

# Sensitivity analysis of spatial models

Application to economic analysis of flood risk management plans

Nathalie Saint-Geours

PhD thesis defended on November 29, 2012

Supervised by: *Christian Lavergne*  
*Jean-Stéphane Bailly – Frédéric Grelot*





# Introduction

# Spatially distributed models

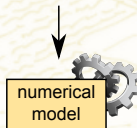


# Spatially distributed models

## Numerical models

- = blackbox
- to describe, understand, support decision making, forecast

inputs



outputs

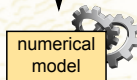
# Spatially distributed models

## Numerical models

- = blackbox
- to describe, understand, support decision making, forecast

## Spatially distributed models

inputs



outputs

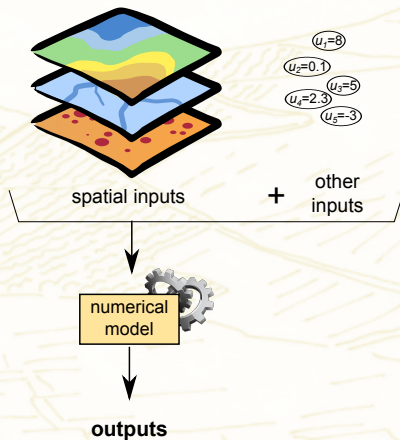
# Spatially distributed models

## Numerical models

- = blackbox
- to describe, understand, support decision making, forecast

## Spatially distributed models

- spatially distributed inputs



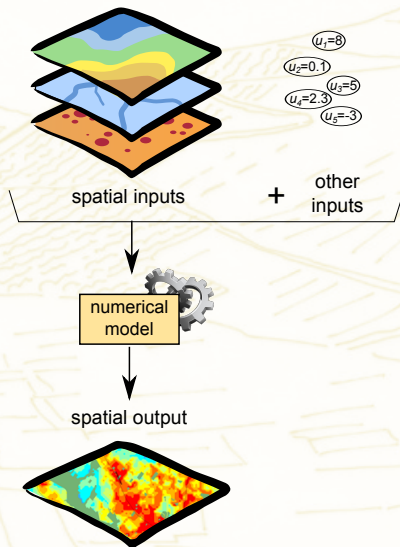
# Spatially distributed models

## Numerical models

- = blackbox
- to describe, understand, support decision making, forecast

## Spatially distributed models

- spatially distributed inputs
- spatially distributed outputs



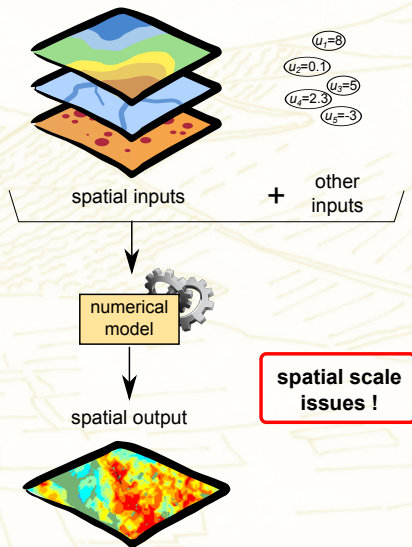
# Spatially distributed models

## Numerical models

- = blackbox
- to describe, understand, support decision making, forecast

## Spatially distributed models

- spatially distributed inputs
- spatially distributed outputs





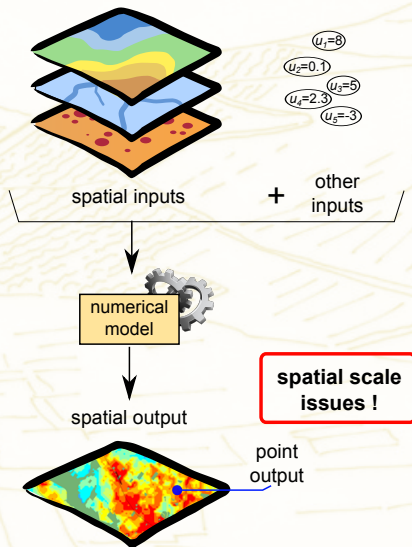
# Spatially distributed models

## Numerical models

- = blackbox
- to describe, understand, support decision making, forecast

## Spatially distributed models

- spatially distributed inputs
- spatially distributed outputs



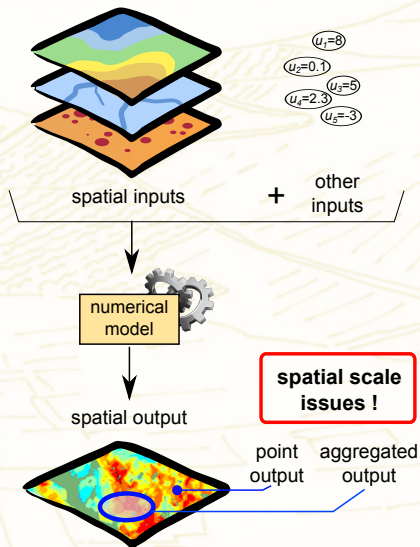
# Spatially distributed models

## Numerical models

- = blackbox
- to describe, understand, support decision making, forecast

## Spatially distributed models

- spatially distributed inputs
- spatially distributed outputs



# Uncertainty in spatial modelling

*All models are wrong, some are useful (G. Box)*



# Uncertainty in spatial modelling

*All models are wrong, some are useful (G. Box)*

## Many sources of uncertainty

- lack of knowledge
- natural variability
- measurement errors
- model assumptions. . .

# Uncertainty in spatial modelling

*All models are wrong, some are useful (G. Box)*

## Many sources of uncertainty

- lack of knowledge
- natural variability
- measurement errors
- model assumptions. . .

## Spatial uncertainty



# Uncertainty in spatial modelling

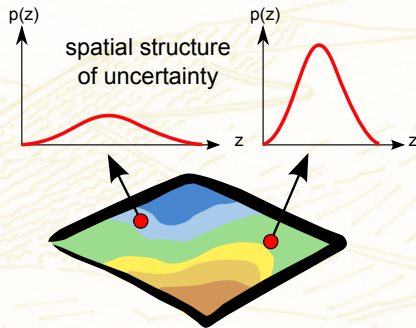
*All models are wrong, some are useful (G. Box)*

## Many sources of uncertainty

- lack of knowledge
- natural variability
- measurement errors
- model assumptions...

## Spatial uncertainty

- spatial structure of uncertainty



# Uncertainty in spatial modelling

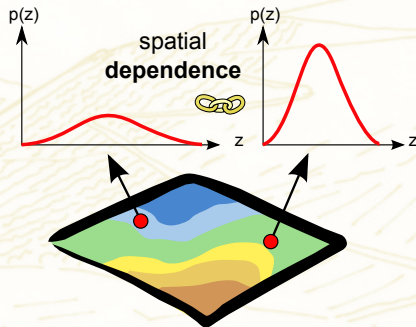
*All models are wrong, some are useful (G. Box)*

## Many sources of uncertainty

- lack of knowledge
- natural variability
- measurement errors
- model assumptions. . .

## Spatial uncertainty

- spatial structure of uncertainty
- spatial dependence



# Uncertainty in spatial modelling

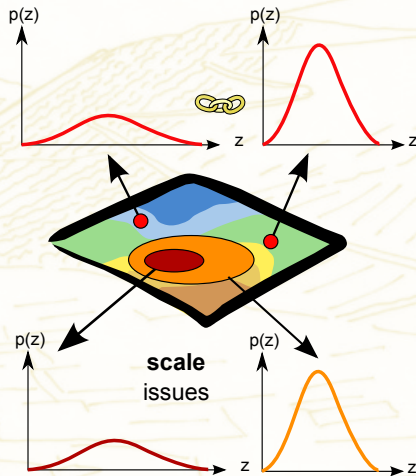
*All models are wrong, some are useful (G. Box)*

## Many sources of uncertainty

- lack of knowledge
- natural variability
- measurement errors
- model assumptions. . .

## Spatial uncertainty

- spatial structure of uncertainty
- spatial dependence
- spatial scale issues





# Uncertainty in spatial modelling

*All models are wrong, some are useful (G. Box)*

## Many sources of uncertainty

- lack of knowledge
- natural variability
- measurement errors
- model assumptions. . .

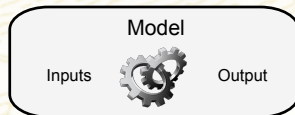
→ what impact on the uncertainty of model outputs?

## Spatial uncertainty

- spatial structure of uncertainty
- spatial dependence
- spatial scale issues

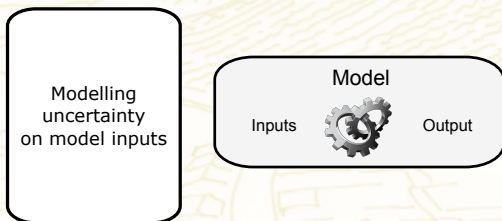
# Sensitivity analysis

How the uncertainty of a model output can be apportioned to different sources of uncertainty in the model inputs



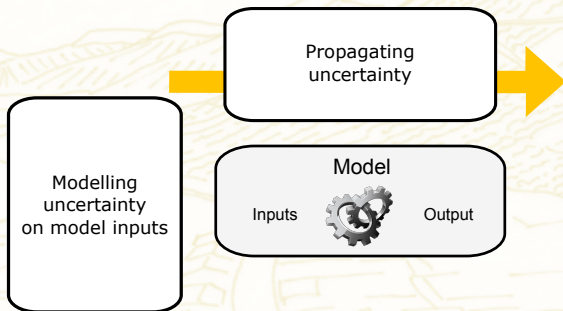
# Sensitivity analysis

How the uncertainty of a model output can be apportioned to different sources of uncertainty in the model inputs



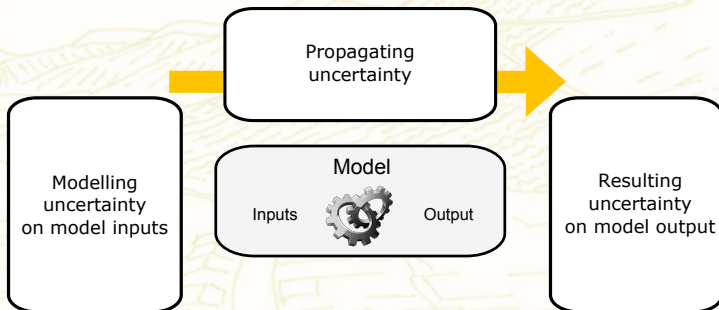
# Sensitivity analysis

How the uncertainty of a model output can be apportioned to different sources of uncertainty in the model inputs



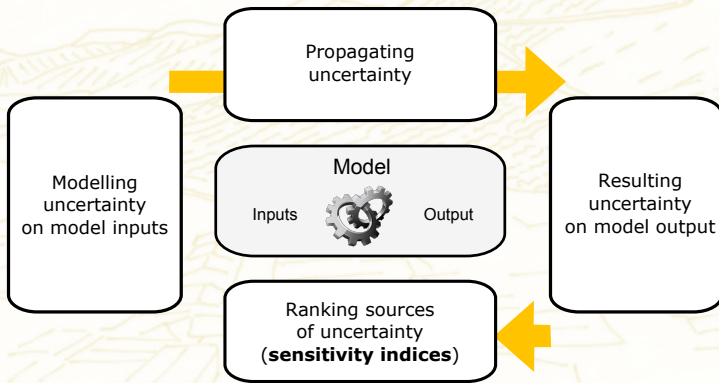
# Sensitivity analysis

How the uncertainty of a model output can be apportioned to different sources of uncertainty in the model inputs



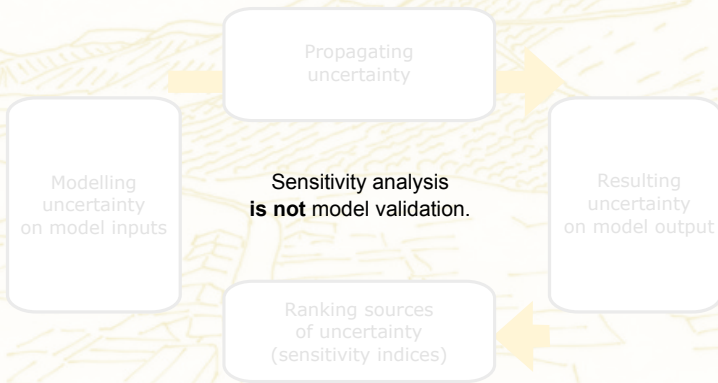
# Sensitivity analysis

How the uncertainty of a model output can be apportioned to different sources of uncertainty in the model inputs



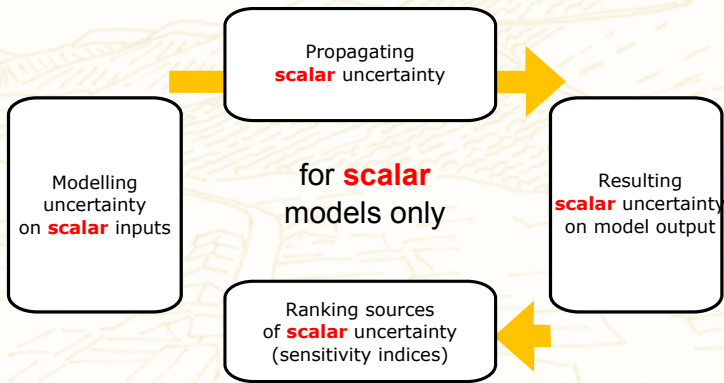
# Sensitivity analysis

How the uncertainty of a model output can be apportioned to different sources of uncertainty in the model inputs



# Sensitivity analysis

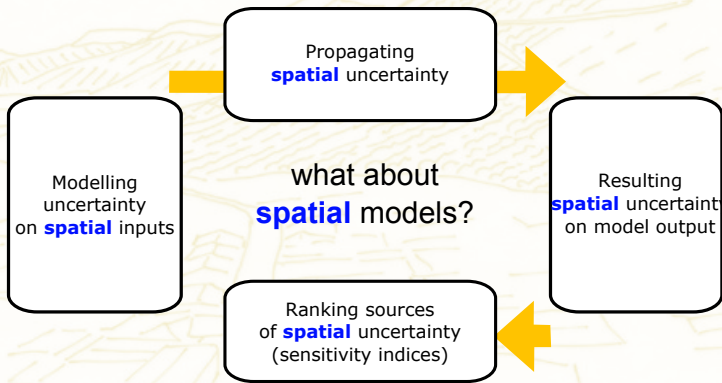
How the uncertainty of a model output can be apportioned to different sources of uncertainty in the model inputs





# Sensitivity analysis

How the uncertainty of a model output can be apportioned to different sources of uncertainty in the model inputs



# Research questions



# Research questions

## Research goal

investigate the use  
of sensitivity analysis  
for spatial models

# Research questions

## Research goal

investigate the use  
of sensitivity analysis  
for spatial models

## Scope

- **VB-GSA**: Variance-Based Global Sensitivity Analysis (*Sobol' 1991*)
- low CPU models

# Research questions

## Research goal

investigate the use of sensitivity analysis for spatial models

## Scope

- **VB-GSA**: Variance-Based Global Sensitivity Analysis (*Sobol' 1991*)
- low CPU models

## 2 research questions

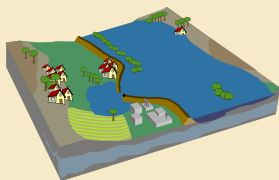
- 1 how to compute sensitivity indices for **spatial inputs**?
- 2 how to account for **scale issues** with VB-GSA?

# An inductive approach



# An inductive approach

## One case-study



flood risk model  
on the Orb river floodplain

# An inductive approach

first tests of  
sensitivity  
analysis

One case-study



flood risk model  
on the Orb river floodplain



# An inductive approach

first tests of  
sensitivity  
analysis

methodological  
problems &  
observations

One case-study



flood risk model  
on the Orb river floodplain

# An inductive approach

first tests of  
sensitivity  
analysis

methodological  
problems &  
observations

One case-study



flood risk model  
on the Orb river floodplain

Theoretical framework



to explain observations  
to elaborate methods

# An inductive approach

first tests of  
sensitivity  
analysis

methodological  
problems &  
observations

One case-study



flood risk model  
on the Orb river floodplain

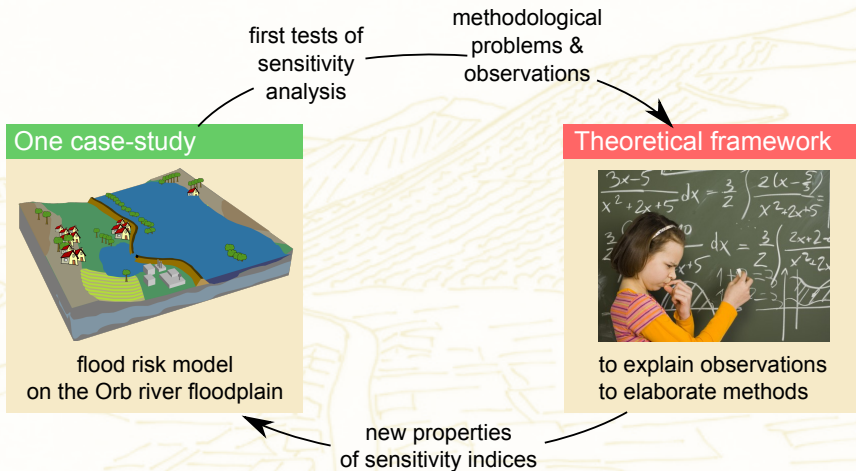
Theoretical framework



to explain observations  
to elaborate methods

new properties  
of sensitivity indices

# An inductive approach



# An inductive approach

first tests of  
sensitivity  
analysis

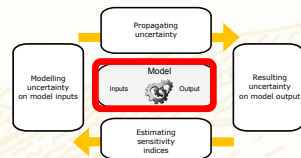
One case-study



flood risk model  
on the Orb river floodplain

# An inductive approach

first tests of  
sensitivity  
analysis



preliminary step:

**delimit** the model under study

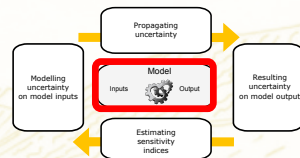
One case-study



flood risk model  
on the Orb river floodplain

# An inductive approach

first tests of  
sensitivity  
analysis



preliminary step:  
**delimit** the model under study

but: initial model **not fully**  
**suitable** to our needs

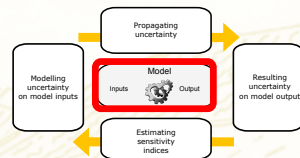
One case-study



flood risk model  
on the Orb river floodplain

# An inductive approach

first tests of  
sensitivity  
analysis



One case-study



flood risk model  
on the Orb river floodplain

preliminary step:  
**delimit** the model under study

but: initial model **not fully  
suitable** to our needs

**better describe the model**  
(not presented)



## Research goal

investigate the use of sensitivity analysis for spatial models

## Scope

- VB-GSA: Variance-Based Global Sensitivity Analysis
- low CPU models

## 2 research questions

- 1 how to deal with **spatial inputs** in VB-GSA?
- 2 how to account for **scale issues** with VB-GSA?

## Applied goals

- design a flood risk model on the Orb floodplain
- perform sensitivity analysis

## Focus on

- maps of flood damages at different scales
- uncertainty in spatial data

## Research goal

investigate the use of sensitivity analysis for spatial models

## Scope

- VB-GSA: Variance-Based Global Sensitivity Analysis
- low CPU models

## 2 research questions

- 1 how to deal with **spatial inputs** in VB-GSA?
- 2 how to account for **scale issues** with VB-GSA?

## Applied goals

- design a flood risk model on the Orb floodplain
- perform sensitivity analysis

## Focus on

- maps of flood damages at different scales
- uncertainty in spatial data

## Research goal

investigate the use of sensitivity analysis for spatial models

## Scope

- VB-GSA: Variance-Based Global Sensitivity Analysis
- low CPU models

## 2 research questions

- 1 how to deal with **spatial inputs** in VB-GSA?
- 2 how to account for **scale issues** with VB-GSA?


## Applied goals

- design a flood risk model on the Orb floodplain
- perform sensitivity analysis

## Focus on

- maps of flood damages at different scales
- uncertainty in spatial data

# Outline of the presentation

- 
- 2 The Orb case-study
  - 3 Variance-based sensitivity indices for spatial inputs
  - 4 Spatial scale issues

## The Orb case-study



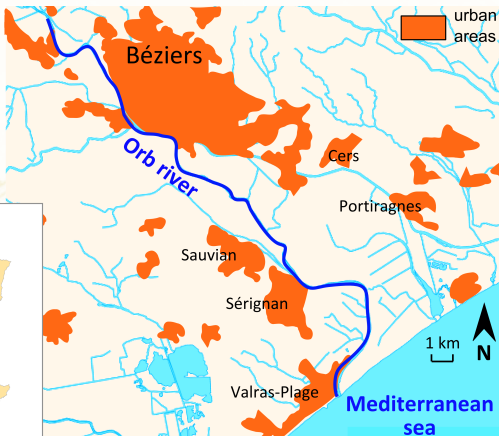
# The Orb floodplain (Hérault)



# The Orb floodplain (Hérault)

floodplain: 100 km<sup>2</sup>

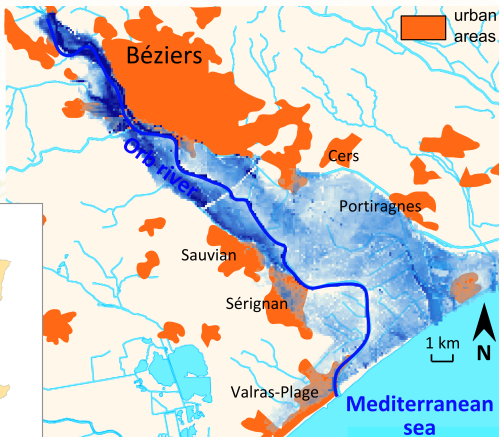
15,000 inhabitants



# The Orb floodplain (Hérault)

floodplain: 100 km<sup>2</sup>

15,000 inhabitants



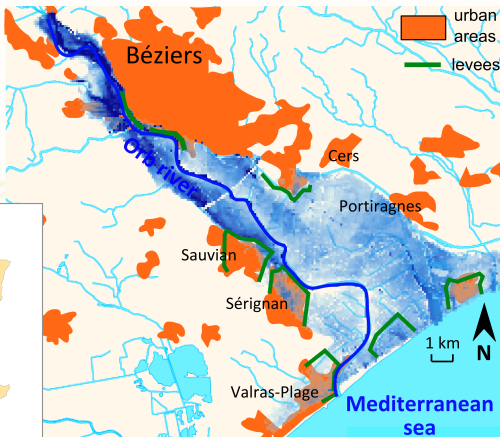


# The Orb floodplain (Hérault)

floodplain: 100 km<sup>2</sup>

15,000 inhabitants

flood risk management  
project (levees)

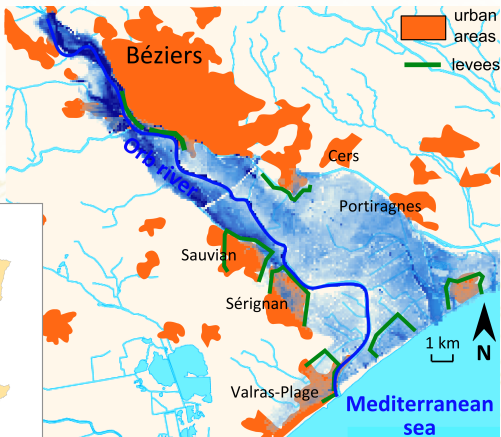


# The Orb floodplain (Hérault)

floodplain: 100 km<sup>2</sup>

15,000 inhabitants

flood risk management  
project (levees)



→ assessment of flood damage reduction  
(required by project funders - EU Floods Directive)



Terrain  
elevation



Hazard map  
(water depths)



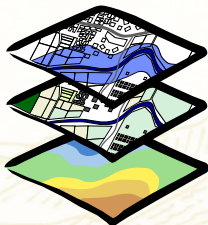
Terrain  
elevation



Hazard map  
(water depths)

Elements at risk  
(landuse map)

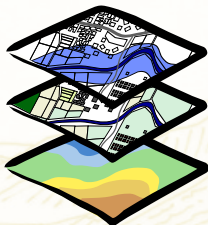
Terrain  
elevation



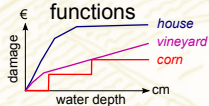
Hazard map  
(water depths)

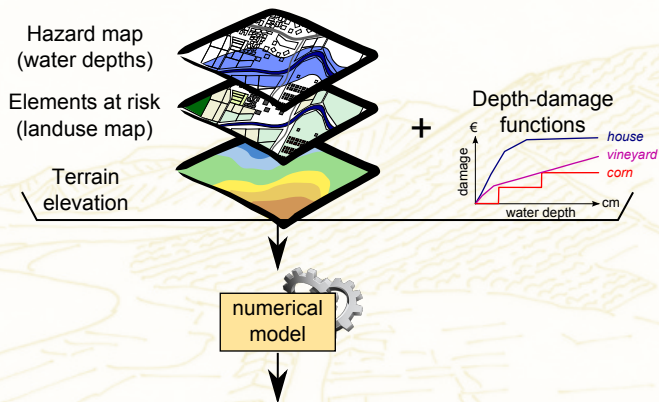
Elements at risk  
(landuse map)

Terrain  
elevation

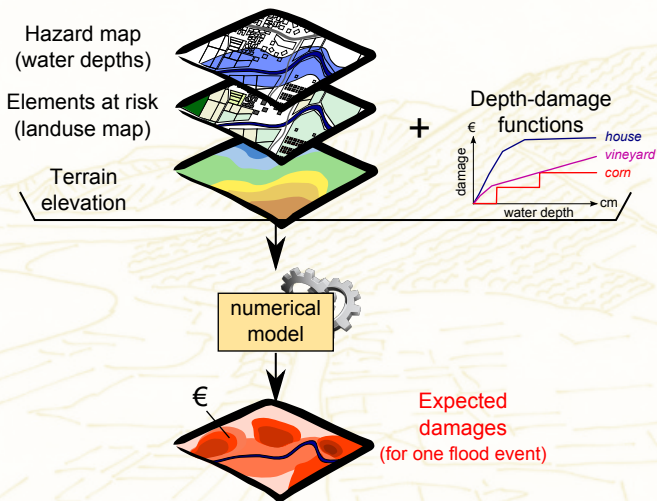


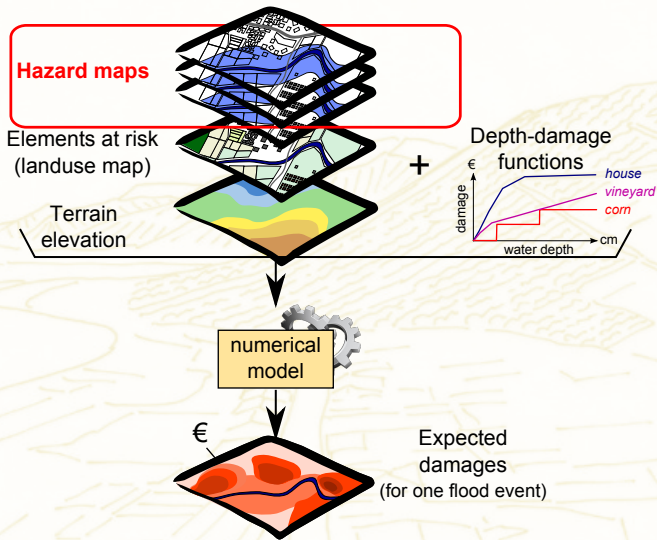
Depth-damage  
functions

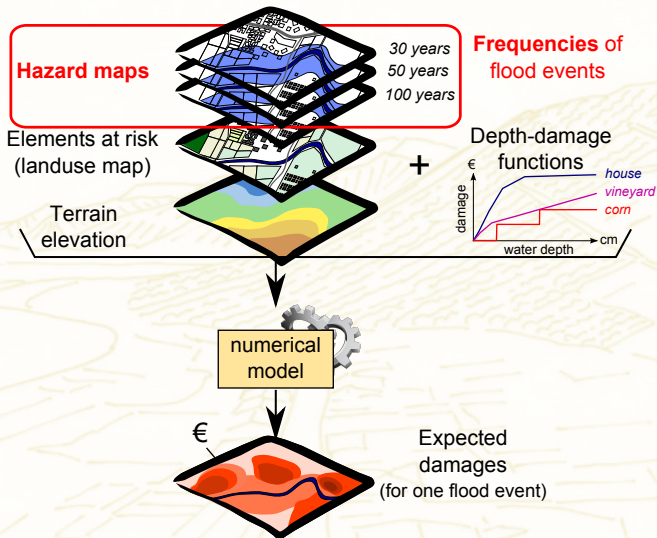


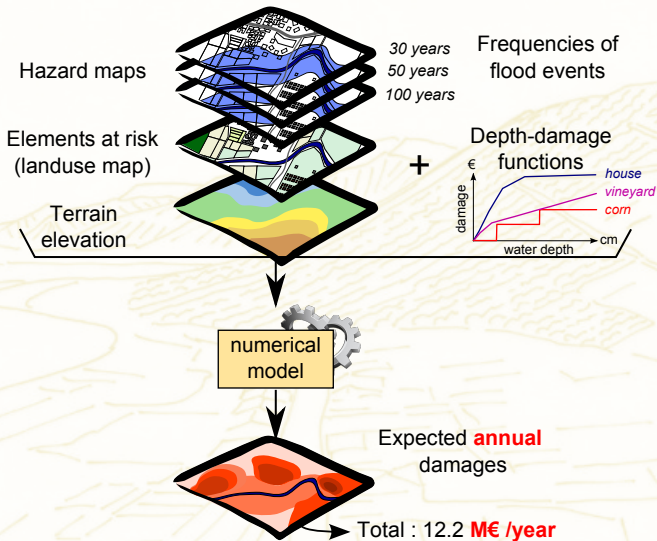


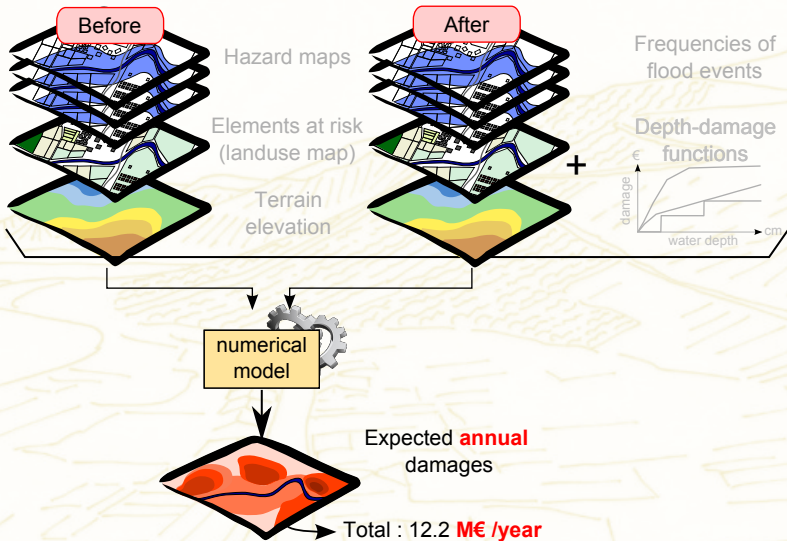


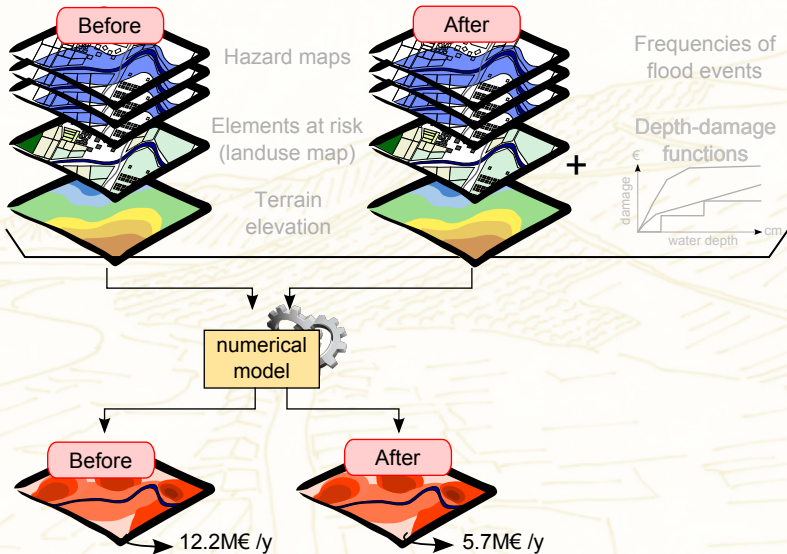


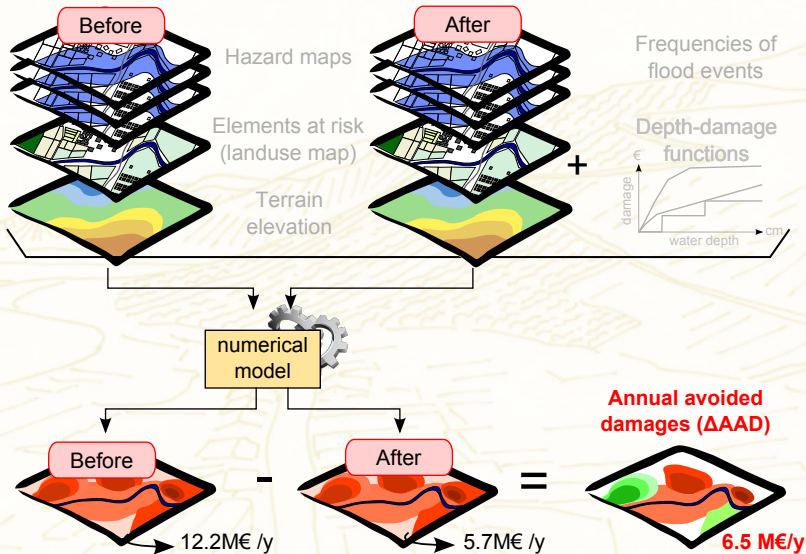








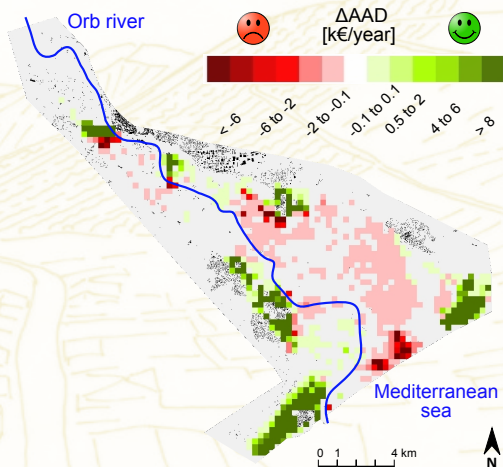




# Orb case-study: model output

Map of **expected annual avoided** flood damages

What spatial **support** (=aggregated area) for model output?

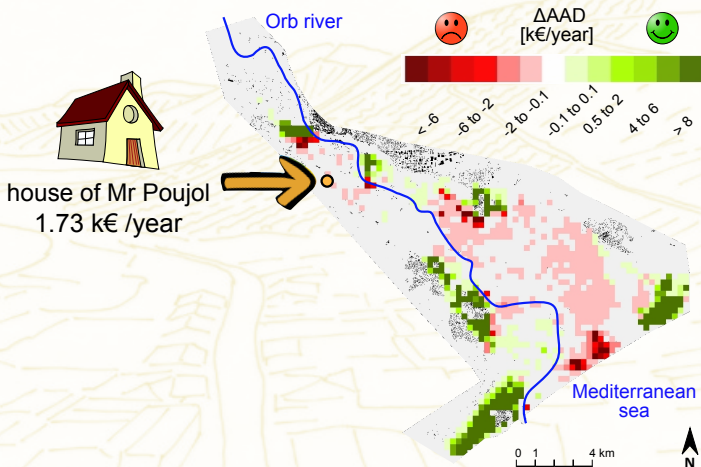




# Orb case-study: model output

Map of **expected annual avoided** flood damages

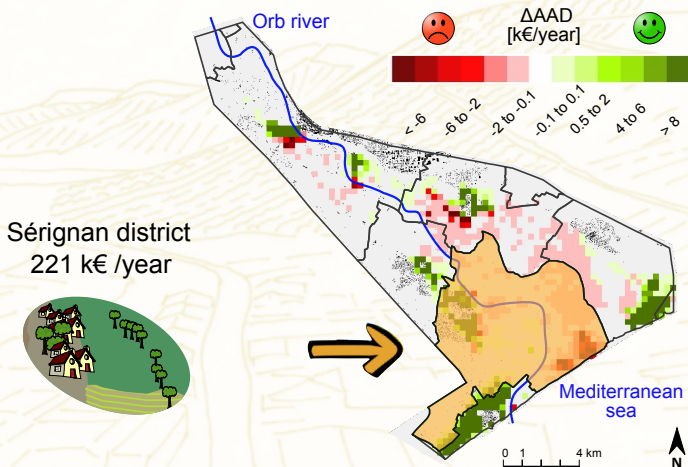
What spatial **support** (=aggregated area) for model output?



# Orb case-study: model output

Map of **expected annual avoided** flood damages

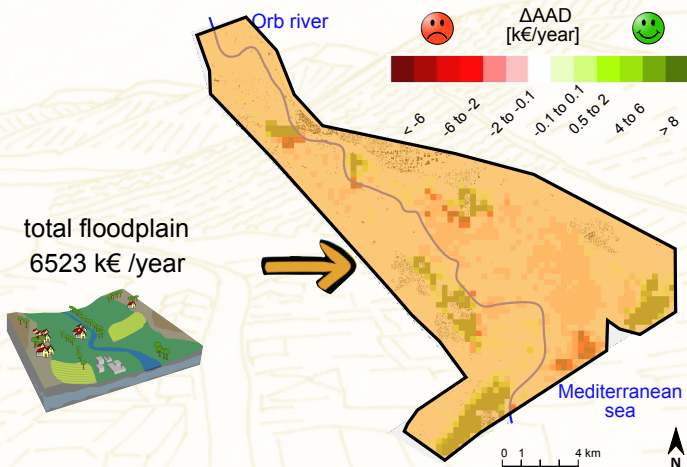
What spatial **support** (=aggregated area) for model output?



# Orb case-study: model output

Map of **expected annual avoided flood damages**

What spatial **support** (=aggregated area) for model output?

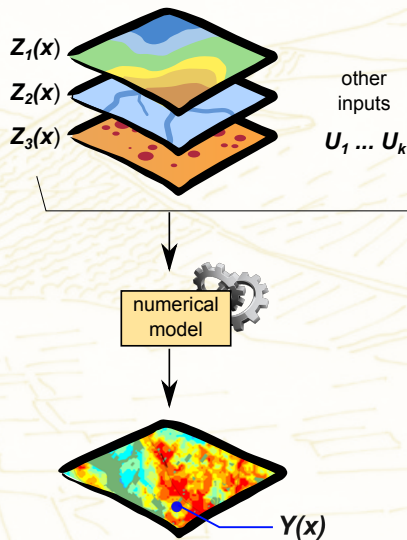


# Main characteristics of the Orb case-study



# Main characteristics of the Orb case-study

## A spatially distributed model

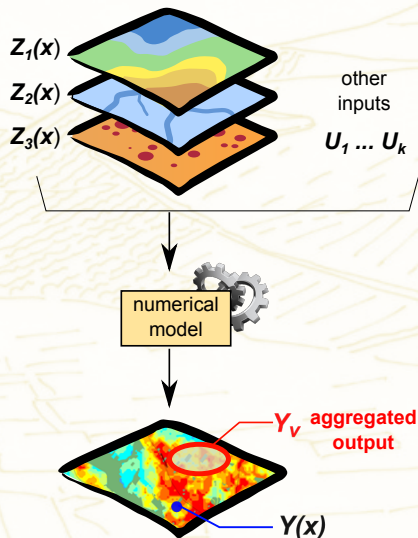


# Main characteristics of the Orb case-study

## A spatially distributed model

- a **spatially additive** model:

$$\forall \nu \subset \mathbb{R}^2, \quad Y_\nu = \int_{\mathbf{x} \in \nu} Y(\mathbf{x}) d\mathbf{x}$$



# Main characteristics of the Orb case-study

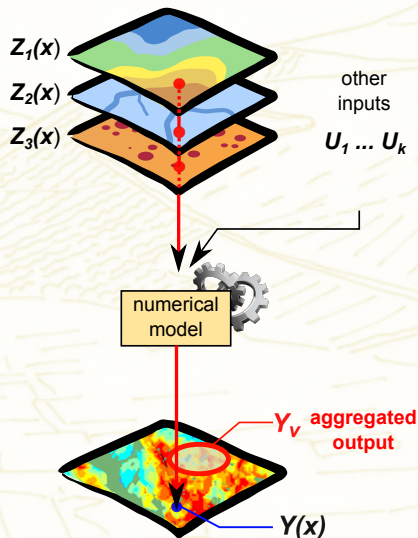
## A spatially distributed model

- a **spatially additive** model:

$$\forall \nu \subset \mathbb{R}^2, \quad Y_\nu = \int_{\mathbf{x} \in \nu} Y(\mathbf{x}) d\mathbf{x}$$

- a **point-based** model:

$$\forall \mathbf{x} \in \mathbb{R}^2, \quad Y(\mathbf{x}) = \mathcal{F}_{\text{loc}}(\mathbf{U}, Z(\mathbf{x}))$$



# Main characteristics of the Orb case-study

## A spatially distributed model

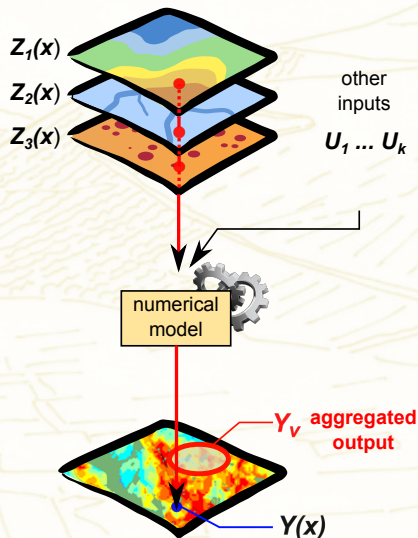
- a **spatially additive** model:

$$\forall \nu \subset \mathbb{R}^2, \quad Y_\nu = \int_{\mathbf{x} \in \nu} Y(\mathbf{x}) d\mathbf{x}$$

- a **point-based** model:

$$\forall \mathbf{x} \in \mathbb{R}^2, \quad Y(\mathbf{x}) = \mathcal{F}_{\text{loc}}(\mathbf{U}, Z(\mathbf{x}))$$

- a **deterministic** model

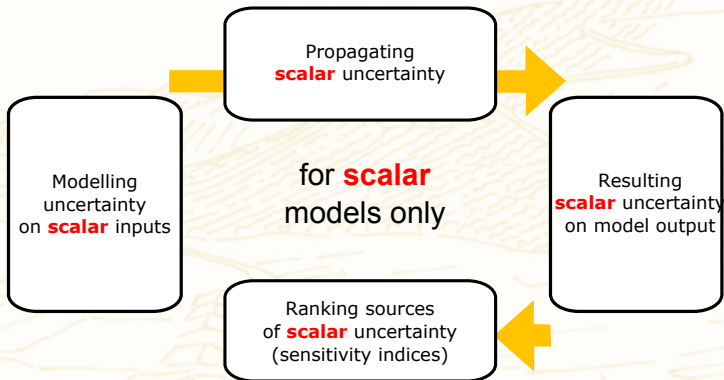




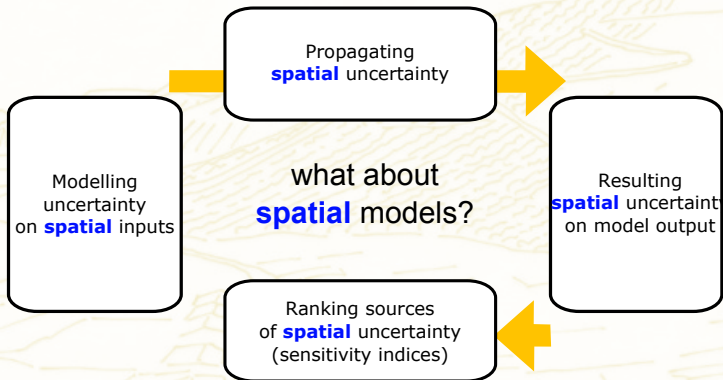
1<sup>st</sup> research question

# Variance-based sensitivity indices for spatial inputs

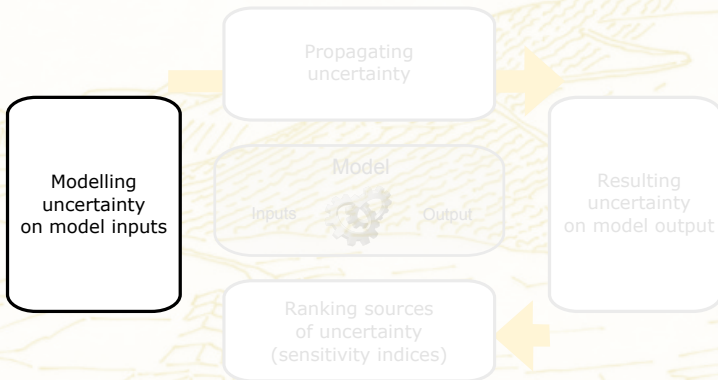
# Problem



# Problem



# Problem



# Modelling uncertainty on spatial inputs

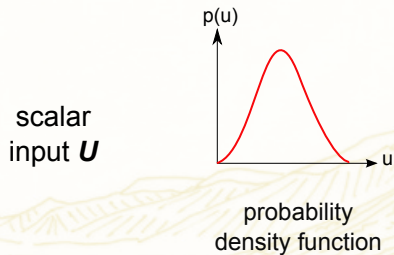


# Modelling uncertainty on spatial inputs

scalar  
input  $U$

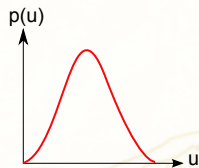


# Modelling uncertainty on spatial inputs



# Modelling uncertainty on spatial inputs

scalar  
input  $U$



probability  
density function

$$u^{(1)} = 12.3$$

$$u^{(2)} = 4.5$$

$$u^{(3)} = 8.9$$

$$u^{(4)} = 7.4$$

$$u^{(5)} = 48.5$$

$$u^{(6)} = 23$$

$$u^{(7)} = 8.5$$

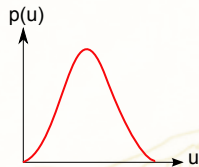
$$u^{(8)} = 10.5$$

random sample



# Modelling uncertainty on spatial inputs

scalar  
input  $U$



probability  
density function

$$u^{(1)} = 12.3$$

$$u^{(2)} = 4.5$$

$$u^{(3)} = 8.9$$

$$u^{(4)} = 7.4$$

$$u^{(5)} = 48.5$$

$$u^{(6)} = 23$$

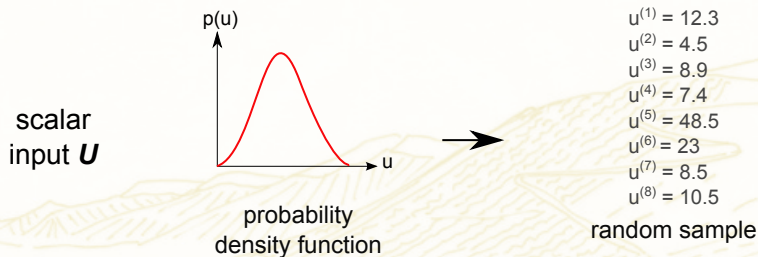
$$u^{(7)} = 8.5$$

$$u^{(8)} = 10.5$$

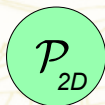
random sample

spatial  
input  $Z(x)$

# Modelling uncertainty on spatial inputs



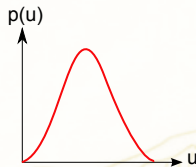
spatial input  $Z(\mathbf{x})$



stochastic process

# Modelling uncertainty on spatial inputs

scalar  
input  $U$



probability  
density function

$$u^{(1)} = 12.3$$

$$u^{(2)} = 4.5$$

$$u^{(3)} = 8.9$$

$$u^{(4)} = 7.4$$

$$u^{(5)} = 48.5$$

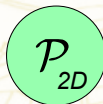
$$u^{(6)} = 23$$

$$u^{(7)} = 8.5$$

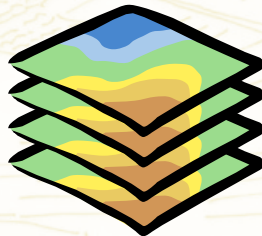
$$u^{(8)} = 10.5$$

random sample

spatial  
input  $Z(x)$



stochastic  
process



$n$  random realisations

# Modelling uncertainty on spatial inputs



# Modelling uncertainty on spatial inputs

## Terrain elevation (DEM)

- grid of 5m resolution
- from aerial photography



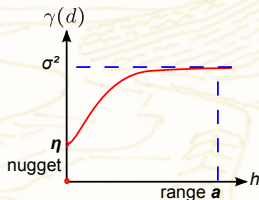
# Modelling uncertainty on spatial inputs

## Terrain elevation (DEM)

- grid of 5m resolution
- from aerial photography

## Modelling uncertainty

- measure & interpolation errors
- 2D Gaussian error field
- variogram model (estimated)



variogram model

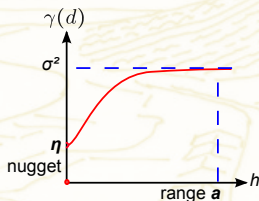
# Modelling uncertainty on spatial inputs

## Terrain elevation (DEM)

- grid of 5m resolution
- from aerial photography

## Modelling uncertainty

- measure & interpolation errors
- 2D Gaussian error field
- variogram model (estimated)



variogram model

geostatistical  
algorithm



**$n=100$  random  
realisations of DEM**

# Modelling uncertainty on spatial inputs





# Modelling uncertainty on spatial inputs

## Landuse map

- Vector data
- from remote sensing  
+ field surveys



# Modelling uncertainty on spatial inputs

## Landuse map

- Vector data
- from remote sensing  
+ field surveys

## Modelling uncertainty

- classification errors
- confusion matrix

	vineyard	corn
vineyard	0.82	0.18
corn	0.18	0.82

confusion matrix

# Modelling uncertainty on spatial inputs

## Landuse map

- Vector data
- from remote sensing  
+ field surveys

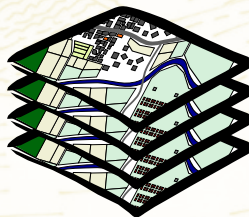
## Modelling uncertainty

- classification errors
- confusion matrix

	vineyard	corn
vineyard	0.82	0.18
corn	0.18	0.82

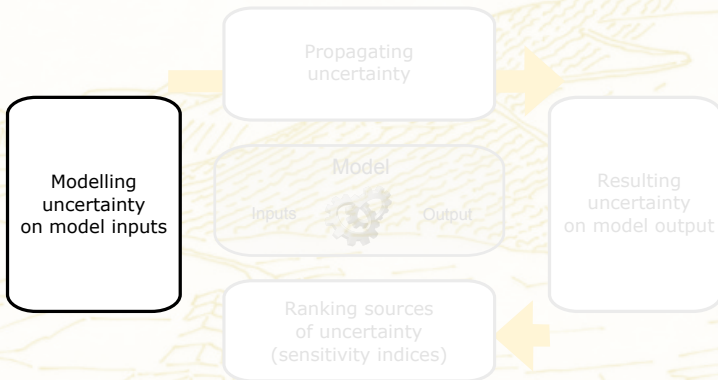
confusion matrix

→  
Monte Carlo  
simulation

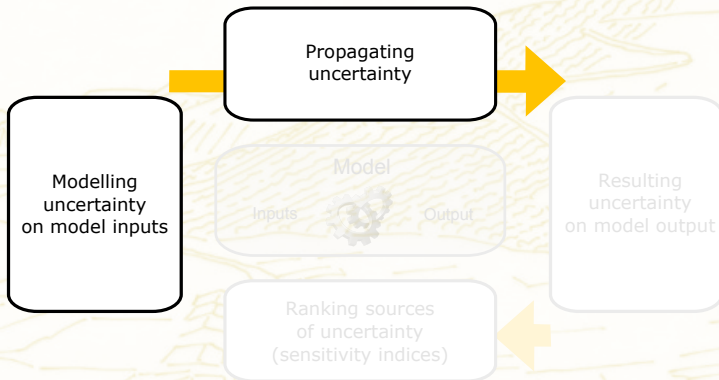


***n=1000 random realisations  
of landuse map***

# Propagating uncertainty



# Propagating uncertainty



# Propagating uncertainty

What uncertainty should be propagated?



stochastic  
process

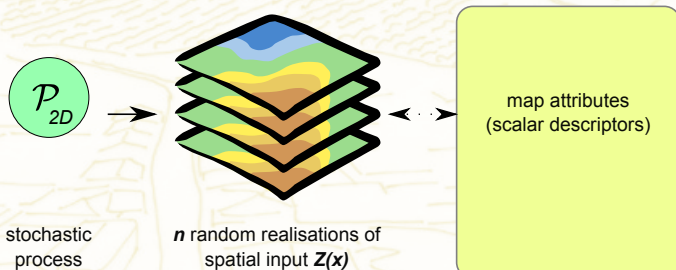


$n$  random realisations of  
spatial input  $Z(x)$

# Propagating uncertainty

What uncertainty should be propagated?

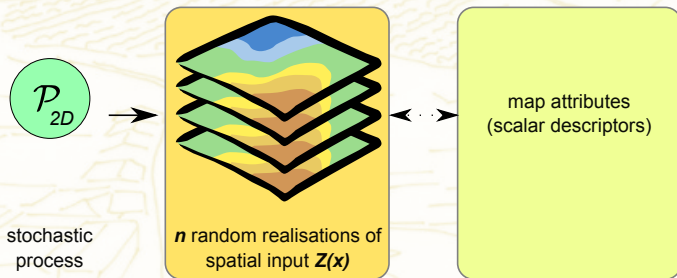
- uncertainty on some **map attributes**



# Propagating uncertainty

What uncertainty should be propagated?

- uncertainty on some **map attributes**
- uncertainty on the set of random realisations (fixed  $\mathcal{P}_{2D}$ )

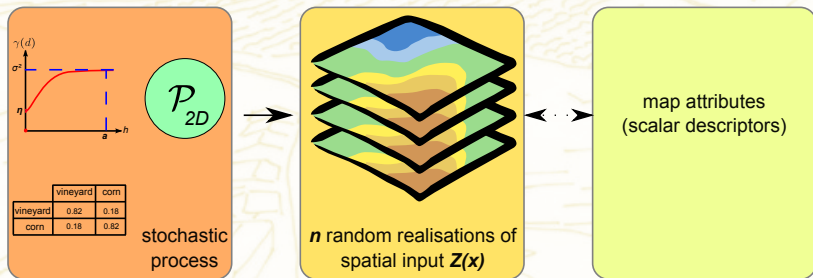




# Propagating uncertainty

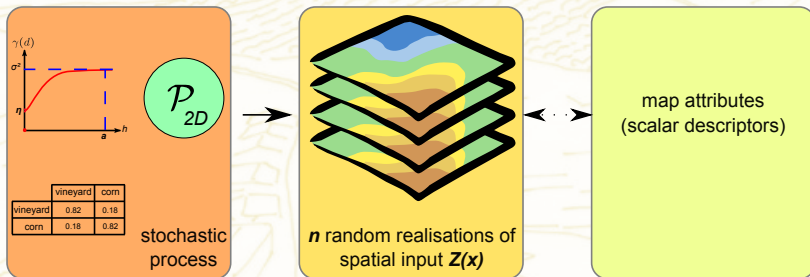
## What uncertainty should be propagated?

- uncertainty on some **map attributes**
- uncertainty on the set of random realisations (fixed  $\mathcal{P}_{2D}$ )
- additional uncertainty on stochastic process  $\mathcal{P}_{2D}$  (**second-level**)



# Propagating uncertainty

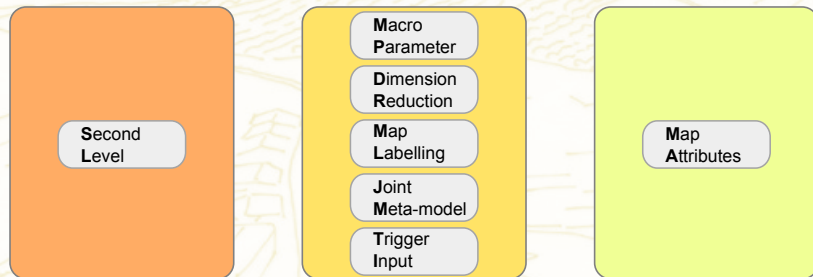
## Benchmark of methods



# Propagating uncertainty

## Benchmark of methods

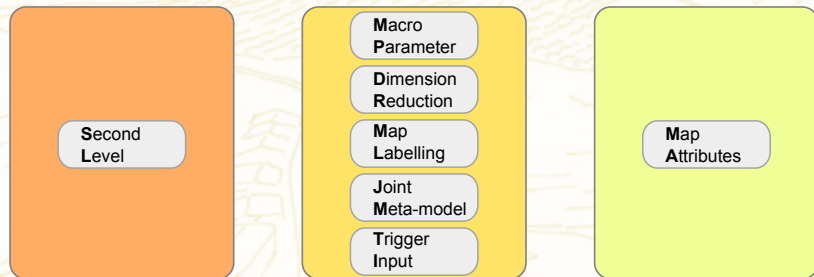
- 7 methods reviewed



# Propagating uncertainty

## Benchmark of methods

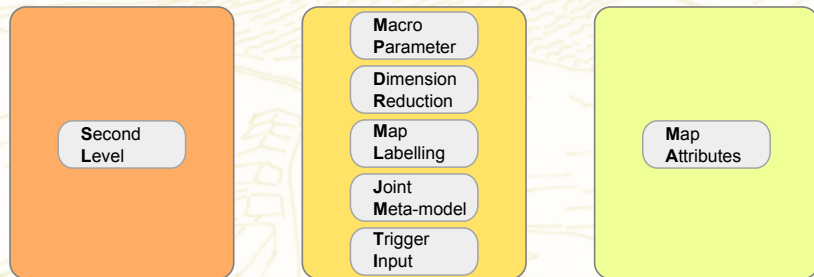
- 7 methods reviewed + **comparison** on analytical test cases:
- CPU cost? / many spatial inputs? / any  $\mathcal{P}_{2D}$ ? / meta-modelling?



# Propagating uncertainty

## Benchmark of methods

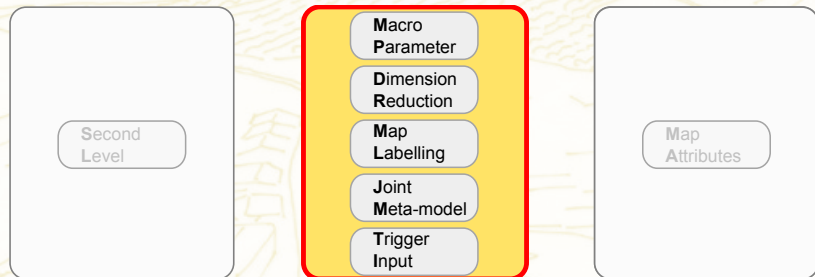
- 7 methods reviewed + **comparison** on analytical test cases:
- CPU cost? / many spatial inputs? / any  $\mathcal{P}_{2D}$ ? / meta-modelling?
- **main idea**: spatial input  $Z(x)$  → some scalar parameters



# Propagating uncertainty

## Benchmark of methods

- 7 methods reviewed + **comparison** on analytical test cases:
- CPU cost? / many spatial inputs? / any  $\mathcal{P}_{2D}$ ? / meta-modelling?
- **main idea**: spatial input  $Z(x)$  → some scalar parameters

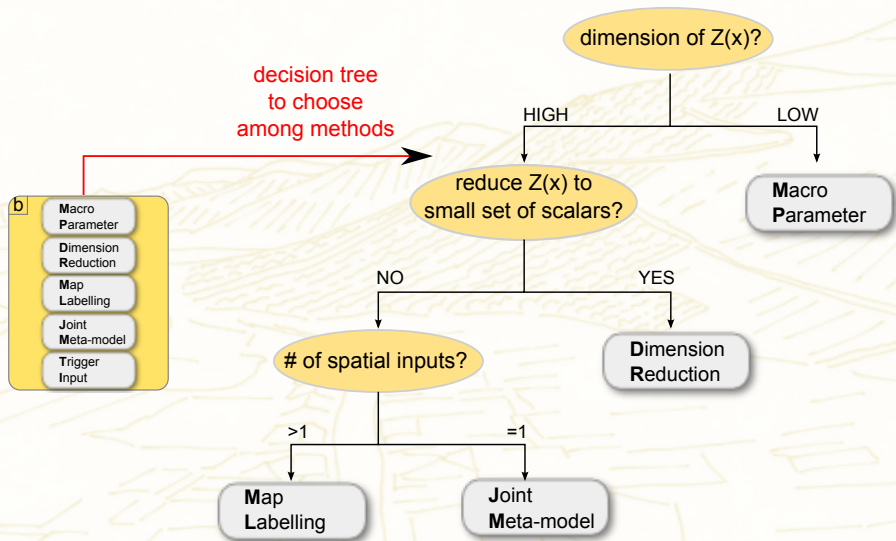


# Propagating uncertainty



# Propagating uncertainty

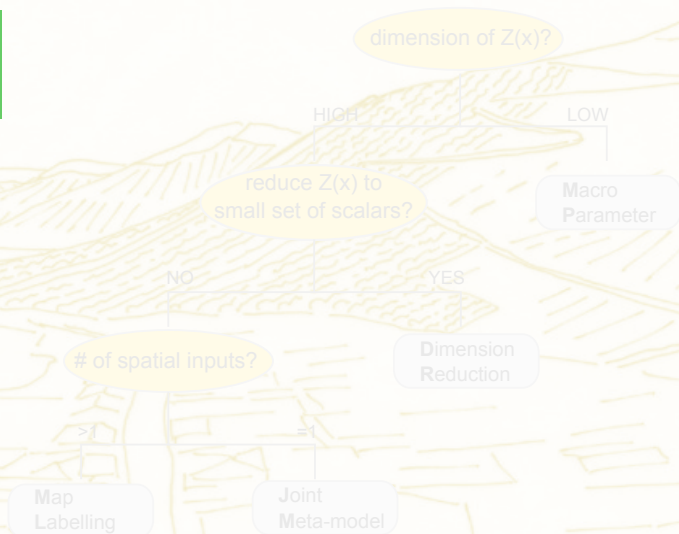
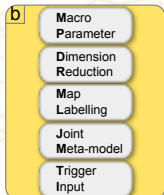
decision tree  
to choose  
among methods





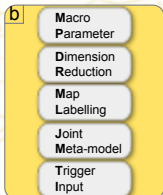
# Propagating uncertainty

Example  
on the  
Orb case-study



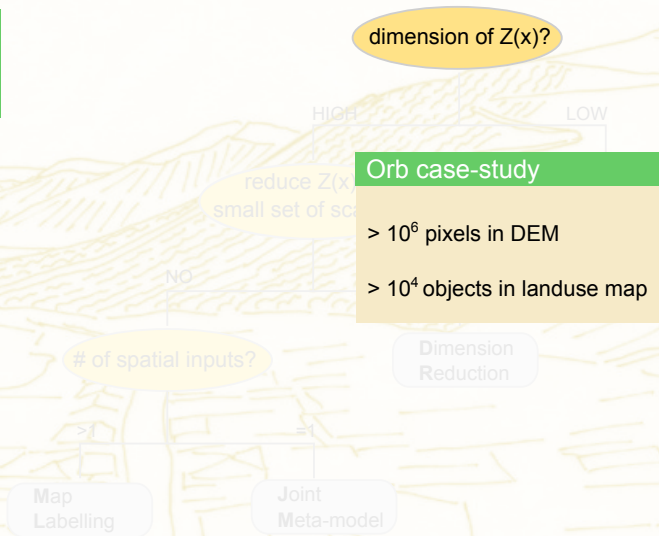
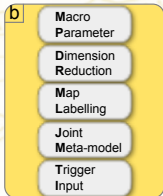
# Propagating uncertainty

Example  
on the  
Orb case-study



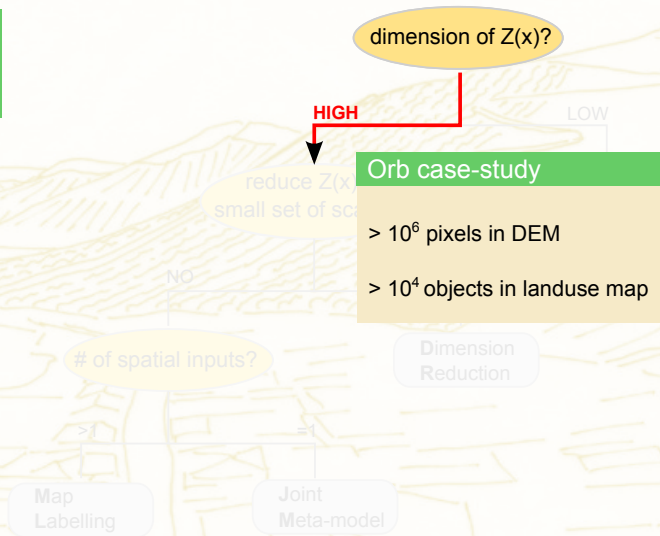
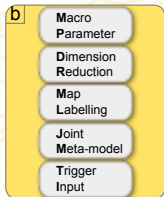
# Propagating uncertainty

Example  
on the  
Orb case-study



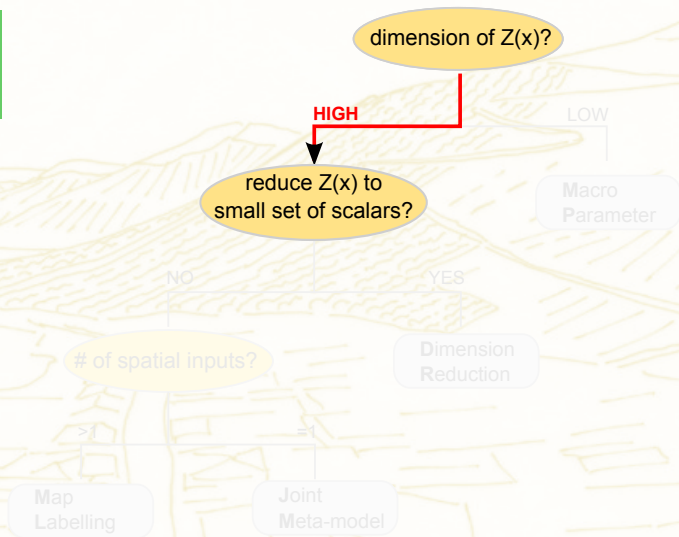
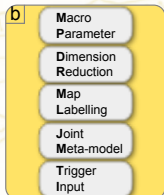
# Propagating uncertainty

Example  
on the  
Orb case-study



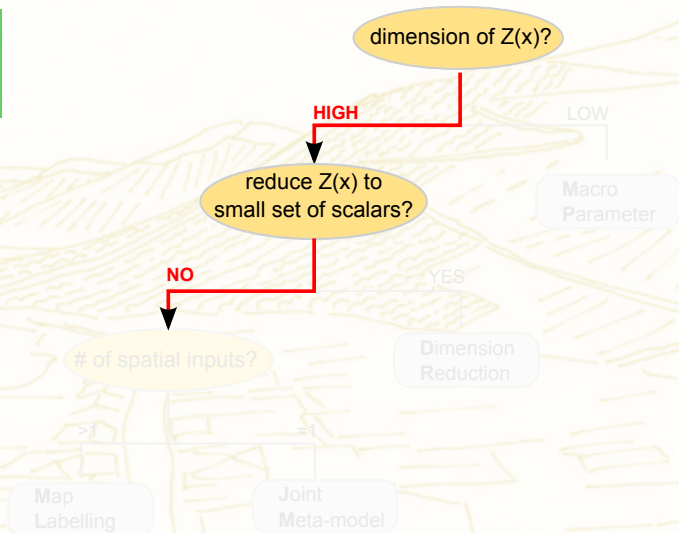
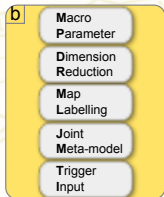
# Propagating uncertainty

Example  
on the  
Orb case-study



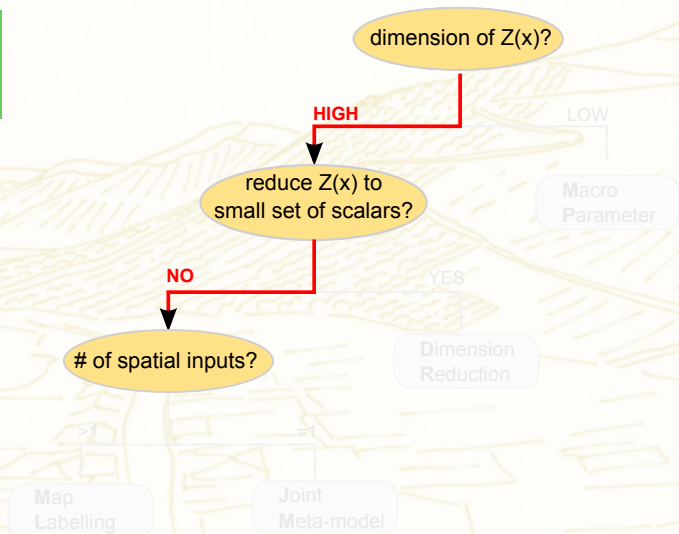
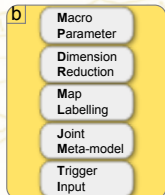
# Propagating uncertainty

Example  
on the  
Orb case-study



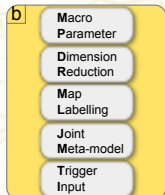
# Propagating uncertainty

Example  
on the  
Orb case-study



# Propagating uncertainty

Example  
on the  
Orb case-study



# of spatial inputs?

NO

reduce  $Z(x)$  to  
small set of scalars?

HIGH

dimension of  $Z(x)$ ?

LOW

Macro  
Parameter

YES

Dimension

Orb case-study

3 spatial inputs:

- landuse map
- terrain elevation (DEM)
- hazard maps

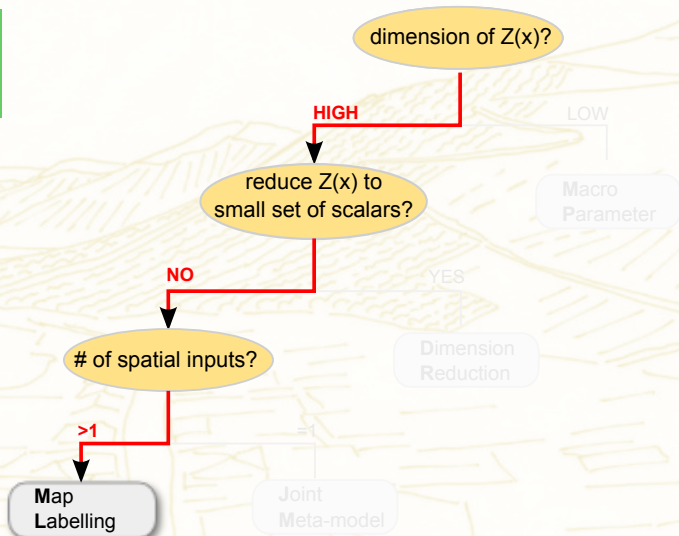
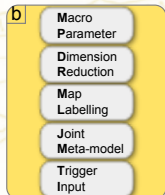
Map  
Labelling

Joint  
Meta-model



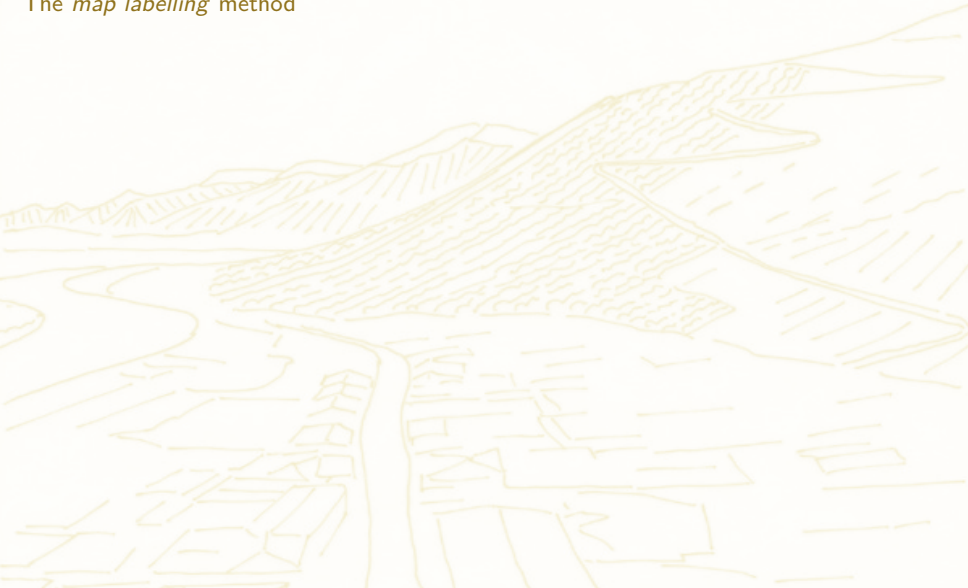
# Propagating uncertainty

Example  
on the  
Orb case-study



# Propagating uncertainty

## The *map labelling* method



# Propagating uncertainty

## The *map labelling* method

### Labelling map realisations



stochastic  
process

$n$  random realisations of  
spatial input  $Z(\mathbf{x})$

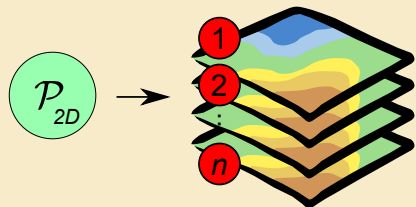
### Model under study

$$Y(\mathbf{x}) = \mathcal{F}(\mathbf{U}, Z(\mathbf{x}))$$

# Propagating uncertainty

## The *map labelling* method

### Labelling map realisations



stochastic  
process

$n$  random realisations of  
spatial input  $\mathbf{Z}(\mathbf{x})$

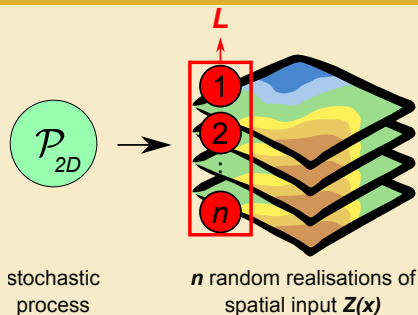
### Model under study

$$Y(\mathbf{x}) = \mathcal{F}(\mathbf{U}, \mathbf{Z}(\mathbf{x}))$$

# Propagating uncertainty

## The *map labelling* method

### Labelling map realisations



### Model under study

$$Y(x) = \mathcal{F}(\mathbf{U}, Z(x))$$

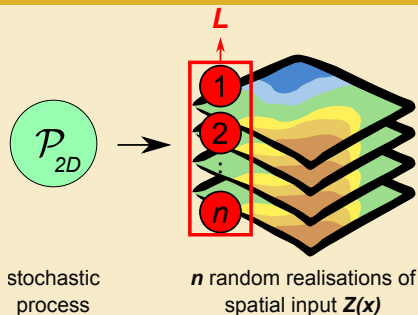
random label:

$$L \sim \text{unif}\{1, \dots, n\}$$

# Propagating uncertainty

## The *map labelling* method

### Labelling map realisations



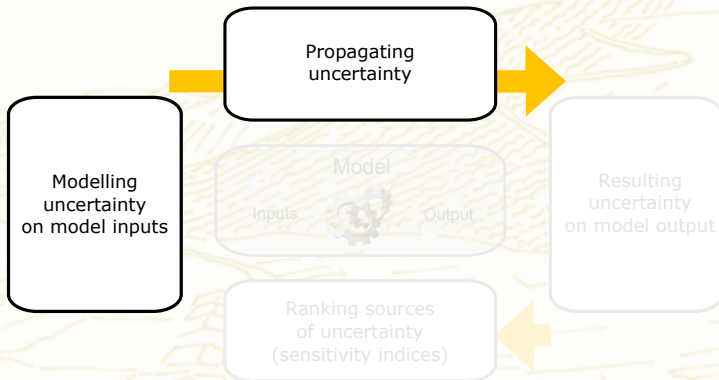
### Model under study

$$Y(\mathbf{x}) = \mathcal{F}^*(\mathbf{U}, \mathbf{L})$$

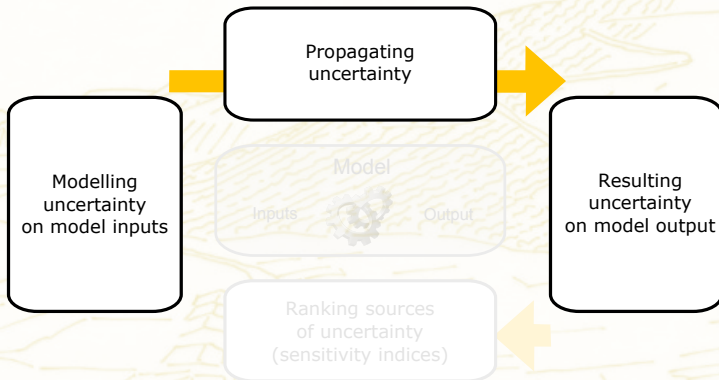
random label:

$$\mathbf{L} \sim \text{unif}\{1, \dots, n\}$$

# Results on the Orb case-study

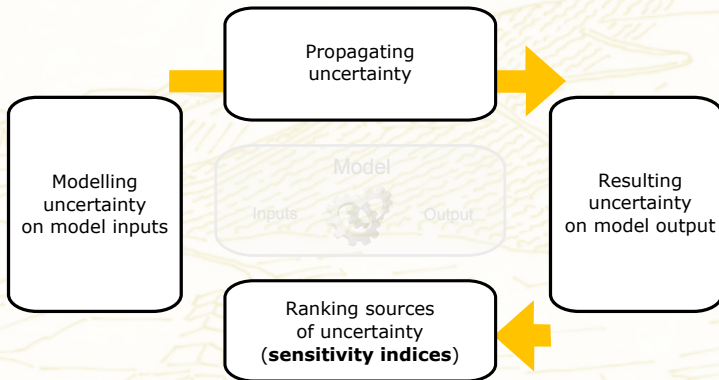


# Results on the Orb case-study

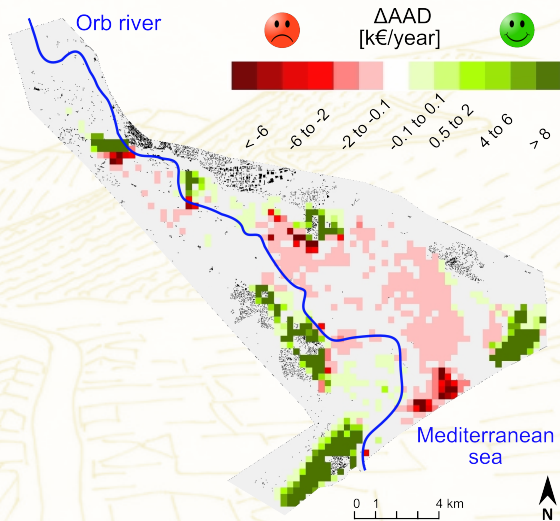




# Results on the Orb case-study

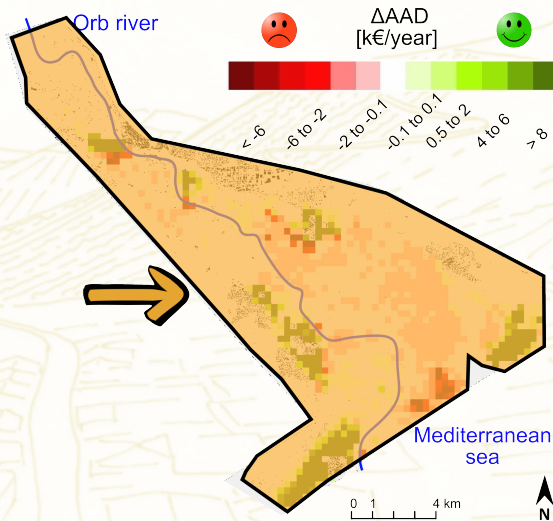


# Results on the Orb case-study



# Results on the Orb case-study

Output of interest:  
annual avoided  
damages ( $\Delta$ AAD)  
over **the floodplain**



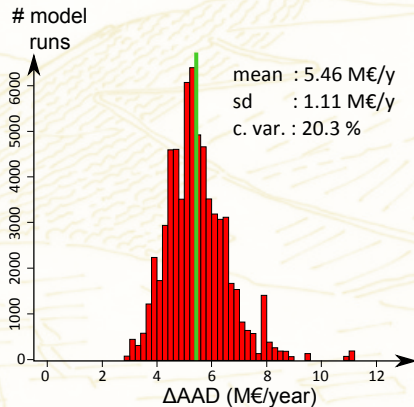
# Results on the Orb case-study



# Results on the Orb case-study

## Variability of $\Delta$ AAD indicator

- empirical distribution over  $N = 28672$  model runs
- pretty large variance



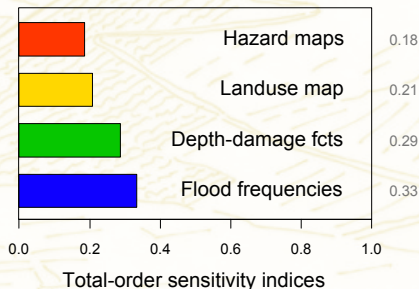
# Results on the Orb case-study

## Variability of $\Delta$ AAD indicator

- empirical distribution over  $N = 28672$  model runs
- pretty large variance

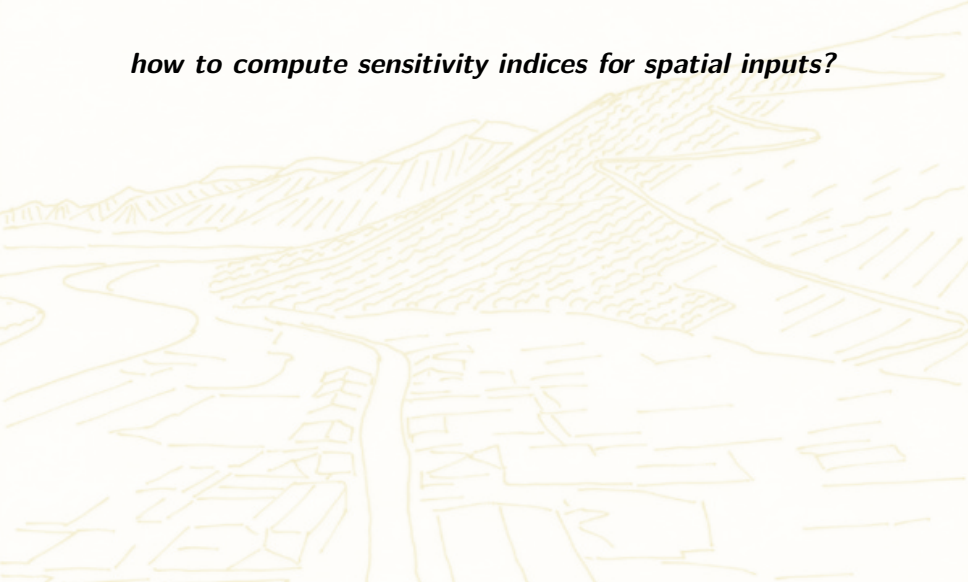
## Sensitivity indices

- all indices are close
- $\rightarrow$  no key source of uncertainty



# Conclusion (first research question)

***how to compute sensitivity indices for spatial inputs?***



# Conclusion (first research question)

***how to compute sensitivity indices for spatial inputs?***

## PhD methodological results

- benchmark of methods
- decision tree
- focus on *map labelling*
  - easy to implement
  - any stochastic process
  - any number of spatial inputs
- investigation of sampling issues



# Conclusion (first research question)

## *how to compute sensitivity indices for spatial inputs?*

### PhD methodological results

- benchmark of methods
- decision tree
- focus on *map labelling*
  - easy to implement
  - any stochastic process
  - any number of spatial inputs
- investigation of sampling issues

### PhD results on the Orb case-study

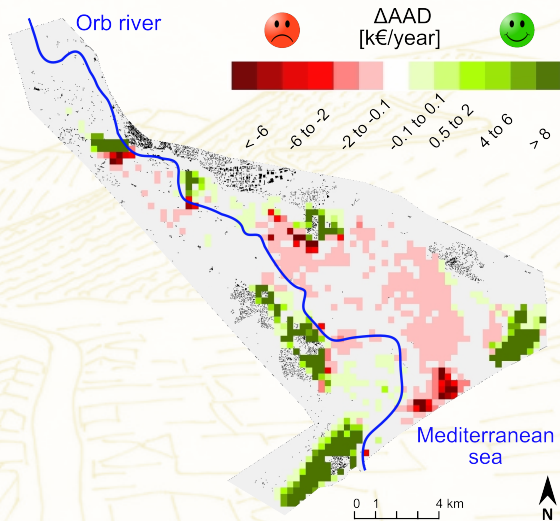
- using *map labelling* to include uncertainties on:
    - hazard maps
    - landuse map
- first ranking of uncertainties



2<sup>nd</sup> research question

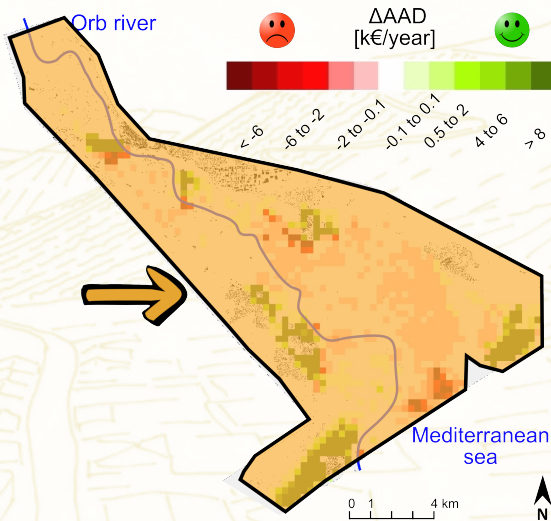
## Spatial scale issues

# Change of support of model output

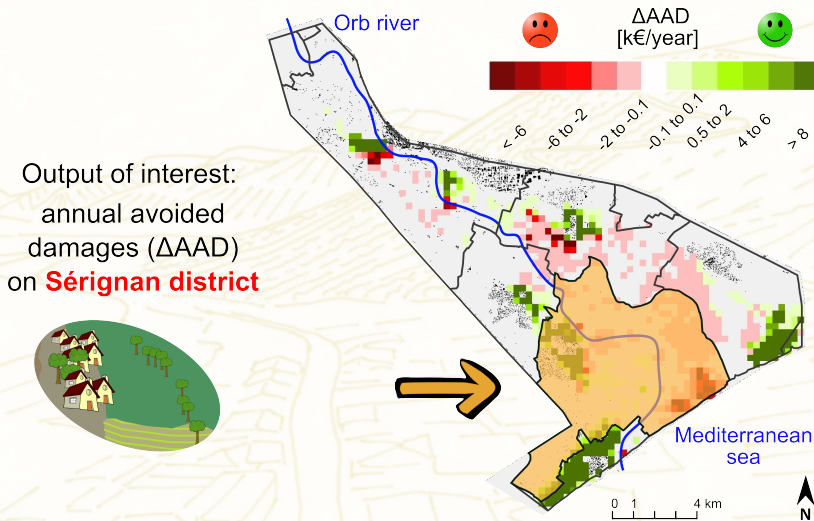


# Change of support of model output

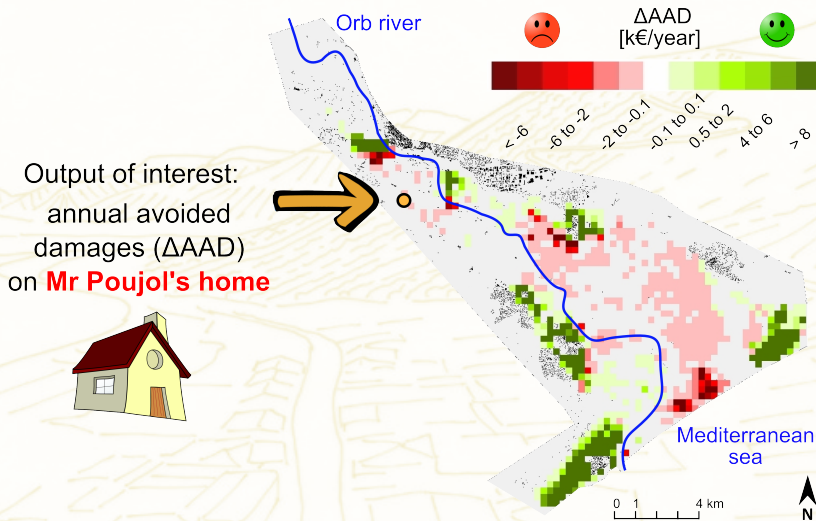
Output of interest:  
annual avoided  
damages ( $\Delta$ AAD)  
over **the floodplain**



# Change of support of model output



# Change of support of model output



# Change of support of model output



**house of Mr Poujol**



**Sérignan district**



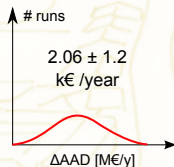
**floodplain**

# Change of support of model output

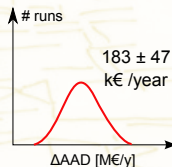
Uncertainty on  $\Delta$ AAD indicator



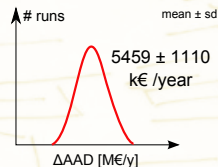
house of Mr Poujol



Sérignan district



floodplain





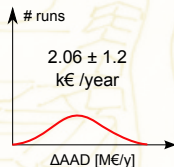
# Change of support of model output

## Uncertainty on $\Delta AAD$ indicator

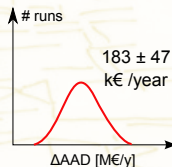
- **smaller** relative uncertainty for **large** supports



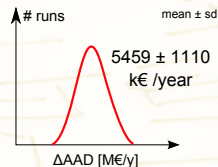
house of Mr Poujol



Sérignan district



floodplain



# Change of support of model output

## Sensitivity indices

- smaller relative uncertainty for large supports



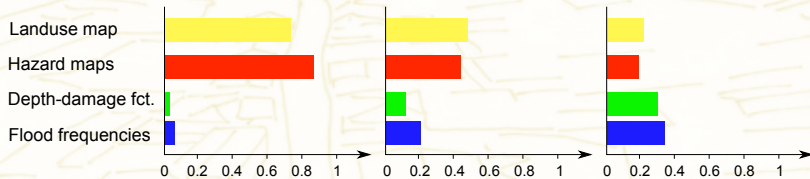
house of Mr Poujol



Sérignan district



floodplain



# Change of support of model output

## Sensitivity indices

- smaller relative uncertainty for large supports
- spatial inputs:  $S_i(\text{floodplain}) < S_i(\text{district}) < S_i(\text{house})$



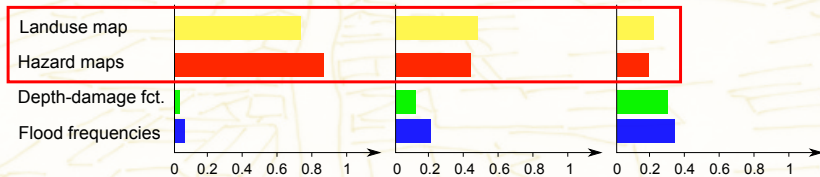
house of Mr Poujol



Sérignan district



floodplain



# Change of support of model output

## Sensitivity indices

- smaller relative uncertainty for large supports
- spatial inputs:  $S_i(\text{floodplain}) < S_i(\text{district}) < S_i(\text{house})$
- non-spatial inputs:  $S_i(\text{floodplain}) > S_i(\text{district}) > S_i(\text{house})$



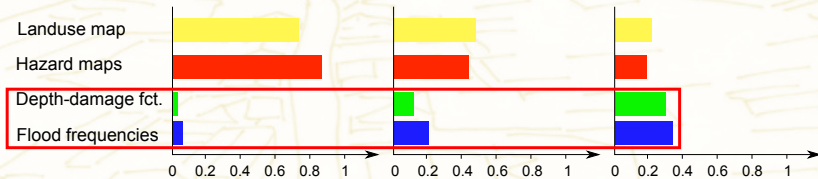
house of Mr Poujol



Sérignan district



floodplain



# Change of support of model output



# Change of support of model output

## Empirical observations

**averaging effect?**

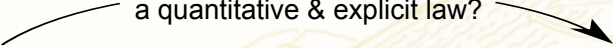
size of spatial support ↗



impact of spatial uncertainties ↘

# Change of support of model output

a general rule?  
a quantitative & explicit law?



Empirical observations

**averaging effect?**

size of spatial support ↗



impact of spatial uncertainties ↘

# Change of support of model output

a general rule?  
a quantitative & explicit law?

## Empirical observations

**averaging effect?**

size of spatial support ↗



impact of spatial uncertainties ↘

## Theoretical framework

**regularisation theory**

(geostatistics)

to show how sensitivity indices  
depend on support of model  
output



# Theoretical framework



# Theoretical framework

## Spatially distributed model $\mathcal{F}$

Let consider a point-based and spatially additive model  $\mathcal{F}$ :

$$\forall x \in \mathbb{R}^2, \quad Y(x) = \mathcal{F}[\mathbf{U}, Z(x)]$$

# Theoretical framework

## Spatially distributed model $\mathcal{F}$

Let consider a point-based and spatially additive model  $\mathcal{F}$ :

$$\forall x \in \mathbb{R}^2, \quad Y(x) = \mathcal{F}[\mathbf{U}, Z(x)]$$

- vector of scalar inputs:  $\mathbf{U} \in \mathbb{R}^d$ , joint pdf  $p_U(\cdot)$

# Theoretical framework

## Spatially distributed model $\mathcal{F}$

Let consider a point-based and spatially additive model  $\mathcal{F}$ :

$$\forall x \in \mathbb{R}^2, \quad Y(x) = \mathcal{F}[\mathbf{U}, Z(x)]$$

- vector of scalar inputs:  $\mathbf{U} \in \mathbb{R}^d$ , joint pdf  $p_U(\cdot)$
- spatial input:  $Z(x)$  **Stationary** Random Field, **covariance**  $C(h)$

# Site and block sensitivity indices



# Site and block sensitivity indices

Definition: **site** sensitivity indices

# Site and block sensitivity indices

## Definition: **site** sensitivity indices

Let  $\mathbf{x} \in \mathbb{R}^2$ .

Output of interest:  $Y(\mathbf{x})$

# Site and block sensitivity indices

## Definition: **site** sensitivity indices

Let  $\mathbf{x} \in \mathbb{R}^2$ .

Output of interest:  $Y(\mathbf{x})$

spatial input  $Z$ : 
$$S_Z(\mathbf{x}) = \frac{\text{var}[E(Y(\mathbf{x}) \mid Z)]}{\text{var}[Y(\mathbf{x})]}$$



# Site and block sensitivity indices

## Definition: **site** sensitivity indices

Let  $\mathbf{x} \in \mathbb{R}^2$ .

Output of interest:  $Y(\mathbf{x})$

spatial input  $Z$ : 
$$S_Z(\mathbf{x}) = \frac{\text{var}[E(Y(\mathbf{x}) \mid Z)]}{\text{var}[Y(\mathbf{x})]}$$

scalar inputs  $\mathbf{U}$ : 
$$S_{\mathbf{U}}(\mathbf{x}) = \frac{\text{var}[E(Y(\mathbf{x}) \mid \mathbf{U})]}{\text{var}[Y(\mathbf{x})]}$$

# Site and block sensitivity indices

## Definition: **site** sensitivity indices

Let  $\mathbf{x} \in \mathbb{R}^2$ .

Output of interest:  $Y(\mathbf{x})$

spatial input  $Z$ :  $S_Z(\mathbf{x}) = \frac{\text{var}[E(Y(\mathbf{x}) | Z)]}{\text{var}[Y(\mathbf{x})]}$

**stationarity (hyp.)**

scalar inputs  $\mathbf{U}$ :  $S_U(\mathbf{x}) = \frac{\text{var}[E(Y(\mathbf{x}) | \mathbf{U})]}{\text{var}[Y(\mathbf{x})]}$

# Site and block sensitivity indices

Definition: **block** sensitivity indices

# Site and block sensitivity indices

Definition: **block** sensitivity indices

Let  $\nu \subset \mathbb{R}^2$ .

Output of interest:  $Y_\nu = \int_{\mathbf{x} \in \nu} Y(\mathbf{x}) d\mathbf{x}$

# Site and block sensitivity indices

Definition: **block** sensitivity indices

Let  $\nu \subset \mathbb{R}^2$ .

Output of interest:  $Y_\nu = \int_{\mathbf{x} \in \nu} Y(\mathbf{x}) d\mathbf{x}$

spatial input  $Z$ : 
$$S_Z(\nu) = \frac{\text{var}[E(Y_\nu | Z)]}{\text{var}[Y_\nu]}$$

scalar inputs  $\mathbf{U}$ : 
$$S_{\mathbf{U}}(\nu) = \frac{\text{var}[E(Y_\nu | \mathbf{U})]}{\text{var}[Y_\nu]}$$

# Site and block sensitivity indices

## Definition: **block** sensitivity indices

Let  $\nu \subset \mathbb{R}^2$ .

Output of interest:  $Y_\nu = \int_{\mathbf{x} \in \nu} Y(\mathbf{x}) d\mathbf{x}$

$$\text{spatial input } Z: \quad S_Z(\nu) = \frac{\text{var}[E(Y_\nu | Z)]}{\text{var}[Y_\nu]}$$

$$\text{scalar inputs } \mathbf{U}: \quad S_{\mathbf{U}}(\nu) = \frac{\text{var}[E(Y_\nu | \mathbf{U})]}{\text{var}[Y_\nu]}$$

# Properties of block sensitivity indices



# Properties of block sensitivity indices

Property: influence of spatial support



# Properties of block sensitivity indices

Property: influence of spatial support

Let  $\nu \subset \mathbb{R}^2$ .

$$\frac{S_Z(\nu)}{S_U(\nu)} = \frac{\nu_c}{|\nu|}$$

# Properties of block sensitivity indices

Property: influence of spatial support

Let  $\nu \subset \mathbb{R}^2$ .

$$\frac{S_Z(\nu)}{S_U(\nu)} = \frac{\nu_c}{|\nu|}$$

Larger zone  $\nu \rightarrow$  averaging of local errors  $\rightarrow$  lower  $S_Z(\nu)$

# Properties of block sensitivity indices

## Property: influence of spatial support

Let  $\nu \subset \mathbb{R}^2$ .

$$\frac{S_Z(\nu)}{S_U(\nu)} = \frac{\nu_c}{|\nu|}$$

Larger zone  $\nu \rightarrow$  averaging of local errors  $\rightarrow$  lower  $S_Z(\nu)$

## Critical surface area $\nu_c$

$$\nu_c = \frac{S_Z}{S_U} \cdot \phi[C(\cdot)]$$

**site** sensitivity  
indices

**spatial covariance**  
of input  $Z(x)$

# Properties of block sensitivity indices



# Properties of block sensitivity indices

Property: influence of covariance of  $Z(x)$

# Properties of block sensitivity indices

Property: influence of covariance of  $Z(x)$

Assume now  $Z(x)$  is a **Gaussian** random field.  
with **range**  $a$  and **nugget**  $\eta$ .

# Properties of block sensitivity indices

Property: influence of covariance of  $Z(x)$

Assume now  $Z(x)$  is a **Gaussian** random field.  
with **range**  $a$  and **nugget**  $\eta$ .

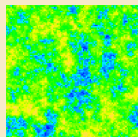
$$\frac{\partial \nu_c}{\partial a} > 0 \quad \text{and} \quad \frac{\partial \nu_c}{\partial \eta} < 0$$

# Properties of block sensitivity indices

## Property: influence of covariance of $Z(x)$

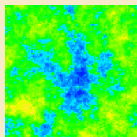
Assume now  $Z(x)$  is a **Gaussian** random field.  
with **range**  $a$  and **nugget**  $\eta$ .

$$\frac{\partial \nu_c}{\partial a} > 0 \quad \text{and} \quad \frac{\partial \nu_c}{\partial \eta} < 0$$



2D error field

range  $a$  ↗  
→



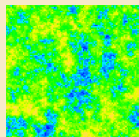


# Properties of block sensitivity indices

## Property: influence of covariance of $Z(x)$

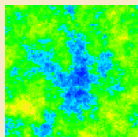
Assume now  $Z(x)$  is a **Gaussian** random field.  
with **range**  $a$  and **nugget**  $\eta$ .

$$\frac{\partial \nu_c}{\partial a} > 0 \quad \text{and} \quad \frac{\partial \nu_c}{\partial \eta} < 0$$



2D error field

range  $a$  ↗  
→



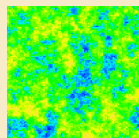
less averaging

# Properties of block sensitivity indices

Property: influence of covariance of  $Z(x)$

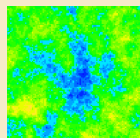
Assume now  $Z(x)$  is a **Gaussian** random field.  
with **range**  $a$  and **nugget**  $\eta$ .

$$\frac{\partial \nu_c}{\partial a} > 0 \quad \text{and} \quad \frac{\partial \nu_c}{\partial \eta} < 0$$



2D error field

range  $a$  ↗



less averaging



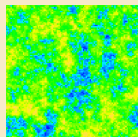
$\nu_c$  ↗  
 $S_Z(\nu)$  ↗

# Properties of block sensitivity indices

## Property: influence of covariance of $Z(x)$

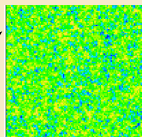
Assume now  $Z(x)$  is a **Gaussian** random field.  
with **range**  $a$  and **nugget**  $\eta$ .

$$\frac{\partial \nu_c}{\partial a} > 0 \quad \text{and} \quad \frac{\partial \nu_c}{\partial \eta} < 0$$



2D error field

nugget  $\eta$  ↗

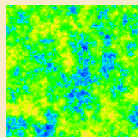


# Properties of block sensitivity indices

## Property: influence of covariance of $Z(x)$

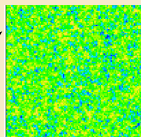
Assume now  $Z(x)$  is a **Gaussian** random field.  
with **range**  $a$  and **nugget**  $\eta$ .

$$\frac{\partial \nu_c}{\partial a} > 0 \quad \text{and} \quad \frac{\partial \nu_c}{\partial \eta} < 0$$



2D error field

nugget  $\eta$  ↗  
→



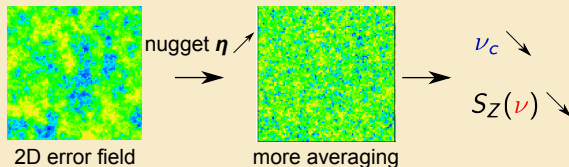
more averaging

# Properties of block sensitivity indices

## Property: influence of covariance of $Z(x)$

Assume now  $Z(x)$  is a **Gaussian** random field.  
with **range**  $a$  and **nugget**  $\eta$ .

$$\frac{\partial \nu_c}{\partial a} > 0 \quad \text{and} \quad \frac{\partial \nu_c}{\partial \eta} < 0$$



# Properties of block sensitivity indices

## Elements of proof

$$\frac{S_Z(\nu)}{S_U(\nu)}$$

# Properties of block sensitivity indices

## Elements of proof

$$\frac{S_Z(\nu)}{S_U(\nu)} = \frac{\text{var}[E(Y_\nu | Z)]}{\text{var}[E(Y_\nu | U)]}$$

# Properties of block sensitivity indices

## Elements of proof

$$\frac{S_Z(\nu)}{S_U(\nu)} = \frac{\text{var}[E(Y_\nu | Z)]}{\text{var}[E(Y_\nu | U)]}$$

$$= \underline{\text{block var. of } E(Y(x) | Z(x))} \quad \underline{\text{(point-based)}}$$



# Properties of block sensitivity indices

## Elements of proof

$$\frac{S_Z(\nu)}{S_U(\nu)} = \frac{\text{var}[E(Y_\nu | Z)]}{\text{var}[E(Y_\nu | U)]}$$

$$= \frac{\text{block var. of } E(Y(x) | Z(x))}{\text{var}[E(Y(x) | U)]} \quad \begin{array}{l} \text{(point-based)} \\ \text{(stationarity)} \end{array}$$

# Properties of block sensitivity indices

## Elements of proof

$$\frac{S_Z(\nu)}{S_U(\nu)} = \frac{\text{var}[E(Y_\nu | Z)]}{\text{var}[E(Y_\nu | U)]}$$

$$= \frac{\text{block var. of } E(Y(x) | Z(x))}{\text{var}[E(Y(x) | U)]} \quad \begin{array}{l} \text{(point-based)} \\ \text{(stationarity)} \end{array}$$

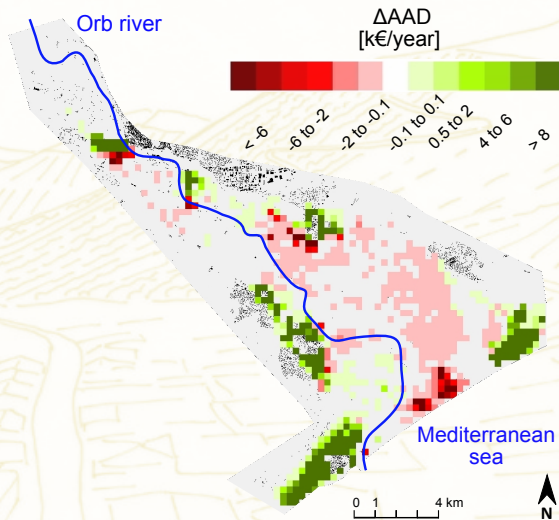
$$= \frac{\text{var}[E(Y(x) | Z(x))] \cdot A/|\nu|}{\text{var}[E(Y(x) | U)]} \quad (A : \text{integral range})$$

# Properties of block sensitivity indices

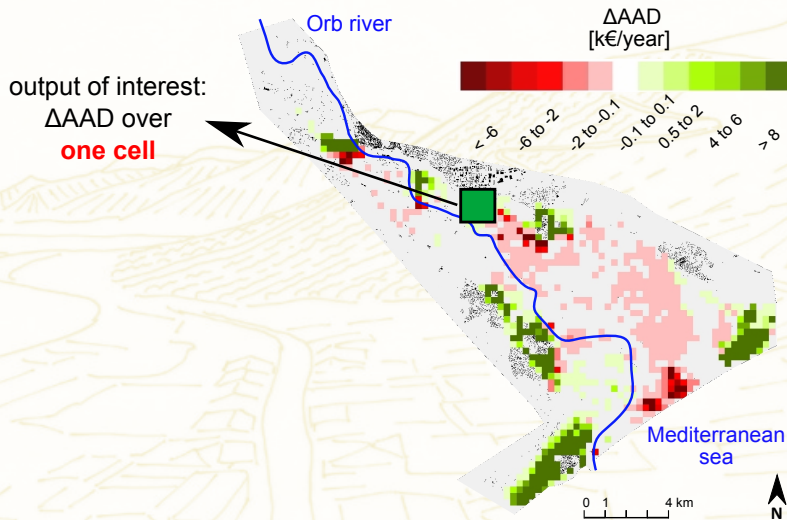
## Elements of proof

$$\begin{aligned}
 \frac{S_Z(\nu)}{S_U(\nu)} &= \frac{\text{var}[E(Y_\nu | Z)]}{\text{var}[E(Y_\nu | U)]} \\
 &= \frac{\text{block var. of } E(Y(x) | Z(x))}{\text{var}[E(Y(x) | U)]} && \begin{array}{l} \text{(point-based)} \\ \text{(stationarity)} \end{array} \\
 &= \frac{\text{var}[E(Y(x) | Z(x))] \cdot A/|\nu|}{\text{var}[E(Y(x) | U)]} && (A : \text{integral range}) \\
 &= \frac{S_Z}{S_U} \cdot \frac{A}{|\nu|}
 \end{aligned}$$

# Illustration on the Orb case-study

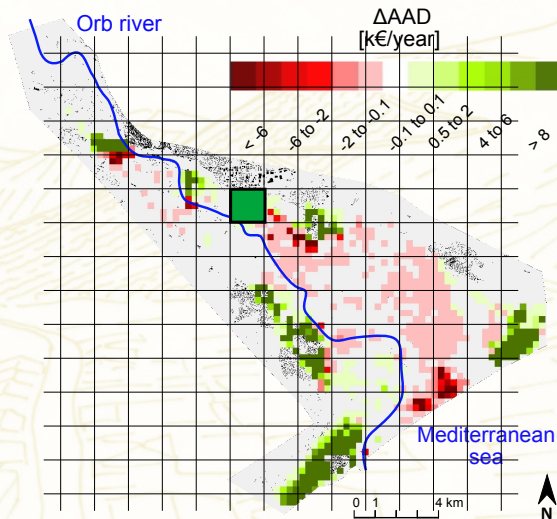


# Illustration on the Orb case-study



# Illustration on the Orb case-study

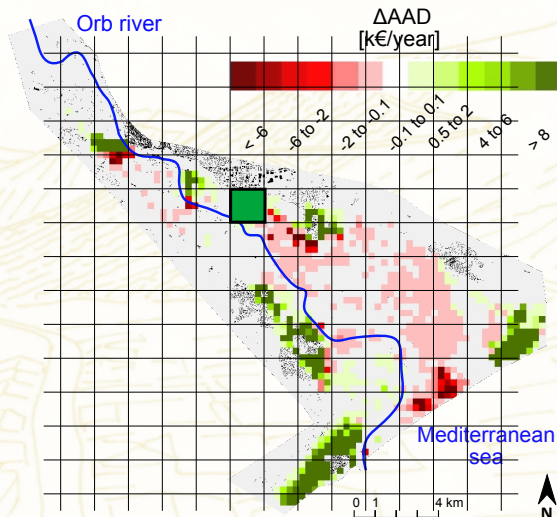
outputs of interest:  
 $\Delta$ AAD over  
 many cells



# Illustration on the Orb case-study

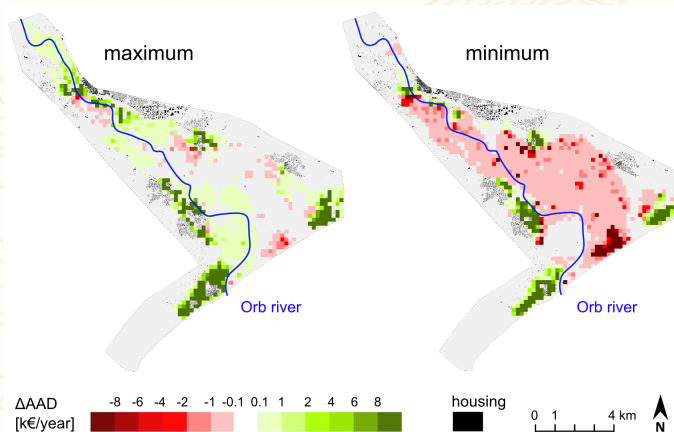
outputs of interest:  
 $\Delta$ AAD over  
many cells

maps of  
uncertainty



# Illustration on the Orb case-study

Maps of uncertainty of  $\Delta AAD$  indicator

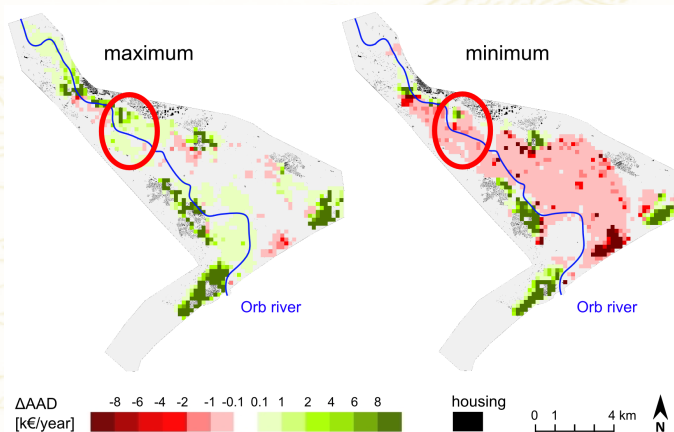




# Illustration on the Orb case-study

Maps of uncertainty of  $\Delta AAD$  indicator

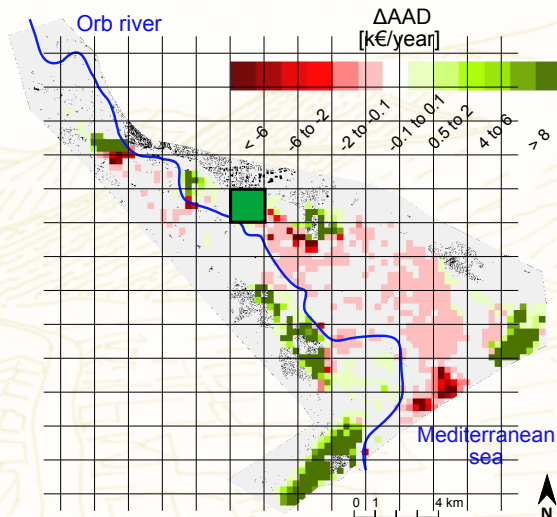
- uncertain signs of  $\Delta AAD$
- spatial variability of uncertainty



# Illustration on the Orb case-study

outputs of interest:  
 $\Delta AAD$  over  
 many cells

↓  
 maps of  
 uncertainty

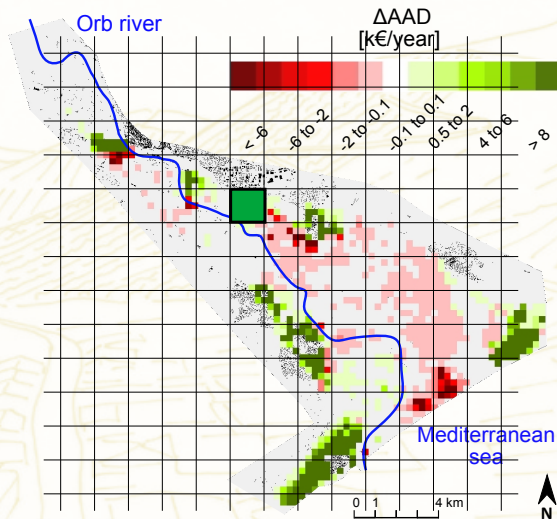


# Illustration on the Orb case-study

outputs of interest:  
 $\Delta$ AAD over  
 many cells

maps of  
 uncertainty

maps of  
 sensitivity indices

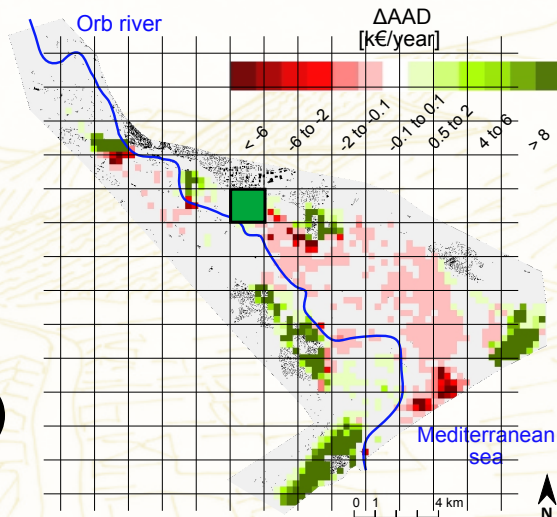


# Illustration on the Orb case-study

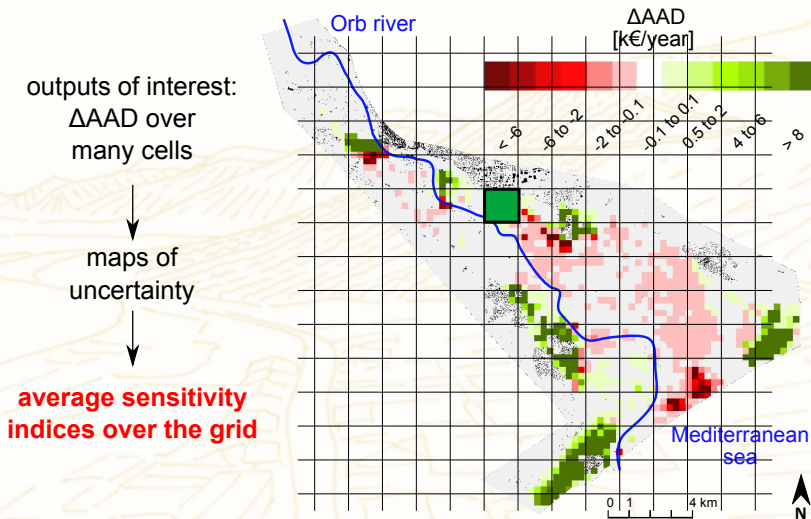
outputs of interest:  
 $\Delta$ AAD over  
 many cells

maps of  
 uncertainty

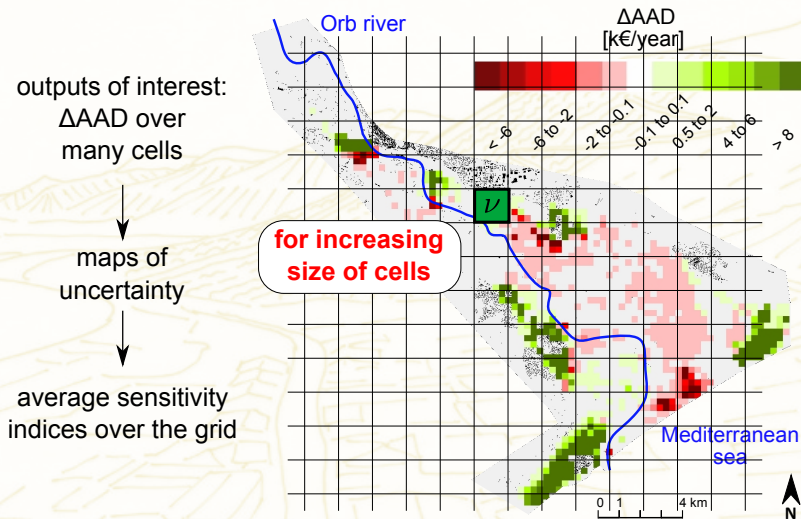
( maps of  
 sensitivity indices )



# Illustration on the Orb case-study



# Illustration on the Orb case-study

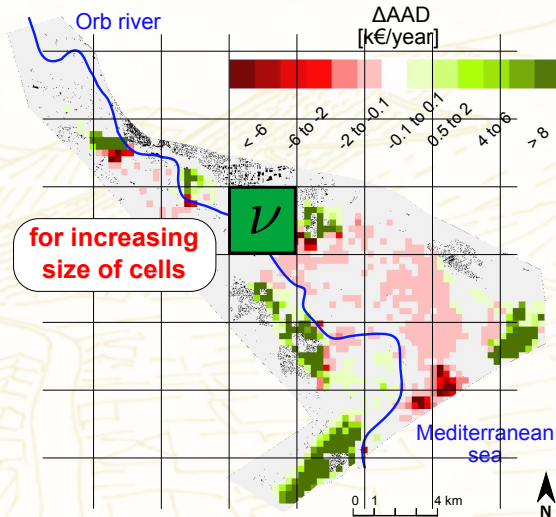


# Illustration on the Orb case-study

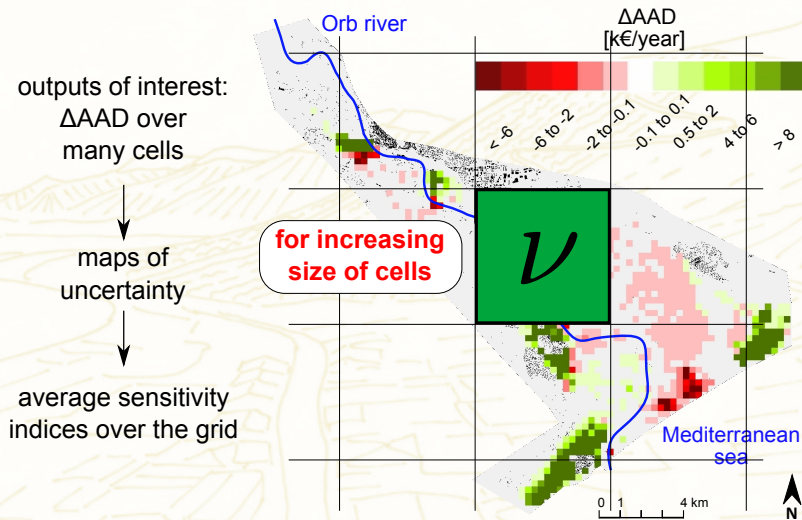
outputs of interest:  
 $\Delta AAD$  over  
 many cells

maps of  
 uncertainty

average sensitivity  
 indices over the grid

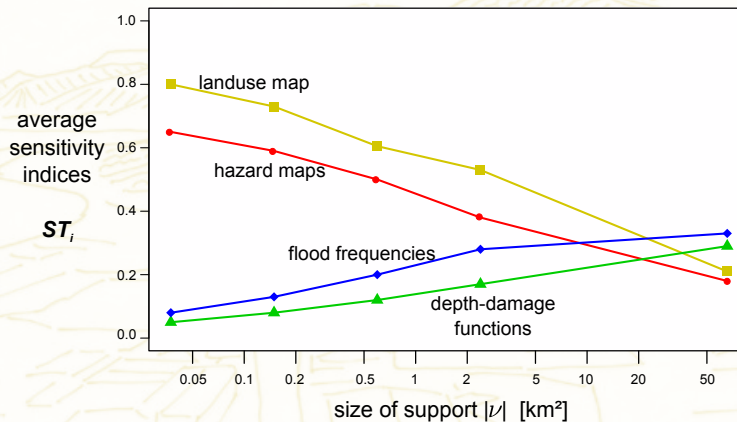


# Illustration on the Orb case-study



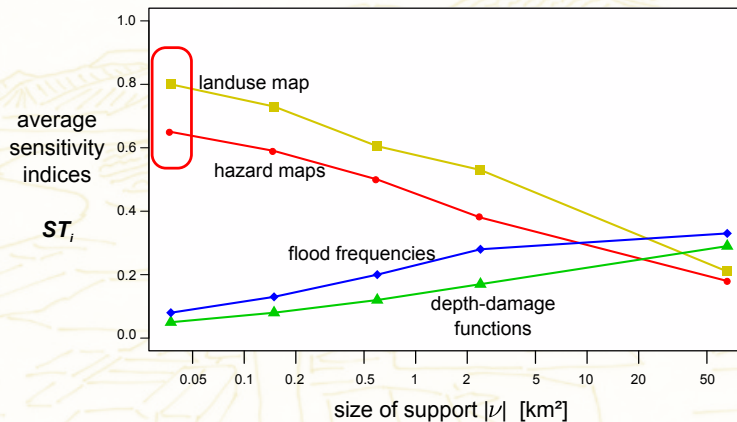


# Illustration on the Orb case-study



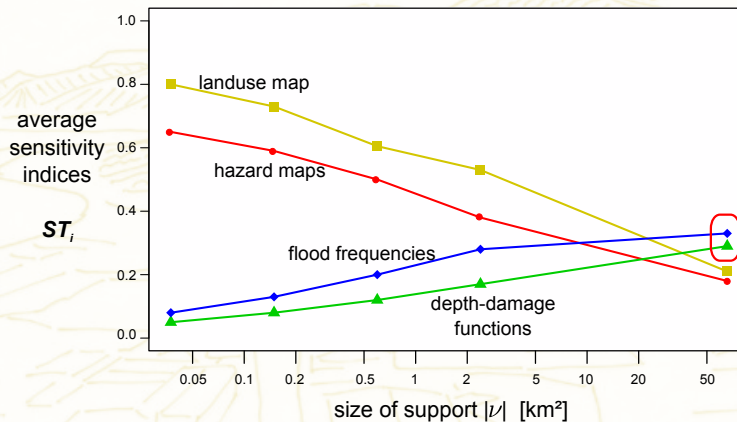
# Illustration on the Orb case-study

- key uncertainty on **small** support: landuse & hazard maps



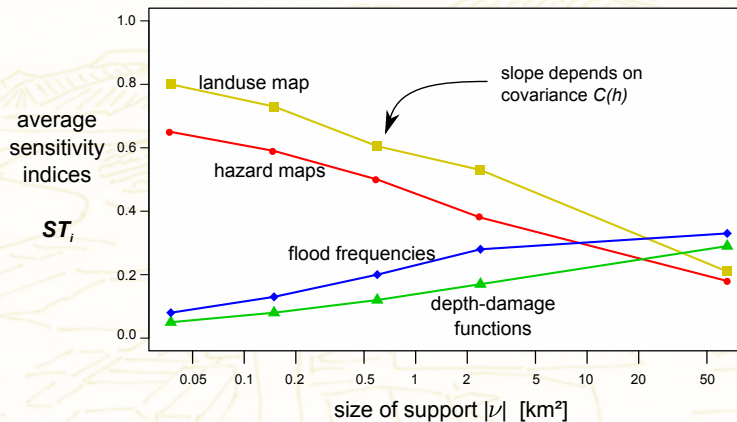
# Illustration on the Orb case-study

- key uncertainty on **small** support: landuse & hazard maps
- key uncertainty on **large** support: flood frequencies

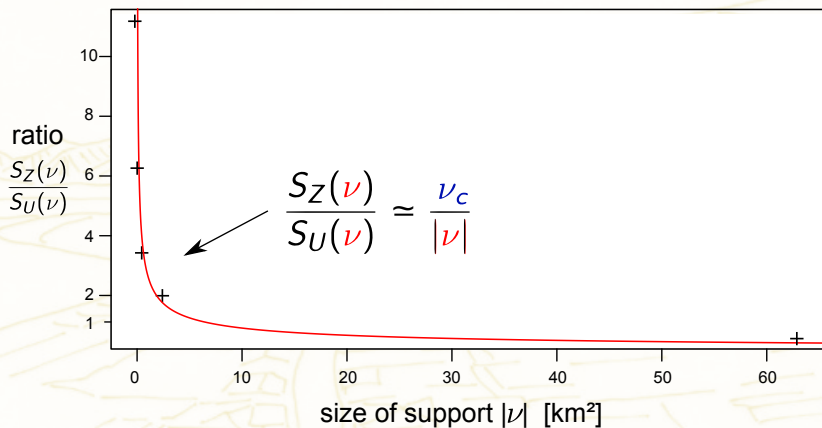


# Illustration on the Orb case-study

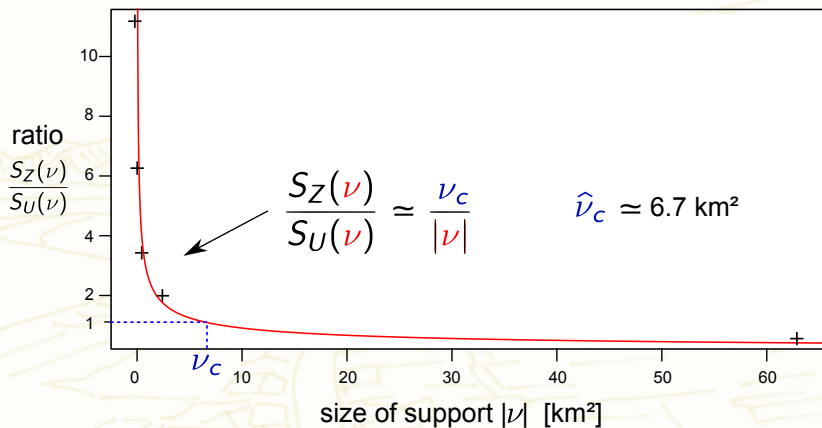
- key uncertainty on **small** support: landuse & hazard maps
- key uncertainty on **large** support: flood frequencies



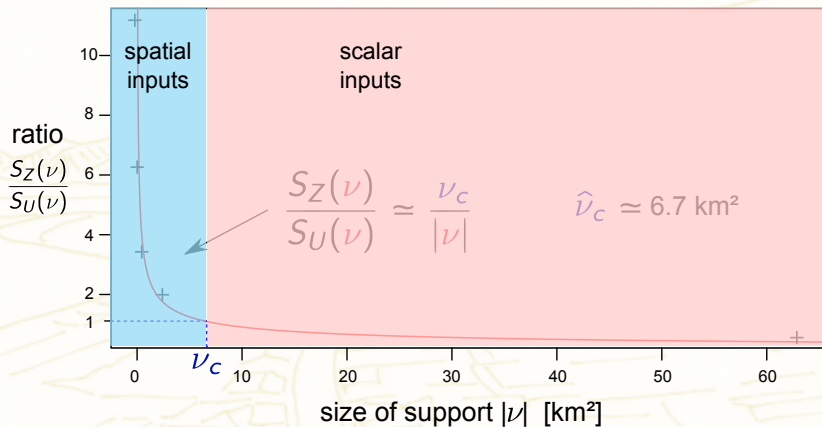
# Illustration on the Orb case-study



# Illustration on the Orb case-study



# Illustration on the Orb case-study



## Conclusion (second research question)

*how to account for **spatial scale** issues  
with variance-based sensitivity analysis?*



## Conclusion (second research question)

*how to account for **spatial scale** issues  
with variance-based sensitivity analysis?*

### PhD methodological results

- site/block sensitivity indices
- block s.i. depend on **both**:
  - support  $\nu$  of model output
  - spatial covariance of inputs
- explicit equalities

## Conclusion (second research question)

*how to account for **spatial scale** issues  
with variance-based sensitivity analysis?*

### PhD methodological results

- site/block sensitivity indices
- block s.i. depend on **both**:
  - support  $\nu$  of model output
  - spatial covariance of inputs
- explicit equalities

### PhD results on the Orb case-study

- key uncertainties on  $\Delta AAD$ :
  - small  $\nu$ : landuse map, hazard maps
  - large  $\nu$ : flood frequencies, depth-damage fct.
- critical area  $\nu_c \simeq 6.7 \text{ km}^2$
- maps of sensitivity indices

The background of the slide features a golden-yellow line drawing of a landscape. It depicts a mountain range in the background, a winding river or path in the middle ground, and a town or village with several buildings in the foreground. The drawing is composed of simple, sketchy lines, giving it a hand-drawn or artistic feel.

## General conclusion

## Research goal

investigate the use of sensitivity analysis for spatial models

## Scope

- VB-GSA: Variance-Based Global Sensitivity Analysis
- low CPU models

## 2 research questions

- 1 how to deal with **spatial inputs** in VB-GSA?
- 2 how to account for **scale issues** with VB-GSA?

## Applied goals

- design a flood risk model on the Orb floodplain
- perform sensitivity analysis

## Focus on

- maps of flood damages at different scales
- uncertainty in spatial data

# Sensitivity analysis for spatial models

## 1. How to compute sensitivity indices for spatial inputs?

- benchmark of available methods
- decision tree
- focus on *map labelling*
- sampling issues (not presented here)

# Sensitivity analysis for spatial models

## 2. Scale issues in sensitivity analysis?

- link with geostatistics theory
- ratio of block s.i.  $S_Z(\nu)/S_U(\nu)$  depends on **both**:
  - spatial support  $\nu$  of model output
  - spatial auto-correlation structure  $C(\cdot)$  of model inputs
- explicit equalities and inequalities
- spatial discretization and approximations errors

# Sensitivity analysis for spatial models

Scope of these results?

# Sensitivity analysis for spatial models

## Scope of these results?

- 1 spatial inputs in sensitivity analysis:



# Sensitivity analysis for spatial models

## Scope of these results?

- 1 spatial inputs in sensitivity analysis:
  - possibly any spatially distributed model. . .

# Sensitivity analysis for spatial models

## Scope of these results?

- 1 spatial inputs in sensitivity analysis:
  - possibly any spatially distributed model. . .
  - . . . with **low CPU cost**

# Sensitivity analysis for spatial models

## Scope of these results?

- 1 spatial inputs in sensitivity analysis:
  - possibly any spatially distributed model. . .
  - . . . with **low CPU cost**
  - also other complex model inputs: time series. . .

# Sensitivity analysis for spatial models

## Scope of these results?

- 1 spatial inputs in sensitivity analysis:
  - possibly any spatially distributed model. . .
  - . . . with **low CPU cost**
  - also other complex model inputs: time series. . .
- 2 scaling properties:

# Sensitivity analysis for spatial models

## Scope of these results?

- 1 spatial inputs in sensitivity analysis:
  - possibly any spatially distributed model. . .
  - . . . with **low CPU cost**
  - also other complex model inputs: time series. . .
- 2 scaling properties:
  - point-based & spatially additive models only

# Sensitivity analysis for spatial models

## Further research

# Sensitivity analysis for spatial models

## Further research

- 1 relaxing **hypotheses** on scaling results?
  - stationarity
  - point-based
  - spatially additive

# Sensitivity analysis for spatial models

## Further research

1 relaxing **hypotheses** on scaling results?

- stationarity
- point-based
- spatially additive

2 methods for **CPU intensive** spatial models?



# Sensitivity analysis for spatial models

## Further research

- 1 relaxing **hypotheses** on scaling results?
  - stationarity
  - point-based
  - spatially additive
- 2 methods for **CPU intensive** spatial models?
- 3 optimal **sampling** of spatial inputs?

## Research goal

investigate the use of sensitivity analysis in spatial models

## Scope

- VB-GSA: Variance-Based Global Sensitivity Analysis
- low CPU models

## 2 research questions

- 1 how to deal with **spatial inputs** in VB-GSA?
- 2 how to account for **scale issues** with VB-GSA?

## Applied goals

- design of a flood risk model on the Orb floodplain
- perform sensitivity analysis

## Focus on ...

- maps of flood damages at different scales
- uncertainty in spatial data

# Uncertainties in flood risk models

What are the key sources of uncertainty?

# Uncertainties in flood risk models

What are the key sources of uncertainty?

**No general ranking of uncertainty sources !**

# Uncertainties in flood risk models

## What are the key sources of uncertainty?

**No general ranking of uncertainty sources !**

- Key uncertainties depend on **spatial averaging effect**:
  - spatial **aggregation level** of the model output
  - spatial **auto-correlation** in water level maps and landuse maps

# Uncertainties in flood risk models

## What are the key sources of uncertainty?

### No general ranking of uncertainty sources !

- Key uncertainties depend on **spatial averaging effect**:
  - spatial **aggregation level** of the model output
  - spatial **auto-correlation** in water level maps and landuse maps
- ... on many other **non spatial** averaging effects

# Uncertainties in flood risk models

## What are the key sources of uncertainty?

### No general ranking of uncertainty sources !

- Key uncertainties depend on **spatial averaging effect**:
  - spatial **aggregation level** of the model output
  - spatial **auto-correlation** in water level maps and landuse maps
- ... on many other **non spatial** averaging effects
- ... and on the **damage category** (housing, agriculture...)

# Uncertainties in flood risk models

What are the key input data?



# Uncertainties in flood risk models

## What are the key input data?

Key input data depends on the **purpose of the model**

# Uncertainties in flood risk models

## What are the key input data?

Key input data depends on the **purpose of the model**

- produce a map of flood damages?  
→ *need for accurate terrain elevation and landuse map*

# Uncertainties in flood risk models

## What are the key input data?

Key input data depends on the **purpose of the model**

- produce a map of flood damages?  
→ *need for accurate terrain elevation and landuse map*
- report total annual avoided damages to project funders?  
→ *flood frequency estimates are probably the key data (among them, events with small return intervals)*

# Uncertainties in flood risk models

## Directions for further work

# Uncertainties in flood risk models

## Directions for further work

- 1 improve description of **input uncertainties**:
  - hydraulic modelling
  - number and repartition of flood events considered
  - model uncertainty (e.g., GIS procedures)

# Uncertainties in flood risk models

## Directions for further work

- 1 improve description of **input uncertainties**:
  - hydraulic modelling
  - number and repartition of flood events considered
  - model uncertainty (e.g., GIS procedures)
- 2 relax some **stationary hypotheses**:
  - climate change, landuse change. . .
  - non point-based risk assesement. . .

# Uncertainties in flood risk models

Was sensitivity analysis useful?

# Uncertainties in flood risk models

## Was sensitivity analysis useful?

- 1 for the **modeller** himself:
  - $\neq$  model validation !
  - better understand the model behaviour
  - find paths for model improvement



# Uncertainties in flood risk models

## Was sensitivity analysis useful?

- 1 for the **modeller** himself:
  - $\neq$  model validation !
  - better understand the model behaviour
  - find paths for model improvement
- 2 for model **end-users** (water managers):
  - what data should I put more money in?
  - do I want to know about uncertainty?

# Uncertainties in flood risk models

## Was sensitivity analysis useful?

- 1 for the **modeller** himself:
  - $\neq$  model validation !
  - better understand the model behaviour
  - find paths for model improvement
- 2 for model **end-users** (water managers):
  - what data should I put more money in?
  - do I want to know about uncertainty?



## Refereed journal publications (as lead author)



Change of support in spatial variance-based spatial sensitivity analysis  
*Mathematical Geosciences*, 2012, 44(8) 945-958.



Analyse de sensibilité de Sobol d'un modèle spatialisé pour l'évaluation économique du risque d'inondation  
*Journal de la Société Française de Statistique*, 2011, 152(1), 24-46.



Ranking sources of uncertainty in flood damage modelling: a case study on the cost-benefit analysis of a flood mitigation project in the Orb Delta, France  
*Journal of Flood Risk Management*, submitted.

## Other works non presented today. . .



LHS sampling of Gaussian Random Field for Sobol' global sensitivity analysis of spatial models  
*Proceedings of Accuracy 2010 conference, Leicester, UK, pp. 81-84.*



Sensitivity of spatial models using geostatistical simulation  
*Proceedings of IAMG 2011 conference, Salzburg, Austria, pp. 178-189.*

# Thank you for your attention

## Partners

- Plan Rhône
- Syndicat Mixte des Vallées de l'Orb et du Libron
- Institution d'Aménagement de la Vilaine
- EC - Joint Research Center

## Funding

- Ministère de l'Agriculture, de l'Agroalimentaire et de la Forêt
- Plan Rhône
- GDR MASCOT-NUM

