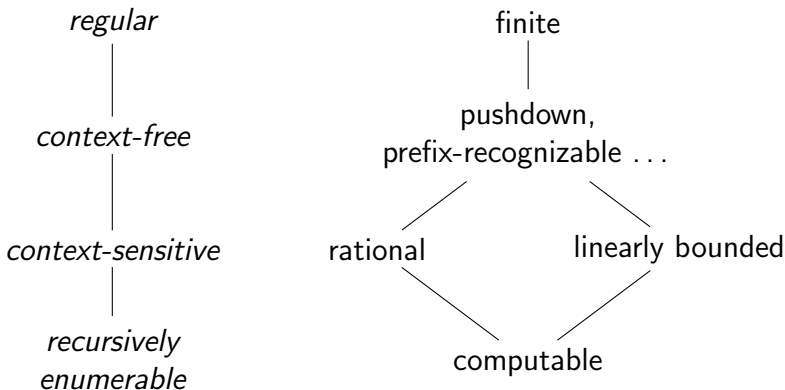


Finitely Presented Infinite Graphs

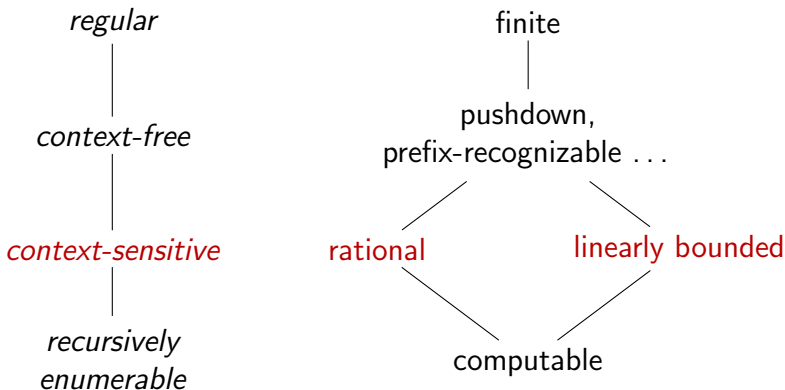
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A hierarchy of infinite automata



A hierarchy of infinite automata



In this talk: infinite automata for **context-sensitive** languages

Graphs and languages

Definition

Graph: set of triples $(u, a, v) \in \Gamma^* \times \Sigma \times \Gamma^*$ (**edges**)
(written $u \xrightarrow{a} v$)

Remarks:

- **Vertices** are identified by **words**
- No isolated vertices
- **Isomorphic graphs considered equal**

Definition (Language of a graph)

$L(G, I, F)$ = set of all path labels of G between I and F (with I, F rational sets of vertices)

Rational graphs

Definition

A Σ -graph G defined by a family of transducers $(T_a)_{a \in \Sigma}$ such that $u \xrightarrow{a} v \iff (u, v) \in T_a$ is called **rational**

Note: **synchronized** transducers define **automatic graphs**
letter-to-letter transducers define **synchronous graphs**

Rational graphs

Definition

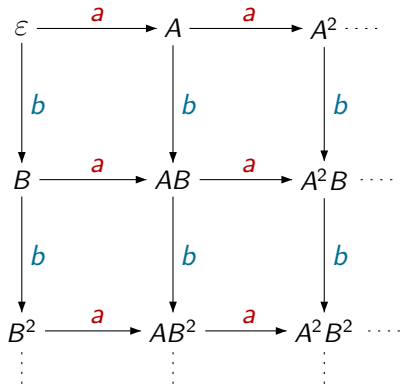
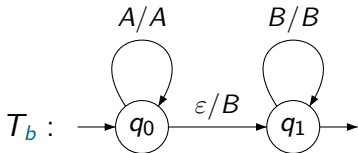
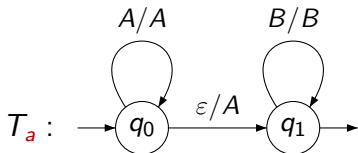
A Σ -graph G defined by a family of transducers $(T_a)_{a \in \Sigma}$ such that $u \xrightarrow{a} v \iff (u, v) \in T_a$ is called **rational**

Note: **synchronized** transducers define **automatic graphs**
letter-to-letter transducers define **synchronous graphs**

Decidability

- Rational graphs have an undecidable FO theory
- Automatic graphs have a decidable FO theory
[Blumensath, Grädel]

A rational graph



Languages of rational graphs

Theorem (Morvan, Stirling, Rispal)

Rational and automatic graphs both accept precisely the context-sensitive languages

- Existing proof uses the Penttonen normal form for context-sensitive grammars
 - Technically non-trivial
 - No notion of determinism
- Our contribution: **syntactical** proof

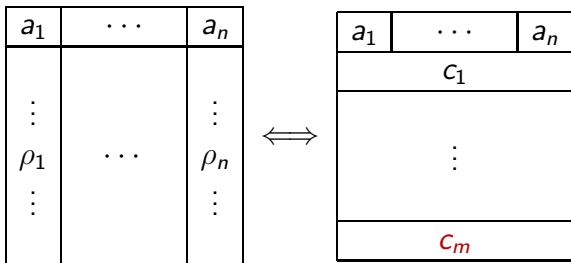
Proof idea

For all rational graph G and sets I, F , there exists a linearly bounded machine (LBM) M accepting $L(G, I, F)$

- 1 Let $u_0 \xrightarrow{a_1} u_1 \dots u_{n-1} \xrightarrow{a_n} u_n$ with $u_0 \in I, u_n \in F$
- 2 For all i , let ρ_i be the corresp. run of T_{a_i} on (u_{i-1}, u_i)
- 3 Build M such that there is a bijection btw. such sequences $\rho_1 \dots \rho_n$ and accepting runs over $a_1 \dots a_n$

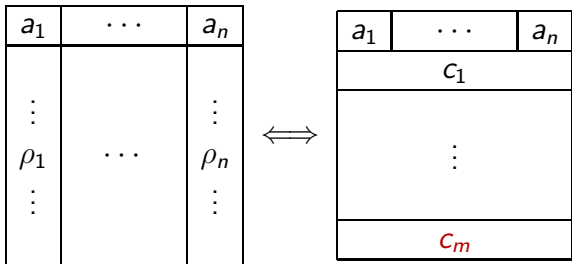
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Proof idea

For all rational graph G and sets I, F , there exists a linearly bounded machine (LBM) M accepting $L(G, I, F)$



Converse: similar idea

(column of a LBM space-time diagram \longrightarrow vertex in a rational graph)

Structural restrictions

Observation

Every CS language can be accepted from a rational set of sources by a synchronous graph of degree 1!

↔ Necessity to consider structural restrictions:

- Graphs with a unique source
- Finite or bounded degree

Structural restrictions

Languages according to transducer class, number of sources and degree

	Rational	Automatic	Synchronous
rat. set of sources	= CS	= CS	= CS

Structural restrictions

Languages according to transducer class, number of sources and degree

	Rational	Automatic	Synchronous
rat. set of sources	= CS	= CS	= CS
1 source, any d	= CS	= CS	= Rat

Structural restrictions

Languages according to transducer class, number of sources and degree

	Rational	Automatic	Synchronous
rat. set of sources	= CS	= CS	= CS
1 source, any d	= CS	= CS	= Rat
1 source, $d < \infty$	= CS	= LRCS	= Rat

LRCS: languages accepted in linear $\#$ of head reversals

Structural restrictions

Languages according to transducer class, number of sources and degree

	Rational	Automatic	Synchronous
rat. set of sources	= CS	= CS	= CS
1 source, any d	= CS	= CS	= Rat
1 source, $d < \infty$	= CS	= LRCS	= Rat
1 source, $d < K$	\subseteq UCS	\subseteq DCS	= Rat

LRCS: languages accepted in linear $\#$ of head reversals

UCS: unambiguous context-sensitive languages

DCS: deterministic context-sensitive languages

Rational graphs and determinism

Families of rational graphs accepting UCS and DCS:

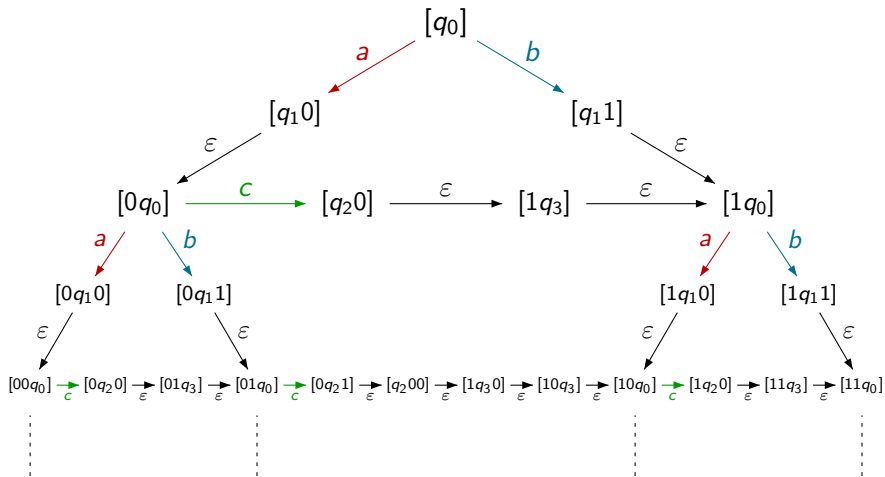
- Unambiguous rational graphs with unambiguous transducers accept UCS
- Synchronous graphs with globally deterministic transducers accept DCS

Note: non-structural criteria!

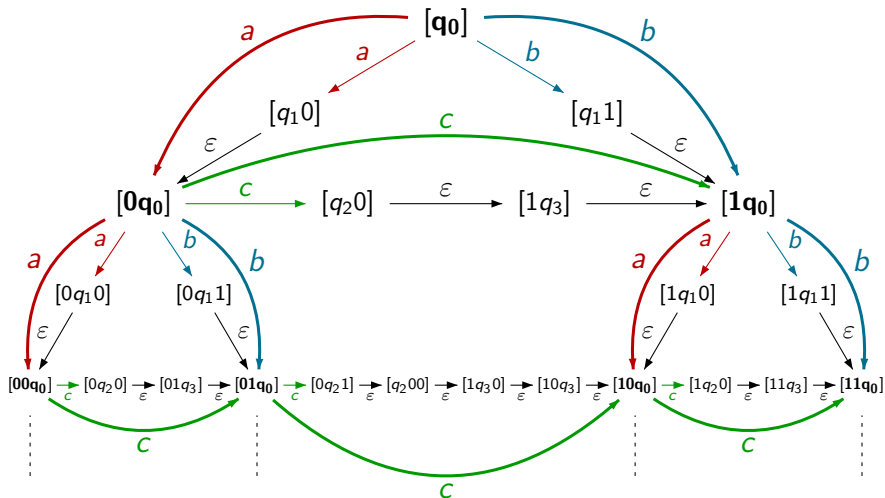
LBM transition graphs

- Idea: associate a **real-time** graph to the computations of a linearly bounded machine
- New LBM model with **labeled** rules:
 - Finite control, input alphabet, tape alphabet
 - Auxiliary list-like memory
 - $(\Sigma \cup \{\varepsilon\})$ -labeled rules for cell insertion, deletion or rewriting, + movement
 - Restriction: **no ε -labeled insertion rule**
- Transition graph of M : $u \xrightarrow{a} v$ iff there exists an $(a\varepsilon^*)$ -run between configs. u and v in M

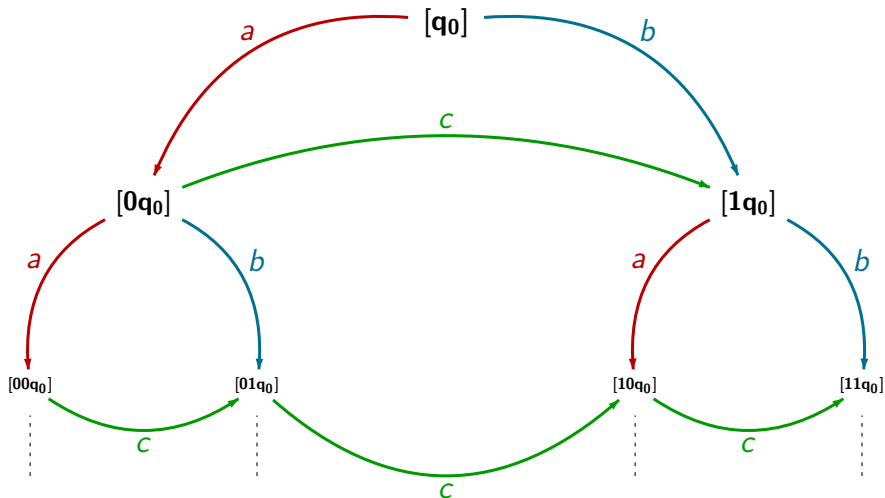
A linearly bounded graph



A linearly bounded graph



A linearly bounded graph



Properties

Languages

- The languages of linearly bounded graphs are the context-sensitive languages
- Every context-sensitive language is accepted by a **deterministic** linearly bounded **tree**
- The transition graph of a **terminating machine** accepts a **deterministic** CS language iff it is deterministic

Properties

Closure properties

Linearly bounded graphs are closed under

- restriction to reachable vertices
- restriction to context-sensitive sets of vertices
- synchronized product

Decidability

At least one linearly bounded graph has an undecidable first-order theory

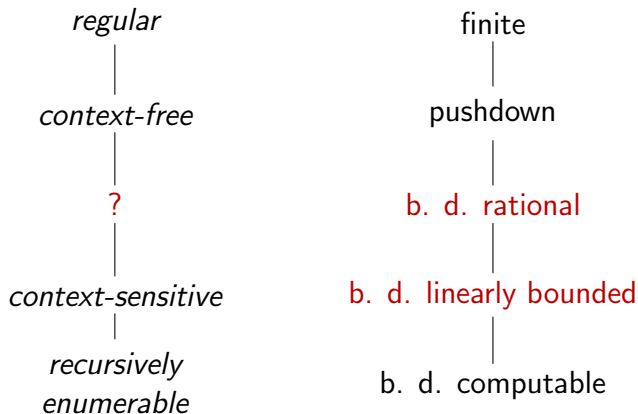
Comparison

- Note: linearly bounded graph have by definition a finite out-degree

Theorem

- ① *Finite degree rational and linearly bounded graphs are **incomparable** (differences in degree growth rate)*
- ② *Every finite degree **automatic** graph is isomorphic to a linearly bounded graph*
- ③ *Every **bounded degree** rational graph is isomorphic to a linearly bounded graph*

A hierarchy of single-source, bounded-degree infinite automata



Perspectives

- More thorough study of bounded-degree and deterministic rational and automatic graphs
- External characterization of linearly bounded graphs (generators + transformations)
- Structural comparison to other families
- Complete family of single-source bounded-degree CS automata with decidable first-order theory?