

# Interactions between Superconductivity and Quantum Criticality in CeCoIn<sub>5</sub>, URhGe and UCoGe

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# My PhD work

## CeCoIn<sub>5</sub>

- Transport: Resistivity under magnetic field.

### **Field induced QCP**

- Analysis of the upper critical field.

### **Effect of magnetic fluctuations on SC**

# My PhD work

## CeCoIn<sub>5</sub>

- Transport: Resistivity under magnetic field.

### **Field induced QCP**

- Analysis of the upper critical field.

### **Effect of magnetic fluctuations on SC**

## Ferromagnetic superconductors URhGe & UCoGe

- First Thermal conductivity measurements

### **Bulk superconducting transition**

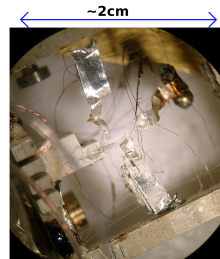
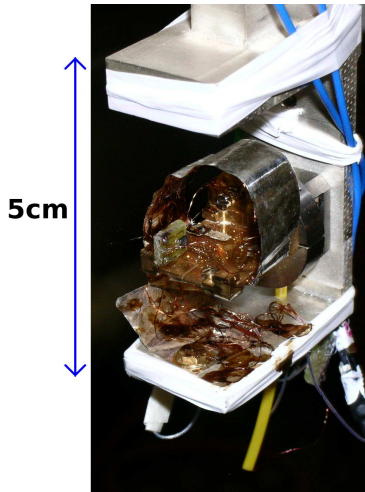
Other low T contributions than  $e^-$ , magnetic fluctuations?

Two band superconductivity?

Large and anisotropic thermoelectric power

## Experimental setups

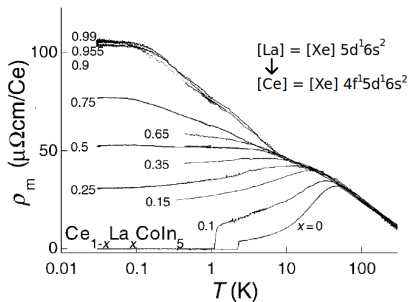
Low temperature (8mK) high field (8.5T) resistivity on CeCoIn<sub>5</sub>  
First low T thermal conductivity measurement on URhGe and UCoGe



Design of 2 new setups with:

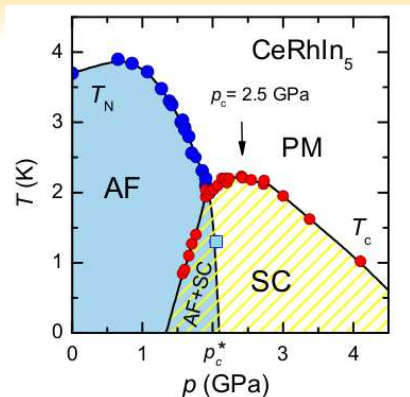
- rotating stage
- sample holder in Ag to allow high field measurements
- low temperature transformer

# Heavy Fermions



S. Nakatsuji *et al.*: Phys. Rev. Lett., 89, 106402 (2002)

- Large effective mass,
- Proximity to magnetic phase transition.



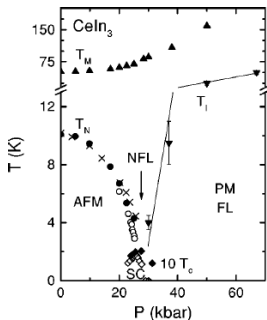
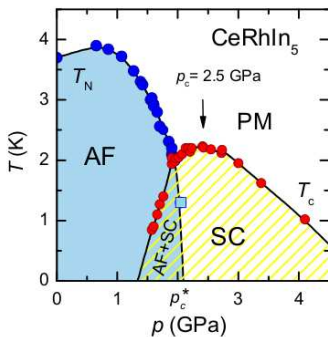
G. Knebel *et al.*: J. Phys. Soc. Jpn., 77, 114704 (2008)

## Quantum Critical Points (QCP)

- second order phase transition at  $T = 0$
- Characterized by critical exponents, effective dimension  $d + z$ ,  $z \in [1, 3]$
- Non Fermi Liquid (Fermi liquid region vanishes at QCP)

Experimentally:

- $\rho(T) = AT^2 + \rho_0$  ( $T < T_{FL}$ ),  $\rho(T) \propto T$  ( $T \gg T_{FL}$ )
- $T_{FL} \rightarrow 0$  at QCP
- $A$  diverges at QCP



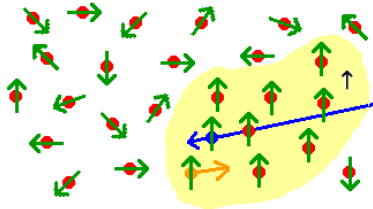
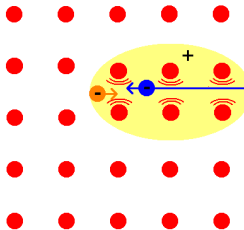
G. Knebel *et al.*: J. Phys. Soc. Jpn., 77, 114704 (2008)

G. Knebel *et al.*: Phys. Rev. B, 65, 024425 (2001)

Interactions between Superconductivity and Quantum Criticality in CeCoIn<sub>5</sub>, URhGe and UCoGe

# Superconductivity

$$T_{SC} = \Omega \exp\left(\frac{-1.04(1+\lambda)}{\lambda - \mu^*(1+0.62\lambda)}\right) \quad \lambda = N(E_F)V$$



$$V(\vec{r}, t) = \underbrace{ee'g_e^2\chi_e(\vec{r}, t)}_{\text{Charges interactions}} + \underbrace{\vec{s} \cdot \vec{s}'g_s^2\chi_s(\vec{r}, t)}_{\text{Spins interactions}}$$

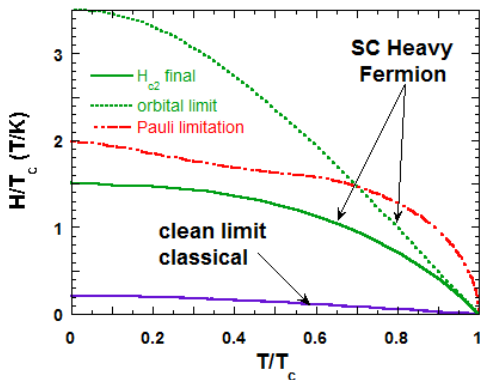
- At a magnetic QCP soft modes re-enforced  $\lambda$
- What is the pairing mechanism? → Experimental probe of  $\lambda$ ?

## Upper critical field $H_{c2}$

$$H_{c2} \rightarrow \lambda$$

$T_{SC}$  under field is limited by two mechanisms:

- Kinetic energy, (given by  $\frac{1}{2m}(p - e\vec{A})^2$ ):  $H_{Orbital} \propto \left(\frac{T_{SC}}{v_F}\right)^2$
- Zeeman splitting:  $H_{Pauli} \cong \frac{\Delta}{g\mu_B}$



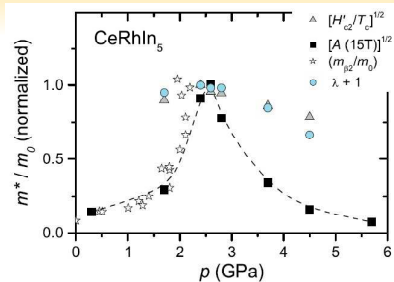
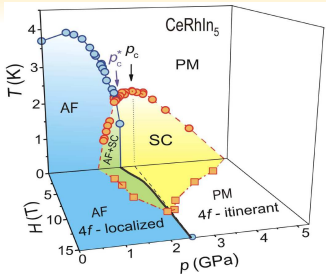
Parameters:

- effective mass:  $v_F \propto 1/m^*$
- gyromagnetic ratio:  $g$
- characteristic energy scale:  $\Omega$
- coupling constant:  $\lambda$ 
  - $T_{SC}$
  - $m^* = m_b(1 + \lambda)$
  - $H_{Pauli} \cong \frac{\Delta}{g\mu_B}$   
 $\Rightarrow H_{Pauli}/T_{SC} \nearrow$  if  $\lambda \nearrow$

J. P. Brison: Habilitation à Diriger des Recherches (1997)

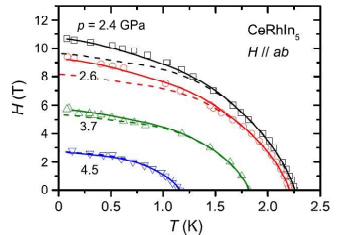


# Phase diagram of CeRhIn<sub>5</sub>

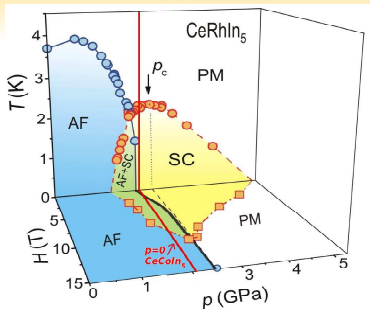


- In CeRhIn<sub>5</sub> one critical pressure  $P_c = 2.5$  GPa.
- $H_{c2}$  can be fitted with:
  - $\lambda$  maximum at  $P_c$ ,
  - $\Omega$  constant,
  - $g$  smoothly evolves with  $p$
  - and  $v_F$  only depend on  $\lambda$ .

G. Knebel *et al.*: J. Phys. Soc. Jpn., 77, 114704 (2008)

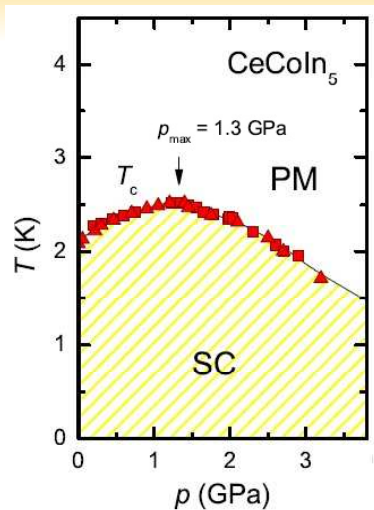


# Phase diagram of CeRhIn<sub>5</sub> & CeCoIn<sub>5</sub>



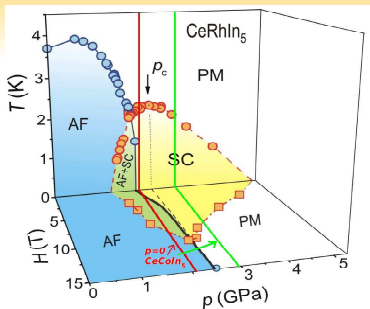
G. Knebel *et al.*: J. Phys. Soc. Jpn., 77, 114704 (2008)

- CeCoIn<sub>5</sub>  $p = 0 \Leftrightarrow$
- CeRhIn<sub>5</sub>  $p \cong 2\text{GPa}$



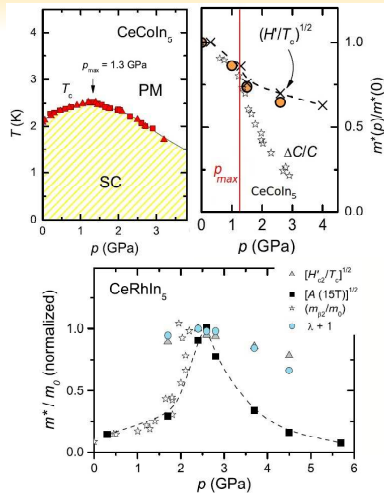
G. Knebel *et al.*: Phys. Status Solidi B, 247, 557 (2010)

# Phase diagram of CeRhIn<sub>5</sub> & CeCoIn<sub>5</sub>



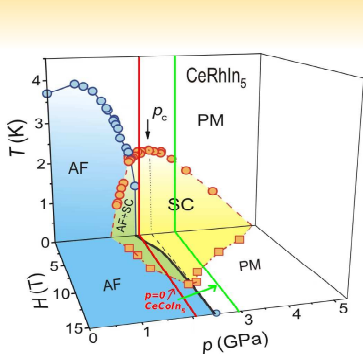
G. Knebel *et al.*: J. Phys. Soc. Jpn., 77, 114704 (2008)

- CeCoIn<sub>5</sub>  $p = 0 \Leftrightarrow$   
CeRhIn<sub>5</sub>  $p \cong 2\text{GPa}$
- No sign of QCP under  $p$



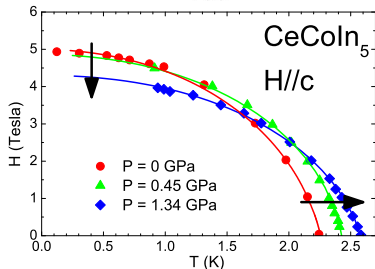
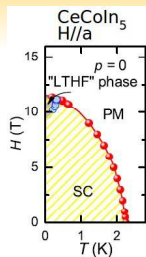
G. Knebel *et al.*: Phys. Status Solidi B, 247, 557 (2010)

# Phase diagram of CeRhIn<sub>5</sub> & CeCoIn<sub>5</sub>



G. Knebel *et al.*: J. Phys. Soc. Jpn., 77, 114704 (2008)

- No AFM phase detected but close to AFM (FFLO/Q-phase, Cd doping, ...)
- Proximity to a field induced QCP  
 $H \parallel \vec{c}$

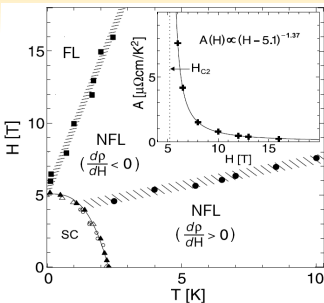


G. Knebel *et al.*: Phys. Status Solidi B, 247, 557 (2010)

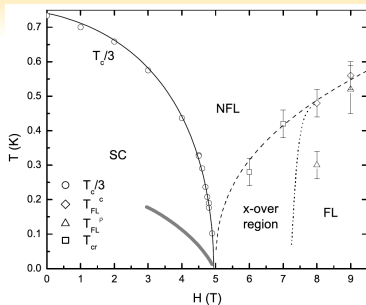
C. F. Miclea *et al.*: Phys. Rev. Lett., 96, 117001 (2006)

Interactions between Superconductivity and Quantum Criticality in CeCoIn<sub>5</sub>, URhGe and UCoGe

## Previous experiments; Field induced QCP



J. Paglione *et al.*: Phys. Rev. Lett., 91, 246405 (2003)

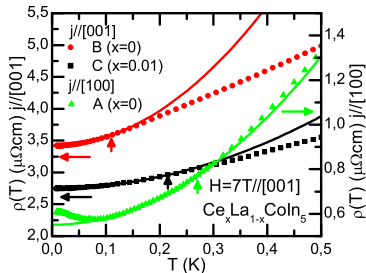
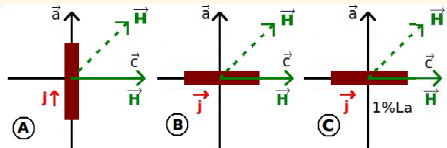


A. Bianchi *et al.*: Phys. Rev. Lett., 91, 257001 (2003)

- QCP obtained from limit of the Fermi-liquid domain. ( $\rho(T) = AT^2 + \rho_0$ )
- $H(\text{QCP}) = H_{c2}$ ?
- magneto-resistance problems at low temperatures ( $\omega_c \tau > 1$ ),
- specific heat data only available down to  $\sim 80\text{mK}$ . At  $100\text{mK}$  70% signal from hyperfine contribution.

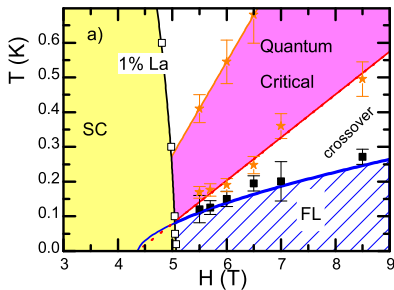
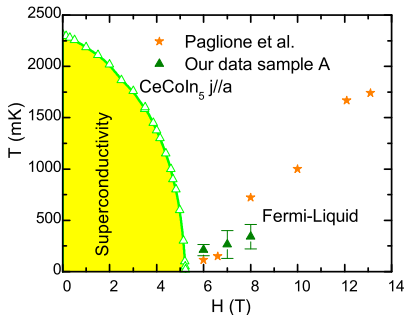
# This experiment

- 3 samples
  - A CeCoIn<sub>5</sub>  $\vec{j} \parallel$  a-axis
  - B CeCoIn<sub>5</sub>  $\vec{j} \parallel$  c-axis
  - C Ce<sub>0.99</sub>La<sub>0.01</sub>CoIn<sub>5</sub>  $\vec{j} \parallel$  c-axis
- 2 fields orientations:
  - H  $\parallel$  c-axis
  - H 45° c-axis
- $\vec{j} \parallel$  c-axis more sensitive to NFL [9]
- $\omega_c \tau < 1$  sample B, C and for 3 samples when H 45° c-axis
- Low noise high resolution
- $\rho(T) = AT^2 + \rho_0$  ( $T < T_{FL}$ )
  - $T_{FL}$  determined from  $\chi^2$
  - A



M. A. Tanatar *et al.*: Science, 316, 1320 (2007)

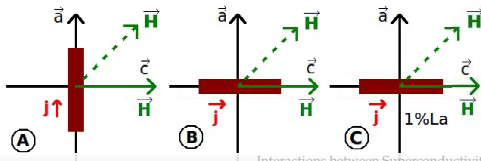
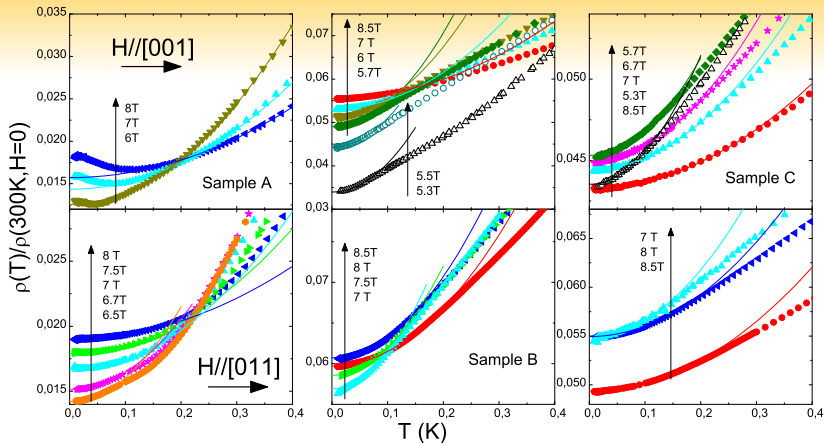
## Phase diagrams $H \parallel c$ -axis



J. Paglione *et al.*: Phys. Rev. Lett., 91, 246405 (2003)

- Previous results reproduced with unfavourable geometry ( $\vec{j} \parallel \vec{c}$ -axis),
- No true coincidence between  $H_{c2}(0)$  and  $H_{QCP}$ .

## All curves

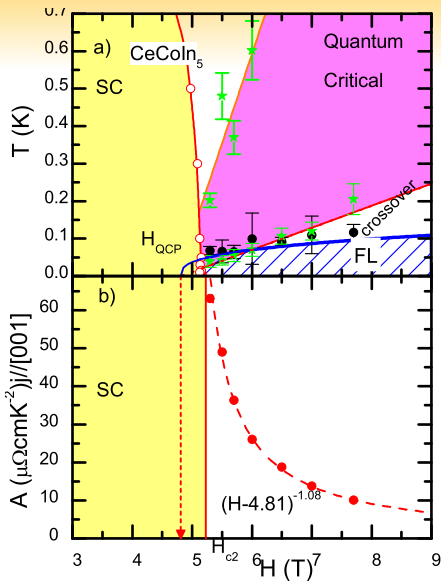


- 5 sets of data can be used to fit

- $A \propto |H - H_{QCP}|^{-\alpha}$
- $T_{FL} \propto |H - H_{QCP}|^{z/2}$

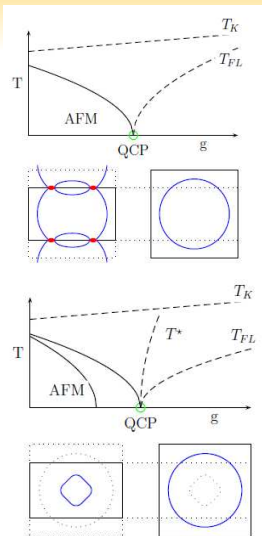


# Dynamical exponent



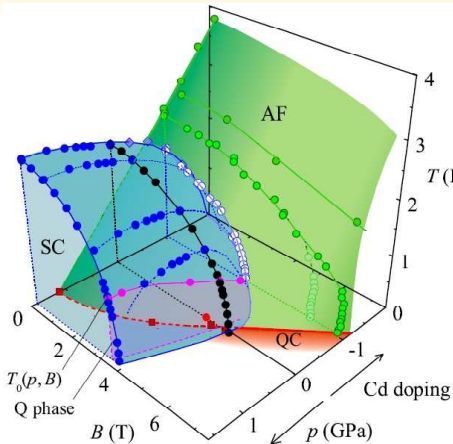
- $A \propto |H - H_{QCP}|^{-\alpha}$ . Fits found  $\alpha = 1.09 \pm 0.37$
- $T_{FL} \propto |H - H_{QCP}|^{z/2}$
- Hertz-Millis theory for AFM  $z = 2$
- for coincidence of divergence of  $A$  coefficient and  $T_{FL} = 0$  we need  $z = 1.16 \pm 0.14$
- Single energy scale:
  - $\rho(T) = a(T/T_0)^2 + \rho_0 \rightarrow A = a/T_0^2$
  - $A \propto |H - H_{QCP}|^{-\alpha}$   
 $\rightarrow T_0 \propto |H - H_{QCP}|^{\alpha/2}$
  - $T_{FL} \propto |H - H_{QCP}|^{z/2}$
  - $T_{FL} \propto T_0 \rightarrow \alpha = z$

## QCP points scenarios



- Divergence of  $m^*$  along hot spot directions
  - Mostly developed theory (Hertz-Millis-Moriya) Predicts  $z = 2$ ,  $\rho(T) \propto T^{3/2}(3d), \dots$
  - Other models: Disorder (Rosch et al.), Kondo Necklace model (Reyes et al.), ...
- 
- Complete reconstruction of the Fermi surface at QCP: divergence of  $m^*$  in all directions.
  - Few theoretical predictions.

# CeCoIn<sub>5</sub> Phase diagram suggested by Zaum et al.



S. Zaum *et al.*: arXiv:1010.3175v1 (2010)  
 F. Ronning *et al.*: Phys. Rev. B, 73, 064519 (2006)

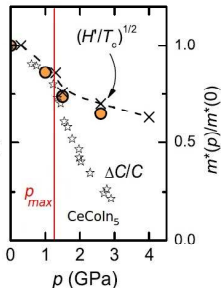
## Conclusion

Proximity between QCP and  $H_{c2}$  at  $p = 0$  is a coincidence

- divergence of  $A$  under  $p$  (Ronning *et al.*)
- Hall effect anomaly (Singh *et al.*)

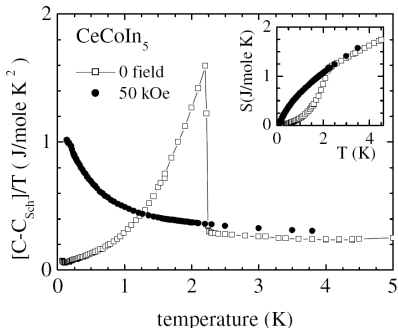
$T$  (K) S. Singh *et al.*: Phys. Rev. Lett., 98, 057001 (2007)

- How to explain  $H_{c2}$ ?



G. Knebel *et al.*: Phys. Status Solidi B, 247, 557 (2010)

# Decoupling between maximum of $T_{SC}$ and maximum of $\lambda$ : Magnetic Pair breaking mechanisms



- In CeCoIn<sub>5</sub> at  $p = 0$ ,  $\Delta C/C \cong 4.5$ , BCS value 1.43,
- Kos et al. and Bang et al. explain this jump with magnetic pair breaking effect. [13]  $T^* \sim 6K \rightarrow T_{SC} = 2.3K$  (coupling between SC and magnetization needed),
- Monthoux et al. show for SC with strong coupling & AFM pairing
  - $\Rightarrow$  pair breaking associated to the QCP
  - $\Rightarrow$  the maximum of  $T_{SC}$  is not at the QCP

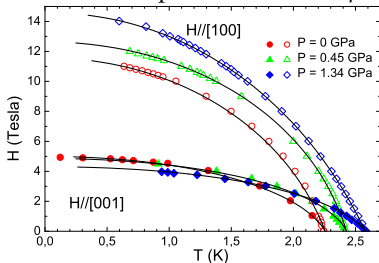
S. Kos *et al.*: Phys. Rev. B, 68, 052507 (2003)

Y. Bang and A. V. Balatsky: Phys. Rev. B, 69, 212504 (2004)

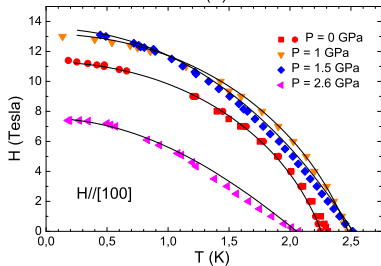
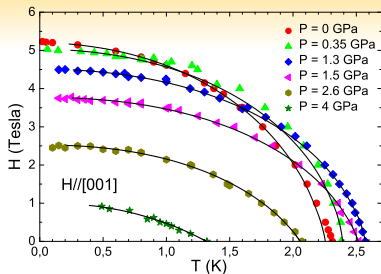
P. Monthoux and G. Lonzarich: Phys. Rev. B, 63, 054529 (2001)

## Data of $H_{c2}$

First measurements from Miclea *et al.* to  $p_{max}$  and recent measurement of Knebel *et al.* up to more than  $2 \cdot p_{max}$



C. F. Miclea *et al.*: Phys. Rev. Lett., 96, 117001 (2006)



G. Knebel *et al.*: J. Phys.: Condens. Matter, 16, 8905 (2004)

## Parameters of the model

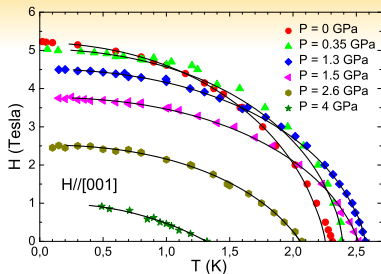
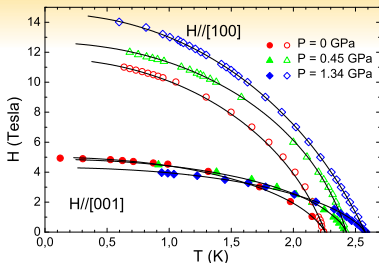
$T_{SC}(p, H)$  fitted with an Eliashberg model

- We include magnetic pair breaking in the calculation:
  - $T_{SC}(H = 0)/\Omega = F(\lambda, \mu^*, T_M)$
  - And defined  $T^* = \Omega F(\lambda, \mu^*, T_M = 0)$
- + Orbital and paramagnetic limit for field dependence.

$\Omega$	const.
$\mu^*$	const. $\cong 0.1$
$\lambda$	vary with $p$
$T_M$	vary with $p$ , $T_M = 0$ at $p = 4\text{GPa}$
$v_F$	vary with $p$ as: $v_F = v_{F0}(1 + \lambda(p = 0))/(1 + \lambda(p))$
$g$	vary with $p$ and field orientation

- $\lambda(p)$  given by  $v_F(p) \propto T_{SC} / \frac{dH_c2}{dT} |_{T=T_{SC}}$
- $\Omega$ ,  $T_M(0)$  and  $\lambda_0$  are related through the condition  $T^*(p = 0) = 6K \cdot (\Delta C/C)$
- $T_M(p)$  given by  $T_{SC}(p)$

L. N. Bulaevskii *et al.*: Phys. Rev. B, 38, 11290 (1988)

Fits of  $H_{c2}$ 

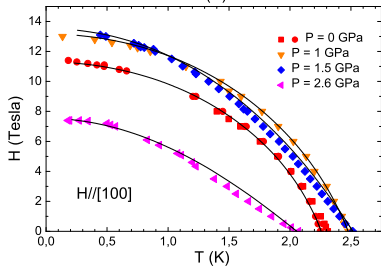
## fixed parameters

$$v_{F0}, \Omega, \mu^*$$

## Pressure dependent parameters

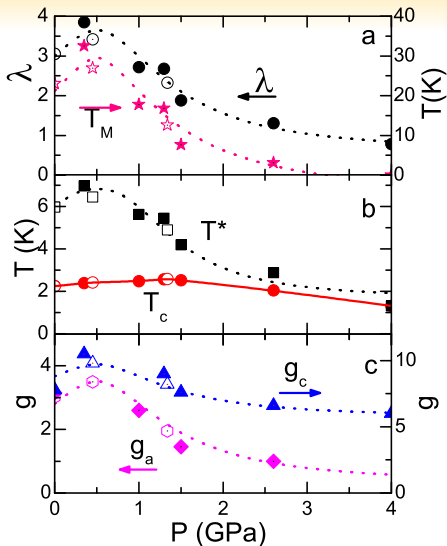
$$\lambda, T_M, g_a, g_c$$

C. F. Miclea *et al.*: Phys. Rev. Lett., 96, 117001 (2006)

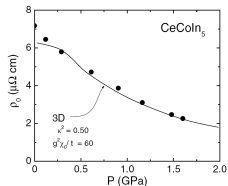
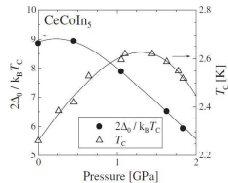


G. Knebel *et al.*: J. Phys.: Condens. Matter, 16, 8905 (2004)

## Resulting parameters



- Maximum of  $g_a$ ,  $g_c$ ,  $T^*$ ,  $\lambda$  and  $T_M$  around 0.4 GPa in agreement with QCP at this pressure,

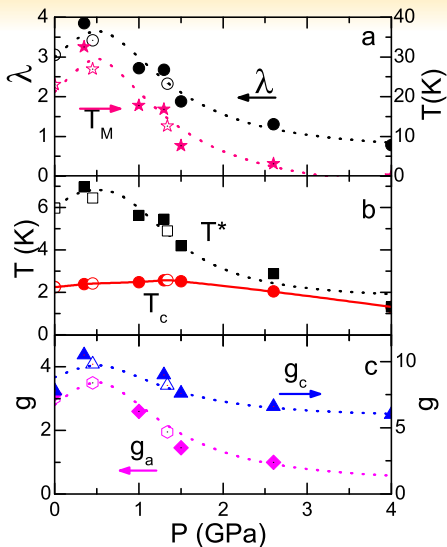


M. Yashima *et al.*: J. Phys. Soc. Jpn., 73, 2073 (2004)

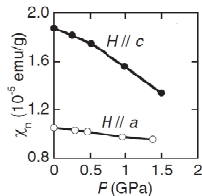
M. Nicklas *et al.*: J. Phys.: Condens. Matter, 13, L905 (2001)



## Resulting parameters

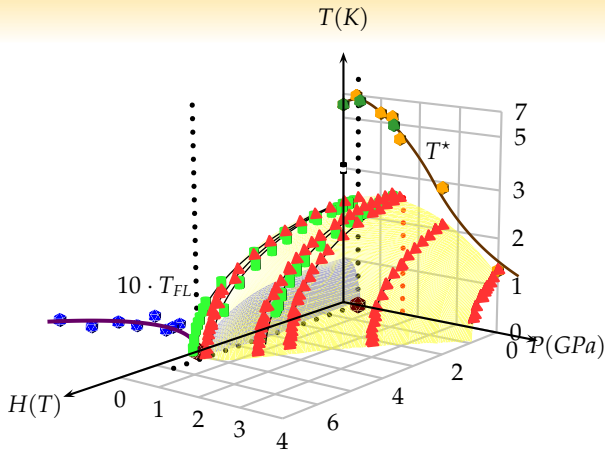


- Maximum of  $g_a$ ,  $g_c$ ,  $T^*$ ,  $\lambda$  and  $T_M$  around 0.4 GPa in agreement with QCP at this pressure,
- Relatively large value of  $g_c \cong 8$  (could be reduced to 6 with a lower value of  $\lambda_0$ )
  - Difference between  $\lambda_z \rightarrow m^*$  &  $\lambda_{\Delta} \rightarrow T_{SC}$ ,
  - Contribution of localized moment may leads to large  $g$ .



T. Tayama *et al.*: Journal of the Physical Society of Japan, 74, 1115 (2005)

## Proposed phase diagram

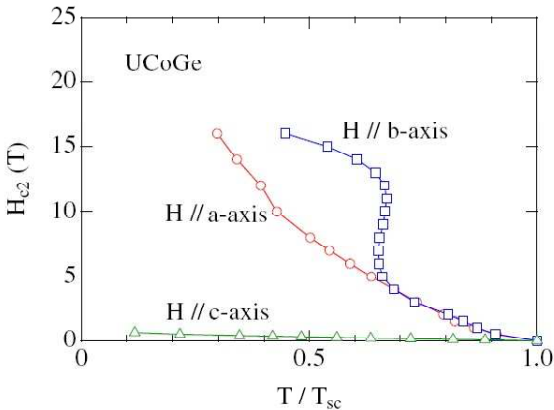


### Conclusion

Features of CeCoIn<sub>5</sub> ( $\Delta C/C$ , pressure dependence of:  $T_{SC}$ ,  $\Delta_0/T_{SC}$ , paramagnetic limit, ...) in this scenario.

Phase diagram of CeCoIn<sub>5</sub> is a paradigm of an (almost 2D) strongly coupled anti-ferromagnetically mediated superconductor.

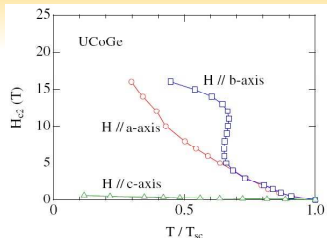
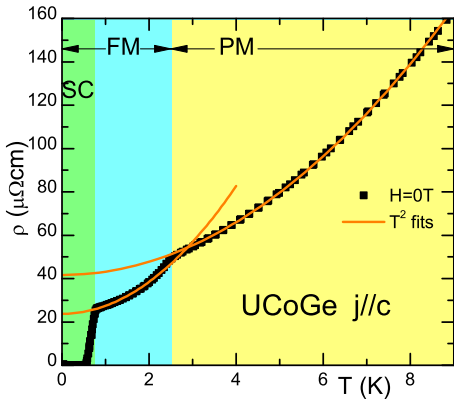
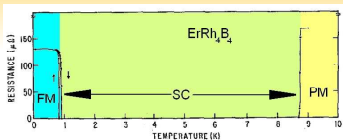
## Ferromagnetic superconductors



- Upper critical field of Ferromagnetic superconductors?

D. Aoki *et al.*: J. Phys. Soc. Jpn., 78, 113709 (2009)

# Introduction

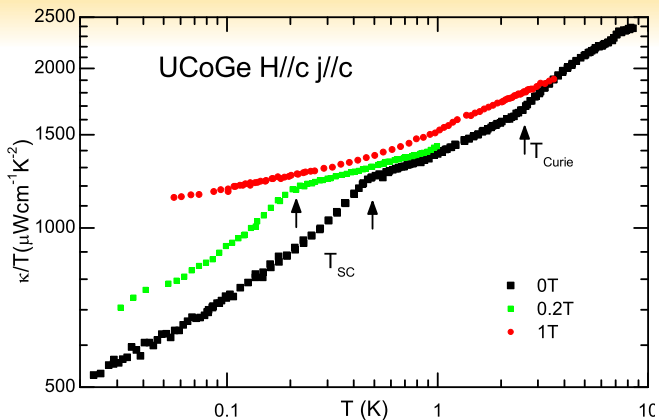


- Co-existence SC+Ferro  $\rightarrow$  Triplet superconductivity,
- Unusual  $H_{c2}$ : Re-entrance, positive curvature, strong angular dependence.

W. A. Fertig *et al.*: Phys. Rev. Lett., 38, 987 (1977)

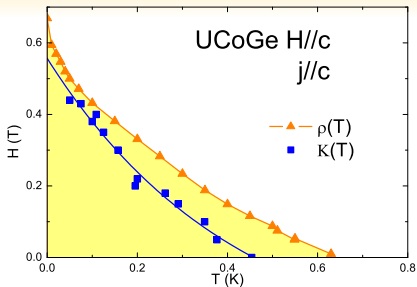
D. Aoki *et al.*: J. Phys. Soc. Jpn., 78, 113709 (2009)

## Measured Thermal Conductivity

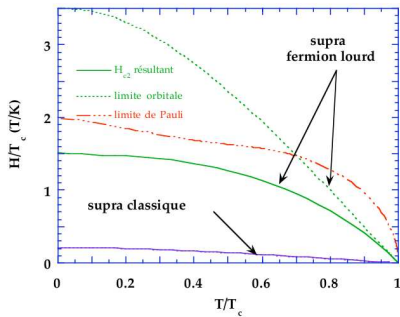


- Large residual term,
- sharp superconducting phase transition,
- Sample Quality?

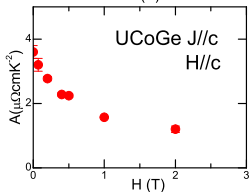
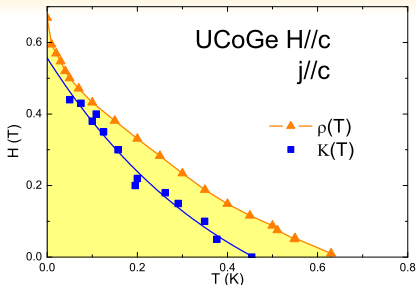
# Superconducting Phase diagram $H \parallel \vec{c}$ -axis



- Unusual shape of  $H_{c2}$



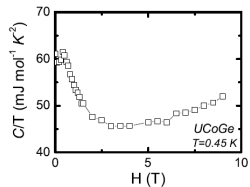
# Superconducting Phase diagram $H \parallel \vec{c}$ -axis



- Unusual shape of  $H_{c2}$  requires either:
  - increase of  $\lambda$
  - decrease of orbital limitation
- Possible explanations:
  - Increase  $m^*$  meta-magnetic transition  
→ increase  $\lambda$  (Miyake et al.),
  - Field  $\perp$  moment superconducting pair → increase  $\lambda$  (Mineev)

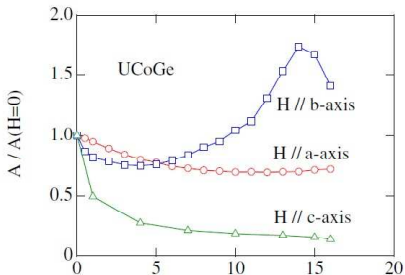
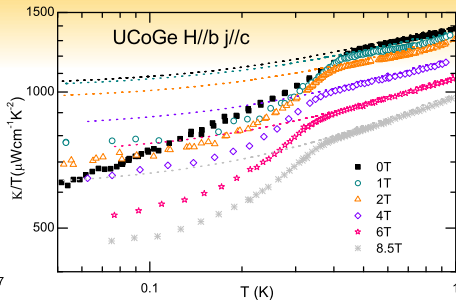
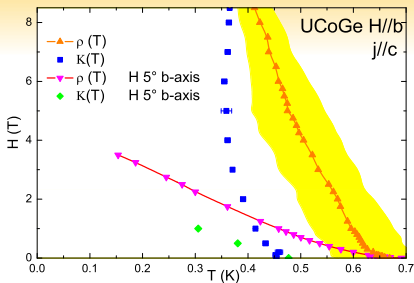
A. Miyake *et al.*: J. Phys. Soc. Jpn., 77, 094709 (2008)

V. P. Mineev: arXiv:1011.3753v1 (2010)



F. Hardy *et al.*: to be published in: J. Phys. Soc. Jpn. (2011)

# Superconducting Phase diagram $H \parallel \vec{b}$ -axis



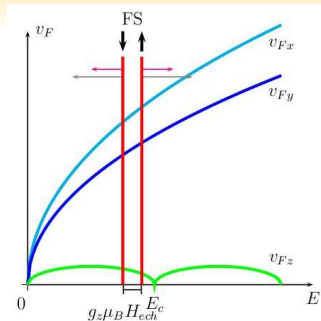
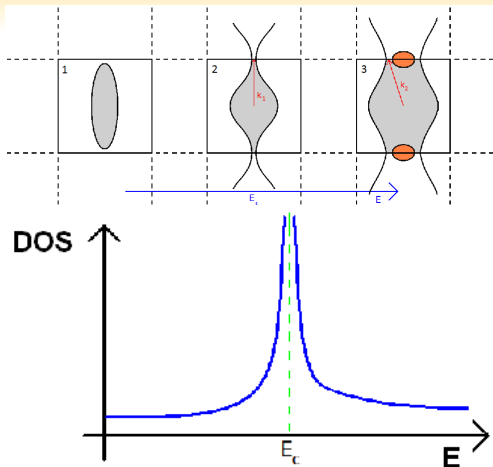
We confirm by bulk measurements:

- strong angular dependence
- re-entrance and positive curvature for  $H_{c2}$ 
  - importance of rotation mechanism

D. Aoki *et al.*: J. Phys. Soc. Jpn., 78, 113709 (2009)

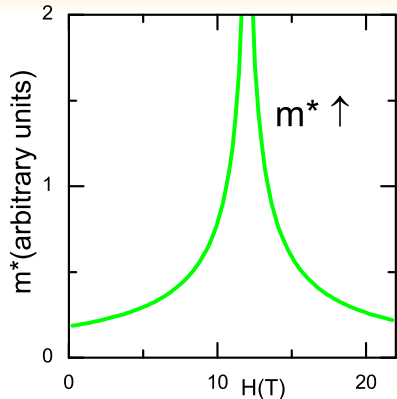
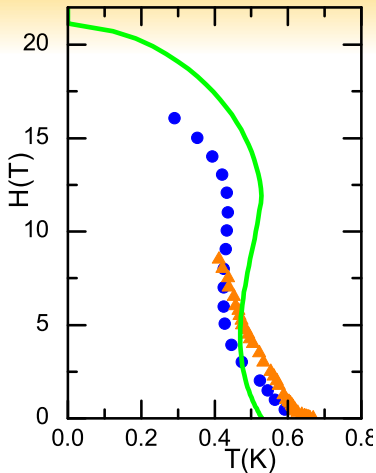


# Model of Lifshitz phase transition



- Developed from an idea of (and with) Vincent Michal and V. Mineev
- Lifshitz phase transition = topological anomaly on FS
- cyclotronic  $v_F \rightarrow 0$  for some H orientations

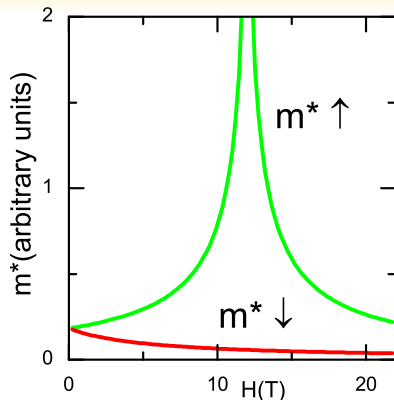
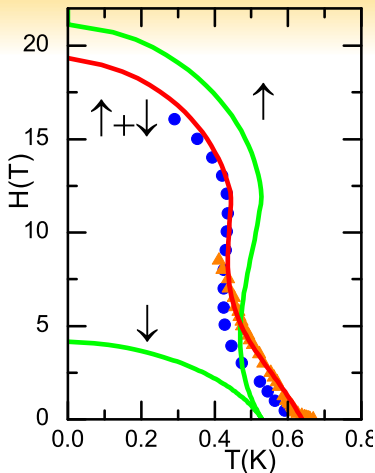
## Fits with a divergence of the effective mass



D. Aoki *et al.*: J. Phys. Soc. Jpn., 78, 113709 (2009)

$$m^* = m_0 \cdot \log\left(1 + \alpha \left| \frac{H_{\text{crit}}}{H - H_{\text{crit}}} \right| \right)$$

## Fits with a divergence of the effective mass



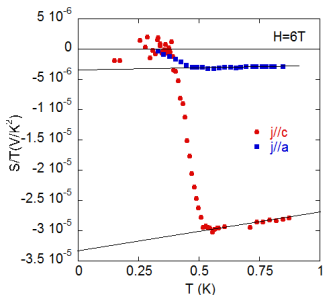
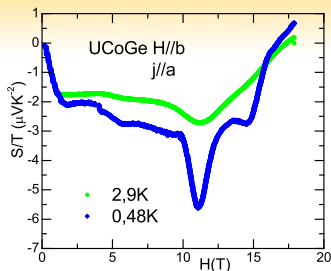
D. Aoki *et al.*: J. Phys. Soc. Jpn., 78, 113709 (2009)

$$m^* = m_0 \cdot \log\left(1 + \alpha \left| \frac{H_{\text{crit}}}{H - H_{\text{crit}}} \right| \right)$$

- $\alpha$  size of the "S" ( $\alpha = 0.2$ )

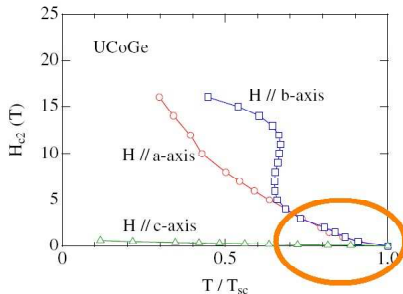
## Other experimental support for a Lifshitz scenario

- Strong increase of thermoelectric power (TEP) at  $H_R$  unrelated to FM phase (measurements done with L. Malone at LNCMI)
- Strong anisotropy on TEP  
 $\vec{j} \parallel \vec{c}\text{-axis} \cong -30, \vec{j} \parallel \vec{a}\text{-axis} \cong -3$   
 (measurements done with L. Malone at LNCMI)



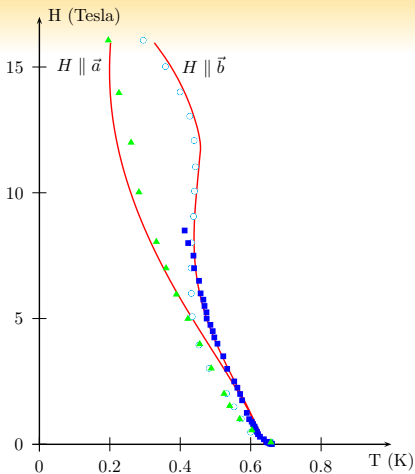
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 (measurements done with L. Malone at LNCMI)
- 2D character of the compound observed from slope  $dH_{c2}/dT$
- Small specific heat  $\gamma \cong 50$  and small Fermi velocity ( $dH_{c2}/dT$  large)  $\rightarrow$  small number of quasi-particles with large effective masses.  $\rightarrow$  Small FS pockets of heavy carriers



D. Aoki *et al.*: J. Phys. Soc. Jpn., 78, 113709 (2009)

## Fits of upper critical field



- Shape of  $H_{c2}$  can be reproduced for:
  - $H \parallel \vec{b}$ -axis  $H_{crit.} = 12\text{T}$
  - $H \parallel \vec{a}$ -axis  $H_{crit.} = 30\text{T}$
  - $\chi_b \cong 2\chi_a$  (Huy et al.)
- Does not explain  $H_{c2}$  for  $H \parallel \vec{c}$ -axis
  - Pair breaking like in CeCoIn<sub>5</sub>?
- However suppression of orbital limitation must happen in case of Lifshitz phase transition and gives an explanation for the shape of  $H_{c2}$ .

N. T. Huy *et al.*: Phys. Rev. Lett., 100, 077002 (2008)

# Conclusion and Prospectives

## CeCoIn<sub>5</sub>

- no true QCP at  $H_{c2}(0)$
- Field dependence of  $T_{FL}$  suggests  $z = 1$  QCP type?
- Inclusion of pair breaking due to magnetic fluctuations explain the SC pressure phase diagram.

L. Howald *et al.*: Journal of the Physical Society of Japan, 80, 024710 (2011)

## Prospectives

- confirm position QCP at 0.4GPa with more measurements of  $H_{c2}$ ,
- dHvA measurements to get more information on the type of QCP,
- Study of the pressure field phase diagram of other compounds with large strong coupling constant: NpPd<sub>5</sub>Al<sub>2</sub>?

## Ferromagnetic superconductors

- First thermal conductivity and first bulk measurements of SC!
- Confirmation of the unusual curvature of  $H_{c2}$  by bulk measurements.
- We propose a new scenario to explain the "re-entrance" of SC.

## Prospectives

- Lots to do... Test different qualities samples / other geometry, ...

# Thank you for your attention