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Florian Ielpo

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THÈSE

pour obtenir

le grade de : DOCTEUR DE L'UNIVERSITÉ DE PARIS 1
Mention ÉCONOMIE

par

Florian IELPO

Équipe d'accueil : CERMSEM
École Doctorale : Centre d'Economie de la Sorbonne
Composante universitaire : CNRS-UMR 8095

Titre de la thèse :

L'INTEGRATION DE L'INFORMATION DANS LE PRIX DES ACTIFS FINANCIERS

soutenue le 19 septembre 2008 devant la commission d'examen

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M. Umberto Cherubini	Rapporteur	Università de Bologna
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M. Roch Heraud	Encadrant industriel	Dexia S.A.

A ma mère, mon père et ma grand-mère.

A Lucie.

*”Statistiquement, tout s’explique,
personnellement, tout se complique.”*
D. Pennac.

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New York, le 8 juin 2008.

Résumé

Le sujet principal de cette thèse est l'intégration de l'information macroéconomique et financière par les marchés financiers. Les contributions présentées ici sont au nombre de cinq. Les trois premières utilisent de récentes avancées de l'économétrie de la valorisation d'actifs. L'objectif est de mesurer les anticipations, l'aversion au risque ou simplement de prévoir le prix des produits dérivés. (1) Tout d'abord, on introduit une nouvelle méthode économétrique permettant d'estimer l'évolution de la distribution subjective à partir des futures sur taux d'intérêt.(2) Ensuite, à partir des cotations d'options et des futures sur le marché européen du Carbone, on met en évidence l'impact de la publication des quotas d'émission attribués par la Commission Européenne sur l'aversion au risque dans ce nouveau marché. (3) Puis, on présente un nouveau modèle d'évaluation de produits dérivés basé sur des rendements suivant une loi hyperbolique généralisée sous la mesure historique. En supposant que le noyau de prix est une fonction exponentielle affine de la valeur future du sous-jacent, on montre que la distribution risque neutre est unique et à nouveau conditionnellement hyperbolique généralisée. Le modèle conduit à de faibles erreurs de prix, lorsqu'on les compare à la littérature existante. Enfin, deux thèmes liés à l'impact des nouvelles macro-économiques sur la courbe des taux sont présentés ici: (4) on montre tout d'abord que la perception de l'impact d'une surprise sur le marché des taux européens est grandement modifiée lorsque l'on tient compte de l'influence américaine. (5) Ensuite, on quantifie l'intuition largement répandue selon laquelle la forme de la structure par terme de l'impact des nouvelles sur la courbe des taux dépend des conditions économiques et monétaires, et ceci dans le cas américain.

Abstract

This PhD dissertation mainly focuses on the information processing of financial markets. It consists in five different contributions. The first three of them focus on the econometrics of asset pricing models. (1) First, I propose a new methodology to estimate the subjective distribution implicit in interest rate futures. I show how to use it to investigate the market participants' perception of the expected monetary policy over 2006. (2) Then, using both options and futures on the European Carbon market, I document the impact of the May 2006 information disclosure regarding allotted carbon emission quotas on the risk aversion implicit in market prices. (3) Next, building on the assumption of an exponential affine pricing kernel, I show how to price options for conditionally Generalized Hyperbolic distributed returns under the historical measure. I provide empirical tests that indicate that the model yields very low mis-pricing errors when compared to the existing literature. Lastly, I present new results regarding the term structure impact of news on the bond market. (4) First, I show how neglecting the US influence over the Euro bond market leads to a misleading diagnostic about European market mover figures. (5) Then, I show how American macroeconomic announcements produce a term structure effect over the US curve that strongly depends on the business cycle.

Contents

Remerciements	v
Résumé	ix
Abstract	xi
Table des matières	xv
Introduction	1
1 Empirical Option Pricing	9
1.1 An estimation strategy for the subjective distribution	13
1.1.1 The informative content of interest rates futures	15
1.1.1.1 Main assumption and notations	15
1.1.1.2 Dataset description	16
1.1.1.3 The predictive content of the Fed fund futures	18
1.1.1.4 The stylized facts of the Fed fund futures	20
1.1.1.5 Adequation tests	21
1.1.2 Flexible time series models	24
1.1.2.1 General settings of the model	24
1.1.2.2 Estimation methodology	26
1.1.3 Results and Event study	27
1.1.3.1 Main estimation results	27
1.1.3.2 Event study	31
1.1.4 Conclusion	35
1.2 Pricing kernels and the Carbon Market	37
1.2.1 Estimation strategy	39
1.2.1.1 Estimation of the Risk Neutral Probability Distribution	40
1.2.1.2 Historical Probability Distribution	41
1.2.2 Estimation Results and Discussion	43

1.2.2.1	The Dataset	43
1.2.2.2	Estimation Results	48
1.2.2.3	Conclusion	54
1.3	Option Pricing under GARCH models with Generalized Hyperbolic innovations	55
1.3.1	GARCH-type models with Generalized Hyperbolic innovations 60	
1.3.1.1	The Generalized Hyperbolic distribution	60
1.3.1.2	Description of the economy under the historical probability P	61
1.3.2	The stochastic discount factor	63
1.3.2.1	Pricing options with exponential affine SDF	65
1.3.2.2	Application to the model	67
1.3.3	Empirical Results	69
1.3.3.1	The dataset	69
1.3.3.2	Methodology	69
1.3.3.3	Results	71
1.3.3.4	Conclusion	72
1.3.4	Proofs	72
2	Assessing the impact of news on the bond market	79
2.1	Yield curve reaction to macroeconomic news in Europe	84
2.1.1	A methodology to deal with the US influence	88
2.1.1.1	Announcements and surprises in a data-rich environment	88
2.1.1.2	General Methodology	90
2.1.1.3	The Dataset	93
2.1.2	Empirical results	95
2.1.2.1	European figures impacting fixed-income markets	95
2.1.2.2	Distortion of the yield curve by announcements	98
2.1.2.3	Conclusion	103
2.2	Toward a classification of the impact of news on the US bond market	103
2.2.1	Assessing the shape of the market reaction function	105
2.2.2	The dataset	105
2.2.3	Assessing the shape of the market reaction function	107

2.2.4	Empirical results	113
2.2.4.1	Bulk effects of the introduction of the threshold variable	113
2.2.4.2	Term structure identification	121
2.2.5	Selected announcements and the underestimation problem . .	125
2.2.5.1	The economic cycle effect	125
2.2.5.2	The outliers effect	127
2.2.6	Conclusion	131
2.3	De l'usage des noyaux de prix en finance	137
2.3.1	Une nouvelle méthode pour l'estimation de la distribution historique	140
2.3.2	L'aversion au risque dans le marché européen du carbone . . .	142
2.3.3	Un modèle de pricing d'options avec hypothèses explicites quant à l'aversion au risque	143
2.4	L'impact des annonces macroéconomiques sur les courbes de taux . .	145
	References	157

Introduction

Financial markets are known to react in a real-time fashion to information flows stemming from data suppliers such as Bloomberg or Reuters. These pieces of information progressively change the market perception of future events, of future expected cash-flows and thus of the fair value of asset prices. My dissertation mainly deals with various measurements of these informational effects on different markets, such as the bond market or the European Carbon Market. It is made of a collection of five working papers, two of which are accepted for publication, the remaining ones being still under submission or about to be submitted. In this introduction, I review the main methodologies and results that have been used in these papers, along with the empirical conclusions associated to each of them.

The two pillars of the dissertation are the following ones. The first chapter is dedicated to the empirics of asset pricing with a special focus on the shape of expectations and risk aversion in financial markets. This chapter presents and uses measures for the subjective and the risk neutral distribution from which the risk aversion is deduced as a by-product. The second chapter is devoted to the assessment of the impact of macroeconomic news on the fixed income market. This assessment focuses on different economic zones and shows how this market impact can vary across time and economic conditions.

Before, providing further details about the comprehensive results presented in this dissertation, I would like to emphasize the particular context in which this work has been achieved. This PhD work has been accomplished during a *contrat CIFRE*¹: this work has been financed by Dexia, a Belgium investment bank. Therefore, the focus of the PhD work and thus of the resulting dissertation is on applied finance. The topics tackled here mainly come from specific requests or interests from Dexia. The consistency given to this dissertation only comes from the second time need to organize ideas and results in a common document.

¹French particular PhD contract for which the PhD student spends half his time in a company and the remaining in a research lab. CIFRE is French for Convention Industrielle de Formation par la Recherche.

The main idea of the first chapter of my dissertation is to measure and use the information implicit in asset prices. Sections are organized as follows:

- In the first section, I present a new methodology to estimate the term structure of the subjective distribution, using the Fed fund futures time series. The short rate is on average very close to the Central Bank target rate. Its subjective distribution is thus likely to provide us the knowledge of the market participants' perception of the future monetary policy. However, measuring subjective distributions is a difficult task. Until now, most of the attempts were designed using options on futures. In this perspective, Brière (2006), Andersen and Wagener (2002), Mandler (2002) and Mandler (2003) use the risk neutral distribution estimated from option prices to measure market expectations. This idea being empirically rejected in the literature – see e.g. Ait-Sahalia and Lo (2000), Jackwerth (2000) and Rosenberg and Engle (2002)– I propose another way around this problem. The solution I propose also deals with the term structure aspect of the fixed income market that makes any equity-based methodology useless. I present empirical evidence that there is a strong link between realized short rates and the historical dynamic of the Fed fund futures. Therefore, I assume that I can use the time series distribution of futures to measure the subjective distribution. I present a new time series model based on the Normal Inverse Gaussian distribution, with time varying parameters. I review the estimation methodology and present the numerical computation of the variance/covariance matrix associated to the estimated parameters. Finally, I show how this time series model behaves over different Central Bank meetings. The model is able to capture the drop in volatility due to Central Bankers' information disclosure. What is more, it also copes with the dramatic change in the skewness observed over the year 2006. At the beginning of the period, Central Bank rate cuts were in store, but not anymore in the end. This type of feature is essential to prove that my approach provides an accurate measure of the subjective distribution.

This contribution has been accepted for publication in an upcoming issue of the *Brussels Economic Review* in 2008. It has been presented at the French Association Annual Meeting (2008) and the Journée d'Economie Monétaire et Bancaire, in Rennes, France (2008). I am thankful to the previous seminars' participants and to two anonymous referees for their comments.

- Starting from the previous futures-based approach, the second section deals with the European Carbon Market. I show how the information disclosure regarding the allotted quotas for 2007 carbon emissions produced a dramatic change in the risk aversion implicit in both option prices and futures. In this perspective, this section should be regarded as an event study focusing on changes in

the risk aversion when comparing it between two periods. The risk aversion can be measured from the estimation of the risk neutral and subjective distribution: on this point, see Leland (1980). Following an approach mixing the Ait-Sahalia and Lo (2000)'s and Rosenberg and Engle (2002)'s approaches, I estimate the subjective distribution from the time series of December 2008 and 2009 futures, in a similar fashion to the previous section. To this end, I use a semi-parametric GARCH-GJR model. The risk neutral distribution is estimated from available option prices, using a non-parametric estimator: following Ait-Sahalia and Lo (1998), I use a Nadaraya-Watson estimator for implied volatility, jointly with a strike-dependent Black-Scholes model. The empirical results are the following: first, the information release decreased both levels of historical volatility and implied volatility. Second, the leverage effects on the Carbon markets are reversed when compared to equity-based ones: the volatility goes up when the futures are going up. Finally, the main empirical results indicate that the information disclosure studied produced an increase in the estimated risk aversion. The main risk in this market is related to the increase in the spot and future prices: consistently, the information released by the European Commission indicated an increase in the probability of the possible rise in the market prices. In this perspective, the rise in risk aversion is clearly not a surprise.

This contribution has been accepted for publication in *Energy Policy*. I am thankful to all the participants of the MBFA Paris 1 seminar in Paris, France (2008), the Environmental Governance seminar in Oxford, UK (2008), the European Meeting of the Econometric Society in Milano, Italy (2008), the French Finance Association seminar in Lille, France (2008), the Journée d'Economie Monétaire et Bancaire in Luxembourg (2008), the International Association for Energy Economics in Istanbul, Turkey (2008) and of the 28th International Symposium on Forecasting in Nice, France (2008) for their comments and remarks. I am also thankful to two anonymous referees for their comments on an earlier draft of this paper.

- The previous section confirmed that the pricing kernel – that is the ratio between the risk neutral density and the historical one – often looks like an exponential affine function of the future value of the underlying asset. This is confirmed by the empirical literature dedicated to the estimation of stochastic discount factors – that is the present value of the pricing kernel: see again Ait-Sahalia and Lo (1998), Ait-Sahalia and Lo (2000), Jackwerth (2000) and Rosenberg and Engle (2002). In fact, this assumption is implicit in many financial asset pricing models, Black and Scholes (1973)'s model to start with. A review of the models built on this implicit assumption can be found in Gouriéroux and Monfort (2007).

The list includes Heston and Nandi (2000)'s, Christoffersen et al. (2006)'s and Bertholon et al. (2007b)'s models for example. In this third section, I present a new option pricing model based on this assumption: in this model, log returns are conditionally Generalized Hyperbolic distributed under the historical measure. I show that assuming the pricing kernel is exponential affine, the risk neutral distribution is unique and conditionally GH again. The difference between the historical and the risk neutral distribution lies in the changed parameters of the distribution: apart from the expected risk free rate, the risk neutral distribution is characterized by a strongly changed skewness. This idea is consistent with the skewness premium usually found in index option datasets. I then propose to empirically test the performances of the model and compare it to the performances of several competitors. This test is performed using a dataset of French CAC 40 call option prices. The GH-based model performs remarkably and the change in parameter produced by the exponential affine pricing kernel decreases the out-of-sample mis-pricing errors, when compared to a simple martingalization of the simulated returns.

This section is taken from a CES working paper and has been submitted to *the Journal of Empirical Finance*. I am thankful to the participants of the MBFA Paris 1 seminar in Paris, France (2008) and of the 28th the International Symposium on Forecasting in Nice, France (2008) for their comments.

The second chapter of this dissertation is devoted to the impact of macroeconomic news on the bond market. I review later the now numerous contributions that can be found in the literature dedicated to this subject to focus on my specific contributions. Interest rates markets are known to react to economic information on the long run. However, many traders take bets on the divergence of economic announcements with what is known as the *market forecast*: because of this kind of behavior, interest rates are tainted by numerous jumps on announcement days. These sharp variations in interest rates produce a term structure impact: the term structure effect of announcements. This chapter contains two different contributions:

- In the first section, I answer the following question: is there any European macroeconomic announcement that moves the Euro bond market? To answer this issue, I need to handle one specific problem of the Euro yield curve: its dependency to the US yield curve and to US macroeconomic news. Since this would be too much information to incorporate in a model, I use the first three factors of a principal component analysis performed over the US rates as instrumental variables. I can compare the estimation results when considering the US

influence or not. The conclusion of this empirical work is that very few domestic or European figures influence the yield curve: the US influence accounts for as much as 50% of the Euro rates's daily changes. Among the indicators which impact the entire yield curve, I note the highly important role of German economic sentiment or activity figures, which carry more weight for the markets than the aggregated European figures. I find little influence from job figures, a result already documented in the existing literature; this is mainly due to the fact that the market has access to Germany's "pre-scheduled release". For price indicators, my results contrast with those previously obtained, and most of the differences appear to be due to the more exhaustive integration of the American influence. I demonstrate that the inflation figures have a relatively weak surprise effect on European interest rates, whereas the M3 money supply has a substantial impact. The latter result fits in with the crucial role of this indicator for the ECB and the testimony of investors. Lastly, I classify the most important figures for the markets for each maturity.

This section is taken from a collective paper of mine that is now a Solvay Business School working paper and that has been accepted for publication in an upcoming collective book. I am thankful to the seminar participants in the 11th International Conference On Finance And Banking: Future of the European Monetary Integration. Karvina, Czech Rep. (2007) where I presented the paper.

- In the last section, I show how the term structure impact of macroeconomic news in the US curve can vary depending on time and on the business cycle. Using several indicators to account for the economic and monetary policy cycles, I propose to use threshold models to capture this dependency. The main estimation results unfold as follows. First, I find that there exist several types of surprises that actually affect the bond market, surprisingly matching the first four factors found when performing a principal component analysis over the daily changes in swap rates. Second, the ranking of market mover figures strongly depends upon the market perception of the economic cycle, measured by publicly available indicators. What is more, it depends upon the stance of monetary policy, measured by the Fed's target rate. Finally, I show that the use of a threshold model when estimating the market's response to macroeconomic news leads to the elimination of outliers within the dataset, yielding different - and often more significant - estimates of the market response to selected figures. The exclusion of these outliers brings about interest rates' reaction functions that are generally higher than the classical ones, and more concave.

This section is taken from a CES Working paper of mine and has not been published until now. I am thankful to the participants of the Journées d'Econométrie de la Finance de Nanterre (2006) and of the 2nd Congress on Econometrics and Empirical Economics in Rimini, Italy (2007) for their comments.

Beyond the different works presented here, I was also involved in three other projects that I was not able to merge within this document. I nonetheless mention them briefly, to present an exhaustive perspective on my PhD work:

- The first one is a paper entitled "An Econometric Specification for Monetary Policy's Dark Art", a pompous name for a paper that shows that the pace of target rate increases of the Fed have explaining factors that are different from those of the level of the target rate itself. This paper is currently under revision, before a re-submission. It is now a CES-Paris 1 working paper. I presented it at the Journée D'Economie Monétaire et Bancaire in Lille, France (2006) and I am thankful to the conference participants for their remarks. The manuscript is under revision before a re-submission.
- I am involved in a second applied research project entitled "Smiled dynamics of the smile". This working paper is devoted to a new stylized fact associated to implied volatilities time series: the implied volatilities' speed of mean reversion is different across moneynesses, displaying a persistence smile. This paper has been presented at the Journée d'Econométrie de la Finance in Nanterre, France (2007), at the French Finance Association in Lille, France (2008), at the Forecasting Financial Markets seminar in Aix-en-Provence, France (2008) and at the Augustin Cournot Doctoral Days in Strasbourg, France (2008).
- Finally, I collaborated to another research project that has yielded two papers until now. This research project focuses on the estimation and financial application of a new stochastic correlation continuous time process named "Wishart Affine Stochastic Correlation" model. The papers were presented at the 11th conference of the Swiss Society for Financial Market Research in Zürich, Switzerland (2008), at the Mathematical and Statistical Methods for Insurance and Finance in Venice, Italy (2008), at the 2nd International Workshop on Computational and Financial Econometrics in Neuchâtel, Switzerland (2008), at the First PhD Quantitative Finance Day, Swiss Banking Institute in Zürich, Switzerland (2008), at the 14th International Conference on Computing in Economics and Finance in Paris, France (2008), at the Inference and tests in Econometrics, in the honour of Russel Davidson in Marseille, France (2008), at the Inaugural conference of the Society for Financial Econometrics (SoFie) in New York, USA

(2008), at the CREST seminar in Malakoff, France (2008), at the 28th International Symposium on Forecasting in Nice, France (2008) and at the ESEM annual meeting in Milano, Italy (2008). I am indebted to participants of all these conferences and seminars for their remarks.

Among the research project I undertook, the papers presented here are the most representative of my personal contributions and work for Dexia. Therefore, the papers presented here can be listed as follows:

- "Flexible Time Series Models for Subjective Distribution Estimation with Monetary Policy in View", with Dominique Guégan, published in a forthcoming issue of the Brussel Economic Review in 2008.
- "Risk Aversion and Institutional Information Disclosure on the European Carbon Market: a Case-Study of the 2006 Compliance Event", with Julien Chevalier and Ludovic Mercier, forthcoming in *Energy Policy*.
- "Option Pricing under GARCH models with Generalized Hyperbolic innovations", with Christophe Chorro and Dominique Guégan. It is a CES-Paris 1 working paper.
- "Yield curve reaction to macroeconomic news in Europe : disentangling the US influence", with Marie Brière, published in an upcoming collective book edited in 2008.
- "Further Evidence on the Impact of Economic News on Interest Rates", with Dominique Guégan. This paper is now a CES-Paris 1 working paper and is in revision before a re-submission.

Of course, each of these articles and papers are collective works. Nonetheless, this D. Phil. dissertation is written using "I" instead of "we" for two reasons. First, all the papers gathered here are the results of my ideas: collective though these contributions may be, I was the one proposing the subject and focus of each paper. Second, I did not copy and paste the bodies of each papers but changed things inside so as to best reflect my personal contribution and to make the papers' merging as natural as possible.

Finally, I must mention that every figure presented in the PhD dissertation is based on Bloomberg data, involving most of the time my own calculations.

Chapter 1

Empirical Option Pricing

The first part of this Ph.D. dissertation¹ is devoted to the empirical understanding of the existing link between subjective, historical and risk neutral distributions. These distributions are well known to both practitioners and academics involved in financial asset pricing. A growing attention is now devoted to their empirical measure and behavior. Here, I briefly review the key concepts and results about asset pricing. The rest of this chapter will focus on the econometrics and interpretation of these distributions.

The value of any derivative based on an underlying asset whose price at time t is X_t , with a payoff function $g(\cdot)$ and a time to maturity set to $T - t$ can be stated as²

$$D_t(T, X_t) = \mathbb{E}^{\mathbb{Q}}[e^{-\int_t^T r_s ds} g(X_T)], \quad (1.1)$$

with r_s the risk-free stochastic rate at time s . This expectation is computed with respect to the \mathbb{Q} -probability distribution, also known as "risk neutral distribution". Several arguments help us understand the origin of this name. First, under this probability distribution, the expected return of any underlying asset is the risk-free rate. Thus, under this probability distribution function, assets are martingale: the expected excess return for any investment is zero. Second, in a more economic framework that will be discussed later, this distribution is assumed to make market participants neutral toward risk. The most simple example of such a derivative is the European call option with strike price K based on an underlying stock with price S_t at time t

$$C_t(T, K) = \mathbb{E}^{\mathbb{Q}}[e^{-\int_t^T r_s ds} (S_T - K)^+]. \quad (1.2)$$

¹This chapter is the result of three working papers and of two Master classes taught at the ENSTA and ESILV. One of the working papers will be published in a forthcoming issue of the Brussels Economic Review.

² $D(t, T)$ is here a pure convention: the price of a derivative is usually not only a function of the time to maturity $T - t$ but also a function of states variables that are model-specific.

Equations (1.1) and (1.2) are model free: relations of this kind are the cornerstone of any asset pricing model (see Cochrane (2002)). The main problem that must be tackled now is to derive a specification for this distribution that is consistent with stylized facts associated with financial markets. Moreover, \mathbb{Q} must be chosen in way that makes in and out of sample mispricing errors minimum. In this section of the dissertation, I focus on the rationale and econometrics hidden behind asset pricing models.

There are two different and fundamental views regarding the building of a dynamic asset pricing model³. First, the more common way to present such a model is to discuss the historical dynamics of asset prices – I should say here “the time series properties of financial asset prices” – most of the time in a continuous time setting. Here, I will focus only on discrete time cases. This approach should be labeled the *historical approach*. Second, another approach is based on the specification of the preferences of a representative agent: the time series properties are now replaced by a specification for the dynamic features of the subjectivity of this “average trader”. Therefore, this distribution is known as the subjective distribution. It is assumed to account for the risk perception and appetite of this economic agent. This approach should be referred to as the *economic approach*. This stems from the fact that this approach is usually based on the specification of a utility function for the representative agent of the market: it allows to quantify the risk perception and appetite associated with this economic agent.

For both these approaches, the key ingredients⁴ to any asset pricing model are the following: (1) a model for the historical or subjective dynamics of the underlying asset, (2) a specification on the way to reach the risk neutral world and (3) the risk neutral distribution itself. As presented in Bertholon et al. (2007a), these ingredients being linked to each other, the knowledge of two of them yields the third one as a by-product. Beyond these conceptual elements, the presentation of the link between these distributions unfolds as follow.

I briefly present here the concept of pricing kernel that is often neglected in financial asset pricing textbooks. Starting with the historical approach, the standard way to proceed is first to set a model under the \mathbb{P} probability measure such that:

$$X_t = m_t + \sigma_t \epsilon_t, \quad (1.3)$$

with m_t and σ_t the possibly time varying conditional expectation and volatility of the X_t process. ϵ_t is an innovation whose distribution is model-dependent. In the case of equity markets, X_t is the log of the price of the asset, whereas, in the case of the bond

³The label “Dynamic Asset Pricing Model” is here taken from the excellent textbook of Singleton (2006).

⁴See Bertholon et al. (2007a).

market, it is simply equal to the short rate (for short rate models). For example, in the discretized Black and Scholes (1973) model, it is easy to see that:

$$m_t = \left(\mu - \frac{1}{2}\sigma^2 \right) \Delta \quad (1.4)$$

$$\sigma_t = \sigma \sqrt{\Delta} \quad (1.5)$$

$$\epsilon_t \sim N(0, 1), \quad (1.6)$$

with Δ being the sampling frequency often expressed on an annual basis.

Then, the second ingredient needs to be stated. This ingredient received several names in the dedicated literature such that *pricing kernel*, *stochastic discount factor* or Radon-Nykodim derivative. The pricing kernel is a function of the state variables of the model $M(X_t)$ such that

$$\mathbb{E}^{\mathbb{P}} \left[e^{-\int_t^T r_s ds} M(X_T) g(X_T) | \mathcal{F}_t \right] = \mathbb{E}^{\mathbb{Q}} \left[e^{-\int_t^T r_s ds} g(X_T) | \mathcal{F}_t \right]. \quad (1.7)$$

Let $f_{t,T}(\cdot)$ and $q_{t,T}(\cdot)$ be respectively the historical and risk neutral distributions of the X_t process conditionally on the information available at time t and for an investment horizon equal to T . Thus, the following relation holds:

$$M_{t,T}(X_T) = \frac{q_{t,T}(X_T)}{f_{t,T}(X_T)}. \quad (1.8)$$

The stochastic discount factor $\psi_{t,T}(X_t)$ is the present value of the pricing kernel, that is:

$$\psi_{t,T}(X_T) = M_{t,T}(X_T) e^{-\int_t^T r_s ds}. \quad (1.9)$$

Once these two elements have been specified, moving from one distribution world to the other is often tedious but straightforward. For example, in the Black and Scholes (1973) case, the pricing kernel is equal to

$$M_{t,T}(X_T) = e^{\frac{(\mu-r)(\mu+r-\sigma^2)}{2\sigma^2}(T-t) + \frac{r-\mu}{\sigma^2}(X_T-X_t)}, \quad (1.10)$$

where r is the constant risk-free rate. It is thus an exponential affine function of the state variable X_T . Moreover, the distribution under the \mathbb{Q} measure is Gaussian again, with a shifted drift.

The "economic" solution to asset pricing can also be analyzed in a similar fashion. Following what is presented in Cochrane (2002), the representative agent's preferences

are summarized by his utility function $U(X_T)$. It is in fact an indirect function of the terminal wealth and thus of the future (random) value of the underlying asset. It is possible to show that the price of any derivative with payoff $g(X_T)$ can be stated as

$$D(t, T) = \mathbb{E}^{\mathbb{S}} \left[\beta \frac{\frac{U'(X_T)}{U'(X_t)}}{\mathbb{E}^{\mathbb{S}} \left[\frac{U'(X_T)}{U'(X_t)} \right]} g(X_T) \middle| \mathcal{F}_t \right], \quad (1.11)$$

with the expectations being computed under the *subjective distribution* \mathbb{S} . This distribution is assumed to summarize the representative agent's view on the future value of the asset. It should be understood in a way as a Bayesian revision of the historical distribution. However, due to the difficulty of measuring the subjective distribution, this revision has not been documented yet in the literature, up to my best knowledge.

With these settings in mind, the stochastic discount factor argument can be used again, insofar as the risk neutral density function is now equal to

$$q_{t,T}(X_T) = s_{t,T}(X_T) \phi_{t,T}(X_T), \quad (1.12)$$

where $s_{t,T}(\cdot)$ is the subjective distribution function and

$$\phi_{t,T}(X_T) = \beta \frac{\frac{U'(X_T)}{U'(X_t)}}{\mathbb{E}^{\mathbb{S}} \left[\frac{U'(X_T)}{U'(X_t)} \right]}. \quad (1.13)$$

Again, all these distributions are considered conditionally to the information at time t and for an horizon equal to T . When assuming that the subjective and historical distributions are alike, $\phi(\cdot)$ is simply the stochastic discount factor $\psi(\cdot)$ advocated in the historical approach. Of course, these distributions are different, though related. Interestingly, the $\phi(\cdot)$ function is related to the representative agent's risk aversion, since

$$-d \log \phi_{t,T}(X_T) = -\frac{U''(X_T)}{U'(X_T)}, \quad (1.14)$$

the right-hand side of the latter equation being the Arrow-Pratt risk aversion coefficient. Here is the main difference in the change in probability measure process between the economic and historical frameworks: in the economic framework, the change in probability mass is linked to a risk aversion correction, whereas in the historical approach, it is simply about a risk premium correction – that is to say changing the drift of the historical distribution. In the Black and Scholes (1973)'s model, risk aversion and risk premium corrections are equal. Within this model, the historical and the subjective distribution are equal in the distribution sense.

The problem is now to be able to estimate these quantities and document their time-varying behavior for different asset classes. In section 1, I present a new method to estimate the subjective distribution implicit in the interest rates futures dynamics. In section 2, I document the dramatic change in the empirical pricing kernel on the European Carbon market after allowances information disclosure. Finally, in Section 3 I present a new option pricing model based on the Generalized Hyperbolic distribution. In this model, I make direct assumption on the shape of the risk aversion and discuss its impact on the pricing errors obtained for an index option dataset.

It is noteworthy to mention that the approaches presented here focus on the univariate setting. The empirical option pricing framework is also of a great help for multivariate contingent claim pricing. It has been successfully used in Cherubini and Luciano (2002a), Cherubini and Luciano (2002b) and Cherubini and Luciano (2003). In such a framework, the dependency in a discrete time setting is handled through copulae. Cherubini et al. (2008) even extent the use of copulae to model the dependency in stock prices time series between two dates. However, I do not make use of copulae here, exploring instead a different way to account for time dependency in various financial time series.

1.1 An estimation strategy for the subjective distribution

The⁵ subjective distribution is the distribution that reflects the market participants' perception of the future value of a financial asset. This distribution carries important information regarding the market participants' risk perception over states and maturities. Unlike the subjective one, the risk neutral distribution⁶ is supposed to make market participants neutral towards risk. This latter distribution should therefore carry no risk aversion component. Since these distributions are equivalent in the probabilistic sense, they are solely related through a risk aversion correction. Following Leland (1980), this relation can be roughly stated as follow:

$$\text{Risk Neutral Probability} = \text{Subjective Distribution} \times \text{Risk Aversion Adjustment.} \quad (1.15)$$

This relation is well-known for empirical finance applications: see the results presented in Ait-Sahalia and Lo (2000), Jackwerth (2000) and Rosenberg and Engle (2002).

⁵"Flexible Time Series Models for Subjective Distribution Estimation with Monetary Policy in View", with Dominique Guégan, published in a forthcoming issue of the Brussel Economic Review in 2008.

⁶The risk neutral distribution is the distribution used to give a price to financial assets.

However, it has scarcely been used to analyze the link between monetary policy and the bond market. For most of the developed countries, the Central Bank directly controls the short rate. In this perspective, the subjective distribution of the short rate process yields pieces of information on the market participants' perception of the future stance of monetary policy. Monetary policy makers need to measure the perception of the Central Bank's policy by financial markets. For example, the information disclosure following the regular meetings of the Federal Reserve Board is known to be followed by a reduction of the bond market volatility. However, the financial market's assessment of the monetary policy goes far beyond the sole changes in conditional variance: the estimation of the conditional distribution is thus essential to Central Bankers. On the market participants' side, monitoring the changes in "market beliefs" is also important for asset management and risk control purposes: monetary policy is known to be the main risk factor in the government bond market. For all these reasons, the design of a model of the kind that is to be discussed here is one of the cornerstones of empirical monetary policy.

Until now, several estimation strategies for the subjective distribution have been proposed. On the one hand, several articles proposed to use the risk neutral distribution estimated from option prices as a proxy for the subjective one: see Mandler (2002), Brière (2006) and the survey presented in Mandler (2003). On the other hand, using the relation presented in equation (1.15), Jackwerth (2000), Ait-Sahalia and Lo (1998) and Rosenberg and Engle (2002) used the historical distribution of index returns as a proxy for the subjective distribution. The conclusion of the latter stream of literature is that the risk neutral and the subjective distribution are very different.

However, these attempts were mainly designed for equity assets, and little attention has been devoted to fixed income securities. For such markets, the additional problem is the term structure aspect of the subjective distribution. Here, I need a kind of financial asset whose historical dynamic carries information about this term structure. I propose to use the existing future contracts to do so: they are actively traded and used to make bets on the future stance of monetary policy. In order to have a reactive estimate of the subjective distribution, I use here a dynamic Normal Inverse Gaussian distribution, building a flexible time series model. This methodology allows for a direct mapping from the past observations space into the parameters' space, without any a priori knowledge about the dependence between them. The estimation of this model can be performed by maximum likelihood. When confronted to the data, this model is accepted. An event study shows that the results obtained with this model are close to what is expected.

In the upcoming subsections, I first present evidences that interest rates futures can be

used as an informative asset regarding the subjective distribution. I then set up a time series model that is able to cope with the time varying volatility, skewness and kurtosis in the dataset. I finally review the key empirical properties and results obtained with this new methodology.

1.1.1 The informative content of interest rates futures

The main contribution of this section is to propose to use the dynamics of the futures contract prices to estimate the subjective distribution of the futures instantaneous rate for various maturities. First, I will explain the reasons for this estimation strategy, both from an empirical and theoretical point of view. I review the hypotheses of this approach and then discuss the informative content of the Fed fund contracts. Finally, key statistics for the dataset will be discussed for modeling purposes.

1.1.1.1 Main assumption and notations

This section reviews the main notations and hypotheses of this work. Let $R(t, T)$ be the spot rate of maturity T at time t . The short or instantaneous spot rate r_t on date t is given by:

$$r_t = \lim_{T \rightarrow t} R(t, T). \quad (1.16)$$

When t is different from today, this rate becomes the future instantaneous spot rate, that is thus unknown at time t and assumed to be a random variable. I denote $F(t, T)$ the instantaneous future rate known at time t for a maturity T from the future contracts. Under no arbitrage restrictions, the link between spot rates and instantaneous forward rates is given by:

$$R(t, T) = \frac{\int_{\tau=t}^T F(t, \tau) d\tau}{T - t} \quad (1.17)$$

The spot yield curve being an average of the forward rates can be used to recover the instantaneous forward rates (using a spline model for example, see e.g. ?). Now, another well-known relation is⁷:

$$E^S[r_T | \mathcal{F}_t] = F(t, T), \quad (1.18)$$

where $E^S[\cdot]$ denotes the expectation under the subjective distribution S , and \mathcal{F}_t being the filtration produced by the information set at time t . I will define this filtration

⁷This is one of the presentation of the expectation hypothesis. See e.g. Jarrow (2002) for more details.

later: it will basically result from the past evolutions of the futures. This relation states that the forward rates can be considered as a market forecast of the future short rate. In many countries, this rate is directed by the Central Banks: in this perspective, the forward rates can be used to investigate the financial markets' understanding of monetary policy and the market's perception of the risk associated to the upcoming Central Bankers' meetings⁸. This link between the bond market and monetary policy has already been used in e.g. [?](#) , in order to measure the market forecast over several Central Bank decision meetings.

The main novelty here is to develop a time series model to recover the distribution of r_T conditionally upon $\overline{F(t, T)} = \{F(t, T), F(t - 1, T), \dots\}$, for each maturity T_i of interest (those of the different future contracts or those of the Central Bank decision meetings). Here, I assume that the conditional historical distribution of $F(t, T)$ is close enough to the conditional subjective distribution of $r_T | \mathcal{F}_t$, hence they can be treated as equal. The main hypothesis is then:

Hypothesis 1 *Let $F_t^S(\cdot)$ be the cumulative distribution function associated to the subjective distribution of r_T conditionally upon the information available at time t . Let $F_t^H(\cdot)$ be the cumulative distribution function associated to the historical distribution of the future rate $F(t, T)$, conditionally upon the information available at time t . Then, the two distributions are equal on any point of their common support, i.e. $F_t^S = F_t^H$.*

Most of the existing literature assumes that the historical distribution of equity returns is a consistent proxy for the subjective distribution: see e.g. Jackwerth (2000), Ait-Sahalia and Lo (2000) and Rosenberg and Engle (2002). This hypothesis seems to hold as long as the working purpose is to recover the short-term subjective distribution of equity indexes. Here, the topic of interest is the term structure of the subjective distribution induced by the time-varying yield curve: this is why I use the futures instead of the short rate. I document this hypothesis in the following subsection.

1.1.1.2 Dataset description

In this section, I present the Fed fund futures contracts that are used here. I first review the motivations to use these future contracts, before I detail the building of the dataset.

⁸Recent papers nonetheless empirically showed that these forecasts may be biased: Piazzesi and Swanson (2008) showed that the difference between the realized target rate and the market forecast implicit in the yield curve is statistically significant and even linked to the economic momentum. This bias in the forecast is often referred to as *term premium*. Even though this premium can be very well explained *in sample*, the out of sample performances are really poor (and biased again). This is why I chose to discard this problem for the time being, considering that given the information on the current date t , the market forecast for the date $T > t$ is *conditionally* unbiased.

There exist several types of future contracts that may be used to recover market probabilities of future rates hikes and cuts. Most of them are three-months contracts, which is not convenient for monetary policy analysis: Central Bank meetings happen more on a monthly basis – this is at least true for the ECB and the Fed. Beyond three-month futures, I focus here on the Federal Fund Future contracts, that are monthly contracts backed on the US refinancing rate.

This contract has a price that is equal to

$$P(t, T) = 100 - 100 \times \frac{1}{n} \sum_{i=T-n+1}^T r_{i,t}, \quad (1.19)$$

that is 100 minus the one-month average of the future refinancing rate $r_{i,t}$ – the future target rate plus a daily cash premium – expected at time t over a one-month period (with n days). The Fed Fund futures are a useful and now quite liquid asset to extract monetary policy stance expectations from the bond market. It has already been widely used in empirical research: see e.g. Krueger and Kuttner (1996), Robertson and Thornton (1997), Kuttner (2000), and Carlson et al. (2003). These contracts are widely used by practitioners to measure the current market’s feeling about the future monetary policy’s stance. The extraction of market-based monetary policy expectations requires some preliminary calculations that are detailed e.g. in Kuttner (2000). I do not report them here, since it merely reduces to particular calendar dates handling.

The global dataset used in this paper is made of 77 Fed Fund future contracts over their whole lifespan. The first contract’s maturity is November 2000 and the final contract’s maturity is March 2007. The time series of future prices is observed on a daily basis: the dataset is actually made of closing future prices. The prices are converted into rates using the relationship state in equation (1.19). This dataset includes rate hikes and cuts periods, which is important for the purpose of this paper, and especially for the examination of the information implicit in these future rates.

Nevertheless, for the ease of the presentation of the empirical results in Section 4, I chose to focus on 4 particular future contracts. These contracts have the following maturities: December 2006, January 2007, February 2007 and March 2007. I retained these contracts because of the period covered. Over the lifespan of these contracts, monetary policy started to change, with the end of a rate hikes period and the starting of an upcoming rate cuts period. This period should result in a dramatic change in the shape of the subjective distribution across the sample that is used here. At least, a progressive change in the skew of the distribution should appear over the period that is considered here: moving from a positive one – which is the sign of upcoming rate hikes – to a negative one, as financial markets will start believing in upcoming rate

cuts. Thus, this dataset should be a challenging one for the modeling that is proposed here.

The sub-sample starts on the August 1, 2006 and ends on the October 30, 2006. It is necessary to suppress at least the last two months of the lifespan of the future contracts for each of them due to the fact that the level of uncertainty for these months is quickly decreasing: thus, during these last two months the volatility is likely to decrease quickly as expectations about the future stance of monetary policy are getting more and more accurate. This decreasing conditional volatility usually leads to diagnose non stationarity⁹.

In the remaining of the paper, I generally refer to the first sample of 77 contracts as the "full-sample" and to the latter restricted sample as the "sub-sample". The full-sample is mainly used in this section and the sub-sample will be used for the empirical results presented in Section 4.

1.1.1.3 The predictive content of the Fed fund futures

From an empirical point of view, the future contracts are actively used by traders to take bets over the upcoming Central Bank decisions – in fact this asset has been explicitly introduced on financial markets to reveal the market expectations regarding the future stance of monetary policy: this point is documented in Krueger and Kuttner (1996), Robertson and Thornton (1997), Kuttner (2000), and Carlson et al. (2003). Thus, the historical dynamics of the Fed fund futures should reflect the changes in the market perception of the future monetary policy.

Beyond these institutional considerations, I propose to test this hypothesis by the following statistical analysis. Building on the previous notations, r_{T_i} is the short spot rate on date T_i , that should be forecast – accordingly to my assumption – by $F(t, T_i)$. If the dynamics of the corresponding future rate is linked to the realized rates¹⁰, then it should be possible to relate the distribution – and thus the moments – of $F(t, T_i)$ to r_{T_i} . I denote

$$\Delta f(t, T_i) = \log \frac{F(t, T_i)}{F(t-1, T_i)}, \quad (1.20)$$

the log increment of the future rate, for a given maturity at time t . This transformation ensures the second order stationarity of $\Delta f(t, T_i)$. Now, the descriptive statistics of

⁹This is easy to check by using stationarity tests, such as the Augmented Dickey Fuller tests. I did the test and ended on the conclusion that excluding the last two months of each futures is a good rule of the thumb.

¹⁰The subjective distribution is a forecasting density of r_{T_i} .

$\Delta f(t, T_i)$ are denoted by $m_1(T_i)$ for the expectation, $m_2(T_i)$ for the volatility, $m_3(T_i)$ for the skewness and $m_4(T_i)$ for the kurtosis. Their estimation is done using the population moments.

Now, I propose to check whether the historical distribution of the futures, using the previous moments as a proxy for it, can be explained by any combination of the realized corresponding spot rates. It would support the fact that the historical dynamic of the future rates carries information regarding the market view of the future realized rates.

I adopt the following notations. $L(T_i) = \log r_{T_i}$ is the log of the realized rate: it is meant to capture a level effect. $\Delta L(T_i) = L(T_i) - L(\tau_i)$ is the variation of the spot rate over the windows defined by the lifespan of the future contracts, i.e. the time elapsed between the issuance τ_i and the delivery T_i dates of each contracts. $|\Delta L(T_i)| = |L(T_i) - L(\tau_i)|$ represents the total variation over the duration of the future contract. Its absolute value accounts for a kind of realized volatility over the future lifetime. Finally, $S(T_i) = |L(T_i) - \bar{L}|$ captures the effect of turning points of the rates' dynamics, when L_T is way above or below its historical average \bar{L} . This is meant to deal with the well known and documented mean reverting behavior of interest rates.

I estimate the following linear model based on the sample moments $m_j(\cdot)$:

$$\hat{m}_j(T_i) = \alpha_0 + \alpha_1 L(T_i) + \alpha_2 \Delta L(T_i) + \alpha_3 |\Delta L(T_i)| + \alpha_4 S(T_i) + \epsilon(T_i), \forall i \quad (1.21)$$

where $\epsilon(T_i)$ is a centered white noise and \hat{m}_j the sample moment. The R^2 is obtained when estimating the linear model defined by equation (1.21) by Ordinary Least Squares with the previously mentioned explanatory variables:

$$L(T_i) = \log \hat{R}(T_i) \quad (1.22)$$

$$\Delta L(T_i) = L(T_i) - L(\tau_i) \quad (1.23)$$

$$|\Delta L(T_i)| = |L(T_i) - L(\tau_i)| \quad (1.24)$$

$$S(T_i) = |L(T_i) - \bar{L}|. \quad (1.25)$$

The estimated R^2 are presented on figure 1.1. The R^2 obtained are very high for a model with 4 explanatory variables, indicating that the moments of the historical distribution of the futures are statistically linked to the realized rates. It seems now natural to relate this historical distribution to market expectations: the subjective distribution is by construction a forecasting density and should verify empirical properties of this kind. When looking at figure 1.1, it is interesting to remark that each independent variable presented above has an explanatory power on its own: even though there may be

some collinearity between these variables, they still explain remarkably well the dependent variable here. What is more, these variables are second order stationary from stationarity tests that are not reported here.

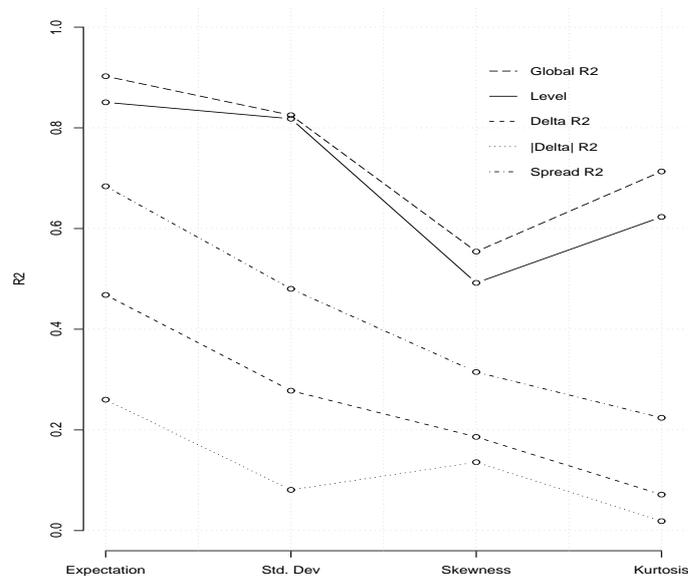


Figure 1.1: R^2 study for Fed fund futures contracts, using 77 contracts of maturities ranging from November 2000 until March 2007.

Global R2 presents the R^2 obtained when using the four variables defined in equations (1.22), (1.23), (1.24) and (1.25). Level presents the R^2 obtained when using only the explanatory variable presented in (1.22). Delta R2 presents the R^2 obtained when using only the explanatory variable presented in (1.23). $|\text{Delta R2}|$ presents the R^2 obtained when using only the explanatory variable presented in (1.24). Spread R2 presents the R^2 obtained when using only the explanatory variable presented in (1.25).

1.1.1.4 The stylized facts of the Fed fund futures

Now, I briefly discuss the stylized facts of the futures dataset used in this sub-section, for time series modeling purposes.

Financial datasets and especially financial returns are known to display time varying volatility and higher order moments. The log increments of future rates display the same characteristics. On the previous dataset, I computed the first four moments estimator for each contract. Figure 1.2 presents the obtained results: each of the moments dramatically vary across the contracts. Thus, I performed $T \times R^2$ tests for autoregressive moments of order 1 to 4 for each contract in the dataset (for this test, see Engle (1982) and for its application to higher order modeling see Jondeau et al. (2006)). Figure 1.3 present the test statistics along with the χ^2 quantile for testing first order autoregressive patterns for each moment across the contracts. It presents the test statistics obtained for each Fed fund futures contracts, with maturity ranging from November 2000 until March 2007. The test statistics were computed over the whole lifespan of the future contracts but the last two months of it, due to the previously mentioned stationarity concerns. For most moments and contracts, there is an autoregressive component that is statistically significant. This supports the idea that the first four moments are time varying.

1.1.1.5 Adequation tests

Finally, I propose here to test the adequacy of four distributions to the data at hand. These distributions are the skewed t-distribution, developed in Hansen (1994), the skewed Laplace distribution (see Kotz et al. (2001) and the references within), the Normal Inverse Gaussian distribution, introduced by Barndorff-Nielsen (1997) and the Gaussian distribution, used as a benchmark. For the explicit expression for the cumulated distribution function, see the references previously mentioned. Before testing the adequation of these distributions, the log-returns were first filtered using a GARCH(1,1) model introduced in Bollerslev (1986): by doing this, I take into account the possible second order dependency in the data at hand.

I performed Kolmogoroff-Smirnoff tests on the GARCH residuals. The results obtained are presented in the table 1.1. The Kolmogorov Smirnoff tests were performed over the Fed fund future contracts for the Skewed Student, the Skewed Laplace, the NIG and the Normal distributions, for Fed fund future log daily increments with maturities ranging from November 2000 until June 2007.

The NIG distribution yields the best results, once compared to the other distributions. Thus, in the remaining of the paper, I will only retain the NIG distribution to model the distribution of the future rates. The adequation tests lead to accept this distribution for 95% of the samples: the regularity of the acceptance is a very interesting feature for my purposes. The purpose of this data mining exercise is to find a distribution that is the most likely to fit any future sample. The fact that this distribution has at

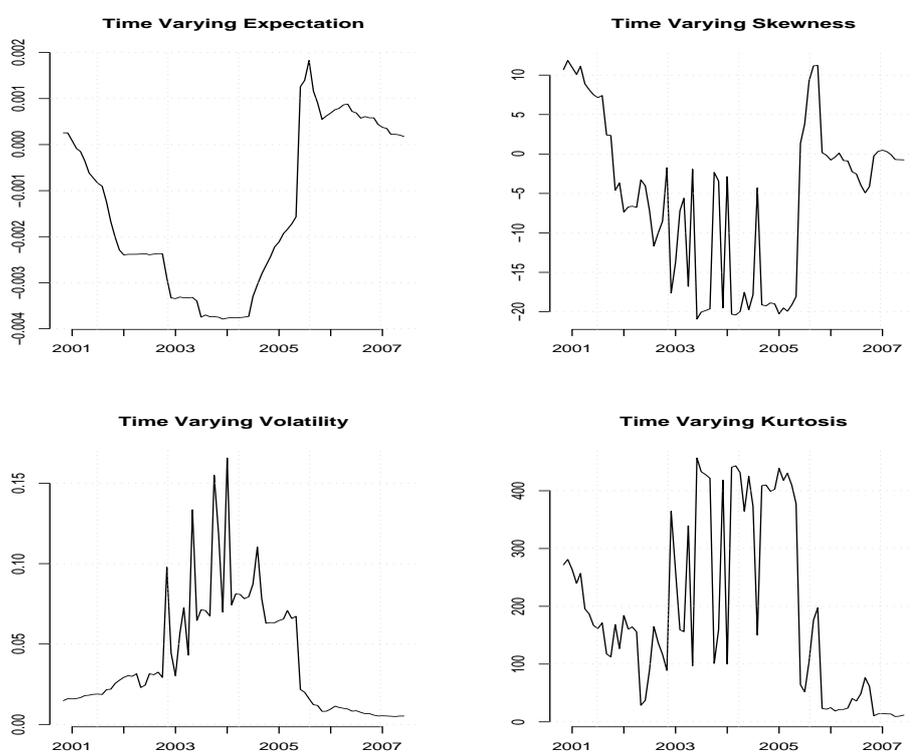


Figure 1.2: Average (annualized), volatility (annualized), skewness and kurtosis of the log-returns of full sample.

	Skewed Student	Skewed Laplace	NIG	Gaussian
# of test acceptance	39	65	73	36
# of test rejection	38	12	4	41
Total	77	77	77	77

Table 1.1: P-value associated to a Kolmogorov Smirnov test.

least one more parameter that the other competitors may explain the quality of the fit. However, this is not a drawback for my approach: what I favor here is the flexibility of the distribution for estimation purposes. For all these reasons, I will only consider the NIG distribution for the models that are to be presented in the following section.

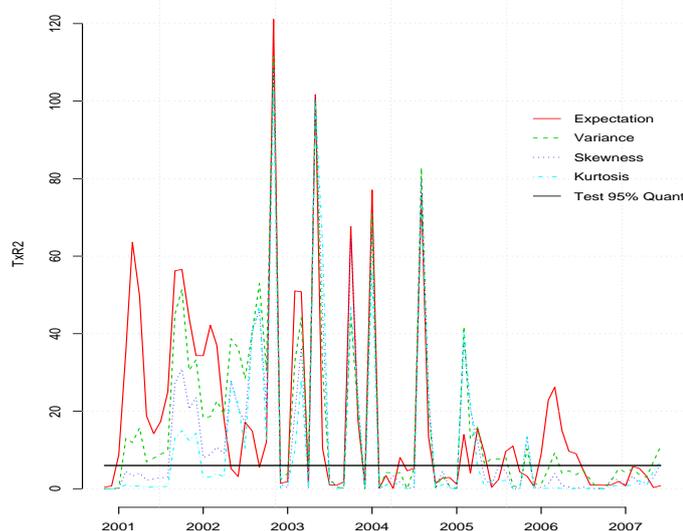


Figure 1.3: Engle (1982)'s $T \times R^2$ test for autoregressive moments. The plain black line indicates the χ^2 quantile used to test the null hypothesis.

1.1.2 Flexible time series models

On the basis of the previous preliminary empirical work, I propose a series of NIG-based new time series models allowing for time varying volatility, skewness and kurtosis. To achieve such an aim, I let the more sophisticated version of the model have dynamic parameters controlling the skewness and the kurtosis. I first present the general settings of the models and then discuss the estimation methodology.

1.1.2.1 General settings of the model

The Normal Inverse Gaussian distribution (NIG hereafter) has been introduced by Barndorff-Nielsen (1997) and used successfully in many financial applications (see among others Jensen and Lunde (2001)). This distribution accommodates the basic stylized facts of financial time series such as asymmetry and excess kurtosis greater than zero. In this section, I build nested time series models based on this distribution to provide estimates of the subjective distribution.

A stochastic variable X is said to be normal inverse Gaussian distributed – $X \sim NIG(\alpha, \beta, \delta, \mu)$ – if it has a probability density function of the following form:

$$f_X(x) = \frac{\alpha\delta \exp[p(x)]}{\pi q(x)} K_1[\alpha q(x)], \quad (1.26)$$

with $K_1(x)$ the modified Bessel function of the second kind, with index 1; $p(x) = \delta\sqrt{\alpha^2 - \beta^2} + \beta(x - \mu)$, $q(x) = \sqrt{(x - \mu)^2 + \delta^2}$ with $\alpha > 0$, $|\beta| < \alpha$ and $\delta > 0$. The expectation, variance, skewness and kurtosis of X have a closed form expression and are:

$$E[X] = \mu - \frac{\delta\beta}{\sqrt{\alpha^2 - \beta^2}} \quad V[X] = \frac{\delta\alpha^2}{\sqrt{\alpha^2 - \beta^2}} \quad (1.27)$$

$$S_k[X] = \frac{3\beta}{\alpha} \frac{1}{\sqrt{\delta\sqrt{\alpha^2 - \beta^2}}} \quad K_u[X] = 3 \left(1 + 4 \left(\frac{\beta}{\alpha} \right)^2 \right) \left(\frac{1}{\delta\sqrt{\alpha^2 - \beta^2}} \right). \quad (1.28)$$

The characteristic function of X is given by:

$$\phi(\omega) = \exp \left\{ i\omega\mu + \delta \left(\sqrt{\alpha^2 - \beta^2} - \sqrt{\alpha^2 - (\beta + i\omega)^2} \right) \right\}, \quad (1.29)$$

and the moment generating function of X is:

$$\psi(\omega) = \exp \left\{ \omega\mu + \delta \left(\sqrt{\alpha^2 - \beta^2} - \sqrt{\alpha^2 - (\beta + \omega)^2} \right) \right\}. \quad (1.30)$$

The Laplace transform is defined whenever $\omega \in]\beta - \alpha : \beta + \alpha[$.

Here, I use this distribution to model the conditional distribution of the log-increments of the future rates $\Delta f(t, T)$ introduced in equation (1.20). It is noteworthy that by doing so, rates are positive with probability 1, which is an essential feature of any interest rates model.

In the following, I consider a fixed maturity and the notation can be simplified such that the log-increments are now denoted by Δf_t . Now, the general settings are:

$$\Delta f_t = \sigma_t \epsilon_t,$$

and I consider the following models:

Model 1: $\sigma_t = \sigma$ and $\epsilon_t | \sigma_t \sim NIG(\alpha, \beta, \delta, \mu)$.

Model 2: $\sigma_t = \sqrt{\omega_0 + \omega_1 \sigma_{t-1}^2 + \omega_2 \Delta f_{t-1}^2}$ and $\epsilon_t | \sigma_t \sim NIG(\alpha, \beta, \delta, \mu)$

Model 3: $\sigma_t = \sqrt{\omega_0 + \omega_1 \sigma_{t-1}^2 + \omega_2 \Delta f_{t-1}^2}$ and $\epsilon_t | \sigma_t \sim NIG(\alpha_t, \beta_t, \delta, \mu)$, with:

$$\begin{aligned} \alpha_t &= \kappa_0 + \kappa_1 \alpha_{t-1} + \kappa_2 \exp\{\chi \epsilon_{t-1}\} \\ \beta_t &= \gamma_0 + \gamma_1 \beta_{t-1} + \gamma_2 \exp\{\chi \epsilon_{t-1}\} \end{aligned}$$

Model 4: $\sigma_t = \sqrt{\omega_0 + \omega_1 \sigma_{t-1}^2 + \omega_2 \Delta f_{t-1}^2}$ and $\epsilon_t | \sigma_t \sim NIG(\alpha_t, \beta_t, \delta, \mu)$, with:

$$\begin{aligned} \alpha_t &= \kappa_0 + \kappa_1 \alpha_{t-1} + \kappa_2 \exp\{\chi_1 \epsilon_{t-1}\} \\ \beta_t &= \gamma_0 + \gamma_1 \beta_{t-1} + \gamma_2 \exp\{\chi_2 \epsilon_{t-1}\}, \end{aligned}$$

where $\epsilon_t | \sigma_t$ means that ϵ_t is taken conditionally upon σ_t . ϵ_t conditionally on $\underline{\epsilon_{t-1}} = \{\epsilon_{t-1}, \epsilon_{t-2}, \dots\}$ is assumed to be a second order stationary process. These models are thus nested, displaying richer and richer dynamics and a growing number of parameters. Model 1 is an homoscedastic NIG model. Model 2 is GARCH(1,1) model with NIG innovations. Model 3 has a GARCH(1,1) variance dynamics and innovations with varying coefficients with the same parameter χ relating the dataset to the parameters' space. Finally, model 4 has $\chi_1 \neq \chi_2$. For all these models, the parameters governing the innovations are chosen conditionally upon σ_t . This way, the focus of the modeling work is on the third and fourth moments, given that the α parameter of the NIG controls the kurtosis and β the skewness.

In an approach with dynamic parameters, the main problem is to choose which power (or functional form) of $\underline{\epsilon_{t-1}}$ is related to each parameter. In model 3 and 4, I propose to relate the parameters to the conditional spectral moments of ϵ_{t-1} , i.e. $\exp\{\chi \epsilon_{t-1}\}$. This allows a direct mapping of the past information $\underline{\epsilon_{t-1}}$ into the parameters space and requires no a priori knowledge on the proper moment to match. What is more,

it may highlight the fact that each parameter of the distribution that is modeled may depend on a different spectral moment, when testing between model 3 and 4 whether it is possible to impose $\chi_1 = \chi_2 = \chi$. In the NIG case, this dependence of the parameters on different values can be documented. For example, Hanssen and Oigard (2001) showed how the NIG parameters are differently related to the cumulant generating function – and thus to the moment generating function. It should be possible to make this dependency over past observation richer, relating α_t and β_t to ϵ_{t-1} and ϵ_{t-2} for example. Due to numerical complexity and to the number of parameters to estimate, I discarded this possibility. My work is obviously related to the work of Jondeau and Rockinger (2003). They propose several ways to map $\underline{\epsilon_{t-1}}$ into the parameter space for the Skewed student t distribution of Hansen (1994) in a GARCH framework again, using several moments or truncated moments; but their approach requires to have preliminary intuitions about the way to truncate series or about the power of the series to consider, which is hardly the case with NIG distribution.

Model 1 and 2 yield second order stationary processes whenever the usual conditions on the existence and stationarity of NIG and GARCH processes are fulfilled. In particular, I need to impose that $\omega_0, \omega_1, \omega_2$ are positive and that $|\omega_1 + \omega_2| < 1$. The conditions for model 3 and 4 are derived using the conditional moments of order 1 and 2 of each dynamic parameter. I thus need to impose $|E[\beta_t]| < E[\alpha_t]$. $E[\beta]$ does not need to be positive. More precisely¹¹, for model 3, the conditions are: $|\omega_1 + \omega_2| < 1$, $\omega_0, \omega_1, \omega_2 > 0$, $E[e^{\chi\epsilon_{t-1}}]$ exists¹², $\kappa_1 \neq 1$, $\gamma_1 \neq 1$, $\text{sgn}(\kappa_0 + \kappa_2\psi(\chi)) = \text{sgn}(1 - \kappa_1)$ and $|E[\beta_t]| < E[\alpha_t]$. Similar conditions can be obtained for model 4: $|\omega_1 + \omega_2| < 1$, $\omega_0, \omega_1, \omega_2 > 0$, $E[e^{\chi_1\epsilon_{t-1}}]$ and $E[e^{\chi_2\epsilon_{t-1}}]$ exists, $\kappa_1 \neq 1$, $\gamma_1 \neq 1$, $\text{sgn}(\kappa_0 + \kappa_2\psi(\chi_1)) = \text{sgn}(1 - \kappa_1)$, and $|E[\beta_t]| < E[\alpha_t]$.

These conditions are obtained when computing the unconditional expectation of α_t and β_t . I chose to let $\kappa_0, \kappa_1, \kappa_2, \gamma_0, \gamma_1, \gamma_2$ to be either positive or negative, provided that the above constraints are satisfied, in so far as it is hard to have a precise idea of the domain of definition of these parameters. Constraining these parameters to be positive may be of no harm for the final results, but some of the estimated parameters may be negative without threatening both the conditional and unconditional existence of the process.

1.1.2.2 Estimation methodology

Now, I present the estimation methodology used to fit the model on a real dataset. I propose to use a sequential maximum likelihood approach to estimate the parameters of

¹¹ $\text{sgn}(x)$ is the sign function: -1 if $x < 0$ and 1 if not.

¹²The conditions of existence for the moment generating function can be found in e.g. Hans (2001).

the dynamic models presented in the previous section. I first consistently estimate the GARCH parameters using a pseudo maximum likelihood approach, assuming that the innovations are Gaussian. Then, once the GARCH parameters have been estimated, I estimate the remaining parameters using the conditional maximum likelihood method, using the filtrated GARCH innovations estimated before. The likelihood function is obtained by assuming that the residuals are NIG distributed. The adequation tests performed in Section 2 clearly point toward the use of this distribution.

The conditional log likelihood function associated to the NIG distribution is:

$$\begin{aligned} \ln L = & (T - 1) \ln \delta + \sum_{t=2}^T \ln \alpha_t + \sum_{t=1}^T \delta \sqrt{\alpha_t^2 - \beta_t^2} + \sum_{t=2}^T K_1 \left(\alpha_t \sqrt{\delta^2 + (\epsilon_t - \mu)^2} \right) \\ & + \sum_{t=2}^T \beta_t \left(\frac{\epsilon_t}{\sigma_t} - \mu \right) - (T - 1) \log(\pi) - \frac{1}{2} \sum_{t=2}^T \ln (\delta \sigma_t^2 + (\epsilon_t - \mu \sigma_t)^2), \end{aligned}$$

with $K_1(\cdot)$ being the previously mentioned Bessel function of the second kind with index 1 and π being approximatively equal to 3.14159. I assume that the usual regularity conditions for the maximum likelihood method to be both consistent and asymptotically Gaussian are verified. The latter expression is maximized using a simulated annealing method, as presented in Belisle (1992), given the possible existence of several local optima.

With models 3 and 4, the derivatives of the log likelihood with respect to the parameters are rather involved, because of the Bessel function. I thus propose to estimate the estimates' variance covariance matrix Σ using bootstrap. This matrix will be estimated using the block bootstrap approach. This bootstrap procedure is described in Efron and Tibshirani (1993).

1.1.3 Results and Event study

This section is devoted to the analysis of the empirical results. I first discuss the bulk results: the significativity of the estimates and the log-likelihood ratio tests that were performed for model selection purposes. Then, I present an event study the aim of which is to test the ability of the model to capture known features of the subjective distribution.

1.1.3.1 Main estimation results

The estimation results are presented in table 1.2 for model 1, in table 1.3 for model 2, in table 1.4 for model 3 and in table 1.5 for model 4.

Contracts	α	β	δ	μ
December 2006	81.99929	-24.2365	0.00238	0.00025
<i>(St. Dev.)</i>	(8.33558)	(11.43168)	(0.00107)	(0.00017)
January 2007	80.81522	-23.80958	0.00243	0.00023
<i>(St. Dev.)</i>	(5.32102)	(13.59215)	(0.00109)	(0.00024)
February 2007	82.61353	-27.50026	0.00294	0.00046
<i>(St. Dev.)</i>	(4.039)	(7.34675)	(0.00126)	(0.00032)
March 2007	82.51478	-25.77417	0.00339	0,00040
<i>(St. Dev.)</i>	(3.96988)	(8.94021)	(0.00135)	-0,00030

Table 1.2: Subjective distribution: Estimation Result for model 1.

Contracts	α	β	δ	μ
December 2006	19.72028	-1.28814	0.00196	0.00014
<i>(St. Dev.)</i>	(5.42572)	(2.41243)	(0.00073)	(0.00013)
January 2007	19.63776	-2.47544	0.00214	0.00013
<i>(St. Dev.)</i>	(2.30486)	(1.53736)	(0.00077)	(0.00012)
February 2007	25.95418	-2.30276	0.00267	0.00022
<i>(St. Dev.)</i>	(17.8399)	(2.95125)	(0.00094)	(0.00021)
March 2007	28.02034	-2.18745	0.00281	9,00E-05
<i>(St. Dev.)</i>	(15.72817)	(2.47819)	(0.00118)	(0.00026)

Table 1.3: Subjective distribution: Estimation Result for model 2.

Contracts	κ_0	κ_1	κ_2	γ_0	γ_1	γ_2	δ	μ	χ
December 2006	0.71189	0.68914	0.71187	-0.08465	-0.09915	-0.08472	0.00202	0.00011	0.04968
(St. Dev.)	(0.32016)	(0.24499)	(0.32013)	(0.07901)	(0.01465)	(0.07896)	(0.00079)	(0.00011)	(0.00111)
January 2007	0.70322	0.64884	0.70319	-0.08771	-0.09917	-0.08781	0.00206	9,00E-05	0.04927
(St. Dev.)	(0.54862)	(0.24375)	(0.54854)	(0.06636)	(0.00915)	(0.06619)	(0.00082)	(0.00013)	(0.00428)
February 2007	0.60165	0.58186	0.60154	-0.11875	-0.09543	-0.11884	0.00235	0.00016	0.04702
(St. Dev.)	(0.30081)	(0.27026)	(0.30055)	(0.2056)	(0.04276)	(0.20608)	(0.00096)	(2.00E-04)	(0.02441)
March 2007	0.83497	0.7271	0.835	-0.12828	-0.06981	-0.12823	0.0028	8,00E-05	0.05253
(St. Dev.)	(0.46218)	(0.22909)	(0.46254)	(0.19541)	(0.16773)	(0.19513)	(0.00113)	(0.00028)	(0.03069)

Table 1.4: Subjective distribution: Estimation Result for model 3.

Contracts	κ_0	κ_1	κ_2	γ_0	γ_1	γ_2	δ	μ	χ_1	χ_2
December 2006	0.67249	0.64738	0.67247	0.02928	0.08713	0.02922	0.00195	0.00013	0.04963	0.04995
(St. Dev.)	(0.32288)	(0.2438)	(0.32285)	(0.04142)	(0.00798)	(0.04146)	(0.00074)	(9,00E-05)	(0.00099)	(5,00E-05)
January 2007	0.7397	0.68358	0.73969	0.03475	0.0818	0.03463	0.00217	0.00014	0.04976	0.04988
(St. Dev.)	(0.485)	(0.29177)	(0.48499)	(0.12857)	(0.03975)	(0.12839)	(0.00076)	(0.00013)	(0.00069)	(4,00E-04)
February 2007	0.81814	0.81833	0.81811	-0.04199	0.09453	-0.04204	0.00273	0.00017	0.04957	0.05001
(St. Dev.)	(0.38033)	(0.14075)	(0.38032)	(0.14463)	(0.09314)	(0.14464)	(0.00094)	(0.00022)	(0.001)	(6,00E-05)
March 2007	0.79315	0.696	0.79314	-0.0342	0.09559	-0.03421	0.00287	9,00E-05	0.04989	0.05014
(St. Dev.)	(0.48094)	(0.27547)	(0.48095)	(0.18922)	(0.09249)	(0.18928)	(0.00116)	(0.00029)	(0.00207)	(0.00114)

Table 1.5: Subjective distribution: Estimation Result for model 4.

Contracts	LL - model 2	LL - model 3	T stat	χ^2 quantile	P-value
December 2006	-1406,6721	-399,4014	2014,5413	11,0705	0,0000
January 2007	-2121,7703	-563,9566	3115,6274	11,0705	0,0000
February 2007	-2939,1257	-590,9558	4696,3399	11,0705	0,0000
March 2007	-3648,5262	-1039,1681	5218,7162	11,0705	0,0000

Table 1.6: Likelihood Ratio tests for model 2 vs. model 3.

Contracts	LL - model 3	LL - model 4	T stat	χ^2 quantile	P-value
December 2006	-399,40145	-353,67602	91,45086	3,84146	0,00000
January 2007	-630,25184	-563,95661	132,59046	3,84146	0,00000
February 2007	-1181,20967	-590,95577	1180,50780	3,84146	0,00000
March 2007	-1039,16812	-934,45203	209,43219	3,84146	0,00000

Table 1.7: Likelihood ratio tests for model 3 vs. model 4.

Most of the parameters estimated are significant up to a 5% risk level. The parameters relating α_t and β_t to past observations are significant, either for models 3 and 4. This is a corner stone for any model with time varying parameters of the kind that is presented here: should it be 0, the parameters α_t and β_t would be degenerated.

The tables 1.6 and 1.7 present Likelihood ratio tests, for model selection purposes. I briefly recall the methodology: for example, let model 3 be the constrained model, with log likelihood denoted $\ln L_c$ and model 4 be the unconstrained model, with a log-likelihood denoted $\ln L_u$. The null hypothesis $\mathbb{H}_0: \chi_1 = \chi_2$ assumes that the constraint in model 3 can statistically be imposed. The test statistics is then:

$$LR = 2(\ln L_c - \ln L_u), \quad (1.31)$$

with the previous notations. Under the null hypothesis, this statistic has a Chi-square distribution, with a degree of freedom equal to the number of constraints imposed in the constraint model. For model 3 vs. model 4, there is only one constraint: $\chi_1 = \chi_2 (= \chi)$. For model 2 vs. model 3, there are 5 constraints: $\kappa_1 = \kappa_2 = \gamma_1 = \gamma_2 = \xi = 0$.

Globally, model 4 is always the one that is favored by the test, bringing support to my approach. As expected, the parameters driving α_t and β_t are not linked to same spectral moments, in so far as χ_1 is statistically different from χ_2 . What is more, the spectral moment to which they are related is close to 0. This finding is not totally surprising: around 0, the derivatives of the moment generating function are known to deliver information regarding any moments of the distribution.

Like when selecting the distribution, I favor again the model that contains the greater number of parameters. This is however not a real problem since I am not interested

in the forecasting ability of the model. My problem is basically a filtration problem: the conditional distribution of the log increments of the futures is unobservable and I propose to estimate it from the time series at hand. Thus, my concern is the in-sample quality of the fit. In this perspective, the result that the more parameter I have, the better the obtained fit is rather intuitive. However, I cannot increase the number of parameters at will for the numerical feasibility of the estimation strategy: this is why I do not propose a more complex model.

Finally, I propose to use the fourth model to perform an event study so as to assess the global ability of the model to capture key facts about the market perception of monetary policy and back the aforementioned empirical results.

1.1.3.2 Event study

I propose here to observe the changes in probabilities for the maturities available in the sub-sample dataset during two particular Central Bank decision meetings. These meetings are the ones that occurred on January, 31st 2006 and August, 8th 2006. During these days, the Fed announced the new target rate applying until the next Central Bank meeting, along with an economic justification of this decision. In the meantime, Mr. Bernanke delivered speeches concerned with the future evolution of the macroeconomic figures and thus of the stance of the future American monetary policy. These two decision meetings took place in an interesting period, that matches the end of a tightening period, i.e. of a rising target rate period. During the meeting of January, no rate cut were forecast by the bond market, and progressively, over the year, a rate cut forecast progressively appeared. It is very important to assess the ability of my model to capture these changes in the market perception of monetary policy.

The change in the Fed's target rate process is very well documented (see Goodfriend (1990) for a detailed analysis stylized facts related to the Fed target rate). The target rate is made of multiple of 25 basis points, i.e. 25/100 percents, and is thus a discrete process. The discreteness of the process is an important feature of monetary policy (see e.g. Guégan and Ielpo (2006) and the references within). I propose here to compute the estimated probabilities associated to these discrete outcomes implied by the continuous and time varying distributions. The computation of these discrete probabilities is made by integrating numerically the continuous distribution over 25 bps sets, using the conditional parameters estimated with model 4. Thus if R_T is the future target rate for the maturity date T , then its support S_T is such that:

$$S_T = \{0.25, 0.5, \dots, 4, 4.25, \dots\}. \quad (1.32)$$

Thus $P(R_T = k)$, with $k \in S_T$ can be computed as:

$$P(R_T = k) = \int_{k-0.125}^{k+0.125} \hat{f}(x) dx, \quad (1.33)$$

where $\hat{f}(\cdot)$ is the subjective distribution density estimated through the historical distribution of the futures rates of maturity T , following my hypothesis 1. Given that the NIG cumulative distribution function has no closed form expression, the expression in equation (1.33) must be numerically computed (I used trapezoidal integration).

The probabilities for these different meetings are presented in figures 1.4 and 1.5. Each figure presents the implicit discrete probabilities before and after the meetings, along with the changes in the probabilities over these decision meetings.

Several observations can be made on the basis of this event study:

- First, it appears that the uncertainty is reduced by the meetings, which is a very classical result of the literature dedicated to the impact of "Central Bank transparency". On this point see e.g. Brière (2006) and the references within. The announcement of the Central Bank decision, along with the chairman's speech brings about a reduction of the volatility in the market. After the meeting, events close to the mode of the distribution are getting higher probabilities. The subjective distribution is thus getting more concentrated around its expectation. This is true for the two decision meetings that are observed.
- Second, over the two meetings, the skew of the distributions changed progressively. In January, the skew of the subjective distributions is positive, underlining the fact that the market is still believing in the increase of the Central Bank target rate over the futures' maturities that are considered. On the contrary, on the August decision meeting, the skew turned out to be negative. At this time, the Central Bank communication made the market believe that no more rate increases were to be feared. Rate cuts were now forecast by the market, underlining the change in economic conditions in the US economy: a slowing economic growth and a slowing inflation.

Beyond this event study, an economic interpretation of several parameters is also possible. First, the parameter controlling the level of persistence of the α_t process (that is κ_1) is higher than that of the β_t (that is γ_1) process over every sample: the kurtosis is a more persistent process than the skewness one. More, for both α_t and β_t the estimated value of κ_1 and γ_1 are remarkably stable across samples. The signs associated to the estimated γ_0 and κ_0 yield interesting pieces of information: κ_0 (resp. γ_0) is a function of

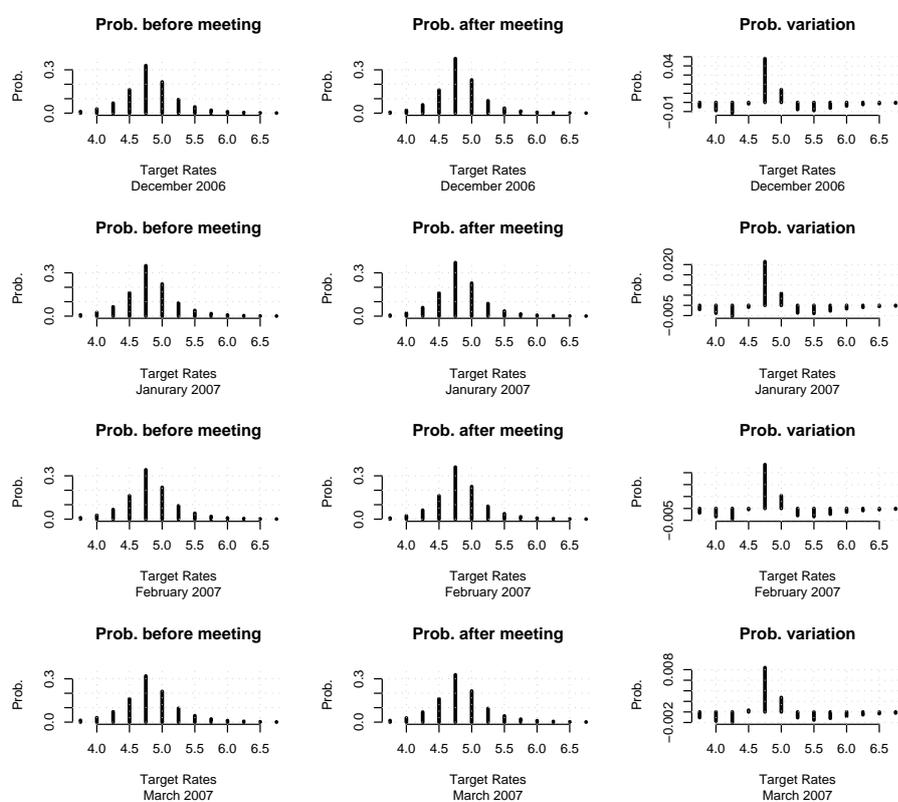


Figure 1.4: Implicit monetary policy scenario probabilities around January, 31st 2006.

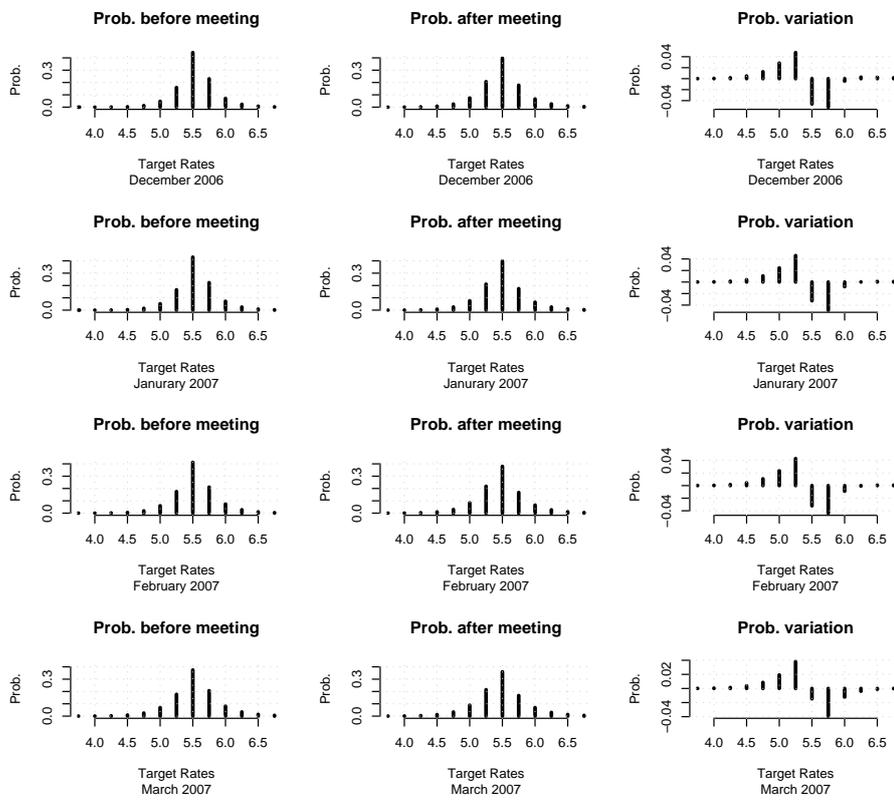


Figure 1.5: Implicit monetary policy scenario probabilities around August, 8th 2006.

the unconditional expectation of α_t (resp. β_t). Over each dataset, κ_0 remains positive and it is increasing from December to March, revealing a growing level of uncertainty in the financial market perception of the upcoming Fed's decisions: α_t is explicitly related to the thickness of the tails of the subjective distribution and thus to the probability of appearance of extreme values, that is extreme monetary policy decisions. On the contrary, γ_0 changes sign when comparing the December-January estimations to the February-March period, moving from a positive sign to a negative one. This is consistent with the change in the skewness sign remarked during the event study: over months, upcoming rate cuts seemed to be more and more likely to happen. These results are globally confirmed by the observation of the time varying conditional moments obtained with model 4. The computation of these moments can be made using the moments closed form expressions previously mentioned. I present it on figure 1.6: the red mark signals the two Central Bank decision meetings studied earlier. The previous observations still holds when observing the figures: the evolution of the skewness changes after the August meeting, confirming what has been presented earlier. Thus, by looking at these results, the approach developed here seems to be likely to provide an accurate measure of the financial markets' view of the future stance of monetary policy. A tool of this kind should be useful to both monetary policy makers and to market analysts.

1.1.4 Conclusion

In this section, I developed a time series model to infer the subjective distribution from the time series of the fed fund future contracts. To do so, I use the link that I introduced between the dynamic parameters and the moment generating function. The empirical results obtained confirm the interest of this approach. What is more, the assumption that the historical distribution of futures should be close to the subjective distribution of the future short rate is empirically realistic and the model proposed presents empirical performances that validate it. This assumption can thus be used for more general study around the estimation of the subjective distribution, and even extended to the econometrics of stochastic discount factors, given that most of the futures are backed to American options. I now present a joint use of futures and options to estimate the risk aversion in the European carbon market.

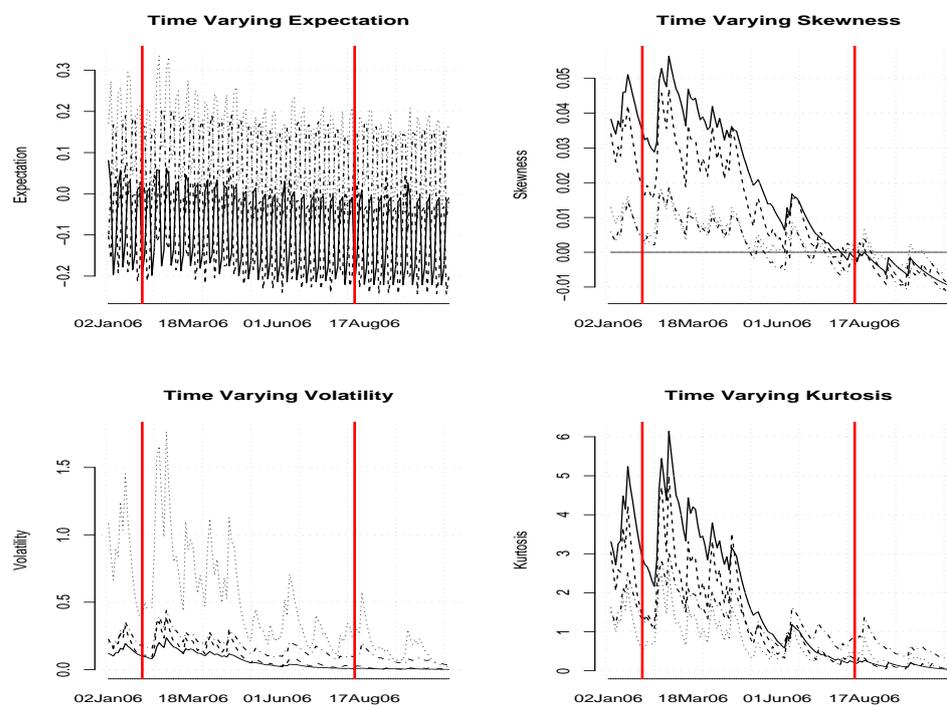


Figure 1.6: Term structure of the time varying moments estimated using model 4.

1.2 Pricing kernels and the Carbon Market

I¹³ now investigate a well known estimation strategy for the pricing kernel on a new commodity market, that is the European Carbon Market. The European Union Emissions Trading Scheme (EU ETS) was created on January 1, 2005 by the European Commission (EC) to foster early compliance with the greenhouse gases emissions reduction targets agreed in the Kyoto Protocol. Its successful implementation is currently being evaluated against its simplicity and the fairly transparency of the trading mechanisms instored. As such, the EU ETS covers up to 46% of CO₂ emissions from European energy-intensive industries. Every year, each industrial plant is allocated EU allowances (EUA)¹⁴ corresponding to its cap and must retribute as many allowances as verified CO₂ emissions. 2.2 billion of allowances were allocated to 10,600 installations across 27 EU Member States in 2005-2007 which are tradable all around Europe on exchanges and by over-the-counter. The next two phases of the scheme are interconnected¹⁵ and will take place during 2008-2012 and 2013-2020.

The role of coordinator, educator and enforcer played by the European Commission is central to the analysis of investors' risk aversion that is about to be developed here. At the start of the EU ETS, most of the information available for trading was deemed as speculative. Consequently, I attempt to characterize investors' hedging strategies for this new carbon commodity by asking the following central question: can we statistically identify a shift in investors' risk aversion around yearly compliance events imposed by the EC?

During Phase I of the EU ETS (2005-2007), spot prices experienced a high level of volatility around each compliance event. Since industrial installations have the obligation to surrender to the EC the exact number of allowances that matches their verified emissions each year around end of March, this institutional compliance event may be used as the cornerstone of each major change in investors' risk aversion. The official report by the European Commission is disclosed by mid-May¹⁶, but installation operators have already a fair amount of information between the publication of their own report and the compilation of verified emissions by the EC to approximate the global level of emissions relative to allowances allocated and to adjust their anticipations. By the end of April 2006, a spot price correction of 54% within four days followed

¹³This section is taken from a paper entitled "Risk Aversion and Institutional Information Disclosure on the European Carbon Market: a Case-Study of the 2006 Compliance Event", with Julien Chevallier and Ludovic Mercier, submitted to a peer-reviewed journal.

¹⁴One EU allowance is equal to one ton of CO₂ emitted in the atmosphere.

¹⁵*i.e.* the transfer of banked or borrowed allowances is allowed between Phase II and III.

¹⁶Indeed, the EC is bound by law to publish the compliance result on May 15 of each year at the latest (see Directive 2003/87/CE).

the announcement by the EC that CO₂ emissions were approximately 3% lower than the allocated allowances during the 2005 compliance period (Ellerman and Buchner (2008)). This particular kind of institutional event led to a shift in investors' risk aversion that I aim at capturing here.

What is more, carbon allowances exhibit strong characteristics of being a nonstandard commodity (Paolella and Taschini (2008)), since installations do not need to physically hold allowances to produce but only to match them with verified emissions for their yearly compliance report to the EC. Consequently, the probability of a potential illiquidity trap exists if market participants face a market squeeze during the compliance event. This specificity of EU allowances adds another line of argument to justify the investigation of the formation of risk aversion and purchasing strategies by market participants around compliance events.

To my best knowledge, this analysis constitutes the first tentative assessment of risk behavior on the EU ETS since it is based on newly available plain vanilla option prices data on the European Climate Exchange which transfer the risk of financial exposure between market agents. Chesney and Taschini (2000) provide an application of CO₂ price dynamics modeling to option pricing, but their study is only based on numerical simulations.

I retrieve empirically investors' risk aversion on the EU carbon market based on the relationship that exists with the risk-neutral and historical probabilities as detailed by Jackwerth (2000). Yet my methodology to recover risk aversion from option and futures prices slightly differs from Jackwerth (2000) whose dataset with monthly frequency would yield to few observations on this new commodity market. Rather, I use non-parametric kernel regression (Ait-Sahalia and Lo (2000), Cont and da Fonseca (2002)) to estimate the risk-neutral probability distribution and an asymmetric GARCH model to estimate the historical probability distribution (Rosenberg and Engle (2002)). Such an approach proved to be useful in documenting changes in implied risk aversion for major equity indices.

I find evidence that shifts in investors' risk aversion are readily observable on the EU ETS following the disclosure of institutional information by the EC on April 30 2006. Moreover, I observe different lower levels of volatility for EUA price series of all maturities after the 2006 compliance period. This latter result suggests that institutional information disclosure has a strong market effect. Besides, I find that periods of increasing markets coincide with periods of higher volatility. This inverted leverage effect reveals that, contrary to equity markets, the risk is associated with increasing allowance prices in a context of a low environmental constraint. What is more, the

main conclusion of this section is that the information disclosure of the 2006 compliance event led to a rise in the market risk aversion. Again, this finding is related to the fact on this particular market, the risk stems from the market fear of an increase in the carbon price.

In the next section, I detail the estimation methodology to recover the risk-neutral and historical probabilities from option and futures prices used in the event study. The final section will be devoted to the presentation of the dataset and the estimation results.

1.2.1 Estimation strategy

As presented in Bertholon et al. (2007a), there are several ways to deal with the risk aversion estimation problem. Absolute risk aversion can be expressed in terms of the historical and risk-neutral probability distributions (Ait-Sahalia and Lo (2000), Jackwerth (2000), Rosenberg and Engle (2002)):

$$RA(x) = \frac{f'(x)}{f(x)} - \frac{q'(x)}{q(x)} \quad (1.34)$$

where $q(x)$ is the risk neutral density and $f(x)$ is the historical density across states. It is easy to see that once two of them are known, the third one is readily available as a by-product. The estimation strategy is thus contingent to the quantity of interest: here, I am interested in the specific estimation of risk aversion. Thus, it will be deduced from my estimation of the risk neutral and historical probability measures. Following the terminology established in Bertholon et al. (2007a), this approach fits the "Risk-Neutral Constrained Direct Modelling" strategy, *i.e.* I make limited assumptions on the risk neutral and historical distributions and no assumption on the pricing kernel.

Following Ait-Sahalia and Lo (2000), I estimate the risk neutral distribution non-parametrically, while the historical distribution is recovered from a semi-parametric GARCH procedure (Barone-Adesi et al. (2007), Rosenberg and Engle (2002)). As explained above, from these estimates I will deduce an empirical estimate of risk aversion. This methodology will then allow me to investigate the empirical characteristics of investors' risk aversion on the European carbon market, along with its potential shifts around yearly compliance events.

Let me first detail how to recover both the risk-neutral and historical probability distributions.

1.2.1.1 Estimation of the Risk Neutral Probability Distribution

Under no arbitrage restrictions, the price of an European call is:

$$C(\tau, K) = B(\tau) \int_{-\infty}^{+\infty} (S_T - K) q(S_T) dS_T = B(\tau) \mathbb{E}^Q[(S_T - K)^+] \quad (1.35)$$

where $C(\tau, K)$ is the premium for a call option of time to maturity τ and strike price K , S is the underlying asset price and r is the risk-free interest rate. $B(\tau)$ is the price of a zero coupon bond with maturity τ and represents the corresponding discount factor, *i.e.* $B(\tau) = e^{-r\tau}$. $\mathbb{E}^Q[\cdot]$ denotes the expectation computed using the risk-neutral distribution. Following Breeden and Litzenberger (1978), we have:

$$\frac{\partial^2 C(\tau, K)}{\partial K^2} = B(\tau) q(S_T | S_T = K). \quad (1.36)$$

Equation (1.36) describes the formal relationship between the second derivative of the call price with respect to the strike price and the risk-neutral density. Since I am mostly interested in recovering the "average" pricing kernel in the option market, I propose to use Ait-Sahalia and Lo (2000) non parametric estimator to the risk neutral density. This estimator uses the link between implied volatility and the risk-neutral distribution: since both implied volatility and risk neutral distribution depend on the moneyness, it is possible to infer the risk neutral distribution from implied volatilities series. When $\sigma(K)$ is a function of the strike that is twice differentiable, using the Black-Scholes model and eq.(1.36) result, Andersen and Wagener (2002) showed that:

$$q(S_T | S_T = K) = e^{r\tau} \frac{\partial^2 C}{\partial K^2} \quad (1.37)$$

$$= N(d_1) \left(\frac{1}{\sigma(K)K\sqrt{\tau}} + \left(\frac{2d_1}{\sigma(K)} \right) \frac{\partial \sigma}{\partial K} + \left(\frac{d_1 d_2 K \sqrt{\tau}}{\sigma} \right) \left(\frac{\partial \sigma}{\partial K} \right)^2 + K \sqrt{\tau} \frac{\partial^2 \sigma}{\partial K^2} \right). \quad (1.38)$$

For $\sigma(\tau, K)$ to be expressed as a function of the strike price, it may be estimated either parametrically with a polynomial¹⁷ or non-parametrically. Ait-Sahalia and Lo (2000) recommend the use of a non parametric estimator of the volatility surface (see also Cont and da Fonseca (2002)). This estimator is particularly adapted to situations where one does not need a day-by-day estimator of this probability density function, but an estimator of the average risk neutral distribution over a large time period. This approach is close to that of Jackwerth (2000). I propose to use the non-parametric approach that also offers the advantage to be more robust to market anomalies which are very likely to occur on such a new commodity market. The non-parametric Nadaraya-Watson estimator with $k = \frac{K}{S}$ defined as the moneyness and τ fixed as in Cont and

¹⁷See Andersen and Wagener (2002) and Brière (2006).

da Fonseca (2002):

$$\sigma(k, \tau) = \frac{\sum_{i,j} K\left(\frac{\tau - \tau_i}{h_1}\right) K\left(\frac{k - k_j}{h_2}\right) \sigma(\tau_i, k_j)}{\sum_{i,j} K\left(\frac{\tau - \tau_i}{h_1}\right) K\left(\frac{k - k_j}{h_2}\right)} \quad (1.39)$$

with

$$K(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \quad (1.40)$$

a Gaussian kernel. $\{h_1, h_2\}$ are bandwidth parameters that determine the degree of smoothing. As pointed out by Cont and da Fonseca (2002), too small values will lead to a bumpy surface while too large ones will smooth away important details. On the optimal choice of these parameters, I follow the methodology used in the latter paper. With this non parametric estimator of implied volatility at hand, it is possible to derive it with respect to the strike price and then use it in equation (1.38) to finally recover the risk neutral distribution for a given horizon τ . It is noteworthy to remark that this methodology imposes martingality restrictions as a by-product. These restrictions are essential to recover the risk neutral distribution (see Jurczenko et al. (2001)).

Finally, I briefly discuss the computation of implied volatility series from option prices. Building on the previous notation, $C(\tau, K)_{obs}$ is the observed call option price and $C(\tau, K, \sigma)_{BS}$ is the Black-Scholes (BS) price computed using the implied volatility σ . By definition, $C(\tau, K)_{obs} = C(\tau, K, \sigma)_{BS}$. The implied volatility of the strike price is obtained by numerically inverting the BS formula, which can be done by solving:

$$\min_{\sigma} (C(\tau, K)_{obs} - C(\tau, K, \sigma)_{BS})^2 \quad (1.41)$$

Then, allowing σ to be a function of the strike price, eq.(1.36) can be used to recover the risk-neutral distribution.

1.2.1.2 Historical Probability Distribution

The first difficulty to be dealt with when using commodity prices is the existence of the so-called convenience yield. On equity market, the price F_t of any future written on an equity stock should be equal to the compounded value of the spot asset S_t . Thus, for any maturity, the following relation holds:

$$F_t = e^{r(T-t)} S_t, \quad (1.42)$$

with r the risk-free rate applying over the convenient time horizon. This relation does not hold for the carbon market. The spread between F_t and $e^{r(T-t)} S_t$ is known as the

convenience yield. This is a very common feature of commodity markets: to account for this term structure aspect, I model each maturity separately.

There exists an emerging body of literature on the spot-forward parity in the EU ETS. Seifert et al. (2007) reveal a very steep spot price volatility increase when coming toward the end of the trading period. Borak et al. (2000) also pointed out that the term structure for allowance prices changed from initial backwardation to contango and is subject to abrupt changes of expectations. This situation provides a first element of justification to use futures instead of spot prices series in my dataset. Besides, futures exhibit a more coherent price pattern since the April 2006 price collapse that occurred for all maturities. This approach is also in line with Paoletta and Taschini (2008) who argue that the study of spot price dynamics is inadequate due to early political uncertainties on the allowance market. That is why I consider futures contract prices of maturity December 2008 and December 2009¹⁸. What is more, the previous considerations around the use of the future prices to assess the changes in the market expectations still applies here. As for the estimation methodology, Benz and Truck (2006) and Paoletta and Taschini (2008) strongly support the use of GARCH specifications to model the returns of CO₂ emission allowances which is also developed below.

Following the literature dedicated to the stock market, I choose to model the historical distribution using a semi-parametric asymmetric GARCH(p, q) model as in Barone-Adesi et al. (2007). I discuss the goodness of fit of the chosen model compared to other specifications for the CO₂ return series in the next subsection. Note the methodology adopted by Jackwerth (2000) is not applicable because the monthly frequency used would lead to few observations. Besides, it is worth underlining that unlike Rosenberg and Engle (2002) and Jackwerth (2000) I use longer term option prices with a 16-month investor horizon¹⁹ to display my results.

The estimated model is:

$$\begin{cases} r_t = \mu + \epsilon_t \\ \sigma_t^2 = \omega_0 + \omega_1 \epsilon_{t-1}^2 + \omega_2 \sigma_{t-1}^2 + \delta \max(0, -\epsilon_{t-1})^2 \end{cases} \quad (1.43)$$

with $\epsilon_t \sim N(0, \sigma_t)$. If $\delta > 0$ then $Cov(r_t - r_{t-1}, \sigma_{t-1}^2 - \sigma_{t-2}^2) < 0$: it thus take into account the asymmetry also known as the skewness effect in stock price dynamics and the leverage effect in financial economics. The model is estimated in a Pseudo Maximum Likelihood (PML) framework by assuming returns are Gaussian. As presented in Gouriéroux et al. (1984), estimating by PML will lead to unbiased estimates

¹⁸Note I rule out contracts that are not liquid such as the contract of maturity December 2007.

¹⁹As explained in Cont and da Fonseca (2002), this investor horizon has been identified as the best fit in my dataset. Note that this choice does not affect the robustness of the results.

even if the true innovation distribution is not Gaussian. Barone-Adesi et al. (2007) and Rosenberg and Engle (2002) proved the robustness of this approach. The estimates covariance matrix is estimated using the BHHH matrix (see Berndt et al. (1974)). The estimated residuals are then bootstrapped to simulate sampled paths for any maturity of interest. This approach is closely related to the one of Barone-Adesi et al. (2007), even if my purpose here is slightly different. Using these simulated returns, I estimate the conditional historical distribution density using the following Gaussian kernel:

$$f(S) = \frac{\sum_{t=1}^T K\left(\frac{S_t - S}{h}\right) S_t}{\sum_{t=1}^T K\left(\frac{S_t - S}{h}\right)}. \quad (1.44)$$

As in Jackwerth (2000), I select the following bandwidth parameters:

$$h = \frac{1.8\sigma}{\sqrt[5]{n}} \quad (1.45)$$

where S is a point of the future value of the asset price support, h is the kernel bandwidth, σ is the standard deviation of the sample returns and n is the number of observations. Using this methodology, I intend to cope with the non normality of future returns as diagnosed in the next section. Thus, my results will not be tainted by any ill-chosen distribution assumption.

The next section presents the results of this estimation strategy.

1.2.2 Estimation Results and Discussion

This section presents the data used in this event study and then discusses the results.

1.2.2.1 The Dataset

The dataset used includes plain vanilla option closing prices in € of maturity December 2008 and December 2009 traded from October 1, 2006 to November 23, 2007 on the European Climate Exchange (ECX), the most liquid trading platform for carbon derivatives with approximately 86.5% of the total exchange-based trades of allowances. Tables 1.8 and 1.9 display the number of available observations for each contract and the average volume for each strike of option prices in the dataset.

Descriptive statistics regarding each contract may be found in tables 1.10 and 1.11. For both contracts, the negative skewness indicates a distribution with an asymmetric tail extending toward more negative values. The positive excess kurtosis suggest a fat tailed empirical distribution and the presence of extreme observations. Thus, as stated

by the Jarque Berra test statistics, residuals are not Gaussian which is characteristic of financial time-series. The Box Pierce statistic reveals residuals are not autocorrelated. I only consider options of moneyness included between 0.5 and 1.5 to remove unreliable observations characterized by a low volume and a low sensitivity to volatility. However, this spectrum of moneynesses is already wide enough to be likely to help me deliver interesting messages about the change in the market risk aversion over the period that is about to be considered.

I also use futures prices drawn from ECX for the same period²⁰. Figures 1.7 and 1.8 show strong reversals of the futures price series depending on the time period, *i.e.* before and after the yearly compliance event. The underlying asset of the contracts are first and second period spot prices. As underlined in the previous section, due to infrequent trading of spot prices, I prefer using future prices which reflect more accurately agents' trading anticipations and hedging allowance strategies. Again, See Guégan and Ielpo (2008) for additional details. The risk free rate is the one year Euro swap rate commonly used by market agents.

Over the period going from October 1, 2006 to November 23, 2007, I choose to split the dataset before and after the yearly compliance event imposed by the EC to evaluate investors' changes in anticipations. Installations need to report by the end of March their verified emissions that occurred during the preceding year. For instance, CO₂ emissions at the installation level for the calendar year 2006 were reported on March 30, 2007. Then, the information becomes publicly available when the EC officially publishes its report between the end of April and mid-May. Thus, to reflect these institutional events and to capture the state of information available to all market participants with most accuracy, Samples #1 and #2 have been split on April 30, 2007, *i.e.* at the time where the EC issued its official report for the 2006 compliance result²¹. For each contract of maturity December 2008 and December 2009, I therefore identify the two subsamples as being "October 1, 2006 - April 30, 2007" and "May 1, 2007 - November 23, 2007".

²⁰I do not include futures prices of maturity December 2007 in the sample since this contract displays fewer observations and is less liquid.

²¹Note the choice of the sample splitting date between end of March and end of April does not affect the stability of the results.

	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
December 2008																		
Strikes	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
Call	285	285	285	285	285	285	285	285	285	285	285	285	285	285	285	285	285	156
Put	285	285	285	285	285	285	285	285	285	285	285	285	285	285	285	285	285	156
Total	570	570	570	570	570	570	570	570	570	570	570	570	570	570	570	570	570	312
December 2009																		
Strikes	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
Call	247	247	247	247	247	247	247	247	247	247	247	247	247	247	247	247	156	156
Put	247	247	247	247	247	247	247	247	247	247	247	247	247	247	247	247	156	156
Total	494	494	494	494	494	494	494	494	494	494	494	494	494	494	494	494	312	312

Table 1.8: Table 1: Number of available option prices. Source: European Climate Exchange

	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
December 2008																		
Strikes	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
Call	-	-	50	50	142	143	73	159	213	58	185	317	96	500	222	158	-	100
Put	185	80	118	172	103	58	58	200	-	-	-	-	-	-	-	-	-	-
Total	185	80	168	222	246	200	132	359	213	58	185	317	96	500	222	158	-	100
December 2009																		
Strikes	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
Call	-	-	-	-	135	-	50	-	-	366	-	-	-	-	217	-	-	-
Put	-	-	-	15	200	-	-	-	-	-	-	-	-	-	-	-	-	-
Total	-	-	-	15	335	-	50	-	-	366	-	-	-	-	217	-	-	-

Table 1.9: Average volume contract for each strike. Source: European Climate Exchange

	Min	Max	Average	Std. Dev.	Skewness	Excess Kurtosis	Jarque Berra	Box pierce
Returns	-0,092	0,094	0,001	0,026	-0,404	1,667	0	0,202
Residual	-2,644	2,027	-0,002	0,91	-0,345	-0,104	0,056	0,265
Residual sample #1	-2,618	1,963	-0,026	0,872	-0,452	0,355	0,091	0,236
Residual sample #2	-2,644	2,027	0,017	0,939	-0,262	-0,347	0,306	0,865

Table 1.10: Descriptive statistics for December 2008 contract

	Min	Max	Average	Std. Dev.	Skewness	Excess Kurtosis	Jarque Berra	Box pierce
Returns	-0,088	0,101	0,001	0,025	-0,259	1,685	0	0,332
Residual	-2,619	2,076	-0,005	0,908	-0,306	-0,149	0,097	0,377
Residual sample #1	-2,373	2,086	-0,034	0,871	-0,337	0,172	0,292	0,467
Residual sample #2	-2,619	1,985	0,018	0,936	-0,272	-0,31	0,307	0,837

Table 1.11: Descriptive statistics for December 2009 contract

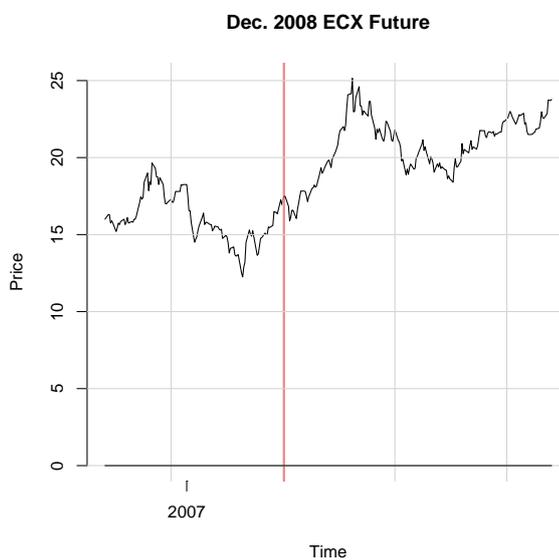


Figure 1.7: Historical price of the ECX future for December 2008 contract

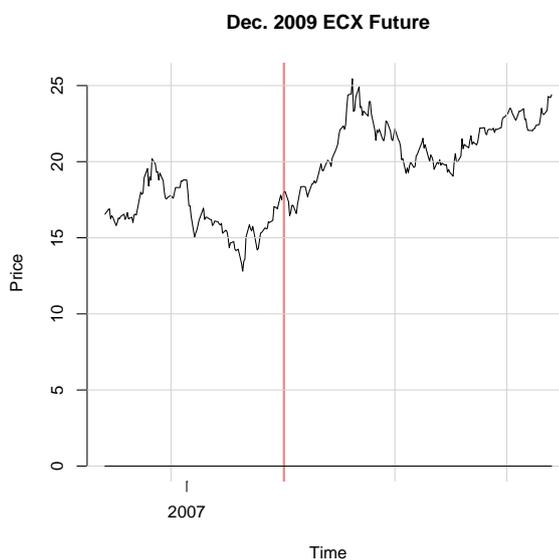


Figure 1.8: Historical price of the ECX future for December 2009 contract

During both split periods, I assume the risk-neutral and historical distributions to be sufficiently stationary to use market prices and recover both of them. This methodology only provides me with estimates of average risk-aversion on the time periods under

consideration²². Finally, for the sake of comparison, I choose to work in a rescaled to maturity of $\tau = 1.3$ on annual basis. This does not affect the result but makes the samples easier to compare.

1.2.2.2 Estimation Results

Now, I review the key estimation results of this event-study. As for the historical probability distribution, tables 1.12 and 1.13 indicate the best model is the asymmetric GARCH(1,1)-GJR to accommodate the leverage effect. The result of the likelihood ratio test confirms the GARCH GJR is the best fit for the historical distribution. The chosen model is the GJR-GARCH model of Glosten et al. (1993) that I re-state below for the ease of the presentation:

$$\begin{cases} r_t = \mu + \sigma_t \epsilon_t \\ \sigma_t^2 = \omega_0 + \omega_1 I_t + \alpha(r_{t-1} - \mu)^2 + \beta\sigma_{t-1}^2 + \delta \max(0, -(r_{t-1} - \mu))^2 \\ \epsilon_t \sim N(0, 1), \end{cases}$$

with r_t the one-day logarithmic return at time t and $I_t = 1$ if t is in Sample #2 (after the compliance result) and 0 for Sample #1 (before the compliance result). ω_1 is statistically different from zero and negative: the European carbon market is characterized by more volatility before the compliance event on April, 2007 than after the compliance event. This finding is consistent with what is expected, *i.e.* that information disclosure is due to reduce uncertainty and thus volatility on financial markets.

Compared to previous literature, these estimates strongly depart from the usual equity-based results. First, while ω_0 and α are higher than the values found in Rosenberg and Engle (2002) and Barone-Adesi et al. (2007), β is systematically lower. However, the degree of persistence of the conditional variance as measured by $(\alpha + \beta)$ is close to the values in the previously cited papers. Second, and most interestingly, δ is negative: periods of increasing market coincide with periods of higher volatility. This increasing feature is the exact opposite of the usual leverage effect found on equity markets²³. In a context of a low environmental constraint on the carbon market, the risk associated to the option contract consists in increasing allowance prices which is the opposite of a standard commodity market²⁴. Thus, beyond the information disclosure effects that led to a lower average volatility in the globally increasing Sample #2, the volatility during increasing periods has been higher than during decreasing periods. Thus, it is

²²See Rosenberg and Engle (2002) on this specific point.

²³Recall that the leverage effect implies a higher level of volatility associated to decreasing prices.

²⁴This logic is conform to the disconnection between first and second period allowance prices described earlier, *i.e.* investors expect increasing allowance prices in a context of increasing allowance scarcity overtime.

possible to disentangle two different asymmetric volatility effects, the first being dependent on information disclosure and the second being an uncovered feature on this new market. However, as explained below when investigating implied volatility, the first effect usually dominates the other.

The rationale behind the second effect may be stated as follows. As shown in figures 1.9 and 1.10, the implied volatilities exhibit smiles with a dramatically different slope depending on the sample considered. For both contracts December 2008 and December 2009, the smile observed in Sample #1 (displayed in blue) is skewed to the right which suggests operators anticipated a decrease of the carbon price before the release of 2006 verified emissions. For Sample #2 (displayed in red), the smile displays a leftward asymmetry which suggests operators anticipated an increase of the carbon price after the confirmation that the number of allocated allowances was higher than verified emissions at the aggregated level²⁵. Between the two contracts, my analysis finds a level of implied volatility in the range of 0.8-0.9 for the second contract of maturity December 2009, which is higher than the values obtained for the first contract of maturity December 2008 comprised between 0.6-0.8. This result may be explained by the fact that the average time to maturity for option prices in the dataset is higher for the second contract compared to the first contract.

The logic at stake to comment the level of implied volatilities obtained is the following. When investors anticipate a sharp price decline, the rationale behind option pricing consists in selling call options with strikes lower than the underlying asset spot value and buying call options with higher strikes. Given the one-to-one relation between option prices and implied volatility, this results in a low implied volatility for low levels of moneyness compared to higher ones. Thus, the implied volatility is lower for levels of moneyness strictly superior to one indicating these declining trends. At every point of the support, the lower the implied volatility, the higher the probability of occurrence of the event. This relationship also explains the changes in the skewness of the risk neutral distribution.

²⁵On April 2007 the second compliance disclosed that verified emissions were about 30 million tons or 1.45% lower than the 2006 allocation. See the EU Environment DG at <http://ec.europa.eu/environment/press/index.htm>. Cited February 2008.

	ω_0	ω_1	α	β	δ	μ	Log-likelihood
ARCH(1)	Estimate	0,079	-0,033	0,088	-	0,129	-666,097
	t-Stat.	6,074	-2,261	12,054	-	6,036	0,000
GARCH(1,1)	Estimate	0,010	-0,006	0,102	0,812	0,150	-659,105
	t-Stat.	3,552	-5,336	54,798	152,358	8,159	0,000
GJR-GARCH(1,1)	Estimate	0,009	-0,006	0,128	0,823	-0,061	-658,519
	t-Stat.	3,799	-5,859	39,471	164,938	-18,046	0,000

Table 1.12: Estimation of the time series models for the December 2008 contracts.

	ω_0	ω_1	α	β	δ	μ	Log-likelihood
ARCH(1)	Estimate	0,081	-0,037	0,078	-	0,136	-659,259
	t-Stat.	5,800	-2,378	12,292	-	6,625	0,000
GARCH(1,1)	Estimate	0,009	-0,006	0,105	0,808	0,150	-652,274
	t-Stat.	3,832	-5,544	54,869	155,761	8,709	0,000
GJR-GARCH(1,1)	Estimate	0,009	-0,006	0,132	0,820	-0,064	-651,645
	t-Stat.	4,238	-6,250	38,876	170,801	-18,370	0,000

Table 1.13: Estimation of the time series models for the December 2009 contracts.

Therefore, the estimation shows a dramatic shift in investors' anticipation around the 2006 compliance event. I expect this result to be of lower magnitude than during the 2005 compliance event²⁶. On April 2007, the EC revealed that verified emissions were about 30 million tons or 1.45% lower than the 2006 allocation²⁷. Two distinct messages are embedded in this diffusion of institutional information. First, the EC confirmed that allocated allowances were higher than the actual level of CO₂ emissions. This first element may explain why market agents were expecting a drop in the carbon price before the 2006 compliance event. Second, the EC revealed that verified emissions were lower than allocated allowances by only 1.45% for the 2006 compliance result, which corresponds to a thinner margin than for the 2005 compliance result²⁸. Thus, market agents have adapted the financial risk of being exposed to a situation of allowance shortage, which may explain why they were expecting an increase in the carbon price after the 2006 compliance event. As developed in Section 2, this futures dynamics is sustained by further EC announcements to restrict allocation and to rely on auctioning during Phase II which have a positive effect on the expected allowance scarcity. As a final line of argument, the decision to maintain the EU ETS at least until 2020 may also contribute to this increasing price pattern.

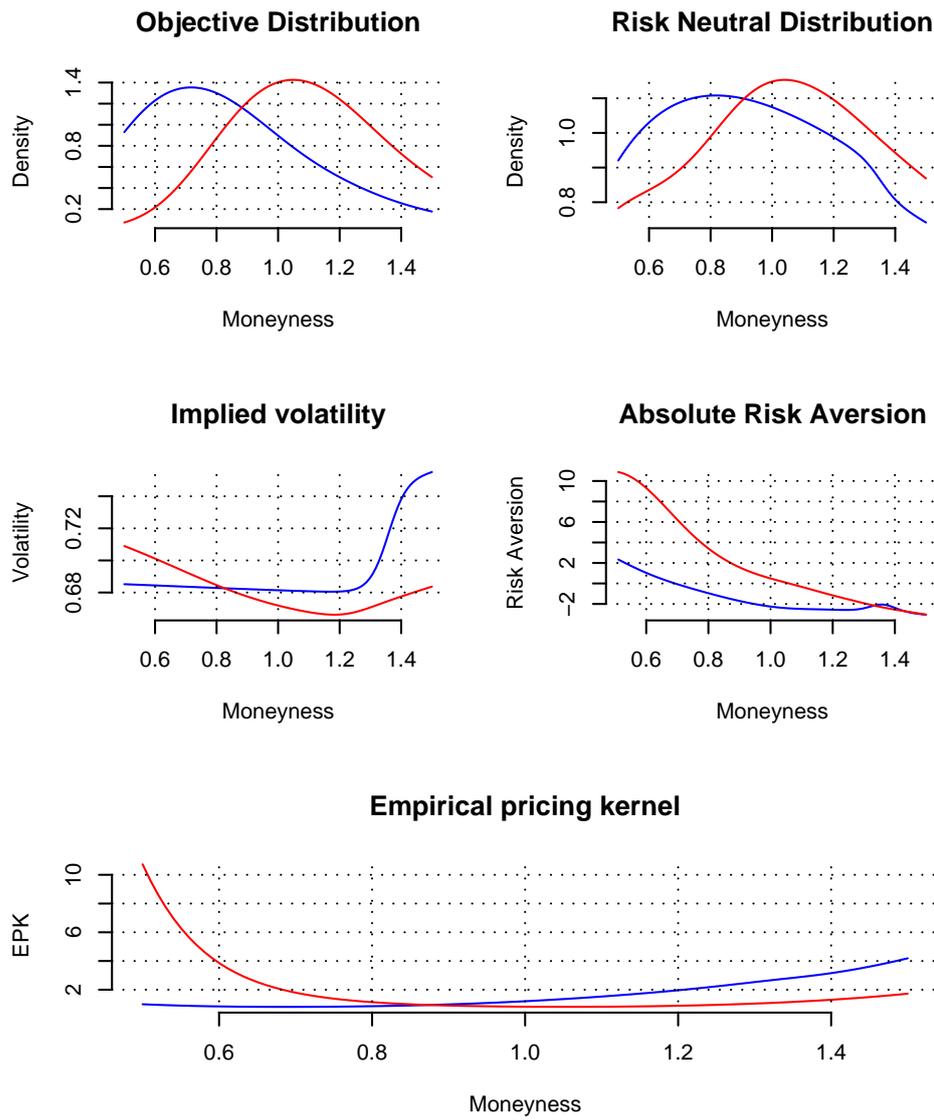
The results obtained for the objective and risk neutral distributions confirm the previously mentioned intuitions. For the risk neutral distribution, in figures 1.9 and 1.10, the blue line which denotes the risk neutral density for Sample #1 has a steeper curve than the red line for Sample #2, which induces more volatility. These results are therefore consistent with what is expected, *i.e.* to obtain lower levels of implied volatility after the EC announcement. The results for the historical density yield the same asymmetries as for the risk neutral density of contracts December 2008 and December 2009.

Turning my attention to risk aversion, the empirical pricing kernels present noticeable shapes that underline the dramatic change in the market risk aversion. The empirical pricing kernel is defined as $\frac{\hat{q}(x)}{\hat{f}(x)}$ for each point of the support of the asset price. Figures 1.9 and 1.10 clearly illustrate this point: before the yearly compliance result (# Sample 1), market agents expect a drop in the spot price whereas after the yearly compliance result (# Sample 2) a sharp price increase is expected. From Figures 1.9 and 1.10, the pricing kernel is countercyclical, *i.e.* it is inversely related to the current market trend. The pricing kernel is decreasing in the context of increasing markets and conversely, as pointed out by Rosenberg and Engle (2002).

²⁶As stated earlier, option prices are not available to capture the magnitude of this effect.

²⁷See the EU Environment DG at <http://ec.europa.eu/environment/press/index.htm>. Cited February 2008.

²⁸Recall that verified emissions were lower than allocated allowances by 3% during the 2005 compliance period (Ellerman and Buchner (2008)).

Figure 1.9: Estimation results for December 2008 contract with $\tau = 1.3$

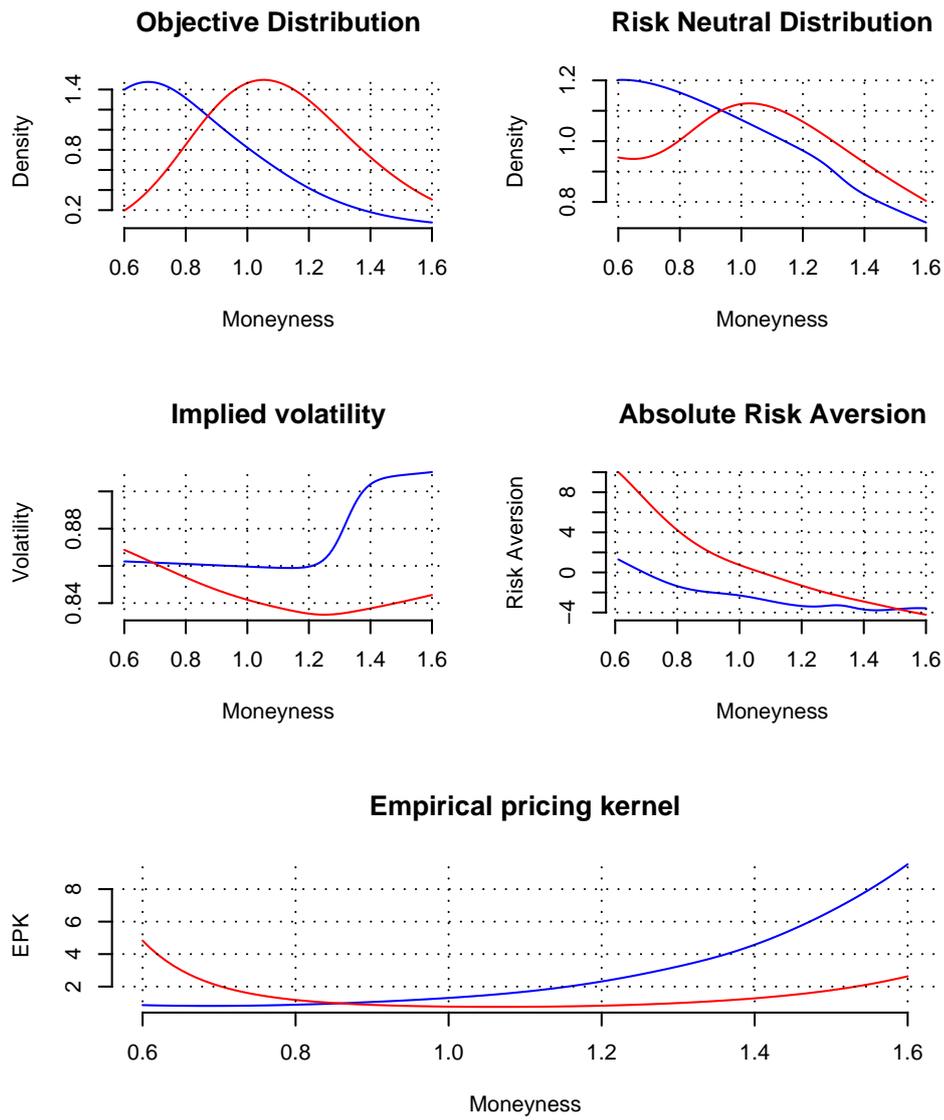


Figure 1.10: Estimation results for December 2009 contract with $\tau = 1.3$

These results should be compared to results obtained on equity markets, using comparable ranges of maturities and moneynesses. Here, the option prices I used display a longer time to maturity than Jackwerth (2000) or Rosenberg and Engle (2002) and the moneyness range discussed here is consequently wider than theirs. Only Barone-Adesi et al. (2007) present empirical results for a comparable range of strikes and maturities. Their estimates are ranging from 2 to 5. As shown in Figures 5 and 6, the estimates range from nearly 0 to 10. These considerably wider estimates suggest that the slope of the pricing kernel is steeper in the Carbon Market. This result applies especially for low moneynesses in Sample #2. Similar comments arise for the graphs of risk aversion where a steep slope for the pricing kernel is associated with a high level of risk aversion.

Thus, one may conclude that risk aversion on the European carbon market is higher than the values typically found on equity markets. It appears consistent with the risk premium associated to the financial exposure on such a new carbon commodity market and the necessity to adopt accurate risk management strategies. Finally, the main conclusion of this event study can be stated as follow: unlike most of the known information disclosure, the 2006 compliance event induced an increase in the market risk aversion. For this compliance event, the message delivered by the European Commission was that upcoming price increases were likely to occur. These price increases being the main fear of the participants of the the Carbon Market, the sharp rise in risk aversion revealed by my event study is easy to understand.

1.2.2.3 Conclusion

The conclusion of this section can be summarized as follow. The information disclosure regarding allotted quotas changed the shape risk aversion in the ECX market. Since rises in the future price is now on the row, risk aversion went up: in quotas markets, the main fear is to see prices go up. Options and futures are thus critical instruments to uncover such shifts: this is essential for the assessment of the developments on this new market. By doing so, it becomes possible to provide information to both risk managers and policy makers regarding the impact of information disclosure on the shared beliefs in the markets.

1.3 Option Pricing under GARCH models with Generalized Hyperbolic innovations

The²⁹ previous sections considerably emphasized the interest of having a discrete time approach to option pricing: it made it possible to document the time varying behavior of the subjective distribution and to test for changes in the risk aversion of the market. There is however a important debate around the use of discrete-time models for option pricing. In this section, I provide a new model of this kind and discuss its ability to produce minimum mispricing errors on equity option books. The discrete time settings and the previous assumption on the shape of the pricing kernel received a large support from empirical datasets. Now, I propose a complete option pricing model based on a discrete time GARCH model, with Generalized Hyperbolic conditional distribution.

This distribution introduced by Barndorff-Nielsen (1977) is known to fit financial dataset remarkably: for example, figures 1.11, 1.12 and 1.13 show its ability to handle the particular tail behavior found in equity indexes returns. More, the distribution clearly passes the usual adequation tests, as presented in table 1. This family of distribution has already been used with empirical successes to model the dynamic of several stock markets (see Barndorff-Nielsen (1995), Eberlein and Keller (1995), Jensen and Lunde (2001), Eberlein and Prause (2002), Fergusson and Platen (2006)) and even to price options in continuous time Lévy type markets (Eberlein and Keller (1995), Eberlein and Prause (2002)).

In the latter article, the authors use the Esscher transform method introduced for option pricing by Gerber and Shiu (1994b) to select a particular equivalent martingale measure with an interesting economic interpretation. For discrete time models this powerful tool has been extended in a general way by Bühlmann et al. (1996) (see also Siu et al. (2004) and Gouriéroux and Monfort (2007) for an equivalent formulation in terms of the exponential affine parametrization of the stochastic discount factor). Note that, although this elegant approach provides a unique martingale measure, there may be other equivalent martingale measures (see e.g Elliott and Madan (1998)). Nevertheless, from a practical point of view, one of the main features of this method with respect to the others is that the conditional distribution of the returns of the risky asset is in general stable under the historical and the risk neutral probabilities (Bertholon et al. (2007b), Siu et al. (2004), Gouriéroux and Monfort (2006), Christoffersen et al. (2006)).

²⁹This section is taken from a paper called "Option Pricing under GARCH models with Generalized Hyperbolic innovations", with Christophe Chorro and Dominique Guégan. It is a CES-Paris 1 working paper.

Similar approaches have already been proposed in the literature, dealing with the incompleteness of the market in a discrete time framework. Through an equilibrium argument Duan (1995) gave an economically consistent approach to price options in GARCH models with Gaussian innovations. Following this methodology, Heston and Nandi (2000) considered a new conditionally Gaussian GARCH model able to cope with the skews in option prices. They derived an almost closed form expression for call option prices and empirically demonstrated its pricing performance. The model being conditionally Gaussian, it usually fails to capture the short term behavior of equity options smiles: Christoffersen et al. (2006) thus extended the Heston and Nandi (2000) model by using the Inverse Gaussian distribution to increase the skewness effect and empirically assessed the higher performances of their approach of option pricing. However, it is not the only way around the skewness effect.

The main novelty that is introduced here is to show how to apply this methodology for GARCH-type models with Generalized Hyperbolic innovations. In particular, the conditional dynamic of the log-returns under the chosen equivalent martingale measure is also of the Generalized Hyperbolic type. Provided that the equivalent martingale measure has been properly selected, no more calibration exercise is required and option prices can be simply computed using Monte Carlo simulations. Since several classical distributions e.g. Normal, Skewed Laplace, Gamma or Inverse Gaussian may be obtained as a limiting case of the Generalized Hyperbolic distribution, many classical dynamics (see Duan (1995), Heston and Nandi (2000), Gouriéroux and Monfort (2006), Siu et al. (2004), Christoffersen et al. (2006)) are recovered from my approach.

	DAX	CAC	UKX	SPX
KS p-value for NIG	0.32	0.3	0.83	0.67
KS p-value for HYP	0.95	0.73	0.53	0.69
KS p-value for GH	0.48	0.31	0.69	0.78
KS p-value for Gaussian	0	0	0	0
AD p-value for NIG	0.28	0.45	0.55	0.5
AD p-value for HYP	0.61	0.5	0.48	0.47
AD p-value for GH	0.45	0.23	0.45	0.55
AD p-value for Gaussian	0	0	0	0

Table 1.14: Kolmogorov-Smirnov and Andersen-Darling adequation tests

This table presents the Kolmogorov-Smirnov and Andersen-Darling adequation tests, testing the adequation of the NIG, Hyperbolic, Generalized Hyperbolic and Gaussian distributions to a dataset of the daily log returns of four major indexes: the French CAC, the German DAX, the US SP500 and the UK FTSE indexes. The sample starts on January 2, 1988 and ends on the December 31, 2007.

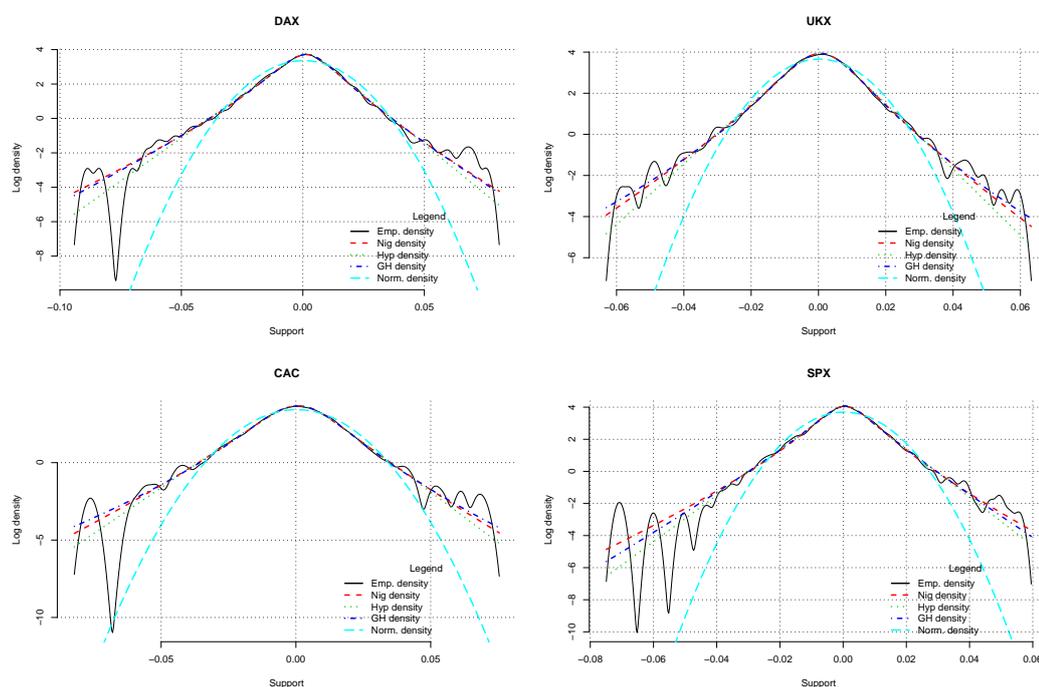


Figure 1.11: Log density

This figure presents the empirical log-density (plain black line) vs. the estimated log density obtained with the NIG (red), Hyperbolic (green), Generalized Hyperbolic (dark blue) and Gaussian (light blue) distributions over different indexes. The estimation has been performed using the daily log returns of four major indexes: the French CAC (bottom left), the German DAX (top left), the US SP500 (bottom right) and the UK FTSE indexes (top right). The sample starts on January 2, 1988 and ends on the December 31, 2007.

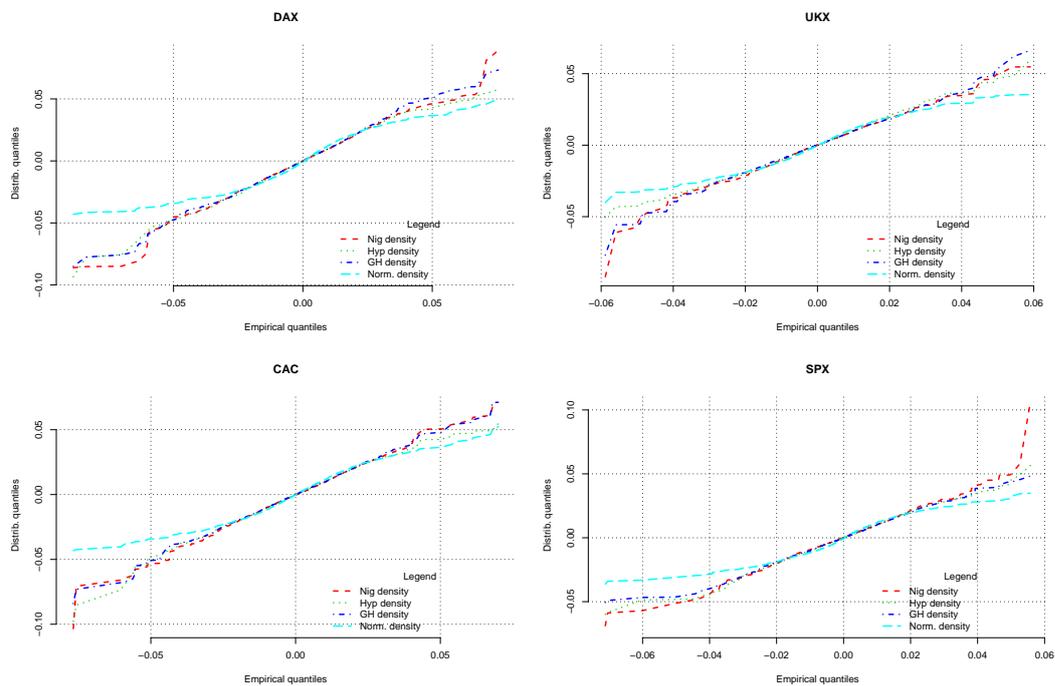


Figure 1.12: QQ-Plot

This figure presents the qq-plots comparing the empirical quantiles over several indexes vs. the estimated quantiles obtained with the NIG (red), Hyperbolic (green), Generalized Hyperbolic (dark blue) and Gaussian (light blue) distributions. The estimation has been performed using the daily log returns of four major indexes: the French CAC (bottom left), the German DAX (top left), the US SP500 (bottom right) and the UK FTSE indexes (top right). The sample starts on January 2, 1988 and ends on the December 31, 2007.

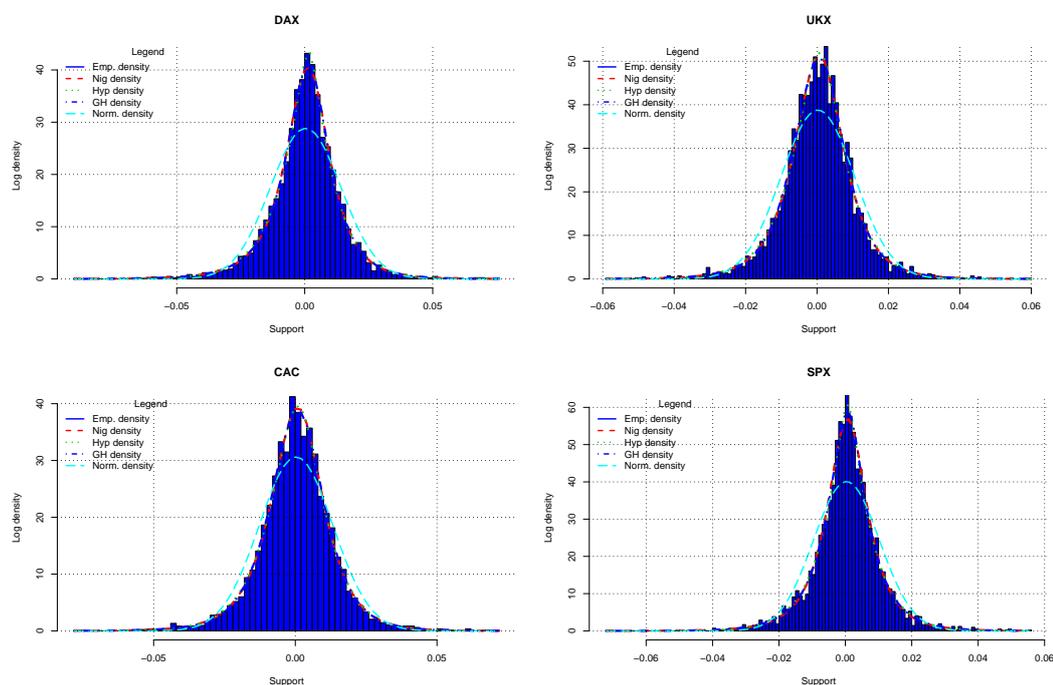


Figure 1.13: Empirical histogram of stock index returns

This figure presents the empirical density of several indexes' returns vs. the estimated densities obtained with the NIG (red), Hyperbolic (green), Generalized Hyperbolic (dark blue) and Gaussian (light blue) distributions. The estimation has been performed using the daily log returns of four major indexes: the French CAC (bottom left), the German DAX (top left), the US SP500 (bottom right) and the UK FTSE indexes (top right). The sample starts on January 2, 1988 and ends on the December 31, 2007.

The remaining of this section is organized as follow. The next subsection defines the Generalized Hyperbolic distribution and the GARCH-type models that will be used in the sequel. Next, the basis of the exponential affine approach for the specification of the stochastic discount factor are presented and I show how to apply it in the Generalized Hyperbolic framework.

1.3.1 GARCH-type models with Generalized Hyperbolic innovations

This section is devoted to the presentation of the asset pricing model. I first review the main features of the Generalized Hyperbolic distribution. I then present the economy under the historical measure.

1.3.1.1 The Generalized Hyperbolic distribution

It is well known that for lower sampling frequencies (e.g. monthly) asset returns empirical distributions are closer to the Gaussian case. Unfortunately, these kind of dataset ignore too much information to be considered especially for pricing purposes. On the contrary, looking at daily data leads to the excess kurtosis phenomena first described by Mandelbrot (1963). Several families of distributions have already been used to reproduce this stylized fact, such as the Paretian distributions or the double Weibull distribution. However, conditional distribution modeling is still under question.

The recently introduced Generalized Hyperbolic (GH) distributions (Barndorff-Nielsen (1977)) have been suggested as a model for financial price processes. Their exponentially decreasing tails seem to fit the statistical behavior of asset returns (Barndorff-Nielsen (1995), Eberlein and Prause (2002)).

For $(\lambda, \alpha, \beta, \delta, \mu) \in \mathbb{R}^5$ with $\delta > 0$ and $\alpha > |\beta| > 0$, the one dimensional $GH(\lambda, \alpha, \beta, \delta, \mu)$ distribution is defined by the following density function

$$d_{GH}(x, \lambda, \alpha, \beta, \delta, \mu) = \frac{(\sqrt{\alpha^2 - \beta^2}/\delta)^\lambda}{\sqrt{2\pi} K_\lambda(\delta\sqrt{\alpha^2 - \beta^2})} e^{\beta(x-\mu)} \frac{K_{\lambda-1/2}\left(\alpha\sqrt{\delta^2 + (x-\mu)^2}\right)}{\left(\sqrt{\delta^2 + (x-\mu)^2}/\alpha\right)^{1/2-\lambda}} \quad (1.46)$$

where K_λ is the modified Bessel function of the third kind. For $\beta = 0$, this distribution is symmetric.

For $\lambda \in \frac{1}{2}\mathbb{Z}$, the basic properties of the Bessel function (see Abramowitz and Stegun (1964)) allow to find simpler forms for the density. In particular, for $\lambda = 1$ I get the Hy-

perbolic distributions (HYP) which log-density is a hyperbola and for $\lambda = -\frac{1}{2}$ I obtain the Normal Inverse Gaussian distributions (NIG) which are closed under convolution. More generally, many important distributions can be found either by constraining the distribution parameters or as a limiting case, e.g Gaussian distribution, Student's t-distribution or the Laplace-distribution (Barndorff-Nielsen and Blaesild (1981)).

Contrary to Paretian distributions, the moment generating function of a GH distribution exists and is given by

$$\mathbb{G}_{GH}(u) = e^{\mu u} \left(\frac{\alpha^2 - \beta^2}{\alpha^2 - (\beta + u)^2} \right)^{\frac{\lambda}{2}} \frac{K_{\lambda}(\delta \sqrt{\alpha^2 - (\beta + u)^2})}{K_{\lambda}(\delta \sqrt{\alpha^2 - \beta^2})}, \quad |\beta + u| < \alpha. \quad (1.47)$$

In particular, moments of all orders are finite (e.g exact values of the skewness and kurtosis are provided in Barndorff-Nielsen and Blaesild (1981)) allowing to apply Central Limit Theorem arguments to ensure the convergence of long time horizon returns towards Normal distributions. Finally, this family is also stable under affine transforms. This property is interesting because in the GARCH setting I will be able to deduce the conditional distribution of the log-returns from the innovations' distribution. More precisely,

Proposition 1 *Let $(M, \Sigma) \in \mathbb{R}^2$. If X follows a $GH(\lambda, \alpha, \beta, \delta, \mu)$ then $M + \Sigma X$ follows a $GH\left(\lambda, \frac{\alpha}{|\Sigma|}, \frac{\beta}{\Sigma}, \delta \mid \Sigma, M + \mu \Sigma\right)$.*

Proof: See Blaesild (1981).□

In particular by defining $\alpha^* = \alpha\delta$ and $\beta^* = \beta\delta$ and if $X \hookrightarrow GH(\lambda, \alpha^*, \beta^*, \delta, \mu)$ then $\frac{X-\mu}{\delta} \hookrightarrow GH(\lambda, \alpha^*, \beta^*, 1, 0)$: the parameters μ and δ respectively describe the location and the scale.

1.3.1.2 Description of the economy under the historical probability P

Empirical evidence suggests that equity return volatility is stochastic and mean reverting. What is more, the response of volatility to positive or negative returns is asymmetric (see e.g. Ghysels et al. (1996)). When returns go down, volatility goes up: effects of this kind are known as leverage effects. In a discrete time setting, the stochastic volatility is often captured using extensions of the autoregressive conditional heteroscedasticity model (ARCH); see for instance Bollerslev (1986). This kind of specification will be classically used in the sequel.

The economy that is discussed here contains two different assets: a risk free one whose price at t is B_t . Its dynamic is given by:

$$B_t = B_{t-1}e^r, \quad B_0 = 1, \quad (1.48)$$

where r is the corresponding risk free rate expressed on a daily basis and supposed to be constant. Under the historical measure, the time series dynamic of the risky asset S_t is:

$$Y_t = \log\left(\frac{S_t}{S_{t-1}}\right) = \underbrace{r + m_t}_{M_t} + \underbrace{\sqrt{h_t}z_t}_{\varepsilon_t}, \quad S_0 = s, \quad (1.49)$$

where $z_t \hookrightarrow f(0, 1)$ (f being an arbitrary distribution with mean 0 and variance 1) and

$$\begin{cases} m_t = F(h_t, \lambda_0); & \lambda_0 \in \mathbb{R}, \\ \Sigma_t^2 = h_t = G(h_{t-1}, \varepsilon_{t-1}). \end{cases} \quad (1.50)$$

In (1.49), I consider a general time varying excess of return m_t that depends on the constant unit risk premium λ_0 . In practice, it will be fixed for the empirical study as in the Duan model (1995):

$$m_t = \lambda_0\sqrt{h_t} - \frac{1}{2}h_t. \quad (1.51)$$

Since the vast majority of articles find very few advantages to work with high order GARCH, I will consider only the first order case. Moreover, to be able to capture the leverage effect, I will favor the EGARCH model that ensures positivity without restrictions on the coefficients (see Nelson (1991))

$$\log(h_t) = a_0 + a_1 \left(\left| \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right| + \gamma \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right) + b_1 \log(h_{t-1}). \quad (1.52)$$

In this model I allow for non Gaussian innovations in order to model extreme returns behavior. Several distributions have been already used to reproduce excess skewness or kurtosis and outperform the Black-Scholes pricing e.g. Generalized Exponential distribution (Nelson (1991)), Gamma distribution (Siu et al. (2004)), Inverse Gaussian distribution (Christoffersen et al. (2006)) or mixture of Normal distributions (Bertholon et al. (2007b)). Here I will focus on the GH distribution presented in the preceding section.

According to Proposition 1, if z_t follows a $GH(\lambda, \alpha, \beta, \delta, \mu)$ distribution then, given \mathcal{F}_{t-1} ,

$$Y_t \hookrightarrow GH\left(\lambda, \frac{\alpha}{\Sigma_t}, \frac{\beta}{\Sigma_t}, \delta\Sigma_t, M_t + \mu\Sigma_t\right). \quad (1.53)$$

Thus, for the estimation of the GARCH model under the historical probability, I favor a two-stage procedure. At the first stage, the Quasi Maximum Likelihood Estimation (QMLE) (see e.g. Franses and van D. Dijk (2000)) is used to determine the parameters $(\lambda_0, a_0, a_1, b_1, \gamma)$. This is an approximation of the exact Maximum Likelihood Estimation obtained by replacing in (1.49) the specific distribution f by a standard normal one. It is well known that under mild technical conditions this method provides unbiased parameter estimates. At the second stage, since I exactly know the form of the density function (1.46) I adopt a classical maximum likelihood approach to estimate the unknown remaining parameters $(\lambda, \alpha, \beta, \delta, \mu)$ using the residuals obtained at the previous stage.

Now, the model is entirely specified under P . Since I want to use it to price contingent claims, I need to postulate an explicit risk premium to perform the change in distribution. This is the aim of the next part.

1.3.2 The stochastic discount factor

I consider the preceding economy with time horizon T consisting of two assets namely a risk-free bond and a risky stock. Note that I denote by $(B_t)_{t \leq T}$ and $(S_t)_{t \leq T}$ the dynamics of the bond and the stock price processes under the historical probability P .

Classically, in a discrete time dynamic equilibrium model (or in an arbitrage free continuous one), the price of any asset equals the expected present value of its future payoffs under an equivalent martingale measure Q . For example, the price P_t at time t of a European asset paying Φ_T at T (Φ_T being \mathcal{F}_T measurable) is given by

$$P_t = E_Q[\Phi_T e^{-r(T-t)} \mid \mathcal{F}_t] \quad (1.54)$$

or equivalently by

$$P_t = E_P[\Phi_T M_{t,T} \mid \mathcal{F}_t]. \quad (1.55)$$

The \mathcal{F}_{t+1} measurable random variable $M_{t,t+1}$ is the stochastic discount factor (SDF) (the quantity $M_{t,t+1}e^r$ is also known as the pricing kernel). In general, the stochastic discount factor depends on several state variables of the economy (e.g. aggregate consumption in a Lucas (1978) economy or past consumptions and equity market returns in Rosenberg and Engle (2002)). Nevertheless, after Rubinstein (1978), Brown and Gibbons (1985) or Cochrane (2002) I suppose that equity market returns are the only variables that must be dealt with for pricing purpose. This is equivalent to assuming that I project the original stochastic discount factor onto the sigma-algebra generated by the payoffs of the risky asset. In such a framework, $M_{t,T}$ is known as the projected pricing kernel (see Rosenberg and Engle (2002)).

In the Black and Scholes (1973) economy, the dynamic of the risky asset under the unique equivalent martingale measure Q given by the Girsanov theorem is

$$dS_t = rS_t dt + \sigma S_t dW_t,$$

where W is a standard Brownian motion. If I denote by μ the drift coefficient under P I obviously have in this case that

$$M_{t,T} = e^{\frac{\mu-r}{\sigma^2} \left[\log\left(\frac{S_T}{S_t}\right) + \frac{(\sigma^2 - r - \mu)(T-t)}{2} \right]}. \quad (1.56)$$

Thus, in this case, the unique SDF is an exponential affine function of the log-returns and pricing may easily be done by closed-form expressions or numerical tools.

In discrete time, it is well known that markets are in general incomplete (see e.g. Elliott and Madan (1998)). Thus the martingale measure Q is not unique and there exists a multiplicity of SDF that are compatible with the previous pricing formulas. Once the dynamics under the historical probability have been specified through statistical modelings, it is possible to overcome this problem adopting one of the two following similar points of view: first, I can impose constraints on the form of the SDF. Second, I can choose a particular martingale measure that fulfills specific economic or risk criteria (e.g. the minimal martingale measure in the sense of Föllmer and Schweizer (1991) that minimizes the variance of the hedging loss (1991)).

When this choice has been made, if I know the dynamic of the risky asset under the new probability, it is possible to price contingent claims from (1.54) or (1.55) using Monte Carlo simulations.

Following the first approach, Rosenberg and Engle (2002) and Gouriéroux and Monfort (2007) consider several parametric specifications for the SDF as power functions or as exponentials of polynomials of the returns. In particular, Gouriéroux and Monfort (2007) show that the power functions case appears naturally in many classical situations. However, their estimation strategies are quite different.

Rosenberg and Engle (2002) is based on a GARCH-type model with empirical innovations. They minimize the classical mean square error criterion between the obtained Monte Carlo prices and the option market quotes in order to estimate the SDF. Contrary to this semi-parametric approach, the knowledge of the distribution of the log-returns is needed in the Gouriéroux and Monfort (2007) framework: conditional Laplace transforms have to be computed to determine the SDF as explained in the next section. A very similar approach using a dynamic Gerber-Shiu's argument can be found in Siu et al. (2004) (see also the seminal paper of Bühlmann et al. (1996)): in this framework,

the conditional Esscher transform is used in order to select a particular martingale measure in a discrete time setting.

The objective of this section is to apply this second point of view for GARCH-type models with Generalized Hyperbolic innovations. This choice may be interesting because Esscher transform has been successfully applied in continuous time to price derivatives when the underlying asset follows an exponential Generalized Hyperbolic Lévy motion (see Eberlein and Keller (1995), Eberlein and Prause (2002)).

1.3.2.1 Pricing options with exponential affine SDF

The methodology unfolds as follows. I assume for the SDF a particular parametric form: $\forall t \in \{0, \dots, T - 1\}$

$$M_{t,t+1} = e^{\theta_{t+1}Y_{t+1} + \beta_{t+1}} \quad (1.57)$$

where $Y_{t+1} = \log\left(\frac{S_{t+1}}{S_t}\right)$ and where θ_{t+1} and β_{t+1} are \mathcal{F}_t measurable random variables.

In a discrete time version of the Black-Scholes economy, the corresponding SDF is given by $\theta_{t+1} = \frac{\mu-r}{\sigma^2}$ and $\beta_{t+1} = \frac{(\sigma^2-r-\mu)(\mu-r)}{2\sigma^2}$ that are independent of t . In particular, the parameter θ corresponds to a constant risk aversion. Here, the specification (1.57) allows for time variations in risk aversion.

Now, I need to compute explicitly $(\theta_{t+1}, \beta_{t+1})$. Considering the bond and the risky asset, the pricing relation (1.55) gives for $T = t + 1$ the following restrictions for the SDF

$$\begin{cases} E_P[e^r M_{t,t+1} | \mathcal{F}_t] = 1 \\ E_P[e^{Y_{t+1}} M_{t,t+1} | \mathcal{F}_t] = 1. \end{cases} \quad (1.58)$$

For all $t \in \{0, \dots, T - 1\}$, I denote by \mathbb{G}_t the conditional moment generating function of Y_{t+1} given \mathcal{F}_t defined on a convex set $\mathcal{D}_{\mathbb{G}_t}$ that is not reduced to $\{0\}$ and I introduce the mapping $\Phi_t : \{\theta \in \mathbb{R}; \theta \text{ and } 1 + \theta \in \mathcal{D}_{\mathbb{G}_t}\} \rightarrow \mathbb{R}$ such that

$$\Phi_t(\theta) = \log\left(\frac{\mathbb{G}_t(1 + \theta)}{\mathbb{G}_t(\theta)}\right).$$

Thus, the preceding system is equivalent to

$$\begin{cases} \mathbb{G}_t(\theta_{t+1}) = e^{-(r+\beta_{t+1})} \\ \mathbb{G}_t(\theta_{t+1} + 1) = e^{-\beta_{t+1}} \end{cases} \quad (1.59)$$

and, with my notations, I have to solve

$$\begin{cases} \Phi_t(\theta_{t+1}) = r \\ \mathbb{G}_t(\theta_{t+1} + 1) = e^{-\beta_{t+1}}. \end{cases} \quad (1.60)$$

The next proposition shows that, under the pricing constraints, there is no ambiguity in the choice of (1.57).

Proposition 2 *Suppose that \mathbb{G}_t is twice differentiable. If there exists a solution to the equation $\Phi_t(\theta) = r$, it is unique.*

Proof: See Gerber and Shiu (1994a).□

In the remaining, I suppose that (1.60) leads for each $t \in \{0, \dots, T-1\}$ to a unique solution denoted by $(\theta_{t+1}^q, \beta_{t+1}^q)$ (see the next section for the proof of the existence in the case of the GH distribution). The SDF

$$M_{t,t+1} = e^{\theta_{t+1}^q Y_{t+1} + \beta_{t+1}^q} \quad (1.61)$$

being explicitly known, I may deduce easily the form of the associated equivalent martingale measure Q . In fact, noting that $\forall k \in \{0, \dots, T-1\}$

$$\frac{M_{k,k+1}}{E_P[M_{k,k+1} \mid \mathcal{F}_k]} = \frac{e^{\theta_{k+1}^q Y_{k+1}}}{\mathbb{G}_k(\theta_{k+1}^q)}, \quad (1.62)$$

I define the stochastic process

$$\left(L_t = \prod_{k=1}^t \frac{e^{\theta_k^q Y_k}}{\mathbb{G}_{k-1}(\theta_k^q)} \right)_{t \in \{1, \dots, T\}} \quad (1.63)$$

that is obviously a martingale under P . Then, I have the following proposition:

Proposition 3 *Let Q be the probability owning the density L_T with respect to P , then*

1. *Q is the unique probability associated to the exponential affine SDF (1.61), in particular, the discounted stock price process $(e^{-rt} S_t)_{t \in \{0, \dots, T\}}$ is a martingale under Q and the price P_t at time t of a European asset paying Φ_T at T is given by*

$$P_t = E_Q[\Phi_T e^{-r(T-t)} \mid \mathcal{F}_t]. \quad (1.64)$$

2. *Under Q , the moment generating function of Y_t given \mathcal{F}_{t-1} is given by*

$$E_Q[e^{uY_t} \mid \mathcal{F}_{t-1}] = E_P[e^{uY_t} \frac{e^{\theta_t^q Y_t}}{\mathbb{G}_{t-1}(\theta_t^q)} \mid \mathcal{F}_{t-1}] = \frac{\mathbb{G}_{t-1}(\theta_t^q + u)}{\mathbb{G}_{t-1}(\theta_t^q)}. \quad (1.65)$$

Under Q , the conditional distribution of Y_t given \mathcal{F}_{t-1} is none other than the conditional Esscher transform of parameter θ_t^q (in the sense of Siu et al. (2004)) of the distribution of Y_t given \mathcal{F}_{t-1} under P . Moreover, for pricing purposes, relation (1.65) is fundamental because it gives explicitly the conditional distribution of the log-returns under Q and allows for Monte Carlo simulation methods. Furthermore, as underlined in the next proposition, under mild assumptions, this conditional distribution under Q belongs to the same family as under the historical one.

Proposition 4 *For all $t \in \{1, \dots, T\}$, if the conditional distribution of Y_t given \mathcal{F}_{t-1} is under P infinitely divisible and if \mathbb{G}_{t-1} is twice differentiable, then, the conditional distribution of Y_t given \mathcal{F}_{t-1} is also infinitely divisible under Q with finite moment of order 2.*

The preceding result is not so surprising because several authors have already shown that it is true for particular subclasses of distributions (see Gouriéroux and Monfort (2006), Siu et al. (2004), Christoffersen et al. (2006)). This point is one of the main features of the exponential affine specification of the pricing kernel that is not fulfilled, for example, in the framework of Elliott and Madan (1998). For the GH distribution, the stability is proved in the next section.

1.3.2.2 Application to the model

Here, I want to apply the methodology of the preceding section in the GH setting of Section 2.2. Thus, I have to identify (if it exists) the unique exponential affine SDF and describe explicitly the dynamics of the log-returns under the associated equivalent martingale measure.

First, I have the following result that ensures, under mild conditions, the existence of a solution $(\theta_{t+1}^q, \beta_{t+1}^q)$ of (1.60) for all $t \in \{0, \dots, T-1\}$.

Proposition 5 *For a GH($\lambda, \alpha, \beta, \delta, \mu$) distribution with $\alpha > \frac{1}{2}$, then,*

1. *If $\lambda \geq 0$, the equation $\log \left(\frac{\mathbb{G}_{GH}(1+\theta)}{\mathbb{G}_{GH}(\theta)} \right) = r$ has a unique solution,*
2. *If $\lambda < 0$, the equation $\log \left(\frac{\mathbb{G}_{GH}(1+\theta)}{\mathbb{G}_{GH}(\theta)} \right) = r$ has a unique solution if and only if $\mu - C < r < \mu + C$ where*

$$C = \log \left(\frac{\Gamma[-\lambda]}{2^{\lambda+1}} \right) - \log \left(\frac{K_\lambda(\delta \sqrt{\alpha^2 - (\alpha - 1)^2})}{\delta \sqrt{\alpha^2 - (\alpha - 1)^2}} \right) \quad (1.66)$$

Remark: The constant C is strictly positive because $\frac{d}{dx} \frac{K_\lambda(x)}{x^\lambda} = -\frac{K_{\lambda+1}(x)}{x^\lambda} < 0$.

Even if I am not able to obtain a closed-form formula for the solution of (1.60), I may apply the risk neutral valuation presented in Section 3.1. For practical cases $(\theta_{t+1}^q, \beta_{t+1}^q)$ may be computed efficiently using a refined bracketing method. The next proposition describes the dynamics of the risky asset under the chosen equivalent martingale measure.

Proposition 6 *Under Q , the distribution of Y_t given \mathcal{F}_{t-1} is a*

$$GH \left(\lambda, \frac{\alpha}{\Sigma_t}, \frac{\beta}{\Sigma_t} + \theta_t^q, \delta \Sigma_t, M_t + \mu \Sigma_t \right) \quad (1.67)$$

where $M_t = r + m_t$ and $\Sigma_t = \sqrt{h_t}$.

Proof: It is a direct consequence of the Proposition 3.b). \square

I deduce from the preceding result that, under Q ,

$$Y_t = r + m_t + \underbrace{\sqrt{h_t} z_t}_{\varepsilon_t}, \quad S_0 = s, \quad (1.68)$$

where the z_t are \mathcal{F}_t measurable random variables such that, conditionally to \mathcal{F}_{t-1} ,

$$z_t \hookrightarrow GH(\lambda, \alpha, \beta + \sqrt{h_t} \theta_t^q, \delta, \mu). \quad (1.69)$$

In particular, the GH distribution is stable under the change of measure, allowing me to simulate easily the sample paths of the risky asset.

Under Q , conditionally to \mathcal{F}_{t-1} , ε_t is no more centered and its variance is not h_t but

$$\text{var}(\varepsilon_t) = h_t \left(\frac{\delta K_{\lambda+1}(\delta \gamma)}{\gamma K_\lambda(\delta \gamma)} + \frac{(\beta + \Sigma_t \theta_t^q)^2 \delta^2}{\gamma_t^2} \left(\frac{K_{\lambda+2}(\delta \gamma_t)}{K_\lambda(\delta \gamma_t)} - \frac{K_{\lambda+1}^2(\delta \gamma_t)}{K_\lambda^2(\delta \gamma_t)} \right) \right)$$

where $\gamma_t = \sqrt{\alpha^2 - (\beta + \Sigma_t \theta_t^q)^2}$. Thus the GARCH structure of the volatility is modified in a nonlinear way from P to Q .

Since several classical distributions e.g. Normal, Skewed Laplace, Gamma or Inverse Gaussian may be obtained as a limiting case (in the sense of the convergence in distribution or for the Wasserstein distance) of the GH distribution (see Eberlein and von Hammerstein (2004) for details) it should be easy to see that the risk neutral dynamics of Duan (1995), Heston and Nandi (2000), Gouriéroux and Monfort (2006), Siu et al.

(2004), Christoffersen et al. (2006) may be recovered with an appropriate choice of F and G in (1.50). In particular, as noticed in Siu et al. (2004), the exponential affine specification of the SDF gives rise, in the normal case, to the same results as the ones obtained using the classical locally risk-neutral valuation relationship of Duan (1995). Moreover, in all these cases, I have explicit solutions for (1.60) due to the special forms of the considered densities.

1.3.3 Empirical Results

In this section, I present the empirical results with this new option pricing model. I first present the dataset, before detailing the methodology used for the tests presented here. Finally, I review the key results obtained.

1.3.3.1 The dataset

The dataset used in the tests presented below contains the following time series. First, I use the closing prices for the French CAC 40 on a daily basis. The sample starts on January 2, 1988 and ends on the December 31, 2007. This sample is the sample used for the figures presented in the introduction, showing qqplots, fitted densities and adequation tests. The risk free rate used here is the zero rate obtained from the European swap rates. I use closing swap rates whose maturities range from 1 month to 3 years. Intermediate maturities required for option pricing are computed using the ? model³⁰.

Beyond this initial dataset, I use the available option contracts written on the French CAC 40. The most liquid contracts available are the quarterly ones, that is the contracts maturing on March, June, September and December for every available years. I focused on these maturities, neglecting the intermediate monthly maturities that are far less liquid on average. The option dataset starts on January 2, 1988 and ends on the December 31, 2007. The strikes are chosen so that the moneynesses used here range from .8 to 1.2, which is the standard moneyness window used in the literature. See for example Cont and da Fonseca (2002).

1.3.3.2 Methodology

I discuss here the methodology used to test the model presented before. First, the time series model is estimated following the previously mentioned sequential approach: Nelson (1991)'s GARCH is first estimated in Quasi Maximum likelihood approach.

³⁰I compared the results obtained with the ? approach and the performances of the models presented here are globally unchanged.

Then, the disturbance's distribution is estimated using the residuals obtained as a by-product at the previous step. Table 1.15 presents descriptive statistics for the estimated parameters of Nelson (1991)'s model. Table 1.16 presents descriptive statistics for the estimated parameters of the GH distribution.

The option pricing methodology used here focuses on out-of-sample-option-pricing errors: for a given current date, I estimate the time series parameters using a dataset of constant size ending on the current date. I used each time 4000 observations. I tried to change this sample size and yet the results remained globally unchanged. The key point here is to maintain as much outliers in the dataset as possible: these extremal events are essential to fit the GH parameters and to control the (especially left) tail behavior. Once these parameters are estimated, I obtain σ_{t+1} from the Nelson's model since it is a measurable function of the available information. I use this forecast volatility to initialize the Monte Carlo simulations.

Once the historical distribution and the initial volatility have been estimated, I compute each theoretical call option price value for each contract, using a Monte Carlo simulation. This MC simulation is indeed path-dependent: each time, I need to solve for α_{t+1} and β_{t+1} , given the simulated volatility σ_{t+1} and the GH parameters. Since this methodology has been presented earlier, I do not present it again. However, the path dependency of the MC simulations is an essential feature of my approach that must be clearly underlined.

So as to reduce the option prices errors linked to the use of Monte Carlo methods, I use the Duan and Simonato (1998) method to impose martingality within the sampled processes. This approach makes it possible to have simulated paths under the risk neutral distribution that are exact martingales. This reduces the pricing errors in a dramatic way: see the tables presented in Duan and Simonato (1998) and the discussion in Barone-Adesi et al. (2007).

Finally, the criterion used to compare option pricing models is the one used in Barone-Adesi et al. (2007) and Heston and Nandi (2000). I use the average absolute relative pricing errors criterion. Let $\hat{C}(t, T_j, K_i)$ be the estimated call option price with a time to maturity equal to $T_j - t$ and a strike price worth K_i . Let $C(t, T_j, K_i)$ be the corresponding quoted market option price. Then the criterion I use here is

$$AARPE = \sum_t \sum_j \sum_i \left| \frac{\hat{C}(t, T_j, K_i) - C(t, T_j, K_i)}{C(t, T_j, K_i)} \right|. \quad (1.70)$$

I chose this criterion for two reasons. First, it is the usual criterion selected in the empirical literature previously mentioned. I will thus be able to compare the range of

errors found in the dataset used with my model with the remaining major models of the literature. Second, this criterion is robust to the well known fact that option pricing errors are proportional to the moneyness: out of the money call option prices are very low, and so are the usual errors found. The converse is true for deep in the money option prices. This criterion rescales the errors using the level of the market option price: it is therefore robust to this effect and the analysis is largely eased. It makes the comparison across moneynesses easier.

1.3.3.3 Results

I present here the pricing errors results obtained with my methodology and the dataset I presented previously. I compare my model to the Black and Scholes (1973), Heston and Nandi (2000), and Bertholon et al. (2007b)'s models. Each model is estimated on an equal basis to make the inter-model comparison easier. Finally, since the NIG and HYP distributions are special cases of my model, I also compare my model performance with the one obtained with these two other distributions.

Table 1.17 presents the AARP criterion for each model, using the exponential affine pricing kernel. These results are graphically presented in figure 1.14 for a time to maturity ranging from .5 to 1 year and in figure 1.15 for time to maturity beyond 1 year³¹. Globally, the worst competitors are the Black and Scholes (1973) and the Heston and Nandi (2000) models, yielding errors beyond 100 percent in this out-of-sample testing strategy. On the contrary, the GH and mixture of Gaussian distribution-based option pricing models behaved fairly well. The main results found here are two-fold. First, the models based on a fine understanding of the tail behaviors (the MN and the GH based models) yield the best results. Second, my model outperform the MN model whenever the moneyness is below 1.1. On the contrary, the GH model is outperformed by the MN model for deep out of the money option prices, that is moneynesses beyond 1.2.

What is more, for the GH and the MN models, the size of the errors obtained is close to what is obtained during a calibration exercise, as presented in Barone-Adesi et al. (2007). Here, I do not perform any optimization exercise to call options: I especially focus on the direct modeling approach (see Bertholon et al. (2007a)): starting from the historical distribution, I impose no arbitrage restrictions based on a specific choice on the shape of risk aversion. I then compute option prices, using an enhanced Monte Carlo method. This results in average absolute relative errors that are very close to what can be found in Barone-Adesi et al. (2007). I must underline that my approach

³¹Given the numerical burden associated to my method, large back-tests as presented here are extremely time consuming, explaining the limited number of maturities I can afford to present now.

is much less time consuming than the one presented in the latter paper, since it avoids the MC based optimization.

Finally, I compare the option pricing errors when using my approach – that is the exponential affine pricing kernel – with a simpler way to perform option pricing. In this simpler way, I only use the Duan and Simonato (1998) method to impose martingality, but the parameters estimated under the historical measure remain unchanged. By doing this, I aim to present evidence that would help accept or reject the sophistication introduced by this exponential affine pricing kernel approach. It is essential to keep in mind that the change in probability measure produced by the method used here does not only change the expectation of the process but every moment of it. This change in the moments implied by the model is not controlled at all. Table 1.18 presents the criterion previously used when call options are priced using this martingalized historical process. Table 1.19 provides a comparison between the two approaches and helps build a global diagnostic. The conclusions are two-folds. First, for time to maturities beyond 1 year, the pricing kernel based option prices are clearly dominated by the simpler method. Second, for time to maturities between .5 and 1 year, the results get trickier to analyze: depending on the moneyness, one method may dominate the other. Globally, the errors obtained for each approach are very close.

1.3.3.4 Conclusion

In this last section, I presented a new model based on two assumptions. First, the pricing kernel is an exponential affine function of the log of the future value of the underlying asset. Second, returns in this economy are conditionally Generalized Hyperbolic distributed. Using these assumptions, I show how to price options and compare the empirical performances of my model with the one found in the existing literature. The performances of the model are found to be close to those found when performing a calibration exercise. Finally, I find that what seems to really matter for option pricing is the degree of fit in the tails of the historical distribution, which is a very interesting message for the econometrics of asset pricing framework.

1.3.4 Proofs

Proof to proposition 3: First, when $s \leq T$, remark that for a \mathcal{F}_s measurable and non negative random variable Z I may deduce from the martingale property of (L_t) that

$$E_Q[Z] = E_P[L_s Z].$$

	a_0	b_1	a_1	γ	λ_0
Average	-5.58	0.47	0.01	0.92	0.06
Std. Dev.	0.00	0.00	0.00	0.00	0.00
Skewness	0.58	0.72	-1.66	0.06	0.62
Excess Kurtosis	-1.06	-1.04	4.56	0.23	-0.70

Table 1.15: Descriptive statistics on the average estimated parameters for the Nelson GARCH model.

	α	β	δ	μ	λ
Average	360.36	0.60	386.19	-0.59	-59.61
Std. Dev.	133.18	0.56	142.68	0.60	36.80
Skewness	1.76	-0.96	1.76	0.96	-2.38
Excess Kurtosis	4.22	2.54	4.23	2.53	7.38

Table 1.16: Descriptive statistics on the average estimated parameters for the GH distribution.

.5 < Maturity < 1					
	[.8-.9]	[.9-1]	[1-1.1]	[1.1-1.2]	> 1.2
BS	0.164854417	0.291341422	0.75001177	2.198779988	6.124823646
HN	0.136259676	0.231888723	0.589886928	1.642235613	4.211025648
Nelson MN	0.073605373	0.038375151	0.091546296	0.223025586	0.358909382
Nelson NIG	0.070926039	0.031306469	0.107912775	0.257212062	0.4972777
Nelson GH	0.072508156	0.034220274	0.08973236	0.211844412	0.473990647
Maturity > 1					
	[.8-.9]	[.9-1]	[1-1.1]	[1.1-1.2]	> 1.2
BS	0.27196985	0.403121978	0.725764514	1.480900027	2.978503687
HN	0.313646807	0.439835965	0.797880773	1.646513312	3.406133441
Nelson MN	0.122027432	0.09213732	0.073386701	0.115907646	0.243910796
Nelson NIG	0.120329339	0.087504852	0.069326101	0.118246221	0.266695327
Nelson GH	0.120757162	0.088165393	0.069513071	0.118837264	0.261635523

Table 1.17: Absolute average pricing errors for CAC 40 option prices.

.5 < Maturity < 1					
	[.8-.9]	[.9-1]	[1-1.1]	[1.1-1.2]	> 1.2
BS	0.187072706	0.344788702	0.907255263	2.595798784	7.457682622
HN	0.202700203	0.35853022	0.920562721	2.730283433	8.558661463
Nelson MN	0.073484336	0.034830098	0.09020335	0.204552092	0.42092125
Nelson NIG	0.073582981	0.036658656	0.096085678	0.216450665	0.48966079
Nelson GH	0.070375688	0.032051441	0.112397312	0.225184538	0.40081137
Maturity > 1					
	[.8-.9]	[.9-1]	[1-1.1]	[1.1-1.2]	> 1.2
BS	0.275909871	0.407205839	0.733059075	1.494468371	3.019062045
HN	0.318259541	0.44205148	0.803332102	1.665532038	3.416788139
Nelson MN	0.122968074	0.094898015	0.07476038	0.113687094	0.23475627
Nelson NIG	0.119712556	0.087404155	0.067942801	0.122847487	0.267499872
Nelson GH	0.119581221	0.087505694	0.067820071	0.119243876	0.266680359

Table 1.18: Absolute average pricing errors for CAC 40 option prices, only using the martingale restriction method.

.5 < Maturity < 1						
		[.8-.9]	[.9-1]	[1-1.1]	[1.1-1.2]	> 1.2
SDF correction	Nelson MN	0.073605373	0.038375151	0.091546296	0.223025586	0.358909382
	Nelson NIG	0.070926039	0.031306469	0.107912775	0.257212062	0.4972777
	Nelson GH	0.072508156	0.034220274	0.08973236	0.211844412	0.473990647
Mart. Correction	Nelson MN	0.073484336	0.034830098	0.09020335	0.204552092	0.42092125
	Nelson NIG	0.073582981	0.036658656	0.096085678	0.216450665	0.48966079
	Nelson GH	0.070375688	0.032051441	0.112397312	0.225184538	0.40081137
Maturity > 1						
		[.8-.9]	[.9-1]	[1-1.1]	[1.1-1.2]	> 1.2
SDF correction	Nelson MN	0.122027432	0.09213732	0.073386701	0.115907646	0.243910796
	Nelson NIG	0.120329339	0.087504852	0.069326101	0.118246221	0.266695327
	Nelson GH	0.120757162	0.088165393	0.069513071	0.118837264	0.261635523
Mart. Correction	Nelson MN	0.122968074	0.094898015	0.07476038	0.113687094	0.23475627
	Nelson NIG	0.119712556	0.087404155	0.067942801	0.122847487	0.267499872
	Nelson GH	0.119581221	0.087505694	0.067820071	0.119243876	0.266680359

Table 1.19: Comparison between absolute average mispricing errors for CAC 40, using the stochastic discount factor restrictions or not.

Moreover, for $T \geq t \geq s > 0$, if Z (resp. X) is \mathcal{F}_s (resp. \mathcal{F}_t) measurable and non negative then

$$E_Q[XZ] = E_P[L_T X Z] = E_P[E_P[X L_T | \mathcal{F}_s] Z] = E_P[E_P[X E_P[L_T | \mathcal{F}_t] | \mathcal{F}_s] Z]$$

thus

$$E_Q[XZ] = E_P[E_P[L_t X | \mathcal{F}_s] Z] = E_P \left[\frac{L_s}{L_t} E_P[X L_t | \mathcal{F}_s] Z \right] = E_Q \left[\frac{1}{L_s} E_P[X L_t | \mathcal{F}_s] Z \right].$$

Hence,

$$E_Q[X | \mathcal{F}_s] = \frac{1}{L_s} E_P[X L_t | \mathcal{F}_s], \tag{1.71}$$

a) and b) easily follow. \square

Proof to proposition 4: From the Kolmogorov representation theorem (see e.g. Mainardi and Rogosin (2006) for an interesting historical approach of this result), I have for all

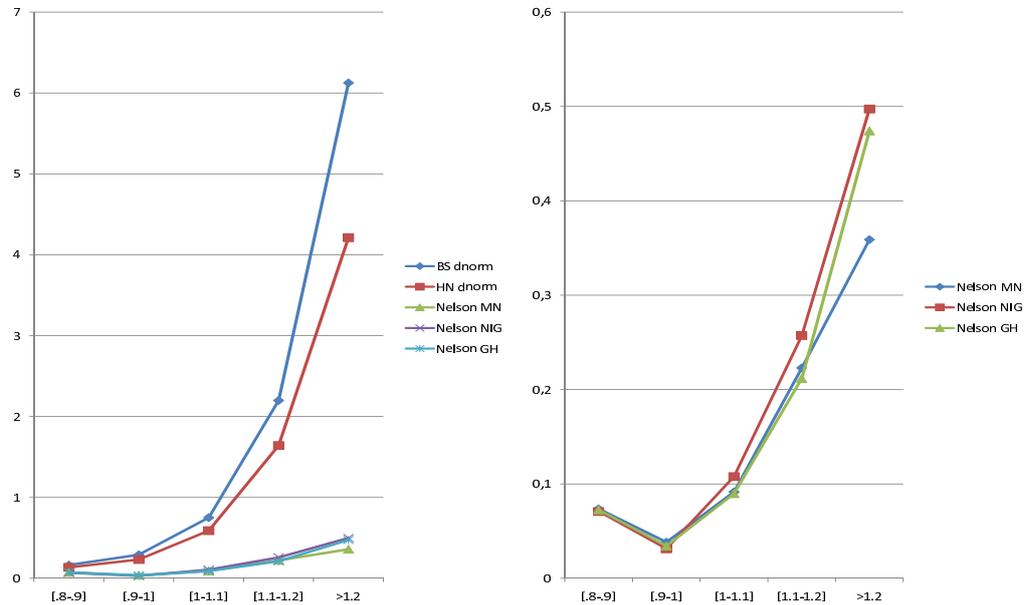


Figure 1.14: Absolute average pricing errors for CAC 40 option prices using the exponential affine pricing kernel for maturities from .5 to 1 year. The left-hand figure presents the criterion across moneynesses for each model. The right-hand figure focuses on the best performing competitors.

$u \in \mathcal{D}_{\mathbb{G}_{t-1}}$,

$$\log(\mathbb{G}_{t-1}(u)) = \gamma_t u + \int_{-\infty}^{+\infty} (e^{zu} - 1 - zu) \frac{dK_t(z)}{z^2}$$

where γ_t is a \mathcal{F}_{t-1} measurable real valued random variable and K_t a \mathcal{F}_{t-1} measurable random variable with values in the space of the non-decreasing and bounded functions with limit zero in $-\infty$. Thus, from Proposition 3, $\forall u \in \{\theta \in \mathbb{R}; \theta + \theta_t^q \in \mathcal{D}_{\mathbb{G}_{t-1}}\}$,

$$\log(E_Q[e^{uY_t} | \mathcal{F}_{t-1}]) = \gamma_t u + \int_{-\infty}^{+\infty} (e^{z(u+\theta_t^q)} - e^{\theta_t^q z} - zu) \frac{dK_t(z)}{z^2}$$

thus

$$\log(E_Q[e^{uY_t} | \mathcal{F}_{t-1}]) = \tilde{\gamma}_t u + \int_{-\infty}^{+\infty} (e^{zu} - 1 - uz) \frac{e^{\theta_t^q z} dK_t(z)}{z^2} \quad (1.72)$$

where

$$\tilde{\gamma}_t = \gamma_t + \int_{-\infty}^{+\infty} (e^{\theta_t^q z} - 1) z \frac{dK_t(z)}{z^2}. \quad (1.73)$$

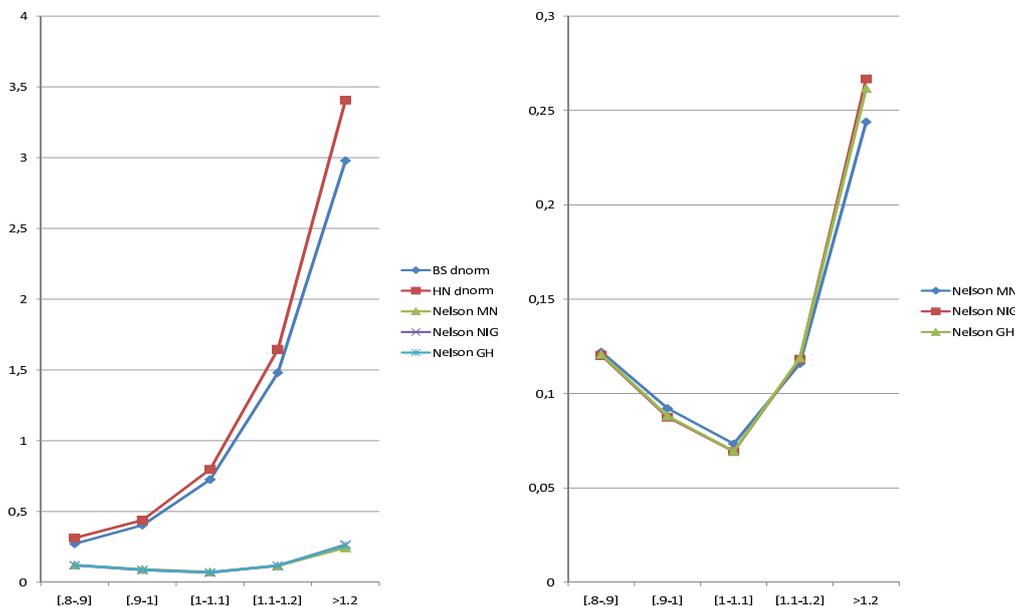


Figure 1.15: Absolute average pricing errors for CAC 40 option prices using the exponential affine pricing kernel for maturities beyond 1 year. The left-hand figure presents the criterion across moneynesses for each model. The right-hand figure focuses on the best performing competitors.

Since \mathbb{G}_{t-1} is twice differentiable, I have in particular that

$$\int_{-\infty}^{+\infty} e^{\theta_t^q z} dK_t(z) < \infty$$

and I may define $\forall x \in \mathbb{R}$,

$$\tilde{K}_t(x) = \int_{-\infty}^x e^{\theta_t^q z} dK_t(z)$$

that is a \mathcal{F}_{t-1} measurable random variable with values in the space of the non-decreasing and bounded functions with limit zero in $-\infty$. The conclusion follows from (1.72) and from the Kolmogorov representation theorem. \square

Proof to proposition 5: For $|\beta + u| < \alpha$,

$$\mathbb{G}_{GH}(u) = e^{\mu u} \left(\frac{\alpha^2 - \beta^2}{\alpha^2 - (\beta + u)^2} \right)^{\frac{\lambda}{2}} \frac{K_\lambda(\delta \sqrt{\alpha^2 - (\beta + u)^2})}{K_\lambda(\delta \sqrt{\alpha^2 - \beta^2})}$$

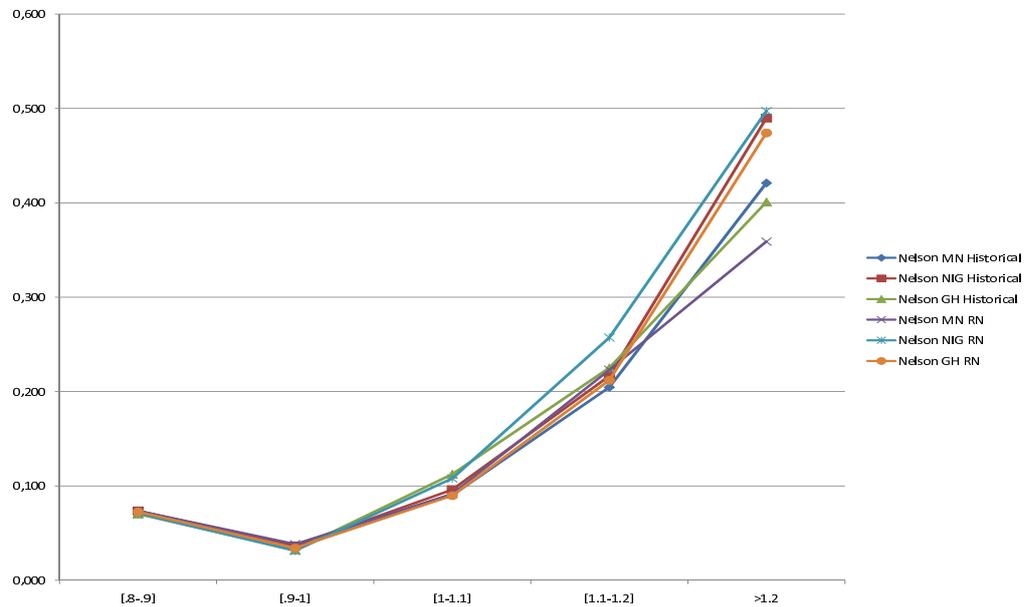


Figure 1.16: Absolute average pricing errors for CAC 40 option prices using the exponential affine pricing kernel (labeled RN) or only using the martingale restriction of Duan and Simonato (labeled historical) for maturities beyond 1 year. I chose to focus on the best competitors here.

hence \mathbb{G}_{GH} is twice differentiable. Moreover, $\Phi(\theta) = \log\left(\frac{\mathbb{G}_{GH}(1+\theta)}{\mathbb{G}_{GH}(\theta)}\right)$ is defined on the interval $]-(\alpha + \beta), \alpha - \beta - 1[$ that is not empty because $\alpha > \frac{1}{2}$. Thus I may apply Proposition 2 and the unicity holds. It remains to prove the existence.

1. For $x > 0$, I define $\Psi(x) = \log\left(\frac{K_\lambda(x)}{x^\lambda}\right)$. Thus,

$$\Phi(\theta) = \mu + \Psi(\delta\sqrt{\alpha^2 - (\beta + 1 + \theta)^2}) - \Psi(\delta\sqrt{\alpha^2 - (\beta + \theta)^2}).$$

For the properties of the Bessel function used in the sequel I refer the reader to Abramowitz and Stegun (1964).

If $\lambda > 0$,

$$\frac{K_\lambda(x)}{x^\lambda} \sim_{x \rightarrow 0^+} \frac{\Gamma[\lambda]2^{\lambda-1}}{x^{2\lambda}}.$$

So I have $\lim_{\theta \rightarrow \alpha - \beta - 1} \Phi(\theta) = +\infty$ and $\lim_{\theta \rightarrow -(\alpha + \beta)} \Phi(\theta) = -\infty$. The conclusion follows from the intermediate value theorem.

When $\lambda = 0$, I may conclude in the same way remembering that $K_0(x) \sim_{x \rightarrow 0^+} -\log(x/2) - \gamma$ where γ is the Euler-Mascheroni constant.

2. When $\lambda < 0$ using the relation $K_\lambda(x) = K_{-\lambda}(x)$ I obtain that $\frac{K_\lambda(x)}{x^\lambda} \sim_{x \rightarrow 0^+} \Gamma[-\lambda]2^{-\lambda-1}$. Thus, $\lim_{\theta \rightarrow \alpha - \beta - 1} \Phi(\theta) = \mu + C$ and $\lim_{\theta \rightarrow -(\alpha + \beta)} \Phi(\theta) = \mu - C$ and I conclude applying again the intermediate value theorem. \square

Chapter 2

Assessing the impact of news on the bond market

It is renowned that the bond market reacts to macroeconomic figures. These macroeconomic figures are in fact announced on a very regular basis and draw the attention of market participants in a very logical way, since they are linked to the expected monetary policy over the upcoming months and years. Moreover, the short rate can be seen as the sum of two components. The first of these components should be the Central Bank's main refinancing rate. The second one is simply a cash spread expressing the temporary divergence between the short rate and the Central Bank's target rate. When r_t is the short rate applying at time t , it can thus be decomposed as

$$r_t = R_t + \epsilon_t, \quad (2.1)$$

with R_t the target rate and ϵ_t the cash premium. Usually this premium is close to zero, except in periods when the quest for liquidity is higher, which is what happened during the recent sub-prime crisis.

The target rate is defined by Central Bankers so as to fight the inflation and/or to foster economic activity. Thus, this rate is endogenous in the economy and extremely forward-looking. In this respect, it can be seen as a combination of the currently available economic information, this combination being chosen in a forecasting manner. This idea is best summarized by the so-called Taylor rule (Taylor (1993)). This monetary policy rule – in a more general manner than the original Taylor (1993) manner – can be stated as follow:

$$R_t = \alpha_0 + \alpha_\pi(\pi_t^e - \bar{\pi}) + \alpha_y y_t^e, \quad (2.2)$$

where $\bar{\pi}$ is the explicit or implicit inflation target of the Central Bank, π_t^e is the current expected inflation for a given horizon and y_t^e is the expected output gap. The output

gap can be roughly defined as the excess economic growth over its long term value. Taylor (1993) managed to show that, with this simple rule, he was able to describe in a convincing manner the past behavior of the Federal Reserve Board. The message here is that the main refinancing rate can be roughly explained by a linear combination of available economics aggregates, such as the Consumer Price Index or the Gross Domestic Product. On Taylor rules, see the classical survey by Clarida et al. (2000).

One of the versions of the expectation hypothesis states that the whole yield curve can be seen as an expectation of the upcoming expected short rates. It seems natural to combine the Taylor rule approach to the short rate modeling with this expectation hypothesis. In this perspective, it is relatively easy to understand how, through no arbitrage arguments, it is possible to relate and explain the behavior of the yield curve, based on a few macroeconomic figures. Works of this kind can be found in the recent macrofinance literature. For example Ang and Piazzesi (2003) propose to describe the economy with a few Vector AutoRegressive (VAR) related macroeconomic factors. They show how these factors under no arbitrage opportunities are able to price zero coupon yields. Despite the simplicity of the approach, the results provided by this literature make this approach exciting: see e.g. Ang and Bekaert (2004), Ang et al. (2005) and Bikbov and Chernov (2006). Ang et al. (2007) extend this way of modeling using switching regimes.

This stream of models aim at capturing the dynamic behavior of the level of interest rates. What is more, since most of the macroeconomic datasets are based on a monthly – if not quarterly – frequency, these macroeconomic affine models can be used to bridge the gap between the global stance of the economy and the overall shape of the yield curve. Unfortunately, these models cannot handle the other way around macroeconomic figures: when released, these figures are known to produce movements in interest rates markets. If several market participants bet on the future stance of the yield curve – using Futures or swap rates – a great number of them also take bets on the intra-day impact of news on the yield curve. The rationale behind these trading strategies stems from the fact that the biggest moves in bond markets are actually produced by the disclosure of economic information. Table 2 presents the events suspected to explain the larger movements in the US swap market over 2006.

Figure 2.1 summarizes the usual schedule of information treatment by financial market participants for monthly releases. Roughly speaking, most of the macroeconomic announcements are either released on a weekly, monthly or quarterly basis. Before the announcement, pools of economists are interviewed and deliver their forecast for the upcoming figure. For Bloomberg users, what is considered as the market's forecast is the median of the pooled forecasts. This figure is then used to analyze the actual

Rank	Date	Variation	Event	Market Consensus	Real Announcement
1	10/03/2006	12.1bp	Non Farm Payroll	123K	92K
2	06/02/2006	-10.8bp	Non Farm Payroll	170K	75K
3	06/14/2006	10.1bp	CPI	0.3%	0.2%
4	12/13/2006	9.4bp	Retail Sales	0.3%	1.1%
5	09/21/2006	-9.3bp	Philly Fed	14	0.4
⋮	⋮	⋮	⋮	⋮	⋮
10	07/19/2006	-8.1bp	B. Bernanke's speech	-	-

Table 2.1: Biggest moves in US swap rates over 2006. Source: Natixis - Economic Research

figure when released: the market analysis of macroeconomic announcements is often performed by comparison with the economist forecast. The market reaction to news is then a function of the spread between the forecast and the disclosed figure. Figure 2.2 presents the time series of the 2-year Euro swap rate in January 2005: the biggest changes in swap rates can be related to scheduled announcements.

The dedicated academic literature empirically demonstrated several, now well-known, results: first, a lot of macroeconomic news produce an impact on the bond market; second, there exists a term structure effect of macroeconomic news. The reality is, of course, much more complex: the term structure effect of news on the bond market is expected to be economic cycle-dependent. The intuition beyond this is simply that the macroeconomic figures and the market's reaction to them is based on the Central Bankers' speech on the upcoming economic threats. When the business cycle is going down, activity and employment figures are closely monitored. On the contrary, when the economy is roaring, the attention is focused on inflation and expected inflation indicators.

Needless to say, the impact of news on financial markets has not only been documented on the bond market side: the equity market is also known to react to many announcements. There is actually a huge literature investigating the effect of information disclosure on either stock indexes or individual components of these indexes. On the reaction of these markets to news, see for example Lardic and Mignon (2003) and the references presented in this article. Even if the methodologies used for this stream of literature are related to what I propose here, bond markets are way more dependent on macroeconomic figures than stocks. Thus, the news that are investigated here strongly differ from stock-market related news: for example, the consensus forecast regarding the upcoming macro-figures makes the analysis of the surprises easier. I will discuss this point later.

What I propose here is two-fold: first, I investigate the impact of news on the European market and show how the naive perception of European news is misleading. It is disturbed by the influence of the US which I account for by using the PCA factors of the US yield curve. This section helps me present the basics of economic news analysis. Second, I present an attempt to classify the different term structure impact depending upon several state variables. I show how the term structure impact of news can vary, depending on the economic and monetary policy cycle.

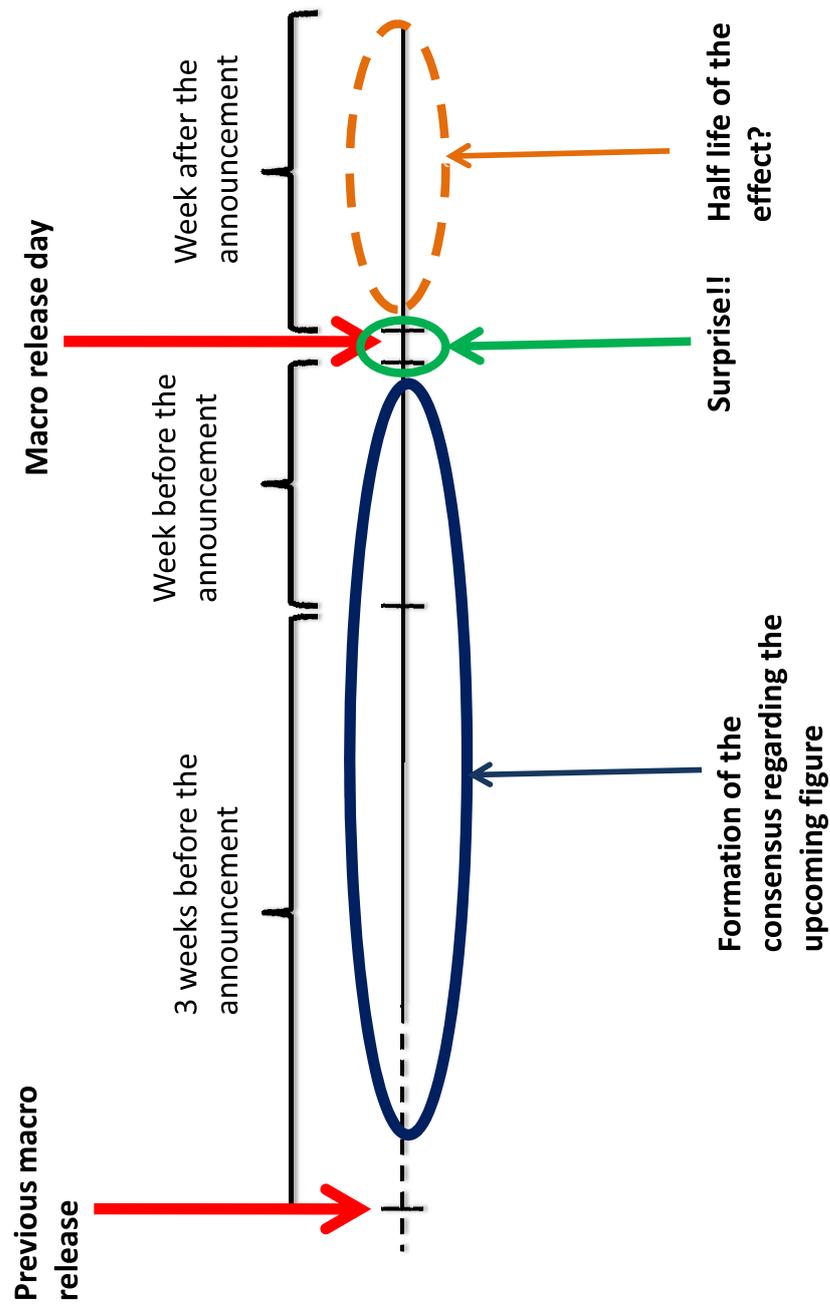


Figure 2.1: Timing of the impact of news on the bond market

2.1 Yield curve reaction to macroeconomic news in Europe

The¹ impact of news about the economy and monetary policy has already been very closely studied in the US (Ederington and Lee (1993), Fleming and Remolona (2001), Balduzzi et al. (2001), Piazzesi (2005) to name a few). In Europe however, given the European Central Bank (ECB)'s relative youth and the shallow historic depth of the European data available to date, these studies remain few and relatively recent (Ehrmann and Fratzscher (2002), Goldberg and Leonard (2003), Andersson et al. (2006)). These papers highlight the fact that the European market's peculiar feature is that it is subject to complex influences: it is impacted not only by European but also American news, by national economic figures and the ECB's aggregated statistics.

As a result, one key difficulty in Europe arises from the fact that it is absolutely crucial to bear in mind the influence of US interest rates on European ones. According to Ehrmann and Fratzscher (2002), Goldberg and Leonard (2003), this influence can be due to several factors: (1) the integration of financial markets and contagion effects; (2) the real macroeconomic integration of the 2 zones; (3) the common perception among markets that the US is the engine of world growth; and (4) the status of "advanced indicators" enjoyed by US figures compared with their European equivalents.

The methodology used by the above authors to take into account the American influence is to include both European and American economic announcements in the regression used to explain European interest rates. Although this methodology is a relatively simple way to take account of American influence, it has the drawback of ignoring part of the information. Notably, the US yield curve changes even in the absence of any economic announcements in the US, and these changes in the American curve certainly exert an influence on European rates. This omission of some information is harmful and, as I will discuss it, can create biases in estimating the impact of economic announcements in Europe. This suggests that it would be a useful improvement to show the influence of the entire American yield curve on European rates, and not merely of announcements in the US alone.

Bernanke and Boivin (2003) demonstrated that in a "data-rich" economic environment, it is optimal to take account of a maximum of available information, rather than concentrate on a small number of variables, even those chosen to be the most relevant. They propose to use Stock and Watson (2002) method of dimension reduction

¹This section is taken from a article named "Yield curve reaction to macroeconomic news in Europe : disentangling the US influence", with Marie Brière, published in an upcoming collective book edited in 2008.

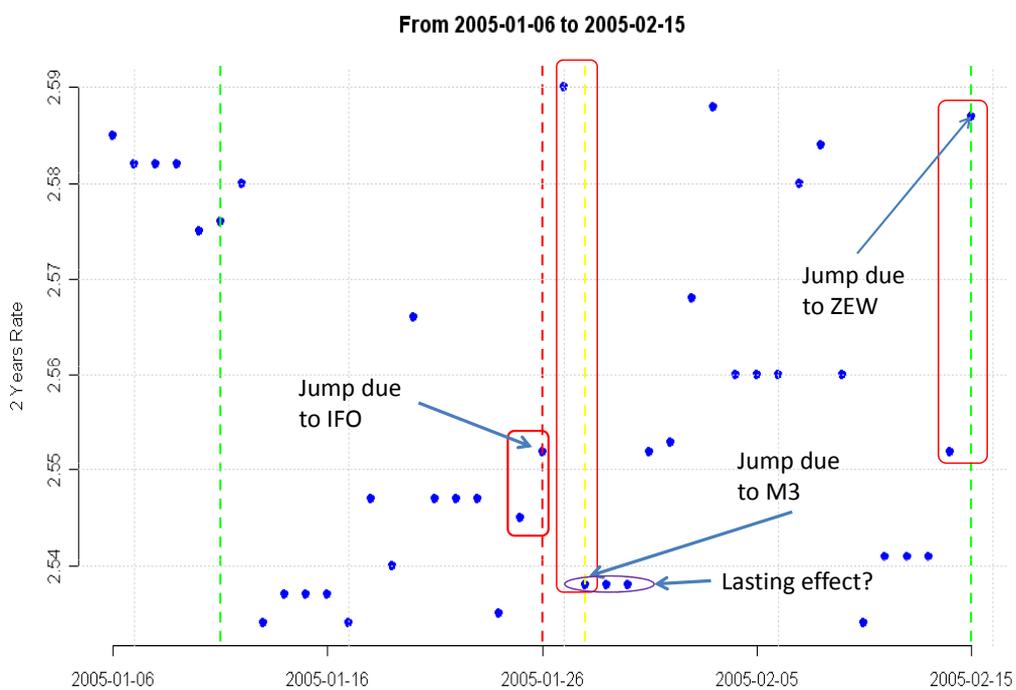


Figure 2.2: Dataset snapshot for European 2-year rate illustrating the impact of announcements on the bond market.

schemes akin to traditional principal component analysis as a way to extract key information from large databases.

My approach in this paper is analogous, though I apply it to a different type of problem. The influence of American interest rates on the European curve passes through multiple channels and cannot be modelled on a simple basis. For example, 10-year European swap rates are as much influenced by the 10-year US swap rate as by the remaining of the US yield curve. More, the US monetary policy is known to impact European interest rates. Figure 2.3 present the estimated correlation – and confidence intervals for the estimate – between the daily variations of the Euro 10-year swap rate and the daily variations of the US swap rates over the whole term structure. Obviously, the influence of the US curve over the Euro curve is due to be important: in this respect, highly informative instrumental variables are required here, so as to control for this link.

One prerequisite is therefore to manage to synthesise all the information coming from day-to-day changes in the whole American yield curve, using a restricted number of factors that can subsequently be incorporated into my regression on each maturity of European interest rates. More, given that there should be numerous information sources that actually enable anybody to control for this integration effect, I am very likely to be faced with the dimensionality curse. I may have to cope with a high number of explanatory variables when compared to the number of observations at hand.

This subsection contributes to the existing literature in several ways. First, it proposes an original methodology that makes it possible to separate, within day-to-day changes in European rates, the impact due to European economic announcements from the influence of the American yield curve. This allows me to measure the impact of announcements more rigorously by limiting the bias caused by the omission of variables. A second contribution is to measure the impact of announcements on the entire swap curve. Whereas many studies focus on a single maturity to demonstrate the impact of news, it is particularly worthwhile to understand how the shape of the entire curve changes and which maturities are most sensitive to the arrival of one piece of information or another. As demonstrated by Fleming and Remolona (2001) and Guégan and Ielpo (2007) for American rates, announcements do not necessarily have the same impact on short rates as on long rates. To my knowledge, no study to date has focused this analysis on Europe.

I examine the impact of 19 macroeconomic and monetary policy announcements on the European swap curve (maturities ranging from 3 months to 30 years) between January 2000 and July 2007, integrating the influence of American swap rates through the

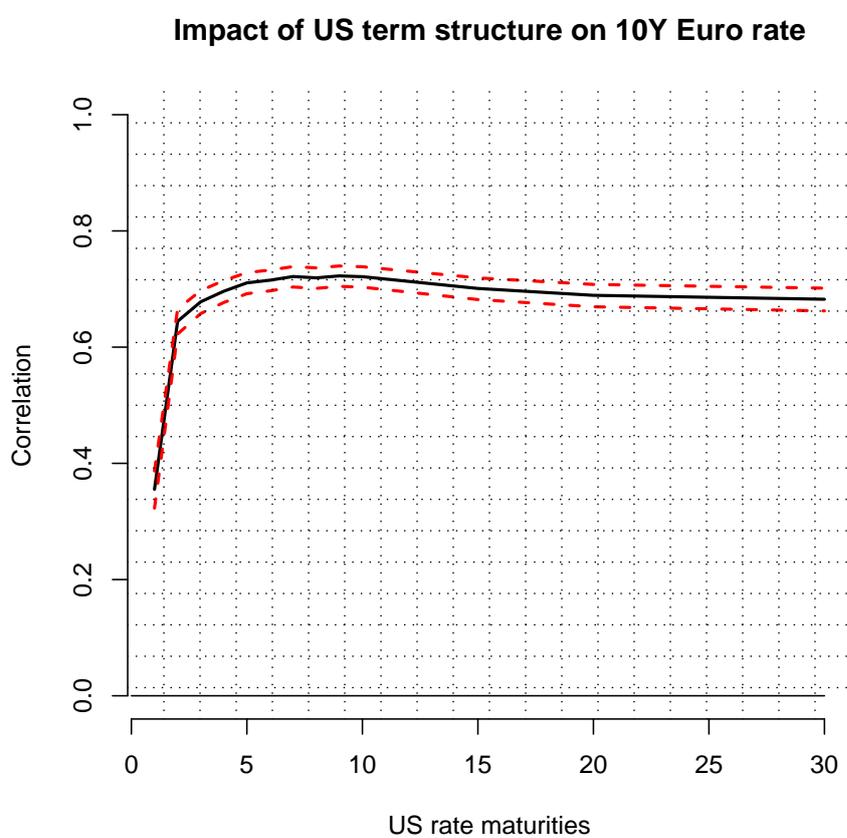


Figure 2.3: Impact of US term structure on 10Y Euro rate

3 first factors extracted from a Principal Component Analysis (PCA). My empirical results highlight the importance of taking the American yield curve into account in measuring the impact of announcements in Europe. Results in fact differ appreciably, depending on whether this influence is integrated or not.

Among the indicators which impact the entire yield curve, I note the highly important role of German economic sentiment or activity figures, which carry more weight for the markets than the aggregated European figures. I find little influence from the job figures, a result already documented in the existing literature; this is mainly due to the fact that the market has access to Germany's "pre-scheduled release". For price indicators, my results contrasts with those previously obtained, and most of the differences appear to be due to my more exhaustive integration of the American influence. I demonstrate that the inflation figures have a relatively weak surprise effect on European interest rates, whereas the M3 money supply has a substantial impact. The latter result fits in with the crucial role of this indicator for the ECB and the testimony of investors. Lastly, I classify the most important figures for the markets for each maturity.

For each figure, I then highlight the shape of the entire yield curve's reaction to announcements. I can very clearly see that the intermediate maturities concentrate the most striking reactions. The short end of the curve (less than 1 year) or the very long end (more than 15 years) generally shows a much weaker reaction, causing a hump shape in the curve. This shape clearly reveals anticipations by traders of the monetary authorities' reaction to economic news.

The remaining of this subsection is organized as follow. First, I review the data and methodology used and then, I present the empirical results obtained.

2.1.1 A methodology to deal with the US influence

This subsection presents the methodology used in this paper as well as the associated dataset. I first discuss the motivations for my methodology, before presenting the methodology used to capture the Euro bond market reaction to macroeconomic news. Finally, I detail the dataset used for the estimation.

2.1.1.1 Announcements and surprises in a data-rich environment

Several US macroeconomic announcements are known to impact the euro yield curve (see Ehrmann and Fratzscher (2002) for example). Moreover, euro rates display a very high correlation with US rates. Therefore, neglecting the US component when analysing variations in euro rates will naturally lead to problems of omitted variables.

The difficulty in treating US information comes from its nature: at present there is an abundance of news on the US market, not all of it quantitative.

Empirical studies such as Balduzzi et al. (2001) and Guégan and Ielpo (2007) present evidence that the number of economic news items that actually impact on the US yield curve can be substantial. Moreover, important US news is not limited to economic figures, but also extends to many other types of intelligence, some of which cannot be reduced to figures, such as central bankers' speeches (Brière (2006)). Thus, the real number of US economic news items to be considered when disentangling US influence from euro rates can be too large to be taken properly into account. To a great extent, US rates can be understood as a summary of the news that is important to the market. Nonetheless, the US yield curve is made of a large number of maturities: choosing those that best account for US yield curve movements can be tricky. Thus, I am confronted with a "data-rich" environment: the number of possible explanatory variables can be larger than the number of observations itself, given the length of the euro rate dataset.

Situations of this kind can be treated with factor-based methods, such as Principal Component Analysis (PCA), as presented in Stock and Watson (2002). For example, when trying to find the factors that actually explain the variations of the Federal Reserve's target rate, Bernanke and Boivin (2003) proposed to summarize the huge amount of economic information with which monetary policy makers are confronted by using a few factors obtained from a PCA. Here, I argue that methods of this kind are particularly suited to my problem. PCA has been applied to interest rates in many papers after the seminal work of Litterman and Scheinkman (1991), which first underlined the existence of three main factors driving most yield curve movements. With this reduced number of factors, I am no longer troubled by the dimensionality curse. On the consistency of the PCA factors, see Bai and Ng (2002).

Thus, I propose to take into account the US component in euro rate movements taking the first three factors of a PCA analysis of the corresponding US rates and introducing them into the usual linear model used to measure the surprise impact. This way, I am likely to capture most of the US influence on the European yield curve. Let me recall that omitting a variable can strongly compromise the estimation results, in so far as it leads to (1) bias within the Generalized Least Square estimates and (2) a loss of estimation efficiency. This type of results can be found in most of the classic textbooks on econometrics.

2.1.1.2 General Methodology

In this paper, I propose to estimate a version of the Balduzzi et al. (2001) model, augmented with the US factors obtained with a PCA performed over a daily yield curve dataset. First, I recall the basic settings of this model and then introduce the modifications I propose.

Suppose that it is possible to observe J types of economic figures announced regularly (on a monthly or quarterly basis). For each of these figures $j = 1, \dots, J$, there is a market consensus given by the median of a survey (see section 2.3 for further details). Let $F_{j,t}$ be this market forecast at time t for the figure j . Let $G_{j,t}$ be the realized figure released during the announcement. Then, I define the surprise in the figure j at time t as:

$$S_{j,t} = \frac{G_{j,t} - F_{j,t}}{\sigma_j} \quad (2.3)$$

where σ_j is defined as the standard error of the economic surprises, i.e.:

$$\sigma_j = \sqrt{V(G_{j,t} - F_{j,t})}. \quad (2.4)$$

This scaling procedure will enable me to compare the results between surprises². I then use these surprises to explain the variation of rates over one day for a given maturity.

Let $R^{EU}(t, \tau)$ be the daily closing swap rate of maturity τ on date t . Then, the linear model presented in Balduzzi et al. (2002) can be stated as follow:

$$\Delta R^{EU}(t, \tau) = R^{EU}(t, \tau) - R^{EU}(t-1, \tau) = \alpha + \sum_{j=1}^J \beta_j \mathbb{1}_{j,t} S_{j,t} + \epsilon_{t,\tau} \quad (2.5)$$

where $\mathbb{1}_{j,t} = 1$ if the surprise j is released on date t and 0 if not. For a given τ , $\epsilon(t, \tau)$ is a Gaussian white noise.

²For this purpose, I assume that each announcement time series are stationary. To check how misleading this hypothesis may be, I tried to estimate GARCH models (Bollerslev (1986)) using these surprises series, so as to be able filtrate a time varying conditional volatility out of the historical behavior of surprises, and globally failed. In fact, surprises really behave as white noises: surprises and centered squared surprises are uncorrelated. More, the Jarque and Berra test cannot be rejected for most of the surprises. See table 2.2. Lardic and Mignon (2004) offers a presentation of each of these tests. These empirical evidence clearly points toward a stable variance for surprises. In this respect, I chose to scale surprises using the in-sample population moment estimator.

	Jarque & Berra pval.	Box pval.	Box pval.for square
Activity and Employment			
Unemployment EC	0	0.78	0.95
Unemployment mom GE	0.07	0.09	0.01
GDP qoq EC	0.68	0.94	0.87
GDP Qoq GE	0.08	0.92	0.93
Retail sales mom EC	0	0.01	0.11
Retail sales mom GE	0.14	0.01	0.36
Industrial prod GE	0.28	0.01	0.77
Manuf. Orders mom GE	0.42	0.11	0.64
Prices			
M3 EC	0.76	0.51	0.25
CPI EC	0.49	0.86	0.49
CPI Flash EC	0.45	0.42	0.73
CPI FR	0.42	0.44	0.34
CPI GE	0	0.27	0.98
Producer Prices GE	0.52	0.2	0.41
Sentiment Indicators			
IFO Business Expect. GE	0.78	0.33	0.98
ZEW Expect. GE	0.89	0.02	0.94
Consumer confidence EC	0.27	0.01	0
INSEE Confidence FR	0.67	0.22	0.16
Monetary Policy			
Target EC	0.46	0.03	0.89

Table 2.2: P values for the usual Jarque and Berra normality test, Box test for absence of autocorrelation and Box test for absence of autocorrelation in square series.

So as to improve this standard model, I add dummy variables to capture daily effects. After running different tests, I elected to use a single "Friday effect", which is highly significant (see Table 5 in part 3.1). The second modification made to the canonical model of Balduzzi et al. (2001) consisted in taking into account the very high autocorrelation identified in the time series of daily variations in swap rates. This was done by adding their lagged increment to the standard linear model, which thus becomes::

$$\Delta R^{EU}(t, \tau) = \alpha + \sum_{j=1}^J \beta_j \mathbb{1}_{j,t} S_{j,t} + \gamma \mathbb{1}_{t=\text{Friday}} + \rho \Delta R^{EU}(t-1, \tau) + \epsilon_{t,\tau}, \quad (2.6)$$

with $\mathbb{1}_{t=\text{Friday}}$ being a dummy variable equal to 1 when date t in a Friday and 0 is not.

Now, to this modified model, I propose to add the factors $f_{i,t}$, $i = 1, 2, 3$, obtained

from the PCA performed on US rates. Let ΔR^{US} be the matrix of the daily variations in interest rates for each of the US maturities available, i.e. :

$$\Delta R^{US} = (\Delta R^{US}(t, \tau) - \Delta R^{US}(t - 1, \tau)). \quad (2.7)$$

Let C be the correlation matrix associated with the series in ΔR^{US} . Given that C meets the standard conditions for the Spectral Theorem to apply, it can be decomposed as:

$$C = D\Delta D, \quad (2.8)$$

where $\Delta = \text{diag}(\lambda_1, \dots, \lambda_p)$ the ordered eigenvalues matrix, when C has rank p , and D is the matrix whose columns contains the associated eigenvectors. I recall that the PCA factors are composed as :

$$f_i = \Delta R^{US}(t, \cdot) D_i, \quad (2.9)$$

where D_i is the i^{th} column of the matrix D and $\Delta R^{US}(t, \cdot)$ is the t^{th} lines of the matrix ΔR^{US} . With these settings, I have $f_i = (f_{i,1}, f_{i,2}, \dots, f_{i,n})^\top$, for a sample of size n and where $f_{i,t}$ is the value at time t for the i^{th} factor. For further details on PCA applied to finance, see Tsay (2002).

With these settings in mind, the model specified in equation (2.6) can be augmented in the following way:

$$\Delta R^{EU}(t, \tau) = \alpha + \sum_{j=1}^J \beta_j \mathbb{1}_{j,t} S_{j,t} + \gamma \mathbb{1}_{t=\text{Friday}} + \rho \Delta R^{EU}(t - 1, \tau) + \sum_{i=1}^3 \delta_i f_{i,t} + \epsilon_{t,\tau}, \quad (2.10)$$

with the previous hypothesis still applying. Thus the three factors $f_{i,t}$ summarise the information in the US yield curve and are the same for all the maturities in the European curve. As I will now show, these three factors, i.e. the level, slope and curvature of the American curve, have a major impact on almost all the maturities in the European curve.

As the interest rate datasets usually display a form of autoregressive variance (Ehrmann and Fratzscher (2002)), I thus allow $(\epsilon(t, \tau))_{t \in \mathbb{Z}}$ to follow a GARCH(1,1) process, making it possible to account for heteroscedasticity in the dataset. The model can be consistently estimated by Generalized Least Squares, thus taking into consideration non-spherical innovations. According to the Zellner theorem, whose assumptions are satisfied here, this is equivalent to estimating these equations either simultaneously or maturity by maturity. I have opted for the second approach because it is numerically easier to implement.

2.1.1.3 The Dataset

The US rate dataset contains daily swap rates with maturities ranging from 1 to 30 years used to compute the PCA factors. It is standard to use swap rates datasets in such a study, when the sampling frequency is daily (Guégan and Ielpo (2007)). My sample period covers the period 2 January 2000 - 14 March 2008, including each business day. Table 2.3 presents descriptive statistics of the daily changes in US swap rates, for selected maturities.

	Min	Max	Mean	Std.Dev.
1 year	-0.48	0.36	-6.44E-05	3.94E-02
2 years	-0.395	0.286	-2.16E-04	4.90E-02
3 years	-0.318	0.303	-2.13E-04	5.22E-02
4 years	-0.28	0.306	-1.95E-04	5.25E-02
5 years	-0.275	0.311	-1.77E-04	5.36E-02
6 years	-0.252	0.293	-1.64E-04	5.28E-02
7 years	-0.251	0.291	-1.49E-04	5.21E-02
8 years	-0.248	0.29	-1.39E-04	5.12E-02
9 years	-0.253	0.277	-1.29E-04	5.14E-02
10 years	-0.259	0.282	-1.20E-04	5.09E-02
15 years	-0.251	0.27	-1.26E-04	4.92E-02
20 years	-0.252	0.277	-1.34E-04	4.70E-02
30 years	-0.248	0.297	-1.35E-04	4.48E-02

Table 2.3: Summary statistics for daily variations of US swap rates

The Euro swap rate dataset is constructed in the same way, including daily swap rates for the same maturities. To these rates, I add cash Euribor rates for maturities of 3 months, 6 months and 9 months to take into account the very short end of the yield curve. Table 2.4 presents summary statistics for selected maturities of the daily changes in euro rates.

It should be noted that in the US and Europe, the change in long rates is negative on average during the period under review because, on the whole, they have declined. For short rates with maturities under three years, the results differ between the two zones: they have declined on average in the US but not in Europe. The reason for this is monetary policy differences, with the Fed cutting its interest rates much more sharply than the ECB during the period 2000-2004.

Table 2.5 presents the list of macroeconomic releases used as explanatory variables

	Min	Max	Mean	Std.Dev.
3 months	-0.358	0.159	4.36E-04	1.48E-02
6 months	-0.266	0.177	3.77E-04	1.69E-02
9 months	-0.211	0.187	3.17E-04	2.11E-02
1 year	-0.123	0.142	2.14E-04	2.44E-02
2 years	-0.145	0.196	8.55E-06	3.36E-02
3 years	-0.154	0.36	-1.16E-04	3.63E-02
4 years	-0.215	0.207	-2.10E-04	3.69E-02
5 years	-0.216	0.21	-2.65E-04	3.68E-02
6 years	-0.205	0.207	-3.14E-04	3.60E-02
7 years	-0.144	0.202	-3.48E-04	3.42E-02
8 years	-0.147	0.2	-3.79E-04	3.36E-02
9 years	-0.155	0.202	-4.00E-04	3.32E-02
10 years	-0.15	0.197	-4.14E-04	3.28E-02
15 years	-0.157	0.184	-4.66E-04	3.25E-02
20 years	-0.164	0.178	-4.77E-04	3.19E-02
30 years	-0.163	0.189	-4.89E-04	3.14E-02

Table 2.4: Summary statistics for daily variations of Euro swap rates

in the linear model specified by equation (2.10). These figures are obtained from the Bloomberg calendar presented in detail in Guégan and Ielpo (2007). This calendar presents the economic figures that are known to be monitored by financial market participants. It includes the release dates of the news, along with the Bloomberg consensus forecast and the economic figure finally released. Table 2.6 provides descriptive statistics on surprises. As discussed in Guégan and Ielpo (2007), I use the actual release figure, as opposed to the revised figure: most economic news is first announced on a crude basis by the statistics agencies, then refined during the following announcements. This data vintage problem is particularly important when estimating reaction functions as presented in equation (3): market participants are known to react to the first release.

Employment	Activity	Monetary figures	Sentiment and survey
Unemployment Rate Euro	GDP Euro QoQ	Money Supply M3	German IFO Forecast
German Unemployment Rate	Retail Sales Main Eurozone mom	HICP Eurozone	German ZEW Eco. Sent.
	German Industrial Production	CPI Flash Estimate	French INSEE Consumer confidence
	German GDP QoQ	German PPI MoM	EC Consumer Confidence
	German Manufacturing Orders	German CPI MoM	
	German Retail Sales	French CPI MoM	
		Euro Target Rate	

Table 2.5: List of macroeconomic announcements investigated

	Min	Max	Mean	Median	Std. Dev.
Activity and Employment					
Unemployment EC	-0.2	0.4	-0.04	-0.1	0.13
Unemployment mom GE	-102.5	89	-4.47	-3	32.91
GDP qoq EC	-0.2	0.1	-0.03	-0.1	0.13
GDP Qoq GE	-0.3	0.5	-0.04	-0.1	0.20
Retail sales mom EC	-1.7	3.1	-0.08	-0.2	0.72
Retail sales mom GE	-3.9	4.6	-0.42	-0.4	1.68
Industrial prod GE	-3.2	2.5	-0.24	-0.2	1.40
Manuf. Orders mom GE	-4.8	6.7	0.22	0.3	2.44
Prices					
M3 EC	-0.7	1.1	0.17	0.2	0.41
CPI EC	-0.3	0.2	0.00	0	0.13
CPI Flash EC	-0.2	0.3	0.01	0.1	0.13
CPI FR	-0.4	0.3	-0.01	-0.1	0.19
CPI GE	-0.9	0.4	0.00	0.1	0.24
Producer Prices GE	-0.6	0.8	0.07	0.1	0.29
Sentiment Indicators					
IFO Business Expect. GE	-2	3.8	0.31	0.3	1.32
ZEW Expect. GE	-21.6	20.6	-1.18	-1.15	8.42
Consumer confidence EC	-2	4	0.33	1	1.64
INSEE Confidence FR	-8	7	-0.42	-1	2.93
Monetary Policy					
Target EC	-0.5	0.25	-0.05	0.25	0.35

Table 2.6: Descriptive statistics on announcement surprises

2.1.2 Empirical results

2.1.2.1 European figures impacting fixed-income markets

For each of the 16 interest-rate maturities, I carried out the regressions presented in the previous section. Table 2.7 summarizes the impact of the 19 economic announcements on 3 key maturities in the yield curve: 2, 5 and 10 rates, with and without introducing the 3 American factors to these regression analyses.

My first observation is that the introduction of the 3 first factors into the American yield curve in the regression changes the impact of European announcements, a sign that the omission of these variables generates a true bias. It also reduces the standard error of the estimated coefficients. This is the case for the European CPI, whose impact

Variable	2 year rate		5 year rate		10 year rate		Number of significant figures	
	Without US	With US	Without US	With US	Without US	With US	Without US	With US
Activity and Employment								
Unemployment EC	0.0057	0.004	0.0063	0.0046	0.0047	0.0033	0/16	0/16
<i>P-value</i>	(0.11)	(0.11)	(0.12)	(0.1)	(0.19)	(0.17)	–	–
Unemployment Mom GE	-0.003	-0.0022	-5,00E-04	-1,00E-04	-0.0013	-0.0012	0/16	0/16
<i>P-value</i>	(0.39)	(0.36)	(0.9)	(0.97)	(0.7)	(0.62)	–	–
GDP qoq EC	0.0025	-0.001	-9,00E-04	-0.0012	-0.0023	-5,00E-04	0/16	0/16
<i>P-value</i>	(0.7)	(0.82)	(0.9)	(0.81)	(0.72)	(0.91)	–	–
GDP Qoq GE	0.0028	0.0032	0.0048	0.0049	0.0033	0.0031	0/16	0/16
<i>P-value</i>	(0.65)	(0.45)	(0.47)	(0.3)	(0.58)	(0.45)	–	–
Retail sales mom EC	-0.0023	0.0021	-0.0039	4,00E-04	-0.0018	0.0019	0/16	0/16
<i>P-value</i>	(0.53)	(0.42)	(0.34)	(0.89)	(0.62)	(0.45)	–	–
Retail sales Mom GE	0.0085**	0.0057**	0.0099***	0.0061**	0.0105***	0.0068***	15/16	15/16
<i>P-value</i>	(0.02)	(0.02)	(0.01)	(0.02)	(0)	(0)	–	–
Industrial prod GE	-0.001	-4,00E-04	-2,00E-04	-7,00E-04	-5,00E-04	-0.0015	0/16	0/16
<i>P-value</i>	(0.78)	(0.87)	(0.96)	(0.8)	(0.88)	(0.51)	–	–
Manuf. Orders Mom GE	0.0021	8,00E-04	0.0055	0.0039	0.0071**	0.0057***	4/16	7/16
<i>P-value</i>	(0.55)	(0.74)	(0.15)	(0.15)	(0.04)	(0.01)	–	–
Prices								
M3 EC	-0.0018	0.0081***	-0.0042	0.0055*	-0.0025	0.0053*	0/16	9/16
<i>P-value</i>	(0.64)	(0)	(0.33)	(0.07)	(0.52)	(0.05)	–	–
CPI EC	0.0055	0.0037	0.0074**	0.0044	0.0068**	0.0036	3/16	0/16
<i>P-value</i>	(0.15)	(0.15)	(0.07)	(0.13)	(0.07)	(0.15)	–	–
CPI Flash EC	0.0032	0.0027	0.0033	0.003	0.0027	0.0025	0/16	0/16
<i>P-value</i>	(0.46)	(0.37)	(0.49)	(0.36)	(0.53)	(0.39)	–	–
CPI GE	0.0066	0.0037	0.0065	0.0028	0.006	0.0023	0/16	0/16
<i>P-value</i>	(0.13)	(0.22)	(0.18)	(0.41)	(0.16)	(0.44)	–	–
Producer Prices GE	-0.001	-2,00E-04	0.0022	0.0026	0.0018	0.002	0/16	0/16
<i>P-value</i>	(0.78)	(0.93)	(0.56)	(0.34)	(0.6)	(0.4)	–	–
Sentiment Indicators								
IFO Business Expect. GE	0.0146***	0.0135***	0.018***	0.0165***	0.0158***	0.0143***	14/16	14/16
<i>P-value</i>	(0)	(0)	(0)	(0)	(0)	(0)	–	–
ZEW Expect. GE	0.0134***	0.0081***	0.0142***	0.0081***	0.0121***	0.0063***	13/16	12/16
<i>P-value</i>	(0)	(0)	(0)	(0.01)	(0)	(0.02)	–	–
Consumer confidence EC	-0.0016	-0.005	0.0012	-0.004	0.001	-0.0044	0/16	2/16
<i>P-value</i>	(0.72)	(0.11)	(0.81)	(0.24)	(0.82)	(0.14)	–	–
INSEE Confidence FR	0.0029	0.0024	0.005	0.0035	0.0083**	0.0062**	4/16	5/16
<i>P-value</i>	(0.42)	(0.34)	(0.21)	(0.21)	(0.02)	(0.01)	–	–
Monetary Policy								
Target EC	0.0034	0.0016	0.0043	0.0018	0.0017	-0.0007	0/16	0/16
<i>P-value</i>	(0.37)	(0.54)	(0.31)	(0.53)	(0.65)	(0.79)	–	–
Week effects								
Friday	-0.0042**	-0.1346***	-0.0036*	-0.1533***	-0.0032*	-0.1394***	2/16	16/16
<i>P-value</i>	(0.02)	(0)	(0.07)	(0)	(0.08)	(0)	–	–
Factors								
Factor 1	–	-0.005**	–	-0.0045***	–	-0.0039**	–	16/16
<i>P-value</i>	–	(0.02)	–	(0)	–	(0.01)	–	–
Factor 2	–	-0.1054***	–	-0.0226***	–	0.0297***	–	12/16
<i>P-value</i>	–	(0)	–	(0)	–	(0)	–	–
Factor 3	–	0.096***	–	0.0279***	–	-0.0285***	–	11/16
<i>P-value</i>	–	(0)	–	(0)	–	(0)	–	–
Diagnostic tests								
R2	0.03	0.55	0.03	0.54	0.03	0.55	–	–
Box Pierce Test	0.93	0.69	0.95	0.55	0.98	0.39	–	–

Table 2.7: Influence of the 19 announcements on 2, 5 and 10 year Euro swap rates. ***, **, * significant up to respective thresholds of 1%, 5% and 10%. (p-values) in brackets. Hit ratio is the number of maturities impacted at the 5% level.

diminishes when I introduce the American factors: it becomes non significant for most of the maturities. For the M3 money supply figure, the difference is even more striking, since the coefficient goes from being non significant (and negative) without counting the American influence to significant when this influence is factored in. I note that the amplitude of the impact is twice as great for the ZEW when the American influence is factored in. Finally, the EC consumer confidence index is significant twice when introducing the US factors.

In all, 6 announcements significantly impact the entire European yield curve (5% threshold): 2 activity figures (German manufacturing orders and retail sales), a price indicator (the M3 money supply) and 4 "sentiment" indicators (IFO and ZEW in Germany, INSEE's measure of consumer confidence in France and the European Commission consumer confidence index).

If I compare my results with those of the existing literature³, I once again find a strong influence from "economic sentiment" figures (ZEW and IFO most of all), plus a very restricted number of activity indicators (Ehrmann and Fratzscher (2002)). In my study, however, I note a much more significant influence of German retail sales. This may be due to the fact that the period I study is slightly more recent, when the weakness of German domestic demand was a genuine market worry. I also find little influence of the job indicators, as noted by Andersson et al. (2006), mainly due to the markets' prior knowledge of German pre-scheduled releases.

Among the national economic figures, the German ones had the greatest impact (the sole exception in this connection being the French consumer confidence figure). These statistics played a much greater role than the aggregated European statistics, for two main reasons: first, the German economy's very important role within the European Union in the eyes of the markets; and second, the fact that the aggregated European statistics are always published later than national statistics, adding only a small amount of additional information when they come out.

The main difference between my results and existing studies has to do with the price indicators. Previous studies highlighted the markets' weak reaction to announcements of the M3 money supply, though this is one of the ECB's pillars. My results are different, showing that taking greater account of the American influence on the yield curve changes this result. It highlights a real impact of the M3 money supply on rates, a much more important one than that of the inflation indicators themselves. This result is in line with the importance of this indicator for the ECB, stressed in most central bankers' speeches, and the testimony of investors on this subject. The weak influence of inflation figures, even if the CPI is one of the explicit targets of the ECB, is not a surprise, given the fact that Euro inflation is pretty well anticipated by economists and markets, who know prior releases of German and Italian data.

Table 2.8 classifies the figures which have a significant impact on rates for four matu-

³Note that Ehrmann and Fratzscher (2002) studied the impact of announcements on monetary rates (ranging from 1 to 12 months), Goldberg and Leonard (2003) examined the rates of German treasury bonds of between 2 and 10 years, and Andersson et al. (2006) looked at the German 10 year future contract.

rities in the curve (selected as the most representative and the most watched among the 16 included in my study). I note the significant figures at or above the 5% and 10% thresholds.

Rank	6M	2Y	5Y	10Y
1	Retail sales Mom GE***	IFO Business Expect. GE***	IFO Business Expect. GE***	IFO Business Expect. GE***
2	IFO Business Expect. GE*	ZEW Expect. GE***	ZEW Expect. GE***	Retail sales Mom GE***
3	M3 EC**	M3 EC***	Economic sentiment EC*	ZEW Expect. GE**
4	–	Economic sentiment EC**	Retail sales Mom GE**	INSEE Confidence FR**
5	–	Retail sales Mom GE**	M3 EC*	Manuf. Orders Mom GE**
6	–	PPI mom EC*	–	M3 EC**

Table 2.8: Classification of the most important figures for 6 months, 2, 5 and 10 year Euro rates. ***, **, * significant up to respective thresholds of 1%, 5% and 10%.

It is worth noting that the determining factors are slightly different for the short and long end of the curve. For virtually all maturities from the 5 year rate, the 4 most important indicators for the markets are the Germany's IFO, ZEW, its retail sales and manufacturing orders. A set of different, much more significant figures appear for short term rates (less than 5 years), namely the money supply and the German retail sales. France's consumer confidence figures have virtually no impact except on long rates (between 7 and 20 years).

Comparison with results obtained in the US in recent years (Fleming and Remolona (2001), Balduzzi et al. (2001), Piazzesi (2005), Guégan and Ielpo (2007)) highlight differences and similarities with the American economy. US activity and economic sentiment indicators take on very great importance, while (for the reason mentioned above) employment indicators, though very important within the US, have no impact in Europe. Another key difference is the fact that inflation figures have barely any impact in Europe over the period under study, whereas they are of crucial importance in the US. They appear to have been replaced in Europe by the money supply, an indicator which the Federal Reserve does not explicitly monitor.

2.1.2.2 Distortion of the yield curve by announcements

For each figure which exerts a significant impact on European interest rates the above Figure 2.5 show the shape taken by the reaction of yield curves. Each graph synthesises the results of the 16 regression analyses by maturity. I represent in grey the impact of European economic figures measured on a "trivial" basis (i.e. without taking account of the American influence), and in black this same impact, taking account of the 3 factors taken from the US yield curve. Dotted lines each time indicate the 95% confidence

interval.

The most striking stylized feature, already noted in the US by Fleming and Remolona (2001), as well as Guégan and Ielpo (2007), is the hump-shaped reaction of rates to most economic announcements. Intermediate maturities concentrate the strongest reactions. This shape is highly revealing of market participants' anticipations of the monetary authorities' reactions (raising or lowering target rates) to economic news. They do not expect the authorities to react at once (accounting for the weak impact on the very short end of the curve), but at a still very near time horizon nonetheless; they expect this reaction to take the form of a gradual cycle of rises or cuts in their target rates. Finally, a stronger than expected economic announcement leads to a steepening of the short end of the curve, then a flattening on longer-end maturities.

Although the overall shape is characterised by this hump, I can more precisely distinguish 2 very distinct types of reaction. For economic sentiment and activity indicators, the impact is weak for very short maturities (less than 1 year); it grows rapidly up to intermediate maturities (which vary by indicator), and then declines for long maturities. The most important impact for the ZEW and IFO is on maturities of around 5 years; that of the French consumer confidence figures and those of German retail sales tend to concentrate around 7-10 years; that of manufacturing orders, however, sees a hump further down the line (15 years). Thereafter, I note a decreasing impact on very long maturities.

In contrast, for price indicators (inflation, M3), the impact grows very steeply between 3 months and 1 years; it is then relatively flat regardless of the maturity, suggesting that the figures influence inflation expectations not only in the short term, but in the long term as well. The M3 indicator exerts its strongest influence on 2 year rates, whereas that of European inflation is around 5 year rates.

Announcements of target rates have the particular feature of influencing yields only in the very short term, at least graphically: their impact only extends to the 3 month rates (significant only up to a 10% risk level), and this reaction's shape slopes downwards with rate maturities, becoming insignificant. The short term rates high volatility is to be blamed for the lack of significance found in my results. However, Figure 1 clearly shows a different impact for very short term rates than for longer term ones. All in all, a target rate increase will lead to a (steeper?) flatter yield curve.

It is very striking to analyse this impact, which underpins the idea that the long end of the curve's reaction to news about monetary policy depends considerably on the markets' view of how credibly the monetary authorities are fighting inflation. In an en-

environment where the authorities enjoy credibility, currently the case in Europe, a rise in target rates can push down long rates provided the markets anticipate that the effects of this decision will be to lower anticipations of long term inflation. This is the well known "paradox" in monetary policy announcements highlighted by Carlstrom (1995), which still holds true, since he demonstrated that a rise in short rates can lead to a fall in long rates, or at least in long rates which are relatively inert to changes in short rates.

Figure 2.4 presents the 3 factors obtained on the American yield curve using PCA. These are the 3 standard factors which account for distortions in the yield curve: parallel shift (factor 1), steepening (2) and changes in the curvature of the yield curve (3). Figure 2 below shows the regression coefficients for each of these factors.

The "level" factor has the greatest impact on the curve, and this impact is quite stable for all maturities starting from 5 years yields. This means that US yield curve parallel shifts tend to drive similar reactions from the Euro yield curve, starting from intermediate maturities. The short end of the two curves, US and Euro, remain independent, and this is easily understandable considering the fact that they mostly react to their own monetary policy. The "slope" factor has a negative impact on the short end of the curve and a positive one on the long end. Lastly, the "curvature" factor's impact on the curve is virtually the opposite to that of the slope factor.

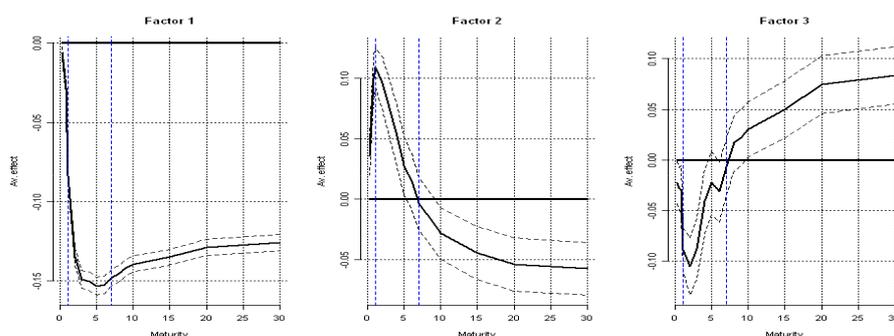


Figure 2.4: Impact of the 3 PCA-based factors, across the entire Euro curve

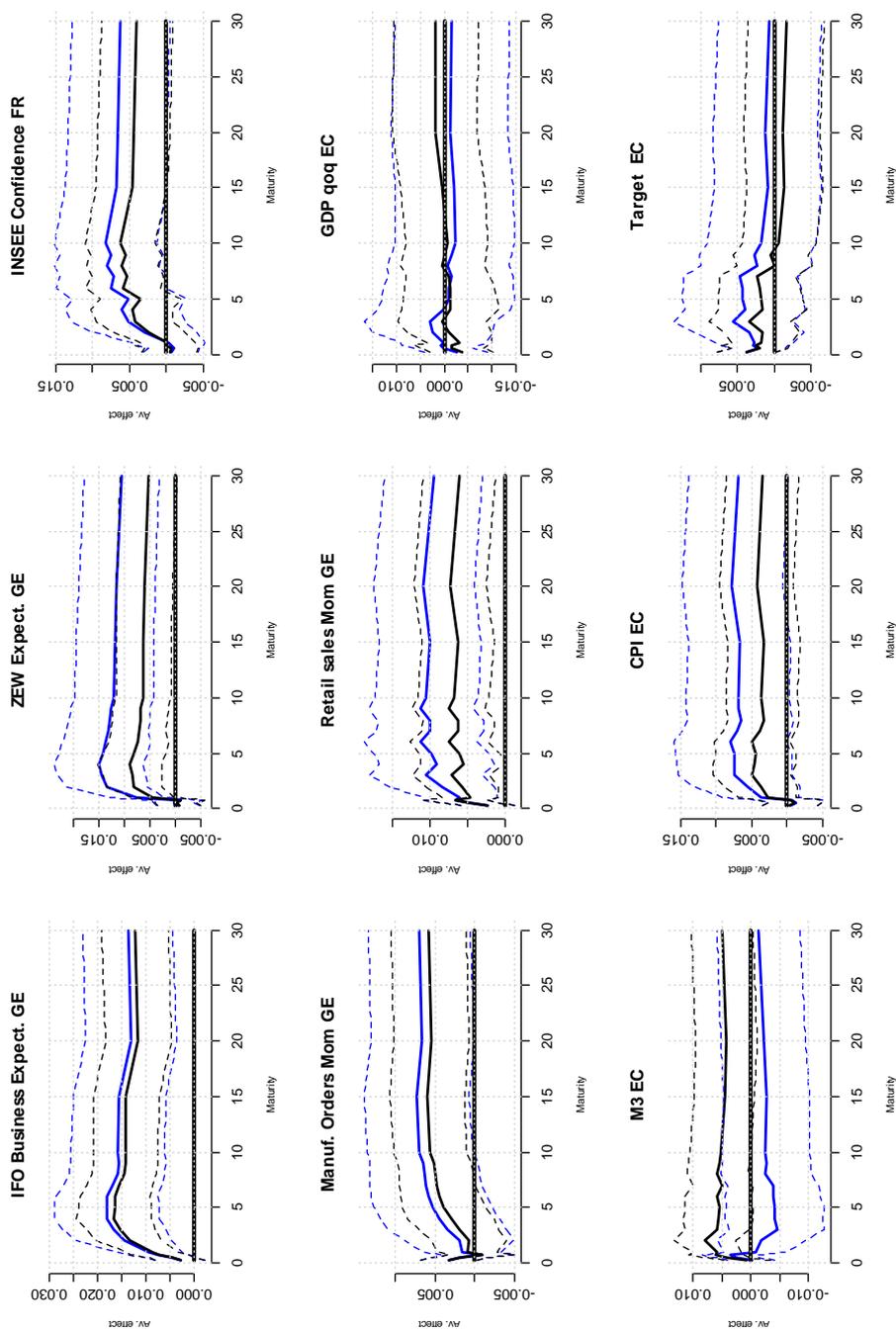


Figure 2.5: Impact of significant announcements on the entire Euro swap curve

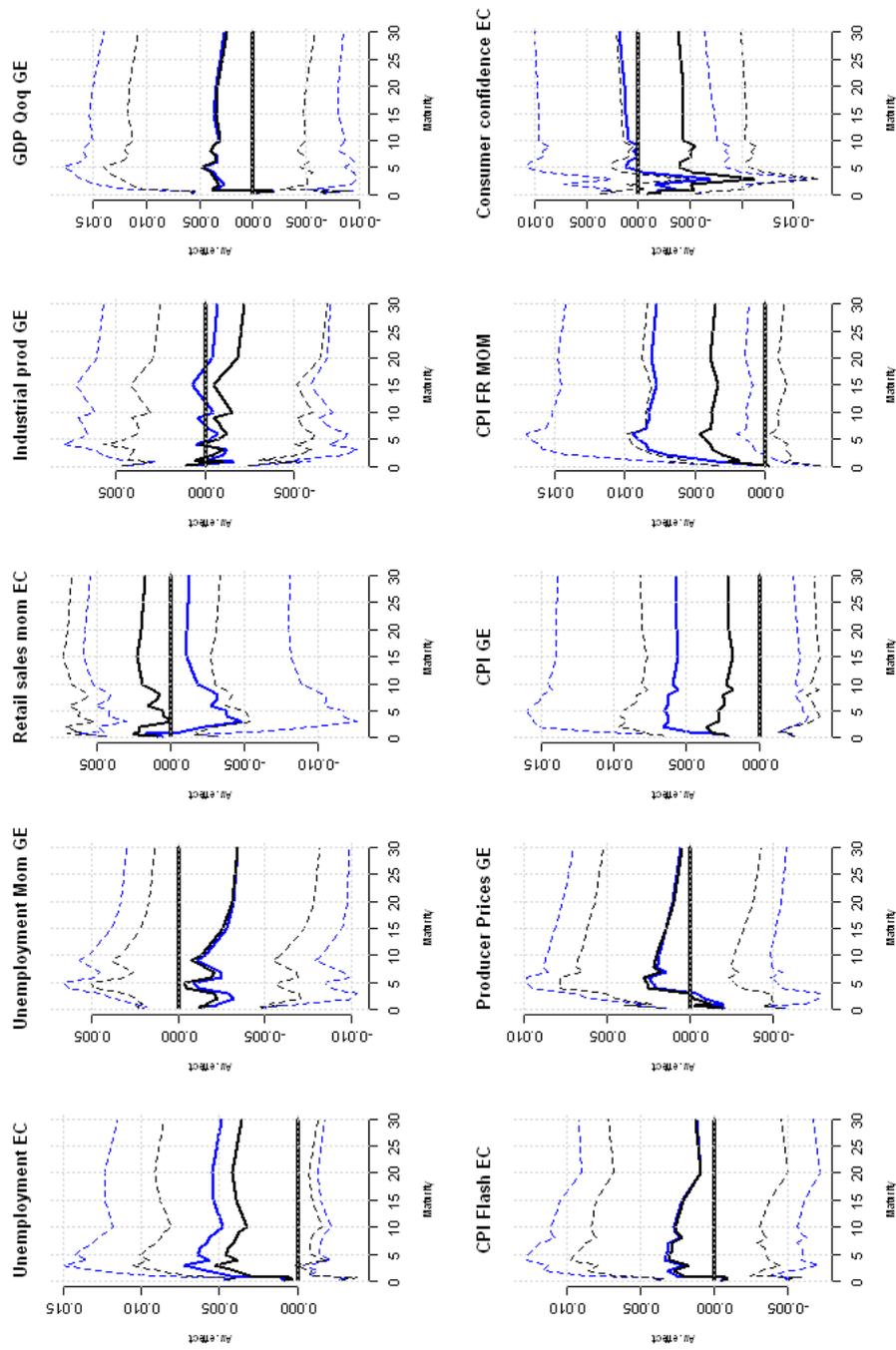


Figure 2.6: Impact of insignificant announcements on the entire Euro swap curve

2.1.2.3 Conclusion

Thus, only a very few European economic figures influence the yield curve in Europe. I note the very important impact of German economic sentiment and activity figures, which the markets view as more important than the European aggregate statistics. Two results contrast with those obtained in the existing literature: the weak impact of inflation figures and the strong influence of the M3 money supply. As noted in the US, the curve's reaction takes the form of a "hump shape", with intermediate maturities generally concentrating the greatest reactions. This shape is highly revealing of market participants' anticipations of the monetary authorities' reactions to economic news. Participants tend to anticipate a reaction by the authorities in the medium term, assuming that a change in the economic environment will lead them to make a gradual cycle of rises or cuts in their target rates, which will not take effect immediately. I draw a more precise distinction between 2 different types of shapes depending on the type of indicator under study: economic sentiment or activity indicators on the one hand, and price indicators on the other. In the next section, I develop an analysis of this kind: I present how the term structure impact of macroeconomic news on US rates dramatically vary. It depends on economic conditions and the type of indicator studied: this idea is logical but has scarcely been documented yet.

2.2 Toward a classification of the impact of news on the US bond market

Thus⁴, the term structure impact of news is not always the same: it clearly depends on the type of news that is investigated. The previous illustrations were based on a European dataset: this dataset is relatively new and thus small. With this kind of information set, it is difficult to move one step forward and answer the question: for a single figure, is the shape of the term structure impact changing, depending on economic conditions? Following what is presented in Dufrénot et al. (2004), I should be able to show that there exists a strong dependence on the business cycle, since the monetary stance itself – and thus the yield curve – depends on the business cycle. In this section, I tackle this issue using an American dataset, whose depth is larger than the European one.

There has been a flourishing literature related to the impact of US news on interest rates that is surveyed in Fleming and Remolona (1997). First, early articles studied

⁴This section is taken from a CES-Paris 1 working paper entitled "Further Evidence on the Impact of Economic News on Interest Rates", with Dominique Guégan. This paper is now in revision before a re-submission.

the impact of a selected number of macroeconomic figures on selected points of the yield curve. For example, Grossman (1981) and Urich and Watchel (1981) chose to focus on money supply surprises for selected maturities of the yield curve. Hardouvelis (1988) and Edison (1996) investigated the impact of employment news along with Consumer Price Index (CPI) and Producer Price Index (PPI) in a similar fashion. Second, while the former studies used daily datasets, the most recent ones made the most of the newly available high-frequency data, assuming that the measurement of the interest rates' reaction to surprises on a narrower window of time was bound to lead to more precise results. The results obtained pointed toward important facts: where studies achieved using daily data only found a few market mover figures, these studies (see for instance Balduzzi et al. (2001), Fleming and Remolona (1997) and Fleming and Remolona (2001)) concluded with the fact that as much as 70 releases actually produce moves within the U.S. bond markets. Finally, recent papers focused on the complete term structure response to macroeconomic news. Using an intraday dataset, Fleming and Remolona (2001) revealed hump shaped term structure effects, similar to what I presented in the previous section for the Euro area case.

Here, I propose different nested time series models to assess the shape of the term structure reaction to macroeconomic announcements. More, I allow this term impact to depend on various state variables: this way, I intend to capture the changes in the shapes depending on the economic or monetary cycles. The main estimation results unfold as follow. First, I find that there exist several types of surprises that actually affect the bond market, surprisingly matching the first four factors found when performing a principal component analysis over the daily changes in swap rates. Second, the ranking of market mover figures strongly depend upon the market perception of the economic cycle, measured by publicly available indicators, and upon the monetary policy stance, measured by the Fed's target rate. Finally, I show that the use of a threshold model when estimating the market response to macroeconomic news leads to the elimination of outliers within the dataset, yielding different - and often more significant - estimates of the market response to selected figures. The exclusion of these outliers brings about interest rates' reaction functions that are generally upper than the classical ones and more concave.

The remaining of this section is organized as follow. First, I present the methodology to estimate the term structure response to macroeconomic news. Then, I review the empirical results obtained.

2.2.1 Assessing the shape of the market reaction function

In this Section, I detail both the dataset and the time series models used to analyze the effect of the announcements on the US swap rate across maturities. The dataset used along the paper and its preliminary treatment is closed to the one used in the main articles investigating the bond market reaction to macroeconomic news, such as Balduzzi et al. (2001) and Fleming and Remolona (2001). The main novelty of this paper being the methodology, I present it in a detailed fashion so as to highlight my contributions.

2.2.2 The dataset

Along this paper I use two types of data. On the one hand, I use the daily changes in the US swap rates from June, 24th of 1996 until March, 1st 2006, for the following maturities: 1- to 10-year, 15-year, 20-year and 30-year swap rates. By daily changes, I mean the difference between two following daily closing rates. Let $\Delta r_t(\tau)$ be this change in the closing swap rate $r_t(\tau)$ for a maturity equal to τ , on a date t . Then, I have:

$$\Delta r_t(\tau) = r_t(\tau) - r_{t-1}(\tau), \quad (2.11)$$

with a time unit equal to one day. One main advantage to use swap rates is that they are generic rates: these rates have a constant time to maturity over the whole sample and thus do not theoretically depend on time. Using such rates means that I do not have to deal with the reduction of the time to maturity. We also had to estimate a few missing rates, which was done using the cubic splines method, like in Bomfim (2003)⁵.

The US swap rates dataset has been extracted from the Bloomberg database. The Bloomberg closing swap rates are gathered from different brokers and financial institutions at the closing of each US bond market trading day. During a trading day, the moments the intraday database is updated is rather random and this randomness extends to the maturities that are updated. On the contrary, for the closing swap rates, the time of the update is rather homogeneous. This is why I propose to use a daily dataset made of these closing swap rates.

From the Bloomberg database, I also extracted the US economic calendar across the dates already mentioned for the swap rates. This calendar contains every economic announcement linked to the US economy which are supposed to be monitored by financial market participants. Several of these figures are well known by economists,

⁵This is a classic method, discussed in classical textbooks, e.g. Martellini et al. (2003).

such as the Non Farm Payroll figure, which is the number of jobs created on a one month period. These figures are issued regularly by office statistics such as the Bureau of Labour Statistics. For example, the Non Farm Payroll figure is issued every first Friday of a month and is usually followed by large moves in the bond market. Other figures are not so well known, and one of the purposes of this paper is to cast some light on the effect of these indicators on the term structure of the US swap rates.

Growth	Conjunctural Indicators	Real estate
Industrial New orders	ISM manuf	Construction Spending
Wholesale Inventory	Philified Index	Housing Start
Industrial Production	Conf. Board Consumer Conf.	Existing Home Sales
GDP	Chicago PMI	New Home Sales
Trade Balance	Non Manuf. ISM	Building Permits
Capacity Utilization Rate	Consumer Conf. Michigan	NAHB Housing Market Index
Durable Good Orders	Empire Manufacturing	Construction Spending
Labor Market	Consumption	Inflation
Unemployment Rate	Household Consumption	Consumer Price Index
Jobless Claims	Personal Income	Producer Price Index
Non Farm Payroll	Consumer Credit	Import Price Index
Employment Cost Index	Retail Sales	
Wages	Personal Consumption (Q)	
Hourly Average Wages		
Weekly Working Hours		
Weekly Jobless Claims		
Indice Help Wanted		

Table 2.9: List of the macroeconomic announcements studied in this paper. These announcements are monthly ones, except for: Weekly Jobless Claims (weekly figure), Personal Consumption (quarterly figure), Capacity Utilization Rate (quarterly figure) and GDP (quarterly figure).

We discarded several series from the Bloomberg database. Table 2.2.2 presents the selected figures used during the estimation process. We eliminated these series for different reasons. First, some of the figures got their names changed over the studied period. In this case, I simply changed the old names into the newer ones so as to avoid having a single figure known under different names. This was the case for the Michigan Consumer Confidence that was reported under several names in the Bloomberg Calendar. Second, some of these figures were ill reported and included a lot of missing values. Finally, some of these figures ceased to be released during the studied period, such as the M3 aggregate and I chose not to include them, to make this study of interest both for academics and practitioners.

Most of the announcements studied are monthly (see table 2.2.2). The series were treated by the Bloomberg calendar the way bond market participants do. For example, the surprise in the Consumer Price Index (CPI hereafter) is a surprise in the month-over-month figure. A month-over-month (m-o-m hereafter) figure is simply the percentage of growth of the index over the month. With an index denoted I_t for the month t , the m-o-m figure will be equal to $\frac{I_t}{I_{t-1}} - 1$, with a time unit equal to one month. The same kind of transformation applies for most of the figures but the sentiment survey such as Purchasing Manager Index (PMI) or Michigan Consumer Confidence. These survey figures are often presented using the value of their index. This is a rather technical knowledge many books are devoted to. Anyone interested in these ways of processing data can get in depth analysis in such books (see e.g. Baumohl (2005)).

In my methodology, I used the first estimates of the macroeconomic news. Most of the macroeconomic figures released in the US are initially preliminary estimates. On the next announcement for the same figure, a revised estimate of the preceding figure is released. Most of the macroeconomic datasets used in empirical papers are made of the revised estimates of every macroeconomic figures. Recently, Orphanides (2001), Bernanke and Boivin (2001) and Kishor and Koenig (2005), among others, took this data revision problem into account, highlighting the importance of this phenomenon on macroeconomic empirical models. For my purposes, the use of the first estimate is of tremendous importance: the first announcement is the one bond market participants had to face with and eventually reacted to.

What is more, the Bloomberg calendar also contains the Bloomberg forecasts regarding each of these figures. Bloomberg forecasts are formed using the 50% empirical quantile of the distribution of a survey made of the forecasts of several bank economists, regarding a precise figure. The use of the median as a measure of the expectations makes the forecast robust to the influence of badly intentionned economists that would want to shift the forecast in order to make the most of it. What is more, this forecast is extensively used by market participants. For each figure that is predicted by Bloomberg's collection of economists' forecasts, the median is regularly updated until every economist answers the survey, which can take up to two weeks. We retained the last median computed by the Bloomberg services, so as to match both the practitioners and academic ways of doing things. Some of the eliminated series were discarded because there was no available forecast.

2.2.3 Assessing the shape of the market reaction function

In this section, I skip to the presentation of the time series models used along the paper. The first model is the classical linear model presented earlier. Let $S_{t,i}$ denote

the surprise at time t in the figures indexed by i as follows:

$$S_{t,i} = \frac{R_{t,i} - F_{t,i}}{\sigma_{S_i}}, \quad (2.12)$$

where $F_{t,i}$ is the market consensus about the upcoming figures i for t , the date of release; $R_{t,i}$ is the real announcement (the first estimate) at time t of the same figure i . To make the surprises comparable, surprises are scaled using their historical standard deviation. This way of proceeding is very common, see e.g. Edison (1996), Fleming and Remolona (1997, 2001) and Balduzzi et al. (2001). We used the Bloomberg forecasts as a measure of the market consensus for a given figure at a given date. Thus, F_t^i will be proxied by the last forecast in the Bloomberg database for each announcement.

Building a time series model to relate the macroeconomic surprises to the changes in the interest rates of maturity τ requires some preliminary considerations, and especially for the dataset building. Even though there seems to be some regularity in the time of arrival of these surprises, they are irregularly spaced in time, preventing the building of a single global model to relate any surprises to the daily changes in rates. For example, the Non Farm Payroll are scheduled to be released on the first Friday of each month: even though this seems to be a regular release pace, it still leads to data that are irregularly spaced in time, in so far as the number of days from the first Friday of a month to the next one is not always the same. What is more, estimating a global model as asserted before would involve the use of 40 exogenous variables which may threaten the robustness of the results. Moreover, the sampling frequency of the exogenous variables can differ: my work involves both quarterly, monthly and weekly news. Finally, the endogenous variable (namely $r_t(\tau)$) depends on the maturity τ of the swap rates. For several maturities, the model to built should be a generalized linear model (a model that encompasses several dependent variables in the meantime), which thus requires to be estimated using the (Quasi) Generalized Least Squares. To solve these difficulties, I built one model for each each surprise and each maturity, in a similar fashion to the *Seemingly Unrelated Regression Models*. This has an obvious consequence over the chosen notations: the subscripts must display the dependency on time, maturity and macroeconomic surprise.

Now, let me denote $\Delta r_{t,i}(\tau)$ the daily change in swap rate of maturity τ on the date t of the release of the figure indexes by $i = \{1, \dots, I\}$, where I is the total number of surprises. The couple (t, i) is somewhat a calendar coordinate in the global dataset. The linear model (model 1 hereafter) assumes for given (fixed) $i = \{1, \dots, I\}$ and $\tau = \{\tau_1, \dots, \tau_m\}$ that:

$$\Delta r_{t,i}(\tau) = \beta_{i,\tau} + \alpha_{i,\tau} S_{t,i} + \epsilon_{t,i,\tau}, \quad (2.13)$$

where $\alpha_{i,\tau}$ and $\beta_{i,\tau}$ are real-valued parameters. $(\epsilon_{t,i,\tau})_t$ is a Gaussian white noise with standard deviation $\sigma_{i,\tau}$, conditionally upon $S_{t,i}$. In the remaining of the paper, I denote these conditions as conditions 2.2.3. This very simple model is usually augmented with the other surprises announced on the same day (t, i) :

$$\Delta r_{t,i}(\tau) = \beta_{i,\tau} + \alpha_{i,\tau} S_{t,i} + \sum_{j=1}^J \gamma_{j,\tau} S_{t,i}^j + \epsilon_{t,i,\tau}, \quad (2.14)$$

where $S_{t,i}^j$ are the scaled surprises j announced on the same day as surprise i . Again, I assume that $\gamma_{j,\tau}, \forall j$ is on the real line. These additional surprises are essential to ensure that the estimated $\alpha_{i,\tau}$ truly isolate the effect of the announcement that is analyzed.

In this section, I build a collection of nested time series models to capture the term structure reaction to macroeconomic news. The linear model defined by equation (2.13) is the first model. For the ease of the presentation, I will get rid of the part of the equation (2.14) that is dedicated to the announcements released on the same date as the announcement studied (that is $\sum_{j=1}^J \gamma_{j,\tau} S_{t,i}^j$), maintaining it during the estimation. What is more, for the sake of simplicity, I do not denote anymore the maturity of each change in the swap rate, skipping from $\Delta r_{t,i}(\tau)$ to $\Delta r_{t,i}$ (the same treatment also applies to the parameters of the model): I present the models for a given and fixed τ .

The immediate consequence of the model 1-like specification is:

$$\mathbb{E}[\Delta r_{t,i} | S_{t,i}] = \beta_i + \alpha_i S_{t,i}. \quad (2.15)$$

This expectation has an important implication: whatever past information and the state of the economy, the conditional expectation of the rates' jump is always the same, for a given surprise, i.e. $\alpha_i S_{t,i}$. This is not in line with what can be observed both by practitioners and academics. We propose two nested non-linear models to account for these facts.

First, with model 1, the market reaction to a given surprise is bound to be the same for each state of the economy. The rates' response to macroeconomic announcements may depend on several factors such as the timeliness of the release - that is the order of release for a one month period -, the degree of surprise, the conditions of market uncertainty or the sign of the surprise. On these points, see Fleming and Remolona (1997) and Hans (2001). Other articles pointed toward the fact that the interest rates' response to macroeconomic announcements may also depend on a threshold variable, such as economic leading indicators or employment figures. For example, Prag (1994) shows that the impact of unemployment surprises on the bond prices may depend on the

current level of unemployment. Veredas (2005) shows that the market response to surprises in macroeconomic releases strongly depends upon the momentum of the cycle: in this framework, bad news have more impact on bond prices during expansion periods than recession ones. Here, I argue that the market response depends on several threshold variables, including indicators for monetary policy stance and economic agent sentiment regarding future activity.

Thus, I propose to use a threshold time series model. Given the small number of observations I have at hand⁶, I will consider a two states economy, say recession/expansion states. Let me define $(\pi_{t,i})_{t \in \mathbb{Z}}$, an observable process that is used as a state variable to capture the conditional reaction to the surprises in the macroeconomic figure i . With this state variable, I measure the state of the economy as follow: this process has to cross a threshold value $\bar{\pi}_i$ for the economy to go through a change in state, say from expansion to recession. For each $i \in \{1, \dots, I\}$, model 2 is then the following:

$$\Delta r_{t,i} = \beta_i + \alpha_{1,i} \mathbb{1}_{\pi_{t,i} > \bar{\pi}_i} S_{t,i} + \alpha_{2,i} \mathbb{1}_{\pi_{t,i} \leq \bar{\pi}_i} S_{t,i} + \epsilon_{t,i}, \quad (2.16)$$

where $\mathbb{1}_{\pi_{t,i} > \bar{\pi}_i}$ takes value 1 if $\pi_{t,i} > \bar{\pi}_i$ and 0 if not. $\mathbb{1}_{\pi_{t,i} \leq \bar{\pi}_i}$ is defined as $1 - \mathbb{1}_{\pi_{t,i} > \bar{\pi}_i}$. $\alpha_{1,i}$ and $\alpha_{2,i}$ are again on the real line. The assumption 2.2.3 applies again. This model belongs to the class of the SETAR models (Self-Exciting Autoregressive models) introduced by Lim and Tong (1980) and developed in Tong (1990).

The estimation of threshold models has been discussed in Chan (1990), Hansen (1997, 2000) and Tong (1990) [chapter 5], and asymptotic estimation results have been derived in it. With these models, the log-likelihood function is not continuous in the threshold parameter. Thus, the threshold cannot be estimated using standard Gradient methods. The estimation can be performed by grid search. This is a standard method in econometrics, as detailed in Greene (2000), in the chapter dedicated to numerical optimization.

The model proposed in equation (2.16) leads to the following conditional expectations:

$$\mathbb{E}[\Delta r_{t,i} | \pi_{t,i} > \bar{\pi}_i, S_{t,i}] = \beta_i + \alpha_{1,i} S_{t,i} \quad (2.17)$$

$$\mathbb{E}[\Delta r_{t,i} | \pi_{t,i} \leq \bar{\pi}_i, S_{t,i}] = \beta_i + \alpha_{2,i} S_{t,i}. \quad (2.18)$$

Thus, the market reaction clearly differs, depending upon the state variable. Once again, each macroeconomic figure can be linked to a proper threshold variable $(\pi_{t,i})_{t \in \mathbb{Z}}$, along with a proper threshold value $\bar{\pi}_i$. Now, I need to select variables to proxy this state variable. Clearly, there is no unique answer: sentiment survey (such as PMI

⁶For monthly figures, I only have one announcement a month, which makes 120 observations with no missing value in the dataset. For the quarterly figures, this makes only 30 observations.

index or Conference Board index) could be a good proxy for this variable. These sentiment survey can be considered as coincident or leading indicators of the stance of the economy and thus reflects the market sentiment better than real aggregates such as industrial indicators or GDP. Monetary policy is also known to play an important part in the psychology of the bond market. This is why I also introduced the Fed's target rate, as a measure of the monetary policy stance.

The table 2.10 presents the different threshold variables that I retained for the estimation of the threshold model. Note that to these variables, I add the first and second factors of a principal component analysis performed over all these variables, so as to get a global economic confidence index. This is a classical method used to build this kind of global economic stance index (see e.g. Stock and Watson (2002)). So as to avoid any data vintage problem, as presented e.g. in Kishor and Koenig (2005), I used the first estimates of every of these series: they were the ones at hand for market participants, at the time of their reactions to the announcements. In the section dedicated to the estimation results, I present the results of the choice of the threshold variable. For each surprise, I retain the threshold value that yielded the highest log-likelihood value or the lowest root mean square error. These results show the benefit from estimating each model for each macroeconomic figure and each maturities: the selected threshold variable can clearly differ depending both on the rates' maturity and the figure that is studied.

Indicator...	... as a measure of	Mean	Std. Deviation
PMI	Future economic activity	53,02	5,26
Conf. Board	Future economic activity	112,24	21,00
Michigan	Future economic activity	96,80	8,90
Fed Target Rate	Monetary policy stance	3,73	1,91
Fed Philadelphie	Future economic activity	9,54	13,52
Factor 1	-	87,30	14,30
Factor 2	-	-128,47	18,38

Table 2.10: Threshold variables used in the estimation process

In the table presenting the results of my estimations, I refer to these threshold variables using the following notations: PMI is for PMI index, CONF is for Conference Board Consumer Confidence, MICH is for Consumer Confidence Michigan, FED is for the Fed Target Rate, PHI is for the Philified Index and FACT1 and FACT2 refer to the first two factors of a principal component analysis performed over all these series.

Finally, I propose to test for path dependency in the dynamics of the rates. By this, I simply mean to specify a model that would link the rates' reaction during two succes-

sive announcements of the same figure. Note that most of the time, a month elapsed between two successive announcements. We propose to test whether a part of $\Delta r_{t_k,i}$ is explained by the rates' reaction at time (t_{k-1}, i) , that is the bonds over- or under-reaction during the former announcement for exactly the same figure i . When model 2 provides consistent estimates of the reaction of the market to announcements, the residuals of this model can be used as a proxy to measure the rates' over or under reaction to a given announcement. Thus, a natural measure of the market absolute overreaction at time (t_{k-1}, i) is $\epsilon_{t_{k-1},i}$. By adding this term to the model proposed in equation (2.16), I obtain model 3:

$$\Delta r_{t_k,i} = \beta_i + \alpha_{1,i} \mathbb{1}_{\pi_{t_k,i} > \bar{\pi}_i} S_{t_k,i} + \alpha_{2,i} \mathbb{1}_{\pi_{t_k,i} \leq \bar{\pi}_i} S_{t_k,i} + \theta \epsilon_{t_{k-1},i} + \epsilon_{t_k,i}, \quad (2.19)$$

where $\theta_i \in \mathbb{R}$ such that $\mathbb{E}[\Delta r_{t_k,i}] < \infty$. Conditions 2.2.3 still apply. By the law of iterated expectations, $\mathbb{E}[\epsilon_{t_k,i}] = \mathbb{E}[\mathbb{E}[\epsilon_{t_k,i} | \pi_{t_k,i}, S_{t_k,i}, \epsilon_{t_{k-1},i}]] = 0$. Thus, I can rewrite equation (2.19) with a mean reverting error process:

$$\Delta r_{t_k,i} = \beta_i + \alpha_{1,i} \mathbb{1}_{\pi_{t_k,i} > \bar{\pi}_i} S_{t_k,i} + \alpha_{2,i} \mathbb{1}_{\pi_{t_k,i} \leq \bar{\pi}_i} S_{t_k,i} - \theta_i (\mathbb{E}[\epsilon_{t_{k-1},i}] - \epsilon_{t_{k-1},i}) + \epsilon_{t_k,i}. \quad (2.20)$$

The interpretation of θ_i in equation (2.20) arises naturally. Let me distinguish three cases. If $\theta_i = 0$, this obviously means that there is no linear link between the past overreaction and the current one. Second, if $\theta_i > 0$, the bond market tends to be self exciting: when an over/undershoot occurs when releasing a figure, then there is a higher probability that the market will over/undershoot again on the next release of the same figure. On the contrary, if $\theta_i < 0$, the market responses to announcements are mean reverting (toward a mean equal to 0). In the latter case, an over/undershoot is likely to be followed by a smoother reaction on the date of the next release of the same figure. Note that from a statistical point of view, if θ_i is significantly different from 0, the estimation of model 1 is likely to be biased.

The conditional expectation of $\Delta r_{t_k,i}$ is path dependent: the rates' response will depend on their former reaction to the announcement of the same figures. Thus I have:

$$\mathbb{E}[\Delta r_{t_k,i} | \pi_{t_k,i} > \bar{\pi}_i, S_{t_k,i}, \epsilon_{t_{k-1},i}] = \beta_i + \alpha_{1,i} S_{t_k,i} + \theta_i \epsilon_{t_{k-1},i} \quad (2.21)$$

$$\mathbb{E}[\Delta r_{t_k,i} | \pi_{t_k,i} \leq \bar{\pi}_i, S_{t_k,i}, \epsilon_{t_{k-1},i}] = \beta_i + \alpha_{2,i} S_{t_k,i} + \theta_i \epsilon_{t_{k-1},i}. \quad (2.22)$$

From this point, I now obtain a collection of nested models that will help me document further the admissible shapes of the bond market reaction function to macroeconomic announcements. This rather simple approach thus entitles me to build *LR* tests, as described in Davidson and MacKinnon (1993). Models 1, 2 and 3 are nested, and likelihood ratio tests can be easily performed so as to chose which is the more interesting model, regarding the data at hand. These elements will be studied within the

next section, along with the analysis of the results obtained with the models defined by equations (2.13), (2.16) and (2.19). In the remaining of the paper I refer to the model defined by equation (2.13) as model 1, to the one defined by equation (2.16) as model 2 and to the model defined by equation (2.19) as model 3. These notations are summarized in the following table :

Model	Equation #	Rates dynamic
Model 1	Equation (2.13)	$\Delta r_{t,i} = \beta_i + \alpha_i S_{t,i} + \epsilon_{t,i}$
Model 2	Equation (2.16)	$\Delta r_{t,i} = \beta_i + \alpha_{1,i} \mathbb{1}_{\pi_{t,i} > \bar{\pi}_i} S_{t,i} + \alpha_{2,i} \mathbb{1}_{\pi_{t,i} \leq \bar{\pi}_i} S_{t,i} + \epsilon_{t,i}$
Model 3	Equation (2.19)	$\Delta r_{t_k,i} = \beta_i + \alpha_{1,i} \mathbb{1}_{\pi_{t_k,i} > \bar{\pi}_i} S_{t_k,i} + \alpha_{2,i} \mathbb{1}_{\pi_{t_k,i} \leq \bar{\pi}_i} S_{t_k,i} + \theta_i \epsilon_{t_{k-1},i} + \epsilon_{t_k,i}$

2.2.4 Empirical results

In this Section, I systematically analyze the results of the estimations of the models presented in the previous section. First, I analyze the results obtained from the likelihood ratio tests performed over the different nested models, using the dataset presented earlier. From these estimation results, I propose a list of the most market mover figures for each maturity and I show that by using model 2 the list of market mover figures significantly increases. We also noticed that model 2 leads to intercepts that are statistically equal to 0, unlike model 1. Third, I propose to identify the shapes of the term structure response with those of the first four factors of a principal component analysis performed over the daily changes in the swap rates. By doing so, I show that there are several kinds of possible shapes for the hump-shaped term structure response to macroeconomic news (see e.g. Fleming and Remolona (2001)). Fourth, I propose a detailed analysis of the term structure response to several announcements, underlying the fact that the inclusion of a threshold variable reveals that model 1 often underestimates the true reaction function. We guess that this can either be due to the economic cycle dependence of the term structure effect or the existence of outliers within the dataset.

2.2.4.1 Bulk effects of the introduction of the threshold variable

The introduction of those threshold variables produced remarkable effects on my estimations, yielding results that I believe are new. We present in tables 2.15, 2.16 and 2.17 the results of the estimation obtained from the models presented in the previous section. We only present the estimates of the model with the higher log-likelihood function, along with the following LR test. For example, let model 1 be the constrained model, with log likelihood denoted $\ln L_c$ and model 2 be the unconstrained model, with

a log-likelihood denoted $\ln L_u$. The null hypothesis \mathbb{H}_0 assumes that the constraint imposed in model 1 statistically holds. Thus, under \mathbb{H}_0 , model 1 is considered as a better model than the unconstrained model. Tables 2.15, 2.16 and 2.17 report the selected threshold variables along with the threshold value, that are estimated for each maturity and macroeconomic figures. We also report the LR test results, testing constrained against the unconstrained models. The test statistics is:

$$LR = 2(\ln L_c - \ln L_u), \quad (2.23)$$

with the previous notations. Under the null hypothesis that the constraint statistically holds, this statistic has a Chi-square distribution, with a degree of freedom equal to the number of constraints imposed in the constraint model. In my case, I have only one constraint, and the statistics is distributed as a χ_1^2 , under the null. We proceed in a similar fashion to test model 3 vs. model 2.

The main result obtained with my methodology is that model 2 is globally the preferred model, regardless of the surprise and the maturity. When testing model 2 vs. model 1, the null is rejected at either a 5% or 10% risk level most of the time for every maturity. The few cases when it is not rejected are reported in table 2.11. This is an essential result for my work: model 2 provides a better explanation of the rates' behavior than model 1. Even though model 1 is the one that is generally proposed in the literature, model 2 better encompasses an important feature of the rates' dynamic: the economic cycle dependence. Note that I do not report the LR test of model 3 against model 2, because the model 3 was almost always rejected at either a 5% or 10% level when compared to model 2.

Economic Announcement	Swap rates maturities
Household Consumption	1,6,7,9 and 10 year
Employment Cost Index	15,20 and 30 year
Empire Manufacturing Index	4,5,6,7,8,9,10,15,20 and 30 year
Personal Consumption	2,3,4,5 and 6 year

Table 2.11: Announcements and maturities for which the null of the LR test is accepted, when testing model 2 vs. model 1.

The introduction of the state variables allowed me to point out more than the usual number of "market movers" figures: I consider that a market mover figure is an announcement for which the estimated impact in models 1 and 3 is significative up to a 5% percent test. Here, almost every announcement that I tested was found to have a significative influence on the yield curve. Fleming and Remolona (2001) assumed that

the use of daily data instead of intra day ones were to bring about an underestimation of the market reaction function. Here, I find that considering the market responses conditionally upon a threshold variable that has been properly selected puts an end to this underestimation. Almost every announcement produces an effect on the yield curve. In appendices, I propose two comparative tables to assess this point. In table 2.12, I present the ranked market mover announcements found when estimating model 1. In table 2.14, I report the ranked market mover announcements obtained when estimating model 2, along with the selected threshold variable and the threshold value. The main point about this table is that the number of market mover figures significantly increases when using model 2: the introduction of the threshold variable leads to the finding of a greater number of market mover figures. The exclusion of this threshold variable seems to bring about an underestimation of the term structure reaction to several announcements. In subsection 2.2.5.2., I detail some of the reasons explaining this new stylized fact.

One other remarkable fact about my methodology is the following: when estimating model 1, most of the intercepts are significant up to a 5% risk level, unlike when estimating model 2. Table 2.13 reports figures and maturities for which this intercept remains significant in model 2. Where the bond market to be efficient, there should be no significant intercept in the estimation of the proposed models. One may think of this constant term as an α in the Capital Asset Pricing Model framework⁷, as presented in Gourioux and Jasiak (2001) and Campbell et al. (1997). Thus, when compared to model 2, model 1 is misspecified and leads to misleading ideas such as the idea that the bond market is not efficient⁸.

⁷The CAPM were initially developed by Sharpe (1964), Lintner (1965) and Mossin (1966).

⁸In a linear model with centered exogenous variables, the intercept can be interpreted as an average of the endogenous variable. In my case, this means that I am looking for regular effects over a given announcement. This effect is not the result of either a positive or a negative surprise, but simply the result of the fact that on this trading day, the announcement produces by itself a regular reaction in the bond market. Note that swap rates are used for many financial applications, such as deriving zero-coupon yield curve, pricing swaps or pricing interest rates derivatives such as swaptions. This kind of regular moves in the whole bond market can have significant implications for the whole bond market.

Rank	2 year	5 year	10 year	30 year
1	Non Farm Payroll	Non Farm Payroll	Non Farm Payroll	Non Farm Payroll
2	ISM manuf	ISM manuf	ISM manuf	ISM manuf
3	Employment Cost Index	Employment Cost Index	Employment Cost Index	Employment Cost Index
4	Philifed Index	Philifed Index	Non Manuf. ISM	Non Manuf. ISM
5	Durable Good Orders	Personal Consumption	Indice Help Wanted	Indice Help Wanted
6	NAHB Housing Market Index	GDP after 1999	Industrial Production	Wholesale Inventory
7	Unemployment Rate	Non Manuf. ISM	Philifed Index	Philifed Index
8	Conf. Board Consumer Conf.	Retail Sales	GDP after 1999	New Home Sales
9	Jobless Claims	Industrial Production	Retail Sales	Retail Sales
10	Industrial Production	Conf. Board Consumer Conf.	Conf. Board Consumer Conf.	Industrial Production
11	Non Manuf. ISM	Jobless Claims	New Home Sales	Jobless Claims
12	New Home Sales	New Home Sales	Trade Balance	Chicago PMI
13	Chicago PMI	Durable Good Orders	Jobless Claims	
14		Chicago PMI	Chicago PMI	
15			Existing Home Sales	
16				
17				

Table 2.12: List of the ranked market mover announcements found when using model 1.

Economic Announcement	Swap rates maturities
Household Consumption	3,4,6,7,8,9,10,15,30
Personal Income	2,3,4,6,7,8,9,10,15,30
ISM Manuf.	4,6,7,8,9,10,15,20,30
Existing Home Sales	8,9,15,20,30
Weekly Jobless Claims	1
Building Permits	1
Empire Manufacturing	1
Personal Consumption	1
Indice Help Wanted	1
NAHB Housing Index	1
Construction Spending	1,7,8,9,10,15,20,30

Table 2.13: Announcements for which the intercept is significant both for model (1) and model (3)

Rank	2 year		5 year		10 year	
	Variable	Condition	Variable	Condition	Variable	Condition
1	Capacity Utilization Rate	FED<3.553	Capacity Utilization Rate	FED<3.553	Capacity Utilization Rate	FED<3.553
2	Trade Balance	PHI<-26.126	Phlified Index	PHI<41.189	Phlified Index	PHI<41.189
3	Phlified Index	PHI<41.189	Unemployment Rate	PHI>60.642	Unemployment Rate	PHI>60.642
4	Existing Home Sales	PHI<49.526	Consumer Price Index	CONF<78.937	ISM manuf	FED>6.211
5	Industrial New orders	PHI<-14.037	Non Farm Payroll	FACT1>97.784	Non Farm Payroll	FACT1>97.784
6	Consumer Price Index	CONF<78.937	Hourly Average Wages	FACT1>97.784	Consumer Price Index	CONF<78.937
7	Non Farm Payroll	FACT1>97.784	Building Permits	PHI<51.968	Hourly Average Wages	FACT1>97.784
8	Unemployment Rate	PHI>60.642	GDP after 1999	PHI<43.968	Building Permits	PHI<51.968
9	ISM manuf	FED>6.211	Industrial Production	PHI<-14.037	Conf. Board Consumer Conf.	PHI<43.968
10	Retail Sales	FED>6.211	Employment Cost Index	FED>5.342	Non Manuf. ISM	FACT1<64.883
11	Hourly Average Wages	FACT1>97.784	Wholesale Inventory	PHI<-10.226	Wholesale Inventory	PHI<-10.226
12	Industrial Production	PHI<-14.037	Non Manuf. ISM	FACT1<64.883	Retail Sales	CONF>122.779
13	Building Permits	PHI<51.968	Existing Home Sales	MICH<100.063	Existing Home Sales	MICH<100.063
14	Personal Consumption	MICH>96.011	Consumer Conf. Michigan	FACT1<61.227	New Home Sales	MICH>102.053
15	Non Manuf. ISM	PHI<-14.037	Personal Income	FACT1>94.128	Employment Cost Index	FED>5.342
16	Construction Spending	MICH<104.758	ISM manuf	MICH>86.137	Personal Income	FACT1>94.128
17	Employment Cost Index	FACT1>86.592	GDP	FACT1<85.497	Trade Balance	FACT2>-109.378
18	NAHB Housing Market Index	FACT2>-101.575	NAHB Housing Market Index	PHI>11.358	Weekly Jobless Claims	PHI<-5.616
19	Wholesale Inventory	PHI<-10.226	Conf. Board Consumer Conf.	PHI<49.526	Indice Help Wanted	FED<3.053
20	Weekly Working Hours	FACT2<-115.275	Trade Balance	FACT2>-112.93	Weekly Working Hours	CONF>92.089
21	Conf. Board Consumer Conf.	FED>3.605	Weekly Working Hours	CONF>92.089	Personal Consumption	PHI>55.253
22	GDP	FACT1<85.497	Indice Help Wanted	FED>3.395	NAHB Housing Market Index	PHI>11.358
23	Productivity	PHI>58.589	Retail Sales	MICH>80.168	Industrial Production	PHI<56.474
24	Consumer Conf. Michigan	MICH>92.105	Personal Consumption	FACT1>71.872	Consumer Conf. Michigan	MICH>92.105
25	Empire Manufacturing	PHI<55.253	Chicago PMI	MICH<94.095	GDP after 1999	MICH>84.147
26	Indice Help Wanted	FED<3.395	New Home Sales	PHI>41.053	Chicago PMI	MICH<94.095
27	Chicago PMI	MICH<94.095	Wages	FACT1<65.128	Wages	FACT1<65.128
28	New Home Sales	PHI>-21.658	Weekly Jobless Claims	FACT1<81.692		
29	GDP after 1999	MICH<108.021	Construction Spending	PHI>1.868		
30	Weekly Jobless Claims	FACT1<81.692				
31	Construction Spending	PHI>1.868				

Table 2.14: List of the ranked market movers figures found when estimating the threshold model for each available maturity, along with the threshold variable used for the estimation and the value of the threshold.

	Intercept	> Threshold	< Threshold	Threshold	Th. value	LR test pvalue
Household consumption	0.008	0.004	-0.023	PMI	42.305	0.04
Personal Income	0.009*	0.004	-0.071	FACT1	61.227	0
ISM manuf	0.011**	0.119**	0.024**	FED	6.211	0
Industrial New orders	-0.001	0.001	0.235**	PHI	-14.037	0.01
Construction Spending	0	-0.021	0.057**	MICH	104.758	0.04
Consumer Credit	0.005	-0.567	-0.003	PHI	31.689	0
Wholesale Inventory	0.001	0.003	0.044**	PHI	-10.226	0.06
Retail Sales	-0.001	0.094**	0.007	FED	6.211	0.01
Industrial Production	-0.002	0.006	0.081**	PHI	-14.037	0
Housing Start	-0.001	0	-0.006	FACT1	68.538	0
Philifed Index	0.001	0.021**	0.59**	PMI	41.189	0
Existing Home Sales	0.006	0.001	0.36**	PMI	49.526	0.04
Conf. Board Consumer Conf.	-0.015	0.038**	0	FED	3.605	0
GDP	-0.001	-0.029	0.034**	FACT1	85.497	0
Chicago PMI	-0.023	0.004*	0.014**	MICH	94.095	0
New Home Sales	-0.011	0.01*	-0.41	PHI	-21.658	0
Consumer Price Index	-0.005	0.007	0.143**	CONF	78.937	0
Unemployment Rate	-0.003	0.128**	-0.022	PMI	60.642	0
Trade Balance	-0.002	0.008	1.077**	PHI	-26.126	0.01
Jobless Claims	0	-0.01	-0.094	PMI	41.189	0
Non Farm Payroll	0.003	0.133**	0.037**	FACT1	97.784	0
Capacity Utilization Rate	-0.008	0.013*	9.031**	FED	3.553	0
Employment Cost Index	-0.001	0.059**	-0.001	FACT1	86.592	0.01
Wages	-0.005	0.007	-0.002	FED	6.224	0.05
Productivity	-0.001	0.038*	-0.011	PMI	58.589	0.02
Durable Good Orders	0.004	-0.046	-0.001	FACT2	-111.393	0.01
Producer Price Index	-0.001	0.002	-0.027	PHI	-14.037	0
Hourly Average Wages	0.005	0.093**	-0.004	FACT1	97.784	0
Non Manuf. ISM	-0.006	0.005	0.067**	PHI	-14.037	0
Weekly Working Hours	0.001	-0.011	0.042**	FACT2	-115.275	0.01
Consumer Conf. Michigan	0.003	0.02**	-0.008	MICH	92.105	0.01
GDP after 1999	-0.009	-0.146	0.015*	MICH	108.021	0.01
Weekly Jobless Claims	0.002	0	0.012*	FACT1	81.692	0.1
Building Permits	-0.004	0.003	0.074**	PMI	51.968	0
Empire Manufacturing	0.011	-0.009	0.02**	PMI	55.253	0.09
Personal Consumption	-0.01	0.071*	0.008	MICH	96.011	0.11
Indice Help Wanted	0.004	-0.013	0.024**	FED	3.395	0.03
NAHB Housing Market Index	-0.001	0.052**	0.004	FACT2	-101.575	0.01
Construction Spending	0.006	0.017**	-0.048	PHI	1.868	0

Table 2.15: Results of the estimation of the threshold model for the 2-year rate, using the best performing threshold variable. * is for significative variable at 10% level and ** is for 5% level.

	Intercept	> Threshold	< Threshold	Threshold	Th. value	LR test pvalue
Household consumption	0.007	0.004	-0.017	PMI	48.568	0.08
Personal Income	0.007	0.042**	0	FACT1	94.128	0.02
ISM manuf	0.008	0.049**	0	MICH	86.137	0
Industrial New orders	-0.001	0.003	-0.106	CONF	78.937	0.01
Construction Spending	0.004	0.015	-0.18	FACT1	68.668	0.01
Consumer Credit	0.003	-0.589	-0.001	PHI	31.689	0
Wholesale Inventory	0	0.004	0.061**	PHI	-10.226	0.02
Retail Sales	0.003	0.025**	-0.021	MICH	80.168	0.03
Industrial Production	-0.004	0.009	0.071**	PHI	-14.037	0
Housing Start	-0.005	0	-0.004	PHI	-6.416	0
Philifed Index	0.003	0.02**	0.587**	PMI	41.189	0
Existing Home Sales	0.006	0.001	0.051**	MICH	100.063	0.01
Conf. Board Consumer Conf.	-0.006	-0.003	0.039**	PMI	49.526	0
GDP	-0.001	-0.025	0.04**	FACT1	85.497	0
Chicago PMI	-0.025	0.004*	0.014**	MICH	94.095	0.01
New Home Sales	-0.006	0.013**	-0.36	PMI	41.053	0.01
Consumer Price Index	-0.005	0.008	0.142**	CONF	78.937	0.01
Unemployment Rate	0.004	0.152**	-0.016	PMI	60.642	0
Trade Balance	-0.004	0.036**	0	FACT2	-112.93	0.01
Jobless Claims	0	-0.01	-0.087	PMI	41.189	0
Non Farm Payroll	0.008	0.149**	0.033**	FACT1	97.784	0
Capacity Utilization Rate	-0.011	0.013*	7.28**	FED	3.553	0
Employment Cost Index	0.002	0.073**	0.015	FED	5.342	0
Wages	-0.009	-0.002	0.012**	FACT1	65.128	0.01
Productivity	-0.007	0.036	-0.009	PMI	58.589	0.07
Durable Good Orders	-0.004	-0.199	-0.01	PMI	64.811	0.01
Producer Price Index	-0.007	0.001	-0.025	PHI	-14.037	0.01
Hourly Average Wages	0.01	0.108**	-0.003	FACT1	97.784	0
Non Manuf. ISM	0.001	0.006	0.063**	FACT1	64.883	0
Weekly Working Hours	0.002	0.038**	-0.029	CONF	92.089	0
Consumer Conf. Michigan	-0.002	0.003	0.05**	FACT1	61.227	0.01
GDP after 1999	-0.01	0.015*	0.088**	PMI	43.968	0.04
Weekly Jobless Claims	0.001	0	0.017**	FACT1	81.692	0.04
Building Permits	-0.009	-0.001	0.103**	PMI	51.968	0
Empire Manufacturing	0.019	0.009	-0.09	MICH	75.758	0.37
Personal Consumption	-0.006	0.024**	-0.044	FACT1	71.872	0.16
Indice Help Wanted	0.01	-0.018	0.033**	FED	3.395	0.01
NAHB Housing Market Index	-0.01	0.042**	-0.019	PHI	11.358	0
Construction Spending	0.013	0.018**	-0.051	PHI	1.868	0

Table 2.16: Results of the estimation of the threshold model for the 5-year rate, using the best performing threshold variable. * is for significative variable at 10% level and ** is for 5% level.

	Intercept	> Threshold	< Threshold	Threshold	Th. value	LR test pvalue
Household consumption	0.007	0.004	-0.017	PMI	48.568	0.08
Personal Income	0.007	0.042**	0	FACT1	94.128	0.02
ISM manuf	0.008	0.049**	0	MICH	86.137	0
Industrial New orders	-0.001	0.003	-0.106	CONF	78.937	0.01
Construction Spending	0.004	0.015	-0.18	FACT1	68.668	0.01
Consumer Credit	0.003	-0.589	-0.001	PHI	31.689	0
Wholesale Inventory	0	0.004	0.061**	PHI	-10.226	0.02
Retail Sales	0.003	0.025**	-0.021	MICH	80.168	0.03
Industrial Production	-0.004	0.009	0.071**	PHI	-14.037	0
Housing Start	-0.005	0	-0.004	PHI	-6.416	0
Philifed Index	0.003	0.02**	0.587**	PMI	41.189	0
Existing Home Sales	0.006	0.001	0.051**	MICH	100.063	0.01
Conf. Board Consumer Conf.	-0.006	-0.003	0.039**	PMI	49.526	0
GDP	-0.001	-0.025	0.04**	FACT1	85.497	0
Chicago PMI	-0.025	0.004*	0.014**	MICH	94.095	0.01
New Home Sales	-0.006	0.013**	-0.36	PMI	41.053	0.01
Consumer Price Index	-0.005	0.008	0.142**	CONF	78.937	0.01
Unemployment Rate	0.004	0.152**	-0.016	PMI	60.642	0
Trade Balance	-0.004	0.036**	0	FACT2	-112.93	0.01
Jobless Claims	0	-0.01	-0.087	PMI	41.189	0
Non Farm Payroll	0.008	0.149**	0.033**	FACT1	97.784	0
Capacity Utilization Rate	-0.011	0.013*	7.28**	FED	3.553	0
Employment Cost Index	0.002	0.073**	0.015	FED	5.342	0
Wages	-0.009	-0.002	0.012**	FACT1	65.128	0.01
Productivity	-0.007	0.036	-0.009	PMI	58.589	0.07
Durable Good Orders	-0.004	-0.199	-0.01	PMI	64.811	0.01
Producer Price Index	-0.007	0.001	-0.025	PHI	-14.037	0.01
Hourly Average Wages	0.01	0.108**	-0.003	FACT1	97.784	0
Non Manuf. ISM	0.001	0.006	0.063**	FACT1	64.883	0
Weekly Working Hours	0.002	0.038**	-0.029	CONF	92.089	0
Consumer Conf. Michigan	-0.002	0.003	0.05**	FACT1	61.227	0.01
GDP after 1999	-0.01	0.015*	0.088**	PMI	43.968	0.04
Weekly Jobless Claims	0.001	0	0.017**	FACT1	81.692	0.04
Building Permits	-0.009	-0.001	0.103**	PMI	51.968	0
Empire Manufacturing	0.019	0.009	-0.09	MICH	75.758	0.37
Personal Consumption	-0.006	0.024**	-0.044	FACT1	71.872	0.16
Indice Help Wanted	0.01	-0.018	0.033**	FED	3.395	0.01
NAHB Housing Market Index	-0.01	0.042**	-0.019	PHI	11.358	0
Construction Spending	0.013	0.018**	-0.051	PHI	1.868	0

Table 2.17: Results of the estimation of the threshold model for the 10-year rate, using the best performing threshold variable. * is for significative variable at 10% level and ** is for 5% level.

2.2.4.2 Term structure identification

We propose to move a step further toward the analysis of my results. When reading tables 2.15, 2.16 and 2.17, one can clearly see that most of the shapes of the term structure responses to macroeconomic news are *hump-shaped*, as already noted by Fleming and Remolona (2001). But even though most of them present this kind of shape, while analysing the results, I found different forms of these term structure responses. What is more, these shapes surprisingly match those of the correlation between swap rates across maturities and the first four factors of a principal component analysis (PCA hereafter) performed over the daily changes in the swap rates. Since Litterman and Scheinkman (1991), using PCA to assess the shape of the factors that are actually moving the yield curve is very classic. The method is still used for the analysis of bond market factors (see e.g. Lardic and Priaulet (2003)). On this preliminary remark, I propose a methodology to build a classification of the term structure responses of the swap rates to macroeconomic announcement using these four factors.

Using the dataset presented in Section 2, I performed a principal component analysis over the daily changes in the swap rates, with maturities ranging from 1- to 30-year. Figure 2.7 presents the correlations between the first four factors of the PCA and the one-day changes in the swap rate across maturities. Let me denote $F_{t,k}$ the value of the k^{th} factor on date t and $\Delta r_t(\tau)$ the change in the swap rate of maturity τ on the same date. For the time being, these notations are independent of the surprises. Then, let me denote $\rho_{k,\tau}$ the correlation:

$$\rho_{k,\tau} = cor(F_k, \Delta r_t(\tau)) \quad (2.24)$$

where $cor(\cdot)$ is the correlation coefficient. We decided to consider⁹ factors 1 to 4, using the classical elbow method to select the number of eigenvalues and eigenvectors to retain for this PCA. By studying the $\rho_{k,\tau}$, I am able to discuss the impact of the factor k on the yield curve. Figure 2.7 presents the correlations between each factor and the jumps in swap rates for a given maturity. Clearly, these factors do not seem to have the same impact on the yield curve. Factor 1 is considered as a level factor and is often related to the monetary policy stance (see e.g. Bomfim (2003), Wu (2001) and Ang et al. (2005)). Factor 2 is extremely well positively correlated (close to one) with the changes in one-year swap rates and thus governs the slope of the beginning of the yield curve. Factor 3 is highly correlated to the swap rates of maturities 2 to 7 years and thus drives the concavity of the curve. Finally, the fourth factor is well correlated to maturities a bit longer than factor 3, that is maturities from 6 till 9 years, and is thus

⁹Most of the studies achieved so far concluded with the fact that three factors were actually driving the pure discount bond yield curve. To my mind, one key explanation for this divergence with the classical literature is due the fact I use a very recent dataset.

again a concavity factor. These results can also be found in other articles such as Steeley (1990), Litterman and Scheinkman (1991), Knez et al. (1994) and more recently Molgedey and Galic (2000) and Blaskowitz et al. (2005).

In this respect, my analysis identifies four types of factors: a first type that seems to be *hump-shaped* and should be theoretically driven by the conduct of monetary policy; a second type affecting mainly the short rate positively; a third type affecting negatively maturities for 2 to 7 years and a fourth one affecting negatively maturities from 6 to 9 years.

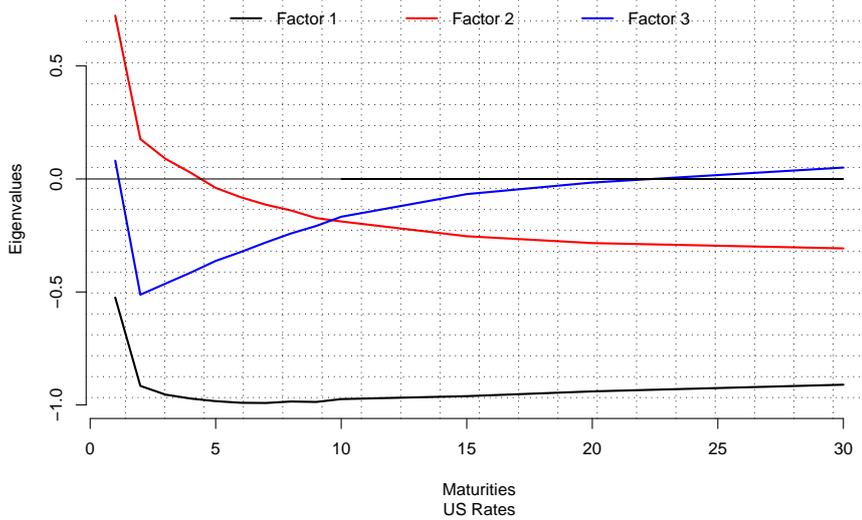


Figure 2.7: Correlations between factors 1 to 4 and the jumps in rates for maturities till 30 years

Noting that the shapes of the impact of the surprises on the yield curve are graphically close to the shapes of the correlations $\rho_{k,\tau}$ across maturities, I propose an identification process to be able to match the effect of the announcements to the factors of the PCA. We propose the following method. Let $\alpha_{i,\tau}$ be the estimate of the impact of the announcement i on the change in swap rate for a maturity τ . Thus, I have:

$$\Delta r_{t,i}(\tau_h) = \beta_{i,\tau} + \alpha_{i,\tau} S_{t,i} + \epsilon_{t,i,\tau}, \quad (2.25)$$

under the assumptions 2.2.3. Now, for a given announcement i , I propose to compare $\rho_{k,\tau}$ and $\alpha_{i,\tau}$ across maturities τ , for each factor k . Note that the $\alpha_{i,\tau}$ can either be estimated with model 1, 2 or 3: I present the methodology using model 1 as an example for the sake of notational simplicity. From now on, I propose to state that an announcement i produces a factor k -like effect on the yield curve when the distance between $\rho_{k,\tau}$ and $\alpha_{i,\tau}$ is the lowest across maturities τ and among the different possible factors. For this purpose, I propose to estimate the following linear model for each factor k and for a given announcement i :

$$\alpha_{i,\tau} = \gamma_0 + \gamma_1 \rho_{k,\tau} + \nu_{k,\tau}, \forall \tau, \quad (2.26)$$

and retain the estimated variance of $\nu_{k,\tau}$ as a distance measure between $\alpha_{i,\tau}$ and $\rho_{k,\tau}$. In equation (2.26), γ_0 and γ_1 are real-valued parameters estimated by OLS. ν_k a Gaussian white noise, with variance σ_k^2 . Now, for example, if σ_1^2 is inferior to σ_2^2 , σ_3^2 and σ_4^2 for a given surprise i , then I say that this surprise produce a factor 1-like effect on the yield curve.

In table 2.18, I report the results of the latter method, using the estimation results obtained with model 2. Table 2.19 provides empirical frequencies regarding the number of announcements per yield curve factor. Most of the announcements seem to match the factor 1 of the yield curve, but I found many other announcements matching the remaining factors. We believe that the results presented here are new, along with the idea that there are several types of shapes for the term structure announcements.

Now, an in depth analysis of the estimation tables yield two different findings: first, each announcement can have a different term effect on the term structure of the interest rates. While reading the estimation tables, what can be clearly noted is that most of the figures lead to a *hump-shaped* reaction function (a factor 1-like effect). Once the PCA is performed, this result should not surprise anyone: the first factor, that is the hump shaped one, is supposed to explain more than eighty percent of the total variance of the overnight change in swap rates sample at hand. Nevertheless, this kind of shape is not the only one that the results pointed out: I found three other shapes that clearly match that of the three remaining factors extracted using PCA. One supporting fact of my findings is that the empirical frequencies associated to this classification are quickly decaying, just like when analyzing the eigenvalues obtained when performing a PCA over the rates. We believe that this fact is new. Second, I found that when modifying the threshold variable and the threshold value, a similar announcement can have different effects on the yield curve, depending upon the state of the US economy for example. A careful reading of table 2.18 should provide important results both to academics and practitioners. We will document this point in the next subsection with well chosen examples.

Indicator	Condition	Pattern
Household Consumption	PMI < 48	Factor 4
Personal Income	FACT1 > 94	Factor 1
Personal Income	FED < 3,25	Factor 1
ISM manuf	PMI < 60	Factor 1
Industrial New orders	PMI < 42	Factor 1
Construction Spending	FACT1 < 69	Factor 4
Construction Spending	PHI < -5	Factor 1
Consumer Credit	MICH > 94	Factor 1
Wholesale Inventory	PMI < 50	Factor 2
Retail Sales	MICH > 80	Factor 2
Industrial Production	PMI < 50	Factor 2
Housing Start	FACT1 < 68	Factor 1
Philifed Index	PMI > 42	Factor 1
Existing Home Sales	PMI > 50	Factor 4
Conf. Board Consumer Conf.	FED > 3,5	Factor 2
GDP	FACT1 < 85	Factor 1
GDP	PMI < 51	Factor 3
GDP	PMI > 51	Factor 1
GDP	FACT1 < 85	Factor 1
Chicago PMI	MICH > 95	Factor 4
Chicago PMI	MICH < 95	Factor 1
New Home Sales	PMI < 60	Factor 1
Unemployment Rate	PMI < 60	Factor 1
Consumer Price Index	MICH > 98	Factor 1
Trade Balance	PMI > 50	Factor 1
Jobless Claims	PMI > 40	Factor 1
Non Farm Payroll	FACT1 > 97	Factor 1
Non Farm Payroll	FACT1 < 97	Factor 1
Capacity Utilization Rate	FED < 3,5	Factor 1
Employment Cost Index	CONF > 110	Factor 1
Wages	PMI > 48	Factor 2
Durable Good Orders	FED > 2	Factor 1
Durable Good Orders	PMI > 52	Factor 3
Durable Good Orders	PMI < 52	Factor 3
Producer Price Index	FED > 3,25	Factor 2
Hourly Average Wages	PMI > 50	Factor 3
Import Price Index	PMI > 50	Factor 3
Non Manuf. ISM	PMI < 60	Factor 2
Weekly Working Hours	CONF > 92	Factor 1
Consumer Conf. Michigan	CONF > 110	Factor 1
GDP after 1999	CONF < 130	Factor 2
GDP after 1999	CONF > 105	Factor 2
Weekly Jobless Claims	PMI < 57	Factor 2
Building Permits	PMI < 50	Factor 1
Empire Manufacturing	PMI < 55	Factor 1
Personal Consumption (Q)	PMI > 55	Factor 4
Indice Help Wanted	PMI < 51	Factor 1
NAHB Housing Market Index	PHI > 11	Factor 1
Construction Spending	PHI < 2	Factor 1
Construction Spending	PMI < 54	Factor 2
Construction Spending	PHI > 2	Factor 3

Table 2.18: Results of the estimation of the model defined by equation (2.26) and identification of the factors of the yield curve

	Factor 1	Factor 2	Factor 3	Factor 4
Number	29	11	6	5
Total number	54	54	54	54
Empirical Frequency	0,54	0,20	0,11	0,09

Table 2.19: Number of announcements matching one of the factors of the yield curve found during the estimation process.

2.2.5 Selected announcements and the underestimation problem

In this subsection, I detail with a greater attention the results I thought of interest, regarding the economic cycle dependence and the effects of the outliers on the estimations.

2.2.5.1 The economic cycle effect

We found several types of statistical effects linked to the introduction of the threshold variables that I thought of equal importance. As I initially used these variables for, I came to be able to separate the bond market reaction function to announcements during expansion and recession cycles. Three types of results arose: first, several announcements were found to have a sharper effect on the yield curve during either the recession or the expansion period, matching in both these cases the same factor pattern. Second, some announcements seemed to have an effect during only one of those periods, and no effect during the other one. Third, a few announcements were found to have a different type of effect on the yield curve, depending upon the threshold variable. In such a case, the global stance of the economy not only influences the strength of the market response to several surprises: it also brings about a change in the type of term structure of the rates' response to surprises. We propose hereafter examples of these statistical effects that I found within my estimations.

First, some of the figures were found to have a sharper effect on the changes in US swap rates when the threshold variable lies below or above the estimated threshold. What is more, the average effect of the announcement usually under- or over-estimates the actual term structure of the swap rates' response. The announcement of Non-farm Payroll is a good example of such a pattern. As presented in figure 2.8, the average effect (i.e. estimated with model 1) of the announcement lies typically below (above) the one obtained when considering the sample for which the threshold variable lies above (below) the estimated threshold. This has important implications for the building of interest rates models, both for professionals of finance and for monetary policy makers: the Non Farm Payroll (NFPR hereafter) figure is not that closely monitored

by financial markets during slowdown periods, but is of tremendous importance during expansion ones. What is more, the term structure reaction matches factor 1 for both cases, suggesting that this variable is interpreted by financial markets as monetary policy driving figure.

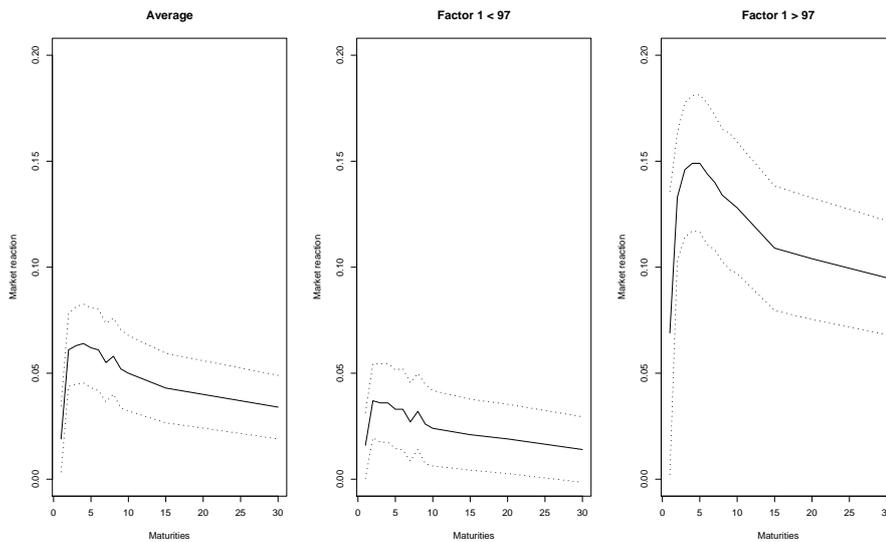


Figure 2.8: Swap rates reaction function to a positive surprise for Non Farm Payroll (plain line) and 95% confidence intervals (dotted lines).

Second, for several figures, only one period includes a significant term structure reaction of the US swap rate. The average effect (estimated with model 1) is not significant and only one of the two states associated to model 3 yields significant estimates. The Capacity Utilization Rate is an example of this phenomenon: when the Fed's target rate is above 3.5%, the term structure effect is globally equal to zero. On the contrary, when the target rate is below 3.5%, one gets an important hump-shaped reaction function. This effect is presented in figure 2.9. Again, this has important implications for the understanding of the reaction of interest rates to macroeconomic announcements. What is more, this type of effect could explain the fact that high-frequency dataset led to the finding of more market mover figures than the daily ones.

Finally, the most striking effect is for figures that lead to different types of shapes of the term structure responses, depending upon the level of the threshold variable. Until

now, I simply underlined figures for which I found the same term structure effect across the different values of the threshold variable. But for several figures, the term structure effect seems to depend on the state of the economy. This means that the interpretation of the signal driven by these variables is state-dependent. One example of such pattern is the Construction Spending figure. Figure 2.10 presents the different patterns of the term structure reaction of the swap rates to positive surprises, depending on whether the Philifed index is above or below 2. Philifed Index is a sentiment survey. Depending upon the threshold variable, I obtain two different patterns: a positive reaction function that is close to the factor 3 shape when the Philifed is above 2 and a negative hump-shaped one that is close to the factor 1 pattern when the Philifed is below 2. This means that the market perception of construction spendings strongly depends on the state of the economy.

2.2.5.2 The outliers effect

Some recent papers using high frequency datasets (e.g. Fleming and Remolona (2001)) found a greater number of market mover figures than usually found in daily datasets. My estimations results produced one possible explanation for this phenomenon. The existence of outliers within the changes in the swap rates across maturities leads to biased estimations of the term structure reaction. This is in line with what has been

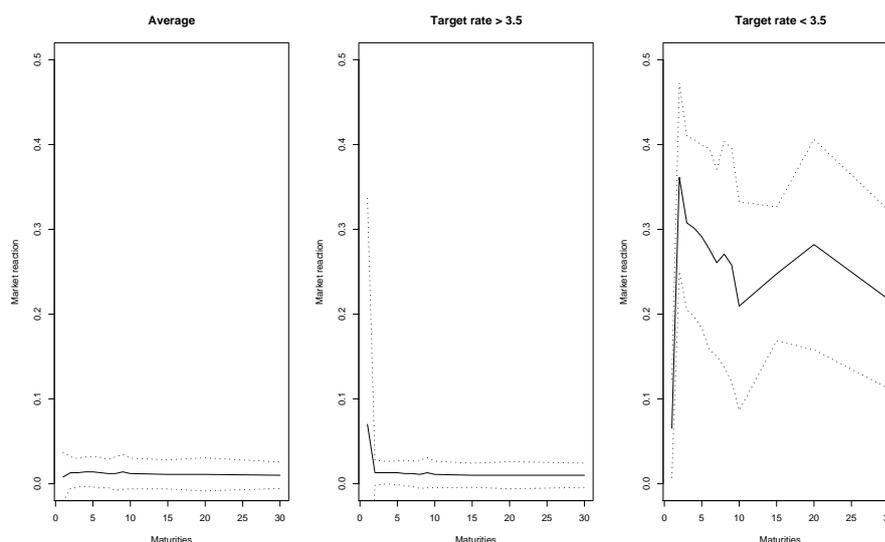


Figure 2.9: Swap rates reaction function to a positive surprise for Capacity Utilization Rate (plain line) and 95% confidence intervals (dotted lines).

said in the previous section: the sample splitting produced by the introduction of a threshold variable led to the assessment of an over- or under-estimation of the bond market reaction function. This phenomenon is often referred to as *aliasing*, and is well known and diagnosed using jump models (see e.g. Andersen et al. (2003)).

These outliers generally appear when the global economic stance of the US is very high or very low, that is to say close to turning points in the economy. Bond markets seem to have odd reactions when getting near these turning points. In fact, one can assume that during these periods, the expectations of bond market participant are very sensitive to any breaking news in the economy. Turning points in the economy are very important in so far as they match the inversion of the central bank policy. When the Fed comes to the end of a tightening cycle, the turning point will trigger the beginning of an easing cycle of the monetary policy and a progressive reduction of the target rate. In this perspective, the forward rates, and thus the spot rates are very sensitive to these changes in economic perspectives.

The estimation results presented in tables 2.15, 2.16 and 2.17 point toward the fact that getting rid of these outliers brings about a reduction of the estimation bias in the bond markets' term structure reaction function. Here again, I found three types of effects: a first one for which I observed an underestimation of the rates' reaction function to macroeconomic announcements, when the effect of the announcement were already

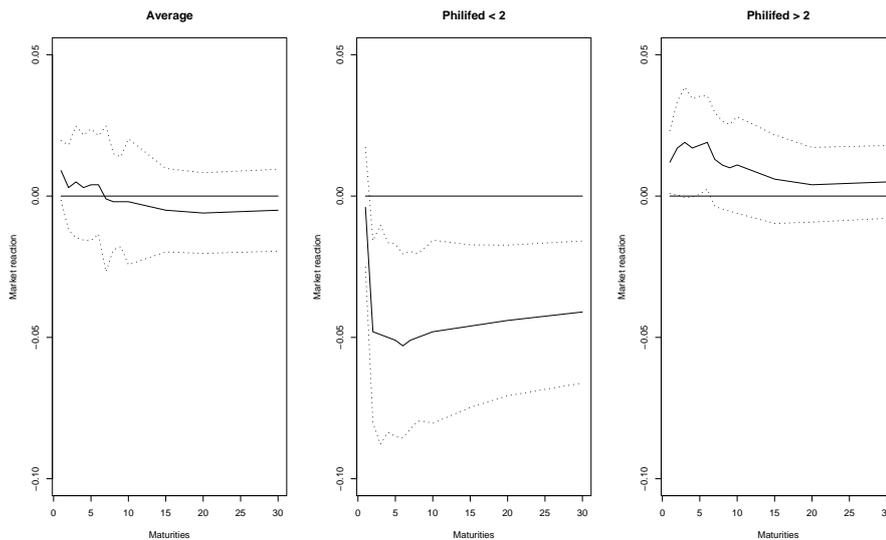


Figure 2.10: Swap rates reaction function to a positive surprise for Construction Spending (plain line) and 95% confidence intervals (dotted lines).

considered significant for model 1; a second one that is related to announcements for which the response is primarily found not to be significant when the outliers are maintained in the dataset, and significant if not; a third one, for which, in case of extreme economic situation, the market seems to have an significant reaction function.

First, when the sample splitting leads to the elimination of a few outliers, the estimated term structure reaction function may be more important for the sample that excludes the outliers. This is for example the case of the Durable Good Orders and of the Philifed Index. When estimating the swap rate reaction function to such announcements with model 1, one would find significant estimates. Nevertheless, the estimates obtained in the threshold model are more significant and present a superior absolute value, when the selected threshold variable is above or below the estimated threshold value. Figure 2.11 presents the term structure reaction to the announcement of the Durable Good Orders, when the Fed fund target rate is below or above 2%.

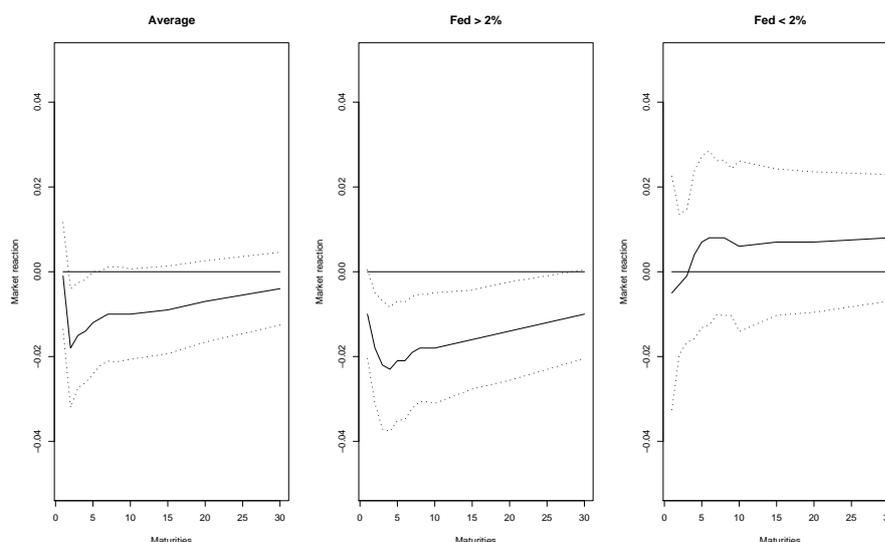


Figure 2.11: Swap rates reaction function to a positive surprise for Durable Goods Orders (plain line) and 95% confidence intervals (dotted lines).

Secondly, the estimation of the impact of some of the studied figures leads to the finding of no remarkable effect on the yield curve when using model 1. The exclusion of the outliers from the dataset then brings about very different estimation results, suggesting that the first estimates were biased because of the presence of these extreme

values. Good examples of this fact are the Unemployment Rate and the Weekly Working Hours. Without the sample splitting process, one would conclude with the fact that these announcements do not have any effect on swap rates. When implementing my methodology, I find that the shape and the significativeness of the term structure's reaction function of the swap rates is clearly very different. In figure 2.12 I present the term structure of the announcement effect of the Weekly Working Hours on the swap rates curve, documenting what has just been said.

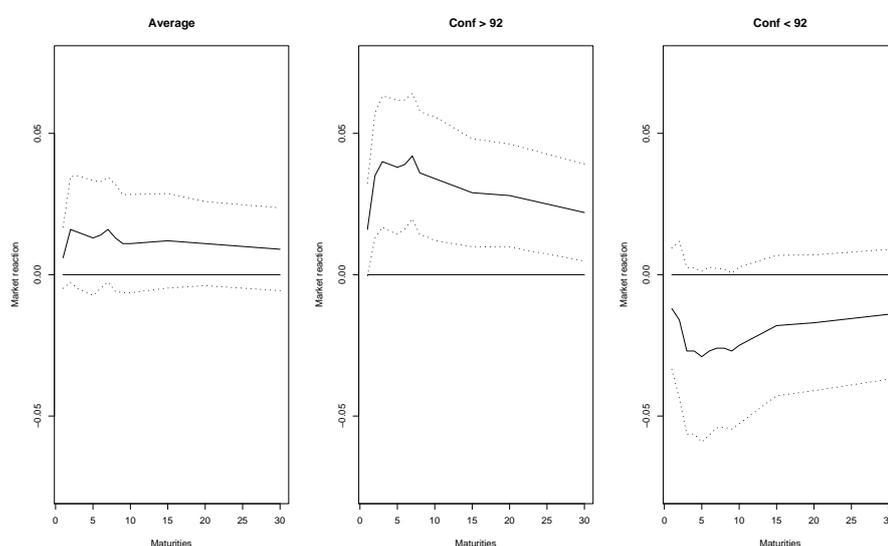


Figure 2.12: Swap rates reaction function to a positive surprise for Weekly Working Hours (plain line) and 95% confidence intervals (dotted lines).

Finally, a last type of effects appeared in the estimation results: some of the studied figures produce no significant effect on the yield curve when estimating model 1, but during very special occasions can have a dramatic impact across maturities. For a few outliers, the response of the swap rates is again important and hump-shaped. The Industrial Orders figure is a good example of such a pattern: the model presented in Section 2 that maximized the log-likelihood was the one using the PMI (Purchasing Manager Index) as a threshold variable. When the PMI index is below 42, which is rarely the case, the term structure of the rates' reaction is significant for each maturity. On the contrary, when the PMI is above 42, I did not find any observable effect. This is presented in figure 2.13. One should remain cautious regarding the interpretation of this finding. The few observations for this type of event makes it hard to be very conclusive. Nevertheless, the fact I have again a hump-shaped reaction function

tends to support the idea that industrial orders are a closely-watched figure in financial markets when getting closer to the end of the slowdown cycle of the economy.

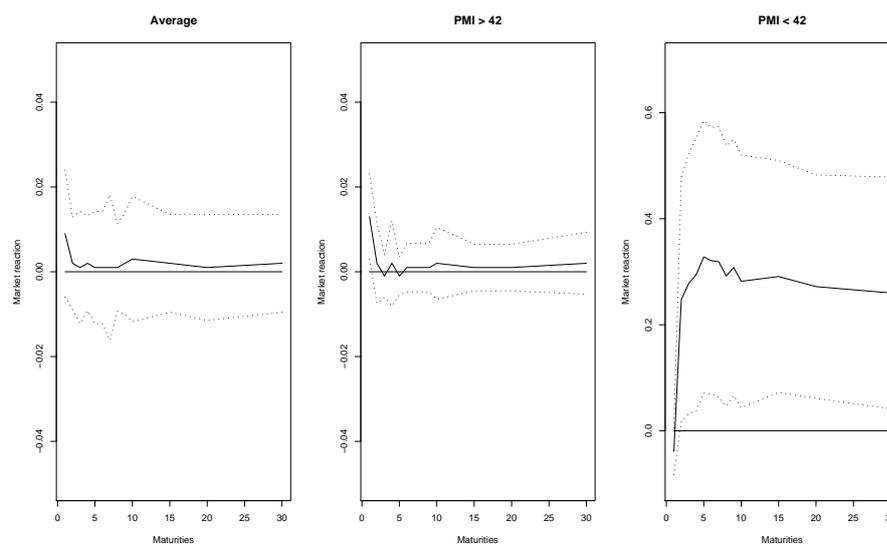


Figure 2.13: Swap rates reaction function to a positive surprise for Industrial Orders (plain line) and 95% confidence intervals (dotted lines).

2.2.6 Conclusion

The aim of this subsection was to estimate a collection of nested time series models for data-mining purposes. I found several new results. First, the use of a threshold model for the analysis of the term structure effect of macroeconomic announcements yields a much longer list of market mover figures. Second, I found that the classical hump-shaped term structure reaction function of interest rates to market mover announcements was not the only existing shape. At least three to four shapes may have to be considered, surprisingly matching that of the first four factors of a PCA performed over the daily changes in the shape rates. I develop a distance measure to build a classification of the term structure effect of announcements on the yield curve. Third, I found that the introduction of a state variable often leads to a better understanding of the reaction function to most of the announcements. When the economy is slowing or roaring, the impact of the surprises in the announcements is obviously not the same. It can even change the shape of the term structure reaction itself. Fourth, the sample splitting used throughout the paper makes it possible to isolate a few outliers and to

analyze the rates' dynamic on each sample separately. The results point toward the fact that these outliers often bring about an underestimation of the reaction function.

Conclusion

Financial markets are thus nothing beyond this huge information treatment machinery. This PhD dissertation showed how these various information flows are integrated in spot, future and option prices. In Chapter 1, I proposed a GARCH-based model to assess the progressive change in the bond market's perception of the upcoming monetary policy. Using a similar approach jointly with a non-parametric estimator for the risk neutral distribution, I showed how the European Commission's information disclosure regarding the carbon allotted quotas changed the risk aversion in the European Carbon Market. To conclude chapter 1, I presented a new option pricing model based on conditionally Generalized Hyperbolic distributed returns and a constant absolute risk aversion function for the representative agent. I finally empirically proved the good pricing performance of my model. In chapter 2, I documented the impact of macroeconomic news on the bond market, both for the US or for the European market. I found that very few Euro figures impact the European bond market, when taking the US influence into account. Finally, I showed how the term structure impact of news on the US market strongly depends on the stance of the economy.

The work presented here could be nonetheless extended in many directions. I now review several of these possibilities.

The method used to estimate the subjective distribution from futures could be jointly used with existing methods for the risk neutral distribution inference: option on futures are likely to contain information about risk aversion. Event studies of the kind presented in section 2 of chapter 1 could be replicated for monetary policy analysis purposes, using both futures and derivatives. Tests for changes in risk aversion in the bond market following Central Bank decision meetings would help us document the macroeconomic information processing of financial markets. However, options on futures are American options: the early exercise premium included in their price needs to be taken into account. A method of this kind is presented in Barone-Adesi and Whaley (1987). This is however beyond the scope of this dissertation.

The results obtained with the Carbon market should still be considered as a "one event"

study: I only assessed the impact of the 2007 information disclosure on risk aversion. Since no break in the futures were observable, the method helped me diagnose the implicit change in the market's perception of the fair value of carbon futures. Should I want to extend this, I would need to wait for a hundred years maybe. It will take a long time before we can answer the question: does each information disclosure affect the carbon prices? As far as the method is concerned, it is now possible to test whether the results are robust to changes in the methodology. For example, using a higher order kernel could be considered as a robustness test.

Beyond the extension of existing methods to new option markets, an empirical investigation on the shape of both risk aversion and pricing kernel would be essential. Both the existing literature and my work seem to indicate that the pricing kernel is an exponentially decreasing function of the future value of the underlying asset. Nevertheless, Ait-Sahalia and Lo (2000) and Golubev et al. (2008) present evidence that the pricing kernel is not a monotonically decreasing function: its exponential shape should be conjugated to a higher order polynomial, as presented in Rosenberg and Engle (2002). An essential improvement would therefore be to assume that the pricing kernel is the exponential of a polynomial of order two. However, the expectation of such a real-valued non linear function is almost never definite. This prevents most of the required explicit computations for an option pricing model building. It is however possible for distributions, such as the Gaussian distribution. In this case, a second difficulty arises: the use of a second order polynomial would require three identifying conditions to replicate a similar change in probability distribution, as performed in section 3, chapter 1. However, until now, only two no arbitrage restrictions are available. Still, this leaves room for future research.

The impact of news on the bond market study can be developed in many ways which I wish I could present here. Beyond stability tests, depending for example on the Central Banker's managing style, I developed for Dexia two other ways around the term structure impact of news. First, using a non parametric estimator, the rates' response to news can be shown to be asymmetric: there is no *a priori* reason to think that the impact of good or bad economic news will produce an equal effect, and empirics clearly confirm this fact. Second, unlike most of the literature, I analyzed the delayed effect of news. For several macroeconomic figures, financial markets are still integrating the surprise during the four days following the day of the announcement. I am currently working on other working papers, which precisely point to these results.

The macro finance models mentioned in the introduction of chapter 2 should be improved by incorporating the various effects of news on the term structure. In a forthcoming paper, I show how affine models for bond pricing can be upgraded to account

for different jump shapes. By being consistent with the different types of term structure effects, the model helps me quantify the frequency of each type of jumps. Moreover, these jumps are no longer limited to those stemming from quantitative news: the model helps me deal with any kind of news on the market.

Résumé

Cette section présente un résumé des travaux conduits au cours de la thèse CIFRE présentée dans ce document. Elle résume le résultat des interactions entre le travail produit pour le compte de Dexia SA et dans les locaux de l'ENS Cachan, puis de l'université de Paris 1 Sorbonne. Cette thèse est composée de cinq travaux empiriques ayant pour point commun le traitement de l'information dans les marchés financiers et les différents usages qu'il est possible d'en faire. Au-delà de cette ligne directrice, il est possible de distinguer globalement deux lignes directrices dans ces différents travaux. La première touche à l'usage du concept financier de noyau de prix – concept défini en détail plus loin – et la seconde à l'impact de la publication des chiffres macroéconomiques sur la courbe des taux, tant aux Etats-Unis qu'en zone Euro.

2.3 De l'usage des noyaux de prix en finance

Cette section repose sur les concepts financiers bien connus de distribution historique, risque neutre et subjective. Avant même de définir ces concepts, il est important de remarquer la chose suivante: l'originalité de cette thèse ne repose pas sur les concepts eux-mêmes. Ceux-ci sont aujourd'hui de connaissance commune, notamment depuis les travaux de Black and Scholes (1973). La nouveauté du travail présenté ici est à mettre au compte de l'usage qui en est fait: ces concepts forment la base de ce que l'on peut aujourd'hui appeler la finance moderne. Néanmoins, le spectre des déclinaisons qu'il est possible de produire à l'aide de ces principes est loin d'être éculé. Prenons néanmoins le temps de présenter une définition littéraire de ces concepts avant de résumer la teneur des travaux qui composent cette thèse.

La finance moderne est basée sur la théorie des probabilités. Le prix et la dynamique des actifs financiers, quels qu'ils soient, sont supposés obéir à certaines lois de probabilité. Dans certains cas, ces lois sont données explicitement. Dans d'autres, seul le caractère probabiliste du comportement des actifs est supposé. Chaque type d'hypothèse est naturellement relié à un modèle particulier: le corpus de ces modèles forme la base

de la finance "orthodoxe" moderne. Néanmoins, par delà les différences éloignant ces différents modèles, tous ont en commun les trois distributions de probabilités évoquées plus haut. Plus encore, Bertholon et al. (2007a) montrent qu'un modèle financier est nécessairement basé sur au moins deux de ces trois concepts. Ces concepts sont donc "model-free" : ils sont indépendants de tout choix de modèle).

La distribution de probabilité dite historique doit être vue comme la loi de probabilité qui gouverne les évolutions du prix des différents actifs. Prenons l'exemple le plus simple possible : celui du prix d'une action. Chaque jour, chaque heure ou chaque minute qui s'écoule voit le prix de cette action varier. La théorie financière suppose que ces évolutions sont aléatoires et que cet aléa peut être capturé au travers d'une distribution de probabilité connue sous le nom de distribution historique.

Cette distribution correspond donc à une quantification du hasard que l'on trouve dans la nature : il s'agit de mesurer le caractère aléatoire de ce qui se passe historiquement à chaque instant dans les marchés financiers. Ceci est très différent de la vision qu'ont les opérateurs de marché des évolutions futures de ce même actif. Si la distribution historique aide les gérants, les traders et les stratégestes à se faire une opinion des évolutions possibles des cours des actifs financiers, il est évident que cette opinion n'a que très peu de chances de correspondre exactement à la distribution historique. Par exemple, si le CAC 40 n'a historiquement que très rarement augmenté de 50% en un mois, cela n'empêchera pas quelques opérateurs de marché optimistes de croire que la probabilité que cet événement survienne dans un avenir proche soit supérieure à ce qu'il en a été historiquement. En agrégeant l'ensemble des avis de ces opérateurs de marché, on obtient ainsi une seconde distribution de probabilité: la distribution subjective. Celle-ci reflète les prédictions de la moyenne des agents de marché : elle synthétise en vérité ce à quoi un trader moyen s'attend dans le futur. Encore une fois, cette distribution est le plus souvent très différente de la distribution historique. Dans le jargon des praticiens, on parle souvent de "vue du marché" ou de "judgemental" pour désigner le fait de donner des probabilités subjectives aux différents événements susceptibles de se produire dans le futur.

Il existe enfin un troisième type de probabilité largement utilisé en finance: la distribution dite "risque neutre". Il s'agit d'une traduction littérale de l'anglais "risk neutral probability". Si la distribution subjective reflète les probabilités qu'un "trader moyen" assigne aux différents événements qu'il anticipe, la distribution risque neutre fait de même dans le cas d'un intervenant de marché dit "neutre vis-à-vis du risque". Là encore, il s'agit d'un abus de langage : cet agent neutre vis à vis du risque est en réalité un agent avec une aversion pour le risque qui est réelle, mais qui – quel que soit l'investissement qu'il réalise – s'attend nécessairement à obtenir une rentabilité

égale à celle d'un actif sans risque. Habituellement, il est naturel de supposer que lorsqu'un investissement est plus risqué qu'un autre, la rentabilité attendue associée à l'actif le plus risqué doit être supérieure. C'est la comparaison des rentabilités attendues en fonction des risques supposés qui forme la base de la finance moderne. Ici, sous cette distribution risque neutre, on suppose que l'intervenant de marché représenté par cette distribution n'a qu'une seule et unique attente, quel que soit le risque associé à cet investissement : le taux sans risque. Concrètement, il s'agit du taux rémunérant généralement les obligations émises par les Etats du G7 afin de financer leur dette. Ce taux est sans risque dans la mesure où on estime que ce type d'acteur sur les marchés financiers ne fera jamais faillite et fera donc toujours face à son engagement à délivrer le taux d'intérêt stipulé lors de l'achat de l'obligation.

La distribution risque neutre est certainement la plus utile de ces trois dernières, eu égard au rôle que lui réserve la finance moderne. Dans la théorie financière moderne – suite à des arguments qu'il est difficile de présenter de façon littéraire ici – le prix à chaque instant de n'importe quel actif est égal – en général – à la somme des versements associés à cet actif auxquels cet agent neutre vis-à-vis du risque est en droit de s'attendre. De façon plus formalisée, plaçons-nous dans le cas d'une économie à une période: à la date initiale, on investit 1 euro dans une action dont le prix initial est S_0 . Le prix de ce titre à la date 1 est S_1 . A la date 0, le prix en date 1 est inconnu. Ce que la théorie financière nous dit alors c'est que le prix de l'actif en date 0 est égal à l'actualisation du flux attendu à recevoir en date 1. De façon plus formelle, on a :

$$S_0 = e^{-r} \mathbb{E}_0^{\mathbb{Q}} [S_1], \quad (2.27)$$

où r est le taux sans risque sur la période en question. \mathbb{Q} désigne le fait que l'espérance $\mathbb{E}_0[\cdot]$ – ce à quoi l'agent de marché est en droit de s'attendre en moyenne – est calculée en fonction de la distribution risque neutre et conditionnellement à l'information dont on dispose en date 0. Ici s'achève cette discussion liminaire de ces concepts de base de la finance moderne. Le lecteur intéressé pour en apprendre davantage lira avec profit Schriever (2004). Les trois premiers travaux de recherche présentés ici sont basés sur les relations qu'entretiennent ces différentes distributions les unes avec les autres. Les liens théoriques existants permettent de mettre à jour des relations utiles pour la gestion d'actif ou bien encore pour l'évaluation par non arbitrage de différents produits financiers. Toujours de façon littéraire, voici les différentes relations qu'il est possible de mettre à jour entre ces différentes distributions de probabilités:

- En neutralisant les primes de risque, il est possible de transformer la distribution historique en distribution risque neutre. On a donc :

$$\text{Distribution historique} = \text{Distribution risque neutre} + \text{Prime de risque}. \quad (2.28)$$

- En neutralisant l’aversion pour le risque, il est possible de transformer la distribution subjective en distribution risque neutre. On a donc :

$$\text{Distribution subjective} = \text{Distribution risque neutre} \times \text{Correction pour l'aversion au risque.} \quad (2.29)$$

- Même si cette dernière relation a été peu étudiée dans la littérature tant théorique qu’empirique, il existe une relation entre la distribution historique et la distribution subjective. Elle correspond à la transformation de l’information historique opérée par les traders ou les gérants : chaque nouvel événement a pour conséquence d’affecter la vision qu’ont les opérateurs de marché des réalisations possibles du futur. Ces événements conduisent nécessairement à modifier la distribution subjective du ”trader moyen” évoquée plus haut. Néanmoins, cet aspect est pour le moment laissé de côté par la recherche académique, dans la mesure où chacune de ces deux distributions sont difficilement mesurables.

Dans ce qui suit, trois travaux basés sur ces relations seront présentés. Le premier présente une méthode d’estimation de la distribution subjective à partir de la dynamique d’un actif financier particulier : les futurs sur taux d’intérêt. Le deuxième présente une utilisation de ces concepts visant à mesurer les variations de l’aversion au risque suite à l’arrivée d’une information essentielle sur le marché du carbone en Europe. Finalement, le dernier de ces travaux présente un modèle d’évaluation d’actif dérivé en temps discret, ce modèle contenant des hypothèses explicites sur l’aversion au risque des acteurs de marché.

2.3.1 Une nouvelle méthode pour l’estimation de la distribution historique

Le premier des travaux empiriques présentés dans cette thèse présente une nouvelle méthode ayant pour but d’estimer la distribution subjective associée au marché de taux d’intérêt américain. Les taux d’intérêt présentent plusieurs particularités que les marchés d’actions n’ont pas. La première de ces singularités est l’existence d’une structure par terme. Emprunter aujourd’hui à dix ans ne se fera pas au même prix qu’à 1 an. La structure ou gamme des taux d’intérêt est composée de l’ensemble des taux d’intérêt pour les différentes maturités imaginables. On se focalisera ici sur les taux d’Etats, ces mêmes taux sans risque évoqués plus haut. Une seconde particularité de ces marchés de taux est que la partie courte de la courbe – les taux d’intérêt allant de 1 jour à 2 ans – est fermement guidée par la politique de la Banque Centrale. Dans le cas des Etats-Unis, comme c’est le cas ici, la politique de la Federal Reserve Board vise à fixer un taux directeur (*target rate* en anglais) en adéquation avec la conjoncture et

l'inflation actuelles et anticipées par les économistes de cette institution. Ce taux cible fixe le loyer de l'argent à très court terme. Plus encore, le reste des maturités constitue une anticipation des actions futures de la banque centrale. Il est dès lors essentiel de parvenir à estimer les probabilités que le marché alloue aux différentes actions possibles de la Fed. Il est d'autant plus facile de se livrer à pareil exercice que les actions de la Fed se limitent généralement à une hausse ou une baisse de 25 points de base¹⁰ ou à un maintien de son taux directeur au niveau courant. La chapitre de cette thèse dédié à cette problématique présente une méthode permettant d'estimer les probabilités que ce "trader moyen" associe à chacun de ces événements – hausse, baisse ou *statu quo* – pour les différentes réunions décisionnaires de la Fed à venir.

Dans ce travail empirique, on propose d'approcher cette distribution subjective à l'aide de la dynamique historique des futures dits "Fed Funds". Ces futures de taux d'intérêt sont utilisées par les acteurs de marché afin de se couvrir ou afin de prendre des paris sur les décisions futures de la Fed. Les variations du taux futur associé à ce produit sont ainsi intimement liées aux anticipations des agents. On propose donc un modèle de série temporelle visant à extraire l'information adéquate à la représentation des probabilités associées aux différentes évolutions possibles du taux directeur, afin de mesurer avec le plus de précision possible les attentes du marché.

Pour ce faire, on propose d'utiliser un modèle dont la distribution est conditionnellement une distribution Normal Inverse Gaussian (NIG), dont certains paramètres évoluent au cours du temps de façon dynamique. L'évolution de ces paramètres est gouvernée par le passé de l'évolution des variations des taux futurs d'une façon novatrice : au lieu de déterminer un lien entre les paramètres de la NIG qui évoluent au cours du temps et un moment d'un ordre entier quelconque, on propose de lier les paramètres à un moment spectral qu'il convient d'estimer. Ainsi, la procédure d'estimation permet de déterminer le lien existant entre les paramètres et les moments spectraux et ce d'une façon naturelle.

En estimant le modèle en utilisant les séries temporelles associées à différents contrats, on montre l'utilité de l'approche. On est en mesure d'estimer les probabilités avant et après les réunions décisionnaires de la Fed : on montre ainsi l'impact des discours de la Fed sur la vue globale du marché quant au futur de la politique monétaire aux Etats-Unis. Cette étude empirique s'appuie enfin sur des tests statistiques visant à confirmer l'intérêt de l'approche. Les tests utilisés nous permettent de conclure que le type de modèle présenté permet de mesurer avec un degré satisfaisant de précision la distribution subjective.

¹⁰Un point de base est égal à 0,01%.

2.3.2 L'aversion au risque dans le marché européen du carbone

Toujours en s'appuyant sur le corpus conceptuel présenté plus haut, nous présentons dans cette section une étude empirique des évolutions de l'aversion au risque dans le marché du carbone. Le marché du carbone est un marché de quotas, visant à déterminer le prix de la tonne d'émission de CO₂. En dépit de la jeunesse de ce marché, il s'y échange quotidiennement des contrats spots, futures ainsi que des options européennes ayant pour sous-jacent le CO₂. Ce marché est intimement lié à l'information distillée par la Commission Européenne : cette dernière collecte chaque année les quantités de quotas disponibles et ainsi que la consommation de CO₂ opérée par le secteur privé. En avril, cette information est rendue publique et fournit un signal de trading essentiel quant aux évolutions futures du marché. En 2005, le signal fourni a conduit à une chute brutale du prix de la tonne de CO₂ spot. En 2006, la lisibilité de l'impact du signal sur la psychologie des marchés fut nettement moindre. Cette étude empirique vise à montrer que la publication de l'adéquation quotas/consommation en 2006 a conduit à une modification profonde de l'aversion pour le risque dans le marché du carbone.

Le corpus théorique sur lequel ce chapitre s'appuie se base principalement sur la relation suivante:

$$\text{Distribution risque neutre} = \text{Distribution historique} \times \text{Correction pour l'aversion au risque.} \quad (2.30)$$

Cette relation est présentée dans Leland (1980) et utilisée dans un but proche du nôtre dans Ait-Sahalia and Lo (2000), Jackwerth (2000) et Rosenberg and Engle (2002). Sachant que cette relation ne dépend pas d'un modèle en particulier, il suffit de rechercher une stratégie d'estimation satisfaisante pour les distributions historiques et risque neutre afin d'en extraire l'aversion au risque comme sous-produit.

Sur la base d'une étude empirique extensive, les stratégies d'estimation retenues sont les suivantes: la distribution historique est modélisée à l'aide d'un modèle GARCH asymétrique semi paramétrique, estimé par Pseudo Maximum de Vraisemblance. Ceci est motivé par une discussion de cette approche dans Barone-Adesi et al. (2007). La distribution risque neutre est quant à elle approchée à l'aide d'un estimateur à noyau, dit de Nadaraya-Watson. La méthodologie d'estimation nous a conduits à découper l'échantillon en deux sous-échantillons: le premier inclut la période précédant l'annonce de la Commission Européenne et le second la période suivant cette annonce. Pour chaque échantillon, on estime les distributions risque neutre et historiques. On obtient ainsi une estimation de l'aversion au risque avant et après l'annonce, nous permettant de discuter l'impact de l'annonce sur la psychologie des marchés.

Les principales conclusions de l'étude sont les suivantes. Il est tout d'abord essen-

tiel de remarquer que les effets leviers dans le marché du carbone sont inversés, par rapport à ce que l'on peut observer dans un marché action. Dans un marché action, une baisse de la rentabilité sur une journée de cotations conduit en général à une plus grande volatilité du marché. Ici, dans le cas du CO₂, il s'agit d'un marché de quotas. La principale peur du marché est de voir le prix des quotas s'envoler : ainsi, dans le marché du carbone, la volatilité a tendance à augmenter lorsque les rentabilités associées aux futures augmentent. Ensuite – et il s'agit de la principale conclusion de notre étude – l'information diffusée par la Commission Européenne a certes conduit à une réduction la volatilité dans le marché, mais elle a également conduit à une augmentation très nette de l'aversion au risque. Il s'agit d'un cas peu courant du traitement d'une publication d'information par un marché financier : la transparence qui naît de la communication de l'information fait baisser la volatilité, mais le message convoyé – pour transparent qu'il soit – n'en a pas moins excité les pires angoisses d'un marché de quotas, la montée attendue du prix de la tonne de CO₂. En effet, le message délivré par la Commission Européenne permettait d'anticiper une tendance croissante du prix du CO₂ pour les années à venir, de façon à mettre en ligne l'évaluation du produit avec le coût social associé à la production de CO₂.

2.3.3 Un modèle de pricing d'options avec hypothèses explicites quant à l'aversion au risque

La précédente étude repose sur l'intuition qu'une fois les distributions historiques et risque neutre estimées, on obtient une mesure de l'aversion au risque comme sous-produit de la précédente stratégie d'estimation. Il est également possible d'invertir les membres de cette relation, de façon à obtenir la dynamique risque neutre comme sous-produit, une fois la dynamique historique et l'aversion au risque correctement spécifiées. Dans cette troisième sous-section, on montre comment construire un modèle d'évaluation d'option empiriquement cohérent avec les faits stylisés observés dans les bases de données financières, en faisant des hypothèses sur la distribution historique et les préférences de l'agent représentatif.

Là encore, notre exposé repose sur la relation indépendante de tout modèle que voici :

$$\text{Noyau de prix} = \frac{\text{Distribution risque neutre}}{\text{Distribution historique}}. \quad (2.31)$$

Dans la littérature théorique, on trouve souvent que ce noyau de prix est une fonction exponentielle affine des variables d'état du modèle d'évaluation, telles que la volatilité, les rendements eux-mêmes ou des facteurs pré-spécifiés. Ceci revient à faire une hypothèse implicite sur l'aversion au risque de l'agent représentatif – le "trader moyen" dont il était question plus haut. Ceci revient à supposer que l'aversion relative au risque

de cet agent est constante. La dérivée du logarithme du noyau de prix est égale au coefficient d'aversion au risque de Arrow-Pratt. Sur ce point, nous renvoyons le lecteur à Gourieroux and Monfort (2007). Si ce type d'hypothèse est par nature irréaliste, il n'en reste pas moins une possibilité de modélisation intéressante : les résultats empiriques obtenus dans cette sous-section montrent que la spécification de l'aversion au risque importe nettement moins que celle de la distribution historique.

On présente un modèle d'évaluation d'options européennes vanille basé sur deux ingrédients principaux. Le premier est un modèle GARCH asymétrique, dont la distribution conditionnelle est une distribution hyperbolique généralisée. Cette distribution permet de répliquer la décroissance linéaire du logarithme des queues de distribution observées dans les rendements des marchés financiers : il s'agit d'un candidat extrêmement intéressant pour la modélisation des rendements des actions. Le second ingrédient est un noyau de prix supposé être une fonction exponentielle affine des variables d'état, ici le rendement de l'actif sous-jacent.

En combinant ces deux éléments, on obtient une distribution risque neutre unique, qui est encore une distribution hyperbolique généralisée. Cependant, la martingalisation nécessaire à l'obtention de la distribution risque neutre conduit à modifier le paramètre de la distribution qui contrôle le degré d'asymétrie de la distribution. Ceci est cohérent avec le phénomène de "skewness premium" (prime de skewness) observée dans le marché : la skewness implicite dans les prix d'option est bien souvent supérieure à celle obtenue dans les séries historiques des rendements des actions.

Au-delà de la nouveauté du modèle, l'une des principales contributions de cette sous-section consiste à présenter les résultats de notre modèle obtenus au terme de tests empiriques. En utilisant une base de données composée des cotations des options sur CAC 40 sur deux ans, on montre que le modèle permet de générer des performances comparables à celles obtenues dans un papier concurrent, reposant sur une calibration des paramètres basée sur une minimisation directe des erreurs de pricing. Dans notre approche, il n'est pas nécessaire de passer par cette étape, et le coût numérique de l'opération s'en trouve considérablement amoindri. Ainsi, la première contribution de cette étude empirique consiste à montrer que les erreurs moyennes obtenues sur les prix d'options sont si faibles qu'il n'est pas nécessaire de se livrer à un exercice de calibration. La seconde conclusion de notre étude empirique est obtenue par une comparaison des erreurs de pricing obtenues avec plusieurs modèles – incluant naturellement le nôtre. Le modèle d'évaluation basé sur une distribution hyperbolique généralisée semble empiriquement fournir les meilleurs résultats, quand on compare ce modèle au modèle de Black and Scholes (1973), Heston and Nandi (2000) ou Bertholon et al. (2007b).

2.4 L'impact des annonces macroéconomiques sur les courbes de taux

Les trois premières contributions de cette thèse avaient pour thématique principale l'incorporation de l'information financière – rendements passés, volatilité, publication d'information institutionnelle – dans le noyau de prix, la distribution historique ou risque neutre. Les deux dernières contributions de cette thèse ont également pour problématique l'incorporation de l'information dans les marchés financiers. Il s'agit cette fois d'aborder la problématique de l'impact de l'information macroéconomique sur la courbe des taux.

Pour ne parler que des économies européennes et américaines, chaque jour, de nombreuses informations économiques quantitatives sont publiées officiellement par divers organismes. Il s'agit des chiffres macroéconomiques de la Banque Centrale Européenne, du Bureau of Labor Statistics ou des chiffres de l'Insee. Quoi qu'il en soit, un nombre important de ces chiffres publiés ont un impact sur la courbe des taux, au moment même de leur annonce, preuve s'il en faut que les marchés financiers sont attentifs aux signaux de l'économie. La mécanique conduisant à ces brusques réajustements des marchés financiers est aisée à comprendre : les chiffres qui revêtent une importance particulière sont "prévus" par différents économistes. Les services de Bloomberg – entre autres – collectent ces prévisions et les agrègent de façon à obtenir une sorte de consensus. Si le véritable chiffre est nettement différent du consensus, les marchés se réajustent brutalement – en une demie heure en général – incorporant ainsi l'information additionnelle apportée par la publication.

On présente un résumé global des deux dernières contributions dans un même temps, dans la mesure où les similarités entre ces études sont nombreuses. Le but de ces études est globalement le même : déterminer les chiffres macroéconomiques qui produisent un impact statistiquement significatif sur la courbe des taux américaine et sur la courbe des taux européenne. Les deux études dressent ainsi une cartographie des chiffres produisant des mouvements dans les marchés de taux. Plus encore, nous y montrons que la forme de la structure par terme de l'impact des surprises – la déviation du consensus par rapport au chiffre réel – varie selon les chiffres ou les périodes : le cycle économique, monétaire ou le type même de chiffre macroéconomique dont il est question conduisent les marchés à réajuster leur façon de réagir face aux erreurs de prévision du consensus.

Dans le cas de l'étude portant sur le marché américain, une extension spécifique à ce travail nous a conduits à construire une classification des formes de la réponse des taux aux chiffres macroéconomiques. Cette classification part des résultats em-

piriques obtenus par Litterman and Scheinkman (1991). En pratiquant une analyse en composantes principales sur les variations de taux, ils montrent qu'en dépit de l'existence d'un grand nombre de maturités pour les taux d'intérêt, il n'existe en réalité que trois types de mouvements principaux de la courbe des taux : une variation de son niveau, une variation de sa pente et une variation de sa concavité. Nous utilisons ici ces types de mouvements de la courbe afin de catégoriser l'impact des nouvelles macroéconomiques sur la courbe américaine. On trouve – comme attendu – que le plus grand nombre de mouvements produits par les annonces macroéconomiques correspondent à une variation du niveau de la courbe des taux. Les variations de pente arrivent en deuxième place, suivi de près par les mouvements de convexité. Cette classification forme l'une des principales nouveautés de cette sous-section.

Dans le cas de la courbe européenne, l'accent est mis sur le traitement de l'influence de la courbe américaine. Etant donné l'importance de l'interdépendance entre les économies américaine et européenne, il est naturel de penser que les variations quotidiennes des taux américains doivent avoir une influence importante sur les taux de la zone Euro. Notre volonté est de parvenir à isoler les chiffres européens qui produisent une influence sur la courbe européenne. Le fait de négliger un facteur aussi important que l'économie américaine peut conduire à des erreurs d'estimation suffisamment conséquentes pour nous empêcher de brosser un tableau réaliste de ces chiffres dits "market movers" et spécifiquement européens. Nous proposons donc d'ajouter au modèle de régression linéaire utilisé généralement dans la littérature les trois premiers facteurs issus d'une analyse en composante principale de la courbe américaine. Notre principale conclusion est qu'il est légitime d'introduire ces trois variables, dans la mesure où elles permettent de rendre significatif l'impact de chiffres bien connus pour être suivis par les marchés financiers avec attention. A cet égard, la création monétaire mesurée à l'aide de M3 constitue certainement le meilleur exemple de ces effets liés à l'omission de variables : lorsque l'influence américaine est négligée, la publication de M3 ne semble pas avoir statistiquement d'effet sur la courbe des taux européenne. Au contraire, quand les facteurs de l'ACP sont ajoutés à la régression, M3 redevient fortement significatif, tel qu'attendu.

Dans les deux cas néanmoins, il est essentiel de remarquer que l'incorporation de l'information dans le prix des actifs est un processus complexe, qui nécessite d'adapter l'approche du modélisateur au cas par cas.

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