Communication stratégique et réseaux

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A mes parents,

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Note de Synthèse

Introduction

Le travail de recherche que j'ai mené à ce jour a été motivé par deux observations extrêmement simples dont l'exposition constitue le point de départ de la présente Note de Synthèse.

Premièrement, dans de nombreuses situations, il nous faut faire des choix sans que notre connaissance des différentes alternatives soit parfaite. Face à une décision, nous utilisons donc nos expériences passées et connaissances propres, mais avons également recours au savoir des autres, surtout de ceux qui nous sont proche. Ainsi, un consommateur souhaitant acheter une voiture peut avoir à choisir une marque sans connaître complètement toutes les options dont il dispose. Comme l'acquisition d'un véhicule constitue un investissement majeur, il est probable que l'acheteur potentiel discute de sa décision avec ses amis et collègues pour obtenir des informations supplémentaires permettant un meilleur choix. De même, les membres d'une équipe peuvent devoir investir dans un nouveau logiciel sans connaître parfaitement la facilité d'utilisation de ceux qui leurs sont proposés. Les magazines spécialisés sont alors un moyen habituel de s'informer. Il y a de grandes chances qu'ils tentent aussi de rassembler des informations dispersées dans l'équipe et permettant d'adapter le choix du logiciel à ses besoins et capacités. Dans de nombreux contextes, parce que les entités individuelles ont accès à des sources d'informations différentes ou parce que celles-ci apprennent des choses variées de leur décisions passées, l'*union fait la force* en matière de connaissances. Dans cette thèse, j'ai considéré une classe générale de situations présentant le caractère suivant: *les informations sur les fondamentaux qui sont pertinents pour un groupe d'agents sont dispersées parmi ses membres.* Par conséquent, le partage des informations détenues de manière privée est le seul moyen dont les agents disposent pour estimer avec précision l'environnement dans lequel ils évoluent et réagir convenablement à celui-ci.

Deuxièmement, il semble désormais admis que la plupart des échanges d'informations utiles à la prise de décisions économiques a lieu via les réseaux informels que forment des entités qui communiquent activement et non via des institutions centralisées. Un réseau de communication peut être défini comme un ensemble de liens à travers desquels les informations des agents circulent. Cette notion est vaste et comprend les relations amicales, les liens virtuels aussi bien que les échanges réguliers que des collègues ont sur leur lieu de travail. Il existe probablement autant de types de réseaux de communication que de façons de les utiliser : les réseaux d'amis sont des canaux important pour se passer le mot concernant les bonnes boutiques et restaurants, ou pour diffuser des opinions politiques ; les réseaux professionnels peuvent servir à rapporter des opportunités liés à des emplois vacants ou bien encore à rassembler des informations sur l'état de la demande à laquelle une firme fait face. Durant la dernière décennie, l'étude des réseaux sociaux a été un champ de recherche très actif en économie. Cette Note de Synthèse détaille comment cette thèse s'inscrit dans la *Théorie des Réseaux Economiques et Sociaux* en plein développement. Pour une vue d'ensemble claire de cette théorie, le lecteur pourra se référer au manuel de M.O. Jackson (Jackson (2008b)).

Concernant la communication décentralisée ayant lieu au sein d'un réseau, deux principales branches sont habituellement distinguées. Dans certaines situations, la transmission des informations entre les agents est *mécanique*. La diffusion de ces dernières est *automatique* au sens où elle ne résulte pas d'un choix fait par les individus. Ainsi, il est probable que les personnes que nous fréquentons régulièrement aient appris quelles sont nos boutiques préférées (et utilisent cette information pour leurs propres achats) sans que nous n'intervenions sciemment dans cette révélation. Ensuite, il est possible que la circulation des informations découle d'un choix fait par les agents mais que celle-ci garde un caractère automatique. En effet, dans un groupe d'agents connectés, si les incitations collectives et individuelles au partage de l'information sont parfaitement alignées, alors il est dans l'intérêt de chaque agent de communiquer rapidement et honnêtement. On peut, par exemple, penser à des agents qui se passeraient le mot concernant l'utilisation d'une nouvelle technologie dont les bénéfices pour chaque utilisateur sont strictement croissants avec le nombre total de ces utilisateurs. Quand la transmission d'informations est automatique ou qu'elle constitue une stratégie dominante pour chaque agent, le processus de circulation peut être lié à l'architecture du réseau auquel ces derniers appartiennent. La communication mécanique dans les réseaux n'a pas été l'objet de mon travail. Une présentation détaillée des mécanismes de *Diffusion* et d'*Apprentissage* dans les réseaux est faite par M.O. Jackson dans les Chapitres 7 et 8 de son manuel (Jackson (2008b)).

Dans d'autres situations, la communication entre les agents a un aspect *stratégique* parce que celle-ci a un impact direct et décisif sur leurs choix. Précisément, c'est en *manipulant* son information ou en *retardant* sa révélation qu'un agent cherche à influencer les comportements de ceux avec qui il interagit. Alors, la structure du réseau a également son importance mais il faut tenir compte du fait que les flux d'informations en son sein résultent de choix réfléchis. Nous considérerons qu'un *Dilemme de Communication* existe dès que l'intérêt collectif d'un groupe nécessite une mise en commun des informations privées, alors que les intérêts individuels poussent ses membres à les cacher ou à les manipuler. Ces dilemmes informationnels occupent une place centrale dans mon travail. En particulier, je me suis intéressée à l'étude de situations dans lesquelles la structure globale des incitations entrave la circulation parfaite des informations parmi des membres modérément informés d'un même groupe. Le travail que j'ai effectué se divise en deux parties. Dans la première partie de cette thèse, constituée du Chapitre 1, le dilemme réside dans le fait que les agents ont un intérêt individuel à garder leurs informations secrètes aussi longtemps que possible, alors qu'une communication rapide est dans l'intérêt du groupe. Dans les Chapitres 2 et 3, qui résultent d'un travail joint avec Frédéric Koessler (Chargé de Recherche au CNRS, affilié à l'Ecole d'Economie de Paris), un dilemme informationnel se pose car la révélation honnête de l'information est efficace socialement alors qu'il est dans l'intérêt des individus de manipuler celle-ci, c'est-à-dire de mentir.

Les dilemmes de communication décrits précédemment correspondent à de nombreuses situations économiques. Par exemple, les différentes divisions d'une organisation ont souvent besoin de mettre en commun leurs informations pour obtenir une vue claire de l'environnement dans lequel la firme évolue et y adapter ses choix. Il est probable que les décisions prises dans la firme au niveau global, que cela soit par un ou plusieurs décideurs, affectent alors chaque division. Par conséquent, chaque division peut souhaiter manipuler les informations qu'elle possède pour pousser ces décisions dans sa direction préférée. En d'autres termes, même si les divisons ont un intérêt commun à maximiser le profit de la firme, leurs actions favorites peuvent différer en raison de leurs capacités propres ou de leurs goûts locaux. De la même manière, les membres d'un parti politique peuvent posséder des informations qui doivent être assemblées afin de bien évaluer la situation et de promouvoir ainsi une politique bien adaptée. Pourtant, il est facile de concevoir que des préoccupations carriéristes puissent avoir un impact sur la communication ayant lieu entre les activistes. Quand, au sein d'un groupe, l'alignement entre les intérêts collectifs et individuels à partager l'information n'est pas parfaite, une question générale se pose de savoir comment ses membres communiquent stratégiquement pour affecter les choix en leur faveur.

Dans les contextes de dilemmes informationnels, cette thèse a donc pour objectif d'examiner la relation entre la façon dont les agents transmettent stratégiquement leurs informations et la structure des réseaux auxquels ils appartiennent. Cette relation peut aller dans deux sens distincts, selon que l'ensemble des connexions entre les agents *influence* ou *résulte de* la communication. Chacune de ces directions correspond à une des deux questions principales qui ont émergées de l'étude récente des réseaux par les économistes : Premièrement, comment la structure des réseaux affecte-t-elle les résultats économiques ? Deuxièmement, pourquoi certaines structures de liens émergent-elles ? La Théorie des Jeux Non-Coopératifs a été appliquée aux deux branches de recherche résultant de ces questions et mon travail contribue à ces deux dernières. D'une part, les participants aux *"Jeux en Réseaux"* (ou *"Network Games"*) sont les membres d'un réseau donné au départ. L'objectif est alors d'analyser comment leurs comportements stratégiques dépendent de l'architecture de ce dernier. Mon premier Chapitre s'inscrit dans l'ensemble de ces modèles. En effet, l'objectif est d'examiner comment la structure fixe des liens affecte la capacité des agents connectés à mettre en commun des informations initialement dispersées. D'autre part, les *"Jeux de Formation de Réseaux"* (ou *"Network Formation Games"*) examinent les incitations des agents à créer des liens. Dans les Chapitres 2[1] et 3, j'étudie comment la nature de la communication stratégique entre les agents façonne leurs relations.

Parce qu'ils font appel à des environnements théoriques très différents, les détails de chacune des deux parties de cette thèse sont présentés séparément. Formellement, les réseaux sont modélisés selon la manière usuelle, c'est-à-dire par des graphes dans lesquels les entités individuelles sont représentées par des nuds et les relations entre ces entités par des arcs.

Chapitre 1 : Centralizing Information in Networks

Motivation, Modèle et Résultats :

C'est une expérience menée en sociologie qui constitue la motivation initiale de ce chapitre. Bonacich (1990) rapporte une expérience dans laquelle le succès d'un groupe d'agents ne peut être assuré que par une bonne circulation des informations entre ses membres. Précisément, chacun des sujets expérimentaux recevait au départ un ensemble de lettres faisant partie d'une citation que le groupe de participants devait identifier. Une fois qu'un des individus avait découvert la citation, et indépendamment de l'identité de cet individu, le groupe recevait une *Récompense Collective* partagée égalitairement entre ses membres. Cette récompense diminuait avec le temps mis par le groupe pour atteindre leur objectif commun. Afin d'assembler les lettres initialement dispersées, les agents disposaient de plusieurs tours de jeu au cours desquels ils pouvaient transmettre les lettres qu'ils détenaient via des canaux de communication. En effet, les participants étaient initialement arrangés dans un réseau donné dont les liens étaient les seuls conduits à travers lesquels les lettres pouvaient circuler. En plus d'être restreinte physiquement, la transmission des lettres avait un aspect stratégique. L'individu qui était le premier à identifier la citation au nom du groupe obtenait une *Récompense Additionnelle* qu'il gardait pour lui. C'est cette

[1]Co-écrit avec F.Koessler, à paraître dans la *Review of Economic Studies*.

récompense qui était à l'origine d'un dilemme de communication.

L'expérience de Bonacich est représentative d'une large classe de situations dans lesquelles le problème de la communication entre les détenteurs d'informations se pose. Prenons par exemple le cas d'une équipe de travailleurs. Même s'il est clair qu'il y a un intérêt collectif pour ses membres à communiquer rapidement les uns avec les autres pour prendre des décisions appropriées, ces derniers peuvent, dans le même temps, se trouver en compétition pour être promu ou pour obtenir toute autre forme de gratitude. Dans ce cas, chaque agent peut souhaiter être finalement celui qui centralise, au nom de son équipe, les informations utiles à la prise de décision. Si cette équipe fait partie d'une organisation plus large, elle peut ne pas avoir la liberté de réorganiser librement la structure des canaux de communication existant entre ses membres. L'analyse de la transmission stratégique des informations privées via les liens d'une réseau donné s'avère alors pertinente.

Le grand intérêt de l'expérience décrite précédemment réside dans ce qu'elle a été menée pour différentes structures de réseau de communication. D'un point de vue individuel, les résultats expérimentaux suggèrent que, parce que les positions dans un réseau sont différenciées, leurs occupants peuvent avoir des incitations différentes à se comporter de manière coopérative - en communiquant librement - ou de manière compétitive - en faisant de la rétention stratégique d'informations. D'un point de vue global, il résulte que l'issue des dilemmes informationnels est affectée par l'architecture des canaux de communication existant entre les joueurs. Mon premier chapitre modélise rigoureusement l'expérience de P. Bonacich et apporte à ses résultats un soutien théorique. En effet, j'articule la relation entre la communication et la structure des connections à travers lesquelles l'information peut circuler autour de deux questions. D'un point de vue collectif, l'objectif est de déterminer s'il existe des réseaux qui améliorent la capacité du groupe à centraliser les informations. La performance collective est évaluée selon que l'équipe réussit ou non à mettre en commun ces dernières à l'équilibre, et par le temps nécessaire pour ce faire. Au niveau individuel, c'est l'impact de la position d'un agent sur sa performance à rassembler les infos qui sera examinée. Enfin, l'on peut noter que, dans ce premier chapitre, le focus est porté sur l'échange stratégique d'informations et que le processus de décision qui succède implicitement à la centralisation des informations est ignoré. Premièrement, je suppose que personne ne décide jamais sans avoir rassemblé la totalité des informations (car il est très couteux de prendre une mauvaise décision par exemple). Deuxièmement, lorsqu'un agent est parfaitement informé, je suppose que la bonne décision à prendre est alors parfaitement déterministe. En permettant à chaque membre du groupe de prendre une décision au nom de tous, ce chapitre fait également abstraction des questions concernant l'allocation du pouvoir décisionnel.

Formellement, j'analyse des jeux dynamiques dans lesquels n joueurs sont les membres d'un graphe donné g et disposent de T périodes pour mettre en commun n informations initialement dispersées. Chacun est, au départ, doté d'une information qu'il est le seul à détenir. A chaque période, chaque joueur choisit stratégiquement de transmettre ou non toutes les informations qu'il détient à tous ses voisins dans le réseau. Les actions sont parfaitement observées par tous les joueurs à chacune des périodes. L'agent qui est le premier à centraliser les n informations est appelé le "gagnant". Dans ce cadre, mon principal résultat consiste en une condition nécessaire et suffisante pour qu'un groupe atteigne l'objectif collectif de mise en commun des informations privées dans chacun des équilibres (parfait en sous-jeux). Cette condition s'avère être indépendante de la structure du réseau dans laquelle se trouvent les agents. Précisément, je montre qu'un groupe de n joueurs n'échoue jamais en matière de centralisation des informations à l'équilibre si et seulement si le nombre des périodes dont il dispose pour ce faire est au moins égal à $n-1$. Cette borne correspond exactement au nombre minimal de liens qui sont nécessaires pour connecter tous les joueurs. Au contraire, l'architecture du réseau affecte le temps mis pour rassembler les informations à l'équilibre. Avant tout, l'on peut noter que, même si la communication était automatique, c'est-à-dire que chaque agent transmettrait ses informations à tous ses voisins à chaque période du jeu, le temps mis pour centraliser les informations dépendrait de la structure des canaux de transmission. En particulier, chaque agent aurait "techniquement" besoin d'un nombre minimal de périodes pour gagner, qui dépendrait de sa position. Ce nombre minimal de périodes correspond à une mesure graphique appelée l'*excentricité*. Je prouve que, pour chaque agent, il existe un équilibre dans lequel il est le seul gagnant à une date égale à son excentricité. Par conséquent, l'on démontre qu'il existe toujours un équilibre dans lequel le jeu prend fin à une date égale à la plus petite excentricité présente dans le groupe, c'est-à-dire à une date égale au rayon du graphe. Enfin, je montre que, pour deux structures particulières de réseaux, il existe une borne supérieure au temps mis par les joueurs pour centraliser les informations à l'équilibre.

Revue de la Littérature :

Parce qu'il tente de lier la performance d'un groupe à sa structure de communication interne, le premier chapitre peut être mis en relation avec une large littérature concernant la *Théorie des Equipes.* En effet, depuis**?**, ce domaine de recherche analyse la prise de décision dans les firmes dans lesquelles les informations sont initialement dispersées et dans lesquelles il existe des contraintes physiques qui rendent la communication et les traitement de ces dernières couteuses. C'est l'arbitrage entre ces coûts (ressources humaines, temps) et les bénéfices de la communication qui est au cur de ces études. Dans ce contexte, Radner (1992), Radner (1993) et Bolton and Dewatripont (1994) (parmi d'autres) ont cherché à identifier la structure interne optimale pour une organisation au sens large. Leurs travaux ont principalement mis en avant le rôle des hiérarchies, ou plus généralement des structures de communication centralisées, pour réduire les coûts associés à la mise en commun des informations privées. Les papiers de Crémer (1980), Aoki (1986), Geanakoplos and Milgrom (1991) and Van Zandt (1999) ont pour objectif d'allouer efficacement les tâches en présence de coûts de traitement des informations mais lorsque leur communication est gratuite. Par conséquent, ces travaux ne traitent pas du design d'un réseau de communication. Un point commun que partagent les papiers associés à la littérature traitant de la théorie des équipes est que ceux-ci font abstraction des problèmes d'incitations. En effet, ils font généralement l'hypothèse que les agents agissent uniquement dans l'intérêt de l'organisation, ce qui n'est pas le cas dans mon premier chapitre. Nous supposons toutefois, en accord avec cette litérature, que les informations ne peuvent être déformées. Les actions des agents consistent simplement à Passer ou à Cacher et l'on peut considérer les objets informationnels transmis comme n'importe quels autres biens dont la valeur vient du fait qu'ils soient rassemblés.

Dans les *Jeux en Réseaux*, les stratégies des joueurs connectés de manière exogène consistent en des choix très variés, qui peuvent évidemment différer de transférer ou non de l'information. Par exemple, Bramoullé and Kranton (2007) examinent un Jeu de Bien Public où les utilités des joueurs dépendent de la somme de leurs propres contributions et de celles de leurs voisins. Ils supposent que les contributions des joueurs sont des substituts stratégiques. Les équilibres du jeu sont multiples et caractérisés à l'aide de concepts issus de la *Théorie des Graphes*. Pour résoudre le jeu proposé dans le Chapitre 1, j'ai également recours à des résultats issus de cette théorie et utilise, en particulier, la notion d'excentricité d'un nud mentionnée précédemment. Ballester, Calvo-Armengol, and Zenou (2006) analysent une classe de jeux à information complète avec des paiements quadratiques et des complémentarités stratégiques entre les actions des agents connectés. Ils montrent qu'à l'équilibre, l'effort exercé par chaque agent dépend de sa position dans le réseau et, plus précisément, d'une mesure de centralité appelée l'indice de Katz-Bonacich (Bonacich (1987)). Dans mon premier chapitre, je relie la position des agents à certains de leurs paiements d'équilibre mais pas directement à leurs stratégies d'équilibres. Concernant l'étude des jeux joués par les membres d'un réseau donné, un cadre général est proposé dans Galeotti, Goyal, Jackson, Vega-Redondo, and Leeat (2009b). Les auteurs considèrent une large classe de paiements, supposant que le paiement d'un joueur dépend de son action ainsi que des décisions prises par ses voisins directs. Leurs résultats montrent comment la structure du réseau, la position d'un individu, la nature des jeux (compléments ou substituts stratégiques, externalités positives ou négatives), et le niveau d'information (complète ou incomplète) affectent les comportements individuels et les paiements. La différence principale entre les travaux cités et notre Jeu en Réseau est que celui que nous proposons est dynamique. Les paiements dépendent de toute l'histoire du jeu, des actions prises par tous les joueurs à toutes les périodes.

D'un point de vue théorique, le premier chapitre de cette thèse est également lié à la littérature traitant des *Guerres d'Usure* à plusieurs joueurs. En effet, pour que notre jeu dynamique finisse avec un gagnant, il faut qu'un joueur parvienne à rassembler toutes les informations dispersées avant que la limite temporelle du jeu ne soit atteinte. Dans un réseau complet, comme chaque agent est directement connecté à chaque autre agent, une information transmise est immédiatement reçue par chaque joueur. Par conséquent, dès que tous les joueurs sauf un seul ont "cédé", le jeu se termine avec, pour gagnant, le joueur ne l'ayant pas fait. En ce sens, le jeu dynamique que nous étudions peut être considéré comme une guerre d'usure en information complète dans laquelle n joueurs sont en compétition, en temps discret et fini, pour remporter un seul prix. Comme dans les guerres d'usure usuelles, chaque joueur préfère strictement gagner à perdre mais préfère perdre tôt à perdre tard. La différence réside dans la fait que l'ordre dans lequel les $n-1$ cessions doivent arriver dépendent de la structure du réseau liant les agents. **?** fournissent une caractérisation complète de tous les équilibres parfaits en sous-jeux pour des guerres d'usure à deux joueurs, en temps continu et information complète. L'extension de leur résultat à notre jeu en information complète est immédiate : pour chaque agent, il existe un équilibre parfait en sous-jeux dans lequel cet agent uniquement gagne immédiatement. Je généralise cette observation à toute structure de réseau en montrant que, pour chaque agent, il existe un équilibre parfait en sous-jeux dans lequel cet agent uniquement gagne à la date la plus petite à laquelle il lui est physiquement possible de gagner, c'est-à-dire à une date qui est égale à son excentricité.

Pour finir, la première partie de cette thèse peut être mise en relation avec des recherches menées plus anciennement en *Psychologie Sociale*. Ces dernières ont mis en lumière le rôle crucial joué par la structure des liens de communication sur les capacités d'agrégation des informations. En particulier, les articles de Bavelas (1950) and Leavitt (1951) ont été à l'origine de nombreux travaux empiriques qui cherchaient à savoir si la performance d'un groupe pouvait être améliorée par un changement de la configuration des chaînes de communication. Ces recherches n'ont pas été accompagnées par beaucoup de développements théoriques. Shaw (1964) propose une revue de cette littérature, qui compare principalement les structures centralisées et décentralisées. Les résultats expérimentaux suggèrent que la relation entre la structure de communication et la performance d'un groupe dépend grandement de la tâche que le groupe doit effectuer. Il semble que si la tâche est relativement simple et nécessite seulement d'assembler des informations initialement dispersées, les structures centralisées sont plus adaptées que les structures décentralisées.

Chapitres 2 et 3 : Strategic Communication Networks

Motivation, Modèle et Résultats :

Dans de nombreuses situations économiques, les agents ont un intérêt à coordonner leurs actions ainsi qu'à adapter ces dernières à un état du monde inconnu. Dans le second et le troisième chapitre, nous allons considérer ce type de contextes mais nous détacher de l'hypothèse courante selon laquelle les agents s'accordent sur le profil optimal de décisions contingent à l'état du monde. Plus précisément, parce que leurs goûts peuvent différer, nous supposons que les entités qui interagissent varient en matière de proximité idéale aux fondamentaux. Comme nous l'avons mentionné précédemment, les différentes divisions d'une organisation doivent souvent coordonner leurs actions pour en maximiser le profit, ainsi qu'adapter celles-ci à l'environnement incertain dans lequel l'organisation évolue. Pour plusieurs raisons, allant des coûts locaux d'adaptation aux préoccupations en matière de carrières, il est probable que chacune de ces divisions cherche, dans le même temps, à adapter ses choix à ses particularités propres. De la même manière, lorsqu'elle investit dans une nouvelle technologie, une firme souhaite, à la fois, faire des choix qui lui permettent de satisfaire les attentes de ses consommateurs et également de rester en accord avec les choix des autres firmes en raison de complémentarités stratégiques. Dans le même temps, chaque firme peut souhaiter investir dans une technologie qui correspond au mieux à ses capacités à l'utiliser. Dans la seconde partie de cette thèse, nous allons examiner ce type de jeu de coordination à information incomplète dans lequel chaque individu cherche à choisir une action à la fois proche des actions choisies par les autres individus et proche de son "action idéale". Chaque action idéale dépend de l'état de la nature et d'un biais idiosyncrasique, comme dans le modèle de *Communication Gratuite* (ou *Cheap-Talk*) de Crawford and Sobel (1982). Ces biais varient d'un individu à l'autre, et le profil de biais dans la population mesure donc le conflit d'intérêt auxquels les agents font face.

Dans ce type de situations, la seconde partie de cette thèse a pour objectif d'analyser comment les agents se transmettent stratégiquement les signaux privés qu'ils détiennent sur les fondamentaux. En effet, avant de choisir les actions qui détermineront leurs paiements, nous offrons aux joueurs la possibilité de communiquer les uns avec les autres de manière *décentralisée et stratégique.* Il n'existe aucune contrainte physique restreignant les possibilités d'échanges d'informations privées. C'est parce que les agents ont des actions idéales différentes qu'ils peuvent avoir un intérêt à mentir à propos de leur type quand ils communiquent avec les agents avec lesquels ils interagiront lors de la phase de décision. Dans cette situation, nous nous focalisons sur la manière dont l'hétérogénéité des préférences influence la transmission stratégique d'informations. Précisément, la question que nous adressons est *"qui parle à qui ?"* durant l'étape de communication étant donnée la diversité des agents en matière d'actions idéales, c'est à dire de préférences. La différence entre les Chapitres 2 et 3 réside dans les protocoles de communication qui sont examinés.

Dans le second chapitre, chaque joueur a la possibilité d'envoyer de manière *privée* à chaque autre joueur un message *gratuit et non vérifiable* sur son signal. L'étape de communication consiste alors en un jeu de communication gratuite, dans lequel chaque agent est à la fois un émetteur et un récepteur de message ainsi qu'un décideur. Une des nouveautés principales de ce chapitre réside dans le fait que nous proposons de caractériser la transmission privée d'informations par ce que nous appelons un *Réseau de Communication Stratégique.* Un tel réseau détermine, pour chaque joueur-émetteur, un ensemble de récepteurs qui constitue en fait son voisinage. Nous dirons qu'un joueur est le récepteur d'un autre joueur si ce dernier lui transmet son information de manière honnête. En d'autres termes, nous considérons qu'une connexion entre deux individus matérialise une révélation d'information sincère entre ces deux protagonistes. La question qui se pose alors est de savoir comment les conflits d'intérêts entre les joueurs affectent la formation des relations matérialisant leur communication. Notre réussite principale réside dans le fait que nous caractérisons complètement, et de manière relativement simple, les réseaux qui émergent à l'équilibre en fonction de l'hétérogénéité des préférences. Cette contribution peut se résumer, grossièrement, de la manière suivante : *plus les préférences des agents sont alignées, plus ces derniers ont tendance à se révéler parfaitement leurs informations.*

Les contraintes d'incitation informationnelles traduisent le fait qu'aucun joueur-émetteur n'a intérêt à mentir à propos de son type à son ensemble (endogène) de récepteurs. Comme dans les jeux standards de communication gratuite, cette condition peut être formulée comme une condition sur la proximité entre les biais de l'émetteur et de ses récepteurs. Dans les modèles existants étendant la communication au cas de décideurs multiples mais stratégiquement indépendants (Farrell and Gibbons (1989), Goltsman and Pavlov (2009), Galeotti, Ghiglino, and Squintani (2009a)), il suffit alors de vérifier que l'émetteur n'a pas intérêt à mentir à aucun de ses récepteurs. Dans le modèle que nous proposons, les contraintes d'incitation informationnelles prennent une forme plus sophistiquée parce que les agents souhaitent coordonner leurs actions lors de la phase de décision qui succède à la phase de communication. En raison des interactions stratégiques existantes entre les récepteurs, la manière dont chacun d'eux réagit au signal d'un émetteur dépend non seulement de son signal mais également du nombre total (anticipé) des récepteurs de ce signal. Dans le même temps, comme l'émetteur souhaite également coordonner son action avec celles de ses récepteurs, chaque déviation de l'émetteur dans la phase de communication entraîne des coûts de coordination qui dépendent à la fois du nombre total de récepteurs et du nombre des ces derniers auxquels il ment. Ces observations, combinées avec l'hypothèse que les fonctions de coûts sont quadratiques, nous mènent au résultat suivant : un émetteur communique de manière honnête - ou, de manière équivalente, se connecte - à un groupe de récepteurs si son biais est assez proche de la moyenne des biais des membres de tout sous-groupe de ce groupe de récepteurs. La mesure exacte de cette proximité est déterminée par un seuil qui dépend à la fois du nombre total des récepteurs et des sous-ensembles potentiels de ceux-ci auxquels l'émetteur peut mentir.

Un élément-clé de notre caractérisation des équilibres de la phase de communication réside dans le fait que la communication entre deux agents ne dépend pas seulement du conflit d'intérêt qui existe entre eux mais également des préférences et du nombre des autres agents à qui ils parlent. En particulier, nous observons que la communication d'un émetteur donné à un large groupe de récepteurs peut émerger à l'équilibre alors même qu'il n'existe aucun équilibre dans lequel cet émetteur communique uniquement avec un sous-groupe de ce large groupe. Pour comprendre cette intuition, considérons une situation simple à 3 agents dont un seul est informé et joue donc le rôle d'émetteur : l'émetteur et un des deux agents non-informé souhaitent choisir une action adaptée exactement au vrai état de la nature ; l'autre agent non-informé est biaisé positivement, c'est-à-dire qu'il souhaite prendre une décision plus grande que l'état de la nature. Nous supposons que chaque joueur cherche également à coordonner sa décision avec celle des deux autres. Lorsque l'émetteur communique seulement avec l'agent biaisé, il a une forte incitation à minimiser, c'est à dire à "sous-rapporter", son type afin de diminuer l'anticipation de l'agent sur l'état de la nature. L'émetteur espère ainsi rapprocher l'action du récepteur de sa propre action idéale. Au contraire, lorsque l'émetteur communique à la fois avec l'agent biaisé et l'agent non-biaisé, il peut n'avoir aucune incitation à mentir conjointement à ces derniers car leur biais moyen est faible, c'est-à-dire proche du sien qui est nul. Il peut également n'avoir aucun intérêt à mentir seulement à l'agent biaisé. En effet, les deux agents sont maintenant plus réactifs au message de l'émetteur que lorsque ce dernier ne communique qu'à un seul agent. Par conséquent, une déviation entraîne une plus grande dispersion des actions des joueurs et donc induit des coûts de coordination plus élevés. L'on parlera donc d'un textitEffet Disciplinant que la Coordination de multiples audiences a sur la Communication.

Finalement, nous établissons des prédictions précises en ce qui concerne les réseaux de communication qui émergent à l'équilibre pour certaines configurations du profil de biais. Premièrement, lorsque les biais sont distribués de manière uniforme dans la population, nous montrons que la tendance d'un émetteur à communiquer augmente avec la proximité de son biais au biais moyen dans la population. En général, la communication est donc asymétrique : les centristes ont tendance à influencer plus fortement les décisions des autres agents que les extrémistes, car ils transmettent leurs informations honnêtement à des agents plus éloignés en terme de préférences. Cet effet est renforcé lorsque le besoin de coordination entre les agents est plus fort. Lorsque ce besoin est très fort, les joueurs aux biais intermédiaires peuvent communiquer avec tous les autres agents, même lorsque la dispersion des préférences est grande, alors que les autres joueurs ne révèlent leurs informations à personne. Deuxièmement, lorsque les joueurs sont organisés en groupes, chaque groupe ayant sa propre action idéale, nous montrons à nouveau que la transmission d'information entre les groupes est asymétrique : les membres du groupe le plus grand ont tendance à communiquer plus facilement avec les gens extérieur à leur groupe que les membres de groupes plus petits. En d'autres termes, ce sont les groupes de grande taille qui influencent les décisions des groupes de petite taille par la transmission crédible d'informations plutôt que le contraire.

Dans le Chapitre 2, nous montrons aussi que les Réseaux de Communication Stratégique ne peuvent pas être parfaitement ordonnés au sens de Pareto, mais que le bien-être social espéré s'accroît quand la communication augmente. Le Chapitre 3 étend ensuite l'analyse du modèle considéré dans le Chapitre 2 et propose trois protocoles de communication différents de la communication privée, gratuite et statique. Tous les trois résultent dans une meilleure transmission de l'information à l'équilibre de la phase de communication. Dans le cas de la *"Communication Publique au Sein de Groupes"*, chaque joueur doit s'exprimer publiquement face à une audience donnée et le message envoyé est, par conséquent, le même pour tous les membres de celle-ci. En comparaison avec la communication privée, cette obligation limite le nombre de possibilités qu'un émetteur a de dévier de la révélation parfaite à l'ensemble du groupe de récepteurs. Dans le cas de la *Communication Dynamique*, plusieurs étapes de communication gratuite sont offertes aux joueurs qui ont alors la possibilité d'utiliser des *Intermédiaires.* Cette opportunité a pour effet qu'un mensonge peut se propager, ce qui change ses conséquences et donc les incitations à manipuler les informations divulguées. Comme pour le cas de la communication publique, la communication dynamique affaiblit donc les contraintes d'incitation informationnelles par rapport au cas où la communication est privée et statique. Finalement, nous considérons le cas où l'information est *Vérifiable* et nous démontrons que la révélation complète d'information est possible même lorsque les conditions pour que cela se produise ne sont pas remplies dans le jeu de communication gratuite. Lorsque les types des joueurs ne peuvent être certifiés que partiellement, les conditions sur cette certification pour obtenir un équilibre avec révélation complète de l'information dépendent du profil de biais.

Revue de la Littérature :

Dans la classe des jeux de coordination à information incomplète, l'hypothèse que les individus diffèrent en matière d'information est assez courante mais pas celle de l'hétérogénéité des préférences. Par conséquent, la question qui est typiquement adressée concerne la manière de disséminer efficacement les informations sur l'état du monde. Depuis Morris and Shin (2002) and Angeletos and Pavan (2007), la manière dont la coordination des agents et leur bien-être est affectée par la structure d'information a ainsi été bien mieux comprise, en particulier en ce qui concerne la nature publique ou privée des signaux des individus. Lorsque les objectifs des agents sont alignés mais qu'il existe des coûts ou des contraintes physiques à l'établissement de liens de communication entre les agents, un autre sujet d'étude concerne l'identification des structures de communication les plus efficaces. Ce problème a été analysé par Morris and Shin (2007), Calvó-Armengol and Martí (2007). Calvó-Armengol and Martí (2009) déterminent ainsi l'architecture du réseau de communication qui améliore la performance de l'organisation. Une caractéristique commune à ces travaux est qu'il n'y a pas de conflit d'intérêts entre les agents en ce qui concerne le profile d'action idéal contingent à l'état du monde, alors que c'est le point central du modèle proposé dans la seconde partie de cette thèse.

Comme nous permettons aux joueurs une communication gratuite dans le second chapitre, ce dernier est lié à la littérature basée sur les travaux de Crawford and Sobel (1982). Notre modèle considère des décideurs multiples et interdépendants, chacun d'eux étant doté d'une information privée. Au contraire, les extensions du modèle émetteur-récepteur de Crawford and Sobel (1982) incluant plus de deux joueurs se sont concentrées sur le cas de plusieurs émetteurs et d'un unique récepteur. Une exception dans cette littérature sur la communication gratuite avec plusieurs récepteurs (mais un seul émetteur) est le papier de Farrell and Gibbons (1989). Dans leur travail, la question principalement adressée concerne la différence qu'il existe entre un envoi privé ou public de messages. Les auteurs identifient en effet une situation qu'ils appellent *Discipline Mutuelle due à la Communication Publique* dans laquelle l'information de l'émetteur n'est pas révélée aux récepteurs lorsque la communication est privée alors qu'il existe un équilibre où la révélation de l'information est parfaite si celle-ci a lieu en public. Contrairement à **?**, les récepteurs que nous considérons ne sont pas des décideurs indépendants dont les actions sont séparables dans la fonction d'utilité de l'émetteur. Nos décideurs interagissent dans lors de la phase de décision, ce qui est à l'origine de notre *Effet Disciplinant de la Coordination sur la Communication* mentionné précédemment. En matière de protocole de communication, le Chapitre 3 est lié à la littérature dans laquelle les joueurs sont capables de fournir des informations verifiables sur leurs informations privées, au sens où les messages disponibles pour un émetteur sont dépendants de son type. La relation entre notre travail et les articles traitant des messages certifiables, comme Milgrom (1981), Green and Laffont (1986), **?** et Seidmann and Winter (1997), est discutée plus en détail dans le Chapitre 3.

Comme nous déduisons l'existence de connections entre les joueurs de l'informativité de leurs stratégies de communication, le Chapitre 2 peut être relié aux *Jeux de Formation de Réseaux*, présentés dans le Chapitre 9 du manuel de M.O.Jackson (Jackson (2008b)). Rappelons que ces jeux, qui ont une place centrale dans la *Théorie des Réseaux Economiques et Sociaux*, examinent comment l'architecture de ceux-ci dépend des stratégies des agents. Cependant, la manière dont les liens de communication sont formés dans la seconde partie de cette thèse diffère complètement de celle proposée dans les Jeux de Formation de Réseaux existants. Dans ce type de jeux, les stratégies des joueurs consistent principalement à lister les contacts désirés, étant donnés, de manière exogène, les coûts et bénéfices des connections directes et indirectes. De plus, comme il est largement admis qu'une grande part des informations nécessaires à la prise de décisions économiques sont échangées dans les réseaux de relations, la valeur des connections est souvent interprétée en terme informationnel. Cependant, la question de savoir si les agents, une fois liés, ont effectivement intérêt à transmettre leurs informations n'a, semble-t-il, pas été étudié. Au contraire, nous nous focalisons sur les incitations à manipuler les informations. Nous fondons ensuite la révélation honnête d'informations avec la construction des canaux à travers lesquelles celle-ci a lieu. Les bénéfices tirés des connections sont alors déterminés de façon endogène par la manière dont les informations transmises seront utilisées dans la phase de décision.

Nous avons mentionné précédemment le lien entre le premier chapitre de cette thèse et la Théorie des Equipes. Dans cette théorie, les agents sont supposés agir dans l'intérêt de leur groupe et la préoccupation principale concerne alors les coûts de communication. En ce sens, l'on peut donc considérer la Théorie des Equipes comme complémentaire à la vision des organisations en terme de Principaux et d'Agents. Un résultat central de la littérature sur les relations entre Principaux et Agents est ce qu'on appelle le *Principe de Révélation*, dont la validité repose sur l'absence de coûts de communication ou de traitement des informations. Sous quelques hypothèses supplémentaires, ce principe établit qu'un mécanisme direct, c'est-à-dire dans lequel les agents rapporteraient leurs informations privées à un organe central, ne peut être dominé par aucun autre moyen de déléguer le rassemblement des informations. En d'autres termes, il démontre que le résultat obtenu dans une structure décentralisée peut toujours être obtenu par un organisme centralisé dans lequel la responsabilité des agents se limite à communiquer leurs informations à une autorité qui leur dicterait ensuite les instructions. En ce sens, les organisations centralisées sont toujours faiblement optimales. Les récents travaux de Alonso, Dessein, and Matouschek (2008) et Rantakari (2008) développent un modèle dans lequel le Principe de Révélation ne peut pas être appliqué puisque les agents ne peuvent s'engager sur les mécanismes mis en place. Ces deux articles s'avèrent être les plus proches de la seconde partie de ma thèse en ce qu'ils considèrent aussi des conflits d'intérêts en matière de décisions et par conséquent endogénéise la communication entre les agents. Ils analysent la communication stratégique dans une organisation à deux divisions, dans laquelle les décisions des divisions doivent répondre à des particularités locales ainsi qu'être coordonnées les unes aux autres. Les paiements des décideurs sont similaires à ceux que nous considérons mais les conflits d'intérêts en matière de décisions sont modélisés différemment. Dans Alonso et al. (2008) and Rantakari (2008), le manager de chaque division a une action idéale qui dépend d'un état idiosyncrasique et cherche à maximiser une somme pondérée du profit de sa division et du profit de l'autre division. Le focus est porté sur la détermination de la meilleure manière d'organiser les échanges d'informations entre les deux divisions en fonctions des biais de ces dernières et de l'importance relative de leur besoin de se coordoner. Comme nous le faisons dans le Chapitre 2, ils caractérisant les équilibres de la phase de communication en fonction des préférences des managers et démontrent qu'une organisation décentralisée peut être strictement optimale.

Conclusion

Dans le premier chapitre, j'analyse un jeu dynamique dans lequel les joueurs sont les membres d'un réseau fixé. A chaque période, chaque agent décide ou non de passer son information privée à ses voisins directs. Il lui est impossible de manipuler l'information qu'il transmet. Etant donné le dilemme informationnel auquel les agents font face, la structure du réseau affecte le temps nécessaire pour atteindre l'objectif commun de centralisation des informations à l'équilibre. Au niveau individuel, la position de chaque joueur a une influence sur la date la plus petite à laquelle il peut gagner à l'équilibre. Dans le second chapitre, nous nous focalisons sur les incitations des agents à manipuler leurs signaux privés. Dans ce cadre, la communication stratégique entre les joueurs dépend de l'hétérogénéité de préférences. Initialement, les joueurs n'appartiennent pas à une réseau donné mais leurs connections sont déduites de l'informativité de leurs stratégies de communication.

Mon travail examine donc la relation entre *communication stratégique* et *réseaux* en considérant deux types distincts d'hétérogénéité des agents : dans la première partie, des joueurs homogènes différent en matière de position dans un réseau ; dans une seconde, les joueurs varient en matière de préférences mais pas en ce qui concerne leur localisation dans une structure de communication. Deux des protocoles de communication que nous proposons dans le Chapitre 3 réconcilient ces deux visions. Dans le Chapitre 2, nous proposons un jeu de communication gratuite avec plusieurs émetteurs et récepteurs et autorisons chaque joueur à envoyer un message différent à chacun des autres joueurs. Dans une certaine mesure, la *Communication Publique au Sein de Groupes* ou la *Communication Dynamique* permettant l'utilisation d'intermédiaires suppose implicitement l'existence d'une structure de communication pré-existante. Dans le Chapitre 3, nous montrons que de tels protocoles facilitent la transmission d'informations par rapport à la communication privée. On peut par conséquent se demander dans quelle mesure une structure de communication donnée permet de forcer les joueurs à faire des annonces publiques à des groupes ou à passer par des intermédiaires. D'une façon plus générale, il semble prometteur de construire un cadre permettant d'analyser les stratégies de joueurs influencés à la fois pas leurs places dans une strcuture sociale et pas leur goûts propres.

Introduction

We often have to make choices without precisely knowing the relative advantage of the different options. In arriving at a decision, we may use our past experiences as well as the experiences of others, especially those who are close to us. For instance, a consumer buying a car may have to choose a brand without being fully informed about all the alternatives. As it is a major purchase, the potential buyer may discuss the pros and the cons of the choices with friends or colleagues. Similarly, workers in a team may have to invest in new pieces of software without complete knowledge of their ease of use. They usually read professional magazines as well as try to gather the information items that are dispersed within the team in order to choose the technology best adapted to the group's ability and needs. In many situations, because individual entities access various sources of information or learn different things from their past decisions, "unity is strength" regarding disseminated knowledge. In the present thesis, I will consider a general class of contexts exhibiting the following common feature: *information about the fundamentals that are relevant for a group of agents is dispersed among the members of that group.* Consequently, sharing privately held information constitutes the only way agents can accurately assess the state of the environment they evolve in, and react appropriately to it.

Instead of taking place via centralized institutions, it seems now largely admitted that many of such information exchanges occur through *informal networks*. A communication network can be defined as a set of links through which agents' information circulates. It is a broad notion that encompasses the relationships of interacting friends, virtual ties that individuals build on the Internet, as well as regular exchanges that a bunch of colleagues have at work. There may be as many kinds of communication networks as sorts of purposes to use them: friendships networks are an important conduit to spread the word about new shops and restaurants or to diffuse political opinions; professional networks may serve as channels for reporting vacant job opportunities or gathering knowledge about the state of a firm's demand. During the past decade, the study of networks has been a very active area of research in economics. As detailed below, the present work is in line with the developing *theory of social and economic networks*, surveyed in Jackson (2008b).

Two main kinds of decentralized communication in networks can be distinguished. In some situations, information transmission between agents is *mechanical*. It can be an automatic diffusion of informational items, in the sense that it is not even a choice made by individuals. For instance, it can be that people we meet on a regular basis have learnt about our favorite shops and use this information for their own purchase. Next, it can result from a choice made by agents and yet still be mechanical. Indeed, in a group of networked members, if collective and individual incentives in sharing information are perfectly aligned, then it is in every agent's interest to communicate truthfully and rapidly. One can think about spreading the word concerning the use of a new communication technology whose benefits for each user always increases with the number of other users. When information transmission is automatic or a dominant strategy for every agent, its process can be related to the architectural properties of the network. Mechanical communication in networks is not the object of the present work. A clear presentation of diffusion and learning in networks is provided by Chapters 7 and 8 of Jackson (2008b).

In other situations, in which the pattern of information flows between agents has a decisive impact on their choices, communication has a *strategic* aspect. Precisely, it is by manipulating his information or delaying its transmission that an agent seeks to influence others' behaviors. Then, network structure also matters but with the added feature that information flow results from calculated choices. Let's consider that a *communication dilemma* exists whenever the collective interest of a group demands that its members perfectly share privately held information, but their individual interests instead motivate them to withhold or manipulate it. Such informational dilemmas are at the core of the present work. Precisely, I study situations in which the overall structure of incentives inhibits information sharing among moderately informed members of a group. In the first part of my thesis, made up of Chapter 1, the dilemma lies in that agents have an individual incentive to keep private information secret as long as possible, while communicating quickly is in the collective interest. Chapters 2 and 3, which result from a joint work with Frédéric Koessler (Researcher at CNRS, affiliated to Paris School of Economics), make up the second part. In this part, an informational dilemma arises because truthful revelation of private information is socially efficient while it may be in the individuals' interest to lie.

I believe that many economic situations can be thought of as communication dilemmas. For example, the different divisions of an organization may need to pool their private information to get a clear view of the whole firm's environment and make appropriate choices. The decisions made in the firm, being by one or several decision-makers, then surely impact every division. Therefore, every division may try to hide or manipulate the information items transmitted to push actions in its ideal direction. Said differently, even if divisions have a common interest in maximizing the firm' profit, they may also vary in their most preferred outcome because of local tastes or abilities. Similarly, activists in a political party may possess different pieces of information that have to be gathered to better evaluate the situation and promote a correct policy. And yet, it is easy to figure out that career concerns may impact the revelation of information occurring between activists. When there is no perfect alignment of the individual and collective interest in sharing information within a group, a question arises about *how members strategically communicate to push outcomes at their own advantage.*

In such contexts of informational dilemmas, this thesis aims at shedding light on *the link between the way agents strategically transmit private information and the structure of the networks they are arranged in.* This link can go in two ways depending on whether the pattern of links "influences" or "is shaped" by communication. Each way corresponds to one of the two main questions which have emerged from the recent study of networks by economists : first, how do network structures affect economic outcomes ? second, why do certain patterns of links emerge ? Non-cooperative game theory has been applied to the two resulting areas of research and this thesis yield new insights in both. One the one hand, players in "Network Games" are the members of a given network. The objective is to investigate how their strategic behaviors are influenced by the architecture of their links. My first chapter is in line with such models. Indeed, I ask how the *fixed* pattern of communication links affects the way networked agents perform in pooling initially dispersed information. On the other hand, "Network Formation Games" analyze agents' incentives to connect to each other. In the second and third chapters, I examine how the informativeness of communication between players *shapes* the structure of their communication ties. Because they use different theoretical frameworks, we now present each part separately. Formally, networks will be presented in the standard way, that is by a graph in which nodes represent individual entities and arcs link nodes when a relationship exists between them.

Strategic Communication *in* Networks: Chapter 1

In this chapter, I introduce a dynamic game whose players are arranged in a fixed network. Everyone is initially endowed with an information item that he is the only player to hold. Players are next offered a finite number of periods to centralize the initially dispersed items. Centralization could arise at any position in the network. In every period, each player strategically chooses whether or not to transmit the items he holds to his neighbors in the graph. The sooner all the items are gathered by an individual, no matter whom, the better it is for the group of agents as a whole. Besides, the player who first gathers all the items, called the "winner", is offered an additional reward that he keeps for himself. The information dilemma lies in that players have a collective interest to share information items as well as an individual interest to hide them and wait for other players' ones to arrive. In this context, free communication among players is restricted both physically - by the network - and strategically - by the reward scheme. An example of such a scheme can be found within many forms of organization : even if it is clear that there is collective interest for members to communicate rapidly with each other to take appropriate decisions, members can, at the same time, be in competition with one another to get promoted or to gain any form of gratitude. In this case, each agent may wish to end up being the one who centralizes the information useful to decision-making.

As suggested by Bonacich (1990), because positions in the network are differentiated, their occupants may have different incentives to behave cooperatively - by communicating freely - or competitively - by hoarding information. Departing from that observation, my investigation of the relationship between group communication and the fixed set of information conduits is organized around two questions. From a collective point of view, the objective is to understand whether the overall network structure affects the group's ability to centralize dispersed items. The collective performance is evaluated regarding whether the team fails or succeeds in pooling information in equilibrium and by the time it needs to do so. At the individual level, I examine the extent to which an agent's position influences his chances to gather information.

The focus of the first chapter is on the strategic information exchange and I abstract from the group's decision-making process following the communication stage. First, I implicitly assume that it is so costly to take a wrong decision that no one ever takes one without having gathered every single piece of information. Second, once one is fully informed, the right decision to make is assumed to be deterministic.[2] Eventually, by allowing each member of the group to take the decision in the name of all, the chapter also puts aside questions about the allocations of decision rights.

In this framework, my main result consists in a necessary and sufficient condition for a group to reach the collective goal of information pooling in every (subgame perfect) equilibrium. Surprisingly, this condition is independent of the network structure in which the agents are arranged. Precisely, I show that a group of n players never fails to pool

[2] A similar focus on communication instead of decision-making process is found in Jehiel (1999). The author considers operating units that communicate their private information regarding decisions to be taken in an organization. He characterizes the optimal communication structures assuming that communication in a group results in the loss of transmitted messages with a probability that solely depends on the group size. In his work, the decision-maker perceives each piece of information as essential as each one has the power to reverse the right decision to take, the very primary concern of the decision-maker being to avoid taking the wrong decision.

information in equilibrium if and only if the number of periods offered to do so is at least equal to $n-1$. This threshold corresponds to the minimal number of links required to connect all the players. On the contrary, the network architecture affects the time needed before information is centralized in equilibrium. First note that, even if communication were automatic, i.e. every agent would transmit his items to all his neighbors in every period, the time needed to pool information would depend on the structure of communication channels. In particular, each player would need a minimal number of periods to centralize information, that depends on his position. This minimal number of periods physically required corresponds to a graph-theoretical measure called a player's eccentricity. It is shown that there always exists an equilibrium in which the game ends at a date that is equal to the smallest eccentricity exhibited in the group. Finally, I show that, for two particular network structures, there exists an upper bound on the duration before success in equilibrium.

Related Literature

By trying to link the performance of a group to its inner communication structure, this first chapter is related to the large literature on *team theory*. Indeed, starting with Marschak and Radner (1972), this research area analyzes decision making in firms in which information is initially dispersed and physical constraints make it costly to communicate and process this information. It is the tradeoff between these costs, in terms of physical resources and professional time, and the benefit of communication that is the core of the investigations. In this context, Radner (1992), Radner (1993) and Bolton and Dewatripont (1994) among others have searched for the optimal inner structure of an organization. These works have mainly highlighted the role of hierarchies, or more generally of centralized structures of communication, to reduce costs associated to the gathering of information items. Work by Crémer (1980), Aoki (1986), Geanakoplos and Milgrom (1991) and Van Zandt (1999) study the efficient allocation of information-processing tasks in the presence of information processing costs but assume that communication is costless. These papers are therefore not concerned with the design of a communication network.

A common feature of the literature on Team Theory is that it abstracts from incentive problems. Indeed, it usually assumes that agents act in the interest of the organization and the main concern is about costs of communication. In this sense, team theory complements the *principal-agent* view of organizations. A central result in principal-agent theory is the so-called Revelation Principle, which relies for its validity on the absence of communication or information processing costs. Under some additional assumptions, this Principle establishes that centralized control cannot be dominated by any delegation arrangement. Specifically, it demonstrates that the outcome of any decentralized organization can be mimicked by a centralized organization in which the responsibility of each agent is merely to communicate their information to a central authority and await instructions on what to do. Centralized organizations are therefore always weakly optimal. In contrast, Alonso et al. (2008) develop a simple model in which the Revelation Principle does not hold since agents are unable to commit to mechanisms. Their work is more closely related to the Chapter 2 of my thesis and is therefore detailed later. The point here is that they endogenize communication as a function of incentive conflicts between the two divisions of an organization and show that decentralized organizations can be strictly optimal. While their focus is on strategic misrepresentation of information, we instead assume that the information transmitted is hard, i.e. items cannot be misrepresented. Agents' actions consist either in Passing On or in Hiding and the items transmitted can be for instance any type of goods that have to be put together to be valuable.

In *network games*, the strategies of the exogenously connected players consist, of course, in a variety of choices that differ from whether or not to transmit information. For instance, Bramoullé and Kranton (2007) consider a Public Good game where players' utilities depend on a sum of their own contributions and the contributions of their neighbors. They assume that contributions of players are strategic substitutes. They find that there is multiplicity of equilibria and use graph-theoretical concepts to characterize them. To solve the game, I also refer to results from graph theory and, in particular, introduce the graphical notion of node's eccentricity. Ballester et al. (2006) analyze a class of complete information games with quadratic payoffs and pairwise-dependent strategic complementarities. They show that, in equilibrium, the effort exerted by each agent strongly depends on his position in the network of relations. In particular, this effort is proportional to his Katz-Bonacich centrality measure (Bonacich (1987)). In Chapter 1, I relate some of the players' equilibrium payoffs to their position, but have not managed to do so with their equilibrium strategies. To analyze games played by members of a fixed network, a more general framework is presented in Galeotti et al. (2009b). The authors allow for a general class of payoffs assuming that the payoff of a player depends on his own action as well as on the actions that his direct neighbors take. They provide a number of results characterizing how the network structure, an individual's position, the nature of games (strategic substitutes versus complements and positive versus negative externalities), and the level of information (incomplete versus complete), shape individual behavior and payoffs. The main difference between previously mentioned works and our network game is that it is dynamic. In my setting, payoffs depend on the history of the game, namely on the actions taken by every player in every period of play.

From a theoretical point of view, this chapter is also related to the literature that analyzes *wars of attrition* with many players. Indeed, for my dynamic network game to end up with a winner, it must be that a player manages to gather all the dispersed items at his position before the deadline is reached. In a complete network, since every agent is directly linked to every other one, an information item which is transmitted is immediately held by every player. As a consequence, as soon as all the players except one have "conceded", the game ends with the player who has not conceded yet winning. To that extent, the dynamic game studied can be viewed as a war of attrition of complete information in which n symmetric players compete for one prize in discrete and finite time. As in the war of attrition, every player strictly prefers to win than to lose but prefers to lose sooner than later. The difference lies in the fact that the order in which the $n-1$ concessions have to happen depend on the structure of the network arranging the player. Hendricks and Wilson (1988) provide a complete characterization of all the subgame-perfect equilibria for two-players wars of attrition in continuous time and complete information. Extending their result to our game played in a complete network is immediate : for every individual, there is a subgame-perfect equilibrium outcome in which only that individual wins immediately. I generalize this statement to any network structure by showing that, for every individual, there is a subgame-perfect equilibrium in which only that individual wins at the earliest date physically possible for him, i.e. at a date that is equal to his eccentricity.

Eventually, this chapter provides theoretical support to Bonacich (1990)'s experimental study that asserts that the outcome of communication dilemmas is affected by the architecture of the links between players.[3] More generally, his experiments are in line with early research in *social psychology* that has documented the crucial role of communication time and communication pattern for information aggregation purposes. It is reported in the seminal works by Bavelas (1950) and Leavitt (1951). They initiated a plethora of empirical works interested in discovering whether group performance could be enhanced through the manipulation of the configuration of communication channels. It was not accompanied by much theoretical development. Shaw (1964) provides a review of this literature, which mainly compares centralized versus decentralized network structures. The experimental findings indicate that the relationship between communication structures and a group's performance highly depends on the task that the group has to perform. It appeared that when the task is relatively simple and requires only the collation of information, centralized structures are likely to be more facilitating than decentralized ones.

Strategic Communication *building* Networks: Chapters 2 and 3

Many economic situations involve agents who share an interest in coordinating their actions as well as in adapting them to an unknown state of the world. In the second and the third chapters, we consider this type of context but depart from the typical assumption that agents agree on the state-contingent optimal profile of decisions. Because their tastes may differ, we let the interacting agents vary in their ideal proximity to the underlying

[3] The author presents two experiments where participants are initially given non-overlapping subsets of letters from a quotation that they have to identify. There are several communication rounds, each being an opportunity for networked subjects to share their letters along given links. If the group identifies the quote, each member receives an identical reward, reduced by a penalty that increases with the time needed to succeed. The first network member who identifies it receives an additional reward that he keeps for himself.

fundamentals. As mentioned previously, it is widely admitted that the different divisions of an organization have to coordinate their actions to maximize the firm's profit, with such actions also corresponding to the uncertain environment of the firm. For a number of reasons, ranging from local adaptation costs to career concerns, it is likely that each of those divisions will also attempt to adapt its choice to some local particularities. Similarly, when investing in a new technology, a firm wishes to make the choice that enables to meet customers' attempts as well as be in line with the other firms' investments because of strategic complementarities. At the same time, every firm may want to invest in the technology that best meets its own capacities to use it. We consider this type of coordination game of incomplete information in which every player incurs losses from any mismatch between his action and both others' actions and his own "ideal action". Every ideal action depends on the state and on an idiosyncratic bias, as in the cheap-talk model of Crawford and Sobel (1982). These biases vary across agents, and the profile of biases in the population is a measure of the conflict of interest they face.

In this type of situations, the second part of my thesis aims at analyzing how agents strategically transmit to each other the signals they privately hold about the fundamentals. Indeed, before taking their payoff-relevant actions, we offer players the opportunity to communicate with each other in a decentralized and strategic manner. There is no physical constraints that restrict the possibilities to exchange private information. It is because agents differ in their ideal decisions, that they may have incentives to lie about their type when sending messages to the players with whom they interact in the decision stage. In this setting, our focus is on the way heterogeneity in preferences shapes strategic information transmission. Precisely, the question we address is *who speaks to whom* during the communication stage given players' heterogeneity in ideal actions. Chapters 2 and 3 only differ in the communication protocols that are examined.

In Chapter 2, we allow players to send costless, non-verifiable, and private messages about their signals. The communication stage therefore consists of a cheap-talk game in which every player is, at the same time, a sender and a receiver. One of the main novelty of this chapter is that we propose to characterize the transmission of private information by what we call a “communication network”, described by a set of “receivers” for every player. A player is said to be a receiver of another player if the latter truthfully reveals his private information to the former. Said differently, we let a connection between two players materialize truthful revelation of information between them. The question asked is then how players’ conflicts of interest influence their communication which shapes the pattern of their links. From the perspective of network theory, this chapter is in line with Network Formation Games in that it examines how the architecture of networks is affected by agents’ strategies. Our main achievement lies in that we provide a complete and tractable characterization of the networks emerging in equilibrium, as a function of players’ heterogeneous preferences. This contribution roughly boils down to the intuitive statement that agents are more prone to perfectly reveal their information, or equivalently to link, when their preferences present some alignments. Precisely, we show that an agent communicates, or connects, to a group as long as his ideal action is close enough to the average ideal action of every subset of agents in this group. A key feature of our equilibrium characterization is that whether communication takes place between two agents depends not only on the conflict of interest between these agents, but also on the number and preferences of the other agents with whom they communicate. In particular, we observe that communication to a large group of recipients may occur in equilibrium even though communication to a small subset of that group may not. Eventually, for two natural configuration of biases, we show that agents who are more central in terms of preferences tend to communicate more and to have a greater impact on decisions.

In Chapter 2, we also show that strategic communication networks cannot be completely ranked in the sense of Pareto, but that expected social welfare always increases when the communication network expands. Chapter 3 extends the analysis of the model considered in Chapter 2 and proposes three variations of the communication protocol which all result in more effective information transmission than private and one-shot cheap talk. Under group communication, every player is required to publicly send the same costless message to all the agents in a given group. Compared to private cheap talk, this requirement reduces the number of possible deviations that a sender has from truthful revelation to the whole group. Under dynamic communication, several cheap-talk communication rounds are offered to players, which enable them to transmit private information using "intermediaries". This opportunity changes the effect of a sender's lie in a way that weakens informational incentive constraints compared to static communication. Finally, we consider the case of verifiable information and prove that complete information revelation is possible even when the conditions for a fully revealing equilibrium to exist in the cheap-talk communication game are not satisfied.

Related Literature

In the same class of coordination games as the one we consider, individuals usually differ in terms of knowledge but not in terms of preferences. In such cases, the question typically asked is about the most efficient way to disseminate information. Since Morris and Shin (2002) and Angeletos and Pavan (2007), it is well-understood how coordination and welfare are affected by the information structure, and in particular by the public or private nature of individuals' signals. With agents' goals aligned but physical or cost constraints on the number of communication links between agents, another object of study is to identify the most efficient communication structures. This problem has been analyzed by Morris and Shin (2007), Calvó-Armengol and Martí (2007). Calvó-Armengol and Martí (2009) single out the geometry of communication links among agents that would improve the organization's performance. A common feature of these papers is that there is no conflict of interests between agents regarding the ideal state-contingent action profile. In contrast, there are *no physical constraints* restricting the possibilities of information revelation in the communication stage we propose.

Since *cheap-talk* communication is offered to players before they take their actions in Chapter 2, our paper is methodologically related to the literature built on Crawford and Sobel (1982). Our model includes multiple and interdependent decision-makers, all of them being endowed with private information. On the contrary, most extensions of Crawford and Sobel (1982)'s sender-receiver game with more than two players involved multiple senders (with no decision) but one uninformed receiver.[4] One exception in the literature on cheap talk with multiple receivers (but only one informed sender) is the paper by Farrell and Gibbons (1989). In their setting, the main question addressed is whether sending private or public messages to the receivers makes a difference. Indeed, in Farrell and Gibbons (1989), a situation called "mutual discipline of public communication" is identified in which information is revealed to neither decision-maker when communication is private but a fully revealing equilibrium is played when communication takes place publicly. Contrary to Farrell and Gibbons (1989), the receivers we consider are not independent decision-makers whose actions are separable in the sender's utility function. The fact that our decision-makers play together in the decision stage is at the origin of a new effect that we called "disciplinary effect of coordination". Regarding communication protocols, Chapter 3 is linked to the literature in which players are able to provide hard, verifiable, or *certifiable information* about their type, starting with Milgrom (1981), Green and Laffont (1986), Okuno-Fujiwara, Postlewaite, and Suzumura (1990) or Seidmann and Winter (1997). How our work is related to these works is discussed more extensively in Chapter 3.

The two papers which are the most closely related to the second part of my thesis in that they consider incentive conflicts over decisions and therefore endogenize communication between agents are Alonso et al. (2008), already mentioned earlier, and Rantakari (2008). They both analyze strategic communication in a two-division organization in which the decisions of the divisions must be responsive to local particularities as well as coordinated with each other. Decision-makers' payoffs are similar to the ones we consider but conflicts of interest regarding decisions are modeled in a different way. In Alonso et al. (2008) and Rantakari (2008), each division manager has an ideal action that depends on an idiosyncratic state and maximizes a weighted sum of his own division's profit and the one of the other division. The focus is on determining the best organizational arrangement driven by these biases and by the relative importance of coordination need.

Since we derive connections between players from the informativeness of their communi-

[4]See, among others, Battaglini (2002), Krishna and Morgan (2001), Ambrus and Takahashi (2008), and Morgan and Stocken (2008).

cation strategies, Chapter 2 is related to *network formation games*, presented in Chapter 9 of Jackson (2008b). However, the way in which communication links are constructed in the second part of this thesis completely departs from usual such games in a number of ways. In typical games of this type, players' strategies mainly consist in listing desired contacts, given the exogenous costs and benefits of direct and indirect connections. In addition, since it is commonly admitted that much of the information required for economic decision-making is exchanged via networks of relationships, the value of these connections is often interpreted as being informational. However, whether agents have an effective interest in transmitting information once a link exists has not yet been investigated to the best of my knowledge. By way of contrast, we focus on the incentives to misrepresent information and merge the truthful communication with the building of the channel trough which that communication occurs. The benefits from linking are then endogenously determined by the way in which the information transmitted is used in the decision stage.

Chapter 1

Centralizing Information in Networks

1.1. Introduction

Bonacich (1990) reports an experiment in which success of a given group depends on an effective flow of information among the members of this group. Precisely, subjects were initially given non-overlapping subsets of letters from a quotation that the group of participants had to identify. Only once an individual identified the quote and independently of who did so, the group received a *Collective Reward*, equally shared between its members. This collective reward was reduced by a penalty that increased with the time needed to reach the common goal. To gather letters, subjects were offered several communication rounds, each being an opportunity for agents to transmit their letters along given communication links. Indeed, participants were arranged in a fixed network, whose links were the only possible channels letters could flow through. In addition to be physically restricted by the architecture of the communication links, the transmission of letters had a strategic aspect. Indeed, the participant who first identified the quotation in the name of the whole group was offered an *Additional Reward* that he kept for himself. Therefore, individuals had a collective interest to share their letters as well as an individual motivation to hoard them while waiting for other players' ones to arrive. Bonacich's experiment was run for different network structures and whether a subject communicated extensively or withheld letters appeared to depend on its network position. At a global level, Bonacich's experimental results support the following hypothesis : the outcome of the experimental game is affected by the architecture of the network players belong to. The present work proposes a model in which this hypothesis can be made precise and given theoretical support.

Bonacich's experiment is representative of a large class of situations in which the problem of communication between information holders arises, communication being physically restricted as well as limited by strategic retention of information. In organizations, the nature of a team's decision is often such that it requires the aggregation of some privately held pieces of information.[1] In this paper, we consider that the team's collective task is to put together *all* the information items that are initially dispersed.[2] As teams often exist as a part of larger organizations, they seldom have the freedom to make adjustments of the stated patterns of communication used to pool information. We therefore examine the transmission of items along the links of a fixed communication structure.[3] We further consider that the agent who first centralizes information in the interest of his team individually benefits from this achievement. For instance, such an additional gain can take the form of a monetary reward, a promotion or gratitude form other members.

In this framework, we investigate how the fixed communication network affects the group's ability to centralize information items in equilibrium. We address the question of whether it may be that among several communication patterns, all physically adequate for the successful completion of the common task, one results in a significantly "better" equilibrium outcome than an other. As there is not a unique definition of what "better" means in this context, we examine the effect of the network structure on the group performance

[1]For instance, in Jehiel (1999), an organization is in charge of a decision and each operating unit of the firm holds a partial and crucial information on the decision to be taken. In this work, communication is not strategic but works through the formation of groups of agents at different levels : employees directly communicate within their group before representatives of each group pool information in some groups of representatives and so on. The author characterizes the optimal communication structure assuming that information transmission within a group fails with a probability that solely depends on the group size.

[2]It is implicitly assumed that centralizing all dispersed information items is useful to take a payoff-relevant action afterwards. However, this paper abstracts from the decision-making part : first, it is assumed that once the items are gathered, a state of the world is identified for which the right decision to take is deterministic; second, we consider that no player ever takes a decision before having gathered *all* the letters cause a decision not adapted to the true state leads to a very large loss.

[3]A view of a firm's internal organization as a communication network can be found in Bolton and Dewatripont (1994) or Radner (1993).

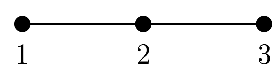
in two ways. First, the performance of a team is evaluated regarding whether there is *failure or success in pooling information in equilibrium.* Next, and if success is ensured, we examine whether the structure affects the *time the group needs to succeed in equilibrium.* If communication within the given network had no strategic aspect, the smaller the distance between a team member and every other member would be, the sooner the collective goal of items centralization could be reached.[4] In such a case, communication networks could be ranked regarding this distance only. From an individual point of view, we also seek to relate a person's position in the network to his ability to win and to the speed of his win.

Formally, we analyze games in which n players are arranged in a network g and have T periods of play to put together n dispersed items. Each agent is initially given a unique item that he is the only player to hold and items are assumed obsolete after date T. In every period of this dynamic game, each player strategically chooses either to *Hide* or to *Pass On* to his neighbors in the network the items that he holds at that time. The game is of perfect information as actions are perfectly observed in every period. Two networks g and g' are compared with respect to the equilibrium outcomes of the two games played in g and g'. Our analysis yields two main insights. First, we provide a necessary and sufficient condition for a group to centralize items at some position in the network in *every* (subgame perfect) equilibrium. Interestingly, this condition is independent of the network structure. Precisely, we show that a group of n players never fails to pool information in equilibrium if and only if the number of periods offered to do so is at least equal to $n-1$, no matter the network players are arranged in. Next, we claim that network structure affects the time needed for the n items to be gathered in equilibrium. Even in the case in which every player Passes On his items to all his neighbors in every period, every player needs a minimal number of periods to win that depends on his position. This minimal number of periods physically required corresponds to a graph-theoretical measure called a player's eccentricity. We prove that, for every player, there exists an equilibrium in which this player is the unique winner at a date that is equal to his eccentricity. It follows that

[4]Ignoring strategic aspects, the impact of the communication structure on group performance is the object of a vaste literature in social psychology mainly based on Bavelas (1953) and Leavitt (1951).

there always exists an equilibrium in which the game ends at the earliest date physically possible for the group. This date is given by the minimal eccentricity in the network, called its radius. Finally, we show that, for two particular networks, namely trees and complete ones, there exists an upper bound on the duration before success in equilibrium.

The game we analyze is a *Network Game* in the sense that non-cooperative players are the members of an exogenous network. It contributes to the economic literature studying games played on social networks extensively surveyed in Goyal (2007) and Jackson (2008b). Galeotti et al. (2009b) present a very general framework for *static* network games. The authors assume that a player's payoff depends on his own action as well as on the actions taken by his direct neighbors in the graph. The same assumption is made in computer sciences models of *Graphical Games* introduced by Kearns, Littman, and Singh (2001). Graphical games literature focuses on finding algorithms to compute equilibria in *one-stage* games played *on* large-scale networks. In the present work, the game played by network members is *dynamic*. Players' payoffs directly depend on the actions taken by *every* member of the network in every period of play *and* on the order of these actions. Indeed, in the game we build, information is pooled not only if every player transmits the items he holds, but also if it happens in a particular order that depends on the network structure. To understand this idea, consider the following network g_{line} :



To get the three dispersed information items privately held by every player at the beginning of the game, player 1 not only needs players 2 and 3 to Pass On but he also needs player 3 to Pass On *before* player 2 does so. In the network g_{line}, player 2 is an intermediary for the transmission of information from agent 3 to 1.

The paper is organized as follows. In the next section, we present the model. The necessary and sufficient condition to ensure information centralization in every equilibrium is provided in Section III. Results on possible sets of winners in equilibrium are presented in Section IV. The focus of Section V is on the time needed to pool information items in equilibrium. Section VI concludes. Proofs are mainly relegated to the Appendix.

1.2. The Model

1.2.1. Set-Up

Players, Actions and Network : The set of agents is $N = \{1, \ldots, n\}$. Agents are arranged in a *connected* network[5] represented by a graph g, with $ij \in g$ if player i is linked to player j. We assume that communication links are undirected so that $ij \in g$ implies $ji \in g$, meaning that information can flow in both ways. For a given network g, the geodesic distance $d_{ij}(g)$ between agents i and j is the length of the shortest path between them. Let $N_i(g)$ be i's neighborhood in g : $N_i(g) = \{\ j \in N\backslash\{i\} : ij \in g\}$. We denote $g\backslash\{i\}$ the subnetwork of g with the set of agents $N\backslash\{i\}$ and all links between these agents which exist in g.

The game is played over discrete time periods $t = 0, ..., T$ with a finite deadline $T \geq 1$. At each date $t \geq 1$, every player i chooses an action a_i^t from the set $A = \{P, H\}$: $a_i^t = P$ means that player i *Passes On* all the information items he holds at time t to every agents in his neighborhood $N_i(g)$ and $a_i^t = H$ means that player i *Hides* all his information items to every player. The way pieces of information are transmitted is exposed in more detail below.

An action profile at time t is a vector $a^t = (a_i^t)_{i \in N} \in A^n$. A history h^t of the game at time t is the observed past sequence of profiles of actions $(a^1, ..., a^{t-1})$, which is an element of the set of histories at date t denoted $\mathcal{H}^t = (A^n)^{t-1}$. At date t, every player perfectly observes the history h^t.

Information Items : We assume that there are n different information items, numbered from 1 to n. Initially, every player is given a unique item, which he is the only player to hold. Player i is given the item numbered i. The state of players' information at date t is given by a matrix $V^t \in \{0, 1\}^{n \times n}$ with the component v_{ij}^t of V^t equal to 1 if player i holds the item j at date t and 0 otherwise. Initially, the matrix of players' information is the identity matrix : $V^0 = Id_n$.

[5]A network is *connected* if there exists a path between any pair of distinct agents. A path is a sequence of agents for which every agent is linked to the next agent in the sequence.

The state of players' information evolves as players *Pass On* or *Hide*. We assume that, once received, an item is never lost, even if Passed On later in the game. Formally, for every $i, j \in N$, the component v_{ij}^t evolves in the following way[6] :

$$v_{ij}^t = Max_{\{k \in N_i(g): a_k^t = P\}} \{v_{ij}^{t-1}, v_{kj}^{t-1}\}. \tag{1.1}$$

Payoffs, Winners and Losers : The payoff structure has common features with the one considered in Bonacich's experimental study. If there is no player who manages to gather the n items before the deadline T is reached, then players earn nothing. On the contrary, if there is at least one player who centralizes the n items in the time offered to do so, then all the players are rewarded. In this case, we denote τ the first period in which the n items are held by an agent. The game ends up at τ. At this date, a *Collective Reward* of value n is equally shared between all the players. Besides, the players who have managed to pool information items, called the *winners*, receive an *Additional Reward* of value $R > 0$. In case there are several winners, the *Additional Reward* is equally shared between them. Players who have not centralized the items are called the *losers*. Payoffs are discounted according to some common discount factor $\delta \in (0, 1]$.

For a given g, each final history h^{T+1} uniquely defines a sequence of matrices representing players' information $(V^0, V^1, ..., V^T)$. Denote ι_n the vector with n components equal to 1. The present value of player i's payoff is given by :

[6]Players' state of information is modeled using the matrix V^t of zeros and ones but it could be modeled using partitions of information as in Chwe (1999). Let's denote the state of the world by $\theta = (\theta_i)_{i \in N} \in \Theta$, where θ_i is player i's initial piece of information. At each date, player i's state of information can be represented by a partition of the set of all possible states of the world. At time $t = 0$, player i's information partition is : $\mathcal{P}_i^0 = \{P_i^0(\theta)\}_{\theta \in \Theta}$ with $P_i^0(\theta) = \{(\theta_i, \theta_{-i}) : \theta_{-i} \in \{0,1\}^{n-1}\}$. In period t, when a player i receives some information from one of his neighbors $j \in N_i(g)$, the latter updates his information partition in the following way : $P_i^{t+1}(\theta) = P_i^t(\theta) \cap P_j^t(\theta)$. The first agent $i \in N$ who manages to identify the true state of the world θ, meaning that at a date t, $P_i^t(\theta) = \{\theta\}$ wins the game. Payoffs can easily be rewritten using information partitions. Indeed, $V_i^t = \iota_n$ is equivalent to $P_i^t(\theta)$ being a singleton.

$$u_i(V^0 ... V^T) = \begin{cases} 0 & \text{if } V_j^T \neq \iota_n,\ \forall\, j \in N, \\ \delta^{t-1} & \text{if } V_i^t \neq \iota_n \text{ and } \exists\, j \neq i,\ j \in N : V_j^t = \iota_n \\ & \text{and } \forall\, k \in N\ ,\ V_k^{t-1} \neq \iota_n, \\ \delta^{t-1}(1+\frac{R}{l}) & \text{if } V_i^t = \iota_n \\ & \text{and } \forall\, k \in N\ ,\ V_k^{t-1} \neq \iota_n\ , \text{ with } l = \#\{k \in N : V_k^t = \iota_n\}. \end{cases}$$

A game involving players in the set N arranged in a network g and lasting T periods is denoted $\Gamma(N, g, T)$.

Strategies : We restrict our attention to pure strategies. A pure strategy of player i is a profile $s_i = (s_i^1, ..., s_i^T)$ with s_i^t: $\mathcal{H}^t \rightarrow A$ for every $t = 0, \ldots, T$. A strategy profile is denoted $s = (s_i)_{i \in N}$.

Example : As an example, consider the one-shot duel $\Gamma(\{1,2\}, g, 1)$ where g is the complete network. Initially, players' states of information V^0 is Id_2. Since $N_1(g) = N_2(g) = N$, if player i Passes On the item he holds initially to player $j \neq i$, then $v_{ji}^1 = v_{ii}^1 = 1$. Let's write players' payoffs in the following way $u_i(a_i, a_j)$ with $a_i, a_j \in \{P, H\}$: $u_i(P,P) = 1 + \frac{R}{2}$, $u_i(P,H) = 1$, $u_i(H,P) = 1 + R$ and $u_i(H,H) = 0$. The static duel $\Gamma(\{1,2\}, g, 1)$ is the well known *Chicken Game*, which has two Nash Equilibria in pure strategies : $(a_1, a_2) = (P, H)$ and $(a_1', a_2') = (H, P)$. Note that every equilibrium outcome is such that the game ends with a winner.

1.2.2. Equilibrium Concept

The game $\Gamma(N, g, T)$ has a multiplicity of Nash Equilibria (NE) and we do not attempt to provide a complete characterization of these. To narrow down the set of NE, the solution concept we use is the Subgame Perfect Nash Equilibrium (SPNE).[7] Since we investigate the way information is pooled in a decentralized way by the members of a fixed network, we find it reasonable to assume that players do not commit themselves to the dates at

[7]Each finite game $\Gamma(N, g, T)$ contains subgames that are uniquely defined by each history h^t and denoted $\Gamma(N, g, T)|h^t$. The strategy profile $s \in S$ is a SPNE if, for every $h^t \in H^t$, the continuation strategy profile denoted $s|h^t$ is a NE of $\Gamma(N, g, T)|h^t$.

which they plan to Pass On. Incorporating subgame perfection therefore makes sense. For every game $\Gamma(N, g, T)$, the set of (SP)NE is denoted $S_{(SP)NE}$.

The way subgame perfection eliminates non-credible threats in the game we propose appears in the following example. Consider $\Gamma(\{1,2,3\}, g_{line}, 2)$ with g_{line} the three-player network presented in the Introduction. The strategy profile that consists in *"every player Hiding in every period, whatever the history"*, is a NE. Indeed, as long as two players out of three Hide in every period, every player receives 0, whatever his strategy. Next, consider the subgame of $\Gamma(\{1,2,3\}, g_{line}, 2)$ that starts at time $t = 2$ after player 1 has Passed On at date $t = 1$ while players 2 and 3 have Hidden. In this subgame, if player 3 Passes On instead of Hiding, he receives δ instead of 0 as player 2 finally holds the three information items. It follows that *"players 2 and 3 Hiding in the second period of play, whatever the history"* is not credible.

1.2.3. Graphical Objects

We define some graph-theoretical concepts that are used in the sequel. First, a classical measure of centrality in graphs is the *eccentricity* :

Definition 1 Player i's *eccentricity* in the network g, denoted $e_i(g)$, is the distance from agent i to the agent furthest away from him : $e_i(g) = \max_{j \in N}\{d_{ij}(g)\}$.

In the game $\Gamma(N, g, T)$, player i's *eccentricity* is equal to the minimal number of periods required for player i to centralize the n items when every player Passes On in every period. Given a network g, the minimal eccentricity is called the radius $r(g)$ and the maximal eccentricity is called the diameter $d(g)$. Obviously, a player i cannot win in a game $\Gamma(N, g, T)$ that lasts strictly less than $e_i(g)$ periods. We define the following set :

Definition 2 In a game $\Gamma(N, g, T)$, the *set of potential winners* is given by $W(g, T) = \{\ i \in N : e_i(g) \leq T\ \}$.

Games of interest are games $\Gamma(n, g, T)$ such that $W(g, T) \neq \emptyset$ or equivalently such that $T \geq r(g)$. We restrict our attention to such games in the present work. Note that every player can potentially win, i.e. $W(g,T) = N$, if and only if $T \geq d(g)$.

Definition 3 In a connected graph g, an agent i is *critical* (respectively *non-critical*) if $g \backslash \{i\}$ is disconnected (respectively connected).[8]

In other words, a non-critical agent can be dropped from a connected graph with the resulting subnetwork still being connected. On the contrary, a critical agent is crucial in maintaining the connectedness of a network. By definition, an agent who is critical in g is on every path between at least one pair of agents in g.

A *complete network*, denoted $g_{complete}$, is a particular architecture in which every agent is linked to every other one, i.e. $N_i(g) = N \backslash \{i\}$ for every $i \in N$. As it implies that a link exists between every pair of distinct agents, every agent is non-critical in $g_{complete}$. A *tree network*, denoted g_{tree}, is such that there is a unique path between every pair of distinct agents. It follows that there is at least one critical agent in every tree involving $n \geq 3$ players. A connected network involving $n = 2$ players is a special structure in that it is both a complete and a tree network. More generally, the following theorem deals with the existence of non-critical agents in connected networks :

Theorem 1 *[Kelly and Merriell (1958)] In a connected network with $n \geq 2$ agents, there are at least two non-critical agents.*

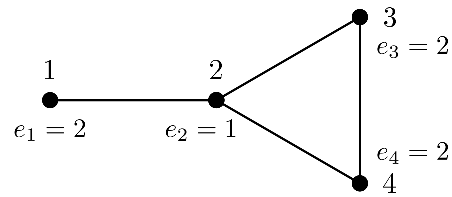
Finally, a particular type of network structure is defined with respect to the existence of a critical agent :

Definition 4 A connected network in which there exists at least one critical agent is *separable*. A connected network in which every agent is non-critical is *non-separable*.

A separable network can be disconnected by removing one agent. Tree networks involving three players or more are separable whereas complete networks are not.

To illustrate the previous definitions, we consider the following network g_{kite} which is neither complete nor a tree :

[8] The term "critical agent" refers to the "critical link" defined by Jackson and Wolinsky (1996) as crucial to maintain the connectedness of a graph.



Players' eccentricities appear near players' labels. We have $r(g_{kite}) = 1$ and $d(g_{kite}) = 2$. The sets of potential winners are $W(g_{kite}, 1) = \{2\}$ and $W(g_{kite}, T) = N$ for every $T \geq 2$. The network g_{kite} is separable with player 2 being critical whereas players 1, 3 and 4 are non-critical.

1.3. Success or Failure in Equilibrium

Our objective is to compare network structures with respect to their efficiency in encouraging information centralization when its transmission is strategic. Note that if information transmission were not strategic but were automatic in every period, network structures could be trivially ranked as $r(g)$ would be the number of periods required to centralize the n dispersed items in a network g.

The first measure of group performance that we consider is the *centralization of dispersed information in every equilibrium*. For every game $\Gamma(N, g, T)$, the set of strategy profiles S is split into two disjoint subsets. Let $S_W \subseteq S$ be the set of strategy profiles such that the game $\Gamma(N, g, T)$ ends with at least one winner, or equivalently, such that the collective goal is reached at a time $t \leq T$. Let $S_L = S \backslash S_W$ be the set of strategy profiles such that the game ends with no winner, or equivalently, such that players have failed to centralize information at some position in the network before the deadline is reached.

Recall that if a game ends with at least a winner, every player earns a strictly positive payoff whereas if the game ends with no winner, every player earns 0. It follows that failure in performing the collective task is an outcome that is Pareto dominated by any outcome in which success is ensured. The following proposition provides a necessary and sufficient condition for success to be ensured in *every* SPNE outcome of $\Gamma(N, g, T)$:

Proposition 1 *$S_{SPNE} \subseteq S_W$ if and only if the game $\Gamma(N, g, T)$ is such that $T \geq n-1$.*

That is, *every* equilibrium yields at least one winner if and only if the game lasts sufficiently many periods. On the contrary, if the deadline is $T \leq n-2$, there exists equilibria that lead to failure in the collective task. Note however that, in such cases, there may also be equilibrium outcomes such that there is a winner.

Interestingly, the condition $T \geq n-1$ is independent of the structure of the connected network g. Given a deadline T and a fixed number of players n, all communication networks are therefore equally efficient with respect to the criterion considered in this section, namely the achievement of the collective goal in every equilibrium. Precisely, a complete network happens to be as efficient as any connected structure that minimizes the number of links such as tree networks do. In settings in which building communication links is costly but neither the identity of the winner nor the time needed to succeed matters, a tree network can be chosen rather than any other structure.

The proof of the fact that $T \geq n-1$ is a sufficient condition to get $S_{SPNE} \subseteq S_W$ is done by induction and is quite constructive. One building block is the following result for two-player games : *every NE of a dynamic duel $\Gamma(\{1,2\}, g, T)$ yields at least one winner.* Indeed, for a duel to end up with a winner, it is sufficient to have one of the two players Pass On before the deadline is reached. It follows that, when two players are offered $T \geq 1$ periods of play, both players loosing cannot be an equilibrium outcome since every player can unilaterally prevent such an outcome. Next, to see how this fact about duels is used, consider the one-shot game $\Gamma(\{1,2,3\}, g_{line}, 1)$. In this game, if two or more players Hide, the game ends with no winner and this is a NE since no player can unilaterally prevent this outcome. Proposition 1 says that, adding a period to $\Gamma(\{1,2,3\}, g_{line}, 1)$ is sufficient to rule out such an equilibrium. This is due to the fact that, in $\Gamma(\{1,2,3\}, g_{line}, 2)$, player 1 or player 3 has the ability to unilaterally make the game evolve into a duel between the two other players. That duel would last at least one period. This happens to be a general feature of non-critical agents whose existence relies on Theorem 1. More precisely, if a non-critical player, say player 1, Passes On at date $t = 1$ while the other players Hide (which means they behave in the worst way regarding items centralization), then the subgame that starts at time $t = 2$ is strategically equivalent to the one-shot duel : players 2 and 3 are directly linked to each other and each player is holding some items that, if transmitted, make the other player win immediately. This is represented as *Situation A* in Figure 1.[9] Next, as mentioned above, once a duel is reached, every equilibrium yields at least a winner. The same reasoning can be applied to the game $\Gamma(\{1,2,3\}, g_{complete}, 2)$ as it is illustrated by *Situation B* on the following Figure.

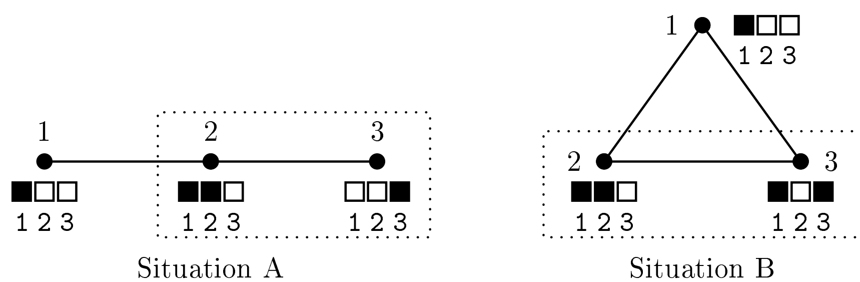


Figure 1.1: Informational situations once player 1 has Passed On and players 2 and 3 have Hidden.

From the previous paragraph, we have that every equilibrium outcome of $\Gamma(\{1,2,3\}, g, 2)$ is such that there is a winner. Next, one can get the same result for games $\Gamma(\{1,2,3,4\}, g, 3)$ by noticing that there always exists a non-critical agent in g who can, by Passing on at time $t = 1$, make the subgame that starts at time $t = 2$ be such that the other three players are in a situation strategically equivalent either to $\Gamma(\{1,2,3\}, g_{line}, 2)$ or to $\Gamma(\{1,2,3\}, g_{complete}, 2)$. This inductive reasoning enables to state that the minimal number of periods sufficient to get a winner in every equilibrium of $\Gamma(N, g, T)$ is $n - 1$. It corresponds to the minimal number of links required to connect n players. It is also the number of periods needed to reduce the game to a duel through successive items transmissions by non-critical agents who are then as "removed" from the network.

[9]On Figure 1, players' labels correspond to numbers written above the line. Every player i's informational situation is represented by three boxes numbered 1,2 and 3 and situated near player i : box numbered j near player i is filled in black if player i holds the item j and is empty otherwise.

1.4. Equilibrium Sets of Winners

The last section stressed the special role played by non-critical players. We next show that, for some particular networks, there is at least one non-critical agent who loses in *every* equilibrium. More generally, in this section, we examine the impact that network architectures have on the set of agents who may manage to centralize information items. Even in the case in which all the players had the opportunity to do so because they were offered a number of periods greater than every eccentricity, the structure prevents some players from winning together in equilibrium.

Proposition 2 *If the game $\Gamma(N, g, T)$ is such that the network g is separable, then every strategy profile $s \in S_W$ is such that there exists at least one non-critical agent who loses.*

Proposition 2 states that it cannot be that all the members of a separable network win together. Taking a look at the separable network g_{kite} presented in section 1.2.3, one can easily get an intuition of why it is so. Assume that non-critical players 1, 3 and 4 win together at time t. To get such an outcome, player 1 must hold, at date t, the items initially held by players 3 and 4 and vice versa. In g_{kite}, player 2 is the intermediary for information transmission between player 1 and players 3 and 4. It follows that players 1, 3 and 4 winning together at time t implies that player 2 already held the four items at a date $t' \leq t-1$, which contradicts the initial assumption.

As stated in definition 2, given an architecture g and a deadline T, a set $W(g, T)$ of potential winners is defined. In particular, a game $\Gamma(N, g, T)$ can be such that every player in $W(g, T)$ is critical. Since a connected structure involves at least one non-critical agent, this directly implies that every strategy profile $s \in S_W$ is such that there exists at least one non-critical agent who loses. Note that this is not the object of the previous Proposition. Indeed, Proposition 2 states that if g is separable, then all the players cannot win together and this, even in the case in which they all potentially could. Proposition 2 imposes no restriction on the set $W(g, T)$.

While the previous proposition relies on the graphical properties of separability, the following result is established for equilibrium strategy profiles. Precisely, we prove the uniqueness of the winner in equilibrium for two particular network structures. The following statement, as Proposition 2, is independent of $W(g,T)$.

Proposition 3 *If the game $\Gamma(N,g,T)$ is such that the network g is either complete or a tree, then every strategy profile $s \in S_{NE} \cap S_W$ is such that the winner is unique.*

From Propositions 2 and 3, the following statement is directly made : if the network is either separable or complete, then every equilibrium strategy profile is such that there exists at least one non-critical agent who loses. On the contrary, we now present an example in which *all* the members of a nonseparable and incomplete network *win together* in equilibrium. In the circle examined, every player is non-critical. Consider the game $\Gamma(\{1,2,3,4\}, g_{circle}, 2)$ with g_{circle} a four-players circle. We start by analyzing the subgame that would result from a first period in which every player Passes On. The informational situation of such a subgame is illustrated by *Situation C* on Figure 2. In *Situation C*, if every player Passes On, the game ends with four winners. At date $t=2$, by Hiding while the other players Pass On, no player can prevent the other three players from winning. Consequently, starting from *Situation C*, *"every player Passing On"* is a NE that yields four simultaneous winners.

Next, we analyze the subgame that would result from a first period of play in which one player, say 1, Hides and the other three players Pass On. *Situation D* on Figure 2 shows the resulting informational situation. In *Situation D*, if player 1 Passes On while the other three players Hide, the game ends up with players 2 and 4 receiving $\delta(1+\frac{R}{2})$ whereas players 1 and 3 earn δ. At date $t=2$, by Hiding instead of Passing On, player 1 makes the game end with no winner. Given *Situation D*, player 3's action has no impact on the outcome of the game. Finally, if player 2 or 4 deviates from Hiding, it makes three players win ($1,2$ and 4) instead of two (2 and 4). Therefore, starting from *Situation D*, *"player* 1 *Passing On while the other players Hide"* is a NE that yields two simultaneous winners, players 2 and 4.

Finally, comparing *Situations C and D*, player 1 has no interest in deviating from Passing On in the first period when the other three players Pass On. The same is true for every player as their positions are symmetric. We conclude that there exists an equilibrium action profile such that every player Passes On in the two periods of play yielding four winners at time $t = 2$.

What makes nonseparable and incomplete networks different from other structures is that they exhibit at least two paths linking any pair of agents. Therefore every information item can flow at least two distinct ways to go from the initial holder to any agent. As a result, for every agent, it can happen that, by Hiding, he is unable to stop some items' transmission because they are transmitted along the other possible path. It follows that a subgame can start in which there does not exist a single player who is able to prevent all the players from winning together.

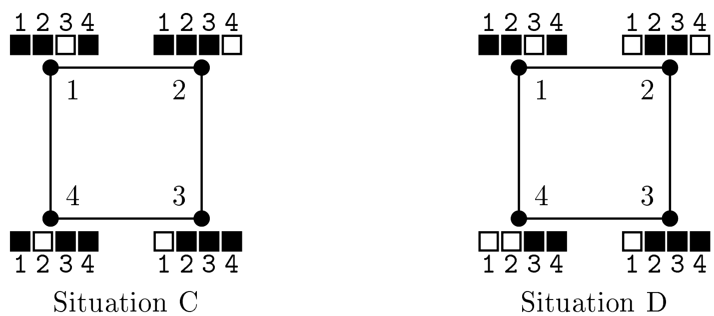


Figure 1.2: Two informational situations at the end of time $t = 1$.

1.5. Equilibrium Duration before Success

Among equilibria that yield success, aggregate payoffs are lower when information is centralized at time t than at any earlier date $t' < t$.[10] This section focuses on *the time needed for the group of players to succeed in equilibrium.* From a global point of view, the minimal duration before success in equilibrium gives the *best* equilibrium outcome and, once success is ensured in every equilibrium, maximal duration before success gives the *worst* equilibrium outcome.

[10]Indeed, if a game ends up with at least a winner at time $t \leq T$, then the aggregate payoffs are equal to $\delta^{t-1}(n + R)$.

1.5.1. *Minimal Duration*

Next proposition relates a person's position in the network to his ability to win and to the speed of his win. Recall that we restrict our attention to games in which $W(g,T) \neq \emptyset$, meaning that there exists at least one agent who is physically able to win. More precisely, it states that, for every player in the set of potential winners, there exists an equilibrium such that this player is the unique winner after a number of periods just equal to the time physically required to centralize the dispersed items at his position.

Proposition 4 *For every player $i \in W(g,T)$, there exists a SPNE such that player i is the unique winner at time $t = e_i(g)$.*

In every game $\Gamma(N,g,T)$, duration before success has a lower bound that depends on the architecture of the network g and corresponds to the radius $r(g)$. The previous Proposition states that an end of the game at time $t = r(g)$ is indeed a SPNE outcome. With respect to the *best* equilibrium outcomes, networks can therefore be ranked according to their radii.

In a connected network g, players' eccentricities range from $r(g)$ to $d(g)$ and there exists at least one player that exhibits each of these eccentricity measures. It follows from Proposition 4 that for games $\Gamma(N,g,T)$ with $T < d(g)$, the equilibrium duration can range from $r(g)$ to T.[11] For games with $T \geq d(g)$, it can range from $r(g)$ to $d(g)$ and may last longer. Proposition 4 says nothing about such outcomes. Macimal duration of the game is the object of the next Section.

Complete networks are particular in that every member's eccentricity is equal to the radius $r(g) = 1$. Interestingly, the previous proposition applied to games $\Gamma(N, g_{complete}, T)$ corresponds to a well-known result of the *war of attrition* literature. In $g_{complete}$, an information item which is Passed On is immediately held by every player. As a consequence, a member of a complete network is the unique winner if and only if he is the last player to Pass On. In other words, as soon as $n-1$ players have "concede", the game ends with the player who has not conceded yet holding the n items and winning. To that extent,

[11] Indeed, there is at least one player i in $W(g,T)$ with $e_i(g) = T$.

the game $\Gamma(N, g_{complete}, T)$ can be viewed as a war of attrition of complete information in which n symmetric players compete for one prize in discrete and finite time. As in the war of attrition, every player strictly prefers to win than to lose but prefers to lose sooner than later.

In Kornhauser, Rubinstein, and Wilson (1988), a concession game with complete information is played in discrete time by two players 1 and 2 with different discount factors. The authors show that "there is an infinity of SPNE outcomes : one of these outcomes is for player 1 to concede immediately, another is for player 1 to wait and for player 2 to concede immediately". Proposition 4 corresponds to the straightforward generalization of the previous statement to n players competing for one prize. Bilodeau and Slivinski (1996) present a n-player continuous-time war of attrition in finite horizon with n players competing for $(n-1)$ prizes. The authors state that "for every individual, there is a SPNE outcome in which only that individual concedes immediatel". On the contrary, since we study a case in which n players compete for one prize, we show that, for every individual, there is a SPNE outcome in which all the individuals except that one concede immediately. If $g \neq g_{complete}$, n players compete for one prize but the order in which $n-1$ players concede is crucial and dependent on the network structure.

1.5.2. Maximal Duration

Among games in which success is ensured in every equilibrium, we further pay attention to the maximal duration of the game in equilibrium. For every game $\Gamma(N, g, T)$, the set of strategy profiles S_W is split into two disjoint subsets. Let $S_{end \leq n-1} \subset S_W$ be the set of strategy profiles such that the game ends with at least a winner at a date $t \leq n-1$. Let $S_{end \geq n} = S_W \backslash S_{end \leq n-1}$ be the set of strategy profiles such that the game ends with at least a winner at a date $t \geq n$.

To start with, let's consider the simple situation of dynamic duels, for which it is easy to state the following : *every equilibrium of a dynamic duel* $\Gamma(\{1,2\}, g, T)$ *is such that the game ends up in the first period of play.* Indeed, a single period is sufficient for every agent to make a duel end. Since Proposition 3 states that the winner is unique in every equilibrium of $\Gamma(\{1,2\}, g, T)$, an equilibrium strategy profile cannot be such that the game lasts strictly more than one period as the loser would have a profitable deviation to a strategy that makes him lose in the first period of play.

Next, let's consider the case of $n = 3$. Since three-players networks are either a tree or complete, we get that, *in a three-player game lasting at least two periods, every equilibrium yielding a winner does so in either one or two shots.* This statement both relies on the fact that, in equilibrium, every duel lasts one period and on Proposition 3. Recall that we assume that every player strictly prefers to lose sooner than later but we do not exclude that an agent may prefer to be a winner (even among many) at date T than to lose earlier. It follows that Proposition 3 is required to find a non-critical player - there are at least two from Theorem 1 - who loses in equilibrium and therefore has a strict incentive to reduce the game to a duel, in which he still loses but more rapidly.

Similarly to the reasoning used in Section 1.3 to establish that $T \geq n-1$ is a sufficient condition to get $S_{SPNE} \subseteq S_W$, an inductive reasoning enables to state the following proposition. It establishes that, for two particular class of networks, there exists an upper bound on the time needed for the game to end with a winner in equilibrium. Note that the inductive reasoning heavily relies on the fact that removing a non-critical agent with all its links from a complete network leaves the subnetwork complete and removing a non-critical agent from a tree leaves the subnetwork still be a tree.

Proposition 5 *Let $\Gamma(N, g, T)$ be such that $T \geq n-1$. In this game, if the network g is either a tree or a complete network, then $S_{SPNE} \subseteq S_{end \leq n-1}$.*

In a game $\Gamma(N, g, T)$ with $T \geq n-1$, there are equilibrium outcomes such that there is a winner at a date $t \leq n-1$. What Proposition 5 shows is that this is true for *every* equilibrium if when g belongs to the class of complete graphs or trees.

1.6. Conclusion

In the dynamic game we propose, the members of a fixed network face a "communication dilemma" in the sense that they have a collective interest to share information items by transmitting them via communication links as well as an individual interest to withhold them. We show that, a group of n players centralizes all the initially dispersed items in every subgame perfect equilibrium, if and only if the game lasts sufficiently many periods, precisely more than $n-1$ periods. It follows that whether or not the collective task is performed in every equilibrium is independent of the network structure, as long as it is physically adequate for the successful completion of this task which means that the network is connected. On the contrary, the architecture of communication links affects the time needed before information items are pooled in equilibrium. For every network, the minimal time needed in equilibrium is given by the radius of the network. For complete networks and trees, once success is ensured in equilibrium, the threshold $n-1$ also corresponds to the maximal number of periods required for items centralization.

One can view the items transmitted by players as any type of goods that have to be pooled to become valuable. To answer the question about the structure that is the most appropriate for the pooling of these goods, we introduce graphical notions and results from graph theory that are used in some areas of operations research [12] but that were not used in economics to the best of our knowledge. For instance, a building block of our analysis is a graphical result stating that, in every connected network, there exists at least two non-critical agents. Since such agents can be dropped from a network without disconnecting the resulting subnetwork, proofs can be done by induction within networks.

Even if Bonacich (1990)'s experimental results stated that the outcome of social dilemmas is affected by the network structure, his study rather examined the influence of an agent's position on his individual behavior. For instance, it seemed that agents with peripheral positions behaved more cooperatively than central agents. In the present work, for agents who are not in the set of potential winners because they are peripheral in the sense that their eccentricities are too large, Passing On in every period of play is a weakly dominating strategy. That is, the effective chances of victory determined by *physical* network positions clearly affect one's communication behavior. Focusing on the effect of positions

[12] For instance, see Buckley (1986) in which the eccentricity measure is used to define and find the center of a tree network. More generally, see *network location theory* that addresses the question of the optimal location of a single-point facility in a graph.

on communication behaviors is left for further research.

1.7. Appendix

For every proposition presented in a previous section, the proof is given in a subsection of the Appendix entitled as the section. We denote $\Gamma(N,g,T)|h^t$ the subgame of $\Gamma(N,g,T)$ that starts at time $t \leq T$ after history h^t. Player i's continuation strategy after history h^t is denoted $s_i|h^t$.

1.7.1. Success or Failure in Equilibrium

Proposition 1 : Sufficient Condition

Lemma 1 *If the game* $\Gamma(N,g,T)$ *is such that* $T \geq n-1$*, then* $S_{SPNE} \subseteq S_W$.

Proof of Lemma 1 is by induction: assume it is true for n players and show it stays true for $n+1$. To do so, fix n and consider three kinds of games. First, games $\Gamma(N,g,T)$ with $|N| = n$. Next, *augmented* games $\Gamma(N',g',T)$ with $|N'| = n+1$. Without loss of generality, let player $(n+1)$ be in $N_n(g')$ and be non-critical in g'. Finally, *augmented modified* games $\tilde{\Gamma}(N',g',T)$ that differ from *augmented* games only in that the initial matrix of information $\tilde{V}'^0 \neq Id_{n+1}$ is such that, for every $i \in N'$, we have $\tilde{v}'^0_{ii} = 1$ and such that $\tilde{v}'^0_{n\,n+1} = 1$ meaning that player n initially holds the item $(n+1)$.

Let two games $\Gamma(N,g,T)$ and $\tilde{\Gamma}(N',g',T)$ form ***a pair*** if the two connected networks g and g' are such that $g = g' \backslash \{n+1\}$. Given either $\Gamma(N,g,T)$ or $\tilde{\Gamma}(N',g',T)$ only, one can always construct a pair. Indeed, a connected g' is built from a connected g by linking agent $(n+1)$ only to agent n. Since agent $(n+1)$ has a unique neighbor in g', he is non-critical in g'. A connected g is built from a connected g' by removing the non-critical agent $(n+1)$ and all its links.

The sets of (SP)NE of games $\Gamma(N',g',T)$ and $\tilde{\Gamma}(N',g',T)$ are denoted $S'_{(SP)NE}$ and $\tilde{S}'_{(SP)NE}$ respectively. The sets of strategy profiles such that games $\Gamma(N',g',T)$ and $\tilde{\Gamma}(N',g',T)$ end with a winner (no winner, resp.) are denoted S'_W and $\tilde{S}'_W$ respectively (S'_L and $\tilde{S}'_L$, resp.). Before proving Lemma 1, we show:

Lemma 2 *For every pair of games* $\Gamma(N,g,T)$ *and* $\tilde{\Gamma}(N',g',T)$*, we have: if* $S_{SPNE} \subseteq S_W$*, then* $\tilde{S}'_{SPNE} \subseteq \tilde{S}'_W$*.*

Proof : Take a pair of games $\Gamma(N,g,T)$ and $\tilde{\Gamma}(N',g',T)$. We show that if there exists a strategy profile $\tilde{s}'$ in $\tilde{S}'_{SPNE} \cap \tilde{S}'_L$, then there exists a strategy profile s in $S_{SPNE} \cap S_L$. In $\tilde{\Gamma}(N',g',T)$, consider a profile $\tilde{s}' \in \tilde{S}'$ such that player $(n+1)$ Hides in every period whatever the history and such that, for every player $i \in N$, player i's action at time t is independent of player $(n+1)$'s actions at dates $t' \in [1, t-1]$. Next, in $\Gamma(N,g,T)$, consider a profile $s \in S$ such that s and $\tilde{s}'$ describe, for every player $i \in N$ and every date $t \leq T$, the same action profile in games $\Gamma(N,g,T)$ and $\tilde{\Gamma}(N',g',T)$ respectively.

Considering the process of items' transmission given by (1.1), it is easy to show that the sequences $(V^0, ...V^T)$ and $(\tilde{V}'^0, ..., \tilde{V}'^T)$ determined by s and $\tilde{s}'$ in $\Gamma(N,g,T)$ and $\tilde{\Gamma}(N',g',T)$ respectively are such that, for every $i \in N$ and every $t \leq T$, we have *(A)* : for each item $j \in N$, $\tilde{v}'^t_{ij} \geq v^t_{ij}$. [13] Next, since $\tilde{s}' \in \tilde{S}'_L$, there exists for every $i \in N$ an item $k \in N'$ such that $\tilde{v}'^T_{ik} = 0$. Given that items n and $(n+1)$ are transmitted together in $\tilde{\Gamma}(N',g',T)$ as $\tilde{v}'^0_{n\,n+1} = 1$, we get that $\tilde{s}' \in \tilde{S}'_L$ implies that there exists for every $i \in N$ an item $k \in N$ such that $\tilde{v}'^T_{ik} = 0$. Using *(A)*, we have that $\tilde{s}' \in \tilde{S}'_L$ implies $s \in S_L$.

Finally, if $\tilde{s}' \in \tilde{S}'_{SPNE}$, the profile of continuation strategy $\tilde{s}'|h'^t$ is a NE of the subgame $\tilde{\Gamma}(N',g',T)|h'^t$ for every $h'^t \in H'^t$. Since $\tilde{s}'$ is such that, for every $i \in N$, player i's action in every period is independent of player $(n+1)$'s past actions and player $(n+1)$'s actions are independent of the history, we directly get: if $\tilde{s}'|h'^t$ is a NE of $\tilde{\Gamma}(N',g',T)|h'^t$, then $s|h^t$ is a NE of $\tilde{\Gamma}(N,g,T)|h^t$ with h'^t and h^t describing the same action profile for every $i \in N$ and every date $t \leq T$. It follows that $\tilde{s}' \in \tilde{S}'_{SPNE}$ implies $s \in S_{SPNE}$ which completes the proof. $\square$

Proof of Lemma 1 : As stated in Section 1.3, every NE of a dynamic duel $\Gamma(\{1,2\},g,T)$ yields at least one winner, which implies that Lemma 1 is true for $n = 2$. We assume that Lemma 1 is true for n agents and prove that it stays true for $n+1$ agents: if the game

[13] Note that we have $\tilde{v}'^t_{ij} \geq v^t_{ij}$ and not $\tilde{v}'^t_{ij} = v^t_{ij}$ because we do not exclude that the initial matrix $\tilde{V}'^0$ of players' information in $\tilde{\Gamma}(N',g',T)$ is such that there exists a pair of players $i,j \in N$, $i \neq j$ such that $\tilde{v}'^0_{ij} = 1$ whereas this is excluded for the initial matrix of players' information $V^0 = Id_n$ of $\Gamma(N,g,T)$.

$\Gamma(N', g', T+1)$ is such that $T+1 \geq n$, then $S'_{SPNE} \cap S'_L = \emptyset$.

First, in $\Gamma(N', g', T+1)$, we consider a strategy profile $s' \in S'_{SPNE}$ such that $a'^1_{n+1} = P$ and we show that $T+1 \geq n$ implies $s' \in S'_W$. By definition of SPNE, the profile of continuation strategy $(s'_i|h'^2)_{i \in N'}$ is a SPNE of the subgame $\Gamma(N', g', T+1)|h'^2$ with $h'^2 = ((a'^1_i)_{i \in N}, P)$. This subgame is equivalent to the *augmented modified* game $\tilde{\Gamma}(N', g', T)$. More precisely, games $\Gamma(N', g', T+1)|h'^2$ and $\tilde{\Gamma}(N', g', T)$ have the same set of players N', the same network g', the same number of periods of play T and the same matrix of players' information : at the beginning of $\Gamma(N', g', T+1)|h'^2$ the matrix V'^1 is such that, for every $i \in N'$, we have $v'^1_{ii} = 1$ and such that $v'^1_{n\,n+1} = 1$ since $n \in N_{n+1}(g')$ and $a'^1_{n+1} = P$. By assumption, if $\Gamma(N, g, T)$ is such that $T \geq n-1$, then $S_{SPNE} \subseteq S_W$. Given $\tilde{\Gamma}(N', g', T)$, we can find a game $\Gamma(N, g, T)$ to get a pair and then deduce from Lemma 2 that $\tilde{S}'_{SPNE} \subseteq \tilde{S}'_W$. Therefore, $(s'_i|h'^2)_{i \in N'} \in \tilde{S}'_{SPNE}$ implies $(s'_i|h'^2)_{i \in N'} \in \tilde{S}'_W$ which implies that $s' = (s'^1_i(h'^1), s'_i|h'^2)_{i \in N'} \in S'_W$.

Next, in $\Gamma(N', g', T+1)$, we consider a strategy profile $s' \in S'_{SPNE}$ such that $a'^1_{n+1} = H$ and we show that $T+1 \geq n$ implies $s' \notin S_L$. By definition of SPNE, the profile of continuation strategy $(s'_i|h'^2)_{i \in N'}$ is a SPNE of $\Gamma(N', g', T+1)|h'^2$ with $h'^2 = ((a'^1_i)_{i \in N}, H)$. As shown in the previous paragraph, if $T \geq n-1$, then every SPNE played in a subgame $\Gamma(N', g', T+1)|h'^2$ that starts after a history $h'^2 = ((a'^1_i)_{i \in N}, P)$ is such that the game $\Gamma(N', g', T+1)$ ends up with a winner. Therefore, if we assume that $s' \in S'_L$, then $T \geq n-1$ implies that player $(n+1)$ has an interest in deviating from s'_{n+1} such that $a'^1_{n+1} = H$ to a strategy s''_{n+1} such that $a''^1_{n+1} = P$. This profitable deviation in the first period of play contradicts $s' \in S'_{SPNE}$ which is why $s' \notin S'_L$.

Proof is completed by noting that every $s' \in S'_{SPNE}$ is either such that $a'^1_{n+1} = H$ or such that $a'^1_{n+1} = P$. $\square$

Proposition 1 : Necessary Condition

Lemma 3 *If the game* $\Gamma(N, g, T)$ *is such that* $T \leq n-2$*, then* $S_{SPNE} \cap S_L \neq \emptyset$.

We prove Lemma 3 for complete networks only and use the the following lemma to get it for every connected network.

Lemma 4 *If $S_{SPNE} \cap S_L \neq \emptyset$ in $\Gamma(N, g_{complete}, T)$, then $S_{SPNE} \cap S_L \neq \emptyset$ in $\Gamma(N, g, T)$.*

Proof : In $\Gamma(N, g_{complete}, T)$, consider a strategy profile $s^c \in S_L$. Next, in $\Gamma(N, g, T)$, consider a strategy profile s such that s and s^c describe the same action profiles for every $i \in N$ and every $t \leq T$, in $\Gamma(N, g_{complete}, T)$ and $\Gamma(N, g, T)$ respectively. It is easy to show that if $s^c \in S_L$, then $s \in S_L$ since $N_i(g) \subseteq N_i(g_{complete}) = N \backslash \{i\}$ for every $i \in N$. Equivalently, we get that if $s \in S_W$, then $s^c \in S_W$. It follows that if there exists a player i who has a strictly profitable deviation from a profile $s \in S_L$ for a history h^t in $\Gamma(N, g, T)$, then the same deviation from $s^c \in S_L$ is strictly profitable in $\Gamma(N, g_{complete}, T)$. We conclude that if the strategy profile $s \in S_L$ is not in S_{SPNE}, then the strategy profile $s^c \in S_L$ is not in S_{SPNE}. $\square$

Lemma 5 *If the game $\Gamma(N, g_{complete}, T)$ is such that $T \leq n-2$, then $S_{SPNE} \cap S_L \neq \emptyset$.*

Proof : We show that if $T \leq n-2$, then there exists a strategy profile $s \in S_{SPNE} \cap S_L$. For every h^t, denote $K(h^t)$ the set $\{i \in N : \forall j \in N \backslash \{i\}, v_{ji}^{t-1} = 0\}$ and let $k(h^t) = |K(h^t)|$. Players in $K(h^t)$ have Hidden in every period $t' \in [1, t-1]$. Note that as soon as a history h^t is such that $K(h^t)$ is a singleton, say $K(h^t) = \{l\}$, the game ends at t with player l being the unique winner. Consider the profile s such that, for every $i \in N$, we have :

- $s_i^t(h^t) = H$ if $i \notin K(h^t)$
- $s_i^t(h^t) = H$ if $i \in K(h^t)$ and $T - t + 1 \leq k(h^t) - 2$
- $s_i^t(h^t) = H$ if $i \in K(h^t)$ and $T - t + 1 > k(h^t) - 2$ and $i = Min_{j \in K(h^t)} j$
- $s_i^t(h^t) = P$ if $i \in K(h^t)$ and $T - t + 1 > k(h^t) - 2$ and $i \neq Min_{j \in K(h^t)} j$.

First, let's show $s \in S_L$. Since for every $i, j \in N$, $i \neq j$, $v_{ij}^0 = 0$, we have that $K(h^1) = N$ and $k(h^1) = n$. If $T - 1 + 1 \leq n - 2$, then, following s, V^1 remains equal to Id_n. Repeating the reasoning directly establishes $s \in S_L$.

Next, let's show $s \in S_{SPNE}$ by showing that s satisfies the one-stage deviation principle. We distinguish two kinds of histories h^t and check that, conditional on h^t reached, no player $i \in N$ has an strict interest in unilaterally deviating from the continuation strategy $s_i|h^t$ at date t and conforming to $s_i|h^t$ thereafter [14].

[14]See one-stage deviation principle for finite horizon games in Fudenberg and Tirole (1991)[pp 108-110].

First, consider a subgame $\Gamma(N, g_{complete}, T)|h^t$ with h^t such that $T-t+1 > k(h^t)-2$. Let $l = Min_{j \in K(h^t)} j$. Following $(s_i|h^t)_{i \in N}$, the action profile $(a_i^t)_{i \in N}$ is such that *(a)* for every $i \notin K(h^t)$, $a_i^t = H$, *(b)* for every $i \in K(h^t) \backslash \{l\}$, $a_i^t = P$ and *(c)* $a_l^t = H$. Therefore, we get $K(h^{t+1}) = \{l\}$ [15]. As a consequence, following $(s_i|h^t)_{i \in N}$ in $\Gamma(N, g_{complete}, T)|h^t$, the game $\Gamma(N, g_{complete}, T)$ ends at t with l being the unique winner. Obviously, player l has no interest in unilaterally deviating from $s_l|h^t$ at time t. In addition, in $g_{complete}$, the action of every $i \notin K(h^t)$ has no effect in $\Gamma(N, g_{complete}, T)|h^t$ [16], so there is no strict interest in deviating from $s_i|h^t$ at t. Finally, consider a deviation of a player $j \in K(h^t)\backslash\{l\}$. A strategy $s'_j|h^t$ that agrees with $s_j|h^t$ except at date t consists in Hiding at t instead of Passing On. If period $t = T$, then player j has no interest in such a deviation as the game would end at T with no winner instead of ending at T with player l winning. If period $t < T$, then at time $t+1$ after player j's deviation, we have $K(h^{t+1}) = \{j, l\}$ and $k(h^{t+1}) = 2$ which implies that $k(h^{t+1})-2 = 0$. Since $t < T$, we have $T-t = T-(t+1)+1 > 0 = k(h^{t+1})-2$. As a consequence, following $(s_i|h^t)_{i \in N}$ in the subgame that starts at $t+1$ after player j's deviation, every agent $i \neq j$ Hides and player j Passes On. It follows that player l is still the unique winner but at time $t+1$ instead of t : if player j deviates, he then receives δ^t instead of δ^{t-1}. Conditional on h^t reached, we conclude that no player $i \in N$ has a strict interest in unilaterally deviating from $s_i|h^t$ at time t only.

Finally, consider a subgame $\Gamma(N, g_{complete}, T)|h^t$ with h^t such that $T-t+1 \leq k(h^t)-2$. Following $(s_i|h^t)_{i \in N}$, the action profile $(a_i^t)_{i \in N}$ is such that, for every $i \in N$, $a_i^t = H$. Therefore, we get $k(h^{t+1}) = k(h^t)$. Since $T-t+1 \leq k(h^t)-2$, we have $T-(t+1)+1 \leq k(h^{t+1})$. As a consequence, following $(s_i|h^t)_{i \in N}$ in $\Gamma(N, g_{complete}, T)|h^t$, we have that for every $i \in N$, $a_i^{t+1} = H$ yiedling $k(h^{t+2}) = k(h^{t+1})$. The same reasoning applies for every $t' \in [t+2, T]$ meaning that following $(s_i|h^t)_{i \in N}$, the game $\Gamma(N, g_{complete}, T)|h^t$ ends at T with no winner. As mentioned in the previous paragraph, in $\Gamma(N, g_{complete}, T)|h^t$, players $i \notin K(h^t)$ have no strict interest in deviating from $s_i|h^t$ at t. For a player $i \in K(h^t)$, a strategy $s'_i|h^t$ that agrees with $s_i|h^t$ except at date t consists in Passing On at t instead of

[15] If a player $i \in K(h^t)$ Passes On at time t, then $i \notin K(h^{t+1})$ since g is complete meaning that $N_i(g_{complete}) = N\backslash\{i\}$ for every $i \in N$.

[16] This is due to the fact that, in $g_{complete}$, a Passed On item immediately reaches every player.

Hiding. If a player i Passes On at time t, we get $k(h^{t+1}) = k(h^t) - 1$. Since h^t is such that $T - t + 1 \leq k(h^t) - 2$, we have that $T - (t+1) + 1 \leq k(h^t) - 1 - 2 = k(h^{t+1}) - 2$. Therefore, following $(s_i|h^t)_{i \in N}$ in the subgame of $\Gamma(N, g_{complete}, T)|h^t$ starting at $t+1$ after history h^{t+1}, we get for every $i \in N$, $a_i^{t+1} = H$. Repeating the reasoning, we get for every $i \in N$, $a_i^{t+2} = H$ and so until date T. Conditional on h^t reached, we conclude that no player $i \in N$ has a strict interest in deviating from $s_i|h^t$ at time t only. $\Box$

Proof of Lemma 3 : Directly from Lemmas 5 and 4. $\Box$

Proof of Proposition 1 : Directly from Lemmas 1 and 3. $\Box$

1.7.2. Equilibrium Sets of Winners

Proof of Proposition 2: Every $s \in S_W$ is either such that all the winners are critical agents [17] or such that there is at least one non-critical agent who wins. We show that if g is separable, then every $s \in S_W$ such that there is at least one non-critical agent who wins is also such that there is at least one non-critical agent who loses. We prove that if g is separable, then there exists a pair of non-critical players who cannot win together.

By definition (chapter 3 in Tutte (2001)), if g is separable, then there exists a pair (g_1, g_2) of connected subnetworks of g such that $g_1 \cup g_2 = g$ and $g_1 \cap g_2$ is a critical agent of g, say k. Letting N_1 be the agents in g_1 and N_2 the agents in g_2, we get $N_1 \cup N_2 = N$ and $N_1 \cap N_2 = \{k\}$. From Theorem 1, there exists at least one agent in $N_1 \backslash \{k\}$ who is non-critical in g_1, say i, and at least one agent in $N_2 \backslash \{k\}$ who is non-critical in g_2, say j. It follows from the fact that $g_1 \backslash \{i\}$ is connected that $(g_1 \backslash \{i\}) \cup g_2 = g_1 \cup g_2 \backslash \{i\} = g \backslash \{i\}$ is connected [18] meaning that i is non-critical in g. The same is true for agent j. We show that i and j cannot win together at a date $t \leq T$.

Assume that i and j win together at t meaning that, at t, player i has every item $l \in N_2$ and player j has every item $l \in N_1$. Since k is on every path between i and j, every item $l \in N_2$ was held by k at least one period before it was held by i and every item $l \in N_1$ was held by k at least one period before it was held by j. Since $N_1 \cup N_2 = N$, there was a

[17] Using Theorem 1, it is obvious that if all the winners are critical agents, there is at least one non-critical agent who loses.

[18] The union of two connected networks is a connected network (Chapter 1 in Tutte (2001)).

period $t' \leq t-1$ in which k held the n items. This contradicts the fact that i and j win together at t. $\square$

Proof of Proposition 3 : Split into the two following lemmas. $\square$

Lemma 6 *In $\Gamma(N, g_{complete}, T)$, every $s \in S_{NE} \cap S_W$ is such that the winner is unique.*

Proof : First, we show that every $s \in S_W$ is either such that the winner is unique or such that there are n winners. Consider a strategy profile $s \in S_W$ such that the game ends with strictly more than one winner, say players i and j win together at t. Since i wins at t, i holds every item $l \in N \backslash \{i\}$ at that date. Since the network is complete, every agent $k \in N$ also holds every item $l \in N \backslash \{i\}$ at t. Applying the same reasoning to j, we get that players i and j both winning at t implies n players winning at that date.

Next, we consider a strategy profile $s \in S_W$ such that there are n winners at t and show that $s \notin S_{NE}$. If n players win at t, every $i \in N$ has Passed On at least at one date $t' \leq t$. Nevertheless, since the n players have not won at $t-1$, at least two agents, say i and j, had not Passed On yet at time $t-1$ but both Pass On at t [19]. Given that i Passes On at t, j has a strict interest in deviating from Passing On so that he can be the only winner at t. $\square$

Lemma 7 *In $\Gamma(N, g_{tree}, T)$, every $s \in S_{NE} \cap S_W$ is such that the winner is unique.*

Proof : We show that there does not exist a strategy profile $s \in S_{NE} \cap S_W$ such that a pair of players, say i and j, win together at t. The proof has three parts depending on the way i and j are linked. Recall that a tree network is such that there is a unique path between any pair of distinct agents.

1st Part : Assume that $ij \notin g_{tree}$. Let a player k be on the unique path between i and j. Since g_{tree} is separable, one can find a pair (g_1, g_2) of connected subnetworks such that $g_1 \cup g_2 = g$ and $g_1 \cap g_2$ is the critical agent k. Using the same reasoning as in the proof of Lemma 2 with $i \in N_1 \backslash \{k\}$ and $j \in N_2 \backslash \{k\}$ [20], we get that there does not exist a strategy

[19] If there is a unique i who has not Passed On yet at time $t-1$, then player i is the unique winner at time $t-1$.

[20] The difference is that i is not necessarily non-critical in g_1 and j is not necessarily non-critical in g_2.

profile $s \in S_W$ such that i and j win together at t.

2nd Part : Assume that $n = 2$ and $ij \in g_{tree}$. The tree network involving 2 players is complete. From Lemma 6, every $s \in S_W \cap S_{NE}$ is such that the winner is unique.

3rd Part : Assume that $n \geq 3$ and $ij \in g_{tree}$. Consider a strategy profile $s \in S_W \cap S_{NE}$ such that i and j win together at t. In a tree with $n \geq 3$ players, two connected agents are either two critical agents or one is critical and the other is non-critical. Assume agent i is critical. It follows that there exists a pair (g_1, g_2) of connected subnetworks such that $g_1 \cup g_2 = g$ and $g_1 \cap g_2$ is agent i. Let N_1 be agents in g_1 and N_2 agents in g_2 and assume that $j \in N_2 \backslash \{i\}$. Since players i and j do not win at $t-1$, they both lack at least one item at that date. Let's show that every item that agent i lacks at $t-1$ is held by j at that time and vice versa. We first focus on items that player $j \in N_2 \backslash \{i\}$ lacks at $t-1$:

First case : j lacks an item $k \in N_1$. As i is on every path between $j \in N_2$ and $k \in N_1$, the item k must be held by i at $t-1$ for j to hold it at t.

Second case : j lacks an item $k \in N_2 \backslash \{i\}$ and j is on the unique path between k and i. Since j is on every path between k and i, if j lacks the item k at $t-1$, it cannot reach i at time t. This contradicts the fact that i and j win together at t.

Third case : j lacks an item $k \in N_2 \backslash \{i\}$ and j is not on the unique path between k and i. First, we show that network g being a tree implies that i is on the unique path between players j and k [21]. We assume it is not and show that this contradicts the fact that j is not on the unique path between i and k : if i is not on the unique path between $j \in N_2 \backslash \{i\}$ and $k \in N_2 \backslash \{i\}$, then the unique path between j and k exists within the subnetwork g_2 and therefore passes trough a player $l \in N_2 \backslash \{i\}$ [22]. Since $ji \in g_{tree}$ and j is linked to k through l, then j is on the path between i and k. Since this path is unique by definition of a tree, j is on the unique path between k and i. We conclude that i is on the unique path between k and j. Therefore, the item k must be held by i at $t-1$ for j to hold it at t.

From the previous cases, we get that i and j winning at t implies that items that j

[21] Note that this cannot be deduced from g's decomposition into g_1 and g_2 as k and j both belong to N_2.

[22] Note that, by definition of the split of the network g into networks g_1 and g_2, the unique path between j and k cannot go through an agent l in $N_1 \backslash \{i\}$ without going through player i since i is on every path between players from the sets $N_1 \backslash \{i\}$ and $N_2 \backslash \{i\}$.

lacks at time $t-1$ are held by i at that time. Using a symmetric reasoning [23], we get that items that i lacks at $t-1$ are held by j at that time. Therefore, the only way i and j can both win at t is that they both Pass On at t. If they both Pass On, each of them wins $\delta^{t-1}(1+\frac{R}{2})$. If one of the two Passes On, the other earns $\delta^{t-1}(1+R)$ by Hiding. A strategy profile $s \in S_W$ such that two players win together at t is not in S_{NE}. $\square$

1.7.3. Equilibrium Duration before Success

Minimal Duration

Let $W(g,T)|h^t$ denote the set of potential winners in $\Gamma(N,g,T)|h^t$. Given h^t, players in $W(g,T)|h^t$ are the ones who can hold the n items at a date $t \leq T$ if every player Passes On in every period of play in $[t,T]$.

Lemma 8 *Consider a history h^t of $\Gamma(N,g,T)$ such that there exists a player $i \in W(g,T)|h^t$ who has Hidden in every period $t' \in [1,t-1]$. There exists a continuation strategy profile $s|h^t$ that is a NE of $\Gamma(N,g,T)|h^t$ and such that player i is the unique winner of $\Gamma(N,g,T)$.*

Proof : Let $s|h^t$ be such that i Hides in every period of play whatever the history and such that every $j \neq i$ Passes On in every period of play whatever the history. Following $s|h^t$, i is the unique winner of $\Gamma(N,g,T)$. It is straightforward to check that no player $i \in N$ has a strict interest in unilaterally deviating from $s|h^t$. $\square$

Proof of Proposition 4 : Pick a player $i \in W(g,T)$. Consider a strategy profile $s \in S$ that results in a final history $h^{e_i(g)+1} = (a^1, \ldots, a^{e_i(g)})$ such that :

- for all $t \leq T$, $a_i^t = H$
- every player $j \neq i$ starts to Pass On in every period at a specific date $t_j = e_i(g) - d_{ij}(g) + 1$ (that depends on his distance to player j) : $a_j^t = H$ for all $t < t_j$ and $a_j^t = P$ for all $t \geq t_j$.

[23] When focusing on items that player $i \in N_1 \backslash \{j\}$ lacks at $t-1$, we also distinguish three cases. The case in which i lacks an item $k \in N_1 \backslash \{i\}$ is similar to the previous *Second case.* The case in which i lacks an item $k \in N_2 \backslash \{i\}$ and j is on the unique path between k and i is similar to the previous *First case.* The case in which i lacks an item $k \in N_2 \backslash \{i\}$ and j is not on the unique path between k and i is similar to the previous *Third case.*

The final history $h^{e_i(g)+1}$ is such that player i Hides in every period and the further a player j is from player i, the earlier this player j starts Passing On in every subsequent period. At every date t, the set of players $N\backslash\{i\}$ can be divided into two sets : $\{j \neq i : t_j \geq t\}$, the set of players who are "t-close" to i, in the sense that they have Hidden in every period $t' < t$, and $\{j \neq i : t_j < t\}$, the set of players who are "t-distant" from i, in the sense that they have started to Pass On at a date $t' < t$. According to $h^{e_i(g)+1}$, at the beginning of every period t, every piece of information initially held by a "t-distant" player is held by at least one"t-close" player. Indeed, every player $j \neq i$ who Passes On at time $t = 1$, i.e. who is such that $d_{ij}(g) = e_i(g)$ is linked to at least one player $k \neq i$ who Passes On at time $t = 2$, i.e. one of his direct neighbor who is such that $d_{ik}(g) = e_i(g) - 1$ and so one. At date $t = e_i(g)$, player i wins the game because he eventually receives the $n-1$ items that he did not hold initially from all his direct neighbors. He is the unique winner as every player lacks at least player i's information item.

Let's prove that $s \in S_{SPNE}$ by showing that s satisfies the one-stage deviation principle. We consider three kinds of histories h^t and check that, conditional on h^t reached, no player has a strict interest in unilaterally deviating from the continuation strategy $s_i|h^t$ at date t and conforming to $s_i|h^t$ thereafter.

(i) First, consider a history $h^t = (a^1, \ldots, a^{t-1})$ with $t \leq e_i(g)$ that describes the same actions as $h^{e_i(g)+1}$ for every $i \in N$ and every date $t' \leq t-1$. Let's examine each kind of player :

- *Player i :* Conditional on h^t, player i has no strict interest in unilaterally deviating from $s_i|h^t$, i.e. Hiding, at date t as being the unique winner at date $e_i(g)$ is player i's best possible outcome.

- *Players in $\{j \neq i : t_j < t\}$:* Conditional on h^t, a "t-distant" player's deviation from $s_j|h^t$ at date t consists in Hiding at that date. Then, the subgame that starts at time $t+1$ is such that player i is still able to win the game[24], i.e. belongs to $W(g,T)|h^{t+1}$, and has Hidden in every period $t' \in [1,t]$. It follows from Lemma 8 that s can be constructed so

[24]At date t, every information item initially held by a "t-distant" player is held by a "t-close" player. If the "t-close" players Pass On in every period from time t on, player i can hold the n item at $t = e_i(g)$.

that $s|h^{t+1}$ is a NE of $\Gamma(N,g,T)|h^{t+1}$ such that player i is the unique winner at a date $t \geq e_i(g)$. Therefore, "t-distant" players have no strict interest in deviating unilaterally from $s_j|h^t$ at the single date t.

- *Players in* $\{j \neq i : t_j > t\}$ *:* Conditional on h^t, such a "t-close" player's deviation from $s_j|h^t$ at date t consists in Passing On at that date time. Then, the subgame that starts at time $t+1$ is such that player i is still able to win the game, i.e. belongs to $W(g,T)|h^{t+1}$, and has Hidden in every period $t' \in [1,t]$. It follows from Lemma 8 that s can be constructed so that $s|h^{t+1}$ is a NE of $\Gamma(N,g,T)|h^{t+1}$ such that player i is the unique winner at a date $t \geq e_i(g)$. Therefore, such "t-close" players have no strict interest in deviating unilaterally from $s_j|h^t$ at the single date t.

- *Player in* $\{j \neq i : t_j = t$ *:* Conditional on h^t, such a "t-close"player's action at date $t = t_j$, namely $s_j|h^t$, consists in Passing On. If one such player unilaterally deviates from $s_j|h^t$ and Hides at time t, it can have two different effects :

1st case, $i \in W(g,T)|h^{t+1}$ *:* it follows from Lemma 8 that s can be constructed so that $s|h^{t+1}$ is a NE of $\Gamma(N,g,T)|h^{t+1}$ such that i is the unique winner at a date $t \geq e_i(g)$. Therefore, player j has no strict interest in deviating from $s_j|h^t$ at t.

2nd case, $i \notin W(g,T)|h^{t+1}$ *:* this case corresponds to a situation in which player i is excluded from potential winners of $\Gamma(N,g,T)|h^{t+1}$ by the fact that player j Hides at t. If this single deviation prevents i from winning, it must be that $d_{ij}(g) > T - t \Leftrightarrow e_i(g) = T$, i.e. the number of remaining periods after date t is too small to enable player i to get player j's information item before the end of the game. Conditional on h^{t+1} reached, player i has never Passed On in $t' \in [1,t]$. It follows from $d_{ij}(g) > T - t$ that j cannot hold the item i at a date $t \in [t+1,T]$ either. Therefore, player j cannot win and we eventually have to check that the outcome of the game is such that player j has no interest in deviating, because the game would end with a player different from i winning earlier than at $t = e_i(g)$ for instance.

First, let players i and j be such that $d_{ij}(g) = 1$. In that case, it follows from $d_{ij}(g) > T - t$ that player j deviates from Passing On at date $t_j = t = T$ and that this deviation yields to the game ending up with no winner - player j has never Passed On at a date

$t' < T$ and does not Pass On at time T - instead of player i winning. Therefore, player j has no strict interest in deviating from $s_j|h^t$ at the single date t.

Next, let players i and j be such that $d_{ij}(g) \geq 2$, which means that there at least one agents, say k, on the shortest path between player i and j. Since we have $d_{ik}(g) < d_{ij}(g)$[25], we have at least three players, namely i, j and k, who have never Passed On at the beginning of the subgame $\Gamma(N,g,T)|h^{t+1}$. It follows that s can be constructed so that $s|h^{t+1}$ is a NE of $\Gamma(N,g,T)|h^{t+1}$ in which these three players Hides in every period whatever the history in the subgame.[26] This leads to the game ending up with no winner instead of player i winning. Therefore, player j has no strict interest in deviating from $s_j|h^t$ at the single t.

(ii) Next, we consider histories $h^t = (a^1, \ldots, a^{t-1})$ with $t \geq e_i(g) + 1$ that describe the same actions as $h^{e_i(g)+1}$ for every $i \in N$ and every $t' \in [1, e_i(g)]$: for any such history, the subgames $\Gamma(N,g,T)|h^t$ do not exist as the game already ended at $t = e_i(g)$.

(iii) Finally, for any other history h^t, s can be constructed so that $s|h^t$ is a NE of $\Gamma(N,g,T)|h^t$. $\square$

Maximal Duration

Proof of Proposition 5 is by induction. As in section 1.7.1, we fix n and consider games $\Gamma(N,g,T)$, *augmented games* $\Gamma(N',g',T)$, and *modified augmented games* $\tilde{\Gamma}(N',g',T)$.

Let two games $\Gamma(N,g,T)$ and $\tilde{\Gamma}(N',g',T)$ form a ***complete pair*** (respectively a ***tree pair***) if the two connected networks g and g' are such that $g = g' \backslash \{n+1\}$ with g and g' two complete networks (respectively two tree networks). Given either $\Gamma(N,g,T)$ with g complete or $\tilde{\Gamma}(N',g',T)$ with g' complete, one can always construct a complete pair. Indeed, a complete g' is built from a complete g by linking agent $(n+1)$ to every agent in g. Since g' is complete, agent $(n+1)$ is non-critical in g'. A complete g is built from a complete g' by removing agent $(n+1)$ and all its links. Given either $\Gamma(N,g,T)$ with

[25] $d_{ij}(g)$ and $d_{ik}(g)$ are the lengths of the shortest paths between player i and players j and k respectively. Since k is one the shortest path between i and j, $d_{ij}(g) = l(i,k) + l(k,j)$ XXX define l XXX. It follows that $l(i,k) = d_{ij}(g) - l(k,j) < d_{ij}(g)$. If $d_{ij}(g) < d_{ik}(g)$, then $l(i,k) < d_{ik}(k)$ which contradicts the definition of $d_{ik}(g)$.

[26] It is straightforward to check that such a profile is a NE of the subgame : given that two players Hide in every period whatever the history, there is no strictly profitable deviation from doing the same for the third agent.

g being a tree or $\tilde{\Gamma}(N', g', T)$ with g' being a tree, one can always construct a tree pair. Indeed, a tree g' is built from a tree g by linking agent $(n+1)$ only to agent n. Since agent $(n+1)$ has a unique neighbor in g', he is non-critical in g'. A tree g is built from a tree g' by removing the non-critical agent $(n+1)$ and all its links [27].

The sets of strategy profiles such that the games $\Gamma(N', g', T)$ and $\tilde{\Gamma}(N', g', T)$ end up with a winner at a date $t \leq n$ (at date $t \geq n+1$, resp.) are denoted $S'_{end \leq n}$ and $\tilde{S'}_{end \leq n}$ respectively ($S_{end \geq n+1}$ and $\tilde{S}'_{end \geq n+1}$, resp.). To prove Proposition 5, we use:

Lemma 9 *For every complete pair or tree pair of games $\Gamma(N, g, T)$ and $\tilde{\Gamma}(N', g', T)$, we have: if $S_{SPNE} \subseteq S_{\leq n-1}$, then $\tilde{S}'_{SPNE} \subseteq \tilde{S}'_{end \leq n'-1}$.*

Proof : Similar to Lemma 2. $\square$

Proof of Proposition 5 : As stated in Section 1.5.2, Proposition 5 is true for $n = 2$ and $n = 3$. We next assume that Proposition 5 is true for n agents and we prove that it stays true for $n+1$ agents : we let $\Gamma(N', g', T+1)$ be such that $T+1 \geq n$. If g' is either tree or complete, then $S'_{SPNE} \subseteq S'_{end \leq n}$. From Proposition 3, we have that if g' is either tree or complete, every $s' \in S'_{SPNE}$ is such that there is one non-critical agent in g' who loses.

First, in $\Gamma(N', g', T+1)$, we consider a strategy profile $s' \in S'_{SPNE}$ such that player $(n+1)$ loses and such that $a'^1_{n+1} = P$. By definition of SPNE, the profile of continuation strategy $(s'_i|h'^2)_{i \in N'}$ is a SPNE of the subgame $\Gamma(N', g', T+1)|h'^2$ with $h'^2 = ((a'^1_i)_{i \in N}, P)$. This subgame is equivalent to the *augmented modified* game $\tilde{\Gamma}(N', g', T)$ in the same sense as in the proof of Lemma 1. By assumption, if g is tree or complete, then $S_{SPNE} \subseteq S_{end \leq n-1}$. Given $\tilde{\Gamma}(N', g', T)$, we can find a game $\Gamma(N, g, T)$ to get a complete or tree pair and then deduce from lemma 9 that $\tilde{S}'_{SPNE} \subseteq \tilde{S'}_{end \leq n-1}$. Therefore, $(s'_i|h'^2)_{i \in N'} \in \tilde{S}'_{SPNE}$ implies $(s'_i|h'^2)_{i \in N'} \in \tilde{S}'_{end \leq n-1}$ which implies $s' = ((s'^1_i(h'^1)), (s'_i|h'^2))_{i \in N'} \in S'_{end \leq n}$.

Next, in $\Gamma(N', g', T+1)$, we consider a strategy profile $s' \in S'_{SPNE}$ such that player $(n+1)$ loses and such that $a'^1_{n+1} = H$. By definition of SPNE, the profile of continuation strategy $(s'_i|h'^2)_{i \in N'}$ is a SPNE of $\Gamma(N', g', T+1)|h'^2$ with $h'^2 = ((a'^1_i)_{i \in N}, H)$. As shown in the previous paragraph, if g is tree or complete, then every SPNE played in a subgame

[27] A critical agent in g' is still critical in g.

$\Gamma(N', g', T+1)|h'^2$ that starts after a history $h'^2 = ((a_i'^1)_{i \in N}, P)$ is such that the game $\Gamma(N', g', T+1)$ ends up with a winner different from player $(n+1)$ at a time $t \leq n$. Therefore, if we assume that $s' \in S'_{end \geq n+1}$, then the fact that g' is complete or a tree implies that player $(n+1)$ has an interest in deviating from s'_{n+1} such that $a'^1_{n+1} = H$ to a strategy s''_{n+1} such that $a''^1_{n+1} = P$. Such a deviation would not make player $(n+1)$ win but make him loose at $t \leq n$ instead of $t \geq n+1$. This profitable deviation in the first period of play contradicts $s' \in S'_{SPNE}$ which is why $s' \notin S'_{end \geq n+1}$.

Proof is completed by noting that every $s' \in S'_{SPNE}$ is either such that $a'^1_{n+1} = H$ or such that $a'^1_{n+1} = P$. $\square$

Chapter 2

Strategic Communication Networks

2.1. Introduction

[1]

Many economic situations involve agents who share an interest in coordinating their actions as well as in adapting them to an unknown state of the world. The analysis presented here considers this type of context but departs from the typical assumption that agents agree on the state-contingent profile of decisions. Because their tastes may differ, we let the interacting agents vary in their ideal proximity to the underlying fundamentals. For example, the different divisions of an organization should coordinate their actions, as well as adjust them to the environment of the firm. But, for a number of reasons, ranging from local adaptation costs to career concerns, it is likely that idiosyncratic considerations will influence each division's actions.[2] Similarly, when advocating policies, members of a political party will wish to best suit the situation, but also to be in line with the announcements made by other members to ensure the cohesion of the party. At the same time, activists may have heterogenous preferences regarding the right policy to implement.[3] We here consider this type of coordination game of incomplete information in which every

[1]This chapter is based on Hagenbach and Koessler (2008), forthcoming in the *Review of Economic Studies*.

[2]A multi-divisional organization in which decisions must be adapted to local conditions and information but also coordinated between divisions is considered in Alonso et al. (2008) and Rantakari (2008).

[3]See, for example, Dewan and Myatt (2008).

player incurs losses from any mismatch between his action and both others' actions and his own "ideal action". Every ideal action depends on the state and on a systematic positive or negative bias, as in the cheap-talk or delegation models of Crawford and Sobel (1982) and Dessein (2002). These biases vary across agents, and the profile of biases in the population is a measure of the conflict of interest that they face.

The aim of this chapter is to analyze how agents strategically transmit to each other the signals they privately hold about the fundamentals in these types of situation. Indeed, *before* taking their payoff-relevant actions, we offer players the opportunity to send costless, non-verifiable, and private messages about their information. Within this stylized framework, the communication stage consists of a cheap-talk game in which every player is, at the same time, a sender and a receiver, and we address the question of *who truthfully speaks with whom*. Our precise focus is on how agents' heterogeneity in ideal actions affects decentralized and strategic communication between them. We propose to characterize the transmission of private information by what we call a *communication network*, described by a set of receivers for every player. A player is said to be a receiver of another player if the latter truthfully reveals his private information to the former. A complete characterization of the information transmission occurring in equilibrium is provided, which roughly boils down to the intuitive statement that agents are more prone to communicate when their ideal actions are more similar, and as the need for coordination becomes larger.

Informational incentive constraints require that no player have an interest in lying about his type to his (endogenous) set of receivers. As in standard cheap-talk games (e.g., Crawford and Sobel, 1982), this condition can be formulated as a condition on the proximity between the sender's and the receivers' biases. In existing models extending communication to multiple but strategically-independent decision-makers (see, e.g., Farrell and Gibbons, 1989, Goltsman and Pavlov, 2009 and Galeotti et al., 2009a), one only has to check that the sender has no incentive to lie to any *single* receiver. In our model, the informational incentive constraints are more sophisticated than in these games since all of the agents want to coordinate their actions. Due to the strategic interaction between receivers, how each receiver reacts to a sender's signal depends not only on this signal but also on (his expectation over) the total number of receivers of this signal. At the same time, since the sender also wants to coordinate his action with the receivers, any deviation by the sender in the communication stage induces coordination costs that depend on both the total number of his receivers and the number of receivers he lies to. Combined with the assumption that loss functions are quadratic, our informational incentive constraints require that the sender's bias be close enough to the average bias of *every subset* of receivers. Exactly how close biases should be is determined by some threshold that depends on both the total number of receivers and the respective subsets of receivers the sender could lie to.

This feature reveals a key insight of our work: communication between two agents depends on not only the conflict of interest between them, but also on the preferences and the size of all the agents with whom they communicate. In particular, one main result is that communication to a large group of recipients may occur in equilibrium even though communication only to a strict subset of that group may not. To understand the intuition, consider a simplified 3-player situation in which there is a unique informed agent (the sender): the sender and one uninformed agent both want to choose an action exactly adapted to the true state of the nature, and another uninformed agent is positively biased, i.e. wants to choose an action higher than the true state. Assume that every player also wants to coordinate his action with that of the two others. When the sender communicates only to the biased agent, he has a strong incentive to under-report his type in order to decrease this agent's expectation about the state so that his action gets closer to the sender's ideal action. On the contrary, when the sender communicates to both the biased and unbiased agents, he may have no incentive to jointly lie to both of them because their average bias is small. He may also have no incentive to misrepresent his information only to the biased agent. Indeed, both agents are now more responsive to the sender's message than when he communicates only to one agent, so this deviation would increase the dispersion of players' actions and thus induce large coordination losses. It is worth noting that the disciplinary effect that the coordination of multiple audiences has on communication is different from the disciplinary effect of public communication identified by Farrell and Gibbons (1989). They show that communication to two independent decision-makers may occur in equilibrium when communication is public, whereas information is revealed to neither decision-maker when communication is private. Our *disciplinary effect of coordination* appears even though communication is *not* public and relies on the fact that the receivers we consider are not independent decision-makers.

We provide sharp predictions regarding equilibrium communication networks for several configurations of preferences. First, when players' biases are uniformly distributed we show that a player's tendency to communicate increases with the proximity of his bias to the average bias in the population. Communication is therefore typically not symmetric: centrists tend to influence the decisions of the other players more because they transmit their information truthfully to more distant players than do extremists, with this effect becoming stronger as the need for coordination increases. When the coordination motive is very strong, middle-biased players may communicate to all of the other players even with a wide dispersion of preferences, while other players may never truthfully report their private information. Second, when players are arranged in groups with the same preferences, we again show that information transmission across groups is typically asymmetric: members from the larger group tend to communicate more easily to members of other groups than do members of a smaller group. That is, large groups of agents tend to influence the decisions of small groups by credibly reporting information, while there is less truthful communication from small to large groups.

The chapter is organized as follows. The model is presented in Section 2.2, which also shows that strategic communication networks cannot be completely Pareto-ranked. Equilibrium communication networks are analyzed and illustrated in Section 2.3, and Section 2.4 concludes. Most of the proofs are relegated to the Appendix.

2.2. Model

2.2.1. A Class of Coordination Games with Incomplete Information

Let $N = \{1, ..., n\}$ be a finite set of agents, with $n \geq 2$. Each agent chooses an action $a_i \in A_i = \mathbb{R}$. The action profile is denoted $a = (a_1, \dots, a_n)$. Each agent's payoff depends on the action profile and a state of nature θ. Before the game starts, nobody knows the state of nature, but each agent $i \in N$ receives a private signal $s_i \in S_i = \{\underline{s}_i, \overline{s}_i\}$ about θ, where $\underline{s}_i < \overline{s}_i$. We assume that agents' types are independent and denote by $q_i \in \Delta(S_i)$ the prior probability distribution over agent i's set of types, for every $i \in N$. When the type profile is $s = (s_1, ..., s_n)$, the underlying state of nature is $\theta(s) \in \mathbb{R}$.

Agent i's payoff function is given by

$$u_i(a_1, \ldots, a_n; \theta(s)) = -(1-\alpha)(a_i - \theta(s) - b_i)^2 - \frac{\alpha}{n-1} \sum_{j \neq i} (a_i - a_j)^2. \qquad (2.1)$$

The first component of agent i's utility function is a quadratic loss in the distance between his action a_i and his ideal action $\theta(s)+b_i$, where $b_i \in \mathbb{R}$. We allow the bias parameter b_i to vary across individuals to reflect agents' conflicts of interest with respect to their ideal actions. The second component is a miscoordination quadratic loss which increases in the average distance between i's action and other agents' actions. The constant $\alpha \in (0,1)$ weights both sources of quadratic loss, i.e. it parameterizes agents' coordination motives arising from the strategic complementarity in their actions. Without loss of generality, players are indexed in increasing order of their biases: $b_1 \leq \cdots \leq b_n$.

We assume that the state of nature is the aggregated term $\theta(s) = \sum_{i \in N} s_i$.[4] The sum of players' private signals is a good approximation to the payoff-relevant state in many situations. In an organizational setting for instance, a signal s_i for division i may represent division i's time, budget or expected benefit from a joint project (which is private information), and the state that matters for the whole organization may be the total time, budget or expected benefit of the project. More broadly, considering a state of nature which is additive in types is a simplifying standard assumption in common-value environments, especially in auction theory (see, amongst many others, Bulow and Klemperer, 2002, Mares and Harstad, 2003 and Levin, 2004), in some models of lobbying with multiple experts (e.g., Wolinsky, 2002) or in organization theory (e.g., Jehiel, 1999). From a theoretical point of

[4] Note that the state can be any *weighted* sum of players' types (since we do not make any assumptions about the two possible values of each signal), and players' signals are not assumed to be i.i.d. (they are only assumed to be independent).

view, assuming an additive state and independent types implies that the impact of player i's signal on the fundamentals, $\overline{s}_i - \underline{s}_i$, which can be interpreted as the value of player i's private information, is independent of others' signals. It follows that in our analysis we abstract from any effects that the correlation and the degree of complementarity between players' signals may have on informational incentive constraints. We focus instead on the effect of players' coordination motives and preference heterogeneity. The robustness of our results to this independence property are discussed and related to the literature at the end of Subsection 2.3.1.

2.2.2. Communication Game

Before the coordination game described above is played, but after each player has learnt his type, a communication stage is introduced in which players can send costless and private messages to each other. More precisely, every player i can send a different message $m_i^j \in M_i$ to every other player $j \neq i$, with M_i denoting the (non-empty) set of messages available to player i. Let $m_i = (m_i^j)_{j \neq i} \in (M_i)^{n-1}$ be the vector of messages sent by player i, and $m^i = (m_j^i)_{j \neq i} \in \prod_{j \neq i} M_j \equiv M_{-i}$ the vector of messages received by player i.

The information transmission occurring during the cheap-talk extension of the game will be characterized by a *communication network*, whose directed links represent revelation of private information from one player to another. In order to focus on the presence or absence of such information-transmission links between the agents, we restrict the analysis to pure communication strategies and abstract from the partial transmission of information generated by random strategies.[5] As we only consider two possible types for each player, it follows that any message from player i to player j will either be fully revealing or non-informative. We consider that a communication link is formed from i to j when i's message to j is fully revealing. Without loss of generality, we assume that message spaces are binary: $M_i = \{\underline{m}, \overline{m}\}$.

[5] We do not exclude the existence of non-trivial mixed equilibria, as in the discrete quadratic version of Crawford and Sobel (1982), but the full characterization of such equilibria is quite difficult since we have to consider the possibility that any combination of players randomize over their messages, for any possible combination of receivers.

Player i's communication strategy is a profile $\sigma_i = (\sigma_i^j)_{j \neq i}$ with

$$\sigma_i^j : S_i \to M_i.$$

Let $\sigma_i^j(m_i^j \mid s_i)$ be the probability (0 or 1) that player i send the message m_i^j to player j according to his strategy σ_i when his type is s_i.

Since each player i's utility function is strictly concave with respect to a_i, his best response is necessarily unique, so we can consider without loss of generality pure second-stage strategies. Player i's second-stage strategy is a mapping

$$\tau_i : S_i \times (M_i)^{n-1} \times M_{-i} \to A_i,$$

where $\tau_i(s_i, m_i, m^i)$ is the action chosen by player i when his type is $s_i \in S_i$, the vector of private messages $m_i = (m_i^j)_{j \neq i} \in (M_i)^{n-1}$ was sent, and the vector of private messages $m^i = (m_j^i)_{j \neq i} \in M_{-i}$ was received. Let $\tau(s, (m_i)_{i \in N}) = (\tau_i(s_i, m_i, m^i))_{i \in N}$ be the corresponding action profile.

As is usual in cheap-talk games, the set of Nash equilibrium outcomes in our model coincides with the set of sequential equilibrium outcomes because messages off the equilibrium path can simply be treated as synonyms of equilibrium messages. Hence we do not have to specify a complete belief system: an equilibrium of the communication game is simply a strategy profile $(\sigma, \tau) = ((\sigma_i)_{i \in N}, (\tau_i)_{i \in N})$ satisfying the following properties:

(i) For all $i \in N$, and $s_i \in S_i$,

$$(\sigma_i^j(s_i))_{j \neq i} \in \arg \max_{m_i \in M_i^{n-1}} \sum_{s_{-i} \in S_{-i}} q_{-i}(s_{-i})\ u_i\left(\tau(s, (\sigma_{-i}(s_{-i}), m_i)); \theta(s)\right),$$

where $q_{-i}(s_{-i}) = \prod_{j \neq i} q_j(s_j)$.

(ii) For all $i \in N$, $m_i \in (M_i)^{n-1}$ and $m^i \in \mathrm{supp}[\sigma_{-i}^i]$,

$$\tau_i(s_i, m_i, m^i) \in \arg \max_{a_i \in A_i} \sum_{s_{-i} \in S_{-i}} \mu_i(s_{-i} \mid m^i)\ u_i\left((\tau_j(s_j, (\sigma_j^{-i}(s_j), m_j^i), (\sigma_{-i}^j(s_{-i}), m_i^j)))_{j \neq i}, a_i; \theta(s)\right),$$

where $\mu_i(s_{-i} \mid m^i) = \prod_{j \neq i} \frac{\sigma_j^i(m_j^i|s_j)q_j(s_j)}{\sum_{t_j \in S_j} \sigma_j^i(m_j^i|t_j)q_j(t_j)}$.

A *communication network*, that characterizes a communication strategy profile $(\sigma_i)_{i \in N}$, is denoted by $(R_i)_{i \in N}$, where, for every player i, the *set of receivers*

$$R_i \equiv \{j \in N \backslash \{i\} : \sigma_i^j(\underline{s}_i) \neq \sigma_i^j(\overline{s}_i)\},$$

is the set of individuals to whom player i truthfully reveals his type. Let $|R_i|$ be the number of individuals who learn player i's type in the communication stage. Using the terminology of network theory, R_i corresponds to player i's (out)neighborhood and $|R_i|$ to player i's (out)degree.

2.2.3. Second-Stage Equilibrium Characterization

The quadratic formulation of players' utility functions, together with the independence of players' types, enable us to obtain a unique and tractable second-stage equilibrium characterization whatever the information structure generated by the communication stage. Indeed, as in Calvó-Armengol and Martí (2009) who consider the same utility functions but without heterogeneity in biases, it can be shown that our payoffs admit a potential that represents common interests for all players. The corresponding common-interest game satisfies the sufficient conditions in Marschak and Radner's (1972) team theory for the equilibrium to be unique and linear. More precisely, we show in Appendix 2.5.1 that, given a profile of types $(s_i)_{i \in N}$ and a communication strategy profile characterized by $(R_i)_{i \in N}$, the second-stage equilibrium strategy of each player $i \in N$ is uniquely given by:

$$a_i = \sum_{j \in I_i} \frac{\alpha(n-1-|R_j|)E(s_j) + (1-\alpha)(n-1)s_j}{n-1-\alpha|R_j|} + \sum_{j \in \overline{I}_i} E(s_j) + B_i, \tag{2.2}$$

where $I_i = \{k : i \in R_k\} \cup \{i\}$ is the set of signals which are known by player i after the communication stage, $\overline{I}_i = \{k : i \notin R_k\} \backslash \{i\}$ is the set of signals which are not known by player i after the communication stage, and

$$B_i = \frac{[(n-1)-(n-2)\alpha]b_i + \alpha \sum_{j \neq i} b_j}{n+\alpha-1}. \tag{2.3}$$

Player i's second-stage equilibrium action has three components. The first component is a weighted sum of j's actual type, s_j, and the expected value of j's type, $E(s_j)$, for each player j whose type is known by player i (including himself). The weight put on the actual type of player j increases with the number of players who know j's type, $|R_j|$. This is because a player who wants to be coordinated with the others has a greater incentive to act according to a signal when many other players act according to the same signal. In other words, the larger the set of receivers, the more the sender and those receivers choose an action which is responsive to the sender's private information. This is one of the key effects that will drive our results regarding efficient and equilibrium communication networks. The second component of player i's equilibrium action corresponds to the sum of the expected values of j's type for each player j whose type is not known by player i. The last component adjusts the action of player i with respect to the bias profile. This increases in all players' biases, with more weight being put on player i's own bias, b_i, as the coordination motive decreases.

2.2.4. Efficient Communication Networks

Before characterizing the networks that arise as equilibria of the communication game, we consider the efficiency of communication networks. The following proposition compares players' ex ante expected payoffs as the communication network expands, assuming that equilibrium actions are played in the second-stage game.[6] While an increase in the size of player i's set of receivers is always strictly beneficial for player i and for these receivers, this increase always makes the players who do not learn player i's type strictly worse off.

Proposition 6 *Consider two communication networks $R = (R_i, R_{-i})$ and $R' = (R'_i, R_{-i})$*

[6]As in Crawford and Sobel (1982), it is not possible to compare players' expected payoffs at the interim stage.

such that $|R_i| < |R_i'|$.

i) Every player $j \in R_i' \cup \{i\}$ *is strictly* better off*, ex-ante, with the communication network* R' *than with the communication network* R*;*

ii) Every player $j \in N \backslash (\{i\} \cup R_i')$ *is strictly* worse off*, ex-ante, with the communication network* R' *than with the communication network* R*.*

Proof. See Appendix 2.5.2. ■

The intuition of this result is the following. Consider one sender's private signal. As we have observed in the previous subsection (see Equation (2.2)), when the number of players informed about this signal increases, they become more responsive to it. While this increase benefits informed players whose actions are now better coordinated and adapted to the state, uninformed players suffer larger miscoordination losses, and are therefore worse off.

This result implies that, in general, communication networks cannot be ranked in the sense of Pareto. In particular, defining a communication network $R' = (R_i')_{i \in N}$ as *more informative* than $R = (R_i)_{i \in N}$ if $R_i \subseteq R_i'$ for every $i \in N$ (with at least one strict inequality), a more-informative communication network does not Pareto dominate, in general, a less-informative communication network. However, using Proposition 6 iteratively, we obtain that the complete communication network ($R_i = N \backslash \{i\}$ for all $i \in N$) Pareto dominates every other communication network.

The next proposition shows that, even if it can be harmful for some players, the overall social-welfare effect of enlarging the set of receivers of every player is always positive. *Social welfare* is defined as the sum of individual utilities: $w(a;\theta) = \sum_{i \in N} u_i(a;\theta)$.

Proposition 7 *If the communication networks* $R' = (R_i')_{i \in N}$ *and* $R = (R_i)_{i \in N}$ *are such that* $|R_i'| \geq |R_i|$ *for all* $i \in N$*, with at least one strict inequality, then welfare is strictly higher, ex ante, with the communication network* R' *than with the communication network* R*.*

Proof. See Appendix 2.5.3. ■

In particular, if a communication network R' is more informative than R, then welfare is strictly higher with R' than with R.

Remark : Our simple informational framework, in which the impact of a player's signal on others' beliefs about the state is independent of others' information, enables us to state that welfare increases with the *size* of receiver sets. Note however that if the impact of a player's signal on others' beliefs decreased with the amount of information they already had, then welfare would depend not only on the number of receivers but also on how evenly receivers are distributed across players (see Galeotti et al., 2009a).

2.3. Equilibrium Communication Networks

In this section we provide a full characterization and the general qualitative features of equilibrium communication networks, and derive a number of comparative-static results. We examine how a large number of receivers with strong coordination motives can discipline communication, which is one main novelty of the present work. In particular, we show that there may exist an equilibrium in which a sender reveals his information to a large group of recipients, whereas there is no equilibrium in which he does so to a strict subset of that group only. We also show that players who are more central in terms of preferences communicate more and have a greater impact on the decisions taken. These features are illustrated in two major configurations of preferences. When players' biases are uniformly distributed we show that an individual communicates more and to more distant individuals as the proximity of his preference to the average preference of the population increases. When preferences are the same within groups, but differ across groups, the impact of group size on communication again produces an interesting result: information transmission across groups is typically asymmetric, since players from the larger group communicate more easily to members of the smaller group than the reverse.

2.3.1. *Full Characterization*

Our main theorem provides a full characterization of the communication networks that arise as equilibrium outcomes of the cheap-talk stage of the game. In short, the theorem states that a player truthfully reveals his information to a group of players if his taste is not too different from the average taste of every subset of that group. More precisely, there exists an equilibrium network in which player i's set of receivers is $R_i \subseteq N\backslash\{i\}$ if and only if, for every subset of players in R_i, the average bias of the players in the subset is close enough to player i's bias.

Theorem 2 *There exists an equilibrium network in which player i's set of receivers is $R_i \subseteq N\backslash\{i\}$ iff for all $R_i' \subseteq R_i$ we have*

$$\left| b_i - \frac{\sum_{k \in R_i'} b_k}{|R_i'|} \right| \leq \frac{(n-1+\alpha)(n-1-\alpha|R_i'|)}{2(n-1)(n-1-\alpha|R_i|)}(\overline{s}_i - \underline{s}_i). \tag{2.4}$$

Proof. See Appendix 2.5.4. ■

To understand the intuition of this characterization, observe that when a sender's bias is close to the average bias of the receivers, a lie about his type may move these receivers' actions too far from the sender's ideal point. On the contrary, if the distance between the sender's bias and the average bias of the receivers is substantial, then the sender has an incentive to over-report or under-report his type so that the receivers' actions become closer to his ideal action. Since private communication allows the sender to lie to any subset R_i' of the set R_i of receivers, informational incentive constraints require that the sender's bias be close enough to the average bias of every subset $R_i' \subseteq R_i$.

The exact proximity between the sender's and the receivers' biases required for truthful communication depends on the threshold given by the RHS of Equation (3.1). It is worth noting that this threshold depends on the numbers of receivers $|R_i|$ and $|R_i'|$. In existing models of cheap talk to multiple audiences who do not interact strategically in the decision stage (e.g., Farrell and Gibbons, 1989; Goltsman and Pavlov, 2009; Galeotti et al., 2009a), the proximity of players' biases required for truthful communication depends only on the information structure and on some game parameters (here, $\overline{s}_i - \underline{s}_i$, α and n). Hence, with independent decision-makers, a *necessary* condition for truthful information transmission from a sender to a set of receivers is that, for every member of this set, there exists an equilibrium in which the sender transmits his information truthfully to this member only. In contrast, in our model, whether communication from a sender to a given receiver can be sustained in equilibrium depends on the *whole* set of players to whom the sender truthfully reveals his information. In particular, the incentive to communicate to a receiver not only depends on the conflict of interest between the sender and this receiver, but also on the number and the preferences of all the receivers to whom the sender sends a truthful message.

To understand why the RHS of Equation (3.1) depends on $|R_i|$ and $|R'_i|$, observe two differences between a cheap-talk model with independent decision-makers and ours. First, since the equilibrium actions of players in R_i depend on i's signal and others' actions, they react differently to i's signal depending on the total number of receivers $|R_i|$. Second, the sender also wants to coordinate his action with his receivers' actions. Hence, when he deviates and sends the wrong signal to a subset of receivers $R'_i \subseteq R_i$ (this deviation cannot be observed by any player different from player i himself), the coordination costs induced by that lie depend on the size of R'_i. Precisely, the threshold of the RHS of Equation (3.1) is decreasing in $|R'_i|$, meaning that it is less costly for the sender to lie to large subsets R'_i of R_i than to small ones.

As an illustration, and for future reference, consider a game with $n = 4$ players, $\alpha = 1/2$, and assume that every player has the same value of private information, given by $\overline{s}_i - \underline{s}_i = \frac{12\times 3}{7}$. The RHS of Equation (3.1) in Theorem 2 simplifies to $3\frac{6-|R'_i|}{6-|R_i|}$. It follows that player i reveals his type to all of the other players if for all $k, l \in N\backslash\{i\}$,

$$\left| b_i - \frac{\sum_{j\neq i} b_j}{3} \right| \leq 3, \quad \left| b_i - \frac{b_k + b_l}{2} \right| \leq 4, \quad \text{and} \quad |b_i - b_k| \leq 5. \tag{2.5}$$

Similarly, player i reveals his type only to players in $\{j,k\} \subsetneq N\backslash\{i\}$ if

$$\left|b_i - \frac{b_j + b_k}{2}\right| \leq 3, \quad \text{and} \quad |b_i - b_j|, |b_i - b_k| \leq 3.75. \tag{2.6}$$

Finally, player i reveals his type only to player $j \neq i$ if $|b_i - b_j| \leq 3$.[7] It appears clearly that the conditions ensuring that player i truthfully communicates with j depend on the whole set of receivers to which j may belong. Given a set of receivers, we can also see that the thresholds on the RHS decrease with the size of the subset considered.

The origin of one main insight of our work is given by the following observation: given $|R_i'|$, the RHS of Equation (3.1) is increasing in $|R_i|$. That is, the conditions given by Theorem 2 on the proximity between i's bias and the average bias of the strict subsets $R_i' \subsetneq R_i$ of receivers become weaker as the set of all receivers, R_i, is larger. The intuition is that, as we had already seen in the second-stage equilibrium characterization (Equation (2.2)), as $|R_i|$ increases, receivers are more responsive to whatever the sender is revealing to them. But the more responsive receivers are to a message by the sender, the less the latter has an incentive to over-report or under-report his information as it may affect the actions of the fixed set of receivers R_i' too much.

This feature implies that when the informational incentive constraints are satisfied for information transmission to a set of receivers, these constraints are not necessarily satisfied for information transmission to a strict subset of these receivers only. In particular, one key effect revealed by our model is that there may exist an equilibrium in which an individual reveals his true type to a group of players whereas there is no equilibrium in which he reveals it only to strict subsets of this group. As an example, consider the bias profile $b = (-4.1, 0, 3.8, 4.1)$ in the previous four-player game. There is then an equilibrium communication network in which player 2's set of receivers is $R_2 = \{1,3,4\}$, but there is no equilibrium communication network in which player 2's set of receivers is a strict subset of $\{1,3,4\}$.

[7] For example, with the bias profile $b = (b_1, b_2, b_3, b_4) = (0, 3.8, 4.8, 9)$, $(R_i)_{i\in N} = (\emptyset, \{3\}, \{1,2,4\}, \emptyset)$ is the most informative equilibrium communication network.

The fact that communication to a large group of recipients may occur in equilibrium even though communication to a small subset of that group may not relies on the receivers' need for coordination. This feature, therefore called the *disciplinary effect of coordination*, will be illustrated for particular configurations of biases in Subsection 2.3.4. Note that this disciplinary effect of coordination differs from the disciplinary effect of public communication identified by Farrell and Gibbons (1989) and further analyzed recently by Goltsman and Pavlov (2009). Indeed, our effect does not rely on the public nature of a sender's message and appears even when communication is private.

To see how coordination motives make large equilibrium communication networks feasible when intermediate communication networks are not, it is instructive to look at extreme situations. When there is almost no coordination motive ($\alpha \to 0$), incentive constraints are as in a model without strategic interactions in the decision stage: the condition for R_i to be an equilibrium set of receivers for player i reduces to

$$|b_i - b_j| \leq \frac{\overline{s}_i - \underline{s}_i}{2}, \text{ for all } j \in R_i.$$

That is, there is an equilibrium in which player i truthfully reveals his information to the players in R_i if and only if, for every agent in R_i, there is an equilibrium in which player i truthfully communicates with that agent only. This is because when the need for coordination vanishes, the responsiveness of the receivers' actions to i's signal and the sender's coordination costs mentioned above no longer depend on the number of receivers.

Consider now the opposite situation in which coordination costs are extremely high ($\alpha \to 1$) and let player i transmit his information to all the other players ($R_i = N \backslash \{i\}$) so that the responsiveness of players' actions to player i's signal is maximal. In that case, the incentive compatibility of Theorem 2 for player i reduces to

$$\left| b_i - \frac{\sum_{j \neq i} b_j}{n-1} \right| \leq \frac{n}{2(n-1)} (\overline{s}_i - \underline{s}_i).$$

That is, the incentive compatibility conditions ensuring that player i does not misrepresent his information to strict subsets of receivers are irrelevant. In particular, full revelation of information from player i to all the other players is possible whenever i's bias b_i is close enough to the average bias of the other players, $\frac{\sum_{j\neq i} b_j}{n-1}$, whatever the distribution of players' biases. The intuition is that, with extremely high coordination costs, player i never wants to lie about his type to only a subset of the other players, as, if he does so, his action cannot be perfectly coordinated with both the players to whom he lies and the players to whom he reveals his true type. This also means that as the weight of the coordination motive tends to one the conditions for full information revelation from any player are equivalent to the conditions for full information revelation were communication to be *public*. Indeed, if communication were public, informational incentive constraints would by definition be weaker than under private communication: the only possible deviation from a message sent publicly would be to jointly lie to the whole audience of players, while private communication enables the sender to lie to any subset of these players.

As shown in Theorem 2, each player's equilibrium communication strategy does *not* depend on other players' communication strategies. To obtain the intuition for this independence property, consider a strategy profile in which some player (say, player 1) reveals his type to players in R_1. This strategy is optimal if, whatever his type, player 1 has no incentive to lie to some or all players in R_1. Now consider the (unobservable) deviation that consists in player 1 lying to all players in R_1 (the intuition is exactly the same when he lies only to a subset of the players in R_1). Player 1's deviation affects player 1's expected utility by changing (i) the second-stage actions of players in R_1, who now act in believing player 1's wrong type instead of the true one and (ii) player 1's best reply to the latter actions. As can be seen from the (linear and additive) form of second-stage equilibrium actions given by Equation (2.2), the change in the actions of the players in R_1 is *independent* of what they learn about the types of players other than 1. This relies on our specific utility functions, on the additivity of the state and also on the independence of players' types.[8] Consequently, player 1's best reply to other players' actions and player 1's expected utility

[8]As an extreme example consider the situation in which some player $j \neq 1$ is almost perfectly informed about player 1's type, i.e. s_1 and s_j are strongly correlated. Then, when players in R_1 know player j's signal, i.e. players in R_1 also belong to R_j, player 1's message only has a limited impact on players' actions (and thus, on player 1's expected payoff), whereas if player j does not reveal his type to players in R_1, then player 1's signal has a stronger impact on the actions of players in R_1.

are affected by the change in the actions of players in R_1 independently of the information transmitted by other senders. In contrast, Austen-Smith (1993, Proposition 1) shows that the incentive for an expert to reveal truthfully his type to a decision-maker *depends* on the communication strategy of the other expert. While we assume that types are independent and the state is additive in types, in Austen-Smith (1993) players' signals are independent *conditional* on the state. Hence, in his model, the effect of the message of an expert on the beliefs and action of the decision-maker depends on how well informed the decision-maker is. More precisely, if the decision-maker is well informed, the expert's message affects his action only slightly, whereas if the decision-maker is poorly informed, the message will affect his action significantly. This implies that communication from a given expert to the decision-maker is more difficult when the other expert communicates with him.[9]

2.3.2. Comparative Statics

The next corollary, easily deduced from Theorem 2, details the effect of the disparity of players' preferences, the coordination motive, and the information structure on equilibrium sets of receivers.

Corollary 1 (Comparative Statics) *The conditions under which information is transmitted from any player i to any set of receivers are relaxed as:*

(i) All biases are reduced by the same factor: $(b_1, \ldots, b_n) \mapsto r(b_1, \ldots, b_n)$, where $r \in [0, 1)$;

(ii) The weight on coordination motives, α, increases;

(iii) The value of private information, $\bar{s}_i - \underline{s}_i$, increases.

Proof. Reducing the absolute values of the biases as in (i) clearly decreases the LHS of Inequality (3.1). The RHS of Inequality (3.1) is also clearly increasing in $\overline{s}_i - \underline{s}_i$. Finally, it is increasing in α because $\frac{\partial}{\partial \alpha} \frac{(n-1-\alpha|R_i'|)}{(n-1-\alpha|R_i|)} = \frac{(n-1)(|R_i|-|R_i'|)}{(n-1-\alpha|R_i|)^2} \geq 0$. ■

[9] A similar statistical structure is used in, e.g., Morgan and Stocken (2008) and Galeotti et al. (2009a), where the willingness of a sender to communicate with a player also declines in the number of senders communicating with that player.

The intuition for (i) is quite clear. As all the biases become more similar to each other, the conflict of interest between all players falls, so informational incentive constraints become weaker as in existing cheap-talk models.

Point (ii) is not as direct as (i) since, in our framework, it is the need for coordination that itself results in incentive conflicts between players. Indeed, were there no need for coordination ($\alpha = 0$), an equilibrium with perfect communication would always exist because a sender would be indifferent between revealing truthfully his information or not. When α is positive his message has an impact on his payoff through the modification of others' actions, and misrepresenting his type may be beneficial when his bias is significantly different from the receivers' biases. However, the higher is α, the more costly it is for the sender to coordinate his action to the actions of the receivers he lies to. The effect of the need for coordination on strategic information transmission is also analyzed in Alonso et al. (2008) who consider a two-division organization in which the decisions of the divisions are both responsive to local conditions and coordinated with each other.[10] Decision-makers' payoffs are similar to those we consider, but the conflict of interest regarding decisions is modeled differently. In Alonso et al. (2008), each division manager has an ideal action that depends on an idiosyncratic state, and maximizes a weighted sum of his own division's profit and that of the other division. Under decentralization, they also show that an increase in the need for coordination facilitates communication between the two divisions. On the contrary, under centralization, when a benevolent principal makes all decisions by relying on cheap-talk statements from the divisions, an increase in the need for coordination worsens communication.

Finally, the intuition of (iii) is standard. As we already observed, the value of player i's information, $\bar{s}_i - \underline{s}_i$, measures the impact of his message on the receivers' belief about the state. So, when $\bar{s}_i - \underline{s}_i$ is small, his influence on the receivers' actions is also small and his incentive to lie about his type is greater. In the extreme case in which $\bar{s}_i - \underline{s}_i$ tends to zero, the incentive constraints of player i would be similar to the condition for full revelation of information in a model with a continuum of types, as in Crawford and Sobel (1982), which

[10] See also Rantakari (2008).

is never satisfied except when players' preferences exactly coincide.

We can also note that, for a given sender and a given set of receivers, increasing the total number of players, n, strengthens the conditions on the proximity between the sender's and receivers' biases, since the RHS of Equation (3.1) is decreasing in n.[11] The intuition of this effect is similar to the intuition of (iii) in the previous corollary: as the total number of players rises, the influence of the actions of a fixed set of receivers is smaller, so the sender's incentive to misrepresent his type is greater. To account for variations in the size of the population, we could describe the state as the *average* of players' signals, instead of the sum (this is irrelevant when n is fixed, since we impose no restrictions on $\bar{s}_i$ and $\underline{s}_i$). Equivalently, we can replace each signal s_i of every player i by $\frac{s_i}{n}$. In this case, the RHS of Inequality (3.1) always tends to zero as n increases, whatever the set of receivers, so that information transmission becomes impossible between any pair of players who do not have the same preferences. This effect is similar to that observed by Morgan and Stocken (2008, Proposition 1) who show that truthful reporting is never an equilibrium for a sufficiently large sample of constituents.

2.3.3. General Properties

By taking a closer look at the way in which the informational incentive constraints given in Theorem 2 intersect, some general properties of the equilibrium sets of receivers can be established. First, we can always construct larger equilibrium sets of receivers by adding agents whose biases are closer to the sender's bias than to those of any of his receivers.

Corollary 2 *If there exists an equilibrium network in which player i's set of receivers is R_i, then there also exists an equilibrium network in which player i's set of receivers is $R_i \cup \{j\}$ for every player j whose bias is closer to i's bias than to those of any player in R_i, i.e.,*

$$|b_i - b_j| \leq |b_i - b_k|, \quad \forall\, k \in R_i. \tag{2.7}$$

[11] The sign of the derivative with respect to n is $2\alpha(n-1)|R_i'| - \alpha^2|R_i||R_i'| - (n-1)^2(|R_i| + 1 - |R_i'|)$, which is always negative.

Proof. See Appendix 2.5.5. ■

In particular, the above corollary implies that there always exists an equilibrium communication network in which, for every player i, his set of receivers R_i includes all the players with the same bias b_i. Note that if condition (2.7) holds for only *some* players in R_i, but not for all of them, then the result may not hold. To see this, consider again the four-player example introduced in Subsection 2.3.1 with the bias profile $b = (0, 2.2, 3.2, 3.7)$. In this case, there is an equilibrium in which player 1's set of receivers is $R_1 = \{2, 4\}$ but no equilibrium in which it is $\{2, 3, 4\}$.

Second, it is obvious from Theorem 2 that the existence of an equilibrium network in which player i's set of receivers is $\{j\}$ implies the existence of an equilibrium in which his set of receivers is $\{k\}$, for every k whose bias is between b_i and b_j. Applying the previous corollary, this yields:

Corollary 3 *If there exists an equilibrium network in which player i's set of receivers is $\{j\}$, $j > i$, then there also exists an equilibrium network in which player i's set of receivers is R_i for every $R_i \subseteq \{i + 1, \ldots, j\}$.*

Proof. Directly from Theorem 2 and Corollary 2. ■

Note that if players i and j have the same value of private information, i.e. $\bar{s}_i - \underline{s}_i = \bar{s}_j - \underline{s}_j$, and if there is an equilibrium network in which player i's set of receivers is $\{j\}$, then there is also an equilibrium network in which player j's set of receivers is $\{i\}$. Combined with the previous corollary, this reciprocity property implies that if all players in $\{i, \ldots, j\}$ have the same value of private information, then any communication network among these players is an equilibrium network. In particular, full revelation of information between all of the players in $\{i, \ldots, j\}$ is an equilibrium.

Under some conditions, larger communication networks can also be constructed by forming the union of existing equilibrium networks. As stated in the following corollary, combining two equilibrium sets of receivers R_i and $\tilde{R}_i$ for player i yields an equilibrium set of receivers $R_i \cup \tilde{R}_i$ for player i if R_i and $\tilde{R}_i$ do not overlap, i.e. $R_i \cap \tilde{R}_i = \emptyset$.

Corollary 4 *If there is an equilibrium network in which player i's set of receivers is R_i, and an equilibrium network in which player i's set of receivers is $\tilde{R}_i$, and if R_i and $\tilde{R}_i$ do not overlap, then there is also an equilibrium network in which player i's set of receivers is $R_i \cup \tilde{R}_i$.*

Proof. See Appendix 2.5.5. ■

More generally, the proof of the previous corollary reveals that a sufficient condition for the result to hold is that the distance between i's bias and the average bias in $R_i \cup \tilde{R}_i$ is smaller than the distance between i's bias and the average bias in R_i or $\tilde{R}_i$. When this condition does not hold, the union of the two equilibrium receiver sets does not necessarily yield another equilibrium receiver set. To see this, consider once more our four-player example with the bias profile $b = (0, 2.2, 3.2, 3.7)$. Here, there is an equilibrium network in which player 1's set of receivers is $R_1 = \{2,3\}$ and an equilibrium network in which it is $\tilde{R}_1 = \{2,4\}$, but no equilibrium network in which player 1's set of receivers is $R_1 \cup \tilde{R}_1 = \{2,3,4\}$. This implies that, in general, there may not exist a "maximal" equilibrium communication network which is more informative than all the other communication networks.

Necessary and sufficient conditions for the complete, Pareto dominant, communication network to be an equilibrium are easily deduced from Theorem 2. More precisely:

Corollary 5 (Complete Network) *The complete communication network is an equilibrium network if and only if for all $i \in N$ and $R_i \subseteq N \backslash \{i\}$,*

$$\left| b_i - \frac{\sum_{j \in R_i} b_j}{|R_i|} \right| \leq \frac{(n-1+\alpha)(n-1-\alpha|R_i|)}{2(n-1)^2(1-\alpha)} (\overline{s}_i - \underline{s}_i). \tag{2.8}$$

Proof. Directly from Theorem 2. ■

Note that when all players have the same value of private information, the set of these conditions is reduced, since we only have to check that the incentive constraints are satisfied for the two extreme players (player 1 and player n). In that case, a simple sufficient condition for the complete network to be an equilibrium is that there exist an equilibrium network in which player 1's set of receivers is $R_1 = \{n\}$ and player n's set of receivers is $R_n = \{1\}$.

2.3.4. Illustrations

In this subsection, we analyze two particular configurations of biases and obtain further results regarding the structure of equilibrium communication networks. Our aim is also to illustrate the disciplinary effect of coordination identified after Theorem 2. First, we consider *uniformly-distributed biases*, assuming that players' biases are equidistant: $b_{i+1} - b_i = \beta \geq 0$ for every $i \in N$. Second, we consider *two-spike biases* situations in which the players are partitioned into two groups, $L = \{1, \ldots, l\}$ and $M = \{l+1, \ldots, n\}$. Players in the first group have a bias b_L and players in the second group have a bias b_M, with $b_M - b_L = \beta > 0$. For both configurations of biases we know from Corollary 1 (i) that in equilibrium the maximal number of receivers of every player falls with the distance β.

To focus on the impact of players' positions on their communication behavior, we assume from now on that they all have the same value of private information: $\bar{s}_i - \underline{s}_i = \Delta$ for all i. In addition, we restrict our attention to equilibrium communication networks R which are maximal (i.e., such that there exists no equilibrium communication network more informative than R) and such that, for every i, players in $\{i\} \cup R_i$ are consecutive.[12]

Uniformly-Distributed Biases

When biases are uniformly distributed, observe that, for any size $|R_i|$, the distance between i's bias and the average bias of the $|R_i|$ players who are the closest to i in the population falls with the proximity of i's bias to the average bias in the whole population. Hence, from the equilibrium characterization of Theorem 1 we have:

Corollary 6 *If players' biases are uniformly distributed, then the maximal number of equilibrium receivers increases with the proximity of a sender's bias to the average bias in the population.*

[12]By consecutive, we mean that there is no player in $N \backslash (\{i\} \cup R_i)$ whose bias lies between the biases of any two players in $\{i\} \cup R_i$. From Theorem 2 it is easy to show that such an equilibrium always exists.

After stating Corollary 5, we noted that full revelation of information between all players is possible whenever the two extreme players reveal their information to all the other players. With uniformly-distributed biases, the previous corollary further asserts that middle-biased players (i.e. players whose biases are close to the average bias in the population) communicate more than extremists (i.e. players whose biases are far from the average bias in the population).

The impact of players' position on their communication behavior is even stronger than that stated in Corollary 6. If every player i, whatever his position, communicates to all players whose biases are less distant than some threshold $d > 0$, i.e., $R_i = \{k \neq i : |b_i - b_k| \leq d\}$, then it is clear that central players communicate more than extremists since $|\{k \neq j : |b_j - b_k| \leq d\}| \geq |\{k \neq i : |b_i - b_k| \leq d\}|$ whenever j is more central than i. The next corollary shows that not only the *number* of receivers increases with the sender's centrality, but so does the distance between the sender's bias and his receivers' biases. In other words, central players can truthfully communicate with agents with whom they have higher conflicts of interest than less central players can do.

Corollary 7 *If players' biases are uniformly distributed and $R_i = \{k \neq i : |b_i - b_k| \leq d\}$ is an equilibrium set of receivers for some player i and some distance $d \geq 0$, then $R_j = \{k \neq j : |b_j - b_k| \leq d\}$ is also an equilibrium set of receivers for every player j who is more central than i. In general, the reverse is not true.*

These results are illustrated by Figure 2.1 which plots the number of receivers $|R_i|$ as a function of the coordination motive α and player i's position, when $n = 7$, $\Delta = 2$ and $\beta = 0.6$. The figure shows how the number of receivers increases with players' centrality whatever the value of α, and with α whatever the players' position. When α is high enough (in this figure, when $\alpha \geq 0.3$) we also see that more central senders communicate to more distant receivers. For example, when $\alpha = 0.5$, the most central player (player 4) communicates to all the other players, while players 1 and 7 only communicate to a single player.

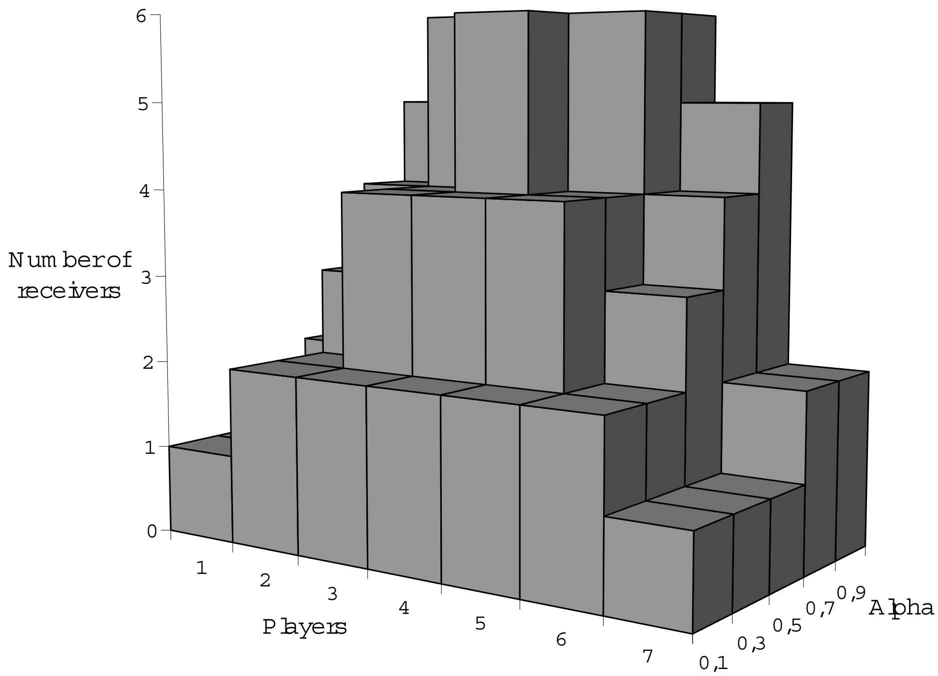


Figure 2.1: Number of equilibrium receivers with $n = 7$ players, uniformly-distributed biases, $\beta = 0.6$ and $\Delta = 2$.

To understand better the role of the coordination motive on the impact of players' position on their incentive to communicate, consider again extreme situations. When $\alpha \to 0$ we have already observed that the equilibrium condition for R_i to be a equilibrium set of receivers for player i reduces to $\max_{j \in R_i} |b_i - b_j| \leq \frac{\Delta}{2}$ whatever player i's position. So, in that case, only the distance between a sender and one receiver matters for incentive compatibility, and the reverse of Corollary 7 is true. More generally, when $\alpha > 0$, the equilibrium condition for a single receiver, namely

$$\beta \leq \frac{n-1+\alpha}{2(n-1)}\Delta,$$

is the same for every sender, but will depend on the sender's position for more than two receivers. To see this, notice that incentive compatibility for the less central players (player

1 and player n) to communicate to $r_i \in \{1, \ldots, n-1\}$ receivers can be written as:

$$\beta \leq \frac{(n-1+\alpha)(n-1-\alpha r_i')}{(n-1)(2r_i - r_i' + 1)(n-1-\alpha r_i)}\Delta, \quad \forall\, r_i' \in \{1, \ldots, r_i\}.$$

It can be shown[13] that for every $\alpha \in (0,1)$ this incentive compatibility condition becomes strictly stronger as r_i increases. That is, it is always more difficult for extremists to communicate to a larger group than to a smaller group, even when the coordination motive is very strong. On the contrary, for strong enough coordination motives, incentive compatibility conditions for more central players are not necessarily monotonic in the number of receivers. That is, large receiver sets may be equilibrium outcomes for more central players, while small receiver sets are not. For example, when n is odd, the incentive compatibility condition for the central player $i = \frac{n+1}{2}$ is always strictly weaker for $r_i + 1$ receivers than for r_i receivers when $r_i \leq n-1$ is odd. As α tends to one the condition always holds for $r_i = n-1$ receivers, but continues to represent a constraint for $r_i < n-1$ receivers. In particular, a central player may reveal his information to all players, while an extreme player may transmit his information to none.

Two-Spike Biases

When players are partitioned into two groups, full revelation of information amongst all players in the same group is always an equilibrium. In addition, Corollary 2 implies that if a player $i \in L$ ($i \in M$, resp.) transmits his information to players in R_i in equilibrium, then there is also an equilibrium in which i reveals his information to players in $R_i \cup (L \backslash \{i\})$ ($R_i \cup (M \backslash \{i\})$, resp.). Hence, there is a unique maximal equilibrium network, such that the set of receivers of each player $i \in L$ includes all players in $L \backslash \{i\}$ and the set of receivers of each player $i \in M$ includes all players in $M \backslash \{i\}$.

Since it is always possible for players to communicate to players with the same bias, and since the RHS of the informational incentive constraint (3.1) is increasing in the total number of receivers, a player's informational incentive constraints are relaxed as the relative

[13] A formal proof is available from the authors upon request.

number of players with the same bias increases. More precisely:

Corollary 8 *In the maximal equilibrium network with two-spike biases, a player's set of receivers increases, and includes more players from the other group, as the relative number of players in his own group increases.*

Proof. Directly from Theorem 2 and the observation above. ■

In particular, this corollary implies that intergroup information transmission is higher for players in the larger group than for players in the smaller group. As a simple example, consider the situation in which $\alpha \to 1$. Then, there is complete communication from players in group L if and only if $\beta \leq \frac{n}{2(n-l)}\Delta$, which becomes easier to satisfy as the size of this group, l, increases.

This property does not extend to more than two groups of players. For example, in the four-player example, when $\alpha = 0.9$ and $\bar{s}_i - \underline{s}_i = 1$ for all i, the condition for player i to reveal his information to all players is

$$\left| b_i - \frac{\sum_{j \neq i} b_j}{3} \right| \leq 0.65, \quad \left| b_i - \frac{b_k + b_l}{2} \right| \leq 2.6, \quad \text{and} \quad |b_i - b_k| \leq 4.55. \tag{2.9}$$

Hence, when $b = (-3, 0, 0, 3)$ there is an equilibrium in which players with zero bias transmit their information to all of the other players, but the first inequality does not hold when $b = (0, 0, 0, 3)$. Actually, when there are more than two groups of players with the same bias, the corollary above only applies, in general, for players in the two extreme groups, i.e., the group with bias b_1 and that with bias b_n.

2.4. Conclusion

In this chapter, we consider a class of economically-relevant coordination games in which information about a common state of nature is distributed among the players. Each of these players chooses an action by trading off the benefit of it being close to his own "ideal action", which depends both on the state and on an idiosyncratic bias, with that of being close to the other players' decisions. Before taking such actions, the players are offered the opportunity to communicate with each other in a decentralized and strategic manner. In this setting, our focus is on the way heterogeneity in preferences shapes strategic information transmission. We provide explicit conditions on the proximity of players' biases for information to be revealed by any sender to any group of receivers. Precisely, we show that an agent reveals his information to a group as long as this group is large enough and his ideal action is close enough to the average ideal action of every subset of agents in this group.

Similar coordination games with incomplete information have already been analyzed in the literature, but under the assumption that there is no conflict of interest between agents regarding the ideal state-contingent action profile (see, for example, Morris and Shin, 2002 and Angeletos and Pavan, 2007). When agents' goals are aligned, but there are physical or cost constraints on the number of communication links between agents, another literature has identified the most efficient communication structures; see, amongst others, Marschak and Radner (1972), Radner (1993), Jehiel (1999), Chwe (2000), Calvó-Armengol and Martí (2007, 2009), and Morris and Shin (2007). In these papers, efficient networks are characterized under *physical* communication constraints. On the contrary, our approach studies the equilibrium communication networks that arise under *strategic* communication constraints. To that extent, our work mainly borrows from the literature on strategic information transmission based on Crawford and Sobel (1982) but then proposes a framework in which every player is at the same time a sender and a receiver.

One key insight that stems from our characterization of equilibrium communication networks is that large networks may be easier to sustain in equilibrium that smaller ones. In other words, we show that the need for coordination of multiple interacting audiences can discipline communication, in the sense that truthful communication may be feasible in a large group but not in strict subsets of this group. A similar phenomenon is obtained in a team-theoretic framework in Dessein and Santos (2006), which also considers a coordination-adaptation situation with quadratic costs, but where communication is non-strategic.[14] Another main result is that agents who are more central in terms of preferences

[14]In that model, a communication link requires "bundling of tasks", which is assumed to be costly.

communicate more and have a greater impact on the decisions taken. Note that such a prediction contrasts with those of models of *costly* communication (e.g., Banerjee and Somanathan, 2001) where there is a tendency for extremists to express more voice.[15]

The way in which communication links have been constructed in the current analysis completely departs from usual non-cooperative network-formation games in a number of ways.[16] In typical games of this type, players' strategies mainly consist in listing desired contacts, given the exogenous costs and benefits of direct and indirect connections. In addition, since it is commonly admitted that much of the information required for economic decision-making is exchanged via networks of relationships, the value of these connections is often interpreted as being informational. However, whether agents have an effective interest in transmitting information once a link exists has not yet been investigated to the best of our knowledge. By way of contrast, we explicitly model agents' informational frameworks and derive the equilibrium links directly from the informativeness of agents' communication strategies. Given that the connection conveys truthful information, the benefits from linking are then endogenously determined by the way in which the information transmitted is used in the decision stage.

2.5. Appendix

2.5.1. Second-Stage Equilibrium Characterization

We first characterize the unique equilibrium action profile under complete information. The utility function of player i (see Equation (2.1)) can be rewritten as (minus a constant):

$$a_i \left[2(1-\alpha)(\theta + b_i) + \frac{2\alpha}{n-1} \sum_{j \neq i} a_j - a_i \right] - \frac{\alpha}{n-1} \sum_{j \neq i} (a_j)^2. \tag{2.10}$$

Bundling a few tasks together may reduce profits relative to stand-alone tasks (no communication links at all), while bundling a lot of them together may actually improve profitability.

[15]In their model, individuals only differ in terms of beliefs about a binary state of nature. Centrists are those who put relatively equal weight on the two states of the world, while extremists firmly believe in one or the other of the states.

[16]See Jackson (2008a) for an extensive survey of such models.

The best response of each player i to a_{-i} is given by:

$$a_i(a_{-i};\theta) = (1-\alpha)(\theta + b_i) + \frac{\alpha}{n-1}\sum_{j\neq i} a_j. \tag{2.11}$$

If a_i is a best response to a_{-i}, then it follows from Equations (3.14) and (2.11) that player i's utility takes the following simple form (minus a constant):

$$u_i(a_i(a_{-i};\theta), a_{-i};\theta) = (a_i(a_{-i};\theta))^2 - \frac{\alpha}{n-1}\sum_{j\neq i}(a_j)^2. \tag{2.12}$$

The system of equations formed by Equation (2.11) can be written as:

$$\begin{pmatrix} a_1 \\ \vdots \\ \vdots \\ a_n \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & -\frac{\alpha}{(n-1)} & \cdots & -\frac{\alpha}{(n-1)} \\ -\frac{\alpha}{(n-1)} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\frac{\alpha}{(n-1)} \\ -\frac{\alpha}{(n-1)} & \cdots & -\frac{\alpha}{(n-1)} & 1 \end{pmatrix}}_{I}^{-1} \begin{pmatrix} (1-\alpha)(\theta+b_1) \\ \vdots \\ \vdots \\ (1-\alpha)(\theta+b_n) \end{pmatrix}.$$

Simple algebra yields:

$$I^{-1} = \frac{1}{(n-1)-(n-2)\alpha-\alpha^2}\begin{pmatrix} (n-1)-(n-2)\alpha & \alpha & \cdots & \alpha \\ \alpha & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \alpha \\ \alpha & \cdots & \alpha & (n-1)-(n-2)\alpha \end{pmatrix}.$$

Therefore, when every player knows the state of nature, equilibrium actions are given by:

$$a_i(\theta) = \theta + \frac{[(n-1)-(n-2)\alpha]b_i + \alpha\sum_{j\neq i} b_j}{n+\alpha-1} \equiv \theta + B_i, \text{ for every } i \in N. \tag{2.13}$$

Since players' best responses are linear, exactly the same algebra shows that, under incomplete information, and whatever the information structure generated by the communication strategy profile, expected equilibrium actions are uniquely characterized by

$$E(a_i) = E(\theta) + B_i, \text{ for every } i \in N. \tag{2.14}$$

The uniqueness of the linear equilibrium identified in (2.2) is proved as in Calvó-Armengol and Martí (2009, Theorem 1). We define the following payoff function:

$$v(a_1, ..., a_n; s) = -(1-\alpha)\sum_{i \in N}(a_i - \theta(s) - b_i)^2 - \frac{\alpha}{2(n-1)}\sum_{i \in N}\sum_{j \neq i}(a_i - a_j)^2. \tag{2.15}$$

The set of equilibria of our second-stage coordination game is the same as that in the corresponding Bayesian game with identical agent preferences in which every player's payoff function is given by (2.15), as the best responses are identical in both games.

Theorem 4 of Marschak and Radner (1972)[167–168] provides a sufficient condition for the equilibrium of a Bayesian game with identical agent preferences to be determined uniquely by a system of linear equations when the set of states of the world is finite and payoff functions are given by:

$$\lambda(s) + 2\sum_{i \in N}\mu_i(s)a_i - \sum_{i,j \in N}v_{ij}(s)a_i a_j, \tag{2.16}$$

where the λ, μ_i and v_{ij} are all real-valued functions of the state of the world, $s \in S$. It is easily checked that the payoff function (2.15) can be written as (2.16), with

$$\lambda(s) = -(1-\alpha)\sum_{i \in N}(\theta(s) + b_i)^2 \ ,$$

$$\mu_i(s) = (1-\alpha)(\theta(s) + b_i) \ ,$$

$$v_{ii}(s) = v_{ii} = 1 \ ,$$

$$v_{ij}(s) = v_{ij} = -\frac{\alpha}{n-1} \ .$$

The sufficient condition in Theorem 4 of Marschak and Radner (1972) then boils down to the n-square matrix $[v_{ij}]_{i,j\in N}$ being positive definite. The determinant of $[v_{ij}]_{i,j\in N}$ is:

$$\begin{vmatrix} 1 & -\frac{\alpha}{n-1} & \cdots & -\frac{\alpha}{n-1} \\ -\frac{\alpha}{n-1} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\frac{\alpha}{n-1} \\ -\frac{\alpha}{n-1} & \cdots & -\frac{\alpha}{n-1} & 1 \end{vmatrix} = (1-\alpha) \begin{vmatrix} 1 & -\frac{\alpha}{n-1} & \cdots & -\frac{\alpha}{n-1} \\ 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\frac{\alpha}{n-1} \\ 1 & \cdots & -\frac{\alpha}{n-1} & 1 \end{vmatrix}$$

$$= (1-\alpha) \begin{vmatrix} 1 & -\frac{\alpha}{n-1} & \cdots & -\frac{\alpha}{n-1} \\ 0 & 1+\frac{\alpha}{n-1} & 0 & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1+\frac{\alpha}{n-1} \end{vmatrix} = (1-\alpha)(1+\frac{\alpha}{n-1})^{n-1}.$$

The first equality results from the replacement of the elements in the first column by the row sum, and then taking out the common factor $(1-\alpha)$. The second equality is obtained by subtracting the first row from every other row. We are left with an upper triangular matrix whose determinant is just the product of the diagonal term, which is positive. Similarly, we deduce that the leading principal minors of $[v_{ij}]_{i,j\in N}$ are positive. The matrix $[v_{ij}]_{i,j\in N}$ is thus positive definite.

Next, by explicitly solving some particular incomplete-information situations as above, it is possible to guess the general form of the unique second-stage equilibrium actions. To check that the solution given by Equation (2.2) is indeed the equilibrium when the communication strategy profile is characterized by $(R_i)_{i\in N}$, fix some player $l \in N$ and suppose that the second-stage equilibrium action of every player $i \neq l$ is given by Equation (2.2). We then show that player l's best response to this profile of second-stage actions $(a_i)_{i\neq l}$ is also of the form of Equation (2.2).

After the communication stage, for all $i \in N$, recall that $I_i = \{k : i \in R_k\} \cup \{i\}$ is the set of players whose signals are known by player i, $\overline{I}_i = \{k : i \notin R_k\}\backslash\{i\}$ the set of players whose signals are not known by player i, and let $E_i(\cdot) = E(\cdot \mid \{s_k : \ k \in I_i\})$ be player i's expectation operator conditional on the set of signals that he knows.

The expected payoff of player l after the communication stage is as follows:

$$-(1-\alpha)E_l\big[(a_l - \sum_{j\in N} s_j - b_l)^2\big] - \frac{\alpha}{n-1}\sum_{i\neq l} E_l\big[(a_l - a_i)^2\big], \tag{2.17}$$

so his best response is given by:

$$a_l = (1-\alpha)\left(\sum_{j\in I_l} s_j \ + \sum_{j\in \overline{I}_l} E(s_j) \ + b_l\right) + \frac{\alpha}{n-1}\sum_{i\neq l} E_l(a_i). \tag{2.18}$$

From now on and for every $i \in N$, we use the notation r_i for $|R_i|$. Using Equation (2.2) for $i \neq l$, player l's conditional expectation of player i's action is given by:

$$\begin{aligned} E_l(a_i) \quad &= \quad \sum_{j\in I_i} \frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j} + \sum_{j\in I_i\cap I_l} \frac{(1-\alpha)(n-1)s_j}{n-1-\alpha r_j} \\ &\quad + \sum_{j\in I_i\cap \overline{I}_l} \frac{(1-\alpha)(n-1)E(s_j)}{n-1-\alpha r_j} + \sum_{j\in \overline{I}_i} E(s_j) + B_i. \end{aligned}$$

Summing over all agents other than l, we can write:

$$\begin{aligned} &\sum_{i\neq l} E_l(a_i) = \\ &= \sum_{i\neq l}\sum_{j\in I_i\cap I_l} \frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j} + \sum_{i\neq l}\sum_{j\in I_i\cap \overline{I}_l} \frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j} + \sum_{i\neq l}\sum_{j\in I_i\cap I_l} \frac{(1-\alpha)(n-1)s_j}{n-1-\alpha r_j} \\ &+ \sum_{i\neq l}\sum_{j\in I_i\cap \overline{I}_l} \frac{(1-\alpha)(n-1)E(s_j)}{n-1-\alpha r_j} + \sum_{i\neq l}\sum_{j\in \overline{I}_i\cap I_l} E(s_j) + \sum_{i\neq l}\sum_{j\in \overline{I}_i\cap \overline{I}_l} E(s_j) + \sum_{i\neq l} B_i. \end{aligned} \tag{2.19}$$

Every signal s_j known by player l is known by r_j players other than l and not known by $n-1-r_j$ players different from l; every signal s_j not known by player l is known by r_j+1 players other than l and not known by $n-2-r_j$ players other than l. This enables us to deduce that:

$$\begin{aligned}
\sum_{i\neq l} E_l(a_i) &= \sum_{j\in I_l} r_j \,\frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j} + \sum_{j\in \overline{I}_l} (r_j+1)\,\frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j} \\
&\quad + \sum_{j\in I_l} r_j \,\frac{(1-\alpha)(n-1)s_j}{n-1-\alpha r_j} + \sum_{j\in \overline{I}_l} (r_j+1)\,\frac{(1-\alpha)(n-1)E(s_j)}{n-1-\alpha r_j} \\
&\quad + \sum_{j\in I_l} (n-1-r_j)\,E(s_j) + \sum_{j\in \overline{I}_l} (n-2-r_j)\,E(s_j) + \sum_{i\neq l} B_i. \\
&= \sum_{j\in I_l} \frac{r_j(1-\alpha)(n-1)s_j + (n-1)(n-1-r_j)E(s_j)}{n-1-\alpha r_j} \\
&\quad + \sum_{j\in \overline{I}_l} (n-1)\,E(s_j) + \sum_{i\neq l} B_i. \qquad (2.20)
\end{aligned}$$

In addition, we have:

$$\sum_{i\neq l} B_i = \frac{\alpha(n-1)b_l + (n-1)\sum_{i\neq l} b_i}{n+\alpha-1}. \qquad (2.21)$$

Plugging (2.21) and (2.20) into (2.18) and simplifying, we obtain player l's optimal action, which takes exactly the same form as that in Equation (2.2).

2.5.2. *Proof of Proposition 6*

The ex ante equilibrium payoff of player $j \in N$ is given by:

$$\begin{aligned}
U_j &= -(1-\alpha)Var(a_j - \sum_{i\in N} s_i - b_j) - (1-\alpha)[E(a_j - \sum_{i\in N} s_i - b_j)]^2 \\
&\quad -\frac{\alpha}{n-1}\sum_{m\neq j} Var(a_j - a_m) - \frac{\alpha}{n-1}\sum_{m\neq j}[E(a_j - a_m)]^2.
\end{aligned}$$

It follows from (2.14) that $E(a_j) = \sum_{i\in N} E(s_i) + B_j$, so we have:

$$U_j = -(1-\alpha)Var(a_j - \sum_{i\in N} s_i) - \frac{\alpha}{n-1}\sum_{m\neq j} Var(a_j - a_m) - (1-\alpha)[B_j - b_j]^2 - \frac{\alpha}{n-1}\sum_{m\neq j}[B_j - B_m]^2.$$

We consider two communication networks $R = (R_k)_{k\in N}$ and $R' = (R'_k)_{k\in N}$ such that $R_i \neq R'_i$ and $R_k = R'_k$ for all $k \in N\backslash\{i\}$. That is, R and R' are identical except that player i has a different set of receivers in R'. Player i is fixed throughout the analysis. The ex ante equilibrium payoff of every player $j \in N$ with the communication network R (R', resp.) is denoted by U_j (U'_j, resp.). Given the communication network R (R', resp.), the second-stage equilibrium action of every player $j \in N$ is denoted by a_j (a'_j, resp.). For all $j \in N$, given a strategic communication network R (R', resp.), let $I_j = \{k : j \in R_k\} \cup \{j\}$ ($I'_j = \{k : j \in R'_k\} \cup \{j\}$, resp.) denote the set of players whose signals are known by player j, and $\overline{I}_j = \{k : j \notin R_k\} \backslash \{j\}$ ($\overline{I'}_j = \{k : j \notin R'_k\} \backslash \{j\}$, resp.) the set of players whose signals are not known by player j.

For every player $j \in N$, we have:

$$\begin{aligned} U_j - U'_j = (1-\alpha) &\left(Var(a'_j - \sum_{i \in N} s_i) - Var(a_j - \sum_{i \in N} s_i) \right) \\ &+ \frac{\alpha}{n-1} \left(\sum_{m \neq j} Var(a'_j - a'_m) - \sum_{m \neq j} Var(a_j - a_m) \right). \end{aligned} \tag{2.22}$$

The second-stage equilibrium action a_j given by (2.2) enables us to write:

$$Var(a_j - \sum_{i \in N} s_i) = Var \left(\sum_{l \in I_j} \frac{\alpha(n-1-r_l)[E(s_l) - s_l]}{n-1-\alpha r_l} + \sum_{l \in \overline{I}_j} [E(s_l) - s_l] + B_j \right).$$

The independence of signals yields:

$$\begin{aligned} &Var(a_j - \sum_{i \in N} s_i) \\ &= \sum_{l \in I_j} Var \left(\frac{\alpha(n-1-r_l)s_l}{n-1-\alpha r_l} \right) + \sum_{l \in \overline{I}_j} Var(s_l) = \sum_{l \in I_j} \left(\frac{\alpha(n-1-r_l)}{n-1-\alpha r_l} \right)^2 Var(s_l) + \sum_{l \in \overline{I}_j} Var(s_l) \\ &= \sum_{l \in I_j \backslash \{i\}} \left(\frac{\alpha(n-1-r_l)}{n-1-\alpha r_l} \right)^2 Var(s_l) + \sum_{l \in \overline{I}_j \backslash \{i\}} Var(s_l) + \mathbf{1}[i \in I_j] \left(\frac{\alpha(n-1-r_i)}{n-1-\alpha r_i} \right)^2 Var(s_i) \\ &\quad + \mathbf{1}[i \in \overline{I}_j]\, Var(s_i), \end{aligned}$$

where $\mathbf{1}[i \in I_j]$ is an indicator function that equals 1 when player j knows the signal s_i, and $\mathbf{1}[i \in \overline{I}_j]$ is an indicator function that equals 1 when player j does not know the signal s_i. A similar equation holds for $Var(a'_j - \sum_{i \in N} s_i)$, when the communication network is

R'.

The two communication networks R and R' that we consider are such that $I_j \backslash \{i\} = I'_j \backslash \{i\}$ and $\overline{I}_j \backslash \{i\} = \overline{I'}_j \backslash \{i\}$, so that for all $j \in N$ we have:

$$Var(a'_j - \sum_{i \in N} s_i) - Var(a_j - \sum_{i \in N} s_i) = Var(s_i)$$
$$\left[\mathbf{1}[i \in I'_j] \left(\frac{\alpha(n-1-r'_i)}{n-1-\alpha r'_i} \right)^2 + \mathbf{1}[i \in \overline{I'}_j] - \mathbf{1}[i \in I_j] \left(\frac{\alpha(n-1-r_i)}{n-1-\alpha r_i} \right)^2 - \mathbf{1}[i \in \overline{I}_j] \right]. \tag{2.23}$$

When the communication network is R, for all $j \in N$ and $m \neq j$, we have, from (2.2):

$$\begin{aligned} Var(a_j - a_m) &= \sum_{l \in I_j \cap \overline{I}_m} \left(\frac{(1-\alpha)(n-1)}{n-1-\alpha r_l} \right)^2 Var(s_l) + \sum_{l \in \overline{I}_j \cap I_m} \left(\frac{(1-\alpha)(n-1)}{n-1-\alpha r_l} \right)^2 Var(s_l) \\ &= \sum_{l \in (I_j \cap \overline{I}_m) \backslash \{i\}} \left(\frac{(1-\alpha)(n-1)}{n-1-\alpha r_l} \right)^2 Var(s_l) + \sum_{l \in (\overline{I}_j \cap I_m) \backslash \{i\}} \left(\frac{(1-\alpha)(n-1)}{n-1-\alpha r_l} \right)^2 Var(s_l) \\ &\quad + \left(\frac{(1-\alpha)(n-1)}{n-1-\alpha r_i} \right)^2 \left[\mathbf{1}[i \in I_j \cap \overline{I}_m] + \mathbf{1}[i \in \overline{I}_j \cap I_m] \right] Var(s_i). \end{aligned}$$

A similar equation holds for $Var(a'_j - a'_m)$ when the communication network is R'.

The two communication networks R and R' are such that $(I_j \cap \overline{I}_m) \backslash \{i\} = (I'_j \cap \overline{I'}_m) \backslash \{i\}$ and $(\overline{I}_j \cap I_m) \backslash \{i\} = (\overline{I'}_j \cap I'_m) \backslash \{i\}$, so for all $j \in N$ and $m \neq j$ we have:

$$Var(a'_j - a'_m) - Var(a_j - a_m)$$
$$= ((1-\alpha)(n-1))^2 \left[\frac{\mathbf{1}[i \in I'_j \cap \overline{I'}_m] + \mathbf{1}[i \in \overline{I'}_j \cap I'_m]}{(n-1-\alpha r'_i)^2} - \frac{\mathbf{1}[i \in I_j \cap \overline{I}_m] + \mathbf{1}[i \in \overline{I}_j \cap I_m]}{(n-1-\alpha r_i)^2} \right] Var(s_i). \tag{2.24}$$

Plugging (2.23) and (2.24) into (2.22), we obtain: $U_j - U'_j =$

$$(1-\alpha)\Bigg[\mathbf{1}[i \in I'_j]\left(\frac{\alpha(n-1-r'_i)}{n-1-\alpha r'_i}\right)^2 + \mathbf{1}[i \in \overline{I'}_j] - \mathbf{1}[i \in I_j]\left(\frac{\alpha(n-1-r_i)}{n-1-\alpha r_i}\right)^2 - \mathbf{1}[i \in \overline{I}_j]$$
$$+\alpha(1-\alpha)(n-1)\sum_{m\neq j}\left(\frac{\mathbf{1}[i \in I'_j \cap \overline{I'}_m] + \mathbf{1}[i \in \overline{I'}_j \cap I'_m]}{(n-1-\alpha r'_i)^2} - \frac{\mathbf{1}[i \in I_j \cap \overline{I}_m] + \mathbf{1}[i \in \overline{I}_j \cap I_m]}{(n-1-\alpha r_i)^2}\right)\Bigg] Var(s_i). \tag{2.25}$$

We next focus on the particular case in which $|R_i| < |R'_i|$ and let $L = R_i \cap R'_i$ be the set of agents who belong both to R_i and R'_i. The set L is fixed throughout the analysis, and $|L| = l$. Let $|R_i| = r_i$, $|R'_i| = r'_i$, $|R_i \backslash l| = r_i - l$ and $|R'_i \backslash l| = r'_i - l$. To evaluate the sign of $U_j - U'_j$, in order to establish who is better off and who is worse off under the networks R and R', the set N of players is divided into the following five types:

- *(i)* Players who belong to R'_i and also to R_i. For every such player $j \in L$, we have $i \in I_j$ and $i \in I'_j$.
- *(ii)* Players who belong to R'_i but not to R_i. For every such player $j \in R'_i \backslash L$, we have $i \in \overline{I}_j$ and $i \in I'_j$.
- *(iii)* Players other than player i who belong neither to R'_i nor to R_i. For every such player $j \in N \backslash (R_i \cup R'_i \cup \{i\})$, we have $i \in \overline{I}_j$ and $i \in \overline{I'}_j$.
- *(iv)* Players who do not belong to R'_i but belong to R_i. For every such player $j \in R_i \backslash L$, we have $i \in I_j$ and $i \in \overline{I'}_j$.
- *(v)* Player i, for whom we have $i \in I_i$ and $i \in I'_i$.

(i) For every player $j \in L$, the set of players other than j can be divided into four disjoint sets of players: $\{i\} \cup L \backslash \{j\}$, $N \backslash (R_i \cup R'_i \cup \{i\})$, $R'_i \backslash L$ and $R_i \backslash L$. We have:

- for every player $m \in \{i\} \cup L \backslash \{j\}$, $i \in I_m$ and $i \in I'_m$;
- for every player $m \in N \backslash (R_i \cup R'_i \cup \{i\})$, $i \in \overline{I}_m$ and $i \in \overline{I'}_m$, and we have $|N \backslash (R_i \cup R'_i \cup \{i\})| = n - 1 - r_i - r'_i + l$;

- for every player $m \in R'_i \backslash L$, $i \in \overline{I}_m$ but $i \in I'_m$;
- for every player $m \in R_i \backslash L$, $i \in I_m$ but $i \in \overline{I'}_m$.

Since $i \in I_j$ and $i \in I'_j$, Equation (2.25) simplifies to:

$$U_j - U'_j \;\; = \;\; \alpha(1-\alpha)\left(\frac{n-1-r'_i}{n-1-\alpha r'_i} - \frac{n-1-r_i}{n-1-\alpha r_i}\right)Var(s_i). \qquad (2.26)$$

Using $r'_i > r_i$, we obtain $U_j - U'_j = -\left(\frac{\alpha(1-\alpha)^2(n-1)(r'_i-r_i)}{(n-1-\alpha r'_i)(n-1-\alpha r_i)}\right)Var(s_i) < 0$. Hence, for all $j \in L$, we have $U_j < U'_j$.

(ii) For every player $j \in R'_i \backslash L$, the set of players other than j can be divided into four disjoint sets of players: $\{i\} \cup L$, $N \backslash (R_i \cup R'_i \cup \{i\})$, $R'_i \backslash (L \cup \{j\})$ and $R_i \backslash L$. We have:

- for every player $m \in \{i\} \cup L$, $i \in I_m$ and $i \in I'_m$, and we have $|\{i\} \cup L| = l + 1$;
- for every player $m \in N \backslash (R_i \cup R'_i \cup \{i\})$, $i \in \overline{I}_m$ and $i \in \overline{I'}_m$, and we have $|N \backslash (R_i \cup R'_i \cup \{i\})| = n - 1 - r_i - r'_i + l$;
- for every player $m \in R'_i \backslash (L \cup \{j\})$, $i \in \overline{I}_m$ but $i \in I'_m$;
- for every player $m \in R_i \backslash L$, $i \in I_m$ but $i \in \overline{I'}_m$.

Since $i \in \overline{I}_j$ and $i \in I'_j$, Equation (2.25) simplifies to:

$$U_j - U'_j \;\; = \;\; -(1-\alpha)^2(n-1)\left(\frac{1}{n-1-\alpha r'_i} + \frac{\alpha\,(r_i+1)}{(n-1-\alpha r_i)^2}\right)Var(s_i) < 0. \qquad (2.27)$$

Hence, for all players $j \in R'_i \backslash L$, we have $U_j < U'_j$.

(iii) For every player $j \in N \backslash (R_i \cup R'_i \cup \{i\})$, the set of players other than j can be divided into four disjoint sets of players: $\{i\} \cup L$, $N \backslash (R_i \cup R'_i \cup \{i,j\})$, $R'_i \backslash L$ and $R_i \backslash L$. We have:

- for every player $m \in \{i\} \cup L$, $i \in I_m$ and $i \in I'_m$, and we have $|\{i\} \cup L| = l + 1$;
- for every player $m \in N \backslash (R_i \cup R'_i \cup \{i,j\})$, $i \in \overline{I}_m$ and $i \in \overline{I'}_m$;

- for every player $m \in R'_i \backslash L$, $i \in \overline{I}_m$ but $i \in I'_m$;
- for every player $m \in R_i \backslash L$, $i \in I_m$ but $i \in \overline{I'}_m$.

Since $i \in \overline{I}_j$ and $i \in \overline{I'}_j$, Equation (2.25) simplifies to:

$$U_j - U'_j = \alpha(1-\alpha)^2(n-1)\left(\frac{r'_i+1}{(n-1-\alpha r'_i)^2} - \frac{r_i+1}{(n-1-\alpha r_i)^2}\right) Var(s_i). \quad (2.28)$$

Using $r'_i > r_i$, we have $\left[\frac{r'_i+1}{(n-1-\alpha r'_i)^2} - \frac{r_i+1}{(n-1-\alpha r_i)^2}\right] > 0$. Hence, for all $N \backslash (R_i \cup R'_i \cup \{i\})$, we have $U_j > U'_j$.

(iv) For every player $j \in R_i \backslash L$, the set of players other than j can be divided into four disjoint sets of players: $\{i\} \cup L$, $N \backslash (R_i \cup R'_i \cup \{i\})$, $R'_i \backslash L$ and $R_i \backslash (L \cup \{j\})$. We have:

- for every player $m \in \{i\} \cup L$, $i \in I_m$ and $i \in I'_m$, and we have $|\{i\} \cup L| = l + 1$;
- for every player $m \in N \backslash (R_i \cup R'_i \cup \{i\})$, $i \in \overline{I}_m$ and $i \in \overline{I'}_m$, and we have $|N \backslash (R_i \cup R'_i \cup \{i\})| = n - 1 - r_i - r'_i + l$;
- for every player $m \in R'_i \backslash L$, $i \in \overline{I}_m$ but $i \in I'_m$;
- for every player $m \in R_i \backslash (L \cup \{j\})$, $i \in I_m$ but $i \in \overline{I'}_m$.

Since $i \in I_j$ and $i \in \overline{I'}_j$, Equation (2.25) simplifies to:

$$U_j - U'_j = (1-\alpha)^2(n-1)\left(\frac{1}{n-1-\alpha r_i} + \frac{\alpha\,(r'_i+1)}{(n-1-\alpha r'_i)^2}\right) Var(s_i) > 0. \quad (2.29)$$

Hence, for every player $j \in R_i \backslash L$, we have $U_j > U'_j$.

(v) The set of players other than i can be divided into four disjoint sets of players: L, $N \backslash (R_i \cup R'_i \cup \{i\})$, $R'_i \backslash L$ and $R_i \backslash L$. We have:

- for every player $m \in L$, $i \in I_m$ and $i \in I'_m$;
- for every player $m \in N \backslash (R_i \cup R'_i \cup \{i\})$, $i \in \overline{I}_m$ and $i \in \overline{I'}_m$;
- for every player $m \in R'_i \backslash L$, $i \in \overline{I}_m$ but $i \in I'_m$

- for every player $m \in R_i \backslash L$, $i \in I_m$ but $i \in \overline{I'}_m$.

Since $i \in I_i$ and $i \in I'_i$, Equation (2.25) yields exactly the same difference as Equation (2.26). Hence, for player i such that $r_i < r'_i$, we have $U_i < U'_i$. This completes the proof of Proposition 6.

2.5.3. *Proof of Proposition 7*

Consider two communication networks $R = (R_i, R_{-i})$ and $R' = (R'_i, R_{-i})$, such that $r'_i > r_i$ and let $L = R_i \cap R'_i$, with $|L| = l$. Ex ante expected welfare is the sum of ex ante expected utilities. When the communication network is R', expected welfare is given by:

$$W' = \sum_{j \in R'_i} U'_j + \sum_{j \in N \backslash (R'_i \cup \{i\})} U'_j + U'_i.$$

When the communication network is R, it is given by:

$$W = \sum_{j \in R_i} U_j + \sum_{j \in N \backslash (R_i \cup \{i\})} U_j + U_i.$$

Using the fact that for all $j \in L$, $U_j - U'_j = U_i - U'_i$, we write the difference $W - W'$ as:

$$W - W' = \sum_{j \in \{i\} \cup L} [U_j - U'_j] + \sum_{j \in N \backslash (R_i \cup R'_i \cup \{i\})} [U_j - U'_j] + \sum_{j \in R'_i \backslash L} [U_j - U'_j] + \sum_{j \in R_i \backslash L} [U_j - U'_j].$$

We have $|\{i\} \cup L| = l + 1$, $|R'_i \backslash L| = r'_i - l$, $|R_i \backslash L| = r_i - l$ and $|N \backslash (R_i \cup R'_i \cup \{i\})| = n - 1 - r_i - r'_i + l$. Using Equation (2.25) in the same way as in the proof of Proposition 6, we obtain:

$$\begin{aligned} W - W' = \;& \alpha(1-\alpha)(l+1)\left[\frac{n-1-r'_i}{n-1-\alpha r'_i} - \frac{n-1-r_i}{n-1-\alpha r_i}\right] Var(s_i) \\ & + \alpha(1-\alpha)^2(n-1)(n-1-r_i-r'_i+l)\left[\frac{r'_i+1}{(n-1-\alpha r'_i)^2} - \frac{r_i+1}{(n-1-\alpha r_i)^2}\right] Var(s_i) \\ & - (1-\alpha)^2(n-1)(r'_i-l)\left[\frac{1}{n-1-\alpha r'_i} + \frac{\alpha\,(r_i+1)}{(n-1-\alpha r_i)^2}\right] Var(s_i) \\ & + (1-\alpha)^2(n-1)(r_i-l)\left[\frac{1}{n-1-\alpha r_i} + \frac{\alpha\,(r'_i+1)}{(n-1-\alpha r'_i)^2}\right] Var(s_i). \end{aligned}$$

After some simplification, we have $W - W' =$

$$\underbrace{-\frac{(1-\alpha)^3(n-1)^2 Var(s_i)}{(n-1-\alpha r_i')^2(n-1-\alpha r_i)^2}}_{<0}\underbrace{\left[\alpha^2(r_i^2(1+r_i') - r_i'^2(1+r_i)) + 2\alpha(n-1)(r_i'-r_i) + (n-1)^2(r_i'-r_i)\right]}_{x}.$$

Solving $x = 0$ in α yields the following discriminant: $4(n-1)^2(r_i'-r_i)^2(1+r_i)(1+r_i') \geq 0$. We have $x \geq 0$ if and only if $\alpha \in [\alpha_1, \alpha_2]$, with $\alpha_1 = \frac{(n-1)[1-\sqrt{(1+r_i)(1+r_i')}]}{(1+r_i)(1+r_i')-1}$ and $\alpha_2 = \frac{(n-1)[1+\sqrt{(1+r_i)(1+r_i')}]}{(1+r_i)(1+r_i')-1}$. From $r_i \geq 1$ and $r_i' \geq 2$, we deduce that $\alpha_1 < 0$. From $r_i \leq n-2$ and $r_i' \leq n-1$, and the fact that α_2 is decreasing in r_i and r_i', we deduce that $\alpha_2 > 1$. Since $\alpha \in (0,1)$, x is always strictly positive. Hence, $W < W'$.

2.5.4. *Proof of Theorem 2*

Consider an equilibrium in which each player i reveals his type to players in $R_i \subseteq N\backslash\{i\}$. Without loss of generality, assume that each player i sends to every player $j \in R_i$ the message $m_i^j = \overline{m}$ when his type is $\overline{s}_i$ and the message $m_i^j = \underline{m}$ when his type is $\underline{s}_i$, and sends the same message whatever his type to players outside R_i. Given $(R_i)_{i \in N}$, the second-stage equilibrium actions are given by (2.2).

Without loss of generality, we look for the conditions under which player 1 does not deviate from his equilibrium communication strategy described above. First, assume that player 1's true type is $s_1 = \overline{s}_1$. In equilibrium, using Equation (2.2), the second-stage action of every player $i \in R_1 \cup \{1\}$ is given by

$$\begin{aligned}\overline{a}_i &= \sum_{j \in I_i \backslash \{1\}} \frac{\alpha(n-1-r_j)E(s_j) + (1-\alpha)(n-1)s_j}{n-1-\alpha r_j} + \sum_{j \in \overline{I}_i} E(s_j) + B_i \\ &\quad + \frac{\alpha(n-1-r_1)E(s_1) + (1-\alpha)(n-1)\overline{s}_1}{n-1-\alpha r_1},\end{aligned} \tag{2.30}$$

and the second-stage action of every player $i \notin R_1 \cup \{1\}$ is given by

$$a_i = \sum_{j \in I_i} \frac{\alpha(n-1-r_j)E(s_j) + (1-\alpha)(n-1)s_j}{n-1-\alpha r_j} + \sum_{j \in \overline{I}_i \backslash \{1\}} E(s_j) + B_i + E(s_1) \tag{2.31}$$

The relevant deviations for player 1 in the communication stage consist in lying to a subset of players $M \subseteq R_1$, i.e. sending message $\underline{m}$ instead of $\overline{m}$ to players in M (and not deviating towards the other players).[17] Let $m = |M|$, and denote by $(a_i')_{i\in N}$ the profile of players' actions after this deviation. Every player $i \in M$ chooses action $a_i' = \underline{a}_i$, which is given by replacing $\overline{s}_1$ by $\underline{s}_1$ in (2.30). The action a_i' of every player $i \in N\backslash(M \cup \{1\})$ is the same as that in the original equilibrium. Player 1's optimal action in the second stage is obtained from the best response of Equation (2.18) to $(a_i')_{i\neq 1}$, and takes the following form:

$$a_1' = (1-\alpha)\left(\sum_{j\in I_1\backslash\{1\}} s_j + \overline{s}_1 + \sum_{j\in \overline{I}_1} E(s_j) + b_1\right) + \frac{\alpha}{n-1}\sum_{i\neq 1} E_1(a_i'). \tag{2.32}$$

Using the same reasoning as that used to obtain expression (2.20), we have:

$$\begin{aligned}\sum_{i\neq 1} E_1(a_i') \quad = \quad & \sum_{j\in I_1} r_j \,\frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j} + \sum_{j\in \overline{I}_1} (r_j+1)\,\frac{\alpha(n-1-r_j)E(s_j)}{n-1-\alpha r_j} \\ & + \sum_{j\in I_1\backslash\{1\}} r_j \,\frac{(1-\alpha)(n-1)s_j}{n-1-\alpha r_j} + \frac{m(1-\alpha)(n-1)\underline{s}_1}{n-1-\alpha r_1} \\ & + \frac{(r_1-m)(1-\alpha)(n-1)\overline{s}_1}{n-1-\alpha r_1} + \sum_{j\in \overline{I}_1} (r_j+1)\,\frac{(1-\alpha)(n-1)E(s_j)}{n-1-\alpha r_j} \\ & + \sum_{j\in I_1} (n-1-r_j)E(s_j) + \sum_{j\in \overline{I}_1} (n-2-r_j)\, E(s_j) + \sum_{i\neq 1} B_i. \end{aligned} \tag{2.33}$$

Plugging (2.33) into (2.32), using (2.21) and simplifying, we obtain:

$$\begin{aligned} a_1' \quad = \quad & \sum_{j\in I_1\backslash\{1\}} \frac{\alpha(n-1-r_j)E(s_j) + (1-\alpha)(n-1)s_j}{n-1-\alpha r_j} + \sum_{j\in \overline{I}_1} E(s_j) \\ & + \frac{\alpha m(1-\alpha)\underline{s}_1 + (n-1-\alpha m)(1-\alpha)\overline{s}_1 + \alpha(n-1-r_1)E(s_1)}{n-1-\alpha r_1} + B_1. \end{aligned} \tag{2.34}$$

We denote by V_1 the expected payoff of player 1 conditional on signal s_1 under the original equilibrium, and by V_1' his expected payoff conditional on signal s_1 when he deviates by lying to players in M (and thus plays action a_1' in the second-stage game). Player 1

[17] In equilibrium, any message off of the equilibrium path is interpreted as exactly $\underline{m}$ or $\overline{m}$.

does not deviate by lying to players in M if $V_1' - V_1 \leq 0$. We have:

$$V_1' - V_1 = (1-\alpha)E\big[(\overline{a}_1 - \sum_{i\in N} s_i - b_1)^2 - (a_1' - \sum_{i\in N} s_i - b_1)^2 \mid s_1\big]$$
$$+\frac{\alpha}{n-1}\Bigg(\sum_{i\in M} E\big[(\overline{a}_1 - \overline{a}_i)^2 - (a_1' - \underline{a}_i)^2 \mid s_1\big]$$
$$+ \sum_{i\in R_1\setminus M} E\big[(\overline{a}_1 - \overline{a}_i)^2 - (a_1' - \overline{a}_i)^2 \mid s_1\big] + \sum_{i\in N\setminus(R_1\cup\{1\})} E\big[(\overline{a}_1 - a_i)^2 - (a_1' - a_i)^2 \mid s_1\big]\Bigg).$$

For the sake of simplicity, we examine separately the elements of the difference $V_1' - V_1$ and use the following notation for $i \neq 1$:

$$z_i = \sum_{j\in(I_1\cap\overline{I}_i)\setminus\{1\}} \frac{(1-\alpha)(n-1)(s_j - E(s_j))}{n-1-\alpha r_j} + \sum_{j\in(\overline{I}_1\cap I_i)\setminus\{1\}} \frac{(1-\alpha)(n-1)(E(s_j) - s_j)}{n-1-\alpha r_j} + B_1 - B_i.$$

Using (2.30), (2.31) and (2.34) and the fact that $E[z_i \mid s_1] = B_1 - B_i$:

$$\sum_{i\in M} E\left[(\overline{a}_1 - \overline{a}_i)^2 - (a_1' - \underline{a}_i)^2 \mid s_1\right] = \sum_{i\in M} E\left[z_i^2 - \left(z_i + \frac{(1-\alpha)(n-1-\alpha m)(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1}\right)^2 \middle| s_1\right]$$
$$= -2\left(\frac{(1-\alpha)(n-1-\alpha m)(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1}\right)\sum_{i\in M}(B_1 - B_i) - m\left(\frac{(1-\alpha)(n-1-\alpha m)(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1}\right)^2 \quad (2.35)$$

$$\sum_{i\in R_1\setminus M} E\left[(\overline{a}_1 - \overline{a}_i)^2 - (a_1' - \overline{a}_i)^2 \mid s_1\right] = \sum_{i\in R_1\setminus M} E\left[z_i^2 - \left(z_i - \frac{(1-\alpha)\alpha m(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1}\right)^2 \middle| s_1\right]$$
$$= 2\left(\frac{(1-\alpha)\alpha m(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1}\right)\sum_{i\in R_1\setminus M}(B_1 - B_i) - (r_1 - m)\left(\frac{(1-\alpha)\alpha m(\overline{s}_1 - \underline{s}_1)}{n-1-\alpha r_1}\right)^2 \quad (2.36)$$

$$
\begin{aligned}
&\sum_{i\in N\backslash(R_1\cup\{1\})} E\left[(\overline{a}_1-a_i)^2-(a_1'-a_i)^2 \mid s_1\right] = \sum_{i\in N\backslash(R_1\cup\{1\})} E\left[\left(z_i+\frac{(1-\alpha)(n-1)(\overline{s}_1-E(s_1))}{n-1-\alpha r_1}\right)^2\right.\\
&\left.-\left(z_i+\frac{(1-\alpha)\alpha m\underline{s}_1+(1-\alpha)(n-1-\alpha m)\overline{s}_1-(1-\alpha)(n-1)E(s_1)}{n-1-\alpha r_1}\right)^2\right| s_1\Bigg]\\
&=2\left(\frac{(1-\alpha)\alpha m(\overline{s}_1-\underline{s}_1)}{n-1-\alpha r_1}\right)\sum_{i\in N\backslash(R_1\cup\{1\})}(B_1-B_i)+(n-1-r_1)\left(\frac{(1-\alpha)(n-1)(\overline{s}_1-E(s_1))}{n-1-\alpha r_1}\right)^2\\
&-(n-1-r_1)\left(\frac{(1-\alpha)\alpha m\underline{s}_1+(1-\alpha)(n-1-\alpha m)\overline{s}_1-(1-\alpha)(n-1)E(s_1)}{n-1-\alpha r_1}\right)^2.
\end{aligned}
\tag{2.37}
$$

In addition, using

$$
\bar{a}_1-a_1'=\frac{(1-\alpha)\alpha m(\overline{s}_1-\underline{s}_1)}{n-1-\alpha r_1},
$$

and

$$
\begin{aligned}
&\bar{a}_1^2-{a_1'}^2=\left(\frac{\alpha(n-r_1-1)E(s_1)+(1-\alpha)(n-1)\overline{s}_1}{n-1-\alpha r_l}\right)^2\\
&-\left(\frac{\alpha m(1-\alpha)\underline{s}_1+(n-1-\alpha m)(1-\alpha)\overline{s}_1+\alpha(n-r_1-1)E(s_1)}{n-1-\alpha r_1}\right)^2\\
&+2\left(\sum_{j\in I_1\backslash\{1\}}\frac{\alpha(n-r_j-1)E(s_j)+(1-\alpha)(n-1)s_j}{n-1-\alpha r_j}+\sum_{j\in\overline{I}_1}E(s_j)+B_1\right)\left(\frac{(1-\alpha)\alpha m(\overline{s}_1-\underline{s}_1)}{n-1-r_1}\right),
\end{aligned}
$$

we obtain:

$$
\begin{aligned}
&E\left[(\bar{a}_1-\sum_{i\in N}s_i-b_1)^2-(a_1'-\sum_{i\in N}s_i-b_1)^2\right| s_1\Bigg]\\
&=E\left[\bar{a}_1^2-{a_1'}^2\right| s_1\Big]-2E\left[(\bar{a}_1-a_1')(\sum_{i\in N\backslash\{1\}}s_i+s_1+b_1)\right| s_1\Bigg]\\
&=\left(\frac{\alpha(n-r_1-1)E(s_1)+(1-\alpha)(n-1)\overline{s}_1}{n-1-\alpha r_1}\right)^2+2\left(B_1-b_1-\overline{s}_1\right)\left(\frac{(1-\alpha)\alpha m(\overline{s}_1-\underline{s}_1)}{n-1-\alpha r_1}\right)\\
&-\left(\frac{\alpha m(1-\alpha)\underline{s}_1+(n-1-\alpha m)(1-\alpha)\overline{s}_1+\alpha(n-r_1-1)E(s_1)}{n-1-\alpha r_1}\right)^2.
\end{aligned}
\tag{2.38}
$$

Next, we plug (2.35), (2.36), (2.37) and (2.38) into $V_1'-V_1$ and simplify. To simplify the part of the difference $V_1' - V_1$ that deals with biases, note that:

$$B_1 - B_i = \frac{(1-\alpha)(n-1)(b_1 - b_i)}{n+\alpha-1} \quad \text{and} \quad B_1 - b_1 = \frac{-\alpha(n-1)b_1 + \alpha\sum_{j\neq 1} b_j}{n+\alpha-1}.$$

Finally, simple but tedious calculus yields:

$$V_1' - V_1 = \frac{2\alpha(1-\alpha)^2(n-1)(\overline{s}_1 - \underline{s}_1)}{(n+\alpha-1)(n-1-\alpha r_1)}\left(\sum_{i\in M} b_i - m b_1\right) - \frac{\alpha(1-\alpha)^2 m(n-1-\alpha m)(\overline{s}_1 - \underline{s}_1)^2}{(n-1-\alpha r_1)^2}.$$

Hence, player 1 of type $s_1 = \overline{s}_1$ does not deviate by lying to players in $M \subseteq R_1$ if $V_1' - V_1 \leq 0$, i.e.:

$$-\left(b_1 - \frac{\sum_{i\in M} b_i}{m}\right) \leq \frac{(n-1+\alpha)(n-1-\alpha m)}{2(n-1)(n-1-\alpha r_1)}(\overline{s}_1 - \underline{s}_1). \tag{2.39}$$

Applying the same reasoning, player 1 of type $s_1 = \underline{s}_1$ has no profitable deviation if, for all $M \subseteq R_1$, the following condition holds:

$$b_1 - \frac{\sum_{i\in M} b_i}{m} \leq \frac{(n-1+\alpha)(n-1-\alpha m)}{2(n-1)(n-1-\alpha r_1)}(\overline{s}_1 - \underline{s}_1). \tag{2.40}$$

Condition (3.1) is obtained from (2.39) and (2.40).

2.5.5. *Other Proofs*

Proof of Corollary 2. We have to show that for every $R_i'' \subseteq R_i \cup \{j\}$ we have:

$$\left| b_i - \frac{\sum_{k\in R_i''} b_k}{r_i''} \right| \leq \frac{(n-1+\alpha)(n-1-\alpha r_i'')}{2(n-1)(n-1-\alpha(r_i+1))}(\overline{s}_i - \underline{s}_i). \tag{2.41}$$

If $j \notin R_i''$, then (3.1) clearly implies (2.41), because the LHS is the same in both inequalities, but the RHS is larger in (2.41). Now, let $R_i'' = R_i' \cup \{j\}$ for some $R_i' \subseteq R_i$. By (2.7), the LHS of (2.41) is smaller than the LHS of (3.1). Since $r_i'' = r_i' + 1$, it remains for us to check that the RHS of (2.41) is larger than the RHS of (3.1), i.e.:

$$\frac{n-1-\alpha(r_i'+1)}{n-1-\alpha(r_i+1)} \geq \frac{n-1-\alpha r_i'}{n-1-\alpha r_i} \iff \alpha^2(r_i - r_i') \geq 0, \tag{2.42}$$

which is satisfied since $r_i \geq r'_i$. ■

Proof of Corollary 4. Let $T_i = R_i \cup \tilde{R}_i$. For $T'_i \subseteq T_i$, let $R'_i \subseteq R_i$ and $\tilde{R}'_i \subseteq \tilde{R}_i$ be such that $T'_i = R'_i \cup \tilde{R}'_i$. Since R_i and $\tilde{R}_i$ do not overlap, we have:

$$\left| b_i - \frac{\sum_{k \in T'_i} b_k}{t'_i} \right| \leq \max \left\{ \left| b_i - \frac{\sum_{k \in R'_i} b_k}{r'_i} \right|, \left| b_i - \frac{\sum_{k \in \tilde{R}'_i} b_k}{\tilde{r}'_i} \right| \right\}.$$

Since R_i and $\tilde{R}_i$ are equilibrium sets of receivers, Theorem 2 implies:

$$\begin{aligned}
\left| b_i - \frac{\sum_{k \in T'_i} b_k}{t'_i} \right| &\leq \max \left\{ \frac{(n-1+\alpha)(n-1-\alpha r'_i)}{2(n-1)(n-1-\alpha r_i)} (\overline{s}_i - \underline{s}_i), \frac{(n-1+\alpha)(n-1-\alpha \tilde{r}'_i)}{2(n-1)(n-1-\alpha \tilde{r}_i)} (\overline{s}_i - \underline{s}_i) \right\} \\
&\leq \max \left\{ \frac{(n-1+\alpha)(n-1-\alpha(r'_i + \tilde{r}_i))}{2(n-1)(n-1-\alpha(r_i + \tilde{r}_i))} (\overline{s}_i - \underline{s}_i), \frac{(n-1+\alpha)(n-1-\alpha(\tilde{r}'_i + r_i))}{2(n-1)(n-1-\alpha(r_i + \tilde{r}_i))} (\overline{s}_i - \underline{s}_i) \right\} \\
&\leq \frac{(n-1+\alpha)(n-1-\alpha t' i)}{2(n-1)(n-1-\alpha t_i)} (\overline{s}_i - \underline{s}_i),
\end{aligned}$$

where the last inequality comes from $r'_i + \tilde{r}_i \geq t'_i$ and $\tilde{r}'_i + r_i \geq t'_i$. Hence, by Theorem 2, $R_i \cup \tilde{R}_i$ is an equilibrium set of receivers for player i. ■

Proof of Corollary 7. The equilibrium conditions for R_i to be a set of receivers for player i can be written as:

$$\max \left\{ \left| b_i - \frac{\sum_{k \in R'_i} b_k}{x} \right| : |R'_i| = x, R'_i \subseteq R_i \right\} \leq \frac{(n-1+\alpha)(n-1-\alpha x)}{2(n-1)(n-1-\alpha r_i)} \Delta, \quad \forall\, x = 1, \ldots r_i.$$

If j is more central than i, then for every $x = 1, \ldots r_i$,

$$\max \left\{ \left| b_i - \frac{\sum_{k \in R'_i} b_k}{x} \right| : |R'_i| = x, R'_i \subseteq R_i \right\} = \max \left\{ \left| b_j - \frac{\sum_{k \in R'_j} b_k}{x} \right| : |R'_j| = x, R'_j \subseteq R_j \right\}.$$

From $r_j \geq r_i$ we also have $\frac{(n-1+\alpha)(n-1-\alpha x)}{2(n-1)(n-1-\alpha r_i)} \Delta \leq \frac{(n-1+\alpha)(n-1-\alpha x)}{2(n-1)(n-1-\alpha r_j)} \Delta$ for every $x = 1, \ldots r_i$, so we obtain:

$$\max \left\{ \left| b_j - \frac{\sum_{k \in R'_j} b_k}{x} \right| : |R'_j| = x, R'_j \subseteq R_j \right\} \leq \frac{(n-1+\alpha)(n-1-\alpha x)}{2(n-1)(n-1-\alpha r_j)} \Delta, \quad \forall\, x = 1, \ldots r_i. \tag{2.43}$$

Next, for every $x = r_i, \ldots r_j$, we have:

$$\max\left\{\left|b_j - \frac{\sum_{k\in R'_j} b_k}{x}\right| : |R'_j| = x, R'_j \subseteq R_j\right\} \leq \left|b_i - \frac{\sum_{k\in R_i} b_k}{r_i}\right| \leq \frac{(n-1+\alpha)}{2(n-1)}\Delta.$$

Using $\frac{(n-1+\alpha)}{2(n-1)}\Delta \leq \frac{(n-1+\alpha)(n-1-\alpha x)}{2(n-1)(n-1-\alpha r_j)}\Delta$ for every $x = r_i, \ldots r_j$ we obtain:

$$\max\left\{\left|b_j - \frac{\sum_{k\in R'_j} b_k}{x}\right| : |R'_j| = x, R'_j \subseteq R_j\right\} \leq \frac{(n-1+\alpha)(n-1-\alpha x)}{2(n-1)(n-1-\alpha r_j)}\Delta, \quad \forall\, x = r_i, \ldots r_j. \tag{2.44}$$

Finally, from Inequalities (2.43) and (2.44) we deduce that R_j is an equilibrium set of receivers for player j. ■

Chapter 3

Strategic Communication Networks : How to Improve Information Transmission ?

3.1. Introduction

[1]

As previously shown in *Strategic Communication Networks* (henceforth SCN), an increase in information transmission is always beneficial, ex ante, in terms of welfare, defined as the sum of individual utilities. We are therefore led to investigate whether other types of strategic and decentralized communication extensions of the game introduced in SCN could result in more effective information transmission than private and one-shot cheap talk. In this chapter, three communication protocols are presented that improve strategic communication between players.

We first show how cheap-talk communication can be enhanced by considering *group communication*, where every player is required to publicly send the *same* costless message to all the players in a given group. Compared to private cheap talk, this requirement

[1] This chapter results from a joint work with F.Koessler and partly borrows from Hagenbach and Koessler (2008).

reduces the number of possible deviations that a sender has from truthful revelation to the whole group. Next, we consider *dynamic communication*, meaning that players are offered several cheap-talk communication rounds, which allow private information to circulate through "intermediaries". This opportunity changes the effect of a sender's lie in a way that weakens informational incentive constraints compared to static communication.

Finally, we consider the case of *verifiable information*. In a situation in which players are able to completely certify their types, we prove that complete information revelation is possible even when the conditions for a fully revealing equilibrium to exist in the cheap-talk communication (private or public) game are not satisfied. When types can only be partially verified, the certifiability requirements for complete information revelation depend on the bias profile.

3.2. Group Communication

It is well known since Farrell and Gibbons (1989) that the credibility of a sender's claim may radically depend on whether this claim is made publicly or privately. In their work, the authors compare public and private information transmission in cheap talk communication games with one sender and two receivers, assuming that decision-makers' actions are separable in the sender's utility function. In particular, they show how public announcements can discipline a privately informed sender. Indeed, they shed light on situations in which no information is revealed when communication to the receivers takes place privately whereas a fully revealing equilibrium is played when communication is public.

In the same model as the one introduced in SCN, we consider *group communication* games in which every player is required to send the same costless message to a fixed subset of players. Precisely, in case every player i is asked to send a common message to players in $\bar{R}_i \subseteq N\backslash\{i\}$, the communication extension of the game is called *group $\bar{R}$-communication game*, where $\bar{R} = (\bar{R}_i)_{i\in N}$. In this game, each player i's communication strategy is a mapping $\sigma_i : S_i \to M_i$, where $\sigma_i(s_i)$ is the message publicly observed by all players in $\bar{R}_i$ when player i's type is s_i. The *public communication* game is a particular group communication game in which every player i is required to send the same message to all the players in $\bar{R}_i = N \backslash \{i\}$.

The definition of the Nash equilibrium for the group communication games is similar to the definition for the private communication game presented in SCN. However, in the group $\bar{R}$-communication game, when focusing on equilibrium in which a player i perfectly reveals his type, the only possible deviation from a common message sent to players in $\bar{R}_i$ is to *jointly lie* to all of them. We recall that player i could lie to any subset of $\bar{R}_i$ if communication were private. Using this observation about the restricted set of possible deviations from truthful revelation to players in $\bar{R}_i$, the following result can be stated :

Proposition 8 *In the group $\bar{R}$-communication game, there exists an equilibrium in which every player i completely reveals his private information iff for all $i \in N$,*

$$\left| b_i - \frac{\sum_{k \in \bar{R}_i} b_k}{|\bar{R}_i|} \right| \leq \frac{(n-1+\alpha)}{2(n-1)} (\overline{s}_i - \underline{s}_i). \tag{3.1}$$

Proof. Similar to the proof of Proposition 3 in SCN. Consider an equilibrium in which player 1 sends to all the players in $\bar{R}_1$ the message $m_1 = \overline{m}$ when his type is $\overline{s}_1$ and the message $m_1 = \underline{m}$ when his type is $\underline{s}_1$. The only possible deviation for player 1 in the communication stage consists in lying to all the players in $\bar{R}_1$, i.e., sending the message $\underline{m}$ instead of $\overline{m}$ to all the players in $\bar{R}_1$. Therefore, Condition (4) (given in Proposition 3 of SCN) for $R'_1 = \bar{R}_1$ is the condition under which player 1 does not deviate from his equilibrium communication strategy above in the group $\bar{R}$-communication game. ■

Note that if there is an equilibrium in which player i perfectly reveals his information to players in R_i in the private communication game introduced in SCN, then there is also an equilibrium in which player i perfectly reveals his information in the group $\bar{R}$-communication game with $\bar{R}_i = R_i$. In other words, informational incentives constraints are weaker when a sender has to talk publicly (to give a speech for instance) in front of a given audience than when communication with the agents of this audience is private. To that extent, a communication protocol such that players' possible deviations are limited by public information revelation improves communication. This can be viewed as a generalization of the *mutual discipline* effect observed by Farrell and Gibbons (1989, Proposition 1).

As an illustration, consider the 4-player example presented in Section 4.3 of SCN. We have seen that player i reveals his type to players in $\{j,k\}$ iff

$$\left|b_i - \frac{b_j + b_k}{2}\right| \leq 3, \quad \text{and} \quad |b_i - b_j|, |b_i - b_k| \leq 3.75. \tag{3.2}$$

In the group $\{j,k\}$-communication game, player i reveals his type whenever the first inequality of Condition (3.2) holds. The *mutual discipline* effect appears when, e.g., $b_1 \in (-9,-5)$ and $b_3 = b_4 = -b_1$, since in that case there is no informative equilibrium from player $i = 2$ in private, while player 2 reveals his type under group $\bar{R}$-communication with $\bar{R}_2 = \{1,3\}$ or $\{1,4\}$.

In particular, the conditions to get a fully revealing equilibrium in the private communication game are stronger than the conditions to get a fully revealing equilibrium in the public communication game. Precisely, in the public communication game, there exists a fully revealing iff for all $i \in N$,

$$\left|b_i - \frac{\sum_{j \neq i} b_j}{n-1}\right| \leq \frac{(n-1+\alpha)}{2(n-1)}(\overline{s}_i - \underline{s}_i). \tag{3.3}$$

Remark : When considering the group $\bar{R}$-communication game, every sender i is required to talk publicly to a set $\bar{R}_i$ which is exogenously given. One could ask whether the set of players to which i decides to talk publicly could be decided by player i himself during a stage of the game that would precede the communication stage, but we abstract from such an issue.

3.3. Multistage Communication

Again, we consider the same class of coordination games with incomplete information as the one presented in SCN. However, in this section, we assume that the information about the state of the world is not dispersed among the players but that θ is in $\{\underline{\theta}, \overline{\theta}\}$ with one player only, say player 1, perfectly informed about it. As in SCN, after player 1 has observed θ but before the players take their payoff-relevant actions, they are offered a cheap-talk communication stage.

Before giving details about the protocol of dynamic communication, we examine the ex-ante effect of an increase in the transmission of the information about θ on welfare. At the end of the communication stage and no matter its rules, let $R \subseteq N \backslash \{1\}$ be the set of players, different from player 1, who are perfectly informed about θ. From Equation (2) of SCN, we have that, given a state θ and a set R, the second-stage equilibrium action of each player $i \in N$ is uniquely given by

$$a_i = \frac{\alpha(n-1-|R|)E(\theta) + (1-\alpha)(n-1)\left(\mathbf{1}_{[i \in R \cup \{1\}]}\theta + \mathbf{1}_{[i \notin R \cup \{1\}]}E(\theta)\right)}{n-1-\alpha|R|} + B_i, \qquad (3.4)$$

with $\mathbf{1}_{[i \in R \cup \{1\}]}$ an indicator function that equals 1 when player i knows the state θ, $\mathbf{1}_{[i \notin R \cup \{1\}]}$ an indicator function that equals 1 when player i does not know the state θ and B_i given by Equation (3) of SCN. Given these actions, Proposition 2 of SCN is directly restated in the case of a unique informed player and we get : If two sets of informed players R and R' are such that $|R'| > |R|$, then welfare is strictly higher, ex ante, with the set R' than with R.

Players are now offered a cheap-talk communication stage that consists in *several* communication rounds. It follows that a player to whom θ has been revealed in some round can further reveal it to some other players in a subsequent round. We examine whether such *dynamic* communication can change players' informational incentive constraints in a way that increases the information transmission compared to the case in which communication is *static*. The conditions to get an equilibrium set of informed players R when communication is one-stage and multistage are compared.

First, note that offering strictly more than one communication round is never harmful for information transmission as player 1 always has the possibility to babble in all the rounds except in one in which he sends exactly the same messages as he would do if there were a unique communication round. In other words, if there exists an equilibrium of the static communication game such that agents perfectly informed about θ are in R, then there always exists an equilibrium of the dynamic communication game that results in the set of informed players R.

Next, under the assumption that player 1 is the only player informed about $\theta \in \{\underline{\theta}, \overline{\theta}\}$, Proposition 3 of SCN still gives the conditions under which player 1 reveals his information to players in a set $R \subseteq N \backslash \{1\}$ with a *single* communication round. Precisely, we get :

Corollary 9 *There exists an equilibrium of the static communication game in which player* 1 *reveals his private information to players in* $R \subseteq N \backslash \{1\}$ *iff for all* $R' \subseteq R$ *we have*

$$\left| b_1 - \frac{\sum_{i \in R'} b_i}{|R'|} \right| \leq \frac{(n-1+\alpha)(n-1-\alpha|R'|)}{2(n-1)(n-1-\alpha|R|)} \Delta \tag{3.5}$$

with $\Delta = \overline{\theta} - \underline{\theta}$.

Proof. Directly from Proposition 3 of SCN. We recall that Condition (3.5) states that an informed player 1 has no incentive to deviate from telling the truth to players in R to lying to players in $R' \subseteq R$. Precisely, it ensures that player 1's expected payoff conditional on θ is higher in the case in which players in R take second-stage actions knowing the true state and player 1 best-responds to these actions than in the case in which players in $R \backslash R'$ take actions knowing the true state, players in R' take actions being wrong about the state and player 1 best-responds to these actions. ■

A 3-player Example of Dynamic Communication : To show how the transmission of information about θ can now be improved by adding a second round of cheap-talk communication, we consider the following example with $N = \{1, 2, 3\}$. We focus on the case in which, at the end of the communication stage, the set of players, different from player 1, who are perfectly informed about θ is $R = \{2, 3\}$. During the decision-stage, the players take their payoff-relevant actions according to Equation (3.4).

With a single communication round, Corollary 9 gives the conditions to get an equilibrium in which player 1 perfectly reveals his type to players in $R = \{2, 3\}$:

$$(a)\ |b_1 - \frac{b_2 + b_3}{2}| \leq \frac{2+\alpha}{4}\,\Delta \quad \text{and} \quad (b)\ \ |b_1 - b_i| \leq \frac{4-\alpha^2}{8(1-\alpha)}\,\Delta \ \text{ for all } i \in \{2,3\}. \quad (3.6)$$

With two communication rounds, the following communication strategy profile can be considered : in the first round, player 1 perfectly reveals θ to a unique player $i \in \{2, 3\}$. During the second round, player i then perfectly reveals θ to player j with $j \neq i, 1$. After two such rounds, the set of players, different from player 1, who are perfectly informed about θ is $R = \{2, 3\}$. Note that such a set of informed players is obtained by the transmission of the information about θ from player 1 to player j *through* player i, who then plays the role of an *intermediary*. The conditions under which this communication strategy profile is an equilibrium of the dynamic communication game are given by :

$$(c)\ \ |b_1 - \frac{b_2 + b_3}{2}| \leq \frac{2+\alpha}{4}\,\Delta \quad \text{and} \quad (d)\ \ |b_i - b_j| \leq \frac{4-\alpha^2}{8(1-\alpha)}\,\Delta. \qquad (3.7)$$

Proof. In the first round, consider that player 1 sends to player i the message $\overline{m}$ when his type is $\overline{\theta}$ and the message $\underline{m}$ when his type is $\underline{\theta}$. In the second round, consider that player i sends to player j the message $\overline{t}$ when he received the message $\overline{m}$ from player 1 and the message $\underline{t}$ when he received the message $\underline{m}$ from player 1. Let player 1's true type be $\overline{\theta}$. In equilibrium, using Equation (3.4), the second-stage action of every player $i \in N$ is therefore given by $\overline{a_i} = \overline{\theta} + B_i$. For every player $i \in N$, let the expected payoff conditional on θ under this equilibrium be V_i. In the two following paragraphs, the reasoning is the one used to prove Corollary 9.

Let's first focus on player 1. In the first communication round, the *only possible deviation* that player 1 has consists in sending $\underline{m}$ to players i instead of $\overline{m}$. However, with the communication profile considered, if player 1 sends $\underline{m}$ to player i, then player j will be sent $\underline{t}$ by player i in the second round. In other words, for player 1, lying to player i implies lying to player j too. As a result of such a deviation, player i's second-stage action will be given by $\underline{a_i} = \underline{\theta} + B_i$ and player j's second-stage action by $\underline{a_j} = \underline{\theta} + B_j$. In the decision stage, player 1's optimal action, denoted a_1', is then obtained from the best-response to

$\underline{a_i}$ and $\underline{a_j}$. It follows that player 1 does not deviate from truthful revelation to player i in the first round if V_1 is strictly higher than his expected payoff conditional on θ with second-stage actions that are a_1', $\underline{a_i}$ and $\underline{a_j}$. This condition corresponds to the one that ensures, under static communication, that player 1 has no incentive to lie to the whole set $R = \{2, 3\}$. It follows that Condition (c) is exactly Condition (a).

Next, let's focus on player i. In the second round, the *only possible deviation* that player i has consists in sending $\underline{t}$ to player j instead of $\overline{t}$. As a result of such a deviation, player 1's second-stage action will still be given by $\overline{a_1}$ whereas player j action will now be given by $\underline{a_j} = \underline{\theta} + B_j$. In the decision stage, player i's optimal action, denoted a_i', is obtained from the best-response to $\overline{a_1}$ and $\underline{a_j}$. It follows that player i does not deviate from truthful revelation to player j in the second round if V_i is strictly higher than his expected payoff conditional on θ with second-stage actions that are $\overline{a_1}$, a_i' and $\underline{a_j}$. This condition corresponds to the one that ensures, under static communication, that an informed player i has no incentive to deviate from telling the truth to a set $R = \{1, j\}$ to lying to the strict subset $R' = \{j\}$.[2] It follows that Condition (d) is given by Equation (3.5) with $R' = \{j\}$ and $r = 2$. ■

Finally, we compare the set of conditions given by (3.6) and (3.7). First, it appears that the set of conditions involving b_1 is smaller with the two communication rounds than with a single one. Indeed, player 1's set of possible deviations is restricted by the fact that player i is an *intermediary* for the transmission of information from player 1 to player j. It follows that it is impossible for player 1 to lie to player i without lying to player j as well.

Next, note that with biases such that $b_1 < b_i$ for all $i \in \{2, 3\}$, the conditions given by (3.7) are strictly weaker than the ones given by (3.6). With a bias profile $b = (0, 0.2, 1)$, we even get that the *unique* informative equilibrium is $R = \{2\}$ when communication is static whereas there exists an equilibrium resulting in $R = \{2, 3\}$ when two communication rounds are offered. With the biais profile b, in the static case, player 1's bias is too far from player 3's which is why he has an incentive to lie to the former only. In the dynamic

[2]This explains why the RHS of Condition (d) is the same as the RHS of Conditions (b) ensuring that player 1 does not lie to player i while telling the truth to player j under static communication.

case, player 3 is informed by player 2 previously informed by player 1, who is better off, ex ante, with 2 and 3 both knowing the true state than both being wrong about it. Note however that with biases such that $b_i \leq b_1 \leq b_j$, the conditions given by (3.6) are weaker than the ones given by (3.7). □

As shown in the 3-player example, adding a second communication round to a single one can facilitates information transmission. More generally, we get :

Proposition 9 *Consider a set $R \subseteq N \backslash \{1\}$ with at least two players j and k such that either $b_j, b_k \geq b_1$ or $b_j, b_k \leq b_1$. If there exists an equilibrium of the static communication game such that the set of informed players is R, then there exists an equilibrium of the dynamic communication game such that the set of informed players is R under weaker conditions.*

Proof. Under static communication, conditions to get an equilibrium in which player 1 perfectly reveals his information to players in R are given by Corollary 9. Next, consider the following communication profile in the dynamic communication game : in the first communication round, player 1 perfectly reveals his information to players in $R \backslash \{k\}$. In the second communication round, player $j \in R \backslash \{k\}$ perfectly reveals his information to player k and every player in $R \backslash \{j, k\}$ babbles. In every other round, every player in R babbles. Such a communication profile is an equilibrium of the dynamic communication game iff, for all $R' \subseteq R$ such that either $\{j, k\} \in R'$ or $j \notin R'$ and $k \notin R'$, with $|R'| = r'$, we have :

$$(A) \left| b_1 - \frac{\sum_{i \in R'} b_i}{|R'|} \right| \leq \frac{(n-1+\alpha)(n-1-\alpha|R'|)}{2(n-1)(n-1-\alpha r)} \Delta \text{ and } (B) \, |b_j - b_k| \leq \frac{(n-1+\alpha)(n-1-\alpha)}{2(n-1)(n-1-\alpha|R|)} \Delta \tag{3.8}$$

with $\Delta = \overline{\theta} - \underline{\theta}$. The proof is similar to the one previously presented in the 3-player example. Condition (A) ensures that player 1 has no incentive to lie to any subset of $R \backslash \{k\}$, taking into account the fact that lying to player $j \in R \backslash \{k\}$ directly implies lying to player k too. It follows that the set of players to whom player 1 may lie is $R' \subseteq R$ such that either $\{j,k\} \in R'$ or $j \notin R'$ and $k \notin R'$. Condition (B) ensures that player j has no incentive to lie to player k, given that players in $\{1\} \cup R \backslash \{k\}$ know the true state. It implies that the RHS of Condition (B) corresponds to the RHS of Equation (3.5) taking $R' = \{k\}$ and $r = |\{1\} \cup R \backslash \{k\}|$. Finally, it is straightforward to check that, with $b_j, b_k \geq b_1$ or $b_j, b_k \leq b_1$, conditions to get an equilibrium resulting in R under dynamic communication, i.e. conditions (8), are weaker than under static communication, i.e. conditions (5). ■

Note that if a set of informed players $R \subseteq N \backslash \{1\}$ includes 3 players, then it includes at least two players j and k such that either $b_j, b_k \geq b_1$ or $b_j, b_k \leq b_1$. In this case, player j can play the role of an intermediary in the transmission of information from player 1 to player k. The transmission of information about θ is then strictly easier than when player 1 directly reveals θ to players in R.

3.4. Certifiable Information

Following the terminology of Milgrom (1981), Green and Laffont (1986), Okuno-Fujiwara et al. (1990) or Seidmann and Winter (1997), we now consider that players are able to provide *hard, verifiable*, or *certifiable* information about their type. Precisely, we change the communication game introduced in SCN by allowing the set of messages available to each player to depend on his private information. Formally, the model is equivalent to the cheap-talk model introduced in SCN, except that each player i can send messages in $M_i(s_i)$, where $M_i(s_i) \neq \emptyset$ is a type-dependent set of messages. This section further differs from previous ones in two regards : first, the set of types S_i of player i is any finite set; second, the function $\theta(s)$ is not required to be additive in types and we only assume that it is weakly increasing with s_i for all $i \in N$. Without further loss of generality, we assume that types in $S_i \subset \mathbb{R}$ are increasingly ordered.

The equilibrium concept used to solve both the private and the public communication games is the perfect Bayesian equilibrium (PBE). Using notations of SCN, a PBE of the communication game analyzed in this section is a strategy profile (σ, τ) and a belief system μ satisfying the following properties:

(i) Sequential rationality in the communication stage, corresponding to property (i) of the Nash equilibrium defined on page 6 of SCN.

(ii) Sequential rationality in the action stage, corresponding to property (ii) of the Nash equilibrium defined on page 6 of SCN, except that it must hold for all $m^i \in M_{-i}$.[3]

(iii) Belief consistency, corresponding to property (iii) of the Nash equilibrium defined on page 6 of SCN. Certifiablity of types leads to the following additional conditions: in the private communication game, for all $i, j \in N$, $i \neq j$, and for all $s_j \in S_j$, $\mu_i^j(s_j \mid m_j^i) = 0$ if $m_j^i \notin M_j(s_j)$; similarly in the public communication game, for all $i, j, k \in N$, $k \neq i \neq j \neq k$, and for all $s_j \in S_j$, $\mu_i^j(s_j \mid m_j) = \mu_k^j(s_j \mid m_j) = 0$ if $m_j \notin M_j(s_j)$.

A type $s_i \in S_i$ is said to be *certifiable* if there exists a message $c_i(s_i) \in M_i \equiv \bigcup_{t_i \in S_i} M_i(t_i)$ such that $M_i^{-1}(c_i(s_i)) \equiv \{t_i \in S_i : c_i(s_i) \in M_i(t_i)\} = \{s_i\}$. The following Proposition shows that no matter the communication protocol, i.e. private or public, if every player can certify his type, then there exists a fully revealing equilibrium in which all players reveal their type to all the other players.[4]

Proposition 10 (Fully revealing equilibrium with certifiable types) *Whatever the communication protocol (private of public) and the bias profile $(b_i)_{i \in N}$, if every type of every player is certifiable, then the communication game has a perfect Bayesian equilibrium which is fully revealing.*

Proof. To support a fully revealing equilibrium, we consider the communication strategy profile in which every player completely certifies his type to all the other players whatever his type. With such a profile in the communication stage, every player perfectly knows the state θ in the decision stage, so the second-stage equilibrium actions are given by Equation (2) of SCN. The public and private communication games are treated separately. It is however important to note that when communication is private, different receivers can

[3]PBE permits the beliefs following messages that are never sent in equilibrium to be any arbitrary beliefs, but requires that second-stage actions maximize expected utilities, taken with respect to these beliefs, even after such messages.

[4]Since we obtain full revelation of information in the private and the public communication games, we do not consider other group-communication games .

make different inferences from the same deviation, while in the public communication all receivers are required to make the same inferences.

• *Public Communication.* We take a fully revealing communication strategy profile $\sigma_i(s_i) = c_i(s_i)$ for all $i \in N$ and $s_i \in S_i$, and consider a deviation by player i to a message $m_i \neq c_i(s_i)$ when his type is s_i. To support this equilibrium, we consider the degenerate *common* belief $\mu^i_j(m_i) = \mu^i(m_i) = \max\{t_i \in S_i : m_i \in M_i(t_i)\}$ for every $j \neq i$ when $b_i \leq \overline{b}$, and $\mu^i_j(m_i) = \mu^i(m_i) = \min\{t_i \in S_i : m_i \in M_i(t_i)\}$ for every $j \neq i$ when $b_i \geq \overline{b}$. According to Equation (12) of SCN which gives a player i's simplified utility function, a sufficient condition for player i's deviation not to be profitable is that for all $s_{-i} \in S_{-i}$,

$$\begin{aligned} &\left[a_i(a_{-i}(\theta(\mu^i(m_i), s_{-i})); \theta(s))\right]^2 - \frac{\alpha}{n-1}\sum_{j\neq i}\left[a_j(\theta(\mu^i(m_i), s_{-i}))\right]^2 \\ &\leq [a_i(a_{-i}(\theta(s)); \theta(s))]^2 - \frac{\alpha}{n-1}\sum_{j\neq i}[a_j(\theta(s))]^2. \end{aligned} \tag{3.9}$$

With player i's best response given by Equation (11) of SCN, this is equivalent to

$$\begin{aligned} &\left[(1-\alpha)(\theta + b_i) + \frac{\alpha}{n-1}\sum_{j\neq i} a_j(\theta(\mu^i(m_i), s_{-i}))\right]^2 - \frac{\alpha}{n-1}\sum_{j\neq i}\left[a_j(\theta(\mu^i(m_i), s_{-i}))\right]^2 \\ \leq &\left[(1-\alpha)(\theta + b_i) + \frac{\alpha}{n-1}\sum_{j\neq i} a_j(\theta(s))\right]^2 - \frac{\alpha}{n-1}\sum_{j\neq i}[a_j(\theta(s))]^2. \end{aligned} \tag{3.10}$$

By replacing the equilibrium action of every player $j \neq i$ given by Equation (2) of SCN in the last inequality we get (after some simplifications):

$$\left[\theta(\mu^i(m_i), s_{-i}) - \theta(s)\right]\left[\theta(s) - \theta(\mu^i(m_i), s_{-i}) + 2\frac{(n-1)b_i - \sum_{j\neq i} b_j}{n+\alpha-1}\right] \leq 0. \tag{3.11}$$

Since $\theta(s)$ is increasing in s_i, a sufficient condition for this inequality to be satisfied is $\mu^i(m_i) = \max\{t_i \in S_i : m_i \in M_i(t_i)\}$ when $b_i \leq \overline{b}$, and $\mu^i(m_i) = \min\{t_i \in S_i : m_i \in M_i(t_i)\}$ when $b_i \geq \overline{b}$.

• *Private Communication.* We consider a fully revealing communication strategy profile $\sigma^j_i(s_i) = c_i(s_i)$ for all $i \in N$, $j \neq i$ and $s_i \in S_i$, and consider a deviation by player i to a vector of messages $m_i \neq (c_i(s_i), \ldots, c_i(s_i))$ when his type is s_i. To support this equilibrium, we consider the degenerate *private* beliefs $\mu_j^i(m_i^j) = \max\{t_i \in S_i : m_i^j \in M_i(t_i)\}$ when $b_i \leq b_j$, and $\mu_j^i(m_i^j) = \min\{t_i \in S_i : m_i^j \in M_i(t_i)\}$ when $b_i \geq b_j$.

The analogue of Equation (3.10) for the private communication game is:

$$\begin{aligned}&\Big[(1-\alpha)(\theta+b_i) + \frac{\alpha}{n-1}\sum_{j\neq i} a_j(\theta(\mu_j^i(m_i^j), s_{-i}))\Big]^2 - \frac{\alpha}{n-1}\sum_{j\neq i}\Big[a_j(\theta(\mu_j^i(m_i^j), s_{-i}))\Big]^2 \\ \leq\ &\Big[(1-\alpha)(\theta+b_i) + \frac{\alpha}{n-1}\sum_{j\neq i} a_j(\theta(s))\Big]^2 - \frac{\alpha}{n-1}\sum_{j\neq i}[a_j(\theta(s))]^2,\end{aligned} \tag{3.12}$$

i.e., by replacing the equilibrium action given by Equation (2) of SCN,

$$\begin{aligned}&\Big[(1-\alpha)\theta + \frac{\alpha}{n-1}\sum_{j\neq i}\theta(\mu_j^i(m_i^j), s_{-i}) + B_i\Big]^2 - \Big[\theta + B_i\Big]^2 \\ &+ \frac{\alpha}{n-1}\Big[\sum_{j\neq i}\theta^2 - (\theta(\mu_j^i(m_i^j), s_{-i}))^2 + 2B_j(\theta - \theta(\mu_j^i(m_i^j), s_{-i}))\Big] \leq 0.\end{aligned}$$

Letting

$$\begin{aligned}T \equiv &\Big(\frac{\alpha}{n-1}\Big)^2\Big(\sum_{j\neq i}\theta(\mu_j^i(m_i^j), s_{-i})\Big)^2 + \frac{2\alpha(1-\alpha)}{n-1}\theta\sum_{j\neq i}\theta(\mu_j^i(m_i^j), s_{-i}) \\ &- \frac{\alpha}{n-1}\sum_{j\neq i}[\theta(\mu_j^i(m_i^j), s_{-i})]^2 - \alpha(1-\alpha)\theta^2,\end{aligned}$$

the condition further simplifies to

$$\begin{aligned}&T + 2\alpha\theta\left(\frac{\sum_{j\neq i} b_j + \alpha b_i}{n+\alpha-1} - B_i\right) + \frac{2\alpha}{n-1}\sum_{j\neq i}\theta(\mu_j^i(m_i^j), s_{-i})(B_i - B_j) \leq 0 \\ \Leftrightarrow\ &T + \frac{2\alpha(1-\alpha)}{n+\alpha-1}\sum_{j\neq i}\big[b_i - b_j\big]\big[\theta(\mu_j^i(m_i^j), s_{-i}) - \theta\big] \leq 0.\end{aligned}$$

By the construction of players' beliefs, and since $\theta(s)$ is increasing in s_i, we have

$$\big[b_i - b_j\big]\big[\theta(\mu_j^i(m_i^j), s_{-i}) - \theta\big] \leq 0, \text{ for all } j \neq i.$$

Finally, to show that the condition for no deviation is satisfied, it suffices to remark that T is always negative. Indeed, solving $T=0$ in θ gives the following discriminant:

$$\frac{4\alpha^2(1-\alpha)}{(n-1)^2}\big([\sum_{j\neq i}\theta(\mu_j^i(m_i^j),s_{-i})]^2-(n-1)\sum_{j\neq i}[\theta(\mu_j^i(m_i^j),s_{-i})]^2\big),$$

which can be checked to be always negative.[5] ■

Proposition 10 extends the results of the literature in several aspects. First, in Okuno-Fujiwara et al. (1990), the class of n-person games with $n>2$ is restricted to the following class of linear-quadratic utility functions for player i:[6]

$$a_i[\beta_i(s)-d\sum_{j\neq i}a_j-a_i], \tag{3.13}$$

where $d\in(0,2)$ and $\beta_i(s_1,\ldots,s_n)$ is increasing with s_i and decreasing with s_{-i}. Developing the utility function that we consider in SCN (given Equation (1) of SCN), we get instead (minus a constant) :

$$a_i[2(1-\alpha)(\theta(s)+b_i)+\frac{2\alpha}{n-1}\sum_{j\neq i}a_j-a_i]-\frac{\alpha}{n-1}\sum_{j\neq i}(a_j)^2. \tag{3.14}$$

Equation (3.14) cannot be rewritten as Equation (3.13) for three reasons :

1. In our model, $\beta_i(s)=2(1-\alpha)(\theta(s)+b_i)$ is increasing with s_i for all $i\in N$;
2. Our model involves strategic complementarities because $d=-\frac{2\alpha}{n-1}$ is negative, while Okuno-Fujiwara et al. (1990) assume strategic substitutes ($d>0$);
3. Equation (3.14) contains the additional term $-\frac{\alpha}{n-1}\sum_{j\neq i}(a_j)^2$ which is absent from Equation (3.13).[7]

Second, Van Zandt and Vives (2007) also prove the existence of a fully revealing equilibrium

[5] By the property $(x_1+\cdots+x_m)^2\leq m((x_1)^2+\cdots+(x_m)^2)\Leftrightarrow(m-1)((x_1)^2+\cdots+(x_m)^2)-\sum_{i\neq j}x_ix_j\geq 0\Leftrightarrow\sum_{i\neq j}(x_i-x_j)^2\geq 0$ for all $(x_1,\ldots,x_m)\in\mathbb{R}^m$ and $m\in\mathbb{N}_+$.

[6] Like us, they consider finite sets of types and assume that players' types are independent.

[7] This term does not modify the second stage equilibrium strategies but may affect players' incentives to communicate.

in a class of games with strategic complementarities, but they assume that each player's utility function is increasing in the actions of the other players. This assumption of positive externalities in actions is clearly not satisfied in our model. Third, our proposition shows that full revelation of information holds in the public and private communication games while Okuno-Fujiwara et al. (1990) and Van Zandt and Vives (2007) only consider public communication.

Finally, with the exception of some sender-receiver games considered, e.g, by Seidmann and Winter (1997), fully revealing equilibria found in the literature are usually robust to a simple inference that either always puts probability one on the lowest type consistent with the sender's report, or always puts probability one on the highest type. Here, as shown in the proof of Proposition 10, to support full revelation of information, the form of players' beliefs off the equilibrium path depends on the parameters of the game (the profile of biases $(b_1, \ldots, b_n)$), on the player who deviates, and on the players who observe this deviation (which depends on whether the communication game is public or private). More precisely, in the private communication game, when player j receives a private message m_i^j from player i and his bias is higher than player i's bias ($b_j \geq b_i$), then his belief off the equilibrium path consists in believing that player i's type is the highest type compatible with i's message (i.e., player j believes that player i's type is $\max\{t_i \in S_i : m_i^j \in M_i(t_i)\}$). On the contrary, when player j's bias is lower than player i's bias, then he believes the lowest type compatible with i's message. In the public communication game, players' inferences depend on whether the bias of the player who deviates is lower or higher than the average bias $\overline{b} = \sum_{i \in N} b_i/n$. When $\overline{b} \geq b_i$, players in $N \backslash \{i\}$ believe the highest type compatible with player i's report, and when $\overline{b} \leq b_i$ they believe the lowest type. The last observation enables to weaken the certifiability requirements for complete information revelation.

Proposition 11 (Fully revealing equilibrium with partially certifiable types) *In the public communication game, if each player i with a lower bias than the average bias (i.e., $b_i \leq \overline{b}$) can certify, whatever his actual type s_i, that his type is at most s_i (i.e., there exists*

$m_i \in M_i$ such that $s_i = \max M_i^{-1}(m_i)$), and if each player i with a higher bias than the average bias (i.e., $b_i \geq \overline{b}$) can certify, whatever his actual type s_i, that his type is at least s_i (i.e., there exists $m_i \in M_i$ such that $s_i = \min M_i^{-1}(m_i)$), then there is a perfect Bayesian equilibrium which is fully revealing.

In the private communication game, if each player i with the lowest bias (i.e., $b_i \leq b_j$ for all $j \in N$) can certify, whatever his actual type s_i, that his type is at most s_i, if each player i with the highest bias (i.e., $b_i \geq b_j$ for all $j \in N$) can certify, whatever his actual type s_i, that his type is at least s_i, and the other players can completely certify their types, then there is a perfect Bayesian equilibrium which is fully revealing.

From Proposition 10, we know that the *mutual discipline* considered by Farrell and Gibbons (1989) is impossible when types are completely certifiable since full revelation of information occurs in both the public and private case. On the contrary, when considering partial certifiablity as in the previous Proposition 11, the sufficient conditions for full information revelation are stronger in the private than in the public communication game. It follows that the *mutual discipline* effect of Farrell and Gibbons (1989) is again possible with partially certifiable information, as in the cheap-talk case considered in Section 3.2. The following example presents a situation in which Proposition 11 applies for the public communication game, whereas there is no fully revealing equilibrium in the private one.

Example of Mutual Discipline under Partial Certifiability Consider the 3-player game in which only player 1 knows the state $\theta \in \{\theta_1, \theta_2, \theta_3\}$. Let players' biases satisfy $b_2 \leq b_1 \leq b_3$ and $b_1 \leq \frac{b_2+b_3}{2}$, and the state-dependent messages available to player 1 be

$$M(\theta_1) = \{m_1, m_2, m_3\}, \quad M(\theta_2) = \{m_2, m_3\}, \quad M(\theta_3) = \{m_3\}.$$

By Proposition 11, these assumptions imply that there exists a fully revealing equilibrium under public communication. Next, consider the fully revealing communication strategy under private communication. When the real state is θ_1 and player 1 deviates by sending message m_2 instead of m_1 to player 2 (without deviating towards player 3), the condition for player 1's deviation to be profitable is given by :

$$b_1 - b_2 > \frac{15(\theta_2 - \theta_1)}{16}.$$

Hence, under this condition there is no fully revealing equilibrium in the private communication game, while a fully revealing equilibrium exists in the public one whatever the distance between the possible fundamentals and the distance between player 1 and player 2's biases (as long as $b_1 \leq \frac{b_2+b_3}{2}$). $\square$

Conclusion

In the first chapter, I analyze a dynamic game in which players are the members of a fixed network. In each period, every agent decides whether or not to pass on his private information to his direct neighbors. He cannot misrepresent the items he transmits. Given the informational dilemma that agents face, the network structure affects the time needed to achieve the common goal of information pooling in equilibrium. At an individual level, every player's position has an influence on the earliest date at which he can "win" the game in equilibrium. In the second chapter, we focus on players' incentives to misrepresent their private signals. In that framework, strategic communication between players depends on preferences heterogeneity. Initially, players do not belong to a given network but we derive connections between the agents from the informativeness of their communication strategies.

In this thesis dealing with the interaction between *strategic communication* and *networks*, two distinct kinds of heterogeneity are thus considered : in one part, homogeneous players differ in their network positions; in another part, players vary in preferences but not in their location in a given communication structure. Two of the communication protocols examined in the third chapter bring these two views together. In Chapter 2, we propose a multi-sender and multi-receiver cheap-talk game in which every player is allowed to send a different message to every other one. To some extent, Chapter 3 implicitly considers that a pre-existing communication structure exists that restricts the set of available conduits for messages.

In Section 2 of Chapter 3, every agent is required to publicly send the same message to an exogenous group of agents, as if there were a pre-existing pattern of communication channels. Whether or not these channels ends up being used for truthful revelation of information is the question which is then addressed.[8] We show that announcements made publicly in groups can enhance information transmission compared to private cheap talk, as it restricts the number of possible deviations from truthful communication to a whole group. Consequently, one could wonder to which extent a given communication pattern could compel players to make public announcement to groups instead of sending private messages.

If this cannot be done, one can also think of a pattern of communication that would generate public communication from senders to groups of receivers by messages sent privately. Indeed, with more than three receivers for instance, a two-stage private communication protocol using a "majority rule" can be used to get any equilibrium that exists when communication to these three players is public : in the first stage, the sender sends a message to the group of three players. In the second stage, each receiver further sends the message he received from the sender to all the other receivers. If all messages received by a receiver during the second stage of communication do not coincide, then he uses a majority rule to form his belief. Clearly, no receiver alone has an impact on others' beliefs, and all possible deviations of the sender generate the same beliefs as in the public communication game. I believe that it is worth extending Chapter 2 by adding heterogeneity in the available conduits of information to the heterogeneity in preferences that we have considered.[9] Next, when introducing several cheap-talk communication rounds in Section 3 of Chapter 3, we stated that the use of intermediaries can improve communication. Precisely, we claimed that it may be that an information item initially held a sender never reaches a given receiver whereas it does reach him in case the item could be passed through an agent whose bias

[8] In Galeotti et al. (2009a) for instance, players are located in a fixed network and the links are said to be "truthful communication links" when they effectively convey truthful information in equilibrium of the communication stage.

[9] By locating players of Chapter 2 in a network, one could also change the coordination game of incomplete information played in the second-stage. Indeed, it may be that every player wants to take an action close to that of his neighbors only. This extension has been mentioned to us quite often, I believe that heterogeneity in the communication possibilities is qualitatively more interesting.

lies between the sender's and the receiver's one. Such a statement raises the question of whether there exists an optimal way to arrange players with heterogeneous preferences in a communication network before letting them exchange costless messages trough the given channels. It seems that intermediaries should lie between the senders and the receivers on the information conduit but also in terms of preferences to play their role in equilibrium.

Finally, I'd like to come back to Bonacich's experiment which initially motivated the present work. My first chapter proposes a way to model the experiment he reports but it differs from his experiment in two main ways. First, his experimental subjects could guess the quotation they had to identify even if they did not have all the letters in their hands. In my work, I have instead restricted attention to the case in which the unknown state of the world is accurately assessed if and only if all the pieces of information initially dispersed were pooled. Naturally, the measure of a player's position that ended up being crucial was the eccentricity, which measures the distance to the player who is furthest away. In case some of the pieces of information would be enough to take appropriate decisions, other measures of locations would obviously matter. It would be interesting to link more generally the way information needs to be pooled to various graphical measures. Secondly, in Bonacich's work participants were allowed to give to their direct neighbors as many letters as they wanted. On the contrary, in my game, players either give or hide all the pieces of information they hold to all their neighbors. Even if it is a common assumption that a player chooses a common action in interaction with all neighbors, one could given more freedom to players regarding the way they would transmit their items.

In the first chapter, the network structure is assumed to be common knowledge among the players. In reality, networks are complex objects and it is likely that individuals will have only partial knowledge about it. Indeed, empirical work suggests that individuals located in social networks generally know their own neighbors and have some idea of the neighbors of their neighbors but usually do not know a great deal more about the network (See for instance Bondonio (1998) and Casciaro (1998)). As reported in Bonacich's experiments, each player's position affects his strategic behavior in the modified war of attrition considered. We could therefore ask what does happen in his experiment if the perception of a player's position is only partial ? Is it better for the group when central players do not realize that they are well-situated ? I believe designing experiments that investigate such questions would be interesting. More generally, I think it is worth examining how accurately agents perceive their chances to win given their position in the network, even under common knowledge of its structure.

Strategic Communication and Networks is a very broad topic. In this work, I propose a way to build networks from communication strategies and one to study whether networks affect the transmission of informational items. I have focused on particular contexts and disconnected the *formation* of communication networks from their *use*. There are several directions to connect them. For instance, one could allow for the removal of a communication links when the information items passed through it was wrong. Further research on communication and networks seems promising and I am enthusiastic about it, all the more so as the truth about information transmission probably lies between strategic and automatic diffusion.

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Contents

COMMUNICATION STRATEGIQUE ET RESEAUX

Jeanne Hagenbach

Abstract

Depuis une dizaine d'années, l'étude des réseaux est une branche très active de la recherche en économie. Il est désormais largement admis que ceux-ci jouent un rôle central dans la transmission décentralisée des informations entre les individus. Les informations communiquées par ces derniers concernent aussi bien les opportunités d'emplois que l'état du marché dans lequel une équipe de travailleurs évolue. Cette thèse propose une nouvelle approche du lien entre la manière dont les agents transmettent stratégiquement leurs informations privées et la structure du réseau dont ils font partie.

La théorie des jeux non coopérative a été appliquée à l'étude des réseaux sociaux et économiques dans les deux branches suivantes: d'une part, les *Jeux en Réseaux* considèrent que les joueurs sont les membres d'un réseau donné et analysent la manière dont les comportements stratégiques et les résultats économiques sont influencés par l'architecture de ce réseau ; d'autre part, les *Jeux de Formation de Réseaux* modélisent la construction stratégique des connections entre les individus. Ce travail apporte une contribution à ces deux domaines de recherche. Dans la première partie de ma thèse, que forme le Chapitre 1 intitulé *Centralisation des Informations dans les Réseaux*, les joueurs appartiennent à un réseau qui affectent leur manière de transmettre leurs informations. Dans la seconde partie, constituée des Chapitres 2 et 3 et intitulée *Réseaux de Communication Stratégique*, la structure des liens entre les agents découle de leur communication stratégique.

MOTS-CLÉS: Réseaux Economiques et Sociaux, Communication, Information, Théorie des Jeux, Graphes.

Abstract

During the past decade, the study of networks has been a very active area of research in economics. It is now largely admitted that they play a central role in the decentralized transmission of information among individuals. Information pieces that agents communicate about range from vacant job opportunities to the state of the market a team of workers is facing. This thesis aims at shedding a new light on the link between the way agents strategically share private information and the structure of the networks they are arranged in.

The application of non-cooperative game theory to the study of social and economic networks has mainly been twofold: on the one hand, *Network Games* consider that players are the members of a given network and investigate how strategic behaviors and economic outcomes are influenced by the architecture of this network; on the other hand, *Network Formation Games* model the strategic building of connections between players. The present work yields new insights in these two research areas. In the first part of my thesis, made up of Chapter 1 entitled *Centralizing Information in Networks*, players belong to a given network which affects the way they transmit private information items. In the second part, made up of Chapters 2 and 3 and entitled *Strategic Communication Networks*, the network structure is derived from strategic communication between players.

KEYWORDS: Social and Economic Networks, Communication, Information, Game Theory, Graphs.