

DEVELOPMENT OF A HYBRID EULERIAN-LAGRANGIAN METHOD FOR THE NUMERICAL MODELING OF THE DISPERSED PHASE IN TURBULENT GAS-PARTICLE FLOWS

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PhD Defense

Introduction

Gas-Particle Flows Applications

- pollutant dispersion,
- combustion chamber, ...
- fluidized bed,

Numerous Physical Issues

- fluid turbulence influence over the particles,
- backward influence of the particles over the fluid,
- inter-particle collisions,
- rebounds on walls,
- chemistry, combustion, evaporation, ...

Numerical Simulation Advantages

- parallel calculation improvement
- production of numerical "experiences"
- cheaper than experience

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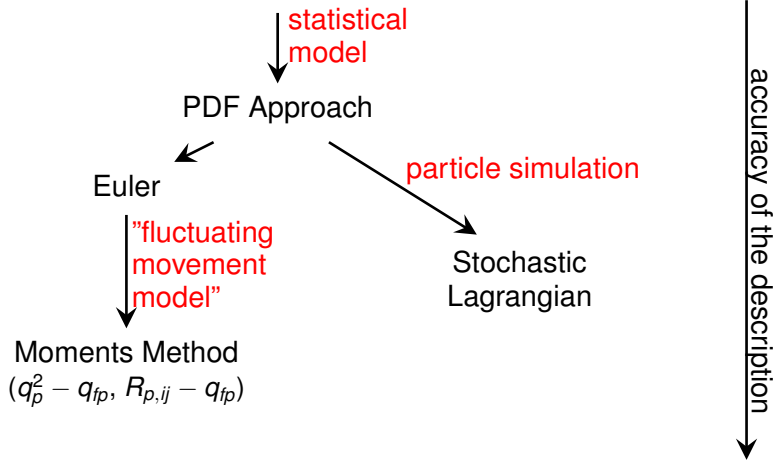
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Particle phase numerical simulation

Discrete Particle Simulation
(DPS, Deterministic Lagrangian)



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statistical
model

PDF Approach

Euler

particle simulation

Stochastic
Lagrangian

"fluctuating
movement
model"

Moments Method

$(q_p^2 - q_{fp}, R_{p,ij} - q_{fp})$

additional hypothesis

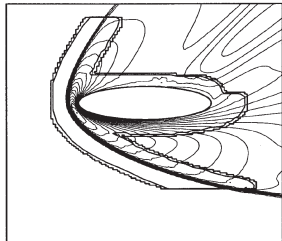
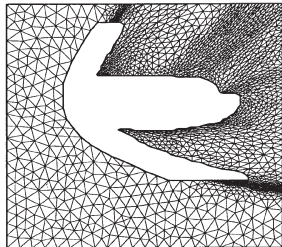
low-cost simulations

high numerical precision

direct simulations
high-cost simulations
low numerical precision

accuracy of the description

Motivation

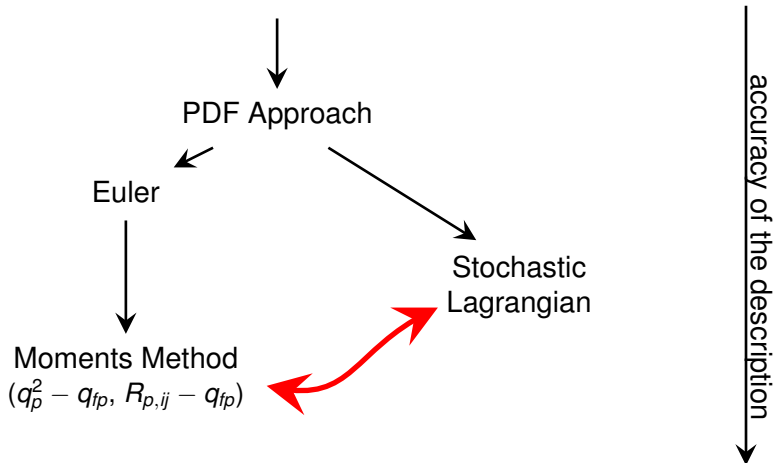


Le Tallec and Mallinger, JCP, 1997

- similarities with atmosphere re-entry flows
- Navier-Stokes cease to be valid near the obstacle, with two possible solutions :
 - extend the domain of validity of the continuous model (Burnett equations. . .)
 - use of a hybrid method coupling Navier-Stokes simulations with particle Boltzmann resolution
- hybridation based on the kinetic derivation of the Navier-Stokes equations
- the fluxes used in Navier-Stokes can be divided into outgoing and ingoing half-fluxes
- the exchange of information between the two domains is done via these half-fluxes

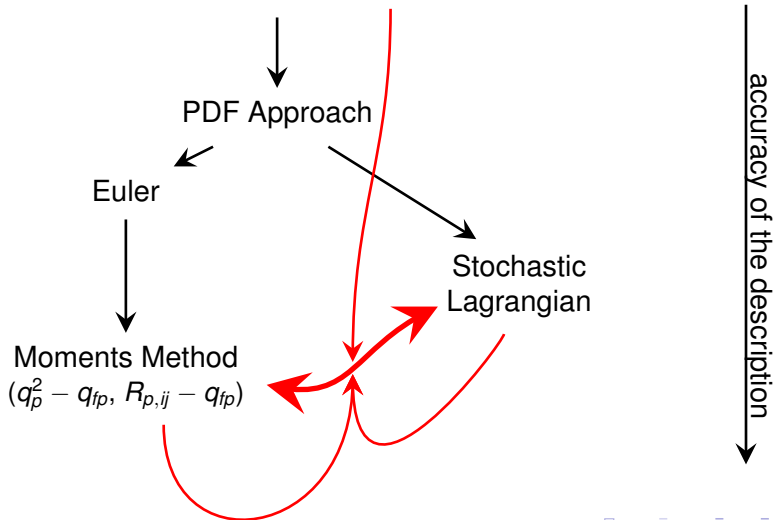
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Outline

- 1 PDF Approach
 - Statistical modeling
 - Stochastic lagrangian
 - Eulerian approach
- 2 Hybrid Methodology
 - Methodology
 - Lagrangian condition
 - Eulerian conditions
- 3 Validation
 - Homogeneous Shear Flow
 - Channel Flow

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Probability Density Function (PDF)

- Joint fluid-particle probability density function f_{fp} [Simonin, VKI, 1996],
- $f_{fp}(t; \underline{x}, \underline{c}_p, \underline{c}_f) d\underline{x} d\underline{c}_p d\underline{c}_f$ is the probable number of particles at instant t with properties $(\underline{x}_p, \underline{u}_p, \underline{u}_{f@p}) \in (\underline{x}, \underline{c}_p, \underline{c}_f)$ "+" $(d\underline{x}, d\underline{c}_p, d\underline{c}_f)$, obtained ideally by averaging over an infinity of realizations of the flow
- the evolution equation of this pdf is similar to the gas kinetic theory's Boltzmann equation :

$$\frac{\partial f_{fp}}{\partial t} + \frac{\partial}{\partial x_k} [c_{p,k} f_{fp}] + \frac{\partial}{\partial c_{p,k}} \left[\left\langle \frac{du_{p,k}}{dt} \middle| \underline{c}_f, \underline{c}_p \right\rangle f_{fp} \right] + \frac{\partial}{\partial c_{f,k}} \left[\left\langle \frac{du_{f@p,k}}{dt} \middle| \underline{c}_f, \underline{c}_p \right\rangle f_{fp} \right] = \left(\frac{\partial f_{fp}}{\partial t} \right)_{coll}$$

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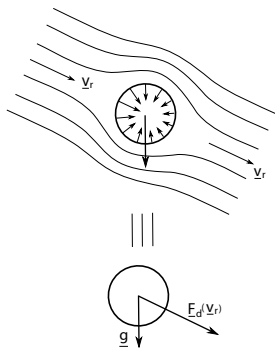
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Forces acting on a particle



- Small rigid spheres with $d_p \leq \eta_K$
- Mass point approximation
- Inertial particles : $\tau_p \gg \tau_K$
- Equation of motion for a particle :

$$\frac{du_{p,k}}{dt} = -\frac{u_{p,k} - u_{f@p,k}}{\tau_p} + g_k$$

- $\tau_p = \tau_p(|\underline{u}_p - \underline{u}_{f@p}|)$ is the particle relaxation time
- \underline{g} is the gravity
- $\underline{u}_{f@p}$ is the "seen" fluid velocity at the particle location :

Langevin scheme for the "seen" fluid velocity

- Inspired by single-phase works [Pope,1983], the fluctuating velocity $u''_{f@p,i} = u_{f@p,i} - \bar{U}_{f,i}$ is predicted using a Langevin equation [Simonin, 1996] :

$$du''_{f@p,i} = -\frac{\partial R_{ff,ik}}{\partial x_k} dt + \left(G_{fp,ik} - \frac{\partial U_{f,i}}{\partial x_k} \right) u''_{f@p,k} dt + H_{fp} \delta W_{fp,i}$$

- $\underline{\underline{A}} = \underline{\underline{G}}_{fp} - \underline{\underline{\nabla}} \cdot \underline{\underline{U}}_f$ is the drift tensor, which simplest model is given by the Simplified Langevin Model (SLM) :

$$G_{fp,ij} = -\delta_{ij} / \tau_{f@p}^t, \quad \tau_{f@p}^t = \tau_f^t$$

- $\underline{\underline{H}}_{fp}$ is the diffusion tensor, *i.e.* the tensor of amplitude of the Wiener process
- all these quantities are given data (and not calculated data as in *fluid* turbulence stochastic lagrangian modeling)

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Collision Operator

- binary collisions (dilute particle flows) of hard spheres (no coalescence)
- collision operator involves the *two points* joint fluid-particle pdf f_{fpfp}
- collision operator is modeled using an extension of the molecular chaos assumption [Bird, 1970]
- this assumption can be called fluid-particle chaos as it assume that the velocities describing colliding particles are uncorrelated :

$$f_{fpfp}(\underline{c}_{pA}, \underline{c}_{fA}, \underline{c}_{pB}, \underline{c}_{fB}) = f_{fp}(\underline{c}_{pA}, \underline{c}_{fA})f_{fp}(\underline{c}_{pB}, \underline{c}_{fB})$$

Stochastic lagrangian resolution

Transport step

- discretization of the pdf in numerical parcels.
- resolution of the transport by the trajectography of the numerical particles :

$$\frac{dx_{p,i}^{(m)}}{dt} = u_{p,i}^{(m)}$$

$$\frac{du_{p,i}^{(m)}}{dt} = -\frac{u_{p,i}^{(m)} - U_{f,i} - u_{f@p,i}''^{(m)}}{\tau_p^{(m)}} + g_i$$

$$du_{f@p,i}''^{(m)} = \frac{\partial R_{ff,ik}}{\partial x_k} dt + A_{ik} u_{f@p,k}''^{(m)} dt + B_{ik} \delta W_{fp,k}^{(m)}$$

Collision step

- discretization of the pdf in collision cells C_k .
- Bird algorithm (DSMC) :
 - decision on probable instants of collision,
 - for each instant, random picking of a particle pair,
 - decision over the occurrence of the collision

Eulerian moments

- the eulerian approach is based on the computation of a few pdf moments such as :

$$n_p = \int \int f_{fp}(\underline{c}_p, \underline{c}_f) d\underline{c}_p d\underline{c}_f$$

$$n_p U_{p,i} = \int \int c_{p,i} f_{fp}(\underline{c}_p, \underline{c}_f) d\underline{c}_p d\underline{c}_f$$

$$n_p V_{d,i} = \int \int (c_{f,i} - U_{f,i}) f_{fp}(\underline{c}_p, \underline{c}_f) d\underline{c}_p d\underline{c}_f$$

$$n_p R_{pp,ij} = \int \int (\underline{c}_{p,i} - U_{p,i})(\underline{c}_{p,j} - U_{p,j}) f_{fp}(\underline{c}_p, \underline{c}_f) d\underline{c}_p d\underline{c}_f$$

$$n_p R_{fp,ij} = \int \int (\underline{c}_{f,i} - U_{f@p,i})(\underline{c}_{p,j} - U_{p,j}) f_{fp}(\underline{c}_p, \underline{c}_f) d\underline{c}_p d\underline{c}_f$$

$$n_p \tilde{R}_{ff,ij} = \int \int (\underline{c}_{f,i} - U_{f@p,i})(\underline{c}_{f,j} - U_{f@p,j}) f_{fp}(\underline{c}_p, \underline{c}_f) d\underline{c}_p d\underline{c}_f$$

- thus, the governing equations of the Eulerian macroscopic variables can be derived from the evolution equation of f_{fp}

Moment equations

- $$\bullet \quad \frac{\partial n_p}{\partial t} + \frac{\partial}{\partial x_k} [n_p U_{p,k}] = 0$$
- $$\bullet \quad \frac{\bar{d}}{dt} U_{p,i} = -\frac{1}{n_p} \frac{\partial}{\partial x_k} [n_p R_{pp,ik}] - \langle F_{d,i}/m_p \rangle_p + g_i + \mathcal{C}(U_{p,i})$$
- $$\bullet \quad \frac{\bar{d}}{dt} R_{pp,ij} = -\frac{1}{n_p} \frac{\partial}{\partial x_k} [n_p S_{ppp,ijk}] - R_{pp,kj} \frac{\partial U_{p,i}}{\partial x_k} - R_{pp,ki} \frac{\partial U_{p,j}}{\partial x_k}$$

$$- \langle \frac{F_{d,i}}{m_p} u''_{p,j} + \frac{F_{d,j}}{m_p} u''_{p,i} \rangle_p + \mathcal{C}(R_{pp,ij})$$
- $$\bullet \quad \frac{\bar{d}}{dt} V_{d,i} = -\frac{\partial}{\partial x_k} [R_{fp,ik} - R_{ff,ik}] - \frac{R_{fp,ik}}{n_p} \frac{\partial n_p}{\partial x_k} + (G_{fp,ik} - \frac{\partial U_{f,i}}{\partial x_k}) V_{d,k}$$
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$$- \langle \frac{F_{d,i}}{m_p} u''_{p,j} + \frac{F_{d,j}}{m_p} u''_{f,i} \rangle_p + G_{fp,ik} R_{fp,kj} + \mathcal{C}(R_{fp,ij})$$
- $$\bullet \quad \tilde{R}_{ff,ij} = R_{ff,ij}$$

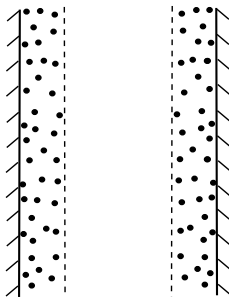
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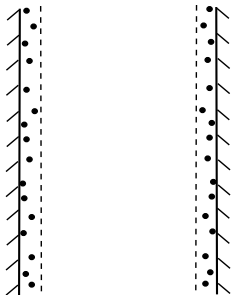
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Domain decomposition



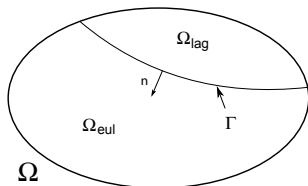
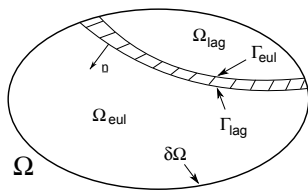
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- the location of the eulerian inner boundary surface is determined according to the validity of the eulerian approach
- static domain decomposition
- two main types of decomposition :
 - overlapping domains : $\Omega_{\text{eul}} \cup \Omega_{\text{lag}}$ is a domain
 - non-overlapping domains : $\Omega_{\text{eul}} \cup \Omega_{\text{lag}}$ is a surface

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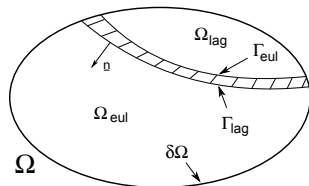
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Notations

Coupling surfaces are oriented from the lagrangian domain towards the eulerian domain ; ingoing and outgoing refer to the lagrangian domain.



- $\mathcal{F}(\Psi, \underline{x}_\Gamma)$ represent the total flux of Ψ across Γ
- $\mathcal{F}^-(\Psi, \underline{x}_\Gamma)$ represent the ingoing half-flux of Ψ across Γ , relative to the lagrangian domain
- $\mathcal{F}^+(\Psi, \underline{x}_\Gamma)$ represent the outgoing half-flux of Ψ across Γ
- Γ_{lag} and Γ_{eul} are the coupling boundary surface of Ω_{lag} and Ω_{eul}

Methodology for homogeneous flows

- non-overlapping domains
- methodology based on flux-splitting [Le Tallec and Mallinger, JCP, 1997] :
- the pdf of ingoing particles in Ω_{lag} is deduced from the eulerian moments, via a presumed pdf assumption [Pialat, Simonin and Villedieu, IJMF, 2007] :

$$\check{f}_p(\underline{c}) = \frac{n_p^{\text{eul}}}{\sqrt{8\pi^3 \det(\underline{\underline{R}}^{\text{eul}})}} \exp\left(-\frac{1}{2} {}^t \underline{c}'' \cdot \underline{\underline{R}}^{\text{eul}^{-1}} \cdot \underline{c}''\right)$$

with $\underline{c}'' = {}^t ({}^t(\underline{c}_p - \underline{U}_p^{\text{eul}}), {}^t(\underline{c}_f - \underline{U}_{f@p}^{\text{eul}}))$ and $\underline{\underline{R}}^{\text{eul}} = \begin{bmatrix} \underline{\underline{R}}_{pp}^{\text{eul}} & \underline{\underline{R}}_{fp}^{\text{eul}} \\ \underline{\underline{R}}_{fp}^{\text{eul}} & \underline{\underline{R}}_{ff}^{\text{eul}} \end{bmatrix}$

- flux boundary conditions are imposed to the eulerian calculation : $\mathcal{F}(\Psi, \underline{x}_\Gamma) = \mathcal{F}^{\text{eul}-}(\Psi, \underline{x}_\Gamma) + \mathcal{F}^{\text{lag}+}(\Psi, \underline{x}_\Gamma)$

$$\mathcal{F}^{\text{eul}-}(\Psi, \underline{x}_\Gamma) = \int_{\underline{c}_p \cdot \underline{n} < 0} (\underline{c}_p \cdot \underline{n}) \Psi \check{f}_p(\underline{c}) d\underline{c}$$

$$\mathcal{F}^{\text{lag}+}(\Psi, \underline{x}_\Gamma, t) = \frac{\kappa}{dS dt} \sum_i \Psi_i$$

Methodology for inhomogeneous flows

- outgoing half-fluxes are still calculated in the lagrangian simulation : $\mathcal{F}^{\text{lag}+}(\Psi, \underline{x}_{\Gamma_{\text{lag}}}) = \frac{\kappa}{dSdt} \sum_i \Psi_i$
- ingoing half-fluxes in the lagrangian region are given by the eulerian fluxes at the lagrangian coupling interface :

$$\mathcal{F}^{\text{lag}-}(\Psi, \underline{x}_{\Gamma_{\text{lag}}}) = \mathcal{F}^{\text{eul}}(\Psi, \underline{x}_{\Gamma_{\text{lag}}}) - \mathcal{F}^{\text{lag}+}(\Psi, \underline{x}_{\Gamma_{\text{lag}}})$$

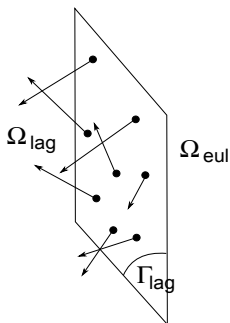
- the injection pdf is presumed as a gaussian pdf which moments \hat{n}_p , \hat{U}_i and \hat{R}_{ij} are given by the resolution of the system ($\Psi = \{1, c_i, \dots\}$) :

$$\int_{\underline{c}_p \cdot \underline{n} < 0} (\underline{c}_p \cdot \underline{n}) \Psi \hat{f}_p(\underline{c}) d\underline{c} = \mathcal{F}^{\text{lag}-}(\Psi, \underline{x}_{\Gamma_{\text{lag}}})$$

- flux boundary conditions are replaced by Dirichlet conditions for the eulerian calculation :

$$\langle \Psi \rangle_{\rho}^{\text{eul}}(\underline{x}_{\Gamma_{\text{eul}}}) = \langle \Psi \rangle_{\rho}^{\text{lag}}(\underline{x}_{\Gamma_{\text{eul}}})$$

Particle injection in the lagrangian domain



- calculate the gaussian presumed pdf \check{f}_{fp} or \hat{f}_{fp} from the eulerian moments :
 - eulerian moments interpolated at the coupling interface in the homogeneous case
 - resolution of a non-linear system to fit the ingoing half-fluxes in the inhomogeneous case
- pdf of injected particles is given by :

$$| \underline{c}_p \cdot \underline{n} | f_{fp}^{inj}(\underline{x}, \underline{c}, t) \text{ for } \underline{c}_p \cdot \underline{n} < 0$$

- the numerical particles are injected via a rejection method

Boundary conditions for the eulerian calculation

flux-type boundary conditions

- homogeneous cases
- $\mathcal{F}(\Psi) = \mathcal{F}^{\text{eul}-}(\Psi) + \mathcal{F}^{\text{lag}+}(\Psi)$
- most natural conditions at the coupling surface
- continuity of the fluxes

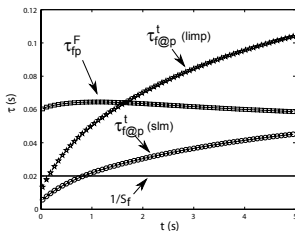
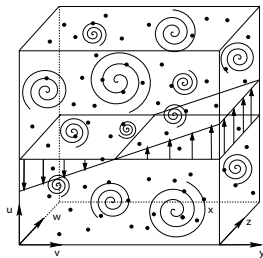
Dirichlet boundary conditions

- inhomogeneous cases
- $\mathcal{F}^{\text{lag}-}(\Psi) = \mathcal{F}^{\text{eul}}(\Psi) - \mathcal{F}^{\text{lag}+}(\Psi)$
- calculation of $\langle \Psi \rangle_{\rho}^{\text{lag}}$ on Γ_{eul}
- continuity of the moments

Outline

- 1 PDF Approach
 - Statistical modeling
 - Stochastic lagrangian
 - Eulerian approach
- 2 Hybrid Methodology
 - Methodology
 - Lagrangian condition
 - Eulerian conditions
- 3 Validation
 - Homogeneous Shear Flow
 - Channel Flow

Homogeneous shear flow [Laviéville, univ. Rouen, 1997]



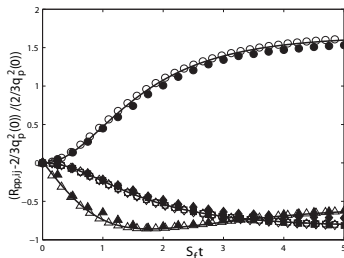
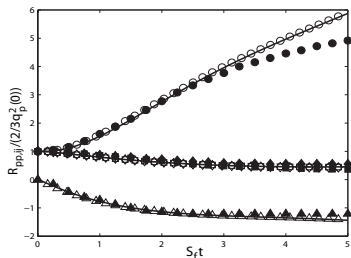
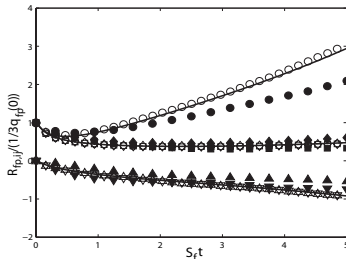
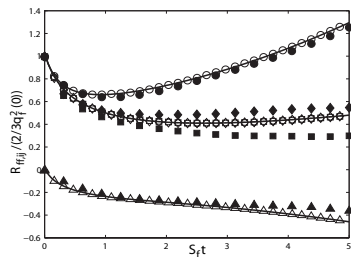
Particle properties	
Particle diameter, d_p (m)	$656 \cdot 10^{-6}$
Particle density, ρ_p ($kg \cdot m^{-3}$)	100.
Mean number density, n_p (m^{-3})	$8.46 \cdot 10^9$
Stokes relaxation time, τ_p (s)	$139 \cdot 10^{-3}$

Fluid properties	
Fluid density, ρ_f ($kg \cdot m^{-3}$)	1.17
Kinematic viscosity, ν_f ($m^2 \cdot s^{-1}$)	$1.47 \cdot 10^{-5}$
Mean velocity gradient, S_f (s^{-1})	50.

Isotropic Homogeneous Turbulence	
Box length, L (m)	0.192
Interaction time scale, $\tau_{f@p}^t$ (s)	$36.7 \cdot 10^{-3}$
Turbulent energy, k_f ($m^2 \cdot s^{-2}$)	0.12

- unsteady turbulent flow
- without collision

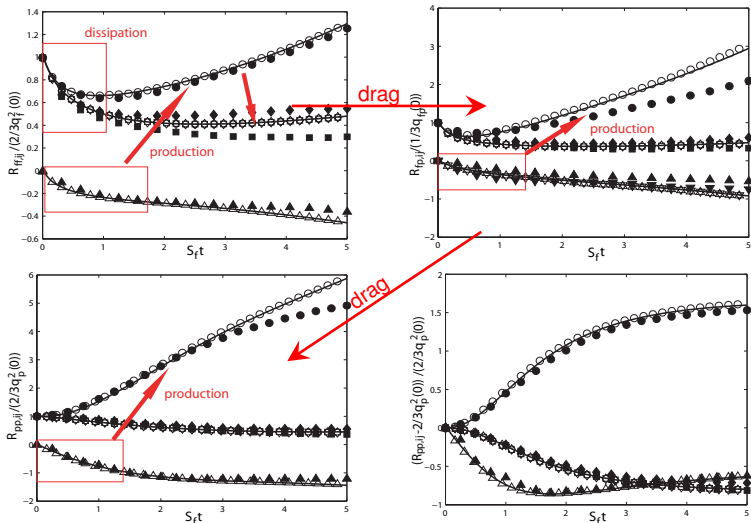
Consistency (without collisions)



● : LES/DPS [Laviéville, univ. Rouen, 1997] (600000 real particles) ;

— : eulerian (Rotta) ; ○ : stochastic lagrangian (SLM) (30000 numerical particles)

Consistency (without collisions)



● : LES/DPS [Laviéville, univ. Rouen, 1997] (600000 real particles) ;

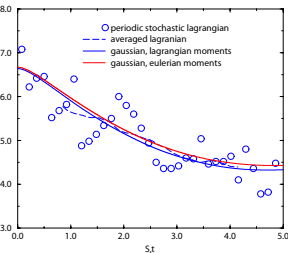
— : eulerian (Rotta) ; ○ : stochastic lagrangian (SLM) (30000 numerical particles)

Half-fluxes study in a periodic box

- the average over particles "passing" through a surface is linked to the half-flux of the variable through this surface :

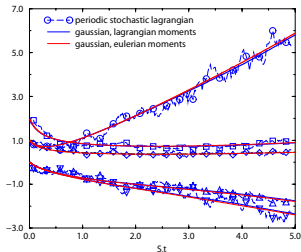
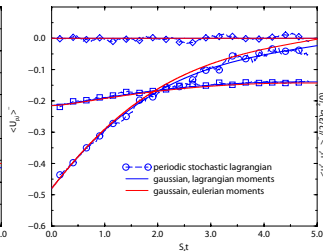
$$\langle \Psi \rangle_{c_p \cdot \underline{n} < 0} \stackrel{\text{def}}{=} \langle \Psi \rangle^- = \frac{\mathcal{F}^-(\Psi)}{\mathcal{F}^-(n_p)}$$

Outgoing Numerical particles Per Time-Step



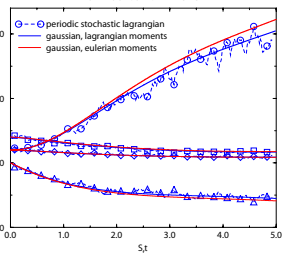
Averaged Particle Velocities

OVER OUTGOING PARTICLES

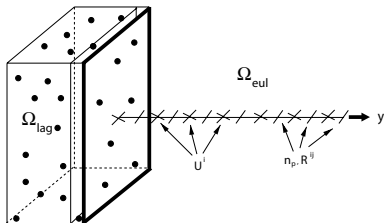


Averaged Kinetic Stresses Evolution

OVER OUTGOING PARTICLES

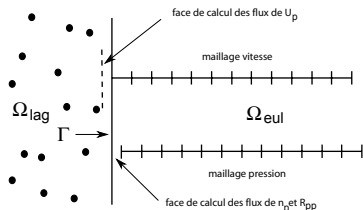


HELM spatial configuration



- coupling surface oriented in the normal direction
- no mean particle normal velocity :

$$\underline{U}_p \cdot \underline{n} = 0$$



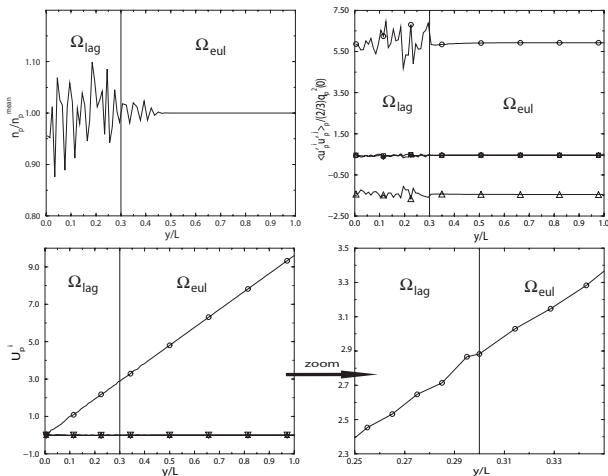
- 30% of the box simulated by the lagrangian approach
- mono-dimensional meshes in the eulerian region
- shifted meshes
- finite-volume discretization

Coupling algorithm

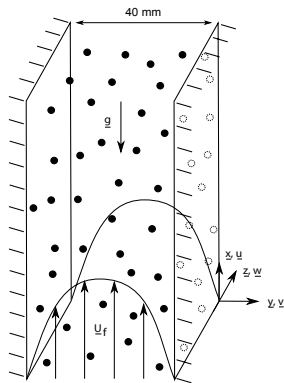
- 1 solve the lagrangian equation set for 5 time-steps dt_{lag} with :
 - regular lagrangian boundary conditions on $\partial\Omega \cap \Omega_{\text{lag}}$,
 - random injection of discrete particle following the presumed ingoing eulerian pdf at the interface,
- 2 compute the outgoing lagrangian half-fluxes $\mathcal{F}_{\Gamma}^{\text{lag}+}$,
- 3 solve the eulerian equation set for 1 time-step $dt_{\text{eul}} = 5dt_{\text{lag}}$ with :
 - regular eulerian boundary conditions on $\partial\Omega \cap \Omega_{\text{eul}}$,
 - flux boundary conditions obtained from
$$\mathcal{F}(\Psi) = \mathcal{F}^{\text{eul}-}(\Psi) + \mathcal{F}^{\text{lag}+}(\Psi),$$
- 4 compute the particle and fluid velocity moments at the interface to presume the ingoing eulerian pdf.

HELM results (particle moments)

- time-development of the moments well predicted
- profiles in the normal direction at $S_f t = 5$



Channel flow [Sakiz, ENPC, 1999]

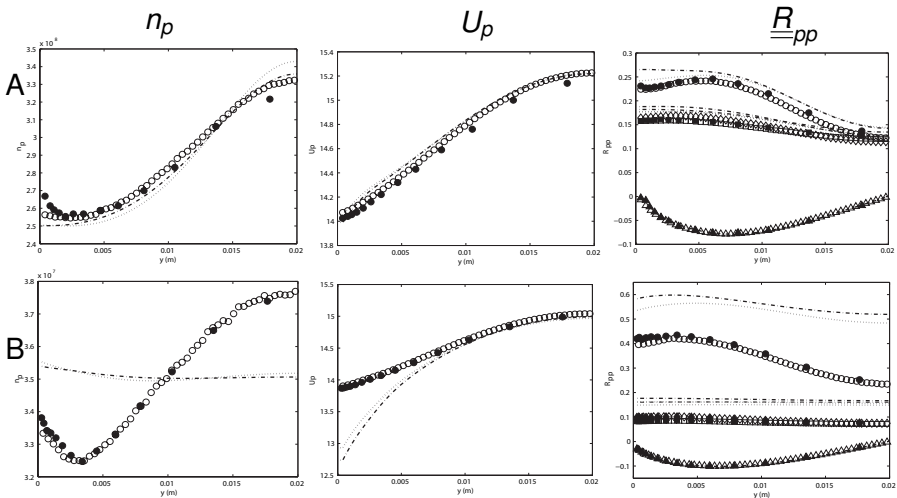


Particle properties	A	B
Particle diameter, d_p (m)	406. 10^{-6}	
Particle density, ρ_p ($kg.m^{-3}$)	1038.	
Mean volumetric fraction, $\bar{\alpha}_p$	1.10^{-2}	$1.23 \cdot 10^{-3}$
Elastic rebounds	yes	no
Mean inter-collision time, τ_p^c (s)	5.10^{-3}	7.10^{-2}
Mean relaxation time, τ_{fp}^f (s)	0.125 – 0.25	

Fluid properties	
Fluid density, ρ_f ($kg.m^{-3}$)	1.205
Kinematic viscosity, ν_f ($m^2.s^{-1}$)	$1.515 \cdot 10^{-5}$
Mean axial velocity, \bar{U}_f ($m.s^{-1}$)	16.
Lagrangian integral time scale, τ_f^t (s)	$0 - 6.10^{-4}$

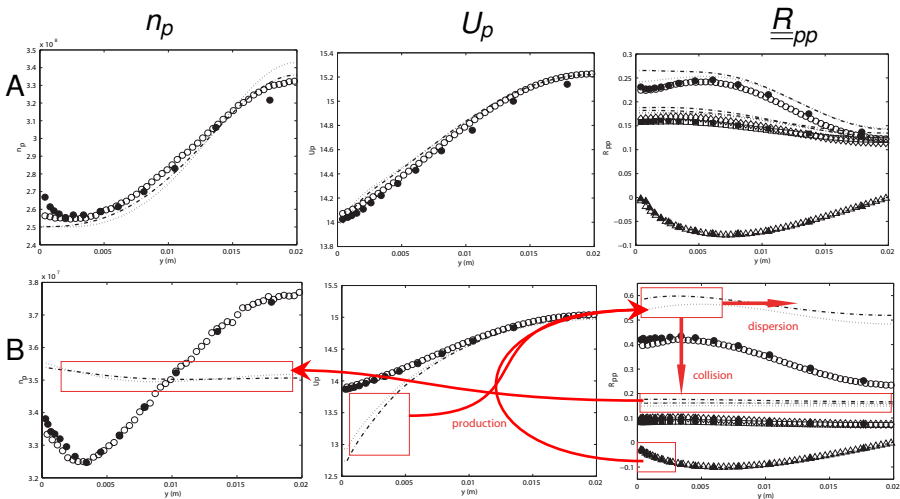
- steady flow
- $St_L = \tau_{fp}^F / \tau_{fp}^t \geq 200$
- no influence of the turbulence over the particles ($\underline{R}_{fp} \simeq \underline{0}$, $\underline{V}_d \simeq \underline{0}$)

Consistency



● : DPS [Sakiz] ; ○ : stochastic lagrangian ;
 - - - : eulerian (Daly-Harlow) ; ··· : eulerian (Hanjalic and Launder)

Consistency



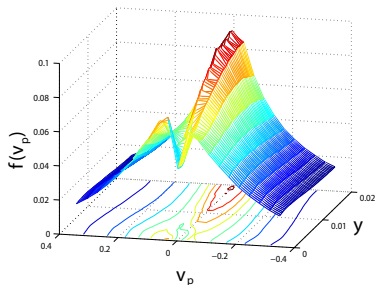
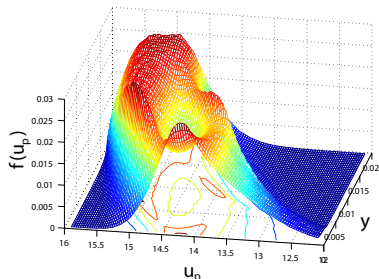
● : DPS [Sakiz] ; ○ : stochastic lagrangian ;
 --- : eulerian (Daly-Harlow) ; ... : eulerian (Hanjalic and Launder)

Particle Knudsen number

- particle pseudo-mean free path :

$$\lambda_p = \sqrt{\frac{\pi R_{pp,vv}}{2}} \left(\frac{2}{\tau_{fp}^F} + \frac{\sigma_c}{\tau_p^c} \right)^{-1}$$

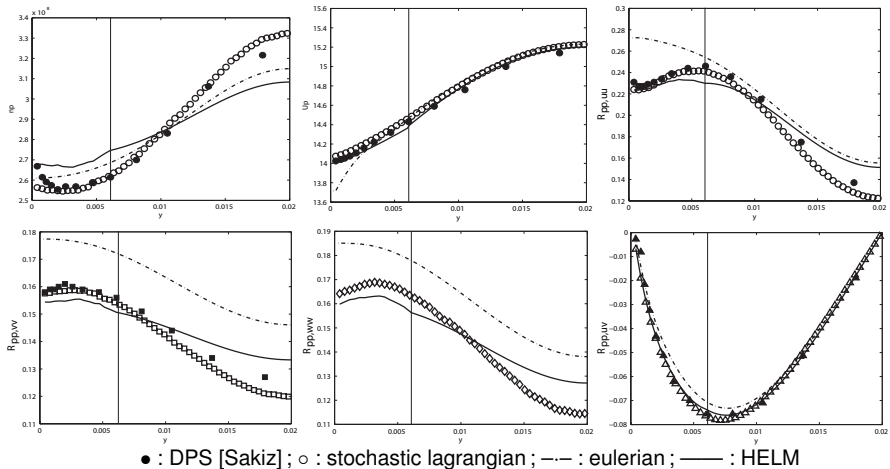
- particle Knudsen number
 $Kn_p = \lambda_p / L_y \approx 0.3$
- Navier-Stokes validity $Kn \leq 10^{-3}$
- flow far from equilibrium
- eulerian predictions can be improved by corrections relative to the particle Knudsen number and corrected drag treatment [Sakiz, ENPC, 1999]



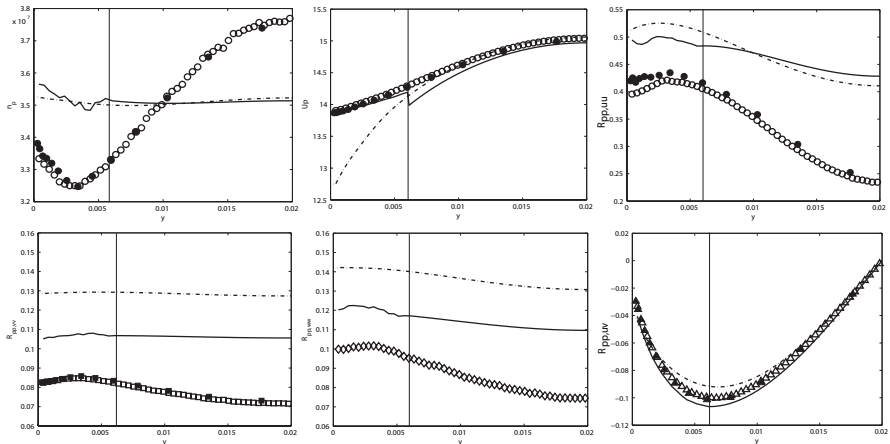
Coupling algorithm

- 1 solve the lagrangian equation set for 10000 time-steps dt_{lag} with :
 - rebounds on the wall,
 - random injection of discrete particle following the presumed ingoing eulerian pdf at the interface,
- 2 compute the outgoing lagrangian half-fluxes on the lagrangian boundary $\mathcal{F}_{\Gamma_{\text{lag}}}^{\text{lag}+}$ and the moments $\langle \Psi \rangle_p^{\text{lag}}$ (Γ_{eul}),
- 3 solve the eulerian equation set for 20000 time-step dt_{eul} with :
 - symmetry conditions at the center of the channel,
 - Dirichlet boundary conditions on Γ_{eul} ,
- 4 compute the total fluxes on Γ_{lag} needed to presume the injection pdf with the same half-fluxes as computed by $\mathcal{F}^{\text{lag}-}(\Psi) = \mathcal{F}^{\text{eul}}(\Psi) - \mathcal{F}^{\text{lag}+}(\Psi)$.
- 5 normalize the solution to keep the number of particles constant.

HELM results (A)

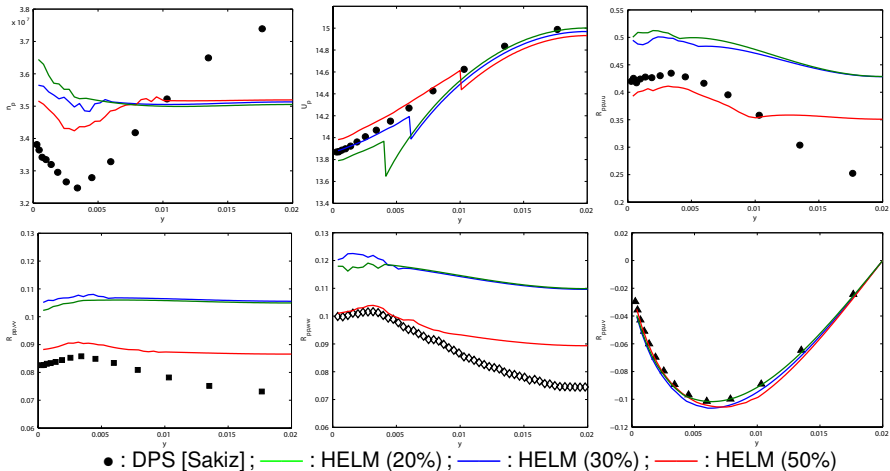


HELM results (B)



• : DPS [Sakiz] ; ○ : stochastic lagrangian ; - - - : eulerian ; — : HELM

Influence of the interface's location



Main contributions

- relations between stochastic lagrangian and eulerian approaches (Langevin equation),
- development of the code solving the two approaches in separate domains and coupling them,
- definition of a coupling methodology in homogeneous flows :
 - random injection of numerical particles in Ω_{lag} ,
 - flux boundary condition for the eulerian calculation,
- assessment in homogeneous shear flow,
- extension of this methodology to inhomogeneous flows :
 - calculated moments of injection to fit the ingoing half-fluxes,
 - Dirichlet boundary conditions,
- validation in channel flow,
- **proved the feasibility of such hybrid method for gas-particles flows.**

Main prospects

PDF approach

- potentialities/limitations of the Langevin equation,
- comparison with LES/DPS in the channel flow [Arcen, Vance].

Coupling methodology

- rough walls, deposition. . .
- turbulent gas-particle channel flow,
- study of the coupling boundary conditions,
- lower-order eulerian approach ($q_p^2 - q_{fp}$),
- validity criterion for the eulerian approach (automatic domain decomposition).

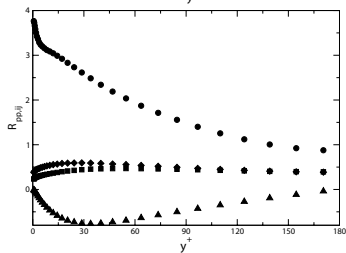
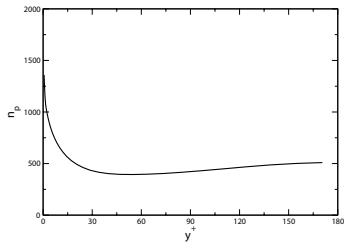
Thank you for your attention !



retour sur innovation



LES/DPS of a channel flow



Richman pdf's ingoing half-fluxes

$$\mathcal{F}_\Gamma^-(\Psi, \underline{x}_\Gamma, t) = \int_{\underline{c}_p \cdot \underline{n} < 0} (\underline{c}_p \cdot \underline{n}) \Psi(\underline{c}_f, \underline{c}_p) f_{fp} d\underline{c}_f d\underline{c}_p$$

- For a Richman joint pdf, the half-fluxes can be explicitly calculated :

$$\check{\mathcal{F}}_\Gamma^-(n_p) = \check{n}_p \left[(\check{U}_p \cdot \underline{n}) g\left(\frac{\check{U}_p \cdot \underline{n}}{\sqrt{\check{R}_{pp,nn}}}\right) + \sqrt{\check{R}_{pp,nn}} h\left(\frac{\check{U}_p \cdot \underline{n}}{\sqrt{\check{R}_{pp,nn}}}\right) \right]$$

$$\check{\mathcal{F}}_\Gamma^-(U_i) = \check{U}_i \check{\mathcal{F}}_\Gamma^-(n_p) + \check{n}_p \Lambda_{ik} Q_{k1} \check{R}_{pp,nn} g\left(\frac{\check{U}_p \cdot \underline{n}}{\sqrt{\check{R}_{pp,nn}}}\right)$$

$$\check{\mathcal{F}}_\Gamma^-(R_{ij}) = \check{R}_{ij} \check{\mathcal{F}}_\Gamma^-(n_p) + \check{n}_p (\Lambda_{ik} Q_{k1}) (\Lambda_{jl} Q_{l1}) \check{R}_{pp,nn} \sqrt{\check{R}_{pp,nn}} h\left(\frac{\check{U}_p \cdot \underline{n}}{\sqrt{\check{R}_{pp,nn}}}\right)$$

- in inhomogeneous cases, we want the injection to give determined half-fluxes

$$\mathcal{F}^{\text{lag}-}(\Psi, \underline{x}_{\Gamma_{\text{lag}}}) = \mathcal{F}^{\text{eul}}(\Psi, \underline{x}_{\Gamma_{\text{lag}}}) - \mathcal{F}^{\text{lag}+}(\Psi, \underline{x}_{\Gamma_{\text{lag}}})$$

- the moments of injection \check{n}_p , \check{U}_i and \check{R}_{ij} are then chosen in order to have

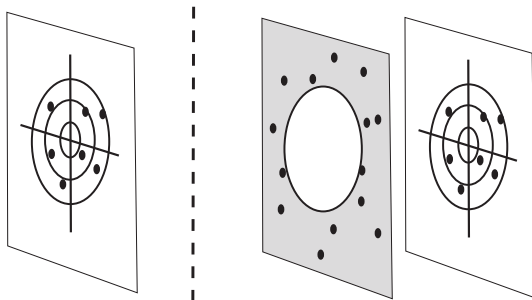
$$\check{\mathcal{F}}_\Gamma^-(\Psi) = \mathcal{F}^{\text{lag}-}(\Psi, \underline{x}_{\Gamma_{\text{lag}}})$$

Rejection method for injection

- The pdf associated with the injected particles is then

$$| \underline{c}_p \cdot \underline{n} | \check{f}_{fp}(\underline{x}, \underline{c}, t) \text{ for } \underline{c}_p \cdot \underline{n} < 0$$

- The half-flux associated to this pdf is simulated by a rejection method
- A majorant pdf can be taken as $f = v_{max} \check{f}_{fp}$

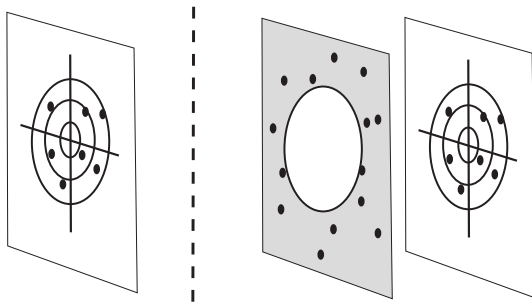


Rejection method for injection

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$$| \underline{c}_p \cdot \underline{n} | \check{f}_{fp}(\underline{x}, \underline{c}, t) \text{ for } \underline{c}_p \cdot \underline{n} < 0$$


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Rejection method for injection

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- The half-flux associated to this pdf is simulated by a rejection method
- A majorant pdf can be taken as $f = v_{max} \check{f}_{fp}$
- Then for each injected particle :
 - compute random fluctuating velocities following $f : (\underline{u}''_p, \underline{u}''_f)$
 - compare the particle normal fluctuating velocity $| \underline{u}''_p \cdot \underline{n} |$ with $v_{max} z$, where z is a random number computed following a uniform law on $[0, 1]$.
 - if superior, the particle is effectively injected in Ω_{lag} with the velocities $(\underline{U}_p + \underline{u}''_p, \underline{U}_f + \underline{u}''_f)$ and advanced in time for a random duration $\delta t^* \in [0, \delta t]$,
 - if inferior, the particle is not injected with these velocities and a new process takes place for that particle. 

Additional eulerian closures (particle motion) ◀

drag terms

$$\begin{aligned}
 - \langle \frac{F_{d,i}}{m_p} \Psi \rangle_p &= \langle \frac{u_{p,k} - u_{f@p,k}}{\tau_p (|\underline{u}_p - \underline{u}_{f@p}|)} \Psi \rangle_p \\
 &\approx \langle \frac{(u_{p,k} - u_{f@p,k}) \Psi}{\tau_p \langle |\underline{u}_p - \underline{u}_{f@p}| \rangle_p} \rangle_p
 \end{aligned}$$

$$\Pi_{p,i} \approx - \frac{U_{p,i} - U_{f@p,i}}{\tau_{fp}^F}$$

$$\Pi_{p,ij} \approx - \frac{2}{\tau_{fp}^F} (R_{pp,ij} - \mathcal{R}_{fp,ij})$$

collision terms

- $f_{fp} = f_{fp} f_{fp}$
- 2nd order Grad's expansion pdf

$$C(U_{p,i}) = 0$$

$$C(R_{pp,ij}) = - \frac{R_{pp,ij} - \frac{2}{3} \alpha_p^2 \delta_{ij}}{5/4 \tau_p^C}$$

dispersion terms

- 2nd order Grad's expansion pdf *and* Gaussian pdf
- fluid element limit consistency

$$S_{ppp,ijk} = -K_{p,kl}^t \frac{\partial}{\partial x_l} R_{pp,ij}$$

$$K_{p,mn}^t = \sigma_p^t \left[\frac{\tau_{fp}^F}{\xi_{fp}^F} R_{pp,mn} + \frac{\xi_p^C}{\tau_p^C} \mathcal{R}_{fp,mn} \right]$$