

Spin-up and spin-down in linearly stratified fluid over flat bottom

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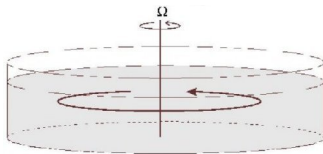
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Overview

def: transient process by which a fluid adjusts to time-dependent boundary conditions



- fluid assumed to be in solid body rotation
- rotation rate of the container modified:
increased (**spin-up**) or decreased (**spin-down**)
- adjustment of the primary circulation
to the new rotation rate

Background

Existence of separated time scales

$$E \equiv \nu/\Omega H^2$$

- | | |
|---|-----------------------------------|
| 1) Ekman boundary layers development: | $\tau_e^* = \Omega^{-1}$ |
| 2) spin-up due to a secondary circulation: | $\tau_s^* = \Omega^{-1} E^{-1/2}$ |
| 3) viscous dissipation of residual motions: | $\tau_v^* = \Omega^{-1} E^{-1}$ |

Large body of work on the axisymmetric stratified spin-up, but not much on the non-axisymmetric counterpart

- numerical models: axisymmetric in nature
- experimental investigations: only in small facilities



Background

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Methodology

$$E \sim O(10^{-5})$$

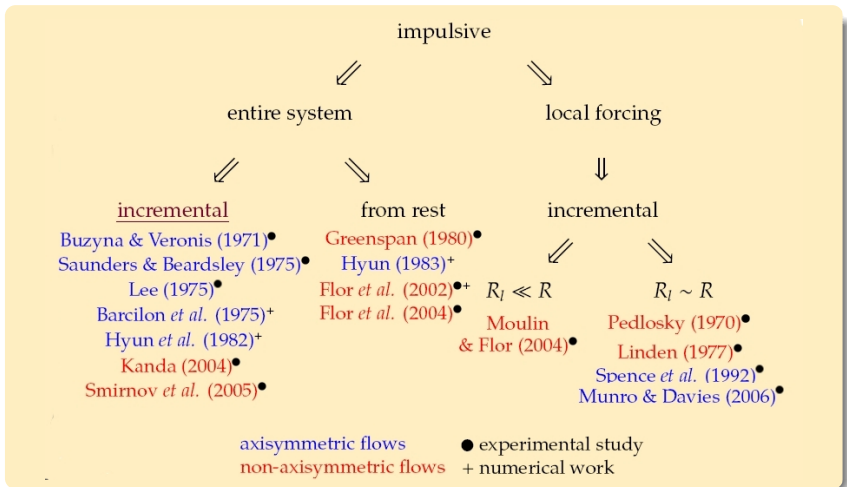
- 1) optimal separation between the time scales
- 2) relatively long spin-up time

Objectives

- a) comparison of the experimental results with the analytical theory of Walin (1969)
- b) study of the effects of non-axisymmetric disturbances on the momentum transport in spin-up time



Stratified Spin-up



R_l = radius of the lid

R = radius of the horizontal boundaries



Stratified Fluid

The presence of stratification introduces physical mechanisms wholly lacking in the homogeneous system

Outstanding effects

- inhibition of vertical motions by buoyancy force
- presence of horizontal density gradients
- baroclinic instability ($Bu < 1$)
- vertical shear of the horizontal velocity

$$Bu \equiv N^2 H^2 / \Omega^2 L^2$$



Stratified Fluid - Holton (1965)

Governing Eq.ns scaled by $\tau_s = E^{-1/2}$

⇒ information on different time scales is lost

- Ekman layers have the same structure as in homogeneous case
- interior region governed by

QGPV Eq.n

$\delta \equiv H/L$

$$\frac{\partial}{\partial t} \left(\nabla_h^2 \psi + \frac{\delta^2}{Bu} \frac{\partial^2 \psi}{\partial z^2} \right) = 0$$

$\psi \equiv p_{(0)}$

- secondary circulation restricted to the rotational shear layer ($h_p \sim H/Bu^{1/2}$) by a stable stratification



Salt-stratified Fluid - Walin (1969)

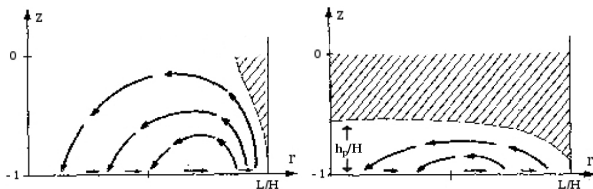
- no transport in the sidewall layer due to buoyancy force
- Ekman flow enters in the interior from corner regions

The adjustment mechanism is localized:

$$H \rightarrow h_p \sim H/Bu^{1/2}$$

$$\tau_{ss} \sim \tau_s/Bu^{1/2}$$

- interior azimuthal velocity field – analytical solution



$$Bu^{1/2} \sim 0.1$$

$$Bu^{1/2} \sim 10$$

The shaded areas are unaffected by the secondary circulation

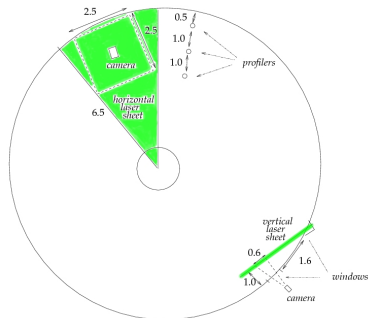


Plan

- Introduction & Motivations
- Theoretical Notions
- **Experimental Results**
- Conclusions & Perspectives



Experimental Set-up

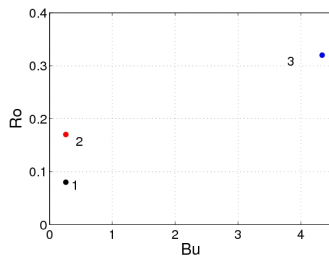


The **Coriolis turntable**, thanks to its large size ($D = 13\text{ m}$), provides access to non-dimensional parameters relevant in geophysical applications, e.g., $\delta \sim O(10^{-1})$, $E \sim O(10^{-5})$

Experimental Runs

A series of incremental spin-up and spin-down run pairs were performed, fixing the final rotation period for each pair

EXP	T [s]	Ro	Bu
UP 1	50	0.08	0.26
DW 1	50	0.08	0.26
UP 2	50	0.16	0.26
DW 2	50	0.16	0.26
UP 3	200	0.32	4.34
DW 3	200	0.32	4.34

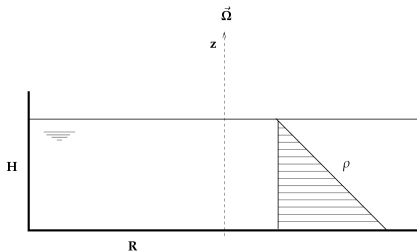


$$\Delta\rho/\rho \simeq 3 \cdot 10^{-2}$$

$$Ro \equiv U/\Omega L = \Delta\Omega/\Omega$$



Physical System



The flow response depends on several dimensional parameters

$$\Omega \quad \Delta\Omega \quad N \equiv \sqrt{-(g/\rho)(\partial\rho/\partial z)} \quad R \quad H \quad \nu \quad \kappa$$

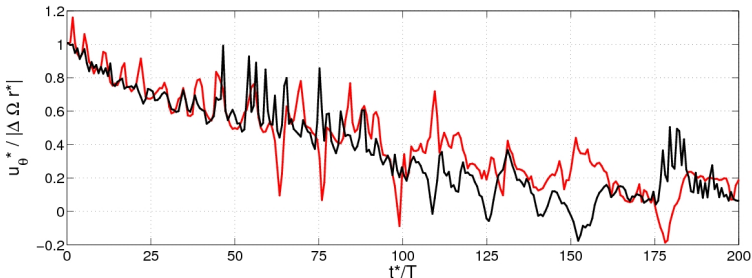
Dimensional Analysis: Ro Bu E δ $\sigma \equiv \nu/\kappa$

$$E \sim \mathcal{O}(10^{-5}) \quad \delta \text{ fixed} \quad \sigma \sim \mathcal{O}(10^3)$$

\Rightarrow flow evolution primarily determined by Ro and Bu

Influence of the Rossby number

Azimuthal velocity decay: UP 1 $\{Ro = 0.08\}$ —●—; UP 3 $\{Ro = 0.16\}$ —●—

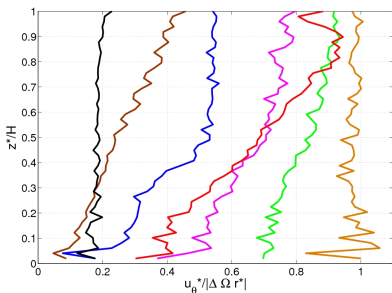


$Bu = 0.26$; $r^*/R = 0.80$; $z^*/H = 0.75$

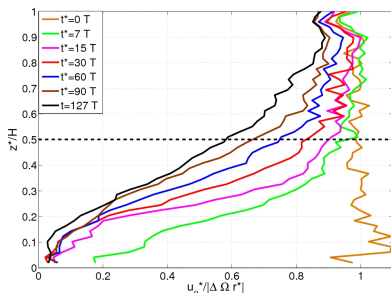
- basic decay not altered by $Ro < 0.20$
- study of the effects of non-vanishing Ro left out

Influence of the Burger number

Azimuthal velocity: temporal evolution of the vertical profile at $r^*/R = 0.85$



(a) UP 1 $\{Bu = 0.26\}$

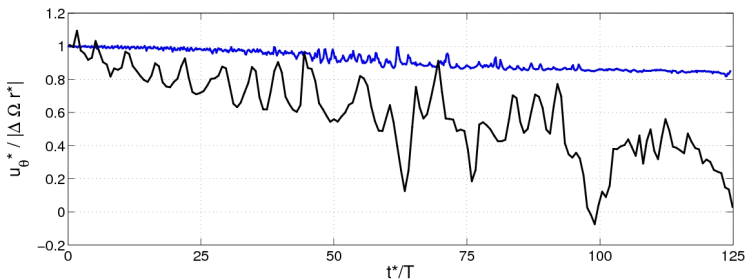


(b) UP 3 $\{Bu = 4.34\}$

- $Bu > 1$: linear vertical profile
- $Bu < 1$: the nature of the instability must be addressed

Influence of the Burger number

Azimuthal velocity decay: UP 1 $\{Bu = 0.26\}$ —●—; UP 3 $\{Bu = 4.34\}$ —●—



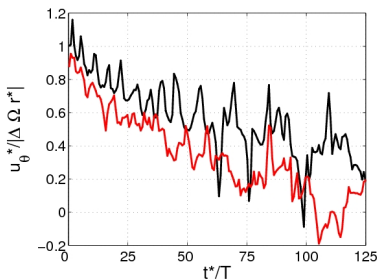
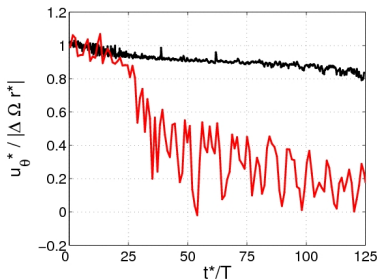
$$r^*/R = 0.80; \quad z^*/H = 0.75$$

Smirnov *et al.* (2005)

- baroclinic instability increases the vertical transport of horizontal momentum

Spin-up —●— Spin-down —●—

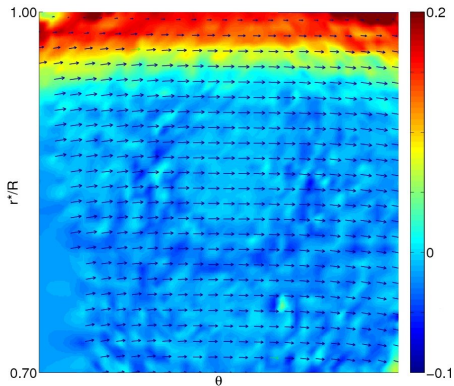
Azimuthal velocity decay

(a) PAIR 1 $\{Bu = 0.26\}$ (b) PAIR 3 $\{Bu = 4.34\}$ $r^*/R = 0.80$; $z^*/H = 0.75$

- centrifugally unstable flow in spin-down

Stable Spin-up

Horizontal vorticity field: UP 3 { $Bu = 4.34$ }

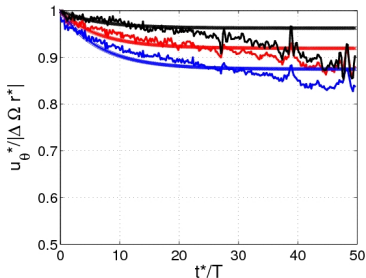


$t^*/T \simeq 100; z^*/H = 0.90$

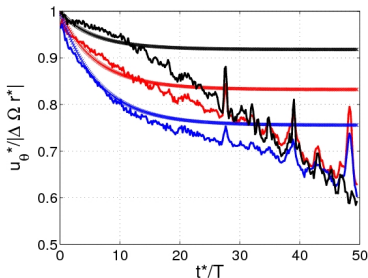


Stable Spin-up

Azimuthal velocity decay: UP 3 $\{Bu = 4.34\}$



(a) $z^*/H \approx 0.65$



(b) $z^*/H \approx 0.43$

Experimental results: —●— $r^*/R = 0.70$; —●— $r^*/R = 0.80$; —●— $r^*/R = 0.90$

Walin's solution: ××××× $r^*/R = 0.70$; ××××× $r^*/R = 0.80$; ××××× $r^*/R = 0.90$

- vortex-stretching stronger at smaller radii
- experimental results decay faster than Walin's theory

Stable Spin-up

Stratified vs Homogeneous spin-up

Viscous effects enhanced in stratified case since the confinement of the meridional circulation produces vertical flow gradients

Spin-up of a stratified fluid controlled by both the inviscid dynamics and viscous-diffusion effects

Any viable model for stratified spin-up should include the viscous-diffusion effects in both the regions (Hyun (1983))

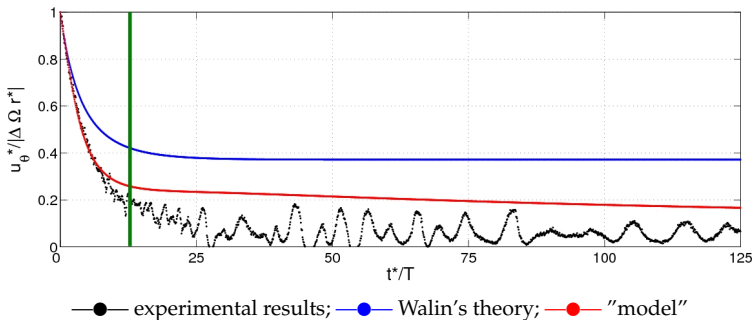
Improvement to Walin's theory

Inclusion of vertical diffusion of momentum



Stable Spin-up

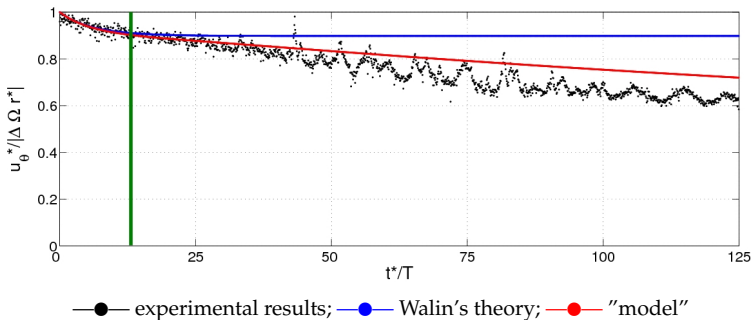
UP 3 $\{Bu = 4.34\}$: azimuthal velocity decay at $r^*/R = 0.85$; $z^*/H = 0.10$



- viscous effects observable in τ_{ss}
- baroclinic instability at large times

Stable Spin-up

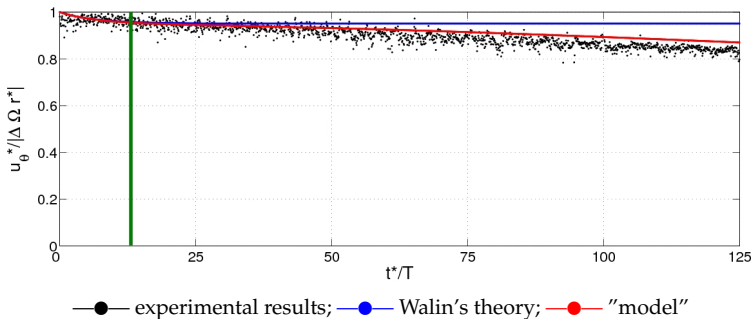
UP 3 $\{Bu = 4.34\}$: azimuthal velocity decay at $r^*/R = 0.85$; $z^*/H = 0.50$



- remarkable penetration depth ($h_p/H \simeq 0.50$)
- general improvement including vertical diffusion

Stable Spin-up

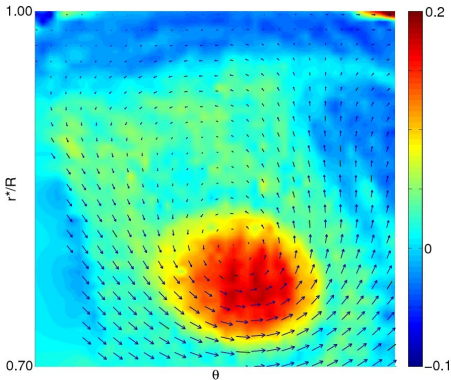
UP 3 $\{Bu = 4.34\}$: azimuthal velocity decay at $r^*/R = 0.85$; $z^*/H = 0.80$



- preeminent role of vertical diffusion
- accordance even at $Re \sim O(10^4)$

Unstable Spin-up

Horizontal vorticity field: UP 1 { $Bu = 0.26$ }

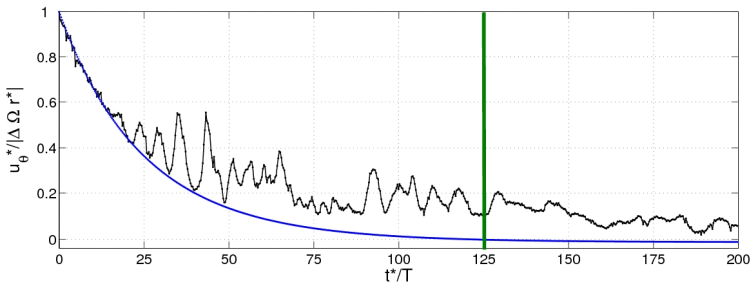


$t^*/T \approx 100$; $z^*/H = 0.90$



Unstable Spin-up

UP 1 $\{Bu = 0.26\}$: azimuthal velocity decay at $r^*/R = 0.85$; $z^*/H = 0.20$

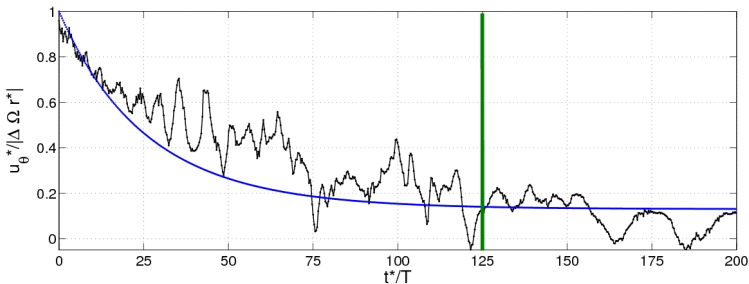


—●— experimental results; —●— Walin's theory

- experimental results decay **slower** than theory
- baroclinic instability **does not** increase the vertical transport of horizontal momentum
- attainment of solid body rotation similar to homogeneous case

Unstable Spin-up

UP 1 $\{Bu = 0.26\}$: azimuthal velocity decay at $r^*/R = 0.85$; $z^*/H = 0.40$

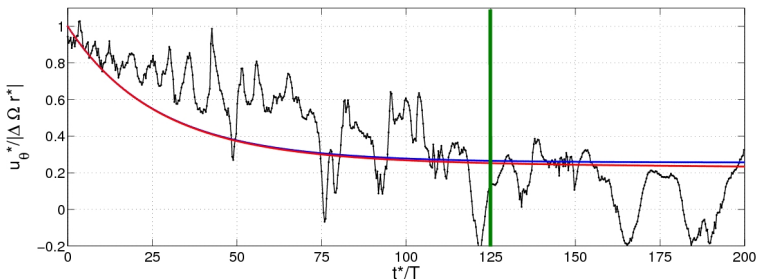


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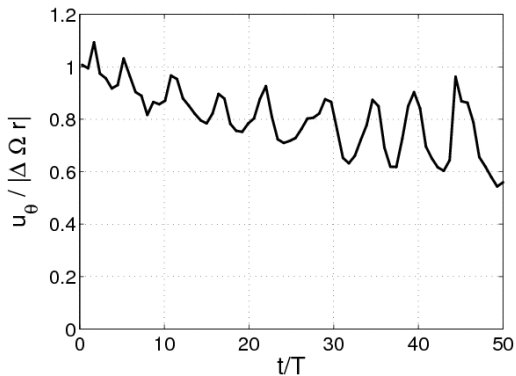


—●— experimental results; —●— Walin's theory; —●— "model"

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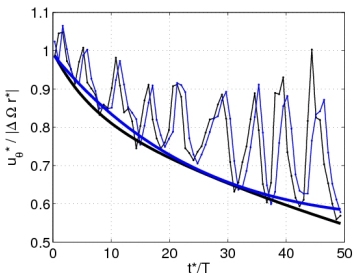


$r^*/R = 0.80$; $z^*/H = 0.75$

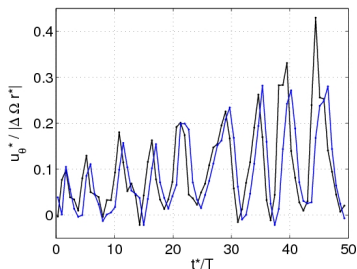


Unstable Spin-up

Azimuthal velocity averaged in small boxes such as they had the same radial coordinate and different azimuthal position



(a) temporal evolution



(b) oscillation growth

UP 1 $\{Bu = 0.26\}$; $r^*/R = 0.80$; $z^*/H = 0.75$

- $f^*/f_0 \approx 0.18$; $\Delta t^*/T \approx 0.70$



Unstable Spin-up

Baroclinic instability: the Eady's model (1949)

- perturbation of the interior pressure

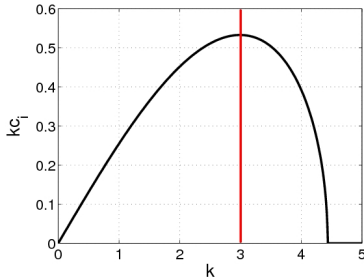
$$\phi = |A(z)|e^{kcit} \cos(k\theta - \omega t + \alpha) \cos l_m r$$

$$\omega = 2\pi f^*/f_0, l_m = (m + 1/2)\pi$$

- at fixed z

$$k\Delta\theta = \omega\Delta t \Rightarrow k \simeq 3$$

number of eddy pairs
in the system



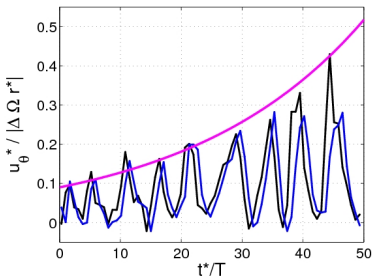
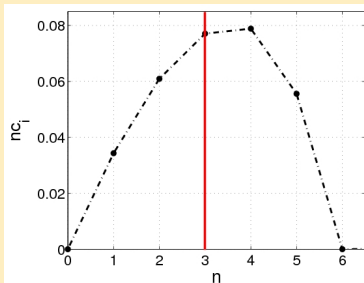
- growth rate for the most unstable radial mode
- experimental zonal wave number



Unstable Spin-up

- growth rate: mismatch with classical Eady's model

Eady's model with reliable initial conditions: Smirnov *et al.* (2005)



● — growth rate ● — exp. wave number ● — growth trend $A = A_0 e^{kc_i t}$

A good agreement is obtained using the model derived by Smirnov *et al.* (2005) with the actual vertical shear

Unstable Spin-up

Velocity field as composition of two mechanisms

- 1) axisymmetric relative current
- 2) long-wave oscillations

Moving average algorithm used as low-frequency band pass filter

this tool removes the short term fluctuations leaving only the long term trend

Horizontal perturbation pattern

reconstructed by shifting and merging the fluctuation velocity fields



Unstable Spin-up

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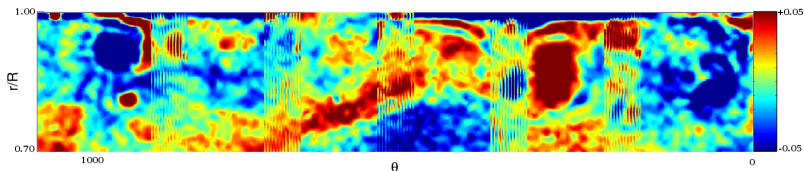
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Unstable Spin-up

Perturbation vorticity field: UP 1 $\{Bu = 0.26\}$

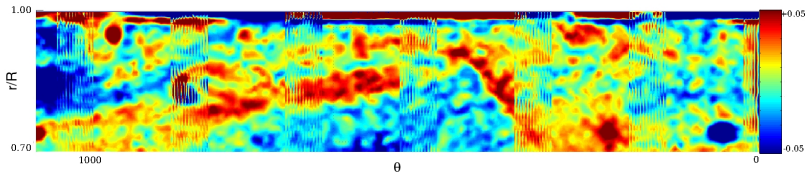


$z^*/H = 0.90$; first oscillation: $t/T \approx 2$

- small vorticity patterns alternated in the flow

Unstable Spin-up

Perturbation vorticity field: UP 1 $\{Bu = 0.26\}$

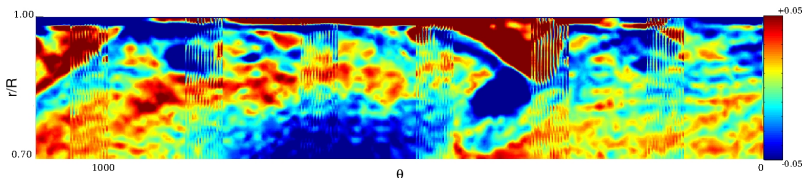


$z^*/H = 0.90$; first oscillation: $t/T \simeq 10$

- positive vorticity arises at the sidewall

Unstable Spin-up

Perturbation vorticity field: UP 1 $\{Bu = 0.26\}$

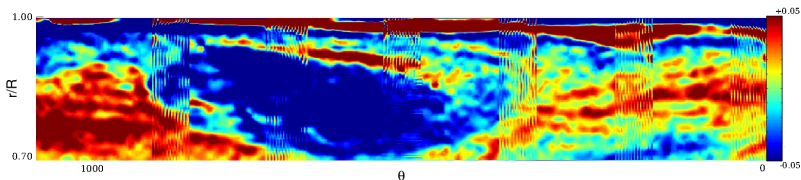


$z^*/H = 0.90$; first oscillation: $t/T \approx 20$

- extension and organization of the eddies

Unstable Spin-up

Perturbation vorticity field: UP 1 $\{Bu = 0.26\}$

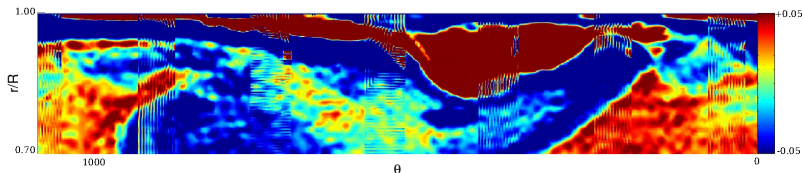


$z^*/H = 0.90$; first oscillation: $t/T \approx 35$

- baroclinic instability wholly developed

Unstable Spin-up

Perturbation vorticity field: UP 1 $\{Bu = 0.26\}$



$z^*/H = 0.90$; first oscillation: $t/T \approx 45$

- radial transport of horizontal momentum

Plan

- Introduction & Motivations
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- Experimental Results
- **Conclusions & Perspectives**



Conclusions

STABLE CASE

- optimal separation between the time scales
- importance of viscous effects captured by the model

UNSTABLE CASE

- baroclinic instability is the most plausible explanation for the existence of long-wave instability
- the claim “baroclinic instability increase the vertical transport of horizontal momentum” is shown to be questionable



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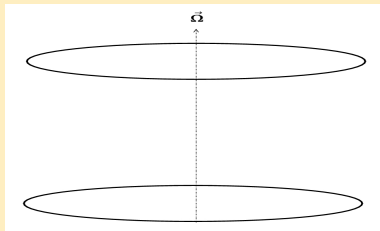


Homogeneous Fluid

Axisymmetric container

Greenspan & Howard (1963): incremental (linear) spin-up

- simplest case: spin-up in double-disc system



The combination of two plane discs rotating in unison produces a motion which typifies all contained spin-up flows

Homogeneous Fluid

- spin-up in double-disc system
- quasi-steady approximation

bottom Ekman layer

(Ω_i frame)

$$\begin{aligned} -2u_\theta &= E \frac{\partial^2 u_r}{\partial z^2} \\ 2u_r &= E \frac{\partial^2 u_\theta}{\partial z^2} \end{aligned} \quad \left\{ \begin{array}{ll} u_\theta = r, u_r = 0 & \text{at } z = 0 \\ u_r, u_\theta \rightarrow 0 & \text{as } z \rightarrow +\infty \end{array} \right.$$

Ekman spiral

(Ω_i frame)

$$\begin{aligned} u_r(r, z) &= r e^{-z/E^{1/2}} \sin(z/E^{1/2}) \\ u_\theta(r, z) &= r e^{-z/E^{1/2}} \cos(z/E^{1/2}) \end{aligned}$$

$E^{1/2}$ boundary layer thickness

Homogeneous Fluid

- spin-up in double-disc system
- quasi-steady approximation

bottom Ekman layer

(Ω_f frame)

$$\begin{aligned} -2u_\theta &= -\frac{\partial \tilde{p}}{\partial r} + E \frac{\partial^2 u_r}{\partial z^2} \\ 2u_r &= E \frac{\partial^2 u_\theta}{\partial z^2} \end{aligned} \quad \left\{ \begin{array}{ll} u_r = u_\theta = 0 & \text{at } z = 0 \\ u_r \rightarrow 0, u_\theta \rightarrow U_\theta & \text{as } z \rightarrow \infty \end{array} \right.$$

Ekman spiral

(Ω_f frame)

$$\begin{aligned} u_r(r, z) &= -U_\theta(r) e^{-z/E^{1/2}} \sin(z/E^{1/2}) \\ u_\theta(r, z) &= U_\theta(r) \left[1 - e^{-z/E^{1/2}} \cos(z/E^{1/2}) \right] \end{aligned}$$

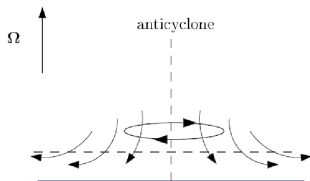
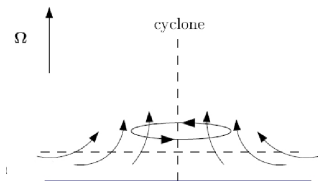
$E^{1/2}$ boundary layer thickness

Homogeneous Fluid

- the Ekman layer acts to convert the no-slip condition on the azimuthal velocity in a normal condition on the interior flow

Ekman pumping/suction

$$w = \frac{E^{1/2}}{2} \omega_z$$

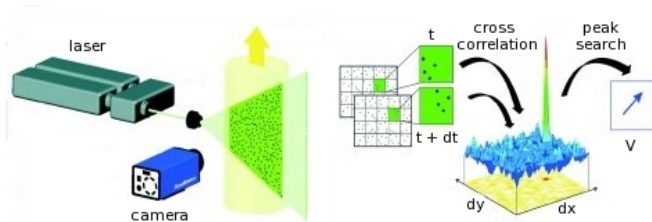


- the effect of the boundary layer is communicated to the interior region through a secondary circulation



P.I.V. technique

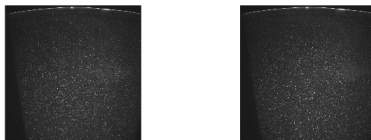
Projection of the fluid velocity in the laser sheet plane



- the flow is seeded with particles
- a laser beam is formed to a light sheet
- the light scattered is recorded as images
- the images subdivided into small boxes
- the particle displacement determined by cross-correlation (C.I.V.)

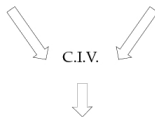


P.I.V. technique

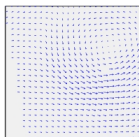


(a) image at t

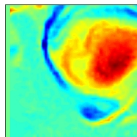
(b) image at $t + \Delta t$



C.I.V.



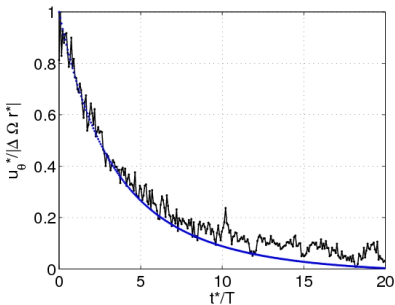
(d) velocity field



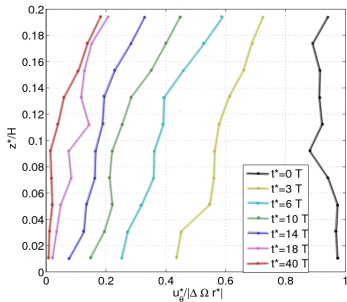
(e) vorticity field



Stable Spin-up - Ekman layer



(a) decay rate at $z^*/H \approx 0.01$



(b) vertical profile

UP 3 $\{Bu = 4.34\}$; azimuthal velocity

- theory accurately predicts the flow evolution in case of locations and times for which the secondary circulation is strong
- no-slip conditions directly satisfied over τ_{ss} without undergoing boundary-like variations

My model

Inclusion of vertical momentum diffusion in Walin's solution

\mathbf{u}_i^t azimuthal velocity at time \mathbf{t} and vertical location \mathbf{i}

\mathbf{u}_{Wi}^t Walin's analytical solution (a priori known)

- at $\mathbf{t} = 0$ $\mathbf{u}_i^0 = \mathbf{1}$ solid body rotation
- at $\mathbf{t} = 1$ $\mathbf{u}_i^1 = \mathbf{u}_{Wi}^1$ non-uniform spin-up due to the secondary circulation
- at $\mathbf{t} = 2, \dots, \mathbf{n}$ (loop)

$$\mathbf{u}_{Di}^t = \mathbf{u}_i^{t-1} + \mathbf{s} \left(\mathbf{u}_{i+1}^{t-1} - 2\mathbf{u}_i^{t-1} + \mathbf{u}_{i-1}^{t-1} \right) \quad \text{vertical diffusion}$$

$$\mathbf{u}_i^t = \mathbf{u}_{Di}^t + \Delta \mathbf{u}_{Wi} \quad \text{non-uniform spin-up due to the secondary circulation}$$

$$\text{being } \mathbf{s} = \mathbf{E}\Delta t / (\Delta z)^2, \quad \Delta \mathbf{u}_{Wi} = \mathbf{u}_{Wi}^t - \mathbf{u}_{Wi}^{t-1}$$



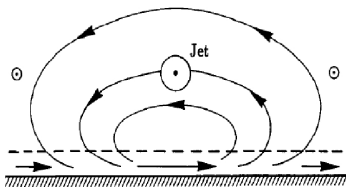
Unstable Spin-up

Recent works

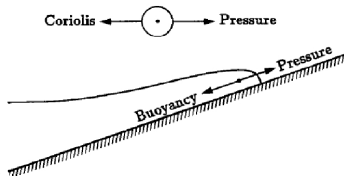
- Flor *et al.* (2004): baroclinic instability at the interfaces between interior and corner regions
- Kanda (2004): front structure near the sidewall ?
classical Eady-type baroclinic instability ?
- Smirnov *et al.* (2005): — instability was not consistent with the classical Eady's model
— extended Eady's model (sidewall boundary layer taken into account)
— the sidewall boundary layers compensates for the narrowness of the cylinder



Ekman layer arrest



(a) Lateral variations in the current cause variations of the flux in the Ekman layer, and force flow into, or out of, the interior. Coriolis force acting on this flow then reduces the initial current in the interior.



(b) The advection of density gives rise to a buoyancy force which opposes cross-slope motion. When the final balance is achieved, the interior flow feels a nearly free-slip condition at the seafloor.



