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# The temporal dimension in search models : horizon, age, experience

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**LA DIMENSION TEMPORELLE DANS LES  
MODELES DE RECHERCHE D'EMPLOI: HORIZON,  
AGE, EXPERIENCE**

soutenance prévue le 5 décembre 2012

devant le jury composé de :

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# Introduction générale

Des questions aussi diverses que, ”comment peuvent coexister emplois vacants et chômage?” ou ”comment des travailleurs a priori identiques peuvent avoir un salaire différent?” ont amené les économistes du travail à dépasser les modèles classiques d’offre et de demande sur un marché du travail et à développer de nouveaux types de modèles: les modèles de recherche d’emploi. Contrairement à la théorie classique, ces modèles supposent que l’information est imparfaite sur le marché du travail et que de ce fait, la recherche d’emploi, du côté du travailleur, et du côté de la firme, y est coûteuse, au moins en termes de temps. Ces modèles sont aujourd’hui très largement utilisés en économie pour modéliser le marché du travail et évaluer les effets de certaines politiques publiques sur ce marché. L’apport significatif à l’analyse économique de cette approche est aujourd’hui reconnu, comme le témoigne le prix de la banque de Suède de 2010 décerné à Diamond, Mortensen et Pissarides, tous trois à l’origine de cette large littérature.

Ces modèles se déclinent en deux grandes familles de modèles en fonction des hypothèses retenues. Le choix de ces hypothèses dépend bien entendu de la problématique abordée. Les modèles dits d’appariement, (Diamond, 1982), (Mortensen, 1982) et (Pissarides, 1985), s’intéressent à la modélisation de la recherche d’emploi afin d’expliquer

les flux sur le marché du travail et le chômage d'équilibre. Ils sont en ce sens des modèles macro-économiques. Les travailleurs au chômage ont une activité de recherche d'emploi et les firmes décident du nombre d'emplois vacants qu'elles veulent créer avant de rechercher, elles aussi, à pourvoir leur emploi. Les travailleurs et les firmes se rencontrent grâce à un processus d'appariement qui peut être plus ou moins long et coûteux en fonction de l'intensité des frictions sur le marché du travail. Les frictions sur le marché du travail permettent ainsi dans ce type de modèles d'expliquer la coexistence d'emplois vacants et de chômage à un même moment dans l'économie. Tout chômage devient alors frictionnel. (Pissarides, 2000) le justifie: " With the exception of a few 'discouraged' workers, unemployed workers are always between jobs or between some other state and a job." La théorie Keynesienne est donc également remise en cause, dans le sens où le chômage ne peut plus être décomposé en deux types de chômage, chômage frictionnel et chômage cyclique. Dans ces modèles d'appariement, le salaire est généralement négocié entre l'employé et la firme, une fois la rencontre établie, en fonction du gain à "l'échange" des deux parties. Ce mode de fixation des salaires a l'avantage d'être en cohérence avec l'idée qu'une rencontre génère des économies de coût de recherche que les partenaires peuvent rationnellement espérer se partager. Il a toutefois été critiqué d'un point de vue quantitatif par (Shimer, 2005) et (Hall, 2005): ce mode de détermination de négociation conduirait à une trop grande flexibilité du salaire, amortissant trop les fluctuations des quantités (chômage, emplois vacants et taux d'embauche) par rapport à leurs contreparties observées. De plus, il suppose que tous travailleurs similaires en termes de coût d'opportunité du travail et de productivité offerte à l'entreprise reçoivent le même salaire. Dans les faits, il existe une grande dispersion de salaires entre travailleurs a priori similaires. Selon (Mortensen, 2003), 70% de la dispersion des salaires ne peut être expliquée par les caractéristiques observables

des travailleurs.

La deuxième branche de cette littérature montre que les frictions sur le marché du travail peuvent expliquer ce paradoxe. L'approche de ces modèles de recherche d'emploi est plutôt micro-économique dans le sens où elle cherche à comprendre les mécanismes de formation d'une distribution des salaires sur le marché du travail. Dans ces modèles, il existe des salaires postés par les entreprises que les travailleurs ne peuvent pas négocier, c'est le principe du "take it or leave it". Dans un marché du travail frictionnel, les travailleurs reçoivent ces offres de salaires une à une et accepteront la première au delà de leur salaire de réservation. Aussi, en fonction du salaire tiré dans cette distribution d'offre, des travailleurs similaires pourront être amenés à recevoir un salaire différent. Si au départ ((Stigler, 1961) ou (McCall, 1970)), ces modèles étaient d'équilibre partiel du fait de la distribution exogène des salaires offerts par les firmes, assez vite, s'est posé le problème du comportement de fixation des salaires du côté des firmes. (Diamond, 1971) explique qu'en effet, dans le contexte décrit ci-dessus, si les travailleurs sont homogènes, les firmes ont l'entière du pouvoir de marché et n'ont en réalité intérêt à ne fixer qu'un seul salaire, le salaire de réservation des chômeurs. La fonction d'offre de salaires offerts est donc dégénérée en un point. La réponse la plus convaincante apportée à cette question sera donnée par une suite de contributions: (Butters, 1977), (Burdett and Judd, 1983), (Mortensen, 1990) puis (Burdett and Mortensen, 1998). Ces auteurs montrent comment la distribution des salaires peut être le résultat d'un équilibre, montrant alors que la recherche sur le marché du travail est une stratégie d'équilibre. Ainsi, (Burdett and Mortensen, 1998) montrent qu'il est suffisant de donner le "pouvoir" aux travailleurs de continuer à chercher de meilleures opportunités en emploi pour que cet équilibre avec recherche existe. En effet, dans ce nouveau contexte, ils montrent que sur un marché du travail frictionnel, dans lequel les

travailleurs sont homogènes, les firmes entrent dans un jeu non coopératif de fixation de salaire qui n'a qu'une seule solution en stratégie mixte ayant la forme d'une distribution non dégénérée de salaires. L'intuition de cet équilibre est la suivante: si toutes les firmes offrent le salaire de réservation des chômeurs comme c'est le cas dans (Diamond, 1971), la recherche en vue de pourvoir un poste étant coûteuse, une firme aura toujours intérêt à dévier en offrant un salaire infinitésimalement supérieur aux autres afin d'accélérer son processus de recrutement. Une offre de salaire supérieure aux autres firmes permet également d'augmenter la rétention de ses employés et d'économiser de cette façon la recherche d'un nouveau travailleur. Les deux marges du poste sont donc positivement affectées par le salaire offert: sa probabilité d'être pourvu et sa durée. L'ensemble de ce travail de thèse s'inscrit dans cette littérature, le chapitre 1 comme extension d'un modèle d'appariement (Mortensen and Pissarides, 1998) et le chapitre, 2, 3 et 4 comme extension du modèle de recherche d'équilibre (Burdett and Mortensen, 1998).

Les modèles de recherche d'emploi d'équilibre ont connu un assez grand nombre d'extensions au cours des dernières années en vue de pouvoir reproduire les données micro-économiques des distributions de salaires. Le plus grand frein à la réalisation de cet objectif fut la forme de la distribution générée par le modèle de recherche d'emploi de (Burdett and Mortensen, 1998). La distribution de salaire observée dans l'économie est une distribution de type log-normale. La distribution générée par le modèle de (Burdett and Mortensen, 1998) est censé représenter le paramètre d'erreur de l'équation de salaire de Mincer; dans ce cas, il serait attendu que cette distribution soit au minimum centrée. Dans les faits, le modèle génère une densité strictement croissante des salaires. C'est en introduisant l'hétérogénéité des productivités des firmes d'abord exogène (Bontemps, Robin, and Van-Den-Berg, 1999), puis endogène (Mortensen, 1998), qu'une distribu-

tion en adéquation avec les faits observés peut être obtenue. En 1998, Mortensen donne la possibilité aux firmes d'investir en capital humain spécifique à la création d'un appariement afin d'en augmenter la productivité au cours de toute sa durée. La dimension investissement du salaire prend alors une plus grande place dans le modèle de part l'intérêt croissant qu'ont les firmes à retenir leur travailleur; en plus de permettre aux firmes d'économiser sur une prochaine recherche de travailleurs, la rétention des travailleurs permet d'amortir un investissement initial en capital humain. La faible densité de très hauts salaires observée dans les données empiriques est obtenue du fait de l'hypothèse de rendement décroissant du capital investi. En outre, dans ce même article, Mortensen réunit pour la première fois les deux familles de modèle de recherche d'emploi pour construire un cadre théorique dans lequel les firmes jouent un rôle aussi bien dans la fixation des salaires que dans la décision de création d'emplois vacants. Deux faits empiriques restent inexpliqués par le cadre théorique développé par (Mortensen, 1998) : le fait qu'un nombre significatif de transition emploi-emploi se fasse vers des salaires plus faibles et le fait que les salaires croissent avec l'ancienneté en emploi. (Postel-Vinay and Robin, 2002) répondent à la première problématique en introduisant la possibilité de contre-offre de la part de la firme d'origine, face à une tentative de débauchage d'un employé par une firme concurrente. En effet, dans le modèle initial, la firme ne peut pas réagir face à la démission d'un de ses employés partant pour occuper un emploi dans une autre firme, alors même que dans un grand nombre de cas, il est optimal pour elle de s'aligner. Cette extension peut être critiquée, car elle fait l'hypothèse que l'information entre la firme concurrente et la firme d'origine est parfaite, ce qui ne semble pas être le cas en réalité. Selon leurs résultats, l'importance de la prise en compte de ces contre-offres est particulièrement grande parmi les cadres. (Burdett and Coles, 2003) expliquent eux, qu'il est optimal pour une firme d'offrir un



profil croissant de salaire avec l'ancienneté afin d'optimiser la rétention de ses employés.

Ces deux récentes extensions ouvrent la voie à une préoccupation dépassant la simple dispersion des salaires, celle de la trajectoire salariale des individus, aussi bien d'un emploi à un autre qu'au cours d'un emploi. Depuis (Becker, 1964) et (Mincer, 1974), les trajectoires salariales sont expliquées par l'accumulation de capital humain. (Becker, 1964) développe la théorie du capital humain selon laquelle les agents peuvent, à l'image du capital physique, investir en capital humain. De plus, au cours de leur vie, ils peuvent accumuler gratuitement ce capital par la pratique, c'est l'apprentissage en emploi. (Mincer, 1974) teste cette théorie en estimant une équation de salaire, dès lors appelée équation de Mincer, dépendant du niveau initial de formation et de l'expérience sur le marché du travail. Il montre que l'expérience a en effet des effets positifs sur le salaire mais que son rendement est décroissant. Le travail de (Bagger, Fontaine, Postel-Vinay, and Robin, 2012) fait une synthèse entre ces deux approches afin d'expliquer la contribution respective de l'accumulation du capital humain et de l'effet de la recherche d'emploi sur la trajectoire salariale des individus. Une partie importante du travail effectué dans cette thèse, le chapitre 2, 3 et 4, s'inscrit dans la continuité de ces travaux.

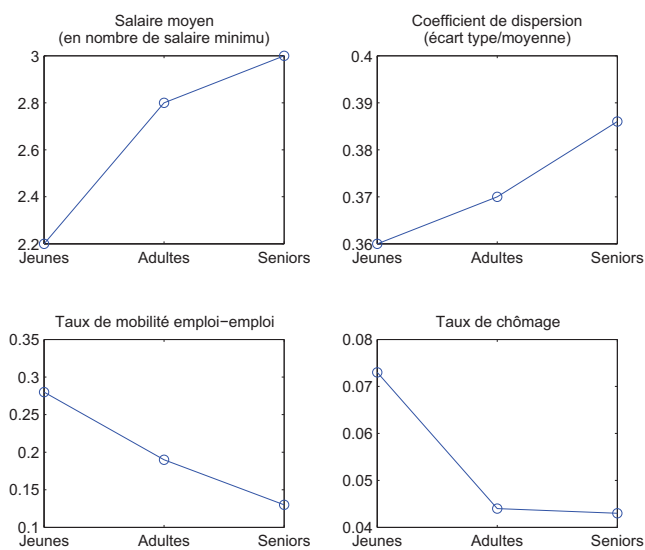
Dans ces trois chapitres, comme dans (Bagger, Fontaine, Postel-Vinay, and Robin, 2012), nous cherchons à dissocier la contribution du jeu de salaire de celle de la productivité, à la trajectoire salariale des travailleurs. Toutefois, contrairement à leurs travaux, qui ne prennent en compte que l'expérience du travailleur, nous prenons en compte l'âge des travailleurs. La prise en compte de l'âge introduit un horizon social fini : l'âge de départ en retraite. La prise en compte de cet horizon dans les modèles de recherche d'emploi constitue en grande partie l'originalité de cette thèse. Prendre en compte l'horizon des travailleurs remet en cause l'ensemble du jeu de salaire décrit

dans (Burdett and Mortensen, 1998) et (Mortensen, 1998). Dans un modèle en horizon infini, l'hypothèse sous-jacente est que les firmes ne peuvent avoir aucune indication sur l'âge des travailleurs, ils ont donc à leurs yeux tous la même probabilité de sortir du marché du travail. L'hypothèse contraire est pourtant plus plausible sachant que l'âge est une caractéristique observable et que l'interdiction de discriminer les travailleurs sur leur âge n'empêche nullement les firmes d'offrir des salaires différents en fonction de leur expérience, mesure hautement corrélée à l'âge. Dans un modèle en horizon fini, dans lequel les firmes peuvent observer au moins les principaux stades de la vie active, la probabilité de sortir du marché du travail dépend de l'âge du travailleur. Aussi, pour une firme, d'une part la forme du surplus espéré dépend de l'âge du travailleur embauché, mais en plus ce surplus n'est donc plus stationnaire au cours de sa durée. Il évolue sans cesse, dépendant en même temps de l'évolution de la productivité du travailleur, de l'évolution de la concurrence des firmes en fonction de l'âge du travailleur et du raccourcissement de l'horizon, jusqu'à être nul au moment où le travailleur a l'âge de se retirer du marché du travail. Cette dynamique est également présente dans (Menzio, Telyukova, and Visschers, 2012), toutefois, en construisant un modèle destiné à être estimé, ils ne peuvent pas l'utiliser pour une analyse théorique de ces mécanismes, sa taille étant trop importante. La prise en compte d'une date terminale introduit en effet des mécanismes riches dans le modèle de recherche d'emploi. D'abord une date terminale agit sur l'intensité de la recherche d'emploi : que ce soit du côté du travailleur, qui abaisse son effort de recherche à proximité de la retraite, ou des firmes qui ouvrent moins de postes pour les travailleurs offrant un court horizon de vie. Ensuite, elle modifie les stratégies salariales des firmes, en confrontant les firmes employant les travailleurs les plus âgés aux risques d'emplois raccourcis. Enfin, elle peut altérer la productivité des emplois créés à proximité de cette date. La durée de vie finie, est

en effet centrale pour rendre compte des investissements en capital humain (voir par exemple (Heckman, 1974)). Ce dernier mécanisme clé de la dynamique des salaires individuels n'est pas étudié dans (Menzio, Telyukova, and Visschers, 2012). Bien sûr, l'effet de l'horizon se confronte à celui de l'expérience dans le jeu de salaire, ce dernier agissant sur l'évolution du salaire de réservation, les gains cumulés issus des mobilités ascendantes connues au cours de la vie, ou encore de l'accumulation de capital humain. Comment l'ensemble de ces décisions individuelles interagissent avec le jeu de formation de la distribution des salaires? Telle est l'une des principales questions de cette thèse. Cette question a évidemment des motivations factuelles, car l'ensemble des performances sur le marché du travail varie avec l'âge: chômage, taux de transition, niveaux de salaire. La figure 1 l'atteste. Comprendre la raison de ces évolutions, et les liens existants entre chacune de ces performances semblent de première importance. En outre, certaines évolutions sociétales placent les problématiques centrées sur l'âge des travailleurs au coeur du débat public, notamment au regard de l'augmentation de la durée de vie, du vieillissement de la population, et du développement rapide de l'utilisation de nouvelles technologies sur le lieu de travail (obsolescence de la formation initiale).

Mais l'horizon bornant la relation d'emploi peut également être technologique : face à un progrès technologique, l'appareil productif d'une entreprise devient progressivement obsolète, le surplus généré par l'appariement décroît jusqu'à ce que le poste cesse d'être rentable. Cette forme d'horizon est centrale dans le modèle utilisé dans le chapitre 1. Dans ce modèle, les calculs économiques réalisés par les agents se font donc en fonction de l'horizon du poste: le partage du surplus entre firmes et travailleurs et les décisions de création d'emploi. Cette date terminale donne à la relation d'emploi une espérance de vie. A l'image de l'âge qui borne l'horizon du côté de l'offre de travail,

Figure 1: Variation du salaire moyen, de sa dispersion, du taux de mobilité d'emploi à emploi et du taux de chômage des travailleurs au cours des trois périodes de vie, 20-34 ans , 35-49 ans et 50-65 ans , en 2002 aux Etats-Unis



cet horizon borne l'appariement du côté de la demande de travail. Cette date terminale est donc manipulable en modifiant le comportement des firmes. Le chapitre 1 abordera cette thématique en introduisant des taxes sur les licenciements.

Les modèles d'appariement ont donné naissance à une abondante littérature cherchant à évaluer comment les flux sur le marché du travail sont affectés par les institutions. Pissarides, dans la réédition de son ouvrage de 2000, présente un chapitre entier sur le rôle des politiques économiques telles que la taxation du salaire, les allocations chômage, les subventions à l'embauche et les taxes sur les licenciements ((Mortensen and Pissarides, 1999) et (Cahuc and Kramarz, 2004)). L'utilisation des modèles de recherche d'emploi d'équilibre (Burdett and Mortensen, 1998) a lui été utilisé afin d'évaluer l'effet du salaire minimum sur la distribution des salaires ((Van-den Berg, 1995), (Bontemps, Robin, and Van-Den-Berg, 1999) et (Robin and Roux, 2002)). L'intérêt des développements théoriques des modèles de recherche d'emploi est en effet in fine de permettre une évaluation des institutions sur les "résultats" du marché du travail, d'un point de vue positif, mais également normatif. La hausse constante du chômage depuis les années 1980 en Europe continentale, regroupant des pays dans lesquels les institutions sont fortes, face à la stagnation du chômage aux États-Unis a constitué la base d'un questionnement sur les effets néfastes de certaines institutions. En outre, face aux imperfections, désormais reconnues, du marché du travail, les politiques économiques sont des leviers pour retrouver l'optimum social. Au cours de ces années, l'analyse de cette comparaison transatlantique en termes de chômage s'est affinée, et le débat contemporain prend en compte désormais largement l'aspect cycle de vie en se concentrant notamment sur certaines catégories de travailleurs particulièrement touchées par le chômage en Europe: les jeunes et les seniors. L'idée que les

institutions peuvent avoir des effets différents en fonction de l'âge et de l'expérience des travailleurs s'est alors développée, suite aux travaux de (Ljungqvist and Sargent, 2008), (Hairault, Langot, and Sopraseuth, 2010), (Chéron, Hairault, and Langot, 2011) et (Chéron, Langot, and Hairault, Forthcoming)). Une partie importante du travail effectué dans cette thèse s'inscrit dans cette problématique. Dans le chapitre 4, dans le cadre d'un modèle de recherche d'emploi du type (Burdett and Mortensen, 1998), nous rendons compte de l'effet d'institutions existant en France, tel que les allocations chômage, l'âge de départ en retraite, et le salaire minimum, sur les mécanismes de formation des salaires aux différents stades du cycle de vie du travailleur. Une telle évaluation ne pourrait pas se faire dans le cadre théorique proposé par (Menzio, Telyukova, and Visschers, 2012). Ces auteurs, se limitant à une analyse d'un équilibre particulier où l'allocation est optimale, éliminent de fait l'impact des institutions sur l'équilibre. Dans le chapitre 1, nous prenons en compte l'évolution possible des politiques publiques avec l'ancienneté de la relation d'emploi. Cette prise en compte a des effets significatifs sur les recommandations de politiques économiques en vue d'atteindre un optimum social.

L'évaluation des politiques publiques étant une préoccupation centrale dans ce travail de thèse, nous avons choisi d'adopter une approche structurelle. En opposition à une approche en forme réduite, cette approche a l'avantage de prendre en compte une grande partie des comportements micro-économiques des agents sur le marché. Dans le cadre d'une évaluation de politiques publiques, l'approche structurelle échappe ainsi à la critique de Lucas. (Lucas, 1976) explique que l'introduction d'une politique publique ne peut pas être évaluée à comportement fixe des agents. Ces derniers prennent en effet en compte le changement d'environnement institutionnel pour prendre

leurs décisions économiques, ce qui modifie les comportements entre avant et après le changement. Mais le recours à l'approche structurelle peut aussi être adapté hors du cadre de l'évaluation de politique publique. Dans le chapitre 3, nous étudions l'effet de l'âge sur la distribution des salaires de façon positive et sans intervention de politiques publiques. Dans cette étude, nous dissocions la contribution du canal de la productivité de celle de l'évolution du jeu de salaire à la progression salariale au cours de la vie. Il semble évident que ces deux canaux interagissent sur le marché du travail. Cette dissociation n'est donc rendue possible que par la modélisation précise des comportements micro-économiques. Dans ce cadre, et dans ce cadre seulement, il est possible d'isoler le canal de la productivité et de construire un monde cohérent sans évolution de productivité. Dans ce nouvel environnement, tous les comportements des agents sont recalculés: ceux des travailleurs, par un salaire de réservation différent, et ceux des firmes, par des décisions d'entrée sur le marché, de stratégies salariales et d'investissement sur les postes différents.

Cette thèse s'articule autour de quatre chapitres. Le chapitre 1 pose la question de l'effet des taxes sur les licenciements en fonction de leur forme. Nous adoptons d'abord l'approche positive, puis normative en calculant la valeur et la forme de cette taxe lorsqu'elle corrige une distorsion induite par la présence d'allocation chômage progressive. Nous utilisons pour ce faire le modèle avec progrès technologique de (Mortensen and Pissarides, 1998). Les chapitres 2, 3, et 4 sont liés par le même cadre théorique, le modèle de recherche d'emploi de (Burdett and Mortensen, 1998) ou son extension de (Mortensen, 1998), et par la même problématique, celle de l'évolution de la distribution des salaires en fonction de l'âge des travailleurs. Le chapitre 2 a d'abord une contribution théorique : comment l'âge modifie le jeu de salaire entre les firmes et les

travailleurs ? Nous choisissons d'étendre le modèle de (Burdett and Mortensen, 1998) en introduisant une hétérogénéité d'âge. Ce modèle, de par sa maniabilité, a l'avantage de pouvoir rendre compte des effets de l'âge algébriquement sur la distribution des salaires. Plusieurs extensions sont proposées afin de comprendre les mécanismes clés de cette évolution : l'ajout d'une classe d'âge, la passivité des travailleurs les plus âgés sur le marché du travail, la variation du coût d'opportunité du travail avec l'âge. Le chapitre 3 a une portée plus empirique. Il cherche à reproduire la distribution de salaires au cours du cycle de vie observée aux États-Unis. La confrontation aux données requiert de faire évoluer le modèle du chapitre 2 en le dotant de canaux expliquant l'évolution de la productivité avec l'âge et d'une fonction d'appariement. La dimension de ce nouveau modèle ne permet plus d'obtenir de résultats algébriquement, nous procédons donc à partir de ce chapitre par simulation numérique. Un des résultats de cette étude est que la part de la progression des salaires attribuable au mécanisme de transition d'emploi à emploi est significative. Ce résultat semble cohérent dans un pays comme les États-Unis où la mobilité professionnelle est forte, cependant, il pose la question : Quels sont les mécanismes dominants de progression salariale dans les pays à faible mobilité professionnelle ? Le chapitre 4 répond à cette question en confrontant le modèle aux données françaises. Là encore, le modèle doit évoluer afin de prendre en compte les institutions fortes existantes en France : allocation chômage, âge de départ à la retraite et salaire minimum. De façon plus générale, ce dernier chapitre évalue l'effet de cet environnement institutionnel sur la progression salariale et ses mécanismes.

Dans le chapitre 1, nous partons du résultat de (Pissarides, 2000) selon lequel les taxes sur les licenciements en association avec les subventions à l'embauche peuvent restaurer l'optimum social lorsque le coût du travail est trop fort. En Europe, les



coûts du travail sont accrus en même temps par la présence d'institutions telles que les allocations chômage et par les taxes nécessaires à leur financement. Cependant, la distorsion induite par les allocations chômage n'est pas constante au cours de la relation d'emploi: en même temps que le travailleur accumule de l'expérience, il accumule des droits supérieurs à l'allocation chômage. Lorsque le salaire est négocié, la croissance du coût d'opportunité avec l'expérience du travailleur au sein d'un poste vient accroître le coût du travail, tout au long de la durée de l'emploi. Dans un monde où la relation d'emploi est borné par un horizon technologique, cette hausse des coûts du travail vient accélérer l'obsolescence du poste et précipiter sa fin. Face à ces destructions précoces, nous calculons, en utilisant le modèle avec progrès technologique de (Mortensen and Pissarides, 1998), la valeur et la forme de la taxe sur les licenciements permettant de restaurer la durée de vie optimale de l'emploi. Cette taxe est alors croissante et concave. En outre, nous montrons de façon positive, que l'allongement de la durée de vie d'un emploi n'est rendu possible que par l'instauration de taxes sur les licenciements dont le taux de croissance est inférieur au taux d'escompte des firmes. Dans le cas contraire, la taxe, soit perd toute efficacité, dans le cas d'égalité, soit a un effet contraire à ce que l'on désire, c'est à dire raccourcit la durée de l'emploi, dans le cas d'une croissance supérieure.

Dans le chapitre 2, nous étendons le modèle de (Burdett and Mortensen, 1998) en introduisant d'abord 2 classes d'âge : les jeunes et les seniors. Ce découpage du cycle de vie est suffisant pour comprendre les deux forces majeures qui modifient le jeu de salaire avec l'âge. Entre le début et la fin du cycle de vie les travailleurs connaissent des transitions, d'abord du chômage vers l'emploi, puisque les travailleurs entrent sur le marché du travail au chômage, puis d'emploi à emploi une fois intégrés à une en-

treprise. Bien sûr au cours de la vie, les risques de chômage existent, mais globalement ces transitions assurent à offre de salaires constante, une croissance des salaires avec l'âge : c'est l'effet de la recherche d'emploi. Cette tendance a également des effets sur le pouvoir de marché des travailleurs dans le jeu d'offre de salaires : les firmes sont incitées à augmenter leurs offres de salaires chez les seniors afin d'augmenter leur probabilité d'embauche. Face à cette force, rémunérant l'expérience des travailleurs, s'oppose une autre force, celle de l'horizon. Dans le modèle de (Burdett and Mortensen, 1998) les deux marges d'un emploi sont affectées positivement par le salaire offert au travailleur : la probabilité d'embauche et la rétention du travailleur. Au début du cycle de vie, les firmes font peu cas de l'horizon social du travailleur, qui est à l'échelle de la durée de vie d'un emploi, très long, toutefois à mesure que le travailleur avance dans sa vie active, cet horizon se raccourcit et peut venir perturber la durée de l'emploi. Les incitations à retenir un travailleur senior en l'empêchant "d'aller à la concurrence" baissent alors puisque le vrai risque est que ce dernier se retire du marché du travail. Selon le modèle de recherche d'emploi à deux âges développé dans ce chapitre, quand firmes et travailleurs sont homogènes en termes de productivité, la combinaison de ces deux forces entraîne une hausse de salaire avec l'âge. Toutefois, l'horizon agit par d'autres canaux : il réduit notamment l'intensité de la recherche. Nous construisons un modèle à trois âges dans lequel les travailleurs les plus proches de leur horizon ne reçoivent plus d'offre d'emploi. Dans ce contexte, nous montrons que la trajectoire salariale est croissante sur les deux premières périodes puis devient stationnaire. La présence d'accumulation de droit à l'assurance chômage au cours de la vie, que nous introduisons dans ce même modèle en augmentant le coût d'opportunité du travail des seniors, permet cependant d'assurer la continuité de la progression salariale. Dans les deux chapitres suivants, nous conserverons le découpage du cycle de vie en trois périodes : les jeunes, les adultes et

les seniors. Ce découpage a l'avantage d'être en cohérence avec les faits stylisés liés au cycle de vie des travailleurs, donnés un peu plus haut dans l'introduction. Nous l'utilisons donc naturellement dans les deux chapitres empiriques qui suivent.

Rappelons que le modèle utilisé dans le chapitre 2 issu de Burdett et Mortensen (1998) n'est pas en mesure de représenter de façon crédible la distribution lognormale des salaires. Dans le chapitre 3, nous partons donc du travail de Mortensen (1998) dans lequel il donne notamment la possibilité aux firmes d'investir en capital humain spécifique à la création d'un appariement afin d'en augmenter la productivité au cours de toute sa durée. De façon à reproduire les données américaines de l'évolution de la distribution de salaires avec l'âge, nous permettons aux travailleurs d'accumuler du capital humain au cours de leur vie et estimons ces paramètres d'accumulation. Les résultats des simulations de ce modèle nous permettent de décomposer la progression salariales en deux canaux : l'évolution du jeu de salaire et l'évolution de la productivité. Le jeu de salaire (quand les travailleurs ont la même productivité *ex ante*) est différent de celui du chapitre 2 puisque il permet l'hétérogénéité des firmes; dans ce cadre là, il permet un légère hausse de salaire au début de la vie active des travailleurs mais génère une forte baisse dans la deuxième partie. En effet, quand la productivité des firmes est issue d'un investissement, les travailleurs jeunes et adultes sont employés dans les firmes les plus productives. La dimension investissement associé à la présence de capital humain spécifique coûteux renforce l'effet négatif du raccourcissement de l'horizon. Seule la plus grande productivité des seniors peut expliquer leurs plus hauts salaires aux États-Unis. Ce canal de productivité vient en effet renforcer en même temps le pouvoir de marché des travailleurs dans le jeu d'offre de salaires et la capacité des travailleurs à sélectionner les emplois les plus productifs et les mieux rémunérés

grâce aux transitions d'emploi à emploi.

La fréquence de ces transitions est bien plus faible en France qu'aux Etats-Unis, alors que la progression salariale y est plus importante. Dans le chapitre 4, nous expliquons ce fait stylisé en introduisant l'effet d'un environnement institutionnel fort, tel qu'il existe en France. Nous introduisons dans le modèle du chapitre 3, des allocations chômage progressives en fonction du salaire : les travailleurs accumulent des droits aux allocation chômage en emploi. Après modification de la durée de vie active et du salaire minimum par rapport aux Etats-Unis, nous calibrons ce modèle sur les données françaises. La présence des allocations chômage expliquent en même temps une augmentation de la progression salariale avec l'âge et une baisse des mobilités entre emploi. En d'autres termes, les allocations chômage viennent dans leurs effets en partie se substituer à la sélection des emplois les mieux rémunérés induite aux Etats-Unis par les forts taux de transition d'emploi à emploi. L'âge de départ à la retraite plus précoce en France (qu'aux Etats-Unis) pénalise assez fortement le salaires des seniors, cet effet est plus fort que l'effet du même raccourcissement de la vie active sur le marché du travail américain. Deux différences l'expliquent, d'abord, la durée des emplois est plus courte aux États-Unis, l'effet de l'horizon est donc moins contraignant pour les firmes, ensuite, la présence d'allocation chômage en France amplifie les évolutions de salaires : la baisse des salaires des seniors est entretenue par la baisse conséquentielle de leurs allocations chômage.



## Chapter 1

# Tenure-dependent Firing Taxes and Labor Market Equilibrium

### 1.1 Introduction

There is a large set of studies that deal with the impact of firing taxes on the labor market equilibrium. (Bertola and Rogerson, 1997), (Mortensen and Pissarides, 1999), (Blanchard and Portugal, 2001), (Ljungqvist, 2002) or all show that firing taxes unambiguously increase job tenure: Firms always prefer to delay the layoff when they are subject to a tax. By using firing taxes combined with hiring subsidies, (Pissarides, 2000) achieves to restore the first best allocation when the Hosios condition is not respected in the standard matching model. When the workers' bargaining power is such that the labor costs are too high<sup>1</sup>, the hiring subsidies give incentives to firms to post more vacancies and the firing taxes discourage them to lay off the workers too soon. The

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<sup>1</sup>In his framework, it means that the workers' bargaining power is higher than the elasticity of the matching function

negative indirect effect of firing taxes on job creation is controlled by the hiring subsidies and the indirect effect of hiring subsidies due to the new opportunities given to the workers which lead them to increase the wage pressure is controlled by the firing taxes.

However, empirical studies show that the workers' bargaining power in a matching model is not necessarily higher than the elasticity of the matching function, which suggests that the search externality is not a source of inefficiency that can account for high labor cost. In reality and especially in European economies, the bargaining power of workers is strengthened by the presence of labor market institutions like unemployment benefits. Explaining the high labor costs by these labor market institutions has crucial consequences on public policy recommendations: Labor market institutions becomes interdependent and the optimal layoff tax depends on the level of the unemployment benefits. Moreover, in most economies the government chooses to implement wage-depending unemployment benefit in order to better protect the workers from a large loss of consumption after a layoff. The labor costs are not simply higher than the optimal labor cost, but also more increasing with seniority. The (OECD, 2004) report underlines that severance pay increases with the job tenure (see tables 6, 7 and 8 in appendix A). This raise comes with a proportional increase in the firing taxes<sup>2</sup>. This acknowledgment incites us to go beyond the limit of the existing studies on firing taxes and consider that optimal firing taxes are not necessarily constant over time. Should they increase or decrease? At an increasing or decreasing rate?

Because the issue of this paper is to study tenure-dependant policies and because tenure is a natural state variable in this framework, we use the (Mortensen and Pissarides, 1998) vintage model where technological progress is embodied to answer these

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<sup>2</sup>see (Cahuc and Kramarz, 2004) for the French example

questions. This model highlights the Schumpeterian creative-destruction effect first discussed by (Aghion and Howitt, 1994). As there is no disembodied technological progress, there is no capitalization effect. This could be a realistic feature for the labor market of the unskilled workers. The firm posts vacancies and searches for a worker. All new jobs are created at the technological frontier, but after this creation date the firm keeps its technology during the whole tenure of the job. The technological choice is irreversible. Yet, as long as the firm continues to use its current technology, its employee accumulates expertise in that technology, and the productivity of the worker rises. We borrow the specification of learning from (Parente, 1994) and used by (Carre and Drouot, 2004). The increase of productivity due to learning is assumed to occur at a decreasing rate. At the opposite, the outside job opportunities grow at a constant rate, the technological progress rate. This implies that after a certain period of time the labor costs grow faster than the job productivity. Hence jobs are destroyed either because they reach the age of obsolescence, which corresponds to the date at which the productivity becomes lower than the labor costs, or because of an exogenous event such as resignation or bankruptcy. Only the endogenous fraction of job destruction bears the firing tax. In this framework, the positive effect of firing taxes on job tenure is based on the time preference of firms: Firms would rather pay the tax as late as possible because the layoff cost will be discounted in the long run. However, if the tax increases too fast, firms could rather terminate the match before the tax becomes a burden. If firing taxes increase with tenure, they must not increase faster than the discounting rate. We also augment this framework with unemployment benefits which depend on the wage earned in the last job. The assumption of learning by doing yields that the wages and therefore the unemployment benefits increase with tenure. The increasing path of the outside options of workers strengthens the bargaining power of



senior workers and consequently weakens their job by making them more subject to creative-destruction. The higher the unemployment benefits are, the larger the employment protection should be. Therefore, firing taxes need to be increasing with tenure yet no more than the discounting rate.

The unemployment benefits are financed by payroll taxes. Firing taxes finance hiring subsidies. At the optimal allocation, the level of hiring subsidies needed to offset the pervert effect induced by firing taxes on job creation is exactly such as the budget is balanced. As beside firing taxes, unemployment benefits and payroll taxes also decrease job creation level, the level of hiring subsidies needed to reach the optimal job creation exceeds the income from firing taxes; this complementary policy yields a deficit. This deficit is financed by a lump-sum paid by everyone in the economy.

Labor market institutions are interdependent. Wage-depending unemployment benefits induce a distortion in the labor costs which increases at a decreasing rate with job tenure. According to our findings to offset this distortion, the government should implement a firing tax which has the same shape as the distortion. Here, the optimal tax is therefore increasing at a decreasing rate with tenure.

Another finding of this paper is that the more increasing with tenure the distortion is, the larger the tax must be. Indeed firms are even more incited to lay off the worker soon if it knows that the labor costs are increasing over time. The increase of the value of the tax guarantees us that its growth rate remains below the actualization rate, whatever the growth rate of the distortion. The firing tax keeps therefore its positive effect on job tenure.

The government can also use a social minimum to protect workers from a loss of income like in the US. In that case, the distortion induced is constant and therefore

requires constant firing taxes to be offset.

In the second section of the paper, we present the theoretical framework. Section three analyzes positively the implications of varying firing taxes with job tenure and of the other labor market institutions on the labor market equilibrium. In the fourth section, we compute the labor market policies (firing taxes and hiring subsidies) allowing to restore the optimal allocation in the economy. Last section concludes.

## 1.2 The model

### 1.2.1 Model Assumptions

We use the framework developed by (Mortensen and Pissarides, 1998). The number of contacts is represented by a standard matching function:

$$m(u, v) = \phi \cdot u^\eta v^{1-\eta} \tag{1.1}$$

where  $u$  stands for the number of unemployed and  $v$  for the number of vacancies. We define market tightness as the vacancy to unemployment ratio  $\theta = \frac{v}{u}$  and make use of the homogeneity of the matching function to write the probability for an unemployed worker to find a job,  $\frac{m(u,v)}{u} = m(1, \theta) \equiv \theta q(\theta)$ , and the probability for a firm to fill its vacancies,  $\frac{m(u,v)}{v} = m\left(\frac{1}{\theta}, 1\right) \equiv q(\theta)$ .

The most advanced known technology is embodied in newly created jobs but then the firm keeps its technology throughout its life because we assume the technological choice is irreversible. As long as firm continues to use its current technology, its employee accumulates expertise in that technology, and the productivity of the worker

risers. We borrow the specification of learning from (Parente, 1994) also used by (Carre and Drouot, 2004). The increase of productivity due to learning is assumed to occur at a decreasing rate. According to the law of motion of productivity, the production of a job created in  $\tau$ , at date  $t$  is:

$$y(\tau, t) = p(\tau)\chi(t - \tau) \tag{1.2}$$

with  $p(\tau) = e^{g\tau}$  and  $\chi(t - \tau) = s - (s - x)e^{-\gamma(t-\tau)}$

$p(\tau)$  is the productivity frontier at date  $\tau$  with  $g$  being the exogenous growth rate of technological progress.  $\chi(t - \tau)$  represents the learning return after  $t - \tau$  years in the job. Any worker starts at a level  $x$  of productivity, and is able to reach at best a level  $s$ . The parameter  $\gamma$  reflects the rhythm of learning within job. This knowledge is a specific skill gained by memorization, routine and automation of tasks, it is therefore not transferable to another job.

In this economy, job destruction takes place either in response to an exogenous event which arrives at rate  $\delta$  (exogenous job destructions) or because the job has ceased to be profitable. Indeed at each date the firm can either keep on being productive or put an end to its match if it has become obsolete. That decision constitutes an endogenous destruction. Outside the firm, everything increases at rate  $g$  and the productivity frontier at date  $t$  is  $p(t) = e^{gt}$ .

### 1.2.2 Labor Market Institutions

We consider two institutions on the labor market.

1. First, the unemployment benefits. They are denoted by  $p(t)b(t - \tau)$ , and are

indexed on the growth rate,  $g$ , in order to insure the existence of an equilibrium growth path. The unemployment benefits we introduce depend on the last wage of the worker. Yet for the sake of simplicity, we assume that it is indexed on the productivity the worker had in the previous job such as:

$$b(t - \tau) = b \cdot \chi(t - \tau) \quad (1.3)$$

The assumption of learning on the job yields that the wage increases with tenure. The unemployment benefits therefore increase in tenure as well. These unemployment benefits are financed by payroll taxes paid by firms  $P_J$  and workers  $P_W$ .

2. Second, the firing taxes and the hiring subsidies. They are also indexed to the growth rate, and are denoted respectively as  $p(T)F(T)$  and  $p(t)H$ , where  $T$  is the job tenure. The value of firing taxes can therefore depend on the job tenure:

- The firing tax can depend positively on tenure and increase with seniority. The longer the firm waits to terminate the job the more expensive the tax it will pay.
- It can depend negatively on tenure and decrease with the seniority. The longer the firm waits to terminate the job the cheaper the tax it will pay.
- Or, it can remain constant. Whenever the firm decides to put an end to the match, it pays the same tax.

If this complementary policy yields a deficit, it is funded by a flat-rate tax which does not affect agents' behavior.

### 1.2.3 Bellman equations

A firm with an empty vacancy participates in the matching process by incurring a cost  $p(t)c$  per period. The interest rate is represented by  $r$ . The asset value of an empty vacancy is given by  $V(t)$  and solves:

$$rV(t) = -p(t)c + q(\theta)[J(t, t) + p(t)H - V(t)] + \dot{V}(t) \quad (1.4)$$

The second term of the right hand side of the expression represents the expected return to participation in the matching process, and the last term is the rate of pure capital appreciation. At equilibrium, firms open vacancies until all rents are exhausted, i.e.,  $V(t) = 0$  for all  $t$ . Combining this free entry job creation condition and equation (1.4) yields :

$$\frac{J(t, t)}{p(t)} = \frac{c}{q(\theta)} - H \quad (1.5)$$

The value of a job created at date  $\tau$ , that produces the output  $y(\tau, t)$  at time  $t$  is represented by  $J(\tau, t)$  and solves the asset pricing equation:

$$rJ(\tau, t) = \max\{y(\tau, t) - (1 + P_J)w(\tau, t) - \delta[J(\tau, t)] + \dot{J}(\tau, t), x\} \quad (1.6)$$

This value depends on the creation date, since this date determines the productivity of the firm, and on the current date since it impacts the outside options and the learning time. With the probability  $\delta$ , the firm experiences an exogenous shock which leads to the job destruction. Job destruction can also be the result of the firms' decision if the asset value of its job in  $t$  falls below  $x$ . Without firing taxes,  $x = 0$ , yet with firing taxes,  $x < 0$ . In our context,  $x$  depends on both the value of the firing tax and its shape.

Analogously, the value of employment for a worker is represented by  $W(\tau, t)$  and depends on the creation and on current date as following:

$$rW(\tau, t) = \max\{(1 - P_W)w(\tau, t) - d - \delta[W(\tau, t) - U(t - \tau, t)] + \dot{W}(\tau, t), rU(\tau, t)\} \quad (1.7)$$

where the right side takes into account the fact that if the instantaneous value of employment falls below the reservation wage, the worker becomes unemployed. The value  $d$  is the flat-rate tax which funds the possible debt of the government. The value of unemployment depends on the tenure noted  $T$  of the last job and is represented by  $U(T, t)$  and solves:

$$rU(T, t) = p(t)b(T) - d + \theta q(\theta)[W(t, t) - U(T, t)] + \dot{U}(T, t) \quad (1.8)$$

#### 1.2.4 Wage determination

As in most of existing literature, we suppose that wages are negotiated according to a Nash bargaining process. The workers' bargaining power is denoted by  $\beta$ . As hiring subsidies and firing taxes are introduced, we assume a tow-tiers contract: one for the outsiders when  $\tau = t$  and one for the insiders when  $t > \tau$ . As unemployed workers receive different levels of compensation, the outsiders' sharing rule depends on  $T$ , the previous seniority of the unemployed workers. The sharing rule for outsiders interiorizes the fact that the firms receive a hiring subsidy at job creation:

$$\max_{w_T(t,t)} \left\{ (W(t, t) - U(T, t))^\beta (J(t, t) + p(t)H)^{1-\beta} \right\} \quad (1.9)$$

The sharing rule of insiders interiorizes the fact that the firm must pay a firing tax if the job is endogenously destroyed:

$$\max_{w(\tau,t)} \left\{ (W(\tau, t) - U(t - \tau, t))^\beta (J(\tau, t) + p(t)F(t - \tau))^{1-\beta} \right\} \quad (1.10)$$

By using these two sharing rules, we deduce the following insiders' bargained wage:

$$w(\tau, t) = \frac{\beta}{1 + P_J} \left[ y(\tau, t) + p(t) \left( (r + \delta - g)F(t - \tau) - \dot{F}(t - \tau) \right) \right] \\ + \frac{1 - \beta}{1 - P_W} p(t) \left( b(t - \tau) + \frac{1 - P_W}{1 + P_J} \frac{\beta}{1 - \beta} c\theta \right) \quad (1.11)$$

The details of this calculation are given in appendix .2, page 181.

The payroll taxes paid by firms and workers reduces the labor's share and increases the firm's share of the surplus. Indeed a unit rise in wages conceded by the firm yields a smaller benefit to the worker than this unit and a higher cost to the firm than this unit. Therefore, the payroll taxes induce a joint loss that can be reduced by keeping wages low. The firing tax pushes the insiders' wage up since workers can use this tax as a threat during the bargaining process. However the growth rate of this firing tax pushes the wage down. Indeed, if this rate, is high the worker is ready to give up a part of his or her current wage, knowing that he or she will enjoy a higher threat in the future.

Here, we only calculate the insiders' wage because except for the starting date, firms pay the insiders' wage and, as time is continuous, this wage is instantaneously renegotiated<sup>3</sup>. We assume this first date compared to the flow of expected wages is insignificant, yet the insiders' wage takes into account the outsiders' surplus since the threat of the outsiders' options occurs during all job tenure.

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<sup>3</sup>There is no shock in this economy so the wage is continuously renegotiated.

### 1.2.5 Equilibrium determination

The firm chooses the optimal scrapping time  $T^o$  by maximizing the value of the match taking into account that at the termination date, the firm will have to pay a firing tax.

$$J(\tau, t) = \max_T \left\{ \int_t^{\tau+T} [y(\tau, s) - (1 + P_J)w(\tau, s)] e^{-(r+\delta)(s-t)} ds - e^{-(r+\delta)(\tau+T-t)} F(T) e^{g(\tau+T)} \right\} \quad (1.12)$$

After simplification, we deduce the following expression of the value of a newly created job:

$$J(t, t) = \int_0^{T^o} \left[ \frac{y(t, s+t)}{p(t)} - (1 + P_J) \frac{w(t, s+t)}{p(t)} \right] e^{-(r+\delta)s} ds - e^{-(r+\delta)T^o} F(T^o) e^{gT^o} \quad (1.13)$$

Using the fact that the value of a new job is proportional to productivity at the technology frontier, i.e.,  $J(t, t) = p(t)J$  and after substitution of the value of the wage and of the unemployment benefits, we deduce the value of the newly created job in function of the labor market institutions.

$$J = (1 - \beta) \int_0^{T^o} \left[ \chi(s) - e^{gs} \left( \frac{1 + P_J}{1 - P_W} b\chi(s) + \frac{\beta}{1 - \beta} ((r + \delta - g)F(s) - \dot{F}(s) + c\theta) \right) \right] e^{-(r+\delta)s} ds - e^{-(r+\delta-g)T^o} F(T^o) \quad (1.14)$$

Setting likewise  $J(t, t) = p(t)J$ , the free entry condition becomes:

$$J = \frac{c}{q(\theta)} - H \quad (1.15)$$

Hiring subsidies allow firms that formerly did not have a sufficient value to enter the market, to do it.



In a  $(J, \theta)$  plan, the free entry condition (1.15) is increasing while the value of a newly created job (1.16) is decreasing. For reasonable values of  $b$ ,  $\beta$  and the payroll tax, we obtain an equilibrium, i.e a value of  $\theta$  noted  $\theta^o$  which allows the intersection. As we use insiders' wage, this integral only stands for values of  $t$  superior to  $\tau^4$ . As the insiders' wage grows at a constant rate when the job's productivity grows at a decreasing rate, from a certain date the profit obtained by the firm decreases and, there comes a time when the technology of a firm with a job created in the past becomes obsolete. The firm must therefore destroy the job, lay off the worker and pay the firing tax. The destruction date chosen by the firm solves the following first order condition of the value of a new job.

$$\chi(T^o)e^{-gT^o} - \frac{1 + P_J}{1 - P_W} b\chi(T^o) - \frac{\beta}{1 - \beta} c\theta + F(T^o)(r + \delta - g) - \dot{F}(T^o) = 0 \quad (1.16)$$

In other words, this equation suggests that firms keep on producing as long as their instantaneous profit flow is positive. The first three terms highlight therefore the creative destruction effect, i.e. the reservation wage increases at least at a constant rate whereas the productivity increases at a decreasing rate. The fourth term represents the firms' earning from not destroying the job today, i.e. the saving of the tax for this period (since discounted). Finally, the last term stresses the firms' loss (resp. the benefit) from not destroying the job now if the tax increases (resp. decreases). We assume from now on that the actualization rate  $r + \delta - g$  is positive so that paying the tax now is more costly than paying it later. We can notice that firing taxes can consequently either strengthen or weaken the creative destruction effect according to the shape of the function that we will discuss later. We can also notice that some shapes of the

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<sup>4</sup>As time is continuous, the integral still stands when  $t$  tends to  $\tau$ .

tax can yield a strictly increasing instantaneous profit according to tenure or a zero profit from the beginning. In the first case, firms have interest to keep on producing forever and  $T^o$  would tend to infinite, in the second case, firms do not have interest to product at all, and  $T^o$  is null. To avoid these scenari which lead to no equilibrium or a degenerated equilibrium, firing taxes must fulfil the following conditions<sup>5</sup>:

**Condition 1.** *Concavity of the intertemporal profit: After a certain period of time, instantaneous profit of a job decreases with tenure until it equals zero (or to say it differently, the intertemporal profit is concave at destruction date)*

$$\iff \exists T \leq T^o / \forall t \in [T; T^o], e^{-gt} (\dot{\chi}(t) - g\chi(t)) - \frac{1+P_J}{1-P_W} b\dot{\chi}(t) + \dot{F}(t)(r + \delta - g) - \dot{F}(t) < 0$$

It is relevant to consider this condition when for instance firing taxes is strongly increasing and concave.

**Condition 2.** *Non negativity of the instantaneous profit: Instantaneous profit must be positive during at least a certain period of time:*

$$\iff \exists T > 0 / \forall t < T, \chi(t)e^{-gt} - \frac{1+P_J}{1-P_W} b\chi(t) - \frac{\beta}{1-\beta} c\theta + F(t)(r + \delta - g) - \dot{F}(t) > 0$$

Given condition 1 and 2 and as the solution for (1.16) is unique on  $\mathfrak{R}, T^o$  is the optimal tenure chosen by the firms. Given the equilibrium values for  $\theta$  and  $T$ , one can deduce the equilibrium labor market flows. Job creations correspond to the number of unemployed workers who find a job  $\theta q(\theta)u$ . Job destructions are composed by the exogenous job destructions  $\delta(1 - u)$  and the endogenous job destructions. The number of firms endogenously destroyed at each date corresponds to the number of firms that

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<sup>5</sup>In Mortensen-Pissarides 1998, the instantaneous profit flow is positive in  $t = 0$  since wage is inferior to productivity at the beginning of the match, and it always decreases with tenure since wage increases, while the productivity does not.

reach  $T^o$ , the optimal destruction date, without having been hit by an exogenous shock  $[\theta^o q(\theta^o)u]e^{-\delta T^o}$ . The equilibrium rate of unemployment solves:

$$\theta^o q(\theta^o)u^o = \delta(1 - u^o) + [\theta^o q(\theta^o)u^o]e^{-\delta T^o}$$

Consequently, under condition 1 and condition 2, the equilibrium is defined by the set of functions  $\{\theta^o, T^o, u^o\}$ , satisfying the following system:

$$\begin{aligned} (JC) : & (1 - \beta) \int_0^{T^o} \left[ \chi(s) - e^{gs} \left( \frac{1 + P_J}{1 - P_W} b\chi(s) + \frac{\beta}{1 - \beta} \left( (r + \delta - g)F(s) - \dot{F}(s) + c\theta^o \right) \right) \right] e^{-(r+\delta)s} ds \\ & - e^{-(r+\delta-g)T^o} F(T^o) + H = \frac{c}{q(\theta^o)} \\ (JD) : & \chi(T^o)e^{-gT^o} - \frac{1 + P_J}{1 - P_W} b\chi(T^o) - \frac{\beta}{1 - \beta} c\theta^o + F(T^o)(r + \delta - g) - \dot{F}(T^o) = 0 \\ (BC) : & u^o = \frac{\delta}{\delta + (1 - e^{-\delta T^o})\theta^o q(\theta^o)} \end{aligned}$$

where  $(JC)$  and  $(JD)$  are respectively the job creation and the job destruction equations, and  $(BC)$  the equilibrium unemployment (Beveridge Curve).

### 1.3 Equilibrium properties

**Property 1.** *The implementation of unemployment benefits and payroll tax decreases the labor market tightness.*

*Proof.* The effects of unemployment benefits and payroll tax on the labor market tightness are given by:

$$\frac{\partial \theta^o}{\partial b} = - \frac{-\frac{1+P_J}{1-P_W} \int_0^{T^o} e^{-(r+\delta-g)s} \chi(s) ds}{-\beta c \int_0^{T^o} e^{-(r+\delta-g)s} ds + \frac{cq'(\theta^o)}{q(\theta^o)}} < 0 \quad (1.17)$$

$$\frac{\partial \theta^o}{\partial P_J} = - \frac{-b \frac{\partial \left( \frac{1+P_J}{1-P_W} \right)}{\partial P_J} \int_0^{T^o} e^{-(r+\delta-g)s} \chi(s) ds}{-\beta c \int_0^{T^o} e^{-(r+\delta-g)s} ds + \frac{cq'(\theta^o)}{q(\theta^o)}} < 0 \quad (1.18)$$

$$\frac{\partial \theta^o}{\partial P_W} = - \frac{-b \frac{\partial \left( \frac{1+P_J}{1-P_W} \right)}{\partial P_W} \int_0^{T^o} e^{-(r+\delta-g)s} \chi(s) ds}{-\beta c \int_0^{T^o} e^{-(r+\delta-g)s} ds + \frac{cq'(\theta^o)}{q(\theta^o)}} < 0 \quad (1.19)$$

As  $\frac{\partial \left( \frac{1+P_J}{1-P_W} \right)}{\partial P_J} > 0$  as well as  $\frac{\partial \left( \frac{1+P_J}{1-P_W} \right)}{\partial P_W} > 0$ , we can deduce that unemployment benefits and both wage taxes decrease labor market tightness at fixed job tenure. The effects of unemployment benefits and payroll tax on job tenure are given by:

$$\frac{\partial T^o}{\partial b} = - \frac{-\frac{1+P_J}{1-P_W} \chi(T^o)}{[\chi(T^o) \dot{x} e^{-gT^o}] + \dot{F}(T^o)(r + \delta - g) - \dot{F}(T^o)} < 0 \quad (1.20)$$

$$\frac{\partial T^o}{\partial P_J} = - \frac{-\frac{\partial \left( \frac{1+P_J}{1-P_W} \right)}{\partial P_J} b \chi(T^o)}{[\chi(T^o) \dot{x} e^{-gT^o}] + \dot{F}(T^o)(r + \delta - g) - \dot{F}(T^o)} < 0 \quad (1.21)$$

$$\frac{\partial T^o}{\partial P_W} = - \frac{-\frac{\partial \left( \frac{1+P_J}{1-P_W} \right)}{\partial P_W} b \chi(T^o)}{[\chi(T^o) \dot{x} e^{-gT^o}] + \dot{F}(T^o)(r + \delta - g) - \dot{F}(T^o)} < 0 \quad (1.22)$$

As the decrease in job tenure decreases the labor market tightness, at equilibrium, the implementation of unemployment benefits and payroll tax unambiguously decreases the labor market tightness.  $\square$

**Property 2.** *If  $F$  is such that  $\frac{\dot{F}(t)}{F(t)} < r + \delta - g, \forall t > 0$  then the implementation of firing taxes lengthens job tenure.*<sup>6</sup>

<sup>6</sup>Obviously, the reciprocal does not stand. There exist firing taxes which do not fulfil this condition for all  $t < T_1$  which still lengthen job tenure.

*Proof.* To assess the effect of firing taxes on labor market tightness, we rewrite (JC) as follows:

$$\begin{aligned}
& (1 - \beta) \int_0^{T^o} \left[ \chi(s) - e^{gs} \left( \frac{1 + P_J}{1 - P_W} \left( b\chi(s) + \frac{\beta}{1 - \beta} c\theta^o \right) \right) \right] e^{-(r+\delta)s} ds + H - \frac{c}{q(\theta^o)} \\
& \underbrace{+\beta F(T^o)e^{-(r+\delta-g)T^o} - \beta F(0)}_{\text{effect on J via the wage}} \quad \underbrace{-e^{-(r+\delta-g)T^o} F(T^o)}_{\text{effect on J via the paiement of the tax}} = 0
\end{aligned} \tag{1.23}$$

The payment of the firing tax at destruction date decreases the value of a job. However, firing taxes have an ambiguous effect on wages. If the firing taxes increase faster than the discounting factor  $e^{-(r+\delta-g)T^o}$  decreases, they push the wage down, otherwise they push the wage up. The sum of this two effects is yet always a decrease in the value of a job and therefore a decrease of labor market tightness. To assess the impact of firing taxes on job tenure, we need to compare the equilibrium job destruction equation with and without firing tax (we assume  $\theta$  constant):

$$\begin{aligned}
& \chi(T_1)e^{-gT_1} - \frac{1 + P_J}{1 - P_W} b\chi(T_1) - \frac{\beta}{1 - \beta} c\theta = 0 \\
& \chi(T_2)e^{-gT_2} - \frac{1 + P_J}{1 - P_W} b\chi(T_2) - \frac{\beta}{1 - \beta} c\theta + F(T_2)(r + \delta - g) - \dot{F}(T_2) = 0
\end{aligned} \tag{1.24}$$

Firing taxes increase job tenure if  $F$  is such that  $T_1 < T_2$ , this occurs when the second equation remains positive longer than the first one. Consequently, to increase job tenure,  $F$  must be such that:

$$\frac{\dot{F}(t)}{F(t)} < (r + \delta - g) + \frac{\chi(t)e^{-gt} - \frac{1+P_J}{1-P_W} b\chi(t) - \frac{\beta}{1-\beta} c\theta}{F(t)}$$

For all  $t \leq T_1$  and for an extra time after  $T_1$ . When  $t < T_1$ , the second term of the right hand side of the expression is positive since the instantaneous profit without tax is positive before  $T_1$ , whereas, when  $t > T_1$ , it is negative. This condition shows that the firing tax lengthens job duration if it does not grow too fast. If we omit the

indirect effect via the labor market tightness, we can deduce from this inequality that the firing tax can grow faster than the actualization rate  $r + \delta - g$  before the date at which firms terminate the job in a tax-free-economy  $T_1$ , yet it must grow slower than the actualization rate after this date if we want it to increase the job tenure.  $\square$

When  $F(t)$  is such that  $\frac{\dot{F}(t)}{F(t)} < r + \delta - g, \forall t > 0$  then the implementation of firing taxes lengthens job tenure at fixed labor market tightness and decreases the labor market tightness at fixed job tenure. As the decrease in labor market tightness increases the job tenure, at equilibrium, the implementation of firing taxes  $F(t)$  such that  $\frac{\dot{F}(t)}{F(t)} < r + \delta - g, \forall t > 0$  lengthens job tenure.

A firing tax which grows slower than the actualization rate necessarily increases job tenure. What happens intuitively? In fact, a firing tax increases job tenure if the savings from waiting before laying off is larger than the tax growth. There are two sources of savings: the financial interests induced by delaying the paiement of the tax, and the indirect savings from the wage cut induced by the tax via the decrease in  $\theta$ <sup>7</sup>. The first source of savings is based on discounting effect, i.e. if the tax is constant, the firm would always prefer to wait before paying it than paying it right now. Therefore if the tax increases too strongly, the firm can have interest to lay off the worker sooner. On the contrary a decreasing tax strengthens the actualization effect.

**Property 3.** *The implementation of hiring subsidies increases labor market tightness and shortens the job tenure.*

*Proof.*

$$\frac{\partial T^o}{\partial H} = - \frac{\frac{\beta}{1-\beta} c \frac{\partial \theta}{\partial H}}{[\chi(T^o) \dot{x} e^{-gT^o}] + \dot{F}(T^o)(r + \delta - g) - \dot{F}(T^o)} < 0 \quad (1.25)$$

<sup>7</sup>The positive direct effect of firing taxes on wages is already taken into account in the first order condition

The implementation of hiring subsidies raises the labor market tightness. Its effect on job tenure only goes through this increase of the labor market tightness, therefore the implementation of hiring subsidies shortens job tenure.  $\square$

The hiring subsidies therefore allow to compensate the decrease in job creation induced by unemployment benefits and payroll taxes. Their implementation has the pervert effect to shortens job tenure. This last effect can be offset by firing taxes under the condition established in the property 2. The following section is dedicated to assess the value and the shape of the firing tax and the value of the hiring subsidy which can restore the optimal allocation.

## 1.4 Social Planner

### 1.4.1 Efficient allocation determination

In the decentralized equilibrium employed workers bargain their wages without taking into account social welfare. They only consider their own interest and forget the outsiders' interests (the unemployed workers). The social planner chooses the values of  $\beta$ ,  $\theta$  and  $T$  that maximize the total population welfare <sup>8</sup> :

$$\begin{aligned} \max_{\beta, \theta, T} rU &= \frac{\beta}{1-\beta} c\theta \\ \text{s.t.} \quad &(1-\beta) \int_0^T \left[ \chi(s) - e^{gs} \left( \frac{\beta}{1-\beta} c\theta \right) \right] e^{-(r+\delta)s} ds = \frac{c}{q(\theta)} \end{aligned}$$

The results from the maximization problem are reported in appendix E. We find that in order to lead the economy to the social efficiency, the social planner must choose the workers bargaining power equal to the elasticity of the matching function ( $\beta = \eta$ ).

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<sup>8</sup>see (Pissarides, 2000)

This restriction is known as the Hosios Condition.

We assume that this restriction is fulfilled<sup>9</sup>. Yet, other distortions such as unemployment benefits and the taxes which finance them are present in the decentralized equilibrium and necessarily entail an inefficient allocation. In order to restore efficiency, the government can implement firing taxes and hiring subsidies.

The efficient values of  $T$  and  $\theta$ , noted  $T^e$  and  $\theta^e$  are the solutions of the following system:

$$\chi(T^e)e^{-gT^e} - \frac{\beta}{1-\beta}c\theta^e = 0 \quad (1.26)$$

$$(1-\beta) \int_0^{T^e} \left[ \chi(s) - e^{gs} \frac{\beta}{1-\beta} c\theta^e \right] e^{-(r+\delta)s} ds = \frac{c}{q(\theta^e)} \quad (1.27)$$

### 1.4.2 Optimal policy

We suppose that  $b > 0$ ,  $P_J > 0$ , and  $P_W > 0$  so that the value of a filled job is necessarily lower at the decentralized equilibrium than at the efficient equilibrium. These distortions in the labor costs induce a shortage of job creation and decrease the labor market tightness so that  $\theta^o < \theta^e$ . The social planner can implement a hiring subsidy policy to promote job creation and restore the efficient value of labor market tightness ( $\theta^o = \theta^e$ ). Yet the reservation wage of a worker is now necessarily higher at the decentralized equilibrium than at the social optimum (higher outside option and same labor market tightness). A higher reservation wage fastens the creative destruction process and therefore:  $T^o < T^e$ . To counteract this behavior, it is possible to implement a firing tax policy which can lengthen job tenure. By combining (JD) and

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<sup>9</sup>Some empirical estimations show that  $\eta$  and  $\beta$  can indeed be rather close



(1.26), we can deduce the firing tax which allows job tenure in decentralized equilibrium to equalize job tenure in the efficient equilibrium. This tax must solve the following differential equation:

$$\frac{1 + P_J}{1 - P_W} b \cdot \chi(T^e) + \dot{F}(T^e) - F(T^e)(r + \delta - g) = 0 \quad (1.28)$$

Solving this equation, we obtain the value of the firing tax as a function of the parameters, of the tenure, and of  $k$  the integration constant:

$$F(t) = \frac{1 + P_J}{1 - P_W} \frac{bs}{r + \delta - g} - \frac{1 + P_J}{1 - P_W} \frac{b(s - x)}{r + \delta - g + \gamma} e^{-\gamma t} + k e^{(r + \delta - g)t} \quad (1.29)$$

The first element of this tax compensates the distortion induced by the unemployment benefits if the productivity of the worker were maximum. The second represents the part to deduct from the firing tax when productivity is less than  $s$ . The longer the tenure, the higher the unemployment benefits, the larger the distortion. The payroll taxes naturally increase this distortion. As in (Pissarides, 2000), if  $b = 0$ , meaning no unemployment benefit or no leisure (depending on the assumptions, here it is no unemployment benefit), the taxes have no impact on the efficiency of the allocation. To offset this increasing distortion, what represents the efficiency of the tax in terms of tenure,  $F(T^e)(r + \delta - g) - \dot{F}(T^e)$  needs to be increasing at the same rate as the unemployment benefits. This requirement leads to the implementation of increasing firing taxes. The last element of this tax is a financial term which does not affect economic behaviors. This term is derived from the integration constant and will be commented it later.

Given this optimal value for the firing tax, the value for the hiring subsidy insuring

$\theta^o = \theta^e$  and  $T^o = T^e$  solves:

$$(1 - \beta) \int_0^{T^e} \left[ \chi(s) - e^{gs} \left( \frac{1 + P_J}{1 - P_W} b\chi(s) + \frac{\beta}{1 - \beta} \left( (r + \delta - g)F(s) - \dot{F}(s) + c\theta^e \right) \right) \right] e^{-(r+\delta)s} ds - e^{-(r+\delta-g)T^e} F(T^e) + H = \frac{c}{q(\theta^e)} \quad (1.30)$$

By substitution of (1.29) and (1.27) in (1.30), we deduce the following value of the hiring subsidies restoring optimal allocation:

$$H = k + \frac{1 + P_J}{1 - P_W} \frac{bx}{r + \delta - g} \quad (1.31)$$

The hiring subsidy offsets three distortions. The distortion induced by the unemployment benefits and the payroll taxes, one induced by the firing taxes on wages and the distortion induced by the payment of the firing tax.

Let's now discuss the term  $k$  that appears in the expression of the firing tax and of the hiring subsidy. This term expresses the fact that the government can give an amount of money  $k$  at creation date and then ask for it back at destruction date once discounted without changing anything in the firm behavior. It shows that  $H = k$  and  $F = ke^{(r+\delta-g)T}$  is a policy which is neutral for agents' behavior. Note, that in Pissarides (2000), the neutral policy is  $H = F$  because there is no actualization rate, no growth, and every job destruction requires the payment of the firing tax.

We can use the expression of the firing tax and of the hiring subsidy to assess the effect of such policy on the budget balance. The hiring subsidy is distributed among all created jobs. The firing tax is collected each time a firm reaches the efficient destruction date  $T^e$ . The government budget is therefore given by:

$$B = e^{-\delta T^e} F(T^e) - H e^{(r-g)T^e} \quad (1.32)$$

$$B = \frac{1 + P_J}{1 - P_W} \frac{b}{r + \delta - g} \left( e^{-\delta T^e} - e^{(r-g)T^e} \right)$$

$r + \delta - g > 0$  therefore this policy necessarily yields a budget deficit. Note that the neutral policy  $H = k$  and  $F = ke^{(r+\delta-g)T}$  fuels a balanced budget as in (Pissarides, 2000). These values correspond to the case where there is no distortion to offset.

### 1.4.3 The Optimal firing tax properties

There is no savings in this economy since agents are risk neutral. The rate  $r$  must be seen as a psychologic discount rate due to agents' time preference. For this matter, it makes sense to set  $k = 0$ . The complementary policy which offsets the distortion becomes therefore:

$$\begin{aligned} H &= \frac{1 + P_J}{1 - P_W} \frac{bx}{r + \delta - g} \\ F(t) &= \frac{1 + P_J}{1 - P_W} \frac{bs}{r + \delta - g} - \frac{1 + P_J}{1 - P_W} \frac{b(s - x)}{r + \delta - g + \gamma} e^{-\gamma t} \end{aligned} \quad (1.33)$$

The properties of the optimal firing tax depend on the properties of the distortion induced by the unemployment benefits and the payroll tax on the job destruction decision. This distortion is given by:

$$D(t) = \frac{1 + P_J}{1 - P_W} b (s - (s - x)e^{-\gamma t}) \quad (1.34)$$

And it is increasing and concave according to tenure.

**Property 4.** *The optimal firing tax has the same shape as the distortion, i.e it is increasing and concave according to tenure.*

*Proof.*

$$\begin{aligned}\dot{F}(t) &= \gamma \frac{1+P_J}{1-P_W} \frac{b(s-x)}{r+\delta-g+\gamma} e^{-\gamma t} > 0 \\ \dot{F}(t) &= -\gamma^2 \frac{1+P_J}{1-P_W} \frac{b(s-x)}{r+\delta-g+\gamma} e^{-\gamma t} < 0\end{aligned}\tag{1.35}$$

□

**Property 5.** *The value of the optimal firing tax is larger than the value of the distortion (1), yet its growth rate is lower than the growth rate of the distortion (2).*

*Proof.* (1) After simplification by  $b$  and  $\frac{1+P_J}{1-P_W}$  we have  $F(t) > D(t)$  since

$$\frac{1}{r+\delta-g} \left( s - \frac{r+\delta-g}{r+\delta-g+\gamma} (s-x)e^{-\gamma t} \right) > s - (s-x)e^{-\gamma t}\tag{1.36}$$

when  $r+\delta-g < 1$  and  $\gamma > 0$ .

(2) According to the following growth rates,

$$\begin{aligned}\frac{\dot{D}(t)}{D(t)} &= \frac{\gamma(s-x)e^{-\gamma t}}{s-(s-x)e^{-\gamma t}} \\ \frac{\dot{F}(t)}{F(t)} &= \frac{\gamma(s-x)e^{-\gamma t}}{s \frac{r+\delta-g+\gamma}{r+\delta-g} - (s-x)e^{-\gamma t}}\end{aligned}\tag{1.37}$$

As  $\gamma > 0$  we can easily deduce that  $\frac{\dot{F}(t)}{F(t)} < \frac{\dot{D}(t)}{D(t)}$ . □

The firing tax is larger than the distortion for two reasons:  $r+\delta-g < 1$  and  $\gamma > 0$ . The first reason refers to the very mechanism of the firing tax. The firing tax achieves to increase tenure if the saving from delaying the payment of the tax compensates the payment of the distortion. For this matter, the value of the tax is necessarily higher than the value of the distortion. Yet if the value of the firing tax naturally depends on the value of the distortion, it also depend on the path of it. That is what the second reason suggests. Indeed, when the distortion is increasing, firms are even more incited

to put an end to the match soon because they anticipate that their situation will become worse, therefore the sharper the distortion increases, the higher the level of the tax must be. However, as the distortion increases at a decreasing rate, this effect slows down with tenure and has for consequence that the tax growth rate remains below the distortion growth rate.

**Property 6.** *The growth rate of the optimal tax, whatever the growth rate of the distortion is always lower than the actualization rate which guarantees the positive effect of the tax on job tenure.*

*Proof.* The tax growth rate is maximum in  $t = 0$  according to property 4, and this growth rate is whatever the value of  $\gamma$  lower than  $r + \delta - g$ :

$$\frac{\dot{F}(0)}{F(0)} = \frac{\gamma(s-x)}{s \frac{r+\delta-g+\gamma}{r+\delta-g} - (s-x)} < r + \delta - g \quad (1.38)$$

□

As the more the distortion increases with tenure, the higher is the tax, the growth rate of the tax always remains below the actualization rate.

#### 1.4.4 When unemployment benefits are social minima

What happens when the government only implements a social minimum? The differential function (1.28) becomes:

$$\frac{1+P_J}{1-P_W} B + \dot{F}(T^e) - F(T^e)(r + \delta - g) = 0 \quad (1.39)$$

with  $B$  the value of the social minimum. Solving this equation, we obtain the value of the optimal firing tax as a function of the parameters:

$$F(t) = \frac{1 + P_J}{1 - P_W} \frac{B}{r + \delta - g} \quad (1.40)$$

**Property 7.** *When the government implements constant unemployment benefits or a social minimum, the optimal firing tax is constant.*

The choice the government makes to insure the unemployed workers has significant consequences on the way it must protect employment. Flat unemployment benefits fuel indeed a flat optimal firing tax.

## 1.5 Conclusion

In this paper, we show that the firing taxes must follow the path of the distortion they are meant to offset. In a model when growth leads to a creative destruction process, we show that increasing firing taxes may have an ambiguous effect on job tenure because firms may not be incited to pay them as late as possible. Notably, if the firing taxes grow faster than the discounting rate, they can indeed shorten job tenure. When it comes to restore the first best allocation, the firing taxes must follow the path of the distortion they offset. In economies with only social minimum allocation, constant firing taxes are enough to restore efficiency. On the opposite, in economies with wage-dependant unemployment benefits, firing taxes need to be increasing since the distortion to correct increases with job tenure. Besides, when the distortion is increasing, firms are even more incited to put an end to the match soon because they anticipate that their situation will worsen, therefore the level of the tax must be higher in that case.

Eventually, the complementary policy of firing taxes and hiring subsidies allows the social planner to restore the optimal allocation, yet this policy yields a deficit.

There are some possible extensions of this paper. Indeed it could be interesting to introduce unemployed worker heterogeneity according to the seniority of their previous job. This would allow to take into account the effect of lengthening job tenure on frictional unemployment. Moreover, this paper does not consider capitalization effect due to technological progress. At empirical level, we observe that only the high skilled jobs are renovated. The analysis proposed in this paper is then well suited for the unskilled labor market jobs. Giving the possibility for firms to renovate their technology when it becomes obsolete as in (Mortensen, 1998) could give firing taxes a new role: When the technology of a firm becomes obsolete, the firm could arbitrate between paying the renovation cost and keeping the worker or, laying the worker off and paying the firing taxes. In an economy where renovation is possible, there exists a level of firing tax which prevents job destructions induced by the creative destruction process.

## Chapter 2

# The Wage Game over the Life Cycle

### 2.1 Introduction

Since the seminal (Burdett and Mortensen, 1998) paper, it is usual to use a wage posting game in a labor market with frictions for explaining the observed wage distribution (see Mortensen03). In this model, firms compete to capture a fraction of the workforce: they have the monopsony power to post wages. Worker can be unemployed or employed but search in each state the best wage offer. Since workers can change jobs to improve their wages, the power of firms is reduced, exerting upward pressure on wages. At equilibrium, the Diamond [1971] paradox is solved: there exists a wage dispersion in the search model explained by the job-to-job mobilities. The level of the wage can result from chance (a initial draw of a wage offer for a unemployed worker higher than the reservation wage) or to job-to-job mobilities (to move up in the wage distribution). This story ensures an increasing wage profile at the steady state of an economy where



nobody in the population has an age, an experience or a tenure duration.

In this paper, we propose to show that the introduction of the worker age can change our understanding of the labor market equilibrium with wage posting game. Thus, we extend the (Burdett and Mortensen, 1998) model by introducing an age-heterogeneity among the population of the workers. And, perhaps more crucially, we introduce a retirement age: young workers have a long horizon, whereas older workers have a short horizon. There exists a large difference between the beginning of the life-cycle of an agent and its end. At the beginning of the life cycle, the agent enters in the labor market as an unemployed worker. In addition, the youngest agents have not had the time to largely improve their careers. At the opposite, at the end of the life cycle, a large majority of workers are integrated in the firms and their experiences have given them the opportunities to find the better wage offers. This clearly suggests that the monopsony power of firms depends on the age of workers. This shift in the firms' monopsony power with the workers' age can explain a part of the age-increasing wage dynamics, without any learning-by-doing mechanism. This backward dynamic must be combined with the forward looking behaviors of agents: the incentives to retain older workers are low because firms expect that seniors will soon retire. At the opposite, on the young's labor market, the incentives for labor hoarding strategy are large. Thus, the forward looking behaviors lead to compress the wage distribution of older workers, whereas they amplify the wage dispersion on the young labor segment.

To analyze analytically the impact of the age on the equilibrium wage distributions, we propose to develop a model with a stylized life cycle: aging is stochastic and there are only two age classes, namely the youth and the seniors. We assume that the labor market is segmented, ie. the search is directed. This assumption is supported by the fact that firm may require or not a minimum of experience when it posts a vacancy.

Thus, even if age discrimination is prohibited, the young can be excluded from the labor market of the older (they have not the minimal experience), whereas the older are physically inapt to insure a task ask to a young worker.

In the benchmark scenario, we assume that the search activity of the younger and the older worker are the same. In this context, we show that young people are more unemployed. Older workers inherit from the careers of the young: thus older workers are less unemployed and part of the initial conditions just comes from the wage distribution of the young. The time spent on the labor market allows therefore workers to be more frequently integrated within a firm and to have selected themselves into better paid jobs. The model shows that thanks to their better initial condition, the seniors are more able to reduce the monopsony power of the firms. Yet, we show that the value of a filled post is greater when it is occupied by a young than by a senior worker. Indeed the higher monopsony power of the firms on the youth's market and the longer duration of jobs occupied by young workers as they are further from retirement raises the expected profit of firms employing a young. Eventually, these "qualities" of young workers do not benefit them in the wage posting game. Indeed, we show that wage offered to them are further down in the wage scale than those available to older workers. Therefore, the initial conditions giving more power to firms on the youth's labor market are crucial in the wage game.

What's going on when the profit on the seniors' market is too low to attract firms on this market? To answer this question, we build a model with 3 ages, the young, the adult and the seniors, which is closer to the main evolutions of the labor market, and in which no wage offers are directed to the seniors. This assumption is also consistent with the view supported by recent empirical and theoretical investigation showing that the short horizon of the older workers leads them to have little incen-

tives to invest in the search process ((Ljungqvist and Sargent, 1998), (Ljungqvist and Sargent, 2008), (Hairault, Langot, and Sopraseuth, 2010), (Chéron, Hairault, and Langot, 2011), (Chéron, Langot, and Hairault, Forthcoming) and (Menzio, Telyukova, and Visschers, 2012)). In this context, the unemployment stops decreasing all along the workers' life-cycle: the adults' unemployment becomes lower than that of senior workers because of the absence of unemployment-employment transitions on the seniors' market. This higher unemployment rate does not affect the wage of the seniors since their labor market is not active, yet the absence of ascendant job to job mobilities prevent seniors from increasing their wage. The wage distribution of the seniors becomes the mere report of the wage distribution of the adult workers.

Thus, in this basic case, the wages increase during the two first life period, and remain stable after. Nevertheless, in countries like France or Belgium the average wage of the older worker is the highest. How to explain this fact in this model? It could be possible if we introduce specific labor market institutions. Indeed seniors also accumulate over their life-cycle rights to certain institutions such as unemployment benefits. If we assume that senior workers are entitled to higher unemployment benefits than other workers, even when they are passive on the labor market, and therefore do not increase their wages through ascendant job-to-job mobility, they keep on being employed by the highest paying firm. Their high unemployment benefits perform indeed a sort: only the best paid workers remain employed at the end of the life-cycle. The source of the wage increase of workers can be therefore different from a stage of the life-cycle to another. When seniors' unemployment benefits are high enough, the distribution of the wage offers cuts in half and becomes discontinuous: the left part of this distribution is composed by low paid jobs which will not survive until the end of the life-cycle and the

right part by the high paid jobs which can survive until the workers' retirement. The increase of wage of the seniors induced by unemployment benefits naturally occurs at the price of an other increase in unemployment. Unemployment benefits therefore allows the coexistence on the seniors' market of a high unemployment rate and high wages.

Recently, (Bagger, Fontaine, Postel-Vinay, and Robin, 2012) have proposed an extended version of the (Postel-Vinay and Robin, 2002) model with a exogenous process of learning-by-doing but without age. The estimation of their model on Danish data shows that human capital accumulation is the most important source of wage growth in early phases of workers' careers, but is dominated by the search process after this first stage in the labor market<sup>1</sup>. Trivially, if we introduce this learning-by-doing mechanism in our model, we reinforce our initial results. This is done by (Menzio, Telyukova, and Visschers, 2012). These authors propose to investigate quantitatively the impact per age of the search process relatively to the human capital accumulation. Life cycle features (finite horizon) and an exogenous learning-by-doing process are introduced in the (Burdett and Mortensen, 1998) model<sup>2</sup>. By restricting the learning-by-doing to be a log-linear process, they show that search process can explain the observed hump-shaped of the average wages: the short horizon of the older workers do not give any incentives them to search for a better job. The large size of the (Menzio, Telyukova, and Visschers, 2012) model does not allow its authors to explain analytically how the solution of a wage-posting game is age-dependent. This last point is the objective of the present paper.

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<sup>1</sup>This decomposition is robust for all levels of education.

<sup>2</sup>Moreover, (Menzio, Telyukova, and Visschers, 2012) introduce endogenous job finding rates *via* a matching function *à la* (Pissarides, 2000).

The paper is organized as follows. In section 2 we spell out the details of the benchmark model with 2 ages. In section 3 and 4 we present two extensions: the first discuss the impact of the low job finding rates of the older workers in a three-age-model, and the second of the impact of the unemployment benefits on the life-cycle wage distribution.

## 2.2 A model with two ages

### 2.2.1 Labor market setup and main notations

We introduce life-cycle in the (Burdett and Mortensen, 1998) theoretical framework. The life-cycle is for now cut in two working life periods. We assume therefore two age classes, namely the youth and the seniors. The workers change age class with the probability  $p$ . As this probability is the same between each age class, in steady state the mass of workers noted  $m$  of each age class is the same. Workers search for a job while unemployed and employed. Workers and firms have a contact with respectively a firm and a worker at the frequency  $\lambda$ . Firms direct their search between the two segments of the labor force, the youth's market and the seniors' market. The employer can therefore observe whether the worker is a young or a senior, yet, he cannot observe the workers' status or the workers' reservation wage: information is not perfect. The firms post wages on the youth's or on the seniors' market and wait to meet a worker with a reservation wage below its proposal. When workers change age class, the contract is not broken unless the worker's value of keeping the contract obtained in the previous age period becomes lower than the value of being unemployed in his current age period. Firms which target the youth can therefore be exposed to employ

senior workers eventually. Technological progress justifies the evolution of a job from a job occupied by a young to a job occupied by a senior. The requirements to perform a certain task necessarily change over a 20 years period. The cumulative distribution function of wage offered by firms is noted  $F_i(w)$  and the cumulative distribution function of wage earned by employed workers is noted  $G_i(w)$ . Each job can be destroyed due to an exogenous event with the probability  $s$ . Time is continuous. An index of the notation used in this chapter is presented in appendix .1.

### 2.2.2 The workers over the life cycle

#### Workers' value functions

The asset values of being employed at a wage  $w$  are noted  $V_i^e(w)$  and solve in each age class:

$$\begin{aligned} rV_y^e(w) &= w + \lambda \int_w^\infty (V_y^e(x) - V_y^e(w)) dF_y(x) \\ &\quad - s(V_y^e(w) - V_y^u) - p(V_y^e(w) - V_s^e(w)) \end{aligned} \quad (2.1)$$

$$\begin{aligned} rV_s^e(w) &= w + \lambda \int_w^\infty (V_s^e(x) - V_s^e(w)) dF_s(x) \\ &\quad - s(V_s^e(w) - V_s^u) - p(V_s^e(w) - V_r) \end{aligned} \quad (2.2)$$

We denote by  $r$  the actualization rate. The expected reward for being employed at a wage  $w$  is first composed by the wage flow  $w$ . Then if the worker meets a firm offering a wage above  $w$ , he resigns and earns in addition the difference between his current asset value and the value associated to this new wage. With the frequency  $s$ , his job is destroyed and he loses the difference between his current asset value and the asset value of being unemployed noted  $V_i^u$ <sup>3</sup>. Eventually with the probability  $p$ , the

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<sup>3</sup>The asset value of being unemployed will be proved to be always below the asset value of being employed later in this subsection

worker changes age class: if he is young, he becomes seniors, if he is senior, he retires. In this case, he earns or loses the difference between the asset value of being employed at the wage  $w$  of the two age classes. We note  $V_r$  the asset value of being retired. This value does not depend on the wage  $w$ .

As unemployed workers are homogenous within the same age class, the asset values of unemployed workers, noted  $V_i^u$ , are the same for all unemployed workers within an age class. They solve for each age class:

$$rV_y^u = b + \lambda \int_{R_y}^{\infty} (V_y^e(x) - V_y^u) dF_y(x) - p(V_y^u - V_s^u) \quad (2.3)$$

$$rV_s^u = b + \lambda \int_{R_s}^{\infty} (V_s^e(x) - V_s^u) dF_s(x) - p(V_s^u - V_r) \quad (2.4)$$

The expected reward for being unemployed is first composed by the flow of the labor opportunity cost that we note  $b$ . This cost is assumed to be the same for all unemployed workers whatever their age class, it can therefore be composed by an unemployment benefit as long as it is constant over time and equal for all workers. Then, if the worker meets a firm offering a wage above  $R_i$ , his reservation wage, he accepts the offer and earns in addition the difference between his current asset value and the value associated to being employed at this new wage. Eventually, as employed workers, with the probability  $p$ , the worker changes age class, and earns or loses the difference between the asset value of unemployed workers of the two age classes.

**Property 8.** *The reservation wage of all unemployed workers is equal to the labor opportunity cost:  $R_y = R_s = b$ .*

*Proof.* The reservation wages  $R_y$  and  $R_s$  of unemployed workers solve:

$$V_y^u = V_y^e(R_y)$$

$$V_s^u = V_s^e(R_s)$$

Using equation 2.2 and 2.4 , we deduce that seniors' reservation wage solves:

$$b + \lambda \int_{R_s}^{\infty} (V_s^e(x) - V_s^u) dF_s(x) - p(V_s^u - V_R) = \\ R_s + \lambda \int_{R_s}^{\infty} (V_s^e(x) - V_s^e(R_s)) dF_s(x) - s(V_s^e(R_s) - V_s^u) - p(V_s^e(R_s) - V_R)$$

By using  $V_s^u = V_s^e(R_s)$ , and as  $V_r$  does not depend on the status or on the wage, we can easily show that:  $R_s = b$ .

Using equation 2.1 and 2.3 , we deduce that youth's reservation wage solves:

$$b + \lambda \int_{R_y}^{\bar{w}} (V_y^e(x) - V_y^u) dF_y(x) - p(V_y^u - V_s^u) = \\ R_y + \lambda \int_{R_y}^{\bar{w}} (V_y^e(x) - V_y^e(R_y)) dF_y(x) - s(V_y^e(R_y) - V_y^u) - p(V_y^e(R_y) - V_s^e(R_y))$$

The youth's reservation wage is therefore given by:

$$R_y = b - p(V_s^e(R_y) - V_s^u)$$

- Assume  $R_y < R_s$ , then  $V_s^e(R_y) - V_s^u < 0$ , then  $R_y > b$ . This is impossible since  $R_s = b$ .
- Assume  $R_y > R_s$ : then  $V_s^e(R_y) - V_s^u > 0$ , then  $R_y < b$ . This is impossible since  $R_s = b$ .

Therefore,  $R_y = R_s = b$ . □

Usually, in all search models, the reservation wage depends on the contact frequencies of workers according to their status. Here we obtain  $R_y = R_s = b$  because we assumed the contact frequencies were similar whatever the workers' status on the labor market. Besides, note that we also assume that the probability to retire does not depend on the workers' status, whereas in reality unemployed workers retire earlier than employed workers ((Hairault, Langot, and Zylberberg, 2012)).



**Workers' flows**

The mass of unemployed workers of each age class is noted  $u_i$ . In steady state, the flows into and out of unemployment in each age class are equal. They are given by:

$$(\lambda + p)u_y = (m - u_y)s + pm \quad (2.5)$$

$$(\lambda + p)u_s = (m - u_s)s + pu_y \quad (2.6)$$

The left side of these flows equations represents the flows out of unemployment in each age class. They are composed by the mass of workers who find a job, and the mass of unemployed workers who change age class. Unemployed workers find a job at the frequency  $\lambda$ . Indeed given property 8, the lowest wage offered by the firms is necessarily equal or above  $b$ , since no firm has interest to offer a wage that nobody can accept, therefore there is no job rejection from unemployed workers. The right side of these flows equations represents the flow in unemployment in each age class. They are composed by the mass of workers who arrive as unemployed in the age class and the mass of employed workers who become unemployed after a lay off. The mass of workers who arrive as unemployed depends on the initial condition of each age class. Young workers all start as unemployed workers ( $pm$ ). On the opposite, only workers who were unemployed as young workers become senior unemployed workers ( $pu_y$ ). The unemployment span is the same whatever the age of the workers (left side), yet the occurrences decrease over life-cycle because with the time spent on the labor market, workers find jobs and progressively integrate within firms. Due to the mobilities of workers over their working life, the unemployment situation of the two age classes is not symmetric.

The mass of workers receiving a wage no greater than  $w$  is given for each age by

$(m - u_i)G_i(w)$ . In steady state, the flows into and out of firms offering a wage no greater than  $w$  for each age class are equal. They are given by:

$$(s + p + \lambda(1 - F_y(w)))(m - u_y)G_y(w) = \lambda F_y(w)u_y \quad (2.7)$$

$$(s + p + \lambda(1 - F_s(w)))(m - u_s)G_s(w) = \lambda F_s(w)u_s + p(m - u_y)G_y(w) \quad (2.8)$$

On the left side of these equations, there is the flow of workers out of firms offering a wage no greater than  $w$ . These workers either experience an exogenous shock with the frequency  $s$ , change age class with the probability  $p$ , i.e. from young to senior or from senior to retired, or resign to be employed by a higher paying job with the frequency  $\lambda(1 - F_i(w))$ . On the right side there is the flow of workers into firms offering a wage no greater than  $w$ . All unemployed workers who have a contact with a firm proposing a wage no greater than  $w$ ,  $F(w)$ , become employed by this firm, since they never refuse jobs. The term  $p(m - u_y)G_y(w)$  of the left side of equation 2.8 means that part of the workers are already employed when they become seniors. It is not the case for the youth who all start as unemployed. The wage distribution of seniors partly depends on the report of the wage distribution of the youth.

### 2.2.3 The firms' expected profit and its maximization

Firms maximize their expected profit by Bertrand competing. Each firm searches the level of wage which maximizes its expected profit given the wage proposed by the other firms and the distribution of workers' reservation wage on each market.

#### Firms' expected profit

In this economy, the expected profit is different on each market. It is given by:

$$\Pi_y(w) = h_y(w)J_y(w) \quad (2.9)$$

$$\Pi_s(w) = h_s(w)J_s(w) \quad (2.10)$$

The term  $h_i(w)$  represents the firms' hiring frequency, that is the frequency with which firms meet a worker who accepts its wage offer on a given market. This frequency naturally depends on the wage proposed. The term  $J_i(w)$  represents the firms' surplus when the firm employs a worker of a given age class, that is the expected flows of income generated by employing worker given its age. The firms' surplus depends on the firms' instantaneous profit and on the expected job duration. This surplus depends on wage since wage affects these both values.

The firms' hiring frequency on each market is given by:

$$h_y(w) = \lambda l_y(w) \quad (2.11)$$

$$h_s(w) = \lambda l_s(w) \quad (2.12)$$

With  $l_i(w)$  the labor supply of a firm when it offers the wage  $w$ .

**Definition 1.** *The labor supply in  $w$  is the mass of workers ready to accept the wage  $w$ . In each age class, it is given by:*

$$l_y(w) = u_y + (m - u_y)G_y(w)$$

$$l_s(w) = u_s + (m - u_s)G_s(w)$$

In each age class, the labor supply is composed of the mass of unemployed workers, who accept any wage offer, and of the mass of employed workers receiving a wage no greater than  $w$ , the wage offered by the firms. Indeed the employed workers only accept wage offer above their current wage.

**Property 9.** *The hiring frequency on both markets raises with the wage offered by the firms.*

*Proof.* Given the definition of a cumulative wage distribution, we can deduce that:

$$h'_y(w) = \lambda(m - u_y)G'_y(w) > 0$$

$$h'_s(w) = \lambda(m - u_s)G'_s(w) > 0$$

□

Offering a wage at the bottom of the distribution allows to hire only the unemployed workers. Indeed given that by definition of the cumulative functions  $G_i(w)$ ,  $G_i(b) = 0$ , the labor supply that faces the firms offering  $b$  is only composed of the mass of unemployed workers:

$$l_y(b) = u_y$$

$$l_s(b) = u_s$$

Whereas, offering higher a wage allows to hire also a part of the employed workers. Notably, at the highest wage in the economy, given that by definition of the cumulative functions  $G_i(w)$ ,  $G_i(\infty) = 1$ , the two labor supplies are equal and given by:

$$l_y(\infty) = m$$

$$l_s(\infty) = m$$

Offering the highest wage allows to hire the entire labor force of the age class.

Given equations 2.5 and 2.6, we can deduce the mass of unemployed workers on

each market. They are given by:

$$u_y = \frac{(s+p)m}{p+s+\lambda} \quad (2.13)$$

$$u_s = \frac{(s+p\frac{u_y}{m})m}{p+s+\lambda} \quad (2.14)$$

Given equations 2.7 and 2.8, we can deduce the mass of workers receiving a wage no greater than  $w$ . They are given by:

$$(m - u_y)G_y(w) = \frac{\lambda F_y(w)u_y}{p+s+\lambda(1-F_y(w))} \quad (2.15)$$

$$(m - u_s)G_s(w) = \frac{\lambda}{p+s+\lambda(1-F_s(w))} \left( F_s(w)u_s + \frac{pF_y(w)u_y}{p+s+\lambda(1-F_y(w))} \right) \quad (2.16)$$

Then, using the expression of the hiring frequency given by 2.11 and 2.12 and the definition 1, we can deduce its value on each market. It is given by:

$$h_y(w) = \lambda m \left( \frac{p+s}{p+s+\lambda(1-F_y(w))} \right) \quad (2.17)$$

$$h_s(w) = \frac{\lambda m}{p+s+\lambda(1-F_s(w))} \left( s + \frac{p(p+s)}{p+s+\lambda(1-F_y(w))} \right) \quad (2.18)$$

The calculation details of these frequencies are given in .2 page 189.

From now on, we assume that the actualization rate tends to zero. The total expected surplus from employing respectively a senior and a young worker is given by:

$$J_s(w) = \frac{y-w}{p+s+\lambda(1-F_s(w))} \quad (2.19)$$

$$J_y(w) = \frac{y-w+pJ_s(w)}{p+s+\lambda(1-F_y(w))}$$

Using equation 2.21, we can rewrite the value  $J_y$  as it follows:

$$J_y(w) = \frac{y-w}{p+s+\lambda(1-F_y(w))} \left( 1 + \frac{p}{p+s+\lambda(1-F_s(w))} \right) \quad (2.20)$$

The firms therefore get surplus  $y-w$  during the job duration expected on the market. For the sake of simplicity, we assume that the actualization rate tends to zero.

**Definition 2.** *The expected job duration is different whether the firm employs a young or a senior. It is respectively given for a senior and a young employee by:*

$$D_s(w) = \frac{1}{p + s + \lambda(1 - F_s(w))}$$

$$D_y(w) = \frac{1}{p + s + \lambda(1 - F_y(w))} + \frac{p}{p + s + \lambda(1 - F_y(w))} \frac{1}{p + s + \lambda(1 - F_s(w))}$$

The job duration of both age classes depends on the exogenous job destruction  $s$ , the aging probability  $p$ , and on the frequency at which the worker finds a better offer  $\lambda(1 - F_i(w))$ , which depends on the wage paid to the worker  $w$ . For firms employing a young worker, this job duration also depends on the probability  $\frac{p}{p+s+\lambda(1-F_y(w))}$  which represents the probability for which a young worker becomes senior within his firm. With this probability the firm keep on receiving the surplus  $y - w$  during the expected job duration of a senior. By affecting the job duration, the horizon affects the surplus that firms can expect from hiring a worker. The total expected surplus therefore differs with the workers' age.

**Property 10.** *The job duration on both markets raises with the wage offered by the firms.*

*Proof.* Given the definition of the job duration given by , it is straightforward that:

$$D'_s(w) > 0$$

$$D'_y(w) > 0$$

□

Finally, using equations 2.9 and 2.10, 2.17 and 2.18, and 2.20 and 2.21, we can deduce the value of the expected profit that firms maximize on each market. These profits are given by:

$$\Pi_y(w) = \lambda m \frac{y - w}{(p + s + \lambda(1 - F_y(w)))^2} \left( s + p + \frac{p(p + s)}{p + s + \lambda(1 - F_s(w))} \right) \quad (2.21)$$

$$\Pi_s(w) = \lambda m \frac{y - w}{(p + s + \lambda(1 - F_s(w)))^2} \left( s + \frac{p(p + s)}{p + s + \lambda(1 - F_y(w))} \right) \quad (2.22)$$

The calculation details are presented in appendix page 189.

### **Profit maximization and equiprofit**

In this subsection, we study how firms in this wage posting game choose the level of the wage they offer. To better understand this game, we can assume firms enter successively on a given market. When there is only one firm on the market, its maximum instantaneous profit would be obtained when it posts the lowest wage of the market. When the other firms enter the market, the intuition is that one firm would have necessarily interest to offer a wage slightly superior to the other to be able to poach all the employed workers. Eventually, (Burdett and Mortensen, 1998) show that at equilibrium, the result of this wage game is a distribution of wage and all the firms reach the same profit. Indeed, when firms increase their offer, their surplus decreases, yet as  $F_i(w)$  increases, so their hiring frequency and expected job duration. As  $F_i(w)$  cannot be superior to 1, there exists in each market a  $\bar{w}_i$  above which firms have no interest to post wages. Firms therefore spread their wage offers along a wage interval. This maximum wage offered by these firms is computed in order to insure the equiprofit with the firms offering the lowest wage of each market, that we note  $\underline{w}_i$ . The wage  $\bar{w}_i$

therefore solves on each market:

$$\begin{aligned}\underline{\Pi}_y(\underline{w}_y) &= \underline{\Pi}_y(\overline{w}_y) \\ \underline{\Pi}_s(\underline{w}_s) &= \underline{\Pi}_y(\overline{w}_s)\end{aligned}\tag{2.23}$$

As the profit is different and can evolve differently from one market to an other with wage, it is likely that the maximum wage would be different on the two markets.

In equation 2.23, note that we specify  $\underline{w}_y$  and  $\underline{w}_s$ . Indeed, the lowest wage offered in the economy can theoretically be different on the two markets and above  $b$ . Without any regulation on the minimum wage, the lowest wage offered by firms on both markets is the wage which maximizes the profit when  $F_i(w) = 0$ , since it is the lowest wage proposed in the economy. These wages can be computed as it follows:

$$\begin{aligned}\underline{w}_y &= \operatorname{argmax}_w \underline{\Pi}_y(w) \\ \underline{w}_s &= \operatorname{argmax}_w \underline{\Pi}_s(w)\end{aligned}\tag{2.24}$$

with  $\underline{\Pi}_y$  and  $\underline{\Pi}_s$  the profit of firms offering the lowest wage on respectively the youth's market and the seniors' market. These profits are given by:

$$\underline{\Pi}_y(w) = \frac{\lambda m(y-w)}{(p+s+\lambda)^2} \left( p+s + \frac{p(p+s)}{p+s+\lambda(1-F_s(w))} \right)\tag{2.25}$$

$$\underline{\Pi}_s(w) = \frac{\lambda m(y-w)}{(s+p+\lambda)^2} \left( s + \frac{p(p+s)}{p+s+\lambda(1-F_y(w))} \right)\tag{2.26}$$

The profit given by 2.25 is equal to the profit presented in equation 2.21 when  $F_y(w) = 0$  and the profit given by 2.26 to the profit presented in equation 2.22 when  $F_s(w) = 0$ . It is tricky to define the wages  $\underline{w}_y$ , and  $\underline{w}_s$  since they depend for each age on the cumulative function of the other market. On the youth's market, the choice of  $\underline{w}_y$  depends on  $F_s(\underline{w}_y)$  which is not equal to zero unless  $\underline{w}_y < \underline{w}_s$ . The wage offered to seniors affects the expected duration of a job created by a firm on the youth's market. On the seniors' market, the choice of  $\underline{w}_s$  depends on  $F_y(\underline{w}_s)$  which is not equal to zero



unless  $\underline{w}_s < \underline{w}_y$ . The wage offered to the youth affects the reservation wage of senior workers and therefore the hiring frequency of firms on the seniors' market. For the sake of simplicity, we assume from now on, that there exists an institutional minimum wage  $\underline{w}$  such as:  $\underline{w} > \underline{w}_y$  and  $\underline{w} > \underline{w}_s$ . Therefore we can rewrite the equations 2.23 as it follows:

$$\Pi_y(\underline{w}) = \Pi_y(\overline{w}_y)$$

$$\Pi_s(\underline{w}) = \Pi_s(\overline{w}_s)$$

With,

$$\Pi_y(\underline{w}) = \frac{\lambda m(y - \underline{w})}{(p + s + \lambda)^2} \left( p + s + \frac{p(p + s)}{p + s + \lambda} \right) \quad (2.27)$$

$$\Pi_s(\underline{w}) = \frac{\lambda m(y - \underline{w})}{(s + p + \lambda)^2} \left( s + \frac{p(p + s)}{p + s + \lambda} \right) \quad (2.28)$$

Eventually, on each age segment, firms spread their wage offer out in order to insure the equiprofit. The distribution of the wages offered by the firms on the youth's market  $F_y$  solves, from  $\underline{w}$  to  $\overline{w}_y$ :

$$\Pi_y(\underline{w}) = \Pi_y(w) \quad (2.29)$$

And the distribution of the wages offered by the firms on the seniors' market  $F_s$  solves, from  $\underline{w}$  to  $\overline{w}_s$ :

$$\Pi_s(\underline{w}) = \Pi_s(w) \quad (2.30)$$

### 2.2.4 Equilibrium wage distribution

From the isoprofit equations 2.29 and 2.30, we can deduce the repartition of the wage offers,  $F_y(w)$  and  $F_s(w)$  as it follows:

$$F_y(w) = \frac{p+s+\lambda}{\lambda} \left( 1 - \left( \frac{y-w}{y-\underline{w}} \left( \frac{s+p + \frac{p(p+s)}{p+s+\lambda(1-F_s(w))}}{s+p + \frac{p(p+s)}{p+s+\lambda}} \right) \right)^{\frac{1}{2}} \right) \quad (2.31)$$

$$F_s(w) = \frac{p+s+\lambda}{\lambda} \left( 1 - \left( \frac{y-w}{y-\underline{w}} \left( \frac{s + \frac{p(p+s)}{p+s+\lambda(1-F_y(w))}}{s + \frac{p(p+s)}{p+s+\lambda}} \right) \right)^{\frac{1}{2}} \right) \quad (2.32)$$

To obtain the equilibrium distribution of the wage earned by workers, we only need to replace  $F_y(w)$  and  $F_s(w)$ , in the expressions of  $G_y(w)$  and  $G_s(w)$  (equations 2.15 and 2.16), given by:

$$G_y(w) = \frac{1}{m-u_y} \frac{\lambda F_y(w) u_y}{p+s+\lambda(1-F_y(w))} \quad (2.33)$$

$$G_s(w) = \frac{1}{m-u_s} \frac{\lambda F_s(w) u_s}{p+s+\lambda(1-F_s(w))} + \frac{p}{m-u_s} \frac{\lambda F_y(w) u_y}{(p+s+\lambda(1-F_s(w)))(p+s+\lambda(1-F_y(w)))} \quad (2.34)$$

$$(2.35)$$

The properties of the equilibrium wage distribution depends on the properties of the wage offered distribution. The difference in the firms' behavior in terms of wage setting between the two markets is induced by the difference between the two expressions of the wage offered distribution presented in 2.31 and 2.32. At the bottom line, the only difference between these two profits lies on the extra  $p$  present in the youth's profit equation. We denote by  $x$  this particular value of  $p$ , and set  $x = 0$  in the wage offered distribution expression of firms that target the seniors and  $x = p$  in the wage offered distribution expression of firms that target the youth. We can rewrite the wage offered

distributions of equation 2.31 and 2.32 as it follows:

$$F_y(w) \equiv F(w, x) = \frac{p+s+\lambda}{\lambda} \left( 1 - \left( \frac{y-w}{y-w} \left( \frac{s+x + \frac{p(p+s)}{p+s+\lambda(1-F_s(w))}}{s+x + \frac{p(p+s)}{p+s+\lambda}} \right) \right)^{\frac{1}{2}} \right)$$

$$F_s(w) \equiv F(w, 0) = \frac{p+s+\lambda}{\lambda} \left( 1 - \left( \frac{y-w}{y-w} \left( \frac{s + \frac{p(p+s)}{p+s+\lambda(1-F_y(w))}}{s + \frac{p(p+s)}{p+s+\lambda}} \right) \right)^{\frac{1}{2}} \right)$$

**Property 11.** *At the equilibrium  $F_y$  is always above  $F_s$ .*

*Proof.* We can show easily that  $\frac{\partial F(w,x)}{\partial x} > 0$ . Therefore  $F_y$  is necessarily always above  $F_s$  and  $\bar{w}_y < \bar{w}_s$ .  $\square$

**Corollary.** *The highest wage offered to senior workers is higher than the highest wage offered to young workers.*

The wage offers directed to the young workers are more concentrated at the bottom of the support than the ones directed to the older workers. Besides the wage support is larger on the seniors' market. The wage game favors therefore the senior workers. Given property 11, we can deduce 14.

**Property 12.** *At the equilibrium  $G_y$  is always above  $G_s$*

*Proof.* To compare the relative position of  $G_s(w)$  and  $G_y(w)$ , we write  $G_s(w)$  and  $G_y(w)$  when  $F_y(w) = F_s(w) \equiv F(w)$  and set  $\psi(w) = G_s(w)/G_y(w)$  as it follows:

$$G_y(w) = \frac{1}{m - u_y} \frac{\lambda F(w) u_y}{p + s + \lambda(1 - F(w))}$$

$$G_s(w) = \frac{1}{m - u_s} \frac{\lambda F(w) u_s}{p + s + \lambda(1 - F(w))} + \frac{p}{m - u_s} \frac{\lambda F(w) u_y}{(p + s + \lambda(1 - F(w)))^2}$$

$$G_s(w) = \psi(w) G_y(w)$$

$$\text{with } \psi(w) = \left[ \frac{u_s}{u_y} + \frac{p}{p + s + \lambda(1 - F(w))} \right] \left( \frac{m - u_y}{m - u_s} \right)$$

Studying the sign and the variation of  $\psi(w)$  allows us to define the relative position of the two distributions. It is straightforward that  $\psi(w) > 0$ . Besides,

1.  $\psi(\infty) = 1$ . Proof in appendix .3.1
2.  $\psi'(w) > 0$ . Proof in appendix .3.2

Consequently, when  $F_y(w) = F_s(w)$ ,  $\psi(w) \in [0, 1]$ , and therefore  $G_s(w) < G_y(w)$ , when  $F_y(w) = F_s(w)$ . Given property 11, it is straightforward that at the equilibrium  $G_s(w) < G_y(w)$ ,  $\forall w > 0$ .  $\square$

Thanks to the search activity (job to job mobility) over the life cycle, when workers face the same wage offers lottery, seniors select themselves among the best paying jobs. At equilibrium, when seniors are offered higher wages, their wage are necessarily more concentrated at the top of the wage distribution than the youth's. Without any learning by doing process, this model allows to account for the increase in wage over the life-cycle. Figure 2.1 illustrates the life cycle evolution of both the workers' market power (effect of  $F_i$ ) and the result of the search activity (gap between  $F_i$  and  $G_i$ ), for reasonable values of the parameters <sup>4</sup>. By comparing  $F_y$  and  $F_s$ , we observe that there is a shift in labor market power over the two periods. Yet it is slight. The shift in workers' wage distribution is larger: the search effect widens the gap between the young's and the seniors' wages<sup>5</sup>.

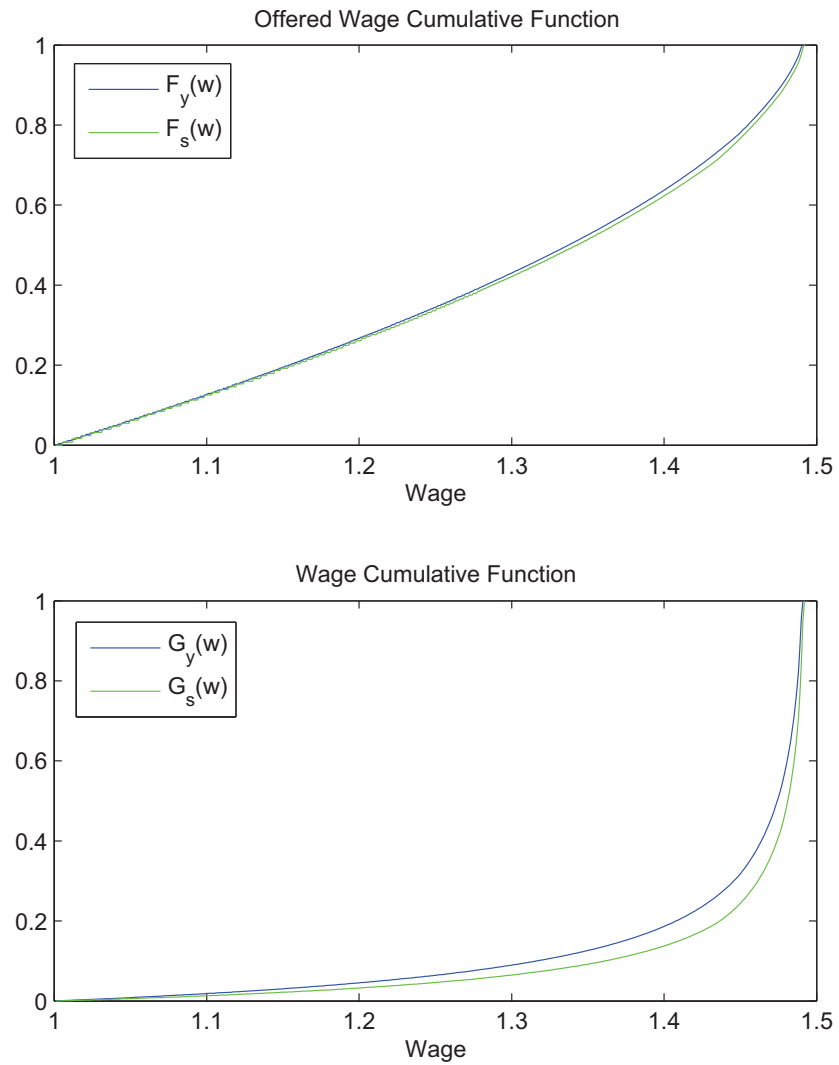
This simple model shows that the wage game evolves over the life-cycle in the favor of seniors. The position of seniors on the labor market allows them to reduce more strongly the firms' monopsony power. The subsection 2.2.5 explains which characteristics of the two labor supplies account for this result.

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<sup>4</sup>In this simulation, we set:  $p = 0.05$ ,  $s = 0.1$ ,  $y = 1.5$ ,  $\lambda = 1$ , and  $\underline{w} = 1$

<sup>5</sup>These results are robust to change in the parameters values.

Figure 2.1: Wage and offered wage cumulative functions for young and seniors



### 2.2.5 Results analysis

At equilibrium, the wage game favors the senior workers. To understand the mechanisms behind this result, we study the behavior of firms in front of the different characteristics of the two labor markets. The two labor markets differ from a backward aspect: senior workers have spent more time on the labor market, and from a forward aspect: the young are much further from retirement. In the subsection 2.2.5, we assess the effect of the difference in the horizon of workers of the two age classes on the level of wage offers, and in the subsection 2.2.5, the effect of the difference in experience on the labor market.

#### The effect of horizon

The difference of horizon between the workers of the two classes affects the expected job duration. To study the impact of this difference on the wage game, we assume in this subsection that the hiring frequency is exogenous and equal for both age classes. In this case, the profit is given on each market by  $hJ_y(w)$  and  $hJ_s(w)$ .

The conditions of equiprofit given for young and senior workers are given by:

$$hJ_y(\underline{w}) = hJ_y(w)$$

$$hJ_s(\underline{w}) = hJ_s(w)$$

From these conditions, we can deduce the wage offered distribution on the youth's and the seniors' market under the assumption of exogenous hiring frequency. They are given by:

$$F_y(w) = 1 - \left( \frac{y-w}{y-\underline{w}} \frac{1 + \frac{p}{p+s+\lambda(1-F_s(w))}}{\frac{1}{p+s+\lambda} + \frac{p}{(p+s+\lambda)^2}} - (p+s) \right) \frac{1}{\lambda} \quad (2.36)$$

$$F_s(w) = 1 - \left( \frac{y-w}{y-\underline{w}} (p+s+\lambda) - (p+s) \right) \frac{1}{\lambda} \quad (2.37)$$

The calculation details are presented in appendix .4.1.

**Property 13.** *When the hiring frequency is equal for workers of both classes,  $F_s$  is always equal or above  $F_y$ .*

*Proof.*

$$\begin{aligned}
 F_s(w) &\leq F_y(w) \\
 \Leftrightarrow p + s + \lambda &\leq \frac{1 + \frac{p}{p+s+\lambda(1-F_s(w))}}{\frac{1}{p+s+\lambda} + \frac{p}{(p+s+\lambda)^2}} \\
 \Leftrightarrow \frac{p}{p+s+\lambda} &\leq \frac{p}{p+s+\lambda(1-F_s(w))}
 \end{aligned}$$

□

When the only difference between the two labor markets is the difference in expected job duration, the wage offered to young workers are more concentrated on the top of the distribution than the wage offered to the seniors. The longer horizon of the youth therefore allows them to limit the monopsony power of firms.

Indeed, on the seniors' labor market, the exogenous job separation rate is higher: beside the exogenous job destructions which occur on both markets at the rate  $s$ , there are on the seniors' labor market the separations due to workers' retirement that occurs at the rate  $p$ . The probability for a senior employee to find a better wage offer is therefore lower than for a young employee. A long horizon reduces the frictions on the labor market of the youth and therefore limits the firms' monopsony power.

### **The effect of experience on the labor market**

The difference of experience on the labor market between the workers of the two classes affects the hiring frequency of firms for a given offered wage. To study the impact of this

difference on the wage game, we assume in this subsection that the job duration is exogenous and equal for both age classes. In this case, the profit is given on each market by  $h_y(w)J(w)$  and  $h_s(w)J(w)$ , with  $J(w) = (y - w)D$ . The constant  $D$  represents the exogenous job duration.

The condition of equiprofit are given for young and senior workers by:

$$h_y(\underline{w})J(\underline{w}) = h_y(w)J(w)$$

$$h_s(\underline{w})J(\underline{w}) = h_s(w)J(w)$$

From these conditions, we can deduce the wage offered distribution on the youth's and the seniors' market under the assumption of exogenous job duration. They are given by:

$$F_y(w) = 1 - \left( \frac{y - w}{y - \underline{w}}(p + s + \lambda) - (p + s) \right) \frac{1}{\lambda} \quad (2.38)$$

$$F_s(w) = 1 - \left( \frac{y - w}{y - \underline{w}} \frac{s + \frac{p(p+s)}{p+s+\lambda(1-F_y(w))}}{\frac{s}{p+s+\lambda} + \frac{p(p+s)}{(p+s+\lambda)^2}} - (p + s) \right) \frac{1}{\lambda} \quad (2.39)$$

The calculation details are presented in appendix .4.2.

**Property 14.** *When the job duration is equal for workers of both classes,  $F_y$  is always equal or above  $F_s$ .*

*Proof.*

$$\begin{aligned} F_s(w) &\leq F_y(w) \\ \Leftrightarrow p + s + \lambda &\leq \frac{s + \frac{p(p+s)}{p+s+\lambda(1-F_y(w))}}{\frac{s}{p+s+\lambda} + \frac{p(p+s)}{(p+s+\lambda)^2}} \\ \Leftrightarrow \frac{p(p+s)}{p+s+\lambda} &\leq \frac{p(p+s)}{p+s+\lambda(1-F_y(w))} \end{aligned}$$

□



When the only difference between the two labor markets is the difference of hiring frequency, as at equilibrium of the benchmark model, the wage offered to young workers are more concentrated on the bottom of the distribution than the wage offered to the seniors. The experience of the seniors on the labor supply limits the monopsony power of firms on their market.

The experience of workers affects the firms' hiring frequency thanks to their mobility: unemployment to employment and job to job.

**Definition 3.** *The size of the labor supply in  $\underline{w}$  represents the mass of workers who accept any wage offered by the firms. This mass is called the reserve army of workers.*

**Property 15.** *The reserve army of the youth is larger than the reserve army of the seniors:  $l_y(\underline{w}) > l_s(\underline{w})$ .*

*Proof.* By definition of the cumulative functions  $G_y$  and  $G_s$ , we have  $G_y(\underline{w}) = G_s(\underline{w}) = 0$ . Therefore,  $l_y(\underline{w}) = u_y$  and  $l_s(\underline{w}) = u_s$ . From equations 2.13 and 2.14, we can deduce that as  $u_y < m$  then  $u_y > u_s$ .

Therefore, the reserve army of the youth is larger than the reserve army of the seniors,  $l_y(\underline{w}) > l_s(\underline{w})$ . □

Thanks to unemployment to employment mobilities, seniors have had time to find a job, this gives them a more favorable initial condition and explains the smaller size of the reserve army on their market. The lowest possible wage offered in the economy,  $b$ , allows firms that target the youth to hire a larger fraction of the labor force than the firms that target the seniors. A large reserve army tends therefore to raise the firms' monopsony power.

The repartition of the reservation wages of employed workers are given by the mass of workers receiving each level of wage. This mass is given by  $(m - u_y)g_y(w)$  and

$(m - u_s)g_s(w)$ , with  $g_y$  and  $g_s$ , the derivatives of  $G_y$  and  $G_s$  according to  $w$ .

**Property 16.** When  $F_y(w) = F_s(w)$ ,

if  $\lambda > p + s$ , there exists a level of wage  $\tilde{w}$  such as  $\tilde{w} < \bar{w}$  above which the mass of senior employed workers who have this reservation wage is greater than the mass of young employed workers.

if  $\lambda < p + s$ , the mass of senior employed workers who have this reservation wage is greater than the mass of young workers for each wage.

*Proof.* We assume that the youth and the seniors face the same wage offers lottery:  $F_y(w) = F_s(w) \equiv F(w)$ . After differentiating 23 and 25, we deduce the mass of workers of each age class receiving a given wage, as it follows:

$$\begin{aligned} (m - u_y)g_y(w) &= \frac{\lambda f(w)u_y(p + s + \lambda)}{(p + s + \lambda(1 - F(w)))^2} \\ (m - u_s)g_s(w) &= \frac{\lambda f(w)u_s(p + s + \lambda)}{(p + s + \lambda(1 - F(w)))^2} + \frac{p(m - u_y)g_y(w)}{p + s + \lambda(1 - F(w))} \\ &\quad + \frac{p\lambda f(w)(m - u_y)G_y(w)}{(p + s + \lambda(1 - F(w)))^2} \end{aligned}$$

Using these two expressions, we deduce:

$$\begin{aligned} (m - u_s)g_s(w) &= \phi(w)(m - u_y)g_y(w) \\ \text{with } \phi(w) &= \frac{u_s}{u_y} + \frac{p}{p + s + \lambda(1 - F(w))} + \frac{p\lambda F(w)}{(p + s + \lambda(1 - F(w)))(p + s + \lambda)} \end{aligned}$$

The mass of young workers is higher than the mass of senior workers if and only if  $\phi(w) < 1$ . It is straightforward that  $\phi(w) > 0, \forall w \in [b, \infty]$ . Besides,

1.  $\phi(\underline{w}) < 1 \Leftrightarrow \lambda > p + s$ . Proof in appendix .5.1 page 196
2.  $\phi(\infty) > 1$ . Proof in appendix .5.2 page 196

3.  $\phi'(w) > 0$ . Proof in appendix .5.3 page 197

□

We assume that the workers of both ages face the same wage offers lottery in order to observe the behavior of firms in front of the two different labor supplies, without any a priori on their behavior. The reservation wages that senior workers have acquired over the time they spent on the labor supply limits the monopsony power of firms on their market.

Two aspects linked to the life-cycle explain this situation. First, as shown in the property 4.3, the better initial condition of seniors lead them to be more largely employed:  $m - u_y < m - u_s$ <sup>6</sup>. This increases the mass of the employed workers on the seniors' market for all wages. Then the ascendant wage mobility allows seniors to be employed on average by higher paying jobs. This can explain that under some conditions, the mass of employed workers on the youth's market can be higher for lower wages. In both cases, when they face the same wage lottery, the properties 4.3 and 16 show that the experience the senior workers have acquired on the labor market is translated by higher reservation wages.

A long horizon constitutes a market power for the young, yet the market power generated by a greater number of mobilities has a larger amplitude and allows seniors to be offered higher wages than the young.

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<sup>6</sup>This result can be directly deduced from property 4.3 since the size of the labor force is similar on both market

## 2.3 A three age classes model with no search on the seniors' market

### 2.3.1 Labor market setup

We can deduce from equations 2.27 and 2.28 of the subsection , that the profit of firms that target the seniors is lower than the profit of firms that target the youth. Indeed seniors have in the same time higher reservation wages and shorter a horizon. Therefore, at equilibrium, firms must be fewer on the seniors' market. Besides, recent empirical and theoretical investigation shows that workers' search effort decreases at the end of the life-cycle. The consequence of both the decrease of job opportunities and the decrease of the search effort is a sharp decrease in the mobility rate of workers at the end of their life-cycle. This phenomenon is largely observed in the data. These mechanisms also demonstrated theoretically by (Hairault, Langot, and Sopraseuth, 2010), (Hairault, Langot, and Zylberberg, 2012), and (Menzio, Telyukova, and Visschers, 2012). For the sake of simplicity, in this model, we actually assume that at the end of the life-cycle, workers are totally passive and there is no more mobility at all.

Yet the strong decrease of the mobility rate of workers is particularly observed at the very end of the life-cycle, when the horizon becomes very short. Taking into account the difference of the mobility rate of workers shows the limit of the 2 ages model. In order to build a model closer to the main characteristics of the life cycle data, we therefore propose a model in which there are three age classes, namely the youth still indexed by  $i = y$ , the adults indexed by  $i = a$  and the seniors still indexed by  $i = s$ . In this model, the seniors are passive and do not search for a job.

All the other assumptions presented in section 2.2 remains unchanged.

### 2.3.2 The workers over the life cycle

#### Workers' value functions

The asset values of being employed at a wage  $w$  now solve for each age class:

$$\begin{aligned} rV_y^e(w) &= w + \lambda \int_w^\infty (V_y^e(x) - V_y^e(w)) dF_y(x) - s(V_y^e(w) - V_y^u) - p(V_y^e(w) - V_a^e(w)) \\ rV_a^e(w) &= w + \lambda \int_w^\infty (V_a^e(x) - V_a^e(w)) dF_a(x) - s(V_a^e(w) - V_a^u) - p(V_a^e(w) - V_s^e(w)) \\ rV_s^e(w) &= w - s(V_s^e(w) - V_s^u) - p(V_s^e(w) - V_r) \end{aligned}$$

As senior workers no longer search for a job, the asset value of being employed as a senior no longer includes the possibility to find a better wage offer and resign for it. The asset value of being employed at a wage  $w$  or unemployed for the youth and the adults are similar to those of the section 2.2.

The asset values of unemployed workers now solve for each age:

$$\begin{aligned} rV_y^u &= b + \lambda \int_{R_y}^\infty (V_y^e(x) - V_y^u) dF_y(x) - p(V_y^u - V_a^u) \\ rV_a^u &= b + \lambda \int_{R_a}^\infty (V_a^e(x) - V_a^u) dF_a(x) - p(V_a^u - V_s^u) \\ rV_s^u &= b - p(V_s^u - V_r) \end{aligned}$$

When a senior becomes unemployed, he remains unemployed until the retirement age. Note that we can consider these unemployed workers as retirees as they do not have search activity. In reality, except is their unemployment benefits are higher than their retirement pension, such workers are retired ((Hairault, Langot, and Zylberberg, 2012)). The same ways as in section 2.2, we can easily show that:  $R_y = R_a = R_s = b$

**Workers' flows**

As a consequence of the behaviors described in the asset values analysis. The flows into and out of unemployment for each age class are given by:

$$u_y \lambda + p u_y = (m - u_y) s + p m$$

$$u_a \lambda + p u_a = (m - u_a) s + p u_y$$

$$p u_s = (m - u_s) s + p u_a$$

And, the flows into and out of firms offering a wage no greater than  $w$  for each age are now given by:

$$(s + p + \lambda(1 - F_y(w)))(m - u_y)G_y(w) = \lambda F_y(w)u_y \quad (2.40)$$

$$(s + p + \lambda(1 - F_a(w)))(m - u_a)G_a(w) = \lambda F_a(w)u_a + p(m_y - u_y)G_y(w) \quad (2.41)$$

$$(s + p)(m - u_s)G_s(w) = p(m - u_a)G_a(w) \quad (2.42)$$

The left side of the last equation shows that once employed at a wage no greater than  $w$  senior workers can no longer move to a firm offering a wage greater than  $w$ . Its right side shows that the only way to be employed as a seniors is to be already employed as an adult.

**The Firms'Expected Profit And Its Maximization**

The principle of the wage game described in section 2.2 remains unchanged. As there is no firms targeting the seniors' market, this game only occurs on the youth's and on the adults' market. The profits of firms targeting the youth and the adults have the same form as those of section 2.2 and are given by:

$$\Pi_y(w) = h_y(w)J_y(w) \quad (2.43)$$

$$\Pi_a(w) = h_a(w)J_a(w) \quad (2.44)$$

The workers'flows of the youth and the adults being similar to those of the youth of the seniors of the section 2.2, so are the hiring frequency. The hiring frequencies  $h_y(w)$  and  $h_a(w)$  have the same shape as the equations 2.17 and 2.18 presented in section 2.2:

$$h_y(w) = \lambda m \left( \frac{p+s}{p+s+\lambda(1-F_y(w))} \right) \quad (2.45)$$

$$h_a(w) = \frac{\lambda m}{p+s+\lambda(1-F_a(w))} \left( s + \frac{p(p+s)}{p+s+\lambda(1-F_y(w))} \right) \quad (2.46)$$

Assuming the actualization rate tends to zero, the surpluses the firms get from employing a worker of each age class are given by:

$$J_y(w) = \frac{y-w+pJ_a(w)}{p+s+\lambda(1-F_y(w))} \quad (2.47)$$

$$J_a(w) = \frac{y-w+pJ_s(w)}{p+s+\lambda(1-F_a(w))} \quad (2.48)$$

$$J_s(w) = \frac{y-w}{p+s} \quad (2.49)$$

Using these equations, we can deduce the expected job duration on each market:

$$D_y(w) = \frac{1}{p+s+\lambda(1-F_y(w))} \left( 1 + \left( 1 + \frac{1}{p+s+\lambda(1-F_a(w))} \frac{p}{p+s} \right) \right)$$

$$D_a(w) = \frac{1}{p+s+\lambda(1-F_a(w))} \left( 1 + \frac{p}{p+s} \right)$$

$$D_s(w) = \frac{1}{p+s}$$

As the seniors cannot resign from a job, the expected duration of the jobs is the same for all wages. Indeed, the only causes of separation on the seniors' market are the exogenous (job destruction and retirement).

**Property 17.** *The passivity of the seniors in this economy raises the job duration of workers of all classes.*

*Proof.* Indeed, in an economy in which the seniors' market is active, the job duration of seniors would be  $\frac{1}{p+s+\lambda(1-F_s(w))}$ , when for all wages under  $\bar{w}_a$ ,  $\frac{1}{p+s+\lambda(1-F_a(w))} < \frac{1}{p+s}$ . This increase of the job duration of seniors affects the job duration of workers of all age classes.  $\square$

Using equations 2.45, 2.46, and 2.47, 2.48 and 2.49, we deduce the expected profit on each market:

$$\begin{aligned}\Pi_a(w) &= \frac{\lambda m(y-w)}{(p+s+\lambda(1-F_a(w)))^2} \left( s + \frac{p(p+s)}{p+s+\lambda(1-F_y(w))} \right) \left( 1 + \frac{p}{p+s} \right) \\ \Pi_y(w) &= \frac{\lambda m(p+s)(y-w)}{(p+s+\lambda(1-F_y(w)))^2} \left( 1 + \frac{p}{p+s+\lambda(1-F_a(w))} \right) \left( 1 + \frac{p}{p+s} \right)\end{aligned}$$

The calculation details are presented in appendix .6. To obtain the equiprofit conditions ruling the wage offered distribution of workers on the two markets, we proceed as in section 2.2.

### 2.3.3 Equilibrium wage distribution

Using the equiprofit conditions, we can deduce the wage offered distribution on the youth's and the adults' market. They are given by:

$$\begin{aligned}F_y(w) &= \frac{p+s+\lambda}{\lambda} \\ &\quad \left( 1 - \left( \frac{y-w}{y-b} \left( 1 + \frac{\frac{p}{s+p+\lambda(1-F_a(w))} + \frac{p^2}{(s+p+\lambda(1-F_a(w))(s+p)}}}{1 + \frac{p}{s+p+\lambda} + \frac{p^2}{(s+p+\lambda)(s+p)}} \right) \right) \right)^{1/2} \\ F_a(w) &= \frac{p+s+\lambda}{\lambda} \\ &\quad \left( 1 - \left( \frac{y-w}{y-b} \left( \frac{s + \frac{ps}{s+p} + \frac{p(p+s)}{s+p+\lambda(1-F_y(w))} + \frac{p^2(p+s)}{(s+p+\lambda(1-F_y(w))(s+p)}}}{s + \frac{ps}{s+p} + \frac{p(p+s)}{s+p+\lambda} + \frac{p^2(p+s)}{(s+p+\lambda)(s+p)}} \right) \right) \right)^{1/2}\end{aligned}$$



To obtain the equilibrium distribution of wage earned by workers, we only need to replace  $F_y(w)$  and  $F_a(w)$  in the expression of  $G_y(w)$ ,  $G_a(w)$  and  $G_s(w)$  deduced from 2.40, 2.41 and 2.42:

$$\begin{aligned}
G_y(w) &= \frac{1}{m - u_y} \frac{\lambda F_y(w) u_y}{p + s + \lambda(1 - F_y(w))} \\
G_a(w) &= \frac{1}{m - u_a} \frac{\lambda F_a(w) u_s}{p + s + \lambda(1 - F_a(w))} \\
&\quad + \frac{p}{m - u_a} \frac{\lambda F_y(w) u_y}{(p + s + \lambda(1 - F_a(w)))(p + s + \lambda(1 - F_y(w)))} \\
G_s(w) &= \frac{m - u_a}{m - u_s} \frac{p}{p + s} G_a(w)
\end{aligned}$$

As in the section 2.2, we deduce the propositions 18 and 19.

**Property 18.** *At the equilibrium,  $F_y$  is always above  $F_a$ .*

*Proof.* We use the same demonstration method as in section 2.2. We need to rearrange the expression of the cumulative functions to compare them more easily. The cumulative function of the youth can be rewritten as it follows:

$$\begin{aligned}
F_y(w) &= \frac{p + s + \lambda}{\lambda} \\
&\quad \left( 1 - \left( \frac{y - w}{y - b} \left( \frac{p + s + \frac{p(p+s)}{s+p+\lambda(1-F_a(w))} + \frac{p^2(p+s)}{(s+p+\lambda(1-F_a(w)))(s+p)}}{p + s + \frac{p(p+s)}{s+p+\lambda} + \frac{p^2(p+s)}{(s+p+\lambda)(s+p)}} \right) \right)^{1/2} \right) \\
F_y(w) &= \frac{p + s + \lambda}{\lambda} \\
&\quad \left( 1 - \left( \frac{y - w}{y - b} \left( \frac{\frac{p^2}{p+s} + s + \frac{p(p+s)}{s+p+\lambda(1-F_a(w))} + \frac{ps}{s+p} + \frac{p^2(p+s)}{(s+p+\lambda(1-F_a(w)))(s+p)}}{\frac{p^2}{p+s} + \frac{p(p+s)}{s+p+\lambda} + \frac{ps}{s+p} + \frac{p^2(p+s)}{(s+p+\lambda)(s+p)}} \right) \right)^{1/2} \right) \\
F_y(w) &= \frac{p + s + \lambda}{\lambda} \\
&\quad \left( 1 - \left( \frac{y - w}{y - b} \left( \frac{\mathbf{A} + s + \frac{p(p+s)}{s+p+\lambda(1-F_a(w))} + \frac{ps}{s+p} + \frac{p^2(p+s)}{(s+p+\lambda(1-F_a(w)))(s+p)}}{\mathbf{A} + \frac{p(p+s)}{s+p+\lambda} + \frac{ps}{s+p} + \frac{p^2(p+s)}{(s+p+\lambda)(s+p)}} \right) \right)^{1/2} \right)
\end{aligned}$$

With  $A = \frac{p^2}{p+s}$ . As now the only difference between the expression of  $F_y$  and  $F_a$  is the presence in  $F_y$  of  $A$ . We need to assess how this value affects the distribution function:

$$\frac{\partial F_y(w)}{\partial A} > 0$$

Therefore, for all  $w < \bar{w}_y$ ,  $F_y(w) > F_a(w)$ . The wage offers directed to the youth are more concentrated at the bottom of the support than the ones directed to the older workers. □

**Property 19.** *At the equilibrium,  $G_y$  is above  $G_a$ .*

*Proof.* As  $F_y(w) > F_a(w)$ , to prove  $G_y$  is above  $G_a$ , we can use the exact same demonstration method that for the property 14. □

There are two main effects of the passivity of the seniors on the seniors' labor market given by property 20 and 21.

**Property 20.** *The mass of senior employed workers  $m - u_s$  is smaller than the mass of adult employed workers  $m - u_a$ .*

*Proof.* Using 2.42, we can deduce:

$$u_s = \frac{sm + pu_a}{s + p}$$

As  $\frac{sm + su_a}{s + p} > u_a \Leftrightarrow m > u_a$ , therefore  $u_s > u_a$  □

The passivity of the seniors on the labor market raises their unemployment duration, and therefore their unemployment rate. In the section 2.2, the mass of unemployed workers on a market affects the workers' wage of the market via its effect on the firms

monopsony power. The higher the unemployment, the larger the reserve army of workers and the lower the wage offers. Yet in this economy, as workers cannot be hired as a senior, the high level of unemployment on the seniors' market has no effect on their wage.

**Property 21.** *The wage distribution between adults and seniors remains stationary.*

*Proof.* From equation 2.42, we can deduce:

$$G_s(w) = \frac{p}{p+s} \frac{m-u_a}{m-u_s} G_a(w)$$

Yet we know that,

$$\frac{p}{p+s} \frac{m-u_a}{m-u_s} = 1$$

The calculation details are presented in appendix .7. Therefore,

$$G_s(w) = G_a(w)$$

□

When the older workers are passive on their market, the wage distributions become stationary at the end of the life cycle. The zero-search assumption for the older workers, reduce the opportunities to find a higher paying jobs. Seniors are therefore paid as adult workers, yet they are fewer to be employed at this age.

## 2.4 The impact of unemployment benefits

During their experience on the labor market, workers can also accumulate entitlement to a system of unemployment benefits. This channel can affect the wage of older

workers. In this section, we extend the model presented in section 2.3, when seniors do not search<sup>7</sup>, to account for the unemployment benefits' effect on the wage distribution over the life cycle. In this extension, the unemployed seniors are entitled to larger unemployment benefits than the younger workers (the young and the adults):  $b_y = b_a = b < b_s$ . Besides, we assume that  $b_s > \underline{w}$ , otherwise, this new assumption would have no effect on equilibrium.

#### 2.4.1 Wage distribution of indemnified seniors

As in the section 2.3, the reservation wage of workers is equal to their benefits consequently,  $R_y = b$ ,  $R_a = b$  and  $R_s = b_s$ . As senior workers do not search for a job, in this economy, this reservation wage corresponds to the wage below which adult workers who become seniors resigns from their jobs to remain senior unemployed. The flows in and out unemployment and in and out firms offering a wage no greater than  $w$  remain the same as in equation 2.40 and 2.40, and 2.40 and 2.41 for young and adult workers. Yet the presence of higher unemployment benefits for seniors affects their flows in and out unemployment and in and out firms offering a wage no greater than  $w$ . They are now given by:

$$(s + p)(m - u_s)G_s(w) = p(m - u_a)[G_a(w) - G_y(b_s)] \quad (2.50)$$

$$pu_s = (m - u_s)s + p(m - u_a)G_a(b_s) + pu_a \quad (2.51)$$

where  $G_a(b_s)$  represents the mass of adult workers paid less than  $b_s$ . These individuals choose to quit their jobs when they become seniors because their reservation wage ( $b_s$ ) is larger than the wage obtained when they were adult.

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<sup>7</sup>This assumption allows us to avoid the problem of the separations of the workers paid less than  $b_s$  when they age. Even if these separations exist in our context, they have no impact on the equilibrium because the seniors do not search.

**Property 22.**  $\forall w > b_s$ , we have  $G_s(w) < G_y(w)$  if, at equilibrium,  $\exists w < b_s$  implying  $G_a(b_s) > 0$ .

*Proof.* The equation (2.51) gives

$$\frac{u_s}{m} = \frac{s + pG_a(b_s) + p[1 - G_a(b_s)]\frac{u_a}{m}}{p + s}$$

By integrating this result in the equation (2.50), we obtain

$$G_s(w) = \frac{1 - \frac{u_a}{m}}{1 - \frac{u_a}{m} - G_y(b_s) \left(1 - \frac{u_a}{m}\right)} [G_a(w) - G_a(b_s)]$$

This equation shows that  $\lim_{G_a(b_s) \rightarrow 0} G_s(w) = G_a(w)$ , whereas

$\lim_{G_a(b_s) \rightarrow G_a(w)} G_s(w) = 0$ . Thus  $G_s(w)$  is bounded by  $G_a(w)$ , implying that  $G_s(w) - G_a(w) < 0, \forall w$ .  $\square$

The property 22 show that seniors are less concentrated at the bottom of the wage distribution than adults. This suggests that their wages are higher than the younger workers'.

## 2.4.2 Labor demand

The surpluses of firms matched with an adult or a senior, are respectively:

$$\begin{aligned} \text{if } w < b_s: J_a(w) &= \frac{y - w}{p + s + \lambda(1 - F_a(w))} \\ \text{if } w \geq b_s: J_a(w) &= \frac{y - w}{p + s + \lambda(1 - F_a(w))} + \frac{p}{p + s + \lambda(1 - F_a(w))} J_s(w) \\ J_s(w) &= \frac{y - w}{p + s} \end{aligned}$$

Given the probability to hire an adult, given by the equation 2.46, the firm's expected profit is:

$$\begin{aligned} \text{if } w < b_s: \Pi_a(w) &= \frac{\lambda(s+p)m(y-w)}{(p+s+\lambda(1-F_a(w)))^2} \\ \text{if } w \geq b_s: \Pi_a(w) &= \frac{\lambda m(y-w)}{(p+s+\lambda(1-F_a(w)))^2} (2p+s) \end{aligned}$$

**Property 23.** *If  $b_s + \frac{p+s}{p}(b_s - \underline{w}) > y$ , then the support of the wage distribution is not continuous:  $w \in [\underline{w}; \tilde{w}] \cup [b_s, \bar{w}_a]$ . At the bottom of the wage distribution, some wages will be rejected when the worker ages.*

*Proof.* The expected profit of the firms leads to:

$$\begin{aligned} \text{if } w < b_s: \Pi_a(\underline{w}) &= \frac{\lambda(s+p)m(y-\underline{w})}{(p+s)^2} \\ \text{if } w < b_s: \Pi_a(\tilde{w}) &= \frac{\lambda(s+p)m(y-\tilde{w})}{(p+s+\lambda(1-F_a(b_s)))^2} \\ \text{if } w \geq b_s: \Pi_a(b_s) &= \frac{\lambda m(y-b_s)}{(p+s+\lambda(1-F_a(b_s)))^2} (2p+s) \end{aligned}$$

These last two equations imply:

$$(s+p)(y-\tilde{w}) = (2p+s)(y-b_s) \Rightarrow \tilde{w} = b_s - \frac{p}{p+s}(y-b_s)$$

The first segment of the wage support ( $[\underline{w}; \tilde{w}]$ ) exists if and only if  $\tilde{w} > \underline{w}$  and  $\tilde{w} < b_s$ .

The first restriction is  $b_s(\frac{2p+s}{p}) - \frac{p+s}{p}\underline{w} = b_s + \frac{p+s}{p}(b_s - \underline{w}) > y$ , whereas the second is simply  $\frac{2p+s}{p+s}b_s - \frac{p}{p+s}y < b_s \Leftrightarrow b_s < y$ .  $\square$

Even if seniors do not search, and therefore can not increase their wages via job-to-job transitions, higher unemployment benefits perform a sort: only high paid workers keep their jobs at the end of their life cycle. The labor market institutions may therefore provide a "power" to seniors, even though they are passive. The average wage for the seniors is higher than that for the younger workers. It is then clear that if seniors

search when unemployed or employed, the effect of unemployment benefits reinforces their market power.

## 2.5 Conclusion

In this chapter, we show that the wage game were affected by the workers' age. Two mechanisms occur: an increase in the workers' market power and a positive effect of the search activity. A natural extension of this work is to introduce the heterogeneity of firms' productivity in order to confront this model to real data. That is the main purpose of chapter 3. Besides, in chapter 2, we show that the search intensity of workers could alter the wage distribution notably at the end of the life cycle. In chapter 3, we therefore endogenize this search intensity. The fact that rights accumulation to an unemployment insurance system could raise workers' wage, even when they do not search is another result given by chapter 2. This result echoes the French, and more generally the European labor market in which both search is low and institutions strong. Following this idea, Chapter 4 propose to assess the effect of some major institutions on the wage progression.





## Chapter 3

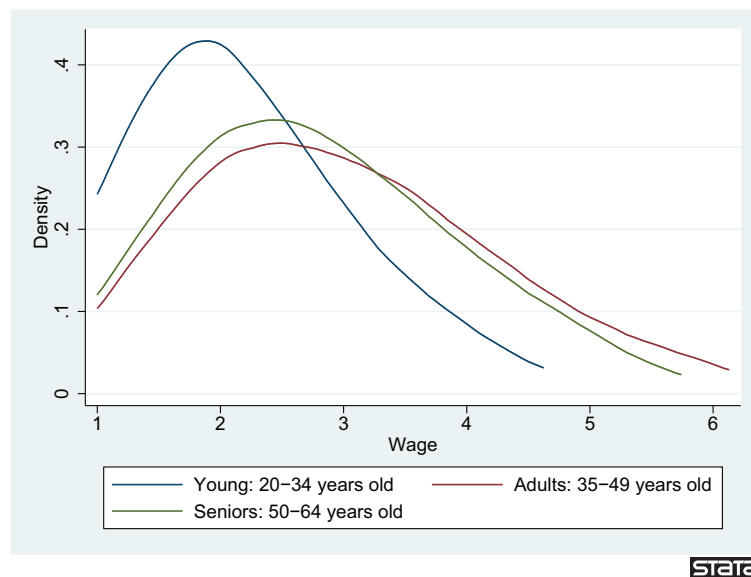
# The U. S. Wage Distribution over the Life Cycle: a Search Model Directed on Age

### 3.1 Introduction

The second chapter of this thesis showed that the non manipulable characteristic of age affects theoretically the workers' wage. The effect of age goes through the selection of good jobs from workers and the accumulation of a market power over their life. Indeed, in the framework developed, the job to job mobilities allow workers to raise their wage over the life-cycle by selecting among the best paid jobs. Age also affects the wage posting game since this selection and the improvement of their initial condition in terms of employment increase globally their reservation wage and therefore their market power: firms tend to offer higher wages to experienced workers. Empirically, it is well known that the distribution of wages also varies significantly with workers' age.

Figure 3.1 displays the U. S. wage distribution for men depending on their age. The mean wage of workers raises by 2% per year in the first half of the working life <sup>1</sup>, and the dispersion coefficient<sup>2</sup> by 0.3%. In the second half of the working life, the wage growth is slower: only 0.4% per year and the dispersion coefficient is almost stationary. The main objective of this third chapter is to construct a structural model able to explain the evolution of the observed wage distribution over workers' life-cycle.

Figure 3.1: U. S. wage distribution of salaried men by age class (First 95%) expressed in US minimum wage



Some changes have to be made to the models developed in the second chapter to do so. Like the model of (Burdett and Mortensen, 1998), these models do not allow

<sup>1</sup>on normalized wage distribution

<sup>2</sup>The dispersion divided by the mean

us to reproduce the data of the observed wage distribution. The model of (Burdett and Mortensen, 1998) is meant to represent the wage game generating the pure wage dispersion, that is the wage dispersion when workers and firms are homogenous. Empirical studies like (Postel-Vinay and Robin, 2002) show that the wage dispersion can be globally decomposed in three main components: the heterogeneity of the firms, the heterogeneity of the workers and a pure wage dispersion. In the second chapter, we show that at equal productivity, workers of different age have a different wage distribution. Yet, the heterogeneity of firms' and workers' productivity is not studied as a source of wage dispersion. In this third chapter we take into account these extra sources of dispersion, and assess the effect of age on wage distribution via its effect on the match productivity. We use for that the wage posting model with friction and on the job search of the second chapter with three age periods and augment it with the heterogeneity of firms' and workers' productivity.

(Bontemps, Robin, and Van-Den-Berg, 1999) and (Mortensen, 1998) shows that the main problems that the (Burdett and Mortensen, 1998) model has in fitting the real data could be solved by adding the heterogeneity of firms' productivity. (Bontemps, Robin, and Van-Den-Berg, 1999) show indeed that an exogenous distribution of firm's productivity allows to generate a single mode wage density consistent with the facts, when the model of (Burdett and Mortensen, 1998) generates a strictly increasing density and too low a dispersion. Yet, quoting (Mortensen, 1998) : "Existing theory neither explains nor restricts the assumed exogenous distribution of employer productivity". In his 1998 paper, Mortensen shows that introducing the endogenous productivity of firms allows to achieve the same objective, with only a very mild restriction on the shape

of the production function<sup>3</sup>. Besides, (Chéron, Hairault, and Langot, 2008) show that the search model with endogenous productivity is not rejected by the data. We choose in this chapter the Mortensen approach of endogenous productivity. In more concrete terms, we assume that firms can now decide of an amount of specific human capital to invest each time they hire a worker. This investment can be associated to a vocational training of the employee at the beginning of the match. This capital is costly and increases the match's productivity during all its tenure. As this investment is costly, the longer the match is expected to be, the more firms invest on it. The investment dimension is therefore not neutral in finite horizon since firms necessarily search to amortize its investment on the long run. Ex ante the firms are homogenous, yet at the equilibrium, the same way firms offer different wages, they also have a different productivity. In this chapter, we refer to this productivity component as the quality of the job. We have seen in chapter 2 that the possibility of on the job search of workers forces firms to compete to get a share of the workforce and to keep it. In this competition game, firms are therefore naturally induced to raise their wage above the monopsony wage. In this chapter firms have now a new lever in the competition game: they can now choose the wage to offer but also the amount of specific human capital they want to invest in the job. Of course, the higher the quality of the job, the bigger the match surplus and therefore the more firms can raise their wage offer and be competitive compared to the other firms. This competition between firms now induces firms to raise their wage yet also to invest in jobs to increase their productivity. The market power of workers becomes the capacity of workers to force firms to raise their wage offers and therefore to create high quality jobs.

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<sup>3</sup>Only a decreasing return of human capital allows to reproduce the hump shape of the wage distribution

On the workers' side, we assume that age affects the workers' productivity by two aspects: first, through their accumulation of human capital, and second through the evolution of the productivity workers have at the workplace (in this chapter, we refer to this two components in these terms). Following the literature on human capital accumulation (Becker, 1964), we assume indeed that aging allows learning by doing: over their lives, workers acquire exogenously corporate knowledge transferable from one job to an other. We yet do not assume the path of this accumulation, the parameter representing it is actually estimated by the model. This accumulation can therefore possibly become negative if the obsolescence of the acquired knowledge is faster than the learning of new one. In our economy, this accumulation of human capital naturally affects negatively the training cost of firms. The level of human capital affects the cost of the specific human capital for firms. The higher the level of human capital of a worker, the more firms are induced to create high quality jobs. The quality of the jobs depends now on the market power of workers and of their level of human capital. Intuitively, age therefore affects this quality in an ambiguous way: on one hand, firms are induced to create higher quality jobs to young workers who have a long working horizon, on the other hand, older workers, who have accumulated more human capital are less costly to train. The productivity of workers at the workplace affects directly the production function of the match. It evolves with the workers' age within a match, or out of a match if the worker is unemployed. The trend of this workers' component of the production function with age is a priori unknown. Indeed, as workers accumulate human capital, we can assume that they are more productive at the workplace, yet, with age their productivity can be affected by health troubles, fatigue etc... The model developed in this chapter allows us to estimate the contribution of these two components to the match productivity and therefore to the wage distribution. Of course they are

correlated with each other.

Age progression over the working life can be now induced by two channels: the wage game channel, which now allows different productivity of firms, and the productivity channel, which represents the increase in wage induced by the accumulation of human capital and the evolution of the workers' productivity at the workplace. Thanks to the introduction of match productivity, it is possible to calibrate the model on empirical data and therefore to assess qualitatively the contribution of each channel to the evolution of the wage distribution with workers' age.

To assess correctly the contribution of these two channels, we need in this chapter to take into account the difference in search intensity of workers of different age classes. Indeed, in chapter 2 we show that when senior workers have no search activity, the wages remain stationary between the adults' and the seniors' period. In the data, age does not affect the mobility rate in such extreme way, it affects the intensity of the search: seniors have indeed fewer opportunities to change jobs yet they still have some. In this chapter, in order to quantify how the mobility rate is affected by the workers' age we assume a matching process in which workers and firms meet at a frequency depending on the number of vacancies on the market and on the number of job searchers. Firms can create freely a vacancy on each market by paying the same vacancy cost. The number of vacancies on each market becomes therefore endogenous and depends on the profit firms can expect from this vacant job. Quantitative results show that due to their shorter horizon, fewer vacancies are created on the seniors' market. By this mechanism, seniors face fewer opportunities on their labor market and the unemployment and job duration increase when workers get closer to their horizon which is consistent with the data. This model does not take into account the search effort of workers. Considering the search effort would go in the same way as the free entry

condition and would reinforce these results ((Hairault, Langot, and Sopraseuth, 2010) and (Chéron, Hairault, and Langot, 2011)).

There is a small yet very recent literature studying the effect of age on the wage distribution. The working paper of (Bagger, Fontaine, Postel-Vinay, and Robin, 2012) explains the wage progression via job-to-job mobilities and human capital accumulation in the theoretical framework developed by (Postel-Vinay and Robin, 2002), yet in a finite horizon. Their decomposition of individual wage growth reveals that human capital accumulation is the most important source of wage growth in early phases of workers' careers, and search-induced wage growth in the second part of the life cycle. (Menzio, Telyukova, and Visschers, 2012) deal with this link in finite horizon and distinguish the same two channels of wage evolution. They study their effects on the job to unemployment, unemployment to job and job to job transitions as well as on the mean wage trajectory over the life-cycle. The authors use a framework very different from the (Burdett and Mortensen, 1998). In their model, all the agents behave efficiently and information is perfect. The labor market is divided in small submarkets in which workers who search for only one wage meet employers who only propose one wage. The job research is perfectly directed on both sides, firms have no monopsony power and the effect of frictions is far weaker than in our framework.

Our model is calibrated on U.S. data and fits rather well the data of the aggregated wage distribution and its evolution with workers' age. In this chapter we therefore show that the framework developed by (Burdett and Mortensen, 1998) augmented with age is able to reproduce the evolution of the wage distribution with age when we take into account the two aspects of the evolution of workers' productivity. We can use the calibrated model to decompose the wage progression with age into the two channels:



the wage game and the productivity channel. These two channels affect the wage progression via two mechanisms in the model: first the evolution of the workers' market power, and second, the search effect. The workers' market power is the capacity of workers to induce firms to post high wages. It can change in function of the age of the worker, via its backward situation like his reservation wage and its forward situation like his horizon. It can also be affected by the productivity of the worker, since the intense competition to hire and retain high productivity workers lead to higher wage posted by the firms. The search effect is the mechanism according which from a given wage offered lottery, workers select into the highest, thanks to job to job transitions.

How does the wage game evolve over the working life? The evolution of the wage game is the pure effect of age on the wage distribution. According to our findings, age affects positively the wage game in the first half of the working life. It accounts therefore for a part of the wage growth in the first half of the working life. This part remains relatively small. Age affects on the contrary negatively the wage game in the second half of the working life. Its evolution generates therefore a wage fall in this period. During this period, both the market power of workers and the search effect decreases. It could seem at first paradoxical that in a context where workers can progressively select into best paid jobs over their life, that wages can decrease at the end of the life cycle: if workers select during all their working life into best paid jobs, we can rightfully think that at the end of their life they are employed in better jobs than at the beginning. Yet it is not the case since, over their life, the wage offer lottery they face also evolves, and in a context where workers are evenly productive, it evolves negatively. When workers are evenly productive, the firms' expected match surplus decreases with workers' age since nothing changes but the progressive decrease of the expected job duration. On the seniors' market, firms are therefore fewer, compete less to retain workers who can

leave exogenously soon, and invest less in specific human capital since it is unlikely that a high investment pays for itself given their short horizon. Firms on this market do not even try to poach employed workers and choose instead low wage strategies which target senior unemployed workers arriving through exogenous job destructions. Facing such wage lottery, the search effect is greatly hampered at the end of the working life: over time workers have increasingly more difficulties to find better opportunities. The effect of search can be totally cut off when the shift in wage offered lottery is unfavorable. Contrary to the results of the chapter 2, we can deduce that when firms can choose their productivity seniors workers get lower wages because they are employed in the lowest productivity jobs.

Calibration of the model reveals the productivity channel parameters. The human capital accumulation of workers allows a constant decrease of the cost of the specific human capital for firms. To achieve the same productivity, a senior worker needs to be less trained than a young. The productivity of workers at the workplace is yet first increasing, then decreasing. This reveal that at the end of the working life, physical condition can alter workers' productivity, yet do not alter the accumulation of human capital. This result is consistent with the observation that senior workers are more valuable in jobs requiring knowledge yet much less in physical jobs<sup>4</sup>. The productivity channel contributes to wage progression in the first half of the working life by its productivity at the workplace component. The increase in this productivity raises the market power of workers since high productive workers are worth competing for (to hire and retain them). The productivity channel accounts fully to the wage progression in the second part of the working life. At these ages, the productivity channel consists

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<sup>4</sup>This fact is justified by a retirement age earlier in physical jobs than in more intellectual ones in France for instance.

mostly in the accumulation of human capital. Thanks to their high level of human capital, firms can now create higher paid jobs since the initial investment is less costly. The human capital accumulation is crucial in the second part of the working life since it must compensate the unfavorable condition of seniors induced by their short horizon. Thanks to this accumulation, some firms can adopt high wage strategies and this allows workers to keep benefiting from the search. As in (Postel-Vinay and Robin, 2002), we find that the workers' productivity accounts for a great part of the wage growth in the first part of the life cycle, yet in our estimation, its contribution to the wage growth in the second part of the life cycle is, although lower, very consequent since without it, wage would decrease. This divergence of results is easily explicable since they do not take into account the finite horizon of workers.

In the second section of this paper we develop our life cycle model and present its main properties in the third section. Section 4 is dedicated to the description of the calibration and of the validation of the model on U.S. data. Section 5 quantifies the contribution of the wage game and productivity channel to the wage growth. Section 7 concludes.

## **3.2 The model**

### **3.2.1 Labor market setup and main notations**

In this chapter, we use the same framework that presented in chapter 2. We choose the three age classes segmentation in order to stay close to the main characteristics of the life-cycle data: the integration to the labor market, the maturity, and the seniority. Between these three age periods, we now allow the workers' productivity to evolve: over their life, workers can first accumulate a transferable human capital, and second see

their productivity at the workplace evolves. The trend of the workers' productivity via these two aspects over the workers' life is a priori unknown and is to be estimated by the model.

Firms can create jobs with different levels of productivity depending on an initial investment which can be seen as a training cost. This investment increases the quality of the job by raising the level of specific (and not transferable) human capital of the worker on the job. The accumulation of general human capital of workers affects the cost of this investment which can therefore vary with the workers' age. Eventually, the match productivity depends now on the workers' age: the workers' productivity at the workplace and the level of general human capital of workers. It is therefore now possible to observe an increase of the match's productivity over the time as the employed worker can become more productive over his life.

Workers search for a job while unemployed and employed. The arrival frequency of wage offers, which results from a matching process, are  $\lambda_i^0$  for the unemployed and  $\lambda_i$  for the employed. On the firms' side, firms have a contact with an unemployed worker at the frequency  $q_i^0$  and with an employed worker at the frequency  $q_i$ . The mobility of workers depends therefore on both their age and status. As the number of vacancies present in the matching process on each of the three markets can be different, mobility of workers may differ in function of their age. As except the difference in mobility frequency, the workers' behavior in this model is similar to that of the model in chapter 2, we choose not to present the workers' behavior in the paper core. You can find the description of the workers' value function in the appendix .1, page 199, and of the workers' flows in the appendix .2, page 201. We assume there exists an institutional minimum wage in this economy which bounds below the wage distribution. It is denoted by  $\underline{w}$ . As in the previous chapter, time is continuous. The following subsection presents since now the

firms'side of the model.

### 3.2.2 Firm's expected profit

As the contact frequency of firms with an unemployed and an employed worker and according the workers'age class can be different, the firms'hiring frequency for each age is now given by:

$$h_y(w) = q_y^0 u_y + q_y(m - u_y)G_y(w)$$

$$h_a(w) = q_a^0 u_a + q_a(m - u_a)G_a(w)$$

$$h_s(w) = q_s^0 u_s + q_s(m - u_s)G_s(w)$$

The property 9 of the chapter 2 still stands when the contact frequency varies<sup>5</sup>. The values of  $u_i$  and of  $(m - u_i)G_i(w)$  are calculated in the appendix .2, page 201.

The expected surplus induced by employing a worker of each age class at a given wage is given by:

$$J_y(w, k) = \frac{y_y(k) - w + pJ_a(w, k)}{r + p + s + \lambda_y(1 - F_y(w))} \quad (3.1)$$

$$J_a(w, k) = \frac{y_a(k) - w + pJ_s(w, k)}{r + p + s + \lambda_a(1 - F_a(w))} \quad (3.2)$$

$$J_s(w, k) = \frac{y_s(k) - w}{r + p + s + \lambda_s(1 - F_s(w))} \quad (3.3)$$

---

<sup>5</sup>The hiring frequency raises with the wage offered by the firms.

With the production function  $y_i(k)$ , given by:

$$\begin{aligned} y_y(k) &= y_y + \left(\frac{q}{\alpha}\right) k^\alpha \\ y_a(k) &= y_a + \left(\frac{q}{\alpha}\right) k^\alpha \\ y_s(k) &= y_s + \left(\frac{q}{\alpha}\right) k^\alpha \end{aligned} \tag{3.4}$$

The parameters  $q$  and  $\alpha$  are strictly positive exogenous parameters and  $y_i$ , the workers' productivity at the workplace, can depend on the age class of the worker. The expected surplus is still composed of the margin of the match and of its duration. Yet, the margin of the match now evolves with the age of the worker. Indeed, the match's productivity function changes over the match's duration. If the firms hire a young, the match's productivity is first:  $y_y + \left(\frac{q}{\alpha}\right) k^\alpha$ , then it becomes:  $y_a + \left(\frac{q}{\alpha}\right) k^\alpha$ , and eventually is:  $y_s + \left(\frac{q}{\alpha}\right) k^\alpha$  <sup>6</sup>.

The expressions of the job duration have the same form as in the chapter two, their values are presented in the appendix .6, page 207. Note that, given this similarity, the property 10 of the chapter 2 still stands here <sup>7</sup>. The surplus depends positively on the quality of the job  $k$ . Yet to acquire this quality firms must pay a certain cost so that the expected profit for each age class is now given by:

$$\begin{aligned} \Pi_y(w, k) &= h_y(w)(J_y(w, k) - \beta_y k) \\ \Pi_a(w, k) &= h_a(w)(J_a(w, k) - \beta_a k) \\ \Pi_s(w, k) &= h_s(w)(J_s(w, k) - \beta_s k) \end{aligned}$$

Firms offering a wage  $w$  hire a worker of the age class  $i$  at the frequency  $h_i(w)$ .

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<sup>6</sup>Naturally, this progression or regression occurs if the job is not destroyed before the worker changes age class

<sup>7</sup>The job duration raises with the wage offered by the firms.

Once the worker hired, the firms get the expected surplus of a job of quality  $k$  net of the training cost induced by this quality. This cost is represented by the parameter  $\beta_i$ , which can be different according to the workers'age.

The decision of firms of the quality of the job is the result of the trade-off between the cost of creating a quality job and the return of it in terms of productivity. The result of this trade-off is age-dependant and is given by:

$$k_s(w) = \left( \frac{\alpha q}{\beta_s(r + p + s + \lambda_s(1 - F_s(w)))} \right)^{\frac{1}{1-\alpha}} \quad (3.5)$$

$$k_a(w) = \left( \frac{\alpha q \left( 1 + \frac{p}{r+p+s+\lambda_s(1-F_s(w))} \right)}{\beta_a(r + p + s + \lambda_a(1 - F_a(w)))} \right)^{\frac{1}{1-\alpha}} \quad (3.6)$$

$$k_y(w) = \left( \frac{\alpha q \left( 1 + \frac{p}{r+p+s+\lambda_a(1-F_a(w))} \left( 1 + \frac{p}{r+p+s+\lambda_s(1-F_s(w))} \right) \right)}{\beta_y(r + p + s + \lambda_y(1 - F_y(w)))} \right)^{\frac{1}{1-\alpha}} \quad (3.7)$$

The calculation details are presented in the appendix .3, page 202. Note that the decision of the quality of the jobs according the wage level depends on age yet not on the workers'productivity at the workplace  $y_i$ .

### 3.2.3 Distribution of matches productivity

In this this chapter, firms can use an extra lever in the Bertrand competition game, the quality of the job created. Therefore the firms'behavior in competition also affects the match productivity. (Mortensen, 1998) shows that the result of this wage game is a distribution of wage and of jobs'productivity. Given equations 3.5, 3.6 and 3.7, we can rewrite the profits of the firms only in function of  $w$ . These profits are therefore given by:

$$\Pi_y(w) = h_y(w)(J_y(w, k_y(w)) - \beta_y k_y(w))$$

$$\Pi_a(w) = h_a(w)(J_a(w, k_a(w)) - \beta_a k_a(w))$$

$$\Pi_s(w) = h_s(w)(J_s(w, k_s(w)) - \beta_s k_s(w))$$

The decision of the quality of the match can be integrated in the expression of the profit. Using these expressions of the profit, we can set the equiprofit conditions that generate the equilibrium wage distributions exactly as in the second chapter. These conditions are presented in the appendix .5 page 205.

At equilibrium, the model generates a distribution of match productivity on each market. Computing this distribution of productivity is not trivial. Indeed, given the existence of the heterogeneity of the quality of the match and of the workers' productivity at the workplace, the productivity of the matches in this economy depends both on the current age of the worker working at the job and on the age of the worker when the job was created. There are therefore six different levels of productivity for a given wage.

1. The productivity of a young, hired when young:  $y_y + \frac{q}{\alpha} k_y(w)^\alpha$ .
2. The productivity of an adult, hired when young:  $y_a + \frac{q}{\alpha} k_y(w)^\alpha$ .
3. The productivity of an adult, hired when adult:  $y_a + \frac{q}{\alpha} k_a(w)^\alpha$ .
4. The productivity of a senior, hired when young:  $y_s + \frac{q}{\alpha} k_y(w)^\alpha$ .
5. The productivity of a senior, hired when adult:  $y_s + \frac{q}{\alpha} k_a(w)^\alpha$ .
6. The productivity of a senior, hired when senior:  $y_s + \frac{q}{\alpha} k_s(w)^\alpha$ .



Each of these functions generate a distribution according to the wage of the workers who belong to each category. The distribution of productivity in the economy is therefore the aggregation of these six different distributions of productivity weighted by their respective distribution of wage. The wage densities of workers according to their production function are presented in appendix .4 page 205.

### 3.2.4 Number of vacancies

In this chapter, firms can move freely from one market to another. As the profit of firms on each market is likely to be different, the number of firms on each market is not the same. The number of firms in each market affects the probability of contact between firms and workers. The number of matches between workers and firms for each age class is indeed given by:

$$M_i = \phi v_i^\eta (u_i + R_\phi(m - u_i))^{1-\eta}$$

with  $\eta$  the elasticity of this matching function,  $v_i$  the number of vacancies,  $u_i$  the number of unemployed workers,  $(m - u_i)$  the number of employed workers and  $R_\phi$  the ratio of the search effectiveness of employed workers  $\phi$  and of the unemployed workers  $\phi^0$ . If we consider that the unemployed workers search more intensively than the employed workers, this ratio will be for instance inferior to 1.

We set  $\theta_i = \frac{v_i}{u_i + R_\phi(m - u_i)}$ , the labor market tightness on each market.

The mobility rate of workers of each age class depends on the frequency for a worker to find a job whether he is employed or unemployed. We can express this frequency in function of the labor market tightness. The frequencies at which an employed and an

unemployed worker has a contact with a firm are given by:

$$\lambda_i = \phi \theta_i^{1-\eta}$$

$$\lambda_i^0 = \phi^0 \theta_i^{1-\eta}$$

The frequencies at which a firm has a contact with an employed and an unemployed worker are given by:

$$q_i = \phi \theta_i^{-\eta}$$

$$q_i^0 = \phi^0 \theta_i^{-\eta}$$

At equilibrium, firms enter each market until the profit in each market is equal to the cost of a vacancy noted  $c$ . We therefore compute the value of  $\theta_y$ ,  $\theta_a$ ,  $\theta_s$ , such that:

$$\Pi_y(\underline{w}_y, \theta_y) = \Pi_a(\underline{w}_a, \theta_a) = \Pi_s(\underline{w}_s, \theta_s) = c \quad (3.8)$$

### 3.3 Properties of the model

This section presents the main properties of the model. We first study of the determinant of quality of the matches and the effect of this quality and of the workers' specific productivity on the wages. Then we study the determinant of the workers' mobility and its effect on wage.

#### 3.3.1 The matches productivity

**Property 24.** *The quality of the job increases with the expected match duration.*

*Proof.* By using the definition of the expected job duration presented in the appendix

.6, page 207, the equations 3.5, 3.6 and 3.7 can be rewritten as it follows:

$$\begin{aligned} k_s(w) &= \left( \frac{\alpha q}{\beta_s} D_s(w) \right)^{\frac{1}{1-\alpha}} \\ k_a(w) &= \left( \frac{\alpha q}{\beta_a} D_a(w) \right)^{\frac{1}{1-\alpha}} \\ k_y(w) &= \left( \frac{\alpha q}{\beta_y} D_y(w) \right)^{\frac{1}{1-\alpha}} \end{aligned}$$

□

**Corollary.** *The quality of the job increases with the wage offered to workers. For each age,  $\frac{\partial k_i}{\partial w} > 0$ .*

*Proof.* The expected match duration increases with the wage proposed by the firms. (See chapter 2) □

This corollary means that the quality of the job increases with the wage offered by the firms. The productivity of the firms are therefore an increasing function of the wage. On the other side, the creation of a high quality job allows firms to offer higher wages, since high wages by their capacity to retain workers induces higher a productivity.

**Corollary.** *The quality of the job decreases with the mobility frequency of the workers. For each age,  $\frac{\partial k_i}{\partial \lambda_i} < 0$ . Yet, this effect decreases when the wage increases, and at the maximal wage in the economy ( $\bar{w}$  such that  $F_i(\bar{w}) = 1$  for each  $i$ ), the mobility rate has no effect anymore.*

*Proof.* Using the expression of the expected match duration presented in the appendix .6, page 207, it is straightforward that the expected duration of a match with a worker of age class  $i$  decreases with the job to job mobility frequency of the workers. Besides,

at the maximum wage on the market, this expected duration (and therefore the level of specific human capital invested) no longer depends on the job to job mobility of workers.  $\square$

When firms anticipate that the worker they hire have a lot of other opportunities, they do not create a high quality job which is a costly investment unless they choose a high wage strategy which protects the firms from the future workers' resignation. Note that when firms decide of the quality of a job on the young's or the adults' market, they take into account the mobility frequency of workers of next periods, since the worker can still be employed in the firm during these periods.

**Corollary.** *When the workers are homogenous in terms of human capital level, mobility and face the same wage offer lottery, ie.  $\beta_y = \beta_a = \beta_s$ ,  $\lambda_y = \lambda_a = \lambda_s$  and  $F_y = F_a = F_s$ , the quality of the job decreases with the workers' age.*

*Proof.* When  $\beta_y = \beta_a = \beta_s$ , the difference between the quality of the job of workers of different age classes only comes from the difference in the expected match duration. Using the expression of the expected match duration presented in the appendix .6, page 207, it is straightforward that, when  $\lambda_y = \lambda_a = \lambda_s$  and  $F_y = F_a = F_s$ , the expected duration of a match decreases with workers' age.  $\square$

When the workers are homogenous in terms of human capital level, mobility, and face the same wage offer lottery, the firms choose to create higher quality jobs on the young's market. Indeed, the young, at equal mobility, have a longer discounting horizon. (Mortensen, 1998), in an infinite horizon model, shows that even if workers are similar in terms of productivity, the Bertrand competition between firms induces firms to create different qualities of jobs. At the equilibrium, similar workers have different level of productivity. This assumption is not neutral in a finite horizon. When we consider

workers' life cycle, even when workers are ex ante similar in terms of productivity, at the equilibrium, the level of productivity of workers depends on their age.

**Property 25.** *For each age, an increase in the human capital level, i.e. a decrease of  $\beta_i$ , increases the quality of job.*

*Proof.* Given equations 3.5, 3.6 and 3.7, it is straightforward that the quality of the job decreases with the training cost  $\beta_i$ .  $\square$

The parameter  $\beta_i$  allows to take into account the heterogeneity of workers' human capital level according to their age class. Notably, if senior workers are less costly to train because they have already worked in several other companies and have accumulated much more human capital than the younger worker, this may compensate the fact that they have a short horizon, and firms can choose to create high quality jobs for these workers.

**Property 26.** *The contribution of the workers' productivity at the workplace to the firms' profit increases with the wage offered by the firms.*

*Proof.* The contribution of the workers' productivity at the workplace to the firms' profit can be represented by the derivative of the profit according to this productivity.

$$\begin{aligned}\frac{\partial \Pi_y(w, k)}{\partial y_y} &= h_y(w) \frac{\partial J_y(w, k)}{\partial y_y} \\ \frac{\partial \Pi_y(w, k)}{\partial y_y} &= \frac{h_y(w)}{r + p + s + \lambda_y(1 - F_y(w))}\end{aligned}$$

$$\begin{aligned}\frac{\partial \Pi_a(w, k)}{\partial y_a} &= h_a(w) \frac{\partial J_a(w, k)}{\partial y_a} \\ \frac{\partial \Pi_a(w, k)}{\partial y_a} &= \frac{h_a(w)}{r + p + s + \lambda_a(1 - F_a(w))}\end{aligned}$$

$$\begin{aligned}\frac{\partial \Pi_s(w, k)}{\partial y_s} &= h_s(w) \frac{\partial J_s(w, k)}{\partial y_s} \\ \frac{\partial \Pi_s(w, k)}{\partial y_s} &= \frac{h_s(w)}{r + p + s + \lambda_s(1 - F_s(w))}\end{aligned}$$

These three derivatives raise with the wage offered since  $h_i(w)$  raises with the wage proposed.  $\square$

The property 26 shows that a high workers' productivity induces firms to offer high wages.

We now study the determinant of the workers' mobility and its possible effect on wages.

### 3.3.2 The workers' mobility

The workers' mobility depends on the number of vacancies on the market.

**Property 27.** *When we assume the institutional minimum wage  $\underline{w}$  is such that  $\underline{w} > \underline{w}_y$ ,  $\underline{w} > \underline{w}_a$  and  $\underline{w} > \underline{w}_s$ <sup>8</sup>, the level of equiprofit of firms on each market decreases with the labor market tightness of the market.*

<sup>8</sup>This assumption allows a greater simplicity in the calculation

*Proof.* See appendix .7, page 207. □

**Corollary.** *When workers are ex ante equally productive, the workers' mobility decreases with workers' age.*

*Proof.* At equal productivity, and mobility, the profit decreases with age (see. Chapter 2). □

At equal productivity, the free entry condition 3.8 accounts for a lower mobility rate of the older workers.

**Corollary.** *An increase in the workers' productivity at the workplace of an age class raises the workers' mobility of this class.*

*Proof.* Using equations 3.1, 3.2 and 3.3, and 3.4, we can easily prove that  $\frac{\partial \Pi_y(w_y, \theta_y)}{\partial y_y} > 0$ ,  $\frac{\partial \Pi_a(w_a, \theta_a)}{\partial y_a} > 0$ , and  $\frac{\partial \Pi_s(w_s, \theta_s)}{\partial y_s} > 0$ . □

The workers' productivity at the workplace, by affecting the productivity of the matches on the market, affects the number of vacancies on this market and eventually the workers' mobility of this market.

**Property 28.** *The reserve army (see definition in the Chapter 2) on each market decreases with the unemployment to employment mobility of workers.*

*Proof.* Given the equation flows presented in appendix .2.1, page 201, we can deduce the mass of unemployed workers on each market as follows:

$$u_y = \frac{(s+p)m}{p+s+\lambda_y^0} \tag{3.9}$$

$$u_a = \frac{sm+pu_y}{p+s+\lambda_a^0} \tag{3.10}$$

$$u_s = \frac{sm+pu_a}{p+s+\lambda_s^0} \tag{3.11}$$

At equal mobility between workers of all age classes, as  $m > u_y$ , necessarily  $u_y > u_a$ , and then  $u_a > u_s$ . Consequently,  $u_y > u_a > u_s$ . Yet, a decrease in the mobility rate of workers of any age class increases the unemployment duration and therefore the unemployment rate of the age period. The extreme case presented in chapter 2 shows that a difference in the mobility frequency can reverse the order of the unemployment mass. Indeed when the mobility rate of unemployed is null for seniors, the unemployment rate of seniors becomes higher than the unemployment rate of adults. The evolution of its size remains therefore ambiguous.  $\square$

The mobility frequency affects the reservation wage of workers and therefore has an impact on the monopsony power of firms in front of these workers.

## 3.4 Calibration and validation of the model

In this section, we present the data we use to calibrate and validate the model. Then we describe the target used for the calibration and the chosen validation criteria.

### 3.4.1 The source of the data

We use the data of the 2002 Annual Social and Economic Supplement (ASEC) to calibrate the model. The ASEC is an annual report of the statistical Current Population Survey (CPS) conducted monthly by the United States Census Bureau for the Bureau of Labor Statistics. Some supplemental questions are added in the ASEC (in March), notably on income received in the previous calendar year, which are used to estimate the data on income and work experience. The ASEC is split in three records, the household record, the family record and the person record. In this chapter, we use the



value of monthly earnings before deduction of the longest job over the last calendar year on the person record. We restrict on men, and exclude self-employed workers and only focus on wage-earner workers. We choose to restrict attention on an homogenous group of workers in terms of educational attainment since in our theoretical framework, workers start their working life with the same productivity and the same level of human capital (they are all ex ante homogenous). The dispersion generated by the heterogeneity of the ex ante workers' productivity cannot therefore be captured by the model<sup>9</sup>. We choose to focus on workers whose educational attainment was the high school degree or equivalent because they constitute the largest group in the sample.

We focus on workers between 20 and 64 years old. Data from OECD show indeed that the actual retirement age in the U. S. was 64 years old in 2002, even if the legal age is fixed to 67 years old. We define as in the theoretical model, three age classes evenly long: the 20<sup>10</sup> to 34 years old, the 35 to 49 years old, and the 50 to 64 years old. We therefore focus on workers between 20 and 64 years old. We compare the wage of these three age classes in cross section. If the real economy were in steady state as in the theoretical economy described in the model, the cross section approach would cause no problem, yet as it is naturally not the case, this approach can show its limits. Notably, by studying the wage of different age classes at a time  $t$ , we mix the notion of age and of generation. Therefore, the wage of an age class depends also of the education level of the generation. For instance, workers between 50 and 65 years old in 2002 have the education standards of the sixties. One could argue that to avoid this critic, we should

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<sup>9</sup>This limit could be overcome by assuming an exogenous distribution of the ex-ante productivity of young workers:  $y_y$ .

<sup>10</sup>From 20 years old, the employment rate is above 50%, oecdstats

follow a cohort of workers over their life cycle. Yet, this approach supposes that we compare wages at periods where the institutional environment is different. Changes in institutions on the labor market can affect the wage setting decision of firms and the actual wages of workers. We therefore choose the cross section approach. Besides, restricting our study to the workers whose educational attainment is similar, protects us partly from the bias generated by the cross section approach. The remaining bias generated by this choice could be a slight underestimation of the trend of human capital accumulation of workers over their life.

The restriction to workers with the same educational attainment causes however another problem: the number of observations decreases significantly with workers' age. Indeed, the number of high school graduates in the 90's is higher than in the 60's. All aggregated moments computed with the data are therefore biased by this composition. For instance, we have twice as many observations for young workers as for senior workers. In order to make realistic comparison between the results computed on the data and the ones computed based on the model, we compute the model aggregated results so that the proportion of each population within the total population is similar to the one found of the data.

To calibrate the model, we also use data on transitions within the labor force: the mobility frequency between unemployment and employment and between jobs. We use the transitions rate computed by (Menzio, Telyukova, and Visschers, 2012) over the life-cycle. Their data comes from the U.S. Census' Survey of Income and Program Participation (SIPP) of the 1996 to 2000 period.

Eventually, the data we use for job tenure are 2002 data which come from a survey published by the Bureau of Labor Statistics of the U.S. Department of Labor on job

tenure in 2012 and the data of 2002 unemployment rate by age are provided by the online database of the OECD.

### **3.4.2 The data presentation**

We convert gross earnings to net earnings and deduce from this net monthly earnings and the hours worked in the job, the net hourly wage. We express the wages received by workers of the three age classes in federal minimum wages (4.33\$ per hour in 2002). It is difficult to pretend that the model developed in the first section can reproduce the extreme wages existing in a wage distribution since in this model workers are ex ante homogenous when they arrive on the labor market. We therefore calibrate this model on a wage distribution corresponding to the first 95 percentiles of the wage distribution of each age. This aggregate distribution is presented in figure 3.4.2 and the distribution of wage by age class in the figure 3.1 (in introduction).

The wage distribution evolves with age by several aspects. The mode of the wage distribution as well as the wage dispersion increase between the first and the second age class. Yet, it remains almost stationary between the adults' and the seniors' class.

Figure 3.3 presents the evolution of the mean wage, the median wage, the standard dispersion and also of the job to job mobility rate, the unemployment duration and the unemployment rate over the three age periods. It is clear on these graphs that the mean wage raises over the life cycle but at a decreasing rate. The dispersion coefficient increase at a slight increasing rate. The workers mobility decreases over the three age periods which explains the decrease in the job to job mobility and the increase in the unemployment duration. The unemployment rate falls at the beginning of the life cycle during the integration of young workers within firms. Then it remains almost still on the second half of the life cycle.

Figure 3.2: Aggregated U. S. wage distribution of salaried men (First 95%) expressed in US minimum wage

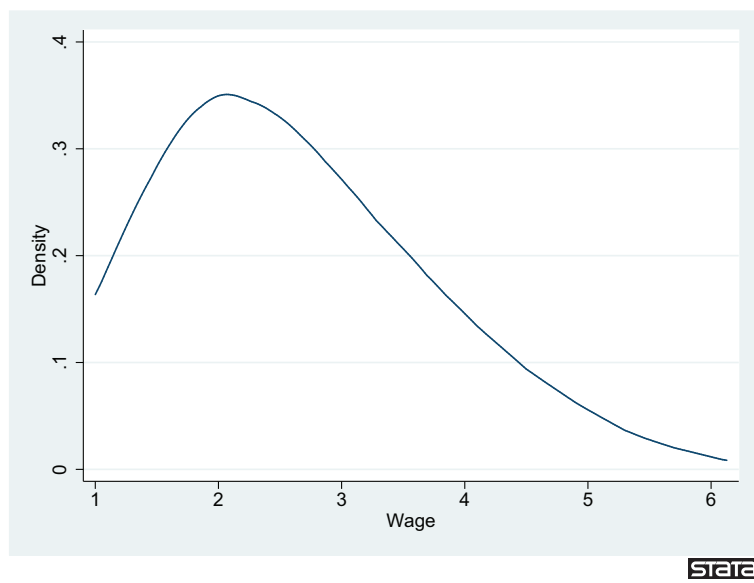
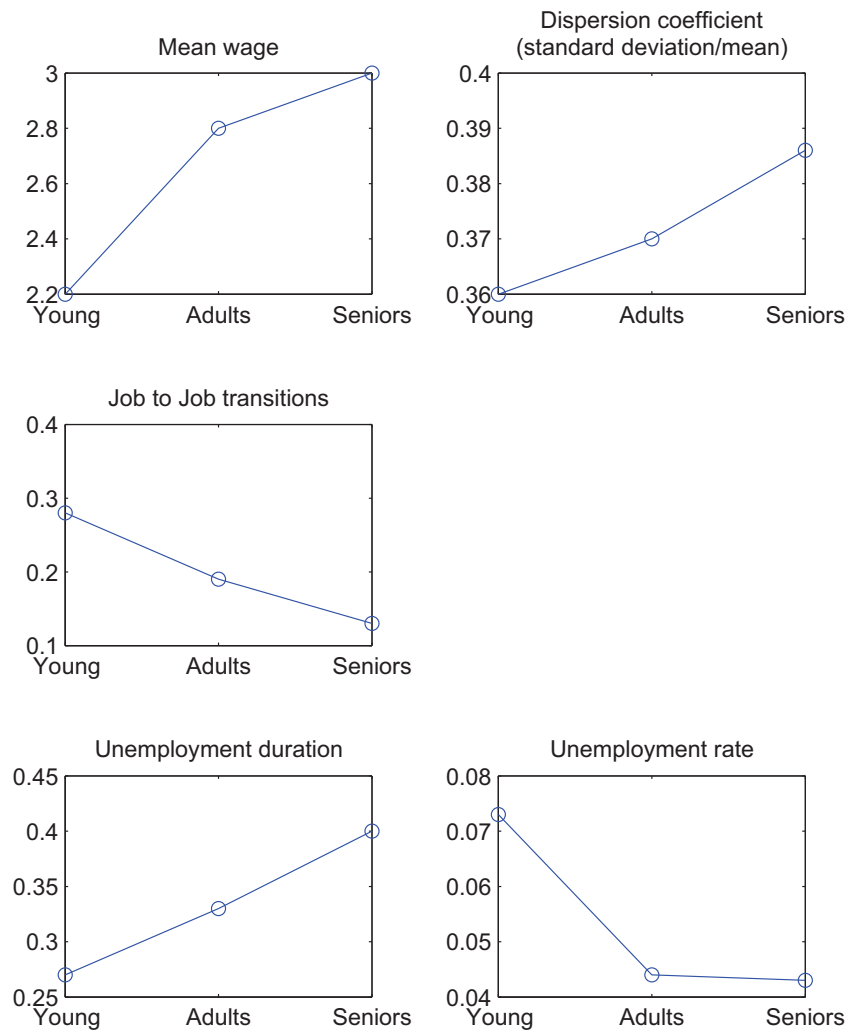


Figure 3.3: Evolution of some U. S. labor market features with age



### 3.4.3 The calibration

We set the model period to be one year. The annual interest rate  $r$  is set to 4% as it is usual in the literature. In the data, we assume three life periods of 15 years, we therefore set the probability to change age class to  $\frac{1}{15}$ . We normalize the institutional minimum wage since all wages are expressed in federal minimum wage. The cost of training for young workers,  $\beta_y$ , is also normalized since only the difference between  $\beta_y$ ,  $\beta_a$  and  $\beta_s$  matters here. We set the elasticity of the matching function to 0.7 as estimated recently by Borowczyk-Martins, Jolivet, Postel-Vinay, (2011)<sup>11</sup>.

The other parameters are calibrated on the data presented above. The exogenous destruction rate is calibrated so that to reproduce the median job tenure of 3.7 years. Our calibration supposes therefore that jobs are exogenously destroyed on average every 7 years. The matching process efficiency parameters for unemployed and employed workers are calibrated respectively on the unemployment duration of 4 months and on the average job to job transition of 0.21 (about one transition every 5 years). The parameters of the production function are calibrated on moments of the wage distribution. The parameter  $q$  is set in order to reproduce the mean wage,  $\alpha$ , the median wage, and the parameter of human capital accumulation  $\beta_a$  and  $\beta_s$ , the ratio between the 75th centile and the median and the 90th centile and the 75th centile. These last two moments allow to capture the shape of the wage distribution of the second half of the distribution since specific human capital investment allows to explain a great part of wages at the top of the distribution. The value of the return to capital  $\alpha$  is equal to the value obtained by (Chéron, Hairault, and Langot, 2008). The workers' productivity at the workplace  $y_y$ ,  $y_a$  and  $y_s$ , are set in order to reproduce the mode of the respective wage distribution. Our calibration suggests that the cost of training decreases over the

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<sup>11</sup>New Estimates of the Matching Function, Working Paper

workers' life cycle, yet the actual productivity on the workplace of workers tends to increase at first, between the first two periods, and then decrease for senior workers. Table 3.1 sums up the annual value of the parameters and the targets used to calibrate them.

Table 3.1: Calibration parameters

Fixed and institutional parameters			Targets' value
$r$	0.04	discounted rate	
$p$	1/15	working life duration	45 years
$\beta_y$	1	Normalized	
$\underline{w}$	1	Normalized	
$\eta$	0.7	fixed	
Calibrated parameters			
$s$	0.12	Median job tenure	3.7 years
$\phi_0$	7.15	Unemployment duration	0.33 years
$\phi_1$	3	Job to job transition	21%
$q$	0.425	Mean Wage	2.6
$\alpha$	0.72	Median Wage	2.4
$y_y$	1.75	Mode of young	1.8
$y_a$	2.26	Mode of adults	2.4
$y_s$	2.14	Mode of seniors	2.5
$\beta_a$	0.88	C75/C50	1.3
$\beta_s$	0.659	C90/C75	1.2

Note that no calibration on these data is possible without the two components of the workers' productivity presented in the theoretical model: the evolution of the workers' productivity at the workplace and the accumulation of human capital. Indeed in a specification without one of these parameters, there is a conflict between an accurate

wage dispersion, the wage distribution shape and the increasing path of wage with age. Indeed, the observed wage dispersion and shape can only be obtained by assuming that firms can have different productivities. Yet firms are naturally induced to create lower quality jobs to seniors because of their shorter working horizon. In order to fit the data, it is therefore necessary to assume the accumulation of human capital of workers. The workers' productivity at the workplace accounts for a part of the translation of the wage distribution with age, it therefore needs to be included too.

#### 3.4.4 Calibration results

The simulation induced by this calibration is the **Simulation 1A: U. S. benchmark economy**<sup>12</sup>. The simulated aggregated wage distribution, the wage distribution over the three age classes and the distribution of productivity over the three age classes are presented on the figure 3.4. Thanks to the calibration and the simulation of the model presented in this chapter, we can infer distributions that are not easily observed in reality and use them to understand the evolution of wage with age. In this section, we therefore compute the distribution of wage and productivity offered to the workers by the firms on each market of simulation 1A. Figure 3.5 presents these distributions, mean figures are given by table 10 presented in the appendix .8.1 page 212. These distributions are largely commented in the next section. On the wage distributions (aggregated and over life cycle), we can observe a step close to the wage 3.5. This step actually appears on the simulated distribution of adults, as we can see it on the second graph in figure 3.4. Looking at the first graph in figure 3.5 which represents the

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<sup>12</sup>To ease the manipulation of the different simulations run in this chapter and in the next chapter, all the simulations of this chapter are identified by a rank number, here 1, followed by the letter A, whereas in the chapter 4, the rank number of simulations is followed by the letter B.



Figure 3.4: Simulated distribution of wage, and of wage and match productivity according to workers' age class- Simulation 1A: U. S. benchmark economy

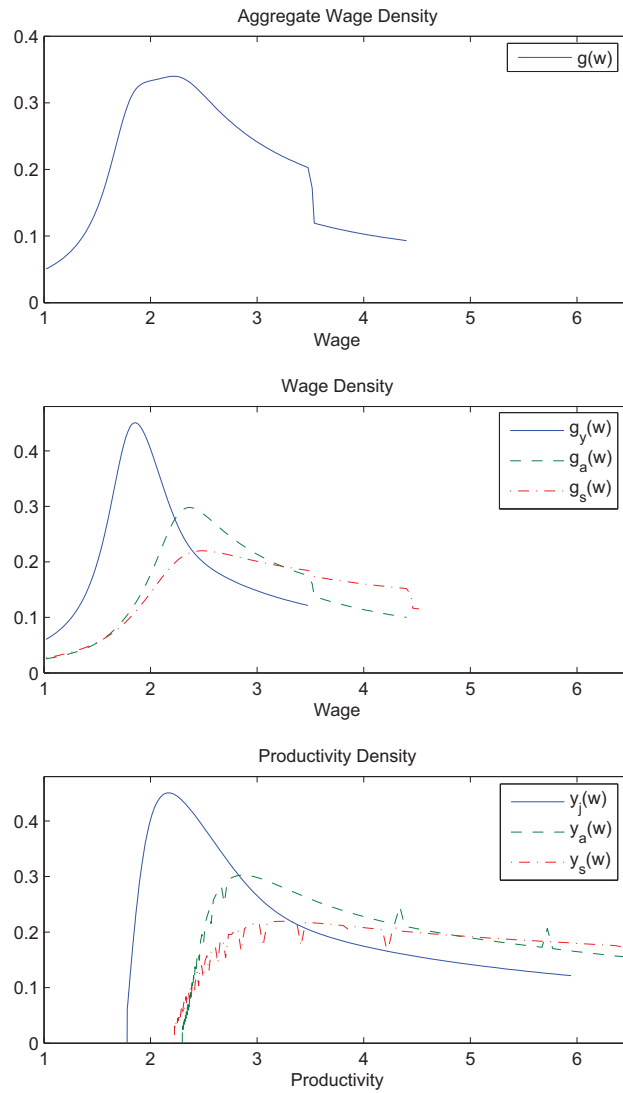
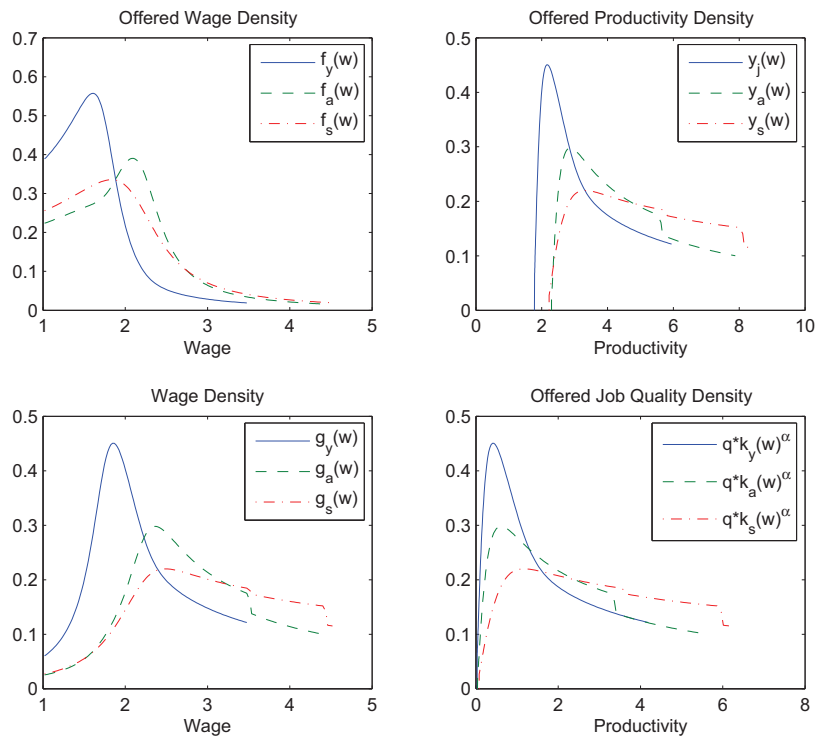


Figure 3.5: Simulated distribution of offered wage, offered productivity, wage and offered job quality - Simulation 1A: U. S. benchmark economy



wage offered to workers according to their age, we can easily explain this phenomenon. Indeed, contrary to the workers employed at a wage no greater than 3.5, all adult workers employed at a wage above 3.5 have necessarily been hired as an adult since young workers are not offered such wages. The adults' wage distribution is therefore composed of a report of young's wage distribution only up to the wage 3.5. The step that we observe on the simulated distribution of seniors after this same wage is the report of this same step. We observe the same phenomenon at the very top of the distribution of the seniors' wage, around 4.5, for the exact same reason, as adults are no longer offered such high wages. Naturally, these discontinuities would fade away if we increased the number of age classes.

Table 3.2 presents the ability of the model to reproduce some extra moments on the labor market: unemployment rate and standard dispersion, and the evolution over the three life periods of the main moments targeted.

We have not searched to reproduce any of these new moments while calibrating the model, we therefore propose to use them to validate the model. The moments used for the calibration are notified in the table in bold letters.

Table 3.2: Validation Results

	Total		Young (20-34)		Adults (35-49)		Senior (50-654)	
	Model	Data	Model	Data	Model	Data	Model	Data
Mean	<b>2.6</b>	<b>2.6</b>	2.2	2.2	2.8	2.8	3	3
Dispersion Coef	30.8%	38.5%	26.8%	36%	27.1%	37%	28.3%	38.6%
Job to job transition	<b>0.21</b>	<b>0.21</b>	0.22	0.28	0.21	0.19	0.18	0.13
Unemployment duration	<b>0.33</b>	<b>0.33</b>	0.25	0.27	0.35	0.33	0.4	0.4
Unemployment rate	5.5%	5.6%	7%	7.3%	4.6%	4.4%	4.7%	4.3%

This model allows to reproduce these new moments and their trend quite well. We

can still notice that the wage dispersion remains underestimated. We can indeed see on figure 3.4 that the simulated wage distributions stop earlier than in reality. It is actually possible to find a calibration in which the rightward tale is longer, yet at the price of a higher mean wage. This model explains therefore quite well most of the wages received in the economy yet it shows its limits in explaining the very top of the distribution. Besides, the mode of each wage distributions is thicker in reality than in the simulation. This supposes that the wage dispersion within each age class is globally slightly underestimated. This underestimation were expectable provided that the model generating this wage distribution assumes workers homogenous within age classes. The trend of the job to job transitions over the three age periods decreases more sharply in the data than in the model. This can be explained by the fact that we do not take into account the search effort of workers in the model. In reality the search effort of workers is decreasing with age since the return of the job search decreases with the shortening of the workers' horizon.

### **3.5 Wage game, productivity channel and wage progression over the life cycle**

In this chapter we take into account two channels to explain the wage progression over the three age classes: the wage game channel and the productivity channel.

What we call the wage game channel is the evolution of the wage game with age at identical ex ante productivity. This channel represents the pure effect of age on the wage game. The productivity channel accounts for the share of the wage progression induced by the productivity increase with age. In the model, the workers' productivity can evolve by two aspects: first the workers' productivity at the workplace can change over time

and second they can accumulate (or loose) human capital. The first component of this productivity channel affects the match productivity directly by affecting the production function, the second affects it by decreasing the cost of specific human capital for firms. As figure 3.4 suggests it, the productivity of the match is strongly correlated to the wage of workers. We explain in this section, by which mechanisms the match's productivity interacts with the workers' wage.

To distinguish the contribution of the two channels to the wage progression over life and to understand the respective role of the two component of the productivity channel, we need to run two new simulations: one in which, *ceteris paribus*, all workers have the same productivity at the workplace, yet keep on accumulating (or loosing) human capital, i.e.  $y_y = y_a = y_s = 1.75$  and  $\beta_i$  as in table 3.1: **Simulation 2A: U. S. benchmark economy with  $y_y = y_a = y_s = 1.75$** , figure 3.6, and table 11 presented in appendix .8.2, page 213), and one in which, *ceteris paribus*, all workers are *ex ante* evenly productive, i.e.  $y_y = y_a = y_s = 1.75$  and  $\beta_y = \beta_a = \beta_s = 1$  **Simulation 3A: U. S. benchmark economy with *ex ante* evenly productive workers,  $y_y = y_a = y_s = 1.75$  and  $\beta_y = \beta_a = \beta_s = 1$** , figure 3.8, and table 12 presented in appendix 3.5, page 105). This last simulation (3A) represents the pure wage game needed to assess the evolution of the wage game over age. The simulation 2A can only be an illustration to understand the mechanism behind the two component of workers' productivity since in reality the workers' specific productivity,  $y_i$ , should naturally be correlated to human capital level,  $\beta_i$ . We only use this simulation to comment the results on the productivity channel, it is yet not taken into account in the wage progression decomposition. The results of this wage progression decomposition is given by the table 3.3.

From this table, we can deduce that the wage progression in the first part of the

working life is generated by both the productivity channel and the wage game channel, yet to a least extent for this latter. At the end of the working life, the wage game channel contributes negatively to the wage progression. The next two subsections are devoted to explain these results. The effect of these two channels goes through two of the model mechanisms: first the evolution of the workers' market power, and second, by the search effect, that is the wage increase workers acquire over their working life by selecting among the best paid jobs given the offered wage distribution. It is important to distinguish the channels from the mechanisms described above. The mechanisms are the ways the channels affect the wage distribution, when the channels correspond to the source of the wage progression. For that matter, it is possible to cut a channel of wage progression, like by assuming there is no increase in productivity, or no wage game (it is therefore the case of the pure monopsony). It is not the case for the described mechanisms since they constitute the model itself.

To explain the results given by the table 3.3, we use extensively the figure 3.7 which draws a comparison between the three simulations (1A, 2A and 3A) in terms of wage, dispersion, productivity and mobility<sup>13</sup>.

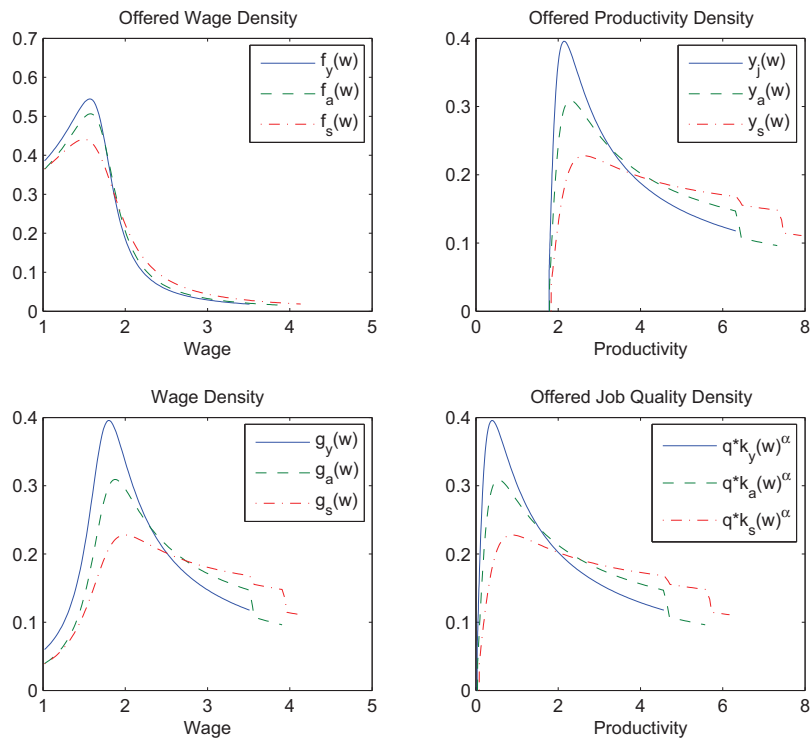
### 3.5.1 The wage game channel

The contribution of the wage game to the wage progression over the life cycle is positive in the first part of the life cycle and negative in the second part. This contribution seems a priori paradoxical. Indeed if workers can select during all their working lives

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<sup>13</sup>The very small decrease in the wage offered to the young when workers' productivity of the other age classes evolves comes from a shift in the firm strategy on the young's market: as firms on the adults' and the seniors' labor market offer low wages when ex ante workers' productivity remain constant over the life cycle, firms on the young's market can create slightly better quality jobs since their retention is improved.

Figure 3.6: Simulated distribution of offered wage, offered productivity, wage and offered job quality - Simulation 2A: U. S. economy with  $y_y = y_a = y_s = 1.75$



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Figure 3.7: Comparison of the U. S. benchmark economy (simulation 1A), the economy with  $y_y = y_a = y_s = 1.75$  (simulation 2A), and the economy with ex ante evenly productive workers,  $y_y = y_a = y_s = 1.75$  and  $\beta_y = \beta_a = \beta_s = 1$  (simulation 3A)

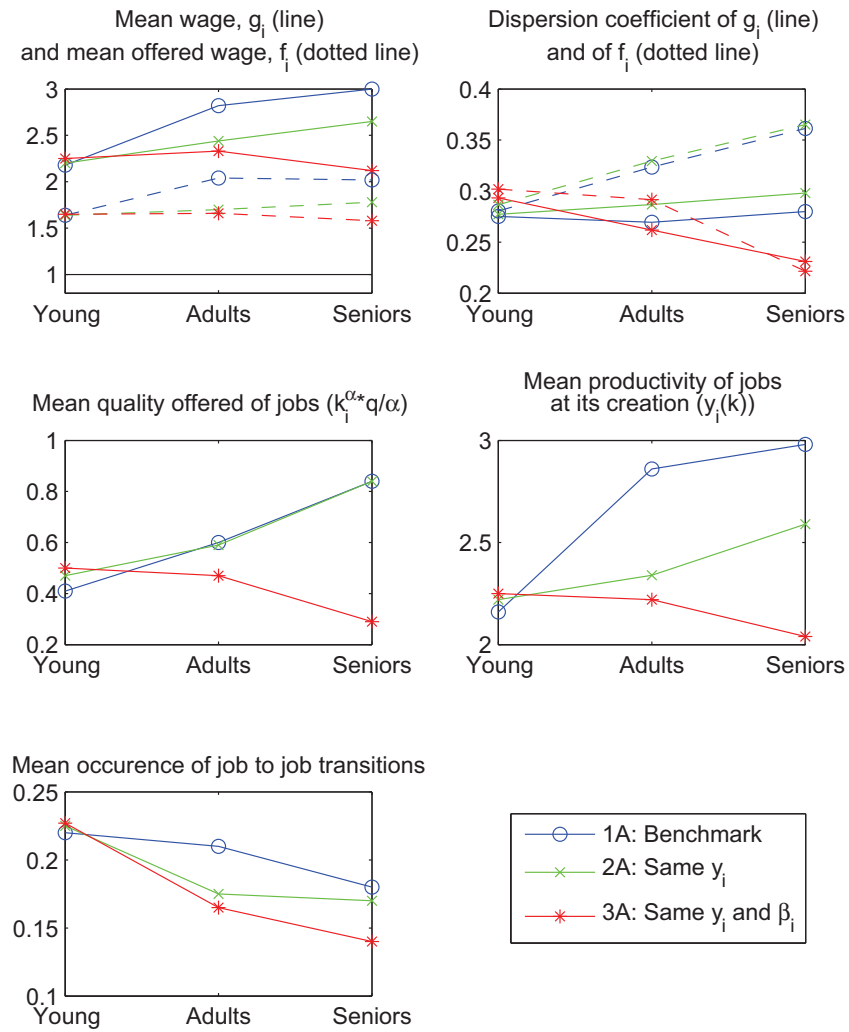




Figure 3.8: Simulated distribution of offered wage, offered productivity, wage and offered job quality - Simulation 3A: U. S. economy with  $y_y = y_a = y_s = 1.75$  and  $\beta_y = \beta_a = \beta_s = 1$

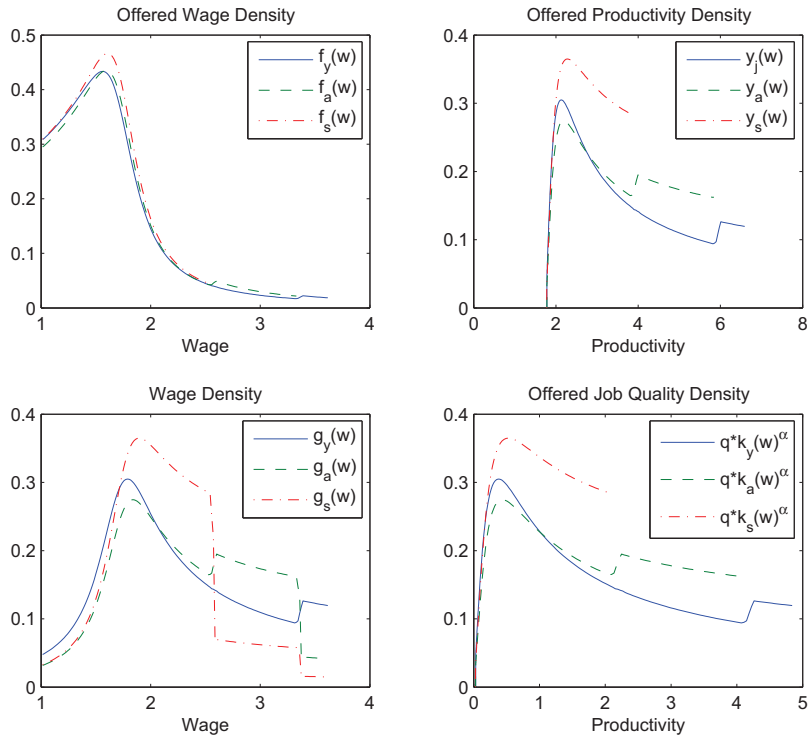


Table 3.3: Decomposition of wage evolution

	Young	Adults	Seniors	Evolution per year Y → A	Evolution per year A → S
Monopsony economy					
Mean wage	1	1	1	0%	0%
With on the job search and endogenous productivity (3A)					
Mean wage ( $g_i$ )	2.25	2.33	2.12	0.24%	-0.60%
Benchmark economy (1A)					
Mean wage ( $g_i$ )	2.18	2.82	3	1.96%	0.43%
Decomposition of wage progression					
Search channel				0.24 points of %	-0.6 points of %
Productivity channel				1.72 points of %	1.03 points of %

among the best paid jobs, one can think that at the end of their life, they are employed in better jobs than at the beginning. This subsection explains this paradox by analyzing the forces which rule the wage game and how they evolve over the working life. We start by explaining the evolution of the wage game by the evolution of the workers' market power and then by the effect of the search.

### **The workers' market power over the life-cycle**

If workers had no market power induced by their possibility to search on the job, whatever the productivity of the workers is, firms would have the entire monopsony power on the labor market and would offer to all workers the monopsony wage, i.e. here the minimum wage equal to 1. The possibility of on the job search of workers forces firms to compete to get a share of the workforce and to keep it. In this competition game, the firms can choose the wage to offer and the productivity of the job they want to create. This competition game naturally induces firms to raise their wages and to

invest in jobs to increase their productivity. The evolution over the life cycle of the workers' capacity to force firms to raise their wage offers (and therefore to create high quality jobs) given they have the same level of ex ante productivity is a measure of the workers' market power evolution. We therefore compare the mean wage offered to workers of each age class  $f_i$  when workers are ex ante evenly productive (simulation 3A) represented by the red dotted line on the first graph in figure 3.7 with the monopsony wage, represented on this same graph by the black line (equal to 1 for each age class). The market power of young workers allows them to raise the wage firms offer on their market by 65%, the market power of the adults by 66%, and the market power of the seniors by 58%, for an ex ante productivity of 1.75.

The market power of workers in such an economy, is at first almost stationary during the first half of the life-cycle and then decreases during the second half of it. The workers' market power does not raise significantly in the first half of the life cycle because the improvement of the employment condition, defined in the chapter 2 as the source of the workers' market power, is now compensated by the shortening of the horizon which gives very little incentive to firms to create high quality jobs. This last effect is particularly strong at the end of the life cycle and dominates the mechanism described in the chapter 2 for this age class. Indeed, in an economy where firms can decide of both the wage and the productivity of the match, the investment dimension of wage is reinforced since retention now allows the initial human capital investment to pay for itself. The distribution of job qualities presented by figure 3.8 shows clearly that without the productivity channel, the firms would be reluctant to create high quality jobs on the seniors' market, and with a least amplitude on the adults' market. The shortening of horizon affects therefore very negatively the quality of proposed jobs, since this quality depends on an investment of the firms that they search to amortize.

This is translated in terms of wage by the absence of high wage offers to adults to a certain extent and to seniors to a much larger extent, as the offered wage distribution shows in figure 3.8. In particular on the seniors' market, firms do not even try to poach employed workers since high wages cannot be amortized on the long run, they choose instead low wage strategies which target senior unemployed workers arriving through exogenous destruction. This last behavior of firms explains why the offered wage dispersion yielded by the competition game decreases with the workers' age class (figure 3.7): from 0.30 for the young, 0.29 for the adults to 0.22 for the seniors.

When workers of all age classes are ex ante evenly productive, given their short horizon, firms are reluctant to create good jobs and offering high wages to older workers: the market power of workers decreases at the end of the life cycle. The distribution of received wage in figure 3.8 shows that eventually in this context senior workers earning high wages are only those hired in previous life periods.

### **The search effect**

The possibility of on the job search allows workers over their life cycle to progressively climb the wage ladder at a given offered wage distribution by resigning from low paid jobs to be employed by high paid jobs. Facing a wage offer lottery, given job to job transitions and after a certain time on the labor market, workers must be able to select themselves among the best paid jobs<sup>14</sup>. We call this workers' behavior the search effect. In our model this selection over the life cycle can be captured by comparing the gap between the distributions  $f_i$  of wage offered to the workers with the distribution  $g_i$  of wage received by the workers. If this gap increases over the workers' life cycle, then

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<sup>14</sup>In our model, this selection is highlighted by the shorter tenure of low paid jobs

workers achieve to raise their wages thanks to job to job transitions over their working life.

The first graph in figure 3.7 in the case of the simulation 3A allows this comparison. As expected, the wage distributions  $g_i$  are composed of higher wages than the distributions of offered wages  $f_i$ . This comparison demonstrates that the selection effect occurs for each age class: within each age class, workers have time to select among the best paid jobs, yet not increasingly as it is expected. To make a more precise comparison of this selection effect in function of the workers' age class, we can refer to the simulation 3A part of the table 3.4. The percentage expressed in this table for "Selection by age" measures the gain of worker of each age class from selecting among good jobs.

For simulation 3A in which workers are ex ante evenly productive, the mean wage of young workers is 36% higher than the mean wage they are offered by the firms, it is 37% on the adults' market and only 19% on the seniors market. Therefore, if we refer to the selection of good jobs contribution to the wage growth, we notice that it contributes positively to the wage progression in the first part of the life cycle, and then negatively in the second part. The increase during the first half of the life cycle seems intuitive: the more time workers spend on the labor market, the more ascendant mobilities they can experience. Indeed, in the wages adult workers earn, there is a part inherited from the first life period during which workers have already had time to select their job. The selection of good jobs should intuitively raise with age.

What happens at the end of the life-cycle? First as shown by the figure 3.7 in the pure wage game (3A), the occurrence of job to job transitions decrease consequently over the working life. Then, the dispersion of offered wage lowers with age. When wage dispersion is low, more firms offer similar wages and the gain from the search is weak. The decrease of job to job transitions comes from a decrease in job openings over the

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Table 3.4: Search effect in the benchmark economy (simulation 1A) and with evenly productive workers (simulation 3A). Effect by age and contribution to the wage progression

	Young	Adults	Seniors	Evolution per year Y → A	Evolution per year A → S
Simulation 1A: U. S. benchmark economy					
Mean wage ( $g_i$ )	2.18	2.82	3	1.96% <sup>†</sup>	0.43%
Mean offered wage ( $f_i$ )	1.64	2.04	2.02	1.63%	-0.07%
Search by age - 1A	32.93%*	38.24%	48.51%		
Simulation 3A: U. S. benchmark economy with evenly productive workers					
Mean wage ( $g_i$ )	2.25	2.33	2.12	0.24%	-0.6%
Mean offered wage ( $f_i$ )	1.65	1.7	1.78	0.2%	0.31%
Search by age -3A	36.36%	37.06%	19.10%		
Progression due to search - 1A				0.33 <sup>‡</sup> points of %	0.49 points of %
Progression due to search - 3A				0.04 points of %	-0.91 points of %
* Young workers achieve to raise their wage by 32.93% by selecting into best jobs in economy 1A ( $\frac{2.18}{1.64} - 1$ )					
† Over the first half of the working life, mean wage raises by 1.96% per year in economy 1A ( $\frac{2.82}{2.18} - 1$ )					
‡ On the 1.96% of yearly wage increase over the first half of the working life, 0.33 points are induced by the selection of good jobs in economy 2B (1.96% - 1.63%)					

life cycle: the expected surplus of firms progressively decreases with workers' age since workers' horizon gets shorter. Fewer firms are created on the market of workers closer from their horizon. Senior workers, as adults, have had time to select among good jobs during the previous periods, yet once on the seniors' market they can no longer experience large wage increases thanks to job to job transitions since first they occur less often and second when they occur, they provide workers with a low gain. Firms on this market do not even try to poach employed workers since high wages cannot be amortized on the long run, they choose instead low wage strategies which target senior unemployed workers arriving through exogenous destruction. As firms offer wages that only unemployed workers can accept, the seniors' market is a two-speed market: already employed workers earn rather high wages even if they cannot progress, while unemployed workers can only find low paid jobs. Even if workers can select among best paid jobs all over their life, as according to the state of their life, the wage offer lottery they face evolves, this selection can be totally hampered. The workers after a certain age have increasingly more difficulties to find better opportunities. As selecting good jobs from a certain moment of the working life becomes ineffective to raise the wage, and given exogenous job destruction, firms can offer low paid jobs, senior workers earn a distribution of wage closer to that they are offered.

The wage game evolves over the workers age. This evolution provokes a wage progression in the first half of the working life and a wage contraction in the second half. The finite horizon of workers penalizes greatly the seniors situation on the labor market. Without any improvement of the workers' productivity over the life-cycle, the wage would stop increasing in the second part of the life cycle.

### 3.5.2 The workers' productivity channel

We can deduce from the comparison of the mean wage of simulation 1A and 2A on figure 3.7 that the workers' productivity at the workplace contributes more to the wage of adults than to the wage of seniors (it actually raises the wage of the adults by 15.6% and of the seniors by 13.2%). The comparison between simulation 2A and simulation 3A shows us that the level of human capital which reduces the price of the specific human capital for firms raises less the mean wage of adults than the mean wage of seniors (it actually raises the wage of the adults by 4.7% and of the seniors by 25%). Note that the value of the calibration parameters of the workers' productivity at the workplace and of the level of the human capital could give us these intuitions, yet they do not allow us to infer precisely the contribution of the two aspects of workers' productivity to the wage progression. Indeed, the productivity channel is the measure of the translation of workers' higher productivity (productivity at the workplace or high level of human capital) into high wages. This translation occurs by two means: the increase of the workers' market power, and the effect of the search.

#### **The evolution of the workers' market power in presence of the productivity channel**

The comparison of the wage offered to seniors on figures 3.8 and 3.6 explains partly the sharp increase in wage of seniors induced by human capital accumulation. Indeed in the last subsection, we have seen that when workers are all ex ante evenly productive, seniors were penalized in terms of wage offered mostly because firms were reluctant to create high quality jobs with workers so close to their horizon. The accumulation of human capital allows to ease the creation of high quality jobs from firms on seniors' market since due to their high level of human capital seniors need less training



to reach the same productivity than workers of the other age classes. The human capital accumulation raises consequently the seniors' market power. The increase in the mean wage offered to workers between simulation 3A and 2A on figure 3.7 represents it. The adults' market power raises only slightly after the introduction of human capital accumulation (see figure 3.7). Yet, it increases consequently after the introduction of productivity at the workplace. Firms are indeed induced to compete more intensively in order to hire and retain highly productive workers.

### **The search effect in presence of the productivity channel**

The productivity channel affects greatly the effect of the search over life cycle. The gap between the mean wage and the mean wage offered in simulation 1A gets greater over the life cycle. Thanks to the productivity channel, workers can select effectively among best paid jobs all over their working life. The table 3.4 gives us the exact measure of this increase: the search raises wage by 0.33% per year in the first part of the working life and by 0.49% in the second part. The search accounts for the wage progression increasingly with wage. Note that with the productivity channel, adults and seniors have the same market power, that is they are on average offered the same wage. The wage progression in the second part of the working life is therefore induced by the search mechanism.

The productivity channel reinforces, or activates (in the second part of the working life) the search channel in several ways: it increases the occurrence of job to job mobilities, the wage offered and the offered wage dispersion. Note that the wide spread offered wage distribution raises the gaps between the wage of two jobs, and therefore without necessarily increasing the occurrence of mobility, increase the gain from mobil-

ity<sup>15</sup>. On the seniors' market, these effects occur via the human capital accumulation, on the adults' market, via the evolution of the productivity at the workplace (except for offered wage dispersion).

The productivity channel explains for a quite large part the wage progression over the life cycle. The wage progression in the first part of the life cycle is mostly affected by the increase in the workers' productivity at the workplace. In the second part of the life cycle, the wage increase results from the higher quality of jobs in which workers are employed. Seniors are employed by high quality jobs for two reasons. First because due to their higher level of human capital, firms are induced to create high quality jobs, and second because the greater dispersion of the offered wage (associated to these high quality jobs, see comparison between simulation 2A figure 3.6 and 3A 3.8), and the more frequent job to job transitions allows them to raise their gain from the selecting among the best paid jobs.

### 3.6 Conclusion

This chapter shows that the framework developed by (Burdett and Mortensen, 1998) augmented with age is able to reproduce the evolution of the wage distribution with age only when we take into account the distribution of the match productivity. The observed wage dispersion and shape can only be obtained by assuming that firms can create jobs with different levels of productivity. Yet firms are naturally induced to create lower quality jobs to seniors because of their short working horizon. Therefore the evolution of wage distribution with age does not fit the data. It is only possible to reproduce the correct evolution of the wage distribution by assuming learning by doing

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<sup>15</sup>Yet, we can think that if workers had a mobility cost, the shift in gain from mobility could in this case affect the occurrence of mobility

of workers.

This structural model allows us to decompose the wage progression into two channels: the wage game channel and the productivity channel. This decomposition reveals that the evolution of the wage game favors workers up to midlife. In the second part of the working life, age, by its horizon dimension affects unfavorably the wage game. The mere evolution of the wage game therefore cannot alone explain the wage progression during the entire working life. In fact without the productivity channel, wages would decrease significantly in the second part of the working life. The productivity channel is therefore crucial in particular at the end of the life cycle. This channel has a positive effect over the life cycle on both the market power of workers and the effect of the search. This last mechanism by which workers select among the best paid jobs all over their life contributes greatly to the capacity of workers to translate their high productivity into high wages. It is well known that the U. S. labor market is characterized by short job tenure and fast job to job transitions. We can therefore wonder rightfully if the wage progression goes by this same mechanism in other countries, in particular in those where workers' mobility is lower. The next chapter answers this question by studying the French case.

## Chapter 4

# Age-dependent Effect Of French Labor Market Institutions: A Quantitative Evaluation

### 4.1 Introduction

According to chapter 3, a great part of wage progression over the working life is induced by the gain of workers from moving to better paid jobs, i.e. the search mechanism. Job to job mobility is therefore crucial for workers to achieve to raise their wage over the life cycle. In France, job to job mobility is particularly low: when a worker has on average 21% of chance to move from one job to an other in one year in the U. S., his French counterpart has only 9%. Yet, average wage progression over the life cycle is slightly higher in France than in the U. S., around 1.34% per year in France for 1.25% in the U. S.. How do French workers achieve to raise their wage with low mobility? French labor market differs greatly from the U. S. labor market by the presence of

strong institutions. In France, over their working life, workers accumulate rights to the unemployment insurance system. The French institutional environment therefore allows workers to improve their outside options over their working life. This improvement could contribute to the wage progression in France without mobility. In this chapter, we firstly propose to test this hypothesis by introducing an unemployment insurance system depending on the workers' previous wage in the framework developed in chapter 3. The French institutional environment is however not only in favor of experienced workers, indeed, a shorter working horizon compared to the existing one in the U. S. (the average retirement age is 59 years old in 2007 in France for 65 years old in the U. S. the same year) also discourages firms to invest on older workers. Besides, the presence of a particularly high minimum wage which bounds below French wages intuitively benefits to a greater extent in terms of wage to new entrants on the labor market. The entire French institutional environment can therefore have ambiguous effect on wage progression. We secondly propose in this chapter to study how the presence of the French institutions taken as a whole affects the worker's wage distribution over their working life. The previous chapter achieves to explain the evolution of the wage distribution in the U. S. by two channels: the wage game channel and the productivity channel. To account for the evolution of the French wage distribution, we therefore use a third channel: the institutional channel. We consider in this chapter three institutions: the unemployment benefits, the retirement age, and the minimum wage. The institutional channel is defined in difference with the U. S. labor market. It therefore represents the presence of the unemployment insurance system that is progressive with wage, the presence of a 23% higher minimum wage and of a 6 year shorter horizon. To do so, we augment the framework developed in chapter 3 with progressive unemployment benefits as modeled in (Chéron, Hairault, and Langot, 2008). The presence of

a minimum wage is already present in this framework, therefore, taking into account a change in its level only requires to change the value of its parameter. Changing the working life duration is trickier, since it supposes to change the age classes size. As firms direct their search on the three age classes, whether we change only the size of the last age class or of each age class depends on our mind on what makes the age of a worker for a firm. If we think that this social age corresponds to the worker's past experience on the labor market, i.e. the number of years since he enters the labor market, then we should only reduce the size of the last age class. Yet, if we think that this social age depends on both his past experience on the labor market, and on his future working horizon, each age class should be reduced. This second approach seems more consistent with the firms' behavior since they consider both backward (productivity) and forward aspects (horizon) before making a hiring decision. We therefore reduce the duration of the three life periods and that way respect the ratio between experience and horizon of the workers of each age class. Note that the terms of young, adult and senior designate now therefore workers in a given state of their working life, not classes of workers between two biological ages.

According to our results, unemployment benefits affect the wage distribution by pushing up the workers' market power. Indeed the reserve army of workers ready to accept any wage sharply decreases when unemployed workers receive compensation. Firms are therefore induced to create higher paid jobs and higher productive jobs to attract workers even when unemployed. Only the new entrants on the labor market now constitute the reserve army of workers. Then over the working life, they accumulate entitlement to the unemployment insurance system and eventually rights to always higher level of benefits as their wage raises. By raising the outside options of workers in an increasing way with age, the unemployment insurance system contributes to wage

progression. Besides, this unemployment insurance system-induced wage progression occurs inhomogenously over the life cycle: on the 1.48% of average wage increase in the first part of the working life they contribute by 0.10 points of percentage and on the 1% of average wage increase in the second part of the working life by 0.25 points of percentage. Eventually seniors are those who benefit the most from the effect of unemployment benefits on wages. Yet, the unemployment insurance system also accounts for both a decrease of the search activity and of its outcome. The unemployment insurance system-induced wage progression in France therefore partly substitute the search-induced wage progression.

The two other institutions studied in this chapter decrease as expected the workers' wage progression over their working life. The higher minimum wage only raises the young's wage. Yet to a small extent since given the presence of an unemployment insurance system, most workers refuse very low wages. The effect of a short horizon penalizes significantly the seniors' market power and to a least extent the adults': firms anticipate lower job tenure on these markets, they invest therefore less on these jobs and stop adopting high wage strategies to retain their workers. The gain from avoiding poaching decreases when workers gets closer to their horizon. This effect is amplified by the unemployment insurance system. As older workers' wages decrease, so their unemployment benefits and their market power. This last mechanism partly explains why the same shift in the horizon decreases in smaller proportion the seniors' wages in the U. S.. Another reason for this difference is the average shorter expected job tenure in the U. S. economy due to a higher rate of job destruction. When the job tenure is short, only workers very close to their horizon can be discriminated by the firms. Eventually, the results of the wage progression decomposition in France shows that the French institutions studied in this chapter contribute positively to wage progression.

The effect of unemployment benefits on the senior's market power dominates the effect of their shorter horizon. Besides, they decrease sharply the wage dispersion within each age class. Globally, in France, institutions therefore raise the wage inequality between the older workers and the young workers yet decrease the wage inequalities within the age classes.

In the second section, we present the assumptions of the model augmented with unemployment benefits and give its main properties in the third section. Section 4 is dedicated to the description of the data, the model's calibration and the validation on French data. Section 5 explains and assesses the effect of unemployment benefits on the wage distribution over the working life. Section 6 first presents the effect of an increase in the minimum wage and of a decrease of horizon and eventually decomposes the French wage progression, taking into account the institutional channel.

## 4.2 Model assumptions

Most of the assumptions of this chapter are similar to the assumptions of chapter 3. We therefore only present in this section the new assumptions added in this chapter and their consequences on the workers', firms' behavior and equilibrium definition. The results used to define this model equilibrium, yet remained unchanged from chapter 3 are recalled in appendix.

### 4.2.1 New assumptions and notations

We assume in this chapter that workers with a working experience are eligible to unemployment benefits depending on the wage they had in their previous job as it exists in France. In this case, workers receive benefits composed of a fixed component *all* and



a progressive one  $\rho$  as follows:

$$b(w) = \rho \times w + all \quad (4.1)$$

As young workers who arrive on the labor market have no working experience, they receive the minimal possible benefits equal to the fixed component of the benefits: *all*. We assume the unemployed workers' income never exceeds his previous wage:  $\rho$  and *all* are such that  $b(w) < w$ . These benefits are financed by a lump tax noted  $\tau$  that all workers whether employed or unemployed pay. The cumulative distribution and density of benefits earned by unemployed workers is respectively noted  $U_i(b)$  and  $u_i(b)$ . The mass of unemployed workers of a period is no noted  $\bar{u}_i$ , in order to avoid confusion. The two other institutions taken into account here are already modeled in the framework of the chapter 3, only the values of the parameter  $\underline{w}$  and  $p$  is affected.

#### 4.2.2 The workers

The unemployment insurance system affects the unemployed workers' behavior. We see in this subsection how and what consequences this change in behavior have on the workers' flows.

##### **Workers' value functions and unemployed workers' reservation wage**

The unemployed workers have now a reservation wage which depends on their unemployment benefits and below which they refuse to work. We can deduce the workers' reservation wage for each age by equalizing the value of being employed noted  $V_i^e$  and the value of being unemployed noted  $V_i^u$ . We therefore search the wages  $R_i$  which allow  $V_i^e(R_i) = V_i^u(b)$  for each level of unemployment benefit  $b$ .

The asset values of being employed at a wage  $w$  for workers of each age class solve:

$$\begin{aligned} rV_y^e(w) &= w - \tau + \lambda_y \int_w^{\bar{w}} (V_y^e(x) - V_y^e(w)) dF_y(x) - s(V_y^e(w) - V_y^u(b(w))) - p(V_y^e(w) - V_a^e(w)) \\ rV_a^e(w) &= w - \tau + \lambda_a \int_w^{\bar{w}} (V_a^e(x) - V_a^e(w)) dF_a(x) - s(V_a^e(w) - V_a^u(b(w))) - p(V_a^e(w) - V_s^e(w)) \\ rV_s^e(w) &= w - \tau + \lambda_s \int_w^{\bar{w}} (V_s^e(x) - V_s^e(w)) dF_s(x) - s(V_s^e(w) - V_s^u(b(w))) - p(V_s^e(w) - V_r) \end{aligned}$$

These asset values are similar to those described of chapter 3 except that now workers finance the unemployment benefits system by paying the lump tax  $\tau$ .

The asset values of unemployed workers who receive the benefit  $b$  are given by:

$$\begin{aligned} rV_y^u(b) &= b - \tau + \lambda_y^0 \int_{R_y(b)}^{\bar{w}} (V_y^e(x) - V_y^u(b(w))) dF_y(x) - p(V_y^u(b) - V_a^u(b)) \\ rV_a^u(b) &= b - \tau + \lambda_a^0 \int_{R_a(b)}^{\bar{w}} (V_a^e(x) - V_a^u(b(w))) dF_a(x) - p(V_a^u(b) - V_s^u(b)) \\ rV_s^u(b) &= b - \tau + \lambda_s^0 \int_{R_s(b)}^{\bar{w}} (V_s^e(x) - V_s^u(b(w))) dF_s(x) - p(V_s^u(b) - V_r) \end{aligned}$$

We can deduce the lowest acceptable wage for a worker receiving the benefits  $b$  by setting for each age class of the workers  $V_i^u(b) = V_s^e(R_i)$ . The level of this reservation wage is therefore given for each age class by:

$$\begin{aligned} R_y(b) &= b + (\lambda_y^0 - \lambda_y) \int_{R_y}^{\bar{w}} (V_y^e(x) - V_y^e(R_y)) dF_y(x) + s(V_y^u(b) - V_y^u(b(R_y))) + p(V_a^u(b) - V_a^e(R_y)) \\ R_a(b) &= b + (\lambda_a^0 - \lambda_a) \int_{R_a}^{\bar{w}} (V_a^e(x) - V_a^e(R_a)) dF_a(x) + s(V_a^u(b) - V_a^u(b(R_a))) + p(V_s^u(b) - V_s^e(R_a)) \\ R_s(b) &= b + (\lambda_s^0 - \lambda_s) \int_{R_s}^{\bar{w}} (V_s^e(x) - V_s^e(R_s)) dF_s(x) + s(V_s^u(b) - V_s^u(b(R_s))) \end{aligned}$$

As in (?), the workers' reservation wages raise with the level of unemployment benefit  $b$ . The second term of these reservation wages shows that the workers also take into account the difference of opportunity between the status of unemployed and employed  $\lambda_i^0 - \lambda_i$  to set their reservation wage. If for example the number of opportunities is higher for unemployed workers  $\phi_0 > \phi$ , the worker will increase his reservation

wage: the accepted wage must compensate this loss. This implication of the heterogeneity of the contact rate between the status of employed and unemployed on the workers' reservation wage is discussed in (Burdett and Mortensen, 1998). The third term accounts for the fact that the worker anticipates a possible job destruction and the loss it would generate  $V_i^e(R_i) - V_i^u(b(R_i))$ . Even if  $\phi_0 = \phi$ , workers will reject a wage equal to  $b$  since this new wage will generate in the case of a job loss benefits lower than  $b$ . This effect is discussed in (?). At last the young and the adult workers take into account the value of being employed in the next period to set their reservation wage of the current period. If for example an adult worker knows that seniors' reservation wage is higher than the adults' one ( $R_s > R_a$ ), he will anticipate that on a long run the status of employed is less valuable and will be more reluctant to accept a job as an adult: his reservation wage will increase. Workers' reservation wages of each age class are therefore co-dependant.

### **Workers' flows**

When the unemployed workers do not receive any unemployment benefits or receive unemployment benefits which do not depend on wage, all the workers of the same age class have the same reservation wage induced by the labor opportunity cost and their allocations if any. In this context, the lowest wage offered by the firms on each market is necessarily greater than this reservation wage, since no firm has interest in offering a wage that no worker can accept. For that matter, in that context, there is no job rejection from unemployed workers.

When workers receive progressive unemployment benefits, wage dispersion generates an heterogeneity of unemployment benefits among workers: unemployed workers have ex post different reservation wages. Firms are therefore exposed to offer wages that

can be rejected by some unemployed workers, yet accepted by some others. The only unemployed workers who never reject any wage for sure are those who receive the lowest benefits in the economy: workers with no working experience. In this framework, the reserve army is from now on only constituted of these workers. In steady state, the mass of these workers is noted  $u_i(all)$  and solve the following flows equations :

$$\begin{aligned}
 [\lambda_j^0 + p]u_y(all) &= p \cdot m \\
 [\lambda_a^0 + p]u_a(all) &= pu_y(all) \\
 [\lambda_s^0 + p]u_s(all) &= pu_a(all)
 \end{aligned} \tag{4.2}$$

All young workers entering the labor market ( $p \cdot m$ ) receive these minimum unemployment benefits. Among the adults and the seniors, the workers who receive these minimum benefits are those who have never worked since they enter the labor market, that is the workers who have not yet found a job when adult or senior. The mass of unemployed workers according to their unemployment benefits for all  $b > all$ , solves in steady state the following flows equations:

$$\begin{aligned}
 [\lambda_y^0(1 - F_y(R_y(b))) + p]u_y(b) &= s(m - u_j)g_y \left( \frac{b - all}{\rho} \right) \\
 [\lambda_a^0(1 - F_a(R_a(b))) + p]u_a(b) &= s(m - u_a)g_a \left( \frac{b - all}{\rho} \right) + pu_y(b) \\
 [\lambda_s^0(1 - F_s(R_s(b))) + p]u_s(b) &= s(m - u_s)g_s \left( \frac{b - all}{\rho} \right) + pu_a(b)
 \end{aligned} \tag{4.3}$$

Unemployed workers who receive a benefit  $b$  accept a job if the wage proposal associated to this job is above  $R_i(b)$ . With the frequency  $\lambda_i^0[1 - F_i(R_i(b))]$ , unemployed workers receiving  $b$  have a contact with a firm offering a wage above his reservation wage, in other terms it is the job finding frequency of unemployed workers receiving

the benefits  $b$ . Note that  $u_i(b)$  depends both on the density of wage in the economy and on the offered wage density by the firms. By using 4.9 and 4.10, we can deduce the unemployment rate on each market, see appendix .1, page 217.

In steady state, the mass of workers earning a wage no greater than  $w$  solves the following flows equations:

$$\begin{aligned}
(p + s + \lambda_y(1 - F_y(w)))(m - u_y)G_y(w) &= \lambda_y^0 \int_{\underline{w}}^w f_y(x)U_y(R_y^{-1}(x))dx \\
(p + s + \lambda_a(1 - F_a(w)))(m - u_a)G_a(w) &= \lambda_a^0 \int_{\underline{w}}^w f_a(x)U_a(R_a^{-1}(x))dx + p(m - u_y)G_y(w) \quad (4.4) \\
(p + s + \lambda_s(1 - F_s(w)))(m - u_s)G_s(w) &= \lambda_s^0 \int_{\underline{w}}^w f_s(x)U_s(R_s^{-1}(x))dx + p(m - u_a)G_a(w)
\end{aligned}$$

The distribution of wages in the economy now depends on the distribution of unemployment benefits among unemployed workers. In chapter 3, an unemployed worker is hired by a firm offering a wage no greater than  $w$  if he has a contact ( $\lambda_i^0$ ) with a firm offering a wage below  $w$  ( $F_i(w)$ ). In this chapter, the unemployed worker must also receive a wage offer greater than their reservation wage. The term  $U_i(R_i^{-1}(x))$  represents the number of unemployed workers of age  $i$  who accept all of offers greater than the wage  $x$ , these workers receive benefits which make them reject all the offers lower than  $x$ . The mass of employed workers earning a wage below or equal to  $w$  becomes:

$$\begin{aligned}
(m - u_y)G_y(w) &= \frac{\lambda_y^0 \int_{\underline{w}}^w f_y(x)U_y(R_y^{-1}(x))dx}{p + s + \lambda_y(1 - F_y(w))} \\
(m - u_a)G_a(w) &= \frac{\lambda_a^0 \int_{\underline{w}}^w f_a(x)U_a(R_a^{-1}(x))dx + p(m - u_y)G_y(w)}{p + s + \lambda_a(1 - F_a(w))} \\
(m - u_s)G_s(w) &= \frac{\lambda_s^0 \int_{\underline{w}}^w f_s(x)U_s(R_s^{-1}(x))dx + p(m - u_a)G_a(w)}{p + s + \lambda_s(1 - F_s(w))}
\end{aligned} \quad (4.5)$$

### 4.2.3 Firms' expected profit

The firms' expected profit depends on the labor supply via their hiring frequency. The hiring probability that the firms face on each market depends on the repartition of workers according to their reservation wage whether they are employed or unemployed. On each market, the hiring frequency therefore becomes:

$$h_y(w) = q_y^0 U_y(R_y^{-1}(w)) + q_y(m - u_y)G_y(w) \quad (4.6)$$

$$h_a(w) = q_a^0 U_a(R_a^{-1}(w)) + q_a(m - u_a)G_a(w) \quad (4.7)$$

$$h_s(w) = q_s^0 U_s(R_s^{-1}(w)) + q_s(m - u_s)G_s(w) \quad (4.8)$$

The frequency at which firms hire an employed worker is similar to chapter 3. In chapter 3, this frequency accounts for the increasing path of the hiring probabilities according to wage, since  $G_i(w)$  increases with wage. The frequency at which firms hire an unemployed worker yet differs from chapter 3. This frequency now depends on the mass of unemployed workers whose reservation wage is lower than  $w$ ,  $U_i(R_i^{-1}(w))$ . The hiring frequency is therefore now increasing with wage because both  $G_i(w)$  and  $U_i(R_i^{-1}(w))$  are increasing with wage. With a higher wage, firms can now in the same time poach a greater number of workers and hire a greater number of indemnified unemployed workers.

The firms' expected surplus and decision in terms of job quality remains identical to those of chapter 3.

#### 4.2.4 Equilibrium

The equiprofit conditions are similar to those of chapter 3. They are given on each market by: From  $\underline{w}_y$  to  $\overline{w}_y$ ,

$$\Pi_y(\underline{w}) = \Pi_y(w)$$

From  $\underline{w}_a$  to  $\overline{w}_a$ ,

$$\Pi_a(\underline{w}_a) = \Pi_y(w)$$

From  $\underline{w}_s$  to  $\overline{w}_s$ ,

$$\Pi_s(\underline{w}_s) = \Pi_y(w)$$

The calculation details are given in chapter 3. These equiprofit conditions allow in chapter 3 to deduce simultaneously the distribution of offered wage  $F_i$ , of wage  $G_i$ , of productivity  $k_i$  and the value of the labor market tightness  $\theta_i$ . The value of the labor market tightness is obtained thanks to the free entry condition presented in chapter 3. In the previous subsections, we saw that both the distribution of offered wage and the distribution of wage depended on a the unemployment benefits' distribution  $U_i$  and on the unemployed workers' reservation wage  $R_i$ . As all distributions and the value of the labor market tightness are interdependent, the entire equilibrium now actually depends on  $U_i$  and  $R_i$ . In this chapter, we therefore define simultaneously, the distribution of offered wage, of wage, of productivity and of unemployment benefits as well as the decision rules of job acceptance from unemployed workers and the value of the labor market tightness.

### 4.3 Properties of the model

In this section, we define some new properties of the model given the introduction of unemployment benefits.

**Property 29.** *The mass of unemployed workers who do not reject any wage offer decreases with age. For all  $\lambda_i^0 > 0$ ,  $u_j(all) > u_a(all) > u_s(all)$ .*

*Proof.*  $\frac{p}{\lambda_i^0 + p} < 1$ , for all  $\lambda_i^0 > 0$  In steady state, for each age, the mass of workers leaving unemployment is equal to the mass of workers becoming unemployed. The number of unemployed workers receiving the minimum benefit for each period is therefore given by:

$$\begin{aligned} u_y(all) &= \frac{p}{\lambda_y^0 + p} \cdot m \\ u_a(all) &= \frac{p^2}{(\lambda_y^0 + p)(\lambda_a^0 + p)} \cdot m \\ u_s(all) &= \frac{p^3}{(\lambda_y^0 + p)(\lambda_a^0 + p)(\lambda_s^0 + p)} \cdot m \end{aligned} \tag{4.9}$$

□

**Property 30.** *On each market, the exit rate from unemployment decreases with the level of unemployment benefit of the worker.*

*Proof.* For workers of each age class, the unemployment duration is given by:

$$\begin{aligned} DU_y(b) &= \frac{1}{\lambda_y^0(1 - F_y(R_y(b))) + p} \\ DU_a(b) &= \frac{1}{\lambda_a^0(1 - F_a(R_a(b))) + p} \\ DU_s(b) &= \frac{1}{\lambda_s^0(1 - F_s(R_s(b))) + p} \end{aligned} \tag{4.10}$$

Since  $F_i$  is increasing in its argument, and as the higher the unemployment benefit, the higher the reservation wage, we can deduce property 30. □



**Property 31.** *If at least one unemployed worker has a reservation wage above the lowest wage proposed to the young  $\underline{w}_y$ , therefore the presence of unemployment benefits decreases the labor market tightness, and consequently the young's mobility.*

*Proof.* On the youths' market, firms offering  $\underline{w}_y$  can only hire workers whose reservation wage is below  $\underline{w}_y$ . If there is at least one worker whose reservation wage is above wage  $\underline{w}_y$ , therefore the hiring frequency of the firm decreases compared to an economy without unemployment benefits. Using property 6 of chapter 3, we can therefore deduce that the labor market tightness of the market is reduced at equilibrium, and that the workers' mobility decreases.  $\square$

For any value of  $\underline{w}_y$ ,  $\underline{w}_a$ , and  $\underline{w}_s$ , this property a priori only stands on the youths' market. To show why, let's take for example the adults' market. If  $\underline{w}_a > \underline{w}_y$ , therefore the report of unemployed workers ready to accept  $\underline{w}_a$  are composed of workers who could have rejected some wage proposals when they were young (those between  $\underline{w}_a$  and  $\underline{w}_y$ ). These job rejections have had the effect of increasing the number of unemployed workers whose reservation wage is between  $\underline{w}_a$  and  $\underline{w}_y$ . It is therefore possible that the number of unemployed workers ready to accept  $\underline{w}_a$  is greater than the total number of unemployed adults in an economy without unemployment benefits. We therefore easily understand that this property stands on the two other labor markets if  $\underline{w}_y \geq \underline{w}_a \geq \underline{w}_s$ . If it is not the case, the smaller the (positive) gap between  $\underline{w}_a$  and  $\underline{w}_y$ , and  $\underline{w}_s$  and  $\underline{w}_y$ , the more likely this property stands on these two other markets too.

## 4.4 Calibration and validation of the model

As in the previous chapter, we present in this section the data we use to calibrate and validate the theoretical model. The calibration and validation's strategy is very close

to the one of chapter 3.

#### 4.4.1 The data source

We use the data of the French Labor Force Survey (Enquête Emploi) of 2002 to calibrate the model. Conducted by the INSEE (Institut National de la Statistique et des Etudes Economiques) since 1950, the French Labor Force Survey provides data such as professions, earnings, and working hours. It is conducted yearly in March on 150 000 people living in 75 000 households. In 2003, the survey evolved and became quarterly, some extra questions were also added. We use the data of this survey just before this change. In this chapter we use the monthly wage after deduction including bonuses (spread monthly). We exclude self employed workers and focus on male wage-earner workers. Contrary to chapter 3, we restrict on a larger range of educational attainments. Since workers whose educational attainment is the high school degree represent too few observations, we also include workers whose attainment is the BEP and the CAP (these are professional degrees considered as below the high school degree level). We restrict on full-time and part-time workers, and exclude workers with variable hours contracts.

According to Eurostat data computed by the DARES in the report of (Lerais and Marioni, 2004), between 2001 and 2003, the average retirement age in France was slightly inferior to 59 years old (58.8) for an average labor market entry age of 21 years old. To ease the comparison with the U. S. situation, we focus on workers between 20 and 58 years old in order to respect the 39 years long working life. As in the previous chapter we cut the life-cycle in three evenly long periods so that the age classes are defined as follows: the 20 to 32 years old, the 33 to 45 years old and the 46 to 58 years old.

The data on the mean 2002 job to job transition<sup>1</sup> and tenure<sup>2</sup> come from the same Labor Force Survey and were reported by (Lemoine and Wasmer, 2010). The figures of life-cycle job to job transition are computed between 1996 and 1999 by the DARES and are based on data from INSEE. These figures are presented by (Lainé, 2004).

#### 4.4.2 The data presentation

We deduce hourly wages from the monthly earnings and the hours worked by workers. From these wage data, we draw two wage distributions expressed in French minimum wage: the aggregated wage distribution and the wage distribution according to age class presented in figure 4.4.2 and 4.4.2. Like on the U. S. data, we restrict our analysis on the first 95 percentiles of the wage distribution. In order to ease the comparison between the French and the U. S. distribution, figure 4.4.2 displays the U. S. wage distribution expressed in French minimum wage. Note that as the U. S. federal minimum wage is lower than the French institutional minimum wage: 4.33 euros per hour in the U. S. (\$4.33 since euro dollar parity in 2002 was equal to 1) for 5.36 euros in France, the distributions of figure 4.4.2 starts at 0.8 (4.33/5.36). The bottom of the French distribution is therefore more contracted than in the U. S. distribution. This contraction is particularly significant on the young's distribution since they earn lower wages. A lower wage dispersion can also be observed at the top of the distributions, in particular on the adults' and seniors' market. The trend of wage over life cycle differs: in the U.S. the wage distribution seems to remain almost stationary at the end of the life cycle, in France it keeps on shifting rightwards. Besides, the wage increase at the beginning of the life-cycle seems weaker in France. This trend is confirmed by the figures of the

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<sup>1</sup>computed by Amossé (2003)

<sup>2</sup>computed by Vandenbrande et al. (2007)

mean and median wage given in figure 4.4 compared to 3.3. Figure 4.4 presents the evolution of the mean wage, the dispersion coefficient, the unemployment duration and the unemployment rate over the three life periods. It is clear on these graphs that contrary to what we observe in the U. S. the mean wage, the dispersion coefficient and the job to job mobility raise at the same path throughout the workers' life. Consequently, seniors have at the same time higher wages and lower wage dispersion and mobility. Besides, unemployment and unemployment duration are much higher in France, and seem to change more over the life cycle. The decrease in unemployment between the young and the adults is sharper in France. The high young's unemployment rate in France is explained by their long unemployment duration compared to the U. S.: As young's unemployment is an new entrants' unemployment, the long unemployment duration observed in France penalizes them particularly. Adults suffer less from these long durations as they are already employed, and benefit from longer job duration compared to U. S. adults. The unemployment duration trend is much sharper in France, yet at the end of the life cycle unemployment rate remains stable like in the U. S. In facts, we know that the trend of unemployment is different over the life cycle in the two countries. Indeed, as showed by (Hairault, Langot, and Zylberberg, 2012), seniors' long term unemployment turns into early retirement. This phenomenon is particularly observed in France as seniors' unemployment duration is high. It has for consequences to underestimate largely seniors' unemployment

#### 4.4.3 The calibration

In the data, we assume three life periods of 13 years, we therefore set the probability to change age class to  $\frac{1}{13}$ . The unemployment benefits are composed of two components, two parameters therefore need to be set:  $all$  and  $\rho$ . The fixed component is the un-

Figure 4.1: Wage distribution of French salaried men by age class (First 95%), expressed in US minimum wage

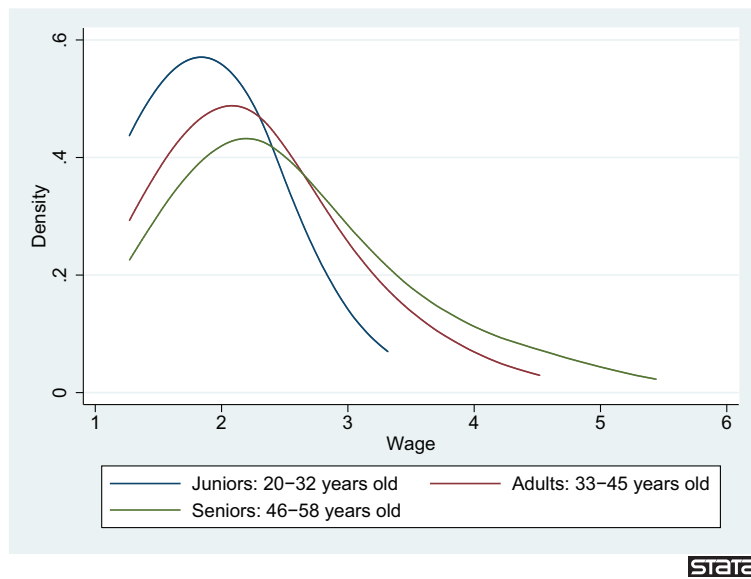
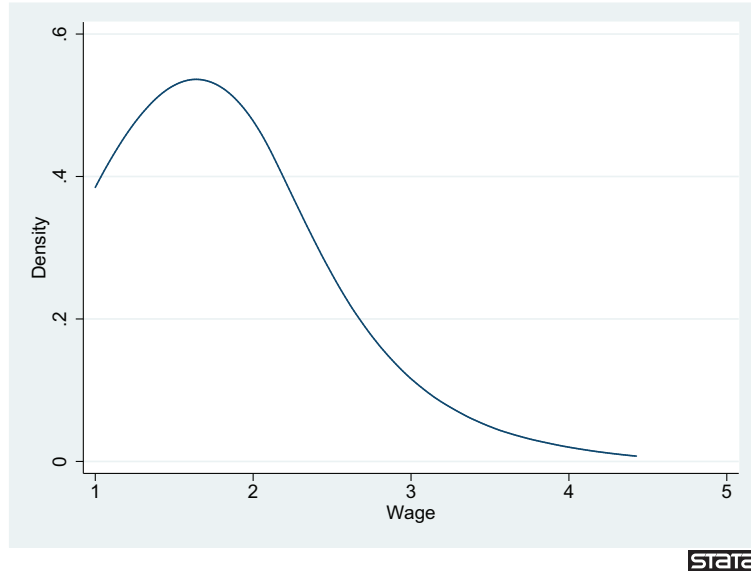


Figure 4.2: Aggregated wage distribution for French salaried men (First 95%)



employment benefits received by the workers with no working experience, it therefore stands for the minimal unemployment benefits that a worker can receive. In France, this minimal income is indexed on the minimum wage and represents about one third of it <sup>3</sup>. We therefore set  $all = 0.33$ . The progressive parameter of indexation on wage is calibrated in order to reproduce the French unemployment rate. Indeed, if we set this parameter to its institutional level of 57.4%, we overestimate the level of unemployment benefits in our economy since in the model workers never lose their eligibility to unemployment benefits. In order to reproduce the right value of being unemployed which induce the right level of job rejection in the economy, this parameter should therefore be below this institutional level. According to our calibration results, this parameter

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<sup>3</sup>In 2012, the daily minimum allocation was the gross amount of 28.21 euros (3.16 euros net and hourly) for a hourly net minimum wage of 9.4 euros

Figure 4.3: Wage distribution for French salaried men by age class (First 95%), expressed in French minimum wage

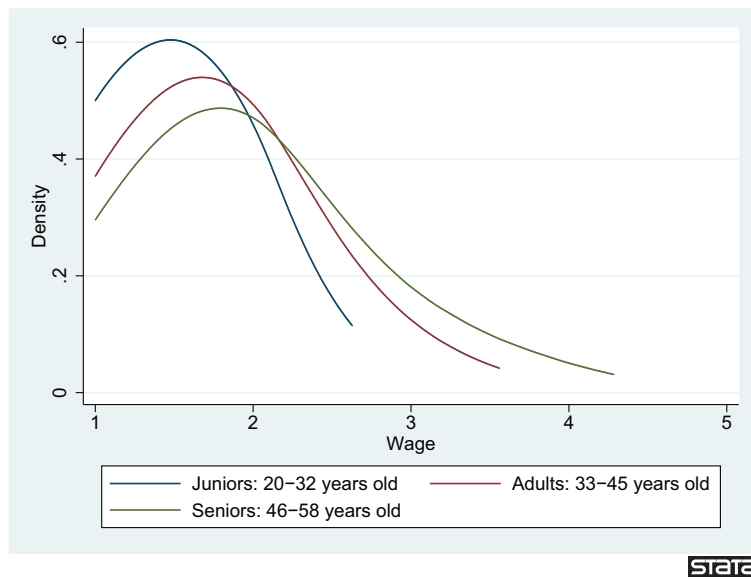
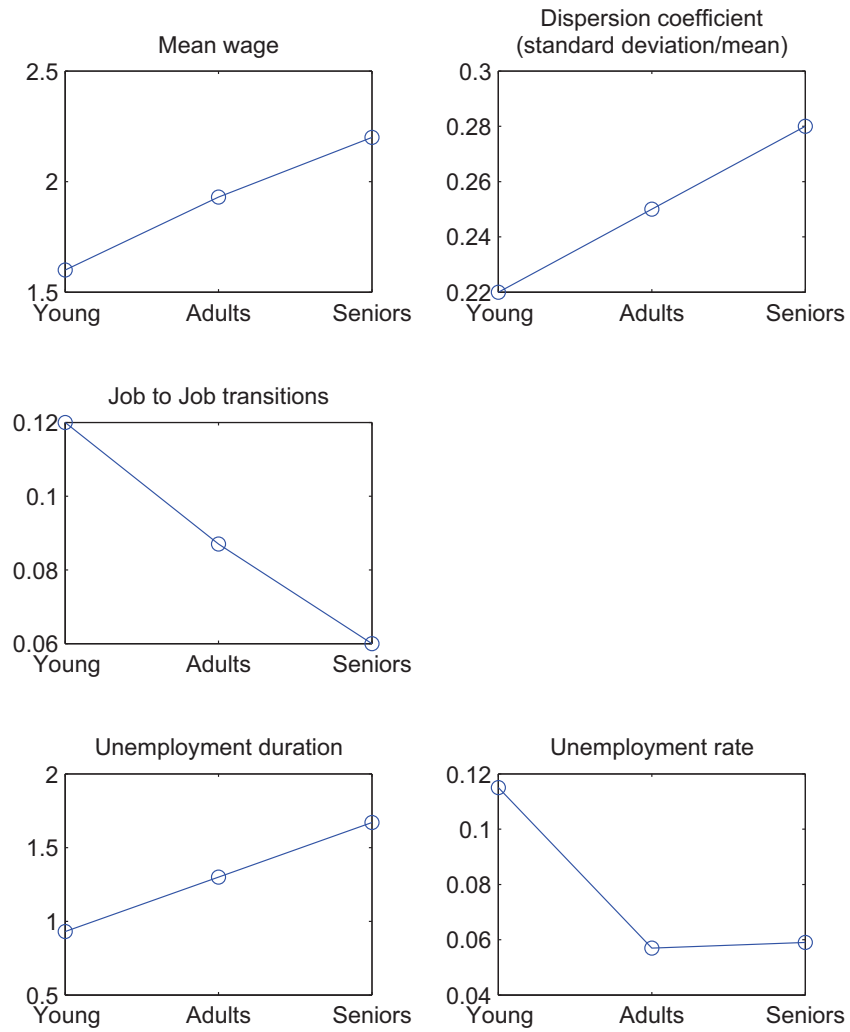


Figure 4.4: Evolution of some French labor market features with age





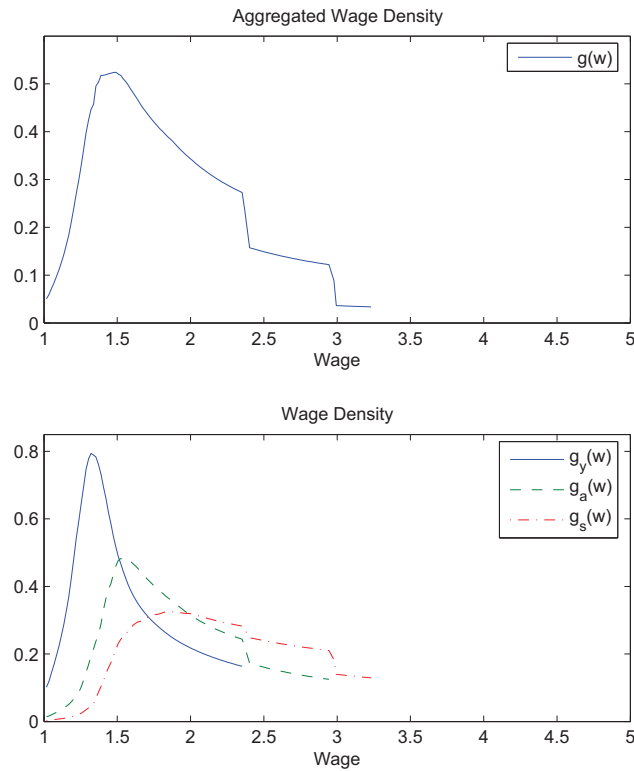
is equal to 0.49. Note that using the two components of the unemployment benefits induces that low paid workers receive a higher part of their wage than high paid workers when they become unemployed. This is consistent with the actual French unemployment benefit system. The other targets used to calibrate the model are similar to the ones of chapter 3 and are summed up in table 4.1.

Table 4.1: Calibration parameters

Fixed and institutional parameters			Targets' value
$r$	0.04	discounted rate	
$p$	1/13	working life duration	59 years
$\beta_y$	1	Normalized	
$\underline{w}$	1	Normalized	
$all$	0.33	A third of minimum wage	
$\eta$	0.7	fixed	
Calibrated parameters			
$s$	0.072	Median job tenure	7 years
$\phi_0$	6.2	Unemployment duration	1.14 years
$\phi_1$	1.4	Job to job transition	9%
$q$	0.26	Mean Wage	1.85
$\alpha$	0.77	Median Wage	1.7
$y_y$	1.4	Mode of young	1.5
$y_a$	1.61	Mode of adults	1.7
$y_s$	1.68	Mode of seniors	1.8
$\beta_a$	0.85	C75/C50	1.27
$\beta_s$	0.573	C90/C75	1.22
$\rho$	0.49	Unemployment rate	7.8%

#### 4.4.4 Validation of the model

Figure 4.5: Simulated distribution of wage, and of wage and match productivity according to workers' age class-Simulation 1B: French benchmark economy



The simulation induced by this calibration is the **Simulation 1B: French benchmark economy**. The aggregated wage distribution and the wage distribution over the three age classes generated by this simulation are presented figure 4.5.

Table 4.2 presents the ability of the model to reproduce the evolution of the main moments targeted over the three life periods. We have not searched to reproduce these

new moments while calibrating the model, we therefore propose to use them to validate the model. The aggregated moments used for the calibration are notified in the table in bold letters.

Table 4.2: Validation Results

	Total		Young (20-32)		Adults (33-45)		Senior (46-58)	
	Model	Data	Model	Data	Model	Data	Model	Data
Mean	<b>1.87</b>	<b>1.87</b>	1.61	1.6	1.92	1.93	2.17	2.2
Coefficient of dispersion	0.22	0.27	0.18	0.22	0.187	0.25	0.19	0.28
Job to job transition	<b>0.09</b>	<b>0.09</b>	0.101	0.12	0.092	0.087	0.066	0.06
Unemployment duration	<b>1.14</b>	<b>1.14</b>	0.93	0.93	1.26	1.28	1.46	1.5
Unemployment rate	7.8%	7.8%	9.65%	11.5%	6.7%	5.7%	7.95%	5.9%

This model allows to reproduce these new moments quite well and their trend over the three age periods. Note that as in chapter 3, the wage dispersion remains underestimated since the simulated wage distributions stop earlier than in reality. There again, the model shows its limits in explaining the very top of the distribution. The trend of unemployment rate and unemployment duration follows the data, at the exception of the unemployment rate of seniors which seems overestimated by the model. Yet it is well known that some seniors are declared as retired when they actually are only unable to find a job. It is difficult to assess the number of inactive people who should be considered as unemployed at the end of the life-cycle in the data, yet it is obvious that the empirical unemployment rate among seniors is largely underestimated.

Thanks to the calibration and the simulation of the model presented in this chapter, we can infer distributions that are not easily observed in reality and use them to understand the evolution of wage with age. As in the chapter 3, we therefore compute the distribution of wage and productivity offered to the workers by the firms on each

market. They are observable on figure 4.6 (the figures are given by table 13 in appendix .2.1, page 218). Figure 4.6 also shows the distribution of unemployed workers according to their previous wage. This distribution seems to follow the wage distribution of employed workers up to a certain wage, and then increases sharply. At the bottom of this distribution, unemployed workers have low unemployment benefits, there is therefore no job rejection from unemployed workers. The number of workers receiving each level of unemployment benefits only depends on the number of job destruction in the economy. As these destructions do not depend on the workers' wage, the two distributions have the same shape. Yet above a certain level of unemployment benefits, workers start to reject job offers and to therefore remain longer unemployed. The tail of the unemployed distribution according to their previous wage highlights the property 30 of the model: the higher the unemployment benefits, the longer the unemployment duration.

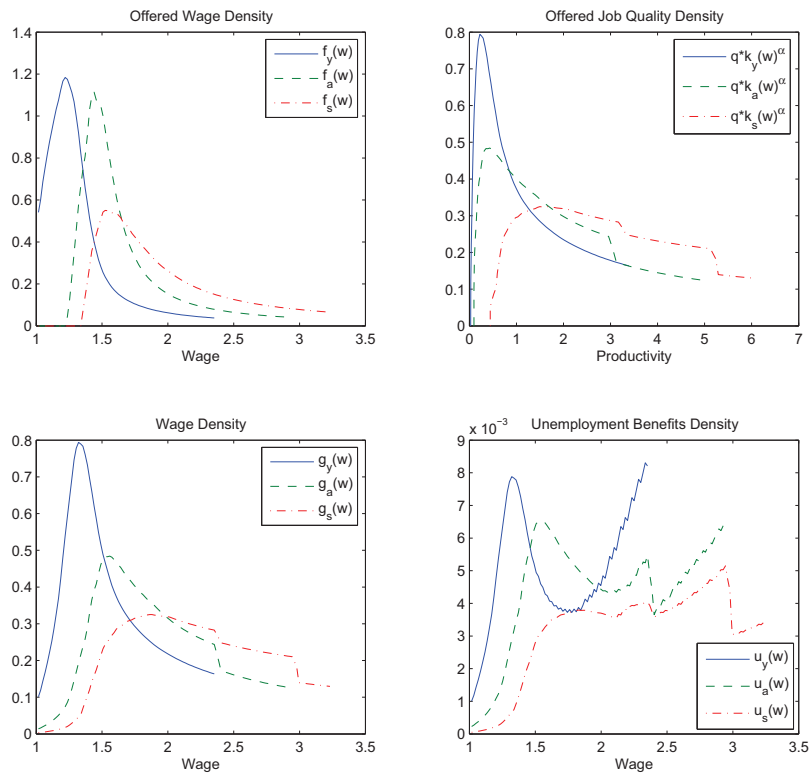
## 4.5 Effect of the unemployment insurance system

In this section, we propose to assess the effect of the unemployment insurance system on wage distribution, through notably its effect on the wage game (workers' market power and search). As the French and the U. S. economies are structurally different, we also show in this section that the same unemployment insurance system would have different effects if it was implemented in the U. S..

### 4.5.1 In France

In this subsection, we run a third simulation with the same parameters values of the benchmark economy given by table 4.1, yet without unemployment benefits: **Sim-**

Figure 4.6: Simulated distribution of offered wage, offered productivity, wage and offered job quality- Simulation 1B: French Benchmark economy



**ulation 2B: French benchmark without unemployment benefits ( $\rho = 0$ )<sup>4</sup>.** This simulation allows to assess in difference the unemployment benefits' effect on the French labor market. The simulation results are given in figure 4.7 (figures are given by table 14 of the appendix .2.2, page 219). Figure 4.8 draws a comparison in terms of wage, productivity and mobility of workers between the economy with unemployment benefits (simulation 1B) and the economy without unemployment benefits (simulation 2B).

From this comparison, four observable points are noteworthy:

1. The unemployment insurance system raises the wage, the wage offered, the quality of the jobs offered and the dispersion of the wage.
2. The unemployment insurance system reduces workers' mobility and therefore raises the unemployment rate and duration.
3. The search is hampered by the unemployment insurance system (its intensity is measured by the gap between the wage and the wage offered)
4. All these effects get stronger with workers' age.

Therefore, while the dispersion within age classes diminishes, the wage dispersion between age classes raises. We now comment the first three points, and explain why these three effects occur in greater proportion on the older workers' market.

(1) The first graph of figure 4.8 shows that the unemployment insurance system affects more significantly wage offered to older workers (and therefore the job quality offered on their market). As between the economy with and without unemployment

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<sup>4</sup>The fixed component of unemployment benefits,  $all = 0.33$ , has no effect on equilibrium since it is below minimum wage

Figure 4.7: Simulated distribution of offered wage, offered productivity, wage and offered job quality- Simulation 2B: French Benchmark economy without UB ( $\rho = 0$ )

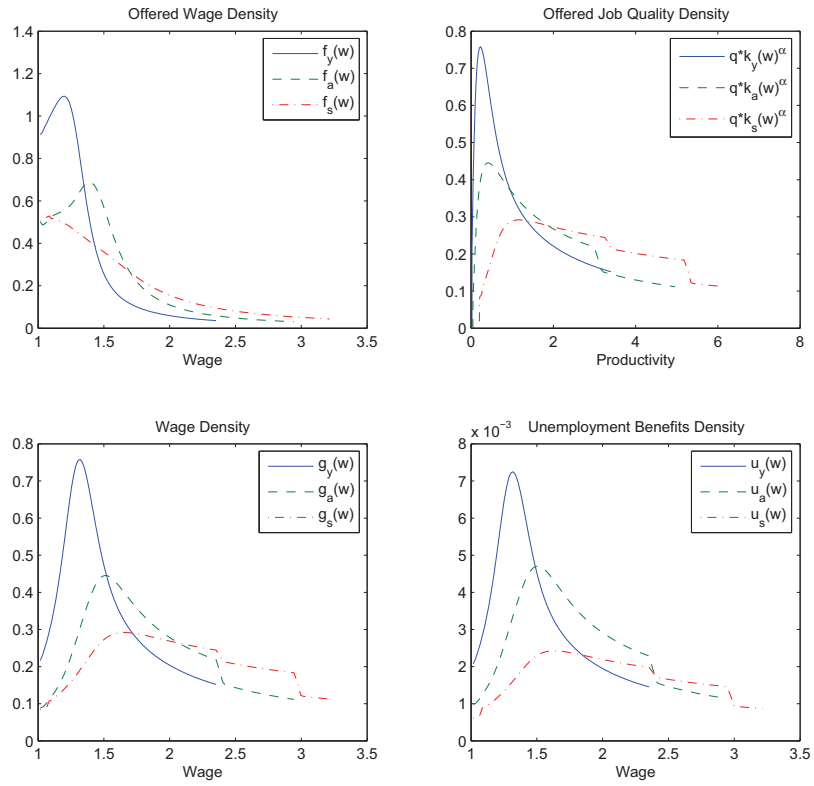
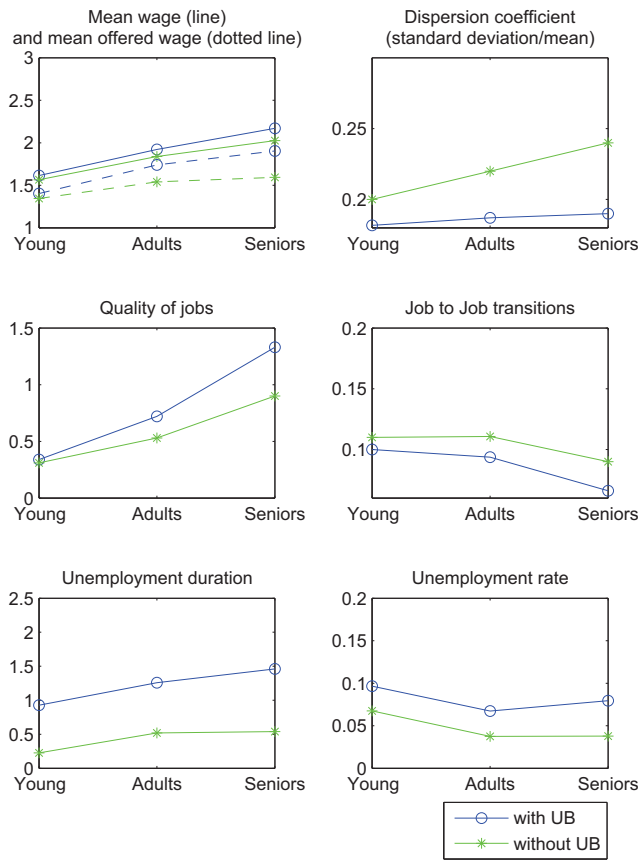


Figure 4.8: Comparison of the French economy with (simulation 1B) and without unemployment benefits (simulation 2B)





benefits, no change in workers' ex ante productivity occurs, the change in wage and job quality offered to workers comes from a shift in the workers' market power. The unemployment insurance system therefore raises the older workers' market power. It is clear on the first graph of figure 4.8 that the young workers' market power is rather close in the two simulations, the wages offered to the young are quite similar in the two simulations. The non homogenous effect of the unemployment insurance system on workers' market power over the life cycle affects wage progression. Table 4.3 displays the contribution of the unemployment insurance system to wage offer progression in the line "Progression due to UIS - Wage offer". Thanks to the unemployment insurance system, wage offer progression is reinforced by 0.65 points of percentage in the first part of the working life and by 0.47 points of percentage in the second part. This non homogenous effect of the unemployment insurance system on wage offer can be explained.

Young workers, when they enter the labor market, are entitled to the minimum benefits. Over their working life, thanks to working experience, they acquire rights to the unemployment insurance system. As the wage globally raises with workers' age, the adults are entitled to higher unemployment benefits than the young, and the seniors higher than the adults. Therefore, when the minimum wage is high, the young's unemployment benefits-induced reservation wages are likely to be below minimum wage. On the adults' and the senior's market on the contrary, higher benefits induce reservation wages above minimum wage. On these last two markets, firms' behavior need to change in order to avoid too many job rejections. The firms' behavior change on adults' and seniors' market can be observed in figure 4.6 and 4.7, by comparing the distribution of wage and of offered wage of simulations 1B and 2B. The unemployment insurance system prevents firms from offering low paid jobs, since they would be exposed to high

rate of job rejection. They therefore concentrate their wage offer. If wages are close to one another, wage offers close to this mode are more likely to be accepted since workers' reservation wages respect  $b(w) < w$ . In other terms, the unemployment insurance system causes no job rejection without wage dispersion. The wage dispersion decrease induced by the unemployment insurance system occurs therefore mostly at the bottom of the wage distribution.

(2) The sharp increase in both unemployment rate and unemployment duration (figure 4.8) is explained by both job rejections and a low labor market tightness on older workers' market. As older workers have higher a reservation wage, more job rejections occur on their market. Besides, as firms' profit is reduced by the unemployment insurance system on older workers' market, fewer are willing to enter those markets (this effect is also observed for the job to job transition rate). Note that, by comparing the figures of unemployment duration in France without unemployment benefits (figure 4.8) with the U. S. values (0.39 in France, and 0.33 in the U. S.), we can observe that the unemployment insurance system accounts for most of the unemployment duration gap between France and in the U. S..

(3) Table 4.3 displays the contribution of the unemployment insurance system to wage progression in the line "Progression due to UIS - Wage". Its contribution to wage progression is positive but smaller than on the wage offer progression. Thanks to the unemployment insurance system, wage progression is reinforced by 0.10 points of percentage in the first part of the working life and by 0.25 points of percentage in the second part. As explained in chapter 3, at equilibrium, the gap between offered wages and workers' wages depends on both the intensity and the gain of the workers' search. However, with an unemployment insurance system, this gap can also be explained by the job selection of unemployed workers (since they reject the less paid ones). The un-

employment insurance system has therefore a priori ambiguous effects on job selection since it decreases both the intensity and the gain of the workers' search. First the unemployment insurance system decreases the firms' profit, fewer vacancies are created and the workers' mobility rate decreases. Second, as wage offers are more concentrated, the workers' mobility at given job to job mobility rate generates lower wage gain. The first graph in figure 4.8 and the table 4.3 show that the gap between mean offered wage and the mean earned wage on the adults and seniors' market is significantly reduced by the unemployment insurance system. The effect of job selection of unemployed workers is therefore dominated by the decrease of the search and of the search-induced wage gain. Table 4.3 gives the extent of this job selection in the economy with and without unemployment benefits. In an economy without unemployment benefits, adults and seniors raise their wage by on average 20.13% and 31.76% by moving from selecting among good jobs, they raise it by only 10.34% and 14.03% in an economy with unemployment benefits. Table 4.3 also computes the wage progression explained by this mechanism (job selection) in the two economies. When, without unemployment benefits, this selection could contribute to the average wage progression by 0.23 points of percentage in the first part of the working life and 0.5 points of percentage in the second part, in an economy with unemployment benefits, they contribute negatively to the wage progression in the first part of the working life, and only by 0.28 points of percentage in the second part. Note that without unemployment insurance system, the contribution of this selection to the wage progression in the second part of the working life could be equal to the one observed in the U. S. economy, 0.5 points of percentage. In the first part, yet, this contribution remains 0.10 points lower.

The unemployment insurance system reinforces wage progression in spite of its negative effect on the intensity and the gain of the workers' search : it contributes to

wage progression by 0.10 points of percentage in the first part of the working life and by 0.25 points of percentage in the second part. Figure 4.8 shows that the unemployment insurance system partly accounts for the lower mobility of French workers, but not entirely. Thanks to this insurance system, wage progression can occur without high level of workers' mobility, in particular in the second part of the working life.

#### 4.5.2 In the U. S.

What would happen if we implemented the French unemployment insurance system in the U. S. ? We simulate the U. S. economy studied in the chapter 3 with the same level of unemployment benefits as there exists in France: **Simulation 3B: U. S. benchmark economy with unemployment benefits** ( $\rho = 0.49$  and  $all = 0.33$ ). Figure 4.9 draws a comparison between the two simulations, 1A and 3B. Note that figure 4.9 is expressed in French minimum wage for the values of mean wage and mean offered wage in order to ease the comparison with the French case (figure 4.8). By comparing figure 4.9 and figure 4.8, we notice that the effect of the unemployment insurance system is stronger on the U. S. economy: the unemployment insurance system induce a wage increase of on average 7.69% (all age classes mixed) instead of 2.19% in France, an unemployment duration increase of 515% instead of 221% and an unemployment rate increase by 144% instead of 89%. This stronger effect of the unemployment insurance system goes in particular through a more significant effect on the youth's labor market: the young are more affected by the increase of unemployment duration and unemployment rate induced by the unemployment insurance system than in France.

The global higher effect of unemployment benefits comes from first a structural (comparison between the two economies without unemployment benefits) higher level of wage dispersion in the U. S.. This dispersion is induced on the top of the distribution

Table 4.3: Selection of good jobs in the economy without unemployment benefits (simulation 2B) and with unemployment benefits (simulation 1B). Effect by age and contribution to the wage progression. Contribution of unemployment insurance system (UIS) to wage and wage offer progression

	Young	Adults	Seniors	Evolution per year Y → A	Evolution per year A → S
Simulation 2B: French benchmark without unemployment benefits ( $\rho = 0$ )					
Mean wage ( $g_i$ )	1.56	1.84	2.02	1.38% <sup>‡</sup>	0.75%
Mean wage offer ( $f_i$ )	1.34	1.54	1.59	1.15%	0.25%
Selection by age - 2B	15.67%*	20.13%	31.76%		
Simulation 1B: French benchmark economy					
Mean wage ( $g_i$ )	1.61	1.92	2.17	1.48%	1%
Mean wage offer ( $f_i$ )	1.4	1.74	1.9	1.85%	0.72%
Selection by age -1B	14.75%	10.34%	14.03%		
Progression due to selection of good jobs					
Progression due to selection of good jobs - 2B				0.23 <sup>†</sup> points of %	0.5 points of %
Progression due to selection of good jobs - 1B				-0.37 points of %	0.28 points of %
Progression due to UIS					
Progression due to UIS - Wage offer				0.65 <sup>‡</sup> points of %	0.47 points of %
Progression due to UIS - Wage				0.10 points of %	0.25 points of %
* Young workers achieve to raise their wage by 15.67% by selecting into best jobs in economy 2B ( $\frac{1.56}{1.34} - 1$ )					
‡ Over the first half of the working life, mean wage raises by 1.38% per year in economy 2B ( $\frac{\frac{1.84}{13} - 1}{1.56}$ )					
† On the 1.38% of yearly wage increase over the first half of the working life, 0.23 points are induced by the selection of good jobs in economy 2B (1.38% - 1.15%)					
‡ On the 1.85% of yearly wage offer increase over the first half of the working life, 0.65 points are induced by the UIS in economy 1B (1.85% - 1.15%)					

by a higher level of job to job mobility and at the bottom of the wage distribution by the lower minimum wage (20% lower). When wages are more spread out, so are the unemployment benefits and more job rejections occur. As a low minimum wage allows a higher dispersion at the bottom of the wage distribution in particular on the young's market, this effect is stronger on their market.

Besides, in the U. S., employment to unemployment and unemployment to employment mobilities are structurally more frequent:  $s = 0.072$  in France and  $s = 0.12$  in the U.S. and the average unemployment duration without unemployment benefits are 0.5 years in France and 0.23 years in the U. S.. U. S. workers therefore go more frequently through unemployment periods in their life and the reservation wage of unemployed workers affects in greater proportion the labor market (figure 4.8).

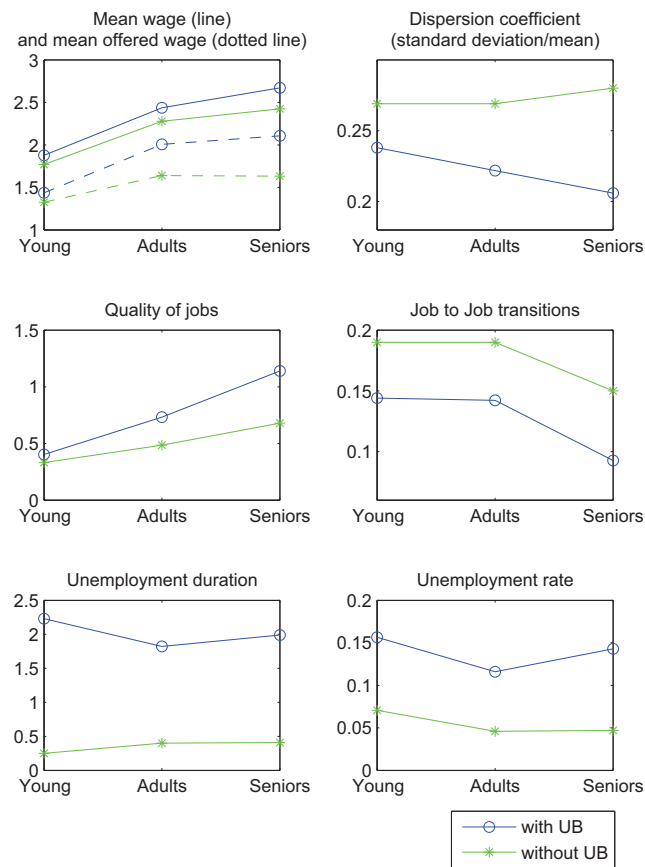
## 4.6 Institutional channel and wage progression over the life cycle

We now assess the global effect of the French institutions on the wage distribution over life cycle. We start by studying the effect of the minimum wage, then the effect of a change in workers' horizon in both France and the U. S., and eventually decompose the wage progression into three channels, the wage game channel, the productivity channel, and the institutional channel.

### 4.6.1 Effect of a high minimum wage

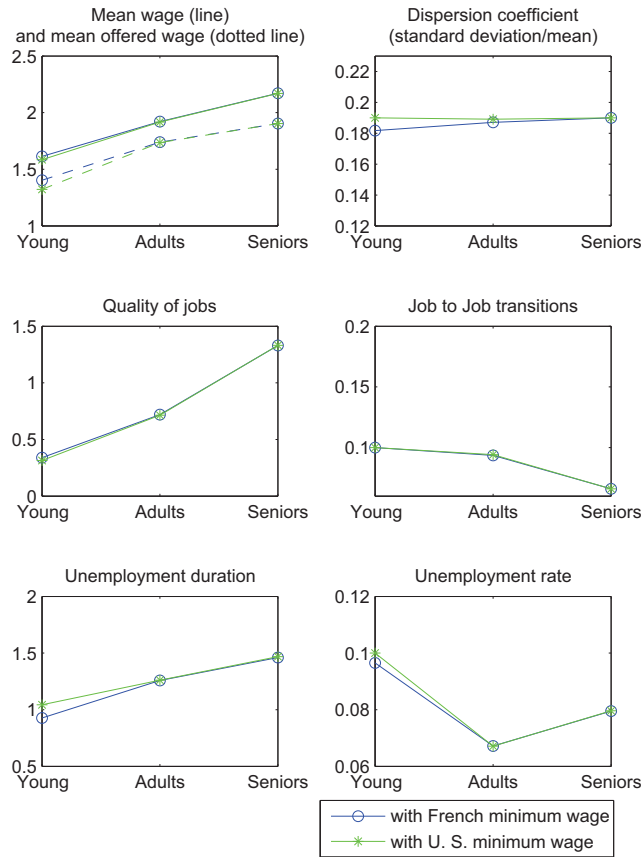
The minimum wage in France is about 24% higher than in the U. S. . To assess how the presence of this high minimum wage affects the wage distribution over the life cycle we simulate the French benchmark with the level of the U. S. minimum

Figure 4.9: Comparison of the U. S. economy with (simulation 1A) and without unemployment benefits (simulation 3B), expressed in French minimum wage for mean wage and mean offered wage



wage: **Simulation 4B: French benchmark with U. S. minimum wage level** ( $\underline{w} = 0.8$ ). The difference between this simulation and the benchmark economy (1B and 4B) informs us on the effect of a decrease in the minimum wage from the French to the U. S. level. This comparison is displayed in figure 4.10.

Figure 4.10: Comparison between the French benchmark economy (simulation 1B) and the French economy with U. S. minimum wage level ( $\underline{w} = 0.8$ ) (simulation 4B)



It is clear according to figure 4.10, that the decrease of the minimum wage mostly af-



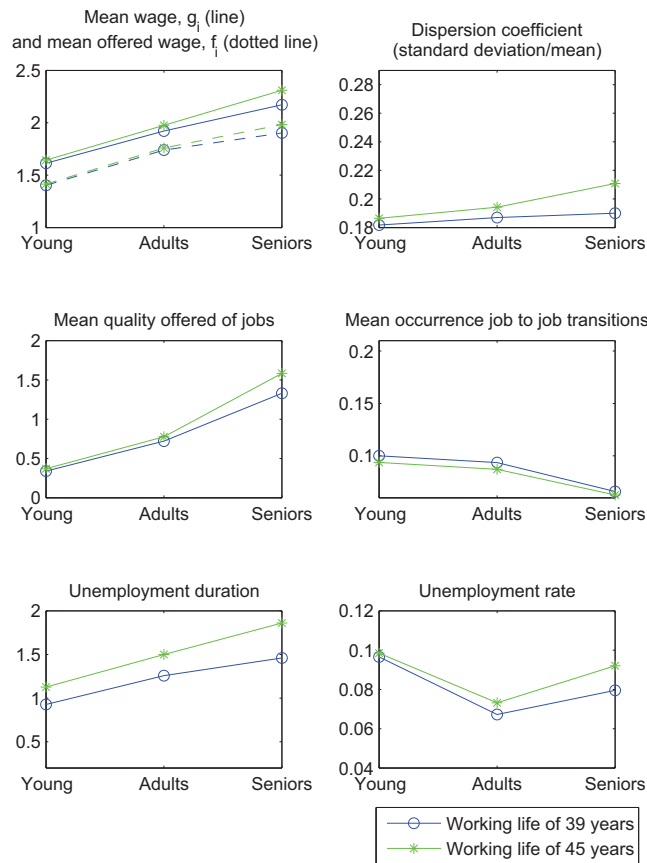
fects the young's labor market: it decreases the wage offered on their market, decreases very slightly their wage, raises their wage dispersion and raises both their unemployment duration and unemployment rate. A lower minimum wage allows some firms to develop very low wage strategies and to offer wages lower than previously, here between 0.8 and 1. These firms have high surplus yet very low hiring frequencies: due to unemployment benefits most workers reject those wage offers, only new entrants can accept them. These job rejections explain the increase in unemployment duration after a decrease of the minimum wage. They also have a very low retention. This explains why the mean wage is weakly affected by this change: new entrants accept these low wages yet dismiss very quickly for higher paid jobs. The increase in wage dispersion is therefore explained by the existence of these very low wages far from the mean wage. Overall, a decrease in the minimum wage only affects negatively the young's wage and to a very small extent in an economy with the French unemployment insurance system.

#### 4.6.2 Effect of the retirement age

The actual average age of retirement in France in 2002 is around 59 years old, that is 6 years earlier than in the U. S. How a lengthening of the French working life as it is progressively planned in France would affect the wage distribution of workers of each age class? To answer this question, we run a simulation in which workers retire after 45 years of activity: **Simulation 5B: French benchmark economy with a 45 years working life** ( $p = \frac{1}{15}$ ). The comparison between this simulation and the benchmark economy is given by the figure 4.11. In order to understand how the structure of the economy affects the way labor market outcomes react to a change of horizon, we compute a simulation of the U. S. economy with an horizon of 39 years as it exists in France: **Simulation 6B: U. S. benchmark economy with a 39 year working life**

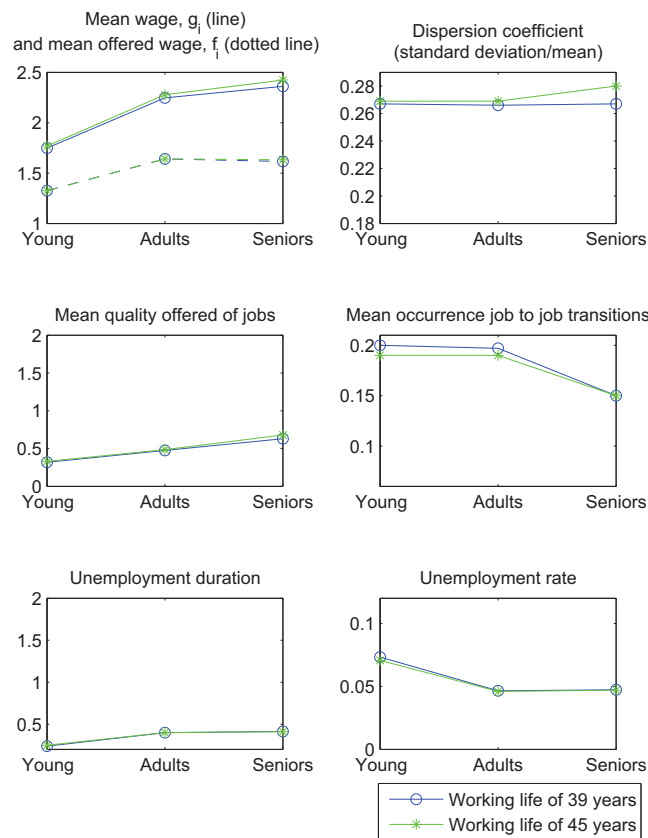
( $p = \frac{1}{13}$ ). The comparison between this simulation and the U. S. benchmark economy is given by the figure 4.12. Note that, here again, figure 4.12 is expressed in French minimum wage for the values of mean wage in order to ease the comparison with the French case (figure 4.11).

Figure 4.11: Comparison between the French economy with a 39 year long (simulation 1B) working life and a 45 year long working life (simulation 5B)



In both economies, lengthening horizon allows seniors to earn higher wages. In

Figure 4.12: Comparison between the U. S economy with a 39 year long (simulation 6B) and a 45 year long working life (simulation 1A), expressed in French minimum wage for mean wage and mean offered wage



France, this seniors' wage increase comes from the increase in offered wage and offered job productivity. Indeed creating high quality jobs on seniors' market becomes more profitable for firms which expect longer job tenure. The seniors' market power raises and firms offer higher wages to these workers whose retention is now manipulable with wage (to avoid poaching). This effect also occurs on the adults' market yet to a very least extent. The higher competition between firms to retain workers on the seniors' market induces also higher a wage dispersion. As seen in 4.5, this wage dispersion induces wage rejections in an economy with unemployment benefits. The unemployment insurance system keeps the wage increase going: a wage increase generates a reservation wage increase that generates higher wage offers, and so on... Besides, they also explain the increase in unemployment duration and unemployment rate among adults and seniors.

Lengthening the working horizon in a country like the U. S. has far smaller consequences: seniors' wage raises by 2.7% after an increase of 6 years in horizon in the U. S. (with the same starting horizon of 39 years), for 6.5% in France. In the U. S. the seniors' market power does not raises (firms propose the same wages whatever the horizon is). The only (small) positive effect of a longer horizon on the seniors' market comes from an increase in the search effect: the gap between offered wages and workers' wages very slightly raises. Workers have more time to select among best paid jobs.

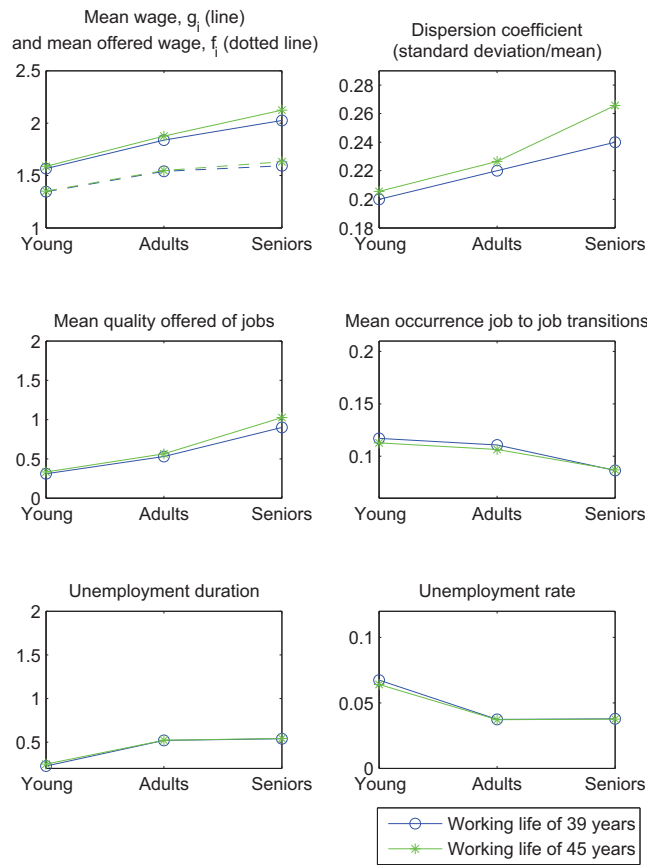
Why does the seniors' market power raise in France with a lengthening of horizon and not in the U. S.? There are two explanations. First, in a country like France, an increase in horizon raises significantly the seniors' job tenure since the exogenous job destruction rate is low. This increase in job tenure induces firms to increase wage offers. In the U. S., the exogenous job destruction rate is high, whatever the workers' social horizon is, firms anticipate short job tenure. Given the difference in exogenous job destructions in the two countries, a U. S. firm anticipates its survival rate over a

15 year period at 20%, when a French firm anticipates it at 40%. The horizon rate matters therefore more for a French firm than for a U. S. one. Secondly, in France, given the unemployment insurance system, a slight increase in wages can reinforce the market power of workers and raises again the wage of workers and so on. This mechanism can also contribute to the higher impact of lengthening the horizon in France. To assess the respective contribution of these two effects, we simulate the French economy without unemployment with a 65 years long horizon: **Simulation 7B: French benchmark economy without unemployment benefits and with a 45 years working life** ( $p = \frac{1}{15}$ ). Figure 4.13 draws a comparison between this simulation and the economy without unemployment benefits yet with a 39 years working life (3B). Even without the effect of unemployment benefits the seniors' market power raises, which means that structurally the French economy by generating longer job tenure react more to a lengthening of horizon. Yet, the unemployment insurance system reinforces this effect. Seniors' wage raises by 4.9% after an increase of 6 years in horizon in the French economy without unemployment benefits, and by 6.5% in the French economy with unemployment benefits.

### 4.6.3 Wage progression and dispersion decomposition

Like in chapter 3, we can decompose the wage progression over workers' life-cycle. We now use three channels of wage progression: the wage game channel, the productivity channel and the institutional channel. The mechanisms by which the first two channels affect the wage trajectory of workers are similar here as in chapter 3, yet their consequences on the wage distributions differ. We simulate the French benchmark economy when workers are ex ante evenly productive and when no labor market institution described in this chapter is present, i.e. the minimum wage is at the U. S. level, there

Figure 4.13: Comparison between the French economy without unemployment benefits and with a 39 year long working life (simulation 3B) and without unemployment benefits and a 44 year long working life (simulation 7B)



is no unemployment benefits, and the working life lasts 45 years : **Simulation 8B: French benchmark economy with ex ante evenly productive workers and no institution** ( $y_y = y_a = y_s = 1.3$  and  $\beta_y = \beta_a = \beta_s = 1$ ), and  $\underline{w} = 0.8$ ,  $\rho = 0$ , and  $p = \frac{1}{15}$ ). This simulation allows us to study the wage game in France. Then we simulate the same economy yet with the evolution of workers' ex ante productivity: **Simulation 9B: French benchmark economy without institution** ( $\underline{w} = 0.8$ ,  $\rho = 0$ , and  $p = \frac{1}{15}$ ). Some key figures of this simulation are given in appendix .2.3, page 220. In difference with simulation 8B, this simulation allows us to assess the productivity channel. In difference with simulation 1B, it allows to assess the institutional channel. Table 4.4 displays the results of this decomposition. Like in the U. S., the wage game channel accounts for part of the wage progression in the first part of the working life and contributes negatively to this progression over the second part. Like in the U. S., the productivity channel contributes greatly to the wage progression. Yet in France this increase occurs at an increasing rate, when it occurs at a decreasing rate in the U. S..

The effect of the institutional channel is theoretically ambiguous since the unemployment insurance system reinforces wage progression but both the minimum wage and the short horizon reduce it, the first one by raising the young's wage and the second by lowering the seniors' wage.

Table 4.4 shows that the global effect of institutions is positive on the wage progression. Viewed in this light, the effect of the unemployment insurance system dominates the effect of the two other institutions. Yet, whereas the unemployment insurance system favors to a greater extent seniors' wages, the institutional channel contributes to wage progression at a decreasing rate. The effect of the short horizon therefore compensates partly the unemployment insurance system effect in the second part of

the working life. On the 1.48% of yearly wage progression that workers experience in the first part of their working life, 0.19 points of percentage is due to the presence of institutions on the labor market. On the 1% of yearly wage increase that workers experience in the second part of their working life, 0.12 points of percentage is due to that presence. Table 4.5 displays the effect of institutions on the wage dispersion. Institutions decrease consequently the wage dispersion in each age class. Indeed the three institutions studied here tend to contract the wage distribution. This decrease is stronger with age since both the unemployment insurance system and the shortening of horizon contract more particularly older workers wage distribution. Institutions in France favor older workers in terms of earnings. Besides, they contract wage distribution in each age class. Therefore, they raise in the same time the wage inequality between age classes and decreases it within age classes.

Table 4.4: Decomposition of wage evolution

	Young	Adults	Seniors	Evolution Y→A	Evolution A →S
Monopsony economy					
Mean wage	1	1	1	0%	0%
With on the job search and endogenous productivity (8B)					
Mean wage ( $g_i$ )	1.6	1.64	1.52	0.18%	-0.50%
With evolution of workers'ex ante productivity (9B)					
Mean wage ( $g_i$ )	1.55	1.85	2.09	1.29%	0.88%
With institutions (1B)					
Mean wage ( $g_i$ )	1.61	1.92	2.17	1.48%	1%
Decomposition of wage progression					
Wage game channel per year				0.18 points of %	-0.5 points of %
Productivity channel per year				1.11 points of %	1.38 points of %
Institutional channel per year				0.19 points of %	0.12 points of %



Table 4.5: Effect of Institutions on wage dispersion

	Young	Adults	Seniors
Without institution (9B)			
Dispersion coefficient	0.23	0.24	0.28
With institutions (1B)			
Dispersion coefficient	0.182	0.187	0.190
Institution effect	-0.048%	-0.053%	-0.09%

## 4.7 Conclusion

This chapter allows to assess the life cycle effect of some major French institutions. A significant share of wage progression in France is fueled by the unemployment insurance system. Besides, this insurance system also accounts for a large decrease of search-induced wage progression, via its negative effect on both mobility rate and wage dispersion. Figure 4.8 shows that the presence of unemployment benefits partly accounts for the lower mobility of French workers, yet not entirely. The unemployment insurance-induced wage contraction could partly explain the remaining gap between the job to job mobility rate in France and in the U. S.. In our framework, workers change jobs as soon as they receive an offer above their current wage, the gap between the two wages does not matter, only does the order of the wages. In this context, the wage dispersion affects weakly the occurrence of mobility (only when firms post the exact same wage). In reality, mobility is costly for workers, and they do not change jobs unless the wage gap between the two jobs compensates this cost. Adding mobility cost on the workers' side could reinforces the negative effect of the unemployment insurance system on the mobility rate found in this chapter. An other result of this chapter is worth exploring. The wage progression decomposition shows that human capital accumulation contributes more to wage progression in France than in the U. S.. This result was expectable and could be explained if we introduce both search effort and heterogenous productivity of workers. In this case, high productivity workers would choose higher a search effort than low productivity workers (because for instance firms direct their search or because job destruction depends on productivity). This scenario is particularly expectable on French seniors' labor market, since they receive high level of unemployment compensation which insures their consumption. Besides, in France, a

low search effort on this market is almost synonymous of an exit from the labor market given the few job opportunities existing on seniors' market. By this mechanism, in countries like France, we can observe a composition effect that explains in this model that the workers' productivity raises significantly at the end of the life cycle. The problematic of low search effort at the end of the life cycle naturally echoes the endogenous retirement issue. We develop this issue in the general conclusion.

# Conclusion générale

L'objectif principal de cette thèse est de montrer comment l'âge affecte tous les "résultats" du marché du travail et en particulier la distribution des salaires. Les deux dimensions fondamentales de l'âge sont prises en compte : l'existence d'une date d'entrée sur le marché du travail et l'existence d'une date terminale. L'écart à la date d'entrée sur le marché du travail constitue l'expérience du travailleur. L'expérience modifie un certain nombre de grandeurs sur le marché du travail, et notamment toutes les variables d'accumulation : les gains cumulés issus des mobilités ascendantes connues au cours de la vie, l'accumulation de capital humain ou encore l'accumulation de droit à l'assurance chômage. A mesure que le travailleur gagne en expérience, la distance à sa retraite diminue. L'écart à cette date terminale sur le marché du travail constitue l'horizon social du travailleur. L'horizon d'un travailleur modifie toutes les variables d'investissement. Du côté des firmes d'abord, il agit sur l'investissement en salaire en vue de retenir le travailleur employé, sur l'investissement en capital humain spécifique à un emploi, et sur l'investissement en recherche d'emploi (l'ouverture d'emplois vacants). Du côté du travailleur, il agit sur l'investissement en effort de recherche. L'effet conjugué de ces deux dimensions de l'âge sur le jeu de salaires est la préoccupation centrale de cette thèse.

Le chapitre 1 est le seul chapitre où l'âge n'est pas pris en compte. Dans ce chapitre, l'horizon est technologique. Dans ce contexte, nous montrons que les taxes sur les licenciements n'allongent pas nécessairement la durée emplois, et qu'il existe, en présence d'allocations chômage progressives, des fondements à ce que cette taxes soit croissante.

Les trois chapitres suivant traitent de l'effet de l'âge sur la distribution des salaires. Dans un monde où tous les emplois ont la même productivité, et où l'intensité de recherche est constante, l'expérience semble être un atout indéniable : les travailleurs seniors accumulent au cours de leur vie un pouvoir de marché sur les entreprises et se voient offrir un plus grand nombre de hauts salaires que lorsqu'ils étaient jeunes. En outre même à offre de salaire identique, l'effet de la recherche d'emploi en emploi leur assure une progression salariale tout au long de leur vie active. L'effet de l'horizon dans un tel monde n'agit que sur la stratégie salariale des firmes à travers la faible rétention des travailleurs les plus âgés. Toutefois, quand l'effet de l'horizon passe également par le canal de l'intensité de la recherche d'emploi, les seniors sont en situation moins favorable et connaissent une stagnation de leurs salaires en fin de vie active.

Que se passe-t-il quand les emplois ont des productivités différentes ? La productivité des emplois est le résultat en même temps du stock de connaissances du travailleur - nous sommes donc dans une logique d'accumulation- et de la volonté de formation de ce travailleur par la firme - nous sommes là dans une logique d'investissement-. Il est possible de dissocier les deux aspects de cette productivité. Dans un monde où les travailleurs n'accumuleraient pas de connaissance au cours de leur vie, et où ainsi l'hétérogénéité de la productivité des emplois ne viendrait que des décisions d'investissement des firmes, les salaires ne pourraient croître qu'au début de la vie active du travailleur ; en deuxième partie, ils chuteraient de façon conséquente. Dans ce contexte, les jeunes et les adultes seraient employés sur les postes les plus productifs des entreprises, ceux requérant une

formation coûteuse (un investissement). A l'inverse, les seniors seraient, à l'exception de ceux embauchés plus jeunes et toujours en emploi, employés dans des emplois faiblement productifs. L'accumulation de capital humain général au cours de la vie change la donne. Elle a pour premier effet de réduire le coût du capital humain spécifique pour les firmes. En d'autres termes, la formation nécessaire à l'obtention d'une même productivité est moins coûteuse chez les travailleurs expérimentés. Les firmes peuvent grâce à cette accumulation continuer de former les travailleurs même si leur horizon est court. L'accumulation de capital humain général des travailleurs agit aussi naturellement sur la productivité directe du travailleur sur son poste de travail (sans formation de la part des firmes, les travailleurs ont des productivités différentes) : cette productivité est plus basse chez les jeunes que chez les adultes et les seniors. Toutefois d'autres facteurs agissent sur cette productivité. En effet, aussi bien aux États-Unis qu'en France, elle décroît dans la deuxième partie de la vie active. Si l'expérience se traduit majoritairement par l'acquisition de connaissances transférables d'un emploi à un autre qui accroît la productivité, elle va aussi de paire avec le vieillissement biologique du travailleur. Aussi des facteurs biologiques peuvent-ils entrer en ligne de compte : baisse de la résistance, douleurs au travail, augmentation de l'occurrence de maladie, etc... Que ce soit via le coût du capital humain spécifique ou la hausse de la productivité du travailleur sur son poste de travail, l'expérience a des effets globalement très positifs sur la productivité de l'appariement. Cette hausse de la productivité permet de renforcer la progression salariale des travailleurs dans la première partie de leur cycle de vie et d'expliquer celle de la deuxième partie. Cette progression passe par deux mécanismes. D'abord la hausse de la productivité accroît le pouvoir de marché des travailleurs : les firmes offrent des salaires supérieurs à des travailleurs plus productifs. Ensuite, elle améliore quantitativement et qualitativement la mobilité entre emplois des travailleurs : un plus

grand nombre de firmes recherchent des travailleurs expérimentés (création d'emplois vacants), et la plus grande dispersion salariales désormais proposée permet des gains issus de la mobilité plus grands. Ce deuxième mécanisme est particulièrement significatif aux États-Unis.

Le constat selon lequel la mobilité entre emploi est crucial pour expliquer la progression salariale dans un pays comme les États-Unis pose la question de l'origine de cette progression dans des pays à faible taux de mobilité. Un pays comme la France présente de faibles probabilités de transition d'emploi à emploi en particulier chez les séniors, toutefois la progression salariale ne semble pas en souffrir. La présence d'un système généreux d'assurance chômage explique en partie cette apparente contradiction. En France, l'expérience permet une troisième forme d'accumulation (après celle des mobilités ascendantes et du capital humain), celle des droits à l'assurance chômage. Un travailleur expérimenté est éligible à des allocations chômage supérieures à celle d'un jeune entrant sur le marché du travail. Cette accumulation a plusieurs effets. D'abord, elle augmente le pouvoir de marché des travailleurs les plus âgés : les firmes se voient contraintes d'offrir à un travailleur proposant la même productivité, un salaire supérieur. Ensuite, en réduisant la marge de manoeuvre des firmes, elle réduit la dispersion de la distribution d'offres de salaires. Enfin, elle réduit le nombre de création postes sur le marché des travailleurs les plus âgés (ceux ci devenant moins rentables car trop coûteux), ce qui a pour effet de réduire l'occurrence des mobilités sur ce marché. Ce système d'assurance chômage explique donc en même temps en partie la faible mobilité des travailleurs les plus âgés et leur progression salariale. Toutefois d'autres institutions agissent sur la distribution des salaires en France : le salaire minimum et l'âge de départ à la retraite. La présence d'un salaire minimum élevé agit uniquement sur le marché du travail des jeunes. En outre en présence d'allocations chômage inchangées, il n'a

pour effet que de diminuer leur taux de chômage, la hausse de leur salaires restant, elle, minime. Un abaissement de l'âge de départ en retraite agit lui sur le marché du travail des seniors. Ces derniers voient leurs salaires baisser de façon conséquente : leur horizon se raccourcissant, les firmes cessent en partie d'investir sur leur marché en salaire, en capital humain et en frais de recherche (en ouvrant des postes). Cette baisse des salaires est auto-entretenu par la présence d'allocations chômage : de faibles salaires génèrent de faibles allocations chômage qui à leur tour réduisent le pouvoir de marché des travailleurs etc... D'autre part, une comparaison transatlantique de l'effet de l'âge de départ en retraite sur la distribution des salaires montre que la durée moyenne des emplois dans l'économie est un facteur essentiel à prendre compte. Des emplois en moyenne courts subissent moins les conséquences néfastes d'un horizon proche, puisque la dimension investissement y est au départ moins présente. Un abaissement (de même ampleur) de l'âge de départ en retraite aux États-Unis aura donc un effet moindre sur les salaires des seniors qu'en France du fait de l'existence d'emplois plus courts et d'absence de système progressif d'assurance chômage. Prises comme un tout, les institutions étudiées en France : système d'assurance chômage, salaire minimum élevé et âge précoce de départ à la retraite favorisent le salaire des travailleurs âgés et accroît donc la progression salariale. L'ensemble de ces institutions a également pour effet une réduction de la dispersion des salaires au sein de chaque classe d'âge. L'environnement institutionnel français accroît les disparités salariales entre âge mais les réduit entre travailleurs de même âge.

L'originalité de ce travail est la prise en compte de l'âge, sous toutes ses dimensions, dans un modèle de recherche d'emploi. De cette prise en compte émergent de nouveaux arbitrages. Les premiers naissent autour de la stratégie salariale des firmes : les salaire de réservation des travailleurs augmentent avec l'âge, ce qui incite les firmes



à accroître leurs salaires afin d'augmenter leur probabilité d'embauche, d'un autre côté l'horizon diminue et avec lui toute incitation à la rétention du travailleur. D'autres naissent autour des décisions d'investissement en capital humain spécifiques : un court horizon freine les décisions d'investissement en capital humain, toutefois il suppose que le travailleur est expérimenté et possède déjà un haut stock de capital humain, ce qui réduit le coût de cet investissement. Enfin les derniers naissent autour des décisions d'ouverture d'emplois vacants : le surplus engendré par un appariement augmente avec la hausse de la productivité du travailleur et diminue avec aussi bien la hausse du salaire de réservation des travailleurs que le raccourcissement de leur horizon.

Outre le fait que la problématique de l'âge dans les modèles de recherche d'emploi réponde à un questionnement très positif : pourquoi les salaires et les flux sur le marché du travail évoluent-ils avec l'âge ?, elle présente aussi un intérêt théorique. De ce point de vue, exceptés les mécanismes de recherche d'emploi des travailleurs qui ont fait l'objet de travaux (Bagger, Fontaine, Postel-Vinay, and Robin, 2012), une grande partie des mécanismes générés par la prise en compte simultanée de l'expérience et de l'horizon sont nouveaux dans le jeu de postage des salaires. Ils sont présents dans (Menzio, Telyukova, and Visschers, 2012), toutefois, ses auteurs ne tiennent pas compte des stratégies d'investissement en capital humain, qui sont une des composantes clé de la dynamique des salaires individuels. En outre, en construisant un modèle destiné à être estimé, ils ne peuvent pas l'utiliser pour une analyse théorique de ces mécanismes, sa taille étant trop importante.

Ce travail est aussi l'amorce d'une réflexion plus profonde sur la prise en compte de l'âge social sur le marché du travail. Dans ce cadre, deux extensions sont imaginables.

Dans cette thèse, les firmes dirigent leur recherche sur des classes d'âge. Étant

donné que nous n'instaurons aucun contrat séparateur permettant de trier dans notre économie, les jeunes, les adultes et les seniors, cette hypothèse de recherche dirigée repose sur la parfaite observabilité par les firmes de l'âge des travailleurs, et de ses deux composantes, l'expérience et l'horizon. A cet égard, nous rejoignons le travail de (Menzio, Telyukova, and Visschers, 2012) et (Burdett, Carrillo-Tudela, and Coles, Forthcoming). Toutefois selon des études empiriques sur l'âge des départs en retraite ((Gruder and Wise, 1999) or (Hairault, Langot, and Sopraseuth, 2010)), l'horizon social d'un travailleur est en réalité assez loin d'être parfaitement observable. Cette inobservabilité tend en outre à croître avec le développement de dispositifs d'assouplissement de l'âge de départ à la retraite dans plusieurs pays européens. Certains pays sont passés d'un âge normal de la retraite fixe à une fourchette d'âge. Au Royaume-Uni par exemple, il n'existe pas d'âge "normal" et les contrats de travail ne prennent pas automatiquement fin à un âge déterminé. Il n'est désormais plus obligatoire de cesser de travailler pour toucher une pension. En Norvège, l'âge de perception de la retraite est de 67 ans, mais les assurés peuvent continuer à travailler. Dans certains pays, l'âge de la retraite est directement lié à la longévité. Cela nécessite souvent d'adopter une réglementation supprimant la cessation automatique des contrats de travail à un âge précis. Dans ce contexte, discriminer les travailleurs sur leur horizon devient plus difficile, seule l'expérience est observable mais elle n'est plus entièrement corrélée à l'horizon du travailleur. Il est par contre assez probable que les travailleurs connaissent leur horizon social, nous sommes donc en présence d'asymétrie d'information. Prendre en compte cette asymétrie d'information sur l'horizon des travailleurs par les firmes pourrait être une extension naturelle du cadre développé dans cette thèse. Cette prise en compte apporterait un regard très différent à la stratégie salariale des entreprises. Les firmes pourraient en effet faire un tri entre travailleurs à

long horizon et à court horizon en mettant en place des contrats séparateurs. Il semble intuitif qu'une progressivité forte du salaire sera préférée par un travailleur ayant alors un horizon long alors qu'un travailleur se sachant plus limité dans le temps en choisira un lui fournissant des flux de salaires importants à court terme. Cette idée a été appliquée au tri entre travailleurs ayant des probabilités de démission différentes par (Salop, 1976). Comment le jeu de salaire est-il affecté par l'évolution de la forme des contrats de salaires avec l'âge des travailleurs? Cette question pourrait être au coeur d'un travail qui prendrait en compte les conséquences de cette évolution en même temps sur la progression salariale des travailleurs et leurs taux de mobilité d'emploi à emploi ((Burdett and Coles, 2003)). En outre, l'intérêt de traiter de l'existence d'un contrat séparateur dans un modèle de recherche permet de se débarrasser du problème de la forte probabilité d'occurrence d'un équilibre avec contrat mélangeant.

Les dispositifs d'assouplissement de l'âge de départ à la retraite posent également de façon assez immédiate la question d'une décision de départ en retraite endogène de la part des travailleurs. C'est ce qui est fait dans le cadre simple d'un modèle de matching, sans distribution endogène de salaire, par (Hairault, Langot, and Zylberberg, 2012) Prise en compte par les entreprises, cette décision pourrait affecter significativement la distribution des salaires des seniors : dans un cadre où les travailleurs ont des productivités hétérogènes, les meilleurs travailleraient plus longtemps que les autres et se verraient offrir des salaires plus importants que si leur départ en retraite était exogène. Les firmes en effet seraient incitées à augmenter leurs salaires qui aurait désormais un effet sur leur décision de départ en retraite. Les décisions d'investissement seraient, elles aussi, remises en cause.

Un autre type d'extension permettrait de rendre compte de façon plus complète de l'effet de certaines institutions sur la distribution des salaires et des productivités au

cours du cycle de vie. Dans les modèles présentés dans le chapitre 3 et 4, l'accumulation du capital humain est exogène, elle ne dépend d'aucun paramètre du modèle. En effet, nous considérons que l'accumulation de ce capital humain est le résultat d'un processus de "learning by doing", qui sans coût vient accroître le stock de connaissance et la productivité du travailleur. Une partie du stock de capital humain général des travailleurs provient d'un investissement, en toute logique opéré avant même l'entrée sur le marché du travail : la formation initiale. Toutefois il existe aussi, du fait des chocs technologiques des possibilités d'investissement en capital humain général au cours de la vie. Les révolutions technologiques peuvent justifier une mise à jour de la formation initiale. Les travailleurs maîtrisent donc en partie le rythme de leur accumulation de capital humain durant leur vie active. Dans les chapitre 3 et 4, nous avons vu que la progression salariale prenait sa source dans des mécanismes différents en France d'aux Etats-Unis. Aux États-Unis, les emplois sont courts et une part importante de la progression salariale se fait grâce aux mobilités entre emplois. A l'inverse, en France, les emplois sont plus longs et l'assurance chômage permet en partie d'assurer la progression salariale. Ces différences agissent grandement sur les incitations d'investissement en capital humain général des travailleurs, aussi bien au début de leur vie qu'au cours de leur vie active. (Wasmer, 2006) explique ces différences dans une comparaison entre le marché du travail européen et américain. Aux États-Unis, un fort capital humain général permet de compenser l'instabilité de l'emploi courant par la possibilité d'obtenir un autre emploi bien rémunéré facilement. En France (ou en Europe continentale), la présence de ce capital humain général n'est pas indispensable à la progression salariale. En effet, la présence en capital humain spécifique à l'entreprise peut-être suffisante : d'une part, elle garantit au cours d'un emploi long un salaire convenable, et d'autre part, même après la destruction de cet emploi, la présence de l'assurance chômage

indexée sur le salaire du dernier emploi, donc prolongeant l'effet du capital humain spécifique, protège le travailleur contre la régression salariale pour les emplois suivants. (Ljungqvist and Sargent, 1998) utilisent un mécanisme similaire en montrant que les allocations chômage en présence d'accumulation de capital humain spécifique sont génératrice de chômage puisque indexé indirectement sur un capital humain non transférable d'un emploi à l'autre. Dans le cadre théorique développé dans cette thèse, il serait possible d'endogénéiser une partie de l'accumulation du capital humain général des travailleurs. Grâce à ces nouveaux comportements, nous observerions l'effet des institutions sur la distribution des salaires au cours du cycle de vie à travers leur impact sur l'investissement en capital humain général et spécifique, des deux côtés du marché.

# Tenure-dependent Firing Taxes

## .1 Tenure and Firing Taxes in OECD

Table 6: Firing taxes and tenure: some stylized facts (source OEDC [1999])

Austria	2 months > 3 years, 3 months > 5 years, 4 months > 10 years, 6 months > 15 years, 9 months > 20 years, 12 months > 25 years.
Canada	0 < 12 months, after which 2 days for each year of tenure, but with a minimum of 5 days (federal jurisdiction).
Denmark	1 month after 12 years, 3 months < 33 months, 2 months after 15 years, 4 months < 68 months, 3 months after 18 years (white collar).
France	1/10th of a month's pay per year of service plus an additional 1/15th after 10 years.
Germany	No legal entitlement, but can be included in collective agreements and social compensation plans.
Greece	<i>Blue collar</i> : 5 days < 1 year, 7 days < 2 years, 15 days < 5 years, 30 days < 10 years, 60 days < 15 years, 90 days < 20 years, 105 days > 20 years. <i>White collar</i> : Half the notice period if written notice is given; otherwise, severance pay according to the schedule for notice.
Hungary	0 < 3 years, 1 month < 5 years, 2 months < 10 years, going up to 5 months > 20 years and 6 months > 25 years.
Ireland	In redundancy cases with at least two years tenure: 1 week pay, plus half a week of pay per year worked under the age of 41, plus one week of pay per year worked over the age of 41, with a maximum of Ir£ 15 600 (as of 1995). Employers are partially reimbursed by redundancy fund.
Italy	2/27 of annual salary per year of service (often higher in collective agreements).

"28 days < 20 years" means 28 days of notice or severance pay is required when length of service is below 20 years.

Table 7: Firing taxes and tenure: some stylized facts (source OECD [1999])

Japan	According to firm surveys, average severance pay (retirement allowance) equals almost 1 month per year of service, although it is not legally required. It is somewhat higher in the case of lay-offs, and lower in case of voluntary quits.
Korea	Notice Retirement allowance of > 30 days per year of service legally required; often more in practice. Although no detailed data are available, difference between allowance for lay-off and voluntary quit was assumed to be somewhat higher than in Japan.
Mexico	3 months.
Netherlands	None by law, and if the dismissal is handled by the employment office. However, if the employer files for permission by a labor court, the court may determine severance pay roughly according to the formula: 1 month per year of service for workers < 40 years of age; 1.5 months for workers between age 40 and 50; 2 months for workers 50 years and over.
New Zealand	None by law; however according to survey data, about three quarters of employees are covered by contracts which provide them with severance pay in case of redundancy (typically 6 weeks for 1st year, and 2 weeks for additional years of tenure).
Norway	None by law; however collective agreements in the private sector may require lump-sum additional payments to long-serving staff who have reached age 50-55, or where the dismissal arises from company reorganisation.
Poland	Usually none, but 1 month in case of termination due to disability or retirement.
Portugal	1 month per year of service (legal minimum 3 months).

“28 days < 20 years” means 28 days of notice or severance pay is required when length of service is below 20 years.



Table 8: Firing taxes and tenure: some stylized facts (source OEDC [1999])

Spain	<p><i>Workers dismissed for “objective” reasons:</i> 2/3 of a month’s pay per year of service up to a maximum of 12 months.</p> <p><i>Workers under fixed-term contracts:</i> None, except for workers under contract with temporary agencies who get 12 days per year of service.</p>
Sweden	No legal entitlement, but occasionally included in collective agreements.
Switzerland	No legal entitlement to severance pay, except for workers over age 50 and with more 20 years seniority, where severance pay cannot be less than 2 months wages.
Turkey	After one year’s employment, one month for each year of service, often extended by collective agreement to 45 days.
United Kingdom	<p>Legally required only for redundancy cases with 2 years tenure:</p> <p>half a week per year of service (ages 18-21); 1 week per year (ages 22 to 40); 1.5 weeks per year (ages 41 to 64), limited to 30 weeks and 220 per week (April 1998).</p> <p>According to a government study, 40% of firms exceed legal minima.</p>
United States	No legal regulations (but can be included in collective agreements or company policy manuals).

“28 days < 20 years” means 28 days of notice or severance pay is required when length of service is below 20 years.

## .2 Wage determination

The insiders' sharing rule is given by:

$$\max_{w(\tau,t)} \left\{ (W(\tau,t) - U(t-\tau,t))^\beta (J(\tau,t) + p(t)F(t-\tau))^{1-\beta} \right\} \quad (11)$$

We deduce from the first order condition of (11) the following sharing rule:

$$W(\tau,t) - U(t-\tau,t) = \frac{\beta(1-P_W)}{\beta(1-P_W) + (1-\beta)(1+P_J)} S(\tau,t) \quad (12)$$

where

$$S(\tau,t) = W(\tau,t) - U(t-\tau,t) + J(\tau,t) + p(t)F(t-\tau) \quad (13)$$

Thanks to (1.6), we can deduce that:

$$\begin{aligned} J(\tau,t) + p(t)F(t-\tau) &= \int_t^{\tau+T} [y(\tau,s) - w(\tau,s)(1+P_J)] e^{-(r+\delta)(s-t)} ds \\ &\quad - e^{-(r+\delta)(\tau+T-t)} F(T) e^{g(\tau+T)} + p(t)F(t-\tau) \end{aligned} \quad (14)$$

Thanks to (1.7), we can deduce that, whenever  $W(\tau,t) > U(t-\tau,t)$ :

$$\begin{aligned} \dot{W}(\tau,t) - \dot{U}(t-\tau,t) &= (r+\delta)(W(\tau,t) - U(t-\tau,t)) + rU(t-\tau,t) + d \\ &\quad - (1-P_W)w(\tau,t) - \dot{U}(t-\tau,t) \end{aligned} \quad (15)$$

$$\begin{aligned} W(\tau,t) - U(t-\tau,t) &= \\ &= \int_t^{\tau+T} \left[ w(\tau,s)(1-P_W) - rU(s-\tau,s) - d + \dot{U}(s-\tau,s) \right] e^{-(r+\delta)(s-t)} ds \end{aligned} \quad (16)$$

By using (14) and (16), we obtain that the value of the surplus is given by:

$$\begin{aligned} S(\tau,t) &= \int_t^{\tau+T} \left[ y(\tau,s) - w(\tau,s)(P_W + P_J) - rU(s-\tau,s) - d + \dot{U}(s-\tau,s) \right] e^{-(r+\delta)(s-t)} ds \\ &\quad - e^{-(r+\delta)(\tau+T-t)} F(T) e^{g(\tau+T)} + p(t)F(t-\tau) \end{aligned}$$

(17)

Unlike the setup of (Mortensen and Pissarides, 1998), the surplus is not only changing overtime along the growth path. Indeed, the firing tax can have any shape, we need therefore to consider the evolution of this surplus and the way it is shared between firms and workers. The evolution of the surplus is given by:

$$\begin{aligned} \dot{S}(\tau, t) = & (r + \delta) [S(\tau, t) - F(t - \tau)p(t)] - y(\tau, t) + w(\tau, t)(P_W + P_J) \\ & + rU(t - \tau, t) + d - \dot{U}(t - \tau, t) + [p(t)F(\dot{t} - \tau)] \end{aligned} \quad (18)$$

By using (1.7), we can deduce:

$$(r + \delta)(W(\tau, t) - U(t - \tau, t)) = (1 - P_W)w(\tau, t) - d - rU(t - \tau, t) + \dot{W}(\tau, t) \quad (19)$$

Substituting (19) into (12), we get:

$$(r + \delta)S(\tau, t) = \frac{\beta(1 - P_W) + (1 - \beta)(1 + P_J)}{\beta(1 - P_W)} \left( (1 - P_W)w(\tau, t) - d - rU(t - \tau, t) + \dot{W}(\tau, t) \right)$$

Now, by using this expression and equation (18), we deduce that:

$$\begin{aligned} \dot{S}(\tau, t) = & \frac{\beta(1 - P_W) + (1 - \beta)(1 + P_J)}{\beta(1 - P_W)} \left( (1 - P_W)w(\tau, t) - d - rU(t - \tau, t) + \dot{W}(\tau, t) \right) \\ & - (r + \delta)F(t - \tau)p(t) - y(\tau, t) + w(\tau, t)(P_W + P_J) \\ & + rU(t - \tau, t) + d - \dot{U}(t - \tau, t) + [p(t)F(\dot{t} - \tau)] \end{aligned}$$

and after simplification:

$$\begin{aligned} \dot{S}(\tau, t) = & \frac{1 + P_J}{\beta} w(\tau, t) - \frac{(1 - \beta)(1 + P_J)}{\beta(1 - P_W)} (rU(t - \tau, t) + d) \\ & + \frac{\beta(1 - P_W) + (1 - \beta)(1 + P_J)}{\beta(1 - P_W)} \dot{W}(\tau, t) \\ & - (r + \delta)F(t - \tau)p(t) - y(\tau, t) - \dot{U}(t - \tau, t) + [p(t)F(\dot{t} - \tau)] \end{aligned}$$

We can now deduce wage bargained between workers and firms according to the surplus evolution:

$$\begin{aligned}
\frac{1+P_J}{\beta}w(\tau,t) &= y(\tau,t) + p(t)(r+\delta)F(t-\tau) + \frac{(1-\beta)(1+P_J)}{\beta(1-P_W)}(rU(t-\tau,t) + d) \\
&+ \dot{S}(\tau,t) - \frac{\beta(1-P_W) + (1-\beta)(1+P_J)}{\beta(1-P_W)}\dot{W}(\tau,t) + \dot{U}(t-\tau,t) \\
&- [p(t)F(t-\tau)]
\end{aligned} \tag{20}$$

We now search to eliminate  $\dot{S}(\tau,t)$ . By manipulating slightly equation (20), we get:

$$\begin{aligned}
\frac{1+P_J}{\beta}w(\tau,t) &= y(\tau,t) + p(t)(r+\delta)F(t-\tau) + \frac{(1-\beta)(1+P_J)}{\beta(1-P_W)}(rU(t-\tau,t) + d) \\
&+ \dot{S}(\tau,t) - \frac{\beta(1-P_W) + (1-\beta)(1+P_J)}{\beta(1-P_W)}\dot{W}(\tau,t) \\
&+ \frac{\beta(1-P_W) + (1-\beta)(1+P_J)}{\beta(1-P_W)}\dot{U}(t-\tau,t) \\
&+ \left(1 - \frac{\beta(1-P_W) + (1-\beta)(1+P_J)}{\beta(1-P_W)}\right)\dot{U}(t-\tau,t) \\
&- gp(t)F(t-\tau) - p(t)F(t-\tau)
\end{aligned}$$

And by using the derivative of the sharing rule (12), we deduce:

$$\begin{aligned}
\frac{1+P_J}{\beta}w(\tau,t) &= y(\tau,t) + p(t)(r+\delta-g)F(t-\tau) \\
&+ \frac{(1-\beta)(1+P_J)}{\beta(1-P_W)}\left(rU(t-\tau,t) + d - \dot{U}(t-\tau,t)\right) - p(t)F(t-\tau)
\end{aligned} \tag{21}$$

We now need to compute the unemployed worker's reservation wage according to his unemployment compensation  $b(T)$ . We know the sharing rule for outsiders are given by:

$$\max_{w_T(t,t)} \left\{ (W(t,t) - U(T,t))^\beta (J(t,t) + p(t)H)^{1-\beta} \right\}$$

We can replace here  $T$  by  $t - \tau$ , which is the seniority within the job at  $t$ . We can deduce from the first order condition of this sharing rule that:

$$W(t, t) - U(t - \tau, t) = \frac{\beta(1 - P_W)}{(1 - \beta)(1 + P_J)} (J(t, t) - p(t)H) \quad (22)$$

Using 1.8, we have:

$$\begin{aligned} \frac{(1 - \beta)(1 + P_J)}{\beta(1 - P_W)} \left( rU(t - \tau, t) + d - \dot{U}(t - \tau, t) \right) &= p(t)b(t - \tau) \\ &+ \theta q(\theta) [W(t, t) - U(t - \tau, t)] \end{aligned}$$

Therefore using the sharing rule (22), we can deduce:

$$\begin{aligned} \frac{(1 - \beta)(1 + P_J)}{\beta(1 - P_W)} \left( rU(t - \tau, t) + d - \dot{U}(t - \tau, t) \right) &= \frac{(1 - \beta)(1 + P_J)}{\beta(1 - P_W)} p(t)b(t - \tau) \\ &+ \frac{(1 - \beta)(1 + P_J)}{\beta(1 - P_W)} \theta q(\theta) \frac{\beta(1 - P_W)}{(1 - \beta)(1 + P_J)} [J(t, t) + p(t)H] \end{aligned}$$

Then, after simplification and by using the free entry condition, we get:

$$\frac{(1 - \beta)(1 + P_J)}{\beta(1 - P_W)} \left( rU(t - \tau, t) + d - \dot{U}(t - \tau, t) \right) = \frac{(1 - \beta)(1 + P_J)}{\beta(1 - P_W)} p(t)b(t - \tau) + p(t)c\theta$$

We substitute this value into (21), and deduce the insiders' bargained wage given by:

$$\begin{aligned} w(\tau, t) &= \frac{\beta}{1 + P_J} \left[ y(\tau, t) + p(t) \left( (r + \delta - g)F(t - \tau) - \dot{F}(t - \tau) \right) \right] \\ &+ \frac{1 - \beta}{1 - P_W} p(t) \left( b(t - \tau) + \frac{1 - P_W}{1 + P_J} \frac{\beta}{1 - \beta} c\theta \right) \end{aligned}$$

### .3 Hosios Condition

The social planner chooses the value of  $\beta$ ,  $\theta$  and  $T$  which maximize the social welfare :

$$\begin{aligned} \max_{\beta, \theta, T} rU &= \frac{\beta}{1 - \beta} c\theta \\ \text{s.t.} \quad &(1 - \beta) \int_0^T \left[ x - e^{gs} \left( \frac{\beta}{1 - \beta} c\theta \right) \right] e^{-(r+\delta)s} ds = \frac{c}{q(\theta)} \end{aligned}$$

Because there is no inefficiency associated to the choice of the optimal scrapping time, the optimal value of  $T = T^e$  satisfies:

$$xe^{-gT^e} - \left( \frac{\beta}{1-\beta} c\theta^e \right) = 0$$

where  $\theta$  is evaluated at its optimal value  $\theta^e$ . Using this result, the problem of the social planner can be reduced as following:

$$\begin{aligned} \max_{\beta, \theta} rU &= \frac{\beta}{1-\beta} c\theta \\ \text{s.t.} \quad &(1-\beta)x \int_0^T [1 - e^{g(s-T)}] e^{-(r+\delta)s} ds = \frac{c}{q(\theta)} \end{aligned}$$

The first order conditions of this problem are:

$$\begin{aligned} \frac{c\theta}{(1-\beta)} + \lambda \left[ x \int_0^T [1 - e^{g(s-T)}] e^{-(r+\delta)s} ds \right] &= 0 \\ \frac{c\beta}{1-\beta} - \lambda \frac{cq'(\theta)}{q(\theta)} &= 0 \end{aligned}$$

Using the constraint of the social planner problem, we get:

$$\begin{aligned} \frac{\theta}{(1-\beta)} + \lambda \frac{1}{q(\theta)} &= 0 \Rightarrow \lambda = -\frac{\theta q(\theta)}{(1-\beta)} \\ \frac{1}{1-\beta} - \lambda \frac{q'(\theta)}{\beta q(\theta)} &= 0 \end{aligned}$$

The combination of these two equations leads to:

$$\frac{1}{1-\beta} = -\frac{\theta q(\theta)}{(1-\beta)} \frac{q'(\theta)}{\beta q(\theta)} \Leftrightarrow \beta = -q'(\theta) \frac{\theta}{q(\theta)} \equiv \eta(\theta)$$



# The Wage Game over the Life Cycle

.1 Notations in order of appearance



Table 9: Notations

$r$	discounted rate
$V_i^e(w)$	asset value of being employed at wage $w$ and in age class $i$
$\lambda$	contact frequency
$F_i(w)$	offered wage cumulative distribution
$s$	exogenous destruction rate
$V_i^u$	asset value of being unemployed in age class $i$
$p$	probability to change age class
$V_r$	asset value of being retired
$b$	labor opportunity cost
$R_i$	reservation wage
$u_i$	unemployment rate in age class $i$
$m$	mass of workers by age class
$G_i(w)$	wage cumulative distribution
$\Pi_i(w)$	expected profit at wage offer $w$
$J_i(w)$	expected surplus at wage offer $w$
$l_i(w)$	labor supply at wage offer $w$
$g_i(w)$	wage density distribution
$y$	match's productivity
$\underline{w}_i$	endogenous minimum wage on the market $i$
$\overline{w}_i$	endogenous maximum wage on the market $i$
$\underline{\Pi}_i(w)$	expected profit of firms posting the lowest wage on the market, in function of this wage
$\underline{w}$	institutional minimum wage
$b_s$	unemployment benefits of seniors

## .2 Calculation of the Hiring Frequency

Given the expression of the hiring frequency given by 2.11 and 2.12 and the definition 1, we deduce easily that the two hiring frequencies can be given by:

$$\begin{aligned} h_y(w) &= \lambda(u_y + (m - u_y)G_y(w)) \\ h_s(w) &= \lambda(u_s + (m - u_s)G_s(w)) \end{aligned}$$

Using equation 2.15, the hiring frequency on the young's market can be written as follows:

$$\begin{aligned} h_y(w) &= \lambda \left( u_y + \frac{\lambda F_y(w) u_y}{p + s + \lambda(1 - F_y(w))} \right) \\ h_y(w) &= \lambda \left( \frac{(p + s + \lambda) u_y}{p + s + \lambda(1 - F_y(w))} \right) \end{aligned}$$

And eventually, by using the expression of unemployment of the youth 2.13, we obtain the expression 2.17:

$$\begin{aligned} h_y(w) &= \lambda \left( \frac{(p + s + \lambda) \frac{(s+p)m}{p+s+\lambda}}{p + s + \lambda(1 - F_y(w))} \right) \\ h_y(w) &= \lambda m \left( \frac{p + s}{p + s + \lambda(1 - F_y(w))} \right) \end{aligned}$$

We proceed as well on the seniors' market. The equation 2.16 allows to rewrite the hiring frequency on the seniors' market as follows:

$$\begin{aligned} h_s(w) &= \lambda(u_s + (m - u_s)G_s(w)) \\ h_s(w) &= \lambda \left( u_s + \frac{\lambda F_s(w) u_s}{p + s + \lambda(1 - F_s(w))} + p \frac{(m - u_y)G_y(w)}{p + s + \lambda(1 - F_s(w))} \right) \\ h_s(w) &= \lambda \left( \frac{(p + s + \lambda) u_s}{p + s + \lambda(1 - F_s(w))} + \frac{p}{p + s + \lambda(1 - F_s(w))} \frac{\lambda F_y(w) u_y}{p + s + \lambda(1 - F_y(w))} \right) \end{aligned}$$

Then, by replacing the unemployment of seniors by its value given by equation 2.14, we get:

$$\begin{aligned}
h_s(w) &= \lambda \left( \frac{sm + pu_y}{p + s + \lambda(1 - F_s(w))} + \frac{p}{p + s + \lambda(1 - F_s(w))} \frac{\lambda F_y(w) u_y}{p + s + \lambda(1 - F_y(w))} \right) \\
h_s(w) &= \lambda \left( \frac{sm}{p + s + \lambda(1 - F_s(w))} + \frac{p}{(p + s + \lambda(1 - F_s(w)))(p + s + \lambda(1 - F_y(w)))} \right. \\
&\quad \left. + (u_y(p + s + \lambda(1 - F_y(w))) + \lambda F_y(w) u_y) \right) \\
h_s(w) &= \lambda \left( \frac{sm}{p + s + \lambda(1 - F_s(w))} + \frac{pu_y(p + s + \lambda)}{(p + s + \lambda(1 - F_s(w)))(p + s + \lambda(1 - F_y(w)))} \right)
\end{aligned}$$

Eventually, we replace the unemployment of young by its value given by equation 2.13 and obtain the expression 2.18:

$$\begin{aligned}
h_s(w) &= \lambda m \left( \frac{s}{p + s + \lambda(1 - F_s(w))} + \frac{p(p + s)}{(p + s + \lambda(1 - F_s(w)))(p + s + \lambda(1 - F_y(w)))} \right) \\
h_s(w) &= \frac{\lambda m}{p + s + \lambda(1 - F_s(w))} \left( s + \frac{p(p + s)}{p + s + \lambda(1 - F_y(w))} \right)
\end{aligned}$$

### .3 Comparison of $(m - u_y)G_y(w)$ and $(m - u_s)G_s(w)$ , when

$$F_y(w) = F_s(w)$$

To compare  $(m - u_y)G_y(w)$  and  $(m - u_s)G_s(w)$  at identical wage offer lottery, we need to study the function  $\psi(w)$ . We first show that  $\psi(\infty) = 1$ , then that  $\psi'(w) > 0$

#### .3.1 $\psi(\bar{w}) = 1$

The function  $\psi(w)$  is given by:

$$\psi(w) = \left( \frac{u_s}{u_y} + \frac{p}{p + s + \lambda(1 - F(w))} \right) \left( \frac{m_y - u_y}{m_y - u_s} \right)$$

At the maximum value of the wage, we have necessarily  $F(w) = 1$ , therefore:

$$\psi(\infty) = \left( \frac{u_s}{u_y} + \frac{p}{p+s} \right) \left( \frac{m-u_y}{m-u_s} \right)$$

We know,

$$u_y = \frac{(s+p)m}{p+s+\lambda}$$

$$u_s = \frac{\left(s + p \frac{u_y}{m}\right)m}{p+s+\lambda}$$

By replacing these two values into  $\phi(\infty)$ , we can deduce,

$$\psi(\bar{w}) = \left( \frac{sm + pu_y}{(s+p)m} + \frac{p}{p+s} \right) \left( \frac{\lambda}{p+\lambda + p \frac{u_y}{m}} \right)$$

$$\psi(\bar{w}) = \frac{s+p + p \frac{u_y}{m}}{s+p} \left( \frac{\lambda}{p+\lambda + p \frac{u_y}{m}} \right)$$

$$\psi(\bar{w}) = \left( \frac{s+p + \frac{p(s+p)}{p+s+\lambda}}{s+p} \right) \left( \frac{\lambda}{p+\lambda + \frac{p(s+p)}{p+s+\lambda}} \right)$$

$$\psi(\bar{w}) = \left( 1 + \frac{p}{p+s+\lambda} \right) \left( \frac{\lambda(p+s+\lambda)}{(p+s+\lambda)(p+\lambda) - p(s+p)} \right)$$

$$\psi(\bar{w}) = \left( \frac{p+p+s+\lambda}{p+s+\lambda} \right) \left( \frac{\lambda(p+s+\lambda)}{(p+s+\lambda)(p+\lambda) - p(s+p)} \right)$$

$$\psi(\bar{w}) = \left( \frac{(p+p+s+\lambda)\lambda}{(p+s+\lambda)(p+\lambda) - (p+s)p} \right)$$

$$\psi(\bar{w}) = 1$$

### .3.2 $\psi'(w) > 0$

Given the definition of  $\psi(w)$ ,

$$\psi(w) = \left( \frac{u_s}{u_y} + \frac{p}{p+s+\lambda(1-F(w))} \right) \left( \frac{m-u_y}{m-u_s} \right)$$

we can deduce easily that  $\psi'(w) > 0$ :

$$\psi'(w) = \frac{p\lambda f(w)}{(p+s+\lambda(1-F(w)))^2} > 0$$

## .4 Result analysis

### .4.1 Effect of Horizon

In this context, on the seniors' market the equiprofit condition is given by:

$$\begin{aligned}\Pi_s(\underline{w}) &= \Pi_s(w) \\ hJ_s(\underline{w}) &= hJ_s(w)\end{aligned}$$

Using the expression of the expected surpluses on the seniors' market at  $w$  and  $\underline{w}$ , we can deduce:

$$\begin{aligned}\frac{y-\underline{w}}{p+s+\lambda} &= \frac{y-w}{p+s+\lambda(1-F_s(w))} \\ \frac{y-\underline{w}}{p+s+\lambda} &= \frac{y-w}{p+s+\lambda(1-F_s(w))} \\ p+s+\lambda(1-F_s(w)) &= \frac{y-w}{y-\underline{w}}(p+s+\lambda) \\ F_s(w) &= 1 - \left( \frac{y-w}{y-\underline{w}}(p+s+\lambda) - (p+s) \right) \frac{1}{\lambda}\end{aligned}$$

Eventually, we obtain the equation 2.37:

$$F_s(w) = 1 - \left( \frac{y-w}{y-\underline{w}}(p+s+\lambda) - (p+s) \right) \frac{1}{\lambda}$$

On the young's market the equiprofit condition is given by:

$$\begin{aligned}\Pi_y(\underline{w}) &= \Pi_y(w) \\ hJ_y(\underline{w}) &= hJ_y(w)\end{aligned}$$

Using the expression of the expected surpluses on the young's market at  $w$  and  $\underline{w}$ , we can deduce:

$$\begin{aligned}\frac{y - \underline{w}}{p + s + \lambda} + \frac{p(y - \underline{w})}{(p + s + \lambda)^2} &= \frac{y - w}{p + s + \lambda(1 - F_y(w))} + \frac{p(y - w)}{(r + p + s + \lambda(1 - F_y(w)))(r + p + s + \lambda(1 - F_s(w)))} \\ \frac{y - \underline{w}}{p + s + \lambda} + \frac{p(y - \underline{w})}{(p + s + \lambda)^2} &= \frac{y - w}{p + s + \lambda(1 - F_y(w))} + \frac{p(y - w)}{(p + s + \lambda(1 - F_y(w)))(p + s + \lambda(1 - F_s(w)))} \\ p + s + \lambda(1 - F_y(w)) &= \left( \frac{1}{\frac{1}{p+s+\lambda} + \frac{p}{(p+s+\lambda)^2}} \right) \left( 1 + \frac{p}{p + s + \lambda(1 - F_s(w))} \right) \frac{y - w}{y - \underline{w}}\end{aligned}$$

Eventually, we obtain the equation 2.36:

$$F_y(w) = 1 - \left( \frac{y - w}{y - \underline{w}} \left( \frac{1 + \frac{p}{p+s+\lambda(1-F_s(w))}}{\frac{1}{p+s+\lambda} + \frac{p}{(p+s+\lambda)^2}} \right) - (p + s) \right) \frac{1}{\lambda}$$

#### 4.2 Effect of Experience

On the young's market the equiprofit condition is given by:

$$\begin{aligned}\Pi_y(\underline{w}) &= \Pi_y(w) \\ h_y(\underline{w})J(\underline{w}) &= h_y(w)J(w)\end{aligned}$$

Using the expression of the hiring frequencies on the young's market at  $w$  and  $\underline{w}$ , we can deduce:

$$\begin{aligned}\lambda m \left( \frac{p+s}{p+s+\lambda} \right) D(y-\underline{w}) &= \lambda m \left( \frac{p+s}{p+s+\lambda(1-F_y(w))} \right) D(y-w) \\ \left( \frac{1}{p+s+\lambda} \right) (y-\underline{w}) &= \left( \frac{1}{p+s+\lambda(1-F_y(w))} \right) (y-w) \\ (p+s+\lambda(1-F_y(w)))(y-\underline{w}) &= (p+s+\lambda)(y-w)\end{aligned}$$

Eventually, we obtain the equation 2.38:

$$F_y(w) = 1 - \left( \frac{y-w}{y-\underline{w}}(p+s+\lambda) - (p+s) \right) \frac{1}{\lambda}$$

On the seniors' market the equiprofit condition is given by:

$$\begin{aligned}\Pi_s(\underline{w}) &= \Pi_s(w) \\ h_s(\underline{w})J(\underline{w}) &= h_s(w)J(w)\end{aligned}$$

Using the expression of the hiring frequencies on the seniors' market at  $w$  and  $\underline{w}$ , we

can deduce:

$$\begin{aligned}
 & \left( \frac{s}{p+s+\lambda} + \frac{p(p+s)}{(p+s+\lambda)^2} \right) D(y-w) \\
 &= \left( \frac{s}{p+s+\lambda(1-F_s(w))} + \frac{p(p+s)}{(p+s+\lambda(1-F_s(w)))(p+s+\lambda(1-F_y(w)))} \right) D(y-w) \\
 & \left( \frac{s}{p+s+\lambda} + \frac{p(p+s)}{(p+s+\lambda)^2} \right) (y-w) \\
 &= \left( \frac{s}{p+s+\lambda(1-F_s(w))} + \frac{p(p+s)}{(p+s+\lambda(1-F_s(w)))(p+s+\lambda(1-F_y(w)))} \right) (y-w) \\
 & \frac{p+s+\lambda(1-F_s(w))}{p+s+\lambda(1-F_s(w))} \\
 &= \left( \frac{1}{\frac{s}{p+s+\lambda} + \frac{p(p+s)}{(p+s+\lambda)^2}} \right) \left( s + \frac{p(p+s)}{p+s+\lambda(1-F_y(w))} \right) \frac{y-w}{y-w}
 \end{aligned}$$

Eventually, we obtain the equation 2.39:

$$F_s(w) = 1 - \left( \frac{y-w}{y-w} \frac{s + \frac{p(p+s)}{p+s+\lambda(1-F_y(w))}}{\frac{s}{p+s+\lambda} + \frac{p(p+s)}{(p+s+\lambda)^2}} - (p+s) \right) \frac{1}{\lambda}$$

## .5 Comparison of $(m - u_y)g_y(w)$ and $(m - u_s)g_s(w)$ , when

$$F_y(w) = F_s(w)$$

To study the two masses of workers  $(m - u_y)g_y(w)$  and  $(m - u_s)g_s(w)$  at identical wage offer lottery, we need to study the function  $\phi(w)$  given by:

$$\phi(w) = \frac{u_s}{u_y} + \frac{p}{p+s+\lambda(1-F(w))} + \frac{p\lambda F(w)}{(p+s+\lambda(1-F(w)))(p+s+\lambda)}$$



**.5.1**  $\phi(w) < 1$  at minimum wage  $\Leftrightarrow \lambda > p + s$

At minimum wage, we have:

$$\phi(\underline{w}) = \frac{u_s}{u_y} + \frac{p}{p + s + \lambda}$$

Using the expression of unemployment given by equations 2.13 and 2.14, we can deduce:

$$\begin{aligned} \phi(\underline{w}) &= \frac{s(p + s + \lambda) + p(p + s)}{(p + s)(p + s + \lambda)} + \frac{p}{(p + s + \lambda)} \\ \phi(\underline{w}) &= \frac{s(p + s + \lambda) + 2p(p + s)}{(p + s)(p + s + \lambda)} \end{aligned}$$

Therefore, we have:  $\phi(\underline{w}) < 1 \Leftrightarrow p + s < \lambda$

**.5.2**  $\phi(\infty) > 1$

At the maximum wage, we have:

$$\phi(\infty) = \frac{u_s}{u_y} + \frac{p}{p + s} + \frac{p\lambda}{(p + s)(p + s + \lambda)}$$

Using the expression of unemployment given by equations 2.13 and 2.14, we can deduce:

$$\begin{aligned} \phi(\infty) &= \frac{u_s}{u_y} + \frac{p}{p + s} + \frac{p\lambda}{(p + s)(p + s + \lambda)} \\ \phi(\infty) &= \frac{s(p + s + \lambda) + p(p + s) + p(p + s + \lambda) + p\lambda}{(p + s)(p + s + \lambda)} \end{aligned}$$

Therefore, we obtain the following condition:  $\phi(\bar{w}) > 1 \Leftrightarrow p(p + s + \lambda) + p\lambda > 0$

This condition is always verified.

**.5.3**  $\phi'(w) > 0$

Using the definition of  $\phi(w)$ , we compute the following derivative.

$$\begin{aligned} \phi'(w) &= \frac{p\lambda f(w)}{(p+s+\lambda(1-F(w)))^2} \\ &+ \frac{p\lambda f(w)(p+s+\lambda(1-F(w)))(p+s+\lambda) + (p+s+\lambda)\lambda f(w)}{((p+s+\lambda(1-F(w)))(p+s+\lambda))^2} > 0 \end{aligned}$$

This derivative is always positive.

**.6 Expected Profit in the model with 3 age classes**

To calculate the firms' expected profit in this framework, we proceed as in the model with two age classes. By using the expected surplus of employing a senior given by equation 2.49, we deduce the following expression of the expected surplus of employing an adult:

$$\begin{aligned} J_a(w) &= \frac{y-w}{p+s+\lambda(1-F_a(w))} + \frac{p}{p+s+\lambda(1-F_a(w))} \frac{y-w}{p+s} \\ J_a(w) &= \frac{y-w}{p+s+\lambda(1-F_a(w))} \left(1 + \frac{p}{p+s}\right) \end{aligned}$$

Then by using this value, we deduce the following expression of the expected surplus of employing a young:

$$\begin{aligned} J_y(w) &= \frac{y-w}{p+s+\lambda(1-F_y(w))} + \frac{p}{p+s+\lambda(1-F_y(w))} \frac{y-w}{p+s+\lambda(1-F_a(w))} \left(1 + \frac{p}{p+s}\right) \\ J_y(w) &= \frac{y-w}{p+s+\lambda(1-F_y(w))} \left(1 + \frac{p}{p+s+\lambda(1-F_a(w))} \left(1 + \frac{p}{p+s}\right)\right) \end{aligned}$$

Now we replace by their values the expected surpluses and hiring frequencies (given by equations 2.45 and 2.46) for each age class, into the expression of the expected profit

given by equations 2.43 and 2.44. The firms' expected profit on both markets becomes:

$$\begin{aligned}\Pi_a(w) &= \frac{\lambda m(y-w)}{(p+s+\lambda(1-F_a(w)))^2} \left( s + \frac{p(p+s)}{p+s+\lambda(1-F_y(w))} \right) \left( 1 + \frac{p}{p+s} \right) \\ \Pi_y(w) &= \frac{\lambda m(p+s)(y-w)}{(p+s+\lambda(1-F_y(w)))^2} \left( 1 + \frac{p}{p+s+\lambda(1-F_a(w))} \right) \left( 1 + \frac{p}{p+s} \right)\end{aligned}$$

## .7 Stationarity

To show  $\frac{m-u_a}{m-u_s} \frac{p}{p+s} = 1$ , we use the expression of the masses of adult and senior employed workers given by:

$$\begin{aligned}m - u_a &= m \frac{(p+s+\lambda)^2 - s(p+s+\lambda) - p(p+s)}{(p+s+\lambda)^2} \\ m - u_s &= m \frac{(p+s+\lambda)^2(p+s) - s(p+s+\lambda)^2 - ps(p+s+\lambda) - p^2(p+s)}{(p+s+\lambda)^2(p+s)}\end{aligned}$$

Then, we can deduce:

$$\begin{aligned}\frac{m - u_a}{m - u_s} \frac{p}{p+s} &= \frac{[(p+s+\lambda)^2 - s(p+s+\lambda) - p(p+s)](p+s)}{(p+s+\lambda)^2(p+s) - s(p+s+\lambda)^2 - ps(p+s+\lambda) - p^2(p+s)} \frac{p}{p+s} \\ \frac{m - u_a}{m - u_s} \frac{p}{p+s} &= \frac{(p+s+\lambda)(p+\lambda) - p(p+s)}{p(p+s+\lambda)(p+\lambda) - p^2(p+s)} \\ \frac{m - u_a}{m - u_s} \frac{p}{p+s} &= 1\end{aligned}$$

# The U. S. Wage Distribution over the Life Cycle

## .1 Workers' value functions

The workers' value functions are similar to those of Chapter 2 except that here all workers search for a job <sup>5</sup> and that workers have different contact frequency. Given these differences, the asset values of being employed at a wage  $w$  for workers of each age class solve:

$$\begin{aligned}
 rV_y^e(w) &= w + \lambda_y \int_w^{\bar{w}} (V_y^e(x) - V_y^e(w)) dF_y(x) - s(V_y^e(w) - V_y^u) - p(V_y^e(w) - V_a^e(w)) \\
 rV_a^e(w) &= w + \lambda_a \int_w^{\bar{w}} (V_a^e(x) - V_a^e(w)) dF_a(x) - s(V_a^e(w) - V_a^u) - p(V_a^e(w) - V_s^e(w)) \\
 rV_s^e(w) &= w + \lambda_s \int_w^{\bar{w}} (V_s^e(x) - V_s^e(w)) dF_s(x) - s(V_s^e(w) - V_s^u) - p(V_s^e(w) - V_r)
 \end{aligned}$$

When workers are already employed, they have a contact with an other firm with the frequency  $\lambda_i$ . The asset values of being unemployed for workers of each age class solve

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<sup>5</sup>In the three age model of Chapter 2, senior workers have no search activity

for each age:

$$\begin{aligned} rV_y^u &= b + \lambda_y^0 \int_{R_y}^{\bar{w}} (V_y^e(x) - V_y^u) dF_y(x) - p(V_y^u - V_a^u) \\ rV_a^u &= b + \lambda_a^0 \int_{R_a}^{\bar{w}} (V_a^e(x) - V_a^u) dF_a(x) - p(V_a^u - V_s^u) \\ rV_s^u &= b + \lambda_s^0 \int_{R_s}^{\bar{w}} (V_s^e(x) - V_s^u) dF_s(x) - p(V_s^u - V_r) \end{aligned}$$

When workers are unemployed, they have a contact with a firm with the frequency  $\lambda_i^0$ . As unemployed workers are homogenous within an age class, the unemployed workers of each age all have the same reservation wage. The reservation wages of workers of each age class are given by:

$$\begin{aligned} R_s &= b + (\lambda_s^0 - \lambda_s) \int_{R_s}^{\bar{w}} (V_s^e(x) - V_s^u) dF_s(x) \\ R_a &= b + (\lambda_a^0 - \lambda_a) \int_{R_a}^{\bar{w}} (V_a^e(x) - V_a^u) dF_a(x) - p(V_s^e(R_a) - V_s^u) \\ R_y &= b + (\lambda_y^0 - \lambda_y) \int_{R_y}^{\bar{w}} (V_y^e(x) - V_y^u) dF_y(x) - p(V_a^e(R_y) - V_a^u) \end{aligned}$$

Contrary to chapter 2, as for each age class,  $\lambda_i^0$  is likely to be different from  $\lambda_i$ , the reservation wage  $R_i$  is likely to be different from  $b$ . As the reservation wage of workers can now differ from an age class to another, when workers change age classes, the contract can possibly be broken: it happens when the wage received by the worker in his previous age period is lower than the reservation wage of his current age period.

## .2 Workers' flows

### .2.1 From and to unemployment

The flows into and out of unemployment for each age are equal and are given by:

$$(\lambda_y^0 + p)u_y = (m - u_y)s + pm$$

$$(\lambda_a^0 + p)u_a = (m - u_a)s + pu_y$$

$$(\lambda_s^0 + p)u_s = (m - u_s)s + pu_a$$

Unemployed workers find a job with the frequency  $\lambda_i^0$ . Indeed, as the unemployed workers of each age class all have the same reservation wage, the lowest wage offered by the firms on each market is necessarily equal or above this reservation wage, and there is no job rejection from unemployed workers.

### .2.2 From and to firms offering a wage no greater than $w$

In steady state, the flows into (on the left side) and out (on the right side) of firms offering a wage no greater than  $w$  for each age are equal and are given by:

$$(s + p + \lambda_y(1 - F_y(w)))(m - u_y)G_y(w) = \lambda_y^0 F_y(w)u_y \quad (23)$$

$$(s + p + \lambda_a(1 - F_a(w)))(m - u_a)G_a(w) = \lambda_a^0 F_a(w)u_a + p(m_y - u_y)G_y(w) \quad (24)$$

$$(s + p + \lambda_s(1 - F_s(w)))(m - u_s)G_s(w) = \lambda_s^0 F_s(w)u_s + p(m_a - u_a)G_a(w) \quad (25)$$

$$(26)$$

### .3 Decision of the job quality

The derivative of the profit of firms targeting each age according to the quality of the job is given by:

$$\begin{aligned}\frac{\partial \Pi_s(w, k)}{\partial k} &= qk^{\alpha-1} - \beta_s(r + p + s + \lambda_s(1 - F_s(w))) \\ \frac{\partial \Pi_a(w, k)}{\partial k} &= qk^{\alpha-1} \left( 1 + \frac{p}{r + p + s + \lambda_s(1 - F_s(w))} \right) - \beta_a(r + p + s + \lambda_a(1 - F_a(w))) \\ \frac{\partial \Pi_y(w, k)}{\partial k} &= qk^{\alpha-1} \left( 1 + \frac{p}{r + p + s + \lambda_a(1 - F_a(w))} \left( 1 + \frac{p}{r + p + s + \lambda_s(1 - F_s(w))} \right) \right) \\ &\quad - \beta_y(r + p + s + \lambda_y(1 - F_y(w)))\end{aligned}$$

The second derivative is given by:

$$\begin{aligned}\frac{\partial^2 \Pi_s(w, k)}{\partial k^2} &= (\alpha - 1)qk^{\alpha-2} \\ \frac{\partial^2 \Pi_a(w, k)}{\partial k^2} &= (\alpha - 1)qk^{\alpha-2} \left( 1 + \frac{p}{r + p + s + \lambda_s(1 - F_s(w))} \right) \\ \frac{\partial^2 \Pi_y(w, k)}{\partial k^2} &= (\alpha - 1)qk^{\alpha-2} \left( 1 + \frac{p}{r + p + s + \lambda_a(1 - F_a(w))} \left( 1 + \frac{p}{r + p + s + \lambda_s(1 - F_s(w))} \right) \right)\end{aligned}$$

When the production function has a decreasing return to the job quality, the second derivatives are negative. The first order condition  $\frac{\partial \Pi_i(w, k)}{\partial k} = 0$ , for  $i = y; a; s$  gives the result of equations 3.7, 3.6, and 3.5.

### .4 Wage distribution at equal productivity function

All young workers are necessarily hired as young. The density of wage of the young hired when young is therefore simply the density of wage of the young workers. It is given by the derivative of the cumulative distribution function of wage of the youth

deduced from equation 23 as it follows:

$$G_y(w) = \frac{\lambda_y^0 F_y(w) u_y}{(m - u_y)(p + s + \lambda_y(1 - F_y(w)))}$$

$$g_y(w) = \frac{\lambda_y^0 f_y(w) u_y}{(m - u_y)(p + s + \lambda_y(1 - F_y(w)))} + \frac{\lambda_y^0 F_y(w) u_y \lambda_y f_y(w)}{(m - u_y)(p + s + \lambda_y(1 - F_y(w)))^2}$$

The density of wage of the adults is given by the derivative of the cumulative distribution function of wage of the adults deduced from 24 as follows:

$$G_a(w) = \frac{\lambda_a^0 F_a(w) u_a + p(m - u_y) G_y(w)}{(m - u_a)(p + s + \lambda_a(1 - F_a(w)))}$$

$$g_a(w) = \frac{\lambda_a^0 f_a(w) u_a}{(m - u_y)(p + s + \lambda_a(1 - F_a(w)))} + \frac{(\lambda_a^0 F_a(w) u_a + p(m - u_y) G_y(w)) \lambda_a f_a(w)}{(m - u_y)(p + s + \lambda_a(1 - F_a(w)))^2}$$

$$+ \frac{p(m - u_y) g_y(w)}{(m - u_y)(p + s + \lambda_a(1 - F_a(w)))}$$

Among the workers employed as adult, some have been employed when young, other when adult. We can distinguish them as follows:

$$g_a(w) = \underbrace{\frac{\lambda_a^0 f_a(w) u_a}{(m - u_y)(p + s + \lambda_a(1 - F_a(w)))}}_{\text{Adults hired from unemployment}} + \underbrace{\frac{(\lambda_a^0 F_a(w) u_a + p(m - u_y) G_y(w)) \lambda_a f_a(w)}{(m - u_y)(p + s + \lambda_a(1 - F_a(w)))^2}}_{\text{Adults poached from other firms}}$$

$$+ \underbrace{\frac{p(m - u_y) g_y(w)}{(m - u_y)(p + s + \lambda_a(1 - F_a(w)))}}_{\text{Adults hired when young}} \quad (27)$$

The adults hired from unemployment, and poached from other firms are hired when adult. The others are hired when they were young.

The density of wage of the seniors is given by the derivative of the cumulative distribution function of wage of the seniors deduced from 25 as follows:



$$\begin{aligned}
G_s(w) &= \frac{\lambda_s^0 F_s(w) u_s + p(m - u_a) G_a(w)}{(m - u_s)(p + s + \lambda_s(1 - F_s(w)))} \\
g_s(w) &= \frac{\lambda_s^0 f_s(w) u_s}{(m - u_s)(p + s + \lambda_s(1 - F_s(w)))} + \frac{(\lambda_s^0 F_s(w) u_s + p(m - u_a) G_a(w)) \lambda_s f_s(w)}{(m - u_s)(p + s + \lambda_s(1 - F_s(w)))^2} \\
&+ \frac{p(m - u_a) g_a(w)}{(m - u_s)(p + s + \lambda_s(1 - F_s(w)))}
\end{aligned}$$

Among the workers employed as senior, some have been employed when young, other adult, and other seniors. We can distinguish them as follows:

$$\begin{aligned}
g_s(w) &= \underbrace{\frac{\lambda_s^0 f_s(w) u_s}{(m - u_s)(p + s + \lambda_s(1 - F_s(w)))}}_{\text{Seniors hired from unemployment}} + \underbrace{\frac{(\lambda_s^0 F_s(w) u_s + p(m - u_a) G_a(w)) \lambda_s f_s(w)}{(m - u_s)(p + s + \lambda_s(1 - F_s(w)))^2}}_{\text{Seniors poached from other firms}} \\
&+ \underbrace{\frac{p(m - u_a) g_a(w)}{(m - u_s)(p + s + \lambda_s(1 - F_s(w)))}}_{\text{Seniors hired when young or when adult}}
\end{aligned}$$

To distinguish the mass of seniors hired when young or when adult, we use the expression 27.

We therefore deduce the wage density of workers according to their production function.

1. The density of wage of the young, hired when young:

$$g_{y,y}(w) = g_y(w)$$

2. The density of wage of the adults, hired when young:

$$g_{a,y}(w) = \frac{p(m - u_y) g_{y,y}(w)}{(m - u_a)(p + s + \lambda_a(1 - F_a(w)))}$$

3. The density of wage of the adults, hired when adult:

$$g_{a,a}(w) = \frac{\lambda_a^0 f_a(w) u_a}{(m - u_a)(p + s + \lambda_a(1 - F_a(w)))} + \frac{(\lambda_a^0 F_a(w) u_a + p(m - u_a) G_y(w)) \lambda_a f_a(w)}{(m - u_a)(p + s + \lambda_a(1 - F_a(w)))^2}$$

4. The density of wage of the seniors, hired when young:

$$g_{s,y}(w) = \frac{p(m - u_a) g_{a,y}(w)}{(m - u_s)(p + s + \lambda_s(1 - F_s(w)))}$$

5. The density of wage of the seniors, hired when adult:

$$g_{s,a}(w) = \frac{p(m - u_a) g_{a,a}(w)}{(m - u_s)(p + s + \lambda_s(1 - F_s(w)))}$$

6. The density of wage of the seniors, hired when adult:

$$g_{s,s}(w) = \frac{\lambda_s^0 f_s(w) u_s}{(m - u_s)(p + s + \lambda_s(1 - F_s(w)))} + \frac{(\lambda_s^0 F_s(w) u_s + p(m - u_a) G_a(w)) \lambda_s f_s(w)}{(m - u_s)(p + s + \lambda_s(1 - F_s(w)))^2}$$

## .5 Equiprofit conditions

Firms spread their wage offers along a wage interval. The firms' maximum instantaneous profit is obtained when the firms post the lowest wage, when firms increase their offer, their instantaneous profit decreases, yet as  $F(w)$  increases, the hiring probability, the retention, and the productivity increase.

The lowest wages that the firms have interest in offering on each market is not necessarily the institutional minimum wage that we note  $\underline{w}$ . They are computed as it

follows:

$$\underline{w}_y = \max\{\underline{w}; \operatorname{argmax}_w \underline{\Pi}_y(w)\}$$

$$\underline{w}_a = \max\{\underline{w}; \operatorname{argmax}_w \underline{\Pi}_a(w)\}$$

$$\underline{w}_s = \max\{\underline{w}; \operatorname{argmax}_w \underline{\Pi}_s(w)\}$$

with  $\underline{\Pi}_i$  the profit of firms offering the lowest wage on the market, ie. the expression of the profit when  $F_i(w) = 0$ . For more details on the definition of this profit, see chapter 2.

As  $F(w)$  cannot be superior to 1, there exists in each market a  $\overline{w}_i$  above which firms have no interest in posting wages. They are computed for each age by:

$$\Pi_y(\underline{w}_y) = \Pi_y(\overline{w}_y)$$

$$\Pi_y(\underline{w}_a) = \Pi_y(\overline{w}_a)$$

$$\Pi_y(\underline{w}_s) = \Pi_y(\overline{w}_s)$$

As the profit is different and can evolve differently from one market to another with wage, it is likely that the maximum wage would be different in each market.

Eventually the distribution of the wages offered by the firms solves:

From  $\underline{w}_y$  to  $\overline{w}_y$ ,

$$\Pi_y(\underline{w}) = \Pi_y(w)$$

From  $\underline{w}_a$  to  $\overline{w}_a$ ,

$$\Pi_a(\underline{w}_a) = \Pi_y(w)$$

From  $\underline{w}_s$  to  $\overline{w}_s$ ,

$$\Pi_s(\underline{w}_s) = \Pi_y(w)$$

## .6 Expected match duration

The form of the expected match duration is similar to that of chapter 2. The respective expected match duration according to the age class of the worker employed is given by:

$$\begin{aligned}
 D_s(w) &= \frac{1}{p + s + \lambda_s(1 - F_s(w))} \\
 D_a(w) &= \frac{1}{p + s + \lambda_a(1 - F_a(w))} \left( 1 + \frac{p}{p + s + \lambda_s(1 - F_s(w))} \right) \\
 D_y(w) &= \frac{1}{p + s + \lambda_y(1 - F_y(w))} \left( 1 + \frac{p}{p + s + \lambda_a(1 - F_a(w))} \left( 1 + \frac{p}{p + s + \lambda_s(1 - F_s(w))} \right) \right)
 \end{aligned}$$

## .7 Effect of the labor market tightness on profit

**Seniors' Labor Market** The effect of labor market tightness on the profit on the seniors' market is given by:

$$\begin{aligned}
 \Pi_s(\underline{w}, \theta_s) &= h_s(\underline{w}, \theta_s) (J_s(\underline{w}, \theta_s) - \beta_s k_s(\underline{w}, \theta_s)) \\
 \frac{\partial \Pi_s(\underline{w}, \theta_s)}{\partial \theta_s} &= \frac{\partial h_s(\underline{w}, \theta_s)}{\partial \theta_s} (J_s(\underline{w}, \theta_s) - \beta_s k_s(\underline{w}, \theta_s)) \\
 &\quad + h_s(\underline{w}, \theta_s) \left( \frac{\partial J_s(\underline{w}, \theta_s)}{\partial \theta_s} - \beta_s \frac{\partial k_s(\underline{w}, \theta_s)}{\partial \theta_s} \right)
 \end{aligned}$$

With,

$$\begin{aligned}
 h_s(\underline{w}, \theta_s) &= q_s^0 u_s \\
 h_s(\underline{w}, \theta_s) &= \phi^0 \theta_s^{-\eta} \frac{sm + pu_a}{p + s + \phi^0 \theta_s^{1-\eta}} \\
 \Rightarrow \frac{\partial h_s(\underline{w}, \theta_s)}{\partial \theta_s} &< 0
 \end{aligned}$$

And with,

$$\begin{aligned}\frac{\partial J_s(\underline{w}, \theta_s)}{\partial \theta_s} &= \frac{q \frac{\partial k_s(\underline{w}, \theta_s)}{\partial \theta_s} k_s(\underline{w}, \theta_s)^{\alpha-1}}{r+p+s+\lambda_s} - \frac{y_s(k_s(\underline{w}, \theta_s)) - \underline{w}}{(r+p+s+\lambda_s)^2} \\ \frac{\partial J_s(\underline{w}, \theta_s)}{\partial \theta_s} &= \frac{\partial k_s(\underline{w}, \theta_s)}{\partial \theta_s} \left( \frac{q k_s(\underline{w}, \theta_s)^{\alpha-1}}{r+p+s+\lambda_s} \right) \\ &\quad - \frac{y_s(k_s(\underline{w}, \theta_s)) - \underline{w}}{(r+p+s+\lambda_s)^2}\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{\partial J_s(\underline{w}, \theta_s)}{\partial \theta_s} - \beta_s \frac{\partial k_s(\underline{w}, \theta_s)}{\partial \theta_s} &= \frac{\partial k_s(\underline{w}, \theta_s)}{\partial \theta_s} \underbrace{\left( \frac{q k_s(\underline{w}, \theta_s)^{\alpha-1}}{r+p+s+\lambda_s} - \beta_s \right)}_{=0 \text{ by definition of } k_s} \\ &\quad - \frac{y_s(k_s(\underline{w}, \theta_s)) - \underline{w}}{(r+p+s+\lambda_s)^2} \\ \Rightarrow \frac{\partial J_s(\underline{w}, \theta_s)}{\partial \theta_s} - \beta_s \frac{\partial k_s(\underline{w}, \theta_s)}{\partial \theta_s} &< 0\end{aligned}$$

Consequently,  $\frac{\partial \Pi_s(\underline{w}, \theta_s)}{\partial \theta_s} < 0$ .

**Adults' Labor Market** We proceed as well on the adults' market. The effect of labor market tightness on the profit on the adults' market is given by:

$$\begin{aligned}\Pi_a(\underline{w}, \theta_a) &= h_a(\underline{w}, \theta_a) (J_a(\underline{w}, \theta_a) - \beta_a k_a(\underline{w}, \theta_a)) \\ \frac{\partial \Pi_a(\underline{w}, \theta_a)}{\partial \theta_a} &= \frac{\partial h_a(\underline{w}, \theta_a)}{\partial \theta_a} (J_a(\underline{w}, \theta_a) - \beta_a k_a(\underline{w}, \theta_a)) \\ &\quad + h_a(\underline{w}, \theta_a) \left( \frac{\partial J_a(\underline{w}, \theta_a)}{\partial \theta_a} - \beta_a \frac{\partial k_a(\underline{w}, \theta_a)}{\partial \theta_a} \right)\end{aligned}$$

With,

$$\begin{aligned}
 h_a(\underline{w}, \theta_a) &= q_a^0 u_a \\
 h_a(\underline{w}, \theta_a) &= \phi^0 \theta_a^{-\eta} \frac{sm + pu_y}{p + s + \phi^0 \theta_a^{1-\eta}} \\
 \Rightarrow \frac{\partial h_a(\underline{w}, \theta_a)}{\partial \theta_a} &< 0
 \end{aligned}$$

And with after simplifications,

$$\begin{aligned}
 \frac{\partial J_a(\underline{w}, \theta_a)}{\partial \theta_a} &= \frac{q \frac{\partial k_a(\underline{w}, \theta_a)}{\partial \theta_a} k_a(\underline{w}, \theta_a)^{\alpha-1} + p \frac{\partial J_s(\underline{w}, \theta_a)}{\partial \theta_a}}{r + p + s + \lambda_a} - \frac{y_a(k_a(\underline{w}, \theta_a)) - \underline{w} + p J_s(\underline{w}, \theta_a)}{(r + p + s + \lambda_a)^2} \\
 \frac{\partial J_a(\underline{w}, \theta_a)}{\partial \theta_a} &= \frac{\partial k_a(\underline{w}, \theta_a)}{\partial \theta_a} \left( \frac{q k_a(\underline{w}, \theta_a)^{\alpha-1}}{r + p + s + \lambda_a} \right) \\
 &+ \frac{p}{r + p + s + \lambda_a} \left( \frac{q \frac{\partial k_a(\underline{w}, \theta_a)}{\partial \theta_a} k_a(\underline{w}, \theta_a)^{\alpha-1}}{r + p + s + \lambda_s} - \frac{y_s(k_a(\underline{w}, \theta_a)) - \underline{w}}{(r + p + s + \lambda_s (1 - F_s(\underline{w})))^2} \right) \\
 &- \frac{y_a(k_a(\underline{w}, \theta_a)) - \underline{w} + p J_s(\underline{w}, \theta_a)}{(r + p + s + \lambda_a)^2} \\
 \frac{\partial J_a(\underline{w}, \theta_a)}{\partial \theta_a} &= \frac{\partial k_a(\underline{w}, \theta_a)}{\partial \theta_a} \left( \frac{q k_a(\underline{w}, \theta_a)^{\alpha-1}}{r + p + s + \lambda_a} + \frac{p}{r + p + s + \lambda_a} \frac{q k_a(\underline{w}, \theta_a)^{\alpha-1}}{r + p + s + \lambda_s} \right) \\
 &- \frac{y_a(k_a(\underline{w}, \theta_a)) - \underline{w} + p J_s(\underline{w}, \theta_a)}{(r + p + s + \lambda_a)^2} \\
 &- \frac{p}{r + p + s + \lambda_a} \frac{y_s(k_a(\underline{w}, \theta_a)) - \underline{w}}{(r + p + s + \lambda_s)^2}
 \end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{\partial J_a(\underline{w}, \theta_a)}{\partial \theta_a} - \beta_a \frac{\partial k_a(\underline{w}, \theta_a)}{\partial \theta_a} &= \frac{\partial k_a(\underline{w}, \theta_a)}{\partial \theta_a} \\
&\times \underbrace{\left( \frac{qk_a(\underline{w}, \theta_a)^{\alpha-1}}{r+p+s+\lambda_a} + \frac{p}{r+p+s+\lambda_a} \frac{qk_a(\underline{w}, \theta_a)^{\alpha-1}}{r+p+s+\lambda_s} - \beta_a \right)}_{=0 \text{ by definition of } k_a(\underline{w})} \\
&- \frac{y_a(k_a(\underline{w}, \theta_a)) - \underline{w} + pJ_s(\underline{w}, \theta_a)}{(r+p+s+\lambda_a)^2} \\
&- \frac{p}{r+p+s+\lambda_a} \frac{y_s(k_a(\underline{w}, \theta_a)) - \underline{w}}{(r+p+s+\lambda_s)^2} \\
\Rightarrow \frac{\partial J_a(\underline{w}, \theta_a)}{\partial \theta_a} - \beta_a \frac{\partial k_a(\underline{w}, \theta_a)}{\partial \theta_a} &< 0
\end{aligned}$$

Consequently,  $\frac{\partial \Pi_a(\underline{w}, \theta_a)}{\partial \theta_a} < 0$ .

**Youth's Labor Market** The effect of labor market tightness on the profit on the youth's market is given by:

$$\begin{aligned}
\Pi_y(\underline{w}, \theta_y) &= h_y(\underline{w}, \theta_y) (J_y(\underline{w}, \theta_y) - \beta_y k_y(\underline{w}, \theta_y)) \\
\frac{\partial \Pi_y(\underline{w}, \theta_y)}{\partial \theta_y} &= \frac{\partial h_y(\underline{w}, \theta_y)}{\partial \theta_y} (J_y(\underline{w}, \theta_y) - \beta_y k_y(\underline{w}, \theta_y)) + h_y(\underline{w}, \theta_y) \left( \frac{\partial J_y(\underline{w}, \theta_y)}{\partial \theta_y} - \beta_y \frac{\partial k_y(\underline{w}, \theta_y)}{\partial \theta_y} \right)
\end{aligned}$$

With,

$$\begin{aligned}
h_y(\underline{w}, \theta_y) &= q_y^0 u_y \\
h_y(\underline{w}, \theta_y) &= \phi^0 \theta_y^{-\eta} \frac{(s+p)m}{p+s+\phi^0 \theta_y^{1-\eta}} \\
\Rightarrow \frac{\partial h_y(\underline{w}, \theta_y)}{\partial \theta_y} &< 0
\end{aligned}$$

And with,

$$\begin{aligned}
 \frac{\partial J_y(\underline{w}, \theta_y)}{\partial \theta_y} &= \frac{q \frac{\partial k_y(\underline{w}, \theta_y)}{\partial \theta_y} k_y(\underline{w}, \theta_y)^{\alpha-1} + \frac{\partial J_a(\underline{w}, \theta_y)}{\partial \theta_y}}{r + p + s + \lambda_y} - \frac{y_y(k_y(\underline{w}, \theta_y)) - \underline{w} + pJ_a(\underline{w}, \theta_y)}{(r + p + s + \lambda_y)^2} \\
 \frac{\partial J_y(\underline{w}, \theta_y)}{\partial \theta_y} &= \frac{\partial k_y(\underline{w}, \theta_y)}{\partial \theta_y} \left( \frac{q k_y(\underline{w}, \theta_y)^{\alpha-1}}{r + p + s + \lambda_y} \right) + \frac{p}{r + p + s + \lambda_y} \\
 &\times \left( \frac{q \frac{\partial k_y(\underline{w}, \theta_y)}{\partial \theta_y} k_y(\underline{w}, \theta_y)^{\alpha-1} + p \frac{\partial J_s(\underline{w}, \theta_y)}{\partial \theta_y}}{r + p + s + \lambda_a} - \frac{y_a(k_y(\underline{w}, \theta_y)) - \underline{w}_y + pJ_s(\underline{w}, \theta_y)}{(r + p + s + \lambda_a)^2} \right) \\
 &- \frac{y_y(k_y(\underline{w}, \theta_y)) - \underline{w} + pJ_a(\underline{w}, \theta_y)}{(r + p + s + \lambda_y)^2}
 \end{aligned}$$

After simplifications, we get:

$$\begin{aligned}
 \frac{\partial J_y(\underline{w}, \theta_y)}{\partial \theta_y} &= \frac{\partial k_y(\underline{w}, \theta_y)}{\partial \theta_y} \left( \frac{q k_y(\underline{w}, \theta_y)^{\alpha-1}}{r + p + s + \lambda_y} + \frac{p}{r + p + s + \lambda_y} \frac{q k_y(\underline{w}, \theta_y)^{\alpha-1}}{r + p + s + \lambda_a} \right) \\
 &+ \left( \frac{p}{r + p + s + \lambda_y} \right) \left( \frac{p}{r + p + s + \lambda_a} \right) \\
 &\times \left( \frac{q \frac{\partial k_y(\underline{w}, \theta_y)}{\partial \theta_y} k_y(\underline{w}, \theta_y)^{\alpha-1}}{r + p + s + \lambda_s} - \frac{y_s(k_y(\underline{w}, \theta_y)) - \underline{w}}{(r + p + s + \lambda_s)^2} \right) \\
 &- \frac{y_y(k_y(\underline{w}, \theta_y)) - \underline{w} + pJ_a(\underline{w}, \theta_y)}{(r + p + s + \lambda_y)^2} \\
 &- \frac{p}{r + p + s + \lambda_y} \frac{y_a(k_y(\underline{w}, \theta_y)) - \underline{w} + pJ_s(\underline{w}, \theta_y)}{(r + p + s + \lambda_a)^2}
 \end{aligned}$$



$$\begin{aligned}
\frac{\partial J_y(\underline{w}, \theta_y)}{\partial \theta_y} &= \frac{\partial k_y(\underline{w}, \theta_y)}{\partial \theta_y} \frac{q}{r+p+s+\lambda_y} \\
&\times \left( k_y(\underline{w}, \theta_y)^{\alpha-1} + \frac{1}{r+p+s+\lambda_a} \left( p k_y(\underline{w}, \theta_y)^{\alpha-1} + \frac{p^2 k_y(\underline{w}, \theta_y)^{\alpha-1}}{r+p+s+\lambda_s} \right) \right) \\
&- \frac{y_y(k_y(\underline{w}, \theta_y)) - \underline{w} + p J_a(\underline{w}, \theta_y)}{(r+p+s+\lambda_y)^2} \\
&- \frac{p}{r+p+s+\lambda_y} \frac{y_a(k_y(\underline{w}, \theta_y)) - \underline{w} + p J_s(\underline{w}, \theta_y)}{(r+p+s+\lambda_a)^2} \\
&- \left( \frac{p}{r+p+s+\lambda_y} \right) \left( \frac{p}{r+p+s+\lambda_a} \right) \frac{y_s(k_y(\underline{w}, \theta_y)) - \underline{w}}{(r+p+s+\lambda_s)^2}
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{\partial J_y(\underline{w}, \theta_y)}{\partial \theta_y} - \beta_a \frac{\partial k_a(\underline{w}, \theta_a)}{\partial \theta_a} &= \frac{\partial k_y(\underline{w}, \theta_y)}{\partial \theta_y} \frac{q}{r+p+s+\lambda_y} \\
&\times \underbrace{\left( k_y(\underline{w}, \theta_y)^{\alpha-1} + \frac{1}{r+p+s+\lambda_a} \left( p k_y(\underline{w}, \theta_y)^{\alpha-1} + \frac{p^2 k_y(\underline{w}, \theta_y)^{\alpha-1}}{r+p+s+\lambda_s} - \beta_y \right) \right)}_{=0 \text{ by definition of } k_y(\underline{w})} \\
&- \frac{y_y(k_y(\underline{w}, \theta_y)) - \underline{w} + p J_a(\underline{w}, \theta_y)}{(r+p+s+\lambda_y)^2} \\
&- \frac{p}{r+p+s+\lambda_y} \frac{y_a(k_y(\underline{w}, \theta_y)) - \underline{w} + p J_s(\underline{w}, \theta_y)}{(r+p+s+\lambda_a)^2} \\
&- \left( \frac{p}{r+p+s+\lambda_y} \right) \left( \frac{p}{r+p+s+\lambda_a} \right) \frac{y_s(k_y(\underline{w}, \theta_y)) - \underline{w}}{(r+p+s+\lambda_s)^2} \\
\Rightarrow \frac{\partial J_y(\underline{w}, \theta_y)}{\partial \theta_y} - \beta_a \frac{\partial k_a(\underline{w}, \theta_a)}{\partial \theta_a} &< 0
\end{aligned}$$

Consequently,  $\frac{\partial \Pi_y(\underline{w}, \theta_y)}{\partial \theta_y} < 0$ .

## .8 Simulations

### .8.1 Simulation 1A

Table 10: Moments of simulated distribution of offered wage, offered productivity, wage and offered job quality - Simulation 1A: Benchmark economy

	Total Population	Young	Adults	Seniors
Mean wage ( $g_i$ )	2.62	2.18	2.82	3
Mean wage offered ( $f_i$ )	1.88	1.64	2.04	2.02
Dispersion of wages ( $\sigma(g_i)$ )	0.3	0.27	0.27	0.28
Dispersion of wages offered ( $\sigma(f_i)$ )	0.39	0.28	0.32	0.36
Mean new hired productivity ( $y_i(k)$ )	2.61	2.16	2.86	2.98
→ With workers' specific productivity ( $y_i$ )	2.04	1.75	2.26	2.14
→ With quality of the job ( $qk_i^\alpha$ )	0.57	0.41	0.60	0.84

## .8.2 Simulation 2A

Table 11: Moments of simulated distribution of offered wage, offered productivity, wage and offered job quality - Simulation 2A: with  $y_y = y_a = y_s = 1.75$

	Total Population	Young	Adults	Seniors
Mean wage ( $g_i$ )	2.40	2.20	2.44	2.65
Mean wage offered ( $f_i$ )	1.7	1.64	1.7	1.78
Dispersion of wages ( $\sigma(g_i)$ )	0.3	0.28	0.29	0.3
Dispersion of wages offered ( $\sigma(f_i)$ )	0.36	0.29	0.32	0.35
Mean new hired productivity ( $y_i(k)$ )	2.34	2.22	2.34	2.59
Workers'specific productivity ( $y_i$ )	1.75	1.75	1.75	1.75
Quality of the job ( $qk_i^\alpha$ )	0.59	0.47	0.59	0.84

### .8.3 Simulation 3A

Table 12: Simulated distribution of offered wage, offered productivity, wage and offered job quality - Simulation 3A: with  $y_y = y_a = y_s = 1.75$  and  $\beta_y = \beta_a = \beta_s = 1$

	Total Population	Young	Adults	Seniors
Mean wage ( $g_i$ )	2.26	2.25	2.33	2.12
Mean wage offered ( $f_i$ )	1.64	1.65	1.66	1.58
Dispersion of wages ( $\sigma(g_i)$ )	0.27	0.29	0.26	0.23
Dispersion of wages offered ( $\sigma(f_i)$ )	0.32	0.3	0.28	0.21
Mean new hired productivity ( $y_i(k)$ )	2.2	2.25	2.22	2.04
→ With workers'specific productivity ( $y_i$ )	1.75	1.75	1.75	1.75
→ With quality of the job ( $qk_i^\alpha$ )	0.45	0.5	0.47	0.29



# Age-dependant Effect of French Institutions

## .1 Unemployment rate

The total unemployment rate of each market is the sum of unemployment rates for each level of unemployment benefit, it depends on the repartition of unemployment benefits in the economy. The total unemployment rate for each age is given by:

$$\begin{aligned}
 \frac{\bar{u}_y}{m} &= \frac{\int_{\underline{b}}^{\bar{b}} \frac{sg_y\left(\frac{b-all}{\rho}\right)}{\lambda_y^0(1-F_y(R_y(b)))+p} db + \frac{p}{p+\lambda_y^0}}{1 + \int_{\underline{b}}^{\bar{b}} \frac{sg_y\left(\frac{b-all}{\rho}\right)}{\lambda_y^0(1-F_y(R_y(b)))+p} db} \\
 \frac{\bar{u}_a}{m} &= \frac{\int_{\underline{b}}^{\bar{b}} \frac{sg_a\left(\frac{b-all}{\rho}\right)}{\lambda_a^0(1-F_a(R_a(b)))+p} db + \int_{\underline{b}}^{\bar{b}} \frac{p \frac{u_j(b)}{m}}{p+\lambda_a^0(1-F_a(R_a(b)))} db}{1 + \int_{\underline{b}}^{\bar{b}} \frac{sg_a\left(\frac{b-all}{\rho}\right)}{\lambda_a^0(1-F_a(R_a(b)))+p} db} \\
 \frac{\bar{u}_s}{m} &= \frac{\int_{\underline{b}}^{\bar{b}} \frac{sg_s\left(\frac{b-all}{\rho}\right)}{\lambda_s^0(1-F_s(R_s(b)))+p} db + \int_{\underline{b}}^{\bar{b}} \frac{p \frac{u_a(b)}{m}}{p+\lambda_s^0(1-F_s(R_s(b)))} db}{1 + \int_{\underline{b}}^{\bar{b}} \frac{sg_s\left(\frac{b-all}{\rho}\right)}{\lambda_s^0(1-F_s(R_s(b)))+p} db}
 \end{aligned} \tag{28}$$

Note that when we set  $\underline{b} = \bar{b} = b$ , we have  $g_i\left(\frac{b-all}{\rho}\right) = 1$  and  $R_i(b) = 0$ , and obtain the same results as in the chapter 3.

It is important to differentiate the mass of unemployed workers according to  $b$  since for each unemployment benefit the exit rate for unemployment is different.

## .2 Simulations

### .2.1 Simulations 1B

Table 13: Moments of simulated distribution of offered wage, offered productivity, wage and offered job quality- Simulation 1B: French Benchmark economy

	Total Population	Young	Adults	Seniors
Mean wage ( $g_i$ )	1.85	1.61	1.92	2.17
Mean wage offered ( $f_i$ )	1.68	1.4	1.74	1.9
Dispersion of wages ( $\sigma(g_i)$ )	0.22	0.18	0.187	0.19
Dispersion of wages offered ( $\sigma(f_i)$ )	0.23	0.16	0.16	0.2
Mean new hired productivity ( $y_i(k)$ )	2.32	1.74	2.33	3.13
→ Workers' specific productivity ( $y_i$ )	1.53	1.4	1.61	1.8
→ Quality of the job ( $qk_i^\alpha$ )	0.79	0.34	0.72	1.33

**.2.2 Simulations 2B**

Table 14: Moments of simulated distribution of offered wage, offered productivity, wage and offered job quality- Simulation 2B: French Benchmark economy without unemployment insurance system

	Total Population	Young	Adults	Seniors
Mean wage ( $g_i$ )	1.81	1.56	1.84	2.02
Mean wage offered ( $f_i$ )	1.5	1.34	1.54	1.59
Dispersion of wages ( $\sigma(g_i)$ )	0.24	0.2	0.217	0.24
Dispersion of wages offered ( $\sigma(f_i)$ )	0.27	0.19	0.23	0.28
Mean new hired productivity ( $y_i(k)$ )	2.11	1.71	2.14	2.7
→ Workers' specific productivity ( $y_i$ )	1.53	1.4	1.61	1.8
→ Quality of the job ( $qk_i^\alpha$ )	0.58	0.31	0.53	0.9



**.2.3 Simulations 9B**

Table 15: Moments of simulated distribution of offered wage, offered productivity, wage and offered job quality- Simulation 9B: French Benchmark economy without institution

	Total Population	Young	Adults	Seniors
Mean wage ( $g_i$ )	1.83	1.55	1.85	2.09
Mean wage offered ( $f_i$ )	1.43	1.27	1.47	1.54
Dispersion of wages ( $\sigma(g_i)$ )	0.28	0.23	0.24	0.28
Dispersion of wages offered ( $\sigma(f_i)$ )	0.33	0.24	0.28	0.36
Mean new hired productivity ( $y_i(k)$ )	2.13	1.7	2.14	2.76
→ Workers' specific productivity ( $y_i$ )	1.53	1.4	1.61	1.8
→ Quality of the job ( $qk_i^\alpha$ )	0.6	0.3	0.53	0.96

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