

---

# Phenomenology of Dark Matter Annihilation and Non-Minimal Flavour Violation in the MSSM

---

Björn Herrmann

Laboratoire de Physique Subatomique et de Cosmologie  
Université Joseph Fourier Grenoble

Ph.D. thesis under supervision of Prof. Dr. Michael Klasen  
and in collaboration with Drs. Giuseppe Bozzi, Benjamin Fuks, and Karol Kovařík

Grenoble, 10 September 2008

**Introduction**

---

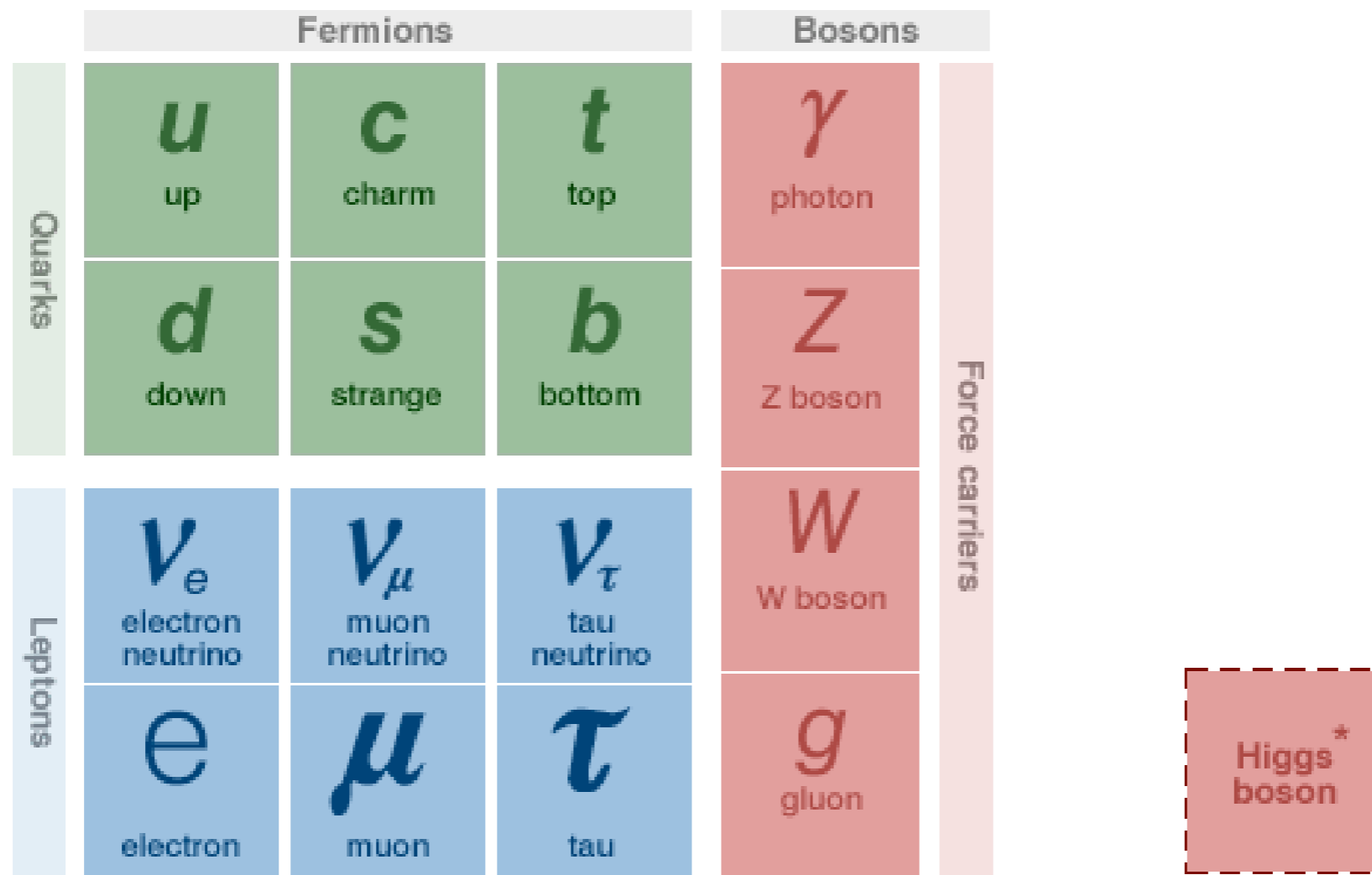
Supersymmetry and the MSSM

---

# The Standard Model of Particle Physics

Based on the gauge group  $SU(3) \times SU(2) \times U(1)$ , includes strong and electroweak interactions

[Glashow 1961, Weinberg 1967 & 1972, Glashow et al. 1970, Gross & Wilczek 1973 & 1974, Politzer 1974]



Experimentally missing piece: Higgs boson originating from scalar isodoublet

[Goldstone 1961 & 1962, Higgs 1964, Englert & Brout 1964]

Strong hints that the Standard Model is not the ultimate theory...

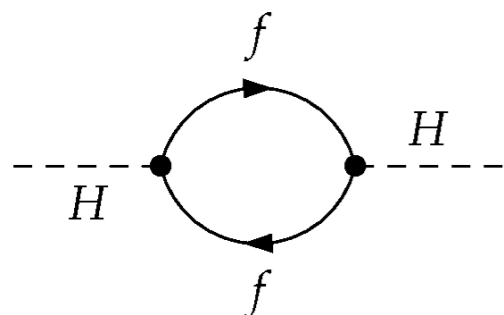
# Motivations for Physics beyond the Standard Model

Particle physics: Standard Model rather “low-energy” limit of a more fundamental theory

## ✓ “Hierarchy problem”

Higgs mass receives quadratically divergent fermionic loop corrections

[Witten 1981, Kaul 1982]



$$\Delta m_H^2 = \frac{|\lambda_f|^2}{16\pi^2} \left[ -2 \Lambda_{UV}^2 + 6m_f^2 \log \frac{\Lambda_{UV}}{m_f} + \dots \right]$$

## ✓ Gauge coupling unification

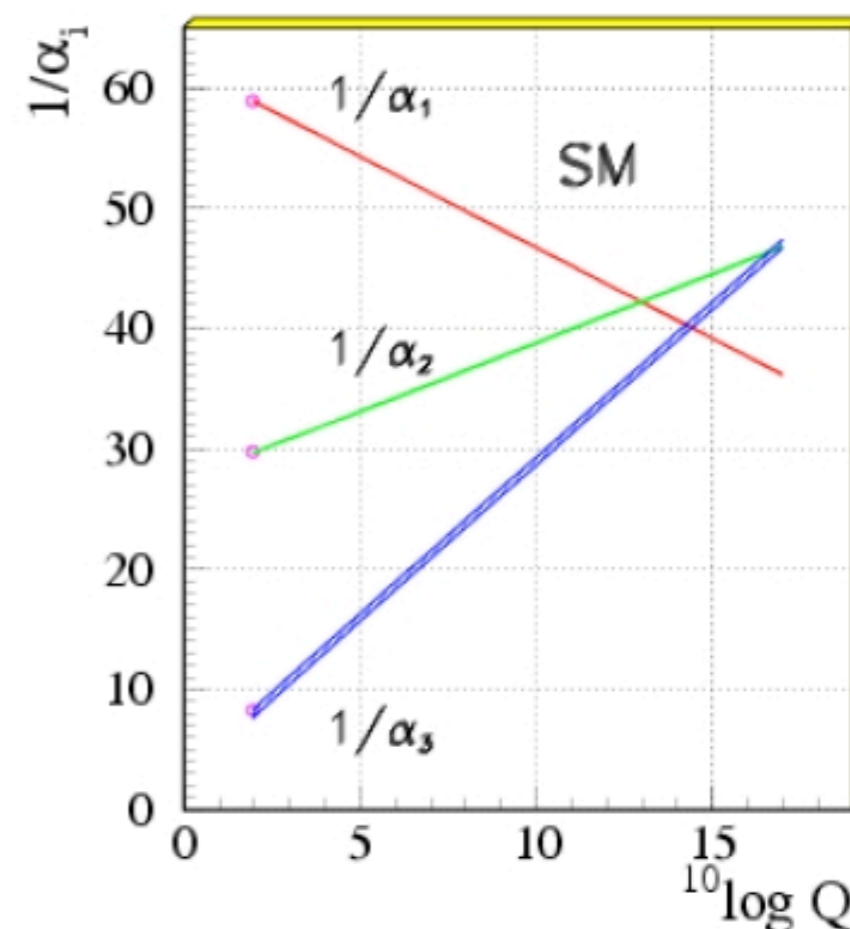
Running coupling constants should meet in single point if corresponding interactions unify

## ✓ Gravity

Cannot be ignored at energies near the Planck scale

## ✓ “Aesthetic” reasons

Number of parameters in Standard Model, ...



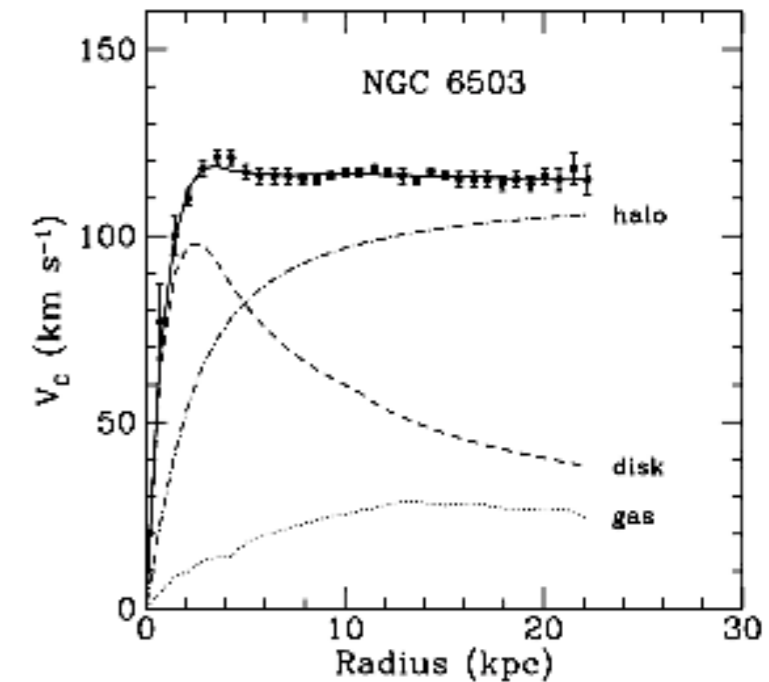
# Motivations for Physics beyond the Standard Model

Cosmology: Standard Model does not include candidate for cold dark matter

## ✓ First observational hints

Velocity dispersion and rotation curves

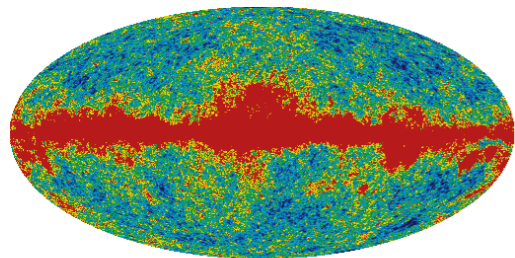
[Zwicky 1933, Rubin *et al.* 1970]



## ✓ CMB anisotropies

Cosmological parameters from WMAP mission

[Komatsu *et al.* (WMAP) 2008]



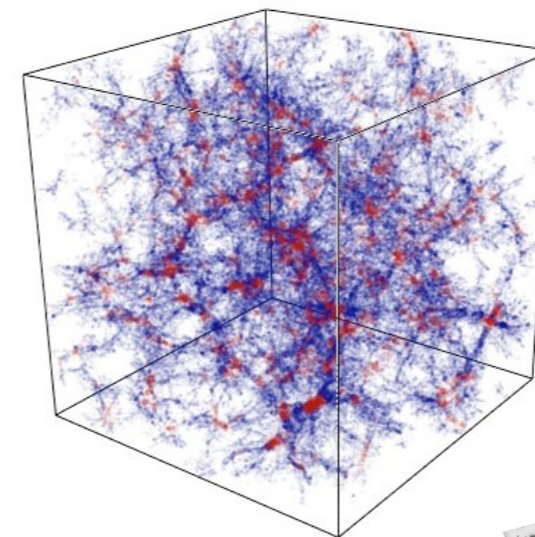
$$\Omega_{\text{tot}} = 1.005 \pm 0.034$$

$$\Omega_{\text{CDM}} = 0.223 \pm 0.013$$

## ✓ Structure formation

Cold dark matter needed to explain large structures

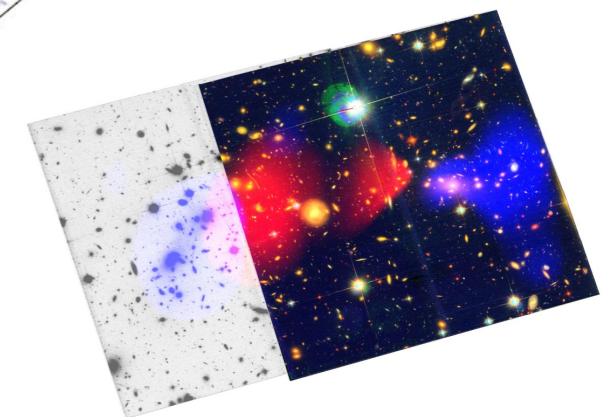
[Blumenthal *et al.* 1984]



## ✓ First direct observation

Observation of “Bullet Cluster” proves existence of dark matter

[Clowe *et al.* 2006]



# The Minimal Supersymmetric Standard Model (MSSM)

Supersymmetry relates bosons and fermions

$$Q|b\rangle = |f\rangle \quad Q|f\rangle = |b\rangle$$

Minimal supersymmetric theory including the Standard Model particles

SM Particles		Spin		Spin	Superpartners	
Quarks	$(u_L \ d_L)$	1/2	$Q$	0	$(\tilde{u}_L \ \tilde{d}_L)$	Squarks
	$u_R^\dagger$	1/2	$\bar{u}$	0	$\tilde{u}_R^*$	
	$d_R^\dagger$	1/2	$\bar{d}$	0	$\tilde{d}_R^*$	
Leptons	$(\nu \ e_L)$	1/2	$L$	0	$(\tilde{\nu} \ \tilde{e}_L)$	Sleptons
	$e_R^\dagger$	1/2	$\bar{e}$	0	$\tilde{e}_R^*$	
Higgs	$(H_u^+ \ H_u^0)$	0	$H_u$	1/2	$(\tilde{H}_u^+ \ \tilde{H}_u^0)$	Higgsinos
	$(H_d^0 \ H_d^-)$	0	$H_d$	1/2	$(\tilde{H}_d^0 \ \tilde{H}_d^-)$	
Gluon	$g$	1		1/2	$\tilde{g}$	Gluino
$W$ bosons	$W^0, W^\pm$	1		1/2	$\tilde{W}^0, \tilde{W}^\pm$	Winos
$B$ boson	$B^0$	1		1/2	$\tilde{B}^0$	Bino
Graviton	$G$	2		3/2	$\tilde{G}$	Gravitino

# Supersymmetry Breaking

No superpartners observed so far: Supersymmetry broken at the electroweak scale

Supersymmetry breaking occurs at scale  $\langle F \rangle$  in some “hidden” sector and is mediated to the “observable” sector through a shared interaction

“Soft” Supersymmetry breaking terms are included in the Lagrangian at the electroweak scale

$$\begin{aligned} \mathcal{L}_{\text{soft}} = & -\frac{1}{2} \left( M_1 \tilde{B} \tilde{B} + M_2 \tilde{W} \tilde{W} + M_3 \tilde{g} \tilde{g} + \text{h.c.} \right) \\ & - \left( \tilde{Q}^\dagger \mathbf{m}_{\tilde{Q}}^2 \tilde{Q} + \tilde{u} \mathbf{m}_{\tilde{u}}^2 \tilde{u}^\dagger + \tilde{d} \mathbf{m}_{\tilde{d}}^2 \tilde{d}^\dagger + \tilde{L}^\dagger \mathbf{m}_{\tilde{L}}^2 \tilde{L} + \tilde{e} \mathbf{m}_{\tilde{e}}^2 \tilde{e}^\dagger \right) \\ & - \left( m_{H_u}^2 H_u^* H_u + m_{H_d}^2 H_d^* H_d + b (H_u H_d + \text{h.c.}) \right) \\ & - \left( \tilde{u} \mathbf{a}_u \tilde{Q} H_u - \tilde{d} \mathbf{a}_d \tilde{Q} H_d - \tilde{e} \mathbf{a}_e \tilde{L} H_e + \text{h.c.} \right) \end{aligned}$$

124 *a priori* free parameters at the electroweak scale,  
often restricted to a few universal parameters at the grand unification scale

Spontaneous breaking of Supersymmetry implies existence of the goldstino  
In local Supersymmetry, it is absorbed by the gravitino which then becomes massive

$$m_{\tilde{G}} = \frac{\langle F \rangle}{\sqrt{3} M_{\text{P}}}$$

# Minimal Supergravity (mSUGRA / cMSSM)

Supersymmetry breaking mediated to observable sector through gravity

Observable sector  
(MSSM particles)

Hidden sector  
(SUSY breaking)

Parameters at the low scale are obtained from universal parameters at the high scale through renormalization group running

$$\begin{aligned} M_3 &= M_2 = M_1 = m_{1/2} \\ m_{\mathbf{Q}}^2 &= m_{\mathbf{u}}^2 = m_{\mathbf{d}}^2 = m_{\mathbf{L}}^2 = m_{\mathbf{e}}^2 = m_0^2 \mathbf{1} \\ m_{H_u}^2 &= m_{H_d}^2 = m_0^2 \\ \mathbf{a}_i &= A_0 \mathbf{y}_i \quad \text{for } i = u, d, e \end{aligned}$$

Five universal parameters at the SUSY breaking scale:  $m_0, m_{1/2}, A_0, \tan \beta, \text{sgn}(\mu)$

Flavour dependent gravitational interaction gives rise to important flavour violating terms in the Lagrangian at low energy (FCNC), in contradiction to current measurements

→ “flavour problem” in Supersymmetry



# Gauge Mediated Supersymmetry Breaking (GMSB)

Supersymmetry breaking mediated to observable sector through gauge interactions

[Giudice & Rattazzi 1999]

Observable sector  
(MSSM particles)

Messenger sector  
(superfield  $S$ , messengers)

Hidden sector  
(SUSY breaking)

Important FCNC naturally suppressed, since gauge interactions are “flavour-blind”

Scalar and auxiliary components of  $S$  acquire vacuum expectation values  $\langle S \rangle, \langle F_S \rangle$

Yukawa couplings give mass to messenger fields  $M_{\text{mes}} \simeq \langle S \rangle$

Masses of MSSM particles stem from messenger loops

$$M_i = \frac{\alpha_i}{4\pi} \Lambda g(x) (N_5 + 3N_{10})$$

$$x = \frac{\Lambda}{M_{\text{mes}}}$$

$$m_{\tilde{j}}^2 = 2 \sum_{i=1}^3 \frac{\alpha_i^2}{16\pi^2} \Lambda^2 f(x) (N_5 + 3N_{10}) C_{ij}$$

Five parameters at the SUSY breaking scale:

$$\Lambda = \frac{\langle F_S \rangle}{\langle S \rangle}, M_{\text{mes}}, N_{\text{mes}}, \tan \beta, \text{sgn}(\mu)$$

No physical solutions to the RGE if  $\Lambda > M_{\text{mes}}$

# Higgs and Gaugino Sectors of the MSSM

Cancellation of chiral anomalies requires two complex Higgs doublets

→ five physical Higgs bosons after electroweak symmetry breaking:  $h^0, H^0, A^0, H^+, H^-$

All parameters of the Higgs sector are related to the key parameters  $\tan\beta$  and  $m_A$

$$m_{h^0, H^0}^2 = \frac{1}{2} \left( m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 + m_Z^2)^2 - 4m_A^2 m_Z^2 \cos 2\beta} \right)$$
$$m_A^2 = \frac{2b}{\sin 2\beta}$$
$$m_{H^\pm}^2 = m_A^2 + m_W^2$$

After breaking of  $SU(2) \times U(1)$ , the neutral higgsinos and gauginos mix, giving rise to four neutralinos

$$\tilde{\chi}_i^0 = \mathcal{N}_{i1} \tilde{B}^0 + \mathcal{N}_{i2} \tilde{W}^0 + \mathcal{N}_{i3} \tilde{H}_d^0 + \mathcal{N}_{i4} \tilde{H}_u^0$$
$$\text{diag}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}) = \mathcal{N}^* \mathcal{M}_{\tilde{\chi}^0} \mathcal{N}^{-1}$$

In the same way, the charged higgsinos and  $W^\pm$  mix, resulting in the charginos  $\tilde{\chi}_{1,2}^\pm$

Being the only fermionic octet, the gluino cannot mix with other particles

# Sfermion Sector and Flavour Violation

## Squared sfermion mass matrices at the electroweak scale

$$M_{\tilde{q}}^2 = \left( \begin{array}{ccc|ccc} M_{L_1}^2 & \Delta_{LL}^{12} & \Delta_{LL}^{13} & m_1 X_1 & \Delta_{LR}^{12} & \Delta_{LR}^{13} \\ \Delta_{LL}^{12*} & M_{L_2}^2 & \Delta_{LL}^{23} & \Delta_{RL}^{12*} & m_2 X_2 & \Delta_{LR}^{23} \\ \Delta_{LL}^{13*} & \Delta_{LL}^{23*} & M_{L_3}^2 & \Delta_{RL}^{13*} & \Delta_{RL}^{23*} & m_3 X_3 \\ \hline m_1 X_1^* & \Delta_{RL}^{12} & \Delta_{RL}^{13} & M_{R_1}^2 & \Delta_{RR}^{12} & \Delta_{RR}^{13} \\ \Delta_{LR}^{12*} & m_2 X_2^* & \Delta_{RL}^{23} & \Delta_{RR}^{12*} & M_{R_2}^2 & \Delta_{RR}^{23} \\ \Delta_{LR}^{13*} & \Delta_{LR}^{23*} & m_3 X_3^* & \Delta_{RR}^{13*} & \Delta_{RR}^{23*} & M_{R_3}^2 \end{array} \right)$$

In the minimal model, flavour violation arises through rotation of gauge eigenstates into physical mass eigenstates and different renormalizations of quarks and squarks

Minimal flavour violation (MFV):  $\Delta_{ij}^{qq'}$  deduced from CKM-matrix

Constrained minimal flavour violation (cMFV): only diagonal entries,  $\Delta_{ij}^{qq'} = 0$

Neglect helicity mixing for 1<sup>st</sup> and 2<sup>nd</sup> generations and introduce mixing angles for 3<sup>rd</sup> generation

$$\begin{aligned} \text{diag}(m_{\tilde{f}_1}^2, m_{\tilde{f}_2}^2) &= \mathcal{R}^{\tilde{f}} \mathcal{M}_{\tilde{f}} \mathcal{R}^{\tilde{f}\dagger} \\ (\tilde{f}_1, \tilde{f}_2)^T &= \mathcal{R}^{\tilde{f}} (\tilde{f}_L, \tilde{f}_R)^T \end{aligned} \quad \mathcal{R}^{\tilde{f}} = \begin{pmatrix} \cos \theta_{\tilde{f}} & \sin \theta_{\tilde{f}} \\ -\sin \theta_{\tilde{f}} & \cos \theta_{\tilde{f}} \end{pmatrix}$$

# Non-Minimal Flavour Violation in the Squark Sector

New sources of flavour violation can appear when embedding SUSY in larger structures

[Gabbiani & Masiero 1989]

Flavour violating entries no longer deduced from the CKM-matrix alone

Convenient parametrization for non-minimal flavour violation (NMFV): 24 free parameters  $\Delta_{ij}^{qq'}$

Normalization to diagonal entries

$$\Delta_{ij}^{qq'} = \lambda_{ij}^{qq'} M_i M_j$$

Non-minimal flavour violation governed by 24 *a priori* free parameters  $\lambda_{ij}^{qq'}$  [Gabbiani et al. 1996]

Flavour mixing between 2<sup>nd</sup> and 3<sup>rd</sup> generations least constrained [Gabbiani et al. 1996, Ciuchini et al. 2007]

In gravity-mediation scenarios:  $\Delta_{LL} \gg \Delta_{LR,RL} \gg \Delta_{RR}$  [Gabbiani & Masiero 1989]

In gauge-mediation scenarios:  $\Delta_{LL,RR} \gg \Delta_{LR,RL}$  due to small A-terms [Dittmaier et al. 2007]

Diagonalization of the mass matrices involves two 6x6 rotation matrices

$$\text{diag}(m_{\tilde{q}_1}^2, m_{\tilde{q}_2}^2, m_{\tilde{q}_3}^2, m_{\tilde{q}_4}^2, m_{\tilde{q}_5}^2, m_{\tilde{q}_6}^2) = \mathcal{R}^{\tilde{q}} \mathcal{M}_{\tilde{q}}^2 \mathcal{R}^{\tilde{q}\dagger}$$

$$(\tilde{q}_1, \tilde{q}_2, \tilde{q}_3, \tilde{q}_4, \tilde{q}_5, \tilde{q}_6)^T = \mathcal{R}^{\tilde{q}} (\tilde{q}_{1L}, \tilde{q}_{2L}, \tilde{q}_{3L}, \tilde{q}_{1R}, \tilde{q}_{2R}, \tilde{q}_{3R})^T$$

All relevant couplings have to be generalized to take into account the 6x6 matrices  $\mathcal{R}^{\tilde{u}}$  and  $\mathcal{R}^{\tilde{d}}$

# R-Parity and Dark Matter

---

Additional symmetry in the MSSM in order to avoid possible proton decay

$$\tau_p \gtrsim 10^{29} \text{ yr} \simeq 10^{19} a_0$$

Definition of R-parity related to baryon and lepton numbers

$$P_R = (-1)^{3(B-L)+2s} = \begin{cases} +1 & \text{for Standard Model particles} \\ -1 & \text{for their superpartners} \end{cases}$$

R-parity conservation has important phenomenological implications

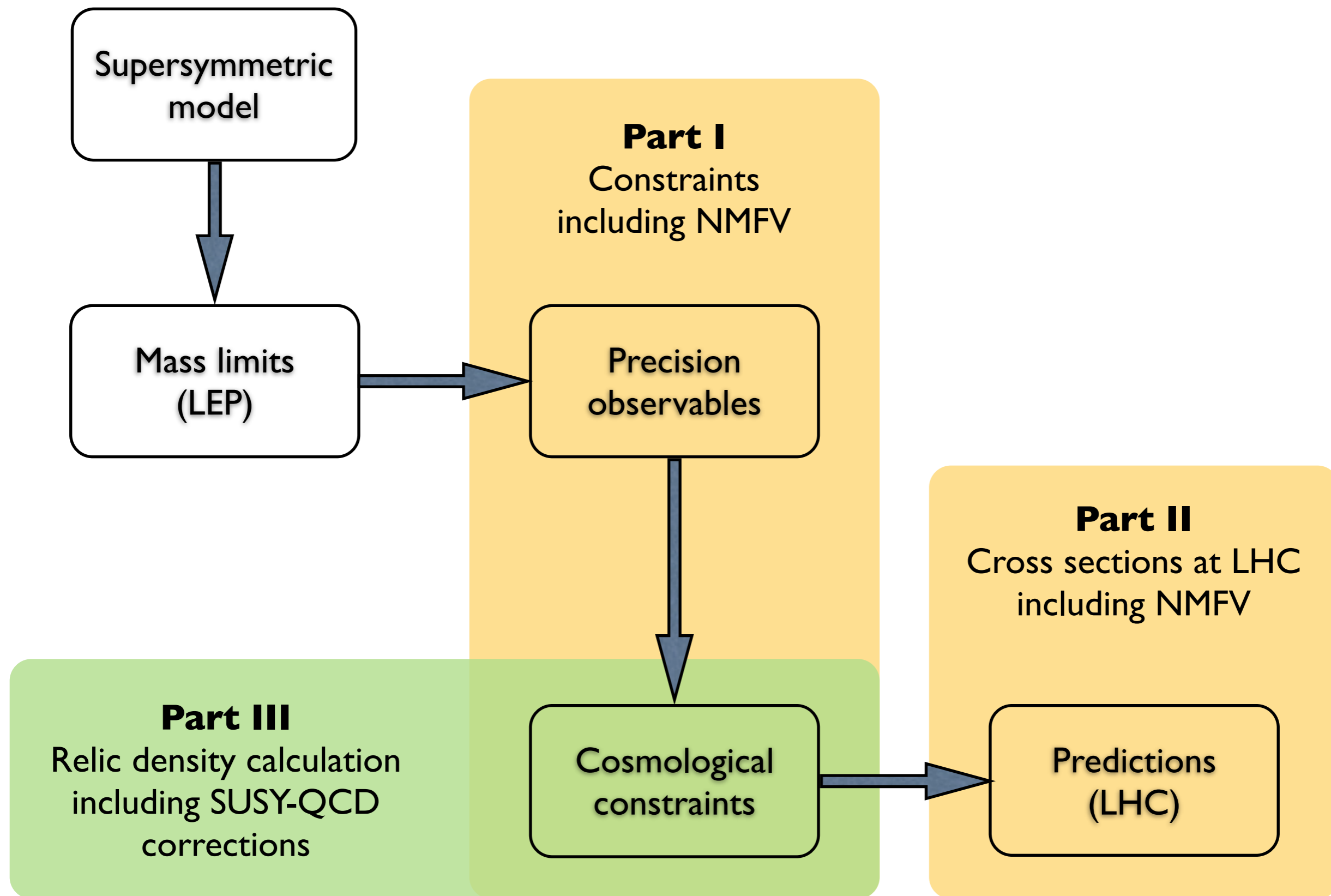
Only vertices involving an even number of superpartners allowed

- supersymmetric particles always produced in pairs
- lightest supersymmetric particle (LSP) cannot decay and must be stable

Dark matter candidate (“WIMP”) must be massive, stable, neutral in electric charge and colour

- Two candidates within the MSSM: Lightest neutralino (mSUGRA)  
Gravitino (GMSB)

# Phenomenological Strategy



**Part I**

---

# Constraints and Benchmark Points for the MSSM

---

# Electroweak precision and low energy observables

---

## Decay $b \rightarrow s\gamma$

Squarks enter calculation already at the one-loop level, sensitive to flavour violation  
Strongly constrained by precision measurements at BaBar, Belle, and CLEO

$$\text{BR}(b \rightarrow s\gamma) = (3.55 \pm 0.26) \cdot 10^{-4} \quad [\text{Barbiero et al. (HFAG) 2006}]$$

Theoretical accuracy: two-loop QCD / one-loop SUSY-QCD [Hahn et al. 2005, Kagan et al. 1998]

## Electroweak $\rho$ -parameter

Relates self-energies of W and Z bosons at zero external momentum  
Sensitive to squark mass splitting and thus also to squark flavour violation

$$\Delta\rho = \frac{\Sigma_Z(0)}{m_Z^2} - \frac{\Sigma_W(0)}{m_W^2} = (1.02 \pm 0.86) \cdot 10^{-3} \quad [\text{Yao et al. (PDG) 2007}]$$

Theoretical accuracy: two-loop cMFV / one-loop NMFV [Heinemeyer et al. 2004]

## Anomalous magnetic moment of the muon

Two-loop squark contribution suppressed with respect to one-loop slepton diagrams

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (29.2 \pm 8.6) \cdot 10^{-10} \quad [\text{Yao et al. (PDG) 2007}]$$

Theoretical accuracy: two-loop level [Heinemeyer et al. 2004]

SUSY one-loop contribution favours positive values of  $\mu$  [Moroi 1996]



# Cosmological Constraints

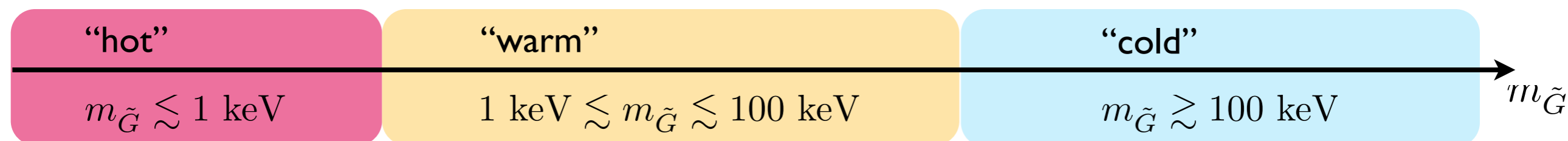
Dark matter relic density required to agree with WMAP+SN+BAO data

$$0.094 \leq \Omega_{\text{CDM}} h^2 \leq 0.136$$

[Hamann et al. 2007]

Further constraints for gravitino dark matter

Nature of dark matter related to gravitino mass



Gravitino relic density receives contributions from thermal production and NLSP decay

$$\Omega_{\tilde{G}}^{\text{th}} h^2 \sim \frac{T_{\text{R}}}{m_{\tilde{G}}} m_{\tilde{g}}^2 \quad \Omega_{\tilde{G}}^{\text{non-th}} h^2 = \frac{m_{\tilde{G}}}{m_{\text{NLSP}}} \Omega_{\text{NLSP}}^{\text{th}} h^2$$

[Bolz et al. 2001]

NLSP lifetime has to preserve abundances of light elements in the Universe

[Pradler & Steffen 2007]

$$\tau_{\text{NLSP}} \sim \frac{m_{\tilde{G}}^2}{m_{\text{NLSP}}^5} \lesssim 2 \cdot 10^3 \text{ s}$$

Leptogenesis may explain the cosmic baryon asymmetry, but favours high reheating temperatures

[Buchmüller et al. 2007]

$$T_{\text{R}} \gtrsim 10^9 \text{ GeV}$$

# Scans of the mSUGRA Parameter Space including NMFV

Impose constraints at  $2\sigma$  on  $m_0$ - $m_{1/2}$  plane for fixed  $\tan\beta$ ,  $A_0$ , and  $\text{sgn}(\mu)$

Renormalization group running:

SPheno 2.2.3

[Porod 2004]

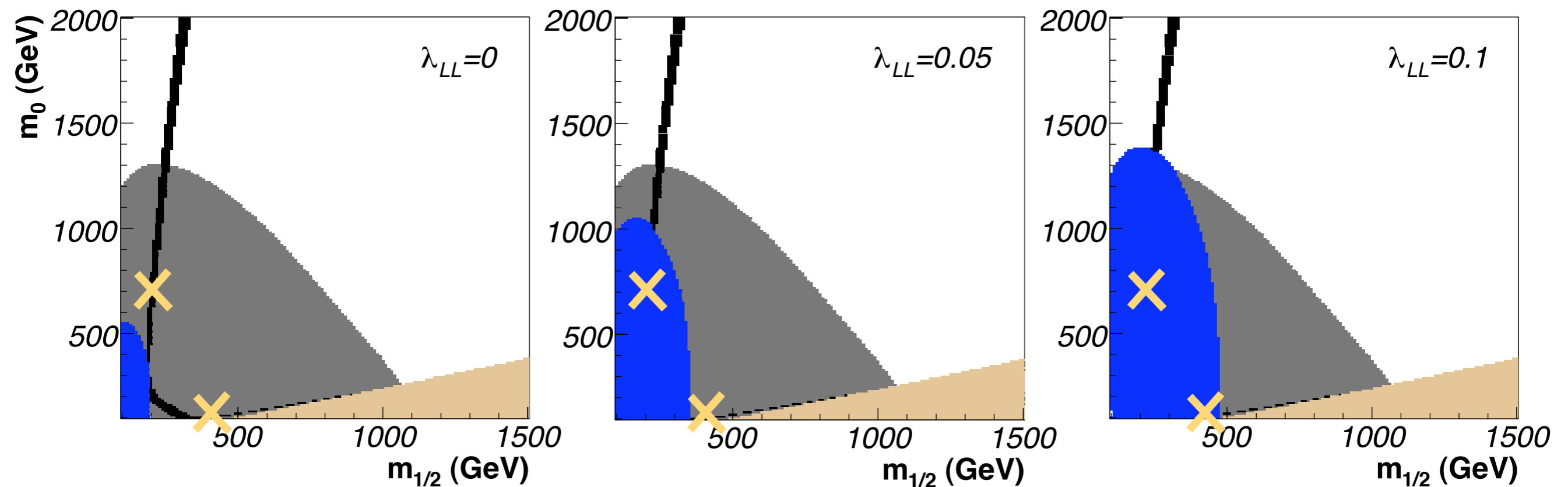
Physical mass spectrum and constraints: FeynHiggs 2.5.1

[Heinemeyer et al. 2000]

Neutralino relic density:

DarkSUSY 4.1 (NMFV-adapted)

Example:  $\tan\beta=10$ ,  $A_0=0$ ,  $\mu>0$ ,  $\lambda_{LL}=0\dots 0.1$



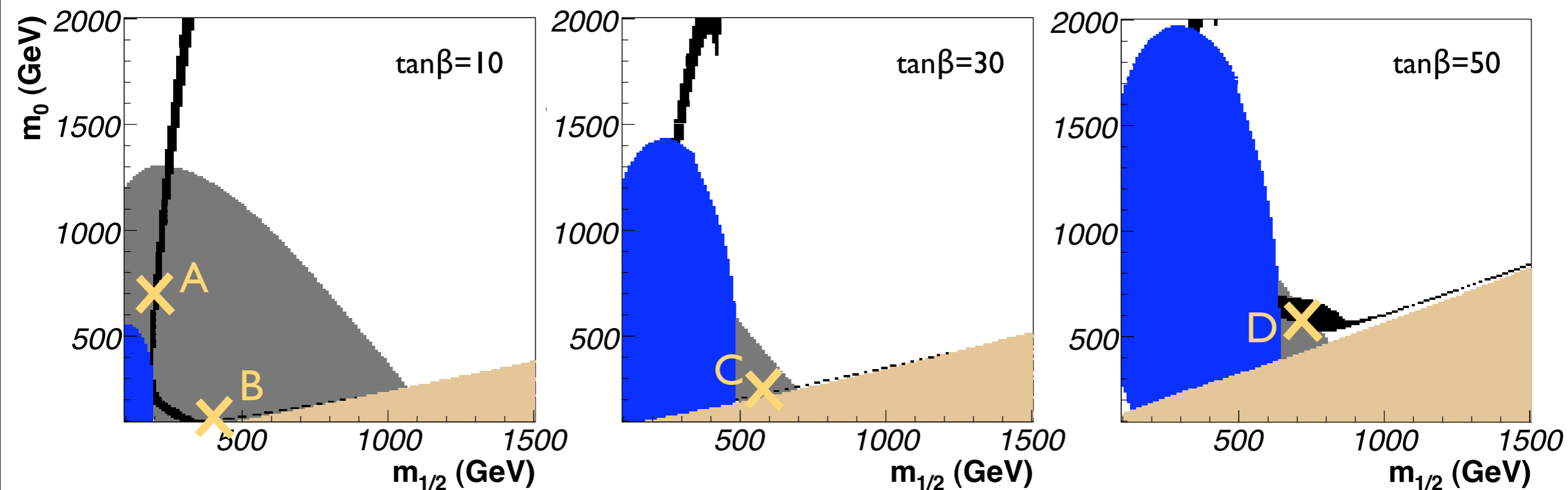
$b \rightarrow sy$  excluded  
 $a_\mu$  favoured

WMAP favoured  
charged LSP

# Benchmark Points for mSUGRA including NMFV

Define four “collider-friendly” benchmark points allowing for flavour violation

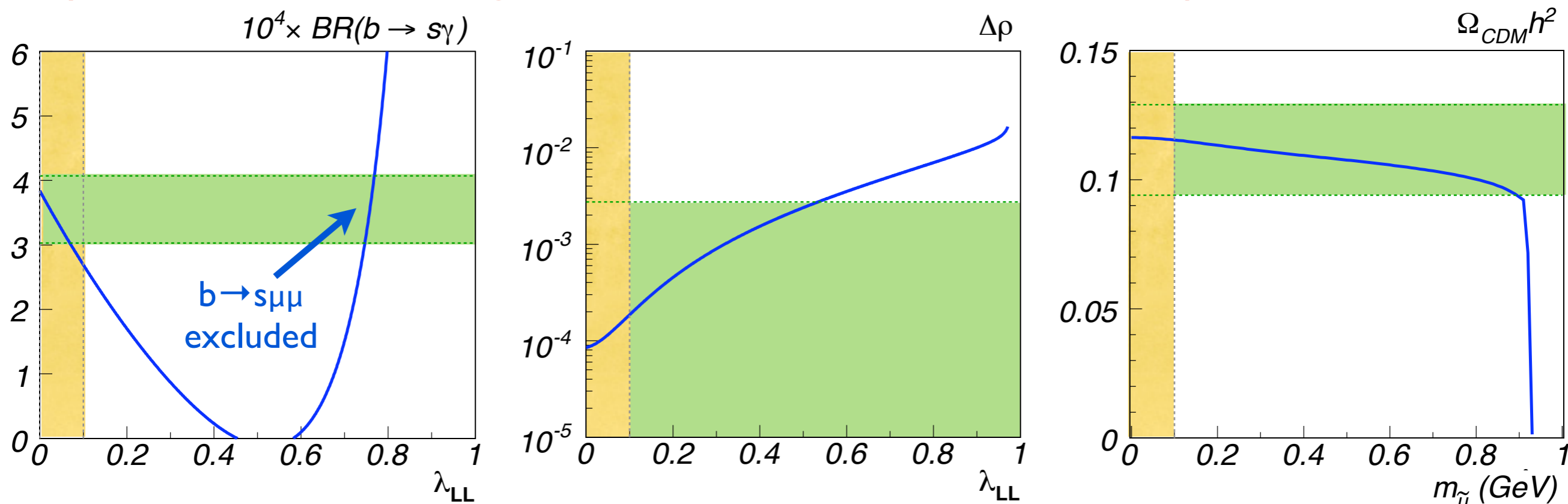
	$m_0$ (GeV)	$m_{1/2}$ (GeV)	$A_0$	$\tan\beta$	$\text{sgn}(\mu)$	SPS
A	700	200	0	10	+	2
B	100	400	0	10	+	3
C	230	590	0	30	+	1b
D	600	700	0	50	+	4



Benchmark points are valid in the vicinity of cMFV ( $\lambda_{LL} \approx 0 \dots 0.1$ )

# Phenomenology of Point B

Study constraints and mass eigenvalues as function of flavour violation parameter  $\lambda_{LL}$



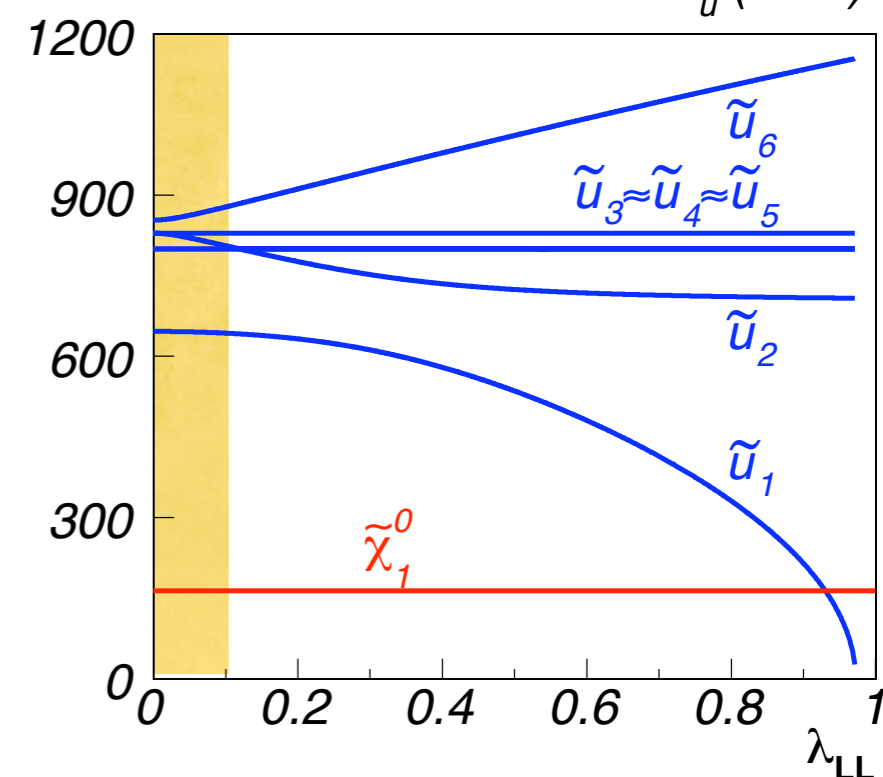
$a_\mu$  independent of  $\lambda_{LL}$  (not shown)

Allowed range for  $\lambda_{LL}$  from  $b \rightarrow s\gamma$ :  $\lambda_{LL} \lesssim 0.1$

Experimental errors of  $\Delta\rho$  allow for large interval

Important splitting of mass eigenvalues for large  $\lambda_{LL}$

Small mass difference between  $\tilde{\chi}_1$  and  $\tilde{u}_1$  at large  $\lambda_{LL}$   
 $\rightarrow \Omega_{CDM} h^2$  falls due to important coannihilations and light squark propagated annihilation processes



# Scans of the Minimal GMSB Parameter Space

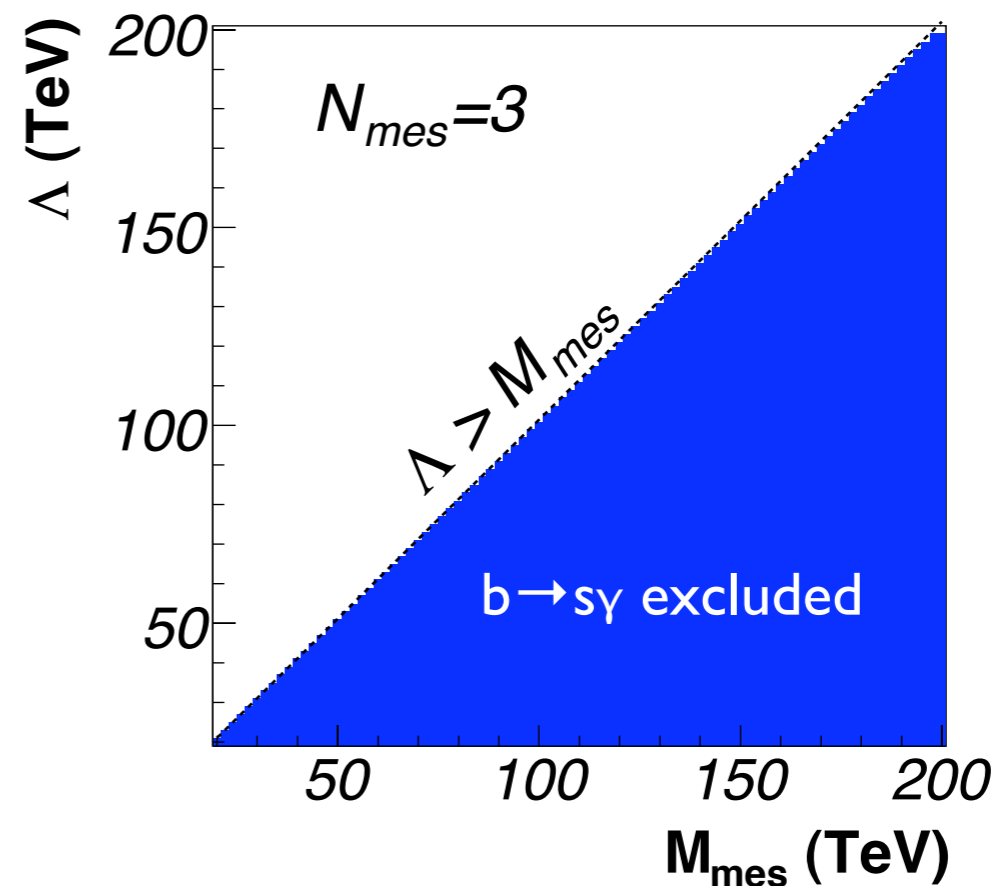
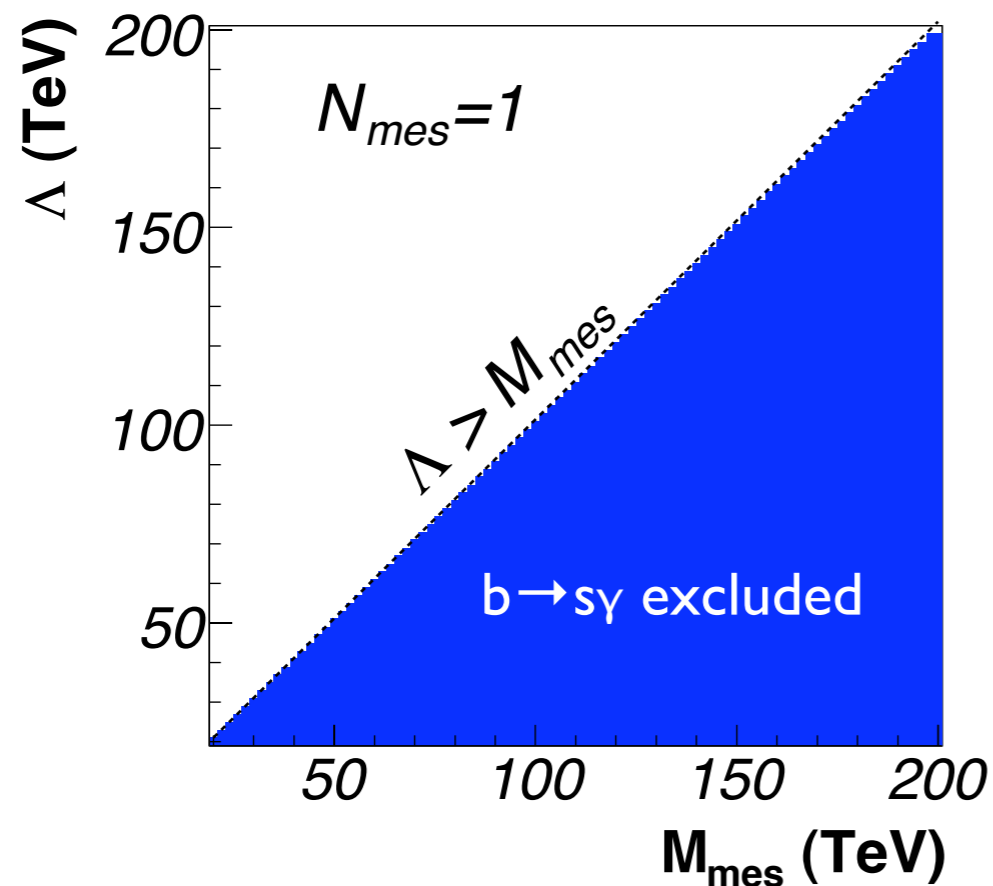
Impose constraints at  $2\sigma$  on  $\Lambda$ - $M_{mes}$  plane for fixed  $\tan\beta$ ,  $N_{mes}$ , and  $\text{sgn}(\mu)$

Renormalization group running: SPheno 2.2.3  
Physical mass spectrum and constraints: FeynHiggs 2.6.4

[Porod 2004]

[Heinemeyer et al. 2000]

Example:  $\tan\beta=15$ ,  $\mu>0$



“Collider-friendly” regions excluded by  $b \rightarrow s\gamma$  constraint for all  $\tan\beta$  and  $N_{mes}$

# Flavour Violation in GMSB Models

## Several possibilities reintroduce flavour violation in GMSB models

[Giudice & Rattazzi 1999, Tobe *et al.* 2004, Dubovsky & Gorbunov 1999]

We consider mixing between messenger and matter fields

→ two possible scenarios depending on nature of messengers

[Dubovsky & Gorbunov 1999]

### ✓ Fundamental messengers (5 and $\bar{5}$ representations of $SU(5)$ )

Carry quantum numbers of left-handed leptons and right-handed quarks

→ induce flavour violation in the sector of right-handed sleptons and **left-handed squarks**

$$\lambda_{LL} > 0 \quad \lambda_{RR} = 0$$

### ✓ Antisymmetric messengers (10 and $\bar{10}$ representations of $SU(5)$ )

Carry quantum numbers of right-handed leptons and left-handed quarks

→ flavour violation in the sector of left-handed sleptons and **left- & right-handed squarks**

$$\lambda_{LL} = \lambda_{RR} > 0$$

For  $SU(2)$  gauge invariance and simplicity:  $\lambda_{LL}^{sb} = \lambda_{RR}^{sb} = \lambda_{LL}^{ct} = \lambda_{RR}^{ct}$

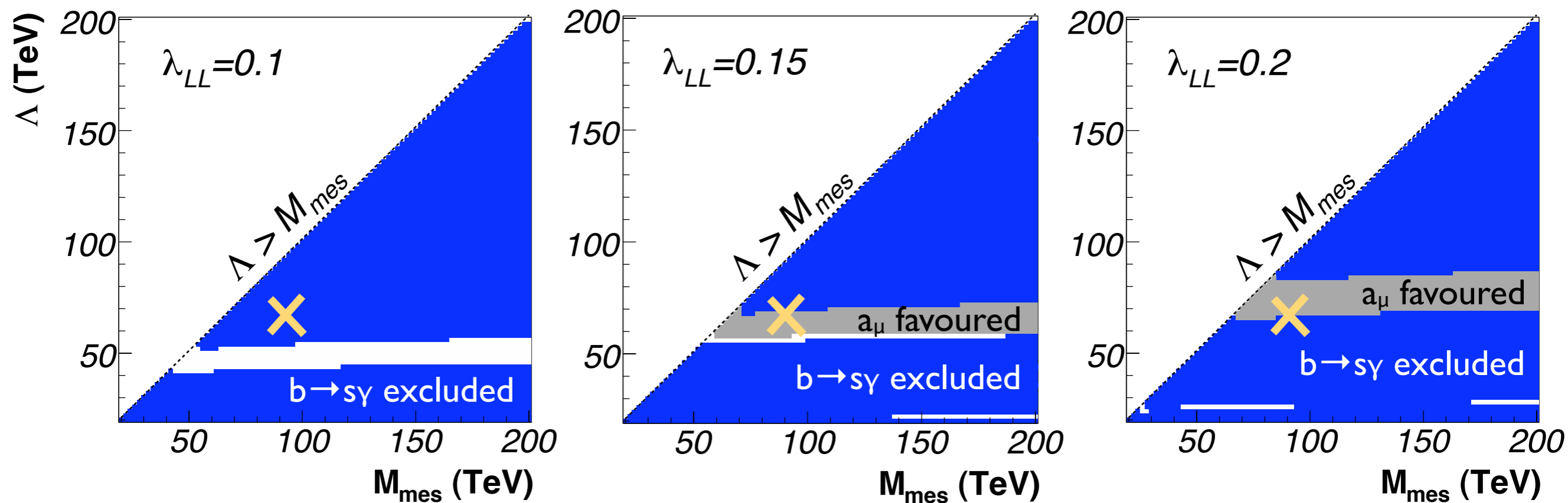
In gauge-mediated scenarios:  $\Delta_{LL,RR} \gg \Delta_{RL,LR}$  due to small A-terms

[Dittmaier *et al.* 2007]

# Scans of the GMSB Parameter Space including NMFV

Re-scan parameter space and impose constraints at  $2\sigma$

Example:  $N_{\text{mes}}=1$ ,  $\tan\beta=15$ ,  $\mu>0$ ,  $\lambda_{\text{RR}}=0$



Flavour mixing between 2<sup>nd</sup> and 3<sup>rd</sup> generation squarks opens allowed and favoured windows in the GMSB parameter space

No significant differences between the two implementations of flavour violation

# Benchmark Points for GMSB including NMFV

Define six “collider-friendly” benchmark points allowing for flavour violation

	$\Lambda$	$M_{\text{mes}}$	$N_{\text{mes}}$	$\tan \beta$	$\text{sgn}(\mu)$	$\lambda_{\text{LL}}$	NLSP	SPS
E	65 TeV	90 TeV	1	15	+	[0.14, 0.20]	$\tilde{\chi}_1^0$	8
F	30 TeV	80 TeV	3	15	+	[0.12, 0.18]	$\tilde{\tau}_1$	7
G	100 TeV	110 TeV	1	30	+	[0.14, 0.20]	$\tilde{\tau}_1$	—
H	45 TeV	100 TeV	3	30	+	[0.12, 0.18]	$\tilde{\tau}_1$	—
I	130 TeV	140 TeV	1	50	+	[0.14, 0.20]	$\tilde{\tau}_1$	—
J	60 TeV	100 TeV	3	50	+	[0.14, 0.20]	$\tilde{\tau}_1$	—

Benchmark points not valid for (constrained) minimal flavour violation

→ indicate valid range for flavour violation parameter  $\lambda_{\text{LL}}$

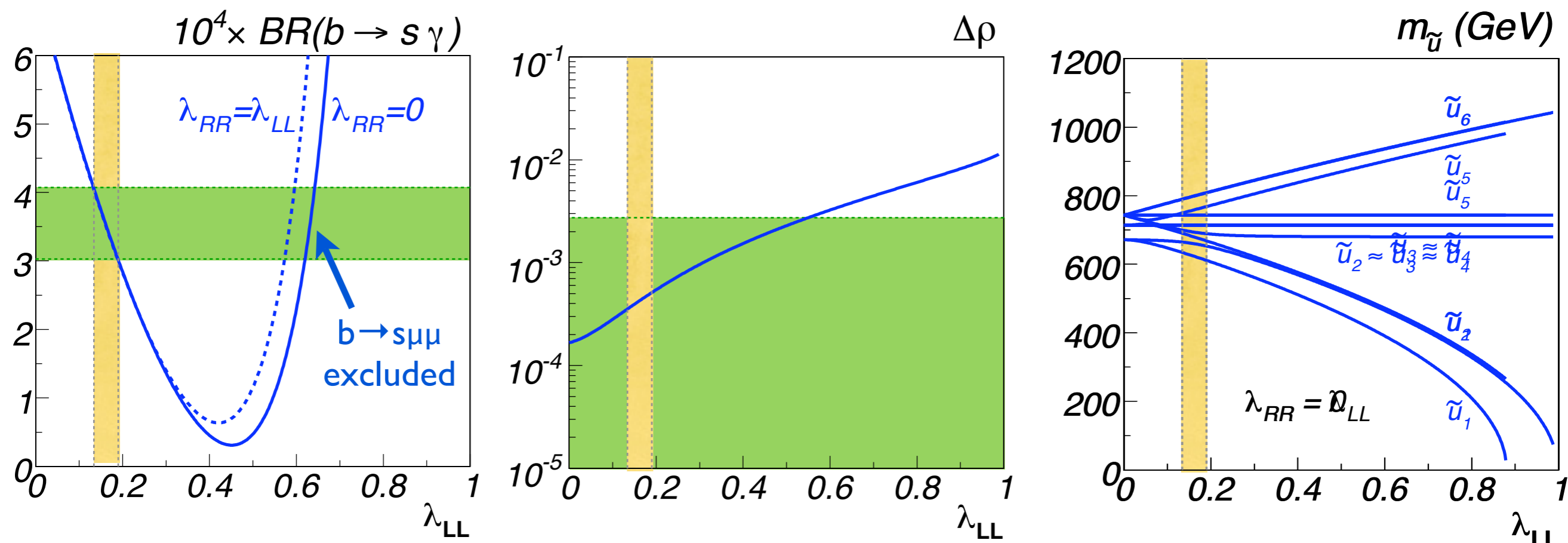
Scenarios differ in nature of NLSP and number of particles that are close in mass

Flavour violation through mixing with either fundamental or antisymmetric messengers



# Phenomenology of Point E

Study constraints and mass eigenvalues as function of flavour violation parameter  $\lambda_{LL}$



Allowed range for flavour violation parameter determined from  $b \rightarrow s \gamma$

$$0.14 \leq \lambda_{LL} \leq 0.20$$

Main difference between the two flavour violation scenarios in the splitting of mass eigenvalues

“Avoided crossings” of mass eigenvalues with corresponding changes in flavour contents

# Cosmological Constraints

Gravitino mass has to lead to cosmologically viable configuration

Constraints involve given masses of gluino and NLSP

Determine allowed regions in the  $m_{\tilde{G}} - T_R$  plane

Example: Point E

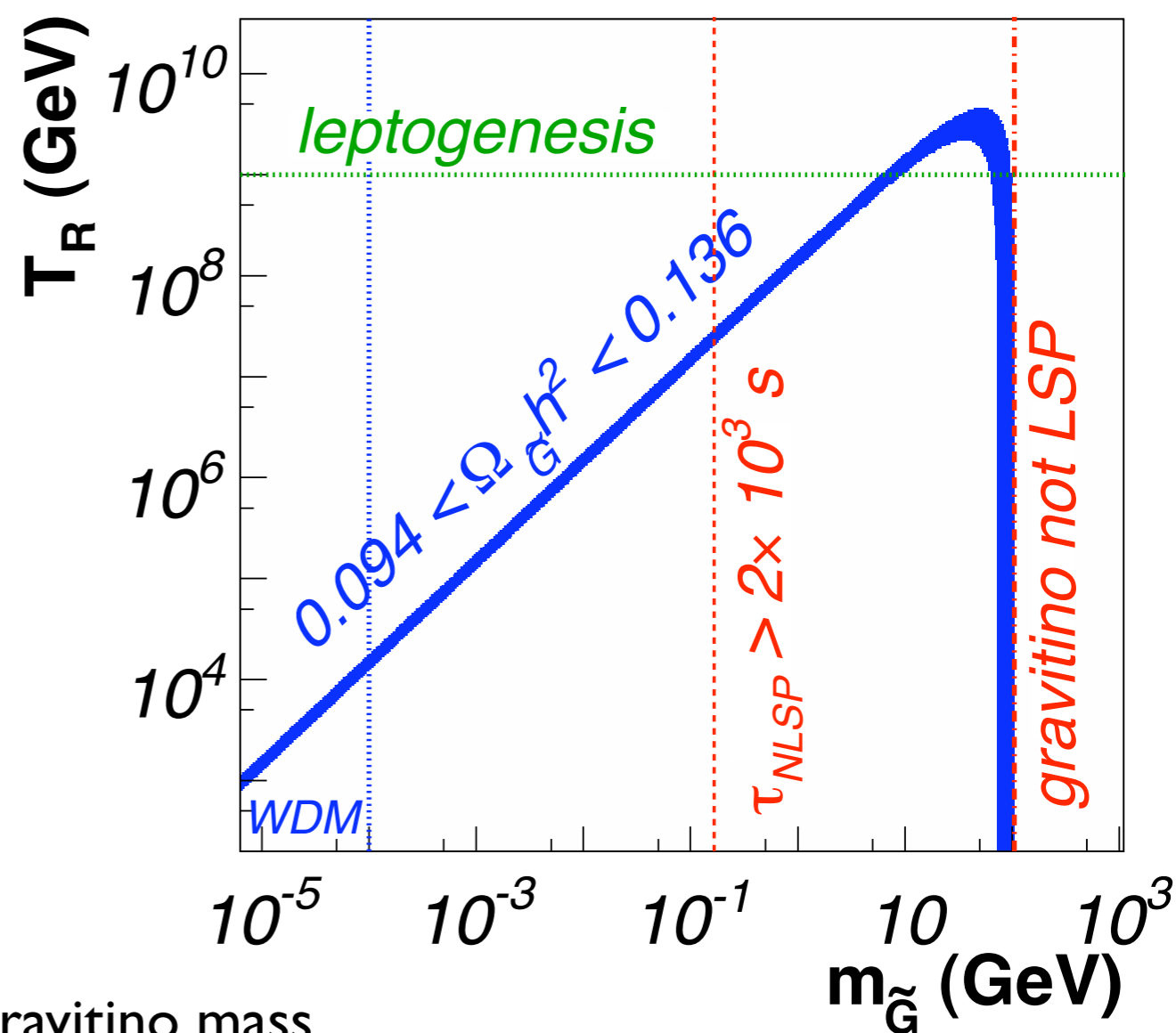
$$\begin{aligned} m_{\tilde{\chi}_1^0} &= 95.4 \text{ GeV} \\ \Omega_{\tilde{\chi}} h^2 &= 0.1275 \\ m_{\tilde{g}} &= 534.2 \text{ GeV} \end{aligned}$$

All constraints cannot be fulfilled together

Allow for lower reheating temperatures and set gravitino mass

$$m_{\tilde{G}} \sim 10^{-1} \text{ GeV}$$

Similar situation for the other benchmark points, leading to the same order of magnitude for the gravitino mass



## **Part II**

---

# Supersymmetric Particle Production at the LHC

---

# Cross Section Calculation

**QCD factorization theorem: Convolution of partonic cross section with parton densities**

$$\sigma = \underbrace{\int_{t_-}^{t_+} dt}_{\text{Phase space}} \underbrace{\int_{4m^2/s}^1 d\tau \int_{-1/2 \ln \tau}^{1/2 \ln \tau} dy}_{\text{Convolution}} \underbrace{f_{a/A}(x_a, M_a^2) f_{b/B}(x_b, M_b^2)}_{\text{Parton density functions (PDF)}} \frac{d\hat{\sigma}}{dt}$$

$x_{a,b} = \sqrt{\tau} e^{\pm y}$

Unpolarized cross sections obtained by averaging over initial state spins

$$d\hat{\sigma} = \frac{1}{4} \left[ d\hat{\sigma}_{1,1} + d\hat{\sigma}_{1,-1} + d\hat{\sigma}_{-1,1} + d\hat{\sigma}_{-1,-1} \right]$$

Analytical expressions for helicity-depending partonic production cross sections in non-minimal flavour violating Supersymmetry [Bozzi, Fuks, Herrmann, Klasen 2007; Fuks, Herrmann, Klasen 2008]

**Numerical examples of squark and gaugino production cross sections for benchmark scenario H**

- “light” quark masses neglected with respect to  $\sqrt{s} = 14$  TeV
- leading order CTEQ6 global parton density fit
- running strong and weak coupling constants
- set scales to average final state mass

$$\mu_R = M_a = M_b = \frac{m_1 + m_2}{2}$$

# Partonic Cross Sections

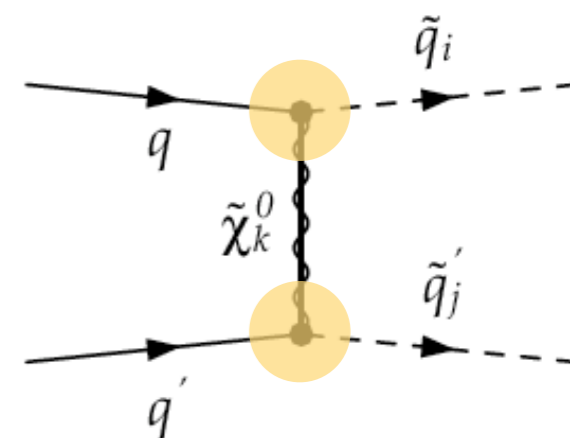
Express cross sections in terms of generalized couplings to include flavour violation

Example: Squark pair production through t-channel neutralino exchange

$$\begin{aligned} \frac{d\hat{\sigma}_{h_a h_b}}{dt} = & (1 - h_a)(1 - h_a) \sum_{k,l=1}^4 \frac{[\mathcal{NT}]_{11}^{kl}}{t_{\tilde{\chi}_k^0} t_{\tilde{\chi}_l^0}} + (1 + h_a)(1 + h_a) \sum_{k,l=1}^4 \frac{[\mathcal{NT}]_{22}^{kl}}{t_{\tilde{\chi}_k^0} t_{\tilde{\chi}_l^0}} \\ & + (1 - h_a)(1 + h_a) \sum_{k,l=1}^4 \frac{[\mathcal{NT}]_{12}^{kl}}{t_{\tilde{\chi}_k^0} t_{\tilde{\chi}_l^0}} + (1 + h_a)(1 - h_a) \sum_{k,l=1}^4 \frac{[\mathcal{NT}]_{21}^{kl}}{t_{\tilde{\chi}_k^0} t_{\tilde{\chi}_l^0}} \end{aligned}$$

Form factors include coupling constants and Dirac traces

$$\begin{aligned} [\mathcal{NT}]_{mn}^{kl} = & \frac{\pi \alpha^2}{\sin^4 \theta_W \cos^4 \theta_W s^2} \mathcal{C}_{\tilde{q}_j q' \tilde{\chi}_k^0}^{n*} \mathcal{C}_{\tilde{q}_i q \tilde{\chi}_k^0}^{m*} \mathcal{C}_{\tilde{q}_j q' \tilde{\chi}_l^0}^n \mathcal{C}_{\tilde{q}_i q \tilde{\chi}_l^0}^m \\ & \times \left[ \left( u t - m_{\tilde{q}_i}^2 m_{\tilde{q}_j}^2 \right) (1 - \delta_{mn}) + m_{\tilde{\chi}_k^0} m_{\tilde{\chi}_l^0} s \delta_{mn} \right] \end{aligned}$$



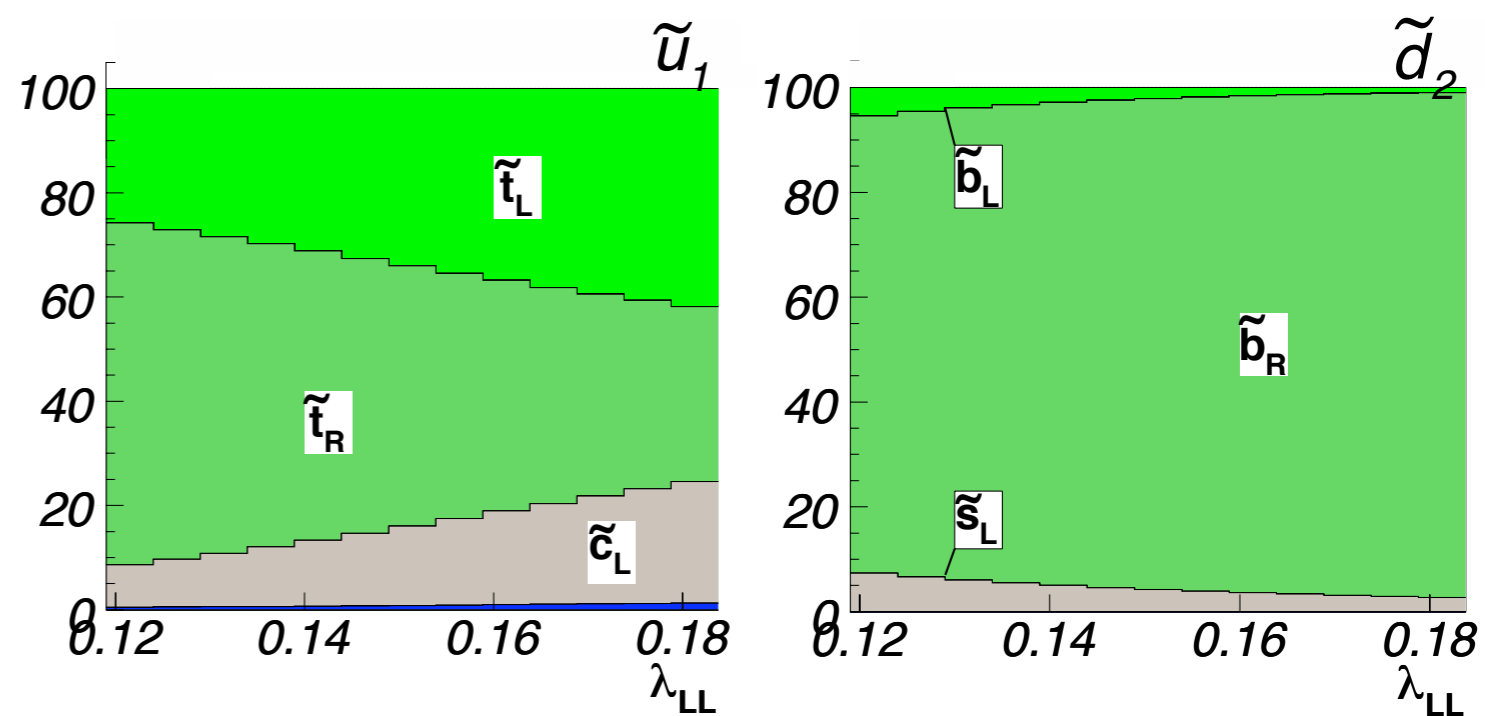
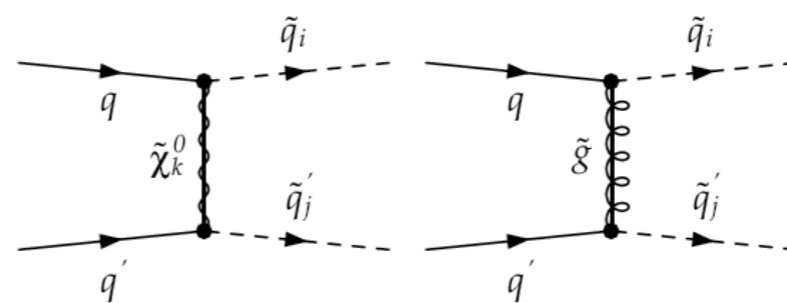
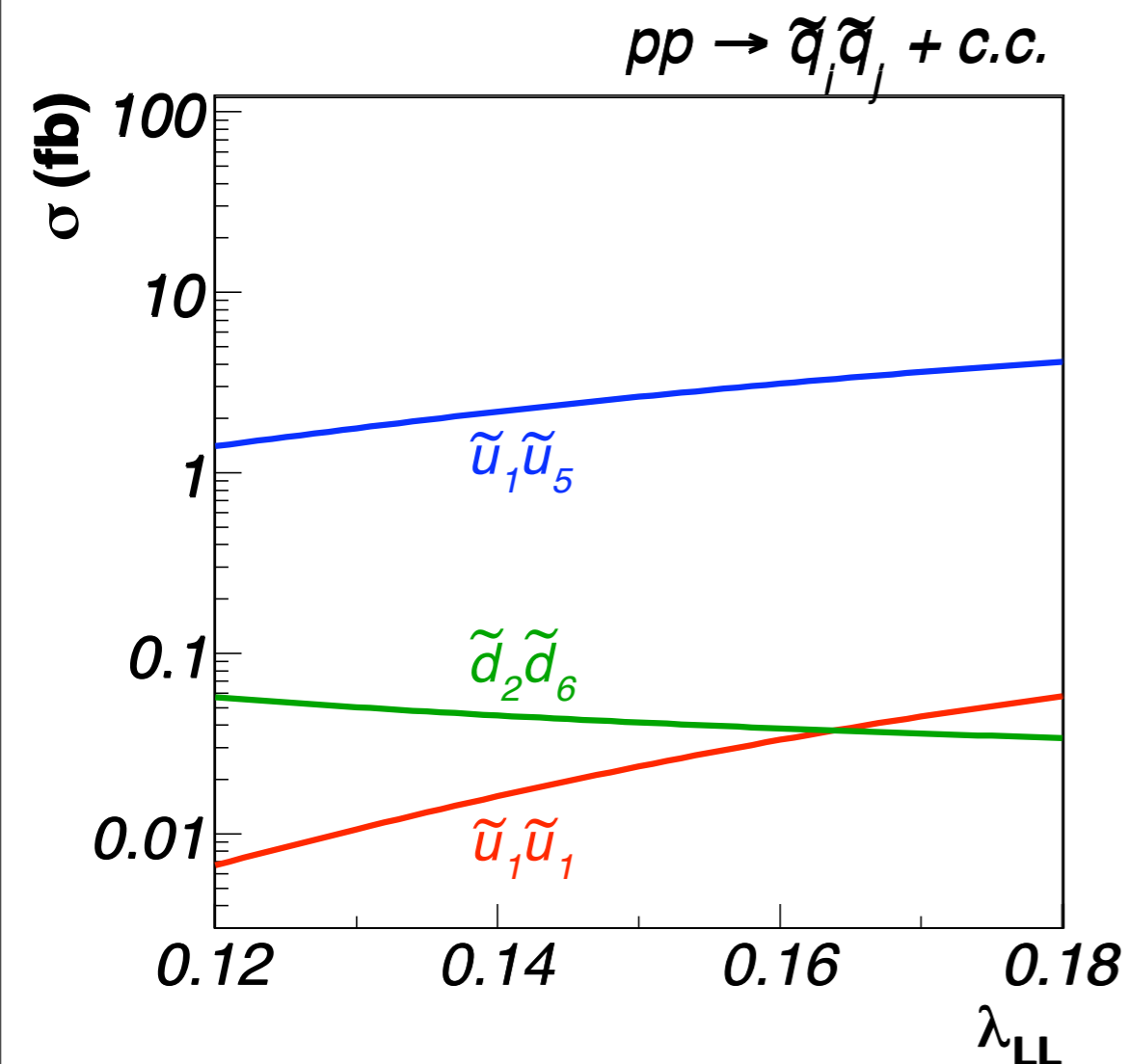
Generic notation for left- and right-handed couplings:  $\{\mathcal{C}_{abc}^1, \mathcal{C}_{abc}^2\} = \{L_{abc}, R_{abc}\}$

Agreement with previous results in constrained minimal flavour violation

[Bozzi, Fuks, Klasen 2005]

# Squark Pair Production

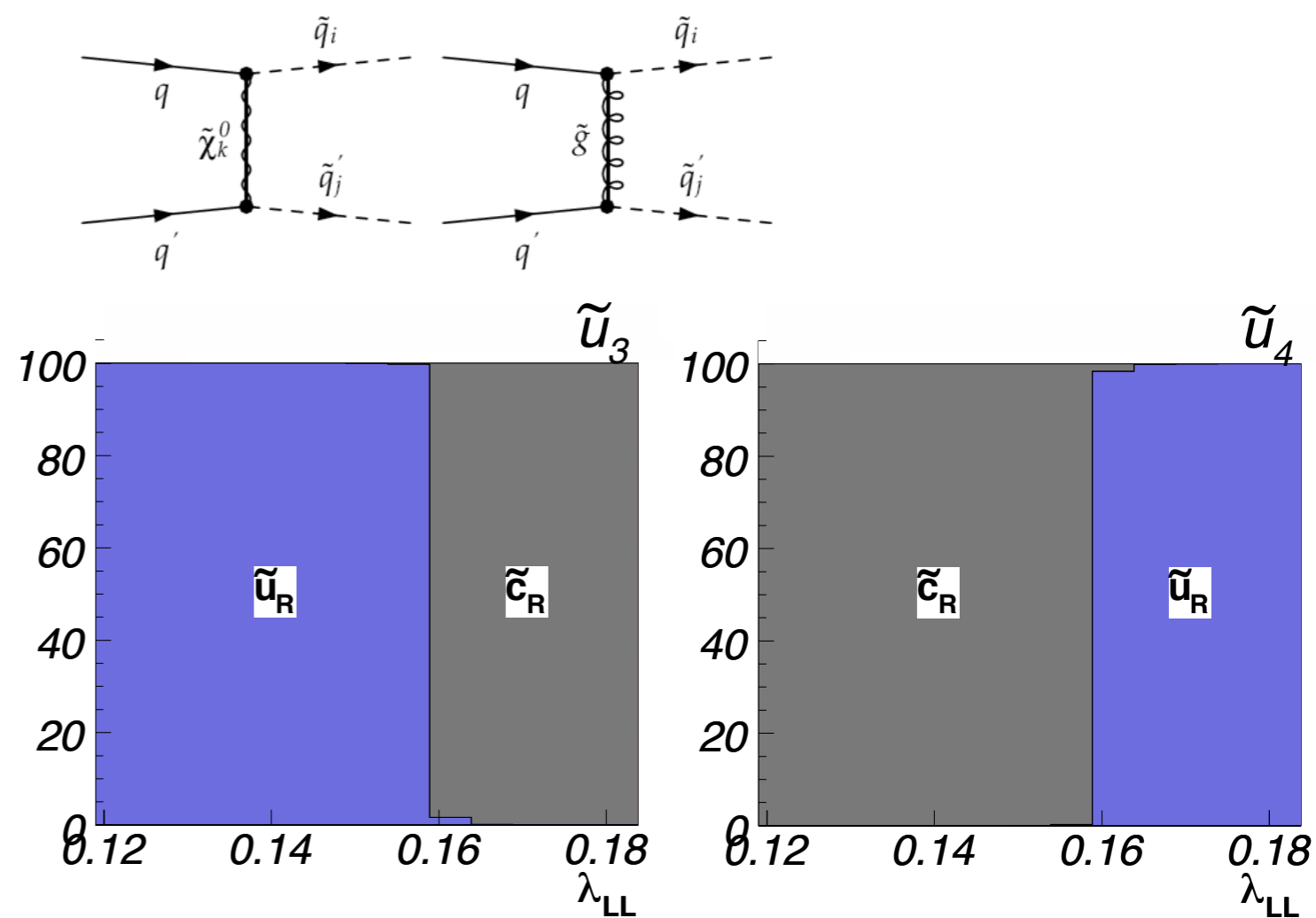
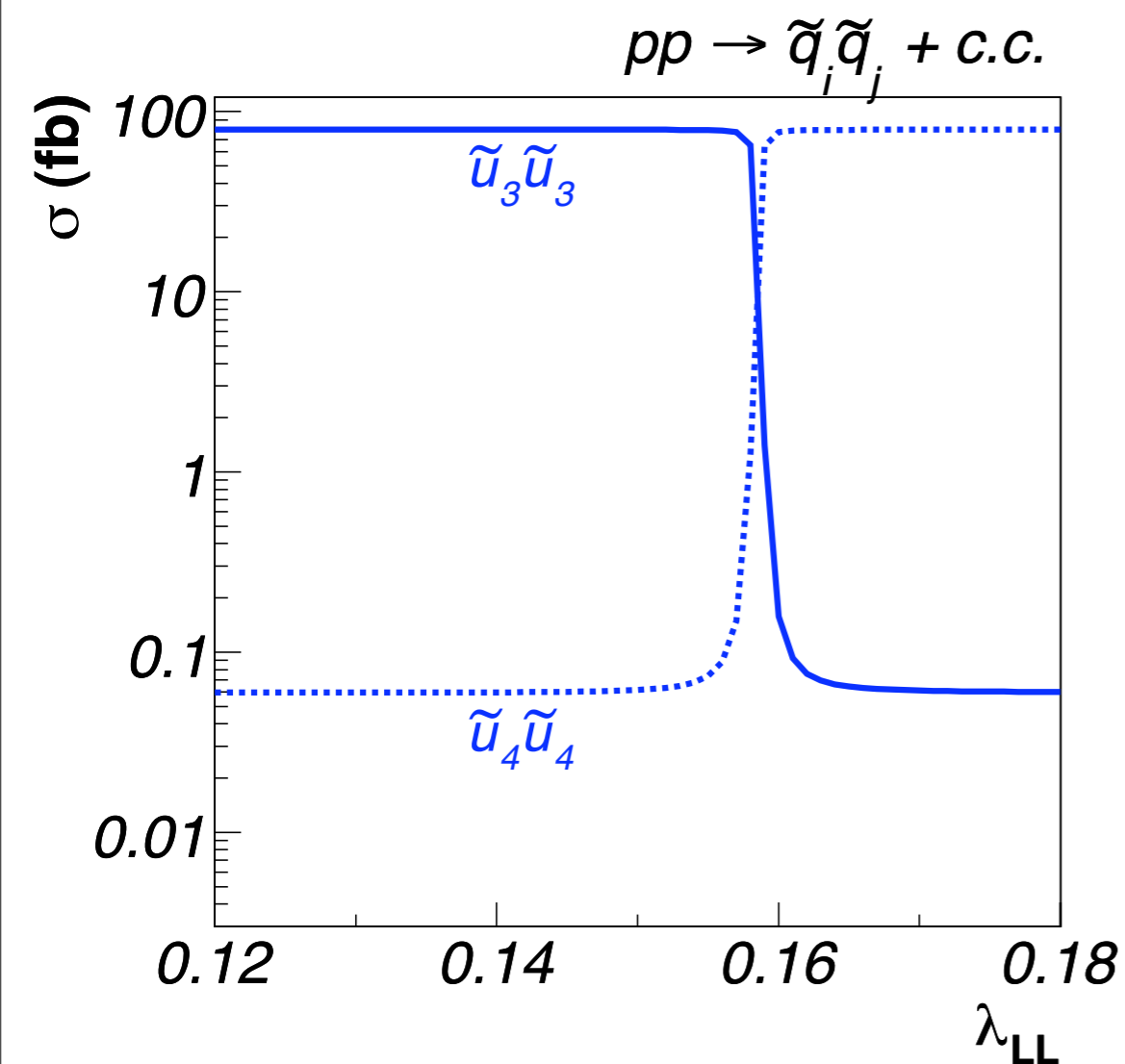
Cross sections sensitive to flavour violation due to  $q\tilde{q}\tilde{g}$ - and  $q\tilde{q}\tilde{\chi}$ -vertices



$\tilde{u}_1 \tilde{u}_5$  and  $\tilde{u}_1 \tilde{u}_1$  production cross sections increase with light flavour content in  $\tilde{u}_1$   
 $\tilde{d}_2 \tilde{d}_6$  production cross section decreases with light flavour content in  $\tilde{d}_2$

# Squark Pair Production

Cross sections sensitive to flavour violation due to  $q\tilde{q}\tilde{g}$ - and  $q\tilde{q}\tilde{\chi}$ -vertices



**Flavour exchanges** between  $\tilde{u}_3$  and  $\tilde{u}_4$  related to “avoided crossings” of mass eigenvalues

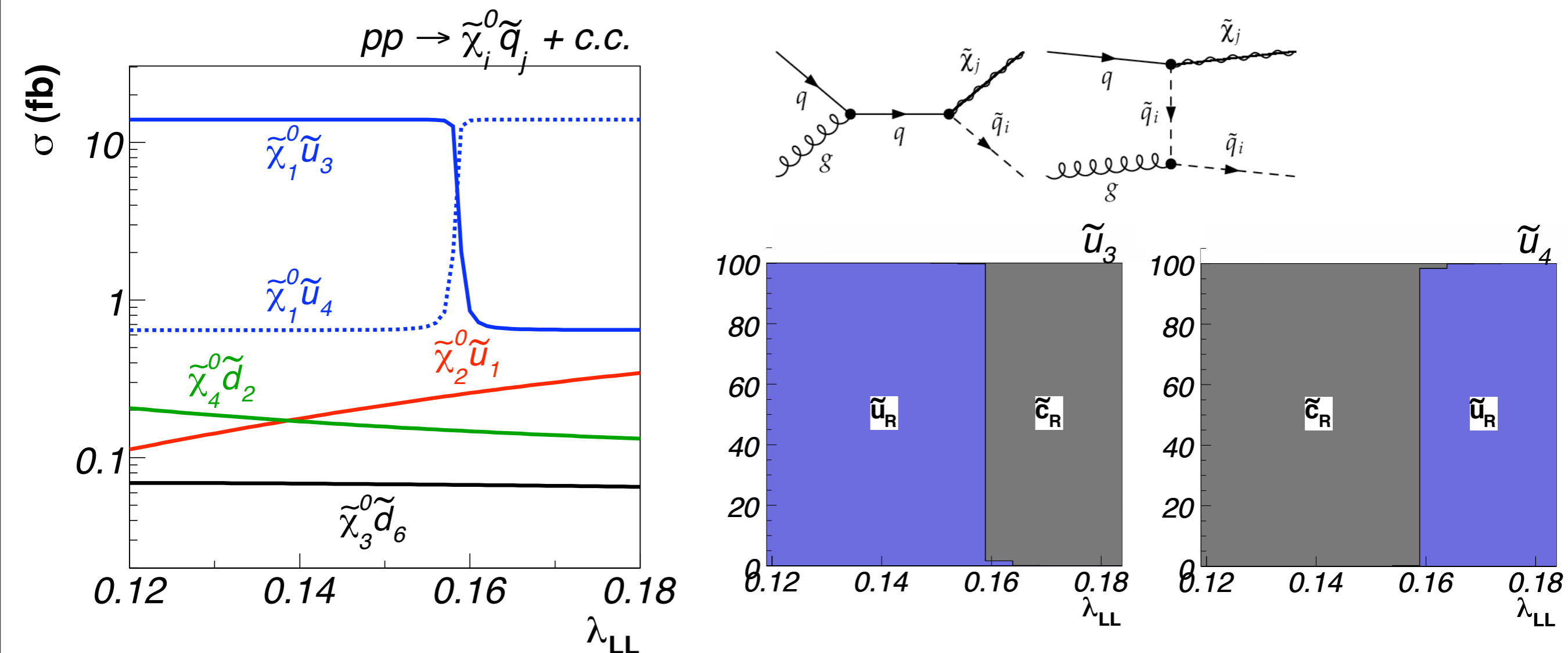
→ flavour exchanges lead to sharp transitions in particular production channels

**Resonance-like behaviour** of  $\tilde{u}_3 \tilde{u}_4$  production cross section due to smooth flavour exchange

→ both squark mass eigenstates receive significant up-quark contributions

# Squark-Neutralino Associated Production

Cross sections sensitive to flavour violation due to  $q\bar{q}\tilde{\chi}$ -vertices



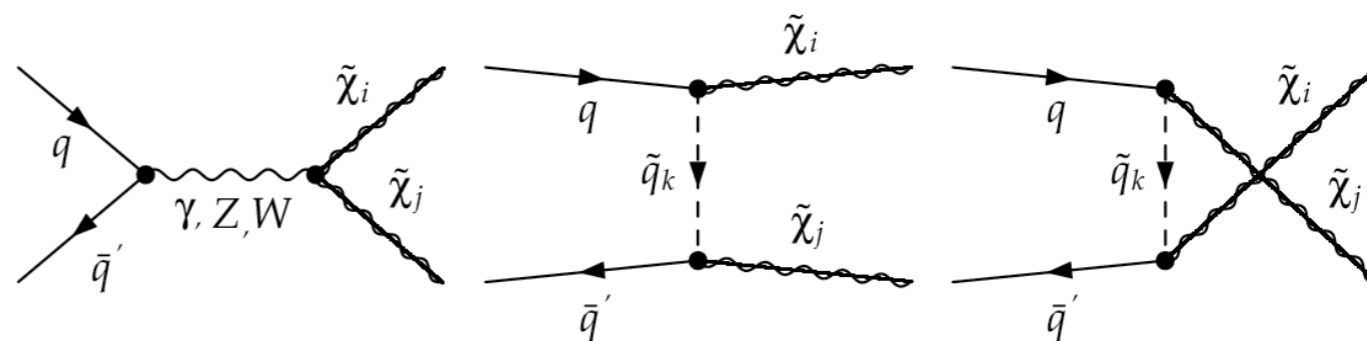
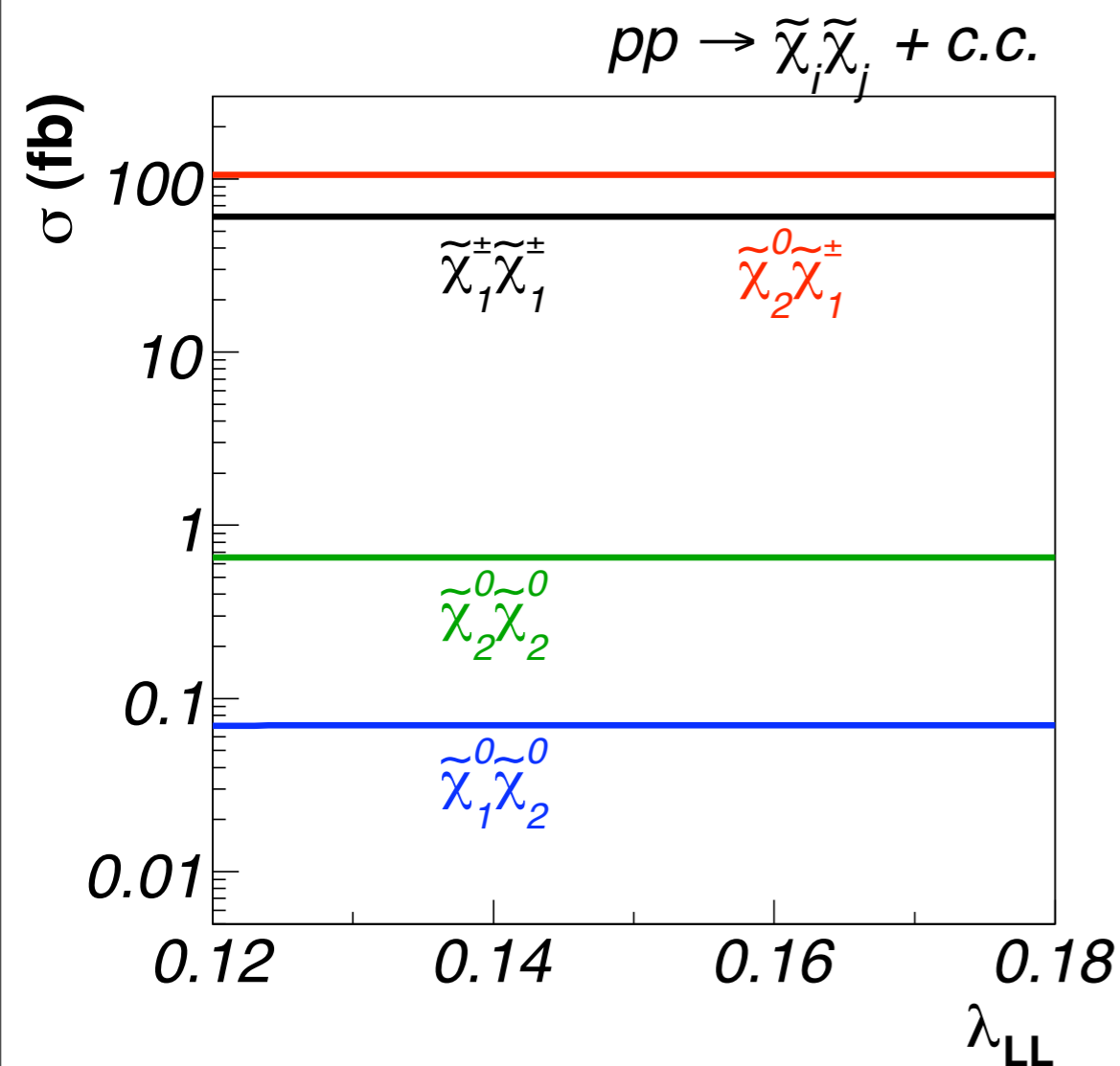
$\tilde{u}_1$  production cross section increases with “light” flavour content in  $\tilde{u}_1$   
 $\tilde{d}_2$  production cross section decreases with “light” flavour content in  $\tilde{d}_2$

Flavour exchanges between  $\tilde{u}_3$  and  $\tilde{u}_4$  related to “avoided crossings” of mass eigenvalues  
 $\rightarrow$  flavour exchanges lead to sharp transitions in particular production channels



# Gaugino Pair Production

Flavour violation involved through  $q\bar{q}\tilde{\chi}$ -vertices



Numerically important due to relatively light gauginos

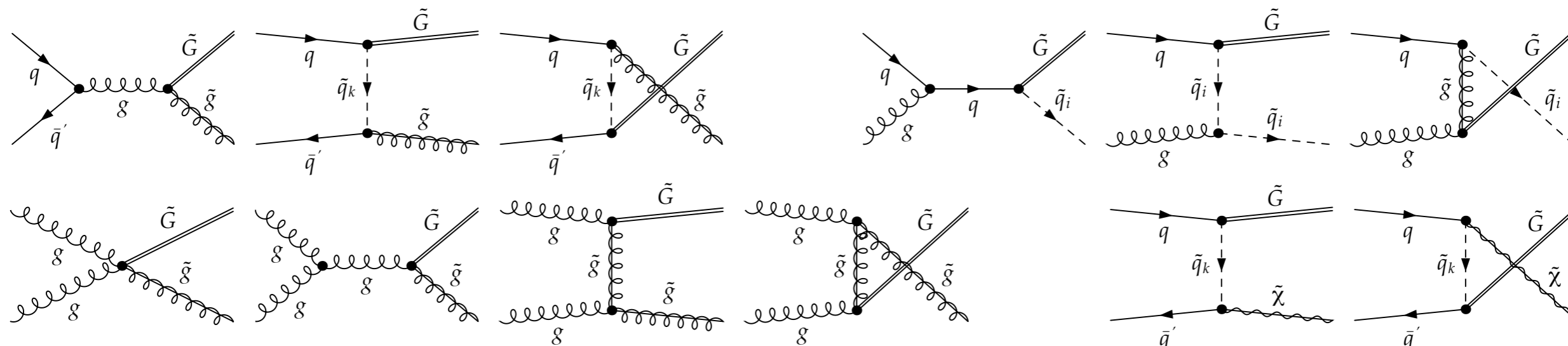
Cross section independent of flavour violation

Squarks only appear in t- or u-channel propagators

→ sum over all physical squark states

# Gravitino Production

Light gravitino production interesting at current hadron colliders, in particular in GMSB scenarios



Flavour violation effects arise through vertices involving quarks and squarks

Cross sections proportional to inverse gravitino mass squared

[Fayet 1986]

$$\mathcal{V}_{q\tilde{q}\tilde{G}} = \pm i \frac{m_{\tilde{q}}^2 - m_q^2}{\sqrt{6}M_{\text{P}}m_{\tilde{G}}} P_{R,L} \quad \mathcal{V}_{g\tilde{g}\tilde{G}} = i \frac{m_{\tilde{g}}}{2\sqrt{6}M_{\text{P}}m_{\tilde{G}}} [\not{p}, \gamma^\mu]$$

Only light gravitinos (i.e. hot dark matter) allow for sizeable cross sections

→ production invisible for  $m_{\tilde{G}} \sim 10^{-1}$  GeV in our benchmark scenarios

Lightest messenger may not be stable due to mixing with matter fields

→ no alternative candidate for cold dark matter

## **Part III**

---

# **SUSY-QCD Corrections to Dark Matter Annihilation**

---

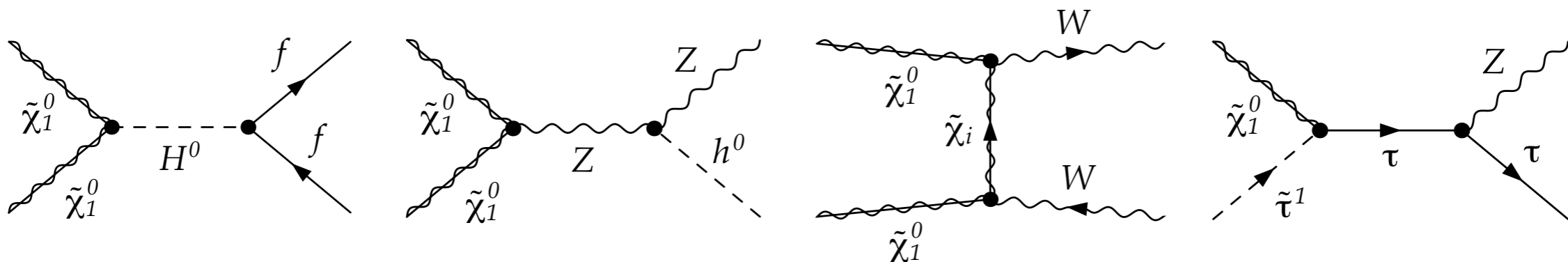
# Relic Density Calculation

Number density of relic particle governed by the Boltzmann equation

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\text{ann}} v \rangle (n^2 - n_{\text{eq}}^2)$$

$$\Omega_{\text{CDM}} h^2 \propto n \propto \frac{1}{\langle \sigma_{\text{ann}} v \rangle}$$

Cross section  $\sigma_{\text{ann}}$  includes all annihilation and coannihilation processes



Thermal average involves velocity distribution of the non-relativistic relic particle

$$\langle \sigma_{\text{ann}} v \rangle = \int dv \tilde{f}(v) \sigma_{\text{ann}} v = \int ds f(s) \sigma_{\text{ann}}(s)$$

Public codes perform a calculation of the relic density for given scenario

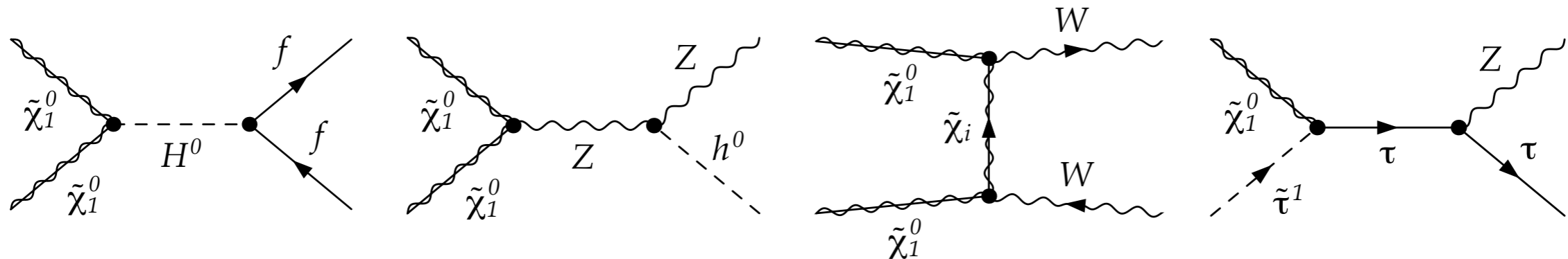
- DarkSUSY (only neutralino)
- micrOMEGAs (all kinds of LSP)

[Gondolo et al. 2004]

[Bélanger et al. 2006]

# Motivation for SUSY-QCD Corrections

Most (co)annihilation processes only implemented at leading order in public codes

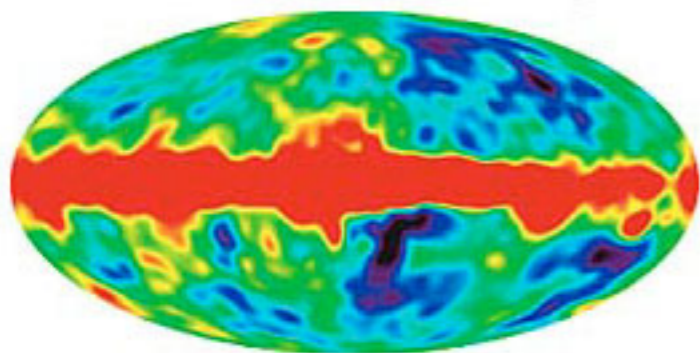


Higher order corrections can have important contributions to cross sections

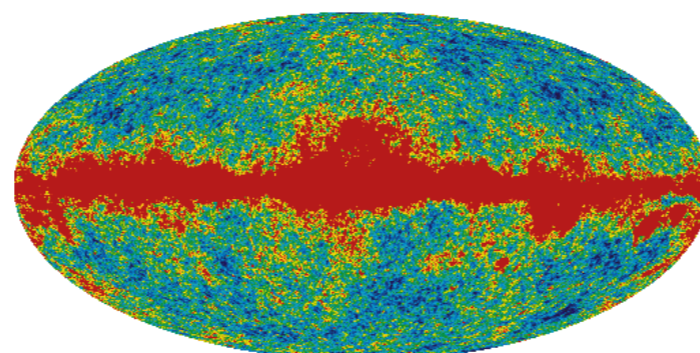
- QCD corrections significant due to strong coupling constant
- modification of preferred regions in parameter space

Planck satellite will deliver new cosmological data in near future

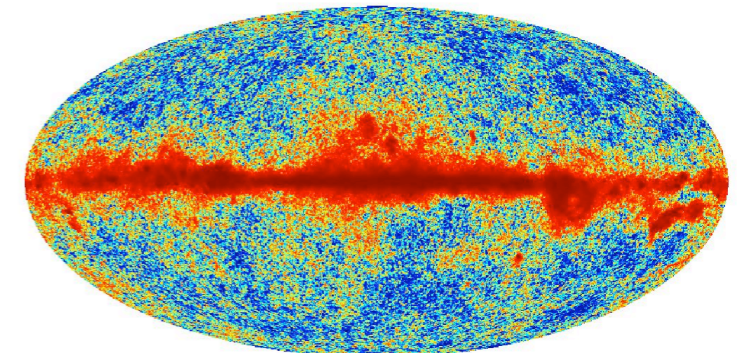
- higher precision in theoretical predictions needed to match experimental improvements



COBE  
1989



WMAP  
2002

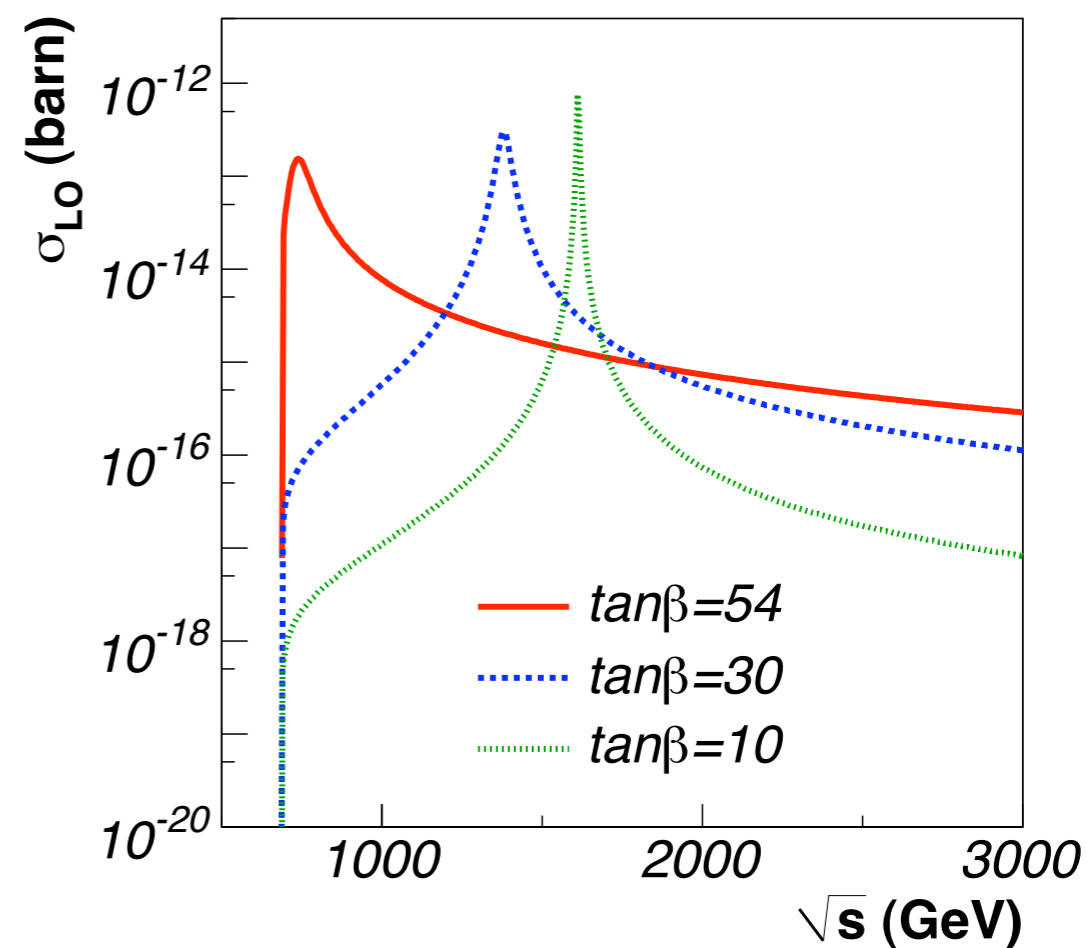
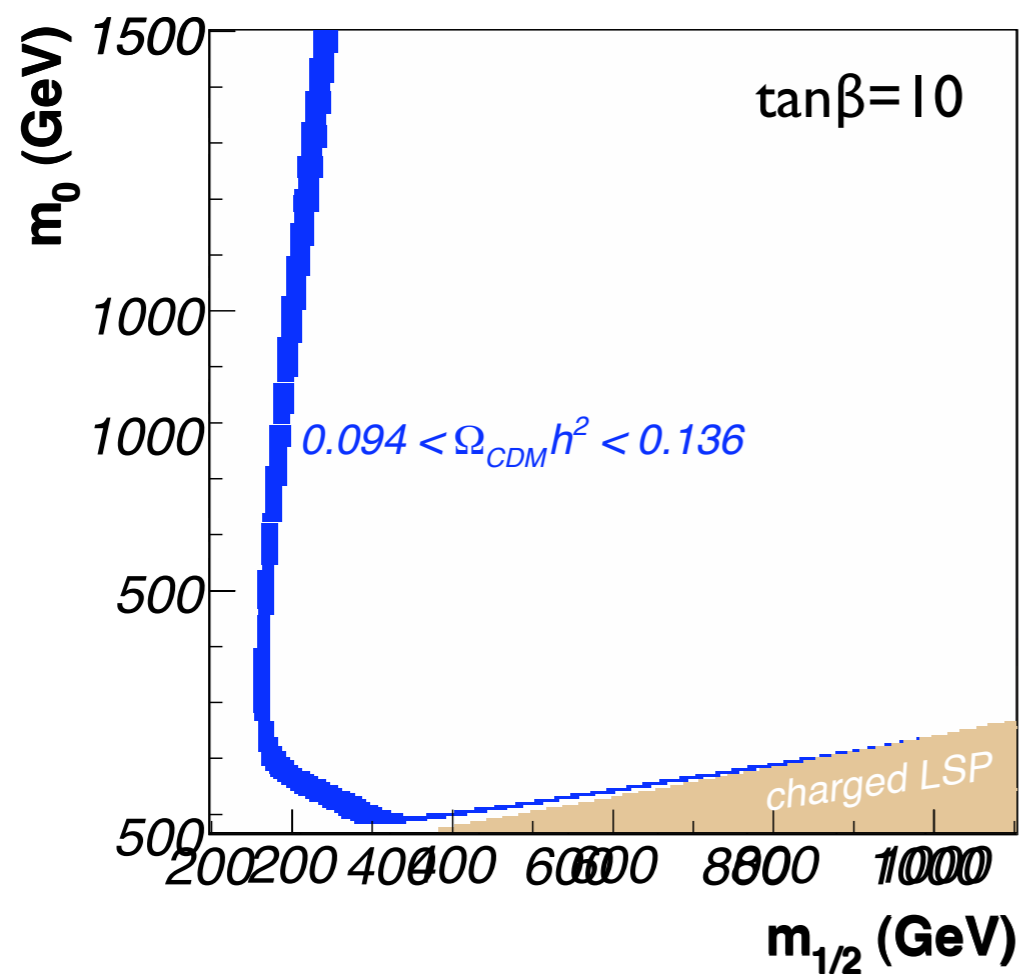
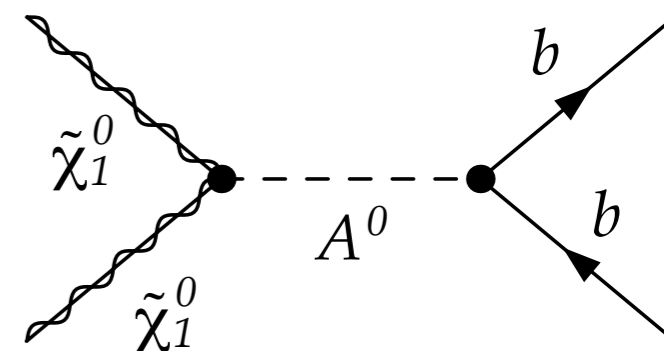


Planck  
2009 ?

# Neutralino Annihilation in the A-Funnel Region

## Annihilation into bottom quarks dominates

- important Yukawa coupling of bottom quarks
- neutralino pair behaves like pseudoscalar particle



Large  $\tan\beta$  favoured by unification of Yukawa couplings in GUT theories

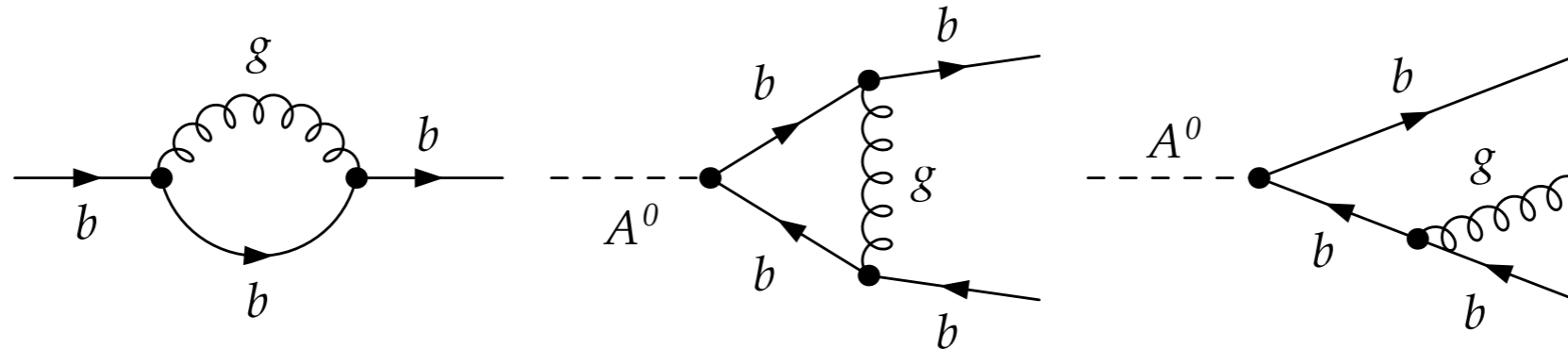
[Carena et al. 1994]

Process might be compatible with the gamma ray excess observed by EGRET

→ problem with observed antiproton flux...?

[de Boer et al. 2004, Bergström et al. 2006]

## Virtual loop corrections and real gluon emission



On-shell renormalization to cancel UV-singularities in virtual corrections

Dipole subtraction method to combine virtual and real emission contributions

[Catani et al. 2000]

$$\sigma_{\text{NLO}} = \left[ \sigma_{\text{V}} + \int d\sigma_{\text{aux}} \right]_{\epsilon=0} + \int \left[ d\sigma_{\text{R}} - d\sigma_{\text{aux}} \right]_{\epsilon=0}$$

## UV- and IR-finite annihilation cross section

$$\sigma_{\text{ann}} = \sigma_{\text{LO}} \left[ 1 + \Delta_{\text{QCD}} \right]$$

$$\Delta_{\text{QCD}} = \frac{\alpha_s(s)}{\pi} C_F \left[ \frac{1 + \beta_b^2}{\beta_b} \left( 4\text{Li}_2 \frac{1 - \beta_b}{1 + \beta_b} + 2\text{Li}_2 \frac{\beta_b - 1}{1 + \beta_b} - 3 \log \frac{2}{1 + \beta_b} \log \frac{1 + \beta_b}{1 - \beta_b} - 2 \log \beta_b \log \frac{1 + \beta_b}{1 - \beta_b} \right) \right. \\ \left. - 3 \log \frac{4}{1 - \beta_b^2} - 4 \log \beta_b + \frac{3}{8} (7 - \beta_b^2) + \frac{1}{16\beta_b} (19 + 2\beta_b^2 + 3\beta_b^4) \log \frac{1 + \beta_b}{1 - \beta_b} \right]$$

# QCD Corrections

Cross section contains logarithmic mass singularity at high energies

$$\Delta_{\text{QCD}}^{(\text{HE})} \simeq \frac{\alpha_s C_F}{\pi} \left[ -\frac{3}{2} \log \frac{s}{m_b^2} + \frac{9}{4} \right]$$

Resummation to all orders using the renormalization group

→ use running bottom quark mass in Yukawa coupling

[Braaten & Leveille 1980]

$$h_{Abb} = -\frac{gm_b \tan \beta}{2m_W} \longrightarrow -\frac{g\bar{m}_b(s) \tan \beta}{2m_W}$$

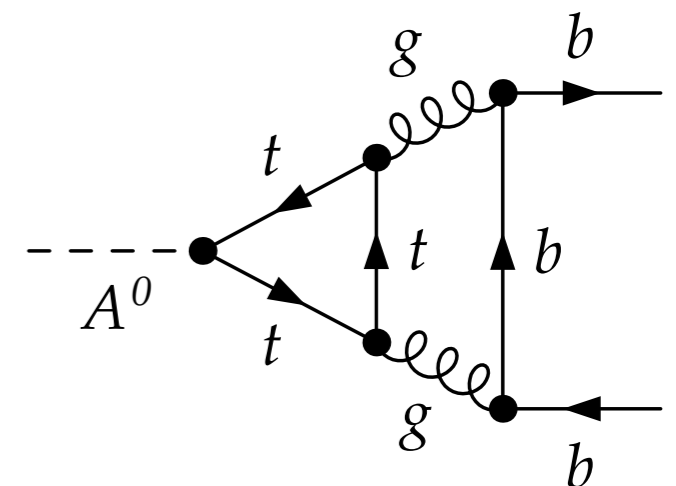
Higher order corrections are known up to  $O(\alpha_s^4)$

[Chetyrkin et al. 1996, Chetyrkin 1997, Chetyrkin et al. 2005]

$$\Delta_{\text{QCD}}^{(n_f=5)} = 29.15 \frac{\alpha_s^2(s)}{\pi^2} + 41.77 \frac{\alpha_s^3(s)}{\pi^3} - 825.6 \frac{\alpha_s^4(s)}{\pi^4}$$

Top quark loop correction negligible at large  $\tan \beta$  [Chetyrkin et al. 1996]

$$\Delta_{\text{top}} = \frac{1}{\tan^2 \beta} \frac{\alpha_s^2(s)}{\pi^2} \left[ \frac{23}{6} - \log \frac{s}{m_t^2} + \frac{1}{6} \log^2 \frac{\bar{m}_b^2(s)}{s} \right]$$



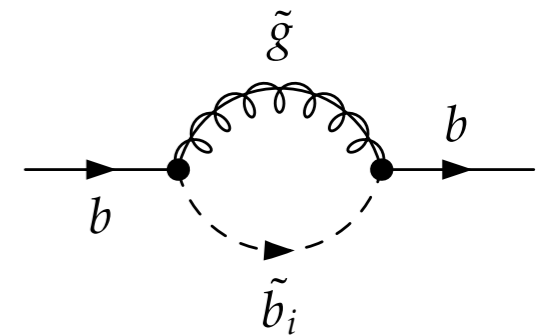


# SUSY-QCD Corrections

Virtual loop corrections include self-energy and vertex correction diagrams

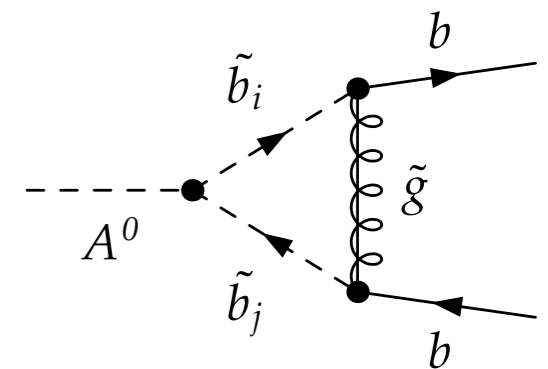
On-shell mass renormalization in “low-energy” limit of large SUSY masses

$$\left(\frac{\Delta m_b}{m_b}\right)_{\tilde{g}\tilde{b}} = \frac{\alpha_s(s)}{\pi} C_F \frac{m_{\tilde{g}}}{2} \left(A_b - \mu \tan \beta\right) I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2);$$



Vertex correction equals mass renormalization up to a factor  $1/\tan^2\beta$

$$\Delta_{\text{SUSY}}^{(\text{LE})} = \frac{\alpha_s(s)}{\pi} C_F \left[1 + \frac{1}{\tan^2 \beta}\right] m_{\tilde{g}} \mu \tan \beta I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2).$$



Mass resummation in Yukawa coupling significant for large  $\tan\beta$

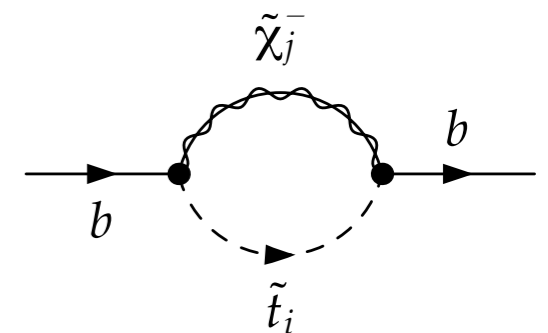
$$\bar{m}_b(s) \rightarrow \frac{\bar{m}_b(s)}{1 + \Delta m_b/m_b}$$

[Carena et al. 2000, Guasch et al. 2003]

Stop-chargino loop contributes at same order of magnitude

[Carena et al. 2000]

$$\left(\frac{\Delta m_b}{m_b}\right)_{\tilde{t}\tilde{\chi}^\pm} = -\frac{\lambda_t^2}{16\pi^2} \left[A_t \mu \tan \beta - \mu^2\right] I(\mu^2, m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2)$$



# QCD and SUSY-QCD Corrections

Final annihilation cross section including corrections

$$\sigma_{\text{ann}} = \bar{\sigma}_{\text{LO}}(s) \left[ 1 + \Delta_{\text{QCD}} + \Delta_{\text{top}} + \Delta_{\text{SUSY}} \right]$$

“Leading order” part contains QCD running bottom mass and SUSY bottom mass resummation

Finite remainders:

$$\Delta_{\text{QCD}} = C_F f_{\text{QCD}}(s) \frac{\alpha_s(s)}{\pi} + 29.15 \frac{\alpha_s^2(s)}{\pi^2} + 41.77 \frac{\alpha_s^3(s)}{\pi^3} - 825.6 \frac{\alpha_s^4(s)}{\pi^4}$$

$$\Delta_{\text{top}} = \frac{1}{\tan^2 \beta} \frac{\alpha_s^2(s)}{\pi^2} \left[ \frac{23}{6} - \log \frac{s}{m_t^2} + \frac{1}{6} \log^2 \frac{\bar{m}_b^2(s)}{s} \right]$$

$$\Delta_{\text{SUSY}} = \frac{\alpha_s(s)}{\pi} C_F \frac{1 + \tan^2 \beta}{\tan \beta} m_{\tilde{g}} \mu \times \left[ C_0(m_b^2, s, m_b^2; m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2) - C_0(0, 0, 0; m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2) \right]$$

# Impact on Annihilation Cross Section

Study numerically the impact of the different corrections for  $\mu > 0$

Main impact due to QCD running mass

→ decrease of about 60% near threshold

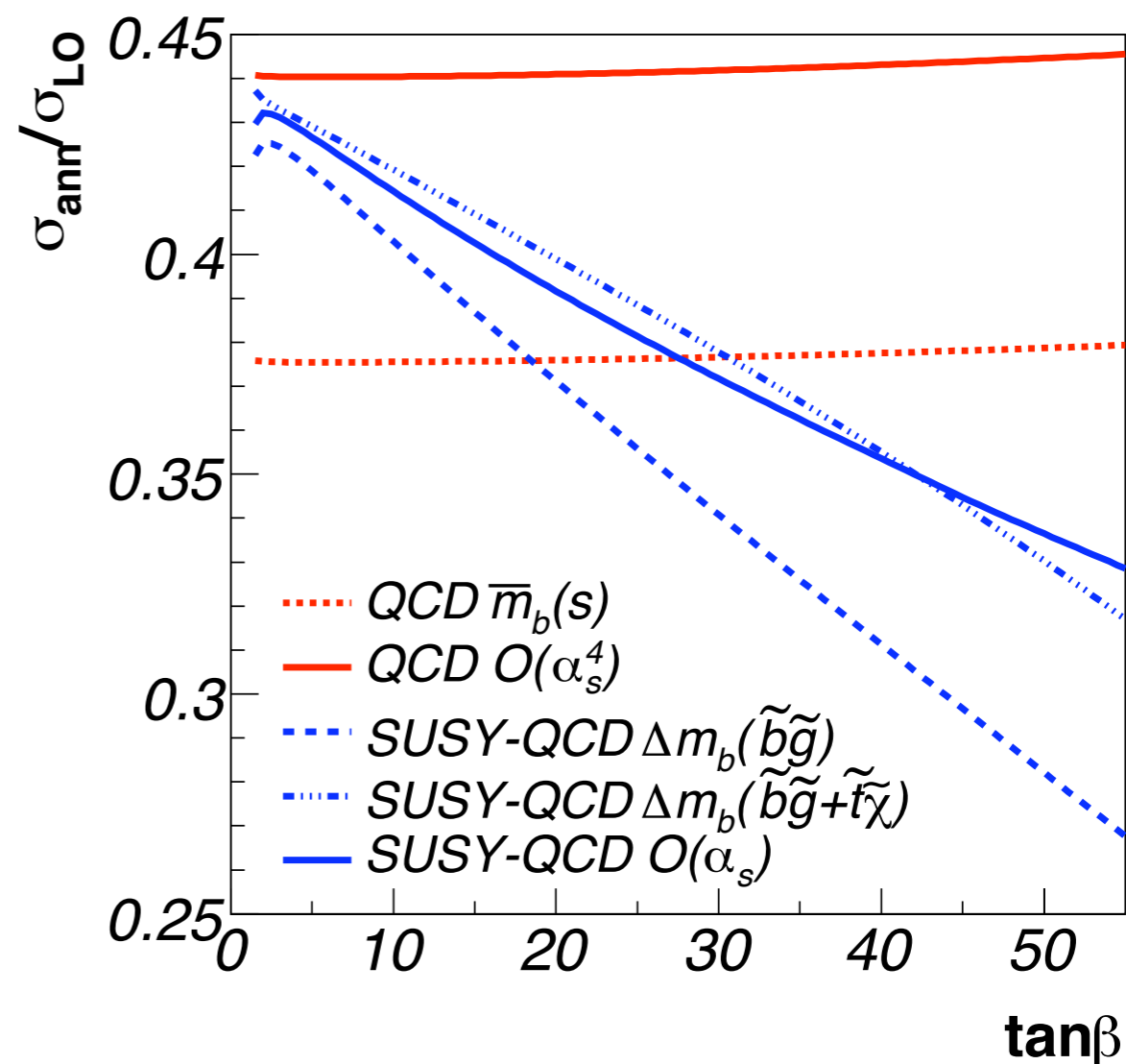
QCD finite parts contribute positively

→ re-increase cross section

Top quark loop negligible at large  $\tan\beta$

→ less than 0.01% of QCD contribution

SUSY-QCD mass resummation decreases cross section by up to 20% at large  $\tan\beta$



Stop-chargino loop and SUSY-QCD finite remainder contribute to a few percent each

Numerical impact of stop-chargino loop is at high  $\tan\beta$  more important than the finite remainder

# Impact on Relic Density

Scan  $m_0$ - $m_{1/2}$  plane with and without including corrections for  $\tan\beta=54$  and  $\mu>0$

Renormalization group running:

SPheno 2.2.3

[Porod 2004]

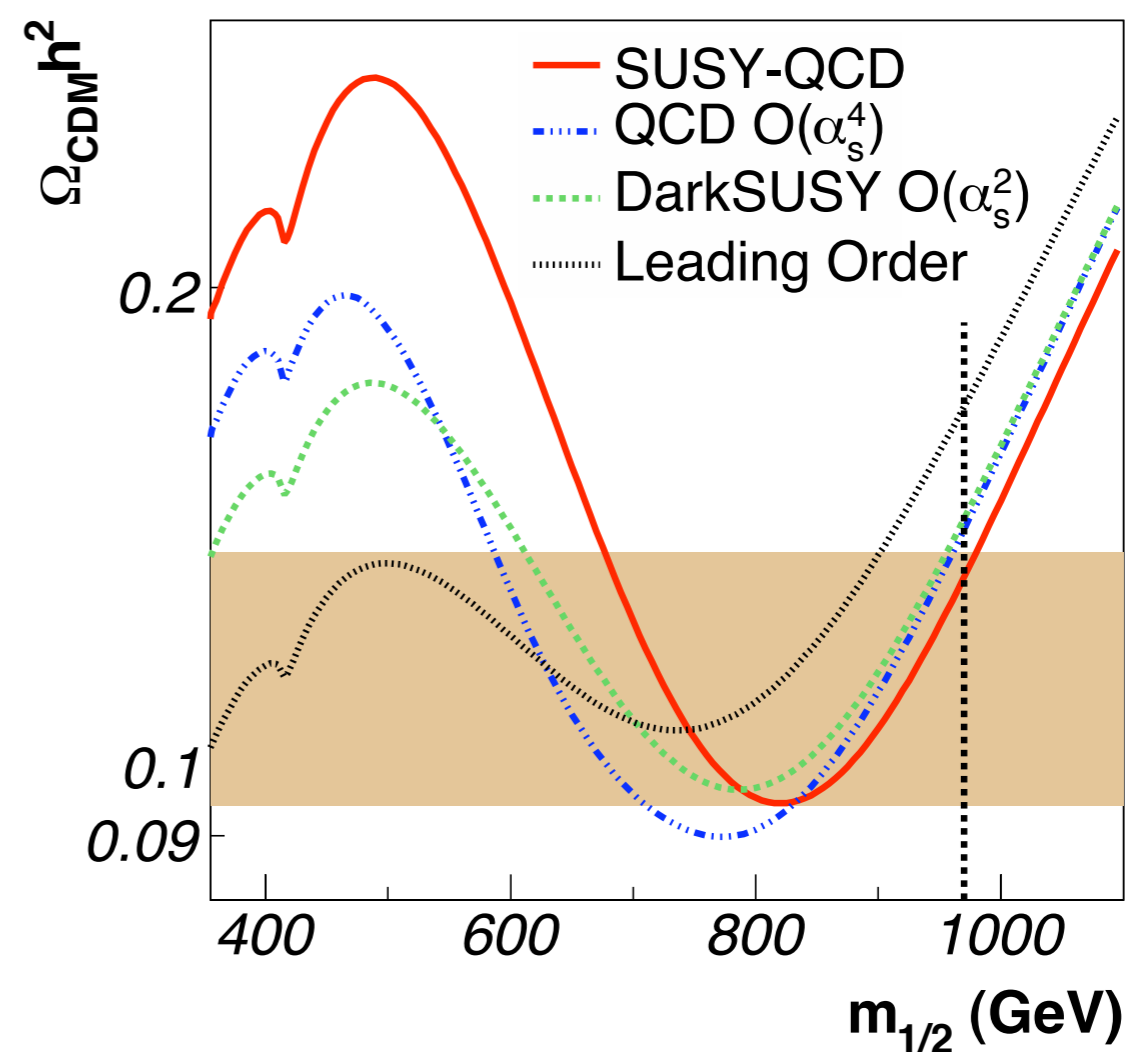
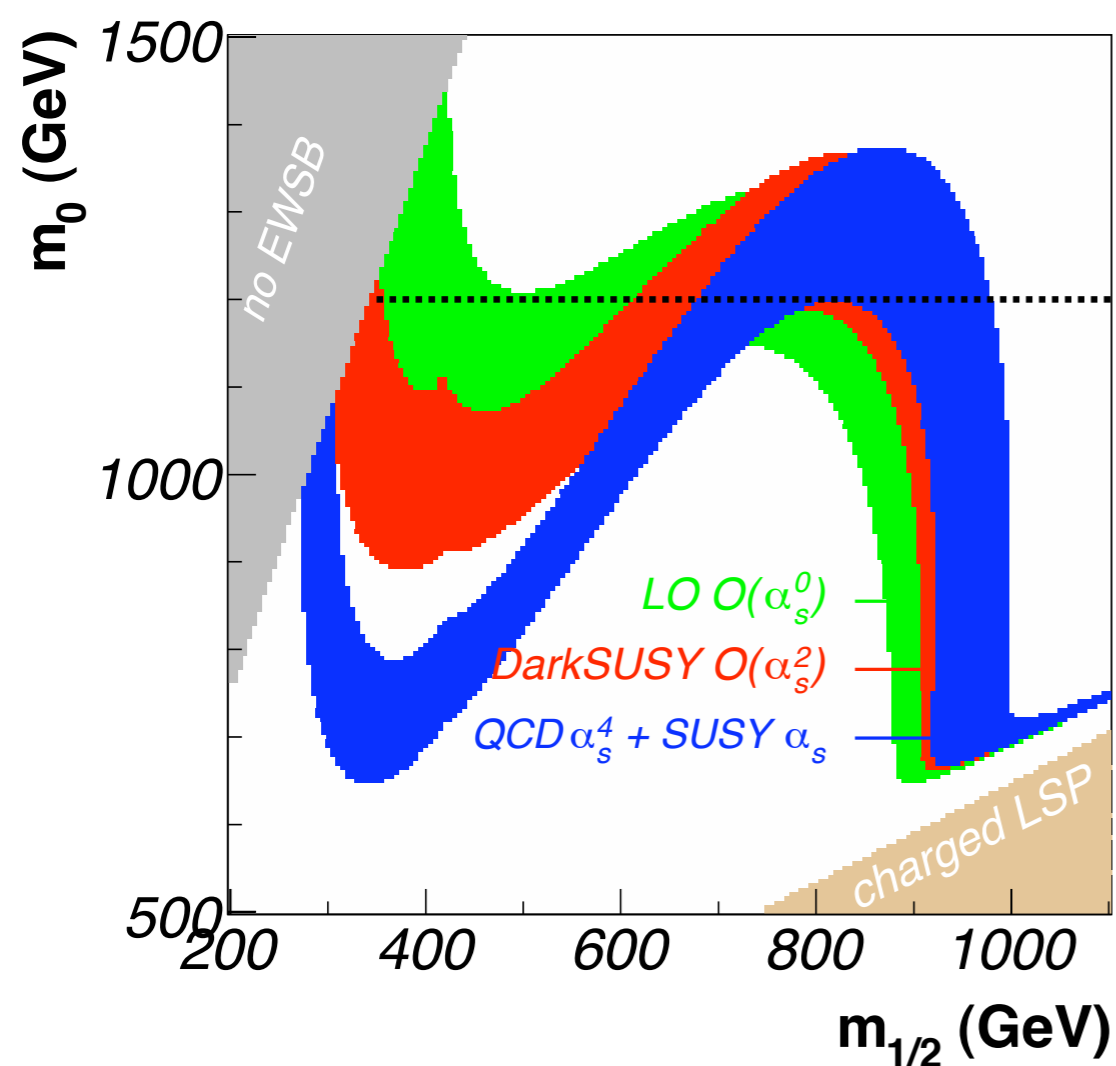
Physical mass spectrum:

FeynHiggs 2.5.1

[Heinemeyer et al. 2000]

Neutralino relic density:

DarkSUSY 4.1 (corrections included)

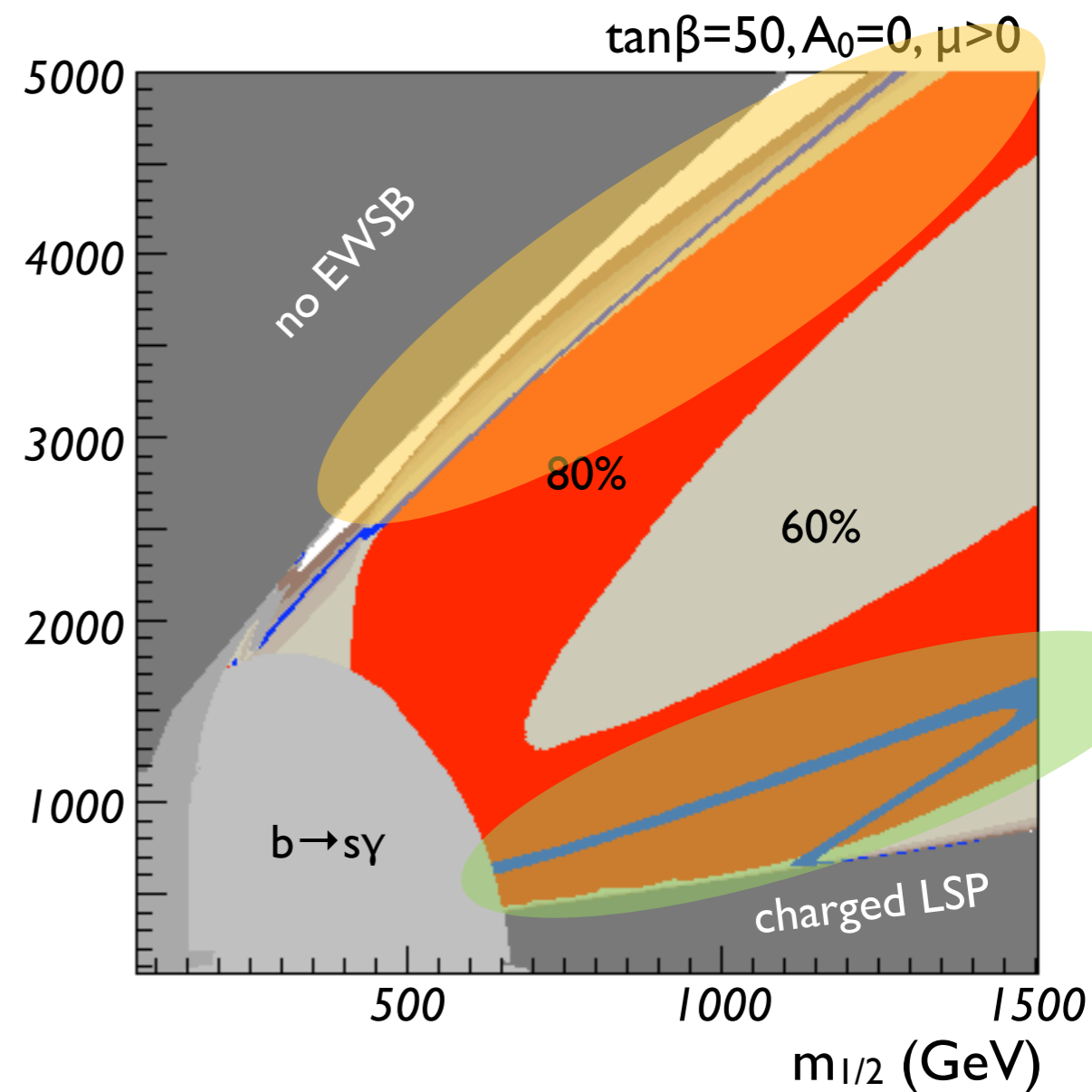
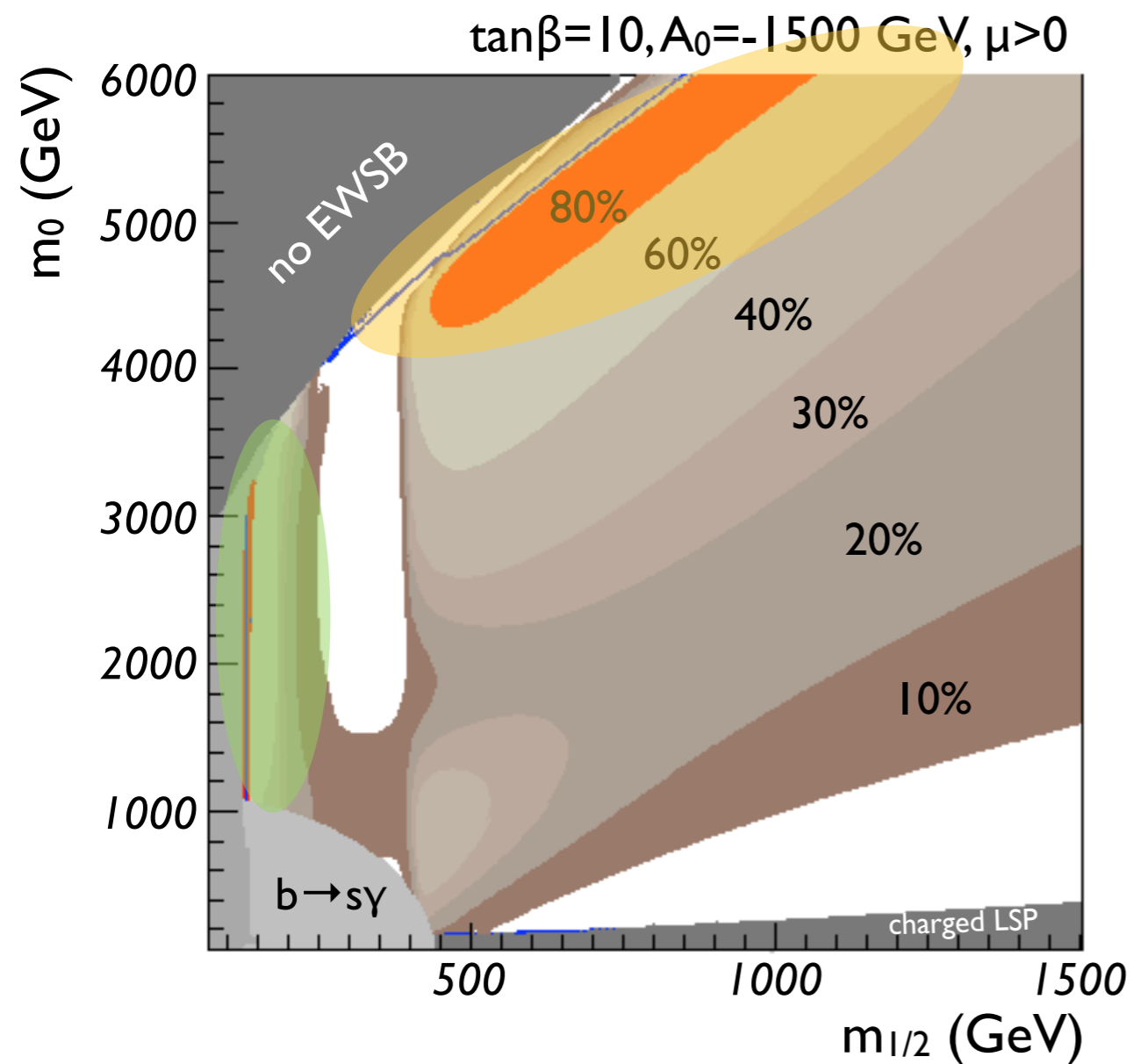


WMAP favoured contour shifted to smaller masses to compensate smaller cross section

Higgs decay width receives same QCD and SUSY-QCD corrections as cross section

# Neutralino Annihilation into Quarks

Determine contribution of quark final states in mSUGRA  $m_0$ - $m_{1/2}$  planes



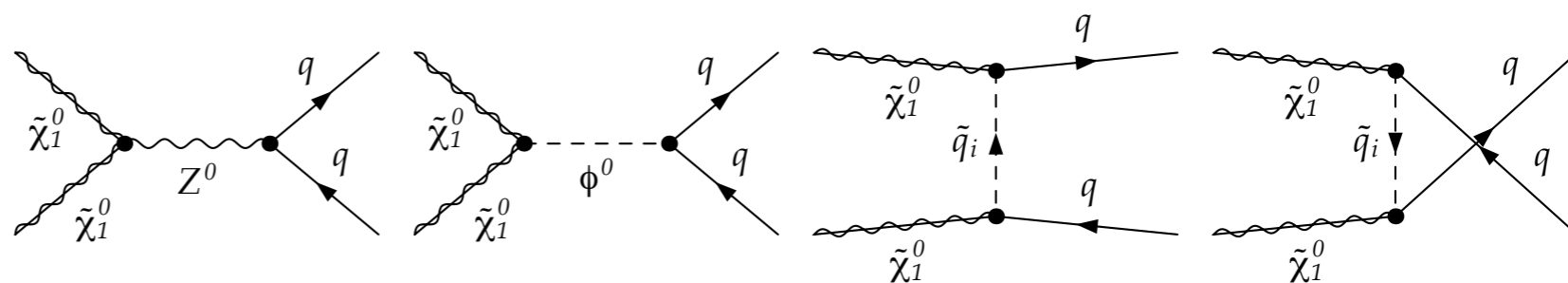
Interesting regions:

Focus point region ( $t\bar{t}$  dominated)

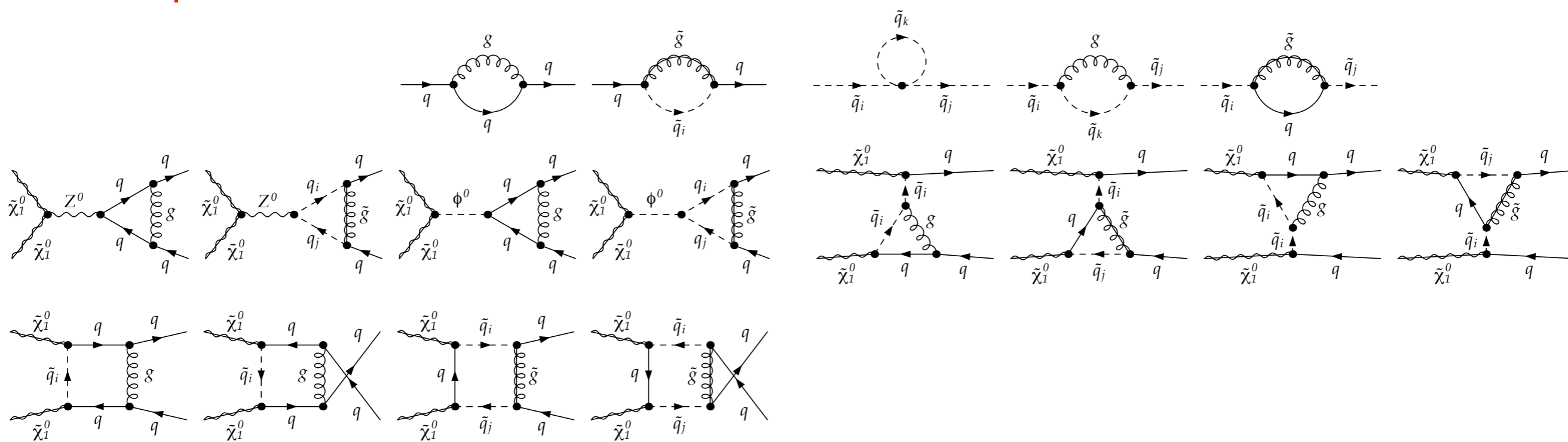
Higgs resonances ( $b\bar{b}$  dominated)

# QCD and SUSY-QCD Corrections

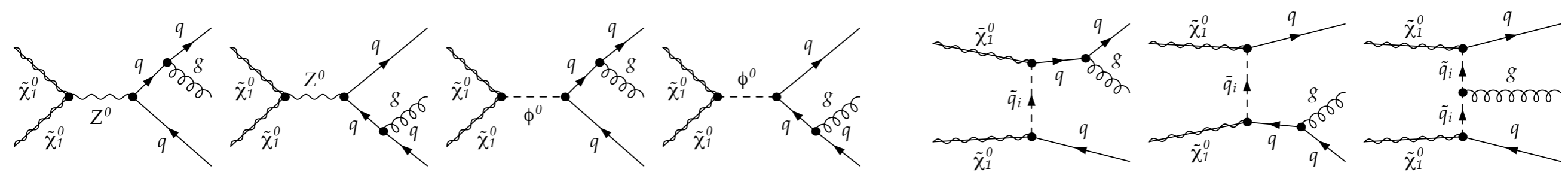
## Tree-level Feynman diagrams



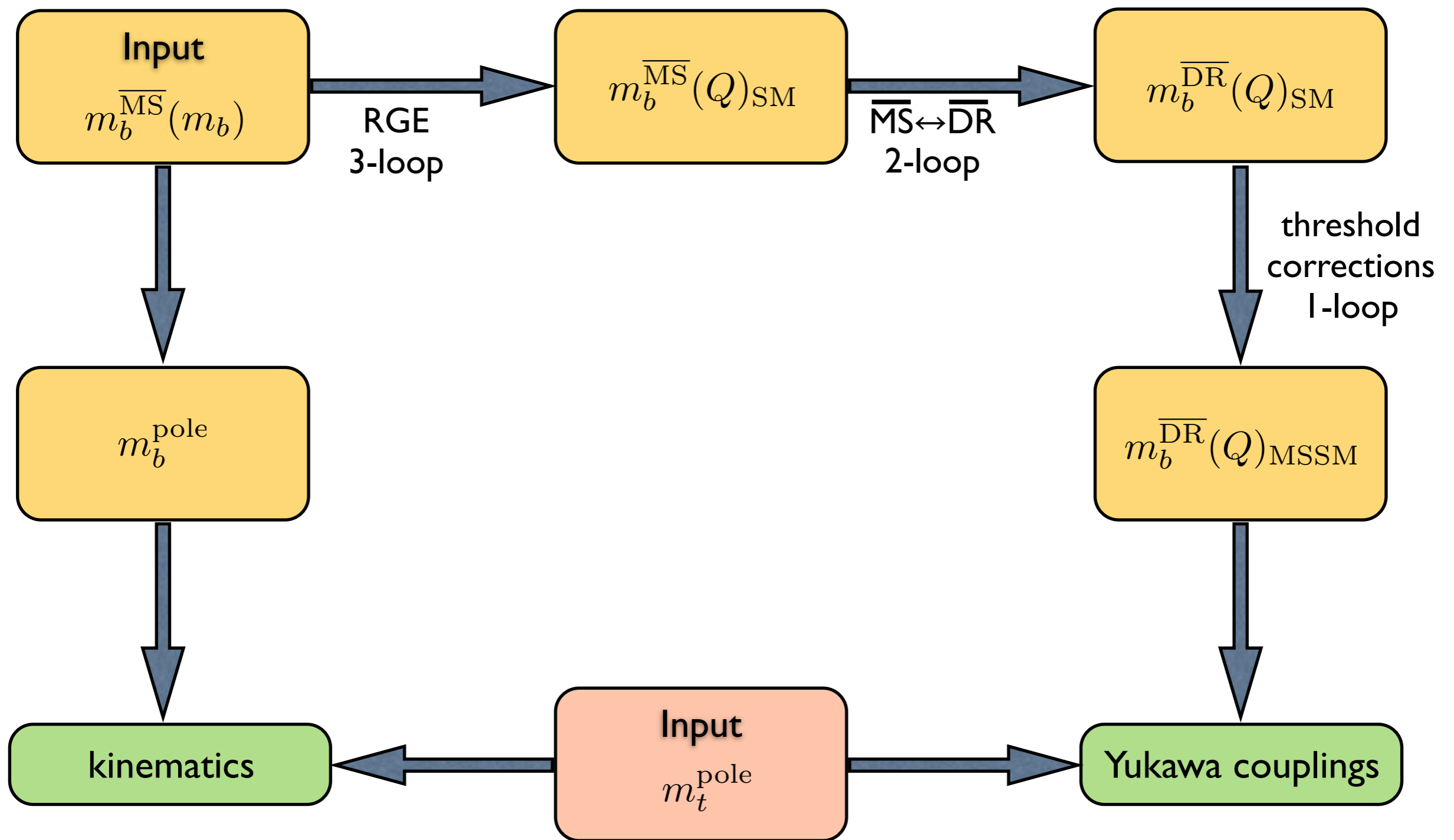
## Virtual loop corrections: On-shell renormalization



## Real emission corrections: Dipole subtraction method

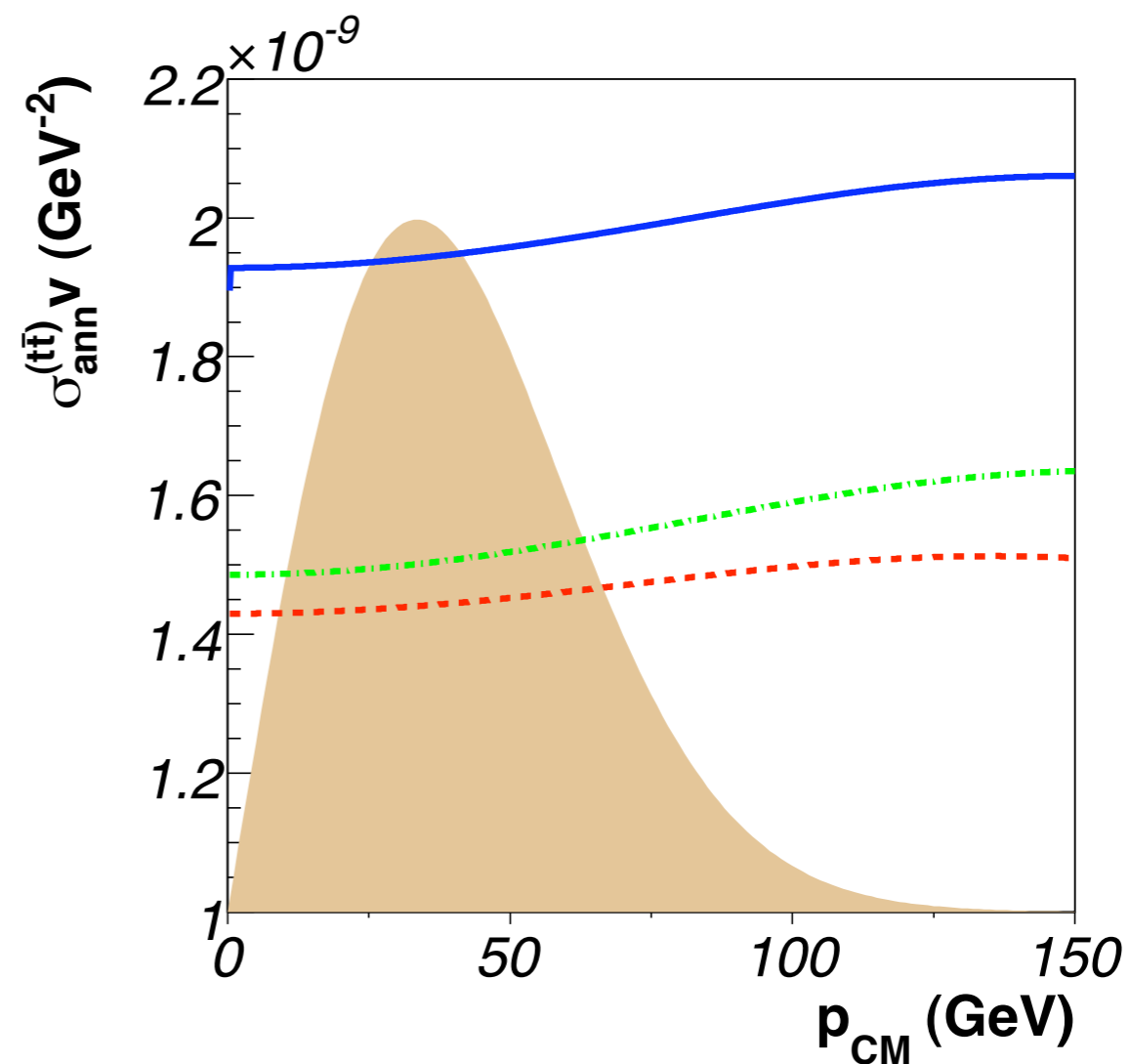
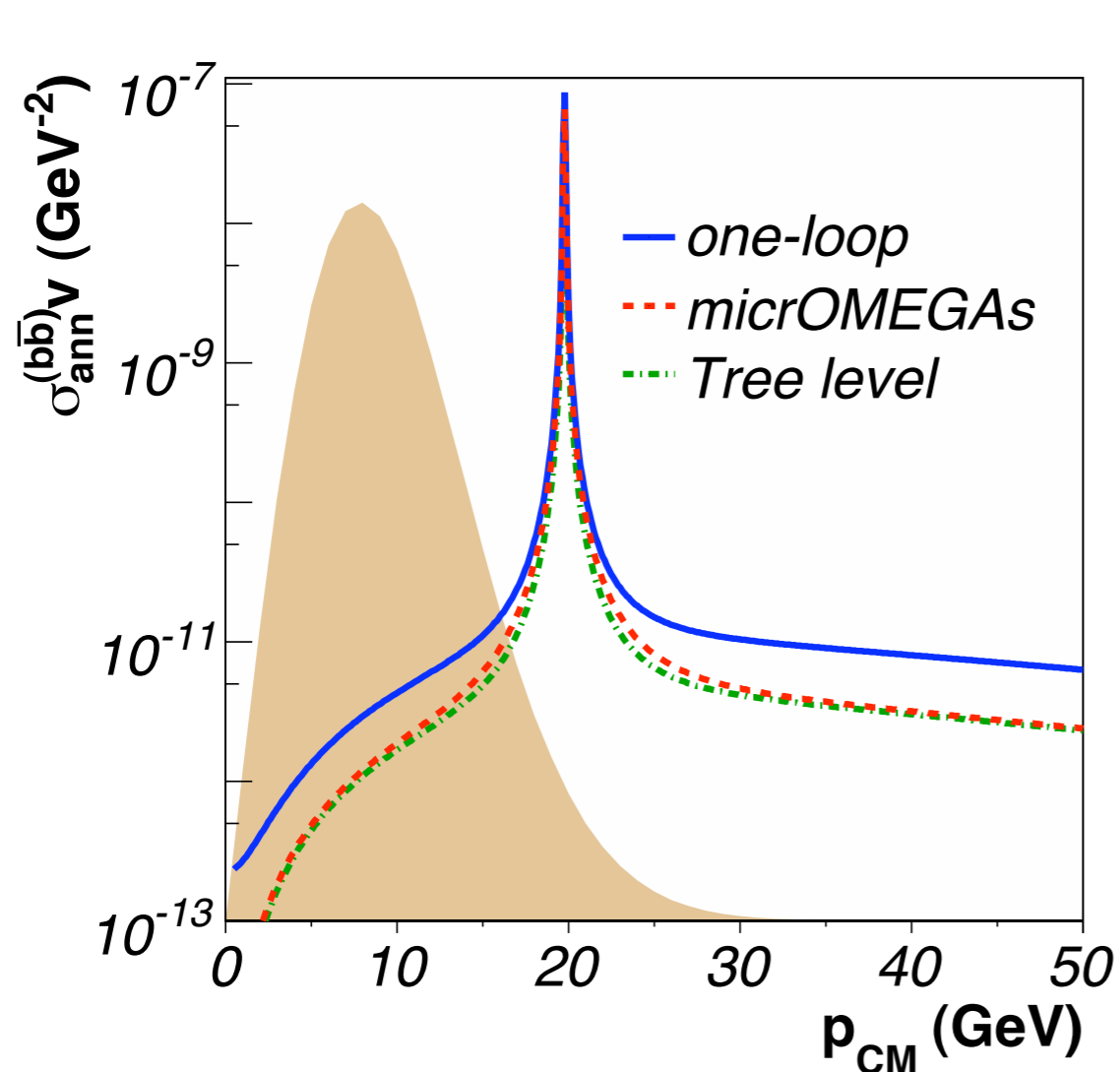


# Treatment of Quark Masses



# Impact on Annihilation Cross Section

Energy dependence for bottom and top quark dominated scenarios



Position of resonance does not coincide with peak of velocity distribution for thermal average  
Differences in “tree-level” predictions are due to different treatments of bottom and top mass  
“Tree-level” for bottom quarks includes effective Yukawa coupling

Cross section enhanced by up to 50% by one-loop QCD and SUSY-QCD corrections



# Impact on Relic Density

Resonance of light Higgs boson at  $\tan\beta=10$  dominated by  $b\bar{b}$  final states

RGE running and spectrum:

SPheno 2.2.3

[Porod 2004]

Neutralino relic density:

micrOMEGAs 2.1 (corrections included)

[Bélanger et al. 2001]

Higgs resonance yields important cross section

- decrease of the relic density
- two allowed intervals around Higgs pole

Disagreement between “tree-level” cross sections

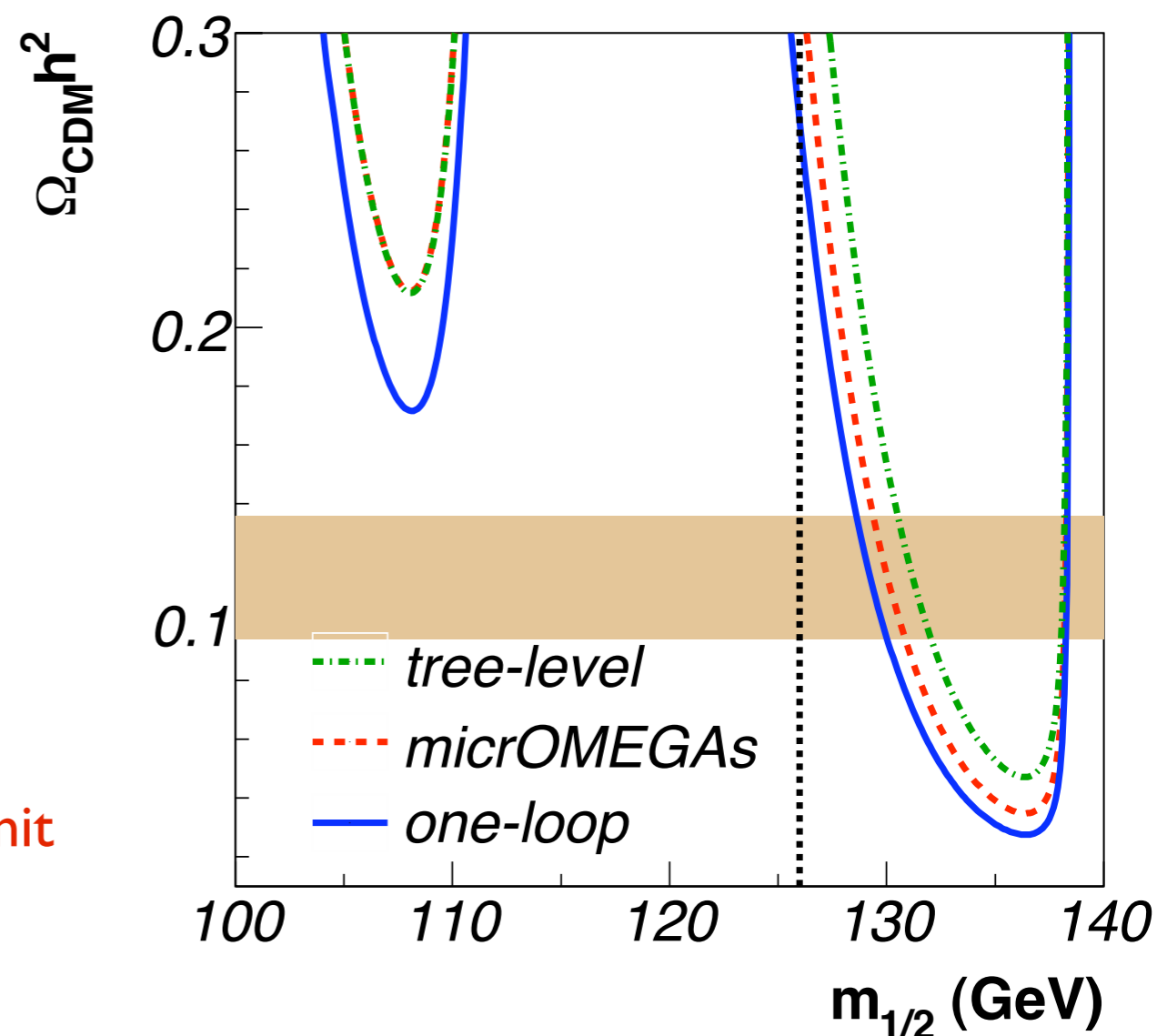
- different predictions of relic density

No Yukawa couplings involved at Z-resonance

- agreement of “tree-level” predictions

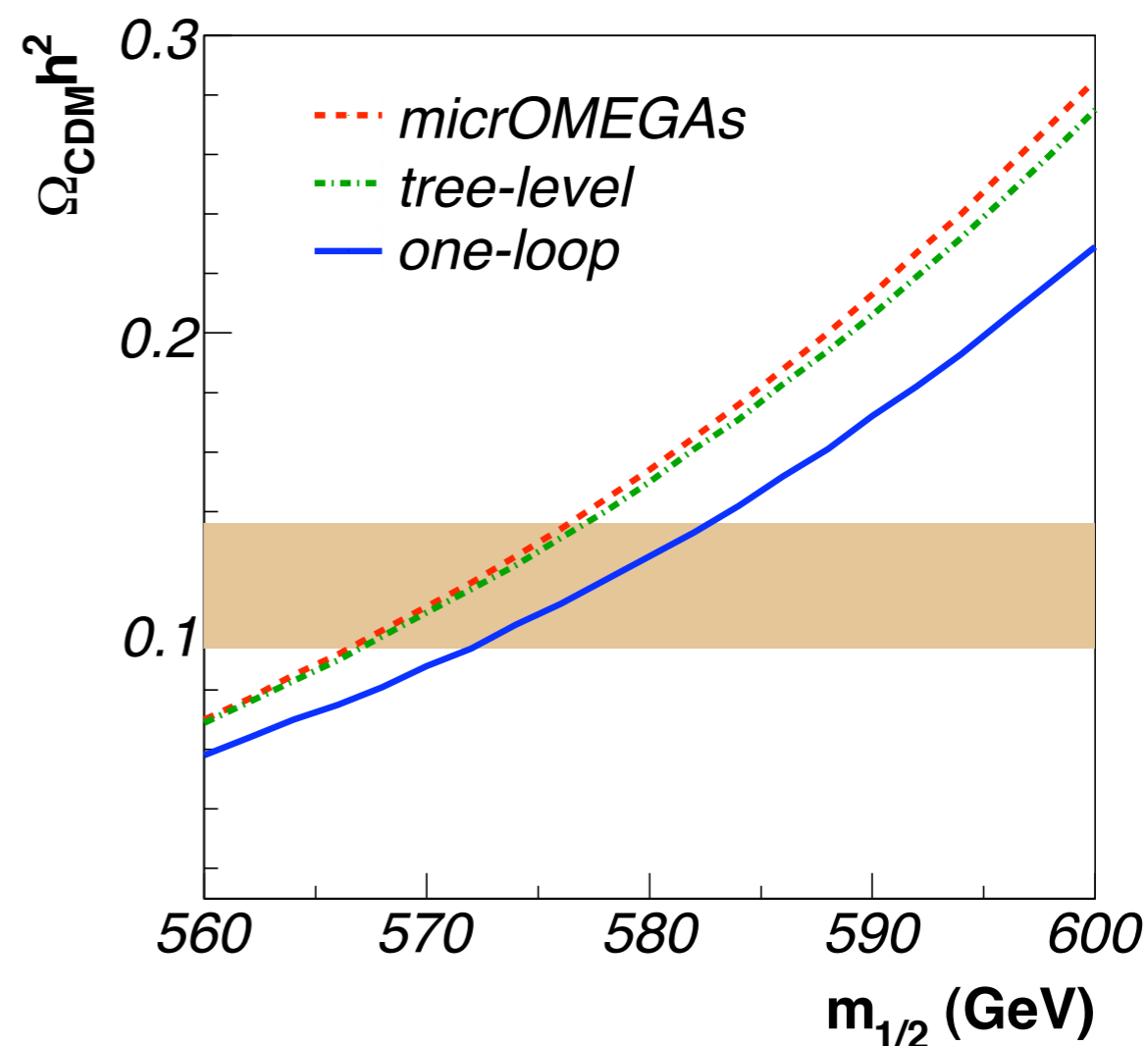
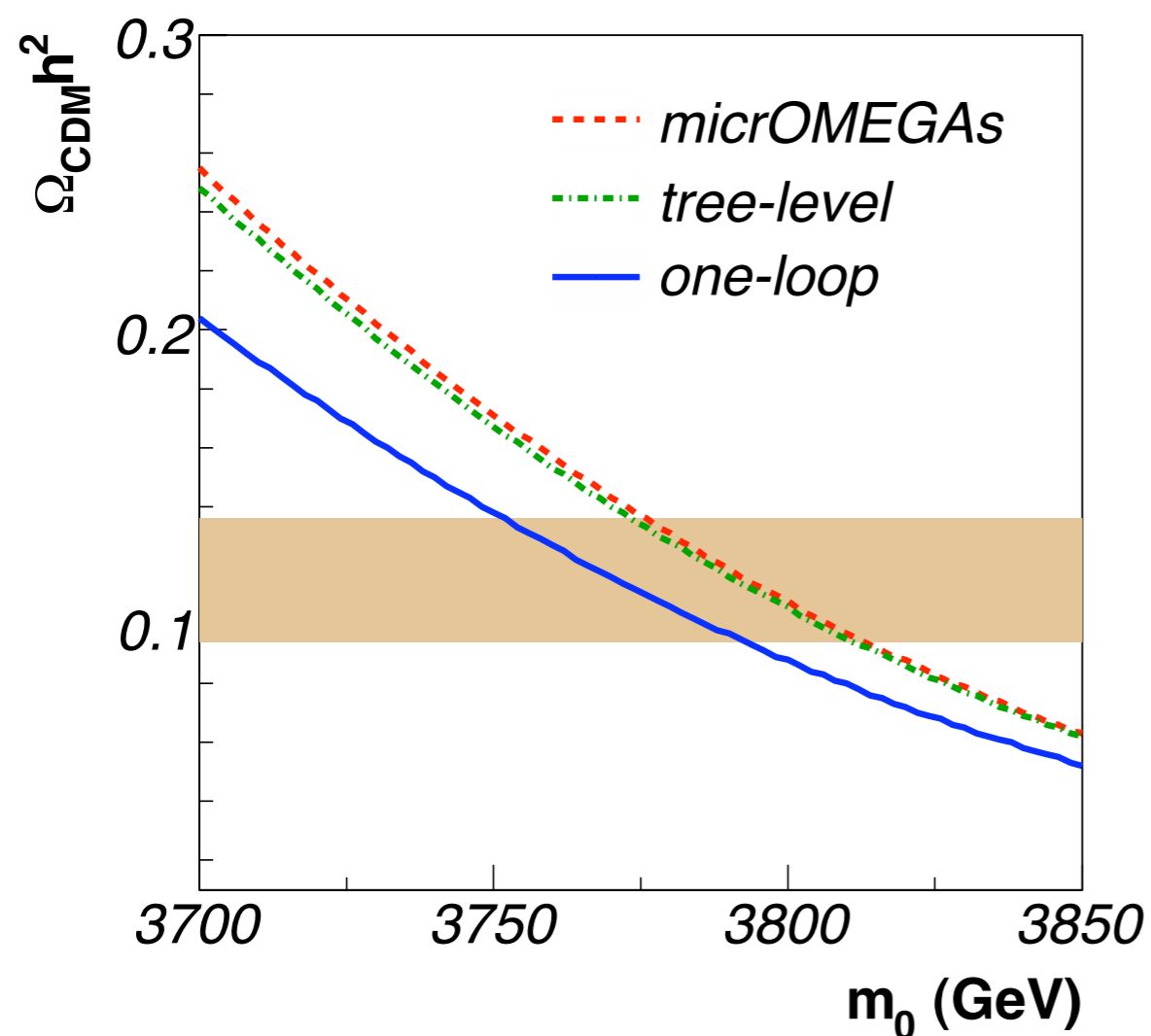
Favoured interval shifted towards chargino mass limit

$$m_{1/2} \geq 126 \text{ GeV}$$



# Impact on Relic Density

Focus point region at  $\tan\beta=10$  dominated by  $t\bar{t}$  final states



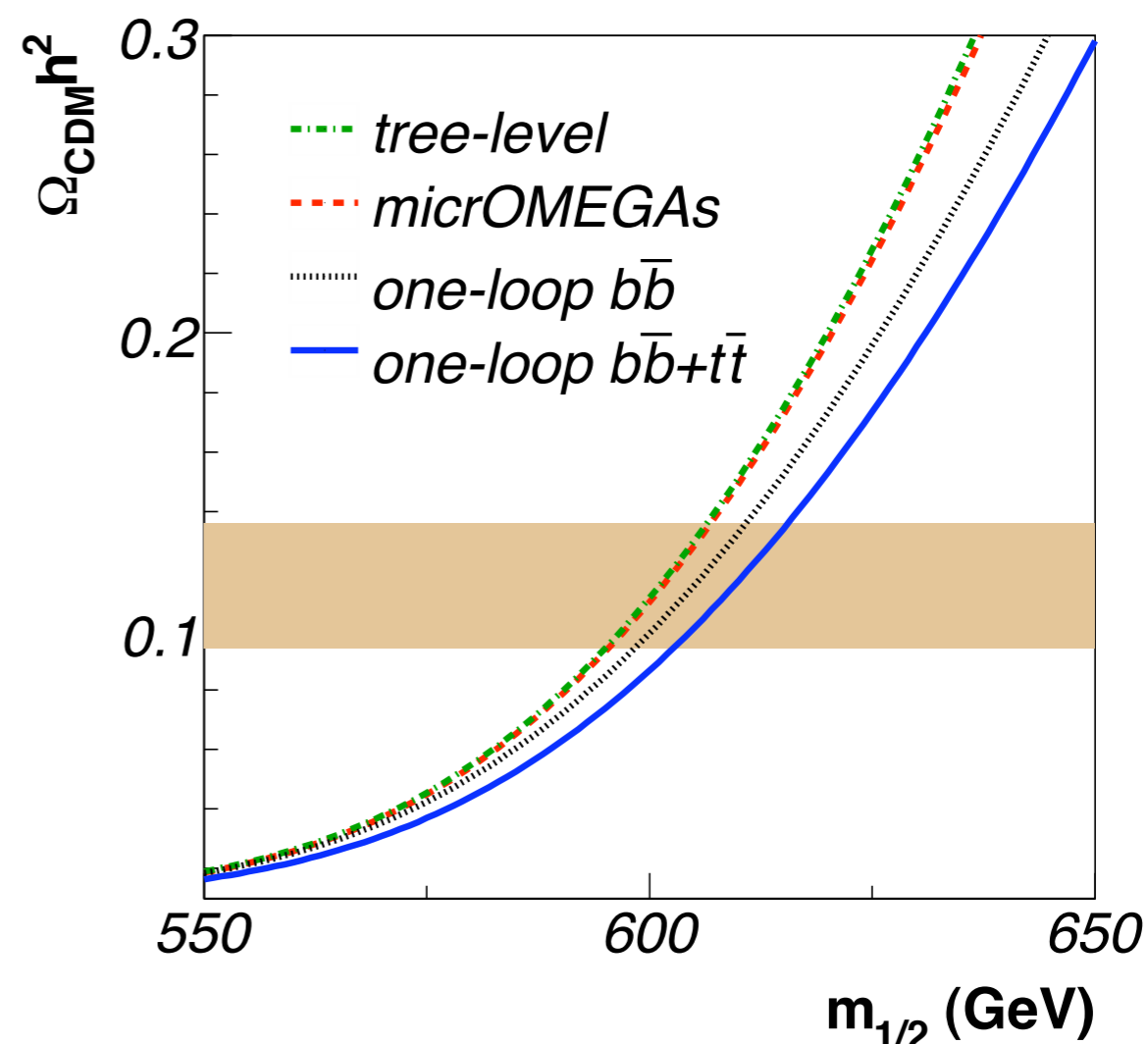
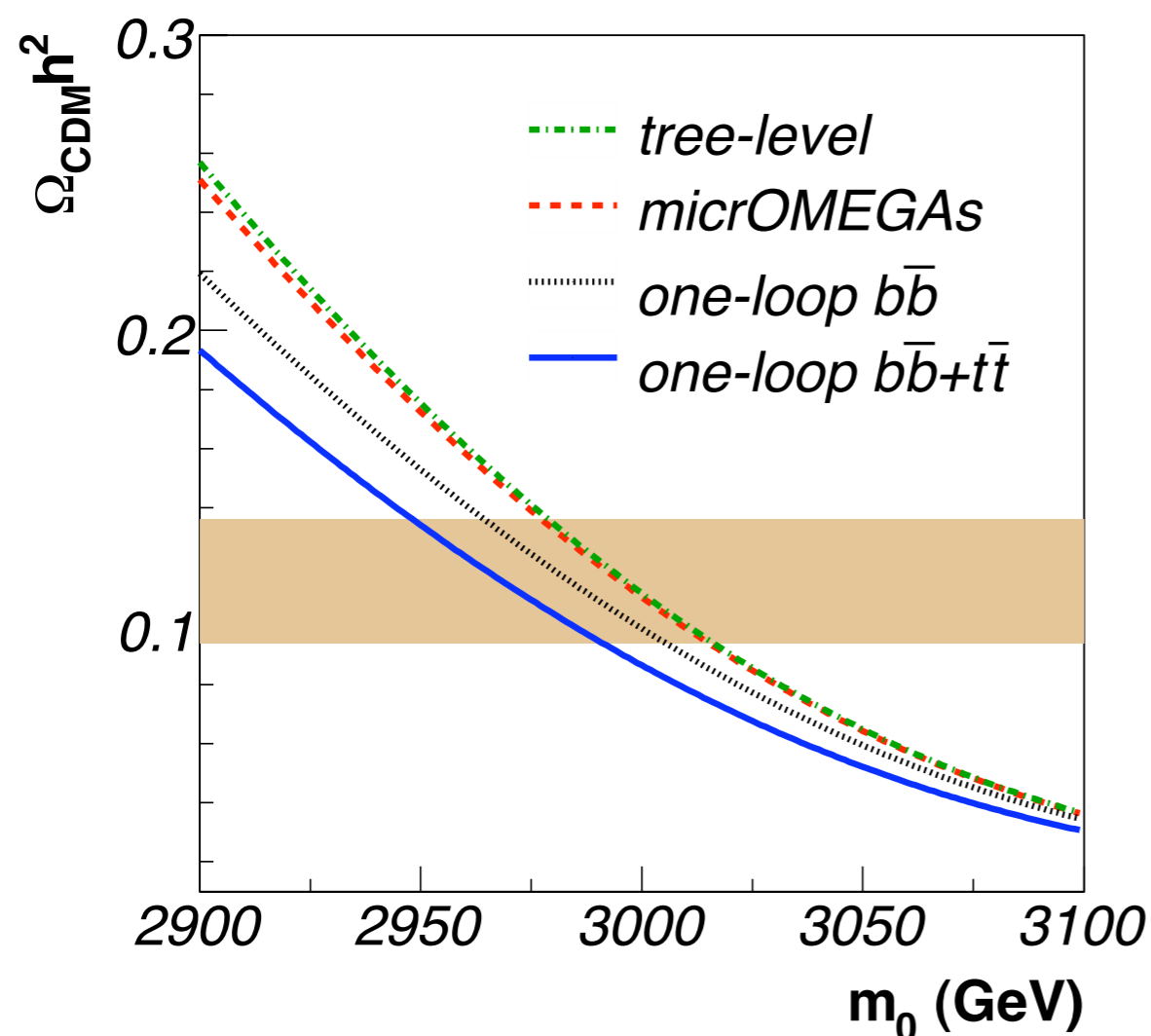
Differences in “tree-level” predictions are due to different mass treatments

Important contribution from QCD and SUSY-QCD corrections

→ favoured region shifted to smaller scalar masses  $m_0$  and higher gaugino masses  $m_{1/2}$

# Impact on Relic Density

Focus point region at  $\tan\beta=50$  with sizeable contributions from  $b\bar{b}$  (15%) and  $t\bar{t}$  (64%)



Important contribution from QCD and SUSY-QCD corrections

- corrections to bottom quark final states already contribute to 50% of total correction
- favoured region shifted to smaller scalar masses  $m_0$  and higher gaugino masses  $m_{1/2}$
- both corrections deform the contour to the same direction

---

## Conclusion and Perspectives

---

# Conclusion and Outlook

---

## Supersymmetry is attractive extension of Standard Model of particle physics

- “hierarchy problem”, gauge coupling unification, dark matter candidates

## Constraining the supersymmetric parameter space is essential

- Benchmark points for mSUGRA and GMSB scenarios
- Phenomenology including non-minimal flavour violation
- Predictions for production cross sections at the LHC

- Perform more general analysis of non-minimal flavour violation
- Include AMSB models
- Evaluate gravitino production numerically
- Compute next-to-leading order corrections

## Dark matter annihilation is an interesting tool to constrain parameter space

- Neutralino annihilation into “heavy” quark-antiquark pairs
- Calculation of full one-loop QCD and SUSY-QCD corrections
- Significant impact on annihilation cross section and neutralino relic density in mSUGRA

- Consider also “light” quarks in a more general MSSM scenario
- Study impact of corrections to (in)direct dark matter detection
- Include coannihilations to provide complete code

