio in the shear measurement from these images. We first define a complex ellipticity for each galaxy

$$e = e_1 + ie_2 \equiv \frac{a-b}{a+b} (\cos(2\theta) + i \sin(2\theta)), \quad (1)$$

where a and b are the major and minor axes, and θ is the orientation of the major axis from the x -axis. This definition is widely used because it is more convenient than a two-component parametrization involving θ . Both the real and imaginary parts are well-defined (zero) for a circular object or, on average, for an unsheared population of objects. In the absence of PSF smearing and shear measurement errors, the observed galaxy ellipticity e^{obs} is related to its intrinsic ellipticity e^{int} by

$$e^{\text{obs}} = \frac{e^{\text{int}} + \gamma}{1 + \gamma^* e^{\text{int}}} \quad (2)$$

(Seitz & Schneider 1997), where $\gamma \equiv \gamma_1 + i\gamma_2$ is the complex shear applied to each image. With only a finite number N of galaxies, all with nonzero intrinsic ellipticity, measurement of the mean shear $\langle \bar{\gamma} \rangle = \langle e^{\text{obs}} \rangle$ is limited by an intrinsic shot noise

$$\text{SN error} \approx \langle e^{\text{int}} \rangle = 0 \pm \sqrt{\frac{\langle (e_i^{\text{int}})^2 \rangle}{N}}. \quad (3)$$

In the STEP2 simulations, $\sqrt{\langle e_i^2 \rangle} \sim 0.1$, about an order of magnitude larger than the shear signal.

Since the morphologies of the simulated galaxies are uncorrelated, this noise can be slowly beaten down by increasing the

² <http://www.physics.ubc.ca/~heimans/step.html>

size of the simulations. But to dramatically improve the efficiency of the simulations, and circumvent the meagre $1/\sqrt{N}$ behaviour, we introduce an innovation in the remaining 64 images. Following a suggestion in Nakajima & Bernstein (2006), the entire sky, including the galaxies, was artificially rotated by 90° before being sheared by the same signals and being convolved with the same PSF as before. This rotation flips the sign of galaxies' intrinsic ellipticities. To measure biases in shear measurement methods, we can then consider matched pairs of shear estimators from the unrotated and rotated version of each galaxy. Averaging these estimators explicitly cancels the intrinsic shape noise, leaving only measurement noise and any imperfections in shear measurement. We thus form a shear estimator for each galaxy pair

$$\tilde{\gamma} = (e^{\text{obs,unrot}} + e^{\text{obs,rot}})/2. \quad (4)$$

Since $e^{\text{int,unrot}} = e^{\text{int}} = -e^{\text{int,rot}}$, we can use equation (2) to find

$$\begin{aligned} \tilde{\gamma} &= \left(\frac{e^{\text{int}} + \gamma}{1 + \gamma^* e^{\text{int}}} + \frac{-e^{\text{int}} + \gamma}{1 - \gamma^* e^{\text{int}}} \right) / 2 \\ &= \frac{\gamma - \gamma^* (e^{\text{int}})^2}{1 - (\gamma^* e^{\text{int}})^2}. \end{aligned} \quad (5)$$

Averaging this shear estimator over $N/2$ galaxy pairs now gives a shot noise error in $\langle \tilde{\gamma} \rangle$ of

$$\text{SN error} \approx \gamma \langle (e_i^{\text{int}})^2 \rangle = 0 \pm \gamma \sqrt{\frac{\langle (e_i^{\text{int}})^4 \rangle}{2N}}, \quad (6)$$

which has been significantly reduced from equation (3). In the STEP2 simulations $\sqrt{\langle (e_i^{\text{int}})^4 \rangle} \sim 0.05$ and $|\gamma| < 0.06$. Nothing is lost by this approach. All 128 images can still be analysed independently – and we do pursue this approach in order to measure the total shape measurement noise in an ordinary population of galaxies.

The Massey *et al.* (2004a) image simulation pipeline required extensive development from previously published versions to mimic ground-based data. We shall therefore now describe its three main ingredients: stars (*i.e.* PSF), galaxies and noise.

2.1 Stars

The simulated images are observed after convolution with a various point-spread functions (PSFs). The PSF shapes are modelled on real stars observed in *Suprime-Cam* images, and are shown in figure 1. They are modelled using shapelets (Refregier 2003; Refregier & Bacon 2003; Bernstein & Jarvis 2002; Massey & Refregier 2005), a (complete) set of orthogonal basis functions that can be used to describe the shape any isolated object. The decomposition of an image into shapelet space acts rather like a localised Fourier transform, with images $f(\mathbf{x})$ being expressed in shapelet space as a set of indexed coefficients $f_{n,m}$ that weight the corresponding basis function

$$f(\mathbf{x}) = \sum_{n=0}^{\infty} \sum_{m=-n}^n f_{n,m} \chi_{n,m}(r, \theta; \beta), \quad (7)$$

with $m \leq n$, and where the Gauss-Laguerre basis functions are

$$\chi_{n,m}(r, \theta; \beta) = \frac{C_{n,m}}{\beta} \left(\frac{r}{\beta} \right)^{|m|} L_{\frac{n-|m|}{2}}^{(|m|)} \left(\frac{r^2}{\beta^2} \right) e^{-\frac{r^2}{\beta^2}} e^{-im\theta}, \quad (8)$$

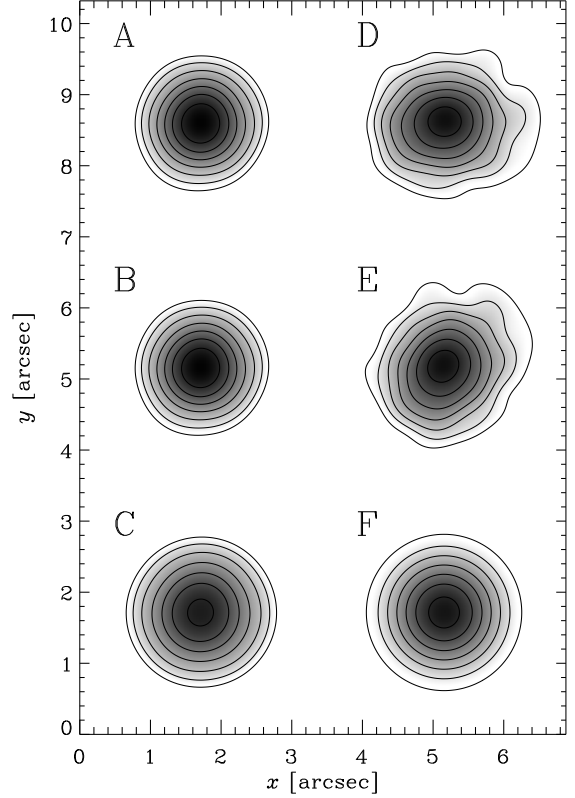


Figure 1. The point spread functions (PSFs) used to generate the six different sets of simulated images. The colour scale is logarithmic, and the contours, which are overlaid at the same absolute value on each PSF, are spaced logarithmically by factors of two. They are designed to target specific aspects of weak lensing measurement that could potentially prove difficult to control. See table 1 and the text for a description of each PSF.

with a normalising constant $C_{n,m}$ and scale size β .

The PSFs can therefore take a complex form. They contain substructure, skewness and chirality. In general, the ellipticity of their isophotes varies as a function of radius. For computational efficiency, the shapelet series is truncated at order $n_{\text{max}} = 12$. The limited wings and the rapid convergence of the PSFs to zero at large radii compared to those used in STEP1 is *not* a consequence of this truncation, but a confirmation of the excellent optical qualities of *Suprime-Cam*.

PSF A is modelled from a fairly typical star towards the centre of a 40 minute long *Suprime-Cam* exposure (which, in practice is likely to be assembled from four 10 minute exposures). It has a full-width at half-max (FWHM) of $0.6''$. PSF B is identical to PSF A. PSF C is the same star, but enlarged to model slightly worse seeing, and has a FWHM of $0.8''$. This is the worst that might be expected in future weak lensing surveys, with nights during poorer conditions typically used to obtain data in additional colours. PSF D is modelled on a star at the edge of the same *Suprime-Cam* exposure. The phases of all of its $m = 2$ shapelet coefficients were adjusted to the same value so that at all radii (and therefore with any radial weight function), its ellipticity derived from quadrupole moments points in exactly the same direction. Substructure and skewness ap-

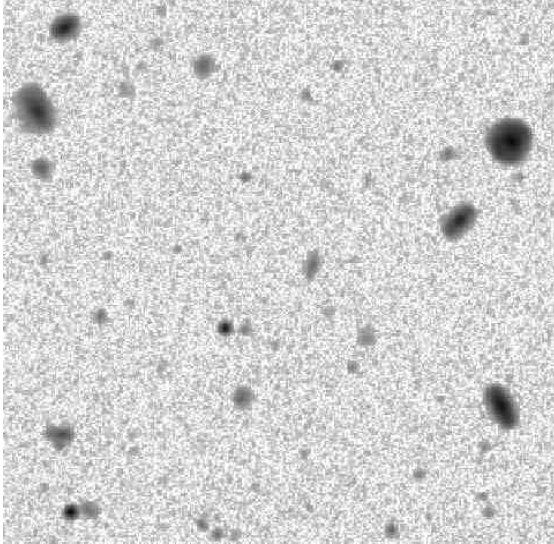


Figure 2. A $1' \times 1'$ section of a simulated image from set A, containing shapelet galaxies with complex morphologies. The colour scale is logarithmic, and the same as that in figure 3.

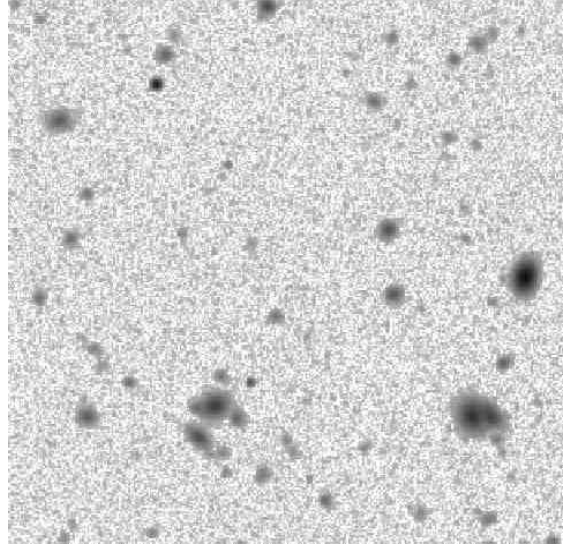


Figure 3. A $1' \times 1'$ section of a simulated image from set B, containing idealised galaxies with exponential radial profiles and simple morphologies. The colour scale is logarithmic, and the same as that in figure 2.

parent in the real *Subaru* PSF is otherwise untouched. As PSF D, the ellipticity is directed parallel to the x -axis of the pixel grid. The star is rotated by 45° to make PSF E. It is an example of extreme ellipticity, which highlights ellipticity-dependent effects. However, it might be possible to limit such ellipticity in weak lensing surveys by improving the optical design of future telescopes or optimising survey tiling and scheduling strategies. PSF F is a circularised version of that star, obtained by setting all of its $m \neq 0$ shapelet coefficients to zero, which is equivalent to averaging the PSF over all possible orientations.

2.2 Shapelet galaxies

Most of the simulated images contain galaxy shapes also constructed from weighted combinations of the shapelet basis functions, using a version of the Massey *et al.* (2004a) image simulation pipeline similar modified to imitate ground-based data. The complex and irregular galaxy morphologies that are possible using this method represent an important advance from the STEP1 analysis using the *SkyMaker* image simulation package (Erben *et al.* 2001). The measurement of weak lensing in STEP1 was considerably simplified by the galaxies' smooth and unperturbed isophotes. Several shear measurement methods are based on the assumption that galaxy shapes and the PSF are concentric, elliptical, and in some cases Gaussian. In addition, the *SkyMaker* galaxies have reflection symmetry about the centroid which could feasibly cause any symmetrical errors to vanish. By contrast, PSF correction and galaxy shape measurement are rendered more challenging in STEP2 by the realistic morphologies that include spiral arms, dust lanes and small-scale substructure. Our analysis is thus designed to test the robustness of weak lensing measurement methods.

The joint size-magnitude-morphology distribution of galaxies was copied from the *Hubble Space Telescope* COSMOS survey (Scoville *et al.* in preparation). This is a uniform, two square degree set of images taken with the *F814W* filter on the *Advanced Camera for Surveys* (ACS), to a depth of 28.7 for a point source at 5σ . It is deeper than our intended simulations, and with a much finer resolution, so provides an ideal source population. The extent of the COSMOS survey also provided sufficient real galaxies to avoid duplication in the simulations without needing to perturb shapelet coefficients, as in section 4 of Massey *et al.* (2004a). We simply used the shapelet models of COSMOS galaxies, randomly rotated, inverted and repositioned. The positions of galaxies in the simulations were chosen at random, without attempting to reproduce higher-order clustering.

Since the galaxy models are inevitably truncated at some level in shapelet space, and since we did not deconvolve the galaxies from the ACS PSF, the smallest simulated galaxies are intrinsically slightly rounder than those in real *Subaru* data. However, this convolution occurs before shearing and does not alter the necessary steps for shear measurement. As in real data, the simulated galaxy ellipticity and morphology distributions do vary with galaxy magnitude and size. We adopt an alternative definition of ellipticity

$$(\varepsilon_1, \varepsilon_2) \equiv \frac{a^2 - b^2}{a^2 + b^2} (\cos(2\theta), \sin(2\theta)), \quad (9)$$

where a and b are the major and minor axes, and θ is the orientation of the major axis from the x -axis. Note the difference from equation (1); this version is closer to the notation used by most shear estimators. Before PSF convolution, the width of this ellipticity distribution

$$\sigma_\varepsilon^{\text{int}} \equiv ((\sigma_{\varepsilon_1}^{\text{int}})^2 + (\sigma_{\varepsilon_2}^{\text{int}})^2)^{1/2} \quad (10)$$

as measured by *SEXTRACTOR* (Bertin & Arnouts 1996) is $\sigma_\varepsilon^{\text{int}} = 0.35 \pm 0.03$ at $r = 22$ and $\sigma_\varepsilon^{\text{int}} = 0.20 \pm 0.02$ at $r = 26$. Note that this ε is a different quantity than the e used in equation (3).

The galaxies were then sheared analytically in shapelet space, using equation (41) of Massey & Refregier (2005). This operation is to first order in γ . Terms of order γ^2 are ignored, but, for typical galaxy shapes, the coefficients by which these are multiplied are also smaller than those multiplying the first order terms. This therefore introduces only a very small error. The galaxies were then convolved with the PSF, also in shapelet space, using equation (52) of Refregier (2003). They were pixellated by analytically integrating the shapelet models within adjoining squares, using equation (34) of Massey & Refregier (2005).

2.3 Idealised galaxies

We have also manufactured one set (B) of simulated images with the same observing conditions but in which the galaxies have simple, exponential profiles and concentric, elliptical isophotes. These idealised galaxies provide a contrast to the morphological sophistication of the shapelet galaxies, and an independent test of the shapelet-based shear measurement methods. We intentionally chose a very simple form for the idealised galaxy shapes, with a sharp cusp and extended wings, to most effectively pronounce any difference to the results from galaxies with realistically complex morphologies. As before, the size-magnitude distribution of un-sheared galaxies was modelled on that observed in the ACS COSMOS images. Galaxy ellipticities were assigned randomly from a Gaussian distribution. Like STEP1, we used a constant distribution of intrinsic ellipticity. This had width $\sigma_\varepsilon^{\text{int}} = 0.3$ for galaxies at all magnitudes.

To add a shear signal, the random ellipticities are then perturbed at the catalogue level. Under a small shear γ_i , the ellipticity ε defined in equation (9) transforms as

$$\varepsilon_i^{\text{obs}} = \varepsilon_i^{\text{int}} + 2(\delta_{ij} - \varepsilon_i^{\text{int}} \varepsilon_j^{\text{int}}) \gamma_j + \mathcal{O}(\gamma^3), \quad (11)$$

(e.g. Rhodes *et al.* (2000)) where δ_{ij} is the Kronecker-delta symbol, and the summation convention was assumed. Similarly, the mean square radius $d \equiv a^2 + b^2$ becomes

$$d^2 = d^2(1 + 2\varepsilon_i^{\text{int}} \gamma_i) + \mathcal{O}(\gamma^2). \quad (12)$$

These two expressions are valid up to first order in the shear. Note that, to this order, the flux F is unaffected by a pure shear. These results are valid for any galaxy with self-similar isophotes (as long as the moments converge).

To create a simulated galaxy image $f(\mathbf{x})$ with a desired ellipticity, we first specify the desired size r_0 and mean radial profile $p(r^2)$, where $r^2 = x_1^2 + x_2^2$ is the square radius and $\mathbf{x} = (x_1, x_2)$ are Cartesian coordinates on the sky, centered on the centroid of the galaxy. For convenience, we choose the normalisation and angular scale of the generic profile such that

$$\iint p(r^2) d^2\mathbf{x} = \iint r^2 p(r^2) d^2\mathbf{x} = 1. \quad (13)$$

The exponential profile used in these simulations is given by

$$p(r^2) = \frac{\sqrt{6}}{2\pi r_0} e^{-\sqrt{6}(r/r_0)^2} \quad (14)$$

(c.f. Refregier 2000 for the alternative case of a Gaussian profile). Using the conventions of equation (13) and a coordinate transformation

$$\mathbf{J} = \mathbf{R}(\theta)^T \begin{pmatrix} a^2 & 0 \\ 0 & b^2 \end{pmatrix} \mathbf{R}(\theta) = d^2 \begin{pmatrix} 1 + \varepsilon_1 & \varepsilon_2 \\ \varepsilon_2 & 1 - \varepsilon_1 \end{pmatrix}, \quad (15)$$

where T denotes transpose and the rotation matrix

$$\mathbf{R}(\theta) \equiv \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \quad (16)$$

it is then easy to show that the elliptical galaxy image should have surface brightness

$$f(\mathbf{x}) = F |\mathbf{J}|^{-\frac{1}{2}} p(\mathbf{x}^T \mathbf{J}^{-1} \mathbf{x}), \quad (17)$$

where the vertical bars denote the matrix determinant. The tails of their exponential profiles were artificially truncated at elliptical isophotes $5 \times r_0$ from the centre. To pixellate the galaxies, the value of the analytic function was computed at the centre of each pixel. The PSF was similarly pixellated, and convolution was then performed in real space to produce the final image $I(\mathbf{x})$. Strictly, these operations should be reversed, and they do not commute. However, the pixels are small and the PSFs are Nyquist sampled, so the error introduced should be minimal.

2.4 Noise

A two-component noise model is then superimposed onto the images. Instrumental performance mimics that attained with a stack of four ten-minute exposures with *Suprime-Cam* on the 8m *Subaru* telescope (Miyazaki *et al.* 2002b). They are complete to $r = 25.5$, and the galaxies selected for lensing analysis are likely to have a median redshift $z_m \approx 0.9$. This is slightly deeper than most existing weak lensing surveys, and is towards the deep end of ground-based surveys planned for the future. The number density of useable galaxies found in these simulated images is therefore unlikely to be greatly surpassed.

The first component of ‘‘photon counting’’ shot noise is first added to the true flux in every pixel. This is drawn from a Gaussian distribution with a width equal to the square root of the photon count. The images are then renormalised to units of counts per second. In the renormalised images, the rms of the Gaussian is 0.033 times the intensity in a pixel.

A second component of sky background is then added throughout each image, with an rms of 4.43 counts per second. The DC background level is assumed to be perfectly subtracted. The model *Subaru* images were combined using DRIZZLE, and the sky background noise is correlated in adjacent pixels. To mimic this effect, we smoothed the sky noise component (but not the flux in objects) by a Gaussian of FWHM 3.5 pixels. After this process, the rms of the sky noise is 1.65 counts per second. A simulated image of a completely blank patch of sky was also available to measure the covariance between pixels. The correlated noise particularly affects the detection of small, faint objects, and impedes the calculation of objects’ weights from their detection S/N. It will be instructive in the future to consider which image resampling kernels and co-addition methods are optimal for shape measurement, or indeed whether we should stack the data at all. Jarvis *et al.* (2003) suggest measuring galaxy ellipticities on individual frames and combining these at the catalogue level. Note that faint simulated galaxies are created to the depth of the COSMOS survey, below the limiting magnitude of the simulated ground-based images, and these unresolved sources will also add slightly to the overall sky background.

3 SHEAR MEASUREMENT METHODS

Sixteen different shear measurement codes have been run on the simulated images, by the authors listed in table 2. Those that have

Author	Key	Method
Bergé	JB	Shapelets (Massey & Refregier 2005)
Clowe	C1	KSB+ (same PSF model used for all galaxies)
Clowe	C2	KSB+ (PSF weight size matched to galaxies ³)
Hetterscheidt	MH	KSB+
Hoekstra	HH	KSB+
Jarvis	MJ	Bernstein & Jarvis (2002)
Jarvis	MJ2	Bernstein & Jarvis (2002) (new weighting scheme)
Kuijken	KK	Shapelets (Kuijken 2006)
Mandelbaum	RM	Reglens (Hirata & Seljak 2003)
Nakajima	RN	Bernstein & Jarvis (2002) (deconvolution fitting)
Paulin-Henriksson	SP	KSB+
Schirmer	MS1	KSB+ (scalar shear susceptibility)
Schirmer	MS2	KSB+ (tensor shear susceptibility)
Schrabback	TS	KSB+
Semboloni	ES1	KSB+ (shear susceptibility fitted from population)
Semboloni	ES2	KSB+ (shear susceptibility for individual galaxies)

Table 2. Table of authors and their shear measurement methods. The key identifies the authors in all future plots and tables.

been used elsewhere on real data, attempt to preserve as similar a pipeline as possible. Each method must first find and measure the shape of stars in each image. It must interpolate the PSF shape across the field, without assuming that it is constant. It must then find and measure the shapes of galaxies, correcting them appropriately for the effects of seeing. Note that we still consider object identification and classification to be part of a shear measurement method, as shape biases can easily be introduced at this point (*e.g.* Bernstein & Jarvis 2002; Hirata & Seljak 2003); however, that task is likely to be separated in future STEP projects.

All of the methods work by obtaining, for each galaxy, a two-component polarisation ε_i that behaves like a generalised ellipticity. Precise definitions of polarisation vary between methods, but it is important to note that easily measurable quantities do not usually change linearly with applied shear, so that $\langle \varepsilon \rangle \neq \gamma^{\text{input}}$ for all values of γ^{input} . To obtain an unbiased shear estimator, methods must determine how their polarisations change under an applied shear, and compute either a shear susceptibility tensor $P_{ij}^\gamma \equiv \delta \varepsilon_i / \delta \gamma_j$ or a shear responsivity factor \mathcal{R} . These are essentially interchangeable concepts, but with the word “susceptibility” used to imply measurement from the higher order shape moments of each galaxy (which are then often averaged or fitted across a galaxy population), and the word “responsivity” to mean an average susceptibility for the population, measured from moments of the galaxy ellipticity distribution. In either case, this quantity can be inverted, and used to form a shear estimator

$$\tilde{\gamma} \equiv (P^\gamma)^{-1} \varepsilon \quad (18)$$

or

$$\tilde{\gamma} \equiv \frac{\varepsilon}{\mathcal{R}}. \quad (19)$$

When computing the mean shear from a limited subset of galaxies, such as those in one size or magnitude bin, we shall investigate two approaches to the calculation of \mathcal{R} . We try using the constant, global value, as has been done in published work, and we also try calculating \mathcal{R} from the statistics of the smaller population. The latter is more noisy, but takes into account the evolution of galaxy morphology between samples (see §5.5).

In table 3, the methods are broadly distinguished by their solutions to the two most important tasks in shear measurement. Some methods correct for the PSF at the catalogue level, by essentially

		Shear measurement method	
		Passive	Active
PSF correction scheme	Subtraction	KSB+ (various) Reglens (RM) RRG* K2K* Ellipto*	BJ02 (MJ, MJ2)
	Deconvolution	Shapelets (JB)	Shapelets (KK) BJ02 (RN) im2shape*

Table 3. Broad classification scheme to distinguish different types of shear measurement methods. Asterisks denote methods not tested in this paper. The top-left quadrant is red; the top-right blue; the bottom-left orange; and the bottom-right green.

subtracting the ellipticities of the PSF from that of each galaxy; others attempt to deconvolve each galaxy from the PSF, and measure the ellipticity of a reconstructed model. To obtain a polarisation, some (“passive”) methods measure combinations of galaxies’ observed shape moments; other (“active”) methods shear a model of an intrinsically circular source until it most closely resembles the observed galaxy. We shall now provide a brief description of each method, starting in the top-left quadrant of table 3. Since the STEP program has dramatically sped the development of new shear measurement methods (Refregier & Bacon 2003; Bernstein & Jarvis 2002; Massey & Refregier 2005; Kuijken 2006; Nakajima & Bernstein 2006, Bridle *et al.* in preparation), we shall particularly concentrate on the latest developments in those algorithms.

3.1 Red class methods

3.1.1 KSB+ (C1, C2, MH, HH, SP, MS1, MS2, TS, ES1 and ES2)

The shear measurement method developed by Kaiser, Squires & Broadhurst (1995), Luppino & Kaiser (1997) and Hoekstra *et al.* (1998) is in widespread use by many current weak lensing surveys. This has led to a high level of optimisation of the basic method. The base IMCAT code is publicly available from the world wide web³. Many variations have been developed, and the ten implementations tested in this paper represent a cross-section of those that have been applied to real data. The details of each method are compared fully in the appendix of STEP1. The differences that STEP2 results reveal to be particularly significant are summarised again in table 4.

The core of the method requires the measurement of the quadrupole moments of each observed galaxy image $I(\mathbf{x})$ weighted by a Gaussian of size r_g . From these are formed a polarisation

$$(\varepsilon_1, \varepsilon_2) \equiv \frac{\iint I(\mathbf{x}) W(\mathbf{x}) r^2 (\cos(2\theta), \sin(2\theta)) d^2\mathbf{x}}{\iint I(\mathbf{x}) W(\mathbf{x}) r^2 d^2\mathbf{x}}, \quad (20)$$

where

$$W(\mathbf{x}) = e^{-r^2/2r_g^2}. \quad (21)$$

The polarisation is corrected for smoothing of the PSF via the smear susceptibility tensor P^{sm} and calibrated as shears via the

³ <http://www.ifa.hawaii.edu/~kaiser/imcat>

shear polarisability tensor P^{sh} : both of which involve higher order shape moments. Using stars to denote measurements from stars (for which a smaller weight function is sometimes used) instead of galaxies, these form a shear estimator

$$\bar{\gamma} = (P^\gamma)^{-1} \left[\varepsilon - P^{\text{sm}} (P^{\text{sm}*})^{-1} \varepsilon^* \right], \quad (22)$$

where

$$P^\gamma = P^{\text{sh}} - P^{\text{sm}} (P^{\text{sm}*})^{-1} P^{\text{sh}*}. \quad (23)$$

The tensor inversions can be performed in full, but these measurements of faint objects are particularly noisy. In practice, since the diagonal elements of P^γ are similar, and its off-diagonal elements are about an order of magnitude smaller, it can be approximated as a scalar quantity. Many implementations of KSB+ therefore simply divide by a shear susceptibility factor. The noise in P^γ is also sometimes reduced by fitting it from the entire population as a function of other observable quantities like galaxy size and magnitude. Reducing noise in any nonlinear aspect of shear measurement is vital, because the lensing signal is so much smaller than both the intrinsic ellipticity and photon shot noise, and must be obtained by linearly averaging away those sources of noise over a large population of galaxies.

Unfortunately, fundamental limitations in the mathematical formalism of KSB+ introduce further decisions that must also be resolved to approximate an ideal scenario in practical implementations. The KSB+ method makes no provision for the effects of pixellisation; assumes that the PSF isophotes are concentric; and is mathematically ill-defined for non-Gaussian or non-concentric PSF and galaxy profiles. The various implementations developed by groups participating in the STEP2 analysis represent a cross-section of those choices.

Since STEP1, the TS method has incorporated a shear calibration factor of 0.91^{-1} , determined from the STEP1 results, but without knowledge of the STEP2 data. STEP2 therefore tests the robustness of this sort of calibration. As in STEP1, the C1 and C2 methods incorporate a calibration factor of 0.95^{-1} to eliminate the effect of close galaxy pairs. The C1 method uses a constant model of the PSF for all galaxies; the C2 method lets the size of the weight function $r_g^* = r_g$ change to match each galaxy. The new SP method numerically integrates weight functions within pixels, uses the trace of P^γ from individual galaxies, and similar galaxy weights to the HH method. The ES1 method is based upon the LV method from STEP1 but, rather than fitting the shear susceptibility from the galaxy population as a function of size and magnitude, it finds the twenty most similar galaxies in terms of those parameters, and uses their average value. This same procedure was used in the Semboloni *et al.* (2006a) analysis of the CFHTLS deep survey. Subsequent tests on STEP1 images suggested that better results could be obtained by using individual measurements of P^γ from each galaxy, and ignoring the galaxy weights. These improvements have been incorporated into the new ES2 method.

One final finesse is required for methods that use weights w_i on each galaxy i that could vary between the rotated and unrotated images. For all N pairs of galaxies, we determine normalised weights

$$w_i = \frac{N w_i}{\sum_{j=1}^N w_j} \quad (24)$$

and then calculate three estimates of the mean shear in each image

$$\langle \bar{\gamma}^{\text{unrot}} \rangle = \frac{1}{N} \sum (w^{\text{unrot}} e^{\text{obs,unrot}}) \quad (25)$$

$$\langle \bar{\gamma}^{\text{rot}} \rangle = \frac{1}{N} \sum (w^{\text{rot}} e^{\text{obs,rot}}) \quad (26)$$

$$\langle \bar{\gamma} \rangle = \frac{1}{2N} \sum (w^{\text{unrot}} e^{\text{obs,unrot}} + w^{\text{rot}} e^{\text{obs,rot}}). \quad (27)$$

Errors on these are estimated using a bootstrap technique.

3.1.2 Reglens (RM)

The Reglens (RM) method consists of two parts: the SDSS data processing pipeline PHOTO (Ivezić *et al.* 2004), followed by the re-Gaussianization pipeline (Hirata & Seljak 2003; Mandelbaum *et al.* 2005). The magnitude cut was adjusted, and one additional subroutine was required for the STEP2 analysis, to properly determine the noise variance in the presence of correlated background noise. The STEP2 images are more crowded than SDSS images, leading to occasional deblending problems. Objects with failed deblending were automatically eliminated, after visual inspection indicated that nearly all of them were really several galaxies very close to each other.

PSF correction is performed via a two-step procedure that addresses KSB+'s limitation of being exact only in the limit of Gaussian PSF and galaxy profile. The PSF is first split into a Gaussian component $G(\mathbf{x})$ plus a small residual $\epsilon(\mathbf{x})$, so that the observed image

$$I = (G + \epsilon) \otimes f = G \otimes f + \epsilon \otimes f, \quad (28)$$

where $f(\mathbf{x})$ is the galaxy image before convolution of the PSF, and \otimes signifies convolution. Assuming knowledge of f , it would be possible to find

$$I' \equiv G \otimes f = I - \epsilon \otimes f, \quad (29)$$

the galaxy image as it would appear when convolved with a perfectly Gaussian PSF. Although f is not known in practice, it is convolved with a small correction ϵ in the final equality, so equation (29) is fairly accurate even with an approximation f_0 . The SDSS and STEP2 analyses used an elliptical Gaussian as f_0 , with its size and ellipticity determined from the difference between the best-fit Gaussians to the observed image and the full PSF. Possible alternatives to this approximation are discussed in Hirata & Seljak (2003).

Correction for the isotropic part of the now Gaussian PSF requires a subtraction similar to that in KSB+ equation (22), except that Reglens directly subtracts moments of the PSF from those of the galaxy (*i.e.* the numerator and denominator of equation (20)) before they are divided (*i.e.* the ratio in equation (20)). Furthermore, the moments are calculated using weight functions $W_{I'}(\mathbf{x})$ and $W_G(\mathbf{x})$ that are the best-fitting elliptical Gaussians to the image and to the PSF respectively. The advantage of these adaptive weight functions is that they do not bias the shape measurement or require later correction. Correction for the anisotropic part of the Gaussian PSF is finally performed by shearing the coordinate system, including I' , until G is circular.

In the absence of galaxy weights, a shear estimate for each galaxy would be computed via equation (19). The shear responsiveness

$$\mathcal{R} = 2 - \sigma_\varepsilon^2 \equiv 2 - \langle \varepsilon_1^2 + \varepsilon_2^2 - s_{\varepsilon_1}^2 - s_{\varepsilon_2}^2 \rangle, \quad (30)$$

is calculated from shape distribution statistics of the entire galaxy

Author	Pixellisation	Galaxy weighting scheme	Cal ⁿ factor	Shear susceptibility
JB	Analytic integration	None	—	Global mean shear responsivity $\mathcal{R} = 2 - \langle \varepsilon^2 \rangle$
C1	Centre of pixel	$\min(\nu, 40)$	1/0.95	$\frac{1}{2} \text{Tr}[P^\gamma]$, fitted as $f(r_g, \varepsilon_i)$
C2	Centre of pixel	$\min(\nu, 40)$	1/0.95	$\frac{1}{2} \text{Tr}[P^\gamma]$, fitted as $f(r_g, \varepsilon_i)$
MH	Numerical integration	$1/(0.15 + \sigma_\varepsilon^2 + \sigma(\frac{1}{2} \text{Tr}[P^\gamma(r_g)])^2)$	1/0.88	$\frac{1}{2} \text{Tr}[P^\gamma]$, from individual galaxies
HH	Numerical integration	$1/(\sigma_\varepsilon^2 + s_\varepsilon^2 / ((1 - \frac{\varepsilon_\varepsilon^2}{2}) \frac{1}{2} \text{Tr}[P^\gamma])^2)$	—	$(1 - \frac{\varepsilon_\varepsilon^2}{2}) \frac{1}{2} \text{Tr}[P^\gamma]$, fitted as $f(r_g)$
MJ	Centre of pixel	$1/\sqrt{\varepsilon^2 + 2.25s_\varepsilon^2}$	—	Global mean shear responsivity \mathcal{R}
MJ2	Centre of pixel	$1/s_\varepsilon^2$	—	Global mean shear responsivity \mathcal{R}
KK	Centre of pixel	$1/(0.1^2 + \sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2)$	—	Global mean shear responsivity $\mathcal{R} = 1 - \langle \varepsilon^2 \rangle$
RM	Centre of pixel	$f(S/N)$	—	Global mean shear responsivity \mathcal{R}
RN	Centre of pixel	$1/\sqrt{\varepsilon^2 + 2.25s_\varepsilon^2}$	—	Global mean shear responsivity \mathcal{R}
SP	Numerical integration	$1/(0.15 + \sigma_\varepsilon^2 + \sigma(\frac{1}{2} \text{Tr}[P^\gamma(r_g)])^2)$	—	$\frac{1}{2} \text{Tr}[P^\gamma]$, Individual galaxies
MS1	Numerical integration	$1/\sigma_\varepsilon^2(r_g, \text{mag})$	—	$\frac{1}{2} \text{Tr}[P^\gamma]$, fitted as $f(r_g, \text{mag})$
MS2	Numerical integration	$1/\sigma_\varepsilon^2(r_g, \text{mag})$	—	Full P^γ tensor, fitted as $f(r_g, \text{mag})$
TS	Numerical integration	None	1/0.91	$\frac{1}{2} \text{Tr}[P^\gamma]$, from individual galaxies
ES1	Numerical integration	$1/(\sigma_\varepsilon^2(r_g, \text{mag}) + 0.44^2)$	—	$\frac{1}{2} \text{Tr}[P^\gamma]$, smoothed from galaxy population $f(r_g, \text{mag})$
ES2	Numerical integration	None	—	$\frac{1}{2} \text{Tr}[P^\gamma]$, from individual galaxies

Table 4. Choices adopted by each of the shear measurement methods that significantly affect their performance in this paper. See the appendix in STEP1 for more details about the differences between the various implementations of KSB+.

population and the error on each polarisation, s_{ε_i} , is calculated by propagating measured photon shot noise in the image. During our analysis, it became apparent that, for the RM, MJ, MJ2 and RN methods, it is necessary to recalculate \mathcal{R} in each bin of galaxy size or magnitude when the catalogue is so split.

To improve the signal to noise, galaxies are each weighted by a factor

$$w = \frac{1}{\sigma_\varepsilon^2 + s_{\varepsilon_1}^2}. \quad (31)$$

An estimate of the mean shear in each image is then simply

$$\langle \bar{\gamma} \rangle = \sum w \frac{\varepsilon}{\mathcal{R}} / \sum w, \quad (32)$$

with a shear responsivity (Bernstein & Jarvis 2002)

$$\mathcal{R} = \sum w (2 - 2k_0 - k_1 |\varepsilon|^2) / \sum w, \quad (33)$$

where $k_0 = \sigma_\varepsilon^2 - w\sigma_\varepsilon^4$ and $k_1 = w^2\sigma_\varepsilon^4$.

Note that this calculation of \mathcal{R} in the STEP2 images is much more uncertain than in SDSS data, because the correlated background noise in the STEP2 images is not as well understood. Consequently, this may introduce some bias into the STEP2 results that does not exist with the real data.

3.1.3 Other methods not tested in this paper

Rhodes *et al.* (2000, RRG) is a modification of the KSB+ method for space-based data in which the PSF is small. In this limit, ε^* becomes noisy. Like Reglens, RRG therefore deals directly with moments rather than polarisations or as long as possible, and performs the subtraction before the division. The moments use a circular weight function, and therefore require correction for this truncation as well as the PSF. RRG uses a global shear responsivity $\mathcal{R} \approx 2 - \langle \varepsilon^2 \rangle$.

Kaiser (2000, K2K) also seeks a resolution of the Gaussian PSF limitation in KSB+. The galaxy image is first convolved by an additional “re-circularising kernel”, which is a modelled version of the observed PSF that has been rotated by 90° . PSF correction and shear measurement is thereafter fairly similar to KSB. However,

particular efforts are made to correct biases that arise from the use of P^γ measured after shear rather than before shear.

Ellipto (Smith, Bernstein, Fischer & Jarvis 2001) also uses a re-circularising kernel to eliminate the anisotropic component of the PSF, following (Fischer & Tyson 1997). It then repeats object detection to remove PSF-dependent selection biases. Galaxy polarisations are derived from moments weighted by the best-fit elliptical Gaussian. It is a partial implementation of BJ02, discussed in the next section, and primarily differs from BJ02 by using a simpler re-circularising kernel.

3.2 Blue class methods

3.2.1 BJ02 (MJ and MJ2)

The remaining methods are based upon expansions of the galaxy and PSF shapes into Gauss-Laguerre (“shapelet”) basis functions. The JB and KK methods use them with a circular basis function, as defined in equations (7) and (8), while the MJ, MJ2 and RN methods use more general elliptical versions. Shapelets are a natural extension of KSB+ to higher order. The first few shapelet basis functions are precisely the weight functions used in KSB+, with r_g reinterpreted as the shapelet scale size β . Generalised versions of the P^{sh} and P^{sm} matrices are derived in Refregier & Bacon (2003). Extending the basis set to higher order than KSB+ allows complex shapes of galaxies and PSFs to be well described, even when the ellipticity varies as a function of object radius. The shapelet basis set is mathematically well-suited to shear measurement because of the simple transformation of shapelet coefficients during typical image manipulation.

The two Jarvis (MJ, MJ2) methods correct for the anisotropic component of the PSF by first convolving the image with an additional, spatially-varying kernel that is effectively 5×5 pixels. This convolution is designed to null both the Gaussian-weighted quadrupole of the PSF as well as its next higher $m = 2$ shapelet coefficient (since it is the $m = 2$ components of the PSF that mostly affects the observed shapes of galaxies). For PSF ellipticities of order ~ 0.1 or less, a 5×5 pixel kernel is sufficient to round a typical PSF up to approximately 50 pixels in diameter: much larger than the PSFs used in this study.

The shapelet basis functions are sheared, to make them elliptical, then pixellated by being evaluated at the centre of each pixel. Shapelet coefficients $f_{n,m} = 0$ are determined for each galaxy in distorted coordinate systems, and the polarisability ε is defined as -1 times the amount of distortion that makes each object appear round (*i.e.* $f_{2,2} = 0$). Some iteration is required to get this measurement to converge. In the distorted coordinate frame where the galaxy is round, the weight function for this coefficient is a circular Gaussian of the same size as the galaxy. Matching the shape of the weight function to that of the galaxy has the advantage that the polarisability no longer requires correction for truncation biases introduced by the weight function.

Finally, a correction for the PSF dilution (the circularising effect of the PSF) is applied by also transforming the PSF into this coordinate system, then using formulae proposed by Hirata & Seljak (2003).

The two methods (MJ, MJ2) differ only in the weights applied to each galaxy. The MJ method is identical to the MJ method used for the STEP1 study. It uses weights

$$w_{MJ} = \frac{1}{\sqrt{\varepsilon^2 + 2.25s_o^2}}, \quad (34)$$

where s_o is the uncertainty in the polarisability due to image shot noise, as measured in the coordinate system where the galaxy is round. STEP1 revealed that this optimised weight gave incorrect responsivities as the input shear became large (≈ 0.1). For this study, method MJ2 was therefore added, which is identical except that it uses weights that are not a function of the galaxies' polarisations

$$w_{MJ2} = \frac{1}{s_o^2}. \quad (35)$$

These weights should be less biased for larger input shears. The MJ weight might be more appropriate for cosmic shear measurements, and the MJ2 weight for cluster lensing.

The shear responsivity \mathcal{R} for the MJ2 method is the same as that in equation (33). For the ellipticity-dependent weight used by the MJ method, this is generalised to

$$\mathcal{R} \equiv \frac{\sum \left[w(2 - 2k_0 - k_1|\varepsilon|^2) + \varepsilon \frac{\partial w}{\partial \varepsilon} (1 - k_0 - k_1|\varepsilon|^2) \right]}{\sum w}, \quad (36)$$

where the summations are over the entire galaxy population, or for each size or magnitude bin. For either method, an estimate of the mean shear in each image is then

$$\langle \tilde{\gamma} \rangle = \sum w \frac{\varepsilon}{\mathcal{R}} / \sum w. \quad (37)$$

Note that, in the absence of shape noise, equation (36) reproduces the extra $(1 - \varepsilon^2/2)$ term multiplying P^γ in the HH implementation of KSB+ (see table 4).

3.3 Orange class methods

3.3.1 Shapelets (JB)

The Bergé (JB) shear measurement method uses a parametric shapelet model to attempt a full deconvolution of each galaxy from the PSF. Deconvolution is an ill-defined operation in general, since information is irrevocably lost during convolution. In shapelet space, however, it is easy to restrict the galaxy model to include only that range of physical scales in which information is expected to survive. Massey & Refregier (2005) describes an iterative algorithm designed to optimise the scale size of the shapelets and to thus

capture the maximum range of available scales for each individual galaxy. A complete software package to perform this analysis and shapelet manipulation is publicly available from the shapelets web site⁴.

To model a deconvolved galaxy shape, the basis functions are first convolved with the PSF in shapelet space, then integrated analytically within pixels: thus undergoing the same processes as real photons incident upon a CCD detector. The convolved basis functions are then fit to the data, with the shapelet coefficients as free parameters. Reassembling the model using *unconvolved* basis functions produces a deconvolved reconstruction of each galaxy. This performs better than a Wiener-filtered deconvolution in Fourier space, because shapelets have a preferred centre. The available basis functions act as a prior on the reconstruction, localising it in real space (and also allowing a slightly higher resolution at the central cusp than at large radii). The deconvolved model can also be rendered free of noise by ensuring that a sufficient range of scales are modelled to lower the residual χ_{reduced}^2 to exactly unity. Unfortunately, achieving exactly this target is hindered by the presence of correlated background noise in the STEP2 simulations. Incorporating the noise covariance matrix is mathematically trivial but computationally unfeasible, and a practical implementation has not yet been developed. Proceeding regardless, the shape of this analytic model can be directly measured (see Massey *et al.* 2004b; Massey *et al.* 2006), including its *unweighted* moments. These can not be measured directly from real data because observational noise prevents the relevant integrals from converging.

Once a deconvolved model is obtained, extraction of a shear estimator is easy. It could mimic the KSB method. However, removing the weight function (like the Gaussian in equation (20)), makes the polarisation itself into an unbiased shear estimator

$$\tilde{\gamma} = \frac{\iint f(\mathbf{x}) r^2 (\cos(2\theta), \sin(2\theta)) d^2\mathbf{x}}{\iint f(\mathbf{x}) r^2 d^2\mathbf{x}}. \quad (38)$$

The numerator of this expression has a shear susceptibility equal to the denominator. But that denominator is a scalar quantity, with explicitly zero off-diagonal elements in the susceptibility tensor, which can therefore be easily inverted. It is also a simple product of a galaxy's flux and size, both low-order quantities that can be robustly measured. The method is intended to be completely linear for as long as possible, and to introduce minimal bias for even faint objects in this final division. Since the denominator also changes during a shear, a population of galaxies acquires an overall shear responsivity factor

$$\mathcal{R} = 2 - \langle \varepsilon^2 \rangle. \quad (39)$$

The method is still under development. The shear responsivity factor has currently been calculated only from the entire galaxy population. No weighting scheme has yet been applied to the shear catalogue when calculating mean shears. Once galaxies have passed crude cuts in size, flux, and flags (which indicate successful convergence of the shapelet series and of the iteration), they are all counted equally. These aspects will be improved in the future.

3.4 Green class methods

3.4.1 Shapelets (KK)

The Kuijken (KK) shear measurement method assumes that each galaxy was intrinsically circular, then shears it, and smears it by

⁴ <http://www.astro.caltech.edu/~rjm/shapelets>

the PSF, until it most closely matches the observed image. The shear required is the stored as the polarisation ε . As described in Kuijken (2006), this approach is desirable, because it is understood precisely how a circular object changes under a shear.

This process could operate in real space; however, the convenient properties of shapelets make the required image manipulations easier and faster in shapelet space. The pixellated image need be accessed only once, when each galaxy is initially decomposed into shapelets (without deconvolution). Models of circular sources can have arbitrary radial profiles, parametrized by shapelet coefficients with $m = 0$ and $n \leq 12$. This is sheared in shapelet space to first order in γ , although, in principle, this could also be increased to accommodate more highly elliptical objects. Also in shapelet space, it is smeared by a model of the PSF. Since there is only one shapelet decomposition overall, and one forward convolution for each object, the code is much faster than the Bergé (JB) method. Furthermore, the decomposition uses completely orthogonal shapelet basis functions, so the errors on shapelet coefficients are also uncorrelated at that stage. To avoid iterating the decomposition, the optimum scale size β for each object is approximated from SEXTRACTOR parameters, and the range of scales is fixed in advance. In the current implementation, the basis functions are evaluated at the centre of each pixel. Since both the PSF and the galaxy are pixellated, its effects ought to drop out. In terms of the orthogonality of the shapelet basis functions, this approach is satisfactory as long so the range of scales is small, and oscillations in the basis functions remain larger than the pixel scale (*c.f.* Berry, Hobson & Withington 2004).

To determine the shear required to make a circular source match each real galaxy, a fit is performed using a numerical recipes Newton-Raphson algorithm, which is quadratic in shapelet coefficients, the centroid and the shear. Since the galaxies are not really all circular, in practice the global population does have a non-trivial shear susceptibility or “responsivity” \mathcal{R} . For an ensemble population of galaxies, this is a scalar quantity. As can be deduced from equation (11), it involves the variance of the intrinsic polarisation distribution

$$\mathcal{R} \equiv 1 - \langle e^2 \rangle. \quad (40)$$

Unlike other methods that use a shear responsivity correction, this quantity was calculated only once for the KK method, from the entire galaxy population. However, the calculation of $\langle e^2 \rangle$ properly takes into account the galaxy weights

$$\langle e^2 \rangle = \frac{\sum [w(e_1^2 + e_2^2 - s_{e_1}^2 - s_{e_2}^2)]}{\sum w} - \left(\frac{\sum w(e_1 + e_2)}{\sum w} \right)^2, \quad (41)$$

where s_{e_i} is the noise on each polarisation calculated by propagating photon shot noise, and the weight for each galaxy is

$$w = \frac{1}{(\sigma_e^{\text{int}})^2 + s_{e_1}^2 + s_{e_2}^2}. \quad (42)$$

Note that the estimates of errors on the polarisations did not take into account the fact that the background noise was correlated between adjacent pixels, and are therefore likely to be underestimated.

Shear estimates for individual galaxies are then computed similarly to equation (37), but where $\tilde{\gamma} \equiv e/\mathcal{R}$ here.

3.4.2 BJ02 (RN)

The “deconvolution fitting method” by Nakajima (RN) implements nearly the full formalism proposed by BJ02, which is further elabo-

rated in Nakajima & Bernstein (2006). Like MJ and MJ2, it shears the shapelet basis functions until they match the ellipticity of the galaxy. The amount of distortion that makes an object appear round (*i.e.* $f_{2,2} = 0$) defines the negative of its polarisability ε .

Since no PSF interpolation scheme has yet been developed, the pipeline deviates from the STEP rules by using prior knowledge that the PSF is constant across each image (but not between images). Deconvolution from the PSF is performed in a similar fashion to the JB method. The Gauss-Laguerre basis functions are convolved with the PSF to obtain a new basis set. These are evaluated at the centre of each pixel. The new basis functions are fitted directly to the observed pixel values, and should fully capture the effect of highly asymmetric PSFs or galaxies, as well as the effects of finite sampling. The fit iterates until a set of sheared Gauss-Laguerre basis functions are obtained, in which the coefficients $f_{2,0} = f_{2,2} = 0$ and hence the deconvolved galaxy appears round. All PSF coefficients were obtained to $n \leq 12$, and galaxy coefficients to $n \leq 8$.

The weights applied to each galaxy are optimised for small shears, using the same prescription as the MJ2 method in equation 35. The shear responsivity \mathcal{R} is similarly calculated using 36, averaged over the entire galaxy population or within size and magnitude bins as necessary.

The evolution of the RN method during the STEP2 analysis highlights the utility of even one set of STEP simulations. In the first submission, it was noticed that a few outlying shear estimates in each field were destabilising the result. These were identified as close galaxy pairs, so an algorithm was introduced to remove these, and the size and magnitude cuts were also gradually adjusted over several iterations to improve stability.

3.4.3 Other methods not tested in this paper

Im2shape (Bridle *et al.* 2001) performs a similar PSF deconvolution, but parametrizes each galaxy and each PSF as a sum of elliptical Gaussians. The best-fit parameters are obtained via a Markov-Chain Monte-Carlo sampling technique. Concentric Gaussians are usually used for the galaxies, in which case the ellipticity is then a direct measure of the shear via equations (1) and (2). For alternative galaxy models using non-concentric Gaussians, shear estimators like that of the JB method could also be adopted. The “active” or “passive” classification of this method is somewhat open to interpretation.

4 RESULTS

Individual authors downloaded the simulated images and ran their own shear measurement algorithms, mimicking as closely as possible the procedure they would have followed with real data. None of the authors knew the input shears at this stage. Their galaxy catalogues were then compiled by Catherine Heymans and Richard Massey. Independently of the other authors, the mean shears in each image were compared to the input values. Galaxies in the measured catalogues were also matched to their rotated counterparts and to objects in the input catalogues, with a $1''$ tolerance. Except for determining false detections or stellar contamination in the measured catalogues (which were removed in the matched catalogues), no results using the input shapes are presented in this paper.

In this section, we present low level data from the analyses, in terms of direct observables. For further discussion and interpretation of the results in terms of variables concerning global survey

Image set	PSF model from TS implementation of KSB+		
	FLUX_RADIUS	ε_1	ε_2
A	0.334''	$-0.68 \pm 0.10\%$	$(1.21 \pm 0.07)\%$
B	0.334''	$-0.66 \pm 0.07\%$	$(1.28 \pm 0.05)\%$
C	0.406''	$-0.47 \pm 0.07\%$	$(0.97 \pm 0.06)\%$
D	0.390''	$(11.49 \pm 0.11)\%$	$(2.20 \pm 0.14)\%$
E	0.390''	$-2.21 \pm 0.14\%$	$(11.29 \pm 0.16)\%$
F	0.392''	$-0.01 \pm 0.12\%$	$(0.01 \pm 0.01)\%$

Table 5. PSF models for the six sets of images used in the STEP2 analysis by the TS implementation of KSB+, averaged over stars in the simulated images. These quantities may be more familiar to some readers. FLUX_RADIUS is directly from SExtractor, and the ellipticities are all measured using a Gaussian weight function of rms size $r_g = 0.6'' = 3$ pixels.

and instrumental performance, see §5. To conserve space, only a representative sample of the many results are displayed here. The rest is described in the text, in relation to the illustrative examples, and is also available from the STEP website². First, we shall describe the measurement of stars; then the number density of galaxies and then shears in each set of images. Finally, we shall split the galaxy catalogues by objects’ observed sizes and magnitudes.

4.1 PSF modelling

The first task for all shear measurement methods is to identify stars and measure the shape of the PSF. Table 5 lists parameters of the PSF model generated by the TS implementation of KSB+. These quantities are more familiar than those derived analytically from the shapelet models, and also demonstrate the differences between measured PSF ellipticities and inputs described in table 1. The few percent polarisations measured for components of PSFs D and E that should be zero are typical of several other methods. These may explain the peculiar residual shear offsets described in §5.3.

4.2 Galaxy number counts and the false detection rate

The methods used a variety of object detection algorithms and catalogue selection criteria. For each method and each PSF, table 6 lists the density of objects per square arcminute, n_{gals} , their mean magnitude, and the percentage of false detections. Clearly, methods that are able to successfully measure the shapes of more (fainter) galaxies, while avoiding false detections, will obtain a stronger measurement of weak lensing, especially because the lensing signal grows cumulatively with galaxy redshift. The false detection and stellar contamination rate is generally low, and the effective survey depth is lowered by less than 0.1 magnitudes for all methods after matching rotated and unrotated catalogues. Nor does matching have a significant effect upon the overall mean polarisation of galaxies, which is always consistent with zero both before and after matching – as might not have been the case in the presence of selection effects (Bernstein & Jarvis 2002; Hirata & Seljak 2003).

Table 6 also shows the measured dispersion of shear estimators σ_γ for each population. This statistic represents a combination of the intrinsic ellipticity of galaxies and the shape measurement/PSF correction noise introduced by each method. Lower values will produce stronger measurements of weak lensing. Since shear measurement is more difficult for smaller or fainter galaxies, and the intrinsic morphology distribution of galaxies varies as a function of magnitude in images other than set B, n_{gals} and σ_γ

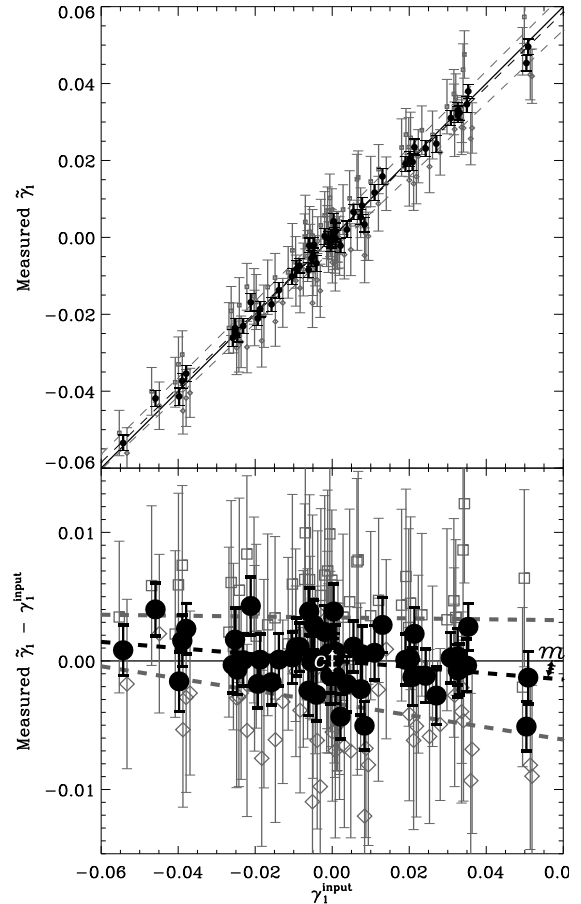


Figure 4. An example of the input vs measured shear for one representative method. This is for the first component of shear measured by the KK method in image set F. It is neither the best method on this image set, nor the best image set for this method, but shows behaviour that is typical of most. The grey squares and diamonds show results from independent analyses of the rotated and unrotated images; the black circles show the effect of matching pairs of otherwise identical galaxies. The bottom panel shows deviations from perfect shear recovery, which is indicated in both panels by solid lines. Linear fits to the data are shown as dashed lines. The fitted parameters m (shear calibration bias) and c (residual shear offset) are plotted for all methods and all for all images sets in figure 5.

are likely to be correlated in a complicated fashion. Galaxy selection effects and weighting schemes are discussed in §5.6 and §5.7.

4.3 Shear calibration bias and residual shear offset

As with STEP1, we assess the success of each method by comparing the mean shear measured in each image with the known input shears γ_i^{input} . We quantify deviations from perfect shear recovery via a linear fit that incorporates a multiplicative ‘‘calibration bias’’ m and an additive ‘‘residual shear offset’’ c . With a perfect shear measurement method, both of these quantities would be zero.

Author	Image set	n_{gals}		mean mag (original)	% mag decrease	$\sigma\gamma$	
		original	matched			original	matched
JB	A	37(0)	25	24.04	1.2	0.012	0.007
	C	28(1)	21	23.50	1.0	0.014	0.008
C1	A	51(2)	45	23.70	0.3	0.008	0.003
	C	46(2)	40	23.64	0.4	0.009	0.003
C2	A	50(2)	45	23.70	0.3	0.008	0.003
	C	45(2)	40	23.64	0.4	0.009	0.003
MH	A	38(0)	35	23.68	0.4	0.008	0.003
	C	33(0)	29	23.56	0.5	0.009	0.004
HH	A	28(0)	26	23.05	0.2	0.010	0.002
	C	24(0)	21	22.97	0.3	0.012	0.002
MJ	A	27(1)	24	23.30	0.3	0.009	0.003
	C	25(0)	22	23.26	0.4	0.009	0.003
MJ2	A	27(1)	24	22.58	0.1	0.014	0.002
	C	25(0)	22	22.48	0.2	0.016	0.002
KK	A	32(0)	26	23.46	0.5	0.009	0.003
	C	27(0)	21	23.35	0.5	0.010	0.003
RM	A	36(0)	32	23.41	0.3	0.009	0.002
	C	27(0)	23	23.21	0.4	0.010	0.003
RN	A	22(1)	19	23.10	0.3	0.009	0.003
	C	16(1)	13	23.03	0.5	0.011	0.004
SP	A	27(11)	15	23.13	0.4	0.014	0.003
	C	25(10)	13	23.10	0.4	0.016	0.004
MS1	A	43(1)	39	23.68	0.3	0.007	0.003
	C	37(1)	33	23.55	0.3	0.008	0.003
MS2	A	41(1)	36	23.46	0.1	0.010	0.004
	C	35(1)	30	23.26	0.1	0.013	0.006
TS	A	40(0)	36	23.74	0.5	0.008	0.004
	C	34(0)	29	23.64	0.6	0.010	0.005
ES1	A	40(0)	34	23.81	0.6	0.008	0.003
	C	35(0)	30	23.71	0.7	0.008	0.003
ES2	A	40(0)	34	23.74	0.6	0.016	0.009
	C	35(0)	30	23.69	0.7	0.017	0.009

Table 6. Number density of galaxies used by each method, and the shear measurement noise from those galaxies. The number of galaxies per square arcminute are listed for the unmatched unrotated/rotated catalogues and after matching. The number in brackets is the percentage of stars or false detections

Since the input shear is now applied in random directions, we measure two components each of m and c , which correspond to the two components of shear,

$$\begin{aligned} \langle \tilde{\gamma}_1 \rangle - \gamma_1^{\text{input}} &= m_1 \gamma_1^{\text{input}} + c_1 \\ \langle \tilde{\gamma}_2 \rangle - \gamma_2^{\text{input}} &= m_2 \gamma_2^{\text{input}} + c_2. \end{aligned} \quad (43)$$

An illustrative example of one typical measurement of the first component of shear is shown in figure 4. The grey points correspond to sets of rotated and unrotated galaxies, and are explained in §4.4. In this example, the negative slope of the black dashed line in the bottom panel (m_1) shows that this method systematically underestimates shear by $\sim 2.5\%$. However, the negligible y -intercept shows that the PSF was successfully corrected and no residual shear calibration (c_1) remained. The measurement of the second component of shear is not shown. Note that the range of input shear values is smaller than STEP1 and, in this weak shear régime, none of the methods exhibit the non-linear response to shear seen with the strong signals in STEP1. We therefore do not attempt to fit a quadratic function to any of the shear v vs shear out results.

4.4 Combining rotated and unrotated galaxies

An important advance in this second STEP project is the simultaneous analysis of galaxies that had been rotated by 90° before the application of shear and convolution with the PSF. This can

largely remove noise due to scatter in galaxies' intrinsic morphology, but complicates the production of a joint shear catalogue, especially where the galaxies are given different weights in the two catalogues.

Taking the rotated and unrotated sets of images individually, we obtain two sets of mean shear estimators $\langle \tilde{\gamma}^{\text{unrot}} \rangle$ and $\langle \tilde{\gamma}^{\text{rot}} \rangle$, which are defined in equations (25) and (26). We typically find that $m_i^{\text{rot}} \approx m_i^{\text{unrot}}$ and $c_i^{\text{rot}} \approx -c_i^{\text{unrot}}$. Such stability to changes in image rotation is to be expected: cross-talk between ellipticity and shear directions are second order in γ according to equation (2), and the mean ellipticity is overwhelmingly dominated by the intrinsic ellipticities of a finite number of galaxies (as demonstrated by the offset between the squares and diamonds in figure 4). Intriguingly, for the MS1 and MS2 methods, the shear calibration bias changes significantly between the rotated and the unrotated catalogues, and when the two are matched. These methods use smaller galaxies than most, including some 10–25% around or below the stellar locus on a size v vs magnitude plane, and this effect may be caused by instabilities in the PSF correction of the smallest. As an alternative explanation, there are also second-order effects inherent in the non-linear lensing equation that involve the dot product of ellipticity and shear, which would become significant in the presence of an ellipticity-dependent selection bias. However, we do not understand why this would affect only this pipeline and not others.

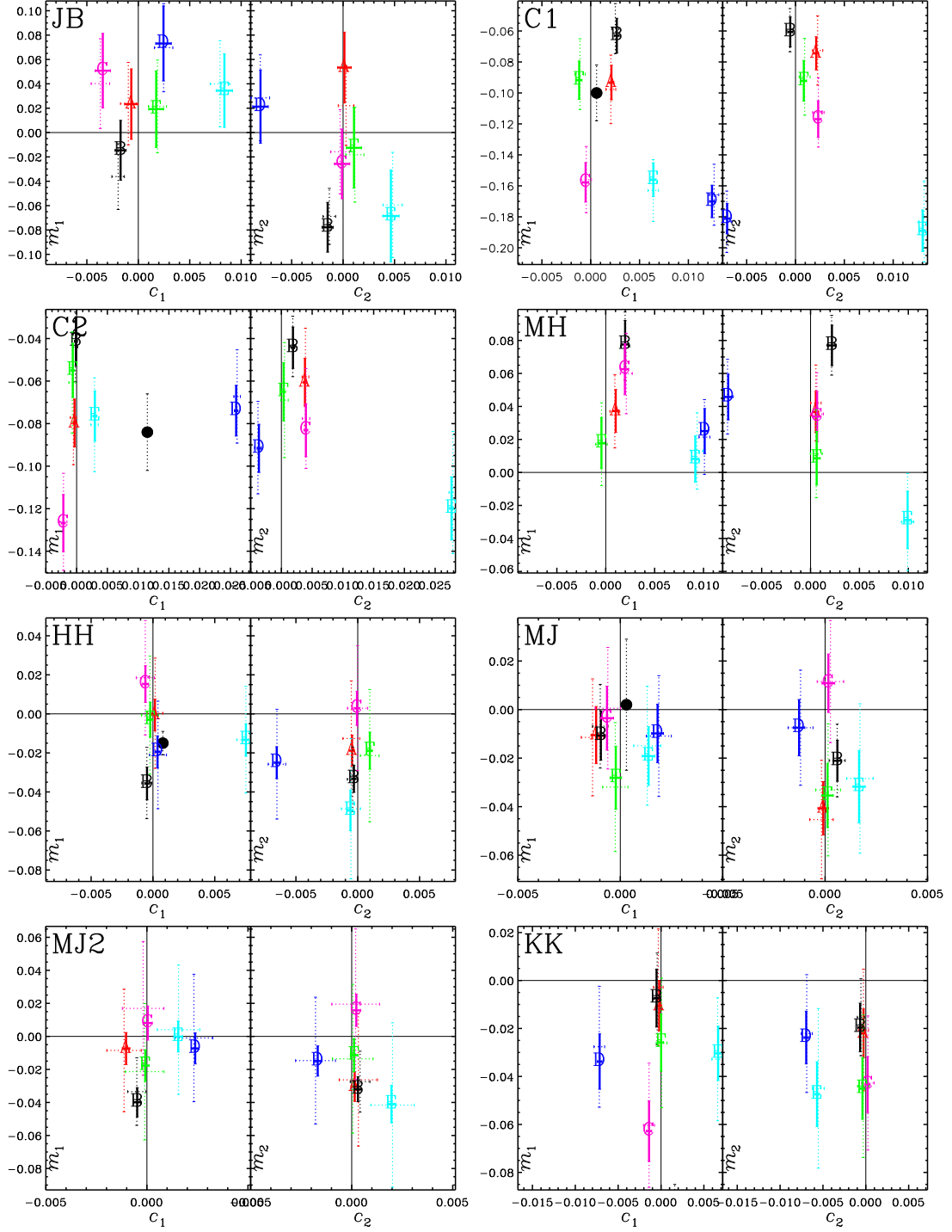


Figure 5. Fitted values of residual shear offset and shear calibration bias for each method and for each PSF. In all cases, the left hand panel shows results for the γ_1 component of shear, and the right hand panel for the γ_2 component. The dotted lines show rms errors after a combined analysis of the rotated and unrotated galaxies, after the two catalogues have been matched (and only common detections kept). The solid lines show the reduced errors after removing...

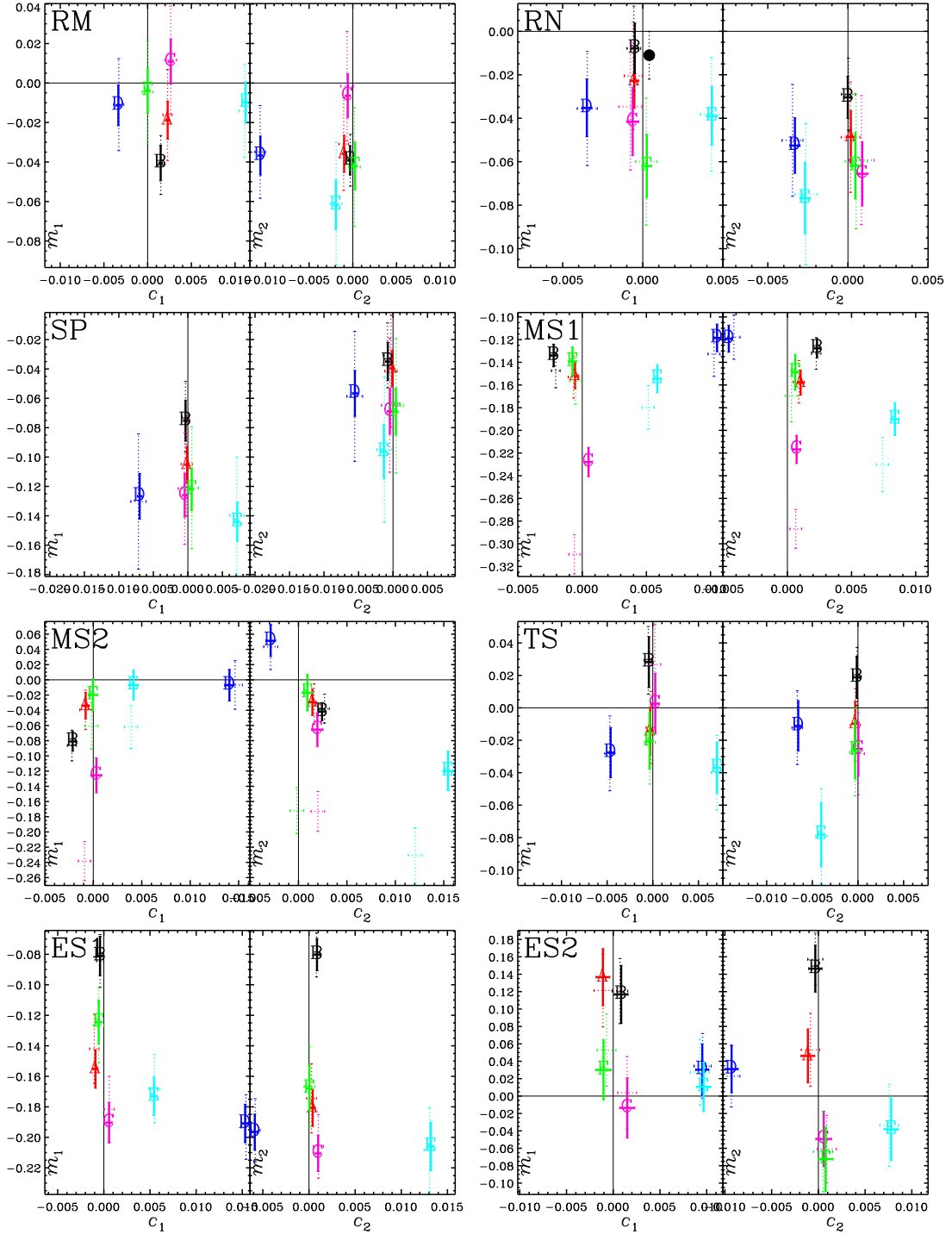


Figure 5 (continued). ...intrinsic galaxy shape noise from the matched pairs of galaxies. Note that the scales on each panel are different, but the frequency of the axis labels is preserved. The red points correspond to image set A. The black points correspond to image set B, and, where available, the filled black circles reproduce results from STEP1. The pink, dark blue, light blue and green points correspond to image sets C, D, E and F respectively.

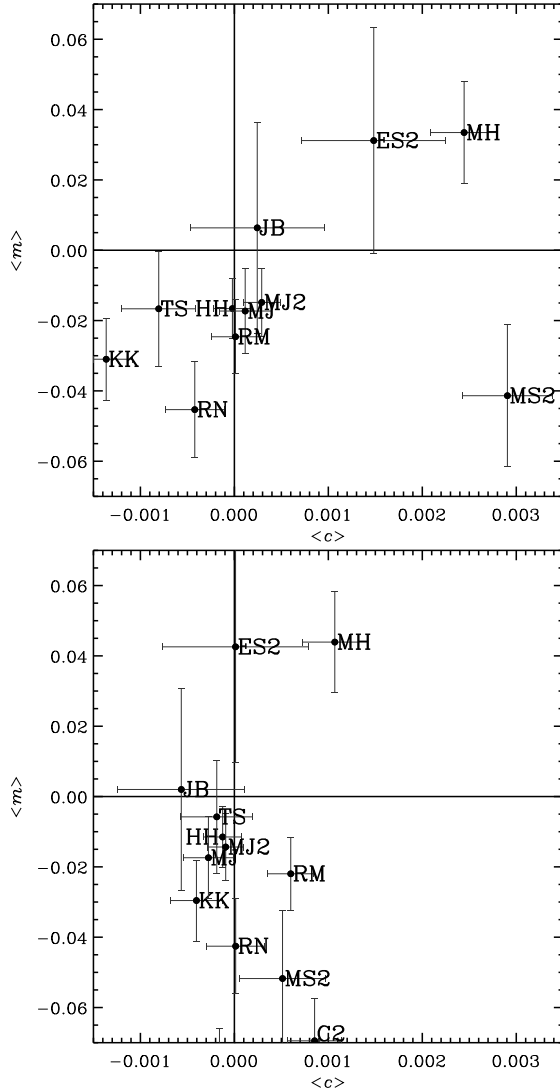


Figure 6. Comparison of shear measurement accuracy from different methods, in terms of their mean residual shear offset $\langle c \rangle$ and mean shear calibration bias $\langle m \rangle$. In the top panel, these parameters have been averaged over both components of shear and all six sets of images; the bottom panel includes only image sets A, B, C and F, to avoid the two highly elliptical PSFs. Note that the *entire region* of these plots lie inside the grey band that indicated good performance for methods in figure 3 of STEP1. The results from methods C1, SP, MS1 and ES1 are not shown here.

We have not attempted to investigate this isolated effect in more detail.

We obtain a third set of parameters m_i and c_i from the matched catalogue with $\langle \gamma \rangle$ defined in equation (27). In general, we find that $m_i \simeq (m_i^{\text{unrot}} + m_i^{\text{rot}})/2$ and $c_i \simeq c_i^{\text{unrot}} - c_i^{\text{rot}}$, with significantly smaller errors in this matched analysis. An example of all three shear estimators for the KK method on image set

F are plotted in figure 4. The fitted parameters for all of the shear measurement methods, on all of the PSFs, are shown in figure 5. Parameters measured from the matched pair analysis are also tabulated in the appendix. Results from the most successful methods are averaged across all of the sets of simulated images and compared directly in figure 6.

4.5 Analysis as a function of galaxy population

It is possible to measure the mean shear correctly from a large population of galaxies, but to underestimate the shears in some and overestimate it in others. This was frequently found to be the case in STEP2 data as a function of galaxy size or magnitude, but correlations could also be present as a function of galaxy morphological type. Anything that correlates with galaxy redshift is particularly important, and figure 7 shows the correlation of shear calibration bias and residual shear offset with galaxy size and magnitude for an illustrative selection of shear measurement methods. Of course, these proxies are not absolute: the fundamental parameters of interest are the size of galaxies relative to the pixel or PSF size, and the flux of galaxies relative to the image noise level. This must be taken into account before drawing parallel conclusions on data sets from shallower surveys or those taken in different observing conditions.

The results for the TS method are fairly representative of most implementations of KSB+. The calibration bias changes by 0.2–0.3 between bright and faint galaxies. The mean shear calibration bias changes between methods by merely raising or lowering this curve. The ES2 curve is least affected, with only a $\sim 5\%$ change. The shear calibration bias also generally changes as a function of galaxy size. The HH method controls this the best, no doubt due to its fitting of P^γ as a function of size only. However, this method still displays significant variation as a function of magnitude; it is not clear in figure 7 because the final point expands the y -axis scale. The fairly constant residual shear offset as a function of galaxy magnitude is typical; as is the dramatic improvements for bigger galaxies in the image sets D and E with highly elliptical PSFs. That demonstrates that it is a PSF-correction problem. The RM method behaves similarly to the implementations of KSB+.

Other methods exhibit more idiosyncratic behaviour. The main difference is between the KK method and the others that use a global shear responsivity \mathcal{R} . This was calculated only once for the KK method, from the entire galaxy population. For the other methods, it was recalculated using a subset of galaxies for each size and magnitude bin. The large trends in the shear calibration bias as a function of size and magnitude merely reflect the evolving distribution of intrinsic galaxy ellipticities. The MJ, MJ2, RM and RN methods also all look like this with a single value of \mathcal{R} , and the KK method would presumably be improved by this step. The JB results are atypical, but their additional noise level represents that in all analyses lacking an optimal galaxy weighting scheme.

5 INTERPRETATION

We shall now revisit the questions posed in the introduction, concerning the accuracy with which current methods can measure shear, and in which régimes that accuracy begins to deteriorate. By noting the variation of results with different PSFs, we shall investigate the effects of changing atmospheric and observing conditions. We shall also investigate the effects of image pixellisation, galaxy morphology and morphology evolution, selection biases and weighting effects. In light of our results, we shall then review the

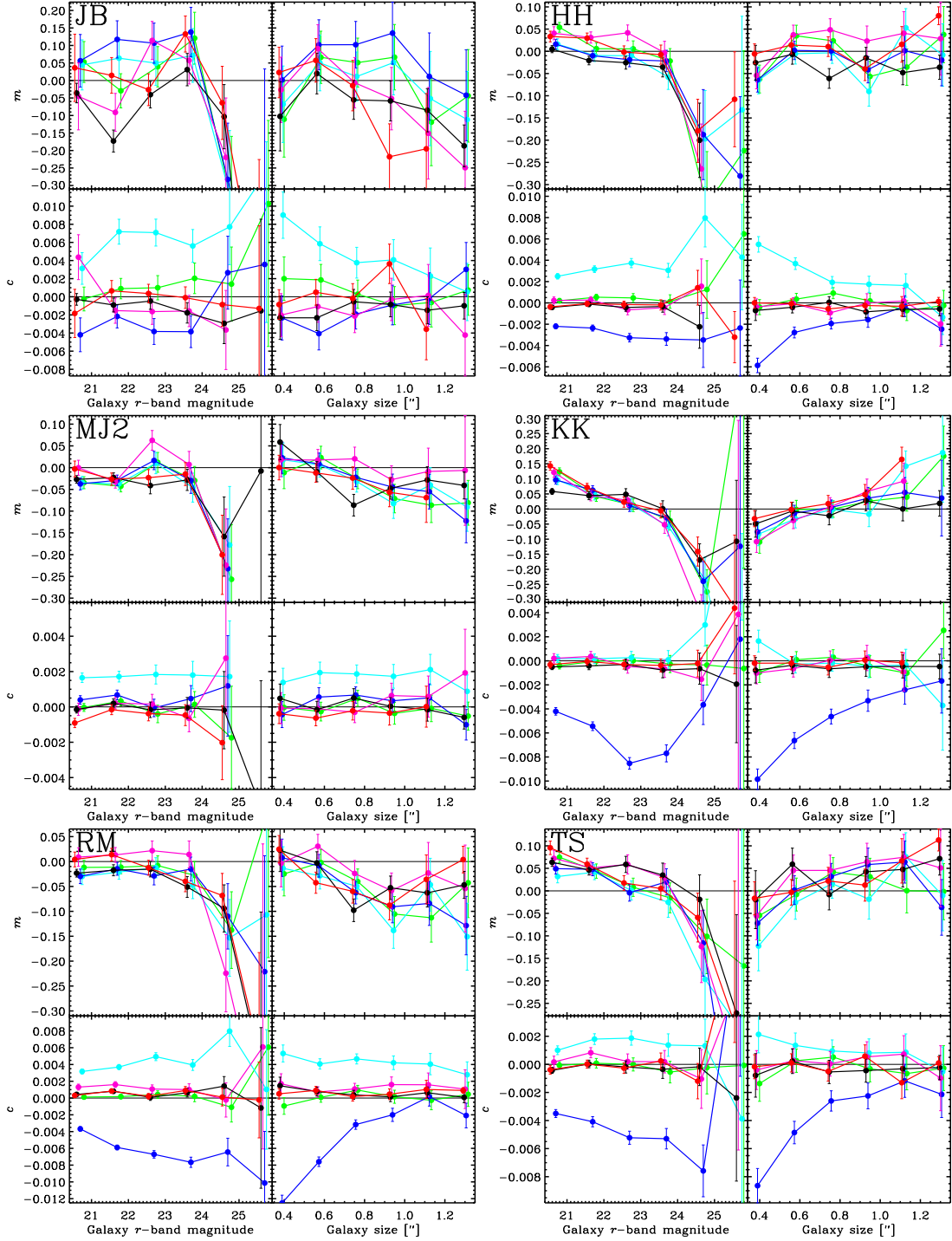


Figure 7. Variation in shear calibration bias and residual shear offset as a function of galaxy magnitude and size, for a representative sample of methods. The input values of these are used, which do not have noise. The “size” on the abscissae is the unweighted rms size of galaxies from equation (53) in Massey & Refregier (2005). The six coloured lines in each plot correspond to the six sets of images, coloured in the same way as in figure 5. In all cases, measurements of the two components of shear have been averaged.

consequences for previously published measurements of cosmic shear.

The rotated pairs of galaxies provide an unprecedented level of discriminatory power, and we can now identify high level causes of shear measurement error. Overall, both the shear calibration (multiplicative) bias and anisotropic PSF correction (additive) errors depend upon the PSF model. From this information, we can deduce that some aspects of shape measurement have been suitably controlled. We can deduce that others still provide difficulty, and it is work in these identified areas that will provide a route to the desired sub-percent level of precision. This section describes various lessons that we have learned from our tests, in terms of high level variables.

5.1 PSF size

Within the precision accessible by this analysis, all of the methods are reassuringly tolerant to reasonable changes in observing conditions. Image set A (0.6'' FWHM PSF) represents typical seeing at a good site, and image set C (0.8'' FWHM PSF) the worst that might be expected for a weak lensing survey after appropriate telescope scheduling.

Differences in the residual shear offsets between the two sets of images with different seeing are generally not significant. The few methods with a significant difference are JB, MH, KK and ES. In all four cases, the $2-3\sigma$ offset is in c_1 but not c_2 . The two KSB+ methods have a positive offset, and the two shapelets methods have a negative one, but no general conclusion seems manifest.

As expected, most methods demonstrate minimal shear calibration bias with image set A, and fare slightly worse on image set C. Shear calibration bias for the JB and RN methods is stable to changes in observing conditions at the $\sim 0.5\%$ level. The MH KSB+ method achieves $\sim 1\%$ consistency, although its applied shear calibration factor is apparently a little overzealous.

No global trends emerge that are able to include all of the KSB+ methods. However, for the generally most successful KSB+ implementations by MH, HH and TS, as well as the BJO2 (MJ, MJ2) methods, m is higher in image set C than in set A. These methods are all on the top row of table 3, and correct for the PSF by subtracting combinations of shape moments. The trend is reversed in the KK deconvolution method on the bottom row, and the calibration bias does not vary in the JB and RN methods. These correct for the PSF via a full deconvolution. Although all implementations of KSB+ do not necessarily fit this trend, it does suggest that the isotropic component of the PSF might be being overcorrected by some moment subtraction schemes. Furthermore, as the PSF moments get larger, this oversubtraction exaggerates pixellisation effects (see §5.3). The best PSF correction is generally attained by methods that model the full PSF and attempt to deconvolve each galaxy – but this currently works on slightly fewer galaxies (see §5.6).

5.2 PSF ellipticity (and skewness)

Image sets D and E demonstrate the ability of methods to correct for highly elliptical PSFs, and can be compared to image set F, which has a circularly symmetric PSF. Imperfect correction for PSF anisotropy will emerge mainly as a residual additive shear offset, c . The method that was most efficient at removing all the different strengths of PSF anisotropy to better than 0.2% accuracy was MJ/MJ2, and all of the PSF deconvolution methods had better

than 1% accuracy. The most successful KSB+ correction was the HH implementation. The residual shear offsets are smallest with large galaxies, and deteriorate only as galaxies get smaller. This behaviour is as expected if the problems are caused by imperfect PSF correction.

Many methods have a spurious residual shear offset in both components of shear, while the PSF is highly elliptical in only the ε_1 or ε_2 direction. This cross-contamination might come from the ignored off-diagonal elements of the P^{sm} tensor in KSB+, and is indeed slightly better controlled in MS2 (with the full tensor inversion) than in MS1. However, this can not explain all of the effect; the off-diagonal elements *are* exactly zero for the circular PSF in image set F, and a few methods (JB, C1, RN, SP, MS1, ES2) have a significantly non-zero residual shear offset for even this set of images.

A more likely source of the contamination lies in the measurement of stellar ellipticities. The non-zero residual shear offsets with image set F probably come from shot noise in the measurement of PSF ellipticity, which is higher than the shot noise for galaxies because of the smaller number of stars. It will therefore be worthwhile to make sure that future methods gather the maximum possible amount of information about the PSF. In particular, small galaxies provide as much information about the PSF as their own shapes, and this is currently discarded. Furthermore, PSFs D and E are not only highly elliptical, but also skewed. The centre of those PSFs therefore depends strongly on the size of the weight function used. While the main direction of ellipticity is not in doubt, changing the centre of the PSF also perturbs its apparent ellipticity. The C1 method, with a fixed stellar weight function and a constant PSF model, removes stellar ellipticity more consistently than the C2 method, in which the size of the stellar weight function is altered to match each galaxy (although matching the galaxy weight function provides a better shear calibration). Methods that involve deconvolution from a full model of the PSF, or correction of PSF non-Gaussianity, and which allow the galaxy centroid to iterate during this process, do indeed seem to be able to better control PSF ellipticity and centroiding errors.

We cannot conclusively explain the cross-contamination of both shear components by a PSF strongly elongated in only one direction, but hypothesise that it is introduced by skewness and substructure in the PSF. Neither of these are addressed by the formalism of KSB+, and they are both controlled more reliably by newer methods that explicitly allow such variation. However, it is also worth noticing the remarkable success of most methods on other image sets with more typical PSF ellipticities, and remarking that this is still a small effect that will not dominate shear measurement for the near future.

Our investigation of PSF effects in the STEP2 images is confused by other competing manifestations of imperfect shear measurement, and the realism of the simulations. The combination of image pixellisation (see §5.3), correlated galaxy sizes and magnitudes, and the evolution of intrinsic galaxy size and morphology as a function of redshift all hinder interpretation. Higher precision tests in the future will counterintuitively require *less* realistic simulated images: for example, ones that are tailored to compare otherwise identical galaxies at fixed multiples of the PSF size.

5.3 Pixellisation effects

This is the first STEP project in which the input shear has been applied in many directions, and in which the two components of shear can be measured independently. In general, residual shear offsets c

are consistent between components. However, we find that the γ_1 component, aligned with the square pixel grid, is typically measured more accurately than the γ_2 component, along the diagonals. This is even observed for image set F, in which the analytic PSF is circularly symmetric. Since there is no other preferred direction, this phenomenon must therefore be an effect of pixellisation. Image pixellisation, which is similar (but not identical) to convolution, slightly circularises galaxies, thereby reducing their ellipticity. Not explicitly correcting for pixellisation may therefore explain both the general 1 – 3% underestimation of γ_1 , and the slightly larger underestimation of γ_2 , in which direction the distance between pixels is exaggerated. For almost all methods, we consistently find that $m_1 > m_2$.

In KSB+, there is no formal mathematical framework to deal with image pixellisation. Two different approaches have been adopted to approximate the integrals in equation (20) with pixelated data. The C1 and C2 implementations calculate the value of the weight functions at the centre of each pixel and then form a discrete sum; all of the others numerically integrate the weight functions by subdividing pixels into a number of smaller regions. Neither approach is ideal. Independent experiments by Tim Schrabback, running objects with Gaussian radial profiles through his implementation of KSB+, have shown that pixellisation can cause a systematic underestimation of ε and P^{sm} , and an overestimation of P^{sh} . This effect can be up to $\sim 10\%$ for small objects. However, as stars and faint galaxies are similarly affected, the error on the shear estimate approximately cancels. Integration using linearly interpolated sub-pixels makes the measurement more stable to the sub-pixel position of the object centroid, but slightly increases the individual bias. Bacon *et al.* (2001) tested a variant of the C1 method, and found a similar $\sim 13\%$ overall calibration bias, which was used to correct subsequent measurements. With hindsight, the different calibration of γ_1 and γ_2 are also already visible in that work.

The MJ2, KK and TS methods are least affected by pixellisation. This might have suggested that the extraction of a shear estimator by shearing circular objects removes the problem, were it not for the peculiar behaviour of the RN method. For this method, image sets A and C follow the usual pattern that $m_1 > m_2$, but that bias is reversed when PSF is circular (image set F and the zero-ellipticity components of PSFs D and E). The SP method is similar. Strangely, the JB method, which ostensibly tries the hardest to treat pixellisation with mathematical rigour, displays the most difference between m_1 and m_2 . However, this method does break a trend by not having an overall negative shear calibration bias. If this bias is indeed caused by pixellisation, this method appears to have most successfully eliminated it.

Pixellisation could also hinder shear measurement, and bring about the observed results, via two additional mechanisms. Firstly, it may exaggerate astrometric errors in the PSF, and produce the consequences described in the previous section. We would be unable to distinguish these effects. Secondly, the undersampling of objects may also fundamentally prevent the measurement of their high order shape moments. All of the STEP2 PSFs (and hence the galaxies) are Nyquist sampled. It would be unfortunate for lensing if Nyquist sampling were theoretically sufficient to measure astrometry, but not shapes. As it happens, for methods other than MJ, the pixellisation bias is more pronounced for image set C (with poor seeing, and therefore better sampled) than on image set A (with good seeing). This suggests that the pixellisation effects are *not* due to undersampling. The STEP1 simulations had the same pixel scale but worse seeing ($\sim 1''$ FWHM), so objects were better sampled there.

We therefore hypothesise that the circularising effects of pixelisation explain the general underestimation of shear and the differential calibration of the γ_1 and γ_2 components. Indeed, a dedicated study of simulated images with varying pixel scales by High *et al.* (in preparation) supports this view. They find that the shear calibration bias of the RRG method tends to zero with infinitely small pixels, grows linearly with pixel scale, and that the bias $m_2 \approx \sqrt{2}m_1$. Because of the isotropy of the Universe, this differential calibration of shear estimators ought not affect two-point cosmic shear statistics. But it can certainly affect the reconstruction of individual cluster mass distributions, and is inherently quite disconcerting. The next STEP project will feature sets of images with varying pixel scales to investigate this effect on a wider scale. In the mean time, dealing properly with pixellisation will provide a promising direction for further improvement in shear measurement methods.

5.4 Galaxy morphology

The introduction of complex galaxy morphologies tends to hinder shear measurement with KSB+ methods. The shear calibration bias is more negative with image set A (shapelet galaxies) than with image set B (simple galaxies) for the C1, C2, MH, SP, MS1, TS and ES1 implementations. Of the implementations of KSB+, only HH and MS2 reverse this trend. This is perhaps not surprising, given the inherent limitation of KSB+ in assuming that the ellipticity of a galaxy does not change as a function of radius.

Many of the newer methods deal with complex galaxy morphologies very successfully. Particularly KK, but also the MJ and MJ2 methods, have no significant difference in the shear calibration bias or residual shear offset measured between image sets A and B. Future ground-based shear surveys are therefore unlikely to be limited at the 0.5% level by complex galaxy morphologies. Indeed, it is apparent in figure 2 that most of the substructure in galaxies that will be used for lensing analysis is destroyed by the atmospheric seeing. Although complex galaxy morphologies may become important at the level of a few tenths of a percent, they do not currently pose a dominant source of error or instability in shear measurement from the ground.

One of the crucial findings of this study, however, concerns the effect of galaxy morphology *evolution*. This could potentially affect the calibration of shear measurement as a function of galaxy redshift, and is investigated further in the next section.

In the next STEP project, which will simulate space-based observations, we shall repeat our investigation of galaxy morphology by comparing three similar sets of image simulations. Galaxy substructure will be better resolved from space and, because the galaxies observed there are likely to be at a higher redshift, their intrinsic morphologies may be both more irregular and more rapidly evolving. Both of these effects will amplify any differences seen from the ground.

5.5 Shear calibration for different galaxy populations

The STEP2 results reveal that the calibration bias of some shear measurement methods depends upon the size and magnitude of galaxies. There seem to be two causes. There is often a sudden $\sim 30\%$ deterioration of performance at very faint magnitudes, due to being noise blown up during the nonlinear process of shear measurement (and exacerbated by ellipticity-dependent galaxy weighting schemes). This is even observed with many methods that are otherwise robust (*e.g.* HH, MJ2, RN), and may urge more caution

in the use of faint galaxies at the limits of detection. There is also a gradual transition in shear calibration between bright and faint galaxies that is probably caused by evolution of the intrinsic morphology distribution as a function of redshift. The observed variation is least pronounced for image set B, in which the galaxies explicitly do not evolve.

Shear calibration bias that changes gradually as a function of galaxy redshift has important consequences for any weak lensing measurement. In a 2D survey, it will change the effective redshift distribution of source galaxies, with all the consequences discussed by Van Waerbeke *et al.* (2006). In a 3D analysis, it will affect the perceived redshift evolution of the matter power spectrum, and the apparent large-scale geometry of the universe. During the STEP2 analysis, we have developed ways to partially control this, as a function of other observables like galaxy size and magnitude. To first order, these act as suitable proxies for redshift, but the underlying causes will need to be well understood, because neither of these are redshift. Even if the mean shear in size/magnitude bins could be made correct, this doesn't necessarily imply that the mean shear would be correct in redshift bins. The techniques could be applied in multicolour surveys as a function of photometric redshift, but this is not perfect either, not least because of the inevitable presence of catastrophic photo- z failures.

The obvious place to start looking for shear calibration errors is in the shear susceptibility and responsivity factors. All of the KSB+ implementations allow variation in P^γ as a function of at least one of galaxy size and galaxy magnitude. However, the behaviour is neither well understood, nor stable at the desired level of precision. Massey *et al.* (2005) have already observed that P^γ fitted from a population ensemble varies for any given object as a function of the catalogue selection cuts. There is less variation in the shear calibration bias of the MS1 method ($\Delta m \approx 0.1$), which fits only the trace of P^γ , than of the MS2 method ($\Delta m \approx 0.2$), which models the entire tensor – except for image set B, in which there is little variation in either. Realistic galaxy morphologies therefore do not have shear susceptibility that is a simple function of these observables; and trying to model the variation of all the components of this tensor merely adds noise. The TS implementation of KSB+, which uses P^γ from individual objects, suffers particularly from this noise, which enters into the denominator equation (18), and has at least as much sudden deterioration at faint magnitudes as other methods. However, this method is about the least affected by gradual variation in shear calibration bias, with $\Delta m \approx 0.05$. Size galaxy size and magnitude are correlated, the variation with galaxy magnitude usually carries over to variation with galaxy size. However, the HH method has notably little variation in m as a function of galaxy size. This is presumably due to the particularly individual form of the function used to model $P^\gamma(r_g)$. Unfortunately, P^γ is not fitted as a function of galaxy magnitude, and the HH method still shows strong ($\Delta m \approx 0.1$) variation with this. The shear susceptibility in this implementation is calculated separately in three magnitude bins, and correction of the faintest galaxies therefore required an extrapolation.

Many of the other shear measurement methods require global calibration via a responsivity \mathcal{R} factor, which is determined from the distribution of galaxy ellipticities. This factor is designed to ensure that the mean shear in a population is unbiased. However, it must be calculated from precisely that population. For the KK method, it was calculated only once, from the entire catalogue. Although it estimated the overall mean shear correctly, it then underestimated the shear in small/faint galaxies, and overestimated that in large/bright galaxies. This bias was addressed for the

MJ, MJ2, RM and RN methods by recalculating \mathcal{R} within each size and magnitude bin. There is no particular reason why this should not, in future, be fitted and allowed to vary continuously like the shear susceptibility in KSB+ methods. The estimates of \mathcal{R} in bins were more noisy, but removed the differential shear calibration (in fact, the variation as a function of galaxy magnitude was slightly overcorrected in the case of the MJ2 and RM methods).

5.6 Galaxy selection effects

There is a marked difference between the depth of the various galaxy catalogues. At one extreme, the C1/C2 catalogues are deeper, and more ambitious, than all others. At the other, the RN catalogue (and to some extent the MJ/MJ2 catalogue) is very shallow. The RN method obtained extremely good results, but only from large and bright galaxies, and it would be interesting to test whether its PSF deconvolution iteration can converge with a deeper sample. The JB catalogue of individual rotated and unrotated images is deeper, but not all of the galaxies at the magnitude limit converged successfully, leading to a relatively shallow matched catalogue. We could conclude from this that the full deconvolution of every galaxy is an overly ambitious goal: it is a panacea for many image analysis problems, but all that we require is one shear estimator. Maximising the number density of useable galaxies will remain crucial in the near future, to overcome noise from their intrinsic ellipticities. However, there has been far less time spent developing the deconvolution methods than the moment subtraction methods, so we reserve judgement for now because of their promise of robust PSF correction. Furthermore, it is not only the methods that require complicated iterations that suffer from catalogue shortcomings: the SP catalogue includes a significant number of spurious detections (10%) and stars (1%). Neither of these contain any shear signal, and their presence partly explains the large, negative calibration bias of the SP method in the rotated and unrotated images (they are removed during the galaxy matching).

Most other methods use a fairly standard density of ~ 30 galaxies per square arcminute in this simulated data. This is unlikely to be increased dramatically by any future weak lensing observations. Since selection effects in the STEP2 analysis must be measured from the individual unrotated and rotated catalogues, rather than the matched catalogues, the results about catalogue selection biases are hardly more profound than those of STEP1.

5.7 Galaxy weighting schemes

The weighting schemes applied to galaxies also vary significantly between methods used in this paper, and these do affect the results in the matched catalogue. Most of the methods increase the contribution to the estimated mean shear from those galaxies whose shapes are thought to be most accurately measured. Such schemes have long been used in the analysis of real 2D data, but the exact form of the weighting scheme as a function of size, magnitude and ellipticity varies widely. Even more sophisticated weighting schemes will also need to be developed for the 3D analyses essential to fully exploit future weak lensing surveys.

In this analysis, the effectiveness of each weighting scheme can be seen in the difference between the size of error bars in the analysis of independent galaxies and of rotated/unrotated pairs of matched galaxies. In the independent analysis, the scatter includes

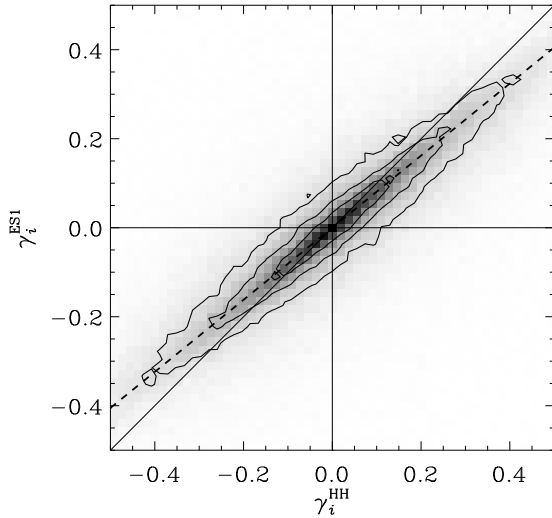


Figure 8. Comparison of shear measurement in real CFHTLS deep data, from a galaxy-by-galaxy comparison of matched catalogues from the ES1 analysis (Semboloni *et al.* 2006a) and a reanalysis using the HH method. The relative calibration of both components of shear are indistinguishable, and are here included in the same plot. A slope of unity would imply perfect agreement. The dashed line indicates the relative calibration of the two methods in simulated image set C, which is the most closely matched to actual observing conditions. Although this should not be regarded as a strict prediction, since there are many image parameters that are not matched, its agreement with the real data is striking.

components from intrinsic galaxy shapes and measurement noise (*e.g.* due to photon shot noise). The former is essentially removed by matching pairs of galaxies. If a set of error bars shrink dramatically by matching, the method was dominated by intrinsic galaxy shapes: this is an ideal situation. If the error bars change little, the measurement was dominated by measurement noise.

The weighting schemes of MJ2 and KK are very effective in this analysis: their error bars shrink by up to 75%. The weighting schemes of HH, SP and MJ are similarly effective – but these methods weight ellipticities using a function of ellipticity, which may be less accurate in regimes where the mean shear is large, such as cluster mass reconstruction. Indeed, the aggressive weighting scheme of MJ was shown in STEP1 to be useful with small input shears, but introduced a non-linear shear response that became important if the shear was high. A new weighting scheme was developed for MJ2 to address this concern; however, the range of input shears in STEP2 does not provide sufficient lever arm to evaluate the potential nonlinear response of any method.

The value of a successful weighting scheme is demonstrated by the lesser performance of methods without one. The JB, TS and ES2 methods apply crude weighting schemes that are merely a step function (cut) in galaxy size and magnitude. Their error bars shrink by only 30–50% during galaxy matching. Their results are also less stable to the sudden deterioration of performance seen in several methods with galaxies fainter than or smaller than a particular limit. This shortfall is easy to correct, and we urge the rapid adoption of a more sophisticated weighting scheme in those methods.

It is important to remember the limitations of the STEP sim-

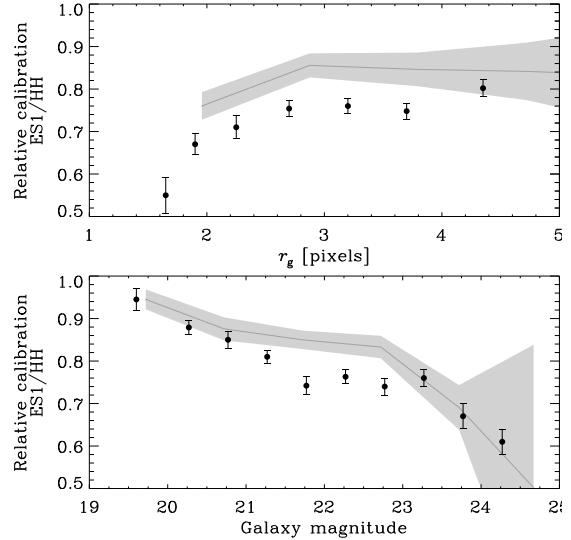


Figure 9. Comparison of shear measurement in real CFHTLS deep data, as a function of galaxy size and magnitude. The relative shear calibration of the ES1 and HH methods is obtained from the ratio of the mean shear calculated in $3' \times 3'$ subfields of each CFHTLS deep field. A value of unity would imply perfect agreement between the catalogues. Note that we have reconciled the different definitions of galaxy size in the simulations compared to real data by approximating $R \approx r_g$. We have dealt with the different relationship between galaxy magnitude and signal-to-noise (*c.f.* §4.5) by offsetting the magnitudes of objects in the deeper simulated data by -1. The grey band indicates the relative calibration of the two methods in simulated image set C, which is the most closely matched to the CFHTLS data.

ulations to optimise a galaxy weighting scheme, because of their inherent simplification that all galaxies are sheared by the same amount. In real data, the lensing signal increases cumulatively with redshift, and the distant galaxies therefore contain the most valuable signal. However, when weighting objects by the accuracy of their shape measurement, it is the contribution of these small, faint sources that is usually downweighted. It would instead be better to set weights that vary as a function of the signal to noise in shear signal – although the exact variation of the signal is of course unknown in advance. A statistically “optimal” weighting scheme verified from the STEP simulations will therefore not be optimal in practice. Weighting schemes can also act like calibration biases as a function of galaxy redshift, exacerbating the problems of differential shear calibration discussed in the previous section.

5.8 Consequences for previously published measurements

The largest cosmic shear survey to date, which has been published since STEP1, comes from the *Canada-France-Hawaii Telescope Legacy Survey* (CFHTLS) *i*-band data. The CFHTLS wide survey (Hoekstra *et al.* 2006) was analysed using the HH shear measurement method, and the CFHTLS deep survey (Semboloni *et al.* 2006a) using the ES1 method. These methods perform very differently on the simulated images.

The HH method recovers shear in the STEP2 images with remarkable success. The seeing in the CFHTLS data is most similar to that in image set C, for which the overall shear calibration

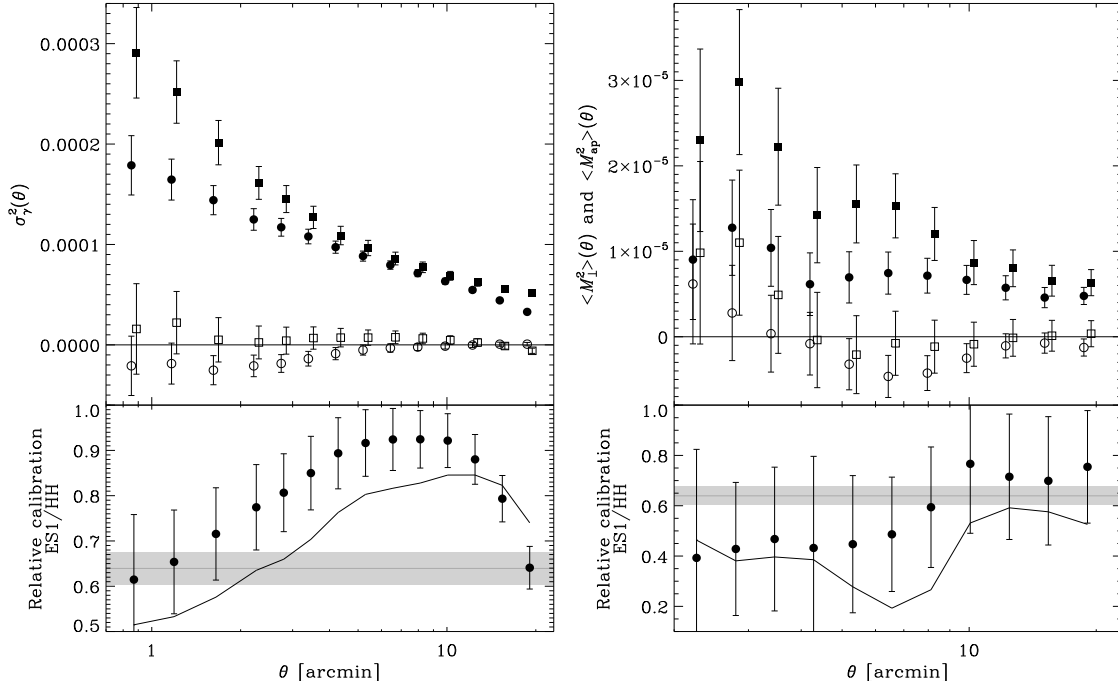


Figure 10. Comparison of shear-shear correlation functions measured from real CFHTLS deep survey data, after HH (squares) and ES1 (circles) analyses. The correlation functions are split into E - and B -modes in two different ways: the variance of the shear in cells is shown on the left as a function of cell radius, and the variance of the mass aperture statistic is shown on the right. In both cases, the solid points show the E -mode, and the open points show the B -mode. The error bars show statistical errors only (*i.e.* no account is made for cosmic variance since the survey region is identical), but note that the difference between the two data sets is in fact more significant than indicated, because the same galaxies are used in each analysis, so noise enters only from the shape measurement process and not from variation in intrinsic galaxy ellipticities. In the lower panels, the points show the ratio of the E -modes calculated from the two analyses, and the lines show the ratio of the E -modes plus B -modes. The grey bands indicate the relative calibration of the two methods in simulated image set C, which is the most closely matched to actual observing conditions.

is within 1%: well within the current error budget. Hoekstra *et al.* (2006) also featured a parallel analysis using an independent KSB+ pipeline, which agreed with the HH results, and also demonstrates the potential robustness of KSB+ at this level of precision (similar comparisons have also been performed by Massey *et al.* (2005) and Schrabback *et al.* (2006), and these also give results consistent with that work). The HH method had difficulty only with the calibration of very faint galaxies, due to its non-smooth fitting of P^γ as a function of magnitude. If a similar bias is present in the CFHTLS analysis, it will have lowered the effective redshift distribution of source galaxies, and slightly diluted the overall signal. Both of these effects would have led to an underestimation of σ_8 , although only by a small amount, due to the low weight given to faint galaxies. As discussed by Van Waerbeke *et al.* (2006), a more significant bias (which acts in the opposite sense) arises from using the Hubble Deep Field to infer the redshift distribution of galaxies. As the survey area of the CFHTLS grows, and the statistical error bars decrease, it may be prudent for this analysis to conservatively use slightly fewer galaxies.

The ES1 method underestimates shear in the STEP2 images by 20% overall, and by as much as 30% for the faintest galaxies. We have verified this result retrospectively in STEP1 simulations, and also confirmed it in real images, by comparing the results of the HH and ES1 shear measurement pipelines on the same CFHTLS

deep data. Of course, the true “input” shear is not known for real data. Figure 8 shows the *relative* calibration of the two methods in real data, with the dashed line indicating their relative calibration in simulated image set C. This should not be interpreted as a strict prediction, since the simulation was not designed to mimic this specific survey: the simulated and real data have very different noise properties, and the only similarity between their PSFs is their size. Nonetheless, the agreement is impressive. Figure 9 shows a further comparison of the methods’ relative calibration, in which galaxies have been split by size and magnitude. Once again, overlaying the performance of ES1 from image set C confirms the results of the STEP simulations with remarkable success. A likely source of the shear calibration bias is in the smoothing of P^γ as a function of r_g and magnitude. Tests indicate that the shear susceptibility is more stable if it is instead fitted as a smooth function of size and magnitude, or even by using the raw values. The strong magnitude dependence is probably related to the sudden drop at small sizes. Note also that both pipelines started from scratch with the individual exposures, reducing them and stacking them independently. All of the available exposures are stacked in both versions, so the two sets of images have effectively the same depth. The full data reduction pipeline of both groups is being tested, and the differences could therefore have been introduced at any stage.

Figure 10 shows the two-point correlation functions of the

matched shear catalogues (using the weights of the individual catalogues), which are normally used to constrain cosmological parameters at the end of a weak lensing analysis. Although the ES1 analysis consistently measures a lower signal than the HH analysis, the discrepancy is not uniform on all scales. The relative bias is most pronounced on small scales when measuring the variance of the aperture mass statistic, and on both small and large scales for the shear variance in cells. Such variation is not seen in the galaxy-by-galaxy comparison of relative shear calibration. For example, the signal in figure 9 is stable to changes in the size of the area over which the shears are averaged.

We hypothesise that there may therefore be an *additional* source of bias in the ES1 CFHTLS analysis, due to PSF anisotropy residuals. Since the PSF anisotropy varies spatially, the residual would average out across the survey, and not affect the overall bias. The correlation functions were calculated using the procedure in Van Waerbeke *et al.* (2005), which deals with an unknown constant of integration in the calculation of $\sigma_\gamma^2(\theta)$ by forcing the B -modes of to zero on large scales. This prior on the B -modes can add spurious power to the E -modes, and could have artificially re-raised the cosmic shear signal. Indeed, the ratio of the sum of the E - and B -modes between analyses is flatter than that of the E -modes alone. Furthermore, the star-star correlation functions (Semboloni *et al.* 2006a) show an excess before PSF correction, on similar scales to that observed in the left-hand panel of figure 10.

A naïve correction for a 20% shear calibration bias in the CFHTLS deep survey (Semboloni *et al.* 2006a) would raise the measured value of σ_8 almost proportionally. This would remain within the estimated error budget for the lensing analysis due to non-Gaussian cosmic variance (Semboloni *et al.* 2006b), but adds tension to an existing discrepancy with the three year results from WMAP (Spergel *et al.* 2006). In practice, a more sophisticated recalibration will probably be required. If our hypothesis of an additional systematic is correct, this would have partially cancelled the shear calibration bias. Judging by the ratio of the observed correlation functions, the net underestimation of σ_8 could have been around 10–15%. More work is needed to test this hypothesis; but it is beyond the scope of this paper. A full reanalysis of the CFHTLS survey, including the latest data, will therefore follow.

The striking confirmation of the STEP results on real data demonstrates the success of our simulation project, and highlights the vital role that artificial images will play in the exploitation of future surveys. Ideally, they ought not be relied upon for simple empirical recalibration, but they will be essential to verify the performance of methods derived from first principles. The STEP images remain publicly available to test future weak lensing analyses. Simultaneously, the complexity of our correlation functions results also highlight the importance of subtleties in weak shear measurement that may arise only within the complex environment of real observational data. To fully understand such effects, we shall pursue further development of the *dataSTEP* project², an ongoing comparison of the output from various shear measurement methods on a common sample of real data.

6 CONCLUSIONS

Performance has improved since STEP1, and the STEP project continues to drive progress and innovation in shear measurement methods. The most accurate methods, with better than $\sim 2\%$ level calibration errors for most of the tested observing conditions, were the MJ2 implementation of BJ02, the TS and HH implementations

of KSB+, the KK and JB implementations of shapelets and the RM implementation of Reglens. Particular advances are apparent in methods that used the results of STEP1 to tune their algorithms, which bodes well for the future of this project. For example, the introduction of a calibration factor to the TS method has proved reassuringly robust with our new, more realistic simulated images. We have also verified the STEP results on real data, finding striking confirmation of methods' relative shear calibration in the CFHTLS deep survey.

There is no one shear measurement method that is doing everything best. With the increased precision possible in this analysis, we can now distinguish all of the methods from *perfect* performance. Since absolute shear calibration can not be directly ascertained from real data, this remains the most important issue. The calibration bias in most methods leads to a slight underestimation of shear. Both the shear calibration (multiplicative) errors and anisotropic PSF correction (additive) errors are also found to depend upon characteristics of the PSF. Technical advances in individual methods will therefore still be required. Ideally, one would attempt to take the most successful aspect of several methods and combine them. The fundamentally different approaches to the two main tasks in shear measurement make this difficult, but there is common ground (*e.g.* object detection algorithms, the shapelet basis functions, and galaxy weighting schemes), so the individual lessons learned with each method may not necessarily be irreconcilable. To this end, we have developed a classification scheme for shear measurement methods, and have described all existing methods in a common language so that their similarities and differences are apparent. Development is continuing in earnest.

We have used our improved simulations to identify various aspects of shear measurement that have been effectively solved at the current level of precision. We have also uncovered other, specific areas that remain problematic. Studying these may provide a route to the most rapid technological advances. Development needs to be focussed towards:

- Pixellisation
- Correlated background noise
- PSF measurement
- Galaxy morphology evolution.

These four points are explained below.

This is the first STEP project in which the input shear has been applied in arbitrary directions relative to the pixel grid. That this direction affects the calibration of shear measurement methods, even for images with a circular PSF and no other preferred direction, implies that pixellisation is not fully controlled. Pixel effects may also explain the general tendency of methods to underestimate shear. Since no explicit provision is made for pixellisation in many methods, this result is not surprising. This work has quantified just how much of an effect it has, and thereby emphasised the importance of a proper treatment in the future. High *et al.* (in preparation) are specifically investigating pixellisation through tailor-made image simulations with varying pixel scales.

Although not all data sets have background noise that is significantly correlated between adjacent pixels, it is particularly apparent in natively undersampled data, for which several exposures dithered by sub-pixel shifts must be co-added. The introduction of correlated background noise to the STEP2 simulations hindered several methods: during the detection of faint objects, the modelling of objects to a specified fidelity, and the weighting of individual shear estimators. Now that this issue has been raised, work is

underway in the context of several of the shear measurement methods.

Various schemes have been developed to improve PSF interpolation across a field of view (Hoekstra 2004; Jarvis & Jain 2004), but some methods seem to be having trouble with the initial measurements of the PSF from individual stars. The measurement of the shape of each star affects shear estimates from many galaxies, and is therefore of vital importance. When the PSF is highly elliptical, this work has revealed some peculiar residual shear offsets, in the directions orthogonal (at 45°) to that ellipticity. We have not yet found a satisfactory explanation for this, but speculate that it might be caused by difficulties measuring the centroid and the ellipticity of stars that have substructure, skewness, and no single, well-defined ellipticity. Methods that model the full PSF, and especially those that attempt PSF deconvolution, are less affected, but at the expense of a having smaller number density of useable galaxies for which the complicated deconvolution algorithms currently converge. This issue will require further investigation, and questions about the residual shears cannot be addressed until this is resolved.

Issues of galaxy morphology evolution become particularly important for those methods whose calibration relies on the overall distribution of galaxies' intrinsic ellipticities. High redshift galaxies are both more elliptical and more irregular; and evolution in the ellipticity variance directly affects the shear calibration. For a 2D cosmic shear survey, even if the mean shear is correctly measured, this can bias the effective redshift distribution of source galaxies and the geometrical interpretation of the lensing signal, with all the consequences discussed in Van Waerbeke *et al.* (2006). For a 3D analysis, it can change the apparent redshift evolution of the signal and hence the apparent cosmological matter distribution.

The next STEP project will analyse a set of simulated space-based images. With their higher spatial resolution, we expect that variation in galaxy morphology will more profoundly affect shear measurement. We will therefore repeat the exercise of comparing the analysis of complex shapelet galaxies with more idealised objects, and also separate the galaxy populations by morphological class. The cuspy space-based PSFs will provide a different (easier) régime in which to test centering, and we shall explicitly avoid PSF interpolation errors by allowing methods to assume that the PSF is constant. This should make interpretation easier. Background noise will also be left intentionally uncorrelated. However, variations in the pixel scale will be introduced, to specifically test methods' robustness to pixellisation effects.

Such ongoing improvements are vital to the success of gravitational lensing as a viable probe of cosmology. Although the measurement of weak lensing is not limited by unknown physical processes, the technical aspect of galaxy shape measurement at such high precision remains computationally challenging. In this paper, we have demonstrated that simulated images can drive progress in this field, and can provide a robust test of shear measurement on real data. Previous cosmic shear measurements would have benefited from access to STEP, and the future exploitation of dedicated surveys relies upon the development of methods that are being tested here first. Both the tools and the collective will are now in place to meet this challenge. The STEP simulations remain publicly available, and the weak lensing community is progressing to the next level of technical refinement in a spirit of open cooperation. We conclude with the hope that, by accessing the shared technical knowledge compiled by the STEP projects, all future shear measurement methods will be able to reliably and accurately measure weak lensing shear.

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REFERENCES

- Bacon D., Massey R., Refregier A., Ellis R., 2003, MNRAS, 344, 673
 Bacon D., Refregier A., Ellis R., 2000, MNRAS, 318, 625
 Bacon D., Refregier A., Clowe D., Ellis R., 2001, MNRAS, 325, 1065
 Bardeau S., Kneib J.-P., Czoske O., Soucail I., Smail I., Ebeling H., 2004, A&A submitted, astro-ph/0407255
 Bartelmann M., Schneider P., 2001, Physics Reports, 340, 291
 Benabed K., Van Waerbeke L., 2004, Physical Review Letters, 70, 123515
 Bernstein G., 2005, ApJ submitted, astro-ph/0503276
 Bernstein G., Jain B., 2004, ApJ, 600, 17
 Bernstein G., Jarvis M., 2002, AJ, 123, 583 (BJ02)
 Bertin E., Arnouts S., 1996, A&AS, 117, 393
 Berry R., Hobson M., Withington S., 2004, MNRAS, 354, 199
 Binney J., de Vaucouleurs G., 1981, MNRAS, 194, 679
 Bonnet H., Mellier Y., 1995, A&A, 303, 331
 Boroson T., 1981, ApJS, 46, 177
 Bridle S., Gull S., Bardeau S., Kneib J. P., 2001, in Scientific N. W., ed., Proceedings of the Yale Cosmology Workshop
 Brown M., Taylor A., Bacon D., Gray M., Dye S., Meisenheimer K., Wolf C., 2003, MNRAS, 341, 100
 Chang T., Refregier A., Helfand D. J., 2004, ApJ, 617, 794
 Clowe D., Gonzalez A., Markevitch M., 2004, ApJ, 604, 596
 Crittenden R., Natarajan R., Pen U., Theuns T., 2002, ApJ, 568, 20
 Dahle H., 2006, ApJ in press, astro-ph/0608480
 Dekel, A. & Lahav, O., 1999, ApJ, 520, 24
 Erben T., Van Waerbeke L., Bertin E., Mellier Y., Schneider P., 2001, A&A, 366, 717
 Fischer P., Tyson J. A., 1997, AJ, 114, 14
 Gray M., Taylor A., Meisenheimer K., Dye S., Wolf C. & Thommes E., 2002, ApJ, 568, 141
 Grosbol P. J., 1985, A&AS, 60, 261
 Hamana T., Miyazaki S., Shimasaku K., Furusawa H., Doi M., Hamabe M., Imi K., Kimura M., Komiyama Y., Nakata F., Okada N., Okamura S., Ouchi M., Sekiguchi M., Yagi M., Yasuda N., 2003, ApJ, 597, 98
 Heavens A., 2003, MNRAS, 343, 1327
 Hettterscheidt M. *et al.*, 2006, A&A submitted, astro-ph/0606571
 Heymans C., Brown M., Heavens A., Meisenheimer K., Taylor A., Wolf C., 2004, MNRAS, 347, 895
 Heymans C., Brown M. L., Barden M., Caldwell J. A. R., Jahnke

- K., Rix H.-W., Taylor A. N., Beckwith S. V. W., Bell E. F., Borch A., Häubler B., Jogee S., McIntosh D. H., Meisenheimer K., Peng C. Y., Sanchez S. F., Somerville R. S., Wisotzki L., Wolf C., 2005, MNRAS, 361, 160
- Heymans C., Heavens A., 2003, MNRAS, 339, 711
- Heymans *et al.*, 2005, MNRAS 368 1323 (STEP1)
- Hillebrandt, W. & Niemeyer, J., 2000, ARA&A, 38, 191
- Hirata C., Seljak U., 2003, MNRAS, 343, 459
- Hirata C. M., Mandelbaum R., Seljak U., Guzik J., Padmanabhan N., Blake C., Brinkmann J., Budavari T. and Connolly A., Csabai I., Scranton R., Szalay A. S., 2004, MNRAS, 353, 529
- Hoekstra H., 2004, MNRAS, 347, 1337
- Hoekstra H., Franx M., Kuijken K., 2000, ApJ, 532, 88
- Hoekstra H., Franx M., Kuijken K., Squires G., 1998, ApJ, 504, 636
- Hoekstra H., Yee H., Gladders M., 2002, ApJ, 577, 595
- Hoekstra H., Yee H., Gladders M., Barrientos L. F., Hall P., Infante L., 2002, ApJ, 572, 55
- Hoekstra H., Yee H. K. C., Gladders M. D., 2004, ApJ, 606, 67
- Hoekstra H., van Waerbeke L., Gladders M., Mellier Y. & Yee H., 2002b, ApJ, 577, 604
- Hoekstra H. *et al.*, 2006, ApJ in press, astro-ph/0511089
- Huizinga J. E., van Albada T. S., 1992, MNRAS, 254, 677
- Huterer & White, 2003, ApJ, 578, L95
- Huterer D., Takada M., Bernstein G., Jain B., 2005, MNRAS submitted, astro-ph/0506030
- Ishak M., 2005, MNRAS submitted, astro-ph/0501594
- Ivezić *et al.*, 2004, Astronomische Nachrichten 325, 583
- Jain B., Taylor A., 2003, Physical Review Letters, 91, 141302
- James, B., Davis, T., Schmidt, B. & Kim, A., 2006, MNRAS in press, astro-ph/0605147
- Jarvis M., Bernstein B. J. G., Dolney D., 2005, ApJ in press, astro-ph/0502243
- Jarvis M., Bernstein G., Jain B., Fischer P., Smith D., Tyson J., Wittman D., 2003, ApJ, 125, 1014
- Jarvis M., Jain B., 2004, ApJ submitted, astro-ph/0412234
- Jarvis, M., Jain, B., Bernstein, G., Dolney D. 2006, ApJ in press, astro-ph/0502243
- Kaiser N., 2000, ApJ, 537, 555
- Kaiser N., Squires G., Broadhurst T., 1995, ApJ, 449, 460
- Kaiser N., Wilson G., Luppino G., astro-ph/0003338
- King L., Schneider P., 2002, A&A, 396, 411
- King L., Schneider P., 2003, A&A, 398, 23
- Kuijken K., 1999, A&A, 352, 355
- Kuijken K., 2006, A&A submitted, astro-ph/0601011
- Lambas D. G., Maddox S. J., Loveday J., 1992, MNRAS, 258, 404
- Luppino G. A., Kaiser N., 1997, ApJ, 475, 20
- Mandelbaum R., Hirata C. M., Seljak U., Guzik J., Padmanabhan N., Blake C., Blanton M., Lupton R., Brinkmann J., 2005, MNRAS, 361, 1287
- Maoli R., Van Waerbeke L., Mellier Y., Schneider P., Jain B., Bernardeau F., Erben T., 2001, A&A, 368, 766
- Margoniner V. E., Lubin L. M., Wittman D. M., Squires G. K., 2005, AJ, 129, 20
- Massey R., Refregier A., Conselice C., Bacon J., 2004, MNRAS, 348, 214
- Massey R., Refregier A., Bacon D., 2004, in Meylan M. (ed.), *Impact of Gravitational Lensing on Cosmology*
- Massey R., Bacon D., Refregier A., Ellis R., 2005, MNRAS, 359, 1277
- Massey R., Refregier A., 2005, MNRAS, 363, 197
- Massey R., Rowe B., Refregier A., Bacon D. Bergé J., 2006, MNRAS submitted
- Miyazaki *et al.* 2002, PASJ, 54, 833
- Miyazaki *et al.* 2002, ApJL, 580, 97
- Pierpaoli E., Scott D. & White M., 2001, MNRAS, 325, 77
- Nakajima R. & Bernstein G., 2006, AJ submitted, astro-ph/0607062
- Refregier A., 2003, ARA&A, 41, 645
- Refregier A., 2003, MNRAS, 338, 35
- Refregier A., Bacon D., 2003, MNRAS, 338, 48
- Refregier A., Massey R., Rhodes J., Ellis R., Albert J., Bacon D., Bernstein G., McKay T., Perlmutter S., 2004, AJ, 127, 3102
- Refregier A., Rhodes J., Groth E. J., 2002, ApJL, 572, L131
- Rhodes J., Refregier A., Collins N. R., Gardner J. P., Groth E. J., Hill R. S., 2004, ApJ, 605, 29
- Rhodes J., Refregier A., Groth E. J., 2000, ApJ, 536, 79
- Rhodes J., Refregier A., Groth E. J., 2001, ApJL, 552, L85
- Schneider P., Kilbinger M., 2006, A&A submitted, astro-ph/0605084
- Schneider P., Van Waerbeke L., Mellier Y., 2002, A&A, 389, 741
- Schimd, C., Tereno, I., Uzan, J.-P., *et al.* 2006, A&A in press, astro-ph/0603158
- Schrabback, T. *et al.* 2006, A&A submitted, astro-ph/0606611
- Seitz C., Schneider P., 1997, A&A, 318, 687
- Semboloni E. *et al.*, 2006, A&A 452, 51
- Semboloni E., van Waerbeke L., Heymans C., Hamana T., Colombi S., White M. Mellier Y., 2006, MNRAS submitted, astro-ph/0606648
- Smith D. R., Bernstein G., Fischer P., Jarvis M., 2001, ApJ, 551, 643
- Smith G., Edge A., Eke V., Nichol R., Smail I. & Kneib J.-P., 2003, AJ, 590, L79
- Spergel D., *et al.*, 2006, ApJ submitted, astro-ph/0603449
- Sullivan M., *et al.*, 2006, ApJ in press, astro-ph/0605455
- Travaglio, C., Hillebrandt, W. & Reinecke, M., 2006, A&A in press, astro-ph/0507510
- Tyson J. A., Wenk R. A., Valdes F., 1990, ApJL, 349, L1
- Vale C., Hoekstra H., Van Waerbeke L. & White M., 2004, ApJL, 613, 1
- Van Waerbeke L., Mellier Y., 2003, Gravitational lensing: a unique tool for cosmology, astro-ph/0305089
- Van Waerbeke L., Mellier Y., Erben T., Cuillandre J. C., Bernardeau F., Maoli R., Bertin E., Mc Cracken H. J., Le Fèvre O., Fort B., Dantel-Fort M., Jain B., Schneider P., 2000, A&A, 358, 30
- Van Waerbeke L., Mellier Y., Hoekstra H., 2005, A&A, 429, 75
- Van Waerbeke L., Mellier Y., Radovich M., Bertin E., Dantel-Fort M., McCracken H., Fèvre O. L., Foucaud S., Cuillandre J., Erben T., Jain B., Schneider P., Bernardeau F., Fort B., 2001, A&A, 374, 757
- Van Waerbeke L., White M., Hoekstra H., Heymans C., 2006, Astropart. Phys. submitted
- Viana P., Nichol R. & Liddle A., 2002, ApJ, 569, 75
- White, M. & Kochanek, C., 2001, ApJ 560, 539
- Wittman D. 2002. *Dark Matter and Gravitational Lensing, LNP Top. Vol.*, eds. F. Courbin, D. Minniti. Springer-Verlag. astro-ph/0208063
- Wittman D., Tyson J., Kirkman D., Dell'Antonio I., Bernstein G., 2000, Nature, 405, 143
- Wittman D., 2005, ApJL, 632, 5
- Weinberg D., Davé R., Katz N. & Hernquist L., 2003, ApJ submitted, astro-ph/0212356

Author	Image set A	Image set B	Image set C	Image set D	Image set E	Image set F
JB	2.34±2.89	-1.46±2.46	5.08±3.06	7.31±3.08	3.44±3.02	1.92±3.14
	5.34±2.91	-7.78±2.05	-2.57±2.87	2.12±3.01	-6.85±3.77	-1.25±3.31
C1	-9.33±1.12	-6.30±1.12	-15.78±1.27	-17.01±1.05	-15.60±1.09	-9.18±1.24
	-7.44±1.07	-6.06±0.97	-11.69±1.19	-18.11±0.97	-18.90±1.35	-9.22±1.32
C2	-7.97±1.13	-4.13±1.14	-12.68±1.35	-7.39±1.19	-7.64±1.19	-5.50±1.28
	-6.05±1.12	-4.43±0.97	-8.31±1.24	-9.16±1.13	-11.99±1.49	-6.50±1.37
MH	3.73±1.30	7.76±1.46	6.26±1.55	2.51±1.37	0.82±1.41	1.78±1.56
	3.67±1.24	7.70±1.23	3.39±1.52	4.57±1.39	-2.88±1.75	0.86±1.63
HH	-0.05±0.81	-3.57±0.84	1.53±0.94	-1.94±0.82	-1.33±0.83	-0.30±0.90
	-1.88±0.79	-3.33±0.70	0.28±0.88	-2.50±0.81	-4.95±1.04	-1.89±0.94
MJ	-1.05±1.18	-1.07±1.01	-0.35±1.31	-0.98±1.20	-1.92±1.21	-2.81±1.30
	-4.07±1.10	-2.11±0.84	1.09±1.21	-0.75±1.16	-3.18±1.49	-3.54±1.33
MJ2	-0.74±0.97	-3.99±0.89	0.81±1.04	-0.73±0.94	-0.01±0.94	-1.77±0.96
	-3.04±0.90	-3.20±0.75	1.58±0.98	-1.49±0.92	-4.10±1.14	-1.14±1.01
KK	-1.06±1.05	-0.74±1.21	-6.28±1.26	-3.38±1.15	-3.04±1.13	-2.58±1.19
	-2.20±1.01	-1.96±1.01	-4.34±1.17	-2.38±1.10	-4.74±1.36	-4.51±1.28
RM	-1.88±0.97	-4.05±0.90	1.08±1.14	-1.13±1.04	-0.99±1.04	-0.39±1.14
	-3.58±0.94	-3.91±0.75	-0.65±1.11	-3.67±0.99	-6.17±1.26	-4.20±1.22
RN	-2.28±1.27	-0.79±1.16	-4.16±1.57	-3.52±1.33	-3.90±1.35	-6.20±1.46
	-4.85±1.21	-3.04±0.96	-6.55±1.48	-5.26±1.28	-7.68±1.66	-6.18±1.53
SP	-10.52±1.25	-7.52±1.40	-12.60±1.49	-12.67±1.55	-14.41±1.34	-12.20±1.44
	-3.96±1.25	-3.49±1.31	-6.89±1.55	-5.66±1.56	-9.62±1.87	-6.91±1.60
MS1	-15.19±1.15	-13.40±1.00	-22.79±1.30	-11.85±1.22	-15.45±1.25	-13.93±1.29
	-15.79±1.11	-12.76±0.85	-21.68±1.24	-11.92±1.19	-19.01±1.45	-14.87±1.56
MS2	-3.40±1.75	-8.09±1.30	-12.55±2.31	-0.70±2.08	-0.68±1.97	-1.99±2.10
	-2.94±1.75	-4.18±1.19	-6.55±2.21	5.13±2.07	-11.98±2.61	-1.70±2.40
TS	-1.43±1.47	2.82±1.57	0.26±1.87	-2.76±1.55	-3.69±1.58	-2.04±1.74
	-0.97±1.38	1.88±1.30	-2.54±1.67	-1.11±1.56	-7.81±1.98	-2.60±1.79
ES1	-15.51±1.27	-8.11±1.29	-19.03±1.34	-19.09±1.26	-17.31±1.26	-12.45±1.45
	-18.07±1.21	-8.02±1.06	-21.05±1.19	-19.65±1.17	-20.60±1.60	-16.80±1.51
ES2	13.66±3.28	11.68±3.34	-1.36±3.47	3.03±2.97	1.06±2.85	3.00±3.47
	4.61±3.10	14.64±2.70	-4.93±3.20	3.10±2.73	-3.82±3.61	-7.25±3.74

Table 1. Tabulated values of shear calibration bias ($\times 10^{-2}$) from figure 5. In each entry, the top line refers to the first component of shear, and the bottom line to the second.

Author	Image set A	Image set B	Image set C	Image set D	Image set E	Image set F
JB	-6.8±6.5	-17.2±5.4	-34.5±7.5	24.5±7.6	83.7±8.0	17.4±7.3
	1.3±6.6	-15.0±5.5	-1.0±7.5	-80.3±7.6	46.5±8.0	10.6±7.5
C1	21.2±2.5	26.7±2.5	-5.2±3.2	124.2±2.6	64.1±2.9	-11.8±2.9
	21.2±2.5	-5.4±2.6	23.2±3.1	-70.0±2.5	130.2±2.9	8.5±2.9
C2	-3.3±2.5	-1.1±2.5	-21.6±3.3	259.4±2.9	29.7±3.2	-6.2±2.9
	38.3±2.6	18.8±2.6	39.8±3.2	-36.6±2.9	276.6±3.2	3.6±3.0
MH	10.2±3.0	19.8±3.3	19.6±3.9	101.2±3.4	91.6±3.8	-4.4±3.6
	5.4±3.0	21.9±3.3	6.7±4.0	-84.2±3.6	99.3±3.8	6.3±3.6
HH	1.6±1.8	-4.8±1.8	-6.1±2.3	3.7±2.0	75.2±2.2	-2.2±2.1
	-4.6±1.8	-3.1±1.8	-0.6±2.2	-65.5±2.0	-5.9±2.1	9.8±2.1
MJ	-11.8±2.5	-9.5±2.2	-6.5±3.2	18.2±2.9	13.8±3.1	-2.2±2.8
	-0.9±2.6	6.0±2.1	1.6±3.1	-12.7±2.8	16.6±3.0	1.2±2.8
MJ2	-10.3±1.9	-4.8±1.7	0.4±2.3	23.9±1.9	15.5±2.2	-0.8±1.9
	1.5±1.9	3.1±1.7	2.3±2.2	-16.8±2.0	19.7±2.1	1.2±1.9
KK	-2.1±2.4	-5.2±2.7	-14.0±3.1	-71.6±2.8	66.6±3.0	0.1±2.8
	-2.7±2.4	-6.6±2.7	2.1±3.0	-69.5±2.7	-56.9±2.9	-3.9±2.8
RM	22.9±2.2	14.9±2.0	26.5±2.9	-33.5±2.5	112.0±2.8	0.1±2.6
	-9.9±2.2	-3.1±1.9	-5.8±2.8	-105.7±2.5	-19.4±2.7	2.4±2.7
RN	-5.3±2.8	-5.0±2.5	-6.3±3.8	-34.9±3.1	43.1±3.4	2.5±3.1
	1.8±2.7	-0.1±2.5	8.9±3.7	-33.1±3.2	-26.8±3.3	4.6±3.2
SP	-1.1±2.5	-3.4±2.9	-4.5±3.3	-69.9±3.3	71.6±3.4	5.5±3.0
	-1.1±2.7	-7.6±3.2	-4.6±3.8	-55.3±3.6	-13.3±3.5	4.1±3.3
MS1	-5.6±2.5	-22.3±2.1	4.9±3.2	105.1±3.0	58.5±3.3	-7.6±3.1
	10.3±2.6	23.1±2.1	7.2±3.2	-45.7±3.0	83.8±3.1	6.1±3.2
MS2	-7.9±3.9	-21.3±2.8	3.2±5.6	140.5±5.0	41.5±5.2	-0.4±4.9
	14.4±4.0	24.3±3.0	19.5±5.9	-28.7±5.1	154.4±5.4	9.2±5.3
TS	-2.9±3.3	-4.3±3.5	2.7±4.5	-46.2±3.9	70.3±4.3	-3.5±4.0
	-3.0±3.2	-1.3±3.6	0.4±4.4	-65.4±3.9	-40.3±4.2	-3.1±4.0
ES1	-9.1±2.8	-4.1±2.9	5.7±3.3	153.1±3.1	54.3±3.4	-5.5±3.4
	4.0±2.8	8.7±2.8	9.9±3.2	-58.7±3.0	132.0±3.4	0.4±3.3
ES2	-11.0±7.4	8.5±7.4	15.0±8.2	95.3±7.1	96.7±7.8	-10.4±8.2
	-11.2±7.4	-3.3±7.2	5.7±8.4	-92.9±7.1	77.9±7.7	7.7±8.1

Table 2. Tabulated values of residual shear offset ($\times 10^{-4}$) from figure 5. In each entry, the top line refers to the first component of shear, and the bottom line to the second.

Cosmic variance of weak lensing surveys in the non-Gaussian regime

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ABSTRACT

The results from weak gravitational lensing analyses are subject to a cosmic variance error term that has previously been estimated assuming Gaussian statistics. In this letter we address the issue of estimating cosmic variance errors for weak lensing surveys in the non-Gaussian regime.

Using standard cold dark matter model ray-tracing simulations characterized by $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.7$, $\sigma_8 = 1$. for different survey redshifts z_s , we determine the variance of the two-point shear correlation function measured across 64 independent lines of sight. We compare the measured variance to the variance expected from a random Gaussian field and derive a redshift-dependent non-Gaussian calibration relation.

We find that the ratio between the non-Gaussian and Gaussian variance at 1 arcminute can be as high as ~ 30 for a survey with source redshift $z_s \sim 0.5$ and ~ 10 for $z_s \sim 1$. The transition scale ϑ_c above which the ratio is consistent with unity, is found to be $\vartheta_c \sim 20$ arcmin for $z_s \sim 0.5$ and $\vartheta_c \sim 10$ arcmin for $z_s \sim 1$. We provide fitting formula to our results permitting the estimation of non-Gaussian cosmic variance errors and discuss the impact on current and future surveys.

A more extensive set of simulations will however be required to investigate the dependence of our results on cosmology, specifically on the amplitude of clustering.

Key words: cosmology: theory - gravitational lenses - large-scale structure

1 INTRODUCTION

Weak lensing by large scale structure, i.e. cosmic shear, offers a direct way of investigating the statistical properties of matter in the Universe, without making any assumptions on the relation between dark and luminous matter. Current surveys are large enough to provide high precision constraints on cosmology and the latest measurements performed with the Canada France Hawaii Telescope Legacy Survey (Hoekstra et al. 2006; Semboloni et al. 2006) is a step in that direction. Most of the cosmological constraints from weak lensing use two-point shear statistics (Réfrégier 2003; Van Waerbeke & Mellier 2003), and a crucial step in these cosmological parameter measurements is the estimate of error bars and systematics. Several papers address, statistically, the issue of systematics from E and B modes (Crittenden et al. 2001; Pen et al. 2002; Schneider & Kilbinger 2006), but only few papers address the estimation of cosmic variance of cosmic shear measurements (White & Hu 2000; Cooray & Hu 2001; Schneider et al. 2002). The lat-

ter assumes that the error on the two-point shear correlation function follows Gaussian statistics. However, we know that this is not the case at small scales where non-linear effects become important. Cooray & Hu 2001 use the dark matter halo model in Fourier space to study non-Gaussian covariance. A tentative calibration of this effect on the aperture mass statistic (Van Waerbeke et al. 2002) showed that departure from Gaussianity is expected to occur at angular scales $\lesssim 10$ arcminutes. The purpose of this *Letter* is to estimate the non-linear covariance of the two-point shear correlation function in real space, such that it can be of direct practical use for weak lensing studies, as in Schneider et al. 2002, without having to calculate high order correlation functions semi-analytically. Using ray-tracing simulations for a model close to the concordance cosmological model (Spergel et al. 2006) at different source redshift slices, we obtain a redshift dependent calibration formula of the Gaussian covariance derived in Schneider et al. 2002. This calibration takes the form of a matrix with which the Gaussian covariance is multiplied by, to obtain the non-Gaussian covariance. This letter is organised as follows. The Section 2 provides the notation relevant for this work, and the theoretical description of the

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Gaussian covariance. Section 3 describes the ray-tracing simulations and Section 4 shows our results. In Section 5 we show their impact on current and future contiguous weak lensing surveys. We conclude by discussing the limitation of our approach and the work that remains to be done in order to achieve percent level accuracy in the non-linear covariance estimate.

2 COSMIC SHEAR AND COVARIANCE

We follow the notation of Schneider et al. 1998. The power spectrum $P_\kappa(k)$ of the convergence κ is given by

$$P_\kappa(k) = \frac{9}{4} \Omega_0^2 \int_0^{w_H} \frac{dw}{a^2(w)} P_{3D} \left(\frac{k}{f_K(w)}; w \right) \times \left[\int_w^{w_H} dw' n(w') \frac{f_K(w' - w)}{f_K(w')} \right]^2, \quad (1)$$

where $f_K(w)$ is the comoving angular diameter distance out to a distance w (w_H is the horizon distance), and $n(w(z))$ is the redshift distribution of the sources. $P_{3D}(k)$ is the 3-dimension non-linear mass power spectrum (Peacock & Dodds 1996; Smith et al. 2003), and k is the 2-dimension wave vector perpendicular to the line-of-sight. We are interested in the non-Gaussian covariance of the two-point shear correlation function, because it can be easily transposed to other two-point statistics (Schneider et al. 2002) by a suitable integration in k -space. The shear correlation function measured at angular scale ϑ can be split into two components, ξ_\pm , where

$$\xi_\pm(\vartheta) = \frac{1}{2\pi} \int_0^\infty dk k P_\kappa(k) J_{0,4}(k \vartheta), \quad (2)$$

and $J_{0,4}$ is a Bessel function of the first kind, of zeroth order for ξ_+ and of fourth order for ξ_- . The covariance matrix $\text{Cov}(\xi_\pm; \vartheta_1, \vartheta_2)$ of the total shear correlation function ξ_\pm can be written as a sum of three different parts:

$$\text{Cov}(\xi_\pm; \vartheta_1, \vartheta_2) = \langle \xi_\pm(\vartheta_1) \xi_\pm(\vartheta_2) \rangle = D \delta_K(\vartheta_1 - \vartheta_2) + q_{++} + < 4^{th} \text{ order correlations} > \quad (3)$$

The first term is the diagonal statistical noise, depending on the intrinsic ellipticity variance, σ_e , the total area of the survey, A , and the density of galaxies, n . In practical units gives:

$$D = 3.979 \times 10^{-9} \left(\frac{\sigma_e}{0.3} \right)^4 \left(\frac{A}{1 \text{ deg}^2} \right)^{-1} \times \left(\frac{n}{30 \text{ arcmin}^{-2}} \right)^{-2} \left(\frac{\theta}{1 \text{ arcmin}} \right) \left(\frac{\Delta\theta/\theta}{0.1} \right)^{-1} \quad (4)$$

where $\Delta\theta$ is the bin size used for the sampling of the correlation function. The second term represents the coupling between the noise and two point shear correlation function:

$$q_{++} = \frac{2\sigma_e^2}{\pi A n} \int_0^\pi d\varphi \xi_+ \left(\sqrt{\vartheta_1^2 + \vartheta_2^2 - 2\vartheta_1\vartheta_2 \cos \varphi} \right) \quad (5)$$

and it can easily be calculated using a prediction for non-linear shear power spectrum (Peacock & Dodds 1996; Smith et al. 2003). The third term requires the knowledge of the fourth order shear correlation function as a function of scale. If we assume Gaussian statistics, it can be expressed as a sum of two terms (Schneider et al. 2002):

$$r_{+0} = \frac{2}{\pi A} \int_0^\infty d\phi \phi \int_0^\pi d\varphi_1 \xi_+(\vartheta_a) \int_0^\pi d\varphi_2 \xi_+(\vartheta_b),$$

$$r_{+1} = \frac{1}{(2\pi)A} \int_0^\infty d\phi \phi \int_0^{2\pi} d\varphi_1 \xi_-(\vartheta_a) \times \int_0^{2\pi} d\varphi_2 \xi_-(\vartheta_b) [\cos 4\varphi_a \cos 4\varphi_b + \sin 4\varphi_a \sin 4\varphi_b], \quad (6)$$

and φ_a, φ_b are the polar angles of ϑ_a, ϑ_b , respectively, $\cos 4\varphi_a = 1 - 8\psi_{a1}^2\psi_{a2}^2/|\vartheta_a|^4$, $\sin 4\varphi_a = 4\psi_{a1}\psi_{a2}(\psi_{a1}^2 - \psi_{a2}^2)/|\vartheta_a|^4$, and the analogous expressions for φ_b .

In this paper we are interested in the last term of eq. (3). At large scales we know that we can use the Gaussian approximation and write it as the sum of r_{+0} and r_{+1} . At small scales the Gaussian statistics break down and this term cannot be calculated with semi-analytical techniques. The rest of the paper discusses our technique to calibrate the Gaussian prediction of this quantity in order to fit the non-Gaussian value measured in ray-tracing simulations. Therefore using ray-tracing simulations, we will measure the covariance of ξ_\pm , $\text{Cov}_{\text{measured}}(\xi_\pm; \vartheta_1, \vartheta_2)$, assuming $\sigma_e = 0$, so $q_{++} = 0$ and $D = 0$ and we will define $\mathcal{F}(\vartheta_1, \vartheta_2)$, the ratio between the measured covariance matrix and Gaussian expectation for the covariance matrix:

$$\mathcal{F}(\vartheta_1, \vartheta_2) = \frac{\text{Cov}_{\text{measured}}(\xi_\pm; \vartheta_1, \vartheta_2)}{\text{Cov}_{\text{Gaussian}}(\xi_\pm; \vartheta_1, \vartheta_2)} \quad (7)$$

where $\text{Cov}_{\text{Gaussian}}(\xi_\pm; \vartheta_1, \vartheta_2) = r_{+0} + r_{+1}$.

3 DESCRIPTION OF THE SIMULATIONS

We performed 16 particle in mesh (PM) dark matter simulations to cover a light cone of angular size 7×7 degrees, from redshift $z = 0$ to $z \simeq 3$, using the tiling technique proposed by White & Hu (2000) and explained in Appendix B of Hamana et al. (2002). We used 7 simulations of size 200 Mpc, 4 of size 400 Mpc, 3 of size 600 Mpc and 2 of size 800 Mpc. Each N -body experiment involved 256^3 particles in a grid of size 1024^3 to compute the forces. The cosmology is a standard Λ CDM model with $\Omega = 0.3$, $\Omega_{\text{baryons}} = 0.04$, $\Lambda = 0.7$ and $H_0 = 70$ km/s/Mpc, closed to the concordance model (Spergel et al. 2006), with a slightly higher value for the normalisation $\sigma_8 = 1$. Combining the simulation data in different ways, we generated 64 different, albeit not fully independent (see below), light cones. Each of them is divided in 64 successive redshift planes separated from each other by 100 Mpc. The ray-tracing method is described in Hamana et al. (2002). The spatial resolution of our simulations translates in an angular resolution of the order of $\theta \simeq 0.5$ arcmin for $z \gtrsim 0.2$. Given the limitations of the PM technique, discreteness effects can be significant at redshift $z \gtrsim 1.5$ (due to transients). Nevertheless, our measurements are reliable at scales larger than the mean interparticle distance, i.e. $\theta \sim 2$ arcmin. and we expect they can still be used with high confidence level down to $\theta \simeq 1$ arcmin.

The size, S , of our light cones matches closely that of the simulations, so using the dispersion among them to compute the covariance matrix would certainly underestimate its amplitude, even at small angular scales. Fluctuations at scales larger than the simulation box size are also missed with these realisations. Furthermore, they are not strictly independent, since they just combine in different ways the 20 simulations. For these two reasons, in the case of $A = S$ the value of $\mathcal{F}(\vartheta_1, \vartheta_2)$ on small scales would be always underestimated, as compared to the cases $A \neq S$, and would not converge to unity at large scales. In order to minimise these limitations and still have a fair estimate of the covariance matrix on the estimator used here, it is thus wise to always keep the angular size

of the survey A to a small fraction of S . In practice, we divide S in 4, 9 and 16 adjacent subsamples, leading to assumed values of $A \simeq 12, 5.4$ and 3.1 square degrees and 256, 576, 1024 realisations respectively, in total. Note that the choice of A is made such that the largest angular scale considered, $\theta = 20$ arcmin, remains small compared to \sqrt{A} . We finally choose $A = 5.44 \text{ deg}^2$.

4 DESCRIPTION OF THE MATRIX CALIBRATION

We measure $\mathcal{F}(\vartheta_1, \vartheta_2)$ according to eq. (7) as follows. The term $\text{Cov}_{\text{measured}}(\xi_+; \vartheta_1, \vartheta_2)$ is given by $\langle (\xi_+ - \langle \xi_+ \rangle)^2 \rangle$, where ξ_+ is measured in each realisation of the survey of size $A = 5.44 \text{ deg}^2$, while the average $\langle \dots \rangle$ is performed over all the realisations. The term $\text{Cov}_{\text{Gaussian}}(\xi_+; \vartheta_1, \vartheta_2)$ is calculated by measuring ξ_+ and ξ_- in the 64 largest samples S of area $A = 49 \text{ deg}^2$, and integrating numerically eqs. (6). This ensures that the numerator and denominator in eq. (7) are self-consistently defined. It is worth noticing that for all cases with $A < S$ the asymptotic behavior of $\mathcal{F}(\vartheta_1, \vartheta_2)$ does not converge to unity. It indeed seems to be even worse than for the case $A = S$. This is a well known effect that results from the measurement of the correlation functions on a limited support that always bias the signal downwards when the scales become comparable with the size of the survey (Peebles 1974). Therefore, at small scales, the measured cosmic variance $\text{Cov}_{\text{measured}}(\xi_+; \vartheta_1, \vartheta_2)$, when $A < S$, is more biased low and decreases faster when the scale increases than for the case $\text{Cov}_{\text{Gaussian}}(\xi_+; \vartheta_1, \vartheta_2)$ and $A = S$. The final result is that the ratio $\mathcal{F}(\vartheta_1, \vartheta_2)$ becomes smaller than unity. Note that in practice, for numerical reasons we have to use $A = S$ to compute $\text{Cov}_{\text{Gaussian}}(\xi_+; \vartheta_1, \vartheta_2)$ using 6. We do not expect this has any impact on our results, within the level of accuracy we can achieve from this set of simulations, provided we rescale the covariance matrix only in the inner part. The left panel of Fig. 1 shows the diagonal elements $\mathcal{F}(\vartheta_1, \vartheta_2)$ for different source redshifts. For a source redshift $z_s \simeq 1$ the calibration factor is ~ 10 at $\vartheta_1 = \vartheta_2 = 1$ arcmin, implying that the cosmic variance has been largely underestimated in previous lensing surveys at scales below ~ 10 arcminutes. The correction factor is larger for lower source redshifts. The transition scale ϑ_c , which defines the angular scale transition from Gaussian and non-Gaussian covariance, is redshift dependent because the non-linear regime starts at larger scales for nearby structures. Therefore, the calibration matrix must be parameterized with an explicit redshift dependence. We choose a generic power law behavior, as suggested by the left panel of Fig. 1, to parameterize $\mathcal{F}(\vartheta_1, \vartheta_2)$:

$$\mathcal{F}(\vartheta_1, \vartheta_2) = \frac{\alpha(z)}{[\vartheta_1 \vartheta_2]^{\beta(z)}}. \quad (8)$$

The two panels in Fig. 2 show α and β as measured in the ray-tracing simulations at nine different source redshifts $z_s = [0.4, 0.5, 0.8, 1.0, 1.2, 1.4, 1.5, 2.0, 3.0]$. These measurements are well fit by the following redshift dependent functions:

$$\begin{aligned} \alpha(z) &= \frac{a_1}{z^{a_2}} + a_3 \\ \beta(z) &= b_1 z^{b_2} \exp(-z^{b_3}) + b_4. \end{aligned} \quad (9)$$

For α , we find $(a_1, a_2, a_3) = (16.90, 0.95, -2.19)$, and for β , $(b_1, b_2, b_3, b_4) = (1.62, -0.68, -0.68, -0.03)$ in the samples with angular size $A = 5.4 \text{ deg}^2$. The fit is performed on scales below 10 arcminutes, which allows us to define also the transition angle ϑ_c as the scale where the fitted function crosses the Gaussian

covariance. The third panel of Fig. 2 represents the measurement of ϑ_c . Using the same functional form as for α , namely $\vartheta_c = \frac{t_1}{z^{t_2}} + t_3$. We find the best fit values $(t_1, t_2, t_3) = (8.07, 0.95, 1.65)$.

Since the normalisation of our simulations is high ($\sigma_8 = 1$), we expect ϑ_c to be slightly overestimated. Several other sources of uncertainty in our measurements may also spoil the estimate of the covariance. In particular, as previously anticipated, there is also a "cosmic error" and a "cosmic bias" that affect our measurements (e.g. Szapudi & Colombi 1996), which are difficult to estimate. Fortunately, such a cosmic bias/error is expected to increase with the survey size A . According to eq. (6), the covariance scales as $\propto 1/A$, so \mathcal{F} should in fact be independent of A , which allows one to use our parametrisation of \mathcal{F} for any (reasonable) angular survey size. This property can also be used to check the convergence between our realisations of various survey sizes as illustrated by right panel of Fig. 1. Surveys with areas $A = 3.1, 5.4$ and 12 square degrees agree with each other, but there is a problem with $A = 49 \text{ deg}^2$, where \mathcal{F} is biased low. In the latter case, this is not surprising since the light cone size is comparable to the simulations size, as discussed in § 3. The convergence between other values of A suggest that the cosmic bias/error on \mathcal{F} measured in these samples is small, i.e. the full set S from which they are extracted, is a fair enough sample. We check this by dividing our $A = 12 \text{ deg}^2$ set of 256 realisations into 4 subsamples of 64 realisations, and measured \mathcal{F} in each of the subsamples. The dispersion between these 4 subsamples is of the order of 10% - 20%, which gives a rough idea of the accuracy of our estimate of $\mathcal{F}(\vartheta, \vartheta)$, in agreement as well with the convergence between the measurements observed on right panel of Fig. 1 for $A \leq 12 \text{ deg}^2$.

While our choice of parametrisation eq. (8) is globally accurate to $\sim 20\%$ along the diagonal of the matrix $\mathcal{F}(\vartheta_1, \vartheta_2)$, it becomes less accurate for very different ϑ_1 and ϑ_2 . One should note that the lack of accuracy in the off diagonal components is not critical because the cross-correlation coefficient is $\lesssim 0.1$ in this region.

5 IMPACT OF NON-GAUSSIANITY ON CURRENT AND FUTURE SURVEYS

Finally, we compare the amplitude of statistical and cosmic variance at small scales for a range of contiguous surveys such as GEMS (Heymans et al. 2005), COSMOS (Massey et al. in prep.), CFHTLS Wide (Hoekstra et al. 2006) and two different versions of SNAP (Réfrégier et al. 2004) whose characteristics are shown in table 1. The statistical noise is computed using eq. (4), assuming a bin size $\Delta\theta = 0.1$. Note that the statistical noise differs if the bin size used to measure the correlation function is different. In addition we choose $\sigma_e = 0.4$ for ground-based surveys and $\sigma_e = 0.3$ for space-based surveys. Fig. 3 shows that by dropping the Gaussian approximation the total noise changes at small scales. The changing due to the non-Gaussian correction depends on the relative amplitude of the three different contributions to the total variance, namely, the shot noise, the sampling noise and the coupling term. For "low density" surveys, such as the CFHTLS Wide, the impact of the non-Gaussian correction is smaller as compared to the one expected for the low noise space based surveys, where the cosmic variance far exceeds the statistical noise. It is worth noticing our results are obtained for a higher σ_8 value than Spergel et al. 2006 ($\sigma_8 \simeq 0.75$) and are likely to be slightly different for this model. A more extensive analysis of simulations made with different cosmologies would be necessary to accurately predict the amplitudes of the non-Gaussianity corrections to the cosmic variance.

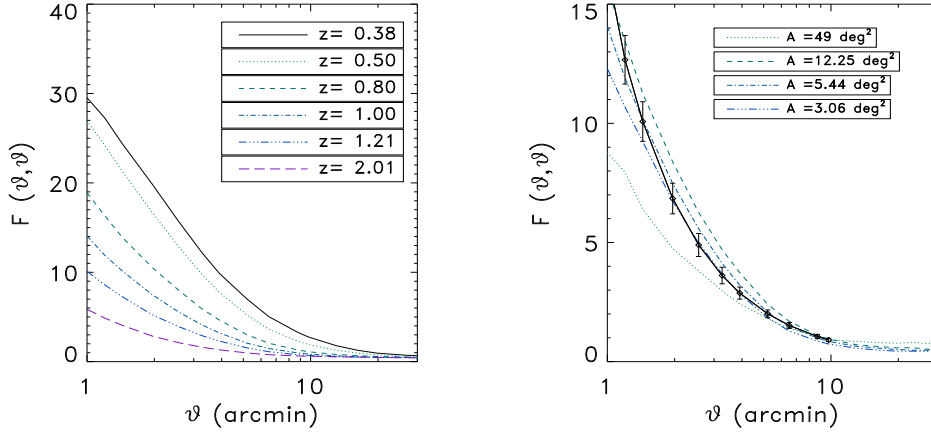


Figure 1. Left panel: Diagonal elements of the matrix $\mathcal{F}(\vartheta_1, \vartheta_2)$ for different source redshift planes. Right panel: Diagonal elements $\mathcal{F}(\vartheta_1, \vartheta_2)$ for different survey sizes and $z_s = 1$. The black solid line represents the best-fit of $\mathcal{F}(\vartheta_1, \vartheta_2)$ using eq. (8). Error bars are computed using bootstrap with 1000 realisations.

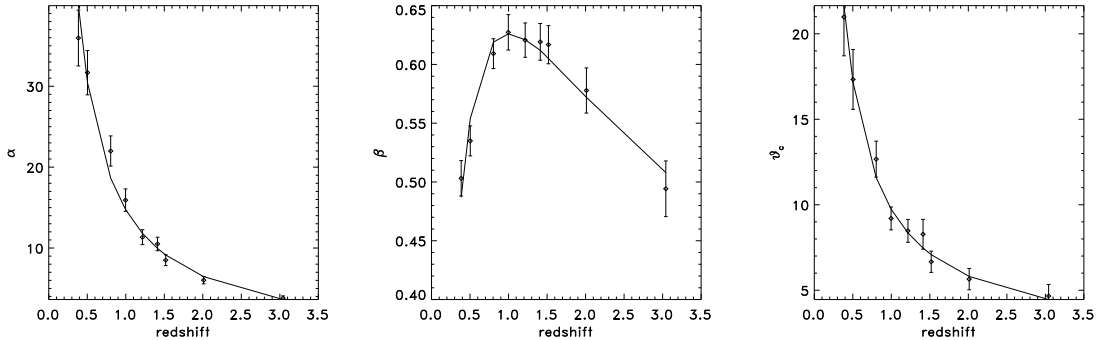


Figure 2. The plots show the measured parameters α , β and ϑ_c of the calibration matrix (see eq. 8) as a function of the redshift. Error bars are computed using bootstrap with 1000 realisations. The solid line shows the best fit from eq. (9).

Table 1. Main Characteristics of surveys used in Fig. 3.

Name	A (deg ²)	n	$\langle z_s \rangle$
GEMS	0.25	65	1
COSMOS	1.6	80	1.2
CFHTLS Wide	50	15	0.8
SNAP deep	15	300	1.4
SNAP wide	260	120	1.2

Unfortunately, only a small set of Λ CDM ray tracing simulations with $\sigma_8 = 0.8$ is available. This set of simulations, whose characteristics are given in Heymans et al. 2006, is composed of two redshift planes each containing 12 simulations of 25 deg^2 which is not enough to find a recalibration fitting formula. Nevertheless, Fig.4 shows that even for a Λ CDM model with $\sigma_8 = 0.8$ the cosmic variance has been widely underestimated. Fig.4 also shows that using a rescaling obtained from $\sigma_8 = 1.0$ gives results which are

in good agreement with the ones obtained for $\sigma_8 = 0.8$ for low redshift surveys and slightly overestimates the cosmic variance as the depth increases. These simulations were also used to confirm the validity of our statements regarding the behavior of the ratio $\mathcal{F}(\vartheta_2, \vartheta_2)$ and the change of the size of \mathcal{A} used for the recalibration.

6 DISCUSSION AND CONCLUSION

We have shown that the non-Gaussian contribution to the covariance in two-point shear statistics cannot be neglected at small angular scales. Using ray-tracing simulations we have calibrated the non-Gaussian covariance with respect to the Gaussian covariance as calculated in Schneider et al. 2002. We have derived a calibration matrix which can be used as a first approximation for cosmological parameter measurements in current lensing surveys and for parameter forecasting.

We found that the correction coefficient could be as high 10

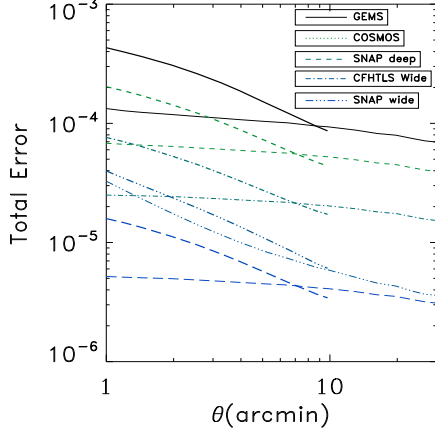


Figure 3. The total, statistical plus cosmic variance noise for each survey of table 1. The noise including the non-Gaussian correction (thick lines) is compared on scales $\vartheta \leq 10$ arcmin, with the noise expected in the case of Gaussian statistics (thin lines). Dropping the Gaussian assumption increases noise on small scales. The impact of the non-Gaussian correction for the CFHTLS Wide is small; the statistical noise $\propto 1/n^2$ and the coupling term $q_{+++} \propto 1/n$ still dominates at small scales for such a density. These same terms become negligible for the space based surveys whose density is much higher.

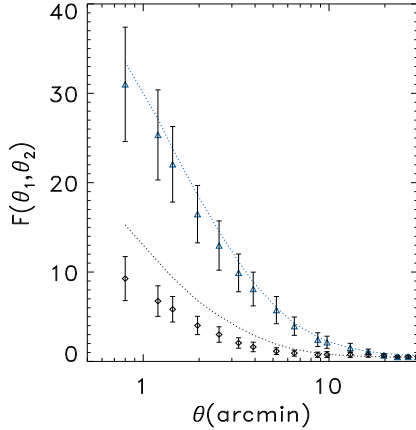


Figure 4. Diagonal elements of the matrix $\mathcal{F}(\vartheta_1, \vartheta_2)$ for plans with $z_s \simeq 0.5$ (blue triangles) and $z_s \simeq 1$ (black diamonds) obtained for the set of simulations Λ CDM with $\sigma_8 = 0.8$. Error bars are obtained using bootstrap with 1000 realisations. For each of the two measurements we compare the obtained value with the predicted value calculated from Λ CDM simulations with $\sigma_8 = 1.$, marked with black and blue dotted lines.

at 1 arcminute for a source redshift of 1, and 30 for source redshift of $z_s = 0.5$. The transition between Gaussian and non-Gaussian covariance occurs around 10 arcminutes for $z_s = 1$ and 20 arcminutes for $z_s = 0.5$. Our work shows that it is important to include this non-Gaussian contribution to the shear estimated errors, and that sub-arcminute resolution ray-tracing simulations are very useful for this purpose. Although this source of error has been neglected in previous lensing analysis, we note that it should not strongly impact the measurement of σ_8 for surveys using the shear signal measured above the transition scale ϑ_c , where the Gaussian covariance is a reasonable assumption. However, it will significantly affect the joined Ω_M - σ_8 constraints, since the degeneracy breaking between these two parameters is based on the relative amplitude of the shear correlation signal between small and large scales (Jain & Seljak 1997). An increased error at small scale, as shown here, will make the degeneracy more difficult to break.

Extension of this work via a thorough analysis of the non-Gaussian covariance based on numerical simulations include shear error calibration with broad redshift distribution (tomography), different two-points statistics and the dependence of the non-Gaussian correction with a varying cosmology. In particular we expect a non-trivial dependence of the calibration matrix with σ_8 , since, for a fixed angular scale, non-linear structures form earlier for higher σ_8 .

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REFERENCES

- Cooray, A., Hu, W., 2001, ApJ, 554, 66
- Crittenden, R. et al., 2001, ApJ, 568, 20
- Hamana, T. et al. 2002, MNRAS, 330, 365
- Heymans, C. et al., 2005, MNRAS, 361, 160
- Heymans, C. et al., 2006, MNRAS, 371, 750
- Hoekstra, H. et al., 2006, ApJ, 647, 166
- Jain, B., Seljak, U., 1997, ApJ, 484, 560
- Peacock, J.A., Dodds, S.J., 1996, MNRAS, 280, L9
- Peebles, P.J.E., 1974, A&A, 32, 197
- Pen, U.L. et al., 2002, ApJ, 567, 31
- Réfrégier, A., 2003, ARAA, 41, 645
- Réfrégier, A. et al., 2004, AJ, 127, 3102
- Schneider, P., Kilbinger, M., 2006, astro-ph/0605084
- Schneider, P. et al., 1998, MNRAS, 296, 873
- Schneider, P. et al., 2002, A&A, 396, 1
- Semboloni, E. et al., 2006, A&A, 452, 51
- Smith R. E. et al., 2003, MNRAS, 341, 1311
- Spergel D. N., et al. 2006, astro-ph/0603449
- Szapudi, I., Colombi, S., 1996, ApJ, 470, 131
- Van Waerbeke, L., Mellier, Y., 2003, astro-ph/0305089
- Van Waerbeke, L. et al., 2002, A&A, 393, 369
- White, M., Hu, W., 2000, ApJ, 537, 1