Microwave imaging of buried objects: two-dimensional numerical modeling and study of the three-dimensional extension

Ph.D. thesis

Ioannis Aliferis

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  UMR CNRS 6071
  Université de Nice – Sophia Antipolis, France

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Greek State Scholarship Foundation (IKY)

Microwave imaging of buried objects

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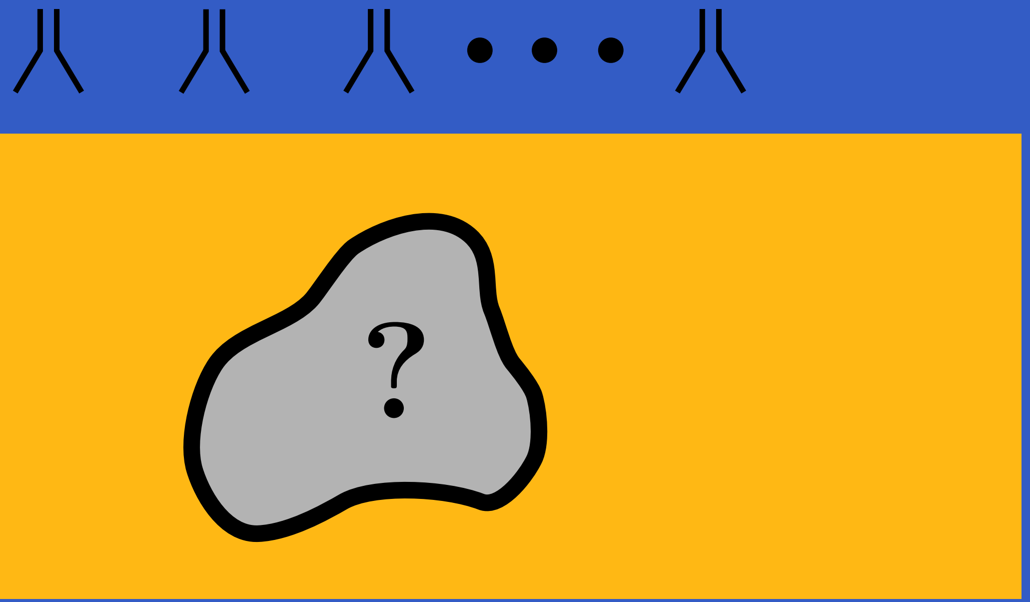

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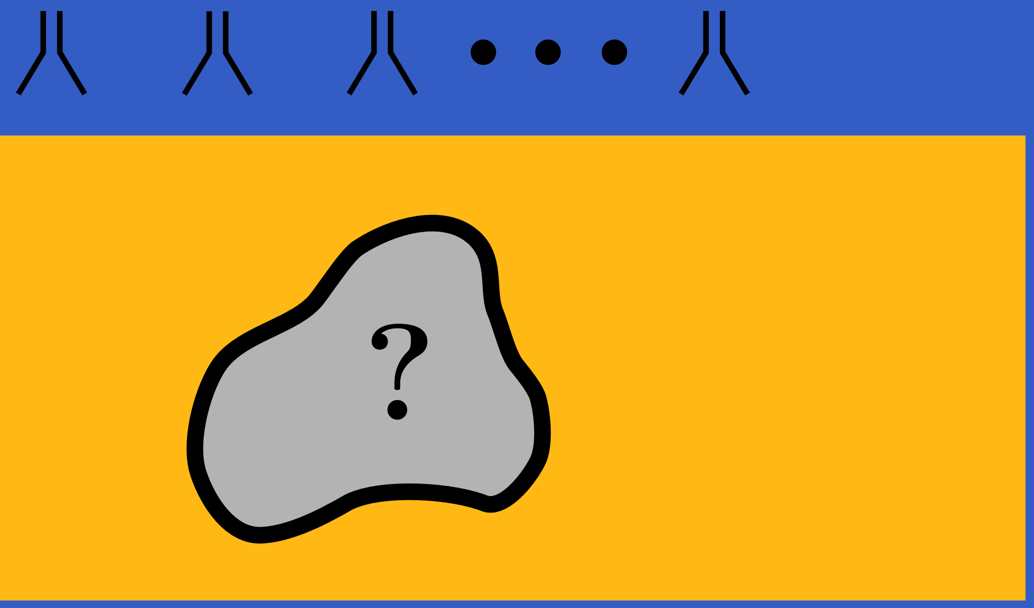
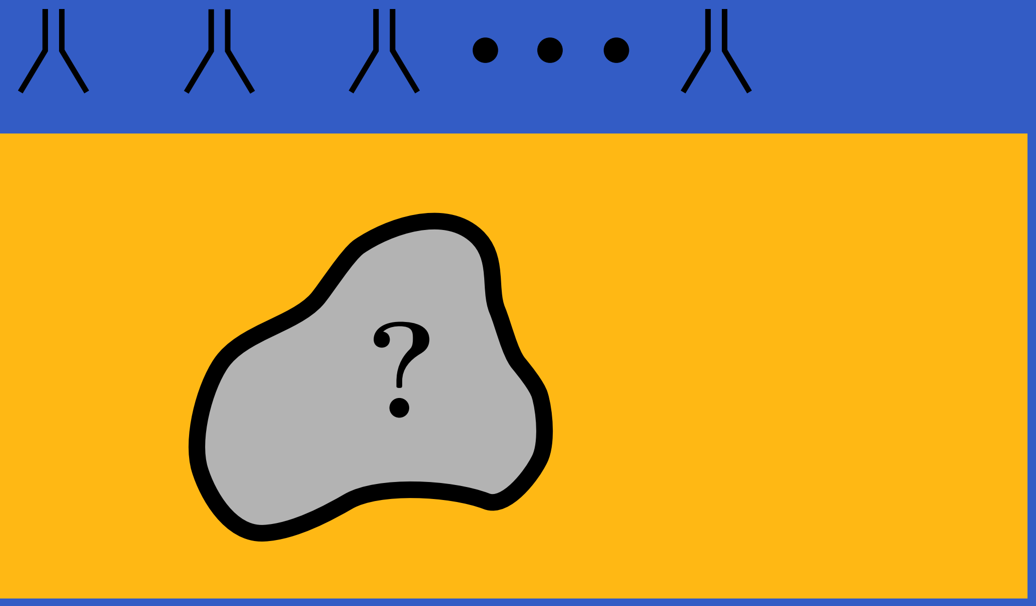


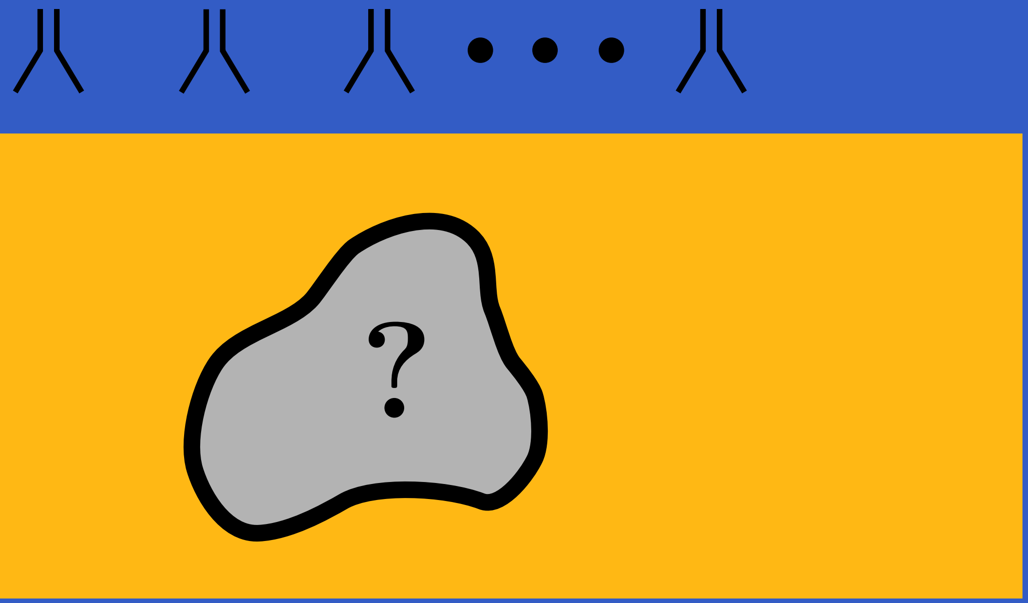
- Incident/Scattered field $\xrightarrow{find}$ object (ε_r, σ)

Microwave imaging of buried objects



- Incident/Scattered field $\xrightarrow{find}$ object (ε_r, σ)
- Limited aspect data

Microwave imaging of buried objects



- Incident/Scattered field $\xrightarrow{find}$ object (ε_r, σ)
- Limited aspect data
- Multifrequency, multiview, multistatic

Iterative algorithm

Iterative algorithm

$E^{(i)}$

Iterative algorithm

$E^{(s)}_{\text{ref}}$

$E^{(i)}$

Iterative algorithm

$E^{(s)}_{\text{ref}}$

$E^{(i)}$

ε_r, σ

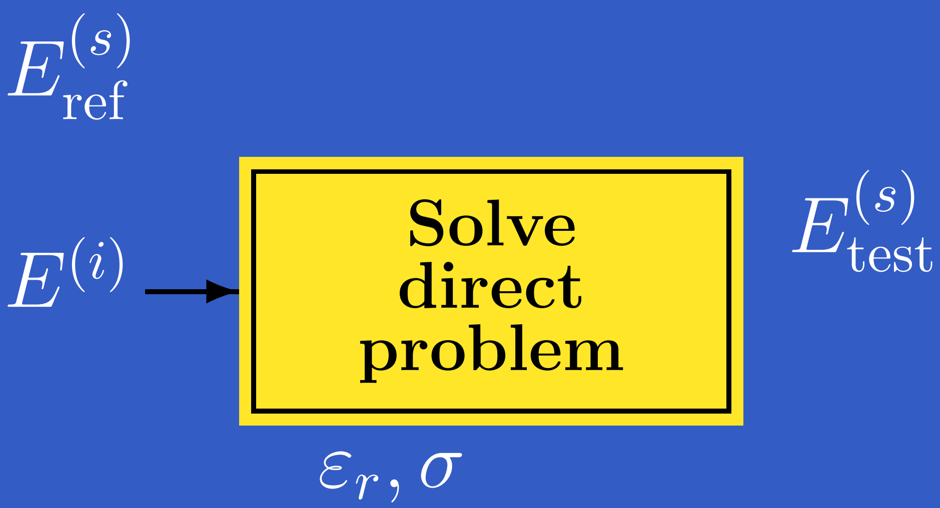
Iterative algorithm

$E^{(s)}_{\mathrm{ref}}$

$E^{(i)} \longrightarrow$ **Solve direct problem**

ε_r, σ

Iterative algorithm



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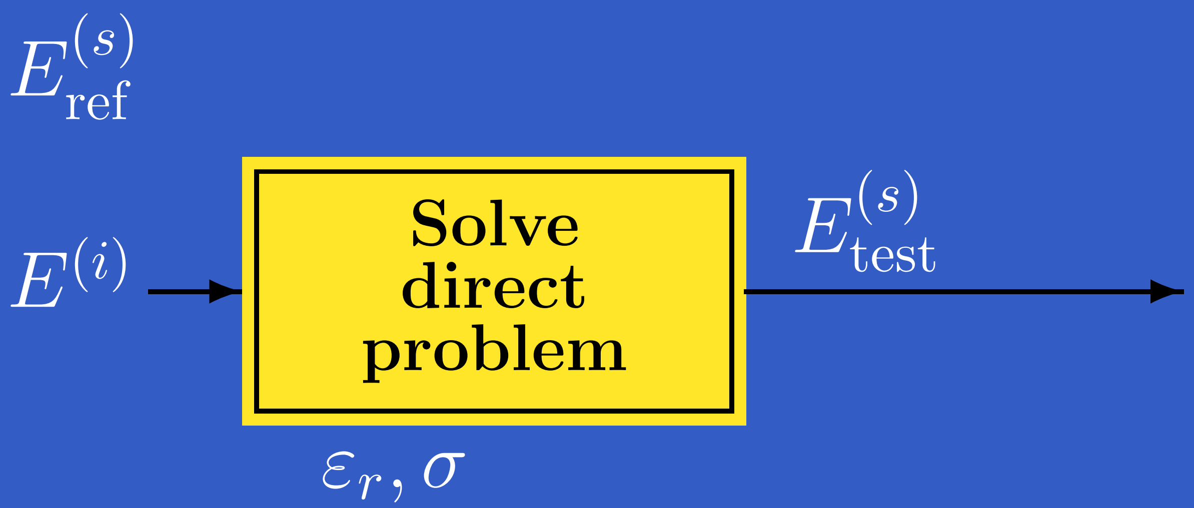

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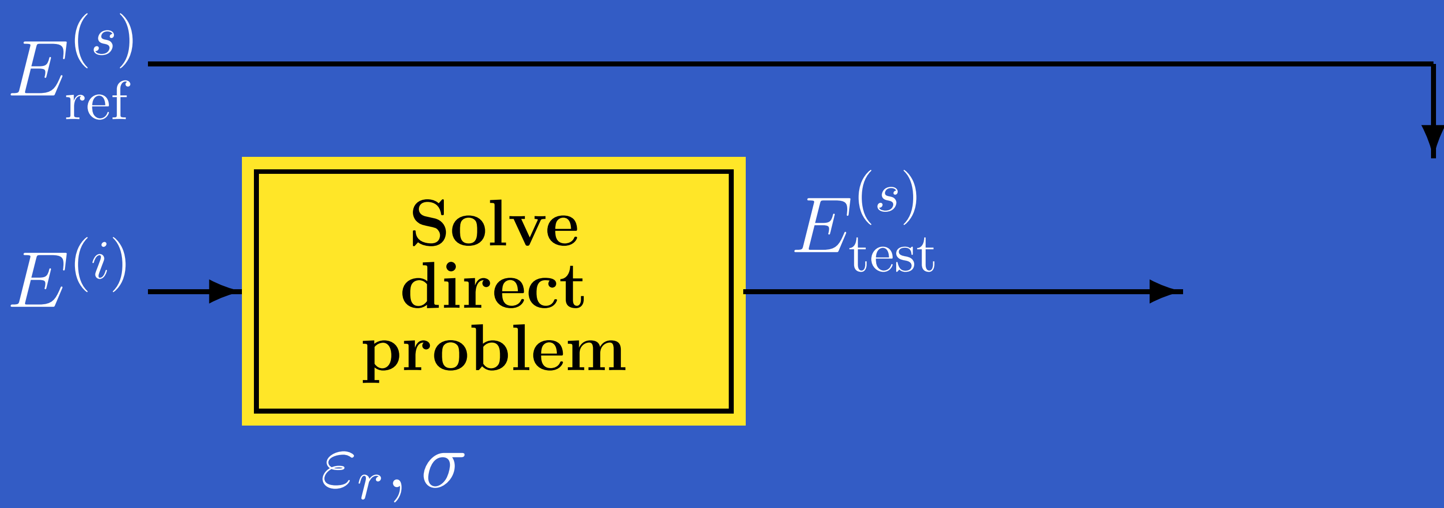

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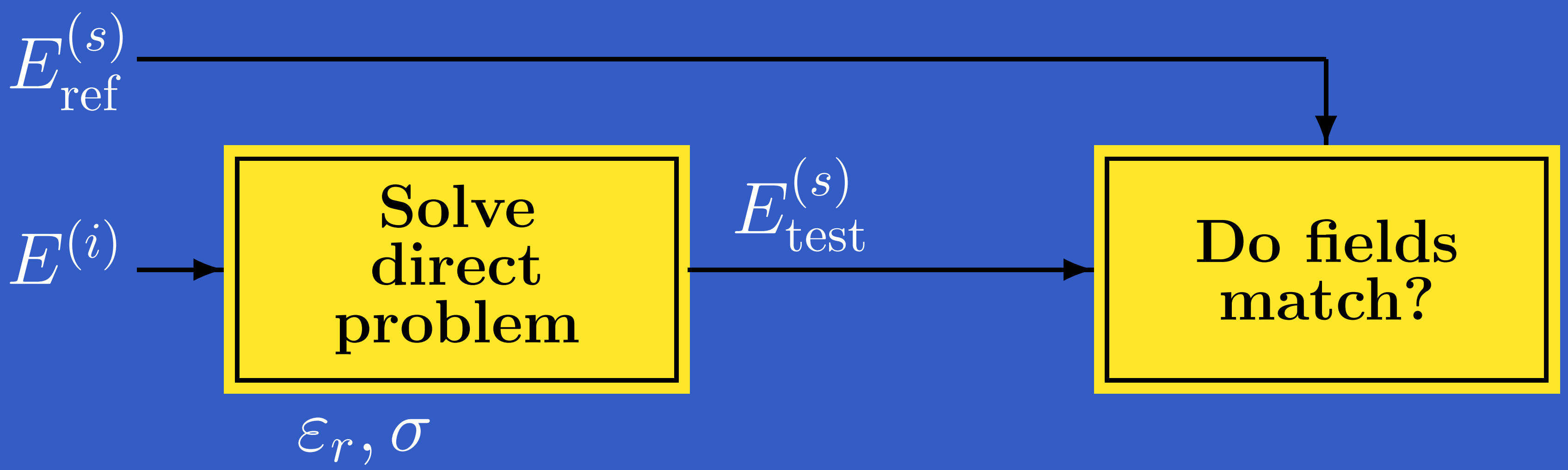

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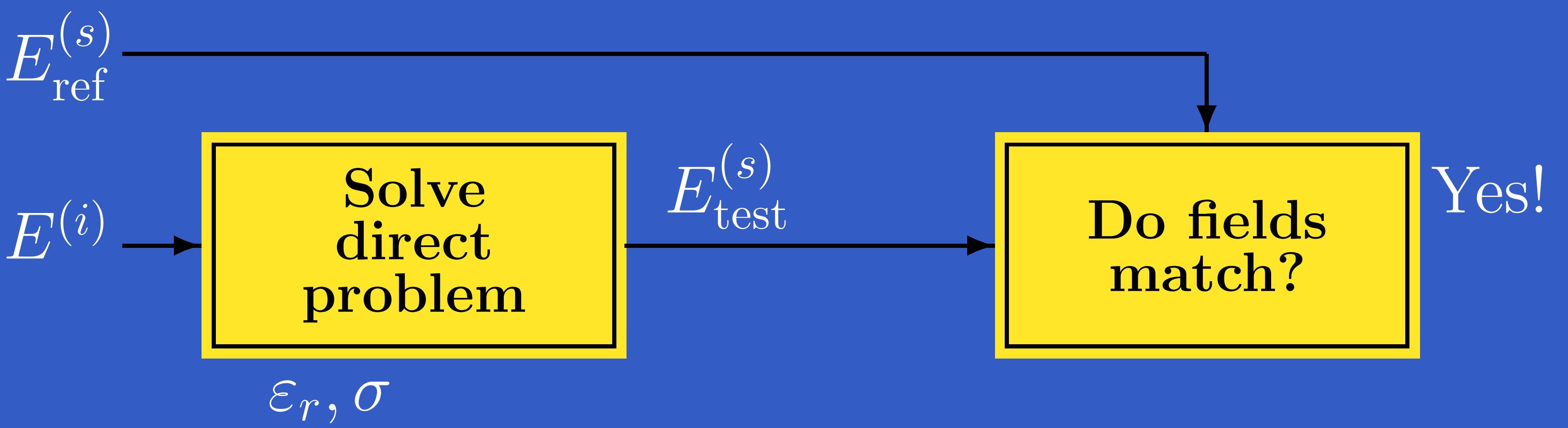

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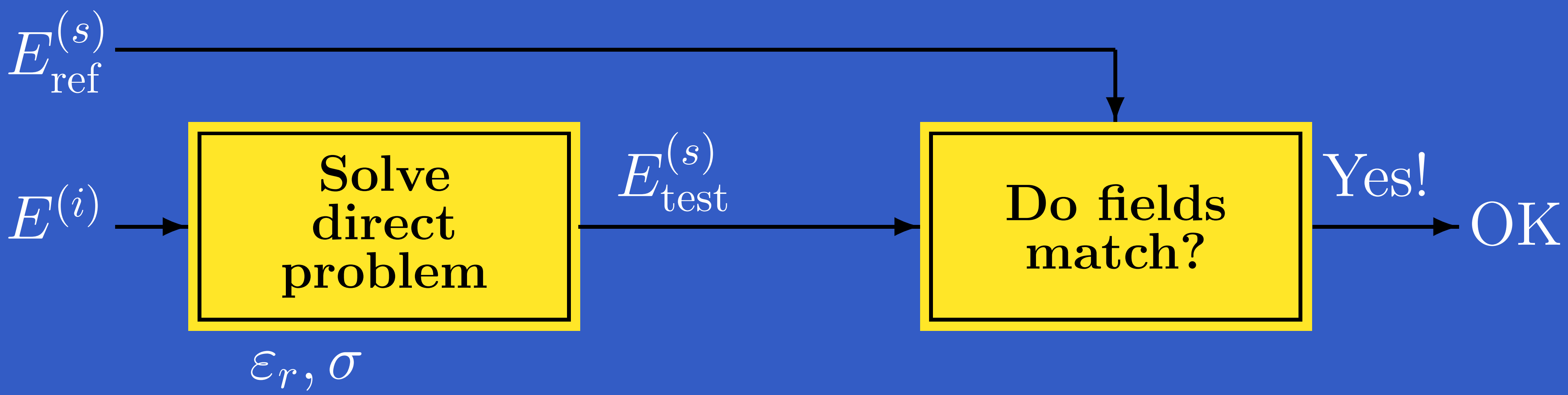

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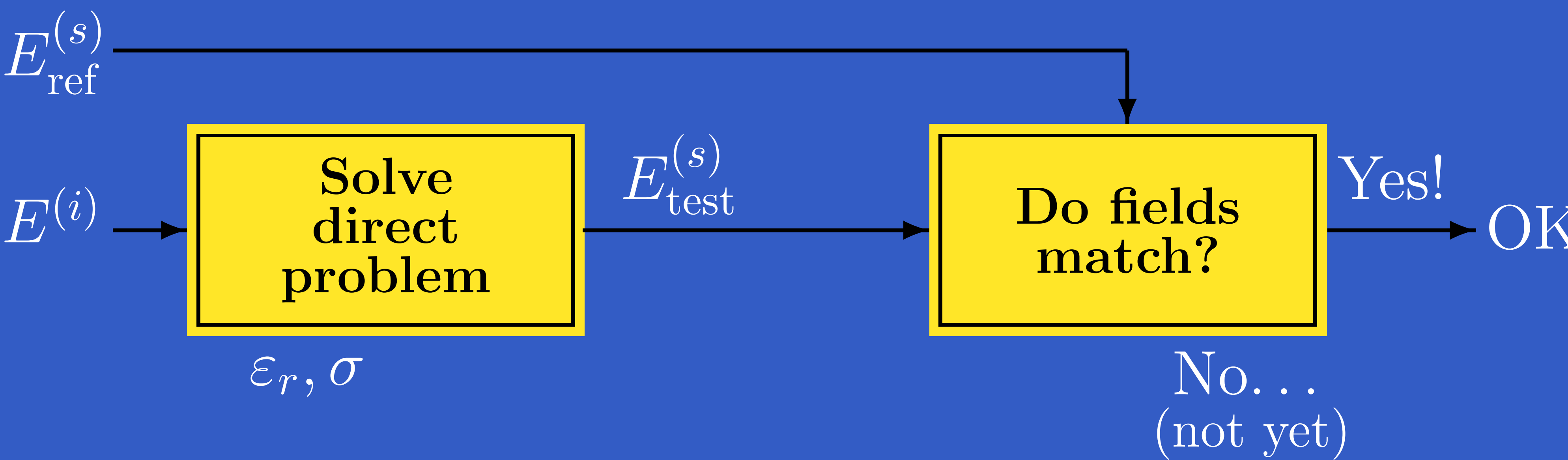

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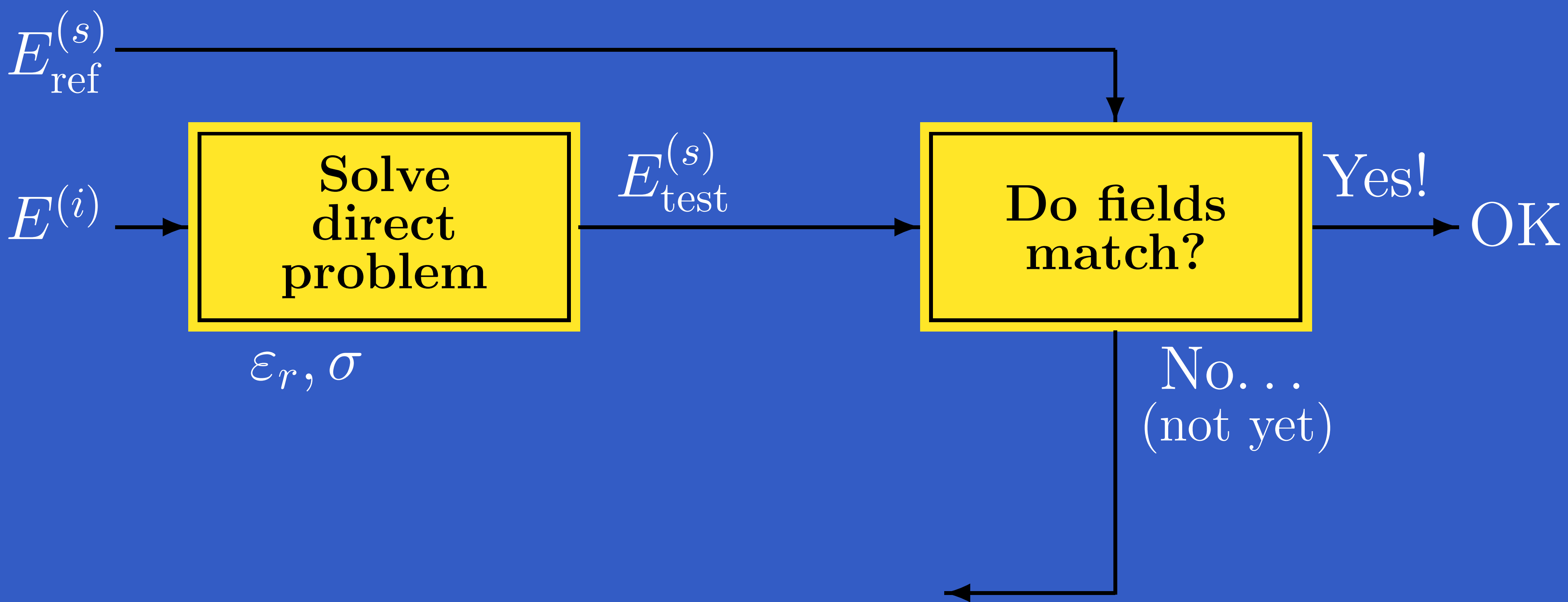

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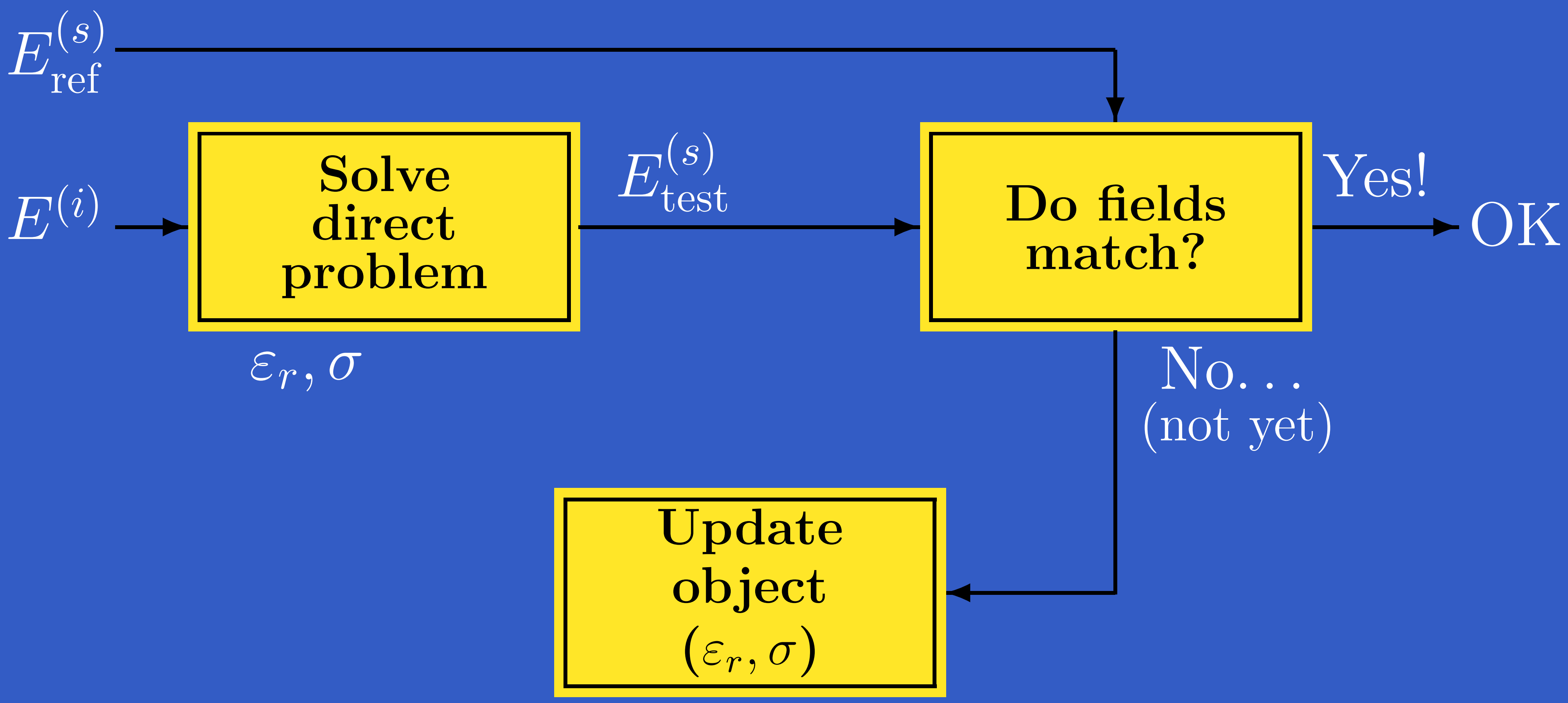

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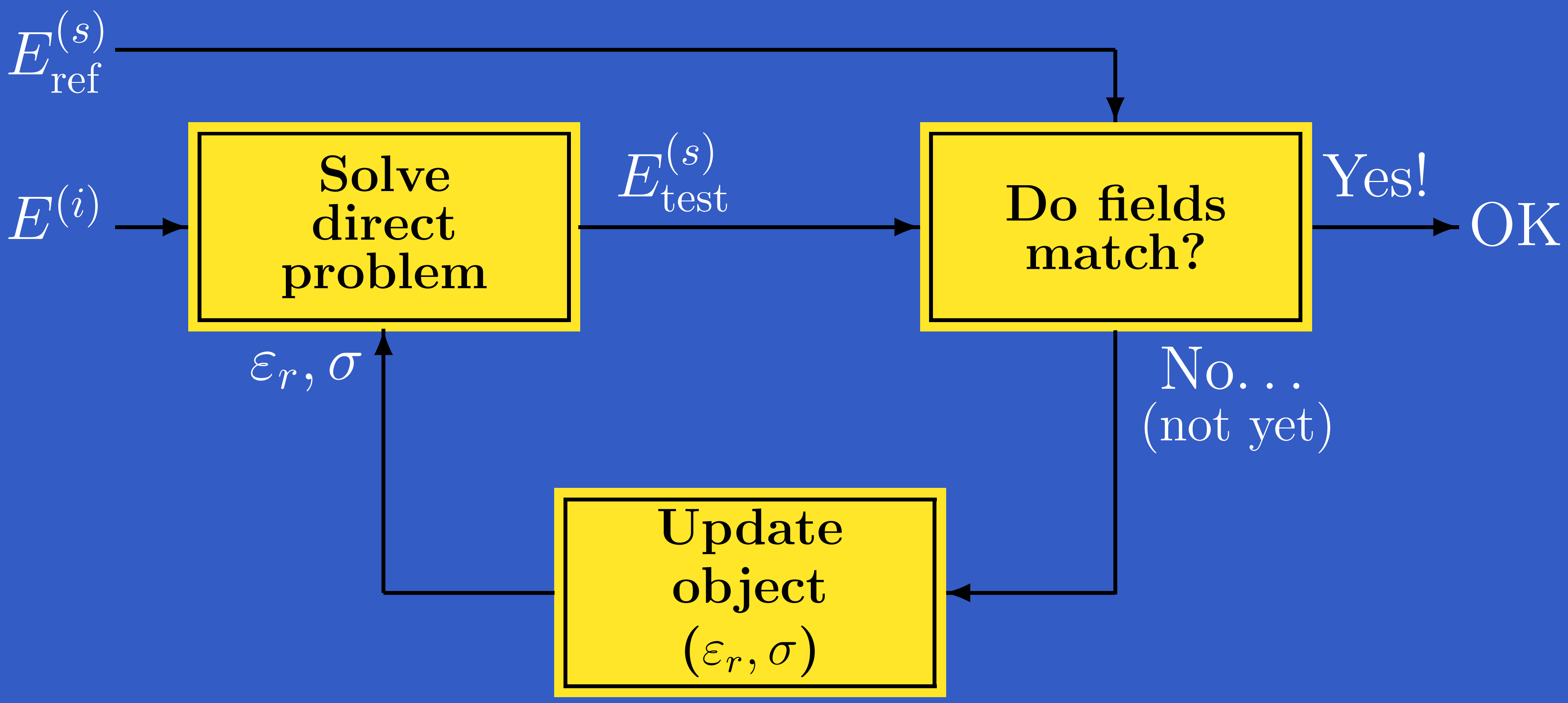

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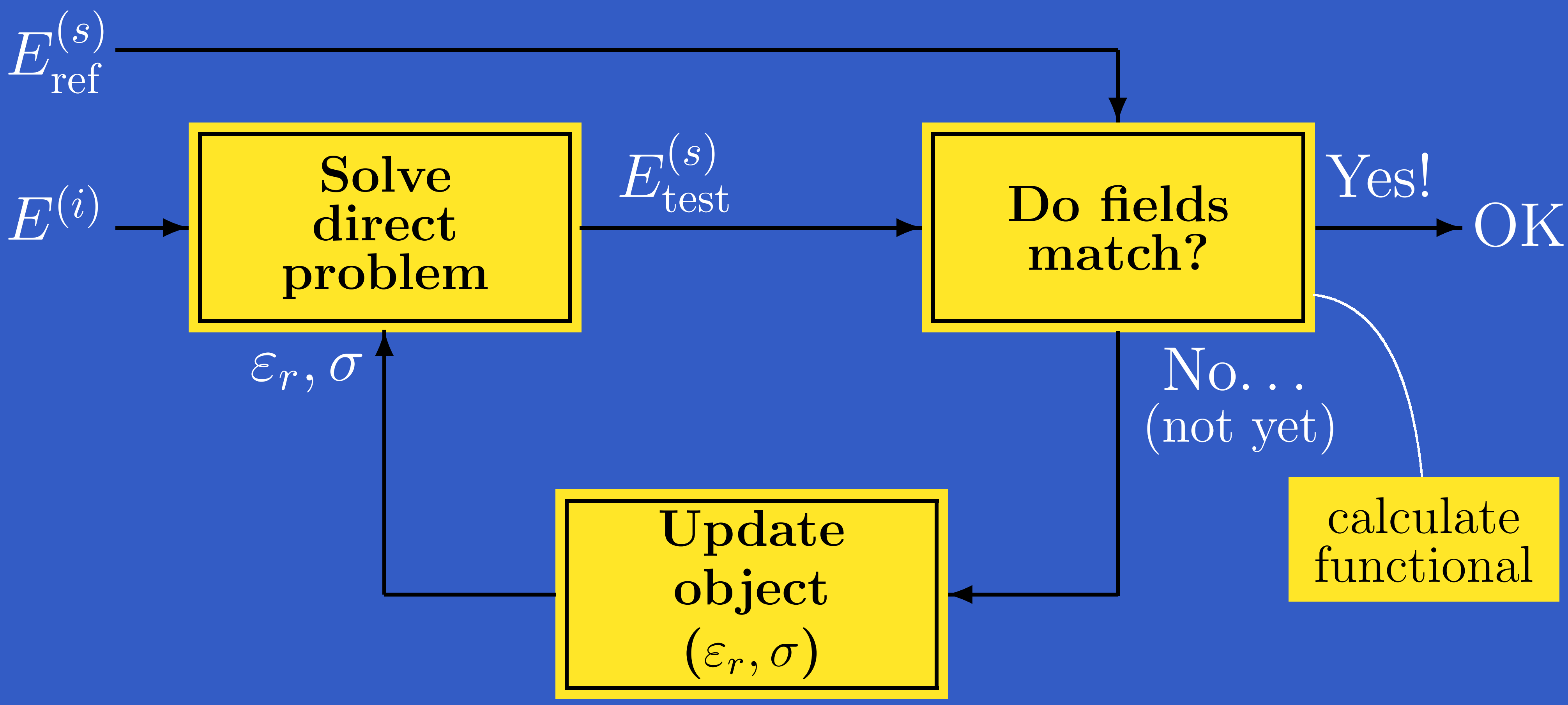

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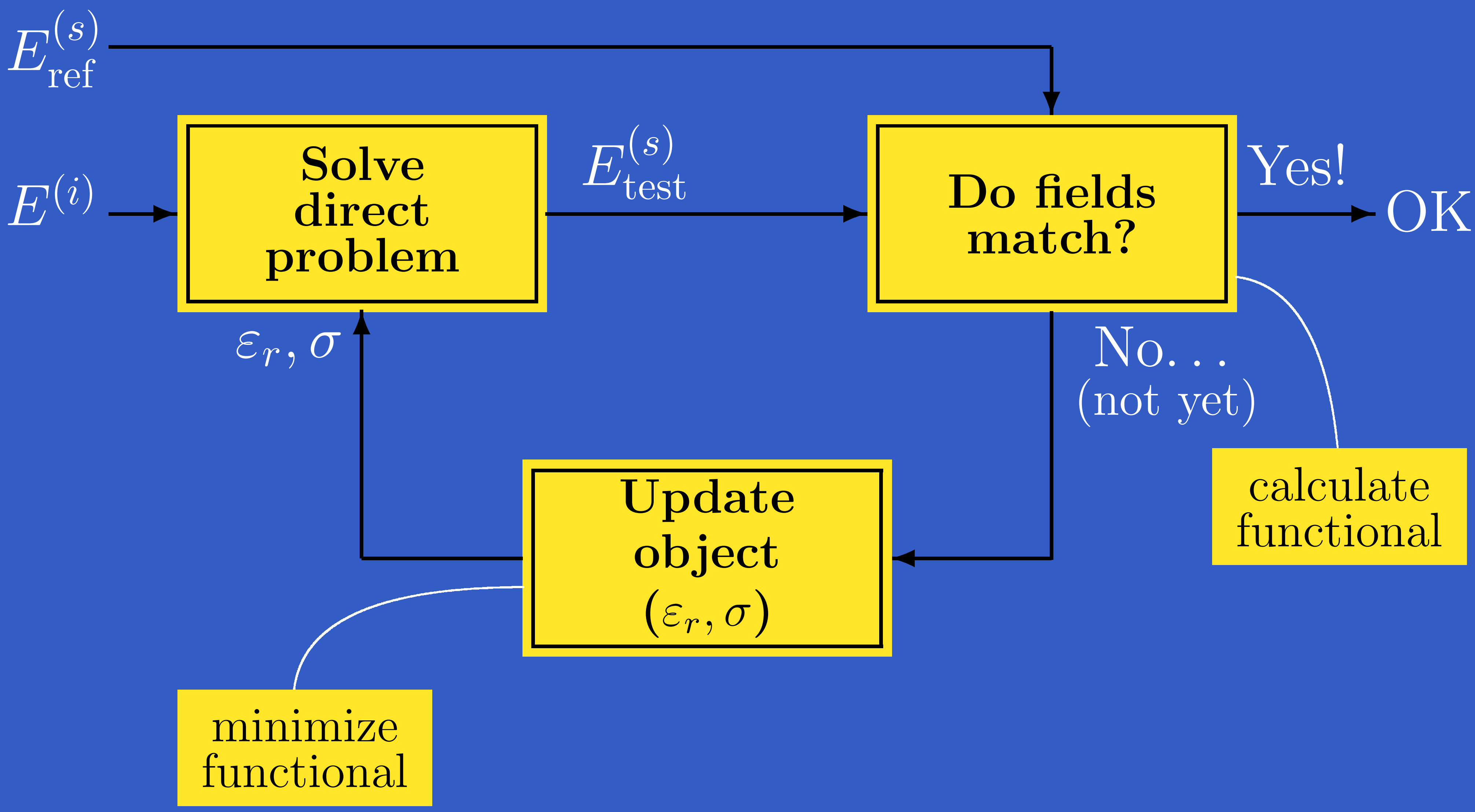

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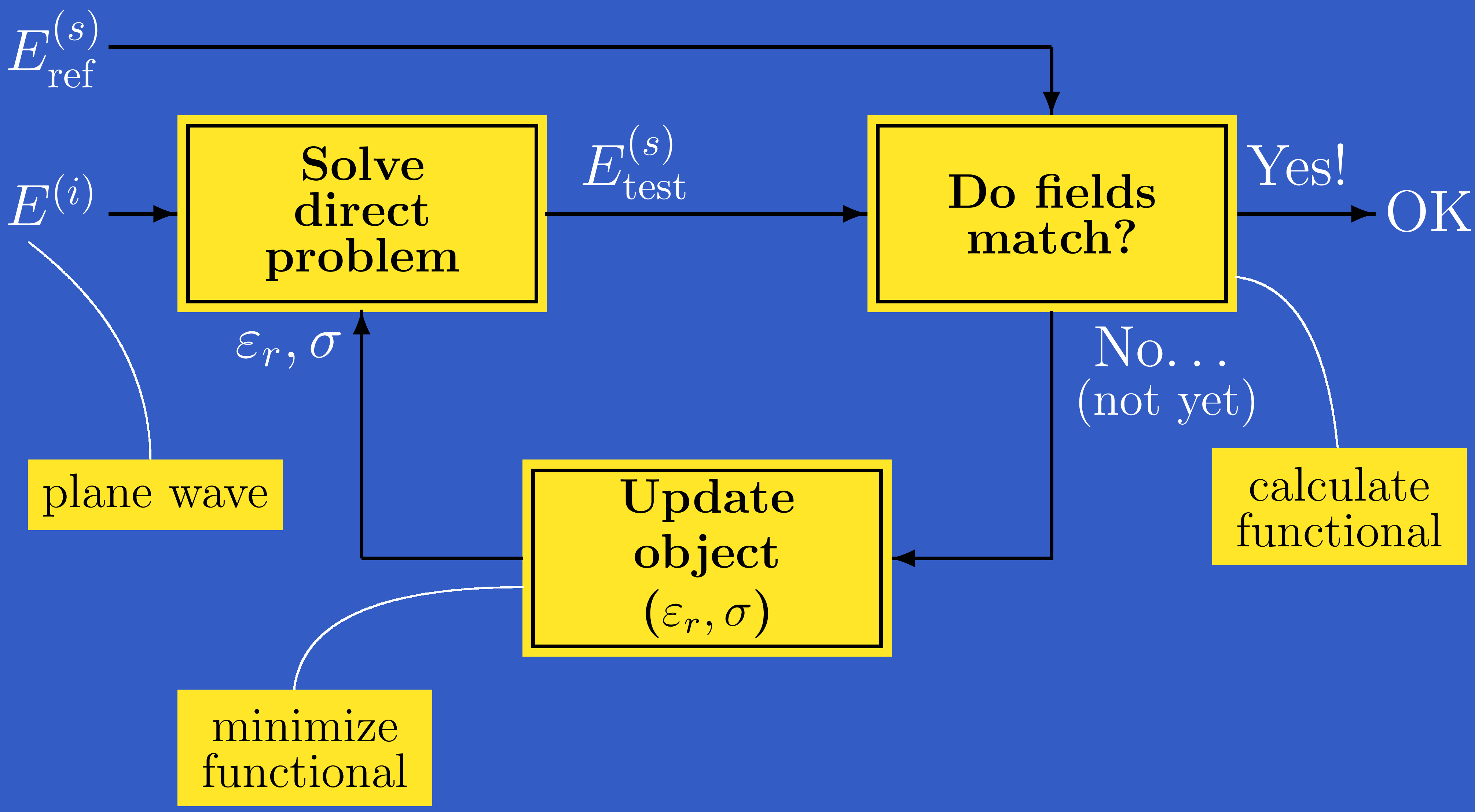

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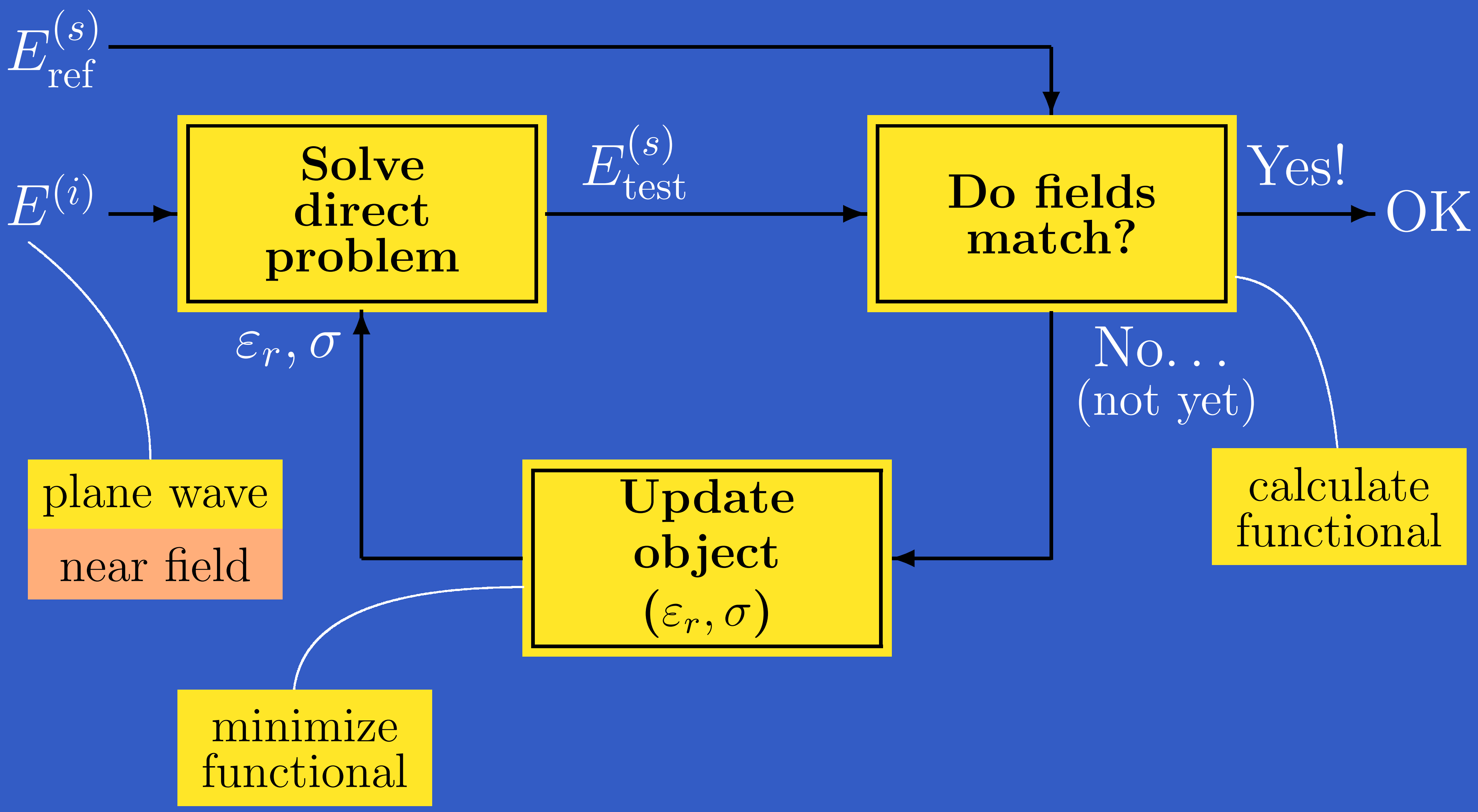

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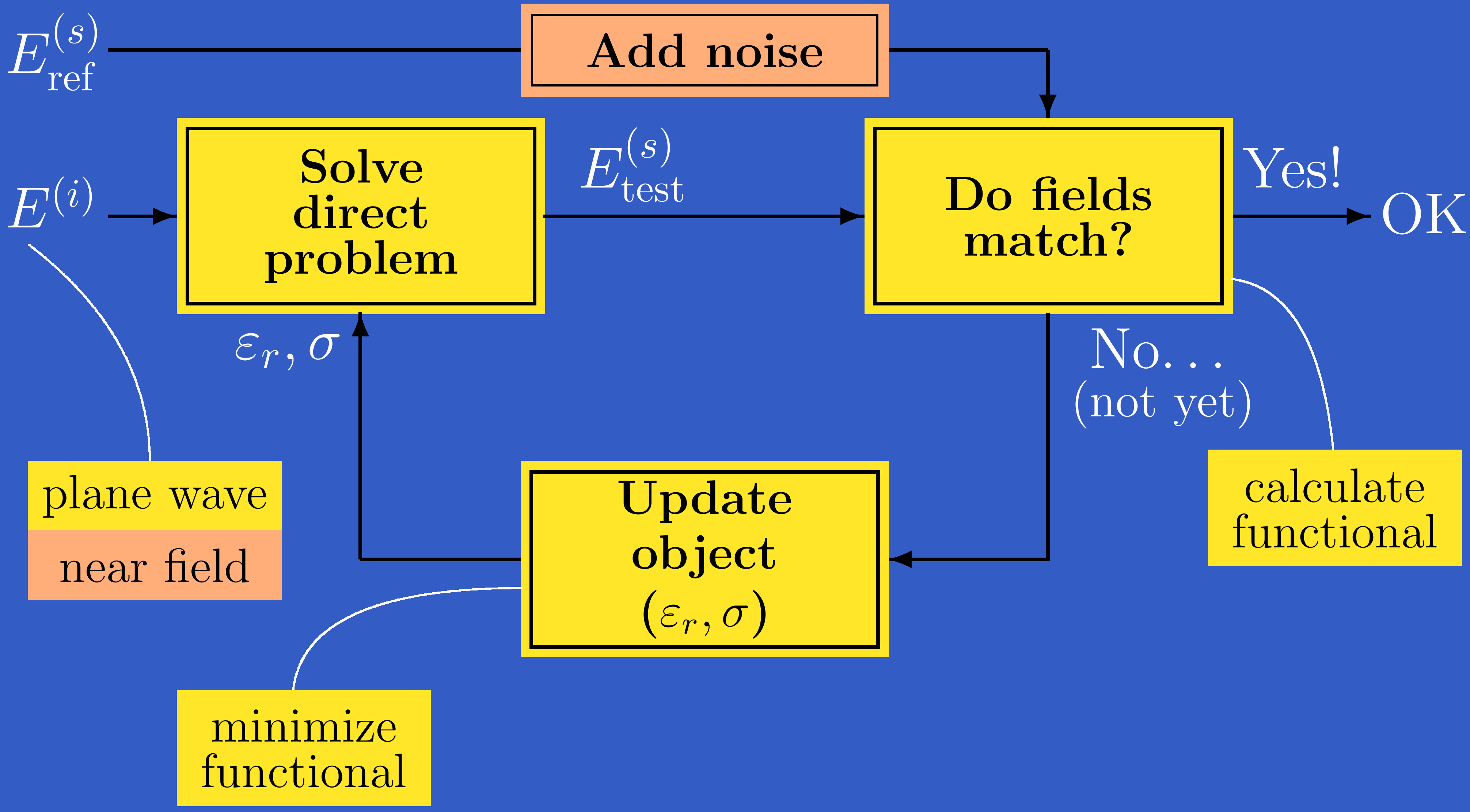

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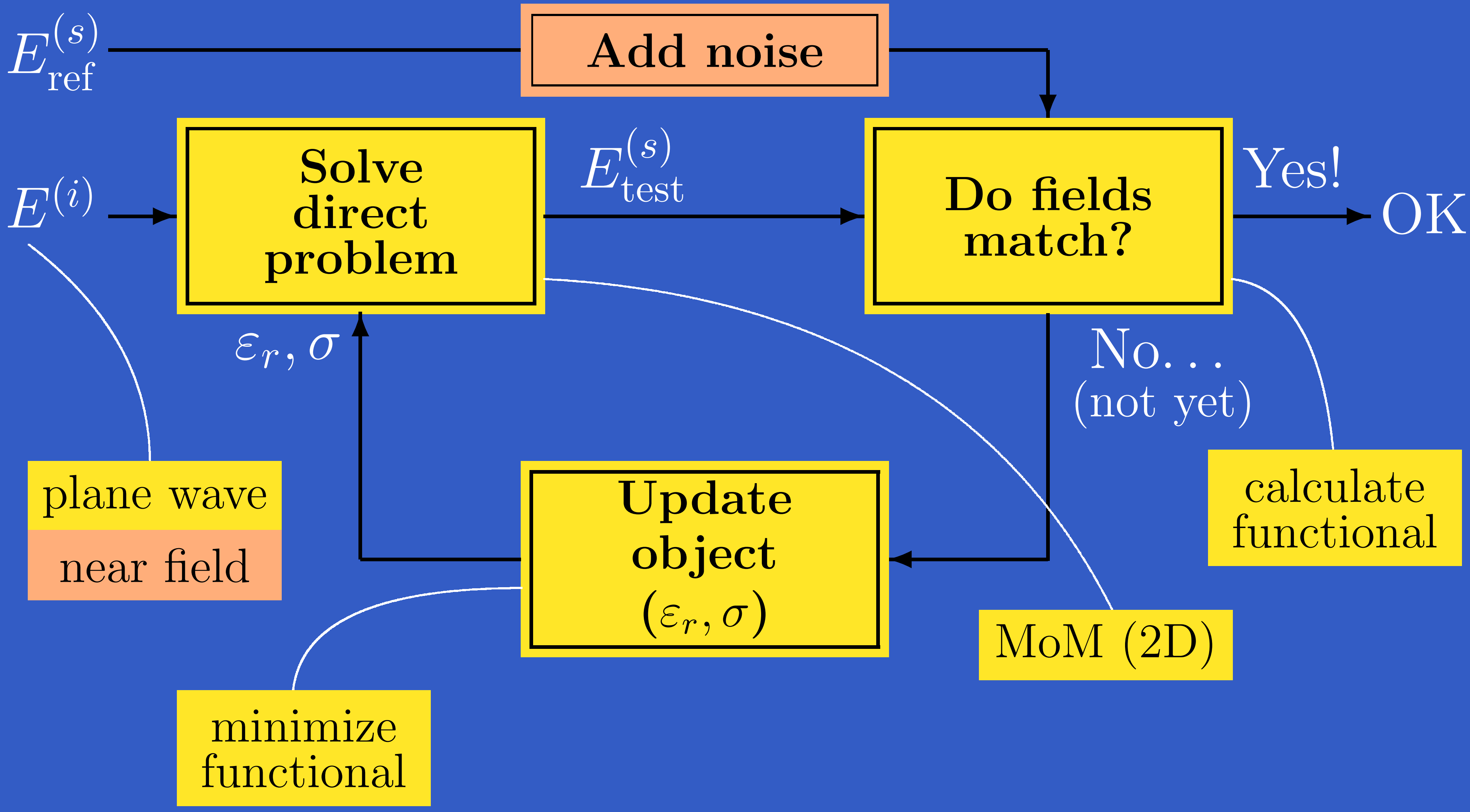

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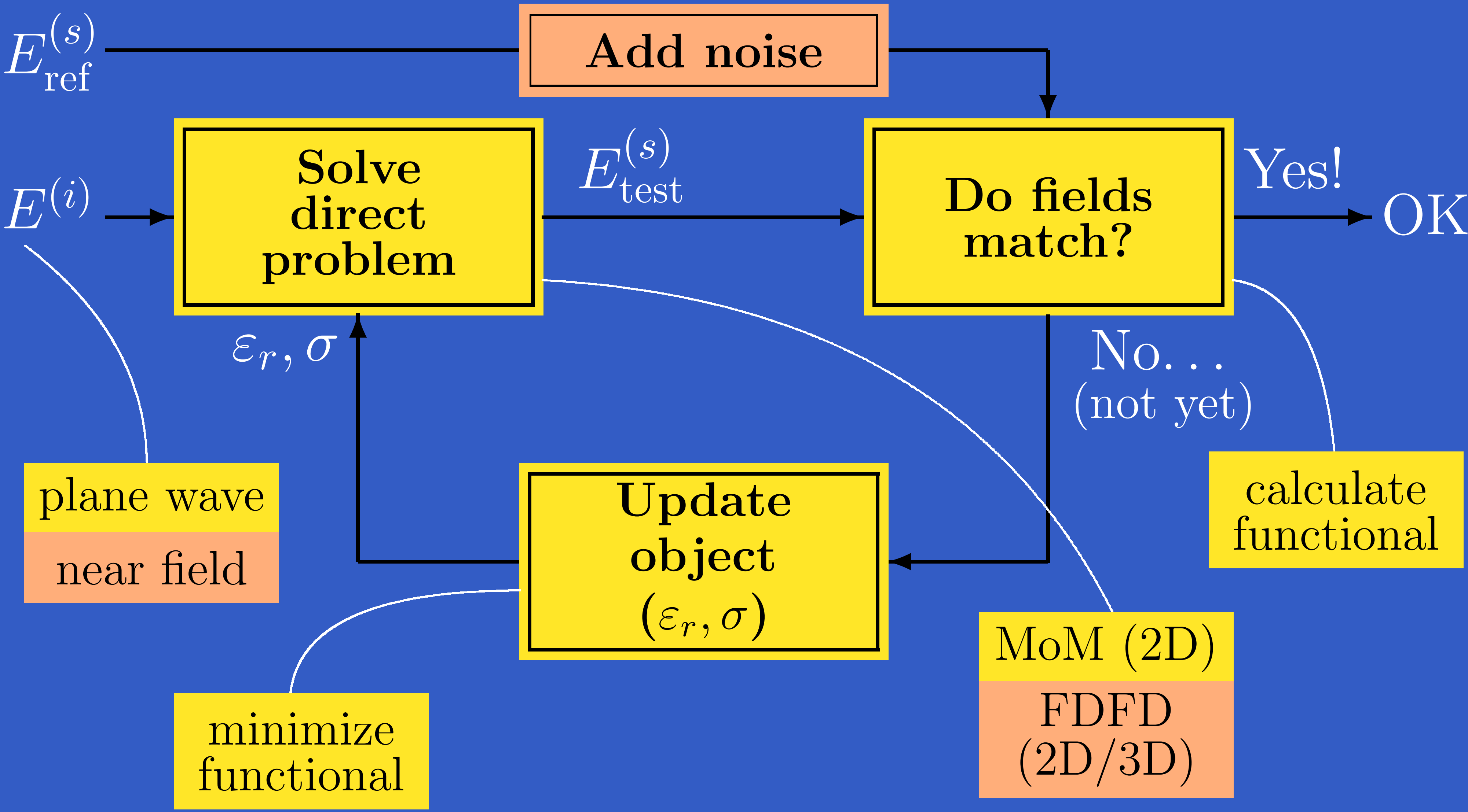

Two-dimensional numerical modeling

Contents I

- Microwave tomography
- Extensions
- Numerical results

Direct problem

- Transverse magnetic (TM) polarization

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- Electric field integral equation (EFIE)

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- Method of moments (pulse basis, point matching)

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$$\left.\begin{aligned} \mathbf{e}^{(i)} &= (\mathbf{I}_{N_C} - \mathbf{G}^{O}\mathbf{C})\mathbf{e} \\ \mathbf{e}^{(s)} &= \mathbf{G}^{R}\mathbf{C}\mathbf{e} \end{aligned}\right\}$$

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$$\mathbf{C} = \mathbf{diag}(\varepsilon_{\mathsf{r}}(\omega)) - \mathrm{j}\,\frac{\mathbf{diag}(\boldsymbol{\sigma}(\omega))}{\omega\varepsilon_0} - \left(\frac{\varepsilon_{N_L}(\omega)}{\varepsilon_0} - \mathrm{j}\,\frac{\sigma_{N_L}(\omega)}{\omega\varepsilon_0}\right)\mathbf{I}_{\mathsf{N_c}}$$

Inverse problem (1/2)

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- Cost functional:

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$$J(\varepsilon_{\mathbf{r}}, \boldsymbol{\sigma}) \triangleq \sum_{f=1}^{N_F} \sum_{s=1}^{N_S} \left\| \mathbf{r}_{\mathbf{f},\mathbf{s}}(\varepsilon_{\mathbf{r}}, \boldsymbol{\sigma}) \right\|_{L_M}^2$$

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- Iterative minimization (biconjugate gradient)

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$$J(\varepsilon_{\mathbf{r}}, \boldsymbol{\sigma}) \triangleq \sum_{f=1}^{N_F} \sum_{s=1}^{N_S} \|\mathbf{r}_{\mathbf{f,s}}(\varepsilon_{\mathbf{r}}, \boldsymbol{\sigma})\|_{L_M}^2$$

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$$\varepsilon_{\mathbf{r}}^{\mathbf{k+1}} = \varepsilon_{\mathbf{r}}^{\mathbf{k}} + \alpha_{\varepsilon_r}^k \boldsymbol{\eta}_{\varepsilon_{\mathbf{r}}}^{\mathbf{k}} \qquad \boldsymbol{\sigma}^{\mathbf{k+1}} = \boldsymbol{\sigma}^{\mathbf{k}} + \alpha_{\sigma}^k \boldsymbol{\eta}_{\boldsymbol{\sigma}}^{\mathbf{k}}$$

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  - Existence?

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  - Smooth *below* threshold

Inverse problem (2/2)

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  - Piece-wise constant profile
  - Smooth *below* threshold

$$\zeta_{\varepsilon_r}^2 \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \phi \left(\frac{\|(\boldsymbol{\nabla} \boldsymbol{\varepsilon}_{\mathbf{r}})_{i,j}\|}{\delta_{\varepsilon_r}} \right) + \zeta_{\sigma}^2 \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \phi \left(\frac{\|(\boldsymbol{\nabla} \boldsymbol{\sigma})_{i,j}\|}{\delta_{\sigma}} \right)$$

Emitting antenna modeling

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- Plane wave

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- ✗ Plane wave
- ✓ Simulated incident field

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- √ Simulated incident field
- ▼ SR3D, France Telecom Research Center, La Turbie

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  Bow-tie antenna (cells of $2.5 \times 2.5\,\mathrm{cm}^2$):

Emitting antenna modeling

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- √ Simulated incident field
- ▼ SR3D, France Telecom Research Center, La Turbie

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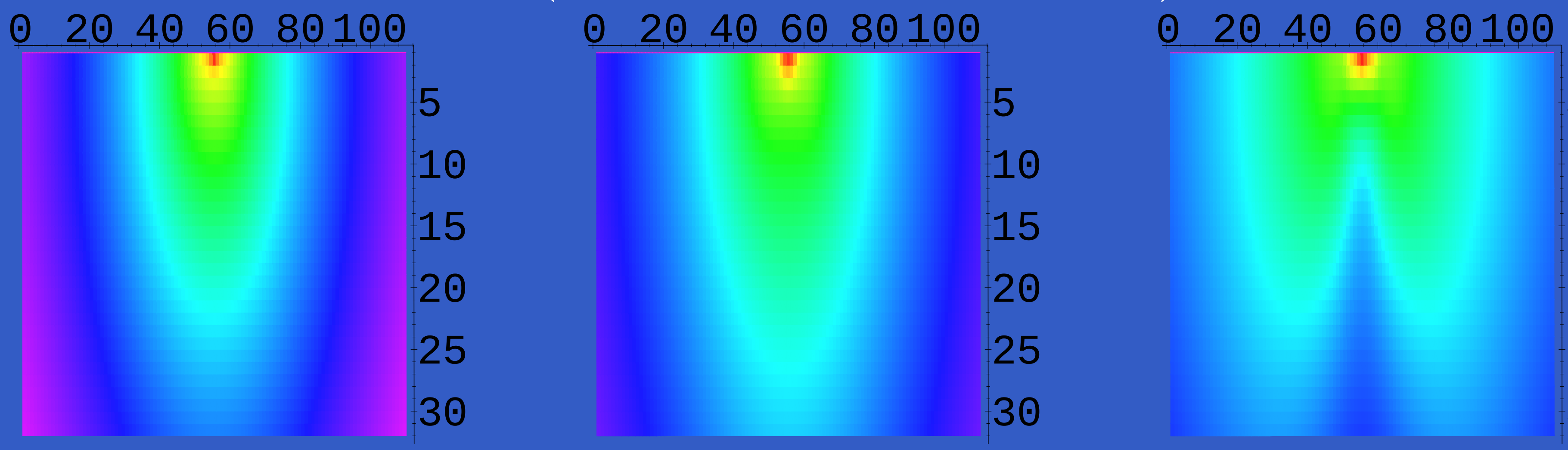

0.3 GHz　　0.8 GHz　　1.3 GHz

Measurement noise modeling

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- White, Gaussian, zero mean

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- Add separately to real/imaginary part

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- Fix SNR; choose σ for each frequency/emitter

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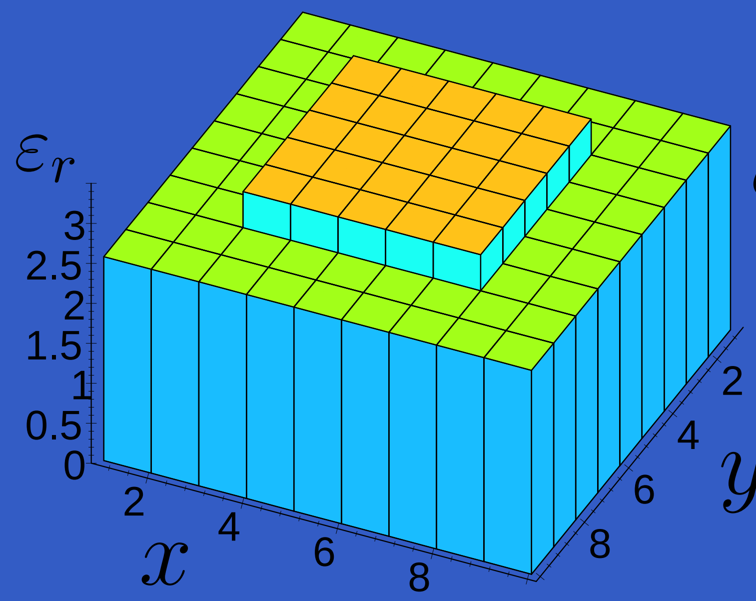
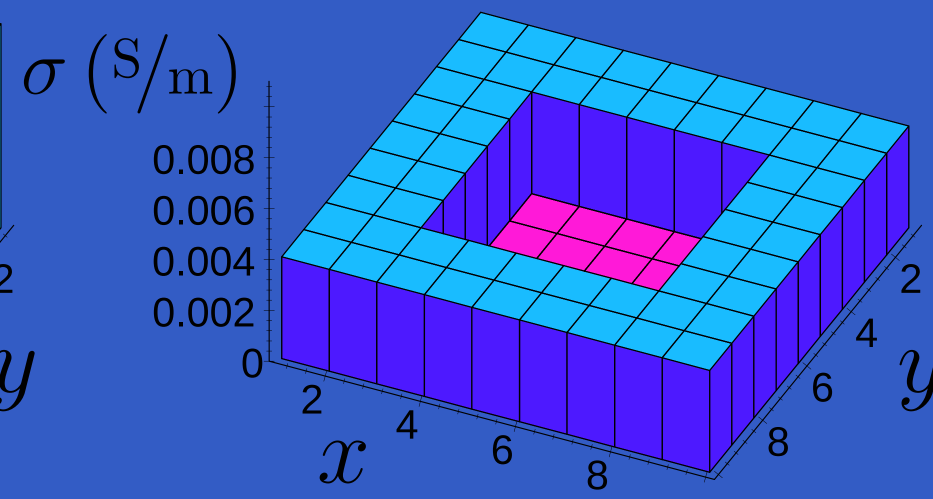
$$E_{\text{signal}} = \frac{1}{N_M} \sum_{i=1}^{N_M} \mathrm{Re}\left[\left(\mathbf{e}_{\mathbf{f},\mathbf{s}}^{(\mathbf{s})} \right)_i \right]^2 \quad \text{real part}$$

$$E_{\text{signal}} = \frac{1}{N_M} \sum_{i=1}^{N_M} \mathrm{Im}\left[\left(\mathbf{e}_{\mathbf{f},\mathbf{s}}^{(\mathbf{s})} \right)_i \right]^2 \quad \text{imaginary part}$$

$$E_{\text{noise}} = \sigma^2$$

Test suite: the real object





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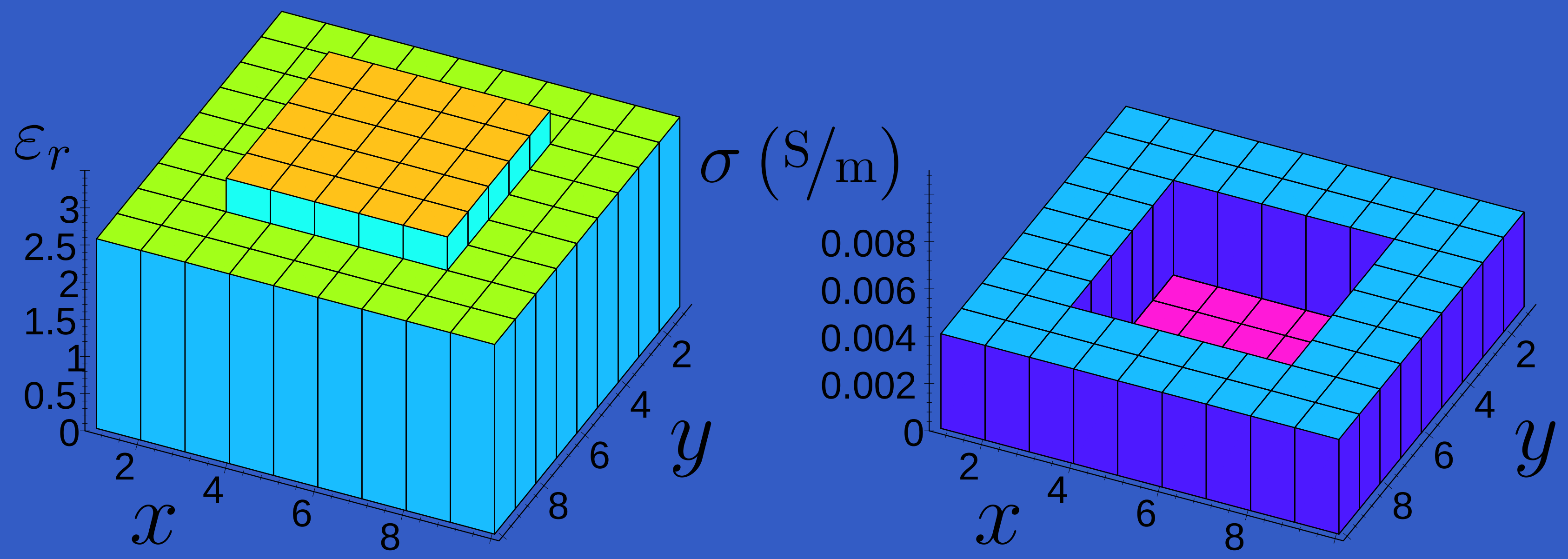
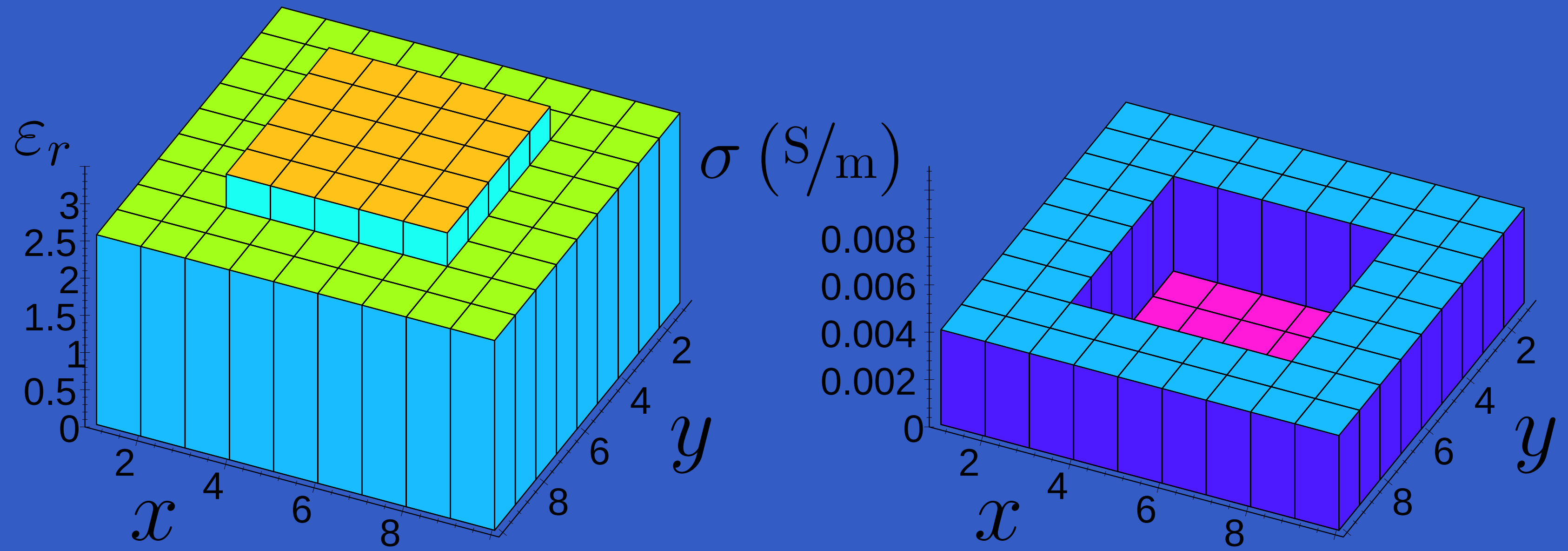


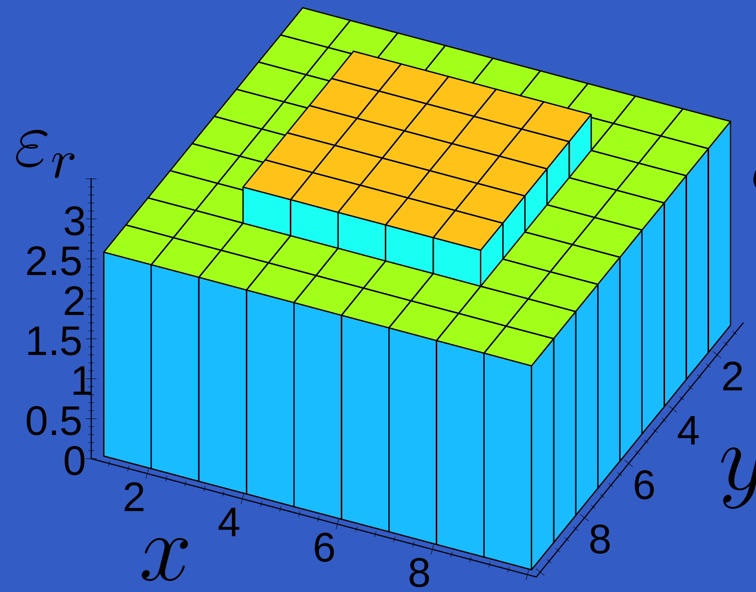
- $\varepsilon_r = 3, \sigma = 0;$ ext: $\varepsilon_r = 2.55, \sigma = 4 \cdot 10^{-3}\ \mathrm{S/m}$

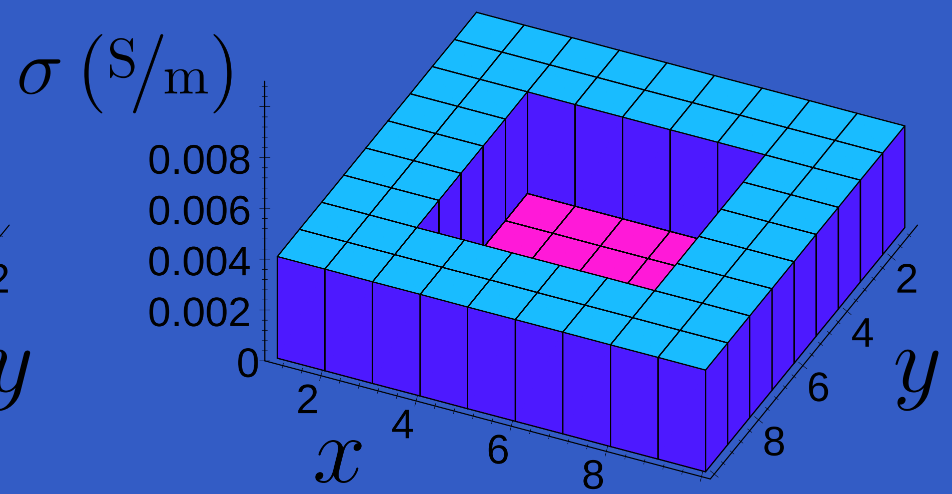
Test suite: the real object



- $\varepsilon_r = 3, \sigma = 0;$ ext: $\varepsilon_r = 2.55, \sigma = 4 \cdot 10^{-3}\,\mathrm{S/m}$
- $L_x = L_y = 22.5\,\mathrm{cm},\ h_d = 23.75\,\mathrm{cm},\ \Delta_x = \Delta_y = 2.5\,\mathrm{cm}$

Test suite: the real object





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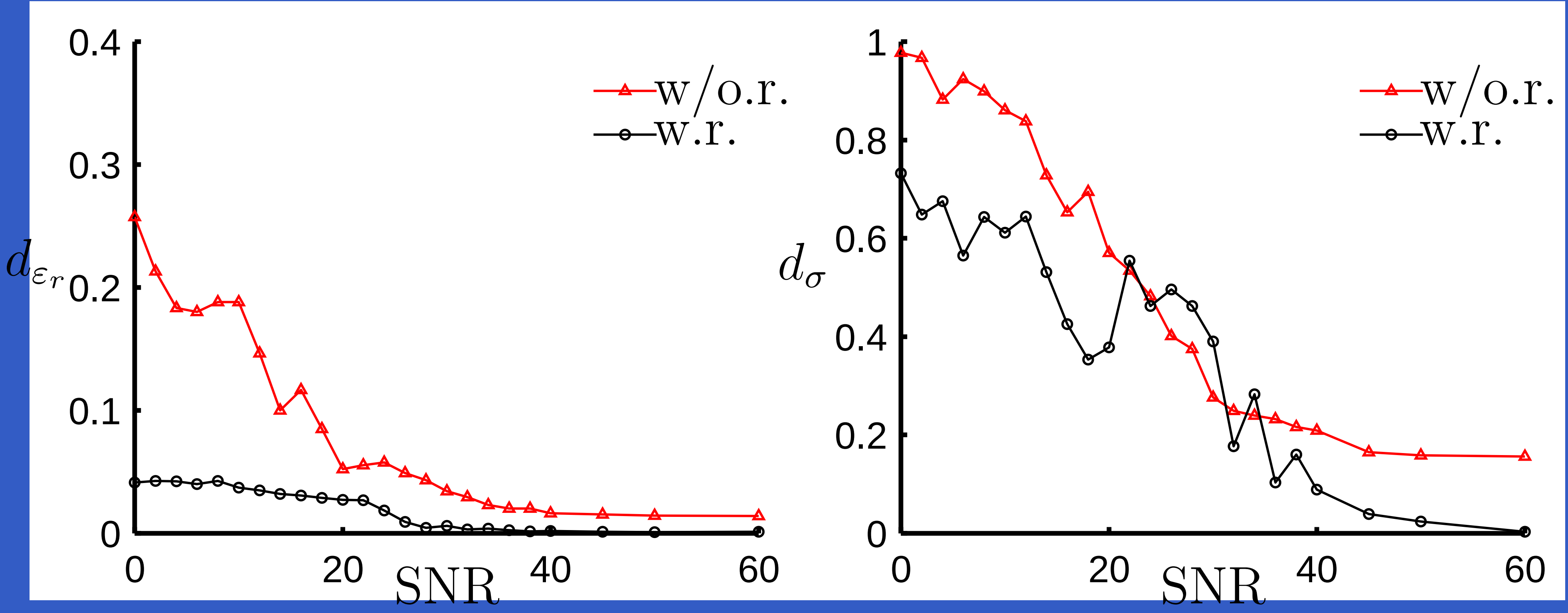
- $L_x = L_y = 22.5\,\mathrm{cm},\ h_d = 23.75\,\mathrm{cm},\ \Delta_x = \Delta_y = 2.5\,\mathrm{cm}$

$$d_{\varepsilon_r} \triangleq \frac{\left\|\boldsymbol{\varepsilon_r} - \boldsymbol{\varepsilon_r^d}\right\|}{\left\|\boldsymbol{\varepsilon_r^d}\right\|} \qquad d_\sigma \triangleq \frac{\left\|\boldsymbol{\sigma_r} - \boldsymbol{\sigma_r^d}\right\|}{\left\|\boldsymbol{\sigma_r^d}\right\|}$$

Robustness to noise (1/3)



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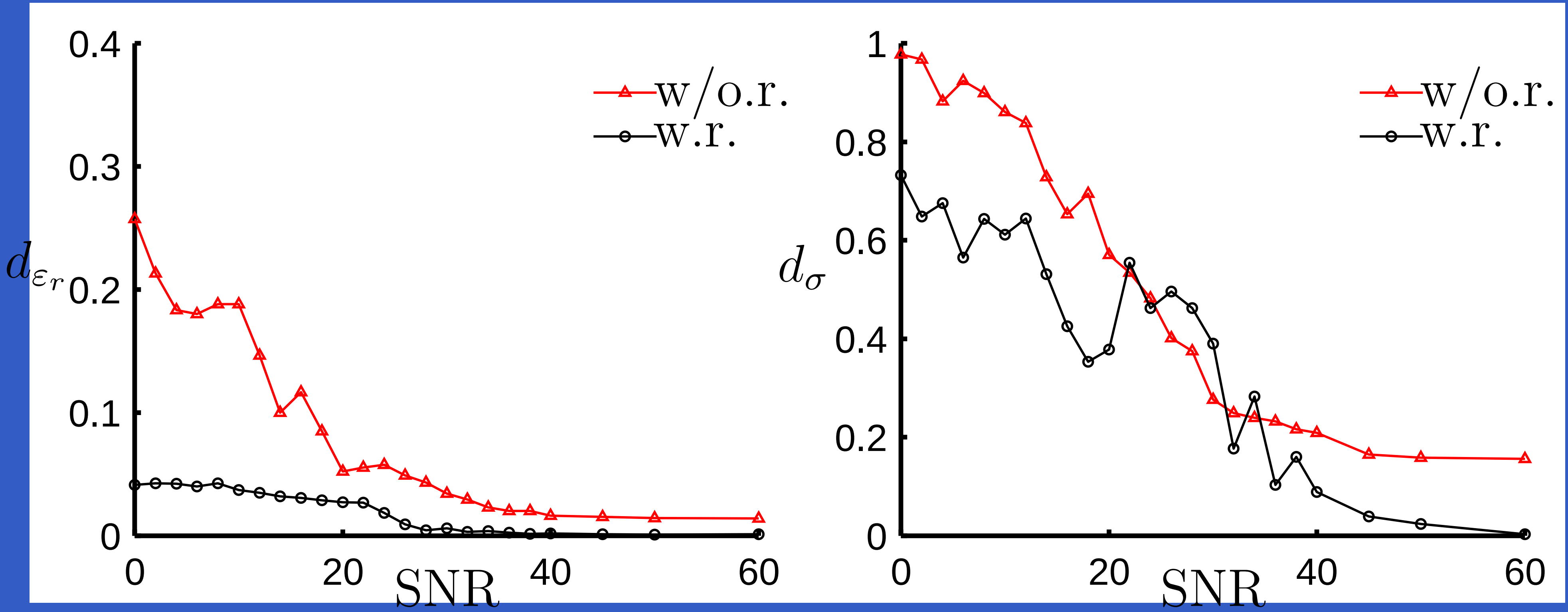
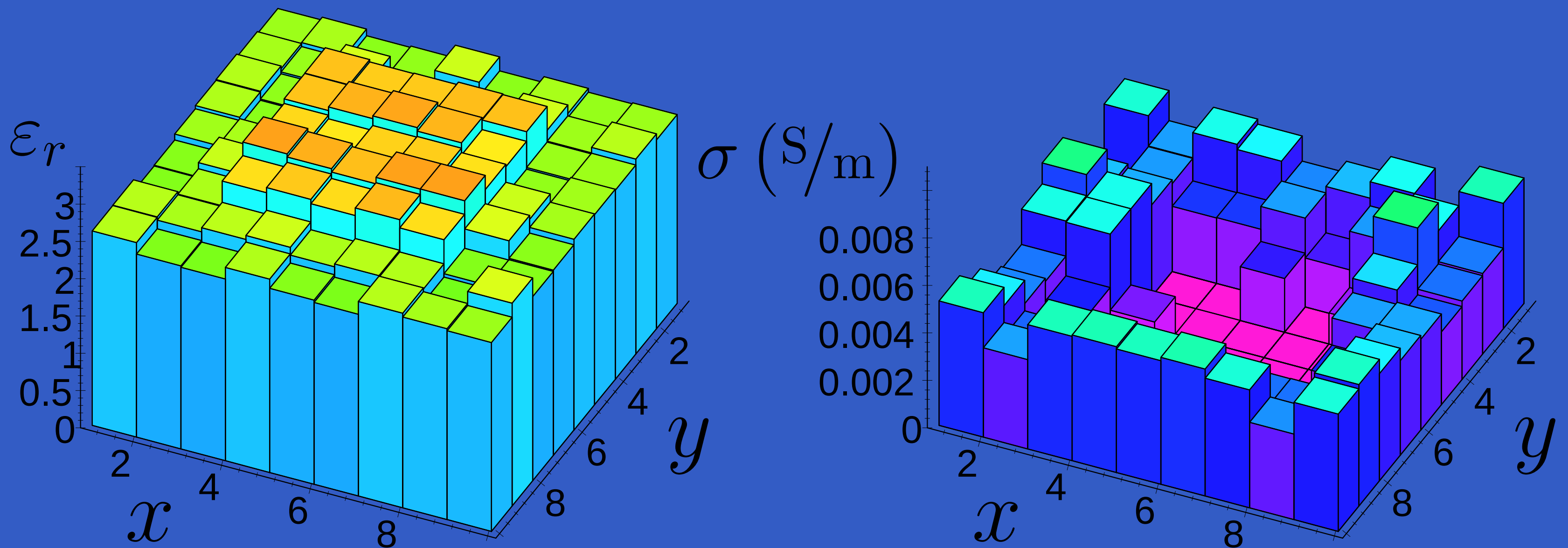


√ Permittivity

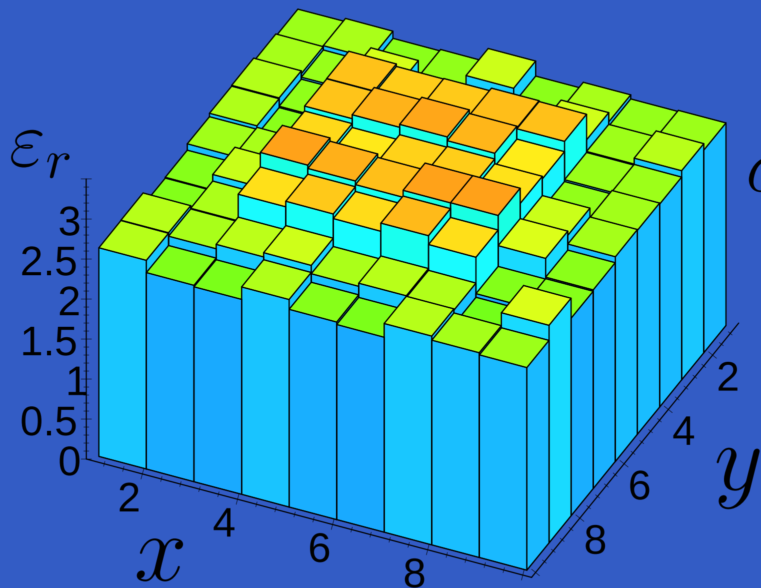
× Conductivity

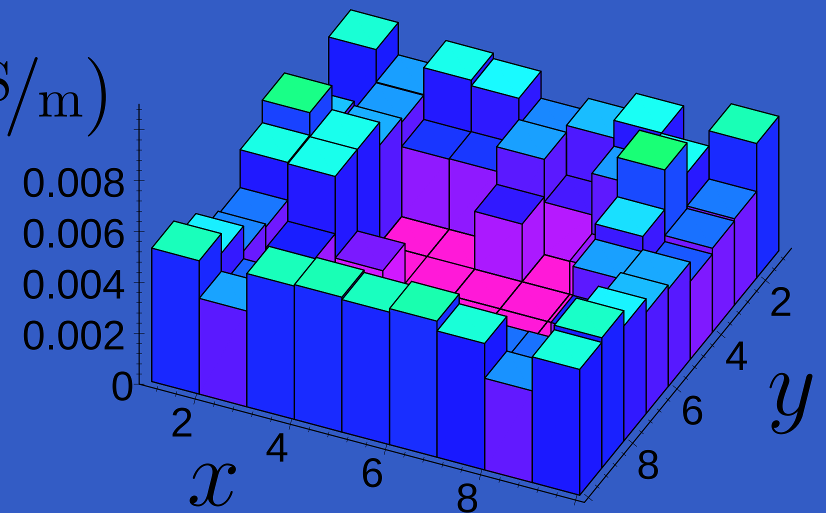
Robustness to noise (2/3)



SNR = 30 dB, without regularization

Robustness to noise (2/3)



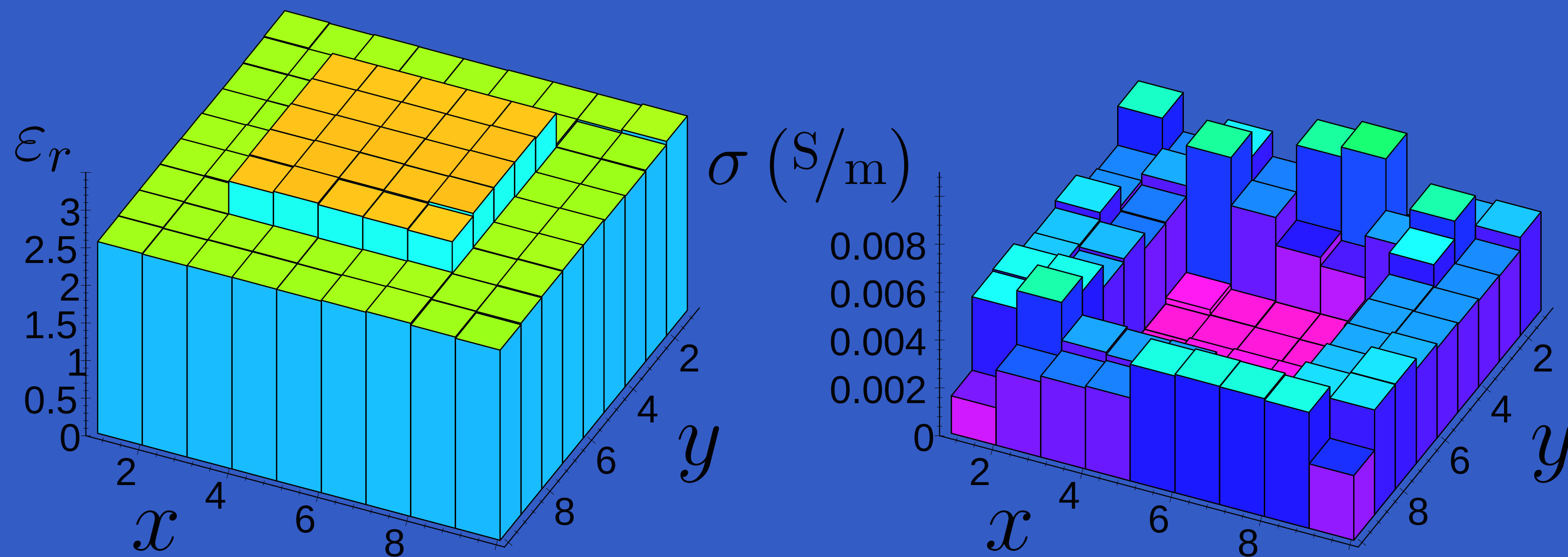


SNR $= 30\,\mathrm{dB}$, without regularization

$L_M = 1.5\,\mathrm{m}$ $\quad N_M = 21$ $\quad \Delta_M = 7.5\,\mathrm{cm}$

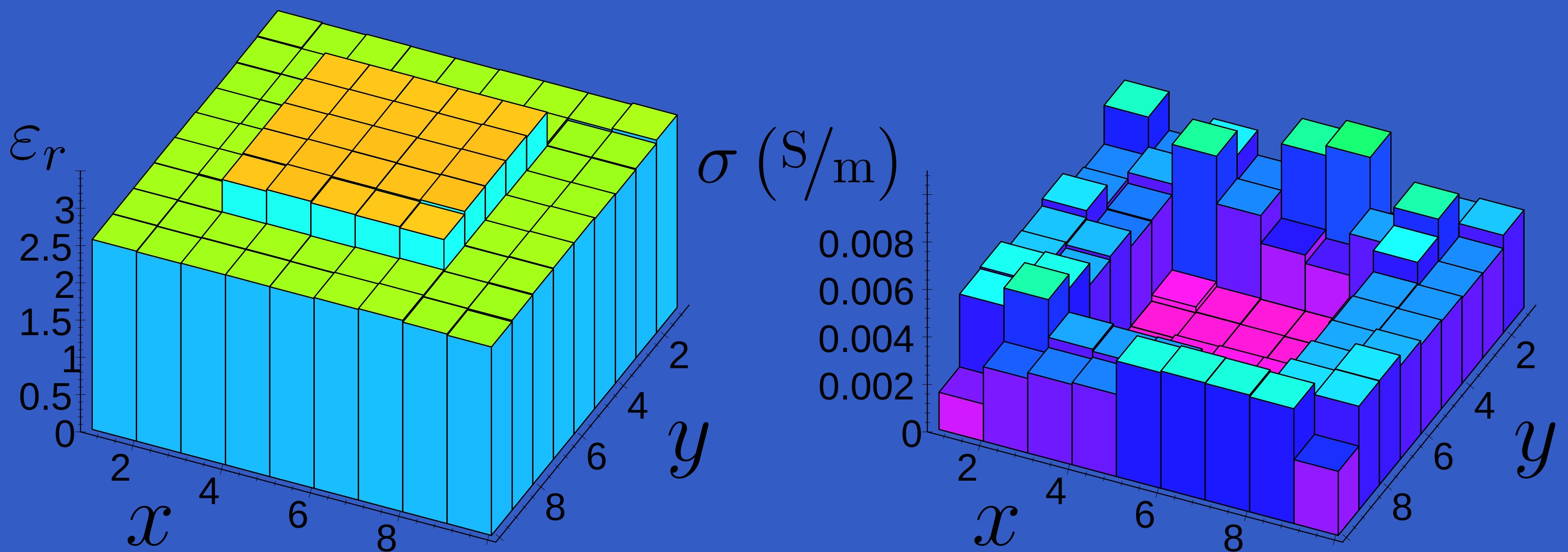
$f_{\min} = 0.3\,\mathrm{GHz}$ $\quad f_{\max} = 1.3\,\mathrm{GHz}$ $\quad N_F = 3$

Robustness to noise (3/3)



$\mathrm{SNR} = 30\,\mathrm{dB}$, *with regularization*

Robustness to noise (3/3)

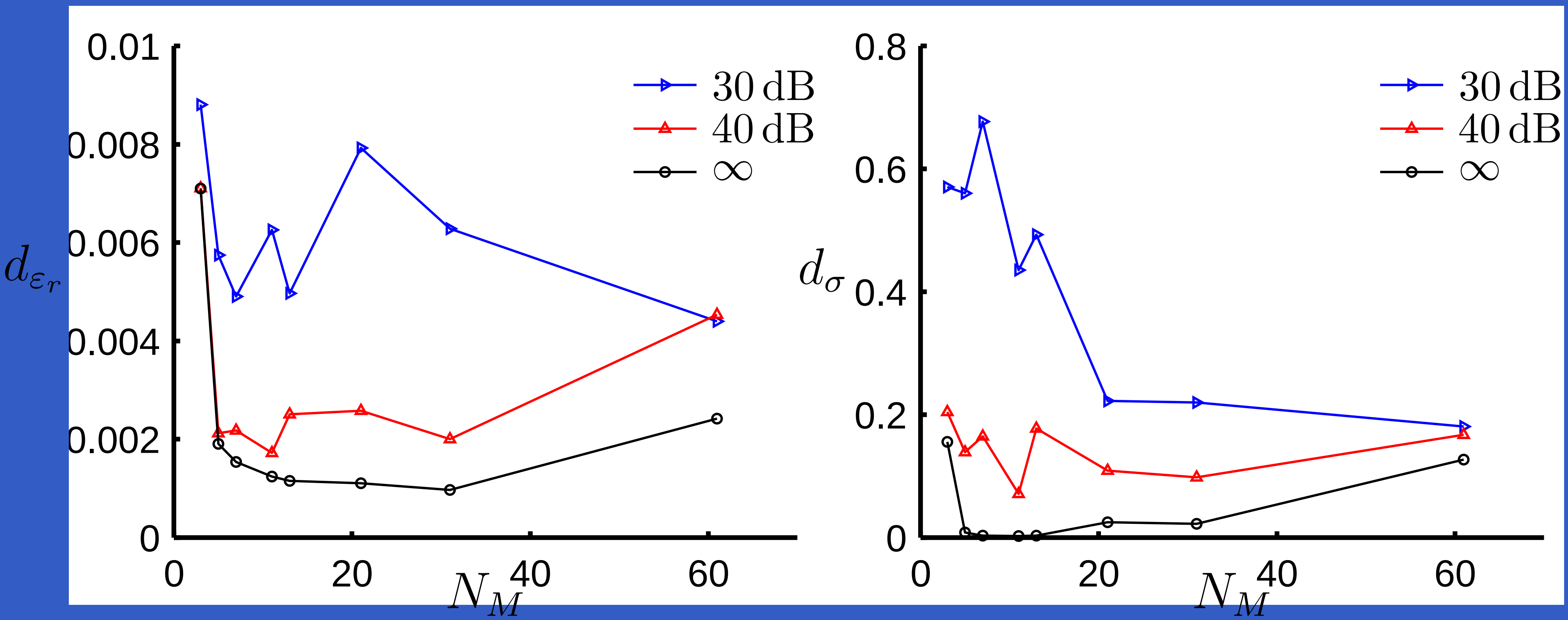


$$\mathrm{SNR} = 30\,\mathrm{dB},\ \textit{with regularization}$$

$$\zeta_{\varepsilon_r} = 10^{-2} \qquad \zeta_\sigma = 10^{-3} \qquad \phi(\cdot) = \phi_{gc}(\cdot)$$

$$\delta_{\varepsilon_r} = 0.4 \qquad \delta_\sigma = 1.25\,10^{-3} \qquad N_{\mathrm{int}} = 10$$

Number of measurement points



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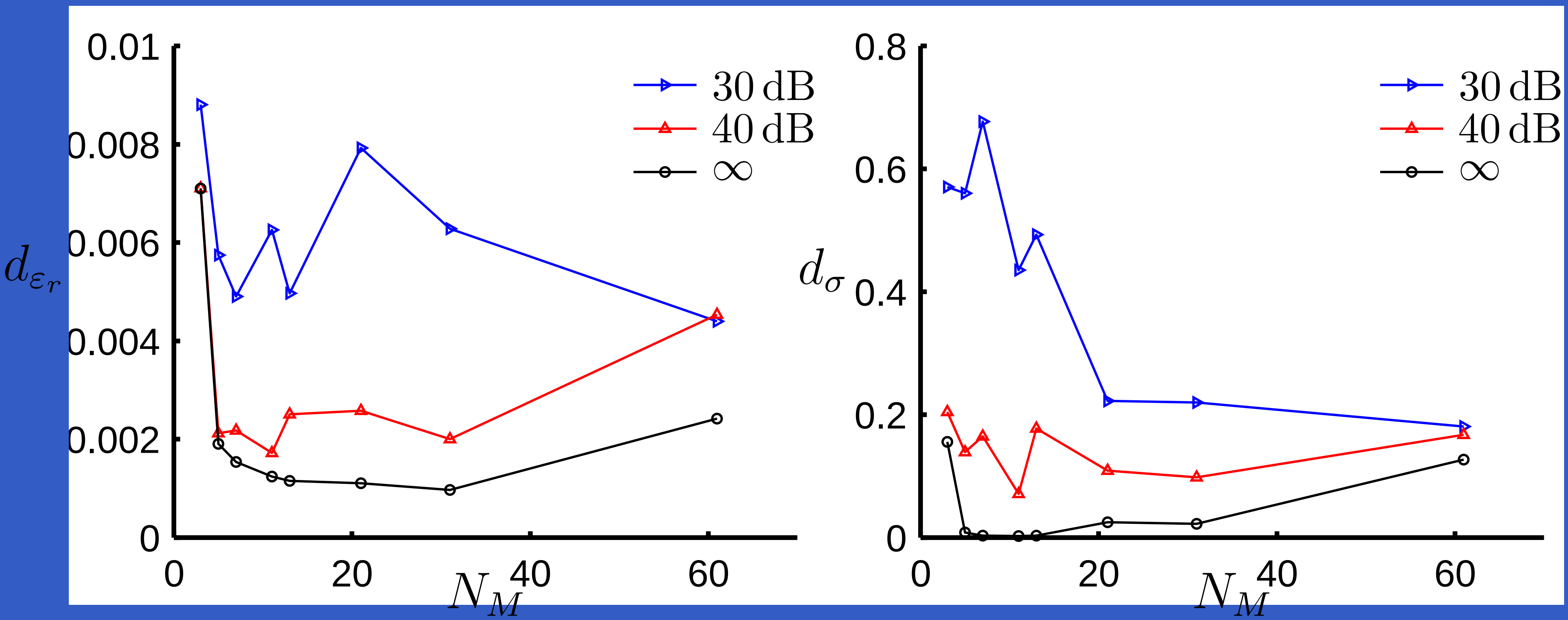
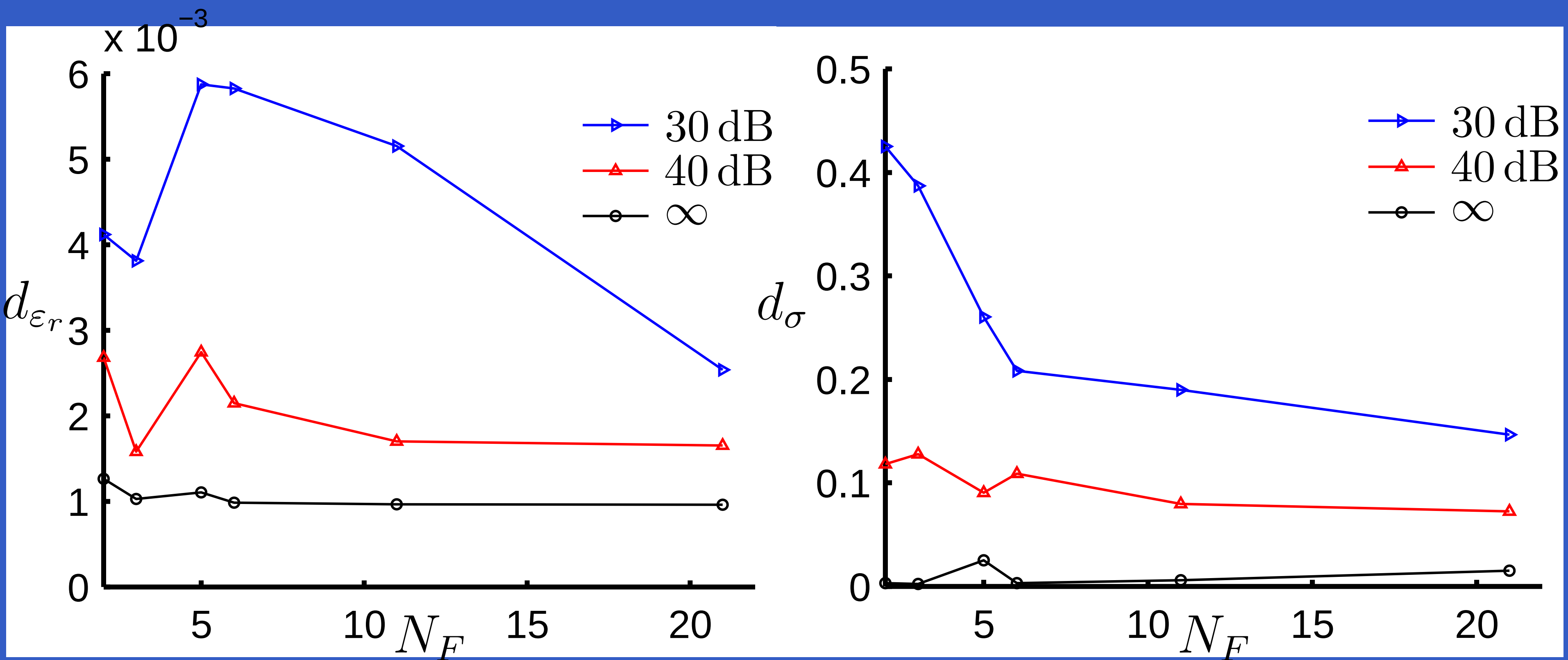


$$(N_F = 5)$$

$$N_M \to 21$$

Number of frequencies



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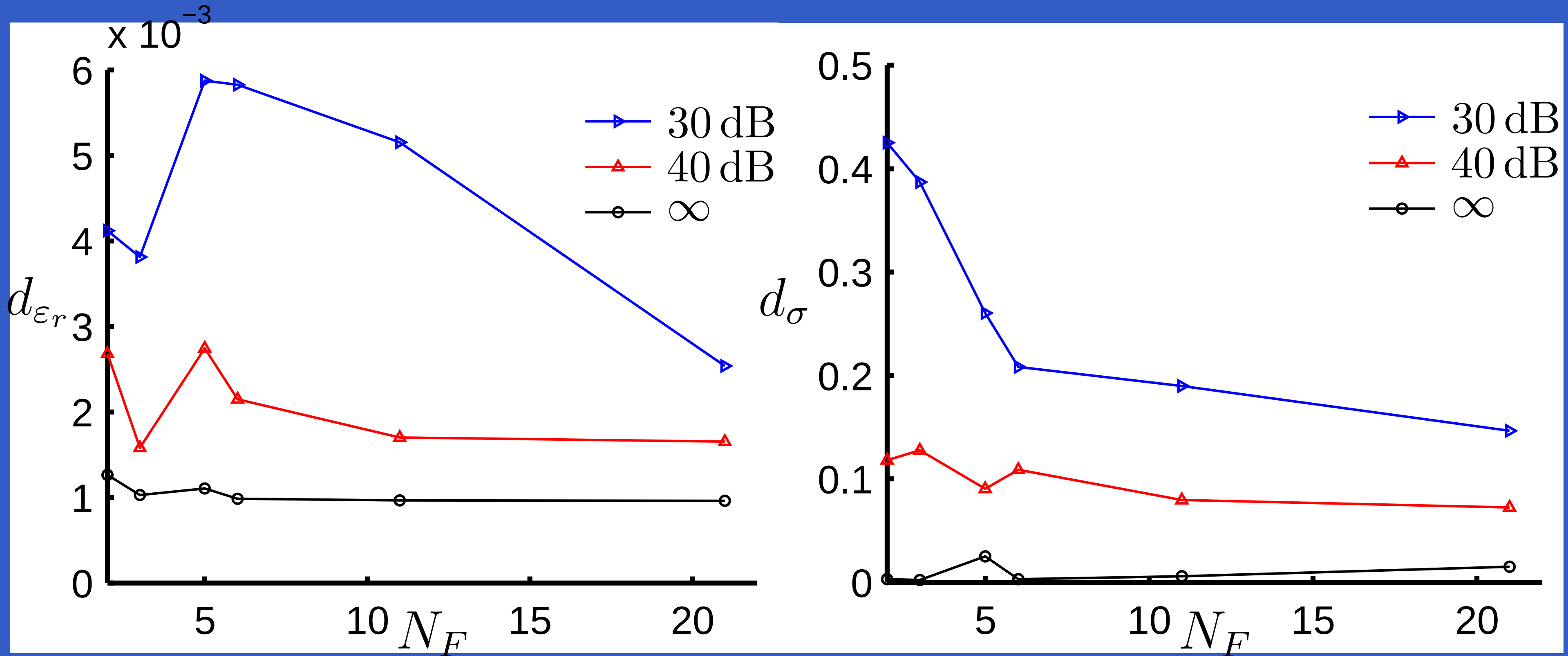
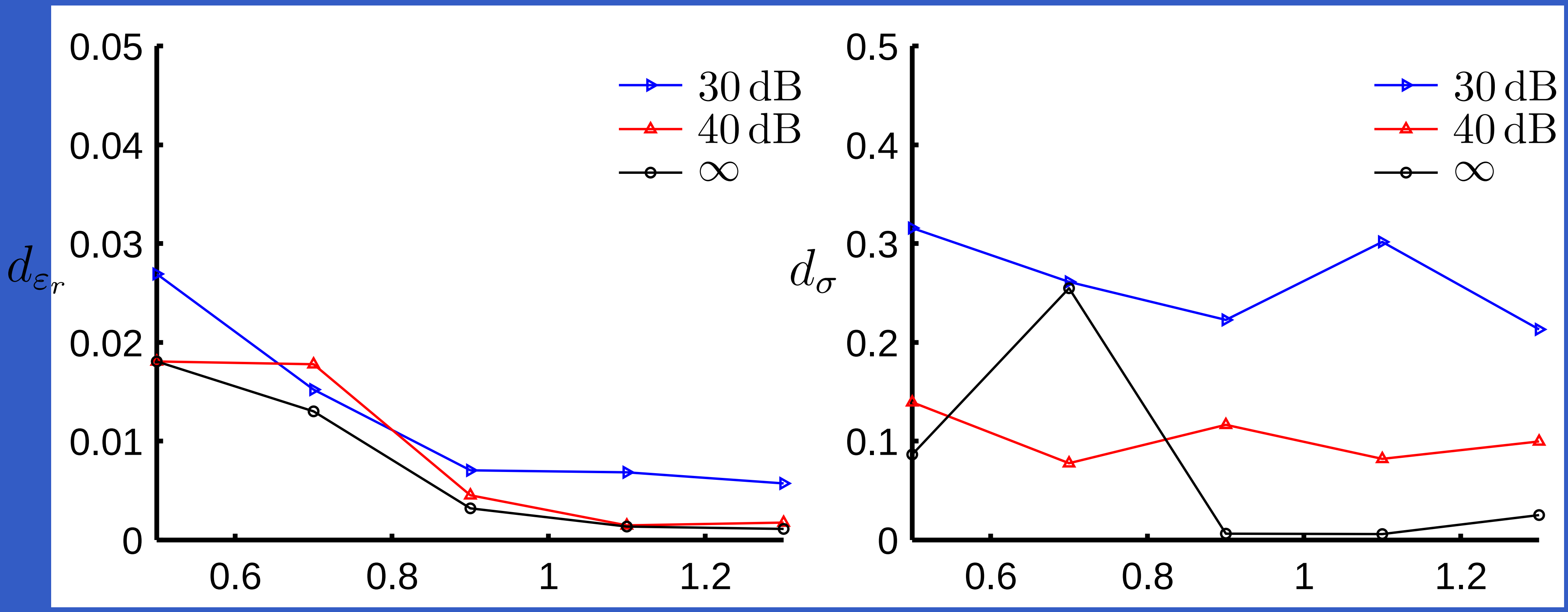


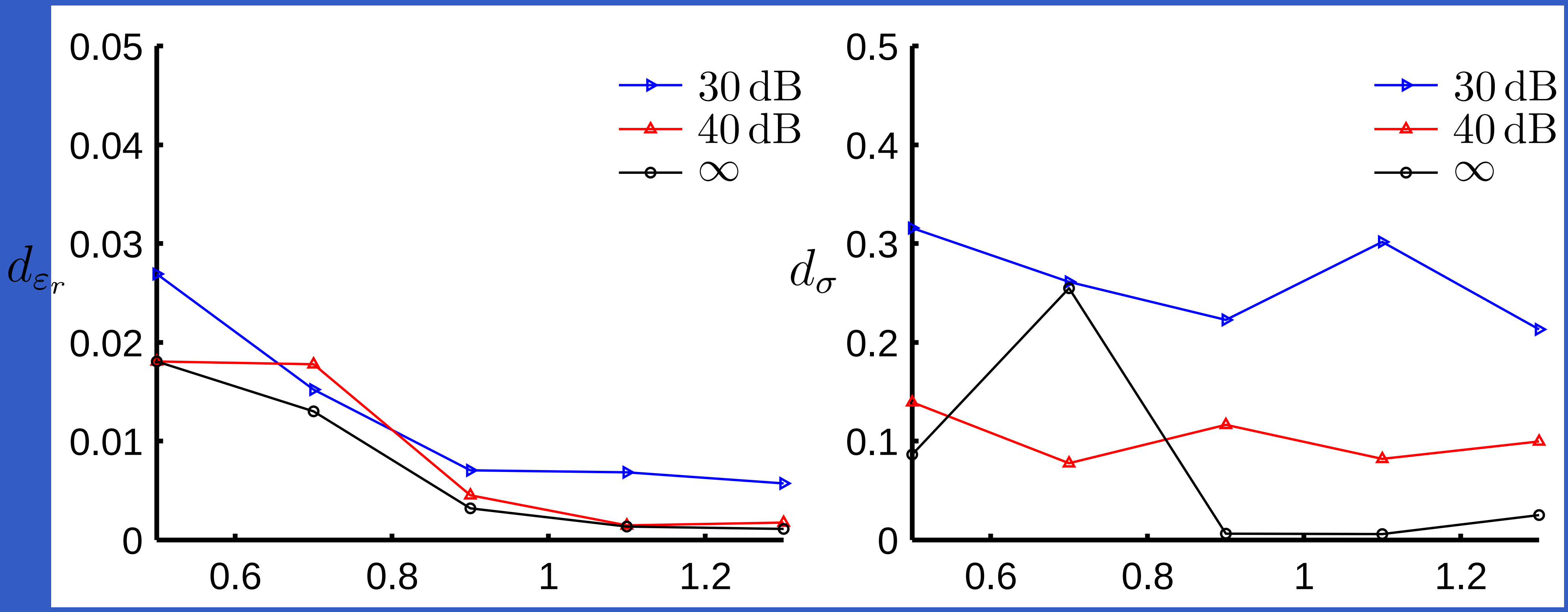
$$N_F \longrightarrow 5$$

Stretch frequency band



$f_{\max}(\mathrm{GHz})$ $f_{\max}(\mathrm{GHz})$

Stretch frequency band



$f_{\max}(\mathrm{GHz})$ $(N_F = 5)$ $f_{\max}(\mathrm{GHz})$

$$f_{\max} \to 1.3\,\mathrm{GHz}$$

Conclusions

Conclusions

- Robustness to noise

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- Quantity of information

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Conclusions

- Robustness to noise
- Quantity of information
- Optimum number of emitting/measurement points
- Frequencies used:
  - Wide band
  - Low: localization
  - High: resolution
  - Intrinsic frequency hopping
  - Optimum number of frequencies: low

Study of the three-dimensional extension

Iterative algorithm (again)

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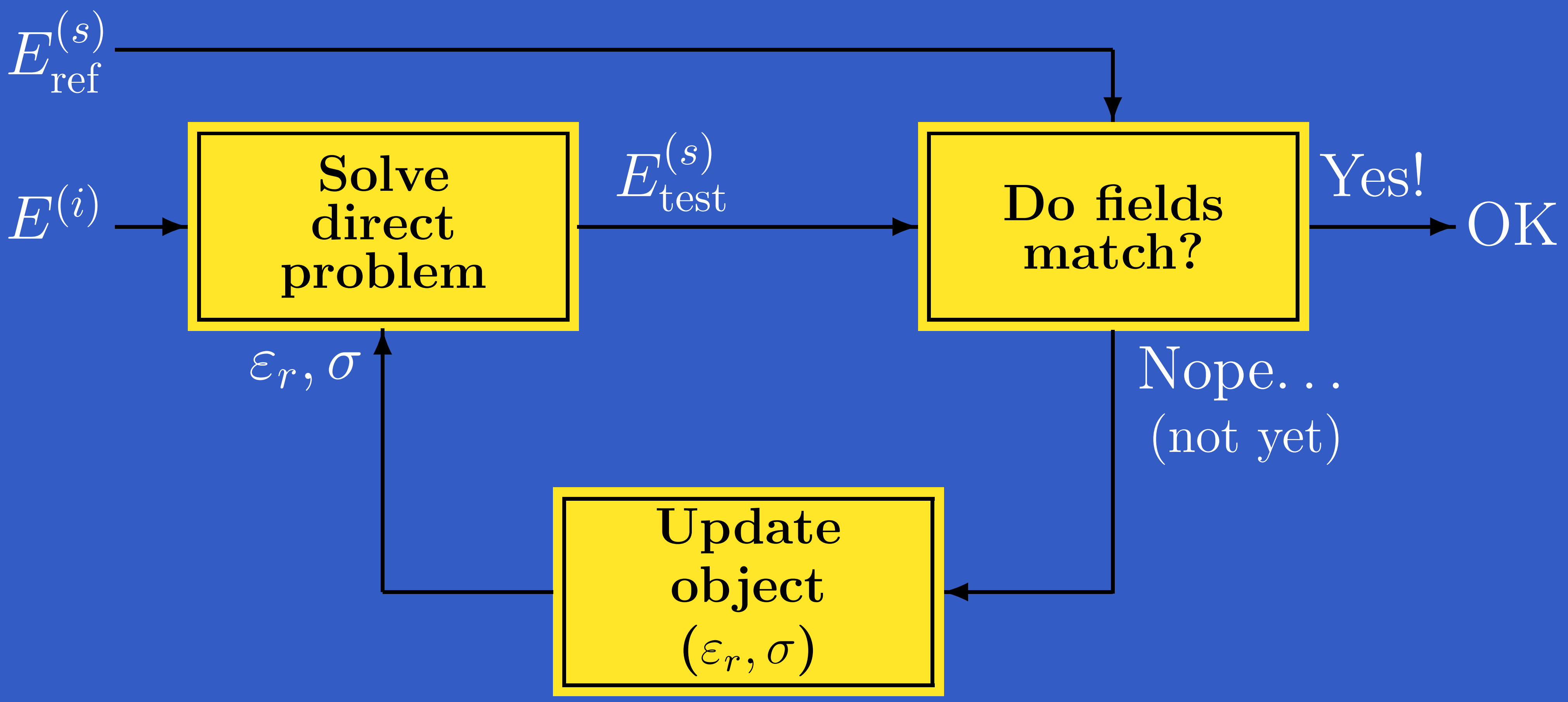
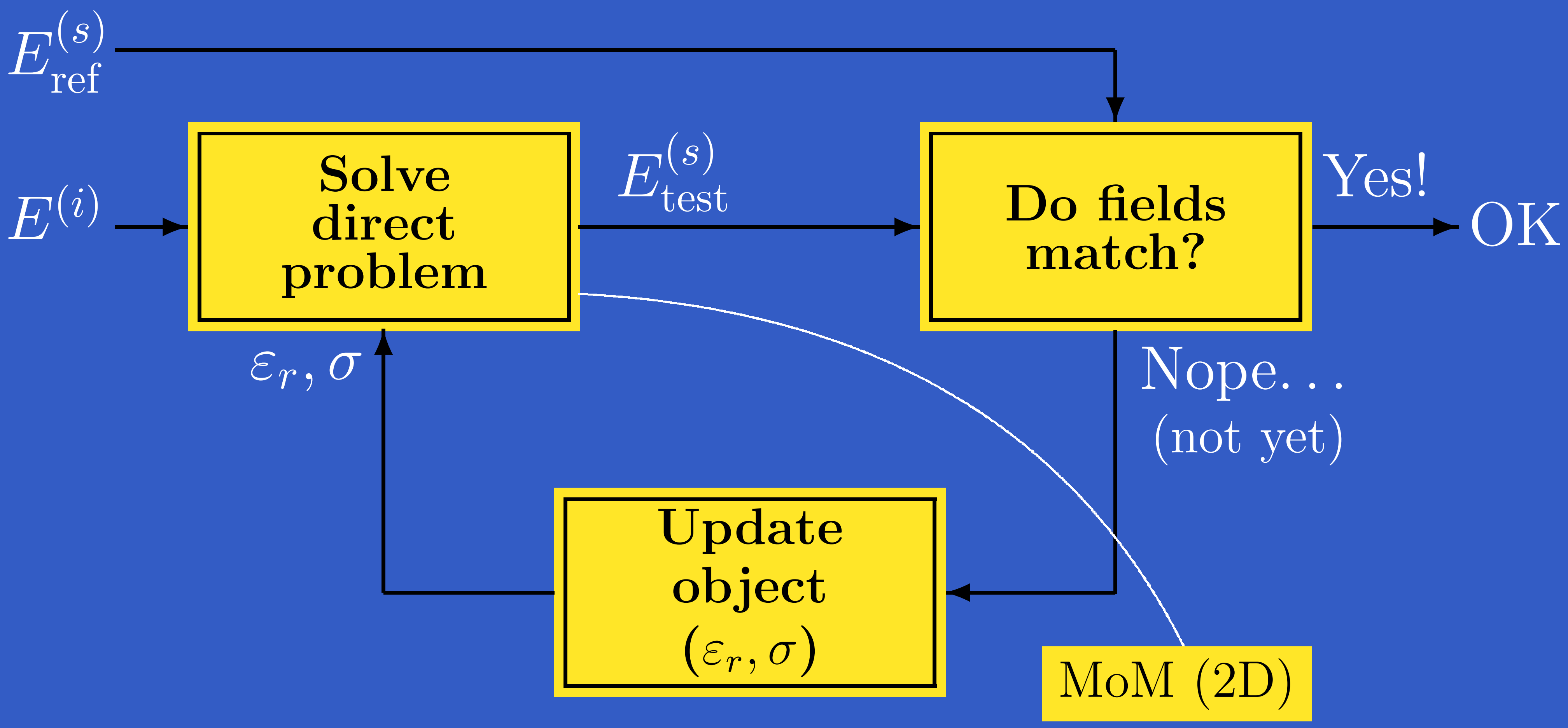
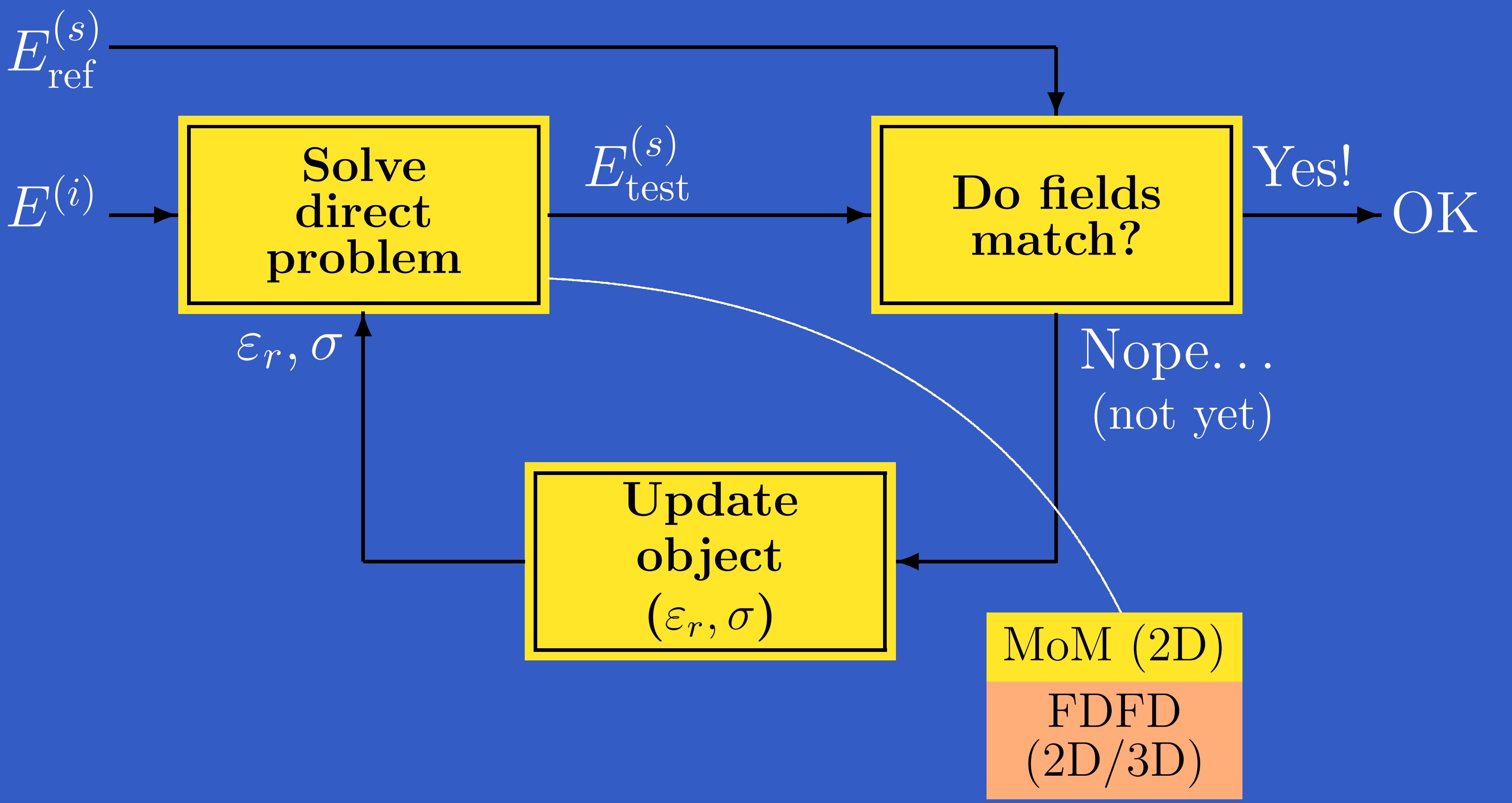


Iterative algorithm (again)



Iterative algorithm (again)



Contents II

- Direct problem
- Finite-Difference Frequency-Domain method
- Numerical results

Total field equations

$$
\begin{aligned}
\boldsymbol{\nabla} \cdot (\bar{\boldsymbol{\varepsilon}} \cdot \boldsymbol{E}) &= \rho \\
\boldsymbol{\nabla} \cdot (\bar{\boldsymbol{\mu}} \cdot \boldsymbol{H}) &= \tau \\
\boldsymbol{\nabla} \times \boldsymbol{E} &= -\mathrm{j}\,\omega \bar{\dot{\boldsymbol{\mu}}} \cdot \boldsymbol{H} - \boldsymbol{M} \\
\boldsymbol{\nabla} \times \boldsymbol{H} &= \quad \mathrm{j}\,\omega \bar{\dot{\boldsymbol{\varepsilon}}} \cdot \boldsymbol{E} + \boldsymbol{J}
\end{aligned}
$$

with

$$
\bar{\dot{\boldsymbol{\mu}}} = \bar{\boldsymbol{\mu}} - \mathrm{j}\,\frac{\bar{\boldsymbol{\sigma}}^*}{\omega} \quad , \quad \bar{\dot{\boldsymbol{\varepsilon}}} = \bar{\boldsymbol{\varepsilon}} - \mathrm{j}\,\frac{\bar{\boldsymbol{\sigma}}}{\omega}
$$

Scattered field equations

$$\boldsymbol{E} = \boldsymbol{E}^{(i)} + \boldsymbol{E}^{(s)}$$

$$\boldsymbol{H} = \boldsymbol{H}^{(i)} + \boldsymbol{H}^{(s)}$$

Scattered field equations

$$\boldsymbol{E} = \boldsymbol{E}^{(i)} + \boldsymbol{E}^{(s)}$$

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$$\left(\boldsymbol{E}^{(i)},\ \boldsymbol{H}^{(i)} \text{ in } \bar{\dot{\boldsymbol{\varepsilon}}}_{(b)}, \bar{\dot{\boldsymbol{\mu}}}_{(b)} \text{ with } \rho^{(i)}, \tau^{(i)}, \boldsymbol{J}^{(i)}, \boldsymbol{M}^{(i)}\right)$$

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$$\begin{aligned}
\boldsymbol{\nabla} \cdot (\bar{\varepsilon} \cdot \boldsymbol{E}^{(s)}) &= \rho^{(ind)} + \rho^{(s)} \\
\boldsymbol{\nabla} \cdot (\bar{\mu} \cdot \boldsymbol{H}^{(s)}) &= \tau^{(ind)} + \tau^{(s)} \\
\boldsymbol{\nabla} \times \boldsymbol{E}^{(s)} &= -\mathrm{j}\,\omega \bar{\dot{\boldsymbol{\mu}}} \cdot \boldsymbol{H}^{(s)} - \boldsymbol{M}^{(ind)} - \boldsymbol{M}^{(s)} \\
\boldsymbol{\nabla} \times \boldsymbol{H}^{(s)} &= \mathrm{j}\,\omega \bar{\dot{\boldsymbol{\varepsilon}}} \cdot \boldsymbol{E}^{(s)} + \boldsymbol{J}^{(ind)} + \boldsymbol{J}^{(s)}
\end{aligned}$$

Definition of charge densities

Exclusively for scattered field:

$$\rho^{(s)} \triangleq \rho - \rho^{(i)}$$

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$$\tau^{(ind)} \triangleq -\boldsymbol{\nabla} \cdot \left\{ \left(\bar{\boldsymbol{\mu}} - \bar{\boldsymbol{\mu}}_{(b)}\right) \cdot \boldsymbol{H}^{(i)} \right\} = -\boldsymbol{\nabla} \cdot \left(\bar{\boldsymbol{c}}_{(\mu)} \cdot \boldsymbol{H}^{(i)} \right)$$

Definition of current densities

Exclusively for scattered field:

$$\boldsymbol{J}^{(s)} \triangleq \boldsymbol{J} - \boldsymbol{J}^{(i)}$$

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Exclusively for scattered field:

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Induced from incident field:

$$\boldsymbol{J}^{(ind)} \triangleq \mathrm{j}\,\omega(\bar{\dot{\boldsymbol{\varepsilon}}} - \bar{\dot{\boldsymbol{\varepsilon}}}_{(b)}) \cdot \boldsymbol{E}^{(i)} = \mathrm{j}\,\omega\bar{\dot{\boldsymbol{c}}}_{(e)} \cdot \boldsymbol{E}^{(i)}$$

$$\boldsymbol{M}^{(ind)} \triangleq \mathrm{j}\,\omega(\bar{\dot{\boldsymbol{\mu}}} - \bar{\dot{\boldsymbol{\mu}}}_{(b)}) \cdot \boldsymbol{H}^{(i)} = \mathrm{j}\,\omega\bar{\dot{\boldsymbol{c}}}_{(m)} \cdot \boldsymbol{H}^{(i)}$$

Similarity of equations

Total field $\longrightarrow$ Scattered field:

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$\surd$ Equivalence between total / scattered field equations

Similarity of equations

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$\surd$ Equivalence between total / scattered field equations

$\surd$ Uniform treatment of electromagnetic problems

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- $\surd$ Equivalence between total / scattered field equations
- $\surd$ Uniform treatment of electromagnetic problems
- $\surd$ Pure scattered field formulation

Finite-difference frequency-domain (1/2)

- Spatial derivatives $\longrightarrow$ finite differences

$$\left.\frac{\partial f}{\partial x_i}\right|_{\mathbf{x_0}} = \frac{f(\mathbf{x_0} + h\mathbf{e_i}) - f(\mathbf{x_0} - h\mathbf{e_i})}{2h} + \mathcal{O}(h^2)$$

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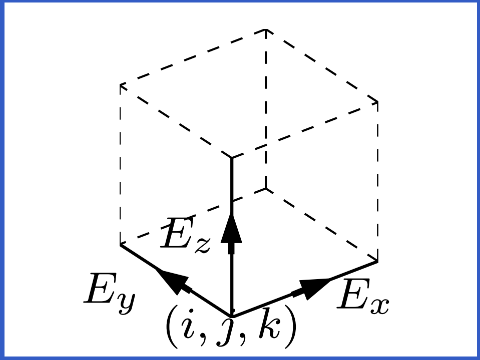
- Uniform cubic grid

$$E_x^{i,j,k} = E_x|_{i+\frac{1}{2},j,k} \quad H_x^{i,j,k} = H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}$$
$$E_y^{i,j,k} = E_y|_{i,j+\frac{1}{2},k} \quad H_y^{i,j,k} = H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}$$
$$E_z^{i,j,k} = E_z|_{i,j,k+\frac{1}{2}} \quad H_z^{i,j,k} = H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}$$



Finite-difference frequency-domain (2/2)

- Average adjacent cell properties

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$$\varepsilon_{xx} E_x \xrightarrow{discretize} \varepsilon_{xx}^{i,j,k} \;\; E_x^{i,j,k}$$

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with

$$\left\langle \varepsilon_{xx}^{i,j,k} \right\rangle \triangleq \frac{1}{4} \left(\varepsilon_{xx}^{i,j,k} + \varepsilon_{xx}^{i,j-1,k} + \varepsilon_{xx}^{i,j-1,k-1} + \varepsilon_{xx}^{i,j,k-1} \right)$$

$$\left\langle \mu_{xx}^{i,j,k} \right\rangle \triangleq \frac{1}{2} \left(\mu_{xx}^{i,j,k} + \mu_{xx}^{i-1,j,k} \right)$$

Faraday: $\boldsymbol{\nabla} \times \boldsymbol{E} = -\mathrm{j}\,\omega \bar{\boldsymbol{\mu}} \cdot \boldsymbol{H} - \boldsymbol{M}$

Faraday: $\nabla \times \boldsymbol{E} = -\mathrm{j}\,\omega \bar{\dot{\boldsymbol{\mu}}} \cdot \boldsymbol{H} - \boldsymbol{M}$

$$H_x^{i,j,k} = \frac{E_y^{i,j,k+1} - E_y^{i,j,k} - E_z^{i,j+1,k} + E_z^{i,j,k} - hM_x^{i,j,k}}{h\,\mathrm{j}\,\omega \left\langle \dot{\mu}_{xx}^{i,j,k} \right\rangle}$$

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$$\mathbf{A_e e} = -\mathrm{j}\,\omega \mathbf{diag}(\langle \dot{\boldsymbol{\mu}} \rangle)\mathbf{h} - \mathbf{m}$$

Maxwell-Ampère: $\nabla \times \boldsymbol{H} = \mathrm{j}\omega\bar{\bar{\boldsymbol{\varepsilon}}} \cdot \boldsymbol{E} + \boldsymbol{J}$

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$$\mathbf{A_h h} = \mathrm{j}\omega \mathbf{diag}(\langle \dot{\varepsilon} \rangle)\mathbf{e} + \mathbf{j} \quad , \quad \mathbf{A_h} = \mathbf{A_e}^T$$

Discretized wave equation (1/3)

The two matrix equations yield:

$$\left[\mathbf{A_e}^T \mathbf{diag}(\langle\dot{\boldsymbol{\mu}}\rangle)^{-1}\mathbf{A_e} - \omega^2 \mathbf{diag}(\langle\dot{\boldsymbol{\varepsilon}}\rangle)\right]\mathbf{e} = \\ -\,\mathrm{j}\,\omega\mathbf{j} - \mathbf{A_e}^T \mathbf{diag}(\langle\dot{\boldsymbol{\mu}}\rangle)^{-1}\mathbf{m}$$

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Compare with the continuous case:

$$\boldsymbol{\nabla}\times\bar{\dot{\boldsymbol{\mu}}}^{-1}\cdot\boldsymbol{\nabla}\times\boldsymbol{E} - \omega^2\bar{\dot{\boldsymbol{\varepsilon}}}\cdot\boldsymbol{E} = -\mathrm{j}\,\omega\boldsymbol{J} - \boldsymbol{\nabla}\times\bar{\dot{\boldsymbol{\mu}}}^{-1}\cdot\boldsymbol{M}$$

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Add gradient of Gauss' law $\boldsymbol{\nabla}\boldsymbol{\nabla} \cdot (\bar{\varepsilon} \cdot \boldsymbol{E}) = \boldsymbol{\nabla}\left(\rho + \rho^{(ind)}\right)$

Discretized wave equation (1/3)

The two matrix equations yield:

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Compare with the continuous case:

$$\boldsymbol{\nabla}\times\bar{\dot{\boldsymbol{\mu}}}^{-1}\cdot\boldsymbol{\nabla}\times\boldsymbol{E} - \omega^2\bar{\dot{\boldsymbol{\varepsilon}}}\cdot\boldsymbol{E} = -\mathrm{j}\,\omega\boldsymbol{J} - \boldsymbol{\nabla}\times\bar{\dot{\boldsymbol{\mu}}}^{-1}\cdot\boldsymbol{M}$$

Add gradient of Gauss' law $\boldsymbol{\nabla}\boldsymbol{\nabla}\cdot(\bar{\varepsilon}\cdot\boldsymbol{E}) = \boldsymbol{\nabla}\left(\rho + \rho^{(ind)}\right)$ in discretized form:

$$\mathbf{A_g}\mathbf{diag}(\langle\varepsilon\rangle)\mathbf{e} = -\mathbf{A_g}\mathbf{diag}(\langle\mathbf{c}_\varepsilon\rangle)\mathbf{e^i} - \mathbf{A}_\rho\langle\rho\rangle$$

Discretized wave equation (2/3)

Sparse linear system ($N \times N$):

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

Discretized wave equation (2/3)

Sparse linear system ($N \times N$):

$$\mathbf{Ax} = \mathbf{b} \qquad N = \begin{cases} N_x N_y & \text{2D-TM} \\ 2N_x N_y & \text{2D-TE} \\ 3N_x N_y N_z & \text{3D} \end{cases}$$

Discretized wave equation (2/3)

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$\surd$ Common formulation for 2D-TM, 2D-TE, 3D

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$$\mathbf{A} = \mathbf{A_e}^T \mathrm{diag}(\mathbf{V_m}\boldsymbol{\mu})^{-1}\mathbf{A_e} + (\mathbf{A_g} - \omega^2 \mathbf{I})\mathrm{diag}(\mathbf{V_e}\boldsymbol{\varepsilon}) + \mathrm{j}\,\omega\,\mathrm{diag}(\mathbf{V_e}\boldsymbol{\sigma})$$

Discretized wave equation (2/3)

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$\mathbf{V_e}$, $\mathbf{V_m}$, $\mathbf{V}_\rho$: averaging matrices, e.g. $\langle \boldsymbol{\varepsilon} \rangle = \mathbf{V_e}\boldsymbol{\varepsilon}$

Discretized wave equation (3/3)

- Closed-geometry and radiation problems

$$\mathbf{x} = \mathbf{e}$$

$$\mathbf{b} = -\left(\mathrm{j}\,\omega \mathbf{j} + \mathbf{A_e}^T \mathbf{diag}(\mathbf{V_m}\dot{\boldsymbol{\mu}})^{-1}\mathbf{m} + \mathbf{A}_\rho \mathbf{V}_\rho \boldsymbol{\rho} \right)$$

Discretized wave equation (3/3)

- Closed-geometry and radiation problems

$$\mathbf{x} = \mathbf{e}$$

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- Scattering problems

$$\mathbf{x} = \mathbf{e^s}$$

$$\begin{aligned}\mathbf{b} = -\Big(\mathrm{j}\,\omega\mathbf{j^s} + {\mathbf{A_e}}^T \mathrm{diag}(\mathbf{V_m}\dot{\mu})^{-1}\mathbf{m^s} + \mathbf{A}_\rho \mathbf{V}_\rho \boldsymbol{\rho}^{\mathbf{s}} + \\ \Big[(\mathbf{A_g} - \omega^2\mathbf{I})\mathrm{diag}(\mathbf{V_e}\mathbf{c}_\varepsilon) + \mathrm{j}\,\omega\mathrm{diag}(\mathbf{V_e}\mathbf{c}_\sigma)\Big]\,\mathbf{e^i} + \\ \mathrm{j}\,\omega{\mathbf{A_e}}^T \mathrm{diag}(\mathbf{V_m}\dot{\mu})^{-1}\mathrm{diag}(\mathbf{V_m}\dot{\mathbf{c}}_\mathbf{m})\mathbf{h^i}\Big)\end{aligned}$$

Enforce boundary conditions

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- Perfect electric conductor $(\in, \parallel)$: $\mathsf{x_n} = 0$ or $\mathsf{x_n} = -\mathsf{e}^{\mathsf{i}}_{\mathsf{n}}$

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$$\mathbf{A}' = \mathbf{I}_{\overline{\mathbf{VMC}}} + \mathbf{I}_{\mathbf{VMC}}\mathbf{A}\mathbf{I}_{\mathbf{C}} \qquad \mathbf{b}' = \mathbf{I}_{\mathbf{VMC}}\mathbf{b} - \mathbf{I}_{\mathbf{V}}\mathbf{I}_{\overline{\mathbf{M}}}\mathbf{e}^{\mathbf{i}} + \mathbf{v}$$

Enforce boundary conditions

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- Applied voltage: $\mathsf{x_n} = \mathsf{v_n}$

$$\mathbf{A}' = \mathbf{I}_{\overline{\mathbf{VMC}}} + \mathbf{I}_{\mathbf{VMC}}\mathbf{A}\mathbf{I}_{\mathbf{C}} \qquad \mathbf{b}' = \mathbf{I}_{\mathbf{VMC}}\mathbf{b} - \mathbf{I}_{\mathbf{V}}\mathbf{I}_{\overline{\mathbf{M}}}\mathbf{e}^{\mathbf{i}} + \mathbf{v}$$

- No Gauss $\perp$ PEC

Enforce boundary conditions

- Perfect electric conductor $(\in, \|)$: $\mathsf{x}_\mathsf{n} = 0$ or $\mathsf{x}_\mathsf{n} = -\mathsf{e}^\mathsf{i}_\mathsf{n}$
- Applied voltage: $\mathsf{x}_\mathsf{n} = \mathsf{v}_\mathsf{n}$

$$\mathbf{A}' = \mathbf{I}_{\overline{\mathbf{VMC}}} + \mathbf{I}_{\mathbf{VMC}}\mathbf{A}\mathbf{I}_{\mathbf{C}} \qquad \mathbf{b}' = \mathbf{I}_{\mathbf{VMC}}\mathbf{b} - \mathbf{I}_{\mathbf{V}}\mathbf{I}_{\overline{\mathbf{M}}}\mathbf{e}^{\mathbf{i}} + \mathbf{v}$$

- No Gauss $\perp$ PEC

$$\mathbf{A}'_{\mathbf{g}} = \mathbf{I}_{\mathbf{M}_n\mathbf{C}_n}\mathbf{A}_{\mathbf{g}} \qquad \mathbf{A}'_{\rho} = \mathbf{I}_{\mathbf{M}_n\mathbf{C}_n}\mathbf{A}_{\rho}$$

Enforce boundary conditions

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- Applied voltage: $\mathsf{x_n} = \mathsf{v_n}$

$$\mathbf{A}' = \mathbf{I}_{\overline{\mathbf{VMC}}} + \mathbf{I}_{\mathbf{VMC}}\mathbf{A}\mathbf{I}_{\mathbf{C}} \qquad \mathbf{b}' = \mathbf{I}_{\mathbf{VMC}}\mathbf{b} - \mathbf{I}_{\mathbf{V}}\mathbf{I}_{\overline{\mathbf{M}}}\mathbf{e}^{\mathbf{i}} + \mathbf{v}$$

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- Condensed system

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- Condensed system

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & 1 & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

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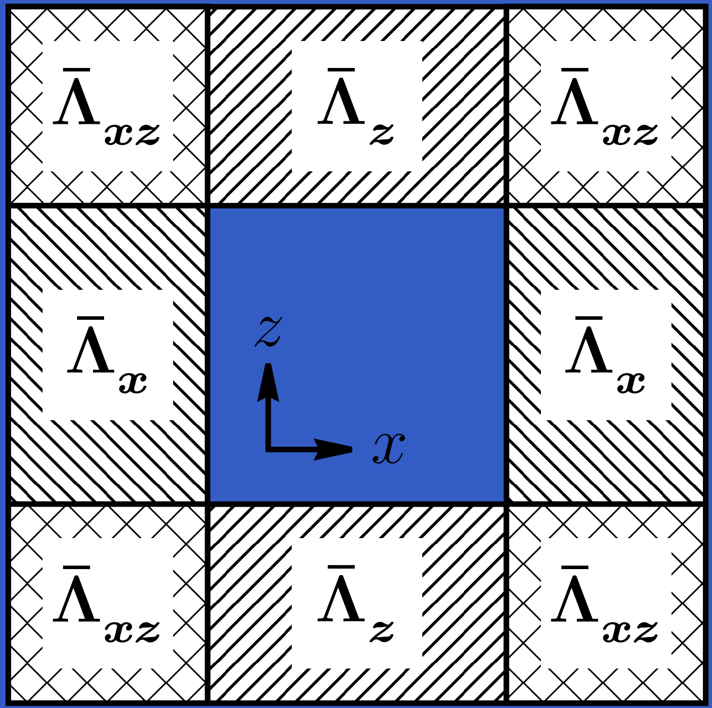
$$x_2 = b_2$$

Perfectly Matched Layers

$$\bar{\boldsymbol{\varepsilon}}_{\mathrm{PML}} = \bar{\dot{\boldsymbol{\varepsilon}}} \cdot \bar{\boldsymbol{\Lambda}}_{\boldsymbol{x}}$$

$$\bar{\boldsymbol{\mu}}_{\mathrm{PML}} = \bar{\dot{\boldsymbol{\mu}}} \cdot \bar{\boldsymbol{\Lambda}}_{\boldsymbol{x}}$$



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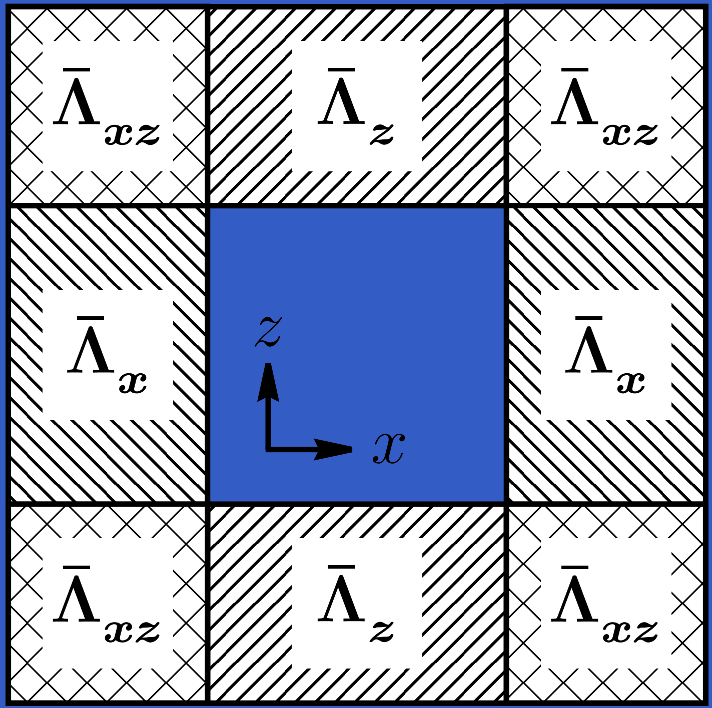
$$\bar{\Lambda}_x = \begin{bmatrix} 1/s_x & 0 & 0 \\ 0 & s_x & 0 \\ 0 & 0 & s_x \end{bmatrix}$$



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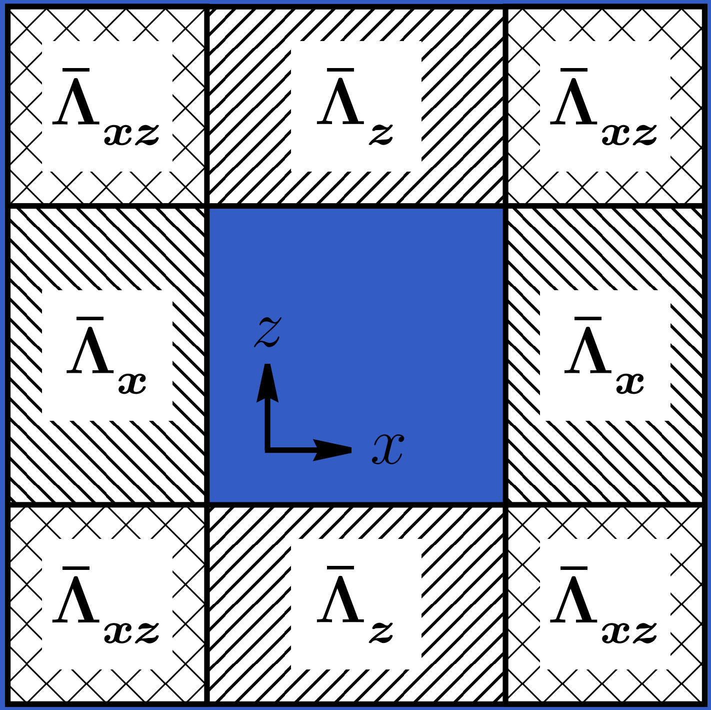

$$s_x = \kappa_x - \mathrm{j}\,\frac{\sigma_x}{\omega\varepsilon_0}, \quad \kappa_x \geq 1, \quad \sigma_x \geq 0$$

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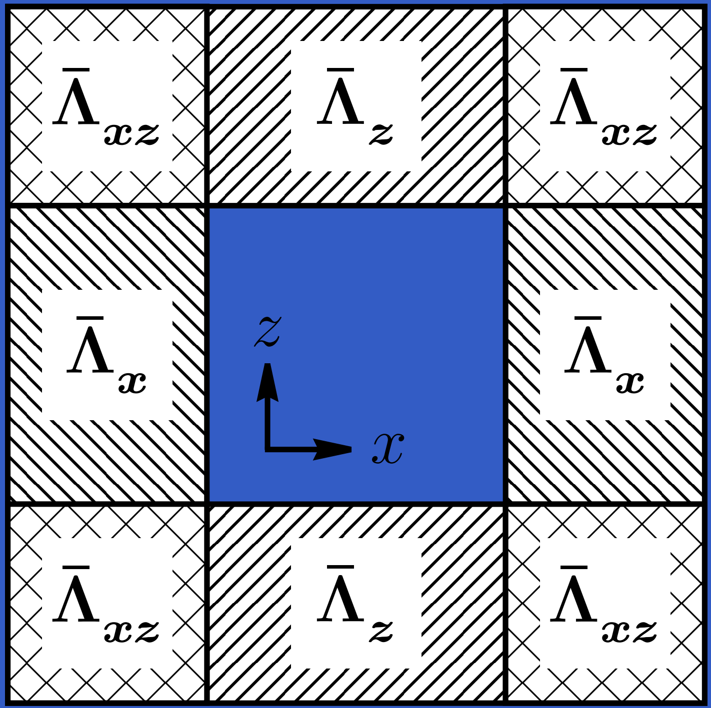
$$\bar{\mathbf{\Lambda}}_{\boldsymbol{xz}} = \bar{\mathbf{\Lambda}}_{\boldsymbol{x}} \cdot \bar{\mathbf{\Lambda}}_{\boldsymbol{z}}$$



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Near- to Far-Field Transform

Near- to Far-Field Transform

$\boldsymbol{r}'$ near-field point

Near- to Far-Field Transform

$\boldsymbol{r}$ far-field (observation) point
$\boldsymbol{r}'$ near-field point

Near- to Far-Field Transform

ψ	any rectangular component of $\boldsymbol{E}$ or $\boldsymbol{H}$
$\boldsymbol{r}$	far-field (observation) point
$\boldsymbol{r}'$	near-field point

Near- to Far-Field Transform

Kirchhoff integral:

$$\psi(\boldsymbol{r}) = -\frac{1}{4\pi} \oint_S \frac{e^{-\mathrm{j}\,kR}}{R} \left[\frac{\partial \psi(\boldsymbol{r}')}{\partial \eta'} - \left(\mathrm{j}\,k + \frac{1}{R} \right) \frac{1}{R} \hat{\boldsymbol{\eta}}' \cdot \boldsymbol{R} \psi(\boldsymbol{r}') \right] \mathrm{d}\boldsymbol{r}'$$

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$$\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}', \quad R = |\boldsymbol{R}|, \quad k = \omega\sqrt{\mu_0 \varepsilon_0}$$

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- $\frac{\partial}{\partial \eta'} \to$ finite differences
- $\oint_S \to \sum_n \sum_m w_{nm} \ldots$ 4th order 2D integration

Near- to Far-Field Transform: Validation (1/6)

Radiation of an elementary dipole (far-field):

$$E_r(r,\theta,\phi) \approx 0, \quad E_\theta(r,\theta,\phi) \sim \sin\theta, \quad E_\phi(r,\theta,\phi) = 0$$

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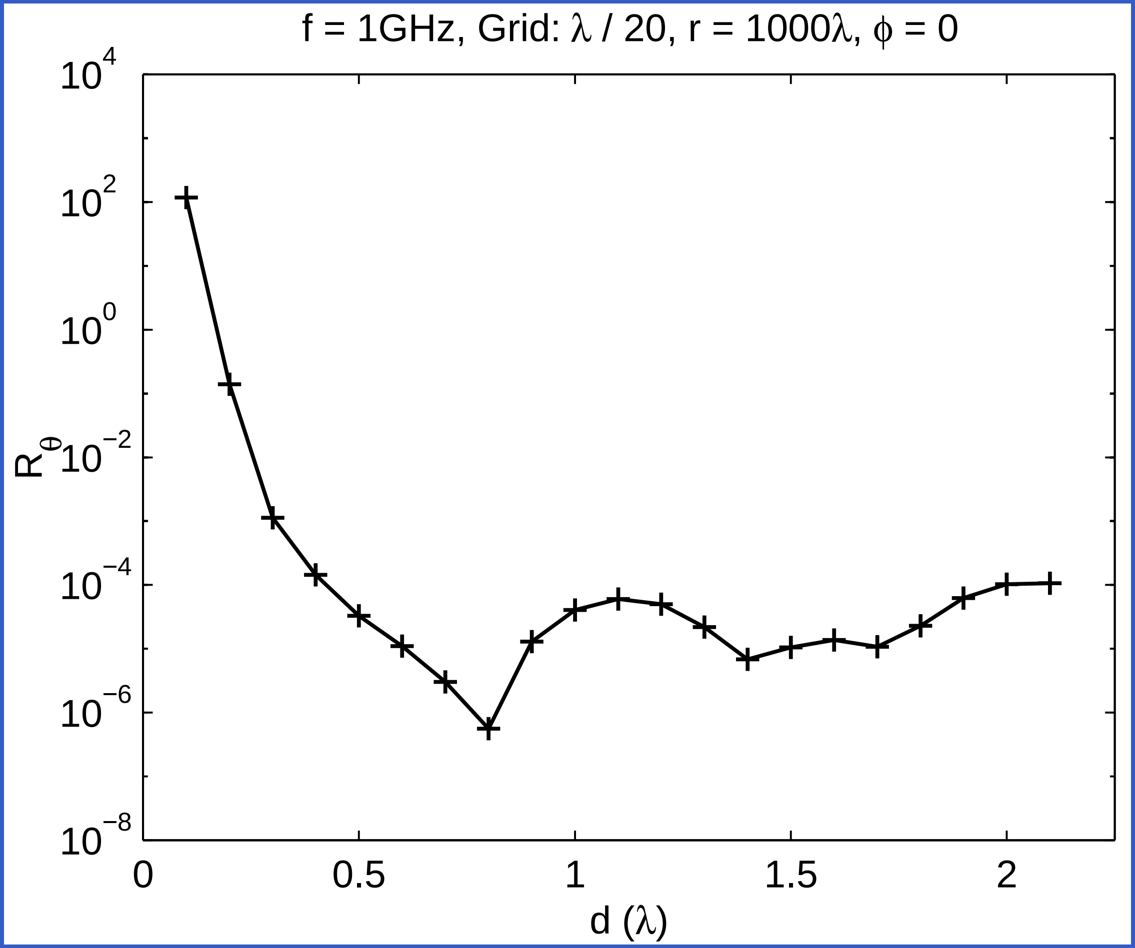
$$R_\theta = \frac{\left\| \mathbf{e}_\theta - \mathbf{e}_{th\theta} \right\|^2}{\left\| \mathbf{e}_{th\theta} \right\|^2}$$

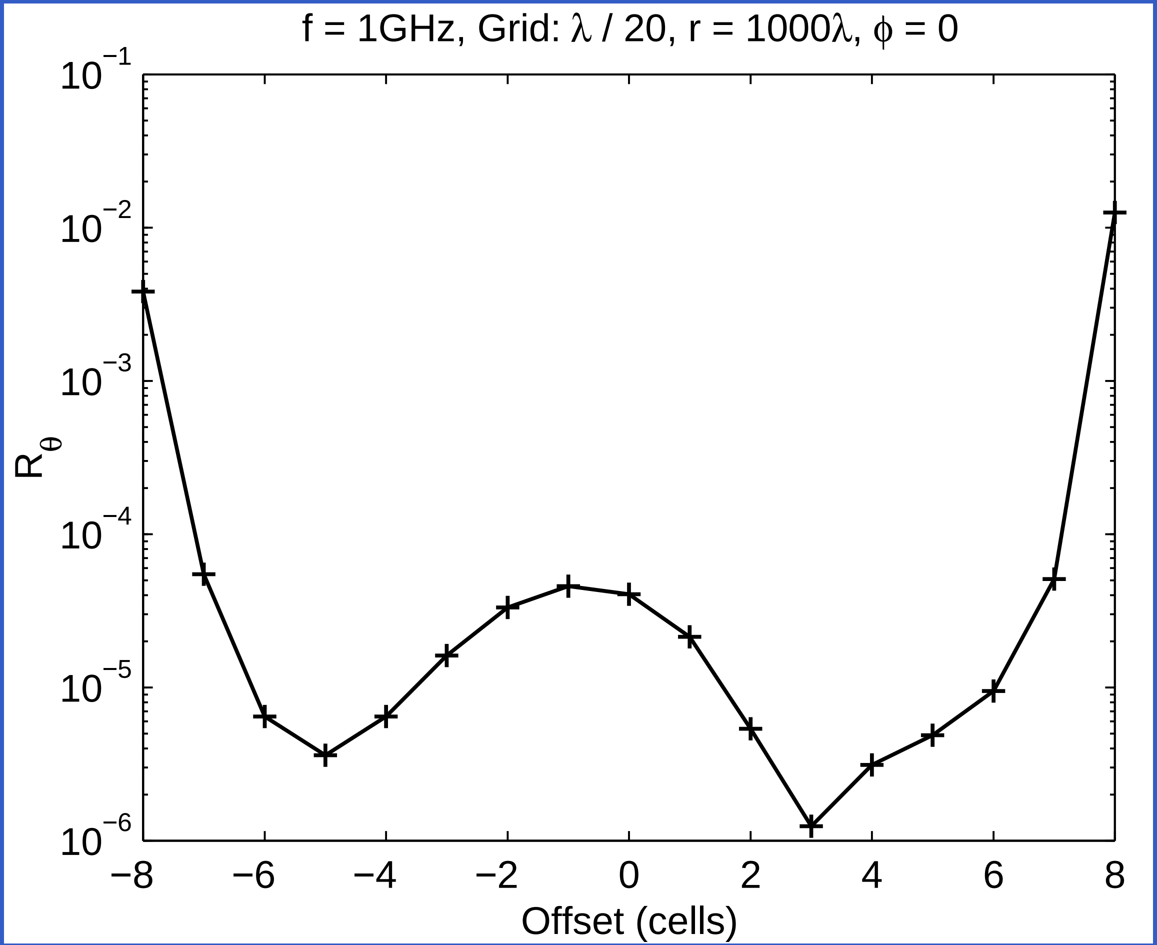
Near- to Far-Field Transform: Validation (2/6)

Size of the integration cube



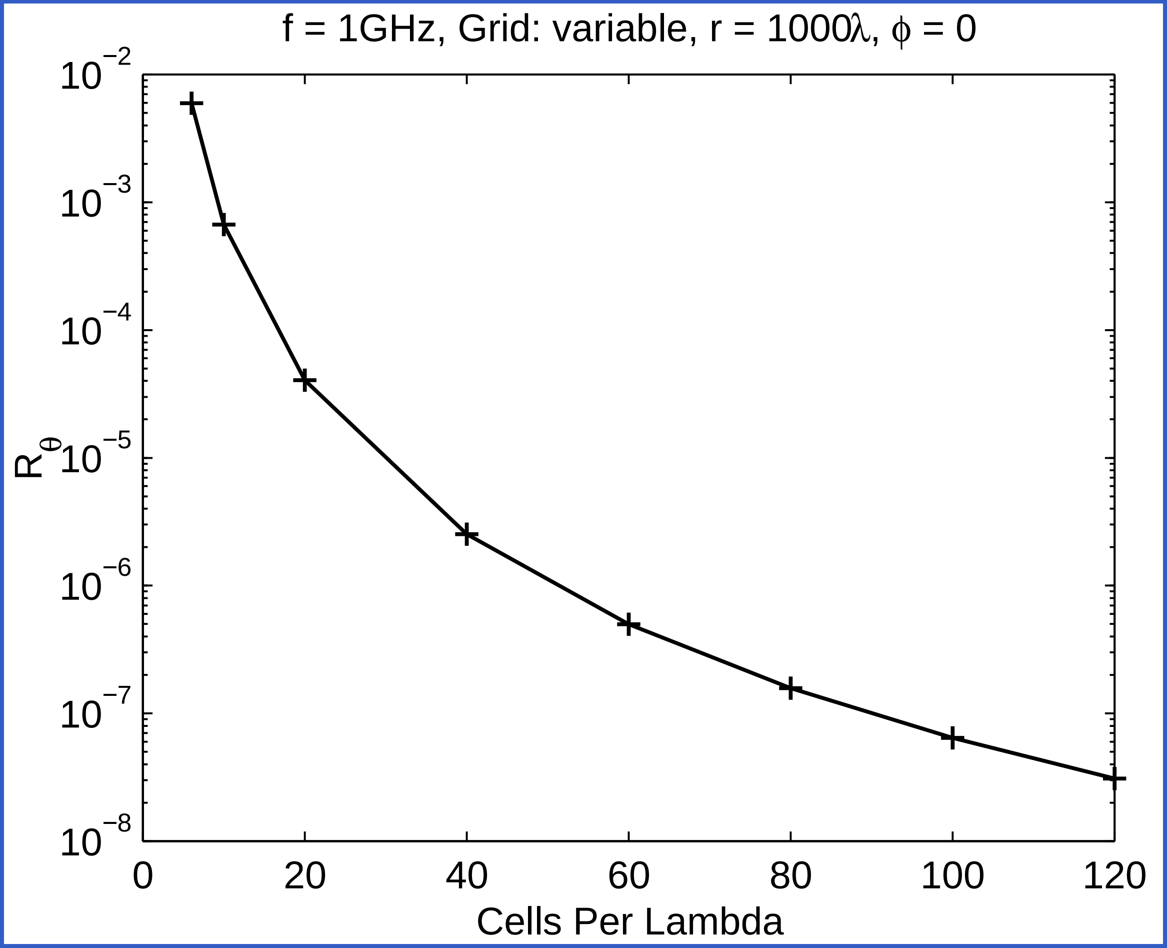
Near- to Far-Field Transform: Validation (3/6)

Position of the integration cube



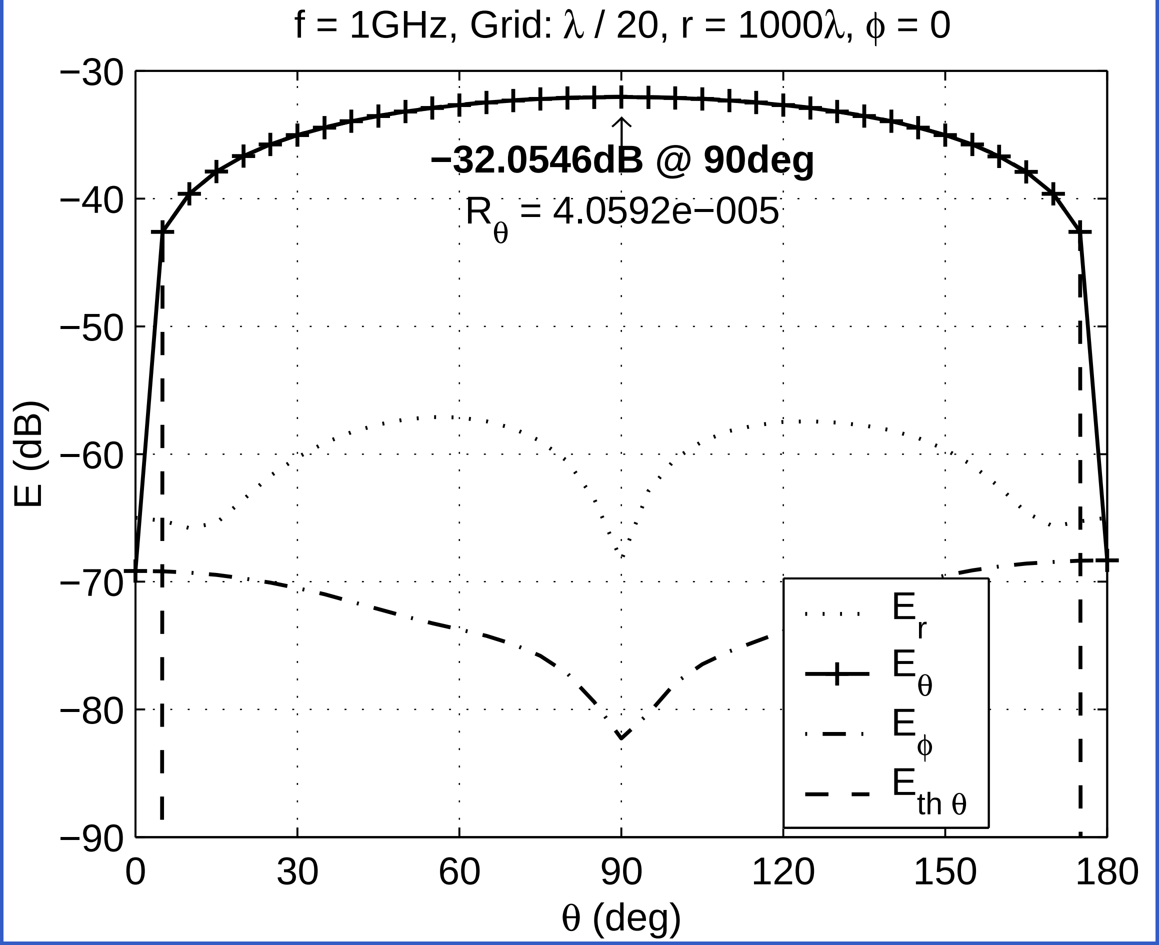
Near- to Far-Field Transform: Validation (4/6)

Coarseness of the discretization grid



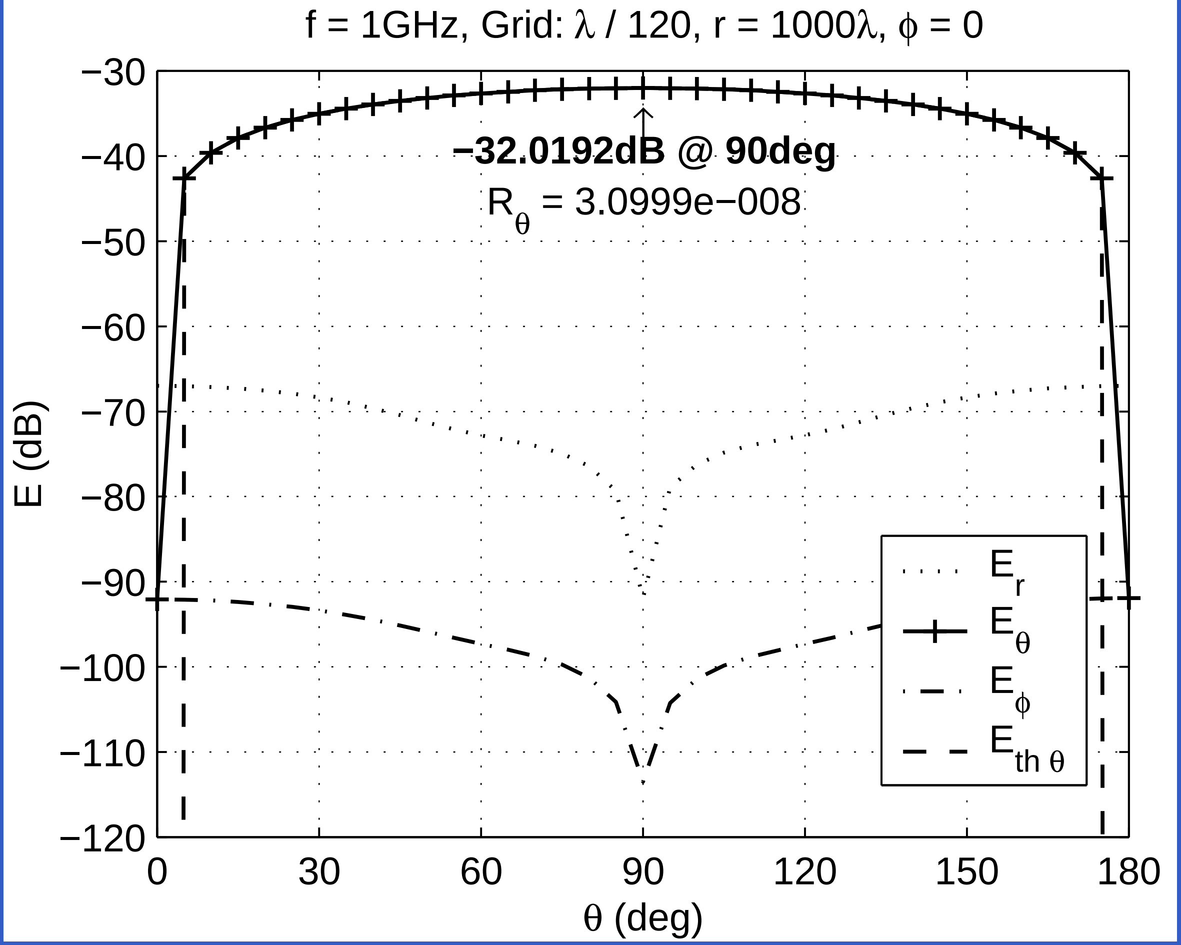
Near- to Far-Field Transform: Validation (5/6)

Grid: $\lambda/20$



Near- to Far-Field Transform: Validation (6/6)

Grid: $\lambda/120$



FDFD: numerical results

Closed-geometry problems

Open-geometry problems

FDFD: numerical results

Closed-geometry problems:

- 2D: waveguide modes
- 3D: cavity modes

Open-geometry problems

FDFD: numerical results

Closed-geometry problems:

- 2D: waveguide modes
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Open-geometry problems:

- 3D: elementary dipole

FDFD: numerical results

Closed-geometry problems:

- $\mathbf{Ae} = \mathbf{0}$

- 2D: waveguide modes
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Open-geometry problems:

- 3D: elementary dipole

FDFD: numerical results

Closed-geometry problems:

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- 3D: cavity modes

- $\mathbf{Ae} = \mathbf{0}$
- $\mathbf{e} \in \text{nullspace } \mathbf{A}$

Open-geometry problems:

- 3D: elementary dipole

FDFD: numerical results

Closed-geometry problems:

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- $\mathbf{Ae} = \mathbf{0}$
- $\mathbf{e} \in$ nullspace $\mathbf{A}$
- find eigenvector(s) for zero eigenvalue

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- $\mathbf{Ae} = \mathbf{b}$ (sparse)

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FDFD: numerical results

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- $\mathbf{Ae} = \mathbf{b}$ (sparse)
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- preconditioning: diagonal, LU, ILU

Waveguide modes

- Dimensions: $a/b = 3$
- Non-degenerate modes: TE^z_{10} $(1\,\mathrm{GHz})$, TE^z_{20}
- Cells: $N_x = 31$, $N_y = 11$

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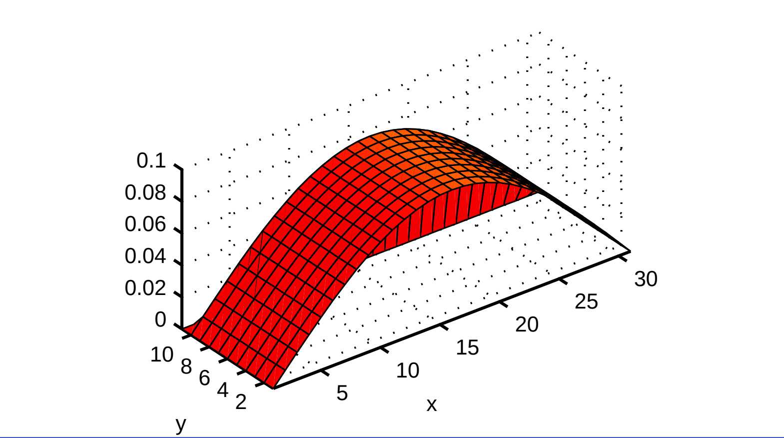
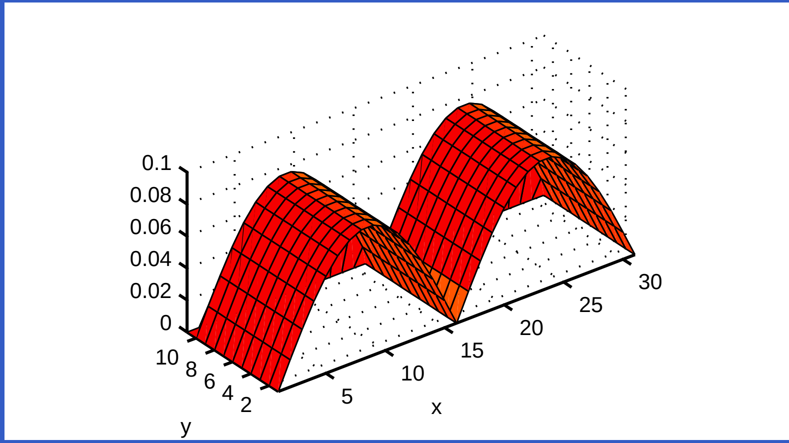


E_y for TE^z_{10}



E_y for TE^z_{20}

Waveguide modes

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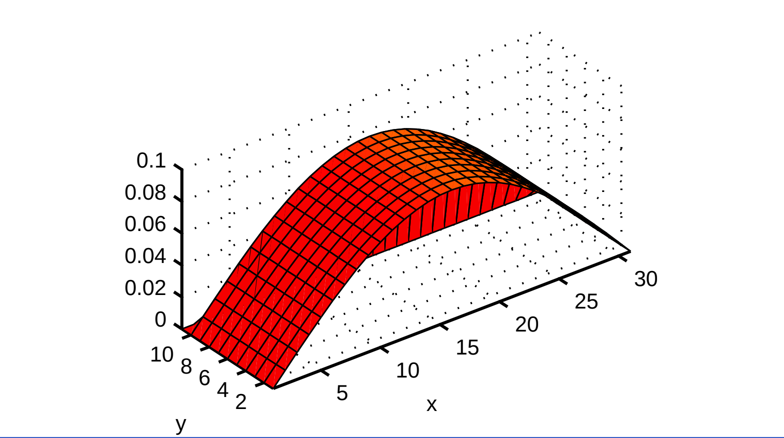
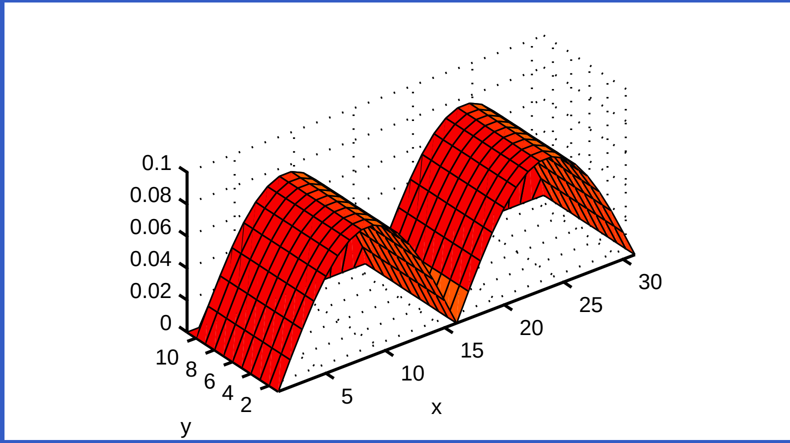


E_y for TE^z_{10}



E_y for TE^z_{20}

$$\checkmark \quad \|\mathbf{e}_{\mathrm{num}}\| = 1, \quad \|\mathbf{e}_{\mathrm{th}} - \mathbf{e}_{\mathrm{num}}\| \approx 10^{-14}$$

Cavity modes

- Dimensions: $a/b = 3$, $a/c = 2$
- Non-degenerate modes: TE^z_{101} $(1\,\mathrm{GHz})$, TE^z_{201}, TM^z_{110}
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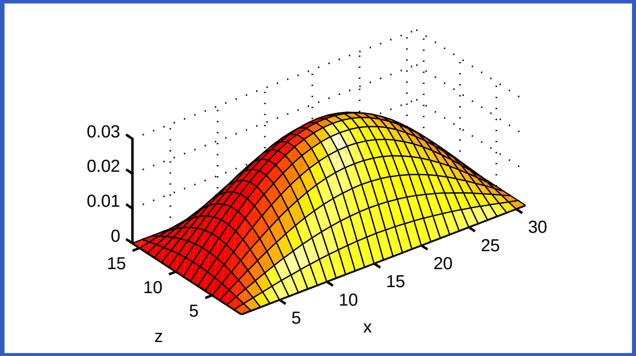
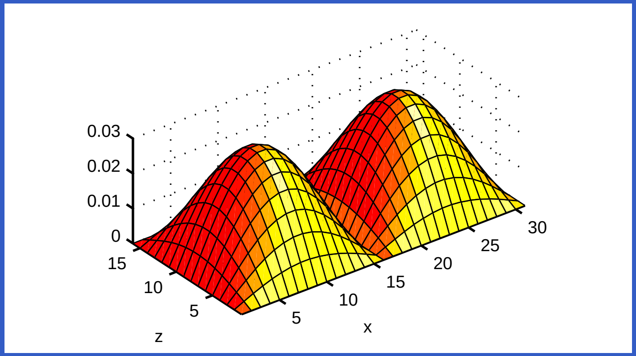
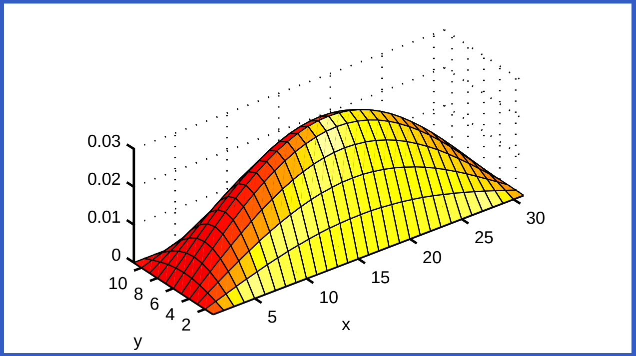


E_y for TE^z_{101}



E_y for TE^z_{201}



E_z for TM^z_{110}

Cavity modes

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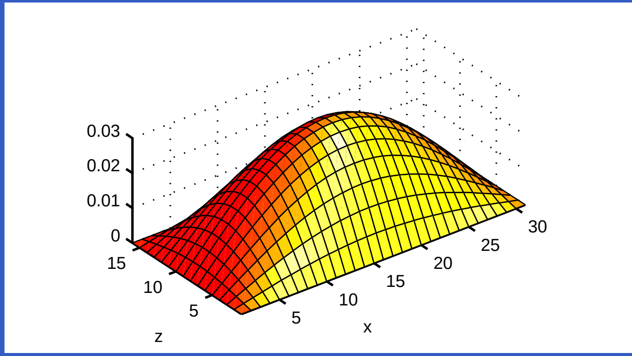
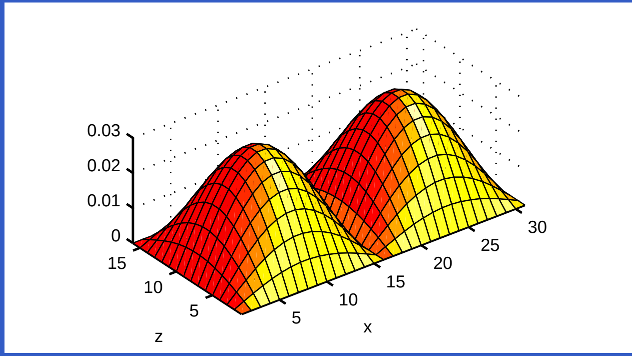
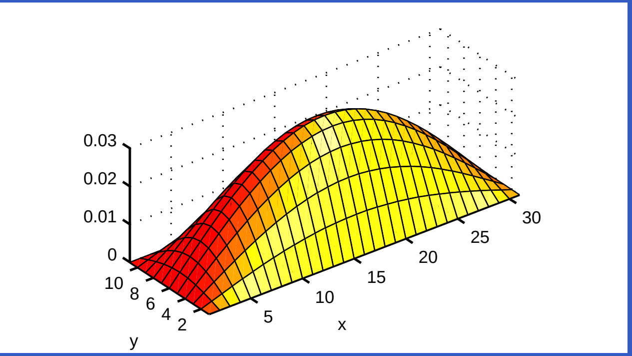


E_y for TE^z_{101}



E_y for TE^z_{201}



E_z for TM^z_{110}

$$\checkmark \quad \|\mathbf{e}_{\mathrm{num}}\| = 1, \quad \|\mathbf{e}_{\mathrm{th}} - \mathbf{e}_{\mathrm{num}}\| \approx 10^{-14}$$

Elementary dipole radiation

- Electric current $\boldsymbol{J}(z) = \hat{\boldsymbol{z}}$ at a single cell (1 GHz)
- Cells: $N_x = 31$, $N_y = 31$, $N_z = 31$

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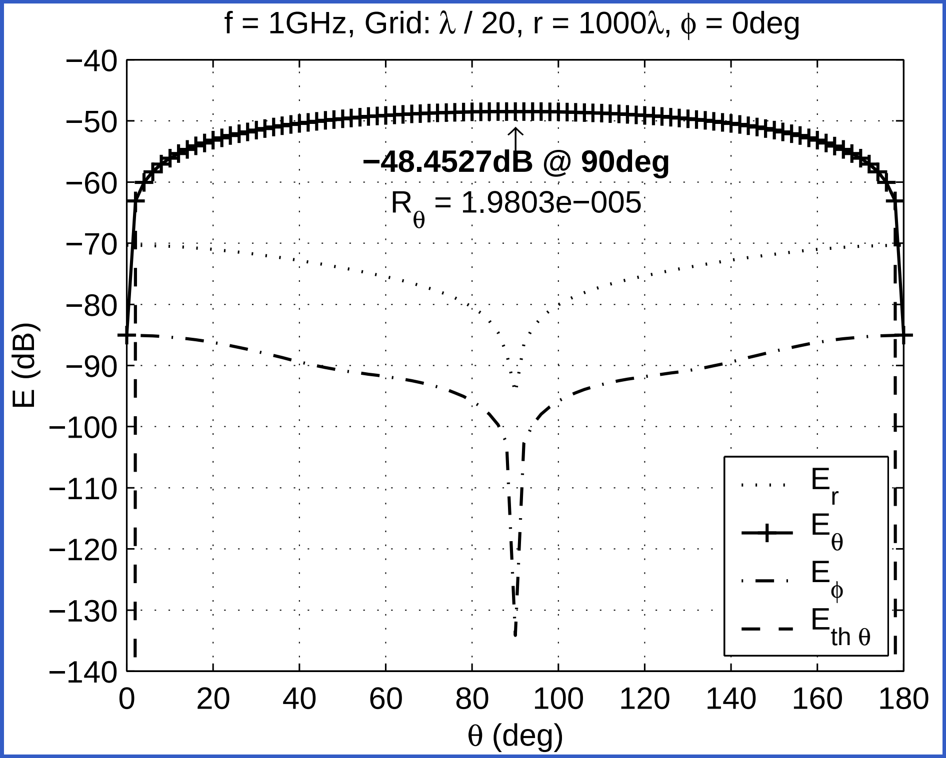

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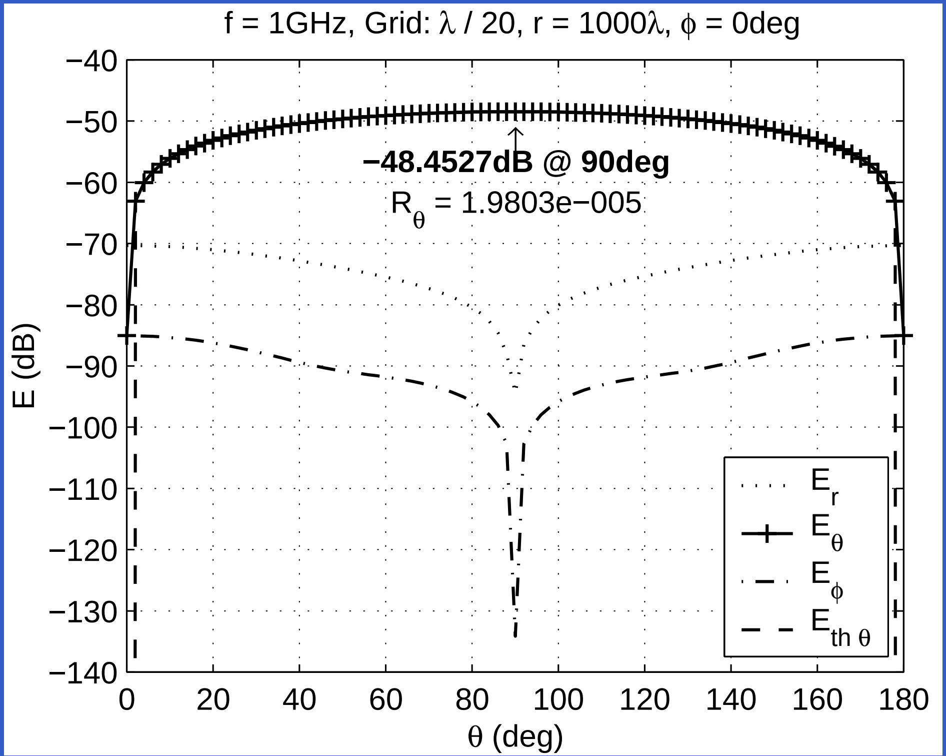

√ Fit theory to data $\longrightarrow$ dipole length $\lambda/44000$

Conclusion and future work

Two-dimensional numerical modeling

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- Enhanced microwave tomography method

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  - Near-field radiation pattern

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- Input information $\longrightarrow$ optimum quantity:
  variety$\Uparrow$ redundancy$\Downarrow$

Study of the three-dimensional extension

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  - Explicit formulation
- Near- to Far-Field transform
- Validations: waveguide, cavity, elementary dipole

Study of the three-dimensional extension

- Finite-difference frequency-domain method
  - Closed-geometry, radiation and scattering
  - 2D-TM, 2D-TE, 3D
  - Explicit formulation
- Near- to Far-Field transform
- Validations: waveguide, cavity, elementary dipole
- FDFD GREC (General-purpose Rectangular-mesh Electromagnetic Code) 20k MATLAB lines

Perspectives

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  - Proceed to the inverse problem
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  - Combine with different approaches
- Use FDFD GREC independently

- 1 in book

- 1 in book
- 7 in proceedings

- 1 in book
- 7 in proceedings
- 2 citations in IEEE Transactions