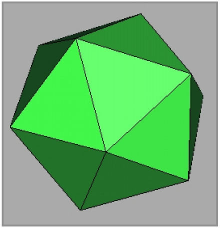
Structuring 3D Geometry based on Symmetry and Instancing Information

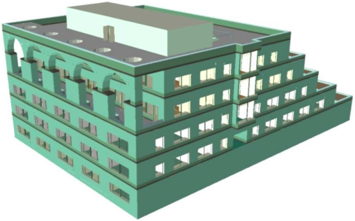
Aurelien MARTINET

May 14, 2007

3D Geometry

Representation



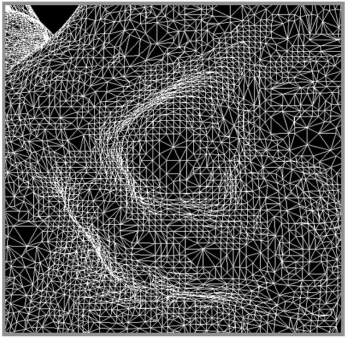


Question

How can these objects be represented in a computer ?

3D Geometry
Representation

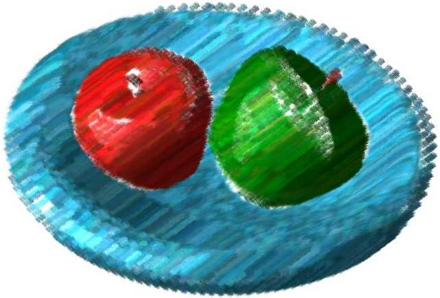
- 3D Geometry represented as a collection of polygons



3D Geometry

Treatements

- Rendering
- Animation
- Editing



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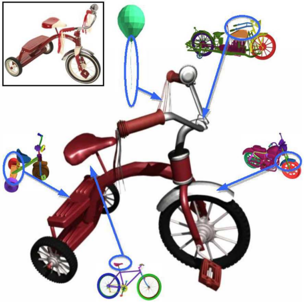
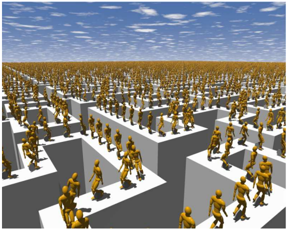


Motivations
Observations

Fact

Structure of Geometry is a key to Efficiency

- *Improve rendering speed*
- *Reduce memory usage*
- ...



Motivations

Observations

Fact

3D Geometry is often unstructured

$\Updownarrow$

Structural Information is not accessible

Raises two important questions:

1. What is Structural Information ?
2. Why is it not accessible ?

Motivations
Observations

Fact

3D Geometry is often unstructured

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Structural Information is not accessible

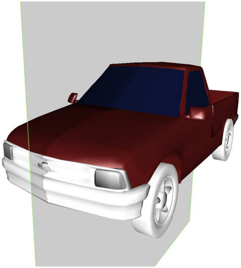
Raises two important questions:

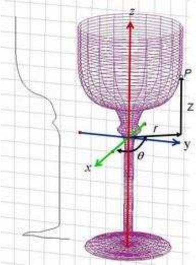
1. What is Structural Information ?
2. Why is it not accessible ?

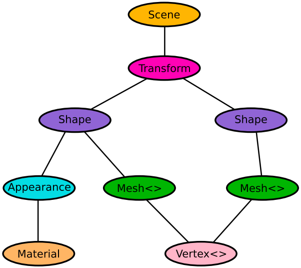
Structural Information

Generalities

- In Computer Graphics:
  - Symmetry Group of a Shape
  - Parameters of a Revolution Surface
  - Scene-Graph
  - ...







Accessibility of Structural Information

Fact

Structural Information is not accessible

Sources of Problems

- Asset Exchange
- Non-Interactive Modeling Techniques

Accessibility of Structural Information

Asset Exchange

Generation of 3D Geometry → Asset Exchange → Utilisation of 3D Geometry: Rendering, Editing, ...

- Exchanging assets is a major problem:
    - Constraints due to multiple platforms, software and file formats
    - Must ideally preserve *the structure* of the geometry

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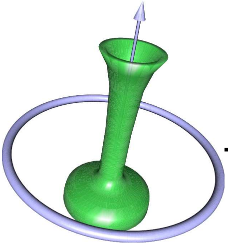
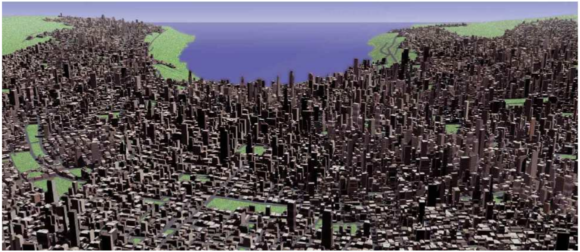


→ Asset Exchange →

Accessibility of Structural Information

Non-Interactive Modeling Techniques



Pros

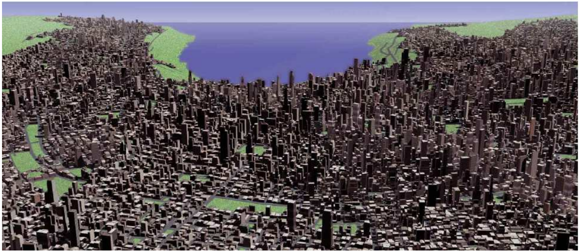
- Reach High-Complexity

Cons

- Unstructured Output

Accessibility of Structural Information

Non-Interactive Modeling Techniques



Pros

- Reach High-Complexity

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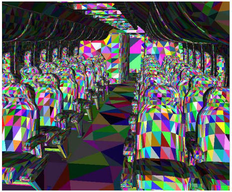
Structural Information
Scene and Objects

Structural Information as a two-scale notion:

- Object Level
- Scene Level

Approach
Pipeline and Outline

- A three-stage pipeline:



Unstructured 3D Geometry

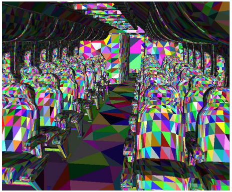
1

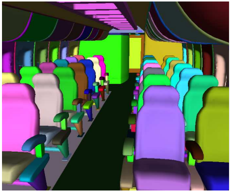
2

3

Approach
Pipeline and Outline

- A three-stage pipeline:





2

3

Unstructured 3D Geometry

Pre-Processing 3D Geometry

Approach
Pipeline and Outline

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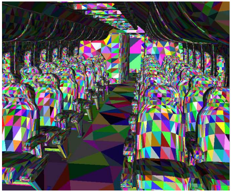

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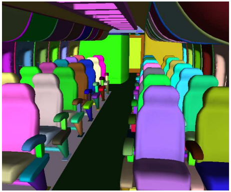

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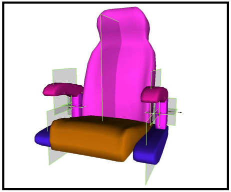
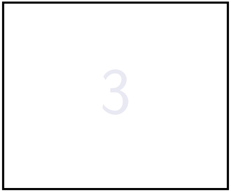


Object Level: Symmetry



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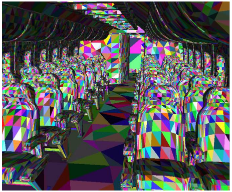

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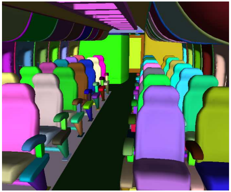

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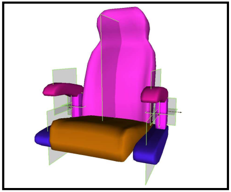

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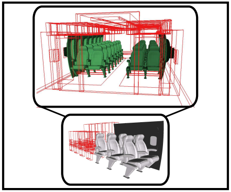
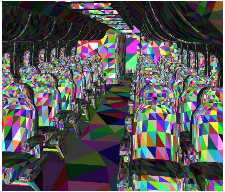


Scene Level: Instancing

Contributions

1. A new way of partitioning unstructured geometry
2. Original methods to compute Symmetries of 3D Shapes
    - Algorithm for single shapes
    - Algorithm for composite shapes
3. A new shape congruency descriptor
4. Original method to represent 3D geometry as a hierarchy of instances

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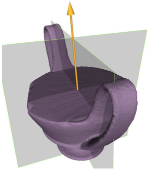
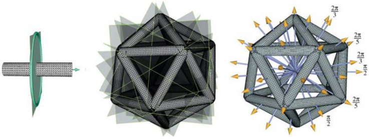


Single Shape



Composite Shape

Contributions

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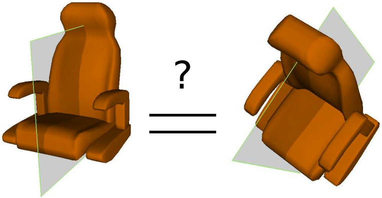

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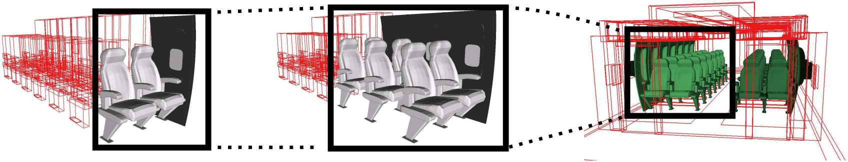

Objective

Question

What is an Object ?

Generalities

- Ill-Defined Notion
- Large Number of Possibilities

In this thesis, we define Objects as Tiles

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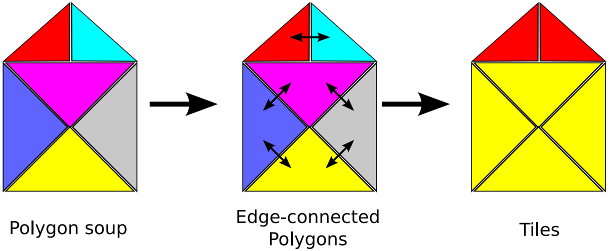
In this thesis, we define Objects as Tiles

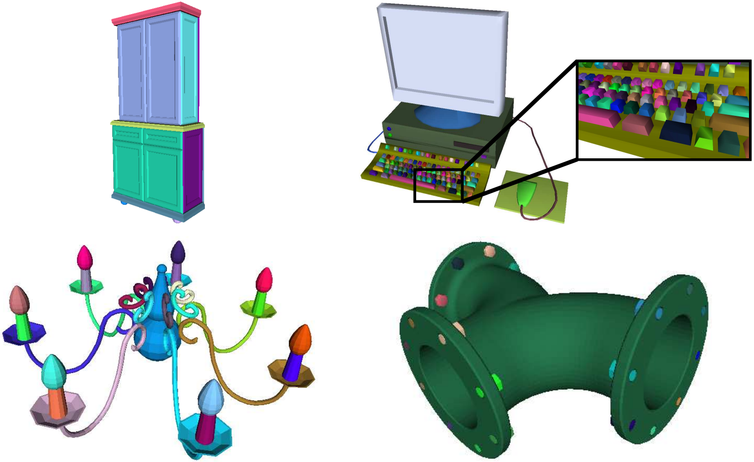
Tiles: Definition and Construction

Definition

A tile is a maximal set of edge-connected polygons

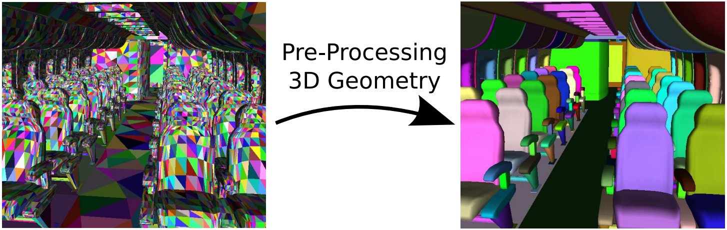


Examples of Tiles Decomposition



Computational-Friendly: Less than 2 minutes for 13M Polygons.

Summary

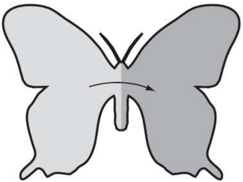


- Polygon Soup is now a Set of Tiles
- Next step is to compute the Symmetries of each Tile.

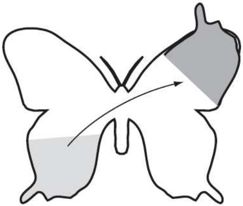
Problem Statement

Objectives

- Detect global symmetries of a 3D Shape
- Independent of Shape Tesselation



Global Symmetry

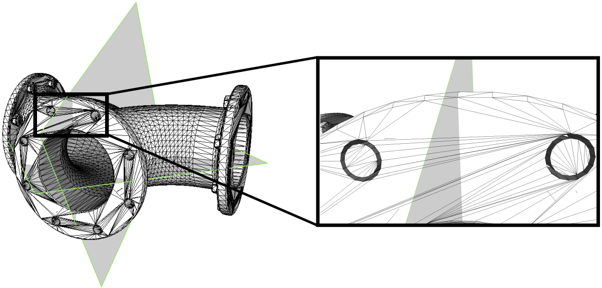


Local Symmetry

Problem Statement

Objectives

- Detect global symmetries of a 3D Shape
- Independent of Shape Tesselation

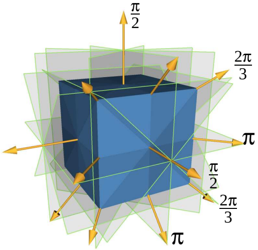


Problem Statement

Definition

Finding a symmetry of a shape $\mathcal{S}$ is equivalent to find an isometry $A = (\mathbf{X}, \alpha)$, such that:

$$A\mathcal{S} = \mathcal{S}$$



Problem Statement

Question

How efficiently found parameters of symmetries of a 3D Shape ?

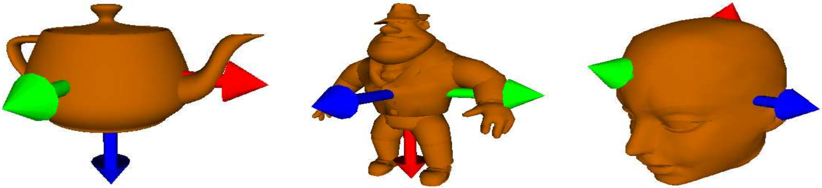
Approach

Our approach *extends* PCA-Based Approach:

1. What is PCA-Based Approach ?
2. What are its limitations ?
3. Our approach: The Generalized Moment Functions

PCA-Based Approach

- Principal Component Analysis (PCA)
  - Used to affect a local frame to a 3D Shape,
  - called Principal Axes



PCA-Based Approach

Fundamental Idea [Minovic, 1993]

ω is a Symmetry Axis of $\mathcal{S}$

$\Downarrow \quad \not\Uparrow$

ω is a Principal Axis of $\mathcal{S}$

The Method

1. Compute principal axes of the shape
2. Check each axis for a symmetry

PCA-Based Approach

Problems

What happends if principal axes are not uniquely defined ?

Properties of Principal Axes

- Along direction of maximum variance
- Unicity only if extrema are strict

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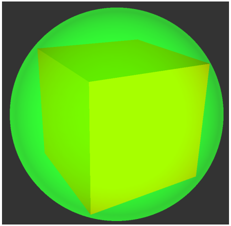


The Generalized Moment Functions

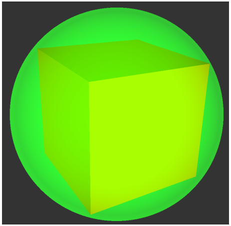
Variance Function *a.k.a.*
Moment of Order 2

$$\mathcal{M}^2(\omega) = \int_{s \in \mathcal{S}} \|\mathbf{s} \times \omega\|^2 \, \mathrm{d}\mathbf{s}$$

$$\Downarrow$$

Generalized Moment of Order k

$$\mathcal{M}^k(\omega) = \int_{s \in \mathcal{S}} \|\mathbf{s} \times \omega\|^k \, \mathrm{d}\mathbf{s}$$



$\mathcal{M}^2$

The Generalized Moment Functions

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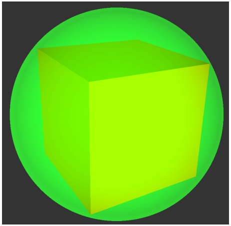
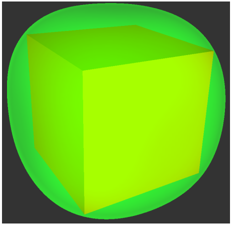
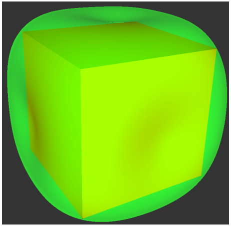


$\mathcal{M}^2$



$\mathcal{M}^4$



$\mathcal{M}^6$

Definition

Property 1 of $\mathcal{M}^k$

Isometry A is a symmetry of $\mathcal{S}$

$$\Downarrow \quad \not\Uparrow$$

Isometry A is a symmetry of $\mathcal{M}^k$, for all k

Strategy

1. Candidates Symmetries are Symmetries of $\mathcal{M}^k$, for all k
2. Check candidates on the shape $\mathcal{S}$

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Determination of the Axis of Symmetry

Property 2

ω is a symmetry axis of $\mathcal{M}^k$

$$\Downarrow \quad \not\Uparrow$$

$$\|\nabla \mathcal{M}^k(\omega)\|^2 = 0$$

Potential Symmetry Axis ω of the Shape by solving:

$$\forall k \quad (\nabla \mathcal{M}^k)(\omega) = \mathbf{0}$$

Determination of the Axis of Symmetry
Efficient Computation

- Closed-form expression of $\mathcal{M}^k$ for k even
- Using Spherical Harmonic (SH) Basis

$$\begin{aligned}\mathcal{M}^{2p}(\omega) &= \int_{\mathbf{s}\in\mathcal{S}} \|\mathbf{s}\times\omega\|^{2p}\,\mathrm{d}\mathbf{s} \\ &= \sum_{l=0}^{p}\sum_{m=-2l}^{2l} C_l^m Y_{2l}^m(\omega)\end{aligned}$$

Determination of the Axis of Symmetry

Efficient Computation

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SH coefficient

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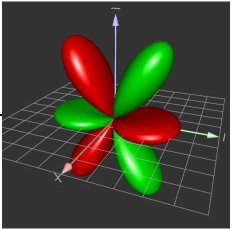
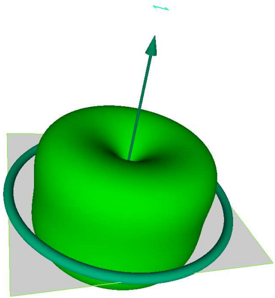


Determination of Symmetry Parameters

Property 3

Symmetries of $\mathcal{M}^{2p}$ are obtained by *testing* SH coefficients.



Determination of Symmetry Parameters

Example: Testing Revolution-Symmetry

Question

Has $\mathcal{M}^{2p}$ a revolution-symmetry around axis $\mathbf{n}$?

We use the following powerful property:

Property

A Moment Function has a revolution-symmetry around $\mathbf{z}-$axis if:

$$\forall l \quad \forall m \quad m \neq 0 \Rightarrow C_l^m = 0$$

Determination of Symmetry Parameters

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Determination of Symmetry Parameters

Example: Testing Revolution-Symmetry

Lead to a simple 2-step method:

1. Rotate $\mathcal{M}^{2p}$ to align axis $\mathbf{n}$ on $\mathbf{z}$
2. Test the nullity of the "new" coefficients C_l^m (up to a threshold)

Determination of Symmetry Parameters

- Equivalent properties exist for:
  - Planar symmetries,
  - fixed-angle rotational symmetries

Last step is to check candidates on the 3D shape S

Determination of Symmetry Parameters

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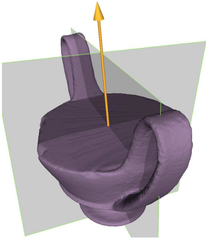
Testing a symmetry on a 3D Shape

- Define a Symmetry Measure
- Symmetries defined up to a threshold
- Allow approximate symmetries.



Fact

- *Testing a symmetry on S is costly.*
- *Symmetry Measure only computed for few candidates*

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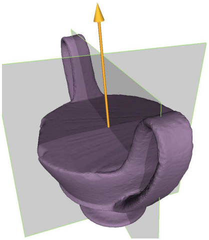

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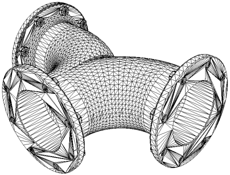
Results

Complete Example



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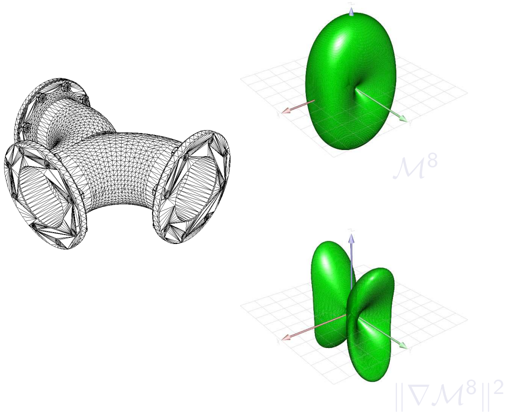

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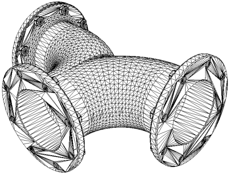
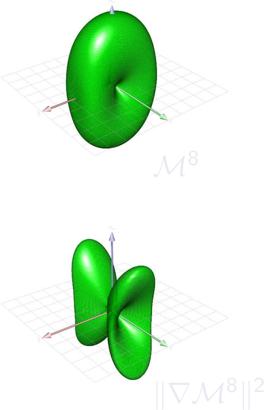
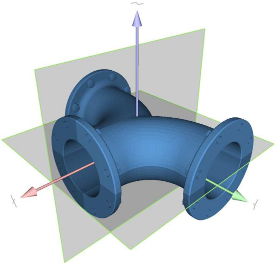
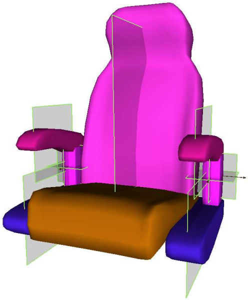
Complete Example







Summary



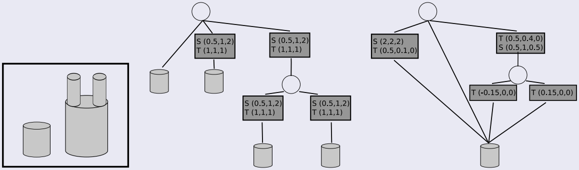
- Each tile (object) is structured using symmetry information
- Last step is to compute a representation of the geometry as a Hierarchy of Instances.

Hierarchical Instantiation

Definition

Instantiation

Factorize repeated geometry



Without Instancing

With Instancing

Hierarchical Instantiation

Extend Instantiation at multiple scales

Hierarchical Instantiation

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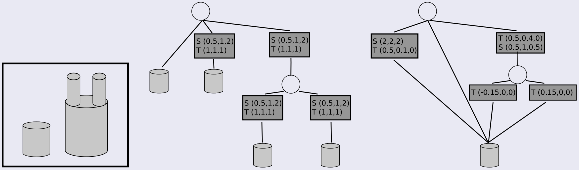


Without Instancing

With Instancing

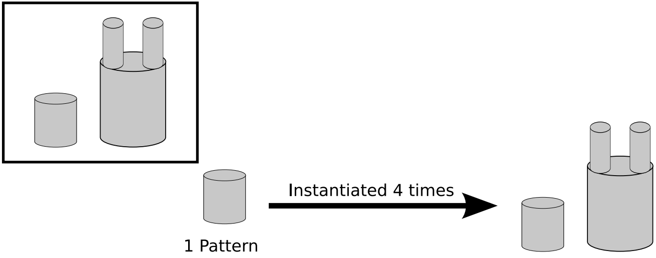
Hierarchical Instantiation

Extend Instantiation at multiple scales

Patterns and Instances

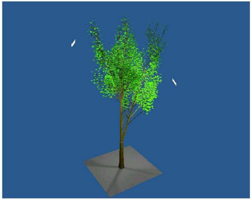
Definition

- *A pattern* is a generic set of objects,
- represented in the scene by *its instances*.
- Per instance attributes: Transformation matrix

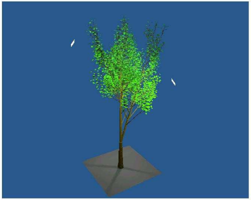


Utility of Instancing Information

- Rendering
- Geometry Editing



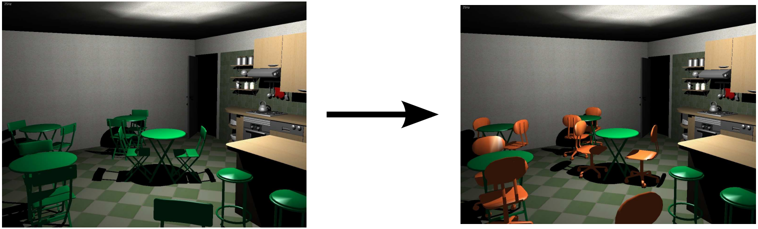
H.Radiosity (**119**mn/**123**MB)



H.Instantiation (**14**mn/**8**MB)

Utility of Instancing Information

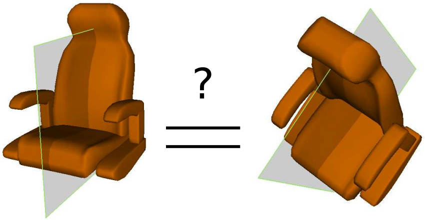
- Rendering
- Geometry Editing

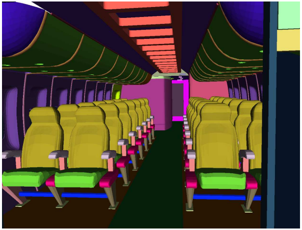


Overview

Input data

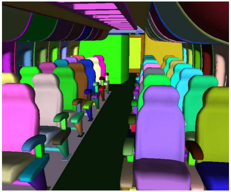
- Set of Tiles
- Class of Congruency
    - Congruent Descriptor (see manuscript)
    - Derived from $\mathcal{M}^{2p}$



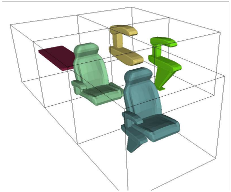


Overview

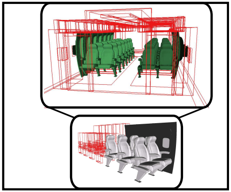
A two-step approach:



Structured 3D Geometry
Object Level



Frequent Patterns
Discovery



Organizing
Frequent Patterns

Problem Definition

Definition

The *frequency* of a pattern is equal to the number of its (possibly overlaping) instances

$\Downarrow$

Objective

Given a threshold f, identify all patterns P which frequency is greater or equal to f.

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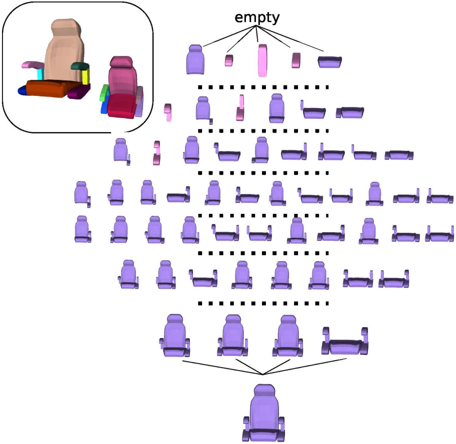
Approach

Two potential approaches

1. Agglomerative Approach
   - Progressively grow up a pattern
   - Efficient traversal of the search space
   - Exponential Complexity
2. Symmetry-Based Approach



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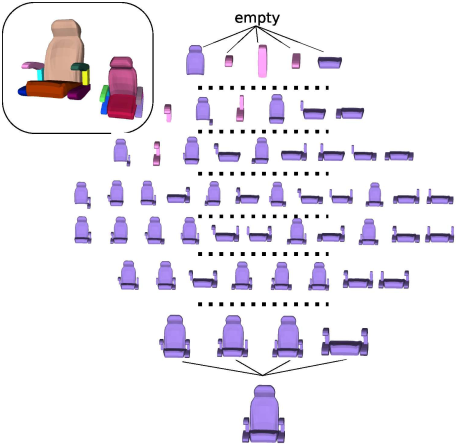
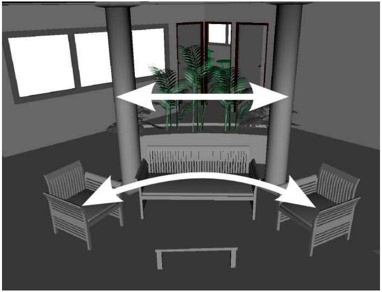


Symmetry-Based Approach

Basic assumption



Fact

Two instances of a pattern form a **local symmetry**.

Symmetry-based approach

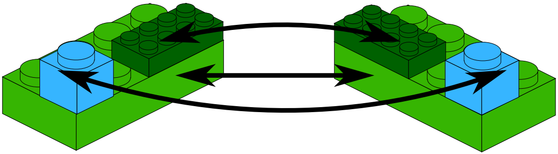
Strategy

Set of Frequent Patterns

$\Updownarrow$

Local Symmetries of the Scene

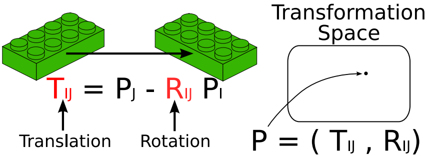
Symmetry-Based Approach



Overview

- Consider each couple of congruent tiles
- Compute the transformation that map one tile to the other
- Add the corresponding point in the Transformation Space

Symmetry-Based Approach



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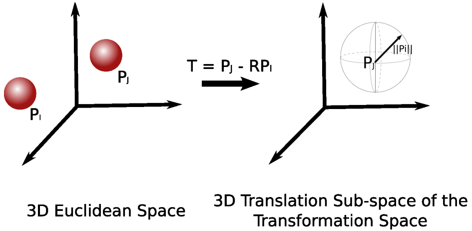
Symmetry-Based Approach

The Transformation Space

- For a couple of tiles, transformation is not unique:
  - Discrete symmetries *i.e.* rotation or planar symmetries.
  - continuous symmetries:
    - spherical symmetries,
    - cylindrical symmetries.



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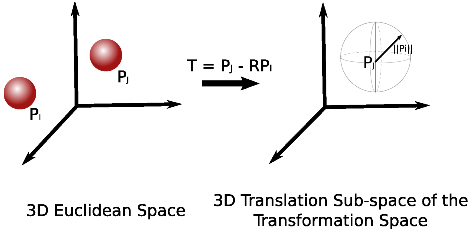

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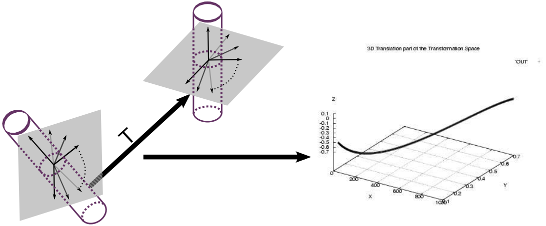
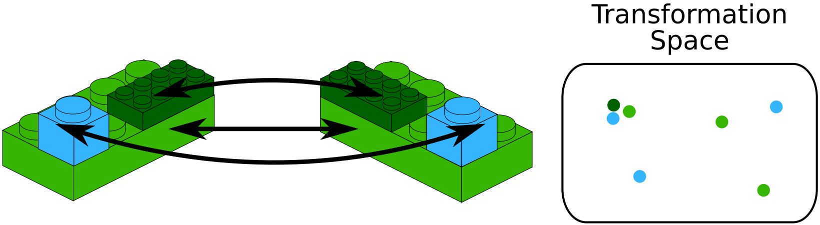


Symmetry-Based Approach

Forming patterns

- Each point of the Transformation Space contains an information of mapping between two tiles,
- Local symmetries are obtained by *clustering points*



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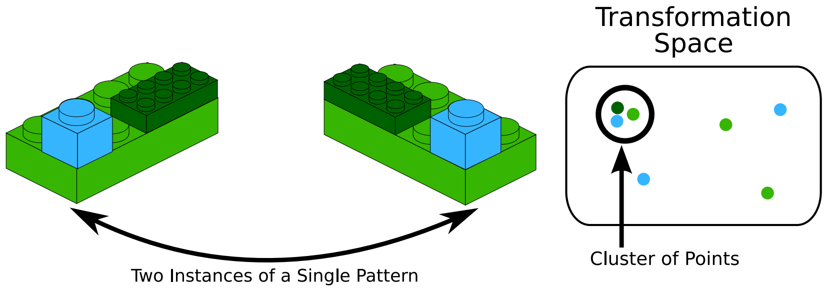
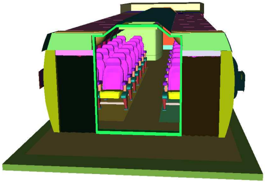
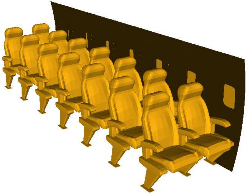


Frequent Patterns: Results





# tiles = 154
Freq = 2

# tiles = 66
Freq = 4

# Tiles	480
Frequency Threshold	2
Runtime (secs)	1.4
# Frequent Patterns	65

Summary of Contributions

1. Identify the "hardness" of the problem
2. A new approach to generate frequent patterns
3. Not presented in this talk (see manuscript):
   - Analytic expression of curve equation for continuous symmetry
   - Method to reduce the number of mappings

Next step is to represent the scene as a Hierarchy of Instances

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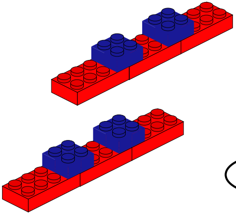
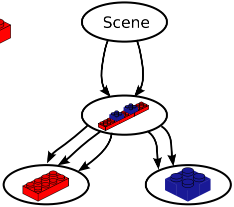
Problem Statement

Goal

- Obtain a hierarchy of instances
- Represented as a **H**ierarchy **A**ssembly **G**raph **(HAG)**



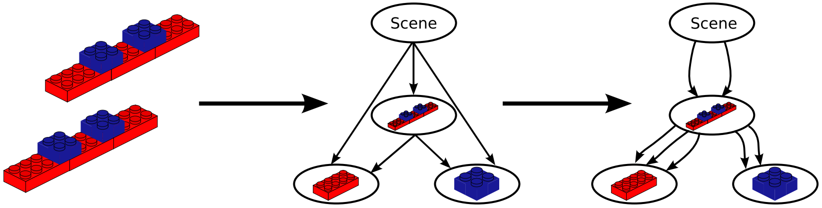


- Directed Acyclic Graph
- Each node is a pattern
- Each edge carries geometric transformation

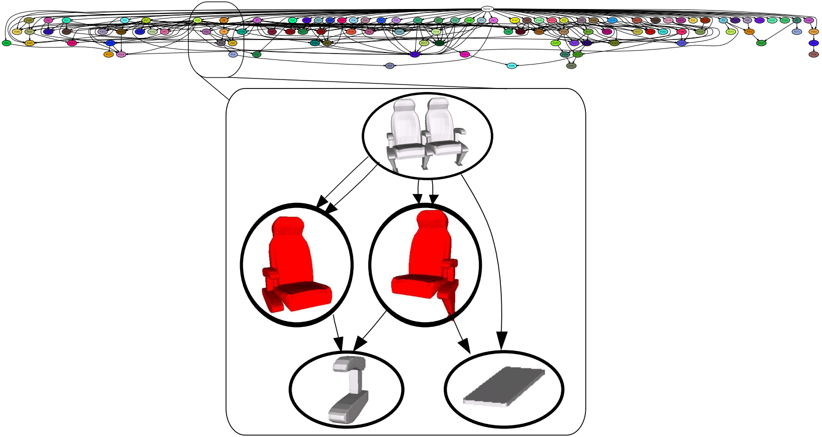
HAG Construction

HAG Construction

- Represent inclusion between frequent patterns
- Pick a *reference instance* for each pattern
- Compute the appropriate transform by iterating through edges



HAG Construction
Example: The Plane Model



HAG Construction

Observation

A HAG with overlap nodes is hardly usable

Overlapping Problems

1. Multiple rendering of overlaped parts
2. Inefficient for memory reduction
3. Geometry editing is much more difficult

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Deriving a usable Hierarchy of Instances

- Usable Hierarchy : Hierarchy with no-overlap
- Some choices must be made
- This process is Application-Dependent

Example

- Hierarchy of Instances optimized for Ray-Tracing

Deriving a usable Hierarchy of Instances

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Example

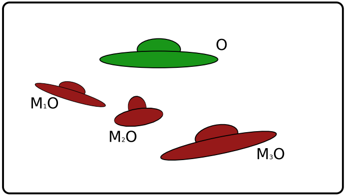
- Hierarchy of Instances optimized for Ray-Tracing

Deriving a usable Hierarchy of Instances

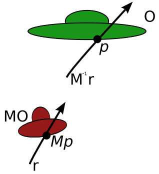
Ray-Tracing

Generalities

- Ray-Tracing naturally allows instancing
- Load a single pattern per instance
- Reduce part of the geometry loaded in memory



Simple Scene with instances



Ray-object intersection

Deriving a usable Hierarchy of Instances
Ray-Tracing

If considering whole scene itself as a Pattern of frequency 1:

Strategy

Hierarchy of Instances optimized for Ray-Tracing

$\Updownarrow$

Reduce storage cost $C(P)$ of each pattern P

Deriving a usable Hierarchy of Instances

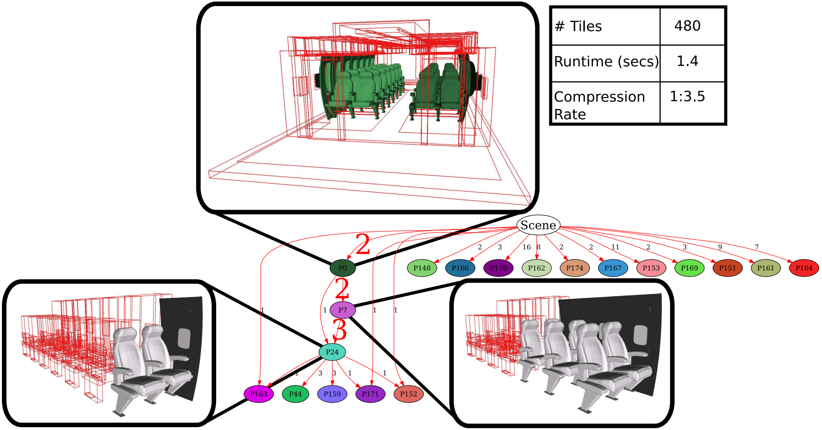
Ray-Tracing

Strategy

- Bottom-Up Approach
- For each pattern P:
  - $\min \mathrm{C}(P)$ constrained by non-overlap
- Such problem is $NP-$complete
- Need an approximation algorithm: greedy approach

Deriving a usable Hierarchy of Instances

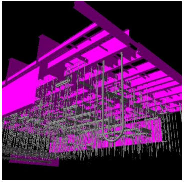
Example: Plane Model

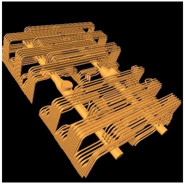


# Tiles	480
Runtime (secs)	1.4
Compression Rate	1:3.5

Deriving a usable Hierarchy of Instances

Example: Powerplant Model

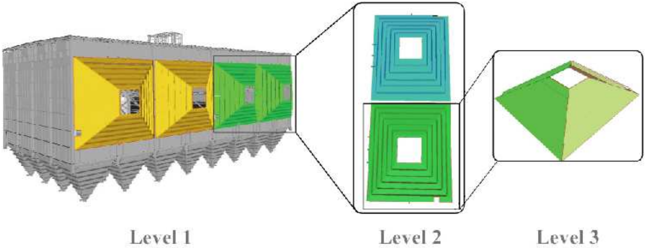




# Polygons	Pre-processing Geometry	# Tiles	Structuring Object Level (Parallel process)	Frequent Patterns	
				Runtime	# Patterns
12,748,510	2 minutes	155,348	Approx. 16 hours	1h45	87,100

Deriving a usable Hierarchy of Instances

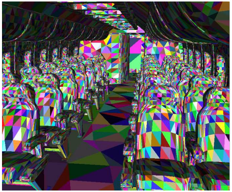
Example: Powerplant Model



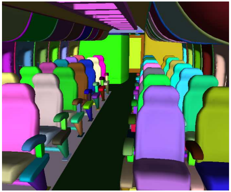
Runtime	Compression Rate
51 seconds	1:5.2

Summary

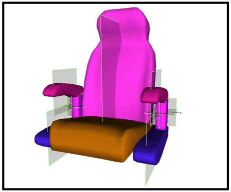
- A whole pipeline for structuring 3D Geometry:



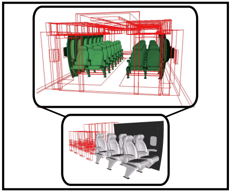
Unstructured 3D Geometry



Pre-Processing 3D Geometry



Object Level: Symmetry



Scene Level: Instancing

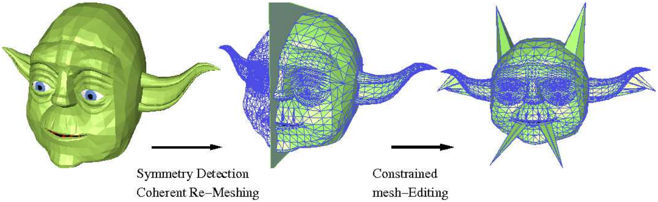
- Potential Applications:
  - Rendering
  - Geometry Editing

Summary of Contributions

Two contributions for Structuring 3D Geometry:

Detection of Symmetries in 3D Shapes

- The Generalized Moment Functions
- Algorithms for Single and Composite Shapes
- Potential Applications: Compression, Geometry Editing, . . .

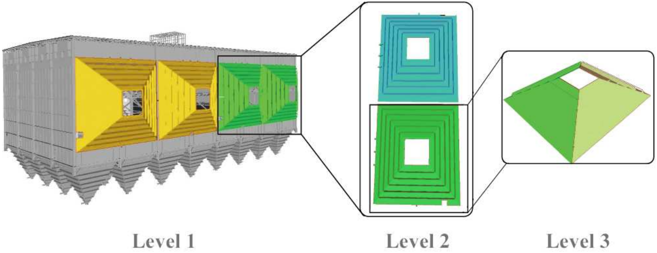


Summary of Contributions

Two contributions for Structuring 3D Geometry:

Hierarchical Instancing of Geometry

- A way of representing geometry as a Hierarchy of Instances
- Potential Applications: Geometry Editing, Compression, rendering, . . .



Future Work

- Most Promising Work: Structuring at the Semantic Level
- Adaptive Display Algorithm [Funkhouser et al. 93]
  - Adapt geometry to render it at interactive frame rates

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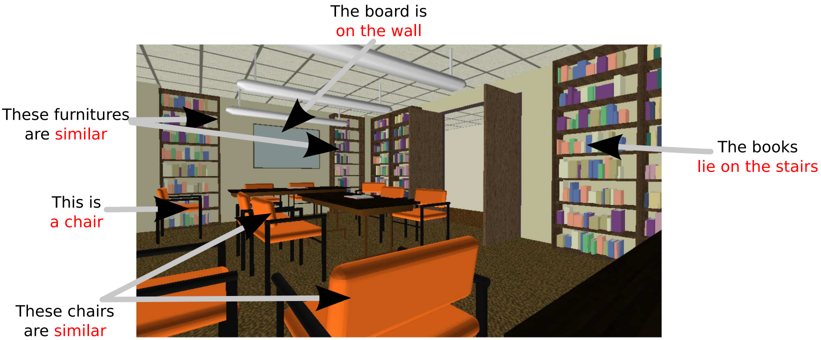

Thank you for your attention