Nucleon structure by lattice QCD computations with twisted mass fermions

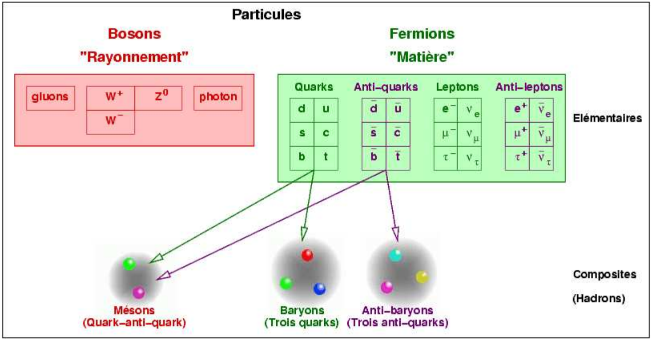
Pierre-Antoine Harraud
LPSC Grenoble

Grenoble
November 3, 2010

Outline

Outline

Standard Model and particles



Gluon fields represented by : $A^i_\nu(x)$.
Gluonic tensor: $F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g f^{ijk} A^j_\mu A^k_\nu$.
Quarks field: $\psi^a_{\alpha f}(x)$ of flavour f, color a and spin indice α.

Quantum ChromoDynamics

- $\mathcal{S}_{QCD} = \int d^4x\Big(-\frac{1}{4}(F^i_{\mu\nu})^2(x) + \bar{\psi}(x)(i\gamma^\mu D_\mu - m_q)\psi(x)\Big)$, with $D_\mu = \partial_\mu - igA^i_\mu\lambda^i$
- only **2 parameters** : g and m_q
- Asymptotic freedom at high energy: perturbative calculation
- Non perturbative regime at low energy: need of numerical resolution, i.e **lattice QCD**

To check QCD theory at low energy, comparison between LQCD and experiments

Hot topic : structure of nucleon

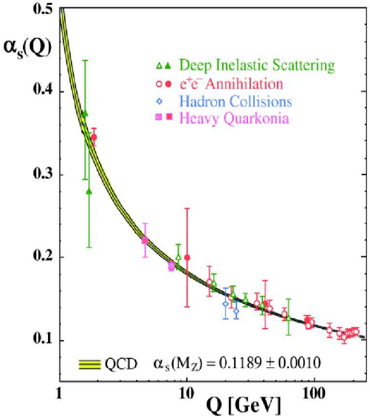
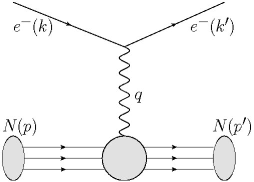


Figure: Running of the strong coupling constant $\alpha_s(Q) = \frac{g^2(Q)}{4\pi}$

Outline

Electric and magnetic form factors

- Elastic electron-proton diffusion
- Depends only on the photon virtuality : $Q^2 = -q^2 = -(p' - p)^2$



$$\langle N, \vec{p'}, s' | J_\mu^{EM}(0) | N, \vec{p}, s \rangle = \bar{u}(p') \left[\gamma_\mu F_1(Q^2) + i\sigma_{\mu\nu} q^\nu \frac{1}{2M} F_2(Q^2) \right] u(p)$$

Electric and magnetic Sachs form factors :

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

with physical meaning in non-relativist limit of
electric/magnetic charge distribution :

$$\rho(\vec{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\vec{q}\vec{r}} \frac{M}{E(\vec{q})} G_E(\vec{q}^2)$$

Electric and magnetic form factors

- For proton at low Q^2, G_E and $G_M \sim$:

$$G_D(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\Lambda^2}\right)^2}, \quad \Lambda^2 = 0.71\,GeV^2$$

- $\langle p|V^3_\mu(0)|p\rangle = \langle p|J^{EM}_\mu(0)|p\rangle - \langle n|J^{EM}_\mu(0)|n\rangle$

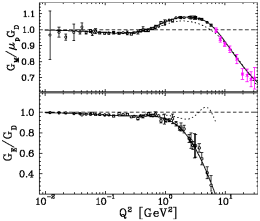


Figure: Experimental values for proton G_E and G_M [Arrington, Melnitchouk and Tjon (2007)]

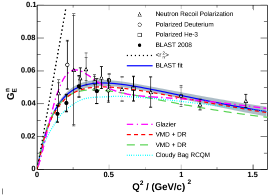


Figure: Experimental values for neutron G_E [BLAS (2008)]

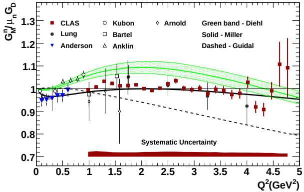
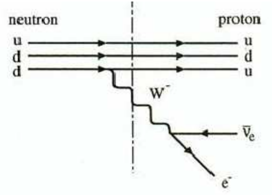


Figure: Experimental values for neutron G_M [CLAS (2009)]

Axial and pseudoscalar form factors

- Desintegration of neutron to proton
- $\langle p | J_\mu^{CC} | n \rangle = \cos\theta_C \langle p | V_\mu^+ - A_\mu^+ | n \rangle$
- $A_\mu^+ = \bar{\psi}_u \gamma_\mu \gamma_5 \psi_d$
- Isospin symetry $\rightarrow \langle N, \vec{p'}, s' | A_\mu^3 | N, \vec{p}, s \rangle$



$$\langle N, \vec{p'}, s' | A_\mu^3 | N, \vec{p}, s \rangle = \bar{u}(p') \left[\gamma_\mu \gamma_5 G_A(Q^2) + i \frac{q_\mu}{2M} \gamma_5 G_P(Q^2) \right] u(p)$$

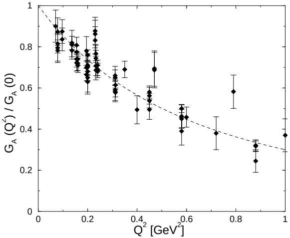


Figure: Experimental values for $G_A(Q^2)$ [Bernard,Elouadrhiri and Meissner (2002)]

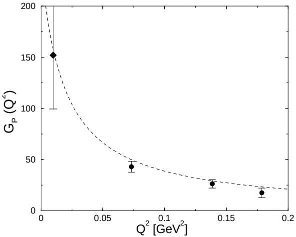


Figure: Experimental values for $G_P(Q^2)$ [Bernard,Elouadrhiri and Meissner (2002)]

Outline

[Wilson (1974)]

Feynmann path integral

Goal

To access matrix element $\langle N, \vec{p'}, s' | \mathcal{J}(0) | N, \vec{p}, s \rangle$
calculate 3 points function $\langle P_{\alpha_f}(x_f) \mathcal{J}(z) \bar{P}_{\alpha_i}(0) \rangle$

Value of an operator given by **Feynmann path integral** (euclidian space-time):

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S_E[\psi,\bar{\psi},U]} O[\psi, \bar{\psi}, U]$$

Goal of lattice QCD:
Evaluate numerically this integral with a discretised QCD action and fields on a lattice. Must verify continuum limit =QCD.

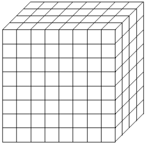
Separation of gauge and fermion integration :

$$S_E = S_G[U] + S_F[U, \psi, \bar{\psi}] \text{ with } S_F[U, \psi, \bar{\psi}] = \bar{\psi} D[U] \psi$$

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}U \langle O(\psi, \bar{\psi}) \rangle_U \ \det \mathrm{D[U]} \mathrm{e}^{-\mathrm{S_G[U]}}$$

Gauge fields discretisation

4D lattice of size $L^3 xT$ with lattice spacing a



- Gauge fields represented by $SU(3)$ matrix : $U_\mu(x,y) = \mathcal{P}\exp(-ig\int_x^y A_\mu(z)dz)$.
- Gauge fields lay on the link of the lattice.
- **One configuration** : ensemble of the gauge links on all the lattice

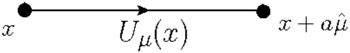


Figure: Representation of a gauge link

Generation of configurations

- Generate an ensemble of configurations U_i, "measure" O on each configuration.
- **Average observable over N configurations** :

$$\langle O \rangle = \frac{1}{N} \sum_{i=1}^{N} \langle O(\psi, \bar{\psi}) \rangle_{U_i} + \mathrm{O}(\frac{1}{\sqrt{\mathrm{N}}})$$

- Give **statistical error** : evaluate by **jackknife method** of resampling.(to get rid of correlation between configurations)
- Difficulty : include the **quark-antiquark pairs in the sea**.
  Our case: light quarks in the sea, $N_f = 2$.

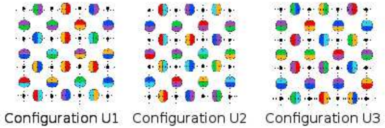


Figure: Illustration of configurations

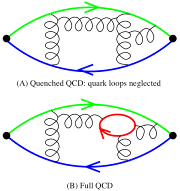


Figure: Illustration of unquenched simulations

Quark propagator

- **Wick theorem** : fermionic observables $\langle O(\psi,\bar{\psi})\rangle_{U_i}$ expressed as combinations of **quarks propagators** :

$$\langle \psi(x_f)\bar{\psi}(x_i)\rangle_{U_i} = S(x_f, x_i)_{U_i}$$

.

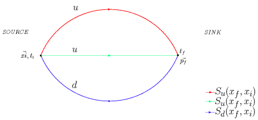


Figure: 2 points function

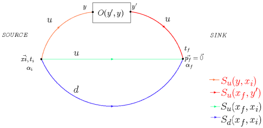


Figure: 3 points function

- Quarks propagator verify :

$$\sum_{y,} D(x_f, y)_{U_i} S(y, x_i)_{U_i} = \delta_{x_f, x_i}$$

where D is the Dirac operator specific to the discretisation scheme.

- By fixing x_i, needs to solve a linear system on each configuration.

Twisted mass fermions in the continuum

[Frezzotti, Grassi, Sint and Weisz (2000)]

$$
\begin{aligned}
S^{\mathrm{F}}_{\mathrm{QCD}}[\chi,\bar{\chi},U] &= \int d^4x \bar{\psi}\left[\gamma_\mu D_\mu + m_q\right]\psi \\
S^{\mathrm{F}}_{\mathrm{tm}}[\chi,\bar{\chi},U] &= \int d^4x \bar{\chi}\left[\gamma_\mu D_\mu + m_q + i\gamma_5\tau_3\mu_q\right]\chi
\end{aligned}
$$

- **Equivalence between QCD and twisted mass QCD** through :

$$
\begin{aligned}
\psi &= \exp(i\omega\gamma_5\tau_3/2)\chi \\
\bar{\psi} &= \bar{\chi}\exp(i\omega\gamma_5\tau_3/2)
\end{aligned}
$$

with $\tan\omega = \dfrac{\mu_q}{m_q}$

- **Maximal twist :** $\omega = \dfrac{\pi}{2}$

Discretised Twisted mass action

- Discretized twisted mass action:

$$S_{\rm tm}^{\rm F} = a^4 \sum_x \left\{ \bar{\chi}_x \left[D_{\rm W} + m_q + i\gamma_5 \tau_3 \mu_q \right] \chi_x \right\},$$

- Parity and isospin breaking, restored in the continuum
- Advantages at maximal twist : **automatic $O(a)$ improvement** [Frezzotti, Rossi (2004) Frezzotti, Martinelli, Papinutto and Rossi (2006)].
- After tuning at maximal twist : 2 parameters β (linked to g) and μ_q

Parameters and errors

- Need 2 observables to fix the 2 parameters, for instance (m_N, m_π).
  Results expressed in term of (a, m_π): lattice spacing and pion mass.

List of errors

• Sample of configurations	$\longrightarrow$	**Statistical error**
• Non zero lattice spacing a	$\longrightarrow$	**Discretisation effects** $(O(a^2))$
• Finite lattice size	$\longrightarrow$	**Finite volume effects**
• Pion mass m_π between 260 and 460 MeV	$\longrightarrow$	**Chiral extrapolation**

To handle with care the errors, needs several simulations with different parameters

Ensembles

Ensemble	β	a (fm)	$(L/a)^3 \times T/a$	L (fm)	$a\mu_q$	m_π (MeV)
B_1	3.9	0.0855	$24^3 \times 48$	2.1	0.0040	304
B_2					0.0064	378
B_3					0.0085	433
B_4					0.0100	468
B_6			$32^3 \times 64$	2.8	0.0040	298
B_7					0.0030	260
C_1	4.05	0.0699	$32^3 \times 64$	2.2	0.0030	293
C_2					0.0060	404
C_3					0.0080	466
D_1	4.2	0.0556	$48^3 \times 96$	2.7	0.0020	263
D_2			$32^3 \times 64$	1.8	0.0065	471

Table: Summary of ETMC used ensembles. Size of lattice $L^3 \times T$, inverse coupling constant β and the corresponding lattice spacing a, twisted mass parameter $a\mu_q$ and the corresponding pion mass m_π.

Outline

Interpolating fields

2 points function: $\langle P_{\alpha_f}(x_f)\bar{P}_{\alpha_i}(0)\rangle$

Proton interpolating field

$$P_\alpha(x) = \epsilon^{abc}(u^a(x)^T C\gamma_5 d^b(x))u^c_\alpha(x)$$

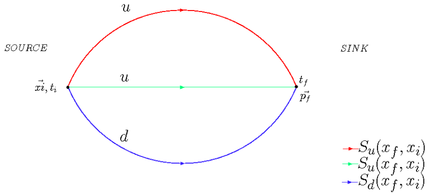
But excitations with the same quantum numbers.

Smearing : create a smeared field to improve the overlap with the ground state

- **APE smearing** of the configuration: one link replaced by an average with its neighbours
- **Gaussian smearing** of the quark propagator : average the quark propagator replacing dirac distribution by a gaussian. Applied at the sink and at the source

2 points Correlator

2 points function $\langle P_{\alpha_f}(x_f)\bar{P}_{\alpha_i}(0)\rangle$: combination of 3 quarks propagators.



$$C^{2pt}(t_f, \vec{p}, \Lambda^0) \quad = \quad Tr(\Lambda^0 \int d^3x_f e^{-i\vec{p}\vec{x_f}} \langle P_{\alpha_f}(x_f)\bar{P}_{\alpha_i}(0)\rangle)$$

- projection on proton spin indices. $\Lambda^0 = \dfrac{1+\gamma_0}{4}$: selected positive parity states
- projection on final momentum

Spectral decomposition

$$C^{2pt}(t_f, \vec{p}, \Lambda^0) \xrightarrow[t_f, T-t_f \to \infty]{} |Z_2^N| \frac{(E_N + M_N)}{2E_N} e^{-E_N t_f}$$

Energy extraction

Effective energy :

$$E_{eff}(t_f,\vec{p}) = \ln \frac{C^{2pt}(t_f,\vec{p})}{C^{2pt}(t_f+1,\vec{p})} \xrightarrow[t_f,T-t_f\to\infty]{} E_N(\vec{p})$$

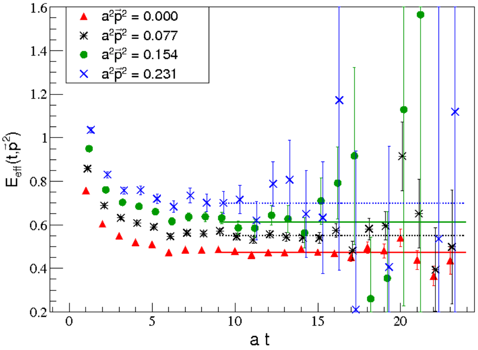


Figure: $E_{eff}(t,\vec{p}^2)$ in function of at.

- excited states present at small t
- determination of a plateau of fit
- weak dependence on fit range
- when $\vec{p}^2 \nearrow$, signal to noise ratio $\searrow$

Dispersion relation

To extract form factors need to know energy of the proton.
Which one ? The one of the continuum : $E_N^2 = M_N^2 + \vec{p}^2$.

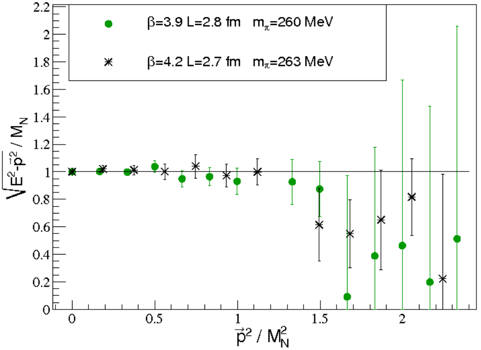


Figure: Difference between continuum relation and fitted energy in function of $(a\vec{p})^2$.

No strong discretisation effects.
Relation valid for range of momenta used.

Effect of smearing

Spectral decomposition:

$$C^{2pt}(t_f, \vec{p}, \Lambda^0) \xrightarrow[t_f,\, T-t_f \to \infty]{} |Z_2^N| \frac{(E_N + M_N)}{2E_N} e^{-E_N t_f}$$

Effect of smearing on Z_2: constant $\to$ gaussian in $\vec{p}^2$, $\equiv$ **wave packet**.

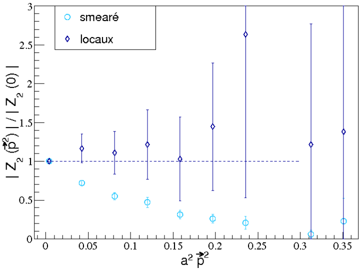


Figure: Comparison of $Z_2(\vec{p}^2)/Z_2(0)$ with and without smearing.

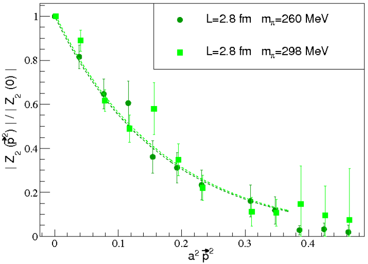


Figure: Fit of $Z_2(\vec{p}^2)/Z_2(0)$ by a gaussian.

Need to compense $Z_2(\vec{p}^2)$ factors for extraction of form factors.

Outline

Definition

$$\text{3 points function}: \langle P_{\alpha_f}(x_f)\mathcal{J}(z)\bar{P}_{\alpha_i}(0)\rangle$$

- Combination of 4 quark propagators.
- Restriction: isovectoriel current to avoid disconnected parts.(except $O(a^2)$ effects)

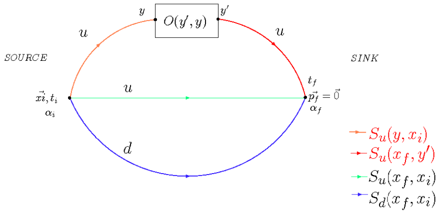


Figure: Illustration of 3 points function

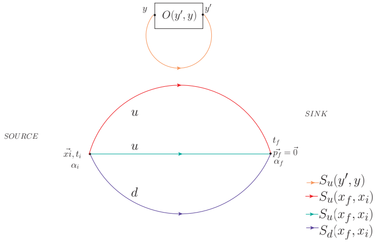
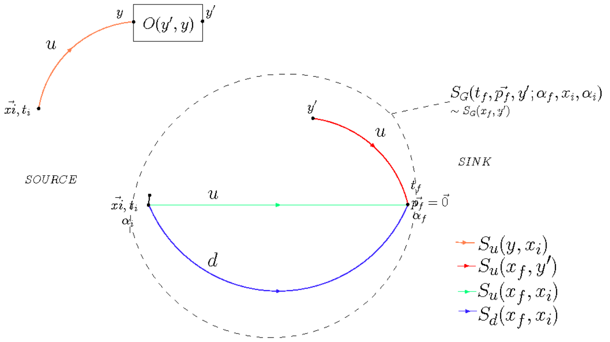


Figure: Illustration of disconnected parts for 3 points function

But needs one propagator $S(x_f, z) \quad \forall z, x_f$.
Problems : we computed propagator $S(x_f, x_i)$ from one fixed point source x_i.

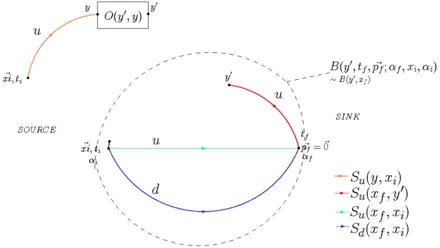
Generalized propagator

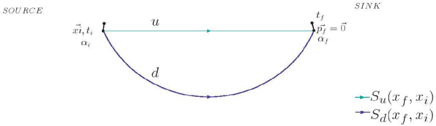
Solution [Maiani and Martinelli (1986)] : Extract the operator and the quark propagator $S(z, x_i)$.



Generalized propagator

- Change the direction of the propagator $S(x_f, z) \to S(z, x_f)$.
- **Fixed the momentum and the time at the sink.**
- **Fixed the spin indices of the proton at source and sink (projection).**
- Define the source of this generalized propagator





By "inverting" this source → generalized propagator → 3 points correlator.

3 points correlators

$$C^{3pt}(t,\vec{q},t_f,\Lambda,\mathcal{J}) = Tr(\Lambda\int d^3x_f d^3z e^{i\vec{q}\vec{z}}\langle P_{\alpha_f}(x_f)\mathcal{J}(z)\bar{P}_{\alpha_i}(0)\rangle)$$

Depends on:

- the operator in study
- the time of current insertion t
- momentum transferred $\vec{q}$
- projector on proton spin indices, Λ : fixed and depend on observables in study.
- time t_f : fixed, source-sink separation ~ 1 fm.
- projection at 0 momentum at the sink

Then construct the following ratio using the 2 and 3 points correlators:

$$R(t_z,\vec{q},t_f,\Lambda,\mathcal{J}) = \frac{C^{3pt}(t,\vec{q},t_f,\Lambda,\mathcal{J})}{C^{2pt}(t_f,\vec{0},\Lambda^0)}\sqrt{\frac{C^{2pt}(t_f-t_z,-\vec{q},\Lambda^0)C^{2pt}(t,\vec{0},\Lambda^0)C^{2pt}(t_f,\vec{0},\Lambda^0)}{C^{2pt}(t_f-t_z,\vec{0},\Lambda^0)C^{2pt}(t,-\vec{q},\Lambda^0)C^{2pt}(t_f,-\vec{q},\Lambda^0)}}$$

Plateaux

Ratio tends to a constant : $R(t_z, \vec{q}, t_f, \Lambda, \mathcal{J}) \xrightarrow[t_f - t_z, t_z \gg 1]{} := \Pi(\vec{q}, \Lambda, \mathcal{J})$

In practice: fit a symetric plateau for all $\vec{q}$

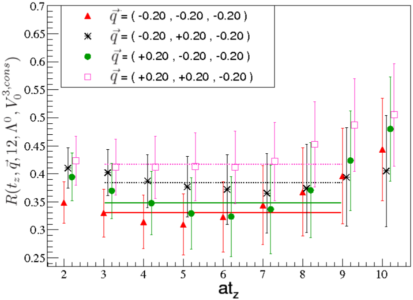


Figure: Examples of plateaux for 3 points functions

Within our statistical errors, no dependence on plateaux range

Extraction of form factors

- Extract the form factors from fitted plateaux
- Use all degenerate momenta and component contributing through Singular Value Decomposition (SVD).
- Consistency check of fit range
  $\longrightarrow$ **plateaux safe from contaminations by excited states.**

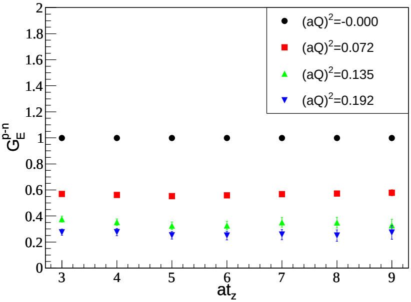


Figure: Plateaux after SVD for $G_E(Q^2)$ for several momenta.

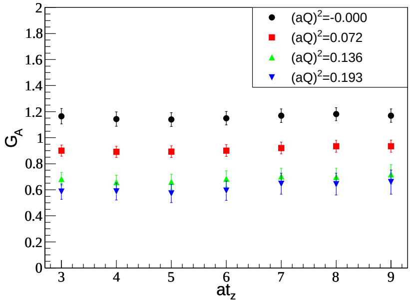


Figure: Plateaux after SVD for $G_A(Q^2)$ for several momenta.

Outline

Electric form factor

- Use of exact conserved current (no renormalization).
- Charge conservation impose $G_E(0) = 1$.
- Dipole fit :

$$G_E(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_E^2}\right)^2}$$

- Fit range : [0-1.2] GeV.

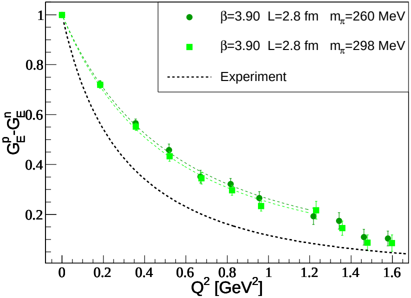


Figure: Examples of $G_E^{p-n}(Q^2)$ with dipole fit. Experimental value given by black curve.

Electric mean square radius

Extract from fit with one parameter : $\langle r_E^2 \rangle = \frac{12}{M_E^2}$

Correspond to the slope at the origin of $G_E(Q^2)$

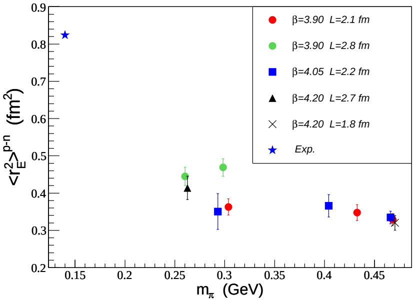


Figure: Chiral dependence of electric mean square radius.

- No evidence of discretisation effects.
- **Evidence of volume effect.**

Electric mean square radius

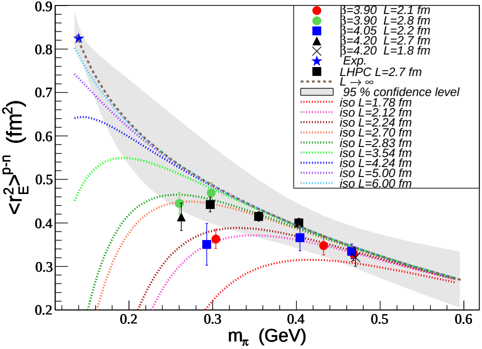
Combined fit :

- Volume effects :

$$M_E(m_\pi L) = A + \frac{B}{m_\pi L} e^{-m_\pi L}$$

- Chiral dependence :

$$\langle r_E^2 \rangle^{p-n} = \frac{c_1 + \chi_N \log \frac{m_\pi^2}{\mu^2 + m_\pi^2}}{1 + c_2 m_\pi^2}$$



Need to correct finite volume effects to get chiral dependence.

Vectorial renormalization constant

Continuum vector current need to be renormalized :

$$\Pi(\vec{q}, \Lambda^0, V_\mu^3) = \frac{G_E(Q^2)^{p-n}}{Z_V}$$

Z_V extracted from the ratio :

$$\frac{\Pi(\vec{q}, \Lambda^0, V_\mu^{3,cons})}{\Pi(\vec{q}, \Lambda^0, V_\mu^3)} = Z_V(Q^2)$$

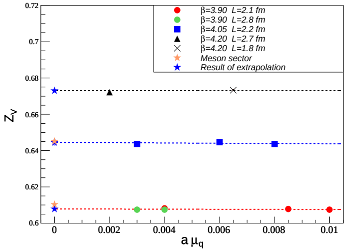
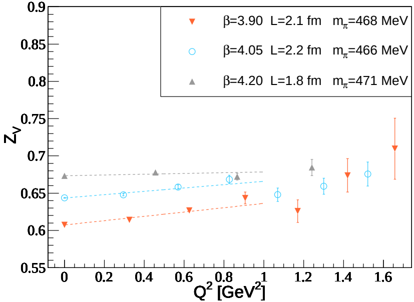


Figure: Dependance of Z_V in function of twisted mass $a\mu_q$

Constitency between Z_V determination in meson and nucleon sector

Vectorial renormalization constant

Q^2 dependence of Z_V.



Dependence comes from discretisation effect.

Evidence of discretisation effects for Q^2 dependence of form factors

Magnetic form factor

- Dipole form :

$$G_M(Q^2) = \frac{\mu^{p-n}}{\left(1 + \frac{Q^2}{M_M^2}\right)^2}$$

- Simultaneous fit of μ^{p-n} and $\langle r_M^2 \rangle^{p-n}$

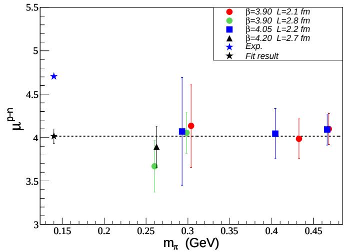


Figure: Chiral dependence of magnetic moment.

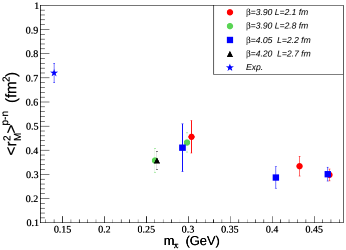


Figure: Chiral dependence of magnetic mean square radius.

No evidence of finite volume neither discretisation effects.

Curvature of chiral extrapolation to the physical point for $m_\pi < 260$ MeV.

Axial form factor

- Dipole form :

$$G_A(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}$$

- $G_A(0) = g_A$: axial coupling.
- Need to be renormalized.

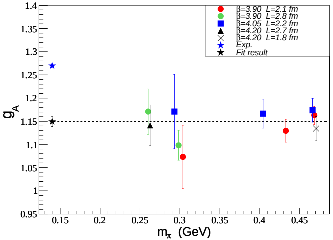


Figure: Chiral dependence of axial coupling g_A.

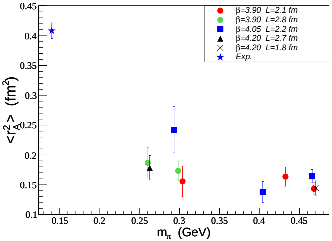


Figure: Chiral dependence of axial mean square radius.

No evidence of finite volume nor discretisation effects.

Curvature of chiral extrapolation to the physical point for $m_\pi < 260$ MeV.

Average momentum carried by quarks

- With $\mathcal{O}_{n,f}^{\mu\mu_1..\mu_{n-1}}$, operator with derivative :

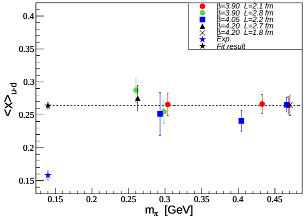
$$\langle N,\vec{p},s|\mathcal{O}_{n,f}^{\mu\mu_1..\mu_{n-1}}|N,\vec{p},s\rangle = A_{n,f}\,\bar{u}(p)\gamma^{\mu}p^{\mu_1}...p^{\mu_{n-1}}u(p)$$

$$A_{n,f} = \langle x^{n-1}\rangle_f = \int_0^1 dx x^{n-1}(f_f(x) - (-1)^{n-1}f_{\bar{f}}(x))$$

- Average momentum carried by quarks :

$$\langle x\rangle_{u-d} = \int_0^1 dx\ x(f_u(x) + f_{\bar{u}}(x)) - (f_d(x) + f_{\bar{d}}(x)) = A_2^{u-d}$$

- Needs renormalization constant.



Conclusion

- Need lattice QCD to get predictions of QCD at low energy.
- Twisted mass fermions at maximal twist : automatic improvement of lattice artefacts.
- Variety of ensembles to get under control systematical errors.
- 3 points function of nucleon challenging due to procedure and large statistical errors.

Conclusion

- Large volume effects in electric mean square radius $\longrightarrow$ combined fit needed to get chiral extrapolation.
- No direct discretisation effects in mean square radius but evidence of their contribution through $Z_V(Q^2)$.
- Accurate results obtained for several quantities: μ^{p-n}, $\langle r_M^2 \rangle^{p-n}$, g_A, $\langle r_A^2 \rangle^{p-n}$, $\langle x \rangle$.
- No finite volume neither discretisation effects in those observables.
- Apparent discrepancy with data in pion mass range and experiments should be explain by chiral extrapolation.
- Agreement with results of others collaborations having different discretisation scheme.

Perspective

- Need lower pion mass.
- Check source-sink time separation to control contamination by excited states.
- Improved statistical error bars: improved algorithms for quarks propagators computations.
- Widen momentum range : for large momenta with different smeared sources; for small momenta with twisted boundaries conditions.

Thanks for your attention !

Smearing for the 3 points function

APE and gaussian smearing at the source and sink

- At the source by using propagator smeared at the source
- At the sink by smearing the source of the generalized propagator : smearing fixed at the sink.

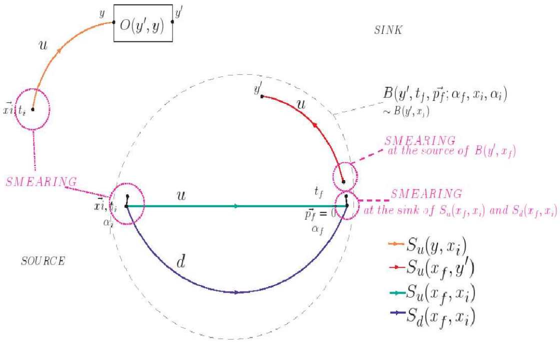


Figure: Illustration of smearing for the 3 points function

Useful tests

Comparing with the 2 points function, we test :

- Of the generalized propagator source :

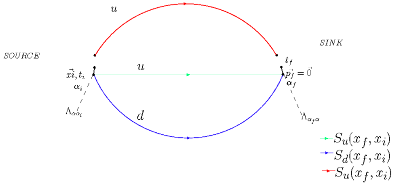


Figure: Illustration of test of the generalized propagator source

- Of the generalized propagator :

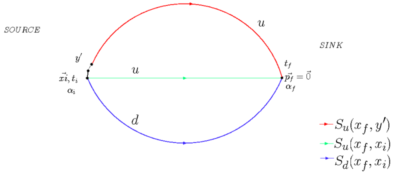
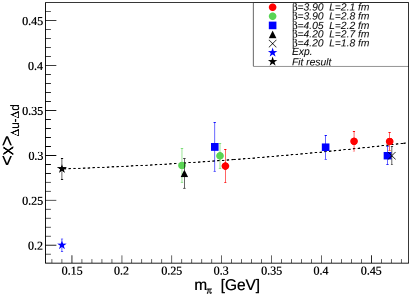


Figure: Illustration of test of the generalized propagator

Average of quark transversity



$\frac{G_P(Q^2)}{G_A(Q^2)}$

For $Q^2 \sim m_\pi^2$:

$$\alpha_{PPD} := \frac{G_P(Q^2)}{G_A(Q^2)} \frac{Q^2 + m_\pi^2}{4M_N^2} = 1$$