



Contributions and perspectives to computer vision, image processing and EEG/MEG data analysis

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Université de Nice-Sophia Antipolis

HABILITATION A DIRIGER DES RECHERCHES

**Contributions and perspectives to
computer vision, image processing and
EEG/MEG data analysis.**

par

THÉODORE PAPADOPOULO

Soutenue le 9 mai 2011 devant le jury composé de

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Introduction

This document presents some contributions in two different fields I have had the chance to explore in the past years.

The first field is that of computer vision, image processing, numerical optimization applied to images, fields that aim at developing tools for recovering informations about a scene from one or more images of it. This activity was developed within the ROBOTVIS project team (1990–2001) and has culminated in 1998 with the creation of the company REALVIZ of which I am one of the co-founders. My research work in these fields can be classified into four main topics:

1. Motion estimation of 3D rigid curves.
2. Estimation of low-level informations from images.
3. Camera self calibration.
4. Using constraints and geometric reasoning for 3D reconstruction.

Chapter 1 presents the work done after 1997, which basically eliminates the first item of that list from this presentation¹. After a short presentation of computer vision and image processing, the chapter presents the contributions grouped in two sections. The first section deals with the “geometrical image”, i.e. with contributions that mainly consider point positions in images or in 3D. The second section deals with the “photometric image”. These are contributions in which pixel values are also important. This part of my activities has led to the creation of the RealViz company which was founded in 1998 with my colleagues of the ROBOTVIS team. The part of this work dealing with the trifocal tensor has also been integrated in the book [1] which was co-written with Olivier Faugeras and Tuan Luong.

After 2001, my interest has moved on to brain imaging and to localisation and modeling of brain activity using the modalities of electro-encephalography (EEG) and magneto-encephalography (MEG). These non-invasive techniques provide complementary measurements related to the electrical activity of the brain. One of the main ideas driving this shift of interest was that better understanding the organisation of the brain and interactions between brain areas, while very interesting in itself, could ultimately (not necessarily on a short time scale) also bring ideas on how to organise and structure artificial visual systems. A few years later, this goal is somewhat buried under all the other interesting problems that we found with EEG and MEG processing. Yet, there is some cross-fertilization at the level of the techniques used, and I still hope that, in the long run, the work presented here will help in better understanding the functioning of the

¹This essentially excludes the work done during the Ph.D. according to the rule of “habilitation à diriger les recherches”.

brain and – obviously – of the visual system. My activities with respect to MEG/EEG can be regrouped in five main topics:

1. Extracting and modeling events of interest.
2. The MEG/EEG forward problem.
3. Geometrical and physical modeling of the head.
4. Localisation of cortical electrical sources from MEG/EEG measurements.
5. Electrode labeling.

Chapter 2 presents this work. Part of this work is the basis of the OpenMEEG software which provides an implementation of the MEG/EEG forward problem using the symmetric BEM technique (details on this technique are provided in chapter 2) which I co-maintain with Maureen Clerc, Alexandre Gramfort and Emmanuel Olivi.

Both these chapters adopt a similar presentation. After a brief introduction of the field and of its main problematics, my contributions are briefly presented. These are grouped by theme. Within each theme, I also describe my view on some short and long term perspectives of the work. The pieces of work I chose to describe in this document are of course those I consider to be the most important ones but they are also intended to show the variety of the topics I studied as well as the diversity of the techniques that have been used for the purpose of this work (geometry, linear algebra, partial differential equations, signal processing, formal methods, numerical analysis, geometrical and biological modeling, inverse problems, visualisation, etc). The work covered by these two chapters involved numerous collaborators over the years. I participated in the supervision of a few Ph.D. students Didier Bondyfalat (with O. Faugeras and B. Mourrain), Sylvain Vallaghé and Alexandre Gramfort (with Maureen Clerc) and Emmanuel Olivi (with Maureen Clerc). With an exemption, I supervised the Ph.D. work of Jérôme Piovano. I also worked with the post-docs Jan Kybic and Christian Bénar on various aspects of M/EEG signal processing and with Manolis Lourakis on some computer vision topics. This work involved many french and international co-authors as well as various INRIA colleagues, with a special mention to Maureen Clerc with whom I work very closely on the topics of MEG/EEG and BCI. A more complete list of these collaborators is provided in the extended resume annexed to this document.

Please note that numbered references correspond to the work in which I have been involved and refer to the bibliography at the end of this document. Background references are provided as footnotes and the citations take the form [Author, year]. Note also that a synthetic summary of all my past research activities is presented at the end of the extended resume.

Over the years, I have been involved in several courses around the topics of this work. The main ones are the master courses “3D computer vision” and “Inverse problems in functional brain imaging” that are given in both “École normale supérieure de Cachan” and University of Nice-Sophia Antipolis. Again, more details on these lectures are given in the appendix. *Please note that numbered references correspond to the work in which I have been involved and refer to the bibliography at the end of this document. Background references are provided as footnotes*

and the citations take the form [Author, year]. Note also that a synthetic summary of all my past research activities is presented at the end of the extended resume.

Following the work presentation, a conclusion develops and organizes some of the perspectives presented in the chapters. Finally, the extended resume of my activities is given in an appendix.

Chapter 1

Computer vision and image processing

Ultimately, computer vision (CV) aims at recovering information about the 3D world from images of that world. In practise, CV is made of many subfields such as camera calibration (recovering the information about the “eyes” that captured the images), 3D reconstruction (recovering the 3D structure of the viewed scene), motion estimation, object segmentation, object recognition, recovering informations about the materials, the light source, etc. Images are full of cues about the world they represent. However, in most cases, these cues only offer an incomplete view of the world and several such cues need to be studied jointly to recover the sought information. As a simple example, with a single image, one cannot, in general, recover the 3D position of a particular image point (assuming a static world). To do so, supplementary information is needed: this supplementary information can come e.g. from a priori knowledge of the scene or from the use of several images. In this late case, the problem of identifying points in two or more images that correspond to the same 3D world point arises. Obviously, such an identification assumes that points can be singled out in images easily, which is not always the case e.g. for smooth untextured shapes. In that case, only techniques such as shape from shading may provide some information about the surface. Finally, in places where there is no texture nor shading, only a priori hypotheses on the world can help by providing default answers. In the general case, all this sources of information (multiple images, shading, a priori knowledge of the scene) must be combined to obtain a robust solution.

Thus, in theory, a “perfect computer vision system” (should one exists) should take into account of all possible cues and integrate the partial information provided by each of those. This is probably the only pathway to obtain a system that would have the perceived¹ reliability of biological vision. Creating such a system is a daunting task. Faced with this dilemma, the scientific community has adopted a divide-and-conquer strategy: several narrow views of the problem (such as stereo reconstruction, shape from motion, shape from texture, shape from shading, etc.) have been selected and developed in relative isolation. Attempting to classify all these approaches is a difficult task (several proposals have been made in the past, none of which is really entirely satisfactory). Still, in the following presentation, I will separate two categories of tasks by – somewhat arbitrarily – separating the photometric and geometric content of an image. Indeed, at the core of computer vision are images which can be seen as:

- **Pure geometric spaces:** in this subfield, the “photometric part” (color, luminance) of the images is not considered at all, only point positions – *pixel coordinates* – are. From

¹We often do not even realise how much we are fooled by our own visual system.

a mathematical point of view, the geometric information of the scene and its relationship with the geometric information in images (through camera parameters such as position and orientation) is quite well understood. It is thus fairly easy to exploit the mathematical framework depicting these relations to obtain geometric information about the 3D world. This probably explains the tremendous development (and success) of the algorithms of that class in the past years [1] [Faugeras, 1993, Hartley and Zisserman, 2000, Forsyth and Ponce, 2003, Ma et al., 2004]. In the following, I will gather all such geometric algorithms under the category “**Geometric image**”.

- **Photometric objects:** a collection of point positions is nothing in itself (all image positions can usually be simply described as a lattice of regularly spaced points). To be useful, selected point positions must be extracted from images. This is where *pixel values* enter the equation. Those pixel values are physical measurements of the imaged world, and they are crucial to single out particular point positions in one or more images. Contrarily to its “geometric” counterpart the **photometric image** is very difficult to model with simple mathematical framework (because such a description will depend on fields such as physics, biology, mechanics, etc.) [Ballard and Brown, 1982, Forsyth and Ponce, 2003, Szeliski, 2010].

Of course, ultimately all CV algorithms use both geometric and photometric image information in some form or the other: even the most geometric information use points that are extracted from images, similarly even the most photometric algorithms rely on some geometric assumptions such as proximity or similarity. Yet, often an algorithm focusses on one of these aspects, the other being considered as some sort of pre- or post-processing step. I entered in the field of computer vision from the point of view of the higher stages of the processing of image information, but as explained above many incursions in the field of the “photometric image” have also been necessary.

1.1 The geometric image

Geometric computer vision finds its root in *photogrammetry*. Photogrammetry studies how to obtain geometric properties about objects from photographic images and can be dated to the mid-nineteenth century. It can be considered as the first remote sensing technology ever developed. Computer vision was born in the mid-60’s with the work of Roberts [Roberts, 1965]

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- [Faugeras, 1993] Faugeras, O. (1993). Cartan’s moving frame method and its application to the geometry and evolution of curves in the euclidean, affine and projective planes. In Mundy, J. and Zisserman, A., editors, *Applications of Invariance in Computer Vision*, pages 1–33. also INRIA Technical report 2053.
- [Hartley and Zisserman, 2000] Hartley, R. and Zisserman, A. (2000). *Multiple View Geometry in computer vision*. Cambridge university press.
- [Forsyth and Ponce, 2003] Forsyth, D. and Ponce, J. (2003). *Computer Vision : a modern approach*. Pearson Education, Prentice Hall.
- [Ma et al., 2004] Ma, Y., Soatto, S., Kov seck’ a, J., and Sastry, S. S. (2004). *An Invitation to 3-D Vision. From Images to Geometric Models*, volume 26 of *Interdisciplinary Applied Mathematics. Imaging, Vision and Graphics*. Springer.
- [Ballard and Brown, 1982] Ballard, D. and Brown, C. (1982). *Computer Vision*. Prentice-Hall, Englewood Cliffs, New Jersey.
- [Szeliski, 2010] Szeliski, R. (2010). *Computer Vision: Algorithms and Applications*. Springer.
- [Roberts, 1965] Roberts, L. (1965). Machine perception of three-dimensional solids. In Tippet, J., Berkowitz, D., Clapp, L., Koester, C., and Vanderburgh, A., editor, *Optical and Electrooptical Information processing*, pages 159–197. MIT Press.

but really started developing in the late 70's with the advent of computers able to process the amount of data involved in images. At that time, computer vision was seen from the artificial intelligence perspective, initially quite inspired by biological vision [Marr and Poggio, 1977, Marr, 1982]. As explained above, geometric computer vision essentially considers the embedding spaces corresponding to images and the 3D scene. Since the basic operation of taking a picture corresponds to fixate on a surface (the retina or the picture) the set of light rays passing through a point (the eye or the camera), the geometry of lines is particularly important to depict the relations between the scene and images of it. This is why projective geometry (the geometry of lines) is so central in geometric computer vision. With geometric computer vision, it is usually supposed that an appropriate preprocessing of the images – automated or manual – has provided a set of labelled geometric primitives (points, lines, segments, contours, etc.) within each image. Each label corresponds to a distinct 3D spatio-temporal object – known or unknown – in the observed scene. This situation gives rise to multiple interesting problems, among which are:

Calibration

If the 3D primitives are known or partially known, then it is possible to – perhaps partially – recover the characteristics of the camera that gave rise to the image: its position and orientation (the external parameters) and sensor characteristics parameters such as the CCD geometry or the focal length (the internal parameters).

Projective invariants

If only image primitives are known in multiple images, the primitives in the various images corresponding to the same label are called correspondences. If the scene is essentially rigid with respect to the cameras, then it is possible to recover from the correspondences *invariant* quantities that characterize the relative positions of the cameras and their internal parameters². These quantities are related to *epipolar geometry* and appear as components of algebraic objects such as the *fundamental matrix* for two views or the *trifocal tensor* for three views.

Self-calibration

From the invariants of the previous section, it is possible in various situations to recover up to a similarity the camera parameters and thus the 3D structure (relative 3D coordinates up to a similarity) of the scene. Starting from the several fundamental matrices, it is possible to partially recover the projection matrices of the cameras associated to the fundamental matrices. This is done using the tool of the absolute conic, absolute quadric [Triggs, 1997], or the absolute quadratic complex [27]. Practically, the matrices are obtained using either the Kruppa equa-

²Camera parameters are divided in two categories: external parameters are the position and the orientation of the camera. Internal parameters are the other ones: they correspond to quantities such as focal length, image resolution, etc.

[Marr and Poggio, 1977] Marr, D. and Poggio, T. (1977). A theory of human stereo vision. Memorandum AIM 451, MIT Artif. Intell. Lab.
[Marr, 1982] Marr, D. (1982). *Vision*. W.H. Freeman and Co.
[Triggs, 1997] Triggs, B. (1997). Autocalibration and the absolute quadric. In *IEEE International Conference on Computer Vision and Pattern Recognition*, pages 609–614.

tions [Maybank and Faugeras, 1992], the modulus constraints [Pollefeys et al., 1996], or linear systems [27].

As depicted, the last two topics are mostly concerned with the geometry of multiple views. The scene geometry is not involved at all besides the hypothesis of being rigid and obeying to the rules of Euclidean or projective geometry. Yet, scene geometry is often much richer and is an important source of information. This scene geometry is used in the *Calibration* topic. It can also be incorporated to the topic of projective invariants for (mostly planar) shapes. Of course, in the most general situation, scene geometry is unknown. But there are also many cases where it obeys to rules. For example, man made environments are often piecewise planar and essentially contain vertical and horizontal elements. Incorporating such rules in vision systems is probably another key to reliability because it has the potential of consequently restricting the “search space” for reconstructions.

The remainder of this section describes the work I was involved into with respect to these topics and provides some perspectives, which to the best of my knowledge are still relevant (remember that this work is about 10 years old). This description is organized along two axes:

- A first section groups the works dealing with projective invariants and self-calibration. This part mostly deals with works on the trifocal tensor and on some related tools often used for self-calibration. These pieces of work mostly deal with the viewing geometry constraints.
- In the second section, some work attempting to incorporate scene geometry information for the reconstruction process is depicted. This work can be viewed as a way to introduce a flexible calibration technique in which objects are not fully specified (as in the strong calibration case). Instead, only object properties such as coplanarity or orthogonality of primitives are used. These properties are manipulated with the tools of geometric reasoning in order to provide constraints on 3D reconstructions.

1.1.1 The trifocal tensor and self-calibration

The trifocal tensors deals with the geometry of three views. The mathematical concept was originally introduced by Spetsakis and Aloimonos [Spetsakis and Aloimonos, 1990] for the problem of structure and motion from line correspondences in 3 views in the calibrated case. The topic was then re-stated in the un-calibrated case by Shashua and Hartley [Shashua, 1994, Hartley, 1994]. Shashua [Shashua, 1994] showed that the coordinates of three corresponding points satisfied a set of trilinear relations. Hartley [Hartley, 1994] later pointed out that those trilinear relations in fact arise from a $3 \times 3 \times 3$ tensor that governs the correspondences of lines between

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- [Maybank and Faugeras, 1992] Maybank, S. J. and Faugeras, O. D. (1992). A theory of self-calibration of a moving camera. *The International Journal of Computer Vision*, 8(2):123–152.
- [Pollefeys et al., 1996] Pollefeys, M., Van Gool, L., and Oosterlinck, A. (1996). Self-calibration with the modulus constraint. Technical Report KUL/ESAT/MI2/9609, Katholieke Universiteit Leuven.
- [Spetsakis and Aloimonos, 1990] Spetsakis, M. E. and Aloimonos, J. (1990). Structure from Motion Using Line Correspondences. *The International Journal of Computer Vision*, 4:171–183.
- [Shashua, 1994] Shashua, A. (1994). Algebraic functions for recognition. *IEEE Transactions on Pattern Analysis and Machine Intelligence*. Use shashua:95.
- [Hartley, 1994] Hartley, R. I. (1994). An algorithm for self calibration from several views. In *Proceedings of the International Conference on Computer Vision and Pattern Recognition*, pages 908–912, Seattle, WA. IEEE.

three views: the trifocal tensor. Two lines in two images always correspond to the projection of a 3D line. The trifocal tensor allows the computation of the image of this line in the third camera. Obviously from this description, given three views, there are three such trifocal tensors, depending upon which view is selected as the one one wants to predict. These three tensors are closely related. Actually, it is possible to define an operation of *change of view* which allows the computation of any of these trifocal tensor given an another one. In some sense, the tensor has to be invertible, which is not possible with all tensors. Hence, a trifocal tensor has to obey to some algebraic constraints. It is important to take account of these constraints when estimating trifocal tensors from image data. This work is encompassed in chapters 8 and 9 of the book [1] in the writing of which I contributed.

Contributions

The trifocal tensor constraints and trifocal tensor non-linear estimation [42, 12, 41, 1]

The 26 (27 up to scale) parameters defining the trifocal tensor are known to depend on only 18 parameters (3×11 parameters for the cameras minus 15 parameters of a 3D projective transform) [Luong and Viéville, 1994]. When we started this work with Olivier Faugeras, only a subset of the algebraic constraints that the trifocal tensor coefficients should satisfy were known [Shashua and Werman, 1995, Heyden, 1995, Avidan and Shashua, 1996]. In the paper [42], we introduced the first complete set of algebraic constraints of the trifocal tensor. This set of constraints arose from the geometric properties of the trifocal tensor, which were described using the formalism of the Grassman-Cayley algebra. This algebra allows the description of the most fundamental projective properties: its two *join* and *meet* operators directly correspond to the geometric operations of summing and intersecting projective spaces. In a second paper [41], we proposed simpler constraints with reduced degrees. Since they characterize the trifocal tensors, these constraints also ensure that the operation of change of view is well defined for tensors that obey these constraints. Originally, this work started with the study by Olivier Faugeras of some geometric properties of three views. While studying these constraints in order to propose a parameterization of the trifocal tensor, I simplified the original constraints using the tools of computer algebra. We then linked the new constraints to the “change of view” operation. Chapter 8 of [1] summarizes most of the algebraical and geometric properties of the trifocal tensor.

Originally, in the works of Hartley and Shashua [Hartley, 1994, Shashua, 1995], the 27 trifocal tensor coefficients were estimated from image correspondences using simple least squares pro-

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- [Luong and Viéville, 1994] Luong, Q. and Viéville, T. (1994). Canonic representations for the geometries of multiple projective views. In Eklundh, J.-O., editor, *Proceedings of the 3rd European Conference on Computer Vision*, volume 1 of *Lecture Notes in Computer Science*, pages 589–599, Stockholm, Sweden. Springer-Verlag.
- [Shashua and Werman, 1995] Shashua, A. and Werman, M. (1995). Trilinearity of three perspective views and its associated tensor. In *Proceedings of the 5th International Conference on Computer Vision*, pages 920–925.
- [Heyden, 1995] Heyden, A. (1995). *Geometry and Algebra of Multiple Projective Transformations*. PhD thesis, Lund University.
- [Avidan and Shashua, 1996] Avidan, S. and Shashua, A. (1996). Tensorial transfer: Representation of $n > 3$ views of 3d scenes. In *Proceedings of the ARPA Image Understanding Workshop*. darpa, morgan-kaufmann.
- [Shashua, 1995] Shashua, A. (1995). Algebraic functions for recognition. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 17(8):779–789.

cedures. Later, Torr and Zisserman [Torr and Zisserman, 1997] introduced the first minimal parameterization of the trifocal tensor. In the papers [42] and [41], we also proposed some minimal parameterizations based on the constraints described in the same papers. Neither the parameterizations in Torr and Zisserman [Torr and Zisserman, 1997] nor the one we proposed in [42] is one to one (each parameter vector provides for 3 or 2 trifocal tensors that need to be differentiated using the data). The one I proposed in [41] does not suffer from this problem. Both the parameterizations we proposed in [42] and in [41] were used in non-linear estimation procedures. These procedures start with the standard linear estimate and refine the coefficients so as to enforce the non-linear constraints by minimizing a geometric criterion over the parameterization of the trifocal tensor. As the operation of change of view is well defined on properly constrained trifocal tensors, the criterion is formulated so as to be symmetric with respect to all the three images, which is not possible with linear techniques. In Chapter 9 of [1], I made a thorough study of the linear and non-linear estimation techniques of the trifocal tensor and provided a systematic study of the linear constraints originating from all combinations of points and lines. Using a simple example (a scene made of two planes), it is also shown in this chapter that the linear algorithms statistically start to fail when point primitives are subject to Gaussian noise of standard deviation as little as 0.4 pixel. While simple, this example is much more realistic than many other statistical studies (or theoretical bounds) where sets of points are taken uniformly in a volume (e.g. the work [Hartley, 1998]), which almost never happens in practise in computer vision. As shown with this simple dataset, the geometric non-linear methods comparatively do much better than linear ones (even though they are based on those for the initialization). Still, this last result is empirical in nature, and with particular configurations of noise the estimation methods can provide poor trifocal tensors even at low levels of noise.

Derivatives of matrix operations [38, 47]

Matrices operations play a fundamental role in many computer science fields. In particular, Singular Value Decomposition (SVD) is a matrix decomposition method closely related to linear least-squares estimation methods. As such, it has been copiously used in computer vision for various purposes: fundamental matrix estimation [Zhang, 1998], epipole and rectification matrix computation [Devernay, 1997], motion computation [Longuet-Higgins, 1981, Hartley,

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- [Torr and Zisserman, 1997] Torr, P. and Zisserman, A. (1997). Robust parameterization and computation of the trifocal tensor. *Image and Vision Computing*, 15:591–605.
- [Hartley, 1998] Hartley, R. I. (1998). Minimizing algebraic error in geometric estimation problems. In *Proceedings of the 6th International Conference on Computer Vision*, Bombay, India. IEEE Computer Society, IEEE Computer Society Press.
- [Zhang, 1998] Zhang, Z. (1998). Understanding the relationship between the optimization criteria in two-view motion analysis. In The Institute of Electrical and Electronics Engineers, I., editor, *International Conference on Computer Vision*, pages 772–777, Bombay, India. IEEE Computer Society, Narosa Publishing HOUSE.
- [Devernay, 1997] Devernay, F. (1997). *Vision stéréoscopique et propriétés différentielles des surfaces*. PhD thesis, École Polytechnique, Palaiseau, France.
- [Longuet-Higgins, 1981] Longuet-Higgins, H. (1981). A computer algorithm for reconstructing a scene from two projections. *Nature*, 293:133–135.

1992, Tomasi and Kanade, 1992, Poelman and Kanade, 1997, Sturm and Triggs, 1996], self-calibration [Hartley, 1997, Lourakis and Deriche, 1999, Lourakis and Deriche, 2000], but also image coding, watermarking, stereo by correlation, face recognition, etc.

When used in optimisation procedures or in some algorithms that need to propagate uncertainty, it is useful to be able to differentiate the SVD operation with respect to the original matrix coefficients. While existing in some not very well known numerical analysis literature [Mathai, 1997], this had never been used in computer vision. Originally, I got interested to the derivatives of the SVD operation for the problem of trifocal tensor estimation [41] (see previous section) and developed the technique of SVD Jacobian computation for that sole purpose. As it appeared useful for many other problems, this technique has been described in [38]. In this paper, Manolis Lourakis (then a postdoc working with R. Deriche) applied the SVD Jacobian technique to the propagation of uncertainty for several computer vision problems. Interestingly, the SVD Jacobian computation only involves: 1) the SVD itself and 2) solving a set of simple 2×2 linear systems based on the SVD coefficients.

The basic technique underlying the derivative computation is very general and can be applied to all matrix transformations based on eigenvalue decompositions (which is very similar to SVD). One such operation is matrix logarithms. Adapting the SVD technique of [38] to the case of matrix logarithms is my main contribution to the paper [47] written with C. Lenglet, O. Faugeras and R. Deriche. The matrix logarithms Jacobian technique is used in a method to register diffusion tensor MR images³. Indeed, tensor comparison involves matrix logarithms and optimizing criteria that contain such comparisons requires such computations of derivatives of matrix logarithms. The method for computing the derivatives of matrix logarithms is faster and more accurate than the previously published method [Dieci et al., 1996].

³Diffusion tensor magnetic resonance (MR) images depict the anisotropic diffusion of water molecules at each point of a biological tissue such as brain white matter. At each point, the anisotropic diffusion is depicted by a 3×3 tensor (a symmetric positive definite matrix). Matrix logarithms are involved in defining a proper distance between two such tensors.

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- [Hartley, 1992] Hartley, R. I. (1992). Estimation of relative camera positions for uncalibrated cameras. In Sandini, G., editor, *Proceedings of the 2nd European Conference on Computer Vision*, pages 579–587, Santa Margherita, Italy. Springer-Verlag.
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Perspectives

Roughly ten years later, trifocal tensors are still not much used in practice. This is mostly because of their complexity and because of their difficult estimation. Expressing the constraints in a simple way and using these to constrain the estimation of trifocal tensor is still a research subject (see e.g. the work of Alzati and Tortora [Alzati and Tortora, 2009]). Computer vision scientists often prefer to use the much simpler fundamental matrix that is both more flexible and easier to estimate. They then rely on an iterative procedure known as *bundle adjustment* to recover coherent sets of projections matrices, which in turn completely define the trifocal tensor. In theory, the trifocal tensor has a small advantage in the case of aligned cameras (in which case the fundamental matrices do not provide all the information about the three view geometry). In my opinion, the real importance of the trifocal tensor is more theoretical than practical and has not been exploited yet by the community as far as I know: self-calibration equations should be expressed (in the case of constant internal parameters where only three views are required) with the trifocal tensor coefficients. This would provide the proper tool to mathematically analyze the properties of self-calibration. One way to see this is to consider Kruppa equations⁴ (which are one way to achieve self-calibration even if it is not the most widely used method nowadays) which are expressed from fundamental matrices. The three fundamental matrices obtained from three views are not independent. Indeed, three fundamental matrices represent 21 (3×7) parameters whereas it depends only on 17 parameters in the constant internal parameter case (5 internal parameters plus 2×6 motion parameters). This means that there are 4 constraints among the 3 fundamental matrices in the constant internal parameter case. Three of these constraints are projective and are easily expressed (see e.g. Eq. (8.3) of [1]), and those are naturally enforced through the trifocal tensor. The remaining constraint is related to the fact that the internal parameters are constant and has never been expressed to my knowledge. Similarly, the trifocal tensor depends in general on 18 independent parameters. Since there are 17 such parameters in the constant internal parameters case, this means that there is one supplementary constraint on the trifocal tensor parameters (in the constant parameter case). This is also consistent with the fact that there are 6 Kruppa equations arising from 3 images, whereas there are only 5 parameters to estimate, so that there is one constraint that needs to be satisfied in order to ensure that this system has a solution. Attempting to find this constraint was my last activity in the computer vision field. At that time, I tried many ways to approach this problem and parameterizing it using the trifocal tensor seemed the most promising approach as it allowed me to have a simple parameterization that enforced the 3 projective constraints on the fundamental matrices. Studying self-calibration from the trifocal tensor perspective should provide some enlightenment on the mathematical structure of the self-calibration problem from Kruppa's equations (notably in the case of aligned cameras). Only a single attempt has been made in this direction by Armstrong, Zisserman and Hartley [Armstrong et al., 1994] in the

⁴Kruppa equations directly relate the fundamental matrix to the internal parameters of a camera [Kruppa, 1913, Maybank and Faugeras, 1992, Luong and Faugeras, 1993, Hartley, 1997]. Each fundamental matrix provides two equations on the internal parameters, so that 3 views – which provide three fundamental matrices – are necessary to recover the 5 internal parameters.

[Alzati and Tortora, 2009] Alzati, A. and Tortora, A. (2009). Constraints for the trifocal tensor. In Aja-Fernández, S., Luis García, R., Tao, D., and Li, X., editors, *Tensors in Image Processing and Computer Vision*, Advances in Pattern Recognition, pages 261–269. Springer London.

[Armstrong et al., 1994] Armstrong, M., Zisserman, A., and Beardsley, P. (1994). Euclidean structure from uncalibrated images. In Hancock, E., editor, *Proceedings of the 5th British Machine Vision Conference*, pages 508–518, York, UK. BMVA Press.

limited case of planar motion. The problem is however difficult because of the complexity of the trifocal tensor object. It might also provide some simplifications in the equations such as the ones obtained in a completely different manner in [27] or those obtained in some specific cases [Ma et al., 2000].

Extensions of the multilinear constraints have also been proposed:

- Higher order tensor for more than three views have been proved to provide no more information than sets of trifocal tensors [Triggs, 1995]. As such, they are not so interesting as they are even more complex than the trifocal tensor and thus even more prone to estimation difficulties in practise. Their sole interest might again be in studying the mathematical properties of extensions of the self-calibration problems such as the case of varying internal parameters.
- Multibody tensors [Wolf and Shashua, 2001, Hartley and Vidal, 2004] are a very interesting mathematical generalization of multiview constraints to the case of a scene constituted of multiple rigid components. But again, the practical difficulties introduced by the estimation of these objects limits severely their practical interest besides the mathematical point of view (more algorithmic approaches have been more successful).

Finally, full self-calibration⁵ (despite its interesting mathematical aspects) is less interesting nowadays since some internal camera parameters are easily extracted from image meta data automatically incorporated by modern digital cameras. But obviously, this last remark applies only to unmodified images: simple operation such as cropping can easily make part of the image meta-data irrelevant. Note that external parameters (relative position between the cameras) will probably be available soon with a good accuracy from the embedded gyroscopes and accelerometers (at least it will provide good initial for those external parameters).

In the context of human vision, it is quite clear that the trifocal tensor is not used (after all, we only have two eyes!). Many studies have been made to check whether or not the brain makes use of something akin to the epipolar geometry in the process of stereo correspondence [Rogers and Bradshaw, 1996, Stevenson and Schor, 1997, Phillipson and vertical disparity influences stereo correspondence Read, 2009]. The topic is the subject of some controversy. In any case,

⁵By “full self-calibration”, I mean self-calibration of all internal and external parameters.

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- [Stevenson and Schor, 1997] Stevenson, S. B. and Schor, C. M. (1997). Human stereo matching is not restricted to epipolar lines. *Vision Research*, 37(19):2717–2723.
- [Phillipson and vertical disparity influences stereo correspondence Read, 2009] Phillipson, G. P. and vertical disparity influences stereo correspondence Read, J. C. A. (2009). Global vertical disparity influences stereo correspondence. In *Proceedings of 32nd of European Conference on Visual Perception*.

it looks like the brain is quite flexible and can find correspondences that do not strictly obey to epipolar geometry (there is some tolerance to vertical disparity). Similarly, it is able to adapt fairly quickly to changes in the viewing geometry (changes of parameters for the cameras – the eyes –). The mechanisms governing this adaptation are not very well known and probably have little to do with the theory of self-calibration.

Besides the pure mathematical aspects evoked previously, the overall subject of viewing invariants has been quite thoroughly explored. On a longer scale, there are, in my opinion, two main perspectives:

- **Integration of algorithms:** Up to now, the self-calibration process has mostly been seen as a preprocessing step that takes a few point correspondences among several images and provides calibration information. Considering it as a pure geometric problem has allowed fundamental advances but is probably now showing its limitations. A tighter integration of geometry-based algorithms with the photometry based ones (see for example section 1.2) is probably mandatory in order to have even more reliable processes. As an example, a tighter integration of epipolar geometry estimation with processes such as image registration or stereo computation seems an interesting path: indeed, as geometry provides constraints on correspondences, correspondences provide constraints on geometry. Working in a pure bottom-up approach (from selected image points, compute epipolar geometry and then a registration) ignores the fact that the newly registered points might influence the extracted geometry. More generally knowledge on the scene (such as correspondences, coplanar patches or constraints on the reconstructed scene) is acquired incrementally, and integrated also incrementally at all the stages of a computer vision algorithm (e.g. scene reconstruction). Each added information can improve considerably the initial information that has been used to determine it. For example, improved point correspondences (after stereo registration) may improve the epipolar geometry which in turn improves the registration, calibration or 3D reconstruction (see e.g. the work [39] depicted hereafter for a simple example of such a situation for stereo matching). Another interesting perspective that has not received much attention as far as I know is that there are basically two “low-level” constraints on correspondences: the epipolar geometry for rigid scenes and homographic correspondences for planar patches. While the relation between the homographies and fundamental matrices (which embodies epipolar geometry) is very well understood, little has been done at the estimation level to integrate those two points of view. This would be particularly interesting for urban scenes.
- **Video and dynamic scenes:** The digital imaging technology has made tremendous advances in the past few years. Nowadays, it is extremely easy to acquire high resolution digital video sequences and with the advent of 3D television, soon there will even be low cost stereo video cameras⁶. The amount of stereo data sequence is likely to explode in the next few years. Providing online algorithms which integrate – preferably in real time – the 3D information along a video sequence and update a 3D reconstruction with each new image is likely to be an even more active research field in the next few years. Similarly, as the average sequence is likely to exhibit multiple rigid motions or even non

⁶http://www.pcworld.com/article/218042-2/the_early_adopters_guide_to_3d_tv_cameras_camcorders_and_everything_else.html

rigid motions, so that the work on dynamic or multi-rigid scenes [Kuettel et al., 2010, Vidal et al., 2006] will take more importance. From this point of view, it is also interesting to note that comparatively very little work has been done on the problem of self-calibration from the differential point of view [Viéville and Faugeras, 1996, Haner and Heyden, 2010] (i.e. for a motion sequence). This is probably because such methods involve not only points but also their first (and potentially second) derivatives, which are quite difficult to extract accurately. Extending these differential methods to contours and more specifically to lines following the work that had been done in the calibrated case [Faugeras et al., 1989, Faugeras and Papadopoulos, 1993, De Ma, 1993] would be meaningful as estimating spatio-temporal derivatives of such objects is much more reliable.

We have just seen that establishing and using constraints is a way to improve computer vision processes. The next chapter will describe some work that attempts to use such constraints for calibration and scene reconstruction.

1.1.2 Geometric constraints

In the standard approaches to calibration or self-calibration, only the viewing constraints of the scene are used: assuming a rigid scene, the only used information is the invariants associated with the viewing geometry (the relative positions of the cameras). In many situations, however, assumptions can be made a priori on the scene structure. This has led me to a second line of work in which we investigated the possibility of using geometric information and geometric reasoning for calibration and scene reconstruction. This work has been done with colleagues (B. Mourrain and D. Wang) from the geometric reasoning/ computer algebra field. I have been involved in this work by co-supervising the PhD of D. Bondyfalat, helping him to adapt the geometric reasoning methods to the requirements and constraints of computer vision algorithms [Bondyfalat, 2000].

Contributions

Parameterization of 3D geometrically constrained scenes [40]

In many situations such as human manufactured objects or city views, it is possible for a user to

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- [Kuettel et al., 2010] Kuettel, D., Breitenstein, M. D., Gool, L. V., and Ferrar, V. (2010). What's going on? discovering spatio-temporal dependencies in dynamic scenes. In *Proceedings of CVPR*.
 - [Vidal et al., 2006] Vidal, R., Ma, Y., Soatto, S., and Sastry, S. (2006). Two-view multibody structure from motion. *International Journal of Computer Vision*, 68:7–25.
 - [Viéville and Faugeras, 1996] Viéville, T. and Faugeras, O. (1996). The first order expansion of motion equations in the uncalibrated case. *CVGIP: Image Understanding*, 64(1):128–146.
 - [Haner and Heyden, 2010] Haner, S. and Heyden, A. (2010). A step towards self-calibration in slam: Weakly calibrated on-line structure and motion estimation. In *Computer Vision and Pattern Recognition Workshops (CVPRW), 2010 IEEE Computer Society Conference on*, pages 59–64.
 - [Faugeras et al., 1989] Faugeras, O., Deriche, R., and Navab, N. (1989). From optical flow of lines to 3-D motion and structure. In *Proceedings IEEE RSJ International Workshop on Intelligent Robots and Systems '89*, pages 646–649. IEEE Industrial Electronics Society, Robotics Society of Japan. Tsukuba, Japan.
 - [Faugeras and Papadopoulos, 1993] Faugeras, O. and Papadopoulos, T. (1993). A theory of the motion fields of curves. *ijcv*, 10(2):125–156.
 - [De Ma, 1993] De Ma, S. (1993). Conics-based stereo, motion estimation, and pose determination. *Int. J. Comput. Vision*, 10:7–25.
 - [Bondyfalat, 2000] Bondyfalat, D. (2000). *Interaction Symbolique et Numérique; Application à la Vision Artificielle*. PhD thesis, Université de Nice Sophia-Antipolis.

state some geometric facts about the scene to be reconstructed. Properties such as parallelism, coplanarity or orthogonality of planes or lines or even equalities of distances are easily hypothesized by a human operator. In such situations, it seems wasteful not to use this knowledge (the human visual system is known to use a priori knowledge of the observed scene coming from memory or from the context). Often, this information is often not provided in a suitable form so as to be used by computer vision algorithms. In [40], the tools of automated geometric proving have been used to transform an unstructured set of geometric constraints on points, lines and planes into a parameterization of the scene. This technique takes the unstructured description of the constrained scene and builds a constructive description of it starting with a minimal set of free parameters. This construction can then be used in various reconstruction or self-calibration tools such as bundle adjustment [Bondyfalat and Bougnoux, 1998] to improve both the reconstructed scene and the camera parameters.

Using a map [36]

One particularly rich source of constraints is given by map of objects, buildings or cities. The paper [36] focusses on the problem of calibration from a single view and such a map of the scene. As above, geometric knowledge of the scene is incorporated within the procedure. As for the trifocal tensor, the Grassman-Cayley algebra has been used to explicit the information on the projection matrix provided by several types of geometric constraints. In the end, the whole process is fairly similar to a calibration procedure where some parameters are kept as variables until some geometric constraint allows to fix its value.

Perspectives

Geometry and geometric reasoning have been identified very early as potential useful tools for scene reconstruction [Brooks et al., 1979, Mundy, 1986, Kapur and Mundy, 1989]. Despite this, this subject has received relatively little interest at least compared to all the studies about viewing geometry. Three phenomena concur toward this relative lack of interest:

- First it is not general enough to be used with all common images. In particular, most natural scenes not involving human made objects have a complex geometric content that is not easily described with simple geometric properties.
- The method needs a human being to organize and annotate the set of primitives seen in the picture to provide the geometric properties. This very boring and sometimes error prone step is limiting the practical use of such methods. To be really useful, systems based on geometric constraints on the scene should be able to be fed with automatically generated hypotheses. While it is fairly easy to detect corresponding points, lines or planes

[Bondyfalat and Bougnoux, 1998] Bondyfalat, D. and Bougnoux, S. (1998). Imposing euclidean constraints during self-calibration processes. In Koch, R. and Gool, L. V., editors, *Proceedings of SMILE Workshop on Structure from Multiple Images*, Lecture Notes in Computer Science. Springer-Verlag, Lecture Notes in Computer Science.

[Brooks et al., 1979] Brooks, R. A., Creiner, R., and Binford, T. O. (1979). The acronym model-based vision system. In *Proceedings of the 6th international joint conference on Artificial intelligence - Volume 1*, pages 105–113. Morgan Kaufmann Publishers Inc.

[Mundy, 1986] Mundy, J. (1986). Reasoning about 3-d space with algebraic deduction. In Faugeras, O. D. and Giralt, G., editors, *Robotics Research, The Third International Symposium*, pages 117–124. MIT Press.

[Kapur and Mundy, 1989] Kapur, D. and Mundy, J. (1989). *Geometric Reasoning*. MIT Press, Cambridge.

in images, automatic generation of geometric properties such horizontalness, verticalness, orthogonality, parallelism, equality of distances has been much less studied. Such systems would definitely generate wrong hypotheses that will need to be filtered by the geometric constraints based algorithms. Clearly, the algorithms based on geometric theorem proving are not yet prepared to this (at least at the time this research was done).

- Geometric reasoning derives from formal algebra. Such systems have difficulty to cope with inaccurate data. There is no such things as for example “three almost aligned points”. Points are aligned or not. In contrast, points measured or reconstructed from images are inaccurate (because they come from a measurement process). Reconciling these two views is quite complicated. In the first of the above contributions, we worked around the difficulty by considering a pure geometric problem from which we derived the parameterization that is used in a second stage by a computer vision algorithm. This is not totally satisfactory.

More generally, geometric constraints are part of a trend of integrating a priori knowledge in computer vision. These last years have seen a major trend of integrating such knowledge through learning techniques with some very nice results for e.g. object recognition [Ponce et al., 2006]. Using hypotheses about the viewed world is probably a key to solve the problem of modeling image intensities and thus to build “robust” computer vision systems. As hinted above, I really think that constraints on the viewed scenes is the way to improve the overall quality of scene reconstructions. At the time when this work was done, I was considering the perspective of having a low-level constraint generator which would propose from low-level primitives a list of possible relatively simple constraints on the image elements (these can be not only colinearities, parallelisms or coplanarities which can be easily hypothesized from images, but also orthogonalities or equal distances which are more heuristic). Those hypotheses would then be incorporated in the calibration/reconstruction problem using numerical soft-constraints. The main challenge of such an approach would have been to find a computational process that would progressively eliminate those generated wrong hypotheses that would make the constraint system unsatisfiable (even approximately). Another difficulty is that such a procedure becomes quickly very non-linear and that a reasonable convergence of the calibration/reconstruction process in a global minimum has to be devised.

1.2 The photometric image

There is more in an image than a set of points. The previous section has already mentioned some perspectives where considering the photometric information might be useful in addition of the pure geometric processing. The works presented in this section make use of this photometric information of the image. As explained in the introduction, this information is not quite well understood: the problem of comparing image (usually color or luminance) values is a complex one because this information depends on the illumination, the orientation, the internal parameters of the camera as well as on the statistics of the surrounding image. Properly modeling it would require to introduce and extract a lot of physical knowledge about the scene (mostly material and illumination source properties), which is usually well beyond the level of detail computer vision is able or willing to achieve. The perspective adopted here – a common one – is that pixel values can be compared directly or almost directly after a linear correction, which is often not

[Ponce et al., 2006] Ponce, J., Hebert, M., C. Schmid, C., and Zisserman, A., editors (2006). *Toward Category-Level Object Recognition*, volume 4170 of *Lecture Notes in Computer Science*. Springer.

the case. Still, this model has provided useful results and is at the roots of most – if not all – the stereo approaches.

1.2.1 Dense image matching and image segmentation

As we have seen in the previous section, most if not all pure geometric computer vision algorithms rely on correspondences between primitives seen in multiple images. The first two contributions depicted in this section are attempts to improve this correspondence process. The last contribution deals with image segmentation and touches at the subtle problem of conciliating local and global decisions.

Contributions

Simultaneous estimation of stereo and epipolar geometry [39, 48]

This first contribution is an effort to integrate epipolar geometry computation with stereo correspondence estimation. This work was done with the PhD student C. Gauclin. Both stereo by correlation and epipolar geometry computation have been largely studied in separation, but much less effort has been put into their simultaneous estimation. The standard view is purely bottom-up: the epipolar geometry (in the form of a fundamental matrix) is first computed from a few sparse matching points across two images either computed automatically or extracted by hand, and then used to reduce the stereo correspondence problem through a procedure known as rectification. Nowadays, many dense techniques have been developed to effectively provide dense stereo matches from rectified images. It is however symptomatic that all those methods are generally compared using perfectly rectified images (see <http://www.middlebury.edu/stereo> even if perfect rectification is sometimes difficult to achieve using uncalibrated images. In [39, 48], we proposed to create a closed loop to estimate simultaneously both the epipolar geometry and the stereo information. The basic idea is that the stereo score map – a map that characterizes the quality of the obtained stereo – is a good criterion to measure the quality of the epipolar geometry for the full image densely (as opposed to standard fundamental matrix criteria that only account for a sparse set of points). This criterion is then optimised using gradient descent to obtain with a refined epipolar geometry and stereo information. Although the method is not limited to a particular registration technique, it used a correlation method based on recursive filtering that has the advantage of allowing weighted and multi-resolution correlation without requiring prohibitive computing time. The method has been tested on pairs of images of human faces and on Scanning Electron Microscope (SEM) images, for which it has been shown that the proposed technique is able to correct an initially inaccurate epipolar geometry.

Symmetric matching with occlusion detection [35, 7]

This second contribution is a collaborative effort with L. Alvarez, J. Sanchez of University of Las Palmas and with R. Deriche from INRIA. Many PDE-based dense matching techniques at that time did not generally yield symmetrical solutions. The results differ if they are applied between two images I_1 and I_2 or between I_2 and I_1 . In contrast, stereo techniques have long established that symmetry helps improving the quality of the registration [Fua, 1993]. PDE-based techniques

[Fua, 1993] Fua, P. (1993). A parallel stereo algorithm that produces dense depth maps and preserves image features. *Machine Vision and Applications*, 6(1). Available as INRIA research report 1369.

such as the work [Alvarez et al., 2000, Alvarez et al., 2002] impose the smoothness of the disparity map (except at image discontinuities) through regularization, but forget the one-to-one property of matches as well as the modeling of occlusions. The main challenge I proposed to these colleagues was to find a variation of their method to recover a dense matching field map from two images, while explicitly taking into account the symmetry across the images as well as possible occlusions. [35, 7] propose such a method. The idea is to consider both displacements vectors from I_1 to I_2 and I_2 to I_1 and to minimise an energy functional that explicitly encodes all those properties. Occlusions are detected as places where symmetry is violated. The variational problem is solved using the gradient flow defined by the Euler–Lagrange equations associated to the energy. Experiments clearly show the added value of these properties to improve the accuracy of the computed registrations.

Image segmentation using local statistics [19, 18, 17]

This last contribution is a joint work with my PhD student J. Piovano [Piovano, 2009]. [19] introduces a framework for image segmentation based on local statistics. Basically, this work is an extension of the work [Chan and Vese, 2001, Rousson and Deriche, 2002] where image mean and variance are computed locally around a point in the image instead of globally for the whole image. Basing segmentation on local contrasts is nice as images (and especially medical images) are known to be subject to drifts. Unfortunately, making local decisions often leads to non-coherent global segmentations. [18] explores the relations between local decisions and global constraints. A form of global constraint is needed to ensure coherence, but locality is needed for adaptation and efficiency (parallelism). Combining these two aspects is probably a key to finding satisfactory solutions to many computer vision problems. The paper identifies several interesting problems in such a situation and proposes some possible solutions. Another interesting side aspect of the paper is the use of the method to segment the head from registered T1- and T2- MRIs. Using these data allows the extraction of the skull interfaces, interfaces that are usually “invented” in EEG forward models (see 2.2.1 for more details). This segmentation method has also been used in a completely different setup in [17] for segmenting lesions in retinal angiography images.

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- [Alvarez et al., 2000] Alvarez, L., Deriche, R., Weickert, J., and Sánchez, J. (2000). Dense disparity map estimation respecting image discontinuities: A pde and scale-space based approach . Research Report 3874, INRIA. Accepted for publication in *Journal of Visual Communication and Image Representation*, Special Issue on Partial Differential Equations in Image Processing, Computer Vision and Computer Graphics, Dec. 2000 (to appear).
- [Alvarez et al., 2002] Alvarez, L., Deriche, R., Weickert, J., and Sánchez, J. (2002). Dense disparity map estimation respecting image discontinuities: A PDE and scale-space based approach. *Journal of Visual Communication and Image Representation, Special Issue on Partial Differential Equations in Image Processing, Computer Vision and Computer Graphics*, 13(1/2):3–21.
- [Piovano, 2009] Piovano, J. (2009). *Image Segmentation and Level Set Method: Application to Anatomical Head Model Creation*. PhD thesis, University of Nice-Sophia Antipolis.
- [Chan and Vese, 2001] Chan, T. and Vese, L. (2001). Active contours without edges. *IEEE Transactions on Image Processing*, 10(2):266–277.
- [Rousson and Deriche, 2002] Rousson, M. and Deriche, R. (2002). A variational framework for active and adaptative segmentation of vector valued images. In *Proc. IEEE Workshop on Motion and Video Computing*, pages 56–62, Orlando, Florida.

Perspectives

There is little unity in the various work presented in this section besides the fact that they are image-based. Yet, they somehow explore at a very small scale some of the problems that a complete computer vision system would have to tackle:

- **Integration of multiple cues:** As explained previously, no single computer vision technique will be able to cope, in general, with all the situations that can be encountered in analyzing images. Biological vision, itself, uses several strategies (pathways) to cope with the various aspects of image understanding. Yet, a single area such as V1 – the primary visual cortex – does much more simultaneous tasks than a single standard computer vision algorithm will do: extracting local contrast, color, orientation, motion, direction, speed (basically spatial or temporal frequencies) at various scales. Current consensus seems to be that early responses of V1 neurons consists of tiled sets of selective spatio-temporal filters, integrated to a level which, to my knowledge, has no computer vision counterpart. There is probably some value in considering all those tasks as a whole and better understanding their inter-relationship. [39] or [7] can probably be considered as (very small) steps in that direction among many other work done in the computer vision community.
- **Local choices, global coherence:** In the idealised view of V1 depicted above, processing is purely local. However, later in time (after 100 ms) neurons in V1 are also sensitive to the more global organisation of the scene [Lamme and Roelfsema, 2000]. This late behaviour probably results from feedbacks from higher level areas which modulate V1 activity. Local processing is crucial for “computational efficiency” as it allows for parallel processing of information. Yet, the solutions sought for must have some global coherence. [18] is a very specific example of how the injection of some very limited global information can drive local decisions to obtain a coherent global behaviour.
- **Knowledge and learning:** There are many reasons to believe that biological visual systems are highly trained to handle common visual tasks. Brain efficiency often decreases with non-natural images which can be experienced very easily with visual illusions, etc. Incorporating prior knowledge is an active current research topic in computer vision. But, at least in the view of an integrated system, this also raises the problems of acquiring, storing and organising prior information.
- **Performance as a whole:** Finally, the performance of each basic block of a computer vision system must be assessed only with respect to the needed accuracy of the (higher level) blocks it interacts with. For example, navigating in an environment often does not need a millimetric precision in the 3D mental model of the scene, while tiny object manipulation may require it. Most computer vision literature focusses on a specific task and aims at achieving the highest possible accuracy for that task. As fruitful as it has been in the understanding of the mathematical and computational aspects of computer vision and image processing, this approach results in quite expensive “single tasks”, which may not be appropriate for an integrated solution.

An interesting question is what mathematical or computational frameworks can be the basis of a system with such properties. A basic assumption made here is that all tasks should be expressed

[Lamme and Roelfsema, 2000] Lamme, V. A. F. and Roelfsema, P. R. (2000). The distinct modes of vision offered by feedforward and recurrent processing. *Trends in Neurosciences*, 23(11):571–579.

in some common language so as to allow tasks to communicate easily with each other. As stated above, a real system should rely on parallel processing to achieve real time. This probably means that computations are based purely on local decisions. Yet several possibilities remain (bayesian approaches, neural networks, PDEs, etc.). In many ways, human technology and especially computer technology, is very different than *equivalent* “biological technology”. This gap is also what makes “imitating” brain functions so difficult (and so interesting). Imitation at the low-level (neurons or below) is probably not achievable with current technology. Yet, better understanding brain organisation and interactions might help in the design of human technology-based visual systems. Following this model, integration of various computer vision algorithms in a global computer vision system is probably a key to provide more robust vision systems. This is a daunting task as a vision system in isolation might not be very meaningful. Yet some steps toward this direction have been made (the work of C. Strecha and colleagues [Strecha and Van Gool, 2002] is a nice such example).

1.3 Conclusion

In this section, I have described my past computer vision contributions in both the geometric and photometric points of view. This work has been done mostly during the years 1998–2001 and 2005–2007. In terms of publications, this represents contributions to one book, 2 international journals and 13 international conferences with proceedings. This work was done in close interaction with 3 PhD students (Didier Bondyfalat, Cyril Gauclin, Jérôme Piovano), one post-doc (Manolis Lourakis) and in collaboration with several INRIA (Olivier Faugeras, Rachid Deriche, Bernard Mourrain) or international (Luis Alvarez, Javier Sanchez, Jean Ponce, Tuan Luong) researchers.

In 2001, the research team ROBOTVIS in which most of this work was done decided to change its research themes and to move towards the goal of understanding the functioning of the brain. From this perspective, I started to work on the topic of MEG and EEG data analysis and processing which is the topic of the next chapter.

[Strecha and Van Gool, 2002] Strecha, C. and Van Gool, L. (2002). Motion-stereo integration for depth estimation. In *European Conference on Computer Vision*, volume 2, pages 170–185.

Chapter 2

Analyzing the electrical activity of the brain from M/EEG measures

Electro-Encephalography (EEG) and Magneto-Encephalography (MEG) are two non-invasive techniques for measuring (part of) the electrical activity of the brain. While EEG is an established technique (Hans Berger, a German neuropsychiatrist, measured the first human EEG in 1929), MEG is a rather new one: the first measurements of the magnetic field generated by the electrophysiological activity of the brain were done in 1968 at MIT by D. Cohen [Cohen, 1968]. The major technical advance that allowed the practical use of MEG was the development and use of SQUIDS (superconducting quantum interference device) by Zimmerman and colleagues in 1969 [Zimmerman et al., 1970]. This type of sensor is extremely sensitive, allowing the measurement of the very low magnetic fields (twelve orders of magnitude lower than the earth magnetic field!) induced by the functioning brain (figure 2).

Nowadays, EEG is relatively inexpensive and is commonly used to detect and qualify neural activity (epilepsy detection and characterisation, neural disorder qualification, BCI, ...). MEG is, comparatively, much more expensive as SQUIDS work in very challenging conditions: they need to be cooled down to 4K by liquid Helium, and a specially shielded room must be used to separate the signal of interest from the ambient noise. However, as MEG reveals a complementary vision to that of EEG and is less sensitive to the head structure, more and more MEG machines are installed throughout the world. INRIA and the Odyssée team have participated to the acquisition of one such machine that has been installed in 2008 in the hospital "La Timone" in Marseille.

MEG and EEG can be measured simultaneously (M/EEG) and reveal complementary properties of the electrical fields. The two techniques have temporal resolutions of about the millisecond, which is the typical granularity of the measurable electrical phenomena that arise in the brain. This high temporal resolution is what makes MEG and EEG attractive for the functional study of the brain. The spatial resolution, on the contrary, is rather poor as only a few hundred of sensors can be placed around the head and acquired simultaneously (about 300-400 sensors for MEG and up to 256 sensors for EEG). MEG and EEG are also somewhat complementary

[Cohen, 1968] Cohen, D. (1968). Magnetoencephalography: Evidence of magnetic fields produced by alpha-rhythm currents. *Science*, 161(3843):784–786.

[Zimmerman et al., 1970] Zimmerman, J., Thiene, P., and Harding, J. (1970). Design and operation of stable RF-biased superconducting point-contact quantum devices and a note on the properties of perfectly clean metal contacts. *J. Appl. Phys*, 41:1572–1580.

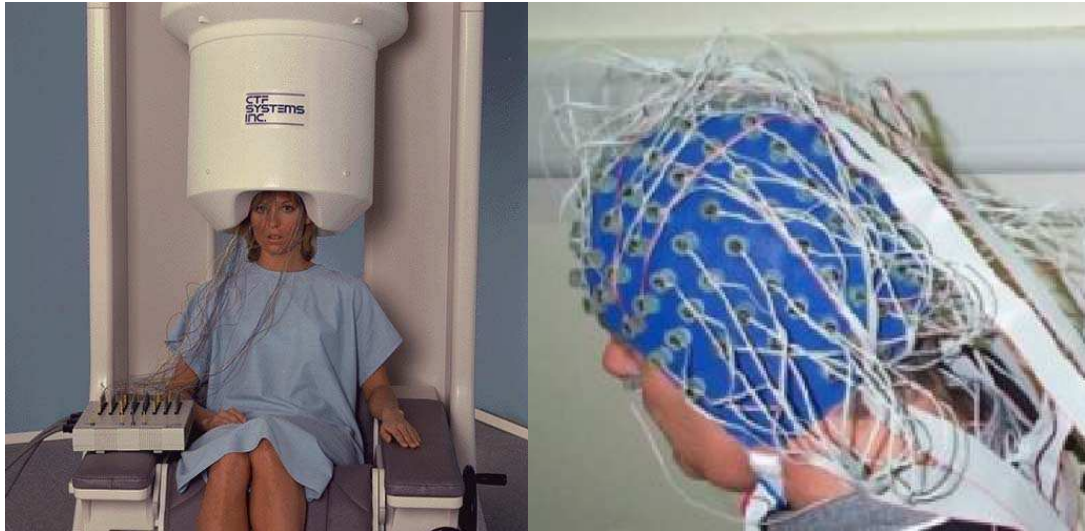


Figure 2.1: MEG (left) and EEG (right) acquisition systems.

with fMRI, PET and SPECT, which provide a very good spatial resolution but a rather poor temporal one (of the order of a second for fMRI and of a minute for SPECT). Contrarily to fMRI, which “only” measures an haemodynamic response linked to the metabolic demand, MEG and EEG measure a direct consequence of the electrical activity of the brain: it is admitted that the MEG and EEG measure signals corresponding to variations of the post-synaptic potentials of the pyramidal cells in the cortex. Pyramidal neurons constitute approximately 80% of the neurons of the cortex (the proportion varies with cortical regions), and at least about 50,000 such neurons are required to be active simultaneously in order to generate some measurable signal.

While the few hundred temporal curves obtained using M/EEG have a clear clinical interest, the measurements are made outside, or on the surface of the head, and hence only provide partial information on the localisation of the sources of activity. With appropriate models and methods, localization of activity from MEG and EEG is nevertheless possible. With a *proper model of the head and the sources* of this electromagnetic activity, it is possible to simulate the electrical propagation (*forward problem*) and to recover the sources corresponding to measurements using an *inverse problem*. Solving the inverse problem is the key to identifying and localizing brain areas responsible for the observed activity. In turn, locations of activities often give some indications on the nature of the corresponding activations.

Reconstructions, however, rely on the modelling elements injected in the inverse problem, which is unavoidable as the inverse problem is an ill-posed one. Consequently, tools are needed to analyse the statistical pertinence of the activities revealed by reconstructions. The statistical assessment of the quality of such solutions is also crucial because in functional neuroimaging in general “ground truth” is difficult to obtain. Solutions using statistical parametric maps, Bayesian learning [Wipf and Rao, 2004] or permutation tests [Nichols and Holmes, 2001] have

[Wipf and Rao, 2004] Wipf, D. and Rao, B. (2004). Sparse Bayesian learning for basis selection. *IEEE Trans Signal Processing*, 52:2153–2164.

[Nichols and Holmes, 2001] Nichols, T. E. and Holmes, A. P. (2001). Nonparametric permutation tests for functional neuroimaging: a primer with examples. *Human Brain Mapping*, 15:1–25.

been proposed.

The use of M/EEG data in practise thus raises various problems some of which are at the core of the work presented hereafter:

- **Exploratory signal analysis:** from a signal processing point of view, detecting and extracting meaningful information from the measurements is a difficult task, because of the low signal to noise ratio and the presence of ongoing cerebral activity (the notion of “noiseless signal” does not exist).
- **Modelling the head and solving the associated forward problem:** as explained above, localizing the sources of the measured activities is highly dependent on the precision of the models relating the sources of electrical activity to the sensors. There are three main ingredients to such models: sources, head tissues with their appropriate conductivity, and sensors.
- **Solving the inverse problem and analysing the results:** as the inverse problem is ill-posed, its solutions are inherently unstable, and in the distributed source case, non-unique. Constraints, or regularization, are necessary in order to guarantee an unique and stable solution [Sarvas, 1987, Hämäläinen et al., 1993]. Choosing the proper type of regularization and constraints is the subject of intense research in the M/EEG community.

2.1 Extracting and modelling events of interest

With data recorded at a rate of about 1000Hz on hundreds of channels simultaneously, analyzing M/EEG signals requires to cope with a huge quantity of data gathered during an experiment. In most cases, only a subset of that piece of data corresponding to “events of interest” is analyzed in depth. These events of interest are all the more complicated to localize that the signal to noise ratio (SNR) of M/EEG is poor. In most traditional MEG or EEG experiments (notable exceptions are some events related to epilepsy which have a high SNR), stimuli are presented multiple times (each repetition is called a trial). The resulting measurements are aligned and averaged in order to improve the signal to noise ratio (evoked potentials).

Such a procedure assumes that signals do not vary across trials, and that they can easily be aligned, usually with respect to some “reference event”. This reference event can be the onset of the stimulus presentation, or a measured subject reaction time. Such alignment is not always possible because latencies of the brain responses may vary across trials (and cumulative latencies make events far from the reference event more difficult to align).

Averaging also smoothes many details in the signal, to the point that some components may disappear. Obviously, this may happen for events that are “not” time-locked to the reference event. Attention and habituation are other sources of variability across trials: while these may not be sufficient to make the events disappear in the average data, they can affect the strength of the signal or its perceived duration. More severe is the case of high frequency events that

[Sarvas, 1987] Sarvas, J. (1987). Basic mathematical and electromagnetic concepts of the biomagnetic inverse problem. *Phys. Med. Biol.*, 32(1):11–22.

[Hämäläinen et al., 1993] Hämäläinen, M., Hari, R., Ilmoniemi, R. J., Knuutila, J., and Lounasmaa, O. V. (1993). Magnetoencephalography— theory, instrumentation, and applications to noninvasive studies of the working human brain. *Reviews of Modern Physics*, 65(2):413–497.

are not phase-locked across trials. Such events, even time-locked to a reference event, tend to cancel out in the average data and thus are difficult to detect. They are usually best detected by averaging the time-power images corresponding to the signal [Tallon-Baudry et al., 1996].

Since the seminal work of Lehmann&al on microstates [Lehmann and Skrandies, 1984], much effort is being devoted in the community in order to be able to analyze single-trial measurements, or to segment continuous strands of data into pieces within which the signals enjoy similar properties. Statistical methods must be adapted to the multidimensionality of the data and heterogeneity of the dimensions (time, 3D space, trials, conditions, subjects) [Miwakeichi et al., 2004]. Blind Source Separation techniques have been applied to M/EEG, in order to separate the data into independent components which may then be easier to interpret. Though well-suited to artefact elimination, the methods rarely prove effective in revealing activities of interest.

2.1.1 Contributions

Moving towards single-trial event analysis is important for many reasons: not only can it better reveal the characteristics of the events (their durations, their shapes, their variability), but it also has the potential of allowing more natural experimental paradigms in which reference events would be less necessary. For the very same reasons, this goal would also bring many improvements for Brain-Computer-Interface (BCI) experiments.

Single trial analysis of brain signals [46, 45, 25, 26]

The body of work [46, 45] attempts to design a general method for modeling and tracking M/EEG events, applicable for both low and high frequencies and taking into account the spatial structure (topography) of the events. The method relies on a spatio-temporal modelling of events as sums of “atoms”: Kronecker products of possible time courses (modelled in the time-frequency space by using a set of Gabor wavelets) by topographical maps in the sensor space. The method can be applied for transient activity (e.g. event-related potentials) as well as for oscillatory activity (e.g. gamma bursts), and for both evoked or induced activity. In order to benefit from all the structure present in the data, the method accounts for (i) spatial structure of the data via multivariate decomposition, (ii) time-frequency structure via atomic decomposition. [25] presents a variation of the method where the topography is obtained by independent component analysis (ICA), in order to separate activities that overlap at the sensor level. Furthermore, a frequency prewhitening procedure is applied as a pre-processing step before ICA to better reveal high frequency activity.

Under the assumption that the event topography is constant across all atoms and trials, [26] introduces a method for estimating an atom for single-trial M/EEG, based on a non-linear fitting procedure. The method uses the same basic tools as the one described in the previous paragraph, but enforces the similarity of atoms across trials via a constraint on the dispersion of the atom

[Tallon-Baudry et al., 1996] Tallon-Baudry, C., Bertrand, O., Delpuech, C., and Pernier, J. (1996). Stimulus specificity of phase-locked and non-phase-locked 40 Hz visual responses in human. *J. Neurosci.*, 16(13):4240–4249.

[Lehmann and Skrandies, 1984] Lehmann, D. and Skrandies, W. (1984). Spatial analysis of evoked potentials in man - a review. *Progr Neurobiol*, 23(3):227–250.

[Miwakeichi et al., 2004] Miwakeichi, F., Martinez-Montes, E., Valdés-Sosa, P. A., Nishiyama, N., and an d Y. Yamaguchi, H. M. (2004). Decomposing EEG data into space-time-frequency components using Parallel Factor Analysis. *NeuroImage*, 22(3):1035–1045.

parameters. An iterative procedure inspired from the Matching Pursuit algorithm [Mallat and Zhang, 1993] but adapted to the multi-trial case is introduced for estimating the initial time-frequency atoms used in the non-linear fit. Numerical experiments show that the method is robust to low signal-to-noise conditions, and that introduction of the constraint on parameter dispersion significantly improves the quality of the fit.

Consensus Matching Pursuit [6]

All the above methods rely, at least in the initialisation of the atom search, on averaging across trials. This can lead to a severe distortion in the initial atom parameters: in the worst case, some atoms may totally disappear in the average signals (e.g. for events which are either too spread in time or non strictly locked oscillatory events). Often, these wrong initialisations cannot be corrected by the per-trial non-linear fit. [6] thus introduces Consensus Matching Pursuit (CMP), which extracts atoms separately for each trial without any averaging. A fuzzy voting procedure (an atom votes for itself and for all its siblings that have a similar time-frequency-oscillation representation) is applied to select relevant atoms (those that repeat across trials). Peaks in the voting map allow for a much more robust recovery of events which would otherwise disappear (or would be under-estimated) in a simple averaging procedure. Examples on toy datasets and real data are given in the articles. Contrarily to our previous work, this work deals only with a single M/EEG signal.

2.1.2 Perspectives

The above work explores various dimensions of single-trial M/EEG event analysis, but never considers the problem in its full generality (simultaneously across time, space, trial, condition, subject, ...). Being the last one developed, the Consensus Matching Pursuit is probably the most accomplished method as it totally departs from the need of considering an averaged signal (the earlier ones were still considering the average signal during their initializations steps). On the other hand, it is also the only method where the topographic information of the signal has not been used. It is thus very tempting to incorporate the framework proposed in [26] within the Consensus Matching Pursuit framework. This is quite natural as both are based on the same ingredients (time-frequency analysis of the signals and Matching Pursuit) and since experience tells that topographic information is often very important. The main challenge in doing so is to keep the computational load as low as possible in spite of the high data dimensionality, to have a method that remains practical for everyday use. Another issue with Consensus Matching Pursuit is that the experimental data has to be separated into trials for which there is a time reference. Suppressing this constraint is a challenging long term goal that would allow more natural experimental or clinical setups in which no “clock” is imposed to the subject. Yet, ways of learning the events of interest – such as comparing different conditions – are needed to distinguish the events related to the task being explored from the brain background activity. This is related to feature identification and learning in BCI.

CMP is a very general tool that makes very few hypotheses on the signals of interest except that those signals must occur repetitively in the measurements (segmented into trials). But there are also some well-known, stereotypic, brain signals for which very specific detectors could be created. Ideally, such detectors should be highly specific and able to work on single trial data.

[Mallat and Zhang, 1993] Mallat, S. and Zhang, Z. (1993). Matching pursuit with time-frequency dictionaries. *IEEE Trans. on Signal Processing*, 41(12):3397–3414.

Examples of such events, are epileptic spikes (with applications to patient monitoring) or “errors potentials” that arise when the brain faces a situation not conform to its expectation. Creating detectors tuned for such specific events could be very useful for patient monitoring or for BCI systems.

2.2 Modelling the head and the sensors

The physics of the propagation of the electric and magnetic fields is governed by the Maxwell equations in quasistatic regime [Hämäläinen et al., 1993]. The electrical field $\mathbf{E}$ over a volume Ω with local conductivity Σ can be described by the electrical potential V ($\mathbf{E} = \nabla V$) is linked to the sources $\mathbf{J}^p$ by the equation:

$$\left\{ \begin{array}{ll} \nabla \cdot (\Sigma \nabla V) = \nabla \cdot \mathbf{J}^p & \text{in } \Omega \\ \Sigma \nabla V \cdot \mathbf{n} = 0 & \text{on } S = \partial\Omega. \end{array} \right. \quad (2.1)$$

where $S = \partial\Omega$ denotes the boundary of the volume Ω . The second part of this equation is a boundary condition that simply states that no current flows outside of the head, which while not strictly true at the neck interface, remains a reasonable assumption as the neck is relatively far from the events interesting us. The magnetic field can be computed either using the Biot-Savart law [Hämäläinen et al., 1993]:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_{\Omega} \mathbf{J}(\mathbf{r}') \times \nabla' \left(\frac{1}{R} \right) d\mathbf{r}' \quad \text{with } R = \|\mathbf{r} - \mathbf{r}'\|, \quad (2.2)$$

or using the potential vector $\mathbf{A}$ ($\mathbf{B} = \nabla \times \mathbf{A}$) as:

$$\nabla \times \nabla \times \mathbf{A} = \mu_0 (\mathbf{J}^p - \Sigma \nabla V), \quad (2.3)$$

with vanishing conditions for $\mathbf{A}$ at infinity.

Forward modelling consists in solving these equations for V (EEG forward problem) and $\mathbf{B}$ (MEG forward problem) given the descriptions of the conductivity Σ of the head and of the sources of activity $\mathbf{J}^p$.

Sources: Isolated dipoles for source modelling were introduced by Scherg and von Cramon [Scherg and Von Cramon, 1986]. Later, Dale and Sereno proposed a distributed source model, on the cortical mantle segmented from MRI [Dale and Sereno, 1993]. Sources are often constrained to

[Hämäläinen et al., 1993] Hämäläinen, M., Hari, R., Ilmoniemi, R. J., Knuutila, J., and Lounasmaa, O. V. (1993). Magnetoencephalography— theory, instrumentation, and applications to noninvasive studies of the working human brain. *Reviews of Modern Physics*, 65(2):413–497.

[Scherg and Von Cramon, 1986] Scherg, M. and Von Cramon, D. (1986). Evoked dipole source potentials of the human auditory cortex. *Electroencephalography and clinical neurophysiology*, 65(5):344–360.

[Dale and Sereno, 1993] Dale, A. and Sereno, M. (1993). Improved localization of cortical activity by combining EEG and MEG with MRI cortical surface reconstruction: A linear approach. *Journal of Cognitive Neuroscience*, 5(2):162–176.

lie in the cortex (often on a surface) and to be orthogonal to the cortical mantle. Imposing the orientation has the advantage of reducing the number of parameters by a factor of 3. But, as it is often quite difficult to estimate the cortex orientation with enough accuracy, relaxing this constraint is sometimes a good idea. Distributed source models can also be constrained this way. There is however a more mathematical constraint: the distribution of current sources $\mathbf{J}^p$ appears in Eq. (2.1) only through its divergence. Indeed, the Helmholtz theorem states that any vector field can be decomposed into the sum of a gradient vector field and a rotational vector field $\mathbf{J}^p = \nabla J_g + \nabla \times \mathbf{J}_r$. Consequently, the potential V only depends on the scalar field J_g (under the hypothesis of a smooth source field which can be accepted in a first approximation). The rotational part $\mathbf{J}_r$ only influences magnetic fields. The discrete vs distributed nature of the source model is important for forward modelling. The tradeoff is between a small number of parameters and a non-linear forward problem (discrete models) vs a huge number of parameters but a linear forward model (distributed sources).

Conductivity is related to the nature of the tissues and is the major physical parameter driving the propagation of electrical and magnetic fields in the head. Conductivities are often considered as being properties of tissues. Describing the conductivity of the head thus requires recovering (at least partly) the geometry of the head. In the simplest model, each tissue (scalp, bone, CSF and brain) is given a single constant isotropic and homogeneous conductivity. This is clearly true for the CSF compartment of the head which is liquid, and it is generally accepted for the scalp and grey matter. This is known not to be true for the bone or the white matter. The bone is a complicated anisotropic structure made of various materials such as compacta or marrow. It is generally accepted that a good conductivity model for it is to have a simple anisotropic model that has different tangential and radial conductivities [Marin et al., 1998]. White matter is even more complicated as it is made of myelinated fibers that behave as a set of “wires” that connect various parts of the grey matter. Diffusion MRI can give an idea of the main bundles of fibers. Since conductivity is intimately related to this fiber organisation, it is quite tempting to use the fiber structure as a template for white matter anisotropy [Wolters et al., 2001]. Furthermore, even for isotropic and homogeneous tissues, scalar conductivity parameters are often given standard values found in the literature. Quite some variability exists between published measurements of conductivities. In practice, it is mostly the ratio of conductivities that is important, as only the relative activation of the various cortical areas is interesting, not their absolute values. Recent studies [Oostendorp et al., 2000, Gonçalves et al., 2003] tend to show that even those ratios of conductivities are not well known (there is a factor of 3 in the published values for the ratio between the conductivities for the scalp and the skull).

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- [Marin et al., 1998] Marin, G., Guerin, C., Baillet, S., Garnero, L., and Meunier, G. (1998). Influence of skull anisotropy for the forward and inverse problems in EEG: simulation studies using FEM on realistic head models. *Human Brain Mapping*, 6:250–269.
- [Wolters et al., 2001] Wolters, C., Anwander, A., Koch, M., Reitzinger, S., M., K., and Svensén, M. (2001). *Influence of Head Tissue Conductivity Anisotropy on Human EEG and MEG using Fast High Resolution Finite Element Modeling, based on a Parallel Algebraic Multigrid Solver*. Forschung und wissenschaftliches Rechnen.
- [Oostendorp et al., 2000] Oostendorp, T., Delbeke, J., and Stegeman, D. (2000). The conductivity of the human skull: Results of *in vivo* and *in vitro* measurements. *IEEE Transactions on Biomedical Engineering*, 47(11):1487–1492.
- [Gonçalves et al., 2003] Gonçalves, S., de Munck, J., Verbunt, J., Heethaar, R., and Lopes da Silva, F. (2003). In vivo measurement of the brain and skull resistivities using an EIT-based method and the combined analysis of SEF/SEP data. *IEEE Transactions on Biomedical Engineering*, 50(9):1124–1128.

Geometrical models of the head are thus necessary to model conductivity and to constrain source locations and orientations. Unfortunately, head geometry varies quite a lot from one subject to another. While in many cases nested spherical models are sufficient (this is more true for MEG than for EEG) to identify the activated brain areas, it is usual, when high accuracy is needed, to design customized head models for the given subject. These models are usually constructed from anatomical MR images. Constructing such models is quite complicated:

- The MR image needs to be segmented into regions corresponding to the various tissues. Some tissues (such as the scalp or the grey and white matter) are quite well segregated using T1 (anatomical) MRIs. It is, however, impossible to distinguish CSF from the skull bone. For this reason, this interface is often “invented” using some simple a priori model. Another difficulty is that some kinds of tissues are deformed by MRI images (e.g. fat shift).
- Then, various domains need to be meshed in order to obtain a computational model of the head. Obtaining good computational meshes remains a subject of research especially for 3D meshes. The head is a quite complicated geometrical domain and meshing it properly is still quite a challenge.

Computational Forward M/EEG methods are compulsory for solving the M/EEG forward problem. Three types of methods have been devised. Analytic models are non-linear and limited to simple head geometries (nested spheres or ellipses and sufficiently simple conductivities) and discrete sources (isolated dipoles). Numerical models are linear and often more at ease with the distributed source model. There are two families of such models:

- **Surface models:** for models with only homogeneous and isotropic conductivities, it is possible to devise integral methods for which the solution is depicted using the potentials and/or currents only at the interfaces separating the various domains of conductivity. The representation theorem directly relates jumps of potentials and currents across these discontinuity interfaces so there is no need to approximate the solution in the volume. This allows to deal with quite general geometries and gives accurate solutions. Depicting the head requires only surface meshes. The discretized problem involves however dense matrices which precludes the use of the method with very fine meshes. The numerical methods arising from such surface models are called Boundary Elements Methods (BEM). Until recently, the state of the art in BEM consisted in using a double-layer formulation [Geselowitz, 1967], with an accuracy improvement provided by the isolated Skull Approach [Hämäläinen and Sarvas, 1989].
- **Volume models** have less constraints. They can deal with anisotropic or inhomogeneous conductivities (interesting for the skull and to a lesser extent for white matter). Volume models usually also involve only local interactions so that even though the matrices are bigger they are also very sparse. Thus the needed memory and computational times grow much slower with mesh sizes than with surface methods. The major drawback of volume

[Geselowitz, 1967] Geselowitz, D. B. (1967). On bioelectric potentials in an inhomogeneous volume conductor. *Biophysics Journal*, 7:1–11.

[Hämäläinen and Sarvas, 1989] Hämäläinen, M. S. and Sarvas, J. (1989). Realistic conductivity geometry model of the human head for interpretation of neuromagnetic data. *IEEE Trans. Biomed. Eng.*, 36(2):165–171.

methods is the need to build volume meshes which is still a difficult problem for domains as complicated as the head. The numerical methods arising from volume models are called Finite Elements Methods (FEM). State-of-the-art FEM use standard tetrahedral meshes with P1 finite element methods [Hauelsen et al., 1997, Wolters et al., 2001, Hauelsen et al., 2002].

While analytical models directly give the potential at the points of interest, both surface and volume models lead to solving large linear systems. As the matrices have very different characteristics, different solving strategies are usually adopted. With surfacic approaches, it is often possible to solve the system using matrix inversion. This is not the case for volume approaches as the inverse of a sparse matrix is usually dense, so that it is usually impossible to store the resulting inverse matrix. For this reason, iterative approaches are used to solve the system for any given distribution of sources.

2.2.1 Contributions

Segmentation of the head using joint T1- and T2- MR images [19, 18]

As stated above, it is currently difficult to obtain the skull geometry as standard anatomical MR images (aka T1-MR images) do not contrast skull and CSF. The work on segmentation using local statistics [19, 18] has been used to extract this structure from combined registered T1- and T2- MR images. Indeed, T1- and T2- images reveal complementary contrasts. In particular, in T2-images, while the skull remains a black area, the CSF gives rise to very white voxels, so that the inner surface of the skull becomes visible. T2-MRI images also allow to distinguish blood vessels from grey/white matters leading to more accurate cortical models. The method proposed in the Ph.D. thesis [Piovano, 2009] extracts hierarchically the different head structures using local statistics on several combinations of T1- and T2- MR images depending on the interface to be extracted. These results can be used for building more realistic head models for the forward M/EEG problem. As interfaces are depicted as levelsets, they can be directly used by Implicit FEM method (see below in this section).

Symmetric BEM formulation for the M/EEG forward problem [34, 32, 31, 10, 9, 30, 28, 8, 24, 16]

In [34], two different methods – one surface and one volume method – for the resolution of the EEG forward problem are compared mainly from the point of view of computational complexity and accuracy. The two compared methods are a classical boundary element method (BEM) using the Gesselowitz formula and a classical finite element method (FEM) using tetrahedral

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- [Hauelsen et al., 1997] Hauelsen, J., Ramon, C., Eiselt, M., Brauer, H., and Nowak, H. (1997). Influence of tissue resistivities on neuromagnetic fields and electric potentials studied with a finite element model of the head. *IEEE Transactions on Biomedical Engineering*, 44(8):727–735.
- [Wolters et al., 2001] Wolters, C., Anwander, A., Koch, M., Reitzinger, S., M., K., and Svensén, M. (2001). *Influence of Head Tissue Conductivity Anisotropy on Human EEG and MEG using Fast High Resolution Finite Element Modeling, based on a Parallel Algebraic Multigrid Solver*. Forschung und wissenschaftliches Rechnen.
- [Hauelsen et al., 2002] Hauelsen, J., Tuch, D., Ramon, C., Schimpf, P., Wedeen, V., George, J., and Belliveau, J. (2002). The influence of brain tissue anisotropy on human EEG and MEG. *NeuroImage*, 15:159–166.
- [Piovano, 2009] Piovano, J. (2009). *Image Segmentation and Level Set Method: Application to Anatomical Head Model Creation*. PhD thesis, University of Nice-Sophia Antipolis.

meshes that I implemented. Both methods are compared using synthetic test cases with concentric spheres where the analytical solution is known. The implication for a realistic head models and the inverse problem (see below) is direct. Compatible surface and volume meshes are constructed and the accuracy of the solution are compared with several electric sources approaching a discontinuity in conductivity, which is known in computational electromagnetics to lead to inaccuracies. This case arises necessarily for the simulation of cortex activation, due to its thinness. The study [34] shows that the FEM method is more accurate than the BEM one. This is somewhat unexpected since, as stated above, the BEM method does not approximate the solution between interfaces as does the FEM method. One possible reason for this is that the FEM has some more degrees of freedom as it uses more discretization points. Nonetheless, the FEM method is both more accurate and faster than the standard BEM one, so everything else being equal, FEM should be preferred.

To cope with this unexpected accuracy discrepancy, a new formulation of the boundary element method has been developed in [31]. Standard BEMs compute the potential as a superposition of a set of harmonic potentials and the potential computed in an infinite homogeneous domain. This symmetric BEM approach keeps the basic ingredients of standard BEMs but decomposes the solution using a different superposition. This leads to a new formulation which involves both the potentials and the normal currents flowing through the interfaces. This leads to symmetrical matrices with many null blocks, bringing two advantages: 1) there are more free parameters to describe the solution, and 2) symmetric matrices are known to have better numerical behaviour when involved in solving linear systems. The resulting method improves drastically the accuracy of the computed potential and gains advantage over the FEM method in the tests used in the work depicted in the previous paragraph. Continuing this work, [10] puts various BEMs (symmetric, single layer and double layer potential) into the same framework (the representation theorem) and compares their accuracies using the spherical benchmark and real geometries. The conclusion is that given a fixed mesh description of the head, the symmetric BEM clearly improves the accuracy over the competing BEMs. Finally, the work [8] explores the generalization of symmetric BEM to non nested geometries. Indeed, nested geometries cannot model the openings present in the skull (e.g. the eyes), or the brain and skull defects caused by brain surgery. The effect of such defects on the localization accuracy can be significant [Bénar and Gotman, 2002, Oostenveld and Oostendorp, 2002] and the proposed method offers new modelling possibilities and promises greater accuracy for MEG/EEG forward and inverse problems notably for patients who underwent brain surgery. It must be observed, though, that such meshed models are even more complicated to build as small topological defects (wrong triangle orientation, non-closed surfaces, almost but not exactly identical points) can have disastrous effects and/or deteriorate numerical stability of the system. Generalized meshes also tend to lead to bigger matrices.

This line of work has been very important in the team as the symmetric BEM has become a central tool for many students and researchers. Several variations of the method have been explored. Some studies are numerical improvements such as in [32, 9] where fast multipole methods are used to accelerate the operation of solving the numerical system involved in the

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- [Bénar and Gotman, 2002] Bénar, C. G. and Gotman, J. (2002). Modeling of post-surgical brain and skull defects in the EEG inverse problem with the boundary element method. *Clinical Neurophysiology*, (113):48–56.
- [Oostenveld and Oostendorp, 2002] Oostenveld, R. and Oostendorp, T. F. (2002). Validating the boundary element method for forward and inverse EEG computations in the presence of a hole in the skull. *Human Brain Mapping*, (17):192–192.

forward problem. Others are using the symmetric BEM to improve head models, e.g. [30, 28] which use it to estimate brain conductivities using electrical impedance tomography. Because of this, a software effort has been made to provide an open source implementation of symmetric BEM as well as several tools necessary to solve the MEG/EEG forward problem. Starting from the source code of the Ph.D. student G. Adde, **OpenMEEG** [24, 16, 3] has been developed. I'm one of the main developers and administrators of this package, planning releases, correcting bugs, adding new functionalities. . . OpenMEEG is central to the research of several students in the group: Sylvain Vallaghé, Alexandre Gramfort, Emmanuel Olivi, Sabir Jacquet, Joan Fruitet have all used it. It is also starting to be used by the group of Jean-Marc Lina at University of Montréal, by Jean-Michel Badier in the hospital La Timone and by Boris Burle at Université de Provence both in Marseille, and by several other groups across the world. It has been directly downloaded about 2000 times since the first release in October 2008.

Implicit Mesh FEM [44, 22, 5, 2]

Compared to Boundary Element Methods (BEM), Finite Element methods (FEM) have many advantages: they lead to sparse symmetric matrices which can be solved efficiently, have a quite good accuracy (as seen with the study on spheres) and allow for the modelling of anisotropy. But obtaining volume tetrahedral meshes for the head revealed to be very difficult. Indeed the 3D meshes needed by FEM methods must have several properties: 1) they must approximate the geometrical domain accurately, 2) they must have good numerical properties, and 3) they must be small enough so that the computations take a reasonable amount of time. These goals are somewhat contradictory and generally make the generation of head meshes quite difficult. Only recent 3D meshing tools (see e.g. the work [Rineau and Yvinec, 2007]) can cope successfully with the kind of meshes that would be required for the EEG forward FEM problem. They are computationally demanding, taking up to 24h of computation in the current state – as of 2007 – of the meshing programs [Olivi, 2007].

With the Implicit Mesh FEM [44, 22], I introduced a technique that bypasses the mesh generation step, assembling the FEM matrices directly from a levelset description of the interfaces separating the various tissues. Using the levelset description is quite convenient as it is already used by many segmentation tools (e.g. obtained by the work [Piovano, 2009]). Other surface descriptions (e.g. meshes) can easily be transformed into levelsets. The proposed method solves the EEG forward problem at the cost of a small accuracy loss (about 3%) and a small speed penalty. This speed penalty is somewhat compensated by the fact that the proposed method transforms the levelset depicting the head interfaces into the FEM matrix in minutes (as opposed to hours for standard meshing techniques). The method, originally proposed for the EEG Forward problem, was generalized to the MEG forward problem in [5] (see below for more details). As such, the Implicit Mesh FEM greatly facilitates the use of FEM models in the contexts of clinical evaluation or cognitive research. However, with the continuous elements used in the Implicit Mesh FEM approach, the solution representation guarantees continuity of potential across interfaces but not that of normal currents. The method has thus been improved by using

[Rineau and Yvinec, 2007] Rineau, L. and Yvinec, M. (2007). A generic software design for delaunay refinement meshing. *Comput. Geom. Theory Appl.*, 38:100–110.

[Olivi, 2007] Olivi, E. (2007). Comparaisons quantitatives de résultats de problème direct eeg. Research Report 6364, INRIA.

[Piovano, 2009] Piovano, J. (2009). *Image Segmentation and Level Set Method: Application to Anatomical Head Model Creation*. PhD thesis, University of Nice-Sophia Antipolis.

non-differentiable elements [2]. In this approach, elements specific to the MEG/EEG forward problem are constructed. These elements ensure that the property that both the potential and normal currents are continuous across the tissues interfaces is preserved up to the numerical implementation. To do so, each element is actually made of two pieces that are designed to have the necessary continuity property. Using such elements, it is possible to recover the 3% accuracy loss introduced by the use of the implicit mesh method at the cost of an increase of computational time.

One of the main difficulties of the Implicit Mesh FEM is the computation of the integrals over the implicit domains. In the original work [44, 22], a semi-numerical approach has been devised: 2D integrals are computed analytically and a numerical integration is done over the third dimension in order to obtain the final integrals. This strategy is both very accurate and very fast, but requires to handle all sorts of degenerate cases related to local topological changes. In the case of the work [2], the supplementary flux integrals needed to compute the non-differentiable elements are computed numerically, which is both slow and has small accuracy issues.

2.2.2 Perspectives

Starting with the work depicted above, various extensions can be considered:

- There are lots of opportunities with the Implicit Mesh FEM. The basic approach is very general and can potentially be extended to many other FEM/BEM problems which need to handle complex geometries. In many cases, this would require relying on numerical computations of the integrals, but the non-differentiable elements used in [2] show that this is realistic. This approach also leads to algorithms that are easier to parallelize because of the underlying regular grids. Simple experiments with the Implicit mesh code and OpenMP has shown that much improved computational times can be attained on machines using the multi-core processors. Yet, the flux integrals of [2] can be computed using similar semi-numerical strategies as those used in [44, 22]. This will solve the speed and accuracy issues mentioned above. Once this is done, the Implicit FEM code will be distributed similarly to OpenMEEG. But this also requires providing a comprehensive set of tools to deal with levelsets-based models, so that users can build the needed head descriptions from either meshes or MR images.
- It is also tempting to combine the various methods depicted in this section in an hybrid model that couples BEM and FEM. In such an implementation, each method will be used for its strength (Symmetric BEM for its accuracy for tissues having an isotropic homogeneous conductivity, FEM for its ability to deal with anisotropy) to provide even better forward problems. Initial work has been made in this direction [15].

2.3 The inverse problem

By comparing the simulated – from the forward problem – and measured fields (V and/or some components of the magnetic field $\mathbf{B}$), it is possible to recover information about the sources $\mathbf{J}^p(\mathbf{r})$ solving an *inverse problem*. Source recovery from sensor measurements is an ill-posed problem: formally, it is unstable, and in the distributed source case, non-unique (due to the relatively low number of simultaneous measurements). Constraints, or regularization, are necessary in order to

guarantee an unique and stable solution [Sarvas, 1987, Hämäläinen et al., 1993]. Choosing the proper type of regularization and constraints is the subject of intense research in the M/EEG community [Hämäläinen and Sarvas, 1989, Mosher et al., 1992, de Munck, 1992, Veen et al., 1997, Gramfort and Kowalski, 2009]. The statistical assessment of the quality of solutions is also a crucial point, because in functional neuroimaging in general “ground truth” is difficult to obtain. Solutions using statistical parametric maps, Bayesian learning [Wipf and Rao, 2004] or permutation tests [Nichols and Holmes, 2001] have been proposed.

2.3.1 Contributions

Adjoint State Approach for leadfield [5, 14]

Most of the methods for the inverse source problem in MEEG use a lead field as an input. The leadfield (the linear mapping from sources to sensors) summarizes the output of the forward problem. Efficiently computing it is the first building block of the MEEG inverse problem. For complex geometries, there is no analytical formula of the leadfield. The common approach is to numerically compute the value of the lead field for a finite number of point sources (dipoles or elementary dipole fields for distributed approaches). There are several drawbacks: the model of the source space is fixed (a set of dipoles), and the computation can be expensive for as much as 10 000 dipoles since for each dipole a forward problem needs to be solved. Using a reciprocity theorem which states that electric current densities (sources) and electromagnetic fields can be interchanged in Maxwell’s equations, it is possible to compute the leadfield from

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- [Sarvas, 1987] Sarvas, J. (1987). Basic mathematical and electromagnetic concepts of the biomagnetic inverse problem. *Phys. Med. Biol.*, 32(1):11–22.
- [Hämäläinen et al., 1993] Hämäläinen, M., Hari, R., Ilmoniemi, R. J., Knuutila, J., and Lounasmaa, O. V. (1993). Magnetoencephalography— theory, instrumentation, and applications to noninvasive studies of the working human brain. *Reviews of Modern Physics*, 65(2):413–497.
- [Hämäläinen and Sarvas, 1989] Hämäläinen, M. S. and Sarvas, J. (1989). Realistic conductivity geometry model of the human head for interpretation of neuromagnetic data. *IEEE Trans. Biomed. Eng.*, 36(2):165–171.
- [Mosher et al., 1992] Mosher, J. C., Lewis, P. S., and Leahy, R. M. (1992). Multiple dipole modeling and localization from spatio-temporal meg data. *IEEE Transactions on Biomedical Engineering*, 39(6):541–553.
- [de Munck, 1992] de Munck, J. C. (1992). A linear discretization of the volume conductor boundary integral equation using analytically integrated elements. *IEEE Trans. Biomed. Eng.*, 39(9):986–990.
- [Veen et al., 1997] Veen, B. D. V., van Drongelen, W., Yuchtman, M., and Suzuki, A. (1997). Localization of brain electrical activity via linearly constrained minimum variance spatial filtering. *IEEE Transactions on Biomedical Engineering*, 44(9):867–880.
- [Gramfort and Kowalski, 2009] Gramfort, A. and Kowalski, M. (2009). Improving M/EEG source localization with an inter-condition sparse prior. In *Proceedings of ISBI*.
- [Wipf and Rao, 2004] Wipf, D. and Rao, B. (2004). Sparse Bayesian learning for basis selection. *IEEE Trans Signal Processing*, 52:2153–2164.
- [Nichols and Holmes, 2001] Nichols, T. E. and Holmes, A. P. (2001). Nonparametric permutation tests for functional neuroimaging: a primer with examples. *Human Brain Mapping*, 15:1–25.

a sensor point of view [Weinstein et al., 2000, Nolte, 2003, Wolters et al., 2004, Schimpf, 2007]. This requires as many forward-like problems as there are sensors (a few hundred) which is a clear computational advantage. Mathematically, this is formulated using the adjoint method [Lions, 1971] (indeed, reciprocity is closely related to the concept of adjoint for Hermitian operators). In [5, 14], the adjoint method is used to derive general EEG and MEG sensor-based lead field equations. Within a simple framework, a complete review of the explicit leadfield equations is provided, and these equations are extended to non-pointlike sensors.

Besides the computational advantage, the adjoint state approach allows for a very compact storage of leadfields without any limitation on the source space (unconstrained sources require essentially the same storage as constrained ones). It is also interesting to note that originally the adjoint variable used in this approach was used as an efficient mean to compute the gradient of the criteria associated with inverse problems [Faugeras et al., 1999]. The leadfield can thus easily be used in gradient-descent like approaches.

Tracking cortical activity [43, 4]

Most source reconstruction methods just provide a map of sources. These maps show the sources distributions either at a chosen time instant or during a given time interval. The work [43, 4] aims at sketching over time the activity of the brain, providing dynamic maps of cortical activations triggered by a stimulus. Raw activations are computed from MEG or EEG using a distributed source model with equivalent current dipoles lying on the folded cortical surface. Exploiting the natural graph structure of the cortical surface and the high time sampling of MEG and EEG recordings, neural currents reconstructions are used to compute, in a robust and efficient way, tracks of cortical activations over time. This problem is cast into a Markov Random Field optimization framework that can be optimized in a few seconds using graphcuts-based algorithms, which guarantee global optimality of the solution. A label is assigned to each vertex in space and time, indicating active vs. non-active condition. Such approach computes a minimum cut on a weighted graph related to a cost function, imposing spatio-temporal regularity constraints on the activations patterns. Nodes of the graph are indexed over space and time. Edges code for the local regularity. The data cost associated to each node describes how likely this node is being active. This information is extracted from statistical maps or from the amplitude of neural activations for overlapping time windows with a duration of a few milliseconds. The method is illustrated and validated on a synthetic dataset and MEG data on the somatosensory cortex for a finger stimulation experiment.

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- [Weinstein et al., 2000] Weinstein, D., Zhukov, L., and Johnson, C. (2000). Lead-field bases for electroencephalography source imaging. *Annals of Biomedical Engineering*, 28(9):1059–1164.
- [Nolte, 2003] Nolte, G. (2003). The magnetic lead field theorem in the quasi-static approximation and its use for magnetoencephalography forward calculation in realistic volume conductors. *Physics in Medicine and Biology*, 48:3637–3652.
- [Wolters et al., 2004] Wolters, C. H., Grasedyck, L., and Hackbusch, W. (2004). Efficient computation of lead field bases and influence matrix for the fem-based eeg and meg inverse problem. *Inverse Problems*, 20:1099–1116.
- [Schimpf, 2007] Schimpf, P. H. (2007). Application of quasi-static magnetic reciprocity to finite element models of the meg lead-field. *IEEE Transactions on Biomedical Engineering*, 54(11):2082–2088.
- [Lions, 1971] Lions, J. (1971). *Optimal control of systems governed by partial differential equations*. Springer.
- [Faugeras et al., 1999] Faugeras, O., Clément, F., Deriche, R., Keriven, R., Papadopoulos, T., Roberts, J., Viéville, T., Devernay, F., Gomes, J., Hermosillo, G., Kornprobst, P., and Lingrand, D. (1999). The inverse eeg and meg problems: The adjoint space approach i: The continuous case. Technical Report 3673, INRIA.

Rational approximation based inverse problems [29]

Most dipole estimation methods (eg MUSIC [Mosher et al., 1992]) extract one dipole at a time. To obtain multiple dipoles the method is iterated after subtracting the contributions of the already localised dipoles. This is known to be error prone in many cases. On the other hand, traditional dipole-fitting methods are highly sensitive to the number of dipoles in the model. The work [29] proposes a method to simultaneously localize multiple dipoles from data at the surface of a homogeneous sphere. The method assumes a spherical geometry with a single conductivity domain. The sphere is cut in slices and in each slice, the anti-harmonic part of the potential can be described analytically as a rational function. Given the potential values and its normal derivatives at the slice border, it is possible to approximate such a rational function. The poles of these rational functions for each slice cluster along trajectories, and it is possible to show that the actual dipoles belong to these trajectories and have some special properties which allow their recovery [Baratchart et al., 2005]. It is thus possible to localise simultaneously multiple dipole.

2.3.2 Perspectives

Software extensions Some of the above developments need to be extended, robustified and made available:

- The implicit mesh and adjoint approaches have been developed only within the FEM framework. In theory, BEM could also benefit from those advances. Implicit meshing could be used to remove the need to mesh interfaces, but completely new integrals would need to be computed (and most certainly those integrals would require a numerical evaluation). Similarly, the adjoint approach could be used to establish leadfields with the BEM (OpenMEEG currently uses matrix inversion). For this, the adjoint approach would need to be extended for normal currents. As the trade-offs between BEM and FEM are not the same, the exact benefit brought by these techniques should be investigated and assessed.
- The FEM code needs to be made widely available as OpenMEEG is. The package needs to be separated from the huge set of libraries it is included into and some parts need to be re-written as they currently contain experimental code.

Patch-based regularisation

Most of the regularisation terms used to constrain the inverse problem are of mathematical nature. It would be interesting to incorporate more anatomical priors. One way to do so is to use the fact that in many cases, activations occur on extended patches of cortex. It is possible to recover such patch descriptions eg using the connectivity patterns provided by diffusion MRI. It thus seems interesting to incorporate such information within the inverse problem. A complementary method for creating patches would be to consider that MEG/EEG measurements are only accurate up to some level, so that there is no need to distinguish cortical areas that give rise to signals which are equivalent up to that accuracy. Cortical positions would not be grouped with respect to their anatomical function but with respect to the capability of the acquisition system. Such a regularisation scheme will thus incorporate the information of

[Mosher et al., 1992] Mosher, J. C., Lewis, P. S., and Leahy, R. M. (1992). Multiple dipole modeling and localization from spatio-temporal meg data. *IEEE Transactions on Biomedical Engineering*, 39(6):541–553.

[Baratchart et al., 2005] Baratchart, L., Ben Abda, A., Ben Hassen, F., and Leblond, J. (2005). Recovery of pointwise sources or small inclusions in 2D domains and rational approximation. *Inverse Problems*, 21:51–74.

the limitations of the acquisition, which is clearly complementary with the anatomy. Ideally, patches should be defined using both anatomical and measurements constraints.

Simultaneous MEG and EEG

Most of the methods localizing sources of brain activity work with either EEG or MEG data (not with both). This is all the more surprising as simultaneous MEG and EEG acquisitions are routinely made. Sources localized separately from either MEG or EEG measurements are compared. The main problem to be solved is the very different units of MEG and EEG measurements. Early localization methods using both MEG and EEG measurements are asymmetric [Cohen and Cuffin, 1983, Cohen and Cuffin, 1987, Purcell and Cuffin, 1989]: sources are first localized using MEG data and their contribution is subtracted from the corresponding EEG measurements. In a second stage, the “remaining EEG measurements” are used to find radial source components (and eventually conductivity values). More recent methods simply combine two independent MEG and EEG least-squares criteria using a noise covariance matrix [Huizenga et al., 2001]. With more and more MEG and EEG data acquired simultaneously, it is quite tempting to explore further localization methods using simultaneously EEG and MEG measurements. This is all the more true as MEG and EEG are known to provide complementary information about sources [Anogianakis et al., 1992, Hämäläinen et al., 1993]. One way to overcome the scale discrepancy of MEG and EEG would be to base criteria on topographies instead of actual measurement values (as it is e.g. done in MUSIC).

Spatio-temporal models of cortical sources

M/EEG localization methods have traditionally very crude ways (if any) of using the spatio-temporal coherence of M/EEG data. At best, they try to enforce that the activations should be stable for a short period of time. On the other hand, there is quite a large literature on cortical source models [Jansen and Rit, 1995, Cosandier-Rimélé et al., 2007, Sotero and Trujillo-

-
- [Cohen and Cuffin, 1983] Cohen, D. and Cuffin, B. N. (1983). Demonstration of useful differences between magnetoencephalogram and electroencephalogram. *Electroencephalography and clinical neurophysiology*, 56(1):38–51.
- [Cohen and Cuffin, 1987] Cohen, D. and Cuffin, B. N. (1987). A method for combining meg and eeg to determine the sources. *Physics in medicine and biology*, 32(1):33–42.
- [Purcell and Cuffin, 1989] Purcell, C. and Cuffin, B. N. (1989). Estimates of skull conductivity using combined meg and eeg inverse solutions. In *International Conference on Biomagnetism (New York)*.
- [Huizenga et al., 2001] Huizenga, H. M., van Zuijlen, T. L., Heslenfeld, D. J., and Molenaar, P. C. (2001). Simultaneous meg and eeg source analysis. *Physics in Medicine and Biology*, 46:1737–1751.
- [Anogianakis et al., 1992] Anogianakis, G., Badier, J., Barrett, G., Erne, S., Fenici, R., Fenwick, P., Grandori, F., Hari, R., Ilmoniemi, R., Mauguiere, F., Lehmann, D., Perrin, J., Romani, G., Rossini, P., and Peters, M. (1992). A consensus statement on relative merits of eeg and meg. *Electroencephalography and Clinical Neurophysiology*, 82(5):317–319.
- [Hämäläinen et al., 1993] Hämäläinen, M., Hari, R., Ilmoniemi, R. J., Knuutila, J., and Lounasmaa, O. V. (1993). Magnetoencephalography— theory, instrumentation, and applications to noninvasive studies of the working human brain. *Reviews of Modern Physics*, 65(2):413–497.
- [Jansen and Rit, 1995] Jansen, B. H. and Rit, V. G. (1995). Electroencephalogram and visual evoked potential generation in a mathematical model of coupled cortical columns. *Biological Cybernetics*, 73:357–366.
- [Cosandier-Rimélé et al., 2007] Cosandier-Rimélé, D., Badier, J., Chauvel, P., and Wendling, F. (2007). A physiologically plausible spatio-temporal model for EEG signals recorded with intracerebral electrodes in human partial epilepsy. *IEEE Transactions on Biomedical Engineering*, 54(3):380–388.

Barreto, 2007]. It is tempting to use such models to spatio-temporally constrain the sources obtained with inverse problems. However, in their current state, these models depend on many parameters (some of them being even redundant). Simplifying and adapting these models so as they become useful for a spatio-temporal MEG/EEG inverse problem is certainly an interesting line of research. The behavior of the spatio-temporal model will certainly be very related to the connections between brain areas (which can be recovered only partially using diffusion MRI). It may also be possible to investigate with such models on the causality between brain activations, or at least the time sequence of activations of different brain areas. Such work might also help refining our understanding of the relations between activities measured by MEG/EEG and other functional modalities such as fMRI or NIRS.

Application to Brain Computer Interfaces (BCI) systems

The least intrusive BCI systems usually capture the brain activity using EEG measurements. BCI is thus a non-clinical application field of choice to apply our methods, foster new developments (which in turn can benefit to clinical applications) and demonstrate our advances in EEG processing. As BCI systems are known to be all the more effective as the user is engaged in the BCI experience [Friedman et al., 2007], it seems quite interesting to develop our system so that it can work within an immersive room. In addition, such a system offers many nice perspectives of fostering collaborations with other INRIA groups more dedicated to visualisation, audio processing, virtual reality, haptic rendering or machine learning. Developing such a system requires the integration of many lines of research (interaction, EEG signal analysis and reconstruction, learning, real time systems, drivers for hardware) which is challenging.

Besides integrating our previous work in such a platform (for example some inverse problems techniques which have been shown to increase mental task discrimination in some preliminary studies [Besserve et al., 2008, Cincotti et al., 2008]), some new lines of research might also be developed:

- **Richer human-computer interactions:** BCI consist of two interacting agents, a user and a computer. As in all human computer interfaces (mouse, screen, ...), in traditional BCI the computer behavior is predefined and the user must adapt to the interaction. EEG measurements contain very specific error signals when a situation does not conform to the human subject's expectations. Extracting these very weak signals would provide a unique opportunity to enrich the human-computer interaction, allowing the computer to gather information that can be used to modify its predefined behavior. The long term idea is that such error signals can be incorporated as rewards/penalties for adapting classifiers which can then continuously improve their performances during BCI experiments.
- **More dynamical cognitive/clinical protocols:** one goal of BCI systems is to be able to assess in real time the activity of the brain. Such a capability opens up the possibility

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- [Sotero and Trujillo-Barreto, 2007] Sotero, R. C. and Trujillo-Barreto, N. J. (2007). Biophysical model for integrating neuronal activity, eeg, fmri and metabolism. *NeuroImage*.
- [Friedman et al., 2007] Friedman, D., Leeb, R., Guger, C., Steed, A., Pfurtscheller, G., and Slater, M. (2007). Navigating virtual reality by thought: What is it like? *Presence*, 16(1):100–110.
- [Besserve et al., 2008] Besserve, M., Martinerie, J., and Garnero, L. (2008). Non-invasive classification of cortical activities for brain computer interface: A variable selection approach. In *Biomedical Imaging: From Nano to Macro, ISBI*, pages 1063–1066.
- [Cincotti et al., 2008] Cincotti, F., Mattia, D., Aloise, F., Bufalari, S., Astolfi, L., Fallani, F. D. V., Toccia, A., Bianchia, L., Marciani, M. G., Gao, S., Millan, J., and Babiloni, F. (2008). High-resolution EEG techniques for brain-computer interface applications. *Journal of Neuroscience Methods*, 167(1):31–42.

to modify stimulation on-the-fly on the basis of the measured activity. This could give rise to some new cognitive protocols to probe the brain functioning.

- **Simplifying the BCI setup:** A major burden in EEG acquisition (at least if source localization is to be effected) is the need to localize the electrodes with reference to the head as given by e.g. MR images. Traditionally, each electrode has to be pointed at by a magnetic or optical localizer. This approach is very inconvenient, is prone to errors and takes a significant amount of time. There is an opportunity of applying our past research of chapter 1 to this problem: obtaining electrode positions can be done automatically from pictures of the cap placed on the head of the subject using self-calibration and stereo reconstruction. Facial head shapes can be obtained with similar techniques. Besides BCI, such a system would be highly interesting for daily usage in hospitals (this application was actually suggested by our partners Patrick Chauvel and Jean-Michel Badier from the hospital “la Timone” in Marseille). The main challenge is to build an integrated system which involves several steps: electrode detection in images, tracking electrodes across the images, self-calibration of the camera positions, 3D reconstruction of electrode positions and facial shape and finally electrode labelling. The integrated system needs to be very reliable and simple to use by non-trained people. Another application of computer vision techniques would be building brain models from MR images (which despite the amount of research it has attracted remains a difficult problem) and registering it to the facial shape. The works [20, 21, 18] can be considered as building blocks for such a system.

2.4 Conclusion

This chapter summarized the work of MEG/EEG in which I have been involved since 2001. In terms of publications, this represents 8 journal articles and 13 articles in international conferences with proceedings (not counting again those articles that were already cited in the previous chapter). This work has been done in collaboration with 4 Ph.D. students (Geoffrey Adde, Sylvain Vallaghé, Alexandre Gramfort and Emmanuel Olivi), 2 post-docs (Jan Kybic and Christian Bénar) as well as with senior researchers such as Olivier Faugeras, Renaud Keriven, Juliette Leblond, Laurent Baratchart, Jean-Paul Marmorat, Alain Dervieux, Bruno Torrèsani, Jean-Michel Badier and last but not least Maureen Clerc.

I would also like to take the opportunity to thank Patrick Chauvel, which has been very helpful in introducing us to the world of MEG and EEG. Even if there is no formal publication with him, we have a very fruitful and long standing collaboration with his team in La Timone hospital in Marseille.

Chapter 3

Research projects

As a final chapter, I will develop some topics I'd like to pursue in the next few years. Of course, this research program is build around my new research topics, so that the perpectives evoked in the chapter 1 are not mentioned in this chapter.

3.1 Improved methods for observing the brain activity

Understanding the brain functioning remains one of the greatest challenges of modern science. The recent years have seen the development of many new means to observe its activity. Along with all these new means of observation is the need to develop tools to integrate the partial information they each provide into a consistent global model. This is a both a neuro-scientific and a computational challenge. On the neuro-scientific part, the challenge is to conceive experiments that are compatible with multiple modalities and to build biological or biophysical models explaining the results of such multiple-modalities experiments. The corresponding computational challenge is to provide tools to help these tasks. **This research program mostly focusses on this last challenge.** Such tools must help in probing the acquired signals to decide whether they are supporting a given model: this spans providing computerized versions of the models, including simplifying them or being able to restrict their parameters' ranges using actual data. Tools helping to analyze and extract information from the various measurements are also necessary. The overall goal is to be able to assimilate in a single global model the diversity of observations that currently exist. Indeed, these observation means are based on very different physical and biological principles. They thus reveal very different aspects of the brain (structural, electrical, haemodynamic, or even chemical) and have very different spatial or temporal resolutions. The challenge is all the more complicate that such computer tools need in the end to be used by non-computer specialists such as neuroscientists, cogniticians or clinicians.

Among these observation means, I will mainly focus on two complementary non-invasive techniques:

- MEG and EEG which provide complementary direct measurements of the electrical activity of the brain with a millisecond temporal resolution (the time scale at which interesting macroscopic electrical events happen in the brain). They are thus of great interest for cognitive experiments studying fine timings of brain activations. Unfortunately, their spatial resolution is quite poor.

- MR images reveal various properties of the brain. Often those properties are structural: standard T1- and T2- MR images are sensitive to head tissue properties, diffusion MR images give access to the connectivity properties of the white matter. Functional MR images (fMRI) use the haemodynamic properties of the brain to recover not only the structure of the functional areas of the brain but also their relative activations in time as well as some sub-area properties of those activations (e.g. in the visual cortex). While the spatial resolution of MR images is very good (of the order of the millimeter), their temporal resolution (of the order of the second) is quite poor.

Combining the measurements obtained by these two types of techniques has the potential of providing a detailed view both in space and time of the functioning brain at a macroscopic level. This research project starts with the MEG and EEG perspective and then proposes some means to integrate various informations obtained using MR images.

3.1.1 MEG and EEG improvements

Analyzing MEG or EEG data remains an important scientific challenge. Replacing some current invasive techniques by the sole use of MEG and EEG requires quite a research effort to improve the current state of the art and validate the proposed techniques. This research program relies on three major ideas:

1. Automating the extraction of interesting events in the acquired data without relying on averaging multiple instances of the activity is important. This allows the study of event variability, and has the potential of revealing finer structures in the event succession.
2. Improving the descriptions of the reconstructed sources in both their spatial and temporal characteristics is a necessary step to retrieve more accurate details on the brain activity. Better modeling, e.g. the head structure or the sources' dynamical properties can reduce the "source reconstruction space" and has the potential of providing clearer and more robust pictures of the reconstructed activity.
3. Simplifying the process of acquiring MEG and EEG data. Developing new tools is useless if those are too complicated to be used in practise.

These are necessary steps **towards the ultimate goal of using MEG or EEG systems to non-invasively and continuously localise brain activity** (i.e. without relying on multiple trials and averaging). This potentially has many implications ranging from better brain computer interfaces (BCI) or game control to improved cognitive protocols: indeed, BCI techniques can be used to define more dynamical M/EEG cognitive protocols where stimulation can change depending on early measured brain signals. The ultimate goal of such a work, as well as the other methodological improvements proposed here, is to work with clinicians or cognitive scientists to better understand some brain processes.

The raw signal level: detecting and aligning events of interest

Several strategies have been developed to cope with the noisy MEG and EEG signals. Some epileptic events (besides crises) can be seen directly due to their good signal to noise ratio. Such events can trigger at any time. On the contrary, signal arising in cognitive tasks are often "clocked" on some stimulus or reaction of the subject but are of low amplitude (almost completely

hidden by background activity). Experiences (a.k.a. evoked potentials) are thus organized in sets of repeated trials: signals arising in various conditions (resting state vs stimuli) can then be averaged to improve the SNR. In both cases (epilepsy or evoked potentials), the model behind the analysis is that **“events of interest” correspond to the part of the signal repeatable over time.**

Currently, manual examination of the signal using the simple operations provided by software packages is often needed to filter out parasite signals, to select the “good” trials, etc. More automated ways of processing the signal and detecting events of interest would be an important improvement over the current state of the art. This would not only reduce the time needed to analyze the data gathered in experiments, but also provide a more objective and systematic way of deciding what are events of interest. The work [6, 26, 45, 46] aims at providing such tools. The main idea is that brain activity corresponds to sparse events that repeat similarly from trial to trial. Events are extracted from time-frequency decompositions of the signals using ideas from the matching pursuit methodology [Mallat and Zhang, 1993] adapted to the multi-trial case. Relevant part of the signal is extracted separately for each trial without relying on averaging. This allows the study of the variability of brain activity across trials (attention, habituation are e.g. known to change activity). Up to now, only partial versions of the problem have been dealt with. Using all the dimensions of the data (time $\times$ space $\times$ trials $\times$ conditions) to **create an automatic multi-trial, multi condition, spatio-temporal, multi-subject event detection algorithm** is still a challenge, I’d like to explore in the next 4 years.

Better detection of repeating events may also require some study on the timing variations of brain activities. Indeed, the succession of events related to a stimulation is subject to some variability across trials (and of course across sessions or subjects). Finding the monotonic time correspondences that best aligns (some specific) events across trials, and studying statistically these temporal alignment curves is probably an interesting way to better model temporal variability of brain signals. This can lead to better event detection algorithms (because delays are better modeled) as well as some better understanding of the temporal organization of the succession of macroscopic time-events within the brain. Another way to use the temporal characteristics of the signal is to use causality. While its effectiveness with MEG and EEG signals is subject to controversy for various reasons (MEG and EEG are mostly blind to deep brain activities, the underlying linear mathematical model is often somewhat crude, interpretation of the results in terms of connected regions is difficult, etc.), I nonetheless believe that causality analysis is one important aspect of brain signals.

On a longer time scale, this also leads to single event detection. In the setup I envision, events of interest are still modelled using the multi-trial approach and are first “learnt” off-line. Single events are then detected during the real acquisition of data using the full spatio-temporal content of the data and the model established in the learning phase. In this way, cognitive or BCI systems could adapt themselves on the fly to the measured data (this also requires improving the computational aspects of current methods). For BCI systems, this could also allow for detection of some specific brain events arising when the brain encounters some unexpected situation (error signal). Detecting such signals would be a key to enable continuous adaptation of BCI systems. This is the subject of the Co-adapt ANR, where I mainly contribute to all these aspects of analyzing and modeling the variability of signals.

[Mallat and Zhang, 1993] Mallat, S. and Zhang, Z. (1993). Matching pursuit with time-frequency dictionaries. *IEEE Trans. on Signal Processing*, 41(12):3397–3414.

Constructing the forward model: “meshless” methods for coupled FEM and BEM

Most EEG reconstruction methods (and to a lesser extent MEG based ones) are limited in their accuracies and ease-of-use by the difficulty of modeling the head. Various head tissues are delimited by complex interfaces exhibiting very thin structures difficult to obtain and to mesh. Some tissues are moreover very anisotropic by nature, a property not yet well handled by the most used current head computational models.

Part of my recent activity has consisted in developing Finite Element Method (FEM) and to make them more accurate and easier to use [44, 22, 5, 2]. At the cost of a small reduction in accuracy, the method removes the costly step of meshing the head domain and constructs in a few minutes (instead of hours with current meshing methods) FEM matrices using directly head segmentations (obtained e.g. using [18]). Another advantage is the use of the natural workspace provided by MR images which is convenient to display computed solutions, and provides easy bounds on the computational resources needed to solve the forward problem. In its last refinement (using piecewise Q1 elements instead of simple Q1 elements [2], so as to allow continuous representations of both potential and normal current across the head interfaces), first experiments show that the accuracy of the method is close to that obtained with a standard tetrahedral FEM. If confirmed by more experiments, this would **remove the last drawback of the FEM methods and open the door to its more widespread use** (only 4 groups in the world develop and use FEM models for M/EEG). Conceptually, the technique could be extended to BEM and to other types of problems.

A **coupled FEM–BEM approach** is currently under investigation by the Ph.D. student Emmanuel Olivi (which I co-supervise) with the idea of combining the strengths of BEM (accuracy for domains with isotropic and homogeneous conductivity) and FEM (ability to deal with the anisotropic conductivity of the skull). White matter anisotropy is another tissue that can be treated with a similar approach.

Source modeling: anatomical and numerical constraints

Source modeling is another interesting problem. The more realistic are the meshes representing the cortex, the more degrees of freedom are available to represent sources (distributed source model). As inverse problems are ill-posed, regularization has to be used. Current methods use simple mathematical criteria such as minimum norm of the solution. Real anatomical constraints are quite different: 1) activations are considered to appear in constant patches corresponding to functional cortical areas, 2) they last for several milliseconds and the constant patch intensity is oriented orthogonally to the cortex.

Using patches to reduce the number of parameters describing sources is thus one possibility for anatomically justified regularization. Methods start to emerge to build patch information from diffusion MR images. Another interesting way to build patch is to consider the intrinsic limitations of forward models and acquisition systems. Sources whose effects in the sensor space differ by less than the noise level can very well be aggregated. Such a criterion associates to each source a map of all undistinguishable sources with respect to the noise. Patches can be created by clustering these maps. But maps already show interesting properties of inverse problems: non-connected components appear which is non-anatomical. These are intrinsic to all distributed solutions. **Better understanding these structures and enforcing spatio-temporal properties such as the connected component constraint can lead to improved regularization schemes and thus to more meaningful solutions.**

Using cortical orientation is another way of reducing source complexity. But, orientation is generally poorly defined, so that it is often more reliable to ignore this constraint. For EEG, another approach would be using source potentials: indeed Helmholtz theorem states that smooth vector fields can be decomposed as the sum of a source potential term and a rotational term. Sources being involved only through their divergence, the rotational term does not contribute to potential and can be eliminated. Using this model provides as many constraints as cortical orientation. This is already used in the FEM method, but incorporating it in the symmetric BEM could be a potential improvement.

3.1.2 Integration of MEG/EEG with other types of measurements

As depicted in the introduction of this section, there are other sources of information on the brain which offer complementary views of the brain organisation and functions. **Better understanding the relationship between the informations provided by the various modalities allowing the observation of the functioning of the brain is a very challenging and interesting topic.**

Better understanding the relationship between the various measurements of activity

Current brain functional measurements are of two broad types:

- Either, they are based on the **electrical properties of the brain**. This is the case of EEG and MEG, but also of many invasive methods such as electrocorticography (ECoG), local field potentials (LFP) or single unit recordings. There are also reports of MR sequences allowing the observation of such phenomena. Voltage-sensitive dye optical imaging methods also fit in this category.
- Or they are based on some of its **physical or chemical properties**. This category includes most MR-based techniques which can measure proton density, diffusion of water (diffusion MRI) or even haemodynamic changes (BOLD in functional MRI) in the brain. Haemodynamic changes is also the phenomenon underlying intrinsic optical imaging.

The precise relationship between electrical activity and haemodynamic measurements such as BOLD is an active current research domain. The aim of the Multimodel ANR research project is to build and analyze a coupled electrical and metabolic model of the brain activity. While separated model – either electric or haemodynamic – have been devised for quite some time, coupling them is an important challenge all the more that the role of some neural constituents (such as glia) is currently not well understood. The goal of this project is thus to propose such a coupled model, to tune its parameters (including using invasive animal measurements) in order to reproduce various experimental data (electrical, haemodynamic, etc.). Such a model could then be simulated and compared to the various non-invasive measurements of brain activity. Initially, the model will probably be very complicated. A very interesting question is its simplification, in order to make it more computationally tractable so that it can be used in a multimodal inverse problem. These are the subjects of my involvement in the Multimodel ANR research project.

Combining structural and functional information

Function (the signals that are transmitted) and structure (the areas that trigger those signals and the wires that connect them) are intrinsically related. There is at least two types of non-

invasive information that is interesting to confront and eventually assimilate with MEG and EEG measurements:

- **Functional areas:** the brain is organized in areas each dedicated to certain functions. As explained above, these functional areas can be used to constraint the MEG/EEG inverse problem. There are two ways of obtaining these areas: 1) functional MR images directly reveal those areas that are related to a specific experimental protocol (as well as the sequence of activated areas along time), 2) diffusion MRI can also provide such functional areas by clustering the cortical areas that display similar connectivity patterns. This last method is purely structural and shows (at a certain scale) all the putative connexions through white matter. It does not state in any way how these connexions are used in practise for a given experimental protocol.
- **Connexions:** diffusion MRI allow the computation of the main white matter pathways between cortical areas. This information can also be inferred – at least partially – from the succession of activations as measured by fMRI. These inferred connexions are putative (it is essentially impossible to decide whether two regions are directly connected or whether there is a third unseen region that is involved). Connexions provides some insight on the causality of events in the brain.

Combining the structural and functional information provided by MR images is a difficult problem for two reasons:

- The spatial and temporal scales of the two types of measures are extremely different (at least one or two orders of magnitude).
- The neuroscience community is usually extremely reluctant – and with reasons – to constrain solutions with a priori knowledge. Introducing such a knowledge must thus be done very carefully and ideally should propose ways of verifying and controlling the influence of this a priori knowledge.

Consequently, the easiest path to integrate some of the information provided by diffusion or functional MR images is to follow the strategy (explained in the previous section) that defines a EEG/MEG reconstruction regularization based on patches built using these modalities. This should be rather consensual given that the spatial resolution of MEG/EEG is rather poor. Yet, the effects of such a regularization term have to be studied carefully. A further possibility would be to use connectivity to better model the spatio-temporal behavior of sources. Such a model would be able to express and use the fact that two connected regions are more likely to have a related activity (with time delays). This is more controversial (as the connectivity model is incomplete and as deep sources are mostly invisible in MEG/EEG signals). Yet, the brain is such a complicated object that, to advance in our knowledge of its functioning, it is probably mandatory to adopt the strategy of building models and confront them to real measurements.

Another possible strategy to combine multimodal data (here mostly MEG, EEG or fMRI) is just to design means of comparing functional maps. There is already this need for sources reconstructions (comparing maps obtained with several methods is rather complex specially for distributed source models). The task here is even more complicated as the spatial and temporal properties of those maps are so different. The path to do so is probably to establish a joint model of electric/metabolic/haemodynamic activity and to build upon it. This is also a topic I contribute to investigate in the ANR project Multimodel.

3.2 Technological developments

Finally, here are some of the future technological developments in which I will insist.

3.2.1 OpenMEEG and MedINRIA-NT

I'm currently one of the co-maintainers of OpenMEEG (with M. Clerc and A. Gramfort). This software is quite mature, but there are still many improvements that could be integrated in it. Some developments such as the adjoint approach to compute M/EEG leadfields are currently being integrated and will be part of next release of OpenMEEG. On a longer term, several other features such better sensor modeling, more elaborated source models might be incorporated within OpenMEEG. But, OpenMEEG will probably remain a command line tool which limits its widespread use.

Having a graphical interface for OpenMEEG would be a nice addition. I'm involved in a project that aims at integrating such an interface to MedINRIA-NT (MedINRIA-NT is the future version of MedINRIA <http://www-sop.inria.fr/asclepios/software/MedINRIA/>) along other tools for visualising MEG and EEG data that we develop in conjunction with J.-M. Badier and B. Colombet at the hospital La Timone in Marseille. MedINRIA-NT will also receive some algorithmic developments developed in the ATHENA team on the processing of diffusion MR images. I am supervising all this effort.

3.2.2 EEG sensor localisation: a photogrammetric method

A major burden in EEG acquisition is the need to localise the electrodes with reference to the head as given by e.g. MR images. Each electrode has to be pointed at by a magnetic or optical localizer. This approach is very inconvenient, prone to errors and takes a significant amount of time.

It seems interesting to use our past stereo reconstruction experience to build a system that **automatically obtains electrode positions from pictures of the cap placed on the head of the subject**. This involves several steps: electrode detection in images, tracking electrodes across images, self-calibration of the camera positions, 3D reconstruction of electrode positions and of head shape and finally electrode labelling. Part of such a system (e.g. electrode labelling [20, 21] or electrode detection) have already been built (but needs some improvements). One important challenge is integrating the various building blocks in a coherent system with an ergonomic graphical interface. But there are several sub-tasks such as a robust method to match electrodes across images that need to be devised for this particular problem.

3.2.3 Implicit FEM

The implicit FEM method is somewhat complementary to OpenMEEG. It is currently being re-written to incorporate some recent developments, but in the end, I hope to provide a package similar to OpenMEEG with this code, with the goal of providing a simple-to-use FEM forward problem to the M/EEG community.

3.2.4 BCI system

Finally, in order to be able to run experiments by ourselves, we constructed a EEG lab at INRIA. This EEG system allows us to do some experiments, but also to put in practise some

of the algorithms developed at ATHENA. There is currently an effort to integrate some of those developments in a BCI system build around the platform OpenViBe (<http://openvibe.inria.fr/>) developed in the INRIA team BUNRAKU.

Conclusion

This document has presented several original contributions to the fields of computer vision, image processing and modeling of brain activity using the modalities of electro-encephalography (EEG) and magneto-encephalography (MEG). Associated to the various contributions are proposed some perspectives for this work. Obviously, these do not have all the same status. Indeed, as my current work mostly focusses on MEG and EEG data processing, the proposed perspectives for that field correspond to work that has already started or that I intend to do in the next future. These perspectives were developed more thoroughly in chapter 3. On the other hand, perspectives for the field of computer vision and image processing correspond more to topics that I would have liked to explore at the time (and that are still open to the best of my knowledge) or to very long term directions that I consider interesting to drive more short term research in that field.

DETAILED CURRICULUM VITAE

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1. *Diplomas*

Ph.D.:

Title : Analysis of the motion of 3D rigid curves from monocular sequences of images.

Date of the defense of the Ph.D. : May 10th, 1995.

Granting institution : University of Paris XI .

Host institution (laboratory, team, etc.) for the preparation of the Ph.D. : INRIA Sophia-Antipolis, équipe ROBOTVIS.

Other diplomas (Master's and higher):

- Graduated from "École Normale Supérieure (ENS) Ulm" (1986–1990).
- "Agrégation" (teaching diploma) in mathematics (Computer Science option) – 1989.
- Magistère de Mathématiques Fondamentale Appliquée et Informatique (MMFAI), ENS Ulm (1986-1990).

2. Professional history

Current professional status

Position and statute: Chargé de recherche 1^{ère} classe

Institution (city -country): INRIA Sophia-Antipolis Méditerranée

Start: 1^{er} décembre 1999

Previous professional experiences

Start	End	Institutions	Positions and status
Sept. 1986	Sept. 1990	École normale supérieure - Paris	Élève fonctionnaire stagiaire
Oct. 1990	Sept. 1994 ¹	Rectorat de Nice	Assistant Moniteur Normalien
Dec. 1994	Feb. 1996	INRIA Sophia-Antipolis	Research assistant
Feb. 1996	Nov. 1996	COGNITECH Inc. (USA)	Research and development engineer
Dec. 1996	Nov. 1999	INRIA	Chargé de recherche 2 ^{ème} classe

3. Supervision of research activities

- Post doctoral supervision:
 - Jan Kybic, co-supervised at 20% with Maureen Clerc and Olivier Faugeras (2001–2003).
 - Christian Bénar, co-supervised at 50% with Maureen Clerc (2005–2006).
- 7 supervised or co-supervised Ph.D. theses:
 - Jérôme Piovano, supervised at 100%: “Image Segmentation and Level Set Method: Application to Anatomical Head Model Creation”(defended at University of Nice-Sophia Antipolis in January 2009).
 - Sylvain Vallaghé, co-supervised at 30% with Maureen Clerc: “EEG and MEG forward modeling: computation and calibration” (defended at University of Nice-Sophia Antipolis in December 2008).
 - Emmanuel Olivi, co-supervised at 50% with Maureen Clerc: “Coupling BEM and FEM approaches for the MEG/EEG forward problem” (started since 2008, University of Nice-Sophia Antipolis).
 - Alexandre Gramfort, co-supervised at 30% with Maureen Clerc and Olivier Faugeras: “Using MEG/EEG inverse problems for retinotopy” (defended in october 2009, Telecom ParisTech. Obtained the best Ph.D. thesis award by “Fondation d’entreprise EADS” in 2010).
 - Didier Bondyfalat, co-supervised at 30% with Olivier Faugeras and Bernard Mourain: “Interaction entre le symbolique et le numérique et son application à la vision artificielle” (defended at University of Nice-Sophia Antipolis in September 2000).
 - José Gomes, co-supervised at 20% with Olivier Faugeras: “Implicit representations of evolving manifolds in computer vision” (defended at University of Nice-Sophia Antipolis in 2001).
 - Cyril Gauclin, co-supervised at 20% with Olivier Faugeras on the aspects of coupling of the estimations of the fundamental matrix and the stereo matching.
- Master thesis or engineers:
 - Nicolas Servant, co-supervised at 50% with M. Clerc: Since December 2009, Nicolas has a young engineer position at INRIA. He works on developing a BCI system that

¹Interrupted for military obligations from aug. 1991 to july 1992.

will work in the immersive room of INRIA Sophia Antipolis. This work is part of the ADT Immersive BCI, which I coordinate.

- Laurent Caraffa: master thesis on EEG skull cap electrode localization and labelisation (March–August, 2010).
- Perrine Landreau, co-supervised at 50% with M. Clerc: Perrine obtained a young engineer position at INRIA and has worked for almost two years (2006–2007) on providing an interface module in Brainstorm/Vrainvisa to OpenMEEG.
- Nicolas Debeissat, after the leave of T. Viéville from the Odyssée project. Nicolas had a young engineer position at INRIA (2006–2007) and has worked on providing various tools for the integration work-package (WP8) of the Facets project. He worked notably on providing a web platform for storing and managing the Facets results and on XML descriptions for neural network simulations.
- Lionel Champalaune, a young engineer in 2002–2003 that has worked on a generic library for implementing levelsets.
- Emmanuel Olivi, co-supervised at 30% with M. Clerc and M. Yvinec during his period (summer 2007).
- Advisor for Frédéric Demoors master thesis entitled “Problème Inverse en EEG”, 2003.
- Advisor of Jacques Bride DEA Aravis (1998).
- Co-advisor (with O. Faugeras and A. Dervieux) of Guillaume Petitjean and (with O. Faugeras) of Gloria Haro-Ortega students from ENST in a long training period (5–6 months).
- Co-advisor (with O. Faugeras) of Robert Stahr (DEA Robotics and Vision trainee).
- Co-advisor (with R. Deriche) of Stéphane Rubino (DEA Robotics and Vision trainee).

4. Responsibilities

- Conference or journal reviews:
 - Reviewer for the most important journals in the field of computer vision and image processing: IJCV (International Journal of Computer Vision), IVJ (Image and Vision Computing Journal), PAMI (IEEE Transactions on Pattern Analysis and Machine Intelligence), JMIV (Journal of Mathematical Imaging and Vision), TGRS (IEEE Transactions on Geoscience and Remote Sensing), TS (Traitement du signal).
 - Reviewer for medical image processing journals: MMB (Mathematical Medicine & Biology), TMI (Transactions on Medical Imaging), SIIMS (SIAM Journal on Imaging Sciences).
 - Regular member of the program committee for various international conferences: ICCV (International Conference on Computer Vision), CVPR (Computer Vision and Pattern Recognition), ECCV (European Conference on Computer Vision), MICCAI (International Conference on Medical Image Computing and Computer Assisted Intervention).
 - Member of the program committee of the national conferences RFIA’04 and RFIA’10 (French conference on Shape Representation and Artificial Intelligence) and TAIMA’03 (Traitement et Analyse d’Images Méthodes et Applications).

- Organisation:
 - Area chair of ECCV’02 (European Conference on Computer Vision) and GRETSI (2009 and 2011) (Symposium on Signal and Image Processing).
 - Local arrangement chair for ICCV (International Conference on Computer Vision), Nice, 2003.
 - Industrial liaison for ECCV (European Conference on Computer Vision), Marseille, 2008.
 - Co-organisation of the two joint JAD–INRIA Sophia-Antipolis Méditerranée seminars “Maths et vivant”: Geometry and medical imaging (November 2007) and Dynamical systems, stochastic and non-linear aspects, and dynamics of populations (December 2007).
- Scientific expertise:
 - Expertise for an ISF (Israelian Science Foundation) proposal.
 - Expertise of an ANR Defis project proposal.
 - Participation to the technical selection committee for the purchase (2008) of the MEG machine in La Timone hospital in Marseille (third modern MEG machine installed in France acquired with INRIA’s financial help).
- Ph.D. reviewing and jurys:
 - Jury member for the Ph.D. thesis of A. J. Salgado de la Nuez entitled "Optical Flow and Disparity Maps Estimation using Variational Methods" (defended at University of Las Palmas in July 2010).
 - Reviewer and jury member for the Ph.D. thesis of Mathieu Salzmann entitled “Learning and recovering 3D surface deformations” (defended at EPFL in December 2008).
 - Jury member of the Ph.D. thesis of José Gomes entitled “Implicit representations of evolving manifolds in computer vision” (defended at University of Nice-Sophia Antipolis in 2001).
- Scientific expertise for INRIA:
 - Jury member for the INRIA Sophia Antipolis-Nice University young researcher chair in 2009.
 - Member of the scientific prospective workgroup for the INRIA prospective committee (COST) (2005-2007).
 - Evaluation of associate teams proposals (2007 and 2008).
 - Evaluation of postdoctoral applications for INRIA.
- Workgroups and committees:
 - Member of the specialists committee in section 27 (computer science) at University of Nice-Sophia-Antipolis (1998–2004)².

²This committee is in charge among other things of the recruitment of the associate professors.

- Member of the INRIA national workgroups for software evaluation and “poste d’accueil”.
- Member of the INRIA Sophia-Antipolis Méditerranée site committee (1998-2004), formation committee (1998–2004), computer users committee (2000-2004) and software development committee (2003–2009).
- Member of several local work groups on several topics such as contractual relations between researchers and the system engineer service, energy economies or disk space usage and rationalisation for the site of Sophia-Antipolis. I was in charge of three such groups.
- Other collective tasks:
 - Technical contact for the ROBOTVIS, ODYSSEE and ATHENA (project) teams since 1998. This involves the management and purchase of the computers for the whole group, a first level of technical help with computers to the group members, the local computer related decisions (disk usage and cleaning, installed tools, etc.).
 - Main contact for students for all the technical aspects of software development.

5. Management

Details on the projects are given in next section.

- Responsible for ATHENA of the MULTIMODEL ANR grant. This grant which will last 3 years involves INRIA and INSERM teams. It aims at studying the relationships between multiple modalities of functional brain imaging. The goal is to build a common model that would encompass both electrical and some metabolic signals and to confront such models to human or animal real data. ATHENA is involved in both the modelisation and the confrontation with real data tasks.
- Responsible for ATHENA of the ADT grant "MedInria-NT" aiming at integrating with the platform MedINRIA tools elaborated over the years in the ATHENA project-team and its partners.
- Responsible the ADT grant "Immersive BCI" which aims at creating within 2 years (2009–2011) a BCI system embedded with an immersive room.
- Workpackage manager of European IP project Facets WP8 (integration of results), since 2008. This involves about 4 active teams within the project. This will end in 2010.
- Workpackage manager (WP3) of Cumuli Esprit LTR project (1996–1999) on using geometrical reasoning for 3D reconstruction. This workpackage involved three of the five teams in the consortium.
- Coordinator for the telemedicine ACI grant “Dir-Inv” (2001–2004): the goal of that grant was the study of the direct and inverse problems in Electro- and Magneto-Encephalography. Partners were Cermics in Marne-la-Vallée (ENPC), "La Timone" hospital in Marseille and the Technologic University in Compiègne. INRIA’s participants were the projects Estime, Gamma, Ondes and Odyssee.

6. Collaborations, visits

Mobility:

- **Until 2001:** my contributions were mainly focussed in the fields of computer vision and image processing within the ROBOTVIS project team. My contributions together with those of all the other researchers of the team has led to the creation of the company REALVIZ of which I am one of the co-founder. The sotware libraries developed within ROBOTVIS were transferred to REALVIZ
- **Since 2002:** following the creation of the ODYSSEE project team, my interests have shifted to the new research domain of computational functional brain imaging, with a particular emphasis on the modeling of the electrical activity of the brain using MEG or EEG measurements.

Collaborations:

European projects:

- Participation to the Facets project (2006–2010). The goal of the Fast Analog Computing with Emergent Transient States (Facets) project was to create a theoretical and experimental foundation for the realisation of novel computing paradigms which exploit the concepts experimentally observed in biological nervous systems. The project involved interaction and scientific exchange between biological experiments, computer modeling and hardware emulations. After the leave of Thierry Viéville from the ODYSSEE project in 2007, I became the WP8 “integration of results” work-package leader.
- Participation in the Mapawamo project (2000–2003). This project aimed at improving the processing of fMRI data, at developing new techniques to generate maps of cortical activity, at studying the functional connectivity between active cortical areas, and at comparing visual perception in man and non-human primates. The main partners were the Leuven medical school (coordinator), the Technical University of Denmark (Lyngby), and INRIA (ROBOTVIS and EPIDAURE groups). I notably participated in the contract proposal and to the transfer to KU-Leuven of the BrainMatcher software (<https://gforge.inria.fr/projects/brain-matcher/>). The BrainMatcher package provides tool to match and warp a 3D image to a 3D target template using a non rigid deformation. It is being used in fMRI studies as a supplementary tool for SPM to better analyse the results.
- Participation to the CogViSys IST project (2001–2004). Its central aim was to build a vision system that is re-usable by introducing self-adaptation at the level of perception and by making explicit the knowledge base at the level of reasoning, and thereby enabling the knowledge base to be changed. The partners were the universities of Oxford, Leuven, Karlsruhe and Freiburg, and the Swiss Federal Institute of Technology in Zurich.
- Participation to the Improofs project “IMage PROcessing Operations for Forensic Support” (1997–2000). The partners were the Katholieke Universiteit Leuven, the Robotics Research Group of Oxford university, the Royal Institute of Technology, KTH, the Nationaal Instituut voor Criminalistiek & Criminologie and the Forensic Science Services Metropolitan Laboratory. The goal of the project was the use of image analysis techniques for forensic studies.

- Participation to the Cumuli Esprit LTR project (1996–1999). The goal of the contract was the understanding of the geometry of multiple images. I was the work package leader of the WP3 task dealing with the use of geometrical reasoning for 3D reconstruction. The partners were the INRIA MOVI project (coordinator), the LUND university, the Fraunhofer-IGD and the companies Innovativ Vision and Imetric.
- Before 1996, participation in the projects ESPRIT-BRA/INSIGHT 1 and 2 (Comparison of artificial and biological vision systems) and VIVA (Use of invariants in computer vision).

National grants:

- Participating (and coordinating for INRIA) in the ANR grant MULTIMODEL (2010–2013) which aims at developing joint electrophysiological and metabolic models of the brain with the aim of better understanding the relations between various modalities of acquisition of the brain activity (MEG, EEG, fMRI, ...). I'm the leader of the data analysis workpackage.
- Participating in the ANR grant co-adapt (2009–2013). The aim of CO-ADAPT is to propose new directions for BCI design, by modeling explicitly the co-adaptation taking place between the user and the system.
- Participating in ANR Vimagine “Multimodal Neuroimaging of Rapid Brain Processes in the Human Visual System” (2008–2011). The main goal of this project is to use MEG to map the cortical areas corresponding to the retina (retinotopy). The other partners are the LENA (CHU Pitié-Salpêtrière), and the Pariétal project-team at INRIA Futurs and Neurospin-Saclay.
- Participating in the EADS grant “Multi-scale investigation of the operating brain with an eye on visual perception” (2007–2009). I notably wrote a large part of the proposal.
- Coordinator for the telemedicine ACI grant “Dir-Inv” (2001–2004): the goal of that grant was the study of the direct and inverse problems in Electro- and Magneto-Encephalography. The partners were Cermics in Marne-la-Vallée (ENPC), the "La Timone" hospital in Marseille and the Technologic University in Compiègne. INRIA's participants are the projects Estime, Gamma, Ondes and Odyssée.
- Participant in the ACI grant “Obs-Cerv” (2003–2006). Its main purpose was to make progresses toward a virtual meta-sensor combining the advantages of the various non-invasive sources of information about the brain activity. This involves manipulating and linking the information provided by some very large heterogenous data sets such as MEG and EEG or various types of MRI images.
- Participant in the ACI RIVAGe pre-project (2003–2004) on the topic of feedback during visual integration.
- Participant in the Priamm project Rotoscoto (2000–2002) on the problem of rotoscoping which is a widely used technique for postproduction applications. Other partners were the companies REALVIZ and DUBOI.

Regular collaborations with:

- Sylvain Baillet, formerly at the LENA laboratory in the hospital of “la Pitié-Salpêtrière” and now in the department of Neurology in the medical college of Wisconsin. This collaboration happens with the framework of the ANR ViMAGINE (see below) through the Ph.D. work of Alexandre Gramfort [43].
- Christian Bénar and Jean-Michel Badier of the hospital of “La Timone” in Marseille: there is a very close relationship with this group on various topics. We are jointly participating in a series of actions such as methodological improvements on MEG/EEG [], buying a MEG machine for Marseille hospital and bringing the new methods developed either in ODYSSEÉ or in our partnership within the hands of clinical technicians and physicians through graphical user interfaces. This collaboration also involved Denis Schwartz of the hospital of “la Pitié-Salpêtrière”. It also introduced us to many Marseille’s teams dealing with the study of the brain through the work-group (gt-signal-meeg), in which we collaborate with researchers such as Bruno Torr  sani or Boris Burl  .
- Thomas Kn  sche and Alfred Anwander from the Max Planck Institute for Human Cognitive and Brain Sciences, Leipzig. This collaboration was initiated in the framework of the Procope project “Multimodal functional imaging of the Brain” (2006–2007). This is an on-going collaboration, which we look at strengthening as the two teams have the same interests in MEG/EEG and DT-MRI. The yet unpublished last part of the Ph.D. work of J  r  me Piovanio [Piovanio, 2009] stems from this collaboration. This project also initiated the contact with Carsten Wolters, which shares many of our interests in the MEG/EEG forward model, notably with FEM techniques.
- Jan Kybic (Center for Machine Perception, at Czech Technical University, Prague). Jan was a former post-doc in the ODYSSE   project. This collaboration has led to numerous publications [32, 33, 34, 31, 10, 9, 30, 28, 8, 23]. Most notably, the symmetrical BEM theory and software (OpenMEEG) has been jointly developed with him. The collaboration continued after his post doc with the framework of a Barrande project: Multimodal functional imaging of the Brain (2006–2007).
- Luis Alvarez and Javier S  nchez (Mathematical Image Analysis group, Las-Palmas University) within the framework of PAI PICASSO (2001–2002 and 2004–2005). The goal was to develop algorithms for use in image analysis based on variational frameworks. This collaboration has led to the publications [35, 7].

Stays in other laboratories:

- In 2004, I was invited by Jean Ponce to spend one month at University of Urbana-Champaign (Beckman Institute). This collaboration has led to the joint publication [27].
- I have spent several stays (usually one week long) at University of Las Palmas (Spain). This collaboration has led to the joint publications [35, 7].
- In 1999, I spent two weeks in Montevideo, Uruguay to teach computer vision course.
- In 1998, I spent two weeks in Siemens SCR, Princeton.

[Piovanio, 2009] Piovanio, J. (2009). *Image Segmentation and Level Set Method: Application to Anatomical Head Model Creation*. PhD thesis, University of Nice-Sophia Antipolis.

- From March to November 1996, I was a Research and development engineer at COGNITECH Inc., Santa Monica, CA, USA doing photogrammetry for forensic applications, co-founded by Leonid Rudin and Stan Osher.

Participations in INRIA grants:

- Responsible for ATHENA of the ADT grant "MedInria-NT" aiming at integrating with the platform MedINRIA tools elaborated over the years in the ATHENA project-team and its partners. This will include both MEG/EEG tools (totally missing in MedINRIA currently) as well as dMRI tools.
- Responsible for the ADT grant "Immersive BCI", which aims at creating a BCI system using the technologies developed in the ATHENA team and to be used in the immersive room recently installed at Sophia-Antipolis. This platform will serve 1) as a validating platform for the research made in ATHENA and 2) as a tool that can be used to evaluate the subject reactions in the immersive environment. One hope is also that this platform will foster some collaborations with other groups using the immersive room at Sophia-Antipolis.
- Medmesh color (2006–2007): the goal of the project was to test the mesh generator developed in Geometrica on real meshing problems. The project participants were Geometrica, Asclépios, Caiman and Odyssée from INRIA Sophia Antipolis and the Neurophysiology and Neuropsychology Laboratory of the hospital La Timone in Marseille. The comparison [Olivi, 2007] of symmetric BEM, traditional FEM and implicit FEM using the most current GEOMETRICA meshing techniques was achieved within this framework.
- “EEG++” color project (2005–2006): the goal of this project was to trigger the collaboration between Odyssée and “La Timone” hospital in Marseille on the subject of conductivity estimation for electro-encephalography.
- MC2 ARC (Cooperative Research Action) (2000–2002). The goal of this grant was a better integration between techniques of magnetic resonance imagery with electro- and magneto-encephalography.
- LSF ARC (Cooperative Research Action) (2001–2002). The objective was the conception of a computer system dedicated to the capture, the recognition and interpretation of the French sign language. The partners were the LIMSI laboratory, and the INT Evry.
- 3D-MEG ARC (Cooperative Research Action) 3D-MEG (1999). The action was the joint effort with the INRIA Gamma, Ondes et Estime teams, which in the end led to all our MEG and EEG work.

7. Teaching

Hours indicate my own involvement in the course, not the total duration of the course (for shared courses).

- **University of Nice-Sophia-Antipolis:**

[Olivi, 2007] Olivi, E. (2007). Comparaisons quantitatives de résultats de problème direct eeg. Research Report 6364, INRIA.

- 3D computer Vision course at Master level in University of Nice-Sophia Antipolis ³ (1998-2010 except 2007) (about 15h per year).
 - Inverse problems in functional brain imaging course in the Master CompBio (Computational Medicine and Biology) since 2009. This course is given jointly with Maureen Clerc (8h).
 - C++ in DESS of Biomedical Engineering, 2003–2004 (about 15h per year).
 - Advanced Introduction to Linux course for the first year Computer Science students preparing a Ph.D., 2005–2006 (two days each year).
 - Practical classes of system programming in Unix (IUT second year), 2000–2001 (32h each year).
 - Maple course in ESSI 2 (École Supérieure en Sciences Informatiques) from 1991 to 1994.
 - Differential geometry in ESSI 2 (École Supérieure en Sciences Informatiques), 1994.
 - Practical classes for the foundation course of the DEA of Robotics and Vision, from 1990 to 1992.
 - Unix initiation lecture in the master of mathematics and computer science in 1990-1991.
- **École Normale Supérieure de Cachan:**
 - 3D computer Vision course in the MVA (Mathématiques, Vision et Apprentissage) Master from 2000 to 2005 (24h per year).
 - Inverse problems in functional brain imaging course in the MVA (Mathématiques, Vision et Apprentissage) Master since 2005. This course is given jointly with Maureen Clerc and Bertrand Thirion (about 10h per year).
 - **École Polytechnique:** Practical classes in computer vision in 1994 and 1995 (18h per year).
 - **Republica University, Montevideo, Uruguay:** computer vision course, June 1999 (30h).
 - **Unesco chair in Tunis :** Inverse problems in functional brain imaging course, February 2007 (15h).

8. Dissemination of scientific knowledge

- Review article on variational, geometric, and statistical methods for modeling brain anatomy and function: [11].
- Article in the wide-audience magazine “La Recherche”: [13].
- Lecture notes for the course “Inverse problems in functional brain imaging” [49].

³From 1998 to 2010, the formation name has changed several times. This course was integrated in several master programs and is now part of a last year engineering program at master level.

SUMMARY OF PAST ACTIVITY

Computer vision was my first scientific field. It aims at developing tools for recovering informations about a scene from one or more images of it. This activity was developed within the ROBOTVIS project team (1990–2001) and has culminated in 1998 with the creation of the company REALVIZ (now part of AutoDesk) of which I was one of the co-founders. After 2001, my research themes have changed radically to the new topics of analysis of the electrical sources of the brain using electro- or magneto-encephalography (EEG or MEG). This new activity has required some time and energy to create new expertise and new collaborations.

1. Computer vision activities [PhD:1, B:1, J:3, C:14, BC:2]⁴

Estimation of low-level informations from images

Extracting useful 2D informations from images is a crucial and difficult step in Computer Vision, given the lack of good mathematical models for images. [58] describes a method to obtain the differential properties up to the second order in space and time of a 2D deforming contour extracted from a sequence of images. [7, 35] proposes a PDE method for establishing point correspondences between a pair of stereo images exploiting their one-to-one properties. Occlusions are detected as places where this property is violated. [39, 48] explores the possibility of a joint estimation of stereo and viewing constraints. Finally, [17, 18, 19] explore the problem of segmenting images into regions using local statistics of the image, while still maintaining a coherent global result (which is difficult with local decisions).

Motion estimation of 3D rigid curves

The goal of my Ph.D. [50] (and [51]) was to estimate the 3D motion parameters and the shape of a moving 3D rigid curve given a sequence of monocular images. While the basic theory is established in the papers [52, 54, 59], [57] proposes an implementation of the method for planar curves. [55] then generalizes this implementation to general 3D curves, which require more complex equations and higher order derivatives to be computed in the images. [58, 60, 56] propose variants for the computation of the derivatives.

Camera self calibration

Recovering cameras parameters (position, orientation and a set of “optical” parameters) is the topic of camera calibration. The so-called fundamental matrix, which formalizes the viewing constraints for two cameras, is the basis of calibration algorithms. [53, 61] deals with the estimation of fundamental matrix. [38] incorporates covariance information in the process of inferring camera parameters from fundamental matrices. This involves the Jacobian of the Singular Value Decomposition of a matrix, for which an algorithm is proposed. A similar methodology is also used in [47] to match images which pixels are tensors obtained from diffusion MRI (dt-MRI). Trifocal tensor (highly non-linear) is the equivalent of fundamental matrix for three views. [42, 41] explore the constraints on tensor coefficients and their usage during trifocal tensor estimation from point matches between three images. This work also formalizes these objects with the general framework of Grassmann-Cayley algebra. Standard self-calibration techniques use only the viewing constraints of the – rigid – scene. The works [36, 40] explore the possibility of using scene constraints and geometric reasoning for 3D reconstruction.

⁴PhD: Ph.D thesis, B: books, J: journal articles, C: articles in peer-reviewed international conferences with proceedings, BC: book chapters

2. MEG/EEG activities [J:10, C:20] ⁴

Extracting and modeling events of interest

MEG/EEG generate a huge quantity of data (hundreds of simultaneous channels recorded at about 1000Hz for several minutes, using multiples trials to improve the signal-to-noise ratio). Only a subset of those data corresponding to “events of interest” is analyzed in practise. [45, 46] attempts to design a general method for modeling and tracking MEG/EEG events, applicable for both low and high frequencies and taking into account the spatial structure (topography) of the events. Events are modeled as sums of Kronecker products of possible time courses by topographical maps in the sensor space. It account for 1) spatial structure of the data via multivariate decomposition, 2) time-frequency structure via atomic decomposition. [26] adds to the framework a non-linear fitting procedure of events in order to better characterize variability across trials. [6] introduces the Consensus Matching Pursuit which aims at extracting events appearing across several trials without relying on averaging which often hides high frequency or jittered events.

The MEG/EEG forward problem

The MEG/EEG forward problem simulates electro-magnetic propagation over the head. It allows, given a head model, the computation of the potential and magnetic fields arising from a set of cortical sources, an essential step for the more interesting problem of localizing sources given measurements. In [34], two implementations of the forward problem are compared: the surfacic Boundary Element Method (BEM) and the volumic Finite Element Method (FEM). It is shown the surprising result that FEM is both more accurate and faster than classic BEM, while also being more general. This led to the development of the symmetrical BEM [9, 10, 31, 32, 8] that gives results better than those obtained with FEM. This has led to the development of the OpenMEEG software [24, 16, 3]. FEM still has the advantage of generality as it can deal with anisotropic conductivities. Its main drawback being the complexity of obtaining 3D meshes for the head (there are some very thin head structures). The Implicit FEM method [5, 22, 44] directly constructs, from MRI segmentations and conductivity descriptions, the linear systems associated to the FEM method, and totally avoids the tedious and complicated step of building a tetrahedral mesh of the head. [5, 14] generalizes the use of the adjoint state approach (introduced in [Faugeras et al., 1999]) within this framework.

Geometrical and physical modeling of the head

The forward model is based on head physical models. The major physical parameter driving the electro-magnetic propagation is conductivity, which depends on head tissue. The main tissues contributing to this propagation are scalp, skull, cerebro-spinal fluid, and the grey and white matters constituting the brain. Anatomical MRIs provide many information about their geometrical structure, except for the skull which is mostly invisible. [18] explores the joint use of T1- and T2- anatomical MR images for recovering full geometrical models of the head including the skull. Finding the actual conductivities corresponding to tissues is another problem. [28, 30] studies the use of electrical impedance tomography to measure the conductivity ratio between the scalp and the skull (instead of using some “standard” values which are subject to quite a discrepancy in the litterature).

[Faugeras et al., 1999] Faugeras, O., Clément, F., Deriche, R., Keriven, R., Papadopoulos, T., Roberts, J., Viéville, T., Devernay, F., Gomes, J., Hermosillo, G., Kornprobst, P., and Lingrand, D. (1999). The inverse eeg and meg problems: The adjoint space approach i: The continuous case. Technical Report 3673, INRIA.

Localisation of cortical electrical sources from MEG/EEG measurements

The inverse problem compares actual measurements with simulations obtained by the forward problem to recover putative sources of the brain activity. As this is an ill-posed problem, regularization is required. In [33, 37] levelset methods are used for regularizing source distributions. A completely different method is developed in collaboration with the APICS project team. Using their rational approximation methods, [29] localizes simultaneously multiple dipoles in “spherical heads” (a model still quite used clinically). Standard discrete dipole methods usually find dipoles one at a time, which is a known source of mis-localisations. The work [43, 4] uses graph-cut based techniques to recover coherent activations during time intervals. This gives a more dynamical vision of the brain activity.

Electrode labeling:

For EEG measurements, a practical problem is finding electrode positions on the head (needed for the inverse problem). Traditional setups require a manual acquisition of each electrode, which takes time and is error prone. A computer vision system could acquire electrode positions simply using images. [20, 21] (joint work with hospital “La Timone” in Marseille) solves one important task for such a system: automatically finding the electrode names associated to 3D electrode positions given a set of two or three hand labelled positions and an approximate model of the EEG cap.

COMPLETE LIST OF CONTRIBUTIONS

<http://www-sop.inria.fr/members/Theodore.Papadopoulos/Publications/>

1. Publications

Books

- [1] O. FAUGERAS, Q. LUONG, T. PAPADOPOULOU, *The Geometry of Multiple Images*, MIT Press, 2001.

International journals

- [2] S. VALLAGHÉ, T. PAPADOPOULOU, « A Trilinear Immersed Finite Element Method for Solving the Electroencephalography Forward Problem », *SIAM Journal on Scientific Computing* 32, 4, 2010, p. 2379–2394, <http://link.aip.org/link/?SCE/32/2379/1>.
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- [10] J. KYBIC, M. CLERC, T. ABOUD, O. FAUGERAS, R. KERIVEN, T. PAPADOPOULOU, « A Common Formalism for the Integral Formulations of the Forward EEG Problem », *IEEE Transactions on Medical Imaging* 24, janvier 2005, p. 12–28, <ftp://ftp-sop.inria.fr/odyssee/Publications/2005/kybic-clerc-etal:05.pdf>.
- [11] O. FAUGERAS, G. ADDE, G. CHARPIAT, C. CHEFD'HOTEL, M. CLERC, T. DENEUX, R. DERICHE, G. HERMOSILLO, R. KERIVEN, P. KORNPORST, J. KYBIC, C. LENGLET, L. LOPEZ-PEREZ, T. PAPADOPOULOU, J.-P. PONS, F. SÉGONNE, B. THIRION, D. TSCHUMPERLÉ, T. VIÉVILLE, N. WOTAWA, « Variational, geometric, and statistical methods for modeling brain

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National journals

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