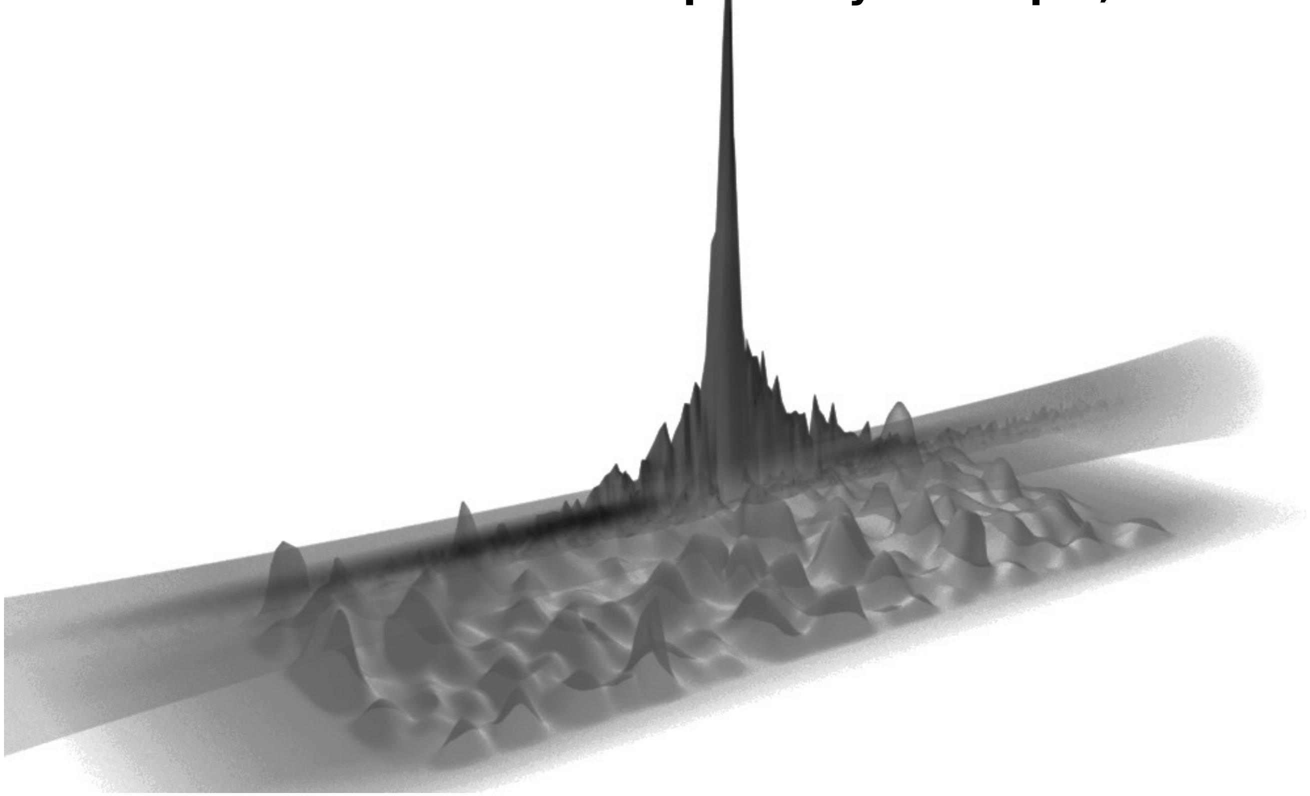
# DISORDERED QUANTUM GASES

## THEORETICAL STUDIES & EXPERIMENTAL PERSPECTIVES

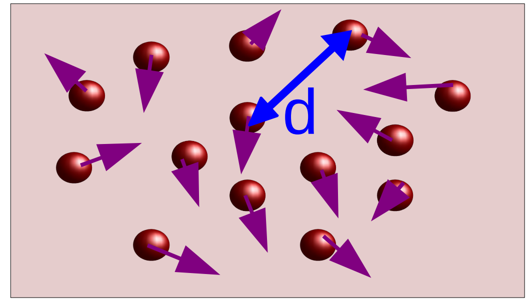
**Laurent Sanchez-Palencia**

**Laboratoire Charles Fabry de l'Institut d'Optique, CNRS and Univ. Paris-Sud, Campus Polytechnique, Palaiseau, France**



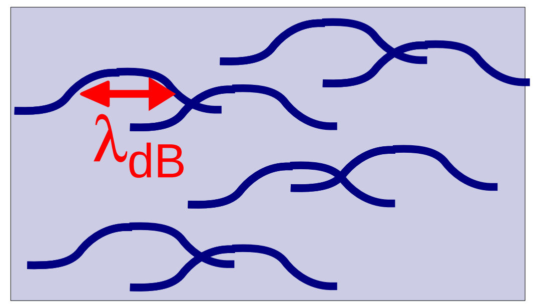
# Condensed Matter and Ultracold Atoms

classical mechanics



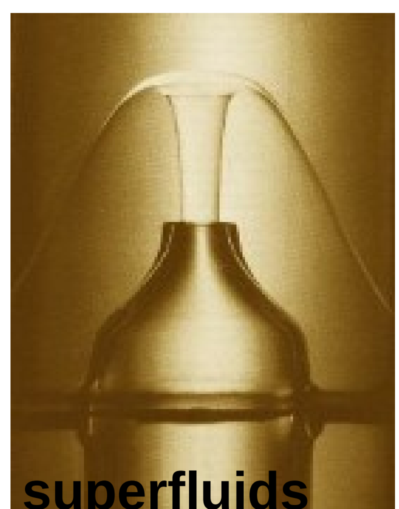

$d \gg \lambda_{dB} \sim 1/T^{1/2}$

quantum mechanics

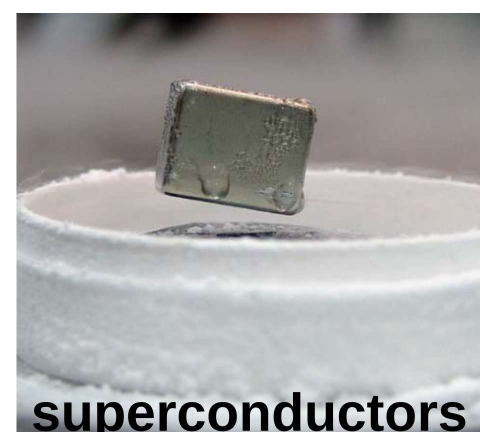



$d < \lambda_{dB}$


T>300K

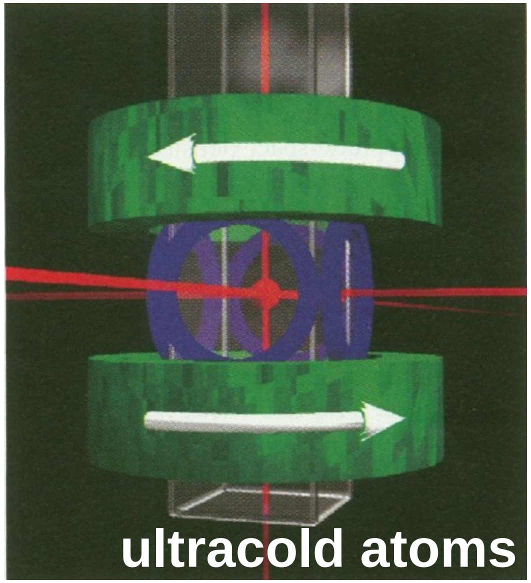



T~1K



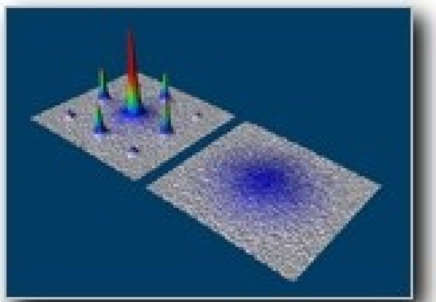
# Ultracold Atoms and Quantum Simulators

T~10nK

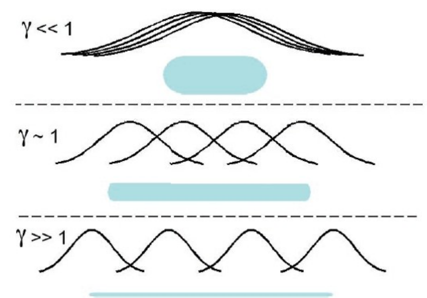



- Flexibility
  - control of parameters (dimensionality, shape, interactions, bosons/fermions/mixtures, ...)
- High-precision measurements
  - complementary to condensed-matter tools
  - variety of diagnostic tools (direct imaging, velocity distribution, oscillations, Bragg spectroscopy, ....)
- Model systems
  - parameters known ab-initio ⇒ direct comparison with theory
  - towards quantum simulators ⇒ artificial, controlled systems to study fundamental questions

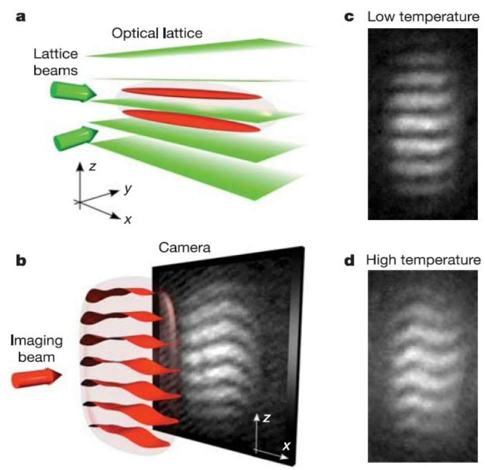
# Ultracold Atoms and Quantum Simulators



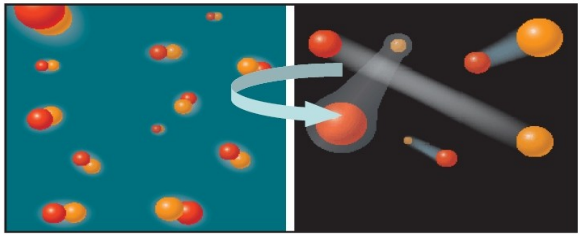
▲ Superfluid-Mott insulator Transition
(Hänsch, Bloch, Esslinger)



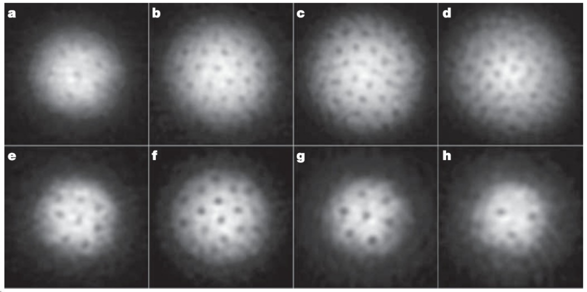

▲ Tonks-Girardeau gas
(Weiss, Bloch)




▲ Berezinskii-Kosterlitz-Thouless crossover
(Dalibard, Phillips,Cornell)

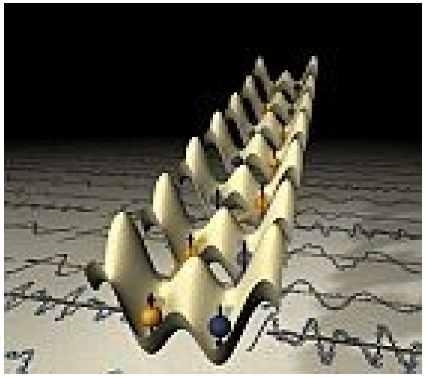


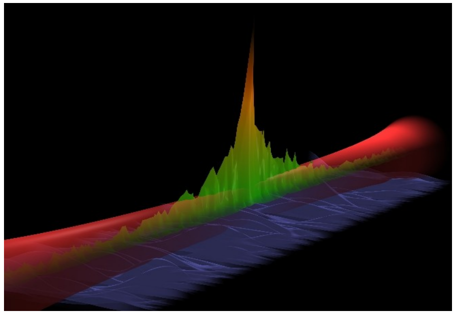
▲ BEC-BCS crossover
(Jin, Ketterle, Grimm, Salomon)




Quantized vortices ▲
in Fermion gases (Ketterle)

Spin exchange ▶
(Phillips, Bloch)

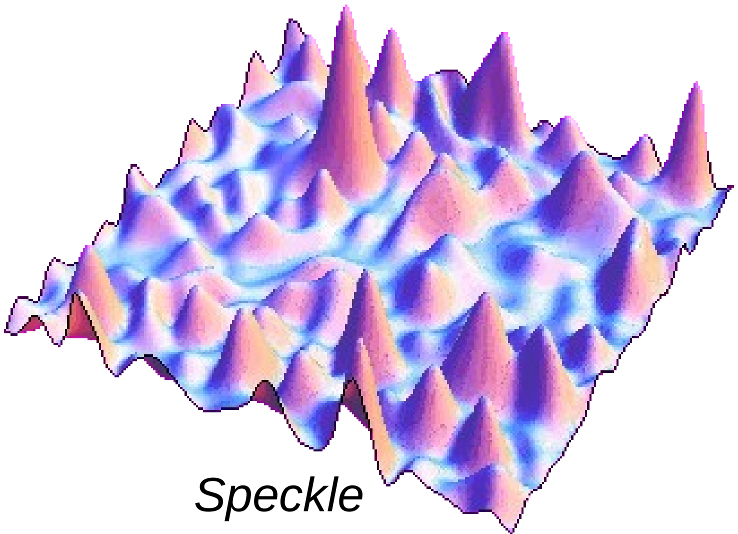




▲ Anderson localization
(Aspect, Inguscio, Hulet)

# Disordered Systems

- Disorder is ubiquitous in physical systems
  - Defects and impurities
    - ➢solid-state physics
    - ➢quantum Hall effect
    - ➢superfluidity / superconductivity
    - ➢propagation of light in dense media

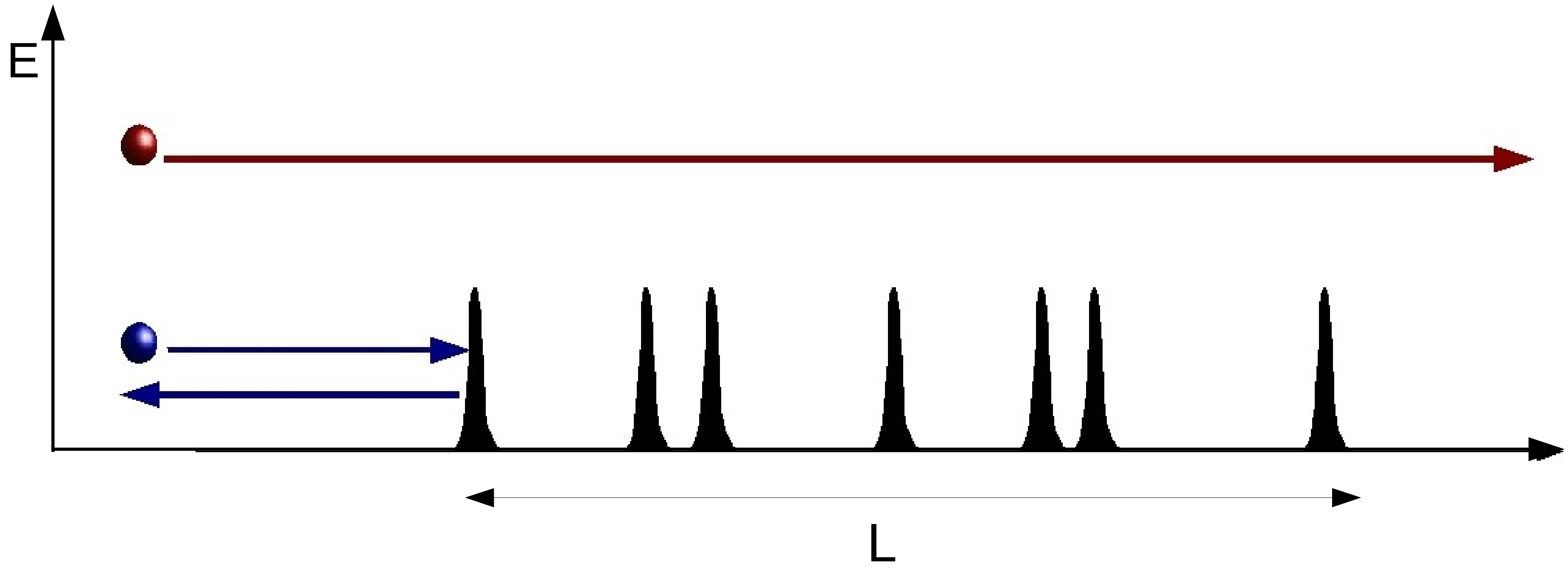


*Speckle Potential (for cold atoms)*

  - Relevant issues
    - ➢Anderson localization (disorder-induced metal-insulator transition)
    - ➢interactions or non-linearities in disordered systems
    - ➢disordered spin systems

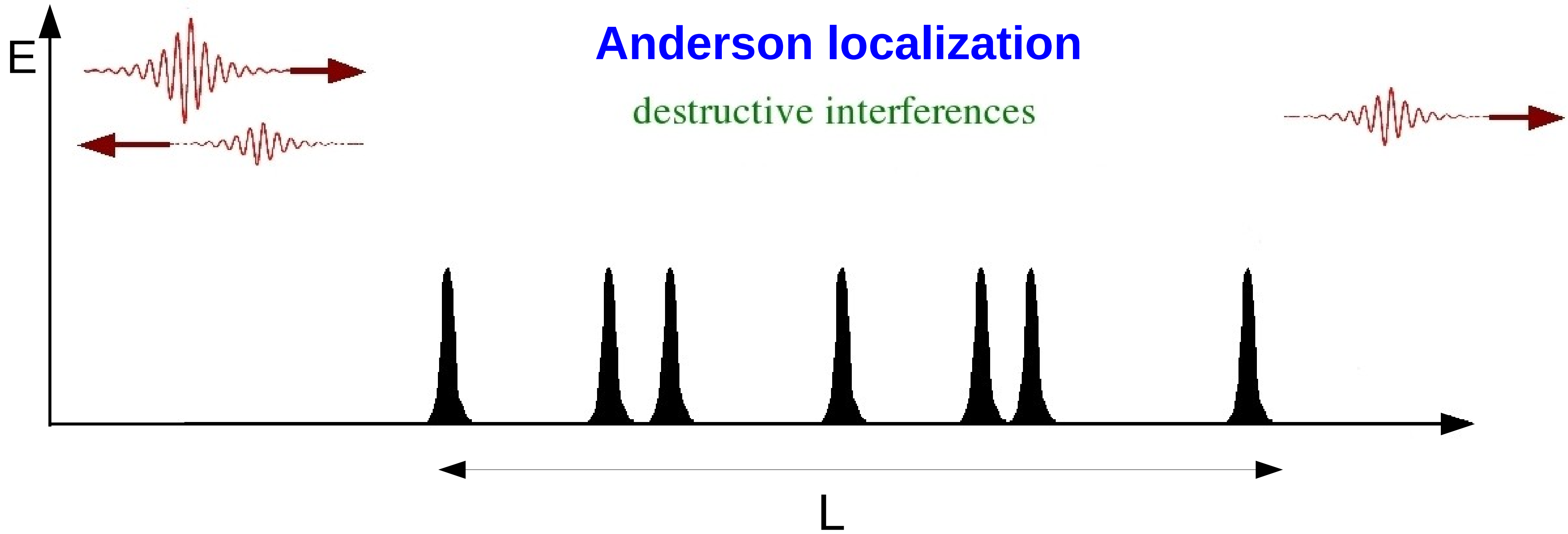
# Anderson Localization (1D picture)

classical particles



# Anderson Localization (1D picture)

**wave physics**




$$\langle \ln[T(L)] \rangle \sim -2L / L_{loc}$$

$$\boxed{L_{loc} = 1/\gamma(E)}$$

# Anderson Localization

- **Theory**
  - P.W. Anderson, Phys. Rev. **109**, 1492 (1958)
  - E. Abrahams *et al.*, Phys. Rev. Lett. **42**, 673 (1979)
  - D. Vollhardt & P. Wölfle, Phys. Rev. Lett. **48**, 699 (1982)
- **Light in diffusive media**
  - D.S. Wiersma *et al.*, Nature **390**, 671 (1997)
  - M. Störzer *et al.*, Phys. Rev. Lett. **96**, 063904 (2006)
- **Photonic crystals**
  - T. Schwartz *et al.*, Nature **446**, 52 (2007)
  - Y. Lahini *et al.*, Phys. Rev. Lett. **100**, 013906 (2008)
- **Microwaves**
  - A.A. Chabanov *et al.*, Nature **404**, 850 (2000)
- **Sound waves**
  - R.L Weaver, Wave Motion **12**, 129 (1990)
  - H. Hu *et al.*, Nature Phys. **4**, 945 (2008)
- **Cold atoms**
  - J. Billy *et al.*, Nature **453**, 891 (2008)
  - G. Roati *et al.*, Nature **453**, 895 (2008)
  - J. Chabé et al., Phys. Rev. Lett. **101**, 255702 (2008)

# Disordered Quantum Gases

- Dynamical localization
  - ➢Zoller *et al.*, PRA (1992)
  - ➢Raizen *et al.*, PRL (1994)
  - ➢Garreau *et al.*, PRL (2008 and 2010)

nature physics | PROGRESS ARTICLE
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**Disordered quantum gases under control**

Laurent Sanchez-Palencia[1]* and Maciej Lewenstein[2]*

**When attempting to understand the role of disorder in condensed-matter physics, we face considerable experimental and theoretical difficulties, and many questions are still open. Two of the most challenging ones—debated for decades—concern the effect of disorder on superconductivity and quantum magnetism. We review recent progress in the field of ultracold atomic gases, which should pave the way towards the realization of versatile quantum simulators, which help solve these questions. In addition, ultracold gases offer original practical and conceptual approaches, which open new perspectives to the field of disordered systems.**

NATURE PHYSICS | VOL 6 | FEBRUARY 2010 | www.nature.com/naturephysics 87

- Disordered molasses
  - ➢Grynberg *et al.*, PRA (1998) ; EPL (2000)

- Classical and Anderson localization
  - ➢Lewenstein & Zoller *et al.*, PRL (2003)
  - ➢Inguscio *et al.*, Aspect, Bouyer *et al.*, PRLs (2005)
  - ➢LSP *et al.*, PRL (2007)
  - ➢Aspect, Bouyer *et al.*, Nature (2008) ; Inguscio *et al.*, Nature (2008)

- Interplay of interactions and disorder
  - ➢Large body of theoretical work; Shultz & Giamarchi (1988), Fisher *et al.* (1989)
  - ➢Inguscio *et al.*, PRL (2007) ; Nature Phys. (2010)
  - ➢DeMarco *et al.*, PRL (2009) ; Nature Phys. (2010)

# Our Theoretical Work

**Quantum Atomic Gases - theory**
LSP

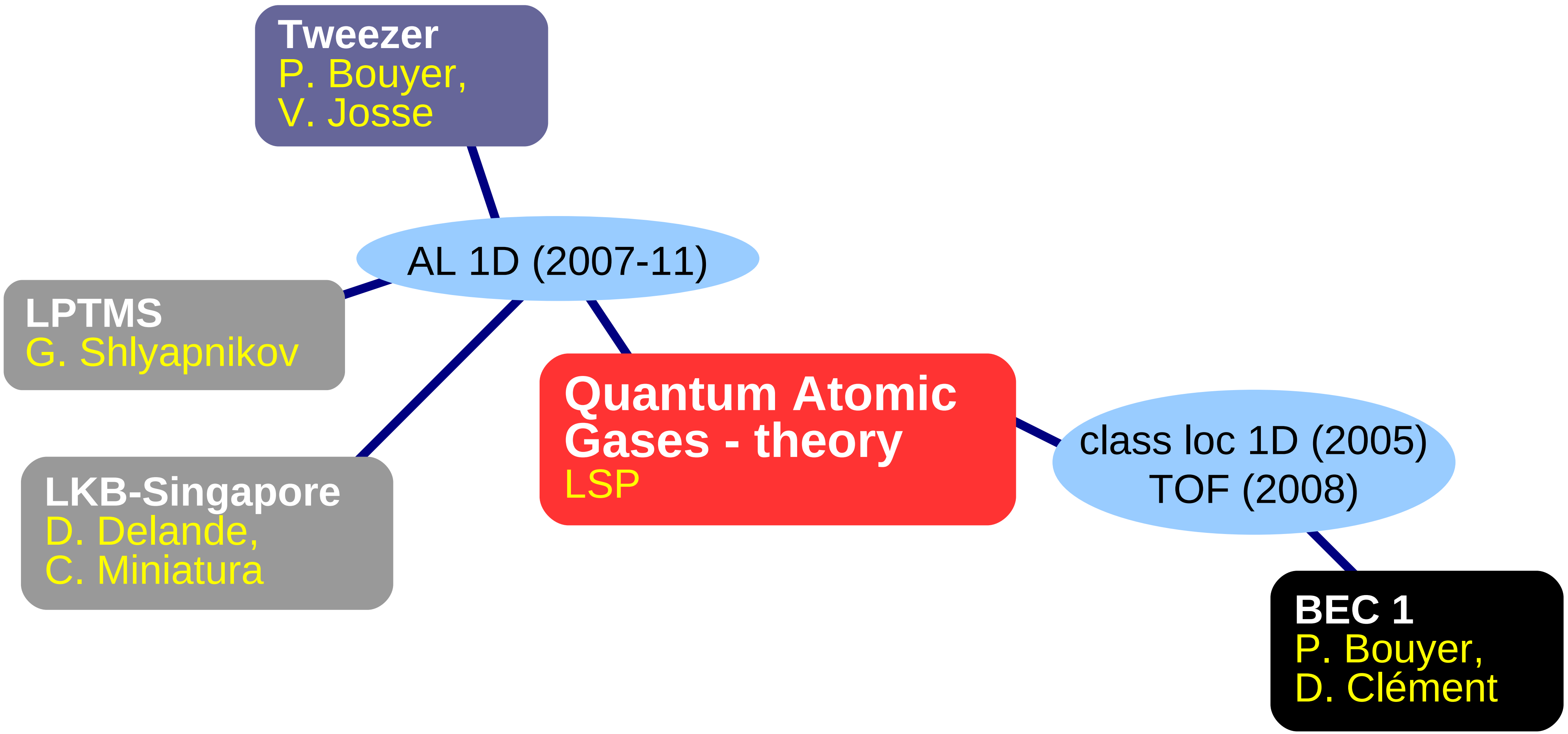
# Our Theoretical Work

**Quantum Atomic Gases - theory**
LSP

class loc 1D (2005)
TOF (2008)

**BEC 1**
P. Bouyer,
D. Clément

# Our Theoretical Work



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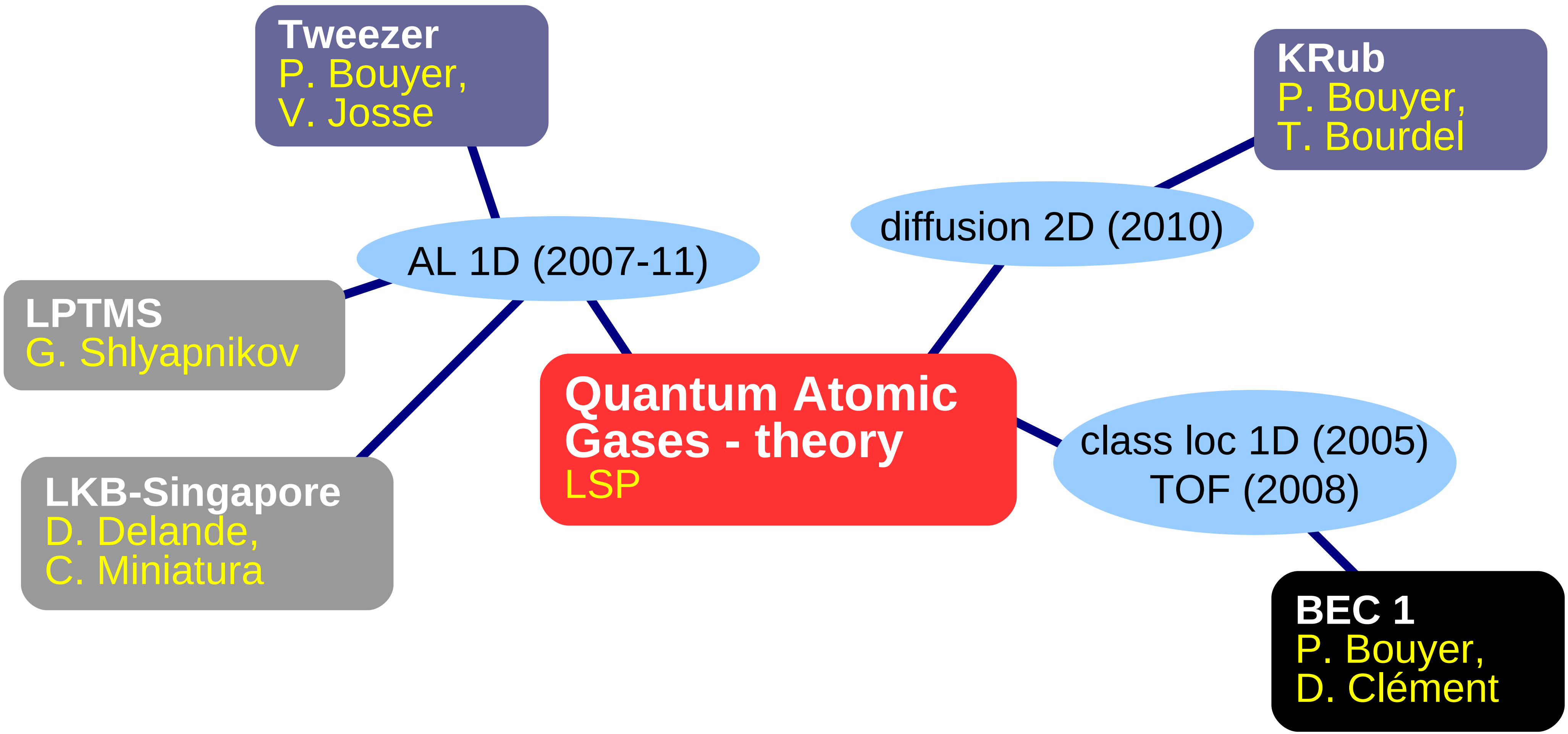

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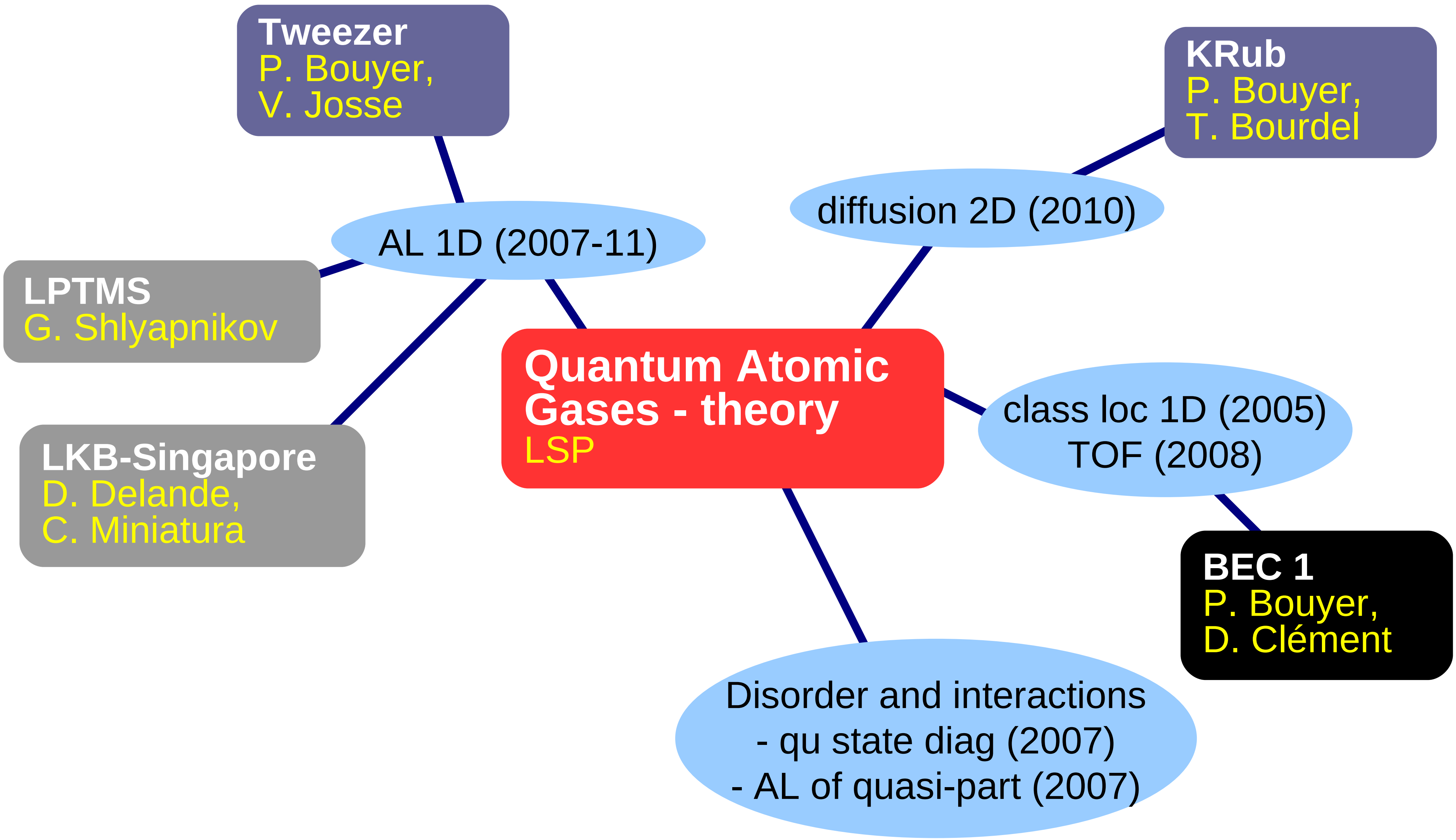

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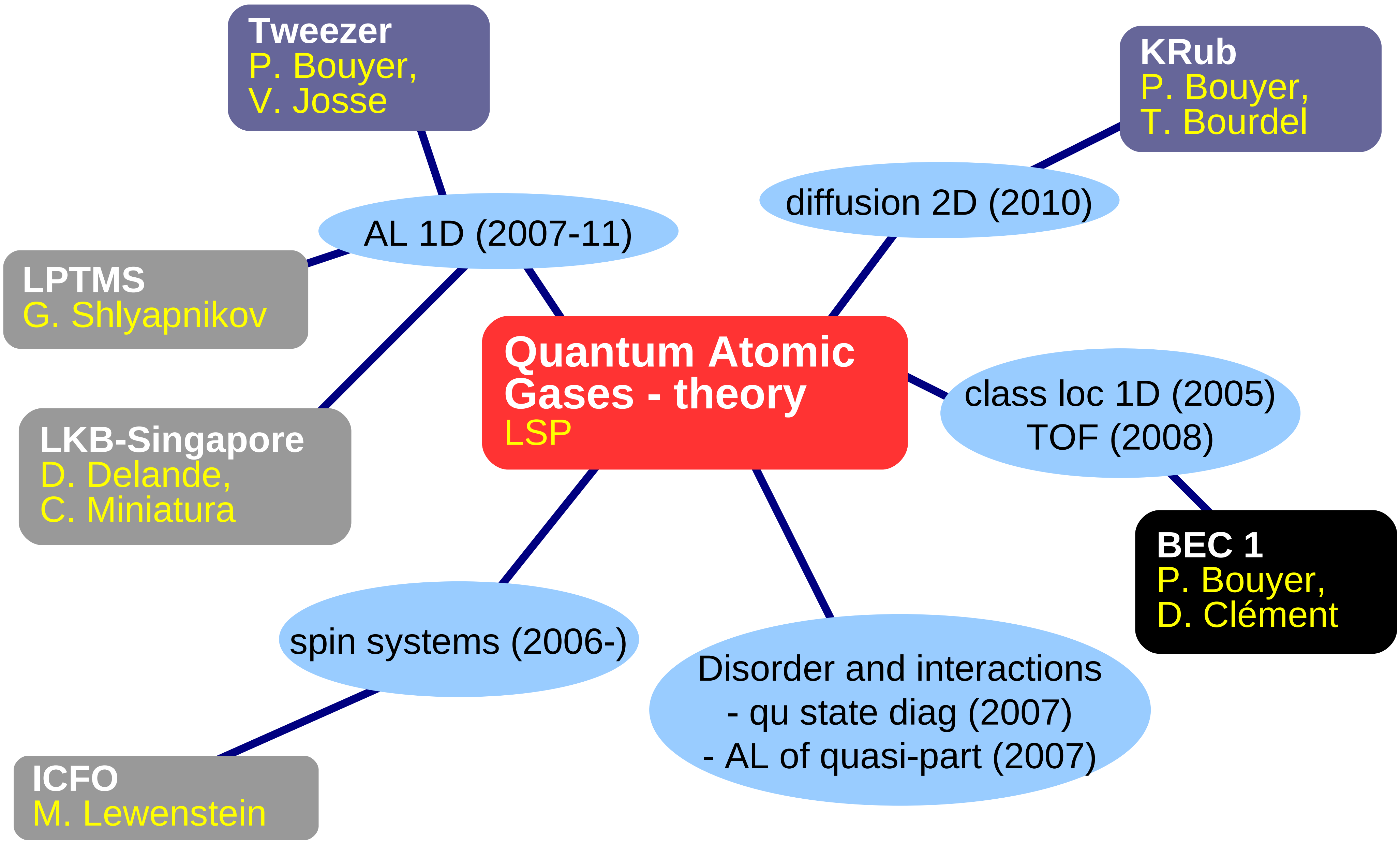

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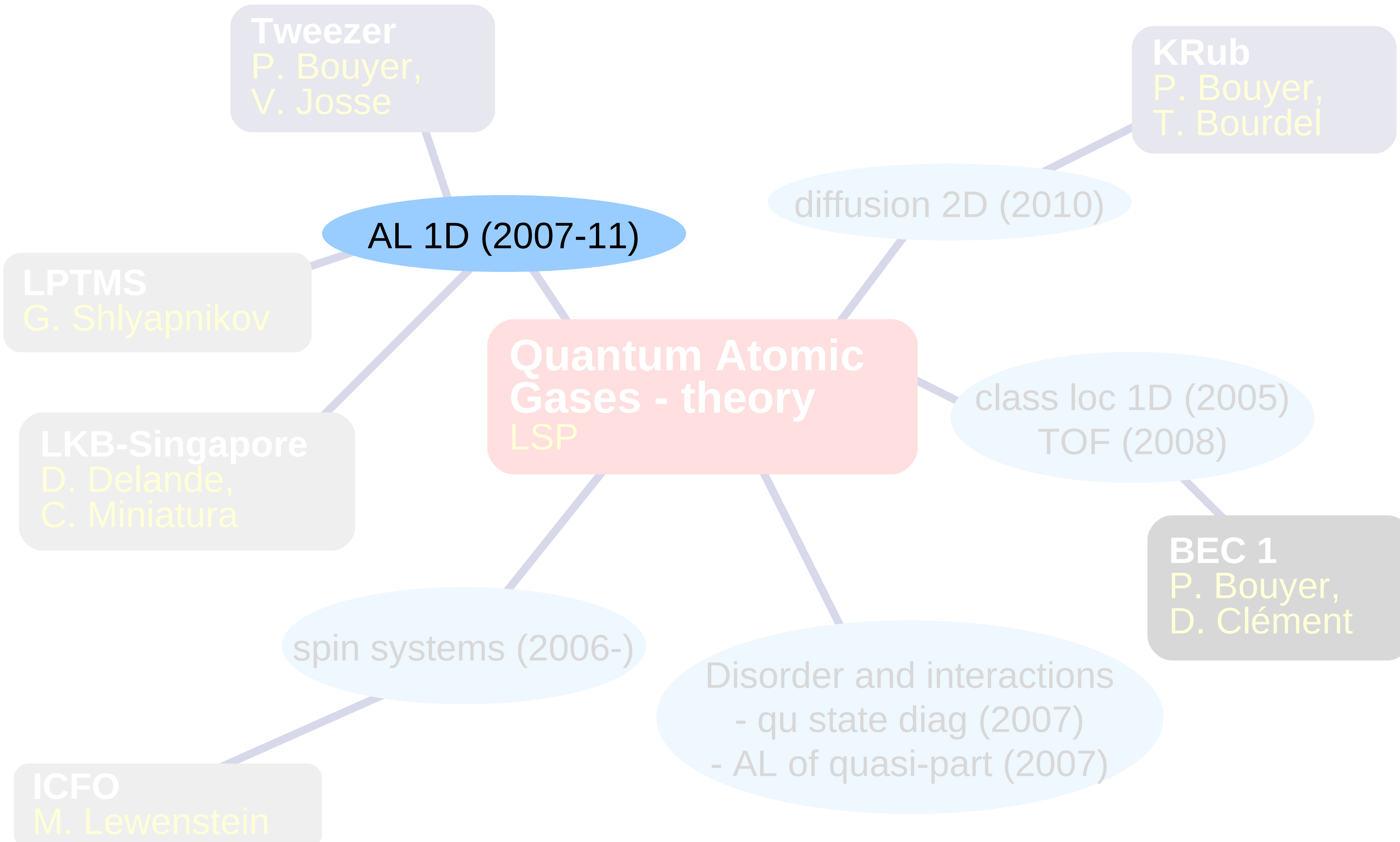


# Outlook

## 1. Anderson localization in a 1D speckle potential

### 1.1 Localization of a matter wave packet

- LSP *et al.*, Phys. Rev. Lett. **98**, 210401 (2007)
- J. Billy *et al.*, Nature **453**, 891 (2008)
- M. Piraud *et al.*, Phys. Rev. A (Rapid Comm.) (2011)

### 1.2 Algebraic localization in a speckle potential

- LSP *et al.*, Phys. Rev. Lett. **98**, 210401 (2007)
- P. Lugan *et al.*, Phys. Rev. A **80**, 023605 (2009)

## 2. Localization in the presence of a trap

- L. Pezzé and LSP, Phys. Rev. Lett. **106**, 040601 (2011)

## 3. Perspectives

# Ultracold Atoms : Simple Simulators or New Systems ?

**Ultracold atoms as ideal simulators**

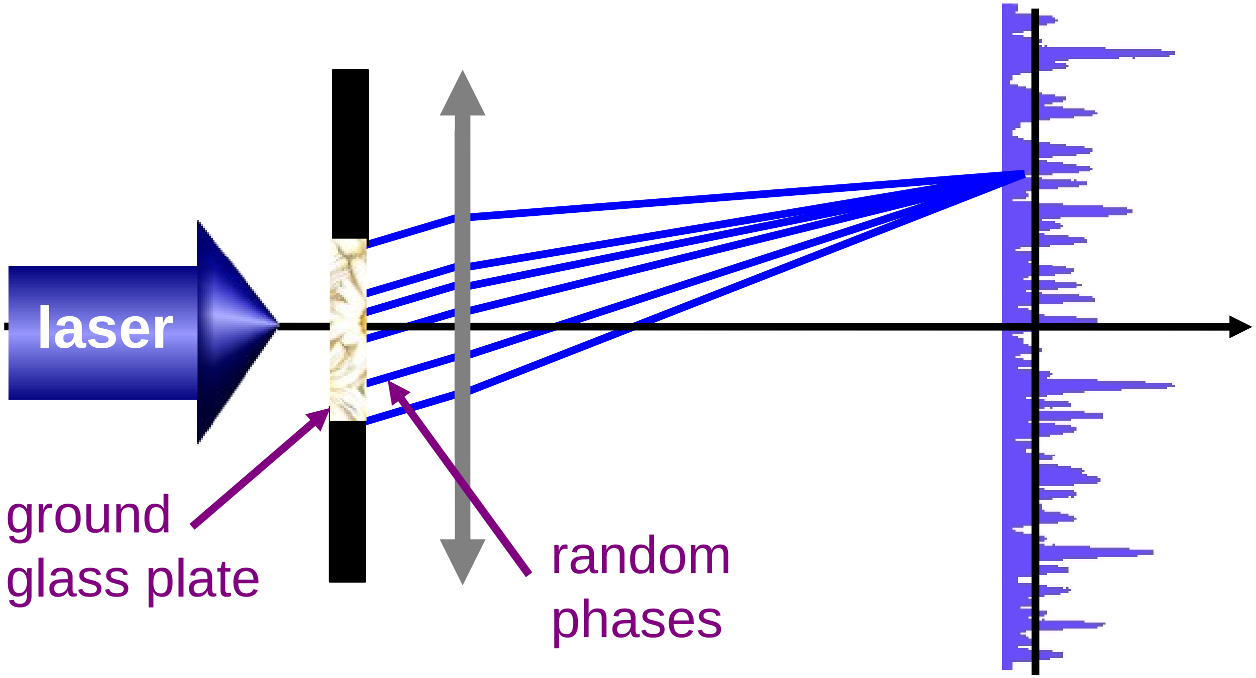
- flexibility
- high-precision measurement tools
- model systems (ab initio calculations)
- control of disorder

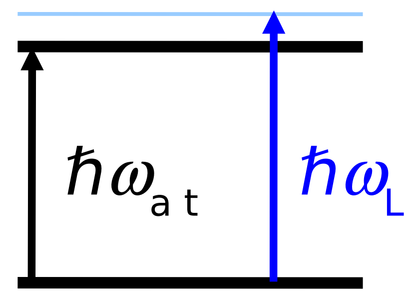
**New issues with ultracold atoms**

- localization of wavepackets
- long-range correlations of the disorder
- what about the trap ?

# Controlled Disorder in Quantum Gases

D. Clément *et al.*,
New J. Phys. **8**, 165 (2006)




<u>*Speckles: an original class of disorder*</u>

- $\mathcal{E}(\boldsymbol{r})$ is a Gaussian random process
- $V(\boldsymbol{r}) \propto |\mathcal{E}(\boldsymbol{r})|^2 - \langle|\mathcal{E}|^2\rangle$ is not Gaussian
  is not symmetric
- $C_n(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_n) = \langle V(\boldsymbol{r}_1) \ldots V(\boldsymbol{r}_n)\rangle$ all
  determined by $C_{\mathcal{E}}(\boldsymbol{r}) = \langle \mathcal{E}(\boldsymbol{r})^* \mathcal{E}(\boldsymbol{0})\rangle$

<u>*Experimental control*</u>

$$C_2(z) = \langle V(z)V(0)\rangle - \langle V\rangle^2$$

- $V_R \propto I_L / \Delta$
- $\sigma_R$: correlation length
  (depends only on the intensity
  profile of the ground-glass plate)

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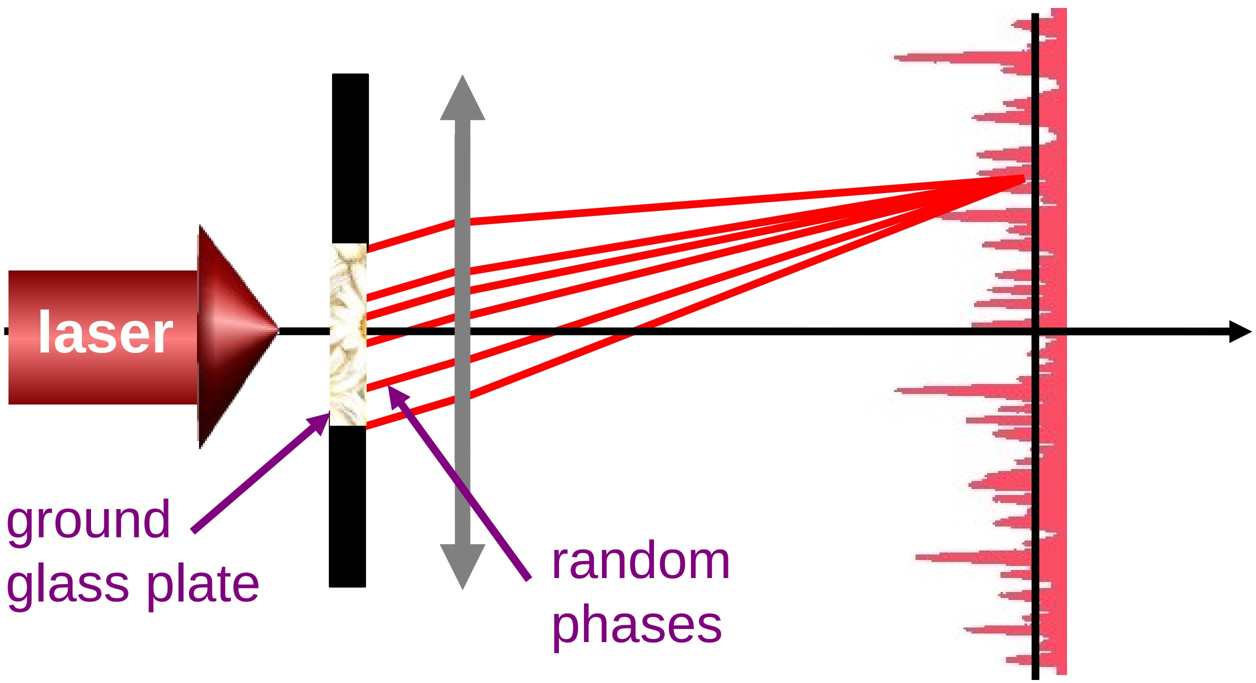


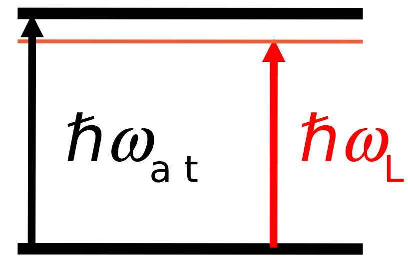


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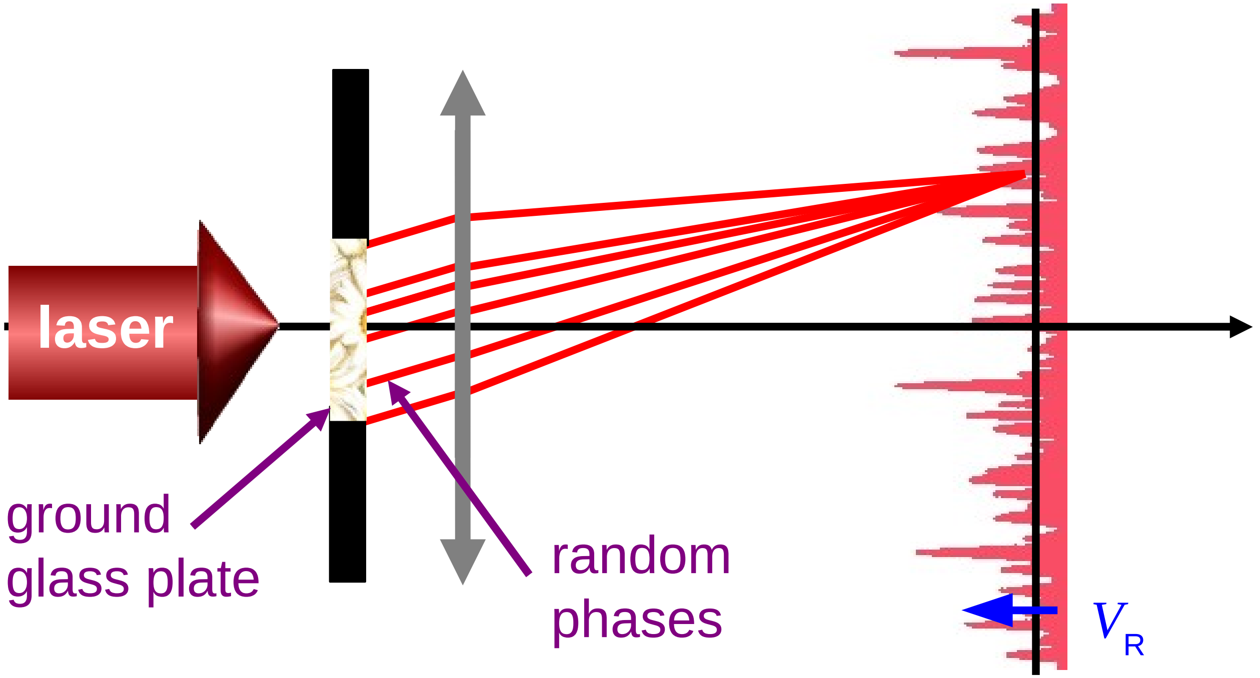


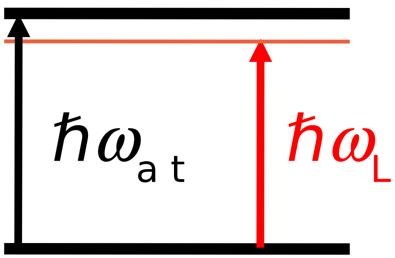


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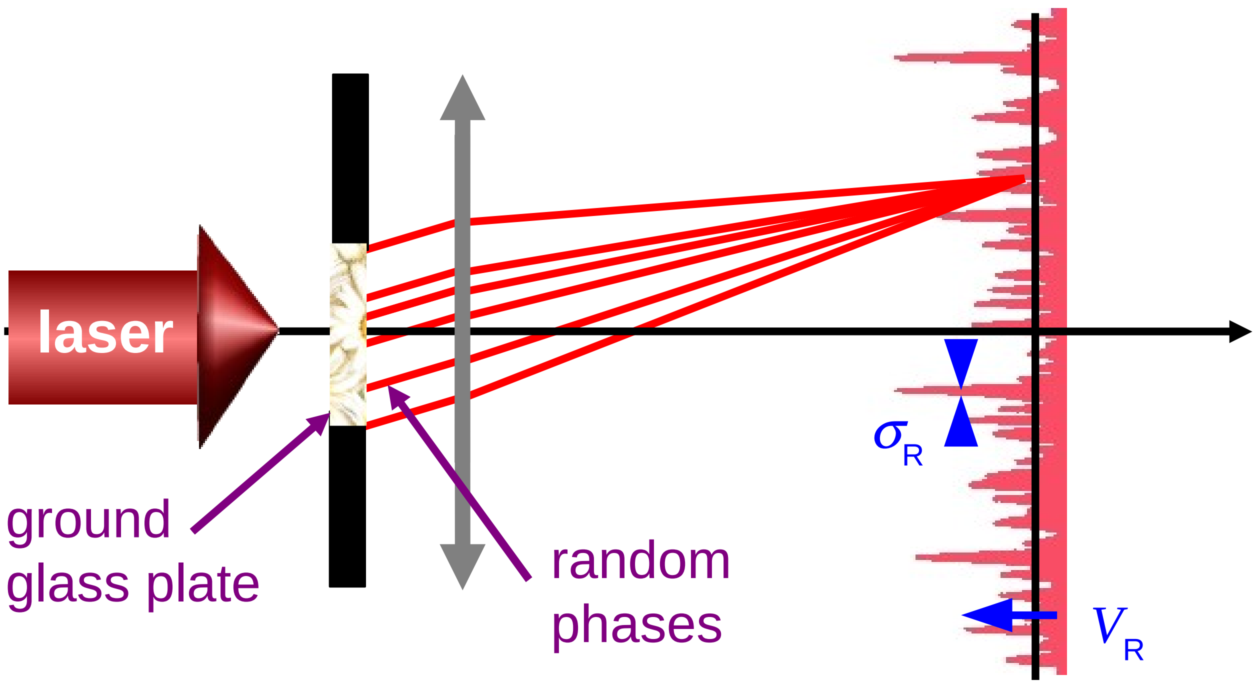


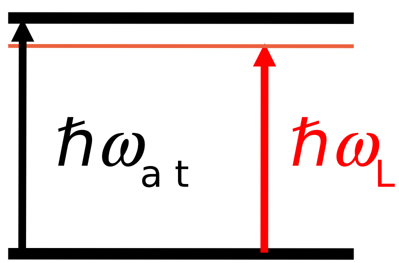


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1.1 Localization of a matter wave packet

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3. Perspectives

# Anderson Localization of a Matterwave : How to Observe It

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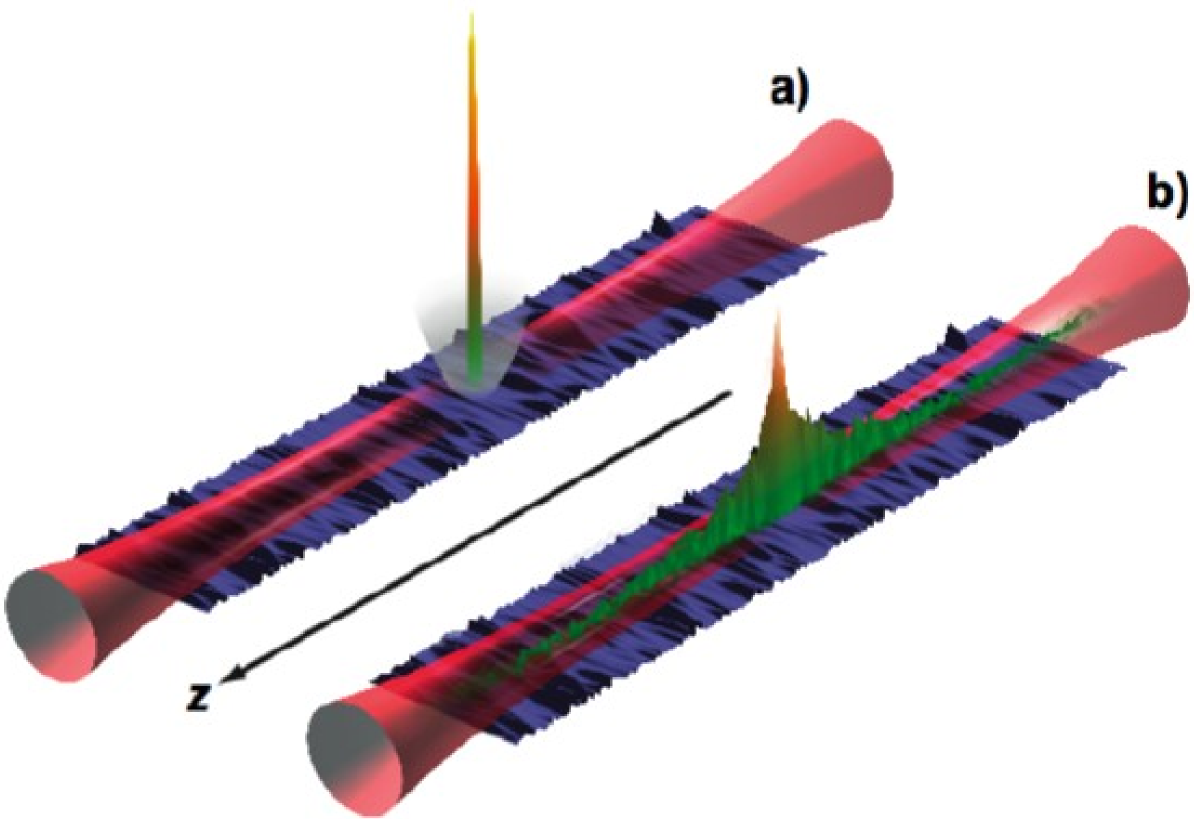
J. Billy *et al.*, Nature **453**, 891 (2008)




*Superposition of matter waves with a broad energy distribution*

$$n(z,t \to \infty) \simeq \int dE \, \mathcal{D}_E(E) \, P(z,t \to \infty | E)$$

see also G. Roati et al., Nature **453**, 895 (2008)

# Anderson Localization of a Matterwave : Theoretical Model

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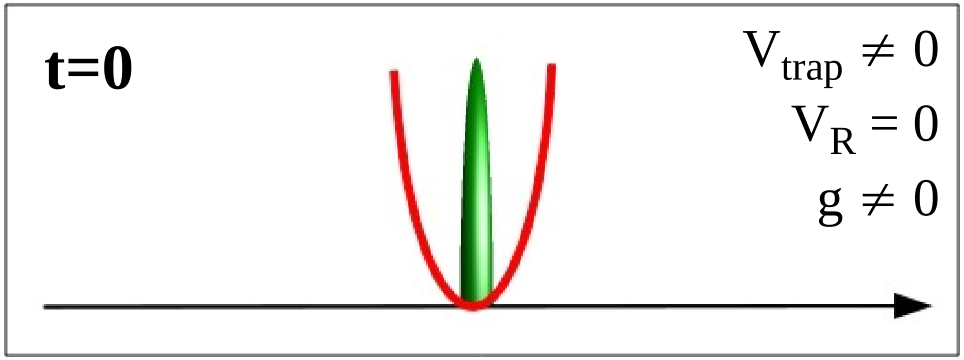
M. Piraud *et al.*, Phys. Rev. A (Rapid Comm.) (2011)




- Bose-Einstein condensate in a harmonic trap
  - Thomas-Fermi regime :

$$n_0(z) \propto 1 - (z/L_{\mathrm{TF}})^2$$

$$\theta(z) \equiv 0$$

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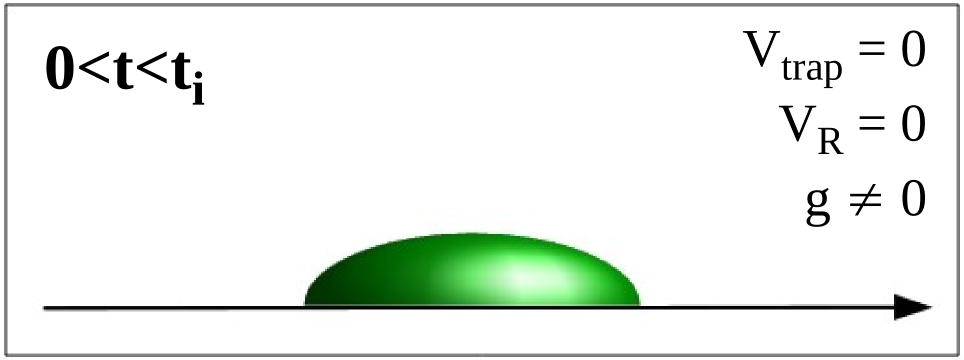


- Interaction-driven expansion of the BEC
  - Scaling theory of expanding BEC (in TF regime)
  - Momentum distribution :

$$\mathcal{D}_{\mathrm{i}}(p) \propto \left[1 - (p\xi_{\mathrm{in}}/\hbar)^2\right]_{\oplus}$$

$$\xi_{\mathrm{in}} \equiv \hbar/\sqrt{4m\mu}$$

Yu. Kagan, E. L. Surkov, and G.V. Shlyapnikov, Phys. Rev. A **54**, R1753 (1996)
Y. Castin and R. Dum, Phys. Rev. Lett. **77**, 5315 (1996)

# Anderson Localization of a Matterwave : Theoretical Model

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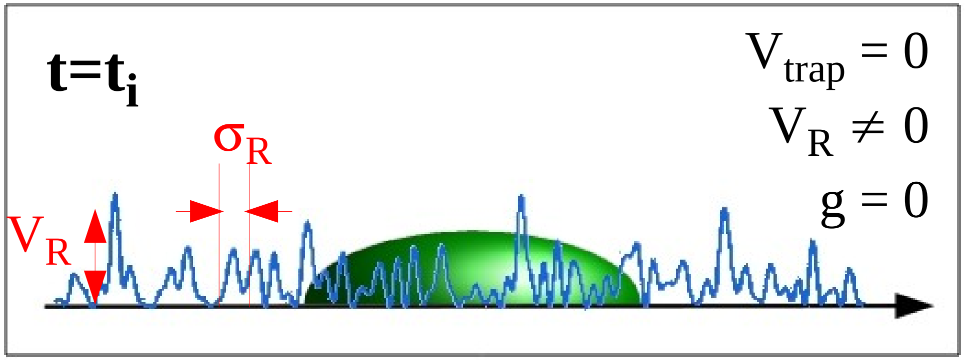


- Interactions off and disorder on
  - « p » is no longer a good quantum number but « E » is !

$$\mathcal{D}_E(E) = \int dp\ A(p, E)\mathcal{D}_{\mathrm{i}}(p)$$

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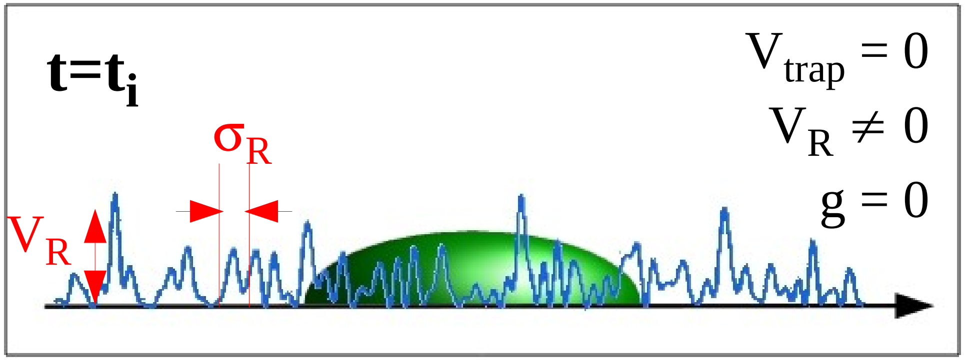


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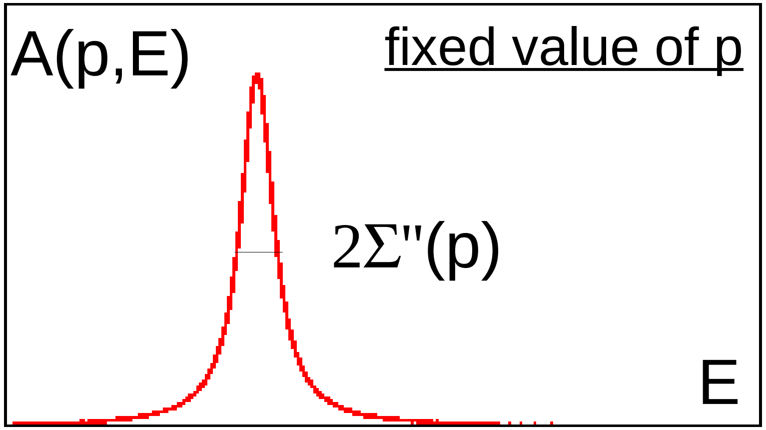
Spectral function




FT of the correlation function of the disorder

$$\Sigma''(p) \simeq -(m/2\hbar p)\{\tilde{C}(0) + \tilde{C}(2p)\}$$

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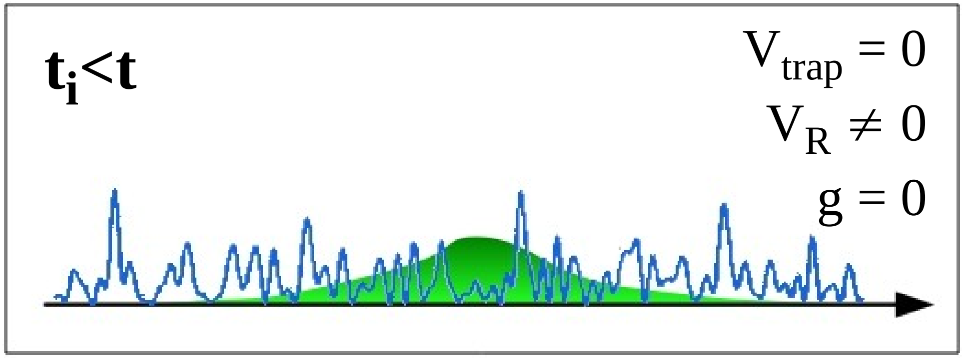


- Localization of the wave packet

$$P_\infty(z|E) = \frac{\pi^2\gamma(E)}{8}\int_0^\infty \mathrm{d}u\ u\sinh(\pi u)\left[\frac{1+u^2}{1+\cosh(\pi u)}\right]$$
$$\times\exp\left\{-(1+u^2)\gamma(E)|z|/2\right\},$$

short distance : $\ln(P) \sim \quad 2\gamma(E)|z|$
long distance : $\ln(P) \sim 0.5\gamma(E)|z|$

V.L. Berezinskii, Sov. Phys. JETP **38**, 620 (1974)
A.A. Gogolin *et al.*, Sov. Phys. JETP **42**, 168 (1976)

# Anderson Localization of a Matterwave : Theoretical Model

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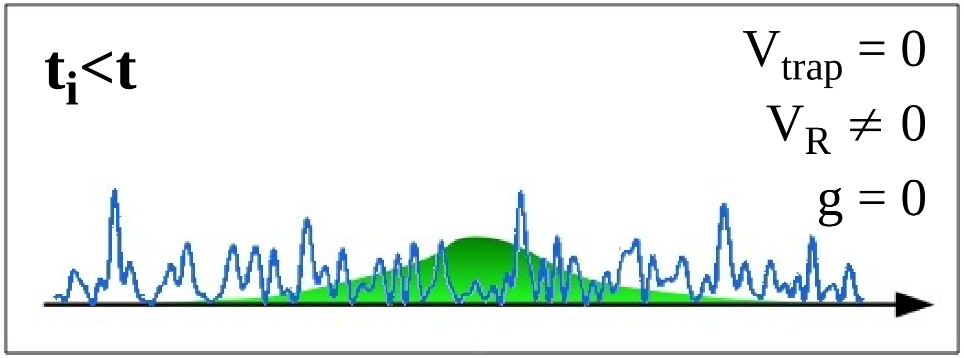


- Localization of the wave packet

$$P_\infty(z|E) = \frac{\pi^2\gamma(E)}{8}\int_0^\infty \mathrm{d}u\; u\sinh(\pi u)\left[\frac{1+u^2}{1+\cosh(\pi u)}\right]^2 \times \exp\left\{-(1+u^2)\gamma(E)|z|/2\right\},$$

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V.L. Berezinskii, Sov. Phys. JETP **38**, 620 (1974)
A.A. Gogolin *et al.*, Sov. Phys. JETP **42**, 168 (1976)

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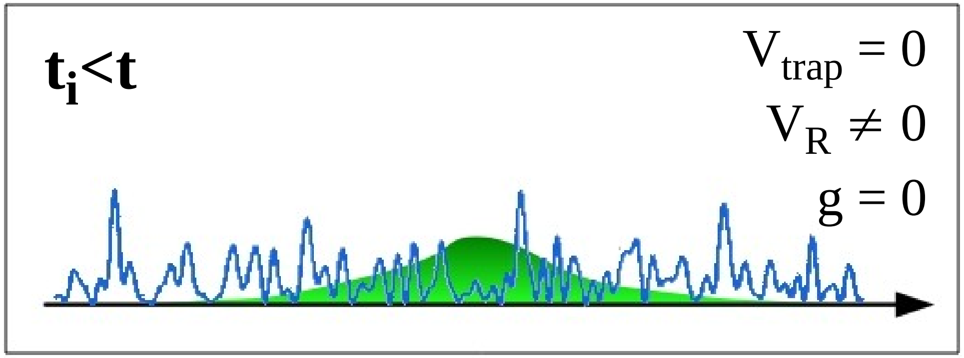


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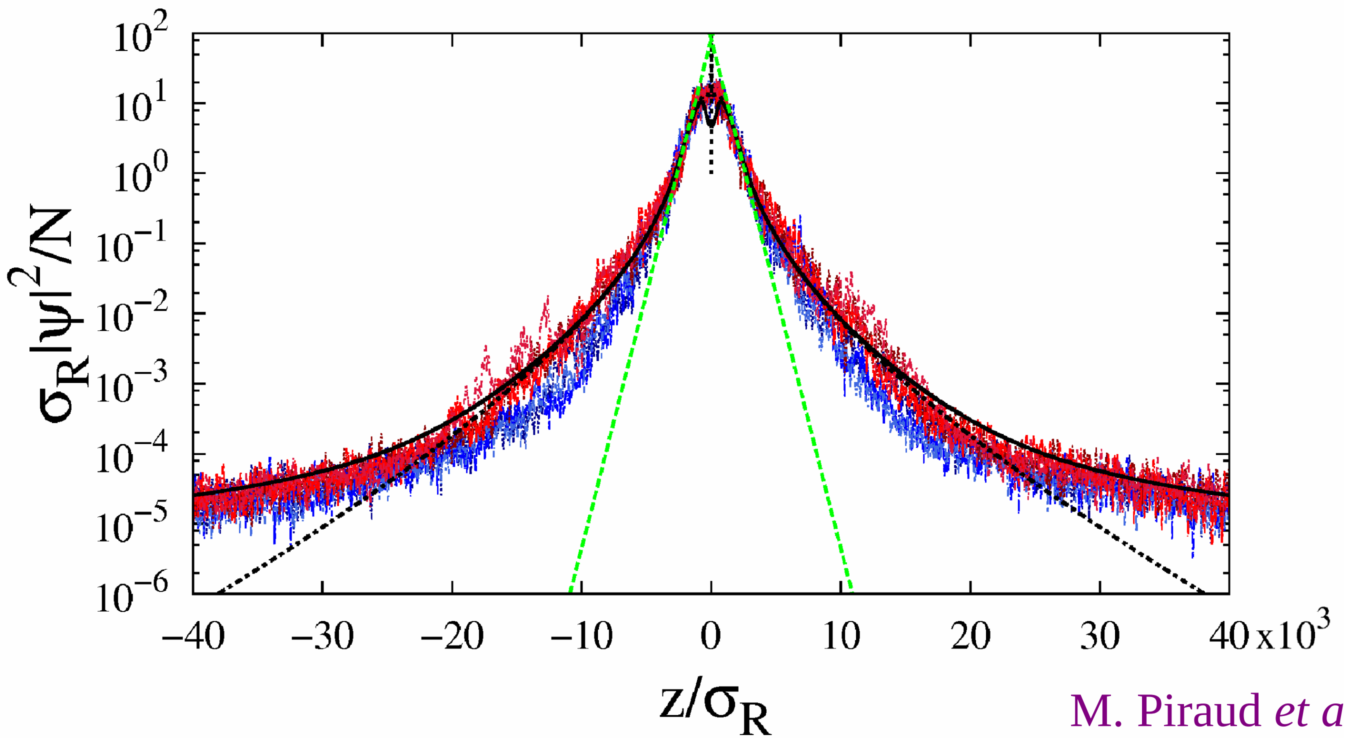


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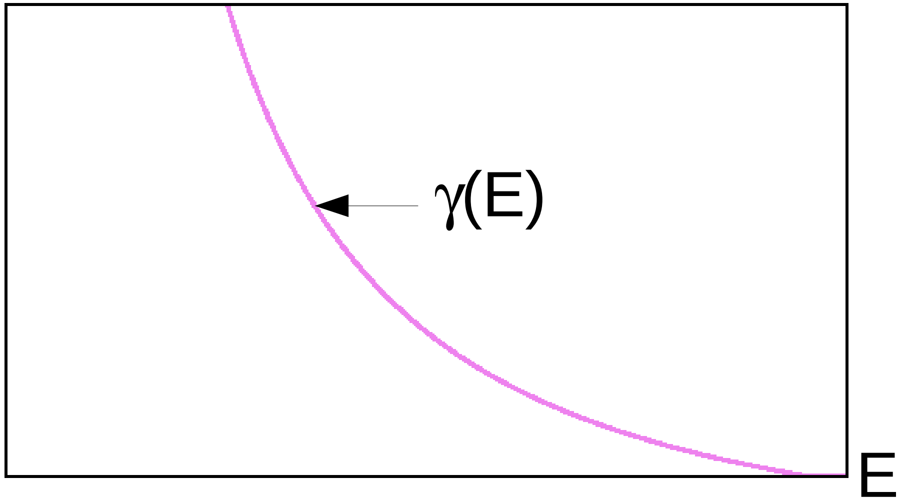


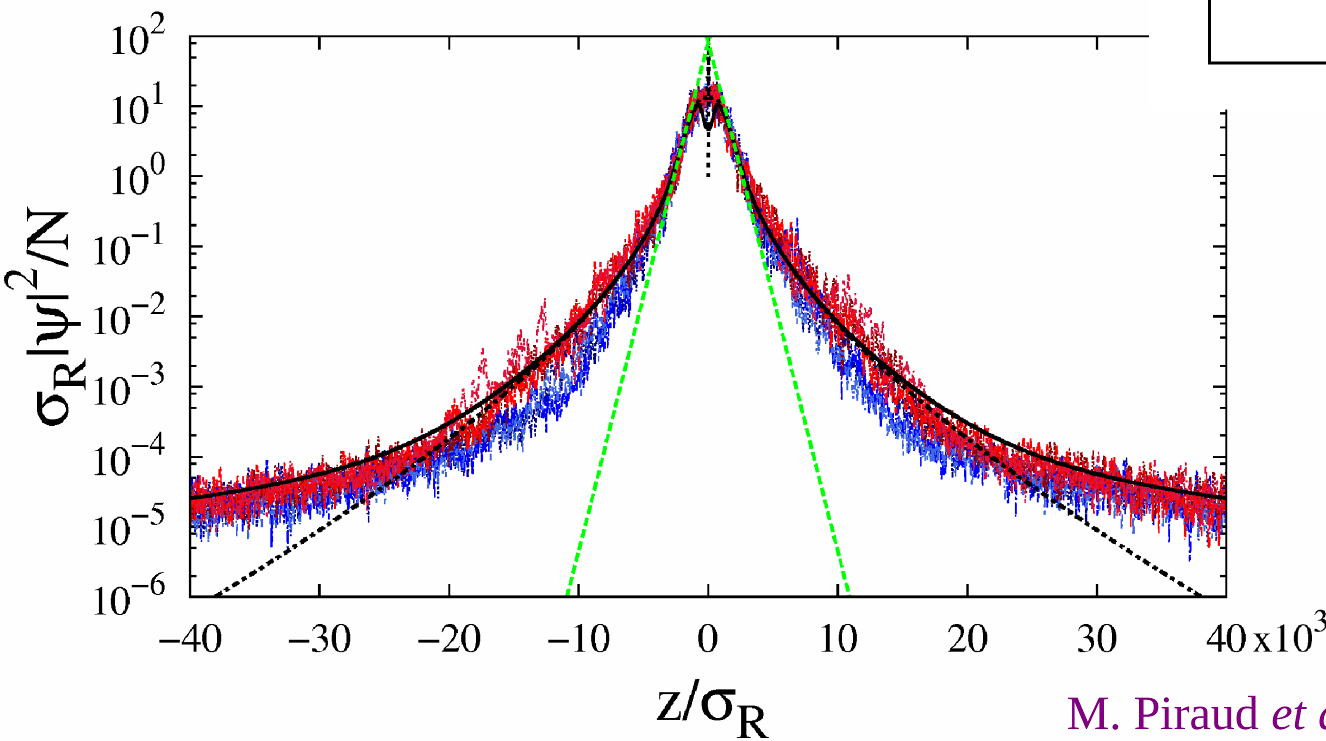


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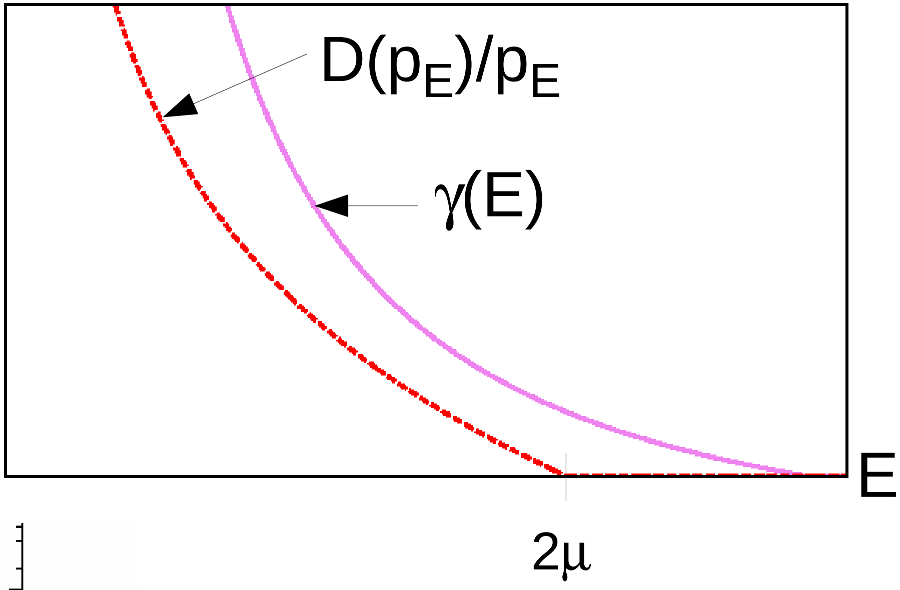


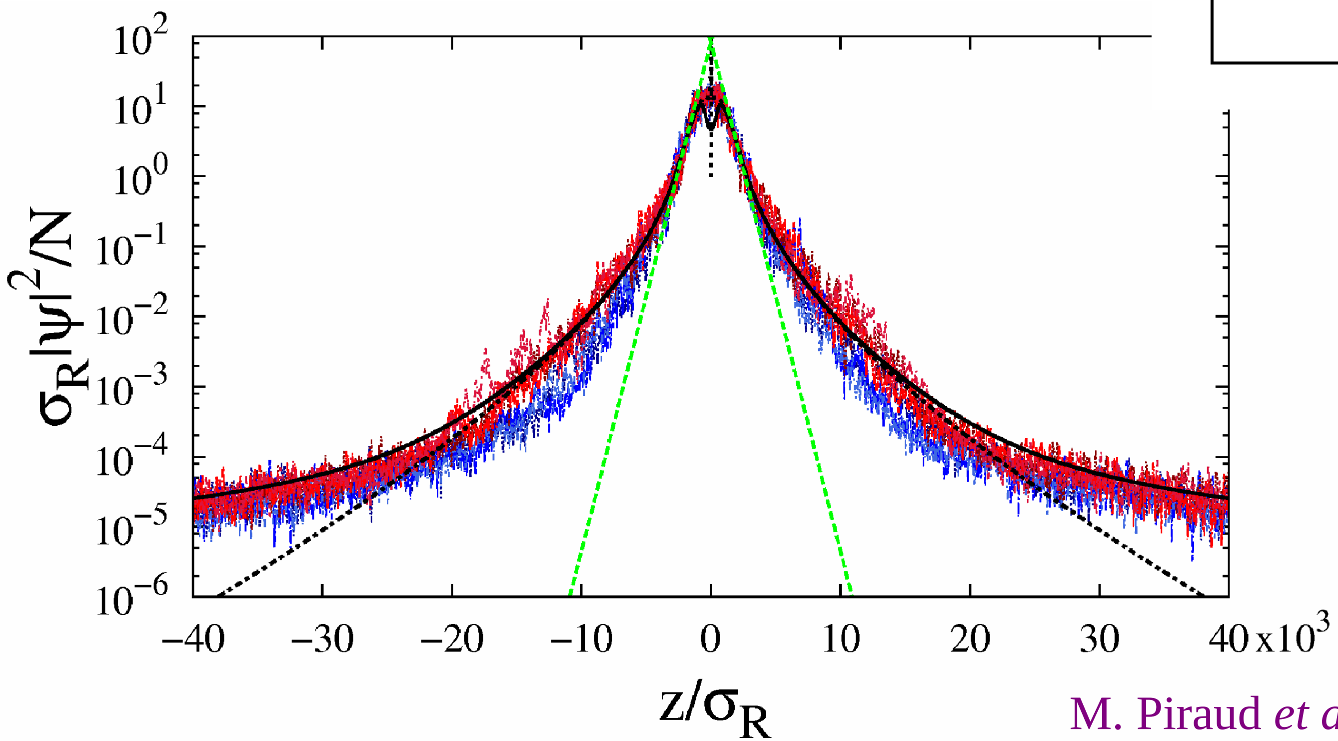


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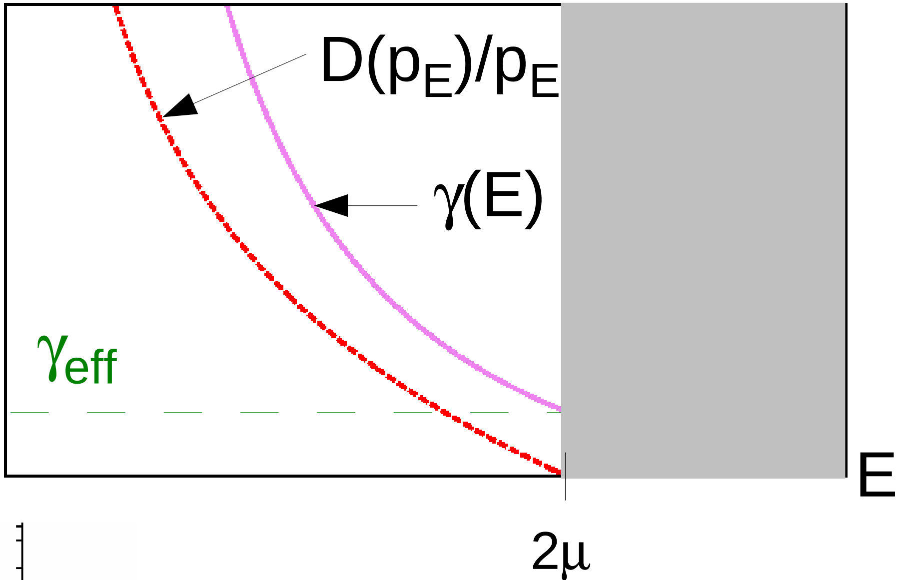


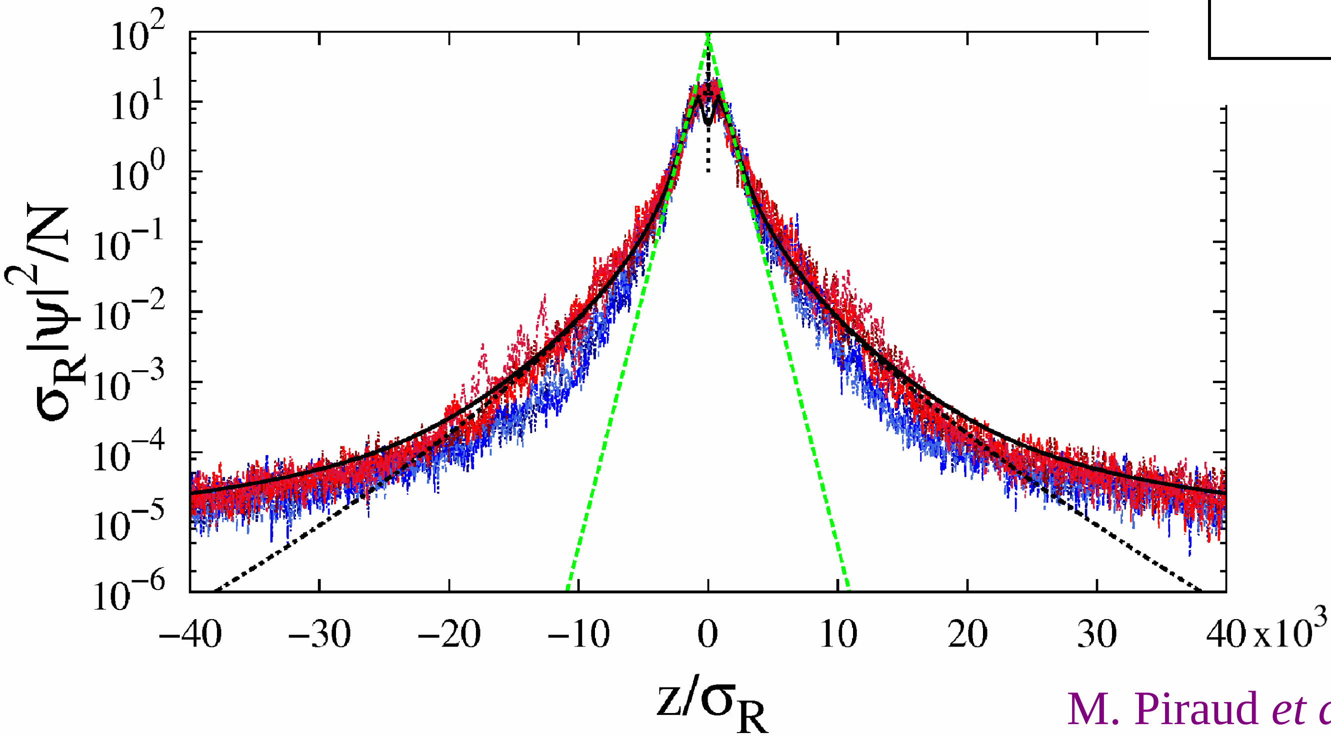


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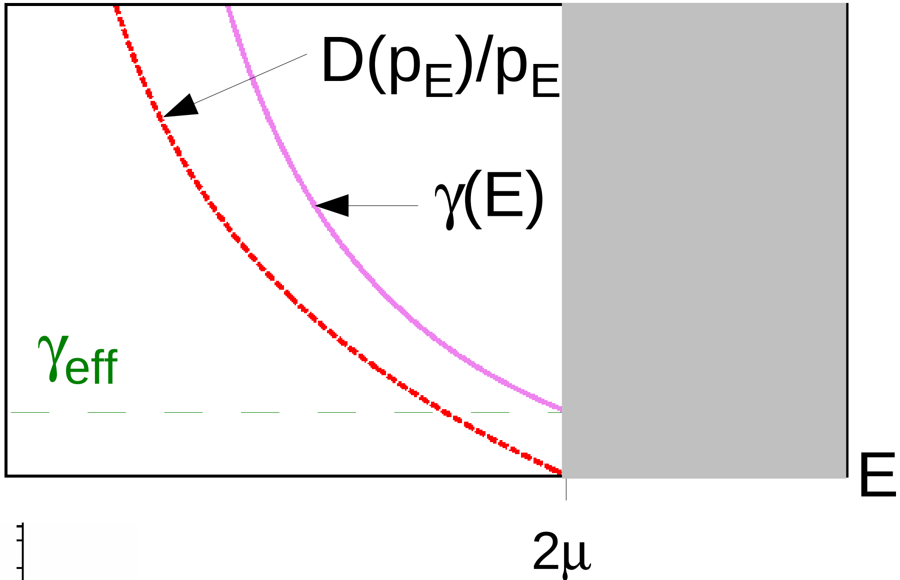


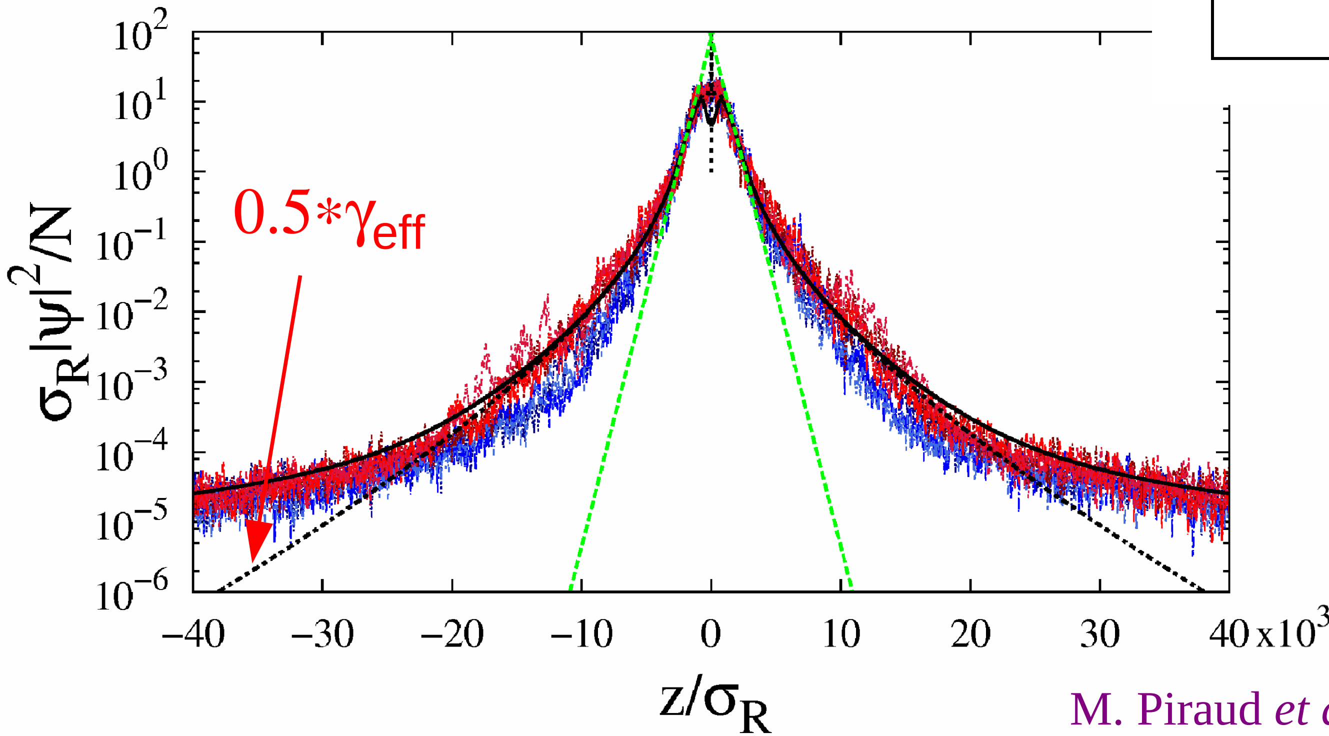


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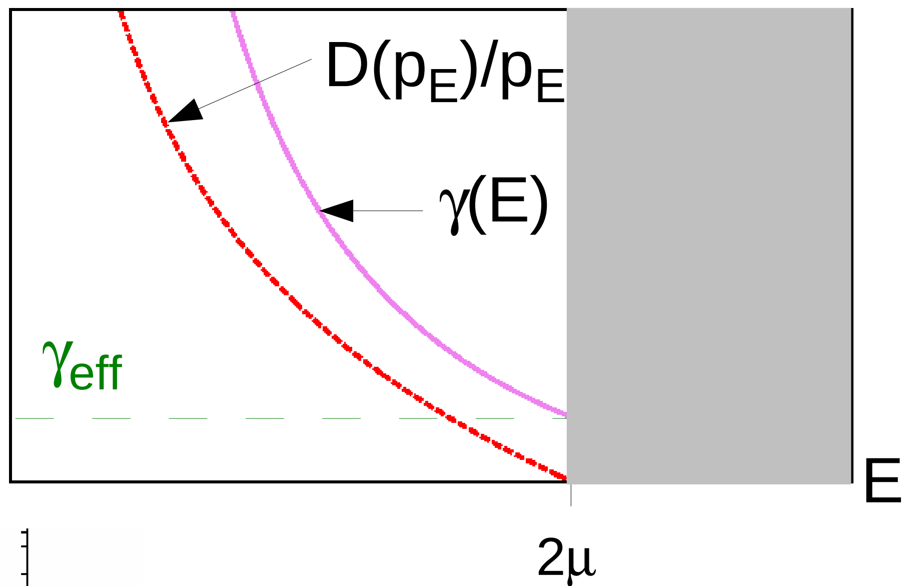


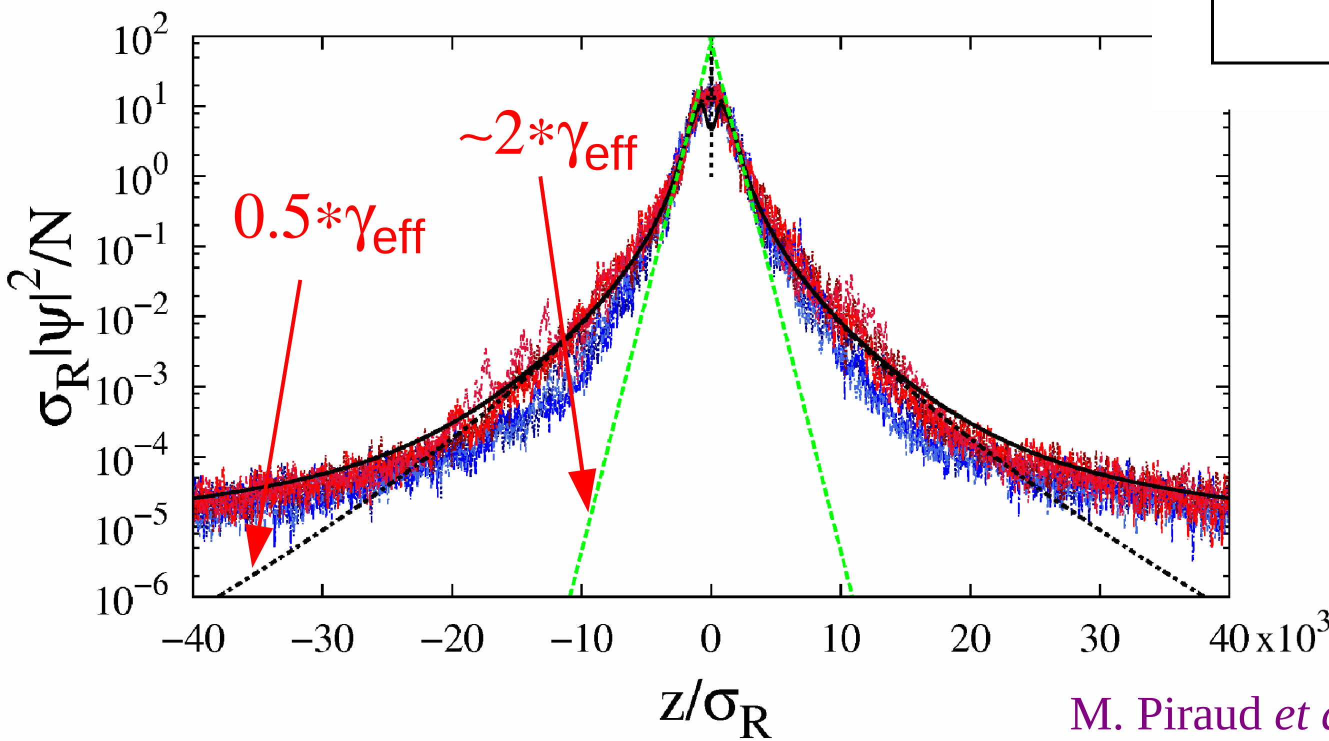


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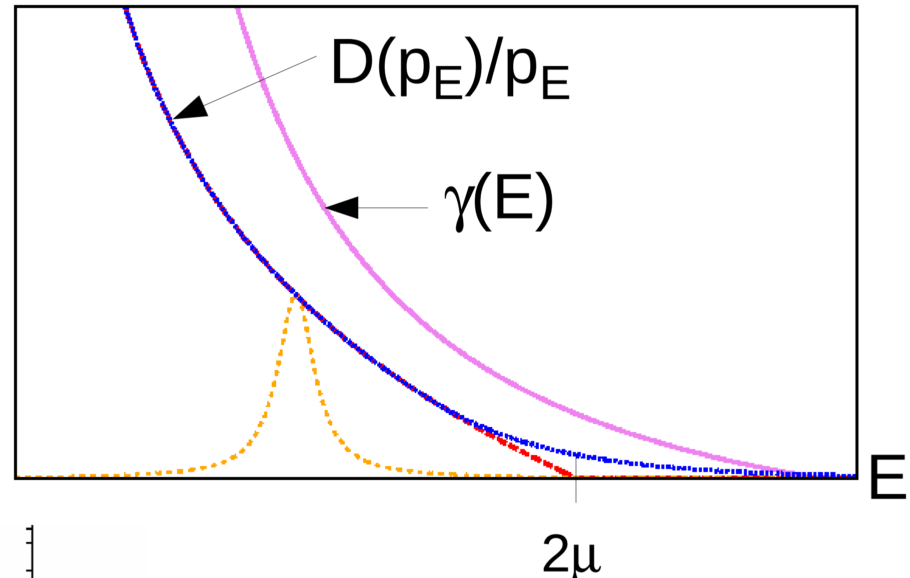


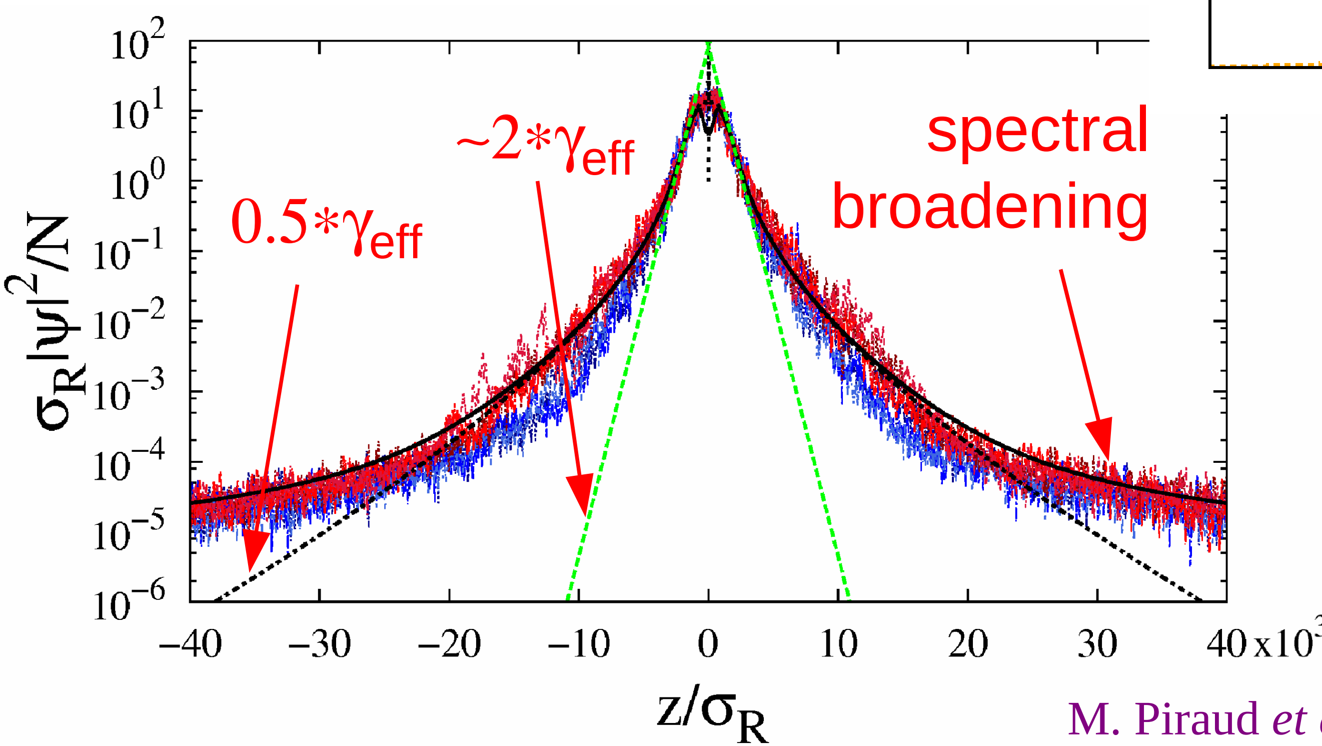


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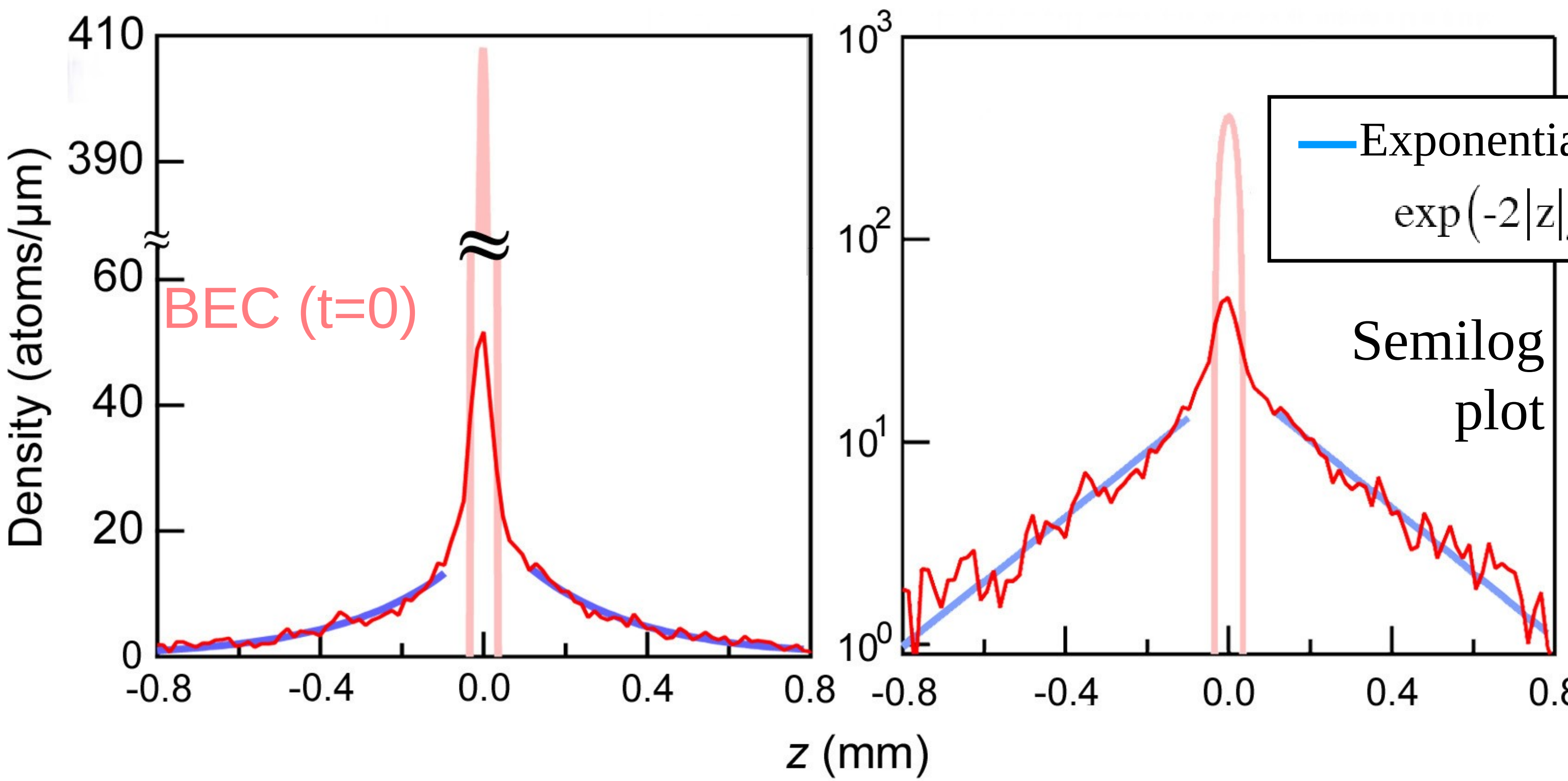
Weak disorder : $V_R/\mu_{in}$ =0.12 << 1




see also G. Roati et al., Nature **453**, 895 (2008)

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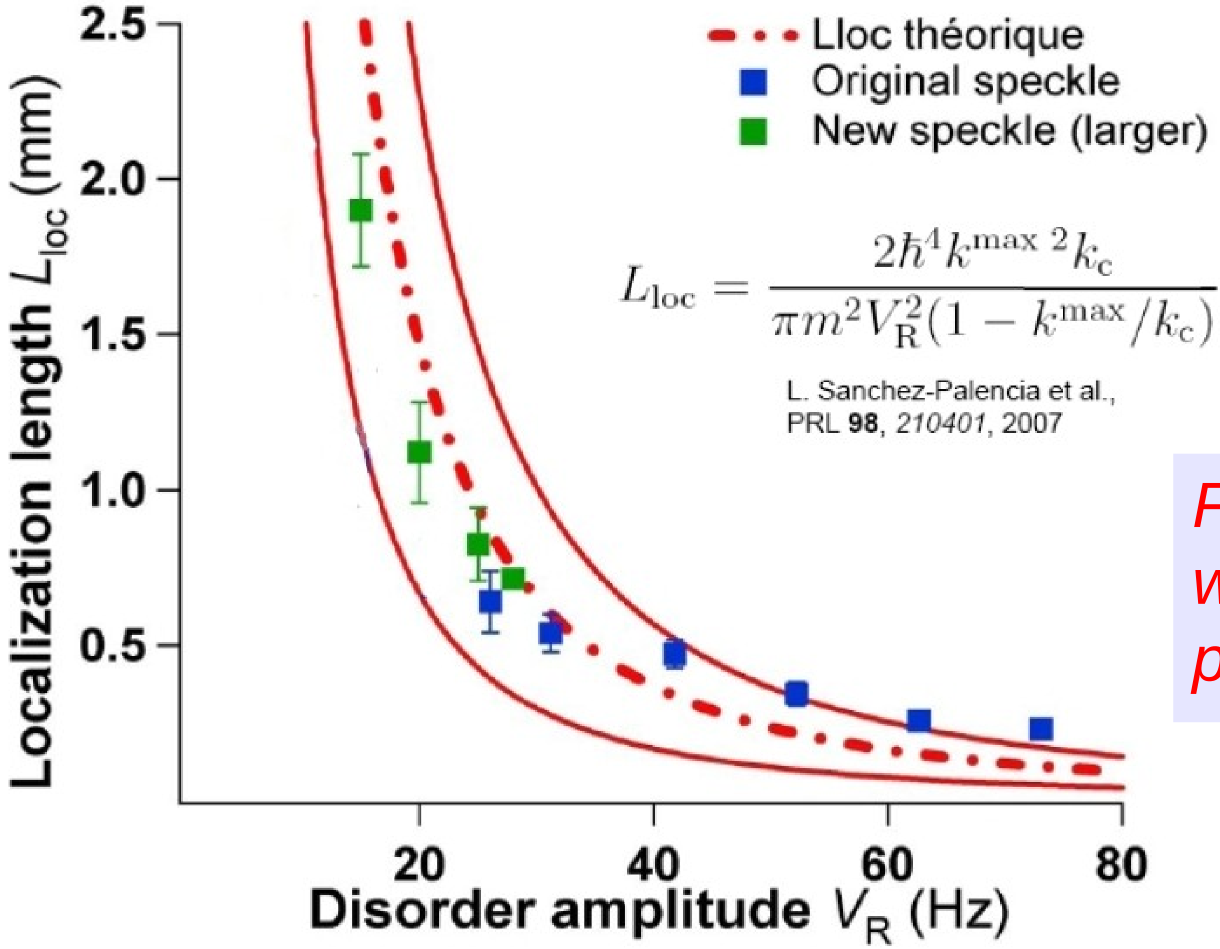


*Fair agreement with theoretical predictions*

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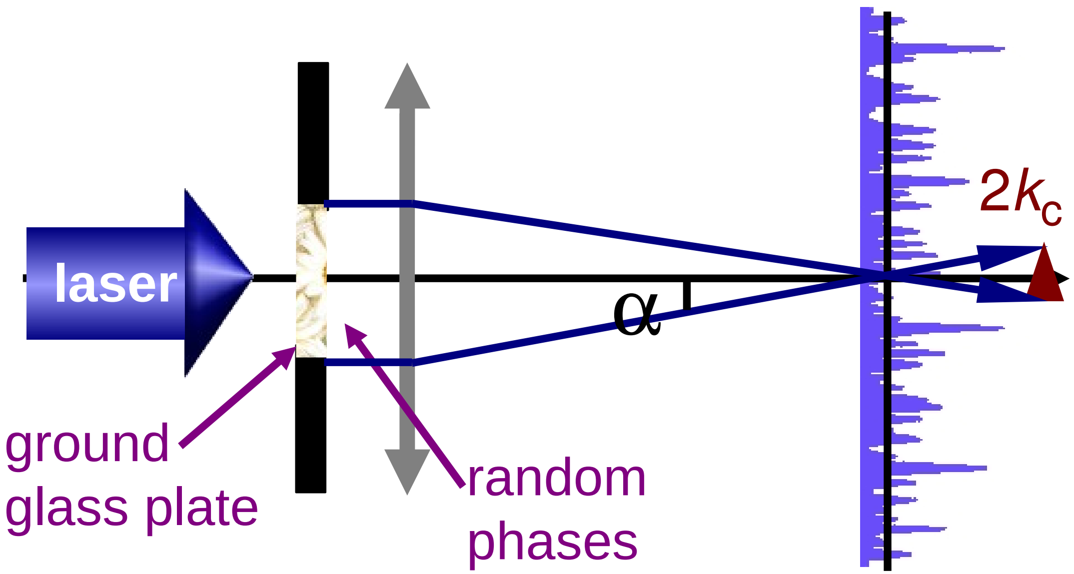


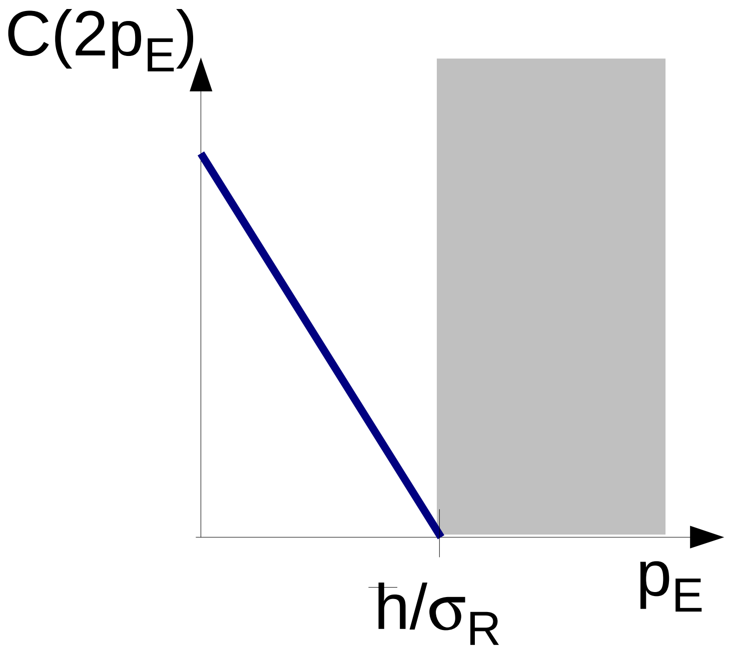

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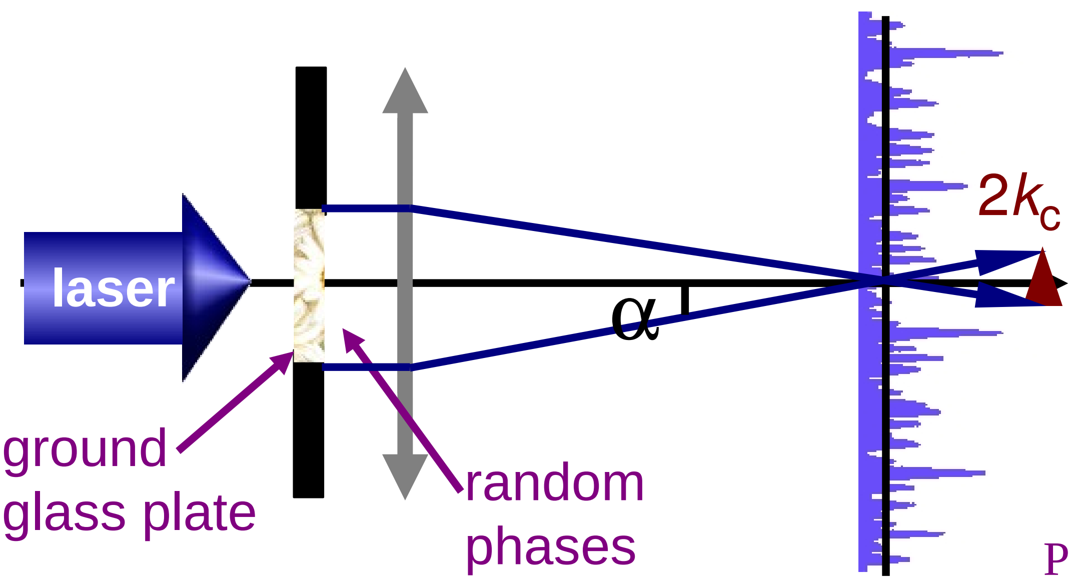


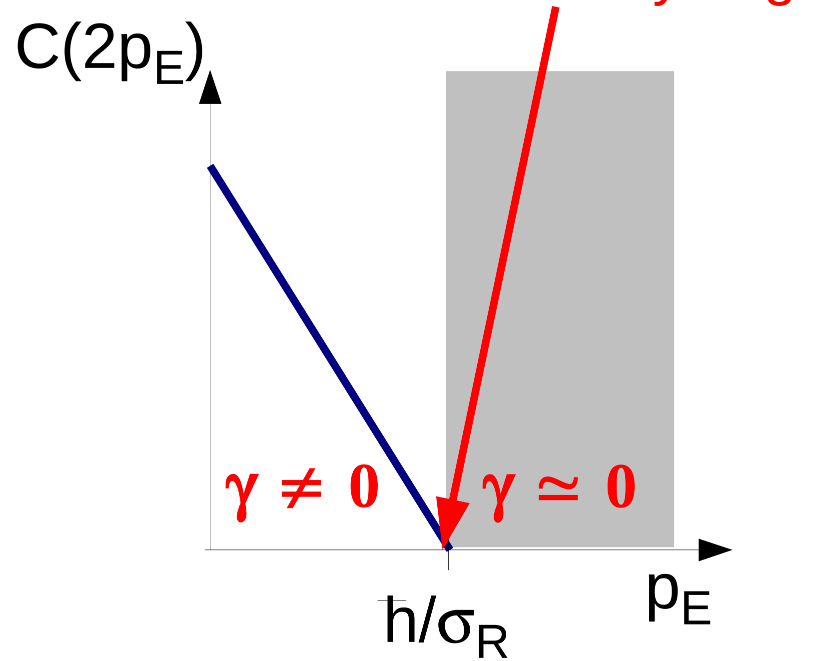


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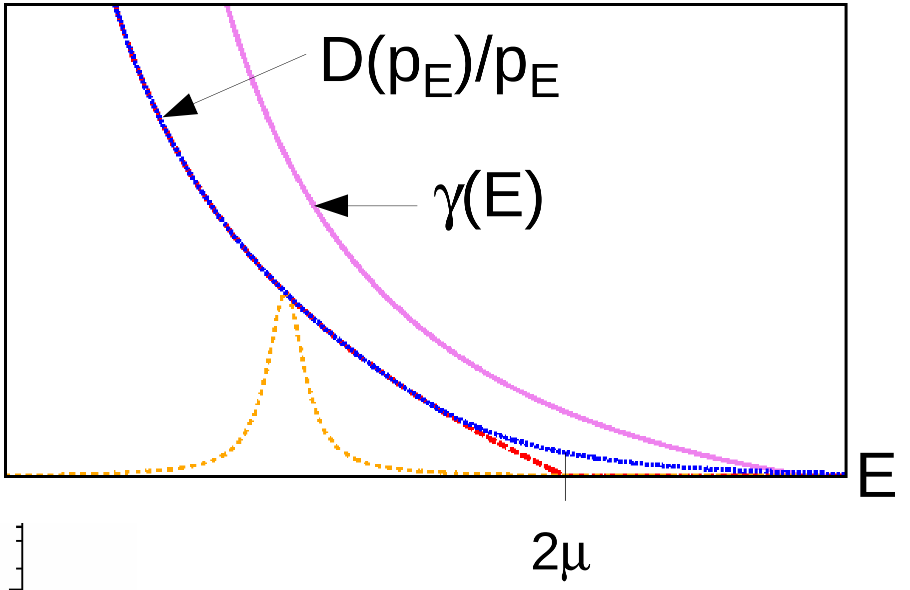


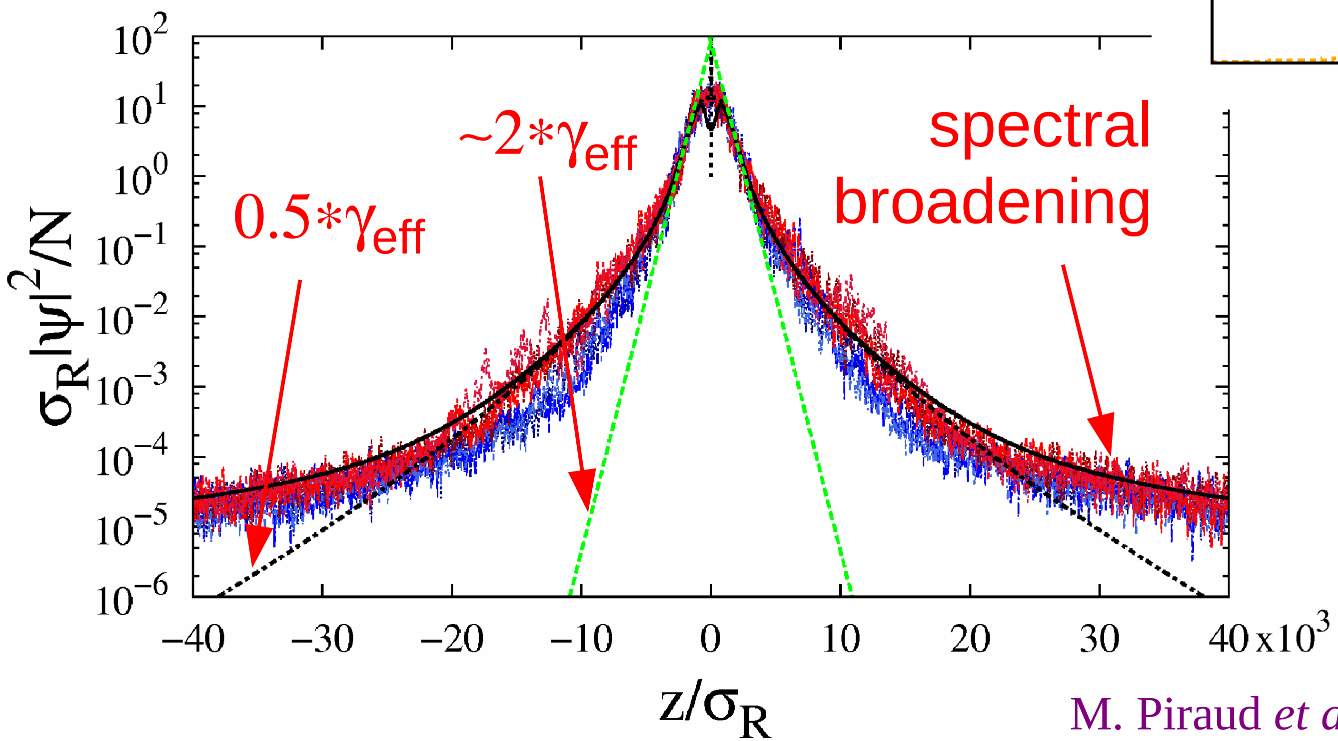


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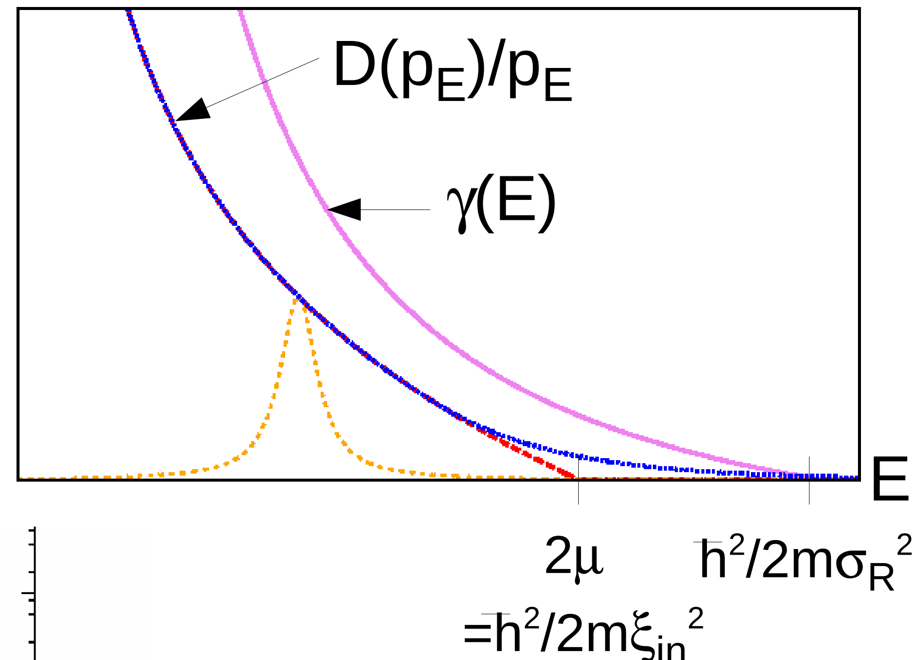


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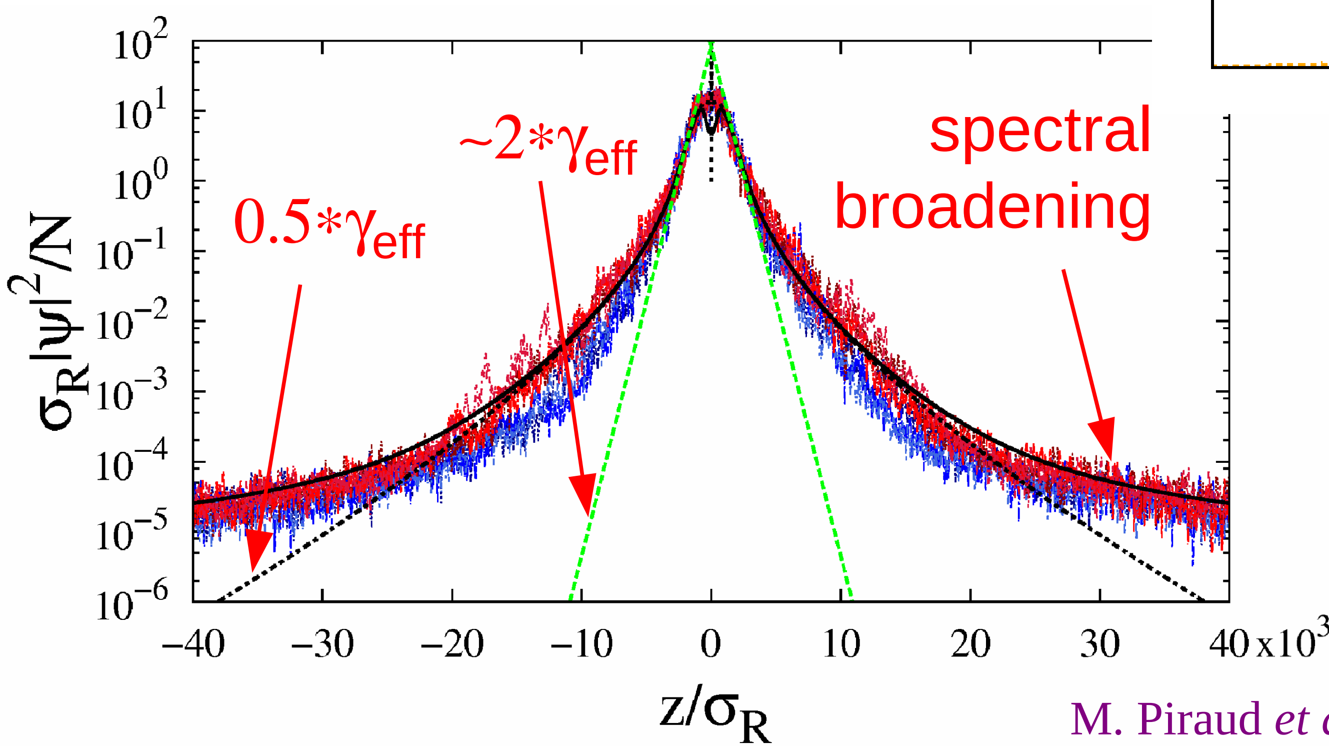


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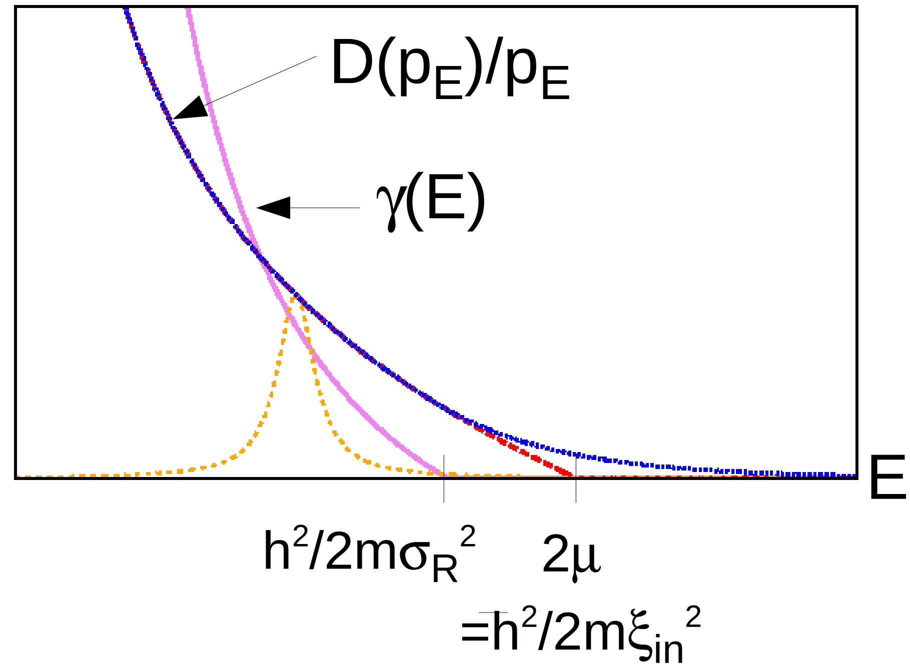


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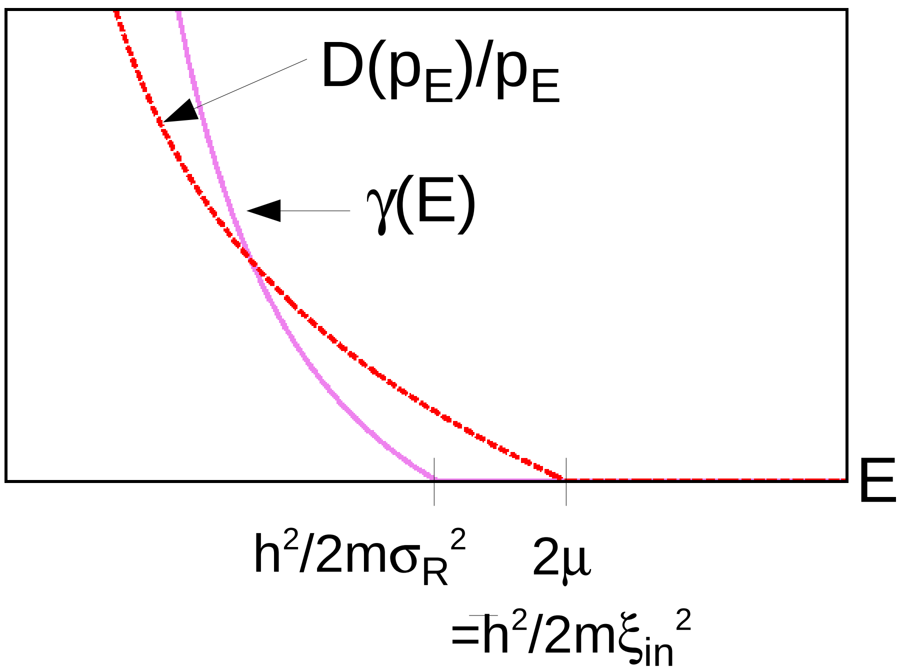


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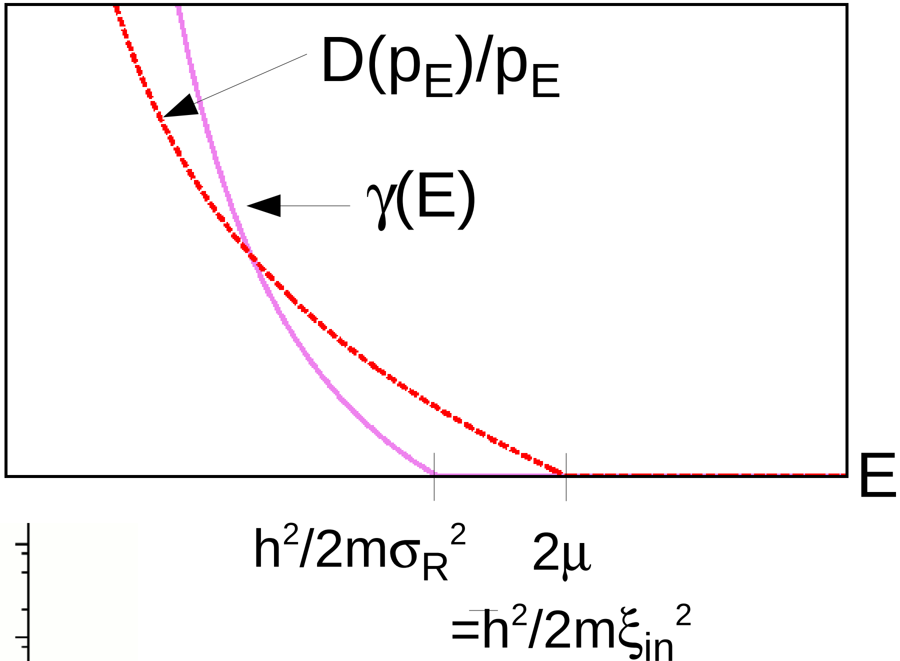


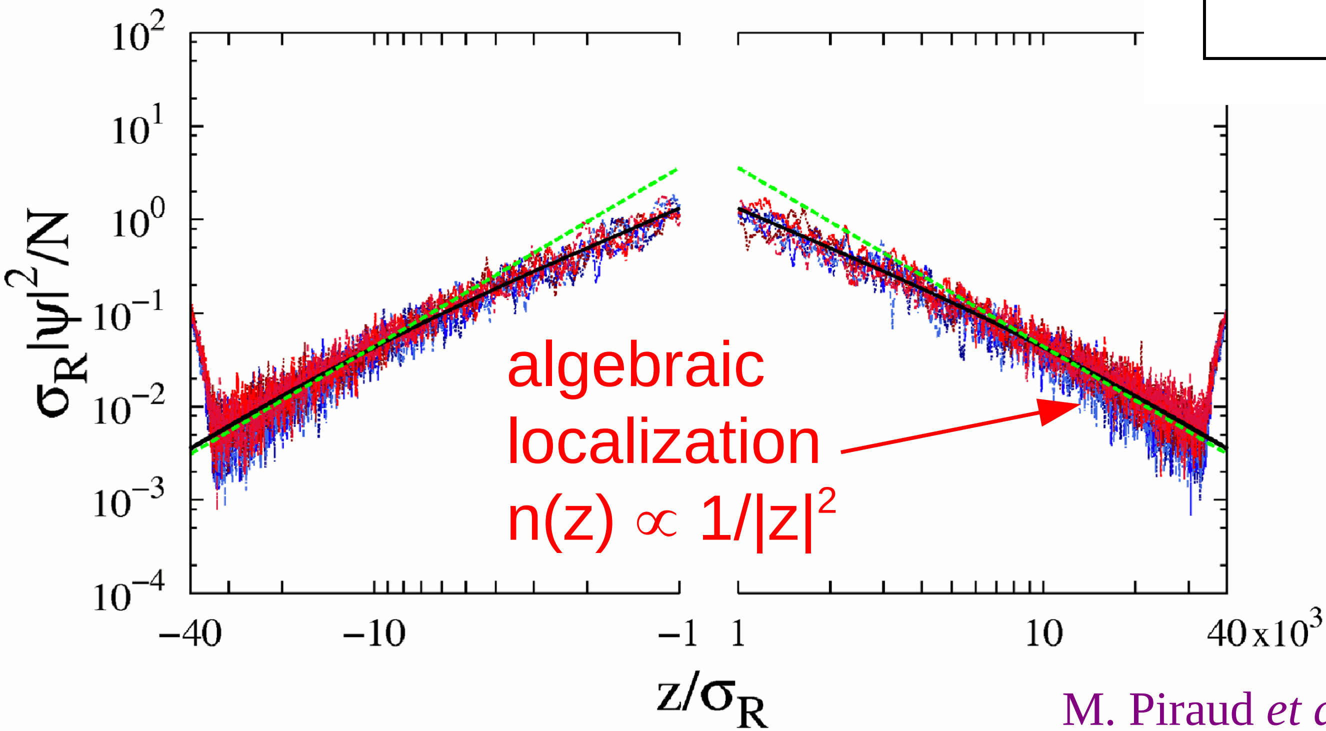


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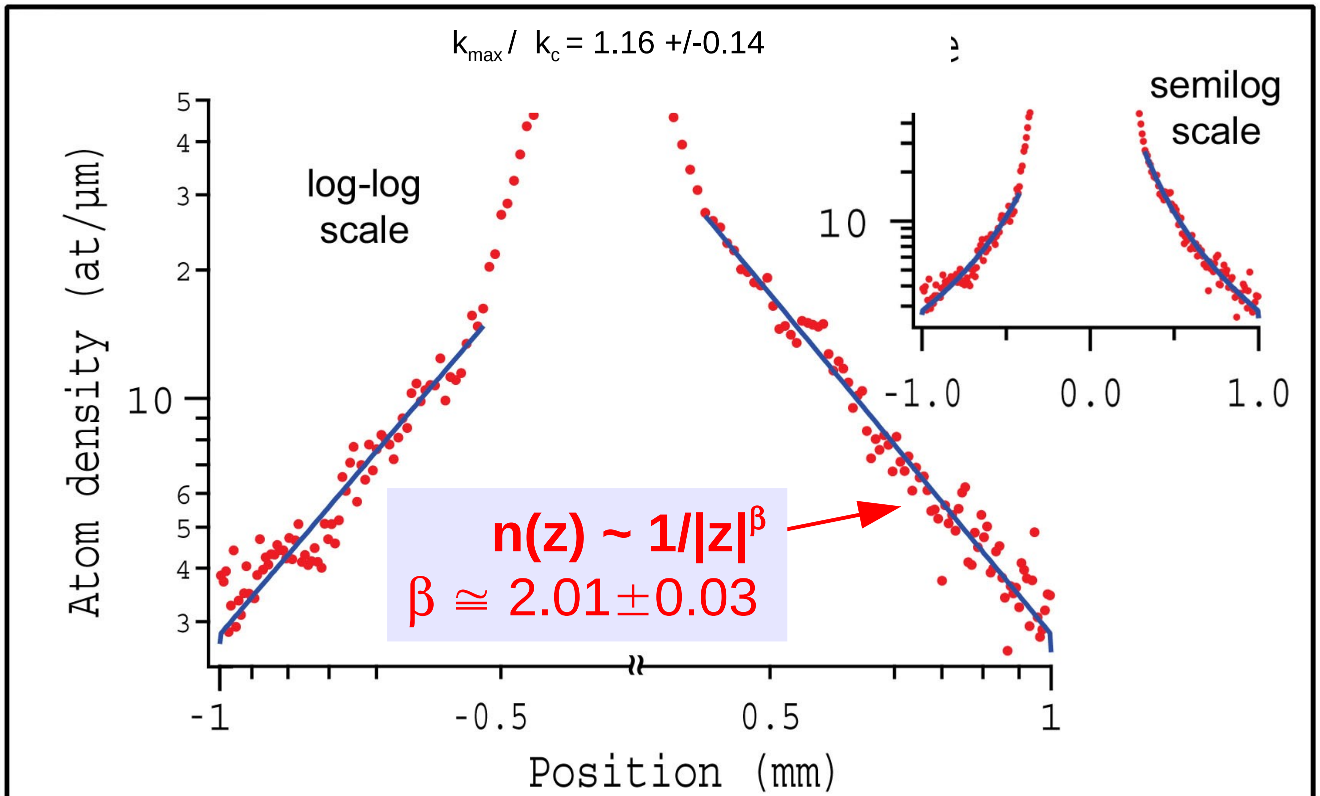

# Anderson localization in a 1D speckle potential : Conclusions

- Model of localization
- For $\xi_{in} > \sigma_R$,
  - short distance : ln[n(z)] ~ -2$\gamma_{eff}$ ➔ observed
  - large distance : ln[n(z)] ~ -0.5$\gamma_{eff}$ ➔ possible signatures (?)

    + deviations from exponential decay

- For $\xi_{in} < \sigma_R$,
  - central role of special long-range correlations in 1D speckle potentials ➔ effective mobility edge
  - algebraic localization, n(z) ~ 1/|z|$^2$ ➔ observed
  - not in contradiction with theorems [P. Lugan *et al.*, PRA (2009)] !

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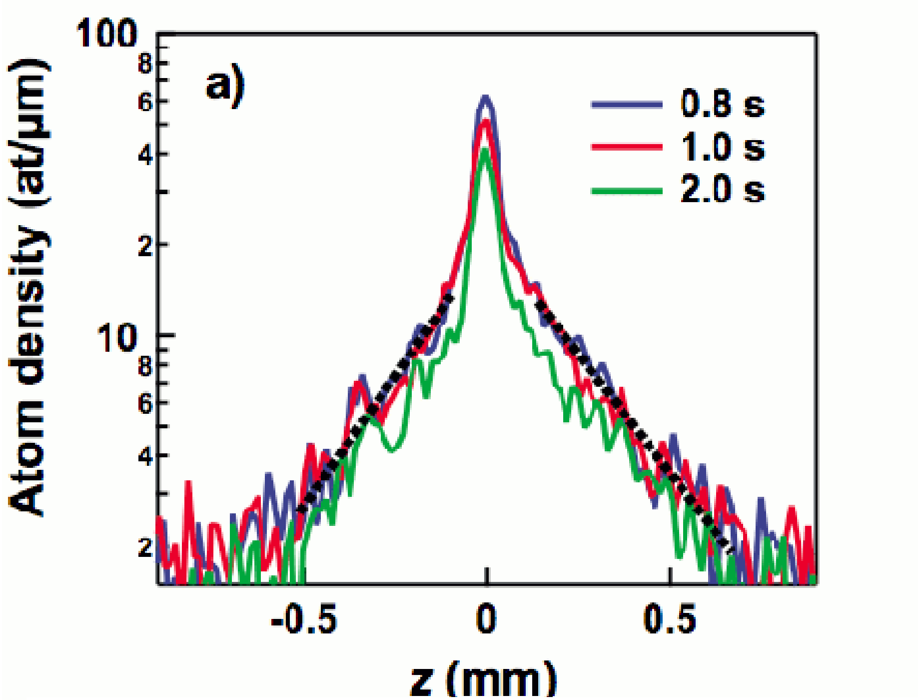


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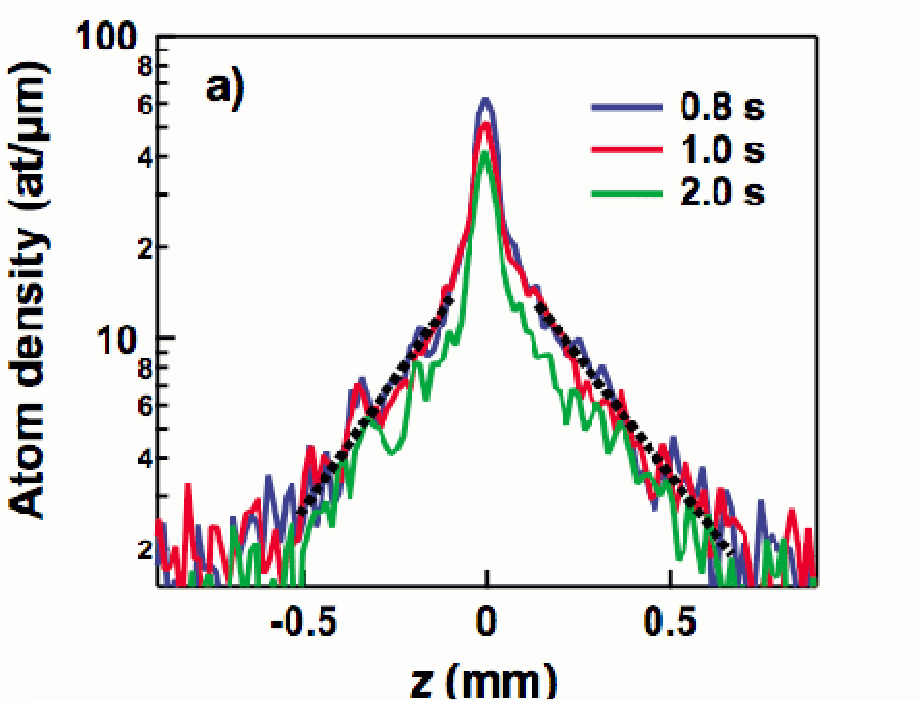


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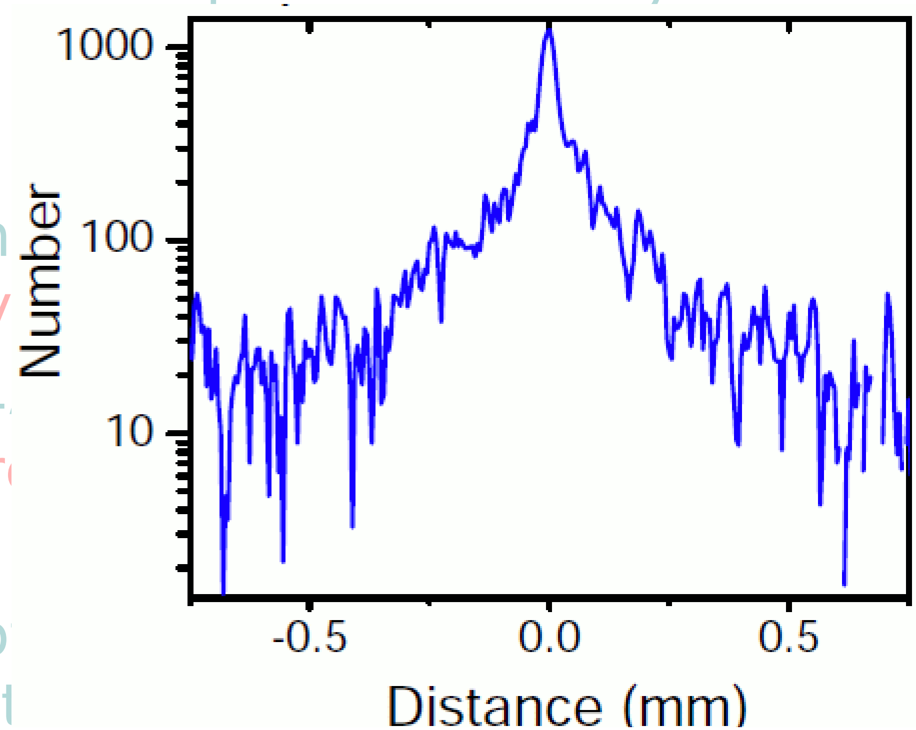


Courtesy of R. Hulet, *unpublished* (2009)

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# Outlook

1. Anderson localization in a 1D speckle potential

   1.1 Localization of a matter wave packet

   - LSP *et al.*, Phys. Rev. Lett. **98**, 210401 (2007)
   - J. Billy *et al.*, Nature **453**, 891 (2008)
   - M. Piraud *et al.*, Phys. Rev. A (Rapid Comm.) (2011)

   1.2 Algebraic localization in a speckle potential

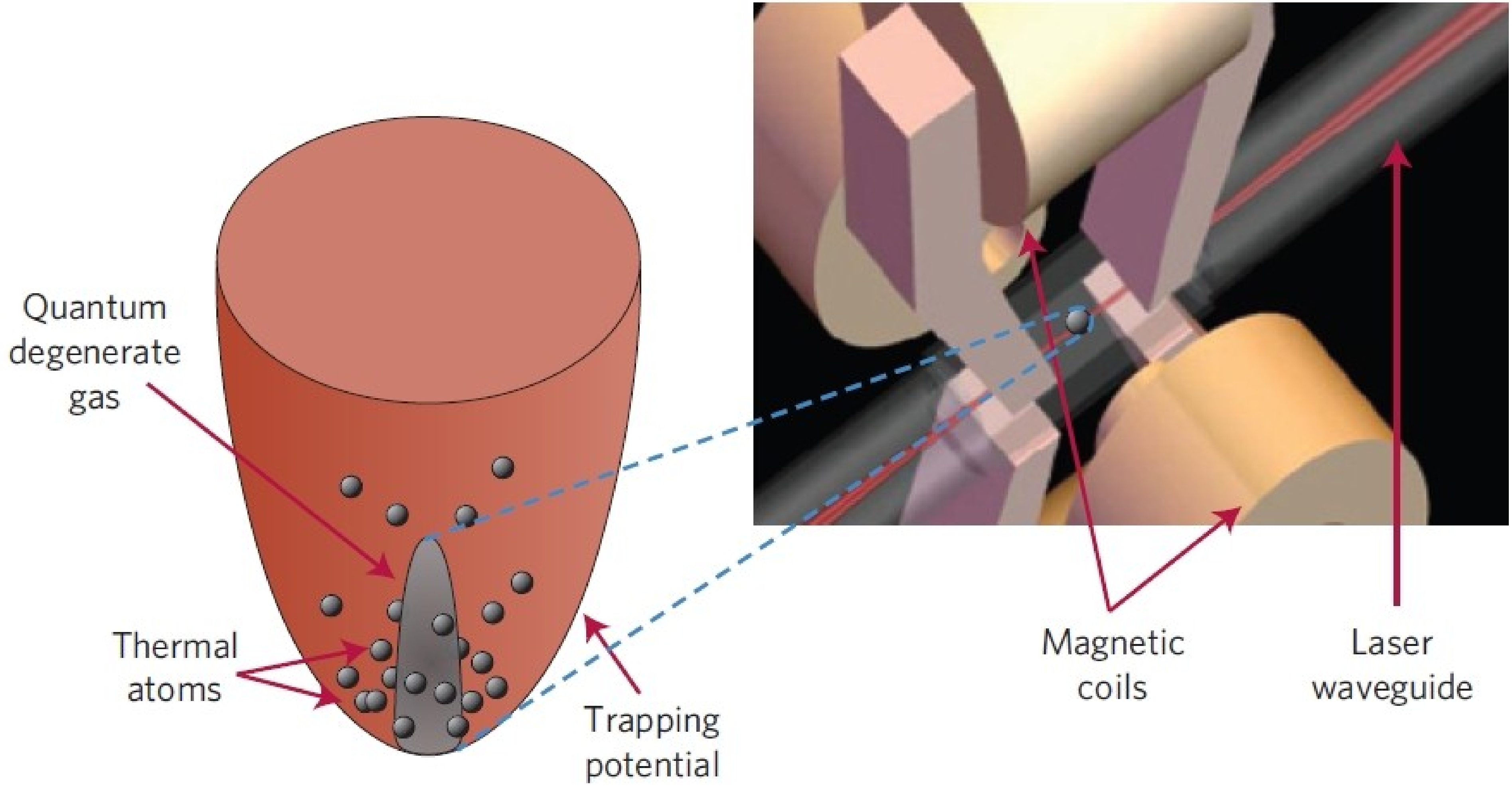
   - LSP *et al.*, Phys. Rev. Lett. **98**, 210401 (2007)
   - P. Lugan *et al.*, Phys. Rev. A **80**, 023605 (2009)

2. Localization in the presence of a trap

   - L. Pezzé and LSP, Phys. Rev. Lett. **106**, 040601 (2011)

3. Perspectives

# Localization in a Disordered Trap




**Effect of trapping on localization ?**

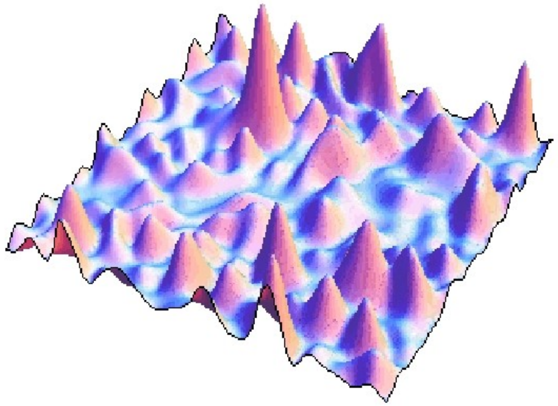
LSP and M. Lewenstein, Nature Phys. **6**, 87 (2010)

# Localization in a Disordered Trap

$$\hat{H} = -\hbar^2 \boldsymbol{\nabla}^2 / 2m + \underline{V(\boldsymbol{r})} + \underline{V_{\mathrm{T}}(\boldsymbol{r})}$$

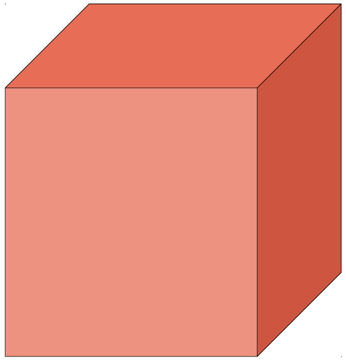
or

**Disordered potential**
(*eg.* speckle field)



**trapping potential**

homog. box



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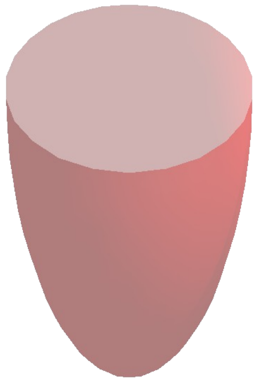

- Numerical diagonalization
  - Center-of-mass position: $\boldsymbol{r}_n \equiv \langle \psi_n | \hat{\boldsymbol{r}} | \psi_n \rangle$
  - Extension: $\Delta r_n \equiv \left( \langle \psi_n | \hat{\boldsymbol{r}}^2 | \psi_n \rangle - \boldsymbol{r}_n^2 \right)^{1/2}$

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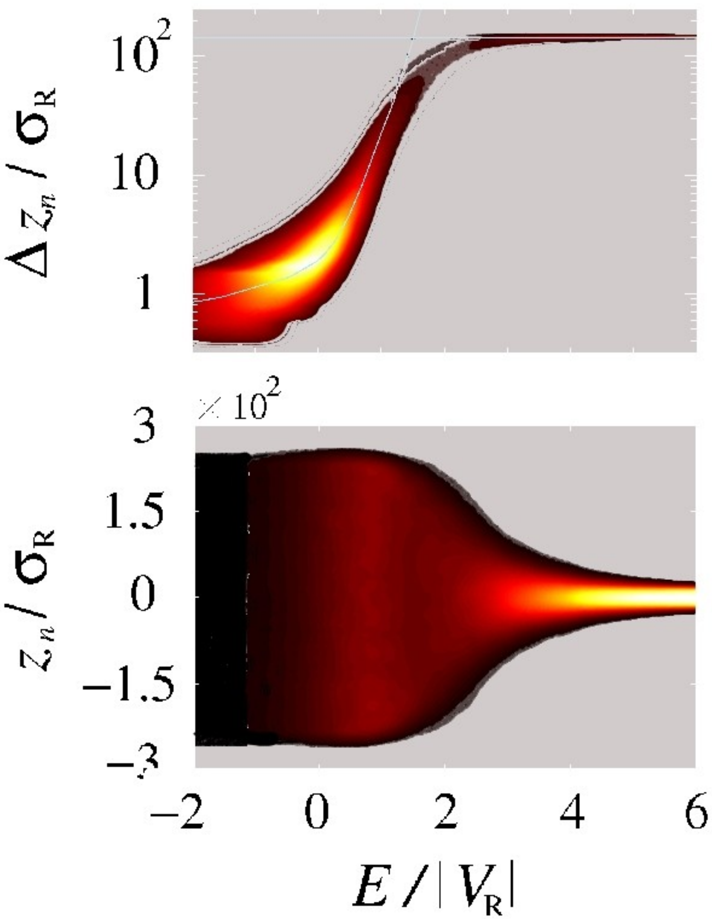


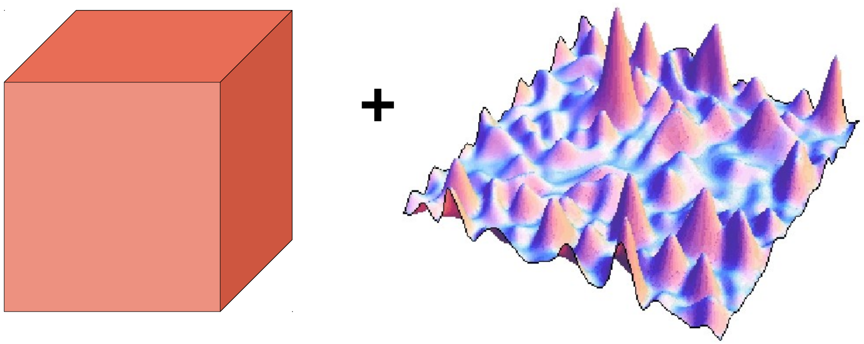

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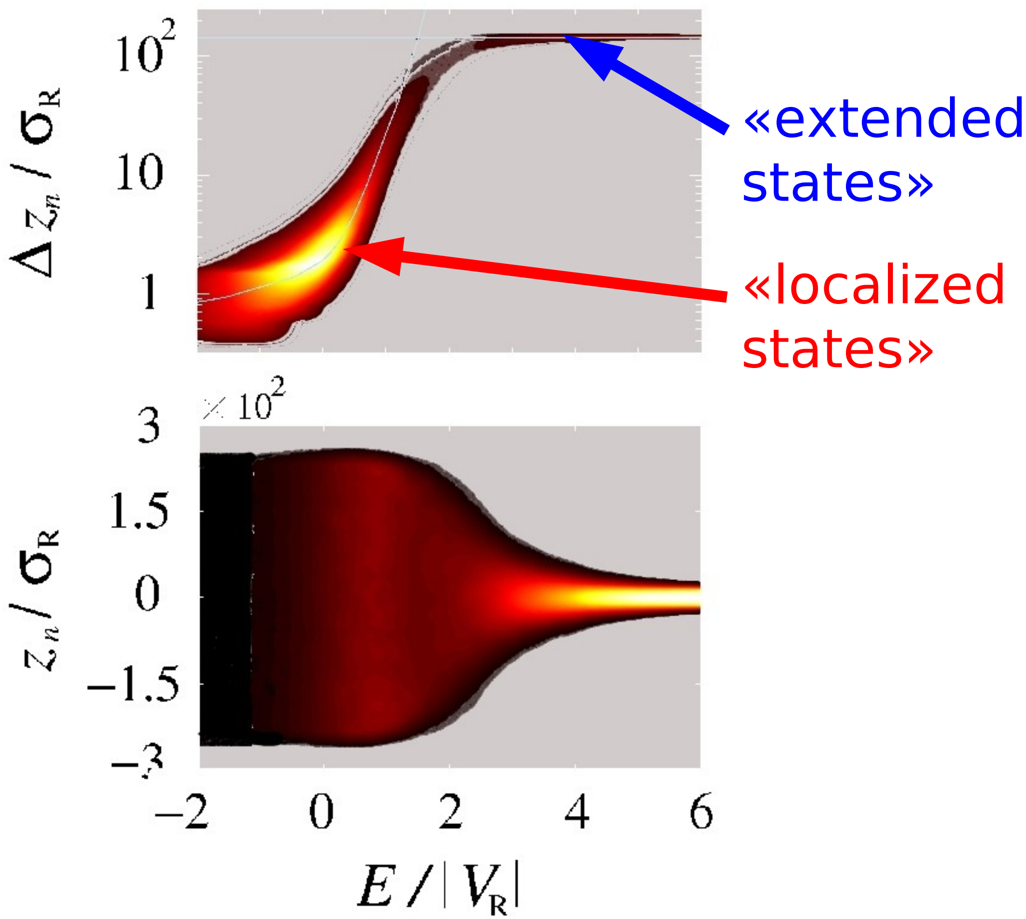


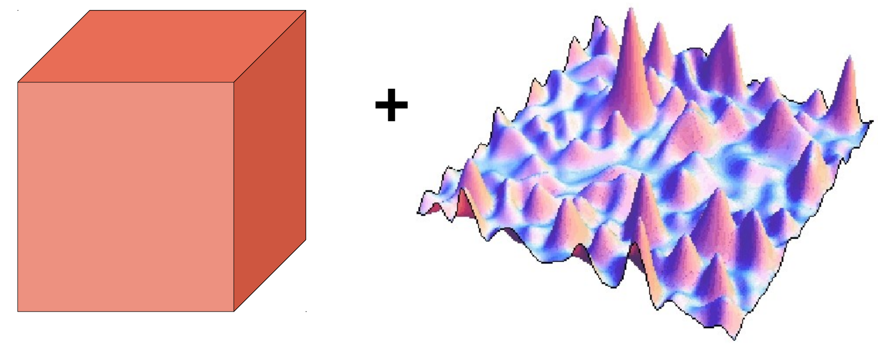


- Smooth crossover from localized states ($\Delta z << L$) and extended states ($\Delta z \sim L$)

  ⇒ **finite-size effect**

- No coexistence of localized and extended states (Mott's argument)

# Localization in a Disordered Trap

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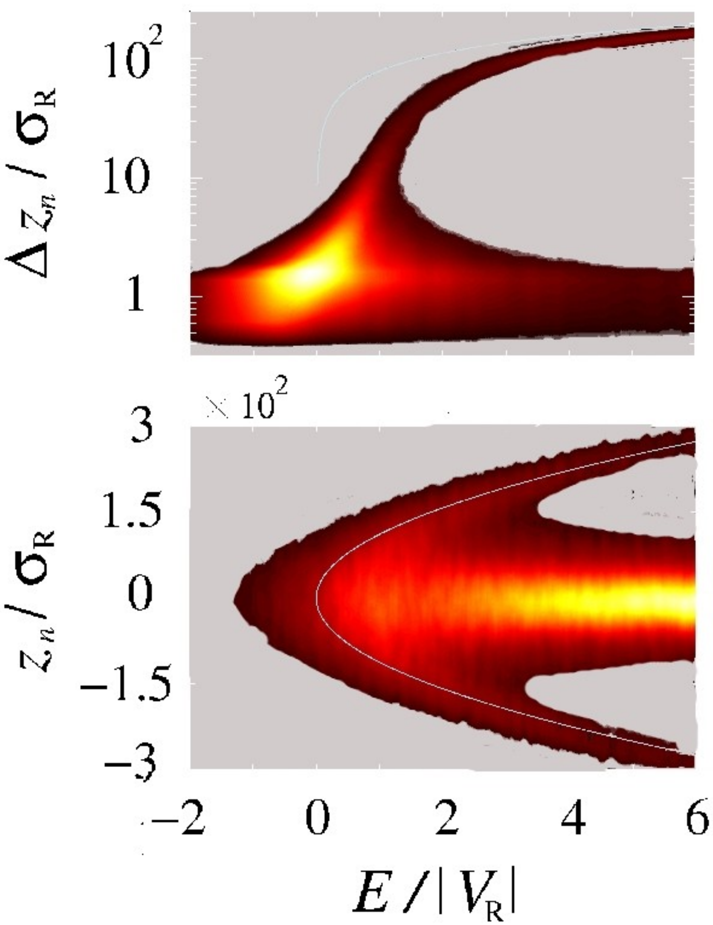


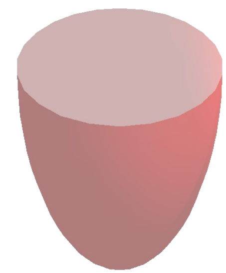

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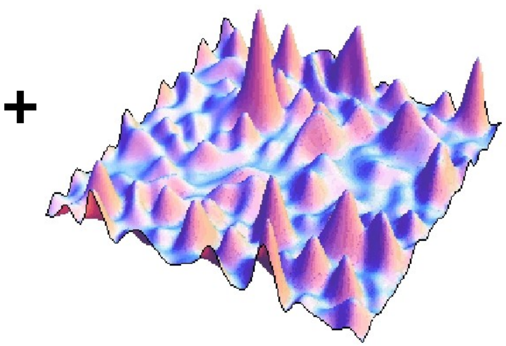

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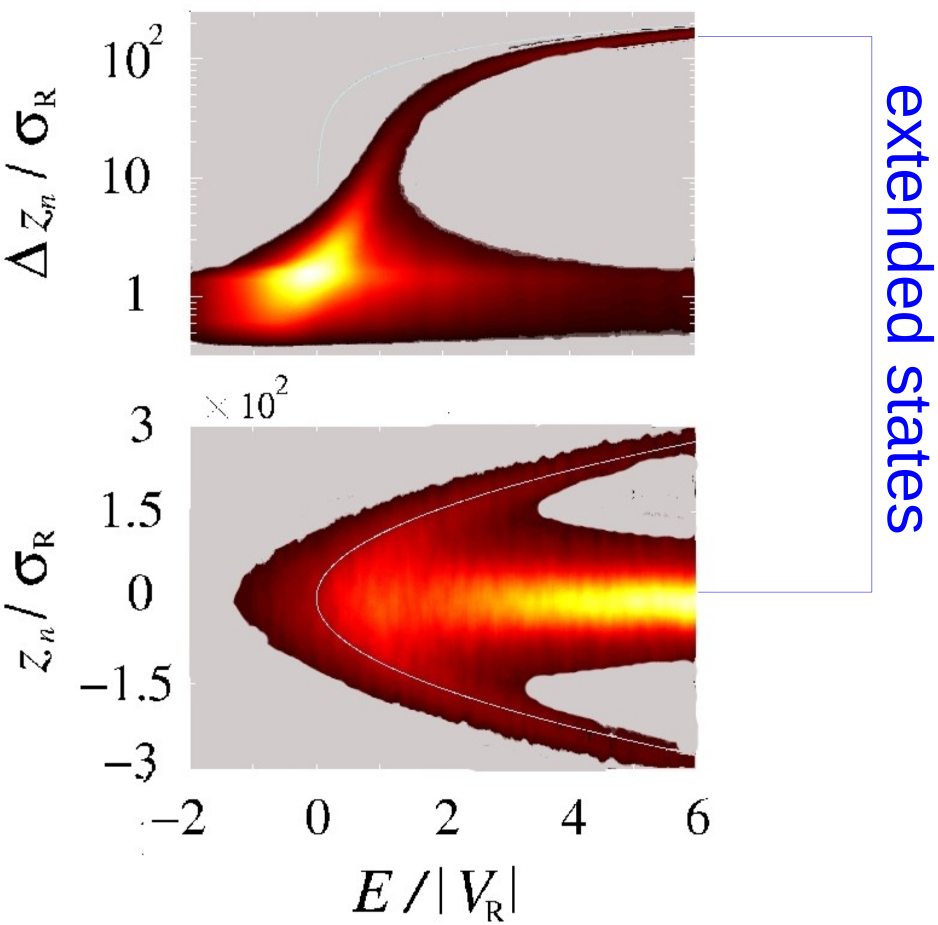


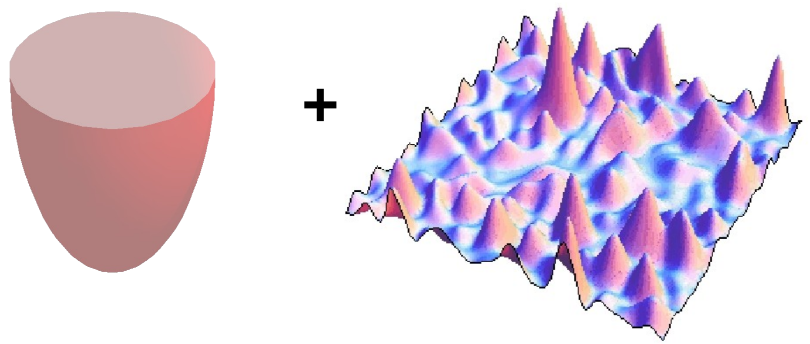


<u>Branch of extended states :</u>

smooth crossover towards $\Delta z_n \sim z_{cl}(E_n)$

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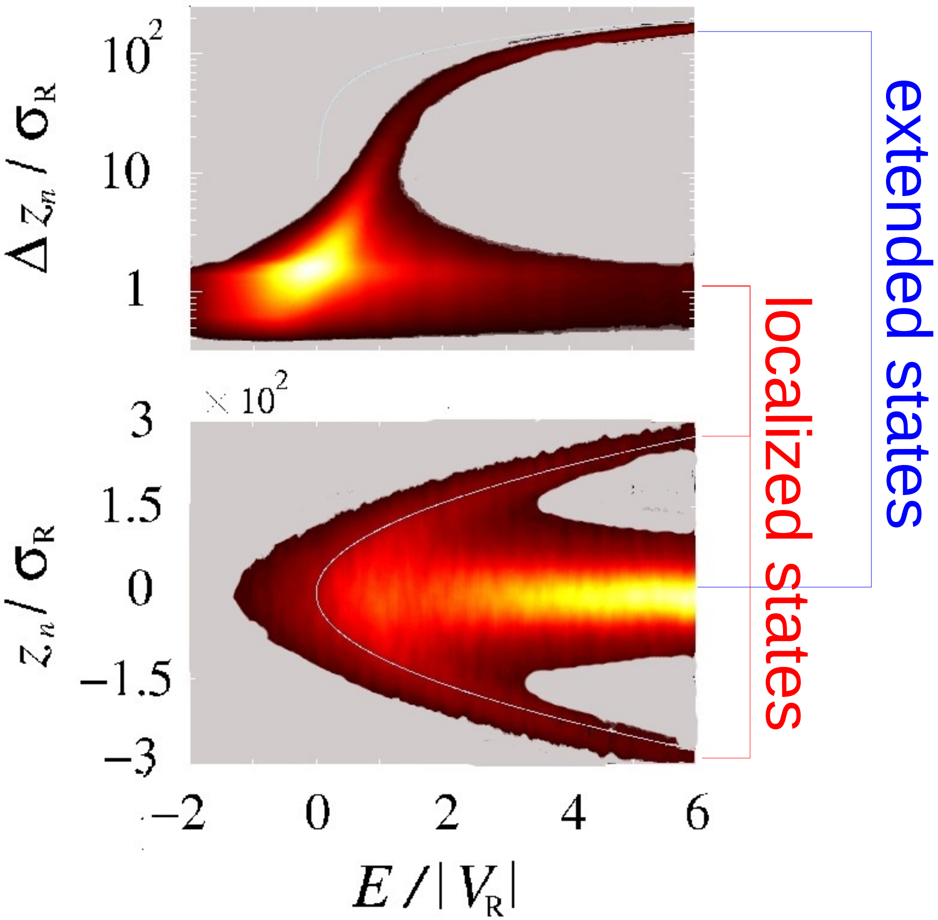


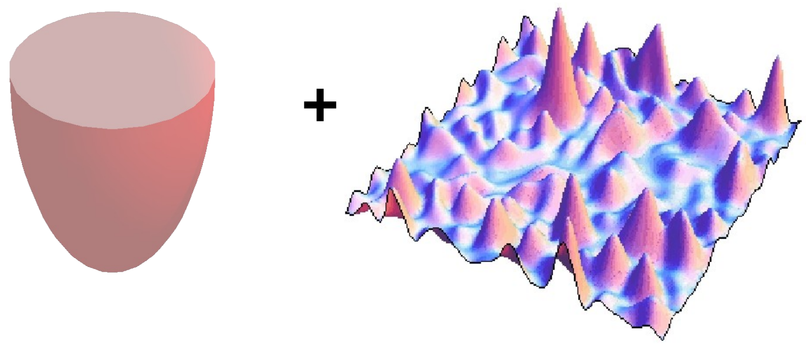


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smooth crossover towards $\Delta z_n \sim z_{cl}(E_n)$

Branch of localized states :

nearly constant $\Delta z$ and localization near the edges of the trap, $z_n \approx z_{cl}(E_n)$

⇒ Coexistence of localized and extended states

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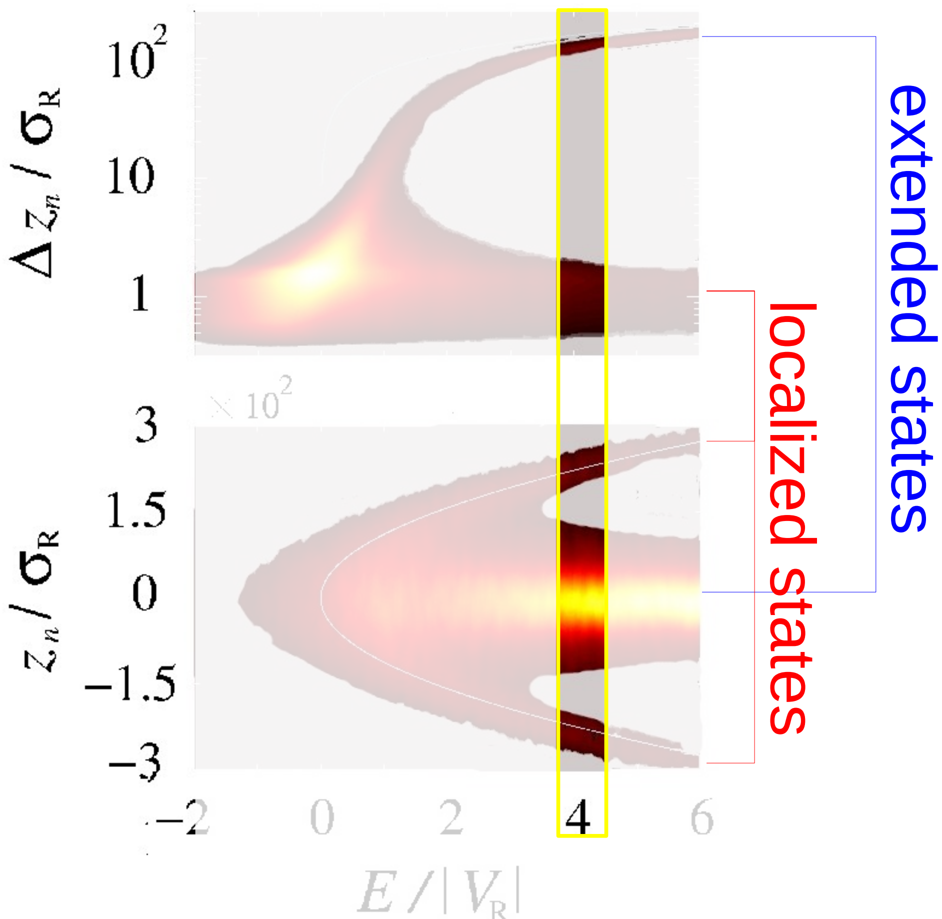


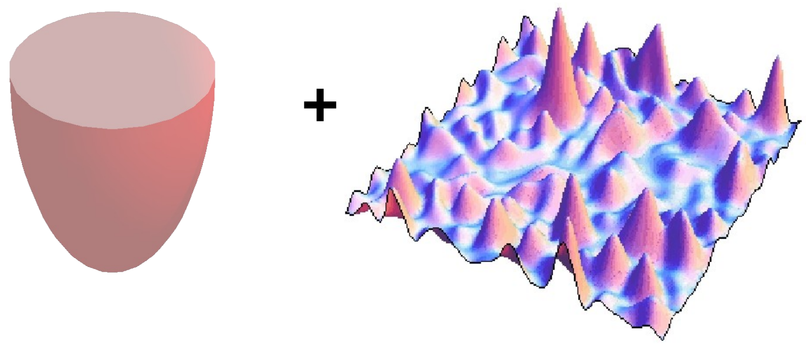


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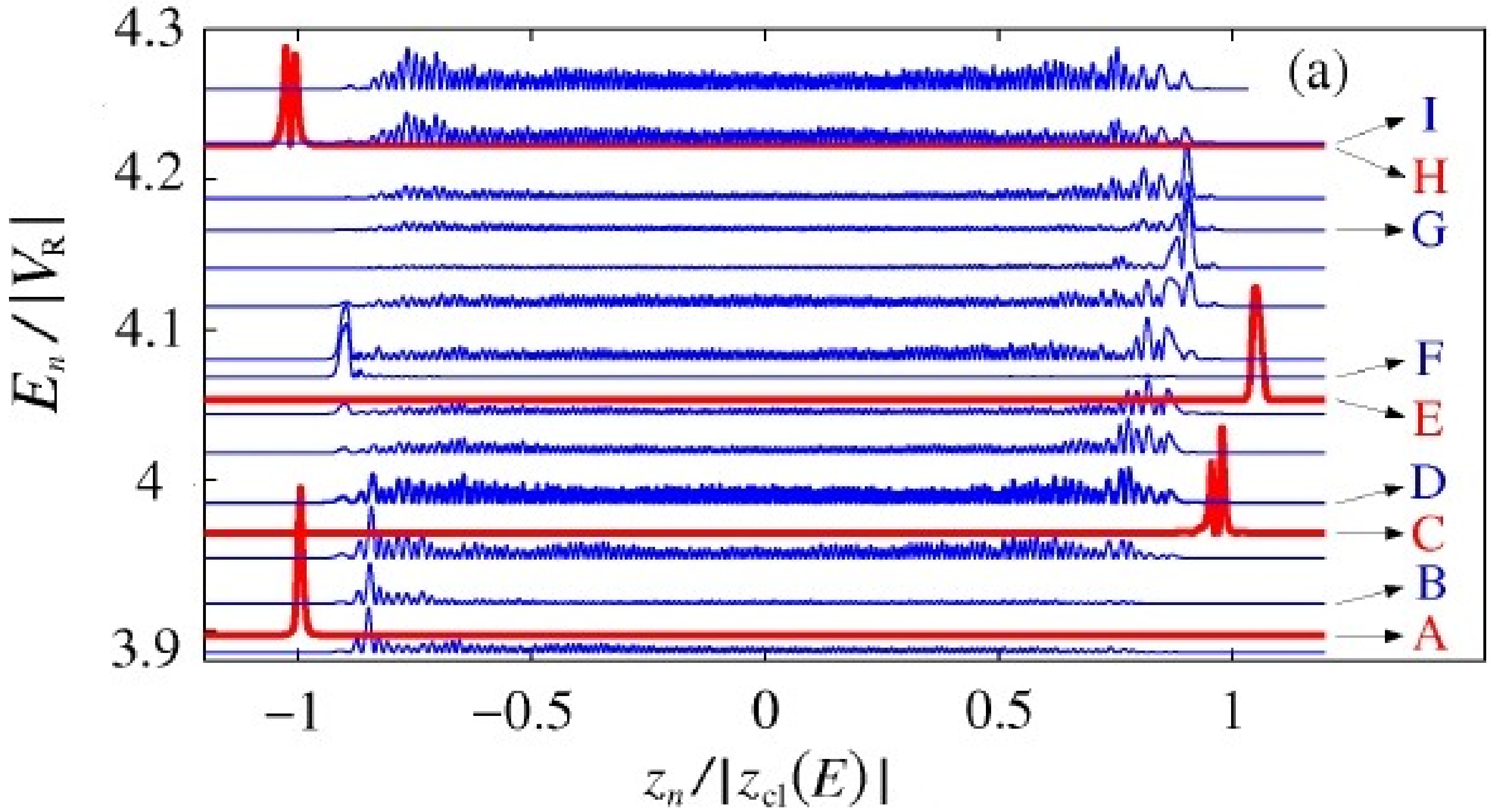


⇒ **Coexistence for each realization of the disorder**

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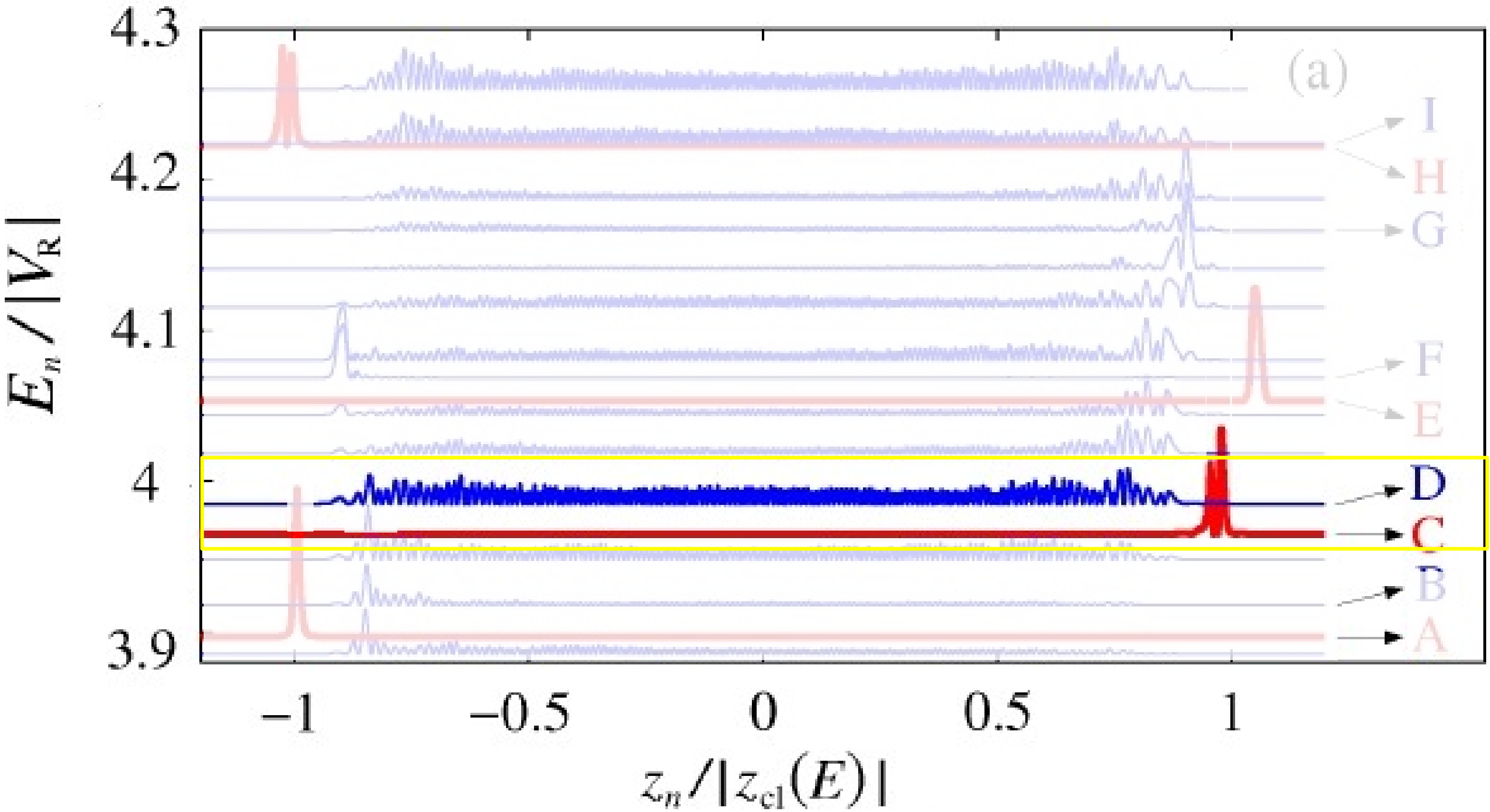


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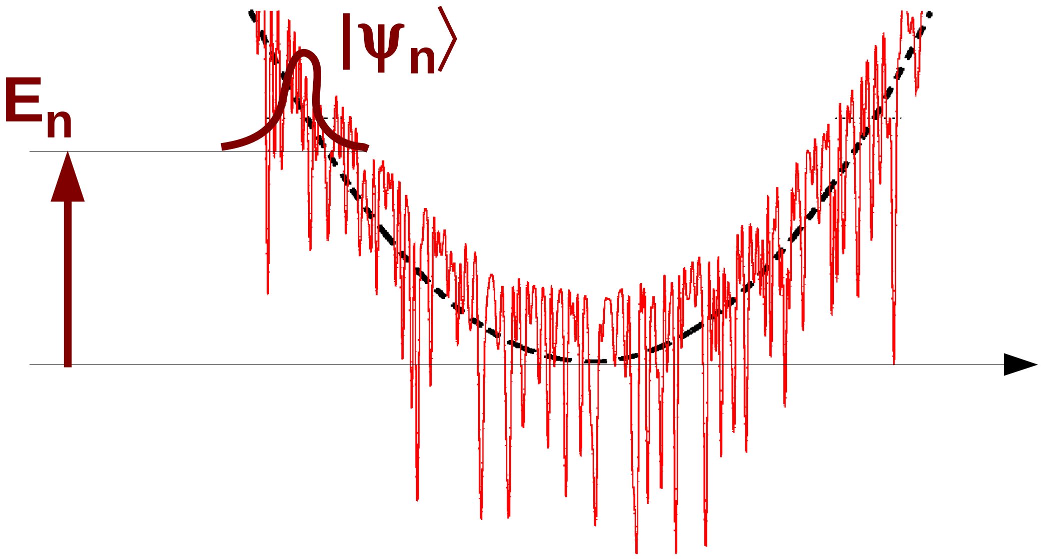

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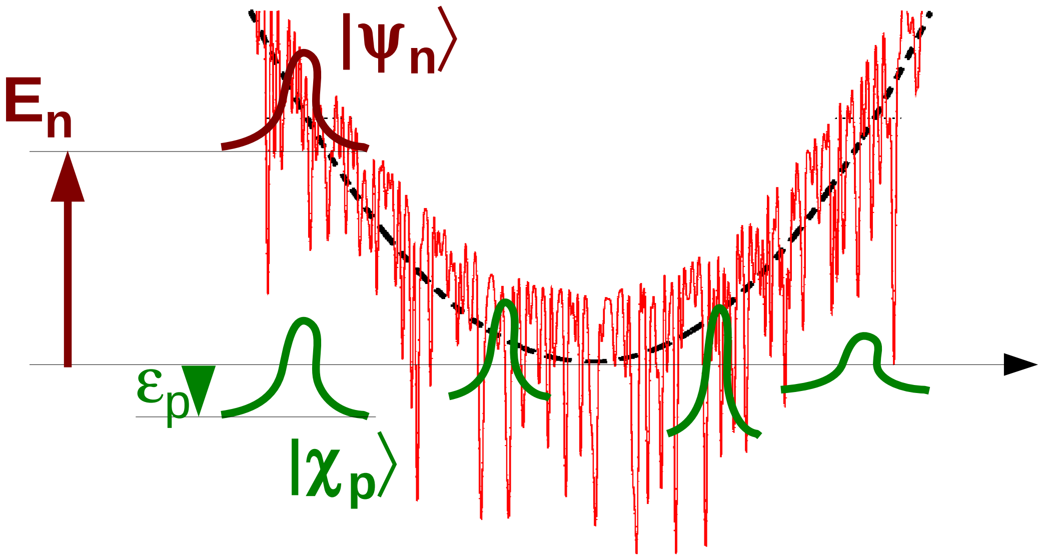

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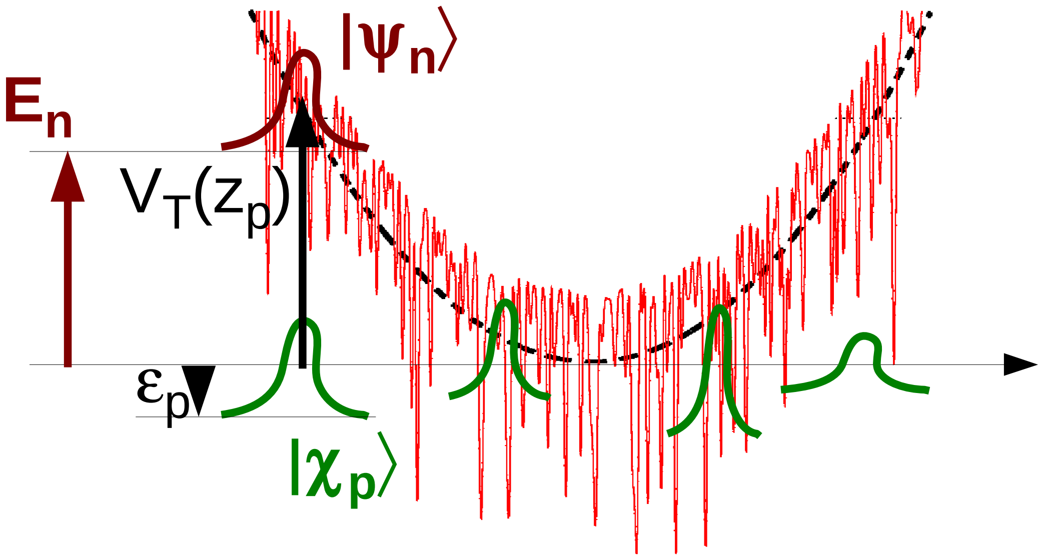


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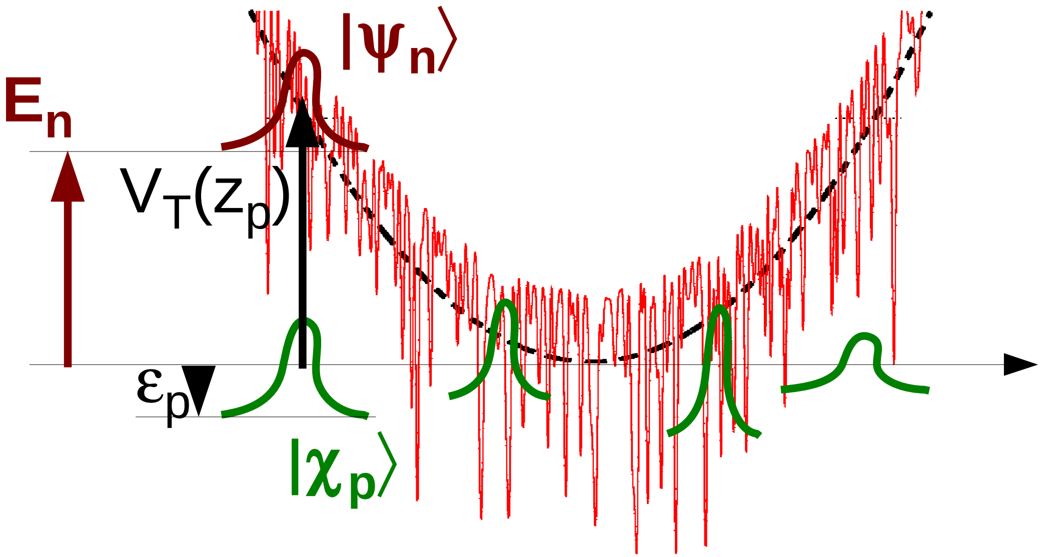


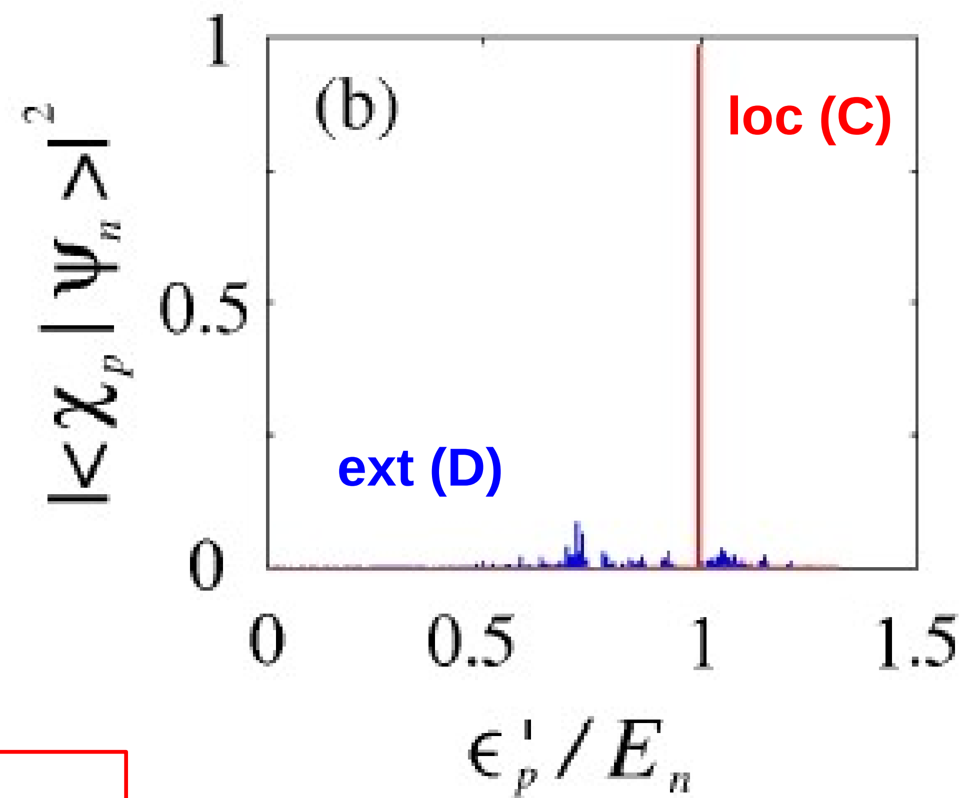


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**Localized states :**

- localized states of the pure disordered potential
- weakly affected by the trap
- energy determined by the trap ⇒ $z_n \approx z_{cl}(E_n)$

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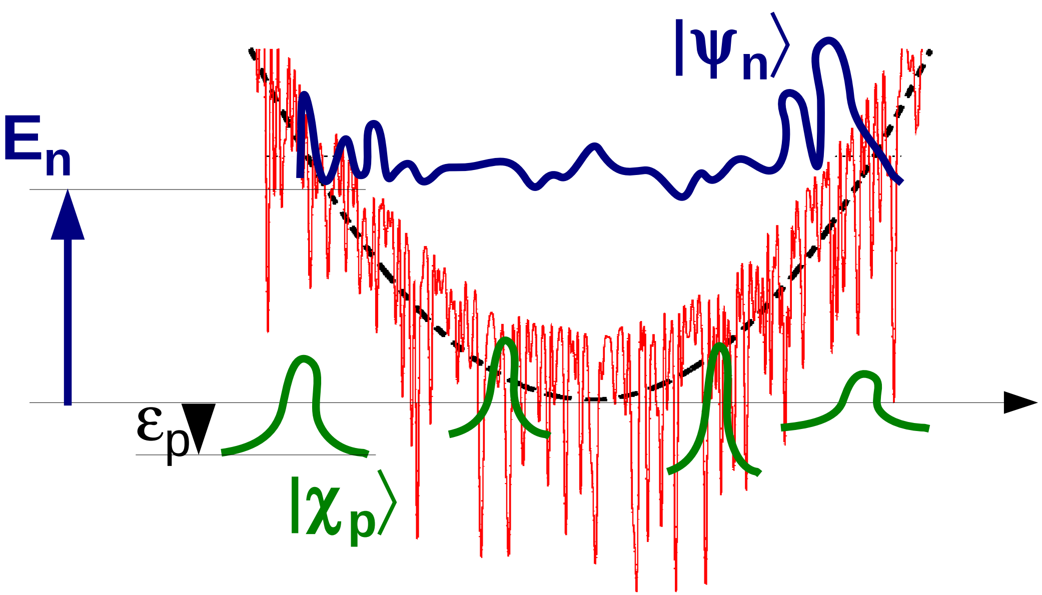


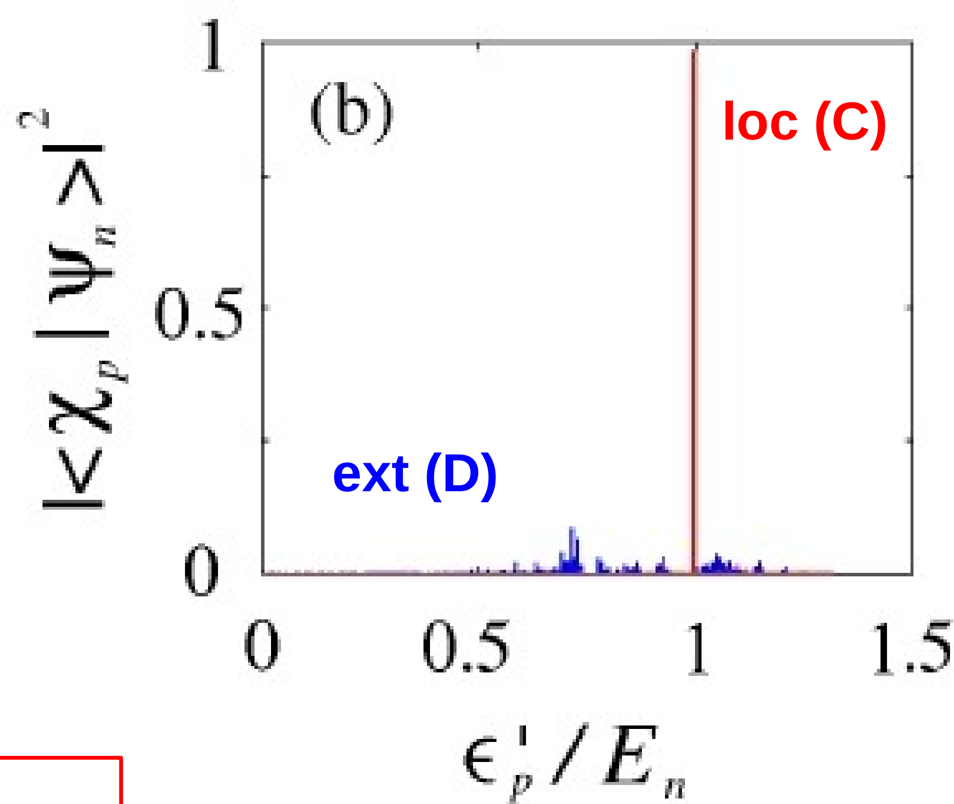


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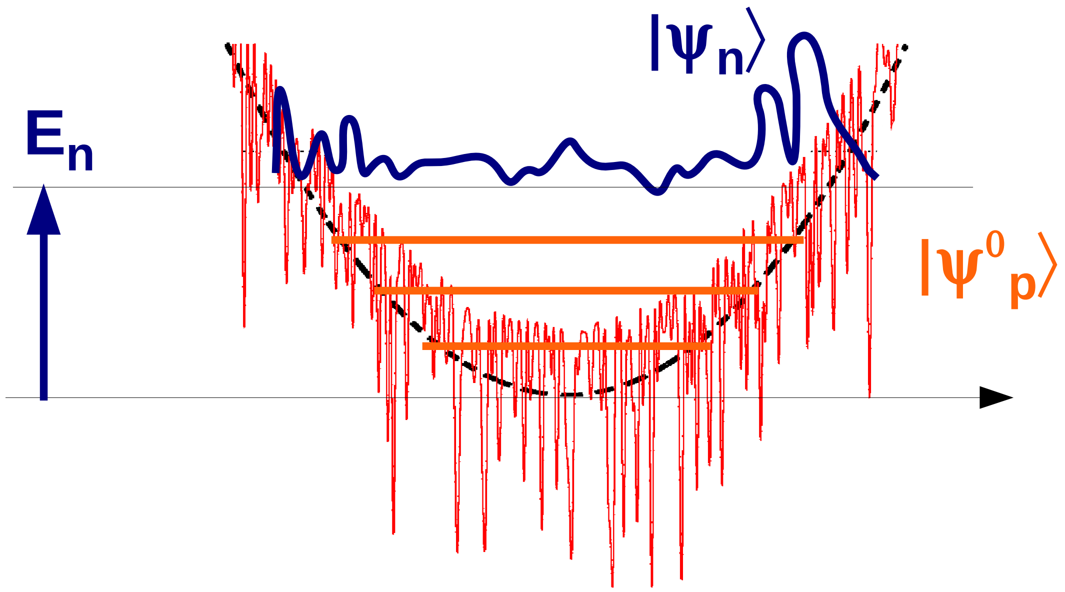

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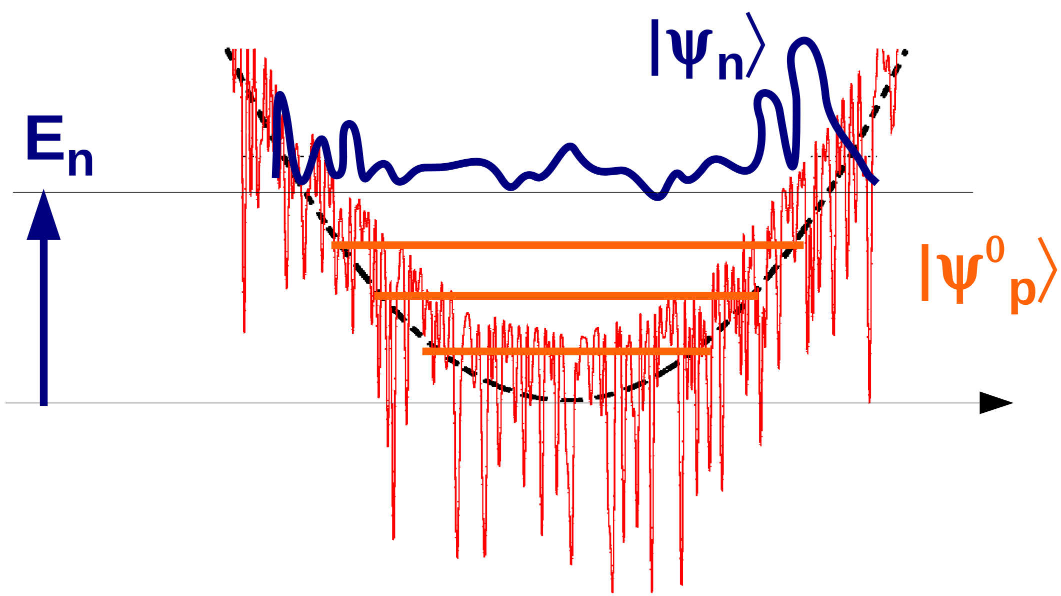


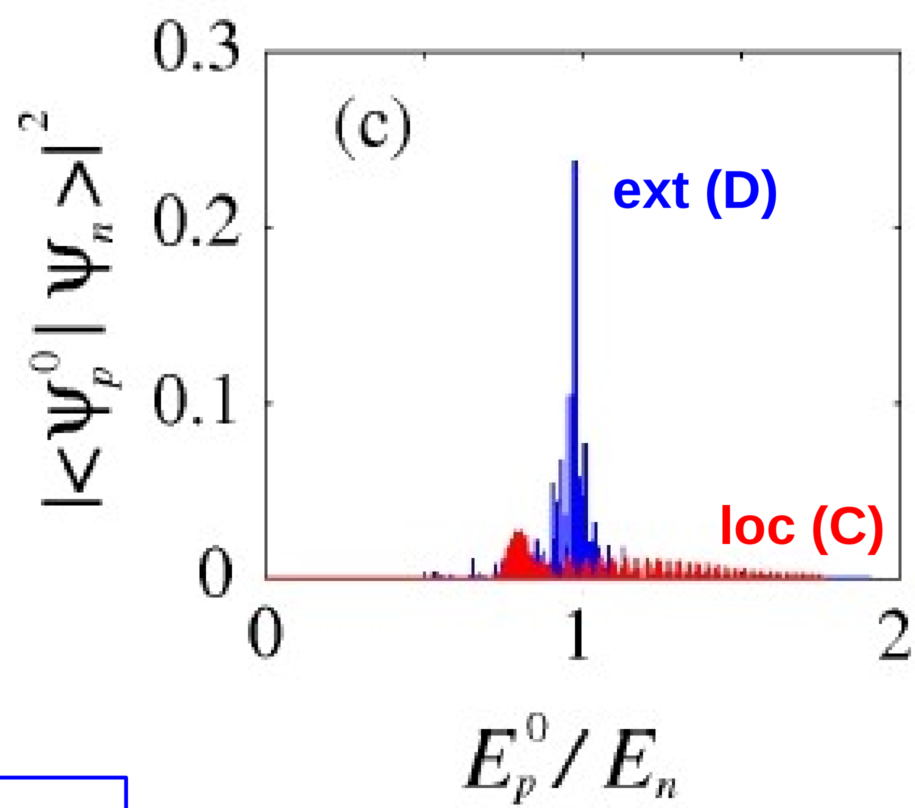


**Extended states :**

- reminiscent of eigenstates of the pure trap
- weakly affected by the disorder ($E_p^0 \approx E_n$)

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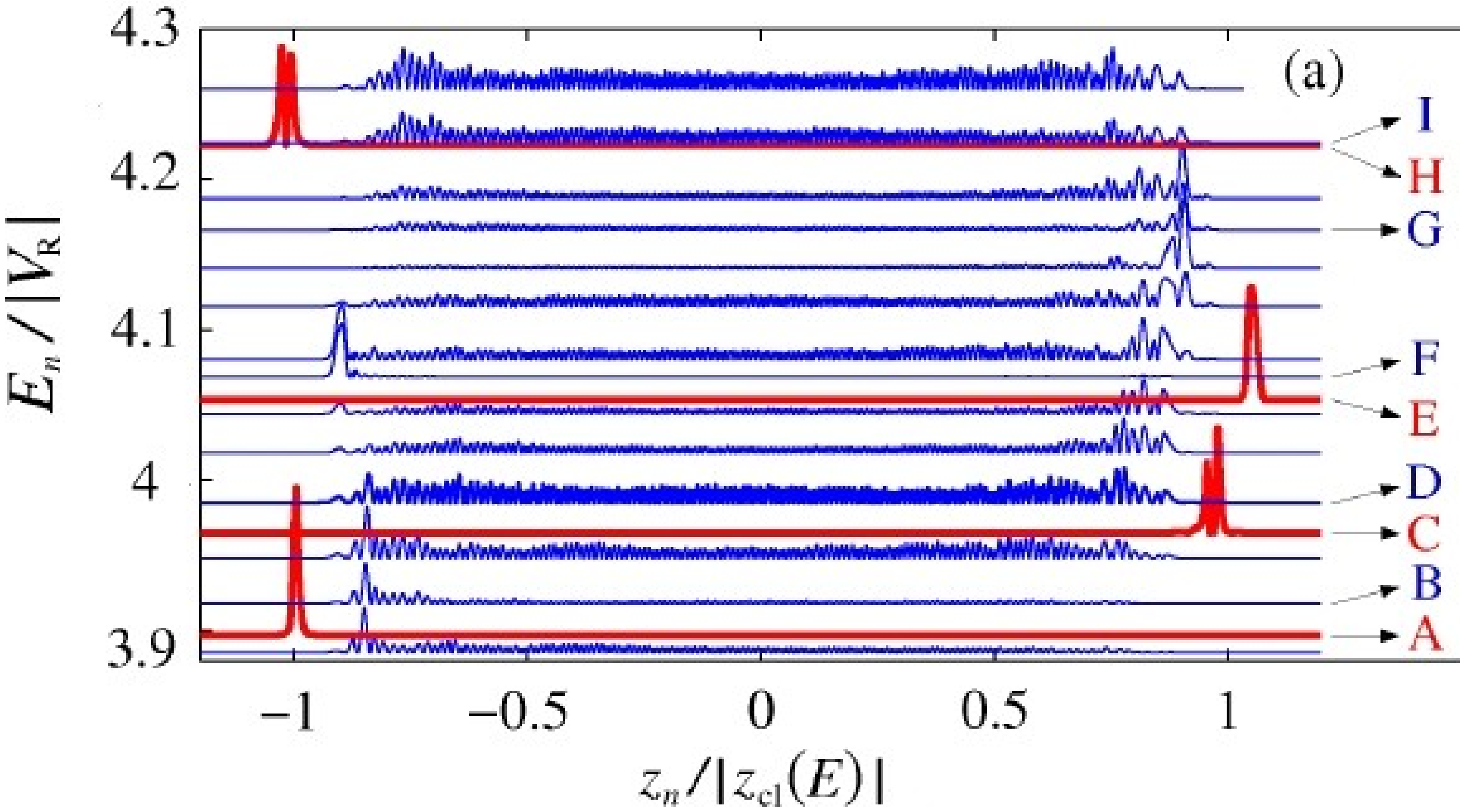



⇒ **Mott argument not applicable (weak spatial overlap)**

# Localization in a Disordered Trap : Conclusions

- Localized and extended states can coexist in energy in a disordered trap
  - Localized states
    - localized by disorder
    - weakly affected by the trap
    - energy determined by their position in the trap
  - Extended states
    - reminiscent of the eigenstates of the trap
    - weakly affected by the disorder

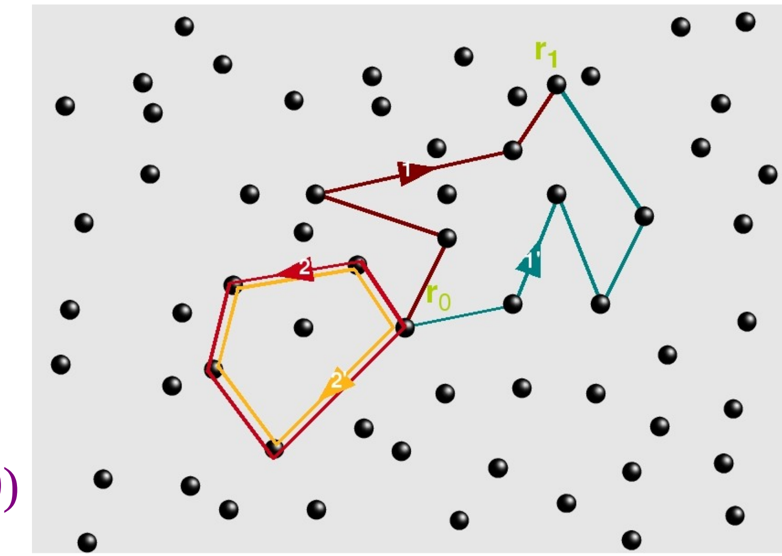
- No hybrydization of the two classes of states (Mott's argument not applicable)

- General feature : relevant to broad class of inhomogeneous traps and in d>1 [see poster by L. Pezzé, tomorrow]

- Perspectives : direct observation (B. DeMarco ?)

# Perspectives ....

- Anderson localization in d>1
  - signatures of localization
  - role of long-range correlations in speckle potentials

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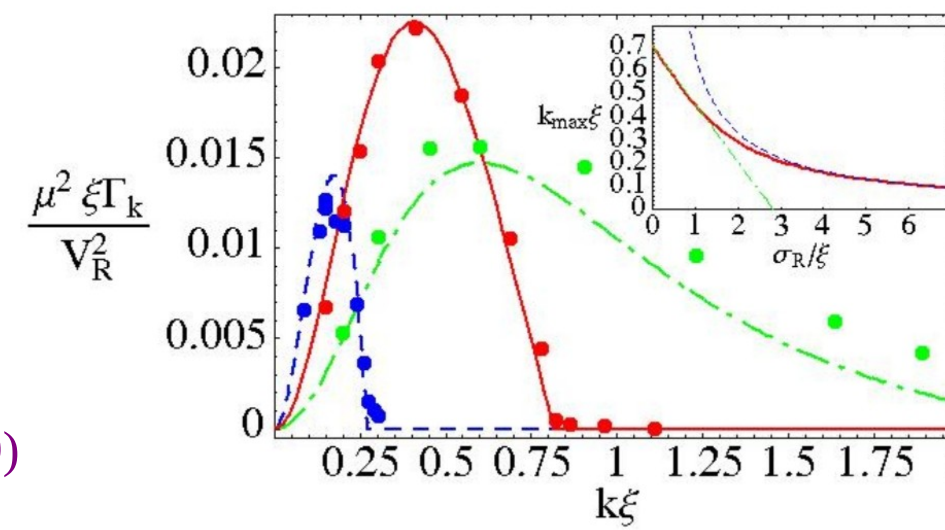
S. Skipetrov *et al.*, Phys. Rev. Lett. **100**, 165301 (2008)
A. Yedjour and B. van Tiggelen, Eur. Phys. J. D **59**, 249 (2010)

- Effects of interactions
  - expansion scheme
  - gas at equilibrium and finite-T metal-insulator transitions

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I. Aleiner *et al.*, Nature Phys. **6**, 900 (2010)

# Acknowledgements


## Theory team at Atom Optics group

Pierre Lugan

(PhD; 2006-2009)

Luca Pezzé

(Post-doc; 2007-)

Marie Piraud

(PhD; 2009-)

Lam Dao

(Post-doc; 2009-)

Thomas Koffel

Samuel Lellouch

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Mondher Besbes

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**Atom Optics group at LCFIO**

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”Ce commerce que tant de motifs puissants nous engagent à former avec les autres hommes, augmente bientôt l’étendue de nos idées, et nous en fait naître de très nouvelles pour nous, et de très éloignées, selon toute apparence, de celles que nous aurions eues par nous-mêmes sans un tel secours.”

*Jean Le Rond d’Alembert*