

Cosmologie et modifications à grandes distances de l'interaction gravitationnelle

Cosmology and modifications of gravity at large distances

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APC, Université Paris 7 Diderot

Sous la direction de
C. Deffayet



Laboratoire APC, Mardi 19 Janvier 2010

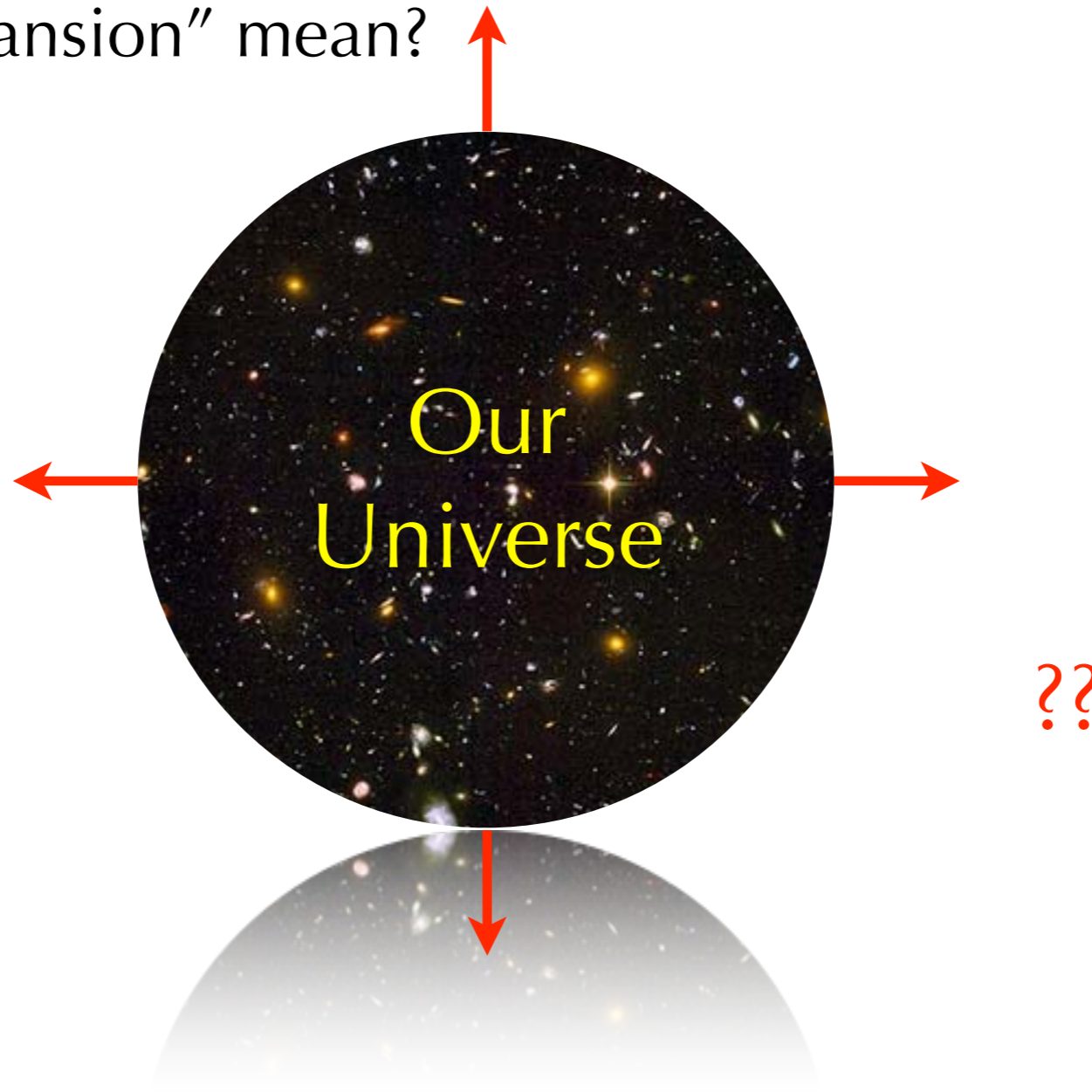
Outline of the presentation

- The acceleration of the Universe: Dark Energy or new gravity?
- Massive gravity and the Vainshtein's mechanism
- Observations: the confrontation with reality

The acceleration of the Universe:
Dark Energy or new gravity?

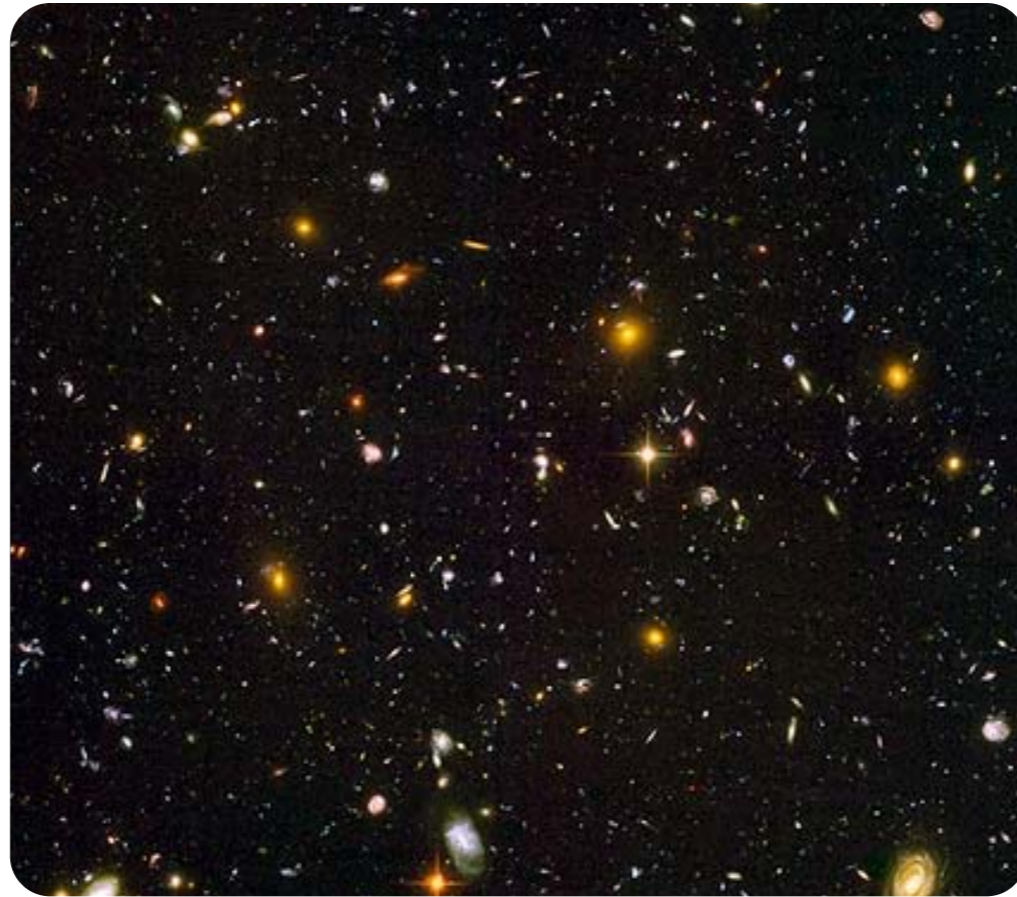
The expansion of the Universe

- What does “expansion” mean?



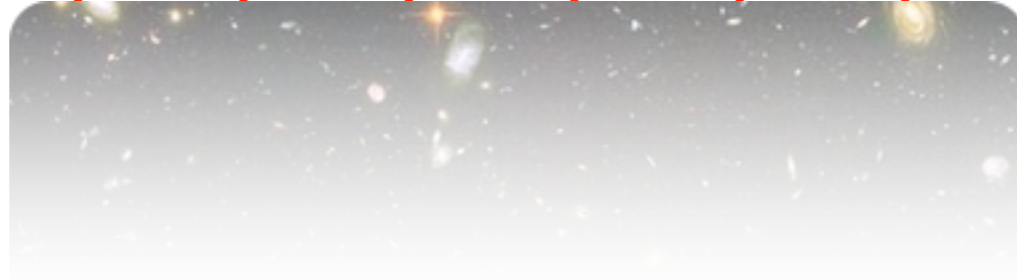
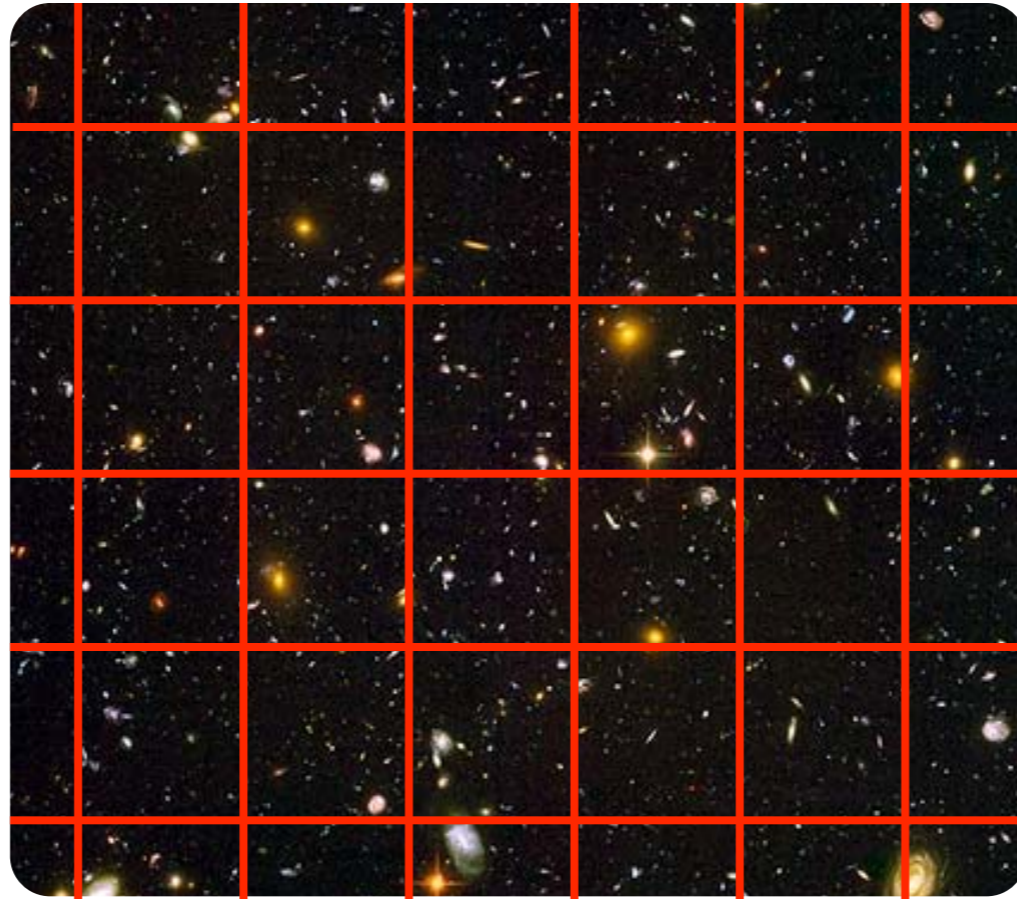
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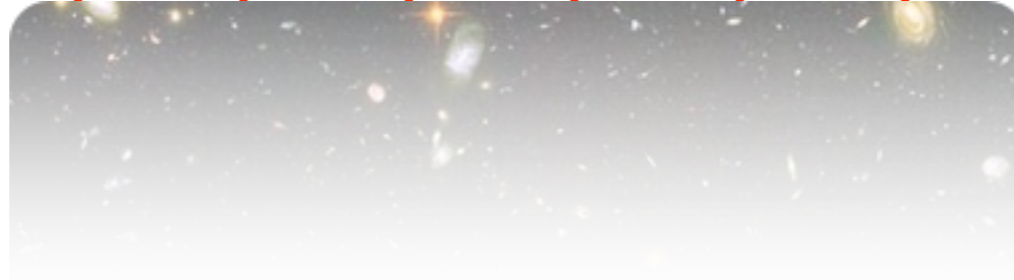
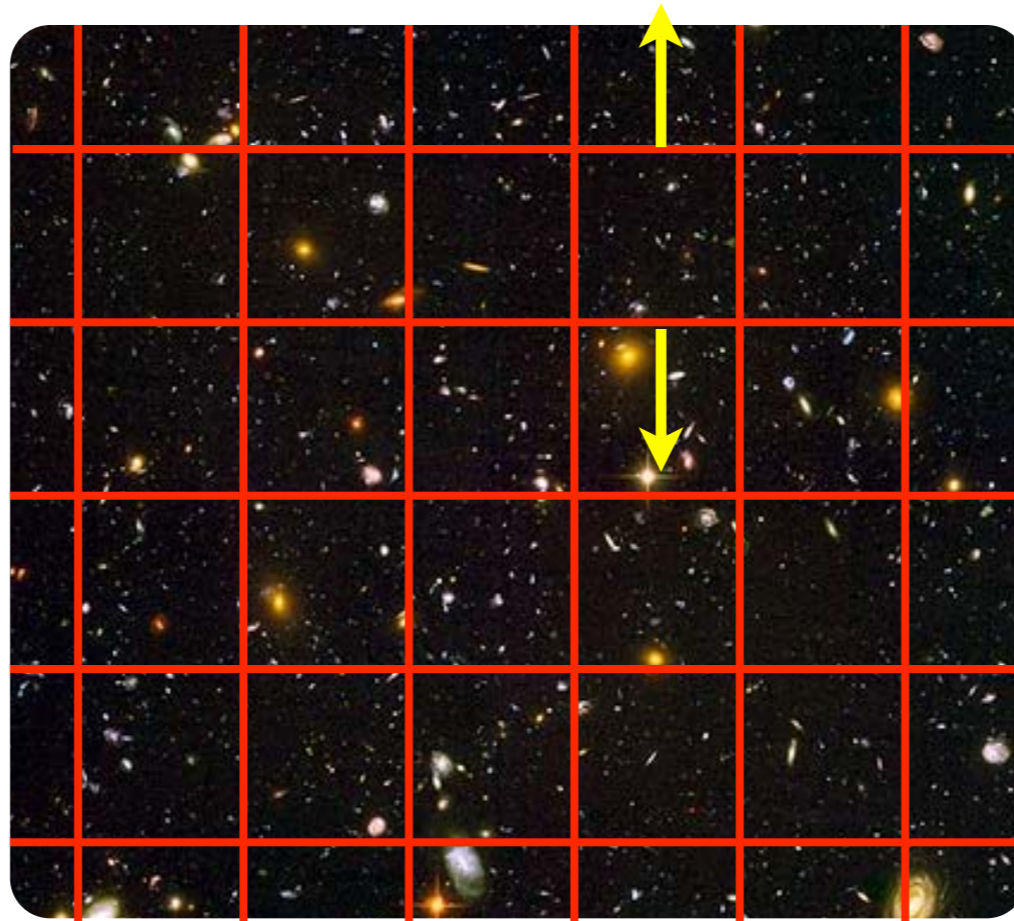
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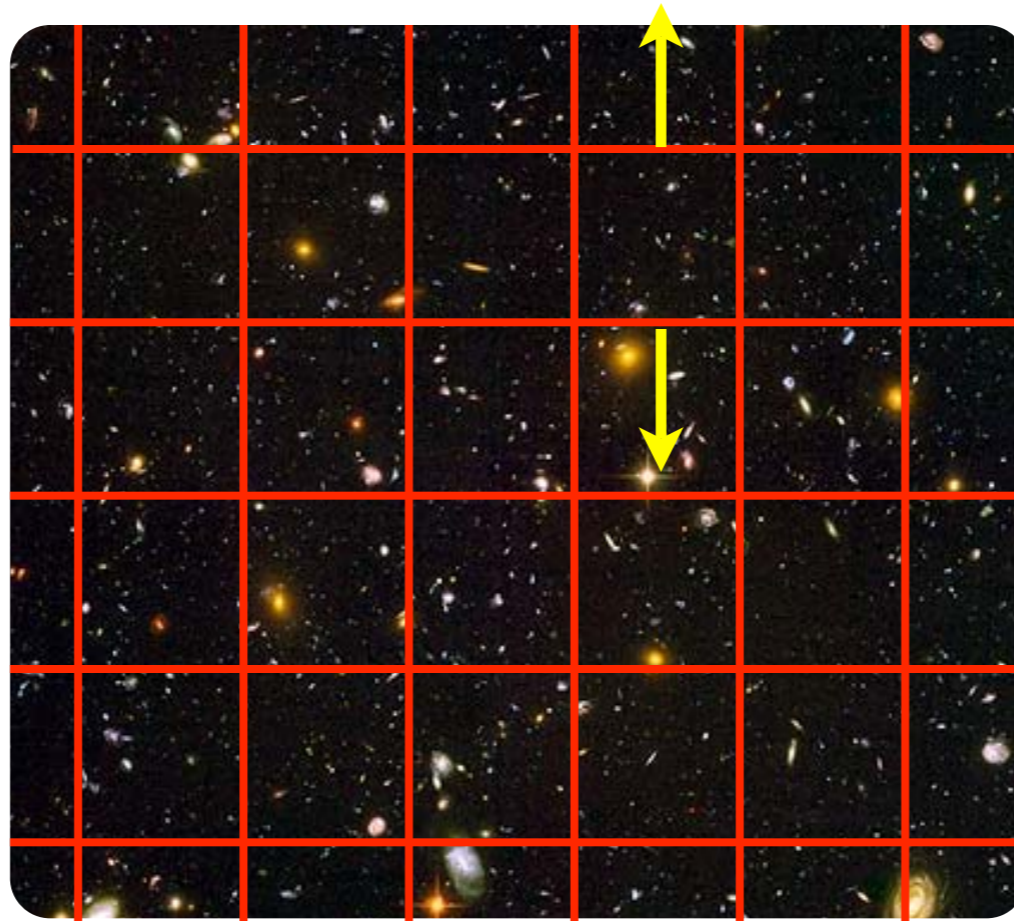
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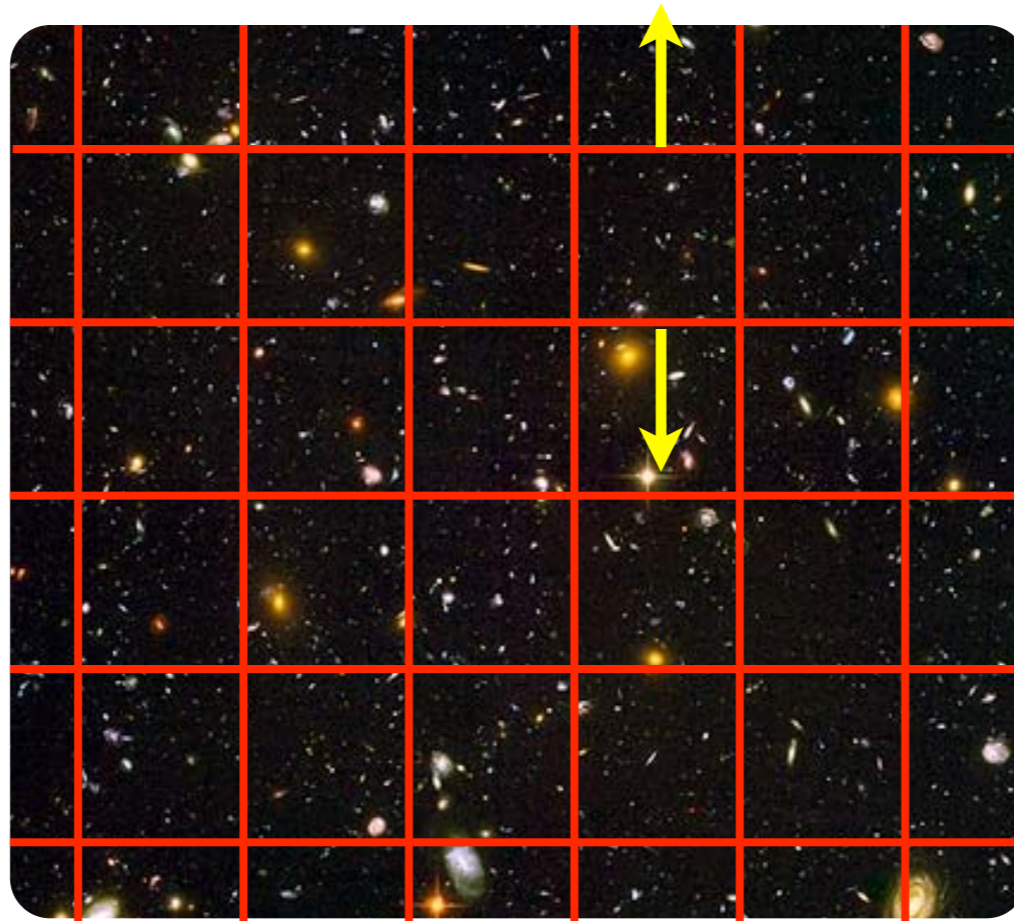
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Two types of units of distances:

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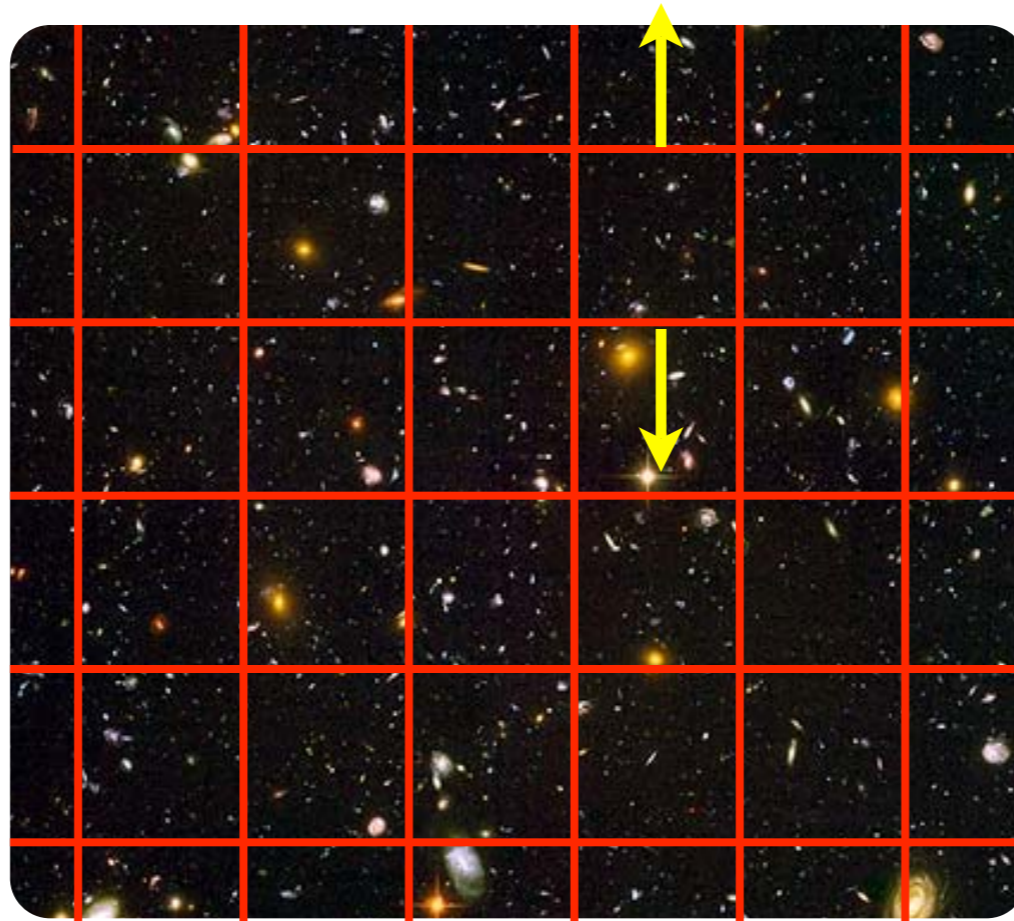


Two types of units of distances:

✓ grid units

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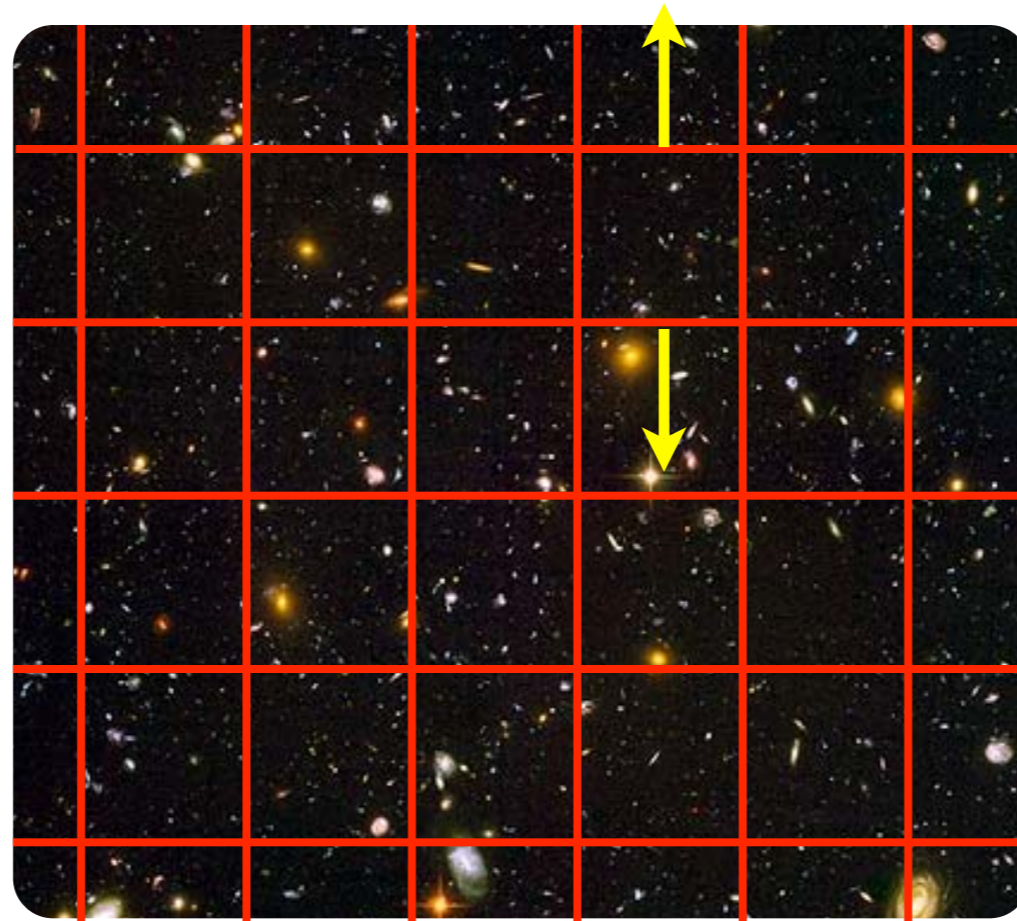
Two types of units of distances:

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✓ physical units

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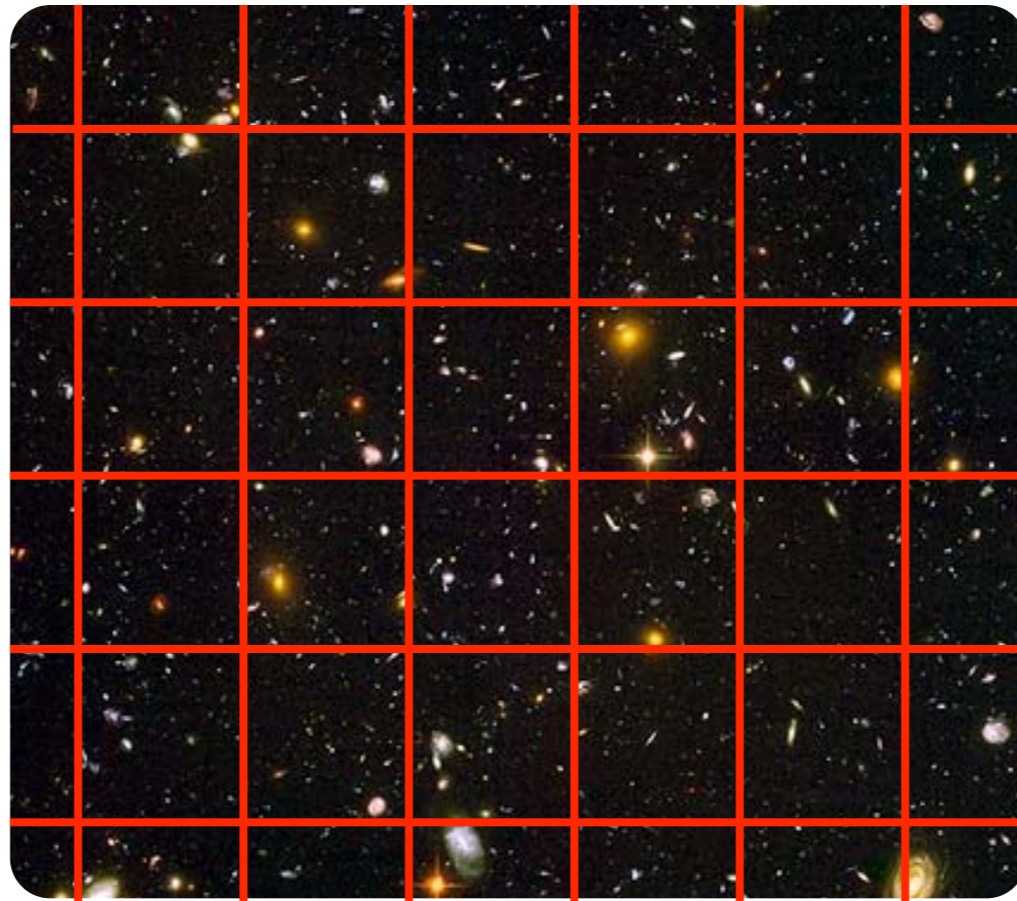
The metric
 $g_{\mu\nu}$



✓ physical units

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Two types of units of distances:

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The metric

$$g_{\mu\nu}$$

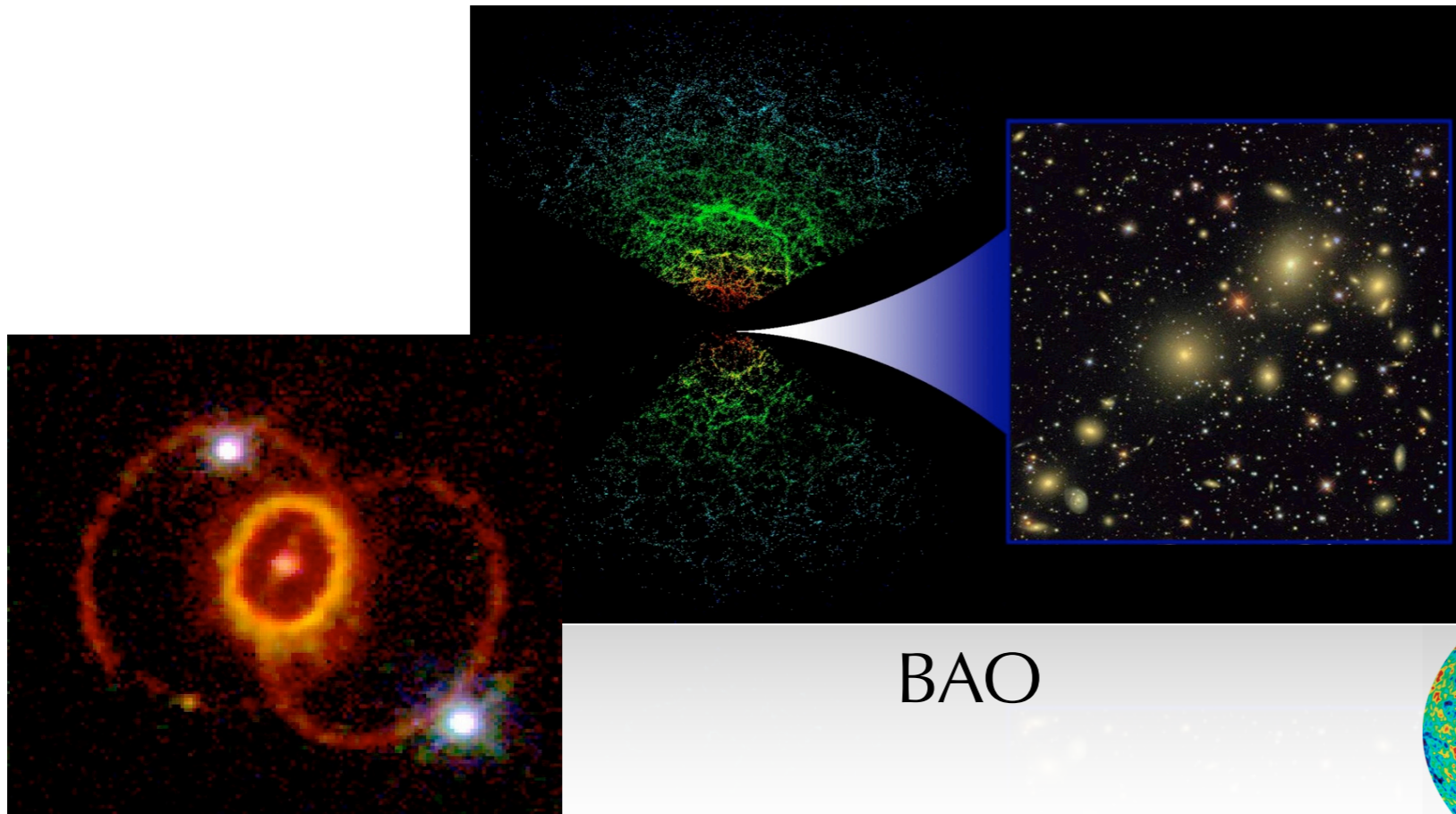


✓ physical units

encodes the space-time

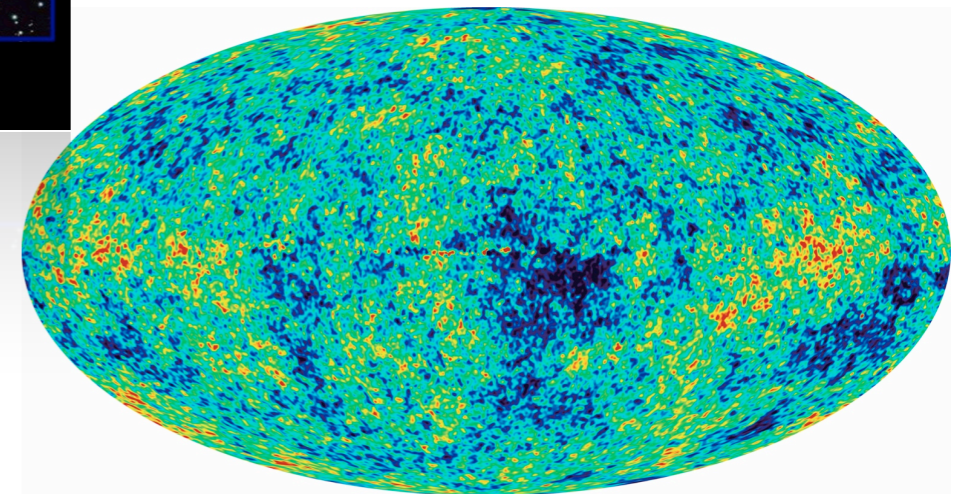
The acceleration of the Universe

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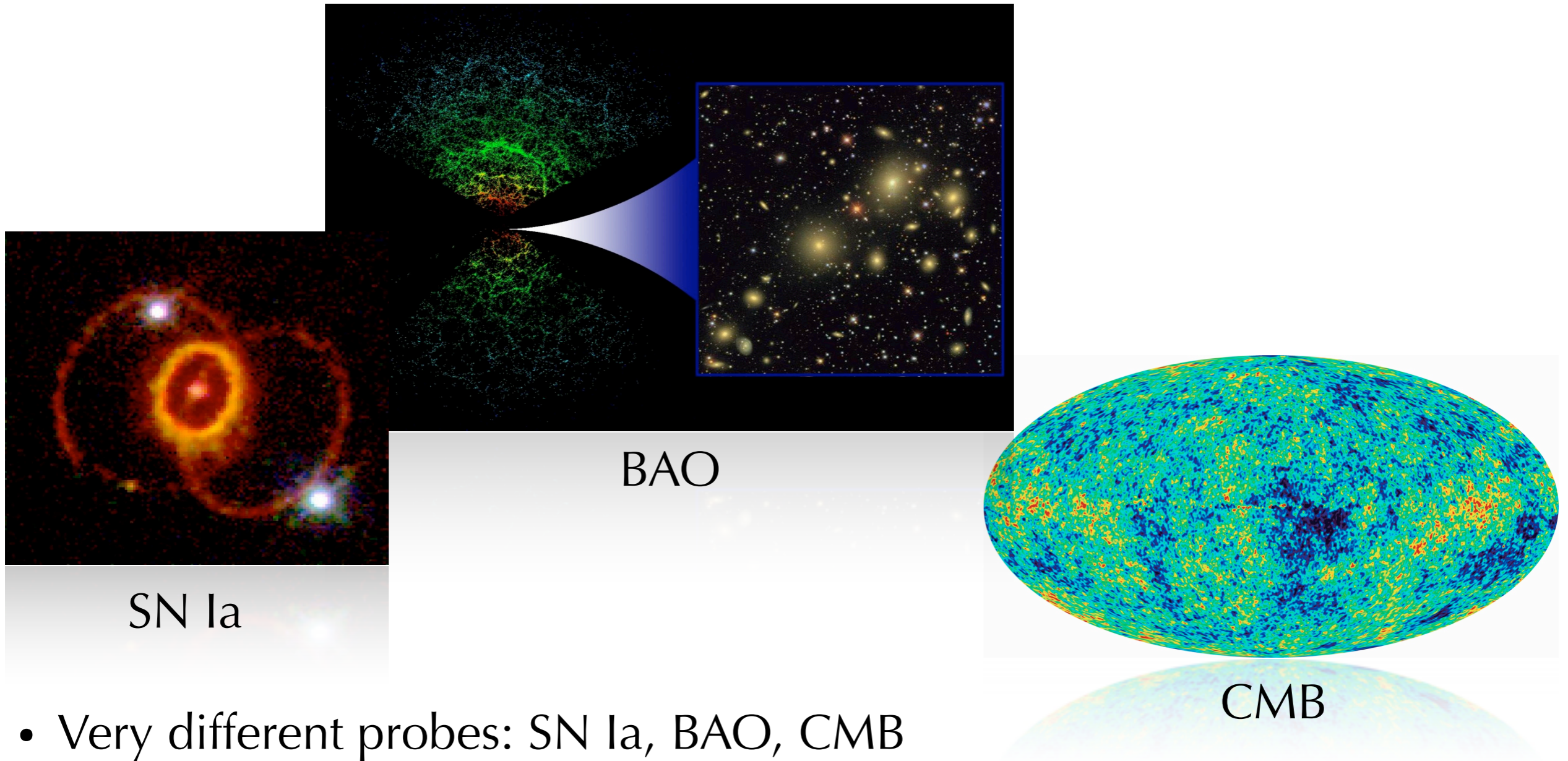
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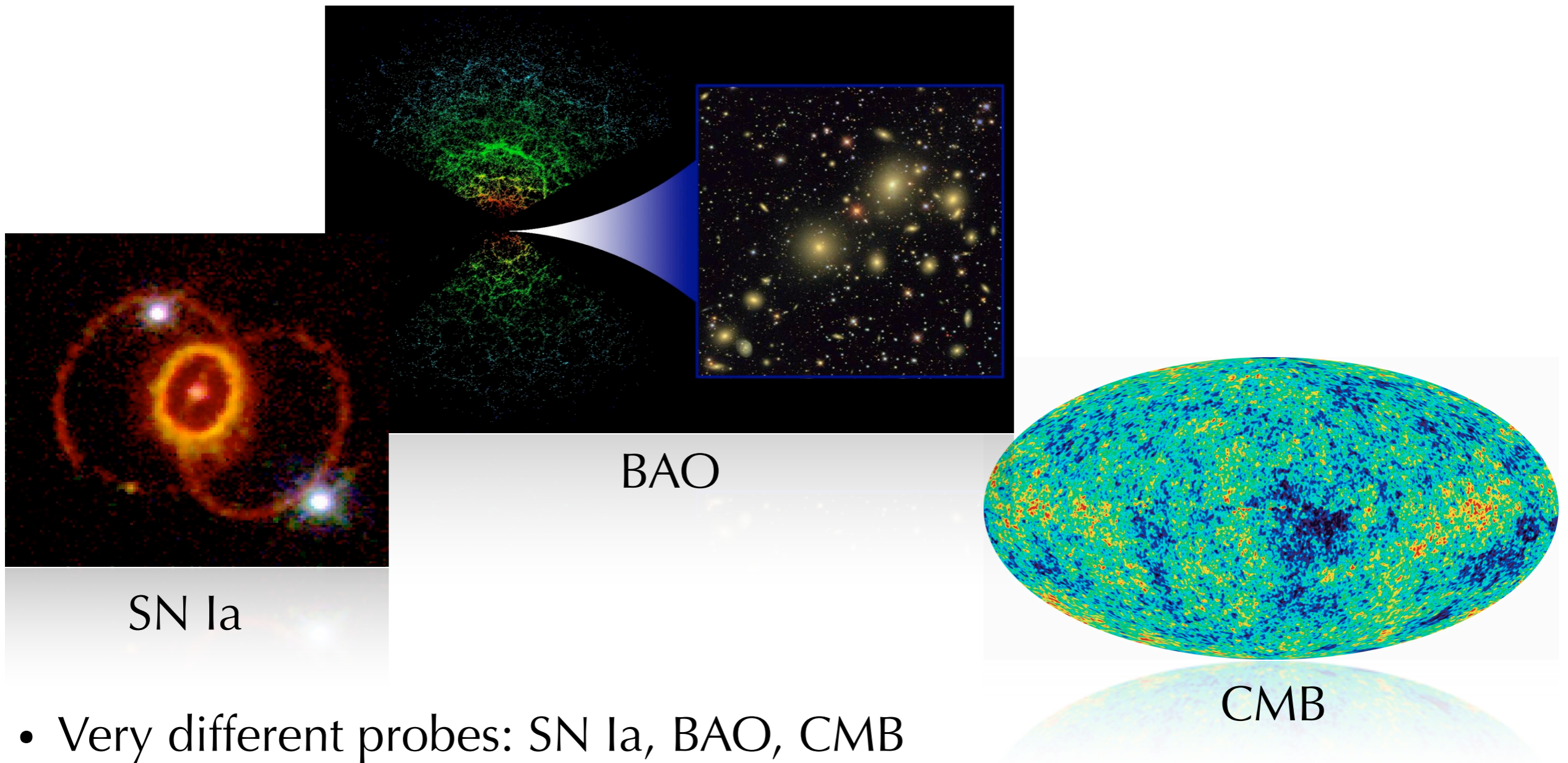


CMB

The acceleration of the Universe



The acceleration of the Universe



They all show that the Universe is accelerating!

The source of the acceleration

The source of the acceleration

- Einstein's equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}^{(m)} - \Lambda g_{\mu\nu}$$

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Geometry

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Geometry

Matter

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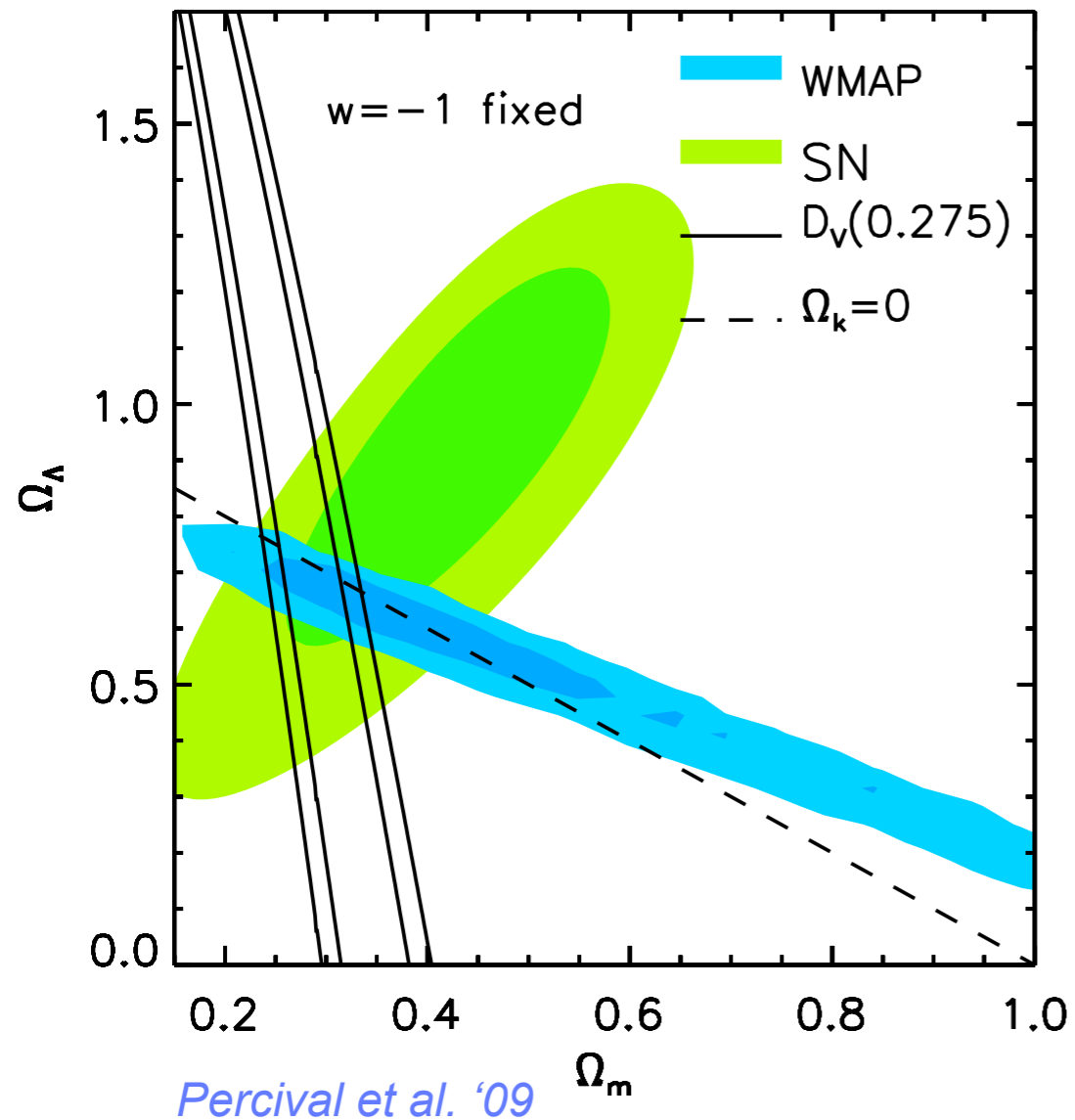
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$$\text{Geometry} \rightarrow G_{\mu\nu} = 8\pi G T_{\mu\nu}^{(m)} \rightarrow \Lambda g_{\mu\nu} \leftarrow \text{Dark Energy}$$

Geometry

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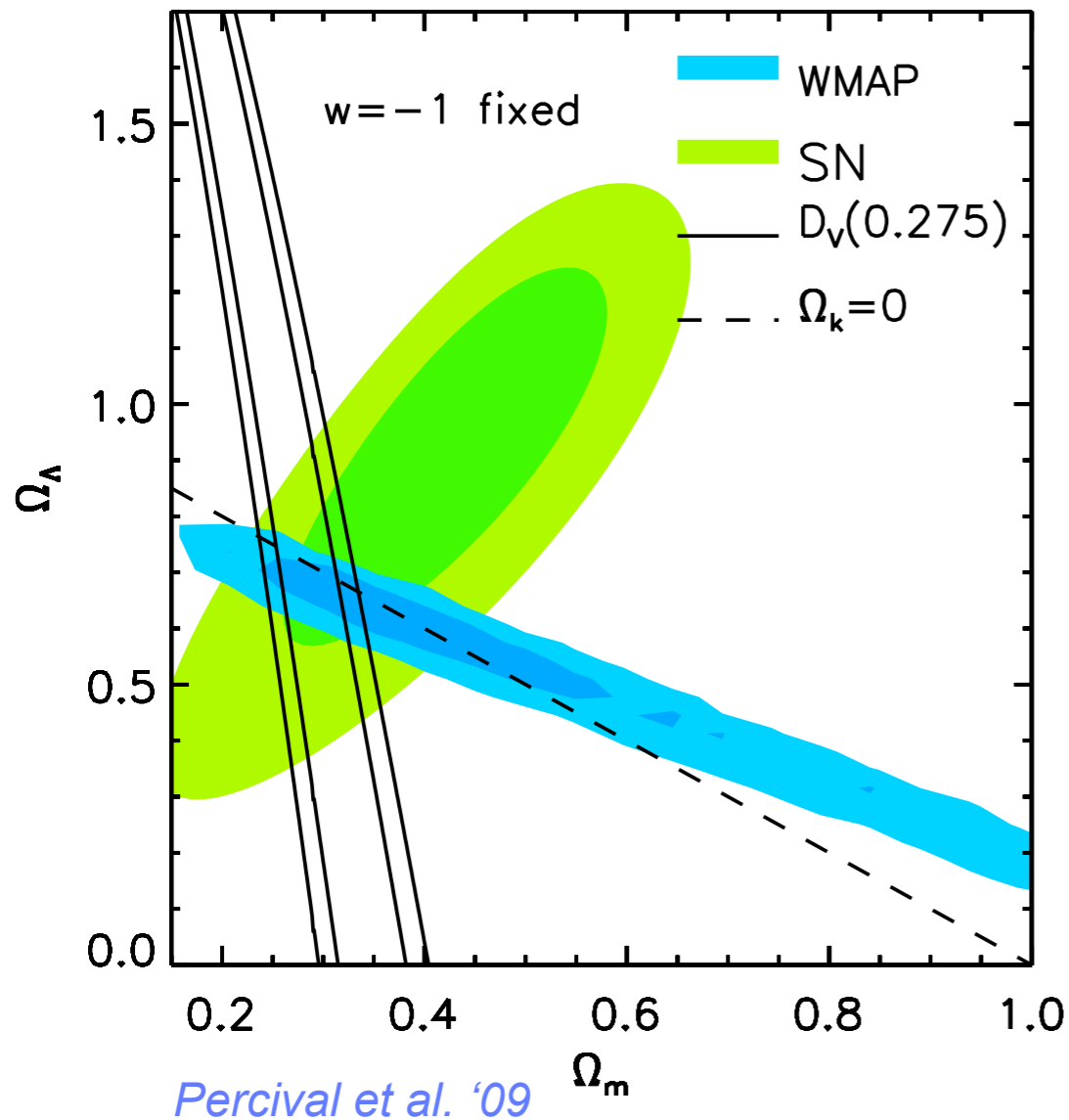


The source of the acceleration

- Einstein's equations

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Geometry Matter Dark Energy



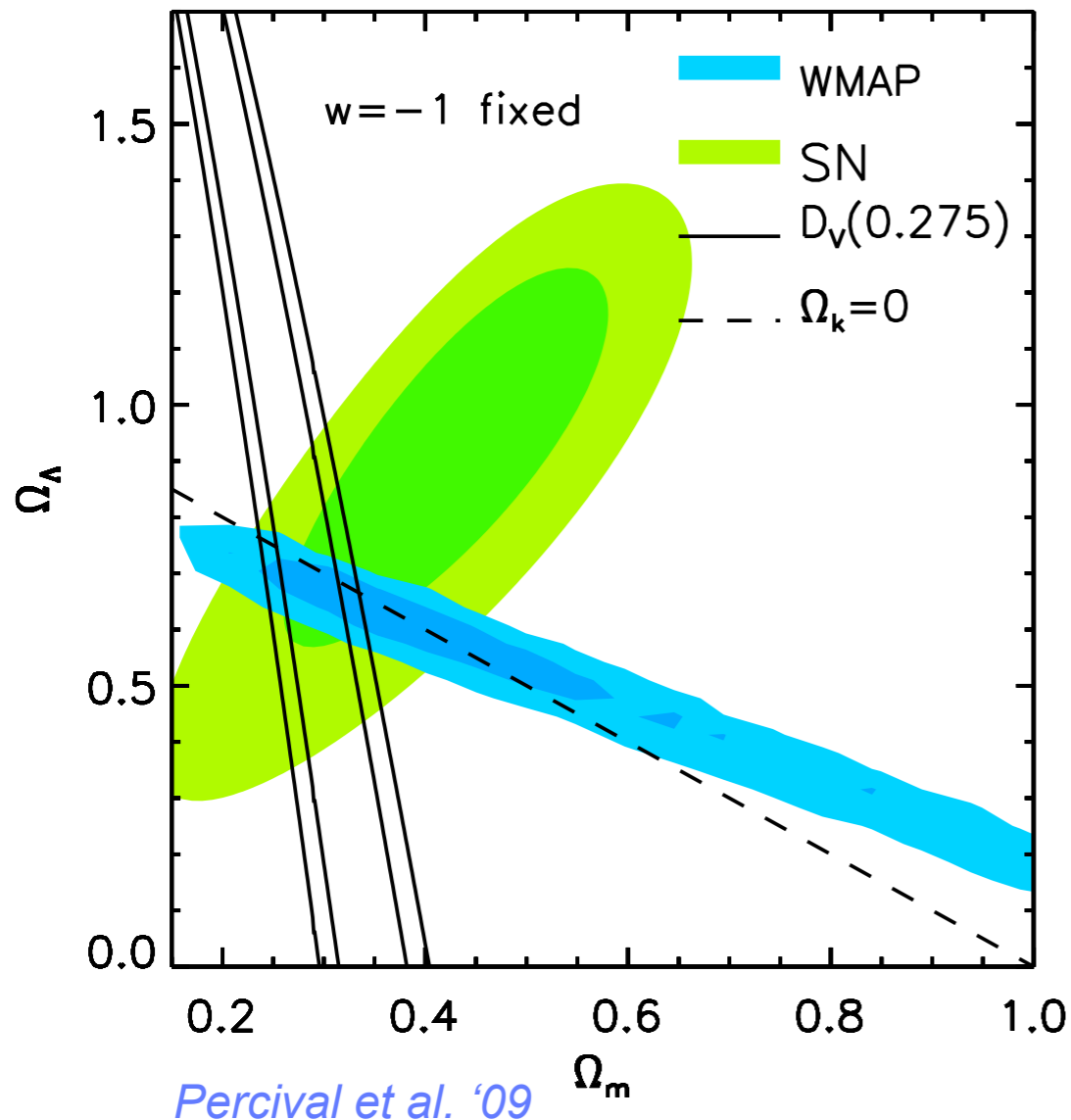
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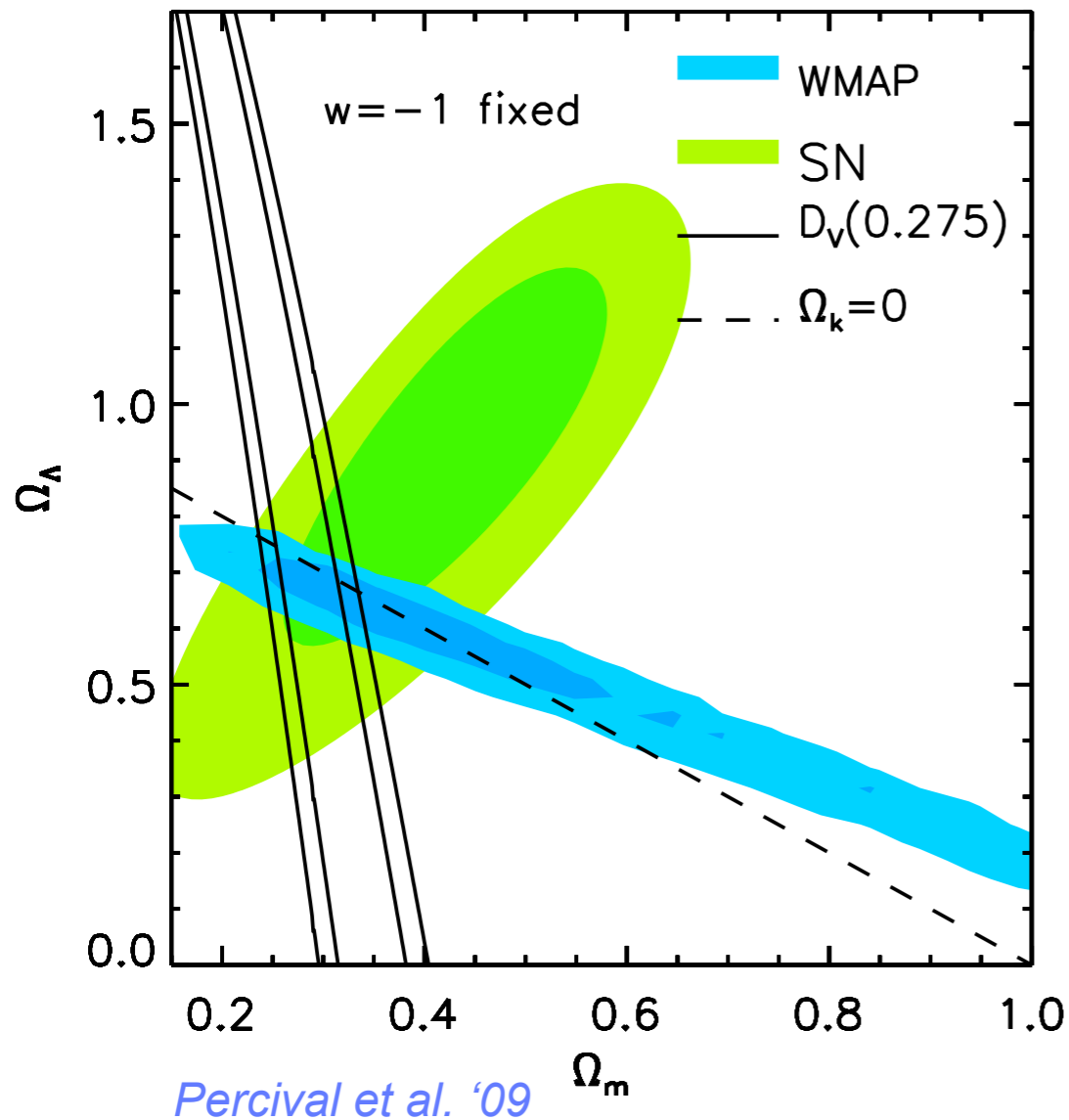
- Coincidence problem
- Fine tuning problem

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What is the nature of the Dark Energy?

Modified gravity theories

Modified gravity theories

Aim: to generalize Einstein's equations to explain the acceleration of the Universe without Dark Energy

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How: by adding extra fields

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→ Chameleon mechanism

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Suppressing the role of these extra fields at short distances:

➡ Chameleon mechanism

➡ Vainshtein's mechanism

Modified gravity theories: a (short) review

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➔ Chameleon mechanism

Khoury & Veltman '00

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- scalar-tensor theories (and $f(R)$ models)

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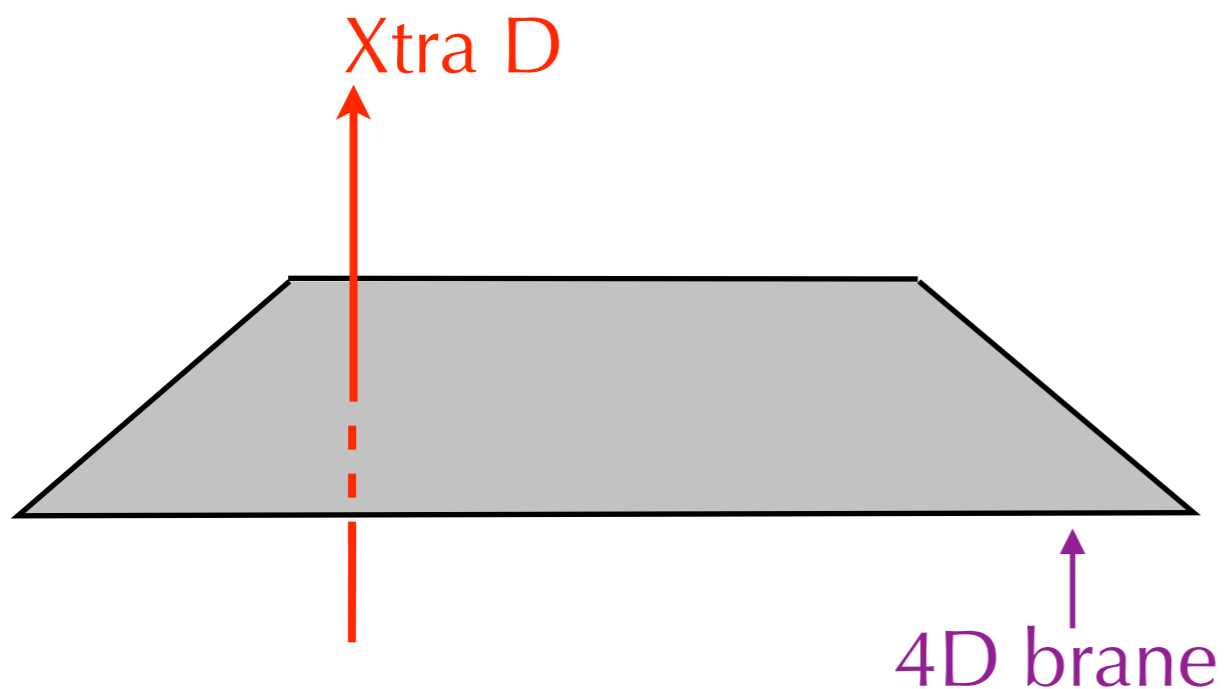
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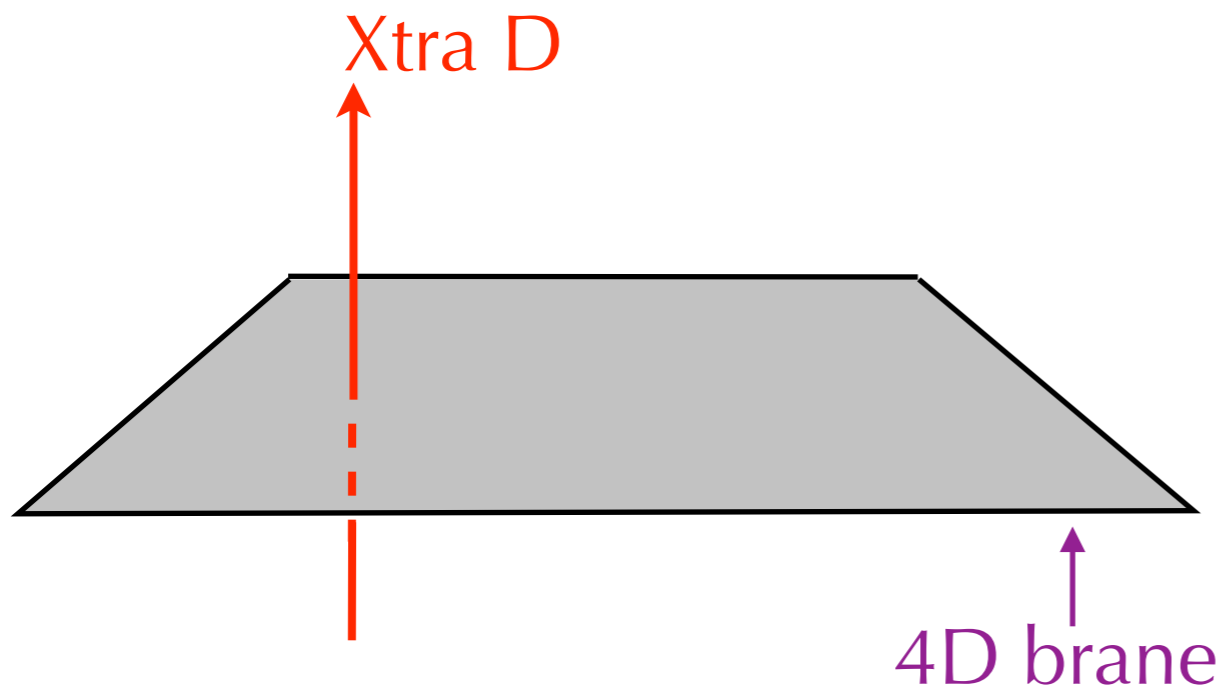
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✓ matter stays on the brane



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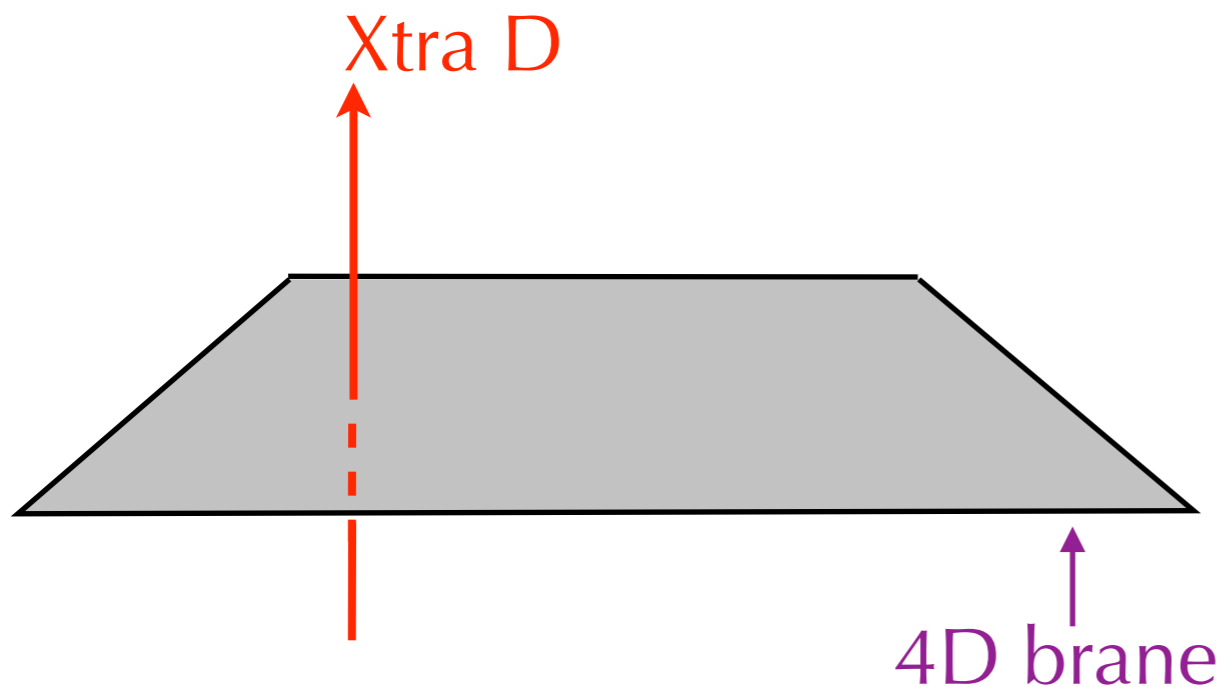
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- ✓ matter stays on the brane
- ✓ 5D + 4D gravity



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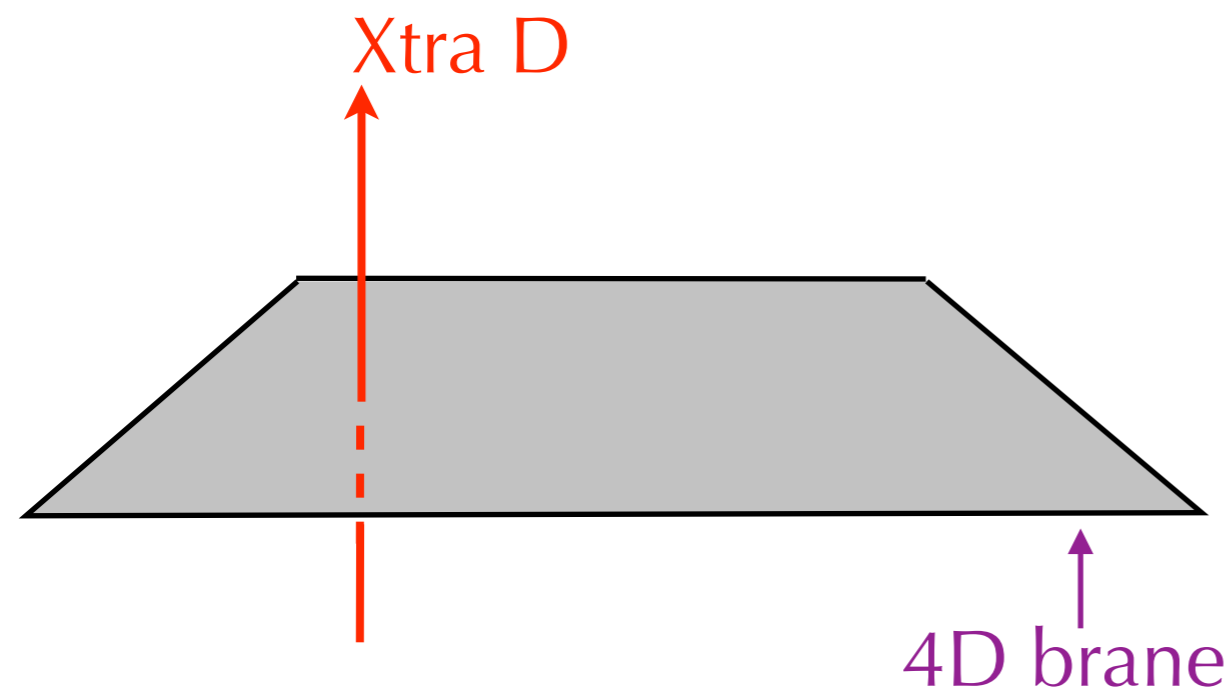
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- ✓ matter stays on the brane
- ✓ 5D + 4D gravity
- ✓ can provide an explanation for the cosmic acceleration

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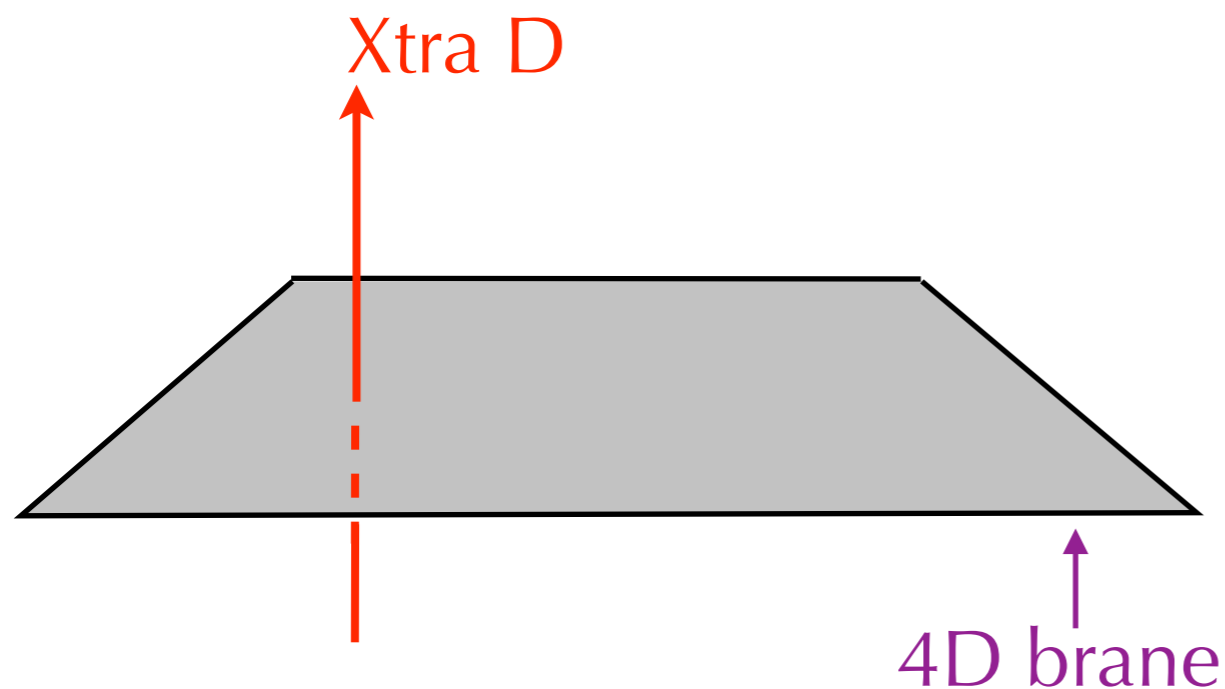
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✓ matter stays on the brane

✓ 5D + 4D gravity

✓ can provide an explanation for the cosmic acceleration

✓ 5D massless graviton = infinite tower of 4D gravitons

Dvali, Gabadadze and Porrati '00

Publications

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- [E. Babichev, C. Deffayet, RZ](#), JHEP 05 (2009) 098 [[arXiv:0901.0393](#)]
The Vainshtein mechanism in the Decoupling Limit of massive gravity
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Massive gravity & the Vainshtein's mechanism

Context & Motivations

- Naive idea: give a mass to the graviton with $m \sim 1/H_0$
- Massive gravity = basic ingredient for models with extra-dimensions (like DGP):
 - in 5D, the gravity is massless,
 - viewed from the 4D brane, the gravity is mediated by a tower a massive Kaluza-Klein gravitons with no massless mode.
- Pathologies: non-bounded by below Hamiltonian and ghosts [Boulware & Deser '72; Creminelli, Nicolis, Papucci, Trincherini '05; Deffayet & Rombouts '05], singular solutions (?) [Damour et al. '03], but interesting toy model for more realistic theories.

What is Massive gravity?

What is Massive gravity?

- The quadratic action for the massive graviton:

f : background metric (often flat) $H_{\mu\nu}$: spin 2 excitation over f

$$S = \frac{M_P^2}{2} \int d^4x \left(\underbrace{\text{“}H\partial^2 H + \dots\text{”}}_{\text{Kinetic term}} - \frac{m^2}{4} \underbrace{[H_{\mu\nu}H^{\mu\nu} - (H^\mu_\mu)^2]}_{\substack{\text{Pauli-Fierz} \\ \text{Mass term}}} \right) + \int d^4x \frac{1}{2} \underbrace{T_{\mu\nu}H^{\mu\nu}}_{\text{Matter coupling}}$$

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- Non-linear completion: dynamical metric $g = f + H$

$$S = \frac{M_P^2}{2} \int d^4x \left(\sqrt{-g}R[g] - \frac{m^2}{4} \underbrace{\mathcal{V}^{(a)}[\mathbf{g}^{-1}\mathbf{f}]}_{\text{Scalar density}} \right) + S_m[g]$$

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- Examples:

$$\mathcal{V}^{(BD)}[\mathbf{g}^{-1}\mathbf{f}] = \sqrt{-f} H_{\mu\nu}H_{\sigma\tau} (f^{\mu\sigma}f^{\nu\tau} - f^{\mu\nu}f^{\sigma\tau})$$

Boulware and Deser '72

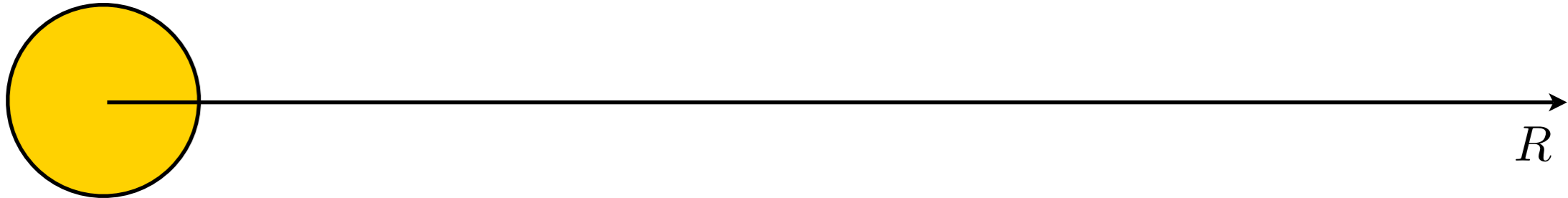
$$\mathcal{V}^{(AGS)}[\mathbf{g}^{-1}\mathbf{f}] = \sqrt{-g} H_{\mu\nu}H_{\sigma\tau} (g^{\mu\sigma}g^{\nu\tau} - g^{\mu\nu}g^{\sigma\tau})$$

Arkani-Hamed, Georgi and Schwartz '03

Static Spherically Symmetric solutions



Static Spherically Symmetric solutions

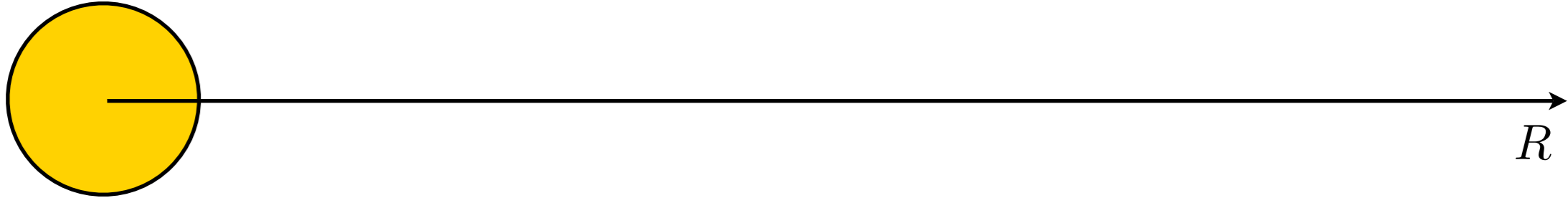


- “Schwarzschild Gauge” (bi-diagonal, asymptotically flat):

$$g_{\mu\nu} dx^\mu dx^\nu = -e^{\nu(R)} dt^2 + e^{\lambda(R)} dR^2 + R^2 d\Omega^2 \quad : \text{“Schwarzschild” like}$$

$$f_{\mu\nu} dx^\mu dx^\nu = -dt^2 + \left(1 - \frac{R\mu'(R)}{2}\right)^2 e^{-\mu(R)} dR^2 + e^{-\mu(R)} R^2 d\Omega^2 \quad : \text{flat}$$

Static Spherically Symmetric solutions



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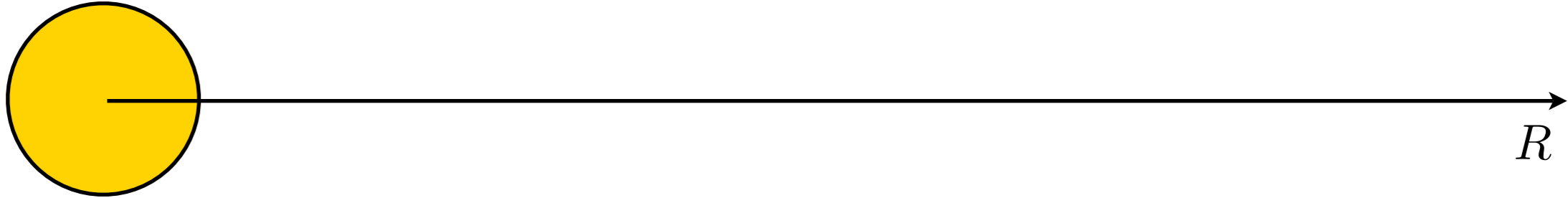
- Equations of motion:

$$e^{\nu-\lambda} \left(\frac{\lambda'}{R} + \frac{1}{R^2} (e^\lambda - 1) \right) = 8\pi G_N (T_{tt}^g + \rho e^\nu),$$

$$\frac{\nu'}{R} + \frac{1}{R^2} (1 - e^\lambda) = 8\pi G_N (T_{RR}^g + P e^\lambda),$$

$$\nabla^\mu T_{\mu R}^g = 0.$$

Static Spherically Symmetric solutions



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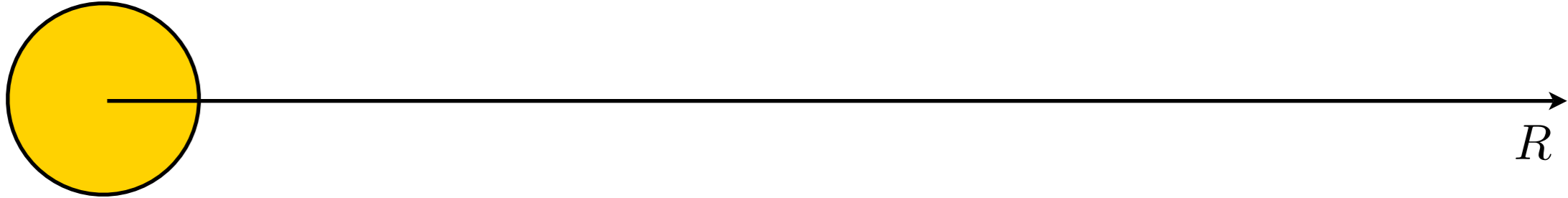
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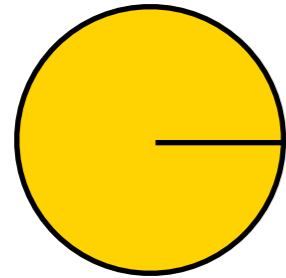
$$\begin{aligned} e^{\nu-\lambda} \left(\frac{\lambda'}{R} + \frac{1}{R^2} (e^\lambda - 1) \right) &= 8\pi G_N (T_{tt}^g + \rho e^\nu), \\ \frac{\nu'}{R} + \frac{1}{R^2} (1 - e^\lambda) &= 8\pi G_N (T_{RR}^g + P e^\lambda), \\ \nabla^\mu T_{\mu R}^g &= 0. \end{aligned}$$

Is it possible to
find a solution
regular
everywhere?

Solution far from the source (I)



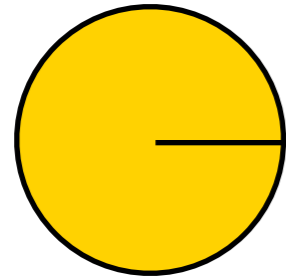
Solution far from the source (I)



Linear massive gravity

R

Solution far from the source (I)



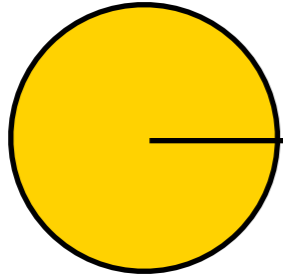
Linear massive gravity

R

- Expansion in the Newton's constant:

$$\lambda = \lambda_0 + \lambda_1 + \dots \text{ etc. , with } \lambda_i, \nu_i, \mu_i \propto G_N^{i+1}$$

Solution far from the source (I)



Linear massive gravity

- Expansion in the Newton's constant:

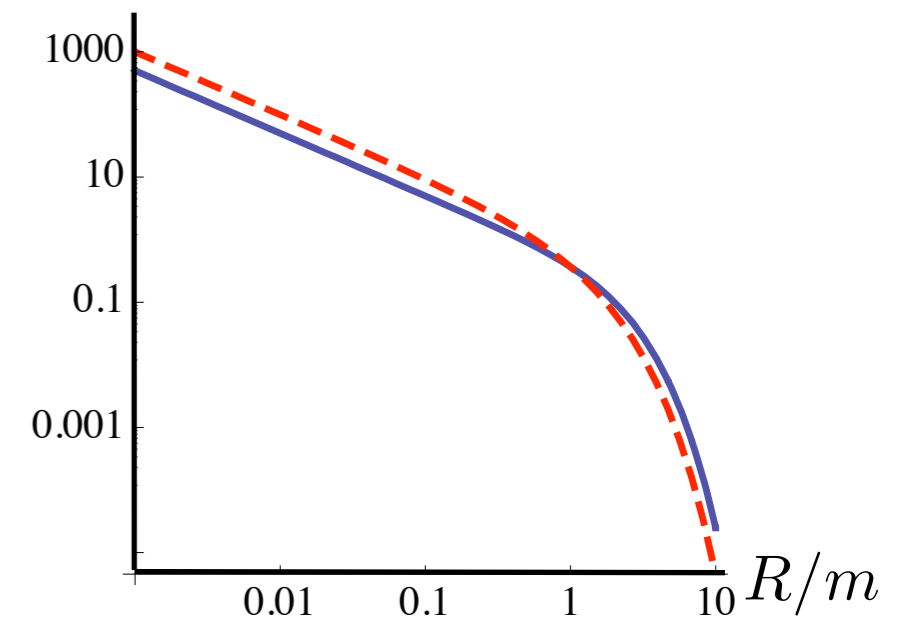
$$\lambda = \lambda_0 + \lambda_1 + \dots \text{ etc. , with } \lambda_i, \nu_i, \mu_i \propto G_N^{i+1}$$

- Linearized solution:

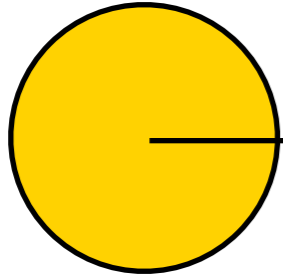
$$\nu_0 = -\mathcal{C} \times \frac{R_S}{R} e^{-mR}$$

$$\lambda_0 = \mathcal{C} \times \frac{R_S}{2R} (1 + mR) e^{-mR}$$

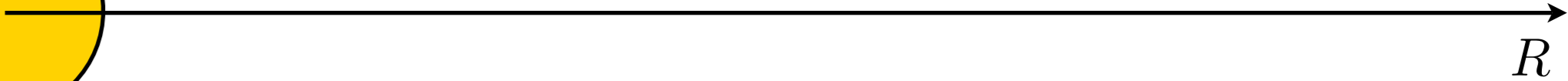
$$\mu_0 = \mathcal{C} \times \frac{R_S}{2(mR)^2 R} (1 + mR + (mR)^2) e^{-mR}$$



Solution far from the source (I)



Linear massive gravity



- Expansion in the Newton's constant:

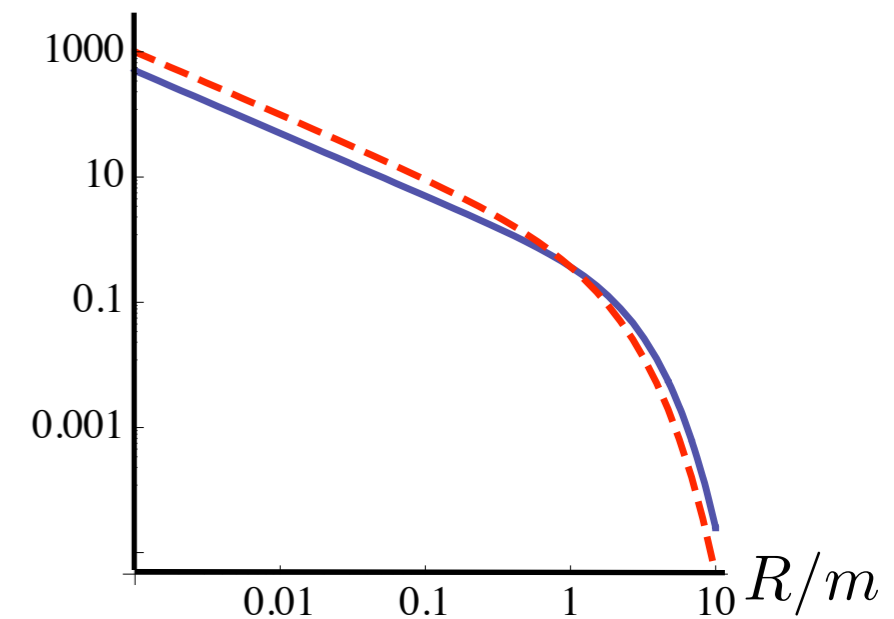
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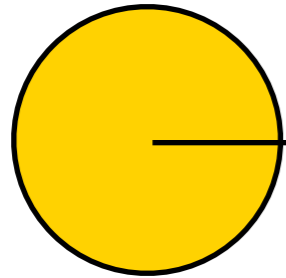
$$\lambda_0 = C \times \frac{R_S}{2R} (1 + mR) e^{-mR}$$

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- for $R \ll m^{-1}$: $\lambda_0 = \frac{C_1}{2R}$, $\nu_0 = -\frac{C_1}{R}$, $\mu_0 = \frac{1}{(mR)^2} \frac{C_1}{2R}$.

Solution far from the source (I)



Linear massive gravity

Non GR

R

- Expansion in the Newton's constant:

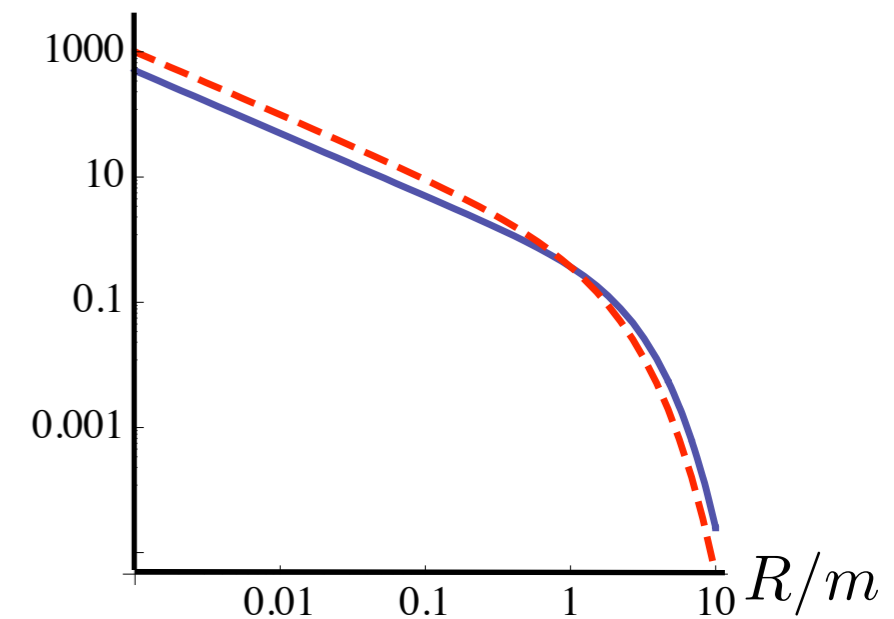
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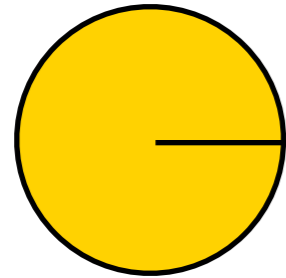
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vDVZ
discontinuity

Solution far from the source (II)



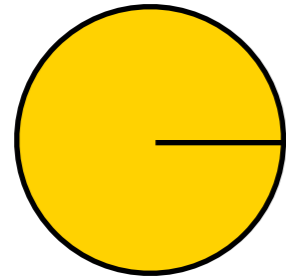
Linear massive gravity

Non GR

R



Solution far from the source (II)



Linear massive gravity

Non GR

R

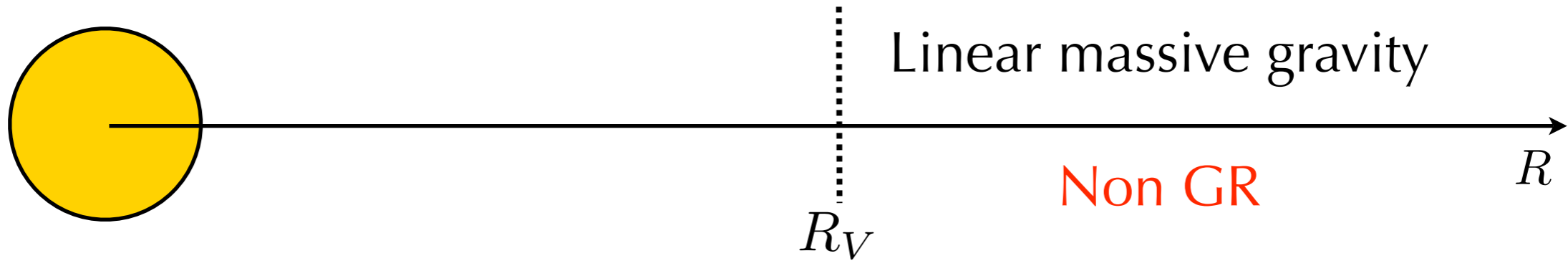
- Nonlinear corrections:

$$\nu = -\frac{2 R_S}{3 R} + \frac{R_S^2}{R^2} \frac{n_1}{(mR)^4} + \mathcal{O}(R_S^3)$$

$$\lambda = \frac{1 R_S}{3 R} + \frac{R_S^2}{R^2} \frac{l_1}{(mR)^4} + \mathcal{O}(R_S^3)$$

$$\mu = \frac{1}{3(mR)^2} \frac{R_S}{R} + \frac{R_S^2}{R^2} \frac{m_1}{(mR)^6} + \mathcal{O}(R_S^3)$$

Solution far from the source (II)



- Nonlinear corrections:

Vainshtein '72

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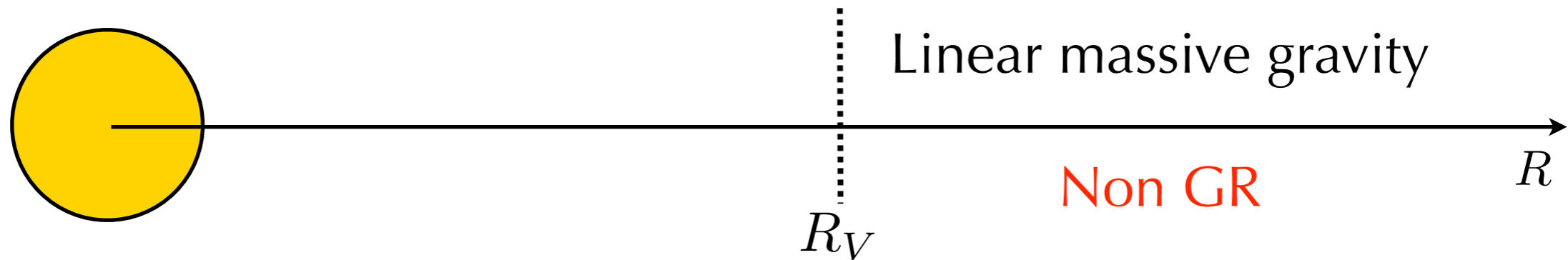
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Relevant at

$$R_V = \left(\frac{R_S}{m^4} \right)^{1/5}$$

New scale:
Vainshtein's radius

Solution far from the source (II)



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Vainshtein '72

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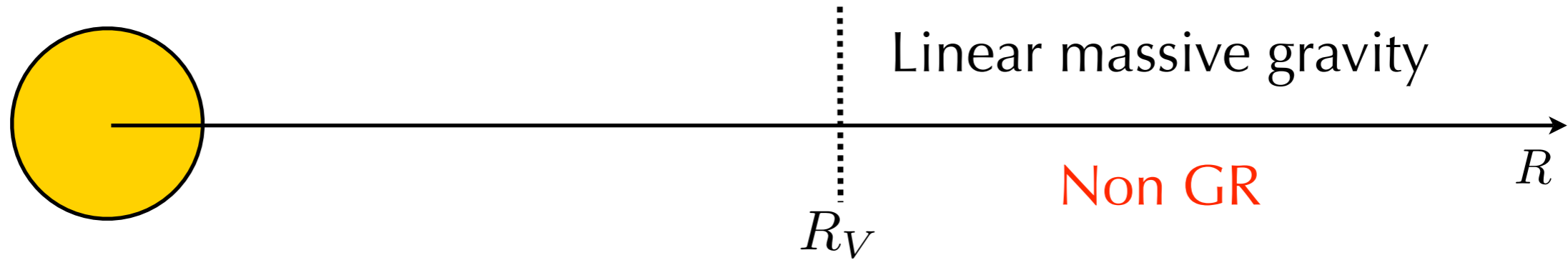
Relevant at

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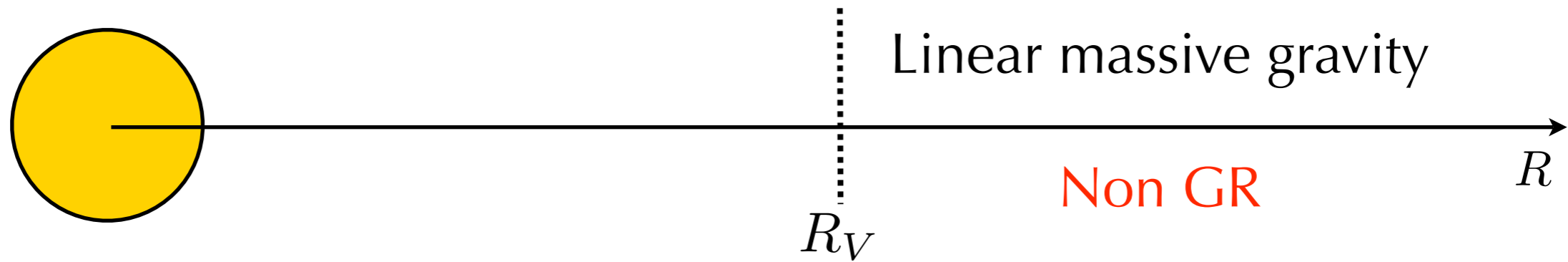
New scale:
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What happens inside the Vainshtein's radius?

Solution close to the source

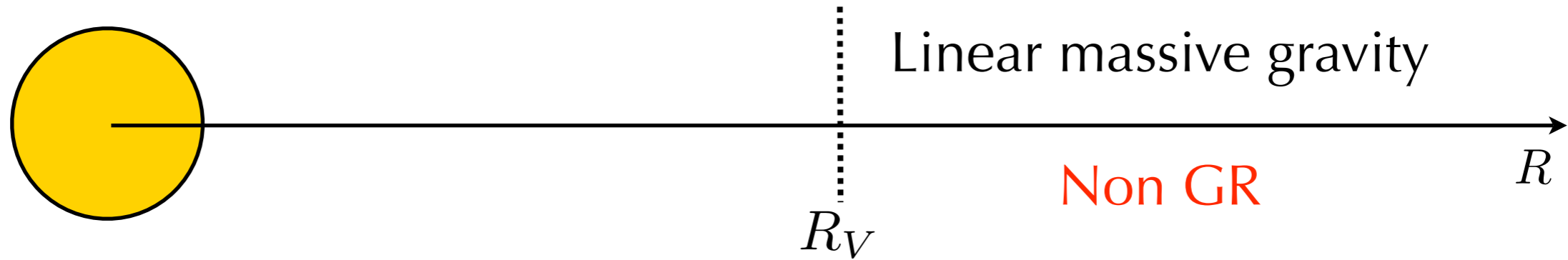


Solution close to the source



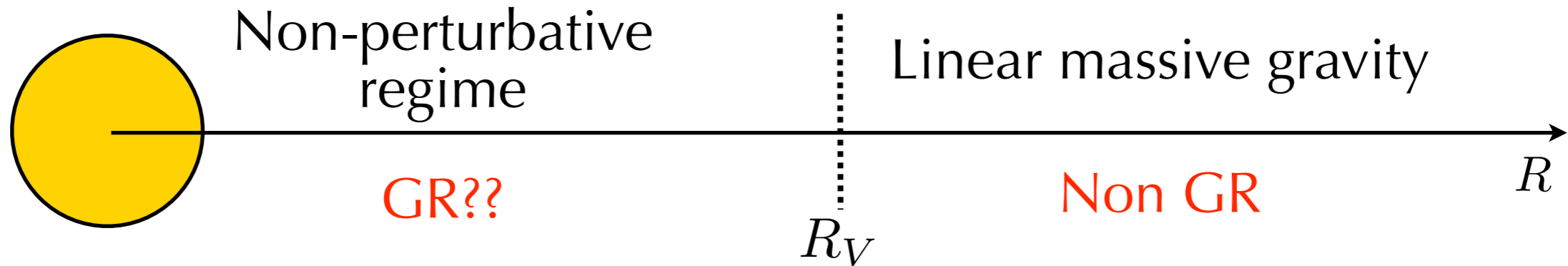
- The functions λ, ν, μ are expanded in m :
$$f(R) = \sum_{n=0}^{\infty} m^{2n} f_n(R)$$

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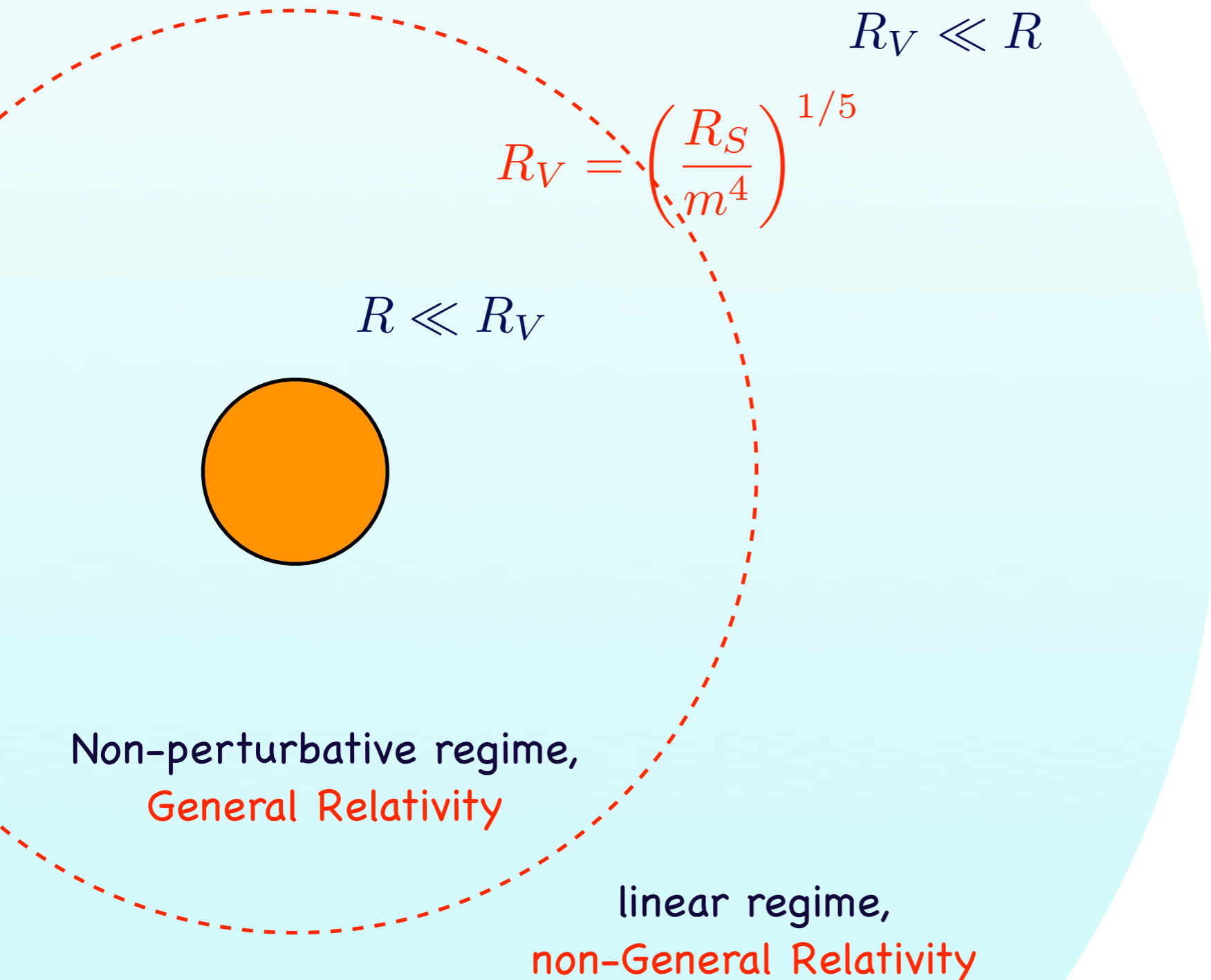
$$\mu = m_0 \sqrt{\frac{R_S}{R}} + m_1 (mR)^2 + \mathcal{O}(m^4)$$

Vainshtein '72

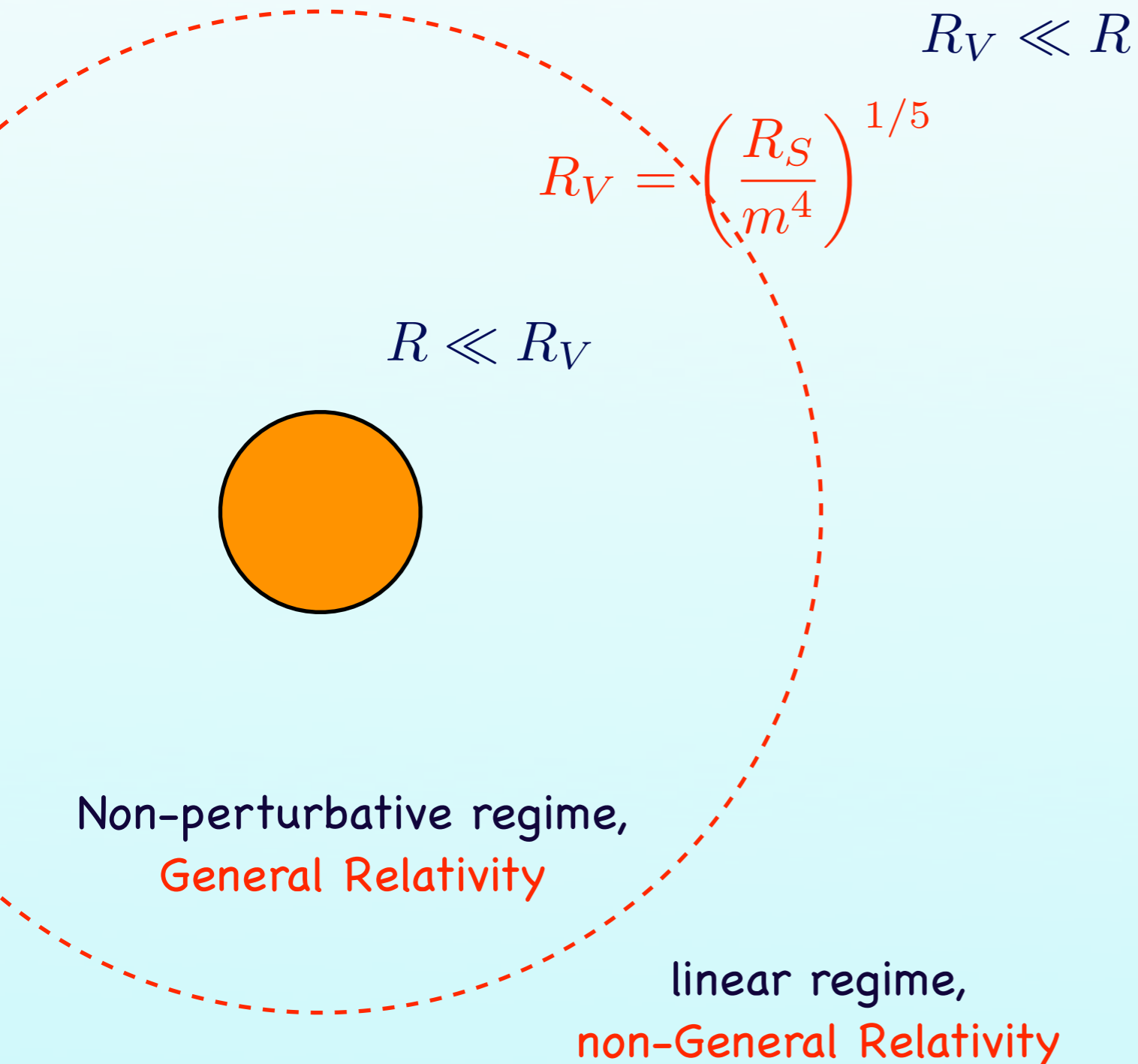
Relevant at

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Existence of a regular solution?



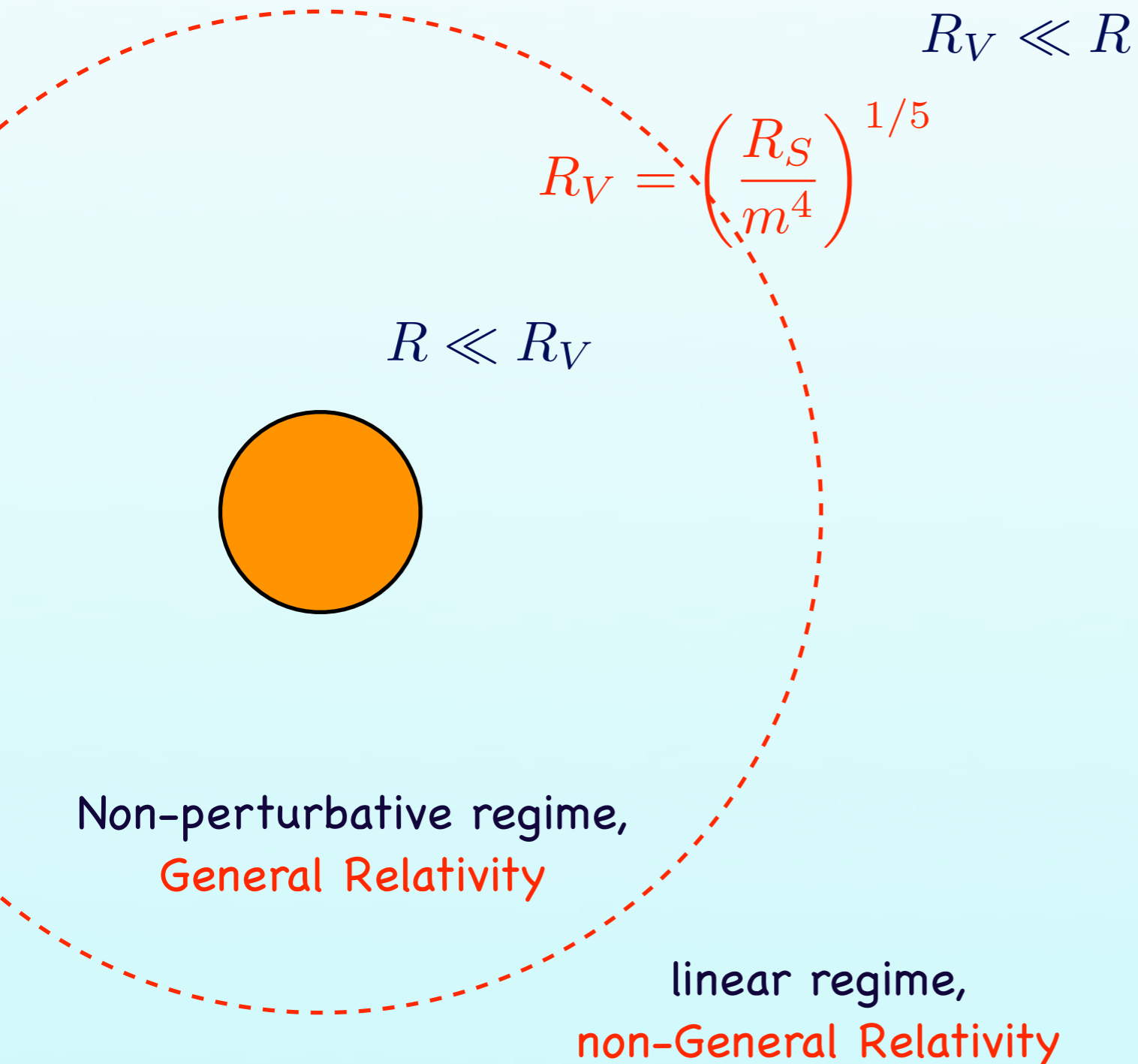
Existence of a regular solution?



Is the Vainshtein's mechanism valid?

Is it possible to find a solution regular everywhere?

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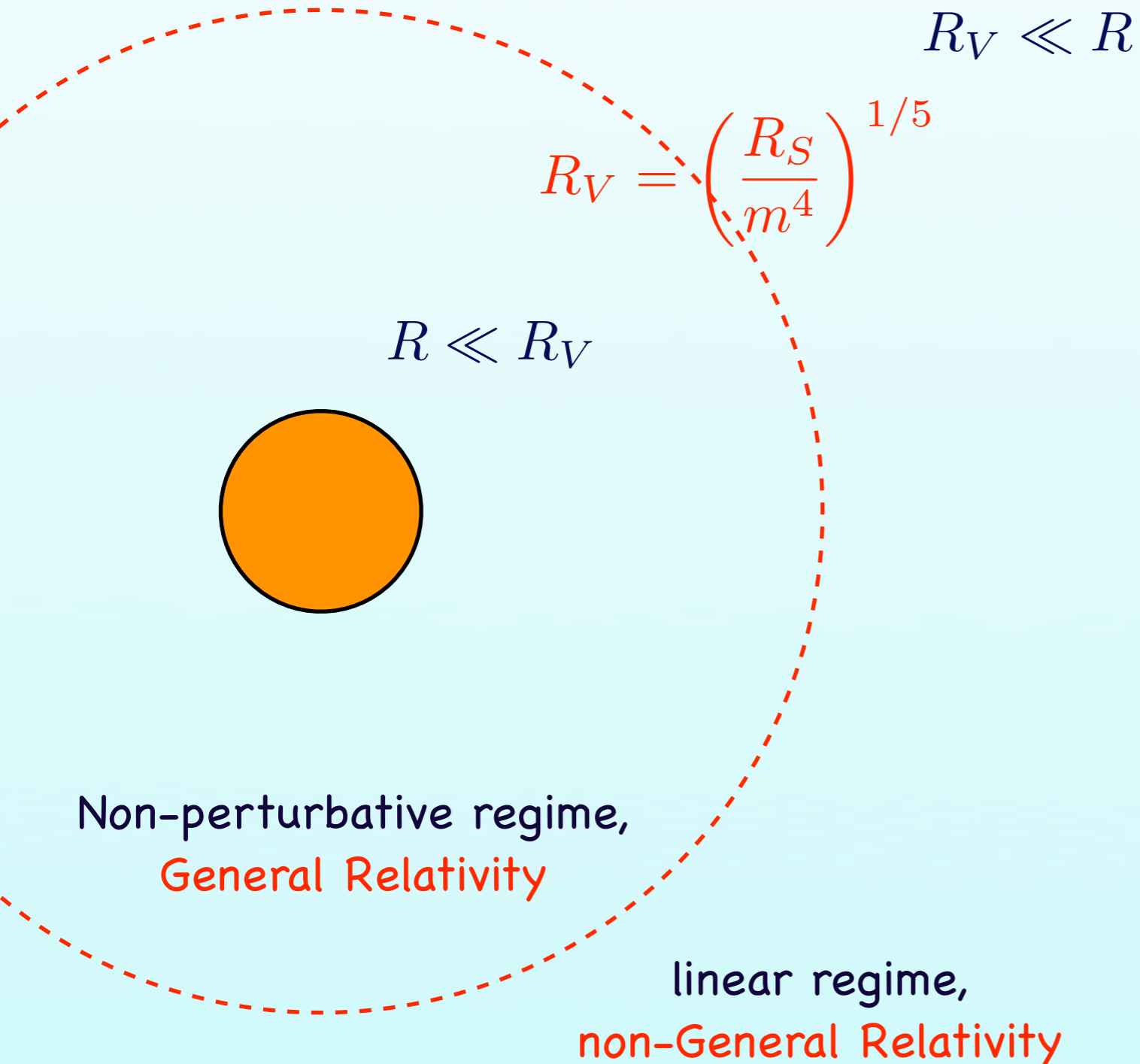


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*Vainshtein '72,
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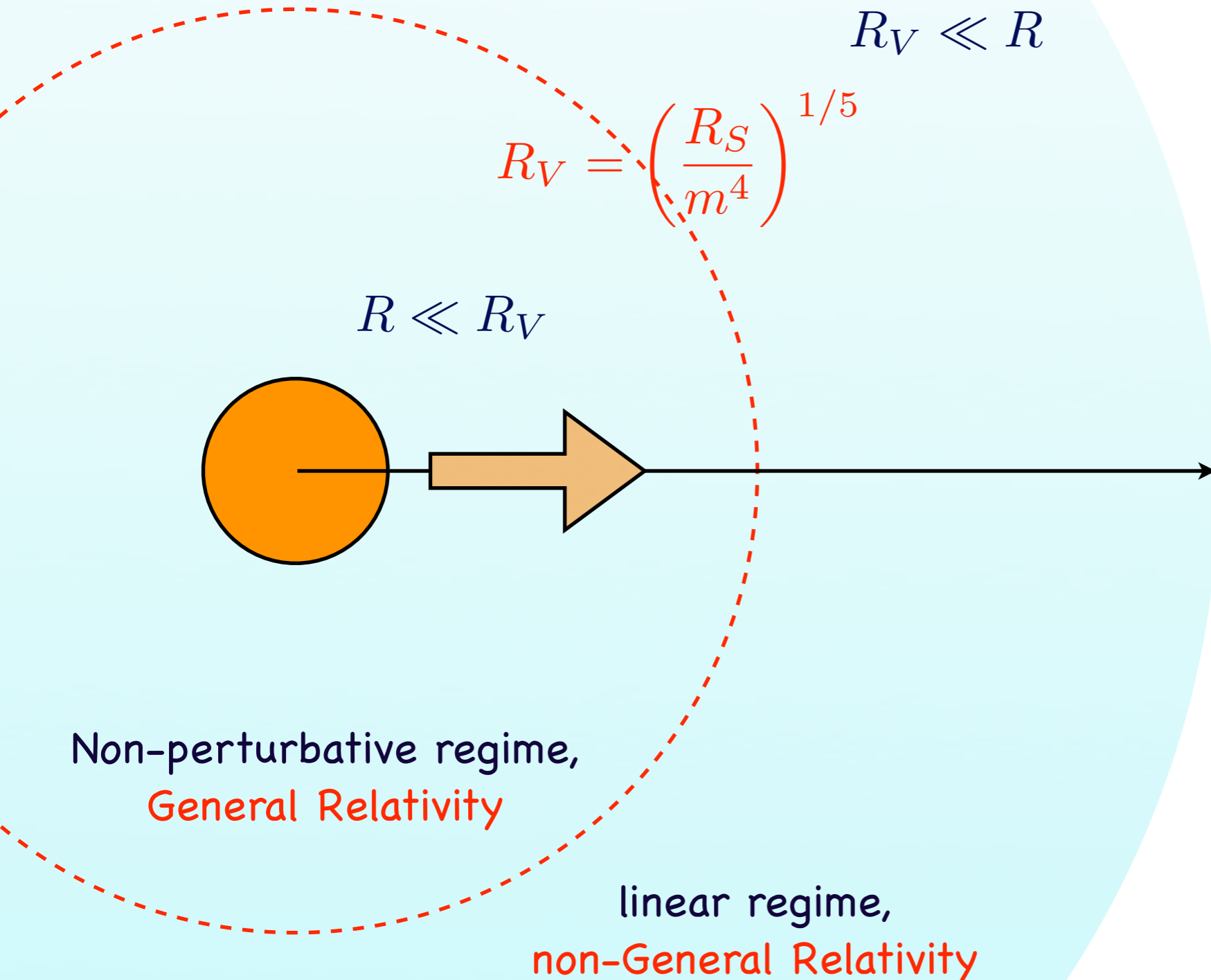
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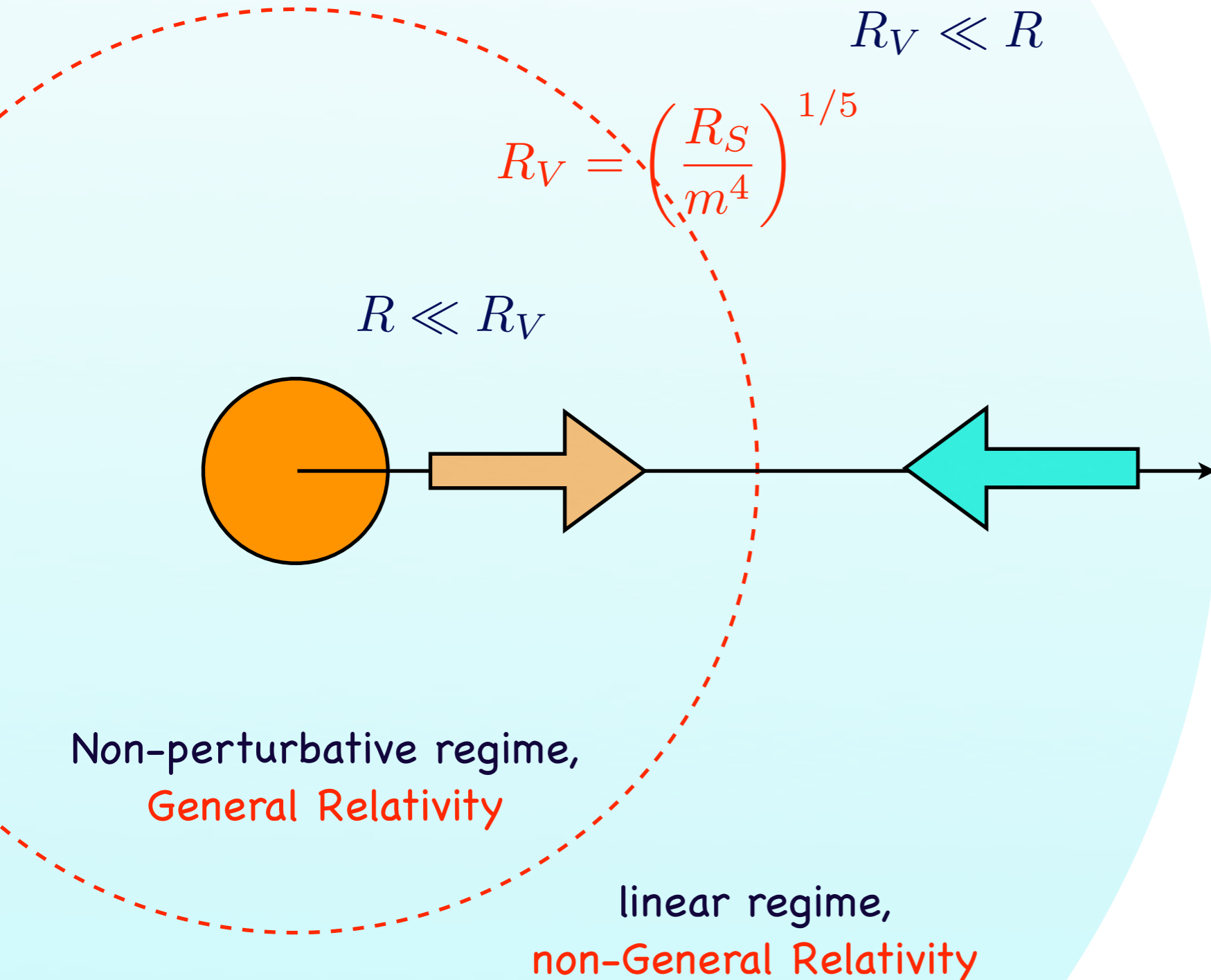
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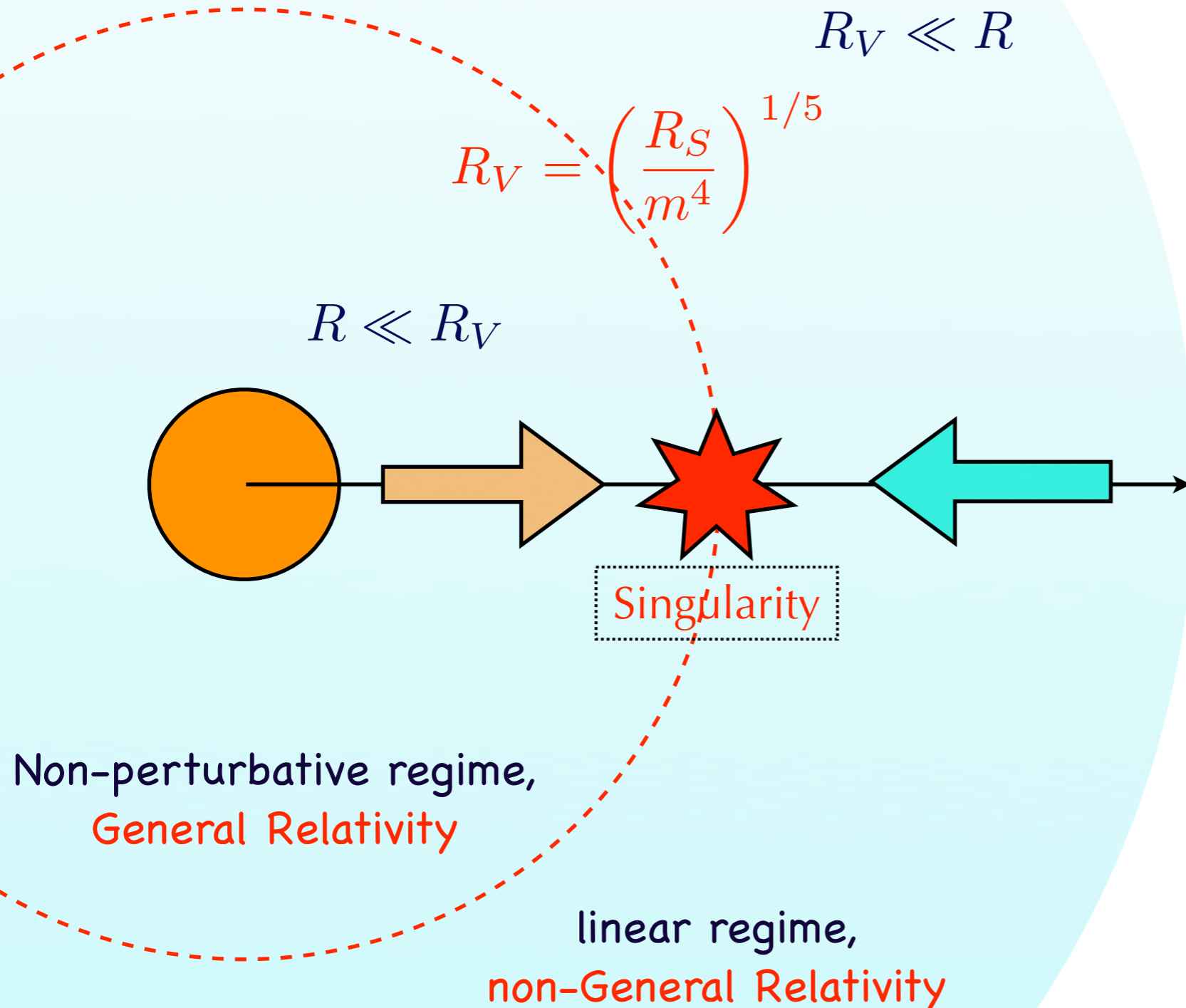
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Our approach

Our approach

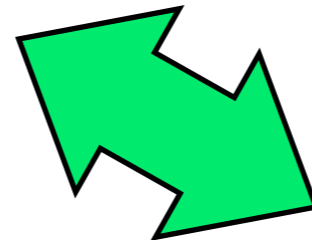
Decoupling Limit

*[E. Babichev, C. Deffayet, RZ,](#)
JHEP 05 (2009) 098*

Our approach

Decoupling Limit

*[E. Babichev, C. Deffayet, RZ,](#)
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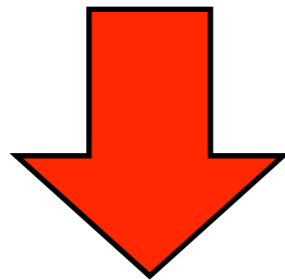


Analytical
expansions
+
adapted numerical
methods

Our approach

Decoupling Limit

*[E. Babichev, C. Deffayet, RZ,](#)
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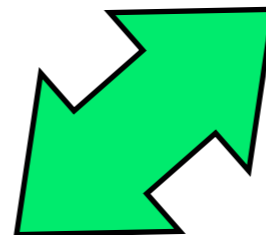
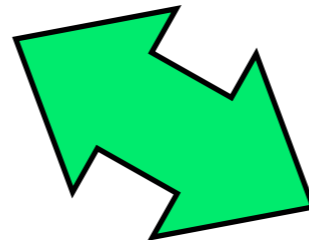


General case

*[E. Babichev, C. Deffayet, RZ,](#)
PRL 103 (2009) 201102*

*[E. Babichev, C. Deffayet, RZ,](#)
in preparation (2010)*

Analytical
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Decoupling Limit

$$\begin{aligned} M_P &\rightarrow \infty \\ m &\rightarrow 0 \\ \Lambda \equiv (M_P m^4)^{1/5} &\sim \text{const} \\ T_{\mu\nu}/M_P &\sim \text{const} \end{aligned}$$

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Arkani-Hamed, Georgi and Schwartz '03

Other point of view:

$$\begin{aligned} h_{\mu\nu} \rightarrow & h_{\mu\nu} - \partial_\mu A_\nu - \partial_\nu A_\mu - 2\partial_\mu \partial_\nu \phi \\ & - \partial_\mu A_\sigma \partial_\nu A^\sigma - \partial_\mu \partial_\sigma \phi \partial_\nu \partial^\sigma \phi \\ & - \partial_\nu A^\sigma \partial_\mu \partial_\sigma \phi - \partial_\mu A^\sigma \partial_\nu \partial_\sigma \phi \end{aligned}$$

$\Lambda =$ strong coupling scale of ϕ

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$$\frac{\nu'}{R} + \frac{1}{R^2} (1 - e^\lambda) = 8\pi G_N (T_{RR}^g + P e^\lambda),$$

$$\nabla^\mu T_{\mu R}^g = 0.$$

Equations of motion

$$g_{\mu\nu} dx^\mu dx^\nu = -e^\nu dt^2 + e^\lambda dR^2 + R^2 d\Omega^2$$

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• but $T_{tt}^g = m^2 M_P^2 f_t$, $T_{RR}^g = m^2 M_P^2 f_R$, $\nabla^\mu T_{\mu R}^g = -m^2 M_P^2 f_g$,

with

$$f_t = \frac{e^{-\lambda-2\mu}}{4}$$

$$\times \left[(3e^{\mu+\nu} + e^\mu - 2e^\nu) \left(1 - \frac{R\mu'}{2}\right)^2 + e^\lambda (2e^\mu - e^\nu) - 3e^{\lambda+\mu} (2e^{\mu+\nu} + e^\mu - 2e^\nu) \right],$$

$$f_R = \frac{e^{-\nu-2\mu}}{4}$$

$$\times \left[(3e^{\mu+\nu} - e^\mu - 2e^\nu) \left(1 - \frac{R\mu'}{2}\right)^2 + e^\lambda (2e^\mu + e^\nu) - 3e^{\lambda+\mu} (-2e^{\mu+\nu} + e^\mu + 2e^\nu) \right],$$

$$f_g = - \left(1 - \frac{R\mu'}{2}\right) \frac{e^{-\lambda-2\mu-\nu}}{8R}$$

$$\times \left[8(e^\lambda - 1) (3e^{\mu+\nu} - e^\mu - e^\nu) + 2R ((3e^{\mu+\nu} - 2e^\nu) (\lambda' + 4\mu' - \nu') - e^\mu (\lambda' + 4\mu' + \nu')) \right. \\ \left. - R^2 \left((3e^{\mu+\nu} - 2e^\nu) (\lambda'\mu' - 2\mu'' - \mu'\nu' + (\mu')^2) - e^\mu (\lambda'\mu' - 2\mu'' + \mu'\nu' + (\mu')^2) - 2e^\nu (\mu')^2 \right) \right].$$

For AGS
potential

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$$\times \left[8(e^\lambda - 1) (3e^{\mu+\nu} - e^\mu - e^\nu) + 2R ((3e^{\mu+\nu} - 2e^\nu) (\lambda' + 4\mu' - \nu') - e^\mu (\lambda' + 4\mu' + \nu')) \right. \\ \left. - R^2 \left((3e^{\mu+\nu} - 2e^\nu) (\lambda'\mu' - 2\mu'' - \mu'\nu' + (\mu')^2) - e^\mu (\lambda'\mu' - 2\mu'' + \mu'\nu' + (\mu')^2) - 2e^\nu (\mu')^2 \right) \right].$$

Simplification of the equations

For AGS potential

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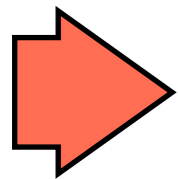
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$$\frac{\lambda'}{R} + \frac{\lambda}{R^2} = -\frac{m^2}{2} (3\mu + R\mu') + \frac{\rho}{M_P^2}$$

$$\frac{\nu'}{R} - \frac{\lambda}{R^2} = m^2 \mu$$

$$\frac{\lambda}{R^2} = \frac{\nu'}{2R} + Q(\mu),$$

$$\text{with } Q = -\frac{1}{2} \left(\frac{\mu'^2}{2} + \mu\mu'' + \frac{4\mu\mu'}{R} \right)$$

*E. Babichev, C. Deffayet, RZ,
JHEP 05 (2009) 098*

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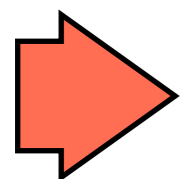
$$\frac{\lambda'}{R} + \frac{\lambda}{R^2} = -\frac{m^2}{2} (3\mu + R\mu') + \frac{\rho}{M_P^2}$$

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$$\frac{\lambda}{R^2} = \frac{\nu'}{2R} + Q(\mu),$$

with $Q = -\frac{1}{2} \left(\frac{\mu'^2}{2} + \mu\mu'' + \frac{4\mu\mu'}{R} \right)$

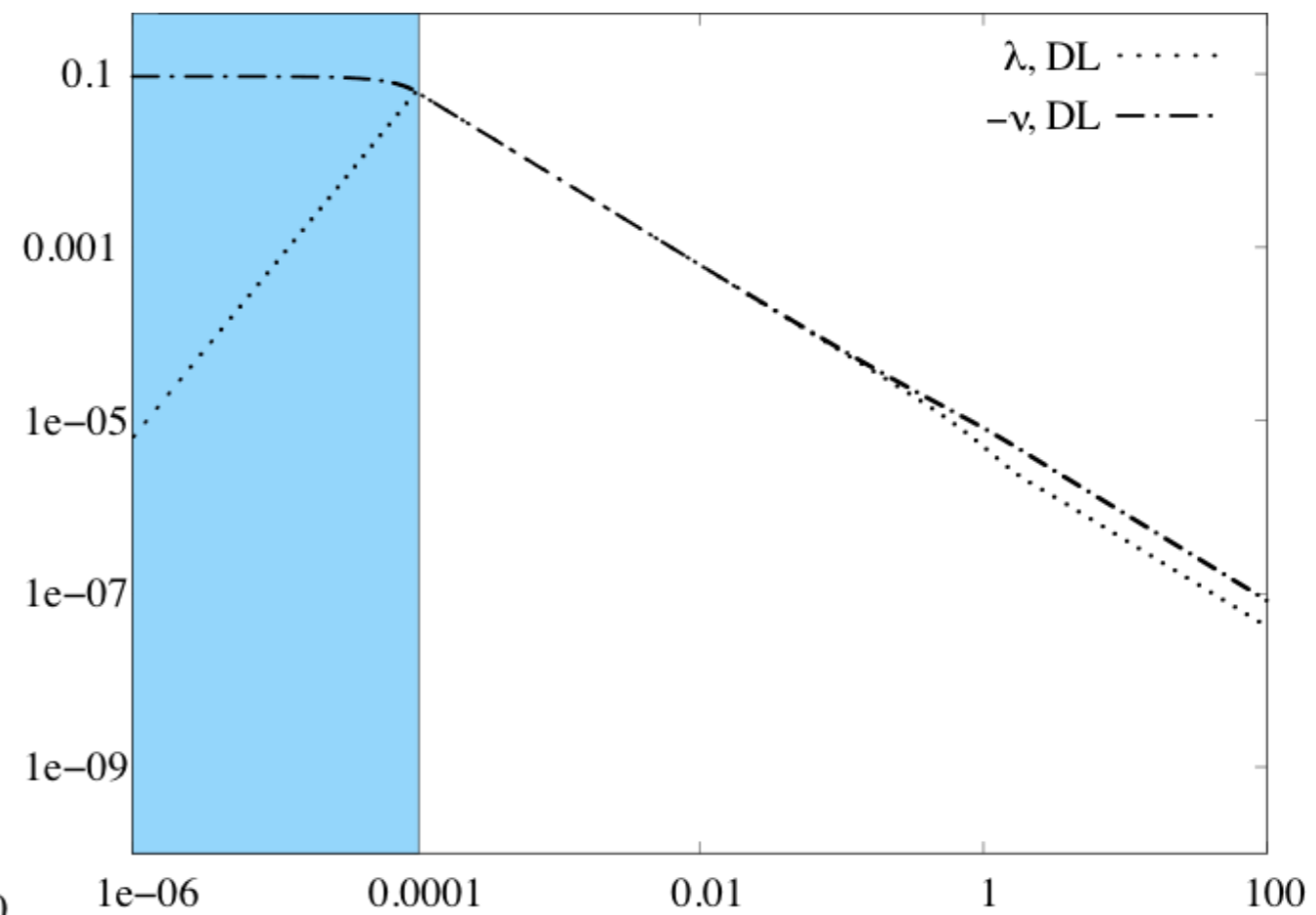
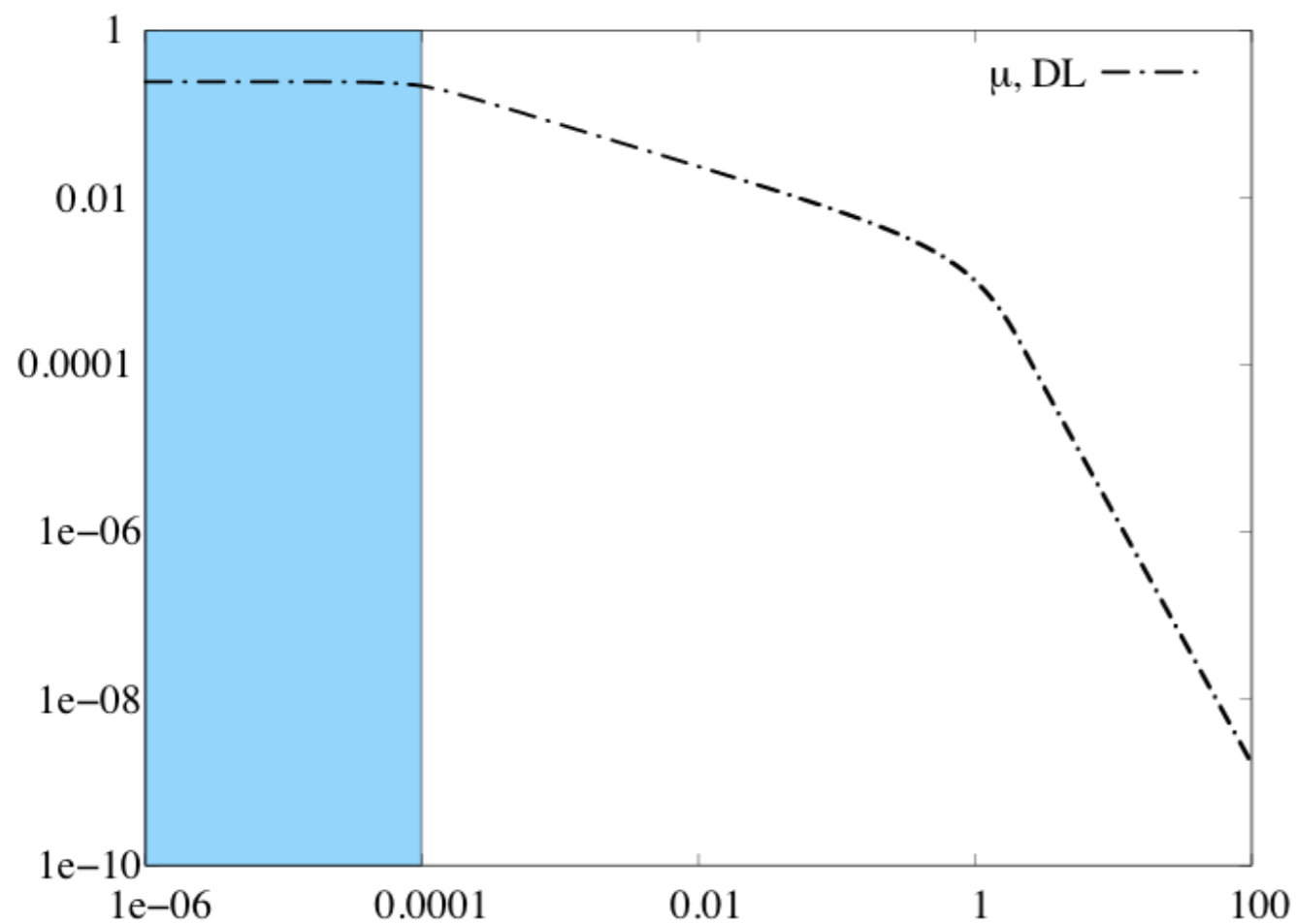
*E. Babichev, C. Deffayet, RZ,
JHEP 05 (2009) 098*



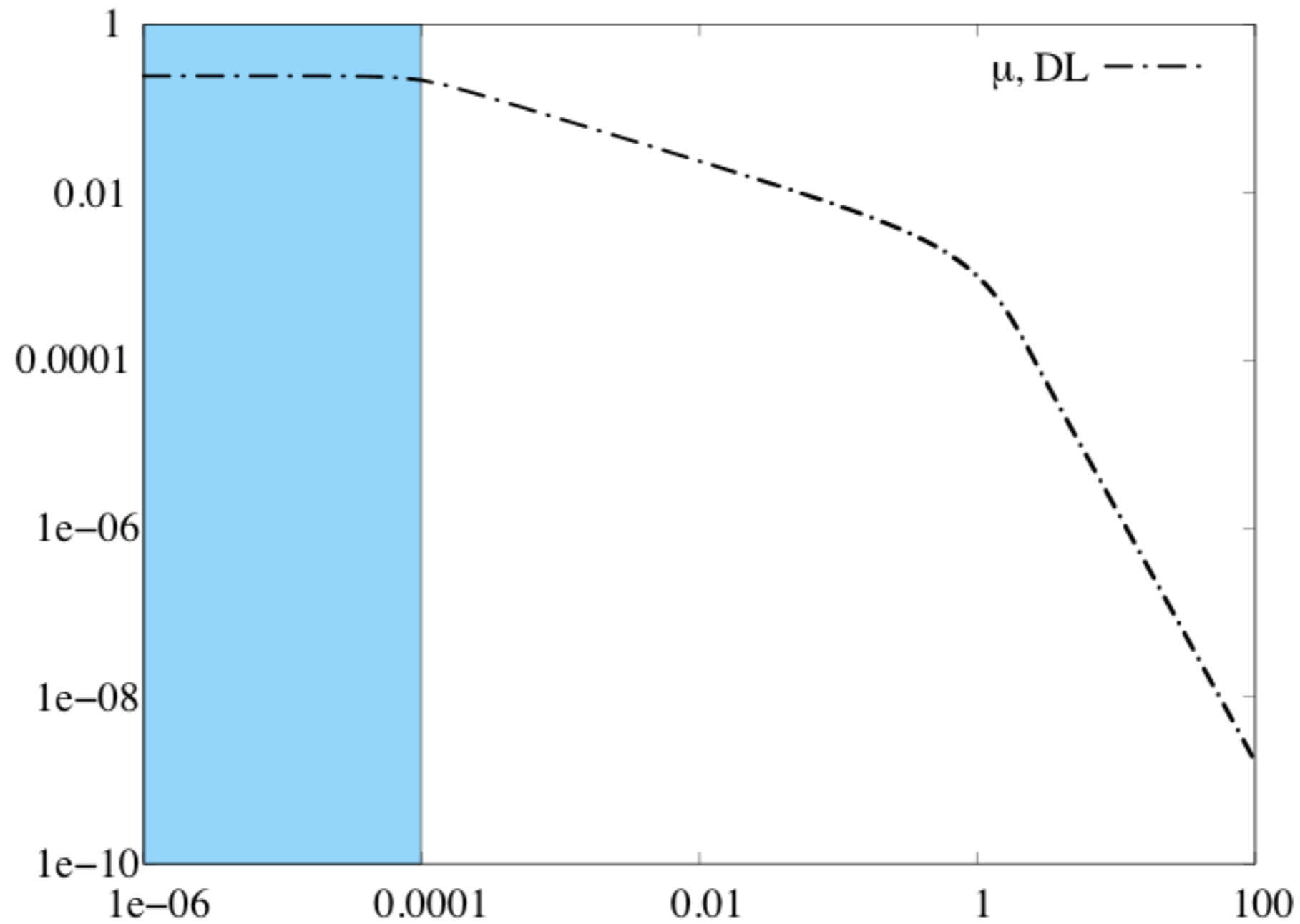
$$2Q(\mu) + \frac{3}{2} m^2 \mu = \frac{R_S}{R^3}$$

Only one equation!

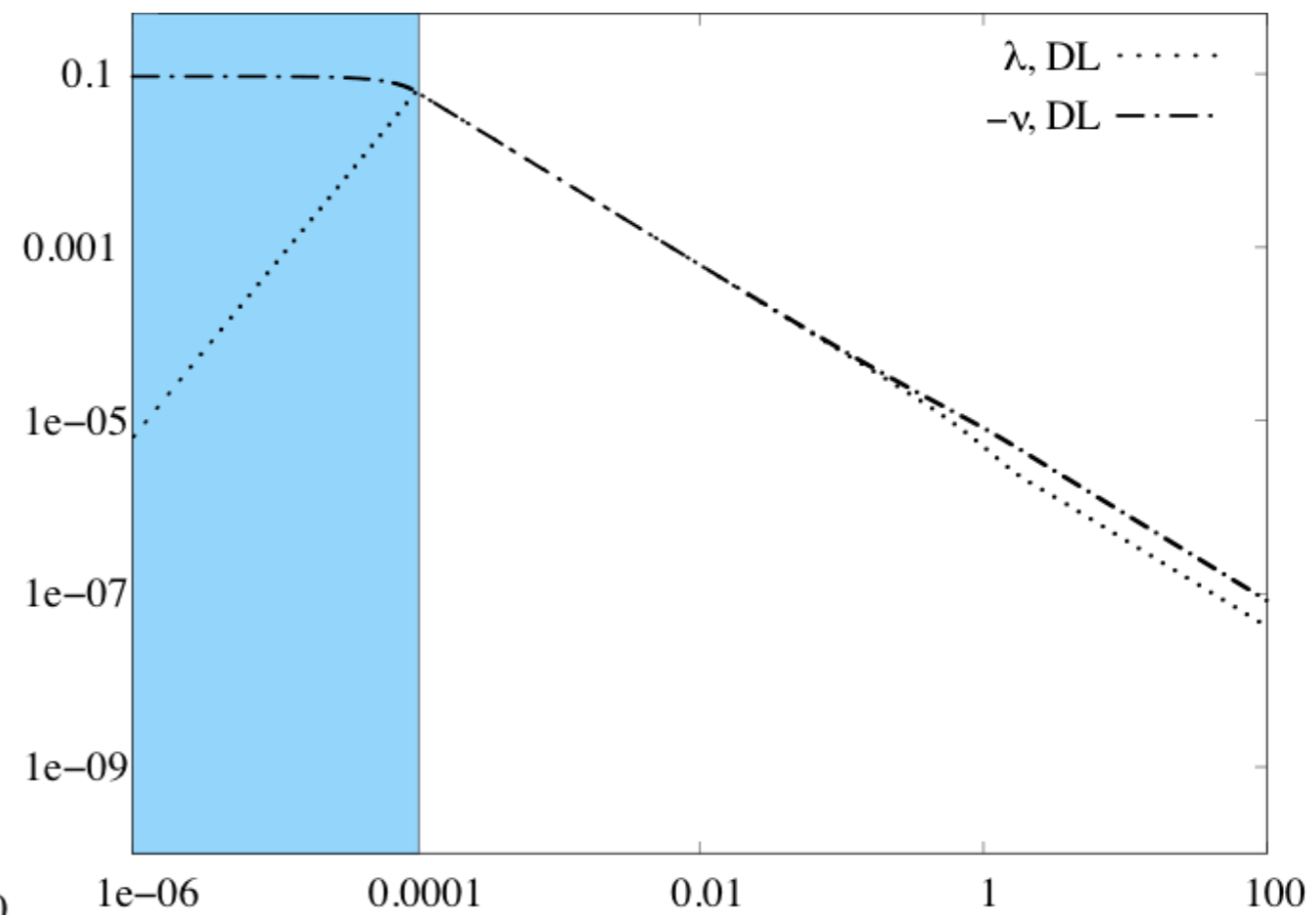
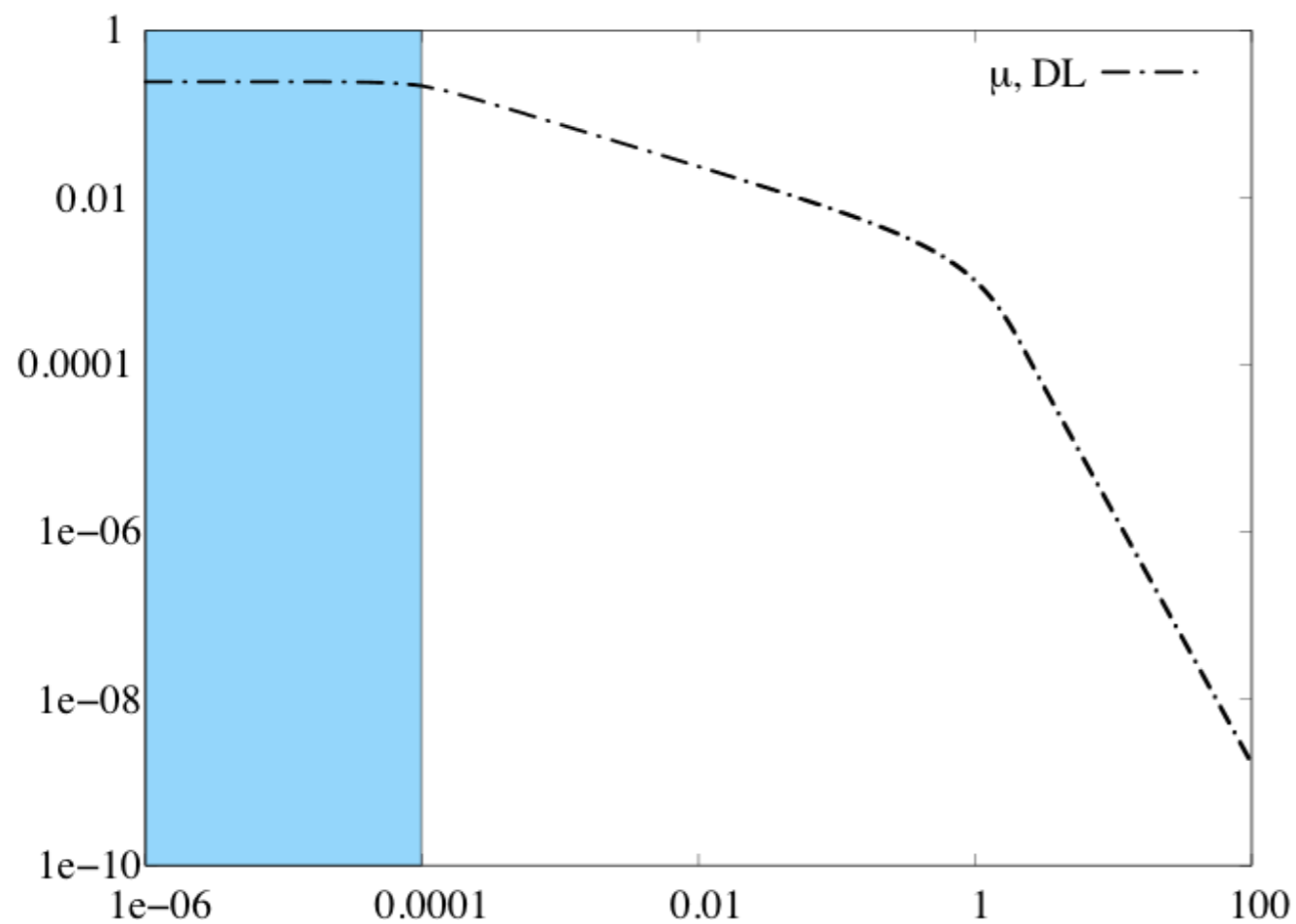
The Decoupling Limit: solution



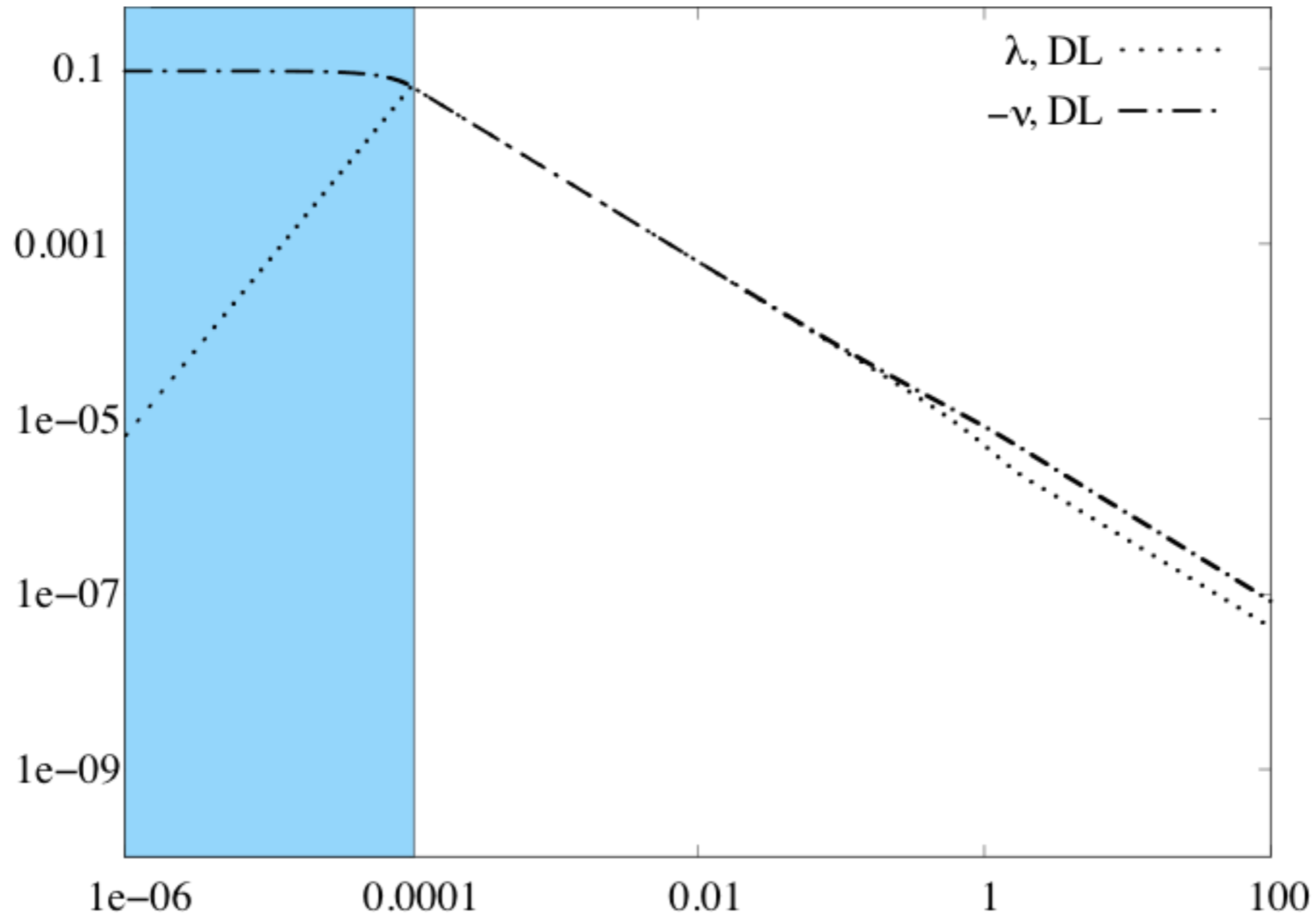
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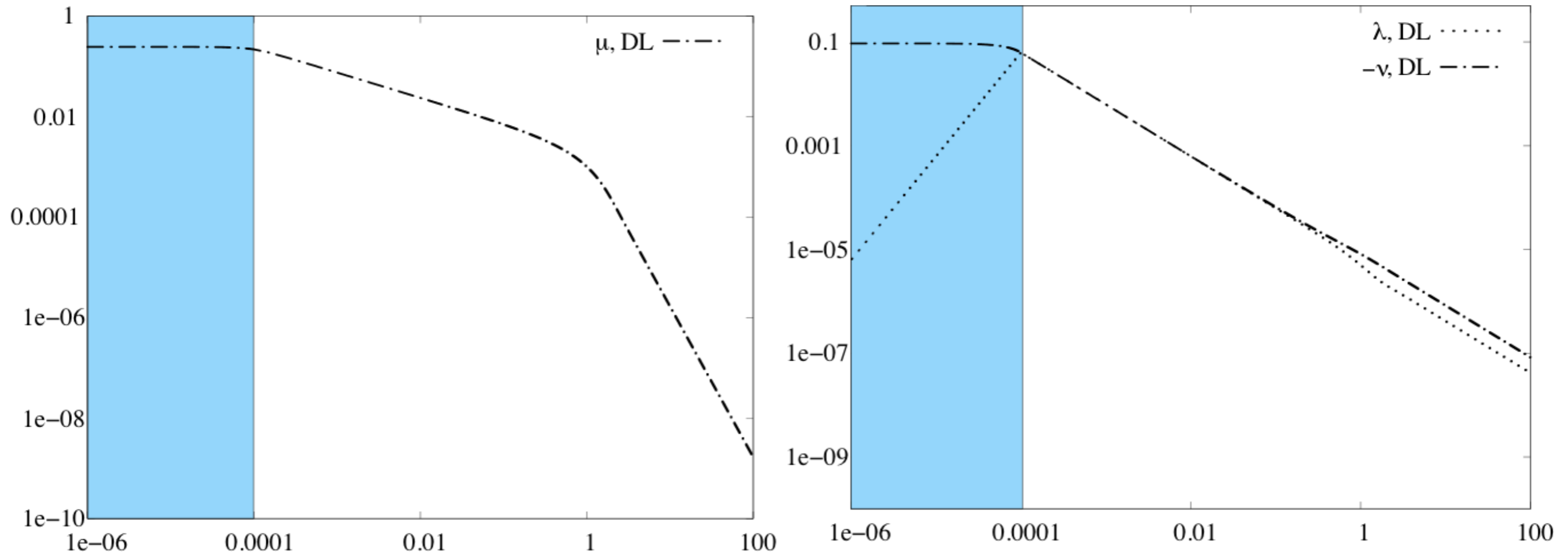
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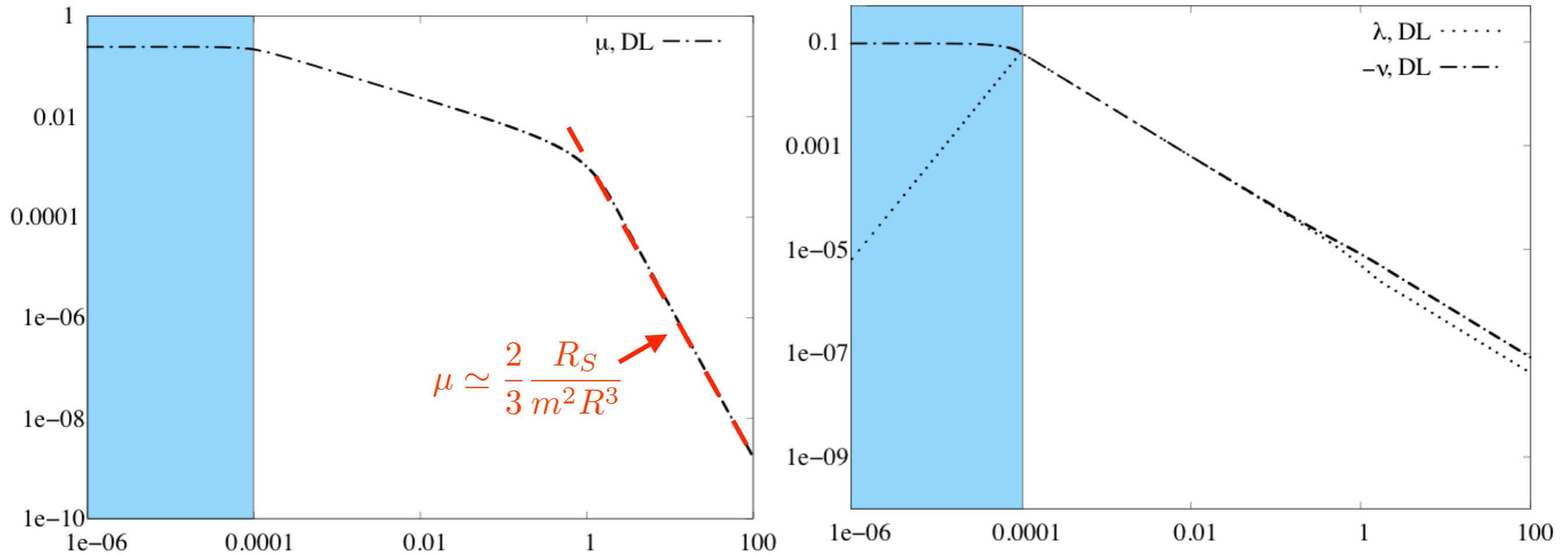
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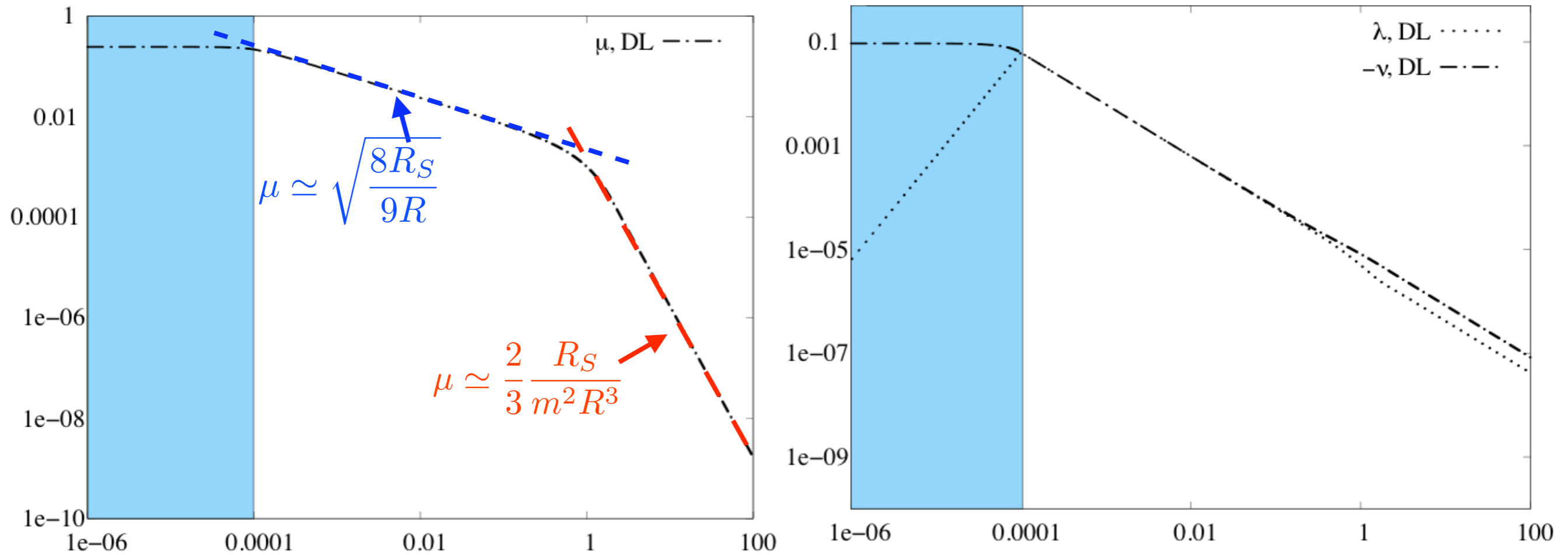
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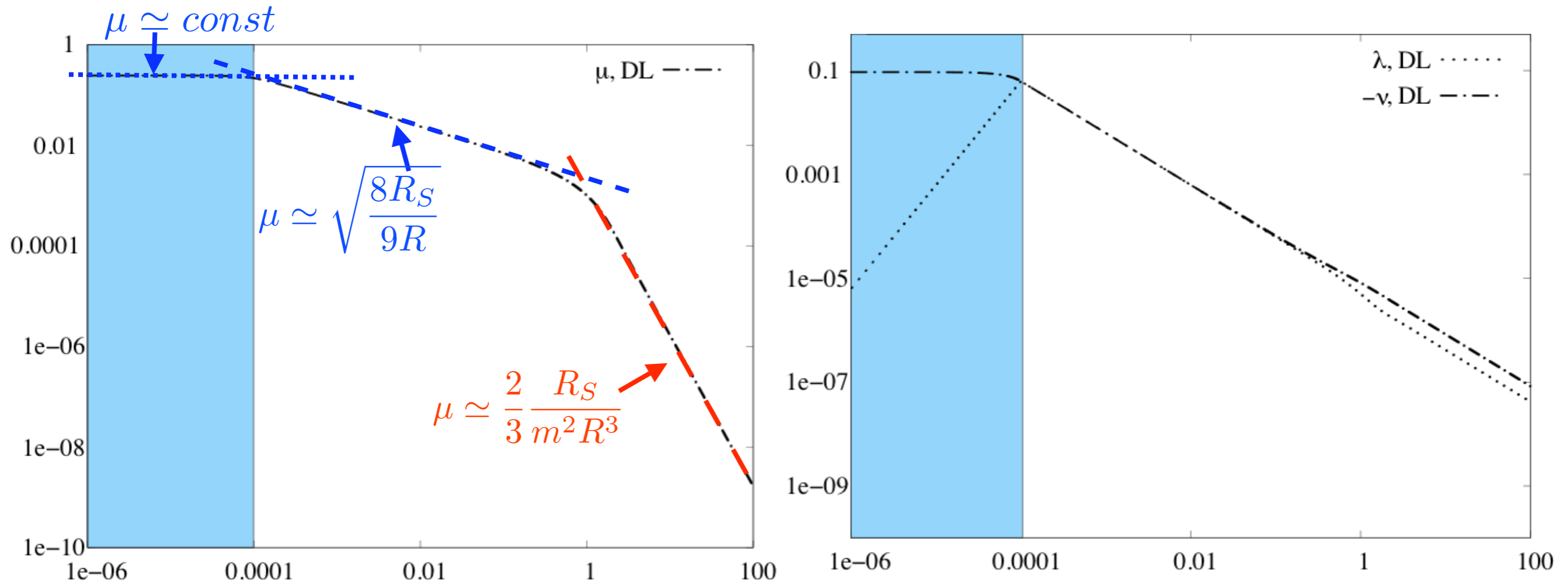
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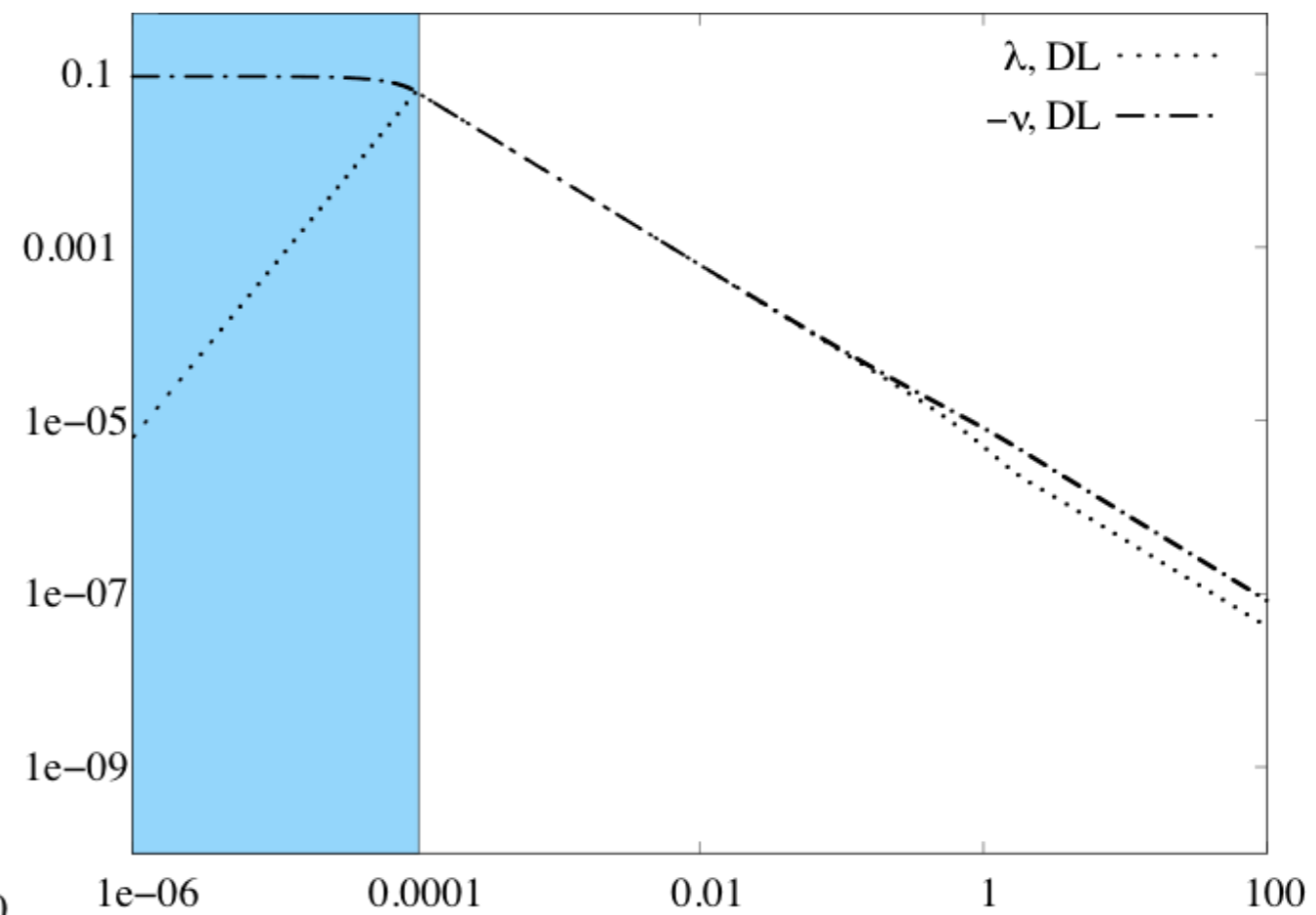
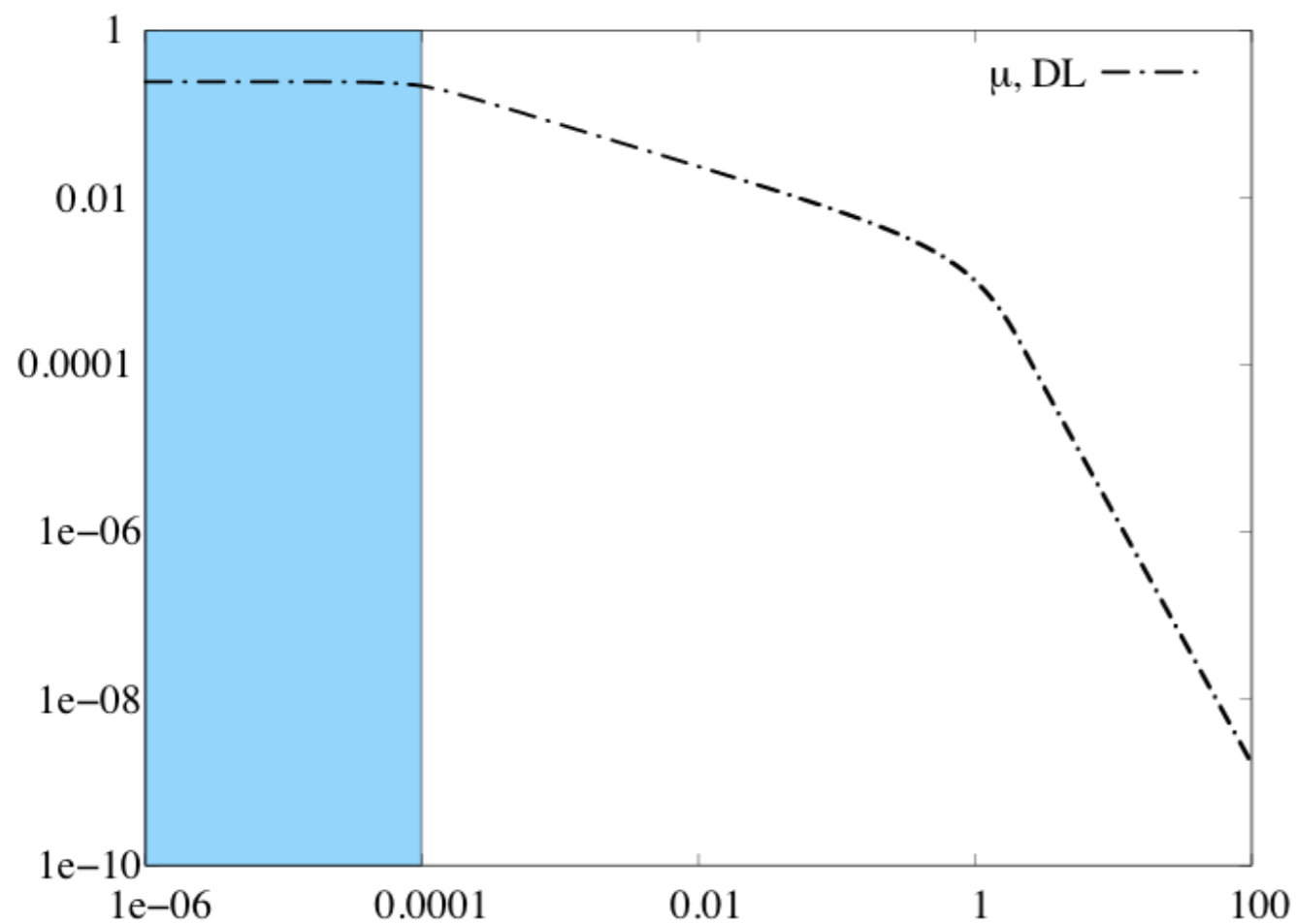
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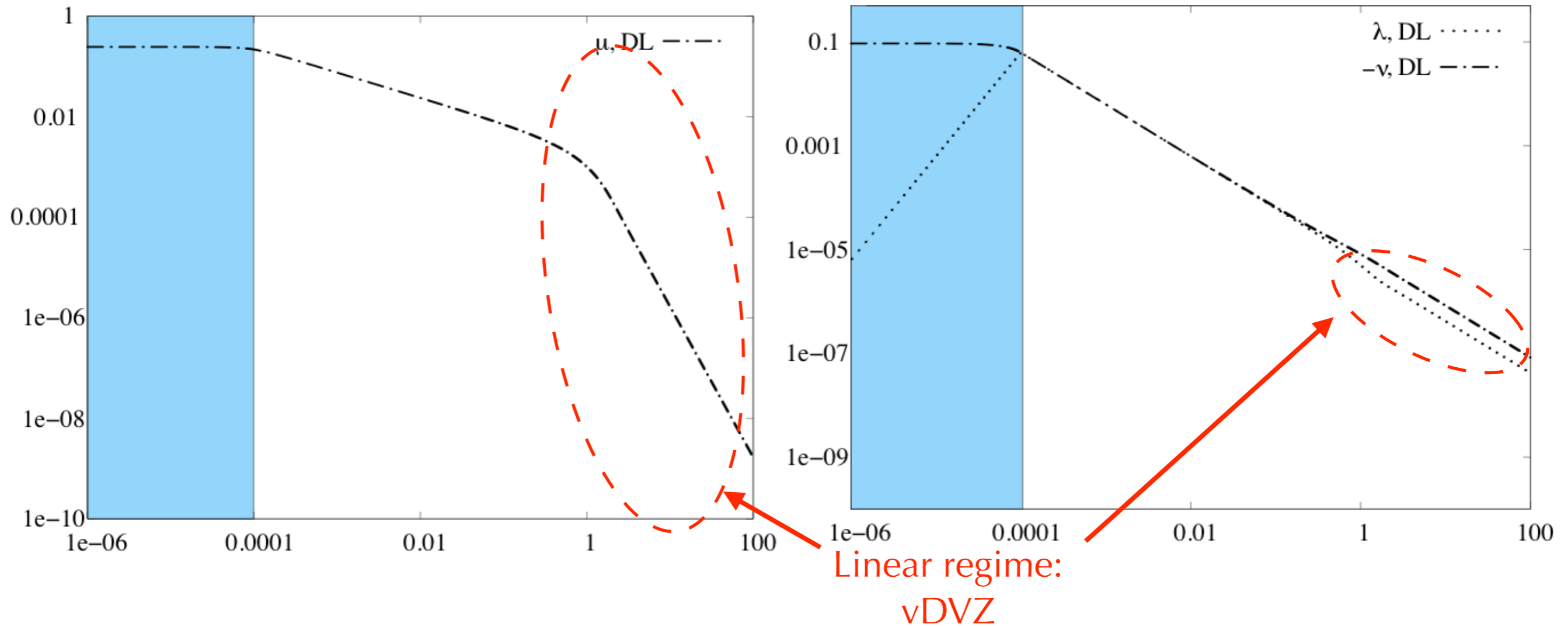
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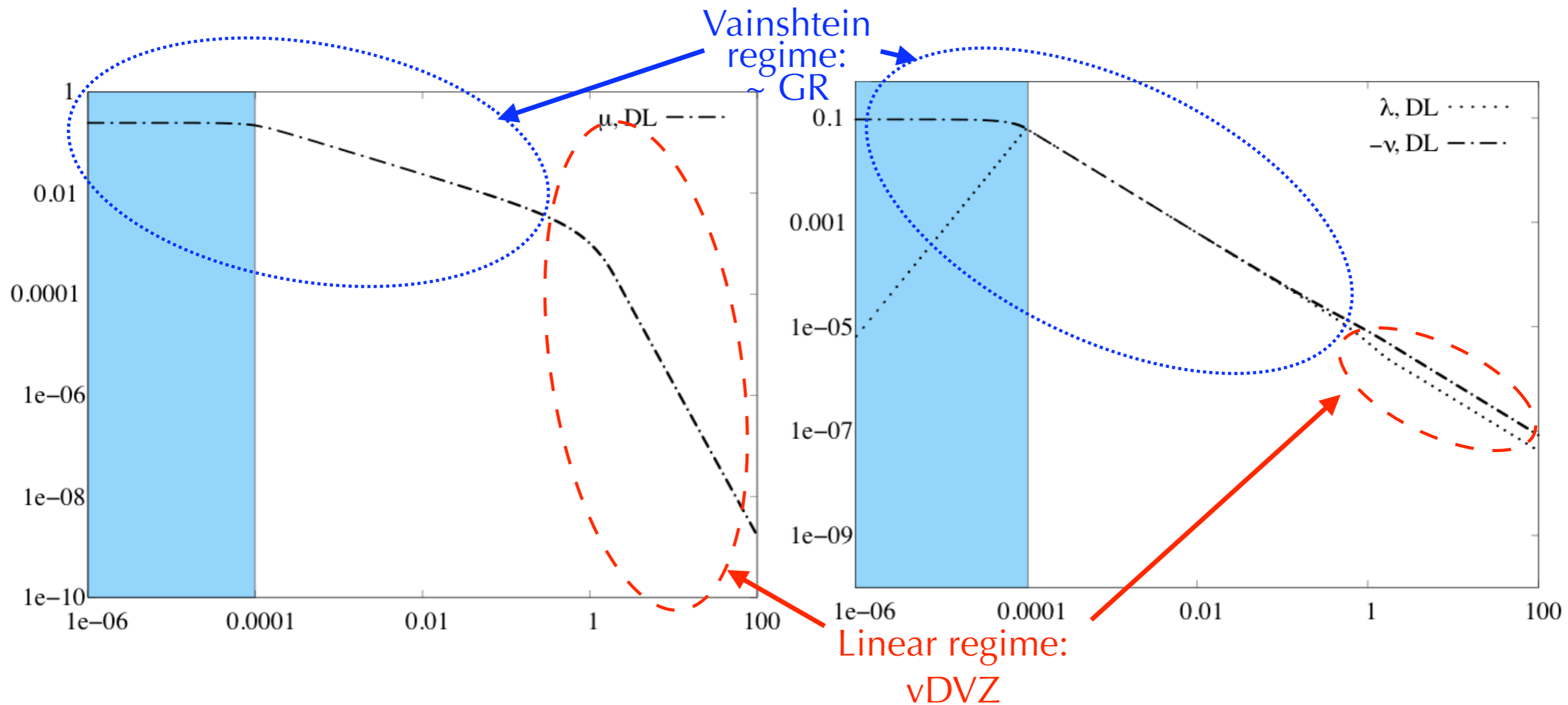
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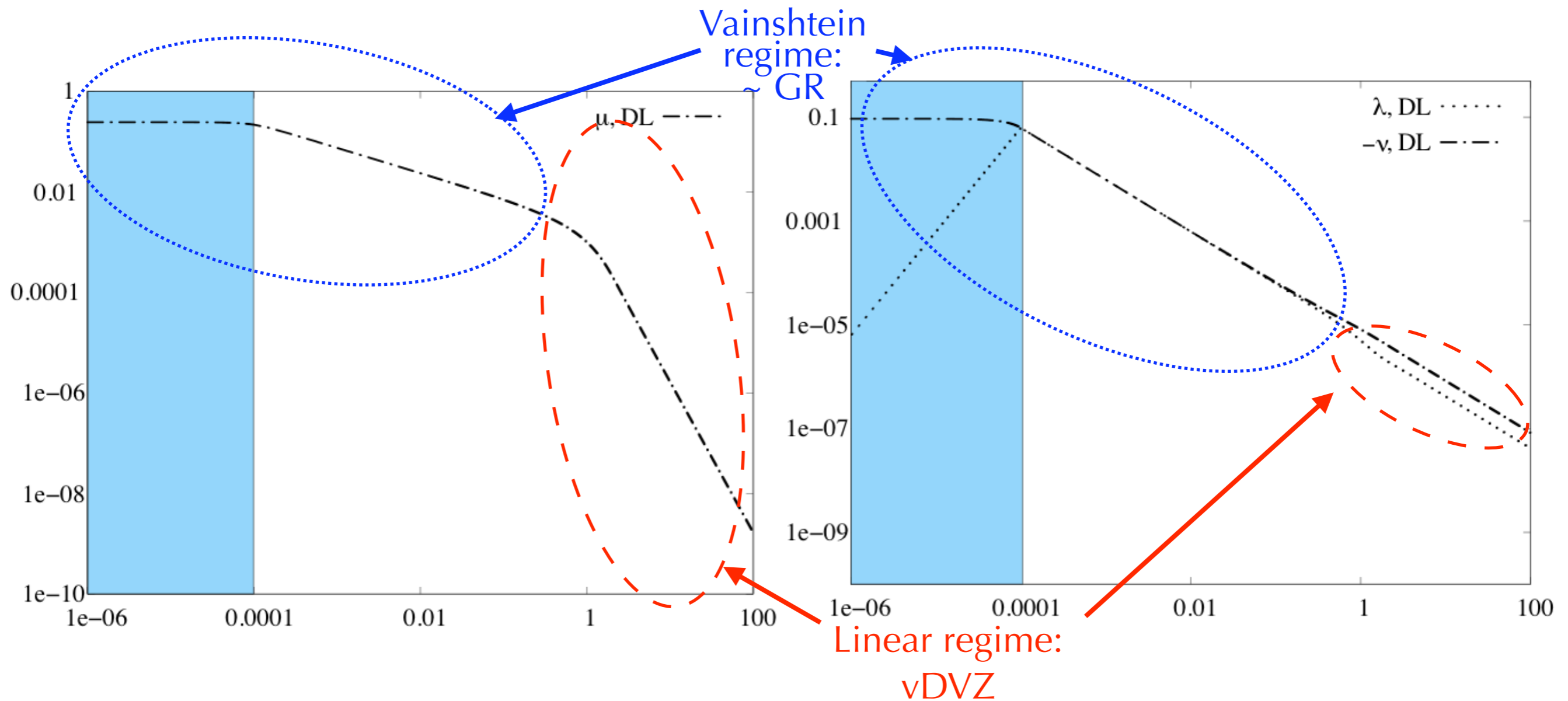
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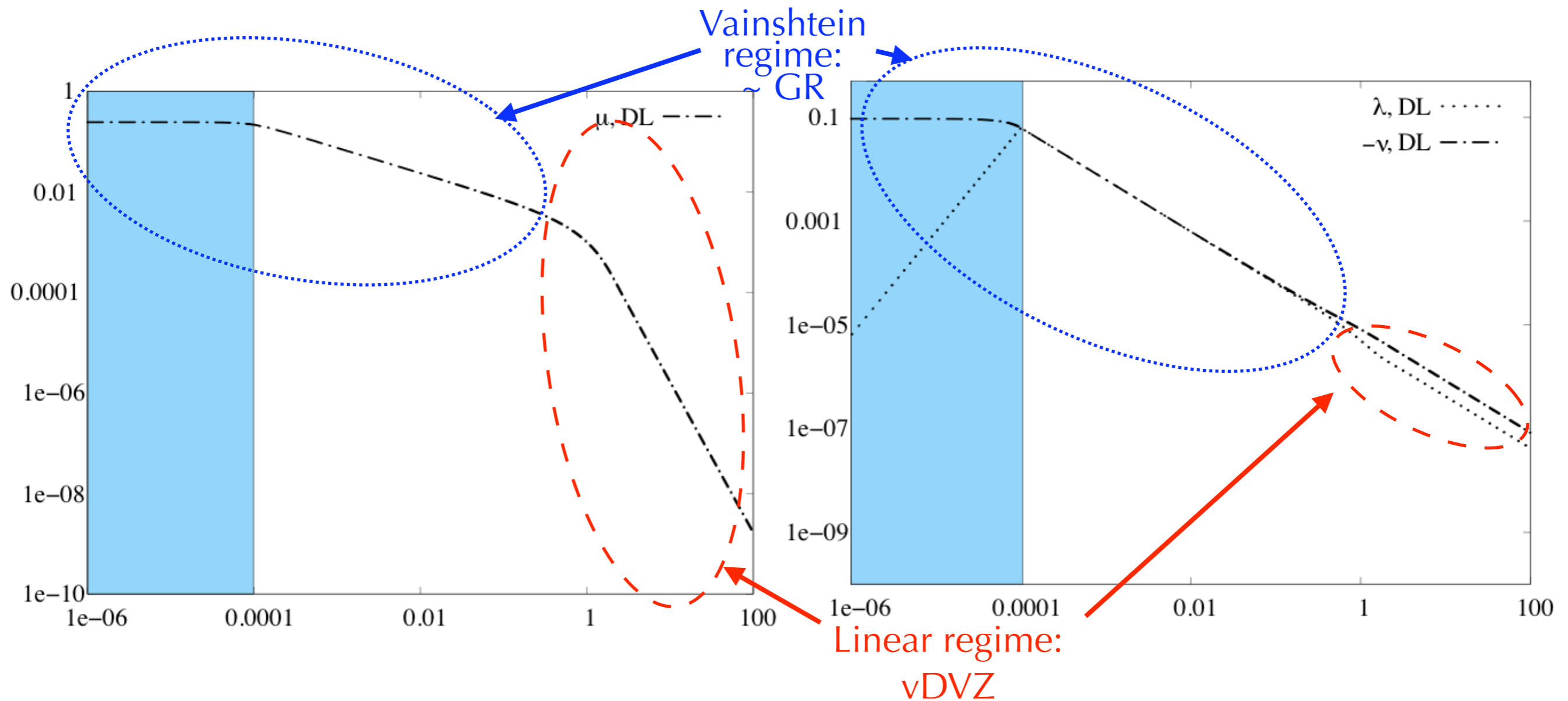


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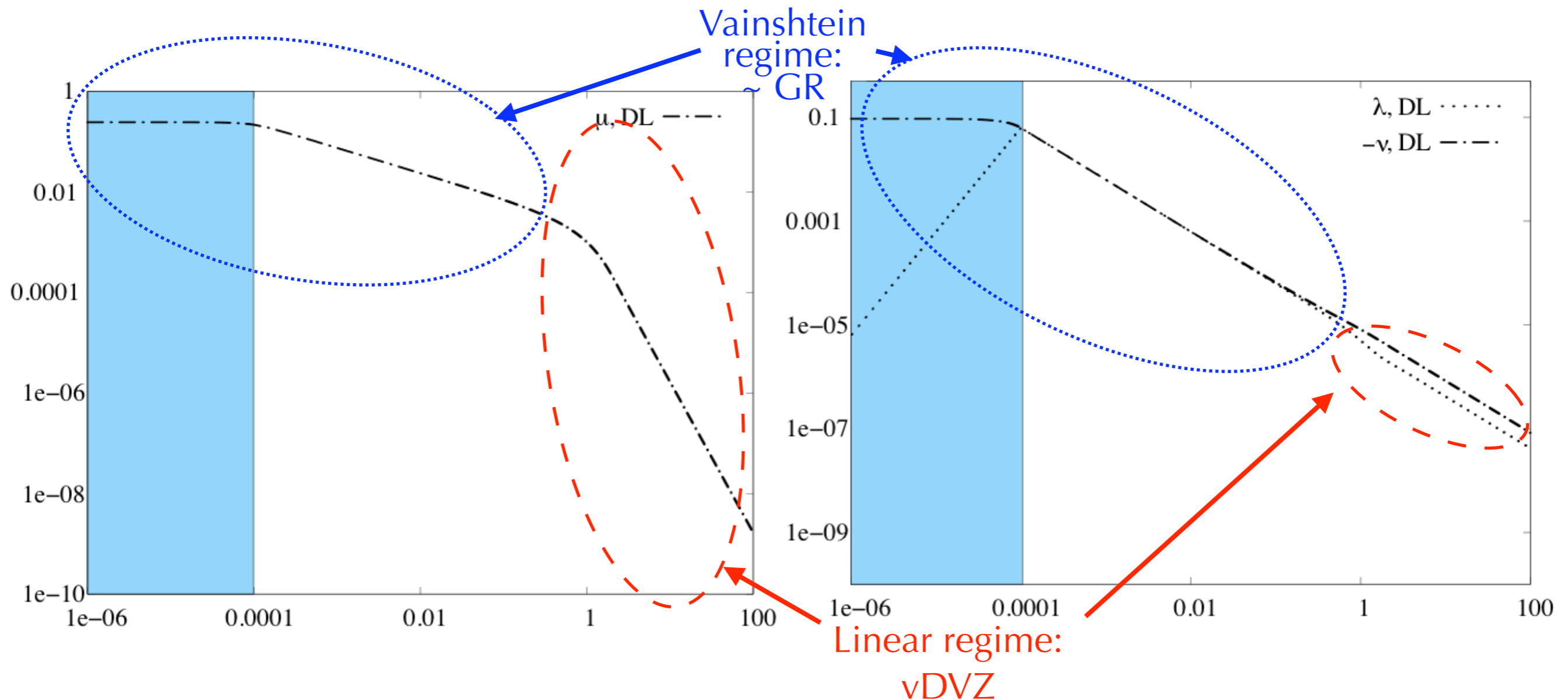
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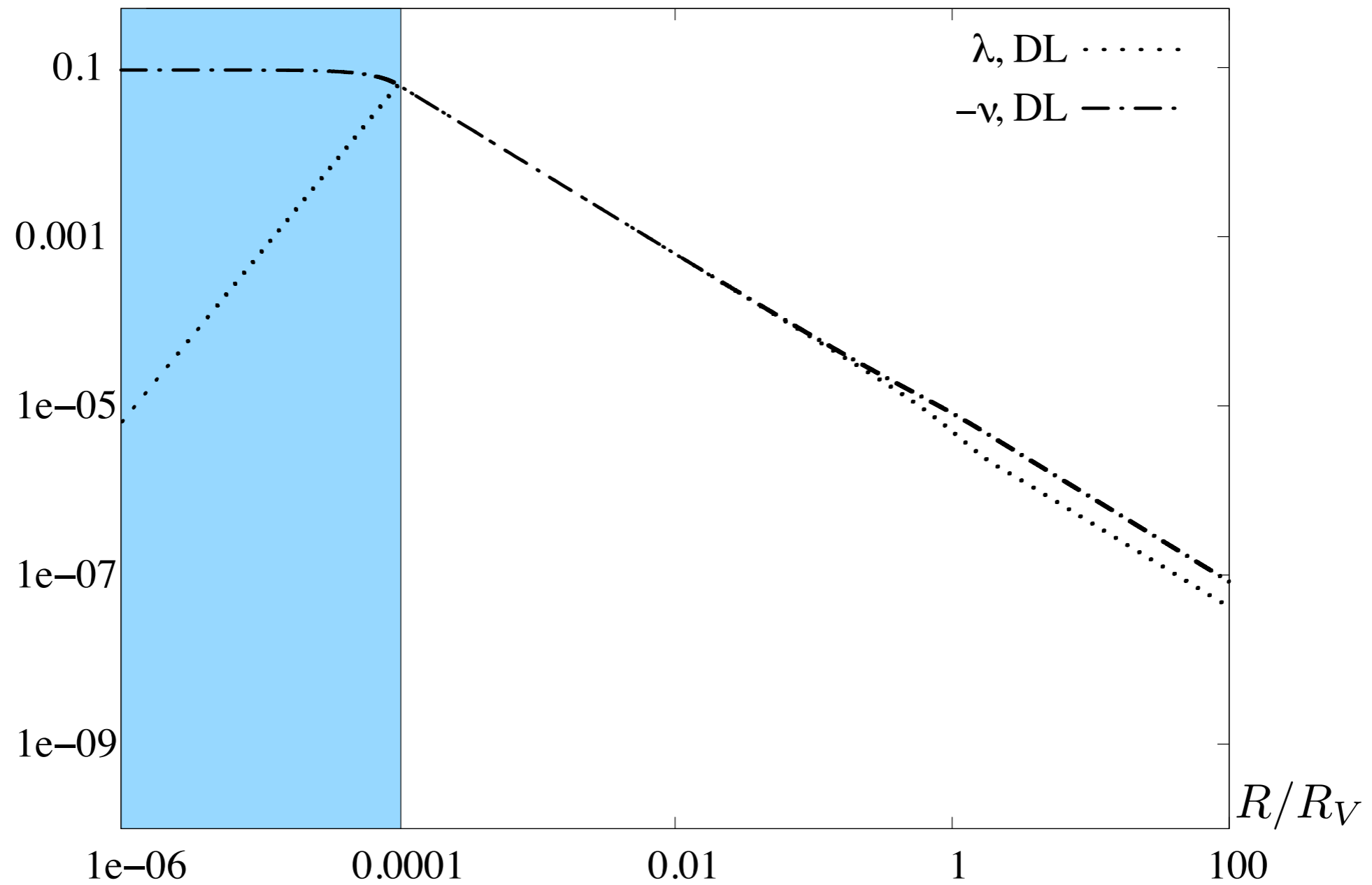


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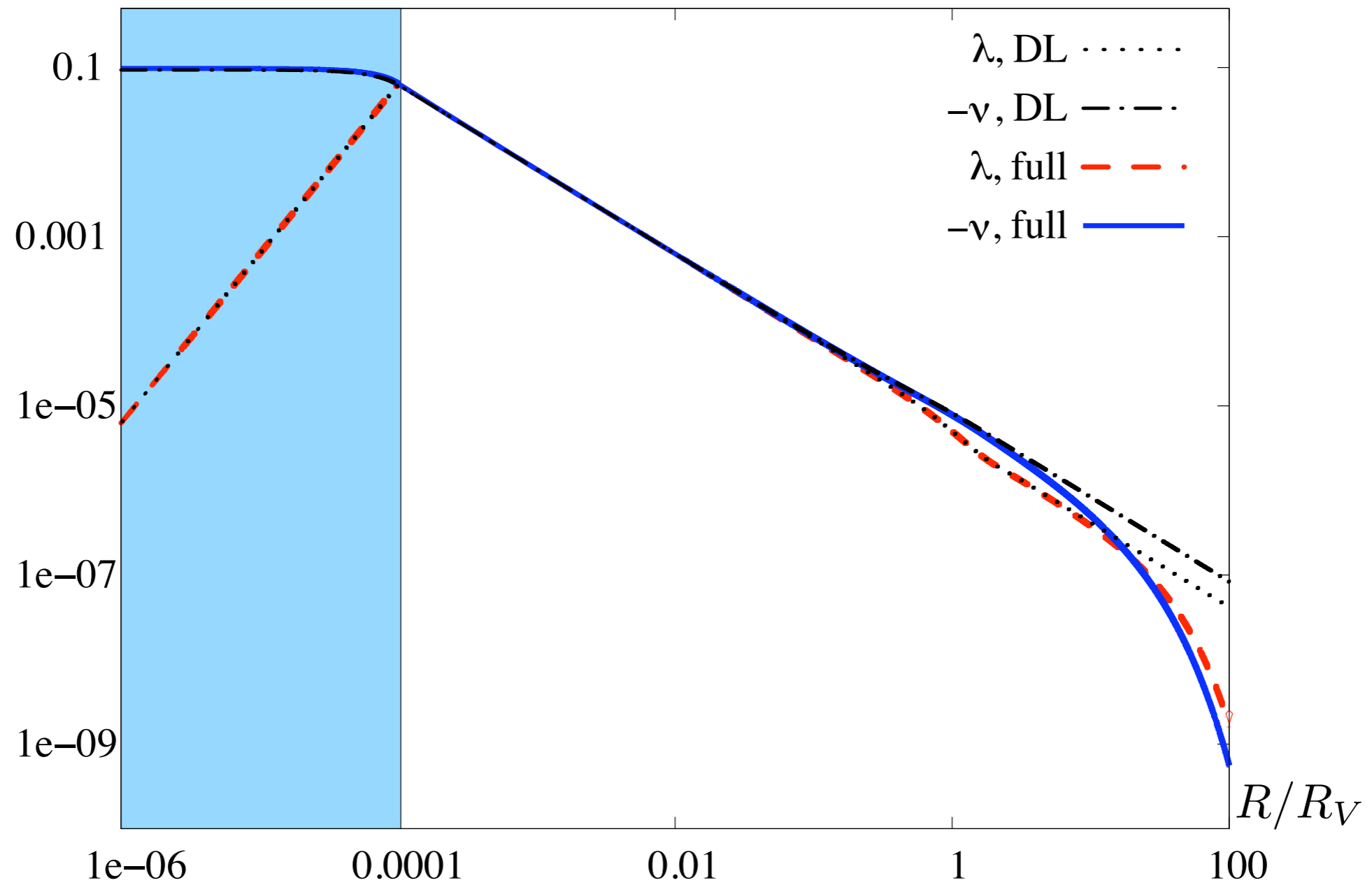
Is it possible to generalize this solution?

The full system solution (I)

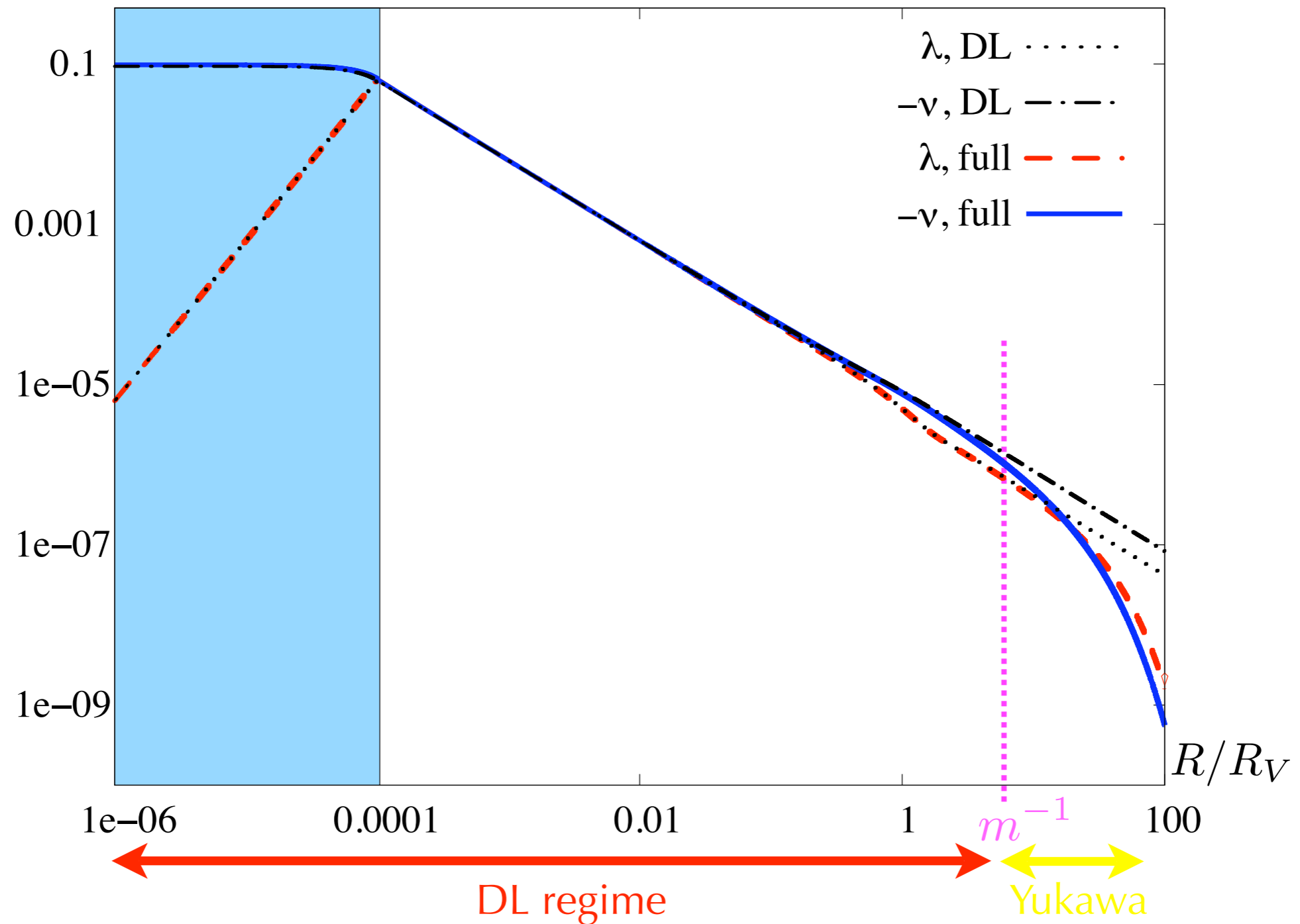
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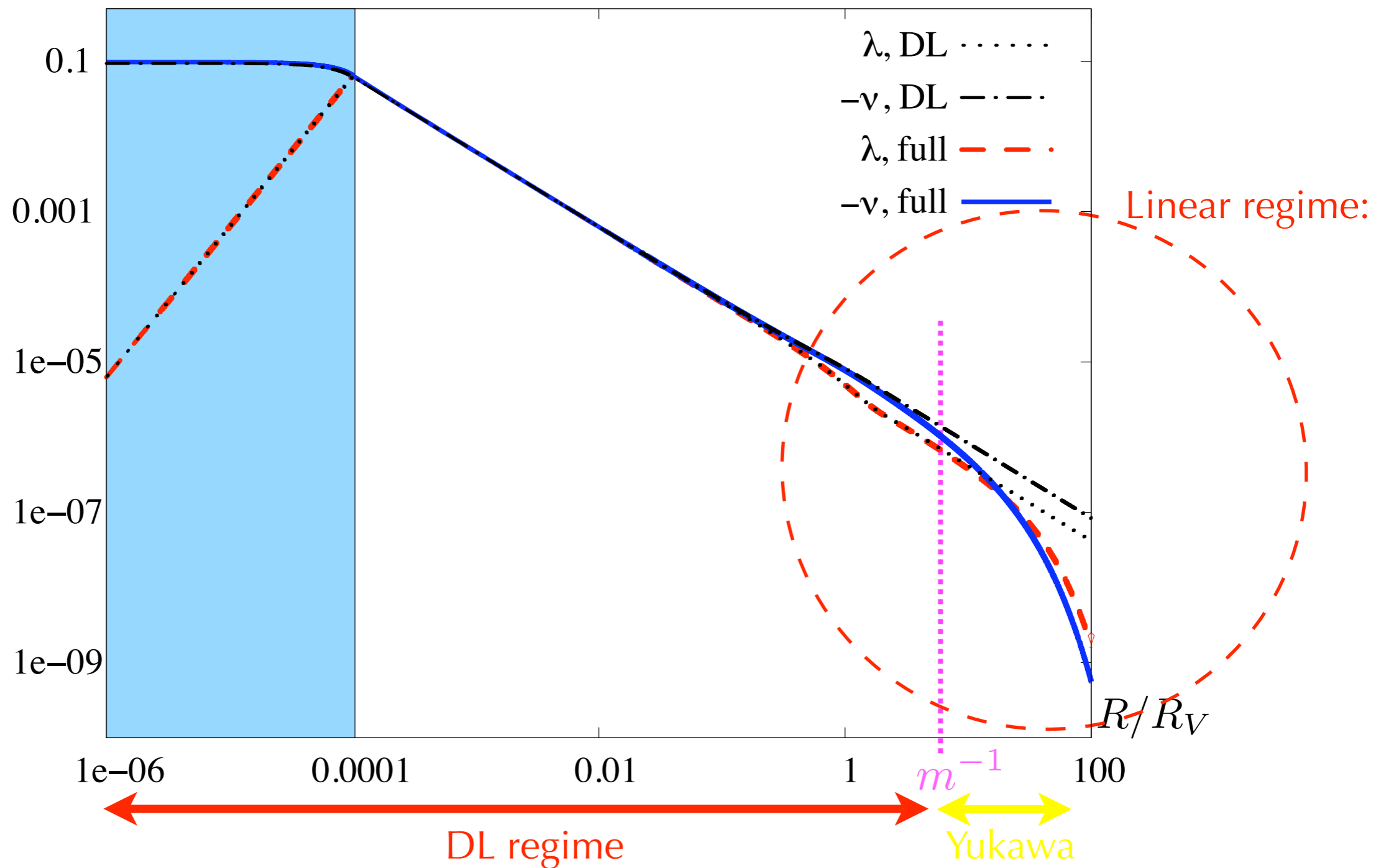
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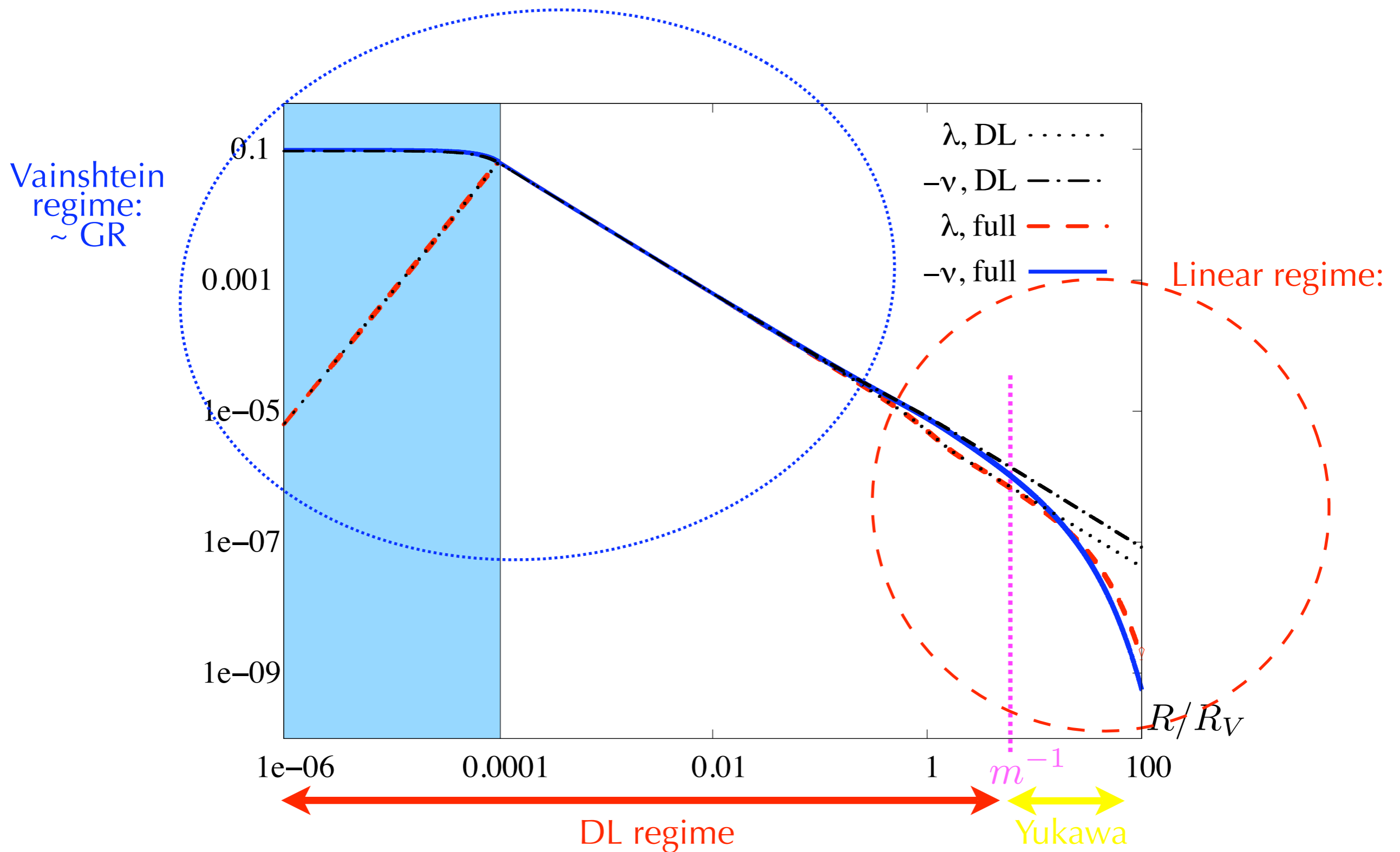
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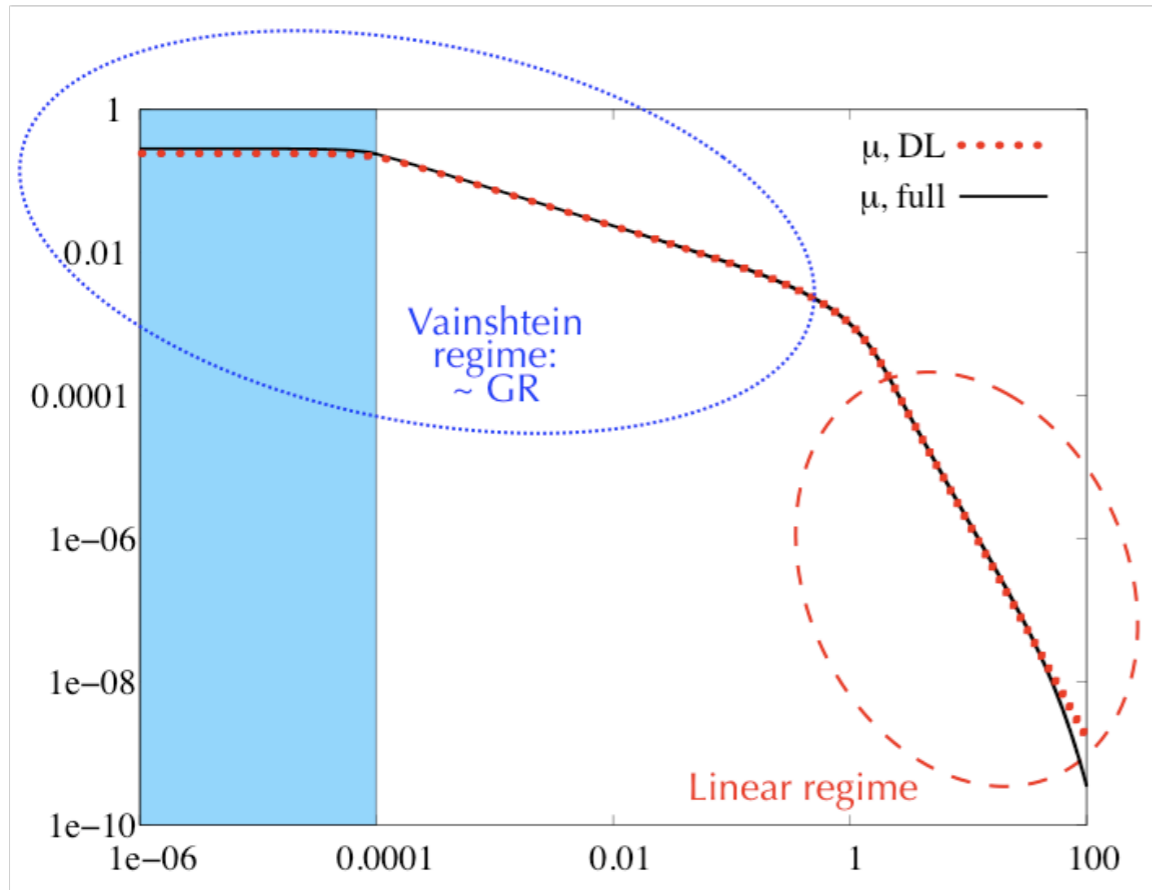
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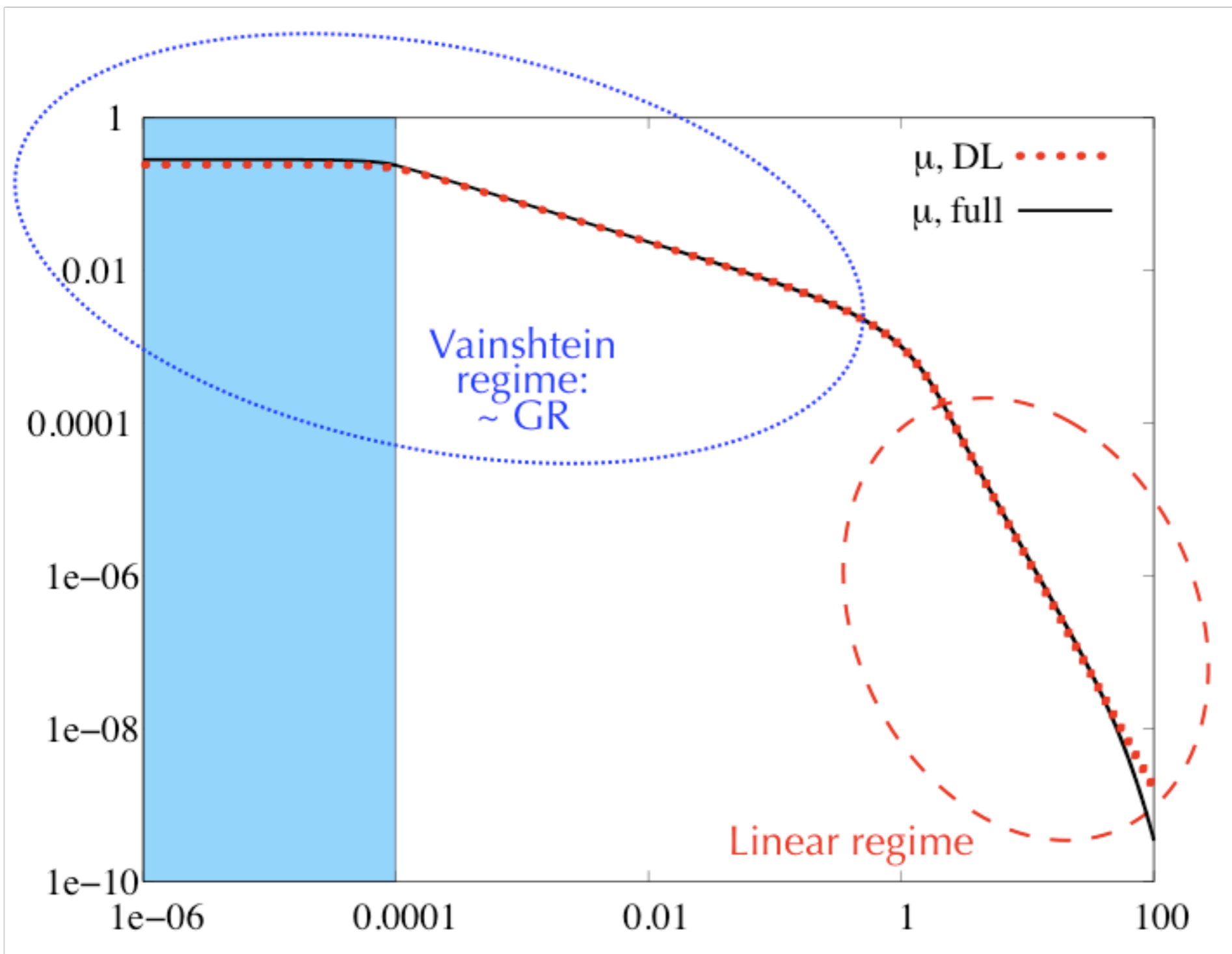


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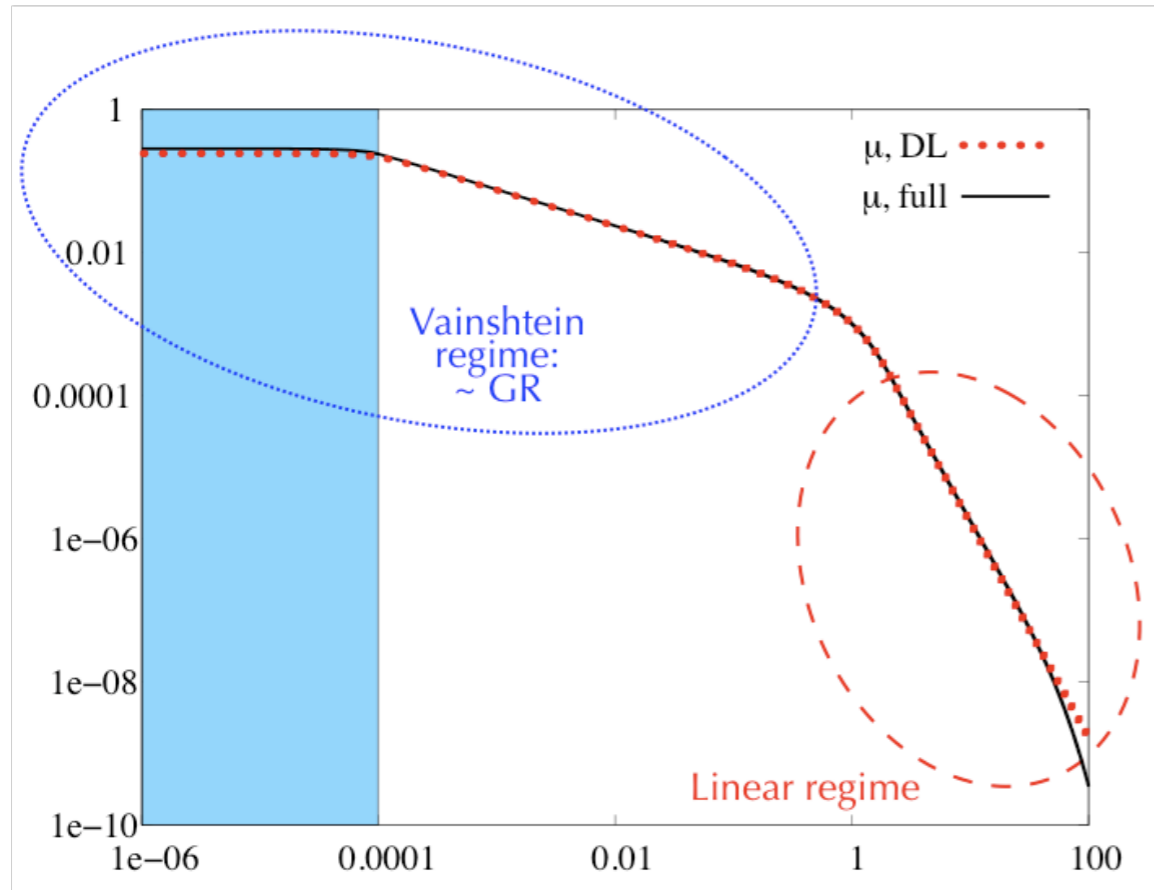


The full system solution (II)



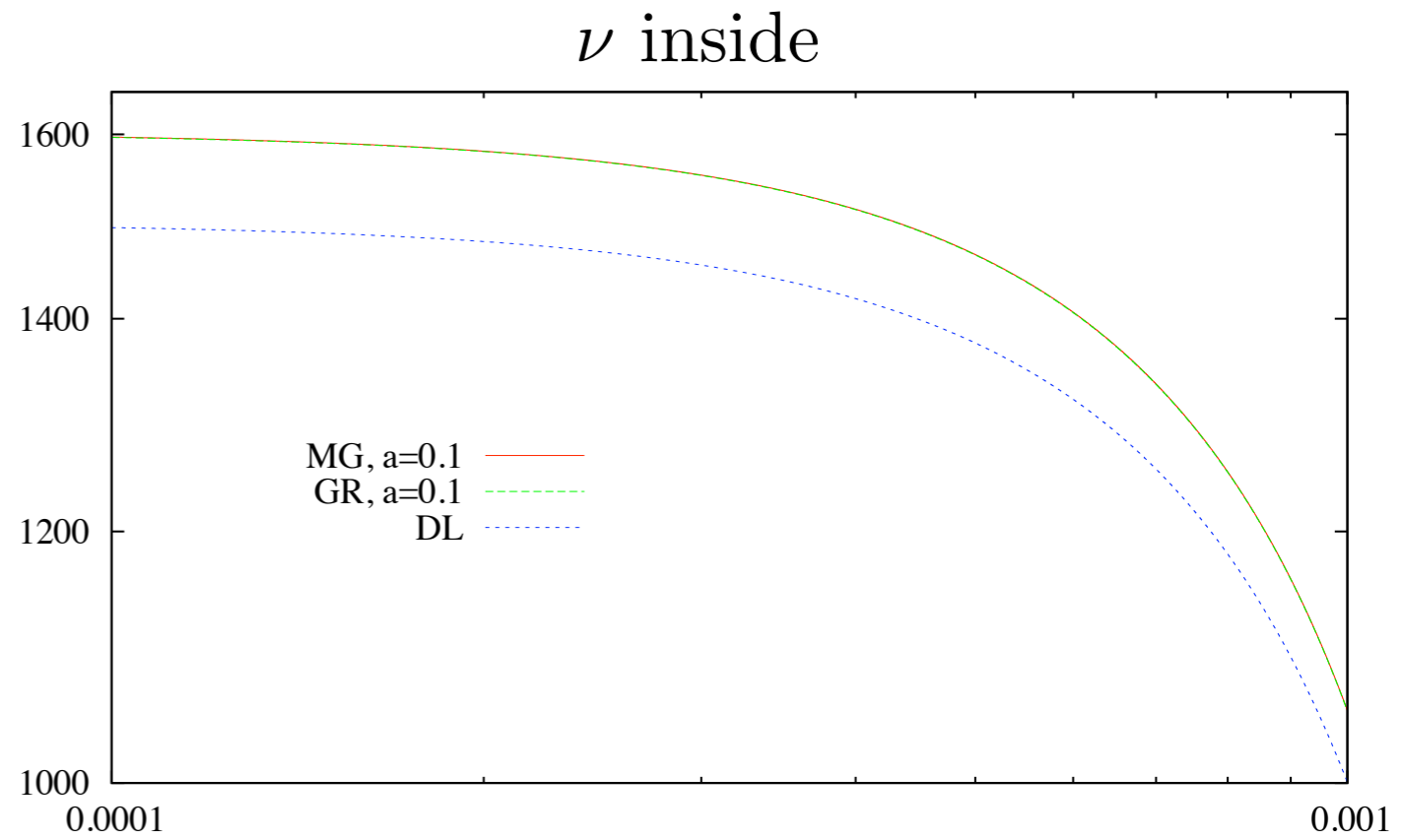
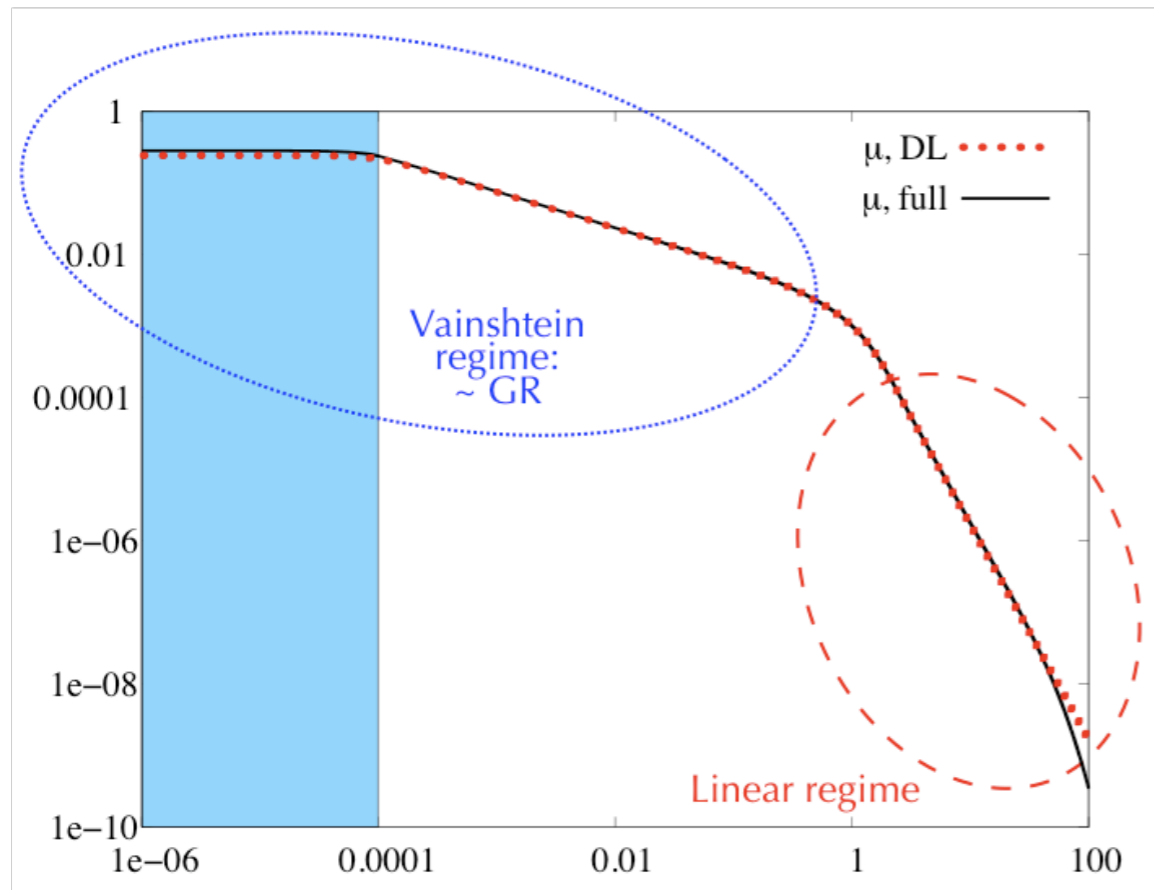


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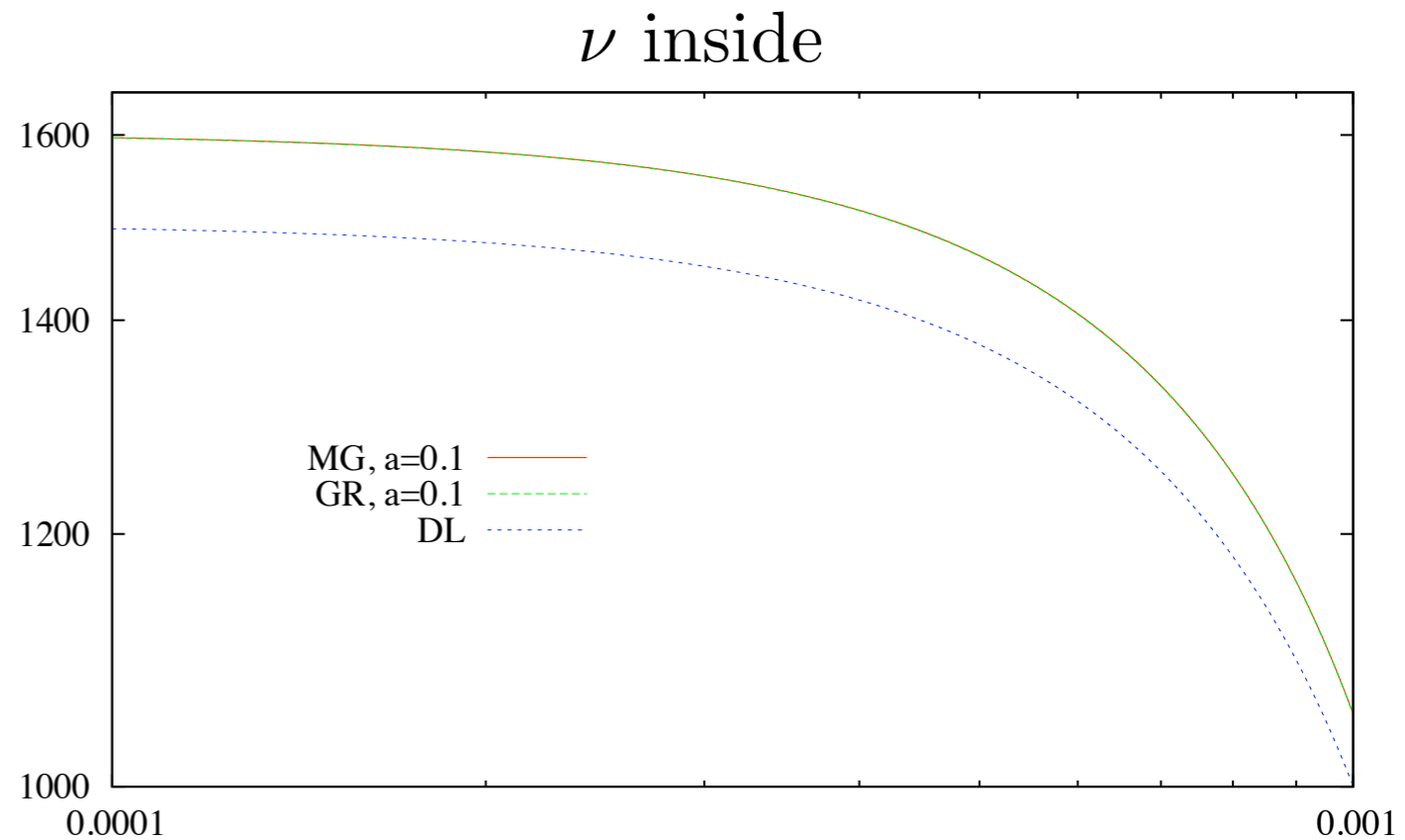
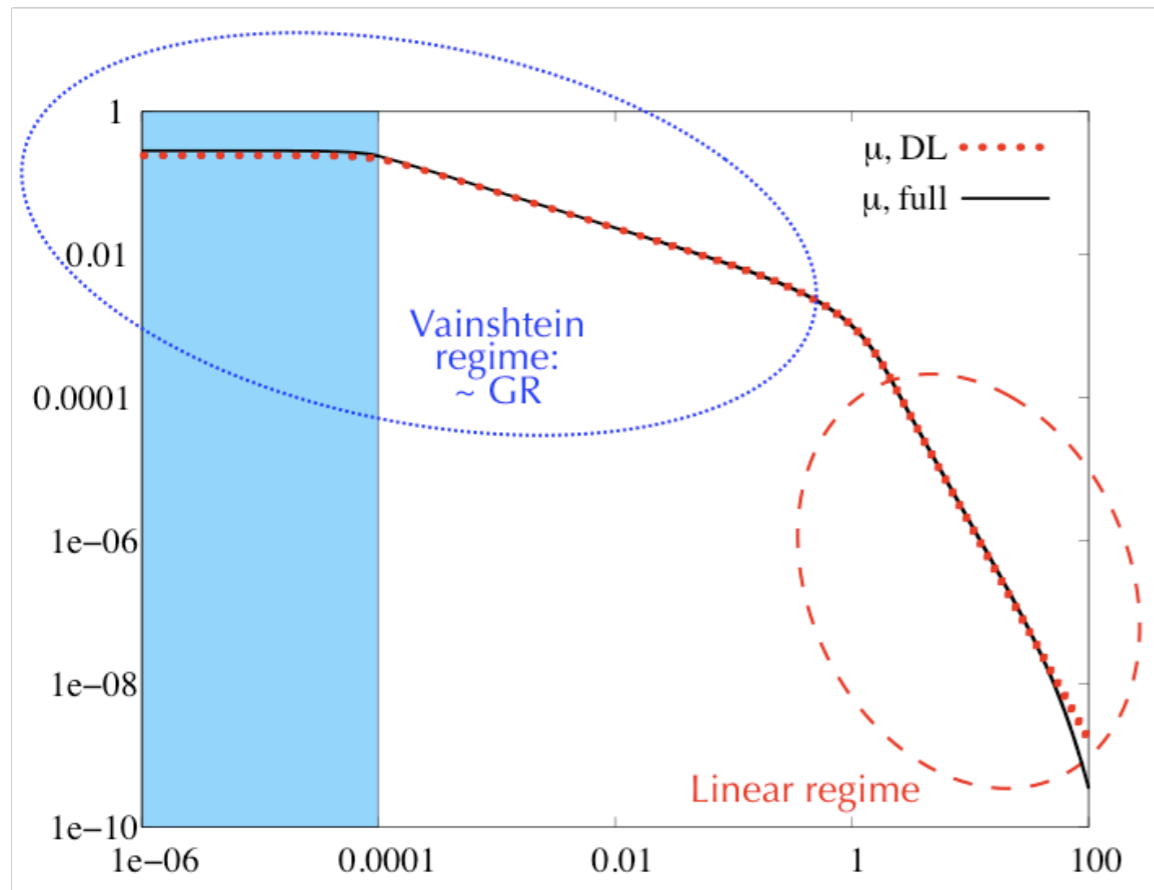


ν inside

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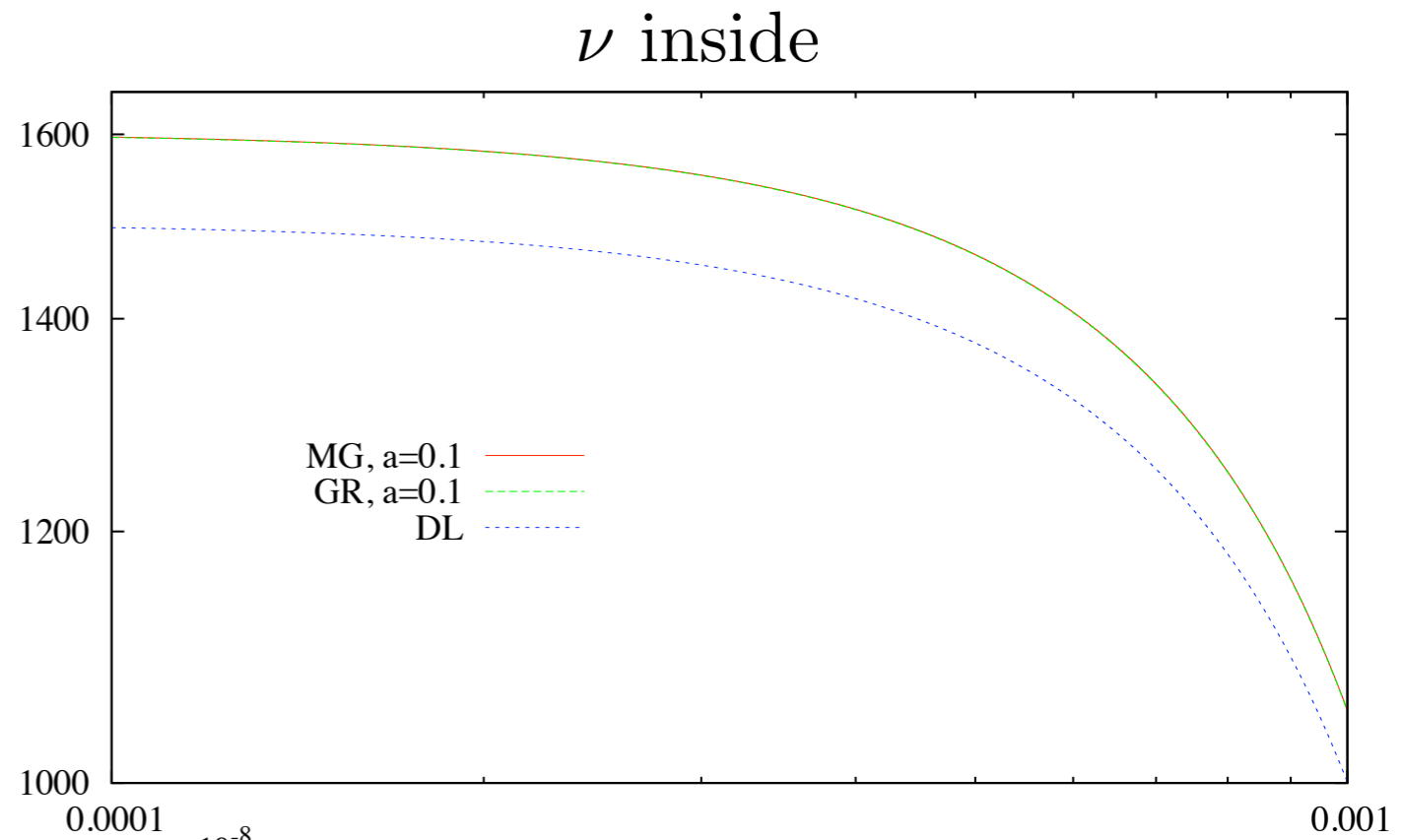
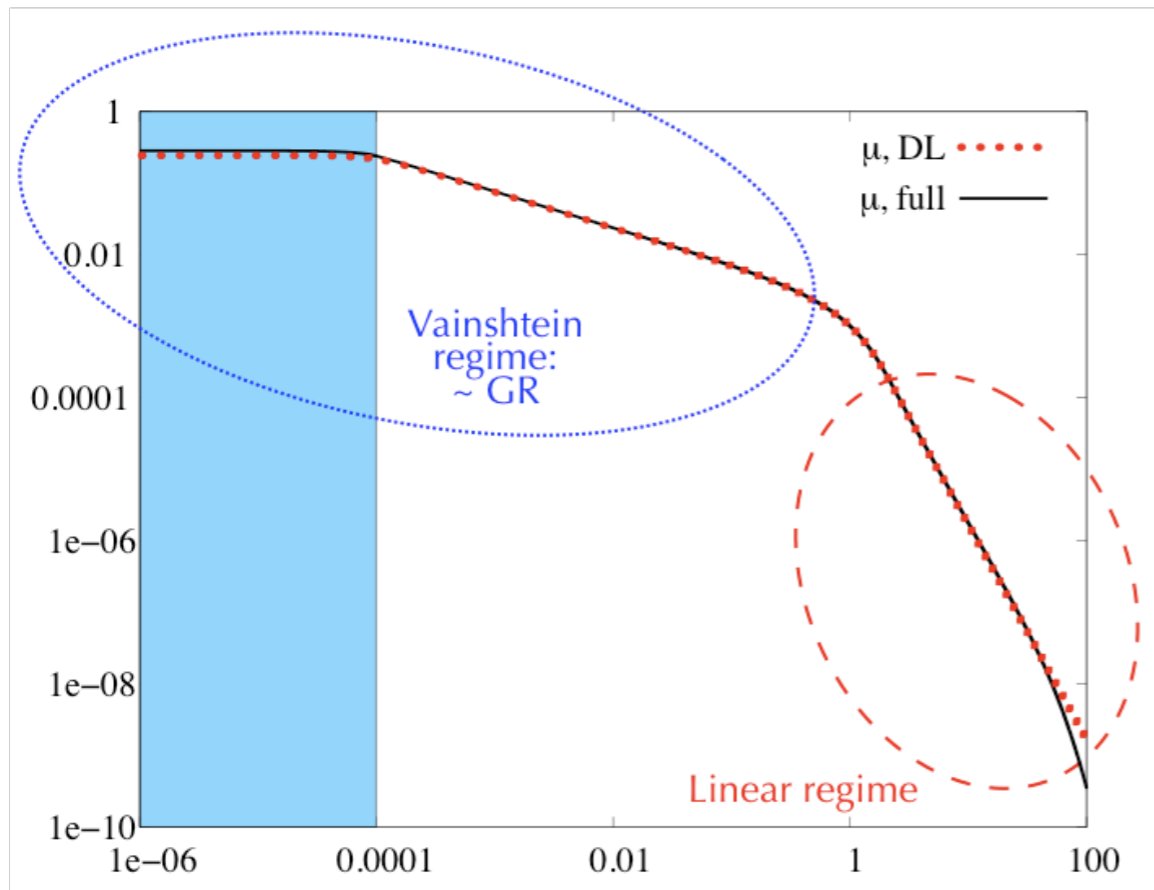
The full system solution (II)



Difference between MG and GR:

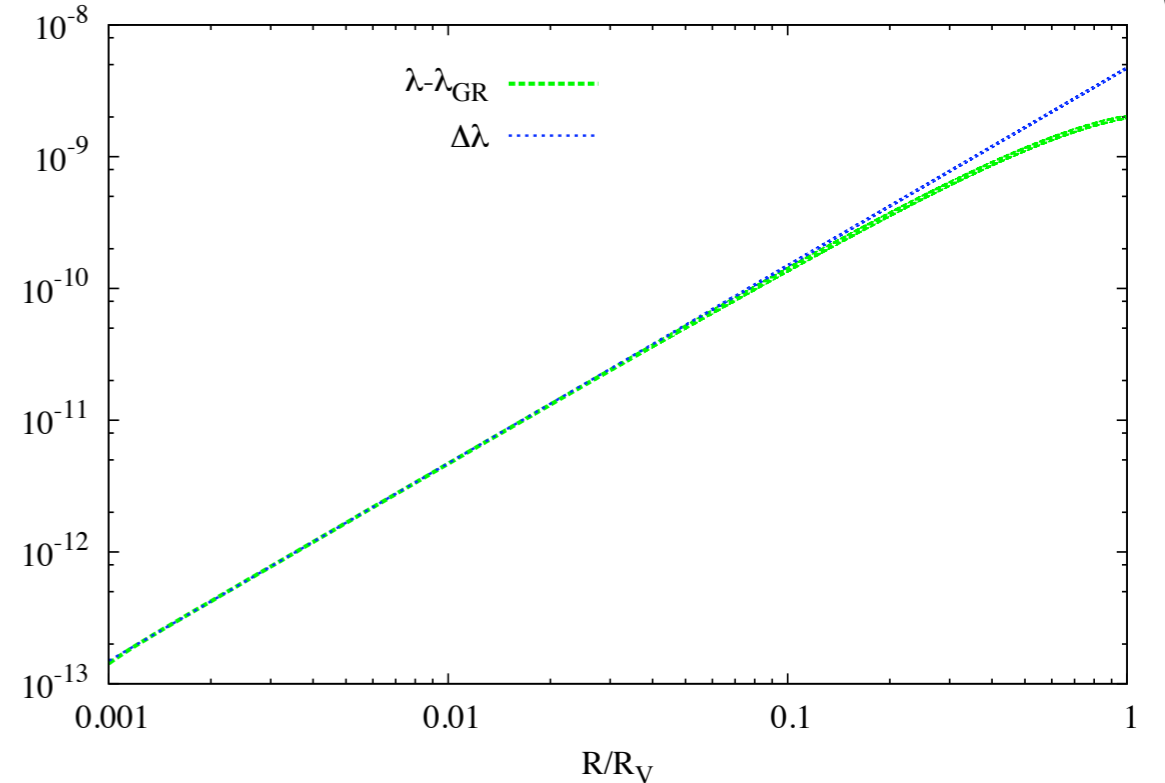
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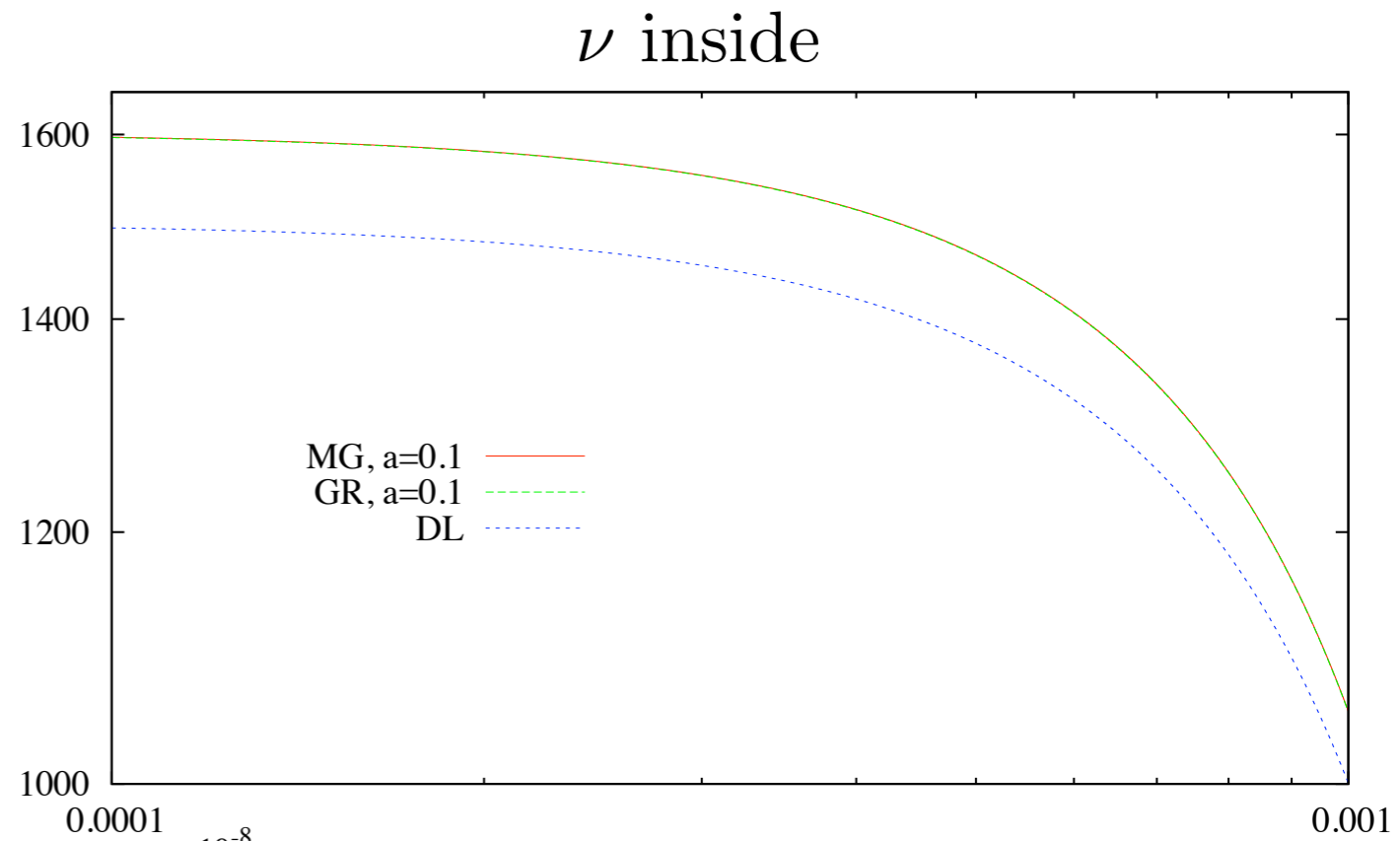
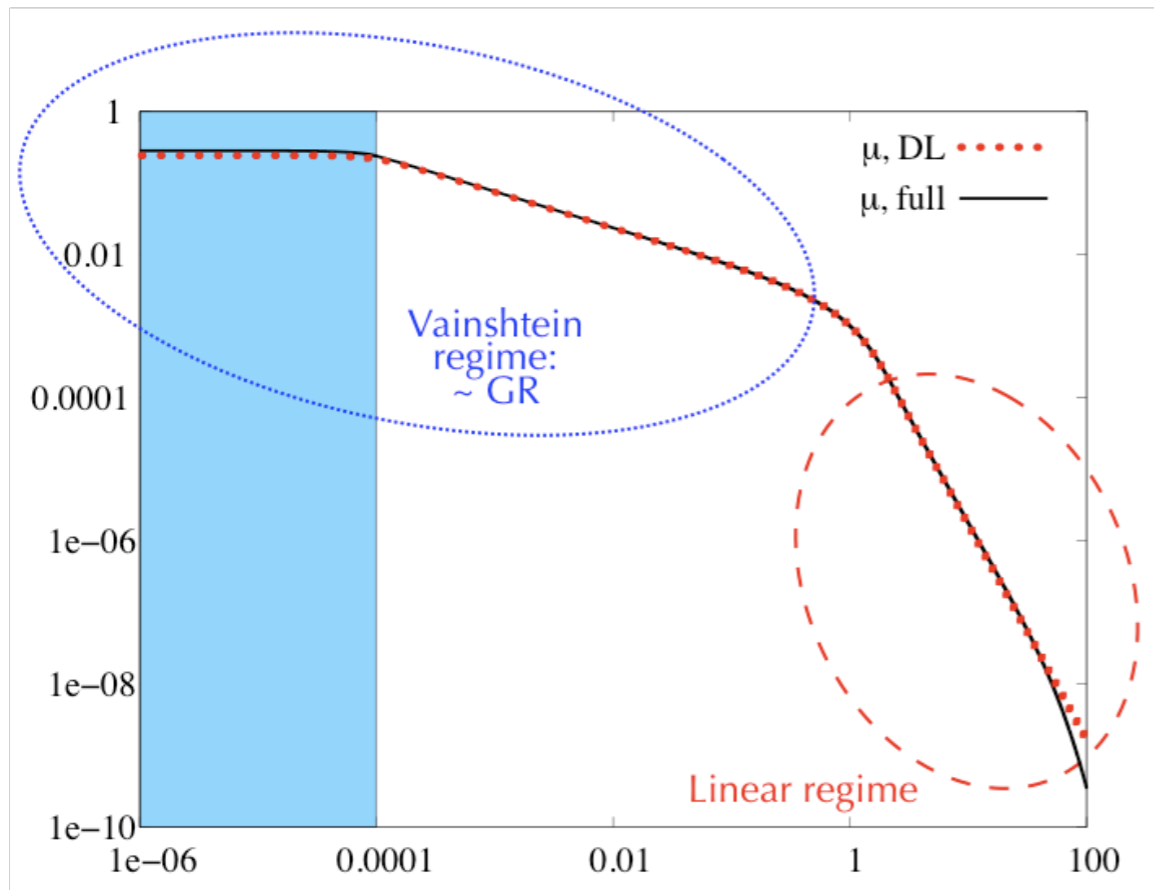


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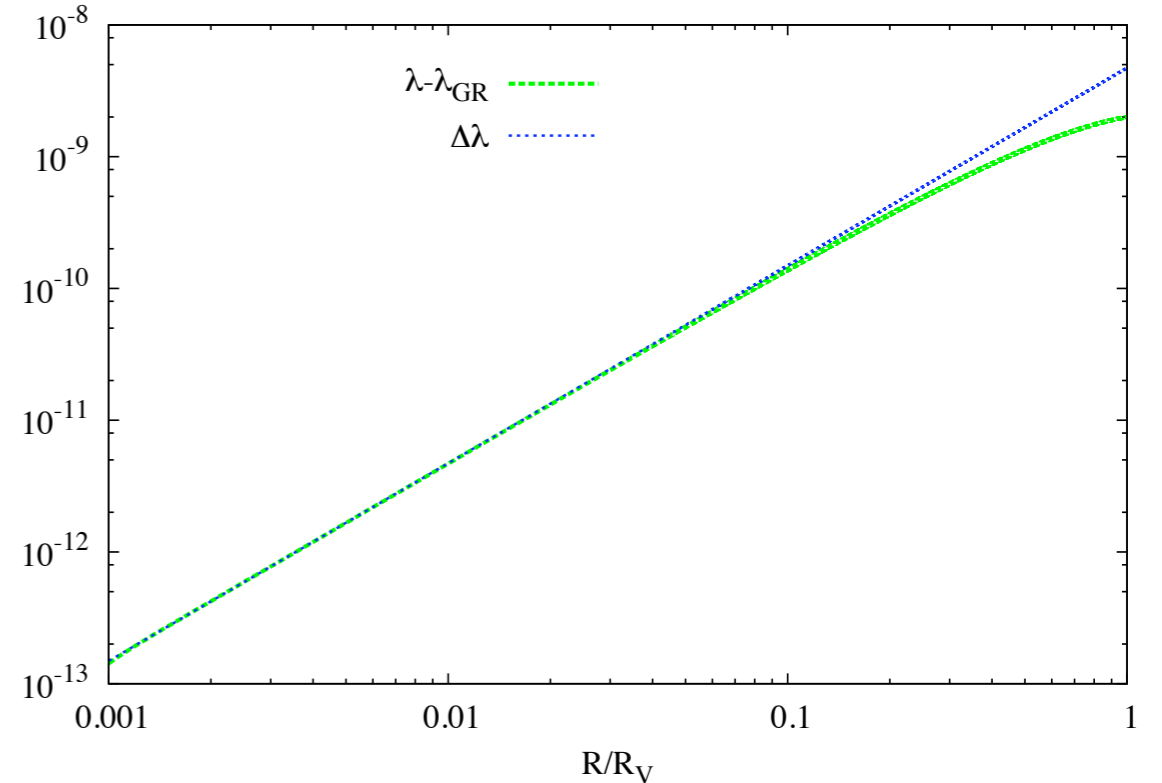
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Excellent agreement with numerics



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All the methods agree: robustness of the results

Beyond Massive Gravity

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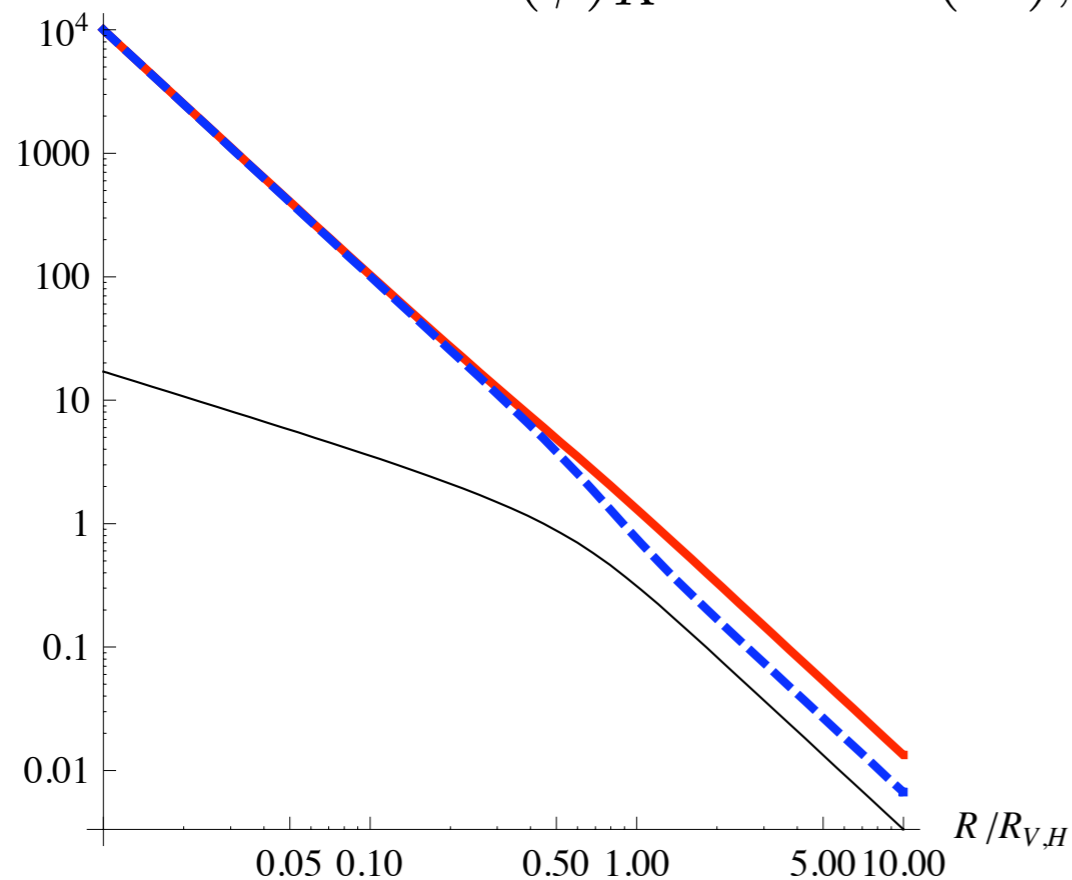
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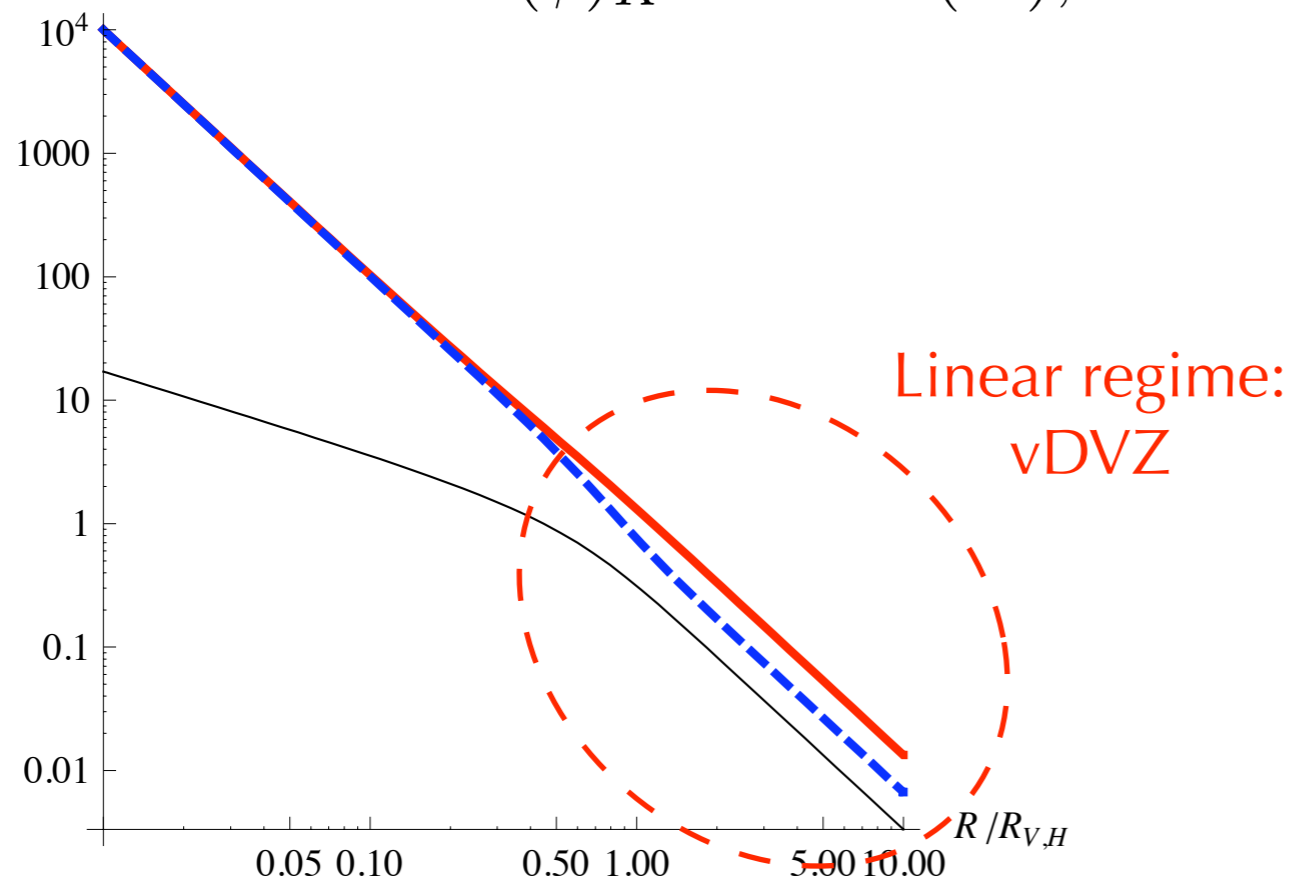
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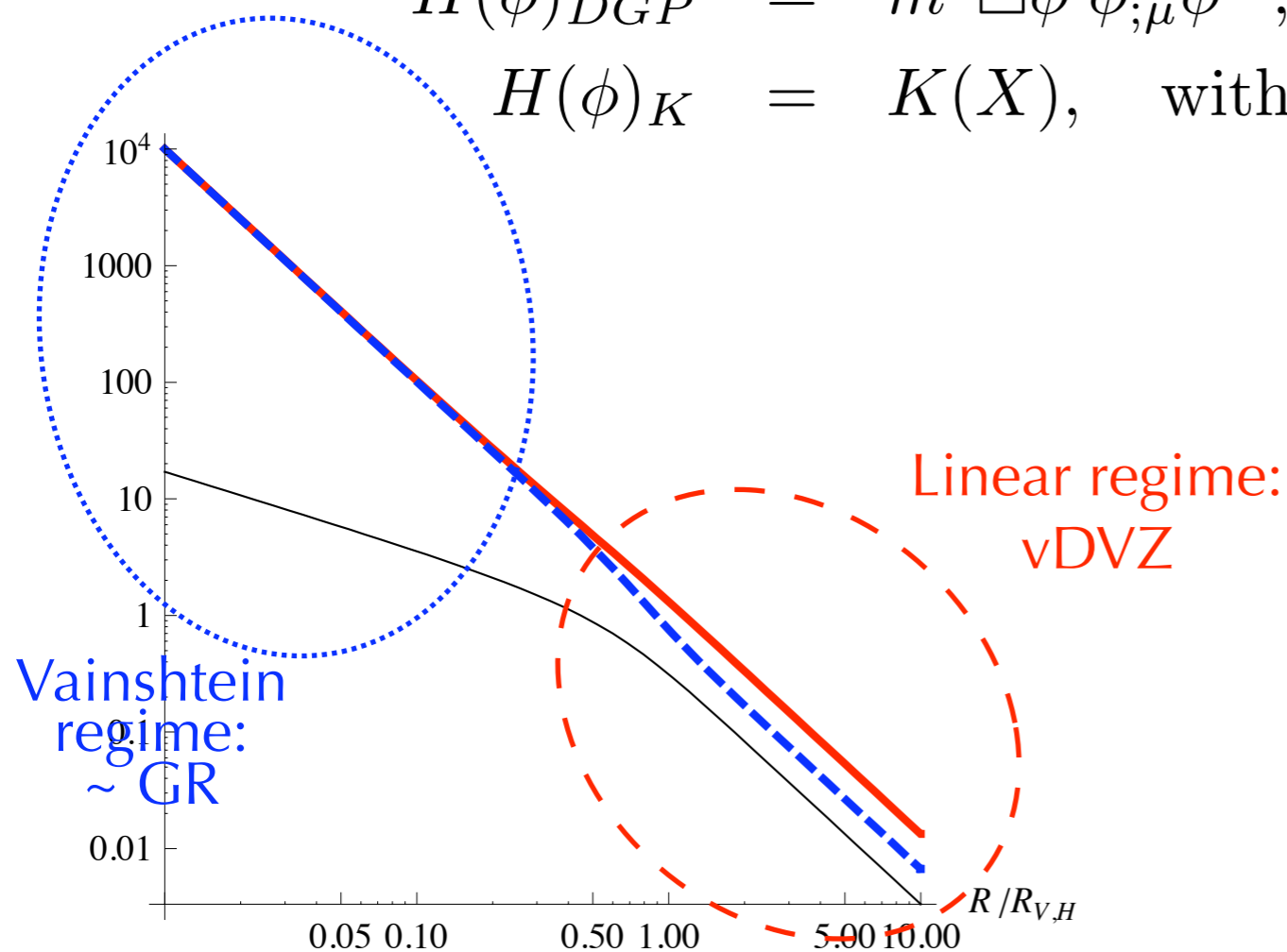
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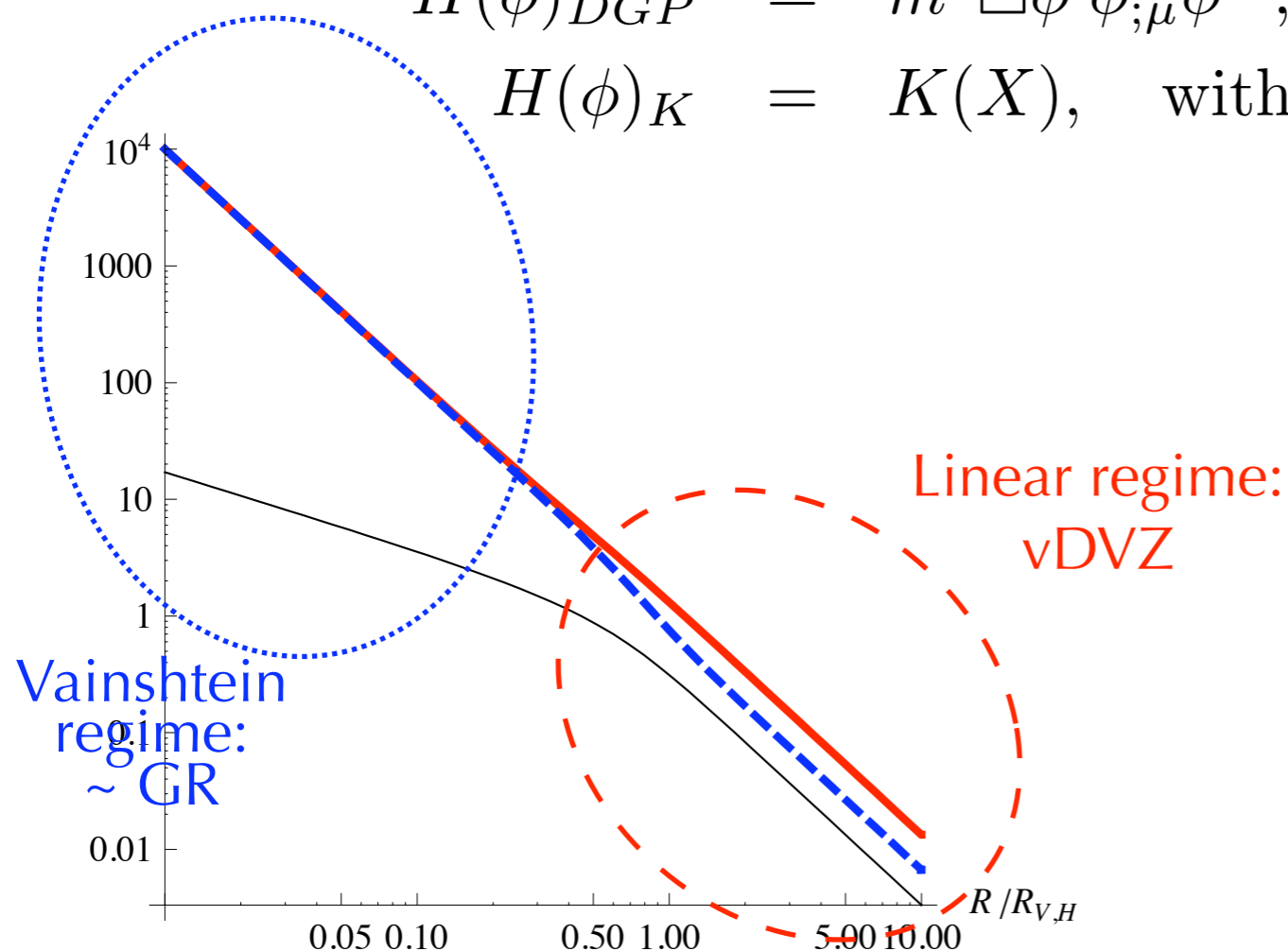
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The Vainshtein mechanism is valid for a broad class of models

Observational constraints

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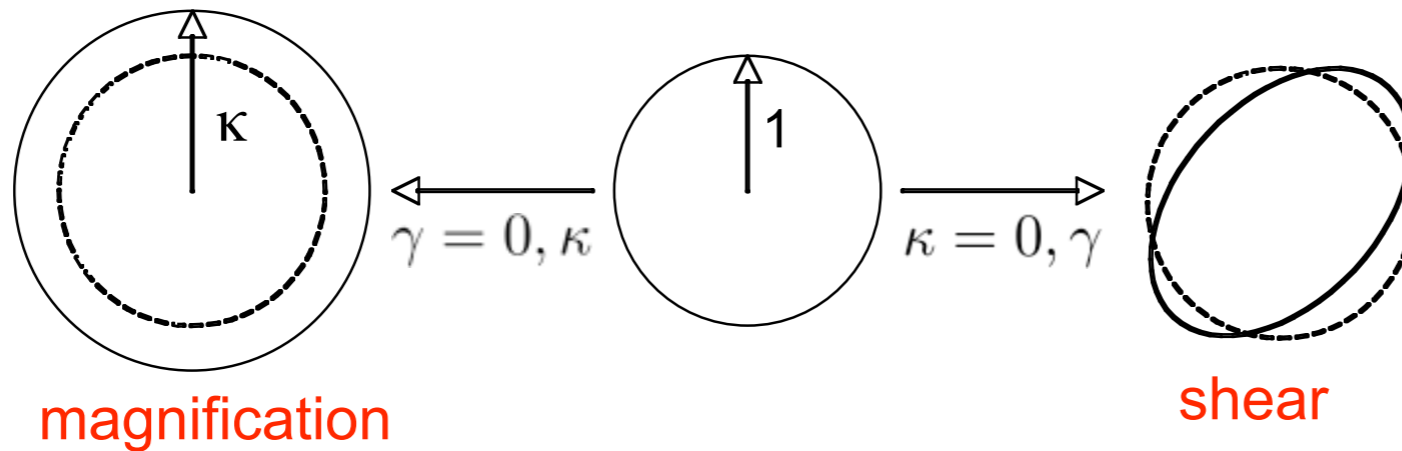
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Combination of all these observations: best constraints

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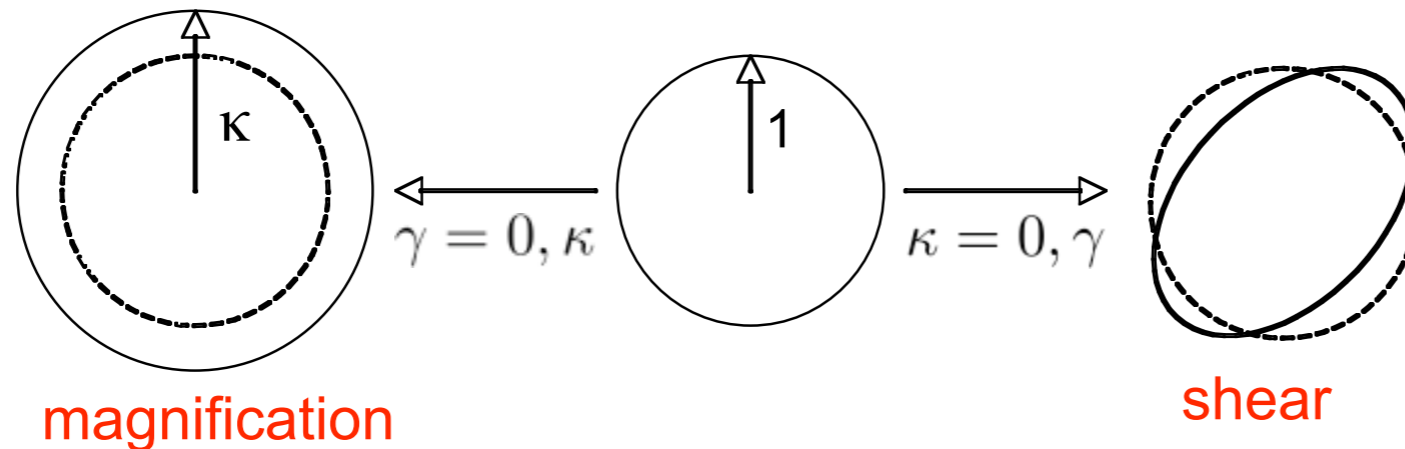
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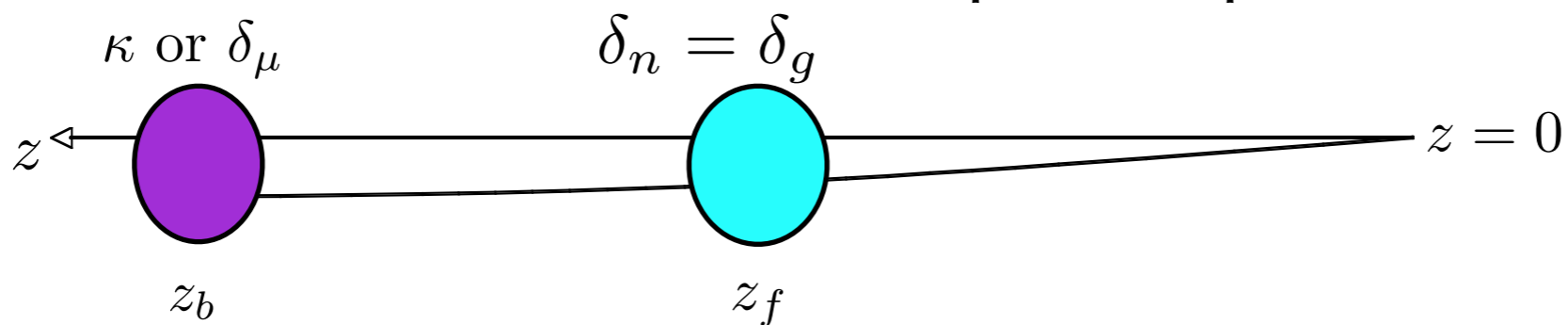
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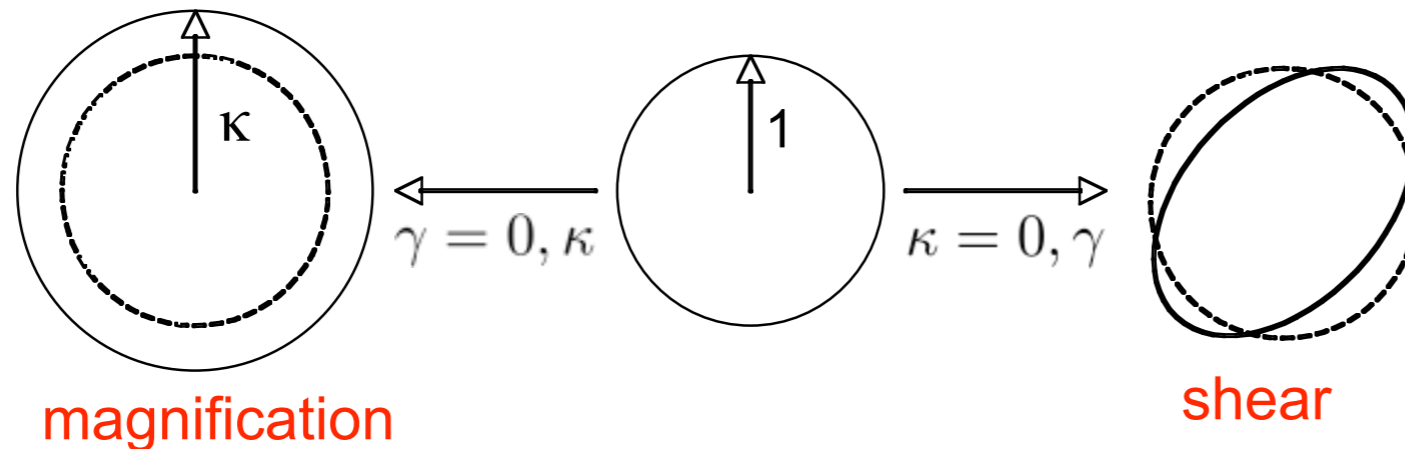
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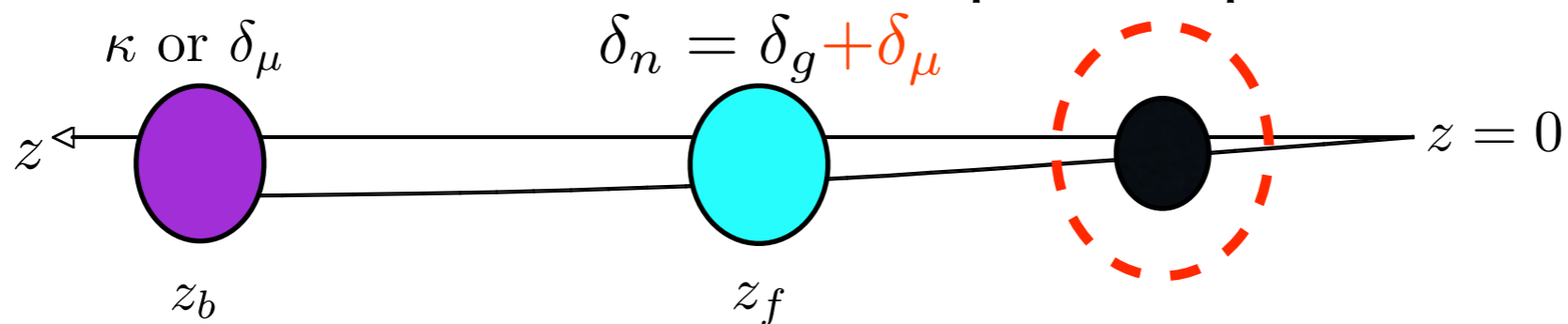
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- A lot of interesting open questions
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 - ✓ stability of the found solutions
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Thank you