

Ph.D. defense

Probabilistic models of impedance matrices

Application to dynamic soil-structure interaction

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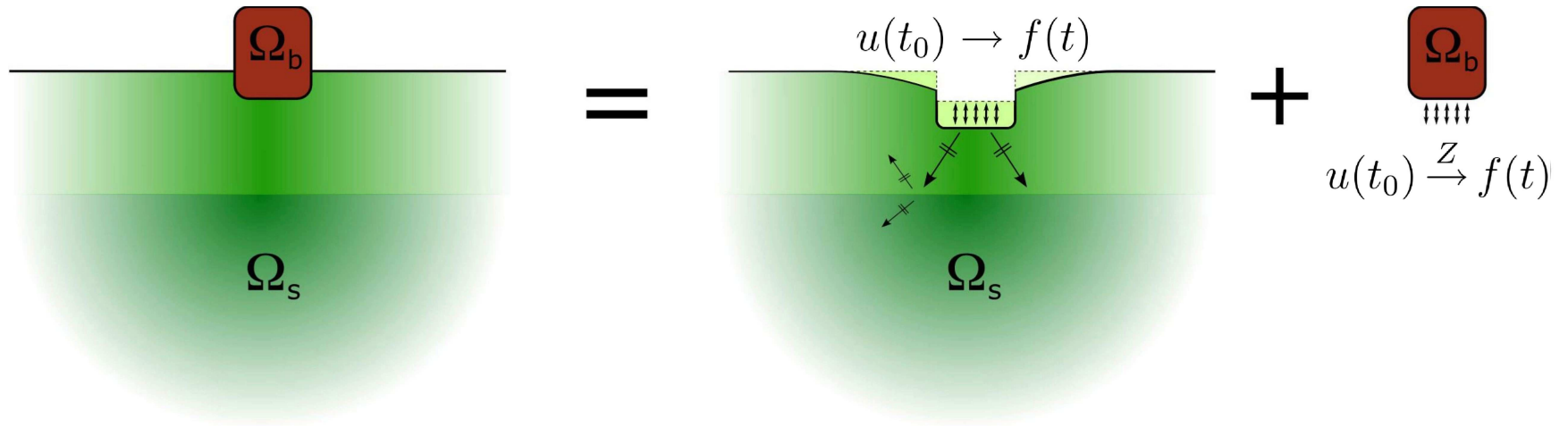
under the supervision of Prof. D. Clouteau¹ and Prof. C. Soize²

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Impedance in the time domain

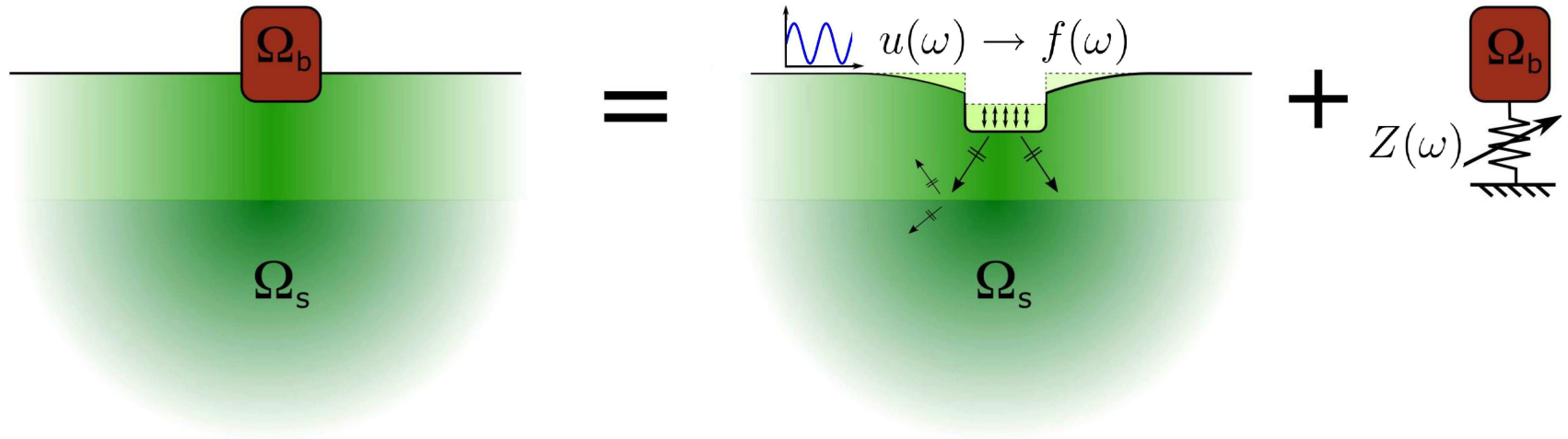
- Introduction
- Det. model
- Prob. model
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- Force $f(t)$ to impose a displacement $u(t_0)$ on the boundary
- Causality $\Rightarrow Z(t < t_0, t_0) = 0$

Impedance in the frequency domain

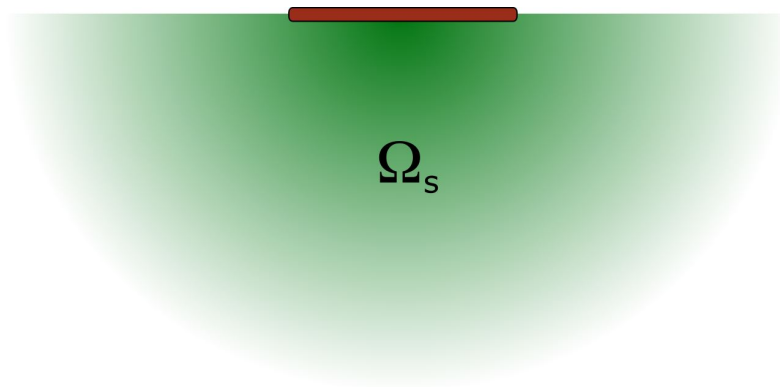
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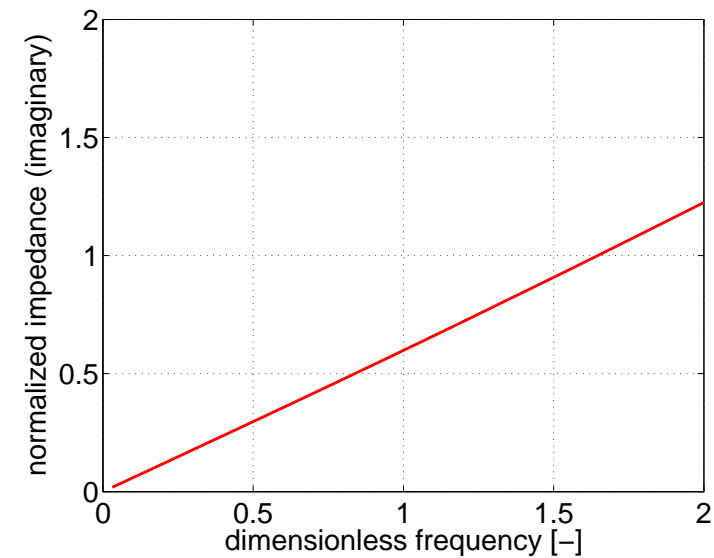
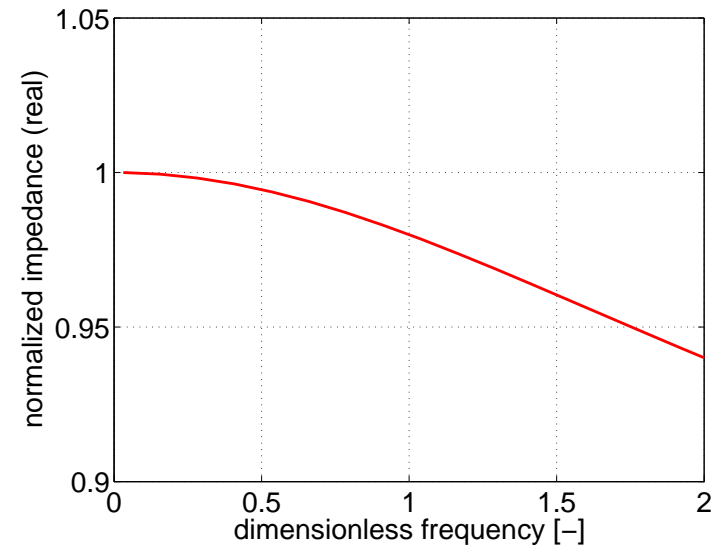
- Linearity
- $\hat{f}(\omega) = \hat{Z}(\omega)\hat{u}(\omega)$
- Complex, frequency-dependent spring.

Rigid surface foundation on homogeneous soil

- Introduction
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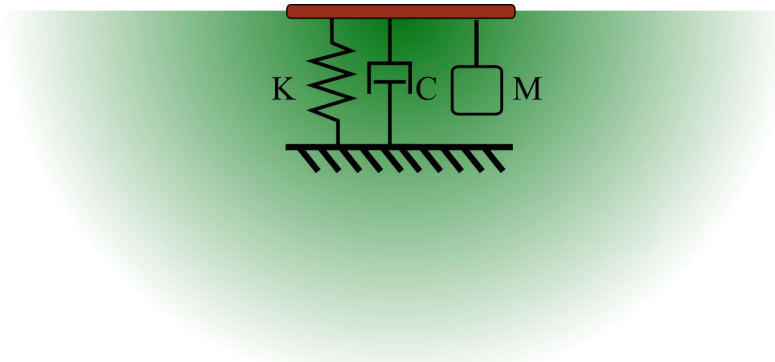


- Additional flexibility
- Damping due to radiation

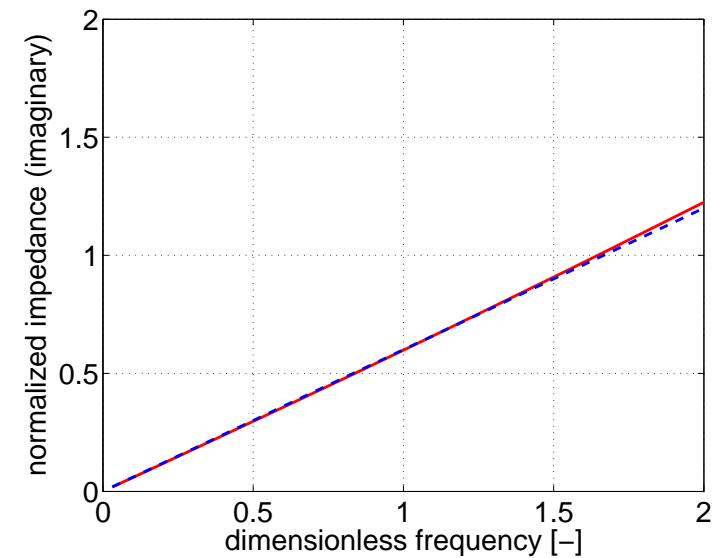
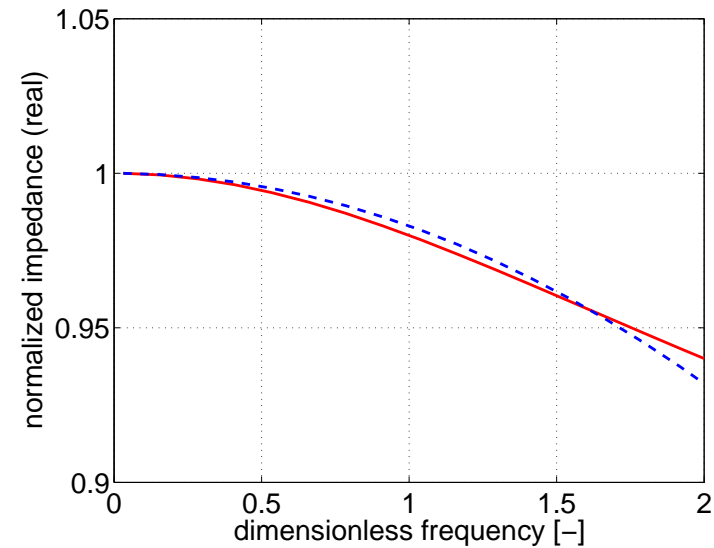


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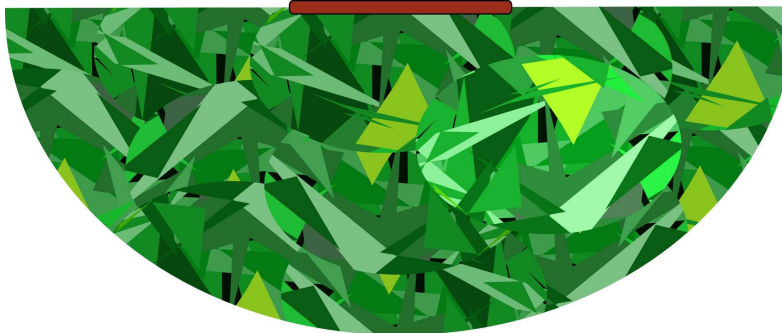


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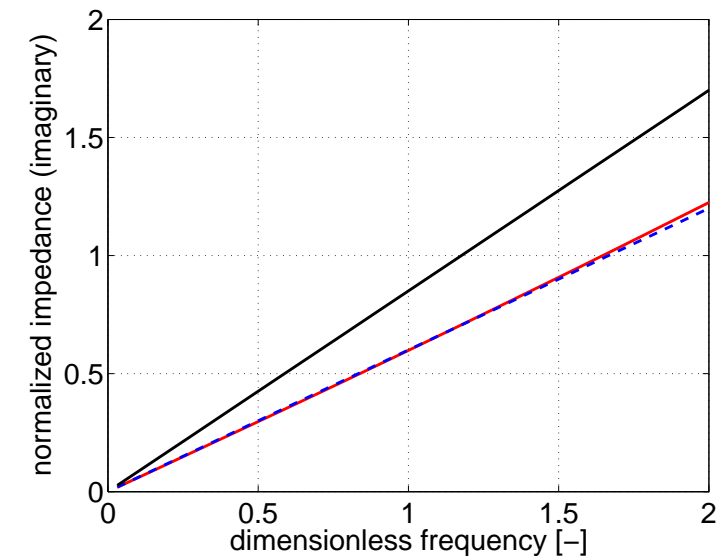
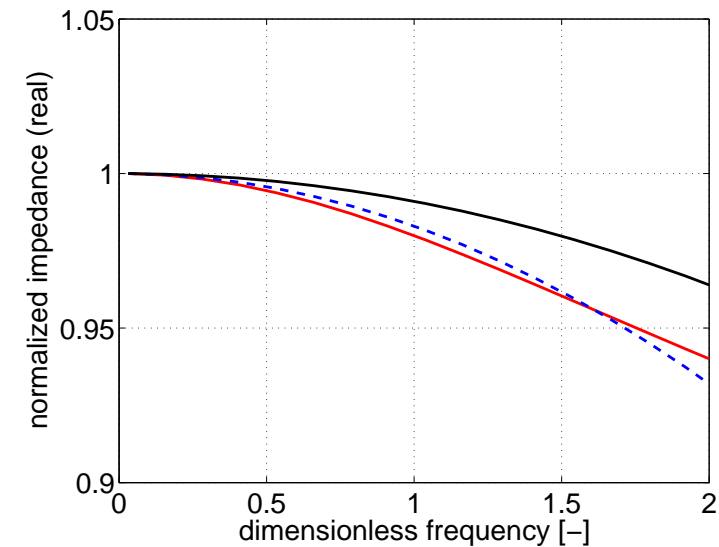
Rigid surface foundation on uncertain soil

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- Det. model
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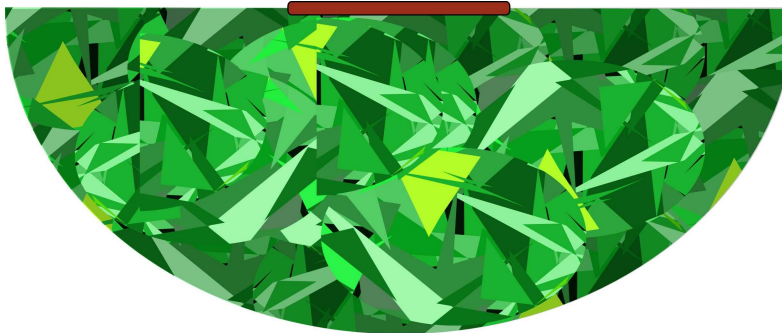
Probabilistic analysis

- Consider all possible situations (?)
- Assign probability weights (?)



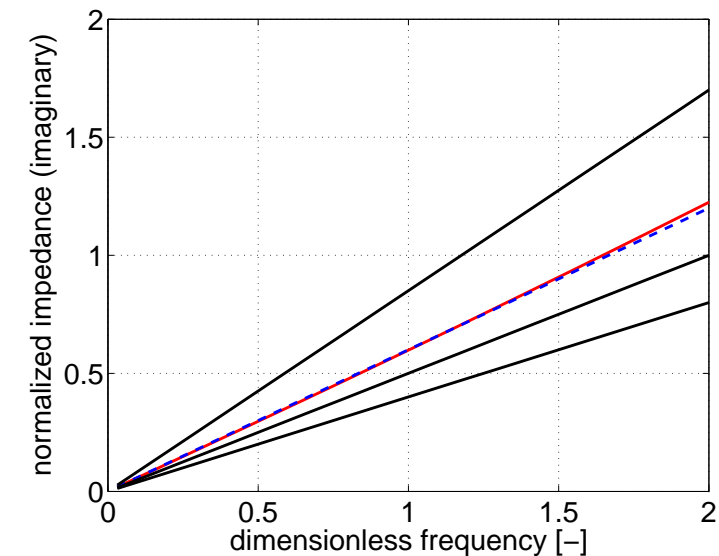
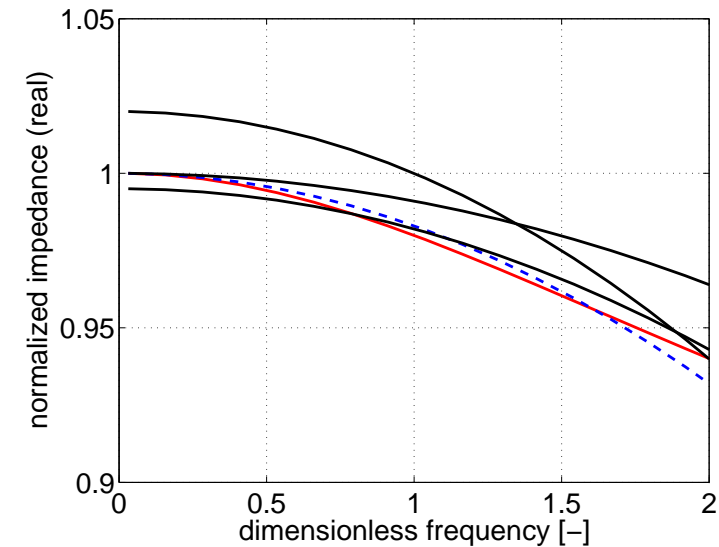
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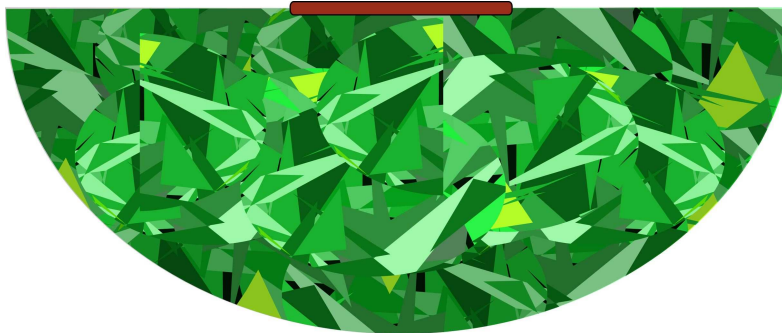
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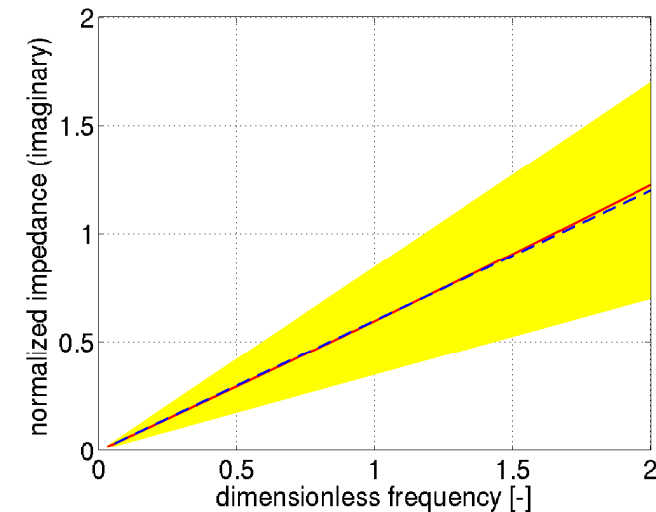
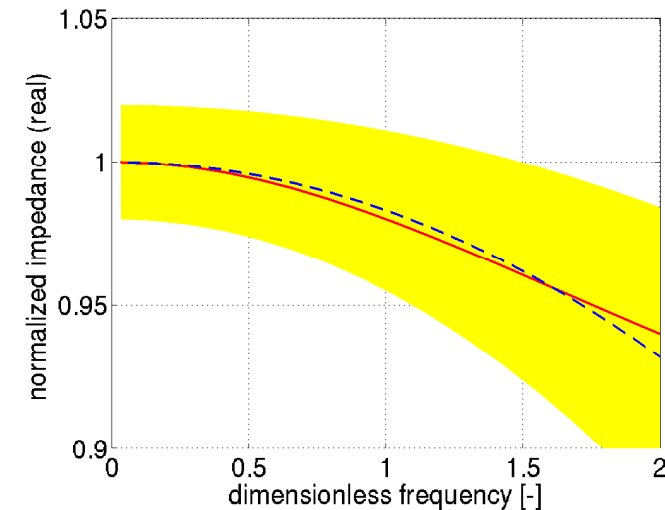
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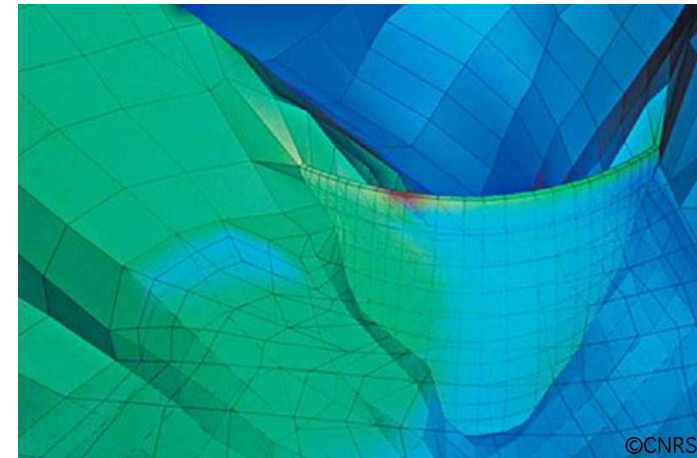
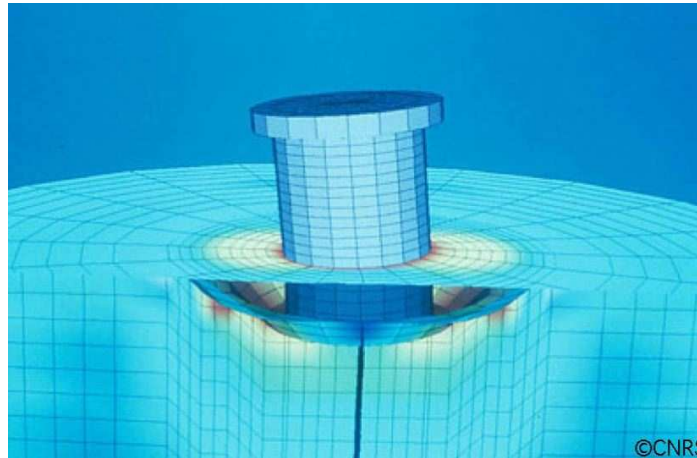


Probabilistic analysis

- Consider all possible situations (?)
- Assign probability weights (?)



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Goal: construct models for the impedance matrix
 → Always causal
 → Accounting for variability

- Challenges:
 - ◆ Unbounded domain
 - ◆ Frequency-dependence

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- **Introduction**
- **Part I: Deterministic model of the impedance matrix $[Z(\omega)]$**
 - ◆ Definition of the impedance matrix
 - ◆ The Hidden Variables (HV) model
 - ◆ Identification of the HV model
- **Part II: Probabilistic model of the impedance matrix $[Z(\omega)]$**
 - ◆ Parametric approach
 - ◆ Nonparametric approach
 - ◆ Comparison
- **Part III: Seismic design of a structure on a random soil**
- **Conclusions and Perspectives**

[Cottureau *et al.*, CMAME, 2006, *accepted*]: Parts I-II

[Cottureau *et al.*, EESD, *in preparation*]: Parts III

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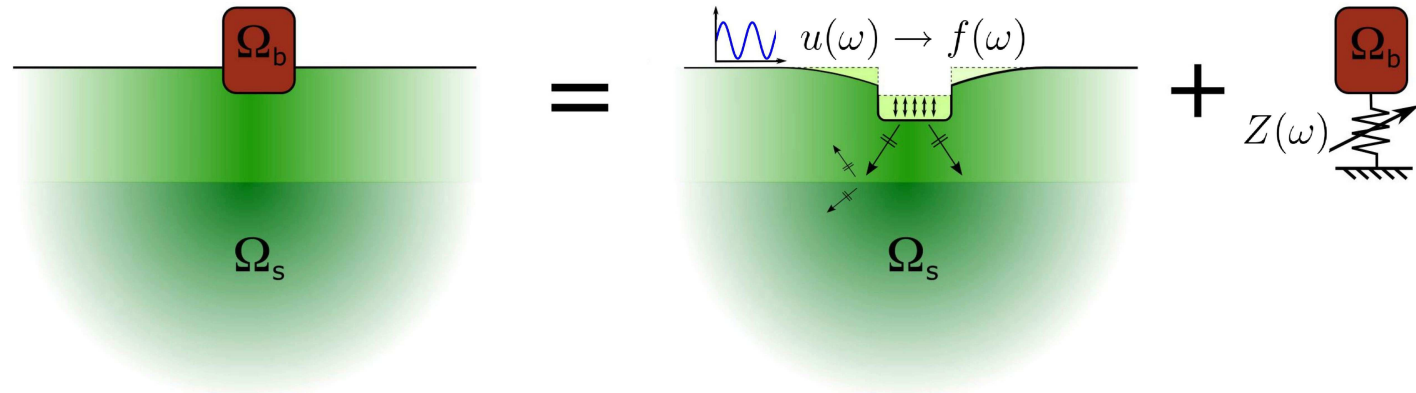
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Simple and general model taking into account causality

- **Definition of the impedance matrix**
 - ◆ Definition
 - ◆ Numerical methods
 - ◆ Properties of the impedance matrix
- **The Hidden Variables (HV) model**
 - ◆ Causality condition
 - ◆ The HV model
- **Identification of the HV model**
 - ◆ Methodology
 - ◆ Interpolation
 - ◆ Identification

Definition of the impedance matrix

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- Harmonic (linear) Boundary Value Problem: find $\mathbf{u}_s \in V(\Omega_s)$ such that:

$$\begin{cases} \underline{\sigma}_{ij,j}(\mathbf{u}_s) + \rho\omega^2 \mathbf{u}_s = \mathbf{0} & \text{in } \Omega_s \\ \underline{\sigma}_{ij}(\mathbf{u}_s)n_j = \mathbf{0} & \text{on } \partial\Omega_s \setminus \Gamma \\ \mathbf{u}_s = \mathbf{u}_\Gamma & \text{on } \Gamma \end{cases} \quad (1)$$

- There exists, $\forall \omega \in \mathbb{R}$, a unique displacement field \mathbf{u}_s solution of (1)
- \mathbf{f}_Γ the corresponding traction field on Γ
- The impedance is defined, $\forall \omega \in \mathbb{R}$, by

$$\mathcal{Z}(\omega)\mathbf{u}_\Gamma(\omega) = \mathbf{f}_\Gamma(\omega) \Rightarrow [\mathcal{Z}(\omega)][u(\omega)] = [f(\omega)].$$

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- **Finite Element method**
 - ◆ Complex
 - ◆ \Rightarrow Absorbing boundary conditions ? [[Givoli, 1992](#)]
- **Boundary Element method**
 - ◆ Complex
 - ◆ \Rightarrow Green's functions ?
- **Lumped-parameter models** [[Wolf, 1988](#)]
 - ◆ Simple
 - ◆ \Rightarrow Limited applicability: network chosen *a priori* ? coupling ?
- **Charts, Winkler spring foundation, Cone models, ...**

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- **Symmetry**
 - ◆ symmetry of elasticity tensors
 - ◆ $[Z(\omega)]^T = [Z(\omega)]$
- **Stability**
 - ◆ bounded $[u(\omega)] \Rightarrow$ bounded $[f(\omega)]$
 - ◆ Poles with negative real value
- **Damped system**
 - ◆ Sink of energy
 - ◆ $\Im\{[Z(\omega)]\}$ positive
- **Causality**
 - ◆ \Rightarrow enforcement in frequency domain ?

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- **Physical approach:** Kramers-Kronig relations

$$\Re\{[Z(\omega)]\} = \frac{1}{\pi} \int_{\mathbb{R}} \frac{\Im\{[Z(\omega')]\}}{\omega - \omega'} d\omega'$$

- **Mathematical approach:** expansion on a Hardy functions basis

$$[Z(\omega)] = -\omega^2[M_{\Gamma}] + i\omega[C_{\Gamma}] + [K_{\Gamma}] + \sum_{n \geq 0} [Z_n]e_b(\omega)$$

- **Alternative approach**

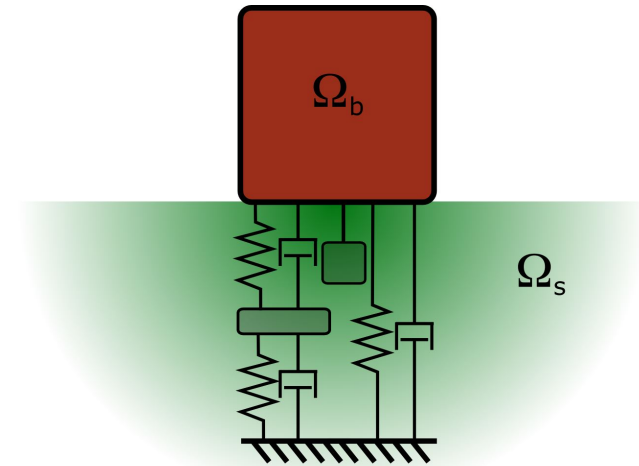
Sufficient condition: **Condensation of an underlying causal system**

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■ Structure of the approximation

Structure of the boundary impedance matrix of a mechanical system whose vibrations in the time domain follow a second-order differential equation with constant coefficients.

[Chabas and Soize, LRA, 1987]



■ Block decomposition

$$\left(\begin{bmatrix} [K_\Gamma] & [K_c] \\ [K_c]^T & [K_h] \end{bmatrix} + i\omega \begin{bmatrix} [C_\Gamma] & [C_c] \\ [C_c]^T & [C_h] \end{bmatrix} - \omega^2 \begin{bmatrix} [M_\Gamma] & [M_c] \\ [M_c]^T & [M_h] \end{bmatrix} \right) \begin{bmatrix} [u_\Gamma] \\ [u_h] \end{bmatrix} = \begin{bmatrix} [Z][u_\Gamma] \\ 0 \end{bmatrix}$$

■ Schur complement (with $[A_\alpha(\omega)] = -\omega^2[M_\alpha] + i\omega[C_\alpha] + [K_\alpha]$)

$$[Z(\omega)] = [A_\Gamma(\omega)] - [A_c(\omega)][A_h(\omega)]^{-1}[A_c(\omega)]^T = \frac{[P(i\omega)]}{Q(i\omega)}$$

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- **Basic properties verified when $[M]$, $[C]$, and $[K]$ positive definite**
 - ◆ Symmetry
 - ◆ Causality
 - ◆ Stability
 - ◆ Damped system
- **Comparison with lumped-parameter models**
 - ◆ Simple - Inexpensive
 - ◆ Network of elements NOT defined *a priori*
 - ◆ Coupling taken into account
 - ◆ \Rightarrow Identification ?

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■ Input impedance matrix $[\tilde{Z}]$

- ◆ discrete set $\{[\tilde{Z}(\omega_\ell)]\}_{1 \leq \ell \leq L}$
- ◆ computed or measured

■ Rational interpolation $[\tilde{Z}] \Rightarrow ([P], Q)$

- ◆ Using existing numerical schemes [van Barel and Bultheel, NM, 1992]
- ◆ Choice of number of HV \Rightarrow degree of approximation

$$[\tilde{Z}(\omega)] \approx \frac{[P(\omega)]}{Q(\omega)} = \frac{\sum_{\ell=0}^{2N+2} (i\omega)^\ell [P_\ell]}{\sum_{\ell=0}^{2N} (i\omega)^\ell Q_\ell}$$

■ Identification $([P], Q) \Rightarrow ([K], [C], [M])$

- ◆ No approximation
- ◆ Non-unique identification

Non-uniquely defined model

$$\left(\begin{bmatrix} K_{\Gamma} & K_c \\ K_c^T & K_h \end{bmatrix} + i\omega \begin{bmatrix} C_{\Gamma} & C_c \\ C_c^T & C_h \end{bmatrix} - \omega^2 \begin{bmatrix} M_{\Gamma} & M_c \\ M_c^T & M_h \end{bmatrix} \right) \begin{bmatrix} u_{\Gamma}(\omega) \\ u_h(\omega) \end{bmatrix} = \begin{bmatrix} Z(\omega)u_{\Gamma}(\omega) \\ 0 \end{bmatrix}$$

$$Z(\omega) = A_{\Gamma}(\omega) - A_c(\omega)A_h(\omega)^{-1}A_c(\omega)^T$$

- Example
- Diagonalization of the hidden part
- Cancellation of the mass coupling

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Non-uniquely defined model

$$\left(\begin{bmatrix} K_{\Gamma} & K_c \\ K_c^T & K_h \end{bmatrix} + i\omega \begin{bmatrix} C_{\Gamma} & C_c \\ C_c^T & C_h \end{bmatrix} - \omega^2 \begin{bmatrix} M_{\Gamma} & M_c \\ M_c^T & M_h \end{bmatrix} \right) \begin{bmatrix} u_{\Gamma}(\omega) \\ u_h(\omega) \end{bmatrix} = \begin{bmatrix} Z(\omega)u_{\Gamma}(\omega) \\ 0 \end{bmatrix}$$

$$Z(\omega) = A_{\Gamma}(\omega) - A_c(\omega)A_h(\omega)^{-1}A_c(\omega)^T$$

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$$\left(\begin{bmatrix} K_{\Gamma} & K_c\Phi \\ \Phi^T K_c^T & \Phi^T K_h \Phi \end{bmatrix} + i\omega \begin{bmatrix} C_{\Gamma} & C_c\Phi \\ \Phi^T C_c^T & \Phi^T C_h \Phi \end{bmatrix} - \omega^2 \begin{bmatrix} M_{\Gamma} & M_c\Phi \\ \Phi^T M_c^T & \Phi^T M_h \Phi \end{bmatrix} \right) \begin{bmatrix} u_{\Gamma} \\ u'_h \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}$$

$$Z(\omega) = A_{\Gamma}(\omega) - A_c(\omega)\Phi\Phi^{-1}A_h(\omega)^{-1}\Phi^{-T}\Phi^T A_c(\omega)^T$$

Non-uniquely defined model

$$\left(\begin{bmatrix} K_{\Gamma} & K_c \\ K_c^T & K_h \end{bmatrix} + i\omega \begin{bmatrix} C_{\Gamma} & C_c \\ C_c^T & C_h \end{bmatrix} - \omega^2 \begin{bmatrix} M_{\Gamma} & M_c \\ M_c^T & M_h \end{bmatrix} \right) \begin{bmatrix} u_{\Gamma}(\omega) \\ u_h(\omega) \end{bmatrix} = \begin{bmatrix} Z(\omega)u_{\Gamma}(\omega) \\ 0 \end{bmatrix}$$

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- Example
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$$\left(\begin{bmatrix} K_{\Gamma} & K_c \\ K_c^T & \Omega_h^2 \end{bmatrix} + i\omega \begin{bmatrix} C_{\Gamma} & C_c \\ C_c^T & 2\Xi_h\Omega_h \end{bmatrix} - \omega^2 \begin{bmatrix} M_{\Gamma} & M_c \\ M_c^T & I_h \end{bmatrix} \right) \begin{bmatrix} u_{\Gamma}(\omega) \\ u'_h(\omega) \end{bmatrix} = \begin{bmatrix} Z(\omega)u_{\Gamma}(\omega) \\ 0 \end{bmatrix}$$

$$Z(\omega) = A_{\Gamma}(\omega) - \sum_{\ell=1}^N \frac{A_c(\omega)A_c^T(\omega)}{-\omega^2 + 2i\zeta_{\ell}\omega_{\ell}\omega + \omega_{\ell}^2}$$

Non-uniquely defined model

$$\left(\begin{bmatrix} \mathbf{K}_\Gamma & \mathbf{K}_c \\ \mathbf{K}_c^T & \mathbf{K}_h \end{bmatrix} + i\omega \begin{bmatrix} \mathbf{C}_\Gamma & \mathbf{C}_c \\ \mathbf{C}_c^T & \mathbf{C}_h \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M}_\Gamma & \mathbf{M}_c \\ \mathbf{M}_c^T & \mathbf{M}_h \end{bmatrix} \right) \begin{bmatrix} u_\Gamma(\omega) \\ u_h(\omega) \end{bmatrix} = \begin{bmatrix} \mathbf{Z}(\omega)u_\Gamma(\omega) \\ 0 \end{bmatrix}$$

$$\mathbf{Z}(\omega) = \mathbf{A}_\Gamma(\omega) - \mathbf{A}_c(\omega)\mathbf{A}_h(\omega)^{-1}\mathbf{A}_c(\omega)^T$$

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$$\left(\begin{bmatrix} \mathbf{K}_\Gamma & \mathbf{K}_c \\ \mathbf{K}_c^T & \Omega_h^2 \end{bmatrix} + i\omega \begin{bmatrix} \mathbf{C}_\Gamma & \mathbf{C}_c \\ \mathbf{C}_c^T & 2\Xi_h\Omega_h \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M}_\Gamma & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_h \end{bmatrix} \right) \begin{bmatrix} u_\Gamma(\omega) \\ u'_h(\omega) \end{bmatrix} = \begin{bmatrix} \mathbf{Z}(\omega)u_\Gamma(\omega) \\ 0 \end{bmatrix}$$

$$\mathbf{Z}(\omega) = -\omega^2\mathbf{M}_\Gamma + i\omega\mathbf{C}_\Gamma + \mathbf{K}_\Gamma - \sum_{\ell=1}^N \frac{(i\omega\mathbf{C}_c^\ell + \mathbf{K}_c^\ell)(i\omega\mathbf{C}_c^\ell + \mathbf{K}_c^\ell)^T}{-\omega^2 + 2i\zeta_\ell\omega_\ell\omega + \omega_\ell^2}$$

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■ Pole-residue expansion

$$[\tilde{Z}(\omega)] \approx \frac{[P(i\omega)]}{Q(i\omega)} = -\omega^2 R_2 + i\omega R_1 + R_0 + \sum_{\ell=1}^N \frac{i\omega R_{-1}^{\ell} + R_{-2}^{\ell}}{-\omega^2 + i\omega p_1^{\ell} + p_0^{\ell}}$$

■ Schur complement

$$Z(\omega) = -\omega^2 M_{\Gamma} + i\omega C_{\Gamma} + K_{\Gamma} - \sum_{\ell=1}^N \frac{(i\omega C_c^{\ell} + K_c^{\ell})(i\omega C_c^{\ell} + K_c^{\ell})^T}{-\omega^2 + 2i\zeta_{\ell}\omega_{\ell}\omega + \omega_{\ell}^2} = \frac{[P(i\omega)]}{Q(i\omega)}$$

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■ Identification

$$\begin{cases} \mathbf{K}_c^\ell \mathbf{K}_c^{\ell T} - 2\zeta_\ell \omega_\ell \mathbf{C}_c^\ell \mathbf{C}_c^{\ell T} = -\mathbf{R}_{-2}^\ell & 1 \leq \ell \leq N \\ \mathbf{C}_c^\ell \mathbf{K}_c^{\ell T} + \mathbf{K}_c^\ell \mathbf{C}_c^{\ell T} - \omega_\ell^2 \mathbf{C}_c^\ell \mathbf{C}_c^{\ell T} = -\mathbf{R}_{-1}^\ell & 1 \leq \ell \leq N \end{cases}$$

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$$[\tilde{\mathbf{Z}}(\omega)] \approx \frac{[\mathbf{P}(i\omega)]}{\mathbf{Q}(i\omega)} = -\omega^2 \mathbf{R}_2 + i\omega \mathbf{R}_1 + \mathbf{R}_0 + \sum_{\ell=1}^N \frac{i\omega \mathbf{R}_{-1}^\ell + \mathbf{R}_{-2}^\ell}{-\omega^2 + i\omega \mathbf{p}_1^\ell + \mathbf{p}_0^\ell}$$

■ Schur complement

$$\mathbf{Z}(\omega) = -\omega^2 \mathbf{M}_\Gamma + i\omega \mathbf{C}_\Gamma + \mathbf{K}_\Gamma - \sum_{\ell=1}^N \frac{(i\omega \mathbf{C}_c^\ell + \mathbf{K}_c^\ell)(i\omega \mathbf{C}_c^\ell + \mathbf{K}_c^\ell)^T}{-\omega^2 + 2i\zeta_\ell \omega_\ell \omega + \omega_\ell^2} = \frac{[\mathbf{P}(i\omega)]}{\mathbf{Q}(i\omega)}$$

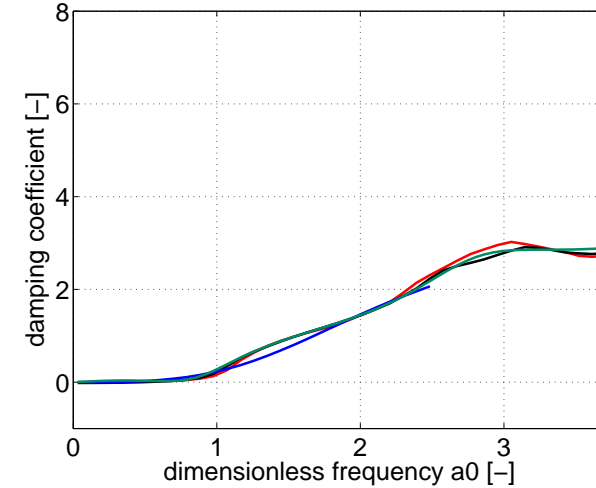
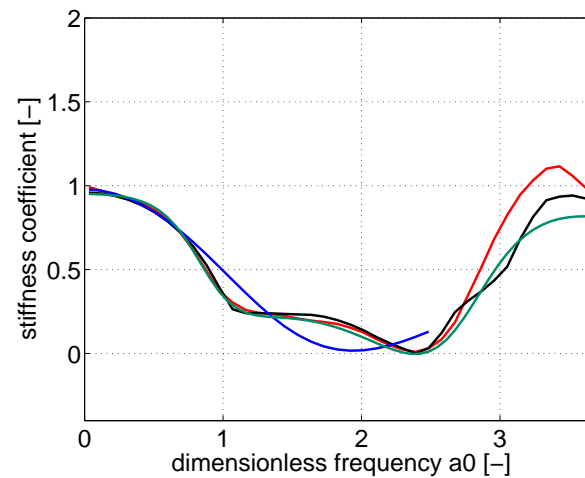
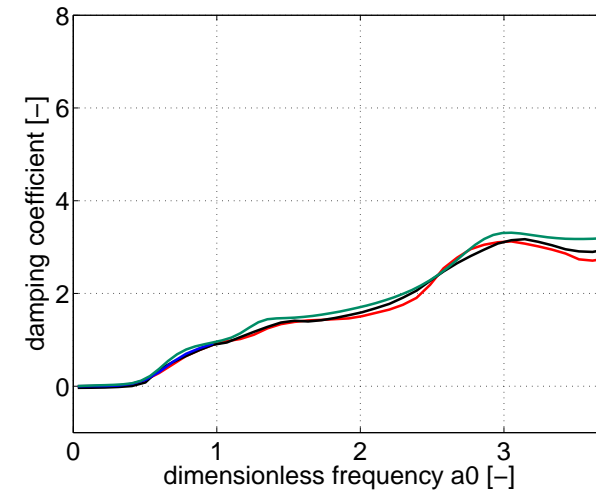
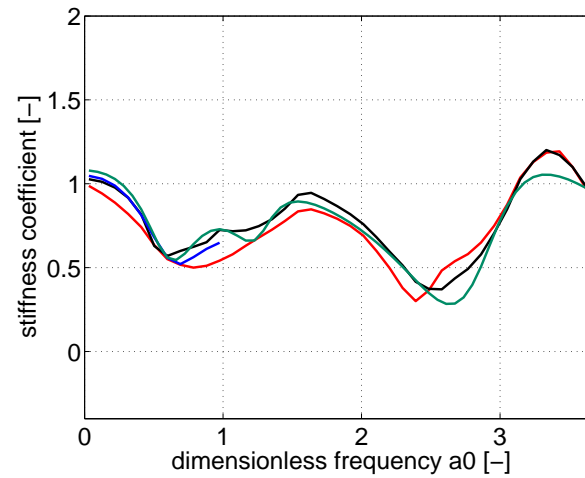
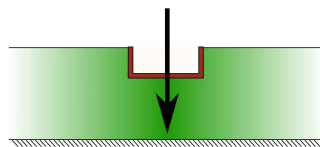
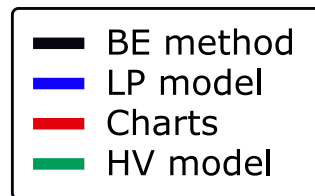
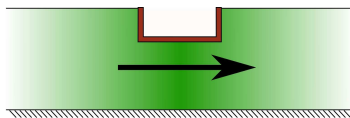
■ Identification [Takagi, JJM, 1925] [Cottureau *et al.*, CMAME, 2006, *accepted*]

$$\begin{cases} \mathbf{K}_c^\ell \mathbf{K}_c^{\ell T} - 2\zeta_\ell \omega_\ell \mathbf{C}_c^\ell \mathbf{C}_c^{\ell T} = -\mathbf{R}_{-2}^\ell & 1 \leq \ell \leq N \\ \mathbf{C}_c^\ell \mathbf{K}_c^{\ell T} + \mathbf{K}_c^\ell \mathbf{C}_c^{\ell T} - \omega_\ell^2 \mathbf{C}_c^\ell \mathbf{C}_c^{\ell T} = -\mathbf{R}_{-1}^\ell & 1 \leq \ell \leq N \end{cases}$$

$$[\mathbf{L}^\ell][\mathbf{L}^\ell]^T = [\mathbf{c}_1] \Rightarrow \begin{cases} \mathbf{K}_c^\ell = \mathbf{c}_2^\ell \Re\{[\mathbf{L}^\ell]\} + \mathbf{c}_3^\ell \Im\{[\mathbf{L}^\ell]\} & 1 \leq \ell \leq N \\ \mathbf{C}_c^\ell = \Im\{[\mathbf{L}^\ell]\} & 1 \leq \ell \leq N \end{cases}$$

Embedded foundation on layer of homogeneous soil over bedrock

- Introduction
- Det. model
- Prob. model
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- Introduction
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Find a simple and general model taking into account soil variability

- **Introduction**
- **Parametric approach: SFE method**
 - ◆ Principle
 - ◆ Example
- **Nonparametric approach**
 - ◆ Principle
 - ◆ Methodology
 - ◆ Application
- **Comparison**

Origin of the uncertainty

- Model uncertainties
 - ◆ Homogeneity within a layer
 - ◆ Nonlinearity (behavior)
 - ◆ ...
- Parameter uncertainties
 - ◆ Measurement errors
 - ◆ Reshuffling
 - ◆ Under-sampling
 - ◆ ...

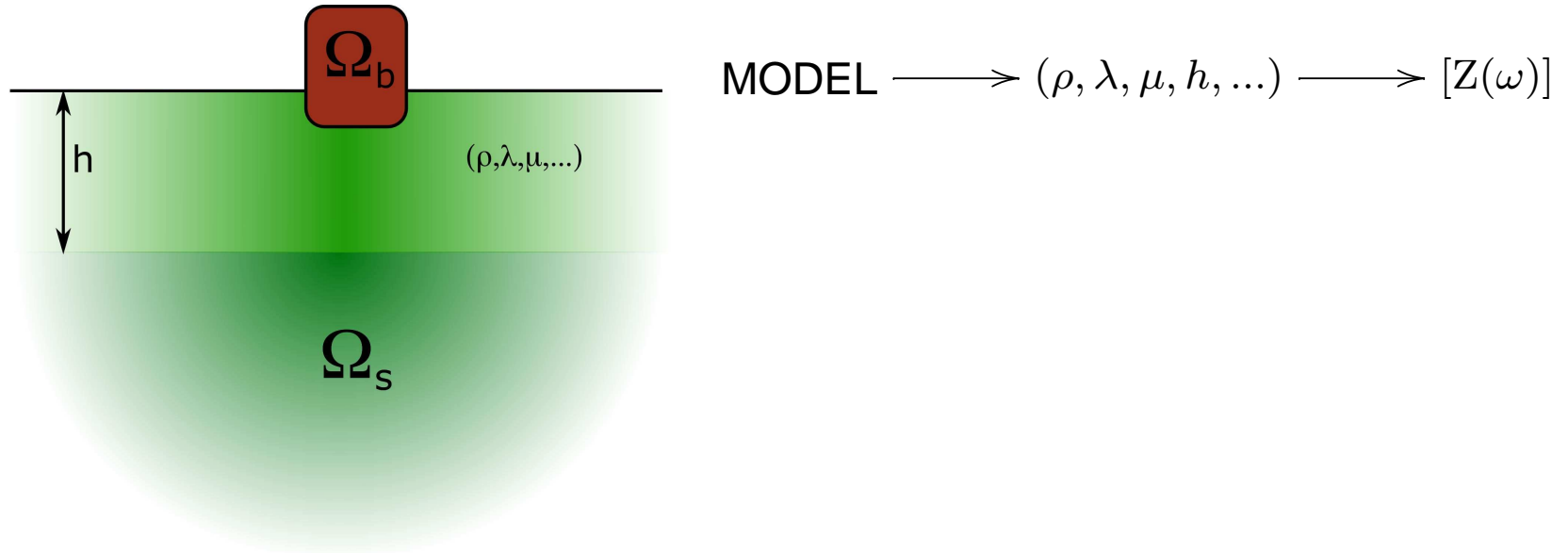
Probabilistic model

$[\mathbf{Z}(\omega, \theta)]$ random process indexed on \mathbb{B}_Ω with values in \mathbb{M}

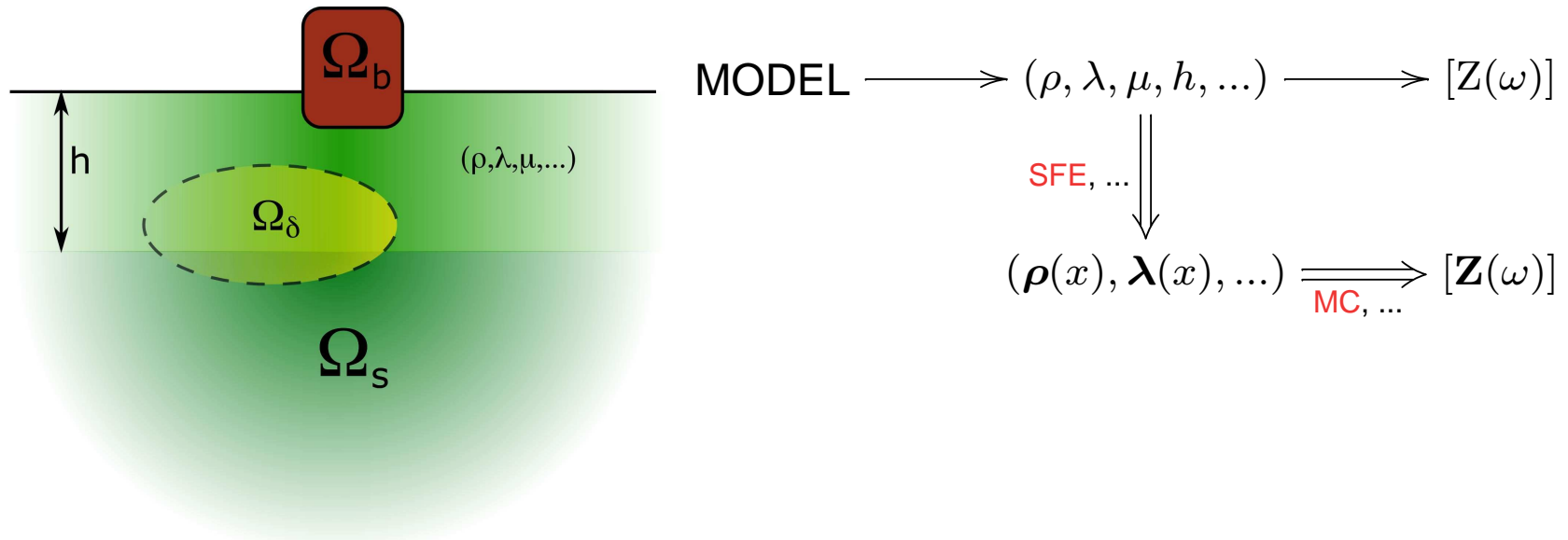
- Space of matrices \mathbb{M}
 - ◆ \Rightarrow Symmetric
 - ◆ \Rightarrow Positive imaginary part
- Trajectories of $[\mathbf{Z}(\omega, \theta)]$
 - ◆ \Rightarrow Causal
 - ◆ \Rightarrow Stable

Parametric approach: Principle

- Introduction
- Det. model
- Prob. model
- Application
- Conclusion



- Introduction
- Det. model
- Prob. model
- Application
- Conclusion



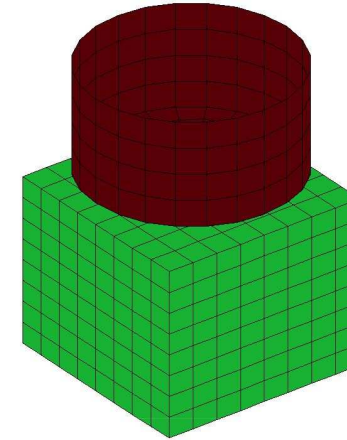
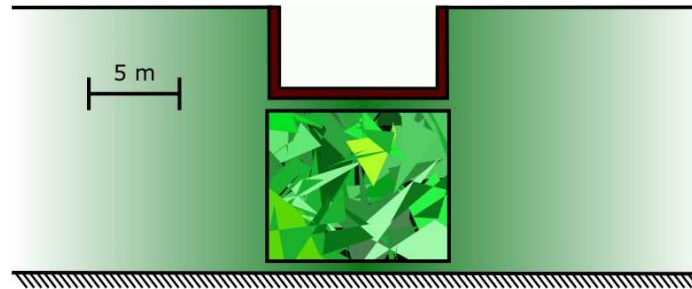
■ Limitations

- ◆ Size of the random domain - cost of the approach
- ◆ Appropriate choice of stochastic model (including correlation model) ?
- ◆ Appropriate representation of random fields

■ Previous studies at ECP

- ◆ [\[Savin and Clouteau, IJNME, 2002\]](#) [\[Lafargue et al., 2003, SIAM CSE\]](#)
- ◆ Software MISSVAR [\[Clouteau, Tech. Rep.\]](#)

- Introduction
- Det. model
- Prob. model
- Application
- Conclusion

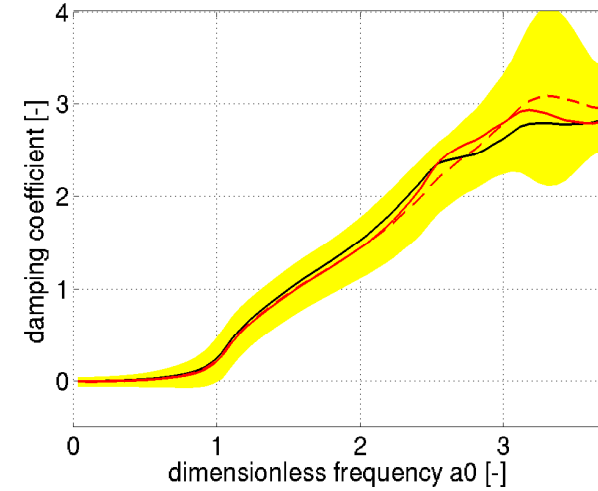
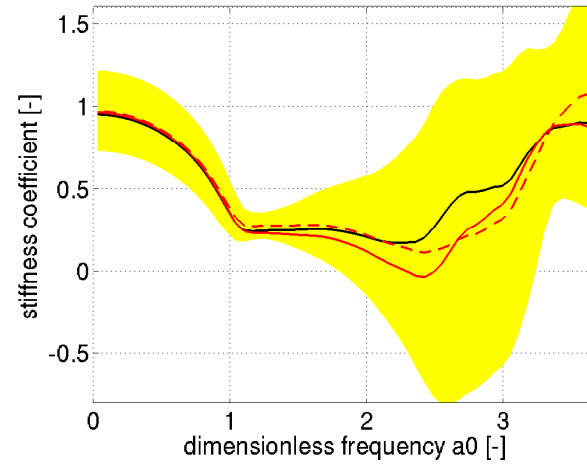
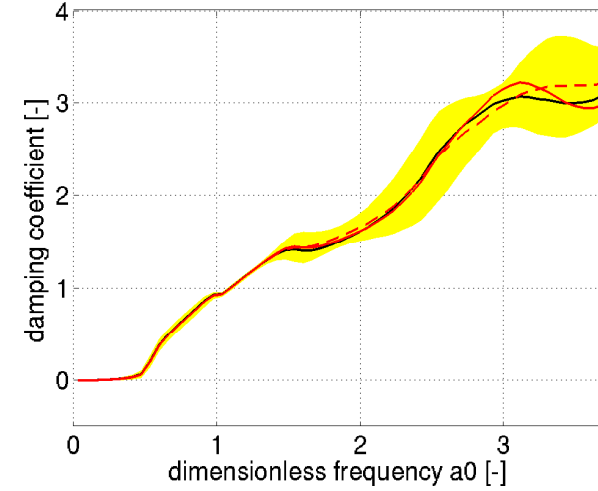
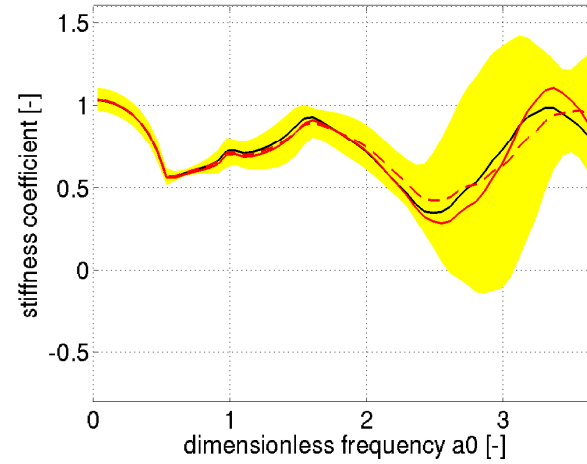
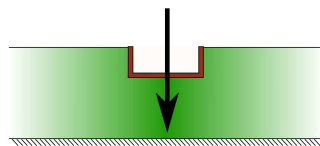
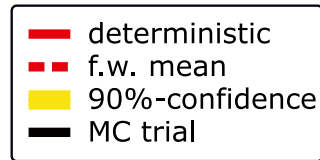
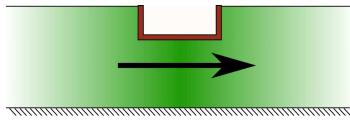


Embedded foundation on layer of heterogeneous soil over bedrock

- Random Lamé coefficients
- Karhunen-Loève decomposition
- Uniform random variables (60% variance)
- Exponential correlation: parameter $L = 5$ m
- Shear-wave velocity $c_s = 200$ m/s

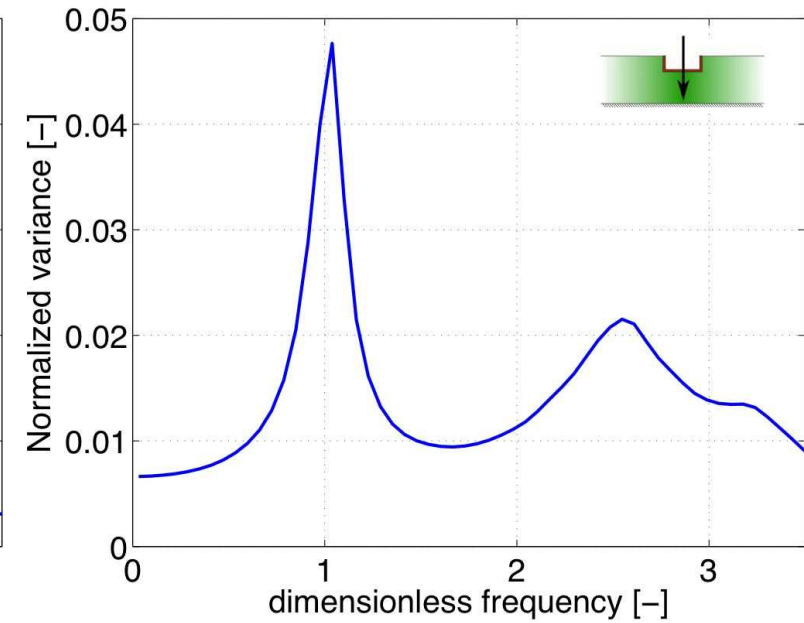
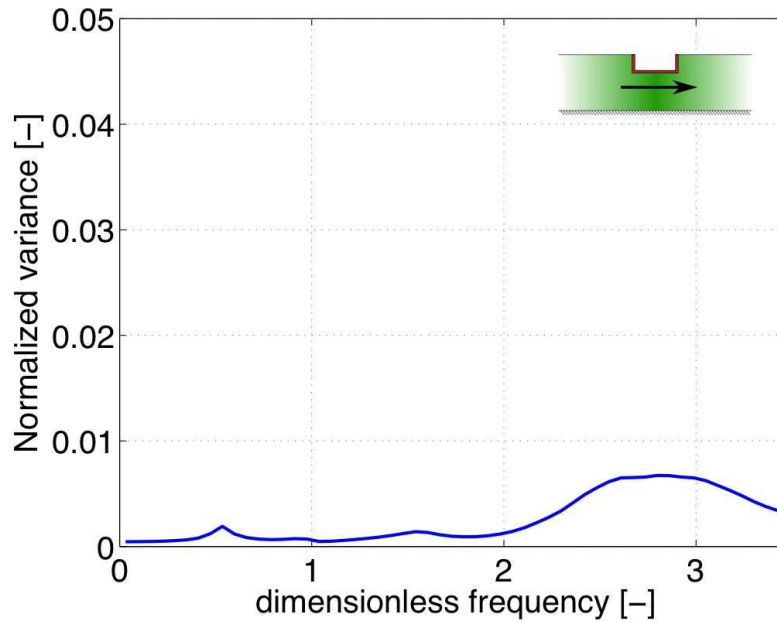
Parametric approach: Results $[Z(\omega)]$

- Introduction
- Det. model
- Prob. model
- Application
- Conclusion



Parametric approach: Variance

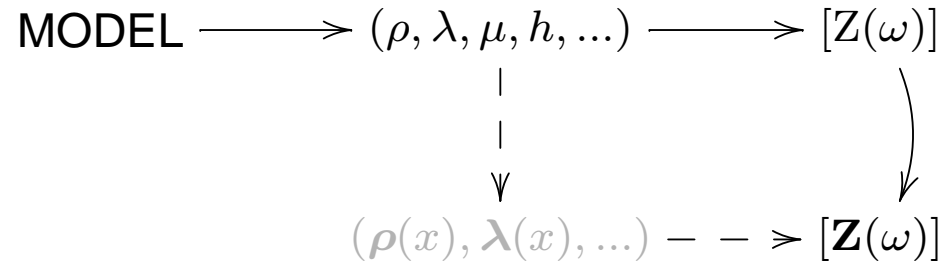
- Introduction
- Det. model
- Prob. model
- Application
- Conclusion



Variability considered mostly on pumping element

Nonparametric approach: Principle (1)

Uncertainty taken into account at a higher level



- Interests
 - ◆ No stochastic model of the soil parameters
 - ◆ No meshing of the soil required
- Parametric uncertainties and some model uncertainties
 - ◆ Heterogeneity of the soil
 - ◆ NO nonlinearities
- Already applied for stochastic reduced dynamic models
[\[Soize, PEM, 2000\]](#) [\[Soize, JASA, 2001\]](#)

Principle

- Identification of the available information
- Construction of the laws of $[\mathbf{Z}(\omega)]$ by Entropy Maximization \Rightarrow HV model

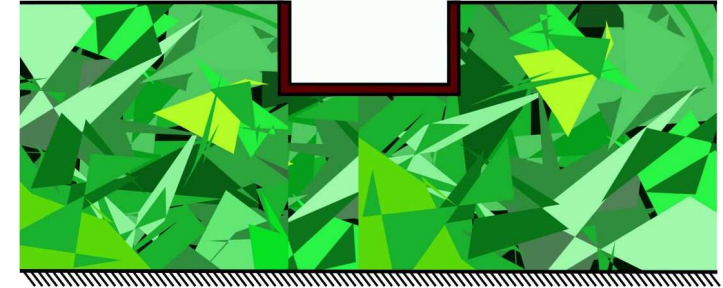
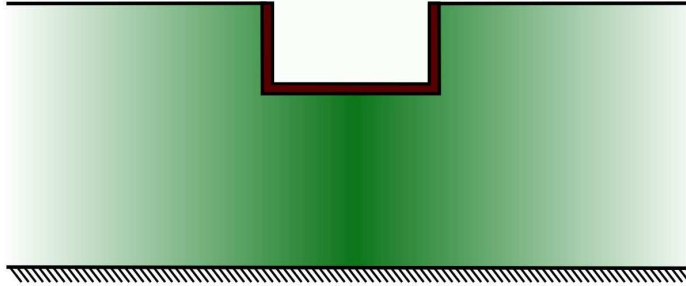
Available information

- Properties of $[\mathbf{Z}(\omega)]$
 - ◆ Causality ensured by the form of the HV model
 - ◆ Algebraic information on \mathbf{K} , \mathbf{C} and \mathbf{M}
- Matrices of the mean model $\underline{\mathbf{K}}$, $\underline{\mathbf{C}}$ and $\underline{\mathbf{M}}$ \Leftarrow Identification
- Dispersion parameters $\delta_{\mathbf{K}}$, $\delta_{\mathbf{C}}$ and $\delta_{\mathbf{M}}$

[Soize, CMAME, 2005] [Arnst *et al.*, PEM, 2005]

- Introduction
- Det. model
- Prob. model
- Application
- Conclusion

- Introduction
- Det. model
- Prob. model
- Application
- Conclusion



Embedded foundation on layer of heterogeneous soil over bedrock

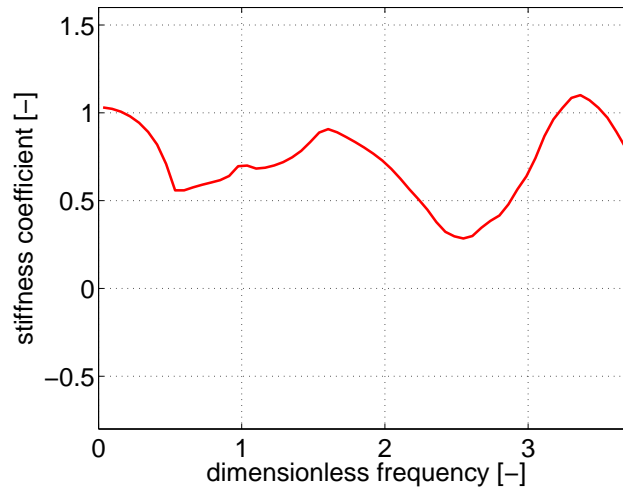
■ Chosen model

- ◆ Mean impedance $[\tilde{Z}(\omega)]$ computed using BE software MISS (deterministic)
- ◆ Identification of \underline{K} , \underline{C} and \underline{M}
- ◆ Choice of $\delta_K = \delta_C = \delta_M = 0.1$

Methodology

- Introduction
- Det. model
- Prob. model
- Application
- Conclusion

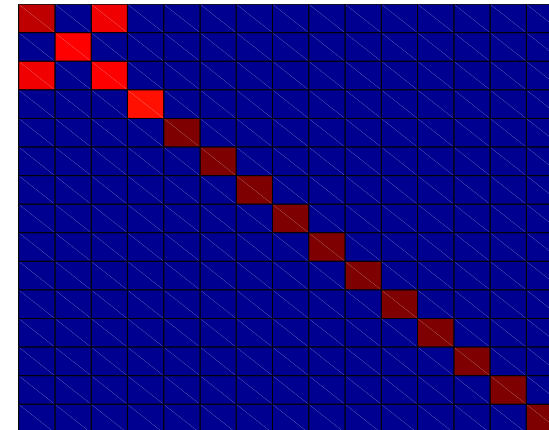
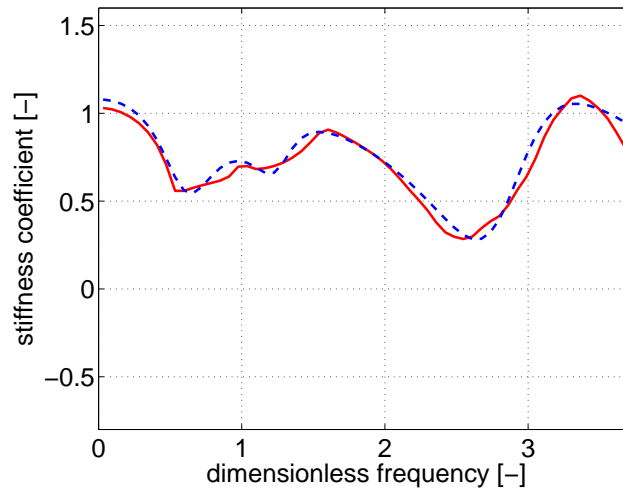
- Computation of an input impedance matrix (set of $\{\tilde{Z}(\omega_\ell)\}_{1 \leq \ell \leq L}$)
- Identification of \underline{K} , \underline{C} and \underline{M}
- Choice of dispersion parameters δ_K , δ_C and δ_M
- Computation of stochastic models for \underline{K} , \underline{C} and \underline{M}
- Computation of stochastic model of $[\underline{Z}(\omega)]$



Methodology

- Introduction
- Det. model
- Prob. model
- Application
- Conclusion

- Computation of an input impedance matrix (set of $\{\tilde{Z}(\omega_\ell)\}_{1 \leq \ell \leq L}$)
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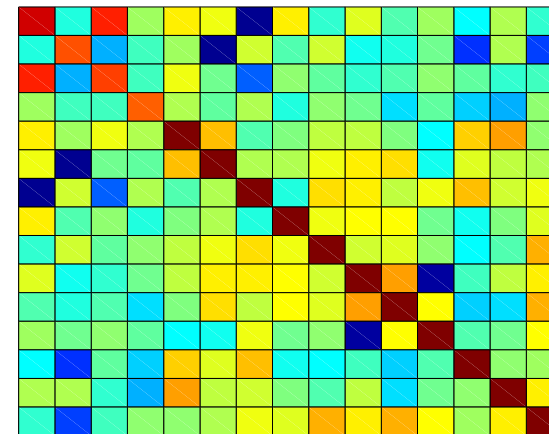
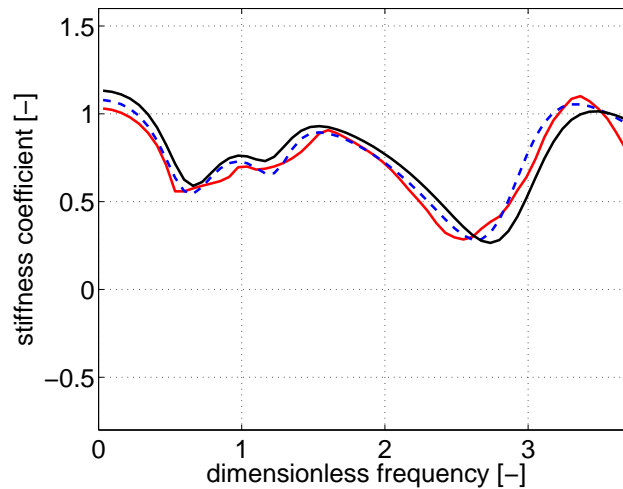


$[\underline{M}]$

Methodology

- Introduction
- Det. model
- Prob. model
- Application
- Conclusion

- Computation of an input impedance matrix (set of $\{\tilde{Z}(\omega_\ell)\}_{1 \leq \ell \leq L}$)
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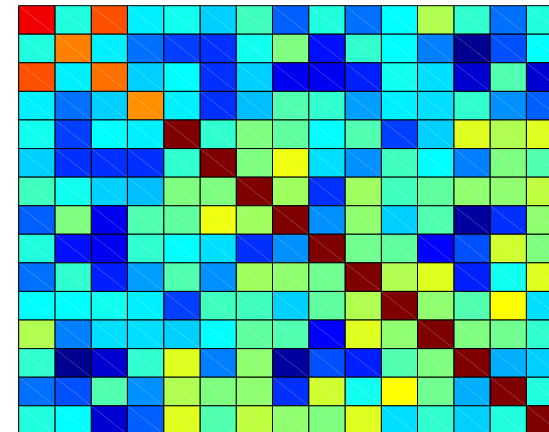
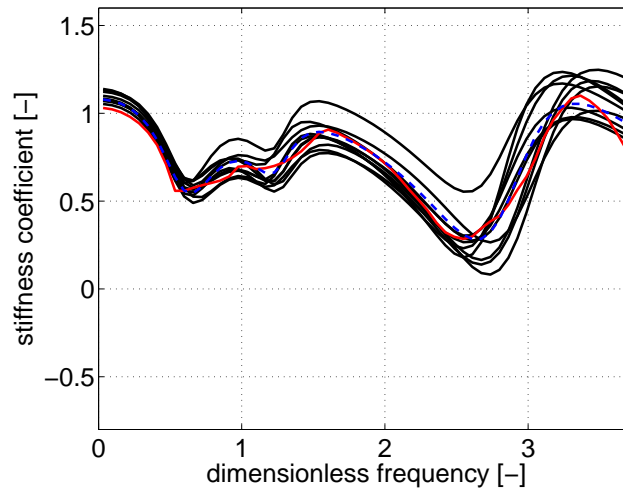


$[\underline{M}]^{(1)}$

Methodology

- Introduction
- Det. model
- Prob. model
- Application
- Conclusion

- Computation of an input impedance matrix (set of $\{[\tilde{Z}(\omega_\ell)]\}_{1 \leq \ell \leq L}$)
- Identification of \underline{K} , \underline{C} and \underline{M}
- Choice of dispersion parameters δ_K , δ_C and δ_M
- Computation of stochastic models for \underline{K} , \underline{C} and \underline{M}
- Computation of stochastic model of $[\underline{Z}(\omega)]$

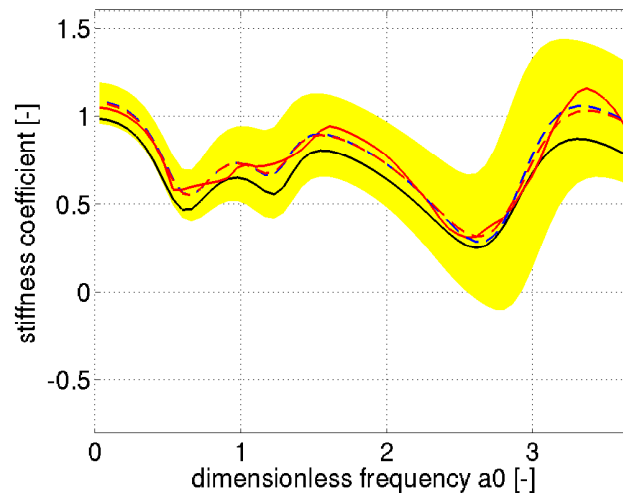


$[\underline{M}]^{(2)}$

Methodology

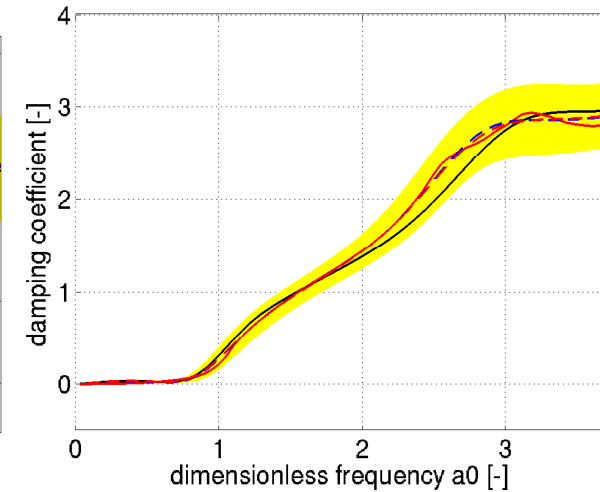
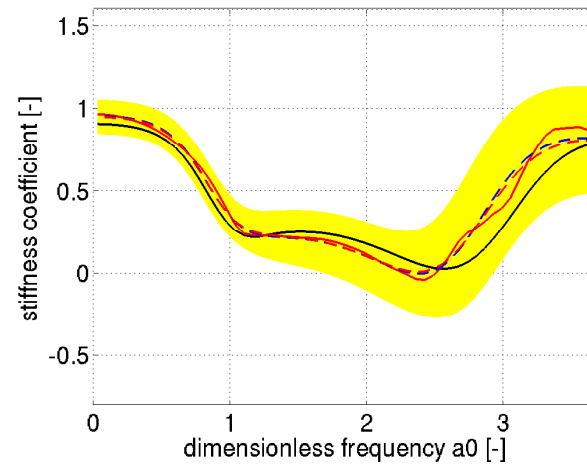
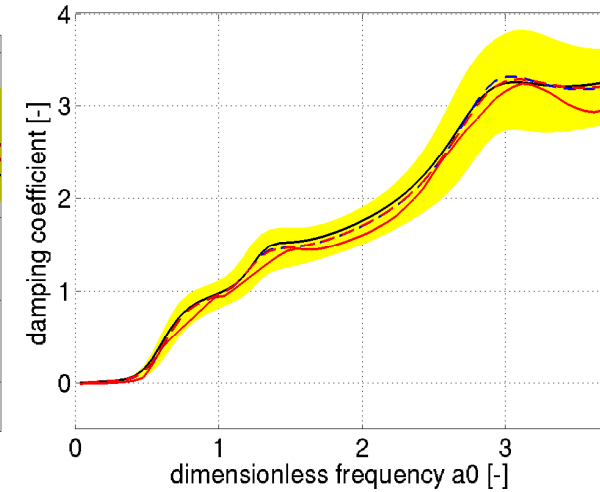
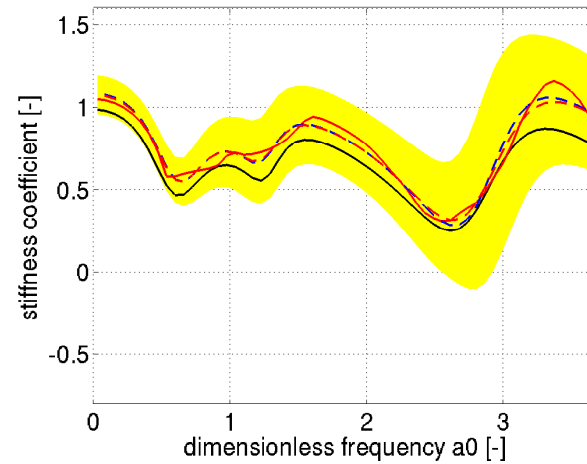
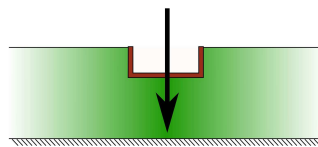
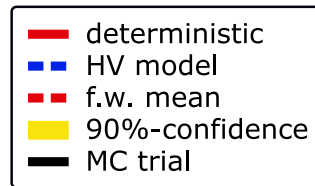
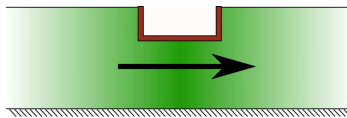
- Introduction
- Det. model
- Prob. model
- Application
- Conclusion

- Computation of an input impedance matrix (set of $\{[\tilde{Z}(\omega_\ell)]\}_{1 \leq \ell \leq L}$)
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- Computation of stochastic model of $[\underline{Z}(\omega)]$



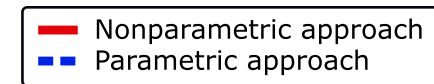
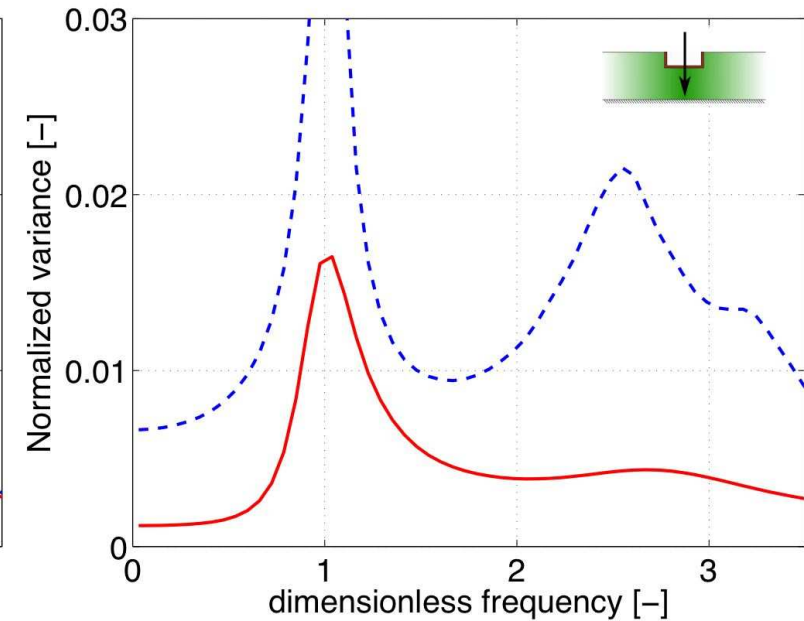
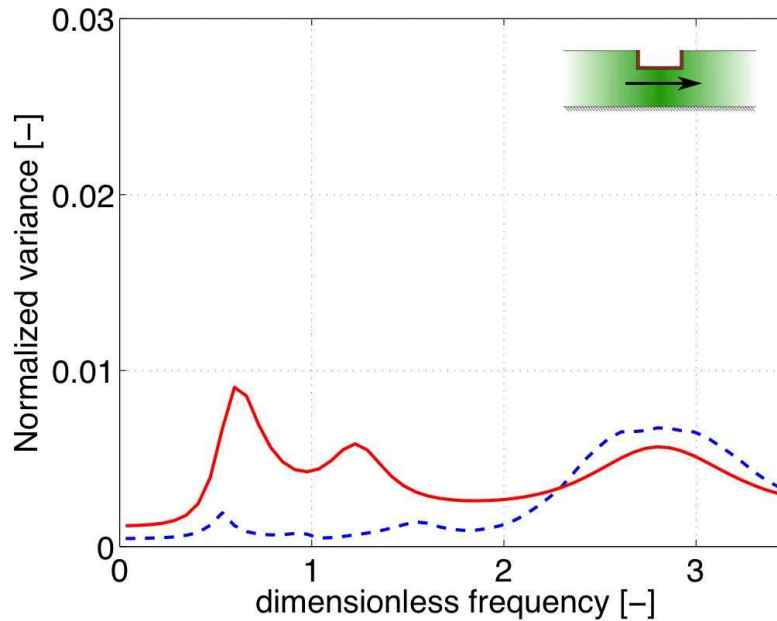
Nonparametric approach: Results

- Introduction
- Det. model
- Prob. model
- Application
- Conclusion



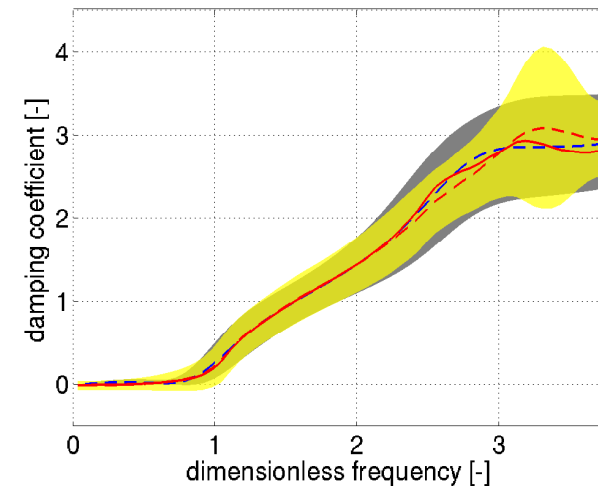
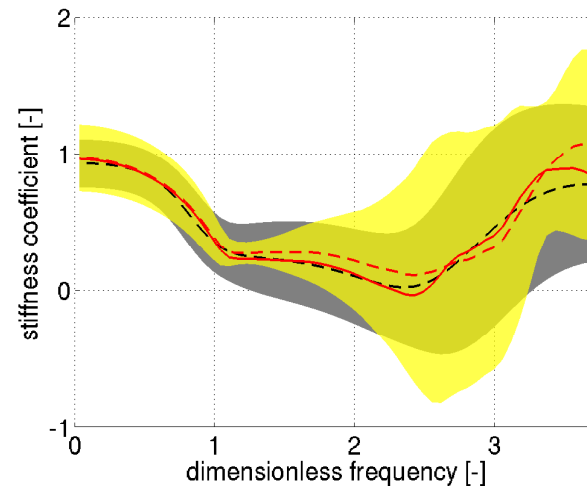
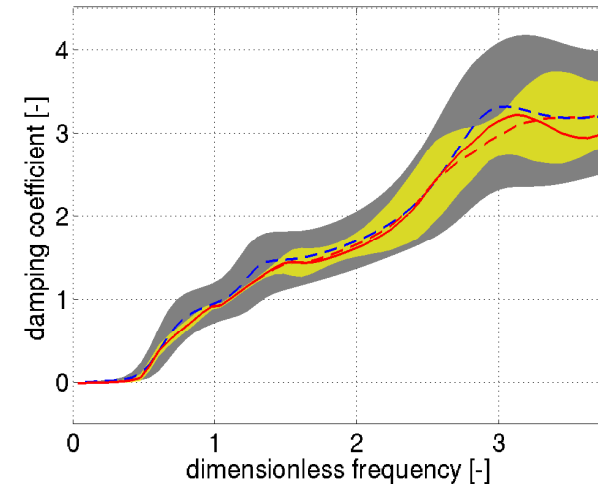
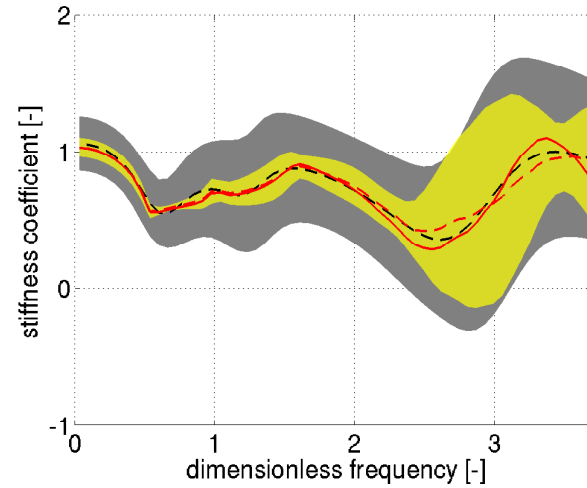
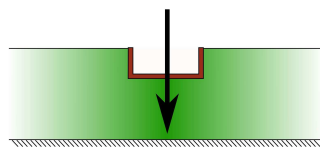
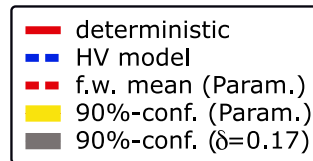
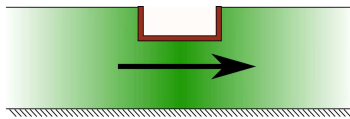
Nonparametric approach: Variance

- Introduction
- Det. model
- Prob. model
- Application
- Conclusion



- Variability on shaking AND pumping
- Identification of δ on pumping term

- Introduction
- Det. model
- Prob. model
- Application
- Conclusion



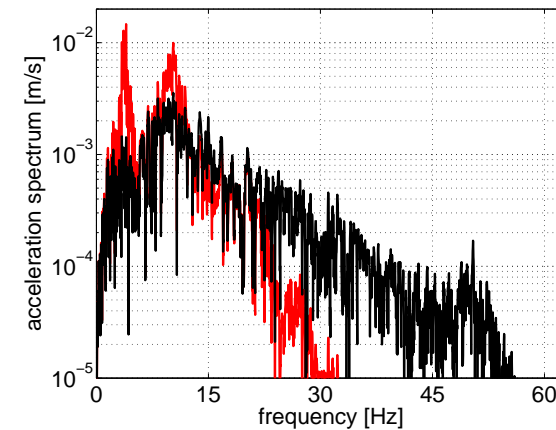
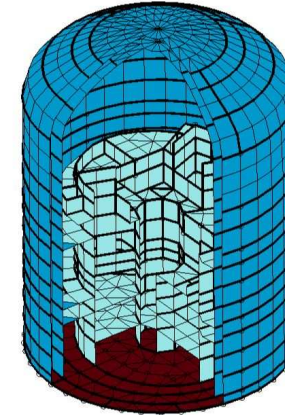
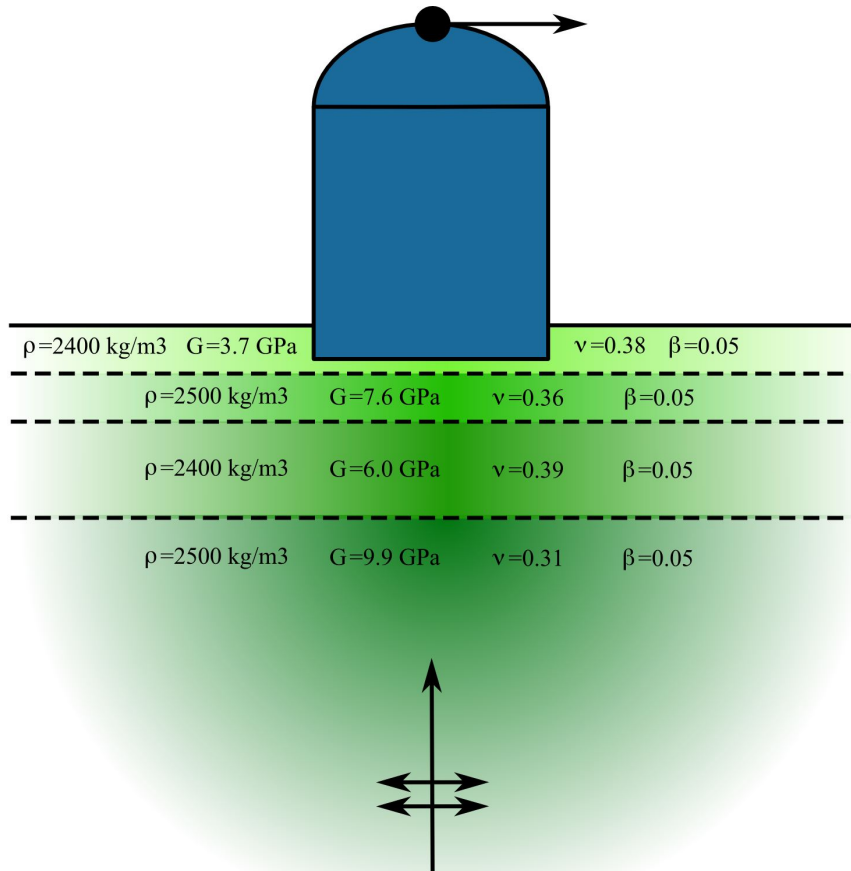
Conclusion on probabilistic models

- Introduction
- Det. model
- Prob. model
- Application
- Conclusion

- Parametric approach
 - ◆ Possibility to control the length of correlation, when available
- Nonparametric approach
 - ◆ No required identification of the variability of the soil parameters
 - ◆ Non-localized variability
 - ◆ Inexpensive

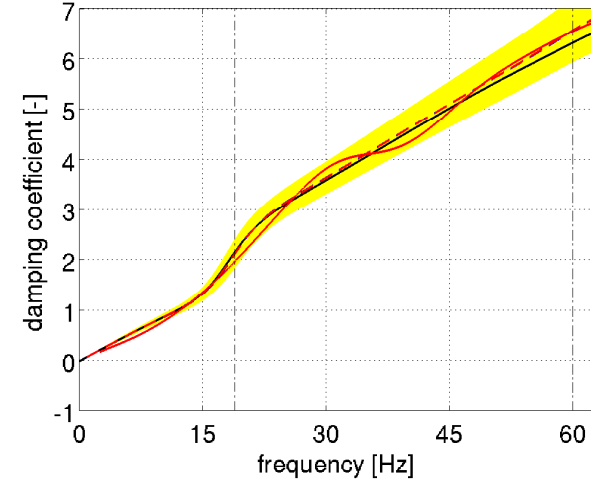
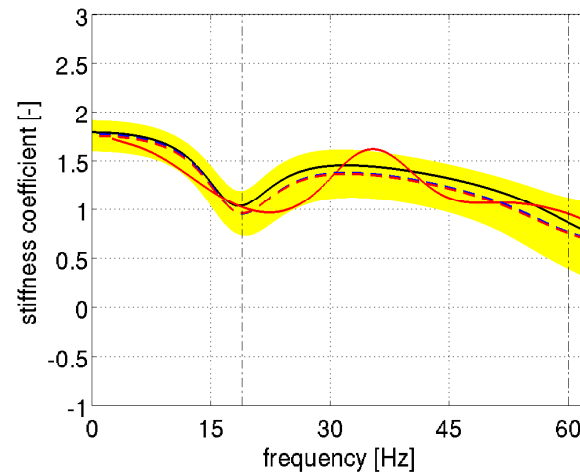
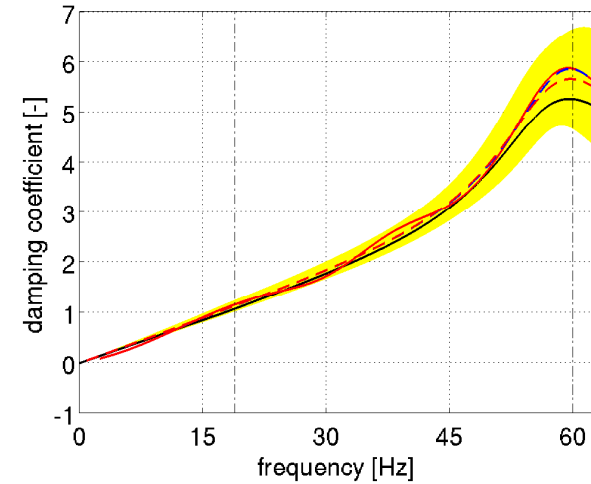
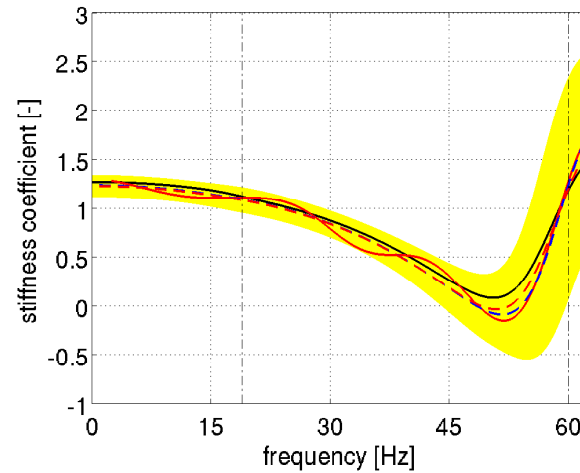
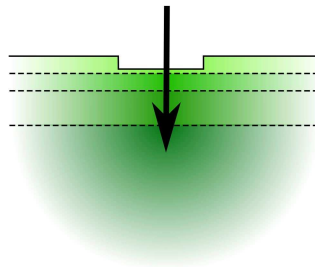
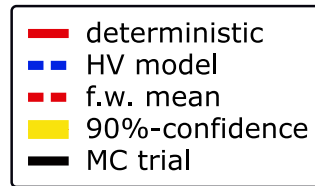
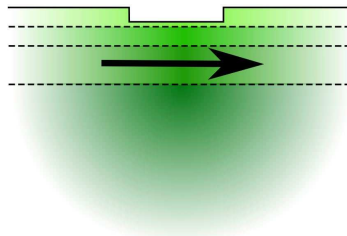
Choice of nonparametric approach for most seismic design problems

- Introduction
- Det. model
- Prob. model
- Application
- Conclusion



[Cottureau *et al.*, EESD, *in preparation*]

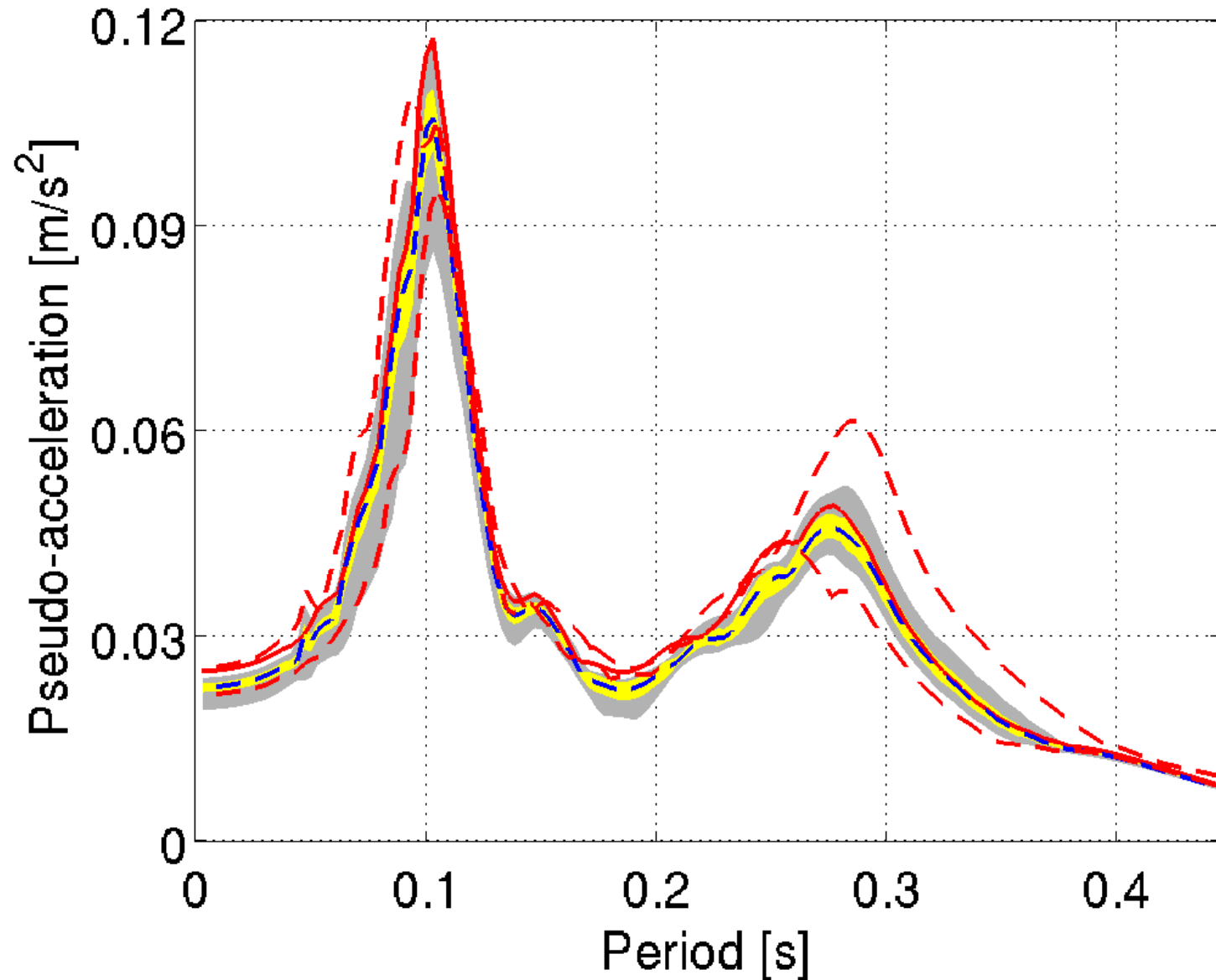
- Introduction
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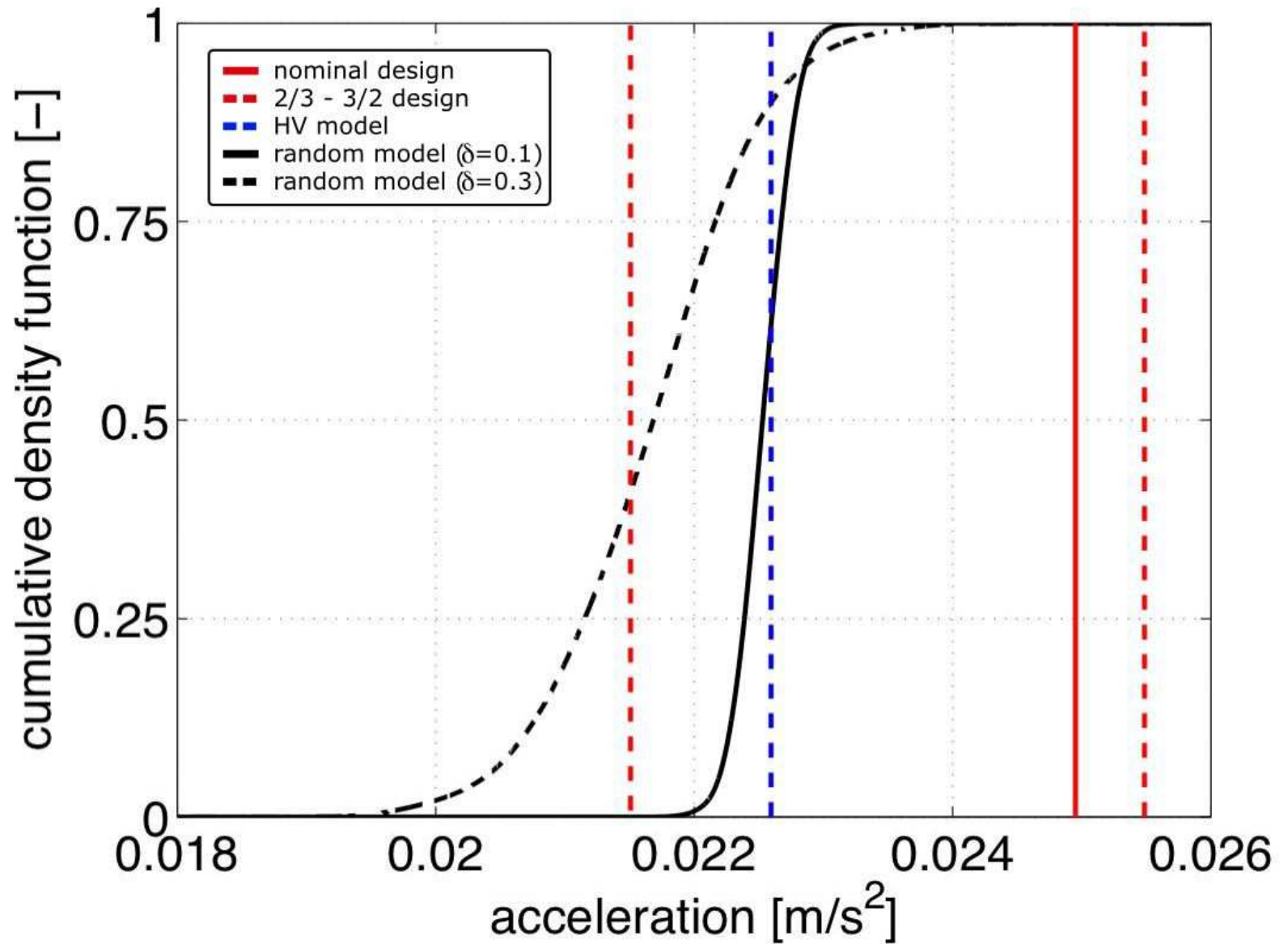
- Problems in identification of mean model
- $\delta = 0.1$

	$l = 1$	$l = 2$	$l = 3$
ω_l [Hz]	18.8	59.9	73.0
ζ_l [-]	0.09	0.14	0.30

- Introduction
- Det. model
- Prob. model
- Application
- Conclusion



- Introduction
- Det. model
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- Introduction
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- **Features of HV model**
 - ◆ Ensures the validation of the basic properties of the impedance
 - ◆ Takes into account matrix-character of the impedance
 - ◆ Usable for stochastic applications
 - ◆ Usable in time domain
- **Features of nonparametric method**
 - ◆ No meshing of the soil
 - ◆ No identification of the soil parameters variability
 - ◆ Inexpensive
- **Correlation between seismic input and impedance**
- **Problems in identification**

- Introduction
- Det. model
- Prob. model
- Application
- Conclusion

- **Problems in identification**
 - ◆ Enforce impedance conditions during interpolation
 - ◆ Stability [[Strelitz, AMM, 1977](#)]
 - ◆ Positivity
- **Extension to experimental impedances matrices**
 - ◆ Measures of wave propagation in Lincent, Belgium [[Arnst *et al.*, Tech. Rep., 2006](#)]
 - ◆ Modification of interpolation
- **Extension to complex and flexible foundations**
[[Taherzadeh and Clouteau, 1st ECEE, 2006](#)]

Thank you...



- Introduction
- Det. model
- Prob. model
- Application
- Conclusion