Théorie des Cordes et Applications Phénoménologiques et Cosmologiques

Soutenance de Thèse

Ioannis G. Florakis

Laboratoire de Physique Théorique Ecole Normale Supérieure Paris

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Papers

Based on published papers:

- I.F., C. Kounnas, Nucl. Phys. B820, 237-268 (2009).
- I.F., C. Kounnas, N. Toumbas, Nucl. Phys. B834, 273-315 (2010).
- I.F., C. Kounnas, H. Partouche, N. Toumbas, Nucl. Phys. B844, 89-114 (2011).
- A. E. Faraggi, I.F., T. Mohaupt, M. Tsulaia, Nucl. Phys. B848, 332-371 (2011).

String Cosmology

The conventional description of spacetime is expected to break down at scales approaching the Planck length.

- Classical Cosmology and General Relativity, if extrapolated backwards in time predict an unphysical Initial Singularity "Big Bang" : $\rho \rightarrow \infty$.
- Effects of quantum gravity become dominant and need to be properly taken into account \rightarrow e.g. QM particle in singular potential.

String Theory is a consistent theory of quantum gravity and can be used to probe this early phase of the universe.

• One needs to study cosmological solutions at the string level.

String Cosmology

Consider vacua with spontaneously broken SUSY, where the 1-loop effective potential $V_{\text{eff.}}(\mu_I) \neq 0$ is finite as a function of the moduli μ_I (or thermal vacua with finite free energy).

- The non-vanishing 1-loop amplitude then triggers a backreaction on the underlying background.
- One is forced to correct the initial background in order to cancel the dilaton tadpole and restore conformal invariance at 1-loop.
- This cancellation of the conformal anomaly at 1-loop gives rise to a cosmological evolution, with the moduli and temperature acquiring time dependence.

In principle, the finiteness of the 1-loop amplitude allows one to study the emerging cosmology within the framework of perturbative string theory.

- This attractive programme constitutes the core of String Cosmology.
- However, even so, there remain two major obstacles to its realization:
 - Hagedorn divergence
 - ► Initial Singularity [Gasperini, Veneziano, ...]

Thermal String Vacua

Well-known observation : a thermal theory at finite temperature is *formally* equivalent to a (cold) theory in one less dimension, with spontaneously broken SUSY.

- In the thermal trace description of the thermal system, Hagedorn divergences occur because of the exponential growth in the density of single-particle states as a function of their mass.
- In the Euclidean picture, where time is compactified on a cycle of radius $R_0 = 1/(2\pi T)$, certain string states winding the Euclidean time cycle become tachyonic when the thermal modulus exceeds a critical (Hagedorn) value.

Thus, the Hagedorn divergence can be interpreted as IR-instability of the Euclidean background and the phase transition is driven by tachyon condensation.

- The presence of such condensates injects non-trivial winding charge ⟨O_n⟩ ≠ 0 into the vacuum.
- This property will be crucial for our attempts to resolve these instabilities.

Hagedorn Phase Transition

Attempts to probe the early, high-temperature (and/or high-curvature) phase of the universe $R \sim \ell_s$, typically hit upon Hagedorn (or tachyonic) instabilities, preventing a perturbative treatment of the backreaction.

- Not true pathologies of string theory \rightarrow rather, signals of phase transition to new vacuum.
- These IR instabilities can be removed by appropriately deforming the Euclidean background.
- One way is to condense the thermal winding tachyon → hard problem! [Antoniadis, Kounnas '91], [Atick, Witten '88]
- An alternative way would be to directly construct stable (non-tachyonic) thermal vacua with non-trivial winding charge (O_n) ≠ 0, that would directly correspond to the resulting vacua describing the new phases after the condensation.

[IF, Kounnas, Toumbas '10] , [Angelantonj, Kounnas, Partouche, Toumbas '08]

Proposal: MSDS Vacua

Recent discovery: interesting class of 2d string vacua, exhibiting a:

Massive Spectral boson-fermion Degeneracy Symmetry (MSDS)

[Kounnas '08], [IF, Kounnas '09], [IF '10]

"almost a supersymmetry"

- Tachyon-finite, non-supersymmetric compactifications on M² × K⁽⁸⁾, with K a particular ĉ = 8 internal CFT.
- They live at special points in the bulk of the moduli space.
- MSDS points correspond to all radii of internal *T*⁸-torus taken at the fermionic point.
- New local superconformal algebra: enhancement of SUSY current algebra.
- Presence of chiral operator $Q_{MSDS} = \oint \frac{dz}{2\pi i} j_{MSDS}(z)$, transforming the massive bosonic into the massive fermionic tower of states, while leaving massless states invariant.

Basic properties of MSDS Vacua

Degeneracy of states as a function of mass M:

$$n_B(M) - n_F(M) = \begin{cases} \neq 0 & , \text{ for } M = 0 \\ 0 & , \text{ for } M > 0 \end{cases}$$

- Massive spectrum is bose-fermi degenerate.
- Similar to the V₈ S₈ = 0 identity between the SO(8) characters in ordinary SUSY compactifications, in MSDS Vacua the degeneracy is manifested in terms of the following chiral identity between the Jacobi θ-characters of SO(24):

$$\frac{1}{2}\sum_{a,b=0,1}(-)^{a+b} \frac{\theta[_{b}^{a}]^{12}}{\eta^{12}} = V_{24} - S_{24} = 24.$$

- Asymptotic supersymmetry requirements are trivially satisfied \rightarrow absence of tachyons at MSDS point. [Kutasov, Seiberg '90]
- Complete classification of Z^N₂-orbifold constructions. [IF, Kounnas '09]

Conformal Origin

Ordinary Supersymmetry s.f. operators:

$$j(z) = e^{-\phi/2 - \frac{i}{2}H_0} C_8(z) , \quad \tilde{j}(z) = e^{-\phi + \frac{i}{2}H_0} S_8(z) .$$

Action of the s.f. operator $S_N(z)$ on spinorial state $C_N(z)$, $(N \in 4\mathbb{Z})$:

$$\begin{split} S_{N}(z) \ C_{N}(w) \ &= \ \frac{1}{\sqrt{2}} \frac{\gamma^{a} \psi^{a}(w)}{(z-w)^{N/8-1/2}} \\ &+ \frac{1}{2\sqrt{2}} \frac{\gamma^{abc} \psi^{a} \psi^{b} \psi^{c}(w) + \gamma^{a} \partial \psi^{a}(w)}{(z-w)^{N/8-3/2}} \\ &+ \frac{1}{4\sqrt{2}} \frac{\gamma^{abcde} \psi^{a} \psi^{b} \psi^{c} \psi^{d} \psi^{e} + \gamma^{abc} \partial (\psi^{a} \psi^{b} \psi^{c}) + \gamma^{a} \partial^{2} \psi^{a}}{(z-w)^{N/8-5/2}} \ + \ \dots \end{split}$$

• Weights match only if: $\frac{N}{16} = \frac{2M+1}{2} + \frac{1}{2}\mathbb{Z}$, where *M*: order of massive descendant $\rightarrow N \in 8\mathbb{Z}$.

• States in C_N fit within single descendant if: $2^{N/2-1} = \sum_{p=0}^{M} {N \choose 2p+1}$.

• Only solutions: (N, M) = (8, 0) and (N, M) = (24, 1).

Soutenance de Thèse

Goals and Motivations

MSDS s.f. operators:

$$j(z) = e^{\phi/2 - \frac{i}{2}H_0}S_{24}(z) , \quad \tilde{j}(z) = e^{\phi/2 + \frac{i}{2}H_0}C_{24}(z) .$$

 $\{Q, Q\}$ anticommutator:

$$\begin{split} j_{\alpha}(z)j_{\beta}(0) &= \dots + \frac{e^{\phi}}{\sqrt{2}z} \Gamma_{\mu} \left[\Psi^{\mu} \left(\gamma^{abcd}_{\alpha\beta} \psi^{a} \psi^{b} \psi^{c} \psi^{d} + \gamma^{ab}_{\alpha\beta} \partial(\psi^{a} \psi^{b}) + \frac{1}{2} c_{\alpha\beta} \partial \psi^{a} \psi^{a} \right) \\ &+ \frac{1}{4} \partial \Psi^{\mu} \gamma^{ab}_{\alpha\beta} \psi^{a} \psi^{b} + \frac{1}{4} \partial^{2} \Psi^{\mu} c_{\alpha\beta} \right] + \dots \end{split}$$

$\{Q,\tilde{Q}\}$ anticommutator:

$$\begin{split} j_{\alpha}(z)\tilde{j}_{\dot{\beta}}(0) &= \dots + \frac{e^{\phi}}{4\sqrt{2}\,z} \left[\left(\Gamma_{\mu\nu}\partial(\Psi^{\mu}\Psi^{\nu}) + \frac{1}{2}\partial\Psi^{\mu}\,\Psi_{\mu} \right) \gamma^{a}_{\alpha\dot{\beta}}\psi^{a} + \Gamma_{\mu\nu}\Psi^{\mu}\Psi^{\nu} \left(\gamma^{abc}_{\alpha\dot{\beta}}\psi^{a}\psi^{b}\psi^{c} + \gamma^{a}_{\alpha\dot{\beta}}\partial\psi^{a} \right) \right. \\ &+ \gamma^{abcde}_{\alpha\dot{\beta}}\psi^{a}\psi^{b}\psi^{c}\psi^{d}\psi^{e} + \gamma^{abc}_{\alpha\dot{\beta}}\partial(\psi^{a}\psi^{b}\psi^{c}) + \gamma^{a}_{\alpha\dot{\beta}}\partial^{2}\psi^{a} \right] + \dots \end{split}$$

MSDS-Preserving Orbifolds

Assume orbifold action G leaves some MSDS charges invariant.

Order-q descendant state in A(z)-conformal family gets mapped into order-q' descendant in B(z) family:

$$j(z) \cdot [A]_{(q)}(w) \sim \sum_{q' \ge 0} \frac{[B]_{(q')}(w)}{(z-w)^{h_A-h_B+q'-q+1}}$$

- Mutual locality implies: $h_A h_B = q' q$, where $q, q' \in \mathbb{N}$.
- E.g. for G = Z^N₂, the conditions for MSDS structure are (in free fermionic notation):

$$n_L(b) = 0 \pmod{8} ,$$

$$n_L(b_i \cap b_j) = 0 \pmod{4} ,$$

$$n_L(b_i \cap b_j \cap b_k) = 0 \pmod{2} .$$

• Generalizations exist for orbifolds of higher order.

MSDS-Preserving Orbifolds

Chiral character identities in \mathbb{Z}_2 -orbifolds:

$$V_{16}O_8 - S_{16}C_8 = 16$$
$$O_{16}V_8 - C_{16}S_8 = 8$$
$$V_{16}C_8 - S_{16}O_8 = 0$$
$$O_{16}S_8 - C_{16}V_8 = 8$$

and in $\mathbb{Z}_2 \times \mathbb{Z}_2$ -orbifolds :

$$V_8 O_8 O_8 - S_8 C_8 C_8 = 8$$

$$O_8 S_8 O_8 - C_8 V_8 C_8 = 8$$

$$V_{12} O_4 O_4 O_4 + O_{12} V_4 V_4 V_4 - S_{12} C_4 C_4 C_4 - C_{12} S_4 S_4 S_4 = 12$$

$$V_{12} O_4 V_4 V_4 + O_{12} V_4 O_4 O_4 - S_{12} C_4 S_4 S_4 - C_{12} S_4 C_4 C_4 = 4$$

$$V_{12} O_4 S_4 S_4 + O_{12} V_4 C_4 C_4 - S_{12} C_4 V_4 V_4 - C_{12} S_4 O_4 O_4 = 0$$

$$V_{12} V_4 S_4 C_4 + O_{12} O_4 C_4 S_4 - S_{12} S_4 V_4 O_4 - C_{12} C_4 O_4 V_4 = 4$$

: : :

MSDS structure may also emerge in other constructions.

- Example: bosonic sector of $E_8 \times E_8$ Heterotic string on T^4/\mathbb{Z}_2 .
- Gepner map from Type II construction.
- N = 4 SCFT gives rise to analogous s.f.
- In the twisted sector: S-V duality of SO(12)-characters.
- MSDS-like identities. [Faraggi, IF, Mohaupt, Tsulaia '11]

Goals and Motivations

The problem of incorporating the backreaction due to thermal and quantum effects in MSDS vacua is highly non-trivial. First task : investigate the moduli space in the simplest class of MSDS vacua.

• Study of tree-level adiabatic marginal deformations away from the MSDS points (for sufficiently small g_s).

Goals:

- Identify classes of thermal and cold MSDS vacua that remain tachyon/Hagedorn-free under arbitrary marginal deformations.
- Establish tachyon-free trajectories continuously connecting the 2d MSDS vacua to higher-dimensional SUSY vacua via large marginal deformations.

Maximally Symmetric MSDS vacuum

Consider the Euclidean version of the 'Maximally' symmetric, Type II, MSDS compactification on T^8 :

$$egin{aligned} Z_{1 ext{-loop}} &= rac{V_2}{(2\pi)^2} \int_{\mathcal{F}} rac{d^2 au}{4 au_2^2} rac{1}{2^2} \sum_{a,b,ar{a},ar{b}} (-)^{a+b} rac{ heta [a]^4}{\eta^{12}} \, \Gamma_{(8,8)}[a,ar{b}]_{b,ar{b}}^{[a,ar{a}]} \, (-)^{ar{a}+ar{b}} rac{ar{ heta} [b]^4}{ar{ heta} [b]^4} \ &= rac{V_2}{(2\pi)^2} \, \int_{\mathcal{F}} rac{d^2 au}{4 au_2^2} \, (V_{24} - S_{24}) \, (ar{V}_{24} - ar{S}_{24}) \, . \end{aligned}$$

• At the MSDS point, the asymmetrically half-shifted $\Gamma_{(8,8)}$ lattice factorizes:

$$\Gamma_{(8,8)}[^{a\,,\,\bar{a}}_{b\,,\,\bar{b}}] = \theta[^{a}_{b}]^{8} \ \bar{\theta}[^{\bar{a}}_{\bar{b}}]^{8}.$$

• The moduli space is given by the 64 massless scalars parametrizing:

$$\frac{SO(8,8)}{SO(8)\times SO(8)},$$

modulo discrete dualities.

Maximally Symmetric MSDS vacuum

First task: identify the precice combinations of Kähler moduli that restore SUSY in the decompactification limit.

Using the triality properties of SO(8) and various discrete dualities that exist at this special point, we can bring the (8,8)-lattice to the standard Lagrangian form:

$$\Gamma_{(8,8)} \begin{bmatrix} a, \tilde{a} \\ b, \tilde{b} \end{bmatrix} = \frac{\sqrt{\det G_{\mu\nu}}}{(\sqrt{\tau_2})^8} \sum_{\tilde{m}^{\mu}, n^{\nu} \in \mathbb{Z}} e^{-\frac{\pi}{\tau_2} (G+B)_{\mu\nu} (\tilde{m}+\tau n)^{\mu} (\tilde{m}+\tilde{\tau} n)^{\nu} + i\pi \mathcal{T}},$$

with the coupling to the *R*-symmetry charges appearing entirely in the phase:

$$\mathcal{T}_{\mathrm{th.}} = \left[\ ilde{m}^0(a+ar{a}) + n^0(b+ar{b}) \
ight] + \left(\ ilde{m}^1n^1 + ilde{m}^1ar{a} + n^1b \
ight).$$

- Deformed $E_8 \times E_8$ lattice with the values of $G_{\mu\nu}$, $B_{\mu\nu}$ completely fixed by the MSDS symmetry.
- X^0 -cycle is "thermally" coupled to $F_L + F_R$, while X^1 -cycle is an independent coupling to F_R .
- Only 2 internal cycles couple to *R*-symmetry charges.

Deforming away from the MSDS point

Perturb the sigma-model by marginal operators of the current-current type:

$$\int d^2 z \,\, \lambda_{\mu
u} J^\mu(z) imes ar J^
u(ar z).$$

- Take infinite-radius limit $R_0, R_1 \to \infty$, where X^0 and X^1 cycles decompactify.
- Only (m
 ⁰, n⁰) = (m
 ¹, n¹) = (0, 0) orbits survive → R-symmetry charges effectively decouple.
- Recover a 4d, $\mathcal{N} = 8$, Type II vacuum

 $\rightarrow~$ gauging of supergravity induced by non-trivial geometrical fluxes responsible for SUSY breaking.

• Two additional large dimensions *emerge* via these deformations.

In a cosmological setting, deformation moduli acquire time-dependence.

 $\rightarrow\,$ It is tempting to consider that a 4d cosmological space is created dynamically from an initially non-singular, 2d MSDS vacuum.

• Problem: Identify such tachyon-free trajectories.

Consider Euclidean version of these models, where time is identified with one of the compact cycles $\subset T^8$ that couple to Q_I .

- Interpretation as a string thermal ensemble.
- Spin-Statistics connection and modular invariance require a specific correlation of winding numbers with spacetime spin.
- Equivalently seen as a "Sherk-Schwarz" freely-acting orbifold (−)^Fδ₀, with δ₀ being a Z₂-shift in the X⁰-direction.
- Add thermal cocycle in the 1-loop thermal amplitude:

 $e^{i\pi(\tilde{m}^0F+n^0\tilde{F})}.$

- This cycle is threaded by non-trivial "gravito-magnetic" fluxes, associated with U(1) graviphoton and axial-vector gauge fields.
- Their presence *refines* the thermal ensemble and renders the free energy finite at the MSDS point, even though it lies beyond the Hagedorn point $R_0 < R_H$.

Decompose lattice into modular orbits - Unfold \mathcal{F} :

$$\int\limits_{\mathcal{F}} \frac{d^2 \tau}{\tau_2^2} \dots [\tilde{m}^0, n^0] \to \underbrace{\int\limits_{\mathcal{F}} \frac{d^2 \tau}{\tau_2^2}}_{=0, \text{ SUSY}} \dots [0, 0] + \int\limits_{||-\text{strip}} \frac{d^2 \tau}{\tau_2^2} \dots [\tilde{m}^0 \neq 0, 0].$$

[McClain, Roth '87]

• $R_0^2 \equiv G_{00} - G_{0I}G^{IJ}G_{J0}$ must be sufficiently large in the strip, in order to ensure absolute convergence.

- Contribution of (0,0)-orbit vanishes because of (4,0)-SUSY.
- Strip integral gives free energy at 1-loop. The integrand becomes:

$$\int_{||} \frac{d^2 \tau}{\tau_2^2} (\dots) \frac{R_0}{\sqrt{\tau_2}} \sum_{\tilde{m}^0 \neq 0} e^{-\frac{\pi}{\tau_2} (R_0 \tilde{m}^0)^2} (-)^{(a+\bar{a})\tilde{m}^0} \\ \times \sum_{m_l, n^l \in \mathbb{Z}} q^{\frac{1}{2} P_L^2} \bar{q}^{\frac{1}{2} P_R^2} (-)^{\bar{b}n^1} e^{2\pi i \tilde{m}^0 (G_{0l} Q_{(M)}^l - B_{0l} Q_{(N)}^l)}$$

• 14 transverse U(1) charges $Q'_{(M)}$, $Q'_{(N)}$ are associated with the graviphoton $G_{I\mu}$ and axial vector $B_{I\mu}$.

Consider a charged scalar/spinor field ϕ at finite temperature T in the presence of a constant U(1) vacuum gauge potential A_0 .

• Theory defined in terms of Euclidean path integral:

$$Z = e^{-eta F} = \int\limits_{(ext{anti-}) ext{periodic}} \mathcal{D}\phi \ e^{-S_E[\phi]} \sim [ext{det}_{\pm}(-D^2 + m^2)]^{\mp 1}$$

with Euclidean action $S_E[\phi] = \int_0^\beta dx_0 \int d^s x \mathcal{L}$ and $D_\mu = \partial_\mu - igA_\mu$.

- Consider constant vacuum gauge potential ⟨A_µ⟩ = (A₀, 0). A v.e.v. ⟨A_i⟩ ≠ 0 can always be absorbed into a momentum integration.
- A₀ cannot be gauged away → Topological obstruction from the boundary conditions. Its v.e.v. has physical meaning as a vacuum parameter.

$$Z \sim \prod_{n,\mathbf{k}} \left[\left(\frac{2\pi n}{\beta} - gA_0 \right)^2 + \mathbf{k}^2 + m^2 \right]^{-1} = \prod_{\mathbf{k}} \left| 2\sinh\frac{\beta(\omega_{\mathbf{k}} + igA_0)}{2} \right|^{-1}$$
$$= \prod_{\mathbf{k}} \frac{e^{-\beta\omega_{\mathbf{k}}}}{\left| 1 - e^{-\beta(\omega_{\mathbf{k}} + igA_0)} \right|^2} = \operatorname{Tr} \left[e^{-\beta H - 2\pi i \mu Q} \right],$$

where Q are integer charges and $\mu = \frac{\beta g A_0}{2\pi}$.

Dropping the infinite vacuum energy, the result is:

$$-\ln Z(\beta,\mu) = \frac{V}{(2\pi)^s} \ (-)^F \int d^s \mathbf{k} \ \Big[\operatorname{Re} \ \ln \left(1 - (-)^F e^{-\beta \omega_{\mathbf{k}} - 2\pi i \mu}\right) + (\mu \to -\mu) \Big].$$

Symmetry Z(β, μ) = Z (β, ½ - μ) implies that the gauge inequivalent vacuum potentials correspond to:

$$0 \leq eta g A_0 \leq \pi \quad \leftrightarrow \quad 0 \leq \mu \leq rac{1}{2}.$$

• They correspond to topological vacuum parameters, characterising the thermal system.

The thermal 1-loop amplitude can be rewritten as a thermal trace over the Hilbert space of the 3d (4, 0)-theory:

$$Z(\beta,\mu',\tilde{\mu}_{I}) = \operatorname{Tr}\left[e^{-\beta H} e^{2\pi i (\mu' \hat{m}_{I} - \tilde{\mu}_{I} n')} \right],$$

with $\beta = 2\pi R_0$. Here \hat{m}_l and n_l are the transverse integral charges:

$$\hat{m}_I \equiv m_I + rac{ar{a}+n^1}{2}\delta_{1I}$$
 , n' .

Thermal ensemble is deformed by the "thermal" fluxes associated to U(1) graviphoton and axial-vector.

- Here $\mu' \equiv \hat{G}_0' \equiv G_{0K} G^{KI}$ and $\tilde{\mu}_I \equiv \hat{B}_{0I} \equiv B_{0I} \hat{G}_0^K B_{KI}$ are scale-invariant, non-fluctuating, thermodynamical parameters of the thermal system.
- These global fluxes can be described as gauge field condensates, of zero field strength (locally), but with non-vanishing value of the Wilson line around the Euclidean time cycle.
- At sufficiently low temperatures, states charged under these U(1) fields become massive and effectively decouple from the thermal system → we obtain a conventional thermal ensemble.
- Would-be winding tachyons can sometimes be lifted, as they are charged under the corresponding gauge fields.

The only sector dangerous of producing tachyons has odd chiral and anti-chiral GSO projections to the lattice:

$$O_8 \bar{O}_8 \frac{1}{2} \sum_{b', \bar{b}'=0,1} (-)^{b'+\bar{b}'} \Gamma_{(8,8)}[^{0}_{b'}, ^{0}_{\bar{b}'}](G_{IJ}, B_{IJ}).$$

- Take lattice in the "temperature basis" in the flat frame $G_{\mu\nu}=e^a_\mu e^a_
 u$.
- Choose $(e^a)_{\mu}$ to be the unique lower triangular matrix in this decomposition.
- The (thermal) mass formula for the BPS-spectrum is:

$$\frac{1}{2}P_{L,R}^{2} = \frac{1}{4}\sum_{a=0}^{7}\left[(e^{*a})^{\mu}(\hat{m}_{\mu} + B_{\mu\nu}n^{\nu}) \pm e_{\mu}^{a}n^{\mu}\right]^{2},$$

with $\hat{m}_{\mu} = m_{\mu} + \frac{1}{2}(\bar{a} + n^1)\delta_{\mu,1}$ is the half-shifted momentum in the direction of right-moving SUSY breaking.

• Decomposition into 8 perfect squares.

Study the first square involving the temperature radius R_0 :

$$\frac{1}{4}\left(\frac{m_0-\mu'\hat{m}_I+\tilde{\mu}_Jn^J+\tilde{\mu}_I\mu'n^0}{R_0}\pm R_0n^0\right)^2.$$

- Entirely determined by the thermodynamical variables of the MSDS phase: β , $\mu' \equiv \hat{G}'_0$, $\tilde{\mu}_I \equiv \hat{B}_{0I}$.
- Unaffected by the fluctuations of the dynamical (transverse) moduli, which give a non-negative contribution to the thermal masses.
- Tachyon-free models for *any* deformations of the dynamical moduli should have at least a *chirally massless* low-lying spectrum protected entirely by the 'frozen' contribution.

Conditions for absence of tachyons for any deformation of the dynamical moduli:

$$egin{array}{l} \hat{G}_0^k \in \mathbb{Z} \ \hat{B}_{0k} \in \mathbb{Z} \end{array}
ight\} \hspace{0.2cm}, \hspace{0.2cm} ext{for} \hspace{0.2cm} k=2,\ldots,7, \ \hat{G}_0^1 \in 2\mathbb{Z}+1 \hspace{0.2cm}, \hspace{0.2cm} \hat{B}_{01} \in \mathbb{Z}+rac{1}{2}. \end{array}$$

It can be shown that whenever these conditions are met, a discrete $O(8,\mathbb{Z}) \times O(8,\mathbb{Z})$ transformation can rotate the fluxes to the form:

$$\hat{G}_0^k = \hat{B}_{0k} = 0$$
 , $\hat{G}_0^1 = 2\hat{B}_{01} = \pm 1$.

This is exactly the case when the temperature cycle *factorizes* from the torus and couples chirally to F_L only. Tachyon-free thermal-like models only if :

$$\Gamma_{(d,d)}[^{a,\,\bar{a}}_{b,\,\bar{b}}] = \Gamma_{(1,1)}[^{a}_{b}](R_{0}) \otimes \Gamma_{(d-1,d-1)}[^{\bar{a}}_{\bar{b}}](G_{IJ},B_{IJ}).$$

In this case :

$$Z = \ln \operatorname{Tr} \left[e^{-eta H} (-)^{F_R}
ight] \ , \ ext{with} \ \mu = 2 ilde{\mu} = 1.$$

Check maximally symmetric MSDS Type II vacuum:

- $\hat{G}_0^{\prime} = \left(\frac{3}{2}, \frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, \frac{1}{4}, 0\right).$
- $\bullet \ \hat{B}_{01} = \big(0, -\tfrac{3}{4}, -\tfrac{1}{4}, -\tfrac{3}{4}, 0, -\tfrac{1}{4}, 0\big).$
- Does *not* satisfy above conditions → naively unstable under arbitrary deformations.
- However, \exists directions along which no tachyonic modes appear (e.g. $R_0 \to \infty$).

Tachyon-free limits:

- Limit $R_0 \rightarrow \infty$ or $T \rightarrow 0$: recover $\mathcal{N}_L = 4$ supersymmetries in +1-dim.
- Limit $\mathcal{M} \to 0$: recover $\mathcal{N}_R = 4$ supersymmetries and +1-dim.
- $\bullet\,$ Continuous connection to cold 4d vacua, with spontaneously broken $\mathcal{N}\leq$ 8 SUSY in presence of well-defined fluxes.
- Analogous deformations are expected to connect Heterotic Z₂ × Z₂ MSDS models to N = 1 4d vacua, with semirealistic gauge groups ~ SO(10).

Hybrid models

Particular class of tachyon-free models in which spacetime SUSY arises from the left-moving side. Right-moving SUSY is broken at the string scale and replaced by MSDS structure. [IF, Kounnas, Toumbas '10]

$$\begin{split} Z &= \frac{V_2}{(2\pi)^2} \int\limits_{\mathcal{F}} \frac{d^2 \tau}{4\tau_2^2} \left[\frac{1}{2} \sum_{a,b} (-)^{a+b} \frac{\theta {a}^{[a]}}{\eta^4} \right] \left[\frac{1}{2} \sum_{\gamma,\delta} \frac{\theta {\gamma}^{[\alpha]}_{\delta}}{\eta^8} \right] \left[\frac{1}{2} \sum_{\bar{a},\bar{b}} (-)^{\bar{a}+\bar{b}} \frac{\bar{\theta} {\bar{b}}^{[\bar{a}]}_{\bar{b}}}{\bar{\eta}^{12}} \right] \\ &= \frac{V_2}{(2\pi)^2} \int\limits_{\mathcal{F}} \frac{d^2 \tau}{4\tau_2^2} \frac{1}{\eta^8} \Gamma_{E_8} \left(V_8 - S_8 \right) \left(\bar{V}_{24} - \bar{S}_{24} \right). \end{split}$$

- (4,0)-SUSYmmetric model with respect to ordinary SUSY, with an MSDS-symmetric right-moving sector.
- 24 \times 8 massless bosons $V_8 \bar{V}_{24}$ and 24 \times 8 massless fermions $S_8 \bar{V}_{24}$.
- Alternatively obtained from maximal model by asymmetric \mathbb{Z}_2 -orbifold $X'_L \to X'_L + \pi$, shifting all 8 transverse left-moving coordinates.
- The shift breaks the left-moving gauge group H_L = [SU(2)_L]⁸_{k=2} down to an abelian U(1)⁸_L, while preserving H_R = [SU(2)_R]⁸_{k=2} unbroken.

Compactify one of the longitudinal directions on a circle $S^1(R_0)$ and mod out by $(-)^{F_L}\delta_0$. Identify the circle with Euclidean time direction relevant for a thermal interpretation.

$$Z = \frac{V_1}{8\pi} \int_{\mathcal{F}} \frac{d^2 \tau}{\tau_2^{3/2}} \left[\frac{1}{2} \sum_{a,b} (-)^{a+b} \frac{\theta[_b^a]^4}{\eta^4} \right] \Gamma_{E_8} (\bar{V}_{24} - \bar{S}_{24}) \Gamma_{(1,1)}[_b^a](R_0),$$

where

$$\Gamma_{(1,1)}[{}^{a}_{b}](R_{0}) = \frac{R_{0}}{\sqrt{\tau_{2}}} \sum_{\tilde{m}^{0},n^{0}} e^{-\frac{\pi R_{0}^{2}}{\tau_{2}}|\tilde{m}^{0}+\tau n^{0}|^{2}} (-)^{\tilde{m}^{0}n^{0}+a\tilde{m}^{0}+bn^{0}}$$

In the Hamiltonian representation:

$$\frac{V_1}{8\pi} \int_{\mathcal{F}} \frac{d^2 \tau}{\tau_2^{3/2}} \Gamma_{E_8}(\bar{V}_{24} - \bar{S}_{24}) \sum_{m,n} \Big(V_8 \Gamma_{m,2n} + O_8 \Gamma_{m+\frac{1}{2},2n+1} - S_8 \Gamma_{m+\frac{1}{2},2n} - C_8 \Gamma_{m,2n+1} \Big),$$

•
$$\Gamma_{m,n} = q^{\frac{1}{2}p_L^2} \bar{q}^{\frac{1}{2}p_R^2}$$
, with $p_{L,R} = \frac{1}{\sqrt{2}} \left(\frac{m}{R} \pm nR \right)$.

• (4,0)-SUSY spontaneously broken by this thermal deformation.

- $V_8 \bar{V}_{24}$ sector contains massless states for any R_0 .
- Initially massless fermions in $S_8 \bar{V}_{24}$ now acquire mass.
- O_8 -sector now carries non-trivial winding and momentum numbers around S^1 .

Lowest mass states in $O_8 \bar{V}_{24}$:

$$2m_{O\bar{V}}^2 = \left(\frac{1}{\sqrt{2}R_0} - \sqrt{2}R_0\right)^2.$$

Extra massless states at $R_0 = 1/\sqrt{2}$. Enhanced gauge symmetry:

$$U(1)_L imes U(1)_R o [SU(2)_{k=2}]_L imes U(1)_R.$$

- Γ_(1,1)-lattice remains factorized, since the mixing moduli (gravitomagnetic fluxes) do not correspond to fluctuating fields → free of tachyons.
- These particular fluxes restore thermal *T*-duality: $R_0 \rightarrow 1/(2R_0)$ together with the interchange $S_8 \leftrightarrow C_8$.
- and inject non-trivial winding charge into the thermal vacuum !

Unfold the fundamental domain for $R_0 > 1/\sqrt{2}$, as before:

$$\begin{split} \frac{Z}{V_1} = & R_0 \sum_{\tilde{m}^0 \neq 0} \int_{||} \frac{d^2 \tau}{8 \pi \tau_2^2} e^{-\frac{\pi (\tilde{m}^0 R_0)^2}{\tau_2}} \frac{1}{2 \eta^{12}} \sum_{a,b} (-)^{a+b} \theta [^{a}_{b}]^4 (-)^{\tilde{m}^0 a} \\ & \times \frac{1}{2 \bar{\eta}^{12}} \sum_{\bar{a}, \bar{b}} (-)^{\bar{a} + \bar{b}} \bar{\theta} [^{\bar{a}}_{\bar{b}}]^4 \Gamma_{(8,8)} [^{\bar{a}}_{\bar{b}}]. \end{split}$$

Integral over the strip becomes the spacetime thermal partition function:

$$Z = \ln \operatorname{Tr} \left[e^{-\beta H} (-)^{F_R} \right] \xrightarrow{MSDS} \ln \operatorname{Tr} |_{m^2=0} e^{-\beta H} \ , \ \mu = 2\tilde{\mu} = 1.$$

Trace over (massless) Hilbert space of the initially (4,0)-SUSYmmetric model.

- This expression is strictly valid for $R_0 > 1/\sqrt{2}$.
- To obtain expression valid for $R_0 < 1/\sqrt{2}$ one must first double-Poisson resum, to ensure absolute convergence. Field theory expression fails to capture thermal duality.
- The fundamental object is the Euclidean path integral, which is valid for all temperatures and which exhibits the dualities manifestly.

MSDS symmetry and level-matching allow the explicit calculation of the thermal 1-loop amplitude:

$$\frac{Z}{V_1} = 24 \times \left(R_0 + \frac{1}{2R_0}\right) - 24 \times \left|R_0 - \frac{1}{2R_0}\right|.$$

- Exact result No α' -approximation !
- Manifestly invariant under $R_0 \rightarrow 1/(2R_0)$.
- Non-analyticity induced by the extra massless states at the dual fermionic point $R_0 = 1/\sqrt{2}$.
- Only massless states contribute to the thermal trace.
- Equation of state $\rho = P = 48\pi T^2 \rightarrow$ thermal massless radiation in 2d.

We can identify three distinct regions:

$$R_0>1/\sqrt{2}$$
: $Z=rac{24}{R_0}~,~eta=2\pi R_0.$

Only initially massless states in $V_8 \bar{V}_{24}$ and $S_8 \bar{V}_{24}$ contribute \rightarrow "pure momenta". • $R_0 < 1/\sqrt{2}$:

$$Z = 24 \times (2R_0)$$
 , $\beta = 2\pi/(2R_0)$.

Light states now occur in $V_8 \bar{V}_{24}$ and $C_8 \bar{V}_{24}$ sectors \rightarrow "pure windings" (vortices).

Accordance with thermodynamics:

 \hookrightarrow thermal partition function should decrease with decreasing temperature.

• $R_0 \sim 1/\sqrt{2}$: 24 (localized) additional massless states from $O_8 \bar{V}_{24}$ carrying non-trivial momentum and winding numbers.

Need for three distinct local effective field theories to describe the stringy thermal system.

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Introduce thermal variable $\sigma \in (-\infty, +\infty)$:

$$R_0=rac{1}{\sqrt{2}}e^{\sigma}.$$

The physical, duality-invariant temperature is then defined in terms of σ :

$$T = T_c e^{-|\sigma|}$$
 , $T_c \equiv rac{\sqrt{2}}{2\pi}$.

The partition function of the theory has the following expression:

$$Z = (24\sqrt{2}) e^{-|\sigma|} = \Lambda T$$
, where $\Lambda = 48\pi$.

As σ increases, the system heats up to the critical temperature, undergoes a phase transition and, subsequently, cools down.

- At low temperature the two dual phases are distinguished by the chirality of the light thermally excited spinors:
 - Hybrid B phase : $\sigma > 0 \rightarrow S_8$.
 - Hybrid A phase : $\sigma < 0 \rightarrow C_8$.

New light modes appear at T_c and are responsible for the transition between the two dual phases.

- Consider pure momentum state in $S_8 \bar{V}_{24}$ sector at the extended symmetry point $\sigma = 0$.
- \exists localized operators inducing transitions between pure momentum \rightarrow pure winding states;

e.g. take
$$J_{-} = \psi^{0} e^{-iX_{L}^{0}}$$
:

$$J_{-}(z)e^{-\phi/2}S_{10,\alpha}e^{\frac{i}{2}X_{L}^{0}+\frac{i}{2}X_{R}^{0}}\bar{V}_{24}(w)\sim\frac{1}{z-w}\gamma_{\alpha\beta}^{0}e^{-\phi/2}C_{10,\dot{\beta}}e^{-\frac{i}{2}X_{L}^{0}+\frac{i}{2}X_{R}^{0}}\bar{V}_{24}(w).$$

 \rightarrow Non-trivial 3-point amplitude producing pure winding state in $C_8 \bar{V}_{24}$.

Effective description at $\sigma = 0 \rightarrow$ one-dimensional non-abelian gauge theory:

$$H_L \times H_R \equiv \left(SU(2) \times U(1)^8 \right)_L \times \left(U(1) \times SU(2)^8 \right)_R$$

- 24 localized complex massless scalars χ_i .
- Localized action:

$$S|_{\sigma=0} = \int dx^1 \sqrt{g_{11}} e^{-2\phi} \left(-g^{11} \frac{\partial \bar{\chi}_i}{\partial x^1} \frac{\partial \chi_i}{\partial x^1} \right) \Big|_{\text{at } \sigma(x^0)=0}$$

- For spatially homogeneous solutions : g_{11} , ϕ independent of x^1 .
- Equation of motion of $\bar{\chi}$ yields:

$$\frac{\partial^2 \chi}{\partial x^{1^2}} = \mathbf{0} \quad \to \quad \chi_i = \alpha_i + \gamma_i \sqrt{g_{11}} \, x^1.$$

Phase transition admits an effective brane description, gluing together the space of momenta and the dual space of windings. Effective action:

$$S=\int d^2x \ e^{-2\phi}\sqrt{-g}\left(rac{1}{2}R+2(
abla\phi)^2
ight)+\int d^2x \ \sqrt{-g} \ P-\kappa\int dx^1d\sigma \ e^{-2\phi}\sqrt{g_{11}}\delta(\sigma).$$

[IF, Kounnas, Partouche, Toumbas '10]

Spacelike brane contribution at the phase transition $\sigma = 0 \rightarrow \text{localized negative pressure.}$

• Sourced by the 24 extra massless bosons at the extended symmetry point.

•
$$\kappa = \sum_{i} |\gamma_i|^2$$
 : positive.

We are assuming:

- Adiabatic evolution.
- Time-scale of transition is assumed short enough \rightarrow " $\delta(\sigma)$ ".

Standard parametrization:

$$ds^2 = -N^2(t)dt^2 + a^2(t)dx^2 \ , \ a = e^{\lambda}.$$

Take brane to be localized at t = 0.

• Equations of motion:

$$\begin{split} \dot{\phi}^2 - \dot{\phi}\dot{\lambda} &= \frac{1}{2} \ N^2 e^{2\phi}\rho, \\ \ddot{\lambda} - \dot{\lambda} \left(2\dot{\phi} - \dot{\lambda} + \frac{\dot{N}}{N}\right) = N^2 e^{2\phi}P, \\ \ddot{\phi} - 2\dot{\phi}^2 + \dot{\phi} \left(\dot{\lambda} - \frac{\dot{N}}{N}\right) &= \frac{1}{2}N^2 e^{2\phi}(P - \rho) - \frac{1}{2}N\kappa\delta(t). \end{split}$$

• Thermal entropy $S = a\beta(\rho + P) = 2\Lambda_{\overline{\beta}}^{a}$ conservation law:

$$\dot{\rho} + \dot{\lambda}(\rho + P) = \frac{2\kappa}{N} e^{-2\phi} \left(\dot{\lambda} - 2\dot{\phi}\right) \delta(t).$$

• Condition of continuity of *thermal entropy* across the transition:

$$\dot{\lambda}_+ + \dot{\lambda}_- = 2\left(\dot{\phi}_+ + \dot{\phi}_-
ight).$$

• Conservation of *S* implies relation:

$$\lambda = |\sigma| + \ln \frac{\beta_c S}{2\Lambda} \quad \rightarrow \quad a \ge \beta_c \frac{S}{2\Lambda}$$

 \hookrightarrow hints at absence of gravitational singularities.

Equations of motion can be simplified in the conformal gauge N = a:

$$\begin{aligned} 2\dot{\phi}^2 - 2\dot{\phi}\dot{\lambda} &= Ce^{2\phi},\\ \ddot{\lambda} - 2\dot{\phi}\dot{\lambda} &= Ce^{2\phi},\\ 2\ddot{\phi} - 4\dot{\phi}^2 &= -\kappa e^{\lambda_0}\delta(t) \end{aligned}$$

where $e^{\lambda_0} \equiv \beta_c S/2\Lambda$ and $C \equiv S^2/4\Lambda$.

Important : only first derivative of the dilaton is discontinuous:

$$\dot{\phi}_+-\dot{\phi}_-=-rac{1}{2}m{e}^{\lambda_0}\kappa.$$

Continuity of $\dot{\lambda}(0)$ gives:

$$\dot{\lambda}(0)=\dot{\phi}_++\dot{\phi}_-$$
 .

Because of conservation of entropy,

$$\dot{\lambda} = \frac{\dot{\beta}}{\beta}$$
,

 $\leftrightarrow \dot{\beta}(0)$ also continuous. Maximal temperature implies $\dot{\beta}(0) = \dot{\lambda}(0) = 0$.

• Dilaton derivative flips sign:

$$\dot{\phi}_-=-\dot{\phi}_+=rac{1}{4}e^{\lambda_0}\kappa > 0$$

...and the dilaton bounces across the transition.

• Eq. of motion give $\ddot{\lambda}=2\dot{\phi}^2.$ At the critical point :

 $\dot{\lambda}(0) = 0$, $\ddot{\lambda}(t) > 0$

...the scale factor bounces as well.

String coupling $g_{str}(0)$ is related to tension κ .

• From eq. of motion :

$$\dot{\phi}_-^2 = \frac{C}{2} e^{2\phi_0}.$$

• Relation between ϕ_0 and κ :

$$g_{
m str}^2(0)\equiv e^{2\phi_0}=rac{eta_c^2\kappa^2}{8\Lambda}$$

 \hookrightarrow Perturbative validity is ensured for sufficiently small κ .

After appropriate rescaling, the solution takes the simple form:

$$ds^{2} = \frac{4}{\kappa^{2}} \frac{e^{|\tau|}}{1+|\tau|} \left(-d\tau^{2}+dx^{2}\right),$$
$$g_{s}^{2} \equiv e^{2\phi(\tau)} = \frac{\pi\kappa^{2}}{192} \frac{1}{1+|\tau|}.$$

The presence of the extended symmetry point (phase transition) induces a bounce and the evolution evades the gravitational singularity, while remaining in the perturbative regime, provided that $\kappa^2 \ll 1$.

Indeed, metric has no essential singularities:

$$R = \left(\frac{\kappa}{2}\right)^2 \frac{\mathrm{e}^{-|\tau|}}{1+|\tau|}.$$

Conical singularity $\ddot{\phi} = -\frac{\kappa}{2}\delta(t)$ resolved by the extra massless states localized at the transition point.

• Higher-derivative (and higher-genus) corrections are suppressed and controllable by a perturbative expansion in $\kappa \sim g_{\rm str} \ll 1$.

To write the solution in the cosmological frame:

$$ds^2 = -d\xi^2 + a^2(\xi)dy^2,$$

perform the change of variables :

$$\begin{split} \xi(\tau) &= \frac{2|\tau|}{\kappa} \int_0^1 \frac{e^{|\tau|u/2}}{\sqrt{1+|\tau|}} du = \sqrt{\frac{8\pi}{e\kappa^2}} \left[\operatorname{erfi}\left(\sqrt{\frac{1+|\tau|}{2}}\right) - \operatorname{erfi}\left(\frac{1}{\sqrt{2}}\right) \right],\\ y &= x/2. \end{split}$$

 $\bullet\,$ For early cosmological times $|\kappa\xi|\ll 1$ we have the asymptotic behaviour:

$$m{a}(\xi) = rac{4}{\kappa} \left[1 + rac{1}{16} (\kappa \xi)^2 + \mathcal{O}(|\kappa \xi|^3)
ight] \,, \, rac{1}{g_{
m str}^2} = rac{192}{\pi \kappa^2} \left[1 + rac{|\kappa \xi|}{2} - rac{1}{8} (\kappa \xi)^2 + \mathcal{O}(|\kappa \xi|^3)
ight] \,.$$

• For late cosmological times $|\kappa\xi| \gg 1$:

$$\mathsf{a}(\xi) = |\xi| \left[1 - \frac{1}{\ln(\kappa\xi)^2} + \ldots \right] \;, \; \frac{1}{g_{\text{str}}^2} = \frac{192}{\pi\kappa^2} \left[\ln(\kappa\xi)^2 + \ln\ln(\kappa\xi)^2 + \text{const} + \ldots \right].$$

Conclusion & Outlook

Conclusions:

- MSDS models turn out to be useful laboratories for studying string thermodynamics close to the Hagedorn point.
- We propose the resolution of Hagedorn divergences in Type II models by deforming the *σ*-model by non-trivial gravito-magnetic fluxes. These refine the thermal ensemble at high temperature.
- Thermal *T*-duality is reinstated by these fluxes. This leads to a duality-invariant definition of the physical temperature.
- Hybrid models indicate a non-analytic contribution to the free energy at the self-dual temperature. Phase transition from winding to momentum states.
- Effective description in terms of a space-like brane. Injects localized negative pressure and bypasses singularity theorems.
- Scale factor and dilaton bounce simultaneously. No gravitational singularity is encountered and the evolution remains perturbative.

Conclusion & Outlook

Further Developments:

- Dynamics of phase transitions and stringy mechanism that avoids initial singularity

 Higher dimensions? [Kounnas, Partouche, Toumbas '11]
- Dynamical exit from MSDS era and emergent spacetime, one-loop quantum instability triggering cosmological evolution in cold vacua.
- Connection of 2d MSDS vacua with 4d semi-realistic vacua at late times \hookrightarrow emergent chirality, \ldots
- Generalization to heterotic MSDS models.

...

Thank You!

Thank You!

Backup Slides

Backup Slides

Gravito-magnetic fluxes

Consider turning on gravito-magnetic fluxes in the Euclidean:

$$\delta S \sim \int d^2 z f_{IJ}^0 \left(\psi' \psi^J - \mathbf{X}' \partial \mathbf{X}^J \right) \times \bar{\partial} \mathbf{X}^0 ,$$

with $X^{I,J} \in T^{d-1}$.

- Not a well-defined conformal field: only quantized f.
- E.g. \mathbb{Z}_2 -case: $f \in 2\mathbb{Z}$: freely-acting orbifold, $e^{i\pi Q}\delta_0$, with $Q = F_L$.
- Technically, can be treated as Wilson line around X^0 , $Y = f/4R_0$.
- Partition function contains:

$$\begin{bmatrix} \frac{1}{2} \sum_{\dots} (-)^{a+b} \theta^{3} \begin{bmatrix} a \\ b \end{bmatrix} \theta \begin{bmatrix} a-2\tilde{n} \\ b-2\tilde{m} \end{bmatrix} e^{-\frac{\pi R_{0}^{2}}{\tau_{2}} |\tilde{m}+\tau\tilde{n}|^{2}} \end{bmatrix}$$
$$= \frac{1}{2} \sum_{a,b} (-)^{a+b} \theta^{4} \begin{bmatrix} a \\ b \end{bmatrix} \left[\sum_{\tilde{m},\tilde{n}\in\mathbb{Z}} e^{-\frac{\pi R_{0}^{2}}{\tau_{2}} |\tilde{m}+\tau\tilde{n}|^{2}} (-)^{\tilde{m}\tilde{n}+a\tilde{m}+b\tilde{n}} \right]$$

Factorized Γ_(1,1)[^a_b](R₀).

Vortex Condensation

Consider simple case, $S^1(R_0) \times S^1(R_1) \times T^{d-2}$: $\Gamma_{(1,1)}[^{a}_{b}](R_0) \times \Gamma_{(1,1)}[^{\bar{a}}_{\bar{b}}](R_1) \times (spectators) .$

• Enhanced gauge symmetry in X^0, X^1 directions:

$$\begin{split} [U(1)_L \times U(1)_R]_0 \times [U(1)_L \times U(1)_R]_1 \times \dots \\ \longrightarrow [SU(2)_{k=2,L} \times U(1)_R]_0 \times [U(1)_L \times SU(2)_{R,k=2}]_1 \times \dots \end{split}$$

• Adding previous gravito-magnetic fluxes amounts to marginal deformation:

$$(G_{01}+2B_{01})\partial X^0 \bar{\partial} X^1 + (G_{01}-2B_{01})\partial X^1 \bar{\partial} X^0$$
.

with $G_{01}/R^2 = 2B_{01}/R^2 = 1$.

Vortex Condensation

At the enhanced symmetry point, $R = 1/\sqrt{2}$, perform $SU(2)_L \times SU(2)_R$ rotation:

$$\partial X^0 \bar{\partial} X^1 \longrightarrow \left(\psi^0 \ \cos X^0_L
ight) imes \left(ar{\psi}^1 \ \cos X^1_R
ight) \; .$$

- Deformation injects non-trivial momentum & winding charges into thermal vacuum (around X^0, X^1).
- Effectively regulates contribution of massive string states.
- Compare with vortex condensation in XY model.

Emergent MSDS Structure

Consider Type II \mathbb{Z}_2 -orbifold on $T^2 \times T^4/\mathbb{Z}_2$ with $N_4 = 4$ supersymmetry. On the worldsheet, $N = (4, 4) \oplus (2, 2)$ compactification.

• Gepner map: embed left-moving spin-connection into gauge connection of $E_8 \times E_8$ Heterotic string (standard embedding):

$$\frac{1}{2}\sum_{a,b} (-)^{a+b+ab} \theta^2 {a \brack b} \theta^{[a+h]}_{[b+g]} \theta^{[a-h]}_{[b-g]} \longrightarrow \frac{1}{2}\sum_{k,\ell} \theta^6 {k \brack \ell} \theta^{[k+h]}_{\ell+g} \theta^{[k-h]}_{\ell-g} .$$

- At Y' = 0, enhanced gauge symmetry $E_7 \times SU(2) \times E_8$.
- N_L = 4 SCFT has s.f. that acts on the twisted matter sector and maps vectorial to spinorial reps of SO(12) ∈ E₇.
- Relevant orbifold block satisfies conditions for MSDS-structure:

$$Z[\begin{smallmatrix}1\\g\end{smallmatrix}] \ = \ \frac{1}{2^2 \eta^{12} \bar{\eta}^4} \left[\sum_{\ell=0,1} (-)^\ell \theta^6[\begin{smallmatrix}k\\ \ell\end{smallmatrix}] \theta^2[\begin{smallmatrix}k+1\\ \ell+g\end{smallmatrix}] \right] \left[\sum_{\gamma,\delta=0,1} \theta^2[\begin{smallmatrix}\gamma\\ \delta\end{smallmatrix}] \theta^2[\begin{smallmatrix}\gamma+1\\ \delta+g\end{smallmatrix}] \bar{\theta}^2[\begin{smallmatrix}\gamma\\ \delta\end{smallmatrix}] \bar{\theta}^2[\begin{smallmatrix}\gamma+1\\ \delta+g\end{smallmatrix}] \right]$$

- Construct explicitly s.f. operators in the twisted sector and verify Spinor-Vector duality map.
- Chiral twisted character identities expose underlying MSDS algebra.

A class of tachyon-free models can be constructed as $\mathbb{Z}_2\times\mathbb{Z}_2$ orbifolds of the maximally symmetric MSDS models.

- Combine (a-)symmetric Z₂ shifts and twists in order to construct stable MSDS vacua under all possible deformations of the remaining (fluctuating) moduli.
- The problem is non-trivial because the orbifold action must respect the MSDS symmetry.

Consider asymmetric $\mathbb{Z}_2 \times \mathbb{Z}'_2$ twists of the maximally symmetric MSDS model, acting on four of the internal coordinates:

 $X_L^{\prime}(z)
ightarrow (-)^g X_L^{\prime}(z) \quad, \quad X_R^{\prime}(ar{z})
ightarrow (-)^{g+g^{\prime}} X_R^{\prime}(ar{z}),$

with $g, g' \in \{0, 1\}$ and I = 5, 6, 7, 8. In addition, introduce independent asymmetric $\mathbb{Z}_2^{(1)} \times \mathbb{Z}_2^{(2)}$ shifts on all 8 internal coordinates:

$$\begin{array}{l} X_L^{I}(z) \to X_L^{I}(z) + \pi G_1 \\ X_R^{I}(\bar{z}) \to X_R^{I}(\bar{z}) + \pi G_2 \end{array} \right\} \quad , \quad \text{for } I = 1, 2, 3, 4. \\ X_L^{J}(z) \to X_L^{J}(z) + \pi G_2 \\ X_R^{J}(\bar{z}) \to X_R^{J}(\bar{z}) + \pi G_1 \end{array} \right\} \quad , \quad \text{for } J = 5, 6, 7, 8.$$

Modular invariant partition function of the model:

$$Z = \frac{1}{2^4 \eta^{12} \bar{\eta}^{12}} \sum_{h,g,h',g'} \sum_{a,b} (-)^{a+b} \theta[^{a}_{b}]^2 \theta[^{a+h}_{b+g}] \theta[^{a-h}_{b-g}] \\ \times \sum_{\bar{a},\bar{b}} (-)^{\bar{a}+\bar{b}} \bar{\theta}[^{\bar{a}}_{\bar{b}}]^2 \bar{\theta}[^{\bar{a}+h+h'}_{\bar{b}+g'}] \bar{\theta}[^{\bar{a}-h-h'}_{\bar{b}-g-g'}] \Gamma_{(8,8)}[^{a,\bar{a},h,h'}_{b,\bar{b},g,g'}].$$

The (8,8)-lattice is factorized into a shifted $\Gamma^{(1)}_{(4,4)}$ lattice and a shifted/twisted $\Gamma^{(2)}_{(4,4)}$:

$$\Gamma_{(8,8)} \begin{bmatrix} a, \bar{a}, h, h' \\ b, \bar{b}, g, g' \end{bmatrix} = \frac{1}{2^2} \sum_{H_i, G_i} \Gamma^{(1)}_{(4,4)} \begin{bmatrix} a, \bar{a}; H_1, H_2 \\ b, \bar{b}; G_1, G_2 \end{bmatrix} \times \Gamma^{(2)}_{(4,4)} \begin{bmatrix} a, \bar{a}; H_1, H_2 \\ b, \bar{b}; G_1, G_2 \end{bmatrix} \overset{h, h'}{}_{g,g'}$$

At the MSDS point, the shifted lattice is expressed in terms of free fermion characters:

$$\Gamma^{(1)}_{(4,4)} \begin{bmatrix} a,\bar{a};H_1,H_2\\b,\bar{b};G_1,G_2 \end{bmatrix}_{\text{MSDS}} = \theta \begin{bmatrix} a+H_1\\b+G_1 \end{bmatrix}^2 \theta \begin{bmatrix} a-H_1\\b-G_1 \end{bmatrix}^2 \times \bar{\theta} \begin{bmatrix} \bar{a}+H_2\\\bar{b}+G_2 \end{bmatrix}^2 \bar{\theta} \begin{bmatrix} \bar{a}-H_2\\\bar{b}-G_2 \end{bmatrix}^2$$

Similarly for the asymmetrically twisted (4, 4)-lattice:

$$\Gamma_{(4,4)}^{(2)} \begin{bmatrix} a,\bar{a};H_1,H_2 \\ b,\bar{b};G_1,G_2 \end{bmatrix} \stackrel{h,h'}{=} \theta \begin{bmatrix} a+H_2+h \\ b+G_2+g \end{bmatrix}^2 \theta \begin{bmatrix} a-H_2 \\ b-G_2 \end{bmatrix}^2 \times \bar{\theta} \begin{bmatrix} \bar{a}+H_1+h+h' \\ \bar{b}+G_1+g+g' \end{bmatrix}^2 \bar{\theta} \begin{bmatrix} \bar{a}-H_1 \\ \bar{b}-G_1 \end{bmatrix}^2 (-)^{h'g'}.$$

At the MSDS point the partition function is simply Z = 208 = constant.

Soutenance de Thèse

Non-vanishing components of the asymmetrically twisted (4, 4)-lattice:

The asymmetric nature of the \mathbb{Z}_2' -orbifold projects out the fluctuations of all moduli associated to the twisted $\Gamma^{(2)}_{(4,4)}$ -lattice.

• The moduli space is reduced to:

$$\frac{SO(8,8)}{SO(8)\times SO(8)} \xrightarrow{\mathbb{Z}_2 \times \mathbb{Z}'_2} \frac{SO(4,4)}{SO(4)\times SO(4)} \ .$$

• Only marginal operators in the shifted $\Gamma^{(1)}_{(4,4)}$ -lattice survive the orbifold and can perturb the sigma model.

All sectors are at least (anti-)chirally massless \rightarrow arbitrary deformations of the shifted $\Gamma^{(1)}_{(4,4)}$ lattice in $\frac{SO(4,4)}{SO(4) \times SO(4)}$ will not produce tachyonic states.

- The vacuum is stable.
- The shifted $\Gamma^{(1)}_{(4,4)}$ lattice can be put in the usual Lagrangian form with phase coupling:

$$\mathcal{T} = \tilde{m}^1 n^1 + (a + \bar{a} + H_2) \tilde{m}^1 + (b + \bar{b} + G_2) n^1 + \dots$$

By decompactifying the X^1 -cycle of T^4 , the coupling to the *R*-symmetry charges is washed out and we obtain a 3d, $N_4 = 2$ Type II vacuum.

- The new non-compact dimension is interpreted as an emergent spatial dimension as it can be generated dynamically.
- Further orbifold twists can be introduced to reduce SUSY down to $N_4 = 1$, without altering the tachyon-free structure.
- In the thermal version of the models, Euclidean time is identified with the compact toroidal cycle coupled to *F*, as in the Hybrid models. However, the present breaking of SUSY is not an arbitrary one.
- It is a very specific breaking dictated by the underlying MSDS algebra.