# Constructive Verification for Component-based Systems 

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le 27 Mai 2010

## Vérification Constructive des Systèmes

à base de Composants (Constructive Verification for Component-based Systems)

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# Constructive Verification for Component-based Systems Vérification Constructive des Systèmes à base de Composants 


#### Abstract

The goal of the thesis is to develop theory, methods and tools for the compositional and incremental verification for component-based systems. We propose a compositional verification method for proving safety properties. The method is based on the use of two kinds of invariants: component invariants which express local aspects of systems and interaction invariants which characterize global constraints induced by synchronizations between components. We provide efficient methods for computing these invariants. We also propose a new technique that takes the incremental design of the system into account. The integration of verification into design process allows to detect an error as soon as it appears. Moreover, the technique helps to avoid redoing all the verification process by reusing intermediate verification results. It takes advantage of the system structure for coping with complexity of the global verification and therefore, reduces significantly the cost of verification in both time and memory usage. The methods have been implemented in D-Finder tool-set. The experimental results obtained on non trivial examples and case studies show the efficiency of our methods as well as the capacity of D-Finder.


Key words: BIP, compositional verification, incremental verification, incremental design, invariant, deadlock detection, static analysis, D-Finder.

## Résumé

L'objectif de la thèse est de développer des méthodologies et des outils pour la vérification compositionnelle et incrémentale des systèmes à base de composants. Nous proposons une méthode compositionnelle pour vérifier des propriétés de sûreté. La méthode est basée sur l'utilisation des deux types d'invariants: invariants de composant qui expriment des aspects locaux des systèmes et invariants d'interaction qui caractérisent les contraintes globales induites par les synchronisations fortes entre les composants. Nous offrons des techniques efficaces pour calculer ces invariants. Nous proposons également une nouvelle méthode de vérification incrémentale qui prend la conception incrémentale du système en compte. L'intégration de la vérification dans le processus de conception permet de déceler une erreur dès qu'elle apparaît. En outre, cette méthode permet d'éviter de refaire tous les processus de vérification par la réutilisation de résultats intermédiaires. Elle prend des avantages des structures de systèmes pour faire face à la complexité de la vérification globale et, par conséquent, réduit significativement le coût de la vérification en temps et en mémoire utilisée. Les méthodes compositionnelles et incrémentales ont été mises en oeuvre dans la chaîne d'outil D-Finder. Les résultats expérimentaux obtenus sur plusieurs études de cas non triviales montrent l'efficacité de nos méthodes ainsi que les capacités de D-Finder.

Mots clés: BIP, vérification compositionnelle, verification incrémentale, conception incrémentale, invariant, détection de blocage, analyse statique, D-Finder.

Several Chapters in this thesis appeared in several papers in form of articles or of Verimag technical reports.

The compositional method in Chapter 2 appeared in the paper [BBNS08] in ATVA 2008 (6th International Symposium on Automated Technology for Verification and Analysis, October 20-23, 2008, Seoul, South Korea). A journal version [BBNS10] appeared in IET Software Journal, Special Issue on Automated Compositional Verification, Vol. 4, Issue 3, June 2010.

The incremental construction and verification methods together with the results in Chapter 3 appeared in two papers: paper [ $\left.\mathrm{BLN}^{+} 10\right]$ appeared in the conference FMCAD 2010 (10th International Conference on Formal Methods in Computer Aided Design) and paper [ $\left.\mathrm{BBL}^{+} 10\right]$ appeared in the conference TASE 2010 (4th IEEE International Symposium on Theoretical Aspects of Software Engineering).

The D-Finder tool-set in Chapter 5 appeared in the tool paper [BBNS09] in CAV 2009 (21st International Conference on Computer Aided Verification).

An application of the method on a robotic system presented in Chapter 6 appeared in the paper $\left[\mathrm{BGL}^{+} 08\right]$ in ECAI 2008 (18th European Conference on Artificial Intelligence, July 21-25, 2008, Patras, Greece) and then the journal version [BGI ${ }^{+} 09$ ] is published in Special Issue on Software Engineering for Robotics of the IEEE Robotics and Automation Magazine Vol. 16, No. 1, March 2009.
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## Part I

Context

## Problems and Needs

Computer science nowadays plays an important role in the development of science and of society. Despite of being a new domain, it appears almost everywhere, from small systems such as a cell phone, a music player to huge systems such as a plane, a spacecraft. The increasing need of computer systems increases also the need of their reliability, correct behaviour, etc.

Constructing correct systems is an essential requirement crossing all areas. A system is correct if it behaves following an expected manner or in other words, it satisfies an explicit set of requirements. The expected behaviour is called formal specification of the system and is often expressed as a set of desired requirements. The correctness of a system is assigned if the system is correct with respect to its specification. Ensuring the correctness is specially important for critical systems since their failure or malfunction may cause a catastrophe such as loss of human life, high economical costs or environmental harm. For example, the crash of Ariane 5, an European expendable launch system, is one of the most infamous computer bugs in history and resulted in the loss of more than 370 millions dollars.

Unfortunately, with the growing of the demand for scalability and complexity of systems, it becomes more and more difficult to design correctly their models. The scale and complexity not only increases potential violations of desired properties but also makes them harder to detect and to handle. The construction of a system that operates reliably despite of complexity is highly desirable but also not always feasible. Therefore the check of the correctness of the system is essential and important to ensure that all requirements are respected. The check process is based on techniques for detecting property violations of the model and the correctness is achieved by the absence of such violations.

There are two main approaches for detecting property violations of a system: formal testing and formal verification. Consider a model of a system, an environment in which the system interacts, and some properties that the designed system is expected to guarantee, one can choose one of the following approaches depending on their goal:

- Formal testing [Bei90, Tre90, Mye04, FFMR07] is a method used to find defects on a system implementation, either during the development or after the complete construction of the system. To do that, testing generates some inputs from environment (test cases) and executes the system to determine whether it produces the required results. Testing, depending on method employed, can be used at any time of development process. It is a quick and direct way to detect bugs or violations in the system. However, testing is not capable of covering all the possibilities that may happen while running the system in reality. The number of possible situations is usually so large that we can test a tiny proportion of them. The absence of property violations provided by testing does not imply the correctness of the system.
- Formal verification [UP83, BM79, QS82, CE81] can both search for input patterns which violate the desired properties or prove the correctness of the system if such input patterns do not exist. In contrast to formal testing, formal verification covers all the possibilities that the system can behave, hence it proves the correctness of the system in the case of the absence of property violation. It relies on the use of mathematical techniques to prove or disprove the correctness of a design with respect to a certain formal specification. Formal verification has been successfully applied to verify both software and hardware systems. The verification of these systems is done by providing a formal proof on an abstract mathematical model of the system. The mathematical objects that are often used to model these systems are: labeled transition systems, petri nets, finite state machines, boolean formula, etc.
The goal of our work is to provide a theory, methods and tools for achieving the correctness of systems. We focus on the formal verification since, as mentioned above, it allows to achieve the correctness of systems. We provide below a brief description of the current state of the art in formal verification.


## State of the Art in Formal Verification

There are basically two main directions in formal verification - theorem proving and model-checking.

Theorem proving [BM79, BM88, GH93, GM93, ORR ${ }^{+} 96$ ] uses mathematical proofs to show that a system satisfies its requirements. In this approach, both system and requirement are expressed in form of formulas in some mathematical logic. This logic defines a set of axioms and inference rules that theorem proving uses, together with intermediate lemmas, to find a proof of a desired property. An advantage of theorem proving is the capacity of dealing directly with infinite systems by using techniques like structure induction. However, one can not get counterexample when the proof fails. Although many theorem prover tools have been developed to support building and checking the proofs, the finding of these proofs is not always feasible and may require a lot of expert intervention from users, that makes the theorem proving process slow and often error-prone. The lack of automation prevents this approach from being largely used in the industrial context.

Model-checking [QS82, CE81, CGP99] is a fully automatic verification technique which relies on building a finite model of the system and checking whether that model
meets its specification. Usually the system is modeled as a finite state machine and the specification is expressed as a temporal logic formula. The temporal logic allows describing the change over time and is therefore suitable for most of the necessary correctness properties such as safety properties (always, i.e. something bad never happens), liveness properties (eventually, i.e. something good happens), etc.. The check is performed by using an efficient search procedure on the exhaustive state space graph of the system's model. Model checking has been extensively developed and used for verifying both software and hardware systems. The automatic nature of model-checking makes it attractive for practical use in the industry. It can be also used for checking partial specification when the system has not been completely specified. Moreover, if a property does not hold, model checking provides a counterexample, a path through the model that reveals this violation.

Despite having been successfully applied in the industrial community, there are several problems which make model-checking difficult for verifying large systems:

- Model checking needs to explore the entire state space of the models of systems, therefore it is not suitable for infinite-state systems. However, a big number of important systems are infinite, and the exploration of the state space of their infinite-state models is not possible or requires extra pre-processing (such as abstraction) to make it feasible.
- Even with finite-state systems, model checking is not always scalable. The systems nowadays become more and more complex with a large number of parallel processes. Model checking techniques examine all possible paths through the system's model to determine whether or not the property being verified is violated, and that is the source of difficulty because the number of possibilities in the global model is exponential in the number of component processes. This is called state space explosion problem in Model-Checking.
- Model checking is not a priori guarantee of correctness. It helps developer detect, understand and then fix inconsistencies, ambiguities, bugs, etc., in the model of the system. However if errors or bugs are found after the complete construction of the system's model, we might have to reconstruct or modify the global model which may cause significant loss of time, money or human resource.
A lot of work has been done to overcome the problems in model-checking, specially the state space explosion problem. The major goal is to make the formal verification scalable in order to increase the size of the systems that can be handled. They can in general be categorized into two approaches: optimization/improvement of model-checking algorithms and compositional reasoning.


## Model-Checking Improvement

## Symbolic Model Checking

A well-known improvement of model-checking algorithms is symbolic model-checking $\left[M c M 93, \mathrm{BCM}^{+} 90\right]$. This method represents implicitly the state graph as a formula in propositional logic instead of building it explicitly. It uses boolean encoding for representing state machine and set of states, therefore allows manipulating a set of states rather than
a single state in the explicit enumeration of states. All operations are handled as boolean functions by using the Binary Decision Diagrams (BDDs).

A symbolic representation based on BDDs provides a canonical form for boolean formula that is more compact than conjunctive and disjunctive normal form. The use of BDDs allows to verify extremely large systems having up to $10^{120}$ states. It is also successful in verifying several systems of industrial complexity. The property to be verified is evaluated recursively by iterative fixed-point computations on the reachable state space. More precisely, the rough procedure for checking a safety property is : initially, the set of initial states is represented as a BDD from which an iterative process starts. At each step $i$, the iteration adds to the BDDs the set of states that can be reached in $i$ steps from the initial states and intersects the new states with the set of states violating the property. If the intersection is not empty, it means that an violation has been detected. The process terminates when there is no more new reachable states or an error is found. If the process terminates without any error, the property holds; otherwise, a counterexample is provided.

In spite of such success, symbolic model checking still has its limitations due to the size of BDDs. In some cases the BDD representation can be exponential in the size of system description. Moreover, BDDs are very sensitive to ordering of variables. Finding a good ordering, which yields the smallest BDD for a given formula is an NP-complete problem.

## Bounded Model Checking

An alternative method that can avoid the state space explosion is Bounded Model Checking (BMC) [BCCZ99, CBRZ01] proposed by Biere et al. in 1999. The basic idea of the method is to search for a counterexample within a bounded number steps of executions. Given a bounded execution length $k, \mathrm{BMC}$ constructs a propositional formula that represents the set of initial states and the states that can be reached from initial states within $k$ steps. It also constructs a formula expressing the violation of a property $P$ in one of these $k$ steps. Then the conjunction of two formulas is checked. If the conjunction is satisfiable, BMC provides a counterexample of length at most $k$. If it is not satisfiable, we can either increase $k$ until a violation is found or stop if time or memory constraints are exceeded.

The BMC problem is reduced to a propositional satisfiability problem that determines whether a propositional formula in conjunctive normal form has a truth assignment that makes the formula true. This problem can be efficiently solved by powerful SAT tools. BMC and BDD-based symbolic model-checking are incomparable. There are several case studies that can not be verified by symbolic model checking but can be verified by BMC and vice-versa. A disadvantage of BMC is the incompleteness of the method. The absence of error after $k$ execution steps does not prove the correctness. A possible solution is the finding of the longest shortest path between any two states (the maximum value $k_{\text {max }}$ of $k)$. The states obtained in $k_{\text {max }}$ steps allows covering all the reachable states since the the path from the initial states to any reachable state is always shorter than or equal to $k_{\max }$. However finding such $k_{\max }$ is extremely hard.

## Partial order reduction

Partial order reduction [God91, GW92, Pel94, CGP99] is a verification method that simplifies the size of state space to be searched by a model-checking algorithm. It exploits the commutativity and concurrently executed transitions, which result in the same state when executed in different orders. Intuitively, if two transitions $t_{1}$ and $t_{2}$ are executed consequently but in any order, the system arrives in the same state, so it is not necessary to consider both the $t_{1} t_{2}$ and $t_{2} t_{1}$ interleavings.

## Compositional Reasoning

The second approach is compositional reasoning [CLM89, CMP94, Lon93] that verifies each component of the system in isolation and allows global properties to be inferred about the entire system. The basic idea is the use of divide-and-conquer approach: the system is decomposed into subsystems and these subsystems are analyzed individually. Since subsystems are smaller than the whole system, the individual analysis of the subsystems reduce the effects of the state space explosion problem. The guarantee of global property is then determined by composing the results of these individual analysis. Since through this thesis, we propose a compositional method for the verification of component-based systems, we will focus on compositional approach by presenting below several existing compositional methods.


#### Abstract

ion Abstraction [CC77, Lon93, CGL94, DF95, LGS $\left.{ }^{+} 95\right]$ is a popular technique which verifies properties on a system by firstly simplifying it. The simplification is often based on the conservative aggregation of states. The simplified system, which is called abstract system, is usually smaller than the original system (concrete system), so the state space is reduced. For a system obtained from the parallel composition of a set of components, i.e $S=B_{1} \|$ $\cdots \| B_{n}$, the compositional abstraction first computes, for each component $B_{a}$, an abstract component $B_{i}^{a}$, then it composes the abstract components $S^{a}=B_{1}^{a}\|\cdots\| B_{n}^{a}$ to obtain an abstract system $S^{a}$ of $S$. The abstraction is required to be sound, i.e. the properties that hold on the abstract system also hold on the concrete system. However, the abstraction is often not complete, i.e. not all true properties of the concrete system are also true on the abstract system so that a process of abstraction refinement may be necessary.


## Assume-guarantee

Assume-guarantee [MC81, Jon83, Pnu85, HQR98, dRdBH ${ }^{+}$00, GPB02, CGP03] is a semi-automatic compositional approach that decomposes properties into two parts. One is an assumption about the global behaviour of the environment and the other is a property guaranteed by the component when the assumption about its environment holds. The assumption is needed since when a subsystem is verified it may be necessary to assume that the environment behaves in a certain manner. Consider a system $S$ which is decomposed into two subsystems $S_{1}$ and $S_{2} . P$ is a property to be verified on the parallel composition
of $S_{1}$ and $S_{2}$, denoted by $S_{1} \| S_{2}$. The basic assume-guarantee rule is as follows:

$$
\begin{gathered}
\langle A\rangle S_{1}\langle P\rangle \\
\langle\text { true }\rangle S_{2}\langle A\rangle \\
\hline\langle\text { true }\rangle S_{1} \| S_{2}\langle P\rangle
\end{gathered}
$$

That is, if under assumption A, subsystem $S_{1}$ satisfies property $P$ and $A$ is satisfied by subsystem $S_{2}$, then the system resulting from the parallel composition $S_{1} \| S_{2}$ satisfies the property $P$. Even though it is widely touted, many issues make the application of assumeguarantee rules difficult. They are discussed in detail in a recent paper [CAC08]. The paper provides an evaluation of automated assume-guarantee techniques. In many cases, the verification based on assume-guarantee is not better than monolithic verification in time and memory usage. The main difficulty is finding decompositions into sub-systems in the case of many parallel sub-systems $S_{1}\|\cdots\| S_{n}$. The verification performance depends on the way of decomposition but finding a good decomposition is not always feasible. Another problem is choosing adequate assumptions for a particular decomposition. The assumption should be weak enough to be satisfied by a sub-system but strong enough to prove the global property.

## Interface processes

[CLM89] proposes a method for reducing the complexity of temporal logic model checking in systems composed of many parallel processes. It minimizes the global state transition graph by focusing on the communication among the component processes. The method models the environment of a process by another process called an interface process. In interface process, only variables involved in the interface between two components are considered and events that do not relate to the communication variables are eliminated. Therefore, the interface process is often smaller than the original process but it preserves properties that refer to interface variables. This method is specially suitable for loosely coupled systems through a small number of interacting variables. However, the method is not very efficient with the tightly coupled systems because the interface process may not be smaller than the original process. And the method has difficulty in handling more general properties involving temporal assertions about several processes.

## Partitioned Transition Relations

In model checking, the set of reachable states from initial states (or co-reachable from bad states) is obtained by computing the set of successors (or predecessors). This process requires the construction of the transition relation of the global systems and that is a source of difficulties. [BCL91] provide methods for computing that set by using the transition relations of each component separately during traversal of the state graph. The set of states in the global graph is then obtained by combining the individual results. More precisely, model-checking requires the computation of the image or pre-image of a set of states under a transition relation. For example, if $S(V)$ is a set of states depending on a set of variables $V$, and $N\left(V, V^{\prime}\right)$ is a transition relation relating the current state variables $V$ and next
state variables $V^{\prime}$, then the image of S is given by $\exists V\left[S(V) \wedge N\left(V, V^{\prime}\right)\right]$. The computation of the value of a large formula with many quantifiers is quite expensive. The method called partitioned transition relations in [BCL91] deals with this problem by decomposing the global formula into sub-formulas and the quantifier elimination is performed on these smaller sub-formulas and therefore reduces the cost of the operation.

## Lazy Parallel Composition

In contrast to partitioned transition relations method, lazy parallel composition presented in $\left[\mathrm{TSL}^{+} 90\right]$ restricts the transition relation of each component before generating the global restricted transition relation. The method is based on the agreement between the restricted transition relation and the global transition relation for "important" states, while other states may behave in a different way. If the original global transition relation $N$ and a set of state $S$, the computation of the set of successors of $S$ can use any restricted transition relation such that $\left.N^{\prime}\right|_{S}=\left.N\right|_{S}$, i.e. $N$ and $N^{\prime}$ agree on transitions that start from states in $S$. The advantage of the method is that the restricted transition relation is often smaller than the global transition relation.

## Lazy Compositional Verification

A compositional method, lazy compositional verification, is presented in [Sha98]. The method allows to prove a global property by showing that it is satisfied by composing a component with an abstract environment and this environment eventually holds of the other components in the system. More concretely, a property $C$ of a component $P$ is satisfied by the composition $P \| E$ where $E$ is an abstract environment specification $E$ that captures the expected behaviour of the environment. Then when $P$ is composed with another component $Q, C$ might not be property of $P \| Q$ but $C$ is property of $P \|(Q \wedge E)$. If $P \|(Q \wedge E)$ can be simplified to $P \| Q$, then $E$ is redundant and can be eliminated. However, in contract to assume-guarantee approach, it is not necessary that $Q$ implies $E$. While $E$ has eventually to be shown to hold of other components in the system, this proof obligation can be discharged lazily as the system design is being refined. The advantage of lazy compositional verification is that the proof that one component meets the expectations of other components can be delayed until sufficient detail has been added to the design.

## Deductive Verification

Deductive methods for proving safety properties, which are also considered as invariance properties, of transition systems are based on a proof rule which can be formulated as follows. To prove that some given predicate $\Phi$ is an invariant of a given program $S$, i.e. every reachable state of $S$ satisfies $\Phi$, it is necessary and sufficient to find an auxiliary predicate $\Phi^{a u x}$ with the following properties:

- $\Phi^{\text {aux }}$ is stronger than $\Phi$,
- $\Phi^{a u x}$ is preserved by every transition of S, i.e., for every states $s$ and $s^{\prime}$, if s satisfies $\Phi^{a u x}$ and $s^{\prime}$ is reachable from $s$ by a transition, then $s^{\prime}$ also satisfies $\Phi^{a u x}$,
- $\Phi^{a u x}$ is satisfied by every initial state of $S$.

As shown e.g., in [MP95], this rule is sound and (relatively) complete for proving invariance properties of transition systems. It is very important to understand that the completeness result/proof of this rule does not give a clue of how to find the auxiliary predicate. Indeed, choosing the set of reachable states $\operatorname{Reach}(S)$ as auxiliary predicate reduces the original problem to the checking of the first premise. Moreover, if $S$ is a finite-state system the predicates can be expressed in propositional logic and checking the premises can be done algorithmically. However, in general, one needs an assertion language which is at least as expressive as integer arithmetic to express predicates, which makes checking the premises of the rule undecidable. Even worse, there are systems for which Reach $(S)$ is expressible using closed formula over integers yet computing such a representation cannot be done effectively. The deductive rule provides only a partial answer to the verification of invariance properties. It leaves open (i) how to find the auxiliary predicate $\Phi^{a u x}$ and (ii) how to prove that $\Phi^{a u x}$ is preserved by every transition of $S$ and satisfied by the initial states. Problem (ii) is related to the problem of proving tautologies of the underlying assertion language.

## Problems in Formal Verification

On exploring the current state of the art in formal verification, it becomes clear that a formal verification method needs to address the following problems:

- Scalability that avoids the state space explosion problem and therefore allows increasing the size of systems to be verified.
- Effectiveness that permits to detect as early as possible errors of systems' models in the design phase.
- Incrementality that integrates verification into the design process.
- Compositionality that allows inferring global properties of a system from the known local properties of its sub-systems.
We have mentioned the problems, the needs in software and hardware engineering to construct correct systems. We have also presented the existing approaches addressing to these needs and its general limitations. We have given a brief presentation on some related work that has been done in the formal verification together with its advantages and disadvantages. In the next section, we will present our methods for the compositional and incremental verification.


## Our Approach

We present, in this thesis, a compositional approach for verifying safety properties of component-based systems. We exploit both local and global aspects of systems. The former is related to the local behaviours of components. The latter represents global constraints between components which are strongly synchronized. These constraints are important, specially for checking deadlock-freedom since the global deadlocks are due to the strong synchronization. Furthermore, the synchronizations restrict the global behavior and therefore these global constraints allow to eliminate product states which are not feasible by the semantics of parallel composition.

Our rule can be seen as an instance of the proof rule of the deductive approach. We describe techniques for generating automatically auxiliary predicates by using two kinds of invariants of systems: component invariants and interaction invariants. Component invariants express constraints on local state space of components. Interaction invariants characterize restrictions enforced by synchronizations between components on the global state space of the systems. They are computed automatically from the set of component invariants and the set of interactions between components.

The general rule of our approach is shown in Equation (1).

$$
\begin{equation*}
\frac{\left\{B_{i}<\Phi_{i}>\right\}_{i}, \Psi \in I I\left(\|_{\gamma}\left\{B_{i}\right\}_{i},\left\{\Phi_{i}\right\}_{i}\right),\left(\bigwedge_{i} \Phi_{i}\right) \wedge \Psi \Rightarrow \Phi}{\|_{\gamma}\left\{B_{i}\right\}_{i}<\Phi>} \tag{1}
\end{equation*}
$$

That is, if $\left\{\Phi_{i}\right\}_{i}$ are respectively component invariants of a set of atomic components $\left\{B_{i}\right\}_{i}$, $\Psi$ are interaction invariants computed from a set of interactions $\gamma$ between $\left\{B_{i}\right\}_{i}$ and $\left\{\Phi_{i}\right\}_{i}$, and if the conjunction of these invariants $\bigwedge_{i} \Phi_{i} \wedge \Psi$ implies a predicate $\Phi$, then $\Phi$ is an invariant of the system resulting from the parallel composition of the set $\left\{B_{i}\right\}_{i}$ by the set of interactions $\gamma$.

The rule allows to prove invariance of a predicate $\Phi$ for a system obtained by using a $n$ ary composition operation parameterized by a set of interactions $\gamma$. It uses global invariants which are the conjunction of individual invariants of components $\Phi_{i}$ and an interaction invariant $\Psi$. Interaction invariants are computed symbolically by solving a set of boolean equations called Boolean Behavioral Constraints obtained from the set of interactions. In the case of system with data, interaction invariants are computed from abstractions of the system to be verified. These are the composition of finite state abstractions $B_{i}^{\alpha}$ of the components $B_{i}$ with respect to their invariants $\Phi_{i}$. Finally, the invariance of $\Phi$ is verified by checking tautology $\left(\bigwedge_{i} \Phi_{i}\right) \wedge \Psi \Rightarrow \Phi$ which can simply be done by using a SAT-Solver tool to check the unsatisfiability of $\left(\bigwedge_{i} \Phi_{i}\right) \wedge \Psi \wedge(\neg \Phi)$.

Figure 1 illustrates the idea of our method for a system composing of two components strongly synchronized. The basic idea is to approximate as precisely as possible the set of reachable states of the system. In the figure, black area is the set of reachable states of the system that we want to approximate, $\Phi_{1}$ and $\Phi_{2}$ are components invariants. Starting from component invariants, the intersection of $\Phi_{1}$ and $\Phi_{2}$ is already an over-approximation of the set of reachable states. However this intersection characterizes the maximal combination of the state spaces of two components. It does not take into account the restrictions by strong synchronizations on two components and therefore is a very weak approximation. By adding some interaction invariant $\Psi$, we can reduce this intersection and get a more precise over-approximation of the set of reachable states.

Our method differs from assume-guarantee methods in that it avoids combinatorial explosion of the decomposition and is directly applicable to systems with $n$-ary interactions. Furthermore, it only needs guarantees for components. It replaces the search for adequate assumptions for each component by the use of interaction invariants. These can be computed automatically from given component invariants (guarantees). Interaction invariants correspond to a "cooperation test" in the terminology of [AFdR80] as they allow to eliminate product states which are not feasible by the semantics of parallel composition.


Figure 1: Idea of the method

We also study and propose methods for incremental construction and verification. Incremental construction aims to deal with the complexity of heterogeneous and large-scale systems. It allows building a composite component from smaller parts. Incremental construction provides flexibility in building systems. During the incremental construction process, the verification is necessary to detect early errors in the constructed system. The verification should take advantages of the incremental construction by integrating verification into construction phase. The idea is that, at each stage of the construction, some properties are established and ideally, they should be preserved in the new system obtained in the next steps of the construction. If they are not preserved, at least they can be used in establishing properties for the new system.

We first provide a systematic methodology for incremental construction of componentsbased systems. We propose rules on invariant preservation from which the already established invariants would not be violated during the incremental construction. However, it is not always possible to apply these rules because many systems do not satisfy them. Moreover, the preserved invariants are often not strong enough to prove safety properties because they do not take into account the new constraints enforced to the system in the incremental construction process. Therefore, we propose methods for incremental computation of invariants from the established invariants.

The idea of the incremental computation of invariants is presented in Figure 2. At some stage of the incremental construction, we have obtained sub-systems with established invariants $I_{1}$ and $I_{2}$. These sub-systems are then composed to build a new system and the invariants of the new system are computed from $I_{1}$ and $I_{2}$ by some function $F$ : $I_{12}=F\left(I_{1}, I_{2}\right)$. Similarly, the new system is then composed with another sub-system with established invariants $I_{3}$. The invariants of the global system are computed in the same way from $I_{12}$ and $I_{3}: I_{123}=F\left(I_{12}, I_{3}\right)$. The incremental verification by computing incrementally invariants reduces significantly the verification cost in both time and memory usage.

The main contributions of the thesis are:

- We propose a compositional method for proving safety properties [BBNS08, BBNS10]. For a given property, the method consists in iteratively conjoining the predicate char-


Figure 2: Incremental Verification Idea
acterizing violations of the property with an over-approximation of the reachable states of the system. The over-approximation is the conjunction of two kinds of invariants: component invariants and interaction invariants. If the conjunction is false, then the property is guaranteed. Otherwise, to eliminate infeasible counterexample, new invariants are computed until either the conjunction becomes false or the method fails to prove invariance of the property. In this case, additional reachability techniques can be used for finer analysis.

- We provide methods for computing two types of invariants. Component invariants are over-approximation of reachable states of the components and are generated by simple forward analysis of their behaviors. Interaction invariants characterize global constraints on the global state space induced by strong synchronizations between components. We propose several techniques for computing interaction invariants from boolean constraints obtained from interactions.
- We also provide incremental construction and verification methods which are based on a construction process leading to a composite component through a sequence of constituent components $\left[\mathrm{BBL}^{+} 10, \mathrm{BLN}^{+} 10\right]$. The sequence starts from a set of atomic components and applies incrementally synchronization constraints. Incremental verification relates the verification process to system construction. It takes advantage of the system structure for coping with complexity of the global verification. We study rules which allow preserving established invariants during the incremental construction. For the general case where a system may not satisfy these rules, we propose methods for computing incrementally invariants of the entire system from the established invariants of its constituents.
- We propose a method for dealing with data transfer on interactions between components. The method is based on a projection of the changes on interactions into components and on a replacement of data transfer by a type of component called interaction component. These projection and replacement enable the construction of an equivalent abstract system without data transfer on which we can apply our verification method.
- The compositional and incremental methods have been fully implemented in the DFinder tool-set [BBNS09]. We have successfully applied these methods to prove deadlock-freedom of non-trivial case studies, some of them are case studies of our projects, described in the BIP language [ $\left.\mathrm{BGL}^{+} 08, \mathrm{BGI}^{+} 09\right]$. Interesting and significant results show the efficiency of the method as well as the capacities of D-Finder.


## Organization of the Thesis

The thesis is split into five parts: the first (Chapter 1) presents an overview of the BIP framework; the second (Chapter 2) describes our compositional verification approach and methods for computing two types of invariants; the third (Chapter 3) presents incremental construction and verification methods; the fourth (Chapter 4) presents a method for dealing with systems with data transfer; the fifth (Chapter 5) describes the D-Finder tool together with the implementation; the sixth (Chapter 6) presents the experimental results on some case studies; and the last part draws the conclusions and future work. The details of all chapters are as follows:

- Chapter 1 presents the basic ideas about component-based methodology, the basic notions about components, their composition using glues, and the necessary properties for component-based construction of systems. It introduces the BIP component framework, describing its architecture, its semantics as well as its properties.
- In Chapter 2 we present our method for verifying safety properties of componentbased systems by using invariants. We also present methods for computing two types of invariants: component invariants and interaction invariants. Component invariants are over-approximations of the set of the reachable states generated by forward propagation techniques. Interaction invariants are derived automatically from component invariants and their interactions. When proving invariance of a property fails, it is possible to find stronger invariants by computing stronger component invariants from which stronger interaction invariants are obtained. An application of the method to deadlock detection is also presented in this chapter.
- Incremental construction and verification methods are presented in Chapter 3. First, we formalize an incremental component-based construction process. A composite component is obtained as the composition of a set of atomic components. Then, we formalize the process of incremental construction based on the operation of increments. The construction is hierarchical: increments can be applied either at the same level or at different levels. We associate with the incremental construction the incremental verification. We study rules which allow preserving already established properties. For the general case where a system may not satisfy these rules, we propose methods for incremental computation of global invariants of a composite component from the invariants of its constituent components.
- In Chapter 4, we propose a method for verifying safety properties of component-based systems with data transfer. The method is based on the transformation from a system with data transfer into an equivalent system without data transfer. The transformation is done by projecting the changes of data on interactions into components and
by replacing the data transfer by a component called interaction invariant.
- In Chapter 5, we present the D-Finder tool and the implementation of the modules in D-Finder: the modules for generating component invariants, for making abstractions, for generating interaction invariants, for checking deadlock-freedom, etc. In the interaction invariant generation module, we implement several methods for computing interaction invariants: an enumerative method using the SMT Sat-Solver Yices and the CUDD package; two symbolic methods based on two symbolic operations: Positive Mapping and Fixed-Point computation. Moreover, two incremental methods are also implemented in D-Finder to support incremental verification.
- In Chapter 6, we provide non trivial case studies showing the capacities of D-Finder as well as the efficiency of the method. First we present results on two systems without data, the Dining Philosopher, a classical problem in detecting deadlocks, and Gas Station where we increase as much as possible the size of the system to show the scalability of the method. Then we present results on an Automatic Teller Machine (ATM) case study which has quite complex structure with a number of variables. And finally, we consider a module in a robotic system which is a case study in one of our projects.
- We conclude the thesis in Chapter 7, with an overview of the work and its future perspectives.


## Problmes et Besoins

L'informatique joue aujourd'hui un rôle important dans le développement de la science et de la société. Malgré le fait d'être un nouveau domaine, elle apparaît presque partout, de petits systèmes tel qu'un portable, un lecteur de musique aux grands systèmes comme un avion, un vaisseau spatial. Le besoin croissant de systèmes informatiques augmente aussi la nécessité de leur fiabilité, de leur comportement correct, etc.

Construire des systèmes corrects est une condition essentielle dans tous les domaines. Un système est correct s'il se comporte de manière attendue ou autrement dit, s'il satisfait un ensemble d'exigences explicites. Le comportement attendu est la spécification formelle du système et est souvent exprimé comme un ensemble d'exigences souhaitées. Assurer qu'un système satisfait sa spécification est particulièrement important pour les systèmes critiques, car leur échec ou mauvais fonctionnement peut entraîner une catastrophe telles que la perte d'humaines, les coûts économiques élevés ou le dommage pour l'environnement. Par exemple, le crash d'Ariane 5, un système de lancement européen, est l'un de défaillances informatiques les plus connues dans l'histoire et a entraîné la perte de plus de 370 millions de dollars.

Malheureusement, avec la croissance de la demande pour la scalabilité et la complexité des systèmes, il devient de plus en plus difficile de concevoir correctement leurs modèles. La scalabilité et la complexité augmentent non seulement les violations potentielles des propriétés, mais les rendent également plus difficiles à détecter et à manipuler. La construction d'un système qui fonctionne de manière fiable, malgré la complexité est hautement souhaitable, mais aussi pas toujours possible. Par conséquent, la vérification de comportement correct du système est essentielle et importante pour s'assurer que toutes les exigences sont respectées. Le processus de vérification est basé sur des techniques de détection des violations de propriétés sur le modèle et le comportement correct est prouvé par l'absence de telles violations.

Il y a deux approches principales pour détecter les violations de propriétés d'un système:
le test et la vérification formelle. Considérons un modèle d'un système, un environnement dans lequel les interactions du système ont lieu, et quelques propriétés que le système conçu est censé garantir, on peut choisir l'une des approches suivantes en fonction de leur objectif:

- Le test formel [Bei90, Tre90, Mye04, FFMR07] est une méthode utilisée pour trouver des défauts sur une mise en oeuvre du système, soit au cours du développement soit après la construction complète du système. Pour cela, le test donne des apports de l'environnement (cas de test) et exécute le système pour déterminer s'il produit les résultats requis. Le test, selon la méthode employée, peut être utilisé à tout moment du processus de développement. C'est une façon rapide et directe de déceler des défaillances ou des violations dans le système. Toutefois, le test n'est pas capable de couvrir toutes les possibilités qui peuvent se produire lors de l'exécution du système dans la réalité. Le nombre de situations possibles est habituellement si grand que nous ne pouvons tester qu'une infime proportion d'entre eux. L'absence de violations de propriétés prévues par le test ne signifie pas le comportement correct du système.
- La vérification formelle [UP83, BM79, QS82, CE81] peut à la fois chercher des modèles d'entrée qui violent les propriétés désirées ou prouver le comportement correct du système si un tel modèle d'entrée n'existent pas. Contrairement au test formel, la vérification formelle couvre toutes les possibilités que le système peut se comporter, d'où elle prouve la justesse du système dans le cas de l'absence de violations de propriété. Elle repose sur l'utilisation de techniques mathématiques pour prouver ou réfuter le comportement correct d'un modèle par rapport à une certaine spécification formelle. La vérification formelle a été appliquée avec succès pour vérifier à la fois logiciel et matériel. La vérification de ces systèmes se fait en fournissant une preuve formelle d'un modèle mathématique abstrait du système. Les objets mathématiques qui sont souvent utilisés pour modéliser ces systèmes sont les suivants: systèmes de transitions étiquetées, réseaux de Pétri, automates à états finis, formules booléennes, etc.
Le but de notre travail est de fournir des théories, des méthodes et des outils pour la vérification de comportement correct des systèmes. Nous mettons l'accent sur la vérification formelle, puisque, comme mentionné ci-dessus, elle permet d'atteindre le comportement correct des systèmes.


## Problmes dans la Vrification Formelle

En explorant l'tat de l'art de la vrification formelle, il devient vident qu'une mthode de vrification formelle a besoin de rsoudre les problmes suivants:

- Scalabilité qui évite le problème d'explosion d'espace d'états et donc permet d'augmenter la taille des systèmes à vérifier.
- Efficacité qui permet de détecter le plus tôt possible les erreurs des modèles de systèmes dans la phase de conception.
- Incrmentalit qui intgre la vrification dans le processus de conception.
- Compositionnalit qui permet de dduire les proprits globales d'un systme partir des proprits locales connues de ses sous-systmes.

Nous avons mentionné les problèmes, les besoins en ingénierie logicielle et matérielle pour construire des systèmes corrects. Nous avons également présenté les approches existantes en s'adressant à leurs besoins et leurs limites générales. Dans la section suivante, nous présenterons nos méthodes de vérification compositionnelle et incrémentale.

## Notre Approche

Nous présentons, dans cette thèse, une approche compositionnelle pour la vérification de propriétés de sûreté des systèmes à base de composants. Nous exploitons les deux aspects locaux et globaux des systèmes. Le premier est lié aux comportements locaux de composants. Le deuxième représente des contraintes globales entre les composants qui sont fortement synchronisés. Ces contraintes sont importantes, particulièrement pour la détection de blocages car les blocages globaux sont dues à la synchronisation forte. En outre, les synchronisations restreignent le comportement global du système, et donc ces contraintes globales permettent d'éliminer les états de produits qui ne sont pas atteignables par la sémantique de la composition parallèle.

Notre règle peut être considérée comme une instance de la règle de l'approche déductive. Nous décrivont les techniques qui permettant de générer automatiquement des prédicats auxiliaires en utilisant deux types d'invariants de système: les invariants de composants et les invariants d'interaction. Les invariants de composants expriment des contraintes sur l'espace locale de composants. Les invariants d'interaction caractrisent les restrictions appliques par les synchronisations entre les composants sur l'espace d'tats globaux des systmes. Ils sont calculs automatiquement partir de l'ensemble des invariants de composants et l'ensemble des interactions entre les composants.

La règle générale de notre approche est illustrée dans l'équation (2):

$$
\begin{equation*}
\frac{\left\{B_{i}<\Phi_{i}>\right\}_{i}, \Psi \in I I\left(\|_{\gamma}\left\{B_{i}\right\}_{i},\left\{\Phi_{i}\right\}_{i}\right),\left(\bigwedge_{i} \Phi_{i}\right) \wedge \Psi \Rightarrow \Phi}{\|_{\gamma}\left\{B_{i}\right\}_{i}<\Phi>} \tag{2}
\end{equation*}
$$

Cette quation revient dire que, si $\left\{\Phi_{i}\right\}_{i}$ sont respectivement des invariants d'un ensemble de composants atomiques $\left\{B_{i}\right\}_{i}, \Psi$ sont des invariants calculs partir d'un ensemble d'interactions $\gamma$ entre $\left\{B_{i}\right\}_{i}$ et $\left\{\Phi_{i}\right\}_{i}$, et si la conjonction de ces invariants $\bigwedge_{i} \Phi_{i} \wedge \Psi$ implique un prdicat $\Phi, \Phi$ est donc un invariant du systme rsultant de la composition paralle de l'ensemble $\left\{B_{i}\right\}_{i}$ par l'ensemble des interactions $\gamma$.

La règle permet de prouver l'invariance d'un prédicat $\Phi$ d'un système obtenu en utilisant une opération de composition $n$-aires paramétrée par un ensemble d'interactions $\gamma$. Elle utilise les invariants globaux qui sont la conjonction de différents invariants de composants $\Phi_{i}$ et d'un invariant d'interaction $\Psi$. Les invariants d'interaction sont calculés symboliquement en résolvant une série d'équations booléennes obtenues à partir de l'ensemble des interactions. Dans le cas d'un système avec données, les invariants d'interaction sont calculés à partir de l'abstraction du système à vérifier. Il s'agit de la composition des abstractions finies $B_{i}^{\alpha}$ de composant $B_{i}$ à l'égard de leurs invariants $\Phi_{i}$. Enfin, l'invariance de $\Phi$ se vérifie en vérifiant la tautologie $\left(\bigwedge_{i} \Phi_{i}\right) \wedge \Psi \Rightarrow \Phi$, ce qui peut être simplement fait en utilisant un sat-solveur pour vérifier la satisfiabilité de $\left(\bigwedge_{i} \Phi_{i}\right) \wedge \Psi \wedge(\neg \Phi)$.


Figure 3: L'idée de notre methode

La Figure 3 illustre l'idée de notre méthode pour un système obtenu à partir de deux composants fortement synchronisés. L'idée de base est d'approximer le plus précisément possible l'ensemble des états atteignables du système. Dans la figure, la zone noire est l'ensemble des états atteignables du système que nous voulons approximer, $\Phi_{1}$ et $\Phi_{2}$ sont des invariants de composants. L'intersection de $\Phi_{1}$ et $\Phi_{2}$ est déjà une sur-approximation de l'ensemble d'états atteignables. Toutefois, cette intersection caractérise la combinaison maximale des espaces d'état de deux composants. Elle ne tient pas compte des restrictions imposées par les synchronisations fortes sur les deux composants et est donc une approximation très faible. En ajoutant l'invariant d'interaction $\Psi$, nous pouvons réduire cette intersection et obtenir une sur-approximation plus précise de l'ensemble des états atteignables.

Notre méthode diffère des méthodes assume-guarantee en ce qu'elle permet d'éviter l'explosion combinatoire de la composition et est directement applicable aux systèmes avec interactions $n$-aires. En outre, elle n'a besoin de garanties que pour les composants. Elle remplace la recherche d'hypothèses adéquates par l'utilisation des invariants d'interaction. Ceux-ci peuvent être calculés automatiquement à partir des invariants de composants donnés (garanties). Les invariants d'interaction correspondent à un "test de coopération" dans la terminologie de [AFdR80] car ils permettent d'éliminer des états de produits qui ne sont pas atteignables par la sémantique de composition parallèle.

Nous avons également étudié et proposé des méthodes incrémentales de construction et de vérification. La construction incrémentale vise à répondre à la complexité des systèmes hétérogènes et à grande échelle. Elle permet de construire un composant complex à partir de plus petites pièces. La construction incrémentale offre une flexibilité dans la construction des systèmes. Lors du processus de construction incrémentale, la vérification est nécessaire pour détecter des erreurs le plus tôt possible. La vérification devrait profiter de la construction incrémentale en intégrant la vérification en phase de construction. L'idée est que, à chaque étape de la construction, certaines propriétés ont été établies, et idéalement, elles doivent être conservées dans le nouveau système obtenu dans les prochaines étapes de la construction. Si elles ne sont pas conservées, au moins elles peuvent être utilisées dans l'établissement de propriétés pour le nouveau système.


Figure 4: L'Idée de la Vérification Incrémentale

Nous avons d'abord fourni une méthodologie systématique pour la construction incrémentale des systèmes à base de composants. Nous proposons des règles sur la préservation d'invariant à partir de laquelle les invariants précédemment établis ne seront pas violés au cours de la construction incrémentale. Cependant, il n'est pas toujours possible d'appliquer ces règles, car de nombreux systèmes ne les satisfont pas. En outre, les invariants conservés ne sont souvent pas assez forts pour prouver des propriétés de sûreté car ils ne prennent pas en compte les nouvelles contraintes appliquées au système dans les différents étapes de construction. Par conséquent, nous proposons des méthodes pour le calcul incrémental d'invariants à partir des invariants précédemment établis.

L'idée du calcul incrémental des invariants est présenté dans la Figure 4. À un certain stade de la construction incrémentale, nous avons obtenu des sous-systèmes mis en place avec des invariants $I_{1}$ et $I_{2}$. Ces sous-systèmes sont alors composés pour construire un nouveau système dont les invariants sont calculés à partir de $I_{1}$ et $I_{2}$ par une fonction $F$ : $I_{12}=F\left(I_{1}, I_{2}\right)$. De même, le nouveau système se compose avec un autre sous-système avec ses invariants établis $I_{3}$. Les invariants du système global sont calculés de la même façon à partir de $I_{12}$ et $I_{3}: I_{123}=F\left(I_{12}, I_{3}\right)$. La vérification par le calcul incémental des invariants réduit considérablement le coût de vérification en temps et en mémoire utilisée.

Les contributions principales de la thse sont:

- Nous proposons une méthode de vérification compositionnelle pour prouver des propriétés de sûreté [BBNS08, BBNS10]. Pour une propriété donnée, la méthode consiste à conjoindre itérativement le prédicat caractérisant les violations de la propriété avec une sur-approximation des états accessibles du système. La sur-approximation est la conjonction de deux types d'invariants: invariants de composants et les invariants d'interaction. Si la conjonction est fausse, alors la propriété est garanti. Sinon, pour éliminer les contre-examples, de nouveaux invariants sont calculés jusqu'à ce que la conjonction devienne fausse ou la méthode échoue à prouver l'invariance de la propriété. Dans ce cas, les techniques d'atteignabilité supplémentaires peuvent être utilisées pour une analyse plus fine.
- Nous fournissons des méthodes pour le calcul de deux types d'invariants. Les in-
variants de composants sont des sur-approximations des états atteignables des composants et sont générés par l'analyse en avant de leurs comportements. Les invariants d'interaction caractérisent les contraintes globales sur l'espace d'états globaux induites par les synchronisations fortes entre composants. Nous proposons plusieurs techniques efficaces pour calculer les invariants d'interaction.
- Nous fournissons également les méthodes incrémentales pour la construction et pour la vérification qui sont fondées sur un processus de construction conduisant à une pièce composite à travers une séquence d'éléments constitutifs $\left[\mathrm{BBL}^{+} 10, \mathrm{BLN}^{+} 10\right]$. La séquence commence à partir d'un ensemble de composants atomiques et applique des contraintes incrémentales de synchronisation. La vérification incrémentale lie le processus de vérification à la construction de système. Elle prend les avantages de la structure du système pour faire face à la complexité de la vérification globale. Nous étudions des règles qui permettent la préservation des invariants établis lors de la construction incrémentale. Dans le cas général où un système ne peut pas satisfaire ces règles, nous proposons des méthodes pour calculer de façon incrémentale les invariants du système entier.
- Nous proposons une méthode pour traiter le transfert de données sur les interactions entre les composants. La méthode est basée sur une projection de changements par le transfert de doonées sur les interactions aux composants et sur le remplacement du transfert de données par un type de composant appelé le composant d'interaction. Ces projections et remplacements permettent de construire un système équivalent abstrait, sans transfert de données auquel nous pouvons appliquer notre méthode de vérification.
- Les méthodes compositionnelles et incrémentales ont été entièrement mises en oeuvre dans l'outil D-Finder [BBNS09]. Nous avons appliqué avec succès ces méthodes pour prouver l'absence de blocage de plusieurs études de cas non triviales, certaines d'entre eux sont des études de cas de nos projets [BGL+ $\left.08, \mathrm{BGI}^{+} 09\right]$. Les résultats intéressants et significatifs montrent l'efficacité de la méthode ainsi que les capacités de D-Finder.


## Organization de la Thèse

La thèse est divisée en cinq parties: la première (Chapitre 1) présente un aperçu de la plate-forme BIP, la seconde (Chapitre 2) décrit notre approche de vérification compositionnelle et les méthodes pour calculer deux types d'invariants, la troisième (Chapitre 3) présente les méthodes incrémentales de construction et de vérification, la quatrième (Chapitre 4) présente une méthode pour traiter les systèmes avec le transfert de données, la cinquième (Chapitre 5) décrit l'outil D-Finder avec la mise en oeuvre, la sixième (Chapitre 6) présente les résultats expérimentaux sur certaines études de cas, et la dernière partie tire les conclusions et décrit les travaux futurs. Les détails de tous les chapitres sont les suivants:

- Le Chapitre 1 présente l'idée de la méthodologie basée sur des composants, les notions de composants, leur composition en utilisant des colles, et les propriétés nécessaires pour la construction des systèmes à base de composants. Il introduit la plateforme de
composants BIP en décrivant son architecture, sa sémantique, ainsi que ses propriétés.
- Dans le Chapitre 2, nous présentons notre méthode compositionnelle basée sur invariants pour vérifier des propriétés de sûreté des systèmes à base de composants. Nous présentons également des méthodes de calcul de deux types d'invariants: les invariants de composants et les invariants d'interaction. Les invariants de composants sont les sur-approximations de l'ensemble d'états atteignables. Ils sont générés par les techniques de propagation en avant. Les invariants d'interaction sont obtenus automatiquement à partir des invariants de composants et des interactions. Lorsque la preuve d'invariance d'une propriété échoue, il est possible de calculer des invariants de composants plus forts à partir desquels des invariants d'interaction plus forts sont obtenus. Une application de la méthode à la détection de blocage est également présentée dans ce Chapitre.
- La construction et la vérification incrémentales sont présentées dans le Chapitre 3. D'abord, nous formalisons un processus incrémental de construction à base de composants. Un composant complex est obtenu par la composition d'un ensemble de composants atomiques. Puis, nous formalisons le processus de construction incrémentale basé sur l'exploitation des couches. La construction est hiérarchique, elle peut être appliquée soit au même niveau soit à différents niveaux. Nous associons la vérification à la construction incrémentale. Nous étudions les règles qui permettent de préserver les propriétés précédemment établis. Dans le cas général où un système ne peut pas satisfaire ces règles, nous proposons les méthodes pour calculer de façon incrémentale des invariants globaux du système entier à partir des invariants de ses éléments constitutifs.
- Dans le Chapitre 4, nous proposons une méthode permettant de vérifier les propriétés de sûreté des systèmes avec le transfert de données. La méthode est basée sur la transformation d'un système avec transfert de données à un système équivalent sans transfert de données. La transformation se fait par la projection de l'évolution des données sur les interactions aux composants et en remplaçant le transfert de données par un composant appelé "composant d'interaction".
- Dans le Chapitre 5, nous présentons l'outil D-Finder et la mise en oeuvre de modules dans D-Finder: les modules pour générer des invariants de composants, pour faire des abstractions, pour générer des invariants d'interaction, pour détecter des blocages, etc. Dans le module de génération des invariants d'interaction, nous mettons en oeuvre plusieurs méthodes de calcul: une méthode énumérative à l'aide de Sat-Solveur Yices et le paquet Cudd; deux méthodes symbolique utilisant deux opérations symboliques: le Mapping Positive et le calcul de points fixes. En outre, deux méthodes incrémentales sont également mis en oeuvre dans D-Finder.
- Dans le Chapitre 6, nous fournissons des études de cas non triviales montrant les capacités du D-Finder ainsi que l'efficacité de nos méthodes. D'abord, nous présentons des résultats sur deux systèmes sans données, le Philosophes, un problème classique dans la détection des blocages, et la Station de Gaz où nous augmenter autant que possible la taille du système pour montrer la scalabilité de la méthode. Ensuite, nous présentons des résultats sur un système de guichet automatique bancaire (Automated Teller Machine) qui a une structure assez complexe avec un nombre de variables. Et
enfin, nous considrons un module dans un systme robotique qui est une tude de cas dans l'un des nos projets.
- Nous concluons la thse au chapitre 7, avec un aperu du travail et des perspectives de futur.

BIP Modeling Framework

## Chapter Abstract

In this chapter, we first present the idea of component-based approach together with its necessary properties for dealing with the complexity of systems in the construction phase. Then we give a short description on the BIP modeling framework developed at Verimag laboratory. BIP (Behavior-Interaction-Priority) is a component-based framework that allows building a complex system by assembling simple components. A system in BIP consists of three layers: Behavior which corresponds to atomic components, Interaction that describes the collaborations between components and Priority which allows to choose an interaction to be executed amongst possible interactions. Finally we provide several important properties of systems in BIP such as deadlock-freedom, invariants, etc.

## Résumé du Chapitre

Dans ce chapitre, nous présentons d'abord l'idée de l'approche basée sur les composants avec ses propriétés nécessaires pour faire face à la complexité de systèmes en phase de construction. Ensuite, nous donnons une brève description de la plate-forme de modélisation BIP (Behavior-Interaction-Priority) développé au laboratoire Verimag. BIP permet de construire un système complexe par l'assemblage de composants simples. Un système de BIP se compose de trois couches: le Comportement qui correspond aux composants atomiques, l'Interaction qui décrit les collaborations entre les composants et la Priorité qui permet de choisir une interaction à exécuter parmi les interactions possibles. Enfin, nous fournissons quelques propriétés importantes des systèmes en BIP telles que l'absence de blocage, les invariants, etc.

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### 1.1 Component-based Design

The design technique and formal verification have mutual influence. A clear and simple design of systems can reduce the cost of verification. The major goal of formal verification is to check the correctness of the system and to provide diagnostics when a requirement is not met. These diagnostics on a clear model can help designers understand easily the problems and locate their source. The modification/correction of the model may be expensive if it requires significant changes of the model, specially with the growing size and complexity of systems. The design techniques are therefore important to reduce this cost by dealing with the complexity of systems.

Component-based design techniques are used for that purpose. The idea is that complex systems can be obtained by assembling components (building blocks). This is essential for the development of large-scale, evolvable systems in a timely and affordable manner. It offers flexibility in the construction phase of systems by supporting the addition, removal, or modification of components without any or very little impact on other components. Components are usually characterized by abstractions that ignore implementation details and describe properties relevant to their composition, e.g., transfer functions, interfaces. Composition is used to build complex components from simpler ones. It can be formalized as an operation that takes, as input, a set of components and their integration constraints and provides, as output, the description of a new, more complex component. This approach mitigates the complexity of systems by offering incrementality in the construction phase. However, for being able to deal with complexity in verification, the component frameworks need to allow constructivity along the design process.

Constructivity is the possibility to build complex systems that meet given requirements by assembling components with known properties. The correctness of the systems is inferred and guaranteed by construction with little computation. Component-based design techniques confer numerous advantages, in particular through reuse of existing components.

A key issue is the existence of composition frameworks ensuring the correctness of composite components. In particular, we need frameworks allowing not only reuse of components but also reuse of their properties for establishing global properties of composite components from properties of their constituent components. Hence, we need theory allowing constructivity and meeting the following requirements:

Incrementality. This means that composite systems can be considered as the composition of smaller parts. Incrementality provides flexibility in building systems by simply adding or removing components and the result of construction is independent of the order of integration. It is necessary for progressive analysis and the application of compositionality rules. Incrementality allows coping with the complexity of the heterogenous and large-scale systems in both construction and verification phases.
Compositionality. Compositionality rules allow inferring global system properties from the local properties of the sub-systems. (e.g, inferring global deadlock-freedom from the deadlock-freedom of the individual components). Compositionality is necessary for obtaining correctness-by-construction.
Composability. Composability rules guarantee that, under some conditions, essential properties of a component will be preserved after integration. Composability means stability of previously established component properties across integration, e. g , a deadlock-free component will remain deadlock-free after gluing together with other components. Composability is essential for incremental construction as it enables the construction of large systems without disturbing the behavior of their components.
Verimag has developed a Component-based Modeling Framework called BIP (Behavior - Interaction - Priority) [BBS06] for modeling heterogeneous real-time components. We will provide in this chapter a short description of the BIP framework. We start by giving notions about component-based framework: the ideas about component, their composition and the necessary properties for component-based construction of systems.

### 1.2 Basic Ideas

Component-based design consists in building a component satisfying a given property from:

- a set of components $B_{1}, B_{2}, \ldots B_{n}$, described by their behavior, and
- a set of glue operators $\mathcal{G} \mathcal{L}=\left\{g l_{1}, g l_{2}, \ldots g l_{n}\right\}$ on components characterizing coordination between components.
A component is an entity with well-defined interfaces for interacting with its environment. It denotes an executable description of which the run can be modeled as sequences of actions. Tasks, processes, threads, functions, blocks of code can be considered as components. Two types of components are considered:
- Atomic component, a basic element that only represents behavior.
- Compound component, a composition of a set of components by using glue.

A component is denoted graphically by a box. The box of an atomic component contains behavior inside and the box of a composite component contains other components and glue.

### 1.3. BIP MODELING FRAMEWORK

Behavior is represented by a labeled transition system (LTS).
Definition 1 (Labeled Transition System) A labeled transition system is a triple $B=$ $(L, P, \mathcal{T})$, where $L$ is a set of locations, $P$ is a set of actions, and $\mathcal{T} \subseteq L \times P \times L$ is a set of transitions, each labeled by an action.

For any pair of locations $l, l^{\prime} \in L$ and an action $p \in P$, we write $l \xrightarrow{p} l^{\prime}$, iff $\left(l, p, l^{\prime}\right) \in \mathcal{T}$. If such $l^{\prime}$ does not exist, we write $l \underset{\rightarrow}{\nrightarrow}$.

When components are composed together, it might be necessary to restrict the product of behaviors in order to meet some global properties. Glue are used for this purpose. The glue is a separate layer that composes the underlying layer of behaviors. It is a set of operators mapping tuples of behaviors into an equivalent behavior. Given $\left\{B_{1}, B_{2}, \ldots, B_{n}\right\}$ a set of atomic components, their composition with the glue $\mathcal{G} \mathcal{L}$ is a composite component $B$ (figure 1.1) represented as follows:

$$
B=\mathcal{G} \mathcal{L}\left(B_{1}, B_{2}, \ldots B_{n}\right)
$$

The new behavior $B$ is obtained by applying restrictions implied by the meaning of the glue to the product of the behaviors of $B_{1}, B_{2}, \ldots B_{n}$. Since glue restrict the product of the behavior, the behavior of $B$ is smaller than the product of $B_{1}, B_{2}, \ldots B_{n}$. This new component $B$ can be further used for composition with other components.


Figure 1.1: Component Composition.

### 1.3 BIP Modeling Framework

BIP is a component framework where the composition of behaviors are performed by two kinds of glue: interactions and priorities. Interactions characterize collaborations between components and priorities allow choosing an interaction to be executed amongst possible interactions. The construction of BIP components is based on a 3-layers architecture (figure 1.2): Behavior, Interaction and Priority. Compound components are built by composition of simpler components and its layers are obtained by composing separately the layers of the constituents. A component is composed of:

- Behavior: behavior is a labeled transition system describing elementary transformations of states. Transitions consist of triggers and local computations. Triggers are conditions depending on local state and port expressions which characterize ability of the component to interact with its environment.
- Interactions: interactions are architecture constraints on behavior. An interaction is a global transformation of the states of different components. Interactions can be


Figure 1.2: Layered component model


Figure 1.3: Three-dimentional space construction
considered as a function allowing computing the interactions of a component from those of its constituents.

- Priorities: priorities provide a mechanism for restricting the global behavior of the layers underneath by filtering amongst possible interactions. They are very useful for enforcing state invariant properties such as mutual exclusion and scheduling policies. A component in BIP can also be viewed as a point in a three-dimensional space represented in Figure 1.3. The dimension Behavior characterizes component behavior and the space Interactions $\times$ Priorities characterizes the overall structure of the system.


### 1.3.1 Atomic Components

A BIP atomic component is a Labeled Transition System extended with data. It consists of a set of ports used for the synchronization with other components, a set of control locations, a set of transitions and a set of local variables. The transitions describe the behavior of the component.

Definition 2 (Atomic Component) An atomic component is a transition system extended with data $B=\left(L, P, \mathcal{T}, X,\left\{g_{\tau}\right\}_{\tau \in \mathcal{T}},\left\{f_{\tau}\right\}_{\tau \in \mathcal{T}}\right)$, where:
$-(L, P, \mathcal{T})$ is a transition system, that is

- $L=\left\{l_{1}, l_{2}, \ldots, l_{k}\right\}$ is a set of control locations,
$-P=\left\{p_{1} \ldots p_{n}\right\}$ is a set of ports,
- $\mathcal{T} \in L \times P \times L$ is a set of transitions,
- $X=\left\{x_{1}, \ldots, x_{n}\right\}$ is a set of variables and for each $\tau \in \mathcal{T}$ respectively, $g_{\tau}$ is a guard, a predicate on $X$, and $f_{\tau}\left(X, X^{\prime}\right)$ is an update relation, a predicate on $X$ (current)
and $X^{\prime}$ (next) state variables.
A transition is of the form $\tau=\left(l, p, g_{\tau}, f_{\tau}, l^{\prime}\right)$ where $l$ (respectively $l^{\prime}$ ) is the source (respectively destination) location, $p$ is a port through which an interaction is sought. The transition $\tau$ can be executed only if its guard $g_{\tau}$, a boolean condition on the set of variables $X$, is true. $g_{\tau}$ is also known as the pre-condition for interaction through the port $p . f_{\tau}$ is a computation step consisting of local state transformations. A transition can be represented in a simple form $\tau=\left(l, p, l^{\prime}\right)$ to insist only the state transformation or in case that the guard $g_{\tau}=$ true and there is no internal computation $f_{\tau}$.

In BIP there are two kinds of ports:

- Complete port: an active port which can initiate an interaction without synchronization with other ports. Complete port is graphically represented by a triangle.
- Incomplete port: a passive port hence it needs a synchronization with other ports to execute its transitions. An incomplete port is denoted by a circle.
A port is enabled if at least one of its transitions is enabled, or disabled if all its transitions are disabled. A transition $\tau=\left(l, p, g_{\tau}, f_{\tau}, l^{\prime}\right)$ is enabled if its source location $l$ is reached and its guard $g_{\tau}$ is true. Conversely, the transition $\tau$ is disabled if the component is not at $l$ or its guard $g_{\tau}$ is false.

The behavior of an atomic component is a labeled transition system with moves of the form $\left(l_{1}, x\right) \xrightarrow{p}\left(l_{2}, x^{\prime}\right)$, where $l_{1}, l_{2}$ are control locations of the automaton and $x, x^{\prime}$ are respectively valuations of the variables at each control location. The move $\left(l_{1}, x\right) \xrightarrow{p}\left(l_{2}, x^{\prime}\right)$ is possible if there exists a transition $\left(l_{1}, p, g_{\tau}, f_{\tau}, l_{2}\right)$, such that $g_{\tau}(x)=$ true. As a result of the move, the set of variables are modified to $x^{\prime}=f_{\tau}(x)$. The semantics of execution of transitions is formally defined as follows:

Definition 3 (Semantics) The semantics of $B=\left(L, P, \mathcal{T}, X,\left\{g_{\tau}\right\}_{\tau \in \mathcal{T}},\left\{f_{\tau}\right\}_{\tau \in \mathcal{T}}\right)$, is a transition system $\left(Q, P, \mathcal{T}_{0}\right)$ such that

- $Q=L \times \mathbf{X}$ is a set of states, where $\mathbf{X}$ denotes the set of valuations of variables $X$,
$-\mathcal{T}_{0}$ is the set including transitions $\left((l, \mathbf{x}), p,\left(l^{\prime}, \mathbf{x}^{\prime}\right)\right)$ such that $g_{\tau}(\mathbf{x}) \wedge f_{\tau}\left(\mathbf{x}, \mathbf{x}^{\prime}\right)$ for some $\tau=\left(l, p, l^{\prime}\right) \in \mathcal{T}$. As usual, if $\left((l, \mathbf{x}), p,\left(l^{\prime}, \mathbf{x}^{\prime}\right)\right) \in \mathcal{T}_{0}$ we write $(l, \mathbf{x}) \xrightarrow{p}\left(l^{\prime}, \mathbf{x}^{\prime}\right)$.

We define here useful notions for later use: given a transition $\tau=\left(l, p, l^{\prime}\right) \in \mathcal{T}, l$ and $l^{\prime}$ are respectively, the source and the target location denoted respectively by ${ }^{\bullet}$ and $\tau^{\bullet}$. We extend this notation for ports: ${ }^{\bullet} p=\left\{\bullet \mid \tau=\left(l, p, l^{\prime}\right)\right\}$ and $p^{\bullet}=\left\{\tau^{\bullet} \mid \tau=\left(l, p, l^{\prime}\right)\right\}$ are respectively the set of source and target locations of the transitions labeled by the port $p$.

We present below the Temperature Control System, a case study in BIP, which is used as an example for illustrating the modeling of components and of system in BIP. This case study will also be used for showing the application of our compositional verification method in the next chapter.

Example 1 (Temperature Control System) [ACH $\left.{ }^{+} 95\right]$ This system controls the coolant temperature in a reactor tank by moving two independent control rods. The goal is to maintain the coolant between the temperatures $\theta_{m}$ and $\theta_{M}$. When the temperature reaches its maximum value $\theta_{M}$, the tank must be refrigerated with one of the rods. The temperature


Figure 1.4: Temperature Control System
rises at a rate $v_{r}$ and decreases at rate $v_{d}$. A rod can be moved again only if $T$ time units have elapsed since the end of its previous movement. If the temperature of the coolant cannot decrease because there is no available rod, a complete shutdown is required.

We provide a discretized model of the Temperature Control System in BIP, decomposed into three atomic components: a Controller and two components Rod1, Rod2 modeling the rods. We take $\theta_{m}=100^{\circ}, \theta_{M}=1000^{\circ}, T=3600$ seconds. Furthermore, we assume that $v_{r}=1^{\circ} / \mathrm{s}$ and $v_{d}=2^{\circ} / \mathrm{s}$. Figure 1.4 presents the BIP model of Temperature Control System. The components Rod1 and Rod2 are identical, up to the renaming of locations and ports. Each one has two control locations and four transitions: two loop transitions labeled by tick and two transitions synchronized with transitions of the Controller. The Controller will be described in detail here as an example for atomic component.

Controller component is composed of a set of locations $L=\left\{l_{5}, l_{6}\right\}$, a set of ports $P=\{$ heat, cool, tick $\}$, a variable $\theta$ representing the temperature of the system and a set of the transitions $\mathcal{T}=\left\{\tau_{1}, \tau_{2}, \tau_{3}, \tau_{4}\right\}$ where:
$-\tau_{1}=\left(l_{5}\right.$, tick, $\left.g=(\theta<1000), f=(\theta:=\theta+1), l_{5}\right):$ from $l_{5}$ location, tick transition can take place if $\theta<1000$ and it increases the temperature $\theta$ by 1. Since this transition is a loop, the component is still at $l_{5}$ location after the transition.
$-\tau_{2}=\left(l_{5}\right.$, cool, $\left.g=(\theta=1000), l_{6}\right):$ from $l_{1}$ location, if the temperature $\theta=1000$, cool transition must take place to refrigerate the system by triggering one of two Rods.
$-\tau_{3}=\left(l_{6}\right.$, tick, $\left.l_{6}\right):$ at $l_{6}$ location, tick loop transition can be executed if $\theta>100$ and it decreases the temperature $\theta$ by 2.
$-\tau_{4}=\left(l_{6}\right.$, rest,$\left.l_{5}\right):$ from $l_{6}$ location, heat transition must be executed if the temperature $\theta$ reaches $100(\theta=100)$. The component returns to location $l_{5}$.

The textual for atomic components in BIP is the following:

```
atomic component::=
    component component id
        {port complete/incomplete port_id+}\mp@subsup{}}{}{+
```


### 1.3. BIP MODELING FRAMEWORK

```
    [data type_id data_id+]}\mp@subsup{]}{}{+
    behavior
        initial do statement to state_id
        {state state_id
            {on port_id [provided guard]
            [do statement] to state_id}++}+
        end
end
```

That is, an atomic component consists of a declaration followed by the definition of its behavior. Declaration consists of ports and data. Ports are identifiers and are specified as complete or incomplete. For data, basic C types (int, double, char, etc.) can be used. In the behavior, guard and statement denote respectively C expressions and statements.

The above description of a component defines a type of atomic component from which a set of component instances can be created and used as atomic components to build a system. The creation of instances will be described in the composite component section.

Behavior is defined by a set of transitions. The initial location of the component together with the initial action specifying initial valuations of variables is described following the keyword initial. The keyword state is followed by a control location and the list of outgoing transitions from this location. Each transition is labeled by a port identifier followed by its guard, its function and a target location.

Example 2 The BIP text of the Controller component in Example 1 is as follows:

```
component Controller
    port incomplete tick, cool, heat
    data int th /* temperature variable theta */
    behavior
        initial do th = 100 to l5
        state l5
            on tick provided th < 1000 do th = th + 1 to l5
            on cool provided th = 1000 to 16
        state l6
            on tick provided th>100 do th = th - 2 to l6
            on heat provided th =100 to l5
    end
end
```


### 1.3.2 Interactions

In BIP, components communicate through a set of interactions. An interaction is a non-empty subset of ports of different components and it synchronizes the executions of the ports.

Definition 4 (Interactions) Given a set of components $\left(B_{1}, B_{2}, \ldots, B_{n}\right)$, where $B_{i}=$ $\left(L_{i}, P_{i}, \mathcal{T}_{i}, X_{i},\left\{g_{\tau}\right\}_{\tau \in \mathcal{T}_{i}},\left\{f_{\tau}\right\}_{\tau \in \mathcal{T}_{i}}\right)$, an interaction a is a set of ports, subset of $\bigcup_{i=1}^{n} P_{i}$, such that $\forall i=1, \ldots, n\left|a \cap P_{i}\right| \leq 1$.

When we write $a=\left\{p_{i}\right\}_{i \in I}, I \in 1 \ldots n$, we suppose that for each $i \in I, p_{i} \in P_{i}$. The interaction model is specified by a set of interactions $\gamma \subseteq 2^{P}$.

In this thesis, to simplify notation, we write for an interaction $\left\{p_{1}, \ldots, p_{k}\right\}$ the expression $p_{1} \ldots p_{k}$. Furthermore, for a set of interactions $\left\{a_{1}, \ldots, a_{n}\right\}$ we write $a_{1}+\cdots+a_{n}$.

We extend interactions with data transfer between the synchronizing components. For an interaction $a$, we use a guard $G_{a}$ (boolean condition) and data transfer function $F_{a}$ to specify data transfer.

Interactions can be enabled or disabled. An interaction is enabled iff its guard is true and all its ports are enabled. Contrarily, it is disabled iff its guard is false or at least one of its ports is disabled.

Example 3 For the Temperature Control System presented in Example 1, the set of interactions between the Controller and two Rods is (Figure 1.4): $\gamma=\operatorname{cool}_{1} \mathrm{Cool}+\operatorname{cool}_{2} \mathrm{cool}+$ rest ${ }_{1}$ heat + rest $_{2} h e a t+$ tick $_{1}$ tick $_{2}$ tick.

We now provide the operational semantics for the composition of a system of behavior with respect to an interaction model.

Definition 5 (Parallel Composition) Given n components $B_{i}=\left(L_{i}, P_{i}, \mathcal{T}_{i}, X_{i},\left\{g_{\tau}\right\}_{\tau \in \mathcal{T}_{i}}\right.$, $\left.\left\{f_{\tau}\right\}_{\tau \in \mathcal{T}_{i}}\right)$ and a set of interactions $\gamma$, we define $B=\gamma\left(B_{1}, \ldots, B_{n}\right)$ as the component $(L, \gamma$, $\left.\mathcal{T}, X,\left\{g_{\tau}\right\}_{\tau \in \mathcal{T}_{i}},\left\{f_{\tau}\right\}_{\tau \in \mathcal{T}_{i}}\right)$, where:

- $L=L_{1} \times L_{2} \times \ldots \times L_{n}$ is the set of control locations,
$-X=\bigcup_{i=1}^{n} X_{i}$ is the set of variables,
- $\mathcal{T}$ is a set of transitions of the form $\tau=\left(\left(l_{1}, \ldots, l_{n}\right), a, g_{\tau}, f_{\tau},\left(l_{1}^{\prime}, \ldots, l_{n}^{\prime}\right)\right)$ obtained by synchronization of sets of transitions $\left\{\tau_{i}=\left(l_{i}, p_{i}, g_{\tau_{i}}, f_{\tau_{i}}, l_{i}^{\prime}\right) \in \mathcal{T}_{i}\right\}_{i \in I}$ such that $\left\{p_{i}\right\}_{i \in I}=a \in \gamma$ and $l_{j}^{\prime}=l_{j}$ if $j \notin I$, for arbitrary $I \subseteq\{1, \ldots, n\}$, the associated guard and function are respectively $g_{\tau}=G_{a} \wedge \bigwedge_{i \in I} g_{\tau_{i}}$ and $f_{\tau}=F_{a} \wedge \bigwedge_{i \in I} f_{\tau_{i}} \wedge \bigwedge_{i \notin I}\left(X_{i}^{\prime}=\right.$ $\left.X_{i}\right)$.

The obtained behavior $B=\gamma\left(B_{1}, \ldots, B_{n}\right)$ can execute a transition $a \in \gamma$, iff for each $i \in I$, the action $a \cap P_{i}$ is enabled in $B_{i}$. The states of the transition system that do not participate in the interaction $a$ remain unchanged. Notice that for $\gamma_{\perp}=\sum_{i=1}^{n} \sum_{p \in P_{i}} p$, the component $\gamma_{\perp}\left(B_{1}, \ldots, B_{n}\right)$ is the transition system obtained by interleaving the transitions of atomic components.

Example 4 Figure 1.5 provides an example on the composition of two components $B_{1}, B_{2}$ by a set of interactions $\gamma=a c+b$. The right side is the composed behavior obtained after the application of the interactions. The overall graph (including solid and dotted arrows) shows the product of the two behaviors. The solid arrows are the transitions allowed by the interactions and the graph with these solid arrows presents the composition of two behaviors by the interactions. The dotted transitions are not legal.

### 1.3. BIP MODELING FRAMEWORK



Figure 1.5: Composition by Interactions

In BIP, interactions are structured by connectors. A connector is a set of ports characterizing a set of interactions. For example, if $p_{1}, p_{2}, p_{3}$ are ports of distinct atomic components, then the connector $\gamma$ consisting of $\left\{p_{1}, p_{2}, p_{3}\right\}$ has seven possible interactions $p_{1}+p_{2}+p_{3}+p_{1} p_{2}+p_{1} p_{3}+p_{2} p_{3}+p_{1} p_{2} p_{3}$. To specify the feasible interactions of a connector $\gamma$, we use two types of synchronizations:

- Strong synchronization or rendez-vous, when the only feasible interaction of $\gamma$ is the maximal one, i. e., it contains all the ports of $\gamma$.
- Weak synchronization or broadcast, when feasible interactions are all those containing a complete port which initiates the broadcast. That is, if $\gamma$ consists of $\left\{p_{1}, p_{2}, p_{3}\right\}$ and the broadcast is initiated by $p_{1}$, then the feasible interactions are $p_{1}+p_{1} p_{2}+$ $p_{1} p_{3}+p_{1} p_{2} p_{3}$.
It is possible to represent any arbitrary interaction through a connector by structured combination of the above two basic synchronization protocols.

To characterize these synchronizations, we associate two types with complete and incomplete ports. A feasible interaction of a connector is a set of its ports such that either it contains some complete port, or it is maximal, i.e, consisting of all the incomplete ports.


Figure 1.6: Connectors and their interactions.

Example 5 Example of sets of connectors and their feasible interactions are shown in figure 1.6:

- In (a), the connector consists of two incomplete ports $p_{1}$ and $p_{2}$, hence the only feasible interaction is $p_{1} p_{2}$. It represents a rendez-vous, meaning that both actions
are necessary for the synchronization.
- In (b), the interaction between $p_{1}$ and $p_{2}$ is asymmetric. $p_{1}$ is a complete port and can occur alone, even if $p_{2}$ is not possible. Nevertheless, $p_{2}$ is an incomplete port and it needs to synchronize with $p_{1}$ to occur. The feasible interactions are $\gamma=p_{1}+p_{1} p_{2}$.
- In (c), the interactions between $p_{1}, p_{2}$ and $p_{3}$ are also asymmetric. The port $p_{1}$ is complete, it can occur alone or synchronize with either or both $p_{2}$ and $p_{3}$, hence the feasible interactions are $\gamma=p_{1}+p_{1} p_{2}+p_{1} p_{3}+p_{1} p_{2} p_{3}$.

The textual for connector description in BIP is as follows:

```
interaction::= port_id+
connector::=
    connector connector_id = port_id+
        [complete interaction]
        behavior
            [on interaction [provided guard] [do statement]]+
    end
```

That is, the keyword connector is followed by a name and a set of ports. The keyword complete define a minimal set of ports to be considered as complete interaction. Then any superset (including) of ports of that minimal set corresponds to a feasible interaction. Since a connector contains a set of interactions, the behavior is defined following the keyword behavior for each interaction: the keyword on is followed by an interaction together with its guard and its function.

If the definition of complete interaction is omitted, then all interactions containing a complete port (if it exists) are feasible, or only the maximal interaction (if all the ports are incomplete) is feasible.

### 1.3.3 Priorities

Priorities are a powerful tool for enforcing a given property by restricting nondeterminism. It allows selecting interactions to be executed amongst the feasible ones based on the current global state of the system. The definition of a priority, followed by the composition of behaviors using the priority glue are provided below.

Definition 6 (Priority) A priority is a relation $\prec \subseteq \gamma \times L \times \gamma$, where $\gamma$ is the set of interactions, and $L$ is the global set of locations. We write $a \prec_{l} a^{\prime}$ for $\left(a, l, a^{\prime}\right) \in \prec$. Furthermore, we require that for all $l \in L, \prec_{l}$ is a strict partial order on $\gamma . a \prec_{l} a^{\prime}$ means that interaction a has less priority than $a^{\prime}$ at location $l$.

The textual for the description of priorities is defined by:

```
priority::=
    priority [priority_id [if cond] interaction < interaction]+
```

That is, priorities are a set of rules, each consisting of an ordered pair of interactions associated with a condition (cond). The condition is a boolean expression in C on the variables of the components involved in the interactions. When the condition holds and both interactions are enabled, only the higher one is eligible for execution. Conditions can be omitted for static priorities.

### 1.3.4 Composite Components

In BIP, a composite component allows defining new components which consist of:

- a set of instances of existing sub-components (atomic or composite).
- a set of connectors between the component instances.
- a set of priorities between the interactions.

The BIP textual of a composite component is defined by:

```
composite component::=
    component component_id
        {contains component_id {instance_id[parameters]}+}\mp@subsup{}}{}{+
        [connector]+
        priority
    end
```

The instances can have parameters providing initial values to their variables through a named association.

Finally, we consider systems defined as parallel composition of components together with an initial state.

Definition 7 (System) $A$ system $\mathcal{S}$ is a pair $\langle B$, Init $\rangle$ where $B$ is a component and Init is the initial state of $B$.

We seperate Init from components because we want to reuse components. A component type is used to build different parts of a system or different systems. And depending on the system, the component might have different initial states. Hence the separation of initial states and components provides flexibility in the reuse of components.

We can consider Init in the form of state, that is the set of initial states of components in $B$, or in the form of a state predicate characterizing the initial state of $B$.
Example 6 The components in Temperature Control presented in Example 1 are composed by using the following set of interactions, indicated by connectors in the Figure 1.4:

$$
\gamma=\text { tick tick } \text { tick }_{2}+\text { cool cool }_{1}+{\text { cool } \text { cool }_{2}+\text { heat rest }}_{1}+\text { heat rest }{ }_{2}
$$

The initial state of the system is Init $=\left(l_{5} \wedge(\theta=100), l_{1} \wedge\left(t_{1}=3600\right), l_{3} \wedge\left(t_{2}=3600\right)\right)$. Init can also be represented in the form of initial condition as Init $=\left(l_{5} \wedge(\theta=100)\right) \wedge$ $\left(l_{1} \wedge\left(t_{1}=3600\right)\right) \wedge\left(l_{3} \wedge\left(t_{2}=3600\right)\right)$.

The Temperature Control System is represented by $S=\langle B$, Init $\rangle$ where $B$ is the Temperature Control composite component described in BIP language as follows:

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| :--- | :--- | :--- |

```
component TemperatureController
    contains Rod rod1, rod2
    contains Controller ctrl
    connector tick = ctrl.tick, rod1.tick1, rod2.tick2
        behavior
            on ctrl.tick, rod1.tick1, rod2.tick2
            provided true do {##}
        end
    connector cooll = ctrl.cool, rod1.cool1
        behavior
            on ctrl.cool, rod1.cool1 provided true do {##}
        end
    connector cool2 = ctrl.cool, rod2.cool2
        behavior
            on ctrl.cool, rod2.cool2 provided true do {##}
        end
    connector heat1 = ctrl.heat, rod1.rest1
        behavior
            on ctrl.heat, rod1.rest1 provided true do {##}
        end
    connector cool1 = ctrl.heat, rod2.rest2
        behavior
            on ctrl.heat, rod2.rest2 provided true do {##}
            end
end
```


### 1.4 Properties of BIP Components

### 1.4.1 Invariants

A state predicate $I$ is an invariant of a system $S$, if every reachable state of the system $S$ satisfies $I$. In other words, each state that is reached during the computation of $S$ satisfies $I$. We first define the set of reachable states of a system.

Definition 8 (Reachable States) Given a system $S=\langle B$, Init $\rangle$ where $B$ is a component and Init is the initial state of $B$. A state $l$ is called reachable (accessible) in $S$ if from the initial state there exists an execution sequence Init $\xrightarrow{p_{1}} l_{1} \xrightarrow{p_{2}} \ldots \xrightarrow{p_{n}} l_{n}$ such that $l_{n}=l$. We denote the set of reachable states of $S$ by Reach $(S)$.

Formally, we have the following definition of system invariants:
Definition 9 (Invariants of System) Given a system $S$ and its set of reachable states Reach $(S)$. A state predicate $I$ is an invariant of $S$, denoted by inv $(S, I)$, if every state of Reach $(S)$ satisfies $I$.

### 1.4. PROPERTIES OF BIP COMPONENTS

In component-based construction, components can be reused to build different systems or different parts of a system. And the initial states of different instances of a component type may be different. Therefore, we define here the notion of invariants of component. Then the invariants of a system are obtained from the invariants of components that are used to build the system depending on the initial state of the system.

Definition 10 (Invariants of Component) Given a component $B=\left(L, P, \mathcal{T}, X,\left\{g_{\tau}\right\}_{\tau \in \mathcal{T}}\right.$, $\left\{f_{\tau}\right\}_{\tau \in \mathcal{T}}$, a state predicate $I$ is an invariant of $B$, denoted by inv $(B, I)$, if for any state $l \in L$ and any port $p \in P, I(l)$ and $l \xrightarrow{p} l^{\prime} \in \mathcal{T}$ imply $I\left(l^{\prime}\right)$, where $I(l)$ means that $l$ satisfies $I$.

That is, a state predicate $I$ is an invariant of a component $B$ if for any state $l$ of $B$ that satisfies $I$, all the states reached from $l$ also satisfy $I$.

For a system $S=\langle B$, Init $\rangle$, all the states reached from the initial state are the set of reachable states of the system. Therefore by Definitions 9 and 10, any invariant of $B$ that is satisfied by the initial state is also invariant of the system $S$.

Proposition 1 Given a system $S=\langle B$, Init $\rangle$ where $B$ is a component and Init is the initial state, then any invariant $I$ of $B$ is also invariant of $S$, denoted by inv $(S, I)$, if the initial state Init satisfies I.

Proof The proposition is proven from the facts that Init satisfies $I$ and because $I$ is an invariant of $B$, according to Definition 10 all the states reached from Init satisfies $I$.

We extensively use the following well-known results about invariants.

Proposition 2 Let $I_{1}, I_{2}$ be two invariants of a component B. Then $I_{1} \wedge I_{2}, I_{1} \vee I_{2}$ are invariants of $B$.

Proof If $l$ is a state of $B$ such that $\left(I_{1} \wedge I_{2}\right)(l)$, we have $I_{1}(l)$ and $I_{2}(l)$. For any successor $l^{\prime}$ of $l$, we have $I_{1}\left(l^{\prime}\right)$ and $I_{2}\left(l^{\prime}\right)$ because $I_{1}$ and $I_{2}$ are invariants of $B$, therefore $\left(I_{1} \wedge I_{2}\right)\left(l^{\prime}\right)$. Similarly for $I_{1} \vee I_{2}$.

Similarly if $I_{1}$ and $I_{2}$ are invariants of a system $S$, then $I_{1} \wedge I_{2}$ and $I_{1} \vee I_{2}$ are invariants of $S$.

The proof that a given predicate $I$ is an invariant of a given component $B$ can be done by finding a stronger invariant, i.e, a predicate that is an invariant of the component $B$ and implies the predicate $I$. We have the following proposition:

Proposition 3 Given a predicate I of a component B. If there exists an invariant $I^{\prime}$ of $B$ such that $I^{\prime} \Rightarrow I$ then $I$ is also an invariant of the component $B$.

### 1.4.2 Local Deadlock-freedom

In the rest of the thesis, we consider control locations of atomic components as boolean variables.

Definition 11 (Deadlock-free States) Given a component $B=\left(L, P, \mathcal{T}, X,\left\{g_{\tau}\right\}_{\tau \in \mathcal{T}}\right.$, $\left\{f_{\tau}\right\}_{\tau \in \mathcal{T}}$ ), we present by DFS the state predicate characterizing deadlock-free states:

$$
D F S=\bigvee_{l \in L} \bigvee_{\tau \in l^{\bullet}} \text { en }(\tau) \text { where en }(\tau)=l \wedge g_{\tau}
$$

The predicate $e n(\tau)$ of a transition $\tau=\left(l, p, g_{\tau}, f_{\tau}, l^{\prime}\right)$ characterizes a set of states from which the transition $\tau$ is enabled, i.e the component is at its source location $l$ and its guard $g_{\tau}$ is true. The following lemma gives a useful characterization of DFS:

Lemma 1 Given a component $B=\left(L, P, \mathcal{T}, X,\left\{g_{\tau}\right\}_{\tau \in \mathcal{T}},\left\{f_{\tau}\right\}_{\tau \in \mathcal{T}}\right)$,

$$
D F S=\bigwedge_{l \in L}\left(l \Rightarrow \bigvee_{\tau \in l} g_{\tau}\right)=\bigvee_{l \in L}\left(l \wedge \bigvee_{\tau \in l \cdot} g_{\tau}\right)
$$

Proof The proof is based on the fact that $\bigvee_{l \in L} l$ and $\neg\left(l \wedge l^{\prime}\right)$ for $l \neq l^{\prime}$.
The predicate $D F S$ characterizes the deadlock-freedom property of a component. A component is deadlock-free if whenever it reaches a location, it always can go out by at least one of its outgoing transitions, i.e the guard of at least one outgoing transition is true.

### 1.4.3 Global Deadlocks

A system has global deadlocks if at some global state, there is no interaction that can be executed. The global deadlocks therefore depend on the enabledness of the interactions. Hence we first define the enabledness condition of an interaction.

Definition 12 (Enabledness) Given a component $B=\gamma\left(B_{1}, \ldots, B_{n}\right)$ we define, for each interaction $a \in \gamma$, an enabledness predicate under which the interaction $a$ is feasible as follows:

$$
\operatorname{en}(a)=\bigwedge_{p \in a} e n(p) \text { where en }(p)=\bigvee_{\operatorname{port}(\tau)=p} e n(\tau)
$$

$\operatorname{port}(\tau)$ for a transition $\tau$ is the port labeling that transition.
That is, en (a) characterizes all the states from which interaction $a$ can be executed. The interaction $a$ can be executed if all its ports are ready for synchronizing and a port is ready if at least one of its transitions is enabled.

We now define a predicate called DIS that characterizes a set of global deadlocks of a system.


Figure 1.7: The BIP Tool-Chain

Definition 13 (Deadlock States) We define the predicate DIS characterizing the set of the states of $\gamma\left(B_{1}, \ldots, B_{n}\right)$ from which all interactions are disabled:

$$
D I S=\bigwedge_{a \in \gamma} \neg \operatorname{en}(a)
$$

Example 7 For the Temperature Control System (see Figure 1.4), we have:
$D I S=\left(\neg\left(l_{5} \wedge \theta<1000\right)\right) \wedge\left(\neg\left(l_{6} \wedge \theta=100\right) \vee \neg l_{2}\right)$
$\wedge\left(\neg\left(l_{6} \wedge \theta>100\right)\right) \wedge\left(\neg\left(l_{5} \wedge \theta=1000\right) \vee \neg\left(l_{3} \wedge t_{2} \geq 3600\right)\right)$
$\wedge\left(\neg\left(l_{5} \wedge \theta=1000\right) \vee \neg\left(l_{1} \wedge t_{1} \geq 3600\right)\right) \wedge\left(\neg\left(l_{6} \wedge \theta=100\right) \vee \neg l_{4}\right)$
The system $\left\langle\gamma\left(B_{1}, \ldots, B_{n}\right)\right.$, Init $\rangle$ is deadlock-free if the predicate $\neg D I S$ is an invariant of the system.

### 1.5 BIP Tool-Chain

The BIP tool-chain provides a set of tools for the modeling, the execution, the verification and the static transformation of BIP models.

The overview of the BIP tool-chain is shown in Figure 1.7. It includes the following tools:

- An editor, for describing textually a system in BIP language.
- A compiler, for generating a BIP model from BIP description source.
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- A code generator, for generating, from a model, $\mathrm{C}++$ code executable on the BIP engine. The code-generator can also produce THINK specification [PPRS06], from which the Think tool-chain can generate code to be executed over a choice of target platforms.
- D-Finder, of which the method and the implementation are presented in this thesis, is a compositional verification tool for component-based systems described in BIP language [BBNS09].
- BIP2BIP transformations, allow useful transformations which generate an efficient monolithic component from a composite component [BJS09].
- An exporter to connect with external tools such as IF toolbox or analysis tools.
- A set of translators from other languages (Lustre, Matlab/Simulink, ect.) to BIP. For example, an $A A D L$-to-BIP translation from Architecture Analysis \& Design Language (AADL) into BIP [CRBS08], allows simulation of systems specified in AADL and application to these systems of formal verification techniques developed for BIP, e.g. deadlock detection.

The editor, compiler and code generator form the front-end of the tool-chain. The backend provides a platform for analyzing and executing the $\mathrm{C}++$ application code which is generated by front-end. The back-end includes an engine and the associated software infrastructure. The engine is a controller which selects and executes interactions between the components. First it considers the states of the components and the interaction model to find all the enabled interactions. Then it applies the priority rules to eliminate lower priority interactions, then chooses one amongst the maximal enabled for execution.

### 1.6 Summary

Component-based approach is aimed to deal with the complexity of systems. It is based on the idea of building a complex system by assembling basic components (blocks). It provides important characteristics for system construction such as reuse, incrementality, compositionality, etc. It allows not only the reuse of components but also the reuse of known properties of constituent components.

We have presented BIP, a component-based framework for modeling heterogeneous systems. The BIP component model is the superposition of three layers: the lower layer describes the behavior of a component as a transition system; the intermediate layer consists of the interactions between transitions of the layer underneath; the upper layer describes the priorities characterizing a set of scheduling policies for interactions. Such a layering offers a clear separation between components' behaviors and the structure of the system (interactions and priorities).

BIP modeling framework allows dealing with complexity of systems by providing incremental composition of heterogeneous components. It also considers correctness-byconstruction for a class of essential properties such as deadlock-freedom [GS05].

We have also presented several important properties of BIP components such as invariants, deadlock-freedom. We have defined predicates characterizing deadlock-freedom of atomic components as well as of systems.

The BIP tool-chain has been developed providing automated support for component integration and generation of glue code meeting given requirements. Efficient model transformations, verification methods have also been studied and implemented in the BIP toolchain.

We are now going to present our compositional verification method for component-based systems. We will also show applications of our method for verifying systems described in the BIP language.

## Part II

## Verification Method

## Compositional Verification

## Chapter Abstract

We present a compositional approach for verifying safety properties for componentbased systems. The method is based on the computation of an over approximation of the reachable state set on which the properties are checked. We use two kinds of invariants: component invariant and interaction invariant. The former expresses the local constraints on atomic components and the latter characterizes the global constraints induced by strong synchronizations between components. We first present our compositional verification rule, then we provide efficient methods for computing those invariants. Since the computation of interaction invariants is done on systems without data, we present an abstraction technique which allows applying our method for systems with data. Finally, we provide an application of our method for checking deadlock-freedom of component-based systems described in BIP.

## Résumé du Chapitre

Nous présentons une méthode compositionnelle pour vérifier les propriétés de sûreté de systèmes à base de composants. La méthode est basée sur le calcul d'une sur-approximation de l'ensemble d'états atteignables. Nous utilisons deux types d'invariants: l'invariant de composant et l'invariant d'interaction. Le premier exprime les contraintes locales de composants atomiques et le deuxième caractérise les contraintes globales induites par les synchronisations fortes entre les composants. Nous présentons d'abord notre règle de vérification compositionnelle, puis nous fournissons des méthodes efficaces pour le calcul de ces invariants. Puisque le calcul d'invariants d'interaction se fait sur des systèmes sans données, nous présentons une technique d'abstraction qui permet d'appliquer notre méthode aux systèmes avec données. Enfin, nous fournissons une application de notre méthode à la vérification de l'absence de blocage des systèmes à base de composants décrits en BIP.

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Formal verification based on Model-Checking nowadays suffers from the state space explosion problem because of the growing size and complexity of systems. Compositional verification approach is used for alleviating this problem. The idea is to apply "divide-andconquer" techniques to infer global properties of a system from properties of its subsystems. Instead of verifying globally the entire system, compositional approach first decomposes it into small subsystems and verifies each of them individually. The size of a subsystem is often quite smaller compared to the size of the whole system, hence there is less risk of explosion of state space. Then, properties of the global system are inferred from the verified properties of its subsystems.

In this chapter, we present a compositional method for the verification of safety properties for component-based systems described in a subset of the BIP language encompassing multi-party interactions. The BIP framework allows to define rich interaction models by using hierarchical interactions extended with data transfer as presented in [Bas08]. However in this chapter, we restrict to pure synchronizations, i.e. synchronizations without data transfer. The absence of hierarchy is not a real limitation, as long as hierarchical interaction models can be statically transformed into equivalent flat interaction models with a potential increased number of interactions [BJS09]. For the systems with data transfer between components, we will present a method to deal with them in Chapter 4.

The organization of this chapter is as follows: Section 1 presents the compositional verification rule together with methods for computing invariants. We then present in Section 2 an abstraction technique that we use to deal with systems with data. Section 3 shows the procedure of the verification method for checking safety properties. In Section 4 we provide an application of the method for checking deadlock-freedom of component-based systems described in BIP. And we finish this chapter by giving conclusions in Summary. We use the Temperature Control System presented in the previous chapter as running example through the chapter for illustrating invariant computation, abstraction and deadlock-freedom checking.

### 2.1 Compositional Verification Method

### 2.1.1 Compositional Verification Rule

We propose a compositional method for verifying safety properties. The idea of the method is based on the inference rules represented in Equations 2.1 and 2.2. The rule
in Equation 2.1 says that if the initial state Init of a system $S$ satisfies a predicate $\Phi$ and $\Phi$ is preserved by every transition $\tau$ of the system, that is every reachable state of $S$ satisfies $\Phi$, then $\Phi$ is an invariant of $S$. Such invariant $\Phi$ is called inductive invariant. Unfortunately, most of invariants are not inductive, that is they are not preserved by every transition. Therefore, an extension rule is proposed in Equation 2.2 which allows proving the invariance property of $\Phi$ by finding an auxiliary predicate $\Phi^{a u x}$ such that: (1) $\Phi^{a u x}$ is stronger than $\Phi$; (2) $\Phi^{a u x}$ is satisfied by the initial state; and (3) $\Phi^{a u x}$ is preserved by every transition of the system.

$$
\begin{gather*}
\text { Init } \models \Phi \\
\{\Phi\} \tau\{\Phi\} \quad \forall \tau \in S  \tag{2.1}\\
S \models \square \Phi
\end{gather*}
$$

$$
\begin{gather*}
\text { Init } \models \Phi^{\text {aux }} \\
\left\{\Phi^{\text {aux }}\right\} \tau\left\{\Phi^{\text {aux }}\right\} \quad \forall \tau \in S \\
\Phi^{\text {aux }} \Rightarrow \Phi \\
S \models \square \Phi \tag{2.2}
\end{gather*}
$$

The best solution as an auxiliary predicate $\Phi^{a u x}$ is the set of reachable states $\operatorname{Reach}(S)$. However, the computation of the reachable state set causes the well-known state space explosion problem. Hence another solution, on which our method is based, is to take an over-approximation $\operatorname{Reach}_{A p p}(S)$ of the reachable state set. If $\operatorname{Reach}_{A p p}(S)$ implies the predicate $\Phi$, then the system $S$ satisfies $\Phi$.

The goal of the method is therefore to compute a strong-enough over-approximation of the set of reachable states to be able to prove invariance properties. Our method for computing such approximation focuses on two aspects of systems: one is the local behavior of atomic components; the other is the global constraints between atomic components based on the interactions between them. Since over-approximation of the reachable state set can be characterized by invariants, we exploit two kinds of invariants:

- Component invariants $\Phi_{i}$ of $B_{i}$ which are over-approximations of components' reachability sets. They are computed by forward propagation techniques.
- Interaction invariants $\Psi$ which are global constraints on the states of atomic components involved in interactions. Interactions force the synchronized atomic components to move together from a set of states to another set of states, hence there are explicit constraints on the states of these atomic components. Interaction invariants captures these constraints by statically exploiting the structure of the interaction set. They are computed from Boolean Behavioral Constraints which are a set of implications obtained from interactions and local behavior.
The main rule of our approach is as follows:

$$
\begin{equation*}
\frac{\left\{B_{i}<\Phi_{i}>\right\}_{i}, \Psi \in I I\left(\|_{\gamma}\left\{B_{i}\right\}_{i},\left\{\Phi_{i}\right\}_{i}\right),\left(\bigwedge_{i} \Phi_{i}\right) \wedge \Psi \Rightarrow \Phi}{\|_{\gamma}\left\{B_{i}\right\}_{i}<\Phi>} \tag{2.3}
\end{equation*}
$$

The rule 2.3 allows to prove invariance of a predicate $\Phi$ for a system obtained by using a $n$-ary composition operation parameterized by a set of interactions $\gamma$ on a set of components $\left\{B_{i}\right\}_{i}$. It uses global invariants which are the conjunction of component invariants $\left\{\Phi_{i}\right\}_{i}$ and interaction invariants $\Psi$. The verification of invariance-property $\Phi$ is then done by checking tautology $\left(\bigvee_{i} \Phi_{i}\right) \wedge \Psi \Rightarrow \Phi$ or equivalently the unsatisfiability of $\left(\bigvee_{i} \Phi_{i}\right) \wedge \Psi \wedge(\neg \Phi)$.

Methods for computing component invariants and interaction invariants will be presented in the next sub-sections.

### 2.1. COMPOSITIONAL VERIFICATION METHOD

### 2.1.2 Component Invariants

Component invariants are over-approximation of the set of reachable states of components. If an atomic component B is finite state with the initial state Init, then we can take $\Phi=\operatorname{Reach}(\langle B$, Init $\rangle)$, the set of reachable states of $B$ or any upper approximation of $\operatorname{Reach}(\langle B$, Init $\rangle)$. If the components are infinite state, Reach $(\langle B$, Init $\rangle)$ can be approximated as shown in [LBBO01]. In this section, we present a lightweight method for the computation of sequences of increasingly stronger inductive invariants for atomic components. Component invariants are computed by using the post predicate transformer which defines the propagation of a predicate through a transition or a transition system. The post predicate transformer allows computing state successors of atomic components. Its formal definition is as follows :

Definition 14 (Post Predicate Transformer w.r.t Transition) Given a component $B=\left(L, P, \mathcal{T}, X,\left\{g_{\tau}\right\}_{\tau \in \mathcal{T}},\left\{f_{\tau}\right\}_{\tau \in \mathcal{T}}\right)$ and a predicate $\varphi$ on the set of variables $X$. We define a post predicate transformer of $\varphi$ w.r.t a transition $\tau=\left(l, p, g_{\tau}, f_{\tau}, l^{\prime}\right) \in \mathcal{T}$, which is a propagation of $\varphi$ by the transition $\tau$, as follows:

$$
\operatorname{post}_{\tau}(\varphi)(X)=\exists X^{\prime} . \varphi\left(X^{\prime}\right) \wedge g_{\tau}\left(X^{\prime}\right) \wedge f_{\tau}\left(X^{\prime}, X\right)
$$

Transition $\tau$ can be executed only if its guard $g_{\tau}\left(X^{\prime}\right)$ ( $X^{\prime}$ is the previous valuation of $X$ at the source location $l$ ) is true. The run of the transition executes function $f_{\tau}\left(X^{\prime}, X\right)$ which updates the value of the predicate $\varphi\left(X^{\prime}\right)$ at the source location $l$ and a new predicate $\operatorname{post}_{\tau}(\varphi)(X)$ is produced at the destination location $l^{\prime}$.

We define in a similar way, the pre predicate transformer for a transition $\tau, \operatorname{pre}_{\tau}(\varphi)(X)=$ $\exists X^{\prime} . g_{\tau}(X) \wedge f_{\tau}\left(X, X^{\prime}\right) \wedge \varphi\left(X^{\prime}\right)$.


$$
\varphi(x)=(x \geq 0) \quad \varphi^{\prime}(x)=\operatorname{post}_{\tau}(\varphi)=(x \geq 2)
$$

Figure 2.1: An example of post predicate

Example 8 Figure 2.1 illustrates a propagation of a predicate $\varphi=(x \geq 0)$ by a transition $\tau$ from $l$ to $l^{\prime}$ with its guard $g_{\tau}=(x \geq 1)$ and its update function $f_{\tau}=(x:=x+1)$. The post condition of $\varphi$ with respect to the transition $\tau$ is $\varphi^{\prime}(x)=\operatorname{post}_{\tau}(\varphi)(x)=\exists x^{\prime} .\left(x^{\prime} \geq 1\right) \wedge(x=$ $\left.x^{\prime}+1\right) \wedge\left(x^{\prime} \geq 0\right)=(x \geq 2) . \varphi^{\prime}(x)$ is the propagation of $\varphi(x)$ through the transition $\tau$.

If $\tau$ is a loop transition, i.e it is of the form $\tau=\left(l, p, g_{\tau}, f_{\tau}, l\right)$, we can use post* predicate transformer which defines the iterative propagation by the loop transition according to the number of iterations. The condition for using post* is that we can find a transitive closure $\mathcal{F}_{\tau}\left(n, X^{\prime}, X\right)$ [CJ98, BIL09] of the function $f_{\tau}\left(X^{\prime}, X\right)$ where $n$ is the number of iterations. For example, if the function $f_{\tau}$ is of the form $x=x^{\prime}+a$ where $a$ is a constant, then


Figure 2.2: Temperature Control System
its transitive closure is $\mathcal{F}_{\tau}\left(n, x^{\prime}, x\right)=\left(x=x^{\prime}+a * n\right)$. It means that, an iteration step increases the value of $x$ by $a$, hence after $n$ times of iterations, the value of $x$ is increased by $a * n$. The predicate transformer of a predicate $\varphi(x)$ over $\tau$ is $\operatorname{post}_{\tau}^{*}(x)=\exists n \exists x^{\prime} .(n \geq$ 1) $\wedge \varphi\left(x^{\prime}\right) \wedge g_{\tau}\left(x^{\prime}+a *(n-1)\right) \wedge\left(x=x^{\prime}+a * n\right)$.

Example 9 Consider the component Controller in Figure 2.2 which has a loop transition $\tau=\left(l_{5}\right.$, tick, $\left.\theta<1000, \theta=\theta+1, l_{5}\right)$. The predicate to be propagated by the loop transition is $\varphi=(\theta=100)$ which is obtained from the post predicate transformer of heat incoming transition. The loop transition $\tau$ can occur if the guard $\theta<1000$ is true and it increases the value of the variable $\theta$ by 1 . The value of $\theta$ after $n$ times of iterations is $(100+1 * n)$. The post $_{\tau}^{*}$ of $\varphi$ w.r.t the transition $\tau$ is $\operatorname{post}_{\tau}^{*}(\varphi)(\theta)=\exists n \exists \theta^{\prime} .(n \geq 1) \wedge\left(\theta^{\prime}=100\right) \wedge\left(\theta^{\prime}+1 *(n-1)<\right.$ 1000) $\wedge\left(\theta=\theta^{\prime}+1 * n\right)=(101 \leq \theta \leq 1000)$.

Consider a predicate $\varphi_{l}$ at each control location $l$ of a component $B=(L, P, \mathcal{T}, X$, $\left.\left\{g_{\tau}\right\}_{\tau \in \mathcal{T}},\left\{f_{\tau}\right\}_{\tau \in \mathcal{T}}\right)$, the global predicate is $\bigvee_{l \in L}\left(l \wedge \varphi_{l}\right)$. The transformation of the global predicate is done by the propagation on all the transitions of the component.

Definition 15 (Post Predicate Transformer w.r.t Transition System) Given a component $B=\left(L, P, \mathcal{T}, X,\left\{g_{\tau}\right\}_{\tau \in \mathcal{T}},\left\{f_{\tau}\right\}_{\tau \in \mathcal{T}}\right)$ and a predicate $\Phi=\bigvee_{l \in L}\left(l \wedge \varphi_{l}\right)$ where $\varphi_{l}$ is a predicate at control location $l$, we define the post predicate transformer of $\Phi$ w.r.t to the transition system of $B$ as follows:

$$
\operatorname{post}(\Phi)=\bigvee_{l \in L}\left(\bigvee_{\tau=\left(l, p, l^{\prime}\right)}\left(l^{\prime} \wedge \operatorname{post}_{\tau}\left(\varphi_{l}\right)\right)\right)
$$

Equivalently, we have that $\operatorname{post}(\Phi)=\bigvee_{l \in L} l \wedge\left(\bigvee_{\tau=\left(l^{\prime}, p, l\right)} \operatorname{post}_{\tau}\left(\varphi_{l^{\prime}}\right)\right)$. This allows computing $\operatorname{post}(\Phi)$ by forward propagation of the assertions associated with control locations in $\Phi$.

### 2.1. COMPOSITIONAL VERIFICATION METHOD

For a component $B=\left(L, P, \mathcal{T}, X,\left\{g_{\tau}\right\}_{\tau \in \mathcal{T}},\left\{f_{\tau}\right\}_{\tau \in \mathcal{T}}\right), \varphi_{l}=$ true is the weakest invariant at each control location of $S$, and $\Phi=\bigvee_{l \in L} l \wedge t r u e ~ i s ~ t h e ~ w e a k e s t ~ c o m p o n e n t ~ i n v a r i a n t . ~$ The iterative forward propagation of this predicate by transition system provides a stronger invariant. The iteration terminates when a fix point is reached. The following proposition provides such means to compute increasingly stronger invariants of a component.

Proposition 4 Given a system $\mathcal{S}=\langle B$, Init $\rangle$ where Init is the initial condition of the component $B$, the following iteration defines a sequence of increasingly stronger inductive invariants:

$$
\Phi_{0}=\operatorname{true} \quad \Phi_{i+1}=\text { Init } \vee \operatorname{post}\left(\Phi_{i}\right)
$$

Proof By induction. $\Phi_{0}$ is an inductive invariant. If $\Phi_{i}$ is an inductive invariant then Init $\vee$ post $_{\tau}\left(\Phi_{i}\right) \Rightarrow \Phi_{i}$. As post is monotonic and distributes over disjunction, post $t_{\tau}\left(\Phi_{i+1}\right)=$ $\operatorname{post}\left(\operatorname{Init} \vee \operatorname{post}\left(\Phi_{i}\right)\right) \Rightarrow \operatorname{post}\left(\Phi_{i}\right) \Rightarrow \Phi_{i+1}$. Moreover, Init $\Rightarrow \Phi_{i+1}$. So $\Phi_{i+1}$ is an inductive invariant.

We use different strategies for producing such invariants. We usually iterate until we find good enough invariants. The good enough invariants mean that they are able to prove some safety properties. This will be explained in Section 2.3.

Example 10 For the Temperature Control System of figure 2.2, the predicates $\Phi_{1}=\left(l_{1} \wedge\right.$ $\left.t_{1} \geq 0\right) \vee\left(l_{2} \wedge t_{1} \geq 3600\right)$ and $\Phi_{2}=\left(l_{3} \wedge t_{2} \geq 0\right) \vee\left(l_{4} \wedge t_{2} \geq 3600\right)$ are respectively inductive invariants of the $\operatorname{Rod}_{1}$ and $R_{2} d_{2}$ components given the initial conditions Init $_{1}=$ $l_{1} \wedge t_{1}=3600$ and Init $_{2}=l_{3} \wedge t_{2}=3600$. Also, the predicate $\Phi_{3}=\left(l_{5} \wedge 100 \leq \theta \leq\right.$ 1000) $\vee\left(l_{6} \wedge 100 \leq \theta \leq 1000\right)$ is a non-inductive invariant of the Controller component, given the initial condition Init $_{3}=l_{5} \wedge \theta=100$. An auxiliary inductive invariant that implies $\Phi_{3}$ is $\Phi_{3}^{a u x}=\left(l_{5} \wedge 100 \leq \theta \leq 1000\right) \vee\left(l_{6} \wedge 100 \leq \theta \leq 1000\right) \wedge(\theta$ is even $)$.

A key issue is efficient computation of such invariants as the precise computation of post requires quantifier elimination. An alternative to quantifier elimination is to compute over-approximations of post based on syntactic analysis of the predicates. In this case, the obtained invariants may not be inductive. We present below lightweight techniques for computing component invariants by avoiding quantifier elimination process.

## Lightweight Computation of Component Invariants

We provide a brief description of a syntactic technique used for approximating post predicate transformer for a transition $\tau$. The idea is that the post can be approximated by predicates which are not affected by the update function of the transition, therefore the methods focuses on finding such predicates. Theses techniques and also some other techniques for generating post predicate transformer and component invariants are proposed and well presented in [BL99].

Given a component $B=\left(L, P, \mathcal{T}, X,\left\{g_{\tau}\right\}_{\tau \in \mathcal{T}},\left\{f_{\tau}\right\}_{\tau \in \mathcal{T}}\right)$, consider a transition $\tau=$ $\left(l, p, g_{\tau}, f_{\tau}, l^{\prime}\right) \in \mathcal{T}$ where its guard $g_{\tau}$ is a predicate of the form $g_{\tau}(Y), f_{\tau}$ is of the form
$Z^{\prime}=e_{\tau}(U) ; Y, Z, U$ are subsets of the set of variables $X$. If $Z \cap U=\emptyset$ which means that variables in the set $U$ are not affected by the associate update function $e_{\tau}$, then the predicate $Z=e_{\tau}(U)$ is preserved by the transition $\tau$. Similarly, if $Z \cap Y=\emptyset$, the predicate $g_{\tau}(Y)$ is also preserved by the transition. Moreover, for an arbitrary predicate $\varphi$ at $l^{\prime}$, we find a decomposition $\varphi=\varphi_{1}\left(Y_{1}\right) \wedge \varphi_{2}\left(Y_{2}\right)$ such that $Y_{2} \cap Z=\emptyset$ i.e. $\varphi_{2}\left(Y_{2}\right)$ is not affected by the update function $f_{\tau}$, then the predicate $\varphi_{2}\left(Y_{2}\right)$ still holds after the execution of the transition.

To formulate the general case, given a transition $\tau$ above, we denote by func $(\tau)$ the predicate $Z=e_{\tau}(U)$ and by $\operatorname{guard}(\tau)$, the guard $g_{\tau}(Y)$. For a transition $\tau=\left(l^{\prime}, p, g(Y), Z^{\prime}=\right.$ $e(U), l)$ we have $Z \cap(Y \cup U)=\emptyset$ and for an arbitrary predicate $\varphi$ at $l^{\prime}$, we find a decomposition $\varphi=\varphi_{1}\left(Y_{1}\right) \wedge \varphi_{2}\left(Y_{2}\right)$ such that $Y_{2} \cap Z=\emptyset$. Then, the post predicate transformer can approximated by $\operatorname{post}_{\tau}^{a}=\varphi_{2}\left(Y_{2}\right) \wedge \operatorname{guard}(\tau) \wedge f u n c(\tau)$.

The required condition for $\Phi_{l}$ to be an invariant is $Z \cap(Y \cup U)=\emptyset$. If this condition is not satisfied, by considering separately the condition for $Z \cap Y$ and $Z \cap U$, we can get post ${ }_{\tau}^{a}$ as follows:

$$
\begin{align*}
& \operatorname{post}_{\tau}^{a}(\varphi)=\varphi_{2}\left(Y_{2}\right) \wedge \begin{cases}\operatorname{guard}(\tau) \wedge \operatorname{func}(\tau) & \text { if } \mathrm{Z} \cap(\mathrm{Y} \cup \mathrm{U})=\emptyset \\
\operatorname{func}(\tau) & \text { if } \mathrm{Z} \cap \mathrm{U}=\emptyset \text { and } \mathrm{Z} \cap \mathrm{Y} \neq \emptyset \\
\operatorname{guard}(\tau) & \text { if } \mathrm{Z} \cap \mathrm{Y}=\emptyset \text { and } \mathrm{Z} \cap \mathrm{U} \neq \emptyset \\
\operatorname{true} & \text { otherwise }\end{cases}  \tag{2.4}\\
& \text { (l) } \begin{array}{l}
y \geq 1 \\
z:=y+2
\end{array} l^{\prime}  \tag{a}\\
& \varphi=(y \geq 0) \wedge(z \geq 0) \quad \operatorname{post}_{\tau}^{a}(\varphi)=(y \geq 1) \wedge(z=y+2) \\
& \text { (l) } \underset{y:=z+2}{y \geq 1} \xrightarrow{l}  \tag{b}\\
& \varphi=(y \geq 0) \wedge(z \geq 0) \quad \operatorname{post}_{\tau}^{a}(\varphi)=(z \geq 0) \wedge(y=z+2) \\
& \text { (l) } \underset{z:=z+2}{y \geq 1} \longrightarrow  \tag{c}\\
& \varphi=(y \geq 0) \wedge(z \geq 0) \quad \operatorname{post}_{\tau}^{a}(\varphi)=(y \geq 1)
\end{align*}
$$

Figure 2.3: Examples of post ${ }_{\tau}^{a}$

Example 11 Figure 2.3 illustrates examples for the cases defined in Equation 2.4. In all the transitions, the predicate at the source location is $\varphi(y, z)=y \geq 0 \wedge x \geq 0$ which can be decomposed into $\varphi(y)=y \geq 0$ and $\varphi(z)=z \geq 0$.

- In example (a), the variable $y$ of the guard is not affected by $f_{\tau}$, hence post $t_{\tau}^{a}(\varphi)=$ $\varphi(y) \wedge \operatorname{func}(\tau) \wedge \operatorname{guard}(\tau)=(z=y+2) \wedge(y \geq 1)$.
- In example (b), the variable $y$ of the guard is changed through the transition but the variable $z$ on the right side of $f$ is unchanged, hence $\operatorname{post}_{\tau}^{a}(\varphi)=\varphi(z) \wedge$ func $(\tau)=$ $(z \geq 0) \wedge(y=z+2)$.
- In the third example (c), y of the guard stays unchanged through the transition but the variable $z$ appears on both sides of the function, therefore post $\tau_{\tau}^{a}(\varphi)=\varphi(y) \wedge$ $\operatorname{guard}(\tau)=(y \geq 1)$.

In the case $Z \cap Y \neq \emptyset$ or $Z \cap U \neq \emptyset$, we still can apply the above rules by decomposing $Y$ and $U$ into two parts: one part is disjoint with $Z$ and the other part is not disjoint with $Z$.

Consider a transition $\tau=\left(l, p, g_{\tau}, f_{\tau}, l^{\prime}\right)$ of a component $B$. Assume that its guard is of the form $g_{\tau}(Y)$ and the associated update function $f_{\tau}$ is of the form $Z_{1}^{\prime}=e_{\tau}(U) \wedge Z_{2}^{\prime}=Z_{2}$ where $Y, Z_{1}, Z_{2}, U \subseteq X . Z_{1}^{\prime}$ and $Z_{2}^{\prime}$ are respectively next valuations of $Z_{1}$ and $Z_{2}$. For an arbitrary predicate $\varphi$, we find a decomposition $\varphi=\varphi_{1}\left(Y_{1}\right) \wedge \varphi_{2}\left(Y_{2}\right)$ such that $Y_{2} \cap Z_{1}=\emptyset$, the post predicate transformer for the transition $\tau$ can be approximated as follows:

$$
\operatorname{post}_{\tau}^{a}(\varphi)=\varphi_{2}\left(Y_{2}\right) \wedge\left\{\begin{array}{cl}
g_{\tau}(Y) & \text { if } Z_{1} \cap Y=\emptyset  \tag{2.5}\\
\text { true } & \text { otherwise }
\end{array}\right\} \wedge\left\{\begin{array}{cl}
Z_{1}=e_{\tau}(U) & \text { if } Z_{1} \cap U=\emptyset \\
\text { true } & \text { otherwise }
\end{array}\right\}
$$

$$
\varphi(x)=(x \geq 0) \wedge(y \geq 2) \quad \operatorname{post}_{\tau}^{a}(\varphi)=(x \geq 2) \wedge(y=z+2)
$$

Figure 2.4: Examples of post ${ }_{\tau}^{a}$

Example 12 Figure 2.4 is an example where we can apply Equation 2.5. The transition $\tau$ has a guard $g_{\tau}=(x \geq 2)$, a function $f_{\tau}=(y:=z+2) ;(t:=t+2)$. Consider a predicate $\varphi=(x \geq 0) \wedge(y \geq 2)$ which can be split into two parts: $\varphi_{1}=(x \geq 0)$ where $x$ is unchanged through the transition and $\varphi_{2}=(y \geq 2)$ where $y$ is affected by function $f_{\tau}$. Similarly, the update function can be decomposed into two parts: $f_{1}=(y:=z+2)$ where variable on the right side is different from the left and $f_{2}=(t:=t+2)$ where the variable $t$ is on both the right and the left sides. Finally we have $\operatorname{post}_{\tau}^{a}(\varphi)=\varphi_{1} \wedge \operatorname{guard}(\tau) \wedge \operatorname{func}_{1}(\tau)=(x \geq$ 2) $\wedge(y=z+2)$.

The following proposition says that, for a transition $\tau$ and a predicate $\varphi, \operatorname{post}_{\tau}^{a}(\varphi)$ is an over approximation of the post predicate transformer $\operatorname{post}_{\tau}(\varphi)$.

Proposition 5 If $\tau$ and $\varphi$ are respectively a transition and a state predicate as above, then $\operatorname{post}_{\tau}(\varphi) \Rightarrow \operatorname{post}_{\tau}^{a}(\varphi)$.


Figure 2.5: Two components with interactions (a) and its composition (b)

Proof We can over-approximate successively $\operatorname{post}_{\tau}(\varphi)$ as follows:

$$
\begin{aligned}
\operatorname{post}_{\tau}(\varphi)\left(X^{\prime}\right) & =\exists X \cdot\left(\varphi(X) \wedge g_{\tau}(X) \wedge f_{\tau}\left(X, X^{\prime}\right)\right) \\
& =\exists Z_{1}, Z_{2} \cdot\left(\varphi_{1}\left(Y_{1}\right) \wedge \varphi_{2}\left(Y_{2}\right) \wedge g_{\tau}(Y) \wedge Z_{1}^{\prime}=e_{\tau}(U) \wedge Z_{2}^{\prime}=Z_{2}\right) \\
& \Rightarrow \exists Z_{2} \cdot\left(\varphi_{2}\left(Y_{2}\right) \wedge Z_{2}^{\prime}=Z_{2}\right) \wedge \exists Z_{1}, Z_{2} \cdot\left(g_{\tau}(Y) \wedge Z_{1}^{\prime}=e_{\tau}(U) \wedge Z_{2}^{\prime}=Z_{2}\right) \\
& =\varphi_{2}\left(Y_{2}^{\prime}\right) \bigwedge \exists Z_{1}, Z_{2} \cdot\left(g_{\tau}(Y) \wedge Z_{1}^{\prime}=e_{\tau}(U) \wedge Z_{2}^{\prime}=Z_{2}\right) \\
& \Rightarrow \varphi_{2}\left(Y_{2}^{\prime}\right) \bigwedge\left\{\begin{array}{ll}
g_{\tau}\left(Y^{\prime}\right) & \text { if } Z_{1} \cap Y=\emptyset \\
\text { true } & \text { otherwise }
\end{array}\right\} \bigwedge\left\{\begin{array}{ll}
Z_{1}^{\prime}=e_{\tau}\left(U^{\prime}\right) & \text { if } Z_{1} \cap U=\emptyset \\
\text { true } & \text { otherwise }
\end{array}\right\} \\
& =\operatorname{post}_{\tau}^{a}(\varphi)\left(X^{\prime}\right) .
\end{aligned}
$$

### 2.1.3 Interaction Invariants

For the sake of clarity, we present methods for computing interaction invariants of systems without data. For systems with data, the methods can be applied by using abstraction techniques which will be presented in the next section.

The idea of our compositional verification method, as explained in the method rule, is that we try to compute as precisely as possible over approximations of the set of reachable states. Given a system consisting of $n$ atomic components $B_{1}, \ldots, B_{n}$ synchronized by a set of interactions $\gamma$, an over-approximation of the global reachable states can be obtained by the intersection of the component invariants of these atomic components. For example, figure $2.5(\mathrm{a})$ illustrates two components $B_{1}$ and $B_{2}$ strongly synchronized by a set of two interactions $\gamma=a_{1} a_{2}+b_{1} b_{2}$. By taking Init $=l_{1} \wedge l_{3}$ as initial condition, we have the set of reachable states $\operatorname{Reach}\left(\left\langle\gamma\left(B_{1}, B_{2}\right)\right.\right.$, Init $\left.\rangle\right)=\left(l_{1} \wedge l_{3}\right) \vee\left(l_{2} \wedge l_{4}\right)$ (Figure 2.5(b)). The component invariants of the components $B_{1}$ and $B_{2}$ are respectively $\Phi_{1}=\left(l_{1} \vee l_{2}\right)$ and $\Phi_{2}=$ $\left(l_{3} \vee l_{4}\right)$. The intersection $\Phi_{1} \wedge \Phi_{2}=\left(l_{1} \vee l_{2}\right) \wedge\left(l_{3} \vee l_{4}\right)=\left(l_{1} \wedge l_{3}\right) \vee\left(l_{1} \wedge l_{4}\right) \vee\left(l_{2} \wedge l_{3}\right) \vee\left(l_{2} \wedge l_{4}\right)$ is an over-approximation of the global reachable states. Unfortunately this over-approximation is often quite large and not strong enough to prove invariance properties because it does not take into account global constraints due to strong synchronizations between atomic components. In this section we present a type of invariants called interaction invariant

### 2.1. COMPOSITIONAL VERIFICATION METHOD

which captures these constraints and therefore represents a more precise over-approximation of the reachable states.

Interactions are used to restrict the global behavior of systems in order to meet given requirements. An interaction consists of a set of ports of different components. The actions of the ports of an interaction must occur simultaneously. There are therefore strong constraints on the moves of the synchronized components. Consider the example in Figure $2.5(\mathrm{a})$, at control locations $l_{1}$ and $l_{3}$, interaction $a_{1} a_{2}$ can take place and enforces two components moving together to control locations $l_{2}$ and $l_{4}$. From $l_{2}$ and $l_{4}$, interaction $b_{1} b_{2}$ can occur and similarly enforces two components back to $l_{1}$ and $l_{3}$. Here we have strong constraints between two components: if $B_{1}$ is at control location $l_{2}$ (respectively $l_{1}$ ), $B_{2}$ must be at control location $l_{4}$ (respectively $l_{3}$ ) and vice-versa. Interaction invariants characterize such constraints on the global state space induced by strong synchronizations between atomic components.

Consider a set of atomic components $B=\left(B_{1}, \ldots, B_{n}\right)$, where $B_{i}=\left(L_{i}, P_{i}, \mathcal{T}_{i}\right)$, synchronized by a set of interactions $\gamma$. Intuitively, an interaction invariant of $\gamma(B)$ is a predicate in the disjunctive form $\Psi=\bigvee_{l \in L_{\Psi}} l$ where $L_{\Psi} \subseteq \bigcup_{1}^{n} L_{i}$ such that if any control location of $L_{\Psi}$ is reached, then there is always at least a control location of $L_{\Psi}$ which is reached by the execution of any interaction of $\gamma$. In other words, $L_{\Psi}$ is a set of locations of atomic components such that for any location $l \in L_{\Psi}$, at least one of its successors by the execution of interactions in $\gamma$ must belong to $L_{\Psi}$. For a system $S=\langle\gamma(B)$, Init $\rangle$, $\Psi=\bigvee_{l \in L_{\Psi}}$ is an invariant of $S$ if it is an invariant of $\gamma(B)$ and it is initially true, that is $L_{\Psi}$ has at least an initial location of an atomic component in $B$. Interaction invariants are computed by solving Boolean Behavioral Constraints which characterize a set of successors of every location of atomic components or by Fixed-point computation.

Example 13 Consider the component $\gamma\left(B_{1}, B_{2}\right)$ in Figure 2.5(a) where $\gamma=a_{1} a_{2}+b_{1} b_{2}$, a set of locations $\left\{l_{1}, l_{4}\right\}$ corresponds to an interaction invariant $l_{1} \vee l_{4}$ because $l_{4}$ is reached from $l_{1}$ by the interaction $a_{1} a_{2}$ and $l_{1}$ is reached from $l_{4}$ by the interaction $b_{1} b_{2}$.

There is a similarity between the notion of interaction invariants and the notion of traps in a type of Petri-net called 1-safe Petri-net, i.e. each place of an 1 -safe Petri-net can not have more than one token. An important property of trap is that if a trap initially has a token, it will always have at least a token. The set of places in a Petri-net corresponds to a set of locations in atomic components, initial tokens correspond to initial locations and a place having a token means that its corresponding location is reached.

In this sub-section, we present methods for computing interaction invariants. We first define Forward Interaction Set of a location according to a set of interactions which is used in the methods.

We recall that we use ${ }^{\bullet} \tau$ (respectively $\left.\tau^{\bullet}\right)$ to denote the source and destination locations of $\tau$. Similarly, for a port $p$ we have $p=\left\{{ }^{\bullet} \tau \mid \tau=\left(l, p, l^{\prime}\right)\right\}$ and $p^{\bullet}=\left\{\tau^{\bullet} \mid \tau=\left(l, p, l^{\prime}\right)\right\}$, for an interaction $a$ we have ${ }^{\bullet} a=\left\{{ }^{\bullet} p \mid p \in a\right\}$ and $a^{\bullet}=\left\{p^{\bullet} \mid p \in a\right\}$.

Definition 16 (Forward Interaction Sets) Given a component $\gamma\left(B_{1}, \ldots, B_{n}\right)$ where $B_{i}=\left(L_{i}, P_{i}, \mathcal{T}_{i}\right)$ are transition systems, we define for every location $l \in \bigcup_{i=1}^{n} L_{i}$ its forward
interaction set as follows:

$$
\overrightarrow{l^{\gamma}}=\{\left\{\tau_{i}\right\}_{i \in I} \mid \forall i .[\left(\tau_{i} \in \mathcal{T}_{i}\right) \wedge(\exists i \cdot \overbrace{i}=l) \wedge\left(\left\{\operatorname{port}\left(\tau_{i}\right)\right\}_{i \in I} \in \gamma\right)]\}
$$



Figure 2.6: Forward interaction sets.

That is, $\vec{l}$ consists of sets of transitions involved in some interaction of $\gamma$ in which a transition $\tau_{i}$ issued from $l$ can participate. A transition is involved in an interaction if the port labeling the transition participates in the interaction. For example, in Figure 2.6, the set $\left\{\tau_{1} \ldots \tau_{i} \ldots \tau_{m}\right\}$ belongs to the Forward Interaction Sets $\overrightarrow{l_{1}^{\gamma}}$ (and also belongs to $\left.\overrightarrow{l_{2}^{\gamma}}, \ldots \overrightarrow{l_{i}^{\prime}}, \ldots \overrightarrow{l_{m}^{\prime}}\right)$.

## Method based on Positive Mapping

We propose a method for computing interaction invariants from Boolean Behavioral Constraints (BBCs). We first give definition of BBCs and show that every solution of BBCs corresponds to an invariant. Then we provide a method for obtaining all the interaction invariants from BBCs by using an operation called Positive Mapping.

Boolean Behavioral Constraint ( BBC ) of a location $l$ of an atomic component can be considered as a constraint enforced by a set of interactions $\gamma$ from that location to a global location. It describes a set of successors of $l$ according to $\gamma$, i.e. a set of locations of atomic components which are reached from $l$ by involved interactions in $\gamma$. The Boolean Behavioral Constraints (BBCs) of a system is the conjunction of the BBC of all its locations.

We use $\operatorname{Bool}[L]$ to denote the free algebra generated by the set of locations $L$. We provide the formal definition of Boolean Behavioral Constraints (BBCs) for a connector $\gamma$ on a set of components as follows:

Definition 17 (Boolean Behavioral Constraints (BBCs)) Let $\gamma$ be a connector over a tuple of components $B=\left(B_{1}, \ldots, B_{n}\right)$ where $B_{i}=\left(L_{i}, P_{i}, \mathcal{T}_{i}\right)$ are transition systems. The Boolean Behavioral Constrains for component $\gamma(B)$ with a set of locations $L=\bigcup_{i=1}^{n} L_{i}$, are
defined by a function $||:. \gamma(B) \rightarrow \operatorname{Bool}(L)$ such that:

$$
|\gamma(B)|=\bigwedge_{l \in L}\left(l \Rightarrow \bigwedge_{\left\{\tau_{i}\right\}_{i \in I} \in \overrightarrow{l^{\gamma}}}\left(\bigvee_{l^{\prime} \in\left\{\tau_{i}^{\bullet}\right\}_{i \in I}} l^{\prime}\right)\right)
$$

If $\gamma=\emptyset$, then $|\gamma(B)|=$ true, which means that no interactions between the components of $B$ will be considered. $|\gamma(B)|$ can be written as the disjunction of monomials, i.e. $|\gamma(B)|=$ $\bigvee_{i \in I} m_{i}$, which we call BBC-solutions.

Example 14 In Figure 2.6, consider the interaction $a=p_{1} \ldots p_{i} \ldots p_{m} \in \gamma$ between transitions $l_{i} \xrightarrow{p_{i}} l_{i}^{\prime}$ for $i=1, \ldots, m$, the corresponding $B B C s$ is

$$
|a(B)|=\bigwedge_{i=1}^{m}\left(l_{i} \Rightarrow \bigvee_{j=1}^{m} l_{j}^{\prime}\right)
$$

In Figure 2.5(a) the BBCs for the the set of interactions $\gamma=a_{1} a_{2}+b_{1} b_{2}$ is $|\gamma(B)|=\left(l_{1} \Rightarrow\right.$ $\left.l_{2} \vee l_{4}\right) \wedge\left(l_{3} \Rightarrow l_{2} \vee l_{4}\right) \wedge\left(l_{2} \Rightarrow l_{1} \vee l_{3}\right) \wedge\left(l_{4} \Rightarrow l_{1} \vee l_{3}\right)$.

The following theorem provides means for computing interaction invariants from BBCs.

Theorem 1 Let $B=\left(B_{1}, \cdots, B_{n}\right)$ be a set of components with $B_{i}=\left(L_{i}, P_{i}, \mathcal{T}_{i}\right)$ and $L=\bigcup_{i=1}^{n} L_{i}, \gamma$ be a connector over $B$, and $v: L \rightarrow\{$ true, false $\}$ be a boolean valuation different from false. If $v$ is a solution of $|\gamma(B)|$, i.e. $|\gamma(B)|(v)=$ true, then $\bigvee_{v(l)=\text { true }} l$ is an invariant of $\gamma(B)$.

Proof According to Definition 17, the constraints are the conjunction of all the implications for interactions of $\gamma$. Consider a valuation $v$ such that $|\gamma(B)|(v)=$ true. In order to prove that $\bigvee_{v(l)=t r u e} l$ is an invariant, assume that for some global state $l=\left(l_{1}, \cdots, l_{n}\right)$, there exists $l_{i}$ such that $v\left(l_{i}\right)=$ true. If from $l_{i}$ there is an interaction $a$ such that $l_{i} \in \bullet$, then there exists $l_{j}^{\prime} \in a^{\bullet}$, such that $v\left(l_{j}^{\prime}\right)=$ true by Definition 17 . So any successor state of $l$ by an interaction $a$ satisfies the invariant.

That is, the disjunction of positive valuations in any solution of BBCs is an interaction invariant of $\gamma(B)$. Concretely, if $v=\bigwedge_{i \in I} l_{i} \wedge \bigwedge_{j \in K} \bar{l}_{j}$ is a solution of BBCs, then the disjunction of the positive valuations $\bigvee_{i \in I} l_{i}$ is an interaction invariant of $\gamma(B)$.

A naive way to compute interaction invariants is to get all the solutions of BBCs, but here there is a risk of explosion if the number of solutions is huge. We therefore provide below a method for computing symbolically interaction invariants based on an operation called Positive Mapping which allows removing all the negative valuations of a set of variables in a boolean formula expressed as the disjunction of monomials.

Definition 18 (Positive Mapping) Given two sets of variables $X, Y$ such that $X \subseteq Y$, and a boolean formula $f$ on $Y$ expressed as the disjunction of monomials. We define an operation, called Positive Mapping, that deletes all the negative variables that do not belong to $X$, denoted by $f^{p(X)}$, as follows:

$$
\begin{aligned}
& \left(\bigwedge_{l_{i} \in Y} l_{i} \wedge \bigwedge_{l_{j} \in X} \overline{l_{j}} \bigwedge_{l_{k} \in Y-X} \overline{l_{k}}\right)^{p(X)}=\bigwedge_{l_{i} \in Y} l_{i} \wedge \bigwedge_{l_{j} \in X} \overline{l_{j}} \\
& \left(f_{1} \vee f_{2}\right)^{p(X)}=f_{1}^{p(X)} \vee f_{2}^{p(X)}
\end{aligned}
$$

Here we use $\bar{l}$ for $\neg l$. When $X$ is empty, the positive mapping will remove all the negative variables in $f$, which is denoted by $f^{p}$. If all the variables are negative in $f$, we have $f^{p}=$ false.

Example 15 Given a boolean function $f=(x \wedge y \wedge \bar{z}) \vee(x \wedge \bar{y} \wedge z)$ and a subset of variables $X=\{x, y\}$, we have Positive Mapping $f^{p(X)}=(x \wedge y) \vee(x \wedge \bar{y} \wedge z)$ and $f^{p}=(x \wedge y) \vee(x \wedge z)$.

The global interaction invariant is obtained by conjunction of all interaction invariants. The following theorem allows computing symbolically the global interaction invariant of $\gamma(B)$ from its Boolean Behavioral Constraints $|\gamma(B)|$. But first let us define the dual operation which is used in the computation of interaction invariants.

Definition 19 (Dual Operation) Given a boolean formula $f(X)$ on a set of variables $X=\left\{x_{1}, \ldots, x_{n}\right\}$. We define the dual operation on $f(X)$, denoted by $\widetilde{f(X)}$, as follows: $\widetilde{f(X)}=\overline{f(\bar{X})}$ where $f(\bar{X})$ is a boolean formula obtained from $f(X)$ by replacing, for each variable $x_{i} \in X$, its positive form $x_{i}$ (respectively its negative form $\bar{x}_{i}$ ) by its negative form $\bar{x}_{i}$ (respectively its positive form $x_{i}$ ).
$\underset{\sim}{\text { Example }} 16$ For the boolean formula $f=(x \wedge \bar{y}) \vee(y \wedge \bar{z}) \vee(x \wedge z)$, we have its dual $\widetilde{f}=(x \vee \bar{y}) \wedge(y \vee \bar{z}) \wedge(x \vee z)$.

Theorem 2 For any connector $\gamma$ applied to a tuple of components $B=\left(B_{1}, \cdots, B_{n}\right)$, the global interaction invariant of $\gamma(B)$ can be obtained as the dual of the positive mapping of $|\gamma(B)|$, denoted by $\widetilde{|\gamma(B)|^{p}}$.

Proof (Sketch). $|\gamma(B)|$ can be written as the disjunction of monomials, that is $|\gamma(B)|=$ $\bigvee_{i \in I} m_{i}$, where $m_{i}$ is of the form $m_{i}=\bigwedge_{j \in I} l_{i_{j}} \wedge \bigwedge_{k \in K} \overline{l_{i_{k}}}$. We have $|\gamma(B)|^{p}=\bigvee_{i \in I} m_{i}^{p}=$ $\bigvee_{i \in I}\left(\bigwedge_{j \in I} l_{i_{j}}\right)$, hence $\widetilde{\mid \widetilde{\left.\gamma(B)\right|^{p}}}=\bigwedge_{i \in I}\left(\bigvee_{j \in I} l_{i_{j}}\right)$ is the global interaction invariant of $\gamma(B)$ according to Theorem 1.

Here we obtain by $\widetilde{\left.\gamma(B)\right|^{p}}$ all the possible interaction invariants of $\gamma(B)$. For a system $S=\langle\gamma(B)$, Init $\rangle$ where Init is the initial state (the set of initial locations of components in $B$ ), the interaction invariants of $S$ are obtained from interaction invariants of $\gamma(B)$ by selecting all the invariants that have at least an initial location. The global interaction invariants of $S$ can be obtained as $\left(|\gamma(B)| \widetilde{\wedge \bigvee_{l \in \text { Init }} l}\right)^{p}$.

Example 17 We use the example in Figure 2.5 to illustrate the computation of invariants, where $B=\left(B_{1}, B_{2}\right)$ and $\gamma=a_{1} a_{2}+b_{1} b_{2}$. The BBCs for $\gamma(B)$, according to Example 14 is:

$$
\begin{aligned}
|\gamma(B)| & =\left(l_{1} \Rightarrow l_{2} \vee l_{4}\right) \wedge\left(l_{2} \Rightarrow l_{1} \vee l_{3}\right) \wedge\left(l_{3} \Rightarrow l_{2} \vee l_{4}\right) \wedge\left(l_{4} \Rightarrow l_{1} \vee l_{3}\right) \\
& =\left(\overline{l_{1}} \wedge \overline{l_{2}} \wedge \overline{l_{3}} \wedge \overline{l_{4}}\right) \vee\left(l_{1} \wedge l_{2}\right) \vee\left(l_{2} \wedge l_{3}\right) \vee\left(l_{1} \wedge l_{4}\right) \vee\left(l_{3} \wedge l_{4}\right)
\end{aligned}
$$

By applying the Positive Mapping operation, we have:

$$
|\gamma(B)|^{p}=\left(l_{1} \wedge l_{2}\right) \vee\left(l_{2} \wedge l_{3}\right) \vee\left(l_{1} \wedge l_{4}\right) \vee\left(l_{3} \wedge l_{4}\right)
$$

Thus the global interaction invariant is:

$$
\widetilde{|\gamma(B)|^{p}}=\left(l_{1} \vee l_{2}\right) \wedge\left(l_{3} \vee l_{4}\right) \wedge\left(l_{1} \vee l_{4}\right) \wedge\left(l_{2} \vee l_{3}\right)
$$

From the global interaction invariant obtained in Example 17, if we take into account the initial state Init $=\left\{l_{1}, l_{3}\right\}$, we have the same global interaction invariant $\Psi=\widetilde{|\gamma(B)|^{p}}$ for the system $S=\langle\gamma(B)$, Init $\rangle$ (because all the interaction invariants of $\gamma(B)$ have at least an initial location). The global reachable states of $S$ are approximated by the global interaction invariant $\Psi=\left(l_{1} \vee l_{2}\right) \wedge\left(l_{3} \vee l_{4}\right) \wedge\left(l_{1} \vee l_{4}\right) \wedge\left(l_{2} \vee l_{3}\right)=\left(l_{1} \wedge l_{3}\right) \vee\left(l_{2} \wedge l_{4}\right)$. In this case, the obtained global interaction invariant represents exactly the set of reachable states.

## Method based on Fixed-point Computation

Fixed-point-based methods are widely used in Model-Checking for computing reachable states. Starting from the global initial states, the global successor states are iteratively computed until no more new global state is generated, i.e, we have reached fixed-points in computing reachable states.

The interaction invariants can be iteratively computed by using fixed-point computation technique. In our fixed-point method, we start from a control location of an atomic component, and then iteratively compute a set of successors of that location by global image vector obtained from the set of interactions. The iteration stops when no more successor is generated, that is we have reached fixed-points. A fixed-point corresponds to a set of locations such that if a location of the set is reached, then at any time, at least one location of the set is reached.

The main difference of our method from the fixed-point method used in Model-Checking is that we do not compute iteratively the global successors of a global state which can be exponential in the size of system. Our fixed-point method computes iteratively successors of every location of atomic components, therefore it does not suffer from the state space explosion problem.

Definition 20 (Image Vector) Let $\gamma$ be a connector over a set of components $B=$ $\left(B_{1}, \ldots, B_{n}\right)$ where $B_{i}=\left(L_{i}, P_{i}, \mathcal{T}_{i}\right)$. The image vector for a set of locations $L=\bigcup_{i=1}^{n} L_{i}$ according to $\gamma$ is defined as follows:

$$
\mathbb{V}_{\gamma}=\left\{\mathbb{V}_{\gamma}(l)=l \wedge f_{l}^{\gamma}(L) \mid l \in L\right\} \text { where } f_{l}^{\gamma}(L)=\bigwedge_{\left\{\tau_{i}\right\}_{i \in I} \in \overrightarrow{l^{\gamma}}}\left(\bigvee_{l^{\prime} \in\left\{\tau_{i}^{\bullet}\right\}_{i \in I}} l^{\prime}\right)
$$

Thesis


Figure 2.7: An example of two components strongly synchronized

The image $\mathbb{V}_{\gamma}(l)$ of a location $l$ according to a connector $\gamma$ defines the set of locations (including $l$ ) that can be reached from $l$ by involved interactions in $\gamma$.

The formula $f_{l}^{\gamma}(L)$ is actually the right side of the implication in the BBC of the location $l$ according to definition 17 . Let $l \Rightarrow f_{l}^{\gamma}(L)$ be an implication for a location $l \in L$, we have $l=l \wedge f_{l}^{\gamma}(L)$.

Example 18 For the example illustrated in Figure 2.7 with $\gamma=a_{1} a_{2}+b_{1} b_{2}$, the Boolean Behavioral Constraints of locations $l_{1}, l_{2}, l_{3}, l_{4}$ are respectively:

$$
l_{1} \Rightarrow l_{2} \vee l_{4} \quad l_{2} \Rightarrow l_{1} \vee l_{3} \quad l_{3} \Rightarrow l_{2} \vee l_{4} \quad l_{4} \Rightarrow l_{1} \vee l_{3}
$$

The corresponding image vector is $\mathbb{V}_{\gamma}=\left\{\mathbb{V}_{\gamma}\left(l_{1}\right), \mathbb{V}_{\gamma}\left(l_{2}\right), \mathbb{V}_{\gamma}\left(l_{3}\right), \mathbb{V}_{\gamma}\left(l_{4}\right)\right\}$ where:

$$
\begin{array}{ll}
\mathbb{V}_{\gamma}\left(l_{1}\right)=l_{1} \wedge\left(l_{2} \vee l_{4}\right) & \mathbb{V}_{\gamma}\left(l_{3}\right)=l_{3} \wedge\left(l_{2} \vee l_{4}\right) \\
\mathbb{V}_{\gamma}\left(l_{2}\right)=l_{2} \wedge\left(l_{1} \vee l_{3}\right) & \mathbb{V}_{\gamma}\left(l_{4}\right)=l_{4} \wedge\left(l_{1} \vee l_{3}\right)
\end{array}
$$

Definition 21 (Image Function) Let $\mathbb{V}$ be an image vector on a set of variables $L$, $\phi$ be a predicate on $L$ in the disjunctive form of monomials $\phi=\bigvee_{i} \varphi_{i}$, the image function of $\phi$ with the image vector $\mathbb{V}$ is defined by:

$$
\operatorname{Image}(\mathbb{V}, \phi)=\bigvee_{i}\left(\bigwedge_{l \in L_{\varphi_{i}}} \mathbb{V}(l)\right)
$$

where $L_{\varphi_{i}}$ is the set of variables in $\varphi_{i}$.
That is, the Image function replaces every variable $l$ of each monomial $\varphi_{i}$ by the corresponding image $\mathbb{V}(l)$.

Definition 22 (Fixed-points) Let $\mathbb{V}$ be an image vector on a set of variables $L$, $\phi$ be a predicate on $L$, we define an iteration process that allows computing fixed-points starting from $\phi$ according to the image vector $\mathbb{V}$ as follows:

$$
\phi^{0}=\phi
$$

$$
\phi^{k+1}=\operatorname{Image}\left(\mathbb{V}, \phi^{k}\right)
$$

When $\phi^{k+1}=\phi^{k}$, the iteration terminates and $\phi^{k}$ is the fixed-points of the computation, denoted by $\mathbb{F}(\mathbb{V}, \phi)$.

Example 19 For the example presented in Figure 2.7 with the image vector obtained in Example 18, and for a predicate $\phi=l_{1}$, the iterations are as follows:

$$
\begin{aligned}
& \phi^{0}=\phi=l_{1} \\
& \phi^{1}=\operatorname{Image}\left(\mathbb{V}_{\gamma}, \phi^{0}\right)=\left(l_{1} \wedge l_{2}\right) \vee\left(l_{1} \wedge l_{4}\right) \\
& \phi^{2}=\operatorname{Image}\left(\mathbb{V}_{\gamma}, \phi^{1}\right)=\left(l_{1} \wedge l_{2}\right) \vee\left(l_{1} \wedge l_{4}\right)
\end{aligned}
$$

The iteration stops because $\phi^{2}=\phi^{1}$ and we obtain the fixed-points $\mathbb{F}\left(\mathbb{V}_{\gamma}, l_{1}\right)=\left(l_{1} \wedge l_{2}\right) \vee$ $\left(l_{1} \wedge l_{4}\right)$. Similarly, from locations $l_{2}, l_{3}, l_{4}$ we obtain respectively the following fixed-points:

$$
\begin{aligned}
& \mathbb{F}\left(\mathbb{V}_{\gamma}, l_{2}\right)=\left(l_{1} \wedge l_{2}\right) \vee\left(l_{2} \wedge l_{3}\right) \\
& \mathbb{F}\left(\mathbb{V}_{\gamma}, l_{3}\right)=\left(l_{2} \wedge l_{3}\right) \vee\left(l_{3} \wedge l_{4}\right) \\
& \mathbb{F}\left(\mathbb{V}_{\gamma}, l_{4}\right)=\left(l_{1} \wedge l_{4}\right) \vee\left(l_{3} \wedge l_{4}\right)
\end{aligned}
$$

The following theorem provides means for computing interaction invariants from fixedpoints.

Theorem 3 Let $B=\left(B_{1}, \ldots, B_{n}\right)$ be a set of components with $B_{i}=\left(L_{i}, P_{i}, \mathcal{T}_{i}\right)$ and $L=\bigcup_{i=1}^{n} L_{i}, \gamma$ be a connector over $B$ with the image vector $\mathbb{V}_{\gamma}$. For any location $l_{i} \in L$, if $m=\bigwedge l_{j}$ is a solution (a fixed-point) of $\mathbb{F}\left(\mathbb{V}_{\gamma}, l_{i}\right)$, then the dual of $m$, that is $\widetilde{m}=\bigvee l_{j}$, is an invariant of $\gamma(B)$.

Proof Let $L_{m}$ be the set of location variables in the solution $m$. We assume that for some global state $l=\left(l_{1}, \ldots, l_{n}\right)$, these exists $l_{k}$ such that $l_{k} \in L_{m}$. If from $l_{k}$ there is an interaction $a$ such that $l_{k} \in{ }^{\bullet} a$, then there exists $l_{k}^{\prime} \in a^{\bullet}$, such that $l_{k}^{\prime} \in L_{m}$ by Definition 20, 21 and 22. So any successor state of $l$ by an interaction $a$ satisfies $\widetilde{m}=\bigvee_{l_{j} \in L_{m}} l_{j}$.

Example 20 According to the fixed-points obtained in Example 19, we have the following interaction invariants:

$$
\begin{array}{ll}
\Psi_{l_{1}}=\left(l_{1} \vee l_{2}\right) \wedge\left(l_{1} \vee l_{4}\right) & \Psi_{l_{2}}=\left(l_{1} \vee l_{2}\right) \wedge\left(l_{2} \vee l_{3}\right) \\
\Psi_{l_{3}}=\left(l_{2} \vee l_{3}\right) \wedge\left(l_{3} \vee l_{4}\right) & \Psi_{l_{4}}=\left(l_{1} \vee l_{4}\right) \wedge\left(l_{3} \vee l_{4}\right)
\end{array}
$$

The global interaction invariant is obtained by conjunction of all interaction invariants. The following theorem allows computing symbolically the global interaction invariant of $\gamma(B)$ by fixed-points starting from the disjunction of locations of atomic components in $B$.

Theorem 4 Let $B=\left(B_{1}, \ldots, B_{n}\right)$ be a set of components with $B_{i}=\left(L_{i}, P_{i}, \mathcal{T}_{i}\right)$ and $L=\bigcup_{i=1}^{n} L_{i}, \gamma$ be a connector over $B$ with the image vector $\mathbb{V}_{\gamma}$, the global interaction invariant of $\gamma(B)$ can be obtained as the dual of fixed-points $\mathbb{F}\left(\mathbb{V}_{\gamma}, \bigvee_{l \in L} l\right)$, denoted by $\widetilde{\mathbb{F}}\left(\mathbb{V}_{\gamma}, \bigvee_{l \in L} l\right)$.


Figure 2.8: Interaction invariants computation for systems with data

Proof We have $\mathbb{F}\left(\mathbb{V}_{\gamma}, \bigvee_{l \in L} l\right)=\bigvee_{l \in L} \mathbb{F}\left(\mathbb{V}_{\gamma}, l\right)$, hence $\widetilde{\mathbb{F}}\left(\mathbb{V}_{\gamma}, \bigvee_{l \in L} l\right)=\bigwedge_{l \in L} \widetilde{\mathbb{F}}\left(\mathbb{V}_{\gamma}, l\right)$ is the global interaction invariant according to Theorem 3.

Example 21 For the example presented in Figure 2.7 with the image vector obtained in Example 18, and for a predicate $\phi=l_{1} \vee l_{2} \vee l_{3} \vee l_{4}$, the iterations are as follows:

$$
\begin{aligned}
& \phi^{0}=\phi=l_{1} \vee l_{2} \vee l_{3} \vee l_{4} \\
& \phi^{1}=\operatorname{Image}\left(\mathbb{V}_{\gamma}, \phi^{0}\right)=\left(l_{1} \wedge l_{2}\right) \vee\left(l_{1} \wedge l_{4}\right) \vee\left(l_{2} \wedge l_{3}\right) \vee\left(l_{3} \wedge l_{4}\right) \\
& \phi^{2}=\operatorname{Image}\left(\mathbb{V}_{\gamma}, \phi^{1}\right)=\left(l_{1} \wedge l_{2}\right) \vee\left(l_{1} \wedge l_{4}\right) \vee\left(l_{2} \wedge l_{3}\right) \vee\left(l_{3} \wedge l_{4}\right)
\end{aligned}
$$

The iteration stops because $\phi^{2}=\phi^{1}$ and we obtain the fixed-points $\mathbb{F}\left(\mathbb{V}_{\gamma}, l_{1}\right)=\phi^{1}$ from which we obtain the global interaction invariant:

$$
\Psi=\widetilde{\mathbb{F}}\left(\mathbb{V}_{\gamma}, l_{1} \vee l_{2} \vee l_{3} \vee l_{4}\right)=\left(l_{1} \vee l_{2}\right) \wedge\left(l_{1} \vee l_{4}\right) \wedge\left(l_{2} \vee l_{3}\right) \wedge\left(l_{3} \vee l_{4}\right)
$$

We call $\mathbb{F}\left(\mathbb{V}_{\gamma}, \bigvee_{l \in L} l\right)$ fixed-points of $\gamma(B)$. For a system $S=\langle\gamma(B)$, Init $\rangle$ where $\gamma$ is a connector on a set of components $B$ and Init is the initial state, since every interaction invariant must contain at least an initial location of a component in $B$, we start the iteration for fixed-point computation from the initial locations of atomic components. That is the global interaction invariant of $S$ is obtained as $\widetilde{\mathbb{F}}\left(\mathbb{V}_{\gamma}, \bigvee_{l \in \text { Init }} l\right)$. This guarantees the existence of the initial locations in all the obtained invariants.

### 2.2 Abstraction

Abstraction techniques have been widely developed and used in verification in order to alleviate the state space explosion problem, specially for the verification of infinite systems. For finite systems, abstraction is also necessary and important to the success of the verification. The goal of abstraction is to build, for each concrete system, an abstract system which preserves properties to verify and is less expensive to analyze or to check by existing verification tools such as Model-Checkers.

We have provided two methods for computing interaction invariants of systems without data. For systems with data, an abstraction technique is needed to abstract away the data
before applying the methods. The process for computing interaction invariants of systems with data is presented in figure 2.8. It consists of three steps:

- First we need to make an abstraction of the system:
- for each atomic component $B_{i}$ with data of the system $S$, an abstraction is made to obtain a corresponding abstract atomic component $B_{i}^{\alpha}$ without data.
- abstract connector $\gamma^{\alpha}$ is obtained from $\gamma$ by generating for each interaction in $\gamma$ a corresponding abstract interaction.
- abstract initial condition Init ${ }^{\alpha}$ is made from Init.
- Then the methods for computing interaction invariants of systems without data is applied for the abstract systems $S^{\alpha}$. We obtain a set of abstract interaction invariants $\Psi^{\alpha}$ of $S^{\alpha}$.
- Finally, interaction invariants $\Psi$ of the concrete system $S$ are obtained by concretizing the set of the abstract interaction invariants $\Psi^{\alpha}$.
The abstraction technique we use is based on the method proposed by Bensalem et al. in [BLO98a]. The basic idea of the method is to use a splitting algorithm to refine an abstract structure in order to preserve properties in the abstract-concrete direction, that is any property satisfied by abstract system will be satisfied by the concrete system. The advantage of this method is that it produces an abstract system which has the same structure as the concrete one. This allows for further application of abstraction and gives a clear correspondence between abstract and concrete transitions which is useful for debugging the concrete system. This method has been implemented in the InVeSt tool [BLO98b].

Given a concrete system $S$, the abstraction method allows to compute an abstract system $S^{\alpha}$ that $S$ simulates $S^{\alpha}$, that is every computation of $S$ can be mapped to a computation of $S^{\alpha}$. Consider a system $\mathcal{S}=\left\langle\gamma\left(B_{1}, \ldots, B_{n}\right)\right.$, Init $\rangle$ and a set of component invariants $\Phi_{1} \ldots \Phi_{n}$ associated with the atomic components. We show below, for each component $B_{i}$ and its associated invariant $\Phi_{i}$, how to define a finite state abstraction $\alpha_{i}$ and to compute an abstract transition system $B_{i}^{\alpha_{i}}$.

Definition 23 (Abstraction Function) Let $\Phi$ be an invariant of a system 〈B, Init〉 written in disjunctive form $\Phi=\bigvee_{l \in L} l \wedge\left(\bigvee_{m \in M_{l}} \varphi_{l m}\right)$ such that atomic predicates of the form $l \wedge \varphi_{l m}$ are disjoint. Given $\Phi$, an abstraction function $\alpha$ is an injective function associating with each atomic predicate $l \wedge \varphi_{l m}$ a symbol $\phi=\alpha\left(l \wedge \varphi_{l m}\right)$ called abstract state. We denote by $\Phi^{\alpha}$ the set of the abstract states.

Example 22 Consider the component Controller in the Temperature Control System (Figure 2.2). The component invariant predicate at each location can be written in the disjunctive form according to the post predicate transformers of its incoming transitions as follows:

$$
\begin{aligned}
& -\Phi_{l_{5}}=l_{5} \wedge(\theta=100 \vee 101 \leq \theta \leq 1000) \\
& -\Phi_{l_{6}}=l_{6} \wedge(\theta=1000 \vee 100 \leq \theta \leq 998)
\end{aligned}
$$

Therefore, we have four abstract states:

$$
\begin{array}{ll}
\Phi_{51}=l_{5} \wedge \theta=100 & \Phi_{61}=l_{6} \wedge \theta=1000 \\
\Phi_{52}=l_{5} \wedge 101 \leq \theta \leq 1000 & \Phi_{62}=l_{6} \wedge 100 \leq \theta \leq 998
\end{array}
$$

Thesis

The abstraction function maps concrete states to abstract states taken from a finite set, hence we obtain a finite state system which can be analyzed algorithmically. This allows us to compute an over approximation of the set of reachable states which is sufficient for the verification of invariants. Using the abstraction function, an abstract system is defined as follows:

Definition 24 (Abstract System) Given a system $\mathcal{S}=\langle B$, Init $\rangle$, an invariant $\Phi$ and an associated abstraction function $\alpha$, we define the abstract system $\mathcal{S}^{\alpha}=\left\langle B^{\alpha}\right.$, Init $\left.{ }^{\alpha}\right\rangle$ where

- $B^{\alpha}=\left(\Phi^{\alpha}, P, \rightsquigarrow\right)$ is a transition system with $\rightsquigarrow$ such that for any pair of abstract states $\phi=\alpha(l \wedge \varphi)$ and $\phi^{\prime}=\alpha\left(l^{\prime} \wedge \varphi^{\prime}\right)$ we have $\phi \stackrel{p}{\rightsquigarrow} \phi^{\prime}$ iff $\exists \tau=\left(l, p, l^{\prime}\right) \in \mathcal{T}$ and $\operatorname{post}_{\tau}(\varphi) \wedge \varphi^{\prime} \neq$ false (or equivalently $\varphi \wedge \operatorname{pre}_{\tau}\left(\varphi^{\prime}\right) \neq$ false),
- Init ${ }^{\alpha}=\bigvee_{\phi \in \Phi_{0}^{\alpha}} \phi$ where $\Phi_{0}^{\alpha}=\left\{\phi \in \Phi^{\alpha} \mid \alpha^{-1}(\phi) \wedge\right.$ Init $\neq$ false $\}$ is the set of the initial abstract states.

The method proceeds by elimination, starting from the universal relation on abstract states. We eliminate pairs of abstract states in a conservative way. To check whether $\phi \stackrel{p}{\sim} \phi^{\prime}$, where $\phi=\alpha(l \wedge \varphi)$ and $\phi^{\prime}=\alpha\left(l^{\prime} \wedge \varphi^{\prime}\right)$, can be eliminated, we check that for all concrete transitions $\tau=\left(l, p, l^{\prime}\right)$ we have $\operatorname{post}_{\tau}(\varphi) \wedge \varphi^{\prime}=$ false or equivalently $\varphi \wedge$ pre $\left(\varphi^{\prime}\right)=$ false.


Figure 2.9: Abstraction of the Temperature Control System

Example 23 The table below provides the abstract states constructed from the component invariants $\Phi_{1}, \Phi_{2}, \Phi_{3}$ of respectively Rod1, Rod2, Controller given in example 10.

$$
\begin{array}{l|l|l}
\phi_{11}=l_{1} \wedge t_{1}=0 & \phi_{51}=l_{5} \wedge \theta=100 & \begin{array}{l}
\phi_{31}=l_{3} \wedge t_{2}=0 \\
\phi_{12}=l_{1} \wedge t_{1} \geq 1 \\
\phi_{21}=l_{2} \wedge t_{1} \geq 3600
\end{array}
\end{array} \begin{aligned}
& \phi_{52}=l_{5} \wedge 101 \leq \theta \leq 1000 \\
& \phi_{61}=l_{6} \wedge \theta=1000 \\
& \phi_{62}=l_{6} \wedge 100 \leq \theta \leq 998
\end{aligned} \quad \begin{aligned}
& \phi_{41}=l_{4} \wedge t_{2} \geq 3600
\end{aligned}
$$

Figure 2.9 presents the computed abstraction of the Temperature Control System with respect to the considered invariants.

We take several transitions of Controller component to illustrate the construction of the abstract transitions, for example:

- transition $\tau^{\alpha}=\left(\phi_{52}, \operatorname{cool}, \phi_{61}\right)$ is established because there exists a concrete transition $\tau=\left(l_{5}, \operatorname{cool}, l_{6}\right)$ and $\operatorname{post}_{\tau}\left(\varphi_{52}\right) \wedge \varphi_{61} \neq$ false where $\operatorname{post}_{\tau}\left(\varphi_{52}\right)=(\theta=1000)$ and $\varphi_{61}=(\theta=1000)$.
- transition $\tau^{\alpha}=\left(\phi_{52}\right.$, cool,$\left.\phi_{62}\right)$ is eliminated since $\operatorname{post}_{\tau}\left(\varphi_{52}\right) \wedge \varphi_{62}=$ false where $\operatorname{post}_{\tau}\left(\varphi_{52}\right)=(\theta=1000)$ and $\varphi_{62}=(100 \leq \theta \leq 998)$.
Consider the concrete initial condition Init $=l_{5} \wedge(\theta=100) \wedge l_{1} \wedge\left(t_{1}=3600\right) \wedge l_{3} \wedge\left(t_{2}=\right.$ 3600), we have the abstract initial condition Init ${ }^{\alpha}=\phi_{51} \wedge \phi_{12} \wedge \phi_{32}$.

By combining well-known results about abstractions, we can compute interaction invariants of $\left\langle\gamma\left(B_{1}, \ldots, B_{n}\right)\right.$, Init $\rangle$ from interaction invariants of $\left\langle\gamma\left(B_{1}^{\alpha}, \ldots, B_{n}^{\alpha}\right)\right.$, Init $\left.{ }^{\alpha}\right\rangle$.

The following proposition says that $\gamma\left(B_{1}^{\alpha_{1}}, \ldots, B_{n}^{\alpha_{n}}\right)$ is an abstraction of $B=\gamma\left(B_{1}, \ldots, B_{n}\right)$
Proposition 6 If $B_{i}^{\alpha_{i}}$ is an abstraction of $B_{i}$ with respect to an invariant $\Phi_{i}$ and its abstraction function $\alpha_{i}$ for $i=1, \ldots, n$, then $B^{\alpha}=\gamma\left(B_{1}^{\alpha_{1}}, \ldots, B_{n}^{\alpha_{n}}\right)$ is an abstraction of $B=\gamma\left(B_{1}, \ldots, B_{n}\right)$ with respect to $\bigwedge_{i=1}^{n} \Phi_{i}$ and an abstraction function $\alpha$ obtained as the composition of the $\alpha_{i}$.

The following proposition says that invariants of the abstract system are also invariants of the concrete system.

Proposition 7 If $B^{\alpha}$ is an abstraction of $B$ with respect to $\Phi$ and its abstraction function $\alpha$, then $B^{\alpha}$ simulates $B$. Moreover, if $\Phi^{\alpha}$ is an invariant of $\left\langle B^{\alpha}\right.$, Init $\left.{ }^{\alpha}\right\rangle$ then $\alpha^{-1}\left(\Phi^{\alpha}\right)$ is an invariant of $\langle B$, Init $\rangle$.

Proof We show that the relation $(l, \mathbf{x}) R \phi$ is a simulation if $\alpha^{-1}(\phi)=l \wedge \varphi$ and $\varphi(\mathbf{x})$ for the valuation $\mathbf{x}$. If $(l, \mathbf{x}) \xrightarrow{p}\left(l^{\prime}, \mathbf{x}^{\prime}\right)$ is a transition of $B$ and $(l, \mathbf{x}) R \phi$ for some abstract state $\phi$, then we show that there exists $\phi^{\prime}=\alpha\left(l^{\prime} \wedge \varphi^{\prime}\right)$ such that $\phi \stackrel{p}{\rightsquigarrow} \phi^{\prime}$. As $\Phi$ is an invariant of $B$, if $\left(l^{\prime}, \mathbf{x}^{\prime}\right)$ is reachable then $\exists \varphi^{\prime} l^{\prime} \wedge \varphi^{\prime} \Rightarrow \Phi$ such that $\varphi^{\prime}\left(\mathbf{x}^{\prime}\right)$ and $\phi^{\prime}=\alpha\left(l^{\prime} \wedge \varphi^{\prime}\right)$. Moreover, as $\varphi(\mathbf{x}) \wedge \varphi^{\prime}\left(\mathbf{x}^{\prime}\right)$, we have $\varphi(\mathbf{x}) \wedge \operatorname{pre} e_{\tau}(\varphi)(\mathbf{x}) \neq$ false for $\tau=\left(l, p, l^{\prime}\right)$ and therefore $\phi \stackrel{p}{\rightsquigarrow} \phi^{\prime}$.

Thus, it is possible to compute from interaction invariants of the abstract system, interaction invariants for the concrete system $\left\langle\gamma\left(B_{1}, \ldots, B_{n}\right)\right.$, Init $\rangle$.

We can show by application of the following proposition that the iteration process gives progressively stronger invariants, in particular that for stronger component invariants we get stronger interaction invariants.

Proposition 8 Let $\langle B$, Init $\rangle$ be a system and $\Phi \Phi^{\prime}$ two non empty invariants such that $\Phi \Rightarrow \Phi^{\prime}$. If $\alpha$ and $\alpha^{\prime}$ are the abstraction functions corresponding to $\Phi$ and $\Phi^{\prime}$ respectively, then $B^{\alpha}$ simulates $B^{\alpha^{\prime}}$.

Proof For two successive component invariants $\Phi_{i}$ and $\Phi_{i}^{\prime}$ for $B_{i}$, we have $\Phi_{i} \Rightarrow \Phi_{i}^{\prime}$. From proposition 8 we deduce that $B_{i}^{\alpha_{i}}$ simulates $B_{i}^{\alpha_{i}^{\prime}}$ where $\alpha_{i}$ and $\alpha_{i}^{\prime}$ are the abstraction functions corresponding to $\Phi_{i}$ and $\Phi_{i}^{\prime}$. As the simulation relation is preserved by parallel composition, we have $\gamma\left(B_{1}^{\alpha_{1}}, \ldots, B_{n}^{\alpha_{n}}\right)$ simulates $\gamma\left(B_{1}^{\alpha_{1}^{\prime}}, \ldots, B_{n}^{\alpha_{n}^{\prime}}\right)$. We can show that for each positive valuation set $L^{\prime}$ of a solution of $\gamma\left(B_{1}^{\alpha_{1}^{\prime}}, \ldots, B_{n}^{\alpha_{n}^{\prime}}\right)$ there exists a positive valuation
set $L$ of a solution of $\gamma\left(B_{1}^{\alpha_{1}}, \ldots, B_{n}^{\alpha_{n}}\right)$ such that $L \subseteq L^{\prime}$. From this we infer that for each interaction invariant of $\gamma\left(B_{1}^{\prime}, \ldots, B_{n}^{\prime}\right)$ there exists a stronger interaction invariant of $\gamma\left(B_{1}, \ldots, B_{n}\right)$.

Below we provide an example on the Temperature Control System for showing the computation of abstract interaction invariants and then the concretization to obtain interaction invariants of the concrete system.

Example 24 For the abstraction of the Temperature Control System given in figure 2.9, we have the following Boolean Behavioral Constraints:

$$
\begin{array}{rlrr}
\phi_{11} \Rightarrow\left(\phi_{12} \vee \phi_{32} \vee \phi_{52}\right) & \wedge\left(\phi_{21} \vee \phi_{32} \vee \phi_{62}\right) & \phi_{61} & \Rightarrow\left(\phi_{12} \vee \phi_{32} \vee \phi_{62}\right) \\
\wedge\left(\phi_{12} \vee \phi_{32} \vee \phi_{62}\right) & \phi_{32} & \Rightarrow\left(\phi_{61} \vee \phi_{41}\right) & \wedge\left(\phi_{21} \vee \phi_{41} \vee \phi_{62}\right) \\
\wedge\left(\phi_{12} \vee \phi_{41} \vee \phi_{52}\right) & \phi_{41} \Rightarrow\left(\phi_{51} \vee \phi_{31}\right) & \wedge\left(\phi_{21} \vee \phi_{32} \vee \phi_{62}\right) \\
\wedge\left(\phi_{12} \vee \phi_{41} \vee \phi_{62}\right) & \phi_{51} & \Rightarrow\left(\phi_{12} \vee \phi_{32} \vee \phi_{52}\right) & \wedge\left(\phi_{12} \vee \phi_{41} \vee \phi_{62}\right) \\
\phi_{12} \Rightarrow\left(\phi_{61} \vee \phi_{21}\right) & \wedge\left(\phi_{21} \vee \phi_{41} \vee \phi_{52}\right) & \phi_{62} & \Rightarrow\left(\phi_{51} \vee \phi_{11}\right) \\
\phi_{21} \Rightarrow\left(\phi_{51} \vee \phi_{11}\right) & \wedge\left(\phi_{21} \vee \phi_{32} \vee \phi_{52}\right) & \wedge\left(\phi_{51} \vee \phi_{31}\right) \\
\phi_{31} \Rightarrow\left(\phi_{12} \vee \phi_{32} \vee \phi_{52}\right) & \wedge\left(\phi_{12} \vee \phi_{41} \vee \phi_{52}\right) & \\
& \wedge\left(\phi_{12} \vee \phi_{32} \vee \phi_{62}\right) & \phi_{52} & \Rightarrow\left(\phi_{61} \vee \phi_{21}\right) \\
& \wedge\left(\phi_{21} \vee \phi_{32} \vee \phi_{52}\right) & & \wedge\left(\phi_{61} \vee \phi_{41}\right)
\end{array}
$$

According to theorem 2, by applying the positive mapping and the dual operation, we obtain the following global abstract interaction invariant of the abstract system:

```
\Psi
    \wedge(\mp@subsup{\phi}{11}{}\vee\mp@subsup{\phi}{12}{}\vee\mp@subsup{\phi}{31}{}\vee\mp@subsup{\phi}{52}{}\vee\mp@subsup{\phi}{61}{}\vee\mp@subsup{\phi}{62}{})\wedge(\mp@subsup{\phi}{12}{}\vee\mp@subsup{\phi}{21}{}\vee\mp@subsup{\phi}{51}{})
    \wedge(\mp@subsup{\phi}{11}{}\vee\mp@subsup{\phi}{12}{}\vee\mp@subsup{\phi}{31}{}\vee\mp@subsup{\phi}{32}{}\vee\mp@subsup{\phi}{61}{}\vee\mp@subsup{\phi}{62}{})}\wedge(\mp@subsup{\phi}{32}{}\vee\mp@subsup{\phi}{41}{}\vee\mp@subsup{\phi}{51}{}
```

The concretization of the global abstract interaction invariant provides the global interaction invariant of the concrete system:

$$
\begin{aligned}
& \Psi=\left(\left(l_{2} \wedge t_{1} \geq 3600\right) \vee\left(l_{4} \wedge t_{2} \geq 3600\right) \vee\left(l_{5} \wedge 100 \leq \theta \leq 1000\right)\right) \\
& \wedge\left(\left(l_{1} \wedge t_{1} \geq 0\right) \vee\left(l_{2} \wedge t_{1} \geq 3600\right) \vee\left(l_{3} \wedge t_{2} \geq 0\right) \vee\left(l_{4} \wedge t_{2} \geq 3600\right)\right) \\
& \Lambda\left(\left(l_{3} \wedge t_{2} \geq 1\right) \vee\left(l_{4}\right) \vee\left(l_{5} \wedge \theta=100\right)\right) \\
& \Lambda\left(\left(l_{1} \wedge t_{1} \geq 0\right) \vee\left(l_{3} \wedge t_{2} \geq 0\right) \vee\left(l_{6} \wedge \theta=1000\right) \vee\left(l_{6} \vee 100 \leq \theta \leq 998\right)\right) \\
& \wedge\left(\left(l_{1} \wedge t_{1} \geq 1\right) \vee\left(l_{2}\right) \vee\left(l_{5} \wedge \theta=100\right)\right)
\end{aligned}
$$

### 2.3 Checking Safety Properties

We have presented the methods for computing component invariants and interaction invariants. We have also presented an abstraction technique to compute interaction invariants of systems with data. In this section, we will show the procedure for the verification of safety properties by using these invariants.

We give a sketch of a semi-algorithm in Algorithm 1 allowing to prove invariance of a safety property $\Phi$ by iterative application of the verification rule (2.3). The semi-algorithm
takes a system $\left\langle\gamma\left(B_{1}, \ldots, B_{n}\right)\right.$, Init $\rangle$ and a predicate $\Phi$. It iteratively computes invariants of the form $\mathcal{X}=\Psi \wedge\left(\bigwedge_{i=1}^{n} \Phi_{i}\right)$ where $\Psi$ is an interaction invariant and $\Phi_{i}$ an invariant of component $B_{i}$. It consists of the following steps:

```
function \(\operatorname{verify}\left(\mathcal{S}=\left\langle\gamma\left(B_{1}, \ldots, B_{n}\right)\right.\right.\), Init \(\left.\rangle, \Phi\right)\)
begin
    for each component \(B_{i}\) do
        \(\Phi_{i}=\) true;
    end
    while true do
        for each component \(B_{i}\) do
            compute component invariants \(\Phi_{i}^{\prime}\);
            \(\Phi_{i}=\Phi_{i} \wedge \Phi_{i}^{\prime} ;\)
            compute abstraction \(B_{i}^{a}=\alpha\left(B_{i}, \Phi_{i}\right)\);
        end
        compute abstract system \(\left\langle\gamma\left(B_{1}^{a}, \ldots, B_{n}^{a}\right)\right.\), Init \(\left.^{a}\right\rangle\);
        compute interaction invariants \(\Psi^{a}\) of \(\left\langle\gamma\left(B_{1}^{a}, \ldots, B_{n}^{a}\right)\right.\), Init \(\left.^{a}\right\rangle\);
        concretize abstract invariants \(\Psi=\alpha^{-1}\left(\Psi^{a}\right)\);
        \(\mathcal{X}=\Psi \wedge\left(\bigwedge_{i=1}^{n} \Phi_{i}\right) ;\)
        if \(\neg \Phi \wedge \mathcal{X}\) is unsatisfable then
            return \(\Phi\) is an invariant;
        end
        else if receive stop or timeout then
            return inconclusive;
        end
    end
end
```

Algorithm 1: Checking Invariance-Property $\Phi$

- Step 0: every component invariant $\Phi_{i}$ is initially true (line 4).
- Step 1: for each component $B_{i}$, a stronger invariant $\Phi_{i}^{\prime}$ is computed (line 8) and $\Phi_{i}$ is updated by conjoining with $\Phi_{i}^{\prime}$ (line 9).
- Step 2: $\Phi_{i}$ is then used, together with $B_{i}$, to compute an abstraction of $B_{i}$ by the abstract function $\alpha$ (line 10).
- Step 3: the abstract system $\left\langle\gamma\left(B_{1}^{a}, \ldots, B_{n}^{a}\right)\right.$, Init $\left.^{\alpha}\right\rangle$ is computed from the set of abstract components, the set of interactions $\gamma$ and the initial condition Init (line 12).
- Step 4: interaction invariants are computed from the abstract system (line 13) and their concretization provides concrete interaction invariants of the concrete system (line 14).
- Step 5: the global invariant $\mathcal{X}=\Psi \wedge\left(\bigwedge_{i=1}^{n} \Phi_{i}\right)$ is used to verify the invariance of $\Phi$ by checking whether $\neg \Phi \wedge \mathcal{X}$ is unsatisfiable. If it is, the verification terminates and results the invariance of $\Phi$ (line 17). If $\mathcal{X}$ is not strong enough for proving that $\Phi$ is an invariant then either a new iteration with stronger $\Phi_{i}$ is started by returning to Step 1 or we can stop. In this case, we cannot conclude about invariance of $\Phi$ and inconclusive result is returned (line 20).


### 2.4 Application for Checking Deadlock-Freedom

We present an application of the method for checking deadlock-freedom. To guarantee that global deadlocks are exclusively due to synchronizations, we use the local deadlockfreedom property of atomic components, that is if an atomic component reaches a state, it is always able to go out by at least one of the out-going transitions from that state. This property is checked for all the atomic components of the system to be verified before checking the global deadlock-freedom property of the system.

According to Definition 13, the predicate DIS characterizes a set of deadlock states, i.e a set of states from which no interaction can take place. A system is deadlock-free if the predicate $\neg D I S$ is an invariant because in that case, all the reachable states of the system satisfy $\neg D I S$ which means that no state in $D I S$ is reachable.

To check that $\neg D I S$ is an invariant, we need a stronger invariant $\Phi$ such that $\Phi \Rightarrow \neg D I S$ or equivalently $\Phi \wedge D I S=$ false. We apply the algorithm 1 and here the invariance property $\Phi$ to be proved is $\neg D I S$.

Example 25 This example illustrates the verification of deadlock-freedom of the Temperature Control System. The DIS predicate of the system is as follows (Example 7):

$$
\begin{aligned}
D I S= & \left(\neg\left(l_{5} \wedge \theta<1000\right)\right) \wedge\left(\neg\left(l_{6} \wedge \theta=100\right) \vee \neg l_{2}\right) \\
& \wedge\left(\neg\left(l_{6} \wedge \theta>100\right)\right) \wedge\left(\neg\left(l_{5} \wedge \theta=1000\right) \vee \neg\left(l_{3} \wedge t_{2} \geq 3600\right)\right) \quad \Phi=\Phi_{1} \wedge \Phi_{2} \wedge \\
& \wedge\left(\neg\left(l_{5} \wedge \theta=1000\right) \vee \neg\left(l_{1} \wedge t_{1} \geq 3600\right)\right) \wedge\left(\neg\left(l_{6} \wedge \theta=100\right) \vee \neg l_{4}\right)
\end{aligned}
$$

$\Phi_{3}$ is the conjunction of the component invariants given in example 10. The predicate $\Phi \wedge D I S$ is satisfiable and it is the disjunction of the following terms:

1. $\left(l_{1} \wedge 0 \leq t_{1}<3600\right) \wedge\left(l_{3} \wedge 0 \leq t_{2}<3600\right) \wedge\left(l_{6} \wedge \theta=100\right)$
2. $\left(l_{1} \wedge 0 \leq t_{1}<3600\right) \wedge\left(l_{4} \wedge t_{2} \geq 3600\right) \wedge\left(l_{5} \wedge \theta=1000\right)$
3. $\left(l_{1} \wedge 0 \leq t_{1}<3600\right) \wedge\left(l_{3} \wedge 0 \leq t_{2}<3600\right) \wedge\left(l_{5} \wedge \theta=1000\right)$
4. $\left(l_{2} \wedge t_{1} \geq 3600\right) \wedge\left(l_{3} \wedge 0 \leq t_{2}<3600\right) \wedge\left(l_{5} \wedge \theta=1000\right)$
5. $\left(l_{2} \wedge t_{1} \geq 3600\right) \wedge\left(l_{4} \wedge t_{2} \geq 3600\right) \wedge\left(l_{5} \wedge \theta=1000\right)$

Each one of the above terms represents a family of possible deadlocks. To decrease the number of potential deadlocks, we find a new invariant $\Phi^{\prime}$ stronger than $\Phi$, such that $\Phi^{\prime}=\Phi \wedge \Psi$, where $\Psi$ is the global interaction invariant obtained in Example 24.

The predicate $\Phi^{\prime} \wedge D I S$ is reduced to:
6. $\left(l_{1} \wedge 1 \leq t_{1}<3600\right) \wedge\left(l_{3} \wedge 1 \leq t_{2}<3600\right) \wedge\left(l_{5} \wedge \theta=1000\right)$
7. $\left(l_{1} \wedge 1 \leq t_{1}<3600\right) \wedge\left(l_{4} \wedge t_{2} \geq 3600\right) \wedge\left(l_{5} \wedge \theta=1000\right)$
8. $\left(l_{2} \wedge t_{1} \geq 3600\right) \wedge\left(l_{3} \wedge 1 \leq t_{2}<3600\right) \wedge\left(l_{5} \wedge \theta=1000\right)$

Finally, it can be checked by using finite state reachability analysis on an abstraction of the system without variables, that only the first term represents feasible deadlocks, the two other being spurious. This term characterizes deadlock configurations leading to complete shutdown.

### 2.5 Summary

We have introduced our compositional method for verifying safety properties of componentbased systems. The method is based on two kinds of invariants characterizing both local and global constraints of systems: component invariants and interaction invariants. Component invariants are over approximation of reachable states sets of components and are computed by using forward propagation. Interaction invariants characterize global constraints related to strong synchronizations between components. We have proposed two methods based on Positive Mapping and Fixed-point for computing symbolically the set of interaction invariants.

The methods for computing interaction invariants are applied for systems without data. For systems with data, we need to abstract away data before applying the methods. The concrete invariants are then obtained by concretizing abstract ones. An abstraction technique based on the component invariants is therefore introduced.

We have also presented an algorithm to verify safety properties by using component invariants and interaction invariants.

Finally, we shown an application of the compositional verification method on checking deadlock-freedom. We illustrated the method on an example, the Temperature Control System. We proved that the system is not deadlock-free and provided potential deadlocks of the system.

The innovation of our compositional verification method is that we use interaction invariants to characterize contexts of individual components. By using component invariants and interaction invariants, we have successfully combined constraints on both local and global aspects of systems. Moreover, the techniques we use to analyze systems are lightweight, hence it is possible to increase the size and the complexity of systems that can be handled.

The next chapter presents incremental construction and verification methods.

## CHAPTER 3

## Incremental Construction and Verification

## Chapter Abstract

Incremental construction is used to deal with the complexity of systems in the construction phase. The idea is that a system can be build from sub-systems by adding interactions. During the construction process, the verification is necessary to detect an error as soon as it appears. Therefore, the verification should be integrated into the construction process. We first provide, in this Chapter, an incremental construction method together with the conditions to preserve already-established properties during the construction. Since these conditions are not always satisfied and the preserved invariants are often not strong enough, we then propose methods for computing incrementally invariants of a new systems from already-computed invariants of its sub-systems. The reuse of invariants reduces significantly the verification cost in both time and memory usage.

## Résumé du Chapitre

La construction incrémentale est utilisée pour faire face à la complexité des systèmes dans la phase de construction. L'idée est qu'un système peut être construit à partir de soussystèmes en ajoutant les interactions. Pendant le processus de construction, la vérification est nécessaire pour détecter une erreur dès qu'elle apparaît. Par conséquent, la vérification doit être intégrée dans le processus de construction. Nous commençons par donner, dans ce Chapitre, une méthode de construction incrémentale ainsi que les conditions de préservation des invariants précédemment établis au cours de la construction. Car ces conditions ne sont pas toujours satisfaites et les invariants conservés ne sont souvent pas assez forts, nous allons ensuite proposer des méthodes pour calculer de façon incrémentale des invariants d'un nouveau système à partir des invariants précédemment calculés de ses sous-systèmes. La réutilisation d'invariants réduit significativement le coût de vérification en temps et en mémoire utilisée.

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Compositional verification approach avoids the state space explosion problem and therefore allows increasing significantly the size and complexity of the systems that can be handled. In the previous chapter, we presented our compositional verification method which is based on the use of invariants. Though we use lightweight techniques for computing invariants, the method may still suffer from the fast increasing size and complexity of systems. The computation of invariants from scratch for a system having thousands of components might be very expensive.

Moreover, nowadays the incremental construction deals with the complexity of the heterogeneous and large-scale systems in the construction phase. The idea is that composite systems can be considered as the composition of smaller parts. The verification should take advantage of the incremental construction process by integrating verification into construction phase in order to detect as soon as possible errors in the model. The verification should also be able to reuse the established properties of sub-systems in the verification of the global system.

The motivation of the work in this chapter is to provide a systematic methodology for the incremental construction and verification of component-based systems. We first formalize the incremental construction of component-based construction, then we propose rules on invariant preservation from which the already established invariants would not be violated during the incremental construction.

However, when the incremental construction is beyond of the invariant preservation rules, the verification process is still required to ensure the system correctness and the generation of new invariants is needed. Therefore, we propose a method for the incremental computation of invariants. It takes advantage of the system structure for coping with complexity of monolithic verification. The incremental method allows reusing the computed invariants from sub-systems which can be considered as the decomposed parts according to concepts of the incremental construction. The reuse of established invariants reduces significantly both time and memory usage in the verification of the system.

Figure 3.1 illustrates the idea of the method for the incremental computation of invariants. First we consider the composite component $B_{12}=\gamma_{12}\left(B_{1}, B_{2}\right)$ which is built from two constituents $B_{1}, B_{2}$ with its established invariants $I_{1}, I_{2}$. The incremental method allows computing invariants of the composite component $B_{12}$ from invariants of its constituents: $I_{12}=F\left(I_{1}, I_{2}\right)$. Similarly, if $B_{12}$ is composed with another constituent $B_{3}$ to build another composite component $B_{123}$, then the invariants of $B_{123}$ are also computed from the estab-


Figure 3.1: Incremental Verification Idea
lished invariants of $B_{12}$ and $B_{3}: I_{123}=F\left(I_{12}, I_{3}\right)$. It means that invariants of a composite components are always computed from the invariants of its constituents.

The chapter is organized as follows: first we give a formal definition of incremental construction based on the operation of increment of a connector. At some stage of the construction, a component can be transformed only by an increment operation which enforces synchronization between interactions of its connectors. The construction is hierarchical: increments can be applied ether at the same level or at different levels. We also provide rules to preserve the invariants during the incremental construction. Since the invariant preservation rules are not always satisfied by the systems, we present, in section 2, a method for incremental computation of invariants of a composite component from invariants of its constituent components. We finish the chapter by some conclusions.

In this chapter, the incremental verification method is considered for systems without data. For systems with data, we need to use the abstraction technique presented in the previous chapter and then apply the method on abstract systems. We also recall that, to simplify notation, for a connector $\gamma=\left\{a_{1}, \ldots, a_{n}\right\}$, we write $\gamma=a_{1}+\cdots+a_{n}$.

### 3.1 Incremental Construction and Invariant Preservation

In component-based systems, the construction of composite component is hierarchical and step-wise. We assume that a system is obtained from a set of atomic components represented by their behavior by adding progressively interactions. It is important to ensure the system correctness by verification during the construction in order to detect early errors. At some stage of the construction we have a component $\gamma(B)$ and a set of established invariants. We want to preserve the already established invariants after adding new interactions - an incremental modification of the behavior. In this section, we present the incremental construction framework and the rules for that the established invariants are preserved.

### 3.1. INCREMENTAL CONSTRUCTION AND INVARIANT PRESERVATION

### 3.1.1 Incremental Construction

In the incremental construction of component-based systems, layers of connectors are applied to build the system bottom-up. $\gamma_{\perp}(B)$ can be viewed as the initial composite component obtained as the interleaving of individual components, where $B=\left(B_{1}, \ldots, B_{n}\right)$. If at some stage of the construction, we have obtained a component $\gamma(B)$, the construction process continues by enforcing new synchronizations on interactions of $\gamma$. We call these interactions generated by enforcing new synchronizations increments. When building a composite system in a bottom-up manner, it is essential that some already enforced synchronizations are not relaxed when increments are added. To guarantee this property, we propose the notion of forbidden interactions.

Definition 25 (Closure and Forbidden Interactions) Let $\gamma$ be a connector.

- The closure $\gamma^{c}$ of $\gamma$, is the set of the non-empty interactions contained in some interaction of $\gamma$. That is $\gamma^{c}=\{a \neq \emptyset \mid \exists b \in \gamma . a \subseteq b\}$.
- The forbidden interactions $\gamma^{f}$ of $\gamma$ is the set of the interactions strictly contained in all the interactions of $\gamma$. That is $\gamma^{f}=\gamma^{c}-\gamma$.

It is easy to see that for two connectors $\gamma_{1}$ and $\gamma_{2}$, we have $\left(\gamma_{1}+\gamma_{2}\right)^{c}=\gamma_{1}^{c}+\gamma_{2}^{c}$ and $\left(\gamma_{1}+\gamma_{2}\right)^{f}=\left(\gamma_{1}+\gamma_{2}\right)^{c}-\gamma_{1}-\gamma_{2}$.

Example 26 Consider a connector $\gamma=p_{1} p_{2}+p_{3}+p_{4}$, we have $\gamma^{c}=p_{1}+p_{2}+p_{3}+p_{4}$ and $\gamma^{f}=\gamma^{c}-\gamma=p_{1}+p_{2}$.

In our theory, a connector describes a set of interactions and, by default, also those interactions in where only one component can make progress. This assumption allows us to define new increments in terms of existing interactions.

Definition 26 (Increments) Consider a connector $\gamma$ over $B$ and let $\delta \subseteq 2^{\gamma}$ be a set of interactions. We say $\delta$ is an increment over $\gamma$ if for any interaction $a \in \delta$ we have interactions $b_{1}, \ldots, b_{n} \in \gamma$ such that $\bigcup_{i=1}^{n} b_{i}=a$.

In practice, one has to make sure that existing interactions defined by $\gamma$ will not break the synchronizations that are enforced by the increment $\delta$. For doing so, we remove from the original connector $\gamma$ all the interactions that are forbidden by $\delta$. This is done with the operation of Layering, which describes how an increment can be added to an existing set of interactions without breaking synchronization enforced by the increment. Formally, we have the following definition.

Definition 27 (Layering) Given a connector $\gamma$ and an increment $\delta$ over $\gamma$, the new set of interactions obtained by combining $\delta$ and $\gamma$, also called layering, is given by the following set $\delta \gamma=\left(\gamma-\delta^{f}\right)+\delta$ the incremental construction by layering, that is, the incremental modification of $\gamma$ by $\delta$.

The above definition describes one-layer incremental construction. By the successive application of increments, we can construct the system with multiple layers.


Figure 3.2: Incremental construction example

Example 27 Consider the example presented in Figure 3.2, let $\gamma=a_{0}+b_{0}+c_{0}+d_{0}+$ $a_{1}+b_{1}+c_{1}+d_{1}$ and $\delta_{1}=a_{0} a_{1}+b_{0} b_{1}$, we have $\delta_{1} \gamma=a_{0} a_{1}+b_{0} b_{1}+c_{0}+d_{0}+c_{1}+d_{1}$.

Besides the fusion of interactions, incremental construction can also be obtained by first combining the increments and then apply the result to the existing system. This process is called Superposition. Formally, we have the following definition.

Definition 28 (Superposition) Given two increments $\delta_{1}, \delta_{2}$ over a connector $\gamma$, the operation of superposition between $\delta_{1}$ and $\delta_{2}$ is defined by $\delta_{1}+\delta_{2}$.

Superposition can be seen as a composition between increments, with looser coupled relation. If we combine the superposition of increments with the layering proposed in Definition 27 , then we obtain an incremental construction from a set of increments. Formally, we have the following proposition.

Proposition 9 Let $\gamma$ be a connector over B, the incremental construction by the superposition of $n$ increments $\left\{\delta_{i}\right\}_{1 \leq i \leq n}$ is given by

$$
\begin{equation*}
\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma=\left(\gamma-\left(\sum_{i=1}^{n} \delta_{i}\right)^{f}\right)+\sum_{i=1}^{n} \delta_{i} \tag{3.1}
\end{equation*}
$$

The above proposition provides a way to transform incremental construction by a set of increments into the separate constituents, where $\gamma-\left(\Sigma_{i=1}^{n} \delta_{i}\right)^{f}$ is the set of interactions that are not tightened during the incremental construction process.

We conclude the subsection with the following example.
Example 28 In the example of Figure 3.2, let $\gamma=a_{0}+b_{0}+c_{0}+d_{0}+a_{1}+b_{1}+c_{1}+d_{1}$. Two increments are $\delta_{1}=a_{0} a_{1}+b_{0} b_{1}$ and $\delta_{2}=c_{0} c_{1}+d_{0} d_{1}$. When we consider two increments together, we have $\left(\delta_{1}+\delta_{2}\right) \gamma=a_{0} a_{1}+b_{0} b_{1}+c_{0} c_{1}+d_{0} d_{1}$.

Notice that $\left(\delta_{1}+\delta_{2}\right) \gamma \neq \delta_{1} \gamma+\delta_{2} \gamma$. For example, $\delta_{1} \gamma+\delta_{2} \gamma=a_{0} a_{1}+b_{0} b_{1}+c_{0} c_{1}+d_{0} d_{1}+$ $a_{0}+b_{0}+c_{0}+d_{0}+a_{1}+b_{1}+c_{1}+d_{1}$ in Example 28. The reason is that $\delta_{1} \gamma+\delta_{2} \gamma$ means the composition of two connectors.

### 3.1. INCREMENTAL CONSTRUCTION AND INVARIANT PRESERVATION

### 3.1.2 Invariant Preservation in Incremental Construction

In Sub-section 3.1.1, we have presented a methodology for the incremental design of composite systems. In this section, we study the concept of invariant preservation. More precisely, we propose sufficient conditions that guarantee that already satisfied invariants are not violated when new interactions are added to the design.

We start by introducing the looser synchronization preorder on connectors, which we will use to characterize invariant preservation. As we have seen, interactions characterize the behavior of a composite component. We observe that if two interactions do not contain the same port, the execution of one interaction will not block the execution of the other interaction. Formally, we have the following definition of conflict-free interactions.

Definition 29 (Conflict-free Interactions) Given a connector $\gamma$, let $a_{1}, a_{2} \in \gamma$, if $a_{1} \cap a_{2}=\emptyset$, we say that there is no conflict between $a_{1}$ and $a_{2}$. If there is no conflict between any interactions of $\gamma$, we say that $\gamma$ is conflict-free.

The conflict-free connector ensures that the execution of one interaction will not disable other interactions. For example, connector $\gamma=p_{1} p_{2}+p_{2} p_{3}$ is not conflict-free. The execution of $p_{1} p_{2}$ makes a transition labeled by $p_{2}$ move to its target location from the source location and $p_{2} p_{3}$ may not be enabled.

We now propose a preorder relation that allows to guarantee the absence of conflicts when new interactions are added. Formally, we have the following definition.

Definition 30 (Looser synchronization Pre-order) We define the looser synchronization pre-order $\preccurlyeq \subseteq 2^{2^{P}} \times 2^{2^{P}}$. For two connectors $\gamma_{1}, \gamma_{2}, \gamma_{1} \preccurlyeq \gamma_{2}$ if for any interaction $a \in \gamma_{2}$, there exist interactions $b_{1}, \ldots, b_{n} \in \gamma_{1}$, such that $a=\bigcup_{i=1}^{n} b_{i}$ and there is no conflict between any $b_{i}$ and $b_{j}$, where $1 \leq i, j \leq n$ and $i \neq j$. We simply say that $\gamma_{1}$ is looser than $\gamma_{2}$.

The above definition requires that the stronger synchronization should be obtained by the fusion of conflict-free interactions. The reason is that the execution of interactions may be disturbed by two conflict interactions, i.e., the execution of one interaction could block the transitions issued from the other interaction. However, if we fuse them together, it means that the transitions of both interactions can be executed, which violates the constraints of the previous behavior.

Example 29 For two connectors $\gamma_{1}=\left\{p_{1} p_{2}, p_{3} p_{4}, p_{5} p_{6}, p_{7} p_{8}\right\}$ which is conflict-free and $\gamma_{2}=\left\{p_{1} p_{2} p_{3} p_{4}, p_{5} p_{6} p_{7} p_{8}\right\}$, we have $\gamma_{1} \preccurlyeq \gamma_{2}$.

It is easy to see that if $\gamma_{1}, \gamma_{2}, \gamma_{3}, \gamma_{4}$ are connectors such that $\gamma_{1} \preccurlyeq \gamma_{2}$, and $\gamma_{3} \preccurlyeq \gamma_{4}$, then we have $\gamma_{1}+\gamma_{3} \preccurlyeq \gamma_{2}+\gamma_{4}$.

Definition 31 (Reachable States) Given a connector $\gamma$ over a component B, $L$ is the set of states of $B$, we define Reach $(l, \gamma(B))=\left\{l_{i} \mid \exists a_{i} \in \gamma \wedge l \xrightarrow{a_{i}}{ }^{*} l_{i}\right\} \cup\{l\}$ the set of reachable states from $l \in L$ by any interaction of $\gamma$.

The above definition provides a notation to record the set of reachable states from a state $l$ through all possible interactions in $\gamma(B)$. If there is no executable interaction from $l$, we have that $\operatorname{reach}(l, \gamma(B))=\{l\}$.
Lemma 2 Given two connectors $\gamma_{1}, \gamma_{2}$ over $B$, if $\gamma_{1} \preccurlyeq \gamma_{2}$, we have Reach $\left(l, \gamma_{2}(B)\right) \subseteq$ Reach $\left(l, \gamma_{1}(B)\right)$ for any $l \in L$,

Proof If there exists a path from $l \in L$ in $\gamma_{2}(B)$, we have $l \xrightarrow{a_{1}} l_{1} \xrightarrow{a_{2}} \cdots \xrightarrow{a_{m}} l_{m}$, where $a_{i} \in \gamma_{2}$. Because $\gamma_{1} \preccurlyeq \gamma_{2}$, for any $a_{i}$, we have a set of interactions $b_{j} \in \gamma_{1}$ such that $a_{i}=\bigcup_{j=1}^{k} b_{j}$. From any state $l_{i}$, there exists a set of interactions $\bigcup_{j=1}^{k} b_{j}$ such that $l_{i} \xrightarrow{b_{1}} \cdots \xrightarrow{b_{k}} l_{i+1}$. Therefore, we conclude that $\operatorname{Reach}\left(l, \gamma_{2}(B)\right) \subseteq \operatorname{Reach}\left(l, \gamma_{1}(B)\right)$ for any $l \in L$.

This lemma shows that from the same state the set of reachable states under a tighter connector is always a subset of reachable states under a looser connector.

We now propose the following proposition which establishes a link between the Looser Synchronization Preorder and invariant preservation.
Proposition 10 Let $\gamma_{1}, \gamma_{2}$ be two connectors over $B$. If $\gamma_{1} \preccurlyeq \gamma_{2}$, we have $\operatorname{inv}\left(\gamma_{1}(B)\right.$, $I) \Rightarrow \operatorname{inv}\left(\gamma_{2}(B), I\right)$.

Proof Let Reach $\left(l, \gamma_{2}(B)\right)$ be the set of reachable states from the path started from $l \in$ $L$ in $\gamma_{2}(B)$. Because Reach $\left(l, \gamma_{2}(B)\right) \subseteq \operatorname{Reach}\left(l, \gamma_{1}(B)\right)$, for any $l^{\prime} \in \operatorname{Reach}\left(l, \gamma_{2}(B)\right)$, $l^{\prime}$ is reachable in $\gamma_{1}(B)$. As $\operatorname{inv}\left(\gamma_{1}(B), I\right)$, we have $I\left(l^{\prime}\right)$. Then we can conclude that $\operatorname{inv}\left(\gamma_{2}(B), I\right)$.

The above proposition, which will be used in the incremental design, simply says that if an invariant is satisfied, then it will remain when combinations of conflict-free interactions are added (following our incremental methodology) to the connector. This is not surprising as the tighter connector can only restrict the behaviors of the composite system.

We now switch to the more interesting problem of providing sufficient conditions to guarantee that invariants are preserved by the incremental construction.

Proposition 11 Let $\gamma$ be a connector over $B$ and $\delta$ be an increment of $\gamma$ such that $\gamma \preccurlyeq \delta$, then we have $\gamma \preccurlyeq \delta \gamma$.

Proof Because $\gamma \preccurlyeq \gamma-\delta^{f}$, we have $\gamma \preccurlyeq\left(\gamma-\delta^{f}\right)+\delta=\delta \gamma$.
The above proposition, together with Proposition 10, says that the addition of an increment preserves the invariant if the initial connector is looser than the increment.

We continue our study and discuss the invariant preservation between the components obtained from superposition of increments and separately applying increments over the same set of components. We use the following definition.

Definition 32 (Interference-free Connectors) Given two connectors $\gamma_{1}, \gamma_{2}$, for any $a_{1} \in \gamma_{1}, a_{2} \in \gamma_{2}$, if either $a_{1}$ and $a_{2}$ are conflict-free or $a_{1}=a_{2}$, we say that $\gamma_{1}$ and $\gamma_{2}$ are interference-free.

### 3.1. INCREMENTAL CONSTRUCTION AND INVARIANT PRESERVATION

This definition considers a relation between two connectors. We observe that two interferencefree connectors will not break or block the synchronizations specified by each other. Though we require that the interactions between $\gamma_{1}$ and $\gamma_{2}$ are conflict-free, $\gamma_{1}$ or $\gamma_{2}$ respectively can contain conflict interactions. For example, consider two connectors $\gamma_{1}=p_{1} p_{2}+p_{2} p_{3}$, $\gamma_{2}=p_{4} p_{5} . \gamma_{1}$ is not conflict-free, but $\gamma_{1}$ and $\gamma_{2}$ are interference-free.

Lemma 3 Given two interference-free connectors $\gamma_{1}, \gamma_{2}$, we have $\gamma_{1} \cap \gamma_{2}^{f}=\emptyset$ and $\gamma_{2} \cap \gamma_{1}^{f}=$ $\emptyset$, and $\left(\gamma_{1}+\gamma_{2}\right)^{f}=\gamma_{1}^{f}+\gamma_{2}^{f}$.

Proof Since $\gamma_{1}$ and $\gamma_{2}$ are interference-free, if $\gamma_{1} \cap \gamma_{2}=\emptyset$, we have $\gamma_{1} \cap \gamma_{2}^{f}=\emptyset$ and $\gamma_{2} \cap \gamma_{1}^{f}=\emptyset$. If $\gamma_{1} \cap \gamma_{2} \neq \emptyset$, for any $a \in \gamma_{1} \cap \gamma_{2}$, we know that $a \notin \gamma_{1}^{f}$ and $a \notin \gamma_{2}^{f} . \gamma_{1} \cap \gamma_{2}^{f}=\emptyset$ and $\gamma_{2} \cap \gamma_{1}^{f}=\emptyset$ are still correct.

According to Definition 25, we have $\left(\gamma_{1}+\gamma_{2}\right)^{f}=\gamma_{1}^{c}+\gamma_{2}^{c}-\left(\gamma_{1}+\gamma_{2}\right)=\left(\gamma_{1}^{c}-\left(\gamma_{1}+\gamma_{2}\right)\right)+$ $\left(\gamma_{2}^{c}-\left(\gamma_{1}+\gamma_{2}\right)\right)$. Because $\gamma_{1}$ and $\gamma_{2}$ are interference-free, $\gamma_{1}^{c}-\left(\gamma_{1}+\gamma_{2}\right)=\gamma_{1}^{c}-\gamma_{1}=\gamma_{1}^{f}$ and $\gamma_{2}^{c}-\left(\gamma_{1}+\gamma_{2}\right)=\gamma_{2}^{f}$. So we have $\left(\gamma_{1}+\gamma_{2}\right)^{f}=\gamma_{1}^{f}+\gamma_{2}^{f}$.

We now present the main result of the section.
Proposition 12 Consider two increments $\delta_{1}$, $\delta_{2}$ over $\gamma$ such that $\gamma \preccurlyeq \delta_{1}$ and $\gamma \preccurlyeq \delta_{2}$, if $\delta_{1}$ and $\delta_{2}$ are interference-free, and $\operatorname{inv}\left(\delta_{1} \gamma(B), I_{1}\right), \operatorname{inv}\left(\delta_{2} \gamma(B), I_{2}\right)$, we have inv $\left(\left(\delta_{1}+\right.\right.$ $\left.\left.\delta_{2}\right) \gamma(B), I_{1} \wedge I_{2}\right)$.

Proof We will show that $\delta_{1} \gamma \preccurlyeq\left(\delta_{1}+\delta_{2}\right) \gamma$ and $\delta_{2} \gamma \preccurlyeq\left(\delta_{1}+\delta_{2}\right) \gamma$, then the conclusion can be obtained Proposition 10.

Because $\delta_{1}$ and $\delta_{2}$ are interference-free, we have $\left(\delta_{1}+\delta_{2}\right)^{f}=\delta_{1}^{f}+\delta_{2}^{f}$, then $\gamma-\left(\delta_{1}+\delta_{2}\right)^{f}=$ $\gamma-\left(\delta_{1}^{f}+\delta_{2}^{f}\right)$. As $\gamma-\left(\delta_{1}^{f}+\delta_{2}^{f}\right) \subseteq \gamma-\delta_{1}^{f}$, we obtain that $\gamma-\delta_{1}^{f} \preccurlyeq \gamma-\left(\delta_{1}^{f}+\delta_{2}^{f}\right)$ and $\gamma-\delta_{1}^{f}+\delta_{1} \preccurlyeq \gamma-\left(\delta_{1}^{f}+\delta_{2}^{f}\right)+\delta_{1}$. Because $\delta_{1}$ and $\delta_{2}$ are interference-free, $\delta_{2} \cap \delta_{1}^{f}=\emptyset$ and $\gamma \preccurlyeq \delta_{2}$, we have $\gamma-\delta_{1}^{f} \preccurlyeq \delta_{2}$. So $\gamma-\delta_{1}^{f}+\delta_{1} \preccurlyeq \gamma-\left(\delta_{1}^{f}+\delta_{2}^{f}\right)+\delta_{1}+\delta_{2}$. The same rule can be applied to $\delta_{2} \gamma$. Therefore, we have $\delta_{1} \gamma \preccurlyeq\left(\delta_{1}+\delta_{2}\right) \gamma$ and $\delta_{2} \gamma \preccurlyeq\left(\delta_{1}+\delta_{2}\right) \gamma$, thus $\operatorname{inv}\left(\left(\delta_{1}+\delta_{2}\right) \gamma(B), I_{1} \wedge I_{2}\right)$.

The above proposition considers a set of increments $\left\{\delta_{i}\right\}_{1 \leq i \leq n}$ over $\gamma$ that are interferencefree. The proposition says that if for any $\delta_{i}$ the separate application of increments over component $\delta_{i} \gamma(B)$ preserves the original invariants of $\gamma(B)$, then the system obtained from considering the superposition of increments over $\gamma$ preserves the conjunction of the invariants of individual increments.

We now briefly study the relation between the looser synchronization preorder and property preservation. Figure 3.3 shows the three ingredients of the BIP toolset, that are (1) priorities, which we will not use here, (2) interactions, and (3) behaviors of components. We shall see that the looser synchronization preorder preserves invariants (Proposition 12). This means that the preorder preserves the so-called reachability properties. On the other hand, the preorder does not preserve deadlocks. Indeed, adding new interactions may lead to the addition of new deadlock conditions. Given two connectors $\gamma_{1}$ and $\gamma_{2}$ over component


Figure 3.3: Invariant preservation for looser synchronization relation


Figure 3.4: Example for the invariant preservation
$B$ such that $\gamma_{2}$ is tighter than $\gamma_{1}$, i.e. $\gamma_{1} \preccurlyeq \gamma_{2}$, we can conclude that if $\gamma_{2}(B)$ is deadlockfree, then $\gamma_{1}(B)$ is deadlock-free. However, we can still reuse the invariant of $\gamma_{1}(B)$ as an over-approximation of the one of $\gamma_{2}(B)$.

Discussion. Though we can reuse invariants to save computation time, the invariants of the system with a looser connector may be too weak with respect to a new system obtained with a tighter connector. Consider the example given in Figure 3.4 and let $\gamma=$ $p_{1}+p_{2}+q_{1}+q_{2}, \delta_{1}=p_{1} p_{2}$, and $\delta_{2}=q_{1} q_{2}$. By using the technique presented in the next section, we shall see that the invariant for $\delta_{1} \gamma(B)$ and $\delta_{2} \gamma(B)$ is $\left(l_{1} \vee l_{2}\right) \wedge\left(l_{3} \vee l_{4}\right)$. By applying Proposition 12, we obtain that this invariant is preserved for $\left(\delta_{1}+\delta_{2}\right) \gamma(B)$. This invariant is weaker than the invariant $\left(l_{1} \vee l_{2}\right) \wedge\left(l_{3} \vee l_{4}\right) \wedge\left(l_{1} \vee l_{4}\right) \wedge\left(l_{2} \vee l_{3}\right)$ that is directly computed on $\left(\delta_{1}+\delta_{2}\right) \gamma(B)$. To overcome the above problem, we will now propose an approach that can be used to compute invariants in an incremental manner.

### 3.2 Incremental Computation of Invariants

In the previous section, we have shown the rules for invariant preservation during incremental construction. However, in general we can not always apply these rules. In this section, we put forward the method to compute incrementally invariants in more general
case of incremental component-based construction which can also be applied for incremental verification. The method is lightweight because we can reuse the computed invariants from constituents that can be considered as the decomposed parts according to concepts of the incremental construction.

### 3.2.1 Incremental Computation of BBCs

From BBC definition and Theorem 1 and 2 in the previous chapter we know that the invariants can be computed from connectors. And Section 3.1.1 shows that the increments will not be modified during the incremental construction. Therefore, we could start from increments to compute incrementally their BBCs and invariants. In this subsection, we provide a method for incremental computation of global BBCs.

Lemma 4 Given two connectors $\gamma_{1}, \gamma_{2}$ over $B$, we have

$$
\left|\left(\gamma_{1}+\gamma_{2}\right)(B)\right|=\left|\gamma_{1}(B)\right| \wedge\left|\gamma_{2}(B)\right|
$$

Proof By Definition 17, we have $\left|\left(\gamma_{1}+\gamma_{2}\right)(B)\right|=\bigwedge_{a \in\left(\gamma_{1}+\gamma_{2}\right)}|a(B)|=\bigwedge_{a \in \gamma_{1}}|a(B)| \wedge$ $\bigwedge_{a \in \gamma_{2}}|a(B)|=\left|\gamma_{1}(B)\right| \wedge\left|\gamma_{2}(B)\right|$.
The following proposition provides a method for obtaining the Boolean Behavioral Constraints taking into account component structure.

Proposition 13 Let $\gamma$ be a connector over B, the Boolean Behavioral Constraint for the system obtained by superposition of $n$ increments $\left\{\delta_{i}\right\}_{1 \leq i \leq n}$ can be written as

$$
\begin{equation*}
\left|\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma(B)\right|=\left|\left(\gamma-\left(\sum_{i=1}^{n} \delta_{i}\right)^{f}\right)(B)\right| \wedge \bigwedge_{i=1}^{n}\left|\delta_{i}(B)\right| \tag{3.2}
\end{equation*}
$$

Proof By Equation 3.1, the union of $\gamma-\left(\sum_{i=1}^{n} \delta_{i}\right)^{f}$ and $\sum_{i=1}^{n} \delta_{i}$ is the set of interactions from the superposition of increments $\left\{\delta_{i}\right\}_{1 \leq i \leq n}$ over $\gamma$. The proof can be concluded by the application of Lemma 4.

Proposition 13 provides a way to decompose the computation of BBCs with respect to increments. The decomposition is based on the fact that different increments describe the interactions between different components. To simplify the notation, $\gamma-\left(\sum_{i=1}^{n} \delta_{i}\right)^{f}$ is represented by $\delta_{0}$. We have the following example.

Example 30 For Example 28, consider the two increments $\delta_{1}=a_{0} a_{1}+b_{0} b_{1}$ and $\delta_{2}=$ $c_{0} c_{1}+d_{0} d_{1}$. The composite component is $\left(\delta_{1}+\delta_{2}\right) \gamma=a_{0} a_{1}+b_{0} b_{1}+c_{0} c_{1}+d_{0} d_{1}$. The BBCs for interactions of $\delta_{1}$ and $\delta_{2}$ respectively are

$$
\begin{aligned}
& \left|\delta_{1}(B)\right|=\left(l_{0} \Rightarrow l_{1} \vee l_{4}\right) \wedge\left(l_{1} \Rightarrow l_{0} \vee l_{3}\right) \wedge\left(l_{3} \Rightarrow l_{1} \vee l_{4}\right) \wedge\left(l_{4} \Rightarrow l_{0} \vee l_{3}\right) \\
& \left|\delta_{2}(B)\right|=\left(l_{0} \Rightarrow l_{2} \vee l_{6}\right) \wedge\left(l_{2} \Rightarrow l_{0} \vee l_{5}\right) \wedge\left(l_{5} \Rightarrow l_{2} \vee l_{6}\right) \wedge\left(l_{6} \Rightarrow l_{0} \vee l_{5}\right)
\end{aligned}
$$

Because $\gamma-\left(\delta_{1}+\delta_{2}\right)^{f}=\emptyset$, we have $\left|\left(\delta_{1}+\delta_{2}\right) \gamma(B)\right|=\left|\delta_{1}(B)\right| \wedge\left|\delta_{2}(B)\right|$ where $\left|\delta_{1}(B)\right|$ and $\left|\delta_{2}(B)\right|$ are the BBCs above.

### 3.2.2 Incremental Computation of Invariants based on Positive Mapping

In the previous sub-section we have shown how to compute incrementally BBCs. The BBCs of a composite component can be obtained as the conjunction of BBCs of constituent components. In this sub-section, we propose a method which allows computing incrementally invariants from invariants obtained from increments. It does not consider the relations between increments which is more flexible.

To distinguish the shared variables between different BBCs, we define the common location variables shared by multiple connectors. We recall that for an interaction $a$, we denote by ${ }^{\bullet}$ (respectively $a^{\bullet}$ ) the set of source locations (respectively destination locations) of the transitions involved in $a$. We also extend this notation for connectors: $\gamma=\bigcup_{a \in \gamma}{ }^{\circ} a$ (respectively $\gamma^{\bullet}=\bigcup_{a \in \gamma} a^{\bullet}$ ).

We propose the following definition that will help in the process of reusing existing invariants.

Definition 33 (Common Location Variables $L_{c}$ ) Given a set of connectors $\left\{\gamma_{1}, \ldots\right.$, $\left.\gamma_{n}\right\}$, we define the set of common location variables by:

$$
L_{c}=\bigcup_{i, j \in[1, n] \wedge i \neq j}\left(\sup \left(\gamma_{i}\right) \cap \sup \left(\gamma_{j}\right)\right)
$$

where $\sup (\gamma)=\boldsymbol{\bullet} \cup \gamma{ }^{\bullet}$ is the set of locations involved in some interaction a of $\gamma$.

When the set of common state variables is empty, the connectors are really disjoint. And there is no common variables between their BBCs.

Example 31 For the example presented in Figure 3.2, consider $\delta_{1}=a_{0} a_{1}+b_{0} b_{1}$ and $\delta_{2}=c_{0} c_{1}+d_{0} d_{1}$, we have $\sup \left(\delta_{1}\right)=\bullet\left(a_{0} a_{1}\right) \cup\left(a_{0} a_{1}\right)^{\bullet} \cup \bullet\left(b_{0} b_{1}\right) \cup\left(b_{0} b_{1}\right)^{\bullet}=\left\{l_{0}, l_{1}, l_{3}, l_{4}\right\}$ and similarly $\sup \left(\delta_{2}\right)=\left\{l_{0}, l_{2}, l_{5}, l_{6}\right\}$, hence $L_{c}=\sup \left(\delta_{1}\right) \cap \sup \left(\delta_{2}\right)=\left\{l_{0}\right\}$.

Given a set of increments $\left\{\delta_{1}, \ldots, \delta_{n}\right\}$ over $\gamma$, according to Proposition 9 and Proposition 13, during the incremental construction, the set of interactions $\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma$ (respectively BBCs $\left.\left|\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma(B)\right|\right)$ can be obtained from the sets of interactions $\gamma-\sum_{i=1}^{n} \delta_{i}$ and $\left\{\delta_{i}\right\}_{i=1}^{n}$ (respectively from the set of BBCs $\left|\gamma-\sum_{i=1}^{n} \delta_{i}(B)\right|$ and $\left.\left\{\left|\delta_{i}(B)\right|\right\}_{i=1}^{n}\right)$. From that we propose a method for computing interaction invariants of $\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma(B)$ by using interaction invariants obtained from $\left|\gamma-\sum_{i=1}^{n} \delta_{i}(B)\right|$ and $\left\{\left|\delta_{i}(B)\right|\right\}_{i=1}^{n}$.

Proposition 14 Consider a composite component $B$. Let $\gamma$ be a connector for $B$ and assume a set of increments $\left\{\delta_{i}\right\}_{1 \leq i \leq n}$ over $\gamma(B)$. Let $\delta_{0}=\gamma-\left(\sum_{i=1}^{n} \delta_{i}\right)^{f}$, $I_{\delta_{i}}=\left\{\phi_{k}\right\}_{k \in \mathcal{I}_{i}}$, for $i=0, \ldots, n$, be the interaction invariants for each $\left|\delta_{i}(B)\right|, S_{\delta_{i}}=\left\{m_{k}\right\}_{k \in \mathcal{I}_{i}}$, for $i=$ $0, \ldots, n$, be the corresponding BBC-solutions, and let

- $L_{\phi}$ be the set of location variables in invariant $\phi$,
- $L_{c}$ be the common location variables between $\left\{\delta_{0}, \delta_{1}, \ldots, \delta_{n}\right\}$.

Then the interaction invariant of $\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma(B)$ is obtained as follows:

$$
I=\left(\begin{array}{cc}
\bigwedge_{i=0}^{n} & \bigwedge_{\substack{k \in \mathcal{I}_{i} \wedge \\
L_{\phi_{k}} \cap L_{c}=\emptyset}} \phi_{k}
\end{array} \bigwedge_{\substack{\left(k_{i 1}, \ldots, k_{i r}\right) \in \mathbb{D}}} \bigvee_{j=1}^{r} \phi_{k_{i j}}\right)
$$

where
$\mathbb{D}=\left\{\left(k_{i 1}, \ldots, k_{i r}\right) \mid\left(k_{i j} \in \mathcal{I}_{i j} \forall j=1 \ldots r\right) \wedge\left(L_{\phi_{k_{i j}}} \cap L_{c} \neq \emptyset\right) \wedge\left(\bigwedge_{j=1}^{r} m_{k_{i j}} \neq\right.\right.$ false $) \wedge$ $\left(\left(k_{i 1}, \ldots, k_{i r}\right)\right.$ is maximal $\left.)\right\}$.

Proof In every $S_{\delta_{i}}$, there exists a solution $m_{0 i}$ without any variables in the positive form, which has no invariant corresponding to. For any $\phi_{k}, k \in \mathcal{I}_{i}$, there exists $m_{k}$ such that $\phi_{k}=\widetilde{m_{k}^{p}}$. According to Proposition 13, the BBC-solution of $\left|\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma(B)\right|$ is $\bigwedge_{i=0}^{n} S_{\delta_{i}}=$ $\bigwedge_{i=0}^{n} \bigvee_{k \in \mathcal{I}_{i}} m_{k}=\bigvee_{k 0 \in \mathcal{I}_{0}, \ldots, k n \in \mathcal{I}_{n}} \bigwedge_{i=0}^{n} m_{k i}$.

- If an $m_{k i}$ does not contain any common location variables, there exists solution $m_{0 j}$ containing only negations in $S_{\delta_{j}}$ such that $i \neq j$ and $\left(\bigwedge_{j=0 \wedge j \neq i}^{n} m_{k i} \wedge m_{0 j}\right)^{p}=m_{k i}^{p}$, so $\phi_{k i}$ is one of the BBC-invariants of $\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma(B)$.
- If there is a maximal set $\left\{m_{k_{i 1}}, \ldots, m_{k_{i r}}\right\}, k_{i j} \in \mathcal{I}_{i j} \forall j=1 \ldots r$ such that all of them contain common location variables, and $\bigwedge_{j=1}^{r} m_{k_{i j}}=$ false, it is not a solution of $\left|\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma(B)\right|$. If $\bigwedge_{j=1}^{r} m_{k_{i j}} \neq$ false, we have $\left(\bigwedge_{j=1}^{r m_{k_{i j}}}\right)^{p}=\widetilde{\bigwedge_{j=1}^{r\left(\overline{\phi_{k_{i j}}}\right.}}=\bigvee_{j=1}^{r} \phi_{k_{i j}}$.

That is, if an interaction invariant obtained from $B B C s$ of $\delta_{i}$ contains only local location variables of $\delta_{i}$, then it is also invariant of the global system after the superposition because it is not affected by others increments. If a set of invariants $\left\{\phi_{0 j_{0}}, \ldots, \phi_{n j_{n}}\right\}$ obtained from $B B C s$ of $\left\{\delta_{0}, \ldots, \delta_{n}\right\}$ which all contain common location variables, they have influence on each other, hence we need to check the conjunction of the corresponding BBC-solutions. If the conjunction is not false, it is a global BBC-solution and then an interaction invariant of the global system is established by the disjunctions of all these invariants. Since each non common variable occurs only in one of the BBCs , and the conjunction of BBC -solutions is false or not depends only on the common location variables, we can delete the noncommon negative variables separately by the positive mapping in every BBC-solutions, which drastically reduces complexity of computation.

Example 32 In Example 30, we illustrate the BBCs for the two increments in the example presented in Figure 3.2 where $\delta_{1}=a_{0} a_{1}+b_{0} b_{1}$ and $\delta_{2}=c_{0} c_{1}+d_{0} d_{1}$. Here we show how to compute the interaction invariants of $\left(\delta_{1}+\delta_{2}\right) \gamma(B)$ from interaction invariants obtained from the increments (for this example $\gamma-\left(\delta_{1}+\delta_{2}\right)=\emptyset$ ).

According to Example 31, we have $L_{c}=\left\{l_{0}\right\}$. Let $S_{\delta_{1}}$, $S_{\delta_{2}}$ be the BBC-solutions for $\left|\delta_{1}(B)\right|$ and $\left|\delta_{2}(B)\right|$ respectively:

$$
\begin{aligned}
& S_{\delta_{1}}=\left(\overline{l_{0}} \wedge \overline{l_{1}} \wedge \overline{l_{3}} \wedge \overline{l_{4}}\right) \vee\left(l_{0} \wedge l_{1}\right) \vee\left(l_{1} \wedge l_{3}\right) \vee\left(l_{0} \wedge l_{4}\right) \vee\left(l_{3} \wedge l_{4}\right) \\
& S_{\delta_{2}}=\left(\overline{l_{0}} \wedge \overline{l_{2}} \wedge \overline{l_{5}} \wedge \overline{l_{6}}\right) \vee\left(l_{0} \wedge l_{2}\right) \vee\left(l_{2} \wedge l_{5}\right) \vee\left(l_{0} \wedge l_{6}\right) \vee\left(l_{5} \wedge l_{6}\right)
\end{aligned}
$$

Thesis
and $I_{\delta_{1}}, I_{\delta_{2}}$ be their interaction invariants:
$I_{\delta_{1}}=\left(l_{0} \vee l_{1}\right) \wedge\left(l_{0} \vee l_{4}\right) \wedge\left(l_{1} \vee l_{3}\right) \wedge\left(l_{3} \vee l_{4}\right) I_{\delta_{2}}=\left(l_{0} \vee l_{2}\right) \wedge\left(l_{0} \vee l_{6}\right) \wedge\left(l_{2} \vee l_{5}\right) \wedge\left(l_{5} \vee l_{6}\right)$
By applying $I_{\left(\delta_{1}+\delta_{2}\right) \gamma(B)}=F\left(I_{\gamma-\left(\delta_{1}+\delta_{2}\right) f}, I_{\delta_{1}}, I_{\delta_{2}}\right)$, we have:

- The invariants $\left(l_{1} \vee l_{3}\right),\left(l_{3} \vee l_{4}\right),\left(l_{2} \vee l_{5}\right),\left(l_{5} \vee l_{6}\right)$ do not contain any common location variables, so they are also interaction invariants of $\left(\delta_{1}+\delta_{2}\right) \gamma(B)$.
- The invariants $\left(l_{0} \vee l_{1}\right),\left(l_{0} \vee l_{4}\right)$ (with the corresponding BBC-solutions $\left.\left(l_{0} \wedge l_{1}\right),\left(l_{0} \wedge l_{4}\right)\right)$ and $\left(l_{0} \vee l_{2}\right),\left(l_{0} \vee l_{6}\right)$ (with the corresponding BBC-solutions $\left(l_{0} \wedge l_{2}\right),\left(l_{0} \wedge l_{6}\right)$ ) contain the common location variable $l_{0}$, and the conjunction between any two monomials from two groups of BBC-solutions are not false, hence the disjunction of any two invariants from two groups of invariants is an invariant of $\left(\delta_{1}+\delta_{2}\right) \gamma(B):\left(l_{0} \vee l_{1} \vee\right.$ $\left.l_{2}\right),\left(l_{0} \vee l_{1} \vee l_{6}\right),\left(l_{0} \vee l_{2} \vee l_{4}\right),\left(l_{0} \vee l_{4} \vee l_{6}\right)$.
Finally, the global interaction invariant of $\left(\delta_{1}+\delta_{2}\right) \gamma(B)$ is:

$$
\begin{aligned}
I_{\left(\delta_{1}+\delta_{2}\right) \gamma(B)} & =\left(l_{0} \vee l_{1} \vee l_{2}\right) \wedge\left(l_{0} \vee l_{1} \vee l_{6}\right) \wedge\left(l_{0} \vee l_{2} \vee l_{4}\right) \wedge\left(l_{0} \vee l_{4} \vee l_{6}\right) \\
& \wedge\left(l_{1} \vee l_{3}\right) \wedge\left(l_{3} \vee l_{4}\right) \wedge\left(l_{2} \vee l_{5}\right) \wedge\left(l_{5} \vee l_{6}\right)
\end{aligned}
$$

### 3.2.3 Incremental Computation of Invariants based on Fixed-point

In the previous chapter, we presented the method for computing interaction invariants based on fixed-point computation. According to Theorem 4, for a component $\gamma(B)$, the interaction invariants of $\gamma(B)$ can be obtained as the dual of the fixed-points $\widetilde{\mathbb{F}}\left(\mathbb{V}_{\gamma}, \bigvee_{l \in L} l\right)$. We call $\mathbb{F}\left(\mathbb{V}_{\gamma}, \bigvee_{l \in L} l\right)$ fixed-points of $\gamma(B)$. In this section, we provide a method which allows computing fixed-points of a composite component from the fixed-points of its constituents.

According to Proposition 9, the set of interactions $\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma$ can be obtained from the sets of interactions $\gamma-\left(\sum_{i=1}^{n} \delta_{i}\right)^{f}$ and $\left\{\delta_{i}\right\}_{i=1}^{n}$. From that, we propose a method which allows computing fixed-points of $\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma(B)$ from the fixed-points obtained from $\gamma-\left(\sum_{i=1}^{n} \delta_{i}\right)^{f}$ and $\left\{\delta_{i}\right\}_{i=1}^{n}$ over $B$.

First, for a set of connectors $\left\{\gamma_{i}\right\}_{i=1}^{n}$ over a component $B$, the following proposition allows getting the global image vector of $\left(\sum_{i=1}^{n} \gamma_{i}\right)(B)$ from the image vectors of $\gamma_{i}(B)$.

Proposition 15 Given a set of connectors $\left\{\gamma_{i}\right\}_{i=1}^{n}$ over a set of components $B=\left\{B_{i}\right\}_{i=1}^{n}$ where $B_{i}=\left(L_{i}, P_{i}, \mathcal{T}_{i}\right)$ and $L=\bigcup_{i=1}^{n} L_{i}$. Let $\mathbb{V}_{\gamma_{i}}$ be image vector for all locations in $L$ according to $\gamma_{i}$, then the image vector $\mathbb{V}_{\gamma}$ for all the locations of $L$ according to $\sum_{i=1}^{n} \gamma_{i}$ can be obtained as follows:

$$
\mathbb{V}_{\gamma}(l)= \begin{cases}\bigwedge_{l \in \boldsymbol{o}_{i}} \mathbb{V}_{\gamma_{i}}(l) & \text { if } l \in \bigcup_{i=1}^{n} \bullet \gamma_{i}  \tag{3.3}\\ l & \text { otherwise }\end{cases}
$$

Proof We have:

$$
\mathbb{V}_{\gamma}(l)=\bigwedge_{a \in \gamma \wedge l \in \bullet_{a}} \mathbb{V}_{a}(l)=\bigwedge_{l \in \boldsymbol{\bullet}_{\gamma_{i}}}\left(\bigwedge_{a \in \gamma_{i} \wedge l \in \bullet_{a}} \mathbb{V}_{a}(l)\right)=\bigwedge_{l \in \bullet_{\gamma_{i}}} \mathbb{V}_{\gamma_{i}}(l)
$$



Figure 3.5: Example for incremental invariant computation

The Proposition 15 allows to compute the image vector of $\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma(B)$ from the image vector of $\delta_{0}=\gamma-\left(\sum_{i=1}^{n} \delta_{i}\right)^{f}$ and $\left\{\delta_{i}\right\}_{i=1}^{n}$ as follows:

$$
\mathbb{V}(l)= \begin{cases}\bigwedge_{l \in \in} \bullet_{i} \mathbb{V}_{\delta_{i}}(l) & \text { if } l \in \bigcup_{i=0}^{n} \bullet_{i}  \tag{3.4}\\ l & \text { otherwise }\end{cases}
$$

Example 33 In the example of Figure 3.5, let $\gamma=a_{1}+b_{1}+c_{1}+a_{2}+b_{2}+c_{2}, \delta_{1}=a_{1} a_{2}+c_{1} c_{2}$, $\delta_{2}=b_{1} b_{2}$, we have:

$$
\left(\delta_{1}+\delta_{2}\right) \gamma=\left(\gamma-\left(\delta_{1}+\delta_{2}\right)^{f}\right)+\delta_{1}+\delta_{2}=a_{1} a_{2}+c_{1} c_{2}+b_{1} b_{2}
$$

The image for every location according to $\delta_{1}$ are as follows:

$$
\begin{array}{ll}
\mathbb{V}_{\delta_{1}}\left(l_{1}\right)=l_{1} \wedge l_{2} \vee l_{1} \wedge l_{5}, & \mathbb{V}_{\delta_{1}}\left(l_{2}\right)=l_{2},
\end{array} \quad \mathbb{V}_{\delta_{1}}\left(l_{3}\right)=l_{1} \wedge l_{3} \vee l_{3} \wedge l_{4}, ~ 子, ~ l_{1}
$$

The image of every location according to $\delta_{2}$ are as follows:

$$
\begin{array}{ll}
\mathbb{V}_{\delta_{2}}\left(l_{1}\right)=l_{1}, & \mathbb{V}_{\delta_{2}}\left(l_{2}\right)=l_{2} \wedge l_{3} \vee l_{2} \wedge l_{6}, \\
\mathbb{V}_{\delta_{2}}\left(l_{4}\right)=l_{4}, & \left.\mathbb{V}_{\delta_{2}}\left(l_{5}\right)=l_{3}\right)=l_{5} \wedge l_{3} \vee l_{6} \vee l_{3} \wedge l_{5}, \\
\mathbb{V}_{\delta_{2}}\left(l_{6}\right)=l_{6}
\end{array}
$$

For $\left(\delta_{1}+\delta_{2}\right) \gamma(B)$, the image vector $\mathbb{V}$ is obtained from those of its increments as follows:

$$
\begin{array}{ll}
\mathbb{V}\left(l_{1}\right)=l_{1} \wedge l_{2} \vee l_{1} \wedge l_{5}, & \mathbb{V}\left(l_{2}\right)=l_{2} \wedge l_{3} \vee l_{2} \wedge l_{6} \\
\mathbb{V}\left(l_{3}\right)=l_{1} \wedge l_{3} \vee l_{3} \wedge l_{4}, & \mathbb{V}\left(l_{4}\right)=l_{2} \wedge l_{4} \vee l_{4} \wedge l_{5} \\
\mathbb{V}\left(l_{5}\right)=l_{5} \wedge l_{6} \vee l_{3} \wedge l_{5}, & \mathbb{V}\left(l_{6}\right)=l_{1} \wedge l_{6} \vee l_{4} \wedge l_{6}
\end{array}
$$

The following proposition allows computing fixed-points for $\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma(B)$ from the fixed-points obtained from $\gamma-\left(\sum_{i=1}^{n} \delta_{i}\right)^{f}$ and $\left\{\delta_{i}\right\}_{i=1}^{n}$ over $B$.

Proposition 16 Given a connector $\gamma$ over a set of components $B$, a set of increments $\left\{\delta_{1}, \ldots, \delta_{n}\right\}$ over $\gamma$, and sets of fixed-points $\left\{S_{i}\right\}_{i=0}^{n}$ where:
$-S_{0}=\mathbb{F}\left(\mathbb{V}_{\delta_{0}}, \bigvee_{l \in \boldsymbol{\circ}_{0}} l\right)$ with $\delta_{0}=\gamma-\left(\sum_{i=1}^{n} \delta_{i}\right)^{f}$,
$-S_{i}=\mathbb{F}\left(\mathbb{V}_{\delta_{i}}, V_{l \in \boldsymbol{o}_{i}} l\right)$ with $1 \leq i \leq n$.
Let $\mathbb{V}$ be the image vector according to $\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma$, the fixed-points $\mathbb{F}\left(\mathbb{V}, \bigvee_{i=0}^{n} S_{i}\right)$ are the fixed-points of $\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma(B)$.

Proof Given two sets of monomials $S_{1}, S_{2}$, we denote $S_{1} \sqsubseteq S_{2}$ if if for all $s_{1} \in S_{1}$ there exists $s_{2} \in S_{2}$ such that $s_{2}$ implies $s_{1}$.

From Proposition 15, for any set of interactions $\gamma^{\prime} \in\left\{\gamma-\left(\sum_{i=1}^{n} \delta_{i}\right)^{f}, \delta_{1}, \ldots, \delta_{n}\right\}$ and for any location $l \in{ }^{\bullet} \gamma^{\prime}$ we have $\mathbb{V}_{\gamma^{\prime}}(l) \sqsubseteq \mathbb{V}_{\left(\sum \delta_{i}\right) \gamma}(l)$. Let $S_{i}$ and $S$ be respectively the fixed-points obtained from $l$ by $\mathbb{V}_{\gamma^{\prime}}$ and $\mathbb{V}_{\left(\sum \delta_{i}\right) \gamma}$, we have $l \sqsubseteq S_{i} \sqsubseteq S$. Therefore by starting from $\bigvee S_{i}$, the fixed-points $\mathbb{F}\left(\mathbb{V}, \bigvee_{i=0}^{n} S_{i}\right)$ are the fixed-points of $\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma(B)$.

The number of iterations to reach fixed-points $\mathbb{F}\left(\mathbb{V}, \bigvee_{i} S_{i}\right)$ starting from $S_{i}$ can be significantly smaller compared to the number of iterations to reach fixed-points $\mathbb{F}\left(\mathbb{V}, \bigvee_{i} l_{i}\right)$ where we start from locations. Therefore the computation cost can be significantly reduced in both time and memory usage.

Example 34 In the example of Figure 3.5 with $\gamma=a_{1}+b_{1}+c_{1}+a_{2}+b_{2}+c_{2}$, $\delta_{1}=$ $a_{1} a_{2}+c_{1} c_{2}, \delta_{2}=b_{1} b_{2}$, since $\gamma-\left(\delta_{1}+\delta_{2}\right)^{f}=\emptyset$, we have two sets of fixed-points corresponding to two increments:

$$
\begin{aligned}
& S_{1}=\mathbb{F}\left(\mathbb{V}_{\delta_{1}}, \bigvee_{l \in \bullet \delta_{1}} l\right)=\left(l_{1} \wedge l_{2}\right) \vee\left(l_{1} \wedge l_{5}\right) \vee\left(l_{4} \wedge l_{5}\right) \vee\left(l_{2} \wedge l_{4}\right) \\
& S_{2}=\mathbb{F}\left(\mathbb{V}_{\delta_{2}}, \bigvee_{l \in \bullet_{2}} l\right)=\left(l_{2} \wedge l_{3}\right) \vee\left(l_{2} \wedge l_{6}\right) \vee\left(l_{5} \wedge l_{6}\right) \vee\left(l_{3} \wedge l_{5}\right)
\end{aligned}
$$

Let $\phi^{0}=S_{1} \vee S_{2}$, the iteration with the image vector $\mathbb{V}$ obtained in Example 33 provides

$$
\begin{aligned}
\phi^{1} & =\left(l_{1} \wedge l_{2} \wedge l_{3}\right) \vee\left(l_{1} \wedge l_{2} \wedge l_{6}\right) \vee\left(l_{1} \wedge l_{5} \wedge l_{6}\right) \vee\left(l_{1} \wedge l_{3} \wedge l_{5}\right) \\
& \vee\left(l_{2} \wedge l_{3} \wedge l_{4}\right) \vee\left(l_{2} \wedge l_{4} \wedge l_{6}\right) \vee\left(l_{4} \wedge l_{5} \wedge l_{6}\right) \vee\left(l_{3} \wedge l_{4} \wedge l_{5}\right)
\end{aligned}
$$

Then $\phi^{2}=\phi^{1}$, so $\mathbb{F}\left(\mathbb{V}, S_{1} \vee S_{2}\right)=\phi^{1}$ are the fixed-points of $\left(\delta_{1}+\delta_{2}\right) \gamma(B)$.

### 3.3 Summary

We have presented methods allowing incremental construction and verification of componentbased systems. Consider a system built from its constituents, we provide conditions in which the established invariants are preserved after the construction. However these conditions are quite limited because many systems do not satisfy these conditions. We therefore proposed two methods for incrementally computing invariants of general systems from the established invariants of their constituent: one is based on positive mapping operation and the other is based on fixed-point computation. The reuse of established invariants (and established fixed-points) reduces significantly the computation cost compared to the operation on the global system from scratch.

We have presented compositional and incremental methods for verification of componentbased systems. However, all these methods are limited to systems without data transfer. Hence we are going to present, in the next chapter, a method for dealing with systems with data transfer.

## CHAPTER 4

## Dealing with Data Transfer

## Chapter Abstract

In this Chapter, we propose a method for dealing with systems with data. The idea is to transform a system with data to an equivalent system without data and then apply our compositional and incremental verification methods presented in the previous Chapters. The transformation is done in two steps: first we need to project the changes caused by data transfer on interactions to components in computing component invariants, then we replace the data transfer on a set of interactions by a component which preserve the behavior of interactions. Finally we provide an application of the method for checking mutual exclusion property on an example that contains data transfer.

## Résumé du Chapitre

Dans ce chapitre, nous proposons une méthode pour traiter le transfert de données sur les interactions. L'idée est de transformer un système avec transfert de données à un système équivalent sans transfert de données, puis d'appliquer nos méthodes compositionnelles et incrémentales présentées dans les chapitres précédents. La transformation se fait en deux étapes. D'abord nous avons besoin de projeter les changements par le transfert de données sur les interactions aux composants dans le calcul d'invariants de composants. Puis on remplace le transfert des données sur un ensemble d'interactions par un composant qui préserve le comportement des interactions. Enfin nous proposons une application de la méthode à la vérification de la propriété d'exclusion mutuelle sur un exemple qui contient le transfert de données.

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### 4.1 Idea and Methodology

The BIP framework allows to define rich interaction models by using interactions extended with data as presented in [Bas08]. However, in the previous chapters, we considered the interaction models without data transfer. Although it allows easier decomposition and compositional reasoning on the system, the absence of data transfer between components is a severe limitation in practice. The difficulty of handling the data transfer is that we can not compute the component invariants of an atomic component in isolation from the others. For example, if a variable involved in a local transition is updated during the execution of an interaction, we should propagate the changes of valuations from the interaction into the transition.

Our current compositional verification method ignores the data transfer in interactions and therefore get a coarser abstraction. Nevertheless, we still expect to approximate the state space by a tighter abstraction. And that motivates us to study the abstraction method toward the interaction with data transfer. The idea is, since we already have a method for dealing with systems containing only pure interactions, that is interactions without data transfer, we want to transform the interaction models with data transfer into the models without data transfer on which we can apply our previous method. In this chapter, we propose methods for computing invariants taking into account the data transfer. For component invariants, we project the changes on interactions to transitions for the computation of post predicate transformers over transitions. For interaction invariants, we propose to generate new components to "replace" the data transfer on interactions and then apply our previous methods to compute interaction invariants. Our method for dealing with data transfer in BIP framework can be applied to other synchronous modeling mechanisms that do not support shared variables.

The organization of the chapter is as follows: first we present a method for computing component invariants in systems with data transfer. Then we present a method for generating interaction invariants based on the replacement of data transfer by a component called interaction component. Finally, we apply the methods on an example, the Bakery Protocol.

Example 35 (Bakery) We consider 2-process Bakery Protocol [Lam74] as an example to illustrate the method presented in this Chapter. Figures 4.1 presents the model of Bakery in BIP. Two components $B_{1}$ and $B_{2}$ model two processes and they are identical up to the renaming of ports and locations. Hence we present the behavior of the component $B_{1}$, the behavior of $B_{2}$ is similar. $B_{1}$ has three locations: $l_{1}$ if $B_{1}$ is in idle section; $l_{2}$ if $B_{1}$ is waiting to enter its critical section; and $l_{3}$ if $B_{1}$ it is in its critical section. It has a variable


Figure 4.1: Modeling of Bakery in BIP
$x_{1}$ ranging over the natural numbers and representing the "tokens" of the process, 3 loop transitions labeled by a allowing accessing the value of "token", three transitions w, e, r for moving between the locations. Initially in location $l_{1}, B_{1}$ can take $w_{1}$ to move to the waiting location $l_{2}$. From $l_{2}, B_{1}$ can enter the critical location $l_{3}$ by $e_{1}$ transition. The return to $l_{1}$ by transition $r_{1}$ resets the "token" $x_{1}$ to 0 .

Two processes $B_{1}$ and $B_{2}$ communicate by a set of interactions $\gamma=a_{1} w_{2}+a_{1} e_{2}+a_{2} w_{1}+$ $a_{2} e_{1}+r_{1}+r_{2}$ with the corresponding guards and functions $g_{a_{1} e_{2}}=\left(x_{1}=0 \vee x_{2} \leq x_{1}\right)$, $g_{a_{2} e_{1}}=\left(x_{2}=0 \vee x_{2}<x_{1}\right), f_{a_{1} w_{2}}=\left(x_{2}:=x_{1}+1\right)$ and $f_{a_{2} w_{1}}=\left(x_{1}:=x_{2}+1\right)$. The function $f_{a_{2} w_{1}}=\left(x_{1}:=x_{2}+1\right)$ allows $B_{1}$ to get the "token" according to the "token" of $B_{1}$ while moving to waiting location $l_{2}$ by transition $w_{1}$. The guard $g_{a_{2} e_{1}}=\left(x_{2}=0 \vee x_{2}<x_{1}\right)$ allows $B_{1}$ to move to the critical location $l_{3}$ only if $B_{2}$ is in idle location $l_{1} \quad\left(x_{2}=0\right)$ or it has higher priority "token" $\left(x_{1} \geq x_{2}\right)$. Similarly for $f_{a_{1} w_{2}}=\left(x_{2}:=x_{1}+1\right)$ and $g_{a_{1} e_{2}}=\left(x_{1}=0 \vee x_{2} \leq x_{1}\right)$ for the process $B_{2}$.

An important property for Bakery system is mutual exclusion: both components can not be at the critical locations $l_{3}$ and $l_{6}$ at the same time, i.e $P=\neg\left(l_{3} \wedge l_{6}\right)$.

### 4.2 Component Invariant Generation

In a system without data transfer, variables are always modified only inside their atomic components, and component invariants are computed by the post predicate transformer w.r.t internal transitions of these components. However, in a system with data transfer, variables can be modified by interactions between components, thus their values depend on variables of other components, and the computation of component invariants should take into account the constraints on the variables of other components. In other words, we need to project the updates on interactions into components for the computation of component invariants. We define below such projection.

Definition 34 (Interaction-to-Component Projection) Given a connector $\gamma(B)$ where $B=\left(B_{1}, \ldots, B_{n}\right)$ with $B_{i}=\left(L_{i}, P_{i}, \mathcal{T}_{i}, X_{i},\left\{g_{\tau}\right\}_{\tau \in \mathcal{T}_{i}},\left\{f_{\tau}\right\}_{\tau \in \mathcal{T}_{i}}\right)$, let $a \in \gamma$ be an interaction with guard $G_{a}(X)$ and function $F_{a}\left(X, X^{\prime}\right)$, let $\varphi(X)$ be a random predicate and let
$\Phi(X)=\bigwedge_{X \cap X_{i} \neq \emptyset} \Phi_{i}\left(X_{i}\right)$ be a predicate where $\Phi_{i}$ is an invariant of component $B_{i}$. The projection $\mathcal{P}$ of $\varphi$ over interaction a to component $B_{i}$ is defined as follows:

$$
\mathcal{P}_{a}(\varphi)\left(X_{i}\right)=\exists X^{\prime}, \exists\left(X \backslash X_{i}\right) \cdot \varphi\left(X^{\prime}\right) \wedge \Phi\left(X^{\prime}\right) \wedge G_{a}\left(X^{\prime}\right) \wedge F_{a}\left(X^{\prime}, X\right)
$$

where $X \backslash X_{i}$ means to remove $X_{i}$ from $X$.
Starting from the weakest invariant $\Phi_{i}\left(X_{i}\right)=$ true of component $B_{i}$, we can increasingly compute tighter predicates.

Example 36 Consider the interaction $w_{1} a_{2}$ of the Bakery example in Figure 4.1 with an update function $x_{1}=x_{2}^{\prime}+1$ where variable $x_{1}$ of component $B_{1}$ is modified according to variable $x_{2}$ of component $B_{2}$. Since $x_{1}, x_{2}$ are natural numbers, the predicate $\Phi_{2}\left(x_{2}\right)=$ $\left(x_{2} \geq 0\right)$ (resp. $\Phi_{1}\left(x_{1}\right)=\left(x_{1} \geq 0\right)$ ) is always true and therefore is an invariant of $B_{2}$ (resp. $B_{1}$ ). The predicate projection of $\varphi=$ true to $B_{1}$ over interaction $w_{1} a_{2}$ is:

$$
\mathcal{P}_{w_{1} a_{2}}(\varphi)\left(x_{1}\right)=\exists x_{1}^{\prime} \exists x_{2}^{\prime} \exists x_{2} . t r u e \wedge \Phi_{1}\left(x_{1}^{\prime}\right) \wedge \Phi_{2}\left(x_{2}^{\prime}\right) \wedge\left(x_{1}=x_{2}^{\prime}+1\right)=\left(x_{1} \geq 1\right)
$$

Similarly, we have $\mathcal{P}_{w_{2} a_{1}}(\varphi)\left(x_{2}\right)=\left(x_{2} \geq 1\right)$.
For an interaction $a$ with a guard $G_{a}$ and a function $F_{a}$, interaction $a$ first executes the interaction function $F_{a}$, then it executes the functions of the transitions of its ports. Since the update function of a transition takes place after the update function of the interaction that its port participates, the post predicate transformer over the transition have to take into account the changes due to the interaction. Here, we extend the definition of post predicate transformer over a transition defined in Chapter 2 by taking into account interaction-tocomponent projection.

Definition 35 (Transition-based Post Predicate Transformer) Given a component $\gamma(B)$ with $B=\left(B_{1}, \ldots, B_{n}\right)$ where $B_{i}=\left(L_{i}, P_{i}, \mathcal{T}_{i}, X_{i},\left\{g_{\tau}\right\}_{\tau \in \mathcal{T}_{i}},\left\{f_{\tau}\right\}_{\tau \in \mathcal{T}_{i}}\right)$, let $\tau \in \mathcal{T}_{i}$ be a transition with guard $g_{\tau}\left(X_{i}\right)$ and function $f_{\tau}\left(X_{i}, X_{i}^{\prime}\right)$, and let $\gamma_{i}=\left\{a \mid a \in \gamma \wedge X_{a}^{u} \cap X_{i} \neq \emptyset\right\}$ be a set of interactions whose update variables $X_{a}^{u}$ involve some variables in $X_{i}$. The post predicate transformer of $\varphi\left(X_{i}\right)$ over $\tau$ is defined as follows:

$$
\operatorname{post}_{\tau}(\varphi)\left(X_{i}\right)=\exists X_{i}^{\prime} \cdot\left(\bigvee_{a \in \gamma_{i}} \mathcal{P}_{a}(\varphi)\left(X_{i}^{\prime}\right)\right) \wedge g_{\tau}\left(X_{i}^{\prime}\right) \wedge f_{\tau}\left(X_{i}^{\prime}, X_{i}\right)
$$

Roughly speaking, we consider all the possible predicate updates by the interactions, in which the port of transition $\tau$ participates, as the predicate before executing transition $\tau$.

Example 37 Let us consider the post predicate computation for all non loop transitions of component $B_{1}$ in Bakery example (Figure 4.1). Initially $\varphi=$ true, we have:

- post $_{w_{1}}($ true $)\left(x_{1}\right)=\exists x_{1}^{\prime} \cdot \mathcal{P}_{w_{1} a_{2}}($ true $)\left(x_{1}^{\prime}\right) \wedge\left(x_{1}=x_{1}^{\prime}\right)=\left(x_{1} \geq 1\right)$. The invariant at $l_{2}$ is therefore $\varphi_{l_{2}}=\left(x_{1} \geq 1\right)$.
- $\operatorname{post}_{e_{1}}\left(\varphi_{l_{2}}\right)\left(x_{1}\right)=\exists x_{1}^{\prime} \cdot \mathcal{P}_{e_{1} a_{2}}\left(\varphi_{l_{2}}\right)\left(x_{1}^{\prime}\right) \wedge\left(x_{1}=x_{1}^{\prime}\right)$ where $\mathcal{P}_{e_{1} a_{2}}\left(\varphi_{2}\right)\left(x_{1}\right)=\exists x_{2} \cdot \varphi_{l_{2}} \wedge$ $\left(x_{2} \geq 0\right) \wedge\left(\left(x_{2}=0\right) \vee\left(x_{1} \leq x_{2}\right)\right)=\left(x_{1} \geq 1\right)$, hence $\varphi_{l_{3}}=\operatorname{post}_{e_{1}}\left(\varphi_{l_{2}}\right)\left(x_{1}\right)=\left(x_{1} \geq 1\right)$ is invariant at $l_{3}$.

Thesis

- $\operatorname{post}_{r_{1}}\left(\varphi_{l_{3}}\right)\left(x_{1}\right)=\left(x_{1}=0\right)$ is invariant at $l_{1}$.

The component invariant of $B_{1}$ is $\Phi_{1}=\left(l_{1} \wedge x_{1}=0\right) \vee\left(l_{2} \wedge x_{1} \geq 1\right) \vee\left(l_{3} \wedge x_{1} \geq 1\right)$. Similarly, the component invariant of $B_{2}$ is $\Phi_{2}=\left(l_{4} \wedge x_{2}=0\right) \vee\left(l_{5} \wedge x_{2} \geq 1\right) \vee\left(l_{6} \wedge x_{2} \geq 1\right)$.

After computing the effects to the local components caused by the updates in the interactions, we can deal with the interactions with data transfer and generate interaction components to replace the data transfer on these interactions. The following subsection will focus on the interaction component generation.

### 4.3 Interaction Invariant Generation

The computation of interaction invariants of a system with data transfer consists of two main steps:

- First, we need to transform the system with data transfer to a corresponding system without data transfer. In this section, we will focus on this transformation.
- Then we use the method presented in Chapter 2 to compute interaction invariants for the system without data transfer.
Based on static analysis between a set of components and a set of interactions, the method proposed in this section "replaces" the effect of data transfer on these interactions by a component called interaction component. Then that interaction without data is connected to the new component which allows to mimic the original behavior of the interaction. The new components encode the guards in the interactions and the updates between the involved components into locations and transition relations. The locations in the new components enumerate all the possible combinations between the guards on the interactions. And the transition relations record the variable updates, which will be synchronized with the updates of the original components.

The choice of the set of interactions from which an interaction component is generated is based on the set of variables involved. Consider a set of variables $X$, we choose a set of interactions $\gamma$ such that its data transfer involves in $X$ or the update functions of its transitions involve in $X$. Moreover, $\gamma$ should include all interactions involving in $X$, that is $X$ is not affected by any interaction outside the set $\gamma$.

Given a set of interactions with data transfer, we need to compute the transition system for the interaction component and the new interactions. Therefore, it is necessary to consider the predicates updated by executing one interaction.

Definition 36 (Interaction-based Post Predicate Transformer) Given a connector $\gamma(B)$ where $B=\left(B_{1}, \ldots, B_{n}\right)$ with $B_{i}=\left(L_{i}, P_{i}, \mathcal{T}_{i}, X_{i},\left\{g_{\tau}\right\}_{\tau \in \mathcal{T}_{i}},\left\{f_{\tau}\right\}_{\tau \in \mathcal{T}_{i}}\right)$ and $X=$ $\bigcup_{i=1}^{n} X_{i}$, let a be an interaction of $\gamma$, the post predicate transformer of a predicate $\varphi(X)$ over the interaction $a$ is defined as follows:

$$
\operatorname{post}_{a}(\varphi)(X)=\bigwedge_{p \in a}\left(\bigvee_{\operatorname{port}(\tau)=p} \operatorname{post}_{\tau}(\varphi)(X)\right)
$$

where $\operatorname{port}(\tau)$ is the port labeling the transition $\tau$.

### 4.3. INTERACTION INVARIANT GENERATION

That is, the post predicate transformer of an interaction $a$ represents the effect of its execution taking into account the effect by the executions of all involved transitions.

The transformation from a system with data transfer to a system without data transfer is based on the replacement of the data transfer by a component called interaction component. If we remove the data transfer from an interaction, we need to know when the interaction can be executed and the effect after the execution of the interaction. The execution condition of the interaction is represented by the guard which is encoded in the location of interaction component. The execution effect of the interaction is presented by the post predicate transformer post defined above and is encoded into transitions of the interaction component.

Definition 37 (Interaction Component) Given $\gamma=\left\{a_{i}\right\}_{i}$ a set of interactions where $G_{a_{i}}(X)$ and $F_{a_{i}}\left(X, X^{\prime}\right)$ are respectively guard and function of interaction $a_{i}$, we define the interaction component $B^{d}=\left(L^{d}, P^{d}, \mathcal{T}^{d}\right)$ over $\gamma$, where:

- $L^{d}=\left\{\varphi^{d} \mid \exists \gamma^{\prime} \subseteq \gamma \cdot \varphi^{d}=\left(\bigwedge_{a \in \gamma^{\prime}} G_{a} \wedge \bigwedge_{a \in \gamma-\gamma^{\prime} \wedge G_{a} \neq t r u e} \bar{G}_{a}\right) \neq\right.$ false $\}$ is a set of the conjunctive combinations of the sets $\left\{\left\{G_{a}, \bar{G}_{a}\right\}\right\}_{a \in \gamma}$. We define the abstraction function $\alpha^{d}$ which associates each atomic predicate $\bigwedge_{a \in \gamma^{\prime}} G_{a} \wedge \bigwedge_{b \in \gamma-\gamma^{\prime}} \bar{G}_{a}$ a symbol $\varphi^{d}$.
- $P^{d}$ is a set of ports $\left\{p_{a}\right\}$ where each $p_{a}$ corresponds to an interaction a $\in \gamma$.
$-\mathcal{T}^{d} \subseteq L^{d} \times P^{d} \times L^{d}$ where for any $\varphi_{1}^{d}, \varphi_{2}^{d} \in L^{d}$, if there exists $a \in \gamma$ such that $\varphi_{1}^{d} \wedge G_{a} \neq$ false and $\varphi_{2}^{d} \wedge \operatorname{post}_{a}\left(\varphi_{1}^{d}\right)(X) \neq$ false, $\left(\varphi_{1}^{d}, p_{a}, \varphi_{2}^{d}\right) \in \mathcal{T}^{d}$.

During the interaction component generation, we can update the interactions by connecting with the ports of the interaction component, and generate an abstract system without data transfer.

Definition 38 (Abstract System of Data Transfer) Given a connector $\gamma(B)$ and $a$ set of interaction components $\left\{B_{1}^{d}, \ldots, B_{k}^{d}\right\}$ with $B_{i}^{d}=\left(L^{d}, P^{d}, \mathcal{T}^{d}\right)$ generated from the corresponding set of interactions $\gamma_{i} \subseteq \gamma$, we define the abstract system $\gamma^{d}\left(B, B^{d}\right)$, where $\gamma^{d}$ is obtained from the following process:

- For any $\tau=\left\{\varphi_{i}^{d}, p_{a}, \varphi_{j}^{d}\right\} \in \mathcal{T}^{d}$, we generate a new interaction by adding the port $p_{a}$ to the corresponding interaction $a: a^{d}=a . p_{a}$, and add a to the replaced interaction list: $\gamma_{r}=\gamma_{r}+a$.
- After no more interaction is generated, add new interactions $\left\{a^{d}\right\}$ to $\gamma$ and remove replaced interactions in $\gamma^{r}$ from $\gamma: \gamma^{d}=\gamma+\left\{a^{d}\right\}-\gamma^{r}$.

Two definitions above provide the method to remove the data transfer in the interactions and to connect the interaction components with other components.

The process for generating an interaction component for a set of interactions is presented in Algorithm 2. It takes as input a set of interactions $\gamma$ on a set of atomic components $\left(B_{1}, \ldots, B_{n}\right)$. These interactions have guards, functions or transitions involving in the same set of variables $X$. It basically works as follows:

- First step is to generate a set of locations according to the interaction guards. For each non-empty subset $\gamma^{\prime} \subseteq \gamma$ such that the predicate $\varphi^{d}=\bigwedge_{a \in \gamma^{\prime}} G_{a} \wedge \bigwedge_{a \in \gamma-\gamma^{\prime} \wedge G_{a} \neq \text { true }} \bar{G}_{a}$

```
function generateInteractionComponent \(\left(\gamma\left(B_{1}, \ldots, B_{n}\right)\right)\)
begin
    \(L^{d}=\emptyset ; P^{d}=\emptyset ; \mathcal{T}^{d}=\emptyset ;\)
    forall \(\gamma^{\prime} \subseteq \gamma\) such that \(\varphi^{d}=\bigwedge_{a \in \gamma^{\prime}} G_{a} \wedge \bigwedge_{a \in \gamma-\gamma^{\prime} \wedge G_{a} \neq \text { true }} \bar{G}_{a} \neq\) false do
        \(L^{d}=L^{d} \cup \varphi^{d} ;\)
    end
    \(\gamma^{r}=\emptyset ; \quad / *\) set of interactions to be removed */
    forall \(\varphi_{i}^{d}, \varphi_{j}^{d} \in L^{d}\) do
        forall \(a \in \gamma\) do
                if \(\left(\varphi_{i}^{d} \wedge G_{a} \wedge \bigwedge_{\text {port }(\tau) \in a} g_{\tau} \neq\right.\) false \() \&\left(\right.\) post \(_{a}\left(\varphi_{i}^{d}\right) \wedge \varphi_{j}^{d} \neq\) false \()\) then
                    if \(p_{a}\) does not exist in \(P^{d}\) then
                                    \(P^{d}=P^{d} \cup p_{a} ;\)
                                    \(\gamma=\gamma \cup\left\{a . p_{a}\right\} ;\)
                                    \(\gamma^{r}=\gamma^{r} \cup a ;\)
                    end
                    \(\mathcal{T}^{d}=\mathcal{T}^{d} \cup\left\{\left(\varphi_{i}^{d}, p_{a}, \varphi_{j}^{d}\right)\right\} ;\)
            end
        end
    end
    \(\gamma^{d}=\gamma \backslash \gamma^{r} ;\)
end
```

Algorithm 2: Interaction Component Generation
is not false, we generate a location $\varphi^{d}$ for the interaction component and add it to the location sets $L^{d}$ (lines 4, 5).

- Second step is to generate the set of ports and the set of transitions. We considered any two locations $\varphi_{i}^{d}, \varphi_{j}^{d}$ of $L^{d}$ and any interaction $a \in \gamma$. If $\varphi_{i}^{d} \wedge G_{a} \wedge \bigwedge_{\text {port }(\tau) \in a} g_{\tau} \neq$ false and $\operatorname{post}_{a}\left(\varphi_{i}^{d}\right) \wedge \varphi_{j}^{d} \neq$ false (line 10), that is there is a transition going out from $\varphi_{i}^{d}$ and coming into $\varphi_{j}^{d}$ and its port participates to the interaction $a$. If $p_{a}$ does not exist in $P^{d}$, we create a port $p_{a}$ (line 12), a new interaction by adding $p^{a}$ to $a$ (line 13) and $a$ is added to the replaced interaction list which will be remove at the end (line 14). Then we generate a transition $\tau=\left(\varphi_{i}^{d}, p_{a}, \varphi_{j}^{d}\right)$ (line 16).
Finally, after the complete construction of the interaction component, the set of interactions is obtained by removing the replaced interaction list $\gamma^{r}$ from $\gamma$ (line 20).

The following proposition shows that the abstract system $\gamma^{\prime}\left(B, B^{\prime}\right)$ by removing data transfer simulates $\gamma(B)$, and invariants of $\gamma^{\prime}\left(B, B^{\prime}\right)$ are also invariants of $\gamma(B)$.

Proposition 17 Given $\gamma(B)$ with data transfer on some of the interactions of $\gamma$, let $B=$ $\left\{B_{i}\right\}_{1 \leq i \leq n}$ be a set of components with $B_{i}=\left(L_{i}, P_{i}, \mathcal{T}_{i}, X_{i},\left\{g_{\tau}\right\}_{\tau \in \mathcal{T}_{i}},\left\{f_{\tau}\right\}_{\tau \in \mathcal{T}_{i}}\right), B^{d}=$ $\left\{B_{j}^{d}\right\}_{1 \leq j \leq k}$ be a set of interaction components obtained from sets of interactions in $\gamma$ with $B_{j}^{d}=\left(L_{j}^{d}, P_{j}^{d}, T_{j}^{d}\right)$, and $\gamma^{d}$ be the corresponding set of interactions without data transfer, then $\gamma^{d}\left(B, B^{d}\right)$ simulates $\gamma(B)$. Moreover, if $\Phi^{\alpha_{d}}$ is an invariant of $S^{d}$ then $\alpha_{d}^{-1}\left(\Phi^{\alpha_{d}}\right)$ is an invariant of $S$.

Proof Let $(l, x)$ be a global state of $\gamma(B)$, where $l \in L_{1} \times \cdots \times L_{n}$, and $\left(l, \varphi_{1}^{d}, \ldots, \varphi_{n}^{d}, y\right)$ be a global state of $\gamma^{\prime}\left(B, B^{\prime}\right)$ where $\varphi_{j}^{d} \in L_{i}^{d}$ and $x, y \in \bigcup_{i=1}^{n} X_{i}$. We show that $(l, x) R\left(l, \varphi_{1}^{d}, \ldots, \varphi_{n}^{d}, y\right)$
is a simulation, if $v(y) \wedge \bigwedge_{j=1}^{n} \varphi_{j}^{d} \models v(x)$, where $v(x)$ is a valuation of $x$. If $(l, x) \xrightarrow{a}\left(l^{\prime}, x^{\prime}\right)$ is a transition of $\gamma(B)$, then we show that there exists $b \in \gamma^{\prime}$ such that $\left(l, \varphi_{1}^{d}, \ldots, \varphi_{n}^{d}, y\right) \xrightarrow{b}$ $\left(l^{\prime}, \varphi_{1}^{\prime d}, \ldots, \varphi_{n}^{\prime d}, y^{\prime}\right)$ where $a \in b$ and $\left(l^{\prime}, \varphi_{1}^{\prime d}, \ldots, \varphi_{n}^{\prime d}, y^{\prime}\right)=\left(l^{\prime}, \varphi_{1}^{d}, \ldots, \varphi_{n}^{d}, y^{\prime}\right)$.

- If $a$ contains no data transfer, we have $a=b$ and $\left(l^{\prime}, \varphi_{1}^{d}, \ldots, \varphi_{n}^{d}, y^{\prime}\right)$. Because no variable is updated on $a,\left(l^{\prime}, x^{\prime}\right) R\left(l^{\prime}, \varphi_{1}^{d}, \ldots, \varphi_{n}^{d}, y^{\prime}\right)$.
- If $a$ contains some data transfer, there exists $B_{j}^{\prime}$ and $a \cup\left\{p_{j}\right\} \subset b \in \gamma^{\prime}$, and $\varphi_{j}^{d} \wedge G_{a} \neq$ false $\wedge \varphi_{j}^{\prime d} \wedge \operatorname{post}_{a}\left(\varphi_{l}\right) \neq$ false such that $\left(l, \varphi_{1}^{d}, \ldots, \varphi_{n}^{d}, y\right) \xrightarrow{b}\left(l^{\prime}, \varphi_{1}^{d}, \ldots, \varphi_{j-1}^{d}, \varphi_{j}^{\prime d}, \varphi_{j+1}^{d}\right.$, $\left.\ldots, \varphi_{n}^{d}, y^{\prime}\right)$. Suppose $z$ is updated in interaction $a$, we have $\operatorname{post}_{a}\left(\varphi_{l}\right)(z) \models v(z)$. In interaction $b$, the transition from $B_{j}^{\prime}$ requires that $\varphi_{j}^{\prime d} \wedge \operatorname{post}_{a}\left(\varphi_{l}\right) \neq$ false. For other variables, they are updated by transitions. So we have $v(y) \wedge \bigwedge_{j=1}^{n} \varphi_{j}^{\prime d} \models v\left(x^{\prime}\right)$.

Example 38 For Bakery example, since the data transfer of all the interactions involve in the set of variables $\left\{x_{1}, x_{2}\right\}$ we will generate an interaction component $B_{3}=\left(L^{d}, P^{d}, \mathcal{T}^{d}\right)$ to replace the data transfer on these interactions.

First we generate the set of locations from the guards of the set of interactions $\left\{g_{a_{1} e_{2}}, g_{a_{2} e_{1}}\right\}$ :
$g_{a_{1} e_{2}} \wedge g_{a_{2} e_{1}}=\left(x_{1}=0 \vee x_{2}<x_{1}\right) \wedge\left(x_{2}=0 \vee x_{1} \leq x_{2}\right)=\left(x_{1}=0 \vee x_{2}=0\right)$
$g_{a_{1} e_{2}} \wedge \bar{g}_{a_{2} e_{1}}=\left(x_{1}=0 \vee x_{2}<x_{1}\right) \wedge\left(x_{2}>0\right) \wedge\left(x_{1}>x_{2}\right)=\left(x_{2}>0 \wedge x_{1}>x_{2}\right)$
$\bar{g}_{a_{1} e_{2}} \wedge g_{a_{2} e_{1}}=\left(x_{1}>0\right) \wedge\left(x_{2} \geq x_{1}\right) \wedge\left(x_{2}=0 \vee x_{1} \leq x_{2}\right)=\left(x_{1}>0 \wedge x_{2} \geq x_{1}\right)$
Since all the combinations are different from false, we generate a set of three corresponding locations $L^{d}=\left\{l_{7}, l_{8}, l_{9}\right\}$ where:

$$
\begin{gathered}
l_{7}=g_{a_{1} e_{2}} \wedge g_{a_{2} e_{1}}=\left(x_{1}=0 \vee x_{2}=0\right) \\
l_{8}=g_{a_{1} e_{2}} \wedge \bar{g}_{a_{2} e_{1}}=\left(x_{1}>0 \wedge x_{1}<x_{2}\right) \\
l_{9}=g_{a_{2} e_{1}} \wedge \bar{g}_{a_{1} e_{2}}=\left(x_{2}>0 \wedge x_{2}<x_{1}\right)
\end{gathered}
$$

The ports and the transition system of the interaction component $B_{3}$ is presented in the figure 4.2. They are specified from the set of locations $L^{d}$ and the set of interactions $\gamma$. For example, there is a port $p_{a_{1} w_{2}}$ together with one of its transition $\tau=\left(l_{7}, p_{a_{2} w_{1}}, l_{8}\right)$ since $l_{7} \wedge g_{w_{1} a_{2}}=l_{7} \neq$ false and post $_{a_{2} w_{1}}\left(l_{7}\right)\left(x_{1}\right)=\operatorname{post}_{a_{2}}\left(l_{7}\right) \wedge$ post $_{w_{1}}\left(l_{7}\right)=\exists x_{1}^{\prime} \exists x_{2}^{\prime} \cdot\left(x_{1}^{\prime}=\right.$ $\left.0 \vee x_{2}^{\prime}=0\right) \wedge\left(x_{1}=x_{2}^{\prime}+1\right)=\left(x_{1}>0\right)$, hence post $a_{a_{2}} w_{1}\left(l_{7}\right) \wedge l_{8}=\left(x_{1}>0\right) \wedge\left(x_{1}>0 \vee x_{1}<\right.$ $\left.x_{2}\right) \neq$ false.

Example 39 (Checking Mutual Exclusion) Mutual exclusion is an important property of Bakery example: two components can not be at the critical locations $l_{3}$ and $l_{6}$ at the same time. The property is formally represented as $P=\neg\left(l_{3} \wedge l_{6}\right)$.

First we compute the component invariants of the system $\Phi=\Phi_{1} \wedge \Phi_{2} \wedge \Phi_{3}$ where $\Phi_{1}=\left(l_{1} \wedge x_{1}=0\right) \vee\left(l_{2} \wedge x_{1}>0\right) \vee\left(l_{3} \wedge x_{1}>0\right)$, $\Phi_{2}=\left(l_{4} \wedge x_{2}=0\right) \vee\left(l_{5} \wedge x_{2}>0\right) \vee\left(l_{6} \wedge x_{2}>0\right)$ and $\Phi_{3}=l_{7} \vee l_{8} \vee l_{9}$ with $l_{7}=\left(x_{1}=0 \vee x_{2}=0\right), l_{8}=\left(x_{2}>0 \wedge x_{1}>x_{2}\right)$ and $l_{9}=\left(x_{1}>\right.$ $0 \wedge x_{2} \geq x_{1}$ ).

From the Bakery abstract system computed in Example 38, we compute the abstract interaction invariants and then the concrete interaction invariants that are respectively:


Figure 4.2: Bakery example with interaction component

$$
\begin{aligned}
\Psi^{d} & =\left(l_{5} \vee l_{7} \vee l_{8}\right) \\
& \wedge\left(l_{2} \vee l_{7} \vee l_{9}\right) \\
& \wedge\left(l_{2} \vee l_{3} \vee l_{5} \vee l_{6} \vee l_{7}\right) \\
\Psi & =\left(\left(l_{5} \wedge x_{2}>0\right) \vee\left(x_{1}=0 \vee x_{2}=0\right) \vee\left(x_{2}>0 \wedge x_{1}>x_{2}\right)\right) \\
& \wedge\left(\left(l_{2} \wedge x_{1}>0\right) \vee\left(x_{1}=0 \vee x_{2}=0\right) \vee\left(x_{1}>0 \wedge x_{2} \geq x_{1}\right)\right) \\
& \wedge\left(\left(l_{2} \wedge x_{1}>0\right) \vee\left(l_{3} \wedge x_{1}>0\right) \vee\left(l_{5} \wedge x_{2}>0\right) \vee\left(l_{6} \wedge x_{2}>0\right) \vee\left(x_{1}=0 \vee x_{2}=0\right)\right)
\end{aligned}
$$

Finally we verify the mutual exclusion property $P$ by using Yices to check $\Phi \wedge \Psi \wedge \neg P$. The unsat output of Yices shows that the property is guaranteed for the Bakery example.

### 4.4 Summary

We have presented a method for dealing with interaction models with data transfer. The idea is that we first project the changes of variables by interactions into transitions, then transform the models with data transfer into the models without data transfer on which we can apply our compositional method for checking safety properties. The transformation is done by replacing the data transfer by interaction components which allow preserving the behavior of the interactions. We have also applied the method for verifying mutual exclusion property of the Bakery example.

Although this method has not been implemented in our tool-set but the obtained result shows the perspectives of the method in dealing with interaction models with data transfer.

## Part III

## Implementation, Tools and Case Studies

## CHAPTER 5

## Implementation

## Chapter Abstract

We present in this Chapter the D-Finder tool-set which implements fully our compositional and incremental methods. First we describe the structure of D-Finder and the process to check deadlock-freedom of a system in BIP. Then we present in detail the implementation of the modules in D-Finder. The implementation of different methods allows D-Finder to be able to verify efficiently different types of systems.

## Résumé du Chapitre

Nous présentons dans ce chapitre l'outil D-Finder qui met en oeuvre nos méthodes de vérification compositionnelles et incrémentales. Nous allons d'abord décrire la structure DFinder et le processus pour détecter des blocages d'un système en BIP. Ensuite, nous allons présenter en détail la mise en oeuvre de modules dans D-Finder. La mise en oeuvre de différentes méthodes permet D-Finder de vérifier efficacement plusieurs types de systèmes.

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### 5.1 The D-Finder Tool

We have implemented the compositional and incremental methods in D-Finder, a tool for verifying safety properties, especially for checking deadlock-freedom for component-based systems described in the BIP language. D-Finder consists of a set of modules interconnected as shown in Figure 5.1. It takes as input a system described in BIP and progressively finds and eliminates potential deadlocks. It basically works as follows:

1. It constructs the predicate characterizing the set of deadlock states (DIS generation module).
2. Iteratively, it constructs increasingly stronger component invariants ( $\Phi_{i}$ generation module). This step might need quantifier elimination that requires collaboration with Omega tool.
3. Component invariants are used to compute finer finite state abstractions and increasingly stronger interaction invariants (Abstraction and $\Psi$ generation module). The computation of interaction invariants is done by collaborating with CUDD package or Sat-solver tool Yices.
4. It verifies deadlock freedom by checking the unsatisfiability of $\bigwedge \Phi_{i} \wedge \Psi \wedge D I S$ (satisfiability module). If it succeeds, the system is proven deadlock-free, else it may continue or give up, according to the user's choice. The unsatisfiability is checked by a Sat-Solver tool Yices in the case of systems with data and by CUDD package in the case of systems without data.


Figure 5.1: The D-Finder tool

It is also connected to the state space exploration tool of the BIP platform, for finer analysis when the heuristic fails to prove deadlock-freedom.

The main programming language used in the implementation of D-Finder is Java. However, since CUDD package is written in C, several parts which are connected to CUDD are
also written in C.
We provide below in detail the description of each module in D-Finder.

### 5.2 DIS Generation

The implementation of $D I S$ generation module is presented in Algorithm 3. The input is a set of interactions $\gamma$ on a set of components $B$. The output is a predicate DIS characterizing deadlock states, i.e a set of states from which no interaction of $\gamma$ can take place. The DIS generation process works as follows: first we compute the enabled condition $e n_{a}$ for each interaction $a$ of $\gamma$ by computing the enabled conditions for all its ports. A port $p$ is enabled if at least one of its transitions is enabled and a transition $\tau=\left(l, p, g_{\tau}, f_{\tau}, l^{\prime}\right)$ is enabled if its source location is reached and its guard is true (line 6). Hence the enabled condition of $p$ is the disjunction of the enabled conditions of all its transitions (line 8). The interaction $a$ is enabled if its guard $G_{a}$ is true and all its ports are enabled (line 10). Finally, the predicate $D I S$ is obtained by the conjunction of the disabled conditions (the negation of enabled conditions) of all the interactions in $\gamma$ (line 12).

```
function generateDIS(Connector \(\gamma\), Component B)
begin
    for each interaction a of \(\gamma\) do
        for each port \(p\) of \(a\) do
                forall transitions \(\tau=\left(l, p, g_{\tau}, f_{\tau}, l^{\prime}\right)\) do
                \(e n_{\tau}=l \wedge g_{\tau} ;\)
                end
                \(e n_{p}=\bigvee_{p o r t(\tau)=p} e n_{\tau} ;\)
        end
        \(e n_{a}=G_{a} \wedge \bigwedge_{p \in a} e n_{p} ;\)
    end
    \(D I S=\bigwedge_{a \in \gamma}\left(\neg e n_{a}\right) ;\)
    return \(D I S\);
end
```

Algorithm 3: DIS Predicate Generation

### 5.3 Component Invariant Generation

The implementation of the Component Invariant Generation module is shown in Algorithm 4. Taking as input an atomic component, we generate the component invariants by computing an invariant predicate, which is initially true, at each control location. Consider a location $l$, we first compute post predicate transformers of all its incoming transitions by calling computePost function (line 5) which returns, for a transition $\tau=\left(l^{\prime}, p, g_{\tau}, f_{\tau}, l\right)$ and an invariant predicate $\Phi_{l^{\prime}}$ at the source location $l^{\prime}$, the propagation of $\Phi_{l^{\prime}}$ by $\tau$. Then the invariant predicate at $l$ is obtained by the disjunction of all these post predicate transformers (line 7).

```
function computeCompInv(Component \(B\) )
begin
    for each control location \(l\) of \(B\) do
        for each transition \(\tau=\left\{l^{\prime}, p, g_{\tau}, f_{\tau}, l\right\}\) do
            post \(_{\tau}=\operatorname{computePost}\left(\tau, \Phi_{l^{\prime}}\right)\);
        end
        \(\Phi_{l}=\bigvee_{\tau \in \bullet l}\) post \(_{\tau} ;\)
        return \(\Phi_{l}\);
    end
end
```

Algorithm 4: Compute Component Invariants

```
function computePost(Transition \(\tau\), Predicate \(\Phi\) )
    /* \(\tau\) is of the form ( \(\left.l^{\prime}, p, g_{\tau}(X), Y=f_{\tau}(Z), l\right)\)
    */;
    /* \(\Phi\) is a predicate on the sets of variables \(\mathrm{X}, \mathrm{Y}, \mathrm{Z}\)
    begin
        post \(=\) true;
        if \(g_{\tau}=\) null then
            \(g_{\tau}=\) true;
        end
        if \(f_{\tau}=\) null then
            post \(=g_{\tau}(X) \wedge \Phi(X, Y, Z) ;\)
            return post;
        end
        if \(X \cap Y=\emptyset\) then
            post \(=\) post \(\wedge g_{\tau} ;\)
        end
        if \(Y \cap Z=\emptyset\) then
        post \(=\) post \(\wedge\left(Y=f_{\tau}(Z)\right) ;\)
        else
            \(\mathrm{f}=\exists X^{\prime}, Y^{\prime}, Z^{\prime} . \Phi\left(X^{\prime}, Y^{\prime}, Z^{\prime}\right) \wedge g_{\tau}\left(X^{\prime}\right) \wedge\left(Y=f \tau\left(Z^{\prime}\right)\right) ;\)
            \(\mathrm{f}=\operatorname{call} 0 \mathrm{mega}(\mathrm{f})\);
            post \(=\operatorname{post} \wedge f ;\)
        end
        return post
    end
```

Algorithm 5: Compute Post Predicate

An important function in generating component invariants is computePost, presented in Algorithm 5, which computes the post predicate transformer of a predicate with respect to a transition. It takes as inputs a transition $\tau$ of the form $\tau=\left(l^{\prime}, p, g_{\tau}(X), Y=f_{\tau}(Z), l\right)$, a predicate $\Phi$ at the source location $l^{\prime} . X, Y, Z$ are subsets of component's variable set. The function works as follows:

- If the guard $g_{\tau}$ is null, the execution condition of $\tau$ is always true, hence $g_{\tau}$ is assigned to true (lines 4, 5).
- If the update function $f_{\tau}$ is null, that is the set of variables is not affected by the transition and therefore $\Phi$ still holds after the transition, the function returns $g_{\tau} \wedge$ $\Phi(X, Y, Z)$ (lines 7, 8, 9) and terminates.
- If $X \cap Y=\emptyset$ which means that the variables in the guard $g_{\tau}$ are not affected by the function, the guard $g_{\tau}$ still holds after the transition. Hence, the post predicate transformer is updated by conjuncting with $g_{\tau}:$ post $=$ post $\wedge g_{\tau}$ (lines 11, 12).
- If $Y \cap Z=\emptyset$, the variables in the right side of the update function are not changed and the predicate $Y=f_{\tau}(X)$ holds after the transition. The post predicate transformer is updated: post $=$ post $\wedge\left(Y=f_{\tau}(Z)\right)$ (line 15).
- If $X \cap Y \neq \emptyset$, the variables in the right side of the update function are affected. The new valuation is computed by taking into account the affect of the update function: there exists some valuation of $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ at the source location $l^{\prime}$ (denoted by $X^{\prime}, Y^{\prime}, Z^{\prime}$ ) such that the predicate $\Phi\left(X^{\prime}, Y^{\prime}, Z^{\prime}\right)$ is true, the guard $g_{\tau}\left(X^{\prime}\right)$ is true (for that the transition $\tau$ can be executed) and the new valuation is computed from the existing valuation according to the update function $Y=f_{\tau}\left(Z^{\prime}\right)$. We call the external tool Omega to eliminate the quantifiers (line 18).
The function callOmega $(f)$ writes the formula $f$ to the input file of Omega. Then it calls Omega to eliminate quantifier and get back the corresponding quantifier-free formula that we might need to convert by a parser to get the expected form. An example of the input file for the post predicate transformer of a predicate $\Phi=(x \geq 0)$ by a transition $\tau$ with $g_{\tau}=(x<10), f_{\tau}=(x:=x+1)$ is as follows:

$$
F:=\{[x]: \operatorname{exists}(\operatorname{tmp}:(\operatorname{tmp} \geq 0) \& \&(t m p<10) \& \&(x=\operatorname{tmp}+1))\}
$$

The output of Omega for this entry is the quantifier-free formula $1 \leq x \leq 10$.

### 5.4 Checking Local Deadlock-Freedom

An assumption of our method for checking deadlock-freedom of a system is that every component is deadlock-free. Hence it is necessary to check the local deadlock-freedom of each atomic component before checking the global deadlock-freedom. An atomic component is deadlock-free if at any location, it is always able to move by taking one of its outgoing transitions. The function for checking local deadlock-freedom takes as input an atomic component together with its component invariants. At each location $l$, the function verifies whether its invariant $\Phi_{l}$ implies at least one of its outgoing transitions' guard, i.e $\Phi_{l} \Rightarrow$ $\bigvee_{\tau \in l} g_{\tau}$. The satisfiability of this condition is checked by Yices tool. If it holds at all the locations of the component, then the function returns deadlock-free. Otherwise, it returns not deadlock-free output together with a set of locations that do not satisfy this condition.

### 5.5 Abstraction

Abstraction is used to transform a system with data into an equivalent system without data. The implementation of abstraction process in the D-Finder tool is presented in Algorithm 6. It takes as input a system consisting of a set of components with their component invariants and a set of interactions. The abstraction process consists of three main steps corresponding to the generations of abstract components, of abstract interactions and of abstract initial condition.

For the generation of an abstract component $B^{\alpha}$ from a concrete component $B=$ $\left(L, P, \mathcal{T}, X,\left\{g_{\tau}\right\}_{\tau \in \mathcal{T}},\left\{f_{\tau}\right\}_{\tau \in \mathcal{T}}\right)$ and its component invariants $\Phi=\bigvee_{l_{j} \in L}\left(l_{j} \wedge \bigvee_{k} \varphi_{j k}\right)$, the abstract function works as follows:

- First, the set of abstract locations $L^{\alpha}$ are generated by splitting concrete locations according to the invariants. For each location $l_{j}$ where the invariant is of the form $\bigvee_{k} \varphi_{j k}$, we create for each predicate $\varphi_{j k}$ an abstract location $l_{j k}^{\alpha}=l_{j} \wedge \varphi_{j k}$ (lines 8 , 9).
- Then, the set of abstract transitions $\mathcal{T}^{\alpha}$ are generated. For each concrete transition $\tau=\left(l_{m}, p, g_{\tau}, f_{\tau}, l_{n}\right)$, we have two sets of abstract source locations $l_{m j}^{a}$ and of abstract destination locations $l_{n k}^{\alpha}$. An abstract transition $\tau_{m j n k}^{a}=\left(l_{m j}^{\alpha}, p, l_{n k}^{\alpha}\right)$ is established (line 15) only if the conditions $\varphi_{m j} \wedge g_{\tau} \neq$ false and $\operatorname{post}_{\tau}\left(\varphi_{m j}\right) \wedge \varphi_{n k} \neq$ false (or equivalently $\varphi_{m j} \wedge \operatorname{pre}_{\tau}\left(\varphi_{n k}\right) \neq$ false) hold (line 14).
- The generation of abstract ports $P^{\alpha}$ is just the creation of an abstract port $p^{\alpha}$ for each concrete port $p$ (lines 21, 22).
The generation of abstract interactions is done by creating, for each concrete interaction $\gamma_{i}=p_{1} \ldots p_{n}$, a new interaction composed of the corresponding abstract ports $\gamma_{i}^{\alpha}=$ $p_{1}^{\alpha} \ldots p_{n}^{\alpha}$ (lines 29, 30).

The generation of initial conditions is to take, for each element $l_{j} \wedge \varphi_{j}^{i n i t}$ of the set Init, all abstract locations $l_{j k}^{\alpha}=l_{j} \wedge \varphi_{j k}$ of $l_{j}$ such that $\varphi_{j}^{i n i t} \wedge \varphi_{j k} \neq$ false (lines 36,37).

### 5.6 Interaction Invariant Generation

We have implemented two sub-modules according to global and incremental computation of interaction invariants as presented in Figure 5.2:

- global computation sub-module takes as input a global system $\langle\gamma(B)$, Init $\rangle$ and computes globally interaction invariants of the system.
- incremental computation sub-module takes as input a connector $\gamma$ over a set of components $B$ with the initial conditions Init, and a set of increments $\left\{\delta_{i}\right\}_{i=1}^{n}$ and computes incrementally the set interaction invariants.


### 5.6.1 Global Computation of Interaction Invariants

Figure 5.3 shows the structure and the data flow for the global computation of interaction invariants. There are several sub-methods for the computation: two enumerative methods using Yices and CUDD; two symbolic methods based on positive mapping and fixed-point using CUDD package. The implementation consists of the following functions:

```
function abstract \(\left(\gamma\left(B_{1}, \ldots, B_{n}\right),\left(\Phi_{1}, \ldots, \Phi_{n}\right)\right.\), Init \()\)
begin
    /* Generate abstract components */
    for each component \(B_{i}\) do
        \(L^{\alpha}=\emptyset ; P^{\alpha}=\emptyset ; \mathcal{T}^{\alpha}=\emptyset ;\)
        for each location \(l_{j}\) of \(B_{i}\) with \(\Phi_{l_{j}}=l \wedge \bigvee_{k} \varphi_{j k}\) do
            for each predicate \(\varphi_{j k}\) do
                \(l_{j k}^{\alpha}=l_{j} \wedge \varphi_{j k} ;\)
            \(L^{\alpha}=L^{\alpha} \cup l_{j k}^{\alpha} ;\)
            end
        end
        for each transition \(\tau=\left(l_{m}, p, g_{\tau}, f_{\tau}, l_{n}\right)\) do
            forall \(l_{m j}^{\alpha}\) and \(l_{n k}^{\alpha}\) do
                if \(\left(\varphi_{m j} \wedge g_{\tau} \neq\right.\) false \() \wedge\left(\right.\) post \(_{\tau}\left(\varphi_{m j}\right) \wedge \varphi_{n k} \neq\) false \()\) then
                \(\tau_{m j n k}^{\alpha}=\left(l_{m j}^{\alpha}, p, l_{n k}^{\alpha}\right) ;\)
                \(\mathcal{T}^{\alpha}=\mathcal{T}^{\alpha} \cup \tau_{m j n k}^{\alpha} ;\)
            end
        end
        end
        for each port \(p_{j}\) do
        create abstract port \(p_{j}^{\alpha}\);
        \(P^{\alpha}=P^{\alpha} \cup p_{j}^{\alpha} ;\)
        end
        \(B_{i}^{\alpha}=\left(L^{\alpha}, P^{\alpha}, \mathcal{T}^{\alpha}\right) ;\)
    end
    /* Generate abstract interactions */
    \(\gamma^{\alpha}=\emptyset ;\)
    for each interaction \(\gamma_{i}=p_{1} \ldots p_{n}\) of \(\gamma\) do
        create \(\gamma_{i}^{\alpha}=p_{1}^{\alpha} \ldots p_{n}^{\alpha}\);
        \(\gamma^{\alpha}=\gamma^{\alpha} \cup \gamma_{i}^{\alpha} ;\)
    end
    /* Generate abstract set of initial locations */
    Init \({ }^{a}=\emptyset\);
    for each \(l_{j} \wedge \varphi_{j}^{\text {init }} \in \operatorname{Init}\) do
        for each \(l_{j k}^{a}=l_{j} \wedge \varphi_{j k}\) do
        if \(\varphi_{j}^{i n i t} \wedge \varphi_{j k} \neq\) false then
            \(\mathrm{Init}^{\alpha}=\operatorname{Init}^{\alpha} \cup l_{j k}^{a} ;\)
                end
            end
        end
    return \(\left\langle\gamma^{\alpha}\left(B_{1}^{\alpha}, \ldots, B_{n}^{\alpha}\right)\right.\), Init \(\left.^{\alpha}\right\rangle ;\)
end
```

Algorithm 6: Abstraction


Figure 5.2: Structure of Interaction Invariant Generation Module


Figure 5.3: Global Computation of Interaction Invariants Module

- A function for generating Boolean Behavioral Constraints (BBCs) from the set of interactions and components.
- Different functions based on different methods for computing interaction invariants:
- two functions based on the enumerative method which computes explicitly the set of interaction invariants. They use the Sat-Solver tool Yices or CUDD package to solve equation systems.
- two functions based on the symbolic methods which compute the set of interaction invariants by using Positive Mapping or Fix-point. The computation is performed using CUDD package.
Below we present in detail the implementation of each function.


## Boolean Behavioral Constraints (BBCs) generation

Boolean Behavioral Constraints can be generated according to Definition 17: for any location $l$, we need to build an implication corresponding to interactions that its outgoing transitions are involved. However, the right sides of the implications in BBCs are different even for the same interaction. For example, consider the interaction $p_{1} p_{2} p_{3}$ in figure 5.4, the transitions $\tau_{1}$ and $\tau_{2}$ are involved in the same interaction but the corresponding implications

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$l_{1} \Rightarrow\left(l_{1}^{\prime} \vee l_{3}^{\prime} \vee l_{5}^{\prime}\right) \wedge\left(l_{1}^{\prime} \vee l_{4}^{\prime} \vee l_{5}^{\prime}\right)$ and $l_{2} \Rightarrow\left(l_{2}^{\prime} \vee l_{3}^{\prime} \vee l_{5}^{\prime}\right) \wedge\left(l_{2}^{\prime} \vee l_{4}^{\prime} \vee l_{5}^{\prime}\right)$ have different right sides. Therefore we have to visit an interaction many times for different implications and the generation of BBCs is not efficient.

These two implications can be rewritten in the form $l_{1} \Rightarrow l_{1}^{\prime} \vee\left(l_{3}^{\prime} \vee l_{5}^{\prime}\right) \wedge\left(l_{4}^{\prime} \vee l_{5}^{\prime}\right)$ and $l_{2} \Rightarrow l_{2}^{\prime} \vee\left(l_{3}^{\prime} \vee l_{5}^{\prime}\right) \wedge\left(l_{4}^{\prime} \vee l_{5}^{\prime}\right)$ or equivalently $l_{1} \Rightarrow l_{1}^{\prime} \vee\left(l_{3}^{\prime} \wedge l_{4}^{\prime}\right) \vee l_{5}^{\prime}$ and $l_{2} \Rightarrow l_{2}^{\prime} \vee\left(l_{3}^{\prime} \wedge l_{4}^{\prime}\right) \vee l_{5}^{\prime}$ from which we can see that for two transitions $\tau_{1}, \tau_{2}$ of the same component, the parts of other components in the implications are the same $\left(l_{3}^{\prime} \wedge l_{4}^{\prime}\right) \vee l_{5}^{\prime}$. Hence, in the implementation, we generate a predicate once for each interaction and then reuse that predicate in generating BBCs corresponding to that interaction.

We recall that for a port $p, p^{\bullet}$ is a set of destination locations of its transitions. For example, in figure 5.4, $p_{1}^{\bullet}=\left\{l_{1}^{\prime}, l_{2}^{\prime}\right\}, p_{2}^{\bullet}=\left\{l_{3}^{\prime}, l_{4}^{\prime}\right\}, p_{3}^{\bullet}=\left\{l_{5}^{\prime}\right\}$.

We define below a predicate called Forward Predicate for an interaction. This predicate is generated once and then will be reused in generating implications corresponding to the interaction.

Definition 39 (Forward Predicate of Interaction) Let $a=p_{1} p_{2} \ldots p_{n}$ be an interaction, we define its forward location predicate as follows:

$$
\vec{a}=\bigvee_{p_{i} \in a} \bigwedge_{l \in p_{i}^{\bullet}} l
$$



Figure 5.4: An example for BBCs

Example 40 For the interaction $a=p_{1} p_{2} p_{3}$ in Figure 5.4, the set of destination locations of each component is respectively $\left\{l_{1}^{\prime}, l_{2}^{\prime}\right\},\left\{l_{3}^{\prime}, l_{4}^{\prime}\right\}$ and $\left\{l_{5}\right\}$. Then we have the forward location predicate of the interaction a is $\vec{a}=\left(l_{1}^{\prime} \wedge l_{2}^{\prime}\right) \vee\left(l_{3}^{\prime} \wedge l_{4}^{\prime}\right) \vee l_{5}^{\prime}$.

Definition 40 (Forward Location Predicate) Given a location $l$ and a set of interactions $\gamma$, we define the forward location predicate of $l$ with respect to $\gamma$ as follows:

$$
\vec{l}^{\gamma}=\bigwedge_{\tau \in l^{\bullet} \wedge l^{\prime} \in \tau^{\bullet}}\left(l^{\prime} \vee \bigwedge_{\operatorname{port}(\tau) \in a \wedge a \in \gamma} \vec{a}\right)
$$

where $\operatorname{port}(\tau)$ is the port labeling the transition $\tau$.

### 5.6. INTERACTION INVARIANT GENERATION

Example 41 Consider again the interaction $a=p_{1} p_{2} p_{3}$ in Figure 5.4 with the forward predicate $\vec{a}$ in Example 40. The forward location predicate of $l_{1}$ corresponding to the transition $\tau_{1}$ can be obtained from $\vec{a}$ by using Definition 40 as follows:

$$
\overrightarrow{l_{1}}=l_{1}^{\prime} \vee \vec{a}=l_{1}^{\prime} \vee\left(l_{1}^{\prime} \wedge l_{2}^{\prime}\right) \vee\left(l_{3}^{\prime} \wedge l_{4}^{\prime}\right) \vee l_{5}^{\prime}=l_{1}^{\prime} \vee\left(l_{3}^{\prime} \wedge l_{4}^{\prime}\right) \vee l_{5}^{\prime}
$$

We can see that ${\overrightarrow{l_{1}}}^{a}$ is the right side of the implication from $l_{1}$ according to the interaction $a$.

The following proposition shows that for a location $l$ and an involved interaction $a$, its Forward Location Predicate $\vec{l}^{a}$ is the right side of the implication according to $a$. Therefore we can save the computation effort by generating once, for every interaction $a$, the Forward Predicate of Interaction $\vec{a}$ and then reuse them for generating the implications of involved locations.

Proposition 18 Let $\gamma$ be a connector over a tuple of components $B=\left(B_{1}, \ldots, B_{n}\right)$. The Boolean Behavioral Constraints for a connector $\gamma(B)$ with a set of locations $L$ of $B$ can be computed as follows:

$$
|\gamma(B)|=\bigwedge_{l \in L}\left(l \Rightarrow \vec{l}^{\gamma}\right)
$$

Proof Given an interaction $a=p_{1} p_{2} \ldots p_{n} \in \gamma$ with the corresponding set of destination locations for port $p_{i}$ is $L_{i}=p_{i}^{\boldsymbol{*}}=\left\{l_{i 1}, \ldots, l_{i m_{i}}\right\}$, we first prove that $\bigwedge_{\phi \in L_{1} \times \cdots \times L_{n}} \bigvee_{l \in \phi} l=$ $\bigvee_{i=1}^{n} \bigwedge_{j=1}^{m_{i}} l_{i j}$ by induction technique:

- For $n=2$, we have:

$$
\begin{aligned}
\wedge_{\phi \in L_{1} \times L_{2}} \bigvee_{l \in \phi} l & =\left[\left(l_{11} \vee l_{21}\right) \wedge \cdots \wedge\left(l_{11} \vee l_{2 m_{2}}\right)\right] \\
& \wedge \ldots \\
& \wedge\left[\left(l_{m_{1} 1} \vee l_{21}\right) \wedge \cdots \wedge\left(l_{m_{1} 1} \vee l_{2 m_{2}}\right)\right] \\
& =\left[l_{11} \vee\left(l_{21} \wedge \cdots \wedge l_{2 m_{2}}\right)\right] \wedge \cdots \wedge\left[l_{12} \vee\left(l_{21} \wedge \cdots \wedge l_{2 m_{2}}\right)\right] \\
& \left.=\left(l_{11} \wedge \cdots \wedge l_{1 m_{1}}\right) \vee\left(l_{21} \wedge \cdots \wedge l_{2 m_{2}}\right)\right] \\
& =\bigvee_{i=1}^{2} \wedge_{j=1}^{m_{i}} l_{i j}
\end{aligned}
$$

- suppose that for $n=k$ we have $\bigwedge_{\phi \in L_{1} \times \cdots \times L_{n}} \phi=\bigvee_{i=1}^{n} \bigwedge_{j=1}^{m_{i}} l_{i j}$, we will prove it is also true for $n=k+1$ :

$$
\begin{aligned}
\bigwedge_{\phi \in L_{1} \times \cdots \times L_{k+1}} \bigvee_{l \in \phi} l & =\bigwedge_{\phi \in\left(L_{1} \times \cdots \times L_{k}\right) \times L_{k+1}} \bigvee_{l \in \phi} l \\
& =\left(\bigvee_{i=1}^{k} \bigwedge_{j=1}^{m_{i}} l_{i j} \vee l_{(k+1) 1}\right) \wedge \cdots \wedge\left(\bigvee_{i=1}^{n} \bigwedge_{j=1}^{m_{i}} l_{i j} \vee l_{(k+1) m_{k+1}}\right) \\
& =\left(\bigvee_{i=1}^{k} \bigwedge_{j=1}^{m_{i}} l_{i j} \vee\left(l_{(k+1) 1} \wedge \cdots \wedge l_{(k+1) m_{k+1}}\right)\right. \\
& =\bigvee_{i=1}^{k+1} \bigwedge_{j=1}^{m_{i}} l_{i j}
\end{aligned}
$$

According to Definition 17, the the right side of the implication for transition $\tau=\left(l_{k t}^{\prime}, p_{k}, l_{k t}\right)$ is as follows:
$\bigwedge_{\phi \in L_{1} \times \cdots \times L_{k-1} \times L_{k+1} \times \cdots \times L_{n}}\left(l_{k t} \vee \bigvee_{l \in \phi} l\right)=l_{k t} \vee\left(\bigwedge_{\phi \in L_{1} \times \cdots \times L_{k-1} \times L_{k+1} \times \cdots \times L_{n}} \bigvee_{l \in \phi} l\right)$. On the other hand, by using Forward Predicate of Interaction $\vec{a}$, we have $l_{k t} \vee \vec{a}=l_{k t} \vee$
$\bigwedge_{\phi \in L_{1} \times \cdots \times L_{n}} \bigvee_{l \in \phi} l=l_{k t} \vee \bigwedge_{\phi \in L_{1} \times \cdots \times L_{k-1} \times L_{k+1} \times \cdots \times L_{n}} \bigvee_{l \in \phi} l$.
So the implication for transition $\tau$ by interaction $a$ can be represented by $l_{k t}^{\prime} \Rightarrow l_{k t} \vee \vec{a}$. Hence, we have:

$$
|\gamma(B)|=\bigwedge_{l \in L}\left(l \Rightarrow \bigwedge_{\tau \in l^{\bullet} \wedge l^{\prime} \in \tau^{\bullet}}\left(l^{\prime} \vee \bigwedge_{\operatorname{port}(\tau) \in a \wedge a \in \gamma} \vec{a}\right)\right)=\bigwedge_{l \in L}\left(l \Rightarrow \vec{l}^{\gamma}\right)
$$

$\vec{l}^{\gamma}$ can be used for generating image vector in the implementation of the method based on fixed-point computation which will be presented later.

The implementation for generating Forward Location Predicate of a location $l$ according to a set of interaction $\gamma$ on a set of components $B$ is presented in Algorithm 7: for every outgoing transition $\tau=\left(l, p, l^{\prime}\right)$ of $l$, we make the conjunction of the Forward Predicates of all the involved interactions (that is the interactions contain $p$ ) and assign the result to a temporary variable $\operatorname{tmp}$ (line 5); then $\vec{l}^{\gamma}$ is updated by the conjunction with $l^{\prime} \vee \operatorname{tmp}$ (line 6 ); finally $\vec{l}^{\gamma}$, the Forward Location Predicate of $l$ according to $\gamma$, is returned (line 8).

```
1 function generateForwardLocationPredicate \((l, \gamma, B)\)
begin
        \(\vec{l}^{\gamma}=\) true;
        for each transition \(\tau=\left(l, p, l^{\prime}\right)\) do
            \(\mathrm{tmp}=\bigwedge_{p \in a \wedge a \in \gamma} \vec{a} ;\)
            \(\vec{l}^{\gamma}=\vec{l}^{\gamma \in a \wedge\left(l^{\prime} \vee \mathrm{tmp}\right) ;}\)
        end
        return \(\vec{l}^{\gamma}\);
end
```

Algorithm 7: Generate Forward Location Predicate
The detail implementation of the function for BBCs generation is presented in Algorithm 8:

```
function generate \(\operatorname{BBC}(\gamma, B)\)
begin
    \(L=\bigcup_{B_{i}=\left(L_{i}, P_{i}, T_{i}\right) \in B} L_{i} ;\)
    for each location \(l\) of \(L\) do
            \(\vec{l}^{\gamma}=\) generateForwardLocationPredicate \((l, \gamma, B)\);
    end
    \(\mathrm{bbc}=\Lambda_{l \in L}\left(l \Rightarrow \vec{l}^{\gamma}\right) ;\)
    return bbc
end
```

Algorithm 8: BBCs Generation

- The input is a set of interactions $\gamma$ on a set of components $B$.
- First we get the union $L$ of locations of components in $B$ (line 3) and for each location $l$ of $L$, we compute the Forward Location Predicate $\vec{l}^{\gamma}$ by calling generateForwardLocationPredicate function (line 5).
- Then, $B B C s$ is computed by the conjunction of the implications $l \Rightarrow \vec{l}^{\gamma}$ of all locations $l$ of $L$ (line 7). The function terminates by returning $B B C s$, the Boolean Behavioral Constraints of $\gamma(B)$.


## Implementation of Enumerative Method

We have implemented two functions for the enumerative method: a function using SMT Sat-Solver tool Yices and a function using CUDD package. The algorithms for the implementations of two functions are similar and are presented in Algorithm 9. Taking as input the $B B C s$ and the set of initial locations Init, the computation process works as follows:

- First, we get a formula $f$ by the conjunction of BBCs with the disjunction of initials locations in Init (line 3) to guarantee that every interaction invariant contains at least an initial location.
- Then, we compute solutions of $f$. For every solution $s=\bigwedge_{i} l_{i} \wedge \bigwedge_{j} \overline{l_{j}}$, we extract all the positive valuations $l_{i}$ in $s$ to generate the corresponding interaction invariant and then add it to $\Psi$ (line 6). We want to avoid getting others solutions of $f$ such that their positive valuation sets are superset of that set of $s$ because they correspond to weaker invariants, hence we update $f$ by adding $\bigvee_{i} \overline{l_{i}}$ before getting another solution (line 7). This step is repeated until $f$ is unsatisfiable.
- Finally, the function returns $\Psi$, the set of interaction invariants.

```
function computeEmumerativeII(BBCs, Init)
begin
    \(\mathrm{f}=\mathrm{BBCs} \wedge \bigvee_{l_{i} \in \operatorname{lnit}} l_{i} ;\)
    \(\Psi=\) true;
    while f has a solution \(s=\bigwedge l_{i} \wedge \wedge \overline{l_{j}}\) do
        \(\Psi=\Psi \wedge \bigvee l_{i} ;\)
        \(\mathrm{f}=\mathrm{f} \wedge\left(\vee \overline{l_{i}}\right) ;\)
    end
    return \(\Psi\);
end
```

Algorithm 9: Enumerative Computation of Interaction Invariants
The enumerative method provides a clear, visual view of interaction invariants of the system. However, there is a risk of explosion of solutions, if exhaustiveness of solutions is necessary in the analyzing process.

## Implementation of the Symbolic Method Based on Positive Mapping

In contract to the enumerative methods which extract positive variables from solutions of BBCs, the method using positive mapping symbolically removes the negative variables from BBCs. The implementation of the method is presented in Algorithm 10. Taking as input BBCs and the set of initial locations Init, the computation process works as follows:

- First, the disjunction of initial conditions is conjuncted with BBCs (line 3) to make sure that every invariant contains at least an initial condition.
- Then, the positive mapping function positiveMapping is called (line 4) which eliminates all the negative forms of the variables in BBCs. The returned result is assigned to $f$.
- Finally, the dual function is called for $f$ (line 5) and the result is assigned to $\Psi$ which is the symbolic interaction invariants.

```
function computeIIByPositiveMapping(bbcs, Init)
begin
    \(\mathrm{f}=\mathrm{bbcs} \wedge \bigvee_{l_{i} \in \text { Init }} l_{i}\);
    \(\mathrm{f}=\) positiveMapping(f);
        \(\Psi=\operatorname{dual}(f)\);
        return \(\Psi\);
end
```

Algorithm 10: Interaction Invariant Computation based on Positive Mapping

The implementation of positiveMapping function is presented in Algorithm 11: given a function $f(X)$ and a subset of variables $Y \subseteq X$, this function eliminates all the negative forms of variables belonging to $X \backslash Y$ by using cofactor function provided in CUDD package. If the subset $Y$ is not provided, this function removes the negative forms of all the variables of $X$.

```
function positiveMapping \((f(X), Y \subseteq X)\)
begin
    for each variable y of \(\mathrm{X} \backslash \mathrm{Y}\) do
            \(\mathrm{f}^{\prime}=\operatorname{cofactor}(\mathrm{f}, \bar{y})\);
            \(\mathrm{f}=\mathrm{f}^{\prime} \vee \mathrm{f} ;\)
        end
        return f;
end
```

Algorithm 11: Positive Mapping Function

The algorithm for dual function is presented in Algorithm 12. Given a symbolic formula $f$ and a set of variables $X=\left\{x_{1}, \ldots, x_{n}\right\}$ in $f$, dual function first creates a set of variables $X^{\prime}=\left\{x_{1}^{\prime}, \ldots, x_{n}^{\prime}\right\}$ such that $x_{i}^{\prime}=\bar{x}_{i}$ (lines $3,4,5$ ). Then, it replaces each variable $x_{i} \in X$ in $f$ by $x_{i}^{\prime} \in X^{\prime}$ and obtains $f^{\prime}$ (line 7). This is done by vectorCompose( $f, X^{\prime}$ ) function provided in CUDD which creates a new BDD by substituting the BDDs for the variables of the BDD $f$. The negative form of $f^{\prime}$ is the dual of $f$ (line 8).

Example 42 Consider a formula $f=x_{1} x_{2}+x_{2} x_{3}$ and its set of variables $X=\left\{x_{1}, x_{2}, x_{3}\right\}$. The dual function first creates a set $X^{\prime}=\left\{x_{1}^{\prime}, x_{2}^{\prime}, x_{3}^{\prime}\right\}$ where $x_{1}^{\prime}=\bar{x}_{1}, x_{2}^{\prime}=\bar{x}_{2}, x_{3}^{\prime}=\bar{x}_{3}$. Then it calls vectorCompose ( $f, X^{\prime}$ ) function which replaces variables in $X$ by the corresponding variables in $X^{\prime}$ and returns $f^{\prime}=\bar{x}_{1} \bar{x}_{2}+\bar{x}_{2} \bar{x}_{3}$. Finally it calls cudd_Neg $\left(f^{\prime}\right)$ function which returns $\tilde{f}=\left(x_{1}+x_{2}\right)\left(x_{2}+x_{3}\right)$, the dual of $f$.

```
function \(\operatorname{dual}\left(f\left(X=\left\{x_{1}, \ldots, x_{n}\right\}\right)\right)\)
begin
    creates \(X^{\prime}=\left\{x_{1}^{\prime}, \ldots, x_{n}^{\prime}\right\} ;\)
    for each variable \(x_{i} \in X\) do
        \(x_{i}^{\prime}=\bar{x}_{i} ;\)
    end
    \(\mathrm{f}^{\prime}=\) vectorCompose \(\left(\mathrm{f}, X^{\prime}\right)\);
    \(\tilde{f}=\) cudd_Neg \(\left(f^{\prime}\right)\);
    return \(\tilde{f}\);
end
```

Algorithm 12: Dual operation

## Implementation of the Symbolic Method Based on Fixed-point

An alternative symbolic method for computing interaction invariants is based on fixedpoint computation. Taking as input a set of interactions $\gamma$ over $B$ and a set of initial locations Init, the implementation of the method based on fixed-point is presented in Algorithm 13:

```
function computeIIByFixedpoint(}(\gamma,B,Init
begin
    V = computeImageVector ( }\gamma,B\mathrm{ );
    f}=\mp@subsup{\}{\mp@subsup{l}{i}{}\in\operatorname{Init}}{}\mp@subsup{l}{i}{\prime}
    f = computeFixedpoint(\mathbb{V},f);
    \Psi = dual(f);
    return \Psi;
end
```

Algorithm 13: Fixed-point-based Computation of Interaction Invariants

- First, we compute functional vector $\mathbb{V}$ for all location variables by calling computeImage Vector function (line 3).
- Then, we compute the fixed-points of the formula $f$ which is initialized by the disjunction of initial location variables $f=\bigvee_{L_{i} \in \operatorname{Init}} l_{i}$ (line 4). Starting from this disjunction guarantees that every interaction invariant contains at least an initial location. The computation of fixed-points is done by computeFixedpoint function (line 5). The returned result is assigned to $f$.
- Finally, the symbolic set of interaction invariants $\Psi$ is computed by calling the dual function for $f$ (line 6).
There are two main functions in computing interaction invariants by fixed-points: computeImageVector and computeFixedpoint.

The function computeImageVector allows computing, for each location variable $l$, an image according to its involved interactions. Taking as input a set of interactions $\gamma$ on a set of components $B$, the implementation of the function is presented in Algorithm 14: first, we get the set of locations $L$ of $B$ (line 3), then for each location $l_{i} \in L$, we call function generateForwardLocationPredicate to compute its Forward Location Predicate ${\overrightarrow{l_{i}}}^{\gamma}$ (lines 4,

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```
function computeImageVector \((\gamma, B)\)
begin
        \(L=\bigcup_{B_{i}=\left(L_{i}, P_{i}, \mathcal{T}_{i}\right) \in B} L_{i} ;\)
    for each location \(l_{i}\) of \(L\) do
        \({\overrightarrow{l_{i}}}^{\gamma}=\) generateForwardLocationPredicate \(\left(l_{i}, \gamma, B\right)\);
        \(\mathbb{V}[i]=l_{i} \wedge{\overrightarrow{l_{i}}}^{\gamma} ;\)
    end
    return \(\mathbb{V}\);
end
```

Algorithm 14: Image Vector Generation
5). The image of $l_{i}$ is obtained by the conjunction of $l_{i}$ and ${\overrightarrow{l_{i}}}^{\gamma}$ (line 6).

```
function computeFixedpoint(\mathbb{V},f)
begin
    while true do
        tmp = Cudd_bddVectorCompose(f, V);
        if Cudd_EquivDC(f, V) then
            return f;
        end
        else
            f = tmp ;
        end
    end
end
```

Algorithm 15: Fixed-point Computation

The function computeFixedpoint allows computing the fixed-point of a given formula $f$ according to an image function $\mathbb{V}$. The implementation of this function is presented in Algorithm 15:

- Step 1: we replace all location variables in $f$ by their images in the image vector $\mathbb{V}$ and the intermediate result is assigned to $\operatorname{tmp}$ (line 4). This operation is done by the function Cudd_bddVectorCompse(f, img) in CUDD which creates a new BDD by substituting the BDDs $\mathbb{V}$ for the variables of the BDD $f$. Then we continue to Step 2.
- Step 2: we compare the intermediate formula $\operatorname{tmp}$ with $f$ by the function Cudd_EquivDC provided in CUDD (line 5). If they are the same, i.e we have reached fix-points, the iteration terminates and the fixed-points are returned (line 6); otherwise, we assign the intermediate formula $\operatorname{tmp}$ to $f$ (line 9) and return to Step 1.


### 5.6.2 Incremental Computation of Interaction Invariants

Similarly to the global symbolic computation of interaction invariants, there are two methods for incremental computation of interaction invariants based on Positive Mapping and Fixed-point Computation. The implementation of the Incremental Computation of Interaction Invariants module therefore consists of:

### 5.6. INTERACTION INVARIANT GENERATION

- A sub-module for the computation based on positive mapping which is composed of two functions: a function for getting common locations involved in different connectors and a function for incremental computation of interaction invariants by using positive mapping.
- A sub-module for the computation based on fixed-point which allows computing the global image vector and the global fixed-points from the sets of image vectors and fixed-points of constituents.
Below we present in detail the implementation of these incremental methods.


## Implementation of the Incremental Positive Mapping-based Method

Given connector $\gamma$ over a set of components $B$, a set of increments $\left\{\delta_{1}, \ldots, \delta_{n}\right\}$ over $\gamma$ and $L_{c}$ the set of common location variables of $\left\{\gamma-\left(\sum_{i=1}^{n} \delta_{i}\right)^{f}, \delta_{1}, \ldots, \delta_{n}\right\}$, the invariants of the system $\left(\sum_{i=1}^{n}\right) \gamma(B)$, according to Proposition 14, can be computed as follows:

$$
I_{\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma(B)}=\operatorname{dual}\left(\bigwedge_{i=0}^{n}\left|\delta_{i}(B)\right|^{p\left(L_{c}\right)}\right)^{p} \text { where } \delta_{0}=\gamma-\left(\sum_{i=1}^{n} \delta_{i}\right)^{f}
$$

The incremental computation of interaction invariants for the superposition $\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma(B)$ therefore consists of the following steps:

- First, we need to get the set of common locations $L_{c}$ of the set $\left\{\delta_{i}\right\}_{i=0}^{n}$.
- Then, we need to build Boolean Behavioral Constraints $\left|\delta_{i}(B)\right|$ for each $\delta_{i}$. The partial positive mapping is used to remove the negative forms of local locations variables, for each $\left|\delta_{i}(B)\right|$ we obtain $\left|\delta_{i}(B)\right|^{p\left(L_{c}\right)}$.
- We integrate $\left|\delta_{0}(B)\right|^{p\left(L_{c}\right)}, \ldots,\left|\delta_{n}(B)\right|^{p\left(L_{c}\right)}$, then remove the negative forms remaining of common variables and apply the dual operation to obtain the global invariant.
The implementation of the function for getting common locations of a set of connectors $\gamma_{1}, \ldots, \gamma_{n}$ is presented in Algorithm 16:
- First, we get the set of components compList ${ }_{i}$ involved in each connector $\gamma_{i}$ (line 3). A component is involved in a connector if its ports participate in any interaction of the connector. Then we get the set of common components commonCompList of $\left\{\text { commpList }_{i}\right\}_{i=1}^{n}$ (lines 5, 6, 7, 8), that is the components that belong to at least two of these sets. $L$ is the union of the set of control locations of the common components (line 12).
- In the second step, for each location $l$ in $L$, we get the set of interactions $\gamma_{l}$ that the incoming transitions and outgoing transitions of $l$ are involved (lines 16, 17). A transition $\tau=\left(l, p, l^{\prime}\right)$ is involved in an interaction $a$ if its port $p$ participates in $a$. For any two different interactions $a_{i}, a_{j}$ of $\gamma_{l}$, if they belong to different connectors, then the location $l$ is common location and is added to the set of common locations $L_{c}$ (lines 19, 20, 21).
The implementation of the function for computing incrementally interaction invariants is presented in Algorithm 17. The input consists of a connector $\gamma$ over a set of components $B=\left(B_{1}, \ldots, B_{m}\right)$, a set of increments $\left(\delta_{1}, \ldots, \delta_{n}\right)$ over $\gamma$, and the initial state Init. The computation process is as follows:

```
function getCommonLocations \(\left(\gamma_{1}, \ldots, \gamma_{n}, B\right)\)
begin
    compList \(_{i}=\left\{B_{j}=\left(L_{j}, P_{j}, \mathcal{T}_{j}\right) \in B \mid P_{j} \cap \gamma_{i} \neq \emptyset\right\} ;\)
    commonCompList \(=\emptyset\);
    for each compList \({ }_{i}\) do
        for each component \(B_{j}\) of compList \(_{i}\) do
            if \(B_{j} \in\) compList \(_{k}, k \neq i\) then
                commonCompList \(=\) commonCompList \(\cup B_{j}\)
            end
            end
    end
    \(L=\bigcup_{B_{i} \in \text { commonCompList }} L_{i} ;\)
    \(L_{c}=\emptyset ; / *\) list of common locations */
    for each location \(l \in L\) do
            \(\gamma_{l}=\emptyset ;\)
            for each transition \(\tau=\left(l^{\prime}, p, l\right)\) or \(\tau=\left(l, p, l^{\prime}\right)\) do
            \(\gamma_{l}=\gamma_{l} \cup\{a \in \gamma \mid p \in a\} ;\)
            end
            for any two interactions \(a_{i}, a_{j}, i \neq j\) of \(\gamma_{l}\) do
                if connectors of \(a_{j}\) and \(a_{j}\) are not the same then
                    \(L_{c}=L_{c} \cup l ;\)
                    break;
                end
            end
        end
    return \(L_{c}\);
end
```

Algorithm 16: Common Location Getting

```
function computeIncrII \(\left(\left(\delta_{1}, \ldots, \delta_{n}\right), \gamma,\left(B_{1}, \ldots, B_{m}\right)\right.\), Init \()\)
begin
    \(L=\cup_{L_{i} \in B_{i}} L_{i} ;\)
    \(\delta_{0}=\gamma-\sum_{i=1}^{n} \delta_{i}^{f} ;\)
    \(L_{c}=\) getCommonLocations \(\left(\delta_{0}, \ldots, \delta_{n}, B\right)\);
    for each \(\delta_{i}\) do
        \(L_{i}=\bigcup_{P_{k} \cap \delta_{i} \neq \emptyset \wedge B=\left(L_{k}, P_{k}, \tau_{k}\right)} L_{k} ;\)
        Init \(_{i}=\bigvee_{l \in\left(\text { Init } \cap L_{i}\right)} l\);
        \(\left|\delta_{i}(B)\right|=\) generateBBC \(\left(\delta_{i}, B\right)\);
        \(\left|\delta_{i}(B)\right|_{\text {init }}=\left|\delta_{i}(B)\right| \wedge I n i t_{i} ;\)
        \(\left|\delta_{i}(B)\right|_{\overline{\text { init }}}=\left|\delta_{i}(B)\right| \wedge \overline{\text { nitit }_{i}} ;\)
        \(\left|\delta_{i}(B)\right|_{i n i t}^{p\left(L_{c}\right)}=\) positiveMapping \(\left(\left|\delta_{i}(B)\right|_{\text {init }}, L_{c}\right)\);
        \(\left|\delta_{i}(B)\right|_{\frac{i n i t}{p\left(L_{c}\right)}}^{i}=\) positiveMapping \(\left(\left|\delta_{i}(B)\right|_{\overline{i n i t}}, L_{c}\right) ;\)
    end
    \(f_{\text {init }}=\) true, \(f_{\overline{\text { init }}}=\) true;
    for each \(\delta_{i}\) do
        \(f_{\text {init }}=\left(f_{\text {init }} \wedge\left|\delta_{i}(B)\right|_{\text {init }}^{p\left(L_{c}\right)}\right) \vee\left(f_{\text {init }} \wedge\left|\delta_{i}(B)\right|_{\overline{i n i t}}^{p\left(L_{c}\right)}\right) \vee\left(f_{\overline{\text { init }}} \wedge\left|\delta_{i}(B)\right|_{i n i t}^{p\left(L_{c}\right)}\right) ;\)
        \(f_{\overline{\text { init }}}=\left(f_{\overline{\text { init }}} \wedge\left|\delta_{i}(B)\right|_{\overline{\text { init }}}^{p\left(L_{c}\right)}\right) ;\)
    end
    \(f_{\text {init }}^{p}=\) positiveMapping \(\left(f_{\text {init }}, L \backslash L_{c}\right)\);
    \(\Psi=\operatorname{dual}\left(f_{\text {init }}^{p}\right)\);
    return \(\Psi\);
end
```

Algorithm 17: Incremental Computation Based on Positive Mapping

- First, we get the set of locations $L$ of all the components $B_{1}, \ldots, B_{m}$ (line 3). We denote $\gamma-\sum_{i=1}^{n} \delta_{i}^{f}$ by $\delta_{0}$ (line 4) and get the set of common locations $L_{c}$ of the set $\left\{\delta_{i}\right\}_{i=0}^{n}$ by the function getCommonLocations (line 5).
- Then, for each $\delta_{i}$, there are the following steps:
- we get $L_{i}$, the set of locations involved in $\delta_{i}$, i.e the union of locations of components that its ports participate in $\delta_{i}$ (line 7),
- we generate the initial condition $\mathrm{Init}_{i}$ from Init for $\delta$ (line 8),
- we generate the Boolean Behavioral Constraints $\left|\delta_{i}(B)\right|$ by function generateBBC (line 9).
- $\left|\delta_{i}(B)\right|$ is then decomposed into two parts: one part $\left|\delta_{i}(B)\right|_{\text {init }}$ satisfying initial condition by adding Init $_{i}$ (line 10) and the other part $\left|\delta_{i}(B)\right|_{\overline{\text { init }}}$ which does not satisfy the initial condition and is obtained by the conjunction of $\left|\delta_{i}(B)\right|$ and $\overline{\text { Init }_{i}}$ (line 11).
- the positive mapping function positiveMapping is called for both $\left|\delta_{i}(B)\right|_{\text {init }}$ (line 12) and $\left|\delta_{i}(B)\right|_{\overline{\text { init }}}$ (line 13) to eliminate the negative forms of non-common location variables. We obtain respectively $\left|\delta_{i}(B)\right|_{\text {init }}^{p\left(L_{c}\right)}$ and $\left|\delta_{i}(B)\right|_{\text {init }}^{p\left(L_{c}\right)}$.
- Now we integrate the above results. $f_{\text {init }}$ (resp. $f_{\overline{\text { init }}}$ ) represents the integrated formula which satisfies (resp. does not satisfy) initial condition $\bigvee$ Init $_{i}$. Since each final invariant must satisfy the initial condition, we have several kinds of integrations for each connector $\delta_{i}$ : three integrations which return a formula satisfying initial
condition: $\left(f_{\text {init }} \wedge\left|\delta_{i}(B)\right|_{\text {init }}^{p\left(L_{c}\right)}\right) \vee\left(f_{\text {init }} \wedge\left|\delta_{i}(B)\right|_{\left.\frac{p\left(L_{c}\right)}{p}\right)}^{\left(L_{i n i t}\right.} \vee\left(f_{\overline{\text { init }}} \wedge\left|\delta_{i}(B)\right|_{\text {init }}^{p\left(L_{c}\right)}\right)\right.$ and are then assigned to $f_{\text {init }}$ (line 17); an integration which returns a formula that does not satisfy initial condition: $\left(f_{\overline{\text { init }}} \wedge\left|\delta_{i}(B)\right| \frac{p\left(L_{c}\right)}{\overline{i n i t}}\right)$ and is then assigned to $f_{\overline{\text { init }}}$ (line 18).
- After the integration, we call positiveMapping function for $f_{\text {init }}$ to remove the remaining negative forms of common location variables (line 20).
- Finally we call the dual function dual for the formula $f_{i n i t}^{p}$ (line 21) to get the global interaction invariants $\Psi$ of the system $\left\langle\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma(B)\right.$, Init $\rangle$.


## Implementation of the Incremental Fixed-point-based Method

First we provide the implementation of the function for incremental computation of image vector from a set of connectors. Given a set of connectors $\gamma_{1}, \ldots, \gamma_{n}$ over a set of components $B$ together with the set of corresponding image vectors $\mathbb{V}_{\gamma_{1}}, \ldots, \mathbb{V}_{\gamma_{1}}$, the function presented in Algorithm 18 computes the image vector $\mathbb{V}_{\gamma}$ for $\left(\sum_{i=1}^{n} \gamma_{i}\right)(B)$ from $\mathbb{V}_{\gamma_{1}}, \ldots, \mathbb{V}_{\gamma_{1}}$.

The computation process is as follows: for every location $l$ of $L$, the set of locations of atomic components in $B$, we initialize the image $\mathbb{V}_{\gamma}(l)$ by $l$ (line 5). Then for any $\gamma_{i}$ such that $l \in{ }^{\bullet} \gamma_{i}, \mathbb{V}_{\gamma}(l)$ is updated by the conjunction with $\mathbb{V}_{\gamma_{i}}(l)$ (lines $\left.6,7,8\right)$. Finally, the function returns $\mathbb{V}_{\gamma}$, the image vector of $\left(\sum_{i=1}^{n} \gamma_{i}\right)(B)$.

```
function computeIncrImageVector \(\left(\left(\gamma_{1}, \ldots, \gamma_{n}\right),\left(\mathbb{V}_{\gamma_{1}}, \ldots, \mathbb{V}_{\gamma_{n}}\right), B\right)\)
begin
    \(L\) is set of locations of atomic components in \(B\);
    for each \(l\) of \(L\) do
        \(\mathbb{V}_{\gamma}(l)=l ;\)
        for each \(\gamma_{i}\) do
            if \(l \in \gamma_{i}\) then
                \(\mathbb{V}_{\gamma}(l)=\mathbb{V}_{\gamma}(l) \wedge \mathbb{V}_{\gamma_{i}}(l) ;\)
            end
            end
    end
    return \(\mathbb{V}_{\gamma}\);
end
```

Algorithm 18: Incremental Computation Of Image Vector
Interaction invariants can be incrementally computed by using fixed-point computation for a system resulting from the superposition of a set of increments. the implementation of the method is presented in Algorithm 19. It takes as input a connector $\gamma$ over a set of components $B$, a set of increments $\left\{\delta_{i}\right\}_{i=1}^{n}$, a set of image vector $\left\{\mathbb{V}_{i}\right\}_{i=0}^{n}$ and a set of fixed-points $\left\{S_{i}\right\}_{i=0}^{n}$ where $\mathbb{V}_{i}=\operatorname{computeImageVector}\left(\delta_{i}, B\right)$ and $S_{i}=\operatorname{computeFixedpoint}\left(\mathbb{V}_{i}, \bigvee_{l \in \bullet \delta_{i}}\right)$ (or $S_{i}=\operatorname{computeFixedpoint}\left(\mathbb{V}_{i}, \bigvee_{l \in \operatorname{Init\cup } \cup \delta_{i}}\right)$ in the case of system with initial state Init) and $\delta_{0}=\gamma-\left(\sum_{i=1}^{n} \delta_{i}\right)^{f}$.

The function compute the fixed-points for $\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma(B)$ starting from the sets of constituent fixed-points above. The computation process is as follows:

- First we compute the image vector $\mathbb{V}$ for $\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma(B)$ from the set of constituent image vector $\left\{\mathbb{V}_{i}\right\}_{i=1}^{n}$ by calling the function computeIncrImageVector (line 3).

```
function computeIncrIIByFixedpoint(\gamma,B,{\mp@subsup{\delta}{i}{}\mp@subsup{}}{i=0}{n},{\mp@subsup{\mathbb{V}}{i}{}\mp@subsup{}}{i=0}{n},{\mp@subsup{S}{i}{}\mp@subsup{}}{i=0}{n})
begin
    \mathbb{V}=\mathrm{ computeIncrImageVector ({ }\mp@subsup{\delta}{i}{}\mp@subsup{}}{i=0}{n},{\mp@subsup{\mathbb{V}}{\mp@subsup{\delta}{i}{}}{}\mp@subsup{}}{i=0}{n},B);
    SO = \ \i=0}\mp@subsup{|}{i}{n}
    S = computeFixedpoint(\mathbb{V},\mp@subsup{S}{0}{});
    return S;
end
```

Algorithm 19: Incremental Computation of Fixed-point

- Then the starting point of the fixed-point iteration is set by the disjunction of the constituent fixed-points $\bigvee_{i=0}^{n} S_{i}$ (line 4).
- Finally, the function computeFixedpoint is called to compute the global fixed-points of $\left(\sum_{i=1}^{n} \delta_{i}\right) \gamma(B)$ using $S_{0}$ and the global image vector $\mathbb{V}$ (line 5).


### 5.7 Checking Satisfiability

For systems without data, since the predicate DIS and component invariants can be symbolically represented, the checking of unsatisfiability of $\bigwedge \Phi_{i} \wedge \Psi \wedge D I S$ can be done by using CUDD package.

For systems with data, the check of unsatisfiability is performed by Yices, a Sat-Solver tool. Given a formula $f$, Yices checks whether $f$ is satisfiable. If it is, Yices provides a sat output together with a solution satisfying $f$, otherwise an unsat output is produced. Since Yices provides just one solution $s$ satisfying the formula $f$, we need to update $f$ by $f \wedge(\neg s)$ to get another solution. And this step is repeated until $f$ becomes unsatisfiable to get all the solutions. An example of Yices input language for a formula $f=(l 1 \wedge(x=0)) \vee(l 2 \wedge(x \geq 1))$ is as follows:

```
(define 11::bool)
(define l2::bool)
(define x::int)
(assert (or (and l1 (= x 0)) (and l2 (>= x 1))))
(check)
```

In the symbolic methods, interaction invariants are generated and stored in a BDD, therefore a transformation is required to convert interaction invariants into a form accepted by Yices. We have implemented this transformation in the Bdd2F function.

## Bdd2F Transformation

We have implemented a function Bdd2F which allows transforming a BDD to a formula. The implementation of the Bdd2F transformation function is described in Algorithm 20. Here we provide a transformation from a BDD to a general formula, for a specific form of formula, we just need to change the form of formula to be printed in an output file. Taking as input a BDD representing a formula $f$ and an output file, this function works as follows:

```
function \(\operatorname{Bdd2F}(B d d f\), File file)
begin
    index \(=0\);
    /* Set value name for each node */;
    for each Node \(n\) of Bdd f do
        n.val \(=\) " \(n\) " " + index ;
        index ++ ;
    end
    /* Convert Bdd to Yices formula */
    String tVal, eVal ;
    for each Node \(n\) of Bdd f do
        /* Get 'then" node of n */
        Node tNode \(=\operatorname{cuddT}(n)\);
        t Val \(=\) if isConst( t Node) then "true" else tNode.val;
        /* Get 'else" node of n */
        Node eNode \(=\operatorname{cuddE}(n)\);
        if isConst(eNode) then
            \(e \mathrm{Val}=\) if isCompl(eNode) then "false" else "true";
        end
        else
            \(\mathrm{eVal}=\) if isCompl(eNode) then "not" + eNode.val else eNode.val;
        end
        \(\mathrm{x}=\) n.var; /* get corresponding variable of \(\mathrm{n} \quad\) */
        print(file, n.val \(+"=\) if \(x\) then " \(+\mathrm{tVal}+\) " else " +eVal );
    end
    /* Assign \(\mathrm{f}=\) true by assigning root node to true */
    Node rNode = f.root; \(\quad\) /* get 'root'" node of f
    print(file, "rNode.val = true");
end
```

Algorithm 20: Bdd-to-formula Transformation

- First, we provide an unique name representing the value of each node of the BDD (lines 6, 7).
- Then, every node of the BDD is considered. A node $n$ corresponds to a variable $x$ and it has two children : a "then" node $t$ Node and an "else" node eNode. The value of the node $n$ is therefore either the value of "then" node ( $n . v a l=t V a l$ ) if $x$ is true or the value of "else" node ( $n . v a l=e \mathrm{eVal}$ ) otherwise. The values of "then" node (tNode) and "else" node (eNode) are specified as follows:
- If tNode is constant, i.e tNode is a true node since "then" node can not be complemented, $t V a l=$ true. Otherwise $t V a l=t$ Node.val (lines 13, 14)
- If eNode is constant, since "else" node can be complemented, the constant can be true or false node. If eNode is complemented $e V a l=$ false, else $e V a l=$ true (lines $16,17,18$ ). If eNode is not constant, we also consider two cases: if eNode is complemented $\mathrm{eVal}=e$ Node.val, otherwise $\mathrm{eVal}=($ not eNode.val $)$ (line 21).
- The formula n.val $=$ if $x$ then $t$ Val else eVal for the node $n$ with the values of tVal and eVal specified as above is written to a file (line 24).
- Finally, we get the root node and assert its value to "true" (lines 27, 28).


Figure 5.5: A bdd example

Example 43 Figure 5.5 is an example of a formula represented in a BDD. The formula generated by Bdd2F function for this BDD is as follows:


### 5.8 Summary

We have provided an overview on the implementation of the D-Finder tool for checking deadlock-freedom of component-based systems. We have first described the structure, the procedure of D-Finder for checking deadlock-freedom together with the corresponding modules, then the detail on the implementation of each module.

An important module is Interaction Invariant Generation where we have implemented several techniques for users to choose. One is based on Yices, a Sat-Solver tool; the others are based on CUDD package and are either enumerative (enumerative method) or totally symbolic (methods based on Positive Mapping and Fixed-point Computation). Moreover, for the symbolic computation of interaction invariants, we has implemented both global and incremental methods. The global method computes directly invariants for the entire systems from scratch and the incremental method permits computing invariants of composite components from their constituents. The implementation of different methods in D-Finder allows it to handle a large range of systems.

In the next chapter, we are going to present the experimental results obtained by DFinder on several non-trivial case studies which show the efficiency of the methods as well as the capacity of the D-Finder tool.

## CHAPTER 6

## Experimentation

## Chapter Abstract

This chapter provides some case studies verified by the D-Finder tool. Two case studies without data are considered to check the scalability of the method. The first is Dining Philosophers, a classical example in detecting deadlocks. The second is Gas Station where we check the scalability by increasing the number of pumps and customers. We also verified two case studies with data: one is Automatic Teller Machine system and the other is a module of a robotic system, robot Dala developed at Laas laboratory. The experimental results show that our methods are scalable and efficient for large-scale systems and in particular, for complex real world systems.

## Résumé du Chapitre

Ce chapitre fournit quelques études de cas vérifiées par l'outil D-Finder. Deux études de cas sans données sont considérées pour montrer la scalabilité de la méthode. La première est Philosophes, un exemple classique dans la détection des blocages. La deuxième est Gas Station où nous vérifions et montrons la scalabilité en augmentant le nombre de pompes et de clients. Nous avons également vérifié deux études de cas avec données: l'une est le système de guichet automatique bancaire (Automatic Teller Machine) et l'autre est un module d'un système robotique, le robot Dala développé au laboratoire Laas. Les résultats expérimentaux montrent que nos méthodes sont très efficaces pour les systèmes complexes et à grande échelle.

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All the experimentations are done on a Linux machine Intel Pentium 4 CPU 3.0 GHz and RAM 1G.

### 6.1 Dining Philosophers

The Dining Philosophers problem, illustrated in Figure 6.1, can be shortly presented as a number of philosophers sitting at a table doing one of two things: eating or thinking. While they are thinking, they are not eating and while they are eating, they are not thinking. They sit at a round table and each philosopher has a spaghetti disk in front of him. Between two any next philosophers, there is a fork. A philosopher can eat if he has both forks from the left and from the right. And a philosopher can put two forks back to the table only if he has finished eating. Here there is a deadlock situation if they all have the same order of taking forks and every philosopher has taken one fork, so everyone can not eat because each has


Figure 6.1: Dining Philosophers only one fork.

However, if a Philosophers changes the order of taking forks, for example, he takes the right one before the left one while the others take the left one before the right one, then there is no deadlock. We have considered both cases in the experimentation.

We have modeled the philosopher and the fork in BIP. Figure 6.2 represents the behavior of a philosopher: initially in state $l_{\text {th }}$, he can think by think transition or prepare for eating by first taking the left fork (take_ $l$ transition) and moves to $l_{1 f}$ location where he has one fork on the left hand, then taking the right fork (take_r transition) and moves to $l_{2 f}$ location where he has two forks on both hands. At $l_{2 f}$ he can eat by taking eat transition and moves to $l_{f i}$ location where he has finished eating, hence he can put two forks by put transition and returns to $l_{t}$ location. The think and the eat transitions are internal, they can be executed without synchronizing with other components and hence their ports are


Figure 6.2: Philosopher Component


Figure 6.3: Fork Component


Figure 6.4: Dinning Philosophers with four Philosophers
complete, represented by triangles in the figure. The other ports (take_l, take_r, put) are incomplete (represented by circles) and their transitions need to synchronize with other components to be executed.

The model of a fork is quite simple (Figure 6.3): it has two locations $l_{f}$ and $l_{u}$. Initially in the location $l_{f}$, a fork can be taken by use transition and goes to $l_{u}$ location. From $l_{u}$, a fork is released if the free transition is executed and the fork returns to $l_{f}$ location. Both ports use and free are incomplete.

### 6.1.1 Dining Philosophers with Deadlocks

In this model, all Philosophers have the same order in taking forks, we assume that all Philosophers take the left fork before the right fork.

Figure 6.4 shows the model for Dinning Philosophers with 4 Philosophers and 4 forks.


Figure 6.5: Verification Time of Dining Philosopher with Deadlock

In general case, given $n$ philosophers and $n$ forks, the interactions are:

$$
\gamma[n]=\sum_{i=1}^{n}\left(t h_{i}+t_{-} l_{i} u_{i}+t_{-} r_{i} u_{(i \bmod n)+1}+e_{i}+p_{i} f r_{i} f r_{(i \bmod n)+1}\right)
$$

where $t h_{i}, t_{-} l_{i}, t_{-} r_{i}, e_{i}, p_{i}$ respectively represent the ports think, take_l,take_r, eat, put of Philosopher philo $_{i}$, and $u_{i}, f r_{i}$ respectively represent the ports use, free of Fork fork $k_{i}$.

In the incremental construction, these interactions can be obtained from the superposition of a set of increments, each increment adds interactions for philosophers from $n_{1}$ to $n_{2}$ with $1 \leq n_{1}, n_{2} \leq n$ as follows:

$$
\delta_{\left[n_{1}, n_{2}\right]}=\sum_{i=n_{1}}^{n_{2}}\left(t h_{i}+t_{-} l_{i} u_{i}+t_{-} r_{i} u_{(i \bmod n)+1}+e_{i}+p_{i} f r_{i} f r_{(i \bmod n)+1}\right)
$$

For example, the set of interactions for a system of 4 Philosophers is the superposition of the two following increments:
$\delta_{[1,2]}=t h_{1}+t_{-} l_{1} u_{1}+t_{-} r_{1} u_{2}+e_{1}+p_{1} f r_{1} f r_{2}+t h_{2}+t_{-} l_{2} u_{2}+t_{-} r_{2} u_{3}+e_{2}+p_{2} f r_{2} f r_{3}$ $\delta_{[3,4]}=t h_{3}+t_{-} l_{3} u_{3}+t_{-} r_{3} u_{4}+e_{3}+p_{3} f r_{3} f r_{4}+t h_{4}+t_{-} l_{4} u_{4}+t_{-} r_{4} u_{1}+e_{4}+p_{4} f r_{4} f r_{1}$.

In Figures 6.5, 6.6 and Table 6.1, we provide experimental results on Dining Philosophers. We increase the number of Philosophers and compare the verification time and memory usage between the different methods implemented in D-Finder. We also compare


Figure 6.6: Memory Usage of Dining Philosopher with Deadlock
our methods with the well-known verification tool NuSmv. In two figures, $x$ axis represents the number $n$ of Philosophers (and also the number of Forks), $y$ axis respectively represents the verification time (in minutes) and the memory usage (in Mb).

All the methods detected a deadlock and that is the real deadlock where all Forks are at the busy location $l_{u}$, i.e. they are being occupied by some Philosopher; all Philosophers are at the location $l_{l}$, i.e. they have taken the left fork and are waiting for the right one (but no fork is available):

$$
\text { deadlock }=\bigwedge_{i=1}^{n}\left(\text { philo }_{i} \cdot l_{1 f} \wedge \text { fork }_{i} \cdot l_{u}\right)
$$

The invariants generated by D-Finder which allows detecting the above deadlock for a system consisting of $n$ Philosophers and $n$ Forks are as follows:

$$
\Psi=\bigwedge_{i=1}^{n}\left(\text { philo }_{i} \cdot l_{t h} \vee \text { fork }_{i} \cdot l_{u} \vee \text { philo }_{(i \bmod n)+1} \cdot l_{t h} \vee \text { philo }_{(i \bmod n)+1} \cdot l_{1 f}\right)
$$

According to the experimentation with a time out of 60 minutes, we have:

- Verification time and memory usage by NuSmv increase exponentially. At the size of 130 Philosophers, NuSmv has time out and uses 900 Mb over 1000 Mb of memory.
- The global verification based on positive mapping can verify up to the size 1800 Philosophers within 60 minutes. The memory used by this method is low: for 1800 Philosophers, it uses less than 200 Mb .

| Component information |  |  | Time (minutes) |  |  |  |  |  | Memory (MB) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| philos | locs | intrs | Smv | Enum | $P M$ | FP | IPM | IFP | Smv | Enum | $P M$ | $F P$ | IPM | IFP |
| 100 | 600 | 500 | 1:32 | 0:06 | 0:13 | 22:41 | 0:4 | 0:19 | 533 | 34 | 46 | 75 | 10 | 32 |
| 500 | 3000 | 2500 | - | 1:51 | 4:01 | - | 0:34 | 10:03 | - | 55 | 61 | - | 29 | 60 |
| 1000 | 6000 | 5000 | - | 7:08 | 17:09 | - | 2:04 | - | - | 90 | 105 | - | 60 | - |
| 1500 | 9000 | 7500 | - | 19:30 | 39:40 | - | 3:09 | - | - | 126 | 148 | - | 74 | - |
| 2000 | 12000 | 10000 | - | 28:44 | - | - | 4:14 | - | - | 163 | - |  | 96 | - |
| 4000 | 24000 | 20000 | - | - | - | - | 8:37 | - | - | - | - | - | 192 | - |
| 6000 | 36000 | 30000 | - | - | - | - | 14:26 | - | - | - | - | - | 382 | - |
| 7000 | 42000 | 35000 | - | - | - | - | 17:34 | - | - | - | - | - | 498 | - |
| 8000 | 48000 | 40000 | - | - | - | - | 20:45 | - | - | - | - | - | 510 | - |
| 9000 | 53000 | 45000 | - | - | - | - | 24:16 | - | - | - | - | - | 581 | - |
| philo locs: intrs | num <br> num <br> nu | er of $p$ er of lo er of in | losop ation racti |  | v: | NuSmv <br> Enume <br> Global | ative <br> ixed-p |  | $\begin{aligned} & \mathrm{PM}: \\ & \mathrm{PM}: \\ & \text { FP: } \end{aligned}$ | Global Increm <br> Increm | Positive <br> tal P <br> tal F | Ma |  |  |

Table 6.1: Comparison between different methods on Dining Philosophers

- The global verification based on fixed-point is not good for this example. The reason is that the Philosophers example has cycle structure which has invariants involved in all the components of the system, hence the number of iterations in fixed-point computation is big since the iteration process has to pass over the whole cycle (all the components) to get these invariants.
- The global enumerative verification is better than the global symbolic methods, it can verify up to the size of 2800 Philosophers within 60 minutes and uses less that 250 Mb of memory. The reason is that the number of interaction invariants of this example does not increase exponentially in the size of the systems. For example, the system of 2000 Philosophers has 10003 interaction invariants and the system of 2800 Philosophers has 13003 interaction invariants.
- The incremental method based on positive mapping consumes almost linearly time according to the size of the system. Each increment adds a set of interactions for 500 Philosophers and 500 Forks. The incremental verification of the system built from 18 increments (corresponding to a system of 9000 Philosophers and 9000 Forks) is done in 25 minutes and uses less than 600 Mb of memory.


### 6.1.2 Dining Philosophers without Deadlocks

In this model, all Philosophers have the same order in taking forks except a Philosopher have different order, we assume that the first Philosopher takes the right fork before the left, and the others take the left fork before the right.

We did the experimentation with the methods implemented in D-Finder and NuSmv. All the methods report that the system is deadlock-free. The interaction invariants generated by D-Finder which allow proving deadlock-freedom of a system having $n$ Philosophers and $n$ Forks are as follows:

$$
\Psi=\bigwedge_{i=2}^{n-1}\left(\text { philo }_{i} \cdot l_{t h} \vee \text { fork }_{i} \cdot l_{u} \vee \text { philo }_{(i \bmod n)+1} \cdot l_{t h} \vee \operatorname{philo}_{(i \bmod n)+1} \cdot l_{1 f}\right)
$$



Figure 6.7: Sketch of Gas Station

$$
\begin{aligned}
& \bigwedge\left(\text { philo }_{1} . l_{2 f} \vee \text { philo }_{1} . l_{f i} \vee \text { fork }_{1} . l_{f} \vee \text { philo }_{2} . l_{2 f} \vee \text { philo }_{2} . l_{f i}\right) \\
& \bigwedge\left(\text { philoo }_{1} . l_{t h} \vee \text { philo }_{1} . l_{1 f} \vee \text { fork }_{1} . l_{u} \vee \text { philoo }_{2} . l_{t h} \vee \text { philo }_{2} . l_{1 f}\right) \\
& \bigwedge\left(\text { philo }_{1} . l_{t h} \vee \text { fork }_{n} . l_{u} \vee \text { philo }_{n} . l_{t h}\right)
\end{aligned}
$$

The performance of the different methods of D-Finder is almost similar to the performance in verifying the Philosophers system with deadlocks presented in the previous subsection.

### 6.2 Gas Station

Gas Station [HL85] consists of an Operator with a computer, a set of pumps, and a set of customers. Each pump can be used by a fixed number of customers. The set of the atomic components involved in a system with $n$ pumps and $m$ customers for each pump is denoted by $\left.B[n, m]=\left\{\text { Operator, }^{\text {pump }}\right\}_{1 \leq i \leq n},\left\{\text { customer }_{i j}\right\}_{1 \leq i \leq n, 1 \leq j \leq m}\right\}$.

Before using a pump, each customer has to prepay for the transaction. Then the customer uses the pump, collects his change and goes to a state from which he may start a new transaction.

Before being used by a customer, the pumps have to be activated by the Operator. When a pump is shut off, it can be re-activated for the next operation.

Figure 6.7 gives the model for Gas Station system for one pump and two customers. The Operator has two control locations and three ports. The transition labeled with prepay accepts a customer's prepay and activates the pump for the customer. When a customer is served, the transition labeled with finish will synchronize the pump and the customer. A pump has three control locations and three ports. Besides the synchronization between

### 6.2. GAS STATION

the Operator and customer through activate and finish ports, a pump and a customer are synchronized through start ports.

We abbreviate port names by using only their first three letters. The ports of Operator are respectively pre, fin, cha; the ports of pump $_{i}$ are respectively $a_{i} t_{i}, s t a_{i}$, fin $_{i}$ and the ports of customer $_{j}$ of $p u m p_{i}$ are $p r e_{i j}, s t a_{i j}, f i n_{i j}$, cha $a_{i j}$. The interactions for a system of $n$ pumps, each one used by $m$ customers, are

$$
\gamma[n, m]=\sum_{i=1}^{n}\left(\sum_{j=1}^{m}\left(\text { pre act }_{i} \text { pre }_{i j}+\text { sta }_{i} \text { sta }_{i j}+\text { fin fin }_{i} \text { fin }_{i j}+\text { cha cha }_{i j}\right)\right)
$$

The system consisting of $n$ pumps and $m$ customers can be incrementally built by the superposition of a set of increments, each increment adds interactions for a set of pumps, a set of customers and Operator. If each pump connects to $m$ customers, the increment that adds interactions for the pumps from $n_{1}$ to $n_{2}$ is:

$$
\delta\left[n_{1}, n_{2}, m\right]=\sum_{i=n_{1}}^{n_{2}}\left(\sum_{j=1}^{m}\left(\text { pre act }_{i} p r e_{i j}+s t a_{i} s t a_{i j}+\text { fin fin }_{i} \text { fin }_{i j}+\text { cha cha } a_{i j}\right)\right)
$$

For example, the system of two pumps and two customers can be built from two increments, each increment contains interactions over a pump, a customer and Operator as follows:

$$
\begin{aligned}
& \delta_{1}[1,1,1]=\text { pre act } \text { pre }_{11}+\text { sta }_{1} \text { sta }_{11}+\text { fin fin } \text { fin }_{1} \text { fin }_{11}+\text { cha cha }{ }_{11} \\
& \delta_{2}[2,2,1]=\text { pre act } \text { pre }_{21}+\text { sta }_{1} \text { sta }_{21}+\text { fin fin } \text { fin }_{1} \text { fin }_{21}+\text { cha cha }
\end{aligned}
$$

D-Finder reports deadlock-freedom of the Gas Station system. The interaction invariants generated by D-Finder which prove the deadlock-freedom of a Gas Station system consisting of $n$ pumps, each pump pump $_{i}$ has $m_{i}$ customers, are as follows:

$$
\begin{aligned}
\Psi & =\bigwedge_{i=1}^{n}\left(\text { pump }_{i} \cdot l_{0} \vee \text { pump }_{i} \cdot l_{1} \vee \bigvee_{j=1}^{m_{i}} \text { customer }_{i j} \cdot l_{2}\right) \\
& \wedge \bigwedge_{i=1}^{n}\left(\text { pump }_{i} \cdot l_{0} \vee \text { pump }_{i} \cdot l_{2} \vee \bigvee_{j=1}^{m_{i}} \text { customer }_{i j} \cdot l_{1}\right) \\
\wedge & \bigwedge_{i=1}^{n}\left({\text { operator. } \left.. l_{1} \vee \text { pump }_{i} \cdot l_{1} \vee \text { pump }_{i} \cdot l_{2} \vee \bigvee_{j=1}^{m_{i}} \text { customer }_{i j} \cdot l_{0}\right)}\right. \\
& \wedge \bigwedge_{i=1}^{n}\left({\text { operator } . l_{0}} \vee \bigvee_{i=1}^{n} \bigvee_{j=1}^{m_{i}} \text { customer }_{i j} \cdot l_{3}\right)
\end{aligned}
$$

Figures 6.8, 6.9 and Table 6.2 present the verification time and memory usage for Gas Station system by different methods implemented in D-Finder and by NuSmv. For the incremental verification methods, $N$ is number of increments and each increment adds interactions for 50 pumps and 500 customers. Hence, the size of a system built from $N$ increments is $N \times 50$ pumps and $N \times 500$ customers. For the global verification method, the number of pumps is $N \times 50$ and the number of customers is $N \times 500$. We also set the time out to 60 minutes. According to the experimentation:


Figure 6.8: Gas Station - Verification Time


Figure 6.9: Gas Station - Memory Usage

| Component information |  |  | Time (minutes) |  |  |  |  | Memory (MB) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| pumps | locs | intrs | NuSMV | PM | $F P$ | IPM | IFP | NuSMV | $P M$ | FP | IPM | IFP |
| 18 | 776 | 720 | 1:43 | 0:18 | 0:8 | 0:9 | 0:9 | 921 | 48 | 35 | 17 | 17 |
| 50 | 2152 | 2000 |  | 0:50 | 0:17 | 0:49 | 0:17 | - | 48 | 53 | 47 | 53 |
| 100 | 4302 | 4000 |  | 2:58 | 0:38 | 1:51 | 0:52 | - | 76 | 65 | 47 | 52 |
| 200 | 8602 | 8000 |  | 11:34 | 1:34 | 2:26 | 1:55 |  | 135 | 107 | 47 | 65 |
| 300 | 12902 | 12000 | - | 26:00 | 2:53 | 4:11 | 2:85 | - | 194 | 160 | 49 | 68 |
| 400 | 17202 | 16000 | - | 47:38 | 5:01 | 5:34 | 3:51 | - | 270 | 215 | 76 | 93 |
| 500 | 21502 | 20000 | - | - | 7:45 | 7:21 | 4:43 | - |  | 261 | 86 | 101 |
| 600 | 25802 | 24000 | - |  | 11:21 | 9:05 | 5:53 | - |  | 316 | 97 | 115 |
| 700 | 30102 | 28000 | - | - | 16:04 | 11:44 | 7:14 | - | - | 350 | 107 | 138 |
| pumps <br> locs <br> intrs <br> Smv |  | : number of pumps <br> number of locations <br> number of interactions <br> : NuSmv |  |  |  | PM <br> FP <br> IPM <br> IFP | Global Positive Mapping Global Fixed-point Incremental Positive Mapping Incremental Fixed-point |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

Table 6.2: Comparison between different methods on Gas Station

- NuSmv reaches timeout at the size ( 18 pumps +180 customers) and uses 920 Mb over 1000 Mb of memory.
- The global symbolic method based on positive mapping can verify up to the size $8 \times$ ( 50 pumps +500 customers) within 50 minutes and uses 300 Mb of memory.
- The incremental method based on positive mapping has almost linear verification time and memory usage according to the size of the system. It can verify up to the size $14 \times$ ( 50 pumps +500 customers) in 12 minutes and uses 107 Mb of memory.
- The global symbolic method and incremental method based on fix-point have very good performance compared to the corresponding methods based on positive mapping. The reason is that there is a small cycle of interactions between the Operator, a Pump and a Customer, hence the number of iterations to reach fixed-points is small (7 iterations) because the iteration process just has to pass over these three components to reach a fix-point.
Figure 6.9 shows that the incremental methods gain not only in verification time but also in memory usage compared to the global methods.


### 6.3 Automatic Teller Machine ATM

Automatic Teller Machine (ATM) is a computerized telecommunication device that provides services to access to financial transactions in a public space without the need for a cashier, human clerk or bank teller.

The structural model of the ATM system [CEFH01] is presented in Figure 6.10. The system is composed of the following components: User, ATM (modeling a cash dispenser) and Bank (modeling some aspects of bank operation). User and Bank interact only with ATM, but not with each other.

Figure 6.11 presents the modeling of ATM in BIP:

- Initially at location $l_{0}$, user can insert the card by insert transition and enter the
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Figure 6.10: ATM Structure


Figure 6.11: Modeling of ATM system in BIP
confidential code (enter transition). Then there are two cases: if the code is invalid, user gets back the card by eject transition and returns to the initial state $l_{0}$; otherwise, user continues by entering the amount of cash he/she wants to withdraw. If the amount is not accepted, the transaction is canceled (cancel transition); else there are two cases: transition fails (fail transition) or is ready (success transition) for user to withdraw the money. Finally user gets back their card.

- Initially at location $l_{0}$, ATM is waiting for user to insert the card (insert transition) and then to enter the confidential code (enter transition). The time-out for entering the code is 5 time units then it validates the entered code. If it receives non-authorized for the code by non_authorized transition, invalid transition takes place and then it ejects the card. If it receives authorized signal by authorized transition, the transition validated takes place and it moves to a location where user can enter the amount of cash. The timeout for entering the amount is 6 time units. If the user cancels the transaction or the amount is not allowed, it returns to $l_{6}$ to eject the card; else it accepts the amount and starts the transaction. If the transaction is forbidden (veto transition), it will announce to user by fail transition; else it will wait for user to withdraw the cash (withdraw transaction) and eject the card to finish the transaction.
- For Bank, there are two components: BankValidation component checks the validity of PIN code and BankTransaction component checks whether the transaction is forbidden (veto) or allowed (fiat). The use of these parallel components allows supporting multi Users and multi ATMs.
We abbreviate port names by using their first three letters except val_ed for validated and non_aut for non_authorized. We also use $[\text { port }]_{i}^{u},[\text { port }]_{i}^{a},[\text { port }]^{b t},[\text { port }]^{b v}$ to represent respectively the ports of user $_{i}$, atm $_{i}$, BankTransaction and BankValidation components. The set of interactions for an ATM system with $n$ ATMs and $n$ Users is as follows (Figure


Figure 6.12: ATM Verification Time
6.11):

$$
\begin{aligned}
& \gamma=\sum_{i=1}^{n}\left(i n s_{i}^{u} i n s_{i}^{a}+e n t_{i}^{u} e n t_{i}^{a}+v a l_{-} e d_{i}^{u} v a l_{-} e d_{i}^{a}+i n v_{i}^{u} i n v_{i}^{a}+a m o_{i}^{u} a m o_{i}^{a}+\operatorname{can}_{i}^{u} \operatorname{can}_{i}^{a}\right. \\
& +a c c_{i}^{u} a c c_{i}^{a}+f a i_{i}^{u} f a i_{i}^{a}+s u c_{i}^{u} s u c_{i}^{a}+w i t_{i}^{u} w i t_{i}^{a}+e j e_{i}^{u} e j e_{i}^{a}+a u t_{i}^{a} a u t^{b v}+\text { val }_{i}^{a} v a l^{b v} \\
& \left.+n o n \_a u t_{i}^{a} n o n \_a u t^{b v}+\text { fia }^{a} \text { fia }^{b t}+\text { vet }^{a} v e t^{b t}+\text { tra }^{a} \operatorname{tra}^{b t}+t i c^{a}+t i c^{b v}+t i c^{b t}\right)
\end{aligned}
$$

In the incremental construction, the ATM system with $n$ ATMs and $n$ Users is built by the superposition of a set of $n$ increments, each increment adds interactions for a set of ATMs and a set of Users. An increment which consists of interactions overs the ATMs from $n_{1}$ to $n_{2}$ is as follows:

$$
\begin{aligned}
& \delta\left[n_{1}, n_{2}\right]=\sum_{i=n_{1}}^{n_{2}}\left(i n s_{i}^{u} i n s_{i}^{a}+e n t_{i}^{u} e n t_{i}^{a}+\text { val_ed }_{i}^{u} v a l_{-} e d_{i}^{a}+i n v_{i}^{u} i n v_{i}^{a}+a m o_{i}^{u} a m o_{i}^{a}\right. \\
& +c a n_{i}^{u} c a n_{i}^{a}+a c c_{i}^{u} a c c_{i}^{a}+f a i_{i}^{u} f a i_{i}^{a}+s u c_{i}^{u} s u c_{i}^{a}+w i t_{i}^{u} w i t_{i}^{a}+e j e_{i}^{u} e j e_{i}^{a}+a u t_{i}^{a} a u t^{b v} \\
& \left.+ \text { val }_{i}^{a} v a l^{b v}+n o n_{\_} a u t_{i}^{a} n o n \_a u t^{b v}+\text { fia }^{a} \text { fia }^{b t}+\text { vet }^{a} \text { vet }^{b t}+t r a^{a}{ }^{a} r a^{b t}+t i c^{a}+t i c^{b v}+t i c^{b t}\right)
\end{aligned}
$$

D-Finder reports deadlock-freedom on the ATM system. Figure 6.12 and Table 6.3 shows the experimental results on ATM system by different methods:

| Component information |  |  |  | Time (minutes) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| atms | locs | intrs | vars | NuSmv | PM | $F P$ | IPM | IFP |
| 5 | 1104 | 902 | 102 | 5:15 | 0:3 | 0:2 | 0:4 | 0:4 |
| 50 | 1104 | 902 | 102 | - | 10:49 | 3:17 | 1:23 | 2:20 |
| 100 | 2204 | 1802 | 202 | - | 43:00 | 6:50 | 1:57 | 6:00 |
| 200 |  |  | 402 | - | - | 14:30 | 4:37 | 14:00 |
| 250 | 5504 | 4002 | 502 | - | - | 17:56 | 4:46 | 17:16 |
| 300 |  |  | 602 | - | - | 21:24 | 8:14 | 22:21 |
| 350 | 7704 | 6302 | 702 | - | - | 39:35 | 8:14 | 27:54 |
| 400 |  |  | 802 | - | - | - | 12:24 |  |
| 500 |  |  | 1002 | - | - | - | 18:05 |  |
| 600 | 13204 | 10802 | 1202 | - | - | - | 24:17 |  |
| S : | number of atms number of locations number of interactions number of variables |  |  | PM | Global Positive Mapping |  |  |  |
| : |  |  |  | FP | Glob | al Fix | -point |  |
| S |  |  |  | IPM | Incr | menta | Positiv | Map |
| : |  |  |  | IFP | Incr | menta | Fixed- | oint |

Table 6.3: Comparison between different methods on ATM system

- Global method based on positive mapping can verify up to (100 atms +100 users) within 50 minutes.
- The time for the verification by the incremental method based on positive mapping is almost linear. We built the ATM system by the superposition of a set of increments, each increment adds interactions for ( $10 \mathrm{atms}+10$ users) and two Bank components. The reason for the good performance is that the number of common components between increments is small (just two Bank components), and therefore the number of common locations is also small.
- The global and incremental methods based on fixed-point have almost the same performance when the size of system is not too big (up to 300 atms +300 users). The reason is that the number of iterations to get fixed-points is the same for any number of atms and users and this number is quite small, 19 iterations. Therefore the global method based on fixed-point is quite fast. Moreover, in the incremental method based on fixed-point, in the final step where we compute global fixed-points starting from fixed-points obtained from increments, the number of iterations is 11 which is not much smaller than the number of iterations in the global method. That is the reason for the same performance of two methods based on fixed-points.


### 6.4 NDD Module of Dala Robot

Robot Dala, an iRobot ATRV, has been developed at LAAS laboratory. It is composed of three layers (Figure 6.13):

- Functional layer includes all the basic built-in robot actions and perception capacities (image processing, motion control, etc.)
- Decisional layer produces the task plan and supervises its execution.
- Control layer is an interface between the decisional and functional layers that controls
the execution of services in the functional layer according to some safety constraints.


Figure 6.13: The Dala Robot Architecture

We have used BIP to model the execution control and functional layers of the robot Dala. The functional layer consists of a set of modules. A module has a set of services, a set of execution tasks and a set of posters where the produced data is stored. A service has a controller and an activity. An execution task is composed of a timer, a scheduler and an activity. We proposed the following grammar which allows building the functional level starting from basic components:

```
Functional level \(::=(\text { Module })^{+}\)
Module \(\quad::=(\text { Service })^{+} .(\text {Execution task })^{+} .(\text {Poster })^{+}\)
Service \(\quad::=\) (Service controller).(Activity)
Execution task \(::=(\) Timer \() .(\) Scheduler activity \()\)
```

where ${ }^{+}$(plus) means the presence of one or more subcomponent and . (dot) means the composition of different components.

We have used D-Finder to check deadlock-freedom of a module in the functional level, module NDD (Figure 6.14), which is one of the most complex modules. It has totally 27 components, 144 control locations, 117 connectors between components, 16 boolean variables and 11 integer variables. NDD module is responsible for the navigation of the robot, that is to reach a goal while avoiding obstacles. It consists of the following control elements:

- InterfaceServer is the interface of the module with the decisional layer. It checks the mailbox which is a shared memory and if there is any message, it will read the content
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and then sends requests to the corresponding service.
- ExecutionControl keeps information about the number of services running in the module. If a service is triggered, it increases the number by 1 , if a service finishes, it decreases the number by 1 .
- ExecutionTask runs periodically to synchronize the executions of different services, that is different services can be executed within a period but a service can not be executed more than one time within a period.


Figure 6.14: NDD Module
and the following services:

- PermanentTask computes the speed of the robot and it is executed periodically during the execution of the robot.
- Init service initializes the module.
- SetParams service sets the necessary parameters of the module.
- SetSpeed service sets the moving speed of the robot which is computed by PermanentTask.
- GoTo service allows the robot moving to a given destination.
- Stop service allows stopping the robot at any time.

NDD also has a set of components called Poster (SpeedPoster, ParamPoster, DiagramPoster, AspectPoster, RefPoster, GoalPoster, AgePoster, MasterPoster) where data produced by the services of the module is stored and exchanged between different services of NDD or with services of other modules.


Figure 6.15: A service

A service basically has two components (Figure 6.15): a Controller and an Activity. The Controller receives requests (trigger transition), checks parameters and execution conditions (control transition) and if everything is fine, the Controller will trigger the Activity (start transition) to perform the request. The Controller can cancel the request if there is an error (error transition) or conflict (abort transition). The Controller updates the status of the Activity by finish, inter and fail transitions. And finally, it sends a report (send_final_report transition) to the ExecutionControl component.

The Activity is triggered by the Controller (start transition) and then it executes its functions to perform the requested task (exec, internal_exec transitions). The execution may finish normally (finish transition), may fail (fail transition) or may be interrupted (inter transition). In any case, the Activity informs the result to the Controller.

We have first used the global symbolic method implemented in D-Finder to check the deadlock-freedom of NDD. We have found potential deadlocks due to the strong synchronization between the timers. We have then fixed these problems and verified again. Finally we obtained the result proving deadlock-freedom of the module. The verification time of the global method by positive mapping is up to 3 hours. However, the incremental verification method based on positive mapping reduces dramatically the verification time: the deadlock-freedom of the module is proven within 20 minutes. In the incremental construction and verification process, we build and check the deadlock-freedom of the NDD module from the following increments:

- increment 1 consisting of interactions between PermanentTask, Init, AspectPoster, InterfaceServer, ExecutionTask, Lock and ExecutionControl,
- increment 2 consisting of interactions between SetParams, SetSpeed, InterfaceServer, ExecutionTask, Lock and ExecutionControl,

| module | comps | locs | intrs | vars | time (minutes) | memory (mb) |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| RFLEX | 56 | 308 | 227 | 35 | $9: 39$ | 165 |
| NDD | 27 | 152 | 117 | 27 | $14: 25$ | 113 |
| SICK | 43 | 213 | 202 | 29 | $6: 07$ | 92 |
| Aspect | 29 | 160 | 117 | 21 | $1: 00$ | 80 |
| Battery | 30 | 176 | 138 | 23 | $4: 34$ | 65 |
| Heating | 26 | 149 | 116 | 19 | $0: 39$ | 57 |
| Platine | 37 | 174 | 151 | 25 |  | $8: 47$ |
| module $:$ | module name | intrs | $:$ | number of interactions |  |  |
| comps | $:$ | number of components | vars | $:$ | number of variables |  |
| locs | $:$ | number of locations |  |  |  |  |

Table 6.4: Time and memory usage for the verification of Dala robot modules

- increment 3 consisting of interactions between GoTo, Stop, InterfaceServer, ExecutionTask, Lock and ExecutionControl,
- increment 4 consisting of interactions between DiagramPoster, RefPoster, ParamPoster, GoalPoster, SpeedPoster, AgePoster and MasterPoster.
Table 6.4 shows the verification time and memory usage for checking deadlock-freedom of other modules in the robot Dala by D-Finder using the incremental positive mapping method. D-Finder detected deadlocks in several modules. Based on the detected deadlocks, we fixed their models and then we successfully proved the deadlock-freedom of these modules.


### 6.5 Summary

We have provided the experimental results on the verification of several case studies by D-Finder. The experimental results have shown the efficiency of the methods and the capacity of D-Finder. D-Finder is able to check deadlock-freedom of systems up to more than ten thousands of components, more than fifty thousands of control locations and about the same number of interactions. Besides the scalability, D-Finder also shows the capacity in dealing with complex systems with a significant number of boolean and integer variables. Moreover, the different methods implemented in D-Finder allows handling many kinds of systems such as cycle-structure or star-structure systems.

We also compare the verification time and memory usage by D-Finder of some case studies with an well-known model checking tool NuSmv. In all the case studies, D-Finder provides much better performance than NuSmv in both verification time and memory usage. Moreover, the experimental results also show the significant gain by the incremental verification methods compared to the global verification.

## Part IV

## Conclusions and Perspectives

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In this chapter, we conclude the thesis describing the main objectives of the work, the goals we have achieved, the future work directions and its perspectives.

### 7.1 Conclusions

Constructing correct systems is always an essential requirement for engineers. However with the growing demand of the complexity and of the size of systems, it becomes more and more difficult to build correctly a system with respect to its specification. The violation of some property in systems, specially in critical systems, might cost expensively. Therefore the check of correctness is essential to guarantee the correct behavior of the system.

In this thesis, we propose a compositional verification method for checking safety properties of component-based systems described in the BIP language. Our method is based on the the use of two kinds of invariants: component invariants which approximate reachable states of atomic components and interaction invariants which characterize global constraints induced by strong synchronizations between atomic components.

There are two key issues in the application of the method. The first is the choice of component invariants depending on the property to be proved. The second is the computation of the corresponding interaction invariants. Here there is a risk of explosion, if exhaustiveness of solutions is necessary in the analysis process. However, this issue is solved by using symbolic computation using BDDs.

For the computation of component invariants, we use lightweight techniques which allow computing increasingly stronger component invariants by using forward propagation over

### 7.1. CONCLUSIONS

the transitions of components.
For the computation of interaction invariants, we have proposed different methods using both SMT Sat-Solver and BDDs. The enumerative method using CUDD package or Yices computes explicitly the set of interaction invariants. This method provides a clear, visual view on the invariants and it is efficient for the systems of which the number of invariants is not too big. The symbolic methods using CUDD package compute interaction invariants based on symbolic operations performed directly on the BDDs. One symbolic method is based Positive Mapping operation which removes symbolically the negative valuations of variables from Boolean Behavioral Constraints. The other symbolic method uses fixedpoint computation technique to compute, starting from the set of single locations, all the locations that can be reached from these locations by any interaction of the system.

The application of our verification method for proving deadlock-freedom of componentbased systems is promising. The class of component invariants that we use captures well enough guarantees for component deadlock-freedom. Their computation does not involve fixed-points and avoids state space explosion. The verification method applies an iteration process for computing progressively stronger invariants. Best precision is achieved when component reachability sets are used as component invariants. This is feasible for finite state components. There are no restrictions on the type of data as long as we stay within theories for which there exist efficient procedures.

We have also improved significantly the compositional verification method by proposing incremental construction and verification method. The incremental method take advantage of properties of the construction process based on the assumption that composite components can be obtained from a set of atomic components by superposition of increments. The verification should be applied in each stage of incremental construction process in order to detect early the violations. Hence the reuse of established properties in the checking of global properties is essential to reduce the verification cost. We proposed the rules on invariant preservation from which the established properties are not violated during the incremental construction. For the general systems where the preservation rules may not hold, we proposed a method for incrementally computing invariants from the established invariants of the increments.

Since the compositional and incremental methods are limited to systems without data transfer over interactions, we have provided a method allowing dealing with systems with data transfer. The method is based on the transformation of systems with data transfer into equivalent systems without data transfer on which the compositional method can be applied. The transformation is done by taking into account the data transfer in computing component invariants and by replacing data transfer over a set of interactions by a component called "interaction component". This method has not been implemented but the results obtained on several examples show the perspectives of the method.

We have fully implemented compositional and incremental methods in the D-Finder tool. D-Finder allows checking safety properties, specially deadlock-freedom, of componentbased systems described in BIP. The implementation of different methods using different techniques make D-Finder efficient in dealing with many kinds of systems, for example "starstructural" systems, "cycle-structural" systems. It also allows users choosing the appropriate method and technique according to their systems to be verified.

We have used D-Finder for checking deadlock-freedom of several non-trivial case studies which showed the capacities of D-Finder as well as the efficiency of the methods. Two case studies without data (Dining Philosopher, Gas Station) are used to check the scalability of the methods. Two other case studies with data (ATM machine, NDD module of Dala robot) are used to show the capacities of the methods in dealing with complex systems. Specially, NDD is a module of Dala robot, a real complex case study of our projects, where D-Finder detected errors in the model. The obtained experimental results by D-Finder are really convincing. D-Finder can handle very large systems with acceptable verification time and memory usage. The experimental results also show significant gain of incremental compositional methods compared to global compositional methods in both verification time and memory usage.

We also compared D-Finder to a well-known monolithic tool NuSmv. In [CAC08], Cobleigh et al. show for a set of finite state benchmarks that only for $30 \%$ of the considered benchmarks, assume-guarantee tools outperform model-checking tools. On the contrary, for all the case studies that we have verified by using D-Finder and monolithic model checkers, D-Finder outperform these tools, in particular for large systems, in both verification time and memory usage. Of course this comparison is not completely balanced because D-Finder uses heuristics and is tuned for checking deadlock-freedom.

### 7.2 Perspectives

The main perspectives of our works can be categorized in two directions:

- The extension of the usability of component verification which involves the following aspects:
- First, the tool should be able to handle richer models, e.g., the method for dealing with data transfer should be implemented in D-Finder.
- Second, the methods should provide counter-example guided abstraction refinement. The idea is to apply the method presented in [BM07] which allows generating an inductive invariant corresponding to a counter-example. If we succeed to find this invariant, the counter-example is excluded. Moreover, we can use this invariant to strengthen the already established invariants and therefore to eliminate many others spurious counter-examples.
- Third, we can develop heuristics for properties other than deadlock-freedom, e.g., partial deadlocks, live-lock. We have exploited the invariants corresponding to notions of traps in Petri-net. However, there is another kind of constraints to be exploited which corresponds to the notion of locks in Petri-net. That is if a set of locations is not reached, it will be not reached forever. By combining the these two kinds of constraints, we can detect local deadlocks of systems.
- Although the experimental results already shows the scalable capacity of the methods, we still want to increase the scalable capacity by using other techniques for generating components invariants, by developing connection with Z3 SMT Solver or by considering compositional abstraction methods.


## Conclusions and Perspectives

Dans ce chapitre, nous concluons la thèse en décrivant les objectifs principales du travail, les résultats que nous avons obtenus, les directions de futur et ses perspectives.

## Conclusions

La construction de systèmes corrects est un élément essentiel pour les ingénieurs. Cependant, avec la demande augmentée de la complexité et de la taille des systèmes, il devient de plus en plus difficile de construire correctement un système par rapport à sa spécification. La violation de certaines propriétés dans un système, en particulier dans un système critique, peut coûter cher. Par conséquent, la vérification est indispensable pour garantir le comportement correct du système.

Dans cette thèse, nous avons proposé une méthode compositionnelle pour vérifier les propriétés de sûreté des systèmes à base de composants décrits dans le langage BIP. Notre méthode est basée sur l'utilisation des deux types d'invariants: les invariants de composants qui se rapprochent les états atteignables de composants atomiques et les invariants d'interaction qui caractérisent les contraintes globales induites par les synchronisations fortes entre les composants.

Il y a deux questions de clés dans l'application de la méthode. La première est le choix d'invariants de composants en fonction de la propriété à prouver. La seconde est le calcul des invariants d'interaction correspondants. Il y a un risque d'explosion si l'exhaustivité des solutions est nécessaire au processus d'analyse. Toutefois, cette question est résolue en utilisant le calcul symbolique.

Pour le calcul des invariants de composants, nous utilisons les techniques légères qui permettent de calculer les invariants de composants de plus en plus forts en utilisant la propagation en avant sur les transitions de composants.

Pour le calcul des invariants d'interaction, nous avons proposé les différentes méthodes en utilisant le Sat Solveur ou le paquet BDD. La méthode énumérative utilisant le paquet Cudd ou l'outil Yices calcule explicitement l'ensemble des invariants d'interaction. Cette

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méthode permet d'obtenir une vue claire et visuelle sur les invariants et elle est efficace pour les systèmes où le nombre d'invariants n'est pas trop grand. Les méthodes symboliques utilisant le paquet Cudd sont basées sur les opérations symboliques effectuées directement sur la BDD. Une méthode utilise une opération symbolique appelée "mapping positive" qui supprime symboliquement les évaluations négatives de variables booléennes dans les contraintes comportementales. L'autre méthode utilise un calcul symbolique de points fixes pour calculer, à partir d'ensemble des locations, toutes les locations qui peut être atteintes par les interactions du système.

L'application de notre méthode de vérification pour prouver l'absence de blocage des systèmes à base de composants est prometteuse. La classe d'invariants de composants que nous utilisons capturent assez bien les garanties pour l'absence de blocage de composant. Leur calcul ne comporte pas de points fixes et évite l'explosion d'états. La méthode de vérification applique un processus d'itération pour calculer des invariants de plus en plus forts. La meilleure précision est obtenue lorsque l'ensemble d'états atteignables est utilisé comme les invariants de composant. Cela est faisable pour les composants à l'état fini. Il n'existe aucune restriction sur le type de données tant que nous restons dans les théories pour lesquelles il existe des procédures efficaces.

Nous avons également amélioré de façon significative la vérification compositionnelle en proposant la construction et la vérification incrémentales. La méthode de vérification incrémentale profite des avantages de la construction incrémentale qui repose sur l'hypothèse que les composants complexes peuvent être obtenus à partir d'un ensemble de composants atomiques par la superposition de tranches. La vérification doit être appliquée à chaque étape de la construction incrémentale afin de détecter rapidement les violations. La réutilisation de propriétés précédemment établies dans le calcul de propriétés globales est essentielle pour réduire le coût de vérification. Nous avons proposé des règles pour la préservation des invariants à partir de lesquelles les propriétés établies ne sont pas violées au cours de la construction incrémentale. Dans le cas général où un système peut violer ces règles, nous avons proposé deux méthodes de calcul incrémental des invariants à partir des invariants précédemment établis dans le processus de construction.

Étant donné que les méthodes compositionnelles et incrémentales sont limitées aux systèmes sans transfert de données sur les interactions, nous avons fourni une méthode permettant de traiter le transfert de données. La méthode est basée sur la transformation des systèmes avec transfert de données aux systèmes équivalents sans transfert de données sur lesquels nos méthodes de vérification peut être appliquées. La transformation se fait en tenant compte le transfert de données dans le calcul des invariants de composants et en remplaçant le transfert de données sur un ensemble d'interactions par un composant appelé "composant d'interaction". Cette méthode n'a pas encore été mise en oeuvre, mais les résultats obtenus sur plusieurs exemples montrent les perspectives de la méthode.

Nous avons complètement mis en oeuvre les méthodes de vérification dans l'outil DFinder. D-Finder permet de vérifier des propriétés de sûreté, particulièrement l'absence de blocage, des systèmes à base de composants décrits dans le langage BIP. La mise en oeuvre des différentes méthodes en utilisant différentes techniques font de D-Finder efficace dans la vérification de nombreux types de systèmes, par exemple les systèmes en structure d'étoile, les systèmes en structure cyclique, etc. Elle permet également aux utilisateurs de choisir la
méthode appropriée en fonction de leurs systèmes à vérifier.
Nous avons utilisé D-Finder pour vérifier l'absence de blocage de plusieurs études de cas non triviales qui ont montré les capacités de D-Finder ainsi que l'efficacité de nos méthodes. Deux études de cas sans données (Philosophes et Gas Station) sont utilisés pour montrer la scalabilité des méthodes. Deux autres études de cas avec des données (ATM, module NDD de robot Dala) sont utilisés pour montrer les capacités des méthodes sur les systèmes complexes. En particulier, le NDD est un module de robot Dala, une étude de cas réel complexe de nos projets, où D-Finder a détecté des erreurs dans le modèle. Les résultats obtenus par l'expérimentation sont vraiment convaincants. D-Finder peut vérifier de très grands systèmes avec le temps de vérification et la mémoire utilisée acceptables. Les résultats expérimentaux montrent également le gain significatif des méthodes incrémentales par rapport aux méthodes compositionnelles globales en temps de vérification et en mémoire utilisée.

Nous avons également comparé D-Finder à un outil monolithique connu NuSMV. Dans [CAC08], Cobleigh et al. ont montré que des outils de l'assume-guarantee sont meilleurs que des outils de model-checking dans $30 \%$ d'ensemble d'études de cas. Au contraire, pour toutes les études de cas que nous avons vérifiées par D-Finder et par l'outil de vérification monolithique, D-Finder est toujours meilleur en temps de vérification et en mémoire utilisé, en particulier pour les grands systèmes. Bien-sûr cette comparaison n'est pas complètement équilibrée, car D-Finder utilise des heuristiques et est réglé pour la détection de blocages.

## Perspectives

Les perspectives principales de nos travaux peuvent être classées dans deux directions:

- L'extension de l'utilisation de la vérification qui implique les aspects suivants:
- D'abord, l'outil D-Finder doit être capable de gérer les modèles plus riches, par exemple, la méthode pour traiter le transfert de données devrait être mis en oeuvre en D-Finder.
- Deuxièmement, les méthodes devraient fournir le raffinement d'abstraction guidé par le contre-exemple. L'idée est d'appliquer la méthode présentée dans [BM07] qui permet de générer un invariant inductif correspondant à un contre-exemple. Si nous réussissons à trouver cet invariant, le contre-exemple est exclu. En outre, nous pouvons utiliser cet invariant pour renforcer les invariants précédemment calculés et donc éliminer des autres faux contre-exemples.
- Troisièmement, nous pouvons développer des heuristiques pour des propriétés autres que l'absence de blocage, par exemple, la détection de blocages partiaux, le lockvivent. Nous avons exploité les invariants correspondants aux notions des pièges dans le Pétri-net. Cependant, il existe un autre type de contraintes à être exploitée, qui correspond à la notion de serrures dans le Pétri-net. C'est le cas si un ensemble de places n'est pas atteint, il sera toujours pas atteint. En combinant les deux types de contraintes, nous pouvons détecter les blocages locaux des systèmes.
- Bien que les résultats expérimentaux montrent déjà les capacités de nos méthodes de vérification, nous voulons encore augmenter la scalabilité en utilisant d'autres
techniques pour générer des invariants, en élaborant avec le SMT Solveur Z3 ou en tenant compte des méthodes d'abstraction compositionnelle.


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1. Saddek Bensalem, Marius Bozga, Thanh-Hung Nguyen and Joseph Sifakis: Compositional Verification for Component-based Systems and Application. IET Software Journal, Special Issue on Automated Compositional Verification, Volume 4, Issue 3, Pages 181-193, June 2010.
2. Saddek Bensalem, Matthieu Gallien, Félix Ingrand, Imen Kahloul and Thanh-Hung Nguyen: Designing autonomous robots, Special Issue on Software Engineering for Robotics of the IEEE Robotics and Automation Magazine Volume 16, Issue 1, Pages 67-77, March 2009.

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3. Saddek Bensalem, Marius Bozga, Axel Legay, Thanh-Hung Nguyen, Joseph Sifakis and Rongjie Yan Incremental Component-based Construction and Verification using Invariants. In FMCAD'10, 10th International Conference on Formal Methods in Computer Aided Design, October 20-23, 2010, Lugano, Switzerland.
4. Saddek Bensalem, Axel Legay, Thanh-Hung Nguyen, Joseph Sifakis and Rongjie Yan: Incremental Invariant Generation for Compositional Design. In TASE'10, 4th IEEE International Symposium on Theoretical Aspects of Software Engineering, Pages 157-167, August 24-27, 2010, Taipei, Taiwan.
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