

Réflexions non-linéaires d'ondes de gravité internes

Nicolas Grisouard

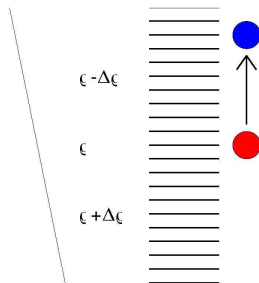
Thèse préparée au sein du Laboratoire des Écoulements Géophysiques et Industriels (UMR 5519), dans l'École Doctorale Terre, Univers, Environnement de l'Université de Grenoble.

Thèse encadrée par Chantal Staquet.



Restoring force: buoyancy

Water parcel in a stably stratified fluid ($d\rho/dz < 0$):

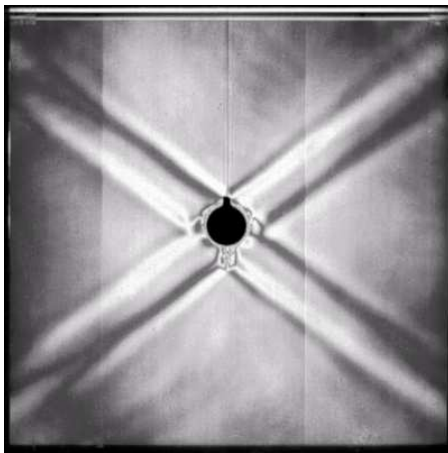


Oscillation at the Brunt-Väisälä frequency

$$N = \sqrt{-\frac{g}{\rho} \frac{d\rho}{dz}}$$

Oscillation of an object in a stably stratified fluid

- No rotation, infinite rod oscillating at frequency $\omega = 0.56N$:



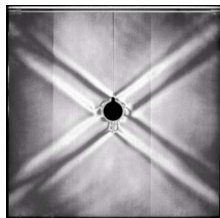
Source: www.gfd-dennou.org

Dispersion of internal gravity waves

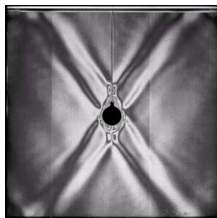
- Dispersion relation: $\omega^2 = N^2 \sin^2 \theta$

θ = angle of propagation of energy:

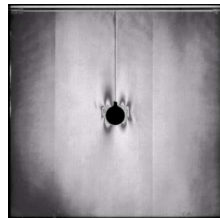
- $\omega \nearrow \Rightarrow \theta \nearrow$ (steeper beams),
- propagative only if $\omega < N$,
- $\vec{c}_g = \vec{\nabla}_{\vec{k}} \omega$, $\partial \omega / \partial |\vec{k}| = 0 \Rightarrow \vec{c}_g \perp \vec{k}$,



$$\omega = 0.56N$$

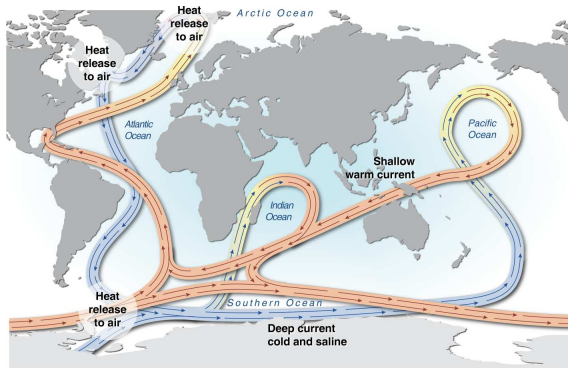


$$\omega = 0.78N$$



$$\omega/N = 1.03$$

Motivation: the meridional overturning circulation

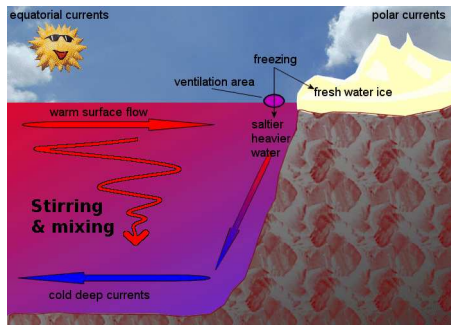


- Global scale oceanic circulation,
- crucial impact on climate.

What drives the meridional overturning circulation? Mixing!

At the poles: surface water sinks down to the bottom \Rightarrow colder than surrounding water: why ?

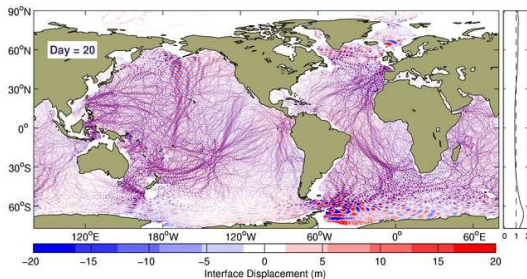
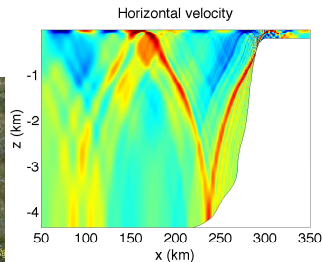
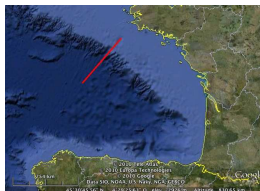
- Because mixing transports solar heat downwards,
- sources of mechanical energy (Munk & Wunsch 1998): $\sim 50\%$ winds, $\sim 50\%$ tides \Rightarrow **internal waves**.



\Rightarrow **Internal waves** play a crucial role in deep-ocean mixing.

Internal tides in the ocean

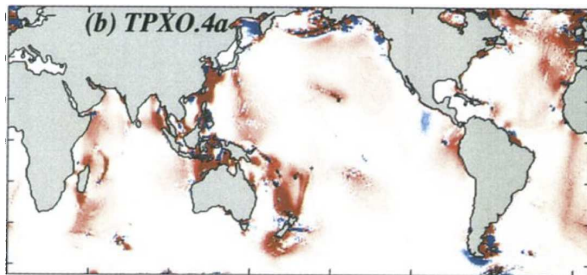
Local scale →
internal wave beam
(↔ fluid ↔
↔ topography)



Global scale:

← Simmons *et al.*
(2004), spin-up of a
tidal model.

Back to mixing: the role of topography



World map of tidal dissipation (Egbert & Ray 2001, from satellite altimetry).

Near topographies:

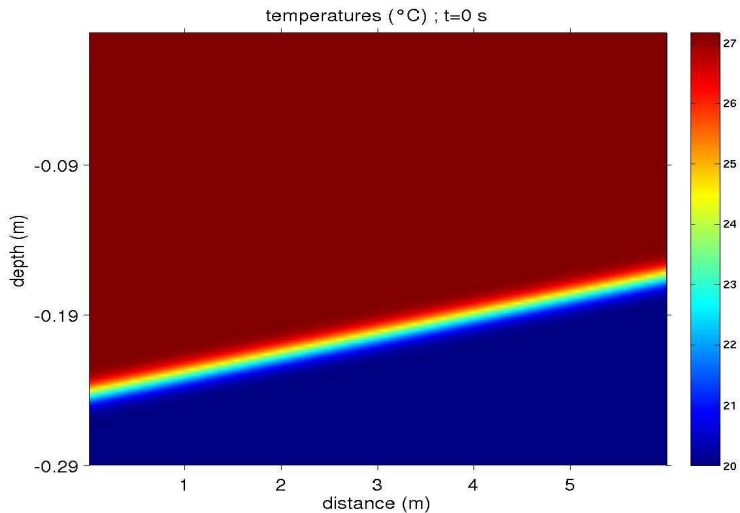
- reflection, scattering
- \Rightarrow enhanced non-linear effects
- \Rightarrow pathway to smaller scales
- \Rightarrow increased mixing due to internal waves.

- 1 Introduction
- 2 Generation of internal solitary waves by internal wave beams
- 3 Internal wave beams and solitary waves in the Bay of Biscay
- 4 Reflection of plane waves on a sloping wall
- 5 Conclusion

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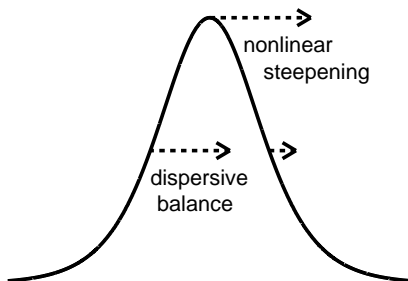
Density jumps and solitary waves

↓ Numerical reproduction of Horn *et al.* (2001)



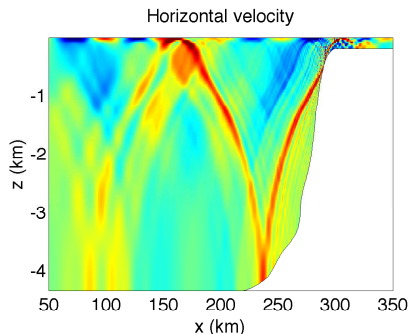
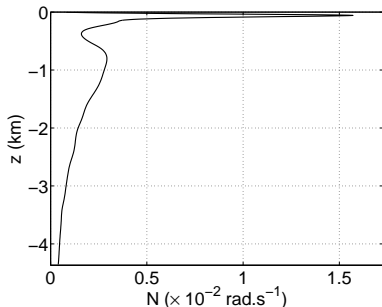
Density jumps and solitary waves

- Solitary waves: balance between nonlinear (finite amplitude) and dispersive (finite width) effects,



- in theory: can propagate without changing shape (in practice: dissipation, etc.),
- described by numerous mathematical models (e. g. KdV, Benjamin-Ono...),

Stratification in the summer Bay of Biscay

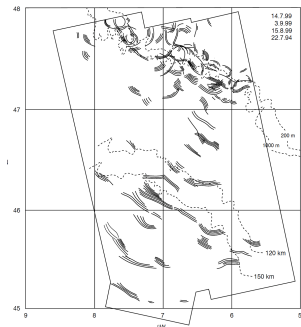


Strong density jump (= pycnocline) 60 m below the surface \Rightarrow solitary waves.

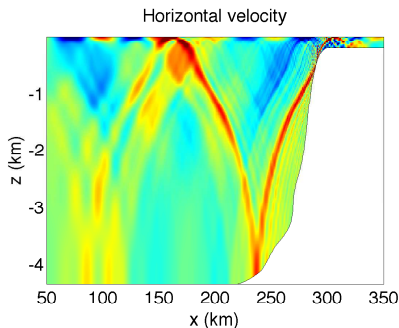
Internal solitary waves in the Bay of Biscay

New & Pingree (1990), Bay of Biscay: solitary waves arise (too) far from the shelf

Explanation (*New & Pingree 1990*): IW beam impinging on the density jump
 \Rightarrow "local" generation.



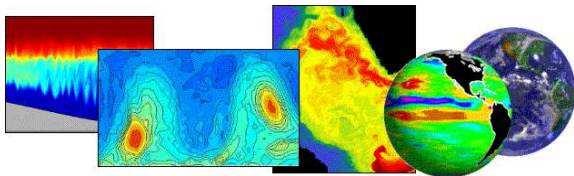
↑ SAR images of the Bay of Biscay (*New & Da Silva 2002*)



Objectives

- We have (too few) observations in the ocean: Bay of Biscay, Portugal (Da Silva *et al.* 2007), Indian Ocean (Konyaev 1995, Da Silva *et al.* 2008),
- we have (very idealized) theoretical models (Gerkema 2001, Akylas *et al.* 2007...)
- we will present direct visualizations of the process using numerical simulations...
- ... and a simple rule to understand the mechanism.

Numerical code: the MITgcm

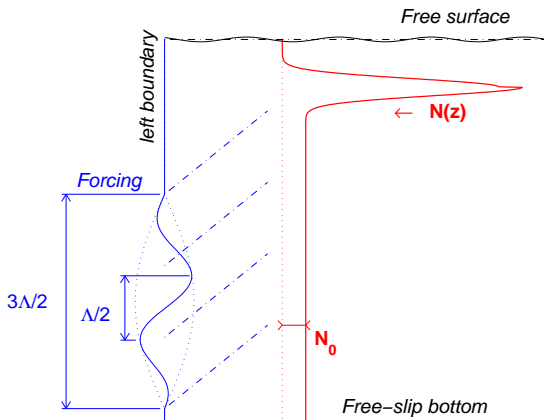


Source: mitgcm.org

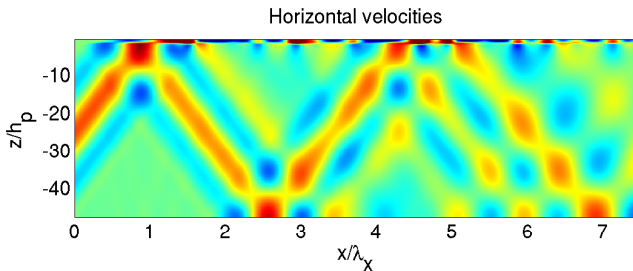
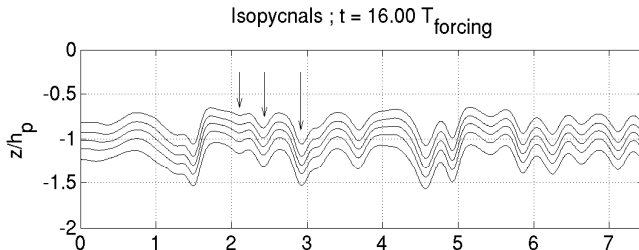
- Primitive equations solved \Rightarrow nonlinear, nonhydrostatic effects,
- approximations: incompressibility, Boussinesq,
- 2D configuration (vertical slice).

Numerical set-up #1

- Stratification:
 - 1 lower layer, constant N_0 ,
 - 2 pycnocline,
 - 3 upper layer, $N = 0$,
- forcing of the velocity on the boundary,
- no rotation.



Generation of mode-1 ISWs



Modal analysis

- Assume for the vertical velocity:

$$w(x, z, t) = W(z) \exp i\Omega(x/c - t),$$

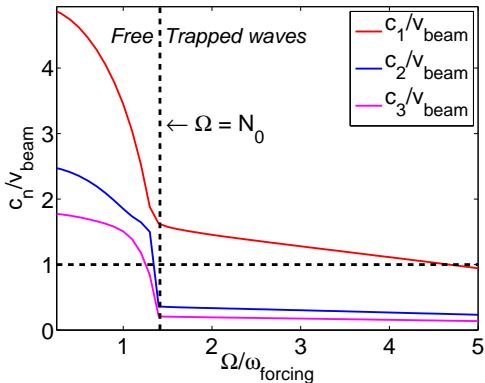
then (Taylor-Goldstein equation):

$$\frac{d^2 W}{dz^2} + \frac{N^2(z) - \Omega^2}{c^2} W = 0 \quad + 2 \text{ B.Cs.},$$

- input: Ω , $N(z)$, rigid-lid B.Cs.,
- output: infinite set of c_n and $W_n(z)$ ($n = 1, 2, 3 \dots$)

Modal analysis

$$\frac{d^2 W_n}{dz^2} + \frac{N^2(z) - \Omega^2}{c_n^2} W_n = 0 \quad + B.C.s. \quad (n = 1, 2, \dots)$$

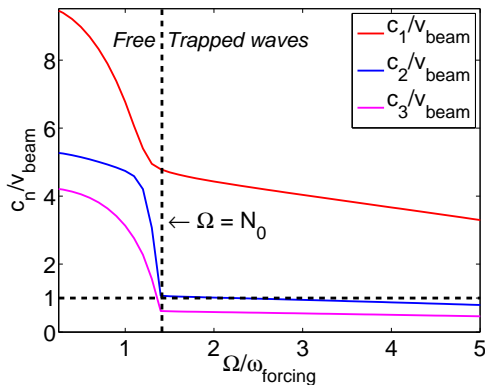


- v_{beam} = phase speed of the IW beam
- mode-1 generated

- v_{beam} closer to c_1 for $\Omega > N_0$

Application to mode-2 ISWs

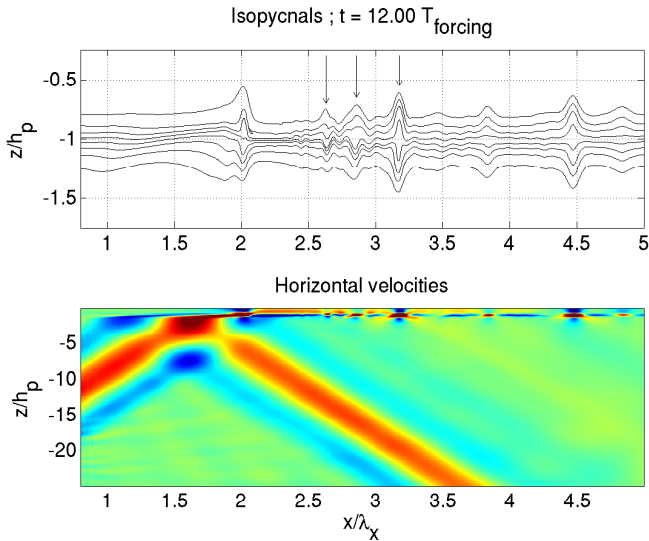
$$\frac{d^2 W_n}{dz^2} + \frac{N^2(z) - \Omega^2}{c_n^2} W_n = 0 \quad + B.C.s. \quad (n = 1, 2, \dots)$$



- v_{beam} = phase speed of the IW beam

- v_{beam} close to c_2 for $\Omega > N_0$

Application to mode-2 ISWs



Summary: a condition for the generation of mode- n ISWs

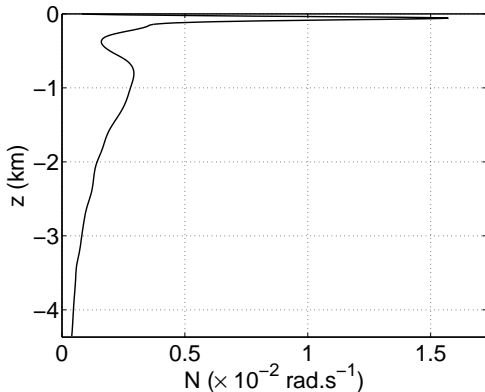
To generate mode- n ISWs in the pycnocline, make sure that these two match (Grisouard *et al.*, *J. Fluid Mech.* 2011):

- horizontal phase speed of the IW beam,
- horizontal phase speed of trapped mode- n IWs.

What about a more realistic case?

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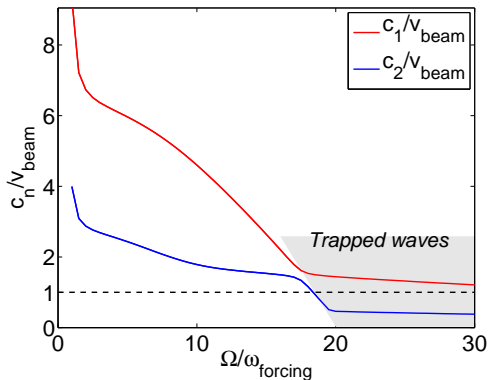
Bay of Biscay: parameters



- Length: 200 km,
- depth: 4300 m,
- semidiurnal tide (period: 12.42 hours),
- rotation: f_{45° ,
- $N(z)$ based on observations,
- again, forcing applied at boundary (no topography).

Bay of Biscay: prediction by the selection condition

$$\frac{d^2 W_n}{dz^2} + \frac{\Omega^2 N^2(z) - \Omega^2}{c_n^2 \Omega^2 - f_{45^\circ}^2} W_n = 0 \quad + B.Cs. \quad (n = 1, 2, \dots)$$

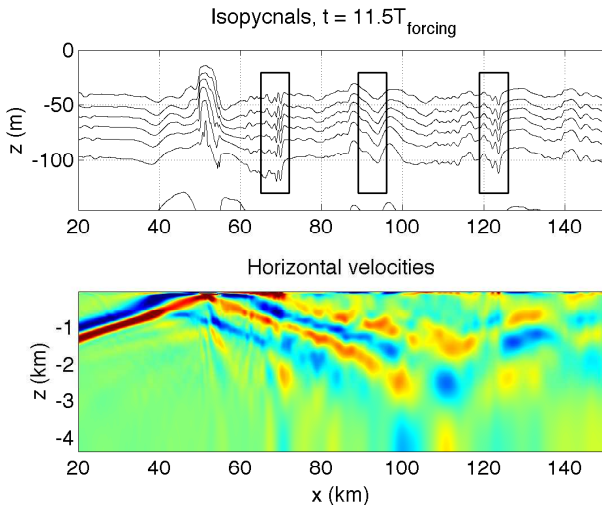


- v_{beam} = phase speed of the IW beam,
- v_{beam} closer to c_1 for $\Omega > N_0$,
- still, v_{beam} quite close to c_2 .

Prediction

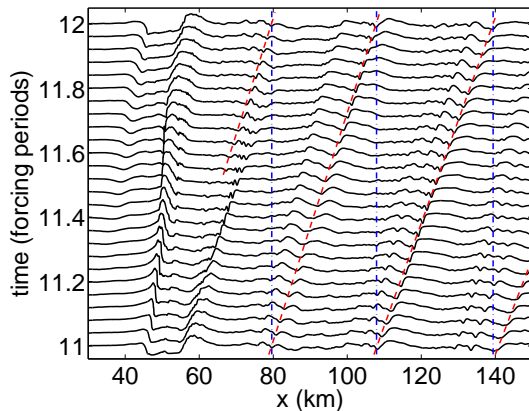
Mode-1 dominant, mode-2 allowed.

Bay of Biscay: IW beam impinging on the pycnocline



Mode-1 dominant, mode-2 barely seen.

Bay of Biscay: space-time displacement



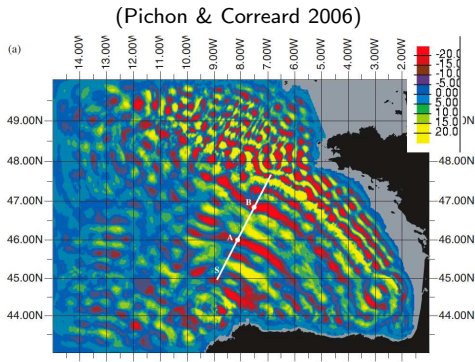
red lines: separated by one forcing period and ≈ 25 km.

Figure: Elevation of the middle of the pycnocline over one forcing period

Contradiction?

In the ocean:

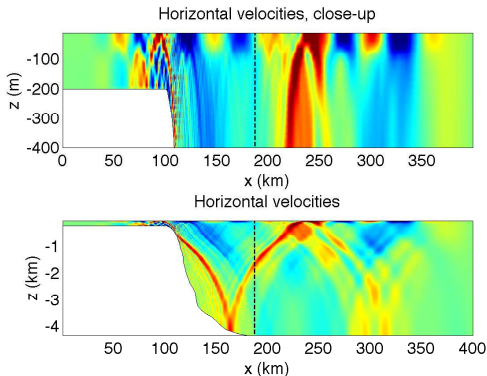
- distance between two trains ≈ 50 km,
- wavelength of the internal tide in the upper ocean ≈ 50 km.



↑ Surface signature of the internal tide
(modeled zonal velocity), Bay of Biscay

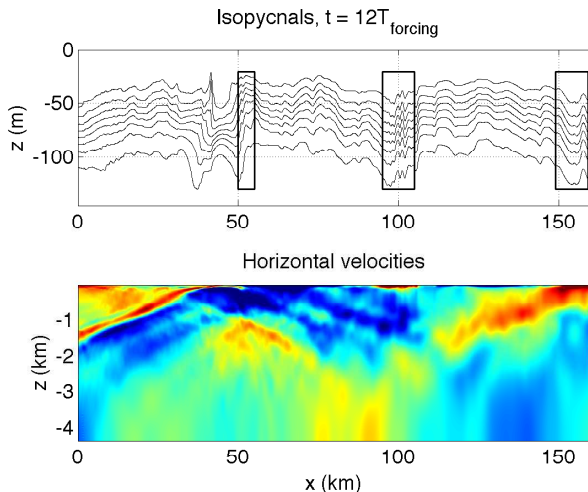
Necessity of taking into account more than the IW beam?

Bay of Biscay: full internal tide, design of the forcing



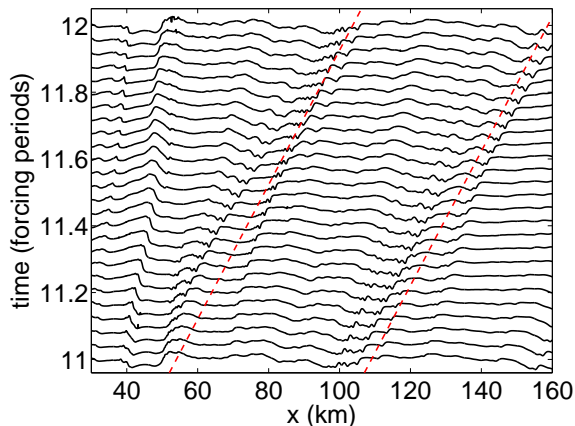
- Idea: to force both IW beam + internal tide in the upper ocean.
- means: running of a low-resolution, low-amplitude simulation with topography ⇒ new forcing.

Bay of Biscay: full internal tide



Distance between two trains of ISWs ≈ 50 km, like in the ocean.

Full internal tide: space-time displacement

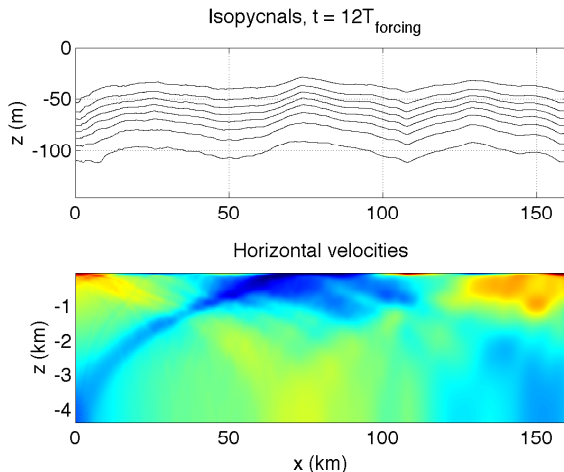


red lines: separated by one forcing period and 52 km, like in the ocean.

Is the IW beam involved ?

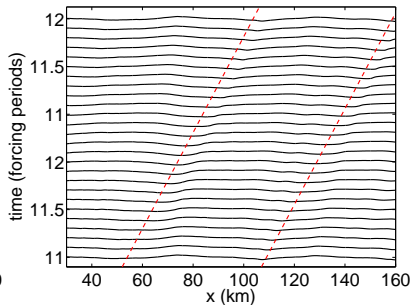
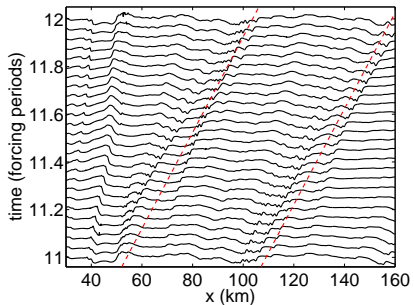
Figure: Elevation of the middle of the pycnocline over one forcing period

Bay of Biscay: without IW beam



IW beam “removed” from the forcing, upper signal kept \Rightarrow dynamics is much more linear, no ISWs.

Bay of Biscay: with and without IW beam



Elevations of the pycnocline with and without IW beam \Rightarrow internal tide in the upper ocean = carrier wave.

Summary

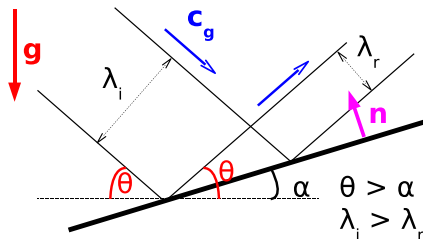
- Selection condition seems to work OK in a realistic setting,
- however, ISWs are very different than in-situ observations (distance between trains),
- interaction with the sub-surface component of the internal tide seems crucial.

Grisouard & Staquet, *Nonlin. Processes Geophys.* (2010).

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Linear reflection of internal waves

- ω is conserved,
 - N : external parameter,
- $$\left. \begin{array}{l} \omega^2 = N^2 \sin^2 \theta \\ \omega \text{ is conserved,} \\ N \text{ is external parameter,} \end{array} \right\} \theta \text{ conserved (instead of } \vec{c}_g \cdot \vec{n} \text{).$$

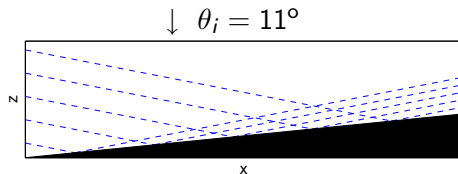


Internal wave reflection is not specular

Reflection of a plane wave on a slope and nonlinearities

- incident + reflected

=



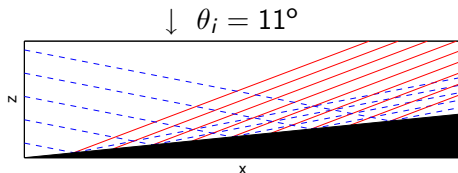
blue: incident & reflected

Reflection of a plane wave on a slope and nonlinearities

- incident + reflected
= forced secondary
wave ("forced 2nd"):

$$\omega_i + \omega_r = 2\omega_i$$

$$\vec{k}_i + \vec{k}_r = \vec{k}_f$$



blue: incident & reflected
red: forced 2nd

Reflection of a plane wave on a slope and nonlinearities

- incident + reflected
= forced secondary
wave ("forced 2nd"):

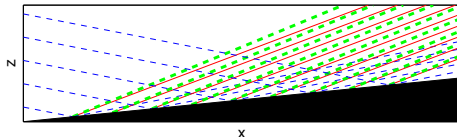
$$\omega_i + \omega_r = 2\omega_i$$

$$\vec{k}_i + \vec{k}_r = \vec{k}_f$$

- free secondary wave
("free 2nd"): dispersion
relation OK

$$(2\omega_i)^2 = N^2 \sin^2 \theta_{2\omega_i}.$$

↓ $\theta_i = 11^\circ$, not resonant.



blue: incident & reflected
red: forced 2nd
green: free 2nd

Reflection of a plane wave on a slope and nonlinearities

- incident + reflected
= forced secondary
wave ("forced 2nd"):
 - $\omega_i + \omega_r = 2\omega_i$
 - $\vec{k}_i + \vec{k}_r = \vec{k}_f$

$$\omega_i + \omega_r = 2\omega_i$$

$$\vec{k}_i + \vec{k}_r = \vec{k}_f$$

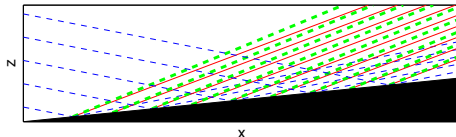
- free secondary wave
("free 2nd"): dispersion
relation OK

$$(2\omega_i)^2 = N^2 \sin^2 \theta_{2\omega_i}$$

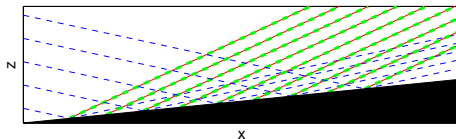
Resonance if

$$\theta_{2\omega_i} = (\vec{k}_f, \hat{e}_z)$$

↓ $\theta_i = 11^\circ$, not resonant.



↓ $\theta_i = 12.17^\circ$, resonant.



blue: incident & reflected

red: forced 2nd

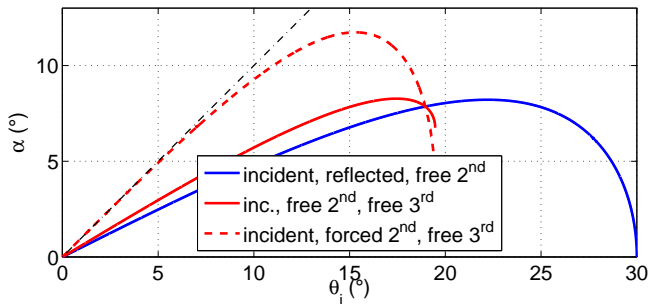
green: free 2nd

Resonance curves (Thorpe 1987)

Other resonances:

- ① incident + free 2nd = free 3rd,
- ② incident + forced 2nd = free 3rd,
- ③ ...

↓ Loci of the resonances in the (θ_i, α) space ($\alpha =$ slope angle)

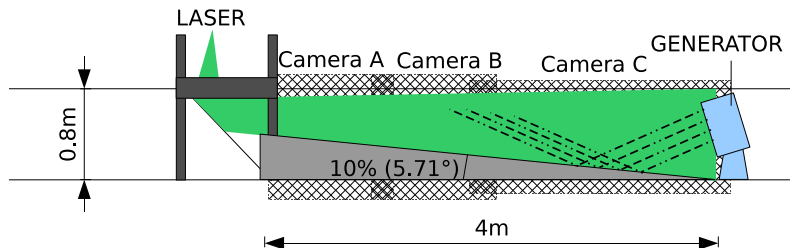


Objectives

- Observation of the resonances (if any) & quantification of their importance,
- quantification of the induced mixing,
- experimental study, continuing Gostiaux's PhD (2006),

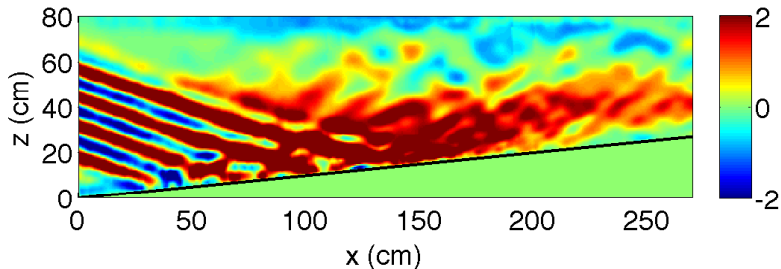
Experimental set-up: overview

Dec. 2009 – Jan. 2010 on the Coriolis platform (Grenoble)



- Wave generator → downward propagating plane wave,
- reflection of the waves on a slope (angle 5.71°),
- fluid seeded with particles, illuminated by laser,
- velocity field measured by PIV,
- no rotation.

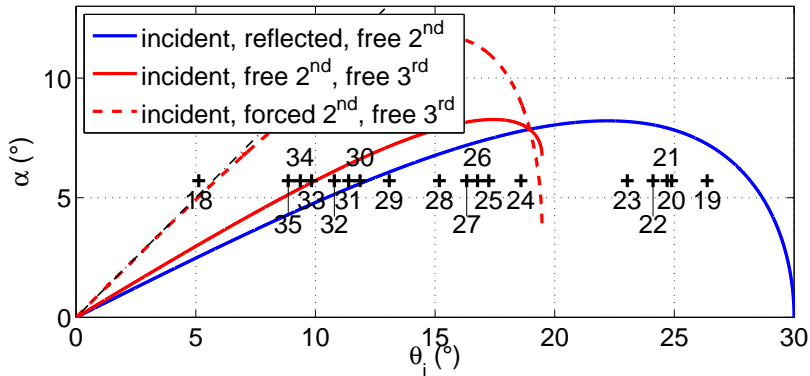
Velocity field, no filtering



2.6 m wide, 3 cameras merged ($\text{cm}\cdot\text{s}^{-1}$).

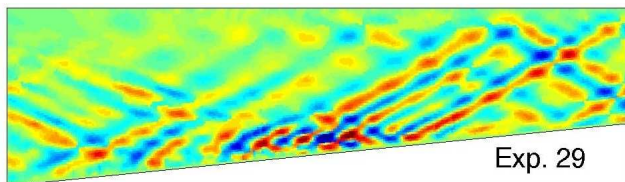
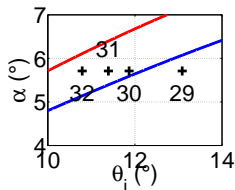
Velocity field, no filtering: the parameter space

Experimental subset: 18 incident angles, $\theta_i = 5.1^\circ \rightarrow 26.4^\circ$.



Are the predicted resonances observed?

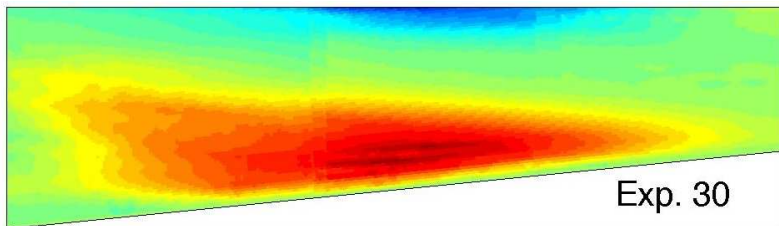
- Harmonic filtering ($2\omega_i$),
- Exp. 30: predicted resonance incident + reflected = free 2nd.



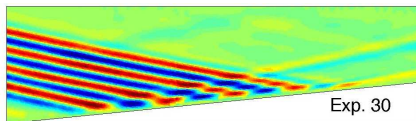
$$\uparrow \text{sgn}(u_x) |\vec{u}|$$

- No obvious resonance here,
- no obvious resonance anywhere.

Instead, a strong mean flow appears

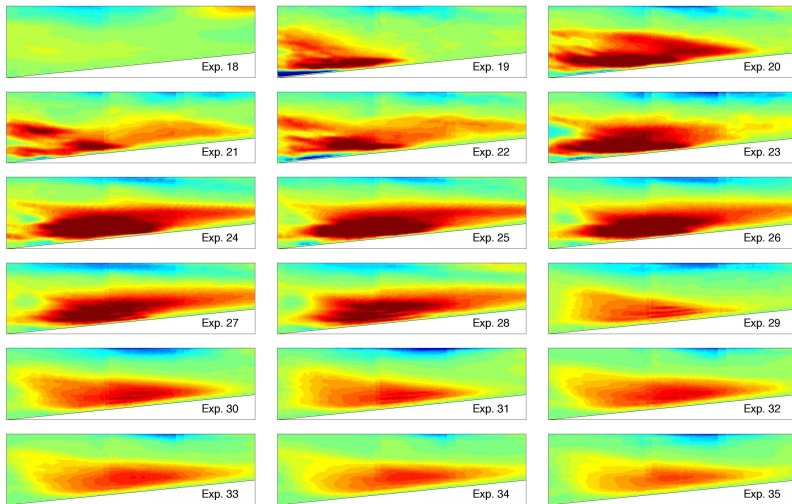


- Very strong mean flow (\uparrow) compared with the forcing (\rightarrow),
- stronger in the interaction zone.



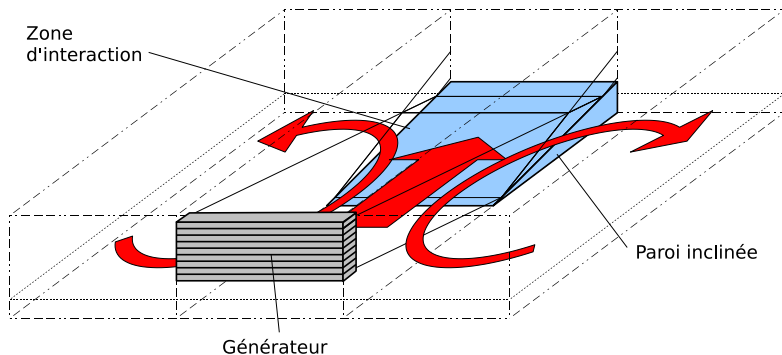
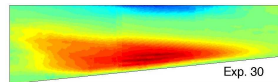
\uparrow Harmonic filtering (ω_i).

Instead, a strong mean flow appears



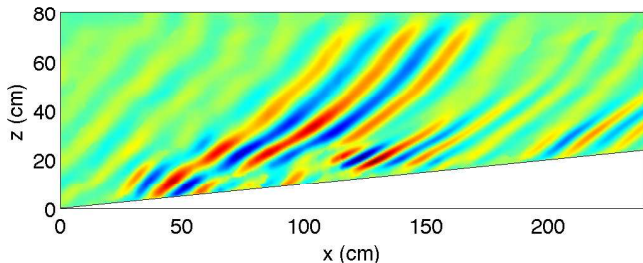
Structure of the mean flow

Stratification + lateral confinement \Rightarrow flow must be horizontal & dipolar.



Is the mean flow a systematic error?

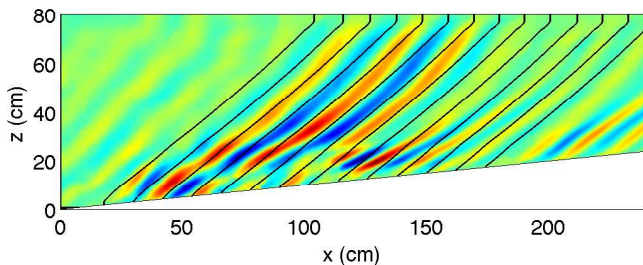
- A physical fact: wave beams are curved \Rightarrow Doppler effect?
- mean flow ref. frame: intrinsic frequency $\omega_D = \omega - \vec{U} \cdot \vec{k}$.



$\uparrow 2\omega_i$, upward component,

Is the mean flow a systematic error?

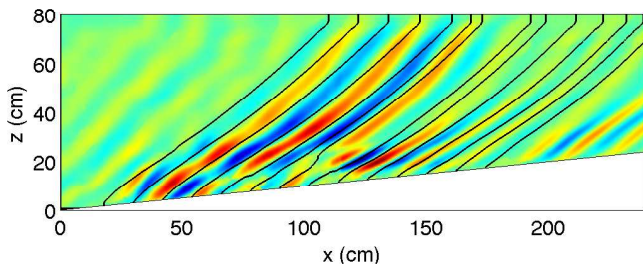
- A physical fact: wave beams are curved \Rightarrow Doppler effect?
- mean flow ref. frame: intrinsic frequency $\omega_D = \omega - \vec{U} \cdot \vec{k}$.



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 $\omega^2 = N^2 \sin^2 \theta$, no Doppler.

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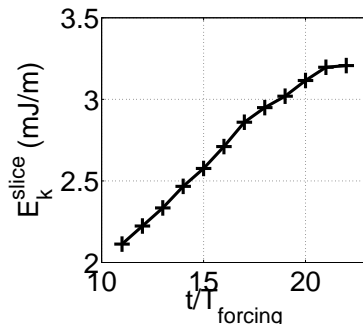
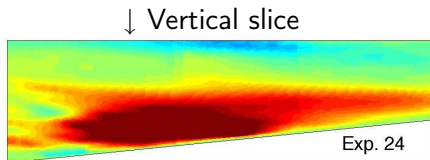
$$\omega_D^2 = N^2 \sin^2 \theta_D, \text{ Doppler included.}$$

The mean flow is a physical effect, with physical consequences.

Energy of the mean flow: temporal evolution

$$E_k^{slice} = \frac{1}{2} \iint_{slice} \rho \vec{u}^2 dx dz \quad (\text{J} \cdot \text{m}^{-1}).$$

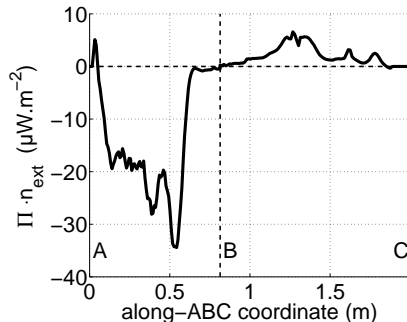
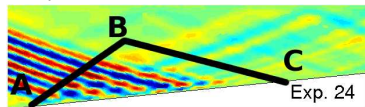
Moving temporal average when wave field is established (Exp. 24):



⇒ Linear evolution first and eventual saturation.

What source of energy accelerates the mean flow?

$\vec{\Pi} \cdot \vec{n}_{ext} = \text{energy flux } (\omega_i, 2\omega_i, 3\omega_i)$ outward of area ABC \rightarrow



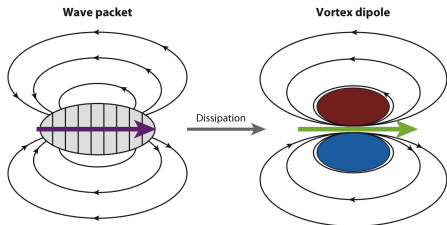
80% of the flux “lost” by the waves in ABC!

Could be transferred to:

- ① mean flow,
- ② viscous dissipation.

Dissipation of gravity waves and mean flow dissipation

- Waves (gravity, Rossby, sound...) can deposit momentum through dissipation and feed mean flows.



← Bühler & McIntyre (2005)
on confined
gravity waves.

A relevant interpretation:

- Intensification in the interaction zone? More wave motion \Rightarrow more dissipation,
- temporal evolution? Effect is cumulative.

Plane waves and sloping walls: summary

- ① Q: are the nonlinear resonances observed?
A: No, instead: strong mean flow,

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Plane waves and sloping walls: summary

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- 4 Q: driving mechanism?
A: dissipative momentum deposit (cumulative effect, intensification in the interaction zone).

- 1 Introduction
- 2 Generation of internal solitary waves by internal wave beams
- 3 Internal wave beams and solitary waves in the Bay of Biscay
- 4 Reflection of plane waves on a sloping wall
- 5 Conclusion**

Take-home results

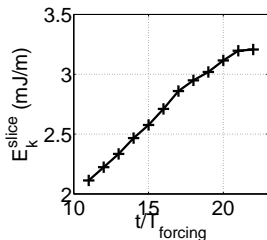
- Reflection of gravity waves on a pycnocline: a selection condition for the vertical structure of the solitary waves,
 - in the Bay of Biscay: latter selection condition insufficient to explain the observations \Rightarrow necessity of taking into account the upper internal tide,
 - reflection of gravity waves on a sloping boundary: generation of a strong mean flow,
- all results show new paths for energy dissipation in the ocean.
 - initial goal of this PhD: to investigate downscale transfers of energy, related to mixing,
 - lab experiments show us that upscale transfers might be very important too (mean flow generation).

Perspectives

- Selection condition: interaction between waves (ω, k) , and waves $(n\omega, nk)$: rather unusual \Rightarrow need of a theoretical model,
- interactions between (solitary) waves and (meso-scale) structures,
- role of mean flows in ocean transport,
- 2D internal wave studies: are the results recovered in 3D?
- in the ocean: $\omega_{forcing} \sim f \Rightarrow$ role of rotation?

La fin !

Energy budget



Possible assumption:

- Work-energy theorem (dipole):

$$\dot{E}_k^{tot} = P_{in}^{(waves)} - P_{out}^{(waves)} - P^{(heat)} \quad (1)$$

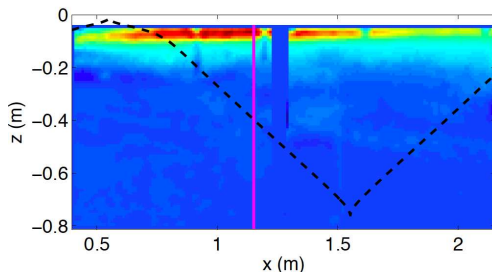
- $E_k^{tot} = \mu E_k^{slice} \quad (2)$, μ constant (m),

3 unknowns - 2 equations = 1 problem.

$$P^{(heat)} = 4\nu \sum_{i=1}^3 k_i^2 E_k^{(i\omega)} + P^{(wall)}.$$

Sensitivity of the local generation viewed from the lab

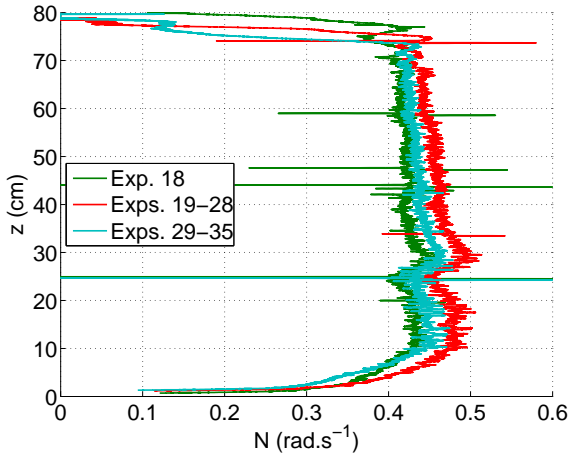
- Gerkema *et al.*, in the lab: mean flow around the pycnocline has to be taken into account to explain the observations,
- confirmation that perturbations have to be taken into account.



Taylor-Goldstein with horizontal mean flow $U(z)$:

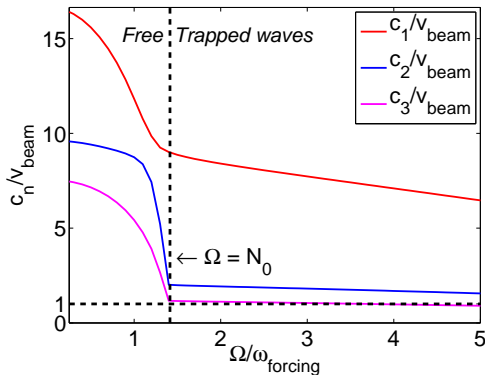
$$\frac{d^2 W}{dz^2} + \left(\frac{N^2(z)}{(U(z) - c)^2} - \frac{\Omega^2}{c^2} - \frac{U''(z)}{U(z) - c} \right) W = 0$$

Stratification of the experiments



Application to mode-3 ISWs

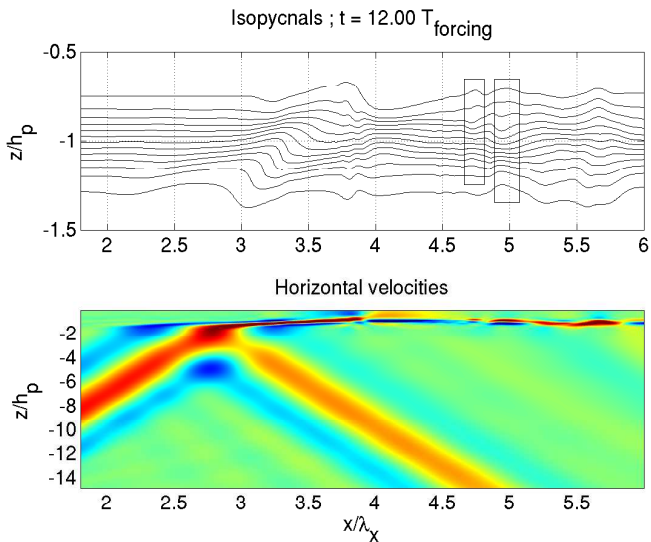
$$\frac{d^2 W}{dz^2} + \frac{N^2(z) - \Omega^2}{c^2} W = 0 \quad + B.C.s.$$



- v_{beam} = horiz. phase speed of the IWB,
- conjecture: if for $\Omega > N_0$, v_{beam} closer to c_n , then mode- n ISWs are generated,

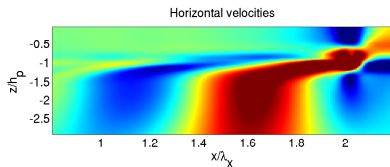
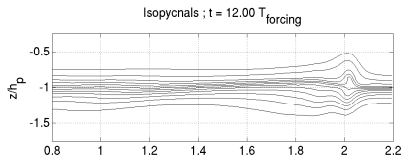
- (left) design of a simulation such that $v_{beam} \approx c_3$ for $\Omega > N_0$

Application to mode-3 ISWs

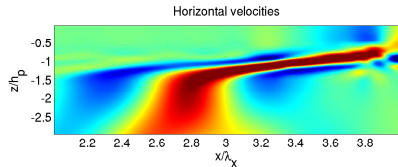
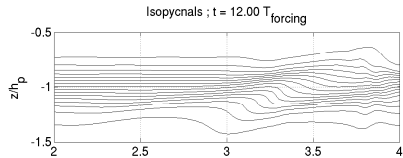


Near-field behavior

Let us have a closer look at the pycnocline:



(a) Mode-2 ISWs (2nd simulation)

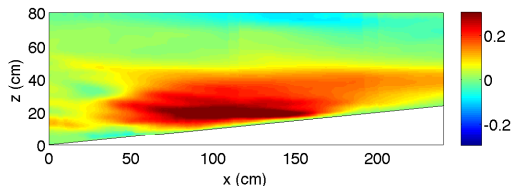


(b) Mode-3 ISWs (3rd simulation)

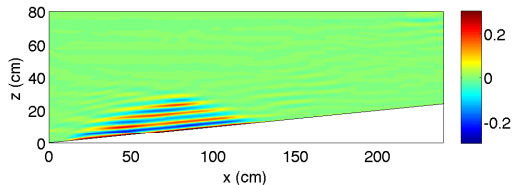
The refracted vertical wavelength matches the vertical wavelength of the mode- n trapped internal waves.

Comparison between experiments and numerics

Experiment 24 →
($\text{cm}\cdot\text{s}^{-1}$)



2D numerical
reproduction →
($\text{cm}\cdot\text{s}^{-1}$)



Two setups, main difference: 3D/2D

⇒ experimental mean flow must be 3D.