

# INFLUENCE OF MECHANICAL AND GEOMETRICAL PARAMETERS ON THE STATIC BEHAVIOR OF A VIOLIN BOW IN PLAYING SITUATION

**Frédéric Ablitzer**

Laboratoire d'Acoustique de l'Université du Maine – UMR CNRS 6613

**PhD defence**

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Examining committee

**A. Askenfelt** | KTH, Stockholm (Examiner)  
**R. Caussé** | IRCAM, Paris (Reviewer)  
**A. Chaigne** | ENSTA ParisTech, Palaiseau (Chairman)  
**G. Chevallier** | SUPMÉCA, Saint-Ouen (Examiner)

**B. Cochelin** | LMA, Marseille (Reviewer)  
**J.P. Dalmont** | LAUM, Le Mans (Supervisor)  
**N. Dauchez** | SUPMÉCA, Saint-Ouen (Supervisor)  
**N. Poidevin** | Bow maker, Dinan (Invited)

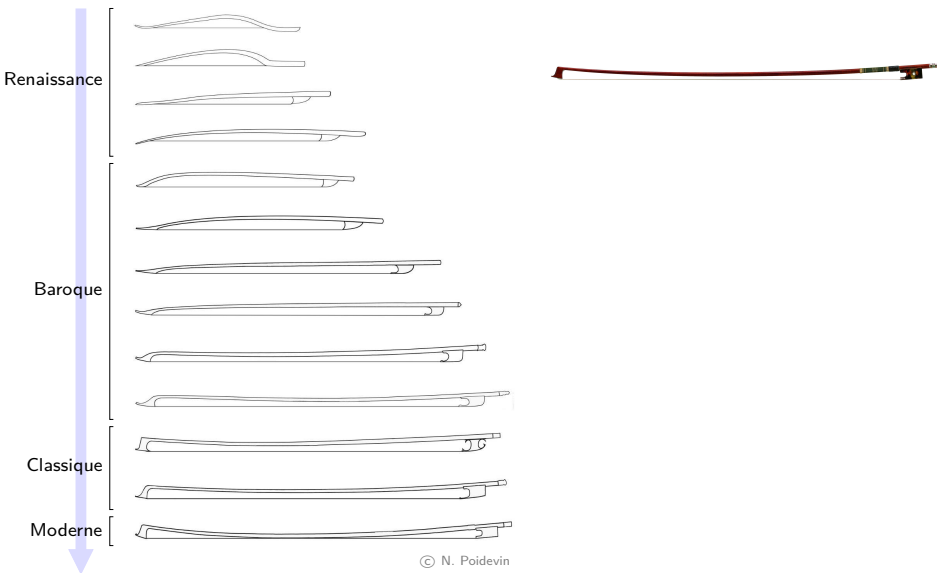


**Paganini's 24th Caprice (1819)**  
played by Alexander Markov

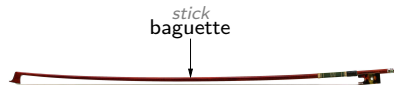
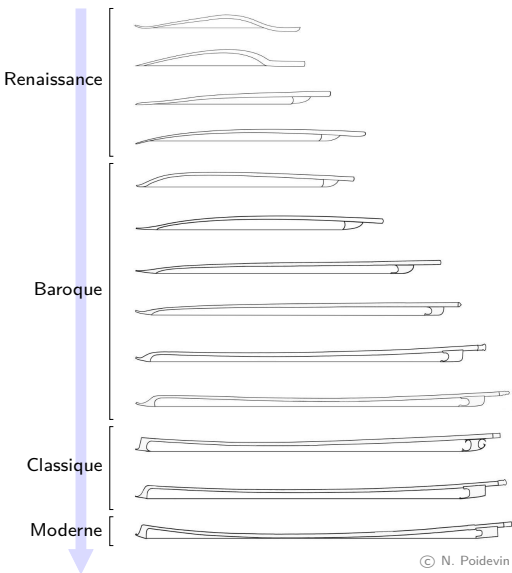


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# Evolution of the bow

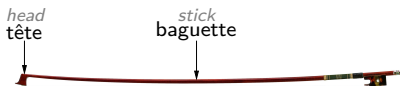
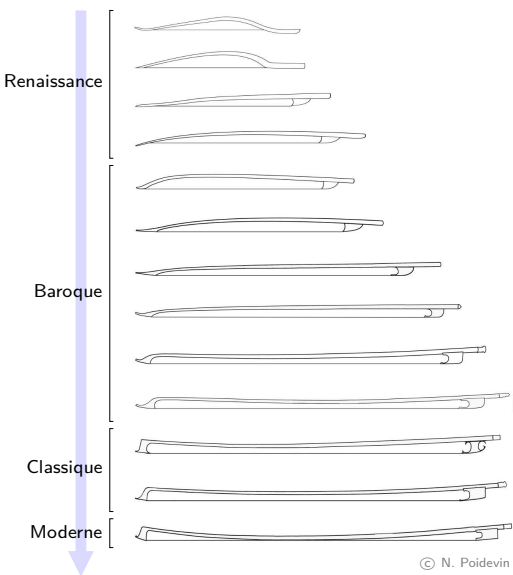


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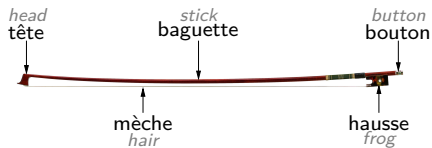
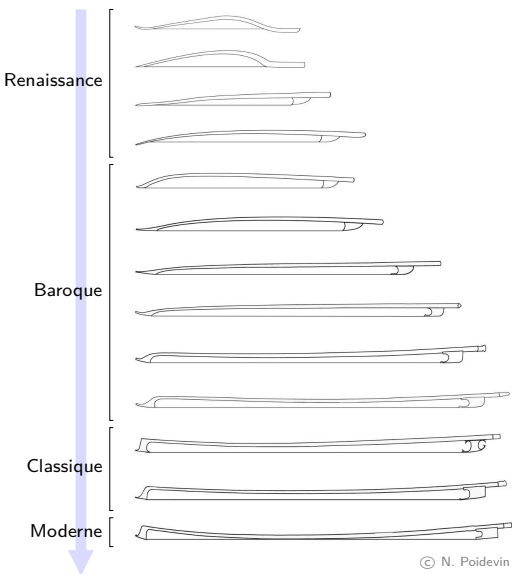
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# Evolution of the bow



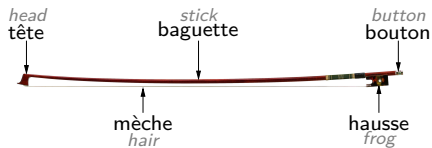
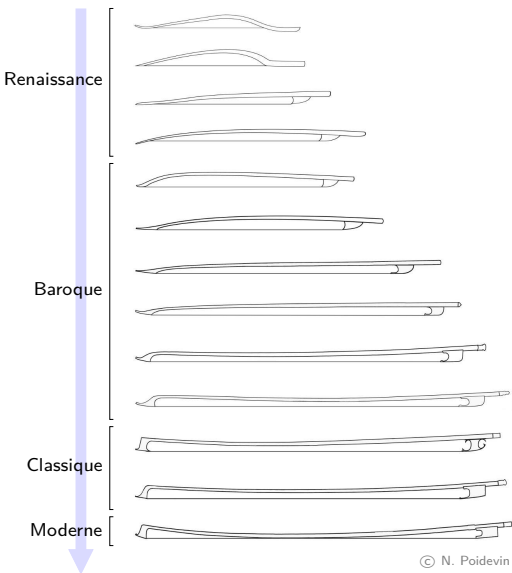
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# Evolution of the bow



- lengthening of the stick
- development of a head
- mechanism to adjust hair tension

# Evolution of the bow



- lengthening of the stick
- development of a head
- mechanism to adjust hair tension
- inversion of the curvature



# The modern bow

Almost the same bow for 200 years



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François-Xavier Tourte  
(1747-1835)

# The modern bow

Almost the same bow for 200 years



François-Xavier Tourte  
(1747-1835)

- Pernambuco wood
- standardized design

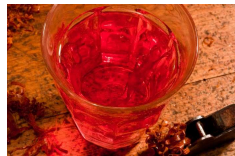
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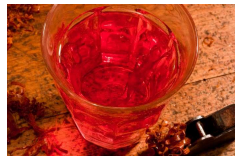
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**An achieved compromise**

# Why study the bow?

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# Why study the bow?

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- Questions from bow makers about the physics behind the bow during “Journées Façture Instrumentale et Sciences” (ITEMM, Le Mans)
- Pernambuco listed as endangered species since 2007 in CITES, Appendix II
- Supply makers with dedicated characterization and simulation tools within the project PAFI supported by ANR (2009-2012) (“Plateforme d’Aide à la Façture Instrumentale”)



# 3 points of view

# 3 points of view

## The player



What does he need?

# 3 points of view

## The player



What does he need?

## The bow maker



How does he meet  
the player's demand?

# 3 points of view

## The player



What does he need?

## The bow maker



How does he meet  
the player's demand?

## The scientist



How can he help  
the maker?

# Player's point of view

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- **playability**  
= allows to play a variety of bow strokes





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- **tonal qualities**  
= allows to achieve a good tone



# Player's point of view

What does the player need?

- **playability**  
= allows to play a variety of bow strokes
- **tonal qualities**  
= allows to achieve a good tone
- **aesthetics**
- **price**
- ...



# Bow maker's point of view

How does the maker meet the player's demand?

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**wood**



- density
- elasticity
- damping

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**wood**



- density
- elasticity
- damping

**tapering**



distribution of mass and stiffness

# Bow maker's point of view

How does the maker meet the player's demand?

## wood



- density
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## tapering



↓  
distribution of mass and stiffness

## camber



↓  
adjustment of playing  
and tonal qualities

# Bow maker's point of view

Making and adjustment mainly empirical...

# Bow maker's point of view

Making and adjustment mainly empirical...

...sometimes combined with a scientific approach



measuring stiffness



*Lucchimeter*



*Lutherie Tools*



# Scientist's point of view

**How to characterize or model a bow?**

# Scientist's point of view

## How to characterize or model a bow?

For the acoustician: bow = vibrating structure

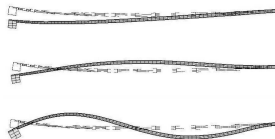
# Scientist's point of view

## How to characterize or model a bow?

For the acoustician: bow = vibrating structure

- eigenmodes  
(modal analysis, FE model)

[*Bissinger 1993, Caussé et al. 2001,*  
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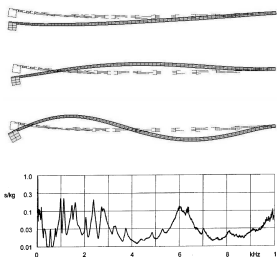


# Scientist's point of view

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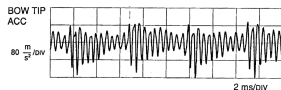
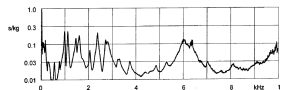
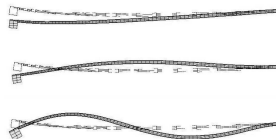


# Scientist's point of view

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- admittance presented to the string  
*[Schumacher 1975, Askenfelt 1995]*
- vibrations during playing  
*[Askenfelt 1993]*



→ may help to better understand and model the bow/string interaction

however, difficult to relate to **bow maker's adjustment** and **player's perception**

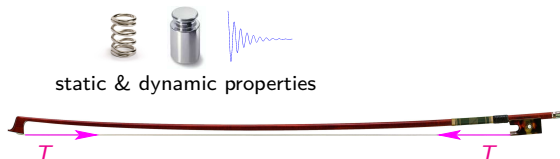
# Axes of investigation



static & dynamic properties

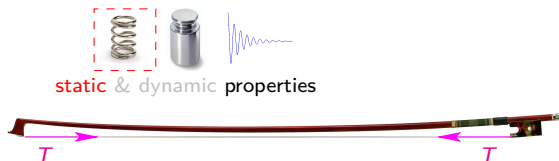


# Axes of investigation



function of the bow:  
**maintain the hair under tension**

# Axes of investigation

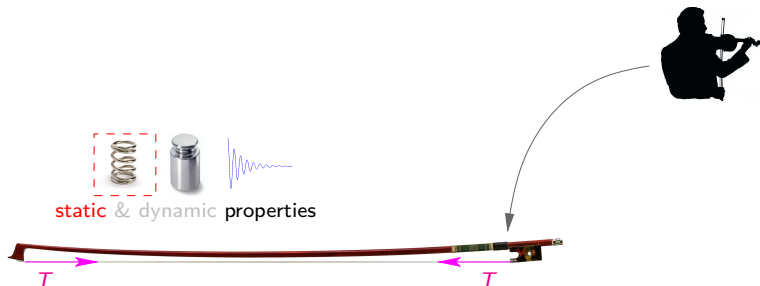


function of the bow:  
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bow = prestressed structure  
→ consequences on the behavior?



# Axes of investigation

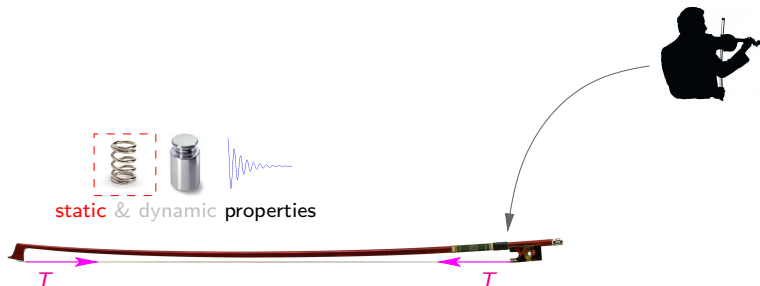


function of the bow:  
**maintain the hair under tension**

- **resist** to player's action

bow = **prestressed structure**  
 → **consequences on the behavior?**

# Axes of investigation



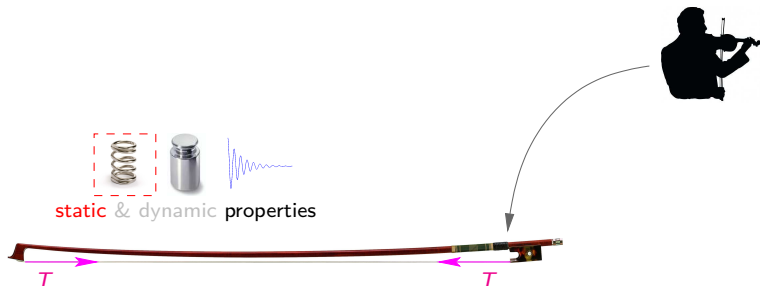
function of the bow:  
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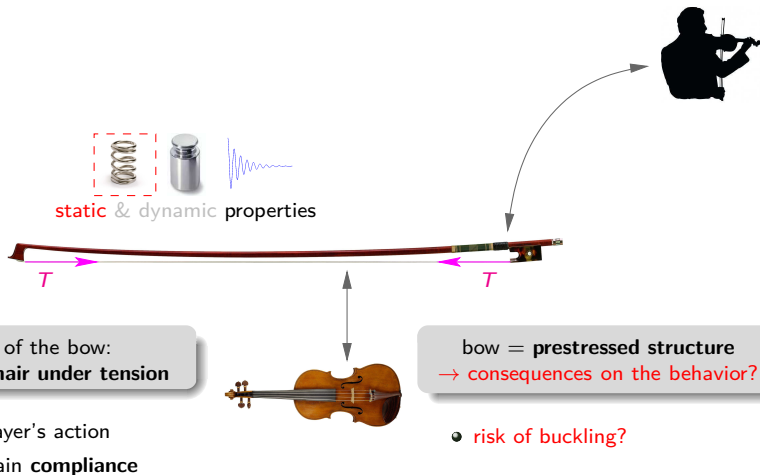
function of the bow:  
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- resist to player's action
- offer a certain **compliance**

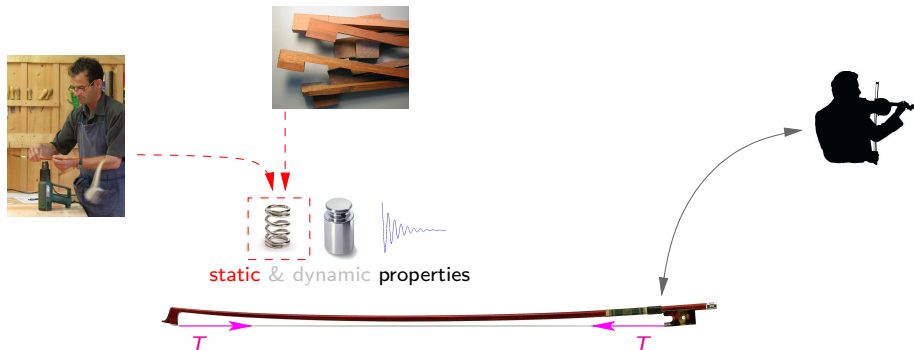
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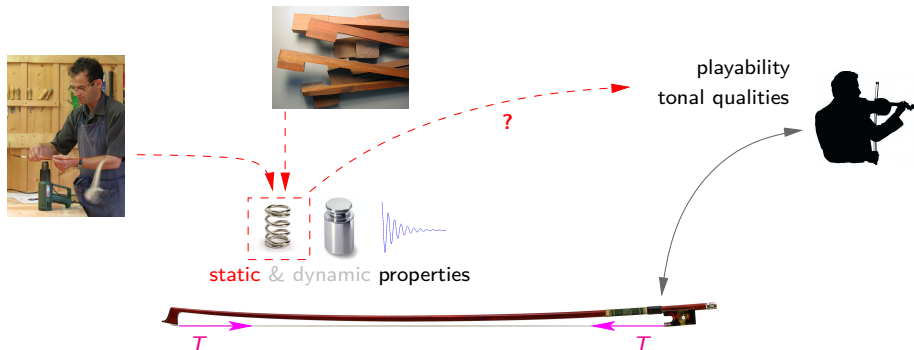
function of the bow:  
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- resist to player's action
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bow = **prestressed structure**  
 → **consequences on the behavior?**

- **risk of buckling?**
- **how to control compliance?**

# Axes of investigation



function of the bow:  
maintain the hair under tension

- resist to player's action
- offer a certain **compliance**

bow = **prestressed structure**  
→ consequences on the behavior?

- risk of buckling?
- how to control compliance?

## Introduction

- 1 Modelling
- 2 Experimental characterization
- 3 Results
  - Static behavior
  - Stability
- 4 Playing tests

## Conclusion

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### 2 Experimental characterization

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# Modelling



bow without hair tension

# Modelling



bow without hair tension

## Assumptions

- stick = Euler-Bernoulli beam

# Modelling

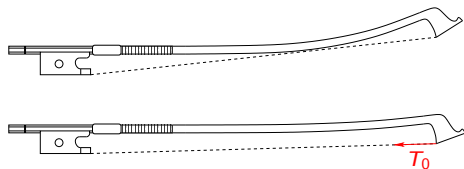


bow without hair tension

## Assumptions

- stick = Euler-Bernoulli beam
- stick oriented along the grain of the wood: longitudinal Young's modulus is considered

# Modelling



bow without hair tension

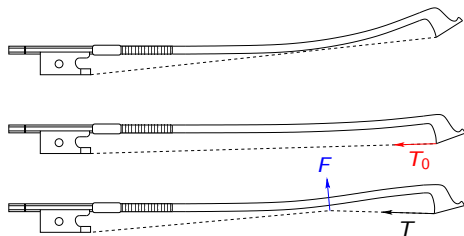


tightened at playing tension  $T_0$   
**prestressed state**

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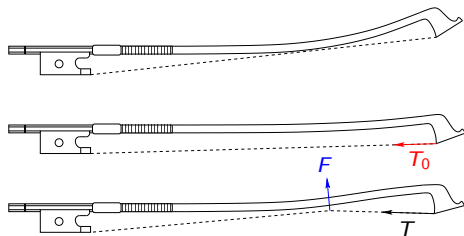


loaded by a force  $F$  on the hair  
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bow without hair tension



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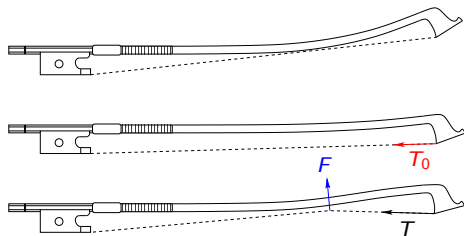


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# Modelling



bow without hair tension



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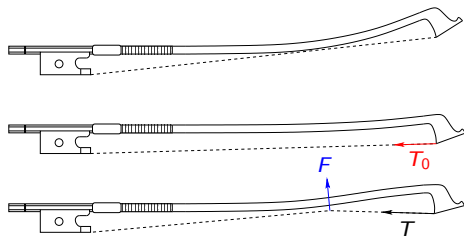


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- hair has longitudinal stiffness
- material is elastic

# Modelling



bow without hair tension

↓ (i)

tightened at playing tension  $T_0$   
prestressed state

↓ (ii)

loaded by a force  $F$  on the hair  
playing situation

## Assumptions

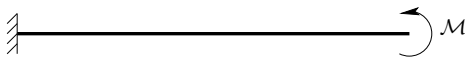
- stick = Euler-Bernoulli beam
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- (i) and (ii) are large transformations → **geometric non-linear model**



# Corotational approach: Illustration

Cantilever beam subject to end moment

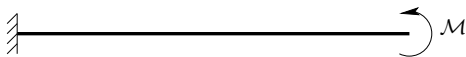
$$\mathcal{M} = \frac{2\pi EI}{L}$$



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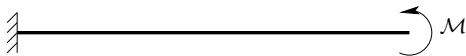
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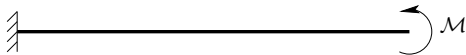


Local deformation (**small**)

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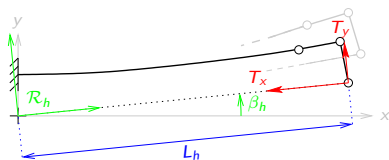
Rigid body-motion (**large**)

Local deformation (**small**)

## 2D model

## Finite element model of the stick

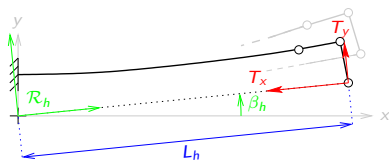
- 2D Euler-Bernoulli beam elements, corotational formulation
- external load : force  $\mathbf{T} = [T_x \quad T_y]^T$ 
  - follower force
  - amplitude depends on displacements



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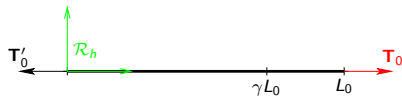
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## Model of the hair

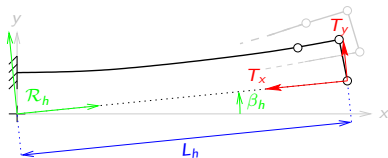
- equivalent single hair
- compliance per unit length  $c_h$
- relationship between  $\mathbf{T}$  and playing force  $F_y$  at relative abscissa  $\gamma$ 
  - $T_y = \gamma F_y$
  - $f(T_x, F_y, L_h, \dots) = 0$



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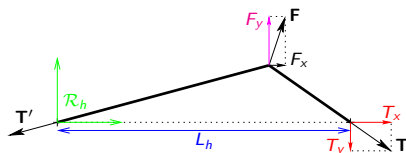
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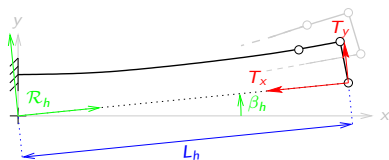
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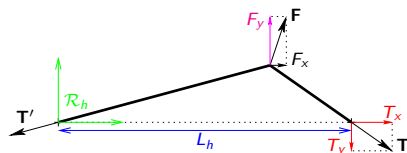
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$$\mathbf{K}(\mathbf{u})\mathbf{u} = \mathbf{f}(\mathbf{u})$$



# 3D model

**Why a 3D model?**

# 3D model

## Why a 3D model?

- player frequently tilts the bow  
→ lateral bending of the stick during playing

# 3D model

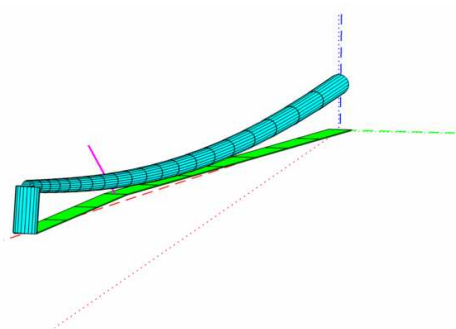
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- bow maker adjusts the lateral compliance of the bow  
(tapering, camber)

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- stick
  - hair
- } 3D Euler-Bernoulli beam
- corotational formulation

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# Measurement of bow shape

**Method to determine the shape of the bow in a given state**

Example: determination of camber



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**Method to determine the shape of the bow in a given state**

Example: determination of camber

- picture of the bow

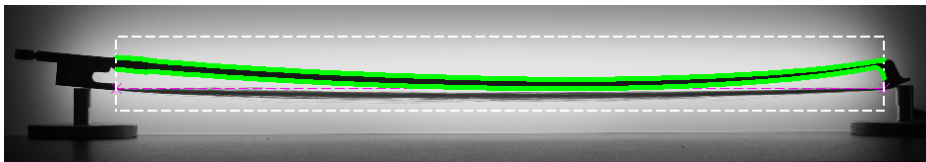


# Measurement of bow shape

**Method to determine the shape of the bow in a given state**

Example: determination of camber

- picture of the bow
- detect lower and upper edges along the bow



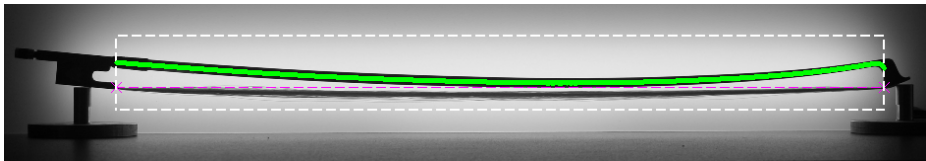


# Measurement of bow shape

## Method to determine the shape of the bow in a given state

Example: determination of camber

- picture of the bow
- detect lower and upper edges along the bow
- approximate neutral curve with polynom of appropriate order



# Determination of bow properties: Step 1

## Procedure in 4 steps

### ① Geometry



# Determination of bow properties: Step 1

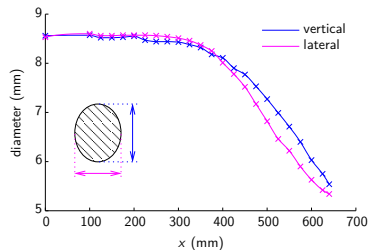
## Procedure in 4 steps

### 1 Geometry



## Tapering

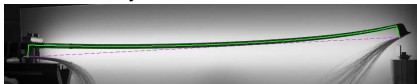
### • measurement with digital caliper



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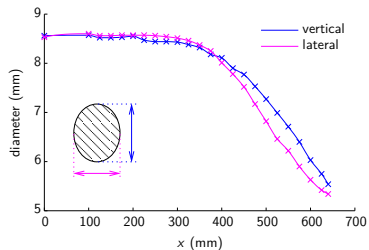
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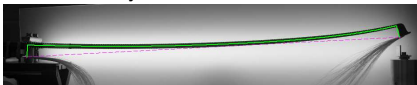
## Camber

- image processing

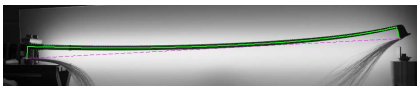
# Determination of bow properties: Step 2

## Procedure in 4 steps

### ① Geometry



### ② Young's modulus of the stick $E$

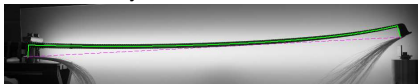


- bow without hair tension

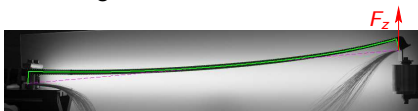
# Determination of bow properties: Step 2

## Procedure in 4 steps

### ① Geometry



### ② Young's modulus of the stick $E$

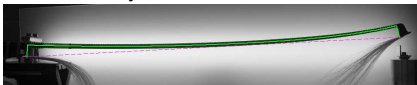


- bow without hair tension
- force  $F_z$  at the tip

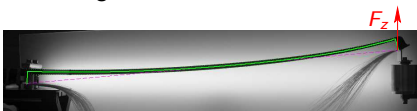
# Determination of bow properties: Step 2

## Procedure in 4 steps

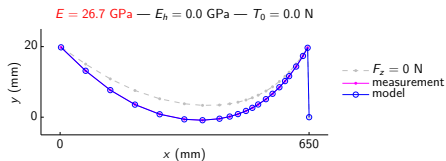
### ① Geometry



### ② Young's modulus of the stick $E$



- bow without hair tension
- force  $F_z$  at the tip
- find  $E$  that minimizes difference between measured and simulated deformed shape



comparison in the hair reference frame



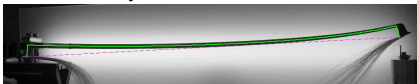
elimination of rigid body motion

$$E = 26.7 \pm 0.7 \text{ GPa (3\%)}$$

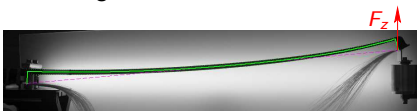
# Determination of bow properties: Step 3

## Procedure in 4 steps

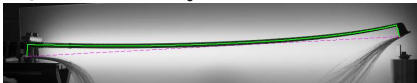
### 1 Geometry



### 2 Young's modulus of the stick $E$



### 3 Hair tension $T_0$



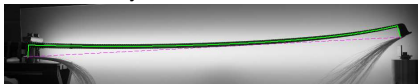
- bow without hair tension



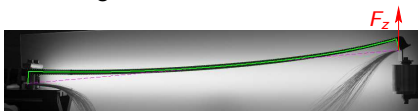
# Determination of bow properties: Step 3

## Procedure in 4 steps

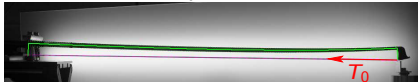
### 1 Geometry



### 2 Young's modulus of the stick $E$



### 3 Hair tension $T_0$

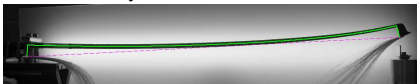


- bow without hair tension
- tighten the bow

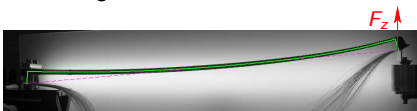
# Determination of bow properties: Step 3

## Procedure in 4 steps

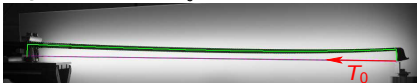
### 1 Geometry



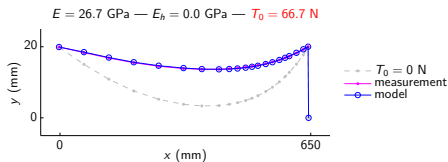
### 2 Young's modulus of the stick $E$



### 3 Hair tension $T_0$



- bow without hair tension
- tighten the bow
- find  $T_0$  that minimizes difference between measured and simulated deformed shape

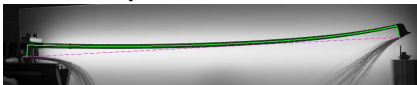


$$T_0 = 66.7 \pm 3.9 \text{ N (6\%)}$$

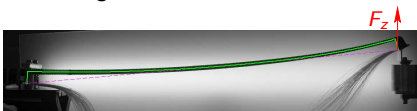
# Determination of bow properties: Step 4

## Procedure in 4 steps

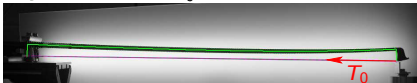
### 1 Geometry



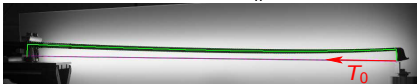
### 2 Young's modulus of the stick $E$



### 3 Hair tension $T_0$



### 4 Stiffness of the hair $E_h$

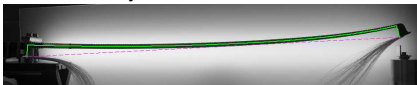


- bow under hair tension

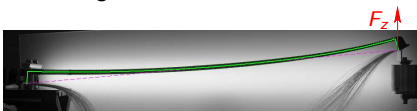
# Determination of bow properties: Step 4

## Procedure in 4 steps

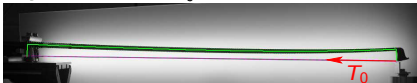
### 1 Geometry



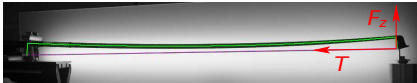
### 2 Young's modulus of the stick $E$



### 3 Hair tension $T_0$



### 4 Stiffness of the hair $E_h$

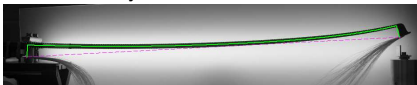


- bow under hair tension
- force  $F_z$  at the tip

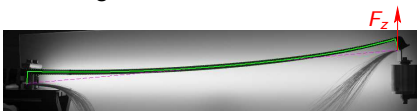
# Determination of bow properties: Step 4

## Procedure in 4 steps

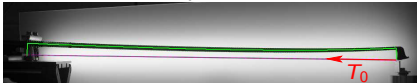
### 1 Geometry



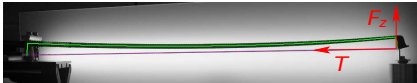
### 2 Young's modulus of the stick $E$



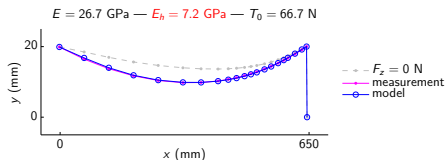
### 3 Hair tension $T_0$



### 4 Stiffness of the hair $E_h$



- bow under hair tension
- force  $F_z$  at the tip
- find  $E_h$  that minimizes difference between measured and simulated deformed shape

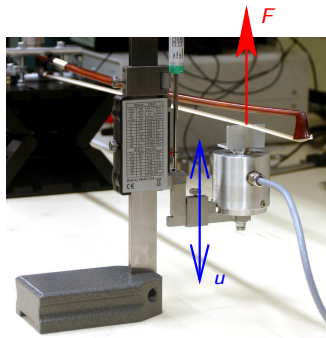


$$E_h = 7.2 \pm 1.7 \text{ GPa} \text{ (24\%)}$$

# Validation: Measurement of compliance

## Distribution of compliance along the bow?

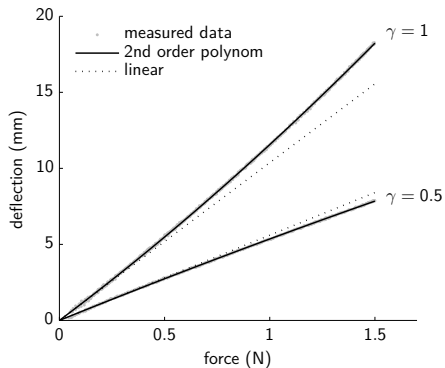
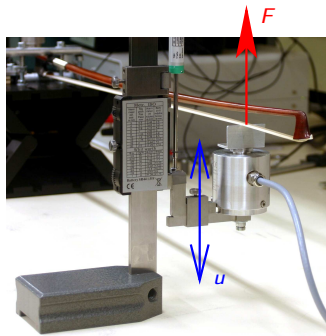
→ simultaneous measurement of force and deflection at several abscissas



# Validation: Measurement of compliance

## Distribution of compliance along the bow?

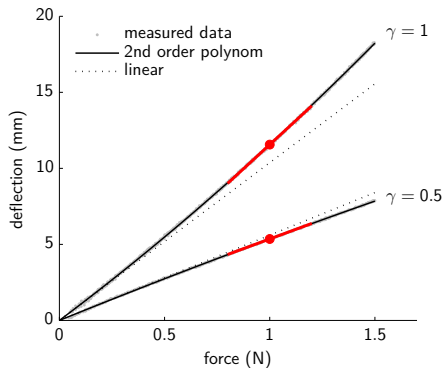
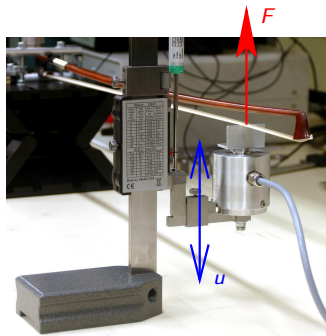
→ simultaneous measurement of force and deflection at several abscissas



# Validation: Measurement of compliance

## Distribution of compliance along the bow?

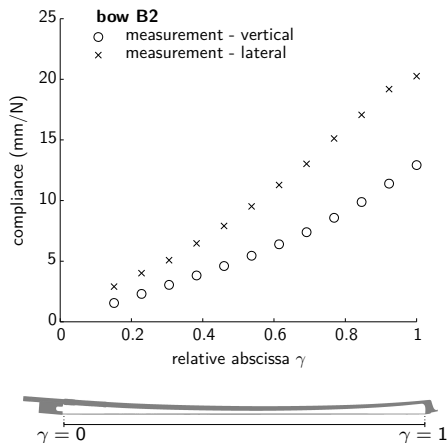
→ simultaneous measurement of force and deflection at several abscissas



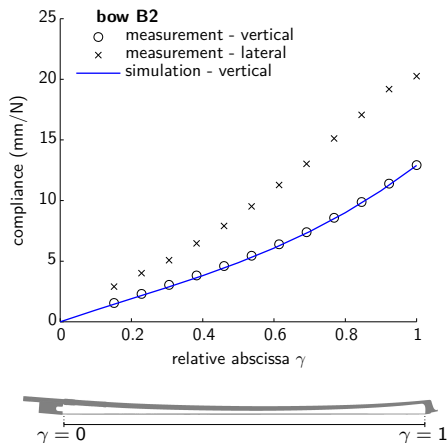
- compliance  $c = \frac{\partial u}{\partial F}$  at  $F = 1 \text{ N}$
- measurement in vertical and lateral directions



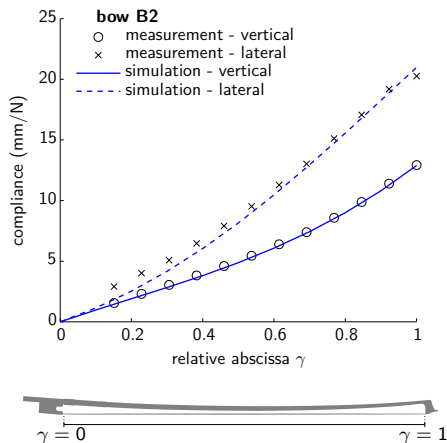
## Comparison between measured and simulated compliance



## Comparison between measured and simulated compliance



## Comparison between measured and simulated compliance



Good agreement between numerical and experimental results → predictive model

[Ablitzer, Dauchez, Dalmont, submitted to Acta Acustica]

## Introduction

1 Modelling

2 Experimental characterization

3 **Results**

- **Static behavior**
- **Stability**

4 Playing tests

## Conclusion

## Introduction

1 Modelling

2 Experimental characterization

3 **Results**

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## Conclusion

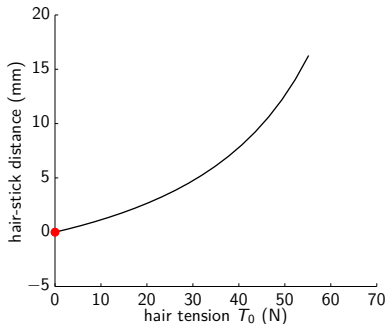
## Adjustment of a bow



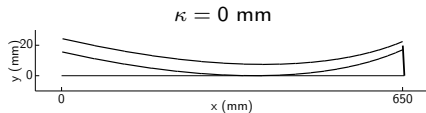
## Adjustment of a bow



Hair tension  $T_0$  vs hair-stick distance  $a_0$



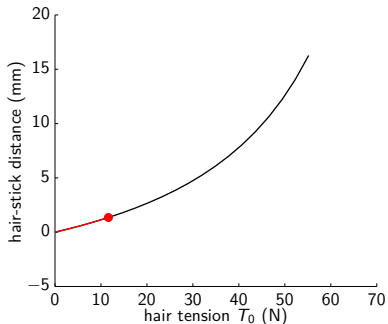
Shape of the bow  
( $\kappa$  initial distance)



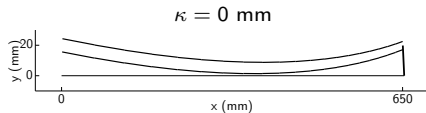
## Adjustment of a bow



Hair tension  $T_0$  vs hair-stick distance  $a_0$



Shape of the bow  
( $\kappa$  initial distance)

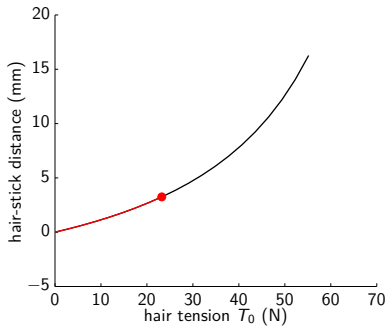




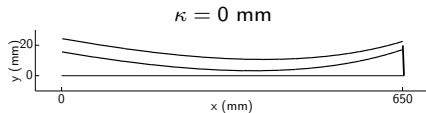
## Adjustment of a bow



Hair tension  $T_0$  vs hair-stick distance  $a_0$



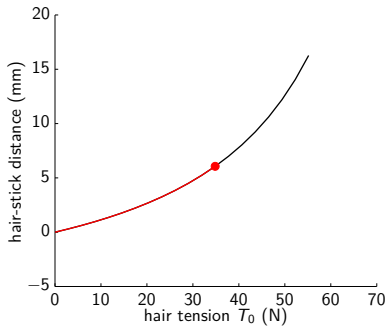
Shape of the bow  
( $\kappa$  initial distance)



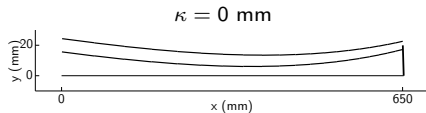
## Adjustment of a bow



Hair tension  $T_0$  vs hair-stick distance  $a_0$



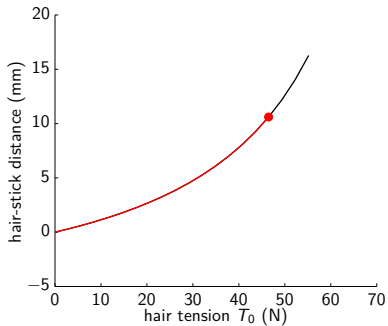
Shape of the bow  
( $\kappa$  initial distance)



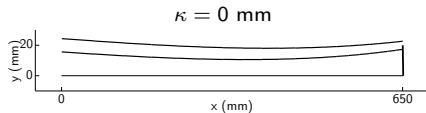
## Adjustment of a bow



Hair tension  $T_0$  vs hair-stick distance  $a_0$



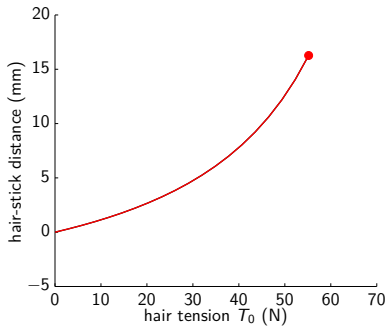
Shape of the bow  
( $\kappa$  initial distance)



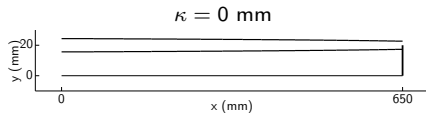
## Adjustment of a bow



Hair tension  $T_0$  vs hair-stick distance  $a_0$



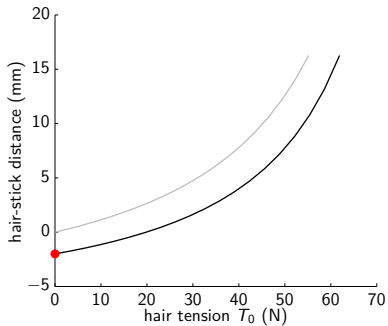
Shape of the bow  
( $\kappa$  initial distance)



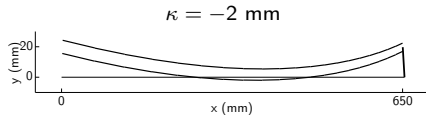
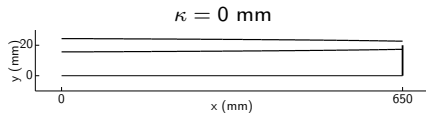
## Adjustment of a bow



Hair tension  $T_0$  vs hair-stick distance  $a_0$



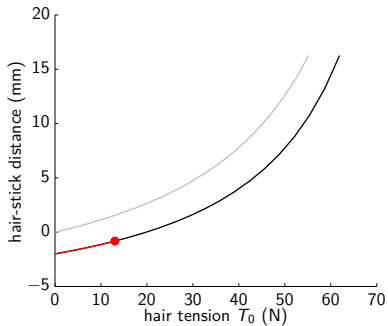
Shape of the bow  
( $\kappa$  initial distance)



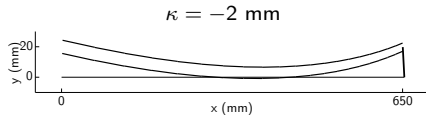
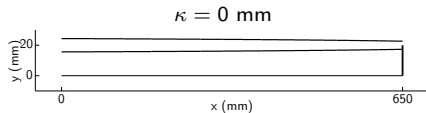
## Adjustment of a bow



Hair tension  $T_0$  vs hair-stick distance  $a_0$



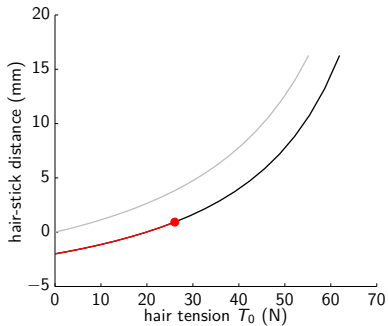
Shape of the bow  
( $\kappa$  initial distance)



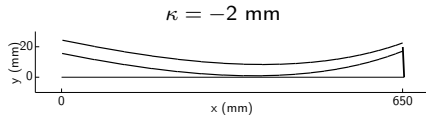
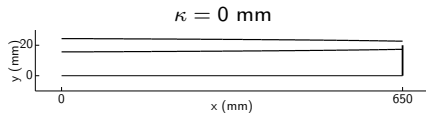
## Adjustment of a bow



Hair tension  $T_0$  vs hair-stick distance  $a_0$



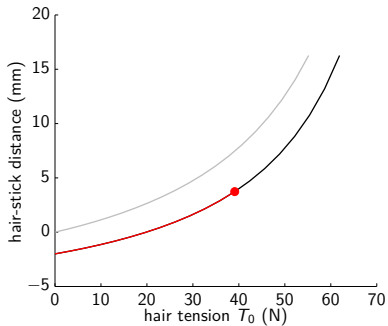
Shape of the bow  
( $\kappa$  initial distance)



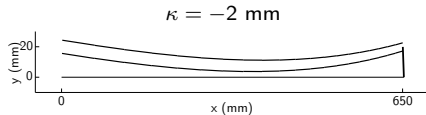
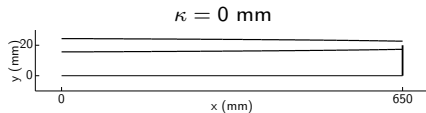
## Adjustment of a bow



Hair tension  $T_0$  vs hair-stick distance  $a_0$



Shape of the bow  
( $\kappa$  initial distance)

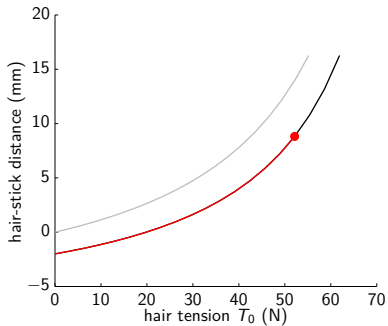




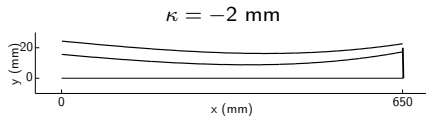
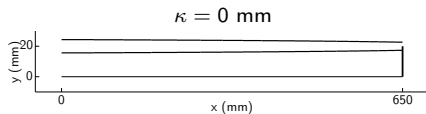
## Adjustment of a bow



Hair tension  $T_0$  vs hair-stick distance  $a_0$



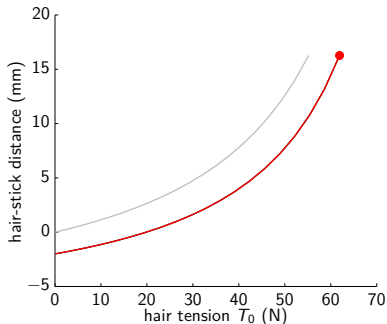
Shape of the bow  
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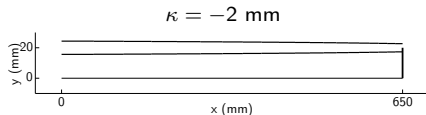
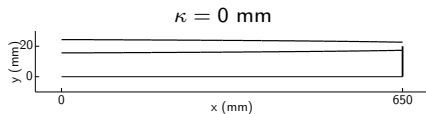
## Adjustment of a bow



Hair tension  $T_0$  vs hair-stick distance  $a_0$



Shape of the bow  
( $\kappa$  initial distance)

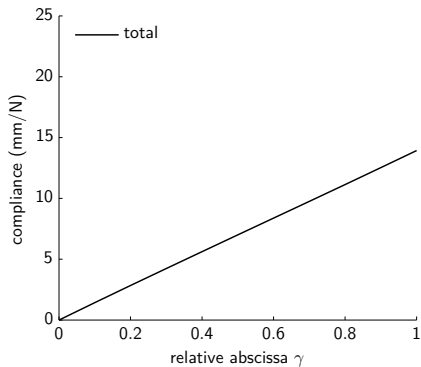


Adjustment of camber allows to reach another hair tension for the same distance

## Compliance of the tightened bow

Vertical compliance along the bow  $c = \frac{\partial u}{\partial F}$

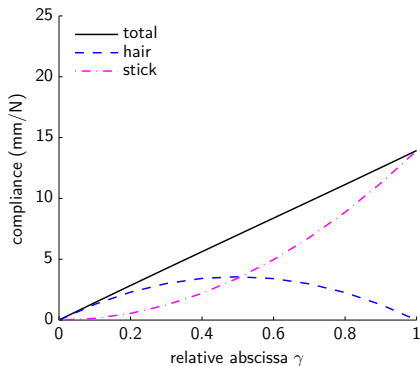
$$F_z = 0 \text{ N}$$



## Compliance of the tightened bow

Vertical compliance along the bow  $c = \frac{\partial u}{\partial F}$

$F_z = 0 \text{ N}$



Two contributions:

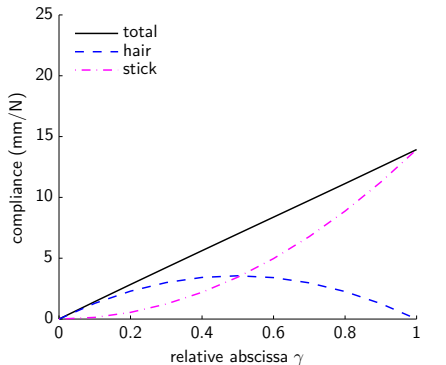
- compliance of the hair
- compliance of the stick



## Compliance of the tightened bow

Vertical compliance along the bow  $c = \frac{\partial u}{\partial F}$

$F_z = 0 \text{ N}$



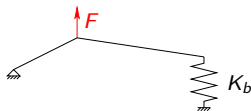
Two contributions:

- compliance of the hair
- compliance of the stick

Pitteroff's model [Pitteroff 1995]

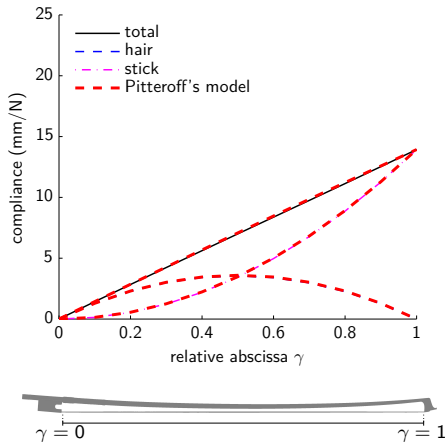
$$c = \underbrace{\frac{\gamma(1-\gamma)L_0}{T_0}}_{\text{hair}} + \underbrace{\frac{\gamma^2}{K_b}}_{\text{stick}}$$

- hair length  $L_0$
- hair tension  $T_0$
- stiffness of the stick at the tip  $K_b$



## Compliance of the tightened bow

Vertical compliance along the bow  $c = \frac{\partial u}{\partial F}$   
 $F_z = 0 \text{ N}$



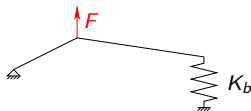
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$$c = \underbrace{\frac{\gamma(1-\gamma)L_0}{T_0}}_{\text{hair}} + \underbrace{\frac{\gamma^2}{K_b}}_{\text{stick}}$$

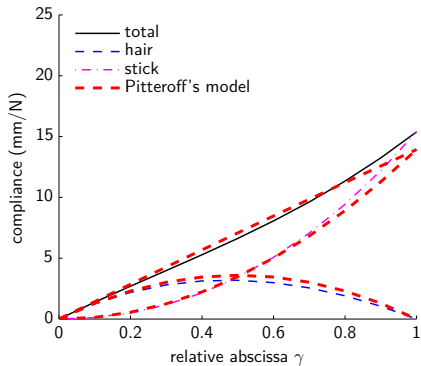
- hair length  $L_0$
- hair tension  $T_0$
- stiffness of the stick at the tip  $K_b$



## Compliance of the tightened bow

Vertical compliance along the bow  $c = \frac{\partial u}{\partial F}$

$$F_z = 0.5 \text{ N}$$



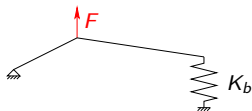
Two contributions:

- compliance of the hair
- compliance of the stick

Pitteroff's model [Pitteroff 1995]

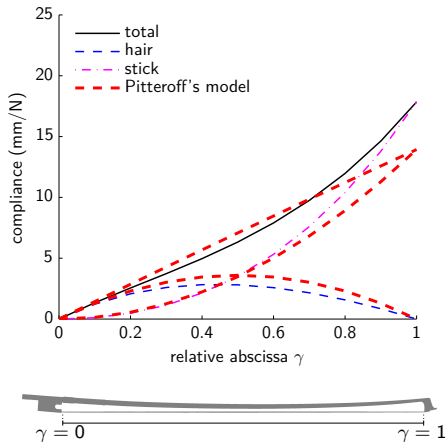
$$c = \underbrace{\frac{\gamma(1-\gamma)L_0}{T_0}}_{\text{hair}} + \underbrace{\frac{\gamma^2}{K_b}}_{\text{stick}}$$

- hair length  $L_0$
- hair tension  $T_0$
- stiffness of the stick at the tip  $K_b$



## Compliance of the tightened bow

Vertical compliance along the bow  $c = \frac{\partial u}{\partial F}$   
 $F_z = 1.0 \text{ N}$



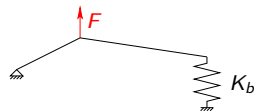
Two contributions:

- compliance of the hair
- compliance of the stick

**Pitteroff's model** [Pitteroff 1995]

$$c = \underbrace{\frac{\gamma(1-\gamma)L_0}{T_0}}_{\text{hair}} + \underbrace{\frac{\gamma^2}{K_b}}_{\text{stick}}$$

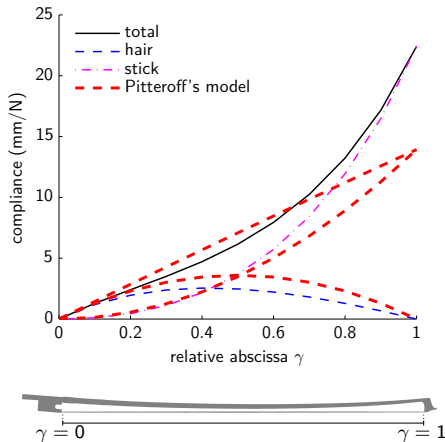
- hair length  $L_0$
- hair tension  $T_0$
- stiffness of the stick at the tip  $K_b$





## Compliance of the tightened bow

Vertical compliance along the bow  $c = \frac{\partial u}{\partial F}$   
 $F_z = 1.5 \text{ N}$



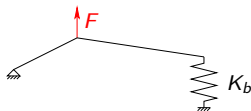
Two contributions:

- compliance of the hair
- compliance of the stick

Pitteroff's model [Pitteroff 1995]

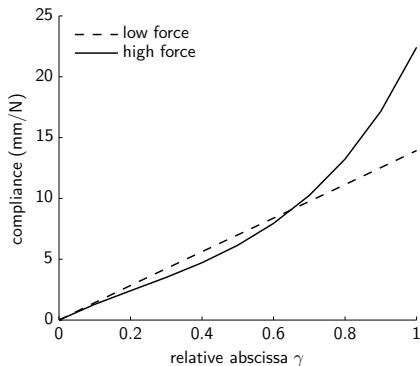
$$c = \underbrace{\frac{\gamma(1-\gamma)L_0}{T_0}}_{\text{hair}} + \underbrace{\frac{\gamma^2}{K_b}}_{\text{stick}}$$

- hair length  $L_0$
- hair tension  $T_0$
- stiffness of the stick at the tip  $K_b$



## Compliance of the tightened bow: Non-linearity

Vertical compliance along the bow  $c = \frac{\partial u}{\partial F}$

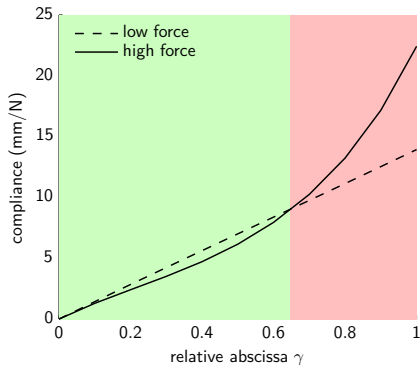


Consider compliance at low forces (0 N) and high forces (1.5 N)



## Compliance of the tightened bow: Non-linearity

Vertical compliance along the bow  $c = \frac{\partial u}{\partial F}$



Consider compliance at low forces (0 N) and high forces (1.5 N)

- near the middle

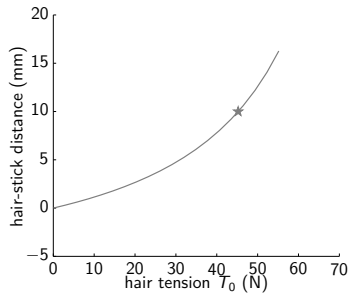
→ stiffening behavior

- near the tip

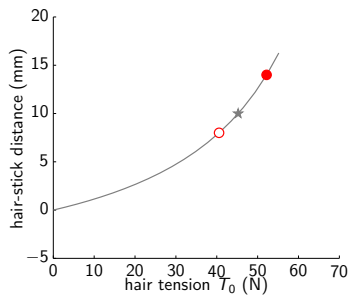
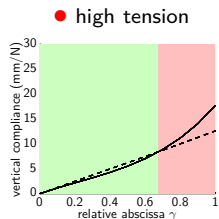
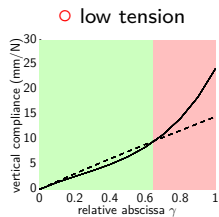
→ softening behavior



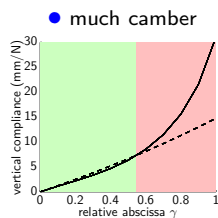
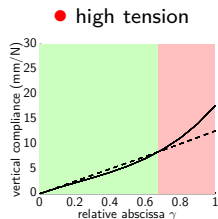
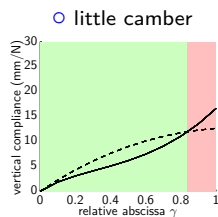
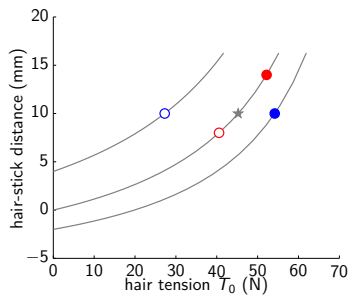
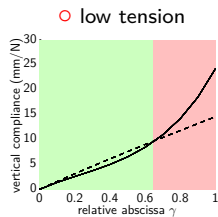
## Compliance of the tightened bow: Effect of hair tension and camber



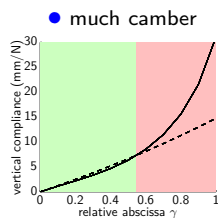
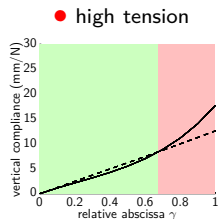
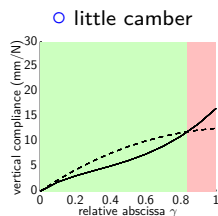
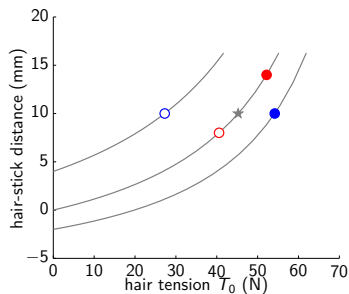
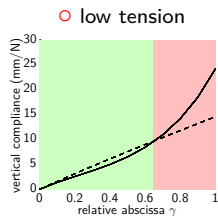
## Compliance of the tightened bow: Effect of hair tension and camber



## Compliance of the tightened bow: Effect of hair tension and camber



## Compliance of the tightened bow: Effect of hair tension and camber

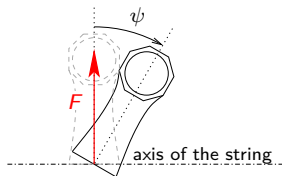


Adjustment of camber strongly affects compliance

[Ablitzer, Dalmont, Dauchez, J. Acoust. Soc. Am. 123 (2012)]

# Effect of bow tilt

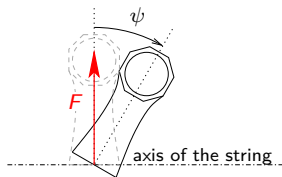
Bow frequently tilted in playing  
(up to about  $30^\circ$ )



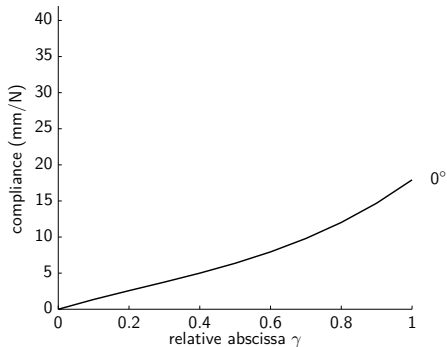


## Effect of bow tilt

Bow frequently tilted in playing  
(up to about  $30^\circ$ )

Evolution of compliance with tilt angle  $\psi$ 

$$F = 1 \text{ N} \mid \kappa = 0 \text{ mm}$$

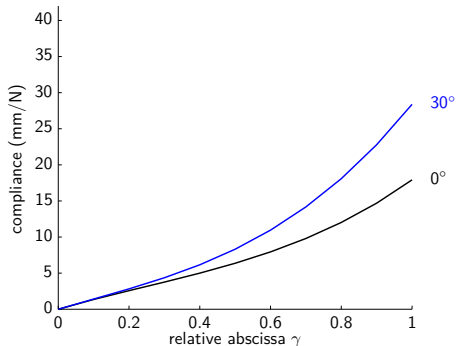
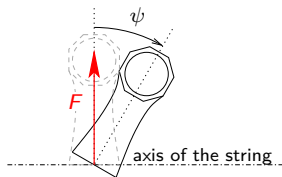


## Effect of bow tilt

Evolution of compliance with tilt angle  $\psi$ 

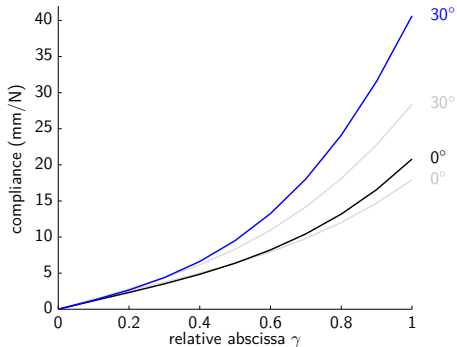
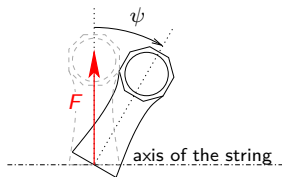
$$F = 1 \text{ N} \mid \kappa = 0 \text{ mm}$$

Bow frequently tilted in playing  
(up to about  $30^\circ$ )



Lateral compliance is higher than vertical compliance

## Effect of bow tilt

Evolution of compliance with tilt angle  $\psi$  $F = 1 \text{ N}$  |  $\kappa = -2 \text{ mm}$  (more camber)Bow frequently tilted in playing  
(up to about  $30^\circ$ )

Lateral compliance is higher than vertical compliance

## Introduction

1 Modelling

2 Experimental characterization

3 Results

- Static behavior
- **Stability**

4 Playing tests

## Conclusion

# Stability of the bow

## Load case



# Stability of the bow

① without perturbation force ( $F_z$  only)

Load case



② with perturbation force ( $F_z + \varepsilon F_y$ )

# Stability of the bow

① without perturbation force ( $F_z$  only)

## Load case

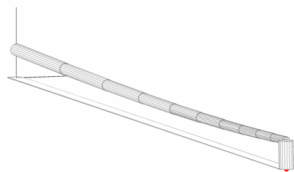
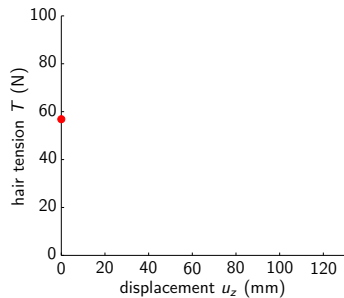
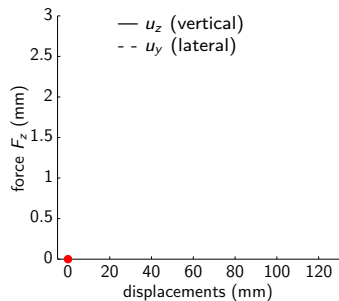


The bow may be unstable in two ways:

② with perturbation force ( $F_z + \epsilon F_y$ )

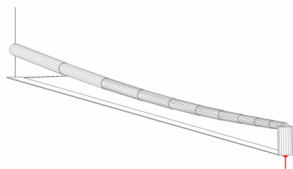
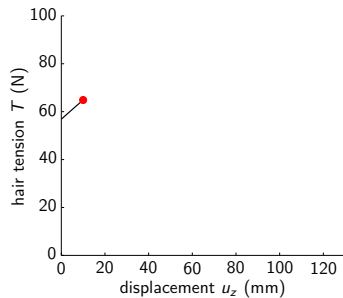
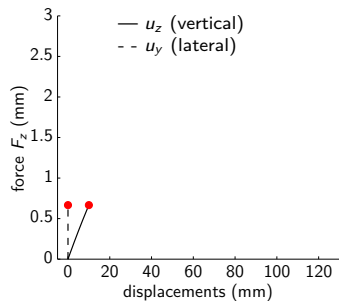
- ① **limit point** instability  
(snap-through)
- ② **bifurcation** instability  
(lateral buckling)

# Limit point instability

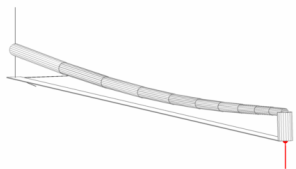
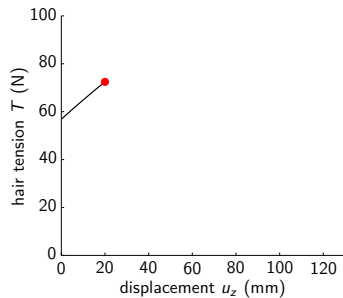
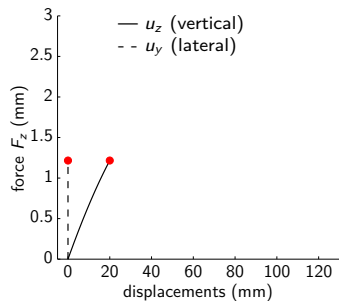




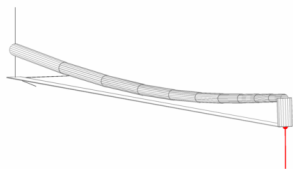
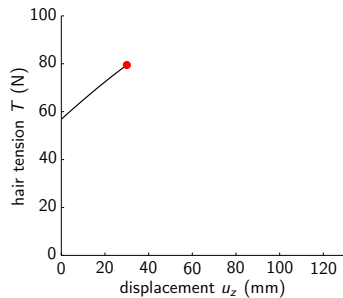
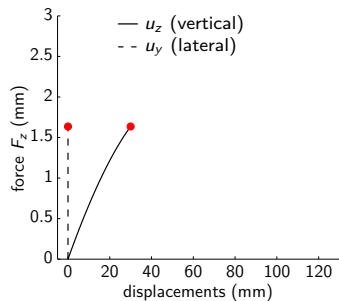
# Limit point instability



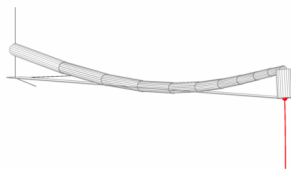
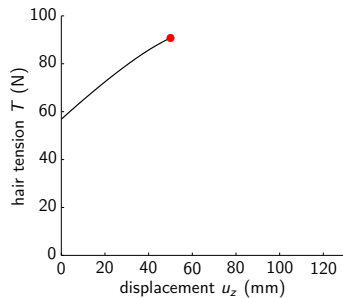
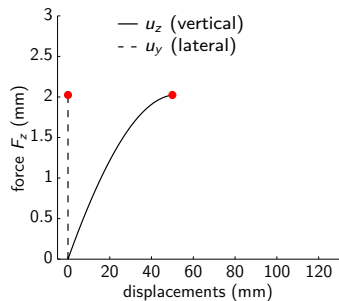
# Limit point instability



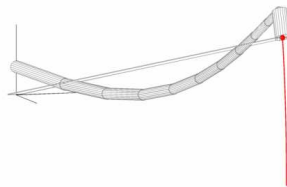
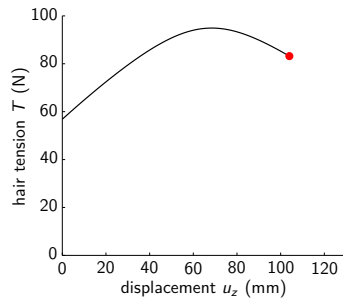
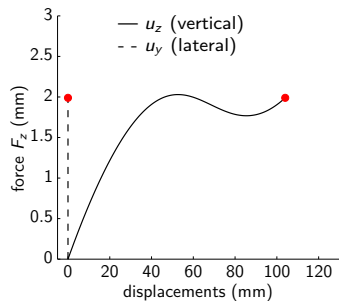
# Limit point instability



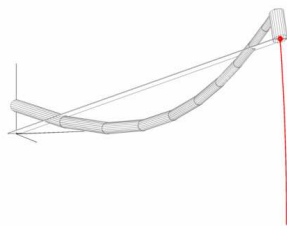
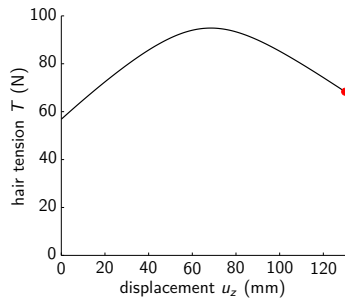
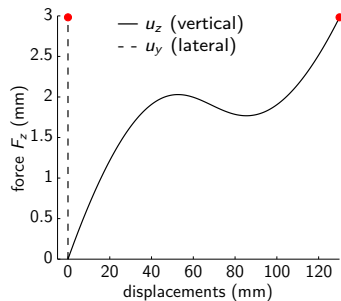
# Limit point instability



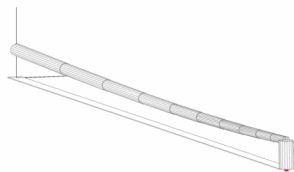
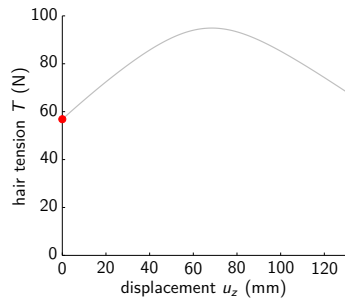
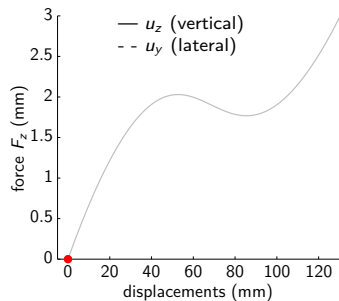
# Limit point instability



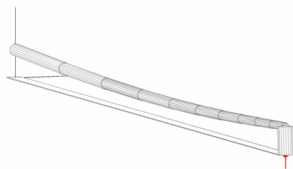
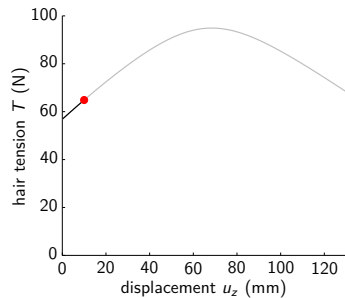
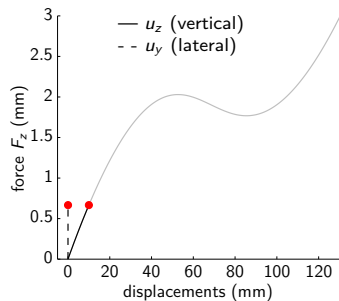
# Limit point instability



## Bifurcation instability

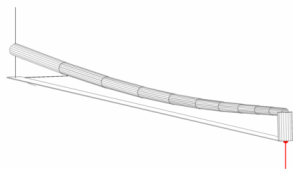
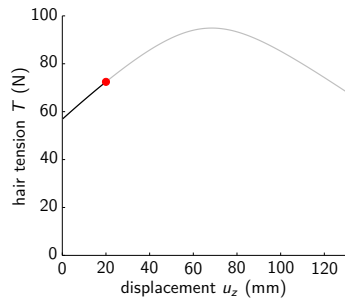
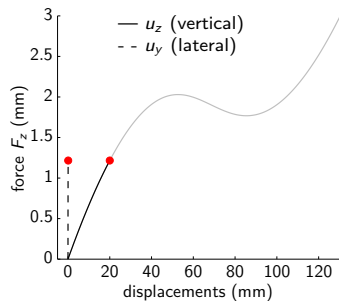


## Bifurcation instability

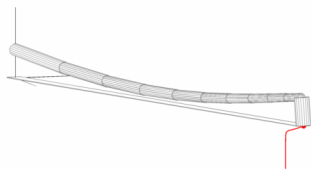
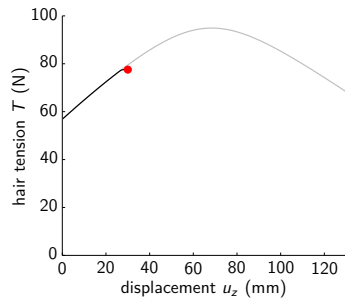
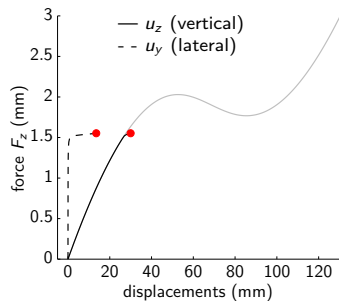




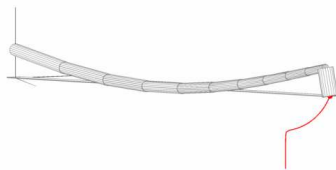
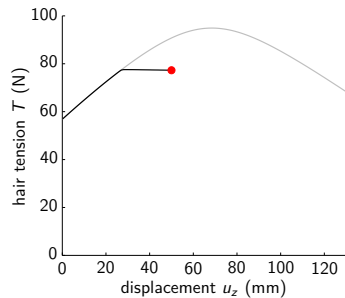
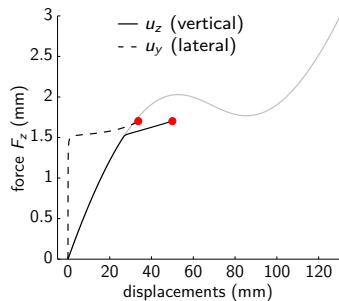
## Bifurcation instability



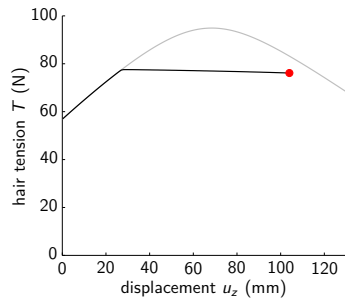
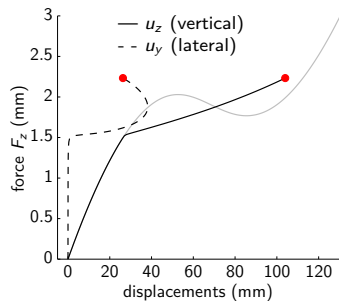
## Bifurcation instability



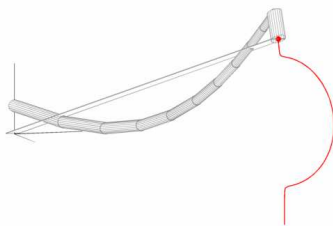
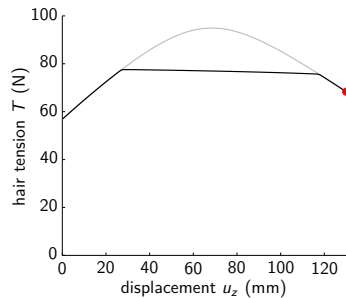
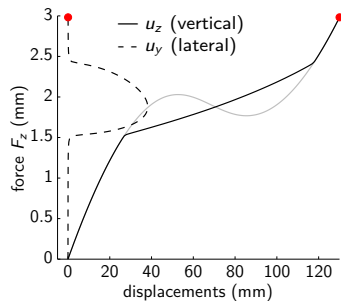
## Bifurcation instability



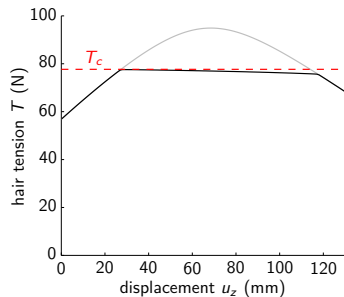
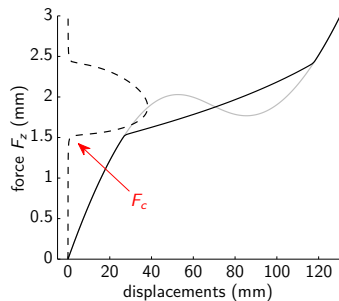
## Bifurcation instability



## Bifurcation instability



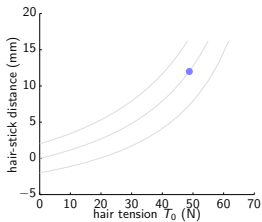
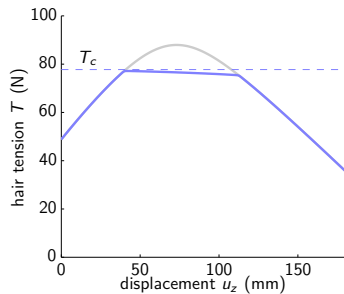
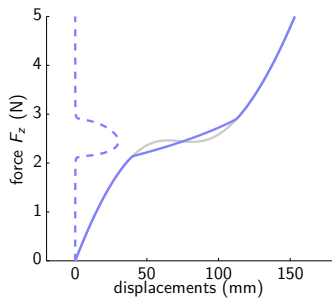
# Critical buckling loads



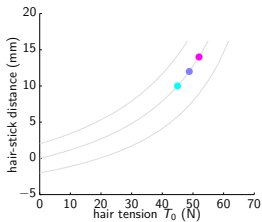
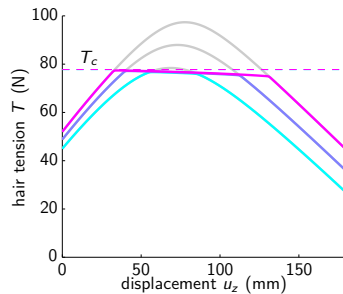
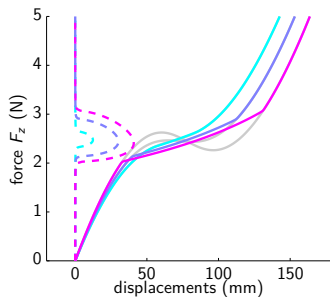
Buckling occurs

- when  $T = T_c$   
critical hair tension
- when  $F_z = F_c$   
critical playing force

## Influence of hair tension



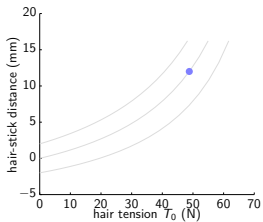
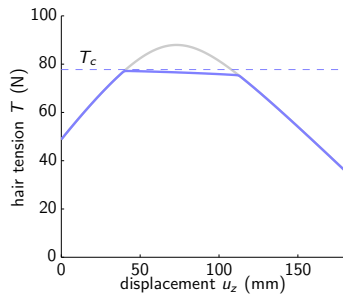
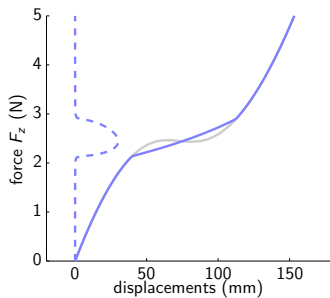
## Influence of hair tension



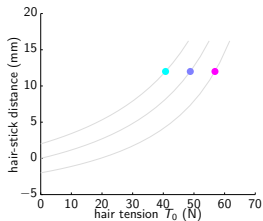
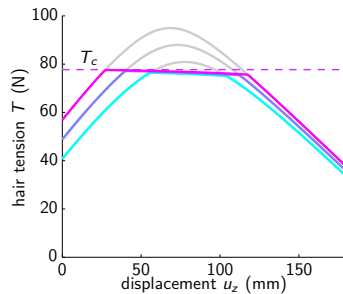
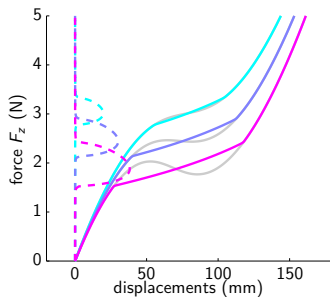
- same critical tension  $T_c$
- critical force  $F_c$  not very sensitive to hair tension



## Influence of camber



## Influence of camber



- same critical tension  $T_c$
- increasing camber  $\Rightarrow$  critical force  $F_c$  decreases

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## Selection and adjustment of bows

Idea: vary only 2 parameters: **camber** and **hair tension**

# Selection and adjustment of bows

Idea: vary only 2 parameters: **camber** and **hair tension**

## ① Selection of 3 bows

- same properties (stiffness, mass, center of inertia...)
- same aspect
- high-quality bows in Pernambuco after Tourte model



×3

# Selection and adjustment of bows

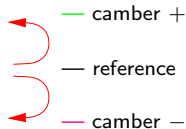
Idea: vary only 2 parameters: **camber** and **hair tension**

## ① Selection of 3 bows

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- one bow with more camber ( $\kappa = -3$  mm)
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selection and adjustment by bow maker Jean-Grunberger

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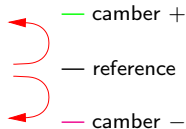
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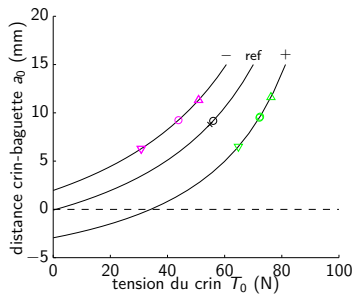
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## ③ Characterization of the bows



- camber +
- reference
- camber -



selection and adjustment by bow maker Jean-Grunberger



# Verbalization

**Expert 1**

**Expert 2**

# Verbalization

## Expert 1

## Expert 2

- ① **Stability** (unstable  $\longleftrightarrow$  stable)  
bow doesn't tremble on long notes

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ease to play at the frog
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timbre on long notes
- ③ **Consonant** (softer  $\longleftrightarrow$  harder)  
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- ④ **Reactivity** (slow  $\longleftrightarrow$  rapid)  
time necessary to produce the tone
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ability to separate notes

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- descriptors relative to **playability** and **tonal qualities**
- 2 descriptors common to both experts: **stability** and **attack**

# Pair-wise comparison task

For each configuration to be tested

prepare the bow



picture



play & compare  
against reference bow



evaluate

		score / 5
com. stabilité		
instabilité	<input type="checkbox"/> <input checked="" type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	→ stable
com. poids des collages		
maie. sides	<input type="checkbox"/> <input type="checkbox"/> <input checked="" type="checkbox"/> <input type="checkbox"/>	→ plus sides
com. intonation		
maie. dec.	<input type="checkbox"/> <input type="checkbox"/> <input checked="" type="checkbox"/> <input type="checkbox"/>	→ plus dec.
com. stabilité au geste		
bat.	<input type="checkbox"/> <input type="checkbox"/> <input checked="" type="checkbox"/> <input type="checkbox"/>	→ rapide
com. sens		
pu.	<input type="checkbox"/> <input type="checkbox"/> <input checked="" type="checkbox"/> <input type="checkbox"/>	→ beaucoup
commentaire		



# Pair-wise comparison task

For each configuration to be tested

evaluate

play & compare  
against reference bow

picture

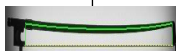


		score / 5
com. stabilité		
instabilité	→ <input checked="" type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> →	stabilité
com. pointe des cotige...		
maise. sides...	→ <input type="checkbox"/> <input type="checkbox"/> <input checked="" type="checkbox"/> <input type="checkbox"/> →	plus maise...
com. direction		
maise. dir.	→ <input type="checkbox"/> <input type="checkbox"/> <input checked="" type="checkbox"/> <input type="checkbox"/> →	plus dir.
com. stabilité au geste		
bat	→ <input type="checkbox"/> <input type="checkbox"/> <input checked="" type="checkbox"/> <input type="checkbox"/> →	rapid
com. sens		
pin	→ <input type="checkbox"/> <input type="checkbox"/> <input checked="" type="checkbox"/> <input type="checkbox"/> →	heavy
commentaire		

prepare the bow



After the test



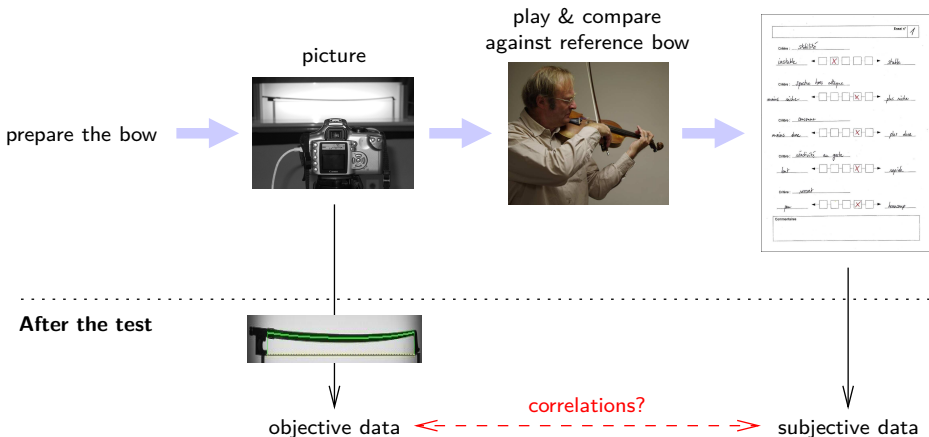
objective data

subjective data

# Pair-wise comparison task

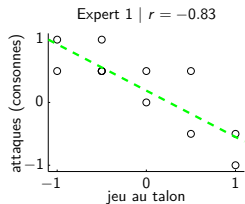
For each configuration to be tested

evaluate



# Results

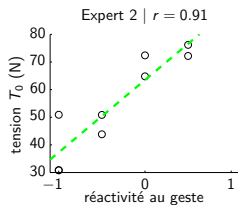
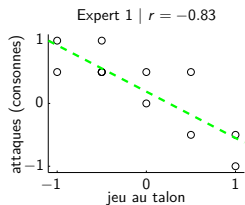
## Significant correlations



- **playing at the frog** | **attack**  
(subjective – subjective)

## Results

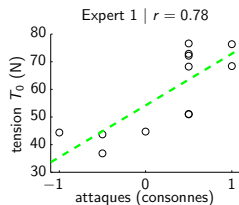
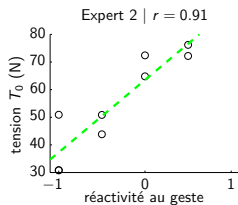
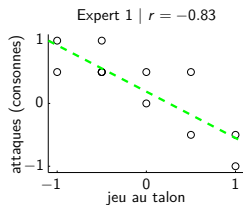
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## Results

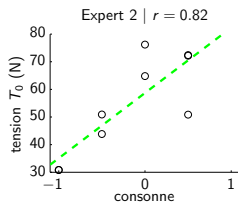
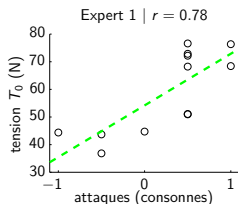
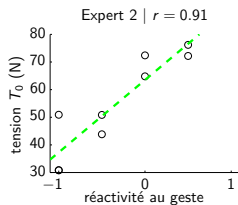
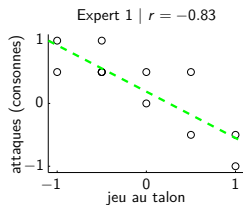
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## Results

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→ result common to both players

# Playing tests: Conclusions

## ✓ Influence of hair tension on player's peception

- reactivity
- attacks

↗ with hair tension

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## ✓ Influence of hair tension on player's peception

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## ✗ Stability

- instability = trembling bow? → find relevant dynamic property



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## ✗ Stability

- instability = trembling bow? → find relevant dynamic property
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## ✓ Characterization of bows for the test

- differences in bow properties
  - state in which the bow is played
- } are well determined

## Introduction

① Modelling

② Experimental characterization

③ Results

- Static behavior
- Stability

④ Playing tests

## Conclusion

# Conclusion



# Conclusion



- static behavior of the bow strongly depends on prestress

# Conclusion



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- static behavior of the bow strongly depends on prestress
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- camber has a strong influence on  $\left\{ \begin{array}{l} \text{playing hair tension} \\ \text{compliance} \\ \text{limit of stability} \end{array} \right.$

# Conclusion



- static behavior of the bow strongly depends on prestress
- bow played near its limit of stability
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{	playing hair tension <span style="color: blue;">↗</span> compliance <span style="color: blue;">↗</span> limit of stability <span style="color: red;">↘</span>
---	---



# Perspectives

**NUMERICAL MODELS**

predictive

+

**PROCEDURE TO DETERMINE BOW PROPERTIES**

using affordable and easy-to-use equipment

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## Assistance to bow making

- **Characterization** in workshop
- **Prediction** upstream from fabrication or adjustment
- Looking for **alternative woods**



# Perspectives

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## Organology

- **Categorization** of bows in museums
- Information on bows **in playing situation**

# Perspectives



playability  
tonal qualities



static & dynamic properties



# Perspectives



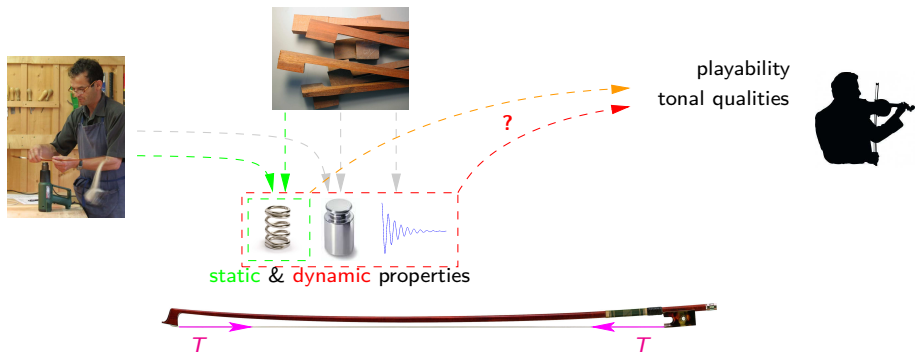
playability  
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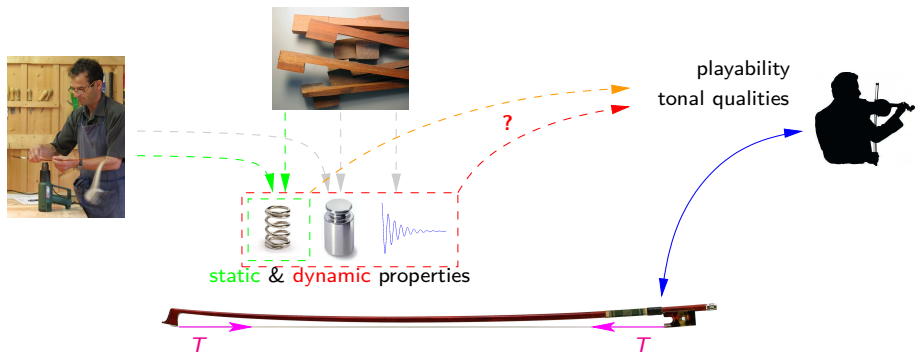
# Perspectives



## Dynamic properties

How do they affect playability?

# Perspectives

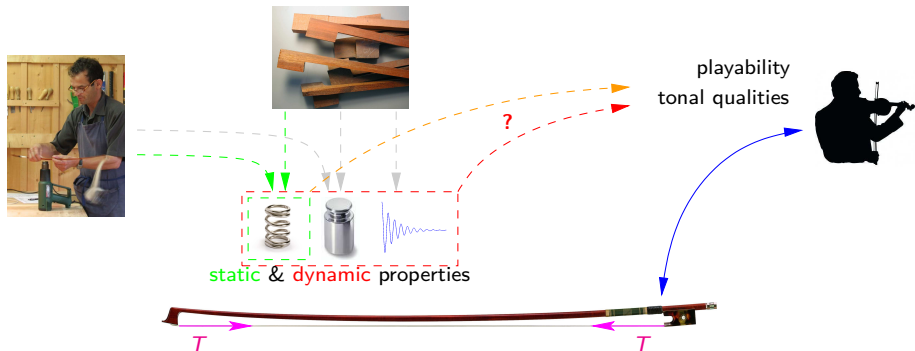


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How do they affect playability?

- Perceptive studies  
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- Measurement of gesture

# Perspectives



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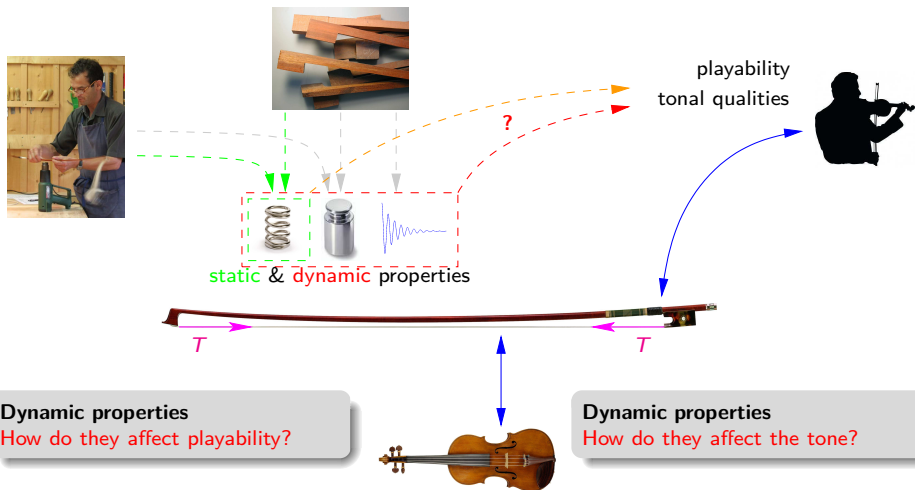
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## Dynamic properties

How do they affect the tone?



# Perspectives



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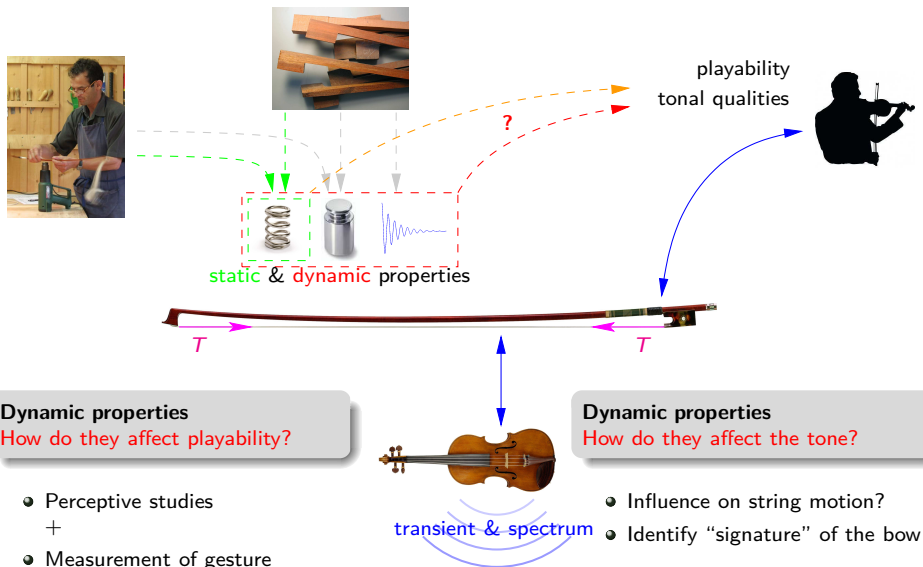
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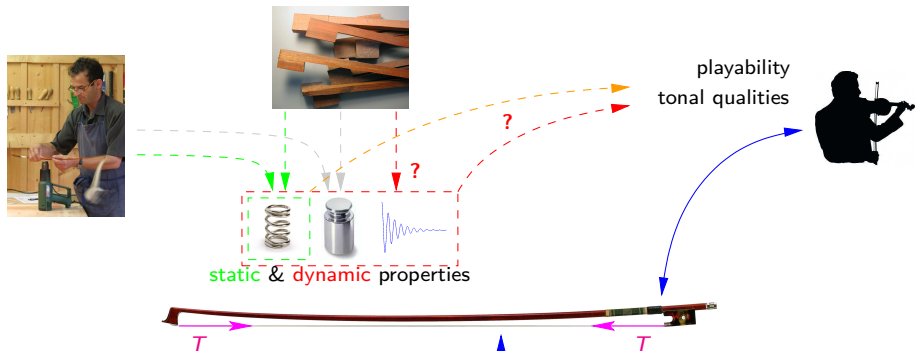
How do they affect the tone?

- Influence on string motion?

# Perspectives



# Perspectives



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transient & spectrum

## Dynamic properties

How do they affect the tone?

- Influence on string motion?
- Identify “signature” of the bow
- Role of damping?

# INFLUENCE OF MECHANICAL AND GEOMETRICAL PARAMETERS ON THE STATIC BEHAVIOR OF A VIOLIN BOW IN PLAYING SITUATION

**Frédéric Ablitzer**

Laboratoire d'Acoustique de l'Université du Maine – UMR CNRS 6613

**PhD defence**

Le Mans, France – December 5th, 2011

Examining committee

**A. Askenfelt** | KTH, Stockholm (Examiner)

**R. Caussé** | IRCAM, Paris (Reviewer)

**A. Chaigne** | ENSTA ParisTech, Palaiseau (Chairman)

**G. Chevallier** | SUPMÉCA, Saint-Ouen (Examiner)

**B. Cochelin** | LMA, Marseille (Reviewer)

**J.P. Dalmont** | LAUM, Le Mans (Supervisor)

**N. Dauchez** | SUPMÉCA, Saint-Ouen (Supervisor)

**N. Poidevin** | Bow maker, Dinan (Invited)

