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**The Anatomy of Inequalities in Attainments:  
An International Investigation on Stratification and Choice**

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être considérées comme propres à leurs auteurs.*



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## General Introduction.

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**R**educing inequalities in educational attainments has become a major preoccupation of educational reforms. Recent studies - especially the OECD's "*Education at a Glance*" - proved the existence of large disparities in outcomes and subsequently triggered a heightened interest for policy evaluations and international comparisons. In fact, education policies have become a main source for political debates. These debates made the headlines in most developed countries. For instance, *Der Spiegel*'s headline on the 22nd of March 2007 was "UN slaps Berlin on the wrist: German schools perpetuate inequality report says;" The *Guardian*'s headline on the 26th of August 2008 was "Is inequality worse then ever?;" *The New York Times*' headline on February the 2nd 2003 was "The tone of inequality in education;" and in France, *Libération*'s headline on the 17th of February 2009 was "Inégalités sociales dans l'éducation: Royal noircit un peu le tableau."

According to the traditional approaches, the level of inequality is defined as the strength of the impact of social background on educational attainments. This definition is implicit in some of the empirical literature and in the international comparative reports on education, such as the "*Education at a Glance*" and the "*PISA Report*." Nevertheless, reality is more complicated. Educational achievements are not the simple direct product of social backgrounds, and in general the latter operate indirectly through intricate stratification mechanisms. For instance, unprivileged households are likely to be stratified into relatively poor communities, due to the functioning of the housing market. These communities are populated with other households of similar type. Under these circumstances, the social mix of the schools operating in these neighborhoods consists mainly of unprivileged students. In such a situation, students' social backgrounds operate directly through their impact on performance and indirectly through school peer effects. Furthermore, certain educational policies can be the source of increased inequalities.

Theoretically, a high level of decentralization (local funding of public schools) paves the way for higher levels of disparities between schools, since the level of funding will be determined by community wealth and the availability of local funds. Schools in relatively rich neighborhoods have higher levels of funding; they can afford higher-quality physical and educational infrastructures as well as higher quality teachers. This concentration of resources and peer effects operates to the detriment of unprivileged households.

At this stage, it should be noted that stratification operates at two levels: first, through the sorting of households between communities with varying wealth, and second, through the sorting of students between schools with varying qualities. The extent of this stratification depends on the institutional characteristics of each education system.

From this discussion, I can say that social backgrounds should not be considered as the unique determinant of inequalities in attainments. In fact, school characteristics can also be seen as a source of inequality, when schools are very heterogeneous and when spatial and educational stratification are considered. In other words, the access to a particular school may have stronger effects on students' performance than their initial endowments in social and economic characteristics.

The literature on stratification is very recent and dates back to the early 1970s and to the founding articles of Barzel (1973) and Stiglitz (1974). The major developments occurred in the 1990s, when two distinct bodies of theoretical literature emerged. The first studied special stratification between jurisdictions differentiated by the price of housing and the level of local taxes. The second studied educational stratification between public and private schools, where school quality was mainly determined by peer effects.

The articles of Westhoff (1977), Rose-Ackerman (1979), De Bartolome (1990), Epple, Filimon, and Romer (1993), Nechyba (1997), Epple and Platt (1998) and Fernandez and Rogerson (1996) constitute the major contributions to the first body. These papers studied the provision of education in a multi-jurisdiction context where education is a locally provided good, funded by local income or property taxes. In most models, hierarchical stratification according to a single factor (such as income) arose at equilibrium. See Fernandez and Rogerson (1996).

The major limitation of these models is their lack of generality. First, they consider public schools as the unique providers of education. This assumption facilitates the application of Black's median voter theorem and avoids non-single peaked preferences at the expense of neglecting the existence of private schools. Second, most models neglect the existence of peer effects, even though the empirical evidence suggests an important role for peer quality. Finally, most of these models generate complete stratification according to a single factor. This is not very realistic, since communities can be stratified according to multiple criteria; see Epple and Platt (1998).

The second body of literature includes the articles of Arnott and Rowse (1987), Epple and Romano (1998, 2000) and Nechyba (2003). In these papers, the authors introduced peer effects as a determinant of school quality and considered private schools as an alternative to the public supply of education. However, a number of limitations remain. First, funding was neglected as a determinant of quality in order to avoid non-single peaked preferences; see Epple & Romano (1998). Second, all models neglected the fact that some private schools may receive public subsidies. Third, most of the papers modeled the public sector as completely passive. In contrast, private schools are considered to be profit maximizers. Finally, the most concerning aspect about these models is the linearity of peer effects in means. In other words, a student whose performance increases average performance by two points has the same effect as several students whose performance increases average performance by two points. This assumption of linearity was criticized in Hoxby and Weingarth (2005, p.2).

The persistence of the aforementioned limitations requires a general approach for the study of stratification and inequalities in attainments. Therefore, a number of questions have to be asked: How does educational and spatial stratification work when public, private and mixed finance schools coexist? How can non-linear in means peer effects be introduced? What is the nature of intercommunity stratification? What are the implications for the empirical analyses? How does stratification work under different institutional arrangements?

In my thesis, I build upon the work of Nechyba (1997 & 2003), Epple and Romano (1998), Epple and Platt (1998) and Glomm and Ravikumar (1998). The objectives are twofold. First, a theoretical framework is constructed in which most of the aforementioned limitations are dealt with. Second, an empirical approach for the study of stratification is developed on the basis of this theory.

On the theoretical level, funding and peer effects are included as simultaneous determinants of school quality and public, private and mixed finance schools are considered as alternative suppliers of education. In this case, Black's median voter theorem does not hold and a number of conditions should be provided. Furthermore, peer effects are not linear in means. In fact, non-linearities are introduced through the inclusion of different moments of the same variable. All schools are allowed to have a strategic behavior on the education market. Private and mixed finance schools maximize their profits and condition admission to student types and public schools receive enough funds to cover the marginal cost of admitting each student. All these maximizations are done under a number of quality constraints. In addition to this, the model is constructed in a multicomunity framework where household movements are subject to a number of constraints. The resulting community stratification is multidimensional and each community has a mix of households with varying endowments.

The main implication from the theoretical analyses is that students are not randomly stratified into schools and communities. In fact, community and school choice depend on household endowments and on the available schooling and residence alternatives. Student achievements depend on their own characteristics and on those of their school, where the latter are determined by sorting mechanisms.

This sorting process is reflected in the empirical analyses through an education production function that accounts for student and school characteristics (including peer quality). Stratification implies that the access to a particular level of quality is not random. In other words, the school social mix, the level of funding and the entire educational experience are determined through stratification. Inequalities can no longer be represented through the mere impact of students' social and economic backgrounds on their performance, since school characteristics are likely to be a source of inequality, too. Hence, inequalities can be channeled through students' own characteristics and through "stratification determined" school characteristics.

The thesis is organized in four chapters, each containing a number of sections. **The first chapter** constitutes a concise examination of the literature on stratification and on inequalities in achievements. This chapter is organized in two sections: the first is a review of the theoretical literature, while the second is a review of the empirical one.

The theoretical review starts with the founding articles of Barzel and Stiglitz and continues through the recent studies of the late 1990s. Two bodies of literature are identified: the first analyzes educational stratification between school types and the second studies spatial stratification between communities. The different specifications, the underlying hypotheses and the results are assessed. School finance and majority voting over tax rates are analyzed and their implications for the existence of equilibrium are made clear.

The second section of this chapter considers the empirical literature on the subject. This literature is very dense; it covers a wide range of areas. These areas include the empirical literature on peer effects, the literature on immigrants and their schooling, the literature on ethnic minorities and segregation, the literature on the effect of social backgrounds on attainments and the literature on the effects of school resources. In addition to the review of the different areas of the literature, the methodologies are discussed in order to assess their applicability.

In the first section of **chapter II**, a simple theoretical model is developed to analyze the equilibrium on the market for schooling where both public and private schools coexist and where individuals are differentiated by income and ability. Non-linearities in peer effects are introduced in order to shed the light on the fact that school quality is not solely dependent on average peer effects, but also on their dispersions. This constitutes an improvement to linear in means models of peer effects, such as Epple and Romano (1998 & 2006).

In the second section of this chapter, a full general equilibrium model is developed in a multi-community framework with public, mixed finance and private schools. Students are differentiated by income, ability and social capital. Local property taxes are used to finance public schools and to subsidize mixed finance ones. Schools maximize their profits under a quality constraint, and the marginal cost of admitting a student is determined by the cost of producing education and the relative position of a student to average ability and average social capital. Income plays an indirect role, since it determines the school and community a student can afford, given his ability and social capital.

In this model, three-dimensional stratification according to student characteristics arises from the interaction between school profit maximization and student utility maximization. Majority voting equilibrium over tax rates is studied, and the necessary conditions for its existence are provided. Finally, the consequences of introducing public school choice on the already existing equilibrium are discussed.

The objective of this chapter is to provide a theoretical groundwork that links stratification to educational achievements. As I have previously mentioned, students are not randomly sorted into schools and the entire educational experience is determined by stratification. This fact is captured through the education production function to be used in the following two chapters. Furthermore, these theoretical models present a very convenient framework for the use of the PISA 2003 dataset, since it contains a large array of student and school characteristics and facilitates the use of multilevel analyses.

**Chapter III** is organized in four sections. The first two explore the PISA 2003 dataset through an overview of the sampling procedures and of the variables. The third section deals with the problem of missing data. Several types of missingness are identified and alternative imputation techniques are discussed. Multiple imputations are then chosen, since they are the most modern and most reliable techniques.

The fourth section presents the selected countries, which are Germany, Finland, The UK, Italy and Japan. These countries have been chosen on the basis of the *Green & all (2006)* typology of education systems. They are intended to represent five distinct geographical areas: the German speaking countries, the Nordic countries, the Anglo-Saxon countries, the Mediterranean countries and the East Asian countries. The major advantage of undertaking a comparative analysis is the possibility of examining stratification under different institutional arrangements.

In the rest of this section, a number of descriptive statistics are computed along several axes, including streaming, decentralization, school choice and stratification. These statistics consist of different moments of a number of key variables, such as national averages and dispersions, and within and between school dispersions. It should be noted that in this chapter the patterns of stratification are studied using the descriptive statistics. However, the impact of stratification on performance scores is assessed in chapter IV.

**Chapter IV** is organized in two sections. In the first, a full examination of multilevel analyses is carried out. I start with a presentation of the arguments in favor of multilevel modeling and then I examine the logic of model building while discussing and comparing the different estimation theories. The underlying assumptions of such models are explored, and a thorough discussion is made on the consequences of the failure of such assumptions.

Furthermore, endogeneity problems are taken into account. This type of problem arises when student variables are correlated with unobserved school characteristics. A solution to the problem was provided by Mundlak (1978) for panel data models. This solution is adapted for multilevel modeling and several models are constructed. In each model, a different set of variables is added. In the most complete one, three categories of variables are controlled for: student characteristics, pure school characteristics (funding, discipline, etc) and peer effects, which are school aggregates of student characteristics.

In the second section of this chapter, the estimation is carried out and the Hausman test is used to assess fixed effects against random effects specifications. The most reliable model is interpreted and a thorough comparison between the selected countries is undertaken.

**At this stage, it is of a great importance to define the objectives of this thesis. These objectives can be summarized through one question: Does the comprehensiveness of education systems leads to more equality in attainments? This question is related to a number of issues, such as: How do inequalities in attainments result from stratification? What are the causes of stratification? How does stratification and inequalities operate under different institutional arrangements?**

**In this thesis, answers to these questions are brought. More precisely, my thesis has the following objectives: to develop educational and spatial stratification models, to examine the effects of stratification on inequalities in achievements, and to elaborate an empirical approach that allows for the quantification of these effects.**





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**The Anatomy of Inequalities in Attainments:  
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**CHAPTER I**

**School Choice and Stratification:  
Review and Perspectives.**

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The debate over the nature of school systems is not a recent one. The application of market principals to the provision of education, pitted against state control of education, has always been at the core of this debate. This debate goes back to the foundations of modern economic thought, with economists such as Adam Smith, John Stuart Mill, Nassau Senior and Edwin Chadwick.

In the mid- to late-18th century, the time of Adam Smith, two competing educational systems coexisted within the same country: those of Scotland and England. Although England had consistently been more prosperous, Scotland had had a better quality education system. According to Smith, this higher quality of education is attributable to the structural organization of the system in Scotland. Since 1696, Scottish law required that teacher salaries were to be paid by students' families. This was the earliest form of private education, where the "consumers" had to pay for the educational services they received. Smith argued that such an organization allowed parents to have more control over the quality of education by bringing the production unit (schools) closer to the final user. Since the end of the 18th century, Scotland experienced ever-widening access to education.

In England the situation was different. The law of 1665 required that English schools were to be controlled by central governments, and teacher hiring and education finance were to be centralized. Smith, in *"The Wealth of Nations,"* criticized this endowments system because it did not allow for any competition between schools. According to him, this type of system disconnected the final consumer from the suppliers of education and did not allow for efficient monitoring of the quality of the product. Smith went further, accusing this system of causing a deficit of private schools, since it would no longer be cost-efficient to operate private schools if public schooling was provided free of charge. Even though Smith was in favor of public education "*among all people of middling or more than middling rank and fortune*" - when he stated that "*the education of common people requires, in a civilized and commercial society, the attention of the state*" - he was still opposed to state training of teachers and to a system of pedagogic licensure operated by a self-governing teaching profession (see West 1964, p. 467).

As a classical economist, Smith argued that a free market for the organization of education would allow higher efficiency and greater achievements, since students and families would have the power of monitoring and scrutinizing the performance of teachers and the content of curricula. Hence, this organization would lead to better coordination between teaching techniques, families' expectations and, ultimately, to a schooling system run according to students' needs. However, Smith also insisted on the role of public education as the means of achieving greater equity.

Similarly to Smith, Thomas Malthus indicated in his "*Letter to Whitbread*" (1807) that the quality of private education will not be affected by the number of enrolled students if each one is able to pay tuition. However, when education is publicly provided, funding problems will arise with the increasing numbers of students.

At the time of Malthus, the organization of the education system started to change in favor of even more governmental control. The English parliament authorized annual subsidies to education in 1833. In 1834, the "*Poor law amendment act*," supported by Nassau senior and Edwin Chadwick, saw the light. This amendment was in favor of a wider public management of the education system and it considered the government as the guardian of national equity. Since 1834, public subsidies increased from 20000 £ to 663435 £ in 1858 (see West 1964).

In 1839, the council on education was established, and later it became the department for education and skills. This council was in charge of the management of the education system, which included the hiring of teachers and the funding and inspection of schools, among other tasks. According to Nassau Senior, a negative aspect of private schooling is the absence of centralized inspection that allows quality to be monitored.

Conversely to Senior and Chadwick, John Stuart Mill was opposed to this system of public control. His argument was based on curricular concerns. According to him, the curricula in public schools would be too rigid to be adapted to the particular and changing needs of the student population. This argument is different than the one made by Smith, where public subsidies would reduce the competition between schools, and hence would reduce the quality of education and the efficiency of the system.

In 1862, Robert Lowe, an admirer of Adam Smith, became the vice president of the committee of the council on education. His opinions on the organization of the education system stirred controversy. According to him, competition on the market for education is necessary to maintain an efficient allocation of resources as well as high quality standards. The system of subsidies and endowments can be considered acceptable in the beginning, but over the long term it will lead to a monopolistic organization of the market for education, something that Lowe considers economically and socially inefficient. Furthermore, Lowe affirmed that parents are rational agents capable of making efficient choices that serve their interests and those of society. This affirmation accords perfectly with the invisible hand of Adam Smith. It is interesting to note that the concept of free choice started to emerge in the economic thought with the writings of Lowe (see R. Lowe 1868, "Middle class education: endowment or free trade"). This choice was not limited to the expansion of private schooling as an alternative to public supply, but it extended to include the right of parents to choose the school for their children.

However, in the same year, the final report of the royal commission on education affirmed that parents cannot be considered perfect decision makers when it comes to the choice of a school for their children. Their decisions are likely to be influenced by their wealth and social standing. On the other hand, this report highlighted the need for private schooling in order to ensure enough competition on the market for education, to grant parents a wider choice and to maintain a system that can cater for the special needs of particular types of students.

After a century, in 1964, the theory of human capital emerged with Gary Becker and Theodore Schultz. The developments made on this theory provided a convenient framework for a wider understanding of stratification and school choice phenomena. Even if the recent studies on stratification and sorting bear little family resemblance to the theory of human capital, it is still possible to see that some of the early works from the 1970s had a clear tie to it.

So far, I have briefly reviewed the historical origins of the debate on school choice. This review will not be carried further and will not be extended to other countries for a number of reasons. This review is intended to put the debate on school choice in its historical context within classical economics and to motivate my further investigations. As it is possible to see, school finance, school organization and school choice were central to the debate over

efficiency and equity of education systems. In the rest of the review a thorough investigation of the literature on school choice and stratification will be undertaken. Hence, a deeper historical review will not serve the purpose of this chapter, since it will be too general and would not cover the issues of concern within the available space.

Before starting with a thorough investigation of the literature on stratification and school choice, it is of a great importance to consider the reasons why the study of stratification and choice has become a major domain of the economics of education in recent years. The analysis of school choice and stratification is concerned with a wide range of issues; these may include: the provision of education and education finance, inequality and equity in the distribution of attainments, geographical inequalities and social segregation, the provision of education to students with special needs and finally the different applications for the labor market.

School choice and stratification are at the intersection of a number of domains: public economics, urban economics, labor economics and industrial organization. Most theoretical analyses are based upon the general equilibrium theory, which seeks to explain the behavior of supply, demand and decision making in a whole economy with several markets. Usually, a bottom-up approach is used where agents have to maximize their utility under a number of constraints and where larger decision making units, such as schools, have a strategic behavior. In what follows, I will present the major domains related to school choice and stratification, while insisting on those relevant to my thesis.

### **1. The structure of education systems and education finance.**

The analysis of the structure of the education system is important for the understanding of the functioning of inequalities. On one hand, different aspects of this structure, such as the availability of public and private alternatives, tuition, the possibility of school choice and the comprehensiveness of the system, constitute a number of constraints that influence the decisions made by students and households. For instance, a selective German education system is likely to affect education decisions on the basis of students' social status. Since selection is operated at the age of 11-12, low social status students have a higher probability of being tracked into professional schools. Conversely, a comprehensive Finnish education system is likely to be more equal, since it has a 9 to 10 years of all-through education.

On the other hand, the structure of the education system also has an impact on education finance. Public schools are usually financed through public funds, while private schools are financed through tuition and sometimes through public subsidies. In some systems, a large private sector is conceived to complement public supply (Japan) and is usually subsidized. While in other countries, such as the UK, the private system is smaller and perceives lower subsidies while being socially elitist. The funding of a school system raises a number of questions: how are the funds going to be collected? How are the funds going to be used, and should private schools be subsidized? How should compensation policies in favor of unprivileged students and communities be operated? And finally, how can the efficiency of the use of funds be monitored?

## **2. Inequality and equity in the access to education.**

Equity in the access to education has become a major challenge for public authorities in most developed countries. Even though the nature of these inequalities is different between education systems, their impact on attainments is important. Inequalities are closely related to stratification which results from constrained school choice. These constraints may include income, ability, social capital and particular legislations on choice and decentralization.

For instance, a high level of decentralization (e.g. local funding of public schools) paves the way for higher levels of disparities between schools. Therefore, schools in relatively rich neighborhoods are expected to have higher levels of funding and hence higher quality physical and educational infrastructures as well as higher quality teachers. In addition to this, rich neighborhoods are usually populated with households with high levels of income, social capital and even ability (since ability might be correlated with the former two factors). The higher funding and the better peer quality tend to have positive effects on performance scores. In other words, a concentration of resources and peer quality resulting from this type of stratification operates in favor of already privileged students. The reverse is true for poor neighborhoods.

As it is possible to see, a number of inequalities will arise from the functioning of the education system and from the way students are allocated into schools at equilibrium. Some of the inequalities may also arise from particular school policies leading to unequal distribution of funds, to disparities in pedagogical practices or even to disparities in school peer quality.

### **3. Geographical stratification and territorial inequalities.**

Geographical stratification and territorial inequalities are directly related to the previous point. Stratification can be seen as the hierarchical ranking of communities according to their wealth. Geographical stratification may take the form of urban segregation between different communities belonging to the same metropolitan area, such as in large cities in the UK, the USA and France, or the form of territorial disparities between regions such as in northern and southern Italy.

As shown in numerous analyses, especially those of Epple et al (1998 & 2006), stratification may arise simply because of the functioning of the market. The interaction between the housing market, school strategic behavior and household's utility maximization gives birth to different patterns of stratification. Stratification may be considered a source of inequality, since students do not have the same opportunities in accessing education.

### **4. Inequality in the distribution of income.**

The level of education is directly related to earnings prospects, to the duration of unemployment, to the ease of acquiring new skills and to economic productivity. Those who face difficulties in acquiring education when they are young are likely to have lower incomes, longer unemployment periods and more difficulties in their professional lives. In general, inequalities in education are carried over an entire lifetime and are passed to next generations.

This relation constitutes a formidable challenge for public policies. Unprivileged households do not have the necessary resources to acquire the level of education that favors social mobility and inequalities are transferred from one generation to the other.

### **5. The Structure and functioning of the labor market.**

The structure of the labor market is largely affected by that of the education system. Even if in some countries this relation is not obvious, in others it is very visible. In Germany, the early selection system is designed to produce the required skills for the functioning of the economic system. Even if early selection is highly controversial, some would still defend it because it reconciles education with the needs of Germany's apprenticeship system.

In comprehensive school systems where the link is not directly visible, the structure of the education system still has some implications for the labor market. Due to the non-coordination between the two, some distortions between labor supply and demand may arise and may lead to unemployment in certain sectors and unavailability of labor force in others.

#### **6. Global competitiveness of economic systems.**

The concept of global competitiveness has emerged as a new paradigm in economic development. It has become a major aspect of modern economic strategy, since it captures the awareness of both the limitations and challenges posed by global competition.

Attainments in scientific domains may have important implications for the role a country will play in tomorrow's world. International competitiveness is highly affected by the advances in the technology sectors and by the ability to adapt to the ever changing-economic constraints (See the WEF's Global competitiveness report 2009). Conversely, deficiencies in the formation of skills may not only lower the competitiveness of a country, but will also negatively affect individuals' earnings prospects and their capacity to fully participate in society. Addressing the increasing demand for skills in key domains is a major challenge for policy makers, to which can be added the quest for a better distribution of those skills. Global competitiveness does not only require the education of the elite, but it also requires an equitable distribution of skills. Wide disparities in performance are not a necessary condition for a country to attain a high level of overall performance. Indeed, some of the best-performing countries (Nordic countries) have achieved their results while displaying a modest dispersion of attainments.

In what follows, a thorough investigation of the theoretical and empirical literature on school choice and stratification is carried out.



## **A. Review of the theoretical literature.**

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In this part of chapter one, an inventory of the theoretical literature on stratification and school choice is made. This literature is articulated around a number of axes, including: the early general equilibrium studies, the educational stratification studies and the geographical stratification ones.

The first category includes the papers written by Barzel (1973), Stiglitz (1974), Clotfelter (1976), Sonstelie (1979) and Epple & Romano (1996). The second category includes the works of Glomm and Ravikumar (1998) and Epple & Romano (1998, and 2006). Finally, the third category includes the works of Fernandez & Rogerson (1996), Nechyba (1997) and Epple & Platt (1998).

Yoram Barzel pioneered the study of the provision of public education when private alternatives exist. Barzel and Stigler (1970) were the first to indicate the existence of an “End against the Middle” phenomenon in the provision of education.<sup>1</sup> According to them, a coalition between the rich and the poor against the middle class will arise at equilibrium and will lead to a decrease in the quality of public education. In other terms, rich households can afford private schooling and have no incentive to pay taxes destined for the funding of public education.<sup>2</sup> Similarly, poor agents do not have enough resources to pay these taxes. Hence, both categories would vote for lower tax rates and lower public funding at the detriment of the middle class.

The article of Barzel was written in the form of a critical analysis of the paper of Robin Barlow (1970). Barlow constructed a model in which he compared the actual level of education with the optimal one, while considering economic agents to be utility maximizers. However, in his analysis he neglected the existence of private schools as an alternative to public supply, something that distorted the results as shown by Barzel.

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<sup>1</sup> Stigler called this phenomenon «director’s law». Stigler 1970 “Director’s Law of public income redistribution” *Journal of Law and Economics* Vol 13 p. (1-10).

<sup>2</sup> Note that public schools are financed through government funds and private schools are financed through tuition. Public subsidies for private schools are not included in this analysis.

Moreover, Barlow considered education to be a public good that is non-rival and non-excludable. Conversely, Barzel considered it to be a private good, since it can be provided by private suppliers and can be subject to congestion when the number of students exceeds a certain threshold. Hence, education is no longer a non-rival good. As we will see later in this chapter, Stiglitz (1974) made a better analysis of the nature of the educational good.

According to Barlow, the quantity of supplied education and the available funds are determined through majority voting over tax rates. When education is only publicly provided, the tax rate will be the one chosen by the median voter, the quantity of education will be equally distributed and the funds will be uniformly collected over the population. However, when Barzel included private schools in the model, the equilibrium allocation changed. He argued that an increase in the quantity of education will face resistance from both the rich and the poor. On one hand, the rich have the opportunity to attend private schools (since they are able to pay the required tuition) and have no incentive to vote for an increased quantity of public schooling. In fact, for them, an increase in the quantity of public education means higher taxes with no additional returns. On the other hand, an increase in the quantity of public schooling will also be met with resistance from the poorer classes, since they cannot afford higher taxes.

The rich and the poor are likely to vote for limited funding for public education in order to lighten the tax burden. This voting will be done to the detriment of the middle class, which is in general favorable towards increased taxes and education supply. This behavior was called the “ends against the middle” phenomenon, through which the two extreme classes act against the middle one. The final impact of such behavior depends on how large the two extreme classes are in comparison with the middle class. (In Barzel’s article this interaction leads to a level of education supply that is inferior to the one chosen by the median voter).

It should be noted that when private schools do not exist, the higher classes have no choice but to accept higher tax rates to maintain public education supply. However, when private schooling emerges, the rich opt for the new private schools and vote for lower tax rates. The funding of public education decreases, along with the number of enrolled students.

In this study, Barzel provided an introduction for the analysis of the provision of education when public and private suppliers coexist. He identified the presence of group behavior that affects the existence or the absence of equilibrium. Even though this study did not consider any factors other than funding, it provided a starting point for the analyses to come (see also Barzel 1969).

## **1. The early general equilibrium studies.**

A number of papers written in the 1970s are of particular interest to my investigation. They include the following: Stiglitz (1974), Clotfelter (1976), Sonstelie (1979) and Epple and Romano (1996).

Stiglitz (1974) studied the demand for education under different institutional arrangements. He considered that education can be seen as an investment as well as a consumption good, and it can be publicly and privately provided. Stiglitz defined education as a publicly provided private good and studied its effect as a screening device. He also introduced the concept of non-single peaked preferences where individual utility curves have more than one maximum. See also Stiglitz (1972), Arrow (1973) and Spence (1973).

Clotfelter (1976) introduced race and ethnic belonging as a factor of school choice. He analyzed the effect of desegregation of the education system in the USA on private school enrollment and concluded that, under some circumstances, a tipping point does exist. This tipping point is defined in terms of the percentage of black minorities; when this point is reached white students will start to flee towards private schools.

Sonstelie (1979) is perhaps the first author to introduce the concept of the quality of education as a determinant of school choice. He adopted a simple representation of quality that only depends on funding. According to his findings, multiple equilibria of the provision of education may exist. In the first equilibrium, public schools have very low quality levels and most students attend private schools. In the second equilibrium, funding as well as public school quality are high and most students attend public schools. These two equilibria are stable.

Epple and Romano (1996) extended the analyses of the “Ends against the Middle” phenomenon introduced by Barzel. They concluded that when only public schools exist, the equilibrium is determined by the median income voter. However, when both school types coexist, the equilibrium is determined by the fraction of the population with below median income levels.

### **1.1. Paretian equilibria with publicly provided private goods.**

A public good is a good that is non-rival and non-excludable. In other words, the consumption of the good by one individual does not reduce the available quantity for consumption by others and no consumer can be effectively excluded from consuming the good. An example of a pure public good would be public lighting or national defense. A publicly provided private good that is also called collective good is a private good that is supplied by the government for several reasons, including social policy and first necessity. This type of goods may include education and to some extent healthcare.

As mentioned previously, there was a debate on the nature of education during the early 1970s. Barlow analyzed education as a pure public good, while Barzel analyzed it as a private one. Joseph Stiglitz defended the idea that education is a publicly provided private good, since the consumption of education can be subject to rivalry when the number of students in a class exceeds a certain threshold. In other terms, when the number of students in a class increases beyond a certain limit, the benefits from education start to fall for all students. Moreover, education is even more complex, since the education of one student may be affected by that of another through peer externalities.

Stiglitz constructed a mathematical model in order to assess whether the resulting majority voting equilibrium is a Pareto optimum or not, while adopting different institutional arrangements. The first part of his model analyzed the supply of a publicly provided good without the presence of private alternatives. He noted  $G$  as the level of funding of the publicly provided good,  $W_i$  as the wealth of student  $i$ ,  $\bar{W}$  as average wealth and  $N$  as the number of agents in the economy. The number of agents is also equal to the number of students and voters.

With  $\bar{W} = \frac{\sum W_i}{N}$  and  $G = t\bar{W}N$ ,  $t$  is the majority voting tax rate.

Each agent in the economy votes for a tax rate that maximizes his utility. This utility depends on two factors: the funding of the publicly provided good  $G$  and the funding of a private good (consumption). The latter is equal to the rest of individual wealth after taxes  $P_i = (1-t)W_i$ .

The utility function is written in the following form.  $U \left[ G, (1 - \frac{G}{\bar{W}N})W_i \right]$ .

After a number of manipulations, the author showed that this utility function is maximized when the marginal rate of substitution between the publicly provided good and the private one is equal to the ratio of individual wealth over total wealth. He also noted that when individual preferences are single peaked, which means that the utility function has only one maximum, the median wealth voter is pivotal.

When comparing his equilibrium to Paretian optimality, the author noted that the quantity of produced education may not be Pareto optimal. Depending on the position of the median voter in comparison to average wealth, education can be over- or under-produced.

In the second part of his model, Stiglitz extended the analysis to include a private alternative to public supply. The Utility function was divided into two parts.

$$U = V(E, \theta) + \mu(W_i - tW_i)$$

$V(E, \theta)$  is the utility resulting from using public education; it depends on the level of public spending  $E$  and student ability  $\theta$ .

$\mu(W_i - tW_i)$  is the utility resulting from the consumption of private education; it is equal to the funds that can be allocated to private education after taxes have been paid.

In his analysis, Stiglitz assumed that the capital markets are imperfect and students cannot borrow money in order to fund their education. He also considered a number of different institutional arrangements of the provision of education, under which different equilibria may exist; these include public provision, private provision and mixed provision.

In his conclusions, Stiglitz noted that most equilibria are not Pareto optimal. The interaction between the two education sectors and the behavior of students usually lead to over- or under-investments in education.

## **1.2. Introducing ethnicity as a determinant of school choice.**

In the late 1960s and the early 1970s, the USA witnessed the abolition of racial segregation in schools. Clotfelter (1976) reflected this change through a study of private school enrollment during this period. He found that when public schools were desegregated, white majorities started to opt out in favor of private ones. This “white flight” phenomenon occurred when the percentage of black minorities exceeded a certain tipping point.

At the time of this study, the abolition of racial segregation in the USA led a lot of white families to opt out from their public schools. Shaffer (1973) indicated that private enrollment rates in Memphis increased of 30% in 1973, while, in North Carolina, private school enrollment increased of 34% in 1972. Another effect of desegregation was the rise of a new form of social and ethnic stratification. Before the desegregation, white and black students were segregated into different public schools; after the desegregation, a lot of white students opted for private schools. A new form of stratification between public and private schools emerged along ethnic lines.

Desegregation was a focus of the American civil rights movement during the 60s and 70s. After the civil war, the Plessy v. Ferguson’s United States Supreme Court decision (1896) affirmed that the fourth amendment of the US constitution only required equality between black and white schools and did not require integration. However, in 1954, the US Supreme Court’s decision on Brown v. Board of Education of Topeka overturned state laws that established separate public schools for black and white students. Effective desegregation started in the early 70s and peaked in 1988; but, since then, schools have become more ethnically stratified. As of 2005, the proportion of black students at majority white schools was at a level lower than in any year since 1968.<sup>3</sup>

Note that the busing policies adopted in the United States required that some non-white students were assigned to white majority schools. These busing policies have been in use during the 70s and 80s; few of them are still in use today.

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<sup>3</sup> See Jonathan Kozol, “Overcoming apartheid”, *The Nation*. December 19, 2005, p.26.

Clotfelter (1976) considered that choice between public and private alternatives is determined by three factors. These include: household income, private school tuition and household preferences for racial segregation. The effect of desegregation is measured through the change in the number of black students enrolled in majority white schools. See Gary Becker (1971), “the economics of discrimination.”

In his analysis, Clotfelter considered the number of white students in a school to be equal to  $w$ , while the number of non-white students is equal to  $n$ . The number of black students assigned to white majority schools is equal to  $cn$ . With  $0 < c < 1$ ,  $c$  is the parameter reflecting the extent of desegregation; it represents the proportion of black students assigned to white schools. Given the demographic composition of the population, the proportion of black students assigned to white majority schools is given by  $D = \frac{cn}{cn + w}$ . Segregation is complete when  $c = 0$  and  $D = 0$ , and desegregation is complete when  $c = 1$  and  $D = \frac{n}{n + w}$ . The latter case means that all black students have been assigned to white majority schools. Hence, the proportion of black students in white majority schools can be between 0 and  $\frac{n}{n + w}$ . At this point, it should be said that the ethnic composition of a school also depends on white flight to private schools. The importance of such flight is given by the proportion of white students in private schools. This factor was included in Clotfelter’s paper in order to give a full image of the effects of desegregation.

The utility function for a white student is given by:

$$U = U(X, R), \text{ with } U_x > 0 \text{ and } U_R < 0.$$

Where  $X$  is consumption goods and  $R$  is the proportion of non-whites in the school attended by the student. In Clotfelter’s paper, an implicit assumption is made that white households prefer segregated schools. Utility is increasing in consumption and decreasing in the percentage of black minorities in a school.

### 1.1.1. Tipping point and white flight.

When segregation was abolished, some black students were bused to white majority schools. Since white students' utility function is decreasing in the percentage of black minorities, busing leads to a decrease in their utility. White students have to compare their utility level in a desegregated public school with the utility resulting from attending private schools. White flight occurs when the latter is higher than the former.<sup>4</sup>

For a given income  $Y$  and private school tuition  $T$ , there exists a percentage of black minorities  $R^*$  for which white students will be indifferent between public and private schools. The indifference condition is the following:

$$U(Y, R^*) = U(Y - T, 0)$$

$U(Y, R^*)$  is utility in the public school where the entire income is consumed and where the percentage of black minorities is equal to  $R^*$ .

$U(Y - T, 0)$  is utility in the private school where the percentage of minorities is equal to zero, and income is not entirely consumed since tuition is to be paid.

$R^*$  is the tipping point; if integration exceeds this level of bused black minorities, white flight will begin. At this point, it is possible to determine the proportion of white students attending private schools for each level of bused black minorities. White demand for private schooling (noted as  $E$ ) is a function of the percentage of black minorities in public schools:

$$E = F(R') = \int_0^{R'} f(R) dR \text{ with } 0 < E < 1$$

$R'$  is the percentage of bused non-whites.

Note that certain levels of  $R'$  are not feasible. If all black students are bused into white majority public schools, we can imagine that most white students will flee to the private sector. Hence, the system will converge again into a state of segregation. An equilibrium has to exist, where  $R$  and  $E$  are relatively small.

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<sup>4</sup> In his paper, Clotfelter considered public and private schools to be two large sectors. He did not differentiate between individual schools.



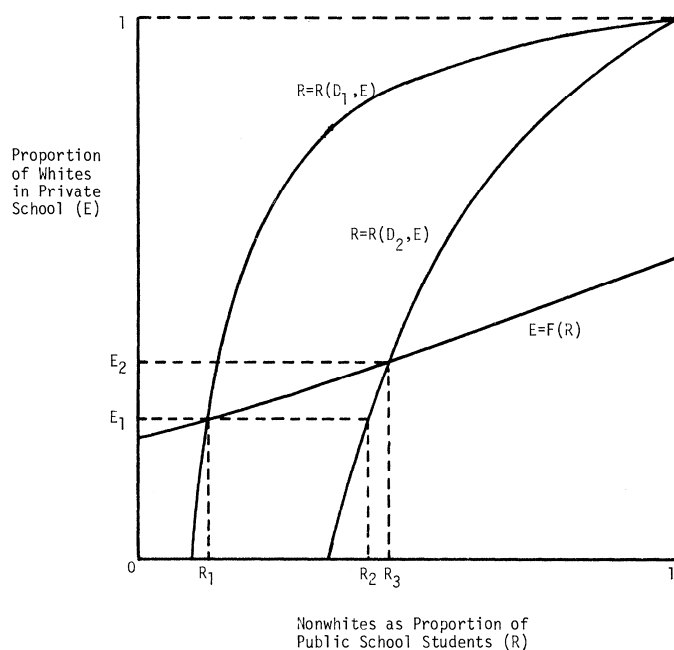
The equilibrium conditions are the following:

$$D = \frac{cn}{cn + w}, \quad E = F(R), \quad \text{and} \quad R = \frac{cn}{cn + (1 - E)w}$$

$D$  is the percentage of non-white students to be assigned to white majority schools.  $E$  is the percentage of white students who will move to private schools (the extent of white flight).  $R$  is the percentage of non-white students in public schools after white flight. In other words,  $D$  is the percentage of bused non-white students that is determined by the educational authorities and  $R$  is the real proportion of non-white students in public schools after white flight has been accounted for.

The equilibrium can be analyzed through figure 1.1, where the demand for private schooling  $E = F(R)$  and the percentage of non-white students in public schools  $R$  are represented. The equilibrium is determined by the intersection of the two curves. If educational authorities decide to increase the proportion of bused students from  $D_1$  to  $D_2$ , the proportion of non-white students in public schools increases from  $R_1$  to  $R_3$ . If white flight did not occur, the increase should have been from  $R_1$  to  $R_2$ .<sup>5</sup> Hence, the increase from  $R_2$  to  $R_3$  can be attributed to white flight after desegregation.

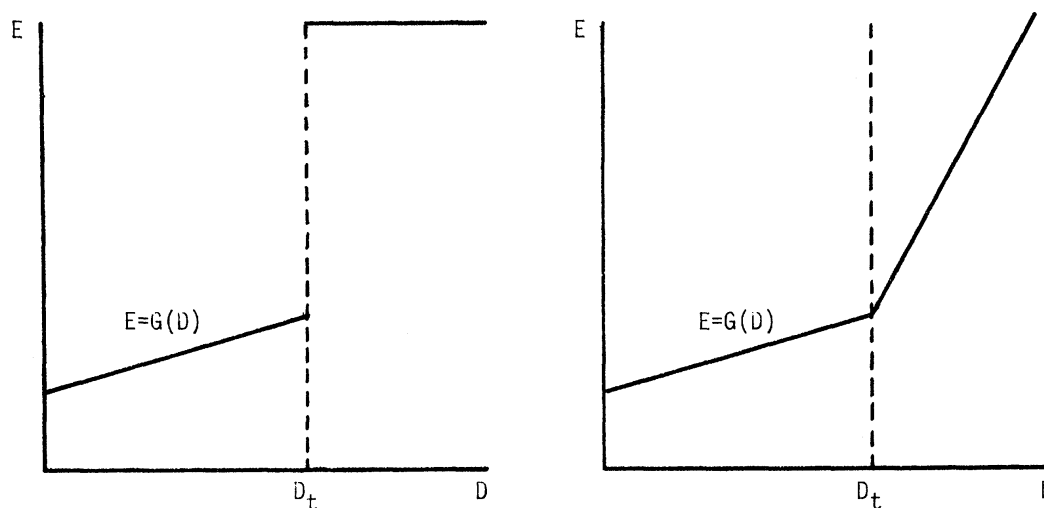
Figure 1.1: The demand for private schooling.



<sup>5</sup> This means that  $E = F(R)$  is non-increasing in  $R$ . In other words,  $E = F(R)$  is horizontal.

A number of authors indicated that the tipping point phenomenon is continuous in the percentage on non-white students (right hand side figure) instead of being discontinuous (left hand side figure). According to the left hand side figure, a tipping point is the point from which all white students will flee public schools. Conversely, according to the right hand side figure, a tipping point is the point from which the enrollment rate in private schools will become much higher. See Stinchcombe et al (1969). This approach is much more realistic.

Figure 1.2: Tipping points.



In this study, it is possible to see that the author has given too much weight to the racial composition of schools. This can only be understood through the historical context of the 70s in the USA, where white flight was associated with desegregation. However, it should be said that stratification depends on a larger set of determinants, especially income and the social characteristics of households. When these are accounted for, ethnicity is unlikely to be the major determinant of stratification.

The second disadvantage of this paper is that it aggregates the education market into two sectors. Hence, schools and geographical jurisdictions do not exist. However, In order to understand school choice, it is of great importance to understand how households are stratified into communities and schools. This category of analysis will be the subject of investigation in section 2 and 3.

### **1.3. Introducing school quality as a determinant of school choice.**

Sonstelie (1979) was the first author to consider the quality of education as a determinant of school choice. This quality was only defined in terms of government expenditure on public education. In his paper, the author concluded that the market for education may have multiple equilibria. The first equilibrium is characterized by a low level of public school quality and high private enrollment rates, while the second is characterized with high public quality and public enrollment. These two equilibria are both stable. See Flowers (1975) and Barzel & Deacon (1975).

The existence of multiple equilibria is caused by the presence of non-single peaked preferences. In other words, the utility function has more than one maximum. This happens when both public and private systems coexist. For very low levels of public education, most households choose private alternatives. A marginal increase in public quality does not enhance utility, since this increase is not sufficient to provoke a movement from private to public schools. Moreover, this marginal increase in public quality may be associated with higher taxes that reduce the utility of those attending private schools.

When the increase in the quality of public education is sufficient, students will opt out from their private school and will join a public one. At this stage, individual utility will increase, since the improvement in public education quality will compensate the losses caused by increased taxes. In this case, the utility function has two maximums, one in each sector. The author noted that, at a certain point, an optimal level of quality will be attained for which the marginal benefit for a household will be equal to the marginal cost. This point is an equilibrium.

One of the objectives that Sonstelie wanted to achieve through this paper was to study the impact of reducing public expenditure on education. He affirmed that in the short term the effect on public quality may be positive, since a large proportion of students will be inclined to attend private schools, and public school enrollment will decrease. In other words, expenditure per pupil in public schools may increase. However, at the long term the quality of public education may start to fall if the decline in public spending is excessive; see also Shapiro & Morgan (1978).

It should be noted that measuring the effect of public expenditure is a little bit complex. For instance, an increase in public expenditure causes public education quality to increase at a first stage. However, at a second stage, more students will get enrolled in public schools and expenditure per pupil, as well as public quality, will decrease.

### **1.3.1. Equilibrium on the education market with quality as a major determinant of choice.**

A household has a choice between the public and the private sectors. His utility function depends on consumption goods notes as  $G$ , and the quality of education noted as  $E$ .  $Y$  is considered to be the net income of the household, and each household has one student.

$$U = U(G, E) \text{ with } U_E > 0 \text{ and } U_G > 0.$$

If the student is enrolled in a public school, the quality he gets is equal to  $\bar{E}$  and the entire net income is consumed (the public system is free of tuition); the level of  $\bar{E}$  is determined by the authorities. The student's utility is given by:

$$\bar{U} = U(Y, \bar{E})$$

If the student is enrolled in a private school, the student gets a quality equal to  $E^*$  and pays tuition that is equal to  $T(E^*)$ . His utility is then given by:

$$U^* = U[Y - T(E^*), E^*]$$

A student chooses a private school if it generates a higher utility,  $U^* > \bar{U}$ . When  $U^* = \bar{U}$ , the student is indifferent between the two sectors. This equality defines the equilibrium on the education market.

### 1.3.2. Analyzing the equilibrium through economic surplus.

This equilibrium can be assessed through the household economic surplus noted as  $S(\bar{E})$ . This surplus is equal to the maximum that a household would accept to pay in order to keep its student enrolled in a public school. The surplus is defined implicitly by the indifference condition  $U[Y - S(\bar{E}), \bar{E}] = U^*$ .

If  $S(\bar{E}) > 0$ , the household earns a positive surplus from enrolling its student in a public school, and hence it will choose a public one.

If  $S(\bar{E}) < 0$ , the household earns a negative surplus and therefore will choose a private school.

This economic surplus is represented through the following figure.  $S'$  is the derivative of  $S$ . In other words, it is the income-compensated demand for education with utility held equal to  $U^*$ .  $T'$  is the marginal cost of private education (marginal tuition). At  $E^*$  the utility-maximizing level of private education is given by  $S' = T'$ . The surplus is then written in the following form:

$$S(\bar{E}) = S(E^*) - \int_{\bar{E}}^{E^*} S'(E) dE$$

In the private sector  $S(E^*) = T(E^*)$ ; in other words, the amount that a family would accept to pay in order to keep its student enrolled in the private school is equal to the tuition it has to pay. The surplus can be written as:

$$S(\bar{E}) = T(E^*) - \int_{\bar{E}}^{E^*} S'(E) dE$$

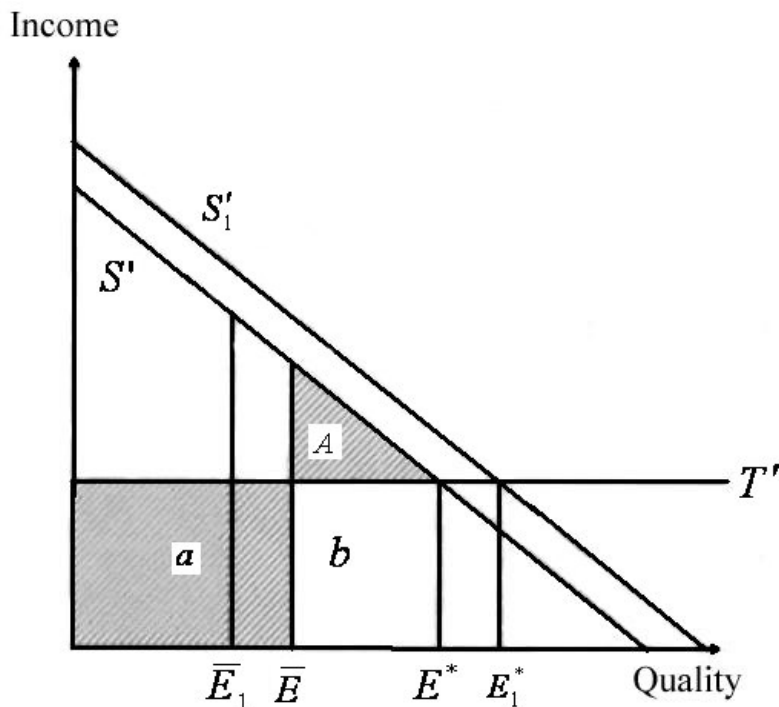
Assuming  $T(0) = 0$ , the surplus finally becomes:

$$S(\bar{E}) = \int_0^{E^*} T'(E) dE - \int_{\bar{E}}^{E^*} S'(E) dE$$

Graphically, the surplus  $S(\bar{E})$  is represented through the difference between the two areas (a+b) and (A+b). Hence, the surplus is equal to the area (a-A). If the income of a household increases, his demand for education will increase from  $S'$  to  $S_1'$ . In other words, the household will be able to afford better quality private schooling. Therefore, the surplus  $S(\bar{E})$  will decrease, since the (a) area is still the same while (A) increases. The same happens when the

government decides to reduce the quality of public education by reducing public expenditure from  $\bar{E}$  to  $\bar{E}_1$ : (a) decreases while (A) increases.

Figure 1.3: Equilibrium and economic surplus.



In conclusion, it is possible to say that the most innovative aspect of this paper is the introduction of the concept of educational quality, instead of the use of quantity, in terms of the number of units of education. However, this quality is only defined in terms of public and private spending on education. It does not include the social or economic peer effects resulting from the composition of the school. These in particular may have very important effects on school choice and on attainments. In chapter 4, it will be shown that these peer effects have a greater impact on performance scores than expenditure.

Similarly to the analysis of Clotfelter, this paper aggregated the schooling market into two sectors. This is unrealistic, since choice occurs between schools and not between sectors. In other words, depending on the geographical location of a household and the number and type of available schooling alternatives, choice will be made.

## 1.4. Majority voting equilibrium with private alternatives.

Majority voting over tax rates in the supply of publicly provided goods has been a major subject in public economics in recent years. The investigation started with the paper of Barzel in (1973), then that of Stiglitz (1974), Ireland (1990) and, more recently, Epple & Romano (1996). In the latter, the authors studied the provision of public goods when private alternatives coexist. Publicly provided goods can include education, healthcare and transport, among others. One difference between education and other publicly provided goods is that education cannot be obtained simultaneously from public and private providers. A student can attend a public or a private school, but not both at the same time. Conversely, healthcare and transport services can be obtained simultaneously from public and private suppliers. Another characteristic of education is the existence of non-single peaked preferences through which utility can have two maximums, one in each sector.

In their paper, Epple and Romano concluded that when education is solely provided by the public sector, the median income voter is pivotal. On the other hand, when education is simultaneously provided by the public and private sectors, the equilibrium tax rate is determined by the below-median fraction of the population.

### 1.4.1. Equilibrium and utility maximization.

In the Epple and Romano (1996) model, the utility function depends on consumption goods and education (in units). The former is noted by  $b$  and the latter by  $x$ . The utility is written in the following form:  $U = U(x, b)$ .

The two authors made the following assumptions:  $U(x, b) > U(0, \bar{b})$ ,  $U(x, b) > U(\bar{x}, 0)$  and  $U'_b < 0$ . The first two mean that individuals will allocate their income between education and consumption goods, neither one of which two can be equal to zero. The last assumption means that the marginal utility resulting from consumption goods is decreasing; each supplementary unit of consumption goods generates less utility.

Households are differentiated by income. Income's distribution is given by  $f(y)$  and its cumulative distribution is given by  $F(y)$ .  $Y$  is aggregate income in the economy,  $Y = \sum y$ .

Public education is funded through taxes that are paid by the entire population, including those who have opted for private schooling. Aggregate public expenditure is given by  $tY = NE$ . With:

$t$ : The tax rate

$Y$ : Aggregate income.

$N$ : The number of households in the economy, given that each household has one student.

$E$ : The expenditure on education per household (or per student).

This equation represents the budget constraint for the public system. The tax rate is determined through majority voting.

Private education is funded by tuition paid by households. The cost of a unit of private education is given by  $p$ . A student can only attend a public or a private school, and not both at the same time.

A student who chooses private education has to maximize his utility  $U = U(x, b)$  under a budget constraint given by  $y(1 - t) = px + b$ . Hence:

$U^*(x^*, b) = \max U[x, y(1 - t) - px]$ , with  $x^*$  the level of education that maximizes this utility.

A student who chooses public education has the following utility:

$$U[E, y(1 - t)]$$

A student is indifferent between the public and the private sectors when the two levels of utility are equal.

$$U^*(x^*, b) = U[E, y(1 - t)]$$

For a given income and tax rate, there exists a level of public expenditure per pupil  $E$  from which households will be inclined to enroll their children in public schools. This level of public expenditure is determined by the equality between utility in the public sector with that in the private one.



### **1.4.2. Interpretation of the findings.**

When private alternatives exist, households with sufficient income are able to choose private schools. These households vote for lower funding for public education, since they are not using it. Similarly, poor households vote for low levels of taxes, since their contributory power is lower. On the other hand, middle class households vote for relatively higher taxes, since they are the main users of public education. At equilibrium, a coalition between the upper and lower classes arise according to the principal of “Ends against the middle;” see Stigler (1970).

In the case of the dual provision of education, preferences are not single peaked and multiple equilibria may exist. The upper class would not accept a marginal increase in the quality of public education and would require a sufficient one to opt out from the private sector, while the middle class would oppose an important increase in taxes because of its lower contributory power. Upper class households have two possible equilibria with different tax rates. When the tax rate is at a low level, they choose private schools, and when it is high enough they choose public ones. The exact position of the equilibrium tax rate depends on how large is each class. The authors, similarly to Barzel (1973), noted that when private alternatives exist, the equilibrium tax rate will be determined by the below-median income fraction of the population.

## **2. Educational stratification studies.**

A number of recent papers tried to assess stratification patterns on the market for education. This category of papers includes the works of Glomm and Ravikumar (1998) and Epple & Romano (1998, and 2006).

Glomm and Ravikumar’s paper can be seen as an extension of Epple and Romano (1996) and Barzel (1973). The authors analyzed majority voting over tax rates when private alternatives coexist. They considered an economy populated with agents differentiated by income. Public education is funded through taxes and private education is funded through tuition and the quality of education is assumed to be an increasing function of public expenditure.

Epple and Romano's papers belong to the same category as Sonstelie (1979). The authors studied stratification on the market for education while considering a utility function that depends on educational quality. This quality is only determined by ability peer effects. This is perhaps one of the first attempts to include ability as a factor of stratification. As we will see in chapters 3 and 4, stratification may as well occur according to a number of different factors such as social status.

## 2.1. Private school choice, majority voting and educational quality.

In the Glomm and Ravikumar model (1998), students were differentiated by income. They have similar preferences and they maximize a utility function that depends on consumption goods and educational quality. The utility is given by:

$$U = U(c_i, q_i)$$

$c_i$ : is the level of consumption goods.

$q_i$ : is the level of educational quality.

$y_i$ : is income of individual  $i$ ; it has a cumulative distribution given by  $F(y)$ .

$Y$ : is aggregate income in the economy.

Each household has one student, and both form a single decision making unit.

The government funds public education using tax revenues. All households pay their taxes even if they do not use publicly provided education. The tax rate is noted by  $t$  and the number of students enrolled in public schools is noted by  $N$ . The level of expenditure per pupil is given by  $\frac{tY}{N}$ . Note that this expenditure is equal for all students enrolled in public schools.

Hence public school quality is homogenous. Public expenditure is converted into educational quality according to the following relation:

$$Q = \frac{tY}{N} \text{ if } t > 0 \text{ and } N > 0.$$

$$Q = 0 \text{ if } t = 0 \text{ and } N = 0.$$

As mentioned above, all households participate in the funding of public education through taxes, even if they have the choice to opt out from public schools. Private education quality is equal to the amount of money that a household can spend on tuition. This amount is equal to a household's disposable income:  $q_i = (1-t)y_i - c_i$

It should be noted that public education can not be supplemented by private alternatives. A household has to choose between public and private alternatives.

If a student chooses to get enrolled in the public sector, he can consume his entire net income, since no tuition is to be paid. In this case  $c_i = (1-t)y_i$ . The indirect utility of individual  $i$  in the public sector is noted:  $V_u(t, y_i, Y, N)$ . The subscript  $u$  is used to denote the public sector. Utility depends on the tax rate, individual income, aggregate income and the number of students enrolled in the public sector. Note that indirect utility is trivial, since there is no maximization.

If a student chooses to get enrolled in a private school, he has to maximize his utility by determining the optimal level of tuition and consumption goods. This maximization can be written in the following form:

$$\max U(c_i, q_i)$$

$$\text{Under the following constraint: } q_i = (1-t)y_i - c_i.$$

$$\text{With } c_i \geq 0, q_i \geq 0 \text{ and } t \in [0,1].$$

The indirect utility of individual  $i$  in the private sector is noted:  $V_r(t, y_i)$ , where  $r$  is a subscript used to denote the private sector. Utility depends only on the tax rate and individual income.

A student chooses the public sector if and only if:

$$V_u(t, y_i, Y, N) \geq V_r(t, y_i).$$

As mentioned above, the number of students choosing the public sector is given by  $N$ . At equilibrium, households will choose the optimum tax rate through majority voting. This tax rate is given by  $t^*$ , and the equilibrium number of students in the public sector is given by  $N^*$ .  $N^*$  should solve the following identity:

$$N = \mu\{V_u(t, y_i, Y, N) \geq V_r(t, y_i)\}$$

A majority voting equilibrium is given by  $(t^*, N^*)$ , which should satisfy the following equations:

1.  $N = \mu\{V_u(t, y_i, Y, N) \geq V_r(t, y_i)\}$ .
2. There does not exist a tax rate  $t'$  that is preferred to  $t^*$ .

There exists a critical level of income  $\hat{y}$ , such that  $V_u(t, y_i, Y, N) \geq V_r(t, y_i)$  if and only if  $y_i \leq \hat{y}$ , with  $t \in (0,1)$ . In other words, at equilibrium, there exists a level of income  $\hat{y}$  for which students are indifferent between the public and private sector. All students with income below this level will choose the public sector, since their utilities are higher there.

The authors have shown through a series of propositions that  $\hat{y}$  is decreasing in  $N$  and increasing in  $Y$  and  $t$ . When  $N$  (the number of students in the public sector) increases, expenditure per pupil  $\frac{tY}{N}$  decreases along with quality. Hence, some students attending public schools will be encouraged to opt out and to choose a private one, and the critical income determining indifference will be lower. The contrary happens when  $Y$  or  $t$  increase. The quality of public education increases and some students opt out of their private school and choose a public one. Hence, the indifference income level will be higher.

In the last section of their paper, the authors analyzed the necessary conditions for the existence of a majority voting equilibrium when individual preferences are non-single peaked. This equilibrium is determined by the median-income voter, and is characterized by a tax rate  $t^*$  and by the equilibrium number of students  $N^*$ . It should be also said that the indifference level of income between the public and private sectors should divide the population into two parts, with  $N^*$  students in the public sector.

In this paper it is possible to see that the authors introduced the concept of educational quality into a majority voting equilibrium model. Educational stratification is characterized by a critical level of income that separates the two schooling alternatives. However, the first disadvantage of this model is that it aggregates the schooling system into two sectors where individual schools do not exist. As I mentioned earlier, school choice, as its name may suggest, occurs between a number of schools and not between two sectors. The second disadvantage is the representation of private school quality as equal to disposable income. In other words, the level of tuition to be paid and hence private quality are determined by the household and schools are completely absent. These problems have been accounted for in the papers of Epple and Romano (1998 and 2006).

## 2.2. Stratification when peer effects are a determinant of quality.

Epple and Romano (1998) was perhaps the first paper to introduce ability peer effects as a determinant of stratification. In their model, households were differentiated by income and ability and quality was solely dependent on ability peer effects. The level of achievements was determined by students' own ability and by average ability in a school. Similarly to the previous models, each household has only one student and both were considered to be a single decision making unit.

Individual net income is given by  $y$ , ability is given by  $b$  and school average ability (peer effects) is given by  $\theta$ .

Achievements are given by  $a$ , and they depend on  $b$  and  $\theta$ , according to the following function:  $a = a(b, \theta)$ . Achievements are increasing in both arguments. The existence of positive ability peer effects is proven in a large number of previous studies, including Hanushek (1986), Summers & Wolf (1977) and Moreland and Levine (1992).

Private schools are tuition-financed; the level of tuition is noted by  $p_{ij}$ . Tuition in public schools is equal to zero. Individual utility depends on consumption goods and educational achievements  $a$ . It is given by the following function:

$$U = U[y - p, a(\theta, b)]$$

The cost of producing education for a school depends on the number of enrolled students. The cost function for both public and private schools is given by:  $C(k) = V(k) + F$  with  $V' > 0$ ,  $V'' > 0$ , and  $F$  is a constant fixed cost. At equilibrium  $k^* = \arg \min [C(k)/k]$ .

In this model, there is no constraint on the access to public schools. Students are free to move (costless migration between schools) and public school quality is homogenous. In fact, the public sector can be seen as a large multi-campus school.

The public system is funded through taxes, and public schools are passive and do not have any strategic behavior on the market for education. Conversely, private schools maximize their profits through the active selection of their students; this selection is done according to income and ability. Private schools charge tuition according to students' types. If a student can afford the price corresponding to his type, then he is admitted by the school. In other words, students are price takers and private schools are non-anonymously crowded clubs; see Scotchmer (1987).

Private school tuition for a student with ability  $b$  and income  $y$  is given by  $p_i(b, y)$ .  $p_0(b, y) = 0$  is tuition in public schools.  $i$  is a subscript used to denote a school,  $i=0$  is used to denote the public school while  $i=1,2,3,\dots,n$  is used to denote private schools.  $\alpha_i(b, y)$  is the proportion of students with ability  $b$  and income  $y$  attending school  $i$ . With  $\alpha_i(b, y) \in [0,1] \forall (b, y)$ .

At equilibrium, private schools maximize their profit under a number of constraints:

$$\max \pi = \iint [p_i(b, y)\alpha_i(b, y)f(b, y)dbdy] - V(k_i) - F$$

$f(b, y)$  is the density function of student types.

The constraints include:

- $\alpha_i(b, y) \in [0,1] \forall (b, y)$ .
- $U = U[y - p_i(b, y), a(\theta, b)] \geq \max U[y - p_i(b, y), a(\theta, b)] \forall (b, y)$ .

This constraint means that the utility level in the chosen school should be higher than in any other school.

- $k_i = \iint \alpha_i(b, y)f(b, y)dbdy$

Is the number of students enrolled in school  $i$ .

- $\theta_i = \iint b\alpha_i(b, y)f(b, y)dbdy$

Is average ability in school  $i$ .

At equilibrium, the level of tuition is given by the following equation:

$$p_i^*(b, y, \theta_i) = V'(k_i) + \eta_i(\theta_i - b) \forall (b, y).$$

Individual utility is maximized:

$$U[y - p_i^*, a(\theta_i, b)] = U^*(b, y)$$

$\eta_i$  is the Lagrangian multiplier that results from school profit maximization.  $\eta_i > 0$ .

As it is possible to see, the price paid for private schooling depends on a fixed part  $V'(k_i)$ , which is determined by the number of students attending school  $i$  and on a variable part  $\eta_i(\theta_i - b)$ , which is determined by the position of a student's ability in relation to school average ability. If a student has an above average ability  $\theta_i < b$ , this fraction of tuition is negative and the total price of private education is reduced. The reverse is also true when  $\theta_i > b$ . As mentioned above, students are price-takers. They maximize their utility by choosing the most convenient school, given the price it charges.

One interesting result from the paper of Epple and Romano is the existence of a hierarchy of school quality at equilibrium, such as  $\theta_0 < \theta_1 < \dots < \theta_{n-1} < \theta_n$ . Public schools always have a lower quality than their private counterparts. This is due to the fact that if a public school has a higher quality than a private one, no student will be encouraged to attend the private school, since he can get a higher quality for free in the public sector. As noted by the authors, the allocation of students into schools at equilibrium is Pareto optimal.

In this paper, the two authors solved a number of issues. First, they introduced schools into the analysis of school choice instead of aggregating the school system into a public and a private sector. Second, they were the first to introduce peer effects as a determinant of stratification; and as we will see in the empirical chapters, peer effects have a greater impact on performance scores than expenditure. It should also be said that this paper can be developed along several axes. First, this paper considered a linear in means function of peer effects. In other words, a student whose ability increases average ability by one point has the same effect as several students whose abilities increase average ability by one point. It is of a great importance to be able to include different measures of peer effects other than the mean; this could be the dispersion of abilities. Second, it would be interesting to expand the model to include multiple communities and to introduce spatial stratification into the analysis. Epple et al. (2006) was a variant of this model where a similar formulation was used for higher education.

### **3. Spatial stratification studies.**

In this part of the theoretical review, I will examine spatial stratification models. Geographical stratification results from the functioning of the housing and the education markets. Households' residential and school choices depend on the interaction between budget, ability, social constraints and the different market actors.

Educational stratification models only consider one community in which students are stratified between schools or between sectors. On the other hand, spatial stratification models consider a number of communities differentiated by housing prices and public goods' quality. A more complete approach should consider both types of stratification. In other words, the economy should include both schools and communities.

The nature of spatial stratification is affected by government policies, such as finance decentralization and public school choice. One can imagine that a high level of decentralization (local funding of public schools) paves the way for high levels of disparities between communities. Schools in relatively rich neighborhoods have higher levels of funding and can afford higher quality physical and educational infrastructures as well as higher quality teachers. Furthermore, rich neighborhoods are usually populated by households with high levels of income, social capital and even ability (since ability may be correlated with the former two factors). Hence, the resulting social and ability peer effects will have a positive impact on performance scores. This concentration of resources and peer quality operates in favor of already advantaged students. The reverse is true for poor neighborhoods.

In this context, public school choice is used to break this educational segregation by allowing students to choose schools outside their neighborhoods. If choice was not constrained, then this solution would lead to more equal opportunities in the access to education. However, choice is not costless; households have ability, social capital and income constraints. Households with low income can not afford transport, low social capital households do not have the required information to make the right choice or even to know that a possible choice exists, and low ability students can not pass selection tests if admittance is granted on the basis of academic records. In chapter 2, we will see that public school choice is likely to benefit the richest households in poor communities by allowing them to select a school of a better quality.



The papers that will be included in this investigation are Fernandez & Rogerson (1996), Nechyba (1997) and Epple & Platt (1998). Related but older papers will be cited but will not be discussed.

Fernandez and Rogerson's paper (1996) developed a multicomunity model of the provision of public education. The authors studied the effect of a number of spending policies on public education. They found that any policy that may increase spending in the poorest communities makes these communities more attractive to wealthier households, and that any policy that redistributes income towards the poorest communities will produce chain reactions in which the quality of education increases and tax rates fall in all communities. One of the major characteristics of this model is the presence of one-dimensional stratification according to income. This paper was motivated by the large spending disparities in the USA, which are at the source of inequalities in the quality of education. It was built on the work of Hamilton (1975), Epple & Zelenitz (1981), De Bartolome (1990), Epple, Filimon, & Romer (1984) and Fernandez and Rogerson (1995).

Nechyba's work (1997) develops a fully closed general equilibrium model of hierarchical and local public goods. Individuals are endowed with income and a house; they are mobile between jurisdictions and vote in local and national elections. Multiple communities finance a local public good through property taxes, which are set through majority voting. A national government produces a national public good financed through income tax that is determined through majority voting. Agents are stratified into communities based on their preferences for public goods and their wealth levels. As it is possible to see, this paper of Nechyba is more complex than previous models, since it involves two goods and two voting mechanisms. This paper is built on the work of Epple, Filimon, & Romer (1993), Dunz (1985) and Rose-Ackerman (1979); it is closely related to Tiebout (1956), Wooders (1978, 1980), Ellickson (1979), Greenberg (1977, 1983), Bewley (1981) and Konishi (1996).

Epple and Platt's paper (1998) studied equilibrium in a multicommodity model where households are differentiated by income and preferences. Households are free to move between jurisdictions that offer different tax expenditure packages. The interaction between individual preferences and community characteristics gives rise to a two-dimensional stratification according to income and preferences. The major finding of this paper is that stratification by income no longer holds, and other factors such as preferences should be taken into account. This paper is closely related to the work of Westhoff (1977), Epple, Filimon, & Romer (1984), Henderson (1985) and Dunz (1985, 1989).

### 3.1. Stratification by income in a multicommodity context.

Fernandez and Rogerson considered an economy comprising several communities differentiated by a proportional income tax rate and by the quality of public education. Individuals are free to choose their community of residence and public education is financed by local income tax. The amount that is spent per student in a community determines the community's quality of education and the tax rate is determined by majority voting. This model only accounts for public education; private alternatives were not included. The resulting equilibria are characterized by complete income stratification and by the positive relation between tax rates and public education quality. In other words, a hierarchy of tax rates and qualities will emerge at equilibrium.

The economy comprises  $J$  communities and  $I$  income groups, with  $I > J$ . There is a continuum of two-period lived individuals with identical preferences given by:

$$U(c_1) + \beta U(c_2)$$

$c_1$  is consumption in period 1 and  $c_2$  is consumption in period 2. Individuals differ in their period one income  $y_i$ , which takes one of  $I$  values indexed by  $i = 1, 2, 3, \dots, I$ . The fraction of the population with income  $y_i$  is given by  $\lambda_i$ , which is assumed to be positive:  $\lambda_i > 0 \forall i$ .

Each community  $j$  is characterized by a proportional tax rate  $t_j$  and by the quality of public education  $q_j$ . The authors assumed that the quality of public education is homogenous within

the same community and cannot be supplemented privately. Hence, education is a locally, publicly provided private good.

An individual's period 1 consumption is equal to his after tax income. Period 2 consumption is given by period 2 income, which is determined by the quality of education. More precisely, the authors considered period 2 income to be an increasing, concave and differentiable function( $f$ ) of the quality( $q$ ). Capital markets are absent and individuals cannot borrow against future earnings.

The sequencing of decision can be described as a two-stage game. In the first stage, individuals choose their community of residence. In the second stage, communities choose tax rates through majority voting and individuals consume their after-tax income and obtain education.

The indirect utility of an individual  $i$  who resides in community  $j$  and has an income  $y_j$  is written in the following form:

$$V_i^j = U[(1-t_j)y_i] + \beta U[f(q_j)]$$

And  $q_j$  is given by:  $q_j = t_j \mu_j$

Where  $\mu_j$  is mean income in community  $j$ .

The equilibrium resulting from majority voting and individual utility maximization can be described through the following points:

- As noted previously, tax rates are chosen through majority voting. At equilibrium, the individual with median income is pivotal. In other words, the tax rate chosen by him will be the tax rate of the community.
- At equilibrium, no community is empty.
- Complete stratification by income results at equilibrium. This means that communities will be ranked according to the quality of public education from highest to lowest and according to the income distribution, with the uppermost partition residing in the highest  $t$  community, the next uppermost partition in the next highest  $t$  community and so forth.

This stratified equilibrium can be written in the following form:

$$(q_1^*, t_1^*) \gg (q_2^*, t_2^*) \gg (q_j^*, t_j^*) \text{ and } \min y \in C_j \geq \max y \in C_{j+1}.$$

In the second section of their paper, the authors studied the implications of a number of policies on social welfare. These policies rely on the transfer of the poorest individuals in the highest income community towards the next highest income community and so forth. This transfer leads to an increase in average income in all communities. Hence, tax revenues and local spending on education would be higher. Note that this transfer will be induced by a number of policies, including policies that attempt to affect residence decisions, policies that attempt to change expenditure on education directly and policies that redistribute income and tax revenues.

The major insights that result from the analysis of these policies can be summarized through the following two points:

- Policies that make poor communities more attractive to wealthier individuals are Pareto improving. These policies generate chain reactions that would finally increase quality of education in all communities as well as welfare.
- Some policies are sensitive to the number of communities and to the residence patterns of individuals, while others are robust to these aspects. Policies that impact directly on the individuals living in the poorest communities are found to be welfare improving.

The advantage of this paper is its simplicity. It has provided a simple framework for the study of the provision of public education in a multicomunity context. However, a number of factors were left out and can be included in future analyses. The first consists of introducing private alternatives; this will make the model more complex, since preferences will be non-single peaked and the majority voting equilibrium may not exist. The second consists of introducing schools within each community; these schools can be differentiated according to expenditure as well as peer effects. The third consists of including an active housing market through which housing prices can be endogenously defined. The final development consists of introducing some interactions between national and local policies (this was done in Nechyba 1997).

### **3.2. Stratification in Tiebout economies with majority voting.**

Tiebout economies or Tiebout models were first described by Charles Tiebout in his article “A pure theory of local expenditures” (1956). Tiebout describes different jurisdictions as offering varying packages of publicly provided goods and tax rates. Given that individuals have different preferences for these goods and different incomes (purchasing power), they will be stratified into communities according to these characteristics. Each individual will choose the community that maximizes his utility. At equilibrium, individuals will be sorted into optimum communities and each one of them will have optimum provision of the local public good.

A number of important assumptions were made in the Tiebout model. Individuals are assumed to be free in the choice of their community of residence; they have perfect mobility and perfect information. This means that mobility between jurisdictions is costless and that individuals are informed about the quality of the local public good and the tax rates in each community. The Tiebout model is most useful to describe an urban economy with a number of independent communities. Moving between these different jurisdictions tends to have a very low cost, and the set of possible choices is very large.

The assumptions made in Tiebout (1956) are the following:

1. Individuals are free to choose where to live and mobility is costless.
2. Individuals have access to complete and perfect information.
3. A wide range of choices is available for individuals to choose from.
4. Local public goods do not have any externalities that may affect other communities than the one in which they are provided.
5. Individuals are optimally stratified into communities according to their types and to the characteristics of the chosen community.

A number of papers followed the steps of Tiebout; these include Rose-Ackerman’s “Market models of local government: exit, voting, and the land market” (1979), Dunz’s “Existence of equilibrium with local public goods and houses” (1985) and Epple, Filimon and Romer’s “Existence of voting and housing equilibrium in a system of communities with property taxes” (1993).

Nechyba (1997) built his model on the work of Rose-Ackerman, Dunz and Epple, Filimon and Romer. The major assumptions used in his paper can be summarized through the following points:

- Local public spending and tax rates are determined through voting.
- Agents are endowed with heterogeneous houses of fixed size and the housing stock is fixed. Fixed house sizes imply that the only way for an agent to change his housing consumption is to move.
- Free mobility: agents differentiated by preferences and endowments are freely mobile between communities.
- There exists a national government that produces a national public good financed with income tax. The tax rate is determined through majority voting.
- There exist multiple communities that finance a local public good through property taxes. The tax rate is determined through majority voting.
- Two tax systems coexist: one at the national level (income tax) and the other at the local level (property tax).
- Community boundaries are arbitrarily fixed.
- Intra-jurisdiction migration is allowed.
- Voters are myopic and vote on each issue separately. Under this assumption, individual preferences are single peaked. Black's median voter theorem can be directly applied in local and national elections.

Note that private alternatives are not included in this article. Hence, preferences are single peaked and the median voter is pivotal. This assumption simplifies the model by allowing the direct application of Black's median voter theorem. This paper included two public goods, one national and one local; the local public good can be seen as public education in the absence of private alternatives. However, as mentioned by Stiglitz (1974), education is a publicly provided private good. If Nechyba's paper is to be extended to include education, then private alternatives should be considered.

The author considered the economy to be populated with a continuum of agents. Each agent is endowed with one of  $H$  different types of houses in one of  $M$  different communities. The utility function depends on four arguments: the community of residence, the type of house the

consumer owns, the amount of national and local public goods and the amount of private consumption good (this private good is called income).

The utility functions are continuous, monotone and strictly quasi-concave in the public and private goods. No amount of public goods and no house can compensate consumers for not owning any private good. Consumers are indifferent between communities and house types whenever they consume no private good. Each consumer is willing to relocate if the compensation in terms of the private good is high enough.

Local and national public goods are produced from the private good (income). The national public good is produced according to one technology, while the local public good is produced according to different local technologies. Houses of the same type in the same community have the same price. At equilibrium, different stratification patterns arise according to preferences, wealth and both aspects. An equilibrium is preference-stratified by house type across communities if agents who live in the same house type sort themselves across communities by their equilibrium marginal willingness to pay for amounts of the local public good. Similarly, an equilibrium satisfies preference stratification across communities if agents sort themselves into communities by their equilibrium marginal willingness to pay for discrete amounts of the local public good.

An equilibrium satisfies wealth stratification by house type if agents sort themselves into house types by their after-tax endowments. An equilibrium is wealth-stratified by community if agents sort themselves into communities by their after-tax endowments. Finally, an equilibrium satisfies complete stratification if agents sort themselves into communities by both their equilibrium marginal willingness to pay for discrete amounts of the local public good and their after-tax endowments.

In this paper, the author proved the existence of a stratified equilibrium in a multicomunity framework under a number of conditions. In doing so, he did not use heavily restrictive assumptions on preferences or utility functions. However, this paper did not include private alternatives or peer effects, and hence did not develop the within-community school choice. In chapter 2, a multicomunity model will be developed to take account of school and residential choices when private schools exist and act strategically on the education market.

### 3.3. Two dimensional stratification in urban economies.

A similar paper to the one developed by Nechyba is Epple and Platt's study of urban stratification in a multicomunity framework. In this paper, the authors studied equilibrium and redistribution in a system of local jurisdictions where households differ by both preferences and income. The major assumptions used in their model are similar to the standard assumptions used in Tiebout economies. However, the authors relied on a number of restrictions on the form of preferences in order to insure single crossing was satisfied. Heterogeneity in income and preferences gives rise to an allocation of households across communities in which complete stratification by income does not occur. More precisely, stratification at equilibrium arises according to both income and preferences.

Similarly to Nechyba (1997), the authors studied the provision of public goods in an urban economy without including any private alternatives. This public good can be seen as public education in the absence of private schools. This assumption simplifies the model, since preferences are not single peaked and Black's median voter theorem holds.

In Epple and Platt's model, individuals are differentiated by income and preferences -  $y$  and  $\alpha$ , respectively - with  $y \in (0, \infty)$  and  $\alpha \in (0, \infty)$ . The economy has two consumption goods: housing and a numeraire. The aggregate distribution of consumers is given by the continuous bivariate density function  $f(y, \alpha)$ . The utility function is given by  $U(h, b; \alpha)$ , where  $h$  is the quantity of housing and  $b$  is the numeraire bundle.

There are  $J$  jurisdictions in the economy, and mobility among communities is costless for consumers.<sup>6</sup> Land is homogenous and the amount of land may vary across jurisdictions. The local community pays a lump sum to each household which is funded through property taxes on the value of housing. This lump sum can be seen as a public good, since it is considered to be non-rivalled and non-excludable. Furthermore, we can imagine that this lump sum can be used to fund a public good (public education) instead of being directly redistributed to households. Each community is characterized by a pair of tax rate and grant levels  $(t_j, g_j)$ , with  $j$  being a subscript that denotes a community.

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<sup>6</sup> This is a standard Tiebout assumption.



In this model two cases were studied: in the first, households were renters, and in the second, they were both renters and owners. However, I will limit the discussion here to the first, simpler case.

The budget constraint of a household can be written in the following form:  $y + g = ph + b$ , where  $p$  is the gross of tax price of housing, with  $p = (1 + t)p_h$ .

Maximization of the individual utility function subject to the budget constraint yields the following indirect utility:  $V(p, g; y, \alpha) = U[h(p, y + g; \alpha), y + g - ph(p, y + g; \alpha); \alpha]$ , where  $h(p, y + g; \alpha)$  is the demand for housing by a consumer of type  $(y, \alpha)$ . Tax rates are chosen through majority voting.

The authors made several assumptions on preferences; they include the following:

- Positive income elasticity of housing demand.
- Housing demand is increasing in the preference parameter  $\alpha$ .
- $\lim_{y \rightarrow \infty} h(p, y + g; \alpha) = \infty$ .

Single crossing between indifference curves arises as a result of these assumptions. This single crossing property is a central condition for the existence of equilibrium.

An equilibrium can be defined as a set of communities  $C_j$  with  $j = 1, 2, 3, \dots, J$  and an allocation of individuals across communities such that each individual lives in one and only one community. The conditions for an equilibrium are the following:

- All consumers choose the community of residence in order to maximize their indirect utility  $V(p, g; y, \alpha)$ .
- $(t_j, g_j)$  is a majority voting equilibrium for all communities.
- The housing market clears in all communities.
- The government budget balances in all communities.

The indifference boundary between communities is determined by the equality between indirect utilities in two communities:

$$V[p_j, g_j + \hat{y}, \alpha] = V[p_{j+1}, g_{j+1} + \hat{y}, \alpha]$$

where  $\hat{y}$  is the level of income that marks the indifference. These boundary loci are strictly decreasing for all communities.

At equilibrium, a strict hierarchy of communities according to tax and grant bundles arises. This hierarchy is accompanied by two-dimensional stratification according to income and preferences. A major consequence of this two-dimensional stratification is the absence of complete income sorting. In other words, individuals of high and low income will be living in the same community.

In this article, as in the previous two, the authors studied the provision of a public good in a multicomunity framework while maintaining the absence of private alternatives. Preferences are single peaked and Black's median voter theorem holds. The major finding of Epple and Platt's paper is the existence of two-dimensional stratification according to income and preferences. This finding reflects the reality in most countries, especially the USA, since households with different income levels live in the same neighborhoods.

In the following section of this chapter, the empirical literature on school choice and stratification will be thoroughly analyzed. This review will consider the methodological aspects, the choice of variables and techniques, the underlying hypotheses and the findings of some relevant empirical studies. Note that, in the conclusion, I will sum up all the discussed points in both the theoretical and empirical reviews and I will assess the possible implications for the theoretical and empirical parts of my thesis.

## **B. Review of the empirical literature.**

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The literature on stratification and inequalities in attainments is very dense; it covers a wide range of areas. These areas include the theoretical and empirical literature on peer effects, the literature on immigrants and their schooling, the literature on ethnic minorities and segregation, the literature on the effect of social and economic backgrounds on attainments and the literature on the effects of school resources. In addition to the review of the different areas of the literature, the methodologies used in the analyses will be assessed.

It is possible to identify two categories of papers from the literature on school choice and inequalities in attainments.

The first category consisted of a direct analysis of the determinants of school choice through discrete choice models (e.g. logits and nested logits mainly) or through the study of school enrollment (e.g. the percentage of students in private schools). This category includes Gustman & Pidot (1973), Sonstelie (1979), Gemello & Osman (1984), Kirby & Darling Hammond (1988), Long & Toma (1988), West & Palsson (1988), Lankford & Wyckoff (1992) and Montgomery (2002).

The second category consisted of an assessment of inequalities in attainments through different sets of variables. These variables covered several areas such as peer effects, immigration, school resources, segregation and teacher quality.

The literature on peer effects is very large; old studies include Winkler (1975), Summers & Wolfe (1977) and Henderson et al. (1978) among others. More recent ones include Hoxby (2000), Sacerdote (2001), Boozer & Cacciola (2001), Angrist & Lang (2002), Zimmerman (2003), Winston & Zimmerman (2004), Betts & Zau (2002), Vigdor & Nechyba (2004), Hoxby & Weingarth (2005) and Kremer & Levy (2008).

The literature on immigration and education for immigrants includes Borjas (1994), Smith & Edmonston (1997), Jasso, Rosensweig, & Smith (2000), Smith (2003), Passel, Capps, & Fix (2004) and Borjas (2004). This literature extends beyond education to include immigrant earnings, the accumulation of skills and human capital, assimilation and segregation.

The literature on school resources is very dense; it covers a large number of aspects such as class size, physical infrastructure, educational infrastructure, school accountability and monitoring and education for students with special needs. This literature includes Fuller (1985), Finn, & Achilles (1990), Card & Kruger (1992a & 1992b), Chaikind, Danielson & Brauen (1993), Akerhielm (1995), Betts (1996), Burtless (1996), Heckman, Layne-Farrar & Todd (1996), Hanushek (1997), Case, Deaton (1999), Hanushek (1999), Hoxby (2000 a), Barro & Lee (2001), Wobmann (2001), Boozer & Rouse (2001), Hanushek, Kain & Rivkin (2002), Dustmann & all (2003), Kruger (2003), Hanushek (2003a, 2003b), Burgess, Propper, Slater & Wilson (2005), Hanushek, Kain, O'Brian & Rivkin (2005), Hanushek & Raymond (2005) and Wobmann & West (2006).

The empirical studies on racial and ethnic segregation are mostly concerned with desegregation in the USA after the 1954's *Brown v. Board of Education* decision. This category include the following: Crain (1970), Clotfelter (1976), Armor (1978), Farley, Richards & Wurdock (1980), Taueber, & James (1982), Cook (1984), Welch (1985), Wilson (1985), Armor (1992), Edwards (1993), Rivkin (1994), Clotfelter (1999), Boozer, Krueger & Wolkon (1992), Ainsworth-Darnell & Downey (1998), Hoxby (2000 b), Angrist & Lang (2004) and Guryan (2004).

The literature on teacher quality covers different areas such as teacher certifications and wages, teacher productivity, teacher hiring and evaluation, teacher mobility and allocation and teachers' unions and their effects on school quality. This literature includes Greenberg, & McCall (1974), Antos & Rosen (1975), Murnane (1981), Ballou & Podgursky (1993), Brewer (1996), Hoxby (1996), Flyer & Rosen (1997), Dolton & Van der Klaun (1999), Goldhaber & Brewer (2000), Stinebrickner (2001), Darling-Hammond, Berry & Thoreson (2001), Wayne & Youngs (2003), Ballou, Sanders & Wright (2004), Corcoran, Evans & Schwab (2004), Dee (2004), Hoxby & Leigh (2004), Podgursky, Monroe & Watson (2004), Rockoff (2004), Boyd, Lankford, Loeb & Wyckoff (2005) and Rivkin, Hanushek & Kain (2005).

In the following subsections, I will examine this literature along several axes: the determinants of school choice, peer effects in the empirical literature, school resources and teacher quality, immigration and its implications and, finally, stratification and sorting across schools and jurisdictions. Note that the first axis analyzes school choice directly, while the rest of the axes analyze the determinants of inequalities in attainments. In other terms, in the

first axis the dependent variable is the actual choice between different alternatives or the level of enrollment in a particular school type, while in the rest of the axes the dependent variable is student performance and the independent variables are student and school characteristics.

**Note that the density of the literature prevents a full assessment of particular studies.**

**Nonetheless, the major implications and findings will be discussed.**

## **1. Empirical analyses of the determinants of School choice.**

In this subsection, I will investigate the empirical literature on school choice. This literature covers two main types of analyses. The first one includes the studies that assessed choice between different alternatives through discrete choice models and the second covers the studies that assessed the level of enrollment in a particular type of schools. These two types are different, since the first is concerned with the micro decision of individuals while the second is concerned with aggregated variables such as the percentage of students in private schools.

These studies covered a wide range of variables and determinants of choice, including:

- Individual and household characteristics such as wealth, the level of parental education, religious belonging and residential choice.
- School characteristics such as physical and educational infrastructures, the quality of teachers, teacher student relations, students' motivation and interest in learning, discipline and the religious orientation of the school.

### **1.1. Variables and methodologies in School choice models.**

One of the first studies to analyze public school enrolment was that of Gustman and Pidot (1973). The authors considered a model of two equations; the dependent variables were public spending per pupil and public school enrollment and the independent variables included the cost of producing education (salaries, the cost of infrastructures, etc) along with student and school characteristics (parental education, household's income, the religious affiliation of the school and its racial composition). The model was first estimated with ordinary least squares and then with two-stage least squares.

This model was followed by a series of articles that tried to analyze school enrolment. Sonstelie (1979) analyzed private school enrolment in the Los Angeles metropolitan area. The author retained a number of student and school level variables, such as family size, income, religious orientation of the household, the percentage of minorities in schools, the percentage of Catholics in schools and the level of spending per pupil. Religion was introduced in order to assess whether it has an effect on private school enrollment or not. This model was estimated with simple OLS. Gemello and Osman (1984) used a similar framework to that of Gustman and Pidot by considering two equations; the first had public spending per pupil as the dependent variable and the second had private school enrollment as its dependent variable. The independent variables included income, tuition and a taste parameter. The authors noted that private school enrollment has contradicting effects on public school spending. On one hand, when private enrollment increases, support and funding for the public sector fall. On the other hand, when private enrollment increases, the number of students in the public system falls, and spending per pupil increases, since public schools would no longer be overpopulated.

The paper of Long and Toma (1988) builds on the previously mentioned papers. However, instead of using the number of enrolled students in public or private schools, the author used school choice between three different alternatives as his dependent variable. In other words, the author assessed school choice through a micro analysis of individual choices (multinomial probit). The dependent variable had the value of zero for public schools, of 1 for private religious schools and of 2 for secular private schools. The independent variables included income, parental education, age of parents, ethnicity, home possessions, public expenditure per pupil, the percentage of private schools, tuition in private schools, the percentage of Catholics in the zone of residence, whether the household is in the southern states of the USA or in a city outside the south, whether the household is in an urban or a rural zone and whether the head of the household is a civil servant. Note that parental education was a key independent variable in previous studies, including Coleman & all (1982), Gemello & Osman (1983) and Martinez-Vazquez & Seaman (1985).

In the paper of Kirby & Darling-Hammond (1988), the authors tried to analyze the effects of education subsidies on school choice. These subsidies are used to enable unprivileged households to make better choices. We can imagine that these subsidies will allow households to pay higher tuition or to fund transportation to better, but farther, schools. Some economists have shown clear opposition to this type of subsidies when they are attributed to all

households independently of their wealth. The state of Minnesota was the first to use deduction in income taxes for households whose children are enrolled in private schools.

The authors assumed that households will maximize a utility function under a number of constraints. These constraints include households' characteristics (income, the number of children, religion etc) and schools' characteristics (tuition, geographical location, public subsidies etc). Note that income is considered to be the most important factor in determining the affordable type of schools, while schools' geographical location represents an additional transportation constraint for the household. Public subsidies are used to render private schools affordable by decreasing the level of tuition for unprivileged households. Households' religious orientation was also included, since Catholics are more likely to attend private religious schools than others.

The paper used data for Minnesota that included urban and rural zones as well as public and private schools. The authors used a Logit to analyze choice between public and private alternatives. The dependent variable was school choice; it took the value of 1 when a private school is chosen and 0 otherwise.

West and Palsson (1988) was one of the first papers to introduce sector characteristics into the analysis of school choice. These characteristics cover several aspects, such as the number and length of strikes, the level of administrative costs and the strength of teacher unions. In this model, the two authors tried to include some teacher-related factors that could be substantially different between public and private sectors. Note that teacher-related factors have been previously included in a number of papers, such as Hanushek (1971 & 1986) and Murnane & Phillips (1981).

Similarly to Kirby & Darling-Hammond (1988), West and Palsson used a Logit regression to estimate the probability of attending private schools. The authors retained the following school characteristics as independent variables: the ratio of teacher to pupil, the proportion of non-whites, the proportion of Catholics, average tuition, the proportion of administrative expenses in the total school budget, the percentage of teachers affiliated to the national education association NEA and the length and number of strikes.

Lankford and Wyckoff (1992) used a random utility model to estimate the probability of attending a private school. They considered that individual utilities depend on two vectors of variables and on an error term. These vectors included the following:

- School characteristics: these include the quality of education, tuition, ability peer effects, the quality of physical and educational infrastructures, the percentage on non-whites, teacher/student ratio, the percentage of lay teachers and discipline.
- Individual and household characteristics: parental education, income, religious affiliation and residential location (rural or urban).

Perhaps the most involved analysis among the discrete choice models is that of Montgomery (2002). The author used a nested Logit in order to explain the choice of graduate business schools across the USA. The model included two levels: the first considered a wide range of alternative business schools and the second considered full- and part-time schooling. The purpose of this paper was to further the simple and Multinomial Logits by considering a large number of alternatives that exhibit certain nesting features. Even though my thesis is only concerned with school choice and stratification in secondary education, this paper presents an overview on how modern discrete choice models can be used.

The independent variables used in this paper included the following vectors of variables:

- School characteristics: average tuition, the percentage of students receiving financial aid, average starting salary upon graduation, school type (public or private) and whether the school is in the same region as residence.
- Student characteristics: age, gender, ethnicity (Black, White, Hispanic, Asian), average score on GMAT, marital status and work experience.



## **1.2. Major findings from school choice models.**

In this subsection, I will examine the major findings in the aforementioned empirical papers. This examination will be followed by a comparison and a number of conclusions will be drawn. Note that the findings from the different articles tend to be similar.

In the model of Gustman and Pidot (1973), the OLS and 2SLS methods generated the same results. Public spending on education is found to be very sensitive to public school enrolment. In other terms, when the number of students attending public schools increases, public funding increases substantially. Household income has a positive and significant effect on the demand for private education. The density of the population has a modest negative effect on public spending. The percentage of Catholics has a negative effect on public school enrolment, since Catholics are more inclined to choose private religious schools. And finally, the number of children has a positive and important effect on public school enrolment.

Similar results have been found by Sonstelie (1979). Spending on public education has a negative and significant effect on private enrolment while household income has an important positive effect on it. Furthermore, the number of children has a substantial negative impact on private school enrolment. Similarly to Gustman and Pidot (1973), the percentage of Catholics in a community increases private school enrolment, except among Hispanics.

The model of Gemello and Osman (1984) is very similar to Sonstelie's paper. The dependent variable in the model was private school enrolment and the findings are very close. Income has a significant positive effect on private school enrolment while the number of children in a family and the percentage of minorities in schools have very negative effects. Public expenditure per pupil has a negative effect on private school enrolment and parental education and being Catholic have positive effects. The most interesting variable included in this model is the size of the school district. In large school districts, parents are more inclined to send their children to private schools. The author explains this fact through the illusion of higher student density or lower spending per pupil in public schools in large districts.

As mentioned earlier, the paper of Long and Toma (1988) studied school choice through a multinomial Logit model where the dependent variable can take three values: zero for public schools, 1 for private religious schools and 2 for secular private schools. Income is found to have a positive effect on the probability of choosing a private school, independent of its religious orientation. White households have higher probabilities of choosing private schools than their non-white counterparts.

The age of the head of the household as well as his education have positive effects on the probability of choosing private schools, but his gender is found to have an insignificant effect. Furthermore, if the head of household is a civil servant, he is more likely to choose public alternatives. Public expenditure per pupil and private tuition have negative effects on the probability of choosing private schools. Similarly to previous papers, Catholics are more likely to choose private religious schools than others.

In Kirby and Darling-Hammond (1988), the authors tried to analyze the effect of public subsidies on school choice. The results are very similar to those from previous studies. On one hand, income has a positive and significant effect on private school enrolment while tuition has a negative effect. On the other hand, the percentage of Catholics is positively and significantly correlated with private school enrolment. It should be noted that when religious affiliation was added to the equation, the effect of tuition became insignificant.

In West & Palsson (1988), the school choice model was estimated using a simple Logit regression with a binary dependent variable taking the value of 1 for private schools and zero otherwise. The most innovative aspect in this paper is the inclusion of factors controlling for the functioning of the education system, such as the percentage of administrative costs in total cost, the number and length of strikes, etc. The teacher/pupil ratio was found to have a positive and significant effect on the probability of choosing private schools. This can be explained through the fact that public schools are more subject to congestion. The percentage of non-whites in the population has an insignificant effect on the probability of choosing private schools, while the proportion of Catholics has a positive and significant effect.

The length and number of strikes has a positive and significant impact on private school choice. This is due to the fact that public schools are subject to longer and more frequent strikes. The most interesting result is perhaps the one on administrative expenses. A higher

level of administrative costs in the public sector decreases the probability of choosing private schools. The authors attributed this finding to the fact that large public administrations can exercise political pressure on policy makers in order to defend public schooling against an expansion of the private system.

In Lankford & Wyckoff (1992), the authors estimated their model using a Logit specification similar to that of Long and Toma (1988). They found that student/teacher ratio has an insignificant effect on the choice of public and religious schools. Average income in a school has a negative effect on its choice, since households would not be willing to send their children to schools where they are the poorest. However, households' income has a positive and significant effect on the choice of private schools. The proportion of lay teachers has a positive effect on the choice of private schools as well as the proportion of Catholics in the population.

The paper of Montgomery (2002) extended the literature on school choice by examining the factors influencing the selection of graduate business schools. It was the first paper to use nested Logits in the analysis of choice between graduate business schools and between full- and part-time attendance. The highly significant coefficient of the inclusive value in the attendance equation demonstrated the importance of nesting the two decisions.

Several interesting findings emerged from this paper. For example, demand for a given school appears to be fairly inelastic with respect to tuition. Students apparently choose a business school mainly on grounds other than cost. On the other hand, location is a powerful determinant of school choice; even as adults, students are reluctant to go to a school outside their geographic region of residence. Finally, the author found that black and Hispanic students were much more likely to attend a prestigious institution than were other students with similar credentials. This was not merely due to Affirmative Action in admission; in fact minority students had stronger preferences for prestigious schools. This may reflect a perceived need by minorities for a stronger educational signal to overcome statistical discrimination.

All these studies have participated in the development of school choice analysis. However, some questions remain unanswered. In the following discussion, I will present the major issues related to school choice studies and some of the implications for my thesis.

Firstly, school choice analyses are designed to assess how student and school characteristics affect the choice of particular schooling alternatives. However, the choice between different types of schools cannot be considered a measure of inequalities. For instance, in the UK, private schools have higher average performances than their public counterparts. Conversely, in Finland, private schools have lower performances and are considered to be the last resort for students who can not be integrated in the public system. Hence, school choice can not be considered a form of inequality in itself, even if it represents a form of stratification. In order to assess educational inequalities, one has to consider performance scores along with student and school characteristics. More precisely, student and school characteristics should explain the variance on individual performance scores in order to explain inequalities. In this context, the most effective methods consist of the estimation of education production functions using multilevel analyses, since the latter acknowledge the nesting of students within schools and permit the inclusion of student and school characteristics. These techniques are thoroughly discussed in chapter 4.

Secondly, most of the previous analyses did not consider peer effects as part of school characteristics. Note that the existence of ability peer effects was proven in numerous papers, such as Summers & Wolfe (1977) and Henderson et al (1978). Peer effects can also be very diverse; they might include ability peer effects, social peer effects and psychological peer effects related to the perception of the school environment. We can expect that such peer effects might be at the centre of the choice of a school, since they reflect its peer composition.

Thirdly, some of the previously mentioned papers have aggregated school choice by using the percentage of students attending a particular type of schools as the dependent variable. This aggregated percentage may explain the impact of particular policies or system characteristics, but it can not explain individual choices. In order to study the determinants of school choice, one has to use micro-level data and discrete choice models such as simple, multinomial, nested and mixed Logits and Probits.

## **2. Peer effects in the Empirical literature.**

Peer effects arise from the composition of the student population in a school. A student belongs to a group; he influences the group and is influenced by it. Peer effects can be thought of as different forms of spillovers and externalities. Students can teach one another; highly performing students can affect their peers through their influence on academic and disciplinary standards. Peer effects can take multiple forms such as ability peer effects, disciplinary peer effects, social, racial, gender and economic peer effects, and even psychological peer effects, related to the levels of motivation and interest in learning. For instance, students with low levels of motivation may negatively influence the ability of others to learn. Racial or gender tensions may interfere with learning and may require a particular intervention from teachers.

The existence of ability peer effects has been proven in several studies, such as: Summers and Wolfe (1977), Henderson et al (1978), Sorensen and Hallinan (1986), Kulik (1992) and Epple and Romano (1998). Ability peer effects have become a major challenge for educational policies. If such effects do exist, a system with an efficient distribution of peers will make the society's investment in education more productive and more equitable. Conversely, a heavily stratified system will certainly lead to inequalities and inefficiencies in the distribution of attainments.

Educational policies have been largely influenced by the idea of an efficient distribution of peers. Tracking policies in the USA, through which students are exposed to peers with similar achievements in all subjects, or setting policies in the UK, through which students are exposed to peers with similar achievements in a limited number of subjects, are at the center of the debate. Even racial desegregation policies can be seen as a quest for an efficient distribution of social and economic peers. As mentioned above, the literature on peer effects is very wide; it includes the following papers, among others: Summers & Wolfe (1977), Henderson et al (1978), Hoxby (2000), Sacerdote (2001), Boozer & Cacciola (2001), Angrist & Lang (2002), Zimmerman (2003), Kremer & Levy (2003), Winston & Zimmerman (2004), Betts & Zau (2004), Vigdor & Nechyba (2004) and Hoxby & Weingarth (2005).

Due to the density of this literature and to the limited space available in this chapter, it is not possible to go through these papers individually. Instead, I will examine a number of issues in the use of peer effects and in the estimation of their impact on performance scores.

## **2.1. Models of peer effects.**

The baseline model described in Hoxby (2000b, p.3) was the most popular model used by researchers to estimate peer effects. The major assumption behind this model was the linearity in means. The linear in means model assumes that each student has the same effect on each other student; that is, in order to give a student a better peer, that peer has to be taken from another student. According to the baseline model, a student's performance in mathematics is affected linearly by the average performance of his class or his school, which means that a single student whose performance raises average performance by one point has the same effect as several students whose performances raise average performance by one point.

Under the baseline model, regardless of how peer effects are allocated, average societal achievement remains the same. Hence, if linearity in means holds, peer effects would have no impact on the global level of achievements, since these effects cancel out. However, many arguments assume that global performance can be enhanced through a redistribution of peers. For instance, the main argument against tracking policies is based on the fact that global performance can be enhanced when high and low achievers are exposed to one another. On the other hand, the argument behind programs for gifted students is based on the fact that high achievers benefit from being among one another. The most appealing feature of the baseline model was its convenience and simplicity: it can be estimated even when the variation in the data is barely sufficient (see Hoxby and Weingarth 2005). In conclusion, it is possible to say that the baseline model can not describe adequately the functioning of peer effects. Furthermore, most applications of peer effects should consider non-linear specifications in order to reflect the reality.

In their paper, Hoxby and Weingarth (2005) examined a number of alternative peer effects models that are defined on the basis of behavior, as opposed to the specification of an equation. The most well-known ones include the following:

The bad apple model of peer effects assumes that the presence of a single student with low performance scores or discipline problems spoils the schooling environment for many other students. Empirically, the plausibility of the bad apple model can be tested through an assessment of the impact of low-achieving students on the rest of the student population. If this impact is found to be disproportionate in comparison with the baseline model, then it is possible to conclude that the bad apple model holds.

The shining light model is the opposite of the bad apple model. It assumes that a single high achiever or disciplined student can inspire his peers to raise their performances. Empirically, if the highest achievers are found to have a disproportionate effect on the rest of the student population, then it is possible to conclude that the shining light model holds.

Another related, but opposing, behavioral model is the invidious comparison model, where the advent of a high achiever depresses the performance of his peers, presumably by depressing their self esteem. By contrast, the advent of a low achiever boosts the performance of his peers.

The boutique model of peer effects assumes that a student benefits from being surrounded by peers with similar characteristics. In other words, this is a model in which students do best when the environment cater to their particular needs. This is the case of tracked programs and classes for gifted and talented students, where teachers organize lessons and instructional materials according to student type.

The focus model is very similar to the boutique model. It assumes that diversity in a classroom is inherently disabling, since instruction can not be well-targeted. In other words, peer homogeneity is good for student's learning, since the environment is designed to cater for their needs.

The opposite of the focus model is the rainbow model. It assumes that all students are better off when they are exposed to students with different characteristics than their own. According to this model, diversity in a school or a class allows students to learn different approaches in solving the same problems.

The subculture model concerns minorities such as high or low achievers or racial minorities. It assumes that the majority will remain supportive of a minority student, such as a low achiever or a black student, as long as he is relatively isolated. However, when the proportion of minorities becomes sufficiently high, they tend to be rejected by majority students, because minority subculture threatens the environment that is designed according to the needs of the majority.

## **2.2. The workings of peer effects.**

As I have previously said, peer effects exist when the behavior and performance of a student are affected by those of his peers. Usually, peer effects are included as an argument of education production functions, where the output is performance scores and the inputs are student, school and peer characteristics. Peer effects can be represented through the following function:  $B_1 = f(B_i, C_i)$ , where  $B_1$  is behavior of student 1, which is a function of the behavior  $B_i$  and characteristics  $C_i$  of other students in his school. Peer effects exist if the partial derivative of this function is different than zero.

Note that peer effects represent an externality for a school. If they are not taken into account, some anomalies will arise, since the marginal cost of admitting a student will not internalize the effect of such externality. Furthermore, peer effects are a major factor of stratification between schools, since they may explain the observed segmentation of peer quality and tuition (see Epple and Romano 1998 & 2006).

Note that education operates like a normal industry that makes a product using a number of inputs that are transformed and sold to students for a price in a quite competitive market. However, some important differences remain:

1. The price of education is not only determined by the quality of the produced output or by the cost of inputs. This price is also determined by the characteristics of the consumer (the student). In reality, this is reflected in scholarships for highly performing students. These scholarships are designed to compensate the positive peer effects that such students generate. Performance-based scholarships may also be considered as a form of income redistribution.



2. School quality is not only measured by the quality of inputs or by the quality of outputs; it is also measured by the aggregate quality of customers. In fact, ability and economic peer effects have been considered to be a determinant of school quality in a number of analyses, such as Epple and Romano (1998 & 2006).
3. Some students may be turned away due to their type or to the peer effects they generate.

These anomalies in the production of education disappear, and education becomes a coherent industry, once peer effects have been controlled for.

In conclusion, a number of implications arise from this discussion. The inclusion of peer effects in any theoretical or empirical model is essential to capture ability, social and income spillovers. If peer effects were linear in means, any re-sorting of students would have no effect on the average performance in a country and, consequently, peer effects would no longer be a major factor of educational policy. Therefore, any specification of peer effects should allow for nonlinearities to be considered. It should be noted that the different aforementioned models of peer effects imply certain forms of nonlinearity. Non-linear peer effects can be captured through the inclusion of different moments of the same variable, such as the average and the dispersion. If the dispersion of students is negatively related to performance scores, it would be possible to say that more homogeneity is beneficial, while the reverse is true for a positive relation; see Schapiro et al. (1991). These points will be thoroughly discussed in chapter 2 and 4.

### **3. School resources in the Empirical literature.**

School resources are considered to be one of the major factors in any education production function. Parents assess a school by its resources and authorities are interested in how resources relate to performance scores (Hanushek & Welch 2006). School resources are also at the center of stratification, since finance decentralization may be the source of disparities in funding and in attainments. In this section of the review of the literature, I will examine the state of knowledge on this subject and the different approaches to the estimation of the impact of resources on educational achievements.

School resources are considered as a major instrument of education policy. They can be directly controlled by public authorities in school systems where funding is mostly publicly provided; see Wobmann (2001, 2003a, 2003b). However, the major challenge is to understand how school resources relate to achievements and how funding can be optimally allocated.

As mentioned above, the literature on school resources is very dense and covers a large number of sub-domains such as class size, teacher quality, the quality of educational and physical infrastructures and education for students with special needs. Hanushek & Welch (2006) noted that two particular subjects are of interest to the study of school resources. The first considers the relation between school resources and performance scores, and the second considers the relation between performance scores and earning prospects. These two subjects relate educational outcomes to labor market outcomes in a lifelong learning framework. In what follows, the investigation is limited to the first subject.

### **3.1. School resources and educational outcomes.**

School resources have been included in a large number of studies in order to assess their impact on performance scores. An important challenge in such analyses is the ability to separate the effect of school resources from those of other determinants of achievements. This dilemma was obvious in a number of studies; Hanushek and Kimko (2000) found a negative relation between spending and performance scores, Wobmann (2001, 2003b) found similar results from TIMSS 1995. This problem was also very apparent in the famous “*Coleman report*” (1966), where school characteristics were found to have limited effects on performance scores.

The Coleman report motivated an in-depth investigation of the education production process. Various conceptual frameworks currently exist; Hanushek (1979, 1986, 2003b), Betts (1996) and Todd & Wolpin (2003) provided an interesting summary of the studies in this domain. Here, the main issues related to the estimation of education production functions are discussed.

The basic education production function can be written in the following form:

$$y_{it} = f(F_i^{(t)}, P_i^{(t)}, S_i^{(t)}, A_i) + \varepsilon_{it}$$

where  $y_{it}$  are performance scores of student  $i$  at an instant  $t$ ,  $F_i^{(t)}$  is the cumulative family characteristics to time  $t$ ,  $P_i^{(t)}$  is the cumulative peer effects to time  $t$ ,  $S_i^{(t)}$  is the cumulative school characteristics to time  $t$ ,  $A_i$  is innate ability and, finally,  $\varepsilon_{it}$  is an error term.

This structure motivated a wide and dense empirical literature. The typical empirical study tries to collect data on the various inputs of the education production function and to assess their impact on educational outcomes using econometric methods. Hanushek & Welch (2006) pointed out that two aspects are very important in the understanding of such structure. First, school resources account for only one type of inputs; other inputs include peer effects and family and individual characteristics. These dimensions have to be controlled for in order to avoid the omitted variable bias. Second, all inputs are cumulative over time. In other words, they summarize the environment of a student until the time of assessment.

The major challenge that arises from these two aspects is the availability of data. In general, aggregated institutional data on schools are widely available from administrative and governmental sources. However, individual data on students' backgrounds and environment are scarcer, especially when cumulative data over time are needed. On the other hand, recent international assessments of individual outcomes, such as PISA, PEARLS and TIMSS, provide a rich source of micro-data suitable for international comparisons; these sources account for student and school characteristics even if they are not cumulative over time.

Hanushek & Welch (2006, p.886) indicated that aggregate student scores for a school are not a suitable measure of its quality. This problem becomes more obvious in econometric analyses, since aggregate scores deny the existence of within-school variations. The solution for this problem resides in the use of individual data that accounts for individual, household and school characteristics. The more challenging aspect is the cumulative nature of achievements, where the learning in any time period builds on previous learning experiences. This places heavy constraints on data requirements, especially when historical information is unavailable.

The second challenge is related to the presence of selection bias in the choice of schools. In fact, families exert preferences for the school that their children attend. In this context, households' social and economic characteristics are likely to influence the choice of the school and hence its characteristics. However, this problem can be solved by controlling for a wide range of individual or nonschool characteristics (Hanushek & Welch 2006, p.887).

A solution for the need for cumulative data is the use of a value-added formulation of the education production function. In this case, the equation can be written in the following form:

$$y_{it} - y_{it^*} = f(F_i^{t-t^*}, P_i^{t-t^*}, S_i^{t-t^*}) + (\varepsilon_{it} - \varepsilon_{it^*})$$

This equation relates the changes in inputs over period  $(t - t^*)$  to the changes in outputs over the same period. This formulation of the education production function lessens the need for historical information and eliminates everything that is constant over time, such as innate ability.

As indicated by Hanushek (1979), it is also possible to move the lagged part of the outcome  $y_{it^*}$  to the right-hand side of the equation. This technique has the advantage of allowing for different scales of measurement in achievement during different years. However, it has the disadvantages of introducing measurement errors on the right-hand side and of complicating the error structure.

Note that in this formulation of the education production function, school characteristics are specific to the individual. In other terms, school characteristics account for the institutional environment a student has experienced over time. The focus of this production function is to relate the variations in achievements to the variations in inputs over a predetermined period of time.

Another approach consists of relating the variations in achievements to the variations in inputs across a number of schools or countries. In this case, the production function can be written in the following form:

$$y_{ij} = f(F_{ij}, P_j, S_j, A_i) + \varepsilon_{ij}$$

where  $y_{ij}$  is performance of student  $i$  in school  $j$ ,  $F_{ij}$  is family characteristics of student  $i$  and  $P_j$  and  $S_j$  are peer effects and school characteristics of school  $j$ .

Note that this formulation eliminates the need for historical information and has the advantage of being more adapted to international databases, such as PISA. The two formulations are very different in their conceptual frameworks. The first analyzes the change in outcomes over time, while the second analyzes the variations in students' performances across schools and students during the same period. In other words, the first works vertically while the second is horizontal. The second formulation is the one used in chapter IV, since it is more suitable for the PISA multilevel data and is more convenient for the analysis of between-student inequalities (it relates disparities in inputs to disparities in achievements).

### **3.2. Teacher quality.**

Another aspect directly related to school resources is teacher quality. Teachers play a fundamental role at school and usually represent the largest budgetary element for governments. Their impact on school quality motivated a wide range of studies that considered teacher salaries, their distribution across schools and their impact on student performance. Moreover, the distribution of teachers across schools can be seen as a source of inequality, since the sorting of teachers usually matches the sorting of students. In fact, schools with higher levels of funding can attract better teachers by offering higher salaries and can provide better educational and physical infrastructures. This higher quality attracts socially and economically advantaged students, and hence leads to better peer quality.

A wide literature studies the effects of salaries on teacher qualifications, teacher flow in and out of the teaching profession and teacher distribution across schools. As indicated by Hanushek and Welch (2006 p.1056), this literature has three major dimensions. The first analyzes the relationship between a particular teacher characteristic on the one hand, (e.g. experience or qualification) and pay, benefits and work conditions on the other. The second analyzes the determinants of teacher transitions and the third analyzes the impact of teacher characteristics on student performance.

In the literature, four teacher characteristics have received much attention; they include the following: experience, specialty, education and skills and certification. Evidence from different studies suggests that experience is positively and significantly related to salaries, but negatively related to the probability of transition, since transitions happen early in the career;

see Hanushek, Kain and Rivkin (2003). Some studies went even further to show that salaries and earning prospects have different impacts on the probability of exiting the teaching profession, depending on experience, e.g., Murnane and Olsen (1989, 1990), Murnane et al (1991), Brewer (1996) and Stinebrickner (2001a & 2001b). Some studies have investigated the impact of measured teacher skills on salaries e.g., Hanushek and Pace (1995) and Podgursky, Monroe and Watson (2004).

Another body of literature examined the changing conditions of the teaching profession over time; see Bacolod (2003) and Corcoran, Evans and Schwab (2004). Furthermore, the impact of salaries and uniform pay on the choice of specialties has also been assessed. For instance, Rumberger (1987) found that uniform pay causes shortages in specialties with high requirements (math in particular). Finally, a last body of research tries to assess the relation between school characteristics and their ability to recruit high quality teachers. This literature reflects teacher stratification concerns.

In my thesis, I am only concerned with the strength of the relation between teacher and school characteristics on the one hand and individual performance scores on the other. An insignificant relation indicates that school characteristics do not affect achievements; or, in other words, the institutional environment is not the source of disparities in performance scores. Furthermore, in this case, individual and household characteristics may be to blame. When individual, household and school characteristics do not have any effect on performance scores, the system can be described as meritocratic, since the only variables that could explain differences in achievements are the ones hiding in the error term, such as innate ability.

The education production function with teacher quality is very similar to the production function discussed in the previous subsection. The production function can be written in the following form:

$$y_{it} = f(F_i^{(t)}, P_i^{(t)}, S_i^{(t)}, T_i^{(t)}, A_i) + \varepsilon_{it}$$

where  $T_i^t$  are teacher characteristics specific to student  $i$  over time,  $F_i^{(t)}$  is the cumulative family characteristics to time  $t$ ,  $P_i^{(t)}$  is the cumulative peer effects to time  $t$ ,  $S_i^{(t)}$  is the cumulative school characteristics to time  $t$ ,  $A_i$  is innate ability,  $y_{it}$  are performance scores of student  $i$  at an instant  $t$  and, finally,  $\varepsilon_{it}$  is an error term.

As mentioned above, the cumulative nature of the education production function places heavy constraints on the needed data. Moreover, the relation that may exist between the characteristics of students and those of their school may generate a selection bias. Similarly, teacher choice of schools can also complicate the estimation of teacher effects. The solution for these two problems is to control for a large array of individual, family, school and teacher characteristics. The problem of selection bias becomes concerning when essential school and teacher characteristics are dropped, or are unaccounted for, because of the unavailability of data. These characteristics are absorbed by the error term and, when they are correlated with one of the included individual variables, such as students' social and economic status, endogeneity problems may arise. Similarly to school resources, this education production function can be transformed into a value-added function where the need for historical data is limited. For a discussion of the estimation approach, see Hanushek (1979), Todd and Wolpin (2003) and Rivkin, Hanushek and Kain (2005).

Another approach consists of relating the variations in achievements to the variations in inputs across a number of schools or countries. In this case, the production function can be written in the following form:

$$y_{ij} = f(F_{ij}, P_j, S_j, T_j, A_i) + \varepsilon_j$$

The interpretation from the previous section applies here. Note that most studies use administrative or survey data in which teacher characteristics are limited to education, experience, salary and certification.

#### **4. Immigration and educational performance.**

The education of immigrants represents a formidable challenge for public authorities in most developed countries. The amount of skills that immigrants are able to accumulate is essential to their integration, to their earning prospects and to their overall success. Furthermore, the impact of immigrant education extends to include intergenerational social and economic mobility, since today's immigrants are the parents and grandparents of tomorrow's labor force; see Smith (2006).

In most developed countries, immigrants have lower achievement levels and earning prospects than their native counterparts. Moreover, the integration of immigrants creates tensions, especially when natives are displaced in the education system and on the labor market. These tensions become more concerning because of the ethnic and religious diversity that immigration might imply. For instance, in a large number of cases, social and economic stratification coincides with ethnic segregation.

In addition to this, high levels of heterogeneity may appear in the distribution of achievements between different ethnic migrant groups. For instance, on average, migrants to the USA have less schooling than native-born Americans. However, when the migrant population is decomposed into different ethnic groups, one may find that recent European and Asian migrants are more educated than native-born Americans and have relatively higher skills, while Hispanics are much less skilled than native-born Americans and other migrants.

In Europe, the situation is similar to that in the USA. Many international studies, such as the OECD's *Program for International Student Assessment (PISA)*, have shown that despite the comprehensiveness of school systems, the educational trajectories of immigrants and first-generation students still deviate from those of native students.

In the following two subsections, I will examine the different possible techniques for the estimation of the effects of migration on individual performance and educational attainments, and I will present some findings from the literature on the USA and Europe.<sup>7</sup>

#### **4.1. Estimating the effect of immigrant background on achievements.**

The estimation of the impact of immigrant background is similar to the estimation of the effect of any other individual characteristic. However, immigrant background is not a continuous variable. Therefore, it is to be included as a dichotomous variable when the distinction is between natives and immigrants, or as a categorical (polytomous) variable when the distinction is between natives, first generation immigrants and recent immigrants.

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<sup>7</sup> Even though the USA is not included in the empirical analyses in chapter 3 and 4, it is still relevant to investigate the US literature on immigration since it can give some indications on the possible effects of immigrant background.



Immigrant background can be directly included in the cumulative education production function. The production function is written in the following form:

$$y_{it} = f(F_i^{(t)}, P_i^{(t)}, S_i^{(t)}, T_i^{(t)}, I_i, A_i) + \varepsilon_{it}$$

where  $I_i$  is immigrant background for individual  $i$  which is time invariant, a value-added formulation of this equation means that immigrant background will automatically drop. Hence this formulation is not the most suitable for the estimation of immigrant background effect.

The second possible formulation tries to relate the variations in performance scores between students to the differences in their origins. This formulation is not time cumulative and facilitates the estimation of such effects. The education production function can be written in the following form:

$$y_{ij} = f(F_{ij}, P_j, S_j, T_j, I_i, A_i) + \varepsilon_j$$

As mentioned above, it is essential to control for a large array of individual and school characteristics to avoid a selection bias in the estimation of the equation. If these variables are not controlled for, they will be absorbed by the error term and may therefore be correlated with the included variables. This unwanted correlation can be the source of endogeneity in the estimation of the effects.

It should be noted that interactions between immigrant background and other variables can be included in the specification of the equation. These variables may include social and economic status and the level of motivation and interest in learning. The interactions indicate that social status may not necessarily work the same way for all immigrant groups. It is also possible to include origins peer effects, such as the percentage of foreign-born students in a school.

## **4.2. Immigration and achievement gaps in the USA and Europe.**

In the USA, the nature of immigration started to change after the 1965 abolition of the national-origin quotas. Before that, about 90% of the foreign-born population was either European or Canadian. However, the situation started to change during the 1970s. More than 75% of the new immigrants were Hispanic or Asian. In 1990, the fraction of foreign-born immigrants from Europe and Canada over the total immigrant population was about 25%. In addition to this, the immigration and reform control act of 1986 created a special program for

the legalization of the situation of about three million illegal residents in the USA; see Smith and Edmonston (1997).

Smith (2006) noted that attainments increased for all categories of the population since the 1940s. The cumulative change in education levels for the foreign-born population was greater than that for native-born Americans, and the gap between the two categories was narrowing down. However, ethnic differences in education remain large. For instance, Hispanics have always lagged behind Asian and European immigrants, and within each ethnic category the native-born tend to have more schooling than their foreign-born counterparts.

In terms of gender differences, the overall levels of education show a gender-specific disadvantage among female immigrants. For instance, in 1970 the average male migrant had 9.2 years of schooling, while females had about 7.6 years, (see Smith 2006). However, the educational advantage that male immigrants had has been eroding over time. In 2002, recent female migrants had slightly higher levels of education than their male counterparts.

In conclusion, two major trends have dominated the last three decades in the USA. The first is the increased flow of mainly Hispanic immigrants who have less schooling than the average native-born American, and the second is the increased number of Asian and European immigrants who have education above the average native-born American.

Intergenerational mobility of human capital (or the lack of it) is an important by-product of the educational disparities that arise between immigrants and the native-born population. In the USA, the different waves of European immigrants were able to successfully integrate into the US education system and economy. However, the integration of Hispanic immigrants has been more problematic. In fact, the levels of education rise by about three years between the first and second generations of Hispanic immigrants, and then drop for the third generation; see Smith (2003).

In Europe, the situation is not very different than in the USA. Since the 1960s, Germany, France, Austria, Denmark, Norway, Sweden, the UK, Spain and Switzerland witnessed a large increase in their immigrant populations. In Belgium, France and The Netherlands, the immigrants were mainly from ex-colonies and in general they spoke the country's language.

Another wave of immigration happened after the fall of the Soviet Union, especially in Denmark and Germany.

The early waves of immigrants were mainly foreign workers with lower levels of education than the native-born population. However, since the 1990s, the level of education of the immigrant population increased dramatically in countries like France, Belgium, Ireland and Sweden. An interesting comparison of the actual situation of immigrants across the European countries was made in the “*Where immigrant students succeed: a comparative review of performance and engagement in PISA 2003*” (Stanat et al, 2006). The use of OECD PISA data makes the comparison easier, since the data and the results are generated using the same procedures.

The results showed that 40% of the first-generation immigrants in Belgium, France, Norway and Sweden have performance scores in mathematics lower than the second level on the PISA scale (the scale has 6 levels). This proportion was 25% in Austria, Denmark, Germany, Luxembourg, The Netherlands and Switzerland. And in all these countries immigrants have lower performance scores than their native-born counterparts.

When the language used at home is accounted for, the differences in performance scores between students who speak the language of their immigration country and those who do not becomes more concerning. It should be noted that, in general, language problems affect the recent immigrants more than the first-generation ones.

The stratification of students into general and vocational schools is also a matter of concern, especially in the German-speaking countries where selection is operated at a very early stage. In Austria, only 9% of the immigrant students are in the general education schools, 20% of them are in schools for special needs and the rest are in vocational schools. The situation is similar in Flemish Belgium, Germany, The Netherlands, even if it is less dramatic. Luciak (2004).

It should also be noted that immigrant status affects many dimensions other than educational performance. These dimensions may include the sense of belonging, identity, social cohesion, motivation, attitudes, ambitions and future social mobility. These issues will not be discussed in this section, due to the lack of space and since they are outside the scope of the analysis.

## **5. Educational and spatial Stratification in the empirical literature.**

As mentioned previously in the theoretical literature, stratification arises from the interaction of students and schools on the education market, the housing market and from the legislation on school choice. I have decided to include the section on stratification at the end of the empirical literature, since stratification is found to be at the intersection of the aforementioned domains: school choice, peer effects, school and teacher quality and student characteristics.

Residential choice is determined by housing prices and by the functioning of the housing market. In most cases, it results in incomplete sorting of households into communities according to wealth, social status and perhaps ability; see Epple and Platt. (1998). Public school quality differs widely between communities, due to peer quality differences and to disparities in resources and inputs, especially in the case of decentralized funding; see Hoxby (2000 & 2001) and Murray, Evans and Schwab (1998). Therefore, the choice of a community of residence has strong implications for school and peer quality. Note that residential choice is subject to income and to social constraints, and that it usually operates along wealth and social class lines.

Moreover, choice is not limited to public schools. In most school systems, private alternatives coexist. Such schools include religious as well as less common secular schools. In addition to this, a large proportion of schools exercise some forms of tracking or setting where students are grouped according to ability.

In conclusion, it is possible to say that the differences in inputs and in peer quality in most countries indicate that the study of stratification is empirically relevant and represents a prime candidate in explaining disparities in achievements; see Nechyba (2006, p. 1329). In the following subsections, I will examine the literature on stratification while emphasizing the different empirical approaches to its analyses.

## **5.1. Different forms of stratification.**

As mentioned in Nechyba (2006), three different types of stratification exist. First, housing markets, local public finance and family characteristics induce residential segregation and self-selection into schools. Second, in the presence of private schools, tuition combined with households' financial constraints may encourage some families to opt out of public schooling and to choose private schools. Third, stratification can be induced by public school policies such as zoning, busing, school choice and the diversification of public school supply.

In the review of the theoretical literature, I have examined the different models of educational and residential stratification. In this subsection, I will summarize the different stratification mechanisms and I will examine the empirical approaches to the analyses of stratification and choice.

The early stratification models, such as Westhoff (1977), Rose-Ackerman (1979), Epple, Filimon and Romer (1994) and Fernandez and Rogerson (1996), provide an analytical framework where jurisdictions are differentiated by the level of local taxes and local public goods whose quality is solely determined by public spending and where complete stratification arises at equilibrium. These models have important implications for public finance. However, they fail to capture the stratification of peers, since they assume that the only input is per capita spending and they impose heavy restrictions on preferences and technologies. Another body of research introduced other inputs; these inputs include housing quality - Dunz (1985) and Nechyba (1997, 1999) - and transportation costs - Epple and Romano (2000).

Nonfinancial stratification determinants have been introduced in the models of Arnott and Rowse (1987), Epple and Romano (1998 & 2006)<sup>8</sup> and Nechyba (2003). These determinants are usually defined as peer effects. In general, peer effects have been modeled as linear in means (e.g. mean ability), even though other moments, such as the variance, can be included. Furthermore, in most previous models, only ability peer effects were considered. However, nonfinancial school characteristics, such as teacher quality, social and income peer effects, psychological peer effects (e.g. interest and motivation) and teacher-student relations, are

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<sup>8</sup> These models included private schooling.

highly relevant as determinants of school quality. In the theoretical model developed in chapter 2, ability and social capital peer effects are considered in a nonlinear in means formulation of school quality. In the empirical analyses in chapter 4, a large array of variables are included to account for social, economic and behavioral peer effects as well as for other nonfinancial school characteristics.

Different constraints have been considered in the literature. In public schooling models, where residence determines the set of possible choices, the housing market defines the natural mechanism through which sorting could arise; see Nechyba (2006). The housing market constraints may include, housing prices, distance and transportation costs and housing quality. When private schooling is considered, another type of constraints is added. These constraints include tuition and ability constraints in cream skimming private schools (see Nechyba 1999). The interaction between family preferences and the different constraints determines the existence or lack of stratification and whether it is one or multidimensional.<sup>9</sup>

Most of the literature on stratification considered that public school quality is homogenous for all students attending the same school. However, this assumption is challenged by tracking and setting policies used for the management of student heterogeneity. The degree of tracking and the grade at which it begins are determined by educational authorities. Most countries have adopted such sorting policies. However, the timing and precise form vary. For instance, in the UK, setting policies have been used where students are tracked into different classes for a limited number of subjects. In the USA, more severe forms of tracking have been used, where students are completely sorted into different tracks according to their abilities.

Other types of policies that may affect stratification and the distribution of peers include school choice policies and zoning. Note that tracking policies do not affect the stratification of students, even though they alter its effects. On the other hand, public school choice policies are designed to alter stratification and to give families the right to choose a school outside their district of residence.

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<sup>9</sup> The theoretical model in chapter 2 considers both public and private schools and the resulting stratification is three dimensional.

## **5.2. Stratification in the empirical literature.**

Much of the discussion in the previous section is reflected in computational versions that follow the theoretical framework. As noted by Nechyba (2006), the need for computational models arose from the complexity of the theoretical approaches and the non-existence of clear closed analytical solutions. Computational models are not only concerned with the impact of policies on individual decision making, but also with the impact of such policies on the economic environment in which individual decisions are made.

Two different computational approaches exist. The first studies stratification in a single homogenous public sector with private alternatives; see Epple and Romano (1998, 2006). The second studies stratification in heterogeneous public school systems supported by spatial stratification. Two types of stratification emerged from the different models; one is hierarchical and the second is horizontal. Some of the papers have also analyzed the impact of vouchers and tracking policies on the sorting of students across schools.

In the paper of Epple and Romano (1998), vouchers are found to increase achievements by allowing families to select private schools. In fact, achievement gains range between 12.9% to 20.2% for a voucher of \$2000 and the largest gains occur for low income, high ability students. In contrast, students who remain in the public sector experience a decline in their achievements of 4.9%. Tracking policies have also been studied in Epple, Newlon and Romano (2002). The authors predicted large within-school disparities in quality for different ability levels. Furthermore, sorting into private schools is also affected by the cream skimming tracks within public schools.

Models with public school heterogeneity, such as Nechyba's (1999), allowed for differences in public school quality between jurisdictions. This model was further extended for computational purposes in Nechyba (2000 & 2003) and in Ferrayra (2002). The introduction of vouchers in the models of Nechyba caused an increase in sorting. In fact, the flight out of public schools increases with the size of the voucher; when the voucher is equal to public spending in poor districts, some public schools will collapse.

Most of the empirical literature on stratification took the form of computational models directly related to analytical formulations of the education market. However, in recent years a growing traditional empirical literature tries to investigate the effect of such stratification on achievements. The main implication of stratification can be seen through disparities in peer quality across schools. The literature on peer effects has been examined previously in this chapter. However, as noted by Nechyba (2006 p.1359), “*the literature investigating the impact of peer sorting on student achievements is still in its early stage.*”

As we have seen previously in the review of the empirical literature, educational achievements are determined by a large array of student and school characteristics. Stratification is at the centre of the mechanisms of inequality, since it determines how students are sorted between schools and which school quality they obtain. In fact, stratification determines the peers with whom a student is studying and socializing as well as the level of spending, teacher quality, the levels of discipline and motivation, and probably the entire educational experience.

**Therefore, any approach towards the study of the effects of stratification should be able to consider all these aspects. More precisely, the education production function should consider individual characteristics, including subjective attitudes such as motivation and interest, different forms of peer effects (economic, social, and motivational) and school characteristics (spending, teacher quality, etc.).**



## Conclusion of chapter I.

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In this chapter, a thorough review of the literature on stratification and choice was carried out. Both the theoretical and empirical bodies of research have been investigated, and a number of conclusions can now be drawn.

**On the one hand**, in the theoretical literature two distinct bodies of research on stratification are identified. The first is concerned with residential stratification between jurisdictions, where school quality is mainly determined by local spending, and the second is concerned with stratification between public and private schools, where ability peer effects are the major determinant of stratification. In addition to these two bodies, some papers even considered educational legislation as a source of stratification.

The papers of Westhoff (1977), Rose-Ackerman (1979), Epple, Filimon and Romer (1993), Fernandez and Rogerson (1996), Nechyba (1997) and Epple and Platt (1998) constitute the major contributions to the first domain. These papers aim at studying the role of public finance in the provision of public goods in a multicomunity framework. This public good can be considered as education in the absence of private schools. In these models, hierarchical stratification usually arises at equilibrium. For instance, in Fernandez and Rogerson (1996), complete stratification by income arose from the functioning of the housing market.

In Nechyba (1997), the author developed a fully closed general equilibrium model in a multicomunity environment with public goods and analyzed voting over national and local tax rates in a hierarchical public sector. In Epple and Platt (1998), the authors constructed an equilibrium in a multicomunity model where individuals have the choice between communities differentiated by housing prices and grant levels. In all these models, the quality of the public good is determined solely by public spending, and private alternatives are not considered. Stratification results from the interaction between individual characteristics (such as income) and housing prices, and leads to self-selection into communities.

In general, these models present a number of limitations. First, spending is considered to be the only determinant of school quality, even though the empirical literature highlights the importance of peer effects. Second, they do not include private schools as an alternative to public supply. This particular simplification facilitates the analytical solutions - since preferences are single peaked and Black's median voter theorem holds - but at a great loss of generality. Finally, in most of these models, complete stratification by income arises at equilibrium. This is not very realistic, since communities are usually stratified along other criteria, such as social class and preferences.<sup>10</sup>

The second body of literature studied stratification between public and private schools. The major articles include Arnott and Rowse (1987), Epple and Romano (1998, 2006) and Nechyba (2003). In these different papers, the authors introduced peer effects in order to study the sorting of students across school types. The inclusion of nonfinancial determinants of school quality is very important, since it emphasizes the fact that sorting may arise not only because of local funding but also because of other inputs that affect school quality.

The major innovation in these models is the introduction of private schools as an alternative to public supply. However, a number of concerns remain. First, these models do not consider funding as a determinant of school quality to avoid non-single peaked preferences over tax rates. Therefore, they limit the determinants of quality to peer effects; see Epple & Romano (1998). Second, only pure public and private schools are considered. Most of the literature neglects the existence of mixed finance schools, which are privately managed but partly publicly funded or subsidized. Third, most of these papers model the public system as completely passive. This system is usually specified as a large homogenous multi-campus school with no active strategy on the market. Finally, peer effects are linear in means. In other words, a student whose performance increases average performance by two points has the same effect as several students whose performance increases average performance by two points.

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<sup>10</sup> Epple and Platt's paper (1998) allows for non single peaked preferences and results in two-dimensional stratification along income and preferences.

From this review of the theoretical literature, a number of implications can be drawn. Any future analysis should include peer effects and funding as simultaneous determinants of school quality. Moreover, peer effects should not be linear in means. Non-linearities can be introduced through different moments of a particular variable, such as the average and the dispersion. Private, public and mixed finance schools should be considered. This will lead to non-single peaked preferences over education and to the inapplicability of Black's median voter theorem. In this context, a number of conditions on the geometric form of preferences should be provided. Schools should be allowed to have a strategic behavior on the market, such as profit maximization under quality constraints. Multiple communities should be considered for public school quality to be differentiated. The specification of the model as well as the assumptions on preferences should allow for multi-dimensional stratification, something that better reflects the reality.

**On the other hand**, the empirical literature included two types of analyses. The first studied school choice directly through discrete choice models (Logits, nested logits...), and the second studied the determinants of achievements using education production functions. The second type was further decomposed into different categories of determinants of achievements, such as peer effects, immigration and ethnic backgrounds, school resources and school and teacher quality.

As we will see in chapter IV, I have opted for the second type of empirical models. The reason behind this decision is that education production functions are better suited for the assessment of inequalities in attainments. In contrast, discrete choice models can only tell why a student chooses a particular school or type of schools. However, this choice does not imply in itself any inequality in performance. In fact, in some countries, such as Finland, the choice of a private school is associated with lower achievements in comparison to public schools. The reverse is true in countries like Britain. Furthermore, most data sources provide information on the school that a student has already chosen without providing any information on other possible alternatives that could have been chosen. Therefore, it is very difficult to reconstruct the choice set of each student.

The estimation of education production functions allows for the assessment of the effects of several categories of student and school characteristics. The individual characteristics discussed in this review included ethnicity, immigrant background, ability, and income, while school characteristics included spending, ability peer effects and teacher quality.

These characteristics can be further extended to include other individual socio-demographic variables and even psychology-related variables, such as motivation and interest in learning. School characteristics can also include different forms of peer effects other than ability peer effects, as well as variables describing the school environment, such as discipline and student-teacher relations.

In addition to the aforementioned concerns, a number of technical issues should be mentioned. The education production function to be estimated is of the following form:

$y_{ij} = f(F_{ij}, S_j)$ , where  $y$  is educational performance,  $F_{ij}$  is a vector of student characteristics and  $S_j$  is a vector of school characteristics.<sup>11</sup> Stratification can be captured through the within- and between-school dispersions on the different variables. A high between-school dispersion combined with within-school homogeneity is an indication of high levels of stratification. Note that my theoretical and empirical specifications of school quality will be non-linear in peer effects and school and student characteristics will include a large array of variables in order to avoid the omitted variable bias and to avoid heteroscedasticity and endogeneity problems.

Moreover, this equation is time invariant, unlike the classical formulations of education production functions. It has the advantage of eliminating the need for historical data and it is more convenient for the use of multilevel PISA data. Furthermore, this equation relates the within- and between-school disparities on the different variables to the variation in individual performance scores. Hence, it is better suited for the assessment of inequalities in attainments.

All these issues will be further analyzed in the corresponding chapters. The next chapter develops a full general equilibrium model of educational and residential stratification and provides a theoretical framework for the empirical analyses undertaken in chapters III and IV.

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<sup>11</sup>  $i$  is a subscript denoting a student, and  $j$  is a subscript denoting a school.

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**The Anatomy of Inequalities in Attainments:  
An International Investigation  
on Stratification and Choice.**

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**CHAPTER II**

**School Quality, Peer Effects,  
Communities, and Public School Choice:  
a Theoretical Analysis.**

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**B**ringing down inequalities, both in the access to education and in its outcomes, has become a major driving force for educational reforms in most developed countries. Unequal access to education, unequal outcomes and disparities in school quality and in earning prospects represent major challenges for educational authorities around the world.

Recent and ongoing international surveys conducted by the OECD “PISA and IALS” showed that student outcomes in major western countries tend to be heavily dispersed. For instance, in PISA2000, Belgium, Germany, the USA, and the UK turned out to be the most unequal states in term of the dispersion of student achievements over reading, numeracy and basic science tests<sup>12</sup>. The debate over these issues as well as the growing literature on school choice, and urban economics provided the motivation for this chapter.

This chapter studies the inequalities in student attainments and in the access to education through an analysis of stratification on the education market. Note that the theoretical literature on this subject can be traced back to Barzel (1973), where the “Ends against the middle phenomenon” was introduced. The article of Barzel constituted an introduction to a more formal theoretical approach to school choice, starting with Joseph Stiglitz (1974).

As we have seen in the previous chapter, there are three different ways in which stratification may arise. First, households may self-select into communities and then into schools according to their ability and income constraints. Second, in the presence of a private system, some households may opt out from their public school and choose a private one. Third, public policies such as zoning, public school choice, busing and within-school tracking may intervene in order to alter the effects of the stratification.

In this chapter two models are developed. First, I start with a simple framework in which I analyze price discrimination on the education market where public and private schools coexist and where individuals are differentiated by income and ability. I introduce a non-linear in means specification of peer effects by shedding the light on the fact that school quality is not solely dependent on average abilities, but also on their dispersion. In both models, school quality is determined by spending and by peer effects.

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<sup>12</sup> Chapter 5 in “Education, Equality and Social Cohesion” by Green, Preston and Janmatt provides interesting results on outcome inequalities in five groups of countries: East Asia, the Nordic countries, the Germanic region, the Mediterranean countries and the predominantly English-speaking countries.

The second model constitutes an expansion of the first through the inclusion of communities. In this model, students are differentiated by income, ability and social capital, and the market includes public, private and mixed finance schools. Public funds are collected through local property taxes and are destined to finance public schools and to subsidize mixed finance ones. Schools maximize their profits under a quality constraint, and the marginal cost of admitting a student depends on the cost of producing education and on the position of a student relative to mean ability and mean social capital. Income plays an indirect role, since it determines the type of schools and communities that can be afforded by a student given his ability and social capital.

Three-dimensional stratification according to student characteristics results from school profit maximization and student utility maximization. In the last section, I study majority voting over tax rates, while providing the necessary conditions for the existence of a majority voting equilibrium determined by the median voter. Finally, I analyze the consequences of introducing public school choice on the already existing equilibrium.

Two major bodies in the theoretical literature are relevant to my analysis. The first is concerned with residential stratification between jurisdictions where school quality is mainly determined by spending. And the second is concerned with stratification between public and private schools where peer effects are introduced as a determinant of school quality.

The papers of Westhoff (1977), Rose-Ackerman (1979), Epple, Filimon, and Romer (1993), Nechyba (1997), Epple and Platt (1998) and Fernandez and Rogerson (1996) constitute the major contributions to the first domain. These papers aim at studying the role of public finance in the provision of public goods. This public good can be thought of as education in the absence of private alternatives. Generally, in these models, hierarchical stratification arises at equilibrium. For instance, in Fernandez and Rogerson (1996), education is treated as a local public good. Complete stratification by income results from the functioning of the housing market and is characterized by a strict hierarchy of communities following educational qualities and aggregate income.

In Nechyba (1997), the author developed a fully closed general equilibrium model in a multicomunity environment with public goods and analyzed voting over national and local

tax rates in a hierarchical public sector. In Epple and Platt (1998), the authors constructed an equilibrium in a multicommodity model where individuals had the choice between communities differentiated by housing prices and grant levels. In all these models the quality of the public good is determined solely by public spending. Stratification results from the interaction between individual characteristics (such as income) and housing prices and local tax rates, and leads to self-selection into communities.

The major disadvantage of such models is their lack of generality. First, they consider spending as the unique determinant of public school quality, even though the empirical literature shows that peer effects are also of major concern. Second, they do not include private schools as an alternative to public supply. Note that the absence of private alternatives facilitates the analytical solutions, since preferences are single peaked and Black's median voter theorem holds. Finally, in most of these models complete stratification by income arises at equilibrium. This is not very realistic, since communities are usually stratified along other criteria, such as social class and preferences.<sup>13</sup>

Another body of literature studied stratification between public and private schools. The major articles include Arnott and Rowse (1987), Epple and Romano (1998, 2000) and Nechyba (2003). In these different papers the authors introduced peer effects in order to study the sorting of students across school types. The inclusion of non-financial determinants of school quality is very important, since it emphasizes the fact that sorting may arise not only because of local funding but also because of other inputs that affect school quality.

The major advantage of these models is their ability to consider private alternatives in the provision of schooling. However, a number of problems remain. First, these models do not consider funding as a determinant of school quality to avoid non-single peaked preferences over tax rates; therefore, they limit the determinants of quality to peer effects; see Epple and Romano (1998). Second, only public and private schools are considered. Most of the literature neglects the existence of mixed finance schools, which can be privately managed but publicly funded or subsidized. Third, most of these papers model the public system as completely passive. This system is usually specified as a single homogenous multi-campus

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<sup>13</sup> Epple and Platt's paper (1998) allows for non single peaked preferences, and results in two-dimensional stratification.



school with no active strategy on the market. Finally, peer effects are mainly linear in means. In other words, a student whose performance increases average performance by two points has the same effect as several students whose performance increases average performance by two points. One reads, for instance, in a paper of Caroline Hoxby and Gretchen Weingarth (Hoxby and Weingarth, 2005 p. 2 and 30):

“The linear-in-means model assumes that each student has the same effect on each other student (a homogeneous treatment effect). It also assumes that a single student whose achievement raises a class's mean achievement by two points has precisely the same effect as several students whose combined achievement raises the class's mean by two points (that is, all effects operate through one moment: the mean of peers)... Moreover, most applications of peer effects—school desegregation, school choice, college choice, urban economics—need to have non-linear peer effects to generate results that are interesting and that mimic the facts.” p.30 “Our finding support for the Boutique and Focus models suggests that schools, colleges, and workplaces should be wary of creating peer groups in which some people are isolated. However, they should also avoid creating critical mass around a certain type of persons. Some focus is good”.

From this small review of the theoretical literature, it is possible to conclude that any further analysis of stratification on the market for schooling should include funding and peer effects as simultaneous determinants of school quality. Furthermore, peer effects need not to be linear in means. In fact, nonlinear peer effects can be introduced through the inclusion of the average and dispersion of a particular variable. Private, public and mixed finance schools should be considered. In this case, preferences over tax rates will no longer be single peaked and the existence of an equilibrium may require a number of conditions. These schools should be allowed to have a strategic behavior. Private schools should maximize their profits, mixed finance schools should maximize their profits given the level of public subsidies and public schools should receive enough funds to cover the marginal cost of admitting each student willing to attend a public school. All these maximizations are done under a number of quality constraints. The model should also consider a multicomunity framework, where household movements are subject to a number of constraints. The inclusion of communities allows for public schools to have differing qualities. The resulting community stratification

should not be one-dimensional, since in reality relatively rich households can live in the same community as relatively poor households. This happens when stratification is multi-dimensional and follows other criteria than income.

The chapter is organized in two sections. The first analyzes price discrimination on the market for schooling, where peer effects are nonlinear in means, and the second develops a full general equilibrium model of the provision of education in a multicomunity environment. The second section considers all the aforementioned remarks.

## A. Price discrimination on the education market.

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The specification of the model in this section of chapter 2 is closely related to the specification used in Epple and Romano (1998). The objective is to analyze how price discrimination by private schools operates when peer effects are not linear in means. Similarly to Epple and Romano, I consider an economy populated with individuals differentiated by income and ability, and which includes public and private schools. However, in contrast with their article, I consider that school quality is determined by per pupil spending and by the average and the dispersion of abilities. Hence, my specification of school quality allows for nonlinearities in peer effects to be considered. In this section, the model is developed within a single community, the housing market does not exist and the analytical structure is simplified. This chapter is intended to construct a simple framework that will be extended in the next section, where communities and mixed finance schools are to be introduced and the tax rate is to be endogenized. Note that in the next section some aspects will be different, since students will be differentiated by social capital, too. In both models, students and households are considered to be a single decision making unit. This is consistent with secondary education where parents and children make education decisions jointly.

The reason behind the introduction of nonlinearities (dispersions) in the specification of peer effects can be understood in two ways<sup>14</sup>. The first is related to the impact of the dispersion on student achievements and the second is related to the signaling effect of education. Schools with low ability dispersions tend to be more appreciated by students, since instruction is more effective in a homogenous environment. In contrast, in a highly dispersed class, instruction may be hindered by large disparities between ability groups<sup>15</sup>. Furthermore, the type of school a student attends has a strong effect on the educational signal he conveys. Belonging to a school with very high dispersions of ability reduces the quality of the signal, since outsiders with limited information on ability will no longer be able to identify student types. In this model, I assume that educational quality is decreasing in the dispersion of abilities.

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<sup>14</sup> M Dertouzos et al (1989; pp. 84-85) underlined the need to take the dispersion in test scores into account. R. Benabou (1996) explored the effect of human capital dispersion on economic growth.

<sup>15</sup> This is consistent with the Boutique and the Focus models. See Hoxby and Weingarth 2005.

## 1. The model.

I consider an economy populated with a continuum of agents; the number of types is largely superior to that of schools, each household has one student and both are considered to be a single decision making unit. Students have identical preferences over consumption, and are differentiated by income and ability. A student, denoted with a subscript  $i$  has an income  $y_i$  and ability  $b_i$ ;  $i$  is also used to denote a type of student. Income and ability are distributed in the population according to a density function  $f(b, y)$ , which is positive and continuous on its support  $S \equiv (0, b_{\max}) \times (0, y_{\max})$ .

Individual utility is assumed to be a function of consumption goods other than education and school quality. It is denoted  $U(c, q)$ ;  $U$  is increasing, strictly quasi-concave and twice continuously differentiable. Education is considered to be a superior good with  $\lim_{c \rightarrow \infty} U_c(c, q) = 0$ . In other words, individuals should increase school quality when disposable income increases; higher consumption is not solely sufficient to increase utility.

School quality, denoted  $q$ , is determined by school expenditure per pupil  $E$ , by school average ability  $\theta_j$  and by the dispersion of ability in a school  $\sigma_j^2$ . Quality is increasing in the first two arguments and decreasing in the last.  $j$  is a subscript denoting a particular school (with  $j = 1, 2, 3, \dots, J$ ). If one of the first two components of quality is equal to zero, then quality is equal to zero. Note that average ability and the dispersion of abilities represent peer effects in a school. **Educational achievements are given by  $a_i = a(b_i, q_j)$ ;  $a$  is continuous and increasing in both arguments. Students' achievements depend on their own ability and on the quality of their school. In other words, the access to higher quality schools is translated through higher achievements. In terms of inequalities, unequal access to education generates unequal outcomes.**

Mobility between schools is costless and no transportation costs are included in the model. The housing market is absent and the economy has only one community. The absence of the housing market and of transportation costs leads to a homogenous, large, multi-campus

public school. In other words, the public sector consists of a single large public school with homogenous quality. This is to be discussed further in the next subsection.

Education production cost depends only on the number of students enrolled in a particular school; it is denoted  $C(K) = V(K) + F$ , with  $V' > 0$  and  $V'' > 0$ .  $K$  is the number of students in the school;  $F$  is a constant representing fixed costs when no students are enrolled. In this analysis, I assume that economies of scale in the production of education do not exist. The absence of economies of scale is translated in the existence of a large number of schools with relatively small number of students.<sup>16</sup>

Each private school retains a fraction of the student population that has chosen the private sector. Admittance in private schools is done according to student type, and the resulting price discrimination tends to be meritocratic. The schooling market is assumed to be competitive.

In a private school, students with ability levels higher than the mean receive a tuition reduction. In contrast, students with ability levels lower than the mean pay a tuition premium. Similarly, students with ability levels close to the mean receive tuition reductions since they limit the dispersion of abilities. In contrast, students with ability levels far from the mean pay a tuition premium, since they disperse the distribution of abilities. This pricing discrimination allows schools to condition tuition according to student type and to internalize any positive or negative externality that a student may generate.

In this model, supplementary educational investments (destined for students with low achievement levels), such as remedial classes, are not considered. Therefore, school expenditure per pupil is the same for all students attending the same school.

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<sup>16</sup> Lawrence Kenny (1982) provided some evidence that economies of scale do exist in the production of education. A counterargument was provided by Ferris and West (2004); it relies on the existence of external costs that increase with size and are related to social problems present in large schools.

### 1.1. The public sector.

Public schools are income tax financed. All households pay income taxes even if they do not use public schools. Public sector resources are determined by  $tY$ ; the expenditure per student is determined by  $\frac{tY}{N}$ . Public schools are free entry, free of tuition schools ( $p_{ju} = 0$ ); they apply an open enrollment policy where all students are accepted independently of their type.

Where  $t$ : The tax rate considered to be exogenous in this section (in the next section  $t$  is endogenized).

$Y$ : Total income in the economy.

$N$ : The number of students in the public sector.  $N \in [0, P]$ .

$P$ : The population or the number of students in the economy.

$ju$ : A subscript used to denote the public school in the model (or the public sector).

The quality of education in the public school is given by:

$$q_{ju} = q\left[\frac{tY}{N}, \theta_{ju}, \sigma_{ju}^2\right]$$

Public school quality is homogenous for all schools and students. This homogeneity results from the assumption that public schools practice an open enrollment policy and from the absence of friction and transportation costs. In other words, if public schools have different qualities, students will always choose the highest quality school without bearing any cost. Therefore, the rest of the schools would disappear since they can not attract any students and the only remaining school will have a homogenous quality. In conclusion, the public sector can be seen as a single, large, multi-campus school. Henceforth, the words “public sector,” “public school” and “public schools” are used interchangeably.

We denote  $\alpha_{ju}(b, y)$  as the proportion of students of type  $(b, y)$  enrolled in the public school, with  $\alpha_{ju}(b, y) \in [0, 1]$ .

$k_{ju} = \iint_s \alpha_{ju}(b, y) f(b, y) db dy$  is the number of students of type  $(b, y)$  enrolled in the public school. Note that  $k_{ju} = N$ .

Individual utility in the public sector depends on the after-tax income and school quality:

$$U_{iu} = U[y_i(1-t), q_{ju}]$$

Indirect utility can be written as:

$$U_{iu} = W_{iu} = W(t, y_i, Y, N, \theta_{ju}, \sigma_{ju}^2, b_i)$$

The subscript  $u$  is used to denote utility in the public sector.

In the public sector, utility maximization is trivial. A student allocates his disposable income (after the deduction of taxes destined for public school finance) to private consumption.

## 1.2. The private sector.

In the private sector, schools are tuition financed. Parents pay tuition equal to  $p_{jr}$ . Private school resources are determined by the sum of tuition in a particular school  $\sum p_{jr}$ . Per pupil expenditure is determined by dividing the production cost of education by the number of students in the school.

Expenditure per student is given by: 
$$\frac{C(k_{jr})}{k_{jr}} = \frac{V(k_{jr}) + F}{k_{jr}}$$

$r$  is a subscript denoting a particular private school.

We denote  $\alpha_{jr}(b, y)$  as the proportion of students of type  $(b, y)$  enrolled in a particular private school. With  $\alpha_{jr}(b, y) \in [0, 1]$ .

$k_{jr} = \iint_s \alpha_{jr}(b, y) f(b, y) db dy$  is the number of students enrolled in school  $r$ .

The quality of education in a private school is given by:

$$q_{jr} = q \left[ \frac{C(k_{jr})}{k_{jr}}, \theta_{jr}, \sigma_{jr}^2 \right]$$

Private sector quality is heterogeneous between schools.

Individual utility in the private sector is given by:

$U_{ir} = U[y_i(1-t) - p_{ijr}, q_{jr}]$ , where  $r$  is a subscript denoting utility in a private school.

Students maximize their utility through the choice of an optimal combination of consumption and school quality according to:

$$\text{Max } U_{ir} = \max U[y_i(1-t) - p_{ijr}, q_{jr}]$$

Subject to a budget constraint  $p_{ijr} = y_i(1-t) - c_i$ , with  $c$  as consumption.

Indirect utility can be written as:  $U_{ir}^* = W_{ir} = W(t, y_i, \theta_{jr}, \sigma_{jr}^2, b_i)$ , which is the only solution to this optimization.

Private schools behave strategically on the market. They maximize profits as utility takers by conditioning admission and tuition according to ability levels. For a student to attend a particular private school, his utility in this school should be at least equal to that obtained elsewhere and he should afford the price corresponding to his type. This behavior is similar to that of private clubs with non-anonymous crowding; see Scotchmer and Wooders (1987). It should be noted that individual ability is perfectly observable by private schools.

Private schools maximize the following profit function:

$$\text{Max } \pi_{jr} = \int \int_s [p_{ijr}(b, y) \alpha_{jr}(b, y) f(b, y) db dy] - V(k_j) - F \quad (1)$$

Subject to these constraints:

$$\alpha_{jr}(b, y) \in [0, 1] \quad \forall (b, y) \quad (1a)$$

A school can not enroll more of a type than exists in the population, and it can not enroll a negative proportion of a particular type.

$$U_r = U[y_i(1-t) - p_{ijr}(b, y), q_{jr}] \geq \text{Max } U[y_i(1-t) - p_{ijr'}(b, y), q_{jr'}] \quad \forall (b, y) \quad (1b)$$

The utility that student  $i$  has in the school of his choice  $jr$ , should be at least equal to the maximum utility that he can obtain elsewhere (in school  $jr'$ , for example). With  $jr \neq jr'$ .



$$k_{jr} = \iint_s \alpha_{jr}(b, y) f(b, y) dbdy \quad (1c)$$

Is the number of students attending school  $jr$ .

$$\theta_{jr} = \frac{1}{k_{jr}} \iint_s b \alpha_{jr}(b, y) f(b, y) dbdy \quad (1d)$$

$$\sigma_{jr}^2 = \frac{1}{k_{jr}} \iint_s (b - \theta_{jr})^2 \alpha_{jr}(b, y) f(b, y) dbdy \quad (1e)$$

These are average ability and the dispersion of abilities in a school. They represent ability peer effects.

## 2. Equilibrium conditions.

Three types of conditions are necessary for the existence of equilibrium:

### A- The conditions for private sector equilibrium are:

- Individual utility maximization:

$$U^*(b, y) = \text{Max}U[y_i(1-t) - p_{ijr}, q_{jr}]$$

- Private schools' profits maximization:

$$[\theta_{jr}, \sigma_{jr}^2, k_{jr}, p_{ijr}(b, y), \alpha_{jr}(b, y)] \text{ Satisfy equation (1).}$$

- New entries are expected as long as private schools make a positive profit. In other words, a positive profit attracts new private schools to the market until the profit is equal to zero and the expenditure per pupil is equal to total resources (tuition). At **equilibrium** we

have:  $\pi_{jr} = 0$  and  $\frac{C(k_{jr})}{k_{jr}} = \sum p_{ijr}$ . The same assumption was used in Epple and Romano (1998).

### B- The conditions for public sector equilibrium are:

- Public schools are free admission schools:  $p_{iju}(b, y) = 0$
- $\alpha_{ju}(b, y) \in [0, 1] \quad \forall (b, y)$ .
- The number of students in the public sector is given by:

$$k_{ju} = \iint_s \alpha_{ju}(b, y) f(b, y) dbdy$$

- Public school's average ability is given by :

$$\theta_{ju} = \frac{1}{k_{ju}} \iint_s b \alpha_{ju}(b, y) f(b, y) db dy$$

- Public school's ability dispersion is given by:

$$\sigma_{ju}^2 = \frac{1}{k_{ju}} \iint_s (b - \theta_{ju})^2 \alpha_{ju}(b, y) f(b, y) db dy$$

**C- All students in the age for schooling are attending one of the two sectors:**

$$k_{ju} + \sum k_{jr} = P \text{ or } \sum \alpha_i(b, y) = 1.$$

This condition represents market clearance; public schools are preferred to no schooling.

### 3. Private school profit maximization and price discrimination.

Private schools maximize the following profit function, which is determined by the sum of tuition minus the cost of producing education.

$$MAX \pi_{jr} = \iint_s [p_{ijr}(b, y) \alpha_{jr}(b, y) f(b, y) db dy] - V(k_{jr}) - F$$

Subject to:

$$k_{jr} = \iint_s \alpha_{jr}(b, y) f(b, y) db dy \quad (1c)$$

$$\theta_{jr} = \frac{1}{k_{jr}} \iint_s b \alpha_{jr}(b, y) f(b, y) db dy \quad (1d)$$

$$\sigma_{jr}^2 = \frac{1}{k_{jr}} \iint_s (b - \theta_{jr})^2 \alpha_{jr}(b, y) f(b, y) db dy \quad (1e)$$

The three constraints can be integrated in two:

$$\theta_{jr} \iint_s \alpha_{jr}(b, y) f(b, y) db dy - \iint_s b \alpha_{jr}(b, y) f(b, y) db dy = 0$$

$$\sigma_{jr}^2 \iint_s \alpha_{jr}(b, y) f(b, y) db dy - \iint_s (b - \theta_{jr})^2 \alpha_{jr}(b, y) f(b, y) db dy = 0$$

The Lagrangian function is of the following form:

$$\Phi = \int \int_s [p_{ijr}(b, y)\alpha_{jr}(b, y)f(b, y)dbdy] - V(k_{jr}) - F - \mu_{jr} \left[ \theta_{jr} \int \int_s \alpha_{jr}(b, y)f(b, y)dbdy - \int \int_s b\alpha_{jr}(b, y)f(b, y)dbdy \right] - \mu'_{jr} \left[ \sigma_{jr}^2 \int \int_s \alpha_{jr}(b, y)f(b, y)dbdy - \int \int_s (b - \theta_{jr})^2 \alpha_{jr}(b, y)f(b, y)dbdy \right]$$

With  $\mu_{jr}$  and  $\mu'_{jr}$  are the Lagrangian multipliers.

Optimization is made through the partial derivation of the Lagrangian function over  $\alpha_{jr}(b, y)$ :

$$\frac{\partial \Phi}{\partial \alpha(b, y)} = p_{ijr}^* - V'(k_{jr}) - \mu_{jr}(\theta_{jr} - b) - \mu'_{jr}[\sigma_{jr}^2 - (b - \theta_{jr})^2] = 0$$

$$p_{ijr}^* = V'(k_{jr}) + \mu_{jr}(\theta_{jr} - b) + \mu'_{jr}[\sigma_{jr}^2 - (b - \theta_{jr})^2]$$

This equation represents price discrimination in private schools.

The Lagrangian multipliers represent the change in price following a change in average ability  $\theta_{jr}$  or in the dispersion of abilities  $\sigma_{jr}^2$ :

$$\mu_{jr} = \frac{1}{k_{jr}} \int \int_s \frac{\partial p_{ijr}^*}{\partial \theta_{jr}} \alpha_{jr}(b, y)f(b, y)dbdy \quad \text{and} \quad \mu'_{jr} = \frac{1}{k_{jr}} \int \int_s \frac{\partial p_{ijr}^*}{\partial \sigma_{jr}^2} \alpha_{jr}(b, y)f(b, y)dbdy$$

$\mu_{jr}$  is positive, since an increase in average ability is associated with an increase in quality and should lead to an increase in price.  $\mu'_{jr}$  is negative, since an increase in the dispersion of abilities is associated with a decrease in quality and should lead to a decrease in price.

The price  $p_{ijr}^*$  is what a student  $i$  of type  $(b, y)$  should pay in order to attend school  $j$ . This price has three parts.

1.  $V'(k_{jr})$  represents the part resulting from the production cost of education. This part is identical for all students attending the same school.
2.  $\mu_{jr}(\theta_{jr} - b_i)$  represents the part resulting from the difference between a student's ability and average ability in a school. This is the price of having an ability above or below the average. Since school quality is increasing in average ability, a student with an ability above the average has a positive externality on the school. This portion of the pricing function internalizes this externality. The reverse is also true for students with below average abilities.

3.  $\mu'_{jr}[\sigma_{jr}^2 - (b - \theta_{jr})^2]$  represents the price of having an ability close or far from school average. Since school quality is decreasing in the dispersion of abilities, a student with an ability far from the average has a negative externality on the school. This portion of the pricing function internalizes this externality. The reverse is also true for students with ability levels close to the average.

Students with ability levels lower than school average ability  $b_i < \theta_{jr}$  will have to pay a tuition premium equal to  $\mu_{jr}(\theta_{jr} - b_i) > 0$ . Students with ability levels higher than school average ability  $b_i > \theta_{jr}$  will receive tuition discounts equal to  $\mu_{jr}(\theta_{jr} - b_i) < 0$ .

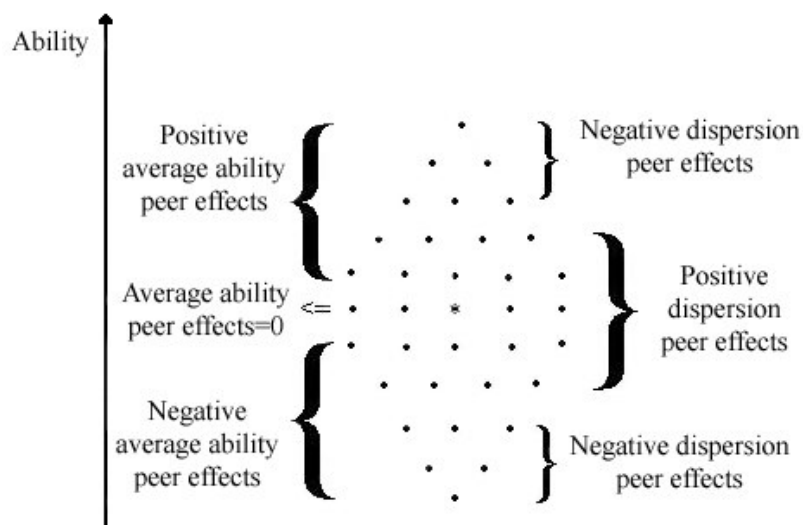
Students with  $(b - \theta_{jr})^2 > \sigma_{jr}^2$  will pay a tuition premium equal to  $\mu'_{jr}[\sigma_{jr}^2 - (b - \theta_{jr})^2] > 0$ . Students with  $(b - \theta_{jr})^2 < \sigma_{jr}^2$  will receive a tuition reduction equal to  $\mu'_{jr}[\sigma_{jr}^2 - (b - \theta_{jr})^2] < 0$ . It should be said that the distance between a student's ability and the average determines the importance of  $(b - \theta_{jr})^2$  relative to  $\sigma_{jr}^2$ . Students with ability equal to the average receive the maximum possible price reduction, which is equal to  $\mu'_{jr}[\sigma_{jr}^2] < 0$ . When student ability moves away from the mean,  $(b - \theta_{jr})^2$  increases and the price discount decreases until it becomes a positive price.<sup>17</sup>

In figure 2.1, the distribution of ability in a private school is illustrated. On the vertical axis we have ability. The dots represent students and the star in the middle represents average ability. The figure gives an idea on how student positions within the distribution of abilities affect “average ability” peer effects and “the dispersion of abilities” peer effects. Note that students with above-average abilities have a positive impact on “average ability” peer effects, and that the reverse is true for below average students. Similarly, students with ability levels close to the average have a positive impact on “the dispersion of abilities” peer effects, and the reverse is true for students with abilities far from the average.

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<sup>17</sup> Note that  $(b - \theta_{jr})^2$  is always positive, and when it increases the price increases.

Figure 2.1. Peer effects and the distribution of ability.



It is important to note that the price of private education can be negative. For some students we may have  $\mu_{jr}(\theta_{jr} - b) < 0$ ,  $\mu'_{jr}[\sigma_{jr}^2 - (b - \theta_{jr})^2] < 0$  and  $p_{ijr}^* = V'(k_{jr}) + \mu_{jr}(\theta_{jr} - b) + \mu'_{jr}[(b - \theta_{jr})^2 - \sigma_{jr}^2] < 0$ . This negative price represents a scholarship intended to compensate the positive externality generated by a student whose ability substantially increases school quality. Note that a private school cannot grant scholarships to all of its students, since in order to grant one student a scholarship another student has to pay a positive price. (Mathematical details are provided in the next section of this chapter).

It should also be noted that income plays an indirect role in determining which school a student attends. More precisely, ability partly determines the price that a school charges for each student and income determines the affordable school given that price. **A private school enrolls every student who is able to pay the price corresponding to his type. And a student selects the school that offers the price that maximizes his utility.**

Furthermore, a subsidization scheme results from price discrimination. Those who are supposed to pay a tuition premium (students with below the average abilities or abilities very far from the mean) can be seen as subsidizing those who will receive a tuition reduction (students with above the average abilities or abilities close to the mean).

The assumption that school quality is increasing in average ability and decreasing in the dispersion of abilities led to a result that is close to the Boutique and Focus models of peer effects, mentioned in the previous chapter, and in Hoxby and Weingarth (2005). In other words, schools tend to look for a high level of average ability and a low level of dispersion. As a result, schools have to make a compromise between the two, since it is no longer possible to admit a student with above-average ability without considering his impact on the dispersion of abilities. This compromise leads to a relative homogenization of the student population without creating a critical mass around the type with average ability.

Note that in the paper of Epple and Romano (1998), school quality is increasing in average ability while the dispersion of abilities is not considered. In this case, a school can admit above average students while disregarding their impact on the dispersion of abilities. Moreover, the subsidization between below- and above-average ability students does not have any limitations. In contrast, under my assumptions, average ability and the dispersion of abilities work in opposite directions, and schools can no longer admit high or low ability students while ignoring their impact on the dispersion of abilities.

## 4. Stratification between public and private schools.

### 4.1. The existence of a tipping point.

I assume that  $\lim_{c \rightarrow \infty} U(c, q) = 0 \quad \forall q > 0$ . This assumption means that when income increases, students can increase their consumption while maintaining school quality constant. However, at a certain point, utility will start to decrease even if consumption is increasing. At this point, students have to increase their school quality, too, in order to get higher utility levels. In other words, an increase in consumption is not sufficient to increase utility; both arguments have to increase with income. A similar assumption has been used in Glomm and Ravikumar (1998).

Given that  $t \in [0, 1]$ ,  $N \in (0, P)$ , there exists a couple  $(\hat{b}, \hat{y})$  with  $\hat{b} > 0$  and  $\hat{y} > 0$  such that  $W_{iu} = W(t, y_i, Y, N, \theta_{ju}, \sigma_{ju}^2, b_i) \geq W_{ir} = W(t, y_i, \theta_{jr}, \sigma_{jr}^2, b_i)$  if, and only if,  $(b_i, y_i) < (\hat{b}, \hat{y})$ .

In this proposition, I demonstrate the existence of a tipping point defined by a couple of ability and income  $(\hat{b}, \hat{y})$ . All students with  $(b_i, y_i) < (\hat{b}, \hat{y})$  prefer the public sector and have higher utility levels  $W_{iu} \geq W_{ir}$ . In contrast, all students with  $(b_i, y_i) > (\hat{b}, \hat{y})$  prefer the private one.

**Remark:**

The comparison between two couples of ability and income may be difficult. For example, it is possible to state that  $(5,4) > (3,2)$ . However it is difficult to compare  $(6,2)$  with  $(4,4)$ .

In order to make the comparison possible, one can imagine that both ability and income enter a function that determines the level of student endowments. For instance, endowments can take the form of a Cobb-Douglass function, such as  $endowments_i = hb_i^\gamma y_i^\beta$ . This function gives different weights for ability and income. It would then be possible to compare any two couples in terms of which one generates more endowments. This function also means that students are differentiated according to a single factor that can be decomposed in two.

In my analysis, I do not specify a particular function in order to make two couples comparable. However, I assume implicitly that such a function exists and that students are able to make comparisons. Henceforth, it is possible to say that  $(b_i, y_i) < (\hat{b}, \hat{y})$  independently of the position of  $b_i$  in comparison to  $\hat{b}$  and of the position of  $y_i$  in comparison to  $\hat{y}$ . Note that each combination of  $(b, y)$  is not unique, since different levels of  $b$  and  $y$  may generate the same level of endowments.

**Proof:**

The public sector is chosen when it generates a higher utility for student  $i$ :

$$W_{iu} = W(t, y_i, Y, N, \theta_{ju}, \sigma_{ju}^2, b_i) \geq W_{ir} = W(t, y_i, \theta_{jr}, \sigma_{jr}^2, b_i)$$

The couple  $(\hat{b}, \hat{y})$  must satisfy the following equation (the indifference between the two sectors):

$$W_{iu}(t, y_i, Y, N, \theta_{ju}, \sigma_{ju}^2, b_i) = W_{ir}(t, y_i, \theta_{jr}, \sigma_{jr}^2, b_i)$$

It should be noted that private school quality is always superior to that of the public sector. In fact, if some private schools have a lower school quality than the public sector, no one would accept to be enrolled in these schools and to pay tuition, since it is possible to obtain a higher quality for free in the public sector. A private school with a quality lower than that of the public sector will be driven out of the market at equilibrium. It is possible to write:

$$q_{ju} < q_{jr}.$$

In order to demonstrate the existence of  $(\hat{b}, \hat{y})$  I have to consider two extreme cases.

**First case:**

For very low income levels, some students would not be able to pay tuition for the private school with the lowest quality. For very low abilities, some students would not benefit from any financial aide or tuition discount; hence  $p_{ijr}^* > 0$  and  $(1-t)y_i - c_{ir}^* < p_{ijr}^*$ . This means that the price of the lowest quality private school exceeds the disposable income.

These individuals can not afford a private school and no private school would admit them. Thus they are obliged to choose the public sector where their utility is higher.  $W_{ir}^* = 0 < W_{iu}$ .

**Second Case:**

For very high income levels, some students would be able to pay tuition in the highest quality private school. For very high abilities, some students would obtain tuition reductions or even scholarships.

If  $p_{ijr}^* > 0$  then  $(1-t)y_i = c_{iu}^* > c_{ir}^* = (1-t)y_i - p_{ijr}^*$ . Consumption while attending the public sector is higher.

If  $p_{ijr}^* \leq 0$  then  $(1-t)y_i = c_{iu}^* \leq c_{ir}^* = (1-t)y_i - p_{ijr}^*$ . Consumption while attending the private sector is higher.



Since  $\lim_{c \rightarrow \infty} (c, q) = 0$ , these students would accept a lower level of consumption if it would allow a sufficient increase in quality. Since private school quality is always higher than that of the public sector, some students will choose private schools and would obtain a utility level  $W_{ir}$ , such as  $W_{ir} > W_{iu}$ .

We have seen that for low levels of income and ability, the public sector is preferred. In contrast, for high levels of income and ability, the private sector is preferred. The continuity of  $f(b, y)$  implies that, when ability and income levels increase (going from one extreme to the other), it is clear that there exists a tipping point for which preferences will be reversed. This equilibrium is determined by a couple  $(\hat{b}, \hat{y})$  for which students are indifferent between the two sectors, and all students with  $(b_i, y_i) < (\hat{b}, \hat{y})$  choose the public sector. **Note that this demonstration represents only a proof of the existence of a tipping point. The actual position of such point depends on the distribution of income and ability in the population.**

#### 4.2. Some properties of the $(\hat{b}, \hat{y})$ couple.

**Three properties of the  $(\hat{b}, \hat{y})$  couple relative to  $N, Y$  and  $t$  can be established.**

- (i) For  $N \in (0, P)$ , the couple  $(\hat{b}, \hat{y})$  is decreasing in  $N$ .
- (ii) For  $Y \in \mathfrak{R}^+$ , the couple  $(\hat{b}, \hat{y})$  is increasing in  $Y$ .
- (iii) For  $t \in [0, 1]$ , the couple  $(\hat{b}, \hat{y})$  is increasing in  $t$ .

The proofs of (i) and (ii) are based on the fact that private school quality is not related to  $N$  and  $Y$ . These factors only influence school choice through public school quality. The proof of (iii) is based on the fact that  $t$  has the same influence on consumption under the two sectors, and has an effect on public school quality without affecting that of private schools.

**Proofs of these properties:**

(i) For  $N \in (0, P)$ , the couple  $(\hat{b}, \hat{y})$  is decreasing in  $N$ . An increase of the number of students in the public sector leads to a decrease in public expenditure per pupil  $\frac{tY}{N}$  along with public school quality. Students on the margin between sectors opt out of the public sector and join the private one. In this case,  $(\hat{b}, \hat{y})$  decreases.

(ii) For  $Y \in \mathfrak{R}^+$ , the couple  $(\hat{b}, \hat{y})$  is increasing in  $Y$ . An increase of  $Y$  implies an increase in public school quality following an increase in public expenditure per pupil  $\frac{tY}{N}$ . Students on the margin between sectors opt out of the private sector and join the public one. In this case  $(\hat{b}, \hat{y})$  increases.

(iii) For  $t \in [0,1]$ , the couple  $(\hat{b}, \hat{y})$  is increasing in  $t$ . On one hand, an increase in  $t$  implies a decrease in consumption. However, this decrease does not affect school choice, since it is identical in both sectors. In other words, the decrease in disposable income caused by higher tax rates is the same whether the student is attending a public or a private school.

On the other hand, an increase in  $t$  leads to an enhancement of public school quality through higher funding. Students on the margin between sectors opt out of the private sector and join the public one. In this case,  $(\hat{b}, \hat{y})$  increases.

From the last two properties, it is possible to see that increased spending makes the public sector more attractive. An increase in per pupil spending may be the result of a higher tax rates or increased aggregate income.

**The existence of a unique  $N$  that determines the size of the public sector.**

For  $t \in [0,1]$  and  $Y \in \mathfrak{R}^+$ , there exists a unique  $N^*$  that determines the number of students in the public sector. The number of students with  $(b_i, y_i) < (\hat{b}, \hat{y})$  must be exactly equal to  $N^*$ .

**Proof:**

We know that public school quality is decreasing in  $N$ , since expenditure per pupil  $\frac{tY}{N}$  is decreasing in  $N$ . In contrast, private school quality is independent of  $N$ .<sup>18</sup> The indifference condition is given by:

$$U_{iu}[(1-t)y_i, q_{ju}] = U_{ir}^*[(1-t)y_i - p_{ijr}^*, q_{jr}^*]$$

The left-hand side of the equation is decreasing in  $N$ , while the right-hand side is independent of  $N$ . For a very low  $N$ , public school quality is very high. For a very high  $N$ , public school quality is very low. While passing from one extreme to the other, the intersection between the decreasing and the non-decreasing functions determines the unique equilibrium number of students in the public sector  $N^*$ .

## **5. Further discussions and concluding remarks.**

### **The existence of school quality hierarchy in the private sector.**

In the paper of Epple and Romano (1998), the authors demonstrated the existence of a vertical hierarchy of school qualities in the private sector. They argued that, if two schools have the same level of quality, they can increase their profits by exchanging a number of students with different levels of income and ability. This exchange leads to a higher quality for one and to a lower quality for the other, while profits increase for both. The use of the single crossing assumption is central to the demonstration. This assumption implies that higher ability, higher income students value increased quality more than lower ability, lower income types.

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<sup>18</sup> Each private school chooses its size and the type of students it admits according to its profit maximization.

Two major differences exist in my model in comparison to Epple and Romano.

The first difference is based on the three-dimensional specification of school quality, which depends on expenditure per pupil, mean ability and the dispersion of abilities. Note that the inclusion of the expenditure per pupil does not affect the applicability of the Epple and Romano demonstration, since exchanging the same number of students does have an effect

on  $k_{jr}$  and therefore expenditure per pupil  $\frac{C(k_{jr})}{k_{jr}} = \frac{V(k_{jr}) + F}{k_{jr}}$  would not change. However,

the inclusion of the dispersion of abilities prevents the use of the Epple and Romano demonstration, since it is no longer possible to exchange the same number of students and to increase profits. In Epple and Romano (1998), private schools were able to differentiate their quality and to increase profits by exchanging a number of students with different abilities. In contrast, in my model, admitting students with higher or lower abilities has mixed effects, since average ability and the dispersion of abilities work in opposite directions. The final effect of this exchange depends on the structural form of school quality. The second difference is that in my model I did not impose a single crossing assumption.

These two differences result in the absence of a strict vertical hierarchy of private school qualities. Different schools may have the same quality through different combinations of

$\frac{C(k_{jr})}{k_{jr}}, \theta_{jr}$  and  $\sigma_{jr}^2$ .<sup>19</sup> In other words, horizontal and vertical forms of stratification may exist.

### **School hierarchy in the public sector:**

As we have seen previously, public school quality is homogenous for all schools and students. This homogeneity results from the assumption that public schools practice an open enrollment policy and from the absence of frictions and transportation costs. In other words, if public schools have different qualities, students will always choose the highest quality school and the rest of the schools would disappear, since they cannot attract any students. In conclusion, the public sector was defined as a single, large, multi-campus school.

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<sup>19</sup> Even if quality is three-dimensional. I implicitly assume that quality levels can be compared between schools. In other words, the three terms enter a function (e.g. a Cobb-Douglas function) and hence quality levels can be compared. This function is not explicitly specified.

However, under different assumptions, the public sector would also have a hierarchy of qualities. For instance the inclusion of a housing market would result in geographical stratification and in public quality differentiation. This assumption will be considered in the next section.

Note that stratification within the public sector may be the natural product of the housing market. When housing prices, local taxes, transportation costs (friction in school choice) and zoning policies are considered, residential location becomes a major determinant of choice.

In this section of chapter II, I have developed a model of the provision of education in a single community where public and private schools coexist. The existence of an indifference point between the two sectors along with the properties of this point have been discussed.

Several major aspects were introduced in this model. First, school quality is considered to be a function of spending per pupil and peer effects. Second, peer effects are considered to be non-linear in means, conversely to Epple and Romano (1998). Third, the price of private schooling was endogenized and was considered to be the result of profit maximization.<sup>20</sup> Fourth, the resulting stratification is not strictly vertical; some private schools may have identical qualities. And, finally, private schools always have higher quality levels than their public counterparts, due to the absence of friction in choice. **Note that inequalities in the access to education lead to inequalities in achievements, since the latter depend on student ability and school quality,  $a_i = a(b_i, q_j)$ . Under the assumptions used in this model, private schools always generate higher achievements (with ability held constant) because they have higher quality levels than their public counterparts. As a consequence, the stratification between the public and the private sectors and within the private sector is a source of inequality, since different school qualities generate different outcomes.**

However, a number of issues have to be dealt with in the next section. So far I have considered the tax rate to be exogenous; in the next section it will be endogenized through majority voting and the conditions for the existence of a majority voting equilibrium will be discussed.

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<sup>20</sup> In Glomm and Ravikumar, private school tuition was exogenous.

Instead of having a single community, a finite number of communities will be introduced, and local taxes, housing prices and transportation costs will lead to three-dimensional stratification between communities. Moreover, public school quality will be differentiated between communities and the impact of public school choice policies will be assessed. Note that, in the next section, students will be differentiated according to social capital in addition to income and ability and public subsidies to private schools will be allowed.

## **B. Residential stratification and public school choice.**

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In this section of chapter 2, a full general equilibrium model of school choice is developed in a multicomunity framework. Private, mixed finance and public schools coexist. Education is a locally provided good funded with local property taxes. The interaction between student utility maximization and school profit maximization leads to three-dimensional stratification across communities.

In this model, the tax rate is endogenized and the necessary conditions for the existence of a majority voting equilibrium determined by the median voter are established. In the last subsection, the impact of free public school choice is assessed to find that only a small fraction of advantaged students is willing to exercise choice due to a transportation constraint. Moreover, when majority voting conditions are verified, at most 50% of the population minus the median voter is able to exercise public school choice. I also find that some private and mixed finance schools will be losing students in favor of public ones, and that public school choice may not necessarily have a negative effect on public school quality.

This model is related to several previous articles, such as Fernandez and Rogerson (1996), Nechyba (1997, 1999), Epple and Platt (1998) and Epple and Romano (2000). In comparison with Fernandez and Rogerson (1996), Nechyba (1997) and Epple and Romano (2000), education can be supplied privately and by mixed finance schools. Therefore, preferences are no longer single peaked and a number of conditions need to be established in order to insure the existence of a majority voting equilibrium over tax rates. Moreover, in my model, I do not assume that houses have a fixed size, such as in Nechyba (1997).

In comparison with Nechyba (1999), I develop a multicomunity model with a finite number of communities, while the author considers multiple neighborhoods nested within multiple school districts. In addition to that, I adopt a school quality that depends on the number of students in a school, mean ability, mean social capital and the dispersion of the latter.

In Fernandez and Rogerson (1996), complete stratification by income with communities having a single public school arises at equilibrium, while in my model stratification is three-dimensional according to income, ability and social capital.

In comparison with Epple and Platt (1998), I introduce the market for schooling and endogenize the price of education. Similarly to them, single crossing properties arise from individual utility maximization.

In comparison with Epple and Romano (2000), the coexistence of private, public and mixed finance schools leads to non-single peaked preferences and to the inapplicability of Black's median voter theorem. Furthermore, in my model I analyze the effect of introducing public school choice on the already existing equilibrium, while in theirs they construct an equilibrium with choice taken as a factor in its construction. Similarly to Epple and Romano (1998) and (2006), the private sector is active and schools maximize profit under a quality constraint. However, in my model social capital is included as a non-linear determinant of school quality.

## **1. The model.**

In our model, we consider an economy populated with a continuum of households differentiated by income, ability and social capital. Social capital is defined to proxy factors such as social origins, the belonging to a social class and the possession of a durable network of relationships of mutual acquaintance, etc. All these aspects are represented through a scalar,  $k$ . Social capital is taken into account to shed the light on how school quality is affected by the social mix of enrolled students. Each household has one student and both form a single decision making unit. Henceforth, "student" and "household" are used interchangeably.

A student  $i$  has an income  $y_i$ , ability  $b_i$  and a social capital  $k_i$ . Note that  $i$  may designate a particular student or a type of students with the same combination of  $y$ ,  $b$  and  $k$ . These students live in a finite number of communities; a community is designated by an index  $C$ ,



with  $(C = 1, 2, 3, \dots, C)$ , and are enrolled in a finite number of schools; a school is designated by an index  $j$  (with  $j=1, 2, 3, \dots, j$ ). Communities' boundaries are arbitrarily fixed.<sup>21</sup>

Income, ability and social capital are distributed in the population according to the density function  $f(b, y, k)$  which is positive and continuous on its support  $S = (0, b_{\max}) \times (0, y_{\max}) \times (0, k_{\max})$ . Correlations between these three endowments are not considered in this model.

Student utility is assumed to be a function of private consumption and school quality. It is noted as  $U(c, q)$ , where  $c$  is consumption and  $q$  is school quality.  $U$  is increasing, strictly quasi-concave and twice continuously differentiable. We assume that  $\lim_{c \rightarrow +\infty} U_c(c, q) = 0$ , as in the previous section. Students can only attend one school and they cannot supplement this education elsewhere. They are myopic when they vote over tax rates and they lack information on how their votes affect the equilibrium in different communities. Educational achievements are given by  $a_i = a(b_i, q_j)$ ;  $a$  is continuous and increasing in both arguments. Students' achievements depend on their own ability and on the quality of their school. In other words, the access to higher quality schools is translated through higher achievements. In terms of inequalities, unequal access to education generates unequal outcomes.

School quality is determined by expenditure per pupil, ability peer effects, social capital peer effects and the dispersion of social capital. Quality is increasing in all its arguments. Ability peer effects are defined to be average ability in a school, social capital peer effects are defined to be average social capital in a school and the dispersion of social capital is its variance in a school. Evidence from several countries suggests that schools make an effort to achieve social diversity. In my model, this is reflected through the acceptance of high dispersions in social capital in order to ensure diversity and democracy in the access to education and in order to reduce social elitism. Note that increasing quality in expenditure per pupil and in ability peer effects is consistent with the literature on the subject. Summers and Wolfe (1977), Henderson et al and (1978), Sorensen and Hallinan (1986) Kulik (1992) and Epple and Romano (1998) provided evidence that ability peer effects exist.

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<sup>21</sup> This assumption is used in Nechyba 1997 (p.5).

Three types of schools are considered: free admission, free of tuition public schools financed entirely through local government subsidies, mixed finance schools financed through local government subsidies and tuition paid by students and private schools financed solely through tuition.<sup>22</sup> All households pay their taxes, even if their children do not attend public and mixed finance schools.

The cost of housing in community  $C$  is given by the housing price  $r^C$  multiplied by the number of units of housing demanded  $H$ .  $r^C$  is the gross of tax price of housing;  $r^C = (1+t)r_{net}^C$ . Note that  $r_{net}^C$  and  $t$  are endogenous. The tax rate is determined by majority voting. Land is homogenous, and the amount of land may vary across communities. In this model I do not make a difference between renters and owners and I neglect the fact that owners may achieve a capital gain or loss from selling their houses. I also assume absentee land lords who receive the rent.

The housing demand in community  $C$  is given by:

$$H_{dem}^C = \iiint_s H_i^C(r^C, b, y, k) f(b, y, k) db dy dk.$$

It depends on the gross of tax price of housing and the characteristics of a household.

The housing supply is given by:  $H_{sup}^C = H_{sup}(r_{net}^C)$ . It depends on the net price of housing.

Equilibrium requires that “market clearance” is satisfied through the equality between demand and supply. The price of housing is determined through this equality.

Mobility is not costless; students face a transportation cost determined by the distance separating a student’s community  $C$  from his school  $j$ . It is noted  $T_j^C$ .  $T_j^C$  is equal to zero for a school  $j$  geographically belonging to community  $C$  (absence of transportation cost within a community). For two schools geographically belonging to the same community, the transportation cost between these schools and any other community is the same:  $T_j^C = T_{j'}^C$  with  $j$  and  $j'$  two schools belonging to a community different than  $C$ . In other words, in this model, we assume the presence of friction between communities and its absence within communities.

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<sup>22</sup> Mixed finance schools represent government-dependent private schools controlled by non-government organizations or with governing boards not selected by a government agency that receive a considerable part of their core funding from government agencies. Mixed finance schools do not have residency requirements.

Note that the number of students is larger than the number of schools and that the latter is larger than the number of communities. The number and type of schools in each community is arbitrarily fixed. It is possible for a community to lack private or mixed finance schools; in this case, students still have a choice between existing types or between schools outside their community. I assume that each community has a public school. In other words, each student has access to a public school within his jurisdiction when residency requirements are imposed.

The proportion of students of type  $(b, y, k)$  in school  $j$  is given by  $\alpha_j(b, y, k)$ , and the number of students in school  $j$  is given by  $l_j$ . With:

$$l_j = \iiint_s \alpha_j(b, y, k) f(b, y, k) db dy dk \quad (1)$$

And  $\alpha_j(b, y, k) \in [0, 1]$

The proportion of students of type  $(b, y, k)$  in community  $C$  is given by  $\alpha_c(b, y, k)$ , and the number of students is given by:

$$l_c = \iiint_s \alpha_c(b, y, k) f(b, y, k) db dy dk \quad (2)$$

And  $\alpha_c(b, y, k) \in [0, 1]$

## 1.1. Schools.

The production cost of education depends on the number of students enrolled in a school; it is given by  $Co(l_j) = V(l_j) + F = n_1 l_j + n_2 l_j^2 + F$ .  $V' > 0$ ,  $V'' > 0$ ,  $F$  is a fixed cost and  $n_1$  and  $n_2$  are positive constants. Technical differences between schools are not included in the model.<sup>23</sup> The absence of economies of scale in the production of education is translated into a large number of schools catering for different types of students.<sup>24</sup>

In this model, schools have perfect information on students' income, abilities and social capital, while students are myopic as mentioned above.

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<sup>23</sup> This assumption was used in Epple and Romano 1998 (p.38).

<sup>24</sup> Ferris and West (2004) provide evidence that large schools suffer from external and invisible costs "such as social problems that prevent the existence of economies of scale."

Schools are assumed to maximize profit under a quality constraint. Funding is provided by three major sources: government subsidies, tuition paid by students and other earnings.

$$R_j = \iiint_s E_{ij} \alpha_j(b, y, k) f(b, y, k) db dy dk + \iiint_s p_{ij} \alpha_j(b, y, k) f(b, y, k) db dy dk + G_j$$

$E_{ij}$  : Local government per student subsidies in school  $j$ .

$p_{ij}$  : Tuition paid by student  $i$  in school  $j$ .

$G_j$  : Other earnings.

The government local subsidies in community  $C$  for school  $j$  are given by  $E_j^C = \iiint_s E_{ij} \alpha_j(b, y, k) f(b, y, k) db dy dk$ , with  $j \in C$ . At equilibrium, the sum of government subsidies in community  $C$  is equal to tax revenues in this community (tax revenues are equal to the property tax rate in community  $C$  multiplied by the net of tax price of housing and by the number of housing units demanded)  $\sum E_j^C = \sum t^C r_{net} H_i$  in community  $C$ .

School quality is given by:

$$q_j = q_j \left[ \frac{R_j}{l_j}, \theta_j, O_j, \sigma_j^2 \right] \quad (3)$$

With:  $\lim_{R_j \rightarrow 0} q_j = 0$ ,  $\lim_{\theta_j \rightarrow 0} q_j = 0$ ,  $\lim_{O_j \rightarrow 0} q_j = 0$ ,  $\lim_{\sigma_j^2 \rightarrow 0} q_j = 0$ .

$q_j$  is increasing in  $\frac{R_j}{l_j}$ ,  $\theta_j$ ,  $O_j$  and  $\sigma_j^2$ .

Where:

Expenditure per pupil is given by  $\frac{R_j}{l_j}$ .

Ability peer effects are given by average ability in a school:

$$\theta_j = \frac{1}{l_j} \iiint_s b_i \alpha_j(b, y, k) f(b, y, k) db dy dk \quad (4)$$

Social capital peer effects are given by average social capital in a school:

$$O_j = \frac{1}{l_j} \iiint_s k_i \alpha_j(b, y, k) f(b, y, k) db dy dk \quad (5)$$

The dispersion of social capital in a school is given by its variance:

$$\sigma_j^2 = \frac{1}{l_j} \iiint_s (k_i - O_j)^2 \alpha_j(b, y, k) f(b, y, k) db dy dk \quad (6)$$

Ability peer effects, social capital peer effects, the dispersion of social capital and the number of enrolled students represent quality constraints under which school profits are maximized.

The profit of a school is equal to the difference between resources and the cost of producing education  $\pi_j = R_j - Co(l_j)$ . At equilibrium, the profit is equal to zero and no new entries on the education market are possible.<sup>25</sup> In this case,  $R_j = Co(l_j)$ . Since the sum of expenditure

per pupil in a school is given by  $\sum \frac{R_j}{l_j} = l_j \frac{R_j}{l_j} = R_j$ , it is possible to write that at equilibrium

$\sum \frac{R_j}{l_j} \equiv Co(l_j)$ . In other words, the sum of expenditure per student is identical to the education production cost.

## 1.2. Students

Since public, mixed finance and private schools coexist in the model, students have a large set of choices, inside and outside their community. First, when public school choice is not allowed, students have a choice between all the schools of their community and the mixed finance and private schools outside their community. Second, when public school choice is allowed, students have a choice between all schools in the economy. Public school choice is introduced in the last subsection.

**Assumption 1:** pinning down preferences over education and housing.

a- I assume a positive  $(b, y, k)$  elasticity of the demand for educational quality. As  $(b, y, k)$  increases, the educational quality demanded increases. Higher  $(b, y, k)$  students prefer higher quality schools.

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<sup>25</sup> As long as profit is positive, new schools will enter the market. See Epple and Romano 1998 (p.39)

b- I assume a positive ability, income and social capital elasticity of housing demand, such as:  $\frac{\partial H}{\partial y} > 0$ ,  $\frac{\partial H}{\partial b} > 0$ , and  $\frac{\partial H}{\partial k} > 0$ . Housing demand is also assumed to be increasing in

$(b, y, k)$ , such as:  $\frac{\partial H}{\partial(b, y, k)} > 0$ . Higher  $(b, y, k)$  students prefer larger houses.

**Remark (similar to the one in the previous section):**

The comparison between two couples of ability, income and social capital may be difficult. For example, it is possible to say that  $(5,4,3) > (3,2,1)$ . However, it is difficult to compare  $(6,2,1)$  with  $(4,4,2)$ .

In order to make the comparison possible, one can imagine that ability, income and social capital enter a function that determines the level of student endowments. For instance, endowments can take the following form:  $endowments_i = hb_i^\gamma y_i^\beta k_i^\omega$ . This function gives different weights for ability, income and social capital. It would then be possible to compare any two couples in terms of which one generates more endowments. This function also means that students are differentiated according to a single factor that can be decomposed in three.

In my analysis, I do not specify a particular function in order to make two couples comparable. However, I assume implicitly that such a function exists and that students are able to make comparisons. Henceforth, it is possible to say that  $(b_i, y_i, k_i) < (\hat{b}, \hat{y}, \hat{k})$  independently of the position of  $b_i$  in comparison to  $\hat{b}$ , the position of  $y_i$  in comparison to  $\hat{y}$  and the position of  $k_i$  in comparison to  $\hat{k}$ . Note that each combination of  $(b, y, k)$  is not unique, since different levels of  $b$ ,  $y$  and  $k$  may generate the same level of endowments.

**The sequencing of decision making for individuals and schools.**

This model has a single period, decisions are simultaneous and no real time elapses between stages. First, schools maximize their profits, students choose their communities and schools and the housing demand of each household is determined. Second, households vote upon property tax rates. At this level, the sum of subsidies needed in the community is determined, such that the community's budget is balanced  $\sum E_j^C = \sum t^C r_{net} H_i$ .<sup>26</sup> Third, local authorities allocate their budget. They attribute subsidies to public schools first, and then the rest of the funding is arbitrarily attributed to mixed finance schools. Note that schools have perfect knowledge of student types, while students are myopic. Students lack information on how their votes affect the equilibrium in different communities; therefore, migration in and out is unlikely and the equilibrium is stable.

**2. School profit maximization and price discrimination.**

Schools maximize profit under several quality constraints. Even if public schools do not charge tuition, local authorities condition the level of subsidies according to student types. For private and mixed finance schools, the level of quality chosen by a school and the level of local subsidies determine the tuition for each type of students. Private and mixed finance schools do not select students directly. They admit any student who is able to pay the price corresponding to his type. Displaying a prohibitive price is equivalent to refusing to admit a student. The quality constraints include average ability, average social capital, the dispersion of social capital and the number of students  $l_j$  enrolled in a school.

Schools maximize profit under several quality constraints:

$$\max \pi_j \equiv R_j - V(l_j) - F$$

Subject to these constraints:

$$l_j = \iiint_s \alpha_j(b, y, k) f(b, y, k) db dy dk \tag{1}$$

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<sup>26</sup> Recall that  $E_j^C = \sum E_{ij}$ . The subsidies attributed to school j in community C are equal to the sum of per student subsidies in this school.

$$\theta_j = \frac{1}{l_j} \iiint_s b_i \alpha_j(b, y, k) f(b, y, k) db dy dk \quad (4)$$

$$O_j = \frac{1}{l_j} \iiint_s k_i \alpha_j(b, y, k) f(b, y, k) db dy dk \quad (5)$$

$$\sigma_j^2 = \frac{1}{l_j} \iiint_s (k_i - O_j)^2 \alpha_j(b, y, k) f(b, y, k) db dy dk \quad (6)$$

These constraints can be transformed by replacing  $l_j$  by its value:

$$1- \theta_j \iiint_s \alpha_j(b, y, k) f(b, y, k) db dy dk - \iiint_s b_i \alpha_j(b, y, k) f(b, y, k) db dy dk = 0$$

$$2- O_j \iiint_s \alpha_j(b, y, k) f(b, y, k) db dy dk - \iiint_s k_i \alpha_j(b, y, k) f(b, y, k) db dy dk = 0$$

$$3- \sigma_j^2 \iiint_s \alpha_j(b, y, k) f(b, y, k) db dy dk - \iiint_s (k_i - O_j)^2 \alpha_j(b, y, k) f(b, y, k) db dy dk = 0$$

The Lagrangian function is then written in the following form:

$$\begin{aligned} \Phi = & R_j - n_1 l_j - n_2 l_j^2 - F - \mu'_j [\theta_j \iiint_s \alpha_j(b, y, k) f(b, y, k) db dy dk - \iiint_s b_i \alpha_j(b, y, k) f(b, y, k) db dy dk] \\ & - \mu''_j [O_j \iiint_s \alpha_j(b, y, k) f(b, y, k) db dy dk - \iiint_s k_i \alpha_j(b, y, k) f(b, y, k) db dy dk] \\ & - \mu'''_j \left[ \sigma_j^2 \iiint_s \alpha_j(b, y, k) f(b, y, k) db dy dk - \iiint_s (k_i - O_j)^2 \alpha_j(b, y, k) f(b, y, k) db dy dk \right] \end{aligned}$$

With  $\mu'_j, \mu''_j, \text{ and } \mu'''_j$  the Lagrangian multipliers.

Optimization is made through the partial derivation of the Lagrangian function over  $\alpha_j(b, y, k)$  ; the following result is obtained:

$$\frac{\partial \Phi}{\partial \alpha_j(b, y, k)} = R_j^* - n_1 - 2n_2 l_j - \mu'_j (\theta_j - b_i) - \mu''_j (O_j - k_i) - \mu'''_j [\sigma_j^2 - (k_i - O_j)^2] = 0$$

The optimal level of per student resources  $R_j^*$  is the following:

$$MC = R_j^* = n_1 + 2n_2 l_j + \mu'_j (\theta_j - b_i) + \mu''_j (O_j - k_i) + \mu'''_j [\sigma_j^2 - (k_i - O_j)^2] \quad (7)$$

Note that  $R_j^*$  represents the marginal cost MC of admitting a student of type  $(b, y, k)$ . Note that  $R_j^* = E_{ij} + p_{ij}^*$ .



$$\mu'_j = \frac{1}{l_j} \iiint_s \frac{\partial R^*}{\partial \theta_j} \alpha_j(b, y, k) f(b, y, k) db dy dk$$

$$\mu''_j = \frac{1}{l_j} \iiint_s \frac{\partial R^*}{\partial O_j} \alpha_j(b, y, k) f(b, y, k) db dy dk$$

$$\mu'''_j = \frac{1}{l_j} \iiint_s \frac{\partial R^*}{\partial \sigma_j^2} \alpha_j(b, y, k) f(b, y, k) db dy dk$$

$\mu'_j$ ,  $\mu''_j$  and  $\mu'''_j$  are positive and they vary between schools.<sup>27</sup>  $\mu'_j$  represents the per student resources change deriving from a change in  $\theta_j$ ;  $\mu''_j$  represents the per student resources change deriving from a change in  $O_j$ .  $\mu'''_j$  represents the per student resources change deriving from a change in  $\sigma_j^2$ .

At equilibrium, profit is equal to zero,  $\pi_j = 0$ . New entries on the market of education are possible as long as  $\pi_j > 0$ . When  $\pi_j = 0$ , no new entries are possible.

In this equation,  $R_j^*$  represents the resources needed to cover the marginal cost of admitting a student. The first term  $n_1 + 2n_2 l_j$  is the part resulting from the education production cost. It is positive and identical for all students attending the same school. The second, third and fourth terms reflect the externality of one's own ability and social capital on the school.

The second term,  $\mu'_j(\theta_j - b_i)$ , represents the impact of one's ability on average ability. Students with  $\theta_j < b_i$  have a negative externality cost on the school. The reverse is true for  $\theta_j > b_i$ . This term is decreasing in  $b_i$  given a value of  $\theta_j$ .  $\lim_{b \rightarrow +\infty} \mu'_j(\theta_j - b_i) = -\infty$ .

The third term,  $\mu''_j(O_j - k_i)$ , represents the impact of one's social capital on average social capital. Students with  $O_j < k_i$  have a negative externality cost on the school. The reverse is true for  $O_j > k_i$ . This term is decreasing in  $k_i$  given a value of  $O_j$ .  $\lim_{k \rightarrow +\infty} \mu''_j(O_j - k_i) = -\infty$ .

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<sup>27</sup> See the definition of Lagrangian multipliers in Epple and Romano (1998) p. 40.

The fourth term,  $\mu_j''[\sigma_j^2 - (k_i - O_j)^2]$ , represents the cost of being too close to the average of social capital. Those who are far from the mean (who create social diversity) represent a negative externality cost for the school. In other words, when  $k_i$  is far enough (higher or lower) from  $O_j$ ,  $(k_i - O_j)^2$  is positive and high. If it is higher than  $\sigma_j^2$ , then the term  $\mu_j''[\sigma_j^2 - (k_i - O_j)^2]$  is negative. The reverse is true for  $k_i$  close enough to  $O_j$ . This term, given constant values of  $\sigma_j^2$  and  $O_j$ , is concave in  $k_i$  and attains its maximum at  $k_i = O_j$ .  $\lim_{k \rightarrow \pm\infty} \mu_j''[\sigma_j^2 - (k_i - O_j)^2] = -\infty$ .

Note that  $R_j^{**}$  might be negative, depending on the level of government subsidies and the position of individual ability and social capital relative to the means. Equation (7) represents a compensation scheme; low ability students subsidize higher ability ones, low social capital students subsidize high social capital ones and students with social capital close to its average subsidize those who are far from the average.

Equation (7) allows us to overcome the strict distinction between public and private schools. Different types of education finance can be considered: we can start with free admission public schools with  $p_{ij} = 0$  for all  $i$  and go through mixed finance schools where both  $p$  and  $E$  are positive and eventually reach purely private schools where  $E_{ij} = 0$ .

When  $R_j^{**}$  is replaced by its value we obtain the following equations:

**For private schools, we have:**

$$p_{ij}^* = n_1 + 2n_2l_j + \mu_j'(\theta_j - b_i) + \mu_j''(O_j - k_i) + \mu_j''[\sigma_j^2 - (k_i - O_j)^2]$$

With  $E_{ij} = 0$ .<sup>28</sup>

Note that  $p_{ij}^*$  can be negative for some students (a scholarship) but not for all, for the following reason.

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<sup>28</sup> Pricing is done according to the marginal cost of admitting a student.

- A private school can not possibly offer scholarships for all its students. In order to offer a scholarship for one student, another one has to pay a positive tuition. This can be seen through the sum of  $p_{ij}^*$  that is always positive.

$$\sum_{i=1}^{l_j} p_{ij}^* = \sum_{i=1}^{l_j} (n_1 + 2n_2 l_j) + \sum_{i=1}^{l_j} [\mu'_j(\theta_j - b_i)] + \sum_{i=1}^{l_j} [\mu''_j(O_j - k_i)] + \sum_{i=1}^{l_j} \left\{ \mu'''_j [\sigma_j^2 - (k_i - O_j)^2] \right\} > 0 \text{ since}$$

$$\sum_{i=1}^{l_j} (n_1 + 2n_2 l_j) > 0 \text{ and}$$

$$\sum_{i=1}^{l_j} [\mu'_j(\theta_j - b_i)] + \sum_{i=1}^{l_j} [\mu''_j(O_j - k_i)] + \sum_{i=1}^{l_j} \left\{ \mu'''_j [\sigma_j^2 - (k_i - O_j)^2] \right\} = 0$$

Mathematical details :

$$\begin{aligned} \sum_{i=1}^{l_j} [\mu'_j(\theta_j - b_i)] &= \mu'_j(\theta_j - b_1) + \dots + \mu'_j(\theta_j - b_{l_j}) = \mu'_j[(\theta_j - b_1) + \dots + (\theta_j - b_{l_j})] \\ &= \mu'_j \left( l_j \theta_j - \int \int \int b_i \alpha_j(b, y, k) f(b, y, k) db dy dk \right) = \mu'_j(l_j \theta_j - \theta_j l_j) = 0 \end{aligned}$$

$$\text{The same applies for } \sum_{i=1}^{l_j} [\mu''_j(O_j - k_i)] = 0 \text{ and } \sum_{i=1}^{l_j} \left\{ \mu'''_j [\sigma_j^2 - (k_i - O_j)^2] \right\} = 0$$

**For mixed finance schools, we have:**

$$p_{ij}^* = -E_{ij} + n_1 + 2n_2 l_j + \mu'_j(\theta_j - b_i) + \mu''_j(O_j - k_i) + \mu'''_j [\sigma_j^2 - (k_i - O_j)^2]$$

For mixed finance schools, the level of per student subsidies  $E_{ij}$  is determined by authorities and not by optimization. Schools can only choose the level of tuition to apply. Pricing is done according to the level of quality, the type and number of enrolled students and the received subsidies. Note that theoretically it is possible that tuition can be negative for all students, if  $E_{ij}$  is positive and very high. However, this is unrealistic, since authorities would not subsidize schools to the extent that they could give scholarships to all students. A necessary

$$\text{condition is: } \sum_{i=1}^{l_j} (-E_{ij} + n_1 + 2n_2 l_j) > 0 \Rightarrow \sum_{i=1}^{l_j} p_{ij}^* > 0.$$

**For public schools, we have:**

$$E_{ij} = n_1 + 2n_2l_j + \mu'_j (\theta_j - b_i) + \mu''_j(O_j - k_i) + \mu'''_j[\sigma_j^2 - (k_i - O_j)^2]$$

With  $p_{ij} = 0$  for all  $i$ .

Given the quality of a public school, student types determine the amount of local subsidies needed to cover the marginal cost of admitting them. In other words, local authorities determine the level of subsidies according to the type of enrolled students. Note that  $E_{ij}$  can

be negative for some students; however,  $\sum_{i=1}^{l_j} E_{ij} > 0$  for the same reasons mentioned above.

Furthermore, all students are admitted, since public schools adopt an open enrollment policy.

**Lemma 1: describing students' feasible choice sets.**

(i) **In the economy**, the general cost “GC henceforth” of acquiring education is defined to be equal to the sum of tuition and transportation cost between the community of residence  $C$  and school  $j$ ,  $GC_{ij}^C = p_{ij} + T_j^C$ .

Consider a student  $i$  living in community  $C$  and two schools 1 and 2 belonging respectively to two communities  $C'$  and  $C''$ , with  $C' \neq C''$ . Suppose that  $GC_{i1}^C = p_{i1} + T_1^C > p_{i2} + T_2^C = GC_{i2}^C$ . If  $q_1 \leq q_2$ , then, only school 2 is part of the feasible choice set. This is simple to explain: a rational student will always choose school 2, since it offers at least an equal quality at a lower general cost.

If  $q_1 > q_2$ , then both schools are part of the student's feasible choice set. The final choice depends on utility maximization (to be seen in the next subsection). In conclusion, it is possible to say that, in the economy, a feasible choice set must present a hierarchy of qualities that follows the hierarchy of general costs.

Remark: if  $q_1 \leq q_2$  and  $GC_{i1}^C = p_{i1} + T_1^C > p_{i2} + T_2^C = GC_{i2}^C$  for all students in the economy, then at equilibrium school 1 is driven out of the market, since it can not attract any student.

(ii) **In the same community**, if school 1 and 2 belong to the same community ( $C' \equiv C''$ ), then  $T_1^C = T_2^C$  (see the assumption on transportation cost). This is a special case of part (i).

$GC_{i1}^C = p_{i1} + T_1^C > p_{i2} + T_2^C = GC_{i2}^C$  is reduced to  $p_{i1} > p_{i2}$ . If  $q_1 \leq q_2$ , then, only school 2 is part of the feasible choice set, and if  $q_1 > q_2$ , then both schools are part of the student's feasible choice set. The same reasons mentioned above apply here. In conclusion, it is possible to say that, in the same community, a feasible choice set must present a hierarchy of qualities that follows the hierarchy of tuition levels.

It is worth noting that a private school can have a lower quality than that of the public school belonging to the same community. However, this private school cannot attract any student from within its community, since no student will be willing to pay more for less quality. For such a school to exist, it should be able to attract students from outside the community.

### 3. Individual utility maximization.

Students have two connected optimization problems; they have to choose a community and a school. They choose communities and schools simultaneously through utility maximization.

The budget constraint is given by:  $y_i = c_{ij}^C + T_j^C + p_{ij} + r^C H_i$  (8)<sup>29</sup>

Maximization of  $U(c, q)$  subject to constraint (8) yields the indirect utility function:

$$W_{ij}^C [(y_i, b_i, k_i), T_j^C, t^C, p_{ij}, r^C, H_i, q_j] = \max U(y_i - T_j^C - p_{ij} - r^C H_i, q_j). \quad (9)$$

**Single crossing properties:** By virtue of the implicit function theorem, we obtain:<sup>30</sup>

$$\begin{aligned} 1-a \quad \frac{\partial(r+p+T)}{\partial q} \Big|_{w=\bar{w}} &= \frac{\partial r + \partial p + \partial T}{\partial q} = \frac{\partial r}{\partial q} + \frac{\partial p}{\partial q} + \frac{\partial T}{\partial q} = \\ &= -\frac{\partial W/\partial q}{\partial W/\partial r} - \frac{\partial W/\partial q}{\partial W/\partial p} - \frac{\partial W/\partial q}{\partial W/\partial T} = \frac{1}{H} + 1 + 1 = \frac{1}{H} + 2 > 0 \end{aligned}$$

Part (a) implies that indifference curves in the  $[q, (r+p+T)]$  plane have positive slopes<sup>31</sup>.

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<sup>29</sup> Notation remark: small letter c is used to denote consumption, capital letter C is used to denote a community.

<sup>30</sup> See Epple and Platt 1998, page 26.

<sup>31</sup> This part of the assumption means that, in order to obtain higher educational qualities, students are willing to spend more on rental, education and transportation combined.

I am using the  $[q, (r+p+T)]$  plane since community choice is not independent from school choice. In other words, the type of chosen community determines the type of schools that can be afforded; and vice versa.

$$1-b \quad \partial \left( \frac{\partial(r+p+T)}{\partial q} \right) / \partial y = -\frac{H_y}{H^2} < 0$$

For students of the same ability and social capital, any indifference curve in the  $[q, (r+p+T)]$  plane of a higher income student cuts any indifference curve of a lower income student from above. Thus for  $b$  and  $k$  held fixed, indifference curves exhibit single crossing in income. In other words, when income increases the slope of the indifference curve decreases.

$$1-c \quad \partial \left( \frac{\partial(r+p+T)}{\partial q} \right) / \partial b = -\frac{H_b}{H^2} < 0$$

For students of the same income and social capital, any indifference curve in the  $[q, (r+p+T)]$  plane of a higher ability student cuts any indifference curve of a lower ability student from above. Thus for  $y$  and  $k$  held fixed, indifference curves exhibit single crossing in ability. In other words, when ability increases the slope of the indifference curve decreases.

$$1-d \quad \partial \left( \frac{\partial(r+p+T)}{\partial q} \right) / \partial k = -\frac{H_k}{H^2} < 0$$

For students of the same income and ability, any indifference curve in the  $[q, (r+p+T)]$  plane of a higher social capital student cuts any indifference curve of a lower social capital student from above. Thus for  $b$  and  $y$  held fixed, indifference curves exhibit single crossing in social capital. In other words, when social capital increases the slope of the indifference curve decreases.

$$1-e \quad \partial \left( \frac{\partial(r+p+T)}{\partial q} \right) / \partial(b, y, k) = -\frac{H_{(b, y, k)}}{H^2} < 0$$

Any indifference curve in the  $[q, (r+p+T)]$  plane of an individual with a higher combination of  $(b, y, k)$  cuts any indifference curve of a student with a lower combination of  $(b, y, k)$  from

above. Thus, indifference curves exhibit single crossing in  $(b, y, k)$ . In other words, when  $(b, y, k)$  increases the slope of the indifference curve decreases.

**Lemma 2:** Community and school choice: boundary indifference.

Consider two communities  $C$  and  $C+1$  and two schools  $j$  and  $j'$  with  $C \neq C+1$  and  $j \neq j'$ .

For  $r^C < r^{C+1}$ , and  $q_j > q_{j'}$ , there exists a combination  $(\hat{b}, \hat{y}, \hat{k})$  such that:

$$W_{ij}^C \left\{ (\hat{y}_i, \hat{b}_i, \hat{k}_i), T_j^C, t^C, p_{ij}, r^C, H_i, q_j \right\} = W_{ij'}^{C+1} \left\{ (\hat{y}_i, \hat{b}_i, \hat{k}_i), T_{j'}^{C+1}, t^{C+1}, p_{ij'}, r^{C+1}, H_i, q_{j'} \right\} \quad (10)$$

$(\hat{b}, \hat{y}, \hat{k})$  marks the indifference between  $(C, j)$  and  $(C+1, j')$ . Note that  $(\hat{b}, \hat{y}, \hat{k})$  is not unique.<sup>32</sup>

$j$  and  $j'$  may or may not belong to  $C$  and  $C+1$ .

Students choose a community and a school as long as their indirect utility is higher than in any other combination of communities and schools.

In this Lemma, I show that, by choosing lower rental communities, higher  $(b, y, k)$  students would have higher numeraire consumption and higher school quality while having no less housing consumption relative to choosing higher rental communities. The continuity of the utility function and of  $f(b, y, k)$  ensures that there is a type of students that is indifferent between each two combinations of communities and schools.

**Proof of Lemma 2:**

We consider a student  $i$  with  $(b_i, y_i, k_i)$ . He consumes the same amount of housing in both communities  $C$  and  $C+1$ .

His budget constraint in school  $j$  and community  $C$  is given by:  $y_i = c_{ij}^C + T_j^C + p_{ij} + r^C H_i$ .

It can be written in the following form:  $c_{ij}^C = y_i - T_j^C - p_{ij} - r^C H_i$

His budget constraint in school  $j'$  and community  $C+1$  is given by:

$y_i = c_{ij'}^{C+1} + T_{j'}^{C+1} + p_{ij'} + r^{C+1} H_i$ . It can be written in the following form:

$c_{ij'}^{C+1} = y_i - T_{j'}^{C+1} - p_{ij'} - r^{C+1} H_i$ .

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<sup>32</sup>  $(\hat{b}, \hat{y}, \hat{k})$  is not unique; students with other equivalent combinations of  $b$ ,  $y$  and  $k$  may also be indifferent between the two sets of communities and schools. See the remark above.

The difference in consumption between the two combinations of communities and schools is given by:

$$c_{ij'}^{C+1} - c_{ij}^C = (T_j^C - T_{j'}^{C+1}) + (p_{ij} - p_{ij'}) + (r^C - r^{C+1})H_i.$$

The sum  $(T_j^C - T_{j'}^{C+1}) + (p_{ij} - p_{ij'})$  can be positive, negative or equal to zero.

Since  $r^C < r^{C+1}$ , we can write  $r^C - r^{C+1} < 0$ .

Even if the distribution of  $(b, y, k)$  has an upper bound determined by  $(b_{\max}, y_{\max}, k_{\max})$  and a lower bound determined by  $(0, 0, 0)$ , it is analytically convenient for the proof to consider  $(b, y, k) \rightarrow +\infty$  and  $(b, y, k) \rightarrow -\infty$ .

We have two cases:

1- According to part (b) of assumption 1, the housing demand is increasing in  $(b, y, k)$ . For  $(b, y, k) \rightarrow +\infty$ , we have  $H_i \rightarrow +\infty$ , and then  $\lim_{(b, y, k) \rightarrow +\infty} (c_{ij'}^{C+1} - c_{ij}^C) = -\infty$ . In other terms, we can always find a  $(b, y, k)$  combination that is sufficiently high such as  $(c_{ij'}^{C+1} - c_{ij}^C) < 0$ .

If this student is to locate to C+1, he will consume the same amount of housing while having lower numeraire consumption. So, student  $(b_i, y_i, k_i)$  strictly prefers community C.

According to part (a) of assumption 1, the demanded for educational quality is increasing in  $(b, y, k)$ . As a consequence, when  $(b, y, k) \rightarrow +\infty$ , the student chooses school  $j$  with  $q_j > q_{j'}$ .

2- According to part (b) of assumption 1, the housing demand is increasing in  $(b, y, k)$ . For  $(b, y, k) \rightarrow -\infty$ , we have  $H_i \rightarrow -\infty$ ,

and then  $\lim_{(b, y, k) \rightarrow -\infty} (c_{ij'}^{C+1} - c_{ij}^C) = +\infty$ . In other terms, we can always find a  $(b, y, k)$  combination that is sufficiently low, such as  $(c_{ij'}^{C+1} - c_{ij}^C) > 0$ . If this student is to locate to C+1, he will consume the same amount of housing while having a higher consumption. So, student  $(b_i, y_i, k_i)$  strictly prefers community C+1.

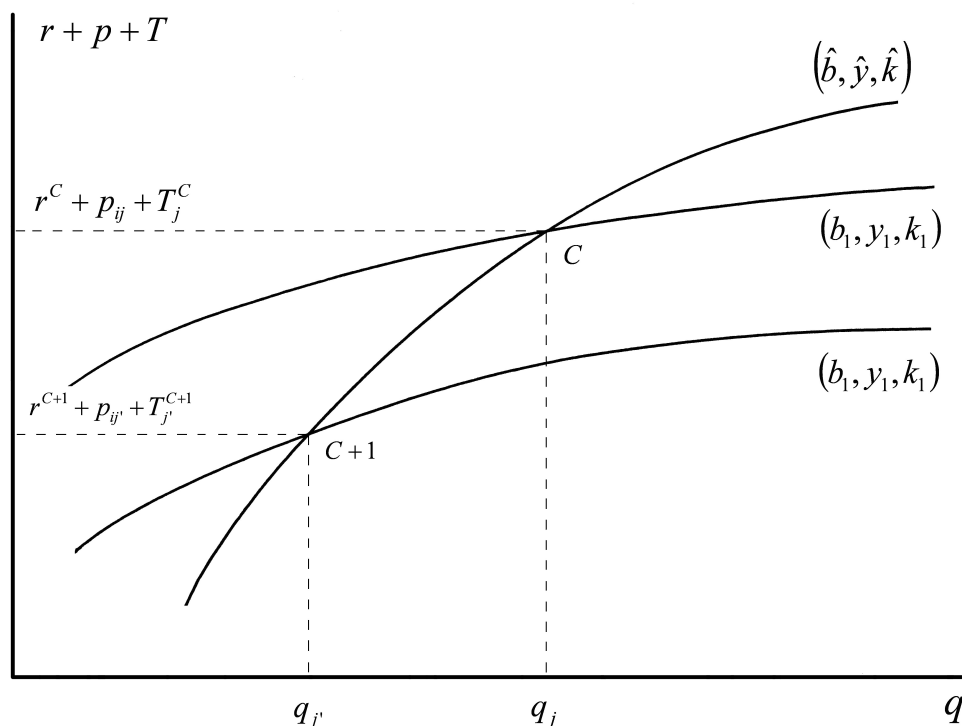
According to part (a) of assumption 1, when  $(b, y, k) \rightarrow -\infty$ , the student chooses school  $j'$  with  $q_j > q_{j'}$ .



We have seen that high  $(b, y, k)$  students prefer communities with lower rental and schools with higher quality and that students with low  $(b, y, k)$  prefer communities with higher rental and schools with lower quality. The continuity of the utility function and of the density function  $f(b, y, k)$  implies that, while going from an extreme to the other, there exists a type of student with  $(\hat{b}, \hat{y}, \hat{k})$  that is indifferent between the two communities and the two schools. The proof that is applied to two communities and two schools can be extended to multiple communities and schools. Between each two consecutive communities and schools there exists a type of student indifferent between them.

**Corollary 1:** The boundary loci  $(\hat{b}, \hat{y}, \hat{k})$  is strictly monotonically decreasing in  $b, y, k$ , and  $(b, y, k) \forall C$ . In this corollary, I show that any student with a better combination than  $(\hat{b}, \hat{y}, \hat{k})$  can not be indifferent between the two combinations of communities and schools,  $(C, j)$  and  $(C + 1, j')$ , as shown in figure 2.2.

Figure 2.2. Existence of Equilibrium.



**Proof of corollary 1:** The proof is by contradiction.

Suppose that  $(\hat{b}, \hat{y}, \hat{k})$  is somewhere nondecreasing in  $b, y, k$  and  $(b, y, k)$ . Then we can find a student with  $(b_1, y_1, k_1)$  indifferent between the two combinations of communities and schools,  $(C, j)$  and  $(C+1, j')$ , such as:

- |  |  |
|--|--|
| 1- $b_1 > \hat{b}, y_1 \geq \hat{y},$ and $k_1 \geq \hat{k}.$  | 4- $b_1 > \hat{b}, y_1 > \hat{y},$ and $k_1 \geq \hat{k}.$ |
| 2- $b_1 \geq \hat{b}, y_1 > \hat{y},$ and $k_1 \geq \hat{k}.$  | 5- $b_1 > \hat{b}, y_1 \geq \hat{y},$ and $k_1 > \hat{k}.$ |
| 3- $b_1 \geq \hat{b}, y_1 \geq \hat{y}$ and $k_1 > \hat{k}.$   | 6- $b_1 \geq \hat{b}, y_1 > \hat{y},$ and $k_1 > \hat{k}.$ |
| 7- $b_1 > \hat{b}, y_1 > \hat{y}, k_1 > \hat{k}$ or $(b_1, y_1, k_1) > (\hat{b}, \hat{y}, \hat{k}).$ |  |

Note that all these alternatives are special cases of  $(b_1, y_1, k_1) > (\hat{b}, \hat{y}, \hat{k})$ . I included them in order to achieve more clarity.

For case 1,

According to property (1-c), the slope of the indifference curve of  $(b_1, \hat{y}, \hat{k})$  is lower than that of  $(\hat{b}, \hat{y}, \hat{k})$ . According to properties (1-b) and (1-d), the slope of  $(b_1, y_1, k_1)$  is equal or lower than that of  $(b_1, \hat{y}, \hat{k})$ .

$$\text{Slope } (b_1, y_1, k_1) \leq \text{Slope } (b_1, \hat{y}, \hat{k}) < \text{Slope } (\hat{b}, \hat{y}, \hat{k})$$

For case 2:

According to property (1-b), the slope of the indifference curve of  $(\hat{b}, y_1, \hat{k})$  is lower than that of  $(\hat{b}, \hat{y}, \hat{k})$ . According to properties (1-c) and (1-d), the slope of the indifference curve of  $(b_1, y_1, k_1)$  is lower or equal to that of  $(\hat{b}, y_1, \hat{k})$ .

$$\text{Slope } (b_1, y_1, k_1) \leq \text{Slope } (\hat{b}, y_1, \hat{k}) < \text{Slope } (\hat{b}, \hat{y}, \hat{k})$$

For case 3:

According to property (1-d), the slope of the indifference curve of  $(\hat{b}, \hat{y}, k_1)$  is lower than that of  $(\hat{b}, \hat{y}, \hat{k})$ . According to properties (1-b) and (1-c), the slope of the indifference curve of  $(b_1, y_1, k_1)$  is lower or equal to that of  $(\hat{b}, \hat{y}, k_1)$ .

$$\text{Slope } (b_1, y_1, k_1) \leq \text{Slope } (\hat{b}, \hat{y}, k_1) < \text{Slope } (\hat{b}, \hat{y}, \hat{k})$$

For Case 4:

According to properties (1-b) and (1-c), the slope of the indifference curve of  $(b_1, y_1, \hat{k})$  is lower than that of  $(\hat{b}, \hat{y}, \hat{k})$ . According to property (1-d), the slope of the indifference curve of  $(b_1, y_1, k_1)$  is equal or lower than that of  $(b_1, y_1, \hat{k})$ .

$$\text{Slope } (b_1, y_1, k_1) \leq \text{Slope } (b_1, y_1, \hat{k}) < \text{Slope } (\hat{b}, \hat{y}, \hat{k})$$

For case 5:

According to properties (1-c) and (1-d), the slope of the indifference curve of  $(b_1, \hat{y}, k_1)$  is lower than that of  $(\hat{b}, \hat{y}, \hat{k})$ . According to property (1-b), the slope of the indifference curve of  $(b_1, y_1, k_1)$  is equal or lower than that of  $(b_1, \hat{y}, k_1)$ .

$$\text{Slope } (b_1, y_1, k_1) \leq \text{Slope } (b_1, \hat{y}, k_1) < \text{Slope } (\hat{b}, \hat{y}, \hat{k})$$

For case 6:

According to properties (1-b) and (1-d), the slope of the indifference curve of  $(\hat{b}, y_1, k_1)$  is lower than that of  $(\hat{b}, \hat{y}, \hat{k})$ . According to property (1-c), the slope of  $(b_1, y_1, k_1)$  is equal or lower than that of  $(\hat{b}, y_1, k_1)$ .

$$\text{Slope } (b_1, y_1, k_1) \leq \text{Slope } (\hat{b}, y_1, k_1) < \text{Slope } (\hat{b}, \hat{y}, \hat{k})$$

For case 7:

According to properties (1-b), (1-c), (1-d) and (1-e), the slope of the indifference curve of  $(b_1, y_1, k_1)$  is lower than that of  $(\hat{b}, \hat{y}, \hat{k})$ .

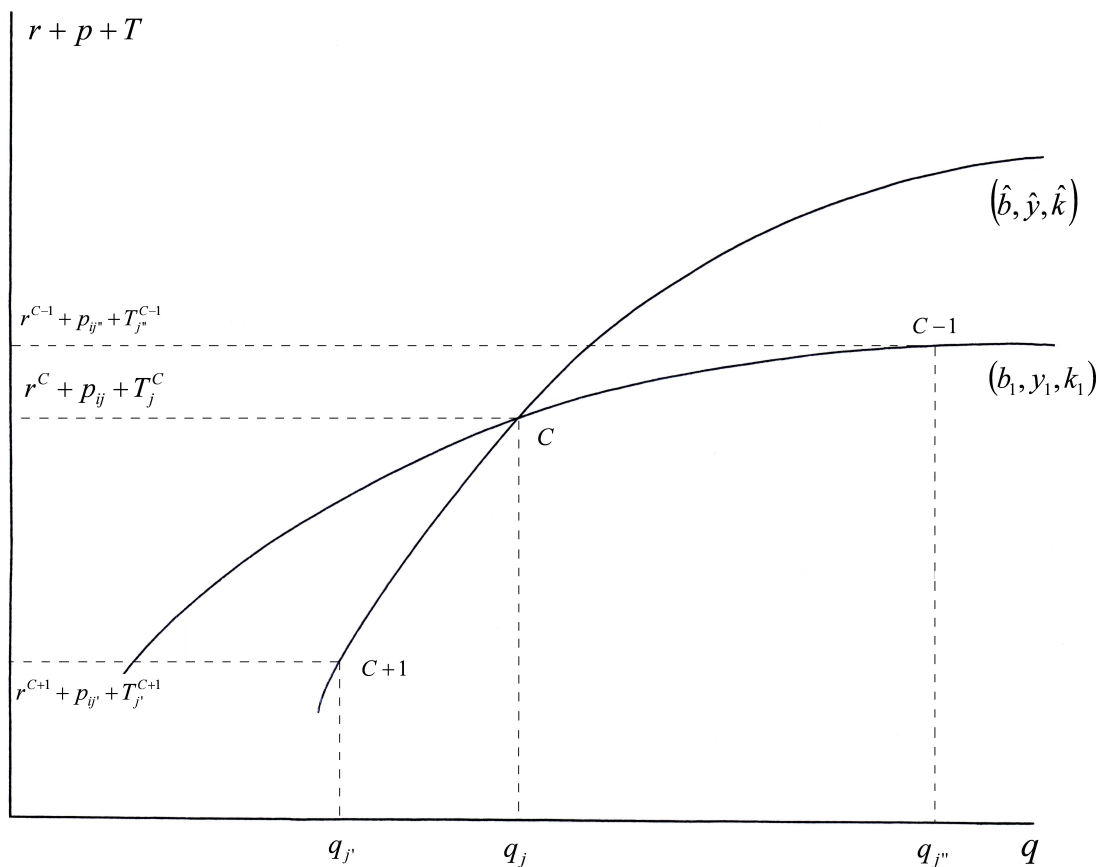
$$\text{Slope } (b_1, y_1, k_1) < \text{Slope } (\hat{b}, \hat{y}, \hat{k})$$

In order to be indifferent between two combinations of communities and schools  $(C, j)$  and  $(C+1, j')$ , the indifference curve must pass through the two points  $[q_j, (r^C + p_{ij} + T_j^C)]$  and  $[q_{j'}, (r^{C+1} + p_{ij'} + T_{j'}^{C+1})]$  (see figure 2.2). Thus, the slope of the indifference curve of  $(b_1, y_1, k_1)$  must be at some point greater than that of  $(\hat{b}, \hat{y}, \hat{k})$  in order to cross it at these two points. Following the 7 cases previously developed, this is a contradiction. The slope of the indifference curve of  $(b_1, y_1, k_1)$  is always smaller than that of  $(\hat{b}, \hat{y}, \hat{k})$  and there is only one intersection between the two indifference curves. The student with  $(b_1, y_1, k_1)$  can not be

indifferent between the two combinations, and the boundary loci between communities and schools are strictly monotonically decreasing in  $b$ ,  $y$ ,  $k$ , and  $(b, y, k)$ .

**Lemma 3:** (i) Students with higher combinations of  $(b, y, k)$  spend at least the same amount on rental, schooling and transportation combined  $(r + p + T)$  and choose at least the same level of educational quality in comparison with lower  $(b, y, k)$  students. (ii) Higher  $(b, y, k)$  students spend at most the same amount on rental, and have a higher housing consumption, in comparison with lower  $(b, y, k)$  students. (iii) Higher  $(b, y, k)$  students spend at least the same amount on the general cost of acquiring education in comparison with lower  $(b, y, k)$  students. Recall that the general cost of acquiring education is equal to the sum of tuition and transportation costs.

Figure 2.3. Three dimensional stratification.



**Proof of Lemma 3:**

Consider two students with  $(b_1, y_1, k_1) > (\hat{b}, \hat{y}, \hat{k})$ . According to property (1-e), the indifference curve of student  $(b_1, y_1, k_1)$  has a lower slope than that of student  $(\hat{b}, \hat{y}, \hat{k})$ . There is only one intersection between their indifference curves, such that they can not be indifferent between the same two combinations of communities and schools. See figure 2.3 and corollary 1.

(i) Note that  $C + 1 \neq C \neq C - 1$  and  $j'' \neq j \neq j'$ .

According to Lemma 2, with  $q_j > q_{j'}$  and  $r^C < r^{C+1}$  we can find a student  $(\hat{b}, \hat{y}, \hat{k})$  indifferent between these two combinations  $(C + 1, j')$  and  $(C, j)$ . His indifference curve passes through these two points  $[q_{j'}, (r^{C+1} + p_{ij'} + T_{j'}^{C+1})]$  and  $[q_j, (r^C + p_{ij} + T_j^C)]$ .

According to Lemma 2, with  $q_{j''} > q_j$  and  $r^{C-1} < r^C$  we can find a student  $(b_1, y_1, k_1)$  indifferent between these two combinations  $(C, j)$  and  $(C - 1, j'')$ . His indifference curve passes through these two points  $[q_j, (r^C + p_{ij} + T_j^C)]$  and  $[q_{j''}, (r^{C-1} + p_{ij''} + T_{j''}^{C-1})]$ .

We can see that there is only one intersection between the two indifference curves at point  $[q_j, (r^C + p_{ij} + T_j^C)]$ .

From the explanation mentioned above, we can write  $q_{j''} > q_j > q_{j'}$ . From Property (1-a) we know that indifference curves have positive slopes in the  $[q, (r + p + T)]$  plane, and thus  $q_{j''} > q_j > q_{j'}$  implies  $(r^{C-1} + p_{ij''} + T_{j''}^{C-1}) > (r^C + p_{ij} + T_j^C) > (r^{C+1} + p_{ij'} + T_{j'}^{C+1})$ .

For a student with  $(\hat{b}, \hat{y}, \hat{k})$  choosing  $(C + 1, j')$ , he would pay  $(r^{C+1} + p_{ij'} + T_{j'}^{C+1})$ . For a student with  $(b_1, y_1, k_1)$  choosing  $(C - 1, j'')$ , he would pay  $(r^{C-1} + p_{ij''} + T_{j''}^{C-1})$ . If both students choose  $(C, j)$ , they would spend the same amount  $(r^C + p_{ij} + T_j^C)$ . With  $(b_1, y_1, k_1) > (\hat{b}, \hat{y}, \hat{k})$ ,  $q_j > q_{j'}$  and  $(r^C + p_{ij} + T_j^C) > (r^{C+1} + p_{ij'} + T_{j'}^{C+1})$ , we deduce that students with a higher combination of  $(b, y, k)$  spend at least the same amount on rental, schooling and transportation combined, while choosing at least the same school quality.

(ii) From part (i) of this Lemma we can write:  $r^{C-1} < r^C < r^{C+1}$ .

Higher  $(b, y, k)$  students spend at most the same amount on rental in comparison with lower  $(b, y, k)$  students.

From Assumption (1-b), we know that:  $\frac{\partial H}{\partial(b, y, k)} > 0$ .

Thus higher  $(b, y, k)$  students have higher housing consumption.

(iii) From part (i) and (ii) of this lemma we can analyze the different possible cases of spending on transportation and tuition. From part (i), we have  $r^{C-1} < r^C < r^{C+1}$  and  $(r^{C-1} + p_{ij'} + T_j^{C-1}) > (r^C + p_{ij} + T_j^C) > (r^{C+1} + p_{ij'} + T_j^{C+1})$ .

We can write  $(r^C + p_{ij} + T_j^C) > (r^{C+1} + p_{ij'} + T_j^{C+1})$ , and  $r^C < r^{C+1}$ .

We can also write  $(r^{C-1} + p_{ij'} + T_j^{C-1}) > (r^C + p_{ij} + T_j^C)$  and  $r^{C-1} < r^C$ .

Thus  $(p_{ij'} + T_j^{C+1}) - (p_{ij} + T_j^C) < r^C - r^{C+1} < 0$ , and  $(p_{ij'} + T_j^C) - (p_{ij''} + T_j^{C-1}) < r^{C-1} - r^C < 0$

This implies that  $(p_{ij'} + T_j^{C+1}) < (p_{ij} + T_j^C) < (p_{ij''} + T_j^{C-1})$ .

For a student with  $(\hat{b}, \hat{y}, \hat{k})$  choosing  $(C+1, j')$ , he would pay  $(p_{ij'} + T_j^{C+1})$ . For a student with  $(b, y_1, k_1)$  choosing  $(C-1, j'')$ , he would pay  $(p_{ij''} + T_j^{C-1})$ . If both students choose  $(C, j)$ , they would spend the same amount  $(p_{ij} + T_j^C)$ . Higher  $(b, y, k)$  students spend at least the same amount on the general cost of education  $(p + T)$ .

Note that tuition and transportation are substitutable. Higher  $(b, y, k)$  students get at least the same school quality and spend at least the same amount on the general cost of education  $(p + T)$  in comparison to lower  $(b, y, k)$  students. However, they may substitute transportation for tuition in getting this quality, and vice versa.

**Proposition 1:** Three-dimensional stratification.

Consider the same two students from Lemma 3. Student  $(\hat{b}, \hat{y}, \hat{k})$  is indifferent between these two combinations,  $(C+1, j')$  and  $(C, j)$ ; while student  $(b_1, y_1, k_1)$  is indifferent between these two combinations,  $(C, j)$  and  $(C-1, j'')$ . Equilibrium exists if  $(b, y, k)$  chooses  $(C, j)$  when  $(\hat{b}, \hat{y}, \hat{k}) < (b, y, k) < (b_1, y_1, k_1)$ .

By virtue of proposition 1, and corollary 1 (Boundary Loci are strictly monotonically decreasing in  $b, y, k$ , and  $(b, y, k)$ ), students are stratified into communities and schools according to their ability, income and social capital. In other words, complete stratification by ability, income or social capital does not occur. Hence, we will have a mixture of individuals with different combinations of  $b, y$  and  $k$  within each community.

**Proof of Proposition 1:**

1- Student  $(b_1, y_1, k_1)$  is indifferent between these two combinations  $(C, j)$  and  $(C-1, j'')$  with  $(r^{C-1} + p_{ij''} + T_j^{C-1}) > (r^C + p_{ij} + T_j^C)$  and  $q_{j''} > q_j$ . Student  $(b, y, k)$  with  $(b, y, k) < (b_1, y_1, k_1)$  spends at most the same amount on rental, schooling and transportation combined, and chooses at most the same school quality, in comparison to  $(b_1, y_1, k_1)$  (see lemma 3). Thus he spends  $(r^C + p_{ij} + T_j^C)$  or less and chooses  $q_j$  or less.

2- Student  $(\hat{b}, \hat{y}, \hat{k})$  is indifferent between these two combinations,  $(C+1, j')$  and  $(C, j)$ , with  $(r^C + p_{ij} + T_j^C) > (r^{C+1} + p_{ij'} + T_j^{C+1})$  and  $q_j > q_{j'}$ .

Student  $(b, y, k)$ , with  $(\hat{b}, \hat{y}, \hat{k}) < (b, y, k)$ , spends at least the same amount on rental, schooling and transportation combined, and chooses at least the same school quality, in comparison to  $(\hat{b}, \hat{y}, \hat{k})$  (see lemma 3). Thus he spends  $(r^C + p_{ij} + T_j^C)$  or more and chooses  $q_j$  or more.

From both parts we can deduce that student  $(b, y, k)$  satisfying  $(\hat{b}, \hat{y}, \hat{k}) < (b, y, k) < (b_1, y_1, k_1)$  must strictly prefer the combination  $(C, j)$ , with  $[q_j, (r^C + p_{ij} + T_j^C)]$ .

#### 4. Majority voting equilibrium.

So far, the tax rate has been considered to be exogenous. In the rest of this section the tax rate is endogenized through majority voting and the necessary equilibrium conditions are established. Tax rates influence educational quality through local subsidies  $E_j^C$ . As mentioned above, students are myopic; they are unlikely to migrate between communities after the equilibrium is established, since they lack the information on the evolution of taxes, rents, school quality and all other variables in the rest of the economy.

In this section, as in the previous ones, students have a choice between all schools of their community and private and mixed finance schools outside their community. In the following figure, I consider a particular community where A, B, and C are utility curves for three different students with 1.2, 1 and 0.8 of the median combination of  $(b, y, k)$  respectively.

In the following paragraphs, I discuss the impact of property tax rates on student behavior. I consider student B as an example. Given the levels of average ability and average social capital in each school, low tax rates lead to low levels of local subsidies and to low public and mixed finance school qualities (within individual B's community). This can be understood since  $\lim_{R_j \rightarrow 0} q_j = 0$ . In this case, student B prefers private schools (inside or outside his community) or mixed finance schools outside his community.

A marginal increase in tax rates does not sufficiently increase the quality of the community's public and mixed finance schools. In contrast, it reduces numeraire consumption and utility. Student B still prefers private schools inside or outside his community or mixed finance schools outside his community. At this stage, student B will not migrate from his community, because he anticipates an increase in utility when the tax rate becomes sufficiently high.

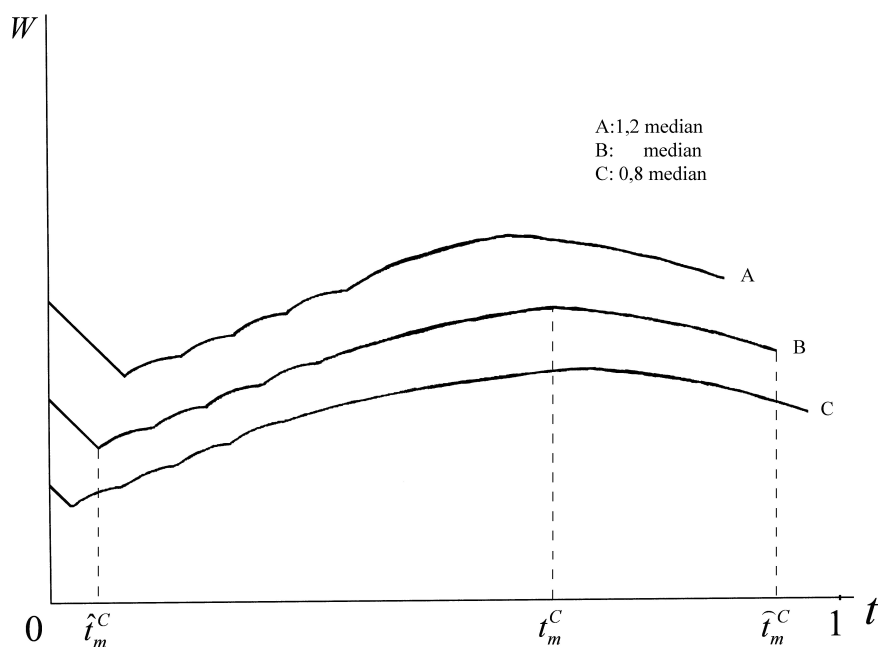
An important increase of the tax rate leads to an important increase of the quality of the community's public and mixed finance schools. At a certain level of the tax rate ( $\hat{t}_m^C$  in figure 2.4), student B is willing to move from his private school (inside or outside his community) or from his mixed finance school (outside his community) to a mixed finance school inside the community. When the tax rate increases further, student B's utility increases until it



reaches its maximum for this school. At this point, student B moves again to the following mixed finance school, and so on. This process continues until the tax rate is high enough to encourage student B to opt for the public school of his community. Utility increases until it reaches its highest level, at  $t_m^C$ . From this point on, any increase in tax rates induces a decrease in utility. Once the tax rate becomes excessive such that the gross of tax price of housing is very high, student B anticipates that any further increase in the tax rate will decrease his utility by more. He decides to migrate when the tax rate reaches  $\hat{t}_m^C$ . Note that preferences are not single peaked and that the “Ends against the Middle” phenomenon is present in this model.

Remark: students enrolled in a private school (inside or outside their community) or a mixed finance school outside their community have a decreasing utility in the tax rate, since they do not use public or mixed finance schools inside their community (this decreasing part of the utility curve can be seen on the left-hand side of figure 2.4). Private school qualities are independent of the tax rate, since  $E_j^C = 0$ , and mixed finance schools’ qualities are independent of the tax rates of communities to which they do not belong.

Figure 2.4: Majority voting over tax rates.



I define  $\hat{t}_i^C$  to be the tax rate at which student  $i$  is indifferent **between** a private school (inside or outside his community) or a mixed finance school (outside his community) **and** a mixed finance school inside his community. I also define  $t_i^C$  to be the tax rate at which the utility of student  $i$  is maximal in the public school and, finally,  $\widehat{t}_i^C$  the tax rate at which student  $i$  is willing to migrate from community  $C$ .  $i = m$  is used to designate the student with the median combination of  $(b, y, k)$ . Note that students are not willing to migrate at  $\hat{t}_i^C$  because they anticipate an increase in utility when the tax rate becomes sufficiently high. In contrast, they are willing to migrate after  $\widehat{t}_i^C$ , because any further increase of the tax rate will reduce their disposable income and their utility by more.

In figure 2.4, some extreme cases can be discussed. For instance, some students with very high  $(b, y, k)$  combinations may always prefer private schools (inside or outside their community) or mixed finance schools outside their community. Their utility curves are everywhere decreasing in the tax rate. For students with very low combinations of  $(b, y, k)$ , their only choice may consist of the public school of the community; in this case these students do not have the decreasing part on the left hand side of the utility curve and they do not have a  $\hat{t}_i^C$ .

Note that the number and type of schools in a community has an important effect on choice. For instance, if the community does not have any mixed finance schools, the student would be indifferent (at  $\hat{t}_i^C$ ) **between** private schools (inside or outside his community) or mixed finance schools outside his community **and** the public school of his community. For the rest of the paper, I adopt the general case where all school types exist. All the results are applicable to particular cases.

**Conditions for the existence of a majority voting equilibrium determined by the median voter:**

- 1- Students with the median combination of ability, income and social capital  $(b, y, k)_m$  prefer a positive tax rate while choosing a public school (noted c) **over** a zero tax rate while choosing a private one (inside or outside their community) or a mixed finance school outside their community (noted a).

$W_{mc}^c[(b, y, k)_m, T_c^c, t_m^c, p_{mc}, r^c, H_m, q_c] > W_{ma}^c[(b, y, k)_m, T_a^c, 0, p_{ma}, r^c, H_m, q_a]$  is true.

$T_c^c = 0$  since  $c \in C$ , while  $T_a^c$  may be different than zero.

This also implies that  $\hat{t}_m^c < t_m^c$ .

2- As seen in figure 2.4, all students with  $(b_i, y_i, k_i) > (b, y, k)_m$  have a migration tax rate  $\hat{t}_i^c$  higher than  $t_m^c$  ( $t_m^c$  is the tax rate that maximizes the utility of the student with  $(b, y, k)_m$  in the public school). If this condition is not verified, at equilibrium some students with  $(b_i, y_i, k_i) > (b, y, k)_m$  may migrate from the community, and the equilibrium is not stable.

3- As seen in figure 2.4,  $t_i^c$  is decreasing in  $(b, y, k)$ .

4- If two students 1 and 2 with  $(b_1, y_1, k_1) > (b_2, y_2, k_2)$  are enrolled in the same school, live in the same community and have the same tax rate, then student 1 has a lower utility such as:

$$W_{2a}^c[(b_2, y_2, k_2), T_a^c, t^c, p_{2a}, r^c, H_2, q_a] > W_{1a}^c[(b_1, y_1, k_1), T_a^c, t^c, p_{1a}, r^c, H_1, q_a]$$

If they are enrolled in two different schools, a and b respectively, with  $q_a > q_b$ , then student 1 has a higher utility such as:

$$W_{2b}^c[(b_2, y_2, k_2), T_b^c, t^c, p_{2b}, r^c, H_2, q_b] < W_{1a}^c[(b_1, y_1, k_1), T_a^c, t^c, p_{1a}, r^c, H_1, q_a]$$

This condition reflects assumption (1-a); higher  $(b, y, k)$  students prefer higher quality schools. If they are enrolled in the same school with lower  $(b, y, k)$  students, they would have lower utilities. In other words, higher  $(b, y, k)$  students always prefer higher qualities even if they would lose on consumption; this can be seen through  $\lim_{c \rightarrow +\infty} U_c(c, q) = 0$ .<sup>33</sup> More intuitively, being enrolled in the same schools and living in the same communities as low  $(b, y, k)$  students would depress high  $(b, y, k)$  students' utilities.

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<sup>33</sup> Epple and Romano (1996) used an assumption where education is considered to be a superior good; they also used a diminishing marginal utility of numeraire consumption; see page 300. Glomm and Ravikumar (1998) used an assumption where utility decreases in consumption with  $\lim_{c \rightarrow +\infty} U_c(c, q) = 0$ . In this chapter I have adopted the same assumption as in Glomm and Ravikumar's paper.

If condition 1 is not satisfied, then  $t = 0$  is a majority voting equilibrium. If 2, 3 and 4 are not satisfied, then majority voting equilibrium may not exist. For the rest of the chapter I consider that these conditions are satisfied. These conditions are necessary in order to pin down the tax rate.

**Definition:** if  $t^C$  is to be a majority voting equilibrium in community C, then there is no other tax rate that is preferred by more than 50% of the population of the community. In other terms, at least 50% and one individual choose this tax rate.

**Lemma 4:**

(i)  $\hat{t}_i^C$  is increasing in  $(b, y, k)$ .

This part can be extended to any indifference tax rate between a private school and a mixed finance school, between a private school and a public school, between two mixed finance schools and between a mixed finance and a public school. It can be explained intuitively: higher  $(b, y, k)$  students need higher tax rates and thus higher school qualities to be encouraged to opt for a mixed finance or a public school within their community.

(ii) In this part of lemma 4 we prove that if the median voter  $(b, y, k)_m$  prefers “**or does not prefer**” a positive tax rate while choosing a public school **over** a zero tax rate while choosing a private one (inside or outside his community) or a mixed finance school outside his community, all students with lower “**or higher**” combinations of  $(b, y, k)$  have the same behavior.

Consider three schools, a, b and c. a and b are two private schools (inside or outside community C) or two mixed finance schools outside community C while c is a public school inside community C.

1- If  $W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] > W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a]$  is true.<sup>34</sup>

Then  $W_{1c}^C[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c] > W_{1b}^C[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b]$  for all  $(b_1, y_1, k_1) < (b, y, k)_m$ .

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<sup>34</sup> Note that since c is a public school  $p_c = 0$ , and for schools belonging to community C,  $T = 0$ .

2- If  $W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] < W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a]$  is true.

Then  $W_{1c}^C[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c] < W_{1b}^C[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b]$  for all  $(b_1, y_1, k_1) > (b, y, k)_m$ .

Note that according to condition 1, the median voter  $(b, y, k)_m$  prefers the tax rate  $t_m^C$  over zero. It will be sufficient to prove that 50% of the population prefers  $t_m^C$  to prove the existence of a majority voting equilibrium determined by the median voter. This has to be done over three intervals:  $[0, \hat{t}_m^C]$ ,  $[\hat{t}_m^C, t_m^C]$  and  $[t_m^C, 1]$ . The type of student with the median combination of  $(b, y, k)$  splits the population into two identical fractions of 50%.

**Proof of lemma 4:**

(i) The proof is by contradiction.

Consider two students 1 and 2 with  $(b_1, y_1, k_1) > (b_2, y_2, k_2)$  living in community C, and three schools a, b and c. a and b are two private schools (inside or outside community C) or two mixed finance schools outside community C with  $q_a > q_b$  and c is a mixed finance school inside community C.<sup>35</sup>

Since higher  $(b, y, k)$  students choose higher educational qualities, as indicated in assumption (1-a), student  $(b_1, y_1, k_1)$  chooses school a while student  $(b_2, y_2, k_2)$  chooses school b. In this lemma, I analyze the position of indifference tax rates of students 1 and 2. Note that student 1 is indifferent between a and c, while student 2 is indifferent between b and c.

For student 1:  $\hat{t}_1^C$  is his indifference tax rate between a and c. We can write:

$$W_{1a}^C[(b_1, y_1, k_1), T_a^C, \hat{t}_1^C, p_{1a}, r^C, H_1, q_a] = W_{1c}^C[(b_1, y_1, k_1), T_c^C, \hat{t}_1^C, p_{1c}, r^C, H_1, q_c] \quad (1)$$

For student 2:  $\hat{t}_2^C$  is the indifference tax rate between b and c. We can write:

$$W_{2b}^C[(b_2, y_2, k_2), T_b^C, \hat{t}_2^C, p_{2b}, r^C, H_2, q_b] = W_{2c}^C[(b_2, y_2, k_2), T_c^C, \hat{t}_2^C, p_{2c}, r^C, H_2, q_c]$$

We suppose that  $\hat{t}_1^C < \hat{t}_2^C$  is true.

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<sup>35</sup> Since school c belongs to community C, then  $T_c^C = 0$ , while  $T_a^C$  and  $T_b^C$  may be different than zero.

By virtue of figure 2.4 and the corresponding discussion, we know that for any  $t < \hat{t}_2^C$  student 2 prefers school b.

$$W_{2b}^C[(b_2, y_2, k_2), T_b^C, t, p_{2b}, r^C, H_2, q_b] > W_{2c}^C[(b_2, y_2, k_2), T_c^C, t, p_{2c}, r^C, H_2, q_c]$$

Since we supposed that  $\hat{t}_1^C < \hat{t}_2^C$ , we can write:

$$W_{2b}^C[(b_2, y_2, k_2), T_b^C, \hat{t}_1^C, p_{2b}, r^C, H_2, q_b] > W_{2c}^C[(b_2, y_2, k_2), T_c^C, \hat{t}_1^C, p_{2c}, r^C, H_2, q_c] \quad (2)$$

Since  $(b_1, y_1, k_1) > (b_2, y_2, k_2)$  and  $q_a > q_b$ , and by virtue of condition 4 we have:

$$W_{1a}^C[(b_1, y_1, k_1), T_a^C, \hat{t}_1^C, p_{1a}, r^C, H_1, q_a] > W_{2b}^C[(b_2, y_2, k_2), T_b^C, \hat{t}_1^C, p_{2b}, r^C, H_2, q_b]. \quad (3)$$

From (1), (2) and (3), we can write:

$$W_{1c}^C[(b_1, y_1, k_1), T_c^C, \hat{t}_1^C, p_{1c}, r^C, H_1, q_c] > W_{2c}^C[(b_2, y_2, k_2), T_c^C, \hat{t}_1^C, p_{2c}, r^C, H_2, q_c]. \quad (4)$$

We know from condition 4 that if  $(b_1, y_1, k_1) > (b_2, y_2, k_2)$  is true then:

$$W_{1c}^C[(b_1, y_1, k_1), T_c^C, \hat{t}_1^C, p_{1c}, r^C, H_1, q_c] < W_{2c}^C[(b_2, y_2, k_2), T_c^C, \hat{t}_1^C, p_{2c}, r^C, H_2, q_c].$$

This is a contradiction with (4). My assumption that  $\hat{t}_1^C < \hat{t}_2^C$  is false. This implies that the reverse is true:  $\hat{t}_1^C > \hat{t}_2^C$ . Thus the indifference tax rate is increasing in the combination  $(b, y, k)$ .

(ii) Consider two students m and 1 with  $(b, y, k)_m$  and  $(b_1, y_1, k_1)$  respectively. a and b are two private schools (inside or outside community C) or two mixed finance schools outside community C and c is a public school inside community C.

According to assumption (1-a), higher  $(b, y, k)$  students choose higher quality schools.

Part 1:

For  $(b, y, k)_m > (b_1, y_1, k_1)$  and  $q_a > q_b$ .

By virtue of condition 4, we can write:

$$W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] < W_{1c}^C[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c].^{36}$$

$$W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a] > W_{1b}^C[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b].^{37}$$

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<sup>36</sup> m and 1 are in the same public school in the same community and have the same tax rate.

<sup>37</sup> m and 1 are in two different schools with  $q_a > q_b$ .

And if  $W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] > W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a]$  is true

We can write:

$$W_{1b}^C[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b] < W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a] < W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] < W_{1c}^C[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c]$$

Thus:

$$W_{1b}^C[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b] < W_{1c}^C[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c]$$

Part 2:

For  $(b, y, k)_m < (b_1, y_1, k_1)$  and  $q_a < q_b$ .

By virtue of condition 4, we can write:

$$W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] > W_{1c}^C[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c].$$

$$W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a] < W_{1b}^C[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b].$$

And if  $W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] < W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a]$  is true

We can write:

$$W_{1b}^C[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b] > W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a] > W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] > W_{1c}^C[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c]$$

Thus:

$$W_{1b}^C[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b] > W_{1c}^C[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c]$$

**Lemma 5:**

(i) Over the interval  $[t_m^C, 1]$ , there does not exist a tax rate different than  $t_m^C$  that is preferred by more than 50% of the population.

(ii) Over the interval  $[\hat{t}_m^C, t_m^C]$ , there does not exist a tax rate different than  $t_m^C$  that is preferred by more than 50% of the population.

An important property used in the proof of Lemma 5 is based on the fact that indirect utility is increasing in the tax rate over the interval  $[\hat{t}_m^C, t_m^C]$  and decreasing over the interval  $[t_m^C, 1]$ .

**Proof of Lemma 5:**

(i) For  $t \in [t_m^C, 1] \Rightarrow t > t_m^C$ .

Consider a student denoted 1 with  $(b_1, y_1, k_1) > (b, y, k)_m$ .

From the third equilibrium condition, we know that for  $(b_1, y_1, k_1) > (b, y, k)_m$  we have  $t_1^C < t_m^C$ .

Two situations should be analyzed:

1- For  $\hat{t}_1^C < t_1^C$ .

We can write:  $\hat{t}_1^C < t_1^C < t_m^C$ . From figure 2.4, we can see that the indirect utility is decreasing in the tax rate for all students with  $(b_i, y_i, k_i) > (b, y, k)_m$  over the interval  $[t_m^C, 1]$ . Student 1 prefers the lowest tax rate over the interval which is  $t_m^C$ .

2- For  $t_1^C < \hat{t}_1^C$  (this is a particular case. It means that student 1 has a maximal utility in the public sector even before being indifferent **between** a private school (inside or outside community C) or a mixed finance school outside community C **and** a mixed finance school within community C). It is obvious that student 1 will never choose a mixed finance or a public school belonging to his community.

For any tax rate lower than the indifference tax rate, such as  $t < \hat{t}_1^C$ , student 1 prefers a private school (inside or outside community C) or a mixed finance school outside community C. This applies for  $t_1^C < \hat{t}_1^C$ . His indirect utility is decreasing in the tax rate over the interval  $[t_m^C, 1]$ . Thus he prefers the lowest tax rate, which is  $t_m^C$ .

Over the interval  $[t_m^C, 1]$  there does not exist a tax rate different than  $t_m^C$  that is preferred by more than 50% of the population.<sup>38</sup>

(ii) For  $t \in [\hat{t}_m^C, t_m^C] \Rightarrow t < t_m^C$ , consider an individual with  $(b_1, y_1, k_1) < (b, y, k)_m$ .

As indirect utility is increasing over this interval, student 1 chooses the highest attainable tax rate over the interval; thus he chooses  $t_m^C$ .

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<sup>38</sup> The second and third equilibrium conditions are essential to the proof.



Over the interval  $[\hat{t}_m^C, t_m^C]$ , there does not exist a tax rate different than  $t_m^C$  that is preferred by more than 50% of the population.<sup>39</sup>

**Proposition 2:**

If  $W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] > W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a]$  is true, then  $t_m^C$  is a majority voting equilibrium.

**Proof of proposition 2:**

We have to demonstrate that over the interval  $[0, \hat{t}_m^C]$ , there does not exist a tax rate different than  $t_m^C$  that is preferred by more than 50% of the population. For student 1 with  $(b_1, y_1, k_1) < (b, y, k)_m$  we have two cases to analyze which correspond to the intervals  $[0, \hat{t}_1^C]$  and  $[\hat{t}_1^C, \hat{t}_m^C]$ .

1- Over the interval  $[0, \hat{t}_1^C]$ .

In Lemma 4 we have proved that if the median voter prefers a positive tax rate while choosing a public school over a zero tax rate or while choosing a private school (inside or outside his community) or a mixed finance school outside his community, all students with lower  $(b, y, k)$  combinations than the median have the same behavior.

If  $W_{mc}^C[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c] > W_{ma}^C[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a]$  is true,<sup>40</sup> all students with  $(b_i, y_i, k_i) < (b, y, k)_m$  have the same behavior, such as:

$W_{1c}^C[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c] > W_{1b}^C[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b]$ . Student 1 prefers a positive tax rate in a public school over a zero tax rate in a private school (inside or outside his community) or a mixed finance school outside his community.

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<sup>39</sup> The third equilibrium condition is essential to the proof.

<sup>40</sup> The median voter prefers a positive tax rate in a public school over a zero tax rate in a private school or a school (private or mixed finance) outside his community.

Note that over the interval  $[0, \hat{t}_1^C]$ , 0 is the most preferred tax rate, since indirect utility is decreasing in the tax rate. But according to the previous discussion,  $t_m^C$  is preferred over 0.

Thus, student 1 prefers the tax rate  $t_m^C$  over the interval  $[0, \hat{t}_1^C]$ .

2- Over the interval  $[\hat{t}_1^C, \hat{t}_m^C]$ .

Indirect utility is increasing in the tax rate over this interval.  $t_m^C$  is higher than any other tax rate  $t \in [\hat{t}_1^C, \hat{t}_m^C]$ , thus individual 1 prefers  $t_m^C$  over any other tax rate.

**From Lemma 5 and proposition 2, it is possible to say that there does not exist a tax rate different than  $t_m^C$  that is preferred by more than 50% of the population.  $t_m^C$  is a majority voting equilibrium.**

**Equilibrium:**

At Equilibrium, all the following conditions are satisfied.

1- Students.

- Utility maximization:

$$W_{ij}^C[(y_i, b_i, k_i), T_j^C, t^C, p_{ij}, r^C, H_i, q_j] = \max U(y_i - T_j^C - p_{ij} - r^C H_i, q_j)$$

- All majority voting conditions hold.

2- Schools.

- Profit maximization yields the optimal level of resources per student:

$$\max \pi_j \equiv R_j - V(l_j) - F .$$

$$MC = R_j^* = n_1 + 2n_2 l_j + \mu'_j (\theta_j - b_i) + \mu''_j (O_j - k_i) + \mu'''_j [\sigma_j^2 - (k_i - O_j)^2]$$

With  $\mu'_j, \mu''_j, \text{ and } \mu'''_j$  the Lagrangian multipliers.

- At equilibrium  $\pi_j = 0$ , no new entries on the market are possible.
- The number of students in a school is given by:  $l_j = \iiint_s \alpha_j(b, y, k) f(b, y, k) dbdydk$ .
- Mean ability in a school is given by:  $\theta_j = \frac{1}{l_j} \iiint_s b_i \alpha_j(b, y, k) f(b, y, k) dbdydk$
- Mean social capital in a school is given by:  $O_j = \frac{1}{l_j} \iiint_s k_i \alpha_j(b, y, k) f(b, y, k) dbdydk$ .
- The dispersion of social capital in a school is given by:

$$\sigma_j^2 = \frac{1}{l_j} \iiint_s (k_i - O_j)^2 \alpha_j(b, y, k) f(b, y, k) dbdydk$$

3- Communities.

- The price of housing is determined by the equality of supply and demand.
- The community budget is balanced:  $\sum E_j^C = \sum t^C r_{net} H_i$  with  $j \in C$ .
- The number of students in a community:  $l_C = \iiint_s \alpha_C(b, y, k) f(b, y, k) dbdydk$ .
- Boundary indifference conditions are satisfied.
- The median voter is pivotal. And  $t_m^C$  is a majority voting equilibrium.

4- Market clearance.

- All students go to school.

## 5. The introduction of public school choice.

So far, I have considered that public school choice across communities was not allowed; for the rest of the analysis this choice is introduced into the already existing equilibrium. I assume that all the aforementioned equilibrium conditions are satisfied. For simplicity, I am only considering two communities C and C+1. From lemma 2 and lemma 3, we know that higher  $(b, y, k)$  students choose community C while lower  $(b, y, k)$  students choose community C+1, with  $r^C < r^{C+1}$ . Students with  $(\hat{b}, \hat{y}, \hat{k})$  determine boundary indifference between communities. When majority voting equilibrium's conditions are satisfied, median voters are pivotal.  $(b, y, k)_m^C$  is the median voter in community C, and  $(b, y, k)_m^{C+1}$  is the median voter in community C+1.  $t_m^C$  and  $t_m^{C+1}$  are majority voting equilibria in each community. We know from condition 1 that median voters prefer positive tax rates ( $t_m^C$  and  $t_m^{C+1}$ ) in a public school **over** a zero tax rate in a private school (inside or outside their community) or a mixed finance school outside their community. Thus, at  $t_m^C$  and  $t_m^{C+1}$ , median voters choose public schools. From lemma 4 (ii), we know that all students with  $(b, y, k)$  below that of the median voter make the same choice, and hence they choose the public school, too. From the figure 2.4, we can see that, at  $t_m^C$  and  $t_m^{C+1}$ , some students with higher than the median combinations of  $(b, y, k)$  may choose public schools. Thus at least 50% and one student choose the public school. Only students with very high  $(b, y, k)$  choose mixed finance or private schools.

When public school choice is allowed, some students will try to get enrolled in the public school of the other community. At equilibrium, those who have chosen the public school in their community have their budget already balanced. The introduction of public school choice would not affect them, since they cannot afford transportation to the other community, and it is assumed that the equilibrium holds and no further utility maximizations are undertaken. Only those who are enrolled in private and mixed finance schools would be able to attend the public school of the other community by using the resources saved on the cost of education to finance transportation. Note that the students enrolled in private and

mixed finance schools are the ones with  $(b, y, k)$  combinations higher than that of the median voter.<sup>41</sup>

Consider a student  $i$  living in community  $C+1$  and enrolled in a private or mixed finance school (inside or outside  $C+1$ ), noted (a). If he is to exercise public school choice, he would still be living in community  $C+1$ , but will attend the public school in community  $C$ , noted (c). In order for student  $i$  to exercise public school choice, he should achieve at least the same level of utility; in other words, he should have at least the same numeraire consumption while having no less educational quality.

His budget constraint in (a) is the following:  $c_{ia}^{C+1} = y_i - T_a^{C+1} - p_{ia} - r^{C+1}H_i$  with  $T_a^{C+1} = 0$ , if  $a \in C+1$  and  $T_a^{C+1} \neq 0$  if  $a \notin C+1$ .

His budget constraint in (c) is the following:  $c_{ic}^{C+1} = y_i - T_c^{C+1} - r^{C+1}H_i$  with  $p_{ic} = 0$ , since  $c$  is a public school and  $T_c^{C+1} \neq 0$  because  $c \notin C+1$ .

For student  $i$  to exercise school choice, these two identities should be verified:

$$c_{ic}^{C+1} - c_{ia}^{C+1} = T_a^{C+1} + p_{ia} - T_c^{C+1} \geq 0 \text{ and } q_c \geq q_a. \quad 42$$

This implies:  $T_a^{C+1} + p_{ia} \geq T_c^{C+1}$  and  $q_c \geq q_a$ . In order to exercise public school choice, the transportation cost between a student's community and his new public school must not exceed the economies made by opting out of his old private or mixed finance school. An interesting result is that prohibitive transportation costs will obstruct the benefits of public school choice. In other words, public school choice is more efficient in an urban economy where multiple neighborhoods exist and are well connected in terms of transportation, while in a rural economy the exercise of choice would be more difficult because of high transportation costs.

We can also see that public school choice is beneficial only to a small fraction of the population. Only high  $(b, y, k)$  students enrolled in private and mixed finance schools (at most 50% of the population minus the median voter) would have access to public school

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<sup>41</sup> All students with  $(b, y, k)$  combinations lower than that of the median voter choose public schools like him.

<sup>42</sup> Note that the student's budget is already balanced and that he has already maximized his utility. It is not possible for him to reduce his consumption; he can only transfer resources from  $T_a^{C+1} + p_{ia}$  to  $T_c^{C+1}$ . In other words, a student cannot reduce his consumption in order to allocate more resources to the general cost of acquiring public education (no further maximizations are to be done). Note that the student should acquire at least the same quality education in his new public school, or it will be senseless to exercise school choice.

choice, and among them only those with high income levels would be able to cover the transportation cost.

This finding is consistent with the fact that in some countries, like New Zealand, Sweden and Ireland, a large fraction of households still choose the public school of their neighborhood even if they have the choice to opt out (see Mons 2004). Note that some additional constraints other than transportation costs can be added to the model. If public schools are to select incoming students by ability, then another ability constraint is added. The final effect on school quality in both communities is ambiguous. While some private or mixed finance schools would be losing students in favor of public schools, the presence of decreasing boundary loci between communities and the absence of complete stratification according to ability, income or social capital, makes the effect on educational quality uncertain; this effect depends on the exact distribution of attributes in the population of each community.<sup>43</sup>

In conclusion, it is possible to summarize what has been done in this part of chapter 2. I have constructed an equilibrium on the education market in a multicomunity framework where students are differentiated by income, ability and social capital. Three-dimensional stratification arose from school profit maximization and from individual utility maximization. Students with higher combinations of ability, income and social capital are found to spend at least the same amount on rental, transportation and education combined, while choosing at least the same level of educational quality, in comparison to students with lower ability, income and social capital. Furthermore, they spend at most the same amount on rental and at least the same amount on the general cost of acquiring education while having no less housing consumption.

Majority voting over tax rates has been studied in a context where preferences are non-single peaked due to the existence of private, mixed finance and public schools. The necessary conditions for the existence of a majority voting equilibrium determined by the median voter were provided. In the final section, public school choice was introduced to find that choice is the most efficient in an urban economy consisting of multiple communities well connected

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<sup>43</sup> For example if a school is losing a student with high ability and low social capital, it would gain in mean social capital and loose in mean ability. Thus, the variation of its quality depends on all variations.

by transportation. I also found that only a small fraction of the most advantaged students in terms of income will exercise their right to choose a public school outside their community.

**Even though the achievement function  $a_i = a(b_i, q_j)$  has not been explicitly used in the model, it is of a great importance to the understanding of the functioning of inequalities in outcomes and to my further empirical analyses. This function summarizes the mechanisms behind inequalities; it implies that achievements depend on student characteristics and on those of his school. Even if student characteristics are exogenously determined, the quality of his school is not. In fact, this quality is determined through a number of elaborate interactions between students and schools. Hence, individual achievements depend on the outcome of stratification. The access to higher levels of quality ensures higher levels of achievements. Furthermore, the mechanisms of the market work in favor of students with high income, ability and social capital endowments. In other words, this achievement function is a generator of inequalities. Students with high endowments have a comparative advantage on both arguments of the equation  $b_i$  and  $q_j$ .**

**In terms of its empirical implications, this equation will be the cornerstone of the analyses undertaken in chapter III and chapter IV. It should be said that this equation can be extended to include other individual socio-demographic characteristics than ability. In fact, this function can even accommodate for psychology-related factors such as motivation and interest in learning. Similarly, school quality can be extended to include different forms of peer effects – represented through different moments - as well as other school characteristics (funding, teacher characteristics, etc.).**

## Conclusion of chapter II.

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In this chapter, I have provided an answer for several of the issues that remained unsolved in the existing theoretical literature. As it was shown in chapter I and in the introduction to this chapter, two distinct bodies of literature on stratification exist. The first is concerned with residential stratification between jurisdictions, where school quality is mainly determined by local spending, and the second is concerned with stratification between public and private schools, where ability peer effects are the major determinant of quality.

The major disadvantage of the first type of models is its lack of generality. First, they consider spending to be the unique determinant of public school quality, even though the empirical literature shows that peer effects are also of major importance. Second, they do not include private schools as an alternative to public supply. Thus, they avoid the complications related to the presence of non-single peaked preferences, something that facilitates the direct application of Black's median voter theorem. And, finally, in most of these models complete stratification by income arises at equilibrium. This is not very realistic, since communities are usually stratified along other criteria, such as social class and other socio-demographic factors.

The second body of models also presents a number of disadvantages. First, these models do not consider funding as a determinant of school quality in order to avoid non-single peaked preferences over tax rates; therefore, they limit the determinants of quality to peer effects.;see Epple and Romano (1998). Second, only pure public and private schools are considered. Most of the literature neglects the existence of mixed finance schools, which can be privately managed but publicly funded or subsidized. Third, most of these papers model the public system as completely passive. This system is usually specified as a single, homogenous, multi-campus school with no active strategy. Finally, peer effects are linear in means. In other words, a student whose performance increases average performance by two points has the same effect as several students whose performance increases average performance by two points; see Hoxby and Weingarth, (2005 p. 2 and 30).



The models that I have constructed in this chapter can be thought of as a development of the previous work of Fernandez and Rogerson (1996), Nechyba (1999), Epple and Platt (1998), Epple and Romano (1998, 2000, 2003, 2006) and Nechyba (2003).

This chapter was intended to bridge the distance between these two bodies of research by including several features that were not considered before. In fact, funding and peer effects were introduced as simultaneous determinants of school quality. Non-linearities in means were considered by including two moments of peer effects: the average and the variance. Private, public and mixed finance schools were also considered and preferences were non-single peaked. The conditions for the existence of a majority voting equilibrium were provided and tax rates were endogenized. All schools are active and maximize a profit function under a number of quality constraints. Hence, the marginal cost of admitting a student and the level of tuition were endogenized. All this analysis is carried out in a multicomunity framework, and the resulting community stratification is multidimensional according to income, ability and social capital.

A number of issues remain and have to be discussed. These issues include the restrictive nature of the majority voting conditions, the existence of school quality hierarchy and the empirical implications of the theoretical framework.

First, the four majority voting conditions seem to be restrictive, since they impose geometric restrictions on the form of the utility function. However, these assumptions are needed to establish the majority voting equilibrium. Glomm and Ravikumar (1998 p. 194) noted that “restrictions on preferences alone do not pin down the tax rate, since the fraction of agents choosing public schools also depends on the income distribution”. Other solutions for this problem may be considered even more restrictive. For instance, it is possible to drop private and mixed finance schools from the model, such as in Fernandez and Rogerson (1996), Nechyba (1997) and Epple and Platt (1998), or to drop public spending as a determinant of school quality, such as in Epple and Romano (1998, 2000) and Nechyba (2003). In both cases, non-single peaked preferences are avoided. However, in the first a major aspect of choice, which is the existence of private alternatives, is lost, and in the second a major determinant of school quality is neglected.

Another solution to the problem is to simply consider that tax rates are exogenous and that individuals do not exercise any influence over them due to the atomistic nature of the economy. Through this assumption the entire fourth section of the second part of this chapter can be dropped. However, this assumption is not realistic, since local property tax rates are not homogenous across communities. It should also be said that the voting process in this chapter, as well as in the theoretical literature, is a simplification of reality. Households do not vote over tax rates, they vote for political parties on the basis of multiple criteria (including tax schemes), and those parties determine the level of taxes through a complicated political process. In this literature, as well as in chapter II, this political process is not modeled, since it lies beyond their scope.

Second, in both models we do not have a strict hierarchy of school qualities, such as in Epple and Romano (1998), and some schools may have the same level of quality. Note that the analytical reason for the existence of a strict hierarchy of qualities in the model of Epple and Romano (1998) and in Fernandez and Rogerson (1996) is the one-dimensional definition of school quality. In Epple and Romano's paper, quality is only determined by ability peer effects, and in Fernandez and Rogerson's it is only determined by public spending.

Finally, these models have strong implications for the empirical analyses in chapter III and IV. In this chapter, I have constructed a framework in which inequalities in achievements are generated through the elaborate interactions between students and schools. However, these mechanisms are not simple to be empirically assessed for several reasons:

1. It is not possible to reconstruct the economy as if we know when stratification arose (period zero). In other words, the education system has always been stratified. This stratification exists and evolves over time due to changes in the economic environment or to changes in national legislations on education. We cannot say that stratification arose at a particular moment as we did in the theoretical model. In addition to that, it is not possible to model school and student strategies - such as profit and utility maximization - using reduced form econometric equations.

2. The availability of data: usually data represent the ultimate limit for any empirical study. For instance, as we will see in the next two chapters, I have decided to use the OECD PISA 2003 data. This data source presents major advantages, such as the existence of a large array of student and school characteristics including economic, social and cultural factors, in addition to school peer effects, several proxies of school funding and student performance scores on mathematics. Hence, it will be possible for the empirical analysis to control for the variables included in the theoretical model (income, social capital, peer effects and school funding). Note that students in PISA are nested within schools, something that facilitates the use of multilevel analyses, which acknowledge the fact that students attending the same school may have some resemblance due to stratification and self-selection into schools (in other words, the PISA dataset has students that are already stratified). The only limitation of PISA data is the unavailability of geographical information on housing prices and local taxes. However, most data sources do not provide this kind of information. It should also be noted that the distinction between achievements  $a_i$  and ability  $b_i$  is not possible in PISA. In fact, performance scores in mathematics can be considered a good measure of achievements. However, no appropriate measure of abilities (such as a certain form of IQ) was provided.

**From this discussion, a number of conclusions can be drawn. In the empirical analyses in chapter III and IV, I will not reconstruct stratification in the education system. Instead, I will use an already stratified sample of students in order to study the nature of stratification and its impact on inequalities in achievements. In other words, I assume that the first step - the rise of stratification - has already happened and that we can only analyze its implications. These implications can be summarized through the achievements function  $a_i = a(b_i, q_j)$ , where achievements depend on a large array of student and school characteristics. This equation allows for great flexibility. Firstly, ability can be extended to include a large array of socio-demographic and even psychological student characteristics,<sup>44</sup> and secondly school quality can include different moments of peer effects and numerous school characteristics.**

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<sup>44</sup> Since the PISA data lacks a reliable measure of ability. Ability is replaced by other student characteristics.

**In this equation, stratification can be captured through the amount of within-school variations on student variables and the between-school dispersions. For instance, if students attending the same school are found to be homogenous, though schools are very heterogeneous, we can deduce that a high level of stratification exists. At this stage another advantage of the PISA data becomes apparent, which is the availability of a number of countries having different institutional structures. In other terms, it will be possible to consider different countries with different levels of stratification and inequalities and to contrast these with the institutional characteristics of each country.**

The rest of the thesis is organized as follows: chapter III presents the data, the selected countries and a number of descriptive statistics on student and school variables. These statistics are then used to study stratification under different institutional arrangements. Chapter IV establishes the link between those characteristics and performance scores through a full multilevel regression for the selected countries.

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**The Anatomy of Inequalities in Attainments:  
An International Investigation  
on Stratification and Choice.**

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**CHAPTER III**

**Stratification under Different Education  
Systems: an International Comparison between Finland,  
Germany, Italy, the UK and Japan.**

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The most important implication of the theoretical models developed in chapter II is that students are non-randomly stratified into schools and communities. This stratification operates along the lines of income, ability and social capital. The impact of stratification on student performances can be seen through the education production function  $a_i = a(b_i, q_j)$ , where achievements depend on student and school characteristics. Stratification implies that the access to a particular level of quality is not random. In other words, school type, the social mix, the funding level and the entire educational experience that a student is confronted with are the results of stratification. Inequalities can no longer be represented through the impact of a student's social and economic background on his performance, since school characteristics are likely to be a source of inequalities, too. Hence, inequalities can be channeled through students' own characteristics and through "stratification-determined" school characteristics.

The education production function  $a_i = a(b_i, q_j)$  can be analyzed in two steps. First, the different sets of student and school variables can be assessed separately through a wide array of descriptive statistics. These statistics should reflect the distribution of the variables and the different forms of stratification. For instance, a low within-school dispersion and a high between-schools variance indicate high levels of stratification. Second, the link between the two categories of variables and performance scores can then be established through econometric regressions. The first step is undertaken in this chapter and the second is to be treated in chapter IV.

As previously mentioned, the empirical analysis in chapters III and IV is based on the OECD's PISA 2003 data. The OECD's program for international student assessment is a triennial survey of student knowledge and skills. It seeks to measure how well students' skills at the end of compulsory schooling are adapted to meet the needs and challenges of today's world. PISA is focused on skills, knowledge, competencies and other attributes that are relevant to personal, economic and social well being, instead of being focused on the mastery of a particular aspect of a curriculum.

PISA covers about 40 countries around the world, mostly OECD ones. These countries represent one-third of the world's population and nine-tenths of its GDP. This large coverage is very convenient for comparative studies, since students are assessed and data are collected using the same techniques in a variety of schooling systems.

The participating countries are the following:

OECD countries: Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Korea, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States.

Partner countries: Brazil, Hong Kong-China, Indonesia, Liechtenstein, Latvia, Macao-China, Russian Federation, Thailand, Tunisia, Uruguay and Serbia.

Different aspects make PISA unique and worthy of being used:

PISA is age and not grade-based; it covers 15 year-old students. This coverage allows the assessment of students at the end of compulsory schooling, where it is essential to determine the extent to which skills and knowledge have been acquired and the degree of inequalities in their accumulation. Note that students that are too young can not reflect the effects of schooling systems, and individuals that are too old can only reflect the effects of schooling systems with a lag in time.

In addition to this, PISA is very rich in terms of the data it collects. PISA 2003 included data on students' social, economic and cultural backgrounds, on their learning strategies and on their motivation and attitudes towards education. This information can further be used to describe schools through different forms of peer effects. PISA also included data on schools' resources, types, teaching strategies and disciplinary climate. The structure of the PISA datasets is adapted for sophisticated empirical analysis. This structure is articulated around three elements: countries, schools and students. Students are nested within schools and schools within countries. This particular structure permits the use of multilevel analyses that are perhaps the most suitable for the study of inequalities in achievements.

PISA mainly assesses reading and mathematical and scientific literacy. In 2003 the major domain was mathematical literacy while the minor domains were reading, science and problem solving. PISA has an innovative literacy concept, which is based on the ability of students to apply their knowledge to key subject areas, to reason and to solve everyday problems. It is based on a dynamic lifelong learning concept in which students acquire knowledge and skills throughout life and adapt them to the changing world.<sup>45</sup> This literacy concept is unique to PISA; in most previous studies, such as the IEA and the IAEP, the assessment is curriculum-based; this means that the assessment is based on parts of the curriculum that are common between countries. Aspects of the curriculum that are particular to one country are dropped, independently of how important they are. The exact definition of mathematical literacy used in PISA is the following: “An individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned and reflective citizen.”

In PISA 2003, the major subject of assessment is mathematical literacy. The use of test scores on mathematics facilitates international comparisons because the assessed concepts tend to be more universal than reading concepts, which have some national particularities.

This chapter is organized as follows. Section A presents the PISA 2003 sample, section B presents the different sets of variables, section C presents missing data and imputation techniques and, finally, section D motivates the selection of countries and presents a number of relevant descriptive statistics.

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<sup>45</sup> The PISA literacy concept is far more reaching than the use of years of schooling and levels completed, previously used as proxies for educational attainments.



## A. The PISA sample.

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The major problem in comparative education studies is the construction of a dataset that permits international comparisons. The nature and importance of pre-primary education, the age of entry to formal schooling and the structural differences among schooling systems do not allow an easy definition of internationally comparable grade levels of schooling. In some previous studies, the sampling of students was based upon a particular grade that ensures maximum coverage of an age group. However, the major problem is that the target grade may not necessarily include the same age groups across countries.

As mentioned above, PISA uses an age-based target population. The definition of the sample does not depend on the structural particularities of education systems. Assessed students are aged between 15 years and 3 months and 16 years and two months, regardless of the grade in which they are enrolled.

In PISA, the term '*yield*' is used to describe the cumulative knowledge and skills that a student has acquired over a certain period of time. Since PISA is age-based, it assesses students of the same generation, but who may have undergone different educational trajectories or who may have belonged to a narrower or a wider range of grades or even to different tracks within a country.

In PISA 2003, a maximum coverage of students of 15 years of age was achieved. The sampling standards required the exclusion of a total of 2% in some countries and up to 5% in others. These exclusions concerned particular students or even entire schools. The reasons for exclusions are the following:

- Some schools are geographically inaccessible.
- The administration of the PISA assessment is not feasible.
- Some schools cater for a particular type of student, such as schools for the blind.
- The percentage of 15 year-old students is less than 2.5%.
- Students having intellectual or functional disabilities or with limited assessment language proficiency.

**The sampling procedure:**

A two stage stratified sampling procedure was used to construct the PISA sample. The first stage consisted of choosing the eligible schools in which 15 year-old students could be enrolled. The probability of selecting a school was proportional to its size, with size being a function of the estimated number of enrolled 15 year-old students. In each country a minimum of 150 schools are sampled as well as replacement schools, in case a sampled school chose not to participate in the survey.

The second stage of the sampling procedure consisted of sampling students within the chosen schools. After the sampling of schools, a list of eligible students (15 years of age) was prepared; then 35 students were randomly selected within each school. If the number of 15 year-old students was less than 35, all students were selected.

**The definition of schools.**

Usually in most countries, schools are administrative units defined as such by the responsible government agency. In some countries, sub-units within schools were selected instead of entire schools. In others, schools were split into different programs and the most convenient program was selected. In countries with multi-campus schools, campuses were sampled as independent units. In countries with multi-language instruction, schools were split into units delivering each language of instruction.

## **B. The variables.**

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In what follows, I will present the variables that will be used in my empirical analyses. Most of these variables are composite indices developed by the PISA program; they summarize responses from students or school principals on a series of related questions. These indices were scaled using IRT (Item Response Theory) scaling or a weighted maximum likelihood estimate (see PISA 2003 technical report and Warm 1985). They can be positive or negative. A negative value on an index does not imply that students responded negatively; actually, it implies that students responded less positively than all students did, on average, across OECD countries. Likewise, a positive value on an index implies that students responded more positively than all students did, on average, across OECD countries.

Three sets of variables are used in my empirical analyses: first, test scores (in mathematics) that account for students' achievements on the standardized PISA tests. Second, two sets of variables that describe students and schools: the first describes students' socio-demographic characteristics, their ethnic background and some psychological factors affecting their learning of mathematics. The second describes schools. These variables can be grouped into two types: the first includes pure school variables related to their financial resources, type (public or private), disciplinary climate and engagement in mathematics; the second includes school aggregates computed from student level data. These represent different forms of peer effects (social, economic, cultural, motivational and ethnic).

I will start with a description of test scores, the assessment methods and the concept of plausible values.

## **1. Test scores.**

### **1.1. The PISA assessment.**

The PISA assessment consisted of a paper and pencil test. The format of the questions is varied. Some of these are multiple choice questions requiring a simple response that could be compared to a single correct answer. Others are more involved and require students to develop their own answers. The PISA assessment usually consists of a stimulus (text, table, chart...) followed by a number of tasks or questions associated with it. Multiple choice questions were either standard multiple choices with a limited number of responses or complex multiple choices offering several statements, for each of which students were required to select one of several possible responses. Short answer questions included closed-constructed response items (requiring a numeric answer) and items requiring a word or a short phrase. These short response items were compared to a wide range of possible correct answers.

In PISA 2003, four domains were assessed: reading, science, problem solving and mathematics, with mathematics being the major domain. The mathematics assessment consisted of 85 test items representing 210 minutes of testing time. The problem solving consisted of 19 items, the reading domain of 28 items and the science domain of 35 items. All combined represented about 60 minutes of testing time. These items were all selected from a large pool of possible ones (about 300) tested in a field trial.

The 167 PISA 2003 items were grouped in 13 clusters, 7 for mathematics and 2 for each minor domain; each cluster represented 30 minutes of testing time. The items were administered in 13 booklets, with each being composed of four clusters. Students were randomly assigned one of the booklets and each student undertook a test of 2 hours (which correspond to the 4 clusters included in a booklet).

## **1.2. The coding of results and the generation of test scores.**

Since PISA is about assessing students' performance in a particular domain, a large number of items and questions had to be used in order to cover the full breadth and depth of this particular domain.

As I have indicated, each student was assessed with a booklet containing 4 of the 13 item clusters, since it was impossible to administer the entire battery of 167 items for every sampled student. The reasons were:

- a) School principals would refuse to free their students for the very long testing time required to answer the 167 items developed by PISA.
- b) After a long extensive testing, students responses start to get affected by fatigue and this would bias the outcomes.
- c) It is very expensive to run the entire battery of items on the entire sample of students for each country.

Due to these problems, each student had to be assessed with only a subset of the item pool. In other words, a sub sample of students responded to each item.

The simplest method of summarizing the responses on the different items or of generating test scores is to compute the percentage of correct responses for each student. This method has been largely used in previous educational studies and it has the advantage of simplicity, since it is possible to compute the percentage of correct answers for each student by domain or even by sub-domains. However, the major disadvantage of this method is that students are tested with different batteries of items; the percentage of correct answers will be affected by the difficulty of the questions. Thus, some bias will be introduced into the test scores, since different difficulty levels will generate different score distributions.

In order to solve this problem, the difficulty of the items had to be taken into account when generating the test scores. All items were field trialed with the purpose of assessing their difficulty. Then they were allocated in a manner that guaranteed that each test has more or less the same average difficulty.

For two tests to be identical, the raw test scores obtained from the same population of students undertaking the two tests should have identical means and dispersions. If this is not the case, then the two tests do not have the same difficulty and some further treatment is needed.

The use of the percentage of correct answers has a second major disadvantage: percentages are on a ratio scale and this limits the use of these raw scores. A student who gets 0 on this scale did not provide any correct answer, but could not be considered as having no competencies. A student with 10% correct answers has twice the score of a student with 5% correct answers, but this does mean that he has twice the competencies. In what follows, two different methods used by PISA to compute robust and fully comparable test scores that solve the problems inherent to the use of raw percentages are examined. These are the Rasch model and plausible values.

### 1.2.1. The Rasch model.<sup>46</sup>

The Rasch model is used to scale the results obtained on the PISA tests. The answer of a student on a particular question can be correct or incorrect, and thus the result is dichotomous. The Rasch model creates a continuum out of these dichotomous results by computing the probability of success conditioned by the difficulty of the question. More precisely, the Rasch model can be depicted through a logistic regression estimating the probability of success (dependent variable) on the difficulty of the question (independent variable); this probability is continuous.

From a mathematical point of view, the probability that a student  $i$ , with ability  $b_i$ , provides a correct answer to a question  $q$  of difficulty  $d_q$  is:

$$P(X_{iq} = 1 | b_i, d_q) = \frac{\exp(b_i - d_q)}{1 + \exp(b_i - d_q)}$$

The probability of failure is:

$$P(X_{iq} = 0 | b_i, d_q) = \frac{1}{1 + \exp(b_i - d_q)}$$

And the sum of the two probabilities is equal to one.

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<sup>46</sup> For a more detailed description of the Rasch model, see chapter 4 in the PISA 2003 data analysis manual.

As we can see when the ability of a student is equal to the difficulty of the question, the probability of success is equal to 0.5. When the ability of a student exceeds the difficulty of a question, the probability becomes higher. In other words, the distance between a student's ability and the difficulty of a question is the only factor influencing the probability of success.

However, the major difficulty that underlies the Rasch model is the building of the Rasch scale which involves the comparison of an item with all other items. Once the Rasch scale has been created and the difficulty of the items has been located on the scale, it is possible to determine the ability of each student. The Rasch model assumes the independence of the responses on each item; this means that the answer provided by a student on a particular item does not affect his answer on another one. This implies that the probability of succeeding a number of items is equal to the product of the probabilities of succeeding each item taken apart.

Having the item difficulty and the responses provided by a particular student, it is possible to compute his ability using the formulas mentioned above. However, Warm (1989) indicated that these estimates tend to be biased. For instance, a difficult item does not provide much information on a low ability student, though it provides more information on a high ability one. Therefore, a difficult item will contribute more in the computation of the ability of high performer and less in that of a low performer. The Warm estimates are computed by applying weights to the maximum likelihood estimates, and they are called weighted likelihood estimates (WLEs).

In conclusion, the raw scores such as the percentage of correct answers do not account for the difficulty of the administered items. However, the MLEs or the WLEs obtained from the Rasch model account for it, and permit a more robust estimation of students' ability. These scores are fully comparable across countries and schools even if students are assessed with different subsets of the item pool.

### 1.2.2. Plausible values.<sup>47</sup>

In order to understand the concept of plausible values, the following example is very helpful. Suppose that a city council decides to levy property taxes based on the total area of houses. Inspectors visit all city houses to take the measurements. They are instructed to round them to 5s or 0s. In other words, if a house has an area of 43m<sup>2</sup> it will be recorded as 45m<sup>2</sup>. Similarly, if a house has an area of 51.5 m<sup>2</sup> it will be recorded as 50m<sup>2</sup>. All houses with a recorded area of 40m<sup>2</sup> are not exactly 40m<sup>2</sup>. Their area varies around 40m<sup>2</sup>; they could be anything between 37.5m<sup>2</sup> and 42.5m<sup>2</sup>. It should be noted that this kind of rounding process introduces some error into the measurement. If the difference between the exact area of a house and the reported area is small then the error is small. For instance, it is unlikely that an area of 41m<sup>2</sup> will be recorded as 45m<sup>2</sup>; thus the error is small. However, an area of 42.5m<sup>2</sup> can be equally recorded as 40m<sup>2</sup> or 45m<sup>2</sup>; thus the error is larger.

As we can see, with the rounding of measurements a continuous variable became discontinuous. This can be translated into the PISA context. The competencies of students are in their nature continuous. However, when a student responds to 8 dichotomous items, his possible scores will be: 1, 2, 3, 4, 5, 6, 7 and 8. In this case, the measured ability of a student is discontinuous and some error is introduced. In comparison with the measurements of areas, education measures encompass substantial measurement errors because:

- a) The concept to be measured is wider.
- b) The ability measures are affected by the mental and physical dispositions of a student on the day of the assessment.
- c) The general conditions of the test may affect the results. Whereas, with the measure of area, the concept is perfectly defined and is not affected by human aspects. In other words, the measurements of the area of a house are independent of the house. However, the measurements of student ability are not independent of the student.

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<sup>47</sup> For a more detailed description of the computation of plausible values, see chapter 5 in the PISA 2003 data analysis manual.



The methodology of plausible values consists of:

1. Mathematically computing distributions around the reported student scores.
2. Assigning to each student a number of random values drawn from this distribution.

Plausible values can be defined as random values from the distribution of scores. In this context, a student with 6 points (answering 6 correct questions over the 8 items) can be assigned any value from the normal distribution such as 6.4 or 5.7. The exact definition of plausible values in PISA 2003 is the following:

“The simplest way to describe plausible values is to say that plausible values are a representation of the range of abilities that a student might reasonably have. ... Instead of directly estimating a student’s ability  $\theta$ , a probability distribution for a student’s  $\theta$ , is estimated. That is, instead of obtaining a point estimate for  $\theta$ , (like a WLE), a range of possible values for  $\theta$ , with an associated probability for each of these values is estimated. Plausible values are random draws from this (estimated) distribution for a student’s  $\theta$  (Wu and Adams, 2002).”<sup>48</sup>

The plausible values methodology has for its objective the construction of a continuum out of discontinuous items.

### **1.2.3. Comparison between Weighted Likelihood Estimates (WLEs) and Plausible Values.**

A comparison between different estimators of student ability (WLEs and plausible values) has been conducted by PISA using real data. This comparison highlighted differences among the estimators. However, it did not identify the best estimator for a particular statistic.

The comparison showed that a good estimate of the population mean was obtained regardless of the ability estimator used (WLEs or plausible values). Actually, a good estimate of the population mean will always be obtained when the test is well targeted and when the average

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<sup>48</sup> See PISA 2003 data analysis manual page 75.

difficulty of the items is equal to zero. However, when the test is too easy or too difficult, the population mean is mis-estimated.<sup>49</sup>

When it comes to the population variance, the comparison showed that the WLEs tend to systematically overestimate it while plausible values give an estimate close to the expected value. In my empirical analyses, as well as in the descriptive statistics developed later in this chapter, I opted for the use of plausible values on mathematics. We have five plausible values for each student, and the choice of mathematics (not reading or science literacy) was motivated by two reasons:

- I. Mathematical concepts are more universal than reading, which is affected by particular aspects of different languages.
- II. Since mathematics were the major domain in PISA 2003, all students undertook the mathematics test, whereas not all of them undertook the science or reading tests.

## **2. Student characteristics.**

A number of indices were developed from the raw data collected through the student questionnaire. These indices reflect certain aspects of student characteristics. The questions used in the construction of these indices were selected from a larger number of possible ones on the basis of theoretical considerations and previous research. Most of these indices were scaled using IRT scaling or a weighted maximum likelihood estimator.

The indices and variables to be used in the descriptive statistics and the econometric models are examined below. Two types of indices are distinguished:

- Simple indices: constructed through the arithmetical transformation of responses obtained from the different questionnaires, such as the student/teacher ratio or the occupational status of parents (ISEI). It should be noted that PISA collected information on the occupation of both parents; however, the responses on these questions had to be recoded. If a student said his father is a university professor and

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<sup>49</sup> See paragraph 1 page 77 in the PISA data analysis manual.

another said that his father is a researcher, these two occupations may correspond to the same category. PISA adopted the International Standard classification of occupation (ISCO 1988) in order to code the occupation of both parents and then to recode the responses into the PISA index of occupational status ISEI. PISA also used the international standard classification of education (ISCED 1997) to code the level of education of both parents and then to recode the responses into the PISA index of education also called ISCED.

- Scale indices: constructed through the scaling of items. Most of these indices are derived through IRT scaling of dichotomous or categorical items, though some use a weighted maximum likelihood estimate.

Scale indices can be positive or negative. A negative value on an index does not imply negative responses; it only implies that students responded less positively than all students did on average across OECD countries. The student variables to be used in further analyses are the following:

## **2.1. Parental Occupation.**

Students were asked to report the occupation of both their parents and to state whether they were in full time employment, part time employment or unemployment. Students answers were first coded using the ISCO (1988), and then the responses were recoded into the international socio-economic index of occupational status (ISEI). This index captured the attributes of an occupation that convert education into income. This method has been used by Ganzeboom et al. (1992).

The occupational data for both parents were obtained from questions ST07Q01 (mother) and ST09Q01 (father) in the student questionnaire. Three simple indices were developed: BMMJ for mother's occupational status, BFMJ for father's occupational status and HISEI for the household's highest occupation status, which is the maximum of BMMJ and BFMJ. Higher values on these indices indicate higher occupational status.

## **2.2. Parental education.**

Parental education is perhaps one of the most important variables in studying educational outcomes and inequalities in performance. Four simple indices have been developed by PISA: the mother's level of education MISCED, the father's level of education FISCED, the household's highest level of education HISCED and finally the household's highest level of education in number of years PARED. These indices were derived from student responses on questions ST11Q01, and ST12 (for the mothers) and on ST13RQ01 and ST14 (for the fathers). Students were asked to identify the level of education of both their parents and the answers were coded according to ISCED (1997).

## **2.3. Home possessions.**

The PISA 2003 scale index of home possessions is derived using students' responses on the following questions:

Which of the following do you have at home?

- a) A desk for study. ST17Q01.
- b) A room of your own. ST17Q02.
- c) A quiet place to study. ST17Q03.
- d) Your own calculator. ST17Q07.
- e) Classic literature. ST17Q08.
- f) Books of poetry. ST17Q09.
- g) Works of art. ST17Q10.
- h) Books to help with your school work. ST17Q11.
- i) A dictionary. ST17Q12.
- j) A dishwasher. ST17Q13.
- k) More than 100 books. ST19Q01

The responses on these questions are binary; they were coded into HOMEPOS using IRT scaling and higher values on this index indicate higher home possessions.

## **2.4. The economic, social and cultural status.**

The economic, social and cultural status index (ESCS) was created to capture the wide effects of the previous three categories of variables; it is constructed using the following indices:

- a. The household's highest international socio-economic index of occupational status HISEI.
- b. The household's highest level of education in number of years of education PARED.
- c. Home possessions HOMEPOS.

The idea behind the construction of the ESCS composite index was that the socio-economic status of a household is determined to a large extent by the occupational status of parents, their level of education and their wealth. Since wealth is not easily assessed, PISA used home possessions as a proxy. Individual scores on the ESCS index are derived using principal component analysis and are standardized to have an OECD mean of 0 and a standard deviation of one. The principal component analysis was performed for each participating country. It showed that all three components are contributing to a similar extent in the ESCS index. Missing values on the three variables were imputed. Higher values on this index indicate higher status.

In the next subsection, as well as in the econometric regressions, only ESCS is to be used, since it encompasses all three other variables. Furthermore, HISEI, PARED and HOMEPOS are strongly correlated. Thus, it is not recommended to use them separately as independent variables.

## **2.5. Computer facilities at home.**

The PISA 2003 scale index of computer facilities at home (COMPHOME) is derived using students' responses on the following questions:

- a) Do you have at home a computer that you can use for school work? ST17Q04.
- b) Do you have at home any educational software? ST17Q05.
- c) Do you have at home a link to the Internet? ST17Q06.

The responses to these questions are binary; they were scaled using IRT scaling and higher values indicate higher computer facilities at home.

## **2.6. Immigration background.**

The simple index on immigration background is derived from student responses on questions ST15Q01 and ST15Q03. Students reported the country of birth of both their parents; then the responses were grouped into three categories:

- a) Native students: those students born in the country of assessment and who had at least one parent born in that country.
- b) First generation students: those students born in the country of assessment but whose parents were born in another country.
- c) Non native students: those students born outside the country of assessment and whose parents were also born in another country.

For the rest of the analyses, I am using a binary variable (ETR) taking the value of one if a student is a first generation student or a non-native. The use of such a variable permits the grouping of students with an immigrant background. This is done for analytical convenience, since in some countries the percentage of students in one of the two categories taken separately can be very close to zero.

## **2.7. Interest in mathematics.**

The PISA scale index of interest and enjoyment of mathematics (INTMAT) was developed to assess the involvement of students in this domain. It is derived from their responses on four items; these items are polytomous with a four point scale: “strongly agree,” “agree,” “disagree” and “strongly disagree.” All items were scaled with IRT scaling and higher values on this index indicate higher levels of interest and enjoyment of mathematics. The items that were used in the construction of this index are the following:

Thinking about your views on mathematics:

- a) I enjoy reading about mathematics. ST30Q01.
- b) I look forward to my mathematics lessons. ST30Q03.
- c) I do mathematics because I enjoy it. ST30Q04.
- d) I am interested in the things I learn in mathematics. ST30Q06.

## **2.8. Anxiety in mathematics.**

The PISA scale index of anxiety in mathematics (ANXMAT) was developed to reflect students' feelings of helplessness and emotional stress when dealing with mathematics. It is derived from their responses on five items; these items are polytomous with a four point scale: "strongly agree," "agree," "disagree" and "strongly disagree." All items were scaled with IRT scaling and higher values on this index indicate higher levels of anxiety. The items that were used in the construction of this index are the following:

Thinking about studying mathematics:

- a) I often worry that it will be difficult for me in mathematics classes. ST32Q01.
- b) I get very tense when I have to do mathematics homework. ST32Q03.
- c) I get very nervous doing mathematics problems. ST32Q05.
- d) I feel helpless when doing a mathematics problem. ST32Q08.
- e) I worry that I will get poor marks in mathematics. ST32Q10.

## **2.9. The school disciplinary climate as perceived by students.**

The scale index of disciplinary climate (DISCLIM) is developed to reflect the subjective perception of discipline by students in a school. This index is computed for each student, and thus reflects the discipline in a school at a micro level. However, in order to obtain a school aggregate variable of disciplinary climate one has to use the school principal's responses on discipline-related questions or the school average of DISCLIM. The index was derived from students' answers on five items; these items are polytomous with a four point scale: "Every lesson," "most lessons," "some lessons" and "never or hardly ever." Higher values on this index indicate higher levels of perceived discipline in a school. The items that were used in the construction of this index are the following:

How often do these things happen in your <mathematics> lessons?

- a) Students don't listen to what the teacher says. ST38Q02.
- b) There is noise and disorder. ST38Q06.
- c) The teacher has to wait a long time for students to <quieten down>. ST38Q08.
- d) Students cannot work well. ST38Q09.

e) Students don't start working for a long time after the lesson begins. ST38Q11.

### 3. School characteristics.

PISA 2003 also constructed a number of school indices describing school characteristics. The questions used in the construction of these indices were selected from a larger combination of possible questions on the basis of theoretical considerations. Most of them were scaled using IRT scaling or a weighted maximum likelihood estimator. Two types of variables are used in the next subsections:

**3.1. Pure school level indices:** these variables are obtained from the responses of school principals on the school questionnaire and are constant for all students enrolled in the same school. There are two types of school indices:

- I. Simple indices: constructed through the arithmetical transformation of items obtained from the different questionnaires. They might include the number of qualified mathematic teachers, the proportion of girls in a school, etc.
- II. Scale indices: constructed through the scaling of items. Most of these indices are derived through IRT scaling of dichotomous or polytomous items; some used a weighted maximum likelihood estimate.

**3.2. Peer effects:** Peer effects are school level aggregates of individual level characteristics. It is possible to represent peer effects through different moments of the same variable.<sup>50</sup> For instance, in order to examine if the influence of ESCS on performance scores is linear or not, we can consider student level ESCS, school average ESCS and the within-school dispersion of ESCS. These issues are thoroughly discussed in chapter IV.

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<sup>50</sup> In my theoretical models, I considered that ability and social capital can influence school quality, stratification and performance through their mean and their dispersions. More precisely, I affirmed that the relation between school quality, ability and social capital is not linear. In empirical terms this can be translated through the use of different moments of the same variable in the estimation procedures.



### **3.1. Pure school level indices.**

#### **3.1.1. Computer facilities at school.**

PISA 2003 developed a number of simple indices assessing the resources available for schools, such as the availability of computer facilities. The indices are derived through arithmetical transformations of the school principals' responses on questions SC02Q01, SC09Q01 and SC02Q02. The indices are:

RATCOMP: is obtained by dividing the number of computers available in a school by the number of students.

COMPWEB: is the proportion of computers connected to the web in a school. It is obtained by dividing the number of computers connected to the web by the total number of computers.

For the rest of the analyses, I will use COMPWEB, since it accounts for two aspects: the availability of computers and of internet.

#### **3.1.2. Teacher shortages.**

The scale index of teacher shortages (TCSHORT) is derived from the principal's responses on five items. These items measure the perception of potential factors hindering instruction; they were administered together with the items on school environment and educational resources. They are polytomous with a four point scale: "not at all," "very little," "to some extent" and "a lot." The index was constructed with IRT scaling. Higher values on this index indicate higher teacher shortages. The items used in the construction of the index are the following:

Is your school's capacity to provide instruction hindered by a shortage or inadequacy of any of the following?

- a) Availability of qualified mathematics teachers. SC08Q01.
- b) Availability of qualified science teachers. SC08Q02.
- c) Availability of qualified <test language> teachers. SC08Q03.
- d) Availability of qualified foreign language teachers. SC08Q05.
- e) Availability of experienced teachers. SC08Q06.

### **3.1.3. Quality of school educational resources.**

The index of quality of educational resources SCMATEDU was derived from seven items from the school questionnaire. These items measure the principal's perception of resource-related factors hindering Instruction. They were coded on a four point scale taking the following values: "not at all," "very little," "to some extent" and "a lot." The items were scaled with IRT and higher values on this index indicate higher qualities of educational resources. The items used in the construction of the index are the following:

Is your school's capacity to provide instruction hindered by a shortage or inadequacy of any of the following?

- a) Instructional materials (*e.g.* textbooks). SC08Q09.
- b) Computers for instruction. SC08Q15.
- c) Computer software for instruction. SC08Q16.
- d) Calculators for instruction. SC08Q17.
- e) Library materials. SC08Q18.
- f) Audio-visual resources. SC08Q19.
- g) Science laboratory equipment and materials. SC08Q20.

### **3.1.4. School type.**

For the classification of school types, PISA 2003 used two aspects about schools: funding and management. The information was obtained through questions SC03Q01 and SC04Q01 to SC04Q04. Schools were classified into three categories:

- a) Public schools controlled and managed by a public education authority or agency.
- b) Government-dependent private schools controlled by non-government organization or with a governing board not selected by a government agency that receive more than 50 per cent of their core funding from government agencies.
- c) Independent private schools controlled by a non-government organization or with a governing board not selected by a government agency that receive less than 50 per cent of their core funding from government agencies.

In my empirical analysis, I have combined category (b) and (c) for analytical convenience, since some countries do not have one of the categories, such as Finland. Thus, school type is a binary variable (PRIVATE) taking the value of 1 when the school is private.

### **3.1.5. Mathematics activities at school.**

The PISA 2003 simple index of mathematics activities (MACTIV) was derived from five items measuring what activities to promote engagement with mathematics are used in a school. The index is equal to the number of activities. The items used in the construction of the index are the following:

In your school, do any of the following activities to promote engagement with Mathematics occur?

- a) Enrichment mathematics. SC17Q01.
- b) Remedial mathematics. SC17Q02.
- c) Mathematics competition. SC17Q03.
- d) Mathematics clubs. SC17Q04.
- e) Computer clubs. SC17Q05.

### **3.1.6. School academic selectivity.**

School principals were asked about admittance policies in their schools through questions SC10Q01 to SC10Q06. The responses were coded on a four point scale taking the following values: “prerequisite,” “high priority,” “considered” and “not considered.” The question asked is the following:

How much consideration is given to the following factors when students are admitted to your school?

- a) Residence in a particular area. SC10Q01.
- b) Student’s academic record. SC10Q02.
- c) Recommendation of feeder schools. SC10Q03.
- d) Parents’ endorsement of the instructional or religious philosophy of the school. SC10Q04.
- e) Student’s need or desire for a special program. SC10Q05.
- f) Attendance of other family members at the school. SC10Q06.

For the rest of the analyses, I am only considering selection based on students' academic records. Selection is depicted through a binary variable ACADEMIC taking the value of 1 if the school considers academic records when selecting its students.

The use of academic selection is motivated by the fact that this type of selection has direct effects on the academic performance of students. The rest of the selection criteria will not be used, because they influence performance indirectly through stratification. For instance, selection through residence requirements may lead to social stratification. However, as we will see in the next chapter, stratification can be captured through peer effects and the use of the above selection criteria will be redundant.

### **3.1.7. School principals' perceptions of teacher morale and commitment.**

The PISA 2003 scale index of teacher morale and commitment (TCMORALE) was developed from school principals' responses on four items measuring principals' perceptions of teachers at their school. The responses were coded on a four point scale taking the following values: "strongly agree," "agree," "disagree" and "strongly disagree." The two categories "disagree" and "strongly disagree" were collapsed into a single category, due to very few responses in the latter one. The index was constructed through IRT scaling and the items used in its construction are the following:

Think about the teachers in your school:

- a) The morale of teachers in this school is high. SC24Q01.
- b) Teachers work with enthusiasm. SC24Q02.
- c) Teachers take pride in this school. SC24Q03.
- d) Teachers value academic achievement. SC24Q04.

### **3.1.8. School principals' perceptions of teacher-related factors affecting school climate.**

The index of school principals' perceptions of teacher-related factors affecting school climate (TEACBEHA) is derived from principals' responses on seven items measuring the potential teacher-related factors hindering instruction at school. The responses were coded on a four point scale taking the following values: "strongly agree," "agree," "disagree" and "strongly

disagree.” The two categories “disagree” and “strongly disagree” were collapsed into one category, due to very few responses in the latter one. The index was constructed through IRT scaling and the items used in its construction are the following:

In your school, to what extent is the learning of students hindered by:

- a) Teachers’ low expectations of students? ST25Q01.
- b) Poor student-teacher relations? ST25Q03.
- c) Teachers not meeting individual students’ needs? ST25Q05.
- d) Teacher absenteeism? ST25Q06.
- e) Staff resisting change? ST25Q09.
- f) Teachers being too strict with students? ST25Q11.
- g) Students not being encouraged to achieve their full potential? ST25Q13.

### **3.1.9. School climate: poor student-teacher relations.**

The simple index of poor student-teacher relations (MSTREL) was developed from students’ responses on questions ST26Q01 to ST26Q05, measuring different aspects of student-teacher relationships. The responses were coded on a four point scale taking the following values: “strongly agree,” “agree,” “disagree” and “strongly disagree.” This variable was obtained by computing the average of individual responses by school. The items used in the computation of this index are:

Thinking about the teachers at your school: To what extent do you agree with the following statements?

- a) Students get along well with most teachers. ST26Q01.
- b) Most teachers are interested in students’ well-being. ST26Q02.
- c) Most of my teachers really listen to what I have to say. ST26Q03.
- d) If I need extra help, I will receive it from my teachers. ST26Q04.
- e) Most of my teachers treat me fairly. ST26Q05.

### **3.2. Peer effects.**

As mentioned before, peer effects are depicted through school averages and other moments of individual level variables. The basic idea behind peer effects relies on the fact that students belong to a group (a school in this case); they influence the group and are influenced by it. The existence of peer effects is well documented in the economic literature and is not controversial. For instance, Summers and Wolfe (1977), Henderson (1978), Sorensen and Hallinan (1986), Kulik (1992), Epple and Romano (1998 and 2006) and Hoxby and Weingarth (2005) provide evidence that peer effects exist. However, in most previous studies, ability peer effects were the most-used form of peer effects. The purpose of these studies is to assess the impact of ability peer effects taken at an instant  $t$  on student performance at  $t+1$ . In the next chapter, I am considering different forms of peer effects such as: social, economic and cultural peer effects, psychological and disciplinary peer effects and immigrant background peer effects.

#### **3.2.1. Economic, social and cultural peer effects.**

The economic, social and cultural peer effects (DESCS) are the school averages of ESCS.

#### **3.2.2. Dispersion of ESCS in a school.**

The dispersion of ESCS, noted (VARESCS), is its within-school variance. This variable in itself does not represent a peer effect. However, when it is combined with school DESCs, it indicates whether these social, economic and cultural peer effects have a linear impact on performance or not. More intuitively, it is to test whether a dispersed social mix (social heterogeneity) has a positive or negative effect on individual performance.

#### **3.2.3. Possession of computer facilities peer effects.**

Possession of computer facilities peer effects (DCOMPH) are the school average of COMPHOME.

#### **3.2.4. Interest in mathematics peer effects.**

Interest in mathematics peer effects (DINTMAT) is the school average of INTMAT. This type of peer effect represents the impact of a generalized interest and enjoyment of mathematics within a school.

**3.2.5. Anxiety in mathematics peer effects.**

Anxiety in mathematics peer effects (DANXMAT) is the school average of ANXMAT. This type of peer effects represents the impact of a generalized feeling of anxiety and helplessness in mathematics within a school.

**3.2.6. Perception of discipline peer effects.**

Perception of discipline peer effects (DDISCL) is the school average of DISCLIM. This type of peer effects represents the impact of a generalized perception of discipline in a school.

**3.2.7. The percentage of immigrants in a school.**

The percentage of immigrants (DETR) is computed as the school average of ETR.<sup>51</sup> It reflects the national diversity in a school.

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<sup>51</sup> Note that for binary variables, frequencies (percentages) and means are the same.

## **C. Missing data and imputation techniques.**

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Economic research is always confronted with the problem of missing data. Improper handling of missing values may distort and even bias the results, especially when a selection bias is introduced to the remaining data set. In what follows, I will examine the different forms of missingness and the techniques used to deal with this problem.

### **1. The nature of missing data.**

Four different types of missing data exist:

#### **1.1. Missing completely at random (MCAR).**

MCAR exists when missing values are randomly distributed across the dataset. This means that the probability that an observation is missing is unrelated to the value of the observation or any other observation. In the PISA dataset, this means that if a student has a missing value for the level of parental education, this “missingness” should not be related to the actual level of parental education. Usually, this is not the case; students who do not answer the question related to the level of parental education often have uneducated parents. When missingness is not completely at random, a selection bias is introduced, since we will lose students whose parents are not educated and the remaining dataset will only contain students with educated parents. In contrast, if missing data results from a student being ill on the day of the interview, we can presume that the data is missing at random. In fact, any piece of data is as likely to be missing as any other piece.

#### **1.2. Missing at random (MAR).**

MAR exists when missing values are not randomly distributed across all observations but are randomly distributed within subsamples. In PISA, this could mean that parental education would be missing more among students with uneducated parents. Hence, missingness is related to the value of parental education for the entire sample. However, within the native and non-native groups of students, the probability of reporting parental education may be unrelated to its actual value. Thus, missingness is random within each subsample. More



precisely, the probability that an observation is missing is unrelated to the value of the observation after controlling for a particular variable (in this case, immigrant status).

### **1.3. Missing not at random (MNAR).**

When data are neither MCAR nor MAR, they are MNAR. This is the case when the probability of missingness is related to the value of the observation for the entire dataset and for subsamples. In PISA, it means that the probability of reporting parental education is related to its actual value, even within subsamples.

When we have a dataset that is MAR or MNAR we have a selection problem that should be solved.

### **1.4. Monotone missingness.**

A data set with a hundred variables is said to have monotone missingness, when the event that the 70<sup>th</sup> variable is missing implies that all subsequent variables are missing for the same individual. This usually happens due to fatigue, boredom, lack of knowledge etc. (e.g. fatigue or boredom occur when long questionnaires are administered. Due to ennui the last questions are systematically unanswered). Monotone missingness is not a problem in the PISA context. Questionnaires were field trialed and non-response and response patterns were thoroughly examined in order to develop the final version of the questionnaires. The length of the administered questionnaires was about 20 minutes (helping to reduce the effects of fatigue).<sup>52</sup>

## **2. Deciding on the technique to deal with missing data.**

In the presence of missing data, the analyst has several options:

### **2.1. Do nothing, or listwise deletion.**

The first option requires working with an incomplete dataset. The problem with this approach is that the results of different analyses performed on the same data file may be inconsistent, depending on the set of variables used for each analysis. The second option, listwise deletion, requires the deletion of all observations containing at least one missing value for all variables

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<sup>52</sup> See the PISA technical report on pages 40 to 43 and the PISA data analysis manual on page 15.

included in the analysis. This approach is perhaps the simplest, as it allows the use of statistical softwares, such as SAS, that only work with a complete dataset (usually SAS performs listwise deletion automatically for regressions). However, the major setback of this approach becomes evident when the missing data is not MCAR. In this case, missingness is related to the value of the missing variable. For instance, we may only be losing students with low parental education and the dataset is biased. To this is added the loss of degrees of freedom and the obligation to work with a smaller dataset. Furthermore, this method can be considered effective when missing values do not account for more than 10% of the actual dataset. With PISA, when combining all the key variables, this method becomes highly unrecommended, since missing values may cause the loss of up to 30% of the dataset.

In addition to this, we know that PISA is not MCAR, since there is a tendency for students belonging to low socioeconomic classes not to report the level of parental education, parental occupation or home possessions.<sup>53</sup>

## **2.2. Coding the missing values as an independent category.**

In some applied econometric studies it is possible to control the belonging of individuals to particular groups. For instance, if questionnaires were handed separately to groups of blue collar and white collar workers, then occupational classification is easy even if there are some non-responses. However, if the questionnaires were handed on the street, then non-responses cannot be classified.

In this case, one can drop all missing values from the dataset and try to convince oneself that they are MCAR. However, it would be hard to defend this decision, since there is no evidence why missingness should be random.

The coding of missing values as an independent category was developed by Sethi and Seligman (1993). This technique requires adding a new category for non-responders. For example, if the occupational status has two categories (white and blue collar workers), the third would be for non-responders. The aim is to compare non-responders to the category of reference to see if there is any difference between the two (through the null hypothesis).

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<sup>53</sup> Note that MCAR is very rare in economic research, since missingness is usually related to some characteristics of the sampled units.

When this is done, the comparison of the rest of the groups with the category of reference can be done with more confidence. This technique seems to be simple when missing values are those of an independent variable. However, it becomes very complicated and even risky when values of the dependent variable are the ones missing.

On page 177 of the PISA data analysis manual, it is suggested to delete any missing values when they do not exceed 10% of the dataset. However, when they do exceed 10%, imputation techniques have to be used or a variable should be dropped. In what follows, I shed light on different imputation techniques and thoroughly examine the one used in my thesis for the imputation of missing values on the PISA dataset.

### **2.3. Hot deck imputation.**

Hot deck imputation requires a random draw from a collection of data to fill in missing values. For example, if a young, female, white collar worker refused to participate in a survey, then hot deck imputation consists of drawing randomly an observation from other young, female, white collar workers to fill in the missing data. In other words, the replacement data is a random draw from similar participants. This technique was used especially in the 1950s and it was the most effective when datasets contained very few missing values and were very large (population surveys, and censuses). This technique is no longer in use today, and more modern imputation approaches have been developed.<sup>54</sup>

### **2.4. Mean imputation.**

The idea behind mean imputation is very simple; it requires substituting missing values by the average of available ones for the same variable. For instance, if the level of parental education is missing for some students, then it can be replaced by average parental education for the rest of the responders. This technique is considered to be old and obsolete for the following reasons. Firstly, it does not add any new information; the overall average of the sampled units will be the same. Secondly, this technique will definitely bias the variance towards zero, and hence underestimate the dispersion of the variables (Rubin 1987, P.13).<sup>55</sup> This technique should be avoided, since it would introduce considerable bias into the dataset.

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<sup>54</sup> See Scheuren (2005) for a presentation of the evolution of multiple imputation techniques.

<sup>55</sup> This technique increases the number of observations, but does not increase their dispersion. In other words, the denominator of the variance increases while the numerator is still the same.

## **2.5. Regression imputation.**

Linear regressions can be used to replace missing values. Under this technique, a missing value is replaced by an estimate obtained on the basis of other, available variables. In comparison to mean imputation, this technique has the advantage of conditioning missing data on available ones. However, the variance of a variable is still biased, since missing values are treated as completely predictable. In other words, if we are regressing Y on X, for every situation where  $X=2$  we will impute the same value of Y. Thus we will have less variability for the imputed data than we would have had if these values were not missing (variance bias).

## **2.6. Maximum likelihood imputation.**

The intuition behind the maximum likelihood imputation is simple, even if the calculations are highly complex. The imputation of missing values is done through the expectation maximization algorithm (EM algorithm). The EM algorithm consists of computing the parameters of the model on the basis of the incomplete data. These parameters are used to estimate the missing values. Then the new complete dataset is used to estimate new parameters of the model, and so on. The EM algorithm alternates the estimation of parameters, and then the estimation of missing values, until the solution stabilizes. At this point, a maximum likelihood estimate is reached and we can obtain a final imputation of the missing values. The EM algorithm is examined in Schaffer (1999, 1997).

In comparison to regression imputations, the EM algorithm does not treat missing values as completely predictable. It adds an error term to the variances it estimates in order to reduce the bias in their estimation and to reflect the uncertainty about the right value to impute. It acknowledges the fact that what is unknown is unknown.

## **2.7. Multiple imputations.**

Multiple imputations are similar to the maximum likelihood imputation technique. However, instead of generating a single imputation through the EM algorithm, multiple imputations will be generated through the use of different error terms randomly drawn from a standard normal distribution.

The multiple imputation process works similarly to the EM algorithm; the first step requires the estimation of the model parameters from the incomplete dataset, and then imputed values can be generated from these parameters. The imputed dataset is used again to estimate new parameters that will be used to impute a new dataset, and so on. The process stops once it reaches a stable solution (the maximum likelihood). This step is equivalent to the maximum likelihood imputation.

This process is repeated several times in order to generate several imputed datasets; these data sets differ slightly from one another, because each is generated using different random errors drawn from the standard normal distribution.

One problem with multiple imputations was the absence of a simple technique permitting the generation of imputed values. However, the development of the *Markov Chain Monte Carlo* simulation techniques rendered the process simpler. The *MCMC* techniques are a type of algorithms for sampling from probability distributions based on a *Markov Chain* that has the desired distribution as its equilibrium distribution. The *Markov Chain Monte Carlo* techniques are used when it is infeasible to compute exact results with a deterministic algorithm. For a complete description of the MCMC method, see Gill 2008 and Robert and Casella 2004.

### **The case of PISA.**

For the PISA dataset, I have decided to apply the Multiple Imputation technique using the expectation maximization algorithm for the following reasons:

1. When combining the different key variables that I have selected, missing values accounted for 30% to 40% depending on the country.
2. There is no reliable evidence to support that missingness is completely at random (MCAR). As mentioned above, in educational research, missingness is usually related to the values of the missing variable.
3. In the case of MNAR or MAR, and with relatively high levels of missingness, the risk of dropping the missing values should be avoided.
4. Multiple imputation techniques are considered to be the most modern and the most reliable. They result in valid statistical inferences that properly reflect the uncertainty due to missing data.

I have used the SAS procedure MI (*Proc MI*) with the following specifications:

- For the EM algorithm, I have selected a convergence criterion of 0.0001. This means that the maximum likelihood iterations will continue until the maximum change in the parameter estimates between two iterations is less than 0.0001. At this point, the iterations are considered to have converged and the final results are obtained.
- The maximum number of iterations is set to 200. However, convergence was achieved after 6 iterations.
- For the Markov Chain Monte Carlo, I have selected full imputation for the replacement of all missing values (in SAS it is possible to impute the dataset until it becomes monotone; however, in PISA, we do not have monotone missingness).
- I have imputed all the continuous variables needed in the analyses.

## **D.The choice of countries.**

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### **1. International typology of schooling systems.**

As I have mentioned earlier in this chapter, PISA 2003 covered 30 OECD countries and 11 partner countries. For my empirical analyses, I have decided to select only 5 among them: Finland, Germany, Italy, the United Kingdom and Japan.<sup>56</sup> This decision is motivated by two reasons. Firstly, the purpose of the empirical part is to illustrate the theory developed in chapter 2, to conduct a comparative analysis on the mechanisms of stratification across major education systems and, finally, to derive some policy implications. It is more reasonable to select few countries representative of major education systems than selecting all countries, with some being irrelevant to the analysis. Secondly, it is difficult to work on the entire sample of countries, because of the lack of space and due to the complexity and the thoroughness of the analyses to be conducted.

These countries were selected according to the Green, Janmaat and Preston typology (Green et al 2006). This typology was constructed using PISA 2000 and IALS (International adult literacy survey, also developed by the OECD). The authors found the existence of regional patterns of educational inequalities. For instance, Nordic countries have almost entirely unstreamed comprehensive schooling. East Asian ones are described as being very comprehensive. English speaking countries have differentiated school systems. And finally Germanic countries have selection in the lower secondary phase.

The use of this typology has a number of advantages. First, instead of reconstructing a new typology we will use countries that we already know that they have different schooling systems. Second, the Green et al typology was constructed using the same type of data (PISA and IALS). Finally, students selected for the PISA study are 15 years old and have not yet finished their schooling. Thus, they reflect the actual state of the schooling system.<sup>57</sup>

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<sup>56</sup> I would have liked to select France. However, the absence of data on schools due to non-responses makes the French sample unusable for the study of stratification.

<sup>57</sup> Adults are not adapted for the study of actual education systems, since they are likely to have experienced other systems in the past.

Using PISA, the authors showed, through a number of descriptive statistics measuring inequalities of performance, that countries can be grouped according to regional patterns. The major findings are the following:

- Using a basic dispersion measure of performances, they found that the most unequal of the advanced states are: Belgium, Germany, New Zealand, the USA and the UK. The most equal are Korea, Finland, Japan, Iceland, Canada and Ireland.
- Another measure of educational inequalities is the strength of the relation between social status and performance. The authors found similar regional patterns, with the Anglo-Saxon and Germanic countries being the most unequal, and the South European, East Asian and Nordic countries being more equal.
- Even when the social background was broken down into separate effects of parental wealth and professional occupation the same patterns result. Parental wealth has the strongest effect in the USA, New Zealand and Germany and the lowest effect in the Netherlands, Japan, Finland, Italy and Norway. Parental occupation has the strongest effect in Germany, the UK and Belgium, and the lowest in Korea, Finland, Canada and Italy.
- The social intake of a school (considered to be the average of ESCS in a school) is found to have a major effect on performance scores. The countries in which school status has a major effect on performance scores are Ireland, New Zealand, Australia, the USA and the UK, and the countries where the effect is found to be low are Japan, Korea and the Nordic countries.
- Using the ratio of between-school performance variance on total variance, the authors showed that Nordic countries tend to have the lowest disparities between schools, followed by the English speaking countries and then the German speaking ones. Southern European countries do not show a clear regional pattern.

In conclusion, 5 regional patterns of educational inequalities were identified. These patterns can be attributed to historical, sociopolitical and regional characteristics of education systems. The 5 regional patterns are:



### **1.1. The East Asian model.**

The East Asian model includes three countries: Japan, Taiwan and South Korea. The Japanese educational system is traditionally characterized by high performances and low disparities among students (White 1987, Dore and Sako 1989). This perception of the Japanese education system is confirmed by major international studies, such as IEA and PISA.

Educational equality in Japan can be attributed to historical factors as well as to specific social and educational elements. Schooling was widespread in Japan even before the *Meiji* reforms of 1868. During the *Bakumatsu* era (also known as *Tokugawa Shogunate*), schools assumed multiple roles; they were the place for the acquisition of knowledge and for the socialization of children. The *Meiji* reforms accentuated the importance of Japanese schools as instruments of ideological unification and as a driving force for the industrialization of the country (Green 1997, 1999). This industrialization led to the emergence of a large middle class that favored equality in education.

In post-World War II Japan, an educational system with characteristics favoring equality emerged. This system was led by the American occupying power and it was characterized by a 6-3-3 organization of elementary, secondary and upper secondary schooling. However, the upper secondary system soon became selective and specialized, contrarily to the elementary and lower secondary ones. Other major characteristics of the Japanese system are the centralization of control and funding, ability mixing and automatic promotion between grades (Ichikawa 1989).

Other East Asian countries, such as Taiwan and South Korea, have similar characteristics; this is perhaps due to long periods of Japanese occupation. On the other hand, Singapore and Hong Kong are influenced by British colonial legacy and are less similar to Japan.

### **1.2. The Nordic model.**

The Nordic model includes 5 countries: Finland, Norway, Sweden, Iceland and Denmark. These countries share the same institutional structure, with nine to ten years of all-through comprehensive schooling in the primary and lower secondary systems and with selection being delayed until the upper secondary phase. The private sector is small and is funded and controlled by the state.

The Nordic model can be described as having a radical form of comprehensive schooling leading to the most equal outcomes in comparison to other countries. This egalitarian structure can be attributed to a number of demographic, sociopolitical and historical characteristics.

Firstly, the common Lutheran religious traditions were favorable to the establishment of universal literacy and state regulation of education. Secondly, the Nordic education system evolved under a political democracy due to historical traditions (Boli 1989). Thirdly, the social class structure in the Nordic countries was favorable to populist politics; the existence of weak landowner and bourgeois classes combined with strong peasant farmers and working classes paved the way towards the comprehensivization of education systems. Finally, low population density across Nordic countries rendered school choice limited, due to the unavailability of different sources of educational supply.

Two major reforms have shaped the Nordic model. The first occurred at the beginning of the twentieth century, when the lower secondary classes from the elite Latin schools were transferred to new middle schools that provided a common ladder from the primary to the upper secondary phase. This reform occurred in Norway in 1895, in Denmark in 1903 and in Sweden in 1905. The second reform occurred after the Second World War, under social democratic governments. It consisted of integrating middle and primary schools in order to create a nine year, all-through comprehensive schooling system. This happened in Norway in 1969, in Sweden by 1962 and in Denmark by 1975.

### **1.3. The Germanic model.**

The Germanic model includes: Germany, Austria, Belgium, Luxembourg, German-speaking Switzerland and the Netherlands. These countries are the only ones in Western Europe to have selective schooling in the lower secondary phase. In Germany, the first selection happens at the age of 11 or 12. Even if parents have the right for school choice, children may be relocated to professional schools if they do not meet the required performance standards. Note that in Germany, the Landers have dropped the old comprehensive schooling system, except in Berlin, and adopted a new selective one.

The persistence of such a selective system can only be understood in the light of a number of characteristics. Firstly, Germany has a federal political system where educational decisions are taken at a regional level, something that does not favor national reforms; however, this argument does not apply to the rest of the Germanic model. Secondly, all the countries have important apprenticeship systems that are orientated towards the labor market, as well as a tradition of cultural particularism associated with the German model of citizenship. Finally, the German labor market can be described as having a high level of coordination leading to the provision of training at a low cost and to mobility between firms (Marsden and Ryan 1995). These factors led to the existence of differentiated schooling tracks destined for particular occupational trajectories.

#### **1.4. The Mediterranean countries.**

The Mediterranean countries include Spain, France, Italy and Greece. Most of these countries share Napoleonic legacies of educational centralization. However, they do not form a distinctive model due to some disparities, (Green et al p.134). The major common characteristics are:

- A relatively old comprehensive lower secondary school system, dating from the 60s for some countries, and a differentiated upper secondary one. E.g. streaming in France was completely abolished in 1978. However, school principals can still use ability groupings for pedagogical reasons.
- Grade repetition for low performing students. Low performers are obliged to repeat a grade in order to meet the required standards to pass.
- Absence or limited school choice. For instance, in France school choice was limited and derogations were controlled by area committees. However, since the end of 2007, France abolished school districts and choice was made possible.

#### **1.5. The English speaking countries.**

Similarly to the Mediterranean countries, the English-speaking ones have large differences in their education systems and do not form a distinctive model. This group includes: the USA, the UK and New Zealand. They share an incomplete comprehensive system and large educational inequalities in comparison to the rest of the world. The major common characteristics are the following:

- The existence of large disparities within each country, between states or even between school districts.
- Incomplete comprehensivization of the schooling system. In the UK, the comprehensivization process was supposed to be achieved by 1988. However, the voluntaristic mode of educational legislation, and the introduction of competition into the education market, were causes for major hindrances.
- The existence of different types of schools, especially a large parallel private system.
- The existence of large disparities in curricula and school structures as well as in the practice of streaming, school choice and diversification policies.

## **2. Towards an integrated treatment of educational stratification.**

Using the Green et al typology of schooling systems as a starting point, I have selected 5 countries representative of each system: Japan for the East Asian model, Germany for the Germanic model, Italy for the Mediterranean countries, Finland for the Nordic model and the UK for English-speaking countries. Despite that, the Mediterranean and English-speaking countries do not form distinctive models; they still share some common characteristics.

In comparison with what Green et al did, a number of issues have to be discussed:

1. Green et al as well as the different PISA reports (2000 and 2003), used a number of descriptive statistics to examine differences between countries. These statistics can give a general perception of schooling systems. However, they are incapable of thoroughly analyzing the mechanisms behind performance inequalities. Even correlation coefficients cannot describe alone the functioning of an education system and they usually suffer from the omitted variable bias.

**The omitted variable bias:** this bias occurs when the specification of the model is incorrect. This means that an independent variable that should be in the model is omitted. This problem becomes more worrying when the omitted variable is correlated with other independent variables. In this case, all estimates are likely to be biased and inconsistent and inference cannot be correctly established. These issues are thoroughly discussed in chapter IV.

2. In relation to point 1: in the rest of this chapter, I will use a number of descriptive statistics based on means and variances of different variables. They are intended to provide a starting point for further analyses as well as a general view of education systems. Correlation coefficients will not be used, since in chapter IV a multilevel estimation approach will provide robust answers on the relation between these variables and performance scores.
3. Green et al, as well as PISA 2000 and 2003 reports, used a linear in mean representations of social peer effects. In other words, the social status of a school is determined only by average ESCS. For example, consider two schools A and B: A has two students with ESCS equal to 9 and 11 and B has two students with ESCS equal to 18 and 2. Obviously, these two schools have the same average ESCS of 10. However, the dispersions are different with school B having a larger variance (school B is more socially diverse). In the PISA reports and in Green et al, average ESCS was used to describe a school's intake mix (see Green et al p. 123). In reality, average ESCS does not tell anything about a school's social mix; an average is not a measure of diversity.
4. In relation to point 3: I am taking into account different moments of ESCS: its mean and variance. The use of the dispersion of ESCS will allow us to determine if the impact of social peer effects on performance is linear or not. Hence, it will provide an answer to the theoretical investigations developed in chapter 2 and it will permit the identification of stratification patterns across the selected countries.

In the following sub-sections, a number of descriptive statistics are used to further the Green et al typology along several axes: streaming, decentralization, school choice policies and stratification.

## **2.1. Selection and tracking policies.**

In most of the OECD countries, schooling till the end of the lower secondary phase was non-selective. However, in recent years the non-selectivity of these systems is being reviewed. The critics of comprehensive schooling argue that comprehensiveness did not lead to the democratization of access to education and that ability mixing is not adapted to students' special needs. As a result, some OECD countries adopted tracking policies and developed differentiated curricula to meet the needs of particular target populations.

Other OECD countries with comprehensive schooling adopted different methods for the management of heterogeneity, such as grade repetition and unofficial streaming. However, these policies were also a subject for debate. Do they allow low achievers to meet performance standards, or do they reinforce inequality by labeling them? Grade repetition has been mostly used in Mediterranean countries, whereas tracking and setting policies were used in English-speaking ones.

The existence of comprehensive schooling has always been challenged by unofficial streaming policies. Most countries officially acknowledge the fact that selection is not made until the end of the lower secondary phase. However, the presence of classes for special needs, of particular study programs and of implicit mechanisms orientating students towards particular options in the upper secondary meant that comprehensive schooling concerned less than 100% of the student population in the OECD countries (actually, about 53%; see Mons 2004 p. 187).

### **2.1.1. Comprehensivization vs. Streaming.**

Comprehensive schooling is considered to be the dominant model in most of the OECD countries. However, in some of them streaming does exist. This is due to unachieved comprehensivization or to the disaggregation of comprehensive schooling.

Within the OECD, 53% of the countries **officially** have comprehensive schooling until the end of the lower secondary phase as part of their legislation. These include Nordic countries, Mediterranean countries, ex-USSR countries and some emerging ones (Thailand and

Indonesia). 18% of the OECD countries have selection at the end of the primary phase and these include Germanic countries (Germany, Austria, Belgium...). 11% of the OECD countries practice selection during the lower secondary phase around the age of 12 or 13, these include (the Czech Republic, Hungary, Luxembourg...). And finally, 18% of the PISA countries officially do not have selection at all (but when different tracking forms are taken into account they do); these include English-speaking countries (the UK, the USA, New Zealand and Canada as well as Brazil and Argentina), see Mons (2004, p.186).

This official version is challenged by a number of considerations: first, some countries use orientation rather than effective selection at the end of the primary phase. They account for 28% of the OECD countries, instead of 18% in the official version. Second, some of the OECD countries start their first selection before the end of the lower secondary phase, through tracking and setting or special study programs that definitely affect future educational choices; they account for 18% versus 11% in the official version. And finally, ability grouping seems to be a common practice across PISA countries; even if officially ability grouping is not allowed, some school principals may still use it to facilitate instruction.

In the following table, the percentages of classes adopting ability grouping was computed using question 16 from the PISA 2003 school questionnaire.

*Table 3.1. Ability grouping.*

<b>Selected countries.</b>	<b>PISA</b>	<b>Absence of ability grouping.</b>	<b>Ability grouping for some classes.</b>	<b>Ability grouping for all classes.</b>
Germany		52.75%	21.21%	26.04%
Finland		43.82%	41.75%	14.42%
Italy		36.25%	43.4%	20.35%
Japan		56.02%	29.85%	14.13%
UK		2.31%	27.79%	69.9%

It is obvious that most countries practice certain forms of ability grouping. For instance, the UK officially has a comprehensive school system. However, 69.9% of the schools have complete ability grouping; this can be considered as a manifestation of setting policies. Even Finland, which is considered to have one of the most comprehensive schooling systems, has

14.42% of complete ability grouping. The presence of high levels of streaming in the PISA sample is perhaps due to the fact that PISA accounts for 15 year-old students at the end of compulsory schooling, when orientation and selection start to be operated even in comprehensive systems.

In general, most PISA and OECD countries have certain levels of ability grouping. In the Nordic countries, one third to one half of the schools affirm using ability grouping for pedagogical reasons, even if the practice is limited. On the other hand, in English speaking countries, the use of ability grouping is very common. About 50% to 90% affirm using ability grouping as a method of managing heterogeneity.

When accounting for these considerations, Mons (2004) reached the following classification of systems.

*Table 3.2. Phase of the first selection.*

<b>Geographical models.</b>	<b>Phase of the first real selection.</b>		
	<b>End of Primary education</b>	<b>During lower secondary phase.</b>	<b>End of lower secondary phase</b>
Germanic model.	100%	0%	0%
Nordic model.	0%	40%	60%
English-speaking countries.	16%	16%	68%
East Asian model.	20%	20%	60%
Mediterranean countries.	0%	0%	100%

As we can see, Germanic countries have a model where students are tracked at the end of the primary phase; this is mostly due to the existence of a strong apprenticeship system. On the other hand, Mediterranean countries have an all-through comprehensive system until the end of the lower secondary phase. However, when we consider grade repetition in the following subsection, the perception will be different. Nordic countries have an all-through comprehensive schooling system, but some of them (Norway and Iceland) start to operate orientation (preselection) by the end of the lower secondary phase. English-speaking as well



as East Asian countries have different tracking systems that start to operate as early as the end of the primary phase. However, most of the selection is done by the end of the lower secondary phase.

In conclusion, we can say that the perfect all-through comprehensive system does not exist. All countries have certain levels of selection, streaming, setting, or at least unofficial orientation, with some being more comprehensive than others.

### **2.1.2. Comprehensivization vs. grade repetition.**

Grade repetition has been used as an instrument of managing heterogeneity of the student population. Students are allocated to classes not according to their date of birth but according to their abilities, with low performers being obliged to repeat a grade until they meet the required performance standards to pass. In other words, instruction is organized in terms of a number of successive curricula that a student has to master. As long as a student does not master the curriculum of a particular grade, he might be obliged to repeat that grade. Grade repetition represents a different form of streaming by ability.

This practice has been used in most of the developed countries, but some have abolished it. Nordic countries (Finland, Denmark, Sweden and Norway) have automatic promotion that is part of their legislation. English-speaking countries have replaced it with different forms of tracking. However, they still have limited grade repetitions that affect mostly ethnic minorities (Mons 2004).

In the following table, I have computed the percentage of students repeating a grade in the lower and upper secondary phases using question 6 from PISA 2003 school questionnaire.

*Table 3.3.* Grade repetition in the lower and upper secondary phases.

<b>Selected countries.</b>	<b>PISA</b>	<b>Percentage of grade repetition in the lower secondary phase.</b>	<b>Percentage of grade repetition in the upper secondary phase.</b>
Germany		4.4%	5.2%
Finland		0.35%	3.43%
Italy		5.95%	8.94%
Japan		4.59%	6.57%
UK		0.16%	0.54%

As we can see, the two countries having the lowest grade repetition for the lower secondary phase are Finland and the UK. In Finland, grade repetition was abolished, whereas, in the UK, setting policies have been used as an alternative. The percentage of grade repetitions becomes higher for Finland in the upper secondary phase; this is due to the fact that, after a long comprehensive phase of 9 years, selection will be operated.

In Germany, the percentage of grade repetitions is higher than that of the previous two countries. Before the age of 14, about one-fourth to one-third of students have already repeated one grade. Italy has the highest levels of grade repetitions in both phases. Italy is known for the particularity of not applying any kind of restrictive policies on grade repetitions, contrarily to other Mediterranean countries that have always allowed repetitions, but under some constraints; (e.g., Greece does not allow repetitions in the primary system and France reduced repetitions in this phase). Note that the percentage of students repeating a grade is always higher in the upper secondary than in the lower secondary systems.

## **2.2. Decentralization policies.**

Decentralization policies have been widely used since the 1980s. They have been implemented by most political parties in most countries around the globe: in the UK, decentralization started with Thatcher and continued under the Labor party. In France, decentralization policies started with the socialist party and continued under the UMP. In Chile and Argentina, decentralization started under authoritarian regimes and continued under democracies. Even in Sweden, decentralization started in the new millennium with the transfer of certain tasks from central government to regions and schools.

In chapter II, I have showed that finance decentralization may lead to disparities among communities (or regions) in terms of funding. When other decentralization policies are accounted for, we can imagine that these disparities would be even more important. Within the same country, a student living in a poor neighborhood would not have the same educational quality as a peer living in a more prosperous one. In fact, if funding is based on local taxes, then schools in poor neighborhoods would have less funding; if teacher hiring and allocation is not centralized, then good schools would attract the most performing teachers by paying them more. These phenomena can even get worse when social stratification is taken into account, since poor neighborhoods tend to have lower quality peers.

We can reasonably say that decentralization policies will decrease territorial homogeneity. Disparities of funding alone may have important effects on several aspects influencing performance, such as teacher quality, the quality of physical and education infrastructures, and the quality of peers.

In this subsection, I will examine the different decentralization policies used in the selected countries.

### **2.2.1. Different decentralization policies across PISA countries.**

In general, the term decentralization is used to describe policies through which some responsibilities are transferred from central government to subunits. These may include the transfer of authority from central government to geographical units such as regions, or the transfer of authority to sub-national entities that are still under governmental supervision (e.g. schools), or even the transfer of authority to autonomous entities that are independent from central government. (see Cheema et al 1984).

The use of decentralization in the management of education systems can be attributed to some historical and sociopolitical factors. For instance, in most of the English-speaking (the UK, the USA, Australia) and Germanic countries (Germany, Switzerland), a historical decentralized management of education was in place, due to the federal nature of these states. However, in the Mediterranean (notably France) and Asian countries (Japan and South

Korea), a centralized management was adopted, due to republican traditions in the first and to the central role played by government in the conception of public education in the second (Ichikawa 1989).

Decentralization policies concern a number of issues: the management of funds, of teaching staff and of curricula. The different combinations of these policies and the level to which responsibilities are transferred define different forms of decentralization.

PISA 2003 accounted for the degree of school autonomy through question 26 in the school questionnaire. This question is used to describe how much autonomy schools have; however, it does not describe how much decentralization is there at a higher level. In other words, if educational management was centralized at a regional level, schools would still have low autonomy but the country may still be considered as decentralized. Thereby, countries having a high score on this question can be considered as the most decentralized, since responsibilities are transferred towards the smallest subunits (schools); however, countries having a low score can still be considered as decentralized at higher levels.

This question covers 4 main areas: the management of staff, of budgets and of curricula, as well as student assessment. The wording of the question is the following:

In your school, who has the main responsibility for?

- a) Selecting teachers for hire?
- b) Firing teachers?
- c) Establishing teachers' starting salaries?
- d) Determining teachers' salary increases?
- e) Formulating the school budget?
- f) Deciding on budget allocations within the school?
- g) Establishing student disciplinary policies?
- h) Establishing student assessment policies?
- i) Approving students for admittance to the school?
- j) Choosing which textbooks are used?
- k) Determining course content?
- l) Deciding which courses are offered?

The options corresponding to each of these questions are: 1) *Not a main responsibility of the school*, 2) *School's governing board*, 3) *Principal*, 4) *Department head* and 5) *Teachers*. The responses on this question were scaled using IRT and an index of school autonomy (SCHAUTON) was created, with higher values indicating higher levels of autonomy (negative values indicate that principals responded less positively than the OECD average).

Table 3.4. Degree of school autonomy.

<b>Selected PISA countries</b>	<b>Degree of school autonomy.</b>
Germany	-0.695
Finland	0.02
Italy	-0.457
Japan	0.114
UK	1.137

In this table, we can see that the UK has the highest score on the degree of school autonomy. This is due to strong decentralization policies that transferred responsibilities from central government to schools.

Japan and Finland have a low level on this indicator. Japan is well known for its centralized organization of education, except in teacher hiring and budget allocation, where local authorities may intervene. Finland as well as other Nordic countries operated a policy of decentralization at a regional level and thus, schools still have a low level of autonomy.

Italy, like other Mediterranean countries, has a tradition of centralized management of education that becomes apparent through this indicator. Finally, Germany, a federal state, seems to have an unexpectedly low level of school autonomy. In fact, only 6% of the federal states in the OECD have applied decentralization policies at a school level and most of them transferred responsibilities to local or regional authorities, such as the Landers in Germany. (see Mons 2004).

Table 3.5. Degree of resource autonomy.

<b>Selected PISA countries</b>	<b>Degree of resource autonomy.</b>
Germany	1.3699
Finland	3.0104
Italy	1.4082
Japan	2.5390
UK	5.4156

Table 3.6. Degree of curricular autonomy.

<b>Selected PISA countries</b>	<b>Degree of curricular autonomy.</b>
Germany	3.0026
Finland	3.891
Italy	3.7004
Japan	3.9329
UK	3.9283

When the degree of school autonomy is broken into resource and curricular autonomy, we can see that the UK still has the highest levels on both. In terms of curricular autonomy, all countries seem to have similar levels; this indicates that most countries have transferred some responsibilities in the definition of curricula and the selection of text books towards schools.

In terms of resource autonomy, Germany has the lowest score, since funding is centralized at a regional level. Italy has a low level as well, since it is a traditionally centralized country. Finland and Japan have a higher level of resource autonomy; that is due to the transfer of some management tasks to schools.

Mons (2004) developed a useful typology of decentralization policies across the OECD countries, following a number of educational tasks that include the following:

- The conception of education systems: 50% of OECD countries conceive their education systems at a central government level, even if 30% of these countries may give certain responsibilities to schools in the definition of certificates. 20% of the countries, mostly

those with federal systems, have completely transferred this task to regional authorities. And 25% of the OECD countries share this responsibility with local or regional entities.

- The management of education systems: most OECD countries have decentralized this task towards regional entities. However, the importance of this decentralization varies between countries. In traditionally centralized ones, like Greece and France, local authorities have been granted the right for the management of educational equipments, transport of students to and from school, etc. Whereas, in countries like Luxembourg, Mexico and Japan, local authorities have been granted the right for the complete management of a particular education level (e.g. primary education). In English-speaking countries (the UK, New Zealand) and Flemish Belgium, a large decentralization was operated in favor of schools, with the latter having the responsibility for organizing curricula, hiring teachers and managing budgets. In federal countries, only 6% of them have adopted decentralization at a school level and most of them have a management system that operates at a higher regional level (Landers, districts).
  
- The evaluation of education systems: another important task that was subject to decentralization is the assessment of students, teachers and schools. In more than 50% of the OECD countries, central authorities have direct control over this task. In France, New Zealand and the UK, inspection entities ensure the assessment of schools and teachers. In Finland and France, an assessment of students is conducted at the end of compulsory education. In the UK and Sweden, standardized tests are frequently administered during the schooling process. And finally, in federal countries, these assessments are conducted by regional authorities.
  
- The management of curricula: even if one-third of the OECD countries still use unique national curricula, the situation started to change in the 1990s. More than one half of these countries started to operate some decentralization, whether by giving local entities limited responsibility over the organization of instruction hours or in the definition of curricula.

Traditionally centralized countries have conserved their unique national curricula, but adopted some decentralization by giving local authorities or even schools the responsibility of defining the local content of these curricula. This local content may refer

to some historical or linguistic characteristics of a region. This has been adopted in Russia, Spain, Sweden and the Czech Republic. In decentralized countries, such as the UK and New Zealand, local authorities and schools have been granted the right to freely define the content of their curricula with the obligation to respect national education objectives. In federal states, a tendency towards a national definition of curricula and performance objectives was adopted due to poor student performances. This is the case of the USA and Brazil.

The choice of text books has been transferred to schools in about 40% of the OECD countries (France, Finland, Sweden and Denmark). However, about 25% of the OECD countries still have direct, central control over the choice of text books, like in Greece, the Czech Republic and Thailand. In some federal countries (Germany and Spain), local or provincial authorities have an important role in this task.

- Teacher hiring: centralized hiring is still in use in about 25% of the OECD countries; these are mostly known for their traditional centralization, such as France and Greece. 43% of the OECD countries have transferred teacher hiring to local authorities; this is the case of federal states and some traditionally centralized ones, like Finland, Denmark and Japan. The two linguistic Belgian communities have transferred this task to schools, even if French-speaking Belgium has a common legislation for the hiring of teachers.<sup>58</sup>

Fixing salaries and working conditions is the responsibility of central governments in more than 60% of the OECD countries (e.g. France, Japan, South Korea and Denmark). Usually, the definition of salary levels is made through negotiations between these central authorities and teacher unions. In 40% of the OECD countries, local authorities as well as schools may intervene in the definition of salary levels or in the attribution of incentives. However, these practices are becoming extinct.<sup>59</sup>

- Funding: the participation of local and provincial authorities in the funding of schools is becoming a common characteristic. Only 40% of the OECD countries have more than 70% central funding. About one-third of these countries have an important collaboration

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<sup>58</sup> This is perhaps attributable to the presence of a very large private system.

<sup>59</sup> In Chile, the local management of teaching staff did not survive the fall of the Pinochet regime. And in New Zealand, Bulk Funding was abolished; both countries went back to centralized management.



between central and local authorities. This collaboration concerns the collection of funds and the use of government subsidies. In Japan, local authorities have the responsibility of managing the different funds destined for education; one-fourth of these funds is collected locally and the rest is from government subsidies. Federal states have completely transferred the management of funds to local authorities. Central governments only intervene in the funding of nationwide policies, such as the “National Literacy and Numeracy Plan” in Australia and the “No Child Left Behind” in the USA.

### **2.2.2. Possible effects of decentralization policies.**

The positive effects that decentralization policies may have on education have been a subject for debate for the last three decades. On one hand, the pros have always affirmed that these policies have a positive effect, since they bridge the distance between the supplier and the consumer. In other words, parents would have more authority and control over suppliers of education than they would have on central governments. This, arguably, must lead to an increase in efficiency and even in educational performance (Winkler 1989 and Oates 1972).

In the economic literature on the provision of public goods, decentralization is depicted as a method to achieve optimal allocation of resources. The Tiebout model (1956) states that, “through the choice process of individuals, jurisdictions and residents will determine an equilibrium provision of local public goods in accord with the tastes of residents, thereby sorting the population into optimum communities.” In fact, the closest the center of decision to the final user (consumer), the better the results should be. Consumers would have direct control through voting, or even indirect voting - “voting with feet”. Another type of literature affirms that decentralization permits the introduction of market competition into the education system. Schools would be competing for funds and for students and will therefore be obliged to achieve higher performances or to leave the market. Hoxby (2000).

On the other hand, the cons argue that decentralization will lead to the multiplication of structures and decision making units, to redundancies and even to contradicting educational trends within the same country. In other terms, decentralization does not acknowledge the global role of education as a driving force for equal opportunities.

Secondly, decentralization is a generator of geographical disparities, since it negatively affects the homogeneity of education systems. In chapter II, I have provided an analytical proof for the presence of social and economic stratification in a context of finance decentralization in a multicomunity set. To this can be added territorial disparities resulting from greater curricular autonomy based on differences in ideology or religious convictions.

In the rest of this subsection, I am examining a number of variables depicting elements related to the quality of infrastructures and to the availability of teachers and computer facilities. Note that all these variables will not be used in the econometric analysis in chapter IV, since some of them may be highly correlated. For instance, a school with a low level of funding would have a low quality of physical infrastructures and a low quality of educational infrastructures. These two variables should not be included in the same regression, since they measure the same thing: funding. As we will see later, I retained three variables: quality of educational infrastructure, teacher shortages and the number of computers connected to the web. These relate to three main areas: funding, availability of teachers and availability of computer facilities (information technologies).

The following descriptive statistics depict the averages and between-school variances on a number of school level variables. The dispersion shows how homogenous or heterogeneous schools are. Negative values on any index do not indicate a negative response; they only indicate a less positive response in comparison with average responses for all PISA countries.

*Table 3.7.* The quality of physical infrastructures.

<b>Selected PISA countries.</b>	<b>Average quality of physical infrastructures.</b>	<b>Dispersion of the quality of physical infrastructures.</b>
Germany	0.1436	1.1521
Finland	-0.2411	0.8981
Italy	-0.0247	1.008
Japan	-0.0902	1.0978
UK	-0.2523	0.8604

The average quality of physical infrastructures can be considered as a proxy of the level of expenditure on factors like buildings, heating/cooling systems, etc. On this index, we see that the UK and Finland have the lowest levels, as well as the lowest dispersions. This means that little funds are allocated for physical infrastructures, but the levels of funding tend to be homogenous across schools. Italy and Japan have slightly higher scores on both moments. Germany comes out with the highest levels of average quality of physical infrastructure and the highest level of dispersions; this means that on average this aspect is well funded across German schools. However, the latter may still have important differences.

*Table 3.8.* The quality of educational infrastructures.

<b>Selected PISA countries.</b>	<b>Average quality of educational infrastructures.</b>	<b>Dispersion of the quality of educational infrastructures.</b>
Germany	0.2031	0.8483
Finland	-0.0244	0.4914
Italy	0.1347	0.9746
Japan	0.0062	1.1561
UK	-0.0685	0.9055

Similarly, the quality of educational infrastructures can be considered as a proxy for expenditure on factors like instructional, library and audio-visual materials, etc. Finland and the UK have the lowest levels on the average of this variable. Italy and Germany have the highest, while Japan is middle-ranking. In terms of the dispersion, Japan, Germany, Italy and the UK have high disparities, contrarily to Finland, which seems to be the most homogenous country.

*Table 3.9.* The ratio of computer per pupil.

<b>Selected PISA countries.</b>	<b>The ratio of computer per pupil.</b>	<b>Dispersion of the ratio of computer per pupil.</b>
Germany	0.0807	0.0042
Finland	0.1683	0.0072
Italy	0.1302	0.01095
Japan	0.1900	0.0757
UK	0.2311	0.0090

The UK has the highest ratio of computer per pupil as well as a low dispersion. Most British schools have a large number of computers, 1 for each 4 students (1/0.23), and this seems to be true across all schools. Finland and Germany have the lowest between-school dispersions; Germany has 1 computer for each 12 students and Finland has 1 computer for each 6 students. Japan and Italy have slightly higher dispersions and higher scores: 1 computer for each 5 students in Japan and 1 computer for each 8 students in Italy. In general, the levels of dispersions for all countries are low. Hence, the ratio of computer per pupil is not a major aspect on which schools differ.

*Table 3.10.* The proportion of computers connected to the web.

<b>Selected PISA countries.</b>	<b>The proportion of computers connected to the web.</b>	<b>The dispersion of the proportion of computers connected to the web.</b>
Germany	0.7087	0.0904
Finland	0.9189	0.0177
Italy	0.7092	0.1083
Japan	0.7386	0.0806
UK	0.8973	0.03838

In Finland, 91% of computers are connected to the web and the dispersion is the lowest. In the UK, 89% of them have internet access and the dispersion is slightly higher. Japan and Germany have similar levels of dispersion, but with lower proportions: 70% for Germany and 73% for Japan. Finally, in Italy, 70% of computers are connected to the web and the dispersion is the highest.

*Table 3.11.* The ratio of students per teacher.

<b>Selected PISA countries.</b>	<b>The ratio of students per teacher.</b>	<b>The dispersion of the ratio of students per teacher.</b>
Germany	17.6436	27.7205
Finland	10.7525	4.0763
Italy	10.0402	29.9051
Japan	14.0017	11.2763
UK	15.5441	5.7883

Germany and the UK come out with the highest level of students per teacher, 17 and 15 respectively, followed by Japan, Finland and Italy. In terms of the dispersion of this ratio, the UK and Finland have low dispersions, indicating that in the UK the number of students per teacher is high across most schools and the reverse is true for Finland. In Germany and Italy, the between-school dispersions are very high. Finally, Japan is closer to the first group than to the second.

*Table 3.12.* The presence of teacher shortages.

<b>Selected PISA countries.</b>	<b>Teacher shortages.</b>	<b>The dispersion of teacher shortages.</b>
Germany	0.1408	0.7806
Finland	-0.5566	0.4205
Italy	0.0778	0.8583
Japan	-0.0369	1.3440
UK	0.2591	0.9977

The index of teacher shortages measures the perception of instruction hindrances related to the impossibility of hiring teachers. The UK has the highest average level of teacher shortages, followed by Germany. Italy and Japan are middle ranking, while Finland has the lowest level on this index. In terms of its dispersion, Japan has the highest level which means that some schools experience high teacher shortages while others do not. The UK, Italy and Germany also have high dispersions, indicating high disparities between schools. And, finally, Finland has the lowest dispersion.

From these descriptive statistics, a number of conclusions can be made:

1. Finland has the lowest level of dispersion on all of the indices mentioned above; this indicates that Finnish schools are homogenous and have the lowest level of between-school disparities. This is mostly due to comprehensive schooling. In terms of the averages on these indices, Finland has low levels of quality of physical and educational infrastructures while having high levels on computer and teacher availability.

2. The UK has the lowest levels on the quality of physical and educational infrastructures, a high ratio of students per teacher and the highest average teacher shortages. The only domain where the UK is doing well is the availability of computers and internet. To these can be added the UK's high levels of decentralization and school autonomy. Nonetheless, this does not seem to be directly reflected in large disparities between -schools. In fact, the UK is middle ranking in terms of the dispersion on these indices; this is perhaps due to equalization policies.
  
3. Germany has high between-school disparities on most of the indicators. It has the highest levels of quality of physical and educational infrastructures, the lowest levels of computer and internet availability and the highest number of students per teacher. Note that, in Germany, these high disparities can be explained by the fact that the PISA sampled schools include general education as well as professional schools.
  
4. In general, Japan and Italy have a middle ranking position among the chosen countries. Japan has the highest between-school disparities on teacher shortages, the ratio of computer per pupil and the quality of educational infrastructure, while Italy has relatively high levels of dispersions on most of the indicators. This can be attributed to differences between the north and the south.

In conclusion, the effects of decentralization are not clearly visible through the dispersions on these indices. However, it is still possible to see that Finland is the most equal country. Germany, Italy and to some extent Japan have high between-school disparities, while the UK has limited disparities but high teacher shortages and low quality infrastructures, this can be the syndrome of low funding. Note that a complete picture of inequalities and school disparities cannot be obtained without considering the social status of schools, the degree of stratification and how these are related to performance scores.

### **2.3. School choice policies.**

In addition to decentralization policies, many countries around the world adopted school choice strategies from the beginnings of the 80s. These policies consisted of the multiplication of school types and the liberalization of choice. Their primary aim was the introduction of competition between and within each type of schools. This competition will, arguably, lead to an increase in performance and in efficiency.

School choice policies may differ between countries. Some of them opted for the abolishment of school districts, allowing for free choice between public alternatives. Others opted for an enlargement of private education through government subsidies, or even the introduction of new school types.

#### **2.3.1. Public school choice.**

The English-speaking countries were the first to introduce school choice into the public system. And, since then, most of the countries around the world started to adopt this kind of policies.

The UK started its school choice policies in 1988 under the Thatcher government. As a consequence, students had the choice between public alternatives and schools had the authority of deciding on the number of students to admit (possibility of selecting students). Under the new legislation, schools were funded on the basis of the number of enrolled students; and hence they were competing to attract a sufficient number while maintaining a high school quality. This policy was combined with the regular publication of the league tables, showing the educational performance of each institution. In theory, school choice should act as an incentive for low performing schools to improve their performance.

Note that, in the UK, schools have the final word over the enrollment of a particular student. As one may imagine, parents would be willing to move into schools with higher performance records. Thereby, highly performing schools would have a high level of demand and would be obliged to reject some of the students. As a consequence, selection will be operated according to criteria such as, geographical proximity or students' academic records. This raises a major

concern, since choice is not unconstrained. The amount of information parents have on different alternatives is subject to their social capital, the likelihood of choosing a school far from home is subject to the availability of financial resources to be allocated to transport, and if schools are to select students according to their academic records, then another layer of constraints will be added. “See chapter II”. It is also worth noting that school quality may be related to the price of housing in a particular neighborhood (Fack and Grenet 2008, Brasington and Haurin 2006). If schools are to give priority to residents of their neighborhood, then highly performing schools will always enroll students with higher economic and social status. All these concerns reduce the effect of school choice as an instrument of social equality.

In addition to school choice policies, the conservative government introduced new types of schools, such as city technology colleges in 1986, which later became City academies under the Labour party (Mons 2004). These schools are funded by public and private funds, and are intended to offer higher quality schooling within poor neighborhoods. Most of the English-speaking countries followed the UK model. New Zealand introduced public school choice in 1991. Canada introduced choice at different points during the 90s. The USA as well used choice policies which were combined with the introduction of new educational alternatives such as charter and magnet schools.

Finland introduced limited school choice in 1998. Households were granted the right to choose among public schools only if free places are available. However, when households opt out from their local public school, they are no longer entitled for transport subsidies. Finland uses a restrictive form of school choice, which can be attributed to low population density and the unavailability of a large set of choices (Green et al 2006).<sup>60</sup>

Other Nordic countries have also adopted choice policies. Denmark was the first to do so in 1989 when students were granted the right to choose among public schools. However, choice was subject to the availability of places and funds. Sweden adopted new school choice policies in 1992 under the conservative party, which granted students the freedom to choose among public schools and led to the development of a wider private system.

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<sup>60</sup> Note that private alternatives are also limited.



Italy adopted school choice in 2000. Students have been granted total freedom in the choice of their schools in the upper secondary system and schools were given larger authority over selection. However, in the primary and lower secondary systems schools are still required to give priority to students living in proximity.

Other Mediterranean countries have adopted public school choice policies. Spain introduced choice into the public system, while maintaining the control over the numbers of applications to be accepted. France has always allowed some expectations in the implementation of zoning, but since September 2007 free choice has been adopted in a number of experimental school districts.

In Japan, nothing has changed in recent years; a strict implementation of zoning is still in application even if some cities started experimenting with school choice recently. Germanic countries did not evolve that much either. School choice is still limited in the primary and lower secondary phases; though, in the higher secondary, more choice is allowed.

In conclusion, most OECD countries have introduced some choice, except East Asian countries and Greece. However, it should be said that choice was not adopted homogeneously within each country. Even when total freedom of choice was granted, students living in rural areas did not have the same opportunities as their urban counterparts. Moreover, In some countries, households did not exercise choice for several reasons: firstly, because educational authorities did not publish the performance records for each school (except in the UK and New Zealand) and as a consequence, households were supposed to choose blindly or at least with incomplete information. Secondly, because transportation services were not available or because parents were not inclined to choose a school outside their district or home town (this is the case in some Nordic countries, where households still choose their local school, even if choice is allowed).

### **2.3.2. School choice between public and private alternatives.**

In addition to school choice among public alternatives, some countries have opted for an expansion of the private schooling system. The aim of such policies was to increase competition between schools, to multiply the number of alternatives and to transfer some responsibilities from the public to the private sector. Usually the expansion of the private system was combined with higher levels of subsidies and control.

In the Nordic countries (except in Denmark), most private schools are government-dependent. They receive funding from public authorities and are subject to their control. In fact, these schools have to follow the same programs as their public counterparts and are not allowed to select their students. Admittance is done according to the “first come first served” method, while free entry is ensured. Furthermore, student assessments and inspections are identical to those in the public sector. However, it is important to note that even with the development of private education in some Nordic countries, the proportion of students in this sector is still very low (Hirsch 2002).

In the developed East Asian countries, such as Japan and South Korea, private education in the upper secondary system was used as a method to complement public supply. For instance, in Japan, the elementary system has 1% of students in private schools; the lower secondary has 5% and the upper secondary has about 55%. In South Korea this percentage attains 25% for the lower secondary and 50% for the upper secondary systems. The expansion of private education was made possible through the combination of public (government subsidies) and private (tuition) funding, even if the level of tuition is still controlled by the ministry of education.

Most of the English-speaking countries have adopted public school choice policies as a method of increasing the number of educational alternatives and of introducing competition into the education market. The expansion of the private education system was also used for the same purpose. In the UK, the USA and New Zealand, vouchers were used as an alternative to direct funding of private schools. Disposing of an amount of financial resources, students have to choose their private school and use their voucher to pay for tuition. This is supposed to increase competition, since schools would compete for students and for funding.

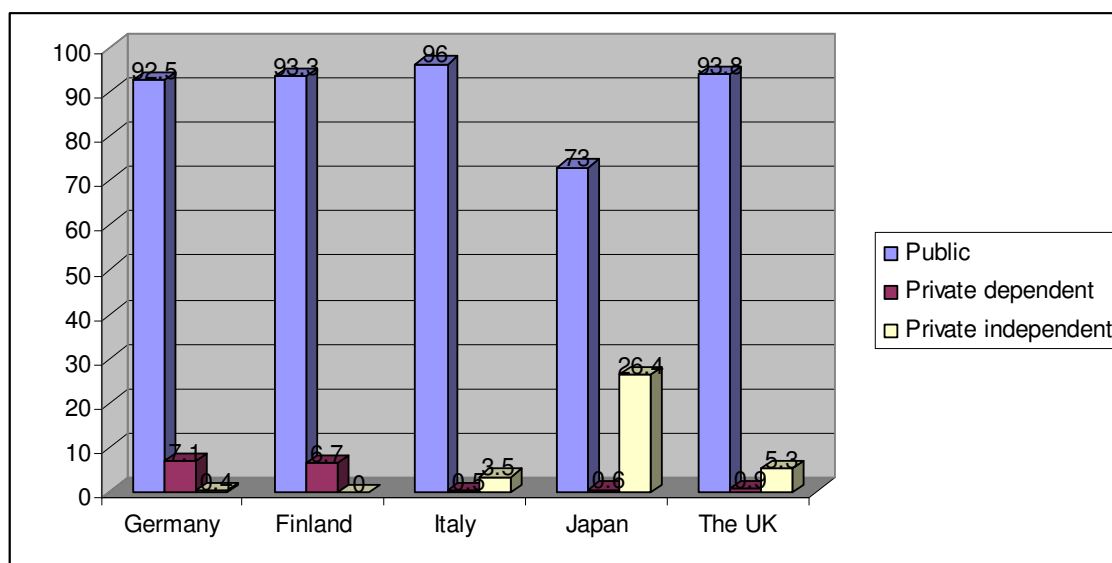
These policies started in the 1980s under the conservative party in the UK and New Zealand but were short-lived, since they were abandoned afterwards.

In the Mediterranean states, the level of development of the private system varies across countries. In Italy, the new legislation of 2000 authorized the public funding of private schools. Historically, the constitution of 1948 did not allow the subsidization of the private sector. However, since 2000, vouchers were given to poor households in order to enable them to attend the school of their choice. These subsidies were combined with higher measures of public control, requiring identical assessments, curricula and inspection as in the public sector.

In Germany, the constitution (article 7 paragraph 4) guaranteed the right to establish private schools. Two types of private schools exist: the Substitute schools (*Ersatzschulen*) and the Auxiliary schools (*Ergänzungsschulen*). The former are ordinary primary or lower secondary schools, controlled by non-government organizations or with governing boards not selected by a government agency. These schools offer the same types of diplomas as public schools and are subjected to government control. They have the same academic standards, teachers with at least the same qualifications and do not apply any selection according to economic and social criteria. Hence, most *Ersatzschulen* have low tuition and perceive important government subsidies. *Ergänzungsschulen* are upper secondary private schools controlled by non-government organizations or with governing boards not selected by a government agency. They are mostly vocational and offer curricula that do not exist in their public counterparts. They are privately funded through tuition and are not subject to public control.

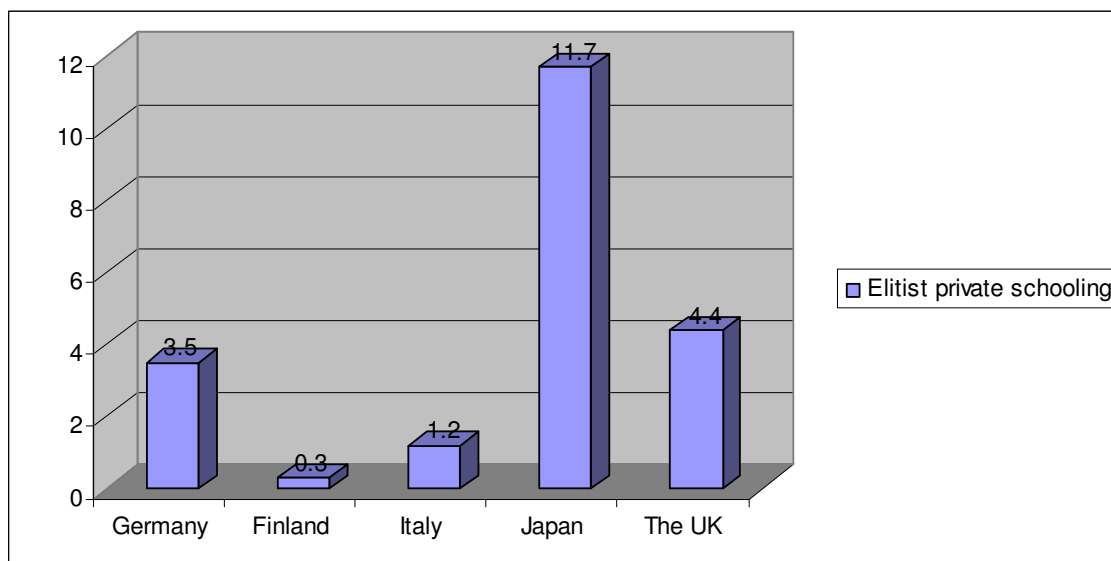
In the following figure, the distribution of students across three types of schools is examined. These types include: public schools controlled, managed and funded by public authorities, government-dependent private schools controlled by non government organizations, or with governing boards not selected by public authorities, which receive more than 50% of their core funding from government agencies, and finally, government-independent private schools controlled by non-government organizations, or with governing boards not selected by public authorities, which receive less than 50% of their core funding from government agencies.

Figure 3.1. The percentage of students across school types.



In all our selected countries, the public system is by far the largest. However, the distribution of students in the private system varies. Germany has a very low percentage of students enrolled in independent private schools; this is due to the fact that most private schools in the lower secondary are government-dependent. In Finland, all private schools are government-dependent. The development of private schools was intended to widen available choices, while public funding and control were maintained (independent private schools are not historically present in Finland, like in other countries). The UK has mostly adopted public school choice policies instead of private system expansions. Therefore, the private dependent sector did not evolve and most private schools in the UK are still independent. Japan has a large private system. Recall that 25% of students in the lower secondary and 55% in the upper secondary are enrolled in private schools. Most of these schools are independent and tuition financed, and tend to be expensive. Finally, Italy has a very small private system. Most private schools were independent and Catholic, since the constitution of 1948 did not allow their subsidization. However, since 2000, some private, dependent schools emerged; but they still account for less than 1%.

Figure 3.2. The percentage of students attending elitist private schools.



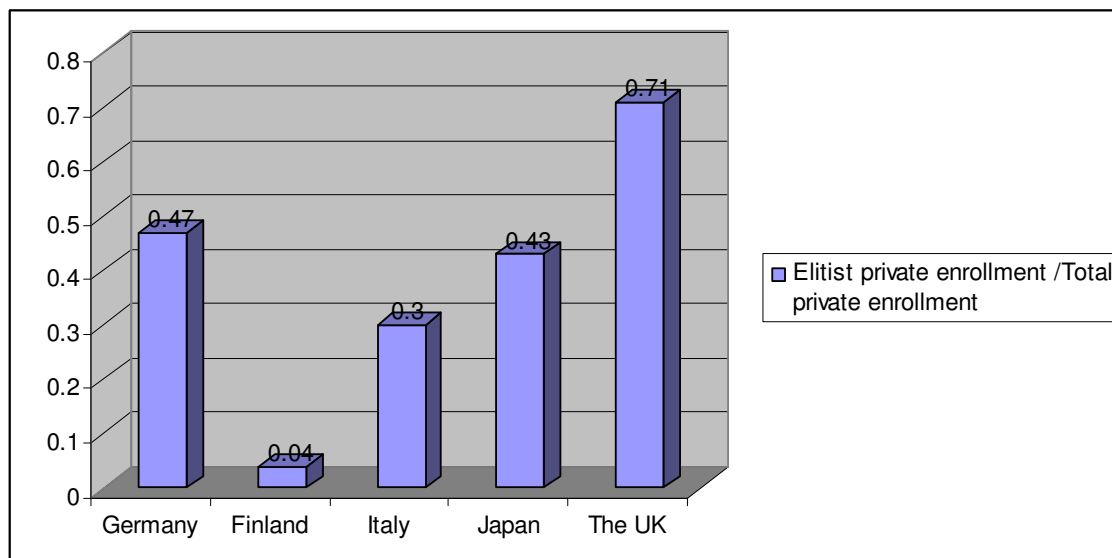
The indicator on social elitism in private schools was constructed using the within-school average and dispersion of ESCS (Economic Social and Cultural Status). Elitist private schools, both dependent and independent, are considered to have a high average ESCS and a low dispersion. The distance between the maximum and the minimum for each country, on both the average and the variance, is divided into 10 units.<sup>61</sup> For a school to be socially elitist, it has to belong to the highest 3 units on average ESCS and to the lowest 3 units on its variance. The percentage of students enrolled in these schools is then computed.

As we may see, Japan has the highest percentage of students enrolled in socially elitist schools; this can be attributed to the importance of the private system. The UK comes second with 4.4%, followed by Germany, Italy and then Finland. However, these simple percentages cannot be used for comparisons, since the size of the private system varies across countries. In other words, it is normal for Japan to have the highest percentage, since it has the largest private sector. The ratio of the percentage of students enrolled in socially elitist private schools over the percentage of students enrolled in the private sector allows for the standardization of the results according to the size of the private sector. This ratio is comprised between 0 and 1. 1 means that all students in the private sector are attending socially elitist schools, and the reverse is true for 0.

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<sup>61</sup>  $unit = \frac{Max - Min}{10}$

Figure 3.3. Ratio of elitist enrollment over total enrollment in the private sector.



When the size of the private sector is taken into account, the hierarchy of countries changes. In the UK, 71% of students enrolled in the private sector attend socially elitist schools. Recall that most private schools in the UK are independent, not funded by the government and apparently they serve the social elite. The UK is then followed by Germany with 47%, by Japan with 43% and then by Italy with 30%. Note that most private schools in these countries charge tuition, and are government-independent, something that explains why they are elitist. On the other hand, in Finland, only 4% of students enrolled in the private sector attend socially elitist schools; this is due to the fact that most Finnish private schools are government-dependent, and do not receive tuition or select students.

In conclusion, it is possible to say that, since the 1980s, most OECD countries have adopted policies for the development of the private sector. These policies were intended to complement public supply as well as to expand choice by developing new alternatives. Usually, private sector expansions were combined with increased public control and funding. However, the expansion of the private system did not definitely lead to an increase in the numbers of students enrolled in private schools, especially in the Nordic countries.

## **2.4. Educational stratification.**

In the following section, I will examine the stratification patterns across the selected countries. Stratification is studied according to ESCS (Economic, Social and Cultural Status), immigrant status and performance scores.

In chapter II, I have established that educational stratification is the result of constrained school choice. The combination between economic and social constraints on the one hand and school behavior and public legislations on the other would lead to different patterns of stratification.

Note that in chapter II I have constructed a stratified equilibrium on the market for education. However, in chapters III and IV, I am using an already stratified sample. In these two chapters, my objective is to study stratification patterns and their effect on performance scores. I am not empirically studying how stratification arises at equilibrium. Hence, chapter II should be seen as the theoretical groundwork for the rest of the analysis.

In what follows, I will examine stratification patterns across the selected countries, using different moments of a number of variables. These moments include means, national variances, between and within-school variances and the dispersion of within-school variances.

- National variance: is the dispersion of a variable around its national average. It indicates how much disparity there is within a country.
- Average within-school variance: is the average within-school dispersion of a variable. With the within-school dispersion being the dispersion of the variable around its school average.
- Between-school variance: is the dispersion of school means around the national average. It indicates how much disparities are there between schools.
- The dispersion of within-school variances: is the dispersion of within-school variances around their mean. (Note that each school has a within-school variance). A high level on this measure indicates that some schools are homogenous while others are not.

### 2.4.1. Stratification according to Economic, Social, and cultural status.

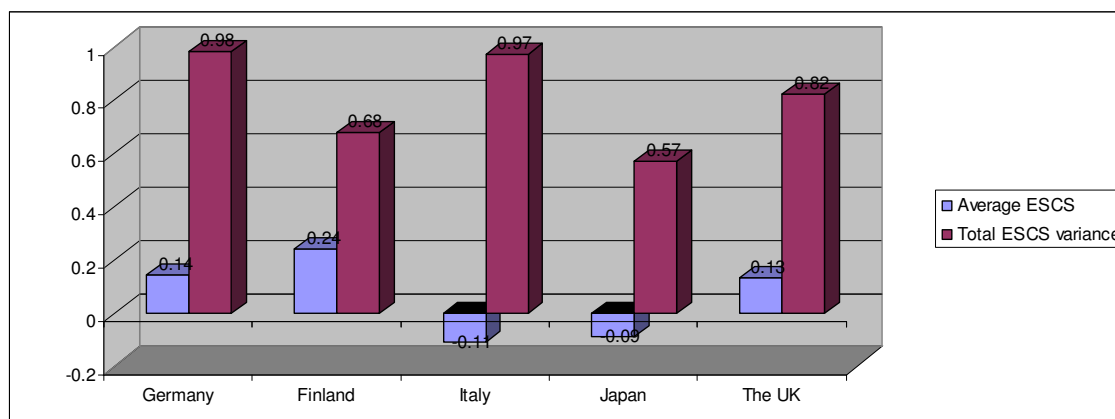
Economic, social and cultural status is perhaps the most important variable according to which stratification operates. ESCS can be considered as a proxy for two of the variables used in the theoretical model in chapter II: income and social capital. The following table summarizes the descriptive statistics computed for ESCS.

Table 3.13. Descriptive statistics for ESCS.

Countries	Average ESCS	Total ESCS variance	ESCS between-school variance
Germany	0.14	0.98	0.31
Finland	0.24	0.68	0.11
Italy	-0.11	0.97	0.33
Japan	-0.09	0.57	0.18
UK	0.13	0.82	0.19

Countries	ESCS Average within-school variance	ESCS dispersion of within-school Variances
Germany	0.67	0.088
Finland	0.57	0.028
Italy	0.64	0.051
Japan	0.39	0.021
UK	0.63	0.044

Figure 3.4. Average and Total variances of ESCS.



On the one hand, it is possible to see that Finland has the highest average ESCS and one of the lowest total dispersions. This is due to a long tradition of social democracy and a tendency for egalitarianism. On the other hand, the modernization process in Japan, which eliminated old elites and led to a less class-divided society, can help understanding the low total dispersion and low average ESCS in this country (see Green et al 2006 for a description of



East Asian countries). Germany, the UK and Italy have high dispersions, indicating wide social disparities within the population. The UK, Germany and Finland have higher values than the OECD average ESCS (which is equal to 0). Italy and Japan scored less than the OECD average and have negative values.

Figure 3.5. Within and between school variances over total variance.

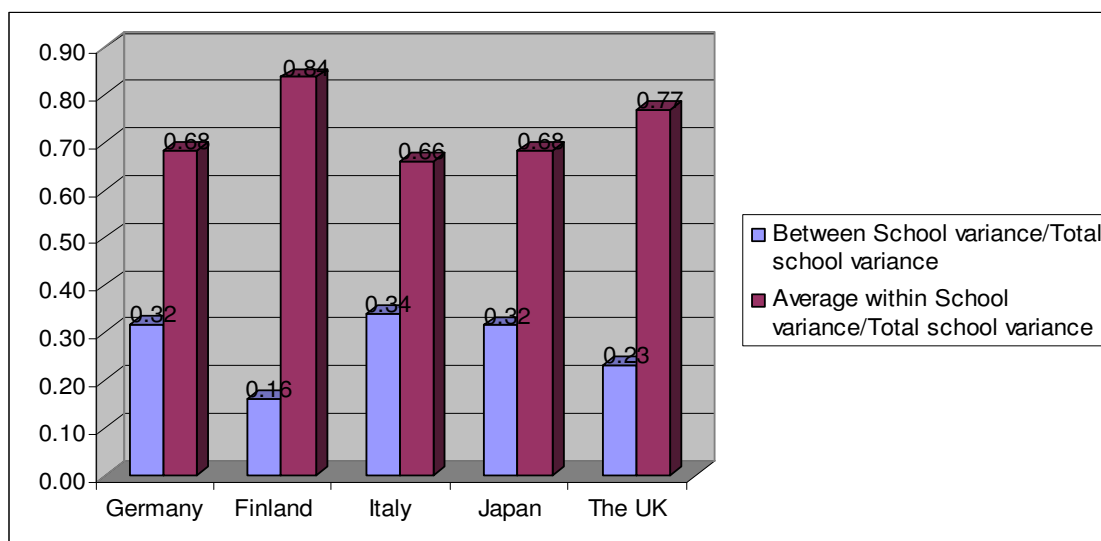


Figure 2.5 shows that Finland has the highest ratio of average within-school variance over total variance, indicating high levels of social diversity within schools and the lowest ratio of between-school variance over total variance. Finland is followed by the UK, then by Germany, Italy and Japan. The last three countries have similar levels on both ratios, with high “between-school variance/Total variance” and relatively low “average within-school variances/Total variance”.

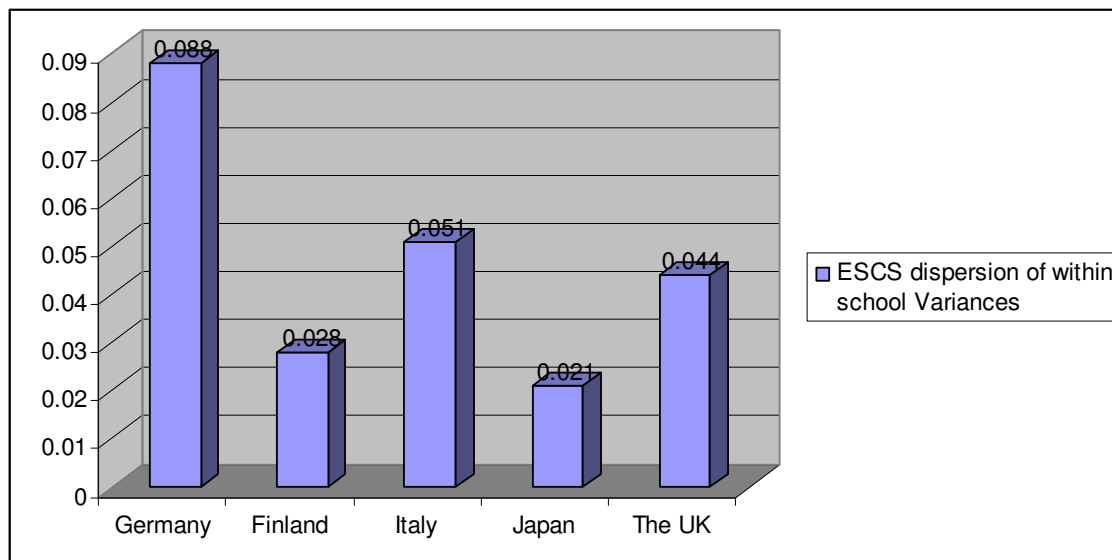
Even if the ratios are equal for two countries, this does not mean that they have the same levels of dispersion. For instance, Finland has one of the lowest average within-school dispersion as well as a very low total dispersion. The result is the highest ratio of average within-school variance over total variance. When the levels of dispersions are not standardized over total variance, we have the following hierarchy of countries. Japan has the lowest average within-school variance, followed by Finland, the UK, Italy and Germany.<sup>62</sup>

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<sup>62</sup> Note that since Japan and Finland have low total dispersions on ESCS, it is normal for them to have low average within-school dispersions.

Moreover, Finland has the lowest between-school variance, followed by Japan, the UK, Germany and Italy.<sup>63</sup>

Figure 3.6. The dispersion of within-school variances.



The interpretation of the dispersion of within-school variances is slightly more involved. The within-school variance indicates how diverse is a social mix within each school; its dispersion indicates if schools are similar or not in terms of their social diversity. Germany has the highest level on this indicator; this means that some German schools have a wide social intake, while others have a more homogenous one (thus the within-school variance is dispersed between schools). Italy and the UK are middle ranking, while Finland and Japan have low levels on this indicator. In conclusion, it is possible to describe stratification in each of our five countries:

Finland is characterized by a high level of average ESCS and a low total dispersion. It has the highest level on the ratio of average within-school dispersion and the lowest level on the ratio of between-school disparities. Schools tend to be homogenous with similar levels of average ESCS and similar dispersions. Finland can be described as a country with very low educational stratification.

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<sup>63</sup> The non-standardized variances do not allow for international comparisons because they do not control for the width of the distribution of ESCS. For the rest of this subsection I will only use the two ratios.

Japan is characterized by a low level of average ESCS and a low total dispersion. It has a low level on the ratio of average within-school dispersion and a high level on the ratio of between-school disparities. Schools tend to diverge on their average ESCS levels while being homogenous on the level of within-school ESCS dispersions. Japan can be described as a socially stratified country; schools have different ESCS averages and each school enrolls students from a particular social class (low and identical within-school dispersions).

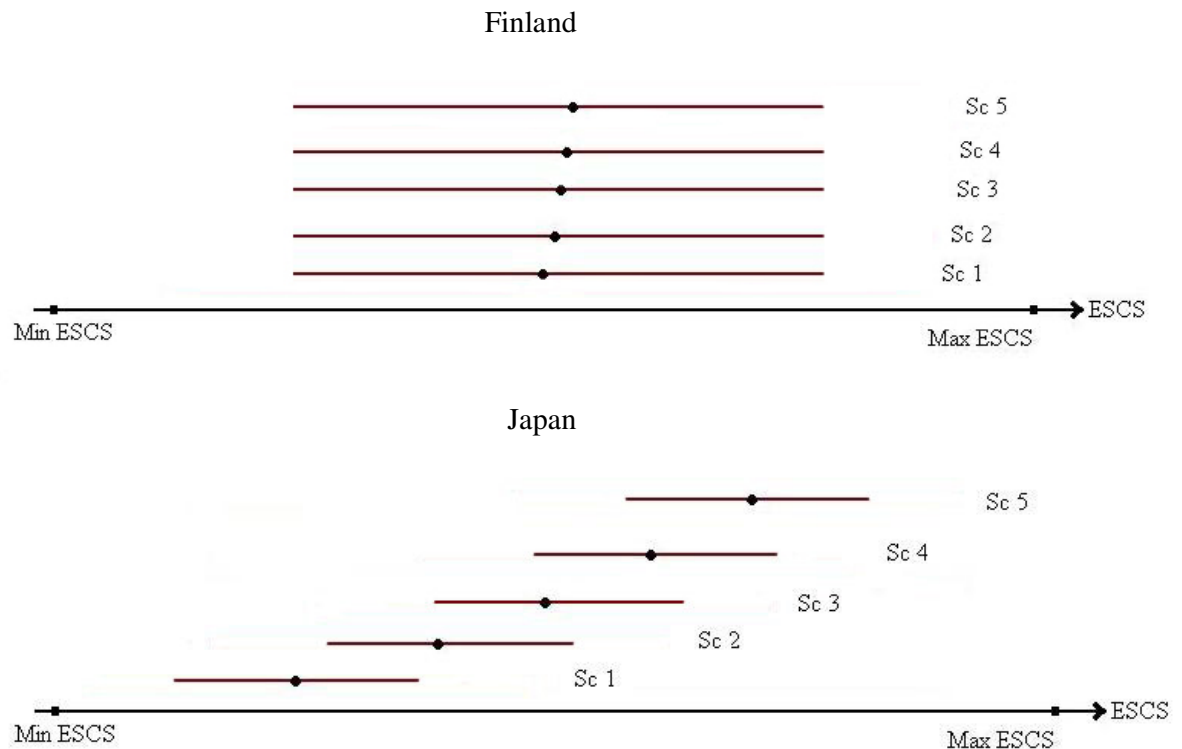
The UK is a middle ranking country; it has a relatively high level of average ESCS and a low total dispersion (the UK ranks after Japan and Finland). It retains a high level on the ratio of average within-school dispersion and a low level on the ratio of between-school disparities. However, even if schools in the UK have relatively similar levels of average ESCS, they differ on the level of within-school ESCS dispersions. Hence, some schools have homogenous social intakes (low dispersions), while others are more diverse (recall that the UK has the highest percentage of elitist private school enrollment). The UK has an average level of stratification; schools have similar means, yet some of them are socially elitist.

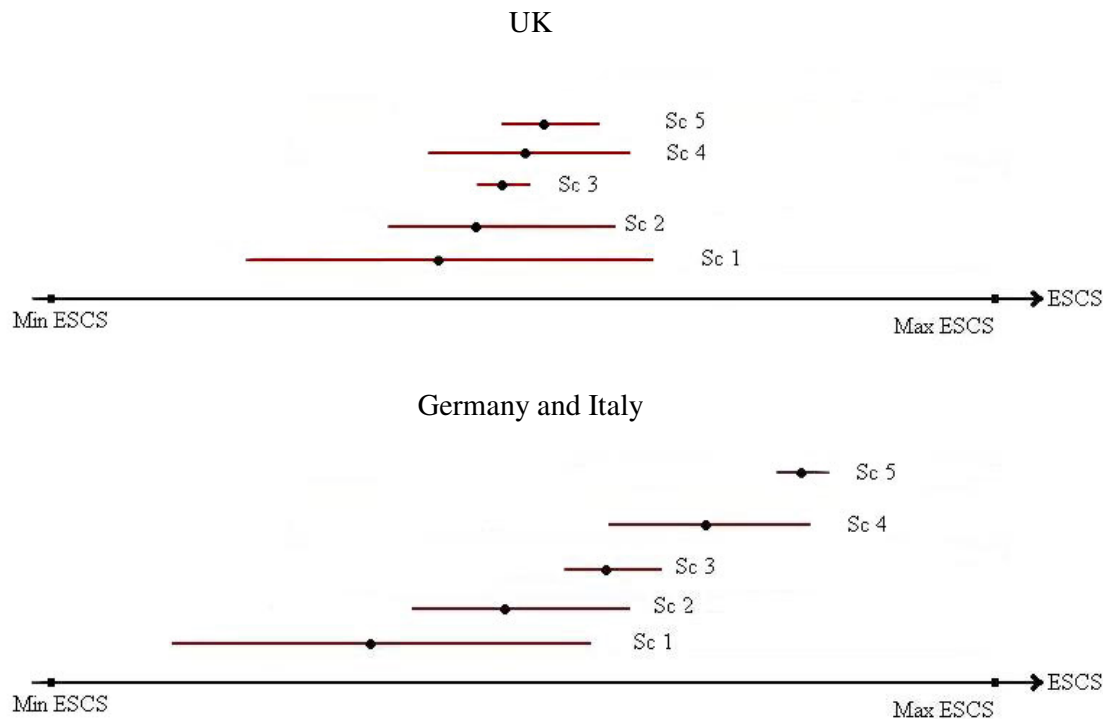
Italy is characterized by a low level of average ESCS and a high total dispersion. It has a low level on the ratio of average within-school dispersion and a high level on the ratio of between-school disparities. Schools are heterogeneous on both average ESCS and its dispersion. Germany is characterized by a relatively high level of average ESCS and the highest level of total dispersion. It has a low level on the ratio of average within-school dispersion and a high level on the ratio of between-school disparities. Schools are heterogeneous on both average ESCS and its dispersion.

Italy and Germany have the highest level of social stratification, since the social status of schools is highly dispersed. Moreover, some schools are elitist (with homogenous social intakes), while others are more diverse. This phenomenon can be explained through the following: Germany has a strong apprenticeship system, students are tracked at the age of 11 and the PISA sample includes both professional and general education schools. Students with low levels of ESCS are more likely to attend vocational schools than those with higher levels of ESCS. Italy has high levels of dispersion between the north and the south as well as some geographical stratification within cities, this helps explaining the high between school differences.

The patterns of stratification are illustrated through the following figures:

- The dots represent average ESCS in five schools. If the projections of the dots on the ESCS axis are close, this indicates low between-school dispersions. The reverse is true when the projections are far apart.
- The length of the lines represents the width of the distribution of ESCS within schools. A short line corresponds to a socially homogenous school, while a long one corresponds to a socially diverse one (Finland and Japan have lines of the same length, even if averages are different for Japan. The UK, Germany and Italy have lines of different lengths). More precisely, Finnish and Japanese schools have relatively the same within-school dispersions, while British, German, and Italian schools differ in the degree of social homogeneity. In some countries the projection of the lines covers the whole range of ESCS, indicating wide social intakes, while in others it does not.





#### 2.4.2. Stratification according to performance scores.

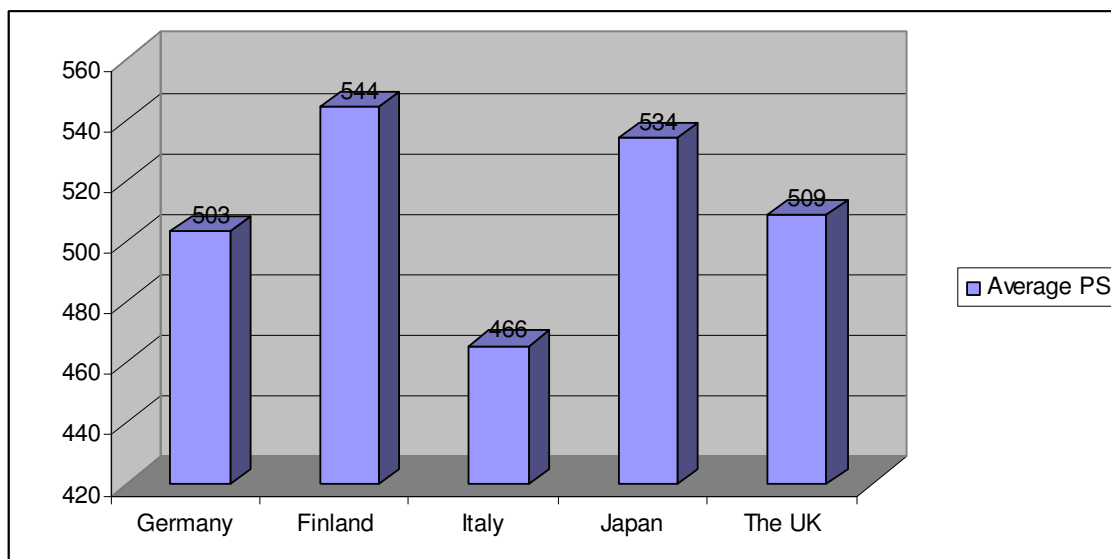
Educational performance is also an important variable according to which stratification operates. In general, performance scores are correlated with the socio-economic characteristics of students. Thus it is reasonable to expect that educational stratification will be similar to social stratification. Table 2.14 summarizes the descriptive statistics for performance scores.

Table 3.14. Descriptive statistics for performance scores.

Countries	Average PS	Total PS' variance	PS' between-school variance
Germany	503	10435	6220
Finland	544	6927	620
Italy	466	9695	5398
Japan	534	10212	5608
UK	509	8380	2836

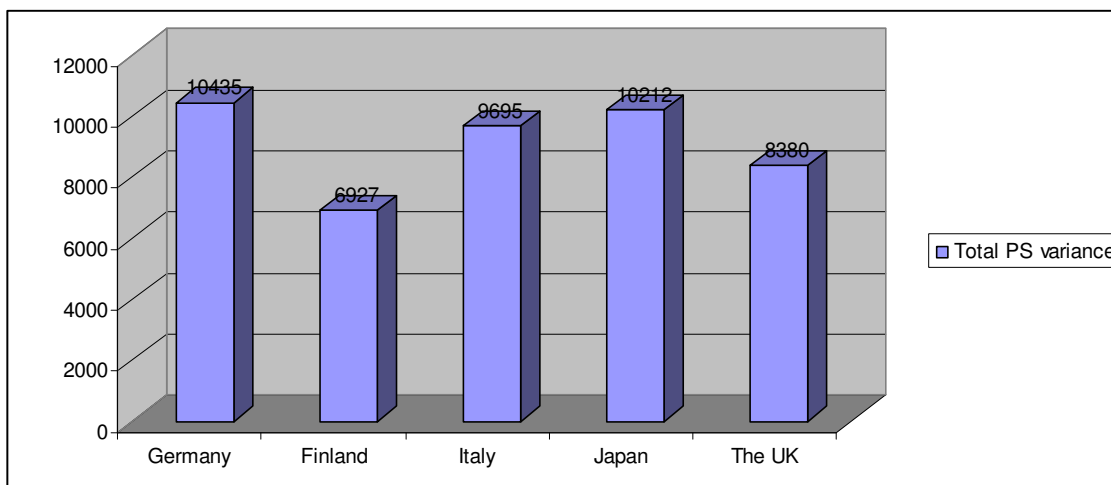
Countries	PS' average within-school Variance	Dispersion of PS' Within-school variances
Germany	4216	2150665
Finland	6306	2892138
Italy	4298	1895955
Japan	4604	2574521
UK	5544	3471417

Figure 3.7. Average performance scores.



Finland scored 44 points above OECD average performance and has the highest score across all PISA countries. It is followed by Japan, the UK, Germany and Italy. The UK and Germany scored very close to the OECD average, while Italy scored 34 points below it.<sup>64</sup>

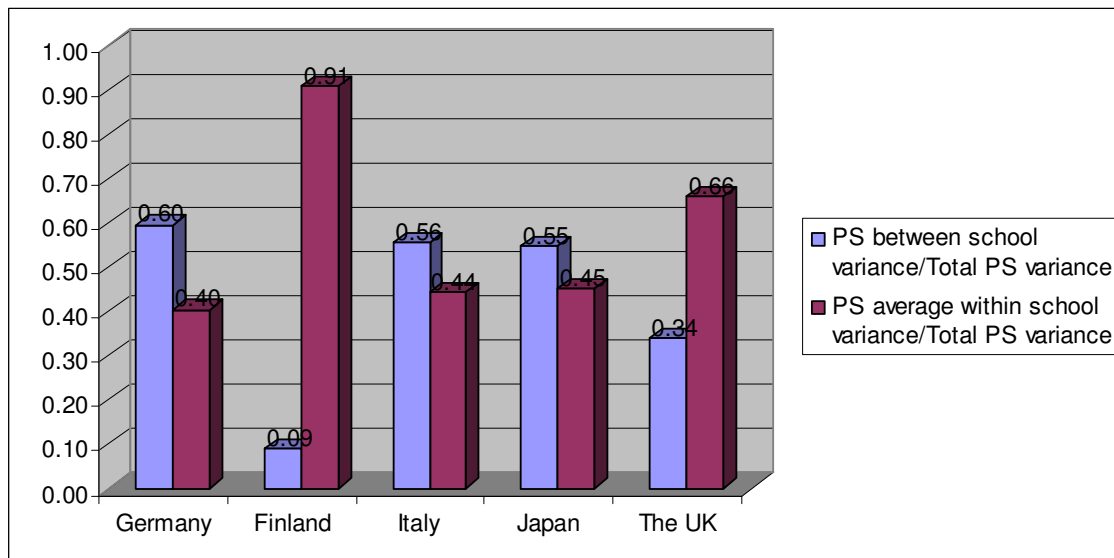
Figure 3.8. Performance scores' total variance.



As expected, Finland has the lowest level of total dispersion on performance scores, followed by the UK. Germany, Japan and Italy have high and relatively equal levels of national dispersions.

<sup>64</sup> The OECD average is equal to 500.

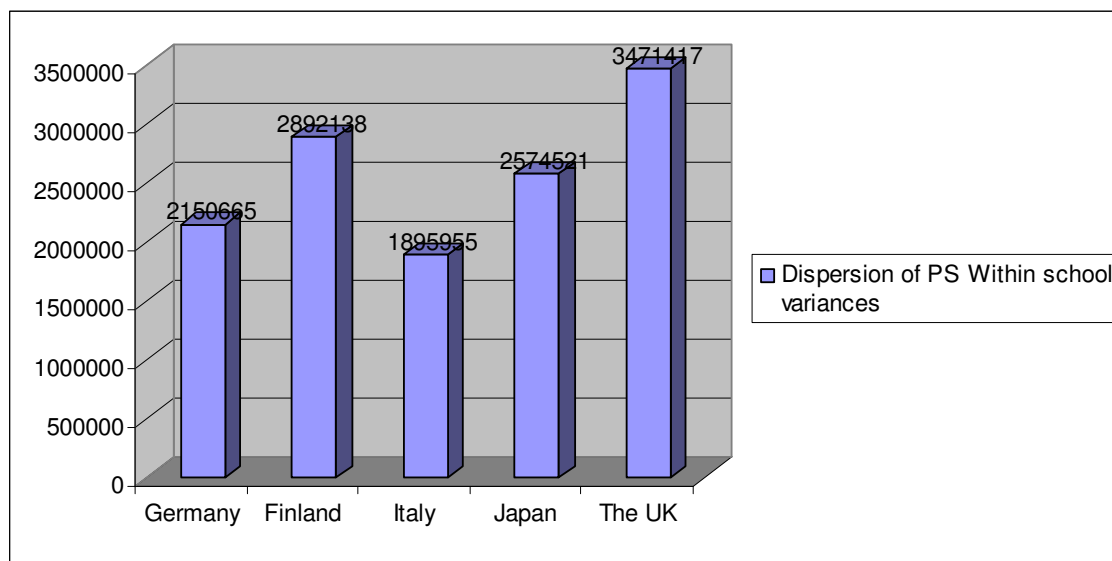
Figure 3.9. Within and between school variances over total variance.



On the one hand, three countries, Germany, Italy and Japan, have a ratio of between-school variance higher than the ratio of average within-school variance. This indicates the presence of large disparities in performance between schools and a relative homogeneity within each one. On the other hand, the UK has a ratio of average within-school variance that accounts for 66% of the total variance, while Finland has a ratio of 91%. Finnish schools are almost identical in terms of their average performance and have high levels of diversity.

When within and between-school variances are not standardized over total variance, the following hierarchy of countries is obtained. For average within-school variance: Germany has the lowest, followed by Italy, Japan, the UK and Finland. For between-school variance: Finland has the lowest, followed by the UK, Italy, Japan and Germany. As mentioned earlier, the standardized ratios are better since they account for the width of the distribution of test scores.

Figure 3.10. The dispersion of within-school variances.



In terms of the dispersion of within-school variances, the UK has the highest level followed by Finland, Japan, Germany and Italy. It should be noted that the values on this indicator are relatively high for all countries, indicating the existence of schools with diverse ability intakes and schools with more homogeneity. A possible reason why within-school variances are higher in Finland and in the UK is that average within-school variance is large for these two countries, allowing for variations between schools. In the rest of the countries, average within-school dispersion is very narrow.

In conclusion, it is possible to describe ability stratification in each of the five countries:

Finland has the highest average performance and the lowest national dispersion. Finnish schools have quasi-identical average performances while having wide ability intakes. However, some schools tend to be less heterogeneous in terms of ability than others. In general, Finland can be described as a country with low performance stratification, something that can be attributed to the comprehensiveness of the system.

The UK follows Finland, but with lower achievements and higher national dispersion. Schools have similar performances and relatively wide ability intakes. They are very dispersed in terms of their within variances; some have wide ability intakes, while others are more homogenous (they may be using some form of academic selection).

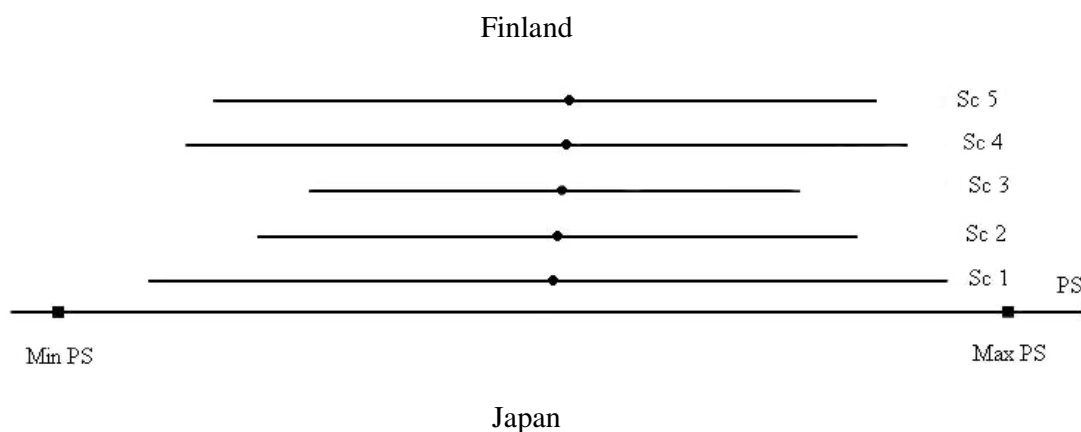


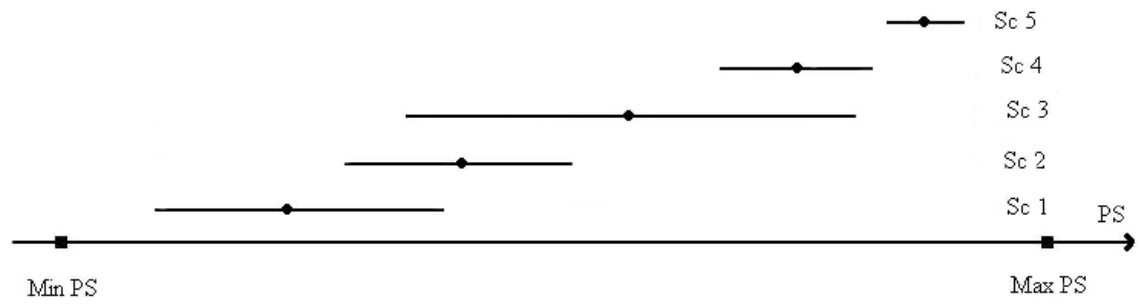
Japan has a high level of average performance as well as a high national dispersion. Japan seems to be strongly stratified, with strong between-school dispersions and limited within-school diversity. Traditionally, Japan is considered to be a very equal country; however, as we will see in the next chapter, school variables such as social peer effects have an important impact on performance scores, something that indicates the presence of inequalities attributable to stratification.

Germany and Italy have the same characteristics as Japan. However, Germany's average performance is closer to the OECD mean, while that of Italy is far below. These two countries have high levels of stratification, with Germany being the one with the highest between-school dispersion and lowest within-school diversity. Germany is closely followed by Italy on these two aspects. As mentioned in the previous subsection, high geographical disparities between the north and the south in Italy and high differences in ability between vocational and general education tracks in Germany may explain the presence of strong performance stratification.

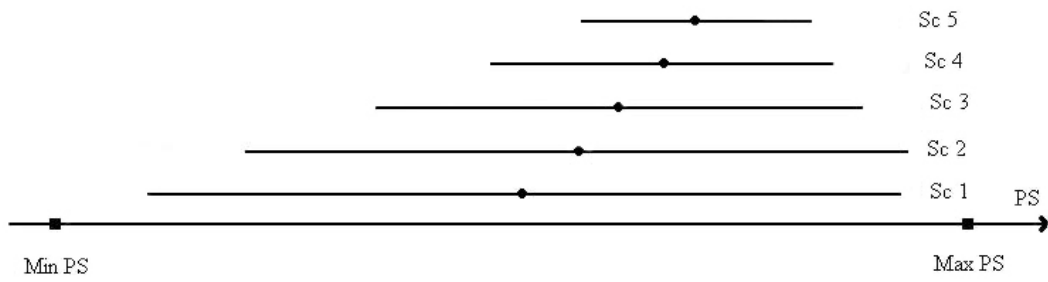
The patterns of stratification are illustrated through the following figures:

- The dots represent average performance scores (PS) in five schools. If the projections of the dots on the PS axis are close, this indicates low between-school dispersions. The reverse is true when the projections are far apart.
- The length of the lines represents the width of the distribution of PS. A short line corresponds to a homogenous school, while a long one corresponds to a school with performance diversity. In some countries, the projection of the lines covers the whole range of PS, while in others it does not (this indicates different performance intakes).

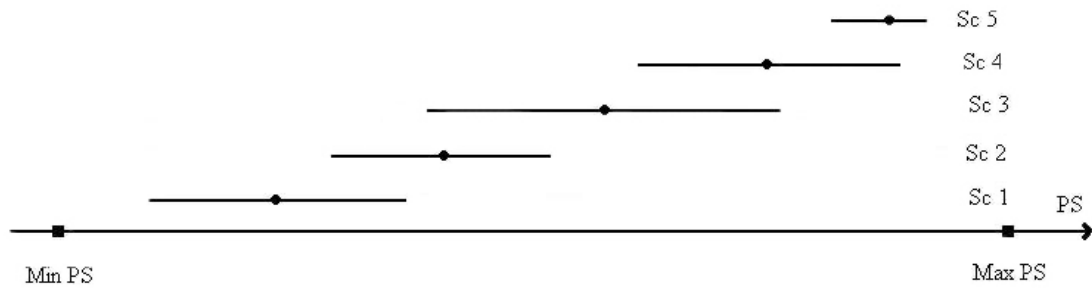




UK



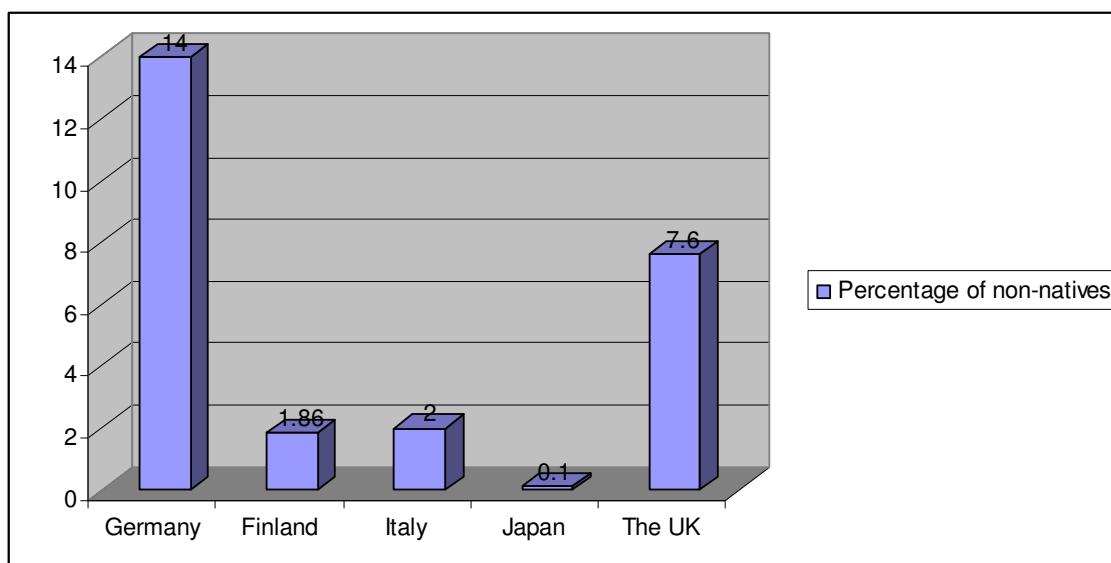
Germany and Italy



### 2.4.3. Stratification according to immigrant status.

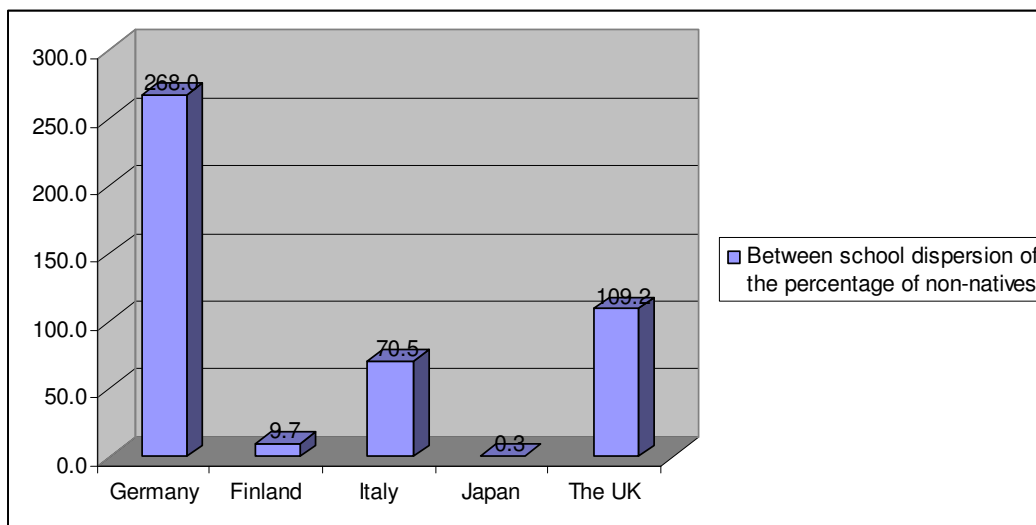
Immigration background is represented through a binary variable taking the value zero for natives and one for immigrants and first generation students.

Figure 3.11. The percentage of non-natives.



The percentage of non-natives is the highest in Germany (14%) and in the UK (7.6%), followed by Italy and Finland with about 2%. The percentage of non-natives in Japan is close to zero, and only 5 schools among 144 have non-native students.

Figure 3.12. The between-schools dispersion of the percentage of non-natives .



The dispersion of the percentage of non-native students indicates if they are stratified into particular schools. In other terms, this indicator has a high value when some schools retain the majority of non-natives, while others only have native students. Germany has the highest dispersion, followed by the UK and Italy. In these countries, non-native students are not uniformly distributed across schools. Conversely, Finnish schools have relatively the same percentage of non-natives.

## **Conclusion of Chapter III.**

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The major objective of this chapter was to examine the dataset, to select a number of relevant countries and to assess the patterns of stratifications using a number of student and school level variables.

In the first two sections, the PISA sample was presented as well as the selected variables. Two types of performance scores, the weighted likelihood estimates and the plausible values were compared, and the latter were chosen due to their better estimation of the population variance. In the third section, missing data were examined. After a presentation of the types of missingness and of imputation methods, the PISA dataset was imputed using multiple imputations with an MCMC procedure.

In the last section of the Chapter, five countries were selected: Finland, Germany, Italy, the UK and Japan. This selection was made according to previous national typologies, in particular Green et al (2006) and Mons (2004). These selected countries reflect five distinct institutional models. Finland is a Nordic country characterized by long social democratic traditions and by a very comprehensive education system. Germany is characterized by high levels of selection and by the presence of an important apprenticeship system. Italy retains some Napoleonic legacies of educational centralization and is characterized by the use of grade repetitions and by the quasi-absence of school choice. Japan is characterized by high levels of achievements and low levels of dispersions. Nevertheless, it has a heavily stratified system. The UK, the Anglo-Saxon country of the analysis, is characterized by incomplete comprehensivization, the presence of multiple school types and the use of decentralization and school choice.

In what follows, I shortly summarize the main findings and I discuss the major implications for chapter IV.

Due to its comprehensive system, Finland has the lowest level of dispersion on most school characteristics. Hence, in a regression, school variables are expected to have a limited effect on performance scores due to the lack of variations. Similarly, Finland has a high level of

average ESCS and a low total dispersion and Schools tend to be homogenous, with respect to average ESCS and its dispersion. Therefore, average ESCS and its within-school dispersion are expected to have limited effects on student performance scores.

The UK has the lowest levels on the quality of physical and educational infrastructures, a high ratio of students per teacher and the highest average teacher shortages, though computers and internet connections are available. Moreover, the UK seems to be a middle ranking country when it comes to the between-school dispersions on school variables. Therefore, it is expected that the UK will have a relatively limited impact of school characteristics on performance scores. In terms of the school social composition, the UK can be described as moderately stratified with schools being relatively homogenous. Thus, it is reasonable to expect mild relations between social peer effects and performance scores. However, it should be said that the presence of a highly elitist private sector is expected to be translated in an important effect of private schooling on test scores.

Germany has high between-school disparities on most of the indicators (pure school variables and peer effects). These disparities are expected to be translated in important and significant effects of school characteristics on performance scores.

In general, Japan and Italy have a middle ranking position among the chosen countries. Japan has the highest between-school disparities on teacher shortages, the ratio of computers per pupil and the quality of educational infrastructure. While Italy has relatively high levels of dispersions on most of the indicators. Thus, Japan is expected to have a mild impact of school characteristics on performance scores, while for Italy the impact should be stronger.

In terms of schools' social composition, Japan can be described as a stratified country. In fact, schools exhibit clear patterns of stratification along social class lines even if they have identical within-school dispersions of ESCS. The impact of social peer effects is expected to be high and significant while the impact of individual level ESCS is expected to be small due to the low total dispersion of ESCS.

The situation of Italy should be similar to that of Germany and to some extent to that of Japan. Italy has high level of between school disparities on school average ESCS and on its within-school dispersion. This is supposed to be reflected through high and significant relations between school peer effects and performance scores.

In Chapter IV, a full multilevel regression analysis of the education production function is carried out and the aforementioned student and school characteristics are included as independent variables. Chapter IV is organized in two sections: first, the presentation of multilevel analyses and second the estimation and interpretation of the results.





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**The Anatomy of Inequalities in Attainments:  
An International Investigation  
on Stratification and Choice.**

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**CHAPTER IV**

**A multilevel Regression Analysis of the Effects of  
Stratification on Inequalities in Attainments.**

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Following the descriptive examination of student and school level variables, one question remains and should be answered: how these different sets of variables relate to individual performance scores, and how their impact on attainments can be seen as a form of inequality. First, from the theoretical framework developed in chapter II, we know that students are not stratified randomly into schools. In fact, this stratification depends largely on their social and economic backgrounds. In other words, in a highly stratified system, students from unprivileged social classes are more likely to be living in poor neighborhoods and to be attending low quality schools. Second, from the descriptive statistics in chapter III, we know that the levels and nature of stratification are different between countries. These differences depend on the institutional context of each one. For instance, Finland, with its comprehensive school system, has limited stratification and important between school homogeneity; while the reverse is true for Germany.

In this chapter, the relation between student and school characteristics on the one hand, and student performance scores on the other, is estimated using a multilevel estimation approach. This estimation constitutes the final step in analyzing the education production function discussed in the theoretical models:  $a_i = a(b_i, q_j)$ .

Note that, in this chapter, I have adopted a statistical definition of inequalities in attainments, which can be used as a benchmark against which countries are compared. According to this definition, when all student and school variables have insignificant effects on performance scores, it is possible to conclude that inequalities do not exist. In other terms, the variable that can explain differences in performance is still hiding in the unobserved component (the error term). In fact, when most student and school characteristics are controlled for, and are found to have no effect, the only possible variable that may explain differences in performance would be student competencies (e.g., IQ). This situation is a perfect meritocracy where the surrounding environment of a student - whether at home or at school - does not affect his achievements. Of course, this situation does not exist, but it is useful to consider it as a benchmark against which countries are compared.

Chapter IV is organized in Three sections. The first presents multilevel regression analyses, model building issues and the different models to be estimated. The second presents the results and their interpretations. Finally, the last section presents some policy implications.

## **A. Multilevel regression analyses.**

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In this part of chapter IV, a concise examination of multilevel techniques is carried out. First, a presentation of multilevel analyses is made and arguments for their use are provided. Second, different estimation theories are assessed while examining the assumptions on error terms. Third, endogeneity problems in multilevel regressions are considered, and a solution according to the approach of Mundlak is provided. Finally, different models are constructed and the estimation procedures are made explicit.

### **1. Introduction to multilevel analyses.**

Multilevel data structures are very common to social sciences, especially educational research. Students are nested within classes, classes within schools, schools within districts and districts within countries. The importance of multilevel analyses becomes apparent when studying the effect of individual, class and school level variables on educational performance. The manner in which students are stratified across schools and classes is not random. Stratification arises as a result of interactions between students and schools and the distribution of students at equilibrium definitely influences their performances through peer effects and other school characteristics.

Multilevel analyses have been previously used in a variety of domains, such as economics, sociology, demographics and biometrics. For instance, in demographics, multilevel regressions were used to estimate the effect of some national and household variables on fertility, with households being nested within countries (Mason, Wong and Entwistle, 1983). In psychology, they were used to investigate the effect of individual variables on the acquisition of vocabulary over a period of time, with different time observations being nested within individuals (Huttenlocher, Haight, Bryk and Seltzer, 1991). In general, multilevel analyses provide a general framework for any research involving a number of entities, with some being nested within others.

Multilevel models were first used in the early 1970s. However, the absence of a general, estimation approach rendered them complex and unfeasible. Since 1977, the development of the expectation maximization algorithm (EM) by Dempster, Laird and Rubin provided the necessary techniques for a feasible estimation.<sup>65</sup> This breakthrough was accompanied with the development of statistical software enabling the treatment of these models.

## **1.1. Improvements made possible through the development of multilevel analyses.**

Raudenbusch and Bryk (2002) discussed three main areas where multilevel analyses have improved our research techniques and furthered their reach.

### **1.1.1. The estimation of individual effects:**

A multilevel model was developed by Braun, Jones, Rubin and Thayer in 1983 to analyze the use of standardized scores in the selection of minority students in business schools. The authors faced two major problems. First, most schools based their admission criteria on equations that use test scores to predict future outcomes. However, since most business schools are attended by white students, their data dominated the estimated equation. The result was an equation that is not adequate for the selection of non-whites. Second, the percentage of minority students in most schools was very low; it did not allow the estimation of an independent equation for them. In general, standard estimation techniques were not feasible in this context. Braun et al (1983) solved the dilemma by using a multilevel approach for the estimation of the model.

In the case of PISA, on the one hand, multilevel analyses allow the identification of stratification effects. Stratification is not random, and the composition of a school does not arise arbitrarily at equilibrium. If a strong relation between school composition (e.g. peer quality) and performance scores exists, then it is possible to confirm the presence of high levels of inequalities resulting from stratification. On the other hand, an independent

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<sup>65</sup> The EM algorithm has been mentioned earlier in the treatment of missing data subsection. For a detailed description, see Schaffer (1997).

regression equation will be obtained for each school, something that is impossible with standard linear models due to the limited number of observations within each school.

### **1.1.2. Interacting cross level variables.**

The use of multilevel analyses permits the modeling of cross level effects. For instance, Mason et al (1983) studied the effect of maternal education and urban residence on fertility while controlling for national variables such as the GDP. The authors found that urban versus rural fertility rates varied across countries as a function of GDP. The same procedures were used in developmental research. The assessment of vocabulary acquisition by infants was studied through an interaction between initial ability (measured at the age of 14 months) and maternal verbal ability (Huttenlocher et al 1991). The results showed that exposure to language during infancy has a more important effect than previously thought. In general, it would have been impracticable to study these effects unless some interactions between different levels were made possible.

### **1.1.3. The decomposition of variances.**

Another improvement made possible through multilevel modeling is the decomposition of variances between levels. A study of performance scores on mathematics illustrated this improvement (Raudenbush, 1988). The authors used performance scores for a sample of 618 students in 86 schools measured at 5 different periods. They found that between-school variance accounted for 83% of the total variance. The use of multilevel modeling allowed the identification of important differences among schools. Standard models would not have allowed such a finding, since they do not permit the decomposition of variations in performance scores. In the case of PISA, the decomposition of the total variance into within- and between-school variances is an essential step in the understanding of inequalities.

## **1.2. Modeling multilevel data.**

In this subsection, a thorough examination of multilevel models will be conducted. First, I compare multilevel and linear regression models to justify the use of the former. Then, I develop a simple framework to be the basis for the full model estimated with PISA data.

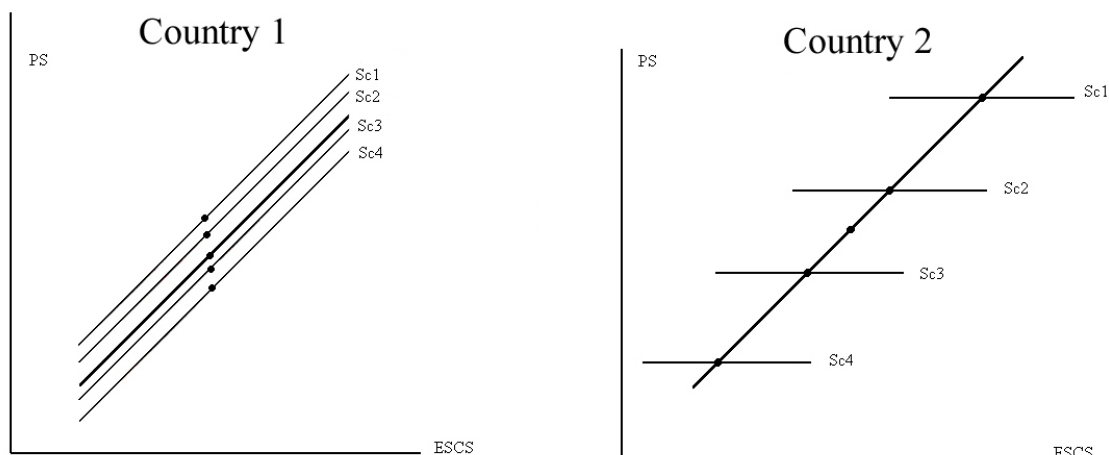
### **1.2.1. Multilevel models versus linear regression models.**

Linear regression models fail to take into account the potential effects arising from the way in which students are assigned to schools or to classes within schools. Hence, they give an incomplete or misleading representation of the efficiency of education systems.

On the one hand, the relation that exists between a dependent and an independent variable on a national level may not necessarily be identical to the one existing on a school level. For instance, the social background of a student may have important effects on his performance on a national level while having no effect on a school level. On the other hand, multilevel models acknowledge the fact that students are nested within schools and that schools are nested within countries. With multilevel modeling, it is possible to identify whether the effect of a variable (e.g., ESCS) on performance is direct or if it is channeled through stratification. As we will see later in this chapter, in Japan, ESCS has little effect on performance. However, average ESCS in a school has an important effect. This indicates that belonging to a school with high ESCS is much more important than the household's background. In other words, social inequalities are channeled through stratification.

In addition to what was mentioned above, multilevel regression analyses permit the differentiation of the effects of each independent variable between schools as well as the decomposition of the total variance into between- and within-school variances. In contrast, simple linear models represent the relation between an independent and a dependent variable through a unique regression line for each country. Hence, this type of models fails to capture the nesting of students within schools.

Figure 4.1. Linear versus multilevel regressions.



The simple linear regression analysis graphically represented through the thick black lines shows that the expected score of a student is increasing in his economic, social and cultural status in both countries. Notice that the slope on both linear regression lines is the same. Based on simple linear regression analyses, one would conclude that the relationship between ESCS and student performance is identical for both countries. However, a multilevel analysis may show the reverse, since it acknowledges the differences between countries at school level.

In country 1, the multilevel regression lines are similar and parallel to the simple linear regression line. This means that:

- Concerning the Economic Social and Cultural Status ( $X$  axis):
  - a. The different schools are all attended by students coming from a wide range of socio-economic backgrounds. All the school regression lines cover the whole range of values on the  $X$  axis. The within-school ESCS variance is large.
  - b. The projections of the dots representing school average ESCS on the  $X$  axis are very close to each others. The between-school ESCS variance is small.
  - c. From points a and b, it is possible to say that these schools have relatively the same social intake.

- Concerning performance scores PS ( $Y$  axis):
  - a. In each school, there are low, middle and top achievers. All the school regression lines cover the  $Y$  axis. The within-school PS variance is large.
  - b. On average, the schools have similar levels of performance. The projections of the dots on the  $Y$  axis are very close to each other. In other words, the between-school PS variance is small.
  - c. From points a and b, it is possible to say that these schools have relatively the same performance intake.
  
- Concerning the relationship between ESCS and performance scores:
  - a. In each school, there is a strong relationship between ESCS and achievements.
  - b. Low ESCS students perform below high ESCS ones. The slope of the regression lines indicates the strength of the relationship.

The opposite of country 1 is country 2:

- Concerning the Economic, Social and Cultural Status ( $X$  axis):
  - a. The school regression lines do not cover the whole range of ESCS that exists at the population level. School 1 is mainly attended by high ESCS students, while school 4 is mainly attended by low ESCS ones. The within-school variance is small.
  - b. The projections of the dots (average ESCS) on the  $X$  axis are far apart. In other words, the between-school variance is high.
  - c. A high level of social stratification exists. The social intake is different between schools, with some being socially elitist.
  
- Concerning performance scores ( $Y$  axis):
  - a. The school regression lines do not cover the whole range of performance. School 1 is mainly attended by high achievers and school 4 is mainly attended by low achievers.



- b. Schools largely differ in their average performance levels, as the projections of the dots (average PS) on the  $Y$  axis would show. The between-school PS variance is therefore very important.
  - c. Schools have a high level of performance stratification and they differ on their performance intakes.
- Concerning the relationship between the socio-economic background and performance:
    - a. In each school, performance scores are independent of the ESCS. The slope is equal to zero and the regression lines are parallel to the  $X$  axis.
    - b. In this country, the ESCS of a student does not determine his performance directly, since the regression coefficient is close to zero. However, ESCS does determine the school he attends, and this choice determines his performance. For instance, a high ESCS student would choose school 1, which is the one with the highest performance level.

In conclusion, it is possible to say that these two countries have different forms of stratification and educational inequalities follow different mechanisms. Linear regressions fail to capture these differences, thereby the use of multilevel analyses is justified.

### 1.2.2 The logic of multilevel modeling.

In the following discussion, I develop a simple framework using one student level variable and a dependent variable. The independent or explanatory variable is noted as  $X$  and the dependant variable is noted as  $Y$ . A subscript  $i$  is used to denote a student and a subscript  $j$  for denoting a school.

**The regression equation of performance scores on ESCS for one school has the following**

**form:**  $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$

$Y_i$ : is the dependent variable for individual  $i$  (in the case of PISA, the dependent variable is performance scores).

$X_i$ : is student's  $i$  independent variable.

$\beta_0$  : is the intercept. It is the level of Y of a student whose X is equal to zero.

$\beta_1$  : is the slope on X; it indicates the change in Y associated with a one unit increase in X everything else held equal. On a graph, it is the angle that the regression line forms with the x axis.

$\varepsilon_i$  : is an error term.  $\varepsilon_i$  follows a normal distribution with a mean of zero and a variance of  $\sigma^2$ ,  $\varepsilon_i \sim N(0, \sigma^2)$ .

**The regression equation of performance scores on ESCS for j schools has the following**

**form:** 
$$Y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + \varepsilon_{ij} . \tag{1.1}$$

With:

$Y_{ij}$  : is the dependent variable for individual i enrolled in school j.

$X_{ij}$  : is the independent variable for student i enrolled in school j.

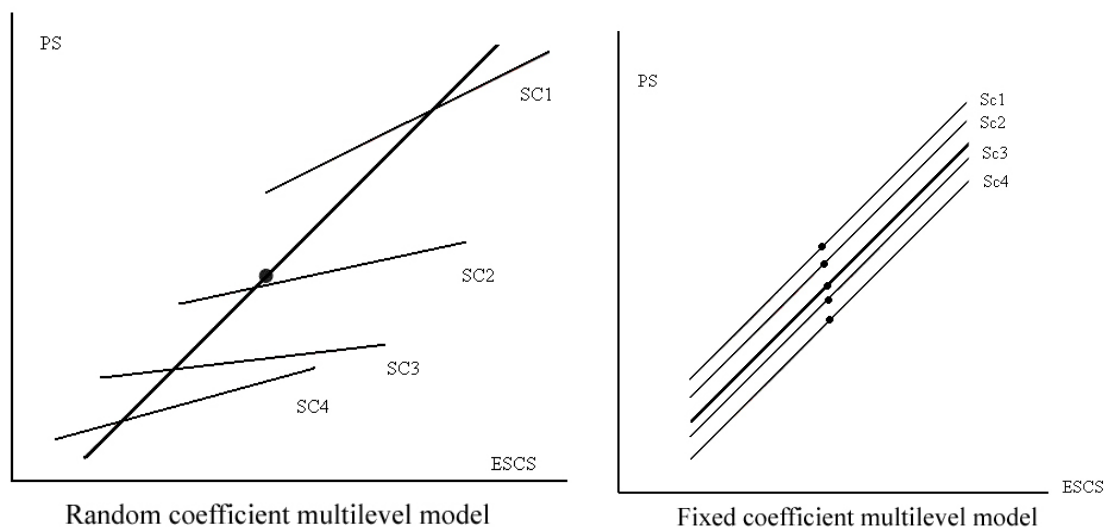
$\beta_{0j}$  : is the intercept for school j. It is the expected level of Y for a student enrolled in school j and whose X is equal to zero.

$\beta_{1j}$  : is the slope on X for school j, it indicates the change in Y associated with a one unit increase in X for this school. On a graph, it is the angle that the regression line forms with the x axis.

$\varepsilon_{ij}$  : is an error term.  $\varepsilon_{ij}$  follows a normal distribution with a mean of zero and a variance of  $\sigma^2$ ,  $\varepsilon_{ij} \sim N(0, \sigma^2)$ .

Notice that each school has an independent equation with its own slope and intercept. This is the general multilevel model, also called the **random coefficient model**, since the slopes on the independent variable vary between schools. When the slopes are constant for all schools, the model is called a **fixed coefficient model** and it has the following form:  $Y_{ij} = \beta_{0j} + \beta_1 X_{ij} + \varepsilon_{ij}$ .  $\beta_1$  is identical for all schools and regression lines are parallel. The only difference between schools is on the intercept. Considering the intercept, too, as a fixed parameter would reduce the multilevel model into a linear regression. The following figures illustrate a random coefficient model versus a fixed coefficient one.

Figure 4.2. Random Coefficients versus Fixed coefficient models.



In random coefficient models, the intercept and the slopes can be further decomposed into fixed and random parts. The original model is  $Y_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + \varepsilon_{ij}$ .

$$\beta_{0j} = c + U_j \tag{1.2}$$

$$\beta_{1j} = \beta + \mu_j \tag{1.3}$$

As shown in the first equation, the intercept is divided into two elements:  $c$  is the overall intercept, which is constant for all schools and equal to the average of the intercepts  $\beta_{0j}$ , and a random part  $U_j$ , denoting school  $j$  departure from the overall intercept, which can also be seen as a unique effect of school  $j$  on the average intercept (Raudenbush and Bryk, 2002).  $U_j$  can be considered as comprising the unobserved school characteristics.  $U_j$  is assumed to have a zero mean and a variance of  $\tau_0^2$ .  $U_j \sim N(0, \tau_0^2)$ .

In equation two, the slope on the independent variable is divided into two elements:  $\beta$  is the overall regression coefficient, equal to the average of regression coefficients  $\beta_{1j}$ , and a random part  $\mu_j$ , denoting school  $j$  departure from the overall regression coefficient, which can also be seen as a unique effect of school  $j$  on the slope of  $X$  (Raudenbush and Bryk, 2002).  $\mu_j$  is assumed to have zero mean and a variance of  $\tau_1^2$ .  $\mu_j \sim N(0, \tau_1^2)$ .

Notice that  $U_j$  and  $\mu_j$  are treated as random errors with normal distributions. When  $\beta_{0j}$  and  $\beta_{1j}$  are replaced by their values, the equation is then written in the following form.

$$Y_{ij} = c + \beta X_{ij} + U_j + \mu_j X_{ij} + \varepsilon_{ij}. \quad (1.4)$$

**Introducing school level variables:**

So far, I have only considered one independent student level variable  $X_{ij}$ . School characteristics have been considered as unobserved and, hence, they were assumed to be absorbed by  $U_j$ . However, in the context of PISA, some school variables are available and can be used. In other terms, these variables can be extracted from  $U_j$ . Hence,  $U_j$  is decomposed in the following manner:

$$U_j = \gamma_1 \bar{X}_{\bullet j} + \gamma_2 K_j + V_j.$$

Where  $\bar{X}_{\bullet j}$  represents peer effects in school  $j$ . It is computed as the school average of an individual level variable  $X_{ij}$  which is constant for all students attending the same school (e.g., average ESCS in a school).  $K_j$  is a pure school level variable that is constant for all students attending the same school and is not an aggregation of student level variables (e.g., quality of physical infrastructures).  $V_j$  is an error term similar to  $U_j$ ; it contains the remaining unobserved school characteristics after controlling for  $\bar{X}_{\bullet j}$  and  $X_{ij}$  and it can also be considered as a unique effect of school  $j$  on the intercept.  $V_j \sim N(0, \tau_0'^2)$ .  $\gamma_1$  and  $\gamma_2$  are the regression coefficients on  $\bar{X}_{\bullet j}$  and  $K_j$  respectively.

Equation (1.4) can be rewritten in the following way:

$$Y_{ij} = c + \beta X_{ij} + \gamma_1 \bar{X}_{\bullet j} + \gamma_2 K_j + V_j + \mu_j X_{ij} + \varepsilon_{ij} \quad (1.5)$$

Equation (1.5) is not a typical linear model that can be fitted with ordinary least squares (OLS). In OLS, random errors are independent, normally distributed and have constant variance. In contrast, the random error on this equation is more complex and has the following form:  $V_j + \mu_j X_{ij} + \varepsilon_{ij}$ . Such errors are not independent within each school, because  $V_j$  and  $\mu_j$  are common to all students belonging to the same school.

For the rest of the analyses, equation (1.1) is labeled the level one model. Equations (1.2) and (1.3) are labeled the level 2 models. And equation (1.5) is labeled the combined model. Besides that, the independent variables are considered to be vectors of variables.

$X_{ij}$ : is a vector of student level variables.

$\bar{X}_{\bullet j}$ : is a vector of peer effects (it may include different moments of the same variable).

$K_j$ : is a vector of pure school variables.

Similarly,  $\beta$ ,  $\gamma_1$  and  $\gamma_2$  are vectors of regression coefficients on  $X_{ij}$ ,  $\bar{X}_{\bullet j}$  and  $K_j$  respectively. The beta is used to denote regression coefficients on students' variables while the two gammas are used to denote regression coefficients on the two different types of school variables.  $V_j$  and  $\mu_j$  are the unique effects of school j on the intercept and regression coefficients, respectively.<sup>66</sup> Notice that I did not add another subscript giving a number for each variable, for notation simplicity.

### 1.3. Estimation procedures.

Even if equation (1.5) cannot be estimated with standard regression methods, a number of solutions have been developed. In this subsection, I will examine different procedures and theories used in the estimation of multilevel models.

In estimating a multilevel model, several aspects have to be taken into account: specification of the model, which determines the variables of interest to be included; the estimation theory, which allows us to make inferences about the model's parameters; and, finally, the computational algorithm, which implements the theory.

The specification of the model, in terms of the choice of the independent variables, the number of levels and the form of the relation between the dependent and independent variables (logarithmic, semi-logarithmic, etc.), should be made according to theoretical considerations. Given the specification of the model, a number of different estimation approaches can be considered. These may include maximum likelihood methods, restricted

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<sup>66</sup>  $V_j$  is a one-dimensional vector, since we have one intercept and j schools.  $\mu_j$  is a two-dimensional matrix, since we have more than one regressor and j schools.

maximum likelihood and Bayesian techniques.<sup>67</sup> Given the estimation technique, a computational algorithm has to be selected in order to implement the estimation procedure. Computational algorithms may include the following: the expectation maximization algorithm (Dempster, Laird and Rubin 1977), Fisher scoring (Longford, 1987), iterative generalized least squares (Goldstein, 1986) and the Newton Raphson algorithm (Lindstrom and Bates, 1989).

Before continuing with the examination of the estimation procedure, it is important to distinguish between two types of multilevel models: linear multilevel models and generalized multilevel models.

In linear multilevel models, the dependent variable can be represented as a linear function of the regression coefficients and random effects can be reasonably assumed to have a normal distribution. As noted by Raudenbush and Bryk (2002, page 291), the normality assumption is widely applicable when the dependent variable is continuous.

In generalized multilevel models, the assumptions of linearity and normality do not hold. This occurs when the dependent variables are not continuous. For instance, the use of standard methods would be inappropriate if, instead of having continuous performance scores, we had a binary variable taking the value of 1 if a student succeeds a grade. In this case, the predicted value of the dependent variable must lie in the interval  $[0,1]$ . Usually, the estimation of generalized multilevel models is more challenging and requires the approximation of the maximum likelihood integral. The accuracy of this approximation will affect the performance of the chosen algorithm. Generalized multilevel models can be estimated using full information maximum likelihood or penalized quasi-likelihood methods. The adapted computational algorithms include the expectation maximization algorithm, Fisher scoring with iteratively reestimated weights or Laplace approximation with Fisher scoring.

Note that in the case of PISA, the dependent variable is continuous; thus there is no need to discuss the generalized multilevel models any further. Standard linear multilevel techniques

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<sup>67</sup> In the PISA context, since the number of observations in each country is very large, the first two techniques should give similar results.

can be applied while assuming linearity and normality. In what follows, I am examining different estimation techniques for standard linear multilevel models with two levels.

### 1.3.1 Maximum Likelihood estimation methods.

The maximization of a likelihood function in multilevel models requires the construction of a likelihood integral and then its maximization. In standard linear multilevel models, this integral has a closed form. In contrast, in generalized multilevel models, the integral has to be approximated.

Let us consider  $Y$  to be a vector representing the dependent variable.  $U$  is a vector of all random effects in the model at level 2 and  $\beta$  is a vector containing all parameters (including the fixed coefficients and variance and covariance elements). The probability distribution of the dependent variable at level 1, given the random effects and parameters, can be written as  $f(Y|U, \beta)$ . The distribution of the random effects, given the parameters, can be written as  $p(U|\beta)$ . The likelihood function, given the parameters, is the following:

$$L(Y|\beta) = \int f(Y|U, \beta)p(U|\beta)dU$$

The second step consists of maximizing the likelihood function. This can be done using different types of algorithms, such as the expectation maximization algorithm, Fisher scoring, Iterative Generalized Least Squares (IGLS) and the Newton-Raphson algorithm. Note that IGLS is equivalent to Fisher scoring for standard linear multilevel models (Raudenbush and Bryk, 2002, p. 440), and that the Newton-Raphson algorithm is preferred to the EM algorithm (Lindstrom and Bates, 1988). Newton Raphson algorithm is the one used in the SAS Mixed procedure, and in my models.

Another estimation approach is the restricted maximum likelihood (RML) which was first described by Patterson and Thompson (1971). In contrast with the full information maximum likelihood, the RML is known to generate unbiased estimates of variance and covariance components. The RML is the default estimation technique used by the SAS mixed procedure. As we will see in the next part of chapter IV, the full information maximum likelihood and the restricted maximum likelihood techniques generates the same results.

### 1.3.2 Bayesian methods for the estimation of multilevel models.

When the needed conditions for the use of maximum likelihood methods do not hold, Bayesian techniques are preferable. These conditions fail when the number of higher level units is limited or when the sample is very small.

Bayesians assume that all unknown parameters have probability distributions. A prior distribution describes the econometrician belief about a parameter. This prior distribution is then combined with available data to produce the posterior distribution. A point estimate of an unknown parameter, from a Bayesian point of view, is an indicator of the central tendency of the posterior distribution of this parameter. This could be its mean or median. A good point estimate is one that minimizes the distance between the unknown parameter and the estimate. The generation of a posterior distribution uses methods such as Gibbs sampling, which is an algorithm that generates a sequence of samples from the joint probability distribution of two or more random variables. The Gibbs sampling algorithm is a special case of the Metropolis Hastings algorithm. Seltzer, Wong and Bryk (1996), Carlin and Louis (1995) and chapters 13 and 14 in Raudenbush and Bryk provide a concise examination of Bayesian estimation methods.

In the case of PISA, a large number of schools (level 2 units) and students (level 1 units) have been sampled for each country. For this reason the use of Bayesian estimation methods is not required. In fact, in chapter 13 of the PISA data analysis manual, it is suggested that maximum likelihood methods with a Newton Raphson algorithm should be used. For the rest of the chapter, the conditions for large-sample estimation theories are assumed to be verified. The following table summarizes the number of students and schools in each country.

*Table 4.1.* The number of students and schools per country.

	Germany	Finland	Great Britain	Italy	Japan
Number of students	4114	5728	9045	11342	4652
Number of schools	216	197	383	406	144



## 2. Model building issues in multilevel analyses.

In multilevel models, a number of assumptions are required at each level. When these are violated, the results and the validity of inference are jeopardized. The assumptions needed in multilevel analyses include the following.

- a) All components in the error term should be uncorrelated with the independent variables. This assumption will be thoroughly discussed in the next section.
- b) Errors are assumed to be independent with equal variances.
- c) Hypothesis tests require the errors to be normally distributed. This assumption is made at both levels. When it fails, inference cannot be properly established.

In this section, I will examine multilevel model building, while taking into account the different assumptions on which they rely.

Recall my model is the following and is given by equation (1.5):

$$Y_{ij} = c + \beta X_{ij} + \gamma_1 \bar{X}_{\bullet j} + \gamma_2 K_j + V_j + \mu_j X_{ij} + \varepsilon_{ij}$$

The assumptions, on which this model relies, are the following:

- a) The independent variables at each level are not correlated with the random effects (error terms) on the other level -  $\text{cov}(X_{ij}, \mu_j) = 0$ ,  $\text{cov}(X_{ij}, V_j) = 0$ ,  $\text{cov}(\bar{X}_{\bullet j}, \varepsilon_{ij}) = 0$  and  $\text{cov}(K_j, \varepsilon_{ij}) = 0$ . In other terms, any unobservable student characteristics relegated to the error term should not be correlated with the observable school characteristics  $\bar{X}_{\bullet j}$ , and  $K_j$ . Similarly, any unobservable school characteristics relegated to the error terms should not be correlated with the observable student characteristics  $X_{ij}$ .
- b) The level one independent variables are not correlated with level one error terms.  $\text{cov}(X_{ij}, \varepsilon_{ij}) = 0$ . In other terms, any unobservable student characteristics relegated to the error term should not be correlated with the observable student characteristics  $X_{ij}$ .

- c) The level two independent variables are not correlated with level two error terms -  $\text{cov}(\bar{X}_{\bullet j}, V_j) = 0, \text{cov}(\bar{X}_{\bullet j}, \mu_j) = 0, \text{cov}(K_j, V_j) = 0$  and  $\text{cov}(K_j, \mu_j) = 0$ . In other terms, any unobservable school characteristics relegated to the error terms should not be correlated with the observable school characteristics  $\bar{X}_{\bullet j}$ , and  $K_j$ .
- d) Each level one error term  $\varepsilon_{ij}$  is independent and normally distributed with a mean of 0 and a constant variance of  $\sigma^2$ .  $\varepsilon_{ij} \sim N(0, \sigma^2)$ .
- e) Each level two random effect (error term) is normally distributed with a mean of 0 and a variance  $\tau^2$ .  $V_j \sim N(0, \tau_0'^2)$  and  $\mu_j \sim N(0, \tau_1'^2)$ . These error terms are independent among the level two schools.
- f) The error terms at level 1 and 2 are independent.  $\text{cov}(\varepsilon_{ij}, V_j) = 0$  and  $\text{cov}(\varepsilon_{ij}, \mu_j) = 0$ .

Assumptions a, b and c focus on the relations that exists between independent variables at different levels and components included in the error term. If these assumptions are violated, then the estimates of regression coefficients may be biased. Assumptions d, e and f focus on the random part of the equation. If these are violated, proper inference cannot be established, since estimates of standard errors will be biased. In what follows, the implications of such assumptions are discussed.

## 2.1 Specifying the multilevel model.

The first step in specifying any econometric model would be the choice of variables. This must be done according to theoretical considerations. In chapter 2, a theoretical framework was developed in which stratification operated through a number of variables, including ability, social capital and income. In chapter 3, the nature of this stratification was described through statistics depicting the moments on different variables. In this chapter, the different econometric models will shed light on the impact of such stratification on test scores, and hence on the formation of inequalities.

The theoretical framework would suggest the variables to be included in the model at both levels. On the student level, these may include socio-demographic variables describing students' economic and social status, ethnicity and attitudes towards schooling. On the school

level, the variables would include school aggregates of student data, depicting different forms of peer effects, and variables depicting discipline, teachers' and students' attitudes and school resources.

### **2.1.1 Specifying the level 1 model.**

After the selection of student level variables, a decision should be made on how to specify their coefficients: random or fixed. Two methods can be contrasted:

- a) The estimation of a full model incorporating all possible independent variables with random coefficients. Then, working backward by deleting nonsignificant variables. Usually, this technique can be useful in ordinary least squares estimation. However, in multilevel analysis it is not recommended. First, such a full model with random effects on each variable requires hundreds of iterations to converge, if it does in the first place. Secondly, it would generate a large amount of insignificant results without giving indications on how to proceed afterwards. In other words, the fixed amount of variations on the dependent variable will be scattered into little pieces, none of which is significant.
- b) The estimation of a simple model, then adding one variable after another according to some theoretical considerations. I have opted for this choice due to the problems inherent to the former and the practicality of the latter. As we will see in the next section, different models have been estimated using different sets of variables.

When estimating a multilevel model, a number of factors may be considered as indicators of the reliability of the specification. These factors include the following:

- a) The number of iterations needed before the convergence of the model. When the model is over-specified and contains too many random effects on the slopes of its independent variables, a large number of iterations is needed. Usually, the Newton-Raphson or the EM algorithms converge after a limited number of iterations when the data is reliable and the specification is sound.
- b) The presence of co-linearity between the independent variables. If some of the included independent variables are heavily correlated, the model may not even

converge. A solution for this problem is to choose variables that are not supposed to be heavily correlated and to constrain some of the random effects to be equal to zero. In fact, random effects should only be used for specific key variables, for which the between-school dispersion of the effects is of particular interest to the analyst. In the PISA context, it is neither possible to interpret the random coefficients nor their significance due to the large number of level two units (schools). The only interesting statistic would be the between-school dispersion of the random effects.

In reality, the maximum number of random effects that could be included in a multilevel model cannot be precisely defined. Some characteristics of the data, as well as theoretical factors, have to be considered when making the decision. In addition to this, a variable should be excluded from the model if the analyst suspects that it is heavily correlated with other variables or when it has no significant fixed effect. As previously mentioned in chapter III, I have chosen to use the economic, social and cultural status in the regression instead of using, home possessions, parental occupation and parental education separately. The latter are definitely correlated since they describe mainly the same aspect: status.

Another concerning issue about level 1 specification is related to the error terms  $\varepsilon_{ij}$ . These errors are supposed to be normal, with a mean of zero and a constant variance  $\sigma^2$ .  $\varepsilon_{ij} \sim N(0, \sigma^2)$ . For the assessment of this assumption, it is essential to make the difference between raw and standardized residuals.

The former are directly obtained from the regression equation:  $\hat{\varepsilon}_i = Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i - \hat{\beta}_q X_{qi}$ , with  $i$  a subscript for individuals and  $q$  a subscript for variables. Two problems arise from the use of raw residuals. First, these residuals have different variances that could be equal to or greater than  $\sigma^2$ . Second, these residuals may not be independent. In general, raw residuals are not reliable in assessing the assumptions made on random errors.

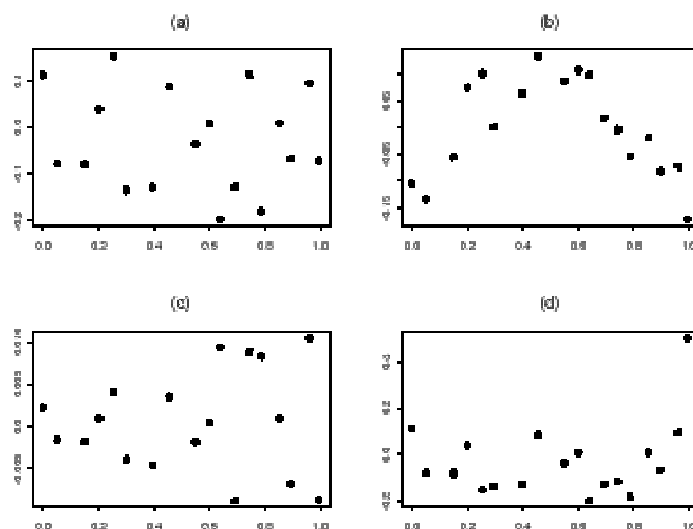
Standardized residuals are designed to overcome the problems inherent to raw residuals. Standardization is done by dividing the residual by an appropriate term which could be an estimate of its standard deviation. This operation would solve the problem of different variances. In general, standardized residuals are better suited for the assessment of the

assumptions on error terms, since they have the same distribution as the error terms  $N(0, \sigma^2)$ . SAS supports two types of standardized residuals, studentized and Pearson. In the next section, studentized residuals are used in order to assess the assumptions on error terms.

**Homoscedasticity:** multilevel models assume that error terms have a constant variance, and when this condition is not satisfied the errors are said to be heteroscedastic. This occurs when error terms are correlated with level 1 or level 2 regressors. Heteroscedasticity may arise for several reasons. These may include the omission of a key variable, the treatment of a variable that should have been included with random effects as fixed and the presence of a large proportion of extreme values (outliers) within the sampled data. Heteroscedasticity can be detected through plots of the residual errors.

The existence of heteroscedasticity can be assessed through a scatterplot combining the fitted values  $\hat{Y}_i$  on the horizontal axis and the standardized residuals on the vertical one.

Figure 4.3. Residual scatterplots.



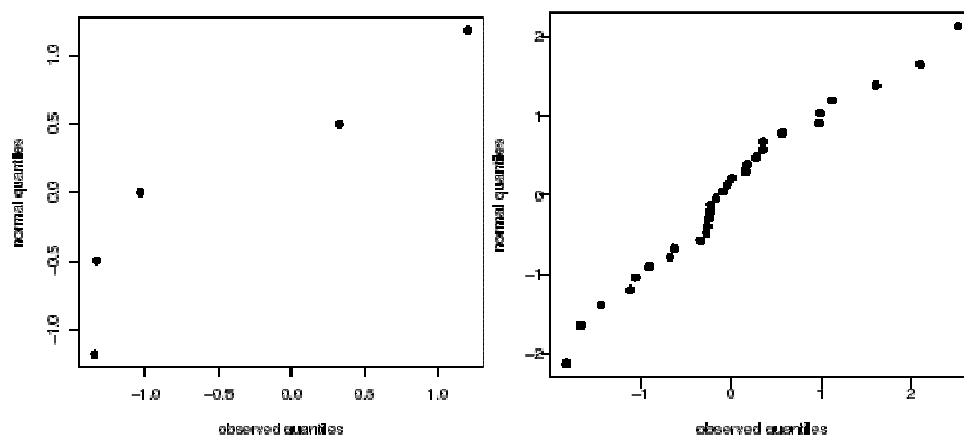
The following figures give an example of residual scatterplots. When considering a scatterplot, one should be looking for peculiar patterns. If residuals do not fluctuate randomly around zero, that would be a sign for the presence of heteroscedasticity. In figure (a), the residuals seem to fluctuate around zero with no particular pattern, indicating the existence of homoscedastic errors. In figure (b), residuals have a curve pattern; they increase, then decrease, with the fitted values. In figure (c), the variations of the residuals seem to increase

with the fitted values, indicating the presence of heteroscedastic errors. Finally, in figure (d), residuals seem to vary randomly around zero except for the dot in the top right-hand side corner. This dot is an outlier representing an extreme observation included in the data. When the assumption of constant variance does not hold, the analyst should change the specification of the model or even drop the extreme observations.

**Normality:** when the dependent variable is binary or ordinal. The assumption of normality does not apply. However, in the case of PISA, the dependent variable is continuous, and normality of the error terms can be assessed using a set of plots.

A normal probability plot of the standardized residuals gives an indication as to whether the normality of the error terms holds or not. Such a plot is obtained by plotting the quantiles of the observed sample against the corresponding quantiles of a standard normal distribution. If the plot shows a straight line, it is reasonable to assume that the observed sample comes from a normal distribution. In contrast, if the dots do not form a straight line pattern, then there is statistical evidence against the assumption of the normality of the random errors.

Figure 4.4. Assessment of normality.



In the left-hand side figure, it is possible to see that the dots diverge from a straight line pattern and that the data is sparse. Hence, the normality assumption might not hold. In contrast, in the right-hand side figure, the dots form a straight line and the normality assumption is likely to hold. When the normality assumption fails, the data require some transformation, or another distribution -different than the normal- has to be chosen.

### 2.1.2 Specifying the level 2 model.

The issues related to the specification of the level two model are similar to those in level 1. As I have mentioned earlier, it is possible to specify a full model with all possible independent variables. However, overloading the model would have diverse effects on the number of iterations needed before convergence, the significance of the coefficients and the presence of multicollinearity. It is preferable to start with a simple model and then to add relevant variables according to some theoretical considerations.

In addition to this, it should be said that the number of level two units should be sufficient to conduct a multilevel regression. In the PISA context, we have a sufficient number of schools in each country. Hence, standard multilevel regression techniques can be used without the need for Bayesian inference. Furthermore, the availability of a large number of level 2 units allows us to control for an important number of variables at this level.

Recall that, at level two, we made the assumption that the random effects  $V_j$  and  $\mu_j$  are normally distributed with a mean of zero and a constant variance. And they are also assumed to be uncorrelated with the level one and the level two independent variables. In other terms, any unobservable school characteristics relegated to the error term should not be correlated with the observable school and student characteristics. If a school level variable correlated with one of the regressors is omitted, then the estimate of one or more betas might be biased. The importance of the bias depends on the predictive power of the omitted variable.

Moreover, a badly specified level two equation may bias the coefficients in another one. This happens because of the correlation between the errors in the properly and improperly specified levels.

Another type of bias that could be introduced into both level 1 and level 2 models is related to the reliability of the data and the sampling procedures. If a variable is measured with error, it is likely that this error will bias the results. The degree of bias depends on the predictive power of the variable, the degree of the measurement error and on the correlations between the regressors.

Similarly to what was mentioned for level 1, the assumptions of normality and homoscedasticity of the random effects (level two error terms) could also be assessed. However, it should be noted that level 2 coefficients would remain unbiased even if homoscedasticity is inappropriately assumed. Furthermore, the fixed effects will not be biased even if the assumption of normality of the level two error terms does not hold. However, a failure of the normality assumption may have some effects on the validity of inference. Checking for normality at level 2 is complicated, since the random effects are not observable. Raudenbush and Bryk suggested the use of standardized residuals or a Mahalanobis distance to assess the degree of departure of random effects from normality and the presence of outliers.

### **3. Endogeneity problems in multilevel analyses and the specification of the models.**

So far, I have examined the failure of 4 of the assumptions used in multilevel models. These assumptions were the normality and homoscedasticity assumptions at each level. In this section, I will examine the cross-levels assumption, where the random effects on the intercept  $V_j$  are correlated with a level one independent variable  $X_{ij}$ . In this case, the assumption that  $\text{cov}(X_{ij}, V_j) = 0$  is violated, and some unobservable school characteristics relegated to the error term, are correlated with the observable student characteristics  $X_{ij}$ . If this assumption is breached, the coefficient estimates might be biased. Usually, this problem is called the level 2 endogeneity problem (Grilli and Rampichini, 2006).

Other forms of endogeneity may arise when level 2 independent variables are correlated with level 1 error terms or when level 1 independent variables are correlated with level two slope random effects  $\mu_j$ . However, these two cases will not be examined within this thesis for the following reason: endogeneity problems have not been widely studied in the economic literature on multilevel analyses. Even if the existence of such problems is well recognized, solutions were not provided. Some of the recent studies on endogeneity problems include: Wooldridge (2002), Skrondal and Rabe Hesketh (2004), Snijders and Berkhof (2006), Fielding (2004), Kim and Frees (2006) and Grilli and Rampichini (2006).



In this section, I will only provide a robust estimation approach for the aforementioned level 2 endogeneity problem through an adaptation of the Mundlak (1978) approach used in panel data.

For this analysis, I will adopt a simplified version of equation (1.5) in which I will consider all random effects to be equal to zero except for the intercept ( $\mu_{ij} = 0$ ). In other words, I will adopt a random intercept model where school regression lines are parallel. The equation becomes:

$$Y_{ij} = c + \beta X_{ij} + \gamma_1 \bar{X}_{\bullet j} + \gamma_2 K_j + V_j + \varepsilon_{ij} \quad (1.6)$$

Mundlak (1978) noted that a straightforward solution to solve endogeneity problems would be to include level 2 means  $\bar{X}_{\bullet j}$  into the equation. Snijders and Berkhof (2006) also noted that the inclusion of such a variable permits the disentanglement of within- and between-clusters effects. In the case of PISA, this has an intuitive interpretation, since school averages represent different forms of peer effects.

In what follows, different models will be specified; the first four are estimated according to the Mundlak approach and the last two are estimated with Full information maximum likelihood and Restricted maximum likelihood techniques. However, in this sub-section, only the specification of the models is discussed. The Hausman test as well as the results are interpreted in the second part of this chapter.

Before proceeding with the specification of the model, it is useful to start with the formulation developed in Maddala (1987). In his paper, the author reviewed some estimation issues that arise when the dependent variable is continuous in a panel data set. Note that a panel data has a number of cross-sectional units (individuals...) observed at several points of time. In other words, different time observations are nested within students. In the context of PISA, we have a similar structure. Students are nested within schools. Hence, it is possible to adapt the endogeneity robust estimation procedures developed by Mundlak (1978) in a panel data context to the PISA multilevel dataset.

Maddala gave an interesting example on the estimation of production functions in firms; his model was the following:  $Y_{it} = \beta X_{it} + \alpha_i + \varepsilon_{it}$ , with  $i = 1, 2, \dots, N$  and  $t = 1, 2, \dots, T$

$i$  is a subscript for a firm and  $t$  is a subscript for a time period.  $Y_{it}$  is the output,  $X_{it}$  is the vector of inputs for firm  $i$  in period  $t$ .  $\beta$  is the regression coefficient,  $\alpha_i$  is the firm specific unobserved inputs assumed to be constant over time. And finally,  $\varepsilon_{it}$  is an error term assumed to be normally distributed with mean 0 and constant variance,  $\varepsilon_{it} \sim N(0, \sigma^2)$ . The model developed by Balestra and Nerlove (1966) treated  $\alpha_i$  as a random variable exactly like  $\varepsilon_{it}$ . When  $\alpha_i$  is treated as random the model is called a random effects model.<sup>68</sup>

Denoting  $\bar{Y}_i = \frac{1}{T} \sum Y_{it}$  to be the within firm average, and  $\bar{Y} = \frac{1}{N} \sum \bar{Y}_i$  to be the population average, it is possible to decompose the total sum of squares (TSS).  $TSS = \sum (Y_{it} - \bar{Y})^2 = \sum (Y_{it} - \bar{Y}_i)^2 + \sum (\bar{Y}_i - \bar{Y})^2 = W_{YY} + B_{YY}$ .

$W_{YY}$  measures the within firm variations and  $B_{YY}$  measures the between firms variations.

Using a similar decomposition of the variance and covariance, we obtain the estimates of  $\beta$ :

$\hat{\beta} = W_{XX}^{-1} W_{XY}$ . Where  $W_{XY} = \sum (X_{it} - \bar{X}_i)(Y_{it} - \bar{Y}_i)$  for the fixed effects model.

And  $\hat{\beta}_{GLS} = (W_{XX} + \Theta B_{XX})^{-1} (W_{XY} - \Theta B_{XY})$  for the random effects model. (Maddala 1971).

Where  $\Theta = \frac{\sigma^2}{\sigma^2 + T\sigma_\alpha^2}$ .

Fuller and Battese (1973), noted that the GLS estimation of the betas used in Maddala (1971) is similar to the OLS estimation with the transformed data:  $Y_{it} - \lambda \bar{Y}_i$  and  $X_{it} - \lambda \bar{X}_i$  where  $\lambda = 1 - \sqrt{\Theta}$ . This is very important for three reasons:

- a) This transformation has been used by Mundlak (1978) in order to solve the level 2 endogeneity problem.
- b) This transformation rearranges the model in a form that is easily estimated with OLS.
- c) When  $\lambda = 1$ , the model is identical to the fixed effects one.

<sup>68</sup> The subscript  $i$  is for the level two units (firms) and  $t$  is for the level one units (time observations). This should not be confused with the notation in the multilevel model (equation 1.6), where  $i$  is the subscript for the level 1 units (students) and  $j$  is the one for the level 2 units (schools).

Maddala noted a number of reasons why the use of random effects models is more appropriate when the data show some nesting features. These can be added to what has been previously said in favor of multilevel models.

- a) When the dataset contains a large number of observations, instead of estimating  $N$  values for  $\alpha_i$  with fixed effects models, it is possible to estimate only the mean and variance with random effects models. This saves a lot of degrees of freedom (Maddala 1987, p. 309).<sup>69</sup>
- b) The treatment of  $\alpha_i$  as random allows us to measure firm-specific effects that we are ignorant about. In other terms, we will be able to estimate the departures from the overall intercept for each firm. These departures reflect the effects of firm unobservable factors.
- c) Another important reason is that if we want to make inferences about the actual set of cross-section units included in the dataset, we should treat  $\alpha_i$  as fixed. However, if we want to make inferences about the population from which these cross section units came,  $\alpha_i$  should be treated as random. Usually the latter is the case (Maddala 1987, p. 309).

In the model of Maddala, it is also possible to add time constant variables. These are similar to our student constant variables, which are school characteristics. His model becomes  $Y_{it} = \beta X_{it} + \gamma Z_i + \alpha_i + \varepsilon_{it}$ .

Mundlak (1978) studied the case where the  $\alpha_i$ 's are correlated with the  $X_{it}$ 's. This is similar to our level 2 endogeneity problem, where the  $X_{ij}$ 's might be correlated with the  $V_j$ 's. The author argued that this endogeneity problem will be solved if  $\alpha_i$  is assumed to depend on the mean value of  $X_{ij}$ , such as  $\alpha_i = \pi \bar{X}_i + w_i$ . With  $w_i$  a random part that has similar properties as  $\alpha_i$ . The equation becomes:  $Y_{it} = \pi \bar{X}_i + \beta X_{it} + \gamma Z_i + w_i + \varepsilon_{it}$ .

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<sup>69</sup> In the case of PISA, if the  $\beta_{0j}$  is treated as fixed and is not decomposed into an overall intercept and a random part, then we have to estimate a  $\beta_0$  for each school (this will cause the loss of  $j$  degrees of freedom). However, when  $\beta_{0j}$  is decomposed in the following manner  $\beta_{0j} = c + U_j$ , only the constant overall intercept  $c$  and the random parts are estimated, thus saving some degrees of freedom.

Using the Fuller and Battese transformation of the model, the estimator of the beta from the random effects model is obtained through OLS estimation of the following equation:

$$Y_{it} - \lambda \bar{Y}_i = \pi(\bar{X}_i - \lambda \bar{X}_i) + \beta(X_{it} - \lambda \bar{X}_i) + \gamma(Z_i - \lambda Z_i) + v_{it}$$

Then, the equation is developed and  $\beta \bar{X}_i$  is added and subtracted from it:

$$Y_{it} - \lambda \bar{Y}_i = \pi \bar{X}_i - \lambda \pi \bar{X}_i + \beta X_{it} - \lambda \beta \bar{X}_i + \beta \bar{X}_i - \beta \bar{X}_i + \gamma(Z_i - \lambda Z_i) + v_{it}$$

Finally, the equation becomes:

$$Y_{it} - \lambda \bar{Y}_i = (\pi + \beta) \bar{X}_i - \lambda(\pi + \beta) \bar{X}_i + \beta(X_{it} - \bar{X}_i) + \gamma(1 - \lambda)Z_i + v_{it}$$

$$Y_{it} - \lambda \bar{Y}_i = (1 - \lambda)(\pi + \beta) \bar{X}_i + \beta(X_{it} - \bar{X}_i) + \gamma(1 - \lambda)Z_i + v_{it}$$

We denote  $\delta = (1 - \lambda)(\pi + \beta)$  and  $\lambda = 1 - \sqrt{\Theta}$  with  $\Theta = \frac{\sigma^2}{\sigma^2 + T\sigma_\alpha^2}$

Since  $(X_{it} - \bar{X}_i)$  and  $\bar{X}_i$  are independent, it is possible to estimate each of  $\delta$ ,  $\beta$  and  $\gamma$  independently through OLS. The estimate of beta is  $\hat{\beta} = W_{XX}^{-1}W_{XY}$ . As it is possible to see, in this case the random effects estimator is identical to the fixed effects estimator mentioned above. The estimate for delta would be  $\hat{\delta} = (\sum \bar{X}_i \bar{X}_i')^{-1}(\sum \bar{Y}_i \bar{X}_i')(1 - \lambda)$  and the estimate for pi is  $\hat{\pi} = (\sum \bar{X}_i \bar{X}_i')^{-1}(\sum \bar{X}_i \bar{Y}_i) - \hat{\beta}$ . Similarly,  $\hat{\gamma}$  can be obtained by regressing the time constant variable  $Z_i$  on the average of  $Y_{it}$ , which is  $\bar{Y}_i$ .

### 3.1 The models to be estimated using PISA data.

Several models are constructed. In the first four, the estimation is carried according to the Mundlak approach. In the last two models, the multilevel equation is directly estimated through more conventional methods, such as the maximum likelihood and the restricted maximum likelihood. Furthermore, in the fifth model I am using a random intercept specification, while in the sixth some random effects on the slopes of key variables are considered.

The general equation is (1.6):

$$Y_{ij} = c + \beta X_{ij} + \gamma_1 \bar{X}_{\bullet j} + \gamma_2 K_j + V_j + \varepsilon_{ij}$$

Where  $Y_{ij}$  are performance scores,  $X_{ij}$  are student characteristics,  $\bar{X}_{\bullet j}$  are school aggregates of student characteristics (peer effects) and  $K_j$  are pure school characteristics.  $c$  is the overall intercept and  $\beta, \gamma_1$ , and  $\gamma_2$  are the regression coefficients on the different sets of variables. Finally,  $V_j$  and  $\varepsilon_{ij}$  are the error terms, with  $V_j \sim N(0, \tau_0^2)$  and  $\varepsilon_{ij} \sim N(0, \sigma^2)$ .

### 3.1.1 Model 1.

In the first specification of the model, school peer effects are dropped from the equation. The model then becomes:

$$Y_{ij} = c + \beta X_{ij} + \gamma_2 K_j + V_j + \varepsilon_{ij}$$

The Hausman test is performed in order to compare the fixed effects model, containing only student characteristics, to the random effects model, containing both  $X_{ij}$  and  $K_j$ . Note that, in the fixed effects model, the parameters  $\gamma_2$  cannot be estimated. Hence, the variables  $K_j$  cannot be included in its estimation.

#### **The Hausman test.**

The Hausman test is a specification test named after Jerry Hausman; it was developed in his article of 1978 and it tests for the presence of level 2 endogeneity. In other words, the null hypothesis is that the random effects  $V_j$  are not correlated with any of the observable students' variables. If the null hypothesis is correct, then the estimates of the coefficients are both consistent and efficient. It should also be said that after the transformation of the model, according to the Mundlak approach, the estimates of the betas are consistent regardless of whether the null hypothesis is valid (see Maddala 1987, page 311).

After the transformation of the model according to the aforementioned Mundlak approach, the model becomes:  $Y_{ij} - \lambda \bar{Y}_{\bullet j} = c + \beta(X_{ij} - \lambda \bar{X}_{\bullet j}) + \gamma_2(K_j - \lambda K_j) + w_{ij}$

I add and subtract  $\beta \bar{X}_{\bullet j}$  from the equation:

$$Y_{ij} - \lambda \bar{Y}_{\bullet j} = c + \beta(X_{ij} - \lambda \bar{X}_{\bullet j}) + \gamma_2(K_j - \lambda K_j) + \beta \bar{X}_{\bullet j} - \beta \bar{X}_{\bullet j} + w_{ij}$$

The equation is developed:

$$Y_{ij} - \lambda \bar{Y}_{\cdot j} = c + \beta X_{ij} - \lambda \beta \bar{X}_{\cdot j} + \gamma_2 K_j - \lambda \gamma_2 K_j + \beta \bar{X}_{\cdot j} - \beta \bar{X}_{\cdot j} + w_{ij}$$

And finally it becomes:

$$Y_{ij} - \lambda \bar{Y}_{\cdot j} = c + \beta(X_{ij} - \bar{X}_{\cdot j}) + (1 - \lambda)\beta \bar{X}_{\cdot j} + (1 - \lambda)\gamma_2 K_j + w_{ij}$$

I denote  $\delta_2 = (1 - \lambda)\gamma_2$

The equation becomes:

$$Y_{ij} - \lambda \bar{Y}_{\cdot j} = c + \beta(X_{ij} - \bar{X}_{\cdot j}) + (1 - \lambda)\beta \bar{X}_{\cdot j} + \delta_2 K_j + w_{ij}$$

Where  $\lambda = \bar{\lambda}_j$ , with  $\lambda_j = 1 - \frac{\sigma_w}{\sqrt{\sigma_w^2 + n_j \sigma_b^2}}$ .

$\sigma_w^2$ : is the within school variance.

$\sigma_b^2$ : is the between school variance.

$n_j$ : is the number of observations in each school for an unbalanced data set (the number of students).

**Remark:**

The within- and between-school variances are the ones on  $\varepsilon_{ij}$  and  $V_j$ , respectively. The variance of  $\varepsilon_{ij}$  is  $\sigma^2$  and the variance of  $V_j$  is  $\tau_0^2$ .  $\sigma_w^2$  and  $\sigma_b^2$  are the estimates of  $\sigma^2$  and  $\tau_0^2$ , respectively.

**Estimation:**

We can assume that  $(X_{ij} - \bar{X}_{\cdot j})$  and  $K_j$  are independent, so they can be estimated separately.

1. I regress  $(Y_{ij} - \bar{Y}_{\cdot j})$  on  $(X_{ij} - \bar{X}_{\cdot j})$ .  $\hat{\beta}$  is obtained as well as the variance components.
2. I regress  $\bar{Y}_{\cdot j}$  on  $\bar{X}_{\cdot j}$ , and  $K_j$ ,  $\hat{\gamma}_2$  is obtained.
3. I compute  $\lambda_j$  with  $\lambda_j = 1 - \frac{\sigma_w}{\sqrt{\sigma_w^2 + n_j \sigma_b^2}}$

Then  $\lambda$  is computed as the national average of  $\lambda_j$ ; hence  $\lambda = \bar{\lambda}_j$ .

4. I multiply  $\hat{\gamma}_2$  by  $(1 - \lambda)$  to obtain  $\hat{\delta}_2$ .

The regression can be fitted through OLS and the regression coefficients would be consistent and efficient.

### 3.1.2 Model 2.

In the second specification of the model, pure school characteristics  $K_j$  are dropped from the education production function. Equation (1.6) becomes:

$$Y_{ij} = c + \beta X_{ij} + \gamma_1 \bar{X}_{\cdot j} + V_j + \varepsilon_{ij}$$

The Hausman test is performed in order to compare the fixed effects model, containing only student characteristics, to the random effects model, containing both  $X_{ij}$  and  $\bar{X}_{\cdot j}$ .

After the transformation of the model according to the aforementioned Mundlak approach, the model becomes:  $Y_{ij} - \lambda \bar{Y}_{\cdot j} = c + \beta(X_{ij} - \lambda \bar{X}_{\cdot j}) + \gamma_1(\bar{X}_{\cdot j} - \lambda \bar{X}_{\cdot j}) + w_{ij}$

We add and subtract  $\beta \bar{X}_{\cdot j}$  from the equation:

$$Y_{ij} - \lambda \bar{Y}_{\cdot j} = c + \beta(X_{ij} - \lambda \bar{X}_{\cdot j}) + \gamma_1(\bar{X}_{\cdot j} - \lambda \bar{X}_{\cdot j}) + \beta \bar{X}_{\cdot j} - \beta \bar{X}_{\cdot j} + w_{ij}$$

We develop the equation:

$$Y_{ij} - \lambda \bar{Y}_{\cdot j} = c + \beta X_{ij} - \lambda \beta \bar{X}_{\cdot j} + \gamma_1 \bar{X}_{\cdot j} - \lambda \gamma_1 \bar{X}_{\cdot j} + \beta \bar{X}_{\cdot j} - \beta \bar{X}_{\cdot j} + w_{ij}$$

And finally it becomes:

$$Y_{ij} - \lambda \bar{Y}_{\cdot j} = c + \beta(X_{ij} - \bar{X}_{\cdot j}) + (\gamma_1 - \lambda \gamma_1 - \lambda \beta + \beta) \bar{X}_{\cdot j} + w_{ij}$$

$$Y_{ij} - \lambda \bar{Y}_{\cdot j} = c + \beta(X_{ij} - \bar{X}_{\cdot j}) + (1 - \lambda)(\gamma_1 + \beta) \bar{X}_{\cdot j} + w_{ij}$$

$$Y_{ij} - \lambda \bar{Y}_{\cdot j} = c + \beta(X_{ij} - \bar{X}_{\cdot j}) + \delta_1 \bar{X}_{\cdot j} + w_{ij}$$

With  $\delta_1 = (1 - \lambda)(\gamma_1 + \beta)$ , and  $\lambda = \bar{\lambda}_j$  with  $\bar{\lambda}_j = 1 - \frac{\sigma_w}{\sqrt{\sigma_w^2 + n_j \sigma_b^2}}$ ,

$\sigma_w^2$ : is the within-school variance.

$\sigma_b^2$ : is the between-school variance.

$n_j$ : is the number of observations in each school for an unbalanced data set (the number of students).

**Estimation:**

Since  $(X_{ij} - \bar{X}_{\cdot j})$  and  $\bar{X}_{\cdot j}$  are independent, they can be estimated separately.

1. I regress  $(Y_{ij} - \bar{Y}_{\cdot j})$  on  $(X_{ij} - \bar{X}_{\cdot j})$ .  $\hat{\beta}$  is obtained as well as the variance components.
2. I regress  $\bar{Y}_{\cdot j}$  on  $\bar{X}_{\cdot j}$ .  $(\gamma_1 + \beta)$  is obtained.
3. I compute  $\lambda_j$  with  $\lambda_j = 1 - \frac{\sigma_w}{\sqrt{\sigma_w^2 + n_j \sigma_b^2}}$

Then  $\lambda$  is computed as the national average of  $\lambda_j$ ; hence  $\lambda = \bar{\lambda}_j$ .

4. I multiply  $(\gamma_1 + \beta)$  by  $(1 - \lambda)$  to obtain  $\hat{\delta}_1 = (1 - \lambda)(\gamma_1 + \beta)$ .
5. We have  $\hat{\beta}$ ,  $(\gamma_1 + \beta)$ , and  $\hat{\delta}_1 = (1 - \lambda)(\gamma_1 + \beta)$ . It is possible to compute  $\hat{\gamma}_1$ .

The estimation is fitted through OLS, as suggested in Fuller and Battese (1973).

**3.1.3 Model 3.**

In the third specification of the model, the full equation (1.6) is estimated.

$$Y_{ij} = c + \beta X_{ij} + \gamma_1 \bar{X}_{\cdot j} + \gamma_2 K_j + V_j + \varepsilon_{ij}$$

The Hausman test is performed in order to compare the fixed effects model, containing only student characteristics, to the random effects model, containing  $X_{ij}$ ,  $\bar{X}_{\cdot j}$  and  $K_j$ .

After the transformation of the model according to the Mundlak approach, the model becomes:  $Y_{ij} - \lambda \bar{Y}_{\cdot j} = c + \beta(X_{ij} - \lambda \bar{X}_{\cdot j}) + \gamma_1(\bar{X}_{\cdot j} - \lambda \bar{X}_{\cdot j}) + \gamma_2(K_j - \lambda K_j) + w_{ij}$

We add and subtract  $\beta \bar{X}_{\cdot j}$  from the equation:

$$Y_{ij} - \lambda \bar{Y}_{\cdot j} = c + \beta(X_{ij} - \lambda \bar{X}_{\cdot j}) + \gamma_1(\bar{X}_{\cdot j} - \lambda \bar{X}_{\cdot j}) + \gamma_2(K_j - \lambda K_j) + \beta \bar{X}_{\cdot j} - \beta \bar{X}_{\cdot j} + w_{ij}$$

We develop the equation:

$$Y_{ij} - \lambda \bar{Y}_{\cdot j} = c + \beta X_{ij} - \lambda \beta \bar{X}_{\cdot j} + \gamma_1 \bar{X}_{\cdot j} - \lambda \gamma_1 \bar{X}_{\cdot j} + \gamma_2 K_j - \lambda \gamma_2 K_j + \beta \bar{X}_{\cdot j} - \beta \bar{X}_{\cdot j} + w_{ij}$$



And finally it becomes:

$$Y_{ij} - \lambda \bar{Y}_{\cdot j} = c + \beta(X_{ij} - \bar{X}_{\cdot j}) + (\gamma_1 - \lambda\gamma_1 - \lambda\beta + \beta)\bar{X}_{\cdot j} + (1 - \lambda)\gamma_2 K_j + w_{ij}$$

$$Y_{ij} - \lambda \bar{Y}_{\cdot j} = c + \beta(X_{ij} - \bar{X}_{\cdot j}) + (1 - \lambda)(\gamma_1 + \beta)\bar{X}_{\cdot j} + (1 - \lambda)\gamma_2 K_j + w_{ij}$$

$$Y_{ij} - \lambda \bar{Y}_{\cdot j} = c + \beta(X_{ij} - \bar{X}_{\cdot j}) + \delta_1 \bar{X}_{\cdot j} + \delta_2 K_j + w_{ij}$$

With  $\delta_1 = (1 - \lambda)(\gamma_1 + \beta)$ ,  $\delta_2 = (1 - \lambda)\gamma_2$ ,

and  $\lambda = \bar{\lambda}_j$  with  $\lambda_j = 1 - \frac{\sigma_w}{\sqrt{\sigma_w^2 + n_j \sigma_b^2}}$ .

$\sigma_w^2$ : is the within-school variance.

$\sigma_b^2$ : is the between-school variance.

$n_j$ : is the number of observations in each school for an unbalanced data set (the number of students).

**Estimation:**

We can assume that  $(X_{ij} - \bar{X}_{\cdot j})$  and  $\bar{X}_{\cdot j}$  are independent as well as  $(X_{ij} - \bar{X}_{\cdot j})$  and  $K_j$ , so they can be estimated separately.

1. I regress  $(Y_{ij} - \bar{Y}_{\cdot j})$  on  $(X_{ij} - \bar{X}_{\cdot j})$ .  $\hat{\beta}$  is obtained as well as the variance components.
2. I regress  $\bar{Y}_{\cdot j}$  on  $\bar{X}_{\cdot j}$  and  $K_j$ .  $(\gamma_1 + \hat{\beta})$  and  $\hat{\gamma}_2$  are obtained.
3. I compute  $\lambda_j$  with  $\lambda_j = 1 - \frac{\sigma_w}{\sqrt{\sigma_w^2 + n_j \sigma_b^2}}$

Then  $\lambda$  is computed as the national average of  $\lambda_j$ , and hence  $\lambda = \bar{\lambda}_j$ .

4. I multiply  $(\gamma_1 + \hat{\beta})$  and  $\hat{\gamma}_2$  by  $(1 - \lambda)$  to obtain:  $\hat{\delta}_1 = (1 - \lambda)(\gamma_1 + \hat{\beta})$  and  $\hat{\delta}_2 = (1 - \lambda)\hat{\gamma}_2$ .
5. We have  $\hat{\beta}$ ,  $(\gamma_1 + \hat{\beta})$  and  $\hat{\delta}_1 = (1 - \lambda)(\gamma_1 + \hat{\beta})$ . It is possible to compute  $\hat{\gamma}_1$ .

**Important remarks:**

- a) In models 1 and 2, different sets of school variables have been dropped, while in model 3 the full (1.6) equation was estimated. The contents of these sets of variables,

- ( $X_{ij}$ ,  $\bar{X}_{\cdot j}$  and  $K_j$ ) is the same across the three models. The selection of each variable within each set has been done previously. I started with very simple models and added one variable after the other, while dropping nonsignificant ones. The result was three sets of variables controlling for student characteristics, peer effects and pure school characteristics. However, these steps are not interpreted within the thesis because of space constraints and since they represent the preliminary work in the development of the models.
- b) Another important aspect to note: the **between-school variance** obtained in these models (and in all multilevel models) is an estimate of the dispersion of the random effects ( $V_j$ ). The **within-school variance** obtained in these models is an estimate of the dispersion of the residuals. **These estimates should not be confused with the between-school variance and average within-school variance examined in chapter III.** The ones from chapter III are directly obtained from the data without estimation, while those from the estimated models are the dispersion of the random parts of the intercept (**between-school variance**) and the dispersion of the residuals (**within-school variance**).
- c) Notice that step one of the estimation procedure enables the estimation of the betas “within effects,” which measure the strength of the relation between student level variables and performance scores. Step two enables the measurement of the “between effects,” which are the effects of school characteristics on school average performance. School characteristics may include averages of student variables or pure school variables.
- d) The transformation of the model according to the Mundlak approach is needed in order to ensure that the within and between parts of the model are independent and can be estimated separately.

#### 3.1.4 Model 4.

Model 4 is identical to model 3, except that a new school level variable which is the within school dispersion of ESCS (VARESCS) is added. This variable can be considered as a complement to average ESCS (social peer effects). The aim of this addition is to fully assess the effect of ESCS on performance scores. If this dispersion is significant, this means that ESCS peer effects in a school are not linear in means. This model will provide an answer to

the theoretical investigations developed in chapter 2. It will also provide an answer on whether social diversity has positive effects on performance. This model is estimated using the same Mundlak transformation mentioned above.

### **3.1.5 Model 5.**

Model 5 has the same specification as model 4, except that the estimation is done using conventional techniques such as the maximum likelihood (ML) and the restricted maximum likelihood (RML). In this model, a random intercept specification is adopted and all coefficients on student variables are considered to be fixed. The results from this estimation are compared to those of model 4 and the results from the different techniques (ML and RML) are also compared.

### **3.1.6 Model 6.**

Finally, the last model is estimated using the same specification from model 5, but with a random slope on ESCS. The aim of this model is not to examine the specific effect for each school - since that would be impossible for more than 200 schools per country - but rather to interpret the between-school dispersions on the random effects. These dispersions are measured through an estimate of the variance of  $\mu'$ , which is  $\tau_1^2$  (see equation 1.5). The model is also estimated using ML and RML.

## **B. Estimation and results.**

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In this part of the chapter, the estimation of the six aforementioned models is carried out and the results are interpreted in light of the characteristics of each education system. On the one hand, a comparison is made between different models to assess their reliability, and then the best models are chosen for the interpretation of the results. On the other hand, a cross-country comparison is undertaken on the basis of these results.

Recall that model 1 included student variables and pure school characteristics as independent variables. Model 2 included student variables and peer effects. Model 3 is the most complete, and included all three categories: student variables, pure school characteristics and peer effects. Model 4 added the within-school dispersion of ESCS (Economic, Social and Cultural status) as an independent variable to reflect nonlinear peer effects. All 4 models were fitted through OLS using a Mundlak transformation. Model 5 uses the exact specification of model 4, but without any transformation. In contrast, it is estimated using the maximum likelihood (ML) and then the restricted maximum likelihood (RML) techniques. At this stage, two comparisons are possible: the first between the results from ML and those from RML and the second between the results from the Mundlak approach and those from the ML and RML methods. Finally, model 6 allows for slope heterogeneity on the student level variables. In other words, a random slope model is used, where each school has a specific effect that departs from the overall regression coefficient. This model is also estimated with ML and RML and both methods are compared.

After the estimation a number of comparisons and interpretations are made:

- a) A comparison across the first four models is made through the Hausman test and the best specification is identified. Model three is interpreted, since it is the most complete and it passes the Hausman test. The results on models 1 and 2 are not interpreted, and are relegated to the appendix.

- b) In model 4, light is shed on the effects of within-school dispersions of ESCS on performance scores. The interpretation is expected to provide an answer to whether achievements are linear in social and economic peer effects.
- c) A comparison is also carried out between the results of model 4 and those of model 5. Notice that the specifications are the same, but the estimation procedures are different. Model 4 is estimated through a Mundlak transformation, while model 5 is estimated with ML and RML techniques.
- d) A comparison is made between the ML and RML results on model 5.
- e) In model 6, the variance components are interpreted but the random coefficients are not. This is due to the fact that each country has more than 200 schools and it is not possible, or even intuitively interesting, to interpret each random effect for each school. However, the between-school dispersions of random effects is interpreted.
- f) A comparison is also made between the ML and RML results on model 6.
- g) My results are also compared to those published by the OECD in the Learning for Tomorrow's World report and differences between the two are assessed.

At this stage, it is useful to recall the **statistical definition of inequalities** in attainments. Inequalities do not exist when - in a regression analysis - all student and school variables have insignificant effects. In other words, the variables that may explain differences in performance are still hiding in the unobserved component (the error term). If most student and school characteristics are controlled for, and are found to have no effect, the only possible variable that may explain differences in performance would be student competencies (e.g., IQ). This situation is a perfect meritocracy, where the surrounding environment of a student - whether at home or at school - does not affect his achievements. Of course, this situation does not exist, but it is useful to consider it as a benchmark against which countries are compared.

A second remark concerns the significance of the coefficients. Note that two different types of significance can be discussed. First, statistical significance assessed through the t statistic, which indicates how high the chances are for the results to be incorrect. Second, economic significance assessed through the value of the regression coefficient. For instance, a particular variable may have a coefficient that is significant at the level of 1%, yet the value of the coefficient itself is so low that it cannot be considered as economically significant.

Before starting with the interpretation of the different models, it is essential to present the content of each set of variables included in the regressions. These variables were previously examined in chapter 3 in a descriptive manner. However, not all variables in chapter 3 will be used in the regressions, since some of them are highly correlated (e.g., parental education and parental occupation).

**Individual level variables  $X_{ij}$  :**

ESCS: The economic, social and cultural status of the student.

COMPHOME: An indicator of computer facilities at home.

INTMAT: An indicator of interest in mathematics.

ANXMAT: An indicator of anxiety in mathematics.

DISCLIM: An indicator of the perception of discipline in a school.

ETR: A binary variable taking the value of one if a student is a first generation student or a nonnative.

**Peer effects, school aggregates of individual level variables  $\bar{X}_{.j}$  :**

DESCS: School average of ESCS, depicting economic, social and cultural peer effects in a school.

VARESCS: The within-school dispersion of ESCS; this variable is added in model 4 onwards.

DCOMPH: School average of COMPHOME, depicting the possession of computer facilities peer effects.

DINTMAT: School average of INTMAT, depicting peer effects resulting from a generalized interest and enjoyment of mathematics within a school.

DANXMAT: School average of ANXMAT, depicting peer effects resulting from a generalized feeling of anxiety and helplessness in mathematics.

DDISCL: School average of DISCLIM, depicting the impact of a generalized perception of discipline in a school.

DETR: The percentage on nonnatives or first generation students in a school.

**Pure school level variables  $K_j$  :**

Compweb: The proportion of computers connected to the web in a school.

Mactiv: The number of activities used to promote engagement with mathematics in a school.

Mstrel: An index measuring poor student teacher relations.

Tcshort: An index measuring principals' perception of potential factors hindering the recruitment of new teachers, and hence instruction.

Tcmorale: An index depicting principals' perception of teacher morale and commitment.

Teacbeha: An index depicting principals' perception of teacher-related factors hindering instruction or negatively affecting school climate.

Private: A binary variable taking the value of one if a school is private.

Scmatedu: The quality of educational infrastructure in a school as perceived by the principal.

Academic: A binary variable taking the value of one if a school selects its students according to their academic records.

## 1. The Hausman test.

Recall, the Hausman test is a specification test developed in 1987 by Jerry Hausman. The test identifies the presence of level 2 endogeneity. The null hypothesis is that the random effects (on the intercept) are not correlated with any of the students' variables,  $\text{cov}(X_{ij}, V_j) = 0$ . If the null hypothesis holds, then the estimates of the coefficients are both consistent and efficient. The Hausman test, applied in the context of the first four models, tests a fixed effects specification of the models against the random effects specification. If the null hypothesis is rejected, then we can conclude that the random effects model suffers from endogeneity and that the fixed effects specification is still better. Furthermore, after the transformation of the model according to the Mundlak approach, the estimates of the betas are consistent regardless of whether the null hypothesis is valid.

The estimation procedure used for each of the first 4 models is the same. The procedure starts with a regression of  $(Y_{ij} - \bar{Y}_{\cdot j})$  on  $(X_{ij} - \bar{X}_{\cdot j})$  in order to obtain the  $\hat{\beta}$  as well as the variance components. These beats were called by Maddala (1987) the "within effects". Then  $\bar{Y}_{\cdot j}$  is

regressed on  $\bar{X}_{\bullet j}$  and  $K_j$  in order to obtain the  $(\gamma_1 + \beta)$  and  $\hat{\gamma}_2$ . These coefficients were called the “between effects”. The parameter  $\lambda_j$  and its national average  $\lambda$  are computed with

$$\lambda_j = 1 - \frac{\sigma_w}{\sqrt{\sigma_w^2 + n_j \sigma_b^2}}.$$

Finally,  $(\gamma_1 + \beta)$  and  $\hat{\gamma}_2$  are multiplied by  $(1 - \lambda)$  to obtain

$\hat{\delta}_1 = (1 - \lambda)(\gamma_1 + \beta)$  and  $\hat{\delta}_2 = (1 - \lambda)\hat{\gamma}_2$ . This estimation procedure is the one used for the complete specification in model 3 and 4. However, model 1 only considered pure school characteristics,  $K_j$ , and model 2 only considered school peer effects -  $\bar{X}_{\bullet j}$ .

Table 4.2. The results on the Hausman test.

	Germany	Finland	UK	Italy	Japan
<b>Model 1</b>	309.07	5.68	125.66	26.60	142.98
<b>Model 2</b>	0.00	0.00	0.00	0.00	0.00
<b>Model 3</b>	0.00	0.00	0.00	0.00	0.00
<b>Model 4</b>	0.00	0.00	0.00	0.00	0.00

As we can see, the Hausman test fails for the first model where the null hypothesis is rejected and succeeds for models 2, 3 and 4. A number of issues should be discussed:

- a) Model one did not control for peer effects (school averages of the  $X_{ij}$  's). These school characteristics were relegated to the error term  $V_j$  and they are correlated with individual level variables. Thus model 1 suffers from endogeneity and the null hypothesis on the Hausman test is rejected. The fixed effects specification is preferred to the random effects one.
- b) The only country that is close to passing the Hausman test in this model is Finland. This indicates that the strength of the correlation between student characteristics and unobserved peer effects is lower. This is perhaps due to the fact that schooling in Finland is comprehensive. Hence, it is unlikely that student characteristics are highly correlated with those of the school. Finland is followed by Italy, the UK, Japan and Germany. The latter has the strongest value on the test; this can be attributed to the German selective system, where student characteristics are likely to determine the characteristics of the attended school.



- c) The failure of the Hausman test is a strong indication that the specification in model 1 is not reliable. Even if the Mundlak transformation solves the endogeneity problem and generates consistent estimates, the model will still be incomplete since it lacks the peer effects. Therefore, I have decided not to interpret the results on this model.
- d) Models 2, 3 and 4 passed the Hausman test. The null hypothesis is accepted and there are no correlations left between students' variables and unobserved school characteristics relegated to the error term.
- e) In model 2, pure school characteristics were omitted. However, the model still passed the Hausman test. This is an indication that student level variables are not correlated with pure school characteristics but with peer effects (since model 1 did not pass the test).
- f) Models 3 and 4 are the most complete; they controlled for peer effects and pure school characteristics and they passed the test. Therefore, these are the ones to be interpreted.
- g) Even if models 2, 3 and 4 passed the Hausman test, the Mundlak transformation is still used in order to obtain the results.

In conclusion, when peer effects were omitted the model did not pass the test. But, when pure school characteristics were omitted, the model passed the test. It is possible to say that it is essential to control for peer effects in any estimation, since these tend to be correlated with individual level variables. Furthermore, peer effects are more important than pure school characteristics, since the omission of the latter did not affect the viability of the model. However, since pure school variables are of particular interest for the interpretations and the cross country comparisons, they are intuitively relevant to be included in the model even if statistically they can be omitted. Models 3 and 4 are the most reliable and the most complete; they passed the Hausman test and included all relevant variables from both levels. The results from these models are interpreted in the next subsections. In contrast, the results from models 1 and 2 are relegated to the appendix due to lack of space and since models 3 and 4 are better specified.

## 2. Interpretation of the results of models 3 and 4.

Several steps were carried out in the estimation of the models using the Mundlak approach. The presentation of the results is done according to those steps. First, the results on the within part of the model (the  $\hat{\beta}$ ) are examined, followed the results on the between part of the model and finally by the variance components (between- and within-school variances).

### 2.1 Interpretation of the coefficients on student level variables.

Recall that the estimation of the coefficients on student level variables is identical for the first 4 models. It consists of regressing  $(Y_{ij} - \bar{Y}_{\bullet j})$  on  $(X_{ij} - \bar{X}_{\bullet j})$  to obtain the betas. The betas are the same across these 4 models; only the coefficients on school variables change. Note that the betas can be interpreted as the variation in performance caused by a one unit increase on the student level variable, everything else held equal. In other words, the betas obtained by regressing  $(Y_{ij} - \bar{Y}_{\bullet j})$  on  $(X_{ij} - \bar{X}_{\bullet j})$  are the same as in equation (1.6).

$$Y_{ij} = c + \beta X_{ij} + \gamma_1 \bar{X}_{\bullet j} + \gamma_2 K_j + V_j + \varepsilon_{ij}$$

The first table depicts the number of observations and the adjusted R squared for the within part of the first 4 models (estimation of student variables' coefficients). These are the results of the regression of  $(Y_{ij} - \bar{Y}_{\bullet j})$  on  $(X_{ij} - \bar{X}_{\bullet j})$ . Note that the adjusted R squared is better than the ordinary R squared, since the latter has a form that is increasing in the number of regressors. Hence, it is likely that the ordinary R squared increases, even if the additional variables are irrelevant.

Table 4.3. The number of observations per country and the adjusted R squared for the within part of the model. Step 1.

	Germany	Finland	UK	Italy	Japan
<b>n</b>	4114	5728	9045	11342	4652
<b>Adjusted R-squared</b>	0.65	0.36	0.44	0.61	0.59

On the one hand, in all countries the number of observations is high enough to support the normality assumption. On the other hand, the values of the adjusted R squared are different between countries. Finland has the lowest value; only 36% of the variations in performance are explained by the variations in student variables. This is a characteristic of comprehensive systems. Finland is followed by the UK, Japan, Italy and Germany. Germany is the opposite of Finland, where 65% of the variations in performance scores are explained by the variations in student variables. The UK, Japan and Italy are middle ranking.

In table 4.4, the coefficients on the betas are presented along their standard errors and level of significance. (\*\*\*) stands for significance at the level of 1%, (\*\*) for significance at the level of 5% and (\*) for significance at the level of 10%.

Table 4.4. The coefficients on individual level variables.  $\hat{\beta}$ .

Variables	Germany		Finland	
	Coefficient	Std Error	Coefficient	Std Error
<b>ESCS</b>	11.6188704(***)	1.23597758	26.8562187(***)	1.23591392
<b>COMHOME</b>	1.28125269	1.21663952	-2.72385848(**)	1.15042696
<b>INTMAT</b>	4.69696341(***)	0.94749482	14.5124817(***)	1.04775242
<b>ANXMAT</b>	-19.0354435(***)	0.90418361	-31.9655508(***)	1.07991781
<b>DISCLIM</b>	2.6305623(***)	0.82418857	1.41710402	1.00131231
<b>ETR</b>	-27.9232916(***)	2.92336538	-63.4902422(***)	6.3707258

Variables	UK		Italy	
	Coefficient	Std Error	Coefficient	Std Error
<b>ESCS</b>	23.2628003 (***)	0.98944402	3.22997241 (***)	0.7133729
<b>COMHOME</b>	8.20944513 (***)	0.98848629	8.89853572 (***)	0.67993139
<b>INTMAT</b>	-1.16124725	0.87482997	6.49337771 (***)	0.65907046
<b>ANXMAT</b>	-25.1125562 (***)	0.8714535	-21.2710943 (***)	0.69486398
<b>DISCLIM</b>	12.3322294 (***)	0.69943581	2.03617407 (***)	0.58092855
<b>ETR</b>	-5.18637109 (*)	2.89924318	-6.81673487 (*)	4.01729008

Variables	Japan	
	Coefficient	Std Error
<b>ESCS</b>	1.14574771	1.61050778
<b>COMHOME</b>	3.3407971 (***)	1.20791729
<b>INTMAT</b>	10.3844416 (***)	1.14305169
<b>ANXMAT</b>	-7.80941069 (***)	1.13521008
<b>DISCLIM</b>	2.68997083 (**)	1.14669476
<b>ETR</b>	-19.5223855	28.5765359

The interpretation of the coefficients is carried out as a cross-country comparison for each included variable. The value of a coefficient indicates the variation in individual performance scores caused by a one unit increase on the corresponding variable, everything else held equal.

### **2.1.1 The economic, social and cultural status ESCS.**

ESCS is the most important student level variable affecting performance. In most countries, it is at the centre of educational policies, since one of the objectives is to ensure equality of opportunity in the access to education. Furthermore, ESCS is the most important dimension according to which stratification operates.

ESCS is statistically significant across all countries except for Japan. Finland has the highest value on the regression coefficient, with an increase of 26.85 points in performance scores caused by an increase of one unit of ESCS. Finland is followed by the UK, Germany, Italy and Japan. The results for Finland seem to be counterintuitive for a comprehensive schooling system. However, when peer effects (especially average ESCS in a school) are taken into account, a full picture will emerge. Comprehensiveness in Finland is associated with high levels of homogeneity between schools. Therefore, the impact of school variables, including peer effects on performance scores, is expected to be small. As a consequence, the only factors that would still explain the variation in performance scores are student characteristics, such as ESCS. Furthermore, this high value on the regression coefficient is not alarming since Finland has the lowest national dispersion of ESCS. In other words, even if the slope is important, there are limited variations on ESCS to cause high inequalities in performance.

The UK as well has an important effect of ESCS on student performance scores. However, the case of the UK is different than that of Finland, since school variables have strong effects, too. This is perhaps the result of the unachieved comprehensivization of the British education system. Germany has a middle ranking value on the coefficient of ESCS, while Italy and Japan have very low values. As we have seen in chapter 3, Italy and Japan have high levels of social stratification and heterogeneity between schools; this causes average ESCS in a school to have a strong impact on performance scores, while the effect of ESCS is low. In other words, inequalities in performance scores are channeled indirectly through the choice of a school. In addition to this, Japan is known to have a less socially divided society (low total

ESCS variance); this may explain why the coefficient on ESCS is neither statistically nor economically significant.<sup>70</sup> Note that when the coefficients on school variables are interpreted, the full picture will immerge.

### **2.1.2 The availability of computer facilities at home COMPHOME.**

The availability of computer facilities at home is statistically significant in all countries except in Germany. The UK and Italy have the highest values. This indicates the presence of disparities between households on the possession of computers; some households have computers while others do not, and those who do are expected to have slightly higher performance scores. The effects in Finland and Japan are low and even negative.

### **2.1.3 Interest in mathematics INTMAT.**

Interest and enjoyment of mathematics can be considered as a motivational factor for the study of mathematics. Previous research showed that interest in mathematics is not necessarily related to the general motivation to learn. Students may only be inclined to learn their subjects of interest, while disregarding other areas of the curriculum. A comparison between PISA 2000 and PISA 2003 showed that students have higher motivation to learn languages than to learn mathematics. Moreover, students tend to lose interest in mathematics after the primary education phase, because of the differentiation of students interests at an older age, the ways mathematics are taught and the content of the curricula that tends to become more complex.

Interest in mathematics has a statistically significant effect on performance scores in all countries except the UK. The effect is positive, indicating higher performance scores for students with higher interest in mathematics. Finland has the highest value; for each additional one unit of interest in mathematics, students are expected to have 14 extra points. Finland is followed by Japan, Italy and Germany. The level on this coefficient for Finland is another indication that, when school variables do not have large effects (since schools are similar), student level variables would have higher effects.

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<sup>70</sup> In Japan, students do not differ largely in terms of their ESCS. However, schools differ on their average ESCS. This is a characteristic of stratified and hierarchical societies.

#### **2.1.4 Anxiety in mathematics ANXMAT.**

Anxiety in mathematics can be considered as a measure of students' difficulties and feelings of helplessness when confronted with mathematics. Students affected by anxiety are more exposed to failure. The likelihood of failing is not necessarily related to their abilities and competencies, but probably to previous failures and emotional stress.

The coefficient on the index of anxiety in mathematics is significant and negative for all countries. Finland has the highest result (in absolute value), indicating that performance scores are highly affected by levels of anxiety and helplessness in mathematics. The UK, Germany and Italy have similar results. But the most interesting finding is for Japan, where students do not seem to be very negatively affected by anxiety. In fact, an increase in anxiety of one unit on its scale causes a decrease of 7 points in performance, while in Finland, the decrease is of 31 points, and in the rest of the countries the decrease is of 20 points. This finding can be explained through an aspect of the Japanese educational culture. Students tend to attribute their low scores to insufficient hard work rather than to factors such as IQ or anxiety (see Green et al 2006, chapter 5).

#### **2.1.5 Perception of discipline in a school DISCLIM.**

In PISA 2003, students were asked to indicate the extent to which school climate and learning were affected by disruptive behavior or discipline related factors. Note that students and principals do not necessarily have the same vision of discipline problems. On the one hand, a principal usually considers objective factors related to the general climate of his school, while on the other hand, a student considers school climate through his own personal experience. As indicated in PISA (learning for tomorrow's world, p. 215), student absenteeism is the most frequent student obstacle to learning, followed by disruptive behavior.

As expected, the perception of discipline by students has a positive effect on their performance across all countries, except in Finland where the effect is not significant. The UK has the highest score on this variable; an increase in the perception of discipline of one unit leads to an increase in performance of 12.33 points. For the rest of the countries, an increase in DISCLIM of one unit leads to an increase in performance of 2 points.

### **2.1.6 Immigration status ETR.**

Most of the selected PISA countries have experienced immigration during the last few decades. The PISA 2000 results showed a large advantage for native students over non-natives in both reading and mathematics. These differences are attributable to the difficulties that face non-native students in adapting to a new school environment and to a new language. These performance gaps are even more concerning in countries where large immigrant populations exist, such as in the UK, Italy and Germany, and where these populations are socially disadvantaged.

In the PISA 2003 report, performance scores were computed for first-generation immigrants and for non-natives using a number of descriptive statistics. First-generation students were found to be lagging behind their native counterparts and non-natives were found to be getting even lower scores. In my thesis, I am not separating first-generation immigrants from those born outside a particular country; the variable ETR is considered to combine both and both are referred to as non-natives. This simplification is motivated by the fact that when the two categories are taken separately, the percentage of students in one of them can be very close to zero. Hence, small frequencies are not convenient for econometric regressions.

Immigration status is perhaps the second-most important variable after ESCS. In all countries, being a non-native has a negative and significant effect on performance, except in Japan. Finland has the highest value; students with an immigrant background are expected to have 63 points less than their native counterparts. In Germany, they are expected to have 27 points less than their native counterparts. Germany is followed by Italy and the UK with 7 and 5 points less for non-natives, respectively. However, it should be said that in Italy and Finland the immigrant population is about 2%. Hence, the proportion of people affected by such a negative relation is small. In contrast, the immigrant populations in the UK and Germany are of 8% and 14% respectively and thus the proportion of people concerned with this negative relation is greater.

## 2.2 Interpretation of the coefficients on school variables.

Recall that step 2 in the estimation procedure, in both models 3 and 4, consisted of regressing schools' average performance scores on school characteristics. The latter include peer effects and pure school characteristics. A number of coefficients are obtained from this regression.

From step 2 ( $\hat{\gamma}_1 + \hat{\beta}$ ) and  $\hat{\gamma}_2$  are obtained, from step 4 the deltas ( $\hat{\delta}_1$  and  $\hat{\delta}_2$ ) are obtained and from step 5  $\hat{\gamma}_1$  is obtained.

At this stage, it is essential to decide which set of coefficients to interpret. I decided to interpret ( $\hat{\gamma}_1 + \hat{\beta}$ ),  $\hat{\gamma}_1$  and  $\hat{\gamma}_2$ , for several reasons. On the one hand,  $\hat{\gamma}_1$  and  $\hat{\gamma}_2$  are the coefficients from equation (1.6); they reflect the effect of school variables on performance scores. Similarly, ( $\hat{\gamma}_1 + \hat{\beta}$ ) reflects the impact of peer effects on average performance in a school.<sup>71</sup> On the other hand, the two computed deltas ( $\hat{\delta}_1$  and  $\hat{\delta}_2$ ) lack the intuitive explanation, since the dependent variable is  $(Y_{ij} - \lambda\bar{Y}_{\cdot j})$ . In other terms, a variation in  $(Y_{ij} - \lambda\bar{Y}_{\cdot j})$  does not have a particular intuitive interpretation. Note that, since the variance of ESCS was added as an independent variable in model 4, the coefficients of models 3 and 4 can be slightly different.

Table 4.5. The number of observations per country and the adjusted R squared for the between part of the model. Step 2 for model 3.

	Germany	Finland	UK	Italy	Japan
n	4114	5728	9045	11342	4652
Adjusted R-squared	0.82955	0.83793	0.8119439	0.74769	0.87164

On the one hand, in all countries the number of observations is high enough to support the assumption of normality and it is equal to the number of observations in the within part of the regression (step 1). On the other hand, the values of the adjusted R squared are relatively the same across all countries; they vary between 74% in Italy and 87% in Japan. In Italy, the included independent variables explain 74% of the variations in school average performance scores. In Japan they explain about 87%. The UK, Finland and Italy have about 82% of the variation in school average performance scores explained by school characteristics.

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<sup>71</sup>  $\hat{\gamma}_2$  also measures the impact of pure school characteristics on school average performance.



Table 4.6. The coefficients on school level variables  $(\gamma_1 + \beta)$  and  $\hat{\gamma}_2$ , obtained from step 2. Model 3.

Variables	Germany		Finland	
	Coefficient	Std error	Coefficient	Std error
<b>DESCS</b>	72.0031316 (***)	1.80950171	29.6779122 (***)	0.95291161
<b>DCOMPH</b>	29.8615439 (***)	2.9357794	-0.95538884	1.42592764
<b>DINTMAT</b>	-19.6216794 (***)	2.07691763	8.49251279 (***)	1.3696608
<b>DANXMAT</b>	-35.5798426 (***)	1.92511369	-29.0953704 (***)	1.58086946
<b>DDISCL</b>	27.6675967 (***)	1.47077184	1.31173003	0.82245084
<b>DETR</b>	-21.5412451 (***)	3.31000001	-7.90148118 (**)	3.13900317
<b>Compweb</b>	10.9687132 (***)	1.79215262	4.83067797 (**)	2.44045551
<b>Mactiv</b>	1.26354484	0.7970734	1.43754715 (***)	0.46728871
<b>Mstrel</b>	12.9503566	11.5664539	-124.763853 (***)	8.01667392
<b>Tcshort</b>	-7.62987123 (***)	0.53944754	-0.34510601	0.36924118
<b>Tcmorale</b>	0.02947631 (***)	0.00220373	1.78160365 (***)	0.31655084
<b>Teacbeha</b>	-4.29821192 (***)	0.78979582	-1.39534487 (***)	0.43129085
<b>Private</b>	-11.3645101 (***)	2.05053538	-18.4228242 (***)	1.46140304
<b>Scmatedu</b>	-0.06662555 (***)	0.00323867	0.24466686	0.36868574
<b>Academic</b>	7.29168301 (***)	0.98092486	11.1504286 (***)	1.01129261
<b>Intercept</b>	466.331487	2.14981428	528.735747	2.43630529

Variables	UK		Italy	
	Coefficient	Std error	Coefficient	Std error
<b>DESCS</b>	66.0920075 (***)	1.08673405	31.5915923 (***)	1.30968108
<b>DCOMPH</b>	1.87066729	1.577258	66.9223187 (***)	1.91117502
<b>DINTMAT</b>	-9.43963671 (***)	1.15532378	-15.8149384 (***)	1.2759081
<b>DANXMAT</b>	-37.7274831 (***)	1.13514029	-72.2539899 (***)	2.82646668
<b>DDISCL</b>	21.7210883 (***)	0.71604657	20.6388854 (***)	1.10809339
<b>DETR</b>	-41.0252885 (***)	2.35621912	-29.5757664 (***)	4.19585689
<b>Compweb</b>	17.7015264 (***)	1.16506971	11.9002776 (***)	1.27633257
<b>Mactiv</b>	-2.97284142 (***)	0.24023833	8.25603192 (***)	0.44599331
<b>Mstrel</b>	-76.8474687 (***)	7.78187943	-105.02929 (***)	10.4742127
<b>Tcshort</b>	-3.78994589 (***)	0.30785461	4.60502306 (***)	0.52481038
<b>Tcmorale</b>	-2.14809054 (***)	0.34017134	0.00995978 (***)	0.00188041
<b>Teacbeha</b>	-1.28290405 (***)	0.33517605	2.21424581 (***)	0.4350672
<b>Private</b>	14.5332994 (***)	1.46133834	-54.6082399 (***)	2.63366558
<b>Scmatedu</b>	2.56334388 (***)	0.26529533	0.0178466 (***)	0.00250079
<b>Academic</b>	4.50833276 (***)	0.97135269	-9.06752336 (***)	0.81870377
<b>Intercept</b>	492.796549	1.37529252	494.561719	1.73240539

Variables	Japan	
	Coefficient	Std error
<b>DESCS</b>	98.5040841 (***)	2.42776504
<b>DCOMPH</b>	18.1582199 (***)	2.32559737
<b>DINTMAT</b>	65.6165343 (***)	2.15458392
<b>DANXMAT</b>	33.4408818 (***)	2.07132094
<b>DDISCL</b>	27.4000697 (***)	1.15821103
<b>DETR</b>	-294.85472 (***)	75.4973397
<b>Compweb</b>	-1.66948555	1.38570822
<b>Mactiv</b>	1.74979551 (***)	0.6125078
<b>Mstrel</b>	-72.1965699 (***)	12.1306528
<b>Tcshort</b>	-1.60200959 (***)	0.48111738
<b>Tcmorale</b>	-2.92096803 (***)	0.51314569
<b>Teacbeha</b>	6.41521725 (***)	0.47399925
<b>Private</b>	-39.9666839 (***)	1.10437921
<b>Scmatedu</b>	-4.59491915 (***)	0.41610399
<b>Academic</b>	17.4910423 (***)	1.2442493
<b>Intercept</b>	552.99961	2.29976755

The interpretation of the coefficients  $(\hat{\gamma}_1 + \hat{\beta})$  and  $\hat{\gamma}_2$  will be carried out as a cross country comparison for each included variable. The value of a coefficient indicates the variation in school average performance scores caused by a one unit increase on the corresponding variable, everything else held equal.  $\hat{\gamma}_2$  can also be interpreted as the variation in individual performance scores caused by a one unit increase on the corresponding variable, everything else held equal. (See equation 1.6).

Table 4.7. The coefficients on peer effects  $\hat{\gamma}_1$ , obtained from step 5. Model 3.

Variables	Germany	Finland	UK	Italy	Japan
	Coefficient	Coefficient	Coefficient	Coefficient	Coefficient
<b>DESCS</b>	60.38(***)	2.82(***)	42.82(***)	28.36(***)	97.35(***)
<b>DCOMPH</b>	28.58(***)	1.76	-6.33	58.02(***)	14.81(***)
<b>DINTMAT</b>	-24.31(***)	-6.01(***)	-8.27(***)	-22.30(***)	55.23(***)
<b>DANXMAT</b>	-16.54(***)	2.87(***)	-12.61(***)	-50.98(***)	41.25(***)
<b>DDISCL</b>	25.03(***)	-0.10	9.38(***)	18.60(***)	24.71(***)
<b>DETR</b>	6.38(***)	55.58(**)	-35.83(***)	-22.75(***)	-275.33(***)

The value of a coefficient  $\hat{\gamma}_1$  indicates the variation in individual performance scores caused by a one unit increase on the corresponding peer effects, everything else held equal.

### **2.2.1 Average economic, social and cultural status in a school (DESCS).**

This variable is the most important among all variables reflecting peer effects. It accounts for factors such as parental education, parental occupation and home possessions. The inclusion of this variable in the model allows us to distinguish between two important effects of ESCS: its direct effect channeled through students' ESCS and its peer effects channeled through average ESCS in a school.

In chapter 3, I showed that in some countries a strong social stratification exists; these countries include Japan, Italy, Germany and, to some extent, the UK. In this part of the analysis, stratification is related to inequalities in performance among schools and students.

The coefficients on school average ESCS are all statistically significant at the level of 1% for all countries. The highest value on the coefficient is for Japan: an increase of 1 unit in DESCES leads to an increase of 98 points in average school performance, and to an increase of 97 points in student performance. Japan is followed by Germany, for which an increase of 1 unit in DESCES leads to an increase of 72 points in average school performance and to an increase of 60 points in student performance. The UK comes third, with 66 points increase on average school performance and 42 points on student performance. The UK is followed by Italy with 31 points increase on average school performance and 28 points on student performance. Finally, Finland has the lowest scores, with 29 points increase on average school performance and 3 points on student performance.

These results are very reflective of the different schooling systems:

Germany: the high levels of stratification and social inequalities in the access to education are translated into inequalities in performance scores. The ESCS of a student determines the school in which he is enrolled as well as a certain proportion of his performance. Hence, a student with a low level of ESCS is likely to be streamed into vocational schools where other students with similar levels of ESCS are enrolled. Since ESCS has an important effect on performance, low ESCS students will get lower results. And since average ESCS in a school also has an important effect on performance, low ESCS students enrolled in schools with low average ESCS are likely to have lower performances. In Germany, the selective school system

is a generator of inequalities, since it allows ESCS to play fully through its direct household effect and through its indirect school effect. However, the German system cannot be understood unless the labor market is considered. Germany retains a strong apprenticeship system through which educational inequalities are channeled and absorbed by the labor market. Furthermore, attending professional schools is not regarded as a sign of failure .

Japan: Japan also has high levels of social stratification. However, inequalities operate differently than in Germany. On the one hand, a student's ESCS has an economically and statistically insignificant direct effect on his performance; while on the other hand, school average ESCS has a very important and significant effect on performance scores. These results reflect the role of schools in the Japanese education system. Schools assume multiple roles; they are the place for the acquisition of knowledge and for the socialization of children. Thus, peer effects are expected to be important (see Green 1997, 1999). This socialization role is very obvious in the results, especially when the effect of ESCS at home is absent. The presence of such a strong impact of social peer effects can be considered as a source of inequality in achievements. In a stratified school system, the access to a school with a high social and economic status can be the source of success. In contrast, a student living in a poor neighborhood is likely to have no choice but his local school, due to residence requirements. The status of this school usually reflects the status of the neighborhood, and this student is likely to be attending a low DESCES school and, hence, is likely to have low performance scores.

Italy: Italy has very similar results to Japan, even if they are quantitatively lower. It can also be described as a country with a high level of stratification, where the social status of a school determines the performance of students. A student's own ESCS has a very low effect on his performance, while the status of the school is much more important. The same interpretations made for Japan apply for Italy, except that the amount of increase in performance caused by a one unit increase in school average ESCS is about 70% lower. In other words, the extent of inequalities in Italy is lower.

The UK: the UK has a particular case, where both individual level ESCS and school average ESCS have high effects on performance. The UK is the closest to Germany, even if it is

considered to have a comprehensive school system and a moderate level of social stratification (see chapter 3). In fact, student's own ESCS has a stronger impact on performance in the UK than in Germany, while average school ESCS has a slightly lower impact. Another difference between the two is that the UK does not have a strong apprenticeship system.

The German system is designed in a way to produce inequalities in achievements that can be adapted to the needs of its apprenticeship system and later to the labor market. Even if low ESCS students are shepherded into vocational tracks, they are likely to be absorbed by the labor force without having to suffer from a socially negative image related to their lower level of education. The UK lacks this coherence. The existing social and economic stratification generates inequalities in performance without providing an adapted channeling mechanism.

Finland: Finland is the opposite of Japan and Italy. Individual level ESCS has an important impact on performance, while school average ESCS is almost absent. This is the reflection of the Finnish comprehensive school system, where schools have similar social intakes in terms of average ESCS. A student may have the choice between different schools; however, this choice is irrelevant, since all schools are almost identical. Finland can be described as the most equal country, since its education system is conceived in a way that does not generate inequalities. Even if student level ESCS still has a high impact on performance, this can be considered as normal. When school characteristics do not have a strong impact on performance, student characteristics are more likely to have a stronger effect. In other words, if the fixed amount of variations in performance is not explained by school characteristics, it will be explained by student level variables. Furthermore, the high level on the coefficient of student level ESCS is not alarming since the national dispersion of ESCS is limited.

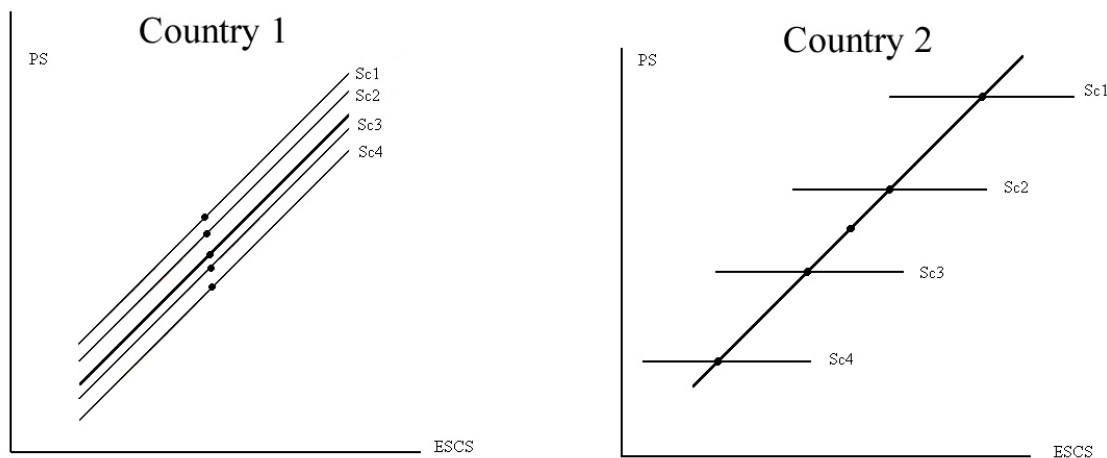
Remark:

It is possible to see from the results on ESCS and DESCES that the effect of school average ESCS is always higher than that of individual level ESCS (except for Finland). This is a strong indication that school status is much more important than household status in determining educational achievements. This finding provides another reason to control for peer effects in a regression analysis and has important policy implications.

Increasing households' ESCS may be outside the reach of a government at the short term;

however, reducing the effect of stratification through the management of school average ESCS (urban or zoning policies) might be more effective, since DESCS is one of the most important factors determining inequalities in performance.

Figure 4.5. Stratified and non-stratified countries.



Using the figure from the first part of this chapter, it is possible to say that Finland is close to country 1 while Italy and Japan are close to country 2. The UK and Germany are in-between. In country one, the projections of the dots on both axes are very close, indicating low levels of between-school dispersion on ESCS and performance scores (PS). All regression lines cover the whole range of ESCS and performance scores, indicating wide social and ability intakes. Finally, the slopes are steep, indicating that performance scores are increasing in ESCS. In country two, the projections of the dots are far apart, indicating wide disparities between schools in their social and ability intakes. The lines do not cover the entire range of ESCS or ability, indicating a high level of stratification and elitism. Finally, the slopes are almost equal to zero, indicating that ESCS does not have an effect on performance. In country 2, the level of students' ESCS determines their position on the horizontal axis and, hence, the school they are attending as well as their performance. In this type of country, inequalities are channeled through school choice and stratification.

The rest of the school variables are included as control variables since they affect performance without being a dimension on which stratification operates. The interpretation will be less elaborate than that of DESCSES.

### **2.2.2 The possession of computer facilities peer effects (DCOMPH).**

The peer effects resulting from the possession of computer facilities at home have a positive and significant impact on performance in Germany, Italy and Japan. In other words, when the proportion of students possessing a computer at home increases, school average performance, as well as individual performance, increases. This variable does not have a significant effect in the UK and Finland, indicating that the proportion of students possessing a computer at home is much the same across schools. However, this variable has the highest effect in Italy, followed by Germany and Japan. This means that schools differ on this aspect. Some schools are attended by students who possess a computer, while others are attended by students who do not. This can also be considered as a reflection of the social stratification present in these countries.

### **2.2.3 Interest in mathematics peer effects (DINTMAT).**

DINTMAT reflects the peer effects resulting from a generalized interest and enjoyment of mathematics. This variable has a significant effect across all countries. However, it does not behave as expected in Germany, the UK and Italy. Only in Japan and Finland does DINTMAT have a positive effect. In Finland, an increase of one point in DINTMAT leads to an increase of 8 points in average school performance, while in Japan the increase is of 65 points. In the rest of the countries, the findings seem to be counterintuitive, since the relation between DINTMAT and performance is negative.

### **2.2.4 Anxiety in mathematics peer effects (DANXMAT).**

DANXMAT reflects the peer effects resulting from a generalized feeling of helplessness and stress when dealing with mathematics. The coefficient on this variable is statistically significant in all countries, and the effect on average school performance is negative, as expected. However, the effect on individual performance is positive in Japan and close to zero in Finland. In Germany, an increase in DANXMAT of one unit leads to a decrease in average school performance of 35 points and to a decrease in individual performance of 16 points. In Finland, a one unit increase in DANXMAT causes a 29 point decrease in school average

performance and an increase in individual performance of 2 points. In Japan, a one unit increase in DANXMAT causes an increase in average school performance of 33 points and an increase in individual performance of 41 points. In Italy and the UK, the effect is negative on both school average performance and student performance. Even if the results for Finland and Japan seem to be counterintuitive, a particular interpretation can be made. When a higher proportion of students are suffering from anxiety in mathematics, this may lower the levels of stress for each one of them, since students will no longer feel alone or different. Hence, a generalized feeling of anxiety in mathematics (average ANXMAT in a school) may have a positive effect on individual performance, even if the impact on school average performance is still negative and high. At this stage, it should be said that the interpretation of such variables related to the psychology of learning is difficult, because we do not have an explicit theory on how they should behave.

### **2.2.5 Perception of discipline peer effects (DDISCL).**

DDISCL is the peer effects resulting from a generalized perception of discipline in a school. This variable has a statistically significant and positive effect across all countries, except in Finland, where it is insignificant. On the one hand, an increase of one unit in DDISCL leads to an increase of 27 points in average school performance in Germany and Japan, while the increase in Italy and the UK is of 21 points. On the other hand, an increase of one unit of DDISCL leads to an increase of 25 points in individual performance scores in Germany, and to an increase of 9, 18 and 24 points in the UK, Italy and Japan, respectively. Hence, in general, when a large proportion of students perceives higher levels of discipline, this is likely to have positive effects on their performance scores. Germany, Italy and Japan have the highest values on this coefficient. This indicates the presence of some disparities between schools. In other terms, some schools have discipline problems while others do not. In Finland, schools are very similar; they all have the same levels of average perceived discipline. Therefore, this variable has an insignificant effect.

### **2.2.6 The percentage of non-natives in a school (DETR).<sup>72</sup>**

DETR is one of the most important variables in explaining differences in performance; it reflects immigration peer effects in a school. We have previously seen that the effect of being

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<sup>72</sup> The results for Japan are not interpreted, because Japan has a very low percentage of non-natives distributed in a very limited number of schools.



a non-native on individual performance scores is negative and significant across all countries. However, when the school percentage of non-natives is considered, the perception of inequalities changes. DETR has a statistically significant effect across all countries. On the one hand, an increase of the percentage on non-natives of 1%, leads to a decrease of 21 points in average school performance in Germany and to a decrease of 41, 7 and 29 points in the UK, Finland and Italy, respectively. On the other hand, an increase of the percentage on non-natives in a school of 1% leads to an **increase** in individual performance of 6 points in Germany and 55 points in Finland. In the UK and Italy, an increase of the percentage on non-natives in a school of 1% leads to a **decrease** of 35 and 22 points in individual performance, respectively.

The UK and Italy: two negative effects add up. Being a non-native reduces a student's own performance scores and being enrolled with non-natives reduces them by more. This is the reflection of ethnic stratification; some schools have very high proportions of non-natives, while others have limited numbers of non-native students. Usually, schools with high proportions of non-natives are located in poor neighborhoods and have low average social intakes. Hence, high concentrations of non-natives can be associated with different kinds of disadvantages and with lower levels of performance scores.

Finland: being a non-native has an important negative effect on a student's performance. However, an increase in the percentage of non-natives in a school has a positive effect on individual performance. In order to understand this finding, two factors should be considered. First, the comprehensive Finnish education system is characterized by an equal distribution of non-natives across schools (about 2% in each). This equal distribution prevents the existence of ethnic concentrations, and thus reduces the social and instructional problems that may arise in such situations. Second, previous research has shown that when the percentage of non-natives in schools is low, a slight increase may have a positive effect, since it reduces the feeling of being an outsider and favors greater solidarity among non-native students.

Germany: Germany has a particular situation; the level of ethnic stratification is the highest among the five countries and is more than twice that in the UK. However, the percentage of non-natives in schools has a slight positive effect on individual performance. This can be explained through the following argument: the impact of being a non-native on individual

performance is negative and very high. Thus, it is possible to say that the effect of immigrant status may be absorbed by the student level ETR. Note that in a highly stratified country such as Germany, non-native students are more likely to be enrolled with other non-natives in particular schools, perhaps vocational ones. In other terms, DETR and ETR are correlated and the entire effect may be absorbed by ETR.

### **2.2.7 The proportion of computers connected to the web (Compweb).**

Compweb is a school indicator that accounts for the proportion of computers connected to the web. It controls for two aspects: the availability of computers and the availability of internet connections. This factor can be considered as a measure of school resources invested in information and communication technologies. This variable has the expected positive effect on student performance and is statistically significant across all countries, except in Japan where its effect is insignificant. In Germany, an increase of one percent in the proportion of computers connected to the web increases performance by 10 points, in the UK the increase is of 17 points, in Italy and in Finland the increase is of 11 and 5 points, respectively. As expected, the UK has the highest value. This is due to the disparities among schools on the availability of computers and internet. Students enrolled in schools where computers and internet are available are likely to have higher performances than their counterparts. This relation can be seen as a source of inequality in a stratified school system. We can imagine that students from higher social classes will be attending schools with higher funding, will have access to better computer facilities and will ultimately get higher results. The findings for Germany and Italy are slightly lower; however, the interpretation is similar. Italy is known for disparities between the North and the South and Germany is known for disparities between Landers and between school types (vocational and general). Finland has the lowest coefficient on this variable. This is another reflection of its comprehensive school system, where schools are almost identical. All schools have relatively the same proportion of computers connected to the web and hence the effect on performance is diluted. Finally, Japan has an insignificant coefficient on this variable. This is an indication that Japanese schools do not differ on this aspect; all schools are equally equipped with information and communication technologies.

### **2.2.8 The number of mathematic activities (Mactiv).**

Mactiv is a discontinuous variable that accounts for the number of activities undertaken by schools to promote engagement with mathematics. This variable has the expected positive impact on performance scores, except in the UK, where the effect is negative, and in Germany, where the effect is insignificant. In Finland and Japan, an increase of one activity leads to an increase of 1.5 points in performance scores. In Italy the increase is of 8.2 points. As it is possible to see, the effect of Mactiv on performance scores is small in comparison with other variables, except in Italy. In Italy, a student attending a school that offers activities such as mathematics clubs, competitions, and remedial classes will have higher performance scores.

### **2.2.9 Poor teacher student relations (Mstrel).**

Mstrel has the largest impact on performance scores among the selected variables. It accounts for factors negatively affecting the learning environment, such as disruption of classes, students lacking respect for teachers and teachers not meeting students' needs. Mstrel has the expected negative and significant impact on performance scores in all countries except for Germany, where the effect is insignificant. In Finland, an increase of Mstrel of one unit leads to a decrease of 124 points in performance scores. In the UK the decrease is of 76 points, in Italy and Japan the decrease is of 105 and 72 points, respectively.

A major characteristic of the Finnish schooling system is the organization of instruction according to the needs of students. A teacher is supposed to cater for small groups of students, while giving attention to their particular needs. This close proximity between students and teachers explains why poor student-teacher relations have a strong negative impact on the functioning of instruction in Finland. In the rest of the countries, the effect is similar, even if it is quantitatively lower.

### **2.2.10 Teacher shortages (Tcshort).**

The recruitment of qualified teachers is considered as a major factor in enhancing performance scores. In OECD countries, the aging teacher population and the rising student participation rates are of major concerns to educational policies. The index Tcshort was constructed using the parts of the school questionnaire that measure to what extent shortages or inadequacy of teacher supply is hindering instruction.

Tcshort has the expected negative and significant effect in all countries except in Italy. Germany has the highest coefficient: an increase in teacher shortages of one unit leads to a decrease in student performance of 8 points, while the decrease in Finland is of 0.3 points, in the UK and in Japan the decrease is of 4, and 2 points, respectively. As expected, Finland has the lowest value on this coefficient. This indicates that elements related to school resources have little effects on performance scores; this is perhaps due to the fact that most schools have equal funding. The results as well as the interpretation are similar for Japan. In the UK, the effect is higher; most British schools have low levels of funding in general, and thus they are more likely to have difficulties recruiting teachers. Furthermore, the disparities in funding between schools are higher in the UK than in Finland, even if the UK uses equalization policies in order to subsidize schools in poor areas. Finally, Germany has the highest value on this coefficient; schools are affected by insufficient teacher supply and there seem to be some disparities between them on this factor.

#### **2.2.11 Teacher morale (Tcmorale).**

The indicator on teacher morale was constructed using principal's responses to a number of questions in the school questionnaire. These questions depict elements related to teachers' morale, enthusiasm and pride. The responses were combined to create the school indicator Tcmorale. Positive values on Tcmorale indicate higher levels of school principals' perceptions of teacher-morale.

The effect of Tcmorale is positive and significant in Germany, Finland and Italy. However, the variable does not behave as expected in the UK and Japan. The effect can be considered as economically insignificant in Germany and Italy, even if it is statistically significant. An increase of teacher morale of one unit leads to an increase in performance scores of 0.02 points in Germany and 0.01 points in Italy. However, the effect is the highest in Finland, even if it is still limited; the increase in performance scores is of 1.8 points.

**Teacher behavior (Teacbeha).**

The indicator on teacher behavior was constructed using principals' responses to a number of questions depicting factors that may affect instruction in a school. These include low expectations of students, absenteeism among teachers, staff resistance to change and students' not being encouraged to achieve full potential. The responses were combined to create the indicator Teacbeha. Positive values indicate higher levels of school principals' perceptions of teacher-related factors hindering students' learning.

The effect of this variable is negative and significant in Germany, Finland and the UK. However, this variable does not behave as expected in Italy and Japan. The effect is the highest in Germany, where an increase in Teacbeha of one unit leads to a decrease in performance scores of 4.3 units. In Finland, the decrease is of 1.4 points while in the UK it is of 1.3 points.

**2.2.12 Private schooling (Private).**

As we have seen in chapter 3, schooling in most countries is mainly a public enterprise. However, some of the selected countries retain large private sectors. Japan has the largest private system, which accounts for 27% of total enrollment, while the UK, Germany, and Finland have private sectors that account for 7% of total enrollment.

In Japan, the private sector was conceived in order to complement public supply. In the UK, the private sector mainly serves the social elite, while in Germany, Italy and Finland private schools are used to maintain an alternative to public schools. Being enrolled in private schools has different effects on performance scores. On the one hand, the coefficient on this variable is significant across all countries, while on the other hand, the effect on performance scores is negative across all of them, except in the UK.

In Finland, students enrolled in private schools are expected to have 18 points less than their counterparts. This is attributable to the fact that Finnish private schools cater for students who fail to be integrated into the public system. In Italy, students enrolled in private schools are expected to have 54 points less than their counterparts. This is due to the fact that Italy has a long tradition of centralization and a predominance of public schooling. Private schools account for only 4% of total enrollment and they are mostly independent, religious and with

very limited public subsidies.<sup>73</sup> In Germany, students enrolled in private schools are expected to have 11 points less than those in public ones. Note that, in Germany, the sampled private schools include both general and vocational ones; thus private schools do not necessarily have a performance advantage in comparison with their public counterparts. In Japan, the impact of private schooling on performance is negative. Students enrolled in private schools are expected to have 39 points less than their counterparts. The only exception is the UK, where students enrolled in private schools are expected to have 14 points more than those enrolled in public ones. This is the result of social elitism in private education. Private schools in the UK combine high levels of funding, high peer effects, high quality physical and educational infrastructure and highly qualified teachers, in addition to being expensive. All these factors favor higher achievements.

### **2.2.13 Quality of educational infrastructure (Scmatedu).**

The quality of educational infrastructure can be considered as a proxy of the level of spending on educational resources. These include elements that are considered vital in guarantying high levels of performance, such as library and teaching materials, textbooks and multimedia resources. The PISA report (p. 250) noted that educational resources were not considered a major factor hindering instruction in most developed countries. However, in most developing partner countries, this factor is concerning.

The effect of Scmatedu is statistically significant across all countries, even if its impact on performance is limited. In Germany Italy and Finland, a one unit increase in Scmatedu has an almost equal-to-zero effect on performance. In the UK, the effect is positive and slightly higher. An increase in Scmatedu of one unit leads to an increase of 2.6 points in performance. Since all my selected countries are developed ones, schools are likely to have equal quality of educational infrastructure. Hence, Scmatedu is not a factor that might explain differences in performance.

### **2.2.14 Academic selection (Academic).**

In order to assess selectivity in schools, principals responded to a number of questions accounting for different admittance policies, such as academic records, recommendations

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<sup>73</sup> Public subsidization of private schools was not allowed in the constitution of 1948. But since 2000, very limited subsidies were allocated to private schools in order to diversify education supply.

from feeder schools, needs for specific programs or residence requirements. As noted in the PISA report (p. 228), academic records are the most reported admittance criteria. In my model, I only retained admittance according to academic records, since the effect of residential and social stratification can be captured through other variables. Usually, selection according to academic records has a positive impact on performance scores. This positive impact indicates that selective schools may perform better simply because they have higher ability intakes and not necessarily because they have better practices. When ACADEMIC was included in the model, it was possible to separate the effect of academic selection from that of other student and school variables.

The variable “Academic” is a binary variable, taking the value of one if a school considers academic records when selecting its students and zero otherwise. As expected, the effect on individual performance scores is positive and significant across all countries except in Italy, and it is relatively high in comparison with other school characteristics. In Germany, students enrolled in schools operating selection according to academic records get 7 extra points more than their counterparts. In Finland, students enrolled in selective schools get 11 extra points, while in the UK and Japan they get 4.5 and 17 extra points, respectively. The positive effect of academic selection on individual performance scores can be explained through the following argument: a student attending an academically selective school is likely to be a high performer and to be enrolled with high performers. Thus, he benefits from his own competencies and from those of his peers.

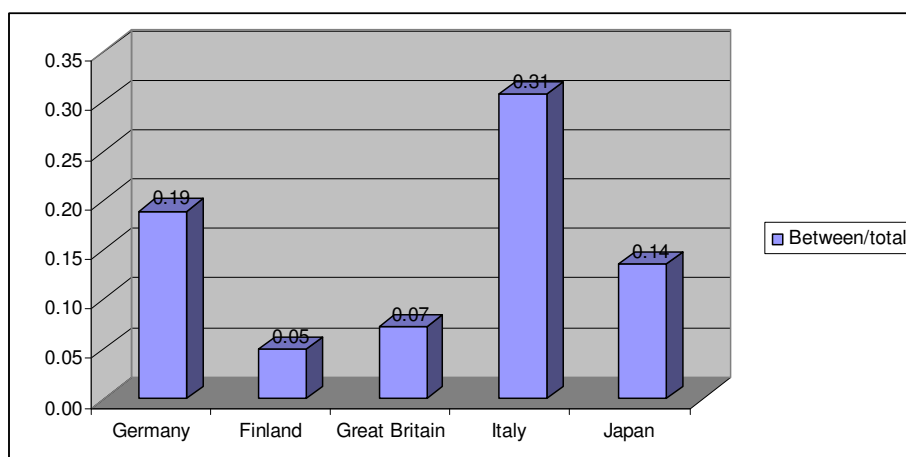
Japan has the highest value on this coefficient, followed by Finland, Germany and then the UK. If students are enrolled in schools with high achievement standards, they probably will have high levels of performance. However, in Finland, even if the effect is high, the number of schools applying academic selection is still limited.

### 2.3 Interpretation of the variance components.

Table 4.8. The variance components for model 3.

	Germany	Finland	UK	Italy	Japan
<b>Total variance</b>	3746.94	4183.22	4642.89906	4602.72	4292.26
<b>Within variance</b>	3038.34	3972.73	4305.25014	3188.13	3707.55
<b>Between variance</b>	708.601	210.485	337.648924	1414.59	584.707
<b>Between/Total</b>	0.189114	0.050317	0.072723727	0.307337	0.136223

Figure 4.6. The between over total variance.



Before interpreting the variance components obtained from the estimation of the model, it is of interest to recall what each component stands for. The between-school variance is the dispersion of school specific effects around zero, or, in other terms, the dispersion of the random parts of the intercept around zero. Note that these random parts  $V_j$  follow a normal distribution, with zero mean and a constant variance of  $\tau_0'^2$ ,  $V_j \sim N(0, \tau_0'^2)$ . The within-school variance is the dispersion of the model residuals around zero; note that these residuals  $\varepsilon_{ij}$  follow a normal distribution, with zero mean and a constant variance of  $\sigma^2$ ,  $\varepsilon_{ij} \sim N(0, \sigma^2)$ . The total variance is the sum of the two variances.<sup>74</sup>

<sup>74</sup> Note that these variance components are obtained from the estimation of the model, and they are conceptually and computationally different from those in chapter 3. The latter were obtained directly from the data without any estimation.



Italy has the highest ratio of between-school variance over total variance, followed by Germany, Japan, the UK and then Finland. The high level of between/total ratio in Italy, Germany and Japan indicates the existence of important between-school disparities, not only in the distribution of the variables but also in their effect on performance scores. In other words, each school tends to have a specific effect that diverges from the average effect (overall intercept). The disparities in Italy may be the reflection of important territorial differences between the North and the South; in Germany they may reflect differences between general and vocational schools, while in Japan they may reflect the hierarchical and stratified nature of the education system.

**Model 4.**

Model 4 is almost identical to model 3; only one school level variable was added. This variable is the within-school dispersion of ESCS (Varescs). Varescs can be considered as a measure of social diversity within a school. I have previously noted that an average in itself cannot measure the width of the distribution of a variable; therefore the dispersion should be included as a measure of diversity. A high level of Varescs indicates the presence of high social diversity, while the reverse is also true.

In the previous literature on peer effects, one of the major assumptions is the linearity in means (see chapter II). The linear in means models assume that a single student whose ESCS raises that of a school by one point has the same effect as several students whose combined ESCS raises that of a school by one point. As noted by Hoxby and Weingarth (2005), if peer effects were linear in means then neither economists nor policy makers would care much about them, because regardless of how peers are arranged, society would have the same average level of outcomes. Furthermore, any assessment of school choice and school desegregation policies should consider non linear in means specifications of peer effects in order to reflect reality.<sup>75</sup> The inclusion of Varescs in model 4 identifies the impact of social diversity on performance scores. This logic can be further extended to include other student characteristics. However, I have only included Varescs because ESCS is the most important variable according to which stratification operates.

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<sup>75</sup> The different models of Epple and Romano (1998, 2006) used a linear in means specification of peer effects.

As mentioned earlier, the objectives of including Varescs are:

1. Firstly, to provide an empirical proof for the assumptions made in chapter 2. If Varescs has a statistically and economically significant effect, then it is possible to say that social peer effects are not linear in means and that the width of the distribution of ESCS within a school has an important effect on achievements.
2. Secondly, to provide a full image on the functioning of peer effects under different education systems.

**Remarks:**

1. Since all independent variables included in model 3 are part of model 4, and since the two models only differ on one independent variable (Varescs), most of the results are identical. For instance, the adjusted R-squared and the variance components are the same for all countries. Furthermore, the coefficients are almost identical. Even if some coefficients have slightly changed, the sign and significance did not.
2. Since the results on all variables are identical to those in model 3, I will only interpret the coefficient on Varescs.

Interpretation of the results of model 4:

1. The first interesting finding from model 4 concerns the statistical significance of the Varescs coefficient. The coefficient is significant across all countries except in Japan. This significance indicates that peer effects are not linear in means and that the inclusion of ESCS and its average is not sufficient. Hence, the within-school distribution of ESCS has a significant effect on achievements and any empirical analysis should take into account a measure of the dispersion of social status. This empirical finding supports my theoretical assumptions that school quality and educational achievements are non-linear in social peer effects. The insignificance of Varescs in Japan is the result of homogeneity between schools on this factor. As we have seen in chapter III, Japanese schools differ on average ESCS; however, the within-school dispersion is almost the same. Hence, a variable that is quasi-constant cannot explain variations in performance scores.
2. The signs on the Varescs coefficient differ between countries. For Germany and the UK, the sign is positive, while it is negative for Finland and Italy. A positive sign indicates that higher levels of social diversity lead to higher levels of performance scores. The reverse is true when the sign is negative.

3. The values on the coefficients are also different between countries. An increase of one unit in the dispersion of ESCS leads to an increase of 16 points in performance scores in Germany and 22 points in the UK. Similarly, an increase of one unit in Varescs leads to a decrease of 4 points in performance scores in Finland and 62 points in Italy. In order to understand these findings, the global context of each country should be considered. As we have seen in chapter III, German and British schools have wide disparities in their social intakes. Some schools are socially diverse while others are elitist. Elitist schools in the UK are mostly private, while in Germany they are the general education ones (note that vocational schools are usually attended by lower social class students). The presence of important between-school disparities in social intakes is reflected through the statistical and economic significance of the coefficient on Varescs. The positive sign indicates that an increase of within-school social diversity has a favorable effect on performance scores. In other words, educational policy should be concerned with fighting social elitism. Conversely, the sign of the coefficient in Finland is negative, indicating that a further increase in social diversity leads to a decrease in performance scores. However, the impact is quantitatively weak (only a 4 point decrease for a one unit increase in Varescs). The negative sign can be explained through the fact that Finnish schools already have a wide social intake; hence, any further increase of social diversity has a slightly negative impact on achievements.

In Italy, the coefficient is significant as expected. This is the reflection of large between-school disparities in social intakes. However, the effect is unexpectedly negative and strong. In other terms, higher levels of social diversity are a source of lower performance scores. One interpretation of this finding is the Focus model of peer effects (Hoxby and Weingarth 2005, p.6). The Focus model suggests that a student has higher achievements whenever he is surrounded by peers with similar characteristics. Diversity is inherently disabling, because instruction cannot be organized according to student types. Moreover, if a school contains two distinct groups of students with low and high levels of ESCS, then it is possible to imagine that this school will suffer from social conflicts between two communities, the rich and the poor. However, in order to assess this hypothesis, one should look at the exact distribution of ESCS within Italian schools. Unfortunately, this lies beyond the scope of my thesis, even if it would be interesting to extend the analysis in this direction.

## 2.4. Comparison with the results from the PISA “learning for tomorrow’s world” report.

In this subsection, the results obtained from model 3 are compared to those presented in the PISA 2003 report “Learning for tomorrow’s world.” Note that the PISA report did not assess the impact of all variables on performance scores. For some of them, the analysis consisted of an interpretation of descriptive statistics without any estimation. Furthermore, the methodology used in the PISA report is different from the one employed in my analyses. Hence, the results tend to diverge on a number of issues.

Table 4.9. The PISA 2003 report results (relation between different variables and performance scores in mathematics).

Variables	Germany	Finland	UK	Italy	Japan
	Coefficient	Coefficient	Coefficient	Coefficient	Coefficient
<b>ESCS</b>	<b>17.00</b>	<b>33.00</b>	<b>31.00</b>	<b>9.00</b>	<b>4.00</b>
<b>INTMAT</b>	<b>10.20</b>	<b>30.50</b>	<b>13.60</b>	<b>10.30</b>	<b>27.60</b>
<b>DESCS</b>	<b>90.00</b>	<b>-2.00</b>	<b>58.00</b>	<b>78.00</b>	<b>145.00</b>
<b>DDISCL</b>	<b>18.60</b>	<b>10.40</b>	<b>24.70</b>	<b>12.50</b>	<b>32.70</b>
<b>Teacbeha</b>	-3.40	1.70	<b>20.30</b>	-6.10	<b>39.50</b>
<b>Scmatedu</b>	11.00	0.20	<b>13.00</b>	<b>14.40</b>	5.60
<b>Tcmorale</b>	7.40	<b>5.00</b>	<b>13.40</b>	1.50	<b>34.30</b>
<b>Private</b>	<b>-66.00</b>	5.00	<b>-87.00</b>	22.00	<b>31.00</b>
<b>Private after accounting for ESCS and DESCS</b>	<b>14.00</b>	<b>16.00</b>	1.00	<b>27.00</b>	<b>64.00</b>

Numbers in bold stand for significant regression coefficients.

In the PISA report, the analyses were carried out along several separate axes. Chapter 3 of the report assessed the distribution and impact on achievements of a number of student characteristics. These characteristics included students’ attitudes, students’ learning strategies and students’ beliefs about themselves. Chapter 4 assessed the impact of student and school ESCS on performance scores along with the impact of immigration background. Chapter 5 assessed the distribution and impact of some school variables. These included school climate, school policies and practices and school resources.

A major characteristic of the analyses undertaken in the PISA report is the use of a very limited number of variables in each regression. Most regressions were bivariate, and the resulting coefficients can be described as correlation coefficients. This technique certainly suffers from the omitted variable bias. When an important variable is omitted, and when this variable is correlated with one or more of the included independent variables, the model may suffer from heteroscedasticity and endogeneity problems. In other words, some of the included independent variables will be correlated with the error term of the model. These problems have been thoroughly assessed in the first section of this chapter, where the different assumptions on multilevel modeling were discussed. If heteroscedasticity or endogeneity problems exist, then the coefficients may be inconsistent and the resulting inference may be distorted. Moreover, we can reasonably expect that the regression coefficients are overestimated when certain variables are omitted. Intuitively, when the effect of important variables is not accounted for, this omission may artificially inflate the effect of included ones.

In comparative studies, another issue is worth discussing. Some researchers assume that if the estimation bias is identical across countries, then it is no longer a problem. However, in reality, there is no explicit empirical or theoretical evidence to support this assumption. Furthermore, even if the hierarchy of countries does not change with the bias, the values and significance of the regression coefficients may be distorted.

**Example:** consider a model containing two variables: student ESCS and school average ESCS (DESCS). These two variables tend to be correlated in countries like Germany and the UK, where students' social class determines the social status of their school. Omitting DESCES or relegating it to the error term generates a heteroscedasticity or endogeneity bias, since ESCS will be correlated with components absorbed by the error term. Conversely, in countries like Finland, the absence of stratification and the presence of a comprehensive school system mean that a student's ESCS does not determine the social status of his school. Hence, ESCS and DESCES are not correlated. If DESCES was omitted, then ESCS would not be correlated with the error term of the model and no bias would be generated. In conclusion, it is possible to say that the nature and extent of the bias depend on the context of each country. Hence, estimation bias cannot be identical for all countries. The results may be affected

differently and the values and significance of regression coefficients, as well as the hierarchy of countries, may be distorted.

A solution for bias problems is to control for all variables of interest, especially those that could be correlated with key independent variables included in the models. This has not been done in the PISA report; for instance, when students' ESCS was controlled for along with school average ESCS, other school characteristics that may be correlated with ESCS and DESCES were not included. In my model, I have controlled for a wide range of variables, including student and school characteristics as well as different forms of peer effects. This helped avoiding any bias resulting from omitted variables. It should also be noted that models 3 and 4 passed the Hausman test.

In bivariate regression analyses (used in the PISA report), the omission of all variables except one leads to an artificial inflation of its effect. In comparison to my findings, the PISA results are systematically overestimated. This can be seen especially through the coefficient of ESCS and DESCES, where the values are about 3 times higher than those obtained in model 3. In addition to this, it is possible to see that some coefficients seem to be inconsistent. DESCES has a negative effect on performance scores in Finland. Similarly, in Japan the coefficient on DESCES is very high: an increase of one unit in DESCES leads to an increase of 145 points in achievements. In other terms, an increase of one unit in average ESCS causes previously low performing students to become high achievers.

Similarly, the coefficients on INTMAT, DDISCL and Tcmorale are in general over estimated, even if the extent of the bias seems to vary between the selected countries. Note that, for some countries, the significance and signs of these coefficients are also different. Another interesting finding concerns the coefficient on Scmatedu; the results are overestimated for all countries except for Finland, where they seem to be identical to those from model 3. This is an indication that the absence of stratification (in the comprehensive Finnish system) reduces the level of correlations between Scmatedu and the variables relegated to the error term (e.g., student characteristics), and thus reduces the estimation bias. As a consequence, the coefficient on Scmatedu for Finland tends to be consistent and very close to the one obtained in model 3.

The results for private schooling are interesting, too. When “Private” was included separately, some of the coefficients were negative. However, when ESCS and DESCS were controlled for, all coefficients became positive even for Finland. In my model, the coefficient on “private” has a negative sign for all countries except the UK. These results are more intuitive and more realistic, especially for Finland, where private schools are attended by students who fail to be integrated in public ones, and the UK, where most private schools are elitist and expected to provide better results.

Moreover, when different forms of peer effects, school and student characteristics are controlled for, private schools do not have an advantage that stems from the fact that they are privately managed or funded. This finding may have very important consequences on educational policy. Private schools do not have higher achievements because they are privately funded and managed, but because they have better peer effects and schooling climate.

In terms of the methodological techniques, PISA used simple bivariate regressions to compute correlation coefficients between a number of key variables and performance scores. In my model, I constructed a full multilevel approach that accounted for a large array of student and school characteristics.<sup>76</sup>

As noted above, biased findings may seriously affect any conclusions or policy implications drawn from the results. This is particularly true in the case of educational policy. If the results are biased, then it would not be possible to identify the sources of inequalities in achievements and to ultimately design adequate policies.

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<sup>76</sup> The advantages of multilevel modeling have been thoroughly discussed earlier in this chapter and have been compared to simple linear regression techniques.

### **3. Interpretation of the results of models 5 and 6.**

In this section, I will compare the results of models 5 and 6 with those obtained from models 3 and 4. Note that model 5 is completely identical to model 4 in terms of the included independent variables. However, the estimation technique is different. In model 5, the estimation is carried out using maximum likelihood and restricted maximum likelihood techniques, as suggested by Raudenbusch and Bryk (2002). In contrast, model 4 used a transformation according to the Mundlak approach.

In what follows, I summarize the results from models 5 and 6:

1. A limited number of iterations were needed before the model converged. This indicates that the model is sound and that the data is reliable.
2. The results from the maximum likelihood (ML) and the restricted maximum likelihood (RML) techniques are identical in terms of the values on the regression coefficients, the significance levels and the values on the variance components. For the rest of this comparison, I will use the results on the RML estimations.
3. Concerning heteroscedasticity and non-normality problems, scatter plots show that residuals do not exhibit any particular pattern in their distribution and that they are randomly distributed around zero. Normal probability plots show that the dots form a relatively straight line except around the tails, where the dots are slightly curved. Hence, homoscedasticity and normality assumptions are respected in models 5 and 6. Note that I have used studentized residuals in these figures, since the use of raw residuals is not recommended (see section 2.1.1). Scatter plots and normal probability plots are included in the appendix.
4. Models 3, 4, 5 and 6 exhibited very close results on the variance components, regression coefficients and significance levels. Even if the estimation technique is different, the hierarchy of countries is preserved. This similarity in the findings can be attributed to the quasi-identical specification of the models. Furthermore, models 3 and 4 passed the Hausman test; this indicates that the random effects estimation is preferred to the fixed effects one. Hence, models 3 and 4 could have been estimated directly as random effects models without the Mundlak transformation. In conclusion, it is possible to say that the Mundlak transformation of the model and the restricted maximum likelihood techniques generate similar results that are robust and consistent.



5. The results from models 5 and 6 will not be interpreted thoroughly, since they are very similar to those obtained in models 3 and 4. However, since model 6 allowed for random coefficients on students' variables, it would be interesting to interpret the variances on these random coefficients. Recall that the coefficient on a student variable is  $\beta_{1j} = \beta + \mu_j$ , where  $\beta$  is the overall coefficient and  $\mu_j$  is the specific effect of school j. These variances represent the between-school dispersion of school specific effects ( $\mu_j$ ) around zero, with  $\mu_j \sim N(0, \tau_0^2)$ . The following table summarizes the dispersion of school specific effects ( $\mu_j$ ).<sup>77</sup>

Table 4.10. Between school variances on the random parameters.

	Germany	Finland	UK	Italy	Japan
<b>Variables</b>	<b>Between school variances on the random parameters.</b>				
<b>ESCS</b>	20.00	56.90	138.65	58.88	50.85
<b>COMPHOME</b>	16.12	22.42	116.20	40.11	27.26
<b>INTMAT</b>	44.17	73.59	158.55	63.45	0.79
<b>ANXMAT</b>	0.00	31.26	134.88	83.77	32.61
<b>DISCLIM</b>	34.06	42.91	93.03	65.81	55.35
<b>ETR</b>	226.14	1184.74	924.26	1452.95	7087.75

Starting with ESCS, it is possible to see that the UK has the highest between-school dispersion, followed by Italy, Finland, Japan and Germany. High dispersions indicate that schools differ on the strength of the impact of ESCS on performance scores. In other terms, some British schools have very low impact of ESCS on performance scores, while others have a very strong one. This result can be seen as a reflection of the unachieved comprehensivization of the education system. It should be noted that these dispersions cannot be considered as measures of inequality, since they only indicate that schools differ on their specific effects without mentioning how strong that effect is. More precisely, if most German schools have a very high impact of ESCS on performance scores, the dispersion of their

<sup>77</sup> The fixed effects and the between- and within-school variances will not be interpreted, since they are similar to those of model 3. Also, the random coefficients  $\mu_j$  will not be interpreted because of space constraints and because the specific effect for a particular school does not have an interesting or intuitive interpretation. Only the dispersions of  $\mu_j$  are interpreted. For detailed results, please refer to the appendix.

specific effects is likely to be small (which is the case here). In conclusion, this measure only indicates that schools differ on their specific effects.

The UK also has the highest dispersion of school specific effects on COMPHOME, INTAMAT, ANXMAT and DISCLIM. On COMPHOME, the UK is followed by Italy, Japan, Finland and Germany. On INTMAT, the UK is followed by Finland, Italy, Germany and Japan. On ANXMAT, the UK is followed by Italy, Japan, Finland and Germany. On DISCLIM, the UK is followed by Italy, Japan, Finland and Germany. And finally, on ETR, Japan is followed by Italy, Finland, the UK and Germany.

## C. Policy implications.

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So far, I have analyzed the determinants of inequalities in achievements in the context of different education systems. In this section of chapter IV, the implications of these findings on educational policy are assessed.

In most countries, achievements in mathematics are central to any educational policy. The performance of a country in mathematics may have important implications for future advances in technology and for its international competitiveness. Similarly, poor performances can have negative consequences on individual earning prospects and on global economic development. Policy makers have attached great importance to increased achievements and to the provision of mathematic skills to a large proportion of the student population.

From the results of chapters 3 and 4, it is possible to see that most countries suffer from wide disparities in performance scores and even from poor achievements, like in Italy and, to some extent, in Germany and the UK. My findings also showed that high levels of achievements can coexist with high levels of equality in performance (Finland). Furthermore, it has been clearly established that comprehensiveness is associated with higher levels of equality.

Social and ability stratification are also of concern in most countries. Students seem to be stratified into schools according to their social status; those with modest social origins are likely to be enrolled with students of the same type, and hence they would not benefit from better peer effects. This social segregation is usually associated with other serious problems, such as disciplinary problems, low peer effects, teacher shortages and low funding. Even if it is difficult to assess to what extent achievements in mathematics at the age of 15 would affect future success, most previous research shows that better educated individuals have better chances on the labor market. For instance, labor force participation rates increase steeply with the level of education (OECD 2004, *education at a glance*). Similarly, earning prospects, labor productivity, economic growth and technological progress are highly and positively correlated with education. Furthermore, when the social benefits of education are accounted

for, education becomes critical for the economic prosperity and well being of any society (McMahon 2000). In what follows, different educational policies are assessed along several axes. These include social stratification, school environment and student attitudes.

## **1. Social stratification.**

The results from model 3 showed that school social peer effects have a very strong impact on performance scores. This impact is even stronger than the one on individual social background (except for Finland). This finding has major implications for educational policy, since it challenges equal opportunities in education. As mentioned in the PISA report (2003), the results for most countries seem to be discouraging. Privileged students have two advantages over their counterparts; first, they benefit from the positive effect of their own social status and they have higher chances of attending schools with better peer qualities. This phenomenon often reinforces the existing patterns of privilege and hinders social mobility.

However, some countries were able to combine high levels of achievements and equality in performance. For instance, Finland has the highest level of performance scores and the lowest dispersion, combined with a very limited impact of social peer effects and school characteristics. In other words, schools have an equalizing effect; students from different origins and social classes can attend the same schools and have similar opportunities. From my comparative approach, it is possible to derive a number of policy implications that could improve equity in the distribution of educational attainments.

### **1.1. Improving educational attainments.**

A number of policies were used in different countries to improve performance scores. These include additional educational resources for particular schools based on their performances and social intakes. For instance, such policies were used in France, where additional funds were provided to schools enrolling students at risk of school failure (ZEP). In the UK, funds were provided for equalization purposes in favor of poor neighborhoods' schools. Another type of policies aims at providing special instruction for students through an organization of the curricula according to their needs, without grouping them according to ability.

As shown previously in chapter III, ability management policies have also been used. These policies can take severe forms, such as streaming and tracking in Germany, or can be more moderate such as setting policies in the UK. Other countries, such as Italy and France, adopted grade repetitions to manage ability heterogeneity. However, these selection policies are highly controversial, since they have important disadvantages. Do they allow an efficient management of ability and hence ameliorate achievements? Or do they label students and have the reverse effects?

## **1.2. Improving equity in the distribution of attainments between social groups.**

Another important challenge for educational policy is the improvement of equity in attainments between social groups. Some policies consisted of spending more on students presenting a specific characteristic, such as belonging to an ethnic minority or being a recent immigrant. The main difference between these policies and the aforementioned ones is that they are oriented towards social groups rather than ability ones, even if the two are usually related.

Another type of policies that could enhance performance scores for unprivileged social groups are related to the geographical organization of educational supply. As we have seen in chapter III, different school choice policies have been used across the OECD countries, ranging from free choice to a strict application of catchment areas. However, a middle solution consists of the use of zoning policies through which districts' boundaries are fixed in a manner that maximizes achievements and enhances their distribution. This type of policy is supported by my findings. The nonlinearity of peer effects in their means suggests that student allocation is not a zero sum game, and that achievements can be enhanced through a better distribution of peers. Hence, the reallocation of students can be the tide that lifts all boats.

In fact, social diversity has a positive effect on performance in countries where some schools tend to be socially elitist (Germany and the UK). Hence, allowing a wider social intake can provide a solution to low quality peers, to social stratification and to low achievements.

Compensation policies can also be used in order to provide additional resources to disadvantaged socio economic groups. The objective is to improve the economic conditions surrounding unprivileged students. Some of these policies consisted of providing free lunch programs to students from poor families, while others consisted of giving them lump sum payments or vouchers in order to enable a wider choice of schools or to provide sufficient resources for the purchase of educational materials.

The comprehensivization of school systems can also be considered as a method of increasing equity and even enhancing achievements. This comprehensivization may consist of improving instructional techniques, as well as introducing full day schooling and extending pre-schooling to cover wider populations. These policies are more concerned with the structure of the education system and can only be implemented through a long-term approach.

The choice of a particular policy depends on the context of each country. On the one hand, countries with high coefficients on ESCS may be inclined to adopt compensation policies to enhance the situation of unprivileged students; this is the case of the UK and Germany. On the other hand, countries with high coefficients on school social peer effects should favor urban policies that enhance the distribution of peers across schools; this is the case of Japan. Countries like the UK, Italy and Germany should do both, since inequalities in attainments originate from students' social backgrounds and schools' peer effects. In Italy, compensation policies should be used in order to achieve more territorial homogeneity between the north and the south.

As indicated in the PISA report, policies can be divided between those aimed at improving the general achievements of low performing schools and those aimed at improving the achievements of low performing students. For countries like Finland, schools have similar levels of achievements; hence the second type of policy is more suitable, while in the rest of the selected countries both types of policies might be used.

## **2. The school environment.**

School characteristics have a strong impact on performance scores. They can be a source of inequalities, especially when the school system is heavily stratified and when a student's social background determines the school he attends. Educational policies aimed at improving the learning environment at school cannot be fully separated from policies aimed at reducing social inequalities in attainments. In four of my selected countries, a significant level of stratification exists. Students from unprivileged families are likely to be enrolled in schools combining different forms of disadvantages, such as low levels of funding and low peer effects.

Some previously mentioned policies, such as compensation policies aimed at providing additional funds for schools operating in poor neighborhoods and attended by unprivileged students, can be applied in this context. Students from disadvantaged backgrounds bring with them discipline problems, lower perceptions of the value of learning and perhaps lower parental pressure towards the enforcement of efficient learning practices. Furthermore, schools attended by unprivileged students and with low levels of funding are less capable of effectively dealing with such problems. In general, schools should be enabled to compensate the negative effects of stratification through better instruction strategies and increased funding. Countries like Germany, Italy, Japan and, to some extent, the UK should adopt policies intended to reduce social stratification. However, since these countries have different schooling systems, different policies can be used. In Germany, unprivileged students are concentrated in professional schools due to the early selection system; thus, policies can be orientated according to the needs of these students. In the rest of the countries, school systems are supposed to be comprehensive. However, stratification exists because of urban (the UK) and territorial disparities (Italy). A solution to this problem would be the use of urban policies that reduce social stratification. Note that social diversity has a positive effect on performance in Germany and the UK.

However, some school characteristics are not directly related to the social origins of students. These may include poor student teacher relations, discipline problems and teacher-related factors hindering instruction. In this context, educational policies should be able to improve discipline as well as teaching practices.

A final concern would be private schooling and public subsidies to private schools. As my results have shown, after controlling for student and school characteristics, private schooling does not have a positive impact on performance scores except in the UK. Hence, the advantages that private schooling may offer are channeled through better peer effects or through funding and not through structural differences between the public and the private systems. Public subsidies to private schools have so far been used to maintain a choice outside the public system. However, such subsidies can be a subject for debate when private schools become the schooling institutions for the social elite (71% of British private schools are socially elitist). In this case, the question that can be asked is: why should private subsidies be maintained when the access to private schools is selective and does not favor equality of opportunities? Perhaps the most coherent policy would be the one applied in Greece, where private schools exist but are not subsidized. In other words, in a comprehensive school system a government should offer equal educational opportunities to all students and families. However, it should be noted that private schools should be maintained and probably subsidized when they provide education to students with particular needs that the public sector cannot satisfy.

### **3. Student attitudes.**

The results from my models show that a number of student attitudes and characteristics are highly correlated with performance scores and even generate very important and significant peer effects.

Interest in mathematics is perhaps the most important among them. Students with high levels of interest in mathematics are likely to invest more time and effort in learning. They are more able to control their learning strategies and to monitor the evolution of their knowledge and how it relates to real life applications. From a learning perspective, this implies the ability to set goals, control learning strategies and monitor progress. In order to achieve these objectives, students should be assisted by their teachers and schools.



In all of our selected countries, interest and anxiety in mathematics have a very strong impact on performance scores and generate very strong peer effects. Since interest and anxiety are largely dependent on previous success and failures, it is essential for any educational policy to be able to teach students how to overcome their failures and to assess the efficiency of their learning strategies. This can be done through different activities, undertaken by teachers and schools and aimed at fostering efficiency in learning. These activities may include professional assistance for students with inefficient learning techniques and general instruction in learning strategies combined with an effective assessment of students' needs.

The fact that the between-school variance on these student characteristics is by far lower than the within-school variance has policy implications. Each school should be able to manage the heterogeneity of the student population by catering for the needs of each student type. Moreover, previous research has shown that females have lower levels of interest and enjoyment of mathematics. This finding reveals gender inequalities in the effectiveness of learning. Hence, educational policy might also be interested in narrowing these differences and in reducing their effect on the long term.

## Conclusion of Chapter IV.

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To conclude this chapter, I will summarize the major estimation procedures and the main findings. In this chapter, six different models were estimated. Model one did not include school peer effects in the equation, model two did not include pure school characteristics, model three included both school level variables and model four introduced the within-school dispersion of ESCS into the education production function. The first model failed the Hausman test and was rejected. Model two passed the Hausman test but was not interpreted, since it neglected an important set of school variables. Models three and four had the best specification, and were interpreted. The first four models were estimated using a Mundlak approach.

On the other hand, models five and six were estimated using conventional maximum likelihood (ML) and restricted maximum likelihood (RML) methods. Model five was exactly identical to model four in terms of its specification, even though the estimation procedure was different. Model six introduced random slopes into the education production function and was estimated using the same variables as in model four. Note that all models exhibited similar results on the variance components, regression coefficients and significance levels.

The main finding is that the comprehensiveness of education systems is found to be a driving force for equality of educational opportunities. Countries like Finland were able to sustain high levels of performance and low levels of inequality, indicating that equality and high performance standards do not necessarily operate in opposing directions. Other countries like Italy have the reverse situation where low achievements are combined with high levels of inequality and stratification.

The detailed findings for each education system are the following:

Germany is characterized by a high level of stratification and inequalities in attainments. Most student and school level variables are significant and have a high effect on performance scores. ESCS is likely to affect students directly through their household environment and indirectly through school peer effects. Furthermore, ESCS determines the school a student is attending. Students with low levels of ESCS are likely to be enrolled with other students with

the same social background. When the effects of ESCS and average ESCS are combined, the impact of social background is substantial. In Germany, the selective school system is a generator of inequalities, since it allows ESCS to play fully through its direct household effect and through its indirect school effect.

Japan also has strong social stratification. However, inequalities operate differently than in Germany. On the one hand, a student's ESCS has an economically and statistically insignificant direct effect on his performance. On the other hand, school average ESCS has a very important and significant effect on performance scores. These results reflect the role that Japanese schools play. Schools are the place for the socialization of students and for the acquisition of knowledge (see Green 1997, 1999). This socialization role is very obvious, especially when the effect of household ESCS is absent. The impact of school average ESCS can be seen as a source of inequality, since the access to a school with high average ESCS in a hierarchically stratified Japanese education system ensures the access to better attainments.

Italy has very similar results to Japan, even if they are quantitatively lower. It can be described as a country with high levels of school-generated inequalities. Moreover, The high levels of stratification and the strong impact of average ESCS on performance scores can be seen as the result of large geographic disparities between schools in Northern Italy and those in the South.

The UK is the closest to Germany, even if it is considered to have a comprehensive school system and moderate levels of social stratification (see chapter III). In the UK, ESCS and its average have a strong and significant effect on performance scores. However, the major difference with Germany is that the UK does not have a strong apprenticeship program to channel these inequalities. In fact, even if the German system shepherds students into vocational tracks, they are likely to be absorbed by the labor force. Furthermore, German students attending vocational schools do not suffer from the negative image that can be associated with manual labor, due to long apprenticeship traditions. The UK lacks this coherence. The existing social and economic stratification generates inequalities in performance without providing an adapted channeling mechanism.

Finland is the opposite of Japan and Italy. Average ESCS as well as other school characteristics are found to have a weak effect on performance scores while being less significant. This finding reflects the comprehensiveness of the Finnish system, where schools are almost identical, even in terms of their social intakes. Nonetheless, household ESCS still has a high impact on performance. However, this can be considered as normal, because when school characteristics do not have a strong effect, student characteristics are more likely to have a strong impact. Furthermore, this finding should not be of major concern since the total dispersion of ESCS in the country is low.

## General Conclusion.

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On the one hand, the research developed within the remit of this thesis shed light on the mechanisms of stratification into schools and communities. On the other, the comparative analysis allowed for a better understanding of the functioning of these mechanisms under different education systems.

The two bodies of literature on educational and spatial stratification served as the foundation for the theoretical models developed in chapter II. The main objective of this chapter was to solve some of the limitations of the previous models. For instance, funding and peer effects were considered as simultaneous determinants of school quality and peer effects were introduced in a non-linear in means form. Private, public, and mixed finance schools coexisted and preferences over tax rates were non-single peaked. This non-single peakedness required the establishment of a number of conditions needed to ensure the existence of a majority voting equilibrium over tax rates. All Schools were allowed to have a strategic behavior on the education market and the resulting marginal cost of admitting a student was determined by the cost of producing education and by the position of a student among his peers. Furthermore, this model was constructed in a multicomunity framework where the resulting stratification was three-dimensional, and the boundary loci between communities were decreasing.

The major implication of the theoretical model is that students are stratified into communities and schools according to their endowments. In other words, student characteristics determine school characteristics and their achievements. This finding was reflected in the education production function (EPF) used in chapter III and IV. This EPF had two determinants: student and school characteristics. The latter had two categories of variables, pure school characteristics and peer effects (aggregates of student variables).

The EPF was estimated using six different models, model 1 (the one where peer effects were dropped) failed the Hausman test. This finding indicated the existence of endogeneity problems. In the following models, peer effects were controlled for and the models passed the Hausman test.

The most consistent were models 3, 4, 5, and 6. Model 4 confirmed that peer effects are not linear in means, since the dispersion of ESCS had a highly significant impact on performance scores across all countries (except in Japan). Moreover, the sign of the coefficient on Varescs indicated whether an increase in social diversity is beneficial or not. For Germany and the UK, a higher level of social and economic diversity is associated with higher levels of performance scores, while the reverse is true for Italy and Finland. All of the four models generated similar results even if model 3 and 4 were estimated with a Mundlak approach and model 5 and 6 were estimated with Maximum likelihood techniques.

**The findings provided a clear answer to the question raised in the introduction. Recall, the major question was: Does the comprehensiveness of education systems leads to more equality in attainments?**

**The results showed clearly that comprehensive schooling is a source of equality.** In fact, Finland can be described as the most equal country with the highest level of performance scores. The homogeneity of Finnish schools in terms of their characteristics and their social intakes is the main aspect of the comprehensiveness of the system. This homogeneity explains the weak impact of school variables on performance scores. The rest of the countries have lower levels of equality and, in general, they retain substantial levels of stratification.

Germany has high levels of inequalities in attainments due to its highly stratified system and to the heterogeneity of its schools. Most student and school variables have a high and significant impact on performance scores. In particular, student social backgrounds generate inequalities in performance through their direct household effects (ESCS) and their indirect school effects (school average ESCS).

The UK is somewhat similar to Germany even if the findings are more modest. In fact, ESCS has a stronger impact on performance scores in the UK than in Germany while school average ESCS has a slightly lower effect. Furthermore, the UK retains the highest percentage of students attending socially elitist private schools, and is the only country where private schools are associated with higher achievements. In addition to this, British schools seem to have the lowest level of funding among the five countries.

Italy is also a highly stratified country with high levels of between-school dispersions and low levels of within-school diversity. All school variables have a strong and highly significant impact on performance scores. However, the main difference with Germany and the UK resides in the fact that student level ESCS has a low impact on performance scores while school average ESCS has a strong effect. This finding indicates that inequalities in attainments are mainly channeled through schools.

Japan is very similar to Italy even though the level of inequalities generated by school variables seems to be higher. In fact, the impact of school average ESCS on performance scores is the highest among the five countries while the impact of student level ESCS is the lowest. This finding reflects the importance of Japanese schools in socializing students and in transmitting knowledge (Green & all 2006). In other words, school quality - especially the quality of its peers - is the most important determinant of achievements in Japan.

At this stage, a number of further developments can be thought of:

On the one hand, the multilevel analyses in chapter IV can be extended to include country level variables that describe education systems and educational policies. The inclusion of such variables allows for a direct and simple assessment of policies across a number of countries. For instance, the coefficient on an interaction term between ESCS and a dummy variable that accounts for the use of completely free school choice measures the supplement of effect that free school choice adds to the direct effect of ESCS. This supplement can be described as additional equality/inequality caused by a particular educational policy.

On the other hand, it is also interesting to undertake country specific analyses. For instance, in the case of Germany, it is possible to estimate different models for professional and general education schools in order to measure the full extent of inequalities in the German education system. Furthermore, it is also possible to estimate a model for each German Lander in order to assess the level of geographical disparities. This can also be done for the different regions in Italy.





## Appendix.

Results on the Hausman test for model 1, 2, 3, and 4.

	Germany	Finland	The UK	Italy	Japan
M1	309.07	5.68	125.66	26.60	142.98
M2	0.00	0.00	0.00	0.00	0.00
M3	0.00	0.00	0.00	0.00	0.00
M4	0.00	0.00	0.00	0.00	0.00

The number of observations per country and the adjusted R squared for the within part of the model. Step 1, for models 1, 2, 3, and 4.

	Germany	Finland	The UK	Italy	Japan
<b>n</b>	4114	5728	9045	11342	4652
<b>Adjusted R-squared</b>	0.6516807	0.369574	0.4467553	0.619669	0.5974499

The coefficients on student level variables for models 1, 2, 3, and 4.  $\hat{\beta}$ .

Variables	Germany		Finland	
	Coefficient	Std Error	Coefficient	Std Error
<b>ESCS</b>	11.6188704(***)	1.23597758	26.8562187(***)	1.23591392
<b>COMHOME</b>	1.28125269	1.21663952	-2.72385848(**)	1.15042696
<b>INTMAT</b>	4.69696341(***)	0.94749482	14.5124817(***)	1.04775242
<b>ANXMAT</b>	-19.0354435(***)	0.90418361	-31.9655508(***)	1.07991781
<b>DISCLIM</b>	2.6305623(***)	0.82418857	1.41710402	1.00131231
<b>ETR</b>	-27.9232916(***)	2.92336538	-63.4902422(***)	6.3707258

Variables	The UK		Italy	
	Coefficient	Std Error	Coefficient	Std Error
<b>ESCS</b>	23.2628003 (***)	0.98944402	3.22997241 (***)	0.7133729
<b>COMHOME</b>	8.20944513 (***)	0.98848629	8.89853572 (***)	0.67993139
<b>INTMAT</b>	-1.16124725	0.87482997	6.49337771 (***)	0.65907046
<b>ANXMAT</b>	-25.1125562 (***)	0.8714535	-21.2710943 (***)	0.69486398
<b>DISCLIM</b>	12.3322294 (***)	0.69943581	2.03617407 (***)	0.58092855
<b>ETR</b>	-5.18637109 (*)	2.89924318	-6.81673487 (*)	4.01729008

Variables	Japan	
	Coefficient	Std Error
<b>ESCS</b>	1.14574771	1.61050778
<b>COMHOME</b>	3.3407971 (***)	1.20791729
<b>INTMAT</b>	10.3844416 (***)	1.14305169
<b>ANXMAT</b>	-7.80941069 (***)	1.13521008
<b>DISCLIM</b>	2.68997083 (**)	1.14669476
<b>ETR</b>	-19.5223855	28.5765359

## Model 1

The variance components for model 1.

	Germany	Finland	The UK	Italy	Japan
<b>Total variance</b>	4923.616546	4182.801062	4911.386033	5910.183043	5745.311087
<b>Within variance</b>	3038.342744	3972.730126	4305.250136	3188.130844	3707.554956
<b>Between variance</b>	1885.273802	210.0709359	606.1358966	2722.052198	2037.756131
<b>Between/Total</b>	0.38290427	0.05022255	0.123414428	0.460569864	0.3546816

The number of observations per country and the adjusted R squared for the between part of the model. Step 2.

	Germany	Finland	The UK	Italy	Japan
<b>n</b>	4114	5728	9045	11342	4652
<b>Adjusted R-squared</b>	0.312569	0.277	0.2697714	0.1284224	0.4610654

The coefficients on school level variables  $\hat{\gamma}_2$ , obtained from step 2.

Variables	Germany		Finland	
	Coefficient	Std error	Coefficient	Std error
<b>Compweb</b>	38.950915 (***)	2.97920799	12.9985991 (***)	4.02494248
<b>Mactiv</b>	29.8028522 (***)	1.53315342	2.73870599 (*)	1.46097872
<b>Mstrel</b>	-137.523677 (***)	24.0063199	-131.336492 (***)	21.2772517
<b>Tcshort</b>	-17.8553618 (***)	1.18839538	1.05977583	1.05093214
<b>Tcmorale</b>	-0.04397213 (***)	0.00476103	5.46759263 (***)	0.95807556
<b>Teacbeha</b>	-12.6414342 (***)	1.40924441	-2.44660331 (***)	0.7800113
<b>Private</b>	34.0733287 (***)	2.10353813	-15.0013691 (***)	3.70374788
<b>Scmatedu</b>	-0.00305388	0.01078516	0.15726668	0.80671334
<b>Academic</b>	25.9062409 (***)	2.01559495	12.6421724 (***)	3.51108515
<b>Intercept</b>	459.180917	3.44340969	532.408647	5.16580539

Variables	The UK		Italy	
	Coefficient	Std error	Coefficient	Std error
<b>Compweb</b>	18.4613213 (***)	3.08706209	27.8214214 (***)	2.46448653
<b>Mactiv</b>	-2.35785254 (***)	0.58767704	24.1833255 (***)	0.88298474
<b>Mstrel</b>	-282.896666 (***)	26.2773722	257.708489 (***)	14.645248
<b>Tcshort</b>	-6.76077728 (***)	0.48925901	2.59832165 (***)	0.8084997
<b>Tcmorale</b>	0.51254716	0.71114741	0.03624807 (***)	0.00274966
<b>Teacbeha</b>	6.82757641 (***)	0.74583468	-1.83376761 (**)	0.72893673
<b>Private</b>	71.6764414 (***)	2.33140349	-1.90226972	3.54332431
<b>Scmatedu</b>	-0.96460478 (*)	0.5605039	0.07427666 (***)	0.00585492
<b>Academic</b>	11.3709952 (***)	1.8220182	-9.0892907 (***)	1.38154952
<b>Intercept</b>	505.68159	3.92624132	387.246899	2.81443825

Variables	Japan	
	Coefficient	Std error
<b>Compweb</b>	10.9265276 (***)	3.18928629
<b>Mactiv</b>	3.82346829 (***)	1.29385611
<b>Mstrel</b>	-480.193658 (***)	13.4419383
<b>Tcshort</b>	1.6499967 (*)	0.93987316
<b>Tcmorale</b>	18.3105829 (***)	0.96458735
<b>Teacbeha</b>	14.5204008 (***)	1.19944801
<b>Private</b>	-20.6334799 (***)	1.87839969
<b>Scmatedu</b>	0.77344645	1.06085788
<b>Academic</b>	4.31191361 (*)	2.6073631
<b>Intercept</b>	580.48937	3.53694947

Coefficient  $\hat{\delta}_2 = (1 - \lambda)\hat{\gamma}_2$ , obtained in step 4.

Variables	Germany		Finland	
	Coefficient	Std error	Coefficient	Std error
<b>Compweb</b>	10.35976 (***)	2.97920799	8.09257 (***)	4.02494248
<b>Mactiv</b>	7.92665 (***)	1.53315342	1.70504 (*)	1.46097872
<b>Mstrel</b>	-36.57711 (***)	24.0063199	-81.76649 (***)	21.2772517
<b>Tcshort</b>	-4.74898 (***)	1.18839538	.65979	1.05093214
<b>Tcmorale</b>	-.01170 (***)	0.00476103	3.40397 (***)	0.95807556
<b>Teacbeha</b>	-3.36224 (***)	1.40924441	-1.52319 (***)	0.7800113
<b>Private</b>	9.06247 (***)	2.10353813	-9.33944 (***)	3.70374788
<b>Scmatedu</b>	-0.00081	0.01078516	.09791	0.80671334
<b>Academic</b>	6.89027 (***)	2.01559495	7.87067 (***)	3.51108515
<b>Intercept</b>	122.12813	3.44340969	331.46299	5.16580539

Variables	The UK		Italy	
	Coefficient	Std error	Coefficient	Std error
<b>Compweb</b>	8.68996 (***)	3.08706209	5.44679 (***)	2.46448653
<b>Mactiv</b>	-1.10987 (***)	0.58767704	4.73453 (***)	0.88298474
<b>Mstrel</b>	-133.16281 (***)	26.2773722	50.45335 (***)	14.645248
<b>Tcshort</b>	-3.18238 (***)	0.48925901	0.50869 (***)	0.8084997
<b>Tcmorale</b>	0.24126	0.71114741	0.00710 (***)	0.00274966
<b>Teacbeha</b>	3.21382 (***)	0.74583468	-0.35901 (**)	0.72893673
<b>Private</b>	33.73895 (***)	2.33140349	-0.37242	3.54332431
<b>Scmatedu</b>	-0.45405 (*)	0.5605039	0.01454 (***)	0.00585492
<b>Academic</b>	5.35246 (***)	1.8220182	-1.77947 (***)	1.38154952
<b>Intercept</b>	238.03032	3.92624132	75.81397	2.81443825

Variables	Japan	
	Coefficient	Std error
<b>Compweb</b>	2.51307 (***)	3.18928629
<b>Mactiv</b>	0.87939 (***)	1.29385611
<b>Mstrel</b>	-110.44323 (***)	13.4419383
<b>Tcshort</b>	0.37949 (*)	0.93987316
<b>Tcmorale</b>	4.21138 (***)	0.96458735
<b>Teacbeha</b>	3.33965 (***)	1.19944801
<b>Private</b>	-4.74564 (***)	1.87839969
<b>Scmatedu</b>	.17789	1.06085788
<b>Academic</b>	.99173 (*)	2.6073631
<b>Intercept</b>	133.51097	3.53694947

## Model 2

The variance components for model 2.

	Germany	Finland	The UK	Italy	Japan
<b>Total variance</b>	3820.053974	4210.12222	4691.950992	4870.581978	4637.950195
<b>Within variance</b>	3038.342744	3972.730126	4305.250136	3188.130844	3707.554956
<b>Between variance</b>	781.7112297	237.3920941	386.7008554	1682.451134	930.3952393
<b>Between/total</b>	0.204633556	0.056386034	0.082417923	0.345431232	0.200604836

The number of observations per country and the adjusted R squared for the between part of the model. Step 2.

	Germany	Finland	The UK	Italy	Japan
<b>n</b>	4114	5728	9045	11342	4652
<b>Adjusted R-squared</b>	0.8135367	0.8206407	0.792117	0.7027571	0.8058363

The coefficients on school level variables ( $\gamma_i + \beta$ ), obtained from step 2.

Variables	Germany		Finland	
	Coefficient	Std error	Coefficient	Std error
<b>DESCS</b>	77.7724 (***)	1.6318546	29.2948194 (***)	1.11767116
<b>DCOMPH</b>	27.35653 (***)	2.7810226	1.23195625	1.41265764
<b>DINTMAT</b>	-19.00788 (***)	2.1914924	11.6773453 (***)	1.4704804
<b>DANXMAT</b>	33.171107 (***)	1.968835	-25.8268527 (***)	1.70006685
<b>DDISCL</b>	30.821549 (***)	1.3156098	4.5312162 (***)	0.79673628
<b>DETR</b>	-17.35668 (***)	3.4055396	-19.3748704 (***)	3.31172805
<b>Intercept</b>	476.89926	1.1309554	533.170741	0.83014285

Variables	The UK		Italy	
	Coefficient	Std error	Coefficient	Std error
<b>DESCS</b>	74.664531 (***)	0.8372547	29.5531918 (***)	1.32791232
<b>DCOMP</b>	-7.569605 (***)	1.3974899	65.3415178 (***)	2.08422025
<b>DINTMAT</b>	-10.04099 (***)	1.1225474	-13.0833227 (***)	1.57982979
<b>DANXMAT</b>	-38.94261 (***)	1.1915494	-73.4694759 (***)	3.04964385
<b>DDISCL</b>	23.718303 (***)	0.6925421	25.2208113 (***)	1.15455344
<b>DETR</b>	-40.32447 (***)	2.3978523	-32.0578935 (***)	4.39329751
<b>Intercept</b>	503.64035	0.6775069	504.076573	0.82173557

Variables	Japan	
	Coefficient	Std error
<b>DESCS</b>	66.775442 (***)	3.3344189
<b>DCOMP</b>	30.629678 (***)	3.3992993
<b>DINTMAT</b>	82.979334 (***)	2.5618507
<b>DANXMAT</b>	45.523617 (***)	2.6647589
<b>DDISCL</b>	44.593617 (***)	1.1903429
<b>DETR</b>	20.525389	82.936079
<b>Intercept</b>	550.90081	2.1910121

Coefficient  $\hat{\delta}_1 = (1 - \lambda)(\gamma_1 + \beta)$ , obtained in step 4.

Variables	Germany		Finland	
	Coefficient	Std error	Coefficient	Std error
<b>DESCS</b>	30.59535 (***)	1.63185	17.5564 (***)	1.11767
<b>DCOMP</b>	10.76195 (***)	2.78102	0.73831	1.41266
<b>DINTMAT</b>	-7.47762 (***)	2.19149	6.99822 (***)	1.47048
<b>DANXMAT</b>	-13.04938 (***)	1.96884	-15.478 (***)	1.70007
<b>DDISCL</b>	12.12507 (***)	1.31561	2.71555 (***)	0.79674
<b>DETR</b>	-6.82805 (***)	3.40554	-11.611 (***)	3.31173
<b>Intercept</b>	187.61024	1.13096	319.529	0.83014

Variables	The UK		Italy	
	Coefficient	Std error	Coefficient	Std error
<b>DESCS</b>	41.42256 (***)	0.83725	7.2648 (***)	1.32791
<b>DCOMP</b>	-4.19948 (***)	1.39749	16.0623 (***)	2.08422
<b>DINTMAT</b>	-5.57057 (***)	1.12255	-3.2162 (***)	1.57983
<b>DANXMAT</b>	-21.60467 (***)	1.19155	-18.06 (***)	3.04964
<b>DDISCL</b>	13.1585 (***)	0.69254	6.19981 (***)	1.15455
<b>DETR</b>	-22.37131 (***)	2.39785	-7.8805 (***)	4.3933
<b>Intercept</b>	279.41077	0.67751	123.913	0.82174

Variables	Japan	
	Coefficient	Std error
<b>DESCS</b>	22.03564 (***)	3.33442
<b>DCOMP</b>	10.10768 (***)	3.3993
<b>DINTMAT</b>	27.38287 (***)	2.56185
<b>DANXMAT</b>	15.02262 (***)	2.66476
<b>DDISCL</b>	14.71573 (***)	1.19034
<b>DETR</b>	6.7733	82.9361
<b>Intercept</b>	181.79519	2.19101

Coefficient  $\hat{\gamma}_1$ , obtained in step 5.

Variables	Germany	Finland	The UK	Italy	Japan
	Coefficient	Coefficient	Coefficient	Coefficient	Coefficient
<b>DESCS</b>	66.15353(***)	2.4386(***)	51.40173(***)	26.32322(***)	65.62969(***)
<b>DCOMP</b>	26.07528(***)	3.95581	-15.77905(***)	56.44298(***)	27.28888(***)
<b>DINTMAT</b>	-23.70484(***)	-2.83514(***)	-8.87975(***)	-19.5767(***)	72.59489(***)
<b>DANXMAT</b>	-14.13566(***)	6.1387(***)	-13.83006(***)	-52.19838(***)	53.33303(***)
<b>DDISCL</b>	28.19099(***)	3.11411(***)	11.38607(***)	23.18464(***)	41.90365(***)
<b>DETR</b>	10.56661(***)	44.11537(***)	-35.1381(***)	-25.24116(***)	40.04777

### Model 3

The variance components for model 3.

	Germany	Finland	The UK	Italy	Japan
<b>Total variance</b>	3746.94	4183.22	4642.89906	4602.72	4292.26
<b>Within variance</b>	3038.34	3972.73	4305.25014	3188.13	3707.55
<b>Between variance</b>	708.601	210.485	337.648924	1414.59	584.707
<b>Between/total</b>	0.189114	0.050317	0.072723727	0.307337	0.136223

The number of observations per country and the adjusted R squared for the between part of the model. Step 2.

	Germany	Finland	The UK	Italy	Japan
<b>n</b>	4114	5728	9045	11342	4652
<b>Adjusted R-squared</b>	0.82955	0.83793	0.8119439	0.74769	0.87164

The coefficients on school level variables ( $\hat{\gamma}_1 + \hat{\beta}$ ) and  $\hat{\gamma}_2$ , obtained from step 2.

Variables	Germany		Finland	
	Coefficient	Std error	Coefficient	Std error
<b>DESCS</b>	72.0031316 (***)	1.80950171	29.6779122 (***)	0.95291161
<b>DCOMPH</b>	29.8615439 (***)	2.9357794	-0.95538884	1.42592764
<b>DINTMAT</b>	-19.6216794 (***)	2.07691763	8.49251279 (***)	1.3696608
<b>DANXMAT</b>	-35.5798426 (***)	1.92511369	-29.0953704 (***)	1.58086946
<b>DDISCL</b>	27.6675967 (***)	1.47077184	1.31173003	0.82245084
<b>DETR</b>	-21.5412451 (***)	3.31000001	-7.90148118 (**)	3.13900317
<b>Compweb</b>	10.9687132 (***)	1.79215262	4.83067797 (**)	2.44045551
<b>Mactiv</b>	1.26354484	0.7970734	1.43754715 (***)	0.46728871
<b>Mstrel</b>	12.9503566	11.5664539	-124.763853 (***)	8.01667392
<b>Tcshort</b>	-7.62987123 (***)	0.53944754	-0.34510601	0.36924118
<b>Tcmorale</b>	0.02947631 (***)	0.00220373	1.78160365 (***)	0.31655084
<b>Teacbeha</b>	-4.29821192 (***)	0.78979582	-1.39534487 (***)	0.43129085
<b>Private</b>	-11.3645101 (***)	2.05053538	-18.4228242 (***)	1.46140304
<b>Scmatedu</b>	-0.06662555 (***)	0.00323867	0.24466686	0.36868574
<b>Academic</b>	7.29168301 (***)	0.98092486	11.1504286 (***)	1.01129261
<b>Intercept</b>	466.331487	2.14981428	528.735747	2.43630529

Variables	The UK		Italy	
	Coefficient	Std error	Coefficient	Std error
<b>DESCS</b>	66.0920075 (***)	1.08673405	31.5915923 (***)	1.30968108
<b>DCOMPH</b>	1.87066729	1.577258	66.9223187 (***)	1.91117502
<b>DINTMAT</b>	-9.43963671 (***)	1.15532378	-15.8149384 (***)	1.2759081
<b>DANXMAT</b>	-37.7274831 (***)	1.13514029	-72.2539899 (***)	2.82646668
<b>DDISCL</b>	21.7210883 (***)	0.71604657	20.6388854 (***)	1.10809339
<b>DETR</b>	-41.0252885 (***)	2.35621912	-29.5757664 (***)	4.19585689
<b>Compweb</b>	17.7015264 (***)	1.16506971	11.9002776 (***)	1.27633257
<b>Mactiv</b>	-2.97284142 (***)	0.24023833	8.25603192 (***)	0.44599331
<b>Mstrel</b>	-76.8474687 (***)	7.78187943	-105.02929 (***)	10.4742127
<b>Tcshort</b>	-3.78994589 (***)	0.30785461	4.60502306 (***)	0.52481038
<b>Tcmorale</b>	-2.14809054 (***)	0.34017134	0.00995978 (***)	0.00188041
<b>Teacbeha</b>	-1.28290405 (***)	0.33517605	2.21424581 (***)	0.4350672
<b>Private</b>	14.5332994 (***)	1.46133834	-54.6082399 (***)	2.63366558
<b>Scmatedu</b>	2.56334388 (***)	0.26529533	0.0178466 (***)	0.00250079
<b>Academic</b>	4.50833276 (***)	0.97135269	-9.06752336 (***)	0.81870377
<b>Intercept</b>	492.796549	1.37529252	494.561719	1.73240539

Variables	Japan	
	Coefficient	Std error
<b>DESCS</b>	98.5040841 (***)	2.42776504
<b>DCOMP</b>	18.1582199 (***)	2.32559737
<b>DINTMAT</b>	65.6165343 (***)	2.15458392
<b>DANXMAT</b>	33.4408818 (***)	2.07132094
<b>DDISCL</b>	27.4000697 (***)	1.15821103
<b>DETR</b>	-294.85472 (***)	75.4973397
<b>Compweb</b>	-1.66948555	1.38570822
<b>Mactiv</b>	1.74979551 (***)	0.6125078
<b>Mstrel</b>	-72.1965699 (***)	12.1306528
<b>Tcshort</b>	-1.60200959 (***)	0.48111738
<b>Tcmorale</b>	-2.92096803 (***)	0.51314569
<b>Teacbeha</b>	6.41521725 (***)	0.47399925
<b>Private</b>	-39.9666839 (***)	1.10437921
<b>Scmatedu</b>	-4.59491915 (***)	0.41610399
<b>Academic</b>	17.4910423 (***)	1.2442493
<b>Intercept</b>	552.99961	2.29976755

Coefficients  $\hat{\delta}_1 = (1 - \lambda)(\gamma_1 + \beta)$  and  $\hat{\delta}_2 = (1 - \lambda)\hat{\gamma}_2$ , obtained in step 4.

Variables	Germany		Finland	
	Coefficient	Std error	Coefficient	Std error
<b>DESCS</b>	29.5115 (***)	1.80950171	18.46556 (***)	0.95291161
<b>DCOMP</b>	12.23918 (***)	2.9357794	-0.59444	1.42592764
<b>DINTMAT</b>	-8.04222 (***)	2.07691763	5.28403 (***)	1.3696608
<b>DANXMAT</b>	-14.5829 (***)	1.92511369	-18.1031 (***)	1.58086946
<b>DDISCL</b>	11.33996 (***)	1.47077184	0.81616	0.82245084
<b>DETR</b>	-8.82898 (***)	3.31000001	-4.91629 (**)	3.13900317
<b>Compweb</b>	4.49568 (***)	1.79215262	3.00564 (**)	2.44045551
<b>Mactiv</b>	0.51788	0.7970734	0.89444 (***)	0.46728871
<b>Mstrel</b>	5.30789	11.5664539	-77.62792 (***)	8.01667392
<b>Tcshort</b>	-3.12721 (***)	0.53944754	-0.21472	0.36924118
<b>Tcmorale</b>	0.01208 (***)	0.00220373	1.10851 (***)	0.31655084
<b>Teacbeha</b>	-1.76168 (***)	0.78979582	-0.86818 (***)	0.43129085
<b>Private</b>	-4.65791 (***)	2.05053538	-11.46266 (***)	1.46140304
<b>Scmatedu</b>	-0.02731 (***)	0.00323867	0.15223	0.36868574
<b>Academic</b>	2.9886 (***)	0.98092486	6.93778 (***)	1.01129261
<b>Intercept</b>	191.13257	2.14981428	328.97873	2.43630529



Variables	The UK		Italy	
	Coefficient	Std error	Coefficient	Std error
<b>DESCS</b>	38.37657 (***)	1.08673405	8.41697 (***)	1.30968108
<b>DCOMPH</b>	1.08621	1.577258	17.83016 (***)	1.91117502
<b>DINTMAT</b>	-5.48116 (***)	1.15532378	-4.21358 (***)	1.2759081
<b>DANXMAT</b>	-21.9066 (***)	1.13514029	-19.25068 (***)	2.82646668
<b>DDISCL</b>	12.61243 (***)	0.71604657	5.49883 (***)	1.10809339
<b>DETR</b>	-23.82148 (***)	2.35621912	-7.87989 (***)	4.19585689
<b>Compweb</b>	10.27846 (***)	1.16506971	3.1706 (***)	1.27633257
<b>Mactiv</b>	-1.72619 (***)	0.24023833	2.19966 (***)	0.44599331
<b>Mstrel</b>	-44.62177 (***)	7.78187943	-27.98303 (***)	10.4742127
<b>Tcshort</b>	-2.20065 (***)	0.30785461	1.22692 (***)	0.52481038
<b>Tcmorale</b>	-1.2473 (***)	0.34017134	0.00265 (***)	0.00188041
<b>Teacbeha</b>	-0.74492 (***)	0.33517605	0.58994 (***)	0.4350672
<b>Private</b>	8.43881 (***)	1.46133834	-14.54931 (***)	2.63366558
<b>Scmatedu</b>	1.48842 (***)	0.26529533	0.00475 (***)	0.00250079
<b>Academic</b>	2.61778 (***)	0.97135269	-2.41587 (***)	0.81870377
<b>Intercept</b>	286.14413	1.37529252	131.76642	1.73240539

Variables	Japan	
	Coefficient	Std error
<b>DESCS</b>	39.73134 (***)	2.42776504
<b>DCOMPH</b>	7.32407 (***)	2.32559737
<b>DINTMAT</b>	26.46624 (***)	2.15458392
<b>DANXMAT</b>	13.48829 (***)	2.07132094
<b>DDISCL</b>	11.05174 (***)	1.15821103
<b>DETR</b>	-118.92882 (***)	75.4973397
<b>Compweb</b>	-0.67338	1.38570822
<b>Mactiv</b>	0.70578 (***)	0.6125078
<b>Mstrel</b>	-29.12028 (***)	12.1306528
<b>Tcshort</b>	-0.64617 (***)	0.48111738
<b>Tcmorale</b>	-1.17816 (***)	0.51314569
<b>Teacbeha</b>	2.58756 (***)	0.47399925
<b>Private</b>	-16.12045 (***)	1.10437921
<b>Scmatedu</b>	-1.85335 (***)	0.41610399
<b>Academic</b>	7.05496 (***)	1.2442493
<b>Intercept</b>	223.05083	2.29976755

Coefficient  $\hat{\gamma}_1$ , obtained in step 5.

Variables	Germany	Finland	Great Britain	Italy	Japan
	Coefficient	Coefficient	Coefficient	Coefficient	Coefficient
<b>DESCS</b>	60.38(***)	2.82(***)	42.82(***)	28.36(***)	97.35(***)
<b>DCOMPH</b>	28.58(***)	1.76	-6.33	58.02(***)	14.81(***)
<b>DINTMAT</b>	-24.31(***)	-6.01(***)	-8.27(***)	-22.30(***)	55.23(***)
<b>DANXMAT</b>	-16.54(***)	2.87(***)	-12.61(***)	-50.98(***)	41.25(***)
<b>DDISCL</b>	25.03(***)	-0.10	9.38(***)	18.60(***)	24.71(***)
<b>DETR</b>	6.38(***)	55.58(**)	-35.83(***)	-22.75(***)	-275.33(***)

## Model 4

The variance components for model 4.

	Germany	Finland	The UK	Italy	Japan
Total variance	3729.36	4183.69	4627.36	4414.73	4292.97
Within variance	3038.34	3972.73	4305.25	3188.13	3707.55
Between variance	691.02	210.96	322.11	1226.60	585.41
Between/total	0.1853	0.0504	0.0696	0.2778	0.1364

The number of observations per country and the adjusted R squared for the between part of the model. Step 2.

	Germany	Finland	The UK	Italy	Japan
<b>n</b>	4114.00	5728.00	9045.00	11342.00	4652.00
<b>Adjusted R-squared</b>	0.83	0.84	0.82	0.78	0.87

The coefficients on school level variables ( $\gamma_1 + \beta$ ) and  $\hat{\gamma}_2$ , obtained from step 2.

Variables	Germany		Finland	
	Coefficient	Std error	Coefficient	Std error
<b>DESCS</b>	72.78 (***)	1.80	29.68(***)	0.95
<b>DCOMPH</b>	26.23(***)	2.98	-1.12	1.43
<b>DINTMAT</b>	-19.28(***)	2.08	8.41(***)	1.37
<b>DANXMAT</b>	-35.39(***)	1.89	-29.58(***)	1.58
<b>DDISCL</b>	27.90(***)	1.47	0.99	0.82
<b>DETR</b>	-34.01(***)	3.38	-6.84(**)	3.15
<b>Compweb</b>	8.85(***)	1.78	5.20(**)	2.43
<b>Mactiv</b>	0.52	0.81	1.56(***)	0.47
<b>Mstrel</b>	6.06	11.45	-122.70(***)	7.99
<b>Tshort</b>	-7.54(***)	0.53	-0.42	0.37

<b>Tcmorale</b>	0.04(***)	0.00	1.76(***)	0.31
<b>Teacbeha</b>	-4.15(***)	0.76	-1.47(***)	0.43
<b>Private</b>	-12.56(***)	2.13	-18.37(***)	1.46
<b>Scmatedu</b>	-0.07(***)	0.00	0.08	0.37
<b>Academic</b>	7.61(***)	0.96	11.13(***)	1.02
<b>Varescs</b>	16.50(***)	1.60	-4.38(***)	1.53
<b>Intercept</b>	461.20	2.15	530.51	2.61

Variables	The UK		Italy	
	Coefficient	Std error	Coefficient	Std error
<b>DESCS</b>	64.90(***)	1.06	33.74(***)	1.24
<b>DCOMPH</b>	4.43(***)	1.53	64.88(***)	1.78
<b>DINTMAT</b>	-9.79(***)	1.16	-14.18(***)	1.26
<b>DANXMAT</b>	-35.11(***)	1.14	-79.20(***)	2.59
<b>DDISCL</b>	22.41(***)	0.70	20.49(***)	1.02
<b>DETR</b>	-45.36(***)	2.37	-24.21(***)	3.86
<b>Compweb</b>	18.63(***)	1.13	10.28(***)	1.19
<b>Mactiv</b>	-3.17(***)	0.23	8.16(***)	0.43
<b>Mstrel</b>	-92.12(***)	7.43	-84.57(***)	10.17
<b>Tcshort</b>	-2.91(***)	0.31	4.83(***)	0.47
<b>Tcmorale</b>	-1.61(***)	0.33	0.02(***)	0.00
<b>Teacbeha</b>	-2.22(***)	0.33	4.41(***)	0.39
<b>Private</b>	22.75(***)	1.55	-52.80(***)	2.41
<b>Scmatedu</b>	2.34(***)	0.25	0.02(***)	0.00
<b>Academic</b>	3.13(***)	0.99	-6.84(***)	0.77
<b>Varescs</b>	21.90(***)	1.09	-62.77(***)	1.90
<b>Intercept</b>	478.27	1.53	538.03	2.04

Variables	Japan	
	Coefficient	Std error
<b>DESCS</b>	98.68(***)	2.41
<b>DCOMPH</b>	18.00(***)	2.30
<b>DINTMAT</b>	65.67(***)	2.15
<b>DANXMAT</b>	33.49(***)	2.08
<b>DDISCL</b>	27.52(***)	1.16
<b>DETR</b>	-294.46(***)	75.87
<b>Compweb</b>	-1.65	1.38
<b>Mactiv</b>	1.79(***)	0.61
<b>Mstrel</b>	-72.58(***)	12.12
<b>Tcshort</b>	-1.57(***)	0.48

<b>Tcmorale</b>	-2.92(***)	0.51
<b>Teacbeha</b>	6.38(***)	0.47
<b>Private</b>	-40.15(***)	1.11
<b>Scmatedu</b>	-4.56(***)	0.42
<b>Academic</b>	17.50(***)	1.27
<b>Varescs</b>	4.67	3.95
<b>Intercept</b>	551.14	2.65

Coefficients  $\hat{\delta}_1 = (1 - \lambda)(\gamma_1 + \beta)$  and  $\hat{\delta}_2 = (1 - \lambda)\hat{\gamma}_2$ , obtained in step 4.

Variables	Germany		Finland	
	Coefficient	Std error	Coefficient	Std error
<b>DESCS</b>	30.14(***)	1.80	18.45(***)	0.95
<b>DCOMPH</b>	10.86(***)	2.98	-0.70	1.43
<b>DINTMAT</b>	-7.98(***)	2.08	5.23(***)	1.37
<b>DANXMAT</b>	-14.66(***)	1.89	-18.39(***)	1.58
<b>DDISCL</b>	11.55(***)	1.47	0.61	0.82
<b>DETR</b>	-14.08(***)	3.38	-4.26(**)	3.15
<b>Compweb</b>	3.66(***)	1.78	3.23(**)	2.43
<b>Mactiv</b>	0.22	0.81	0.97(***)	0.47
<b>Mstrel</b>	2.51	11.45	-76.29(***)	7.99
<b>Tcshort</b>	-3.12(***)	0.53	-0.26	0.37
<b>Tcmorale</b>	0.02(***)	0.00	1.09(***)	0.31
<b>Teacbeha</b>	-1.72(***)	0.76	-0.91(***)	0.43
<b>Private</b>	-5.20(***)	2.13	-11.42(***)	1.46
<b>Scmatedu</b>	-0.03(***)	0.00	0.05	0.37
<b>Academic</b>	3.15(***)	0.96	6.92(***)	1.02
<b>Varescs</b>	6.83(***)	1.60	-2.72(***)	1.53
<b>Intercept</b>	191.00	2.15	329.86	2.61

Variables	The UK		Italy	
	Coefficient	Std error	Coefficient	Std error
<b>DESCS</b>	38.27(***)	1.06	9.60(***)	1.24
<b>DCOMPH</b>	2.61(***)	1.53	18.46(***)	1.78
<b>DINTMAT</b>	-5.77(***)	1.16	-4.03(***)	1.26
<b>DANXMAT</b>	-20.70(***)	1.14	-22.53(***)	2.59
<b>DDISCL</b>	13.21(***)	0.70	5.83(***)	1.02
<b>DETR</b>	-26.75(***)	2.37	-6.89(***)	3.86
<b>Compweb</b>	10.98(***)	1.13	2.93(***)	1.19
<b>Mactiv</b>	-1.87(***)	0.23	2.32(***)	0.43
<b>Mstrel</b>	-54.32(***)	7.43	-24.06(***)	10.17

<b>Tcshort</b>	-1.71(***)	0.31	1.37(***)	0.47
<b>Tcmorale</b>	-0.95(***)	0.33	0.01(***)	0.00
<b>Teacbeha</b>	-1.31(***)	0.33	1.25(***)	0.39
<b>Private</b>	13.41(***)	1.55	-15.02(***)	2.41
<b>Scmatedu</b>	1.38(***)	0.25	0.01(***)	0.00
<b>Academic</b>	1.84(***)	0.99	-1.94(***)	0.77
<b>Varescs</b>	12.91(***)	1.09	-17.86(***)	1.90
<b>Intercept</b>	282.01	1.53	153.04	2.04

Variables	Japan	
	Coefficient	Std error
<b>DESCS</b>	39.78(***)	2.41
<b>DCOMPH</b>	7.26(***)	2.30
<b>DINTMAT</b>	26.47(***)	2.15
<b>DANXMAT</b>	13.50(***)	2.08
<b>DDISCL</b>	11.09(***)	1.16
<b>DETR</b>	-118.71(***)	75.87
<b>Compweb</b>	-0.66	1.38
<b>Mactiv</b>	0.72(***)	0.61
<b>Mstrel</b>	-29.26(***)	12.12
<b>Tcshort</b>	-0.63(***)	0.48
<b>Tcmorale</b>	-1.18(***)	0.51
<b>Teacbeha</b>	2.57(***)	0.47
<b>Private</b>	-16.19(***)	1.11
<b>Scmatedu</b>	-1.84(***)	0.42
<b>Academic</b>	7.05(***)	1.27
<b>Varescs</b>	1.88	3.95
<b>Intercept</b>	222.19	2.65

Coefficient  $\hat{\gamma}_1$ , obtained in step 5.

Variables	Germany	Finland	The UK	Italy	Japan
	Coefficient	Coefficient	Coefficient	Coefficient	Coefficient
<b>DESCS</b>	61.16(***)	2.82(***)	41.64(***)	30.51(***)	97.53(***)
<b>DCOMPH</b>	24.95(***)	1.60	-3.78(***)	55.98(***)	14.66(***)
<b>DINTMAT</b>	-23.98(***)	-6.10(***)	-8.62(***)	-20.67(***)	55.28(***)
<b>DANXMAT</b>	-16.36(***)	2.38(***)	-10.00(***)	-57.93(***)	41.30(***)
<b>DDISCL</b>	25.27(***)	-0.43	10.08(***)	18.45(***)	24.83(***)
<b>DETR</b>	-6.09(***)	56.65(**)	-40.18(***)	-17.39(***)	-274.93(***)

## Model 5

**With Maximum Likelihood estimation method.**

The variance components for model 5.

	Germany	Finland	The UK	Italy	Japan
<b>Total variance</b>	4033.62	4232.81	4777.71	4443.82	4446.92
<b>Within variance</b>	3049.228705	4019.061	4411.621	3193.726	3764.733
<b>Between variance</b>	984.3940824	213.7479	366.0881	1250.089	682.1831
<b>% Between/Total</b>	24.40	5.05	7.66	28.13	15.34

The coefficient on all independent variables.

Germany					Finland				
Variable	Estimate	StdErr	tValue	Probt	Variable	Estimate	StdErr	tValue	Probt
<b>Intercept</b>	452.66	9.90	45.72	0.00	<b>Intercept</b>	526.44	11.74	44.86	0.00
<b>ESCS</b>	10.33	1.16	8.87	0.00	<b>ESCS</b>	27.33	1.23	22.16	0.00
<b>COMPHOME</b>	2.03	1.16	1.75	0.08	<b>COMPHOME</b>	-2.39	1.15	-2.08	0.04
<b>INTMAT</b>	3.82	0.91	4.18	0.00	<b>INTMAT</b>	13.97	1.05	13.35	0.00
<b>ANXMAT</b>	-18.96	0.88	-21.57	0.00	<b>ANXMAT</b>	-32.01	1.08	-29.73	0.00
<b>DISCLIM</b>	2.32	0.80	2.91	0.00	<b>DISCLIM</b>	1.57	1.00	1.57	0.12
<b>ETR</b>	-25.10	2.86	-8.79	0.00	<b>ETR</b>	-63.43	6.41	-9.89	0.00
<b>DESCS</b>	67.45	8.54	7.90	0.00	<b>DESCS</b>	3.60	5.91	0.61	0.54
<b>VARESCS</b>	-0.57	8.58	-0.07	0.95	<b>VARESCS</b>	-4.15	9.08	-0.46	0.65
<b>DCOMPH</b>	23.94	12.69	1.89	0.06	<b>DCOMPH</b>	-2.01	8.80	-0.23	0.82
<b>DINTMAT</b>	-29.32	9.60	-3.05	0.00	<b>DINTMAT</b>	-10.02	8.08	-1.24	0.22
<b>DANXMAT</b>	-27.26	10.10	-2.70	0.01	<b>DANXMAT</b>	2.33	8.85	0.26	0.79
<b>DDISCL</b>	30.74	6.72	4.58	0.00	<b>DDISCL</b>	-2.00	4.88	-0.41	0.68
<b>DETR</b>	39.13	19.48	2.01	0.04	<b>DETR</b>	29.00	49.68	0.58	0.56
<b>Compweb</b>	8.59	7.71	1.11	0.27	<b>Compweb</b>	5.19	10.27	0.50	0.61
<b>Mactiv</b>	3.68	3.70	1.00	0.32	<b>Mactiv</b>	2.65	2.60	1.02	0.31
<b>Mstrel</b>	20.49	52.43	0.39	0.70	<b>Mstrel</b>	-119.59	54.60	-2.19	0.03
<b>Tcshort</b>	-7.19	3.25	-2.21	0.03	<b>Tcshort</b>	-0.81	2.34	-0.35	0.73
<b>Tcmorale</b>	3.76	3.12	1.21	0.23	<b>Tcmorale</b>	1.98	2.01	0.99	0.32
<b>Teacbeha</b>	-10.72	4.40	-2.44	0.01	<b>Teacbeha</b>	-2.03	2.25	-0.90	0.37
<b>Private</b>	-13.73	10.33	-1.33	0.18	<b>Private</b>	-19.65	6.56	-2.99	0.00
<b>Scmatedu</b>	7.85	2.97	2.64	0.01	<b>Scmatedu</b>	-0.15	2.22	-0.07	0.95
<b>Academic</b>	6.35	5.25	1.21	0.23	<b>Academic</b>	11.53	4.93	2.34	0.02

The UK					Italy				
Variable	Estimate	StdErr	tValue	Probt	Variable	Estimate	StdErr	tValue	Probt
<b>Intercept</b>	484.87	8.55	56.73	0.00	<b>Intercept</b>	520.80	9.78	53.27	0.00
<b>ESCS</b>	21.74	0.98	22.26	0.00	<b>ESCS</b>	3.45	0.71	4.87	0.00
<b>COMPHOME</b>	8.06	0.97	8.32	0.00	<b>COMPHOME</b>	8.80	0.68	12.98	0.00
<b>INTMAT</b>	-1.18	0.87	-1.37	0.17	<b>INTMAT</b>	6.69	0.65	10.25	0.00
<b>ANXMAT</b>	-23.57	0.86	-27.26	0.00	<b>ANXMAT</b>	-21.56	0.69	-31.23	0.00
<b>DISCLIM</b>	12.45	0.69	18.03	0.00	<b>DISCLIM</b>	2.49	0.58	4.29	0.00
<b>ETR</b>	-3.10	2.92	-1.06	0.29	<b>ETR</b>	-4.40	3.99	-1.10	0.27
<b>DESCS</b>	42.93	5.97	7.19	0.00	<b>DESCS</b>	35.59	6.11	5.83	0.00
<b>VARESCS</b>	16.45	6.80	2.42	0.02	<b>VARESCS</b>	-32.53	8.25	-3.94	0.00
<b>DCOMPH</b>	5.88	8.24	0.71	0.48	<b>DCOMPH</b>	48.48	8.51	5.70	0.00
<b>DINTMAT</b>	-17.00	6.70	-2.54	0.01	<b>DINTMAT</b>	-17.50	6.34	-2.76	0.01
<b>DANXMAT</b>	-13.65	7.05	-1.94	0.05	<b>DANXMAT</b>	-53.40	9.23	-5.78	0.00
<b>DDISCL</b>	13.54	4.39	3.09	0.00	<b>DDISCL</b>	18.41	5.21	3.53	0.00
<b>DETR</b>	-38.68	13.95	-2.77	0.01	<b>DETR</b>	-3.16	21.11	-0.15	0.88
<b>Compweb</b>	4.56	6.44	0.71	0.48	<b>Compweb</b>	10.79	5.81	1.86	0.06
<b>Mactiv</b>	-2.01	1.47	-1.37	0.17	<b>Mactiv</b>	7.47	2.40	3.11	0.00
<b>Mstrel</b>	-25.76	40.90	-0.63	0.53	<b>Mstrel</b>	-63.17	47.52	-1.33	0.18
<b>Tcshort</b>	-2.61	1.73	-1.51	0.13	<b>Tcshort</b>	3.56	2.57	1.38	0.17
<b>Tcmorale</b>	-1.86	1.74	-1.07	0.29	<b>Tcmorale</b>	-1.37	2.45	-0.56	0.58
<b>Teacbeha</b>	0.40	2.27	0.17	0.86	<b>Teacbeha</b>	1.95	2.38	0.82	0.41
<b>Private</b>	1.89	8.26	0.23	0.82	<b>Private</b>	-38.27	9.13	-4.19	0.00
<b>Scmatedu</b>	3.03	1.67	1.82	0.07	<b>Scmatedu</b>	6.15	2.22	2.77	0.01
<b>Academic</b>	14.05	4.07	3.45	0.00	<b>Academic</b>	-6.76	4.21	-1.60	0.11

Japan				
Variable	Estimate	StdErr	tValue	Probt
<b>Intercept</b>	605.63	29.50	20.53	0.00
<b>ESCS</b>	1.76	1.59	1.11	0.27
<b>COMPHOME</b>	3.01	1.21	2.49	0.01
<b>INTMAT</b>	10.48	1.14	9.21	0.00
<b>ANXMAT</b>	-7.86	1.13	-6.96	0.00
<b>DISCLIM</b>	2.58	1.14	2.26	0.02
<b>ETR</b>	-18.23	28.78	-0.63	0.53
<b>DESCS</b>	91.51	13.46	6.80	0.00
<b>VARESCS</b>	39.39	19.71	2.00	0.05
<b>DCOMPH</b>	16.39	13.84	1.18	0.24
<b>DINTMAT</b>	42.66	11.69	3.65	0.00
<b>DANXMAT</b>	34.87	12.27	2.84	0.00
<b>DDISCL</b>	32.07	6.54	4.90	0.00

<b>DETR</b>	-189.93	445.76	-0.43	0.67
<b>Compweb</b>	-2.21	8.47	-0.26	0.79
<b>Mactiv</b>	5.19	4.59	1.13	0.26
<b>Mstrel</b>	-107.32	47.86	-2.24	0.02
<b>Tcshort</b>	-0.61	2.87	-0.21	0.83
<b>Tcmorale</b>	-2.11	3.08	-0.68	0.49
<b>Teacbeha</b>	7.38	3.69	2.00	0.05
<b>Private</b>	-38.01	6.37	-5.97	0.00
<b>Scmatedu</b>	-5.05	3.02	-1.67	0.10
<b>Academic</b>	-53.59	23.09	-2.32	0.02

**With Restricted Maximum Likelihood estimation method.**

The variance components for model 5.

	<b>Germany</b>	<b>Finland</b>	<b>The UK</b>	<b>Italy</b>	<b>Japan</b>
<b>Total variance</b>	4140.01	4274.00	4810.79	4510.46	4559.01
<b>Within variance</b>	3053.81597	4023.236	4415.746	3195.81	3769.705
<b>Between variance</b>	1086.19651	250.7646	395.0445	1314.65	789.3034
<b>% Between/Total</b>	26.24	5.87	8.21	29.15	17.31

The coefficient on all independent variables.

<b>Germany</b>					<b>Finland</b>				
<b>Variable</b>	<b>Estimate</b>	<b>StdErr</b>	<b>tValue</b>	<b>Probt</b>	<b>Variable</b>	<b>Estimate</b>	<b>StdErr</b>	<b>tValue</b>	<b>Probt</b>
<b>Intercept</b>	452.93	10.28	44.04	0.00	<b>Intercept</b>	526.35	12.26	42.92	0.00
<b>ESCS</b>	10.33	1.16	8.86	0.00	<b>ESCS</b>	27.33	1.23	22.15	0.00
<b>COMPHOME</b>	2.03	1.16	1.75	0.08	<b>COMPHOME</b>	-2.39	1.15	-2.08	0.04
<b>INTMAT</b>	3.82	0.92	4.17	0.00	<b>INTMAT</b>	13.97	1.05	13.35	0.00
<b>ANXMAT</b>	-18.96	0.88	-21.56	0.00	<b>ANXMAT</b>	-32.01	1.08	-29.72	0.00
<b>DISCLIM</b>	2.32	0.80	2.90	0.00	<b>DISCLIM</b>	1.57	1.00	1.57	0.12
<b>ETR</b>	-25.10	2.86	-8.78	0.00	<b>ETR</b>	-63.43	6.42	-9.89	0.00
<b>DESCS</b>	67.60	8.90	7.59	0.00	<b>DESCS</b>	3.68	6.19	0.59	0.55
<b>VARESCS</b>	-0.53	8.96	-0.06	0.95	<b>VARESCS</b>	-4.15	9.56	-0.43	0.66
<b>DCOMP</b>	23.77	13.24	1.80	0.07	<b>DCOMP</b>	-2.19	9.25	-0.24	0.81
<b>DINTMAT</b>	-29.37	10.02	-2.93	0.00	<b>DINTMAT</b>	-10.15	8.47	-1.20	0.23
<b>DANXMAT</b>	-27.24	10.54	-2.58	0.01	<b>DANXMAT</b>	2.64	9.31	0.28	0.78
<b>DDISCL</b>	30.80	7.00	4.40	0.00	<b>DDISCL</b>	-2.10	5.12	-0.41	0.68
<b>DETR</b>	39.29	20.31	1.93	0.05	<b>DETR</b>	28.90	52.24	0.55	0.58
<b>Compweb</b>	8.32	7.99	1.04	0.30	<b>Compweb</b>	5.07	10.70	0.47	0.64
<b>Mactiv</b>	3.57	3.85	0.93	0.35	<b>Mactiv</b>	2.75	2.73	1.01	0.31
<b>Mstrel</b>	19.80	54.01	0.37	0.71	<b>Mstrel</b>	-118.85	57.52	-2.07	0.04
<b>Tcshort</b>	-7.20	3.40	-2.12	0.03	<b>Tcshort</b>	-0.85	2.46	-0.34	0.73
<b>Tcmorale</b>	3.75	3.25	1.15	0.25	<b>Tcmorale</b>	1.99	2.11	0.94	0.35



<b>Teacbeha</b>	-10.73	4.59	-2.34	0.02	<b>Teacbeha</b>	-2.05	2.37	-0.87	0.39
<b>Private</b>	-13.85	10.77	-1.29	0.20	<b>Private</b>	-19.85	6.90	-2.88	0.00
<b>Scmatedu</b>	7.86	3.10	2.53	0.01	<b>Scmatedu</b>	-0.14	2.33	-0.06	0.95
<b>Academic</b>	6.37	5.48	1.16	0.25	<b>Academic</b>	11.47	5.18	2.21	0.03

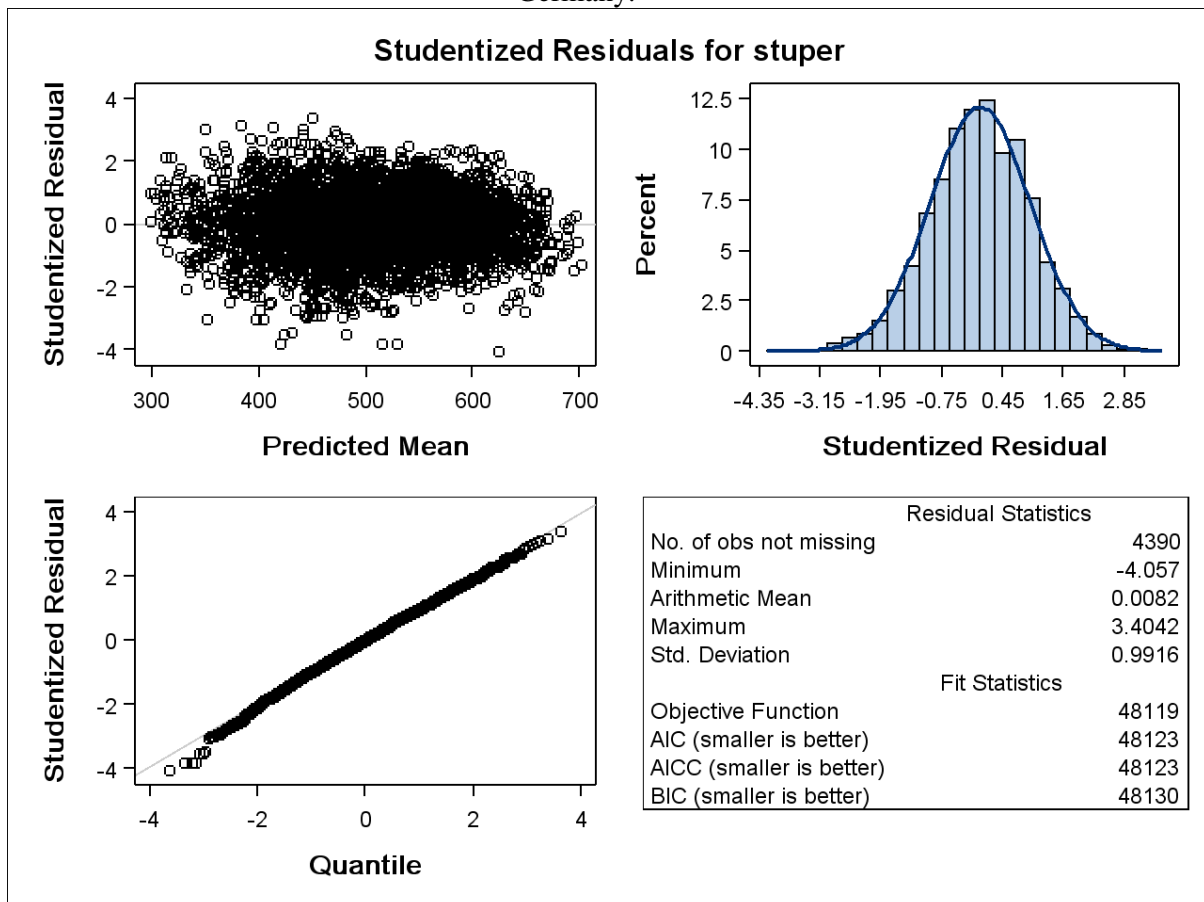
The UK					Italy				
Variable	Estimate	StdErr	tValue	Probt	Variable	Estimate	StdErr	tValue	Probt
<b>Intercept</b>	484.86	8.73	55.52	0.00	<b>Intercept</b>	520.62	9.97	52.24	0.00
<b>ESCS</b>	21.74	0.98	22.25	0.00	<b>ESCS</b>	3.45	0.71	4.87	0.00
<b>COMPHOME</b>	8.06	0.97	8.32	0.00	<b>COMPHOME</b>	8.80	0.68	12.97	0.00
<b>INTMAT</b>	-1.19	0.87	-1.37	0.17	<b>INTMAT</b>	6.69	0.65	10.25	0.00
<b>ANXMAT</b>	-23.57	0.86	-27.25	0.00	<b>ANXMAT</b>	-21.56	0.69	-31.22	0.00
<b>DISCLIM</b>	12.45	0.69	18.02	0.00	<b>DISCLIM</b>	2.49	0.58	4.29	0.00
<b>ETR</b>	-3.10	2.93	-1.06	0.29	<b>ETR</b>	-4.40	4.00	-1.10	0.27
<b>DESCS</b>	42.93	6.12	7.02	0.00	<b>DESCS</b>	35.62	6.24	5.71	0.00
<b>VARESCS</b>	16.46	6.97	2.36	0.02	<b>VARESCS</b>	-32.25	8.42	-3.83	0.00
<b>DCOMPH</b>	6.00	8.44	0.71	0.48	<b>DCOMPH</b>	48.36	8.69	5.57	0.00
<b>DINTMAT</b>	-17.19	6.86	-2.51	0.01	<b>DINTMAT</b>	-17.47	6.47	-2.70	0.01
<b>DANXMAT</b>	-13.71	7.22	-1.90	0.06	<b>DANXMAT</b>	-53.41	9.40	-5.68	0.00
<b>DDISCL</b>	13.62	4.49	3.03	0.00	<b>DDISCL</b>	18.38	5.32	3.46	0.00
<b>DETR</b>	-38.43	14.31	-2.69	0.01	<b>DETR</b>	-3.38	21.45	-0.16	0.87
<b>Compweb</b>	4.27	6.56	0.65	0.52	<b>Compweb</b>	10.88	5.91	1.84	0.07
<b>Mactiv</b>	-1.95	1.50	-1.29	0.20	<b>Mactiv</b>	7.44	2.45	3.04	0.00
<b>Mstrel</b>	-22.63	41.71	-0.54	0.59	<b>Mstrel</b>	-63.04	48.55	-1.30	0.19
<b>Tcshort</b>	-2.60	1.77	-1.47	0.14	<b>Tcshort</b>	3.53	2.63	1.34	0.18
<b>Tcmorale</b>	-1.83	1.79	-1.03	0.31	<b>Tcmorale</b>	-1.37	2.51	-0.55	0.58
<b>Teacbeha</b>	0.47	2.33	0.20	0.84	<b>Teacbeha</b>	1.93	2.43	0.79	0.43
<b>Private</b>	1.35	8.48	0.16	0.87	<b>Private</b>	-38.15	9.32	-4.09	0.00
<b>Scmatedu</b>	3.02	1.70	1.77	0.08	<b>Scmatedu</b>	6.16	2.26	2.72	0.01
<b>Academic</b>	14.35	4.17	3.44	0.00	<b>Academic</b>	-6.77	4.30	-1.57	0.12

Japan				
Variable	Estimate	StdErr	tValue	Probt
<b>Intercept</b>	605.75	31.40	19.29	0.00
<b>ESCS</b>	1.75	1.59	1.10	0.27
<b>COMPHOME</b>	3.01	1.21	2.49	0.01
<b>INTMAT</b>	10.48	1.14	9.20	0.00
<b>ANXMAT</b>	-7.86	1.13	-6.95	0.00
<b>DISCLIM</b>	2.58	1.14	2.26	0.02
<b>ETR</b>	-18.23	28.80	-0.63	0.53
<b>DESCS</b>	91.79	14.30	6.42	0.00
<b>VARESCS</b>	39.39	20.99	1.88	0.06

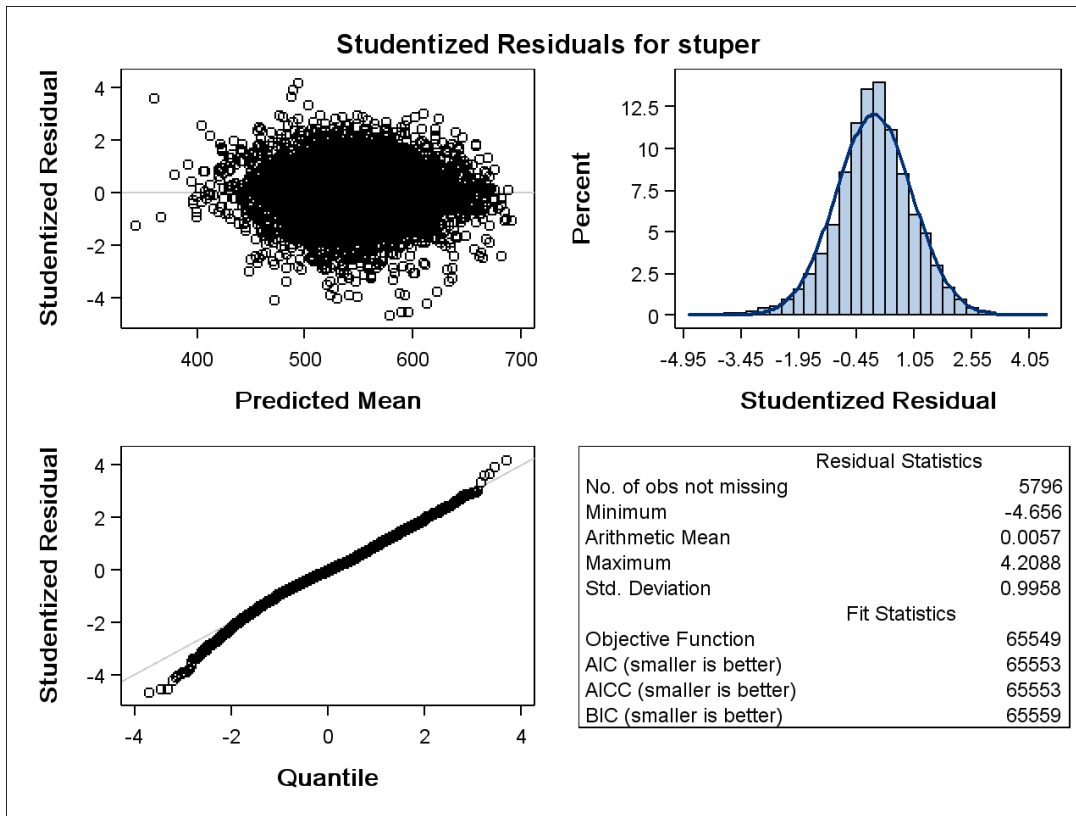
<b>DCOMPH</b>	16.17	14.73	1.10	0.27
<b>DINTMAT</b>	42.13	12.42	3.39	0.00
<b>DANXMAT</b>	34.48	13.05	2.64	0.01
<b>DDISCL</b>	31.95	6.94	4.61	0.00
<b>DETR</b>	-190.33	474.49	-0.40	0.69
<b>Compweb</b>	-2.11	8.97	-0.23	0.81
<b>Mactiv</b>	5.54	4.86	1.14	0.25
<b>Mstrel</b>	-109.32	50.64	-2.16	0.03
<b>Tcshort</b>	-0.59	3.05	-0.19	0.85
<b>Tcmorale</b>	-2.07	3.28	-0.63	0.53
<b>Teacbeha</b>	7.33	3.93	1.86	0.06
<b>Private</b>	-38.07	6.78	-5.62	0.00
<b>Scmatedu</b>	-5.01	3.22	-1.56	0.12
<b>Academic</b>	-53.78	24.59	-2.19	0.03

Residuals' scatter plots and normal probability plots for model 5. (Stuper is the dependent variable: student performance).

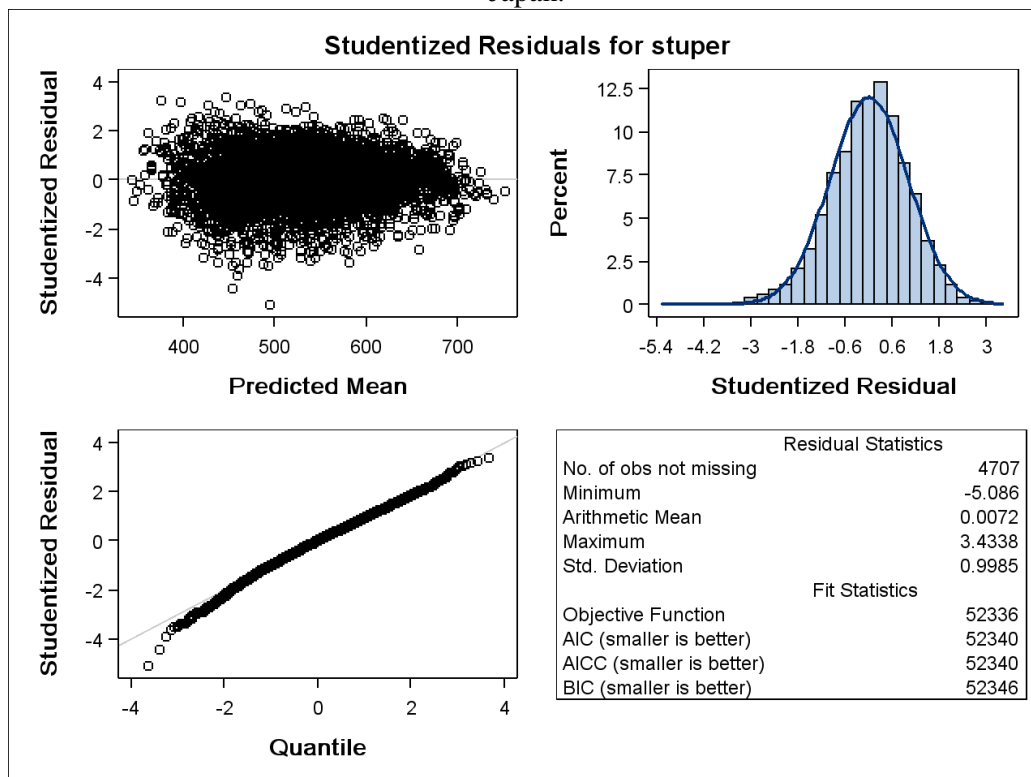
Germany.



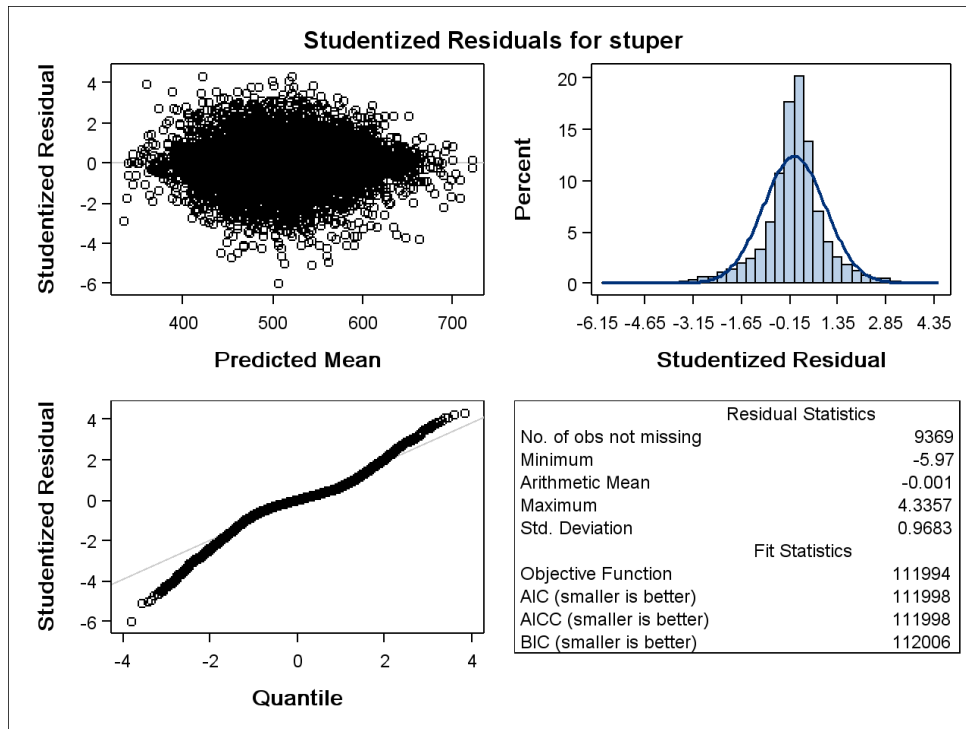
Finland.



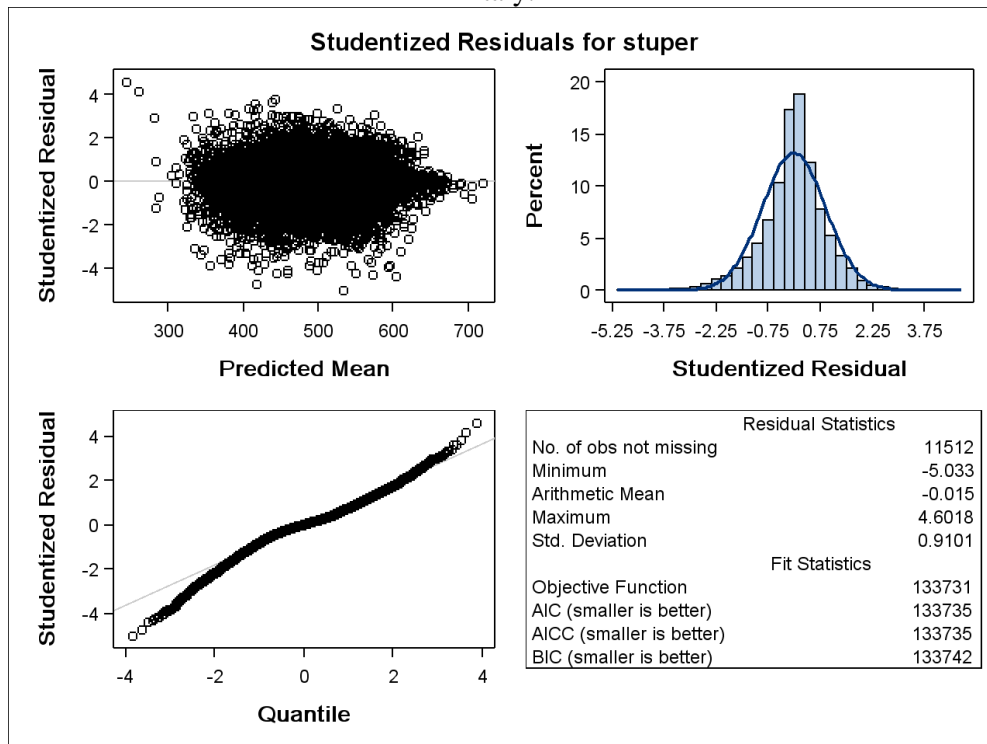
Japan.



The UK.



Italy.



## Model 6

With Maximum Likelihood estimation method.

The variance components for model 6.

	Germany	Finland	The UK	Italy	Japan
<b>Total variance</b>	3905.81	4038.67	4250.75	4080.72	4321.13
<b>Within variance</b>	2911.164	3834.167	3820.766	2936.006	3650.406
<b>Between variance</b>	994.6468	204.5045	429.9868	1144.718	670.7192
<b>% Between/Total</b>	25.47	5.06	10.12	28.05	15.52

	Germany	Finland	The UK	Italy	Japan
<b>Variable</b>	<b>Between school variance</b>				
<b>ESCS</b>	18.60	54.55	135.75	58.81	47.74
<b>COMPHOME</b>	14.52	21.59	113.34	39.71	27.14
<b>INTMAT</b>	43.15	71.62	157.28	62.78	0.00
<b>ANXMAT</b>	0.00	30.50	133.90	82.91	31.34
<b>DISCLIM</b>	32.72	40.59	91.90	65.31	51.26
<b>ETR</b>	215.39	1118.42	901.87	1412.33	4848.88

The coefficient on all independent variables.

Germany					Finland				
Variable	Estimate	StdErr	tValue	Probt	Variable	Estimate	StdErr	tValue	Probt
<b>Intercept</b>	453.11	10.02	45.21	0.00	<b>Intercept</b>	525.65	11.86	44.31	0.00
<b>ESCS</b>	10.18	1.21	8.41	0.00	<b>ESCS</b>	27.07	1.36	19.84	0.00
<b>COMPHOME</b>	1.98	1.20	1.66	0.10	<b>COMPHOME</b>	-2.46	1.21	-2.04	0.04
<b>INTMAT</b>	3.86	1.04	3.73	0.00	<b>INTMAT</b>	14.02	1.24	11.32	0.00
<b>ANXMAT</b>	-18.80	0.88	-21.35	0.00	<b>ANXMAT</b>	-32.02	1.16	-27.60	0.00
<b>DISCLIM</b>	2.40	0.90	2.66	0.01	<b>DISCLIM</b>	1.36	1.13	1.20	0.23
<b>ETR</b>	-25.78	3.18	-8.12	0.00	<b>ETR</b>	-64.89	8.08	-8.03	0.00
<b>DESCS</b>	68.75	8.66	7.94	0.00	<b>DESCS</b>	4.08	5.99	0.68	0.50
<b>VARESCS</b>	-0.97	8.70	-0.11	0.91	<b>VARESCS</b>	-2.92	9.18	-0.32	0.75
<b>DCOMPH</b>	23.81	12.89	1.85	0.06	<b>DCOMPH</b>	-2.27	8.90	-0.26	0.80
<b>DINTMAT</b>	-29.12	9.73	-2.99	0.00	<b>DINTMAT</b>	-9.65	8.17	-1.18	0.24
<b>DANXMAT</b>	-27.65	10.21	-2.71	0.01	<b>DANXMAT</b>	4.28	8.92	0.48	0.63
<b>DDISCL</b>	31.01	6.81	4.55	0.00	<b>DDISCL</b>	-1.26	4.96	-0.25	0.80
<b>DETR</b>	42.55	19.83	2.15	0.03	<b>DETR</b>	40.76	50.41	0.81	0.42
<b>Compweb</b>	7.67	7.79	0.98	0.32	<b>Compweb</b>	5.52	10.36	0.53	0.59
<b>Mactiv</b>	4.03	3.74	1.08	0.28	<b>Mactiv</b>	2.77	2.62	1.06	0.29
<b>Mstrel</b>	13.75	53.06	0.26	0.80	<b>Mstrel</b>	-120.66	55.23	-2.18	0.03
<b>Tcshort</b>	-7.11	3.29	-2.16	0.03	<b>Tcshort</b>	-0.55	2.36	-0.23	0.82
<b>Temorale</b>	3.83	3.15	1.22	0.22	<b>Temorale</b>	2.10	2.03	1.03	0.30

<b>Teacbeha</b>	-10.68	4.45	-2.40	0.02	<b>Teacbeha</b>	-2.10	2.28	-0.92	0.36
<b>Private</b>	-14.04	10.45	-1.34	0.18	<b>Private</b>	-20.87	6.71	-3.11	0.00
<b>Scmatedu</b>	7.91	3.01	2.63	0.01	<b>Scmatedu</b>	0.23	2.25	0.10	0.92
<b>Academic</b>	6.62	5.32	1.24	0.21	<b>Academic</b>	12.33	5.00	2.46	0.01

<b>The UK</b>					<b>Italy</b>				
<b>Variable</b>	<b>Estimate</b>	<b>StdErr</b>	<b>tValue</b>	<b>Probt</b>	<b>Variable</b>	<b>Estimate</b>	<b>StdErr</b>	<b>tValue</b>	<b>Probt</b>
<b>Intercept</b>	485.30	9.16	52.96	0.00	<b>Intercept</b>	520.11	9.60	54.19	0.00
<b>ESCS</b>	22.55	1.25	18.06	0.00	<b>ESCS</b>	2.83	0.88	3.21	0.00
<b>COMPHOME</b>	7.82	1.22	6.43	0.00	<b>COMPHOME</b>	8.72	0.81	10.76	0.00
<b>INTMAT</b>	-1.13	1.20	-0.94	0.35	<b>INTMAT</b>	6.41	0.84	7.60	0.00
<b>ANXMAT</b>	-23.77	1.17	-20.25	0.00	<b>ANXMAT</b>	-22.39	0.91	-24.47	0.00
<b>DISCLIM</b>	12.80	0.94	13.60	0.00	<b>DISCLIM</b>	2.67	0.79	3.38	0.00
<b>ETR</b>	-0.88	4.14	-0.21	0.83	<b>ETR</b>	-6.26	5.48	-1.14	0.26
<b>DESCS</b>	43.16	6.44	6.70	0.00	<b>DESCS</b>	35.96	5.98	6.01	0.00
<b>VARESCS</b>	14.55	7.34	1.98	0.05	<b>VARESCS</b>	-33.90	8.19	-4.14	0.00
<b>DCOMPH</b>	3.34	8.87	0.38	0.71	<b>DCOMPH</b>	52.74	8.39	6.29	0.00
<b>DINTMAT</b>	-23.00	7.20	-3.19	0.00	<b>DINTMAT</b>	-18.54	6.30	-2.94	0.00
<b>DANXMAT</b>	-14.24	7.62	-1.87	0.06	<b>DANXMAT</b>	-48.18	9.15	-5.27	0.00
<b>DDISCL</b>	16.19	4.72	3.43	0.00	<b>DDISCL</b>	19.27	5.14	3.75	0.00
<b>DETR</b>	-46.57	15.29	-3.05	0.00	<b>DETR</b>	-7.56	23.76	-0.32	0.75
<b>Compweb</b>	6.55	6.78	0.97	0.33	<b>Compweb</b>	11.89	5.68	2.09	0.04
<b>Mactiv</b>	-1.60	1.58	-1.01	0.31	<b>Mactiv</b>	7.72	2.33	3.31	0.00
<b>Mstrel</b>	-45.14	43.48	-1.04	0.30	<b>Mstrel</b>	-68.60	46.35	-1.48	0.14
<b>Tcshort</b>	-2.75	1.86	-1.48	0.14	<b>Tcshort</b>	2.90	2.51	1.16	0.25
<b>Tcmorale</b>	-1.67	1.86	-0.90	0.37	<b>Tcmorale</b>	-1.98	2.40	-0.83	0.41
<b>Teacbeha</b>	0.65	2.43	0.27	0.79	<b>Teacbeha</b>	1.69	2.32	0.73	0.47
<b>Private</b>	5.56	9.20	0.60	0.55	<b>Private</b>	-38.88	8.90	-4.37	0.00
<b>Scmatedu</b>	2.60	1.78	1.46	0.15	<b>Scmatedu</b>	6.06	2.16	2.81	0.00
<b>Academic</b>	18.98	4.35	4.36	0.00	<b>Academic</b>	-6.89	4.10	-1.68	0.09

<b>Japan</b>				
<b>Variable</b>	<b>Estimate</b>	<b>StdErr</b>	<b>tValue</b>	<b>Probt</b>
<b>Intercept</b>	600.43	29.98	20.03	0.00
<b>ESCS</b>	1.43	1.69	0.84	0.40
<b>COMPHOME</b>	3.08	1.29	2.39	0.02
<b>INTMAT</b>	10.01	1.14	8.77	0.00
<b>ANXMAT</b>	-8.35	1.23	-6.78	0.00
<b>DISCLIM</b>	2.55	1.30	1.97	0.05
<b>ETR</b>	-23.95	42.33	-0.57	0.60
<b>DESCS</b>	92.20	13.70	6.73	0.00
<b>VARESCS</b>	39.25	20.17	1.95	0.05

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<b>DCOMPH</b>	15.18	14.09	1.08	0.28
<b>DINTMAT</b>	44.61	11.91	3.74	0.00
<b>DANXMAT</b>	37.39	12.49	2.99	0.00
<b>DDISCL</b>	31.19	6.66	4.68	0.00
<b>DETR</b>	-197.99	454.00	-0.44	0.66
<b>Compweb</b>	-1.58	8.60	-0.18	0.85
<b>Mactiv</b>	5.74	4.65	1.23	0.22
<b>Mstrel</b>	-102.38	48.79	-2.10	0.04
<b>Tcshort</b>	-0.51	2.92	-0.18	0.86
<b>Tcmorale</b>	-2.08	3.12	-0.66	0.51
<b>Teacbeha</b>	7.63	3.75	2.04	0.04
<b>Private</b>	-37.99	6.46	-5.88	0.00
<b>Scmatedu</b>	-4.83	3.07	-1.57	0.12
<b>Academic</b>	-49.64	23.57	-2.11	0.04

## With Restricted Maximum Likelihood estimation method.

The variance components for model 6.

	Germany	Finland	The UK	Italy	Japan
<b>Total variance</b>	4008.42	4075.06	4283.42	4142.11	4432.38
<b>Within variance</b>	2911.1337	3833.185	3820.851	2936.647	3649.407
<b>Between variance</b>	1097.2867	241.879	462.5656	1205.464	782.9725
<b>% Between/Total</b>	27.37	5.94	10.80	29.10	17.66

	Germany	Finland	The UK	Italy	Japan
<b>Variable</b>	<b>Between school variance</b>				
<b>ESCS</b>	20.00	56.90	138.65	58.88	50.85
<b>COMPHOME</b>	16.12	22.42	116.20	40.11	27.26
<b>INTMAT</b>	44.17	73.59	158.55	63.45	0.79
<b>ANXMAT</b>	0.00	31.26	134.88	83.77	32.61
<b>DISCLIM</b>	34.06	42.91	93.03	65.81	55.35
<b>ETR</b>	226.14	1184.74	924.26	1452.95	7087.75

The coefficient on all independent variables.

Germany					Finland				
Variable	Estimate	StdErr	tValue	Probt	Variable	Estimate	StdErr	tValue	Probt
<b>Intercept</b>	453.43	10.40	43.58	0.00	<b>Intercept</b>	525.40	12.40	42.38	0.00
<b>ESCS</b>	10.17	1.21	8.37	0.00	<b>ESCS</b>	27.06	1.37	19.76	0.00
<b>COMPHOME</b>	1.97	1.20	1.64	0.10	<b>COMPHOME</b>	-2.46	1.21	-2.04	0.04
<b>INTMAT</b>	3.87	1.04	3.72	0.00	<b>INTMAT</b>	14.02	1.24	11.27	0.00
<b>ANXMAT</b>	-18.80	0.88	-21.32	0.00	<b>ANXMAT</b>	-32.02	1.16	-27.54	0.00
<b>DISCLIM</b>	2.40	0.91	2.65	0.01	<b>DISCLIM</b>	1.36	1.14	1.19	0.23
<b>ETR</b>	-25.81	3.19	-8.08	0.00	<b>ETR</b>	-64.89	8.17	-7.95	0.00
<b>DESCS</b>	68.94	9.03	7.63	0.00	<b>DESCS</b>	4.16	6.28	0.66	0.51
<b>VARESCS</b>	-0.94	9.08	-0.10	0.92	<b>VARESCS</b>	-2.90	9.67	-0.30	0.76
<b>DCOMPH</b>	23.64	13.44	1.76	0.08	<b>DCOMPH</b>	-2.45	9.35	-0.26	0.79
<b>DINTMAT</b>	-29.15	10.15	-2.87	0.00	<b>DINTMAT</b>	-9.77	8.56	-1.14	0.25
<b>DANXMAT</b>	-27.64	10.65	-2.59	0.01	<b>DANXMAT</b>	4.54	9.38	0.48	0.63
<b>DDISCL</b>	31.07	7.10	4.38	0.00	<b>DDISCL</b>	-1.40	5.20	-0.27	0.79
<b>DETR</b>	42.73	20.67	2.07	0.04	<b>DETR</b>	41.00	52.99	0.77	0.44
<b>Compweb</b>	7.35	8.07	0.91	0.36	<b>Compweb</b>	5.52	10.80	0.51	0.61
<b>Mactiv</b>	3.92	3.89	1.01	0.31	<b>Mactiv</b>	2.87	2.76	1.04	0.30
<b>Mstrel</b>	12.79	54.66	0.23	0.81	<b>Mstrel</b>	-119.93	58.18	-2.06	0.04
<b>Tshort</b>	-7.12	3.43	-2.08	0.04	<b>Tshort</b>	-0.59	2.49	-0.24	0.81
<b>Tcmorale</b>	3.83	3.29	1.17	0.24	<b>Tcmorale</b>	2.11	2.13	0.99	0.32
<b>Teacbeha</b>	-10.69	4.64	-2.30	0.02	<b>Teacbeha</b>	-2.13	2.40	-0.89	0.38



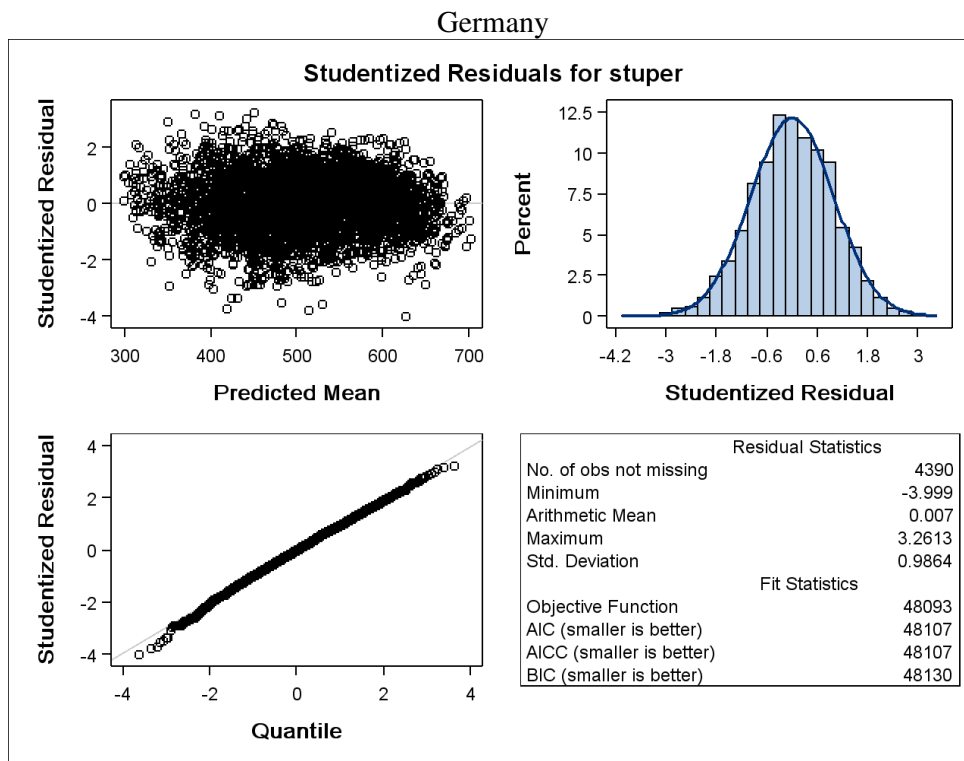
<b>Private</b>	-14.19	10.90	-1.30	0.19	<b>Private</b>	-21.02	7.04	-2.99	0.00
<b>Scmatedu</b>	7.91	3.14	2.52	0.01	<b>Scmatedu</b>	0.24	2.36	0.10	0.92
<b>Academic</b>	6.66	5.55	1.20	0.23	<b>Academic</b>	12.28	5.26	2.33	0.02

The UK					Italy				
Variable	Estimate	StdErr	tValue	Probt	Variable	Estimate	StdErr	tValue	Probt
<b>Intercept</b>	485.30	9.34	51.93	0.00	<b>Intercept</b>	519.97	9.78	53.16	0.00
<b>ESCS</b>	22.54	1.25	17.98	0.00	<b>ESCS</b>	2.83	0.88	3.21	0.00
<b>COMPHOME</b>	7.82	1.22	6.39	0.00	<b>COMPHOME</b>	8.72	0.81	10.75	0.00
<b>INTMAT</b>	-1.13	1.20	-0.94	0.35	<b>INTMAT</b>	6.41	0.85	7.58	0.00
<b>ANXMAT</b>	-23.77	1.18	-20.22	0.00	<b>ANXMAT</b>	-22.39	0.92	-24.43	0.00
<b>DISCLIM</b>	12.80	0.94	13.56	0.00	<b>DISCLIM</b>	2.67	0.79	3.38	0.00
<b>ETR</b>	-0.90	4.16	-0.22	0.83	<b>ETR</b>	-6.33	5.52	-1.15	0.25
<b>DESCS</b>	43.24	6.58	6.57	0.00	<b>DESCS</b>	35.99	6.11	5.89	0.00
<b>VARESCS</b>	14.45	7.51	1.92	0.05	<b>VARESCS</b>	-33.70	8.36	-4.03	0.00
<b>DCOMPH</b>	3.40	9.07	0.37	0.71	<b>DCOMPH</b>	52.61	8.56	6.14	0.00
<b>DINTMAT</b>	-23.22	7.36	-3.16	0.00	<b>DINTMAT</b>	-18.49	6.43	-2.88	0.00
<b>DANXMAT</b>	-14.29	7.79	-1.84	0.07	<b>DANXMAT</b>	-48.13	9.32	-5.17	0.00
<b>DDISCL</b>	16.25	4.83	3.36	0.00	<b>DDISCL</b>	19.24	5.24	3.67	0.00
<b>DETR</b>	-46.43	15.66	-2.97	0.00	<b>DETR</b>	-7.74	24.14	-0.32	0.75
<b>Compweb</b>	6.35	6.90	0.92	0.36	<b>Compweb</b>	11.97	5.78	2.07	0.04
<b>Mactiv</b>	-1.53	1.61	-0.95	0.34	<b>Mactiv</b>	7.69	2.38	3.23	0.00
<b>Mstrel</b>	-42.45	44.27	-0.96	0.34	<b>Mstrel</b>	-68.30	47.34	-1.44	0.15
<b>Tcshort</b>	-2.73	1.90	-1.43	0.15	<b>Tcshort</b>	2.87	2.56	1.12	0.26
<b>Tcmorale</b>	-1.64	1.90	-0.86	0.39	<b>Tcmorale</b>	-1.98	2.45	-0.81	0.42
<b>Teacbeha</b>	0.71	2.49	0.28	0.78	<b>Teacbeha</b>	1.65	2.37	0.70	0.49
<b>Private</b>	5.07	9.42	0.54	0.59	<b>Private</b>	-38.78	9.09	-4.27	0.00
<b>Scmatedu</b>	2.58	1.82	1.42	0.16	<b>Scmatedu</b>	6.08	2.20	2.76	0.01
<b>Academic</b>	19.26	4.45	4.33	0.00	<b>Academic</b>	-6.93	4.19	-1.65	0.10

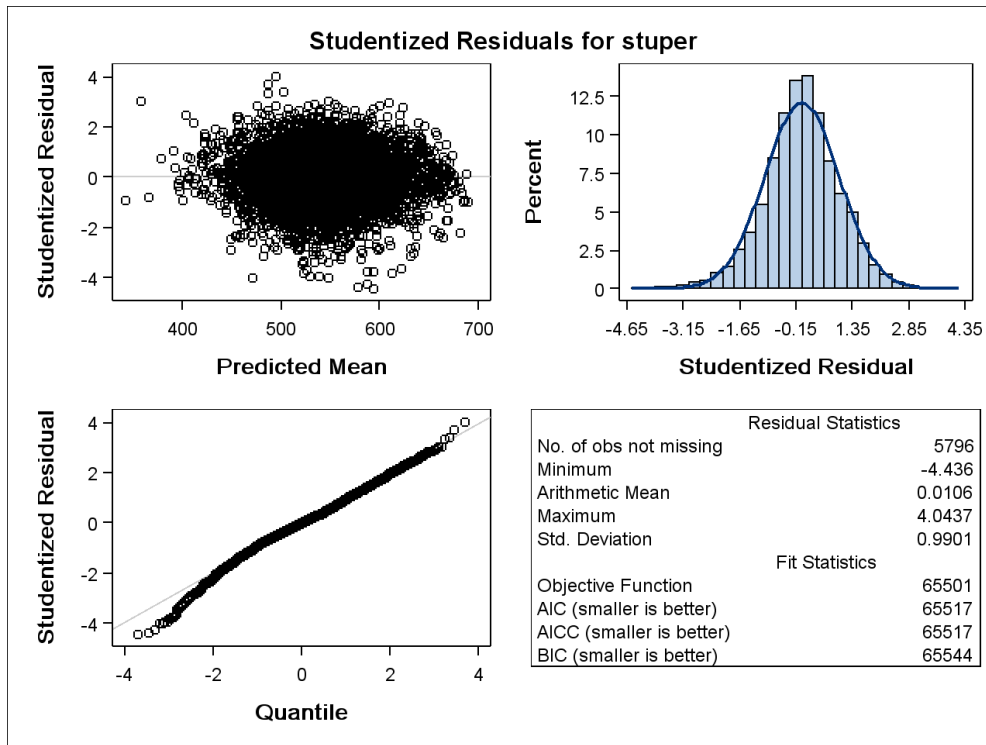
Japan				
Variable	Estimate	StdErr	tValue	Probt
<b>Intercept</b>	600.97	31.97	18.80	0.00
<b>ESCS</b>	1.41	1.70	0.83	0.41
<b>COMPHOME</b>	3.08	1.29	2.39	0.02
<b>INTMAT</b>	9.99	1.14	8.72	0.00
<b>ANXMAT</b>	-8.37	1.24	-6.77	0.00
<b>DISCLIM</b>	2.55	1.31	1.95	0.05
<b>ETR</b>	-24.41	47.34	-0.52	0.63
<b>DESCS</b>	92.51	14.58	6.34	0.00
<b>VARESCS</b>	39.23	21.50	1.82	0.07
<b>DCOMPH</b>	14.98	15.02	1.00	0.32

<b>DINTMAT</b>	44.08	12.68	3.48	0.00
<b>DANXMAT</b>	37.14	13.30	2.79	0.01
<b>DDISCL</b>	31.07	7.07	4.39	0.00
<b>DETR</b>	-194.42	483.92	-0.40	0.69
<b>Compweb</b>	-1.45	9.12	-0.16	0.87
<b>Mactiv</b>	6.07	4.94	1.23	0.22
<b>Mstrel</b>	-103.81	51.73	-2.01	0.04
<b>Tcshort</b>	-0.47	3.11	-0.15	0.88
<b>Tcmorale</b>	-2.05	3.33	-0.61	0.54
<b>Teacbeha</b>	7.62	4.00	1.91	0.06
<b>Private</b>	-38.04	6.89	-5.52	0.00
<b>Scmatedu</b>	-4.77	3.27	-1.46	0.15
<b>Academic</b>	-50.37	25.14	-2.00	0.05

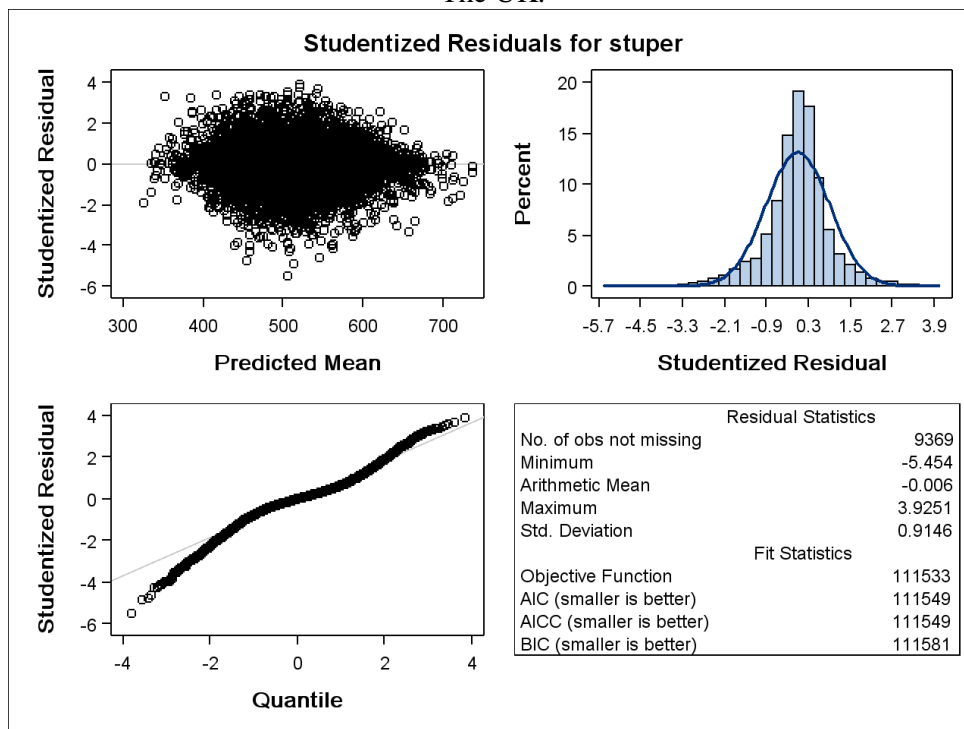
Residuals' scatter plots and normal probability plots for model 6. (Stuper is the dependent variable: student performance).



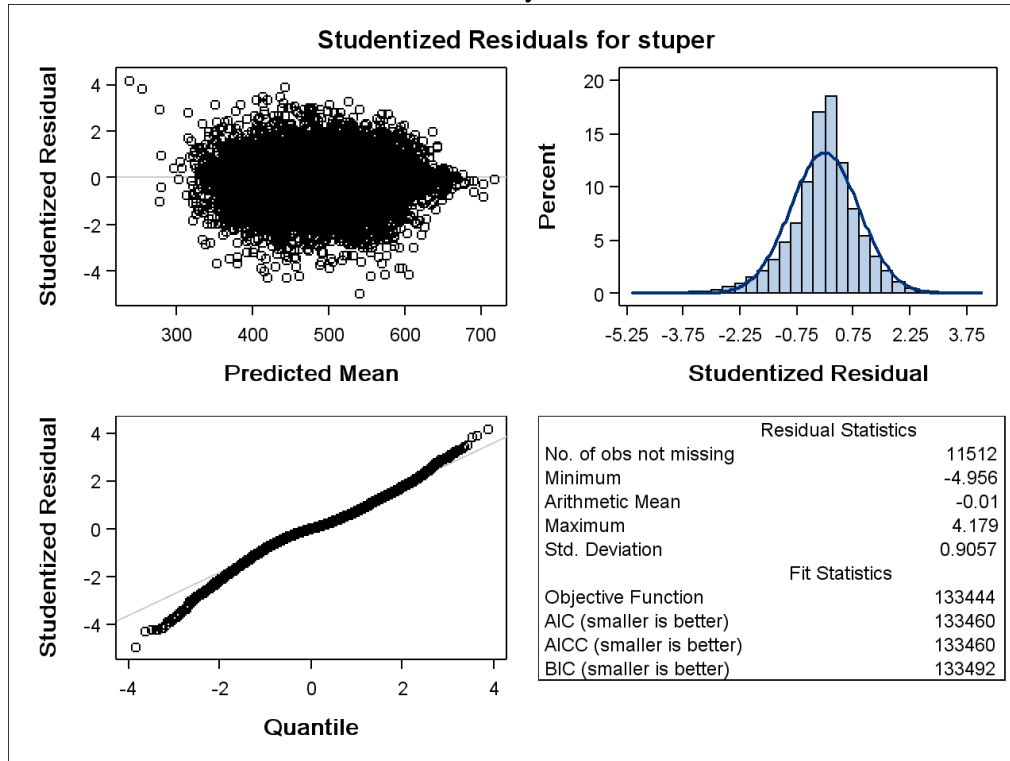
Finland.



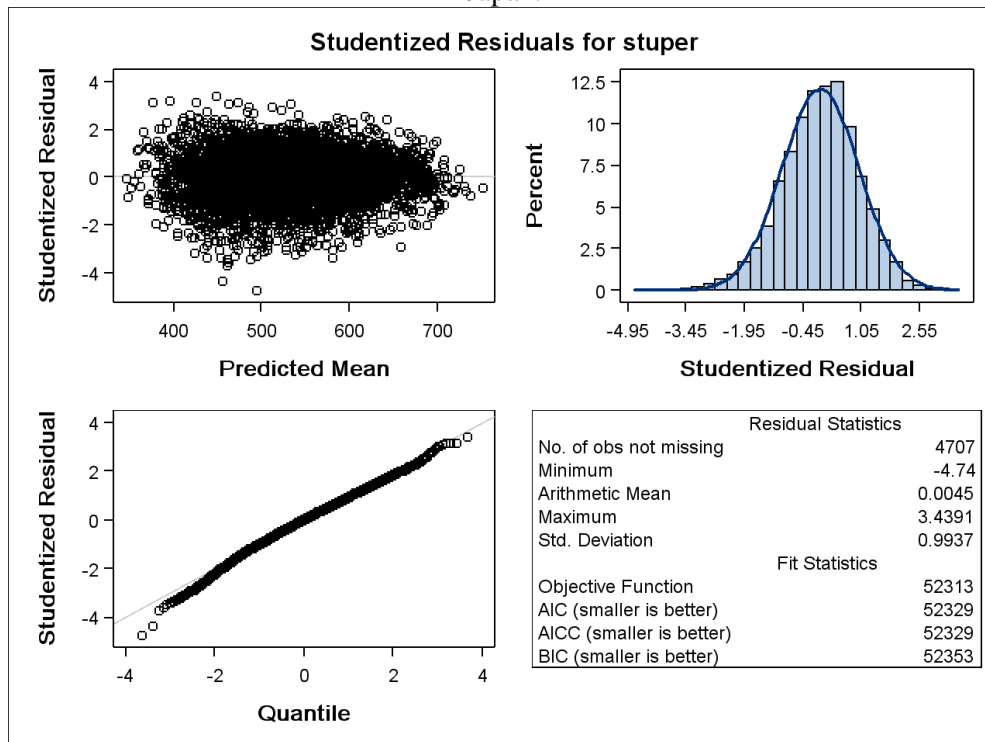
The UK.



Italy.



Japan.



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**L'Anatomie des Inégalités dans les Performances Scolaires :  
Une Analyse Internationale de la Stratification.**

**Résumé :** Cette thèse propose d'étudier les mécanismes de stratification et des inégalités dans les performances. L'objectif principal est de savoir si la 'compréhensivisation' des systèmes éducatifs engendre plus d'égalité dans les performances.

Dans un premier temps, une étude théorique basée sur les travaux de Epple et Romano (1998), Glomm et Ravikumar (1998), et Nechyba (1997, 2003) est développée pour mettre en évidence les mécanismes de stratification et pour évaluer leurs impacts sur les performances scolaires. Dans un second temps, une analyse empirique est élaborée pour quantifier cet impact dans le cadre de plusieurs systèmes éducatifs différenciés par leurs caractéristiques institutionnelles.

**Mots clefs :** Stratification scolaire et spatiale, inégalités dans les performances, comparaison internationale entre les systèmes éducatifs, modèles multi-niveaux.

**The Anatomy of Inequalities in attainments:  
An International Investigation on Stratification and Choice.**

**Abstract:** This thesis studies the mechanisms of stratification and inequalities in attainments. The main objective is to determine if the comprehensivization of education systems leads to higher levels of equality in achievements.

On the one hand, a theoretical analysis is built upon the work of Epple and Romano (1998), Glomm and Ravikumar (1998), and Nechyba (1997, 2003) in order to assess the mechanisms of stratification and their impact on achievements. On the other hand, an empirical approach is elaborated in order to quantify this impact in the context of five education systems differentiated by their institutional characteristics.

**Keywords:** Educational and spatial stratification, achievement inequalities, comparative analysis of education systems, multilevel modeling.

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