

Indecomposability in field theory and applications to disordered systems and geometrical problems

Romain Vasseur

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Examineurs : D. Bernard (ENS), N. Read (Yale) & J.-B. Zuber (UPMC)

PhD Defense, Sept. 27, 2013



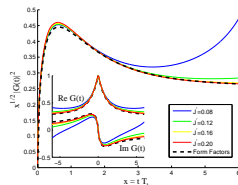
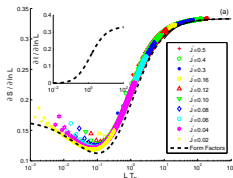
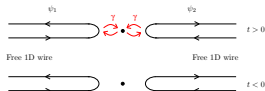
This talk : Logarithmic Conformal Field Theory

Collaborators : A.M. Gainutdinov, J. Jacobsen, N. Read & H. Saleur.

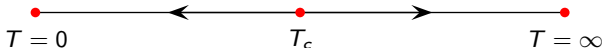
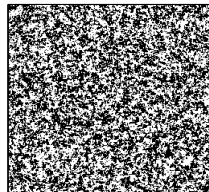
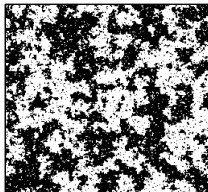
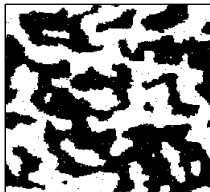
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Entanglement and quenches in quantum impurity systems

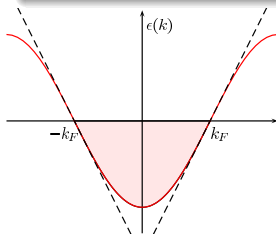
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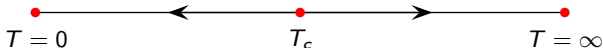
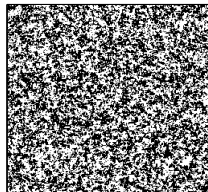
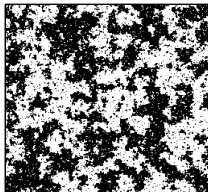


Indecomposability, non-unitarity and logarithms in critical disordered/geometrical systems

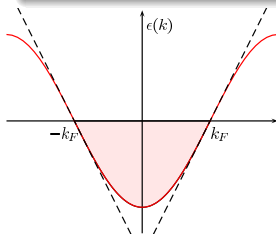


- 2D critical statistical systems
 - gapless spin chains $\epsilon(k) \sim v_F k$
- $\} \implies \text{Conformal Field Theory}$

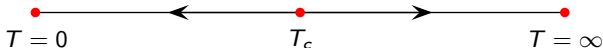
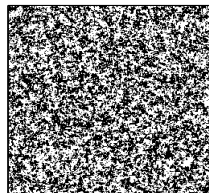
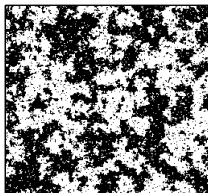




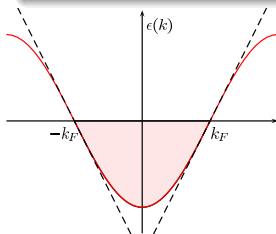
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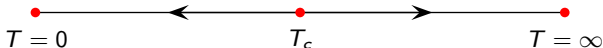
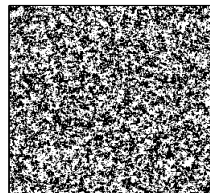
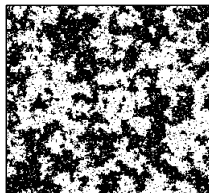
- Conformal symmetry \Rightarrow Virasoro algebra [BPZ '84]



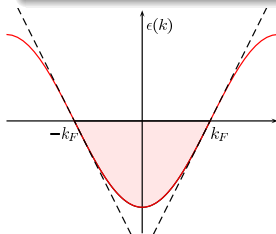
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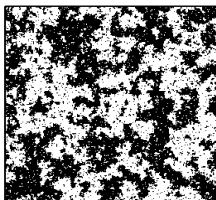
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- Irreducible representations \Rightarrow Exact results : critical exponents, correlation functions, *etc* for stat phys/cond mat systems.



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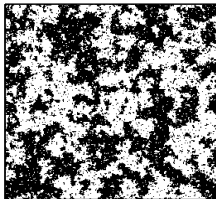


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- Irreducible representations \Rightarrow Exact results : critical exponents, correlation functions, *etc* for stat phys/cond mat systems.
- Algebraic decay of two-point correlation functions $\langle \phi(r_1) \phi(r_2) \rangle \sim |r_1 - r_2|^{-2\Delta}$



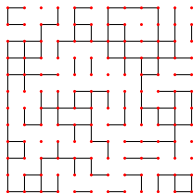
2D Ising Model

Quantum spin chain in 1+1D : $H = - \sum_i (\sigma_i^x \sigma_{i+1}^x + \mu \sigma_i^z)$
 H hermitian, Field Theory (Majorana Fermion) unitary



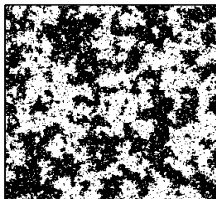
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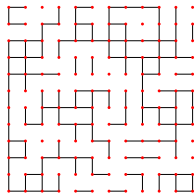
Percolation

Quantum version in 1+1D : $H \neq H^\dagger$
 Field theory **non-unitary** : $\langle \phi(r) \phi(0) \rangle$ may be negative



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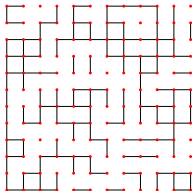
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⇒ 'in general' we expect logarithms in correlation functions

$$\langle \psi(r) \psi(0) \rangle = \frac{\text{Cst} - 2b \log r}{r^{2\Delta}}$$

⇒ **Logarithmic** Conformal Field Theories [Rozansky & Saleur '92, Gurarie '93]



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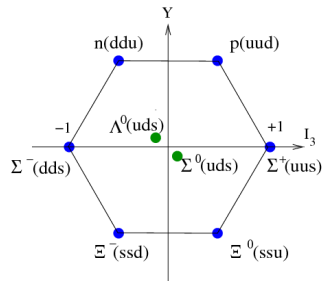
\Rightarrow **Logarithmic** Conformal Field Theories [Rozansky & Saleur '92, Gurarie '93]

Non-diagonalizability of the scale transformations generator = **Indecomposability**

Ordinary CFT	Log CFT
$\phi(\Lambda r) = \Lambda^{-\Delta} \phi(r)$	$\begin{pmatrix} \psi(\Lambda r) \\ \phi(\Lambda r) \end{pmatrix} = \Lambda^{-\begin{pmatrix} \Delta & 1 \\ 0 & \Delta \end{pmatrix}} \begin{pmatrix} \psi(r) \\ \phi(r) \end{pmatrix}$

- Symmetry is crucial in modern theoretical/quantum physics
→ representation theory [Wigner, Weyl, ...]
- Usually, physicists need to worry about irreducible representations only

What is indecomposability?

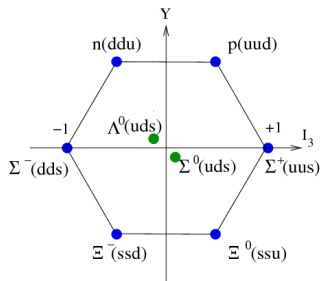


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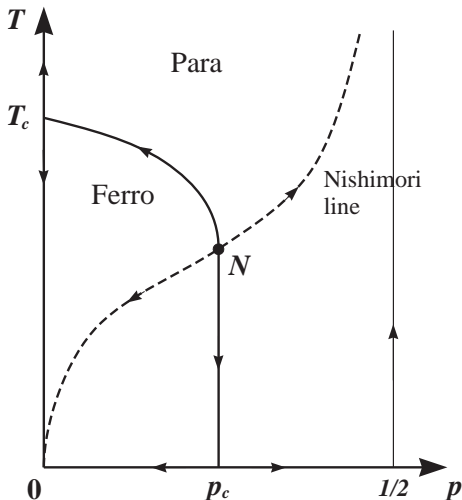
Mathematical concept of representation theory :

Indecomposable, yet reducible representation $V \Leftrightarrow \exists$ subrepresentation $V_1 \subset V$, but V cannot be decomposed into smaller representations ($V \neq V_1 \oplus V_2$)



Disordered fixed points are described by $c = 0$ (L)CFTs [Gurarie 93', Caux, Kogan & Tsvetik 96', Cardy 99', Gurarie & Ludwig 02', ...]

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Random-bond Ising model

$$\beta H = - \sum_{\langle i,j \rangle} K_{ij} S_i S_j$$

$$K_{ij} = \begin{cases} K & \text{proba } 1 - p \\ -K & \text{proba } p \end{cases}$$

SUSY formulation [Gruzberg, Read & Ludwig 01']

At the fixed point [Cardy 01'] :

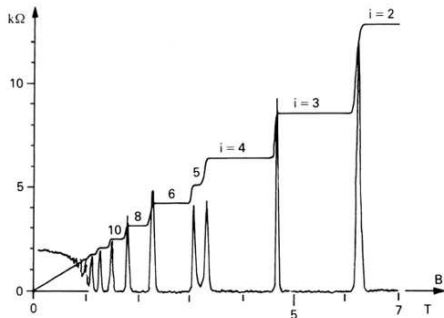
$$\frac{\overline{\langle \epsilon(r) \rangle \langle \epsilon(0) \rangle}}{\langle \epsilon(r) \epsilon(0) \rangle - \langle \epsilon(r) \rangle \langle \epsilon(0) \rangle} \sim \log r$$

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Integer Quantum Hall Effect

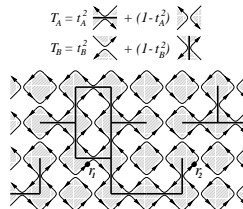
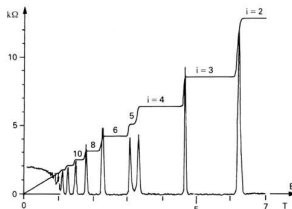
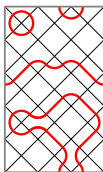
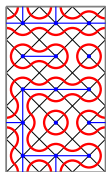
[von Klitzing et al 85']

- Sigma model formulation [Pruisken, Weidenmuller, Zirnbauer] on coset $U(2n)/U(n) \times U(n)$, $n \rightarrow 0$ (or SUSY equivalent).
- Plateaux transition $\equiv c = 0$ LCFT



Logarithmic CFTs

- Condensed Matter applications :
 - Geometrical properties of statistical models, geometrical models (percolation, SAW, loop gas ...)
 - Statistical systems with quenched disorder (e.g. Random-bond Ising model)
 - Integer Quantum Hall Effect transition, Spin Quantum Hall Effect (\sim percolation [Gruzberg, Ludwig & Read 99'])
 - ...

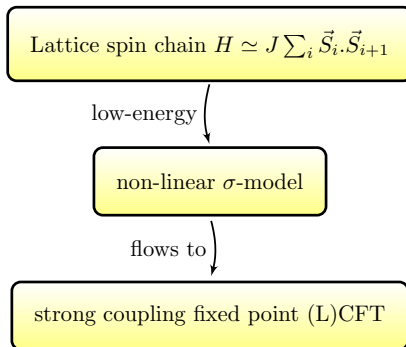


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 - ...
- High energy/String Theory applications :
 - AdS/CFT correspondence
 - σ -model and WZW models on supergroups

Questions

- Main features of LCFTs ? Where are the logs in stat phys/cond mat systems ?
What kind of physical observables are they related to ?
⇒ **geometrical systems** (Section 2)
- How to tackle these complicated Quantum Field Theories ? Classification ?
⇒ **algebraic lattice approach** (Section 3)



Logarithmic correlations in critical geometrical models and 'replica' approach

$c \rightarrow 0$ catastrophe and b number [Cardy '01, Gurarie & Ludwig '02, ...]

- Let $\Phi_{\Delta}(r)$ be a spinless field. Conformal invariance enforces

$$\Phi_{\Delta}(r)\Phi_{\Delta}(0) \simeq \frac{a_{\Phi}}{r^{2\Delta}} \left(1 + \frac{\Delta}{c} z^2 T(0) + \dots \right)$$

Limit $c \rightarrow 0$?

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- Correlation functions

$$\langle T(z)T(0) \rangle = 0$$

$$\langle T(z)t(0) \rangle = \frac{\textcolor{red}{b}}{z^4}$$

$$\langle t(z)t(0) \rangle = \frac{\theta - 2\textcolor{red}{b} \log z}{z^4}$$

Pattern completely general for other values of c [VJS '11, GV '12]

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 \Rightarrow Coincidence of exponents as $n \rightarrow 0$

$$\frac{1}{n} \left(r^{-2\Delta_1(n)} - r^{-2\Delta_2(n)} \right) \sim 2 \left. \frac{d(\Delta_2 - \Delta_1)}{dn} \right|_{n=0} r^{-2\Delta_1(0)} \log r$$

Minus sign \leftrightarrow non-unitarity

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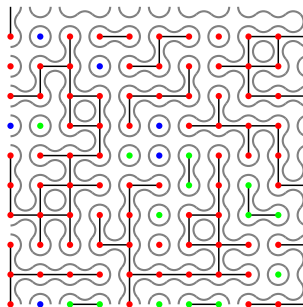
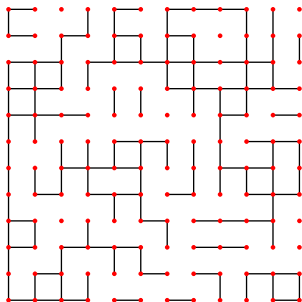
Appealing idea [Cardy '01, Gurarie & Ludwig '02, ..., VJS '11, GV '12, ...] :

log CFTs as limits of ordinary CFTs : 'replica' approach

Let's apply this to geometrical systems

- Geometrical problems such as percolation or Self Avoiding Walks (Polymers, $O(n \rightarrow 0)$ model) are described by logarithmic CFTs
- ...yet many aspects are of course well understood
- What makes the continuum theory logarithmic? What kind of physical observables are logarithms related to?

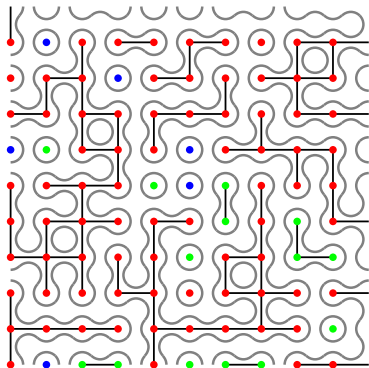
[Cardy '99 (disordered systems, polymers), VJS '12 (Potts model, percolation), VJS in preparation]



Logarithms in the Potts model

- Q -state Potts model (S_Q symmetry)

$$Z = \sum_{\sigma} e^{K \sum_{\langle ij \rangle} \delta_{\sigma_i, \sigma_j}}$$

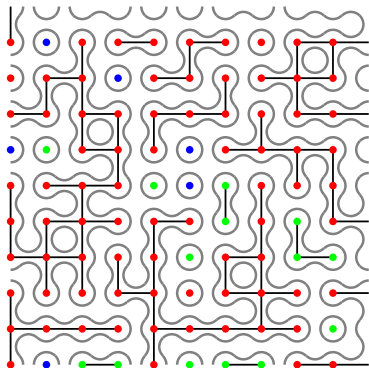


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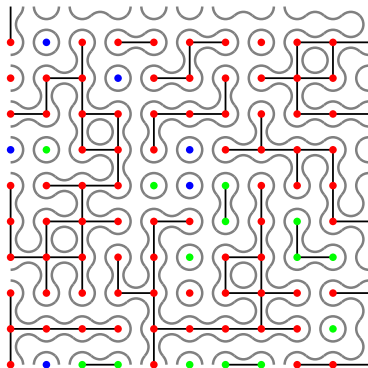


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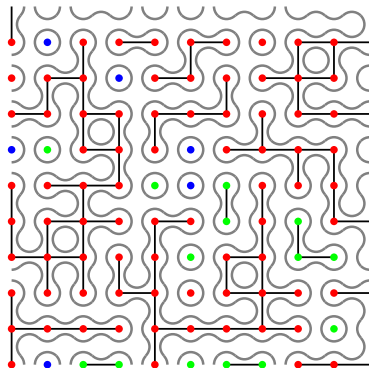


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- Classification of the operators with respect to S_Q symmetry



Use S_Q symmetry to infer the form of two-point correlators

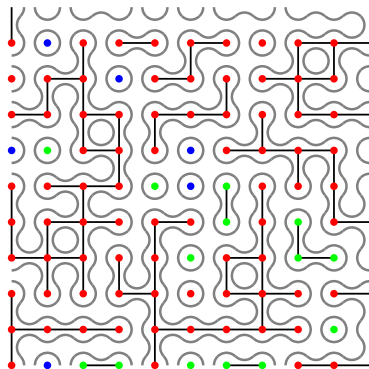
$$\langle \psi_{ab}(r) \psi_{cd}(0) \rangle = A(Q) \left(\delta_{ac} \delta_{bd} + \delta_{ad} \delta_{bc} - \frac{1}{Q-2} (\delta_{ac} + \delta_{ad} + \delta_{bc} + \delta_{bd}) + \frac{2}{(Q-1)(Q-2)} \right) \times \underbrace{r^{-2\Delta(Q)}}_{\text{CFT part}},$$

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- Similar to $O(n=0)$ model for polymers
- Classification of the operators with respect to S_Q symmetry

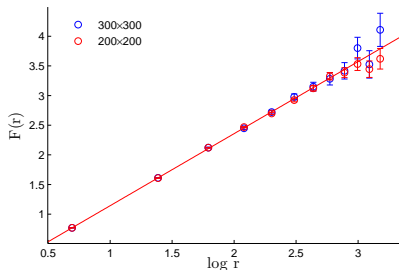


Linear combinations of correlators to cancel the divergences

$$\frac{1}{Q-1} \left(r^{-2\Delta_1(Q)} - r^{-2\Delta_2(Q)} \right) \sim 2 \frac{d(\Delta_2 - \Delta_1)}{dQ} \Big|_{Q=1} r^{-2\Delta_1(Q=1)} \log r$$

Similar ideas in [Cardy 99', Gurarie & Ludwig '02, ...]

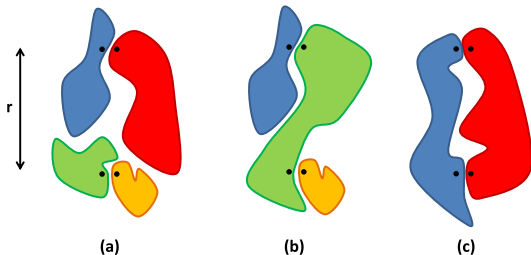
Logs in percolation [VJS '12]

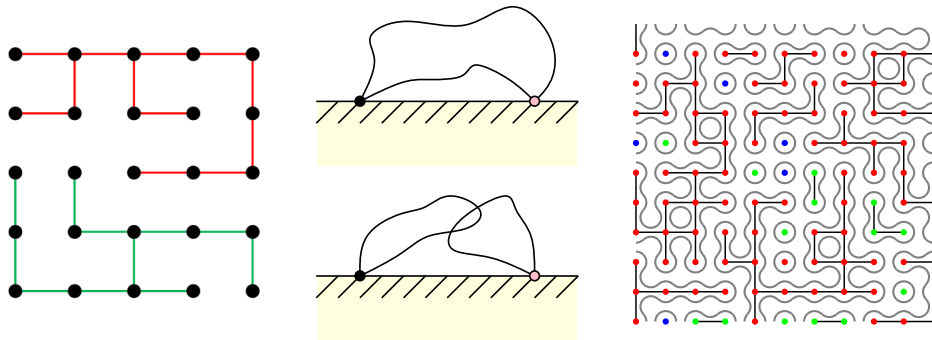


- Logarithmic correlation functions have a clear geometrical interpretation

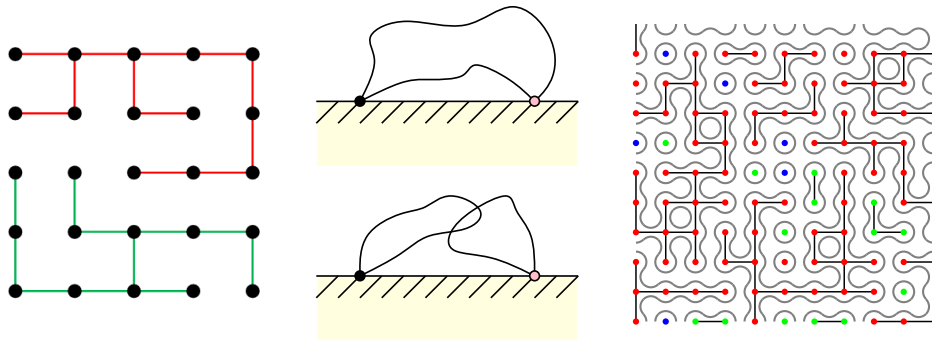
$$F(r) \sim \frac{2\sqrt{3}}{\pi} \log r$$

- → Logarithmic behavior of geometrical probabilities

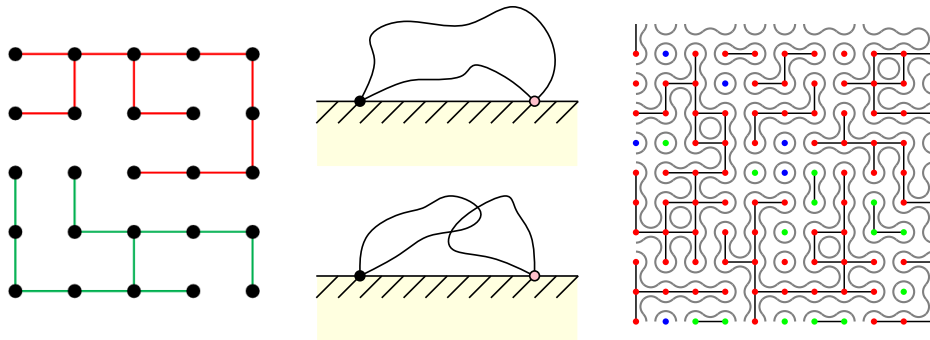




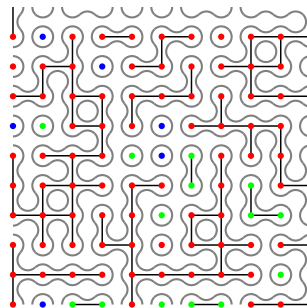
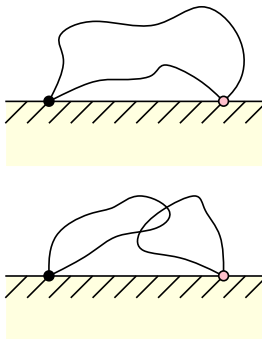
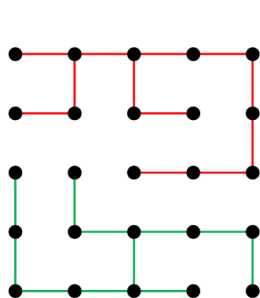
- Family of 2-point and 3-point functions with logarithms in many models [VJS in preparation] : Percolation, spanning trees, resistor networks, Ising, dilute and dense phases of polymers, ...



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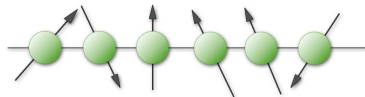


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- Clear physical meaning, can be checked numerically.
- Universal amplitudes in front of the logs, exact expressions for $d = 2$.
- Results also valid in higher dimensions ($d = 3$). Universal coefficients can be computed thanks to $\epsilon = 6 - d$ and $\epsilon = 4 - d$ expansions.

Lattice algebraic approach to strongly interacting QFTs

How to understand more the intricate structure of LCFTs?

- Idea : thorough study of lattice 'regularizations'
- Think of $SU(2)$ Heisenberg spin chain [Haldane, Affleck, Read, Sachdev, ...]



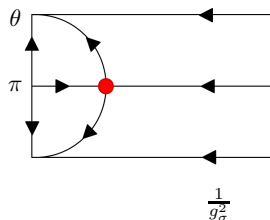
Lattice spin chain $H \simeq J \sum_i \vec{S}_i \cdot \vec{S}_{i+1}$

low-energy

non-linear σ -model

flows to

strong coupling fixed point (L)CFT



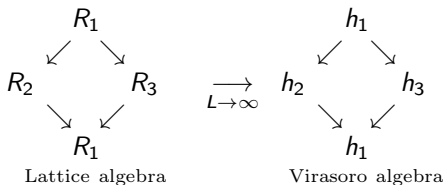
$\mathbb{CP}^{n+m-1|m}$ Sigma model with $\theta = \pi$

[Read & Saleur '01]

[Read & Saleur '07]

Main idea :

Much information about the continuum limit is already encoded in a finite-size lattice model : emergent symmetries, universal numbers, fusion rules, representation theory, ...



with the lattice Virasoro generators [Koo & Saleur '94]

$$\frac{1}{L}(L_n + \bar{L}_{-n})^{(L)} \sim \sum_{j=1}^L e^{2\pi i j n / L} (h_j - h_\infty)$$

Main idea :

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Universal amplitudes

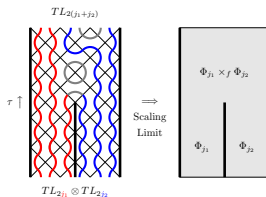
$$\langle T(z)T(0) \rangle = 0$$

$$\langle T(z)t(0) \rangle = \frac{b}{z^4}$$

$$\langle t(z)t(0) \rangle = \frac{\theta - 2b \log z}{z^4}$$

L	b
12	-0.606403
14	-0.607775
16	-0.609226
18	-0.610561
20	-0.611738
22	-0.612764
∞	-0.6249 ± 0.0005
Exact	$-5/8 = 0.625$

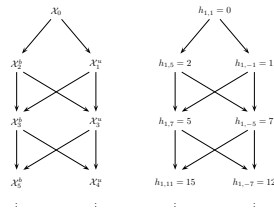
Fusion rules



$$\phi_\alpha \times \phi_\beta = \sum_\gamma C_{\alpha\beta}^\gamma \phi_\gamma$$

[Read & Saleur '07, GV 13']
Similar approach [Pearce,
Rasmussen & Zuber '06]

Representation Theory



From the lattice to Virasoro

'Classification' of indec. reps. ?

[GJSV 13']

- At $c = 0$, the stress energy tensor has a log partner t [Gurarie & Ludwig '02]

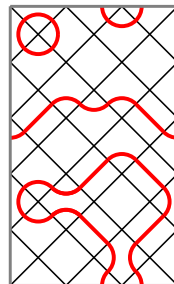
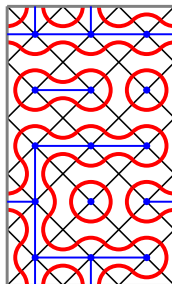
$$\langle T(z)T(0) \rangle = 0$$

$$\langle T(z)t(0) \rangle = \frac{b}{z^4}$$

$$\langle t(z)t(0) \rangle = \frac{\theta - 2b \log z}{z^4}$$

- Same structure for many other fields. How to compute the b numbers ?
- Direct measure from lattice models ! [Dubail Jacobsen Saleur '10, VJS '11]
- $b = -5$ in the bulk \neq usual boundary values $b_{\text{perco}} = -\frac{5}{8}$, $b_{\text{poly}} = \frac{5}{6}$ [VGJS '12]

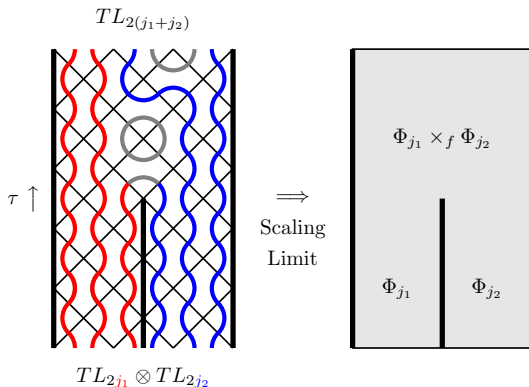
L	b
8	-0.609088
10	-0.605858
12	-0.606403
14	-0.607775
16	-0.609226
18	-0.610561
20	-0.611738
22	-0.612764
∞	-0.6249 ± 0.0005
Exact	$-5/8 = 0.625$

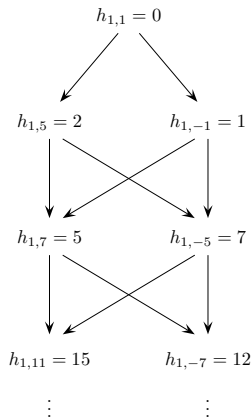
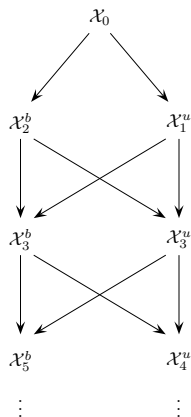


- Another crucial CFT information : fusion rules

$$\phi_\alpha \times \phi_\beta = \sum_{\gamma} C_{\alpha\beta}^{\gamma} \phi_{\gamma}$$

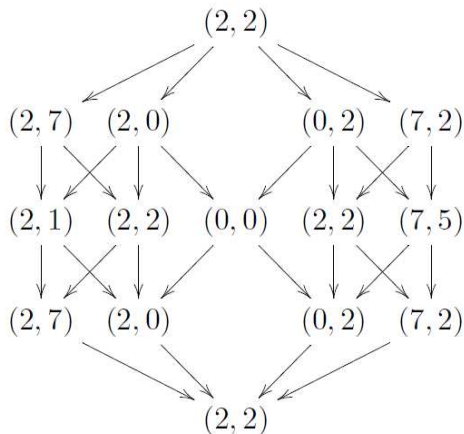
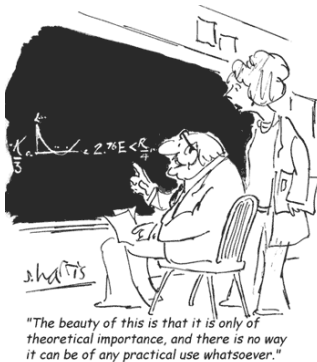
- Direct computation from the lattice ! [Pearce, Rasmussen & Zuber '06, RS '07, GV '12]
Fusion \equiv joining two 'spin chains'
- Some fields are mixed together (indecomposability)





- Lattice algebras representation theory mimics what is known from Virasoro [Pasquier & Saleur '87, Read & Saleur '07] + Lattice Virasoro generators [Koo & Saleur '94]
- Lattice models easier to handle (finite size) \Rightarrow deduce results for Virasoro representations [GJSV '13]
- Active branch of mathematics [J. Germoni, K. Iohara, P. Martin, O. Mathieu, G. Lehrer, ...]

- Boundary vs Bulk very important for CFT [Cardy '89], way more complicated for LCFT [Gaberdiel & Runkel '08]
- Bulk LCFT : indecomposable representations over $\text{Virasoro} \times \overline{\text{Virasoro}}$
- Richer structure : different b -numbers, Jordan cells of arbitrary rank (ex. rank 3 for $T\bar{T}$ at $c = 0$), no simple relation to the boundary case
[Gainutdinov, Read & Saleur '12, GRSV in preparation]



Conclusion

Conclusion

- Low energy excitations of critical disordered systems (quantum or classical) and of geometrical models are described by *logarithmic* CFTs
- Unusual features : non-unitarity, non-diagonalizability of Renormalization Group transformations, no Mermin-Wagner theorem ...
- ... \Rightarrow very hard to solve but lattice models can help
- Geometrical meaning of logarithms slowly getting under control (what about disordered systems ?)
- Logarithmic features occur in higher dimensions too $d = 3, 4, \dots$!

Thank you !

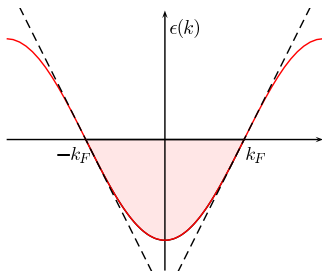
- Virasoro algebra

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c}{12}n(n^2 - 1)\delta_{n+m,0}$$

- How does this algebra emerge from the lattice spin chain $H = \sum_i h_i$?

$$\begin{aligned} E_0(L) &= h_\infty L - \frac{v_F \pi c}{6L} + \dots \\ E_\phi(L) - E_0(L) &= \frac{2\pi v_F}{L} \Delta_\phi + \dots \end{aligned}$$

[Blöte, Cardy & Nightingale '86]



Koo-Saleur Formula '94, Kadanoff & Ceva '71

$$\begin{aligned} L_n + \bar{L}_{-n} &= \frac{L}{2\pi v_F} \sum_{j=1}^L e^{inj2\pi/L} (h_j - h_\infty) \\ L_n - \bar{L}_{-n} &= \frac{iL}{2\pi v_F^2} \sum_{j=1}^L e^{inj2\pi/L} [h_j, h_{j+1}] \end{aligned}$$

\Rightarrow Lattice regularization of Virasoro

- Algebra of hamiltonian densities 'tends to' Virasoro!

Deep correspondence even at the level of representation theory