



# Brisure de symétries en théorie des supercordes : applications en cosmologie et en physique des particules

Tristan CATELIN-JULLIEN

30 octobre 2008

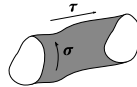
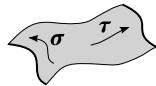
# Outline

- 1 A brief introduction to string theory
  - From particles to strings
  - Free fermionic constructions
  - Supergravity theories
- 2 Explicit breakings vs. spontaneous breakings
- 3 Spinor-vector duality in a class of heterotic  $\mathbb{Z}_2 \times \mathbb{Z}_2$  models
  - Introduction
  - General construction
  - Partition function and spectrum
  - Duality
- 4 Cosmological evolutions from superstring theories
  - Context
  - Construction of the model
  - Range of validity of the ansatz; additional terms in  $V_{1\text{-loop}}$
  - The case of the other moduli

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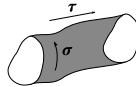
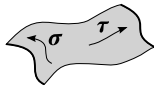
- Elementary excitations modeled by open or closed strings



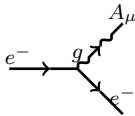
- Interactions are computed by summing over world-sheets



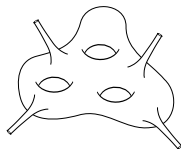
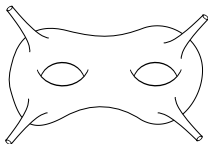
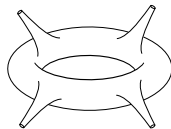
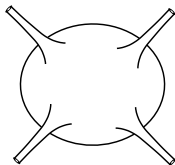
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Loop expansion  $\Leftrightarrow$  Genus expansion



# Conformal anomaly cancellation

Absence of conformal anomaly  $\Rightarrow$

- $c = 26$  for bosonic theories
- $c = 15$  for supersymmetric theories

Compactified extra dimensions  
or  
additional internal degrees of freedom

Many analogies between the two scenarii.

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# General setup

Conformal anomaly set to zero by the addition of internal free fermions.

- 18 fermions  $(\chi^I, y^I, \omega^I)_{I=1\dots 6}$  for SUSY theories.

Bosonisation :

$$i\partial X^I \equiv y^I \omega^I.$$

- 44 fermions for bosonic theories (or bosonic side of heterotic).

$Z_{1\text{-loop}}$  is built by summation  
over the spin-structures  $(a_i, b_i)$ .

Antoniadis-Bachas-Kounnas '86

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# Rules of construction (heterotic theories)

- Parity group basis  $\Xi = \text{Vect}(b_0 \equiv F, b_1 \equiv S, b_2, \dots, b_n)$
- $\alpha \in \Xi$  : sector  $R^\alpha NS^{F+\alpha}$
- GGSO projections associated to each basis vector ; given by the coefficients

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# Correspondance with orbifolds

$\mathbb{Z}_2$ ,  $\mathbb{Z}_2 \times \mathbb{Z}_2$ , freely- or non-freely-acting orbifolds correspond to vectors  $b \in \Xi$ .

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- Equivalence at the *fermionic point*; however, twisted sectors are insensitive to the internal geometry...

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# $\mathcal{N} = 1$ supergravity

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- a Kähler function  $G = K + \log |w|^2 \Rightarrow$  scalar kinetic terms, superpotential
- a gauge kinetic function  $f_{ab} \Rightarrow$  gauge couplings

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# No-scale supergravities

$\mathcal{N} = 1$  scalar potential :  $F$ -term +  $D$ -term

- $F$ -term :  $e^G (G_I G^I - 3)$
- $D$ -term :  $(f^{-1})_{ab} (G_I T_{I\bar{J}}^a Z_{\bar{J}}) (Z_I T_{I\bar{J}}^b G_{\bar{J}})$

$D$ -term  $> 0$ ; vanishes for flat directions.

Cosmological constant :

$$\Lambda = V_F + V_D \Big|_{min.}$$

No-scale models :  $G_I G^I = 3 \Rightarrow V_F \equiv 0$  identically.

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$\frac{SU(1,1)}{U(1)}$  model :

$$K = -3 \log(T + \bar{T}); \quad \mathcal{L}_{cin.} = -3 \frac{\partial T \partial \bar{T}}{(T + \bar{T})^2}$$

$T$  flat direction at the SUGRA level.

Dimensional reductions of superstring theories give no-scale models.

Witten '85 ; Ferrara-Kounnas-Porrati '86

Ex :

$$\frac{SU(1,1)}{U(1)} \times \frac{SO(6,6+n)}{SO(6) \times SO(6+n)}; \quad \frac{SU(1,1)}{U(1)} \times \frac{SU(1,n)}{U(1) \times SU(n)};$$
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# Orbifolds

- Non-free action :

$$T^4/\mathbb{Z}_2 : X^{6,7,8,9} \rightarrow -X^{6,7,8,9}$$

SUSY, gauge **explicitly** broken. Non-invariant states are projected out.

- Free action :

$$S^1/\mathbb{Z}_2 : X \rightarrow X + \pi R$$

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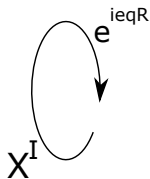
## Stringy Scherk-Schwarz

In field theory, internal momentum modes are shifted for charged fields :

$$m/R \rightarrow m/R + eq$$

$\Rightarrow$  mass shift in the presence of fluxes.

SUSY breaking : Charge = helicity  
 $\Rightarrow$  “Geometrical Fluxes”.



Scherk-Schwarz '79

In string theory :

$\Leftrightarrow$  freely-acting orbifolds / discrete Wilson lines.

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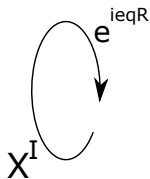
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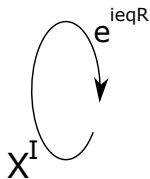
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# $SO(10)$ GUT

The observed spectrum (+ Higgs, right-handed neutrino) fits in the  $SO(10)$  representations :

$$S : \mathbf{16} \quad \bar{S} : \overline{\mathbf{16}} \quad V : \mathbf{10}$$

$$\begin{aligned} \mathbf{16} &= (\mathbf{3}, \mathbf{2})_{1/6} \oplus (\overline{\mathbf{3}}, \mathbf{1})_{-2/3} \oplus (\mathbf{1}, \mathbf{1})_1 \oplus (\overline{\mathbf{3}}, \mathbf{1})_{1/3} \oplus (\mathbf{1}, \mathbf{2})_{-1/2} \oplus (\mathbf{1}, \mathbf{1})_0. \\ \mathbf{10} &= (\mathbf{3}, \mathbf{1})_{-1/3} \oplus (\overline{\mathbf{3}}, \mathbf{1})_{1/3} \oplus (\mathbf{1}, \mathbf{2})_{1/2} \oplus (\mathbf{1}, \mathbf{2})_{-1/2}. \end{aligned}$$

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$\mathbb{Z}_2 \times \mathbb{Z}_2$  generic models :  $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$  orbifold

- $F = \{\psi^\mu, \chi^{1\dots 6}, y^{1\dots 6}, \omega^{1\dots 6} \mid \bar{y}^{1\dots 6}, \bar{\omega}^{1\dots 6}, \bar{\psi}^{1\dots 5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1\dots 8}\}$
  - $S = \{\psi^\mu, \chi^{1\dots 6}\}$
  - $b_1 = \{\chi^{3,4,5,6}, y^{3,4,5,6} \mid \bar{y}^{3,4,5,6}, \bar{\psi}^{1\dots 5}, \bar{\eta}^1\}$ ;  $b_2, b_3$ .
- $\Rightarrow T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$  orbifold.

Remarks :

- $\mathbb{Z}_2 \times \mathbb{Z}_2$  (or Calabi-Yau)  $\Rightarrow E_8 \rightarrow E_6 \times U(1)^2$ .
- Right-handed fermions  $\bar{\psi}^{1,2,3,4,5} \Rightarrow SO(10)$  algebra.

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# $\mathbb{Z}_2 \times \mathbb{Z}_2$ generic models : translation orbifolds

- $e_i = \{y^i, \omega^i \mid \bar{y}^i, \bar{\omega}^i\}, \quad i \in \{1 \dots 6\}$

$$\Rightarrow X^i \rightarrow X^i + \pi R$$

- $z_1 = \{\bar{\phi}^{1\dots 4}\} \quad z_2 = \{\bar{\phi}^{5\dots 8}\}$

$\Rightarrow$  Hidden  $E_8$  breaking.

- NB :  $x = \{\bar{\psi}^{1\dots 5}, \bar{\eta}^{1,2,3}\}$  belongs to the parity set.

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- $e_i = \{y^i, \omega^i \mid \bar{y}^i, \bar{\omega}^i\}, \quad i \in \{1 \dots 6\}$

$$\Rightarrow X^i \rightarrow X^i + \pi R$$

- $z_1 = \{\bar{\phi}^{1\dots 4}\} \quad z_2 = \{\bar{\phi}^{5\dots 8}\}$

$\Rightarrow$  Hidden  $E_8$  breaking.

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$\mathbb{Z}_2 \times \mathbb{Z}_2$  partition function : fermionic blocks

*Spin structure* for each element of the parity basis :

$$(a, b); \quad (a + h, b + g); \quad (\epsilon, \xi); \quad \dots$$

(Sums over spin-structures are implicit)

$$\begin{aligned} Z = & [\text{osc.}] \times (-)^{a+b+ab} \vartheta \begin{bmatrix} a \\ b \end{bmatrix} \vartheta \begin{bmatrix} a+h_1 \\ b+g_1 \end{bmatrix} \vartheta \begin{bmatrix} a+h_2 \\ b+g_2 \end{bmatrix} \vartheta \begin{bmatrix} a-h_1-h_2 \\ b-g_1-g_2 \end{bmatrix} \\ & \times \bar{\vartheta} \begin{bmatrix} \epsilon \\ \xi \end{bmatrix}^5 \bar{\vartheta} \begin{bmatrix} \epsilon+h_1 \\ \xi+g_1 \end{bmatrix} \bar{\vartheta} \begin{bmatrix} \epsilon+h_2 \\ \xi+g_2 \end{bmatrix} \bar{\vartheta} \begin{bmatrix} \epsilon-h_1-h_2 \\ \xi-g_1-g_2 \end{bmatrix} \\ & \times (-)^{H_1 G_2 + H_2 G_1} \bar{\vartheta} \begin{bmatrix} \epsilon+H_1 \\ \xi+G_1 \end{bmatrix}^4 \bar{\vartheta} \begin{bmatrix} \epsilon+H_2 \\ \xi+G_2 \end{bmatrix}^4 \\ & \times \Gamma_{6,6} \begin{bmatrix} h_i | s_i \\ g_i | t_i \end{bmatrix} \times e^{i\Phi} \end{aligned}$$

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# Matter spectrum : twisted planes

$3 \times 16$  twisted sectors, in 3 planes :  $B^1, B^2, B^3$ .

- **Spinorials** of  $SO(10)$  :  $\text{Spin}(\bar{\psi}^{1\dots 5})$ .

$$b_1 + \lambda_3 e_3 + \lambda_4 e_4 + \lambda_5 e_5 + \lambda_6 e_6 \quad \lambda_i \in \{0, 1\}$$

- **Vectorials** of  $SO(10)$  : weight  $-1/2$  excitations

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# Principle

Twisted sectors  $\Rightarrow$  representations

$$S : \mathbf{16} \quad \bar{S} : \overline{\mathbf{16}} \quad V : \mathbf{10}.$$

Duality exchanges :

$$n(S + \bar{S}) \leftrightarrow n(V)$$

$E_6$  :

$$\mathbf{27} = \mathbf{10} \oplus \mathbf{16}$$

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$N = 2_R$  superconformal

Spectral flow  $\Leftrightarrow \alpha \xrightarrow{x\text{-map}} \alpha + x$ .

- SCFT breaking  $\Leftrightarrow$  “ $x$ -map” breaking  $\Leftrightarrow E_6 \rightarrow SO(10) \times U(1)$ .
- $\epsilon, \xi$  spin-structures for  $x : N = 2_R$  broken by insertion of

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# Effect of the different cocycles

Twisted plane  $h = 1$

$(-)^{\epsilon t + \xi s + st} \Rightarrow \epsilon = 1$  massive  $SO(10)$  spinors

$(-)^{ht + gs} \Rightarrow h = 1$  massive whole sector is lifted

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# Constructing the dual of a given model

In the  $B_1$  plane;  $N = 2_R$  is broken iff :

$$\exists e_i, [e_i|x] = -1 \quad (i = 1, 2) \quad \text{ou} \quad \exists z_i, [z_i|x] = -1.$$

Dual model :

Reverse  $[e_i|b_1]$  and/or  $[z_i|b_1]$ .

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# Example : sector-by-sector duality in the first plane

$$[b_1|e_1] = 1$$

$$[x|e_1] = 1$$

$$[x|e_2] = -1$$

et

$[\cdot \cdot]$	$e_1$	$e_2$
$e_3$	-1	1
$e_4$	1	-1
$e_5$	1	1
$e_6$	-1	1

$[\cdot \cdot]$	$e_1$	$e_2$	rep.	$e_1$	$e_2$	rep.
$B_{0000}^1$	1	1	$\emptyset$	1	-1	$\emptyset$
$B_{0001}^1$	-1	1	$V$	-1	-1	$S$
$B_{0010}^1$	1	1	$\emptyset$	1	-1	$\emptyset$
$B_{0100}^1$	1	-1	$\emptyset$	1	1	$\emptyset$
$B_{1000}^1$	-1	1	$V$	-1	-1	$S$
$B_{1100}^1$	-1	-1	$S$	-1	1	$V$
$B_{1010}^1$	-1	1	$V$	-1	-1	$S$
$B_{1001}^1$	1	1	$\emptyset$	1	-1	$\emptyset$
$B_{0101}^1$	-1	-1	$S$	-1	1	$V$
$B_{0110}^1$	1	-1	$\emptyset$	1	1	$\emptyset$
$B_{0011}^1$	-1	1	$V$	-1	-1	$S$
$B_{1110}^1$	-1	-1	$S$	-1	1	$V$
$B_{1101}^1$	1	-1	$\emptyset$	1	1	$\emptyset$
$B_{1011}^1$	1	1	$\emptyset$	1	-1	$\emptyset$
$B_{0111}^1$	-1	-1	$S$	-1	1	$V$
$B_{1111}^1$	1	-1	$\emptyset$	1	1	$\emptyset$

Left side :  $[b_1|e_2] = 1$  ; right side :  $[b_1|e_2] = -1$ .

## Conclusion and interests

- Large class of natural  $SO(10)$  GUT models; large proportion of 3-families models.
- Intuitive proof of the duality, based on the lifting of  $SO(10)$  representations by freely-acting orbifolds.
- Information about the structure of the vacua space (analogous to mirror symmetry).
- Hint for a selection mechanism?

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# Stringy cosmologies

At the supergravity level :

Difficulties to construct realistic stringy cosmologies.

Compactifications with fluxes  $\Rightarrow$  AdS or flat.

Maldacena - Nuñez '01

Corrections to supergravity :

- Flux compactifications :  $H_3$ ,  $F_3$  stabilize  $S$  and  $U^i$ ;  $T^i$  runaway

Graña, Minasian, Petrini, Tomasiello '06

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Thermal effects? SUSY breaking scale dynamics?



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# Thermal heterotic strings

## In field theory :

Temperature  $\Leftrightarrow$  insertion of  $(-)^F$  in the path integral  
Euclidean time compactification, radius  $R_0 = \frac{1}{2\pi T}$

## In superstring theory :

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- Temperature  $\Rightarrow$  boson/fermion mass gap

Temperature breaks supersymmetry.

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Examples :

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# Heterotic partition function

- $(R_0, R_5) \Rightarrow$

Kähler modulus  $t = \text{Im } T \equiv R_0 R_5$

complex structure modulus  $u = \text{Im } U \equiv R_0 / R_5$

- Finite temperature thermodynamics :

$$\frac{Z}{V_4} = F_{\text{Str.}} = -V_{\text{Str.}} = P_{\text{Str.}}$$

(quantities obtained in the **string frame**)

- energy density  $\rho = T \frac{\partial P}{\partial T} - P.$

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# Potential in the Einstein frame

$S = a + ie^{-2\phi}$  “axio-dilaton”.

Rescaling of the metric

$$\bullet V_{\text{Ein.}} = \frac{V_{\text{Str.}}}{(\text{Im } S)^2}, \quad P_{\text{Ein.}} = \frac{P_{\text{Str.}}}{(\text{Im } S)^2}, \quad \dots$$
$$\Rightarrow V_{\text{Ein.}}, P_{\text{Ein.}} = \frac{1}{(stu)^2} g(u)$$

•  $z^3 = (\text{Im } S)(\text{Im } T)(\text{Im } U) = stu$ . Ansatz :

- $u$  is frozen
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•  $K = -\log(S - \bar{S}) - \log(T - \bar{T}) - \log(U - \bar{U}) = -3\log(Z - \bar{Z})$

$\Rightarrow Z$  is a no-scale modulus.

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•  $K = -\log(S - \bar{S}) - \log(T - \bar{T}) - \log(U - \bar{U}) = -3\log(Z - \bar{Z})$

$\Rightarrow Z$  is a no-scale modulus.

# Potential in the Einstein frame

$S = a + ie^{-2\phi}$  “axio-dilaton”.

Rescaling of the metric

$$\bullet V_{\text{Ein.}} = \frac{V_{\text{Str.}}}{(\text{Im } S)^2}, \quad P_{\text{Ein.}} = \frac{P_{\text{Str.}}}{(\text{Im } S)^2}, \quad \dots$$
$$\Rightarrow V_{\text{Ein.}}, P_{\text{Ein.}} = \frac{1}{(stu)^2} g(u)$$

$\bullet z^3 = (\text{Im } S)(\text{Im } T)(\text{Im } U) = stu$ . Ansatz :

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# Effective supergravity vs. FRW equations

In the Einstein frame, the supergravity action reads :

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{2} R - g^{\mu\nu} K_{I\bar{J}} \partial_\mu \Phi^I \partial_\nu \bar{\Phi}^{\bar{J}} - V_{\text{Ein.}} \right)$$

Define :

$$M_{3/2}^2 = e^G = \frac{1}{z^3} = \frac{1}{sR_5^2} := e^{2\alpha\Phi}$$

$\Phi$  : scalar coupled to gravity, in a FRW evolution

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# Einstein and SUGRA equations compatibility

- $\dot{H} + 3H^2 = -\frac{2k}{a^2} + \frac{1}{2}(\rho - P)$   
(Hubble + variation of the spatial components)
- $\ddot{\Phi} + 3H\dot{\Phi} = -\frac{\partial V}{\partial \Phi}$

Ansatz :  $(T, M_{3/2}) \propto \frac{1}{a} \Rightarrow H = -\alpha\dot{\Phi}$ .

Identification of the  $1/a^4$  terms :

$$\rho = \frac{6\alpha^2 - 1}{2\alpha^2 - 1} P$$

$\alpha^2 = 3/2 \Rightarrow \rho = 4P$  : analogous to 5D radiation.

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# Temperature $\leftrightarrow$ SUSY breaking duality

Only temperature :

$$P_{\text{Ein.}} = \frac{1}{(stu)^2} u^4 f(u)$$

Temperature + SUSY breaking :

$$P_{\text{Ein.}} = \frac{1}{(stu)^2} \left( n_T^* u^4 f(u) + n_V^* \frac{f(1/u)}{u} \right).$$

- $n_T^*$  : number of boson-fermion pairs.
- $n_V^*$  : counting of pairs is weighted by  $(-)^Q$  (massive fermion/massive boson).



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Numerical/theoretical treatment : a solution  $u_0$  exists for

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# Additional terms for $V_{1\text{-loop}}$

Generic form of the one-loop potential in spontaneously broken SUGRA :

$$\frac{1}{32\pi^2} \text{Str} \mathcal{M}^2 \Lambda^2 + \frac{1}{64\pi^2} \text{Str} \left( \mathcal{M}^4 \log \frac{\mathcal{M}^2}{\mu^2} \right) + \dots$$

Spontaneously broken  $\mathcal{N} = 4 \Rightarrow$  corrections only  $\propto M_{3/2}^4$ .

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## “Spectator” moduli

- Exponentially small contributions in  $V_{1\text{-loop}}$  (away from Hagedorn transition)  
 $\Rightarrow$  remain flat at 1-loop.
- Soft breaking terms  $\sim M_{3/2} \Rightarrow$  stabilisation (twisted *and* untwisted moduli)

# Additional SUSY-breaking moduli

Example : 2 radii case. Spontaneous breaking along  $R_0, R_5, R_6$ .

2 equations :

- $\mathcal{U} \equiv R_5/R_6$  stabilization;
- Ansatz consistency : state equation  $\rho = 5P$  (analogous to 6D radiation)

Explicit construction of cosmological solutions :

Heterotic  $\mathcal{N} = 4, E_8 \times E_8$  gauge,

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# Conclusion

- Explicit construction of thermal early cosmologies, with dynamical SUSY-breaking scale.
- Supergravity corrections obtained by  $Z_{1\text{-loop}}$ , controlled by the no-scale structure.
- Stabilization of  $M_{3/2}$  when the no-scale structure breaks down.
- $M_{3/2} \propto T$  is a critical solution for the evolution (attractor).

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