Quelques modèles homogénéisés pour la modélisation du comportement mécanique du parenchyme pulmonaire

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Overview

1. Motivations

2. Modeling the mechanics of the parenchyma during ventilation
   - The model
   - Convergence of the tree operator
   - Two–scale convergence for the ventilation model

3. Modeling sound propagation through the parenchyma
   - A first model and its homogenization
   - A second model with memory effects
   - Numerical study

4. Conclusion

Motivation: anatomy of the parenchyma

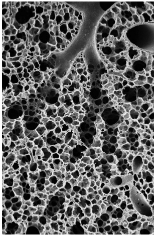
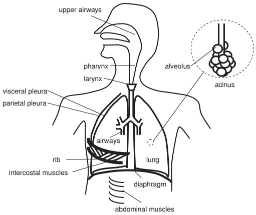
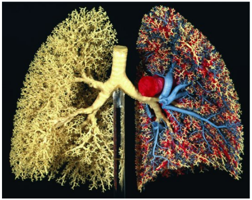


Image of the alveolar region, realized by Weibel



Bates, *Lung mechanics* (2009)



Cast of the human lung, realized by Weibel

- A few numbers:
  ⇝ Around 500 million alveoli,
  ⇝ Mean diameter .3mm,
  ⇝ Bronchial tree: around 23 generations.
- Geometry of the bronchial tree, [Weibel '63, '84], [Mauroy, Filoche, Weibel, Sapoval '04]...
- Parenchyma mechanics, [Mead '63], [Fung '75], [Suki '11]...
- Acoustic properties of the lung, [Rice, Hamlin, Donnerberg '76], [Dunn '86]...

Motivation: mechanical modeling of the lung

Macroscopic measures:

- Spirometry:
  - ⇝ Measures of flow, expired volume at the mouth,
  - ⇝ Flow–volume curves...

Models of the lung mechanics

- ODE models, [Bates '09]...
- Airflow in the bronchial tree, [Baffico, Grandmont, Maury '10]...
- Parenchyma models as a porous media, [Rice '83], [Owen, Lewis '02], [Siklosi, Jensen, Tew, Logg '08], [Baffico, Grandmont, Maday, Osses '08]...
- One–dimensional parenchyma models, [Grimal, Waitzki, Naili '02], [Grandmont, Maury, Meunier '06]...

Our goal:

⇝ Rigorously upscale models based on mechanics at the microscopic (alveolar) scale.

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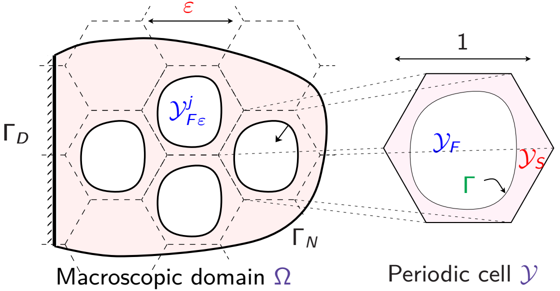
Geometry of the parenchyma model

Hypothesis: the alveoli $\mathcal{Y}^j_{F,\varepsilon}$ are periodically distributed in the lung parenchyma.
The parenchyma fills the perforated domain $\Omega_{S,\varepsilon} = \Omega \setminus \bigcup_j \mathcal{Y}^j_{F,\varepsilon}$.

The period ε is characteristic of the size of the alveoli (around .3mm).



- structure, fluid domain $\Omega_{S,\varepsilon} \cup \Omega_{F,\varepsilon} = \Omega$.
- structure, fluid cells $\mathcal{Y}_S \cup \mathcal{Y}_F = \mathcal{Y}$.

Modeling the displacement of the tissue

- Collaboration with Céline Grandmont.
- Generalization of the model in [Grandmont, Maury, Meunier '06], based on a model in [Vannier '09].
- **Compressible, linearized elastic material.**

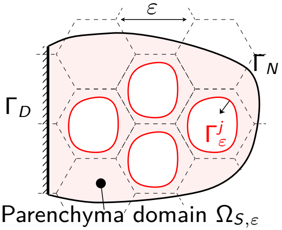
$$\begin{cases} \rho\partial_{tt}\mathbf{u}_\varepsilon - \mathbf{div}\left(\underline{\sigma}(\mathbf{u}_\varepsilon)\right) = \mathbf{f} & \text{in } (0,T)\times\Omega_{S,\varepsilon}, \\ \mathbf{u}_\varepsilon = 0 & \text{on } (0,T)\times\Gamma_D, \\ \underline{\sigma}(\mathbf{u}_\varepsilon)\cdot\mathbf{n} = 0 & \text{on } (0,T)\times\Gamma_N, \\ \underline{\sigma}(\mathbf{u}_\varepsilon)\cdot\mathbf{n}_\varepsilon = -p_j\mathbf{n}_\varepsilon & \text{on } (0,T)\times\Gamma_\varepsilon^j. \end{cases}$$

$\mathbf{u}_\varepsilon$ tissue displacement,

p_j alveolar pressures,

$\underline{\sigma}(\mathbf{u}_\varepsilon) = \lambda\,\mathrm{div}(\mathbf{u}_\varepsilon)\underline{\mathrm{Id}} + 2\mu\underline{e}(\mathbf{u}_\varepsilon)$, stress tensor,

$\underline{e}(\mathbf{u}_\varepsilon) = 1/2\left(\nabla\mathbf{u}_\varepsilon + \nabla\mathbf{u}_\varepsilon^T\right)$, strain tensor.

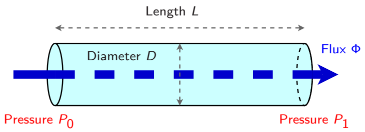


The pressures p_j are unknowns determined by coupling with the tree model.

Modeling the airflow in the bronchial tree

[Grandmont, Maury, Meunier '06]

Poiseuille law



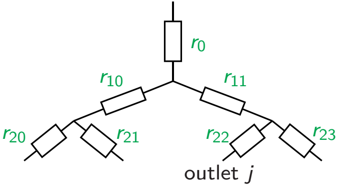
- Linear relationship between flux and pressures at the ends

$$P_0 - P_1 = r\Phi$$

- Viscous flow in a cylindrical tube

$$r = \nu \frac{8L}{\pi D^4}$$

Dyadic resistive tree



- Linear relationship between vectors of fluxes and pressures at the outlets

$$\vec{p} = A^\varepsilon \vec{\phi}$$

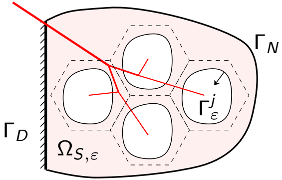
- A^ε is symmetric, definite positive

Coupling tissue displacement and airflow by the boundary conditions

The air is assumed to be **incompressible.**

$\rightsquigarrow$ Computing the fluxes

$$\phi_k = \int_{\Gamma_\varepsilon^k} \partial_t \mathbf{u}_\varepsilon \cdot \mathbf{n}_\varepsilon .$$



$$\begin{cases} \rho \partial_{tt} \mathbf{u}_\varepsilon - \mathbf{div}\,(\underline{\sigma}(\mathbf{u}_\varepsilon)) = \mathbf{f} & \text{in } (0,T) \times \Omega_{S,\varepsilon}, \\ \mathbf{u}_\varepsilon = 0 & \text{on } (0,T) \times \Gamma_D, \\ \underline{\sigma}(\mathbf{u}_\varepsilon) \cdot \mathbf{n} = 0 & \text{on } (0,T) \times \Gamma_N, \\ \underline{\sigma}(\mathbf{u}_\varepsilon)\mathbf{n}_\varepsilon = - p_j \mathbf{n}_\varepsilon & \text{on } (0,T) \times \Gamma_\varepsilon^j, \\ \left(\int_{\Gamma_\varepsilon^k} \partial_t \mathbf{u}_\varepsilon \cdot \mathbf{n}_\varepsilon \right) = \phi_k & \text{for all } k, \\ p_j = \sum_k A_{j,k}^\varepsilon \phi_k & \text{for all } j, \\ + \text{ initial conditions.} & \end{cases}$$

- Dissipative effect due to viscous losses in the airflow through the bronchial tree.

Coupled parenchyma model

$$
\begin{cases}
\rho \partial_{tt} \mathbf{u}_\varepsilon - \mathbf{div}\,(\underline{\sigma}(\mathbf{u}_\varepsilon)) = \mathbf{f} & \text{in } (0, T) \times \Omega_\varepsilon, \\
\mathbf{u}_\varepsilon = 0 & \text{on } (0, T) \times \Gamma_D, \\
\underline{\sigma}(\mathbf{u}_\varepsilon) \cdot \mathbf{n} = 0 & \text{on } (0, T) \times \Gamma_N, \\
\underline{\sigma}(\mathbf{u}_\varepsilon)\mathbf{n}_\varepsilon = -\sum_k A^\varepsilon_{j,k} \left(\int_{\Gamma^k_\varepsilon} \partial_t \mathbf{u}_\varepsilon \cdot \mathbf{n}_\varepsilon \right) \mathbf{n}_\varepsilon & \text{on } (0, T) \times \Gamma^j_\varepsilon, \\
+ \text{ initial conditions.}
\end{cases}
\tag{1}
$$

Existence and uniqueness [Vannier '09]

For any $T > 0$, under some regularity conditions, the system (1) has a unique weak solution

$$\mathbf{u}_\varepsilon \in L^\infty(0, T; H^1(\Omega_\varepsilon)) \cap W^{1,\infty}(0, T; L^2(\Omega_\varepsilon)).$$

$\rightsquigarrow$ In addition, we have shown:

A priori bounds

The solution $\mathbf{u}_\varepsilon$ of system (1) satisfies **uniform** *a priori* bounds in ε.

Asymptotic study as ε goes to zero

$\rightsquigarrow$ the number of alveoli goes to infinity as their size ε goes to zero,
$\rightsquigarrow$ the number of generations of the tree goes to infinity.

To do :

- **Asymptotic analysis** of the non–local boundary conditions
- **Two–scale homogenization**

State of the art

- **Limit of a spring–mass system coupled with a tree in a one–dimensional setting:** [Grandmont, Maury, Meunier '06].
- **Study of finite and infinite resistive trees and their embedding in a domain:** [Maury, Salort, Vannier '09], [Bernicot, Maury, Salort '10]...
- **Study of fractal trees and traces:** [Achdou, Sabot, Tchou '08], [Achdou, Tchou '10]...

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Dyadic multiscale decompositions

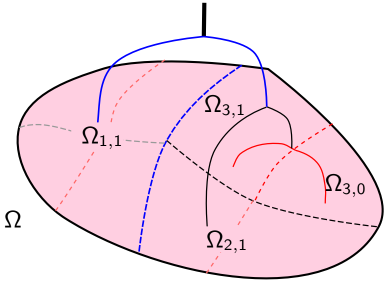
[Maury, Salort, Vannier '09]

Definition

Dyadic multiscale domain decomposition:

$$\mathcal{O} = (\Omega_{n,k})_{n\in\mathbb{N},k\in\{0,\dots,2^n-1\}}$$

- $\Omega_{0,0} = \Omega$;
- $\overline{\Omega_{n+1,2k}} \cup \overline{\Omega_{n+1,2k+1}} = \overline{\Omega_{n,k}}$;
- $\Omega_{n,k_1} \cap \Omega_{n,k_2} = \emptyset$ if $k_1 \neq k_2$.



Action of the tree as a non–local operator in $L^2(\Omega)$

[Grandmont, Maury, Meunier '06]

Rewrite the non–local boundary conditions:

Extend formally $\mathbf{u}_\varepsilon$ in the alveoli as $\widetilde{\mathbf{u}_\varepsilon}$,

$$p_j = \sum_k A^\varepsilon_{j,k} \left(\int_{\Gamma^k_\varepsilon} \partial_t \mathbf{u}_\varepsilon \cdot \mathbf{n}_\varepsilon \right) = \mathcal{R}_\varepsilon \left(1_{\Omega_{F,\varepsilon}} \operatorname{div} \partial_t \widetilde{\mathbf{u}_\varepsilon} \right) \quad \text{in } \mathcal{Y}^j_\varepsilon.$$

Proposition: $\mathcal{R}_\varepsilon$ as an integral kernel operator

$$\mathcal{R}_\varepsilon \phi(x_1) = \int_\Omega K_\varepsilon(x_1, x_2)\phi(x_2)\mathrm{d}x_2 \quad \forall \phi \in L^2(\Omega)$$

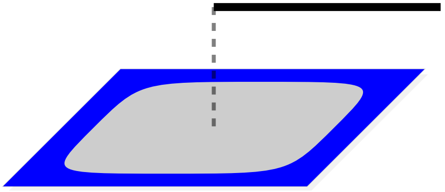
where $$K_\varepsilon(x_1, x_2) = \sum_{(n,k)\in\mathcal{B}_\varepsilon} r_{n,k} 1_{\Omega_{n,k}}(x_1) 1_{\Omega_{n,k}}(x_2),$$

$\mathcal{B}_\varepsilon$: indexes of the branches of the finite tree.

- Image of $\mathcal{R}_\varepsilon$: the functions constant on each cell $\mathcal{Y}^j_\varepsilon$.
- $\mathcal{R}_\varepsilon$: linear operator of finite rank, symmetric and positive.

An explicit construction

Geometry: the H–tree construction



- $\Omega = \mathcal{Y}$.
- 1 square: $\Omega_{\varepsilon=1} = \mathcal{Y}_S$.

$$\varepsilon = 1$$

- 4 squares, 2 generations.
- 16 squares, 4 generations.

Resistances: geometric trees

$$r_{n,k} = r_0 \alpha^n.$$

In the human lungs, $\alpha \approx 1.63$ [Weibel '63].

Proposition

If $\alpha < 2$, $\mathcal{R}_\varepsilon$ converges strongly as $n \to \infty$ and $\varepsilon \to 0$ in $\mathcal{L}(L^2(\Omega))$.

Adaptation of the 1D case, [Grandmont, Maury, Meunier '06].

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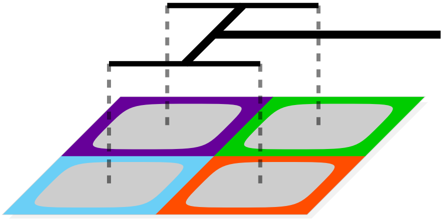

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$$\varepsilon = \frac{1}{2}$$

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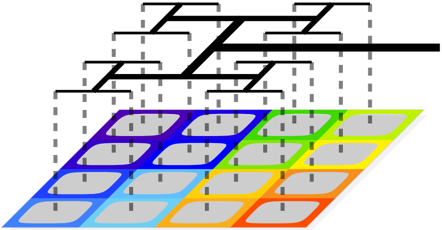

- $\Omega = \mathcal{Y}$.
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$$\varepsilon = \frac{1}{4}$$

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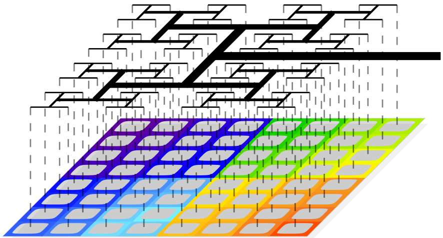

At step n:

- 2^{2n} squares, $2n$ generations.
- **Organisation** by a dyadic tree.
- **Scaling**:

$$\varepsilon = \frac{1}{2^n}.$$

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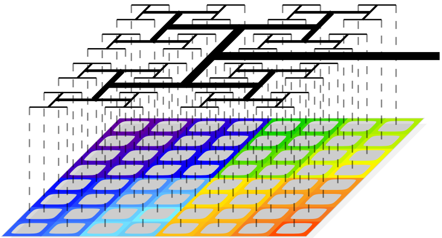

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Back to the parenchyma model

$$\begin{cases} \rho \partial_{tt} \mathbf{u}_\varepsilon - \mathbf{div}\left(\underline{\sigma}(\mathbf{u}_\varepsilon)\right) = \mathbf{f} & \text{in } (0,T) \times \Omega_\varepsilon, \\ \mathbf{u}_\varepsilon = 0 & \text{on } (0,T) \times \Gamma_D, \\ \underline{\sigma}(\mathbf{u}_\varepsilon) \cdot \mathbf{n} = 0 & \text{on } (0,T) \times \Gamma_N, \\ \underline{\sigma}(\mathbf{u}_\varepsilon) \cdot \mathbf{n}_\varepsilon = -\mathcal{R}_\varepsilon \left(1_{\Omega_{F,\varepsilon}} \operatorname{div} \, \partial_t \mathbf{u}_\varepsilon \right) \cdot \mathbf{n}_\varepsilon & \text{on each } (0,T) \times \Gamma^j_\varepsilon, \\ + \text{ initial conditions.} \end{cases} \tag{2}$$

State of the art:

- **Periodic homogenization techniques**:
  - Asymptotic developments, energy method ('70, '80): [Bensoussan, Lions, Papanicolaou '78], [Tartar '80]...
  - Two–scale convergence ('90): [Nguetseng '89], [Allaire '92]...
  - Periodic unfolding ('00): [Cioranescu, Damlamian, Griso '02, '08]
- **Homogenization of small displacements in fluid–solid mixtures**:
  - Using asymptotic developements: [Lévy '79], [Sanchez–Hubert '79], [Fleury '80], [Sanchez–Palencia '80, '86]...
  - Using two–scale convergence: [Nguetseng '90]...
- **Homogenization of a foam in the static case:** [Baffico, Grandmont, Maday, Osses '08]...

Taking the two–scale limit

- **Extend** carefully $\mathbf{u}_\varepsilon$ to the whole domain Ω as $\widetilde{\mathbf{u}_\varepsilon}$ [Cioranescu, Donato '89].
- Obtain *a priori* estimates independent of ε.
- Identify **two–scale limits** $\mathbf{u}_0(x)$ and $\mathbf{u}_1(x, y)$ by compacity [Nguetseng '90], [Allaire '92]

$$\widetilde{\mathbf{u}_\varepsilon} \twoheadrightarrow \mathbf{u}_0 \qquad \text{and} \qquad \nabla \widetilde{\mathbf{u}_\varepsilon} \twoheadrightarrow \nabla_x \mathbf{u}_0 + \nabla_y \mathbf{u}_1.$$

- Write the **variational formulation** for well–chosen test functions:

$$\mathbf{v}_\varepsilon(x) := \mathbf{v}_0(x) + \varepsilon \mathbf{v}_1 \left(x, \frac{x}{\varepsilon} \right).$$

- Find and study the **two–scale problem** solved by $\mathbf{u}_0$, $\mathbf{u}_1$.

Convergence of the non–local term

$$\sum_{j,k} A^{\varepsilon}_{j,k} \left(\int_{\Gamma^j_\varepsilon} \partial_t \mathbf{u}_\varepsilon \cdot \mathbf{n}_\varepsilon \right) \left(\int_{\Gamma^k_\varepsilon} \mathbf{v}_\varepsilon \cdot \mathbf{n}_\varepsilon \right)$$

$$= \int_\Omega \frac{\partial}{\partial t} \mathcal{R}_\varepsilon \left(1_{\Omega_{F,\varepsilon}} \operatorname{div} \widetilde{\mathbf{u}_\varepsilon} \right) \Pi^\varepsilon \left(1_{\Omega_{F,\varepsilon}} \operatorname{div} \widetilde{\mathbf{v}_\varepsilon} \right)$$

Π^ε : L^2 projector on the functions constant over each $\mathcal{Y}^j_\varepsilon$.

Key points:

- $\mathcal{R}$ is a compact operator in $\mathcal{L}(L^2(\Omega))$,
- *a priori* bounds.

$\rightsquigarrow$ $\mathcal{R}_\varepsilon(1_{\Omega_{F,\varepsilon}} \operatorname{div} \widetilde{\mathbf{u}_\varepsilon})$ converges **weakly** in $H^1(0, T; L^2(\Omega))$ to $\mathcal{R} \left(\int_{Y_F} \operatorname{div}_x \mathbf{u}_0 + \operatorname{div}_y \mathbf{u}_1 \right)$.

Lemma [Allaire, Conca '96], [Baffico, Grandmont, Maday, Osses '08]

$$\Pi^\varepsilon \left(1_{\Omega_{F,\varepsilon}} \operatorname{div} \widetilde{\mathbf{v}_\varepsilon} \right) \to \left(\int_{Y_F} \operatorname{div}_x \mathbf{v}_0 + \operatorname{div}_y \mathbf{v}_1 \right) \textbf{ strongly in } L^2(\Omega).$$

The non–local term converges to

$$\int_\Omega \frac{\partial}{\partial t} \mathcal{R} \left(|\mathcal{Y}_F| \operatorname{div}_x \mathbf{u}_0 - \int_\Gamma \mathbf{u}_1 \cdot \mathbf{n} \right) \left(|Y_F| \operatorname{div}_x \mathbf{v}_0 - \int_\Gamma \mathbf{v}_1 \cdot \mathbf{n} \right).$$

Limit of the system at the microscopic scale

Non–standard microscopic cell problem

$$\begin{cases} \mathbf{div}_y \left(\lambda \operatorname{div}(\mathbf{u}_1)\underline{\mathrm{Id}} + 2\mu \underline{\mathrm{e}}_y(\mathbf{u}_1)\right) = 0 & \text{in } (0, T) \times \Omega \times \mathcal{Y}_S, \\ \left(\lambda \operatorname{div}_y(\mathbf{u}_1)\underline{\mathrm{Id}} + 2\mu \underline{\mathrm{e}}_y(\mathbf{u}_1)\right) \cdot \mathbf{n} + \dfrac{\partial}{\partial t}\mathcal{R}\left(|Y_F| \operatorname{div}_x \mathbf{u}_0 - \displaystyle\int_\Gamma \mathbf{u}_1 \cdot \mathbf{n}\right) \cdot \mathbf{n} & \\ \qquad = -\left(\lambda \operatorname{div}_x(\mathbf{u}_0)\underline{\mathrm{Id}} + 2\mu \underline{\mathrm{e}}_x(\mathbf{u}_0)\right) \cdot \mathbf{n} & \text{on } (0, T) \times \Omega \times \Gamma, \\ \mathbf{u}_1 \text{ periodic on } (0, T) \times \Omega \times \partial\mathcal{Y}. & \end{cases}$$

- **Additional macroscopic variable**

$$\pi = \mathcal{R}\left(|Y_F| \operatorname{div}_x \mathbf{u}_0 - \int_\Gamma \mathbf{u}_1 \cdot \mathbf{n}\right).$$

- Now x is only a parameter in this problem.
- **Linear relation**:

$$\mathbf{u}_1 = \mathcal{C}\underline{\mathrm{e}}_x(\mathbf{u}_0) + \partial_t \pi \chi_0.$$

- **New differential equation**:

$$\pi + \left(\int_\Gamma \chi_0 \cdot \mathbf{n}\right) \partial_t(\mathcal{R}\pi) = -\mathcal{R}\left(|Y_F| \operatorname{div}_x \mathbf{u}_0 - \int_\Gamma \mathcal{C}\underline{\mathrm{e}}(\mathbf{u}_0) \cdot \mathbf{n}\right) \quad \text{on } (0, T) \times \Omega.$$

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Limit of the system at the macroscopic scale

Macroscopic system solved by $\mathbf{u}_0$ and π

$$\begin{cases} \theta\rho\partial_{tt}\mathbf{u}_0 - \mathbf{div}\left(\underline{\sigma}^{hom}(\mathbf{u}_0,\pi)\right) = \theta\mathbf{f} & \text{in } (0,T)\times\Omega, \\ \underline{\sigma}^{hom}(\mathbf{u}_0,\pi) = \mathcal{A}^{hom}\underline{\mathrm{e}}(\mathbf{u}_0) - \partial_t\pi\mathcal{B}^{hom} & \text{in } (0,T)\times\Omega, \\ \pi + \tau_{hom}\partial_t(\mathcal{R}\pi) = -\mathcal{R}\left(\mathcal{B}^{hom} : \underline{\mathrm{e}}(\mathbf{u}_0)\right) & \text{on } (0,T)\times\Omega, \\ + \text{ boundary and initial conditions.} \end{cases} \tag{3}$$

$\mathcal{A}^{hom}$, $\mathcal{B}^{hom}$, τ_{hom}: Homogenized elastic tensor, matrix, scalar
$\rightsquigarrow$ computed by solving **cell problems** on the periodicity cell;

θ: proportion of parenchyma tissue.

Proposition

The homogenized system (3) has a **unique** solution.

Remarks : analysis of the homogenized system

A non–local viscoelasticity problem

$$\begin{cases} \theta\rho\partial_{tt}\mathbf{u}_0 - \mathbf{div}\left(\underline{\sigma}^{hom}(\mathbf{u}_0,\pi)\right) = \theta\mathbf{f} & \text{in } (0,T)\times\Omega, \\ \underline{\sigma}^{hom}(\mathbf{u}_0,\pi) = \mathcal{A}^{hom}\underline{e}(\mathbf{u}_0) - \partial_t\pi\mathcal{B}^{hom} & \text{in } (0,T)\times\Omega, \\ \pi + \tau_{hom}\partial_t(\mathcal{R}\pi) = -\mathcal{R}\left(\mathcal{B}^{hom} : \underline{e}(\mathbf{u}_0)\right) & \text{in } (0,T)\times\Omega, \\ + \text{ boundary and initial conditions.} \end{cases}$$

- Same elastic tensor $\mathcal{A}^{hom}$ as in a standard perforated medium.
- No added mass effect from the fluid (air) on the structure,
- $\mathcal{B}^{hom}$: action from the fluid on the structure,
- τ_{hom}: relaxation time for the fluid.

Remarks: physical considerations

A non–local viscoelasticity problem

$$\begin{cases} \theta\rho\partial_{tt}\mathbf{u}_0 - \mathbf{div}\left(\underline{\sigma}^{hom}(\mathbf{u}_0,\pi)\right) = \theta\mathbf{f} & \text{in } (0,T)\times\Omega, \\ \underline{\sigma}^{hom}(\mathbf{u}_0,\pi) = \mathcal{A}^{hom}\underline{\mathrm{e}}(\mathbf{u}_0) - \partial_t\pi\mathcal{B}^{hom} & \text{in } (0,T)\times\Omega, \\ \pi + \tau_{hom}\partial_t(\mathcal{R}\pi) = -\mathcal{R}\left(\mathcal{B}^{hom} : \underline{\mathrm{e}}(\mathbf{u}_0)\right) & \text{in } (0,T)\times\Omega, \\ \qquad + \text{ boundary and initial conditions.} \end{cases}$$

- Strongly coupled viscoelastic system
  - $\rightsquigarrow$ $\boldsymbol{\partial_t\pi}$ represents the **air pressure** in the material.
  - $\rightsquigarrow$ **diphasic medium**,
- **Non–local viscous effects both in space and time**.
- Effect of **pathologies**:
  - Affecting **resistances** of the tree: **asthma**...
  - Affecting the **elastic properties** of the parenchyma: **fibrosis**...

Remarks : the incompressible case

- Incompressibility constraint:

$$\mathrm{div}\ \mathbf{u}_\varepsilon = 0.$$

- After two–scale convergence:

$$\mathrm{div}_x\,\mathbf{u}_0 + \mathrm{div}_y\,\mathbf{u}_1 = 0 \text{ on } \Omega \times \mathcal{Y}_S \quad \rightsquigarrow \quad \pi = \mathcal{R}(\mathrm{div}_x\,\mathbf{u}_0).$$

- Macroscopic limit problem

$$\begin{cases} \theta\rho\partial_{tt}\mathbf{u}_0 - \mathbf{div}\left(\underline{\sigma}^{hom}(\mathbf{u}_0)\right) = \theta\mathbf{f} & \text{in } (0,T)\times\Omega, \\ \underline{\sigma}^{hom}(\mathbf{u}_0) = \mathcal{A}^{hom}\underline{\mathrm{e}}(\mathbf{u}_0) - (1-\theta)\partial_t\mathcal{R}(\mathrm{div}_x\,\mathbf{u}_0) & \text{in } (0,T)\times\Omega, \\ + \text{ boundary and initial conditions.} & \end{cases}$$

- The viscous effects are now **non–local only in space.**

Observations:

$\rightsquigarrow$ Similar to the one–dimensional case [Grandmont, Maury, Meunier '06],

Numerical approach

Numerical method:

- Difficulty: to avoid assembling the **full matrix**,
- Use of an **iterative method**,
- Use the **tree structure.**

Discretization:

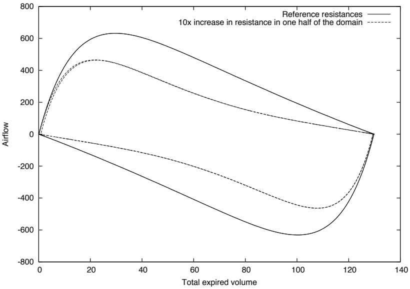
- Test geometry: Ω is a square irrigated by a H–tree,

Realistic parameters (reference case):

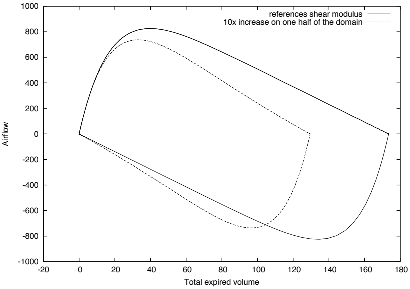
- $\rho = 1e3$ kg/m^3: parenchyma density,
- $\lambda = 1e9$ Pa, $\mu = 1e5$ Pa: Lamé parameters of the parenchyma,
- Geometric resistances: $\alpha \approx 1.63$:
$$r_{n,k} = r_0 \alpha^n.$$

Resolution: using FreeFem++.

Numerical point of view: flow–volume curves



Obstructive lung disease



Restrictive lung disease

Overview

1. Motivations

2. Modeling the mechanics of the parenchyma during ventilation
   The model
   Convergence of the tree operator
   Two–scale convergence for the ventilation model

3. Modeling sound propagation through the parenchyma
   A first model and its homogenization
   A second model with memory effects
   Numerical study

4. Conclusion

Why model sound propagation in the lungs?

Medical applications:

- Sound is used as an important diagnostic tool by medical doctors,
- Potentially useful as a cheap medical lung imaging or monitoring method.

State of the art:

- One–dimensional models:
  [Grimal, Waitzki, Naili '02]...
- Formal homogenization:
  [Owen, Lewis '02],
  [Siklosi, Jensen, Tew, Logg '08]...
- Vibrations of porous media:
  [Biot '56],
  [Sanchez–Palencia '86],
  [Nguetseng '90],
  [Gilbert, Mikelic '00],
  [Alouges, Augier, Graille, Merlet '11],
  [Augier '10]

First sound propagation model

Collaboration with Céline Grandmont and Yvon Maday.

Main differences with our previous model:

- **Closed pores**: no coupling with the bronchial tree.
- Air: **compressible, inviscid, irrotational** fluid.
- Harmonic dependence in time:
  $\rightsquigarrow$ frequency dependence of the behavior?

$$\mathbf{u}_\varepsilon(x,t) \equiv \mathbf{u}_\varepsilon(x)e^{i\omega t} \text{ (displacement)},$$
$$\phi_\varepsilon(x,t) \equiv \phi_\varepsilon(x)e^{i\omega t} \text{ (air velocity potential)}.$$

Equations of the model

Coupled system in the frequency variable

$$\begin{cases} -\rho_S \omega^2 \mathbf{u}_\varepsilon - \mathbf{div}\underline{\sigma}(\mathbf{u}_\varepsilon) = \mathbf{f} & \text{in } \Omega_{S,\varepsilon}, \\ -\omega^2 \phi_\varepsilon - c^2 \Delta \phi_\varepsilon = i\omega \dfrac{C_\varepsilon}{\rho_A} & \text{in } \Omega_{F,\varepsilon}, \\ i\omega \mathbf{u}_\varepsilon \cdot \mathbf{n}_\varepsilon = \nabla \phi_\varepsilon \cdot \mathbf{n}_\varepsilon & \text{on } \Gamma_\varepsilon^j, \\ \underline{\sigma}(\mathbf{u}_\varepsilon) \cdot \mathbf{n}_\varepsilon = -\left(-i\omega\rho_A \phi_\varepsilon + C_\varepsilon\right) \mathbf{n}_\varepsilon & \text{on } \Gamma_\varepsilon^j, \\ \mathbf{u}_\varepsilon = 0 & \text{on } \partial\Omega. \end{cases}$$

Parameters:

C_ε: constant by cell such that ϕ_ε has zero mean in each $\mathcal{Y}^j_{F,\varepsilon}$,

ρ_S, ρ_A: density of the parenchyma and the air respectively,

c: velocity of sound in the air.

Equations of the model

Coupled system in the frequency variable

$$
\begin{cases}
-\rho_S \omega^2 \mathbf{u}_\varepsilon - \mathbf{div} \underline{\sigma}(\mathbf{u}_\varepsilon) = \mathbf{f} & \text{in } \Omega_{S,\varepsilon}, \\
-\omega^2 \phi_\varepsilon - c^2 \Delta \phi_\varepsilon = i\omega \dfrac{C_\varepsilon}{\rho_A} & \text{in } \Omega_{F,\varepsilon}, \\
i\omega \mathbf{u}_\varepsilon \cdot \mathbf{n}_\varepsilon = \nabla \phi_\varepsilon \cdot \mathbf{n}_\varepsilon & \text{on } \Gamma_\varepsilon^j, \\
\underline{\sigma}(\mathbf{u}_\varepsilon) \cdot \mathbf{n}_\varepsilon = -\left(-i\omega \rho_A \phi_\varepsilon + C_\varepsilon\right) \mathbf{n}_\varepsilon & \text{on } \Gamma_\varepsilon^j, \\
\mathbf{u}_\varepsilon = 0 & \text{on } \partial\Omega.
\end{cases}
\tag{4}
$$

Difficulties

- System of **Helmholtz type**.
- We cannot apply the usual steps to obtain the two–scale limit problem.

Two–scale convergence result

Theorem

- There exists $\Lambda = \{\lambda_n\}_{n\in\mathbb{N}}$ **a discrete set**, $\lim_{n\to\infty} \lambda_n = \infty$,
- $\forall\ \omega \in \mathbb{R} \setminus \Lambda$, ε small enough, (4) is well–posed.
- Then

  $$\widetilde{\mathbf{u}_\varepsilon} \twoheadrightarrow \mathbf{u}_0, \qquad \widetilde{\phi_\varepsilon} \twoheadrightarrow 0, \qquad \nabla\widetilde{\phi_\varepsilon} \twoheadrightarrow i\omega\mathbf{u}_0,$$

  where $\mathbf{u}_0$ is the unique solution of the homogenized problem:

  $$\begin{cases} -\rho^{hom}\omega^2\mathbf{u}_0 - \mathbf{div}(\mathcal{A}^{hom}\underline{\mathrm{e}}(\mathbf{u}_0)) = |\mathcal{Y}_S|\mathbf{f} & \text{on } \Omega, \\ \mathbf{u}_0 = 0 & \text{on } \partial\Omega. \end{cases} \tag{5}$$

- The set Λ is the set of eigenvalues of the homogenized problem.

Sketch of the proof of convergence

[Bouchitté, Felbacq '04], [Ávila, Griso, Miara, Rohan '06], [Alouges, Augier, Graille, Merlet '11].

First: suppose we have uniform *a priori* bounds for $\mathbf{u}_\varepsilon$.

- **Identify the homogenized problem** (5).
- Show that (5) has a **unique solution** for $\omega \notin \Lambda$.

Next: argument by contradiction.

- By **linearity**, build
  - $(\varepsilon_n)_{n\geq 0}$ converging to zero,
  - $(\mathbf{f}_n)_{n\geq 0}$ converging to zero in L^2,
  - Solutions $(\mathbf{u}_n, \phi_n)$ of (6) with

$$\|\mathbf{u}_n\|^2_{H^1(\Omega_{S,\varepsilon_n})} + \|\phi_n\|^2_{H^1(\Omega_{F,\varepsilon_n})} = 1.$$

- Pass to the two–scale limit.
- $=>$ **contradiction** with the uniqueness for problem (4).

Remarks on the homogenized system

$$\begin{cases} -\rho^{hom}\omega^2\mathbf{u}_0 - \mathbf{div}(\mathcal{A}^{hom}\underline{\mathrm{e}}(\mathbf{u}_0)) = |\mathcal{Y}_S|\mathbf{f} + |\mathcal{Y}_F|\nabla g & \text{on } \Omega, \\ \mathbf{u}_0 = 0 & \text{on } \partial\Omega. \end{cases}$$

- **Linearized elastodynamics** in the frequency domain.
- **Added mass effect** of the fluid on the structure with

$$\rho^{hom} = \theta\rho_S + (1-\theta)\rho_A.$$

- $\mathcal{A}^{hom}$ can be computed by solving cell problems.
  $\rightsquigarrow$ **Isotropy** recovered with an hexagonal cell.
- **No memory effects.**
- No band structure as in [Ávila, Griso, Miara, Rohan '06].

Remark

Static case $\omega = 0$: we recover the foam model [Baffico, Grandmont, Maday, Osses '08].

Velocity of sound

$$\begin{cases} -\rho^{hom}\omega^2\mathbf{u}_0 - \mathbf{div}(\mathcal{A}^{hom}\underline{e}(\mathbf{u}_0)) = |\mathcal{Y}_S|\mathbf{f} + |\mathcal{Y}_F|\nabla g & \text{on } \Omega, \\ \mathbf{u}_0 = 0 & \text{on } \partial\Omega. \end{cases}$$

- Numerical computations of the **sound velocity** with:

θ	λ	μ	ρ_A	ρ_S	c
0.15	$1e9$ Pa	$1e5$ Pa	1.2 kg/m^3	$1e3$ kg/m^3	350 m/s

We obtain for **pressure waves**

$$c^{hom} \approx 35\text{m/s}.$$

- Velocity of sound propagation in the lungs:

between 30 and 50m/s depending on inflation [Rice '84].

An unexplained frequency dependence

Note: little data for ultrasound propagation in the human parenchyma in the litterature.

References: [Rice '84], [Dunn '86]...

Experiments described in [Rueter, Hauber *et al.* '10].

Sound frequency (Hz)

	10^1 10^2 10^3	10^4	10^5 10^6 10^7	
Experimental	Sound transmission	No sound	Ultrasound efficient	Ultrasound inefficient
Sound speed	30–50 m/s	Filter band–cut	$>$ 1000 m/s	
Applications	Auscultation, percussion		lung monitoring Imagery	

Dynamic behavior

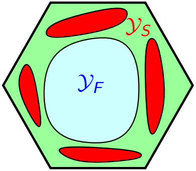
- This first model captures the acoustic behavior **only at low frequency**.
- It does not capture the **frequency–dependent** macroscopic acoustic properties of the parenchyma.

A second model

Collaboration with Jan Hesthaven at Brown University.

Main difference:

- Incompressible, **viscoelastic parenchyma**,
  $\rightsquigarrow \mathcal{G}(t,y)$, $\mathcal{Y}$–periodic relaxation function,
  [Fabrizio, Morro '92]



Example of periodic cell:
- Mostly elastic material
- Viscous inclusions
- Air bubble

Equations of the model:

$$\begin{cases} \rho_s \partial_{tt}\mathbf{u}_\varepsilon - \mathbf{div}\ \underline{\sigma}_\varepsilon(\mathbf{u}_\varepsilon, q_\varepsilon) = \mathbf{f} & \text{in } (0,T)\times\Omega_{S,\varepsilon}, \\ \underline{\sigma}_\varepsilon(\mathbf{u}_\varepsilon, q_\varepsilon) = -q_\varepsilon + \displaystyle\int_0^t \mathcal{G}(t-\tau, \mathbf{x}/\varepsilon)\underline{e}(\mathbf{u}_\varepsilon(\tau))\,\mathrm{d}\tau & \text{in } (0,T)\times\Omega_{S,\varepsilon}, \\ \mathrm{div}\ \mathbf{u}_\varepsilon = 0 & \text{in } (0,T)\times\Omega_{S,\varepsilon}, \\ \underline{\sigma}_\varepsilon(\mathbf{u}_\varepsilon, q_\varepsilon)\cdot\mathbf{n}_\varepsilon = -p_j\mathbf{n}_\varepsilon & \text{on } (0,T)\times\Gamma_\varepsilon^j, \\ p_j = -C_{air}\left(\dfrac{1}{\varepsilon^{dim}}\displaystyle\int_{\Gamma_\varepsilon^j}\mathbf{u}_\varepsilon\cdot\mathbf{n}_\varepsilon\right), & \\ +\ \text{boundary and initial conditions.} & \end{cases} \tag{6}$$

Two–scale homogenization

Denote the Fourier transform by $\widehat{\cdot}$.

- **Viscosity** $\rightsquigarrow$ existence, uniqueness and uniform *a priori* bounds.
- Two–scale convergence:

$$\widehat{\widetilde{\mathbf{u}}_\varepsilon} \twoheadrightarrow \widehat{\mathbf{u}_0}(x), \qquad \widehat{\nabla \widetilde{\mathbf{u}}_\varepsilon} \twoheadrightarrow \nabla_x \widehat{\mathbf{u}_0}(x) + \nabla_y \widehat{\mathbf{u}_1}(x,y), \qquad \widehat{\widetilde{q}_\varepsilon} \twoheadrightarrow \widehat{q_0}(x,y).$$

Cell problem (frequency domain)

$$\begin{cases} -\mathbf{div}_y \underline{\sigma_y^\omega}\left(\widehat{q_0}, \widehat{\mathbf{u}_1}\right) = 0 & \text{in } \Omega \times \mathcal{Y}_S, \\ \operatorname{div}_y \widehat{\mathbf{u}_1} = -\operatorname{div}_x \widehat{\mathbf{u}_0} & \text{in } \Omega \times \mathcal{Y}_S, \\ \underline{\sigma_y^\omega}\left(\widehat{q_0}, \widehat{\mathbf{u}_1}\right) \cdot \mathbf{n} = \left(C_{air} \operatorname{div}_x \widehat{\mathbf{u}_0}\right)\mathbf{n} - \underline{\sigma_x^\omega}\left(0, \widehat{\mathbf{u}_0}\right) \cdot \mathbf{n}, & \text{on } \Omega \times \Gamma, \end{cases}$$

- $\underline{\sigma_y^\omega}\left(\widehat{q_0}, \widehat{\mathbf{u}_1}\right) = -\widehat{q_0}\,\mathrm{Id} + \widehat{\mathcal{G}}(\omega, y)\underline{e}_y(\widehat{\mathbf{u}_1})$,
- $\underline{\sigma_x^\omega}\left(\widehat{q}\,, \widehat{\mathbf{u}_0}\right) = -\widehat{q}\ \mathrm{Id} + \widehat{\mathcal{G}}(\omega, y)\underline{e}_x(\widehat{\mathbf{u}_0})$,

- Cell problem **parameterized by the macroscopic variable** x **and** ω,
- Linear relation:

$$\widehat{\underline{\sigma}^{hom}}\left(\widehat{\mathbf{u}_0}\right) = \widehat{\mathcal{G}^{hom}}(\omega)\underline{e}_x\left(\widehat{\mathbf{u}_0}\right).$$

- $\mathcal{G}^{hom}(\omega)$: **homogenized relaxation function.**

Viscoelastic macroscopic problem

Macroscopic homogenized system (time domain)

$$\begin{cases} \rho^{hom}\partial_{tt}\mathbf{u}_0 - \mathbf{div}\left(\underline{\sigma}^{hom}(\mathbf{u}_0)\right) = \theta\mathbf{f} & \text{on } (0,T)\times\Omega, \\ \underline{\sigma}^{hom}(\mathbf{u}_0) = \displaystyle\int_0^t \mathcal{G}^{hom}(t-\tau)\underline{\mathrm{e}}\left(\mathbf{u}_0(\tau)\right)\mathrm{d}\tau & \text{on } (0,T)\times\Omega, \\ \qquad + \text{ boundary and initial conditions.} & \end{cases}$$

Remarks:

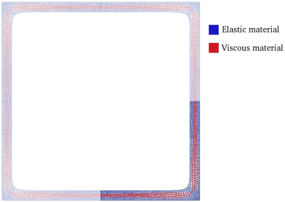
- Homogenization of fluid–structure mixtures often leads to integrodifferential problems.

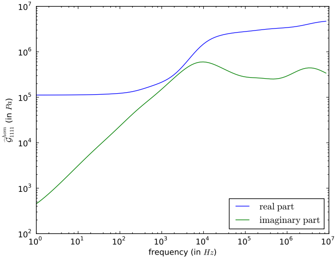
  [Sanchez–Palencia '80], [Boutin, Auriault '90], [Gilbert, Mikelic '00], [Griso, Blasselle '11]

- **New memory effects**.

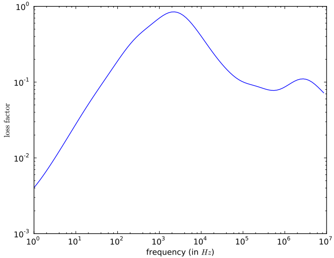
Numerical study: new memory effects

- Very simple 2D test geometry
- Realistic parameters:
  - Linear elastic fibers, $\mu = 10^5$ Pa
  - Viscous gel inclusions (ground substance...)
    with viscosity $\nu = 10^3$ Pa·s
  - Air at atmospheric pressure: 10^5 Pa
- Numerical result:
  $\rightsquigarrow$ **Absorption peak around 10 kHz** in the homogenized medium, **as in the parenchyma.**



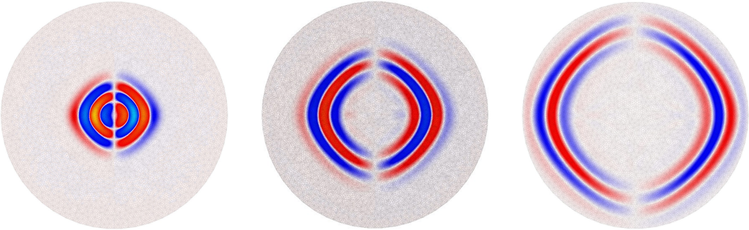


Complex viscoelastic modulus profile



Loss factor profile

Numerical simulation of low–frequency waves (10Hz)



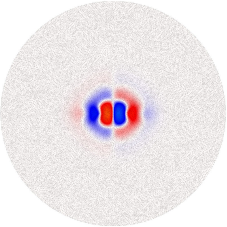
$t = .15$ s $\quad$ $t = .28$ s $\quad$ $t = .4$ s

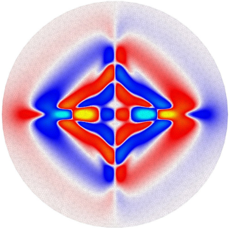
Propagation in the viscoelastic homogenized medium.

- Wave propagation
- Little absorption
- Sound speed around 30 m/s

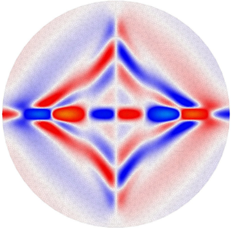
$\rightsquigarrow$ **We recover the low–frequency behavior of the parenchyma, as with the first model.**

Numerical simulation of high–frequency waves (2000Hz)





color scale $\times 200$



color scale $\times 200$

$t = .6$ ms $\quad t = 1.4$ ms $\quad t = 2$ ms

Propagation in the viscoelastic homogenized medium.

- Strong wave dissipation.
  $\rightsquigarrow$ **We recover the cut-off behavior of the parenchyma for frequencies above** 1000 **Hz.**
- Anisotropic behavior, the waves propagate mainly along the axis.
  $\rightsquigarrow$ Square periodic cell.
- **Question:** what are the factors determining the macroscopic behavior?

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Perspectives

Coupled tree–parenchyma model:

- Influence of the resistances on inflation, flow, energy dissipation? [Vannier '09]
- Other assumptions on the alveolar geometry: stochastic, fractal...
- Incorporate some non–linearities in the problem:
  $\rightsquigarrow$ boundary conditions, one–dimensional airflow models, elasticity model...

Sound propagation models:

- 3D simulations,
- Inverse problems using the homogenized dispersion curve profile?
  $\rightsquigarrow$ proportion of constituents, shape of the inclusions, inflation state...
- Interaction between airways and parenchyma in sound propagation.

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Merci de votre attention !