Efficient Use of Local and Regional Information for Flood Frequency Analysis

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The Aim of an Extreme Value Analysis

"To know or predict some statistical properties of events that haven't been seen yet (or only a few)..."

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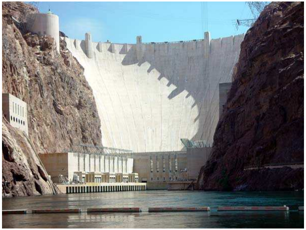

The Hoover dam (Colorado)

The Taichai river (Taiwan)

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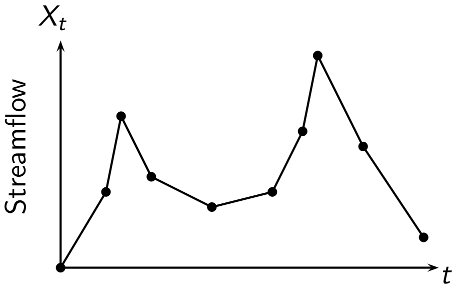
Figure: The Seine river at Paris, France (January 1910).

Question

How unpredictable is an event such as the one depicted above?

- We need a theoretical framework to answer this question
- This is the aim of the Extreme Value Theory (EVT)

AM/POT Asymptotic Distributions



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- Define $Y_j = \{X_t : X_t > u, t = 1, \ldots, n\}$
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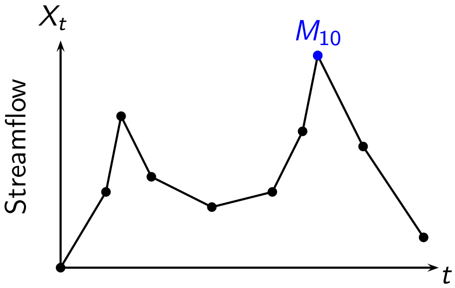

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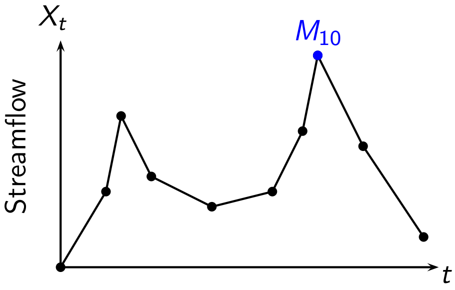

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where $G(x) = \exp\left[-\left(1 + \xi \frac{x-\mu}{\sigma}\right)_+^{-1/\xi}\right]$ is the Generalized Extreme Value distribution (GEV)

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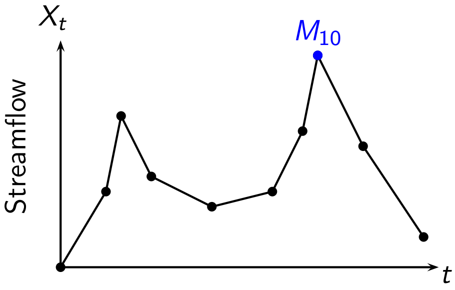

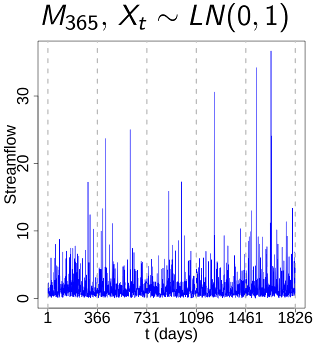

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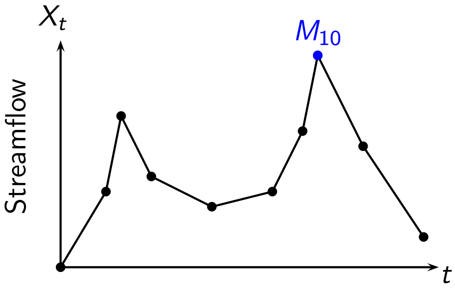

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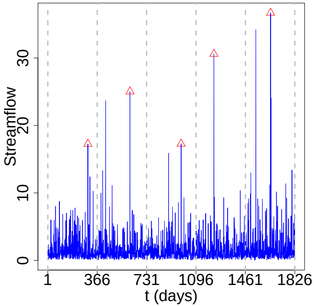

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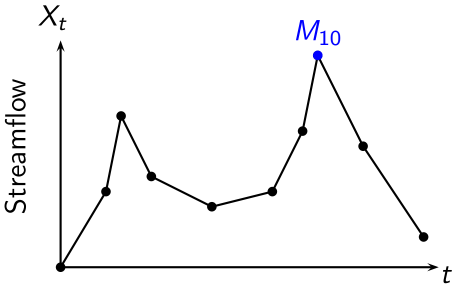

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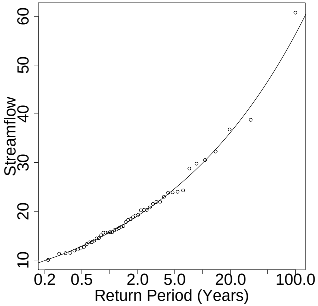

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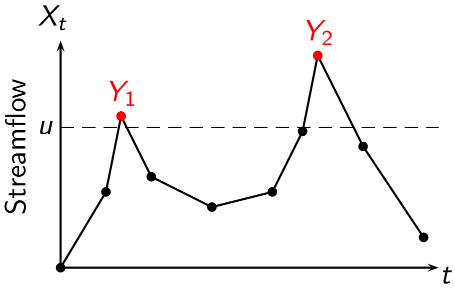

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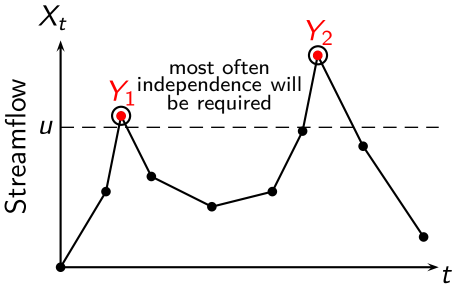

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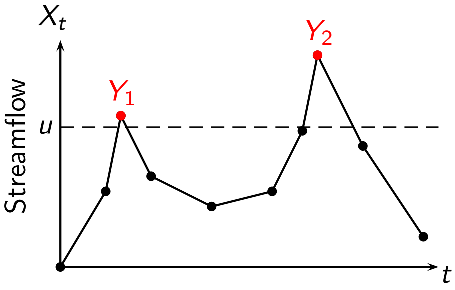

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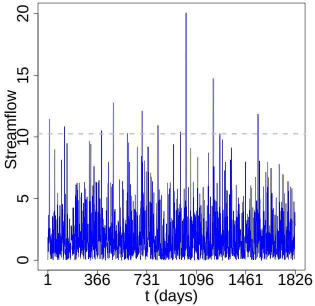

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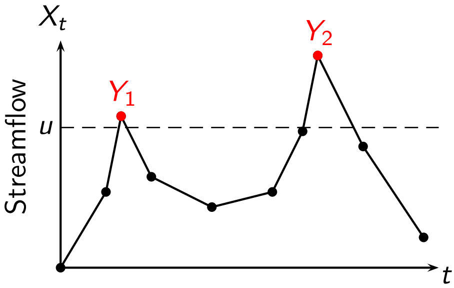

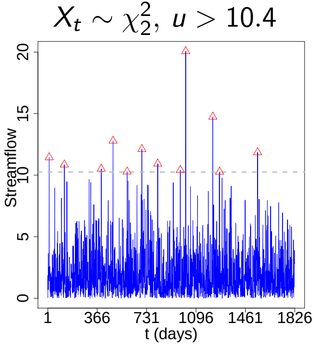

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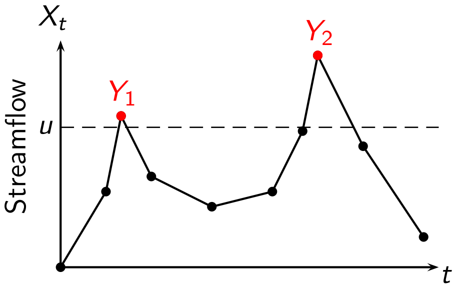

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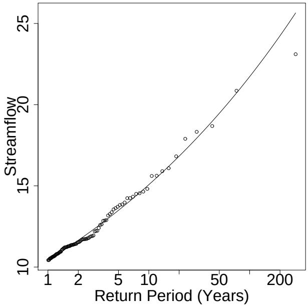

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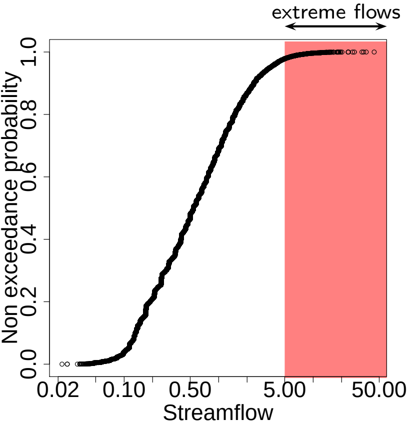
The GPD needs 3 parameters to be estimated:

location u defines which levels are extreme?

scale σ controls how "smallest extremes" increase?

shape ξ controls how "largest extremes" evolve?



Link with conventional distributions

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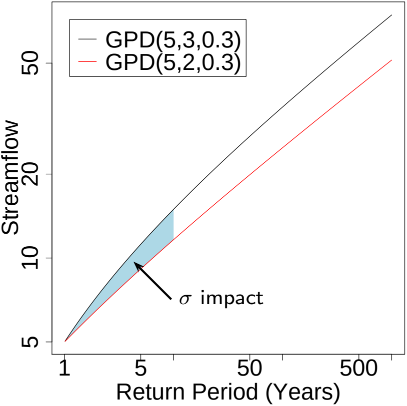

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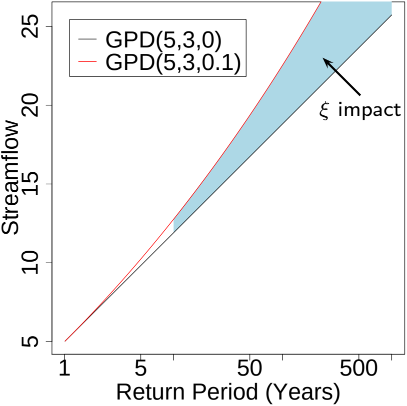

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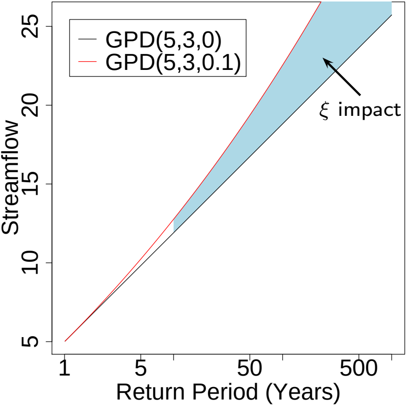

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- Using the EVT, we do not need to know X_t's distribution
  (we did not suppose X_t were LN or χ^2 distributed)
- We only need to get closer to the asymptotic hypothesis
- However, as we focus on the tail area,
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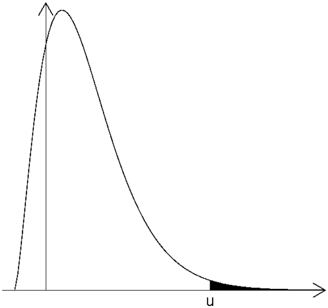

Figure: A probability density function and a right-tail area.

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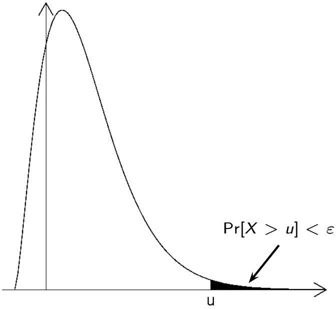

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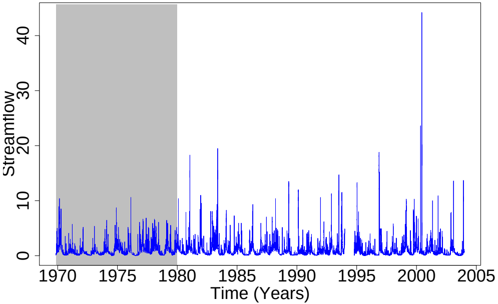

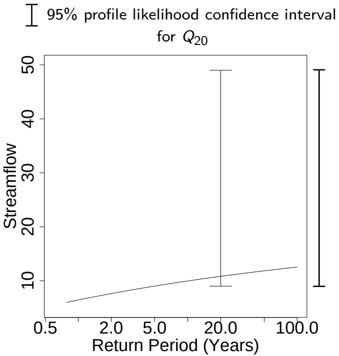

Figure: An Illustration with the Ardières River at Beaujeu

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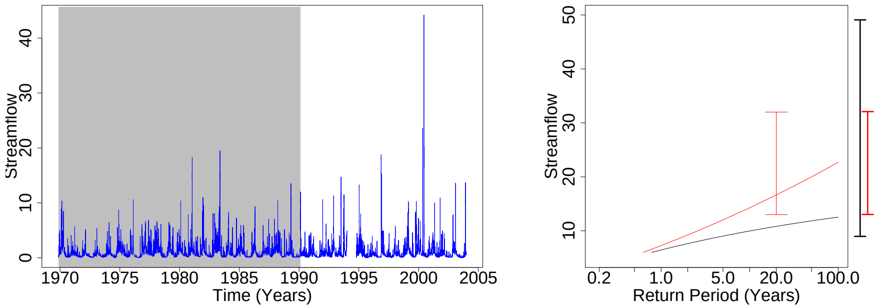

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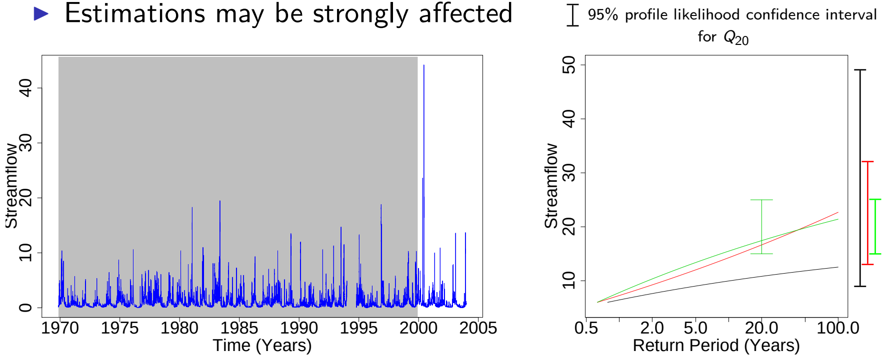

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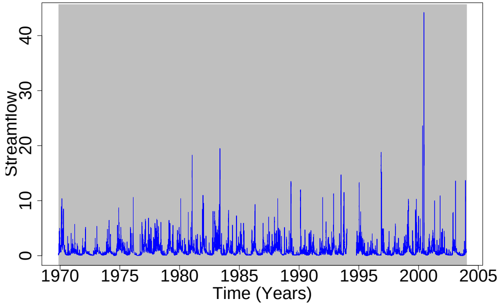

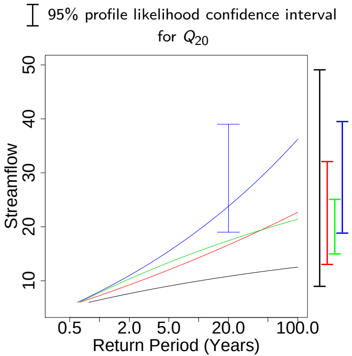

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- EVT states which distribution has to be fitted to the data
- But estimations may be still unreliable even with relatively large record lengths
- There is a need to provide models which are robust enough for partially gaged stations (less than 10 years record length)

Objectives

- Provide statistical models which use the EVT potency
- Prove their accuracies on partially gaged stations
- Use efficiently all the data by:
  - using regional information: data from other gaged stations
  - using the data from the target site in a new way: modeling all excess

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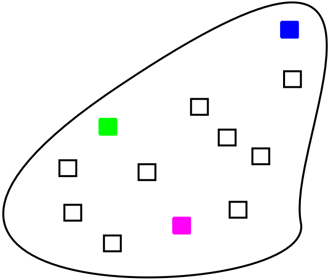

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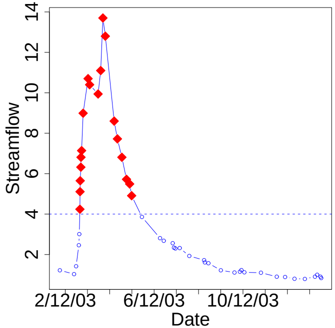

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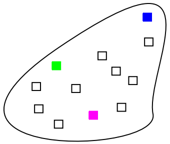

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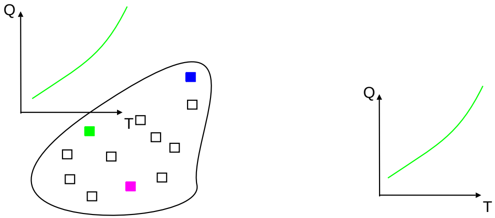

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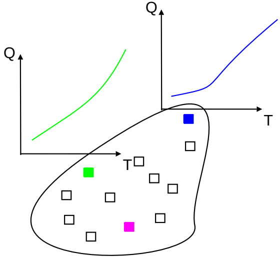

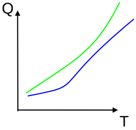

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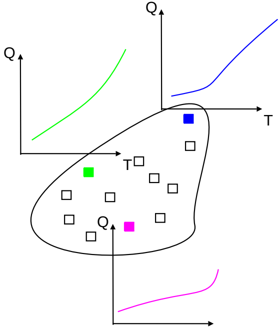

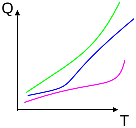

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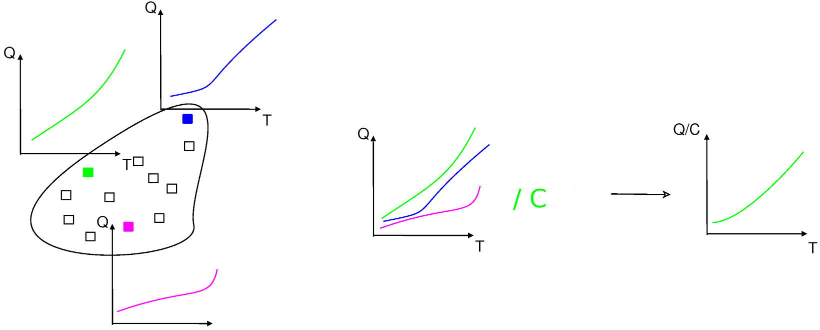

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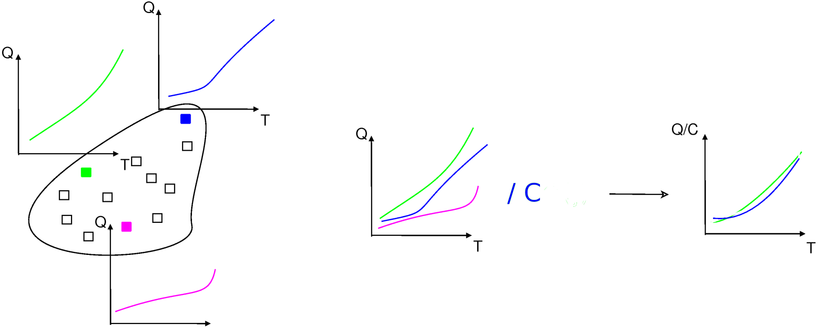

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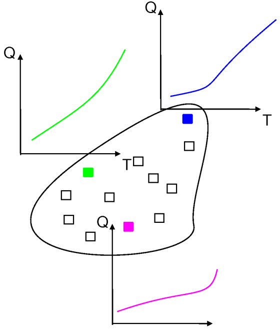

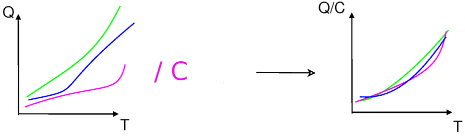

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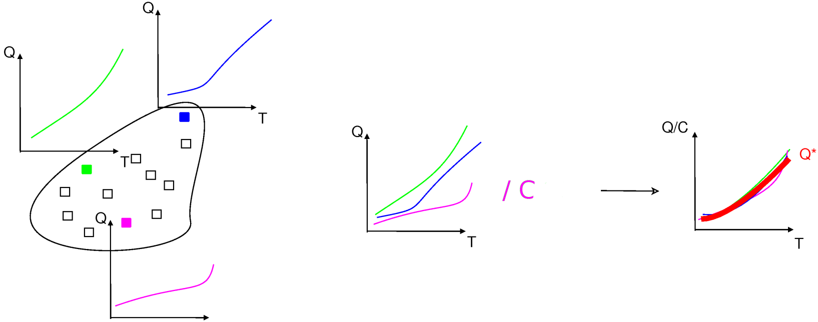

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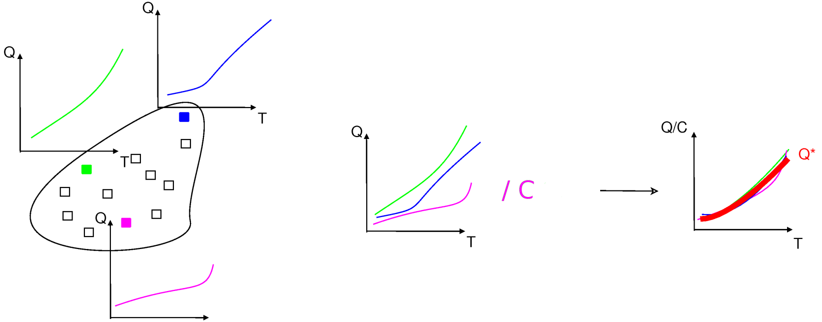

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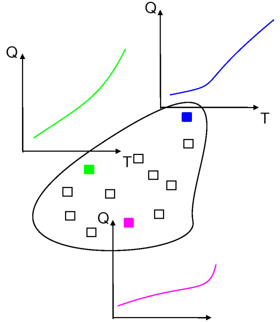
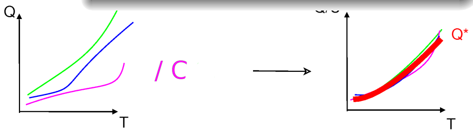
C estimation

- Using the target site sample, e.g, sample mean, median, . . .
- Using physiographic variables - ungaged sites.





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The Index Flood Has Drawbacks

Several authors point out some problems with the IFL approach:

- No likelihood [Katz et al., 2002]
  $\Rightarrow$ What about estimate uncertainties?
- Wrong scale invariance properties [Gupta et al., 1994; Robinson and Sivapalan, 1997]
  $\Rightarrow$ Bias, artificial variance reduction
  - By definition, each observation within the pooling group has the same weight
  - Target site observation may have poor influence

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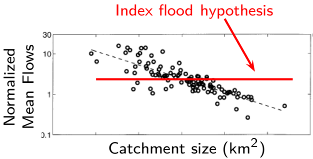

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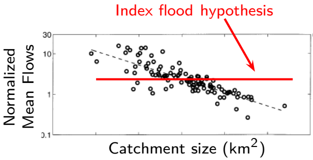

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Outline

Use Regional Information in Another Way

Our Goals

- The whole information must be used differently
- Especially, particular attention must be paid to the target site
- Need to relax scale invariance properties
- Need to have a rigorous statistical model

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Presentation of the BAY Model

The BAY model is based on the so-called Bayes' theorem:

$$\pi(\theta \mid x) = \frac{\pi(\theta)\pi(x;\theta)}{\int_\Theta \pi(\theta)\pi(x;\theta)d\theta}$$

where $\pi(\theta|x)$ is the *posterior* distribution, $\pi(\theta)$ is the *prior* distribution and $\pi(x;\theta)$ the likelihood function.

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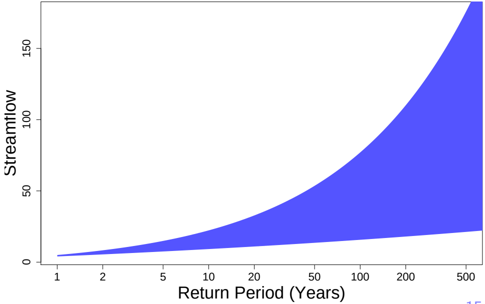

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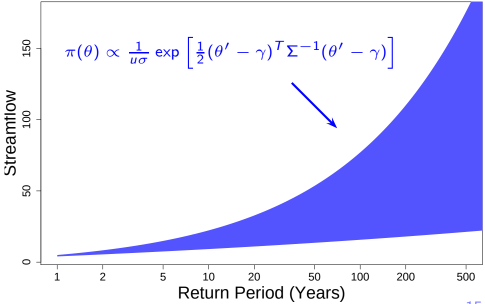

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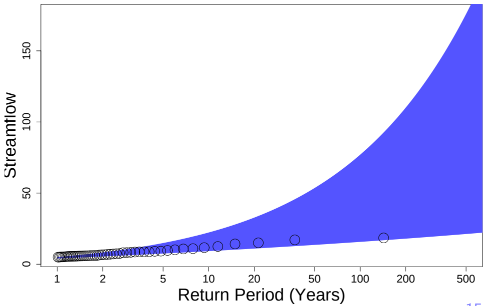

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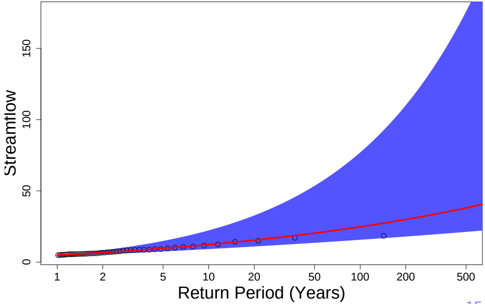

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- IFL supposes that there is a unique regional distribution
- Thus, a unique 3-uplet GPD parameters (u, σ, ξ)
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Figure: Scale invariance properties relaxation.

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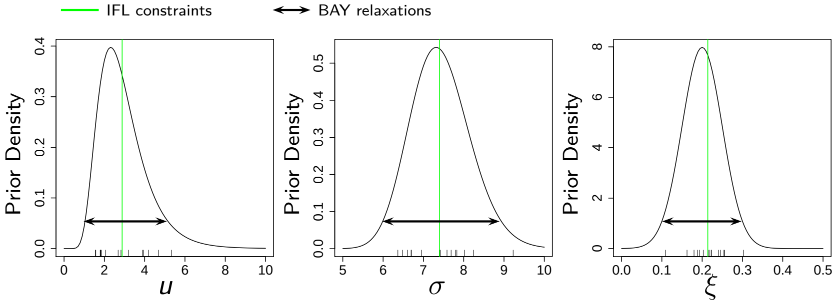

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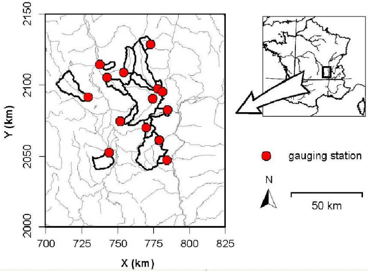
Outline

The Case Study

Characteristics of the Pooling Group

- 14 homogeneous stations
  (DIREN Rhône Alpes)
- Record length: 22 to 37 years
- Drainage areas: 32 to 792 km^2
- 3 particular target sites (37 years)



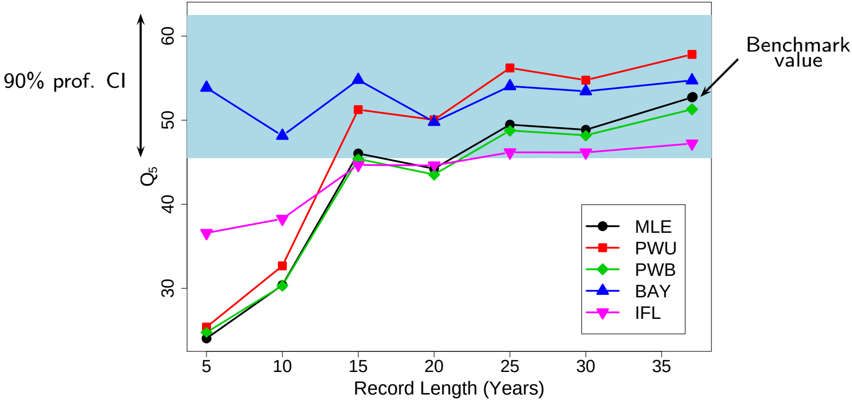
Outline

Performance for Partially Gaged Stations Assessment

Methodology

- Benchmark Values: MLE on the whole time series
- Estimators: MLE, PWU, PWB, IFL and BAY
- Inferences on several "sub-time series"
  (5, 10, 15, 20, 25, 30 and 37 years)
- Quantiles of interest: Q_2, Q_5, Q_{10} and Q_{20}

Results



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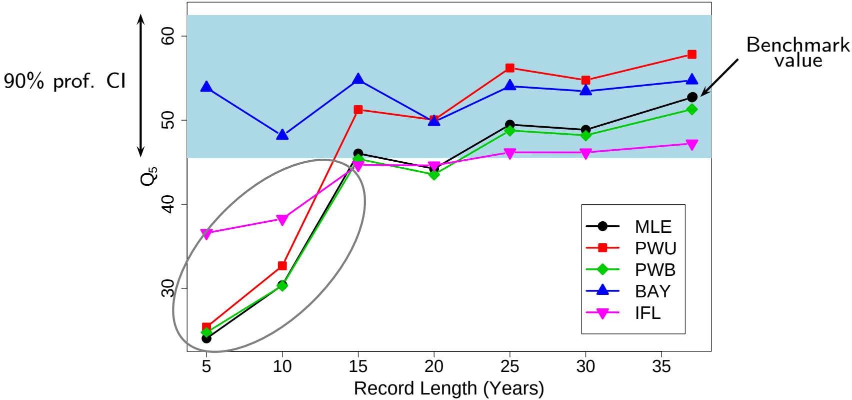

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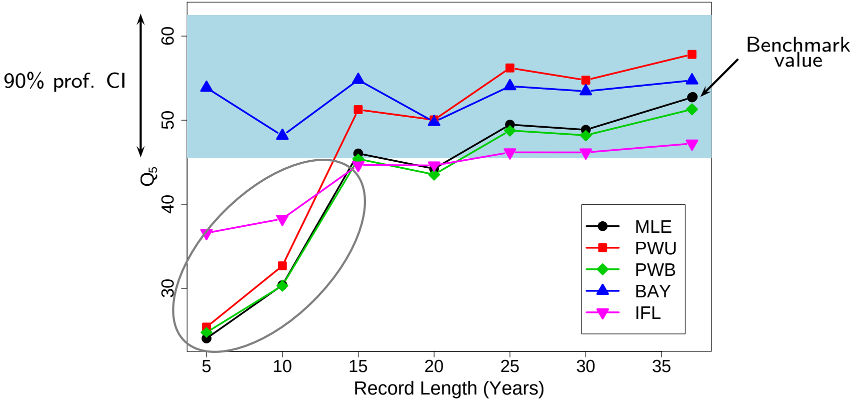

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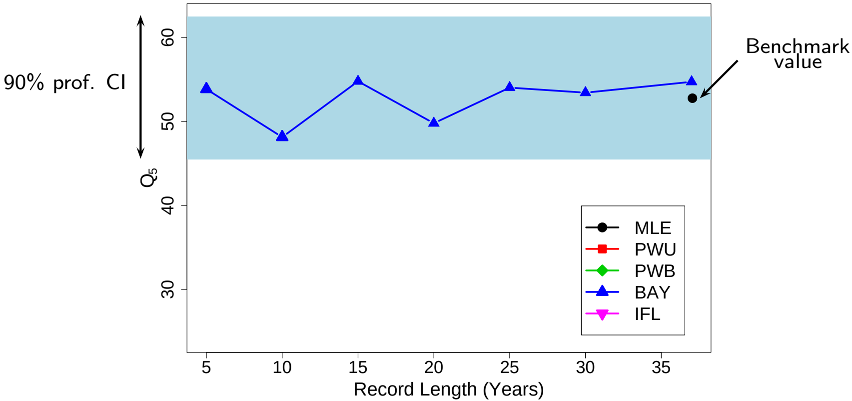

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Outline

Assessment of the Homogeneity Degree Impact

Methodology

- We now consider 4 distinct pooling groups:

  He^+ Definitively heterogeneous region
  He Probably heterogeneous region
  Ho Probably homogeneous region
  Ho^+ Definitively homogeneous region

- Inferences are performed on several “sub-time series”
  (5, 10, 15, 20, 25 and 30 years)
- Studied estimators: MLE, IFL and BAY
- Comparison using a score approach
  (measure of both bias and variance for all estimates)
  - Score ≈ 1: good estimator
  - Score ≈ 0: poor estimator

Results

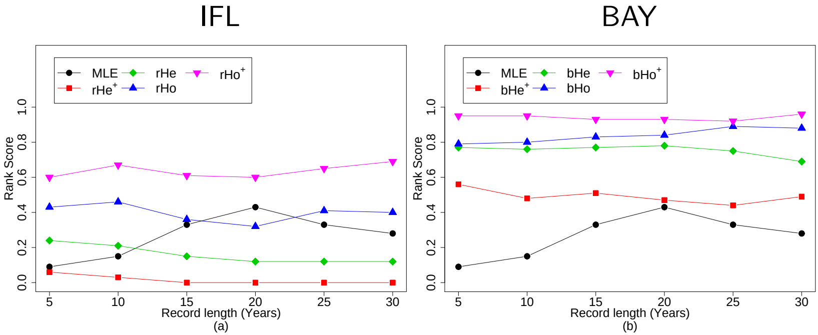

Figure: Evolution of the score as the record length increases.

- To increase homogeneity is more relevant for IFL
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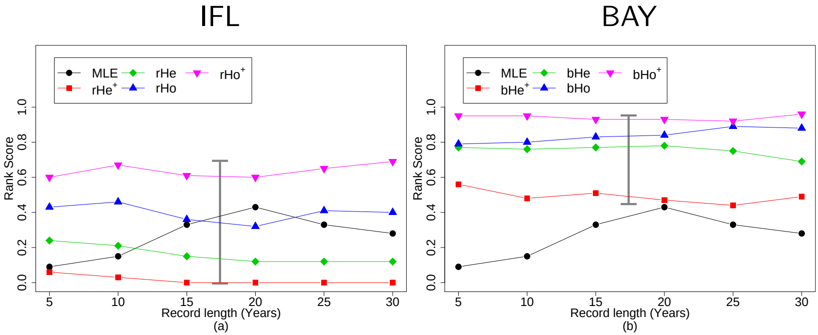

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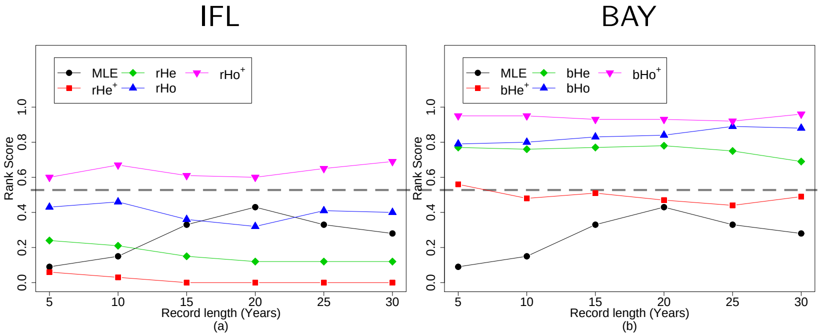

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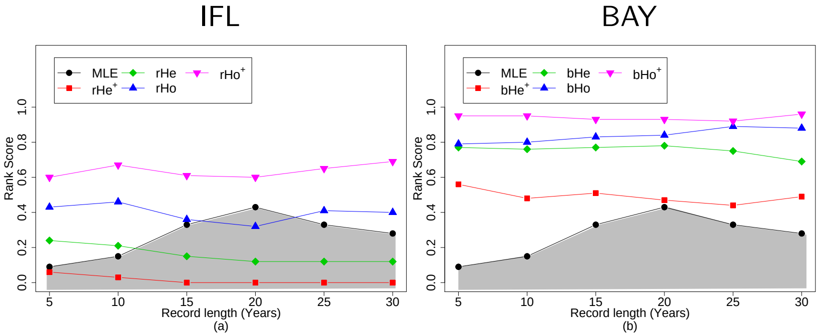

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- The empirical Index Flood concept was relaxed using a rigorous Bayesian model
- BAY performance was assessed on a specific French and homogeneous region (for Q_T, $T \leq 20$ years)
- Results show that:
  - BAY is more accurate than IFL (especially when the record length ≤ 10 years)
  - BAY seems more robust than IFL comparing:
    - the sample variability (presence/absence of "extreme extremes")
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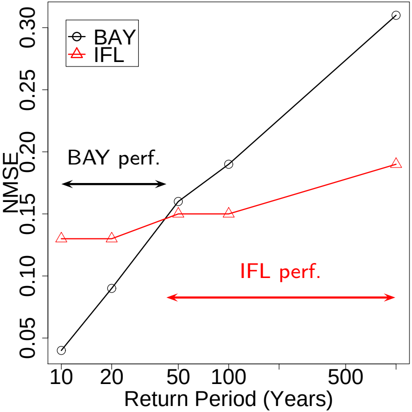

Figure: Evolution of the NMSE as the return period increases.

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- Mainly because of the estimation of the GPD shape parameter, i.e., large variance

Table: *NMSE* for the GPD parameter estimates for the BAY and IFL models.

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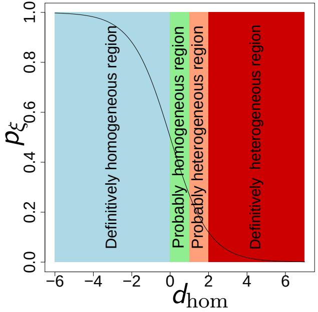
p_ξ parametrization

$$p_\xi = \frac{\exp(-d_{\text{hom}})}{1 + \exp(-d_{\text{hom}})}$$



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New Prior Asset

- $\pi(\theta)$ allows to consider ξ_{Fix} with a non-null probability
- but we do not impose $\xi = \xi_{\mathrm{Fix}}$
- Only the target site data will corroborate the ξ_{Fix} relevancy (by sampling in Θ_0 within the MCMC stage.)

Outline

Impact of the Regional Configuration on Estimations

A Study on Simulated Data

Methodology

- Two regional distributions:
  - Heavy tail: Conf1–3 ($\xi > 0$)
  - near Expo.: Conf4–6 ($\xi \approx 0$)
- Several region configurations:
  - Conf1,Conf4: small regions (10 sites) with many data (450 obs.)
  - Conf2,Conf5: large regions (20 sites) with few data (450 obs.)
  - Conf3,Conf6: medium regions (15 sites) with many data (700 obs.)
- Comparison between 3 regional estimators:
  - IFL: Index Flood
  - BAY: First Model
  - REV: Reversible jump approach (i.e. mixture)
- Targeted quantiles $Q_{0.75}$, $Q_{0.95}$ and $Q_{0.995}$ are aimed (e.g. return periods $\approx$ 2, 10 and 100 years)
- Sample sizes of the target site: 10, 25, 40

Finally, 18,000 homogeneous regions were simulated

BAY/IFL Comparison

Table: Evolution of the NRMSE as the region configuration changes.

	$\xi > 0$				$\xi \approx 0$		
	$Q_{0.75}$	$Q_{0.95}$	$Q_{0.995}$		$Q_{0.75}$	$Q_{0.95}$	$Q_{0.995}$
		CONF1		Small Regions		CONF4	
BAY	0.015	0.035	0.101		0.011	0.022	0.054
IFL	0.037	0.038	0.053		0.025	0.027	0.037
		CONF2		Large Regions		CONF5	
BAY	0.015	0.063	0.326		0.012	0.037	0.144
IFL	0.035	0.037	0.049		0.029	0.031	0.039
		CONF3		Medium Regions		CONF6	
BAY	0.012	0.030	0.085		0.011	0.023	0.050
IFL	0.037	0.039	0.049		0.029	0.032	0.042

- $Q_{0.75}$: *BAY is the most accurate model* (Confirms previous results)
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BAY	0.015	0.035	0.101		0.011	0.022	0.054
IFL	0.037	0.038	0.053		0.025	0.027	0.037
		CONF2		Large Regions		CONF5	
BAY	0.015	0.063	0.326		0.012	0.037	0.144
IFL	0.035	0.037	0.049		0.029	0.031	0.039
		CONF3		Medium Regions		CONF6	
BAY	0.012	0.030	0.085		0.011	0.023	0.050
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ξ_{Fix} suggestion leads to more accurate estimations for large quantiles while preserving the same accuracy for smaller quantiles

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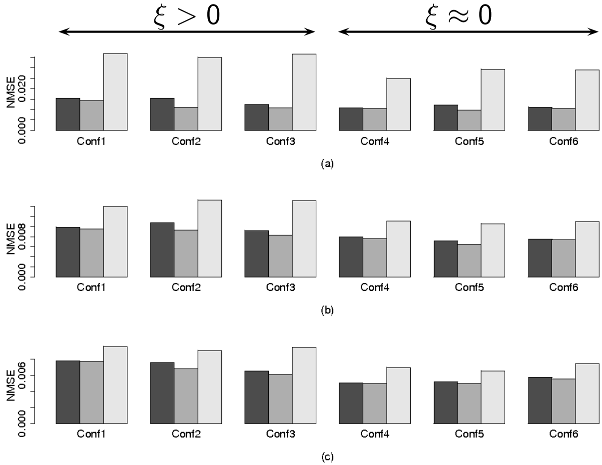

Target Sample Size: 10

Target Sample Size: 25

Target Sample Size: 40

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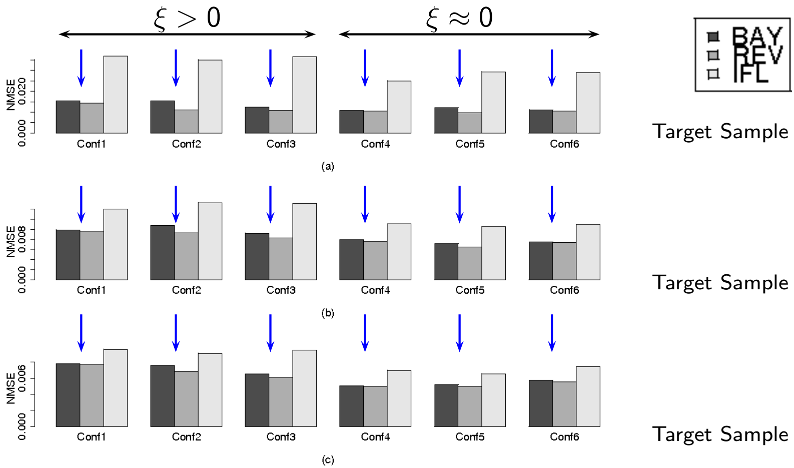

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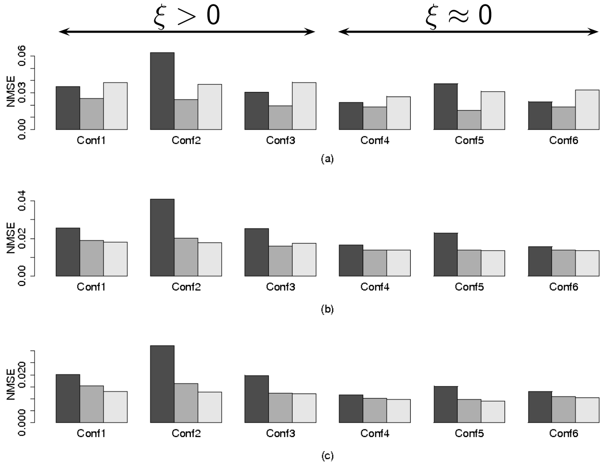

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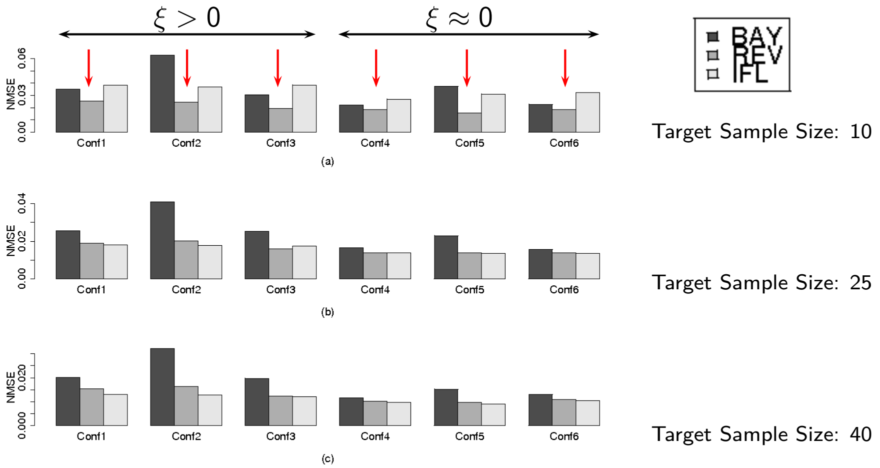

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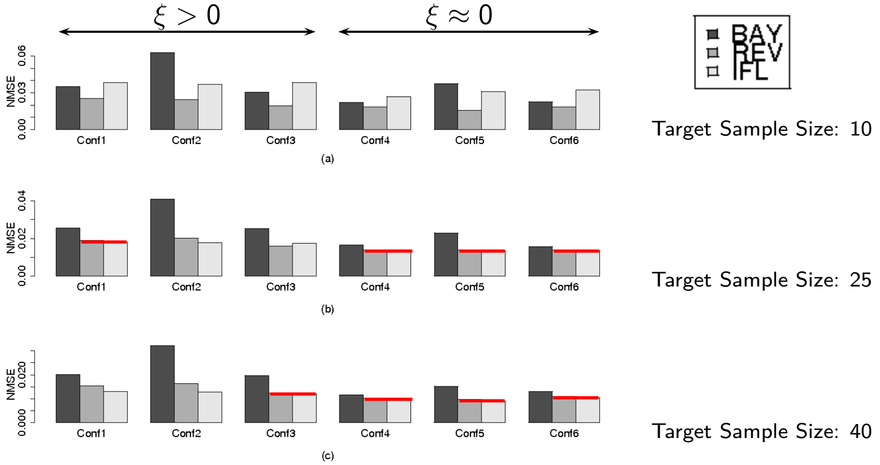

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Conclusions

- REV generalizes BAY to derive more robust extrapolations
- A relevant regional shape parameter ξ_{Fix} was proposed
- For Q_T, $T \leq 20$ years, REV has the same performance as BAY
- For Q_T, $T > 20$ years, there are two cases:
  - Record length < 10 years: REV is the most accurate model
  - Otherwise, REV and IFL yield to the same results

Outline

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Motivations

- Previous models distinguish the target site from the other gaging stations
- Such procedures lead to more accurate estimations
  $\Longrightarrow$ Focusing on the data from the target site has a large impact on quantile estimations
- It seems to be a logical stage before looking at other kind of information (e.g. regional, historical information)

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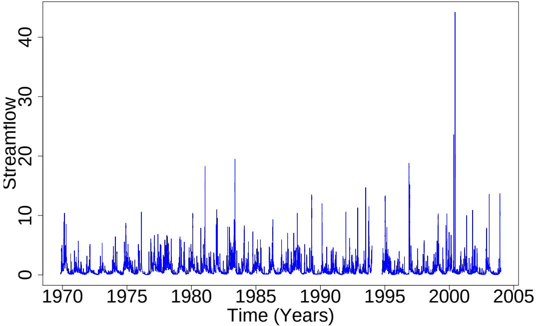
Has the information from the target site been efficiently used?



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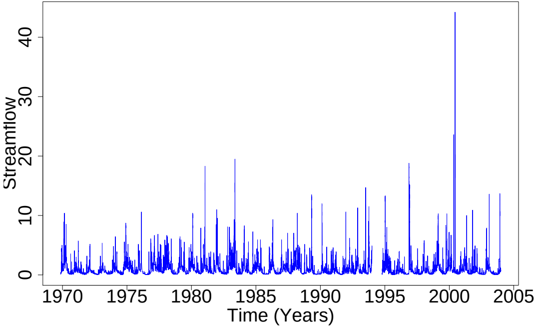

How local information is used?

From a time series to...

- Annual Maxima
- POT
- Suggestion

- Time series $\Rightarrow$ 1 obs/year
- Time series $\Rightarrow$ λ obs/year
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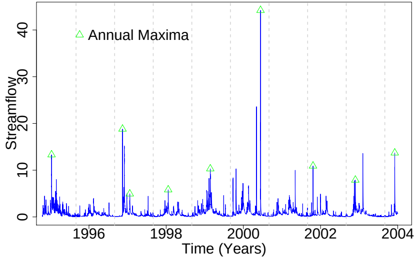

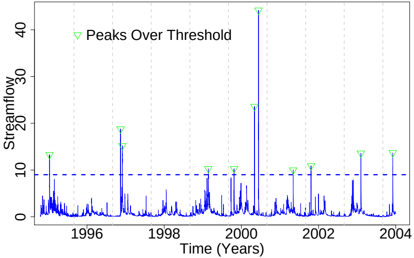

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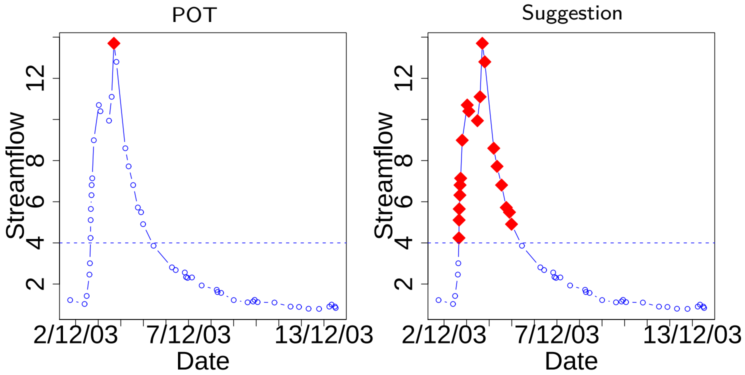

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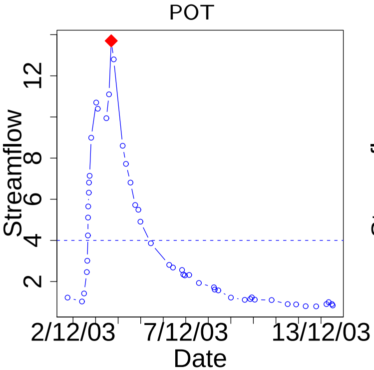

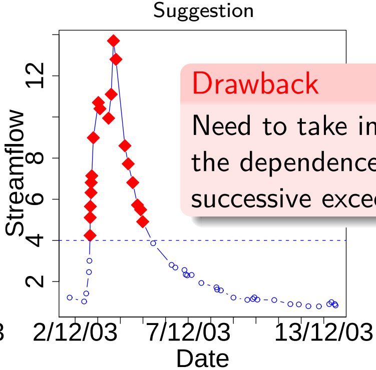

Drawback

Need to take into account the dependence between successive exceedances

Outline

Presentation of the Model

X_t gives information about the X_{t+1} value.

Hypothesis

- First order Markov chain approach

$$L(y_1, \ldots, y_n; \phi, \psi) = \frac{\prod_{i=2}^{n} f(y_i, y_{i-1}; \phi, \psi)}{\prod_{i=2}^{n-1} f(y_i; \phi)}$$

- $f(\cdot; \phi) \sim GPD$
- $f(\cdot, \cdot; \phi, \psi) \sim MEVD$ (bivariate)
  $f(\cdot, \cdot; \phi, \psi)$ depends on a dependence structure V
- V governs "how Y_t and Y_{t+1} are related"

Outline

Comparison between all Markovian Models

- 50 daily discharge stations (B. Renard PhD)
- Record lengths: 39–97 years
- Catchment Areas: 13–5660 km^2
- 6 different dependence structures V:

  symmetric: *log*, *nlog*, *mix*

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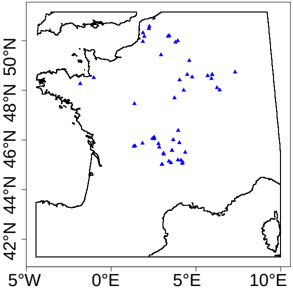

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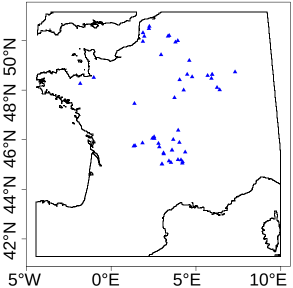

Table: Evolution of the Q_{50} *NMSE* as the record length increases.

	5 years	10 years	15 years
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nlog	0.24	0.15	0.12
mix	0.14	0.08	0.06
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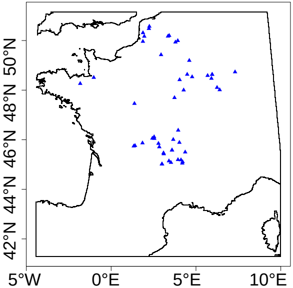

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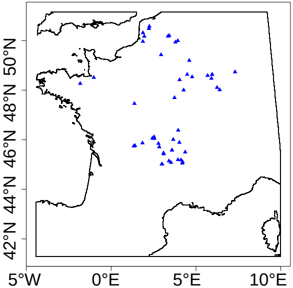

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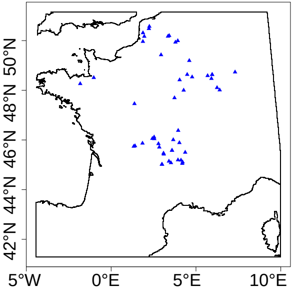

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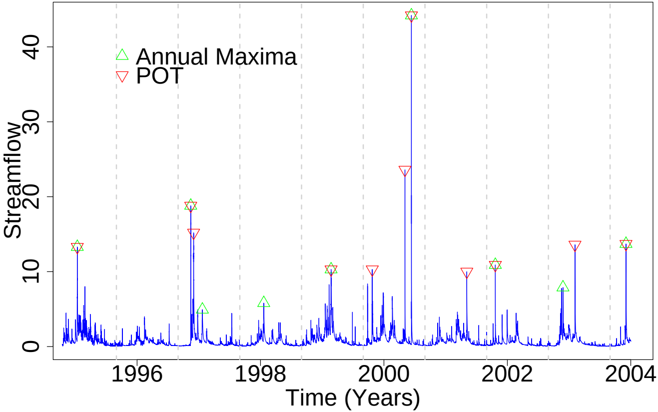

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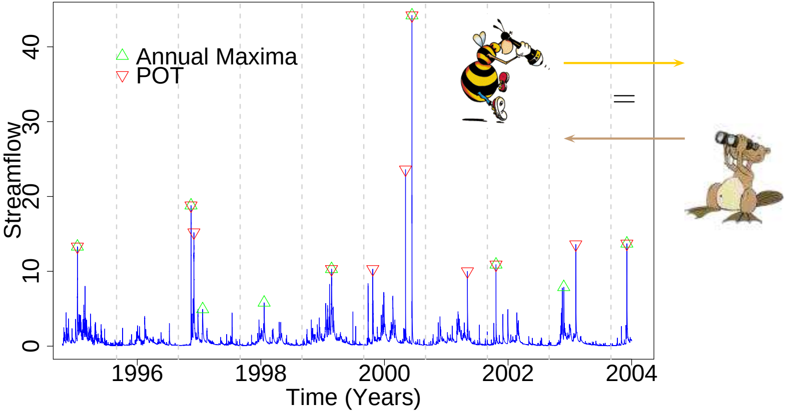

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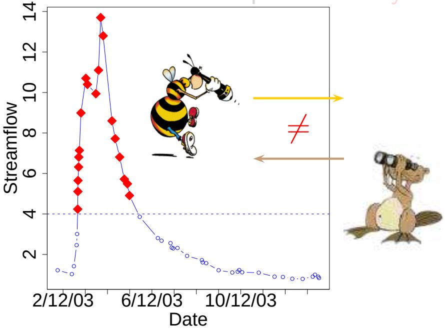

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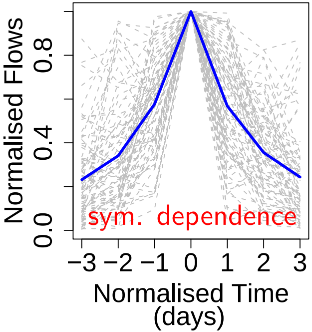

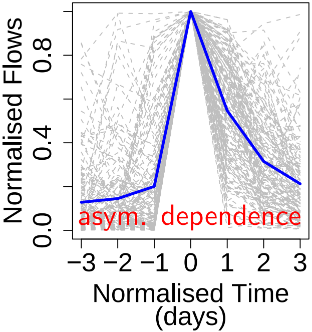

Comparison between *amix* and classical estimators

Table: Several characteristics of the *amix*, *MLE*, *PWU* and *PWB* estimators for Q_{50} estimation as the record length increases.

Model	5 years			10 years			15 years		
	NBIAS	*SD*	*NMSE*	*NBIAS*	*SD*	*NMSE*	*NBIAS*	*SD*	*NMSE*
amix	−0.06	0.33	0.12	−0.05	0.25	0.07	−0.04	0.21	0.05
MLE	−0.13	0.50	0.27	−0.14	0.36	0.14	−0.13	0.29	0.10
PWU	0.08	0.55	0.31	−0.01	0.39	0.15	−0.03	0.31	0.10
PWB	−0.07	0.45	0.21	−0.10	0.33	0.12	−0.11	0.27	0.09

- *amix* has the same biases than classical estimators
- but *amix* has the smallest variance (as more data are inferred)
- Thus, *amix* has the smallest MSE
- This is true whatever the return period is!

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MLE	−0.06	0.50	0.27	−0.14	0.36	0.14	−0.13	0.29	0.10
PWU	0.08	0.55	0.31	−0.01	0.39	0.15	−0.03	0.31	0.10
PWB	−0.07	0.45	0.21	−0.10	0.33	0.12	−0.11	0.27	0.09

- *amix* has the same biases than classical estimators
  $\Longrightarrow$ *amix* dependence structure seems to be suited to model two consecutive flood observations
- but *amix* has the smallest variance (as more data are inferred)
- Thus, *amix* has the smallest MSE
- This is true whatever the return period is!

Comparison between *amix* and classical estimators

Table: Several characteristics of the *amix*, *MLE*, *PWU* and *PWB* estimators for Q_{50} estimation as the record length increases.

Model	5 years			10 years			15 years		
	NBIAS	*SD*	*NMSE*	*NBIAS*	*SD*	*NMSE*	*NBIAS*	*SD*	*NMSE*
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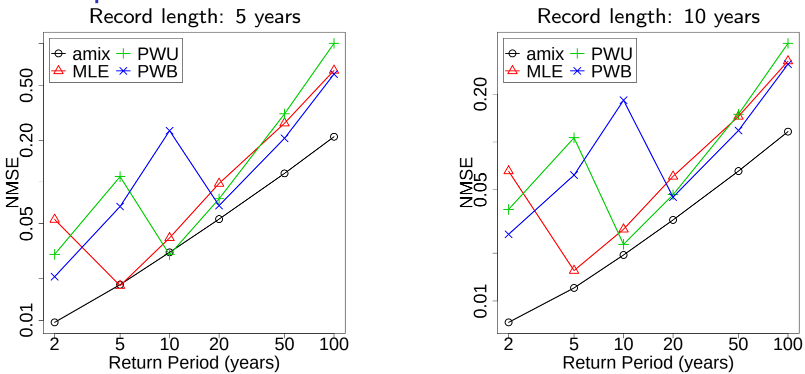

Figure: Evolution of the NMSE as the return period increases.

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An Attempt to Estimate Flood Hydrograph

- Flood peak were accurately estimated BUT...
- Severity of a flood event is not only defined by its flood peak
- Volume, duration and the shape of the hydrograph play a major role
- The proposed model can estimate design-flood hydrographs

Figure: Estimation of the normalized flood hydrograph.The Rance river at Guenroc (left) and the Loire river at Villerest (right).

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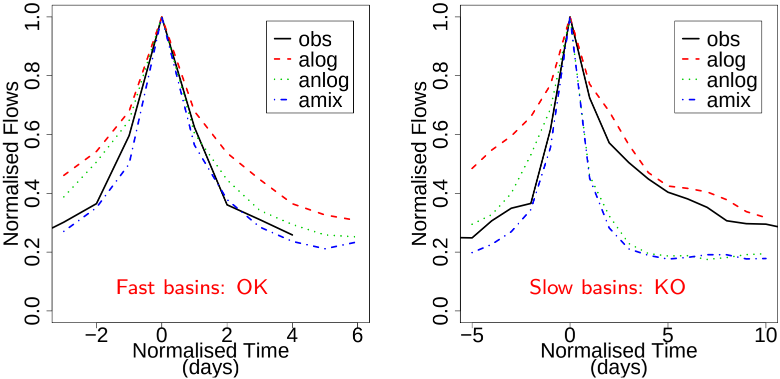

Figure: Estimation of the normalized flood hydrograph.The Rance river at Guenroc (left) and the Loire river at Villerest (right).

Conclusions

- A purely local model was proposed
- Most of the data from the target site are used
- MEVT can be applied in practice
- Our results show that it is worthy
  (especially when $T > 50$ years)
- Markovian models are attractive as they can predict flood peaks, durations, volumes...
- But there is still a lot of work to be done
  (for flood hydrograph estimation)

Outline

Regional Models

Conclusions

- The whole information is used more rationally than the IFL does (distinguishing the target site data)
- Models are based on well-established theory (Bayes)
- Bayesian models are definitively adapted to partially gaged stations
- BAY is accurate for return periods such as $T \leq 20$ years
- REV generalizes BAY
- And allows accurate estimations for $T > 20$ years

Perspectives

- Take into account the potential inter-site dependence (copula, MEVT, pairwise likelihood, ...)
- Test other parametrizations for p_ξ as a function of d_{hom} (maybe try relationships that are more clear-cut)
- Extend these models to the non-stationary case

Purely Local Model

Conclusions

- Need not to waste the target site data
- Uses all exceedances by applying a theoretically based structure for the extremal dependency
- This approach seems to be more accurate than conventional local estimators (mse divided by 3... when $T > 50$ years)
- Modeling flood volumes and duration is now available

Perspectives

- Improve the estimation of flood volume and duration (test other V choices)
- Link catchment characteristics to the V functions
- Maybe consider higher order Markov chains
- Or other "lag conditioning" values (time step adapted to the basin dynamic)


Thank you very much for your attention!