Modèles DSGE Nouveaux Keynésiens, Monnaie et Aversion au Risque.

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Numéro attribué par la bibliothèque

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THÈSE
Pour obtenir le grade de
Docteur de l'Université Paris 1 Panthéon Sorbonne
Discipline : Sciences Economiques
Présentée et soutenue publiquement par

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le 9 décembre 2011

MODÈLES DSGE NOUVEAUX KEYNÉSIENS, MONNAIE ET AVERSION AU RISQUE.

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Doctorat de Sciences Economiques

New Keynesian DSGE models, Money and Risk Aversion

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Remerciements

Je tiens en tout premier lieu à adresser mes remerciements les plus respectueux à mes directeurs de thèse, Messieurs les professeurs Christian Bordes et André Fourçans, pour la confiance et l'aide qu'ils m'ont apporté tout au long de ces cinq années de thèse. Leur soutien, leurs encouragements, leurs précieux conseils et le temps qu'ils m'ont consacré m'ont été d'un grand profit.

Je remercie Laurent Clerc et Marc-Alexandre Sénégas qui ont accepté d'être rapporteurs de cette thèse.

J'exprime ma profonde gratitude à Radu Vranceanu, Abraham Lioui et Nesim Fintz. Leur expérience, leur disponibilité et l'attention qu'ils m'ont porté depuis le début de cette thèse ont été favorable à l'avancement de mes travaux.

Les sympathiques et non moins avisés Thomas Baron et Mathilde Peyrat ont aussi joué un rôle important dans le bon déroulement de cette thèse. Ce travail fut d'autant plus agréable en leur compagnie. Je les remercie vivement.

J'ai également une pensée chaleureuse pour tous ceux que j'ai eu le plaisir de côtoyer à la MSE et à l'ESSEC Business School.

Je souhaite témoigner ici toute ma reconnaissance à deux personnes qui ont fait preuve d'une confiance envers moi débordante.

Mr Christian Bordes, qui m'a soutenu depuis le premier jour de mon Master Recherche à la MSE. Sa présence et surtout nos conversations m'ont toujours données des ailes et une grande motivation.

Mr André Fourçans, pour ses qualités scientifiques et humaines, ainsi que son dynamisme, rendent notre collaboration agréable et enrichissante. Nos projets communs m'ont conduit à un épanouissement inattendu. Sa confiance, depuis le début de mon entreprise, dans le cadre de l'ESSEC Business School jusqu'à nos dernières recherches, restera à jamais gravée dans ma mémoire.

Je suis évidement très reconnaissant envers mes parents. Tout est toujours plus facile avec des parents comme les miens. Depuis toujours, leurs petits mots, plus ou moins agréables, ont toujours guidé mes performances avec réussite.

Enfin, je tiens à remercier mon épouse, Noémie, pour son soutien, son amour et son regard qui me donne toujours envie d'aller plus loin dans mes recherches.

Mes derniers mots s'adressent à mes grands parents, et plus particulièrement à mes grands parents paternels, qui ne sont plus là et qui j'en suis sur, seraient aujourd'hui fiers de moi.

C'est à ces derniers que je dédie ce travail.
A ceux qui partent, et à ceux qui arrivent.
A mon fils, Moïse.

Résumé

Cette thèse présente trois modèles théoriques et empiriques de la Zone Euro, mettant en perspective l'influence de l'aversion au risque et de la monnaie sur différentes variables. Ces modèles d'équilibre général intertemporels et stochastiques (DSGE) s'inscrivent dans le cadre de la théorie des Nouveaux Keynésiens.

Dans un premier modèle de base, nous montrons que l'aversion au risque influence la production, contribuant à sa baisse, notamment en période de crise. Pendant ces périodes de crise (Système Monétaire Européen, 1992; Internet, 2000; Subprimes, 2007), l'aversion au risque impacte significativement la détention de monnaie réelle.

Dans un second modèle, dans lequel la monnaie est considérée comme un facteur de production, cette dernière n'a pas d'implication significative sur les dynamiques des autres variables. L'hypothèse de rendements d'échelle constants est par là même rejetée.

Dans un troisième modèle, en utilisant une fonction d'utilité non-séparable entre la consommation et les encaisses réelles, nous montrons que le rôle de ces dernières sur la production dépend du degré d'aversion au risque des agents, devenant significatif lorsque celui-ci est deux fois plus élevé que la normale.

Enfin, nous testons et comparons ce modèle avec le modèle de base pendant les trois périodes susmentionnées. La monnaie explique alors une partie significative des variations de la production pendant ces crises. De plus, notre analyse montre qu'un modèle non-séparable entre la consommation et les encaisses réelles a de meilleures capacités prédictives qu'un modèle séparable en période de crise.

Résumé général

Cette thèse présente trois modèles théoriques et empiriques de la Zone Euro, mettant en perspective l'influence de l'aversion au risque et de la monnaie sur différentes variables telles que l'inflation, la production et le taux d'intérêt. Ces modèles d'équilibre général intertemporels et stochastiques (DSGE) s'inscrivent dans le cadre des « Nouveaux Keynésiens », la monnaie étant introduite dans la fonction d'utilité des ménages sous une forme séparable et non-séparable ainsi que dans la fonction de production. En testant ces modèles selon la méthode Bayésienne et en effectuant des simulations, nous expliquons les dynamiques de la production et de l'inflation, mais aussi du taux d'intérêt, des encaisses réelles, de la production en prix flexibles et des encaisses réelles en prix flexibles.

Dans un premier article, nous développons un modèle de base microfondé pour la Zone Euro, incluant un choc sur l'aversion au risque (inverse de l'élasticité de substitution intertemporelle). Cette modélisation DSGE, ainsi que l'estimation Bayésienne et la simulation du modèle estimé, nous permet de montrer que l'aversion au risque a un impact significatif sur la production, contribuant négativement à celle-ci, notamment en période de crise. Grace à une décomposition des variables historiques, et de leurs variances, en fonction des chocs microfondés, nous montrons également que pendant ces périodes de crise (Système Monétaire Européen, 1992; Internet, 2000; Subprimes, 2007), le choc d'aversion au risque joue un rôle important dans la dynamique des encaisses réelles. Nous démontrons aussi que, conformément à la littérature, la monnaie n'a aucun rôle dans les dynamiques des autres variables dans ce type de modèle.

Dans un second article, en considérant la monnaie comme un facteur de production et comme un motif d'utilité des ménages (détention de monnaie), nous montrons que la monnaie n'a que très peu d'influence sur les dynamiques des autres variables économiques en Zone Euro. De plus, nous apportons une solution concernant le choix de l'hypothèse de rendements d'échelle lorsque la monnaie fait partie des facteurs de production, en excluant l'hypothèse de rendement d'échelle constant, longtemps évoquée dans la littérature des années 1960-70.

Dans un troisième article, nous supposons que l'arbitrage de l'agent représentatif entre consommer tout de suite ou détenir de la monnaie n'est pas sans lien. Nous cherchons à savoir si la monnaie peut jouer un rôle en fonction du niveau d'aversion au risque des ménages. Lorsque l'aversion au risque augmente, les mé-

nages peuvent choisir de détenir plus de monnaie pour faire face à l'incertitude implicite et optimiser leur consommation dans le temps, en arbitrant entre la dépense présente et la dépense future. Ainsi, nous développons un modèle DSGE avec une fonction d'utilité non-séparable entre la consommation et les encaisses réelles et nous testons cette hypothèse. Nous montrons que le rôle de la monnaie dans la dynamique de la production en Zone Euro dépend du degré d'aversion au risque des agents, qu'il augmente avec ce degré, et qu'il devient significatif quand l'aversion au risque intertemporelle est deux fois plus élevé que la normale. L'impact direct de la monnaie est en revanche très limité pour expliquer la variance de l'inflation, la politique monétaire, via le taux d'intérêt, constituant le facteur dominant.

Dans un dernier article, nous testons et comparons ce modèle avec le modèle de base. Nous analysons le rôle de la séparabilité entre la monnaie et la consommation, avec un accent particulier sur l'influence de la politique monétaire et de la monnaie en Zone Euro pendant les trois dernières crises (Système Monétaire Européen, 1992; Internet, 2000; Subprimes, 2007). Nous constatons que la monnaie explique une grande partie des variations de la production et de la production en prix flexibles pendant les crises et, dans le même temps, que le rôle de la politique monétaire sur la production diminue de manière significative. Par ailleurs, nous constatons que le modèle supposant la non-séparabilité entre la consommation et les encaisses réelles a de meilleures capacités prédictives qu'un modèle de base (séparable) en période de crise.

Introduction générale

Les autorités monétaires reconnaissent qu'à long terme, l'inflation est déterminée par la politique monétaire. Toutefois, il existe des désaccords sur le rôle des agrégats monétaires dans la conduite de la politique monétaire. Ainsi, l'ancien membre de la Réserve Fédérale américaine (Fed), Laurence Meyer, a déclaré que « la monnaie ne joue aucun rôle dans le modèle macroéconomique d'aujourd'hui » permettant de guider le consensus de la Fed, « et pratiquement aucun rôle dans la conduite de la politique monétaire, au moins aux Etats-Unis ». En revanche, Otmar Issing, ancien membre du Conseil exécutif de la Banque Centrale européenne (BCE) a déclaré que « la monnaie ne doit jamais être ignorée, ni dans la conduite de la politique monétaire, ni dans la recherche ».

Tant dans l'histoire de ces deux institutions que dans leur éventuelle prise en compte des agrégats monétaires, sous différentes formes (taux de croissance, monnaie réelle, nominale, par tête etc.), le débat sur le rôle de la monnaie dans les décisions de politique économique reste ouvert depuis plusieurs décennies dans la littérature économique.

En parallèle, l'analyse de cette littérature, notamment la littérature des Nouveaux Keynésiens, montre une absence quasi totale de référence aux agrégats monétaires. En effet, dans ce cadre théorique désormais classique, les agrégats monétaires n'apparaissent pas explicitement dans les équations descriptives des dynamiques macroéconomiques, que ce soit dans l'écart de production, dans la dynamique de l'inflation ou encore dans la détermination des taux d'intérêt.

L'inflation y est expliquée par le taux d'inflation anticipé ainsi que par l'écart de production. De son coté, l'écart de production dépend principalement de son anticipation et du taux d'intérêt réel (Clarida, Galí et Gertler, 1999; Woodford, 2003; Galí et Gertler, 2007; Galí, 2008). Enfin, le taux d'intérêt est fixé par une règle de Taylor traditionnelle en fonction de l'écart d'inflation et de l'écart de production.

Dans ce cadre, la politique monétaire, via une modification du taux d'intérêt réel, influence la demande globale et par voie directe de conséquence, l'inflation et la production: une augmentation du taux d'intérêt fait diminuer la production, ce qui augmente l'écart de production, diminue l'inflation, et ce jusqu'à ce qu'un nouvel équilibre soit atteint.

La masse monétaire et la demande de monnaie ne figurent donc pas d'une façon clairement et explicitement formulée dans cette théorie et c'est la banque centrale qui fixe le taux d'intérêt nominal de manière à satisfaire la demande de monnaie (Woodford, 2003; Ireland, 2004).

Cette approche du mécanisme de transmission néglige le comportement des encaisses monétaires réelles, ces dernières pouvant avoir des effets sur la demande globale suite à une modification des prix. Comme les individus réaffectent leur portefeuille d'actifs, le comportement des encaisses monétaires réelles induit des ajustements des prix relatifs des actifs financiers et immobiliers.

Dans le processus, la demande globale est modifiée, ce qui modifie donc la production. En affectant la demande globale, les encaisses réelles font partie du mécanisme de transmission. Ainsi, nous pouvons affirmer que le seul taux d'intérêt ne suffit pas à expliquer l'implication de la politique monétaire sur les marchés et le rôle actif ou rétroactif joué par les marchés financiers (Meltzer, 1995, 1999, Brunner et Meltzer, 1968).

En parallèle, un facteur est aussi digne d'intérêt pour notre analyse: l'aversion au risque, autrement dit, l'attitude mentale et psychologique face au risque (la peur du lendemain).

Un rôle spécifique des encaisses réelles peut exister en fonction de l'aversion au risque des ménages. Lorsque l'aversion au risque augmente, les ménages peuvent choisir de détenir plus de monnaie pour faire face à l'incertitude implicite et optimiser leur consommation à travers le temps, en arbitrant entre la dépense présente et la dépense future.

Milton Friedman a fait allusion à ce processus dès 1956 (Friedman, 1956). Si cette hypothèse était fondée, l'aversion au risque pourrait modifier l'impact des encaisses réelles sur les prix relatifs des actifs financiers et des actifs réels, et par conséquent, sur la demande globale et la production.

En guise d'introduction à ces développements, nous montrons dans une première partie que la monnaie, dans un cadre Nouveau Keynésien classique, ne joue aucun rôle dans les dynamiques de l'économie (Ireland, 2004; Andrés, López-Salido et Vallés, 2006). Nous en profiterons pour développer un modèle DSGE de référence principalement inspiré par Smets et Wouters (2003) et Galí (2008). Cette modélisation sera centrée sur le rôle de l'aversion au risque dans l'économie afin de répondre à la question suivante : quelles sont les conséquences d'un choc d'aversion au risque sur les dynamiques des principales variables macroéconomiques ?

Au sein d'un modèle DSGE Nouveau Keynésien, lorsque la monnaie est introduite de façon séparable dans la fonction d'utilité des ménages, cette dernière n'a pas de rôle, à première vue, sur les dynamiques des variables macroéconomiques utilisées (inflation, production et taux d'intérêt).

Nelson (2008) a montré que ces modèles standards Nouveau Keynésien sont construits sur l'hypothèse controversée de banques centrales pouvant contrôler le taux d'intérêt à long terme, alors même que cette variable est de fait déterminée par une équation de Fisher dont l'inflation attendue dépend de l'évolution monétaire. Reynard (2007) a constaté qu'aux États-Unis et dans la Zone Euro, les évolutions monétaires fournissent des informations qualitatives et quantitatives sur l'inflation. Assenlacher-Wesche et Gerlach (2006) ont confirmé que la croissance monétaire contenait des informations sur les pressions inflationnistes et pouvait jouer un rôle sur l'état des différentes variables non observées, ou difficiles à observer, qui influent sur l'inflation ou la production.

Comment la monnaie est-elle généralement introduite dans les modèles DSGE Nouveaux Keynésiens ? La méthode standard est de recourir à une fonction d'utilité non-séparable, où les encaisses monétaires réelles sont censées influer sur l'utilité marginale de la consommation. Kremer, Lombardo, et Werner (2003) corroborent cette hypothèse de non-séparabilité pour l'Allemagne, et ont montré que les encaisses monétaires réelles contribuaient à la détermination de la production et de la dynamique de l'inflation. Une contribution récente a introduit le rôle de la monnaie avec des coûts d'ajustement de détention de monnaie réelle, et a montré que les encaisses réelle devraient contribuer à expliquer les variations futures du taux d'intérêt naturel aux États-Unis et dans la Zone Euro (Andrés, López-Salido, et Nelson, 2009). Nelson (2002) a constaté que la monnaie était un déterminant significatif de la demande globale, à la fois aux États-Unis et au Royaume-Uni. Cependant, les travaux empiriques entrepris par Ireland (2004), Andrés, López-Salido, et Vallés (2006) et Jones et Stracca (2008) suggèrent qu'il y'a peu d'éléments concernant le rôle significatif de la monnaie dans le cas des États-Unis, de la Zone Euro et du Royaume-Uni.

Dans les années cinquante, une autre méthode consistait à considérer les encaisses réelles comme un facteur de production à part entière, contribuant ainsi à introduire la monnaie dans la fonction de production (Sinai et Stokes, 1972; Ben-Zion et Ruttan, 1974; Short, 1979). La controverse de la monnaie comme facteur de production était née, notamment à la suite des travaux de Levhari et Patinkin (1968), Friedman (1969), Johnson (1969) et Stein (1970). Elle s'est éteinte quelques années plus tard, avec les travaux de Boyes et Kavanaugh (1978) qui ont montré que l'inclusion d'encaisses réelles dans une fonction de production n'était pas significative.

Dans la deuxième partie de la thèse, nous avons choisi d'explorer à nouveau ce champ délaissé de l'économie monétaire, récemment remis en avant par Petrucci (1999) et Chang (2002), avec les outils de l'analyse des Nouveaux Keynésiens actuels. Cette étude aura pour conséquence d'apporter une réponse à une controverse historique concernant l'hypothèse de rendements d'échelle constants ou décroissants dans ce nouveau genre de fonction de production. Short (1979), Startz (1984), Benzing (1989) et Chang (2002) considèrent que les rendements d'échelle sont constants entre la monnaie et le travail dans la fonction de production alors que nos recherches confirment l'hypothèse de Khan et Ahmad (1985) de rendements d'échelle décroissants entre la monnaie et le travail dans la fonction de production. Bien que la monnaie soit introduite de façon explicite dans la fonction de production, et après en avoir conclut que l'hypothèse de rendements d'échelle constant n'a pas lieu d'être maintenue, nous en concluons que la monnaie n'a qu'un rôle négligeable dans les dynamiques de la production, de l'inflation et des taux d'intérêt.

Dans une troisième partie, nous nous distinguons de la littérature tant dans les conclusions empiriques, donnant un rôle plus important à la monnaie, du moins dans la Zone Euro, que dans la modélisation théorique. Comme dans la littérature, nous avons eu recours à une fonction d'utilité incluant la monnaie (MIU) avec une hypothèse de non-séparabilité entre la consommation et les encaisses réelle. Nous spécifions tous les paramètres microfondés, permettant d'extraire les caractéristiques et implications de ce type de modèle qui ne pourraient être extraites si seuls les paramètres agrégés étaient utilisés. Nous montrons, entre autres, que le coefficient d'aversion au risque peut jouer un rôle important dans l'explication du rôle de la monnaie.

Notre modèle se distingue également par son expression de l'inflation et de la dynamique de la production. Les modèles standards DSGE Nouveaux Keynésiens donnent un rôle important à l'inertie endogène, à la fois sur la production (habitudes de consommation des ménages) et sur l'inflation (indexation des prix affichés par les entreprises). En fait, les deux dynamiques peuvent avoir une plus forte composante prospective que leur composante inertielle. Et cela semble être le cas au moins dans la Zone Euro, s'il ne l'est pas clairement aux États-Unis (Galí, Gertler et Lopez-Salido, 2001). Ces composantes inertielles peuvent cacher une partie du rôle de la monnaie. Par conséquent, notre choix méthodologique a été de simplifier autant que possible la modélisation en question afin de pouvoir mettre en exergue un éventuel rôle des encaisses réelles sur les autres variables.

Nous nous distinguons également par les analyses empiriques de la Zone Euro en utilisant des techniques d'analyse Bayésienne, comme dans Smets et Wouters (2007), tout en introduisant la monnaie dans le modèle, comme plus récemment dans Barthélemy, Clerc et Marx (2011). Nous estimons aussi les microparamètres du modèle, alors que les tentatives d'introduction de la monnaie dans la littérature actuelle, par agrégation de certains paramètres et sans spécification explicite de la fonction d'utilité non-séparable utilisée, laissent de côté des informations potentiellement pertinentes.

Nous simulons cette modélisation afin d'analyser les conséquences de chocs structurels sur l'économie. L'analyse dynamique des modèles met en outre en lumière les changements dans le rôle de la monnaie pour expliquer les fluctuations à court terme de la production liés aux changements de l'aversion des ménages pour le risque. Ils montrent que plus l'aversion au risque est grande, plus le rôle de la monnaie dans le processus de transmission augmente. Nous mettons également en évidence le rôle prépondérant de la politique monétaire sur la variabilité de l'inflation.

Enfin, dans une quatrième et dernière partie, nous nous intéressons aux capacités prédictives de tels modèles, notamment en période de crise. En effet, la justesse des prévisions des trajectoires futures de séries macroéconomiques telles que la production ou l'inflation est une information cruciale pour le gouvernement et la banque centrale dans leur processus décisionnel. Les modèles DSGE fournissent des informations précieuses sur la dynamique des cycles économiques et sur les effets de divers chocs sur l'économie (Smets et Wouters, 2007; Christiano, Motto et Rostagno, 2008). Pour toutes ces raisons, les modèles DSGE sont de plus en plus utilisés par les banques centrales et autres institutions économiques et financières comme aide aux décisions de politique monétaire (Tovar, 2008).

Dans l'analyse des politiques économiques, la politique monétaire engendre des effets variables sur l'ensemble de l'économie. Afin de rendre compte de telles interactions, aussi complexes soient-elles, entre les variables économiques et l'ensemble de l'économie, la prévision macroéconomique est devenu un outil indispensable dans l'élaboration des politiques économiques actuelles (Kohn 1995; Blinder 1997; Diebold, 1998). Sims et Zha (1998) ont introduit les techniques Bayésiennes dans l'estimation de vecteurs autorégressifs (BVAR) afin d'améliorer l'exactitude des prévisions hors échantillons dans un cadre dynamique multidimensionnel. Ils ont montré comment calculer des distributions de probabilité Bayésiennes et des bandes d'erreur autour de prévisions hors échantillon. Plus récemment, certains ont commencé à examiner la performance prévisionnelle de ces modèles: Smets et Wouters (2007) montrent de plus que les modèles DSGE peuvent générer des prévisions qui ont un faible écart de variance (Root Mean Squared Deviation, RMSD) par rapport aux modèles autorégressifs vectoriels Bayésiens (BVAR).

D'autre part, Edge et al. (2010) montrent que la performance des prévisions hors échantillon de la Reserve Fédérale des nouveaux modèle DSGE pour l'économie américaine (Estimated, Dynamic, Optimization-based, EDO) est dans de nombreux cas meilleure que leurs grands modèles macro-économétriques (FRB / US).

Après avoir montré que la monnaie joue un rôle important lorsque l'aversion au risque est assez élevée, nous choisissons de tester cette modélisation durant des périodes de crises. Comme ces périodes sont généralement limitées dans le temps, nous utilisons des échantillons relativement petits pour estimer le modèle de base (fonction d'utilité séparable) et le modèle à fonction d'utilité non-séparable entre la monnaie et la consommation, afin de saisir l'influence des paramètres à très court terme sur les dynamiques des variables des deux modèles.

En utilisant des techniques d'estimation Bayésienne, nous estimons ces deux modèles avec des données sur la Zone Euro sur trois crises différentes: lors des attaques spéculatives sur les monnaies dans le mécanisme de change européen (MCE), début 1992; après l'éclatement de la bulle internet, début 2001; et durant la crise des Subprimes de 2007 à 2010.

Nous analysons les variations des paramètres microéconomiques et macroéconomiques pendant ces périodes de crise, ainsi que la décomposition de la variance des variables par rapport à des chocs structurels microfondés. Nous étudions et comparons les capacités prédictives de ces deux modèles pendant ces périodes de crises pour remarquer que le modèle incluant la monnaie dans la fonction d'utilité sous une forme non séparable est meilleur en période de crise que le modèle séparable de base, en termes de capacité prédictive (erreur de prévision minimale). Nous accordons une attention particulière à l'impact de la politique monétaire et de la monnaie durant ces crises. Nous démontrons notamment que le rôle de la monnaie est plus important pendant les crises qu'en dehors des crises.

Les résultats de cette thèse mettent en évidence des mécanismes originaux dans le cadre de la modélisation actuelle, utilisée notamment au sein des banques centrales. Ces dernières peuvent, dans le cadre de leurs décisions de politique monétaire, considérer de tels développements en vue d'optimiser leur politique en temps de crise.

Chapter 1

Baseline New Keynesian DSGE model

Jonathan Benchimol

1.1 Abstract

We propose a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model where real money balances enter a separable utility function. By using a Bayesian analysis and simulations, we show that money has no role to play in the dynamics of the system. Our analysis also focuses on the role of risk aversion and suggests that risk aversion is an important component of the decomposition of output and real money balances dynamics, as well as flexible-price output and flexible-price interest rate. Risk aversion explains a part of the decrease of output during crises.

Keywords: Risk Aversion, New Keynesian DSGE models, Bayesian Analysis.

JEL Classification Number: E27, E30, E47.

1.2 Introduction

The development of the forward-looking, microfounded, New Keynesian DSGE model stands as one of the most significant achievements in new macroeconomics. The New Keynesian model consists of just three equations in its simplest form. The first corresponds to the log-linearization of an optimizing household's Euler equation, linking consumption and output growth to the inflation-adjusted return on nominal bonds, that is, to the real interest rate. The second, a forward-looking version of the Phillips curve, describes the optimizing behavior of monopolistically competitive firms that either set prices in a randomly staggered fashion, as suggested by Calvo (1983), or face explicit costs of nominal price adjustment, as suggested by Rotemberg (1982). The third equation, a monetary policy rule of the kind proposed by Taylor (1993), dictates that the central bank should adjust the short-term nominal interest rate in response to changes in inflation and output.

The New Keynesian model brings these three equations together to characterize the dynamic behavior of three key macroeconomic key variables: output, inflation, and the nominal interest rate.

> *Thus, the New Keynesian model places heavy emphasis on the behavior of nominal variables, calls special attention to the workings of monetary policy rules, and contains frequent allusions back to the traditional IS-LM framework. All this makes it easy to forget that the New Keynesian models of today share many basic features with, and indeed were originally derived as extensions to, a previous generation of dynamic, stochastic, general equilibrium models: the real business cycle models of Kydland and Prescott (1982), Prescott (1986), Cooley and Prescott (1995), and many others. In real business cycle models, technology shocks play the dominant role in driving macroeconomic fluctuations. Monetary policy either remains absent altogether or has minimal effects on the cyclical behavior of the economy, as in Cooley and Hansen (1989).*
>
> Peter N. Ireland (2002)

Yet, even if money or real balances are included in the utility (MIU) in a separable way or/and in the production function, money is an irrelevant variable in this framework (Woodford, 2003; Ireland, 2004; Chapter 2). In this baseline framework, under separable preferences, we will show how real money balances are completely recursive from the rest of the system. This article proposes to estimate a New Keynesian DSGE model so as to serve as a baseline model.

It also focuses on the role of risk aversion in the economy. Relative risk aversion measures the willingness to substitute consumption over different periods. The smaller risk aversion is, the more households substitute consumption over time. Bekaert, Engstrom, and Grenadier (2005) and Wachter (2006) show that a rise in risk aversion involves an increase in equity and bond premiums, and may increase the real interest rate through a consumption smoothing effect, or decrease it through a precautionary savings effect. Our model explores the role of risk aversion on inflation, output, interest rate and real money balances. A specific emphasis will be paid on how risk aversion could modify the dynamics of these key variables and could contribute to magnify crises.

This baseline model is used to simulate the behavior of the Eurozone economy. Bayesian estimation and dynamic analysis of the model, with impulse response functions following structural shocks and shock decompositions, yield different relationships between risk aversion and other structural variables. It sheds light on the importance of risk aversion, and its impact on the effectiveness of the monetary policy as well as its influence on output during crisis periods. It also sheds light on the role of risk aversion on real money balances, flexible-price output and flexible-price interest rate.

Section 1.3 describes the theoretical set up. In section 1.4, the model is calibrated and estimated with Bayesian techniques with Euro area data, and impulse response functions and variance decomposition are analyzed. Section 1.5 presents an alternative model with a risk aversion shock. Section 1.6 concludes and Section 1.7 presents additional results.

1.3 The model

The model consists of households that supply labor, purchase goods for consumption, hold money and bonds, and firms that hire labor and produce and sell differentiated products in monopolistically competitive goods markets. Each firm sets the price of the good it produces, but not all firms reset their price during each period. Households and firms behave optimally: households maximize the expected present value of utility, and firms maximize profits. There is also a central bank that controls the nominal rate of interest. This model is inspired by Smets and Wouters (2003) and Galí (2008).

1.3.1 Households

We assume a representative infinitely-lived household, seeking to maximize

$$E_t\left[\sum_{k=0}^{\infty}\beta^k U_{t+k}\right] \tag{1.1}$$

where U_t is the period utility function and $\beta < 1$ is the discount factor.

We assume the existence of a continuum of goods represented by the interval $[0;1]$. The household decides how to allocate its consumption expenditures among the different goods. This requires that the consumption index C_t be maximized for any given level of expenditures[1]. Furthermore, and conditional on such optimal behavior, the period budget constraint takes the form

$$P_tC_t + M_t + Q_tB_t \leq B_{t-1} + W_tN_t + M_{t-1} \tag{1.2}$$

for $t = 0, 1, 2...$, P_t is an aggregate price index, M_t is the quantity of money holdings at time t, B_t is the quantity of one-period nominally riskless discount bonds purchased in period t and maturing in period $t+1$ (each bond pays one unit of money at maturity and its price is Q_t where $i_t = -\log Q_t$ is the short term nominal rate), W_t is the nominal wage, and N_t is hours of work (or the measure of household members employed). The above sequence of period budget constraints is supplemented with a solvency condition[2].

Preferences are measured with a common time-separable utility function (MIU). Under the assumption of a period utility given by

$$U_t = e^{\varepsilon_t^P}\left(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\gamma e^{\varepsilon_t^M}}{1-\nu}\left(\frac{M_t}{P_t}\right)^{1-\nu} - \frac{\chi N_t^{1+\eta}}{1+\eta}\right) \tag{1.3}$$

consumption, labor supply, money demand and bond holdings are chosen to maximize (1.1) subject to (1.2) and the solvency condition. This MIU utility function depends positively on the consumption of goods, C_t, positively on real

[1] See Appendix 1.7.1

[2] Such as $\forall t \lim_{n\longrightarrow\infty} E_t\left[B_n\right] \geq 0$. It prevents engaging in Ponzi-type schemes.

money balances, $\frac{M_t}{P_t}$, and negatively on labour N_t. σ is the coefficient of relative risk aversion of households (or the inverse of the intertemporal elasticity of substitution), ν is the inverse of the elasticity of money holdings with respect to the interest rate, and η is the inverse of the elasticity of work effort with respect to the real wage. The utility function also contains three structural shocks: ε_t^P is a general shock to preferences that affects the intertemporal substitution of households (preference shock), and ε_t^M is a money shock. γ and χ are positive scale parameters.

This setting leads to the following conditions[3], which, in addition to the budget constraint, must hold in equilibrium. The resulting log-linear version of the first order condition corresponding to the demand for contingent bonds implies that

$$c_t = E_t\left[c_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right] - \rho_c\right) - \sigma^{-1} E_t\left[\Delta\varepsilon_{t+1}^P\right] \tag{1.4}$$

where the lowercase letters denote the logarithm form of the original aggregated variables, $\rho_c = -\log(\beta)$ and where Δ is the first-difference operator.

The demand for cash that follows from the household's optimization problem is given by

$$\varepsilon_t^M + \sigma c_t - \nu\left(m_t - p_t\right) - \rho_m = a_2 i_t \tag{1.5}$$

where $mp_t = m_t - p_t$ are the log linearized real money balances, $\rho_m = -\log(\gamma) + a_1$, a_1 and a_2 are resulting terms of the first-order Taylor approximation[4] of $\log(1 - Q_t) = a_1 + a_2 i_t$.

Real cash holdings depend positively on consumption with an elasticity equal to $\frac{\sigma}{\nu}$ and negatively on the nominal interest rate[5]. In what follows we take the nominal interest rate as the central bank's policy instrument[6].

The resulting log-linear version of the first order condition corresponding to the optimal consumption-leisure arbitrage implies that

$$w_t - p_t = \sigma c_t + \eta n_t - \rho_n + \varepsilon_t^N \tag{1.6}$$

[3] See Appendix 1.7.2

[4] More precisely, if we compute our first-order Taylor approximation around $\frac{1}{\beta}$, the steady state interest rate, we obtain: $a_1 = \log\left(1 - \exp\left(-\frac{1}{\beta}\right)\right) - \frac{\frac{1}{\beta}}{e^{\frac{1}{\beta}} - 1}$, $a_2 = \frac{1}{e^{\frac{1}{\beta}} - 1}$.

[5] Because $\frac{1}{\beta} > 1$, then $a_2 > 0$.

[6] In the literature, due to the assumption that consumption and real money balances are additively separable in the utility function, cash holdings do not enter any of the other structural equations: accordingly, the above equation becomes recursive to the rest of the system of equations.

where $\rho_n = -\log(\chi)$.

Finally, these equations represent the Euler condition for the optimal intratemporal allocation of consumption (equation (1.4)), the intertemporal optimality condition setting the marginal rate of substitution between money and consumption equal to the opportunity cost of holding money (equation (1.5)), and the intratemporal optimality condition setting the marginal rate of substitution between leisure and consumption equal to the real wage (equation (1.6)).

1.3.2 Firms

We assume a continuum of firms indexed by $i \in [0,1]$. Each firm produces a differentiated good, but they all use an identical technology, represented by the following production function[7]

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{1.7}$$

where A_t represents the level of technology, assumed to be common to all firms and to evolve exogenously over time.

All firms face an identical isoelastic demand schedule, and take the aggregate price level P_t and aggregate consumption index C_t as given. As in the standard Calvo (1983) model, our generalization features monopolistic competition and staggered price setting. At any time t, only a fraction $1-\theta$ of firms, with $0 < \theta < 1$, can reset their prices optimally, while the remaining firms index their prices to lagged inflation[8].

1.3.3 Price dynamics

Let's assume a set of firms not reoptimizing their posted price in period t. Using the definition of the aggregate price level[9] and the fact that all firms resetting prices choose an identical price P_t^*, leads to $P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(P_t^*\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$. Dividing both sides by P_{t-1} and log-linearizing around $P_t^* = P_{t-1}$ yields

$$\pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right) \tag{1.8}$$

[7] We assume a very simple production function in order to simplify the model without losing explanatory power of the production function and its key variable (N_t).

[8] Thus, each period, $1-\theta$ producers reset their prices, while a fraction θ keep their prices unchanged.

[9] As shown in Appendix 1.7.1

In this setup, we don't assume inertial dynamics of prices. Inflation results from the fact that firms reoptimizing in any given period their price plans, choose a price that differs from the economy's average price in the previous period.

1.3.4 Price setting

A firm reoptimizing in period t chooses the price P_t^* that maximizes the current market value of the profits generated while that price remains effective. We solve this problem to obtain a first-order Taylor expansion around the zero inflation steady state of the firm's first-order condition leads to

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\widehat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})\right] \tag{1.9}$$

where $\widehat{mc}_{t+k|t} = mc_{t+k|t} - mc$ denotes the log deviation of marginal cost from its steady state value $mc = -\mu$, and $\mu = \log(\varepsilon/(\varepsilon-1))$ is the log of the desired gross markup.

1.3.5 Equilibrium

Market clearing in the goods market requires $Y_t(i) = C_t(i)$ for all $i \in [0,1]$ and all t. Aggregate output is defined as $Y_t = \left(\int_0^1 Y_t(i)^{1-\frac{1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$; it follows that $Y_t = C_t$ must hold for all t. One can combine the above goods market clearing condition with the consumer's Euler equation (1.4) to yield the equilibrium condition

$$y_t = E_t[y_{t+1}] - \sigma^{-1}(i_t - E_t[\pi_{t+1}] - \rho_c) - \sigma^{-1}E_t\left[\Delta\varepsilon_{t+1}^P\right] \tag{1.10}$$

Market clearing in the labor market requires $N_t = \int_0^1 N_t(i)\,di$. Using (1.7) leads to

$$\begin{aligned} N_t &= \int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}} di \\ &= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} di \end{aligned} \tag{1.11}$$

where the second equality follows from the demand schedule and the goods market clearing condition. Taking logs leads to

$$(1-\alpha)\,n_t = y_t - \varepsilon_t^a + d_t$$

where $d_t = (1-\alpha)\log\left(\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} di\right)$ and di is a measure of price (and, hence, output) dispersion across firms. Following Galí (2008), in a neighborhood of the zero inflation steady state, d_t is equal to zero up to a first-order approximation.

Hence, one can write the following approximate relation between aggregate output, employment, real money balances and technology as

$$y_t = \varepsilon_t^a + (1-\alpha)\, n_t \tag{1.12}$$

An expression is derived for an individual firm's marginal cost in terms of the economy's average real marginal cost. With the marginal product of labor

$$\begin{aligned} mpn_t &= \log\left(\frac{\partial Y_t}{\partial N_t}\right) \\ &= \varepsilon_t^a + \log(1-\alpha) - \alpha n_t \end{aligned}$$

we obtain an expression of the marginal cost

$$\begin{aligned} mc_t &= (w_t - p_t) - mpn_t \\ &= w_t - p_t - \frac{1}{1-\alpha}(\varepsilon_t^a - \alpha y_t) - \log(1-\alpha) \end{aligned}$$

for all t, where the second equality defines the economy's average marginal product of labor, mpn_t, in a way consistent with (1.12). Using the fact that $mc_{t+k|t} = (w_{t+k} - p_{t+k}) - mpn_{t+k|t}$ then

$$\begin{aligned} mc_{t+k|t} &= (w_{t+k} - p_{t+k}) - \frac{1}{1-\alpha}\left(\varepsilon_{t+k}^a - \alpha y_{t+k|t}\right) - \log(1-\alpha) \\ &= mc_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}(p_t^* - p_{t+k}) \end{aligned} \tag{1.13}$$

where the second equality follows from the demand schedule $C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t$ combined with the market clearing condition ($y_t = c_t$).

Substituting (1.13) into (1.9) and rearranging terms yields

$$\begin{aligned} p_t^* - p_{t-1} &= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\Theta\widehat{mc}_{t+k} - (p_{t+k} - p_{t-1})\right] \\ p_t^* - p_{t-1} &= (1-\beta\theta)\,\Theta\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\widehat{mc}_{t+k}\right] + \sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\pi_{t+k}\right] \end{aligned} \tag{1.14}$$

where $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \leq 1$.

Finally, combining (1.8) in (1.14) yields the inflation equation[10]

$$\pi_t = \beta E_t \left[\pi_{t+1}\right] + \lambda_{mc}\widehat{mc}_t \tag{1.15}$$

where $\lambda_{mc} = \Theta\frac{(1-\theta)(1-\beta\theta)}{\theta}$ is strictly decreasing in the index of price stickiness θ, in the measure of decreasing returns α, and in the demand elasticity ε.

Next, a relation is derived between the economy's real marginal cost and a measure of aggregate economic activity. Notice that independent of the nature of price setting, average real marginal cost can be expressed as

$$\begin{aligned} mc_t &= (w_t - p_t) - mpn_t \\ &= \left(\sigma + \frac{\eta+\alpha}{1-\alpha}\right) y_t - \frac{1+\eta}{1-\alpha}\varepsilon_t^a - \log(1-\alpha) - \rho_n + \varepsilon_t^N \end{aligned} \tag{1.16}$$

where derivation of the second equality make use of the household's optimality condition (1.6) and the (approximate) aggregate production relation (1.12).

Furthermore, and as shown previously, the flexible-price real marginal cost is constant and given by $mc = -\mu$. Defining the natural level of output, denoted by y_t^f, as the equilibrium level of output under flexible prices

$$mc = \left(\sigma + \frac{\eta+\alpha}{1-\alpha}\right) y_t^f - \frac{1+\eta}{1-\alpha}\varepsilon_t^a - \log(1-\alpha) - \rho_n + \varepsilon_t^N \tag{1.17}$$

thus implying

$$y_t^f = \upsilon_a \varepsilon_t^a + \upsilon_0 \tag{1.18}$$

where $\upsilon_a = \frac{1+\eta}{\sigma(1-\alpha)+\eta+\alpha}$ and $\upsilon_0 = \frac{(1-\alpha)\left(\log(1-\alpha)+\rho_n-\log\left(\frac{\varepsilon}{\varepsilon-1}\right)\right)}{\sigma(1-\alpha)+\eta+\alpha}$. Subtracting (1.17) from (1.16) yields

$$\widehat{mc}_t = \left(\sigma + \frac{\eta+\alpha}{1-\alpha}\right)\left(y_t - y_t^f\right) \tag{1.19}$$

where $\widehat{mc}_t = mc_t - mc$ is the *real marginal cost gap* and $y_t - y_t^f$ is the *output gap*. Then, combining the above equation with (1.15), one can obtain our first equation relating inflation to its one period ahead forecast and output gap

$$\pi_t = \beta E_t \left[\pi_{t+1}\right] + \psi_x \left(y_t - y_t^f\right) \tag{1.20}$$

[10] Notice that the above discounted sum (1.14) can be rewritten more compactly as the difference equation

$$p_t^* - p_{t-1} = \beta\theta E_t \left[p_{t+1}^* - p_t\right] + (1-\beta\theta)\,\Theta\widehat{mc}_t + \pi_t$$

by adding $-\beta\theta E_t\left[(1.14)_{t+1}\right]$ to (1.14).

where $\psi_x = \frac{(1-\theta)(1-\beta\theta)(\sigma(1-\alpha)+\eta+\alpha)}{\theta(1-\alpha+\alpha\varepsilon)}$ and $\psi_n = -\frac{(1-\alpha)(1-\theta)(1-\beta\theta)}{\theta(1-\alpha+\alpha\varepsilon)}$.

From (1.10), we obtain an expression for the *natural interest rate*

$$i_t^f = \rho_c + \sigma E_t \left[\Delta y_{t+1}^f \right] \tag{1.21}$$

The second key equation describing the equilibrium of the New Keynesian model is obtained from (1.10)

$$y_t = E_t \left[y_{t+1} \right] - \sigma^{-1} \left(i_t - E_t \left[\pi_{t+1} \right] - \rho_c \right) - \sigma^{-1} E_t \left[\Delta \varepsilon_{t+1}^P \right] \tag{1.22}$$

Henceforth (1.22) is referred to as the dynamic IS equation.

The third key equation describes behavior of the *real money balances.* Rearranging (1.5) yields

$$mp_t = \frac{\sigma}{\nu} y_t - \frac{a_2}{\nu} i_t - \frac{\rho_m}{\nu} + \frac{1}{\nu} \varepsilon_t^M \tag{1.23}$$

The last equation determines the interest rate through a standard smoothed Taylor-type rule

$$i_t = (1 - \lambda_i) \left(\lambda_\pi \left(\pi_t - \pi^* \right) + \lambda_x \left(y_t - y_t^f \right) \right) + \lambda_i i_{t-1} + \varepsilon_t^i \tag{1.24}$$

where λ_π and λ_x are policy coefficients reflecting the weight on inflation and on the output gap; the parameter $0 < \lambda_i < 1$ captures the degree of interest rate smoothing. All structural shocks are assumed to follow a first-order autoregressive process with an *i.i.d.* normal error term, such as $\varepsilon_t^k = \mu_k \varepsilon_{t-1}^k + \omega_{k,t}$, where $\varepsilon_{k,t} \sim N(0; \sigma_k)$ for $k = \{P, M, i, a\}$. To simplify, we assume that the target inflation rate is equal to zero, i.e. $\pi^* = 0$.

1.4 Results

1.4.1 DSGE model

Our macro model consists of six equations and six dependent variables: inflation, nominal interest rate, output, real money balances, flexible-price output and the natural interest rate. Flexible-price output and the natural interest rate are completely determined by shocks: flexible-price output is mainly driven by technology shocks (whereas fluctuations in the output gap can be attributed to supply and demand shocks) whereas the natural interest rate is mainly driven by flexible-price output.

$$y_t^f = \frac{1+\eta}{\sigma(1-\alpha)+\eta+\alpha}\varepsilon_t^a + \frac{(1-\alpha)\left(\log(1-\alpha)+\rho_n-\log\left(\frac{\varepsilon}{\varepsilon-1}\right)\right)}{\sigma(1-\alpha)+\eta+\alpha} \quad (1.25)$$

$$i_t^f = \rho_c + \sigma E_t\left[\Delta y_{t+1}^f\right] \quad (1.26)$$

$$\pi_t = \beta E_t[\pi_{t+1}] + \frac{(1-\theta)(1-\beta\theta)(\sigma(1-\alpha)+\eta+\alpha)}{\theta(1-\alpha+\alpha\varepsilon)}\left(y_t - y_t^f\right) \quad (1.27)$$

$$y_t = E_t[y_{t+1}] - \sigma^{-1}(i_t - E_t[\pi_{t+1}] - \rho_c) - \sigma^{-1}E_t\left[\Delta\varepsilon_{t+1}^P\right] \quad (1.28)$$

$$mp_t = \frac{\sigma}{\nu}y_t - \frac{a_2}{\nu}i_t - \frac{\rho_m}{\nu} + \frac{1}{\nu}\varepsilon_t^M \quad (1.29)$$

$$i_t = (1-\lambda_i)\left(\lambda_\pi(\pi_t - \pi^*) + \lambda_x\left(y_t - y_t^f\right)\right) + \lambda_i i_{t-1} + \varepsilon_t^i \quad (1.30)$$

where $\rho_m = -\log(\gamma) + a_1$, $\rho_n = -\log(\chi)$, $\rho_c = -\log(\beta)$, $a_1 = \log\left(1 - e^{-\frac{1}{\beta}}\right) - \frac{\frac{1}{\beta}}{e^{\frac{1}{\beta}}-1}$ and $a_2 = \frac{1}{e^{\frac{1}{\beta}}-1}$.

1.4.2 Euro area data

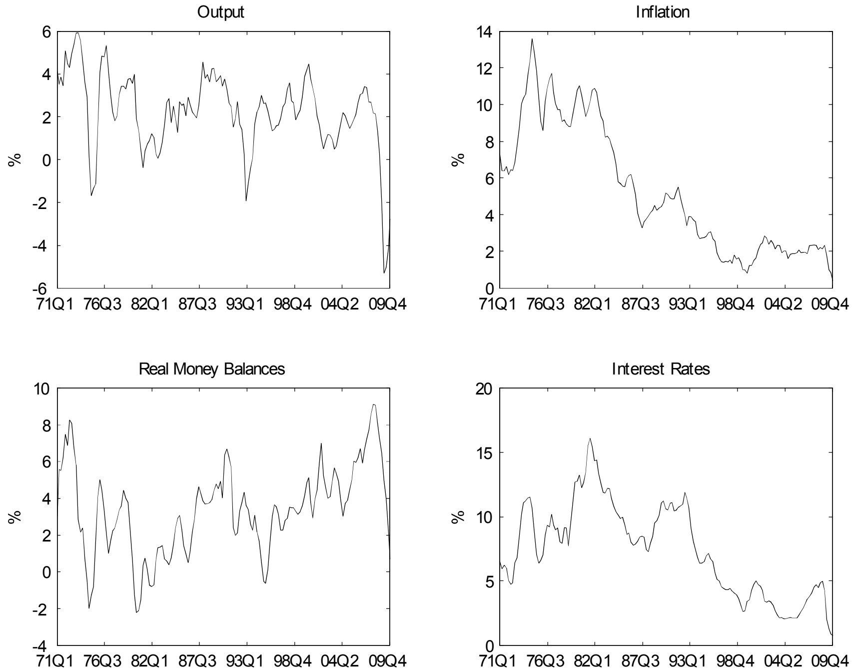


Figure 1.1: Euro area data

In this model of the Eurozone, π_t is the log-deviation of prices such as inflation rate is measured as the quarter to quarter change in the GDP Deflator, y_t is the log-deviation of output measured as the quarter to quarter change in the GDP, and i_t is the short-term (3-month) nominal interest rate. These Data are extracted from the Euro area Wide Model database (AWM) of Fagan, Henry and Mestre (2001). mp_t is the quarter to quarter log-deviation rate of real money balances. We use the $M3$ monetary aggregate from the OECD database. y_t^f, the trend of the log-deviation of output, and i_t^f, the trend of the nominal interest rate, are completely determined by structural shocks.

1.4.3 Calibration and results

We calibrate our model following Galí (2003), such as $\sigma = 2$ (risk aversion coefficient), $\eta = 1$ (inverse of the Frisch elasticity of labor supply), and $\varepsilon = 6$ (elasticity of demand of households for consumption goods). The others parameters are estimated with Bayesian techniques.

Calibration and estimation of structural parameters

	Prior			Posterior				
	Law	Mean	Std.	Mean	t-stat	Std.	5%	95%
β	beta	0.99	0.005	0.9939	425.0	0.0023	0.9893	0.9987
α	beta	0.33	0.050	0.3750	7.296	0.0513	0.2933	0.4595
θ	normal	0.66	0.050	0.8056	34.43	0.0238	0.7643	0.8469
υ	normal	1.50	0.100	1.7988	20.80	0.0864	1.6525	1.9371
γ	beta	0.05	0.010	0.0500	4.883	0.0098	0.0337	0.0659
χ	beta	0.05	0.010	0.0506	4.935	0.0099	0.0340	0.0663
λ_i	beta	0.50	0.100	0.4890	9.643	0.0512	0.4069	0.5745
λ_π	normal	3.00	0.200	3.2125	16.55	0.1926	2.9022	3.5295
λ_x	normal	1.50	0.200	1.8050	10.43	0.1736	1.5172	2.0912
ρ_a	beta	0.75	0.100	0.9162	30.33	0.0301	0.8729	0.9620
ρ_p	beta	0.75	0.100	0.9116	62.82	0.0145	0.8880	0.9358
ρ_i	beta	0.75	0.100	0.9920	324.6	0.0031	0.9865	0.9975
ρ_m	beta	0.75	0.100	0.9479	59.35	0.0160	0.9229	0.9742
σ_a	invgamma	0.02	2.000	0.0096	13.91	0.0007	0.0085	0.0108
σ_i	invgamma	0.02	2.000	0.0061	7.497	0.0008	0.0048	0.0074
σ_p	invgamma	0.02	2.000	0.0991	6.913	0.0139	0.0750	0.1219
σ_m	invgamma	0.02	2.000	0.0215	14.31	0.0015	0.0190	0.0239

Table 1.1: Bayesian estimation (baseline model)

The estimations of the implied posterior distribution of the parameters (Table 1.1) are done using the Metropolis-Hastings algorithm (10 chains, each of 100000 draws; see Smets and Wouters, 2007, and Adolfson et *al.*, 2007). The final acceptance rate is about 30% for all chains[11], which is a good value given that

[11] For the baseline model, the resulting acceptance rates are: 0.3037; 0.3048; 0.3035; 0.3027; 0.3030; 0.3084; 0.3033; 0.3023; 0.3021; 0.3076.

the literature[12] has settled on a value between 20% and 30%. We verify the convergence towards the target posterior distribution via the convergence checks proposed by Brooks and Gelman (1998). As typically done in the literature, we discarded all the draws not implying a unique equilibrium of the system.

See Appendix 1.7.3 for the model validation analysis and Appendix 1.7.4 for the prior and posterior distributions.

1.4.4 Simulations

Impulse response functions

According to the literature, Figure 1.2 shows that a preference shock increases the inflation rate, the output and the output gap, the nominal and the real interest rate, and the real money balances.

Figure 1.2 also presents the response to an interest rate shock. Inflation, the nominal interest rate, the output and the output gap fall. The real interest rate rises. A positive monetary policy shock induces a fall in interest rates due to a low enough degree of intertemporal substitution (i.e. the risk aversion parameter is high enough) which generates a large impact response of current consumption relative to future consumption. This result has been noted in, *inter alia*, Jeanne (1994) and Christiano et *al.* (1997).

The responses of output, real money balances and real money growth to a positive technology shock are positive. Notice that the improvement in technology is partly accommodated by the central bank, which lowers nominal interest and real interest rates, while increasing the quantity of money in circulation. However, that policy is not sufficient to close a negative output gap, which is responsible for the decline in inflation. Finally, Figure 1.2 shows the inexistent impact of money on the other variables of the system. It confirms the fact that, in this baseline model, money has no role to play on output, inflation, and interest rate.

[12] If the acceptance rate were too high, the Metropolis-Hastings iterations would never visit the tails of a distribution, while if it were too low, the iterations would get stuck in a subspace of the parameter range.

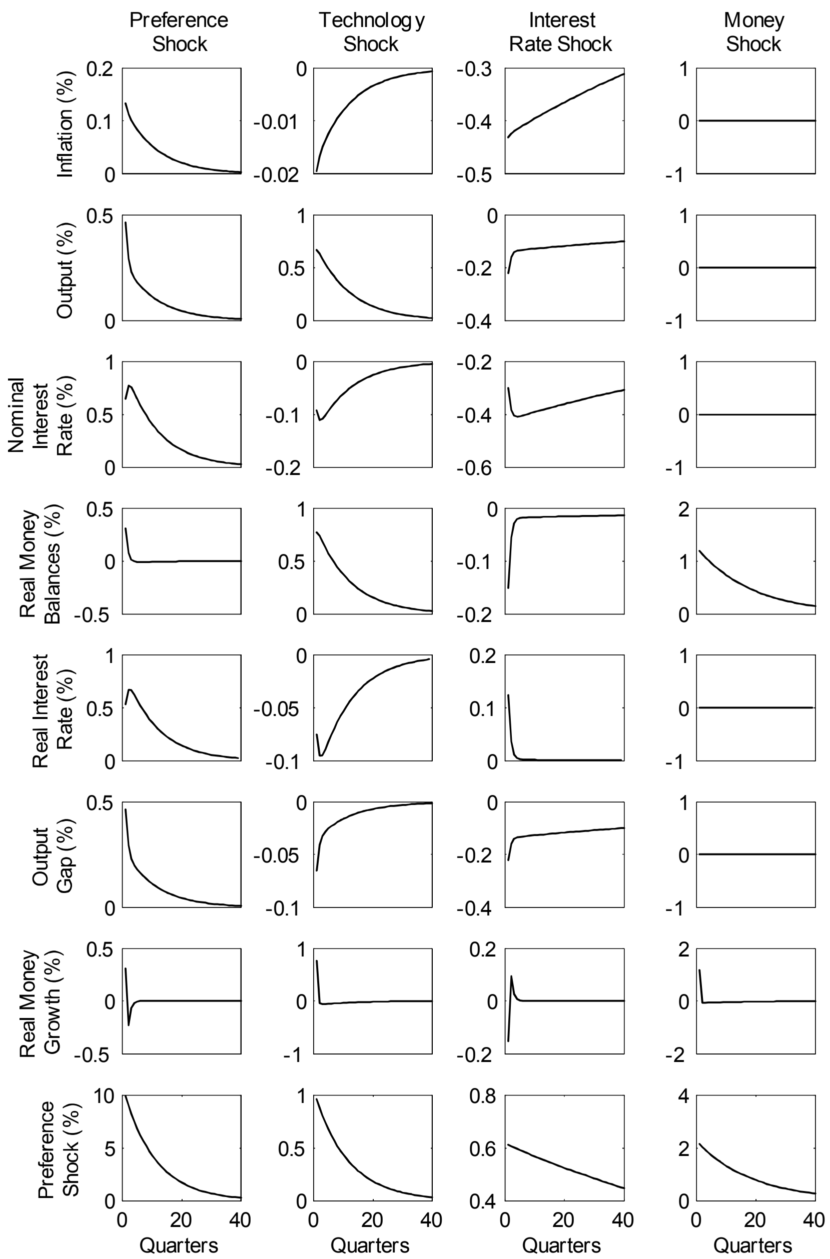


Figure 1.2: Impulse response functions (baseline model)

Variance decompositions

We analyze the forecast error variance decomposition of each variable following exogenous shocks. The analysis is conducted first via an unconditional variance decomposition (Table 1.2), and second via a conditional variance decomposition (Figure 1.3).

Unconditional variance decomposition (%)

	ε_t^P	ε_t^i	ε_t^M	ε_t^a
y_t	12.24	25.29	0.0	62.47
π_t	0.79	99.19	0.0	0.02
i_t	29.4	69.97	0.0	0.63
mp_t	0.57	0.25	76.86	22.32
y_t^f	0.0	0.0	0.0	100
i_t^f	0.0	0.0	0.0	100

Table 1.2: Variance decomposition (baseline model)

Unconditional variance decompositions (Table 1.2) show that the money shock has no role to play in this model, and is in turn completely recursive to the rest of the system of equations. It also shows that the flexible-price output and the natural interest rate are completely determined by the technology shock.

Table 1.2 shows that output is mainly explained by the technology shock (around 62%) and by the interest rate shock (around 25%), the rest of the output's variance is explained by the preference shock (around 12%). It also shows that inflation is mainly explained by the interest rate shock (monetary policy) and the interest rate is mainly explained by the interest rate shock (almost 70%) and the preference shock (almost 30%). Furthermore, most of the variance of real money balances is impacted by the money shock (almost 77%) and the technology shock (around 22%).

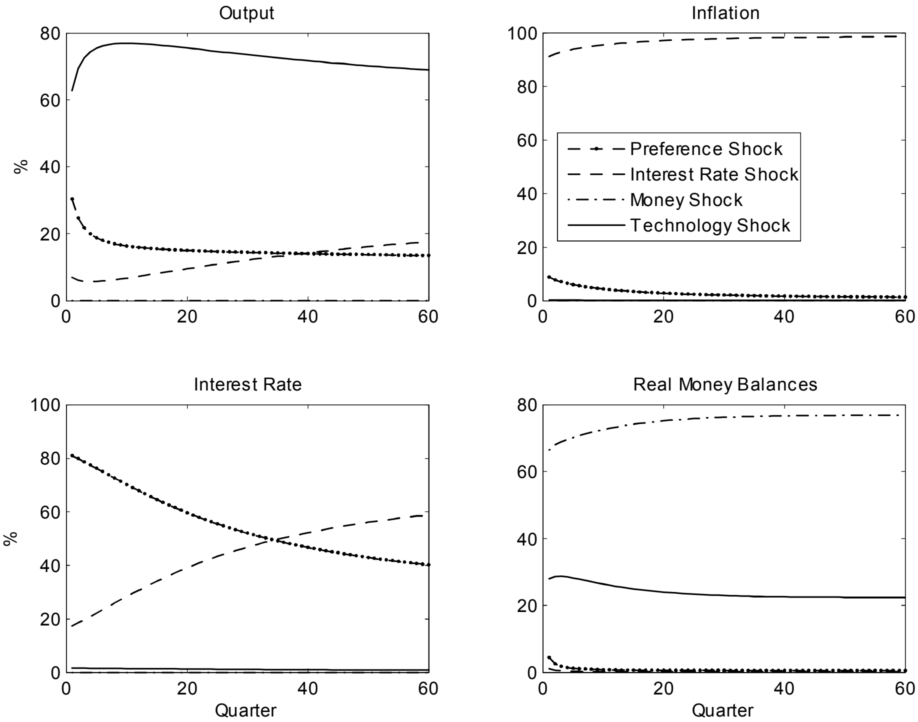


Figure 1.3: Variance decomposition through time (baseline model)

Variance decomposition through time of variables with respect to shocks (Figure 1.3) also shows that money has no role to play in the dynamics of output, inflation and interest rate. However, we remark that the preference shock has a significant role on output in the first periods, decreasing from about 30% to almost 15%. The role of interest rate on output increases from about 7% in the first periods to almost 20% at a longer horizon. Inflation is explained by the interest rate shock and real money balances are explained by the money shock, whereas the technology shock has a non-negligible role (about 20%) on money balances. Interest rate is mainly explained by the preference shock in the first periods and at a longer horizon, is mainly explained by the monetary policy shock.

The presence of sticky prices is shown to make monetary policy non-neutral. This is illustrated by analyzing the economy's response to two types of shocks: an exogenous monetary policy shock (interest rate shock) and a technology shock. It is also interesting to note that inflation is mainly explained by the monetary policy shock. Finally, the introduction of money in the utility by a separable way leads to an inexistent role for it in the dynamics of the other variables of the system. Here, money has no role to play.

1.5 Risk aversion shock

We have computed previously a baseline model with a calibrated risk aversion level ($\sigma = 2$) and we concluded that the money shock has no role to play.

In the following model, and in order to have as many shock as variables, we don't consider the money shock, which is irrelevant in the baseline framework (i.e. in a common New Keynesian DSGE model with separable preferences), and we include instead a risk aversion shock.

Then, we replace σ in equation 1.3 by $(\sigma + \varepsilon_t^r)$ where $\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \omega_{r,t}$ and $\varepsilon_{r,t} \sim N(0; \sigma_r)$. In other words, preferences are now measured with a time-separable utility function (MIU) such as

$$U_t = e^{\varepsilon_t^P} \left(\frac{C_t^{1-(\sigma+\varepsilon_t^r)}}{1-(\sigma+\varepsilon_t^r)} + \frac{\gamma}{1-\nu} \left(\frac{M_t}{P_t} \right)^{1-\nu} - \frac{\chi N_t^{1+\eta}}{1+\eta} \right) \tag{1.31}$$

All structural shocks are assumed to follow a first-order autoregressive process with an *i.i.d.* normal error term, such as $\varepsilon_t^k = \mu_k \varepsilon_{t-1}^k + \omega_{k,t}$, where $\varepsilon_{k,t} \sim N(0; \sigma_k)$ for $k = \{P, r, i, a\}$.

1.5.1 Calibration and results

The following table presents the results of the Bayesian estimation of the model with risk aversion shock

Calibration and estimation of structural parameters

	Priors			Posteriors				
	Law	Mean	Std.	Mean	t-stat	Std.	5%	95%
β	beta	0.99	0.005	0.9945	465.4	0.0021	0.9905	0.9986
α	beta	0.33	0.050	0.3424	7.812	0.0432	0.2723	0.4142
θ	beta	0.66	0.050	0.8154	37.03	0.0224	0.7767	0.8544
υ	normal	1.50	0.100	1.5757	18.50	0.0848	1.4386	1.7163
γ	beta	0.05	0.010	0.0498	4.883	0.0098	0.0332	0.0656
χ	beta	0.05	0.010	0.0479	4.781	0.0095	0.0321	0.0633
λ_i	beta	0.50	0.100	0.3911	7.380	0.0524	0.3054	0.4756
λ_π	normal	3.00	0.200	3.1641	16.26	0.1930	2.8572	3.4892
λ_x	normal	1.50	0.200	1.7833	10.32	0.1731	1.5020	2.0686
ρ_a	beta	0.75	0.100	0.9441	50.27	0.0188	0.9158	0.9738
ρ_r	beta	0.50	0.100	0.9113	54.51	0.0168	0.8846	0.9400
ρ_p	beta	0.75	0.100	0.9112	66.14	0.0138	0.8883	0.9345
ρ_i	beta	0.75	0.100	0.9929	385.4	0.0026	0.9883	0.9976
σ_a	invgamma	0.02	2.000	0.0066	13.74	0.0005	0.0058	0.0074
σ_r	invgamma	0.02	2.000	0.0345	7.318	0.0045	0.0266	0.0419
σ_i	invgamma	0.02	2.000	0.0069	8.480	0.0008	0.0055	0.0082
σ_p	invgamma	0.02	2.000	0.0928	7.116	0.0128	0.0714	0.1137

Table 1.3: Bayesian estimation (risk aversion shock model)

The estimation of the implied posterior distribution of the parameters (Table 1.3) is done using the Metropolis-Hastings algorithm (10 chains, each of 100000 draws; see Smets and Wouters, 2007, and Adolfson et *al.*, 2007). The final acceptance rate is about 30% for all chains[13], which is a good value given that the literature has settled on a value between 20% and 30%. We verified the convergence towards the target posterior distribution via the convergence checks

[13] The acceptance rates, for each chain, for the baseline model with risk aversion shock are: 0.3066; 0.3056; 0.3064; 0.3051; 0.3077; 0.3090; 0.3063; 0.3088; 0.3037; 0.3093.

proposed by Brooks and Gelman (1998). As typically done in the literature, we discarded all the draws not implying a unique equilibrium of the system.

See Appendix 1.7.3 for the model validation analysis and Appendix 1.7.4 for the prior and posterior distributions.

1.5.2 Simulations

Impulse response functions

Figure 1.4 and Figure 1.2 are very similar for the preference shock, the technology shock and the interest rate shock.

This case is very interesting because Figure 1.4 shows that a shock in risk aversion leads to a decrease in output and an increase in inflation. This implies a monetary policy tightening (because of the strong weight on inflation putted by the central banker). The risk aversion shock also implies an increase in the real money balances.

Households consumption is reduced (decreasing output) and companies increase their price (to face high risk aversion and maybe low consumption), which implies an increase of the inflation rate, contained by a monetary policy tightening.

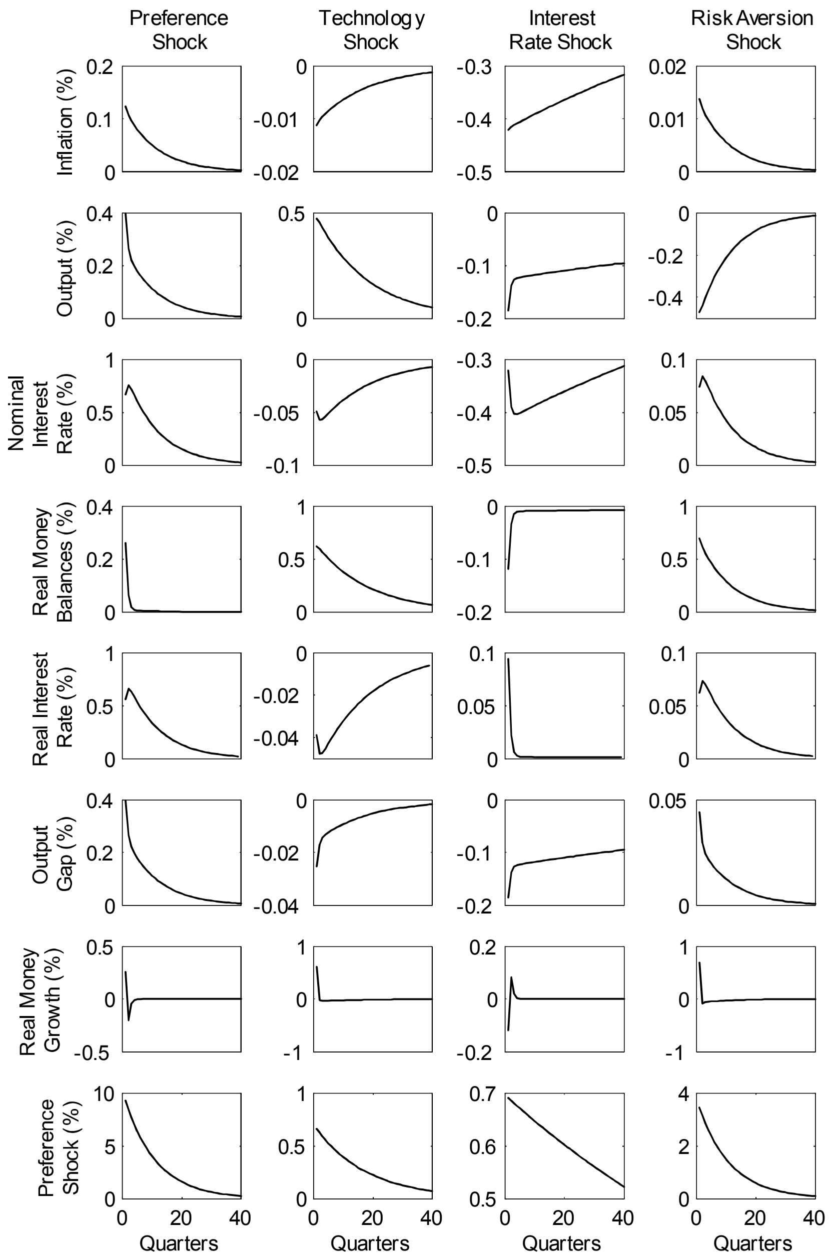


Figure 1.4: Impulse response functions (risk aversion shock model)

Variance decompositions

Unconditional variance decomposition (%)

	ε_t^r	ε_t^P	ε_t^i	ε_t^a
y_t	26.75	9.83	22.0	41.42
π_t	0.01	0.66	99.32	0.01
i_t	0.32	25.88	73.58	0.22
mp_t	41.48	1.13	0.33	57.06
y_t^f	40.64	0.0	0.0	59.36
i_t^f	63.29	0.0	0.0	36.71

Table 1.4: Variance decomposition (risk aversion shock model)

Table 1.4 shows that flexible-price output and the natural interest rate are completely determined by the technology shock and the risk aversion shock. This finding shows the leading role of relative risk aversion in the dynamics of these two important variables. Although flexible-price output and the natural interest rate are strong components of output, risk aversion has a minor role to play in the variance of inflation and interest rate.

As in the baseline model (Table 1.2), output variance is mainly driven by the technology shock. However, the variance of output depends also on the risk aversion shock (about 27%). Table 1.4 shows that inflation and interest rate variances are quasi unaffected by the introduction of the risk aversion shock, letting these variables mainly explained by, respectively, the interest rate shock, and the preference and the interest rate shock.

However, real money balances variance is not explained by the money shock, as in Table 1.2. In fact, Table 1.4 shows that real money balances are mainly driven by the technology shock (about 57%) and also by the risk aversion shock (about 41%).

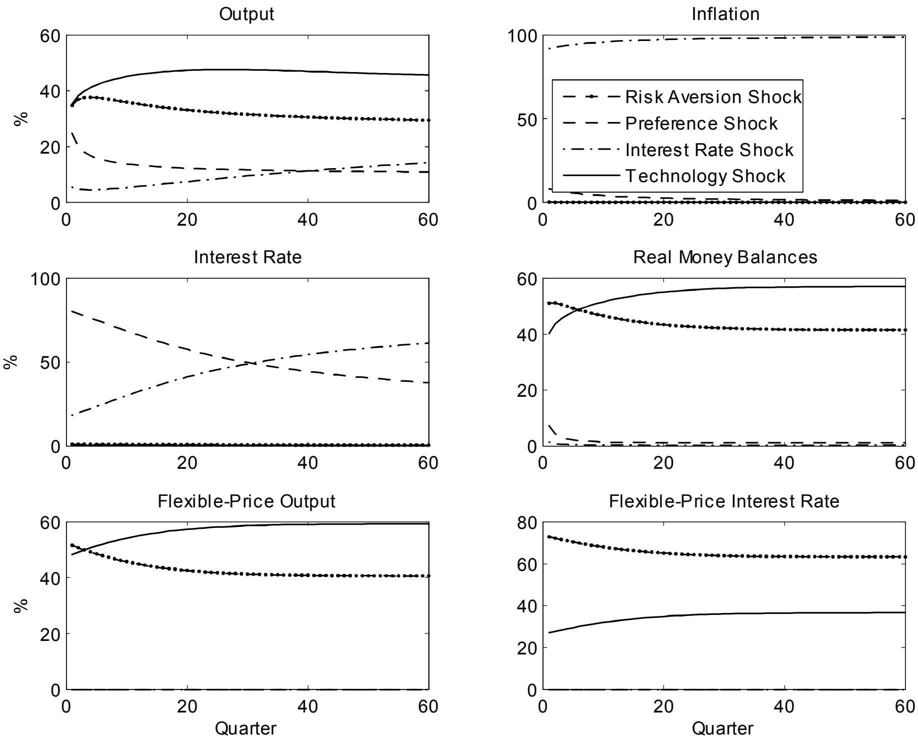


Figure 1.5: Variance decomposition through time (risk aversion shock model)

Figure 1.5 shows that interest rate plays an increasing role on output through time, reducing the role of preferences and risk aversion at longer horizon. Yet, introducing a risk aversion shock implies no consequences on the variance decomposition of inflation and interest rate through time.

Figure 1.5 clearly shows that at shorter horizons, risk aversion has an important role on real money balances dynamics, whereas at longer horizons, the variance of real money balances are mainly determined by the technology shock.

Last but not least, Figure 1.5 shows that risk aversion has a significant role in the distribution over the time of the flexible-price output and the natural interest rate variance decompositions.

Shock decompositions

We compute the shock decompositions of the key variables of the baseline model with a risk aversion shock.

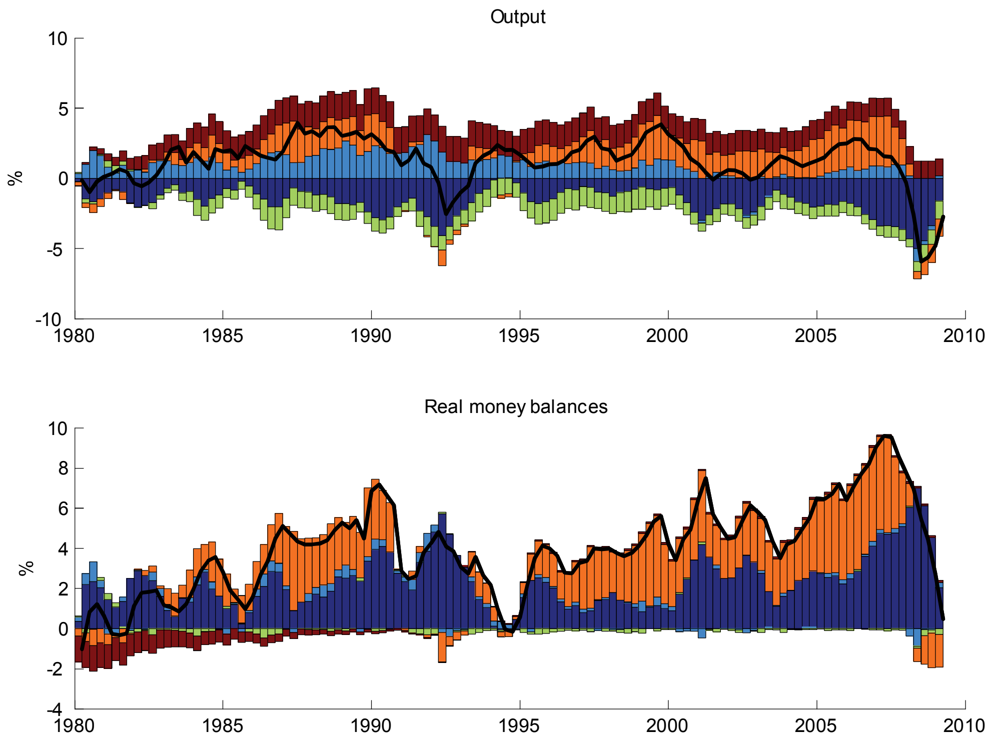
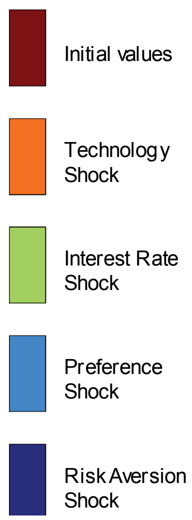


Figure 1.6: Output and real money balances shock decompositions

Figure 1.6 shows that the risk aversion shock contribution to output is more important during crisis periods than during normal periods (ε_t^r in dark blue, ε_t^P in blue, ε_t^i in green and ε_t^a in orange). We can clearly identify the European Monetary System crisis (1990-1993), the Dot-com crisis (2001-2003) and the beginning of the Subprime crisis (2007). Figure 1.6 also shows the significant role of risk aversion in the determination of historical real money balances, especially during financial crisis, where risk aversion plays an important role.



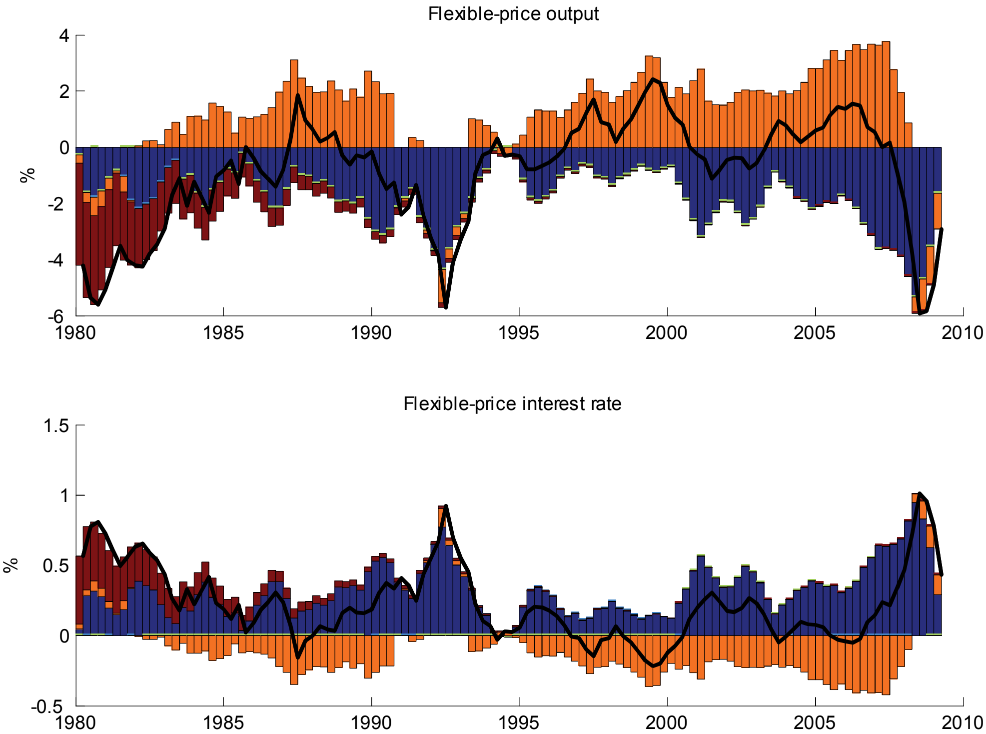


Figure 1.7: Flexible-price output and flexible-price interest rate shock decompositions

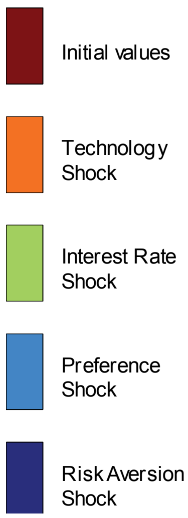


Figure 1.7 confirms the conclusions of Figure 1.6: risk aversion plays a significant role in crisis periods, also in the determination of the flexible-price output and the natural interest rate.

In this baseline model including a relative risk aversion shock, we show that risk aversion plays an important role on output and real money balances. This role means that risk aversion could modify the dynamics of output and real money balances. Furthermore, it confirms a potential link between money, output and risk aversion.

In addition, by analyzing the log marginal densities of the two resulting estimations, we obtain a log marginal density of -528.19 for the baseline model and a log marginal density of -505.43 for the model with a risk aversion shock. This confirms that the model with a risk aversion shock is better in terms of log marginal density than the baseline model. Moreover, in terms of model validation (Appendix 1.7.3), the risk aversion shock model is more valid than the baseline model.

Finally, and more descriptively, Figure 1.6 shows the negative role of risk aversion on output dynamics and a potential link between money, output and risk aversion.

1.6 Conclusion

This paper presents a baseline model and compares it with the same model including a risk aversion shock. Risk aversion is a concept in economics and finance based on the behavior of consumers and investors, whilst exposed to uncertainty. It is the reluctance of a person to accept a bargain with an uncertain payoff rather than another bargain with a more certain, but possibly lower, expected payoff.

Our last model shows the involvement of the risk aversion shock in the economy: it increases inflation, decreases output (Figure 1.4) and diminishes the impact of the central bank on output variance (Table 1.4), by comparison to the baseline model. Risk aversion plays also an important role on output and real money balances dynamics as well as on flexible-price output and flexible-price interest rate. The negative role played by risk aversion on output is clearly identified during crisis periods (Figure 1.6).

Moreover, this enhanced baseline model shows the importance of such parameter (risk aversion parameter) on the economy, and especially on the influence of monetary policy. And it shows how it is important to control it, by communication for example.

Last but not least, it will be interesting to analyse the potential link between output, real money balances and risk aversion by building a non-separable model with money in the utility and to test it with high risk aversion levels or during crises periods.

1.7 Appendix

1.7.1 Aggregate consumption and price index

Let $C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$ be a consumption index where $C_t(i)$ represents the quantity of good i consumed by the household in period t. This requires that C_t be maximized for any given level of expenditures $\int_0^1 P_t(i) C_t(i) di$ where $P_t(i)$ is the price of good i at time t. The maximization of C_t for any given expenditure level $\int_0^1 P_t(i) C_t(i) di = Z_t$ can be formalized by means of the Lagrangian

$$\mathcal{L} = \left[\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} - \lambda \left(\int_0^1 P_t(i) C_t(i) di - Z_t\right) \tag{1.32}$$

The associated first-order conditions are $C_t(i)^{-\frac{1}{\varepsilon}} C_t^{\frac{1}{\varepsilon}} = \lambda P_t(i)$ for all $i \in [0,1]$. Thus, for any two goods (i,j),

$$C_t(i) = C_t(j) \left(\frac{P_t(i)}{P_t(j)}\right)^{-\varepsilon} \tag{1.33}$$

which can be substituted into the expression for consumption expenditures to yield $C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} \frac{Z_t}{P_t}$ for all $i \in [0,1]$ where $P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$ is an aggregate price index. The latter condition can then be substituted into the definition of C_t to obtain

$$\int_0^1 P_t(i) C_t(i) di = P_t C_t \tag{1.34}$$

Combining the two previous equations yields the demand schedule equation $C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t$ for all $i \in [0,1]$.

1.7.2 Optimization problem

Our Lagrangian is given by

$$L_t = E_t \left[\sum_{k=0}^{\infty} \beta^k U_{t+k} - \lambda_{t+k} V_{t+k}\right]$$

where

$$V_t = C_t + \frac{M_t}{P_t} + Q_t \frac{B_t}{P_t} - \frac{B_{t-1}}{P_t} - \frac{W_t}{P_t} N_t - \frac{M_{t-1}}{P_t}$$

and

$$U_t = e^{\varepsilon_t^P} \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\gamma e^{\varepsilon_t^M}}{1-\nu} \left(\frac{M_t}{P_t} \right)^{1-\nu} - \frac{\chi e^{\varepsilon_t^N} N_t^{1+\eta}}{1+\eta} \right)$$

The first order condition related to consumption expenditures is given by

$$\lambda_t = e^{\varepsilon_t^P} C_t^{-\sigma} \tag{1.35}$$

where λ_t is the Lagrangian multiplier associated with the budget constraint at time t.

The first order condition corresponding to the demand for contingent bonds implies that

$$\lambda_t \frac{Q_t}{P_t} = \beta E_t \left[\frac{\lambda_{t+1}}{P_{t+1}} \right] \tag{1.36}$$

The demand for cash that follows from the household's optimization problem is given by

$$\gamma e^{\varepsilon_t^P} e^{\varepsilon_t^M} \left(\frac{M_t}{P_t} \right)^{-\nu} = \lambda_t - \beta E_t \left[\lambda_{t+1} \frac{P_t}{P_{t+1}} \right] \tag{1.37}$$

which can be naturally interpreted as a demand for real balances. The latter is increasing in consumption and inversely related to the nominal interest rate, as in conventional specifications.

$$\chi e^{\varepsilon_t^P} e^{\varepsilon_t^N} N_t^{\eta} = \lambda_t \frac{W_t}{P_t} \tag{1.38}$$

We obtain from (1.35)

$$\lambda_t = e^{\varepsilon_t^P} C_t^{-\sigma} \Leftrightarrow U_{c,t} = e^{\varepsilon_t^P} C_t^{-\sigma} \tag{1.39}$$

where $U_{c,t} = \left. \frac{\partial U_{k,t}}{\partial C_{t+k}} \right|_{k=0}$. Equation (1.39) defines the marginal utility of consumption.

Hence, the optimal consumption/savings, real money balances and labor supply decisions are described by the following conditions:

- Combining (1.35) with (1.36) gives

$$Q_t = \beta E_t \left[\frac{e^{\varepsilon_{t+1}^P} C_{t+1}^{-\sigma}}{e^{\varepsilon_t^P} C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \right] \Leftrightarrow Q_t = \beta E_t \left[\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right] \tag{1.40}$$

  where $U_{c,t+1} = \left. \frac{\partial U_{k,t}}{\partial C_{t+k}} \right|_{k=1}$. Equation (1.40) is the usual Euler equation for intertemporal consumption flows. It establishes that the ratio of marginal utility of future and current consumption is equal to the inverse of the real interest rate.

- Combining (1.35) with (1.37) gives

$$\gamma \frac{e^{\varepsilon_t^M}}{C_t^{-\sigma}} \left(\frac{M_t}{P_t}\right)^{-\nu} = 1 - Q_t \Leftrightarrow \frac{U_{m,t}}{U_{c,t}} = 1 - Q_t \tag{1.41}$$

where $U_{m,t} = \left.\frac{\partial U_{k,t}}{\partial (M_{t+k}/P_{t+k})}\right|_{k=0}$. Equation (1.41) is the intertemporal optimality condition setting the marginal rate of substitution between money and consumption equal to the opportunity cost of holding money.

- And combining (1.35) with (1.38) gives

$$\chi \frac{\varepsilon_t^N N_t^\eta}{C_t^{-\sigma}} = \frac{W_t}{P_t} \Leftrightarrow \frac{U_{n,t}}{U_{c,t}} = -\frac{W_t}{P_t} \tag{1.42}$$

where $U_{n,t} = \left.\frac{\partial U_{k,t}}{\partial N_{t+k}}\right|_{k=0}$. Equation (1.42) is the condition for the optimal consumption-leisure arbitrage, implying that the marginal rate of substitution between consumption and labor is equated to the real wage.

1.7.3 Model validation

The diagnosis concerning the numerical maximization of the posterior kernel indicate that the optimization procedure was able to obtain a robust maximum for the posterior kernel. A diagnosis of the overall convergence for the Metropolis-Hastings sampling algorithm is provided in Figure 1.8 and Figure 1.9.

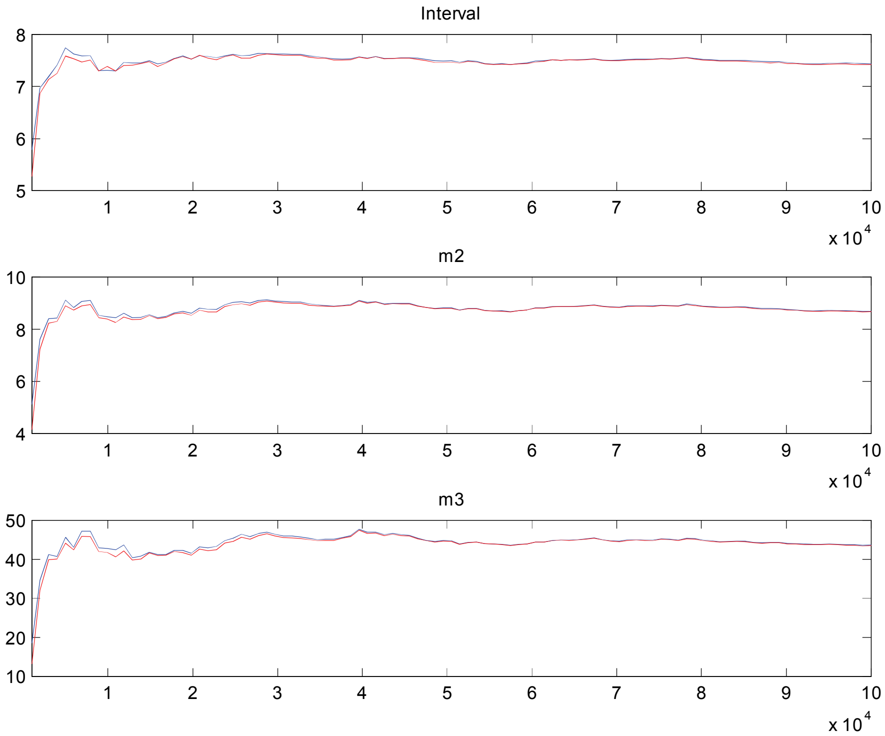


Figure 1.8: Multivariate MH convergence diagnosis (baseline model)

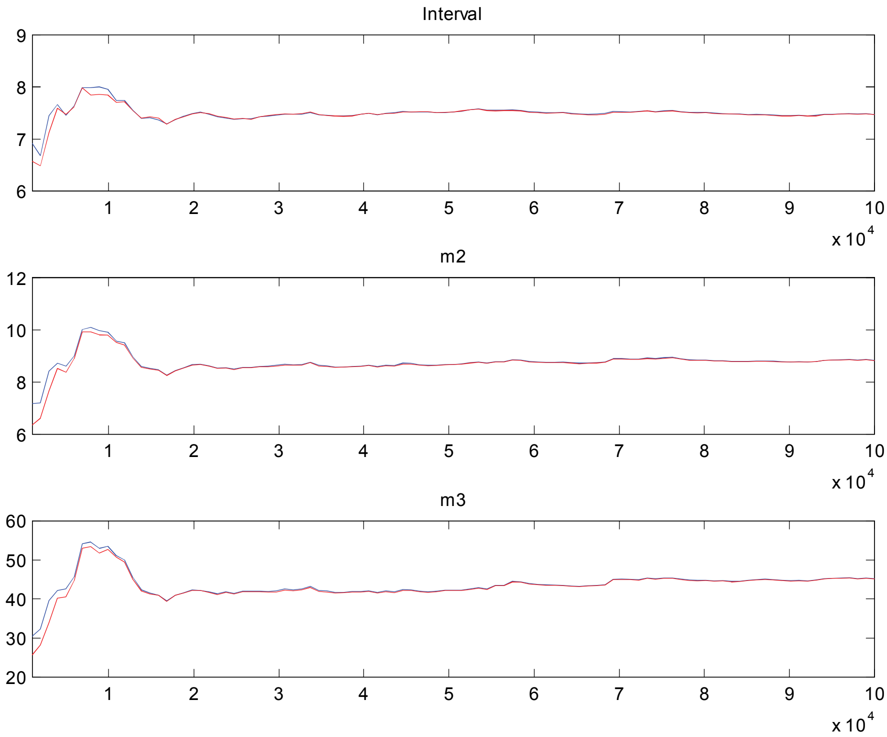


Figure 1.9: Multivariate MH convergence diagnosis (risk aversion shock model)

Each graph representing specific convergence measures and having two distinct lines that represent the results within and between chains. Those measures are related to the analysis of the parameters mean (interval), variance (m2) and third moment (m3). Convergence requires that both lines, for each of the three measures, become relatively constant and converge to each other.

Diagnosis for each individual parameter were also obtained, following the same structure as the overall. Most of the parameters don't seem to exhibit convergence problems, notwithstanding the fact that for some of them this evidence is stronger than for others.

Figure 1.10 and Figure 1.11 displays the estimates of the innovation component of each structural shock, respectively for the baseline model and for the model with a risk aversion shock.

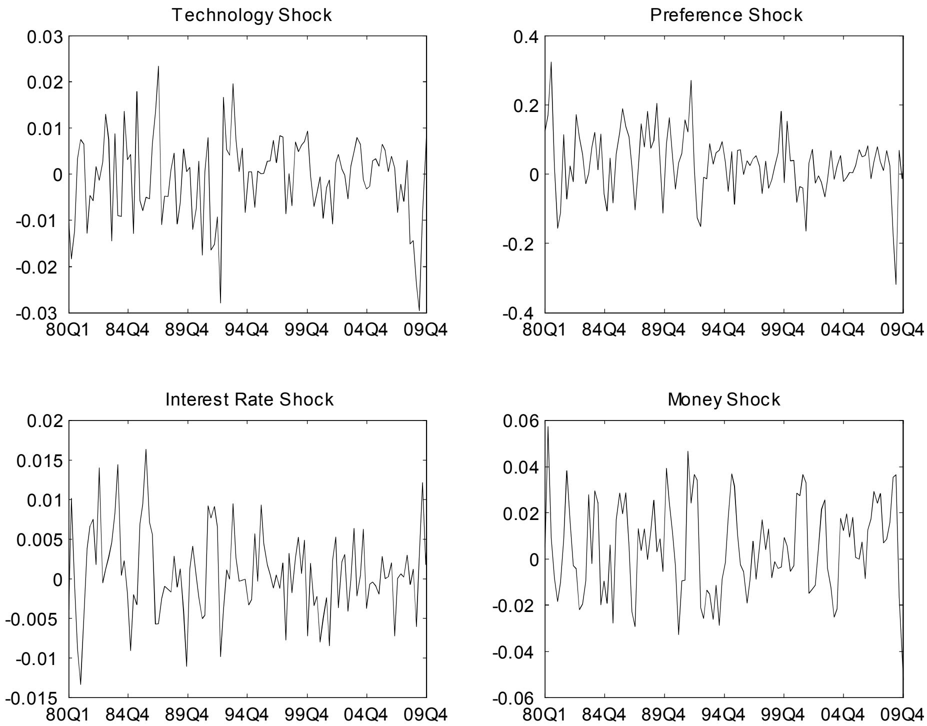


Figure 1.10: Estimated shocks (baseline model)

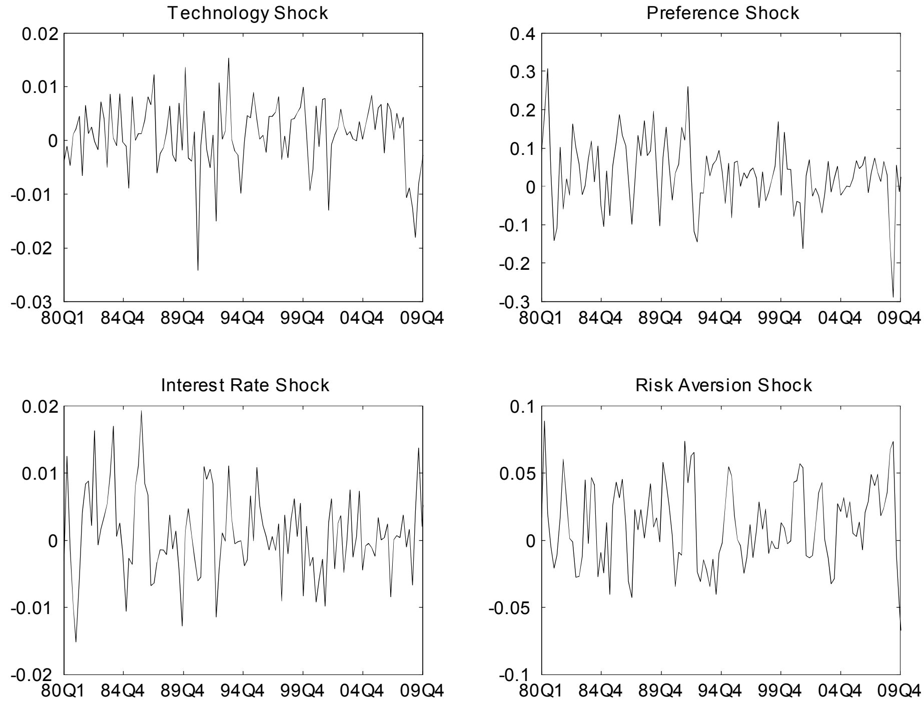
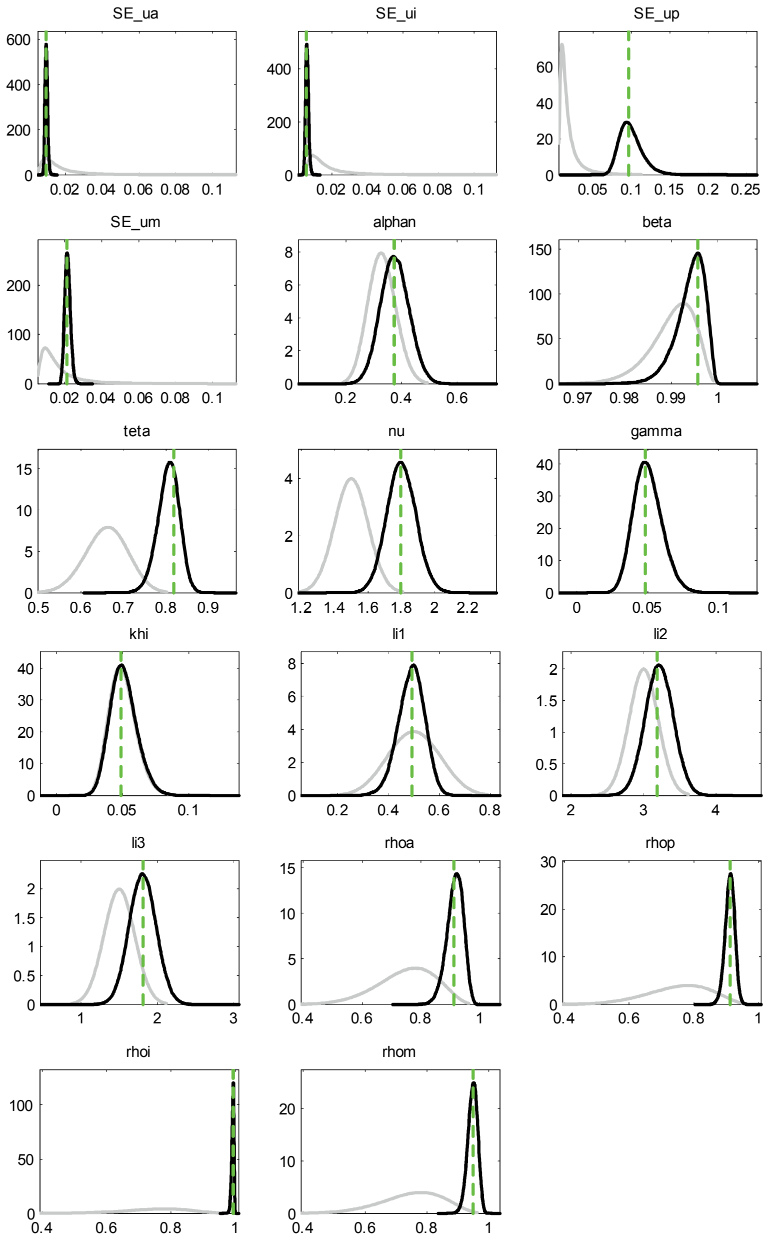


Figure 1.11: Estimated shocks (risk aversion shock model)

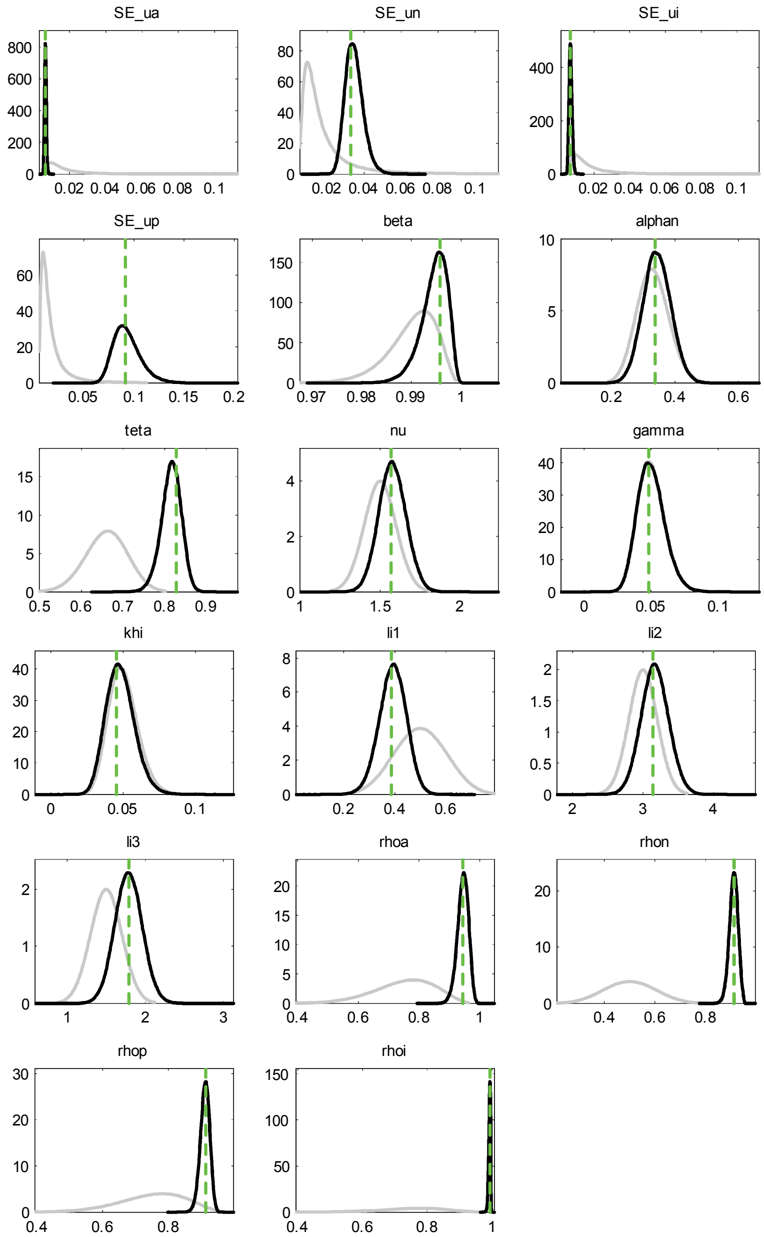
These appear to respect the i.i.d. properties and are centered around zero, which gives some positive indication on the statistical validity of the estimated model.

1.7.4 Priors and posteriors

Baseline model



Risk aversion shock model



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Chapter 2

Money in the production function

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2.1 Abstract

This paper proposes a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model where real money balances enter the production function. By using a Bayesian analysis, our model shows that money is not an omitted input to the production process and rejects the decreasing returns to scale hypothesis. Our simulations suggest that money plays a negligible role in the dynamics of output and inflation, despite its inclusion in the production function.

Keywords: Money, Production Function, DSGE, Bayesian Analysis.

JEL Classification Number: E23, E31, E51.

2.2 Introduction

The theoretical motivation for empirical implementations of money in the production function originates from monetary growth models of Levhari and Patinkin (1968), Friedman (1969), Johnson (1969) and Stein (1970), which include money directly in the production function. Firms hold money to facilitate production, on the grounds that money enables them to economize the use of other inputs, and spares the cost of running short of cash (Fischer, 1974).

> *Real cash balances are at least in part a factor of production. To take a trivial example, a retailer can economize on his average cash balances by hiring an errand boy to go to the bank on the corner to get change for large bills tendered by customers. When it costs ten cents per dollar per year to hold an extra dollar of cash, there will be a greater incentive to hire the errand boy, that is, to substitute other productive resources for cash. This will mean both a reduction in the real flow of services from the given productive resources and a change in the structure of production, since different productive activities may differ in cash-intensity, just as they differ in labor - or land - intensity.*
>
> Milton Friedman (1969)

In an old article, Sinai and Stokes (1972) present a very interesting test of the hypothesis that money enters the production function, and they suggest that real balances could be a missing variable that contributes to the attribution of the unexplained residual to technological changes. Ben-Zion and Ruttan (1974) conclude that money as a factor of demand seems to play an important role in explaining induced technological changes.

Short (1979) develops structural models based on Cobb-Douglas and generalized translog production functions, both of which provide a more complete theoretical framework for analyzing the role of money in the production process. The empirical results obtained by estimating these two models indicate that the relationship between real cash balances and output, even after correcting for any simultaneity bias, is positive and statistically significant. The results suggest that it is theoretically appropriate to include a real cash balances variable as a factor input in a production function in order to capture the productivity gains derived from using money.

You (1981) finds that the unexplained portion of output variation virtually vanishes with real balances included in the production function. Besides labor and capital, real money balances turn out to be an important factor of production, especially for developing countries. The results in Khan and Ahmad (1985) are consistent with the hypothesis that real money balances are an important factor of production. Sephton (1988) shows that real balances are a valid factor of production within the confines of a CES production function. Moreover, the results in Hasan and Mahmud (1993) support the hypothesis that money is an important factor in the production function and that there are potential supply side effects of a change in the interest rate.

Petrucci (1999) analyzes the consequences of the money supply growth rate on capital intensity and economic growth in an overlapping-generations model where real balances enter the production function. Chang (2002) proposes an alternative view of the Wang and Yip (1992) model to allow for real money balances as an input into human capital production. Shaw, Lai and Chang (2005) introduce adjustment costs of investment and the role of money in the production function and highlight the dynamic behavior of an open economy in an endogenous growth model.

Recent developments in econometrics regarding co-integration and error correction models provide a rich environment in which the role of money in the production function can be reexamined. In a co-integrated space, Moghaddam (2010) presents empirical evidence indicating that different definitions of money play an input role in the Cobb-Douglas production function.

At the same time, Clarida, Galí and Gertler (1999), Woodford (2003) or Galí (2008) develop New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models to explain the dynamics of the economy. However, no studies use money as an input in the production function in New Keynesian DSGE models.

This article departs from the existing theoretical and empirical literature by specifying a New Keynesian DSGE model where money enters the production function. This feature generates a new inflation dynamics where money could play a significant role. We also analyze the dynamics of the economy by using Bayesian estimations and simulations to confirm or reject the potential role of money in the dynamics of the Eurozone. Moreover, this paper intends to solve the now-old controversial hypothesis about constant returns to scale of money in the production function.

After describing the theoretical set up in Section 2.3, we calibrate and estimate the model for the Euro area using Bayesian techniques in 2.4. We analyze impulse response functions and variance decomposition are analyzed in Section 2.5, and we solve the choice of the returns to scale hypothesis by comparing the two models of this paper in Section 2.6. Section 2.7 concludes, and Section 2.8 presents additional results.

2.3 The model

The model consists of households that supply labor, purchase goods for consumption, and hold money and bonds, and firms that hire labor and produce and sell differentiated products in monopolistically competitive goods markets. Each firm sets the price of the good it produces, but not all firms reset their respective prices each period. Households and firms behave optimally: households maximize the expected present value of utility, and firms maximize profits. There is also a central bank that controls the nominal interest rate. This model is inspired by Galí (2008), Walsh (2003) and Smets and Wouters (2003).

2.3.1 Households

We assume a representative infinitely-lived household, seeking to maximize

$$E_t\left[\sum_{k=0}^{\infty}\beta^k U_{t+k}\right] \tag{2.1}$$

where U_t is the period utility function, and $\beta < 1$ is the discount factor.

We assume the existence of a continuum of goods represented by the interval $[0;1]$. The household decides how to allocate its consumption expenditures among the different goods. This requires that the consumption index, C_t, be maximized for any given level of expenditures[1]. Furthermore, and conditional on such optimal behavior, the period budget constraint takes the form

$$P_tC_t + M_t + Q_tB_t \leq B_{t-1} + W_tN_t + M_{t-1} \tag{2.2}$$

for $t = 0, 1, 2...$, where P_t is an aggregate price index, M_t is the quantity of money holdings at time t, B_t is the quantity of one-period nominally riskless

[1]See Appendix 2.8.1

discount bonds purchased in period t and maturing in period $t+1$ (each bond pays one unit of money at maturity and its price is Q_t, so that $i_t = -\log Q_t$ is the short term nominal rate), W_t is the nominal wage, and N_t is hours of work (or the measure of household members employed). The above sequence of period budget constraints is supplemented with a solvency condition, such as $\forall t$ $\lim_{n \longrightarrow \infty} E_t[B_n] \geq 0$. It prevents engaging in Ponzi-type schemes.

Preferences are measured with a common time-separable utility function. Under the assumption of a period utility given by

$$U_t = e^{\varepsilon_t^P} \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\gamma e^{\varepsilon_t^M}}{1-\nu} \left(\frac{M_t}{P_t} \right)^{1-\nu} - \frac{\chi N_t^{1+\eta}}{1+\eta} \right) \tag{2.3}$$

consumption, labor supply, money demand and bond holdings are chosen to maximize (2.1) subject to (2.2) and the solvency condition. This MIU utility function depends positively on the consumption of goods, C_t, positively on real money balances, $\frac{M_t}{P_t}$, and negatively on labor N_t. σ is the coefficient of relative risk aversion of households or the inverse of the intertemporal elasticity of substitution, ν is the inverse of the elasticity of money holdings with respect to the interest rate, and η is the inverse of the elasticity of work effort with respect to the real wage. The utility function also contains two structural shocks: ε_t^P is a general shock to preferences that affects the intertemporal substitution of households (preference shock) and ε_t^M is a money demand shock. γ and χ are positive scale parameters.

This setting leads to the following conditions[2], which, in addition to the budget constraint, must hold in equilibrium. The resulting log-linear version of the first order condition corresponding to the demand for contingent bonds implies that

$$c_t = E_t[c_{t+1}] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - \rho_c) - \sigma^{-1} E_t\left[\Delta \varepsilon_{t+1}^P\right] \tag{2.4}$$

where the lowercase letters denote the logarithm of the original aggregated variables, $\rho_c = -\log(\beta)$, and where Δ is the first-difference operator.

The demand for cash that follows from the household's optimization problem is given by

$$\varepsilon_t^M + \sigma c_t - \nu(m_t - p_t) - \rho_m = a_2 i_t \tag{2.5}$$

[2] See Appendix 2.8.2

where $mp_t = m_t - p_t$ are the log-linearized real money balances, $\rho_m = -\log(\gamma) + a_1$, a_1 and a_2 are resulting terms of the first-order Taylor approximation[3] of $\log(1 - Q_t) = a_1 + a_2 i_t$.

Real cash holdings depend positively on consumption, with an elasticity equal to $\frac{\sigma}{\nu}$, and negatively on the nominal interest rate[4]. Below, we take the nominal interest rate as the central bank's policy instrument[5].

The resulting log-linear version of the first order condition corresponding to the optimal consumption-leisure arbitrage implies that

$$w_t - p_t = \sigma c_t + \eta n_t - \rho_n + \varepsilon_t^N \tag{2.6}$$

where $\rho_n = -\log(\chi)$.

Finally, these equations represent the Euler condition for the optimal intratemporal allocation of consumption (equation (2.4)), the intertemporal optimality condition setting the marginal rate of substitution between money and consumption equal to the opportunity cost of holding money (equation (2.5)), and the intratemporal optimality condition setting the marginal rate of substitution between leisure and consumption equal to the real wage (equation (2.6)).

2.3.2 Firms

We assume a continuum of firms indexed by $i \in [0,1]$. Each firm produces a differentiated good, but they all use an identical technology, represented by the following money-in-the-production function[6]

$$Y_t(i) = A_t \left(\frac{M_t}{P_t}\right)^{\alpha_m} N_t(i)^{1-\alpha_n} \tag{2.7}$$

[3] More precisely, if we compute our first-order Taylor approximation around the steady-state interest rate, $\frac{1}{\beta}$, we obtain: $a_1 = \log\left(1 - \exp\left(-\frac{1}{\beta}\right)\right) - \frac{\frac{1}{\beta}}{e^{\frac{1}{\beta}}-1}$, $a_2 = \frac{1}{e^{\frac{1}{\beta}}-1}$.

[4] $\frac{1}{\beta} > 1$ which implies that $a_2 > 0$.

[5] In the literature, due to the assumption that consumption and real money balances are additively separable in the utility function, cash holdings do not enter any of the other structural equations: accordingly, the above equation becomes a recursive function of the rest of the system of equations. However, equation (2.5) will be useful to solve for the equilibrium of the model. Because of the presence of real money balances in the aggregate supply, as in Khan and Ahmad (1985), Subrahmanyam (1980) or Sinai and Stokes (1972), we will use this money demand equation. See Ireland (2001), Jones and Stracca (2008) and Benchimol and Fourçans (2010) for models in which money balances enter the aggregate demand equation without entering the production function.

[6] We assume a very simple money-in-the-production-function in order to simplify the model without losing the explanatory power of the production function and its key variable (N_t).

where $A_t = \exp(\varepsilon_t^a)$ represents the level of technology, assumed to be common to all firms and to evolve exogenously over time.

All firms face an identical isoelastic demand schedule, and take the aggregate price level, P_t, and aggregate consumption index, C_t, as given. As in the standard Calvo (1983) model, our generalization features monopolistic competition and staggered price setting. At any time t, only a fraction, $1-\theta$, of firms, with $0<\theta<1$, can reset their prices optimally, while the remaining firms index their prices to lagged inflation[7].

2.3.3 Price dynamics

Let's assume a set of firms that do not reoptimize their posted price in period t. Using the definition of the aggregate price level[8] and the fact that all firms that reset prices choose an identical price, P_t^*, leads to $P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$. Dividing both sides by P_{t-1} and log-linearizing around $P_t^* = P_{t-1}$ yields

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}) \tag{2.8}$$

In this set up, we do not assume inertial dynamics of prices. Inflation results from the fact that firms reoptimizing their price plans in any given period, choose a price that differs from the economy's average price in the previous period.

2.3.4 Price setting

A firm reoptimizing in period t chooses the price P_t^* that maximizes the current market value of the profits generated while that price remains effective. We solve this problem to obtain a first-order Taylor expansion around the zero inflation steady state of the firm's first-order condition, which leads to

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\widehat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})\right] \tag{2.9}$$

where $\widehat{mc}_{t+k|t} = mc_{t+k|t} - mc$ denotes the log deviation of marginal cost from its steady state value $mc = -\mu$, and $\mu = \log(\varepsilon/(\varepsilon-1))$ is the log of the desired gross markup.

[7] Thus, each period, $1-\theta$ producers reset their prices, while a fraction θ keep their prices unchanged.

[8] As shown in Appendix 2.8.1

2.3.5 Equilibrium

Market clearing in the goods market requires $Y_t(i) = C_t(i)$ for all $i \in [0,1]$ and all t. Aggregate output is defined as $Y_t = \left(\int_0^1 Y_t(i)^{1-\frac{1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$; it follows that $Y_t = C_t$ must hold for all t. One can combine the above goods market clearing condition with the consumer's Euler equation to yield the equilibrium condition

$$y_t = E_t[y_{t+1}] - \sigma^{-1}(i_t - E_t[\pi_{t+1}] - \rho_c) - \sigma^{-1}E_t\left[\Delta\varepsilon_{t+1}^P\right] \tag{2.10}$$

Market clearing in the labor market requires $N_t = \int_0^1 N_t(i)\,di$. Using (2.7) leads to

$$\begin{aligned} N_t &= \int_0^1 \left(\frac{Y_t(i)}{A_t\left(\frac{M_t}{P_t}\right)^{\alpha_m}}\right)^{\frac{1}{1-\alpha_n}} di \\ &= \left(\frac{Y_t}{A_t\left(\frac{M_t}{P_t}\right)^{\alpha_m}}\right)^{\frac{1}{1-\alpha_n}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha_n}} di \end{aligned} \tag{2.11}$$

where the second equality follows from the demand schedule and the goods market clearing condition. Taking logs leads to

$$(1-\alpha_n)\,n_t = y_t - \varepsilon_t^a - \alpha_m mp_t + d_t$$

where $d_t = (1-\alpha_n)\log\left(\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha_n}} di\right)$, and di is a measure of price (and, hence, output) dispersion across firms. Following Galí (2008), in a neighborhood of the zero inflation steady state, d_t is equal to zero up to a first-order approximation.

Hence, one can write the following approximate relation between aggregate output, employment, real money balances and technology as

$$y_t = \varepsilon_t^a + (1-\alpha_n)\,n_t + \alpha_m mp_t \tag{2.12}$$

An expression is derived for an individual firm's marginal cost in terms of the economy's average real marginal cost. With the marginal product of labor,

$$\begin{aligned} mpn_t &= \log\left(\frac{\partial Y_t}{\partial N_t}\right) \\ &= \log\left(A_t\left(\frac{M_t}{P_t}\right)^{\alpha_m}(1-\alpha_n)\,N_t^{-\alpha_n}\right) \\ &= \varepsilon_t^a + \alpha_m mp_t + \log(1-\alpha_n) - \alpha_n n_t \end{aligned}$$

and the marginal product of real money balances,

$$\begin{aligned} mpmp_t &= \log\left(\frac{\partial Y_t}{\partial \frac{M_t}{P_t}}\right) \\ &= \log\left(A_t \alpha_m \left(\frac{M_t}{P_t}\right)^{\alpha_m - 1} N_t^{1-\alpha_n}\right) \\ &= \varepsilon_t^a + \log(\alpha_m) + (\alpha_m - 1)\, mp_t + (1-\alpha_n)\, n_t \end{aligned}$$

we obtain an expression of the marginal cost

$$\begin{aligned} mc_t &= (w_t - p_t) - mpn_t - mpmp_t \\ &= w_t - p_t + \frac{2\alpha_n - 1}{1-\alpha_n} y_t + \frac{1-\alpha_m-\alpha_n}{1-\alpha_n} mp_t \\ &\quad - \frac{1}{1-\alpha_n}\varepsilon_t^a - \log(\alpha_m(1-\alpha_n)) \end{aligned}$$

for all t, where the second equality defines the economy's average marginal product of labor, mpn_t, and the economy's average marginal product of real money balances, $mpmp_t$, in a way that is consistent with (2.12). Using the fact that $mc_{t+k|t} = (w_{t+k} - p_{t+k}) - mpn_{t+k|t}$,

$$\begin{aligned} mc_{t+k|t} &= (w_{t+k} - p_{t+k}) + \frac{2\alpha_n - 1}{1-\alpha_n} y_{t+k|t} \\ &\quad + \frac{1-\alpha_m-\alpha_n}{1-\alpha_n} mp_{t+k} - \frac{1}{1-\alpha_n}\varepsilon_{t+k}^a - \log(\alpha_m(1-\alpha_n)) \\ &= mc_{t+k} + \frac{2\alpha_n - 1}{1-\alpha_n}\left(y_{t+k|t} - y_{t+k}\right) \\ &= mc_{t+k} - \varepsilon \frac{2\alpha_n - 1}{1-\alpha_n}(p_t^* - p_{t+k}) \end{aligned} \tag{2.13}$$

where the second equality follows from the demand schedule, $C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t$, combined with the market clearing condition ($y_t = c_t$).

Substituting (2.13) into (2.9) and rearranging terms yields

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t \left[\begin{array}{c} \widehat{mc}_{t+k} - \varepsilon\frac{2\alpha_n - 1}{1-\alpha_n}(p_t^* - p_{t+k}) \\ +(p_{t+k} - p_{t-1}) \end{array}\right]$$

$$p_t^* - p_{t-1} = (1-\beta\theta)\,\Theta\sum_{k=0}^{\infty}(\beta\theta)^k E_t[\widehat{mc}_{t+k}] + \sum_{k=0}^{\infty}(\beta\theta)^k E_t[\pi_{t+k}] \tag{2.14}$$

where $\Theta = \frac{1-\alpha_n}{1-\alpha_n+\varepsilon(2\alpha_n - 1)} \leq 1$.

Finally, combining (2.8) in (2.14) yields the inflation equation[9]

$$\pi_t = \beta E_t [\pi_{t+1}] + \lambda_{mc} \widehat{mc}_t \tag{2.15}$$

where $\lambda_{mc} = \Theta \frac{(1-\theta)(1-\beta\theta)}{\theta}$ is strictly decreasing in the index of price stickiness, θ, in the measure of decreasing returns, α_n, and in the demand elasticity, ε.

Next, a relation is derived between the economy's real marginal cost and a measure of aggregate economic activity. Notice that, independent of the nature of price setting, average real marginal cost can be expressed as

$$\begin{aligned} mc_t &= (w_t - p_t) - mpn_t - mpmp_t \\ &= \left(\sigma y_t + \eta n_t - \rho_n + \varepsilon_t^N\right) + \frac{2\alpha_n - 1}{1-\alpha_n} y_t + \frac{1-\alpha_m-\alpha_n}{1-\alpha_n} mp_t \\ &\quad - \frac{1}{1-\alpha_n}\varepsilon_t^a - \log\left(\alpha_m\left(1-\alpha_n\right)\right) \\ &= \frac{\sigma(1-\alpha_n)+\eta+2\alpha_n-1}{1-\alpha_n} y_t + \frac{1-(1+\eta)\alpha_m-\alpha_n}{1-\alpha_n} mp_t \\ &\quad - \frac{1+\eta}{1-\alpha_n}\varepsilon_t^a - \log\left(\alpha_m\left(1-\alpha_n\right)\right) - \rho_n + \varepsilon_t^N \end{aligned} \tag{2.16}$$

where derivation of the second[10] and third[11] equalities makes use of the household's optimality condition (2.6) and the (approximate) aggregate production relation (2.12).

[9] Notice that the above discounted sum (2.14) can be rewritten more compactly as the difference equation

$$p_t^* - p_{t-1} = \beta\theta E_t \left[p_{t+1}^* - p_t\right] + (1-\beta\theta)\,\Theta \widehat{mc}_t + \pi_t$$

by adding $-\beta\theta E_t\left[(2.14)_{t+1}\right]$ to (2.14).

[10] We use the following equations:

$$y_t = a_t + (1-\alpha_n)\,n_t + \alpha_m mp_t$$

and

$$\begin{aligned} mpn_t &= a_t - \alpha_n n_t + \alpha_m mp_t + \log(1-\alpha_n) \\ mpmp_t &= a_t + (1-\alpha_n)\,n_t + (\alpha_m - 1)\,mp_t + \log(\alpha_m) \end{aligned}$$

[11] We use the following equation

$$n_t = \frac{1}{1-\alpha_n}\left(y_t - a_t - \alpha_m mp_t\right)$$

from (2.12).

It is obvious that $\sigma(1-\alpha_n)+\eta+2\alpha_n-1>0$, but the inequality $1-(1+\eta)\alpha_m-\alpha_n>0$ coming from (2.16) appears unusual. In fact, it confirms some studies from Sinai and Stokes and others where they conclude that the weight on labor is more important than the weight on money (or real money balances).

Furthermore, and as shown previously, under flexible prices the real marginal cost is constant and given by $mc=-\mu$. Defining the natural level of output, denoted by y_t^f, as the equilibrium level of output under flexible prices

$$\begin{aligned} mc &= \frac{\sigma(1-\alpha_n)+\eta+2\alpha_n-1}{1-\alpha_n}y_t^f+\frac{1-(1+\eta)\alpha_m-\alpha_n}{1-\alpha_n}mp_t^f \\ &\quad -\frac{1+\eta}{1-\alpha_n}\varepsilon_t^a-\log(\alpha_m(1-\alpha_n))-\rho_n \end{aligned} \tag{2.17}$$

thus implying

$$y_t^f=\upsilon_a\varepsilon_t^a+\upsilon_m mp_t^f+\upsilon_c \tag{2.18}$$

where

$$\begin{aligned} \upsilon_a^y &= \frac{1+\eta}{\sigma+\eta+(1-\sigma)\alpha_n-1+\alpha_n} \\ \upsilon_m^y &= \frac{\alpha_n+\alpha_m(1+\eta)-1}{\sigma+\eta+(1-\sigma)\alpha_n-1+\alpha_n} \\ \upsilon_c^y &= \frac{(1-\alpha_n)(\log(\alpha_m(1-\alpha_n))+\rho_n-\mu)}{\sigma+\eta+(1-\sigma)\alpha_n-1+\alpha_n} \end{aligned}$$

From (2.10), we obtain an expression for the *natural interest rate*,

$$i_t^f=\rho_c+\sigma E_t\left[\Delta y_{t+1}^f\right] \tag{2.19}$$

Then, by using (2.5) and (2.19), we obtain an expression of flexible-price *real money balances*

$$mp_t^f=-\frac{a_2\sigma}{\nu}E_t\left[\Delta y_{t+1}^f\right]+\frac{\sigma}{\nu}y_t^f-\frac{a_2\rho_c+\rho_m}{\nu}+\frac{1}{\nu}\varepsilon_t^M \tag{2.20}$$

Subtracting (2.17) from (2.16) yields

$$\widehat{mc}_t=\frac{\sigma(1-\alpha_n)+\eta+2\alpha_n-1}{1-\alpha_n}\left(y_t-y_t^f\right)+\frac{1-(1+\eta)\alpha_m-\alpha_n}{1-\alpha_n}\left(mp_t-mp_t^f\right) \tag{2.21}$$

where $\widehat{mc}_t=mc_t-mc$ is the *real marginal cost gap*, $y_t-y_t^f$ is the *output gap*, and $mp_t-mp_t^f$ is the *real money balances gap*. Then, combining the above equation with (2.15), we obtain our first equation relating inflation to its next period forecast, output gap and real money balances gap,

$$\pi_t = \beta E_t\left[\pi_{t+1}\right] + \psi_x\left(y_t - y_t^f\right) + \psi_m\left(mp_t - mp_t^f\right) \tag{2.22}$$

where $\psi_x = \frac{(1-\theta)(1-\beta\theta)(\sigma(1-\alpha_n)+\eta+2\alpha_n-1)}{\theta(1-\alpha_n+\varepsilon(2\alpha_n-1))}$ and $\psi_m = \frac{(1-\theta)(1-\beta\theta)(1-(1+\eta)\alpha_m-\alpha_n)}{\theta(1-\alpha_n+\varepsilon(2\alpha_n-1))}$.

The second key equation describing the equilibrium of the New Keynesian model is obtained from (2.10):

$$y_t = E_t\left[y_{t+1}\right] - \sigma^{-1}\left(i_t - E_t\left[\pi_{t+1}\right] - \rho_c\right) - \sigma^{-1}E_t\left[\Delta\varepsilon_{t+1}^P\right] \tag{2.23}$$

Henceforth (2.23) is referred to as the dynamic IS equation.

The third key equation describes behavior of the *real money balances.* Rearranging (2.5) yields

$$mp_t = \frac{\sigma}{\nu}y_t - \frac{a_2}{\nu}i_t - \frac{\rho_m}{\nu} + \frac{1}{\nu}\varepsilon_t^M \tag{2.24}$$

The last equation determines the interest rate through a standard smoothed Taylor-type rule,

$$i_t = (1-\lambda_i)\left(\lambda_\pi\left(\pi_t - \pi^*\right) + \lambda_x\left(y_t - y_t^f\right)\right) + \lambda_i i_{t-1} + \varepsilon_t^i \tag{2.25}$$

where λ_π and λ_x are policy coefficients reflecting the weight on inflation and on the output gap, and the parameter $0 < \lambda_i < 1$ captures the degree of interest rate smoothing. ε_t^i is an exogenous *ad hoc* shock accounting for fluctuations of nominal interest rate.

All structural shocks are assumed to follow a first-order autoregressive process with an *i.i.d.* normal error term, such as $\varepsilon_t^k = \mu_k\varepsilon_{t-1}^k + \omega_{k,t}$, where $\varepsilon_{k,t} \sim N\left(0;\sigma_k\right)$ for $k = \{P, M, i, a\}$.

2.4 Results

2.4.1 DSGE model

Our model consists of six equations and six dependent variables: inflation, nominal interest rate, output, flexible-price output, real money balances and its flexible-price counterpart. Flexible-price output and flexible-price real money balances are completely determined by shocks: flexible-price output is mainly driven by technology shocks (fluctuations in the output gap can be attributed to supply and demand shocks), whereas the flexible-price real money balances is mainly driven by money shocks and flexible-price output.

$$y_t^f = \upsilon_a^y \varepsilon_t^a + \upsilon_m^y m p_t^f + \upsilon_c^y \tag{2.26}$$

$$mp_t^f = \upsilon_{y+1}^m E_t \left[\Delta y_{t+1}^f \right] + \upsilon_y^m y_t^f + \upsilon_c^m + \frac{1}{\nu} \varepsilon_t^M \tag{2.27}$$

$$\pi_t = \beta E_t \left[\pi_{t+1} \right] + \kappa_x \left(y_t - y_t^f \right) + \kappa_m \left(mp_t - mp_t^f \right) \tag{2.28}$$

$$y_t = E_t \left[y_{t+1} \right] - \sigma^{-1} \left(i_t - E_t \left[\pi_{t+1} \right] - \rho_c \right) - \sigma^{-1} E_t \left[\Delta \varepsilon_{t+1}^P \right] \tag{2.29}$$

$$mp_t = \frac{\sigma}{\nu} y_t - \frac{a_2}{\nu} i_t - \frac{\rho_m}{\nu} + \frac{1}{\nu} \varepsilon_t^M \tag{2.30}$$

$$i_t = (1 - \lambda_i) \left(\lambda_\pi \left(\pi_t - \pi^* \right) + \lambda_x \left(y_t - y_t^f \right) \right) + \lambda_i i_{t-1} + \ \varepsilon_t^i \tag{2.31}$$

where

$$\begin{array}{ll}
\upsilon_a^y = \frac{1+\eta}{\sigma+\eta+(1-\sigma)\alpha_n-1+\alpha_n} & \kappa_x = \frac{(1-\theta)(1-\beta\theta)(\sigma(1-\alpha_n)+\eta+2\alpha_n-1)}{\theta(1-\alpha_n+\varepsilon(2\alpha_n-1))} \\
\upsilon_m^y = \frac{\alpha_n+\alpha_m(1+\eta)-1}{\sigma+\eta+(1-\sigma)\alpha_n-1+\alpha_n} & \kappa_m = \frac{(1-\theta)(1-\beta\theta)(1-(1+\eta)\alpha_m-\alpha_n)}{\theta(1-\alpha_n+\varepsilon(2\alpha_n-1))} \\
\upsilon_c^y = \frac{(1-\alpha_n)\left(\log(\alpha_m(1-\alpha_n))+\rho_n-\log\left(\frac{\varepsilon}{\varepsilon-1}\right)\right)}{\sigma+\eta+(1-\sigma)\alpha_n-1+\alpha_n} & \rho_m = -\log(\gamma) + a_1 \\
\upsilon_{y+1}^m = -\frac{a_2\sigma}{\nu} & \rho_n = -\log(\chi) \\
\upsilon_y^m = \frac{\sigma}{\nu} & \rho_c = -\log(\beta) \\
\upsilon_c^m = -\frac{a_2\rho_c+\rho_m}{\nu} & a_1 = \log\left(1 - e^{-\frac{1}{\beta}}\right) - \frac{\frac{1}{\beta}}{e^{\frac{1}{\beta}}-1} \\
 & a_2 = \frac{1}{e^{\frac{1}{\beta}}-1}
\end{array}$$

2.4.2 Euro area data

In order to make output and real money balances stationary, we use first log differences, as in Adolfson and *al.* (2008). In our model of the Eurozone, $\hat{\pi}_t$ is the log-linearized inflation rate measured as the yearly log difference of GDP Deflator between one quarter and the same quarter of the previous year, $\hat{y}_t$ is the log-linearized output measured as the yearly log difference of GDP between one quarter and the same quarter of the previous year, and $\hat{\imath}_t$ is the short-term (3-month) nominal interest rate. These data are extracted from the Euro area

Wide Model database (AWM) of Fagan, Henry and Mestre (2001). $\widehat{mp}_t$ is the log-linearized real money balances measured as the yearly log difference of real money between one quarter and the same quarter of the previous year, where real money is measured as the log difference between the money stock and the GDP Deflator. We use the $M3$ monetary aggregate from the OECD database. $\hat{y}_t^f$, the flexible-price output, $\widehat{mp}_t^f$, the flexible-price real money balances, and $\hat{\imath}_t^f$, the flexible-price (natural) interest rate, are completely determined by structural shocks.

2.4.3 Calibration and estimations

We calibrate our model following Galí (2003), such that $\beta = 0.99$ (discount factor), $\eta = 1$ (inverse of the Frisch elasticity of labor supply), and $\varepsilon = 6$ (elasticity of demand of households for consumption goods). These parameters are calibrated while other parameters are estimated with Bayesian techniques.

Calibration and estimation of structural parameters

	Priors			Posteriors				
	Law	Mean	Std.	Mean	t-stat	Std.	5%	95%
α_n	beta	0.50	0.15	0.6430	12.85	0.0494	0.563	0.721
α_m	beta	0.50	0.15	0.0646	2.156	0.0256	0.022	0.105
θ	beta	0.66	0.05	0.8029	29.26	0.0278	0.755	0.848
υ	normal	2.00	0.10	2.1395	22.66	0.0941	1.987	2.294
σ	normal	2.00	0.10	1.8545	19.09	0.0970	1.692	2.010
γ	beta	0.25	0.10	0.2507	2.096	0.1041	0.085	0.406
χ	beta	0.25	0.10	0.2565	2.220	0.1020	0.095	0.410
λ_i	beta	0.50	0.10	0.4750	9.323	0.0513	0.388	0.557
λ_π	normal	3.00	0.20	3.2021	16.40	0.1937	2.886	3.523
λ_x	normal	1.50	0.20	1.8060	10.27	0.1764	1.507	2.094
ρ_a	beta	0.75	0.10	0.9251	37.53	0.0247	0.886	0.964
ρ_p	beta	0.75	0.10	0.9135	64.30	0.0142	0.889	0.936
ρ_i	beta	0.75	0.10	0.9914	301.7	0.0033	0.985	0.997
ρ_m	beta	0.75	0.10	0.9412	54.10	0.0174	0.912	0.969
σ_a	invgamma	0.02	2.00	0.0072	14.09	0.0005	0.006	0.008
σ_i	invgamma	0.02	2.00	0.0065	8.092	0.0008	0.005	0.007
σ_p	invgamma	0.02	2.00	0.0992	6.943	0.0138	0.075	0.122
σ_m	invgamma	0.02	2.00	0.0230	13.57	0.0017	0.020	0.025

Table 2.1: Bayesian estimation (decreasing return to scale)

The estimation of the implied posterior distribution of the parameters (Table 2.1) is conducted using the Metropolis-Hastings algorithm (10 chains, each of 100000 draws; see Smets and Wouters, 2007, and Adolfson et *al.*, 2007).

The real money balances parameter (α_m) of the "augmented" production function is estimated to 0.064. This result is in line with Sinai and Stokes (1972), who obtain a value of 0.087 for the same parameter (and and also considering

M3). The prior and posterior distributions are in Appendix 2.8.4 and estimates of the macro-parameters (aggregated structural parameters) are provided in Appendix 2.8.5.

As in Table 2.1, we use Bayesian techniques to estimate our model with money in the production function and a supplementary restriction. This restriction is adopted from Short (1979) and involves the hypothesis of constant returns to scale of the production function. Then, we assume that $\alpha_n = \alpha_m$ and we test our model with this hypothesis.

Calibration and estimation of structural parameters

	Priors			Posteriors				
	Law	Mean	Std.	Mean	t-stat	Std.	5%	95%
α_n	beta	0.50	0.15	0.5744	17.63	0.0320	0.519	0.625
θ	beta	0.66	0.05	0.8388	35.52	0.0239	0.799	0.878
υ	normal	2.00	0.10	2.3026	25.88	0.0887	2.158	2.450
σ	normal	2.00	0.10	1.7821	18.06	0.0986	1.619	1.944
γ	beta	0.25	0.10	0.2493	2.096	0.1041	0.085	0.403
χ	beta	0.25	0.10	0.2998	2.529	0.1086	0.134	0.463
λ_i	beta	0.50	0.10	0.5432	10.74	0.0513	0.458	0.629
λ_π	normal	3.00	0.20	3.2002	16.27	0.1955	2.876	3.515
λ_x	normal	1.50	0.20	1.7647	10.09	0.1756	1.473	2.053
ρ_a	beta	0.75	0.10	0.9169	32.79	0.0280	0.874	0.960
ρ_p	beta	0.75	0.10	0.9056	58.20	0.0156	0.880	0.931
ρ_i	beta	0.75	0.10	0.9907	271.7	0.0037	0.984	0.997
ρ_m	beta	0.75	0.10	0.9480	57.74	0.0165	0.921	0.974
σ_a	invgamma	0.02	2.00	0.0076	13.91	0.0005	0.006	0.008
σ_i	invgamma	0.02	2.00	0.0059	7.101	0.0008	0.004	0.007
σ_p	invgamma	0.02	2.00	0.0966	7.034	0.0132	0.073	0.118
σ_m	invgamma	0.02	2.00	0.0238	13.75	0.0017	0.021	0.026

Table 2.2: Bayesian estimation (constant return to scale)

The resulting log marginal density for the model with decreasing returns to scale (-512.93) and for the model with constant returns to scale (-557.52) indicates that, if we admit that money enters the production function, this production function should have decreasing returns to scale.

2.5 Simulations

2.5.1 Impulse response functions

Figure 2.1 presents the response of key variables to structural shocks. The thin solid line represents the decreasing return-to-scale model responses and the dashed line represents the constant return-to-scale model responses.

In response to a preference shock, the inflation rate, the output, the output gap, the real money balances, the nominal and the real interest rates rise; real money growth displays a little overshooting process in the first periods, then returns quickly to its steady-state value.

After a technology shock, the output gap, the inflation rate, the nominal and the real interest rates decrease, whereas output as well as real money balances and real money growth rise.

Following a money shock, inflation, output, real and nominal interest rates and the output gap dynamics differs depending on the model. The model of decreasing returns to scale (thin solid line) displays more coherent results than that of constant returns to scale.

In response to an interest rate shock, the inflation rate, the nominal interest rate, the output and the output gap fall. The real interest rate rises. A positive monetary policy shock induces a fall in interest rates due to a low enough degree of intertemporal substitution (i.e., the risk aversion parameter is high enough), which generates a large impact response of current consumption relative to future consumption. This result has been noted in, *inter alia*, Jeanne (1994) and Christiano et *al.* (1997).

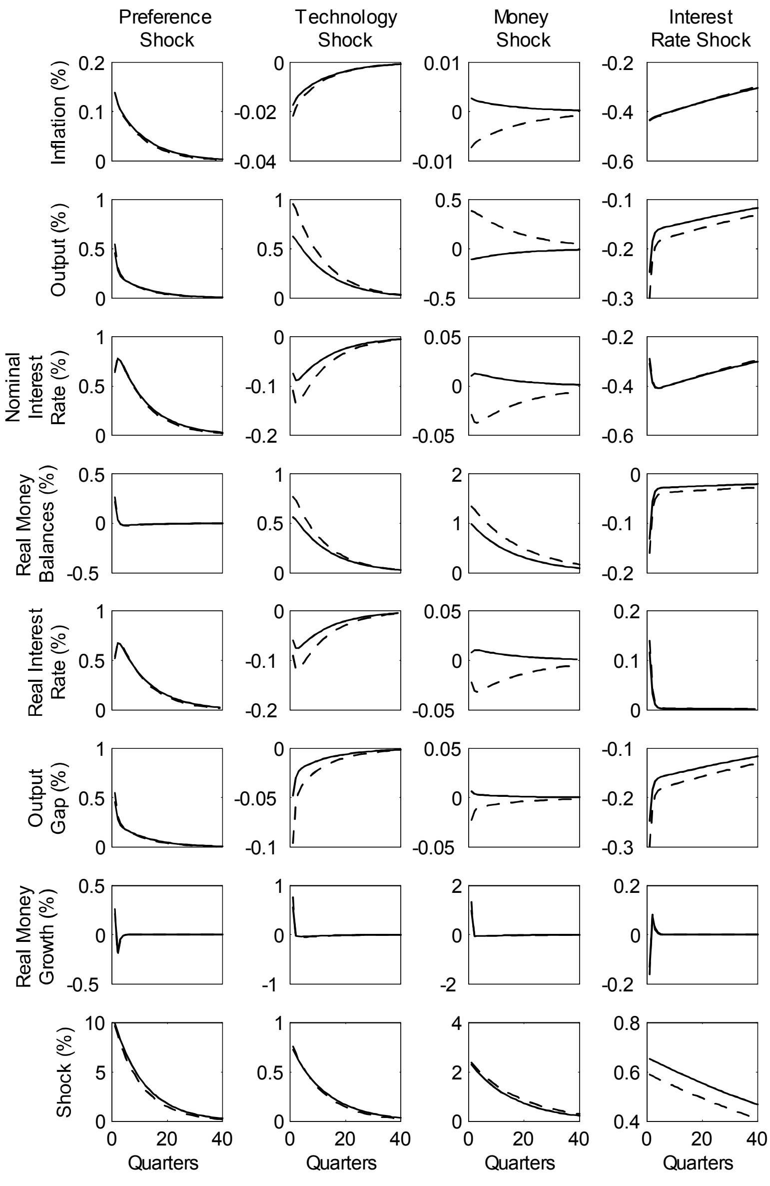


Figure 2.1: Impulse response functions

2.5.2 Variance decompositions

We analyze in two different ways the forecast error variance of each variable following exogenous shocks: using constant returns to scale and decreasing returns to scale. The analysis is conducted first via an unconditional variance decomposition (Table 2.3) and second via a conditional variance decomposition (Figures 2.2 to 2.4) to compare the two models' dynamics of variance decomposition over time.

Unconditional variance decomposition (%)

	Decreasing returns to scale				Constant returns to scale			
	ε_t^P	ε_t^i	ε_t^M	ε_t^a	ε_t^P	ε_t^i	ε_t^M	ε_t^a
y_t	10.92	31.29	2.07	55.72	6.5	19.08	15.06	59.37
π_t	0.9	99.08	0	0.02	0.9	99.07	0	0.02
i_t	31.08	68.45	0.01	0.46	31.83	67	0.12	1.05
mp_t	0.47	0.64	77.14	21.76	0.34	0.52	80.94	18.2
y_t^f	0	0	3.5	96.5	0	0	19.37	80.63
mp_t^f	0	0	76.45	23.55	0	0	80.12	19.88

Table 2.3: Variance decomposition

Adding the constant returns-to-scale restriction gives real money balances a larger role in explaining the variance of output. Moreover, its radically changes the variance decomposition of output and its flexible-price counterpart, whereas the variance decomposition of other variables are almost unchanged.

For the two models, most of the output's variance comes from the technology shock, about a quarter of the variance of output results from the interest rate shock (31.29% for the decreasing returns case, and 19.08% for the constant returns case) and the remaining quarter from the other shocks. For the decreasing returns case, if money plays a role, its role is rather minor (an impact of less than 2.1%) and insignificant.

Moreover, Table 2.3 shows that assuming constant returns to scale gives money a role. As in Moghaddam (2010), the variance decomposition indicates that money supply is capable of explaining some of the forecasting error variance of real output[12]. This is the case only for the constant returns to scale model.

As Table 2.3 shows, the money shock contribution to the business cycle depends on the returns to scale hypothesis.

[12] Moghaddam (2010) also finds a stronger relation between real output and cyber cash than that explained by traditional paper money (M2).

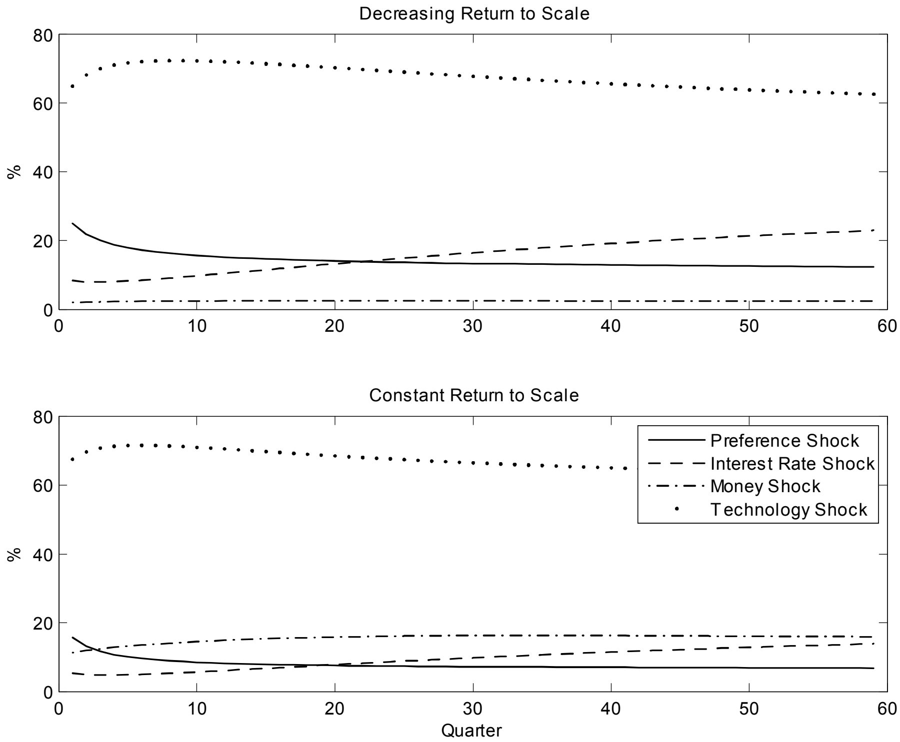


Figure 2.2: Variance decomposition through time of output

If approximately half of the variance of output is still explained by the technology shock, the role of the preference shock decreases notably, whereas the impact of the interest rate shock increases over time. Figure 2.2 also confirms the significant role of money in the dynamics of output, and its increasing role over periods, under the constant returns to scale hypothesis.

A look at the conditional and unconditional inflation variance decompositions shows the overwhelming role of the interest rate shock which explains more than 96% of the inflation rate's variance. As there is no significant change over the two models, we don't represent this decomposition.

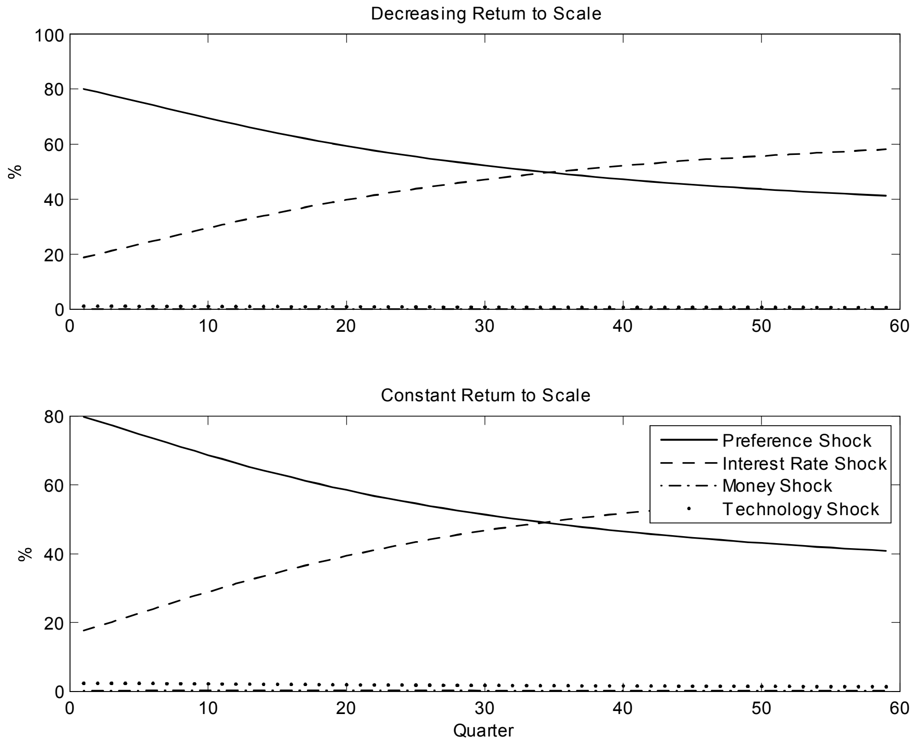


Figure 2.3: Variance decomposition through time of interest rate

The variance of the nominal interest rate is dominated by the direct shock to the interest rate in the long run. The relative importance of each of these shocks changes through time (Figure 2.3). Over short horizons, the preference shock explains almost 70% of the nominal interest rate variance, whereas the interest rate shock explains less than 20%. For longer horizons, there is an inversion: the nominal interest rate shock explains close to 70% of the variance, and the preference shock explains a bit more than 20%.

Table 2.3 as well as the conditional variance decomposition of real money balances shows that real money balances are mainly explained by the real money balances shock and the technology shock. As there is no significant change over the two models, we don't represent this decomposition.

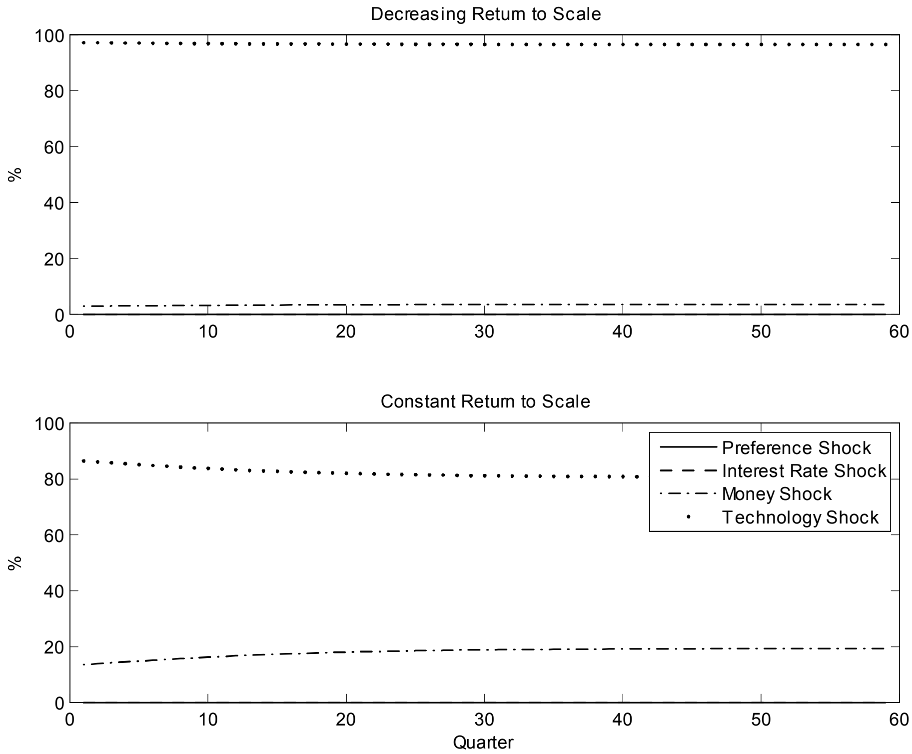


Figure 2.4: Variance decomposition through time of flexible-price output

It is also interesting to note that the same type of analysis applies to the flexible-price output variance decomposition (Figure 2.4). Technology, with a weight greater than 85%, is the main explanatory factor, while other shocks play minor roles.

As the flexible-price real money balances variance is mainly explained by the money shock, with a significant impact of the technology shock, the impact of each of these shocks does not change through time, so we don't represent this decomposition either.

2.6 Interpretation

The constant returns-to-scale hypothesis gives money a more important role than does the decreasing returns to scale hypothesis. Following the log-marginal density criteria, the decreasing returns to scale hypothesis is preferred to the constant returns to scale hypothesis.

This result disproves the hypothesis of Short (1979), Startz (1984), Benzing (1989), Chang (2002) of constant returns to scale for money in the production function and confirms the hypothesis of Khan and Ahmad (1985) of decreasing returns.

As a consequence, this criterion gives no significant role to money in the dynamics of the variables, despite its introduction in the production function.

The simulation results are close to those obtained in the baseline model (Galí, 2007) and provide interesting results about the potential effect of money on output and flexible-price output under the constant returns to scale hypothesis. Interestingly, and even if money enters the inflation equation, the variance decomposition of inflation with respect to shocks is unaffected under the two hypotheses.

2.7 Conclusion

One of the most controversial issues of the postwar economic literature involves the role of money as a factor of production. The notion of money as a factor of production has been debated both theoretically and empirically by a number of researchers in the past five decades. The question is whether money is an omitted variable in the production process.

However, empirical support for money as an input along with labor (and capital) has been mixed and, thus, the issue appears to be unsettled. Recent developments involve a reexamination of the role of money in the production function. One of these development is the New Keynesian DSGE theory mixed with Bayesian analysis.

We depart from existing theoretical (and empirical) New Keynesian literature by building a New Keynesian DSGE model that includes money in the production function, displaying money in the inflation equation.

Despite their inclusion in the production function, real money balances do not play a significant role in the dynamics of the system. The only way to ascribe a role for real money balances in the dynamics of the system is to assume constant returns to scale to factors of production, which is a strong and controversial hypothesis.

Moreover, we confirm that the model with decreasing returns to scale is better than the model with constant returns to scale. Under decreasing returns to scale, real money balances do not play a significant role in the dynamics of the economy. We also show that adding a money component to the system does not necessarily create a role for it.

2.8 Appendix

2.8.1 Aggregate consumption and price index

Let $C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$ be a consumption index where $C_t(i)$ represents the quantity of good i consumed by the household in period t. This requires that C_t be maximized for any given level of expenditure, $\int_0^1 P_t(i) C_t(i) di$, where $P_t(i)$ is the price of good i at time t. The maximization of C_t for any given expenditure level, $\int_0^1 P_t(i) C_t(i) di = Z_t$, can be formalized by means of the Lagrangian

$$\mathcal{L} = \left[\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} - \lambda\left(\int_0^1 P_t(i) C_t(i) di - Z_t\right) \tag{2.32}$$

The associated first-order conditions are $C_t(i)^{-\frac{1}{\varepsilon}} C_t^{\frac{1}{\varepsilon}} = \lambda P_t(i)$ for all $i \in [0,1]$. Thus, for any two goods (i,j),

$$C_t(i) = C_t(j)\left(\frac{P_t(i)}{P_t(j)}\right)^{-\varepsilon} \tag{2.33}$$

which can be substituted into the expression for consumption expenditures to yield $C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} \frac{Z_t}{P_t}$ for all $i \in [0,1]$ where $P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$ is an aggregate price index. The latter condition can then be substituted into the definition of C_t to obtain

$$\int_0^1 P_t(i) C_t(i) di = P_t C_t \tag{2.34}$$

Combining the two previous equations yields the demand schedule equation $C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t$ for all $i \in [0,1]$.

2.8.2 Optimization problem

Our Lagrangian is given by

$$L_t = E_t\left[\sum_{k=0}^{\infty} \beta^k U_{t+k} - \lambda_{t+k} V_{t+k}\right]$$

where

$$V_t = C_t + \frac{M_t}{P_t} + Q_t\frac{B_t}{P_t} - \frac{B_{t-1}}{P_t} - \frac{W_t}{P_t}N_t - \frac{M_{t-1}}{P_t}$$

and

$$U_t = e^{\varepsilon_t^P}\left(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\gamma e^{\varepsilon_t^M}}{1-\nu}\left(\frac{M_t}{P_t}\right)^{1-\nu} - \frac{\chi e^{\varepsilon_t^N} N_t^{1+\eta}}{1+\eta}\right)$$

The first order condition related to consumption expenditures is given by

$$\lambda_t = e^{\varepsilon_t^P} C_t^{-\sigma} \tag{2.35}$$

where λ_t is the Lagrangian multiplier associated with the budget constraint at time t.

The first order condition corresponding to the demand for contingent bonds implies that

$$\lambda_t \frac{Q_t}{P_t} = \beta E_t\left[\frac{\lambda_{t+1}}{P_{t+1}}\right] \tag{2.36}$$

The demand for cash that follows from the household's optimization problem is given by

$$\gamma e^{\varepsilon_t^P} e^{\varepsilon_t^M}\left(\frac{M_t}{P_t}\right)^{-\nu} = \lambda_t - \beta E_t\left[\lambda_{t+1}\frac{P_t}{P_{t+1}}\right] \tag{2.37}$$

which can be naturally interpreted as a demand for real balances. The latter is increasing in consumption and is inversely related to the nominal interest rate, as in conventional specifications.

$$\chi e^{\varepsilon_t^P} e^{\varepsilon_t^N} N_t^{\eta} = \lambda_t \frac{W_t}{P_t} \tag{2.38}$$

We obtain from (2.35)

$$\lambda_t = e^{\varepsilon_t^P} C_t^{-\sigma} \Leftrightarrow U_{c,t} = e^{\varepsilon_t^P} C_t^{-\sigma} \tag{2.39}$$

where $U_{c,t} = \left.\frac{\partial U_{k,t}}{\partial C_{t+k}}\right|_{k=0}$. Equation (2.39) defines the marginal utility of consumption.

Hence, the optimal consumption/savings, real money balances and labor supply decisions are described by the following conditions:

- Combining (2.35) with (2.36) gives

$$Q_t = \beta E_t\left[\frac{e^{\varepsilon_{t+1}^P} C_{t+1}^{-\sigma}}{e^{\varepsilon_t^P} C_t^{-\sigma}}\frac{P_t}{P_{t+1}}\right] \Leftrightarrow Q_t = \beta E_t\left[\frac{U_{c,t+1}}{U_{c,t}}\frac{P_t}{P_{t+1}}\right] \tag{2.40}$$

  where $U_{c,t+1} = \left.\frac{\partial U_{k,t}}{\partial C_{t+k}}\right|_{k=1}$. Equation (2.40) is the usual Euler equation for intertemporal consumption flows. It establishes that the ratio of marginal utility of future and current consumption is equal to the inverse of the real interest rate.

- Combining (2.35) with (2.37) gives

$$\gamma \frac{e^{\varepsilon_t^M}}{C_t^{-\sigma}} \left(\frac{M_t}{P_t} \right)^{-\nu} = 1 - Q_t \Leftrightarrow \frac{U_{m,t}}{U_{c,t}} = 1 - Q_t \tag{2.41}$$

  where $U_{m,t} = \left. \frac{\partial U_{k,t}}{\partial (M_{t+k}/P_{t+k})} \right|_{k=0}$. Equation (2.41) is the intertemporal optimality condition setting the marginal rate of substitution between money and consumption equal to the opportunity cost of holding money.

- And combining (2.35) with (2.38) gives

$$\chi \frac{\varepsilon_t^N N_t^\eta}{C_t^{-\sigma}} = \frac{W_t}{P_t} \Leftrightarrow \frac{U_{n,t}}{U_{c,t}} = -\frac{W_t}{P_t} \tag{2.42}$$

  where $U_{n,t} = \left. \frac{\partial U_{k,t}}{\partial N_{t+k}} \right|_{k=0}$. Equation (2.42) is the condition for the optimal consumption-leisure arbitrage, implying that the marginal rate of substitution between consumption and labor is equal to the real wage.

2.8.3 Calibration

We estimate all parameters, except the discount factor (β), the inverse of the Frisch elasticity of labor supply (η), and the elasticity of demand of households for consumption goods (ε).

Following standard conventions, we calibrate beta distributions for parameters that fall between zero and one, inverted gamma distributions for parameters that need to be constrained to be greater than zero and normal distributions in other cases.

The calibration of σ is inspired by Rabanal and Rubio-Ramírez (2007) and by Casares (2007). They choose, respectively, a risk aversion parameter of 2.5 and 1.5. In line with these values, we consider that $\sigma = 2$ corresponds to a standard risk aversion. As our goal is to compare two models, we adopt the same priors in the two models with the same calibration.

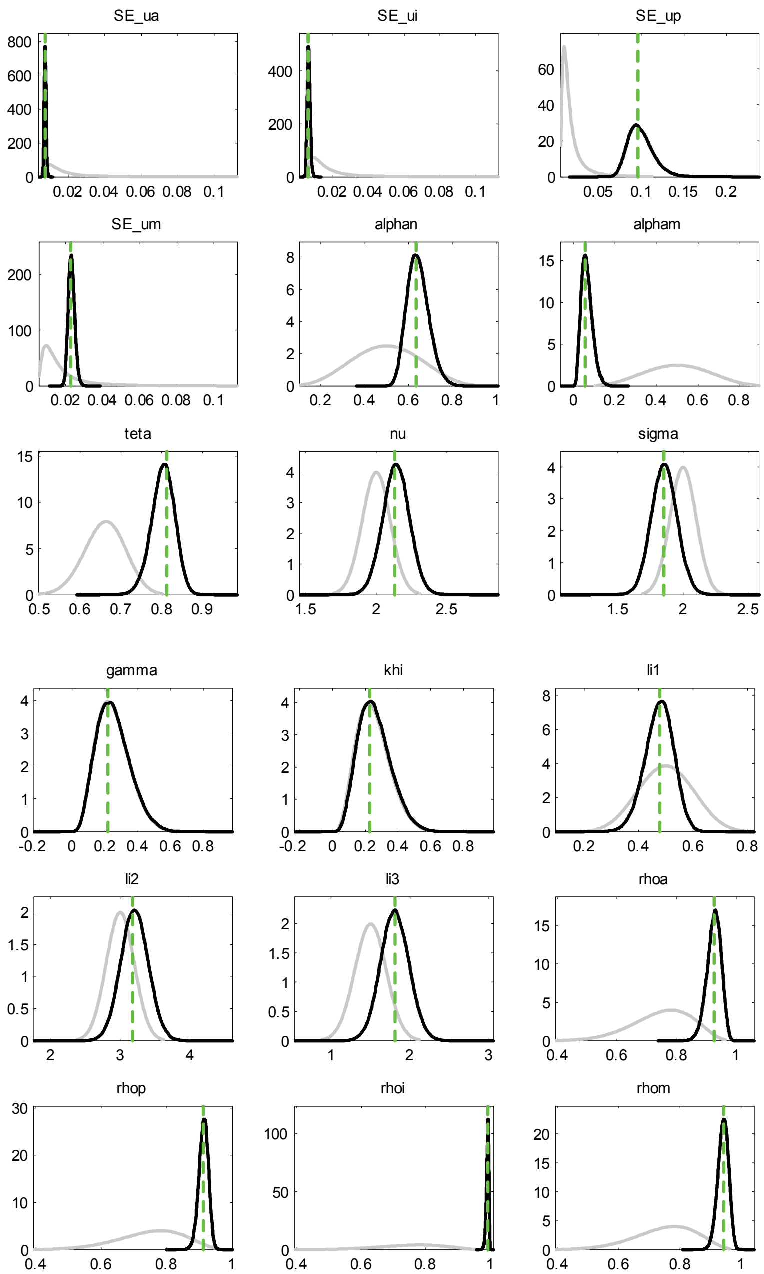
As in Smets and Wouters (2003), the standard errors of the innovations are assumed to follow inverse gamma distributions, and we choose a beta distribution for shock persistence parameters (as well as for the backward component of the Taylor rule, scale parameters, γ and χ, price stickiness index, θ, and output elasticities of labor, α_n, and of real money balances, α_m, of the production function) that should be lesser than one.

The calibration of α, β, θ, η and ε comes from Galí (2007) and Casares (2007). The smoothed Taylor rule (λ_i, λ_π, and λ_x) is calibrated following Gerlach-Kristen (2003), with priors analogous to those used by Smets and Wouters (2003). In order to take into consideration possible behaviors of the central bank, we assign a higher standard error for the Taylor rule's coefficients.

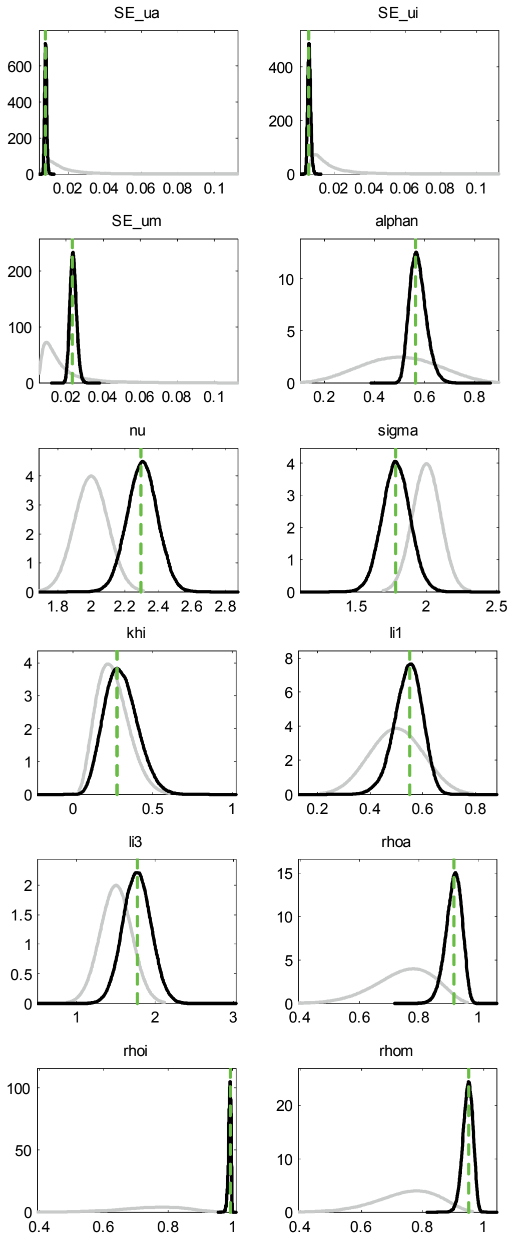
The calibration of the shock persistence parameters and the standard errors of the innovations follows Fève et *al.* (2010), where a much lower mean is adopted for ρ_a. All the standard errors of shocks are assumed to be distributed according to inverted Gamma distributions, with prior means of 0.02. The latter law ensures that these parameters have a positive support. The autoregressive parameters are all assumed to follow Beta distributions. Except for technology shocks, all these distributions are centered around 0.75. We take a common standard error of 0.1 for the shock persistence parameters, as in Smets and Wouters (2003). We allow for a lower standard error for the prior distribution of ρ_a.

2.8.4 Priors and posteriors

Decreasing returns to scale



Constant returns to scale



2.8.5 Macro parameters

	Decreasing returns to scale	Constant returns to scale
υ_a^y	1.026665	1.048629
υ_m^y	-0.116906	0.379180
υ_c^y	-0.085881	0.221747
υ_{y+1}^m	-0.496467	-0.443291
υ_y^m	0.866771	0.773933
υ_c^m	0.167187	0.157893
$\frac{1}{\nu}$	0.467391	0.434287
κ_x	0.047323	0.047126
κ_m	0.005532	-0.017869
σ^{-1}	0.539232	0.561143
ρ_c	-0.010050	-0.010050
$\frac{\sigma}{\nu}$	0.866771	0.773933
$\frac{a_2}{\nu}$	0.164496	0.155393
$\frac{\rho_m}{\nu}$	0.467391	0.434287
$\lambda_\pi(1-\lambda_i)$	1.681117	1.461802
$\lambda_x(1-\lambda_i)$	0.948134	0.806101
λ_i	0.475001	0.543219

2.9 Bibliography

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Chapter 3

Money and Risk Aversion

Jonathan Benchimol

3.1 Abstract

We present and test a model of the Eurozone, with a special emphasis on the role of risk aversion and money. The model follows the New Keynesian DSGE framework, money being introduced in the utility function with a non-separability assumption. By using Bayesian estimation techniques, we shed light on the determinants of output, inflation, money, interest rate, flexible-price output and flexible-price real money balance dynamics. The role of money is investigated further. Its impact on output depends on the degree of risk aversion. With a "traditional" level of risk aversion money plays a minor role. A higher level of risk aversion implies that money plays a non-negligible role.

Keywords: Euro area, Bayesian Estimation, Money, DSGE.

JEL Classification: E31, E51, E58.

3.2 Introduction

Standard New Keynesian literature analyses monetary policy practically without reference to monetary aggregates. In this now traditional framework, monetary aggregates do not explicitly appear as an explanatory factor neither in the output gap and inflation dynamics nor in interest rate determination. Inflation is explained by the expected inflation rate and the output gap. In turn, the output gap depends mainly on its expectations and the real rate of interest (Clarida, Galí and Gertler, 1999; Woodford, 2003; Galí and Gertler, 2007; Galí, 2008). Finally, the interest rate is established via a traditional Taylor rule in function of the inflation gap and the output gap.

In this framework, monetary policy impacts aggregate demand, and thus inflation and output, through change in the real interest rate. An increase in the interest rate reduces output, which increases the output gap, thus decreases inflation until a new equilibrium is reached. The money stock and money demand do not explicitly appear. The central bank sets the nominal interest rate so as to satisfy the demand for money (Woodford, 2003; Ireland, 2004).

This view of the transmission mechanism neglects the behavior of real money balances. First, there may exist a real balance effect on aggregate demand resulting from a change in prices. Second, as individuals re-allocate their portfolio of assets, the behavior of real money balances induces relative price adjustments on financial and real assets. In the process, aggregate demand changes, and thus output. By affecting aggregate demand, real money balances become part of the transmission mechanism. Hence, interest rate alone is not sufficient to explain the impact of monetary policy and the role played by financial markets (Meltzer, 1995, 1999, Brunner and Meltzer, 1968).

This monetarist transmission process may also imply a specific role to real money balances when dealing with risk aversion. When risk aversion increases, individuals may desire to hold more money balances to face the implied uncertainty and to optimize their consumption through time. Friedman alluded to this process as far back as 1956 (Friedman, 1956). If this hypothesis holds, risk aversion may influence the impact of real money balances on relative prices, financial assets and real assets, affecting aggregate demand and output.

Other considerations as to the role of money are worth mentioning. In a New Keynesian framework, the expected inflation rate or the output gap may "hide" the role of monetary aggregates, for example on inflation determination. Nelson (2008) shows that standard New Keynesian models are built on the strange assumption that central banks can control the long-term interest rate, while this variable is actually determined by a Fisher equation in which expected inflation depends on monetary developments. Reynard (2007) found that in the U.S. and the Euro area, monetary developments provide qualitative and quantitative information as to inflation. Assenlacher-Wesche and Gerlach (2006) confirm that money growth contains information about inflation pressures and may play an informational role as to the state of different non observed (or difficult to observe) variables influencing inflation or output.

How is money generally introduced in New Keynesian DSGE models ? The standard way is to resort to money-in-the-utility (MIU) function, whereby real money balances are supposed to affect the marginal utility of consumption. Kremer, Lombardo, and Werner (2003) seem to support this non-separability assumption for Germany, and imply that real money balances contribute to the determination of output and inflation dynamics. A recent contribution introduces the role of money with adjustment costs for holding real balances, and shows that real money balances contribute to explain expected future variations of the natural interest rate in the U.S. and the Euro area (Andrés, López-Salido and Nelson, 2009). Nelson (2002) finds that money is a significant determinant of aggregate demand, both in the U.S. and in the U.K. However, the empirical work undertaken by Ireland (2004), Andrés, López-Salido, and Vallés (2006), and Jones and Stracca (2008) suggests that there is little evidence as to the role of money in the cases of the United States, the Eurozone, and the UK.

Our paper differs in its empirical conclusion, resulting in a stronger role to money, at least in the Eurozone, when risk aversion is high enough. It differs also somewhat in its theoretical set up. As in the standard way, we resort to money-in-the-utility function (MIU) with a non-separability assumption between consumption and money. Yet, in our framework, we specify *all* the micro-parameters. This specification permits to extract characteristics and implications of this type of model that cannot be extracted if only aggregated parameters are used. We will see, for example, that the coefficient of relative risk aversion plays a significant role in explaining the role of money. As risk aversion could be very high in short periods of time, but cannot be estimated over such short periods of time, we test the model with a standard and a high risk aversion calibration. This strategy allows us to compare the impact of two levels of risk aversion on the dynamics of the variables.

Our model differs also in its inflation and output dynamics. Standard New Keynesian DSGE models give an important role to endogenous inertia on both output (consumption habits) and inflation (price indexation). In fact, both dynamics may have a stronger forward-looking component than an inertial component. And this appears to be the case, at least in the Euro area if not clearly in the U.S. (Galí, Gertler, and López-Salido, 2001). These inertial components may hide part of the role of money. Hence, our choice to remain as simple as possible on that respect in order to try to unveil a possible role for money balances.

We differ from existing theoretical (and empirical) analyses by specifying the flexible price counterparts of output and real money balances. This imposes a more elaborate theoretical structure, which provides an improvement on the literature and enriches the model.

Finally, we differ from the empirical analyses of the Eurozone by using Bayesian techniques in a New Keynesian DSGE framework, like in Smets and Wouters (2007), while introducing money in the model. Current literature attempts to introduce money only by aggregating model parameters, therefore leaving aside relevant information. Here we estimate all micro-parameters of the model under standard and high risk aversion. This is an important innovation and leads to interesting implications.

In the process we unveil transmission mechanisms generally neglected in traditional New Keynesian analyses. The framework highlights in particular the non-negligible role of money in explaining output variations, given a high enough risk aversion. It also highlights the overwhelming role of monetary policy in inflation variability.

The dynamic analysis of the model sheds light on the change in the role of money in explaining short run fluctuations in output as risk aversion changes. It shows that the higher the risk aversion, the higher the role of money in the transmission process.

Section 3.3 of the paper describes the theoretical set up. In Section 3.4, the model is calibrated and estimated with Bayesian techniques and by using Euro area data. Variance decompositions are analyzed in this section, with an emphasis on the impact of the coefficient of relative risk aversion. Section 3.5 concludes and the Appendix presents additional theoretical and empirical results.

3.3 The model

The model consists of households that supply labor, purchase goods for consumption, hold money and bonds, and firms that hire labor and produce and sell differentiated products in monopolistically competitive goods markets. Each firm sets the price of the good it produces, but not all firms reset their price during each period. Households and firms behave optimally: households maximize the expected present value of utility, and firms maximize profits. There is also a central bank that controls the nominal rate of interest. This model is inspired by Galí (2008), Walsh (2003) and Smets and Wouters (2003).

3.3.1 Households

We assume a representative infinitely-lived household, seeking to maximize

$$E_t\left[\sum_{k=0}^{\infty}\beta^k U_{t+k}\right] \tag{3.1}$$

where U_t is the period utility function and $\beta < 1$ is the discount factor.

We assume the existence of a continuum of goods represented by the interval $[0;1]$. The household decides how to allocate its consumption expenditures among the different goods. This requires that the consumption index C_t be maximized for any given level of expenditures. Furthermore, and conditional on such optimal behavior, the period budget constraint takes the form

$$P_tC_t + M_t + Q_tB_t \leq B_{t-1} + W_tN_t + M_{t-1} \tag{3.2}$$

for $t = 0, 1, 2...$, where W_t is the nominal wage, P_t is an aggregate price index (see Appendix 3.6.1), N_t is hours of work (or the measure of household members employed), B_t is the quantity of one-period nominally riskless discount bonds purchased in period t and maturing in period $t+1$ (each bond pays one unit of money at maturity and its price is Q_t where $i_t = -\log Q_t$ is the short term nominal rate) and M_t is the quantity of money holdings at time t. The above sequence of period budget constraints is supplemented with a solvency condition, such as $\forall t \lim_{n\longrightarrow\infty} E_t[B_n] \geq 0$.

In the literature, utility functions are usually time-separable. To introduce an explicit role for money balances, we drop the assumption that household preferences are time-separable across consumption and real money balances. Preferences are measured with a CES utility function including real money balances.

Under the assumption of a period utility given by

$$U_t = e^{\varepsilon_t^P}\left(\frac{1}{1-\sigma}\left((1-b)\,C_t^{1-\nu} + be^{\varepsilon_t^M}\left(M_t/P_t\right)^{1-\nu}\right)^{\frac{1-\sigma}{1-\nu}} - \frac{\chi}{1+\eta}N_t^{1+\eta}\right) \quad (3.3)$$

consumption, labor, money and bond holdings are chosen to maximize (3.1) subject to (3.2) and the solvency condition. This CES utility function depends positively on the consumption of goods, C_t, positively on real money balances, M_t/P_t, and negatively on labour N_t. σ is the coefficient of relative risk aversion of households (or the inverse of the intertemporal elasticity of substitution), ν is the inverse of the elasticity of money holdings with respect to the interest rate, and can be seen as a *non separability* parameter, and η is the inverse of the elasticity of work effort with respect to the real wage.

It must be noticed that ν must be lower than σ. If $\nu = \sigma$, equation (3.3) becomes a standard separable utility function whereby the influence of real money balances on output, inflation and flexible-price output disappears. This case has been studied in the literature. In our model, the difference between the risk aversion parameter and the separability parameter, $\sigma - \nu$, plays a significant role.

The utility function also contains two structural shocks: ε_t^P is a general shock to preferences that affects the intertemporal substitution of households (preference shock) and ε_t^M is a money demand shock. All structural shocks are assumed to follow a first-order autoregressive process with an *i.i.d.*-normal error term. b and χ are positive scale parameters.

As described in Appendix 3.6.1, this setting leads to the following conditions, which, in addition to the budget constraint, must hold in equilibrium. The resulting log-linear version of the first order condition corresponding to the demand for contingent bonds implies that

$$\begin{aligned} \hat{c}_t &= E_t\left[\hat{c}_{t+1}\right] - \left(\hat{\imath}_t - E_t\left[\hat{\pi}_{t+1}\right]\right)/\left(\nu - a_1\left(\nu-\sigma\right)\right) \\ &\quad - \frac{\left(1-a_1\right)\left(\nu-\sigma\right)}{\nu - a_1\left(\nu-\sigma\right)}\left(E_t\left[\Delta\hat{m}_{t+1} - E_t\left[\hat{\pi}_{t+1}\right]\right]\right) + \xi_{t,c} \end{aligned} \quad (3.4)$$

where $\xi_{t,c} = -\frac{1}{\nu - a_1(\nu-\sigma)}E_t\left[\Delta\varepsilon_{t+1}^P\right] - \frac{(1-a_1)(\nu-\sigma)}{(1-\nu)(\nu-a_1(\nu-\sigma))}E_t\left[\Delta\varepsilon_{t+1}^M\right]$ and by using the steady state of the first order conditions $a_1^{-1} = 1 + \left(\frac{b}{1-b}\right)^{\frac{1}{\nu}}(1-\beta)^{\frac{\nu-1}{\nu}}$. The lowercase $(\hat{\ })$ denotes the log-linearized (around the steady state) form of the original aggregated variables.

The demand for cash that follows from the household's optimization problem is given by

$$-\nu\left(\hat{m}_t-\hat{p}_t\right)+\nu\hat{c}_t+\varepsilon_t^M=a_2\hat{\imath}_t \tag{3.5}$$

with $a_2=\frac{1}{\exp(1/\beta)-1}$ and where real cash holdings depend positively on consumption with an elasticity equal to unity and negatively on the nominal interest rate. In what follows we will take the nominal interest rate as the central bank's policy instrument. In the literature, due to the assumption that consumption and real money balances are additively separable in the utility function, cash holdings do not enter any of the other structural equations: accordingly, the above equation becomes recursive to the rest of the system of equations.

The first order condition corresponding to the optimal consumption-leisure arbitrage implies that

$$\eta\hat{n}_t+\left(\nu-a_1\left(\nu-\sigma\right)\right)\hat{c}_t-\left(\nu-\sigma\right)\left(1-a_1\right)\left(\hat{m}_t-\hat{p}_t\right)+\xi_{t,m}=\hat{w}_t-\hat{p}_t \tag{3.6}$$

where $\xi_{t,m}=-\frac{(\nu-\sigma)(1-a_1)}{1-\nu}\varepsilon_t^M$.

Finally, these equations represent the Euler condition for the optimal intratemporal allocation of consumption (equation (3.4)), the intertemporal optimality condition setting the marginal rate of substitution between money and consumption equal to the opportunity cost of holding money (equation (3.5)), and the intratemporal optimality condition setting the marginal rate of substitution between leisure and consumption equal to the real wage (equation (3.6)).

3.3.2 Firms

We assume a continuum of firms indexed by $i\in[0,1]$. Each firm produces a differentiated good but uses an identical technology with the following production function,

$$Y_t\left(i\right)=A_tN_t\left(i\right)^{1-\alpha} \tag{3.7}$$

where $A_t=\exp\left(\varepsilon_t^a\right)$ is the level of technology, assumed to be common to all firms and to evolve exogenously over time, and α is the measure of decreasing returns.

All firms face an identical isoelastic demand schedule, and take the aggregate price level P_t and aggregate consumption index C_t as given. As in the standard Calvo (1983) model, our generalization features monopolistic competition and staggered price setting. At any time t, only a fraction $1-\theta$ of firms, with

$0 < \theta < 1$, can reset their prices optimally, while the remaining firms index their prices to lagged inflation.

3.3.3 Central bank

The central bank is assumed to set its nominal interest rate according to a generalized smoothed Taylor rule such as:

$$\hat{\imath}_t = (1-\lambda_i)\left(\lambda_\pi \hat{\pi}_t + \lambda_x\left(\hat{y}_t - \hat{y}_t^f\right)\right) + \lambda_i \hat{\imath}_{t-1} + \varepsilon_t^i \tag{3.8}$$

where λ_π and λ_x are policy coefficients reflecting the weight on inflation and on the output gap; the parameter $0 < \lambda_i < 1$ captures the degree of interest rate smoothing. ε_t^i is an exogenous *ad hoc* shock accounting for fluctuations of the nominal interest rate.

3.3.4 DSGE model

Solving the model (Appendix 3.6.1) leads to six micro-founded equations and six dependent variables: inflation, nominal interest rate, output, flexible-price output, real money balances and its flexible-price counterpart.

Flexible-price output and flexible-price real money balances are completely determined by shocks. Flexible-price output is mainly driven by technology shocks (whereas fluctuations in the output gap can be attributed to supply and demand shocks). The flexible-price real money balances are mainly driven by money shocks and flexible-price output.

$$\hat{y}_t^f = \upsilon_a^y \varepsilon_t^a + \upsilon_m^y \widehat{mp}_t^f - \upsilon_c^y + \upsilon_{sm}^y \varepsilon_t^M \tag{3.9}$$

$$\widehat{mp}_t^f = \upsilon_{y+1}^m E_t\left[\hat{y}_{t+1}^f\right] + \upsilon_y^m \hat{y}_t^f + \frac{1}{\nu}\varepsilon_t^M \tag{3.10}$$

$$\hat{\pi}_t = \beta E_t\left[\hat{\pi}_{t+1}\right] + \kappa_x\left(\hat{y}_t - \hat{y}_t^f\right) + \kappa_m\left(\widehat{mp}_t - \widehat{mp}_t^f\right) \tag{3.11}$$

$$\begin{aligned}\hat{y}_t \quad = \quad & E_t\left[\hat{y}_{t+1}\right] - \kappa_r\left(\hat{\imath}_t - E_t\left[\hat{\pi}_{t+1}\right]\right) + \kappa_{mp} E_t\left[\Delta\widehat{mp}_{t+1}\right] \\ & + \kappa_{sp} E_t\left[\Delta\varepsilon_{t+1}^P\right] + \kappa_{sm} E_t\left[\Delta\varepsilon_{t+1}^M\right]\end{aligned} \tag{3.12}$$

$$\widehat{mp}_t = \hat{y}_t - \kappa_i \hat{\imath}_t + \frac{1}{\nu}\varepsilon_t^M \tag{3.13}$$

$$\hat{\imath}_t = (1 - \lambda_i)\left(\lambda_\pi \hat{\pi}_t + \lambda_x \left(\hat{y}_t - \hat{y}_t^f\right)\right) + \lambda_i \hat{\imath}_{t-1} + \varepsilon_t^i \tag{3.14}$$

where

$\upsilon_a^y = \frac{1+\eta}{(\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha}$
$\upsilon_m^y = \frac{(1-\alpha)(\nu-\sigma)(1-a_1)}{(\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha}$
$\upsilon_c^y = \log\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{1-\alpha}{(\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha}$
$\upsilon_{sm}^y = \frac{(\nu-\sigma)(1-a_1)(1-\alpha)}{((\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha)(1-\nu)}$
$\upsilon_{y+1}^m = -\frac{a_2}{\nu}\left(\nu - (\nu-\sigma)a_1\right)$
$\upsilon_y^m = 1 + \frac{a_2}{\nu}\left(\nu - (\nu-\sigma)a_1\right)$
$\kappa_x = \left(\nu - (\nu-\sigma)a_1 + \frac{\eta+\alpha}{1-\alpha}\right)(1-\alpha)(1-\beta\theta)\frac{(1-\theta)}{\theta(1-\alpha+\alpha\varepsilon)}$
$\kappa_m = (1-\alpha)(1-\beta\theta)(\sigma-\nu)(1-a_1)\frac{(1-\theta)}{\theta(1-\alpha+\alpha\varepsilon)}$
$\kappa_r = \frac{1}{\nu-a_1(\nu-\sigma)}$
$\kappa_{mp} = \frac{(\sigma-\nu)(1-a_1)}{\nu-a_1(\nu-\sigma)}$
$\kappa_{sp} = -\frac{1}{\nu-a_1(\nu-\sigma)}$
$\kappa_{sm} = -\frac{(1-a_1)(\nu-\sigma)}{(\nu-a_1(\nu-\sigma))(1-\nu)}$
$\kappa_i = a_2/\nu$
with $a_1 = \frac{1}{1+(b/(1-b))^{1/\nu}(1-\beta)^{(\nu-1)/\nu}}$ and $a_2 = \frac{1}{\exp(1/\beta)-1}$.

Structural shocks, ε_t^M and ε_t^P, the exogenous component of the interest rate, ε_t^i, and of the technology, ε_t^a, are assumed to follow a first-order autoregressive process with an *i.i.d.*-normal error term such as $\varepsilon_t^k = \rho_k \varepsilon_{t-1}^k + \omega_{k,t}$ where $\varepsilon_{k,t} \sim N(0;\sigma_k)$ for $k = \{P, M, i, a\}$.

As can be seen, σ and ν influence all macro-parameters. This influence highlights the fact that separability and risk aversion are prominent factors involved in output, inflation, real money balances and nominal interest rate dynamics, as well as in flexible-price output and flexible-price real money balances. Moreover, as far as money is concerned, it is the three macro-parameters, υ_m^y, κ_m and κ_{mp}, that are essential to highlight its possible role in the dynamics of the model: these coefficients determine the *weight* of money in equation (3.9), (3.11) and (3.12).

3.4 Empirical results

As in Smets and Wouters (2003) and An and Schorfheide (2007), we apply Bayesian techniques to estimate our DSGE model. Contrary to Ireland (2004) or Andrès et *al.* (2006), we did not choose to estimate our model by using the maximum of likelihood because such computation hardly converges toward a global maximum.

3.4.1 Euro area data

In our model of the Eurozone, $\hat{\pi}_t$ is the log-linearized inflation rate measured as the yearly log difference of GDP Deflator from one quarter to the same quarter of the previous year, $\hat{y}_t$ is the log-linearized output measured as the yearly log difference of GDP from one quarter to the same quarter of the previous year, and $\hat{\imath}_t$ is the short-term (3-month) nominal interest rate. These Data are extracted from the Euro area Wide Model database (AWM) of Fagan, Henry and Mestre (2001). $\widehat{mp}_t$ is the log-linearized real money balances measured as the yearly log difference of real money from one quarter to the same quarter of the previous year, where real money is measured as the log difference between the money stock and the GDP Deflator. We use the $M3$ monetary aggregate from the OECD database, which is the broadest monetary aggregate. $\hat{y}_t^f$, the flexible-price output, and $\widehat{mp}_t^f$, the flexible-price real money balances, are completely determined by structural shocks. To make output and real money balances stationary, we use first log differences, as in Adolfson and *al.* (2008).

3.4.2 Calibration and results

As explained in Appendix 3.6.2, we consider two model versions. In the first version, we set $\sigma = 2$, the standard level of risk aversion. This model version is considered as a benchmark specification. In the second version, we set $\sigma = 4$, twice as much as the standard level and close to Rabanal and Rubio-Ramírez (2007), i.e., a high risk aversion level. This set-up is motivated by Holden and Subrahmanyam (1996). They show that acquisition of short-term information is encouraged by high risk aversion level, and that the latter can cause all potentially informed investors in the economy to concentrate exclusively on the short-term instead of the long-term. Risk aversion is generally low in the medium and long run while it could be very high in short periods. As we can't estimate risk aversion in the short run, we choose to estimate our model with our two calibrations of σ.

In both model versions, all the parameters but σ are re-estimated. The choice of parameters priors and distributions is summarized in Table 3.1 and explained in Appendix 3.6.2.

Bayesian estimation of structural parameters

	Priors			Posteriors					
				$\sigma = 2$			$\sigma = 4$		
	Law	Mean	Std.	5%	Mean	95%	5%	Mean	95%
α	beta	0.33	0.05	0.346	0.434	0.524	0.421	0.516	0.614
θ	beta	0.66	0.05	0.750	0.789	0.829	0.780	0.816	0.855
υ	normal	1.25	0.05	1.368	1.431	1.492	1.436	1.492	1.550
b	beta	0.25	0.10	0.088	0.251	0.410	0.089	0.249	0.405
η	normal	1.00	0.10	0.881	1.041	1.201	0.929	1.088	1.251
ε	normal	6.00	0.10	5.844	6.007	6.170	5.848	6.010	6.175
λ_i	beta	0.50	0.10	0.422	0.507	0.592	0.468	0.549	0.632
λ_π	normal	3.00	0.20	2.848	3.167	3.479	2.816	3.132	3.446
λ_x	normal	1.50	0.20	1.507	1.796	2.081	1.483	1.765	2.054
ρ_a	beta	0.75	0.10	0.873	0.914	0.957	0.907	0.937	0.966
ρ_p	beta	0.75	0.10	0.848	0.880	0.913	0.795	0.840	0.885
ρ_i	beta	0.75	0.10	0.985	0.991	0.997	0.983	0.989	0.996
ρ_m	beta	0.75	0.10	0.910	0.938	0.967	0.914	0.941	0.969
σ_a	invgamma	0.02	2.00	0.009	0.011	0.012	0.014	0.017	0.021
σ_i	invgamma	0.02	2.00	0.004	0.006	0.007	0.004	0.005	0.007
σ_p	invgamma	0.02	2.00	0.057	0.076	0.093	0.060	0.077	0.092
σ_m	invgamma	0.02	2.00	0.014	0.016	0.018	0.015	0.017	0.019

Table 3.1: Bayesian estimations

We estimate the model with 117 observations from 1980 (Q4) to 2009 (Q4) in order to avoid high volatility periods before 1980 and to take into consideration the core of the global crisis. The estimation of the implied posterior distribution of the parameters under the two configurations of risk (Table 3.1) is done using the Metropolis-Hastings algorithm (10 distinct chains, each of 100000 draws; see Smets and Wouters, 2007, and Adolfson et *al.*, 2007). Average acceptation rate per chain for the benchmark model ($\sigma = 2$) are included in the interval $[0.2601; 0.2661]$ and for ($\sigma = 4$) in the interval $[0.2587; 0.2658]$. The literature has settled on a value of this acceptance rate between 0.2 and 0.3.

Priors and posteriors distributions are presented in Appendix 3.6.3.

3.4.3 Model validation

To assess the model validation, we first insure convergence of the proposed distribution to the target distribution (Appendix 3.6.4). Moreover, to assess the model fit, Figure 3.1 shows the actual and one-side Kalman filter fitted data evaluated at the posterior mean for the model under normal risk aversion and high risk aversion.

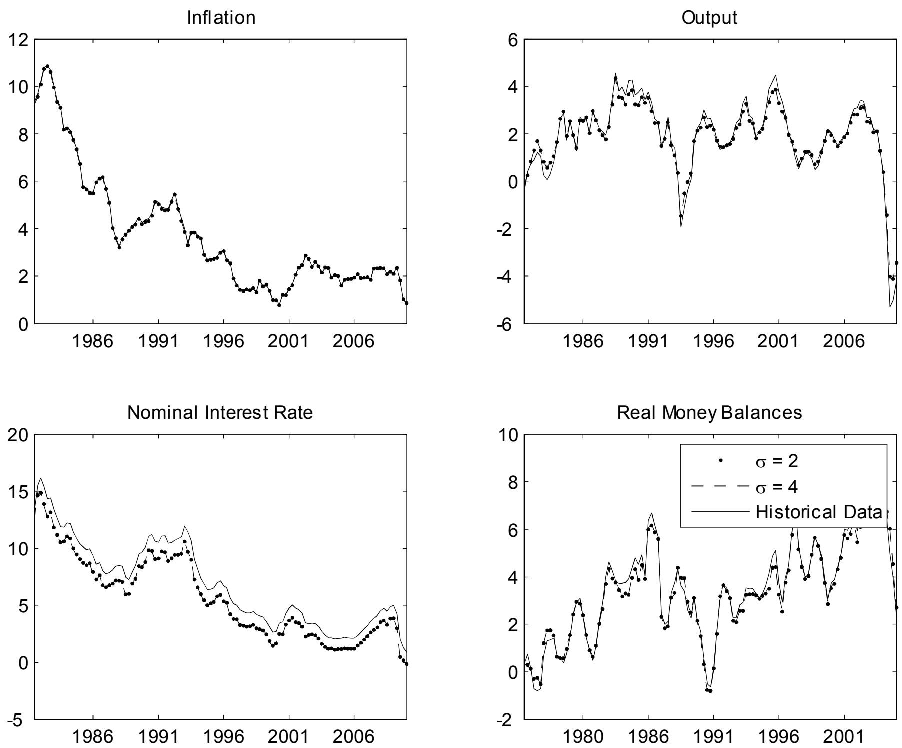


Figure 3.1: Historical data and one-side Kalman filter fitted data evaluated at the mean of the posterior.

The thin solid line of Figure 3.1 represents the actual data; the dotted line represents the outcome from the model with normal risk aversion ($\sigma = 2$); the dashed line represents the results under high risk aversion ($\sigma = 4$) and follows very closely the solid line. Our model fits well with the data and, prima facie, there is no big differences between the two calibrations.

3.4.4 Variance decompositions

Here we analyze in two different ways the forecast error variance of each variable following exogenous shocks. The analysis is conducted first via an unconditional variance decomposition, and second via a conditional variance decomposition.

Unconditional variance decomposition (%)

	with $\sigma = 2$				with $\sigma = 4$			
	ε_t^P	ε_t^i	ε_t^M	ε_t^a	ε_t^P	ε_t^i	ε_t^M	ε_t^a
$\hat{y}_t$	8.59	26.92	4.47	60.02	5.7	19.81	14.76	59.73
$\hat{\pi}_t$	0.44	99.53	0	0.03	0.27	99.54	0.01	0.18
$\hat{\imath}_t$	24.72	74.16	0.06	1.05	25.85	68.2	0.58	5.37
$\widehat{mp}_t$	0.43	0.15	75.75	23.68	0.41	0.96	69.16	29.46
$\hat{y}_t^f$	0	0	5.6	94.4	0	0	15.29	84.71
$\widehat{mp}_t^f$	0	0	74.75	25.25	0	0	68.83	31.17

Table 3.2: Variance decomposition

The unconditional variance decomposition (Table 3.2) shows that with a standard calibration of the model ($\sigma = 2$), more than half of the variance of output results from the technology shock, more than a quarter from the interest rate shock, the remaining from the other shocks. If money plays some role, this role is rather minor (an impact of less than 4.5%).

Yet, as the table shows, the money shock contribution to the business cycle depends on the value of agents' risk aversion. The estimation with the higher risk aversion ($\sigma = 4$) gives interesting information as to the role of money, and more generally to the role of each shock.

These results indicate that a higher coefficient of relative risk aversion increases significantly the impact of money on output. Yet it does not change significantly the impact of money on inflation dynamics. The very small role of the money shock in inflation dynamics is a consequence of the low value of κ_m in equation (3.11), whatever the level of risk aversion, even though κ_m increases with σ. By comparison the value of κ_{mp} in equation (3.12) is significantly higher, and increases no less significantly with σ (see Table 3.3 in Appendix 3.6.5).

If more than half of the variance of output is still explained by the technology shock with the high risk calibration ($\sigma = 4$), the role of the interest rate shock and the role of preference decrease whereas the impact of the money shock increases from about 4% to 15%, i.e. is multiplied by a factor of four. Similarly, the impact of the money shock on the flexible-price output variance increases from 5.6% to 15.29%.

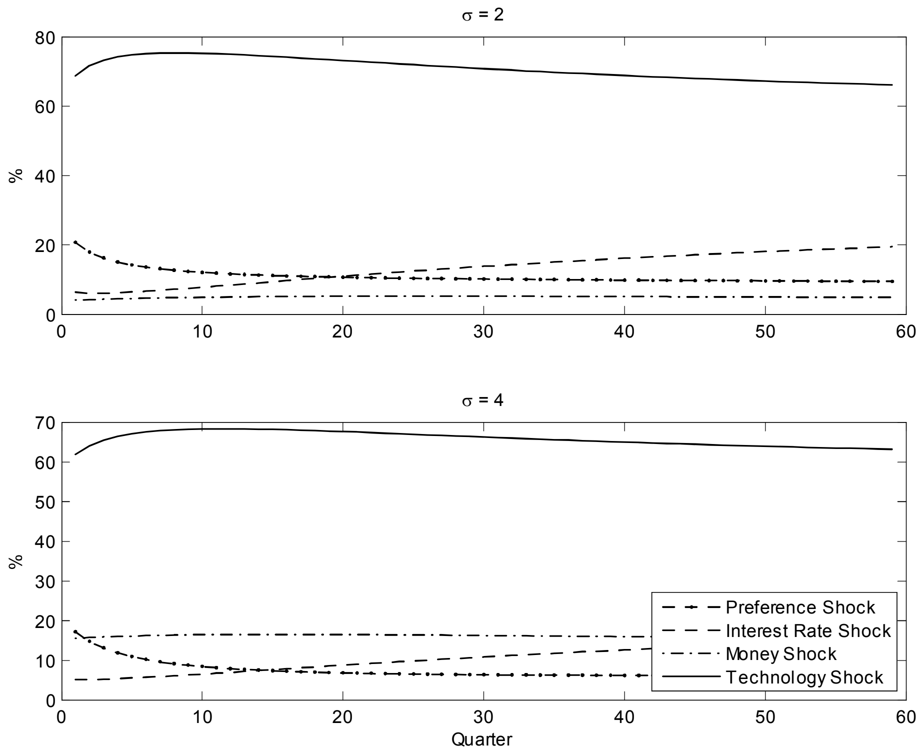


Figure 3.2: Variance decomposition through time of output

The analysis through time (conditional variance) of the different shocks on output (Figure 3.2) also shows that the impact of the money shock, and especially of the interest rate shock, increases a bit with the time horizon whereas it is the reverse for the preference shock[1].

The unconditional and conditional inflation variance decompositions show the overwhelming role of the interest rate shock (the monetary policy shock)

[1] The conditional variances decompositions figures for the other variables are not shown here but are available upon request.

which explains most of the variance. It must be noted that the change in risk aversion does not affect this result, and there is no significant change of the respective impacts through time.

The interest rate variance is dominated by the direct shock on the interest rate. Yet, as risk aversion increases, the role of the technology shock increases. The relative importance of each of these shocks changes over time: in the short term, the preference shock explains almost 70% of the nominal interest rate variance whereas the interest rate shock explains less than 20%. For longer terms, there is an inversion: the nominal interest rate shock explains close to 70% of the variance and the preference shock a bit more than 20%.

Unsurprisingly, real money balances are mainly explained by the money shock and the technology shock, with a small increase in the role of the technology shock as risk aversion increases. The respective role of these two shocks barely changes through time.

It is interesting to notice that the same type of analysis applies to the flexible-price output variance decomposition: technology is the main explanatory factor with a weight of almost 95%, the role of money increasing with the relative risk aversion coefficient (from a weight of less than 6% to more than 15%) whereas monetary policy plays no role.

The flexible-price real money balances variance is mainly explained by the money shock, with a significant impact of the technology shock. The impact of each of these shocks does not vary much through time, but when risk aversion increases the impact of the technology shock also increases.

3.4.5 Shock decomposition

To evaluate further the model, we also conducted a shock decomposition analysis, i.e. we evaluated the impact of each shock on the observed trajectories of the series.

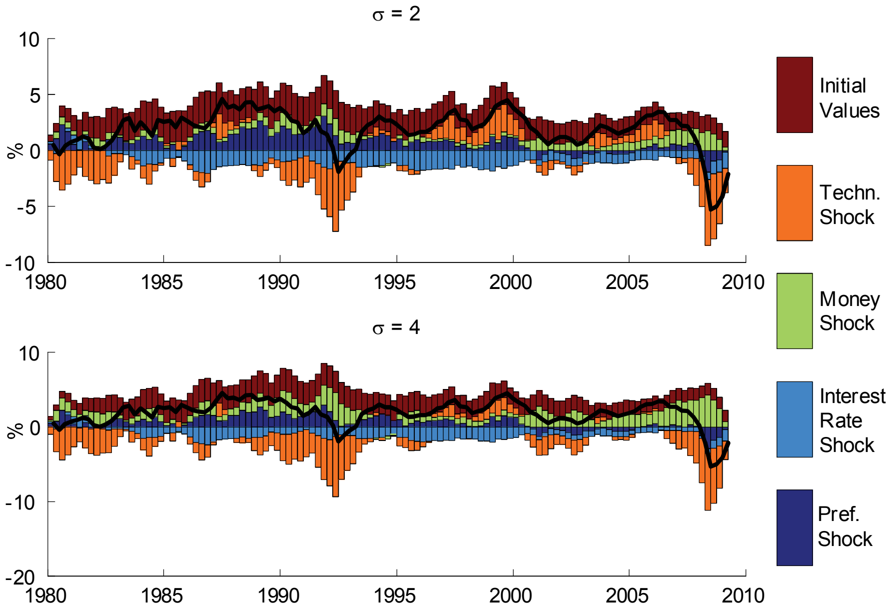


Figure 3.3: Output shock decompositions

Figure 3.3 illustrates for example the resulting decomposition of output fluctuations according to their structural shock contributions (ε_t^P in dark blue, ε_t^M in green, ε_t^i in azure and ε_t^a in orange). It shows that real money balances have a greater role during crises than in other periods. Moreover, the technology shock has an important negative impact on output dynamics in these crisis periods whereas the real money shock has a positive impact during the same periods.

This is also true for the flexible-price output shock decomposition[2] (Figure 3.4).

[2]The remaining decompositions for the other variables are less revealing. They can be provided upon request.

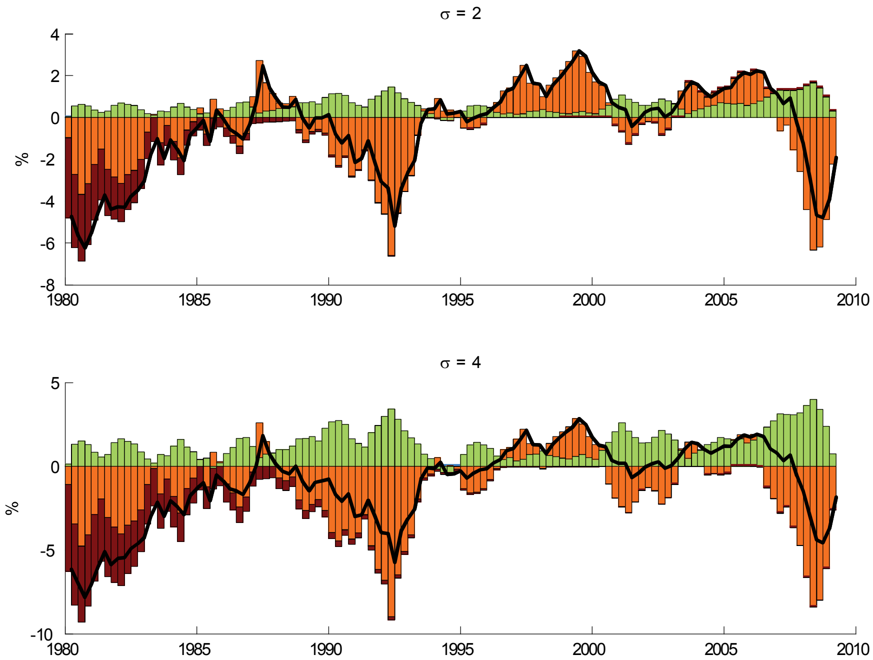


Figure 3.4: Flexible-price output shock decompositions

3.4.6 Interpretation

The estimates of the macro-parameters (aggregated structural parameters) for standard and high risk aversion are reported in Appendix 3.6.5 (Table 3.3). These estimates suggest that a change in risk aversion implies significant variations in the value of several macro-parameters, notably υ_m^y, κ_m and κ_{mp} - respectively the *weight* of money in the flexible-price output, inflation and output equations. Moreover, the *weight* of the money shock on output dynamics, κ_{sm}, and on flexible-price output, υ_{sm}^y, increases with risk aversion, thus reinforcing the role of money in the dynamics of the model.

It is also worth mentioning that the smoothing parameter in the Taylor rule equation, λ_i, increases with risk aversion. This reflects the idea that the central bank strives for financial stability in crisis periods.

The comparison between the variance decompositions (Table 3.2) of the two model versions illustrates the fact that the role of the money shock on output and flexible-price output depends crucially on the degree of agents' risk aversion, increases accordingly, and becomes significant when risk aversion is high enough. This result highlights the role of money balances to smooth consumption through time, especially when risk aversion reaches certain levels.

Another interesting outcome is that the higher the risk aversion, the higher the role of the technology shock in the conditional and unconditional variance decomposition of the nominal interest rate.

The decompositions of historical output (Figure 3.3) and of flexible-price output (Figure 3.4) explicitly show that the money shock contributes positively to output variations, especially during crisis periods (Black Monday, 1987; Scandinavian banking crisis, 1990; Speculative attacks on currencies in the European Exchange Rate Mechanism, 1992; Bursting of dot-com bubble, 2001; Subprime crisis, 2007). This result does not manifest itself in "standard" analyses, i.e. where the level of risk aversion is not taken into account.

Impulse response functions for the two configurations of risk (Appendix 3.6.6) highlights the role of risk aversion on the dynamics of several of the model's variables. These results also demonstrate the predominant role of the risk aversion level on the impact of the money shock and that of the technology shock on output, inflation, and real and nominal interest rates. The higher the risk aversion level, the greater the reactions to the shocks.

3.5 Conclusion

We built and empirically tested a model of the Eurozone, with different levels of risk aversion and with a particular emphasis on the role of money. The model follows the New Keynesian DSGE framework, with money in the utility function whereby real money balances affect the marginal utility of consumption.

By using Bayesian estimation techniques, we shed light on the determinants of output and inflation dynamics, but also on the interest rate, real money balances, flexible-price output and flexible-price real money balances variances. We further investigated how the results are affected when intertemporal risk aversion moves, especially as far as money is concerned. With a "traditional" level of intertemporal risk aversion, more than half of the variance of output is explained by the technology shock, the rest by a combination of labor, preference, interest rate and money shocks.

The first calibration of the model with a standard risk aversion shows that money plays a minor role in explaining output variability, a result in line with current literature (Andrès et *al.*, 2006; Ireland, 2004). Another calibration with a higher risk aversion implies that money plays a non-negligible role in explaining output and flexible-price output fluctuations. This outcome differs from existing literature using New Keynesian DSGE frameworks with money, insofar as it neglects the impact of a high enough risk factor.

On the other hand, the explicit money variable does not appear to have a notable direct role in explaining inflation variability. The overwhelming explanatory factor is the interest rate (monetary policy) whatever the level of risk aversion. Another outcome concerns monetary policy. The higher the risk aversion, the stronger the smoothing of the interest rate. This reflects probably the central bankers' objective not to agitate markets.

Based on these results, one may infer that by changing economic agents' perception of risks, the last financial crisis may have increased the role of real money balances in the transmission mechanisms and in output changes.

3.6 Appendix

3.6.1 Solving the model

- **Price dynamics**

Let's assume a set of firms not reoptimizing their posted price in period t. Using the definition of the aggregate price level and the fact that all firms resetting prices choose an identical price P_t^*, leads to $P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(P_t^*\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$. Dividing both sides by P_{t-1} and log-linearizing around $P_t^* = P_{t-1}$ yields

$$\pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right) \tag{3.15}$$

In this setup, we don't assume inertial dynamics of prices. Inflation results from the fact that firms reoptimizing in any given period their price plans, choose a price that differs from the economy's average price in the previous period.

- **Price setting**

A firm reoptimizing in period t chooses the price P_t^* that maximizes the current market value of the profits generated while that price remains effective. This problem is solved and leads to a first-order Taylor expansion around the zero inflation steady state:

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\widehat{mc}_{t+k|t} + \left(p_{t+k} - p_{t-1}\right)\right] \tag{3.16}$$

where $\widehat{mc}_{t+k|t} = mc_{t+k|t} - mc$ denotes the log deviation of marginal cost from its steady state value $mc = -\mu$, and $\mu = \log\left(\varepsilon/\left(\varepsilon-1\right)\right)$ is the log of the desired gross markup.

- **Equilibrium**

Market clearing in the goods market requires $Y_t(i) = C_t(i)$ for all $i \in [0,1]$ and all t. Aggregate output is defined as $Y_t = \left(\int_0^1 Y_t(i)^{1-\frac{1}{\varepsilon}}\, di\right)^{\frac{\varepsilon}{\varepsilon-1}}$; it follows that $Y_t = C_t$ must hold for all t. One can combine the above goods market clearing condition with the consumer's Euler equation (3.4) to yield the equilibrium condition

$$\begin{aligned}\hat{y}_t \;=\;& E_t\left[\hat{y}_{t+1}\right] - \frac{1}{\nu - a_1(\nu-\sigma)}\left(\hat{\imath}_t - E_t\left[\hat{\pi}_{t+1}\right]\right) \\ &+ \frac{(\sigma-\nu)(1-a_1)}{\nu - a_1(\nu-\sigma)}\left(E_t\left[\Delta\hat{m}_{t+1}\right] - E_t\left[\hat{\pi}_{t+1}\right]\right) + \xi_{t,c}\end{aligned} \tag{3.17}$$

Market clearing in the labor market requires $N_t = \int_0^1 N_t(i)\, di$. By using the production function (3.7) and taking logs, one can write the following approximate relation between aggregate output, employment and technology as

$$y_t = \varepsilon_t^a + (1-\alpha)\, n_t \tag{3.18}$$

An expression is derived for an individual firm's marginal cost in terms of the economy's average real marginal cost:

$$\begin{aligned} mc_t &= (\hat{w}_t - \hat{p}_t) - \widehat{mpn}_t & (3.19)\\ &= (\hat{w}_t - \hat{p}_t) - \frac{1}{1-\alpha}\left(\varepsilon_t^a - \alpha \hat{y}_t\right) & (3.20) \end{aligned}$$

for all t, where $\widehat{mpn}_t$ defines the economy's average marginal product of labor. As $mc_{t+k|t} = (\hat{w}_{t+k} - \hat{p}_{t+k}) - mpn_{t+k|t}$ we have

$$mc_{t+k|t} = mc_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}\left(p_t^* - p_{t+k}\right) \tag{3.21}$$

where the second equality follows from the demand schedule combined with the market clearing condition $c_t = y_t$. Substituting (3.21) into (3.16) yields

$$p_t^* - p_{t-1} = (1-\beta\theta)\,\Theta \sum_{k=0}^{\infty} (\beta\theta)^k E_t\left[\widehat{mc}_{t+k}\right] + \sum_{k=0}^{\infty} (\beta\theta)^k E_t\left[\pi_{t+k}\right] \tag{3.22}$$

where $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \leq 1$.

Finally, (3.15) and (3.22) yield the inflation equation

$$\pi_t = \beta E_t\left[\pi_{t+1}\right] + \lambda_{mc}\widehat{mc}_t \tag{3.23}$$

where β, $\lambda_{mc} = \Theta\frac{(1-\theta)(1-\beta\theta)}{\theta}$. λ_{mc} is strictly decreasing in the index of price stickiness θ, in the measure of decreasing returns α, and in the demand elasticity ε.

Next, a relation is derived between the economy's real marginal cost and a measure of aggregate economic activity. From (3.6) and (3.18), the average real marginal cost can be expressed as

$$\begin{aligned} mc_t \;=\; & \left(\nu - (\nu-\sigma)\, a_1 + \frac{\eta+\alpha}{1-\alpha}\right)\hat{y}_t - \varepsilon_t^a\left(\frac{1+\eta}{1-\alpha}\right) \\ & + (\sigma-\nu)(1-a_1)(\hat{m}_t - \hat{p}_t) + \xi_{t,m} \end{aligned} \tag{3.24}$$

Under flexible prices the real marginal cost is constant and equal to $mc = -\mu$. Defining the natural level of output, denoted by y_t^f, as the equilibrium level of output under flexible prices leads to

$$\begin{aligned} mc &= \left(\nu - (\nu - \sigma)\, a_1 + \frac{\eta + \alpha}{1 - \alpha}\right) \hat{y}_t^f - \varepsilon_t^a \left(\frac{1 + \eta}{1 - \alpha}\right) \\ &\quad + (\sigma - \nu)\,(1 - a_1)\, \widehat{mp}_t^f + \xi_{t,m} \end{aligned} \tag{3.25}$$

where $\widehat{mp}_t^f = \hat{m}_t^f - \hat{p}_t^f$, thus implying

$$\hat{y}_t^f = \upsilon_a^y \varepsilon_t^a + \upsilon_m^y \widehat{mp}_t^f + \upsilon_c^y + \upsilon_{sm}^y \varepsilon_t^M \tag{3.26}$$

where

$$\begin{aligned} \upsilon_a^y &= \frac{1 + \eta}{(\nu - (\nu - \sigma)\, a_1)\,(1 - \alpha) + \eta + \alpha} \\ \upsilon_m^y &= \frac{(1 - \alpha)\,(\nu - \sigma)\,(1 - a_1)}{(\nu - (\nu - \sigma)\, a_1)\,(1 - \alpha) + \eta + \alpha} \\ \upsilon_c^y &= -\frac{\mu\,(1 - \alpha)}{(\nu - (\nu - \sigma)\, a_1)\,(1 - \alpha) + \eta + \alpha} \\ \upsilon_{sm}^y &= \frac{(\nu - \sigma)\,(1 - a_1)\,(1 - \alpha)}{(\nu - (\nu - \sigma)\, a_1)\,(1 - \alpha) + \eta + \alpha} \frac{1}{1 - \nu} \end{aligned}$$

We deduce from (3.17) that $\hat{\imath}_t^f = (\nu - (\nu - \sigma)\, a_1)\, E_t \left[\Delta \hat{y}_{t+1}^f\right]$ and by using (3.5) we obtain the following equation of real money balances under flexible prices

$$\widehat{mp}_t^f = \upsilon_{y+1}^m E_t \left[\hat{y}_{t+1}^f\right] + \upsilon_y^m \hat{y}_t^f + \frac{1}{\nu} \varepsilon_t^M \tag{3.27}$$

where $\upsilon_{y+1}^m = -\frac{a_2(\nu - (\nu - \sigma) a_1)}{\nu}$ and $\upsilon_y^m = 1 + \frac{a_2(\nu - (\nu - \sigma) a_1)}{\nu}$

Subtracting (3.25) from (3.24) yields

$$\widehat{mc}_t = \phi_x \left(\hat{y}_t - \hat{y}_t^f\right) + \phi_m \left(\widehat{mp}_t - \widehat{mp}_t^f\right) \tag{3.28}$$

where $\widehat{mp}_t = \hat{m}_t - \hat{p}_t$ is the log linearized real money balances around its steady state, $\widehat{mp}_t^f$ is its flexible-price counterpart, $\phi_x = \nu - (\nu - \sigma)\, a_1 + \frac{\eta + \alpha}{1 - \alpha}$ and $\phi_m = (\sigma - \nu)\,(1 - a_1)$.

By combining (3.28) with (3.23) we obtain

$$\hat{\pi}_t = \beta E_t \left[\hat{\pi}_{t+1}\right] + \kappa_x \left(\hat{y}_t - \hat{y}_t^f\right) + \kappa_m \left(\widehat{mp}_t - \widehat{mp}_t^f\right) \tag{3.29}$$

where $\hat{y}_t - \hat{y}_t^f$ is the *output gap*, $\widehat{mp}_t - \widehat{mp}_t^f$ is the *real money balances gap*,

$$\kappa_x = \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\left(\frac{1-\theta}{\theta}\right)(1-\beta\theta)\left(\nu-(\nu-\sigma)\,a_1+\frac{\eta+\alpha}{1-\alpha}\right)$$

and

$$\kappa_m = \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\left(\frac{1-\theta}{\theta}\right)(1-\beta\theta)\,(\sigma-\nu)\,(1-a_1)$$

Then (3.29) is our first equation relating inflation to its one period ahead forecast, the output gap and real money balances.

The second key equation describing the equilibrium of the model is obtained by rewriting (3.17) so as to determine output

$$\hat{y}_t = E_t\left[\hat{y}_{t+1}\right] - \kappa_r\left(\hat{\imath}_t - E_t\left[\hat{\pi}_{t+1}\right]\right) + \kappa_{mp}E_t\left[\Delta\widehat{mp}_{t+1}\right] + \xi_{t,c} \tag{3.30}$$

where $\kappa_r = \frac{1}{\nu-(\nu-\sigma)a_1}$, $\kappa_{mp} = \frac{(\sigma-\nu)(1-a_1)}{\nu-a_1(\nu-\sigma)}$ and $\xi_{t,c} = \kappa_{sp}E_t\left[\Delta\varepsilon_{t+1}^P\right] + \kappa_{sm}E_t\left[\Delta\varepsilon_{t+1}^M\right]$ where $\kappa_{sp} = -\frac{1}{\nu-a_1(\nu-\sigma)}$ and $\kappa_{sm} = -\frac{(1-a_1)(\nu-\sigma)}{\nu-a_1(\nu-\sigma)}\frac{1}{1-\nu}$. (3.30) is thus a dynamic IS equation including the real money balances.

The third key equation describes the behavior of the money stock. From (3.5) we obtain

$$\widehat{mp}_t = \hat{y}_t - \kappa_i\hat{\imath}_t + \frac{1}{\nu}\varepsilon_t^M \tag{3.31}$$

where $\kappa_i = a_2/\nu$.

3.6.2 Calibration

Following standard conventions, we calibrate beta distributions for parameters that fall between zero and one, inverted gamma distributions for parameters that need to be constrained to be greater than zero, and normal distributions in other cases.

The calibration of σ is inspired by Rabanal and Rubio-Ramírez (2007) and by Casares (2007). They choose, respectively, a risk aversion parameter of 2.5 and 1.5. In line with these values, we consider that $\sigma = 2$ corresponds to a standard risk aversion while values above that level imply higher and higher risk aversion, hence our choice of $\sigma = 4$ to represent a high level of risk aversion. As our goal is not to estimate risk aversion (see Section 3.4.2) but to analyze two different configurations of risk, we adopt the same priors in the two models with different risk aversion calibration.

As in Smets and Wouters (2003), the standard errors of the innovations are assumed to follow inverse gamma distributions and we choose a beta distribution for shock persistence parameters (as well as for the backward component of the Taylor rule) that should be lesser than one.

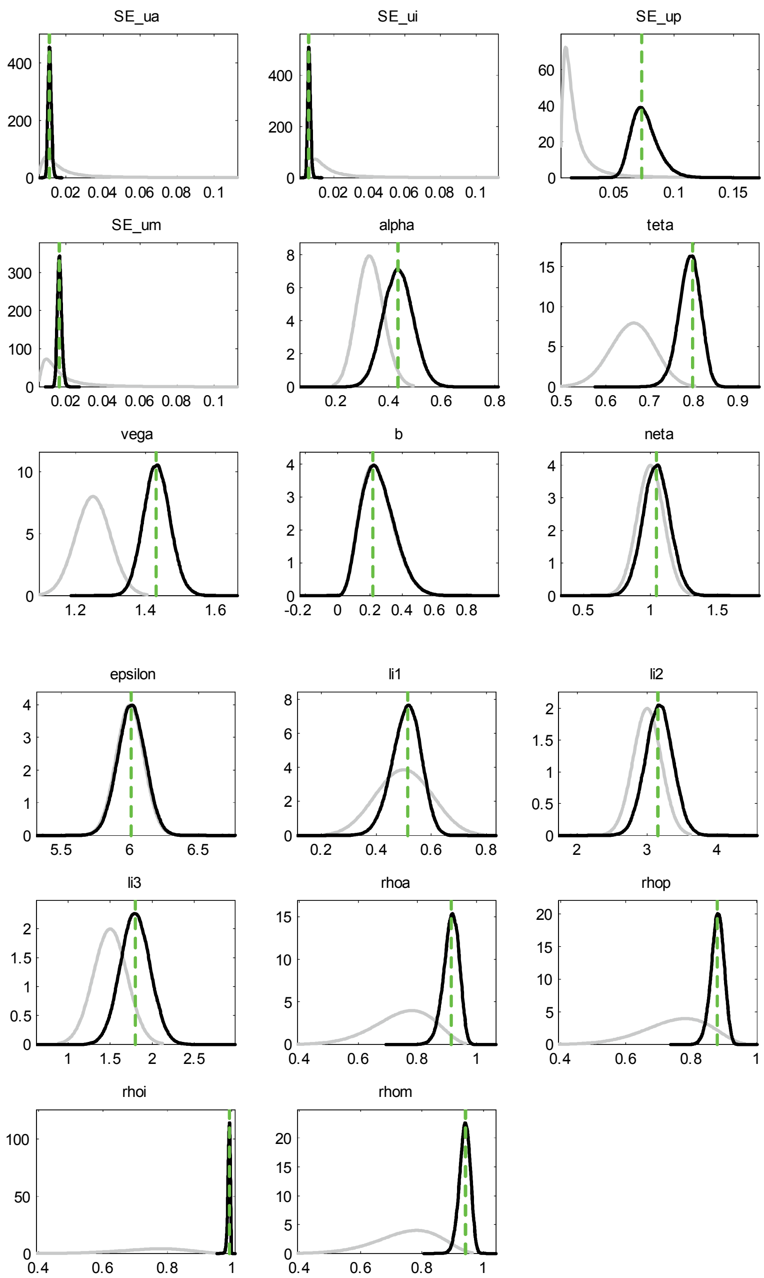
The calibration of α, β, θ, η, and ε comes from Galí (2007) and Casares (2007). The smoothed Taylor rule (λ_i, λ_π, and λ_x) is calibrated following Gerlach-Kristen (2003), analogue priors as those used by Smets and Wouters (2003) for the monetary policy parameters. In order to observe the behavior of the central bank, we assign a higher standard error for the Taylor rule's coefficients. v (the non-separability parameter) must be greater than one. κ_i (equation 3.13) must be greater than one as far as this parameter depends on the elasticity of substitution of money demand with respect to the cost of holding money balances, as explained in Söderström (2005); while still informative, this prior distribution is dispersed enough to allow for a wide range of possible and realistic values to be considered (i.e. $\sigma > v > 1$).

Our prior distributions are not dispersed to focus on the role of risk aversion. The calibration of the shock persistence parameters and the standard errors of the innovations follows Fève et *al.* (2010). All the standard errors of shocks are assumed to be distributed according to inverted Gamma distributions, with prior means 0.02. The latter ensures that these parameters have a positive support. The autoregressive parameters are all assumed to follow Beta distributions. All these distributions are centered around 0.75. We take a common standard error of 0.1 for the shock persistence parameters, as in Smets and Wouters (2003).

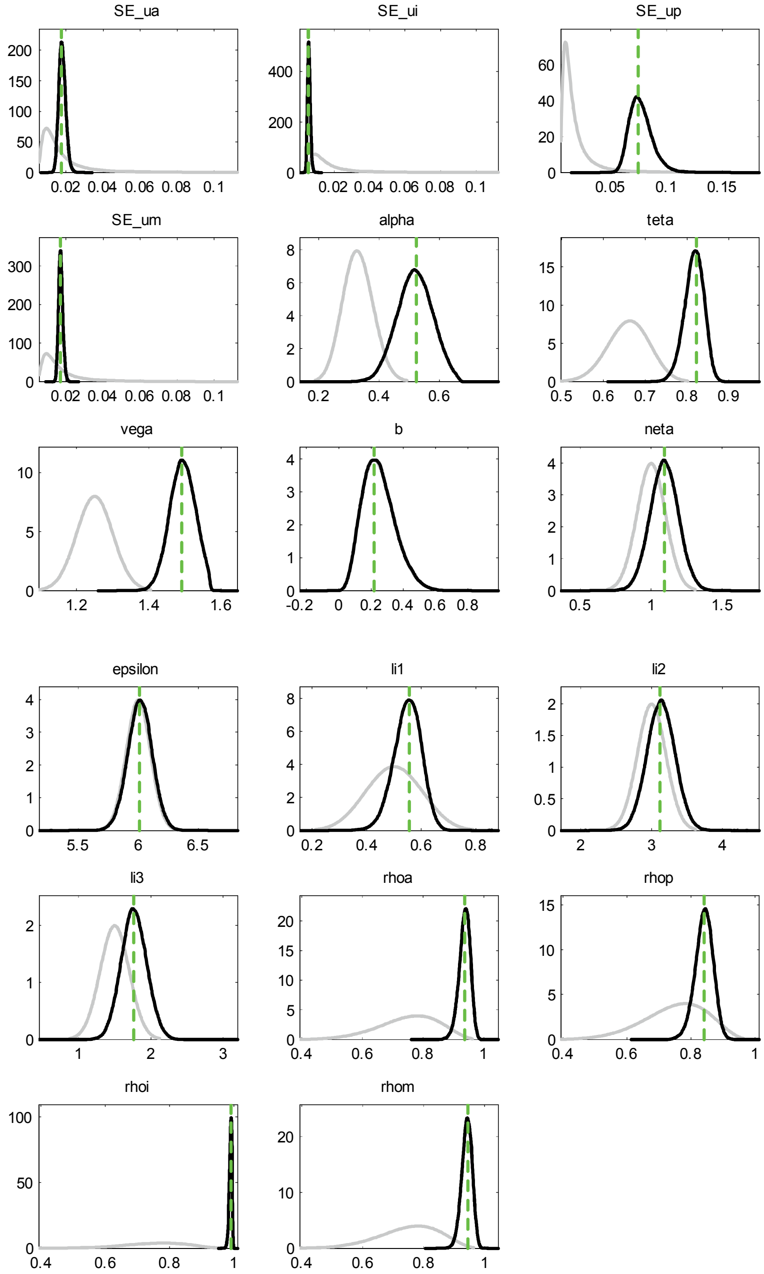
3.6.3 Priors and posteriors

The vertical line of Figures 3.6.3 and 3.6.3 denotes the posterior mode, the grey line the prior distribution, and the black line the posterior distribution.

Priors and posteriors ($\sigma = 2$)



Priors and posteriors ($\sigma = 4$)



3.6.4 Model validation

The diagnosis concerning the numerical maximization of the posterior kernel indicates that the optimization procedure leads to a robust maximum for the posterior kernel. The convergence of the proposed distribution to the target distribution is thus satisfied.

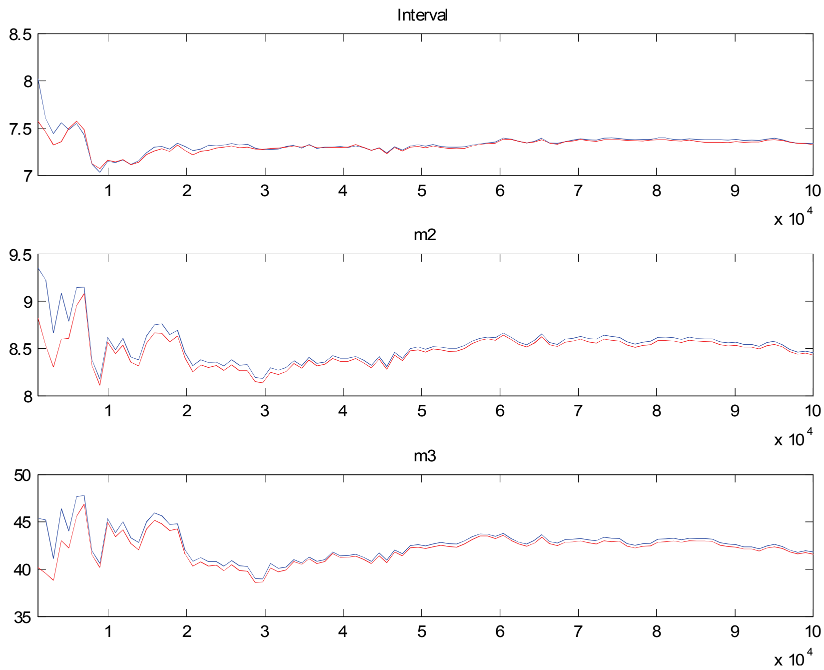


Figure 3.5: Multivariate MH convergence diagnosis ($\sigma = 2$)

A diagnosis of the overall convergence for the Metropolis-Hastings sampling algorithm is provided in Figure 3.5 and Figure 3.6.

Each graph represents specific convergence measures with two distinct lines that show the results within (red line) and between (blue line) chains (Geweke, 1999). Those measures are related to the analysis of the parameters mean (interval), variance (m2) and third moment (m3). For each of the three measures, convergence requires that both lines become relatively horizontal and converge to each other in the two models.

From Figure 3.5, it can be inferred that the model with standard risk aversion needs more chain to stabilize m3 (third moment), in comparison with the case where risk aversion is high (Figure 3.6).

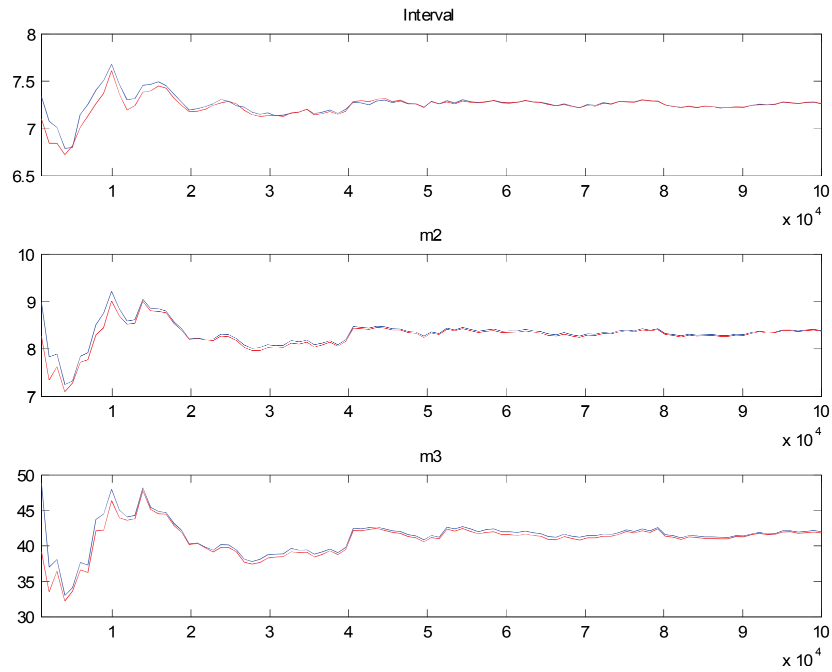


Figure 3.6: Multivariate MH convergence diagnosis ($\sigma = 4$)

Diagnosis for each individual parameter (not included but it can be provided upon request) indicates that most of the parameters do not exhibit convergence problems.

Moreover, a BVAR identification analysis (Ratto, 2008) suggests that all parameter values are stable.

The estimates of the innovation component of each structural shock, respectively for $\sigma = 2$ and $\sigma = 4$, respect the i.i.d. properties and are centered around zero. This reinforces the statistical validity of the estimated model (the corresponding figures can be provided by the authors).

3.6.5 Macro-parameters

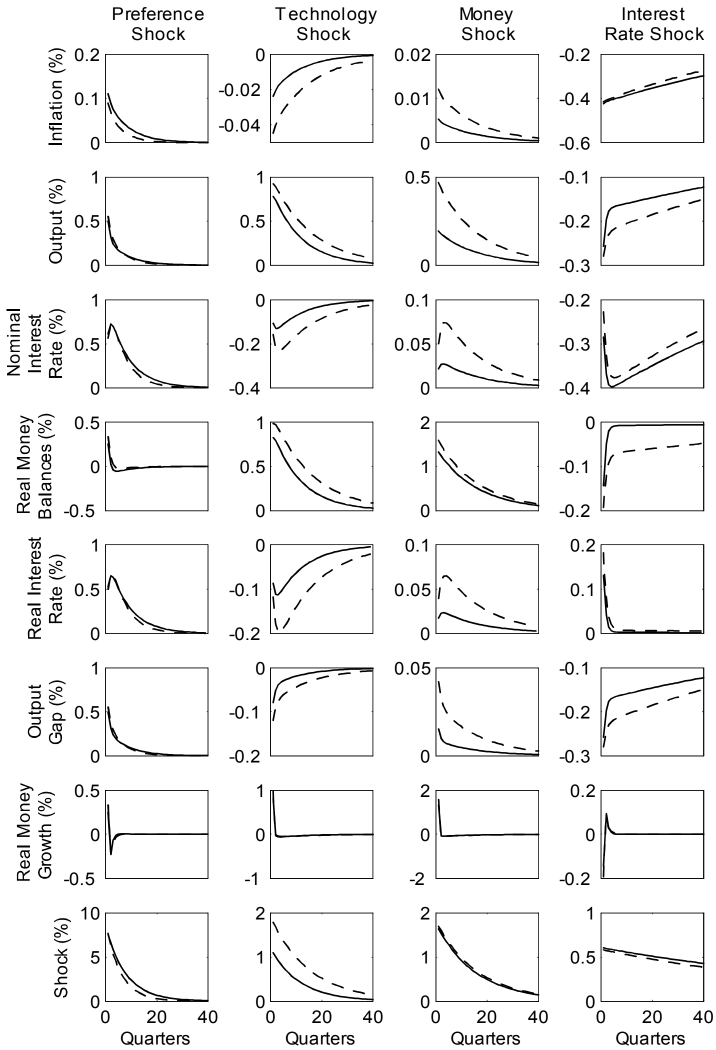
Aggregated structural parameters

	$\sigma = 2$	$\sigma = 4$
υ_a^y	0,8439	0,7444
υ_m^y	-0,0770	-0,2551
υ_c^y	0,0424	0,0309
υ_{sm}^y	0,1778	0,5166
υ_{y+1}^m	-0,6669	-0,9589
υ_y^m	1,6669	1,9589
κ_x	0,0407	0,0306
κ_m	0,0031	0,0078
κ_r	0,5990	0,3998
κ_{mp}	0,1980	0,5993
κ_{sp}	-0,5990	-0,3998
κ_{sm}	-0,4568	-1,2133
κ_i	0,3995	0,3834
λ_i	0,5117	0,5553
$(1-\lambda_i)\,\lambda_\pi$	1,5530	1,4006
$(1-\lambda_i)\,\lambda_x$	0,8724	0,7791

Table 3.3: Aggregated structural parameters

3.6.6 Impulse response functions

Figure 3.7: Impulse response functions with both risk configurations



The thin solid line of Figure 3.7 represents the impulse response functions of the model with normal risk aversion ($\sigma = 2$) while the dashed line represents the impulse response functions of the model with high risk aversion ($\sigma = 4$).

After a preference shock, the inflation rate, the output, the output gap, real money balances, the nominal and the real rate of interest rise, then gradually decrease toward the steady state. Real money growth displays an overshooting/undershooting process in the first periods, then rapidly returns to its steady state value. After an interest rate shock, inflation, the nominal rate of interest, output and the output gap fall. The real rate of interest rises.

For the two model versions, a positive monetary policy shock induces a fall in interest rates due to a low enough degree of intertemporal substitution (i.e. the risk aversion parameter is high enough) which generates a large impact response of current consumption relative to future consumption. This result has been noted in, *inter alia*, Jeanne (1994) and Christiano et *al.* (1997).

After a technology shock, the output gap, the inflation, the nominal and the real interest rate decrease whereas output as well as real money balances and real money growth rise. After a money shock, inflation, the nominal and the real rate of interest, the output and the output gap rise.

All these results are in line with the DSGE literature, especially with Galí (2007).

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Chapter 4

Role of money and monetary policy in crisis periods

Jonathan Benchimol

4.1 Abstract

In this paper, we test two models of the Eurozone, with a special emphasis on the role of money and monetary policy during crises. The role of separability between money and consumption is investigated further and we analyse the Euro area economy during three different crises: 1992, 2001 and 2007. We find that money has a rather significant role to play in explaining output variations during crises whereas, at the same time, the role of monetary policy on output decreases significantly. Moreover, we find that a model with non-separability between consumption and money has better forecasting performance than a baseline separable model over crisis periods.

Keywords: Euro area, Money, DSGE forecasting.

JEL Classification: E31, E51, E58.

4.2 Introduction

The ability to accurately forecast the future path for macroeconomic series such as output or inflation is crucial information for the business sector, government and central bank in their decision-making process. Dynamic Stochastic General Equilibrium (DSGE) models provide valuable information about business cycle dynamics and the effects of various economic shocks on the economy (Smets and Wouters, 2007 and Christiano *et al.*, 2010). For all those reasons, DSGE models are increasingly being utilized by central banks and other policy-making institutions to assist with policy decisions (Tovar, 2008).

In policy analysis, it is believed that monetary policy has long and variable effects on the overall economy. To capture such complex interactions between policy variables and the economy as a whole, macroeconomic forecasting becomes indispensable in actual policy making (Kohn 1995, Blinder 1997, and Diebold 1998). Sims and Zha (1998) introduced Bayesian methods to vector autoregressive (VAR) models to improve the accuracy of out-of-sample forecasts in a dynamic multivariate framework. They showed how to compute Bayesian probability distributions or error bands around out-of-sample forecasts. More recently, researchers have started to examine the forecasting performance of these models. In one such investigation, Smets and Wouters (2007) show that a DSGE model can generate forecasts that have a lower root mean-squared deviation (RMSD) than a Bayesian Vector Autoregression (BVAR).

On the other hand, Edge *et al.* (2010) show that the out-of-sample forecasting performance of the Federal Reserve Board's new DSGE model for the U.S. economy (EDO) is in many cases better than their large-scale macro-econometric model (FRB/US).

In a DSGE framework applied to the Eurozone, Chapter 3 shows that the role of money with respect to the economy, especially on output dynamics, increases with risk aversion. Yet we don't develop a complete analysis of the role of money and of monetary policy during crisis periods. The purpose of this paper is to fill this gap by focusing on the fluctuations of micro and macro parameters; on variance decompositions of variables with respect to shocks; and on output and inflation forecasts.

Chapter 3 shows that money has an explicit role when risk aversion is high enough, which may be the case during crisis periods. Yet, these periods do not last long. And other parameters changes in the very short term may also affect the role of money and monetary policy. That is why, in the present article, we use relatively small samples for the estimations, in order to capture the impact of very short term parameters changes on the dynamics of the model.

First, we compare two types of New Keynesian models in a DSGE framework. The first model is a standard one whereby money is included in the utility function with a separability assumption (Chapter 1). The second model introduces money in the utility function by assuming non-separability between real money balances and consumption (Chapter 3). By using Bayesian techniques, we estimate these two models with Eurozone data over three different crises: during speculative attacks on currencies in the European Exchange Rate Mechanism (ERM) at the beginning of the 1990s (Black Wednesday crisis); following the bursting of the Dot-com bubble at the beginning of 2001 (Dot-com crisis); and during the Subprime crisis from 2007 to 2010 (Subprime crisis).

Second, we analyze the results on the dynamics of the model, by studying the variations of micro and macro parameters during these crisis periods as well as the variance decomposition of the variables (notably current output, flexible-price output and inflation) with respect to structural shocks (preference shock, technology shock, money shock and interest rate shock). We also study the forecasting performances of the two models over the periods under scrutiny. Focusing on these three periods sheds light on the specific role of money and monetary policy in crisis situations and leads to interesting results as to output and inflation dynamics during these periods.

The results show that the role of money increases during crises. It also demonstrates that a New Keynesian model with non-separable preferences between money and consumption is able to better forecast output than a simple New Keynesian model with separable preferences during crises.

The study leads to policy implications as to the conduct of monetary policy, especially during crisis periods.

In Section 2, we present the models used for the empirical analysis presented in Section 3. We analyze the ERM crisis in Section 4, the Dot-com crisis in Section 5, and the Subprime crisis in Section 6. For each crisis, we provide estimated parameters, variance decompositions and analyse predictive properties of each model over time. We discuss the results and compare the three crises in Section 7. Section 8 concludes.

4.3 The models

These two models consist of households that supply labor, purchase goods for consumption, hold money and bonds, and firms that hire labor and produce and sell differentiated products in monopolistically competitive goods markets. Each firm sets the price of the good it produces, but not all firms reset their price during each period. Households and firms behave optimally: households maximize the expected present value of utility, and firms maximize profits. There is also a central bank that controls the nominal rate of interest. These models are inspired by Galí (2008), Walsh (2003) and Smets and Wouters (2003).

4.3.1 The separable baseline model

The following New Keynesian DSGE model comes from Chapter 1, Section 1.3, and serves as a baseline model.

Households

We assume a representative infinitely-lived household, seeking to maximize

$$E_t \left[\sum_{k=0}^{\infty} \beta^k U_{t+k} \right] \tag{4.1}$$

where U_t is the period utility function and $\beta < 1$ is the discount factor.

We assume the existence of a continuum of goods represented by the interval $[0;1]$. The household decides how to allocate its consumption expenditures among the different goods. This requires that the consumption index C_t be maximized for any given level of expenditures. Furthermore, and conditional on such optimal behavior, the period budget constraint takes the form

$$P_t C_t + M_t + Q_t B_t \leq B_{t-1} + W_t N_t + M_{t-1} \tag{4.2}$$

for $t = 0, 1, 2...$, P_t is an aggregate price index, M_t is the quantity of money holdings at time t, B_t is the quantity of one-period nominally riskless discount bonds purchased in period t and maturing in period $t+1$ (each bond pays one unit of money at maturity and its price is Q_t where $i_t = -\log Q_t$ is the short term nominal rate), W_t is the nominal wage, and N_t is hours of work (or the measure of household members employed).

The above sequence of period budget constraints is supplemented with a solvency condition[1]. Preferences are measured with a common time-separable utility function (MIU). Under the assumption of a period utility given by

$$U_t = e^{\varepsilon_t^P}\left(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\gamma e^{\varepsilon_t^M}}{1-\vartheta}\left(\frac{M_t}{P_t}\right)^{1-\vartheta} - \frac{\chi N_t^{1+\eta}}{1+\eta}\right) \tag{4.3}$$

where consumption, labor, money and bond holdings are chosen to maximize (4.3) subject to the budget constraint (4.2) and the solvency condition. σ is the coefficient of relative risk aversion of households (or the inverse of the intertemporal elasticity of substitution), ϑ is the inverse of the elasticity of money holdings with respect to the interest rate, and η is the inverse of the elasticity of work effort with respect to the real wage. The utility function also contains two structural shocks: ε_t^P is a general shock to preferences that affects the intertemporal substitution of households (preference shock accounts for changes in the marginal rate of substitution between goods, real money balances and work) and ε_t^M is a money shock (it accounts for changes in households' money holdings). γ and χ are positive scale parameters.

Firms

We assume a continuum of firms indexed by $i \in [0,1]$. Each firm produces a differentiated good but uses an identical technology with the following production function,

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{4.4}$$

where $A_t = \exp(\varepsilon_t^a)$ is the level of technology assumed to be common to all firms and to evolve exogenously over time, ε_t^a is the technology shock, and α is the measure of decreasing returns.

All firms face an identical isoelastic demand schedule, and take the aggregate price level P_t and aggregate consumption index C_t as given. As in the standard Calvo (1983) model, our generalization features monopolistic competition and staggered price setting. At any time t, only a fraction $1-\theta$ of firms, with $0 < \theta < 1$, can reset their prices optimally, while the remaining firms index their prices to lagged inflation.

[1] Such as $\forall t \lim_{n \longrightarrow \infty} E_t[B_n] \geq 0$. It prevents engaging in Ponzi-type schemes.

Central bank

Finally, the model is closed by adding the monetary policy smoothed Taylor-type reaction function:

$$\hat{\imath}_t = (1-\lambda_i)\left(\lambda_\pi(\pi_t - \pi^*) + \lambda_x\left(y_t - y_t^f\right)\right) + \lambda_i \hat{\imath}_{t-1} + \varepsilon_t^i \tag{4.5}$$

where λ_π and λ_x are policy coefficients reflecting the weight on inflation and on the output gap; the parameter $0 < \lambda_i < 1$ captures the degree of interest rate smoothing. ε_t^i is an exogenous *ad hoc* shock accounting for fluctuations of nominal interest rate.

Solution

The solution of this model leads to six equations with six variables: flexible-price output $(\hat{y}_t^f)$, flexible-price interest rate $(\hat{\imath}_t^f)$, inflation $(\hat{\pi}_t)$, output $(\hat{y}_t)$, real money balances $(\widehat{mp}_t)$, and nominal interest rate $(\hat{\imath}_t)$; and four structural shocks which are assumed to follow a first-order autoregressive process with an *i.i.d.*-normal error term such as $\varepsilon_t^k = \rho_k \varepsilon_{t-1}^k + \omega_{k,t}$ where $\varepsilon_{k,t} \sim N(0;\sigma_k)$ for $k = \{P, M, i, a\}$. The lowercase superscript $(\hat{\ })$ denotes the log-linearized (around the steady state) form of the original aggregated variables.

$$\hat{y}_t^f = \frac{1+\eta}{\sigma(1-\alpha)+\eta+\alpha}\varepsilon_t^a + \frac{(1-\alpha)\left(\log(1-\alpha) + \rho_n - \log\left(\frac{\varepsilon}{\varepsilon-1}\right)\right)}{\sigma(1-\alpha)+\eta+\alpha} \tag{4.6}$$

$$\hat{\imath}_t^f = \rho_c + \sigma E_t\left[\Delta \hat{y}_{t+1}^f\right] \tag{4.7}$$

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \frac{(1-\theta)(1-\beta\theta)(\sigma(1-\alpha)+\eta+\alpha)}{\theta(1-\alpha+\alpha\varepsilon)}\left(\hat{y}_t - \hat{y}_t^f\right) \tag{4.8}$$

$$\hat{y}_t = E_t[\hat{y}_{t+1}] - \sigma^{-1}(\hat{\imath}_t - E_t[\hat{\pi}_{t+1}] - \rho_c) - \sigma^{-1}E_t\left[\Delta\varepsilon_{t+1}^P\right] \tag{4.9}$$

$$\widehat{mp}_t = \frac{\sigma}{\vartheta}\hat{y}_t - \frac{a_2}{\vartheta}\hat{\imath}_t - \frac{\rho_m}{\vartheta} + \frac{1}{\vartheta}\varepsilon_t^M \tag{4.10}$$

$$\hat{\imath}_t = (1-\lambda_i)\left(\lambda_\pi\hat{\pi}_t + \lambda_x\left(\hat{y}_t - y_t^f\right)\right) + \lambda_i\hat{\imath}_{t-1} + \varepsilon_t^i \tag{4.11}$$

where
$\rho_m = -\log(\gamma) + a_1$
$\rho_n = -\log(\chi)$
$\rho_c = -\log(\beta)$
with $a_1 = \log\left(1 - e^{-\frac{1}{\beta}}\right) - \frac{\frac{1}{\beta}}{e^{\frac{1}{\beta}}-1}$ and $a_2 = \frac{1}{e^{\frac{1}{\beta}}-1}$.

ε_t^M is the shock on real money balances, ε_t^P is the shock preferences, ε_t^i is the exogenous component of the interest rate and ε_t^a is the technology shock.

4.3.2 The non-separable model

As in the previous model the representative infinitely-lived household seeks to maximize

$$E_t\left[\sum_{k=0}^{\infty}\beta^k U_{t+k}\right] \tag{4.12}$$

Now, the period utility function U_t is such as:

$$U_t = e^{\varepsilon_t^P}\left(\frac{1}{1-\sigma}\left((1-b)\,C_t^{1-\nu} + be^{\varepsilon_t^M}\left(\frac{M_t}{P_t}\right)^{1-\nu}\right)^{\frac{1-\sigma}{1-\nu}} - \frac{\chi}{1+\eta}N_t^{1+\eta}\right) \tag{4.13}$$

where consumption, labor, money and bond holdings are chosen to maximize (4.12) subject to the same budget constraint and the same solvency condition as in the baseline model. This CES utility function depends positively on the consumption of goods, C_t, positively on real money balances, M_t/P_t, and negatively on labour N_t, as in the baseline model. ν is the inverse of the elasticity of money holdings with respect to the interest rate, and can be seen as a *non separability* parameter. b and χ are positive scale parameters. We use the same production function and Taylor rule as in the baseline model.

This New Keynesian DSGE model was developed in Chapter 3. As in the first model, it leads to six equations with six macro variables: flexible-price output ($\hat{y}_t^f$), flexible-price real money balances ($\widehat{mp}_t^f$), inflation ($\hat{\pi}_t$), output ($\hat{y}_t$), nominal interest rate ($\hat{\imath}_t$), and real money balances ($\widehat{mp}_t$), such as

$$\hat{y}_t^f = \upsilon_a^y \hat{a}_t + \upsilon_m^y \widehat{mp}_t^f - \upsilon_c^y + \upsilon_{sm}^y \varepsilon_t^M \tag{4.14}$$

$$\widehat{mp}_t^f = \upsilon_{y+1}^m E_t\left[\hat{y}_{t+1}^f\right] + \upsilon_y^m \hat{y}_t^f + \frac{1}{\nu}\varepsilon_t^M \tag{4.15}$$

$$\hat{\pi}_t = \beta E_t\left[\hat{\pi}_{t+1}\right] + \kappa_x\left(\hat{y}_t - \hat{y}_t^f\right) + \kappa_m\left(\widehat{mp}_t - \widehat{mp}_t^f\right) \tag{4.16}$$

$$\begin{aligned} \hat{y}_t \quad = \quad & E_t\left[\hat{y}_{t+1}\right] - \kappa_r\left(\hat{\imath}_t - E_t\left[\hat{\pi}_{t+1}\right]\right) + \kappa_{mp}E_t\left[\Delta\widehat{mp}_{t+1}\right] \\ & + \kappa_{sp}E_t\left[\Delta\varepsilon_{t+1}^P\right] + \kappa_{sm}E_t\left[\Delta\varepsilon_{t+1}^M\right] \end{aligned} \tag{4.17}$$

$$\widehat{mp}_t = \hat{y}_t - \kappa_i\hat{\imath}_t + \frac{1}{\nu}\varepsilon_t^M \tag{4.18}$$

$$\hat{\imath}_t = (1-\lambda_i)\left(\lambda_\pi\hat{\pi}_t + \lambda_x\left(\hat{y}_t - \hat{y}_t^f\right)\right) + \lambda_i\hat{\imath}_{t-1} + \varepsilon_t^i \tag{4.19}$$

where

$\upsilon_a^y = \frac{1+\eta}{(\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha}$

$\upsilon_m^y = \frac{(1-\alpha)(\nu-\sigma)(1-a_1)}{(\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha}$

$\upsilon_c^y = \log\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{1-\alpha}{(\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha}$

$\upsilon_{sm}^y = \frac{(\nu-\sigma)(1-a_1)(1-\alpha)}{((\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha)(1-\nu)}$

$\upsilon_{y+1}^m = -\frac{a_2}{\nu}\left(\nu-(\nu-\sigma)a_1\right)$

$\upsilon_y^m = 1+\frac{a_2}{\nu}\left(\nu-(\nu-\sigma)a_1\right)$

$\kappa_x = \left(\nu-(\nu-\sigma)a_1+\frac{\eta+\alpha}{1-\alpha}\right)(1-\alpha)(1-\beta\theta)\frac{(1-\theta)}{\theta(1-\alpha+\alpha\varepsilon)}$

$\kappa_m = (1-\alpha)(1-\beta\theta)(\sigma-\nu)(1-a_1)\frac{(1-\theta)}{\theta(1-\alpha+\alpha\varepsilon)}$

$\kappa_r = \frac{1}{\nu-a_1(\nu-\sigma)}$

$\kappa_{mp} = \frac{(\sigma-\nu)(1-a_1)}{\nu-a_1(\nu-\sigma)}$

$\kappa_{sp} = -\frac{1}{\nu-a_1(\nu-\sigma)}$

$\kappa_{sm} = -\frac{(1-a_1)(\nu-\sigma)}{(\nu-a_1(\nu-\sigma))(1-\nu)}$

$\kappa_i = \frac{a_2}{\nu}$

with $a_1 = \frac{1}{1+(b/(1-b))^{1/\nu}(1-\beta)^{(\nu-1)/\nu}}$ and $a_2 = \frac{1}{\exp(1/\beta)-1}$.

As we assume that households get utility from holding money, these two models include money in the utility function. The baseline model considers separability between consumption and real money balances, as is generally the case in the literature. In this case, real money balances are irrelevant in explaining the dynamics of the model, due to this separability condition. Consequently the money equation (4.10) becomes completely recursive from the rest of the system. In that case, money has no role to play in the equations explaining the other variables of the model (equations 4.6, 4.7, 4.8, 4.9 and 4.11).

The second model introduces non-separability between consumption and real money balances in order to analyse the situation where the marginal rate of substitution between current and future consumption depends on current and future real money balances. In that case money enters explicitly in the equations that determine output (current output and its flexible-price counterpart) and inflation (equations 4.14, 4.16 and 4.17). This results from the fact that consumption and money being linked in the agents utility function, money enters the equations where output appears, insofar $Y_t = C_t$ at equilibrium.

4.4 Empirical results

4.4.1 Data

To make output and real money balances stationary, we use first log differences, as in Adolfson and al. (2008). In our Eurozone model, $\hat{\pi}_t$ is the log-linearized inflation rate measured as the yearly log difference of GDP deflator from one quarter to the same quarter of the previous year, $\hat{y}_t$ is the log-linearized output measured as the yearly log difference of GDP from one quarter to the same quarter of the previous year, and $\hat{\imath}_t$ is the short-term (3-month) nominal interest rate. These data are extracted from the Euro area Wide Model database (AWM) of Fagan, Henry and Mestre (2001). $\widehat{mp}_t$ is the log-linearized real money balances measured as the yearly log difference of real money from one quarter to the same quarter of the previous year, where real money is measured as the log difference between the money stock and the GDP Deflator. We use the $M3$ monetary aggregate from the OECD database. $\hat{y}_t^f$, the flexible-price output, $\widehat{mp}_t^f$, the flexible-price real money balances, and $\hat{\imath}_t^f$, the flexible-price (natural) interest rate, are completely determined by structural shocks.

4.4.2 Bayesian estimations

We study three different crisis periods: 1990Q1 to 1993Q4, during the speculative attacks on currencies in the European Exchange Rate Mechanism (Black Wednesday crisis); 2000Q1 to 2003Q4, during the burst of the Dot-com bubble (Dot-com crisis); and 2006Q1 to 2009Q4, during the Subprime's crisis.

Calibration of the models is explained in Appendix 4.10.1 and all the marginal densities are presented in Appendix 4.10.2.

Each period is of 16 quarters. For every quarter of each period we run a Bayesian estimation by using the 25 observations before each respective quarter. We thus obtain 16 Bayesian estimations for each period of analysis.

Our purpose here is not to present all these results, a very cumbersome task indeed[2]. What is of interest is to draw from these estimations the evolution of the micro and macro parameters, the unconditional variance decomposition of variables with respect to shocks and the forecasting performance of the two models.

4.4.3 Methodology

The above estimations provide the values of micro and macro parameters through time. These parameters explain the dynamics of the different variables during the crises under consideration.

The estimated micro parameters on which we concentrate are the risk aversion coefficient, the Taylor rule coefficients, the measure of decreasing returns, and the probability of firms that reoptimize optimally their price every period. The other parameters are calibrated.

Several key macro-parameters of the non-separable model are also analyzed, such as: the parameter of the gap between real money balances and its flexible-price counterpart on inflation (κ_m); the expected real money growth shock parameter on output (κ_{sm}); the expected real money growth parameter on output (κ_{mp}); the flexible-price real money parameter on flexible-price output (υ_m^y); and the money shock parameter on flexible-price output (υ_{sm}^y).

Other common macro-parameter of the two models are analyzed, notably the real interest rate parameter on output (κ_r) and the technology shock parameter on flexible-price output (υ_a^y).

The successive estimations and simulations lead to variance decompositions of variables with respect to shocks. In order to study the role of each shock on the variance of the variables, we analyze the unconditional variance decomposition of output, inflation, interest rate and real money balances with respect to the preference shock (ε_t^P), the technology shock (ε_t^a), the money shock (ε_t^M) and the interest rate shock (ε_t^i). This analysis reveals the potential role of money and monetary policy, but also of technology and preferences on the dynamics of the

[2] All the results can however be provided upon request.

system.

Finally, after each estimation, we run out-of-sample (over four periods, i.e. one year) DSGE forecasts in order to compare the forecasting performance of the two models. To conduct these forecasting exercises, we simulate our estimated models from a given state and analyze the trajectories of the forecasted endogenous variables for the baseline model and the non-separable money model. These forecasting exercises are done following the Metropolis-Hastings iterations, on the basis of the posterior means of each estimated variables. The main objective of this exercise is to compare the forecasts to the actual data. Then, a comparison of the two models' forecasts is provided by the calculation of the Root Mean Square Deviation (RMSD).

To illustrate the prediction performance of our DSGE models, we perform sixteen out-of-sample prediction over the three crises.

In order to evaluate the forecasting performances, the models are re-estimated every quarter, by taking twenty five observations before a given quarter, and this for each of the sixteen quarters of a crisis period. For each four out-of-sample forecast, we calculate the sum of the corresponding RMSD values, and compare these values between the two models.

4.5 European Exchange Rate Mechanism crisis

4.5.1 Parameters analysis

The results of the Bayesian estimates are summarized in the following figures, where each date corresponds to the end of each estimation sample (of twenty five observations).

In all the figures, the dashed line refers to the non-separable model while the solid line refers to the baseline model.

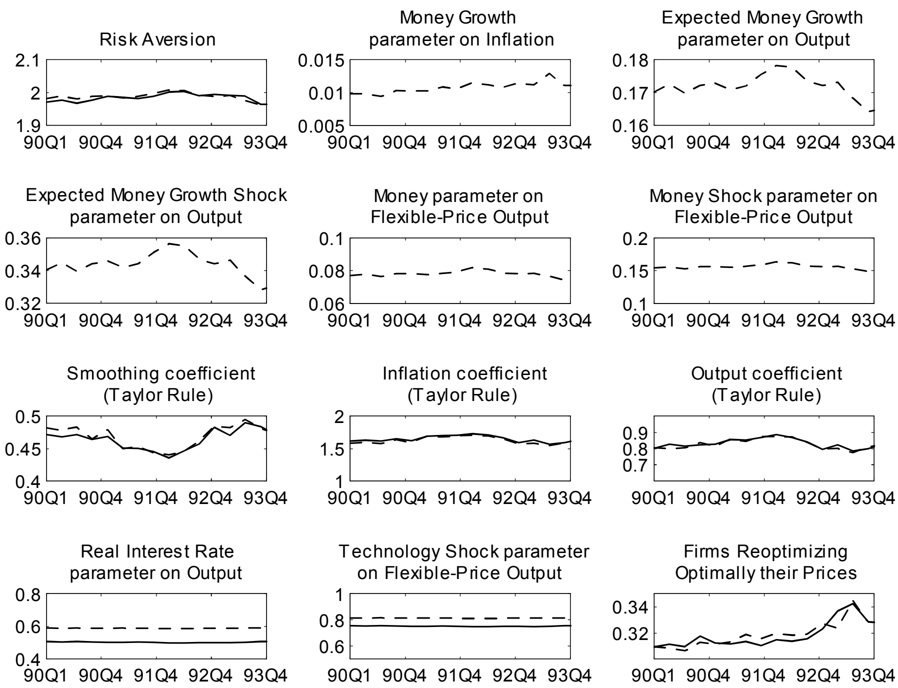

Figure 4.1: Parameters variations (1990Q1 to 1993Q4)

Figure 4.1 shows the evolution of micro and macro-parameters. Even though these values do not change much in absolute terms, such small variations may be enough to explain changes in the dynamic impact of shocks on variables and on the overall interdependent system of equations over time.

A closer look at the value of these parameters shows that risk aversion, the expected money growth parameter on output, the expected money growth shock parameter on output, and the inflation and output coefficients of the Taylor rule, display a small peak in 1992Q1.

Moreover, after the Black Wednesday crisis (i.e. after 1993Q1), more firms choose to reoptimize optimally their prices. During the period, the real interest rate parameter on output and the technology shock parameter on flexible-price output are rather constant.

There does not exist wide differences between both models as to the dynamics of the parameters estimations.

4.5.2 Variance decomposition

For each Bayesian estimation of the two models, we compute the unconditional variance decomposition of the variables. This variance decomposition gives interesting information as to the role of each shock.

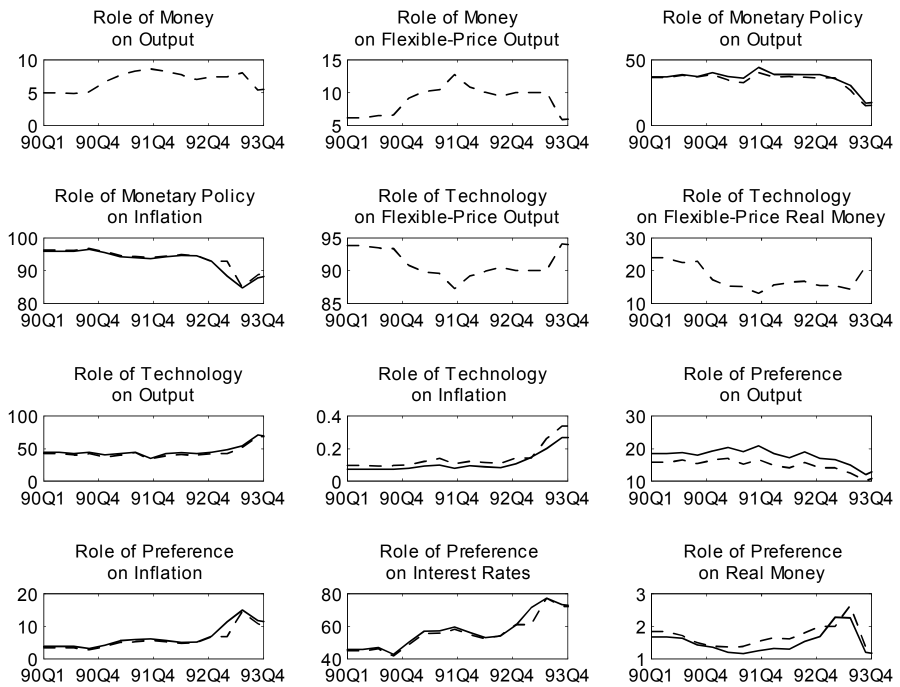

Figure 4.2: Variance decomposition in percent (1990Q1 to 1993Q4)

Figure 4.2 shows that money plays a non-negligible role in output variations during the European Monetary System crisis. This role reaches its maximum in 1991Q4, where about 10% of the output's variance is explained by the money shock. This result is mainly due to the variation in the expected money growth parameter and in the expected money growth shock parameter on output (see

Figure 4.1). After and before the crisis, money seems to play a less significant role. We don't present the role of money on inflation as far as it is almost nil (less than 0.1%).

Figure 4.2 also shows that the role of monetary policy on output and inflation decreases by the end of the crisis, and that technology plays a lower role during the crisis, especially as far as flexible-price output is concerned.

Shocks on output from money, monetary policy, technology and preferences explain 100% of the output variance. The role of technology increases after 1993Q1, at the same time as the role of monetary policy, of money and of preferences decrease.

The role of preferences on interest rates and inflation increases after the crisis (this role is negligible on real money balances) whereas it decreases on output.

4.5.3 Forecasting performances

From each Bayesian estimation, we simulate the out-of-sample forecasts of output and inflation over the next four periods (one year).

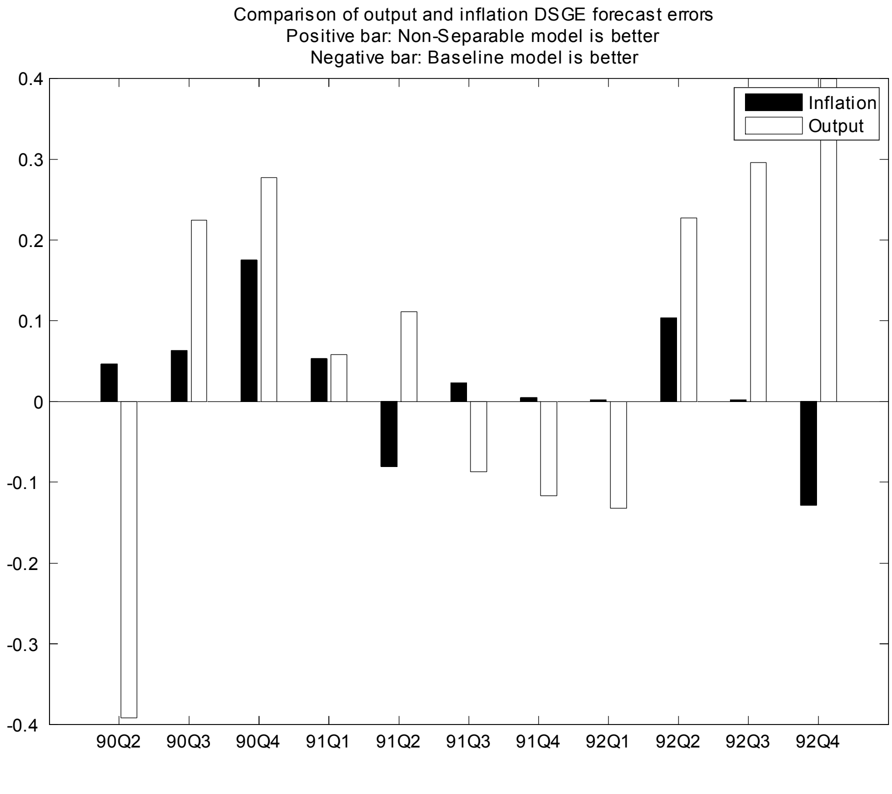

Figure 4.3: Out-of-sample forecasting errors (DSGE Forecast)

A negative number implies that the non-separable money model's RMSD is higher than the baseline model's RMSD. In that case the baseline model has better forecasting performances than the non-separable model. Figure 4.3 shows that the model with non-separability demonstrates a better predictive power for output dynamics than the baseline model between 1990Q2 and 1991Q2 and during the speculative attacks on currencies in the European Exchange Rate Mechanism between 1992Q2 and 1992Q4.

From 1990Q1 to 1991Q2, the non-separable model has a better predictive power of inflation dynamics whereas the performances are mixed during the other periods.

4.5.4 Interpretation

Black Wednesday refers to the events of 16 September 1992 when the British Conservative government withdrawed the pound sterling from the European Exchange Rate Mechanism. Yet other crises occurred during our period of analysis.

From 1990Q2 to 1991Q2, an oil crisis followed the first Gulf war[3]. From 1992Q2 to 1992Q4, the Russian crisis[4] and the French Real Estate crisis[5] could have affected the Eurozone business cycle. Figure 4.1 shows that during this period several parameters reach a peak.

Figure 4.2 shows that from 1990Q3 on the impact of money on (current and flexible-price) output increases and remains at a higher level until about the end of the period. This impact is higher than what Ireland (2004) and Andrès *et al.* (2006) found. The reason for this result seems mainly due to the variation of the expected money growth parameter and of the expected money growth shock parameter on output (Figure 4.1).

Figure 4.2 also indicates that, since 1992Q4, the beginning of the ERM crisis, the role of monetary policy on output and inflation has decreased and reached its lowest level at the top of the crisis (1993Q3).

The RMSD errors comparison (Figure 4.3) and the business cycles of the period are anticorrelated, showing that the non-separable model has a better predictive power of output than the baseline model during the lower part of the

[3]The 1990 oil price spike occurred in response to the Iraqi invasion of Kuwait on August 2, 1990. The war lasted until February 28, 1991.

[4]The constitutional crisis of 1993 was a political stand-off between the Russian president and the Russian parliament that was resolved by using military force.

[5]From 1992 to 1996, real estate prices declined up to 40%.

cycle. Moreover, during the Black Wednesday period (after 1992Q2), and during the other crises mentioned above (from 1990Q2 to 1991Q2), the non-separable model demonstrates a better predictive power of output than over the other periods.

These findings confirm the predictive abilities of the non-separable model during crisis periods, whereby the role of money on output increases.

4.6 Dot-com crisis

4.6.1 Parameters analysis

The following figure presents the Bayesian estimation results of micro and macro parameters through time during the Dot-com crisis (2000Q1 to 2003Q4).

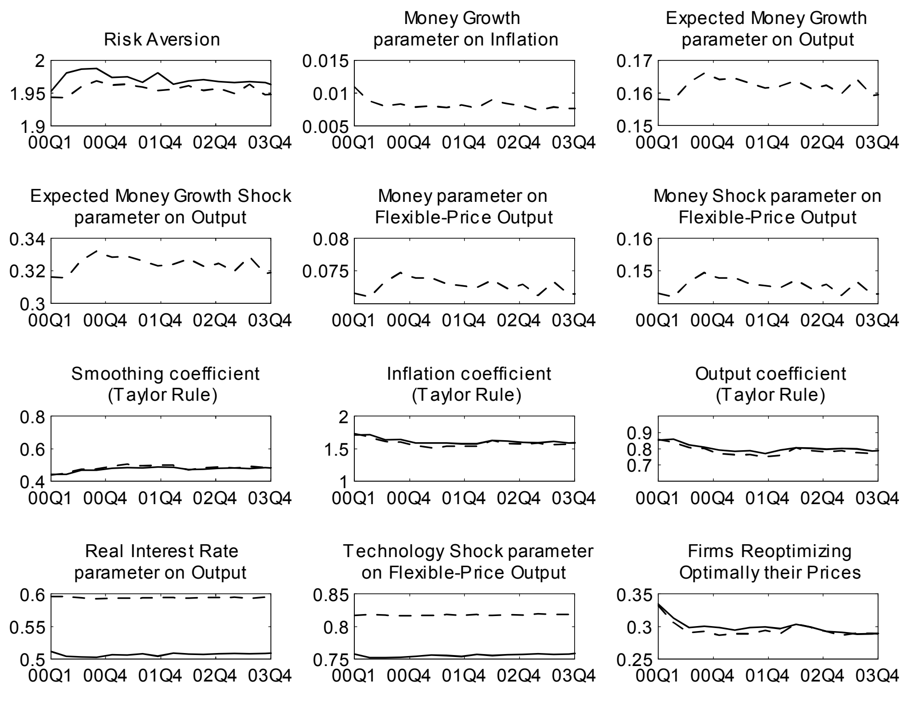

Figure 4.4: Parameters variations (2000Q1 to 2003Q4)

Figure 4.4 shows that if the risk aversion parameter does not change much over the period, at least in absolute terms, it gets to a peak whenever troubles happen: around 2000Q4 when the internet bubble started to burst and in 2001Q3 around the time of the 9/11 terrorist attacks.

In absolute terms the other parameters do not change much either even though the money related parameters on output (current and its flexible-price counterpart) show also a peak in 2000Q4 and remain at a somewhat higher level over the rest of the period.

4.6.2 Variance decomposition

The following figure presents the variance decomposition of variables through time during the Dot-com crisis (2000Q1 to 2003Q4).

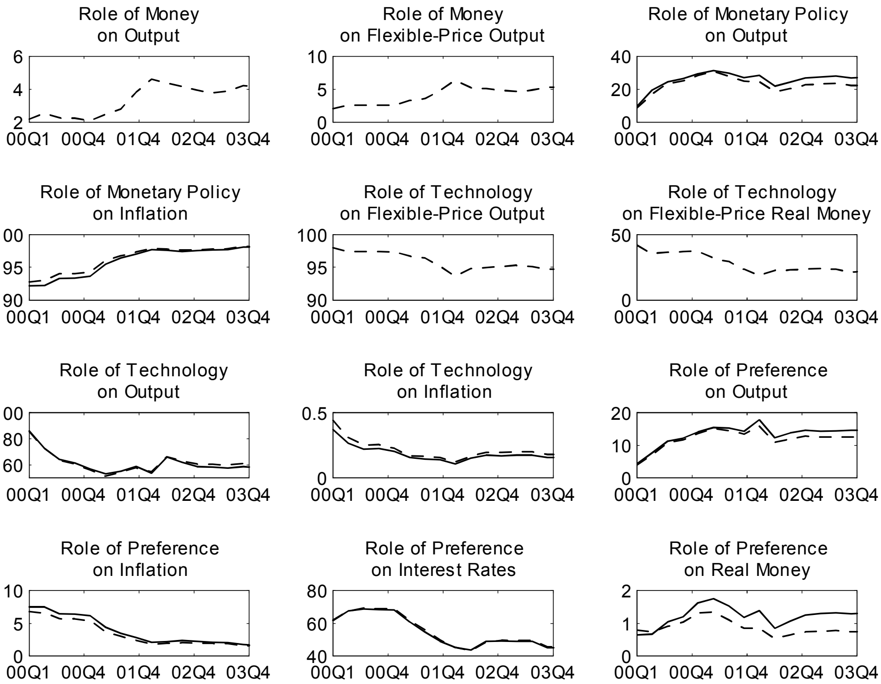

Figure 4.5: Variance decomposition in percent (2000Q1 to 2003Q4)

Figure 4.5 shows that since 2001Q2, the role of money on output and on flexible-price output has increased. However, this role is rather minor. As in the ERM crisis, the role of money on inflation is negligible (not presented).

After the bubble bursting, i.e. mainly after 2000Q4, the role of monetary policy on inflation increases whereas the change is not visible on output.

As in Figure 4.2, Figure 4.5 also shows that technology plays a lower role in explaining current and flexible-price outputs during the crisis.

4.6.3 Forecasting performances

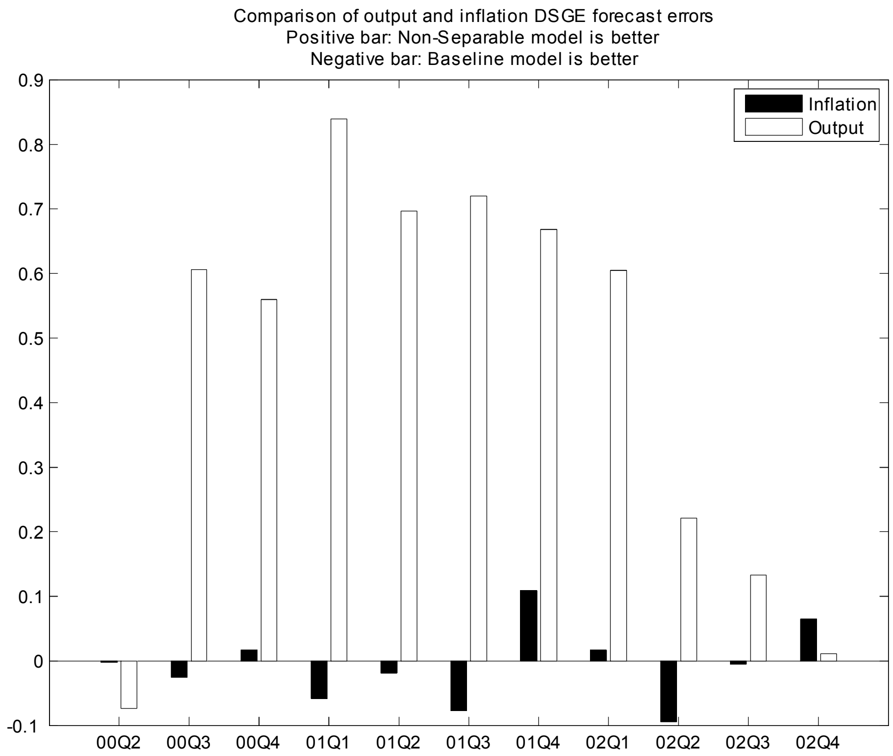

Figure 4.6: Out-of-sample forecasting errors (DSGE Forecast)

Figure 4.6 shows that the non-separable model is significantly better than the baseline model over the whole period (except in 2000Q2), in terms of predicting output. In terms of inflation, it is difficult to discriminate which model is the best (the RMSD differences of the inflation forecasts are rather small).

4.6.4 Interpretation

The bursting of the Dot-com bubble occurs in the Eurozone approximately two quarters (2000Q4) after the United States (2000Q2). Even if the role of money on output before 2001Q4 is small, it increases after this date. Between 2001Q1 and 2001Q4, the core of the Dot-com crisis, the role of technology on output reaches its minimum. This result confirms the decreasing role of technology on output dynamics during this crisis.

The percentage of the variance of output and flexible price-output explained by the money shock is small and close to the value found by Andrès *et al.* (2006).

Moreover, although the Taylor rule coefficients are rather constant (Figure 4.4), the role of monetary policy on output and inflation has increased since the beginning of the period (Figure 4.5).

The clear dominance of the non-separable model over the baseline model in terms of output forecasting errors confirms the predictive abilities of the non-separable model during crisis periods.

4.7 Subprime crisis

4.7.1 Parameters analysis

The following figures present the Bayesian estimation results through time over the Subprime crisis (2006Q1 to 2009Q4).

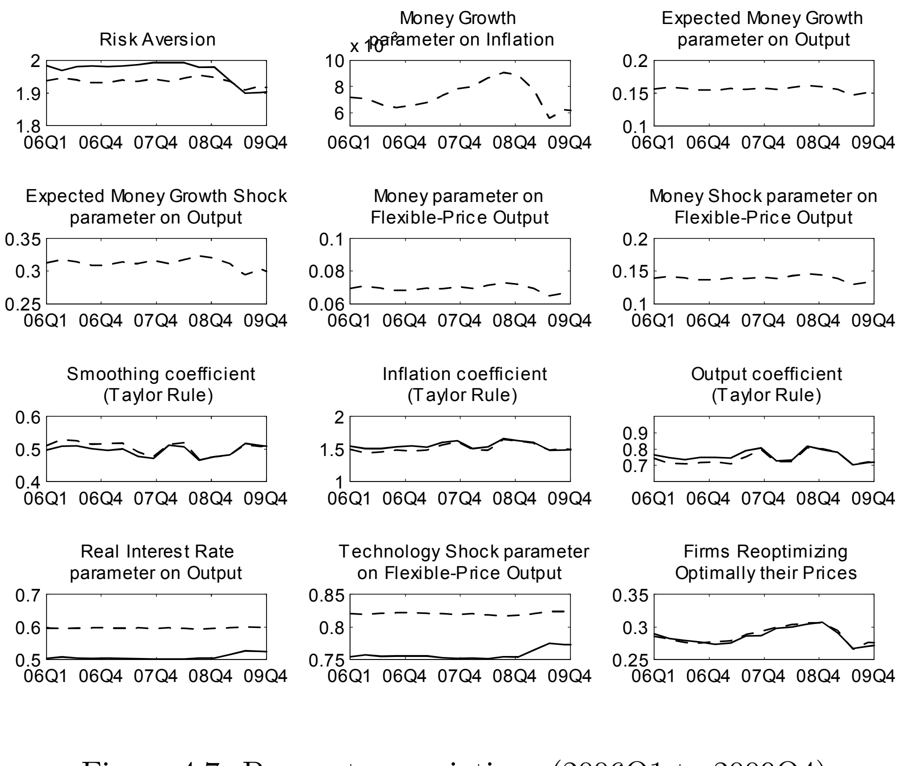

Figure 4.7: Parameters variations (2006Q1 to 2009Q4)

Figure 4.7 shows that after the Lehman Brother's bankruptcy (2008Q4), the risk aversion parameter decreases, whatever the model.

The expected money growth shock parameter on output reaches its maximum in 2008Q3 whereas the other parameters of output and flexible-price output remain rather stable during the period.

The money growth parameter on inflation also reaches its maximum in 2008Q3, but its variations are not large.

Even if the variations are small, the weights on inflation and on output in the Taylor rule reach a peak in 2007Q4 and in 2008Q4, while the smoothing parameter reaches on the contrary its lowest values.

4.7.2 Variance decomposition

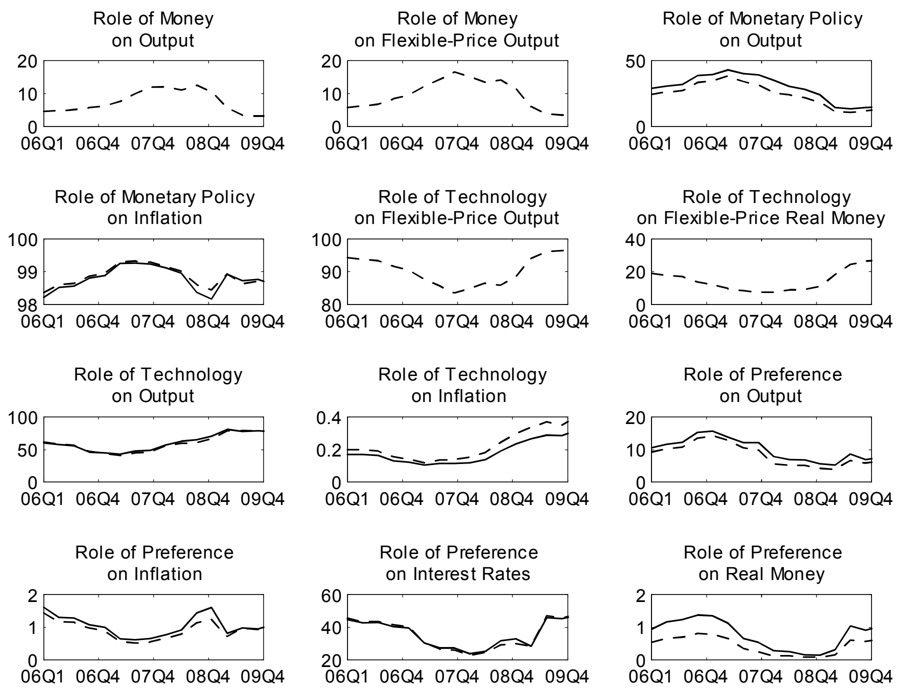

Figure 4.8: Variance decomposition in percent (2006Q1 to 2009Q4)

Figure 4.8 shows that the role of the money shock on output increases in 2007 and reaches a peak in 2008Q2. This shock explains around 5% of the variance in 2006Q4, whereas the percentage increases to 12% in 2008Q2, and goes back to 4% in 2009Q4. The impact of money on the flexible-price output follows about the same dynamic path.

As in the other crises, and because it is insignificant, we don't represent the role of money on inflation.

It is interesting to notice that the impact of monetary policy on output and inflation follows the same pattern as with the money shock, but it gets its higher level a little earlier (2007Q1). Monetary policy explains most of the inflation variance, with a maximum in 2007Q3.

Figure 4.8 also shows that the role of preferences on output, interest rates, and real money balances is lower during the crisis than before and after the crisis. Technology has a lower role in explaining flexible-price output variance at the beginning of the crisis than before and after the crisis. The role of technology on output and inflation grows up significantly after the crisis.

A simultaneous analysis of all the shocks indicates that the increasing role of money on output is associated with a decline in the role of monetary policy and of preferences. These declining impacts start at the beginning of the Subprime crisis.

4.7.3 Forecasting performances

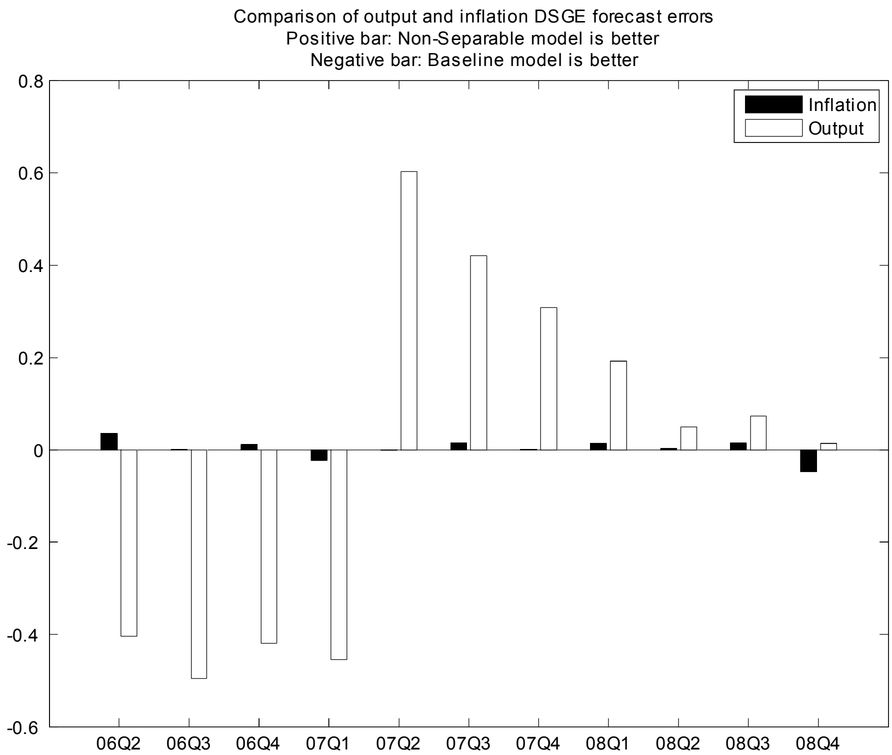

Figure 4.9: Out-of-sample forecasting errors (DSGE Forecast)

Figure 4.9 shows that the non-separable model provides better forecasts of output than the baseline model at the core of the financial crisis (2007Q2 to 2008Q3). The inflation RMSDs are about the same over the period.

4.7.4 Interpretation

The Subprime crisis can be attributed to a number of factors pervasive in both housing and credit markets, factors which emerged over a number of years. For Cecchetti (2008) and Mishkin (2010), a complete chronology of the crisis might starts in 2007Q1 when several large subprime mortgage lenders started to report losses. The real trigger of the crisis was in 2007Q3 when the French bank BNP Paribas temporarily halted redemptions from three of its funds that held assets backed by U.S. subprime mortgage debt.

As a direct consequence, credit spreads began widening, overnight interest rates in Europe shot up, and the European Central Bank immediately responded with the largest short-term liquidity injection in her nine year history. Furthermore, if the global financial crisis began in 2007 in the US, the Eurozone entered its first official recession in 2008Q3.

The Euro Group heads of states and governments and the European Central Bank (ECB) held an extraordinary summit in October 2008 to define a joint action for the Eurozone. They agreed on a bank rescue plan which would involve hundreds of billions of euros: governments would enter banks capital and guarantee interbank lending. That may explain the decrease in risk aversion after 2008Q4 (Figure 4.7), and the decreasing role of money on output variations after this date.

These results also suggests that at the top of the crisis, the role of money is at its highest. Contrary to other studies (Ireland, 2004; Andrès and al., 2006;), it shows that money had a significant role to play during the financial crisis.

To understand better the relationship between the role of money and financial risk, it is interesting to introduce the evolution of the interest rate spread over the period. This spread[6] provides an assessment of counterparty risk from one bank lending to another, reflecting both liquidity and credit risk concerns.

As Figure 4.10 shows, the dynamics of the role of money on output during the crisis is positively related to the widening of the spread between the Euribor

[6] The spread is measured as the difference between the 3-month Euribor and a short maturity bond. As an European bond does not exist, we choose the 3-month BTF (France) and the 1-Year Bubill (Germany) as short-term Treasury bills.

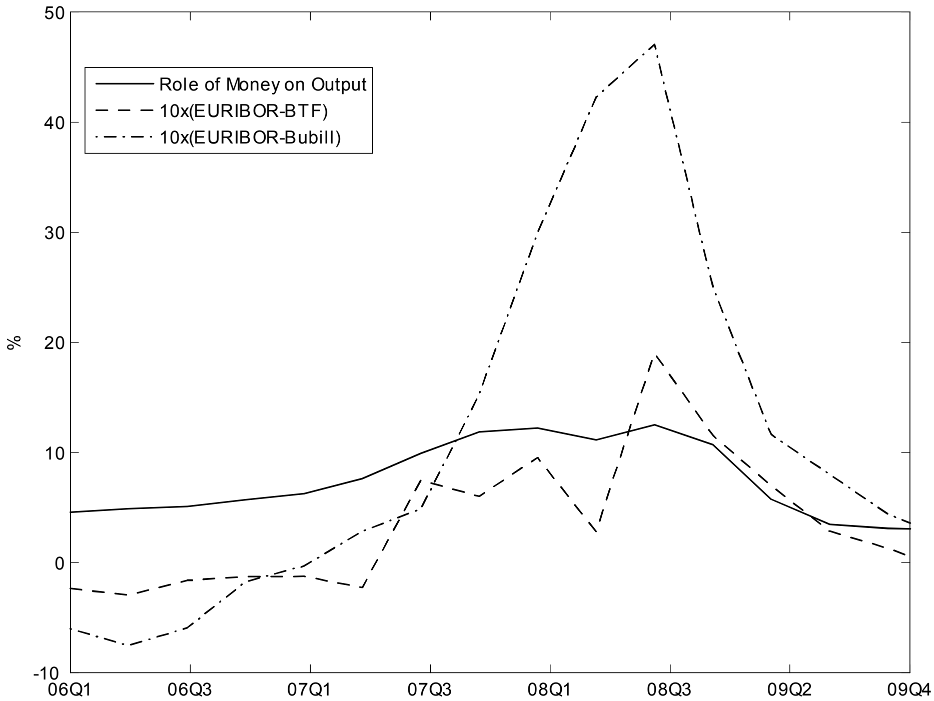

Figure 4.10: Comparison between the role of money on output (variance decomposition) and the spreads between the Bubill/BTF and the Euribor

and both baselines interest rates (Bubill and BTF). In the same vein, Figure 4.11 shows that the role of monetary policy decreases as the same time as the spread increases. If the role of monetary policy is at its maximum before the crisis starts (2007Q2), it diminishes quickly after: its impact on output is divided by four, declining from 40% to about 10%.

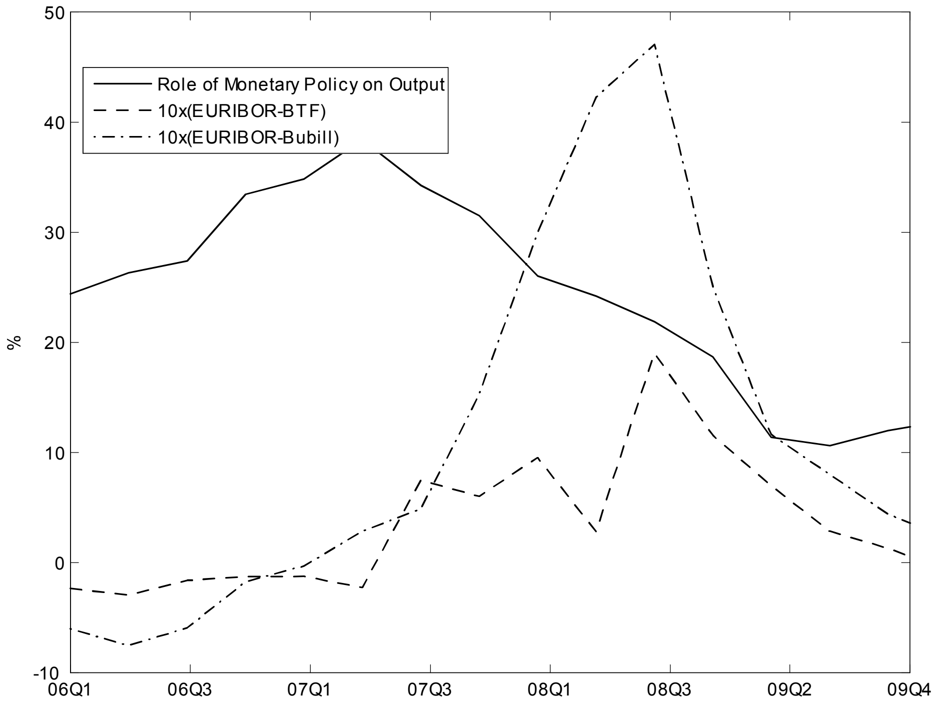

Figure 4.11: Comparison between the role of monetary policy on output and the spreads between Bubill/BTF and the Euribor

The RMSD analysis also reveals that the DSGE model with non-separable utility fares quite well against the baseline model after 2007Q2, that is after the beginning of the Global Financial Crisis (GFC). Given the stability of the macro parameters (Figure 4.7), it can be inferred that this increasing role is mainly due to micro parameters variations such as variations of the risk aversion parameter (σ) or of the percentage of firms reoptimizing their prices ($1-\theta$).

4.8 A comparison of the three crises

To better assess the relationship between money, monetary policy and output during the crises under consideration, and to better understand the respective role of the shocks, a comparison of variance decompositions between the different crises is useful. For the Subprime crisis and the European Exchange Rate Mechanism crisis, money plays a more significant role on output (more than 10%) than during the Dot-com crisis (less than 4%). These values must be compared to the literature (Ireland, 2004; Andrés, López-Salido and Vallés, 2006; Andrés, López-Salido and Nelson, 2009;), which found that money's role in the business cycle appears limited. Money plays a stronger role during the Subprime crisis (12.5%) than during the other crises. Similarly, money plays a stronger role on flexible-price output at the beginning of the Subprime crisis (16.5%) than during the other crises, where the money shock contribution to the flexible-price output variance is about 13% in 1992Q1 (beginning of the ERM crisis) and about 5% in 2002Q1 (Dot-com crisis).

The period where the role of technology on flexible-price output is at its minimum seems to be at the peak of the crisis, and this for the three crises. This result means probably that technology has a lower role to play in explaining flexible-price output during crisis than during more normal periods where flexible-price output variability is completely determined by the technology shock. The decline of the role of technology on flexible-price output is furthermore associated with a corresponding increase of the role of money on flexible-price output.

Preferences seems to play a role on all variables only during the Subprime crisis, and this for both models. The preference shock has a significant impact on output and interest rates before the Subprime crisis, yet this impact diminishes during the crisis (divided by almost two). This behavior may be due to the importance of the role of money, especially during the Global Financial Crisis.

It is also interesting to note that over the three crisis, the impact of monetary policy on output is different. It is around 35% at the top of the Dot-com crisis whereas it reaches almost 50% at the top of the ERM and of the Subprime crisis.

The impact of monetary policy on inflation is also different over the three crises. It appears to be lower during the ERM crisis and the Dot-com crisis than during the Subprime crisis.

Finally, in terms of forecasting, the non-separable model performs generally better than the separable one, as shown in Figure 4.3, 4.6 and 4.9.

4.9 Conclusion

The goal of this paper was to study the role of money and monetary policy during crises periods. To achieve this goal, we compared the performance of two DSGE models, one baseline model with separable preferences (Chapter 1) and one with non-separable preferences between consumption and real money balances (Chapter 3); the study is carried over three crisis periods: European ERM crisis (1992), Dot-com crisis (2001) and Subprime crisis (2007).

We tested the two models by using successive Bayesian estimations, so as to obtain empirical estimates of the evolution of parameters, variance decomposition and forecasting performances of both models over the three crises. Our analysis shows that the role of money on output variations increases during crises. Yet this role was higher during the ERM and the Subprime crises than during the Dot-com crisis. It also demonstrates that the model with non-separable preferences provides better forecasts of output than with the baseline model over these crisis periods.

Moreover, our results show that the impact of monetary policy on output variability diminishes significantly during the Subprime crisis, at the same time as the impact of money increases. Inflation does not seem to be affected directly by money variables, it is mainly explained by monetary policy over the three crises.

Our findings support the view that New Keynesian DSGE models with non-separability between consumption and real money balances should be preferred to separable models, as far as macroeconomic forecasting is concerned, at least during crisis periods.

Our results provide also interesting clues regarding the structural dynamics of the economy that may help inform central banks, markets and policy regulators. For example, the more significant role played by real money balances than generally expected during financial crises.

All in all, our analysis has highlighted the importance of money during crises, and showed that the hypothesis of non-separability between money and consumption leads to better forecast during these periods than when money and consumption are taken as separable.

4.10 Appendix

4.10.1 Calibration

We calibrate all parameters, excepted shocks' parameters (ρ_k and σ_k for $k = \{P, M, i, a\}$), the risk aversion parameter (σ), the price adjustment parameter (θ), the decreasing return parameter (α) of the production function and the Taylor rule's parameters (λ_i, λ_π, λ_x). The monetary policy rule is an ad-hoc reaction function and completely dependent on the monetary authority.

Following standard conventions, we calibrate beta distributions for parameters that fall between zero and one, inverted gamma distributions for parameters that need to be constrained to be greater than zero, and normal distributions in other cases.

The calibration of σ is inspired by Rabanal and Rubio-Ramírez (2007) and by Casares (2007). They choose, respectively, a risk aversion parameter of 2.5 and 1.5. In line with these values, we consider that $\sigma = 2$ corresponds to a standard risk aversion. We adopt the same priors in both models with the same risk aversion calibration.

As in Smets and Wouters (2003), the standard errors of the innovations are assumed to follow inverse gamma distributions and we choose a beta distribution for shock persistence parameters (as well as for the backward component of the Taylor rule) that should be lesser than one.

The calibration of α, β, θ, η, and ε comes from Galí (2007) and Casares (2007). The smoothed Taylor rule (λ_i, λ_π, and λ_x) is calibrated following Gerlach-Kristen (2003), analogue priors as those used by Smets and Wouters (2003). In order to take into consideration possible changes in the behavior of the central bank, we assign a higher standard error for the Taylor rule's coefficients. υ (the non-separability parameter) must be greater than one. κ_i (equation 4.18) must be greater than one as far as this parameter depends on the elasticity of substitution of money with respect to the cost of holding money balances, as explained in Söderström (2005); while still informative, this prior distribution is dispersed enough to allow for a wide range of possible and realistic values to be considered (i.e. $\sigma > \upsilon > 1$).

The calibration of the shock persistence parameters and the standard errors of the innovations follows Fève *et al.* (2010), where a much lower mean is adopted for ρ_a. All the standard errors of shocks are assumed to be distributed according to inverted Gamma distributions, with prior means of 0.02. The latter law ensures that these parameters have a positive support. The autoregressive parameters are all assumed to follow Beta distributions. Except for technology shocks, all these distributions are centered around 0.75. We take a common standard error of 0.1 for the shock persistence parameters, as in Smets and Wouters (2003).

Priors			
	Law	Mean	Std.
α	beta	0.33	0.05
θ	beta	0.66	0.05
σ	normal	2.00	0.10
β	calibrated	0.99	
υ	calibrated	1.50	
b	calibrated	0.25	
χ	calibrated	0.05	
γ	calibrated	0.05	
η	calibrated	1.00	
ε	calibrated	6.00	
λ_i	beta	0.50	0.10
λ_π	normal	3.00	0.20
λ_x	normal	1.50	0.20
ρ_a	beta	0.75	0.10
ρ_i	beta	0.75	0.10
ρ_P	beta	0.75	0.10
ρ_M	beta	0.75	0.10
σ_a	invgamma	0.02	2.00
σ_i	invgamma	0.02	2.00
σ_P	invgamma	0.02	2.00
σ_M	invgamma	0.02	2.00

Table 4.1: Calibration for the two models

4.10.2 Marginal densities

In what follows, we present all the marginal densities of our estimates. The dashed line refers to the non-separable model while the solid line refer to the baseline model.

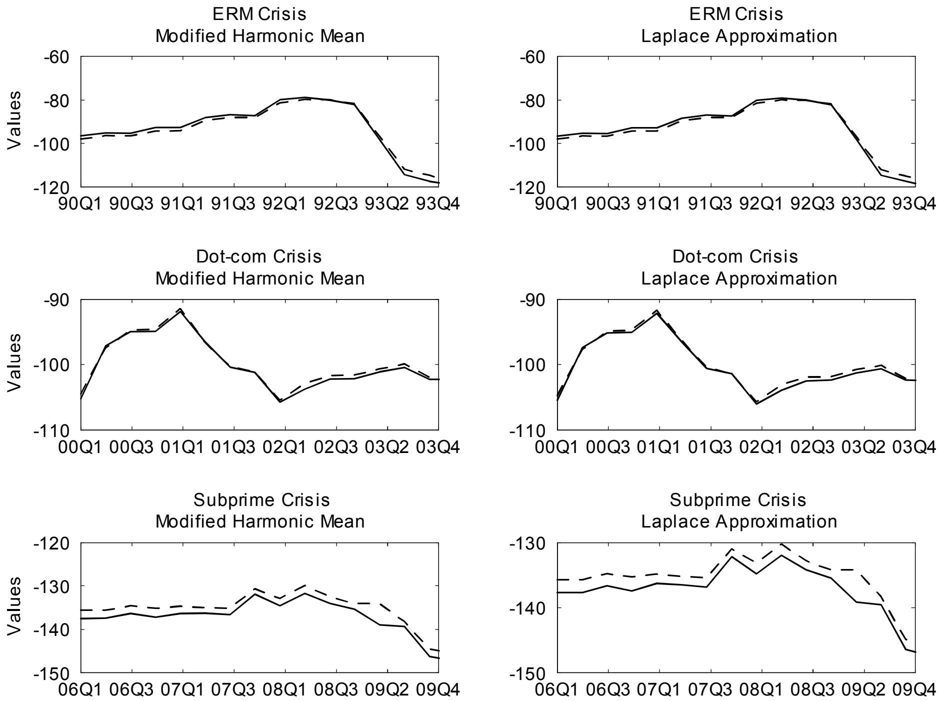

Figure 4.12: Log-marginal densities

For the three crises, both models have approximately the same log marginal densities.

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Conclusion générale

Dans cette thèse, nous avons analysé l'influence de l'aversion au risque et de la monnaie dans les dynamiques macroéconomiques.

Dans un premier chapitre, nous avons présenté et testé un modèle de base inspiré de Galí (2007) et Smets et Wouters (2007). Nous avons comparé celui-ci à un modèle mettant en évidence un choc d'aversion au risque. Ce dernier modèle montre l'implication significative du choc d'aversion au risque dans les dynamiques macroéconomiques. Un choc positif d'aversion au risque augmente l'inflation, réduit la production et diminue le rôle de la politique monétaire sur la production, par comparaison au modèle de base. L'aversion au risque joue également un rôle significatif dans la variance de la production, des encaisses réelles, de la production en prix flexibles ainsi que sur le taux d'intérêt en prix flexibles. Le rôle négatif joué par l'aversion au risque sur la production permet d'identifier clairement les périodes de crise.

Ce modèle de base augmenté d'un choc d'aversion au risque montre l'importance d'un tel paramètre sur l'économie, et plus particulièrement sur l'influence de la politique monétaire. Et il montre comment il est important de le contrôler, par la voie de la communication par exemple.

Dans un deuxième chapitre, nous avons approché un des sujets les plus controversées de la littérature économique d'après-guerre : la monnaie comme facteur de production. La question est de savoir si la monnaie doit être prise en compte ou non dans le processus de production.

Toutefois, les récents développements de l'analyse des Nouveaux Keynésiens DSGE, et les techniques d'estimation Bayésienne, poussent à un réexamen du rôle de la monnaie dans la fonction de production.

Après avoir construit un modèle original inspiré de ces développements, et développé les équations macroéconomiques microfondées qui en découlent, il en résulte une équation d'inflation inédite : l'inflation est expliquée par l'inflation anticipé, le taux d'intérêt réel et la différence entre les encaisses réelles et les encaisses réelle en prix flexibles.

De plus, en dépit de l'inclusion de la monnaie dans la fonction de production, cette dernière variable ne joue pas un rôle significatif dans les dynamiques du système. La seule façon d'attribuer un rôle à la monnaie dans ces dynamiques est de supposer des rendements d'échelle constants entre les facteurs de production (monnaie et travail), ce qui est une hypothèse forte et très controversée dans la littérature.

D'ailleurs, nous confirmons que le modèle avec rendements d'échelle décroissants est meilleur que le modèle avec rendements d'échelle constants en termes de densité marginale de vraisemblance. Avec des rendements d'échelle décroissants, les encaisses réelles ne jouent pas de rôle significatif dans les dynamiques macroéconomiques. L'introduction de la monnaie ne confère pas nécessairement un rôle significatif à celle-ci.

Dans un troisième chapitre, nous avons construit et testé empiriquement un modèle avec différents niveaux d'aversion au risque, en mettant un accent particulier sur le rôle de la monnaie. Le modèle suit le cadre Nouveau Keynésien DSGE, avec la monnaie dans la fonction d'utilité où les encaisses réelles affectent l'utilité marginale de la consommation.

En utilisant des techniques d'estimation Bayésienne, nous mettons en lumière les déterminants de la production et de la dynamique d'inflation, mais aussi des taux d'intérêt, des encaisses réelles, de la production en prix flexibles et des encaisses réelle en prix flexible. Nous avons également étudié la façon dont les résultats sont affectés lorsque l'aversion au risque change. Avec un niveau d'aversion au risque "classique", plus de la moitié de la variance de la production s'explique par le choc technologique, le reste par une combinaison de chocs de préférences et de chocs de politique monétaire (ce qui est conforme à la théorie des cycles réels).

Une première calibration du modèle avec une aversion au risque standard montre que la monnaie ne joue presque pas de rôle dans l'explication de la variabilité de la production, un résultat en ligne avec la littérature actuelle (Andres et al, 2006; Ireland, 2004). Une autre calibration avec un niveau d'aversion au risque plus grand confère à la monnaie un rôle non négligeable dans l'explication de la production et des fluctuations de la production en prix flexibles. Ce résultat diffère de la littérature existante sur le rôle de la monnaie dans la mesure où cette dernière néglige l'impact d'un facteur de risque assez élevé.

D'autre part, bien que la monnaie apparaisse explicitement dans l'équation de l'inflation, la monnaie n'a pas de rôle notable dans l'explication directe de la variabilité de l'inflation. L'inflation étant déterminée essentiellement par le choc de politique monétaire.

Un autre résultat concerne la politique monétaire. Plus l'aversion au risque augmente et plus le lissage du taux d'intérêt augmente. Ceci reflète probablement l'objectif des banquiers centraux de ne pas agiter les marchés.

D'après ces résultats, on peut en déduire que, en changeant la perception des agents économiques pour le risque, la dernière crise financière pourrait avoir accru le rôle des encaisses réelles dans la dynamique de la production.

Dans un quatrième et dernier article, nous avons étudié le rôle de la politique monétaire et de la monnaie pendant des périodes de crise. Pour atteindre cet objectif, nous avons comparé les performances des deux modèles DSGE développés plus haut : le modèle de base avec des préférences séparables (Chapitre 1) et le modèle avec des préférences non séparables entre la consommation et les encaisses réelles (Chapitre 3). Cette étude est réalisée sur trois périodes de crise: la crise du système monétaire européen (1992), la crise des High tech (2001) et la crise des Subprimes (2007).

Nous avons testé ces deux modèles en utilisant des estimations Bayésiennes récursives sur de petits échantillons afin d'obtenir des estimations empiriques de l'évolution des paramètres, de l'évolution des décompositions de la variance des variables en différents chocs, et des performances prédictives hors échantillon des deux modèles. Notre analyse montre que le rôle de la monnaie sur les variations de la production augmente pendant les crises. Pourtant, ce rôle a été plus élevé pendant la crise du SME et des Subprimes que pendant la crise des valeurs internet. Nous démontrons également que le modèle avec des préférences non séparables fournit de meilleures prévisions de la production que le modèle de référence au cours de ces périodes de crise.

Par ailleurs, nos résultats montrent que l'implication de la politique monétaire sur la variabilité de la production diminue de manière significative pendant la crise des Subprimes. Nous montrons que les modèles Nouveaux Keynésiens DSGE avec des préférences non-séparables entre la consommation et les encaisses réelles devraient être préférés aux modèles séparables, en ce qui concerne les prévisions macro-économiques, du moins pendant les périodes de crise.

Nos résultats fournissent également des indications intéressantes quant à la dynamique structurelle de l'économie qui peuvent aider à informer les banques centrales, les marchés et les régulateurs. Par exemple, le rôle plus important joué par les encaisses réelles en période de crise plutôt qu'entre les crises.

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