



# Analyse des effets spatiaux et aspects économiques dans les réseaux de communications

Manjesh Kumar Hanawal

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This dissertation is carried out in the framework of the INRIA and LIA, University of Avignon joint project on Efficient Cooperating Small Cells (EcosCells).

To my parents, family, friends and teachers.



# Abstract

In this thesis we analyze the performance of communication networks using game theoretic approaches. The thesis is in two parts. The first part studies the performance of Ad hoc, cellular and transportation networks taking into consideration spatial effects. The second part studies economic issues in the communication networks related to the ‘net neutrality’ regulations. Here we study price competition and revenue sharing mechanisms between the network service providers.

In the first part, we use Medium Access Control (MAC) game and Jamming game models to study the performance of a Mobile Ad hoc NETWORK (MANET), and routing games to study the performance of a transportation network. In the cellular networks, we study the effect of reducing the spatial density of base stations on the amount of radiation to human body (green networking). We use tools from stochastic geometry to model spatial characters.

We begin with the study of MAC game in a MANET where the nodes are non-cooperative, and their locations form a Poisson point process. The nodes use Aloha protocol to access the channel, and choose their Medium Access Probability (MAP) such that it optimizes their utility. The utility of each node is defined as weighted difference between a performance metric (expected goodput or expected delay) and transmission costs. We first consider a scenario in which nodes can be priced for channel access. We show that by using a simple (linear) pricing mechanism and setting the price ‘appropriately’, the selfish behavior of the nodes can be used to achieve the same performance as social optima at equilibrium. In the case where channel access is free, we consider transmission energy costs and analyze the Price of Anarchy (PoA). For the game with goodput based utility, we show that the PoA is infinite at the price that achieves the global optimal goodput.

We then study the performance of MANET in the presence of a Jammer while all the nodes cooperate by using a MAP that is assigned by a network an Operator. The objective of the Jammer is to degrade the spatial performance of the MANET by causing interference, whereas the objective of the Operator is to set a MAP to optimize it. We model the interaction between the Jammer and the Operator taking into account the transmission energy costs. We transform the resulting non zero sum game into a zero sum game by constructing an anti-potential, and show that for a given Jammer’s transmission cost, if the nodes transmit at a power higher than certain threshold, then the Jammer has no incentive to jam, i.e., the nodes can operate as if there is no Jammer.

Next, we consider cellular networks. We study energy saving by switching off a fraction of the base stations (BSs). This saving comes at some cost: the coverage is reduced, and the uplink transmission power of mobiles may increase. This implies exposure of the human body to stronger electromagnetic fields. We quantify this through the deactivation of base stations under the assumptions that the random location of base stations and mobiles form a Poisson processes. We consider the case of sparse network where the interference is negligible, and the case where it is non negligible. We observe that when the mobiles have no power constraints, unlike in the case of negligible interference, switching off base stations reduces the uplink power.

Finally, we study a dynamic routing game by adding temporal dimension to the classical routing problem. We consider a scenario where each user have to ship a fixed demand on a shared link. The users can delay shipping their demand, but need to ship within  $T$  days, and each time they delay, a delay cost is incurred. We study the effect of delay costs on the user strategies by translating the time dimension into space dimension. Considering both atomic and non-atomic game models and polynomial congestion costs, we show that there exists a threshold time  $T^* \leq T$  before which all users ship their demand at equilibrium, and during this period total demand on the link is decreasing in time. Further, we extend the analysis to the case when the demand of each user arrives randomly.

In the second part of the thesis, we study economics aspects in communication networks. Representatives of several Internet access providers (ISPs) have expressed their wish to see a substantial change in the pricing policies in the Internet. In particular, they would like to see content providers (CPs) pay for use of the network, given the large amount of resources they use. This would be in clear violation of the “network neutrality” principle that had characterized the development of the wireline Internet.

Our first goal here is to propose and study possible ways of implementing payments in a nonneutral network. We introduce a model that includes the users’ behavior, the utilities of the ISP and of the CPs, and the monetary flow that involves the users, an ISP and CPs, and CP’s revenues from advertisements. We consider various game models and obtain the resulting equilibrium prices. We show that if a regulator decides the amount of money that CPs pays to the ISP, rather it is decided by the ISP or the CPs, then it results in a favorable situation for all the players. Thus, we demonstrate the necessary for regulation of the monetary transactions between the ISP and the CPs in the nonneutral regime. The mechanism we propose for monetary interaction is based on the Nash bargaining solution.

One of the central issues in the debate on network neutrality has been whether one should allow or prevent preferential treatment by an ISP. This raised the question of whether to allow an ISP to have exclusive agreement with a content provider (CP). We study the impact of exclusive contracts between a CP and an ISP in a nonneutral network. We consider a simple case of collusion where a CP and an ISP aim to maximize their sum of revenues, and show that such a collusion may not always be beneficial. We derive an explicit condition in terms of the advertisement revenues of the CPs which tells when a collusion is profitable to the colluding entities.

Finally, we consider discrimination in the opposite direction, i.e., CP discriminating the ISPs. We derive models for studying the impact that the CP can have on the utilities of the ISPs by favoring one of them through exclusively revealing its private information on the demand. For a special case of linear demand function, we show that such preferential treatment always benefits the ISPs. However, if the CP charges the ISPs for providing the private information, then the ISP obtaining preferential treatment may not always gain. We then propose mechanisms based on *weighted proportional fairness* for deciding the payments between the CP and the ISP and compare them by proposing a new metric termed *Price of Partial Bargaining*.





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# Chapter 1

## Introduction

### 1.1 Motivation

The telecommunication networks has elevated human comforts to new heights. Today, communication networks is used from the simple purpose of message delivery to doing business remotely. With the growing number of activities that people use the communication networks, the demand for better connectivity, higher quality of service, and higher security is increasing as expected. While the researchers are constantly thriving to improve the performance and security of communication networks, policy makers are debating how the communication network infrastructure can be made available to all the users in a fair way.

In the following we first briefly discuss various communication techniques used to enhance the performances of communication networks. We focus on the *spatial effects* on the performance of communication networks which is the main topic of study in this thesis. We then discuss *economical aspects* of the communication networks and highlight the issues discussed in the ongoing *net neutrality* debate.

Telecommunication technologies have constantly evolved to meet the ever increasing demand for improved performance. This evolution has resulting in a series of cellular standards. Table 1.1 summaries various cellular stands and different services they support.

Many techniques have been developed that overcome inherent problems on a wireless channel like, randomness, interference, fading, shadowing to achieve higher throughput. Thanks to the advancement in the very large scale integration (VLSI) technologies, many of these techniques are now a reality. Turbo codes [1], Gallager's LDPC codes [2], fountain codes [3] that achieve throughput close to Shannon capacity limit are now realizable in practice. The current operational 4G standards supports peak download rate of 300 mega bits per seconds (Mbps), and has helped achieve quality of services on the mobile devices that is so far available only on wired networks.

The advancement in signal processing in culmination with the gains of narrow

Generation	Period	Transmission	Services	Examples
1G	1980's	Analog FM FDMA / FDD	Voice	AMPS
2G	1990's	Digital modulation TDMA/CDMA	Voice, SMS	GSM, IS-95 GPRS, EDGE
3G	2000+	Wideband modulation	Internet, email, Multi-media streaming, etc.	WCDMA HSDPA, HSUPA
4G	2005+	OFDMA	Mobile internet Mobile video, etc.	LTE, WiMAX

**Table 1.1:** Evolution of wireless generations

band signalling has made *multiple access techniques*, like Code Division Multiple Access (CDMA), Orthogonal Frequency division Multiplexing (OFDM) resulting in higher spectral efficiency.

There has been lots of efforts to leverage the inherent properties of the wireless channel to further improve the achievable throughput : fading and interference, which form the basic ingredients of *Opportunistic communication*. Opportunistic communication aims at scheduling and allocating users which have good channel conditions. Since fading is a time/frequency varying phenomenon, everyone stands to benefit over some averaging duration. Further, fair schedulers [4] can be employed to guarantee certain quality of service (QoS), even to disadvantaged users (cell edge or non line-of-sight). The other interesting idea of opportunistic communication is the principle of Cognitive radio [5]. This exploits unused spectrum and transmission opportunities of primary subscribers to schedule secondary users, resulting in better usage of scarce radio spectrum. Relaying [6], yet another aspect of Opportunistic communication, manages interference and improve end to end QoS. Signals from transmitters to receivers are routed through relays which offer the best channel conditions.

*Multiple Input Multiple Output* (MIMO), based on Multiple antennas at the transmitter and / or receiver is a technique which uses the diversity in channel gain combined with multiple access and / or multiplexing gains. This enables higher data rates between transmitters and receivers. The trade-off between diversity and multiplexing has been well captured by the landmark paper of Tse [7].

The spatial component of the wireless channels has seen tremendous advancement in recent, starting from Telatar's landmark paper [8] and became popular with the famed Alamouti [9] code. We will get back to spatial component in the following section.

Of late, co-operative as well as distributive strategies are gaining popularity. Co-operative strategies like multi cell co-operation [10, 6] aim at managing interference actively. At any given time the channel state of the entire system is known and a central controller can now decide the most optimal way the communication happens between individual transmitters and receivers.

As the size of the network grows, managing the network centrally gets complicated. Decentralized and distributed processing is becoming popular to manage the complexity of central control. Also, distributed techniques scale easily. Here, each agent (base station / mobile), decides the parameters, for example, power, medium access rate, transmission rate, etc., on its own. In a decentralized control, each agent has to learn and adopt to the environment. This is where the concepts of learning and adaptive algorithms come in. Reinforcement learning [11, 12], stochastic approximation [13], etc. are becoming an integral part of new generation base stations.

Though decentralized schemes are easy to implement and scale easily, they come with the inherent problem of agents being selfish. A selfish agent can tune his parameters to take undue advantage and can cause harm to others. To analyze such scenario Game theory [14] has been extensively used to understand the equilibrium behavior of the system. Further, many pricing schemes, incentivizing mechanisms are proposed that lead to improved system performance in presence of selfish agents.

## 1.2 Spatial effects in communication networks

The spatial structure observed in communication networks is usually far from being regular, and the network geometry and its structural fluctuations are critical parameters that greatly influence the performance of random networks. Specifically, the interference and the signal strength at a receiver critically depends on the distribution of the interfering transmitters. Mathematical techniques that explicitly model the node distribution, and characterize performance measures as a function of signal-to-interference-and-noise ratio (SINR, or SIR when the noise is neglected) are much needed.

Cellular networks are being extensively deployed and upgraded in order to cope with the steady rise of user-traffic. In the context of an actual deployment of cellular networks, lattice (e.g. hexagonal) models for the base station placement are usually thought of as more pertinent. However, perfect lattice models do not seem to allow analytic techniques for the study of the SINR-based characteristics. This has created the need for new and robust analytic techniques to study the network performance.

Recently, tools like stochastic geometry surged as a useful tool to analyze performance of wireless networks [15],[16]. The traditional methods of network analysis assumes linear or hexagonal networks. Stochastic geometry takes into account the random location (distribution) of transmit and receive nodes, which is a more realistic assumption and many a times, it is possible to obtain explicit expressions for important system metrics. For example, one can compute explicit expressions for the total interference at a base station with simultaneous transmissions from randomly located nodes that are distributed according to a Poisson process [16]. This can be plugged into the SINR equations to get a more realistic value as compared to a idealized value with regular placement of nodes. These techniques based on point process theory, and percolation theory were instrumental in recent breakthroughs and have shed light to the fundamental limits of wireless networks.

In using stochastic geometric tools it is assumed that the nodes (base station of mobiles) are located according to a Poisson process in the plane. The Poisson assumption means that

- The numbers of mobiles in disjoint sets are independent;
- The number of mobiles in any given set follows a Poisson distribution.

This class of point processes maximizes entropy.

The Poisson model is used and justified by positioning ‘irregularities’ of the network. It is also considered as a ‘worst-case’ scenario due to its complete randomness property. Stochastic geometric tools have been applied to study performance of ALOHA, carrier sense multiple access (CSMA) protocols in Ad hoc networks [17] and to evaluate throughput and outage probabilities in cellular networks [18] [19]. Further, in a recent paper [20] it is shown that as the variance of log-normal shadowing increases, the resulting propagation losses between the stations and the typical user form a stochastic process that converges to the non-homogeneous Poisson process. Thus validating the use of Poisson assumption to model irregularities in a random network.

Stochastic geometric tools are also used in strategic planning and economical modeling of telecommunication networks [21].

### 1.3 Economic Aspects of communications networks

The Internet over time has evolved from a simple, small-scale, yet robust communication system carrying low data rate traffic into a very large scale and highly complex network carrying heterogeneous traffic in diverse environments. Its impact on economy, social and cultural influence is significant. Thus, in the design of future internet architecture and protocol development one needs to take both social and economic aspects into consideration.

**The influence on economy:** Since the commercialization of the Internet in 1995, many information and communication technologies have emerged enabling variety of online commercial activities. The rise of near instant communication by electronic mail (e-mail), instant messaging, Voice over Internet Protocol (VoIP) "phone calls", two-way interactive video calls, and online shopping sites (Amazon, Netflix, eBay, Flipkart, etc.) has created a huge electronic commerce. The information and communication technologies have a central economic role. Indeed, The information and communication technologies sector already generates revenue of 2,700 billion euros, or close to 7% of global Gross Domestic Product (GDP), and could account for 20% of GDP within the next 10 years [22]. To give an example, in December 2009 ComScore reported that 5.4 billion videos had been watched in France that year (a 141% increase over the year before), of which 1.8 billion on YouTube.com between January and September.

**Social and cultural influence.** The emergence of global internet has not only redefined the way business is done, but also the way people interact with each other. Online

discussion forums, blogs, social networking (Facebook, Twitter, Orkut, etc.), has significant effect on the social and cultural aspects. The Internet has emerged as a platform for freedom of expression and has immense influence on the social and cultural activities.

In 2009, French Constitutional Council went back to the time of the French revolution (two hundred years before the Internet was born) and recognized that the Internet is an instrument for exercising the freedom of speech. Similar relations between the Internet and the American constitution (and amendments) have also been made in the USA [23].

Recent events in the Arab world have shown us that people use Internet as a source not only to express their opinion on governments, but also to coordinate actions that allows overthrowing them. Governments have reacted, predictably, by blocking not only the Internet but also the mobile phone service. However, the effectiveness of these measures has been compromised with the release in media outlets throughout the world, of videos recorded in those countries.

## 1.4 Future of the Internet

The global Internet is a interconnection of several smaller networks that are owned and operated by multiple, independent, profit-making entities. We hence forth refer to these entities as network service providers or simply service providers. In today's competitive market, improving network QoS (reduced latency, error rate) is just a part of the story. Service providers also need to increase profit margins to ensure their competitiveness. Past history has shown that sound technologies based on pure engineering considerations often fail to get adopted. Economic factors do influence successful adaptation of a technology. Recent debates, such as over *network neutrality*, is on regulating the economic interests of the service providers (economic aspects) and study of its impact on the end users (social aspects).

It is important for the scientists from research institutes and universities and who do not have direct economic interests to understand various aspects in the ongoing net neutrality debate and put forward their opinions. Such neutral opinions will be helpful to maintain the principle of *neutrality* in the Internet based on which it has evolved since its inception. The main issues debated in net neutrality debate are

- Quality of service to be guaranteed to the end users.
- Taxation of the service providers by the governments.
- The monetary interaction between internet service providers (ISPs) the Content Providers (CPs). In particular, should last mile ISPs be allowed to charge CPs for accessing the users on their network?
- Allowing exclusive agreements between ISP and CPs (vertical monopoly).

Since 2002 the broadband Internet is classified as 'information service'. This classification does not forbid the service providers from using discriminatory policies in

the Internet [24]. It also does not forbid the last mile ISPs demanding compensation from the CPs for using their infrastructure. However, many argue that such openness is a threat to *neutrality* of the Internet and feel the need for regulation to maintain the Internet as a neutral platform.

#### 1.4.1 The impact of the government position

Both the U.S. government led by President Obama and the FCC headed by the Commissioner Genachoswki are strong proponents of the consecration of the principle of net neutrality either by an amendment of the Telecommunications Act, or by an administrative mandate issued by the FCC itself. This has re-launched the debate on net neutrality. In December 2010, the FCC issued a Report and Order [25] as the conclusive and regulatory document obtained from the Notice of Proposed Rule Making (NPRM). In it, the FCC keeps maintaining its authority to adopt rules on the open Internet [25][Part IV], but not unanimously, as two of its members believe that it does not have such authority [25][148-150, 188-193].

The European Union, both through the statement issued by the Commission under the Telecom Package<sup>1</sup> as well as through Commissioners Reding and Kroes, made it clear that the intention of the European government is to protect the neutrality of the network.

#### 1.4.2 Net Neutrality Debate

Several public consultations on network neutrality policies have taken place in 2010. From them, regulation was introduced by the Federal Communications Commission (FCC) in the USA [25], while the European authorities feel there is no need for it [27] at this moment. France's Assembly, meanwhile, is discussing a bill on net neutrality [28]. The growing economic and social role of the Internet along with the fast evolution of its performance and of the services it offers, have triggered evolution of the legal status of the access to the Internet. Already on 2002, access to the Internet has become an universal service in the European Union (EU), which should imply guarantees on QoS.

From the debate on network neutrality we learn that new indicators of QoS are sought, and that we should expect an intensive work of regulating and standardization bodies on defining requested minimum values of performance measures. By actually requiring QoS to be guaranteed to the end user's actual experience (see e.g. [22] page 19), we may expect new legislation to create a new reliable and yet more expensive Internet, that would be different than the best effort type network that we have known.

Another new comer issue is that of imposing taxes on CPs by the government, and ISPs charging the CPs for reaching the end users using their infrastructure.

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<sup>1</sup>The Commission acknowledges in a declaration attached to the Telecom Package that net neutrality is "a policy objective and regulatory principle to be promoted by national regulatory authorities" [26].

### 1.4.3 Taxation issues

Among the issues that net neutrality is concerned with are relations between ISPs and CPs along with related pricing issues, as well as the possibility of a ISP to have exclusive agreement with some CP or some other ISPs.

In Europe, these issues take another dimension, due to the fact that many large CPs (such as google, facebook etc) are non-european: these are mainly American companies. These companies make large benefits from advertisement. It is known that google pays very little taxes on this income elsewhere, in contrast to what google pays in the USA. In a context where all other actors related to the Internet do pay taxes, applying network neutrality would mean favoring or subsidizing these providers that do not pay. To be more precise, Google managed to cut 3.1 billion American dollars of taxes in the three last years by declaring its foreign profits (made in Europe) in Bermuda. This enabled Google to reduce its overseas tax rate to 2.4% [30].

The so-called “Zelnick Report” [31], which came out in France in January 2010, proposed to impose a tax on advertising revenue generated by the use of online services from France. According to estimates put forward by the authors, between 10 to 20 million euros would be collected mainly from U.S. CPs (Google, Microsoft, AOL, Yahoo and Facebook). It is pertinent to note that the report expresses concerns about the drop in advertising revenues of the French CPs, citing the poor state of competition in the French market for search engines, and certain behaviors (never clarified in the text) of Google.

Many last mile ISPs are not proponents of net neutrality. They argue that returns on their investments in the neutral regime is not sufficient. More specifically, those opposing the neutrality argue that

- some applications (such as peer to peer (P2P) streaming applications) require a lot of costly resources, and
- if a neutral policy is pursued, there would be no incentive for investing in the infrastructure of the network in order to upgrade it.

In an interview<sup>2</sup> in 2005, Edward Whitacre, the then CEO of AT&T, said: ‘....Now what they would like to do is use my pipes free, but I amn’t going to let them do that because we have spent this capital and we have to have a return on it. So there’s going to have to be some mechanism for these people who use these pipes to pay for the portion they’re using. Why should they be allowed to use my pipes? The Internet cant be free in that sense....’ This has sparked a huge debate on nonneutral network vs neutral network.

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<sup>2</sup>Business Week 2005, November 7



## 1.5 Organization of thesis

The thesis is presented in two parts. The first part studies the spatial effects on the performance of the communication networks, and it consists of 4 chapters. In Chapter 2, we study performance of a mobile ad hoc network with selfish users. We use stochastic geometric tools to model the location of the nodes and derive performance metrics that involve costs for the channel access (spectrum license fee or transmission energy costs). We propose pricing schemes to improve the system performance at equilibrium. In chapter 3, we study a jamming game in a similar setup. We derive the system performance in presence of a jammer, and study equilibrium performance as a function of the transmission cost associated with both the legitimate nodes and that of the jammer. In chapter 4, we study the effect of switching off a fraction of base stations (distributed as homogeneous Poisson process) on the amount of uplink power emitted by a mobile node. We use stochastic differential equations to derive closed form expressions for the interference seen at a base station. In chapter 5, we study a routing game where the users can delay shipping their goods, but incur cost for delaying. We transform the time dimension into space and derive equilibrium flow on the links.

In the second part of the thesis, we study economic aspects of the communication networks, and it consists of 3 chapters. In chapter 6, we study the monetary interaction in a nonneutral network. We propose mechanisms based on Nash bargaining solution to decide amount paid by the CPs to ISPs for using their infrastructure. In chapter 7, we consider the scenario where a CP makes exclusive contract with an ISP to receive preferential treatment. We study the effect of such collusions on both the colliding and non colliding players. In chapter 8, we consider another form on nonneutral network where the CPs discriminate the ISP on sharing its private information. We study how such discrimination affects the revenues of the ISPs. Finally, we end with conclusion in Chapter 9.

## 1.6 Publications

**The list of publications related to the first part of this thesis is as follows:**

- M. K. Hanawal, E. Altman and F. Baccelli, "Stochastic Geometry based Medium Access Games in Mobile Ad hoc Networks," *IEEE Journal of Selected Areas in Communications*, Special issue on Economics of Communication Networks and Systems, vol. 30, no. 11, pp. 2146 - 2157, December 2012.
- M. K. Hanawal, E. Altman and F. Baccelli, "Stochastic Geometry based Medium Access Games," in *Proceedings of the 31<sup>st</sup> International Conference on Computer Communication (INFOCOM 2012)*, held at Orlando, Florida, USA, March 2012.
- M. K. Hanawal and E. Altman, "Stochastic Geometry based Jamming Games in Mobile Ad hoc Networks," in *Proceedings of the 9<sup>th</sup> International Conference on Wireless on-demand Networks and Services (WONS 2012)*, held at Courmayeur, Italy,

January 2012.

- E. Altman, M. K. Hanawal, R. El-Azouzi and S. Shamaï, "Tradeoffs in Green Cellular Networks," presented at *GreenMetrics* Workshop, held in conjunction with *SIGMETRICS*, San Jose, USA, June 2011. To appear in Performance Evaluation Review (PER).
- M. K. Hanawal, E. Altman, R. El-Azouzi and B. Prabu, "Spatio-temporal control for Dynamic Routing Games," in Proceedings of the 2nd *International ICST Conference on Game Theory for Networks* (GameNets 2011), Shanghai, China, April 2011.

**The list of publication related to the second part of this thesis as follows:**

- M. K. Hanawal and E. Altman, "Network Non-Neutrality through Preferential Signaling," accepted for presentation at the 11th Intl. Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt 2013), to be held in Tsukuba Science City, Japan, May 2013.
- M. K. Hanawal, E. Altman and R. Sundaresan, "A Game Theoretic Analysis of Collusions in Nonneutral networks," presented at *W-PIN* Workshop, held in conjunction with *SIGMETRICS*, London, UK, June 2012. To appear in Performance Evaluation Review (PER).
- E. Altman, J. Rojas, S. Wong, M. K. Hanawal and Y. Xu, "Net Neutrality and Quality of Service," *invited paper* in Proceedings of the 2nd *International ICST Conference on Game Theory for Networks* (GameNets 2011), Shanghai, China, April 2011.
- E. Altman, M. K. Hanawal and R. Sundaresan, "Nonneutral network and the role of bargaining power in side payments," in Proceedings of the Fourth Workshop on *Network Control and Optimization* (NETCOOP 2010), pp. 66-73, Ghent, Belgium, November 2010.



## **Part I**

# **Spatial Effects in communication networks**



## Chapter 2

# Stochastic Geometry based Medium Access Games in Wireless Ad hoc Networks

### Contents

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## 2.1 Introduction

Wireless terminals in an ad hoc network compete with each other to gain access to the shared resources. When several terminals access the channel simultaneously, they interfere with each other, resulting in transmission failures. Medium access control is thus necessary to increase network efficiency. In ad hoc networks, the presence of selfish terminals can degrade the performance of the network. In this chapter we study competition for network resources at the medium access control (MAC) layer when the nodes are selfish. Game theory is extensively used to study the selfish behavior in ad hoc networks. We henceforth refer to a wireless terminal as node or player.

It is well known that computing the Nash equilibria in games is in general a hard problem. Indeed, this problem falls into a class of problems introduced by Christos Papadimitriou in 1994, called PPAD (Polynomial Parity Arguments on Directed graphs). In view of this complexity, it becomes attractive to identify classes of games for which one can compute the equilibria at a low complexity. We thus study a MAC game under some statistical assumptions on the location of the nodes. These assumptions are on one hand reasonable in many real scenarios, and on the other hand, allow for tractable and in several cases, even explicit expressions for the Nash equilibria.

The basic assumptions on our model are

- Time is slotted and all transmitters are synchronized;
- The location of the transmitters at each time slot forms a realization of a homogeneous Poisson point process (P.p.p.);
- Medium access is controlled using Aloha;
- Transmission success is based on signal to interference and noise ratio (SINR) being larger than some threshold;
- We assume saturated sources, i.e., every mobile always has a packet to send.

We consider a non cooperative scenario in a non cellular, self organizing wireless network. Each node in the network is selfish and rational. The nodes share a common channel and choose a Medium Access Probability (MAP) so as to maximize their own performance metric. However, the nodes may need to take other costs into account while optimizing their performance. For example, if licensed frequencies are used, one may expect extra costs to be imposed on the users of wireless terminals. In novel peer-to-peer ad hoc networks such as FalshLinQ [69] the nodes operate in a licensed spectrum. The owner of the proprietary spectrum can charge the nodes for spectrum usage and aim to optimize the overall network performance. We assume the players choose a tradeoff between performance and transmission costs as they may value them differently. We thus define utility functions that model the tradeoff between quality of service indicators (such as goodput or expected delay) and costs related to transmissions for each player. In these we represent the relative propensity of a player between the performance metric and the transmission costs by a tradeoff factor.

The transmission costs incurred by the players are influenced by the pricing scheme used in the network. A simple pricing scheme is to charge the terminals per transmission attempt. Such a pricing mechanism could involve simple counter embedded in the terminal. Since we assume constant transmission power per packet, this type of pricing simply adds to the dis-utility for transmission energy consumption. We relate the tradeoff factor to a pricing scheme. Thus by regulating the price, the relative preference of the players for the performance metrics and transmission costs can be influenced, which in turn affects the network performance at equilibrium.

We also consider the case when transmission costs can't be influenced through pricing. For example when the nodes are operating in an unlicensed spectrum no pricing can be imposed. The nodes still incur costs related to transmission energy and take

them into account in optimizing their performance. In this case we study the degradation in the perceived performance of the nodes due to lack of cooperation by computing the Price of Anarchy (PoA) as a function of the tradeoff factor.

Our goals are to obtain symmetric Nash equilibria (SNE), possibly mixed, and to study their properties. By restricting our study to symmetric games, and by restricting to choices of MAP that are not functions of the locations of players or interferers, we have to consider the utility evaluated only at points where the strategies of all other players are the same, say  $p$ . We search for a  $p$  such that if all use it, then a player that deviates unilaterally will not improve its utility. Thus we can focus on a much simpler two person game, and any equilibrium in the two player game is a symmetric equilibrium in the original game. The SNE computed in our game is a function of the product of the tradeoff factor and the pricing factor. When we consider the game with a pricing scheme, we fix the tradeoff factor and analyze the SNE as a function of the pricing factor. When we study the PoA, we fix the pricing factor and analyze the SNE as a function of the tradeoff factor.

Our main findings are:

1. Considering the goodput as the quality of service to be maximized, we observe the tragedy of the commons [53]: the utility at equilibrium is zero for large values of the price factor. Thus the PoA is infinite for these parameters. We show that there exists a pricing factor for which the goodput at equilibrium coincides with the one obtained under global cooperative throughput maximization. Further, we prove that when each node uses the replicator dynamics to update its MAP, the system converges to the SNE.
2. Considering the delay as the quality of service to be minimized, we observe that the PoA is bounded uniformly in the tradeoff factor. Here too, there exists a pricing that results in the global optimum at equilibrium. We show that the SNE is not unique. The range of pricing factors that result in two SNE is characterized.

We shall compute the SNE but not address the question of how equilibrium is reached. Our results can be viewed as follows: In the licensed case, if the nodes are appropriately priced and the corresponding equilibrium MAP is made public, then all nodes access the channel with the announced MAP as they have no incentive to deviate unilaterally, thus achieving the global optimal performance in a decentralized way.

Our Aloha assumption on the MAC protocol and the Poisson assumption on the location of nodes allow us to obtain utilities in a surprisingly simple explicit form, which in turn allows us to obtain much insight on the property of the equilibria and on the role of the pricing. Such Poisson assumptions are often justified. For instance, in [18], in the context of cellular networks, the authors obtain explicit expressions for the coverage and throughput with the Poisson assumption on base station locations. Their numerical investigation show that these expressions closely capture the real behavior of the network. We leave the study of other, less tractable, CSMA type of MAC protocols for future study.

**Related Work:** There are several papers that model the nodes in the Aloha system as



selfish users. Here we discuss a few that are most relevant to our work. In [56], a game theoretic model is proposed to analyze the performance of the Aloha network. A payoff for each player is defined with a fixed cost on each transmission attempt. Assuming that each player knows exactly how many nodes are backlogged in the system, the authors analyze the equilibrium MAP. They show that for a particular value of the transmission cost, the throughput achieved in the system can be made equal to the social optimum. In [57], the authors relax the assumption that the players know the number of backlogged nodes in the system. It is shown numerically that by adding a transmission cost, the throughput at equilibrium can be made equal to the optimal team throughput. Both [56] and [57] assume that the players are indistinguishable and incur the same amount of transmission cost to compute the symmetric Nash equilibria. The authors in [55] consider a heterogeneous Aloha network. They assume that each player always has a packet to send, and incurs a transmission cost (not necessarily the same for all) on each failed transmission. By computing the resulting Nash equilibria as a function of the cost factors, they show that any optimal performance achieved in a centralized scheme can also be achieved in a decentralized scheme by appropriately setting the costs. [67] introduces a generalized random access games to study contentions in medium access games. The authors introduce a general utility function, and give several examples explaining how to design a utility function by forward engineering from the desired operating point. They also study the convergence of various dynamic algorithms. For a nice survey on how game theory is used to model various aspects of wireless networks see [68]. Pricing is also used in the context of the power control games in [59] to improve the performance at equilibrium.

None of these papers account for the geometric aspects of the node location, which is important in the wireless context. In this thesis we consider the geometric aspects of the wireless ad hoc network and analyze the network performance at equilibrium. Also, unlike in all other papers, where simultaneous transmissions lead to transmission failure, we assume here that the probability of successful transmission depends on SINR at the receiver. The effect of spatial aspects on the performance of wireless networks were studied in [58, 71, 72]. In [58], the authors study the problem of mobile association to base stations based on spatial SINR considerations. [71] studies spectrum sharing mechanisms in a cognitive radio network considering spatial reuse.

The chapter is organized as follows. In Section 3.2, the Poisson bipolar ad hoc network model is set up and performance metrics of interest are discussed. Section 2.3 considers the team case in which all nodes use the same MAP. Section 2.4 considers a scenario in which all nodes are selfish. A medium access game is defined among the nodes. Section 2.5 studies this game with goodput as performance metric. Section 2.6 studies the medium access game with potential delay as performance metric. Section 2.7 considers the case where pricing is not possible and analyzes the price of anarchy for these games. We conclude with some remarks and future work in Section 8.9.

## 2.2 Model and Setup

Consider the simplified mobile ad hoc network (MANET) model called the Poisson bipolar model proposed in [32]. Assume that each node follows the slotted version of the Aloha medium access control (MAC) protocol. Each dipole of the MANET consists of a transmitter and an associated receiver. Let the sequence  $\{X_i, y_i\}_{i \geq 1}$  denote the location of the transmitters and receivers, where  $y_i$  is the location of receiver associated with the transmitter at  $X_i$ . We assume that the transmitters  $\Phi = \{X_i\}_{i \geq 1}$  are scattered in the Euclidian plane according to an homogeneous P.p.p of intensity  $\lambda$ . In this chapter we consider a scenario in which each receiver is at a fixed distance<sup>1</sup>  $r > 0$  from its transmitter, i.e.,  $|X_i - y_i| = r$  for all  $i$ . Consider a realization of the location of the transmitters. Let  $n = 0, 1, 2, \dots$  denote the index of time slots with respect to which all nodes are synchronized. We associate with each node a multi dimensional mark that carries information about the MAC status and the fading condition at each time slot. We follow the notation of [17][Chap. 17]. Let the sequence  $M_i = \{e_i(n), F_i(n)\}_{n \geq 0}$  denote the marks associated with node  $i$ , where

- $e_i = \{e_i(n)\}_{n \geq 0}$  denotes the sequence of MAC decisions of node  $i$ .  $e_i(n)$  is an indicator function that takes value 1 if node  $i$  decides to transmit in time slot  $n$ ; otherwise it takes value zero. The random variables  $e_i(n)$  are assumed to be independently and identically distributed (i.i.d.) in  $i$  and  $n$ , and independent of everything else.
- $F_i = \{F_i^j(n) : j \geq 1\}_{n \geq 0}$  denotes the sequence of channel conditions between the transmitter of node  $i$  and all receivers (including its own receiver). It is assumed that channel conditions are i.i.d. across the nodes and time slots, with a generic distribution on  $\mathbb{R}^+$  denoted by  $F$  with mean  $1/\mu$ .
- The marks are assumed to be independent in space and time.

The probability that the  $i$ th node transmits in time slot  $n$  is  $p := \Pr\{e_i(n) = 1\} = \mathbb{E}[e_i(n)]$  (MAP). This defines a pair of independent Poisson processes at each time slot  $n$ , one representing transmitters  $\Phi^1(n) = \{X_i, e_i(n) = 1\}$  and the other non-transmitters  $\Phi^0(n) = \{X_i, e_i(n) = 0\}$  with intensities  $p\lambda$  and  $(1 - p)\lambda$  respectively. All the nodes transmit at a fixed power denoted by  $P$ .

Let  $l(x, y)$  denote the attenuation function between any two given points  $x, y \in \mathbb{R}^2$ . We assume that this function just depends on the distance between the points, i.e.,  $|x - y|$ . With a slight abuse of notation we denote this function as  $l(x, y) = l(|x - y|)$ . We assume the following form for this attenuation function

$$l(x, y) = (B|x - y|)^{-\beta} \text{ for } B > 0 \text{ and } \beta > 2. \quad (2.1)$$

<sup>1</sup>Our analysis continues to hold when the distances between transmitters and their receivers are i.i.d. See [70].

A signal transmitted by a transmitter located at  $X_i$  is successfully received in time slot  $n$  if the SINR at the receiver is larger than some threshold  $T$ , i.e.,

$$\text{SINR}_i(n) := \frac{PF_i^l(r)}{I_{\Phi^1(n)}(y_i) + W(n)} > T, \quad (2.2)$$

where  $W(n)$  denotes the thermal noise power at the receiver and  $I_{\Phi^1(n)}$  denotes the shot noise of the P.p.p.  $\Phi^1(n)$  in time slot  $n$ , namely,  $I_{\Phi^1(n)}(y_i) = \sum_{X_j \in \Phi^1(n)} PFl(|X_j - y_i|)$ . We assume that the noise is an i.i.d. process.

Consider a typical node at the origin,  $X_0 = 0$  with mark  $M_0(0) = (e_0(0), F_0(0))$  at  $n = 0$ . The typical node is said to be covered in slot  $n = 0$  if (2.2) holds given that it is a transmitter. Then the coverage probability of the typical node is

$$\mathbf{P}^0 \left\{ \frac{PFl(r)}{I_{\Phi^1(0)}(y_0) + W(0)} > T \mid e_0(0) = 1 \right\}, \quad (2.3)$$

where  $\mathbf{P}^0$  denotes the Palm distribution [16][Chap. I] of the stationary marked P.p.p.  $\tilde{\Phi}$ . Note that due to time-homogeneity this conditional probability does not depend on  $n$ . By using Slivnyak's theorem [16], the coverage probability of a typical nodes when all other nodes use the same MAP is evaluated in [50]. Continuing the notation used in [33] we denote this coverage probability (non-outage probability) as  $p_c := p_c(r, p\lambda, T)$ . Consider a tagged node that uses MAP  $p'$ . Then the tagged node is a transmitter with probability  $p'$  and a non transmitter with probability  $(1 - p')$ . We refer to the product of the MAP and the coverage probability of the tagged node as *goodput* and denote it as  $g(p', p) := p' p_c(r, p\lambda, T)$ . We shall be interested in performance metrics of the form

$$\frac{g(p', p)^{1-\alpha}}{1-\alpha},$$

where  $\alpha \geq 0$  denotes the fairness parameter. In this chapter we consider the performance metric corresponding to  $\alpha = 0$  and  $\alpha = 2$ . For  $\alpha = 0$ , the performance metric corresponds to the goodput, and for  $\alpha = 2$ , it corresponds to the negative of *potential delay* introduced in [63], which is defined as the reciprocal of the rate. Potential delay may be interpreted as the delay incurred by a node in successfully delivering its packets at the receiver. Indeed, it has been shown in [17][Sec. 17] that the *local delay* is given as reciprocal of goodput when the node locations form an i.i.d. process across time slots. In all other cases it is a lower bound. Local delay is defined as the number of time slots needed for a node to successfully transmit a packet (with retransmissions). We denote the potential delay by  $t(p', p) = 1/g(p', p)$ .

In the following sections we consider two scenarios. First we assume that the nodes of the MANET cooperate, i.e., use the same MAP that is assigned to them in each time slot. The value of a MAP that optimizes the spatial network performance is evaluated. We then consider a game scenario in which each node is selfish and chooses a MAP that optimizes its own performance taking into account the transmission costs. We study

the effect of the transmission costs on the network performance at equilibrium and look for a pricing mechanism that results in improved spatial network performance at equilibrium. When pricing is not possible we study the degradation in network performance.

### 2.3 Rate Control: The Team Case

In this section we assume that all the nodes cooperate by agreeing to transmit using the same MAP. The following proposition immediately follows from [32][Lemma 3.2]:

**Proposition 2.3.1.** *Let each node in the Poisson bipole transmit with MAP  $p$  and the channel be Rayleigh fading (exponentially distributed with mean  $1/\mu$ ); then the goodput is*

$$g(p) = p \exp \left\{ -2\pi\lambda p \int_0^\infty \frac{u}{1 + l(r)/(Tl(u))} du \right\} \times \psi_w(\mu T / Pl(r)), \quad (2.4)$$

where  $\psi_w(\cdot)$  denotes the Laplace transform of the noise power  $W$ .

**Corollary 2.3.2.** *For  $F$  as above, zero noise power  $W \equiv 0$ , and the path loss model in (2.1), the goodput of a typical node is*

$$g(p) = p \exp \{ -2\pi\lambda p r^2 T^{2/\beta} K(\beta) \}, \quad (2.5)$$

where  $K(\beta) = \frac{\Gamma(2/\beta)\Gamma(1-2/\beta)}{\beta}$  and  $\Gamma(x) = \int_0^\infty z^{x-1} e^{-z} dz$ .

Henceforth we adopt the assumptions of Corollary 2.3.2 in all the subsequent calculations. However, our results hold when  $W$  has any distribution with finite mean, and this only requires adding a constant multiplicative factor in (2.5). For notational convenience we write  $C := C(\beta, T, r) = 2\pi r^2 T^{2/\beta} K(\beta)$ .

The cooperating users are interested in optimizing the social performance of the network. In particular, we assume that they aim to maximize the mean goodput per unit area or minimize the mean delay per unit area. The performance seen by a typical node can be used to derive the spatial performance of the Poisson bipolar MANET. The mean goodput per unit area is the product of the intensity of the P.p.p. and the goodput, i.e.,  $\lambda g(p)$ . This quantity is referred to as the density of success and denoted by  $d_{suc}(r, p\lambda, T)$  in [17][Chap. 16]. We denote this term simply by  $d_s(p)$ . Similarly, the mean delay per unit area is given by  $\lambda t(p)$ . We denote this spatial performance metric by  $d_t(p)$  and refer to it as spatial delay density. Note that the spatial delay density is the reciprocal of the density of success multiplied by a factor  $\lambda^2$ . Hence the MAP that maximizes  $d_s(p)$  also minimizes  $d_t(p)$ . The MAP that optimizes the density of success is given in [17][Prop 16.8] and [17][Corol. 16.9]:

**Proposition 2.3.3.** *Under the assumption of Corollary 2.3.2, the MAP that maximizes the density of success and minimizes the density of delay is given by*

$$p_m = \min\{1, 1/\lambda C\}, \quad (2.6)$$

the corresponding optimal density of success is given by

$$d_s(p_m) = \begin{cases} 1/(e\lambda C), & \text{if } \lambda C > 1 \\ \lambda \exp\{-\lambda C\}, & \text{if } \lambda C \leq 1, \end{cases} \quad (2.7)$$

and the corresponding optimal delay density is given by

$$d_t(p_m) = \begin{cases} \lambda^2 eC, & \text{if } \lambda C > 1 \\ \lambda \exp\{\lambda C\}, & \text{if } \lambda C \leq 1. \end{cases} \quad (2.8)$$

## 2.4 Rate Control: The Non-Cooperative Case

In this section we assume that each node of the Poisson bipolar network is a selfish player. We consider a non-cooperative case, and model it as a game with an infinite number of players as follows. We use node and player interchangeably.

Each node can take two actions: transmit ( $A$ ) or no-transmit ( $\bar{A}$ ). We assume that when taking the decision whether to transmit or not, a player does not know the positions of the other mobiles nor the level of the SINR. A mixed strategy chosen by a node corresponds to its MAP. A player chooses its MAP once for all and always uses the same MAP. The choice is done without knowledge of the realization of the interference.

We shall restrict ourselves to symmetric Nash equilibria (SNE), in which all nodes use the same MAP  $p$  at equilibrium. The utility of a *tagged* player can then be written as a function of his strategy  $p'$  and of the strategy  $p$  of all the others. We denote the utility of the tagged player by  $U(p', p)$ . The objective of each player is to maximize its utility<sup>2</sup>. Let  $U(p^*, p^*)$  denote the value of the utility function at SNE  $p^*$ .

**Definition 2.4.0.1.**  $p^* \in [0, 1]$  is said to be a symmetric Nash equilibrium if for any node, the following holds

$$U(p^*, p^*) = \max_{p \in [0, 1]} U(p, p^*).$$

In the rest of this section we describe utility functions of an individual player. We begin with computing the goodput of a tagged node. The next lemma follows from Lemma 3.1 and Corollary 3.2 in [32].

**Lemma 2.4.1.** Consider a Poisson MANET with the assumptions in Corollary 2.3.2. Let a tagged node transmit with MAP  $p'$ , while all other players transmit with MAP  $p$ . Then the goodput of the tagged node is

$$g(p', p) = p' \exp\{-p\lambda C\}. \quad (2.9)$$

where  $C = 2\pi r^2 T^{2/\beta} K(\beta)$  is defined earlier.

We note the following:

---

<sup>2</sup>The game has infinitely many players, hence the utility should be defined on a infinite product strategy space. However, since we are restricting ourselves to the case where all players other than the tagged node use the same strategy  $p$ , we write the utility as a function of two arguments.

1. The goodput is monotone increasing in  $p'$ . Hence, if the objective of each mobile is to maximize its goodput, then the only equilibrium is  $p = 1$  for all nodes.
2. Under the conditions of the above lemma, the expected transmission energy consumption of the tagged node per time slot is proportional to  $p'$  and does not depend on the MAP  $p$  of the other nodes. Thus the ratio between the goodput and the expected energy consumption does not depend on  $p'$ . We conclude that any  $p$  is an equilibrium point when the criterion of each node is that of minimizing the ratio between goodput and energy.

Keeping the above remarks in mind and noting that the utility related to goodput is not a linear function of the MAP, see for example [64], we shall be interested in utilities<sup>3</sup> of the form

$$U(p', p) = f(p', p) - \alpha w(p'), \quad (2.10)$$

for each player, where  $f$  gives the performance measure of interest and  $w$  is related to dis-utility. The multiplicative factor  $\alpha$  denotes the tradeoff factor, i.e., the importance each node assigns to its transmission cost relative to the performance metric. For reasons explained in the introduction, we consider a dis-utility proportional to the expected transmission energy costs incurred by each node. Let  $\rho$  denote the price per unit transmission energy for each node. Then the expected transmission cost for the tagged node that uses MAP  $p'$  is  $\rho p' P$ . Without loss of generality we assume  $P = 1$ , and define  $w(p') = \rho p'$ .

Assume that the function  $f$  is a concave function in  $p'$  and continuous in  $p$ ; then the arguments in [49][Thm. 1] can be used to show the existence of SNE. We state this result in the following lemma.

**Lemma 2.4.2.** *Assume that the utility function  $U(p', p)$  is concave in  $p'$  and continuous in  $p$ . Then SNE exist.*

In this chapter we assume that  $\rho$  is set by the owner of the proprietary spectrum, a regulator who can regulate its value. In the next section, with some abuse of notation, we denote the product  $\alpha\rho$  simply as  $\rho$  and refer to it as the price factor. The aim of the regulator is to optimize the performance of the network at equilibrium.

In the next two sections we consider two utility functions defined in terms of goodput and potential delay as the performance measure. We evaluate the MAP at equilibrium and the corresponding system performance. This system performance is then optimized by searching for the best price factor. The best achievable system performance at equilibrium is then compared with that evaluated when nodes cooperate as in Section 2.3.

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<sup>3</sup>With some abuse of notation we use the notation  $U(p)$  to denote the expected system utility when the same  $p$  is used by all players. We consider utilities which attain optimal values for some  $p$  and denote the latter, again with some abuse of notation, by  $p_m$ . It will be clear from the context whether  $p_m$  is an optimizer of the spatial performance metric or the team utility.

## 2.5 Goodput Based Utility

Assume that each node of the MANET is interested in maximizing its goodput taking into account the cost it incurs. Then by taking  $f(p', p) = g(p', p)$  in (2.10), we define the utility as

$$U(p', p) = g(p', p) - \rho p' = p' \{ \exp\{-p\lambda C\} - \rho \}. \quad (2.11)$$

The objective of a each node is to choose a MAP that maximizes its utility, i.e.,

$$p' \in \operatorname{argmax}_{0 \leq p' \leq 1} U(p', p).$$

This utility function is a linear function in  $p'$  and convex in  $p$ . Then, from Lemma 2.4.2, a SNE exists. We proceed to calculate the SNE of this game.

When  $\rho \geq 1$ , the slope of the utility of the tagged node is non positive irrespective of the MAP of the other nodes. Then the optimal strategy for each node is to choose  $p = 0$ , which is a dominant strategy and hence the SNE.

When  $\rho < 1$ , consider the following two cases.

Assume  $\rho < \exp\{-\lambda C\}$ : In this case the slope of the tagged node is always positive. Then the optimal strategy for the tagged node is to choose  $p = 1$  irrespective of the MAP chosen by the other nodes. Thus  $p = 1$  is a dominant strategy and hence also is the SNE.

Assume  $\rho \geq \exp\{-\lambda C\}$ : If each node other than the tagged node chooses a MAP such that

$$\exp\{-p\lambda C\} = \rho, \quad (2.12)$$

then the utility of the tagged node in (2.11) is set to zero and is not affected by its strategy, i.e., the tagged node becomes indifferent to its own strategy. Further, if any node deviates from the MAP that satisfies (2.12), it will not gain anything given that all other nodes set their MAP value as in (2.12). Hence the MAP satisfying (2.12) constitutes the SNE. We summarize the above observations in the following proposition.

**Proposition 2.5.1.** *For any given  $C, \lambda$ , and  $\rho > 0$*

- *if  $\rho \geq 1$  then  $p^* = 0$  is the SNE;*
- *if  $\exp\{-\lambda C\} > \rho$  then  $p^* = 1$  is the SNE;*
- *if  $\exp\{-\lambda C\} \leq \rho < 1$  then  $p^* = \frac{-\log \rho}{\lambda C}$  is the SNE.*

The goodput of each node at equilibrium is given by

$$g(p^*, p^*) = \begin{cases} 0, & \text{if } \rho \geq 1 \\ \exp\{-\lambda C\}, & \text{if } \rho \leq \exp\{-\lambda C\} \\ \frac{-\rho \log \rho}{\lambda C}, & \text{if } \exp\{-\lambda C\} < \rho < 1. \end{cases} \quad (2.13)$$

We can now identify the value of  $\rho$  that maximizes the above expression for goodput. We assume that the objective of the regulator is to maximize the density of success at



equilibrium. The optimization problem of the regulator is given by

$$\begin{aligned} & \underset{\rho}{\text{maximize}} && \frac{-\lambda\rho \log \rho}{\lambda C} \\ & \text{subject to} && \exp\{-\lambda C\} < \rho < 1. \end{aligned} \quad (2.14)$$

The objective function in (2.14) is a concave function of  $\rho > 0$  attaining its maximum at  $\rho = 1/e$ . If  $\lambda C > 1$ , the optimal price factor lies within the constraint set and the operator can set  $\rho^* = 1/e$ . Suppose  $\lambda C \leq 1$ ; then the objective function is decreasing in the interval  $\exp\{-\lambda C\} < \rho < 1$  and the maximum is attained at  $\rho^* = \exp\{-\lambda C\}$ . The maximum density of success at equilibrium with the optimal price factor is

$$d_s(p^*, p^*) = \begin{cases} 1/(e\lambda C), & \text{if } \lambda C > 1 \\ \lambda \exp\{-\lambda C\}, & \text{if } \lambda C \leq 1. \end{cases} \quad (2.15)$$

Comparing (2.15) with the global optimal density of successful transmissions given in (2.7), we have the following result.

**Theorem 2.5.1.1.** *The global optimal density of successful transmissions can be attained at equilibrium if the pricing factor  $\rho$  satisfies the following:*

$$\rho^* = \begin{cases} 1/e, & \text{if } \lambda C > 1 \\ \exp\{-\lambda C\}, & \text{if } \lambda C \leq 1. \end{cases} \quad (2.16)$$

With the pricing factor  $\rho^*$  in (2.16), the MAP of each node at equilibrium is the same as that needed to achieve the global optimum given in (2.6). Thus by pricing appropriately, one can use the selfish behavior of the players to reach an equilibrium at which the global optimal performance is attained.

In practice, the pricing factor  $\rho$  may reflect direct sensitivity to power or pricing by some provider (a relation with a provider is needed whenever a terminal uses licensed spectrum). The result of the above theorem are useful in the latter case.

**Replicator Dynamics:** In this subsection we briefly discuss how we can use tools from population games and evolutionary dynamics to study the aggregate behavior in the network. The aim of this subsection is to touch upon the dynamic aspects of the game studied in this section. The discussion is concise, and the results discussed here are not used elsewhere in the chapter. We begin by mapping the MAC game of this section into a population game. One can view the set of nodes in the Poisson bipolar MANET as a single population, where each node can take two actions: either to transmit ( $A$ ) or not ( $\bar{A}$ ). Let each node decide to transmit with probability  $p$ , then by our assumption that nodes decisions are i.i.d in time and space, a fraction  $p$  of the population will be transmitting and the complementary fraction will be non-transmitting.

One of the most frequently used dynamics to describe the evolution of the behavior in population games is the replicator dynamics [65]. It describes the evolution of the fraction of the population that take a particular action. In terms of population games, we can interpret the probability of successful transmission minus the transmission cost as the fitness function for each node. Let us denote the fitness of a node that chooses



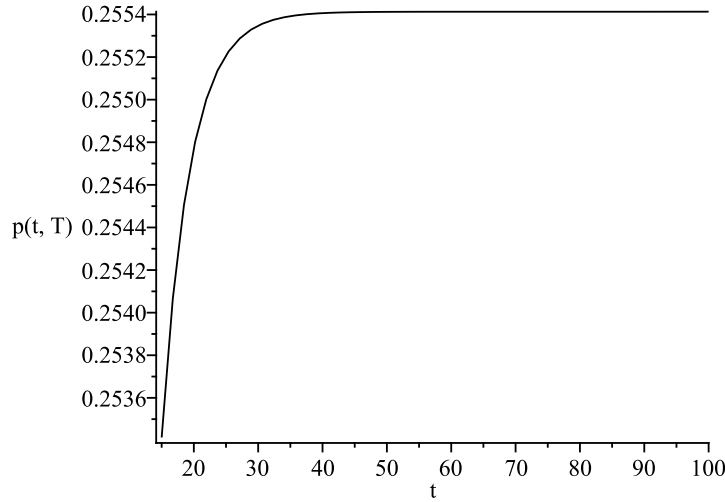
action  $X \in \{A, \bar{A}\}$  while the rest of the population transmits with probability  $p'$  by  $V(X, p')$ . The utility of a node using mixed strategy  $p$ , as given in (2.11), and its fitness function are related as  $U(p, p') = pV(A, p') + (1 - p)V(\bar{A}, p')$ . The fraction of nodes that uses action  $A$  evolves according to the replicator dynamics as follows [65][Ch. 5,6]:

$$dp(t, A)/dt = p(t, A)[V(A, p(t, A)) - \bar{V}(t)],$$

where  $\bar{V}(t)$  denotes the mean fitness of the population given by  $\bar{V}(t) = p(t, A)V(A, p(t, A)) + (1 - p(t, A))V(\bar{A}, p(t, A))$ . Let each node incur a transmission cost of  $\rho$  units per transmission attempt. Then the fitness function of a transmitting node is  $V(A, p') = p_c(r, p'\lambda, T) - \rho$ . Note that in our medium access game the fitness of a node that is not transmitting is zero. The replicator dynamics is given by

$$\frac{dp(t, A)}{dt} = p(t, A)(1 - p(t, A)) \left\{ e^{-\lambda C p(t, A)} - \rho \right\}.$$

From the above equation the stationary point is clearly 0, 1, or  $\frac{\log \rho}{\lambda C}$  depending on the value of  $\rho$ , which is in agreement with Proposition 2.5.1. Figure 2.1 shows the convergence of replicator dynamics to the SNE starting from any interior point.



**Figure 2.1:** Convergence of Replicator dynamics, with  $\lambda C = 2, \rho = 0.6$  and starting value=.2

## 2.6 Delay Based Utility

In this section we define the utility of each player in terms of the delay associated in delivering a packet successfully at its receiver, and the transmission costs. As earlier, let a tagged node incur a cost of  $\rho$  units per unit energy dissipated. By taking  $f(p', p) = -t(p', p)$  in (2.10), where  $t(p', p)$  denotes the potential delay, we get the following utility for the tagged node:

$$U(p', p) = -\frac{1}{p' \exp\{-p\lambda C\}} - p'\rho. \quad (2.17)$$

where  $p'$  is the MAP of the tagged node, and  $p$  is the MAP used by all other nodes.

$\lambda\bar{C}$	Price factor	SNE ( $p^*$ )
$\lambda\bar{C} < 1$	$\rho \leq \rho_0$	1
	$\rho > \rho_0$	$-\mathbb{W}_0(-\lambda\bar{C}/\sqrt{\rho})/\lambda\bar{C}$
$\lambda\bar{C} \geq 1$	$\rho < (e\lambda\bar{C})^2$	1
	$\rho_{-1} \geq \rho \geq (e\lambda\bar{C})^2$	$-\mathbb{W}_0(-\lambda\bar{C}/\sqrt{\rho})/\lambda\bar{C}$ or $-\mathbb{W}_{-1}(-\lambda\bar{C}/\sqrt{\rho})/\lambda\bar{C}$
	$\rho \geq \rho_{-1}$	$-\mathbb{W}_0(-\lambda\bar{C}/\sqrt{\rho})/\lambda\bar{C}$

**Table 2.1:** Characterization of symmetric Nash equilibria

The objective of each node is to choose a MAP that maximizes its utility function (2.17). Or equivalently it can be given by the following minimization problem:

$$\begin{aligned} & \underset{p'}{\text{minimize}} \quad \frac{\exp\{p\lambda C\}}{p'} + p'\rho \\ & \text{subject to} \quad 0 \leq p' \leq 1. \end{aligned}$$

For a given value of  $p$ , the utility function in (2.17) is a concave function in  $p'$  and continuous in  $p$ . Then by Lemma 2.4.2, SNE exist. We proceed to calculate the SNE by a direct computation.

Differentiating the utility function with respect to  $p'$ , equating to zero, and simplifying yields  $p' = \exp\{p\lambda\bar{C}\}/\sqrt{\rho}$ , where  $\bar{C} = C/2$ . This equation gives the best response of the tagged node when all other nodes use MAP  $p$ . If there exists a  $p \in [0, 1]$  such that  $p' = p$ , then  $p$  is the SNE of the game. Hence we look for the conditions when the following fixed point relation holds

$$p = \frac{\exp\{p\lambda\bar{C}\}}{\sqrt{\rho}}. \quad (2.18)$$

**Lemma 2.6.1.** *If  $\lambda\bar{C}e > \sqrt{\rho}$ ,  $p^* = 1$  is the unique SNE. If  $\lambda\bar{C}e \leq \sqrt{\rho}$ , the SNE is*

$$p^* = \min \left\{ \frac{-1}{\lambda\bar{C}} \mathbb{W} \left( -\frac{\lambda\bar{C}}{\sqrt{\rho}} \right), 1 \right\}, \quad (2.19)$$

where  $e$  denotes the base of the natural logarithm and  $\mathbb{W}(\cdot)$  denotes the Lambert function [51].

*Proof.* By using the relation  $e^x \geq xe$  for all  $x \geq 0$ , it is easy to see that  $p^* = 1$  is the SNE when  $\lambda\bar{C}e > \sqrt{\rho}$ . Assume  $\lambda\bar{C}e \geq \sqrt{\rho}$ ; rearranging the terms in (2.18), we can write  $-p\lambda\bar{C}\exp\{-p\lambda\bar{C}\} = \frac{-\lambda\bar{C}}{\sqrt{\rho}}$ . Using the definition of the Lambert function [51], we obtain

$$p = \frac{-1}{\lambda\bar{C}} \mathbb{W}\left(-\frac{\lambda\bar{C}}{\sqrt{\rho}}\right). \quad (2.20)$$

□

With this explicit expression for the SNE we can characterize the effect of the price factor on the SNE. Before we proceed to characterize the SNE let us briefly recall the properties of the Lambert function:

- The Lambert function is two-valued in the interval  $[-1/e, 0]$ . The two branches of the Lambert function denoted by  $\mathbb{W}_0(\cdot)$  and  $\mathbb{W}_{-1}(\cdot)$  meet at  $-1/e$  and take value  $-1$  at this point.
- In the interval  $[-1/e, 0]$   $\mathbb{W}_0(\cdot)$  is a continuous, increasing function taking values in  $[-1, 0]$ .
- In the interval  $[-1/e, 0]$   $\mathbb{W}_{-1}(\cdot)$  is a continuous, decreasing function taking values in  $[-\infty, -1]$ .

For some values of  $\rho$  the resulting SNE is not unique as shown in the following lemma. For notational simplicity we write  $\rho_t = (e\lambda\bar{C})^2$ .

**Lemma 2.6.2.** *Assume  $\lambda\bar{C} \geq 1$ ; then there exists a  $\rho_{-1} \geq \rho_t$  such that for all  $\rho$  satisfying  $\rho_{-1} \geq \rho \geq \rho_t$  two SNE are possible. One resulting from the  $\mathbb{W}_0$  and the other from the  $\mathbb{W}_{-1}$  branch of the Lambert function. For all  $\rho$  satisfying  $\rho > \rho_{-1}$ , the SNE is unique and lies on the  $\mathbb{W}_0$  branch of the Lambert function.*

Note that for  $\rho \in [\rho_t, \rho_{-1}]$ , the equilibrium MAP computed on the  $\mathbb{W}_{-1}$  is larger than that computed on the  $\mathbb{W}_0$  branch. If nodes reach the equilibrium that is resulting from the  $\mathbb{W}_{-1}$  branch, then they will be transmitting aggressively at equilibrium. Later we will see that this leads to inefficiency (See Remark 2.6.6).

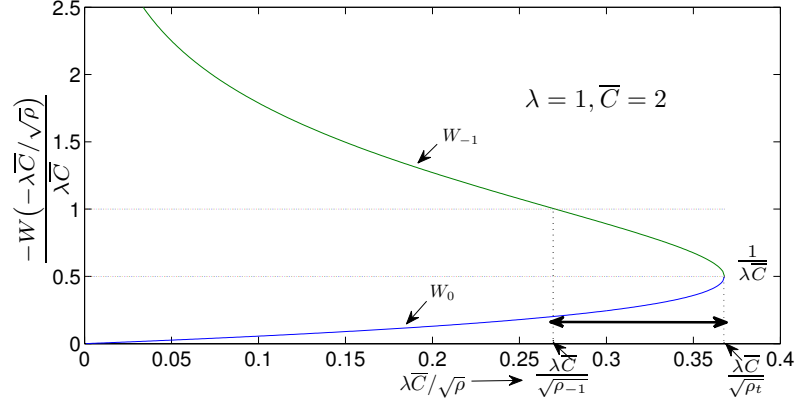


Figure 2.2: SNE on both branches.

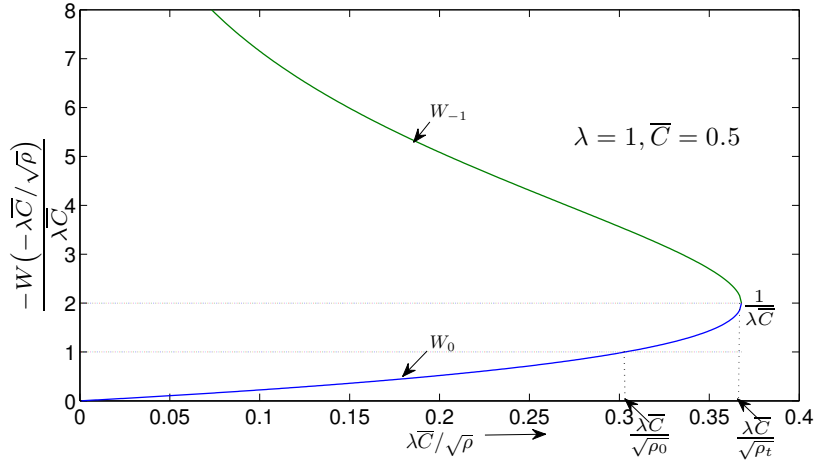


Figure 2.3: Unique SNE.

**Lemma 2.6.3.** Assume  $\lambda \bar{C} < 1$ ; then there exists a  $\rho_0 \geq \rho_t$  such that for all  $\rho$  satisfying  $\rho_0 \geq \rho \geq \rho_t$ , the SNE is  $p^* = 1$ . For all  $\rho$  satisfying  $\rho \geq \rho_0$ , the SNE is unique and lies on the  $W_0$  branch of the Lambert function.

Lemma 2.6.1 together with 2.6.2 and 2.6.3 completely characterize all possible equilibria. We summarize all the SNE in Table 2.1. Figures 2.2 and 2.3 illustrate the Lambert function and its properties used in the proof of Lemma 2.6.2 and 2.6.3. In Figure 2.2, the  $y$  axis marked with a double arrow denotes the range of price factor that results in two equilibrium points.

**Stability of SNE:** In Figure 2.4 we plot the best response of the tagged node and that of all nodes against the tagged node. This example demonstrates the existence of two SNE. The smaller SNE among the two results from the principal branch of the Lambert function in (2.20), and the larger one from the  $W_{-1}$  branch. When two equilibrium points exist, we denote the SNE resulting from the  $W_0$  branch of the Lambert function by  $p_0^*$  and that resulting from the  $W_{-1}$  branch as  $p_{-1}^*$ . When we do not need to make this distinction or it is unique, we write the SNE as  $p^*$ . From Figure 2.4 we see that, at the equilibrium point  $p_0^*$ , a slight increase in the MAP  $p$  results in a decrease in  $p'$ . This is a stabilizing behavior and we conclude that  $p_0^*$  is stable. In contrast, at the equilibrium point  $p_{-1}^*$ , a slight increase in  $p$  is seen to cause an increase in the value of  $p'$ . Thus the second equilibrium point is unstable.

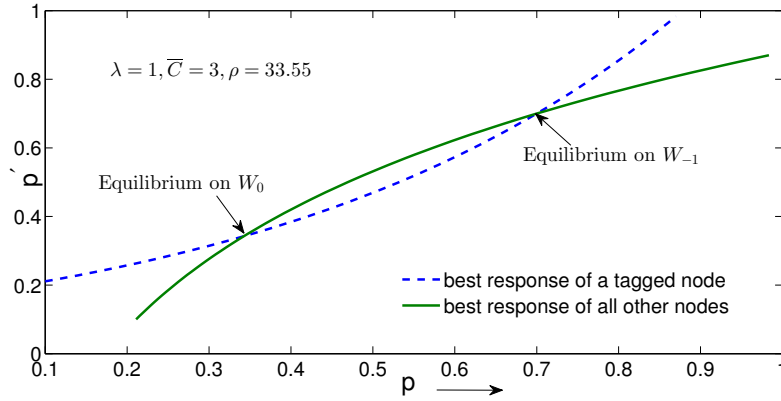


Figure 2.4: Best Response.

**Optimal Pricing:** Assume that  $p^*$  is an equilibrium point for a given value of  $\rho$  that satisfies the conditions in Lemma 2.6.2 or 2.6.3 for a given value of  $\lambda$  and  $\bar{C}$ . The potential delay experienced by the packets of a tagged node at equilibrium can be computed as

$$\begin{aligned} t(p^*, p^*) &= \frac{1}{p^* \exp\{-p^* \lambda \bar{C}\}} \\ &= \frac{1}{p^* \exp\{-p^* \lambda \bar{C}\} \exp\{-p^* \lambda \bar{C}\}} \end{aligned} \quad (2.21)$$

$$= \frac{\sqrt{\rho}}{\exp\{\mathbb{W}(-\lambda \bar{C} / \sqrt{\rho})\}} \quad (2.22)$$

$$= \frac{\sqrt{\rho} \mathbb{W}(-\lambda \bar{C} / \sqrt{\rho})}{\mathbb{W}(-\lambda \bar{C} / \sqrt{\rho}) \exp\{\mathbb{W}(-\lambda \bar{C} / \sqrt{\rho})\}} \quad (2.23)$$

$$= \frac{\sqrt{\rho} \mathbb{W}(-\lambda \bar{C} / \sqrt{\rho})}{(-\lambda \bar{C} / \sqrt{\rho})} = -\frac{\rho \mathbb{W}(-\lambda \bar{C} / \sqrt{\rho})}{\lambda \bar{C}}. \quad (2.24)$$

In the above chain of equalities the relation  $C = 2\bar{C}$  is used in (2.21). (2.22) follows from (2.18). Equation (2.24) follows by applying the definition of the Lambert function to the denominator term in (2.23) and rearranging. From (2.24) we see that the delay experienced by the tagged node is not unique for some price factors. It is larger when the SNE results from the  $W_{-1}$  branch of the Lambert function.

With the expression for the delay of a tagged node at equilibrium, one can look for the value of the price factor that minimizes the delay experienced by each node at equilibrium. We assume that the objective of the regulator is to minimize the average delay per unit area, i.e., spatial delay density, at equilibrium. The optimization problem of the regulator is:

$$\begin{aligned} & \underset{\rho}{\text{minimize}} && -\frac{\rho W(-\lambda\bar{C}/\sqrt{\rho})}{\bar{C}} \\ & \text{subject to} && \sqrt{\rho} \geq \lambda\bar{C}e \\ & \text{and} && -W(-\lambda\bar{C}/\sqrt{\rho}) \leq \lambda\bar{C}. \end{aligned} \tag{2.25}$$

The first constraint in this optimization problem results from Lemma 2.6.1 and the second ensures that the resulting value of  $p$  in (2.18) lies in the interval  $[0, 1]$ .

Let  $h(\rho) := -\rho W(-\lambda\bar{C}/\sqrt{\rho})$  denote the objective function in the above optimization problem without the multiplicative factor  $1/\bar{C}$ .  $h(\rho)$  is defined for  $\sqrt{\rho} \geq e\lambda\bar{C}$ . In the following lemma we state some of its properties.

**Lemma 2.6.4.** *On the principal branch  $W_0$ ,  $h(\rho)$  is a quasi convex function in  $\rho$  and the global minimum is attained at  $\rho^* = 4e(\lambda\bar{C})^2$ . On the  $W_{-1}$  branch  $h(\rho)$  is a monotonically increasing function taking value  $(e\lambda\bar{C})^2$  at  $\rho = (e\lambda\bar{C})^2$ .*

Consider the optimization problem in (2.25) on the  $W_0$  branch of the Lambert function. The value of  $\rho^*$  at which  $h(\rho)$  achieves minima satisfies the first condition in (2.25) as  $\rho^* = 4e(\lambda\bar{C})^2 \geq e^2(\lambda\bar{C})^2$ . The value of  $p$  in Equation (2.18) at  $\rho = \rho^*$  is  $1/(2\lambda\bar{C})$ . This implies that whenever  $2\lambda\bar{C} = \lambda C > 1$ , the resulting value of  $p$  lies inside  $(0, 1)$ , thus satisfying the second condition of the optimization problem. Hence under the assumption  $\lambda C > 1$ , the global minimizer of  $h(\rho)$  lies in the constraint set of (2.25) and the value of the objective function at this point is

$$-\frac{\rho^* W(-\lambda\bar{C}/\sqrt{\rho^*})}{\lambda\bar{C}} = -\frac{(4e(\lambda\bar{C})^2)(-1/2)}{\lambda\bar{C}} = e\lambda C. \tag{2.26}$$

When  $\lambda C \leq 1$ , any value of  $\rho$  such that  $\rho < \rho^*$  violates the second condition, as  $-W(-\lambda\bar{C}/\sqrt{\rho}) > 1/2$ . From Lemma 2.6.4, the minimum value of  $h(\rho)$  is achieved at a  $\rho$  satisfying  $-W(-\lambda\bar{C}/\sqrt{\rho}) = \lambda\bar{C}$ . This implies that under the condition  $\lambda C \leq 1$ , the value of the SNE is  $p^* = 1$  and the delay experienced by each node is given by  $\exp\{-\lambda C\}$ . We summarize these observations in the following result.

**Proposition 2.6.5.** *The value of the price factor  $\rho^*$  that minimizes the delay at equilibrium is as follows:*

$$\rho^* = \begin{cases} (2\lambda\bar{C}\sqrt{e})^2 & \text{if } \lambda C > 1 \\ -2W(-\lambda\bar{C}/\sqrt{\rho^*}) = \lambda C, & \text{if } \lambda C \leq 1 \end{cases} \tag{2.27}$$

and the corresponding delay at equilibrium is

$$d_t(p^*, p^*) = \begin{cases} \lambda^2 e C & \text{if } \lambda C > 1 \\ \lambda \exp\{\lambda C\}, & \text{if } \lambda C \leq 1. \end{cases} \quad (2.28)$$

Comparing (2.28) and (2.8) we conclude the following:

**Theorem 2.6.5.1.** *The spatial delay density in the game problem at equilibrium equals the global optimal spatial delay density, i.e.,  $d_t(p_m) = d_t(p^*, p^*)$ , if the price factor is chosen as in (2.27).*

Again, by appropriately pricing the nodes, the selfish behavior can be used to attain the global optimal performance in the game problem. If the SNE is not unique at the optimal price factor  $\rho^*$ , then one needs to ensure that the nodes reach the equilibrium that is computed on the principal branch of the Lambert function to realize the global optimal performance at equilibrium. Indeed, if  $\lambda C > 1$  and  $-\mathbb{W}_{-1}(-1/2\sqrt{e}) \leq \lambda \bar{C}$  or equivalently  $\rho_{-1} \geq 4e(\lambda \bar{C})^2$ , at the optimal price  $\rho^*$ , an equilibrium point exists on the  $\mathbb{W}_{-1}$  branch of the Lambert function. It would be interesting to learn about the way to reach a favorable equilibrium at the optimal price factor. However we do not pursue this question in this chapter.

**Remark 2.6.6.** *If  $\lambda \bar{C} > 1$  and optimization is restricted to the  $\mathbb{W}_{-1}$  branch in (2.25), then by Lemma 2.6.4, the objective function is minimized by choosing a  $\rho$  satisfying  $\sqrt{\rho} = e\lambda \bar{C}$ , which results in the equilibrium probability  $1/(\lambda \bar{C})$ . The spatial density of delay at this equilibrium point is given by*

$$-\frac{\rho \mathbb{W}(-\lambda \bar{C}/\sqrt{\rho})}{\lambda \bar{C}} = \frac{(e\lambda \bar{C})^2}{\lambda \bar{C}} = (e/2)e\lambda \bar{C}.$$

Comparing this value with (2.8), we see that spatial density of delay increased by a factor of  $e/2$  by the selfish behavior of the nodes.

## 2.7 Non Cooperative case without regulator

In the previous section we considered the non cooperative case in the presence of a regulator. The aim of the nodes was to optimize their performance metric taking into account the transmission costs. The regulator was interested in the spatial performance and not in the utility of the nodes. In this section we consider a scenario in which there is no regulator who influences the transmission costs through pricing. We revisit the game dynamics studied in the previous section assuming that each node incurs a fixed energy cost of  $\rho$  units per transmission attempt. With some abuse of notation we denote the product  $\alpha\rho$  simply as  $\alpha$ , the tradeoff factor, and compute the SNE<sup>4</sup>. In this setting we study the Price of Anarchy (PoA) of the resulting game.

The PoA compares the social utility at the worst equilibrium with the optimal social utility [60]. For our Poisson bipolar MANET with infinitely many players, we define

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<sup>4</sup>The SNE depends on the product  $\alpha\rho$ , not on their individual values. In the previous section we computed the SNE as a function of  $\rho$  for a given  $\alpha$ , in this section we compute the SNE as a function of  $\alpha$  for a given  $\rho$ .

the PoA as the ratio of the optimal spatial average performance that can be achieved, to the spatial average performance at the worst SNE. Recall that we denoted the system utility by  $U(p)$  when we considered the team problem, with each node using the same MAP  $p$ . For the game problem we denoted the utility of a tagged node by  $U(p', p)$ . In the game problem, the spatial average performance at equilibrium is evaluated by multiplying the utility of the tagged node and the intensity of the P.p.p. Then the PoA is given by

$$PoA = \frac{\max_{p \in [0, 1]} U(p)}{\min_{p^* \in S} \lambda U(p^*, p^*)}, \quad (2.29)$$

where  $S \subset [0, 1]$  denotes the set of symmetric Nash equilibria.

We study the PoA as a function of  $\alpha$  for a given value of  $\lambda$  and  $C$ . The utilities studied in Sections 2.5 and 2.6 are considered below.

### 2.7.1 Goodput

In this subsection we consider the utility defined in Section 2.5. Let us begin by considering the team utility. When all the nodes use the MAP  $p$ , then from Equation (2.11), the team utility is given by

$$U(p) = \lambda p \exp\{-p\lambda C\} - \lambda p\alpha. \quad (2.30)$$

If  $\alpha > 1$ , the maximum value of the utility is zero and the maximum is attained at  $p_m = 0$ . Let  $p_m := p_m(\alpha)$  denote the MAP that maximizes the team utility in (2.30).

**Lemma 2.7.2.** *Let  $\alpha \in [0, 1]$  and  $W(\alpha e) \geq 1 - \lambda C$ , then the MAP value that maximizes the team utility (2.30) is given by*

$$p_m = \frac{1 - W(\alpha e)}{\lambda C},$$

and the corresponding maximum team utility is given by  $U(p_m) = \frac{\alpha(1 - W(\alpha e))^2}{CW(\alpha e)}$ .

By using the definition of the Lambert function, one can verify that  $U(p_m)$  is a decreasing function in  $\alpha$ . Indeed, differentiating  $U(p_m)$  with respect to  $\alpha$  we have

$$\begin{aligned} \frac{\partial U(p_m)}{\partial \alpha} &= \frac{(1 - W(\alpha e))^2}{CW(\alpha e)} - \\ &\quad \alpha e \frac{(1 - W(\alpha e))}{CW(\alpha e)} \frac{(1 + W(\alpha e))W'(\alpha e)}{CW(\alpha e)} \end{aligned} \quad (2.31)$$

$$= \frac{(1 - W(\alpha e))^2}{CW(\alpha e)} - \frac{(1 - W(\alpha e))}{CW(\alpha e)}. \quad (2.32)$$

In (2.31)  $W'$  denotes the derivative of the Lambert function. Equation (2.32) follows by applying the formula for the derivative of the Lambert function. The last expression is negative for all  $\alpha \in [0, 1]$ . Thus the optimal utility is a decreasing function in  $\alpha$ .



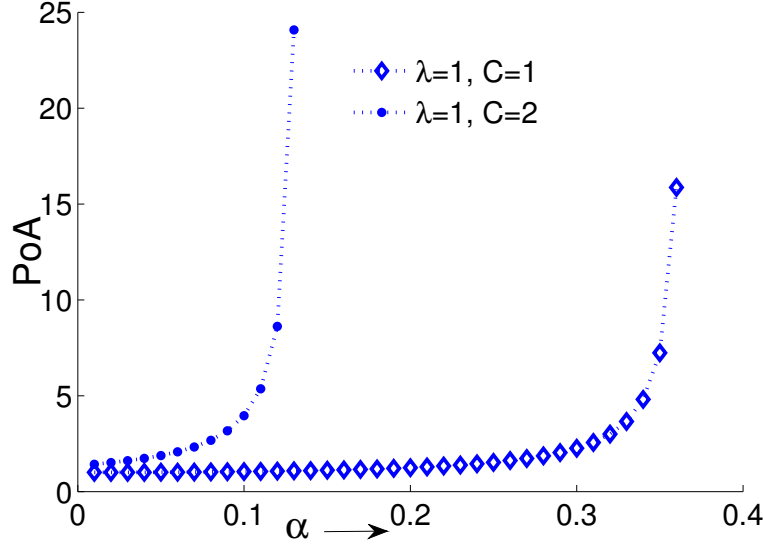


Figure 2.5: PoA for Goodput

Let us look at the utility of the tagged node at equilibrium. From Proposition 2.5.1 we have

$$U(p^*, p^*) = \begin{cases} \exp\{-\lambda C\} - \alpha, & \text{if } \alpha \leq \exp\{-\lambda C\} \\ 0, & \text{if } \alpha \geq \exp\{-\lambda C\}. \end{cases} \quad (2.33)$$

The utility at equilibrium is also a decreasing function in  $\alpha$  for all  $\alpha \leq \exp\{-\lambda C\}$ . Using the expression for utility at equilibrium and global optimum we have the following result for PoA:

**Theorem 2.7.2.1.** *The value of the PoA is*

$$PoA(\alpha) = \begin{cases} \frac{\alpha(1 - W(\alpha e))^2}{\lambda C W(\alpha e) \{\exp\{-\lambda C\} - \alpha\}}, & \text{if } \alpha < \exp\{-\lambda C\} \\ \infty, & \text{if } \alpha \geq \exp\{-\lambda C\}. \end{cases} \quad (2.34)$$

The PoA is shown as a function of  $\alpha$  in Figure 2.5. From this figure we see that as  $\alpha$  increases, the PoA grows unboundedly. Thus the PoA is optimal when the tradeoff factor is zero. If  $\lambda C \geq 1$ , the PoA is infinite by definition at  $\alpha = e^{-1}$ . However we noted in Section 2.5 that the optimal performance of the spatial density of success is achieved at equilibrium with the same price factor. If  $\lambda C < 1$  then the PoA is infinite at  $\alpha = \exp\{-\lambda C\}$ . But again we noted in Section 2.5 that at this price factor, the optimal performance of the spatial density of success is achieved at equilibrium.

In Figure 2.6 the equilibrium MAP and global optimal MAP are shown. For all values of  $\alpha$ , the equilibrium MAP is larger than the global optimal MAP. Hence the nodes transmit more aggressively at equilibrium. But we note from Figure 2.6 that the gap between the global optimal MAP and the equilibrium MAP reduces with the tradeoff factor.

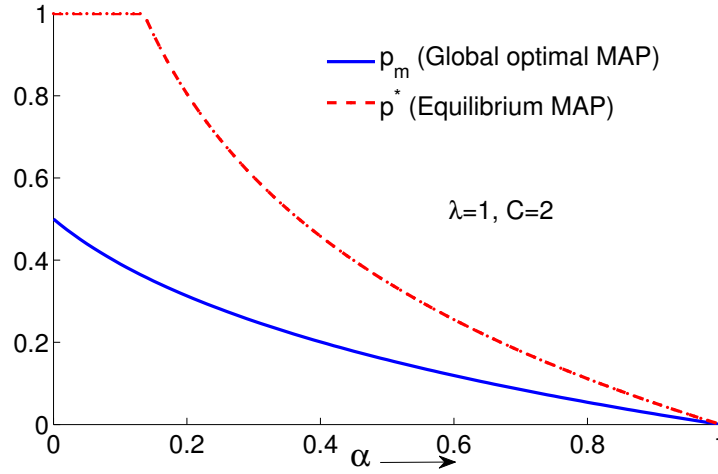


Figure 2.6: Equilibrium and Optimal MAP

### 2.7.3 Delay

Consider the utility function in Equation (2.17). The team utility for this game, when each node transmits with MAP  $p$  is

$$U(p) = \frac{-\lambda}{p \exp\{-p\lambda C\}} - \lambda \alpha p. \quad (2.35)$$

It is easy to verify that the above utility function is concave in  $p$ . Assume that  $\lambda C > 1$ . Then the unique MAP, denoted by  $p_m := p_m(\alpha)$  that maximizes the social utility satisfies

$$\exp\{p_m \lambda C\} (1 - p_m \lambda C) = \alpha p_m^2. \quad (2.36)$$

We obtain this by differentiating Equation (2.35) and setting to zero. Note that any  $p_m$  that satisfies Equation (2.36) also satisfies  $p_m \lambda C \leq 1$ , hence  $p_m \in [0, 1]$ . Also, it can be easily verified that  $p_m$  is decreasing in  $\alpha$ .

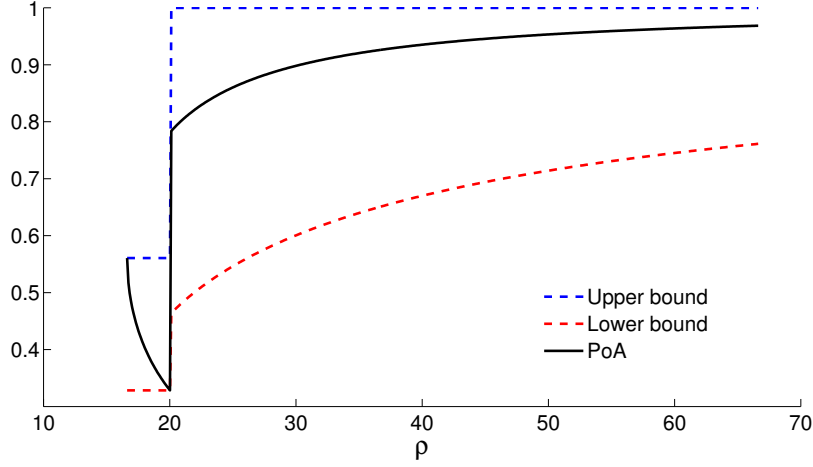


Figure 2.7: PoA for delay,  $C=3$ ,  $\lambda=1$

The utility at equilibrium can be obtained by using the equilibrium MAP in (2.20) and (2.24), as<sup>5</sup>

$$U(p^*, p^*) = (-2\lambda\alpha/\lambda\bar{C})\mathcal{W}\left(\frac{-\lambda\bar{C}}{\sqrt{\alpha}}\right). \quad (2.37)$$

When  $\lambda C > 1$ , from Lemma 2.6.2, two symmetric Nash equilibria are possible for the tradeoff factor  $\alpha \leq \rho_{-1}$ . Hence the above utility function can take two values, one corresponding to each equilibrium. Recall that we denoted by  $p_0^*$  the SNE computed on the principal branch, and by  $p_{-1}^*$  that computed on the other branch of the Lambert function. Also recall that  $p_0^* \leq p_{-1}^*$ .

**Theorem 2.7.3.1.** Assume  $\lambda\bar{C} > 1$ ; For the Poisson bipolar MANETs with utility as in (2.17),

$$\begin{aligned} \frac{p_m(\rho_{-1})(2 - p_m(\rho_{-1})\lambda C)}{2(1 - p_m(\rho_{-1})\lambda C)} &\leq \text{PoA}(\alpha) \\ &\leq \frac{p_m(\rho_t)\lambda\bar{C}(2 - p_m(\rho_t)\lambda C)}{2(1 - p_m(\rho_t)\lambda C)} \end{aligned} \quad (2.38)$$

for  $\alpha \in [\rho_t, \rho_{-1}]$ . In addition

$$\frac{p_m}{p_0^*} \leq \text{PoA}(\alpha) \leq 1 \text{ for } \alpha \geq \rho_{-1}. \quad (2.39)$$

The PoA as a function of  $\alpha$  and the bounds obtained in Theorem 2.7.3.1 are shown in Figure 2.7. The jump in the figure at  $\alpha = \rho_{-1}$  is due to two possible SNE for  $\alpha \leq \rho_{-1}$  and a unique SNE for  $\alpha > \rho_{-1}$ . In the interval  $[\rho_t, \rho_{-1}]$  the PoA is decreasing in  $\alpha$ . Thus the

---

<sup>5</sup>Recall that we are denoting the product  $\alpha\rho$  as  $\alpha$  in this section, where as we denoted it as  $\rho$  in the previous section

best PoA value is achieved at  $\alpha = \rho_t$ . This results from the bad Nash equilibrium that occurs on the  $W_{-1}$  branch of the Lambert function which increases in  $\alpha$ . For  $\alpha > \rho_{-1}$ , SNE is unique, and decreasing<sup>6</sup> in  $\alpha$ . Thus as  $\alpha$  increases above the value  $\rho_{-1}$ , the value of  $p_0^*$  approaches the value of optimal solution  $p_m$  resulting in improved price of anarchy.

## 2.8 Conclusions

Geometric considerations play a very central role in wireless communications, since the attenuation of wireless channels strongly depend on the distance between transmitter and receiver. Models that take into account the exact location of mobiles are often too complex to analyze or to optimize. Our objective in this chapter is to model the competition between mobiles as a game in which the locations of players is given by a Poisson point process.

In this chapter we considered a competition between individual mobiles each taking its own selfish decisions. We saw that the equilibrium of the game results in a more aggressive access (larger access probabilistically). We identified pricing parameters that induce an equilibrium achieving the social optimal performance.

In the future, we plan to study other games within this framework: for instance games with finitely many operators, each taking decisions for all its subscribers.

## 2.9 Appendices

### 2.9.1 Proof of Lemma 2.4.2

Consider a point to set map  $\delta : [0, 1] \rightarrow [0, 1]$  defined by

$$\delta(p) = \left\{ p' \mid U(p', p) = \max_{q \in [0, 1]} U(q, p) \right\}.$$

This defines the set of best responses of the tagged node when all the other nodes use the MAP  $p$ . It follows from the continuity of  $U(p', p)$  in  $p$  and concavity in  $p'$  for a fixed  $p$ , that  $\Gamma$  is an upper continuous mapping that maps each point of the set  $[0, 1]$  into a subset of  $[0, 1]$ . By the Kakutani fixed point theorem, there exists a point  $p^* \in [0, 1]$  such that

$$U(p^*, p^*) = \max_{p' \in [0, 1]} U(p', p^*).$$

Then  $p^*$  is the symmetric Nash equilibria by Definition 2.4.0.1.

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<sup>6</sup>The principal branch of the Lambert function is decreasing in  $\alpha$ .

### 2.9.2 Proof of Lemma 2.6.2

First consider the  $W_0$  branch of the Lambert function. As  $\rho$  takes value in the interval  $[\rho_t, \infty]$ ,  $W_0\left(\frac{-\lambda\bar{C}}{\sqrt{\rho}}\right)$  increases continuously from  $-1$  to  $0$ . Thus (2.18) has a solution in the interval  $[0, 1]$  if  $\lambda\bar{C} \geq 1$ . This implies that equilibrium point exists on the  $W_0$  branch for all  $\rho$ , satisfying  $\rho \geq \rho_t$ .

The  $W_{-1}\left(\frac{-\lambda\bar{C}}{\sqrt{\rho}}\right)$  branch decreases continuously from  $-1$  to  $-\infty$  as  $\rho$  takes value in the interval  $[\rho_t, \infty]$ . This implies that there exists a  $\rho_{-1} \geq \rho_t$  such that  $W_{-1}\left(\frac{-\lambda\bar{C}}{\sqrt{\rho_{-1}}}\right) = -\lambda\bar{C}$ , and for all  $\rho$  such that  $\rho_{-1} \geq \rho \geq \rho_t$  satisfies  $-W_{-1}\left(\frac{-\lambda\bar{C}}{\sqrt{\rho}}\right) \leq \lambda\bar{C}$ , resulting in a  $p \in [0, 1]$  that is a solution of (2.18). Hence there exists an equilibrium point on the  $W_{-1}$  branch for all  $\rho$  satisfying  $\rho_{-1} \geq \rho$ . This concludes the proof.

### Proof of Lemma 2.6.3

As in Lemma 2.6.2 we can argue that on the  $W_0$  branch, there exists  $\rho_0 \geq \rho_t$  such that  $W_0\left(-\frac{\lambda\bar{C}}{\sqrt{\rho_0}}\right) = -\lambda\bar{C}$  and for all  $\rho$  such that  $\rho \geq \rho_0$  satisfies  $-W_0\left(\frac{-\lambda\bar{C}}{\sqrt{\rho}}\right) \leq \lambda\bar{C}$  as  $-W_0$  is decreasing in  $\rho$ .

### 2.9.3 Proof of Lemma 2.6.4

Differentiating  $h(\rho)$  with respect to  $\rho$

$$\begin{aligned} \frac{d}{d\rho}h(\rho) &= -W\left(-\lambda\bar{C}/\sqrt{\rho}\right) - W'\left(-\lambda\bar{C}/\sqrt{\rho}\right) (\lambda\bar{C}/2\sqrt{\rho}) \\ &= -W\left(-\lambda\bar{C}/\sqrt{\rho}\right) \left(1 - \frac{1}{2(1 + W\left(-\lambda\bar{C}/\sqrt{\rho}\right))}\right), \end{aligned} \quad (2.40)$$

where  $W'(\cdot)$  denotes the derivative of the Lambert function which is given as [51][eqn. 3.2]

$$W'(x) = \frac{W(x)}{x(1 + W(x))} \quad \text{for } x \neq 0, x \neq -1/e. \quad (2.41)$$

Equation (2.40) is obtained by substituting the derivative in (2.41), evaluated at  $x = -\lambda\bar{C}/\sqrt{\rho}$ . Recall that on the principal branch of the Lambert function  $W(-\lambda\bar{C}/\sqrt{\rho})$  is a negative valued increasing function in  $\rho$ . Then the term within parenthesis in (2.40) is a increasing function of  $\rho$  passing through the origin at  $\rho^*$  that satisfies  $W(-\lambda\bar{C}/\sqrt{\rho^*}) =$

$-1/2$ . Thus  $h(\rho)$  is decreasing for  $\rho \leq \rho^*$  and increasing for  $\rho \geq \rho^*$ . From [52][sec. 3.4.2] we conclude that  $h(\rho)$  is a quasi convex function in  $\rho$ .

Further by the definition of the Lambert function

$$\begin{aligned} -\lambda\bar{C}/\sqrt{\rho^*} &= \mathbb{W}(-\lambda\bar{C}/\sqrt{\rho^*}) \exp\{\mathbb{W}(-\lambda\bar{C}/\sqrt{\rho^*})\} \\ &= -\frac{1}{2} \exp\{-1/2\}. \end{aligned} \quad (2.42)$$

Rearranging Equation (2.42), we get  $\rho^* = 4e(\lambda\bar{C})^2$ . The other part of the Lemma follows by noting that  $-\mathbb{W}(-\lambda\bar{C}/\sqrt{\rho})$  is an increasing function in  $\rho$  on the  $\mathbb{W}_{-1}$  branch.

#### 2.9.4 Proof of Theorem 2.7.3.1

From Equation (2.37) and (2.35) we have

$$PoA(\alpha) = \frac{\exp\{p_m\lambda C\}/p_m + \alpha p_m}{(-2\alpha/\lambda\bar{C})\mathbb{W}\left(-\frac{\lambda\bar{C}}{\sqrt{\alpha}}\right)} \quad (2.43)$$

$$= \frac{p_m/(1 - p_m\lambda C) + p_m}{(-2/\lambda\bar{C})\mathbb{W}\left(-\frac{\lambda\bar{C}}{\sqrt{\alpha}}\right)} \quad (2.44)$$

$$= \frac{p_m(2 - p_m\lambda C)}{(-2/\lambda\bar{C})\mathbb{W}\left(-\frac{\lambda\bar{C}}{\sqrt{\alpha}}\right)(1 - p_m\lambda C)} \quad (2.45)$$

$$\geq \frac{p_m}{(-1/\lambda\bar{C})\mathbb{W}\left(-\frac{\lambda\bar{C}}{\sqrt{\alpha}}\right)}. \quad (2.46)$$

We arrive at equality (2.44) by dividing both numerator and denominator in (2.43) by  $\alpha$ , and applying the relation in (2.36). Equality (2.45) is obtained by simple rearrangement of terms in the previous step. To derive the bounds in (2.38), we consider the equilibrium computed on the  $\mathbb{W}_{-1}$  branch of the Lambert function as it leads to the worst case equilibrium. This equilibrium is an increasing function in  $\alpha$  in the interval  $\rho_t \leq \alpha \leq \rho_{-1}$  as discussed in the proof of Lemma 2.6.2. Also, recall that the value of  $p_m$  is decreasing in  $\alpha$ . Thus the numerator in (2.44) is decreasing in  $\alpha$ . Which implies that the ratio in (2.45) is also decreasing in  $\alpha$ . The upper bound in (2.38) now follows by noting that  $-\mathbb{W}(-\lambda\bar{C}/\sqrt{\rho_t}) = 1$ . To obtain the lower bound we use the relation  $-\mathbb{W}(-\lambda\bar{C}/\sqrt{\rho_{-1}}) = \lambda\bar{C}$  in (2.46).

For values of  $\alpha$  larger than  $\rho_{-1}$  the SNE is unique, resulting from the principal branch of the Lambert function. The upper bound in (2.39) follows directly by the definition of PoA, and the lower bound follows from the Inequality (2.46). Note that the lower bound is a function of  $\alpha$ .

#### 2.9.5 Proof of Lemma 2.7.2

Assume  $\alpha \leq 1$ , then  $p_m > 0$  and satisfies the relation  $\exp\{-p_m\lambda C\}(1 - p_m\lambda C) = \alpha$ . By rearranging,  $p_m$  can be expressed  $p_m = (1 - \mathbb{W}(\alpha e))(\lambda C)$ . Recall that  $\mathbb{W}(\cdot)$  is a

monotonically increasing function taking values  $\mathbb{W}(0) = 0$  and  $\mathbb{W}(e) = 1$ . If  $\lambda C \geq 1$  the  $p_m$  lies in the interval  $[0, 1]$  for all  $\alpha \in [0, 1]$ . If  $\lambda C < 1$ , then  $p_m$  lies in the interval  $[0, 1]$  for all  $\alpha$  such that  $\mathbb{W}(\alpha e) \geq 1 - C\lambda$ . Thus whenever  $\lambda C < 1$  we assume that  $\alpha$  satisfies  $\mathbb{W}(\alpha e) \geq 1 - C\lambda$ . The optimal value of the utility function is

$$\begin{aligned} & p_m \exp\{-p_m \lambda C\} \\ &= \frac{1 - \mathbb{W}(\alpha e)}{\lambda C} \exp\{-1 + \mathbb{W}(\alpha e)\} \end{aligned} \quad (2.47)$$

$$= \frac{1}{e\lambda C} \{\exp\{\mathbb{W}(\alpha e)\} - \alpha e\} \quad (2.48)$$

$$= \frac{1}{e\lambda C} \left\{ \frac{\alpha e}{\mathbb{W}(\alpha e)} - \alpha e \right\} = \frac{\alpha}{\lambda C} \left\{ \frac{1}{\mathbb{W}(\alpha e)} - 1 \right\}, \quad (2.49)$$

where (2.47) is obtained by substituting the value of MAP that maximizes the team utility. Equations (2.48) and (2.49) follow by application of the definition of the Lambert function. The maximum utility for the team case can be now computed as a function of  $\alpha$

$$U(p_m) = \frac{\alpha(1 - \mathbb{W}(\alpha e))^2}{C\mathbb{W}(\alpha e)}.$$

## Chapter 3

# Stochastic Geometry based Jamming Games in Mobile Ad hoc Networks

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### 3.1 Introduction

In this chapter, we study an adversarial situation in wireless Mobile Ad hoc NETWORKS (MANETs) called the jamming game consisting of two players, namely, an Operator and a Jammer. The Operator owns the MANET and his objective is to maximize a suitably defined spatial performance metric and that of the Jammer is to minimize it. In addition, the players take into account the power consumption disutility incurred in order to achieve the throughput. The jamming game we consider this chapter amounts to choosing appropriate medium access probabilities in the presence of the adversary jammer while the nodes of the MANET cooperate among themselves. Viewing the channel and the available transmit power as resources, this jamming game can also be viewed as a resource allocation problem in the presence of an adversary.

Game theory is extensively used to study resource allocation problems in networks [43]. These games often include information theoretic aspects and/or communication theory. In that context, game theory has not only been applied to situations of competition over resources but also to adversarial situations, which can often be modeled as zero-sum games. As in the previous chapter, our objective in this chapter is to study



jamming game taking into account spatial aspects of nodes distribution and that of the jammer. We model the locations of both the nodes of the MANET and that of the Jammer as a Poisson point process.

**Literature on Jamming Games:** Jamming are among the first problems involving conflicts in networks that have been modeled and solved using game theory. The first publications on these games go thirty years back with the pioneering work [34, 35]. The question of the capacity achievable in channels prone to jamming was one of the main concerns, and was thus naturally studied within the information theory community, see e.g. [36, 37, 40]. For a recent survey on wireless games that includes jamming games, see [43]. Not only abstract jamming models have been studied using game theory, but also jamming of specific wireless local area networks, see [42] that study the jamming of IEEE 802.11. In some cases an adversary jammer may have access to signalling or information channels, and may be able to harm more by jamming these than by jamming the data transfer itself. Examples are jammers that interfere the signalling protocols, see e.g. [47] who study jamming signalling channels in a cellular network. [41] studies the jamming game in multihop radio networks with ALOHA multiple access scheme and Poisson assumption on the node distribution. The paper considers a scenario in which the network operator aims to maximize the expected forward progress of packets in a given direction, where as a Jammer aims to minimize it. The resulting Nash equilibria of the zero sum game are investigated numerically.

In [18], in the context of cellular networks explicit expression are obtained for coverage and throughput with the Poisson assumption on the base stations. In [46], we study the Poisson MANET where each node is selfish. The Nash equilibria are characterized by considering the transmission energy costs.

**Our Contributions:** The main contributions of the paper are the following:

- We model a jamming game in the Poisson MANETs with constraints on the transmission energies for both the Jammer and Operator. We introduce an anti-potential that allows us to study the jamming game as a zero-sum game.
- Considering the Poisson bipolar model, we obtain explicit expression for the saddle point of the game by considering utilities based on throughput and delay. We then study the effect of the Jammer transmission power on the equilibrium. We shown that by transmitting at a power higher than certain level, the Operator can operate without the Jammer being active at equilibrium.
- In the receiver selection model, we again obtain explicit expression for the saddle point considering throughput as the performance metric. We observe similar behavior as in the case of bipolar model with fixed receiver.

The chapter is organized as follows: Section 3.2 we discuss the performance metrics of interest. In Section 3.3 we model the jamming game and characterize the resulting equilibria of the Poisson bipolar model with density of successful transmission and density of delay as performance metrics. In Section 3.4 we consider cases where the distance between a node's transmitter and its receiver is not fixed. Finally, we end with some concluding remarks in Section 8.9.

## 3.2 Model and Setup

We use the same Poisson bipolar model introduced in the previous chapter. Below we recall the relevant spatial performance metrics.

### 3.2.1 Spatial Performance metrics

Let us assume that all the nodes belong to a single Operator. If the nodes have information about other nodes in their vicinity, they can achieve a better performance by cooperating: Nodes moving from dense clusters to sparse clusters can increase their MAP as chances of collision reduce. Nodes moving from sparse clusters to dense clusters can reduce their MAP as chances of collision increase. We assume that, because of mobility, nodes cannot gather such information in each time slot and transmit with a MAP  $q_1$  that is set by the Operator. Then the typical node is a transmitter with probability  $q_1$  and a non-transmitter with probability  $(1 - q_1)$ . We call the product of the MAP and the coverage probability of the typical node the *goodput* and denote it as

$$g(q_1) := q_1 p_c(r, q_1 \lambda_1, T). \quad (3.1)$$

where  $T$  is the SINR threshold for successful reception and  $r$  is the distance between a transmitter and its receiver. The set of nodes that transmit in a given slot form another Poisson point process of intensity  $\lambda_1 q_1$ . Let  $d_s(q_1)$  denote the average number of successful transmissions in a unit area. By Campbell's formula [16][Sec. 2.1.2] for stationary Poisson point processes this spatial performance metric is given as the product of goodput and the intensity of the Poisson point process, i.e.,  $d_s(q_1) = \lambda_1 q_1 p_c(r, q_1 \lambda_1, T)$ . We shall also consider the mean delay experienced by the nodes in successfully transmitting the packets to their associated receiver. We assume that if a packet transmission fails then the packet is retransmitted till success. We also assume that nodes can resample the channel in each slot. If we further assume that that locations of the nodes are independently and identically distributed (i.i.d) across the time slot, then by Little's theorem [54] the mean delay for a typical node is given by reciprocal of its goodput given in (3.1). Let  $d_t(q_1)$  denote the spatial mean delay of nodes per unit area. Then again using Campbell's formula we get  $d_t(q_1) = \lambda_1 / q_1 p_c(r, q_1 \lambda_1, T)$ . We refer to this quantity as *spatial density of delay*. Note that we need the i.i.d assumption on the nodes realization only to derive the spatial mean delay, and not on the performance metric goodput.

In the following sections we analyze the spatial performance of the network in the presence of a Jammer. In the next section we introduce our game model. We study two jamming games, one with density of success as the spatial performance metric and the other with density of delay.

### 3.3 Jamming Game

In this section we consider a jamming game in the Poisson bipolar MANET. Assume that there is a Jammer who also has nodes that are scattered according to a Poisson point process in the same geographical area. The Jammer aims to degrade the performance of the Operator by causing interference.

Let  $\lambda_2$  denote the intensity of the nodes of the Jammer. The transmitters of the Jammer can re-sample the channel in each time slot and transmit at a fixed power  $P_2$ . If a transmitter of the Jammer is surrounded by a dense cluster of nodes belonging to the Operator, then the degradation in performance due to the interference from the Jammer may not be significant; as the success probability is already low. On the contrary, if a transmitter of the Jammer is surrounded by a sparse cluster of nodes belonging to the Operator then, the degradation in performance due to interference from the Jammer could be significant. However, we assume that, due to random mobility, the Jammer cannot gather such information. Also the Jammer incurs costs from the energy transmissions. Thus we assume that the Jammer keeps each of its transmitters turned ON with probability  $q_2$  independently of its other transmitters. The transmitters of the Jammer form a Poisson point process of intensity  $q_2\lambda_2$ .

Recall that  $q_1$  denotes the MAP set by the Operator. A transmission of a typical node of the Operator gets interference from all the other nodes that are transmitting. Thus a typical node gets interference from the nodes that form a Poisson point process of intensity  $q_1\lambda_1 + q_2\lambda_2$ . In this section we assume that the channel between the receiver of the typical node and transmitters of all other nodes, both the Jammer and the Operator, is distributed according to the random variable  $S$  and take the same path loss model in (2.1). The probability that a transmissions by the typical node is successful in the presence of the Jammer can be easily computed following the same steps as in [32][Lemma 3.2]. With some abuse of notation we represent this quantity by  $p_c(q_1, q_2)$ .

**Lemma 3.3.1.** *Let each node of the Operator in the Poisson bipolar MANET transmit with MAP  $q_1$  and let the Jammer turns ON each of its transmitters with probability  $q_2$ . Assume that  $S$  is Rayleigh distributed with parameter  $\mu$  and noise power is zero, then*

$$p_c(q_1, q_2) = \exp\{-C(q_1\lambda_1 + q_2\lambda_2Q)\}, \quad (3.2)$$

where  $C = 2\pi r^2 T^{2/\beta} K(\beta)$ , and  $Q = (P_2/P_1)^{2/\beta}$

In the following, we consider performance metrics that are a monotonically increasing functions of this success probability. Let  $d(q_1, q_2)$  denote the generic performance metric of the Poisson MANET that the Operator is interested in. In the remaining of the paper when we write a performance metric with two arguments  $q_1$  and  $q_2$ , it is understood that it is calculated in the presence of the Jammer who turns ON each of its node with probability  $q_2$ , and the nodes of the Operator uses MAP  $q_1$ .

From Equation (3.2) it is clear that the Operator can improve the performance of its nodes by increasing their transmission power, whereas the jammer can decrease it by increasing the transmitting power of its nodes. Thus it is beneficial for both the Operator and the Jammer to increase the transmission power of their nodes. However,

we assume that each node incurs a transmission cost due to limited power available to them. The average density of power dissipated among the nodes of the Operator is  $\lambda_1 q_1 P_1$  and that among the nodes of the Jammer is  $\lambda_2 q_2 P_2$ . For jamming games with energy constraints in cellular networks see [45]. Let  $\rho_1$  and  $\rho_2$  denote the cost incurred by the Operator and the Jammer per unit power transmitted, respectively. More formally, we define the strategy, utility, and objective of the game between the Jammer and the Operator as following:

**Strategy:** The strategy of the Operator is to choose a MAP  $q_1 \in [0, 1]$  with which each of its nodes can access the channel. The strategy of the Jammer is to set a probability  $q_2 \in [0, 1]$  with which a transmitter is turned ON in a given slot. Denote the joint strategy as  $\mathbf{q} \in (q_1, q_2) = [0, 1] \times [0, 1]$ .

**Utilities and objective:** The objective of the Operator is to choose a MAP  $q_1$  that maximizes the density of success of its nodes taking into account average transmission costs among its nodes. We define the utility of the Operator as

$$U_1(q_1, q_2) = d(q_1, q_2) - \rho_1 \lambda_1 q_1 P_1. \quad (3.3)$$

The objective of the Jammer is to set a probability  $q_2$ , with which each of its transmitters is turned ON in a given slot, that minimizes the density of the successful transmission of the Operator taking into account the average transmission costs incurred among its transmitters. We define the utility of the Jammer as

$$U_2(q_1, q_2) = -d(q_1, q_2) - \rho_2 \lambda_2 q_2 P_2. \quad (3.4)$$

In terms of the optimization problems, we can write the objective of the Operator as to choose a  $q_1^*$  such that

$$q_1^* \in \operatorname{argmax}_{q_1 \in [0, 1]} U_1(q_1, q_2), \quad (3.5)$$

and for the Jammer it is to choose a  $q_2^*$  such that

$$q_2^* \in \operatorname{argmax}_{q_2 \in [0, 1]} U_2(q_1, q_2). \quad (3.6)$$

Note that the jamming game with the utilities in (3.3) and (3.4) is not a zero sum game. This game can be studied as a zero sum game with an anti-potential function obtained by modifying the utilities of the players as below.

**Anti potential:** If we add the term  $\rho_2 \lambda_2 q_2 P_2$  to the utility function of the Operator, then the set of optimal values  $q_1^*$  in equation (3.5) does not change. We define the modified utility function of the Operator as

$$\bar{U}_1(q_1, q_2) = d(q_1, q_2) - \rho_1 \lambda_1 q_1 P_1 + \rho_2 \lambda_2 q_2 P_2$$

Similarly, if we add the term  $\rho_1 \lambda_1 q_1 P_1$  to the utility function of the Jammer, then the set of optimal values  $q_2^*$  in equation (3.6) does not change. We define the modified utility function of the Jammer as

$$\bar{U}_2(q_1, q_2) = -d(q_1, q_2) - \rho_2 \lambda_2 q_2 P_2 + \rho_1 \lambda_1 q_1 P_1.$$

If we consider a new game with this modified utilities, then the resulting equilibrium, if exists, will be same as in the original game. Notice that the modified utilities are such that  $\bar{U}_1(\mathbf{q}) = -\bar{U}_2(\mathbf{q})$  for all  $\mathbf{q}$ . Hence, the new game constitutes a zero sum game. In the following subsections we consider performance metrics discussed in Subsection 3.2.1, and analyze the resulting equilibria.

### 3.3.2 Spatial density of Success as performance metric

In this subsection we assume that the Operator is interested in the number of successful transmission per unit area. We refer to this performance metric as density of successful transmissions, and can be computed as

$$d(q_1, q_2) = \lambda_1 q_1 p_s(q_1, q_2).$$

The anti-potential of the jamming game with this performance metric is given by

$$\begin{aligned} \bar{U}_1(q_1, q_2) = & \lambda_1 q_1 \exp\{-C(q_1 \lambda_1 + q_2 \lambda_2 Q)\} \\ & - \lambda_1 q_1 \rho_1 P_1 + \lambda_2 q_2 \rho_2 P_2. \end{aligned}$$

The following lemma states relevant properties of this anti-potential function  $\bar{U}_1(\mathbf{q})$ .

**Lemma 3.3.3.**  $\bar{U}_1(q_1, q_2)$  is a quasi concave function in  $q_1$  and convex in  $q_2$ . Assume that  $\rho_1 P_1 < \exp\{-\lambda_2 C Q\}$ , then  $\bar{U}_1(\cdot, q_2)$  achieves the maximum in the interval  $(0, 1]$  if  $\lambda_1 C \geq 1$ .

The utility function  $\bar{U}_1(q_1, q_2)$  is continuous in both the arguments. The strategy space of both the players is convex and compact. From Lemma 3.3.3, and using the Sion's minmax theorem [48] we can readily conclude that the jamming game has a saddle point, i.e., there exists a  $\mathbf{q}^* = (q_1^*, q_2^*)$  such that for all  $(q_1, q_2)$

$$\bar{U}_1(q_1, q_2^*) \leq \bar{U}_1(q_1^*, q_2^*) \leq \bar{U}_1(q_1^*, q_2).$$

By transforming the jamming game into a zero-sum game we easily concluded that the saddle point of the jamming game exists. We can also explicitly obtain the saddle point as given in the following proposition. The saddle points are characterized in terms of the Lambert function [51] which we denote as  $W(x)$ .

**Proposition 3.3.4.** Assume that  $\lambda_1 C \geq 1$ . The Nash equilibrium of the jamming game are as following. If  $\rho_1 P_1 \geq 1$ , then  $(0, 0)$  is an equilibrium. If  $\rho_1 P_1 < \exp\{-\lambda_2 C Q\}$  we have the following cases.

- If  $\rho_2 P_2 \geq \bar{q}_1 \lambda_1 C Q \exp\{-\bar{q}_1 \lambda_1 C\}$  then  $(\bar{q}_1, 0)$  is an equilibrium, where  $\bar{q}_1$  is given by

$$\bar{q}_1 = \frac{1 - W(e \rho_1 P_1)}{\lambda_1 C}. \quad (3.7)$$

- If  $\rho_2 P_2 \leq \bar{q}_1 \lambda_1 C Q \exp\{-\bar{q}_1 \lambda_1 C\} \exp\{-C \lambda_2 Q\}$  then  $(\bar{q}_1, 1)$  is an equilibrium, where  $\bar{q}_1$  is given by

$$\bar{q}_1 = \frac{1 - W(e \rho_1 P_1 \exp\{\lambda_2 C Q\})}{\lambda_1 C}. \quad (3.8)$$

If  $(q_1^*, q_2^*)$  is an interior equilibrium point then

$$q_1^* = \frac{1}{\lambda_1 C (1 + Q(\rho_1 P_1 / \rho_2 P_2))}, \quad (3.9)$$

$$q_2^* = \frac{-1}{CQ} \log \left\{ \frac{\rho_2 P_2 \exp\{\lambda_1 q_1^* C\}}{\lambda_1 q_1^* CQ} \right\}. \quad (3.10)$$

One can interpret the results of the above proposition as follows. Under the condition  $\rho_1 P_1 \geq 1$  the utility of the Operator is always negative, i.e., the total transmission energy cost for the Operator is high, hence it is optimal for the Operator not to allow any transmissions from its nodes. Once the nodes of the MANET are not active then the Jammer will also turn OFF its transmitter. The condition in the first bullet give a scenario in which the operating cost for the Jammer is too high to allow any transmissions from its nodes. Hence the Jammer keeps all of its nodes turned OFF at equilibrium. The condition in the second bullet gives a scenario in which the transmission costs for the Jammer is low and all the transmitters are turned ON at equilibrium. Equation (3.10) results in an interior point if

$$P_2 \rho_2 < q_1^* \lambda_1 Q C \exp\{-q_1^* \lambda_1 C\} \quad \text{and}$$

$$P_2 \rho_2 > Q C q_1^* \lambda_1 \exp\{-q_1^* \lambda_1 C\} \exp\{-\lambda_1 C Q\},$$

where  $q_1^*$  is given by Equation (3.9).

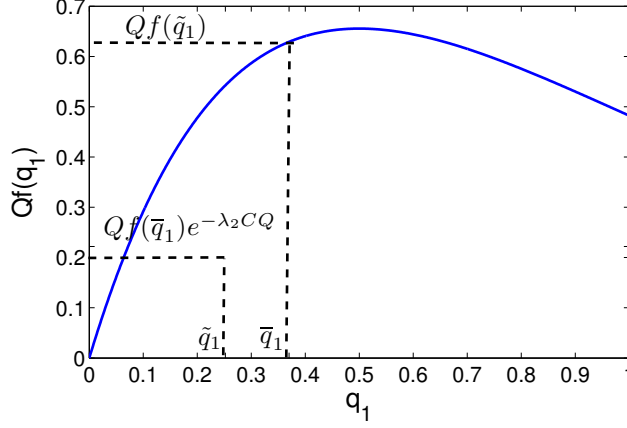
Let us proceed to study the effect of power and cost factors on the equilibria. Let  $q_1^*(q_2)$  denote the best response of the Operator when the Jammers' strategy is  $q_2$ . From Equation (3.34) it is easy to note that the best response of the Operator is a decreasing function in  $q_2$ . Hence  $\bar{q}_1 \leq q_1^*(q_2) \leq \tilde{q}_1$  for all  $q_2 \in (0, 1)$ . Where  $\tilde{q}_1$  is the best response when  $q_2 = 0$ , and  $\bar{q}_1$  is the best response when  $q_2 = 1$ . To understand the bounds on the product  $\rho_2 P_2$  given in Proposition 3.3.4 more clearly, let us define a function  $f : [0, 1] \rightarrow \mathbb{R}_+$  given by  $f(q) = q \lambda_1 C \exp\{-q \lambda_1 C\}$ . The function  $f$  attains maxima at  $q = 1/\lambda_1 C$ , and is increasing in the interval  $[0, 1/\lambda_1 C]$ . Thus it is clear that

$$f(\tilde{q}_1) \geq f(q_1^*(q_2)) \geq f(\bar{q}_1) \quad \text{for all } q_2 \in (0, 1).$$

This verifies the consistency of the bounds on  $P_2 \rho_2$  given in Proposition 3.3.4 and are shown in Figure 3.1.

Note that the upper bound and lower bound on  $\rho_2 P_2$  in Proposition 3.3.4 that determines the value of equilibrium  $(q_1, q_2)$  are functions of the power transmitted by the Jammer and the operator. In Figure 3.2, we show the variations of  $Q f(\bar{q}_1) \exp\{-\lambda_2 C Q\}$  and  $Q f(\tilde{q}_1)$  as a function of  $P_2$  for a given value of system parameters, price factors, and the power  $P_1$  transmitted by the nodes of the Operator. In the region marked A,  $P_2$  satisfies the relation

$$\rho_2 P_2 \leq Q C \tilde{q}_1 \lambda_1 \exp\{-\tilde{q}_1 \lambda_1 C\} \exp\{-\lambda_1 C Q\}.$$



**Figure 3.1:** Price Bounds:  $\lambda_1 = 2, \lambda_2 = .5, C = 1, \rho_2 = .8, \rho_1 = 0.41, P_1 = 0.3, P_2 = 0.6, \beta = 2.4$

If the power transmitted by the Jammer lies in this region, then the Jammer turns ON all of his nodes at equilibrium. In the region marked  $C$ ,  $P_2$  satisfies the relation

$$P_2 \rho_2 \geq \bar{q}_1 \lambda_1 Q C \exp\{-\bar{q}_1 \lambda_1 C\} = Qf(\bar{q}_1).$$

If the power transmitted by the Jammer lies in this region, then, at equilibrium, all the nodes of the Jammer are turned OFF. In the region marked  $B$ ,  $P_2$  satisfies the relation

$$P_2 \rho_2 < \bar{q}_1 \lambda_1 Q C \exp\{-\bar{q}_1 \lambda_1 C\} \quad \text{and}$$

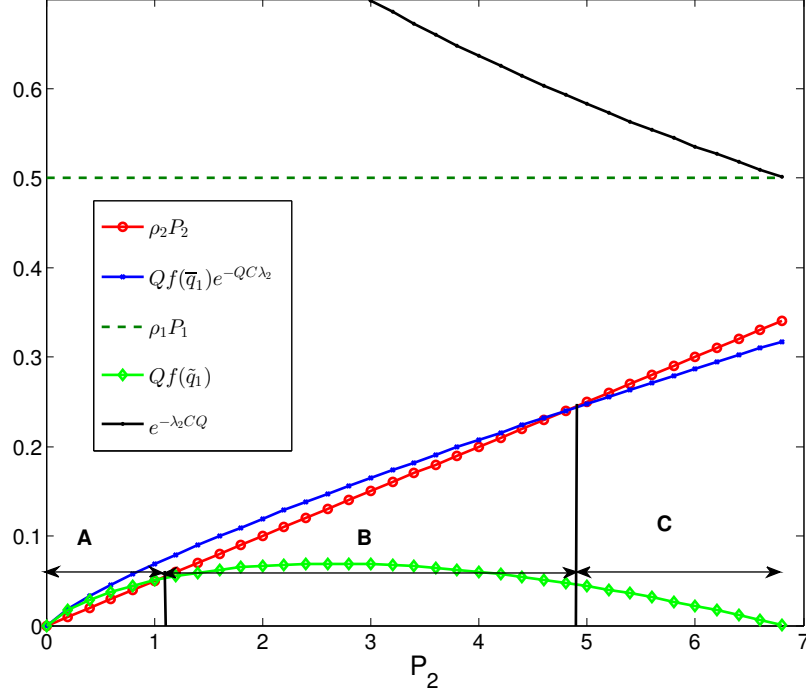
$$P_2 \rho_2 > Q C \tilde{q}_1 \lambda_1 \exp\{-\tilde{q}_1 \lambda_1 C\} \exp\{-\lambda_1 C Q\}.$$

If the power transmitted by the Jammer lies in this region  $B$ , then the Jammer turns ON each of its nodes with probability that is an interior point at equilibrium.

In Figure 3.3, we plot the best response (BR) of the Jammer and the Operator. We choose one power level,  $P_2$ , for the Jammer from each of the region  $A$ ,  $B$ , and  $C$ , with the power level for the Operator fixed at the same value as that used in Figure 3.2. In Figure 3.3 the blue lines correspond to the best response of the Operator, and the red lines correspond to that of the Jammer. The equilibrium for the corresponding value of  $P_2$  are marked.

*Nullifying the Jamming effect:* For a given power level and the cost factor of the Jammer if the condition  $P_2 \rho_2 \geq Qf(\bar{q}_1)$  holds at equilibrium, then the Operator sees no Jamming effect as all the nodes of the Jammer are turned OFF. Thus it is interesting for the Operator to choose a power level such that the Jammer turns OFF its nodes at equilibrium. It is easy to note the  $Qf(\bar{q}_1)$  is a decreasing function in  $P_1$ . Indeed,  $f(\tilde{q}_1)$  is a decreasing function in  $P_1$  as  $\tilde{q}_1$  is less than  $\leq 1/\lambda_1 C$  and decreasing in  $P_1$  (see





**Figure 3.2:** Range of  $P_2$ - (At equilibrium) In Region A Jammer is always ON. In region B Jammer is ON with some probability. In region C Jammer is OFF :  $\lambda_1 = 2, \lambda_2 = .5, \rho = .11, \rho_2 = 0.05, \beta = 2.5, P_1 = 4.54$

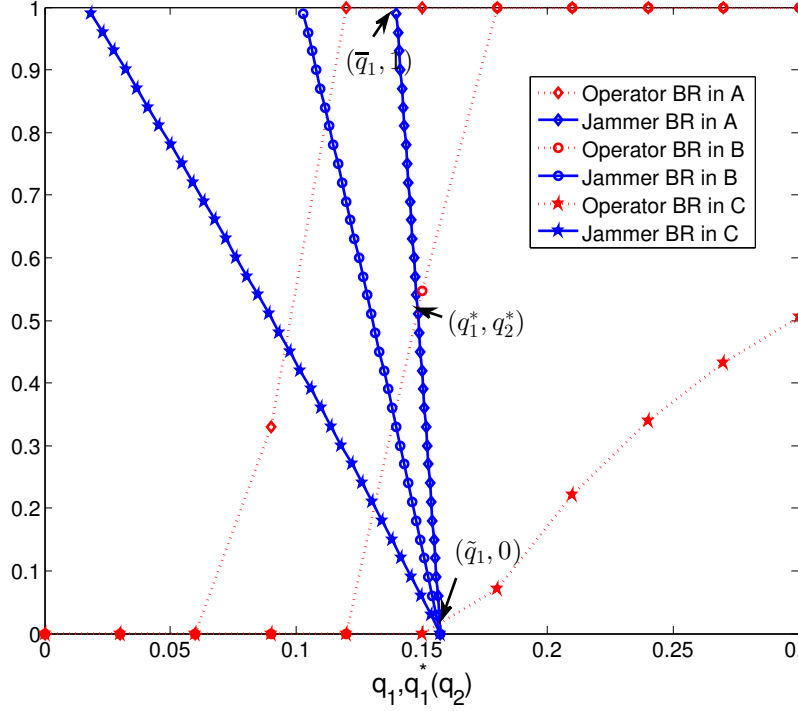
Equation (3.7)). Hence the product  $Qf(\tilde{q}_1)$  is also decreasing in  $P_1$ . Thus the Operator can increase his power level, say to  $P_1^*$ , such that  $P_2 \rho_2 \geq Qf(\tilde{q}_1)$ , and operate without the Jamming effect at equilibrium. However, the Operator can do so provided its cost factor is such that  $\rho_1 P_1^* \leq 1$ .

### 3.3.5 Spatial density of delay as performance metric

In this section we assume that the Operator aims to minimize the spatial density of delay, whereas the Jammer aims to maximize it. From the arguments in Subsection 3.2.1, we can write the spatial density of delay as the reciprocal of the spatial density of success. Continuing the set up used in the jamming game model, we write the performance metric and the resulting anti-potential as

$$d(q_1, q_2) = -\frac{\lambda_1 \exp\{C(\lambda_1 q_1 + \lambda_2 q_2 Q)\}}{q_1} \quad (3.11)$$





**Figure 3.3:** Best Response:  $\lambda_1 = 2, \lambda_2 = .5, \rho = .11, \rho_2 = 0.05, \beta = 2.5, P_1 = 4.54, P_2 = 0.5$

and

$$\bar{U}_1(q_1, q_2) = -\frac{\lambda_1 \exp\{C(\lambda_1 q_1 + \lambda_2 q_2 Q)\}}{q_1 - \rho_1 \lambda_1 q_1 P_1 + \rho_2 \lambda_2 q_2 P_2}. \quad (3.12)$$

The following lemma states the properties of this anti-potential function.

**Lemma 3.3.6.**  $\bar{U}_1(q_1, \cdot)$  is a concave function for all  $q_1 \in [0, 1]$ .  $\bar{U}_1(\cdot, q_2)$  is a concave function for all  $q_2 \in [0, 1]$ , and attains maximum at an argument within the interval  $q_1 \in [0, 1]$  if  $\lambda_1 C \geq 1$ .

From the above lemma we see that the hypothesis of Sion's minmax theorem [48] holds. Hence the Nash equilibrium exists. The following proposition characterizes the Nash equilibria.

**Proposition 3.3.7.** Assume that  $\lambda_1 C \geq 1$ .

- Let  $\bar{q}_1$  be the solution of

$$\frac{\lambda_1}{(\bar{q}_1)^2} \exp\{(\lambda_1 \bar{q}_1 + \lambda_2 Q)\} (1 - C \lambda_1 \bar{q}_1) = \rho_1 P_1,$$

and  $(\lambda_2 / \lambda_1) \rho_2 P_2 \bar{q}_1 \exp\{-\lambda_1 C \bar{q}_1\} < \exp\{\lambda_2 C Q\} - 1$ . Then  $(\bar{q}_1, 1)$  is the Nash equilibrium.

- Let  $\tilde{q}_1$  be the solution of

$$\frac{\lambda_1}{(\tilde{q}_1)^2} \exp\{\lambda_1 C \tilde{q}_1\} (1 - C \lambda_1 \tilde{q}_1) = \lambda_1 \rho_1 P_1,$$

and  $(\lambda_2 / \lambda_1) \rho_2 P_2 \tilde{q}_1 \exp\{-\lambda_1 C \tilde{q}_1\} > \exp\{\lambda_2 C Q\} - 1$ . Then  $(\tilde{q}_1, 0)$  is an equilibrium.

*Proof.* The proof follows by noting that  $-\bar{U}_1(q_1, q_2)$  is convex in  $q_2$  and hence maximized at the extreme points.  $\square$

### 3.4 Receiver Selection Models

In the Poisson bipolar model of the last section we assumed that the receiver of a node is at a fixed distance. In this section we relax this condition. We consider two models for receiver selection introduced in [17][Ch 17], namely, independent nearest receiver (INR) model and MANET nearest receiver (MNR) model.

#### 3.4.1 Independent Nearest Receiver model

In the INR model we assume that the potential receivers form a Poisson point process  $\Phi_0$  of intensity  $\lambda_0$  and is independent of the Poisson point process  $\Phi$ . Each active node of the Operator aims to transmit to the receiver that is nearest to it. Let the random variable  $R$  denote the distance between a typical node at the origin and to its nearest receiver. The probability density function of  $R$ , denoted as  $F_R$ , is given by

$$F_R(r) = 2\pi\lambda_0 r \exp\{-\lambda_0 \pi r^2\}, \quad r \geq 0.$$

Now consider the jamming game of Section 3.3. Recall that the Jammer turns ON each of its nodes with probability  $q_2$  and uses the power  $P_2$  to cause interference to the nodes of the Operator. Let  $d_s(\text{INR}, q_1, q_2)$  denote the density of the successful transmissions of the Operator. Following the steps of Lemma 3.3.1 and Proposition 17.2 in [17], we have

$$\begin{aligned} d_s(\text{INR}, q_1, q_2) &= \lambda_1 q_1 \int_{r \geq 0} r \exp\{-\pi r^2 C_1 (q_1 \lambda_1 + q_2 \lambda_2 Q)\} dF_R(r) \\ &= \frac{\lambda_1 \lambda_0 q_1}{C_1 (q_1 \lambda_1 + q_2 \lambda_2 Q) + \lambda_0}, \end{aligned} \quad (3.13)$$

where  $C_1 = 2T^{2/\beta} K(\beta)$ . Let  $\rho_1$  and  $\rho_2$  denote the cost factor of the Operator and Jammer respectively. We define the utility of the Operator and the Jammer with the energy cost as

$$\begin{aligned} U_1(\text{INR}, q_1, q_2) &= d_s(\text{INR}, q_1, q_2) - \lambda_1 q_1 \rho_1 P_1 \\ &= \frac{\lambda_1 \lambda_0 (1 - \rho_1 P_1) - \lambda_1 \rho_1 P_1 C_1 (q_1 \lambda_1 + q_2 \lambda_2 Q)}{C_1 (\lambda_1 + q_2 / q_1 \lambda_2 Q) + \lambda_0 / q_1} \end{aligned} \quad (3.14)$$

and

$$U_2(\text{INR}, q_1, q_2) = -d_s(\text{INR}, q_1, q_2) - \lambda_2 q_2 \rho_2 P_2, \quad (3.15)$$

respectively. As in Section 3.3, by adding  $\lambda_2 q_2 \rho_2 P_2$  to Equation (3.14) and  $\lambda_1 q_1 \rho_1 P_1$  to Equation (3.15), we have an anti-potential function given by

$$\bar{U}_1(\text{INR}, q_1, q_2) = U_1(\text{INR}, q_1, q_2) + \lambda_2 q_2 \rho_2 P_2 \quad (3.16)$$

$$= -(U_2(\text{INR}, q_1, q_2) + \lambda_1 q_1 \rho_1 P_1). \quad (3.17)$$

### 3.4.2 MANET nearest receiver model

In the MNR model we assume that each transmitter picks the nearest node of  $\Phi$  that is not active in the considered time slot. Let the Operator choose the MAP  $q_1$ . Then the receivers form a Poisson point process  $\Phi^0$  of intensity  $(1 - q_1)\lambda_1$  which is independent of the set of transmitters. Let  $d_s(\text{MNR}, q_1, q_2)$  denote the density of successful transmissions of the nodes of the Operator when the Jammer turns ON each of its nodes with probability  $q_2$ . We have [17][Eq. 17.6]

$$d_s(\text{MNR}, q_1, q_2) = d_s(\text{INR}, q_1, q_2)$$

with  $\lambda_0 = (1 - q_1)\lambda$ . We define the utility of the Operator with transmission cost as

$$\begin{aligned} U_1(\text{MNR}, q_1, q_2) &= d_s(\text{MNR}, q_1, q_2) - \lambda_1 q_1 \rho_1 P_1 \\ &= \frac{\lambda_1(1 - q_1)\lambda_1 q_1}{C_1(q_1 \lambda_1 + q_2 \lambda_2 Q) + (1 - q_1)\lambda_1} - \lambda_1 q_1 \rho_1 P_1 \\ &= \frac{\lambda_1(1 - \rho_1 P_1) - q_1 \lambda_1(\lambda_1 \rho_1 P_1 C_1 + (1 - \lambda_1 P_1))}{C_1(1 + q_2 \lambda_2 / q_1 \lambda_1 Q) + 1/q_1 - 1} \\ &\quad + \frac{\lambda_1 \rho_1 P_1 C_1 q_2 \lambda_2 Q}{C_1(1 + q_2 \lambda_2 / q_1 \lambda_1 Q) + 1/q_1 - 1}. \end{aligned} \quad (3.18)$$

Comparing Equation (3.14) with Equation (3.18) we observe that both the utility functions have the same structure. Henceforth we restrict our analysis of the jamming problem to the case of INR model.

*Saddle Point:* It is clear that the anti-potential function  $\bar{U}_1(\text{INR}, q_1, q_2)$  is a concave function in the strategy of the Operator and a convex function in the strategy of the Jammer. Then we can directly apply Sion's minimax theorem to conclude that Saddle point of the Jamming game exists. The next proposition characterizes the equilibrium in the INR receiver selection model.

**Proposition 3.4.3.** *The Nash equilibrium of the jamming game are as following. If  $\rho_1 P_1 \geq 1$ , then  $(0, 0)$  is an equilibrium. If  $\rho_1 P_1 < \lambda_0 / (C_1 \lambda_2 Q + \lambda_0)$  we have the following cases.*

- If  $\rho_2 P_2 \geq \frac{\lambda_1 \lambda_0 \tilde{q}_1 C_1 Q}{(C_1 \tilde{q}_1 \lambda_1 + \lambda_0)^2}$  then  $(\tilde{q}_1, 0)$  is an equilibrium, where  $\tilde{q}_1$  is given by

$$\tilde{q}_1 = \min \left\{ 1, \frac{(\lambda_0 / \sqrt{\rho_1 P_1} - \lambda_0)}{\lambda_1 C_1} \right\}. \quad (3.19)$$

- If  $\rho_2 P_2 \leq \frac{\lambda_1 \lambda_0 \bar{q}_1 C_1 Q}{(C_1 \bar{q}_1 \lambda_1 + C_1 \lambda_2 Q \lambda_0 + \lambda_0)^2}$  then  $(\bar{q}_1, 1)$  is an equilibrium, where  $\bar{q}_1$  is given by

$$\bar{q}_1 = \min \left\{ 1, \frac{\sqrt{(C_1 \lambda_2 Q \lambda_0 + \lambda_0^2) / \rho_1 P_1 - \lambda_2 Q C_1 - \lambda_0}}{\lambda_1 C_1} \right\}. \quad (3.20)$$

If  $(q_1^*, q_2^*)$  is an interior equilibrium point, then

$$q_1^* = \frac{\lambda_0 Q \rho_2 P_2}{\lambda_1 C (\rho_2 P_2 + Q \rho_1 P_1)^2} \quad (3.21)$$

$$q_2^* = \frac{1}{\lambda_2 C_1 Q} \left\{ \sqrt{(\lambda_0 \lambda_1 q_1^* Q C_1) / \rho_2 P_2 - C_1 \lambda_1 q_1^* - \lambda_0} \right\}. \quad (3.22)$$

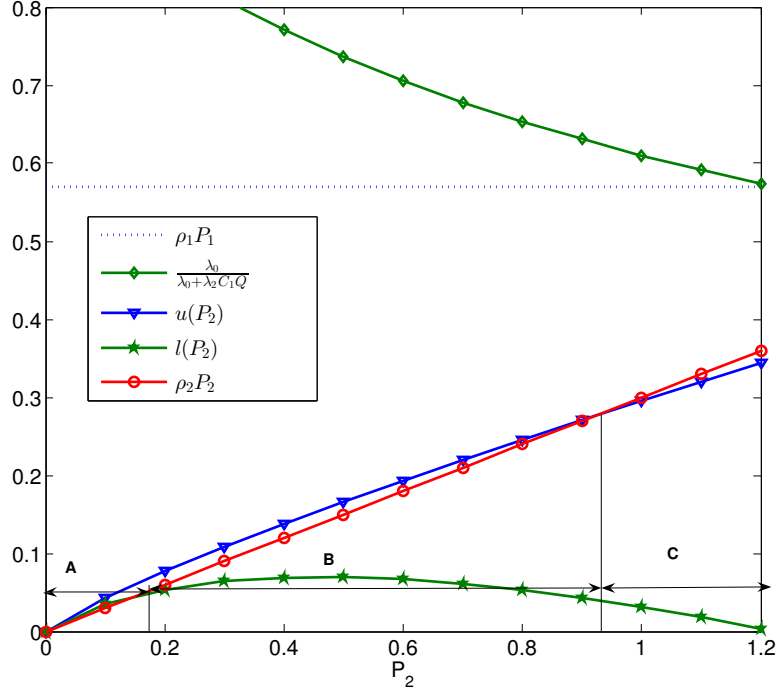
Note that, unlike in Proposition 3.3.4 we did not put the restriction  $\lambda_1 C_1 \geq 1$  in the above proposition. However, for Equation (3.21) and (3.22) to result in an interior point we need the conditions  $\lambda_0 Q \rho_2 P_2 \leq C_1 \lambda_1 (\rho_2 P_2 + Q \rho_1 P_1)^2$ , and

$$\frac{\lambda_1 \lambda_2 \bar{q}_1 C_1 Q}{(C_1 \bar{q}_1 \lambda_1 + C_1 \lambda_2 Q \lambda_0 + \lambda_0)^2} \leq \rho_2 P_2 \leq \frac{\lambda_1 \lambda_2 \tilde{q}_1 C_1 Q}{(C_1 \tilde{q}_1 \lambda_1 + \lambda_0)^2}.$$

Figure 3.4 shows the bound on  $\rho_2 P_2$  as given in Proposition 3.4.3. This has similar shape as that in 3.2. Hence, we can infer the same kind of results for the INR receiver selection model as in the fixed distance receiver model.

## 3.5 Conclusions

In this chapter we analyzed the performance of Poisson MANET in the presence of an adversarial Jammer considering the costs incurred in energy transmission. Our assumption of Poisson distribution on node locations lead to explicit expression for Nash equilibria. We analyzed the performance of the Network at equilibrium as a function of the transmission costs.



**Figure 3.4:** Range of  $P_2$ - (At equilibrium) In Region A Jammer is always ON. In region B Jammer is ON with some probability. In region C Jammer is OFF:  $\lambda_1, \lambda_2, \lambda_0 = 1, C_1 = .4, \rho_1 = 1, \rho_2 = .3, \beta = 2.4, P_1 = .57$

## 3.6 Appendices

### 3.6.1 Proof of Proposition 3.3.4

If  $\rho_1 P_1 \geq 1$ , then the slope of  $\bar{U}(\cdot, q_2)$  is negative for all values of  $q_2$ . Thus  $q_1 = 0$  is the unique maximizer of  $U(0, q_2)$ . If  $q_1 = 0$  then  $q_2 = 0$  is the unique minimizer of  $\bar{U}(0, q_2)$ . Hence  $(0, 0)$  is an equilibrium.

Differentiating  $\bar{U}_1$  with respect to  $q_1$  we have

$$\begin{aligned} \frac{\partial \bar{U}(q_1, q_2)}{\partial q_1} &= \lambda_1 \exp\{-C(q_1 \lambda_1 + q_2 \lambda_2 Q)\} \\ &\quad \times (1 - q_1 \lambda_1 C) - \lambda_1 \rho_1 P_1 \end{aligned} \quad (3.23)$$

Assume that  $\rho_1 P_1 < \exp\{-\lambda_2 C Q\}$ . Note that from Lemma 3.3.3,  $\bar{q}_1$  and  $\tilde{q}_1$  lies in the interval  $[0, 1]$ . Let  $q_2 = 0$ , then it follows from Equation (3.34) and the definition of the Lambert function [51], that the maximizer of  $\bar{U}_1(\cdot, 0)$  is (3.7). Let  $q_1 = \tilde{q}_1$ , the slope

of  $\bar{U}_1(\tilde{q}_1, \cdot)$  is

$$\begin{aligned} \partial \bar{U}_1(\tilde{q}_1, q_2) / \partial q_2 = \\ - q_1 \lambda_1 C Q \exp\{-\tilde{q}_1 \lambda_1 C\} \exp\{-C Q q_2 \lambda_2\} + \rho_2 P_2, \end{aligned}$$

which is positive for all values of  $q_2$  under the hypothesis of the first bullet. This implies that  $\bar{U}_1(\tilde{q}_1, \cdot)$  is an increasing function in  $q_2$ . Then  $q_2 = 0$  is the optimal strategy for the Jammer. This verifies the claim in the first bullet. Assume  $q_2 = 1$ , then it follows from Equation (3.34) and the definition of the Lambert function, that the maximizer of  $\bar{U}_1(\cdot, 1)$  is (3.8). Let  $q_1 = \bar{q}_1$ , the slope of  $\bar{U}_1(\bar{q}_1, \cdot)$  is

$$\begin{aligned} \partial \bar{U}_1(\bar{q}_1, q_2) / \partial q_2 = \\ - q_1 \lambda_1 C Q \exp\{-\bar{q}_1 \lambda_1 C\} \exp\{-C Q q_2 \lambda_2\} + \rho_2 P_2, \end{aligned}$$

which is negative for all values of  $q_2$  under the hypothesis of the second bullet. This implies that  $\bar{U}_1(\bar{q}_1, \cdot)$  is a decreasing function in  $q_2$ . Then the optimal value for the Jammer is to set  $q_2 = 1$ . This verifies the claim in second bullet. Let  $(q_1^*, q_2^*)$  be an interior equilibrium point, then the following simultaneous equations hold.

$$q_1^* \lambda_1 C Q \exp\{-q_1^* \lambda_1 C\} \exp\{-C Q q_2^* \lambda_2\} = \rho_2 P_2.$$

$$\exp\{-C(q_1^* \lambda_1 + q_2^* \lambda_2 Q)\} (1 - q_1^* \lambda_1 C) = \rho_1 P_1.$$

Solving these simultaneous equation we get the values in (3.9) and (3.10).

### 3.6.2 Proof of Lemma 3.3.1

We calculate the probability that a transmission by a typical node of the Operator is successful in the presence of Jammer.

*Proof.* Let the independently marked Poisson point process  $\tilde{\Phi} = \{\bar{X}_i, \bar{e}_i\}_{i \geq 1}$  denote the locations and the marks associated with the transmitters of the Jammer. We assume that  $\tilde{\Phi}$  is independent of  $\Phi$ . The marks  $\{\bar{e}_i\}_{i \geq 1}$  indicate if a transmitter at location  $\bar{X}_i$  is turned ON or OFF in a given time slot. A given transmitter is turned ON with probability  $\mathbb{E}[\bar{e}_i] = q_2$ . Let  $\bar{\Phi}^1 = \{\bar{X}_i, \bar{e}_i = 1\}$  denote the set of transmitters of the Jammer that are active. Then the shot noise seen by a typical node that is placed at the origin due to the active transmitters of the Jammer is given by  $I_{\bar{\Phi}^1} = \sum_{\bar{X}_i \in \bar{\Phi}^1} P_2 S l(|\bar{X}_i|)$ . For notational

simplicity write  $I_1 = I_{\bar{\Phi}^1}$  and  $I_2 = I_{\bar{\Phi}^0}$ . The total interference at the typical node is  $I_1 + I_2$ . Let  $p_c(q_1, q_2)$  denote the success probability of the typical node when the Operator uses the MAP  $q_1$  and the Jammer turns ON its transmitters with probability  $q_2$ . Then from the definition of success probability in Equation (2.2) we have

$$p_c(q_1, q_2) = \Pr \{ Sl(r)P_1 \geq T(I_1 + I_2 + W) \} \quad (3.24)$$

$$= \mathbb{E} \left\{ e^{\frac{-\mu T I_1}{l(r)P_1}} \right\} \mathbb{E} \left\{ e^{\frac{-\mu T I_2}{l(r)P_1}} \right\} \mathbb{E} \left\{ e^{\frac{-\mu T W}{l(r)P_1}} \right\} \quad (3.25)$$

$$= \mathbb{E} \left\{ e^{\frac{-\mu T W}{l(r)P_1}} \right\} \mathcal{L}_{I_1} \left( \frac{-\mu T}{l(r)P_1} \right) \mathcal{L}_{I_2} \left( \frac{-\mu T}{l(r)P_1} \right), \quad (3.26)$$

where Equation (3.25) follows from the assumptions that the  $I_1$ ,  $I_2$  and  $W$  are independent, and  $S$  is Rayleigh distributed with mean  $1/\mu$ . In Equation (3.26)  $\mathcal{L}_{I_1}(s)$  and  $\mathcal{L}_{I_2}(s)$  denotes the Laplace of shot noise  $I_{\Phi_1}$  and  $I_{\Phi_2}$  respectively, evaluated at  $s$ . We compute each term in the last expression as following:

$$\begin{aligned} & \mathcal{L}_{I_1} \left( \frac{-\mu T}{l(r)P_1} \right) \\ &= \exp \left\{ -2\pi\lambda_1 q_1 \int_{u \geq 0} \left( 1 - \mathbb{E} \left[ e^{\frac{-l(u)\mu T}{l(r)}} \right] \right) u du \right\} \end{aligned} \quad (3.27)$$

$$= \exp \left\{ -2\pi\lambda_1 q_1 \int_{u \geq 0} \left( \frac{u du}{1 + l(r)/Tl(u)} \right) \right\} \quad (3.28)$$

$$= \exp \left\{ -2\pi\lambda_1 q_1 r^2 T^{2/\beta} K(\beta) \right\} \quad (3.29)$$

where Equation (3.27) follows by applying the definition of Laplace functional of Poisson point process. Equation (3.28) follows by noting that expectation in Equation (3.27) is over the Rayleigh distribution.  $K(\beta) = \frac{\Gamma(2/\beta)\Gamma(1-2/\beta)}{\beta}$ . Similarly we calculate

$$\begin{aligned} & \mathcal{L}_{I_2} \left( \frac{-\mu T}{l(r)P_1} \right) \\ &= \exp \left\{ -2\pi\lambda_2 q_2 \int_{u \geq 0} \left( 1 - \mathbb{E} \left[ e^{\frac{-P_2 l(u)\mu T}{l(r)P_1}} \right] \right) u du \right\} \end{aligned} \quad (3.30)$$

$$= \exp \left\{ -2\pi\lambda_2 q_2 \int_{u \geq 0} \left( \frac{u du}{1 + P_1 l(r)/P_2 Tl(r)} \right) \right\} \quad (3.31)$$

$$= \exp \left\{ -2\pi\lambda_2 q_2 r^2 T^{2/\beta} (P_2/P_1)^{2/\beta} K(\beta) \right\} \quad (3.32)$$

Substituting Equations (3.29), (3.32) in Equation (3.26) we get

$$p_c(q_1, q_2) = \mathbb{E} \left\{ e^{\frac{-\mu T W}{l(r)P_1}} \right\} \exp \left\{ -C(\lambda_1 q_1 + \lambda_2 q_2 (P_2/P_1)^{2/\beta}) \right\}. \quad (3.33)$$

The intensity of the nodes of the Operator is given by  $\lambda_1 q_1$ . From the Campbell's formula [16] for the stationary point processes the density of successful transmission is given by multiplying Equation (3.33) by  $\lambda_1 q_1$ . This concludes the proof by taking  $W \equiv 0$ .  $\square$

### 3.6.3 Proof of Lemma 3.3.3

It is easy to observe that the  $\bar{U}_1(q_1, \cdot)$  is a convex function. Differentiating  $\bar{U}_1$  with respect to  $q_1$  once and twice we have

$$\begin{aligned} \frac{\partial \bar{U}(q_1, q_2)}{\partial q_1} &= \lambda_1 \exp\{-C(q_1 \lambda_1 + q_2 \lambda_2 Q)\} \\ &\quad \times (1 - q_1 \lambda_1 C) - \lambda_1 \rho_1 P_1 \end{aligned} \quad (3.34)$$

and

$$\begin{aligned} \frac{\partial^2 \bar{U}(q_1, q_2)}{\partial q_1^2} &= -\lambda_1 \exp\{-C(q_1 \lambda_1 + q_2 \lambda_2 Q)\} \\ &\quad \times (2 - q_1 \lambda_1 C) \end{aligned} \quad (3.35)$$

respectively. First consider the case  $\lambda_1 C \geq 1$ . For  $q_1 \geq 1/\lambda_1 C$  the slope in (3.34) is negative. From Equation (3.35), it is clear that Equation (3.34) is a decreasing function in  $q_1$  in the interval  $[0, 1/\lambda_1 C]$ , taking values  $\lambda_1 \exp\{-Cq_2 \lambda_2 Q\} - \lambda_1 \rho_1 P_1 > 0$  at  $q_1 = 0$ , and  $-\lambda_1 \rho_1 P_1$  at  $q_1 = 1/\lambda_1 C$ . Thus there exists a unique  $q'_1 \in [0, 1/\lambda_1 C]$  such that

$$\exp\{-C(q'_1 \lambda_1 + q_2 \lambda_2 Q)\} (1 - q'_1 \lambda_1 C) = \rho_1 P_1.$$

For all  $q_1 \leq q'_1$  the slope of  $\bar{U}_1(\cdot, q_2)$  is positive and for all  $q_1 \geq q'_1$  it is negative. Hence we conclude that  $\bar{U}_1(\cdot, q_2)$  is a quasi concave in  $q_1$  [52].

For the case  $\lambda_1 C < 1$ , the second derivative in (3.35) is negative for all  $q_1 \in [0, 1]$ . Hence  $\bar{U}(q_1, q_2)$  is concave in  $q_1$ .

### 3.6.4 Proof of Lemma 3.3.6

It is easy to note that  $\bar{U}_1(q_1, \cdot)$  is concave in  $q_2$ . The concavity in argument  $q_1$  can be verified by taking the second derivative. The first and second order derivative of  $\bar{U}_1(q_1, \cdot)$  with respect to  $q_1$ , after simplification, are as following.

$$\begin{aligned} \frac{\partial \bar{U}_1(q_1, q_2)}{\partial q_1} &= \frac{\lambda_1}{q_1^2} \exp\{C(\lambda_1 q_1 + \lambda_2 q_2 Q)\} \times \\ &\quad (1 - C\lambda_1 q_1) - \lambda_1 \rho_1 P_1 \end{aligned} \quad (3.36)$$

and

$$\begin{aligned} \frac{\partial^2 \bar{U}_1(q_1, q_2)}{\partial q_1^2} &= -\frac{\lambda_1}{q_1} \exp\{C(\lambda_1 q_1 + \lambda_2 q_2 Q)\} \times \\ &\quad ((C\lambda_1 + 1/q_1)^2 + 1/q_1) \end{aligned} \quad (3.37)$$

respectively. Equation (3.37) is negative valued for all  $q_1 \in [0, 1]$ . Hence  $\bar{U}_1(\cdot, q_2)$  is a concave function. Now assume that  $\lambda_1 C \geq 1$ . As (3.37) is negative valued,  $\partial \bar{U}_1(q_1, q_2)/\partial q_1$  is a decreasing function in  $q_1$  taking the value  $\partial \bar{U}_1(q_1, q_2)/\partial q_1 = -\lambda_1 \rho_1 P_1$  at  $q_1 = 1/\lambda_1 C$ . Hence maximum is achieved in the interval  $[0, 1/\lambda_1 C]$ .



### 3.6.5 Proof of Proposition 3.4.3

If  $\rho_1 P_1 \geq 1$  the slope of  $\bar{U}(\text{INR}, \cdot, q_2)$  is negative for all values of  $q_2$ . Thus  $q_1 = 0$  is the unique maximizer of  $\bar{U}(\text{INR}, q_1, q_2)$ . If  $q_1 = 0$  then  $q_2 = 0$  is the unique minimizer of  $\bar{U}(\text{INR}, 0, q_2)$ . Hence  $(0, 0)$  is an equilibrium. Differentiating anti-potential (3.16) with respect to  $q_1$  and  $q_2$ , we have

$$\begin{aligned} & \partial \bar{U}(\text{INR}, q_1, q_2) / \partial q_1 \\ &= \frac{(C_1 \lambda_2 q_2 Q + \lambda_0) \lambda_0}{(C_1 \lambda_1 q_1 + C_1 \lambda_2 q_2 Q + \lambda_0)^2} - \rho_1 P_1 \end{aligned} \quad (3.38)$$

and (3.16) with respect to  $q_1$

$$\begin{aligned} & \partial \bar{U}(\text{INR}, \tilde{q}_1, q_2) / \partial q_2 = \\ & - \frac{\lambda_1 \lambda_0 q_1 C_1 Q}{(C_1 \lambda_1 q_1 + C_1 \lambda_2 q_2 Q + \lambda_0)^2} + \rho_2 P_2, \end{aligned} \quad (3.39)$$

respectively. Assume that  $\rho_1 P_1 < \lambda_0 / (\lambda_2 C Q + 1)$ . Let  $q_2 = 0$ , then it follows from Equation (3.38) that the maximizer of  $\bar{U}(\text{INR}, \cdot, 0)$  is (3.19). Let  $q_1 = \tilde{q}_1$ , the slope of  $\bar{U}(\text{INR}, \tilde{q}_1, \cdot)$  is

which is positive for all values of  $q_2$  under the hypothesis of the first bullet. This implies that  $\bar{U}(\text{INR}, \tilde{q}_1, \cdot)$  is an increasing function in  $q_2$ . Then  $q_2 = 0$  is the optimal strategy for the Jammer. This verifies the claim in the first bullet. Assume  $q_2 = 1$ , it follows from Equation (3.38) that the maximizer of  $\bar{U}(\text{INR}, \cdot, 1)$  is (3.20). Let  $q_1 = \bar{q}_1$ , from Equation (3.39) the slope of  $\bar{U}(\text{INR}, \bar{q}_1, \cdot)$  is negative for all values of  $q_2$  under the hypothesis of the second bullet. This implies that  $\bar{U}(\text{INR}, \bar{q}_1, \cdot)$  is a decreasing function in  $q_2$ . Then the optimal value for the Jammer is to set  $q_2 = 1$ . This verifies the claim in second bullet. Let  $(q_1^*, q_2^*)$  be an interior equilibrium point, then from (3.38) and (3.39) the following simultaneous equations hold.

$$\frac{(C_1 \lambda_2 q_2^* Q + \lambda_0) \lambda_0}{(C_1 \lambda_1 q_1^* + C_1 \lambda_2 q_2^* Q + \lambda_0)^2} = \rho_1 P_1 \quad (3.40)$$

$$\frac{\lambda_1 \lambda_0 q_1^* C_1 Q}{(C_1 \lambda_1 q_1^* + C_1 \lambda_2 q_2^* Q + \lambda_0)^2} = \rho_2 P_2 \quad (3.41)$$

Taking the ratio of the above equations we obtain

$$(C_1 \lambda_2 q_2^* Q + \lambda_0) = \frac{\rho_1 P_1 \lambda_1 q_1^* C_1 Q}{\rho_2 P_2}.$$

Substituting this relation in (3.41) we get (3.21). Using (3.21) and the relation (3.41) we obtain (3.22).

# Chapter 4

## Tradeoffs in Green Cellular Networks

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### 4.1 Introduction

What is green networking? The following definition is given in <http://searchnetworking.techtarget.com> “Green networking is the practice of selecting energy-efficient networking technologies and products, and minimizing resource use whenever possible”.

Why is green networking relevant? Increasing the data rate for supporting multimedia data is one of the key issue in information and communication technology (ICT). Hence, the volume of traffic is expected to increase dramatically in the coming future and the energy consumed in mobile networks is around 2% of total carbon emission [74]. Moreover, more than 50% of the energy consumption is directly attributed to base station equipment and 30% more to mobiles switching and core transmission equipment [81]. From the economical perspective of network operator, a significant portion of their operational expenditure goes to pay the electricity bill. For instance, it is estimated that the mobile network operational expenditure for electricity globally will increase up to \$22 billion in 2013. According to Juniper Research [80], total base station emissions would peak in 2011 to 22 megatons and fall to 15.6 megatons by 2014, reaching a 30 percent reduction under the transformational model, which calls for operators to invest in power reduction in the base station, and place greater emphasis on addressing issues such as cooling, network planning and power management. Therefore, there is a growing consensus on the need to develop more energy-efficient networks.

A growing awareness to the dangers related to large scale energy consumption and drafting of many international agreements as well as legislation have reduced energy consumption in several sectors. From the economical perspective of network operator, there is also a growing willingness to reduce energy consumption in wireless networks.

We study another aspect of what we consider as green networking, that of minimizing the average uplink transmitted power, as the latter is proportional to the amount of energy that our body is exposed in communications by wireless terminals. Standards on the maximum amount of permitted radiation to the humans exist (see [82]) due to the awareness that the radiation can cause health problems [76]. The energy saving obtained by switching off base stations can results in larger uplink energy and poor coverage. In this paper we quantify the tradeoff between these aspects of green networking: total energy saving and uplink energy transmission.

Several works focus on the base station deployment in order to reduce power while taking into account the Capital Expenditure (CapEX) and Operational Expenditure (OpEX) [79, 78]. Other literature deals with improving the energy efficiency in order to accomplish the same task with less energy. Several solutions aiming at reducing power from base station may be divided into different types as following

- Increasing the number of cells in order to reduce the cell size leading to a reduction in the average transmitted power. This approach is more efficient for indoor network [85, 84]
- Femtocells and indoor distributed antenna systems using MIMO channel: This architecture is used to reduce co-channel interference introduced by frequency reuse among the femto cells and maintain high spectral efficiency [73].
- Cooperation at the base stations level: In [83], the authors show how the degree of redundancy of a network may reduce the power. The authors propose an approach based on cooperation between base stations in order to minimize the active number of base stations while satisfying the minimum required quality of service and minimum coverage.

In this paper we study a sleep mode where a fraction of base stations can be shut down when possible to save energy. Our goal is to provide some insight on the cost of switching off base stations on the uplink energy. We chose to illustrate this trade-off by an example that involves particular assumptions on the distribution of the base stations and on the interferences: the location of the base stations is assumed to form a homogeneous Poisson point process (Sec 4.2), and the radio interference is assumed to be negligible. The latter is a feature of operating at a light traffic, which is usually the one in which it is proposed to switch off base stations. Under those assumptions we obtain explicit expressions for the impact of switching off base stations on the total expected power consumption, on the coverage, and on the amount of radiation to the human's body. In section 4.3, we consider another example which does account for the interference. We derive expression for the expected interference using stochastic difference equation method. We calculate again the expected uplink power and study the impact of switching off base stations. Section 4.4 contains the concluding remarks and

discussions.

## 4.2 Model & Performance analysis

We consider an OFDMA cellular network. We shall focus on one (arbitrary) resource unit (a given time slot at a given frequency band). Assume that mobiles are Poisson distributed with parameter  $\beta$  and base stations are Poisson distributed with parameter  $\lambda$ . These two processes are assumed to be independent. We assume that each mobile connects to its closest base station. Let the random variable  $L$  denote the distance between a mobile and its nearest base station. We first assume that the mobiles are quite sparse or traffic is very light so that we can neglect the interferences. We later provide a more complex model that takes into account interference.

We assume that in order to save power of the base stations at times with low load, one keeps only a fraction  $0 < q < 1$  of base stations turned on and the remaining are turned off. The stations that are switched off are chosen at random. Indeed, we assume that the duration of a call is much shorter than the duration of the period during which the base station is turned off, so it is not beneficial to use mechanisms that take into account the current state of the network in order to decide which station should go to sleep. We note that the distance between a mobile to the closest base station is greater than  $l$  if and only if within a  $d$ -dimensional ball of radius  $l$ , there is no base station. The probability of the latter is  $\exp(-\lambda V(l))$  where  $V(l)$  is the volume of a  $d$ -dimensional ball with radius  $l$ . In particular, if we consider the problem on a line, i.e., ( $d = 1$ ), then  $V(l) = 2l$ . For  $d = 2$  it is  $V(l) = \pi l^2$ .

Let  $p$  be the transmission power of any mobile. We assume that there is a limit  $p_m$  on the transmitted power  $p$ . We call it green limit threshold; its size is determined by health considerations: it is the limit amount of power absorbed by the brain that is allowed. We assume that  $p = p(l)$  is controlled such that a target SNR,  $\eta$ , is reached at the closest base station given at a distance  $l$ .  $p(l)$  is the smallest transmitted power that guarantees the required SNR.

We consider here attenuation due to path loss only so that

$$\frac{p(l)l^{-a}}{\sigma^2} = \eta, \quad (4.1)$$

where  $\sigma^2$  denotes noise variance and  $a$  denotes the path loss exponent. If  $p(l)$  exceeds  $p_m$  we assume that there is an outage. When dealing with the line, we assume throughout that  $a > 1$ , and when dealing with the plane, we assume throughout that  $a > 2$ . Inverting equation (4.1), we obtain  $p(l) = \sigma^2 \eta l^a$ . The distance at which  $p_m$  is reached is denoted by  $l_m$  and is given by

$$l_m = \left( \frac{p_m}{\sigma^2 \eta} \right)^{\frac{1}{a}} \quad (4.2)$$

We consider the following frameworks to react to outage:

- **(i) No transmission (NT):** there is no transmission when a mobile is not covered, or

$$p_{nt}(l) = \begin{cases} \sigma^2 \eta l^a & \text{if } l \leq l_m \\ 0 & \text{otherwise} \end{cases}$$

- **(ii) Always transmit (AT):** Transmission occurs at the maximum power when  $l > l_m$  resulting in bad quality of service. Thus

$$p_{at}(l) = \begin{cases} \sigma^2 \eta l^a & \text{if } l \leq l_m \\ p_m & \text{otherwise} \end{cases}$$

This is equivalent to  $p_{at}(l) = \min(\sigma^2 \eta l^a, p_m)$ .

#### 4.2.1 Uplink power and coverage probability

Let  $\Delta(\lambda, l_m)$  denote the expected uplink power, i.e.,

$$\Delta(\lambda, l_m) := \mathbf{E}[p(L)] = \int_{B(l_m)} p(s) dP(s),$$

where  $B(l)$  is the ball of radius  $l$  at the origin. We compute the expected power transmitted by a mobile in the following proposition.

**Proposition 4.2.2.** *In the case of no transmission at outage, the expected power that a mobile node transmits, on a line, is given by*

$$\begin{aligned} \Delta_{nt}(\lambda, l_m) &= 2\lambda \int_0^{l_m} \sigma^2 \eta l^a \exp\{-2\lambda l\} dl \\ &= \frac{\sigma^2 \eta 2^{-\frac{a}{2}} l_m^a (\lambda l_m)^{-\frac{a}{2}} \exp\{-\lambda l_m\} \text{WM}\left(\frac{a}{2}, \frac{a}{2} + \frac{1}{2}, 2\lambda l_m\right)}{a + 1}, \end{aligned}$$

where  $\text{WM}(\cdot, \cdot, \cdot)$  denotes the WhittakerM function. On the two dimensional plane, it is given by

$$\begin{aligned} \Delta_{nt}(\lambda, l_m) &= \frac{\sigma^2 \eta \pi^{-\frac{a}{4}} l_m^a (\lambda l_m^2)^{-\frac{a}{4}} \exp\left\{-\frac{\pi \lambda l_m^2}{2}\right\} \text{WM}\left(\frac{a}{4}, \frac{a}{4} + \frac{1}{2}, \pi \lambda l_m^2\right)}{\frac{a}{2} + 1} \end{aligned}$$

In the case of always transmit, the expected transmitted power is given by

$$\begin{aligned} \mathbf{E}[p(L)] &= \Delta_{nt}(\lambda, l_m) + P(L > l_m) p_m \\ &= \Delta_{nt}(\lambda, l_m) + p_m \exp\left(-\lambda V(l_m)\right). \end{aligned}$$

The proof of the above proposition and that of the next corollary is direct, except for the expressions for  $\Delta(\lambda, l_m)$  for which we thank Maple.

Assume that there is no bound on power transmitted by mobiles  $p$ , and denote the expected power in this regime as  $\bar{\Delta}(\lambda)$ . Then we have the following corollary.

**Corollary 4.2.3.** *As  $p_m \rightarrow \infty$ , we have for the line:*

$$\bar{\Delta}(\lambda) := \lim_{l \rightarrow \infty} \Delta(\lambda, l) = \sigma^2 \eta (2\lambda)^{-a} \Gamma(a+1).$$

*For the plane we get:*

$$\bar{\Delta}(\lambda) = \sigma^2 \eta (\pi \lambda)^{-\frac{a}{2}} \Gamma\left(\frac{a}{2} + 1\right).$$

A mobile is connected to a base station if it is within a distance of  $l_m$  from any base station, otherwise it will not be covered. The following proposition gives the expression for coverage probability a given mobile is covered.

**Proposition 4.2.4.** *The coverage probability at the target SNR is given by*

$$c(\lambda, l_m) = 1 - \Pr\{L > l_m\} = 1 - \exp(-\lambda V(l_m))$$

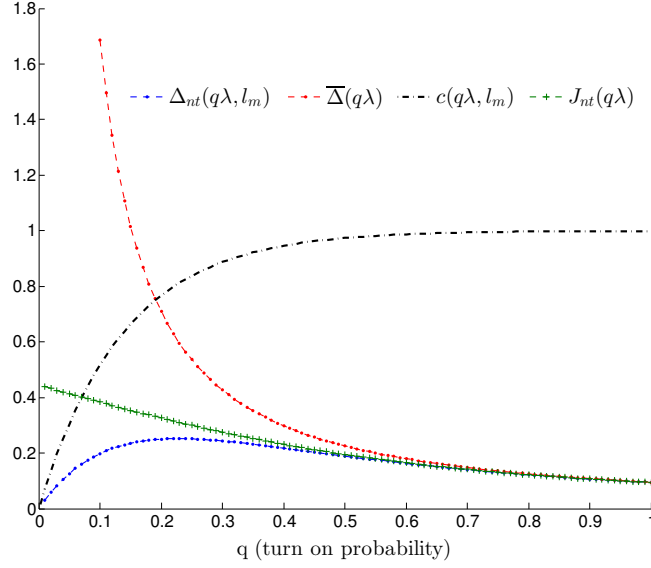
*in both regimes.*

#### 4.2.5 Effect of Base station deactivation

The aim of the network operator is to minimize the total power spend in the system. We consider a scenario in which operator tries to achieve this goal by turning off those base station that are not loaded heavily. For example, turning off those base stations when a number of mobiles served by them is small. However, it is not possible for the network operator to know a priori which base stations are lightly loaded. So, the operator can decide to switch them off randomly. Recall that  $q$  denotes the probability that a given base station turned on. By the thinning property of Poisson point process it is clear that the resulting point process is still Poisson with intensity  $q\lambda$ . Then the expected power transmitted by any mobile is given by  $\Delta(q\lambda, l_m)$ .

The figure 4.1 shows the variations of  $\Delta(q\lambda, l_m)$  in  $q$  for the ‘no transmit’(NT) scenario in a plane. Note that when a large fraction of base stations are turned off, i.e.,  $q \ll 1$ , the probability that a mobile connected to a base station is small and most of the mobiles do not transmit any power in the NT case. This leads to decrease in the expected power near origin in the above plot. However, the coverage is very poor in this region. This is also shown in figure 4.1.

The term  $\Delta(\lambda, l_m)$  averages (with respect to the distance to the base station) over all potential calls, including those that are in outage conditions. We shall be more interested in measures that characterizes successful calls. We thus define for the “no transmission” regime:  $J_{nt}(\lambda) = \frac{\Delta_{nt}(\lambda, l_m)}{c_{nt}(\lambda, l_m)}$ . For the case of “always transmit” we have



**Figure 4.1:** Expected Uplink Power, Coverage and Successful calls:  $\sigma^2 = 0.01, \eta = 35, \lambda = 1, a = 2.5, p_m = 1$

$J_{at}(\lambda) = \Delta_{at}(\lambda, l_m)$ . The variation of  $J_{nt}(q\lambda)$  as function in  $q$  is also depicted in the figure 4.1.

#### 4.2.6 Exponential Attenuation

In this subsection we consider the absorbing channel model instead of path loss model, i.e., the power received at a distance of  $D$  from an antenna is given by  $\exp(-\xi D)$  times the transmitted power, where  $\xi$  is the attenuation factor. (e.g. very humid air: the attenuation is thus exponential in distance). Again we assume that each mobile connects to a nearest base station and transmits power just enough to meet the target SNR  $\eta$ , i.e., a mobile at a distance  $l$  from base station transmit power  $p(l)$  such that

$$\frac{p(l) \exp\{-l\xi\}}{\sigma^2} = \eta.$$

If  $p_m$  is the maximum power that mobile can transmit, then inverting the above equation the corresponding maximum distance is  $l_m = (1/\xi) \log(p_m/(\sigma^2\eta))$ . Repeating the calculation for expected power we obtain following expressions. On the line it is

given by

$$\begin{aligned}\Delta(\lambda, l_m) : &= \mathbb{E}[p(L)] = 2\lambda \int_0^{l_m} \sigma^2 \eta \exp\{\xi l\} \exp\{-2\lambda l\} dl \\ &= \frac{2\lambda \sigma^2 \eta}{\xi - 2\lambda} \left\{ \exp\{(\xi - 2\lambda)l_{max}\} - 1 \right\},\end{aligned}$$

and on the plane

$$\begin{aligned}\Delta(\lambda, l_m) &= 2\pi\lambda \int_0^{l_m} \sigma^2 \eta l \exp\{\xi l\} \exp\{-\lambda\pi l^2\} dl \\ &= \sigma^2 \eta \left\{ 1 - e^{(\xi l_m - \lambda\pi l_m^2)} + \frac{\xi \exp\{\xi^2/(4\lambda\pi)\}}{2\sqrt{\lambda}} \right. \\ &\quad \left. \left( \operatorname{erf}\left\{\frac{\xi}{2\sqrt{\lambda\pi}}\right\} + \operatorname{erf}\left\{\frac{2\pi\lambda l_m - \xi}{2\sqrt{\lambda\pi}}\right\} \right) \right\}.\end{aligned}$$

The coverage probability remains unchanged.

The behavior of expected transmitted power when a fraction of base stations are switched off is same as in the case of path loss model. In the next section we take the effect of interference into consideration. However, we consider only the absorbing model (exponential attenuation) for analytic tractability.

### 4.3 Accounting for the interference

In this section we take into account the interference in a simple linear model. We make the following assumptions:

- The mobiles and base stations are scattered on a line at locations given by a Poisson process with parameter  $\beta$  and  $\lambda$  respectively, and are independent
- We focus on one resource (in time/frequency)
- Power control: Consider the absorbing channel as in subsection 4.2.6, but assume that there are no power limitations. Each mobile transmits at a power that guarantees a target SINR of  $\eta$ . Thus for an interference  $I$  and a noise variance  $\sigma^2$  at a mobile, the transmission power should be

$$p(y) = (\sigma^2 + I)\eta \exp(\xi y) \quad (4.3)$$

where  $y$  is the distance between mobile and its base station.

- Each BS has a directional antenna. Assume all antennas transmit towards the east. (For example, in order to communicate with vehicles that go in that direction.) It is then natural to consider also directional receiving antennas at the base stations: they would receive signals sent from the west.



- We assume that if some resource is used at a given cell then the resource is reserved so that within some radius  $R$  of the base station, no other call is accepted with this resource. (We allow for  $R = 0$  in which case there is no resource reservation). Such reservation is useful in pico-cells as it facilitates fast switching between neighboring pico-cells.
- A base station is restricted to receive one call at a time on a given resource (frequency or time). Therefore if a base station is closest to a mobile at  $x$  and to a mobile at  $y$  at the same time, then we have to decide which of them will be chosen to be first to transmit.

**Blocking Rate.** Let  $d_n$  be the location of some mobile that transmits. Let  $d_n + y_n$  be the location of the base station that receives the transmission. We assume that this is the base station which is the closest to the mobile on its east. Because of the memoryless property of the exponential random variables,  $y_n$  are i.i.d. exponential distributed random variables with parameter  $\lambda$ . Since only one source can transmit to the base station, all other calls whose location is between  $d_n$  and  $d_n + y_n + R$  are blocked and are thus assumed not to transmit. The next mobile that can transmit is then the one located at  $d_{n+1} = d_n + y_n + R + x_n$ , where  $x_n$  are i.i.d. exponentially distributed random variables with parameter  $\beta$  and independent of  $y_n$ . We conclude that the expected distance between two consecutive transmitting mobiles is  $E[y_n] + E[x_n] + R = 1/\lambda + 1/\beta + R$ . Hence the density of mobiles that transmit (and that are not blocked) is

$$\gamma = \frac{1}{\frac{1}{\lambda} + \frac{1}{\beta} + R} \quad (4.4)$$

which is the harmonic mean of  $\lambda$  and  $\beta$  (when  $R = 0$ ). The blocking rate is then  $\beta - \gamma$ .

**The interference.** The interference  $I_n$  of a mobile  $n \in \mathbb{Z}$ , where  $\mathbb{Z}$  denotes the set of integers, is the sum of powers received at its base station located at  $d_n + y_n$  from mobiles transmitting from  $d_i$  over all  $i < n$ . Note that it satisfies the recursion:

$$I_n = \left( I_{n-1} + p_{n-1}(\exp(-\xi y_{n-1})) \right) \exp(-\xi(x_{n-1} + y_n + R)).$$

Recall that we use power control so that the SINR of a mobile will equal a target value  $\eta$ . Substituting equation (4.3) we get for any  $n$ ,

$$\begin{aligned} I_n &= \left( I_{n-1} + \eta(\sigma^2 + I_{n-1}) \right) \exp(-\xi(x_{n-1} + y_n + R)) \\ &= A_{n-1} I_{n-1} + B_{n-1} \end{aligned} \quad (4.5)$$

where

$$A_{n-1} = (1 + \eta) \exp(-\xi(x_{n-1} + y_n + R)), \quad (4.6)$$

and

$$B_{n-1} = \eta \sigma^2 \exp(-\xi(x_{n-1} + y_n + R)). \quad (4.7)$$

We note that the two component random vectors  $(A_n, B_n)$  are i.i.d. and that

$$E[A] = (1 + \eta) \exp(-\zeta R) \frac{\beta \lambda}{(\zeta + \beta)(\zeta + \lambda)} \quad (4.8)$$

$$E[B] = (\eta \sigma^2) \exp(-\zeta R) \frac{\beta \lambda}{(\zeta + \beta)(\zeta + \lambda)}. \quad (4.9)$$

**Theorem 4.3.0.1.** *The stationary solution of (4.5) satisfies the following iteration*

$$I_n = \sum_{j=0}^{n-1} \left( \prod_{i=n-j}^{n-1} A_i \right) B_{n-j-1} + \left( \prod_{i=0}^{n-1} A_i \right) I_0 \quad (4.10)$$

If the following condition

$$1 + \eta < \exp(\zeta R) \frac{(\zeta + \beta)(\zeta + \lambda)}{\beta \lambda}. \quad (4.11)$$

is satisfied, then,

$$I_n^* = \sum_{j=0}^{\infty} \left( \prod_{i=n-j}^{n-1} A_i \right) B_{n-j-1} \text{ for all } n \in \mathbb{Z} \quad (4.12)$$

is the finite stationary solution of (4.5).

*Proof.*  $(A_n, B_n)$  are a sequence of i.i.d, non negative and finite valued random variables. Under the assumption (4.11), we obtain

$$\log E[A] = \log(1 + \eta) \exp(-\zeta R) \frac{\beta \lambda}{(\zeta + \beta)(\zeta + \lambda)} < 0.$$

Further Jenson's inequality yields  $E[\log A] \leq \log E[A] < 0$ . Also  $E[\log B] < \infty$ . Hence we have verified that condition (6) in [77][Thm 2A] holds and the result follows.  $\square$

**Corollary 4.3.1.** *Under assumption (4.11), we have in stationary regime*

$$E[I^*] = \frac{E[B]}{(1 - E[A])} = \frac{\eta \sigma^2}{\exp(\zeta R) \left( \frac{(\zeta + \beta)(\zeta + \lambda)}{\beta \lambda} \right) - (1 + \eta)} \quad (4.13)$$

For the case where there is no resource reservation, i.e.,  $R = 0$ , the condition (4.11) becomes

$$\eta < \frac{\zeta^2}{\lambda \beta} + \frac{\zeta}{\lambda} + \frac{\zeta}{\beta} \quad (4.14)$$

and the expected of interference is obtained as follows

$$E[I^*] = \frac{\sigma^2 \eta}{\frac{\zeta^2}{\lambda \beta} + \frac{\zeta}{\lambda} + \frac{\zeta}{\beta} - \eta} \quad (4.15)$$

Next we obtain the expression for expected power as stated in the following corollary.

**Corollary 4.3.2.** *The expected power is given by*

$$E[p_n] = \eta \left( \frac{\sigma^2 \lambda}{\lambda - \xi} + E[I_{n-1}] (1 + \eta) \frac{\beta}{\beta + \xi} \exp(-\xi R) \right. \\ \left. + \eta \frac{\sigma^2 \beta}{\xi + \beta} \exp(-\xi R) \right)$$

where  $E[I_{n-1}]$  is given by (4.13).

*Proof.* For any given  $I_n$  and  $y_n$  from eq. (4.3) we have

$$\begin{aligned} p_n &= \eta(\sigma^2 + I_n) \exp\{\xi y_n\} \\ &= \eta(\sigma^2 + A_{n-1} I_{n-1} + B_{n-1}) \exp\{\xi y_n\} \end{aligned} \quad (4.16)$$

$$\begin{aligned} &= \eta(\sigma^2 \exp\{\xi y_n\} + (1 + \eta) I_{n-1} \exp\{-\xi(x_{n-1} + R)\}) \\ &\quad + \sigma^2 \eta \exp\{-\xi(x_{n-1} + R)\}) \end{aligned} \quad (4.17)$$

where equality (4.16) follows from (4.5), and equality (4.17) follows from (4.6). Note that  $I_{n-1}$  does not depend on  $x_{n-1}$ . Taking expectation on both sides in (4.17) and recalling that  $y_n$  and  $x_n$  are exponentially distributed with parameter  $\lambda$  and  $\beta$  respectively, we get the desired result.  $\square$

With the above expression we can study the effect of switching off a fraction of base stations. The Figure 4.2 shows the expected power as a function of turn on probability  $q$ , for different values of  $\beta$ . The interesting point to note is that the expected power is increasing when more and more base stations are turned on whereas in the case of no interference, as in figure 4.1, it is decreasing. However, it can be easily see from equation (4.4) that blocking rate improves (i.e., lesser mobiles are blocked) as  $q$  increases.

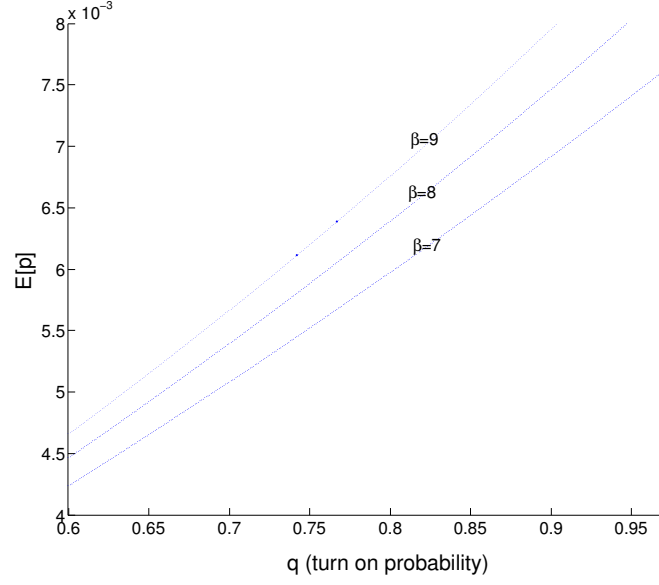
## 4.4 Conclusion and Discussion

We studied tradeoffs arising when turning off base stations. We presented two simple scenarios that allow us to quantify the tradeoffs. In both examples we derived several performance measures related to the network and investigated their dependence on the fraction of base stations that remains operational (not turned off). Our main contribution was to consider the cost of the energy saving obtained by switching off base stations on the uplink. This cost, seldom studied, is relevant to green networking as it is known that the uplink power is the main source of electro-magnetic energy to which humans are exposed.

## Appendix A: WhittakerM function

WhittakerM function arises as one of the solution to the following differential equation

$$W''' + \left( \frac{-1}{4} + \frac{k}{z} + \frac{\frac{1}{4} - m^2}{z^2} \right) W = 0,$$



**Figure 4.2:** Expected power as function of  $q$ : with  $\lambda = 3, \eta = 0.2315, \xi = 1, \sigma = 0.01, R = 0$

where  $W' := dW/dz$ , and it is defined as the following hypergeometric series

$$\text{WhittakerM}(k, m, z) = z^{m+1/2} \exp\{-z/2\} \sum_{n=0}^{\infty} \frac{(m-k+1/2)_n}{n!(2m+1)_n} z^n,$$

where  $(m)_n := m(m+1) \cdots (m+n-1)$ .

## Appendix B: LommelS2 function

LommelS2 function arises as one of the solution to the following differential equation

$$z^2 W'' + zW' + (z^2 - \nu^2)z = z^{\mu+1},$$

where  $W' := dW/dz$ .



# Chapter 5

## Spatio-Temporal Control for Dynamic Routing Games

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### 5.1 Introduction

Routing games are concerned with one or more classes of individuals, where each class is characterized by a source-destination pair and a demand function. A given network is shared by the users. Routes are chosen by the players so as to minimize the delays. In road traffic engineering, routing games appeared in 1952 [90]. Non-cooperative routing has long been studied both in the framework of road-traffic as well as in the framework of telecommunication networks. Such frameworks allow us to model the flow configuration that results in networks in which routing decisions are made in a non-cooperative and distributed manner between the users. There can be finite or infinite number of users.

In the case of a infinite number of players each player is assumed to be atomless. By atomless we mean that the impact of routing choices of a single player on the utilities of other players is negligible. The resulting flow configuration corresponds to the Wardrop equilibrium [90]. This concept, has long been studied in the context of road traffic where there is an infinite number of players (drivers) [88]. In the telecommunication community, the mostly used routing game model introduced by Orda, Rom and

Shimkin [87] is that in which the number of players are finite, where a player (typically corresponding to a service provider) takes the routing decisions for the whole class of users that it controls. It then decides on how to split the demand it controls between various possible routes. They establish existence and uniqueness of Nash equilibrium over large class of general cost functions. This approach also appeared in the road traffic literature (e.g. [86]) but was not much used there. Such a routing game may be handled by models similar to [91] in the special case of a topology of parallel links.

An alternative class of routing games is the one in which a player has to route all the demand it controls through the same path. A special case of such framework is the “congestion games” introduced by Rosenthal in [89]. All the above works have been well studied in time-invariant networks.

In this paper, we study a dynamic routing game where the decision of a user is spatio-temporal. The demand has to be split not only over space but also over time. As an example, assume that  $N$  players have each its own demand which should be shipped within a week from a given source to a destination. Thus a player has to split its demand into that corresponding to each of the days of the week. At each day, the route corresponding to the daily demand of each player should be determined. Examples of such games in road traffic appear in [98].

A dynamic routing game over  $T$  days can often be transformed into an equivalent static one. Indeed, in the equivalent static model, we make  $T$  replicas of the original network, one corresponding to each day. The source node for a player in each of the replicated networks is connected to a node corresponding to the source for that player (and we do the same with the destination). The fact that the game is repeated over time allows often to observe what other players did in the past.

The paper is organized as following. In section 5.2 we briefly overview some results in dynamic games that are relevant to the dynamic routing game: the existence of equilibria within open-loop as well as closed-loop strategies and procedures for computing them. We obtain explicit expression for equilibrium considering polynomial cost functions and study its structure. In section 5.3 a simple stochastic congestion game is modeled in which only one unit of demand can arrive to each player with some probability. The state of a player evolves according to whether they decide to transmit or not. We study the existence and structure of stationary policies. In section 5.4 we consider another model of stochastic congestion game in which the fixed demand arrives each day that has to be shipped within two days. We end with conclusion in section 8.9.

## 5.2 Dynamic game with fixed demand

Assume that there is a fixed amount of demand that has to be shipped over a link within a fixed time, say  $T$  days. We consider two non-cooperative scenarios. In the first case the demand is considered as infinitely many users. In other case we consider finite number of players, where each tries to ship their demand over the shared link in a selfish way so that the total cost incurred by them is minimized. We study structure of

spatio temporal equilibrium flows in both cases using notion of Wardrop equilibrium in the former case and Nash equilibrium in the latter.

### 5.2.1 Non-atomic routing games

In the context of road traffic, Wardrop [90] proposed the following definition of equilibrium

*“The journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route”*

Consider an amount  $d$  of demand that has to traverse a common shared link. Each player has to determine at what day within a common finite set  $S$  to leave. Let  $x_i$  be the amount of players that leave at day  $i$ . A player leaving in day  $i$  pays a congestion cost  $f(x_i)$  and a delay cost of  $i$  units. The vector  $x = (x_1, \dots, x_T)$  is said to be feasible if  $\sum_{i=1}^T x_i = d$ . It is a spatio temporal Wardrop equilibrium if there is some constant  $\alpha > 0$  such that for each  $i = 1, \dots, T$ , we have

$$f(x_i) + i \geq \alpha \text{ and } x_i(f(x_i) + i - \alpha) = 0$$

Equivalently,

$$\begin{aligned} &\text{all days for which } x_i > 0, \quad f(x_i) + i = \alpha \\ &\text{and for all other } i\text{'s}, \quad f(0) + i \geq \alpha. \end{aligned} \tag{5.1}$$

and for all  $i \in I$  we have  $x_i = f^{(-1)}(\alpha - i)$ .

Thus if  $f$  is increasing then so is  $f^{(-1)}$  and then  $x_i$  is decreasing in  $i$  at equilibrium. Thus there is a threshold policy such that till day  $i^*$ , some traffic is shipped everyday, and after that threshold nothing is shipped.

### 5.2.2 Atomic routing games

In this section we consider finite number of players  $N$  each competing for the link. The decision of each player influences the cost of the others. Let the demand of each player  $i = 1, 2, \dots, N$  be  $d_i > 0$  that has to be shipped to destination over a period of  $T$  days. The amount of flow sent by player  $i$  in the  $j$ th day is denoted by  $x_j^i$  and the vector  $\mathbf{x}^i = (x_1^i, x_2^i, \dots, x_T^i)$  denotes the flow of user  $i$ . The vector  $\mathbf{x}^i$  is said to be feasible if  $\sum_{j=1}^T x_j^i = d_i$ . Let the vector  $\mathbf{x}_j = (x_j^1, x_j^2, \dots, x_j^N)$  denote the amount of flow sent by

each user on day  $j$ . The total flow on day  $j$  is denoted as  $x_j = \sum_{i=1}^N x_j^i$ . For a given flow configuration of users  $(\mathbf{x}^1, \mathbf{x}^2, \dots, \mathbf{x}^N)$ , user  $i$  pays a congestion cost of  $f(x_j)$  and delay



cost of  $j$  per unit of its flow on day  $j$ . The objective of each user is to minimize his cost given by

$$J^i(\mathbf{x}^1, \mathbf{x}^2, \dots, \mathbf{x}^N) = \sum_{j=1}^T x_j^i (f(x_j) + j)$$

subjected to its demand constraints. A feasible vector  $\mathbf{x}^* = (\mathbf{x}^{*1}, \mathbf{x}^{*2}, \dots, \mathbf{x}^{*N})$  is said to be Nash equilibrium if for each user  $i = 1, 2, \dots, T$ .

$$J^i(\mathbf{x}^{*1}, \mathbf{x}^{*2}, \dots, \mathbf{x}^{*i}, \dots, \mathbf{x}^{*N}) \leq J^i(\mathbf{x}^{*1}, \mathbf{x}^{*2}, \dots, \mathbf{x}^i, \dots, \mathbf{x}^{*N}) \quad \text{for all feasible } \mathbf{x}^i$$

.

The above spatio temporal dynamic game is completely equivalent to a static parallel link problem studied by [87] with link cost  $J_i(x_i)$  taking value  $J_i(x_i) = f(x_i) + i$ . When  $f$  is a positive, strictly increasing, convex and continuously differentiable all the assumptions in type-B functions in [87] are satisfied. Hence the existence and uniqueness of Nash equilibrium hold. Further the following property hold for the equilibrium flow.

**Proposition 5.2.3.** *The total flow  $x_j$  is non-increasing in each day.*

*Proof.* Let  $\mathcal{L}_j$  denote the set of all users who put a positive flow on day  $j$ , i.e.,  $\mathcal{L}_j = \{i : x_j^i > 0\}$ . From the KKT conditions for all  $i \in \mathcal{L}_j$  there exists a  $\lambda^i$  such that (see, [87][eq. 2,3])

$$f(x_j) + x_j^i f'(x_j) + j = \lambda^i \quad \text{and} \quad f(x_k) + x_k^i f'(x_k) + k \geq \lambda^i \quad \text{for all } k \neq j.$$

Summing over the set  $\mathcal{L}_j$  ( $j > 1$ ) we get

$$\begin{aligned} f(x_j)|\mathcal{L}_j| + x_j f'(\bar{x}_j) &\leq f(x_{j-1})|\mathcal{L}_j| + \sum_{i \in \mathcal{L}_j} x_{j-1}^i f'(x_{j-1}) \\ &\leq f(x_{j-1})|\mathcal{L}_j| + x_{j-1} f'(\bar{x}_{j-1}) \end{aligned}$$

By the assumption that  $f$  and  $f'$  are monotonically increasing we obtain  $x_{j-1} \geq x_j$  for all  $j$ .  $\square$

Ayesta *et al.* [92] showed while studying the load balancing non-cooperative game that the worst case Nash equilibrium occurs when each user have the same amount of demand, i.e, when users are symmetric. Next we will calculate the Nash equilibrium for this case.

*Symmetric Users:* Consider that every user has a fixed demand  $d_i = d$  to route. Cominetti, Correa, and Stier-Moses [93][Sec. 4.2] showed that the game with symmetric users is a potential game [94] and the Nash equilibrium can be obtained as the solution of a single optimization problem as following. If  $\mathbf{x}^* = (x_1^*, x_2^*, \dots, x_T^*)$  is the solution of the following optimization problem

$$\begin{aligned} & \underset{\mathbf{x}=(x_1, x_2, \dots, x_T)}{\text{minimize}} && \sum_{i=1}^T x_i(f(x_i) + i) + (N-1) \sum_{i=1}^T \int_0^{x_i} (f(x) + i) dx \\ & \text{subject to} && -x_i \leq 0, \quad i = 1, 2, \dots, T \quad \text{and} \quad \sum_{i=1}^T x_i = d. \end{aligned}$$

then flow profile at Nash equilibrium is given by  $\mathbf{x}^i = \{\frac{x_1^*}{N}, \frac{x_2^*}{N}, \dots, \frac{x_T^*}{N}\}$  for each user  $i = 1, 2, \dots, N$

We consider the special case when the congestion cost is polynomial, i.e., of the form  $f(x) = ax^p + b$ , for some  $a, b > 0$  and  $p > 1$ . Polynomial functions were introduced for congestion cost originally in the context of road traffic. In [97] existence of equilibrium and its uniqueness are studied for the general network topology with polynomial congestion cost. The following proposition gives values of the flow at equilibrium.

**Proposition 5.2.4.** *For polynomial congestion cost the equilibrium flow is*

$$x_j^* = \begin{cases} \left( \frac{\beta - j - b}{a + \frac{ap}{N}} \right)^{\frac{1}{p}} & \text{if } j < \beta - b \\ 0 & \text{otherwise} \end{cases}$$

where  $\beta$  is such that it satisfies

$$\sum_{j: j < \beta - b} \left( \frac{\beta - j - b}{a + \frac{ap}{N}} \right)^{\frac{1}{p}} = d$$

*Proof.* See Appendix 5.6. □

Next we consider the case when players co-operate among themselves and try to minimize the total cost.

### 5.2.5 Global Optimum

Consider the scenario as in the previous sub section 5.2.2. The objective of each user when the total demand  $D$  is

$$\begin{aligned} & \underset{(x_1, x_2, \dots, x_T)}{\text{minimize}} && \sum_{j=1}^T x_j(f(x_j) + j) \\ & \text{subject to} && -x_j \leq 0, \quad j = 1, 2, \dots, T. \quad \text{and} \quad \sum_{j=1}^T x_j = D. \end{aligned}$$

Let  $(x_1^*, x_2^*, \dots, x_T^*)$  be the optimal value of flows that achieve minima. Then by the necessary KKT conditions there exist a  $\alpha > 0$  such that the following hold

$$\begin{aligned} f(x_j^*) + x_j^* f'(x_j^*) + j &= \alpha \quad \text{if } x_j^* > 0 \quad \text{and} \\ f(x_j^*) + x_j^* f'(x_j^*) + j &\geq \alpha \quad \text{if } x_j^* = 0 \end{aligned} \tag{5.2}$$

Since we assumed that  $f$  is monotonically increasing and convex, the function  $F(x) = f(x) + xf'(x)$  is monotonically increasing. Also, by writing  $x_j^* = F^{-1}(\alpha - j)$  we observe that flow is monotonically decreasing till day  $j^*$  and after that nothing is shipped.

From equations (5.2) and (5.1) the following lemma is easy to verify

**Lemma 5.2.6.** *Let  $\mathbf{x}^* = (x_1^*, x_2^*, \dots, x_T^*)$  denote the minimum of the global optimization problem  $\sum_{i=1}^T (g(x_i) + i \cdot x_i)$  subjected to constraints, where  $g$  is a differentiable function. Define  $f(\cdot) = g'(\cdot)$  then  $\mathbf{x}^*$  is Wardrop equilibrium of the game with cost function  $f$  if and only if it is a global optimum.*

More general versions of the above results appear in [96][Chapter 18].

Again, considering the polynomial cost structure, optimal flows can be characterized as in the following proposition

**Proposition 5.2.7.** *For the polynomial congestion cost optimal flow is*

$$x_j^* = \begin{cases} \left( \frac{\beta - j - b}{a + ap} \right)^{\frac{1}{p}} & \text{if } j < \beta - b \\ 0 & \text{otherwise,} \end{cases}$$

where  $\beta$  is such that it satisfies

$$\sum_{j: j < \beta - b} \left( \frac{\beta - j - b}{a + ap} \right)^{\frac{1}{p}} = D.$$

With the Nash equilibrium flow and optimal flow in proposition 5.2.4 and proposition 5.2.7, we can compute social cost at equilibrium and optimal cost and hence price of anarchy(PoA). For the polynomial cost function we obtain an upper bound of  $\left( \frac{1+p}{1+p/N} \right)^{1+1/p}$ . For detailed proof see Appendix 5.6.1. The proof method is similar to that in [92].

## 5.3 Stochastic Congestion game

### 5.3.1 Motivating examples

The game is motivated by the following scenario:  $N$  transport companies share a network of roads over which they want to schedule their traffic on a day-to-day basis. The traffic to be routed by each company varies on a day-to-day basis according to a given stochastic process. The objective of each company is to route its traffic in such a way so as to minimize the average delay of its traffic. The cost incurred by a player, which represents the delay due to congestion on the roads over which they schedule the traffic, depends not only on the amount it routes but also on the amount the other players route thereby leading to a game.

Another practical scenario where this game is played is that of a set of universities who share a cluster of processors over which they want to schedule tasks which arrive according to a given stochastic process. At the beginning of each day, each player has to decide the tasks it schedules over each of the processors with the knowledge that the delay incurred by its tasks depends also on the scheduling policy of the other players.

In this section we assume that there is only one shared resource (road or processor), and that the traffic is *indivisible*, that is, a player cannot divide its traffic over two or more slots. It should either route the entire traffic or wait. Unlike in the previous section we consider the case where game is repeated infinitely many times. Description of our game model is as following. We largely follow the notation used in [95].

### 5.3.2 Model

1. **State space:** At the beginning of a slot, player  $i$  can be either *active* or *inactive*. In the active state, the player has some amount of traffic to ship from some source to some destination. It stays in this state until it routes this traffic and then moves to the inactive state. The player then stays in the inactive state until a new unit of traffic arrives.

We shall assume that, when player  $i$  is in the inactive state, a new unit of traffic arrives with probability  $p_i$  as a result of which the player becomes active. Let  $x_t^i$  denote the state of player  $i$  in slot  $t$ , and let  $\mathbf{x}_t$  denote the state vector at the beginning of slot  $t$ . We denote state space of  $i$ th player as  $X^i = \{0, 1\}$ . Active state corresponds to 1 and inactive state to 0.

2. **Action set :** At the beginning of a slot, an active player can decide either to route its traffic or to postpone. The actions available to player  $i$  in slot  $t$ ,  $a_t^i$ , is thus a subset of  $\{0, 1\}$ . A player sends all of its traffic when it decides to route in the active mode. We shall denote the action set of player  $i$  when it is in state  $x$  as  $A^i(x)$ . Specifically,  $A^i(x = 0) = 0$  and  $A^i(x > 0) = \{0, 1\}$ . Let  $\mathbf{a}_t = (a_t^1, a_t^2, \dots, a_t^N) \in \prod_{i=1}^N A^i(x_t^i)$  denote the action vector at the beginning of slot  $t$  when the state vector is  $\mathbf{x}_t$ . Define the local set of state-action pair for player  $i$  as  $\mathcal{K}^i = \{(x^i, s^i) : x^i \in X^i, s^i \in A^i(x^i)\}$ , and  $\mathcal{K}^{-i}$  denote the state-action set of all other users, i.e.,  $\mathcal{K}^{-i} = \prod_{j \neq i}^N \mathcal{K}^j$ .

3. **Dynamics of the state :** The state of player  $i$ ,  $x_t^i$  evolves according to a Markov chain whose transition matrix in slot  $t$  depends on the action  $a_t^i$  and the current state. For each user we denote the transition probability from state  $x \in X^i$  to  $y \in X^i$  when user takes action  $a \in S^i(x)$  as  $P_{xay}^i$ . When a player  $i$  is in inactive mode it can get a unit of traffic with probability  $p_i$ , hence takes values  $x_{t+1}^i = 1$  with probability  $p_i$  or remains in the inactive mode with probability  $1 - p_i$ . Similarly, in the active mode if the action is to transmit then a new state becomes active with

probability  $p_i$  and inactive with probability  $1 - p_i$ . If the state is active and action is not to transmit then state remains active. Transition probability matrix is given by

$$P_{(\cdot)a(\cdot)}^i = \begin{bmatrix} 1 - p_i & p_i \\ (1 - p_i)a & 1 - (1 - p_i)a \end{bmatrix} \quad (5.3)$$

4. **Cost functions** : In a given slot  $t$ , an active player who routes its traffic incurs a cost which is a function of the number of players who routed their traffic in that slot. On the other hand, if an active player decides to postpone, then it incurs a fixed cost  $d$  which can be inferred as a penalty for the increased delay incurred by the traffic. Let  $c_t^i : \prod_{i=1}^N \mathcal{K}_i \rightarrow \mathcal{R}$  be the cost incurred by player  $i$  in slot  $t$ , then

$$c_t^i(\mathbf{x}_t, \mathbf{a}_t) = f\left(\sum_i x_t^i a_t^i\right) + d\left((1 - a_t^i)x_t^i\right). \quad (5.4)$$

5. **Policies and Information** Define the history of player  $i$  at time  $t$  to be a sequence of its past state and actions including the current state  $\mathbf{h}_t^i = (x_1^i, a_1^i, x_2^i, a_2^i, \dots, x_{t-1}^i, a_{t-1}^i)$ . Let  $H_t^i$  denote the collection of all such histories. The policy of a players  $i$  is a sequence of maps  $(u_1^i, u_2^i, \dots)$ , where the maps are given by  $u_t^i : H_t^i \rightarrow M(A^i)$  with  $M(A^i)$  denoting the probability vector on the set  $A^i$ . Set of all policies of player  $i$  is denoted as  $U^i$  and the collection  $U = \prod_{i=1}^N U^i$  is called multi-policies. A **stationary policy** for a player  $i$  is a function  $u^i : \mathbf{X}^i \rightarrow M(A^i)$  so that  $u^i(\cdot|x^i) \in M(A^i(x^i))$ . Set of all stationary policies for player  $i$  is denoted as  $U_S^i$  and the collection of all users stationary policies as  $U_S = \prod_{i=1}^N U_S^i$ . Note that when the user is in inactive mode then only possible action is 0, hence  $u^i(0|x^i = 0) = 1$ . In our model a stationary policy is completely characterized by the probability of transmission in active state, i.e.,  $q_i := u^i(1|1)$ . We use both  $u^i$  and its associated  $q_i$  to denote a stationary policy interchangeably. The transition probability matrix when the  $i$ th users stationary policy is  $q_i$  is given by

$$P_i(q_i) = \begin{bmatrix} 1 - p_i & p_i \\ q_i(1 - p_i) & 1 - q_i(1 - p_i) \end{bmatrix} \quad (5.5)$$

For any multi policy  $u \in U$ , let  $u^{-i}$  denote the multi policy without the policy  $u^i$  and  $[u^{-i}|v^i]$  denote the multi policy with the policy  $u^i$  of  $i$ th user replaced by  $v^i$ . We assume that each user  $i$  has information of its own state and makes decision only based on this information.

## 6. Utility and the objective

The objective of player  $i$  is to select its strategies so as to minimize the total cost it incurs over the horizon. Let  $\beta$  denote the distribution of the initial state.

Let  $\{X_t, A_t\}_{t>0}$  denote the sequence of random variable on state and action sets, where  $X_t = \{X_t^1, X_t^2, \dots, X_t^N\}$  and  $A_t = \{A_t^1, A_t^2, \dots, A_t^N\}$ . The state action pair  $\{X_t, A_t\}$  evolve according to the distribution which is determined by multi policy  $u \in U$ , the transition probabilities and  $\beta$ . We denote this distribution by  $P_\beta^u$  and expectation with respect to this distribution as  $\mathbb{E}_\beta^u$ . Each user  $i = 1, 2, \dots, N$  seeks to minimize his average expected cost.

$$C^i(U, \beta) = \limsup_{T \rightarrow \infty} \frac{1}{T} \mathbb{E}_\beta^u \sum_{t=1}^T c_t^i(X_t, A_t) \quad (5.6)$$

Below we restate the definition in [95][Def 2.1] for Nash equilibrium and optimal response for a stochastic game.

**Definition 5.3.2.1.** 1. A multi policies  $u \in U$  is said to be Nash equilibrium if  $i = 1, 2, \dots, N$  and for any  $v^i \in U^i$

$$C^i(\beta, u) \leq C^i(\beta, [u^{-i}|v^i]) \text{ for any } v^i \in U^i, i = 1, 2, \dots, N. \quad (5.7)$$

2. For any multi policy  $u$ , policy  $u^i$  of user  $i$  is said to be optimal response against  $u^{-i}$  if (5.7) holds for any  $v^i \in U^i$ .
3. For any multi polices  $u, v \in U$ ,  $v$  is said to be optimal response against  $u$  if for each user  $v^i$  is the optimal response against  $u^{-i}$ .

Next we will study the existence of stationary Nash equilibrium and its properties.

### 5.3.3 Equilibrium and properties

Let  $\pi(q_i) = (\pi(1|q_i), \pi(0|q_i))$  denote the stationary distribution on the states of user  $i$  when he/she uses the stationary policy  $q_i \in U_S^i$ , where the component  $\pi(x|q_i)$  denotes the stationary probability of state  $x \in A^i$ . Markov chain of each user is irreducible with the stationary distribution as a function of policy given by

$$\pi(1|q_i) = \frac{q_i(1-p_i)}{q_i + p_i(1-q_i)} \text{ and } \pi(0|q_i) = \frac{p_i}{q_i + p_i(1-q_i)}. \quad (5.8)$$

The stochastic congestion game defined above satisfies all the assumptions in Theorem 2.1 in [95]. Hence, we have the following existence result.

**Proposition 5.3.4.** The stochastic game has a stationary multi policy  $u \in U_S$  which is a Nash equilibrium.

The optimal response of the  $i$ th user when the other user use stationary policies can be computed from the linear programming.

### 5.3.5 Linear Programming

For any state action pair  $(x, a) \in \mathcal{K}_i$  the expected cost incurred by the the  $i$  player when other user use stationary policy  $u^{-i} \in U_S^{-i}$  is

$$c^{i,u}(x, a) = \sum_{(x^{-i}, a^{-i}) \in \mathcal{K}^{-i}} \left( \prod_{j \neq i} u^j(a^j | x^j) \pi(x^j | q_j) \right) c^i(\mathbf{x}, \mathbf{a}), \quad (5.9)$$

where  $q_j$  denotes the probability associated with stationary strategy  $u^j$  and  $\mathbf{x} = [x^{-i} | x^i]$   $\mathbf{a} = [a^{-i} | a^i]$ .

Let  $z_{i,u}^* = \{z_{i,u}^*(x, a); (x, a) \in \mathcal{K}_i\}$  be such that it minimizes

$$\begin{aligned} & \sum_{(x,a) \in \mathcal{K}_i} c^{i,u}(x, a) z_{i,u}(x, a) \quad \text{subjected to} \\ & \sum_{(x,a) \in \mathcal{K}_i} z_{i,u}(x, a) [\delta_r(x) - P_{xar}^i] \quad \forall r \in X^i \\ & z_{i,u}(x, a) \geq 0 \quad \forall (x, a) \in \mathcal{K}_i \quad \text{and} \quad \sum_{(x,a) \in \mathcal{K}_i} z_{i,u}(x, a) = 1 \end{aligned} \quad (5.10)$$

Then the optimal policy is given by

$$u^{i*}(a|x) = \frac{z_{i,u}^*(x, a)}{\sum_{a \in A^i} z_{i,u}^*(x, a)}. \quad (5.11)$$

Next, we will consider symmetric users and characterize the Nash equilibrium.

### 5.3.6 Symmetric users

Let  $p$  denote the probability that a packet arrives to each players. From equation (5.9) observe that  $c^{i,u}(1, 0) = d(1)$  and  $c^{i,u}(0, 0) = 0$ . By substituting the the expected cost of  $i$ th player when the others use stationary strategy  $u^{-i}$  in (5.10) the best response of  $i$ th player is given by the following linear programming below. Let  $x := z_{i,u}(1, 1)$ ,

$$\begin{aligned} & \text{minimize} \quad c^{i,u}(1, 1)x + c^{i,u}(1, 0)\left(1 - \frac{x}{p}\right) \\ & \text{subjected to} \quad 0 \leq x \leq p \end{aligned} \quad (5.12)$$

If  $x^*$  is the optimal value of the above optimization problem than from equation (5.11) the best stationary response of  $i$ th player is given by  $q_i = x^* / (x^* + (1 - x^*/p))$ .

Now, considering two player case we characterize the Nash equilibrium as following.

**Proposition 5.3.7.** *For any given functions  $f, d$  and packet arrival probability  $p$ , if there exists  $q \in [0, 1]$  that satisfies*

$$q\pi(1|q)f(2) + (1 - q\pi(1|q))f(1) - \frac{d(1)}{p} = 0,$$

then  $(q, q)$  is the stationary symmetric Nash equilibrium (SSNE). Further, if  $f(1) > \frac{d(1)}{p}$  then 'no-transmit' for both player is SSNE. If  $(1 - p)f(2) + pf(1) < \frac{d(1)}{p}$  than 'transmit' for both players is SSNE.

*Proof.* See Appendix 5.6.4. □

## 5.4 Stochastic Congestion game with fixed demand

In this section we consider another version of stochastic congestion games with one player every day. Each day the arriving player has to decide how much of the traffic to send that day and how much to send the next day. The model is as following.

- One player arrives each day with a fixed demand of  $\phi > 0$  units. This demand has to be shipped to the destination using a shared link.  $\phi - \lambda$  units of demand is urgently required at the destination and hence needs to be shipped the same day by every player, where  $0 \leq \lambda \leq \phi$ . The remaining  $\lambda$  units of demand can be shipped either entirely in that same day or entirely at the next day, but no later. The  $i$ th player ships the remaining  $\lambda$  units of demand with probability  $p_i$  the same day. We denote the amount of demand shipped by the  $i$ th player at its arrival date as  $X_i$  and it takes values as following

$$X_i = \begin{cases} \phi, & \text{w.p } p_i \\ \phi - \lambda, & \text{w.p } 1 - p_i \end{cases} \quad (5.13)$$

- **Cost function:** Each player has to pay for the usage of the shared link. We assume that the amount paid depends on the total units of demand shipped on that day (*congestion cost*). Let  $f$  denote the congestion function. If the player decides to send  $\lambda$  units of demand the next day he/she has to pay a *storage cost* of  $d$  units per demand and the congestion cost the next day. Note that the cost incurred by the  $i$ th player depends only on shipping policy of the players of the previous day and that of the next day. We denote the total cost incurred by the  $i$ th player as  $J_i$  and it is given by

$$J_i(X_{i-1}, X_i, X_{i+1}) = X_i f(X_i + \phi - X_{i-1}) + (\phi - X_i)(d + f(\phi - X_i + X_{i+1}))$$

- **Strategy and utility:** Each player has to decide whether to ship the entire demand in one shot or ship only  $(\phi - \lambda)$  at its arrival day and the remaining  $\lambda$  units of demand the next day. The strategy of the  $i$ th player is the probability with which it transmits the entire demand in one shot. We denote it by  $p_i \in [0, 1]$ . We are interested in symmetric equilibria, so we assume that there is some constant  $p_{-i}$  such that any player other than  $i$  ships its entire demand with the same probability  $p_{-i}$ . The objective of each player is to choose a strategy  $p_i \in [0, 1]$ , for day  $i$ , such



that it minimizes his/her expected cost given that other players use  $p_{-i}$ . Player  $i$  faces the following optimization given that all other days the strategy used is  $p_{-i}$ .

$$\min_{0 \leq p_i \leq 1} \mathbf{E}_{p_i, p_{-i}}[J_i(X_{i-1}, X_i, X_{i+1})] \quad i = 1, 2, \dots$$

$\mathbf{E}_{p_i, p_{-i}}$  denotes that expectation is taken by assigning probability  $p_i$  to  $X_i$ .

### 5.4.1 Equilibrium strategies

We will be interested in studying the stationary equilibrium of the above stochastic congestion game. In particular, we will study the stationary symmetric Nash equilibrium (SSNE) assuming linear congestion cost, i.e.,  $f(x) = \beta x$ , for some  $\beta > 0$ . For any given strategy  $\{p_i, p_{-i}\}$  profile of the players, the utility function for the  $i$ th player is given by

$$\begin{aligned} & \mathbf{E}_{p_i, p_{-i}}[J_i(X_{i-1}, X_i, X_{i+1})] \\ &= p_i \phi \mathbf{E}_{p_{i-1}}[f(2\phi - X_{i-1})] + (1 - p_i) \mathbf{E}_{p_{i-1}}[(\phi - \lambda)f(2\phi - \lambda - X_{i-1})] \\ & \quad + (1 - p_i) a \mathbf{E}_{p_{i+1}}[d + f(\lambda + X_{i+1})] \\ &= p_i \left( 2\beta\phi^2 - \beta\phi \mathbf{E}_{p_{i-1}}[X_{i-1}] - \beta(\phi - \lambda)(2\phi - \lambda - \mathbf{E}_{p_{i-1}}[X_{i-1}]) - \lambda d - \beta\lambda^2 \right. \\ & \quad \left. - \beta\lambda \mathbf{E}_{p_{i+1}}[X_{i+1}] \right) + \beta(\phi - \lambda)(2\phi - \lambda - \mathbf{E}_{p_{i-1}}[X_{i-1}]) + \lambda d + \beta\lambda^2 + \beta\lambda \mathbf{E}_{p_{i+1}}[X_{i+1}] \end{aligned}$$

Now assume that all player  $j \neq i$  use the strategy  $p_j = \bar{p}$ . Then it is clear that  $\mathbf{E}[X_{i+1}] = \mathbf{E}[X_{i-1}] = (\phi - \lambda) + \lambda\bar{p}$ . Substituting in the above equations and continuing the chain of equalities we have

$$\begin{aligned} & \mathbf{E}_{p_i, p_{-i}}[J_i(X_{i-1}, X_i, X_{i+1})] \\ &= p_i(-2\beta\lambda^2\bar{p} + \beta\lambda\phi - \lambda d) + \beta\phi^2 + \lambda d + \beta\lambda(2\lambda - \phi)\bar{p} \end{aligned} \quad (5.14)$$

SSNE are characterized in the following proposition.

**Proposition 5.4.2.**

- When  $d \leq \beta\phi$ 
  - $p_i^* = 0$  for all  $i$  is SSNE.
  - $p_i^* = (\beta\phi - d)/2\lambda\beta$  for all  $i$  is also SSNE.
- When  $d > \beta\phi$ 
  - $p_i^* = 1$  for all  $i$  is SSNE.

*Proof.* First assume that  $d \leq \beta\phi$ . Suppose  $p_i > 0$  for player  $i$  and  $p_j = 0$  for all other players  $j \neq i$ . Then from equation (5.14) it can be seen that the utility of the  $i$ th player is given by

$$\mathbf{E}_{p_i, 0}[J_i] = p_i(\beta\lambda\phi - \lambda d) + \beta\phi^2 + \lambda d,$$

which is strictly larger than the case when  $p_i = 0$  is chosen. Hence  $p_i^* = 0$  for all  $i$  is a SSNE. To see that  $p_i^* = (\beta\phi - d)/2\lambda\beta$  is also an equilibrium, substitute this value in equation (5.14) and notice that it becomes independent of  $p_i$ . For the case  $d > \beta\phi$  the claim can be verified by substituting  $\bar{p} = 1$  in equation (5.14) and noticing that the coefficient of  $p_i$  is negative.  $\square$

## 5.5 Conclusion

We studied the dynamic routing game in which there is both congestion and delay cost. We established the existence of equilibrium and gave its properties considering both cases of atomic and non-atomic players. When the demand of players is random, we considered a simple stochastic congestion game and investigated the existence of stationary equilibrium policies and its properties. Its interesting to consider more general model in stochastic game scenario and study its equilibrium properties. For example, when buffer length is more than one. We also considered a simple congestion game in which the demand that arrives is not random but fixed and characterized its equilibrium.

## 5.6 Appendices

### Proof of Proposition 5.2.4

$$\begin{aligned} & \underset{\mathbf{x}=(x_1, x_2, \dots, x_T)}{\text{minimize}} && \sum_{j=1}^T \frac{x_j}{N} (ax_j^p + b + j) + \frac{N-1}{N} \sum_{j=1}^T \int_0^{x_j} (ax^p + b + j) dx \\ & \text{subject to} && -x_j \leq 0, \quad j = 1, 2, \dots, T \text{ and } \sum_{j=1}^T x_j = d. \end{aligned}$$

If  $\mathbf{x}^* = (x_1^*, x_2^*, \dots, x_T^*)$  is the optimal solution than by the necessary conditions of the KKT theorem there exists  $\lambda_j \geq 0$  for  $j = 1, 2, 3 \dots T$  and  $\beta \in \mathbb{R}$  such that the following holds

$$ax_j^{*p} + b + j + \frac{ap}{N} x_j^{*p} = \beta + \lambda_j \quad \text{for } j = 1, 2, \dots, T \quad (5.15)$$

$$x_j^* \lambda_i = 0 \quad \text{for } i = 1, 2, \dots, T \quad (5.16)$$

$$\sum_{j=1}^T x_j^* = d \quad (5.17)$$

substituting the value of  $\lambda_j$  from equation (5.15) into equation (5.16) we get

$$x_j^* \cdot (ax_j^{*p} + b + j + \frac{ap}{N} x_j^{*p} - \beta) = 0.$$

if  $b + j \geq \beta$  than it is clear that  $x_j^* = 0$ . Now consider the case  $b + j < \beta$ , we have either  $x_j^* = 0$  or  $x_j^* > 0$ . Now suppose that  $x_j^* = 0$ , then from equation (5.15) and the fact that

$\lambda_j \geq 0$  we get  $b + j \geq \beta$ , which is a contradiction. Hence, for the case  $b + j < \beta$ ,  $x_j^*$  is strictly positive and can be obtained as the solution of

$$ax_j^{*p} + b + j + \frac{ap}{N}x_j^{*p} - \beta = 0 \quad , \text{i.e., } ax_j^{*p} = \left( \frac{\beta - j - b}{a + \frac{ap}{N}} \right)^{\frac{1}{p}}.$$

Substituting the above in equation (5.17) we get

$$\sum_{j: j < \beta - b} \left( \frac{\beta - \alpha i - b}{a + \frac{ap}{N}} \right)^{\frac{1}{p}} - d = 0.$$

This concludes the proof.

### 5.6.1 Bound on Price of Anarchy

We began by studying the properties of the lagrangian multiplier associated with equality constraints ( $\beta$ ) as a function of number of user. In this regard define a function  $W_i$  for each  $i = 1, 2, \dots, T$  as following

$$W_i(N, \gamma) = \mathbb{I}_{\{i+b < \gamma \leq (i+1)+b\}} \left( \frac{\gamma - i - b}{a + \frac{ap}{N}} \right)^{\frac{1}{p}}.$$

Also define  $W(N, \gamma) = \sum_{i=1}^T W_i(N, \gamma)$ . Some of the properties of the function  $W$  are summarized in the following lemma.

**Lemma 5.6.2.** *The function  $W$  is such that*

1. *For a fixed  $N$ ,  $W$  is continuous and monotonically increasing in  $\gamma$*
2. *For a fixed  $\gamma$ ,  $W$  is monotonically increasing  $N$ .*
3. *For a fixed  $N$ ,  $W(N, \gamma) = 0$  has a unique solution denoted as  $\gamma(N)$  in the interval  $(1 + b, \infty)$*

*Proof.* First, notice that for each  $i$ ,  $W_i$  has a right limit at the point  $i + b$  given by  $W_i(N, i + b)$ . It is clear that  $W$  is continuous in each interval  $(i + b < \gamma < (i + 1) + b)$ , we need to verify its continuity only at the boundaries. Fix  $j$  and consider the difference

$$\begin{aligned} & \lim_{\gamma \rightarrow ((j+1)+b)^+} W(N, \gamma) - \lim_{\gamma \rightarrow ((j+1)+b)^-} W(N, \gamma) \\ &= W_{j+1}(N, (j+1) + b) - W_j(N, (j+1) + b) \\ &= \sum_{i=1}^{j+1} \left( \frac{(j+1) + b - i - b}{a + \frac{ap}{N}} \right)^{\frac{1}{p}} - D - \sum_{i=1}^j \left( \frac{(j+1) + b - i - b}{a + \frac{ap}{N}} \right)^{\frac{1}{p}} + D = 0 \end{aligned}$$

Hence  $W$  is continuous. Also,  $W_i$  is strictly increasing in the interval  $(i + b, (i + 1) + b)$  for each  $i$  which implies that  $W$  is strictly increasing. The second claim is straightforward. To prove the third part of the lemma notice that  $W$  starts from  $-D < 0$  at  $\gamma = 1 + b$ , then the claim follows from the first part of the lemma.  $\square$

We next show that the unique zero of the function  $W$  is increasing in  $N$ .

**Lemma 5.6.3.**  $\gamma(N)$  is monotonically decreasing in  $N$

*Proof.* Let  $\gamma(N_1)$  and  $\gamma(N_2)$  denote the unique roots of  $W(N_1, \gamma)$  and  $W(N_2, \gamma)$  respectively, and assume that  $N_1 < N_2$ . We have

$$W(N_1, \gamma(N_1)) = 0 = W(N_2, \gamma(N_2)) \geq W(N_1, \gamma(N_2)),$$

where the last inequality follows from second part of lemma 5.6.2. Now the claim directly follows from first part of the lemma 5.6.2.  $\square$

From the previous lemma 5.6.3 it is easy to observe that as the number of symmetric users increases (with total demand held constant), positive flow is sent on lesser number of days at equilibrium. Let  $L_N$  denote set of days on which total flow sent by  $N$  users is positive, i.e.,  $L_N = \{i : i + b < \gamma(N)\}$ . Then the observation made from the previous lemmas can be written as  $L_{N+1} \subset L_N$ . In particular, when there is just one user, i.e., centrally controlled system, the positive flow is sent on maximum number of days.

Equilibrium social cost with  $N$  users is given by

$$\begin{aligned} D_N &= \sum_{i \in L_N} x_i^* (a(x_i^*)^p + b + i) = \sum_{i \in L_N} \left( \frac{\gamma(N) - i - b}{a + \frac{ap}{N}} \right)^{\frac{1}{p}} \left( a \left( \frac{\gamma(N) - i - b}{a + \frac{ap}{N}} \right) + b + i \right) \\ &= \sum_{i \in L_N} \left( \frac{\gamma(N) - i - b}{a + \frac{ap}{N}} \right)^{\frac{1}{p}} \left( \frac{\gamma(N) + \frac{p}{N}(i + b)}{1 + \frac{p}{N}} \right), \end{aligned}$$

and optimal social cost is given by

$$\begin{aligned} D_1 &= \sum_{i \in L_1} x_i^* (a(x_i^*)^p + b + i) = \sum_{i \in L_1} \left( \frac{\gamma(1) - i - b}{a + ap} \right)^{\frac{1}{p}} \left( a \left( \frac{\gamma(1) - i - b}{a + ap} \right) + b + i \right) \\ &= \sum_{i \in L_1} \left( \frac{\gamma(1) - i - b}{a + ap} \right)^{\frac{1}{p}} \left( \frac{\gamma(1) + p(i + b)}{1 + p} \right) \end{aligned}$$

$$\begin{aligned} \text{PoA} &= \frac{\sum_{i \in L_N} \left( \frac{\gamma(N) - i - b}{a + \frac{ap}{N}} \right)^{\frac{1}{p}} \left( \frac{\gamma(N) + \frac{p}{N}(i + b)}{1 + \frac{p}{N}} \right)}{\sum_{i \in L_1} \left( \frac{\gamma(1) - i - b}{a + ap} \right)^{\frac{1}{p}} \left( \frac{\gamma(1) + p(i + b)}{1 + p} \right)} \\ &\leq \frac{a^{\frac{1}{p}} (1 + p)^{1 + \frac{1}{p}} \sum_{i \in L_1} \left( \gamma(1) - i - b \right)^{\frac{1}{p}} \left( \gamma(1) + \frac{p}{N}(i + b) \right)}{a^{\frac{1}{p}} \left( 1 + \frac{p}{N} \right)^{1 + \frac{1}{p}} \sum_{i \in L_1} \left( \gamma(1) - i - b \right)^{\frac{1}{p}} \left( \gamma(1) + p(i + b) \right)} \leq \left( \frac{1 + p}{1 + \frac{p}{N}} \right)^{1 + \frac{1}{p}} \end{aligned}$$

Where the first inequality follows as  $\gamma(N) \leq \gamma(1)$  and  $L_N \subset L_1$ .

#### 5.6.4 Proof of proposition 5.3.7

Let  $q$  be denote the stationary strategy of player-1, from (5.9) the value of the expected cost incurred by player-2 when his/her action is to transmit in active state is given by

$$c^{2,u}(1,1) = q\pi(1|q)f(2) + (1-q)\pi(0|q)f(1) + \pi(0|q)f(1).$$

Substituting the above in the objective function in (5.12), and differentiating with respect to  $x$  we get the following first order optimality condition

$$q\pi(1|q)f(2) + (1-q\pi(1|q))f(1) - \frac{d(1)}{p} = 0. \quad (5.18)$$

Suppose player-1 chooses  $q$  such that the above equation is satisfied than any values of  $x \in [0, p]$  is optimal and any values of  $q_2 \in [0, 1]$  is optimal response for player-2. Hence  $(q, q)$  is the stationary symmetric Nash equilibrium. To see the other part of the proposition, substitute the expression for stationary distributions in (5.8), when the stationary policy is  $q$ , in (5.12) to get

$$\left( \frac{q^2(1-p)}{q+p(1-q)}f(2) + \left( 1 - \frac{q^2(1-p)}{q+p(1-q)} \right) f(1) - \frac{d(1)}{p} \right) x - d(1).$$

If player-1 chooses  $q_1 = 0$ ,  $x^* = 0$  is the minimizer of the above equation when  $f(1) > \frac{d(1)}{p}$ , hence  $(0, 0)$  is the SSNE. By similar arguments we can show that  $(1, 1)$  is the SSNE

when  $(1-p)f(2) + pf(1) < \frac{d(1)}{p}$ .

## **Part II**

# **Economic aspects in communication networks**



# Chapter 6

## Nonneutral Network and the Role of Bargaining Power in Side Payments

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### 6.1 Introduction

Since its inception, the Internet is based on the following principle of providing end-to-end connectivity: end users need not know how their packets are transported in the network, but are guaranteed best effort delivery without any discrimination, i.e, irrespective of their identity, the application, or the protocol they use. This encouraged rapid innovation at the edge of the network without any interference from the network operators and creation of content that is accessible in a non-discriminatory fashion. Also, the end users paid only the internet service providers that connects them to the Internet and not any intermediate network operators. This lead to a huge growth of the Internet and related electronic commerce and businesses. The growth is thus characterized by a nondiscriminatory or neutral (egalitarian) policy for accessing the global Internet.

Innovations at the edge of the network have resulted in a variety of applications and content development. These application have enabled a wide range of services on the Internet, including the basic voice services offered by regular telephone companies. In 2002 Federal Communications Commission (FCC) reclassified broadband Internet as



‘information service’ from the earlier status of ‘telecommunications service’ [121], and in 2005 this decision was upheld by the courts in the USA. According to this ruling the internet service providers (ISPs) are no more bound to follow the non-discriminatory policy required for the ‘telecommunication services’. Many last-mile ISPs expressed their desire to move away from the neutral policy that has thus far characterized the Internet arguing that returns on their investments are insufficient in this regime. More specifically, those opposing neutrality argue that (i) some applications (such as peer-to-peer (P2P) streaming applications) require a lot of costly resources, and (ii) if a neutral policy is pursued, there would be no incentive for investing in the network infrastructure upgrade. In an interview<sup>1</sup> in 2005, Edward Whitacre, the then CEO of AT&T, said “...Now what they would like to do is use my pipes free, but I ain’t going to let them do that because we have spent this capital and we have to have a return on it. So there’s going to have to be some mechanism for these people who use these pipes to pay for the portion they’re using. Why should they be allowed to use my pipes? The Internet cannot be free in that sense...” This sparked a huge debate on whether the internet should be neutral or not.

While there is no commonly accepted definition for a neutral policy, one could associate *neutrality* with no *off-network pricing* or *quality of service (QoS) differentiation*, and *nonneutrality* with either off-network pricing or QoS differentiation or both.

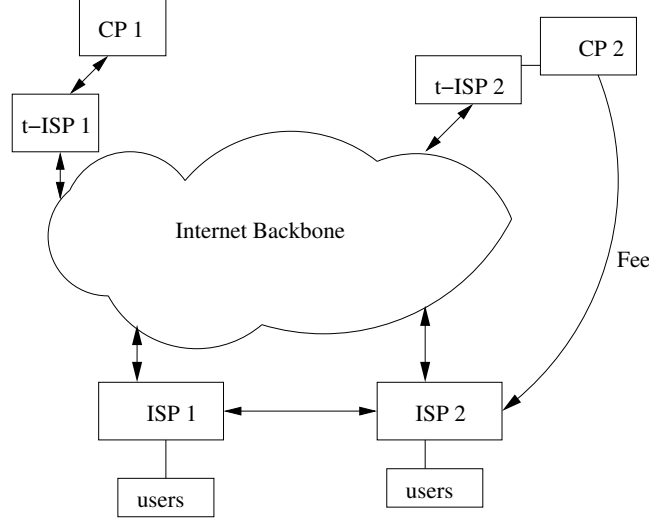
In the neutral regime ISPs charge the users only once, i.e., users pay only to the ISP that connects them to the Internet and no other intermediate network operator, including the last-mile ISP that serves its destination. Here a user can be a content provider (CP) or an end user who is a consumer of content (termed *internauts* in this paper). Through off-network pricing, the last-mile ISPs may propose to charge the CPs that are not directly connected to them for enabling access to their customers. In other words, companies like YouTube, Skype, Amazon, etc. have to pay the last-mile ISPs to reach their customers. In literature different terminologies are used for this criteria - in [113], it is termed ‘zero-fee’, and in [112] ‘user discrimination’. We borrow the term off-network pricing from [103].

In QoS differentiation ISPs may propose to introduce prioritized service through multi-tiering [117]. Users paying higher fees get better QoS (for example, higher bandwidth or lesser delay), while the users paying lower fees get only a minimal QoS guarantee. This means, the ISPs can discriminate packets of the users based on payment. In the neutral regime all packets are treated equally without any priorities or discrimination. Service differentiation may involve selective blocking of packets, selective throttling of flows (allocating less throughput to some flows). Blocking and throttling P2P traffic has been a common practice in several countries, and not just during congestion epochs [99]. In the USA, such a practice was criticized by the FCC telecom regulation body.

Figure 6.1 shows the connection between ISPs, CPs and end users in the Internet. The end users are connected to the Internet backbone by the last-mile ISPs. Usually, users have only option to use the services of a monopoly or duopoly ISP (ISP 1 or ISP

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<sup>1</sup>Business Week 2005, November 7



*Figure 6.1: Users-ISP-CP connections in the Internet*

2). The content providers are connected to the Internet backbone by the transit ISPs<sup>2</sup> denoted as t-ISP in Figure 6.1. The CPs usually have agreements with the t-ISPs and pay them based on the amount of the bandwidth used. In the neutral regime, CP 1 only pays t-ISP 1 for connectivity to the end users, and not any other intermediate ISPs (ISP 1 or ISP 2). In the nonneutral off-network pricing regime a last-mile ISP (ISP 1 or ISP 2) can ask the CPs to pay for the connection to its end customers.

We wish to focus on a version of a nonneutral network where the last-mile ISPs can charge the content providers. For this purpose, let us consider the abstracted architecture in Figure 2, where there is a last-mile ISP monopoly (i.e., ISP 1 and ISP 2 of Figure 1 are the same entity, and where the combination of CP  $i$ , t-ISP  $i$  and the internet backbone is abstracted out as a single entity. We shall call this combined entity CP  $i$ . There may be one or several CPs, and in case there are several (in Section 7.2), each has a dedicated clientele (users  $i$ ). We note that our focus is to study a nonneutral with only off-network pricing in this paper. We do not take into account other QoS, prioritization, investment aspects into our study.

<sup>2</sup>ISPs that connects the smaller ISPs to the internet backbone are referred to as transit-ISPs. The last-mile ISPs connect to internet backbone through transit-ISPs. This is not shown in Figure 6.1

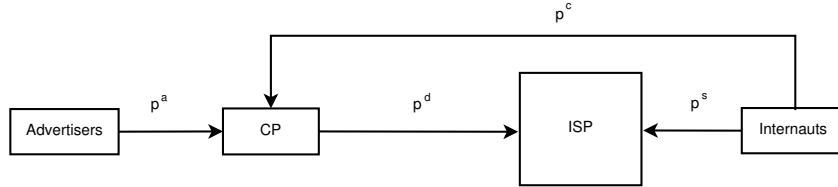


Figure 6.2: Monetary flow in a nonneutral network.

### 6.1.1 Related work

The papers on economics aspects of the net neutrality through mathematical models<sup>3</sup> can be broadly classified into two categories depending on what they consider as violation of network neutrality policy: One group of papers consider off-network pricing as the criteria, and the others QoS differentiation. Here we only discuss the papers that consider former as a violation of net neutrality principle as we do in this paper. Analysis of nonneutral networks with QoS differentiation can be found in [117, 114, 115, 118].

In [113], the authors model the nonneutral network as a two sided market, with a continuum of users and CPs connected by a monopoly ISP. They show that if the ISP charges the CPs its profit margin increases, whereas the CPs profit reduces with lesser number of CPs being active at equilibrium. However, the social welfare can be higher or lower compared to the case with the zero-fee case depending on model parameters. Further, if a social planner is to decide the payment from CPs to the ISP, it will be set at a price smaller than that set by the monopoly ISP. In particular, the ISP will be made to subsidize CPs for reaching their costumers. In a similar setting, [116] studies investment incentives for the ISPs and CPs. The paper concludes in favor of the the neutral network arguing that CP and consumer participation will increase in this regime with improved investment and higher social welfare. In [103], the authors consider a duopoly ISP market and bring in several aspects such as investments by ISPs, pricing of CPs, CPs connection decision, consumer pricing and their connection decision etc. Analyzing it as a hierarchial 6-stage game, they conclude that in the non-neutral regime the investments will be higher with increased participation of consumer and CPs. The authors in [112] consider finite number of CPs and ISPs. They conclude that social welfare is higher in nonneutral regime if the ratio of the advertisement revenues to user price sensitivity is either or low, and for intermediary values neutral regime is preferable.

The literature on economics of net neutrality is not conclusive, they argue in favor of off-network pricing or against it or have mixed opinion depending on the market

<sup>3</sup>There is huge literature discussing the legal and policy implication of network neutrality regulation. We do not discuss these papers here. See [143],[119],[120].

scenarios. Also, methods for payment between ISPs and CPs is less explored. In [111], the authors study revenue sharing mechanisms between interconnected network operation based on weighted proportional fairness criteria, but not in the context of net neutrality. The side payment from one provider to another is expected to be financed by income from internauts and from advertisers (publicity income). Cooperative game theory provides us with useful tools for designing revenue sharing mechanisms that possess some fairness properties. In [101]-[102] the Shapley value (which is known to have some fairness properties [106]) was used for deciding how revenues from internauts should be split between the service and the content providers. In [99] it is shown that if one provider, say the internet service providers (ISP), has the power to impose payments on the other provider (the CP), not only does the content provider lose control over how much they can pay, the end users suffer, and moreover, the ISP's performance degrades. More precisely, [99] showed that the only possible equilibrium would be characterized by prices that will induce zero demand from the end users. This phenomenon does not occur if the price that one provider is requested to pay to the other were fixed by some regulator.

### 6.1.2 Our Objective and Contributions

Our objective in this paper is to study mechanisms for determining which provider should pay the other and how much. We are in particular interested in the impact of such mechanisms on the equilibrium. We shall focus in this paper on mechanisms based on the Nash bargaining paradigm (which is known in the network engineering context as the proportional fair assignment). It is the unique way of transferring utilities that satisfies a well known set of four axioms [104] related to fairness. We note that assigning the side payments fairly is just part of the story. In practice one provider may have more weight than the other one in the decision on the amount of side payment. We then say that the provider has a larger bargaining power (we shall make this precise in the next section). As an example, the Spanish ISP "Telefonica" announced on 8 February 2010 that it considered charging Google, indicating perhaps that the bargaining power of Google is weaker than that of Telefonica in the Spanish telecommunications market. Our work will allow to determine exactly how much payment would go from one provider to another as a function of the bargaining powers of each provider. For quantifying the bargaining power of each side, we follow the approach presented in [100].

Revenue sharing mechanisms were studied in [111] and [112]. But our models are very different from theirs. While they assume smooth demand functions, this is not the case in our models. Furthermore, the two-sided pricing model of [112] does not allow the content providers to charge the customers directly, while our models do. A third difference is related to bargaining and timing of actions, as discussed next.

Our goal in this paper is to understand a very simple and tractable model of a two player game with one agent being a content provider (CP) and another agent being an internet service provider (ISP). Several ancillary parties are also involved – the users

who respond to prices via a demand function, advertisers who are providers of revenue, and an arbitrator who regulates the side payments taking into account the bargaining power of the players. In Section 6.2 we study the single-CP single-ISP game in two settings – bargaining before and after setting the prices. We then extend the results to the case when there are multiple content providers in Section 7.2. The demand function we consider in Section 6.2 (single-CP case) is a simple, linear, decreasing function of the net price. In the multiple-CP case, demand for content from a CP is linear and decreasing in the price of that CP’s content, but linear and increasing in the price of other CPs’ contents, reminiscent of the Bertrand oligopoly [133]; see (7.1). In other words, demand for content from CP  $i$  is negatively correlated with the price of CP  $i$  content, but positively correlated with others’ prices. This positive correlation may be anticipated as due to increased demand for CP  $i$  content as a consequence of increased capacity arising from decreased demand for CP  $j$  content on account of a CP  $j$  price increase. However, the positive correlation ceases as soon as the demand for content from a CP goes to zero, a generalization that extant literature does not consider. We examine the structure of the demand function in the general case in Section 6.3.2, and then study the equilibria for bargaining before the actions and after the actions in subsequent sections (Sections 6.3.4 and 6.3.5). In the multiple-CP case, due to combinatorial complexity, we give a complete characterization of all equilibria for the case of two CPs. We anticipate that the results will extend straightforwardly when there are more than two CPs. In Section 6.4, we study two dynamic models of the game studied in Section 7.2 and study their convergence. The paper concludes with a discussion in Section 6.5 and proofs of main results given in the appendices.

## 6.2 The Case of a Single CP and a Single ISP

We first begin with the simple case of a single CP and a single ISP. All the internauts are connected to the ISP, and can access the content of the CP only through the ISP. See Figure 8.1 for a payment flow diagram. The various parameters of the network neutrality game are as follows.

We consider two interesting games. The timing for the first game is as follows.

- The ISP and the CP bargain over the payment  $p^d$  from the CP to the ISP. This can be positive or negative.
- The CP sets the price  $p^c$ . The ISP sets the price  $p^s$ . Both set their prices simultaneously.
- The internauts react to the prices and set the demand.

In the second game, bargaining is done later:

- The CP and the ISP set their respective access prices  $p^c$  and  $p^s$  simultaneously.

Parameter	Description
$p^s$	Price per unit demand paid by the users to the ISP. This can be positive or negative.
$p^c$	Price per unit demand paid by the users to the CP. This too can be positive or negative.
$d(p^s, p^c)$	Demand as a function of prices. We shall take this to be $d(p^s, p^c) = [D_0 - \alpha(p^s + p^c)]_+$ , where $[x]_+ = \max\{x, 0\}$ is the positive part of $x$ .
$p^a$	Advertising revenue per unit demand, earned by the CP. This satisfies $p^a \geq 0$ .
$p^d$	Price per demand paid by the CP to the ISP. This can be either positive or negative.
$U_{\text{ISP}}$	The revenue or utility of the ISP, given by $d(p^s, p^c)(p^s + p^d)$ .
$U_{\text{CP}}$	The revenue or utility of the CP, given by $d(p^s, p^c)(p^c + p^a - p^d)$ .
$\gamma$	Bargaining power of the ISP with respect to the CP. This satisfies $0 < \gamma < 1$ .

Table 6.1: Single CP and ISP net neutrality game

- The ISP and the CP bargain over the payment  $p^d$  from the CP to the ISP. This can be positive or negative.
- The internauts react to the prices and set the demand.

The first game arises when the charges per unit demand can change over a comparatively faster time-scale while the CP-ISP price changes over a slower time-scale. The second one is an interesting case that may arise in a regulatory setting where the prices per unit demand charged to the internauts varies over a slower time-scale, but the ISP and the CP can quickly renegotiate their prices. We analyze both models via backward induction and identify the equilibria.

For a fixed  $p^s$  and  $p^c$ , bargaining results in an agreement of a payment  $p^d$  from the CP to the ISP, determined by

$$p^{d*} \in \arg \max_{p^d} U_{\text{ISP}}^\gamma \times U_{\text{CP}}^{1-\gamma}.$$

The parameter  $\gamma$  determines the bargaining power of the ISP with respect to the CP. When  $p^{d*}$  is negative, the ISP is the one that makes a payment to the CP.

We now discuss some properties of the bargaining solution.

If we take  $\gamma = 1/2$  then the maximization is equivalent to that of the product of the utilities of the ISP and the CP. This is then the standard Nash bargaining approach [104] for resource allocation, known in networking as the proportional fair allocation [135]. (It is known to be the unique assignment satisfying a set of four axioms.) To understand the case of general  $\gamma$ , consider first the problem of maximizing  $U_{\text{ISP}}^{m_1} U_{\text{CP}}^{m_2}$  where  $m_1$  and  $m_2$  are integers. The form of the objective function suggests that we are simply searching for a standard proportional fairness solution but where there are  $m_1$  ISP's and  $m_2$  content providers<sup>4</sup>. Thus the ISP is said to have a bargaining power equivalent to that of  $m_1$  players, and the CP equivalent to that of  $m_2$  players. Finally, we note that  $\arg \max(U_{\text{ISP}}^{m_1} U_{\text{CP}}^{m_2})$  is the same as that of  $(U_{\text{ISP}}^\gamma U_{\text{CP}}^{1-\gamma})$  where  $\gamma := m_1/(m_1 + m_2)$ .

We next observe that the game problem is equivalent to replacing all utilities by the log of the utilities. We may imagine that the bargaining is done by another player, the regulator, whose (log) utility equals

$$\bar{U}_{\text{regulator}} := \gamma \bar{U}_{\text{ISP}} + (1 - \gamma) \bar{U}_{\text{CP}}, \quad (6.1)$$

where  $\bar{U}_{\text{ISP}} = \log U_{\text{ISP}}$  and  $\bar{U}_{\text{CP}} = \log U_{\text{CP}}$ . At least one previous work has already used as utilities the linear combination of performance measures of other players and interpreted  $\gamma$  as some measure of the “degree of cooperation”, see [105], in other contexts.

Let us now return to our games. In the first game, the CP and the ISP bargain over Nash equilibria. In the second, they choose  $p^a$  and  $p^c$  knowing that they will bargain subsequently. A *summary of the results* for the single-CP single-ISP games are as follows.

1) In both cases, there exists a pure strategy Nash equilibrium, in a sense that will be made precise, with strictly positive demand and strictly positive utilities for the agents. In the pre-bargaining problem there are other zero-demand equilibria. In the post-bargaining problem, the aforementioned pure strategy Nash equilibrium is unique.

2) In all cases with strictly positive demand, users pay the ISP. But users pay the CP only if the advertising revenue is small. Otherwise the CP subsidizes the users.

3) If either of the agents have control over  $p^d$ , the equilibrium demand is zero. None of the parties benefit from this situation. On the contrary, if  $p^d$  is under the control of a disinterested arbitrator, there is an equilibrium where every one benefits. This is the key insight gained from our analysis, that some sort of regulation can bring benefits to all.

4) Interestingly, if the agents bargain beforehand and the strictly positive demand equilibrium ensues, the payments by the users and resulting utilities of all agents are independent of the actual value of  $p^d$ .

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<sup>4</sup>The optimization problem involved in computing the proportional fair solutions, that of maximizing the product of utilities, may have some constraints. We do not modify the constraints when altering  $m_1$  or  $m_2$ . The change in  $m_1$  or  $m_2$  merely corresponds to a re-weighting of the utilities and not of addition or removal of users.

5) If the agents bargain beforehand, over Nash equilibria, then demand settles at a lower value than if the agents bargain after setting their prices.

6) If the agents bargain beforehand, they end up with equal revenues. If they bargain afterward, they share the net revenue in the proportion of their bargaining power.

7) Finally, if  $\gamma \in \left[\frac{4}{9}, \frac{5}{9}\right]$ , then both agents prefer to fight it out after setting their prices. For  $\gamma > 5/9$ , the ISP prefers post-arbitration, and for  $\gamma < 4/9$ , the CP prefers post-arbitration.

While the above appear to suggest that post-arbitration may prove to be good to the internauts, there are no pure-strategy Nash equilibria in the post-arbitration game when there are two or more CPs under a model where the demand for content from a CP is positively correlated with a competing CP's price. We shall return to this in a later section.

With these motivating remarks, we shall now proceed to state these claims in a precise fashion and to prove them. In subsequent sections we shall study the extension of the above results to the case of multiple CPs and to the case of an exclusive contract between one of the CPs and the ISP.

### 6.2.1 Bargaining over Nash equilibria

We first consider the case where the agents bargain over Nash equilibria. Both agents bargain over the choice of  $p^d$ , knowing that they will subsequently play a simultaneous action game where the ISP and CP will choose  $p^s$  and  $p^c$ , respectively. Our main result here is summarized as follows.

**Theorem 6.2.1.1.** *When the CP and the ISP bargain beforehand over Nash equilibria, we have the following complete characterization of all pure strategy Nash equilibria.*

(a) *Among profiles with strictly positive demand, there is a unique pure strategy Nash equilibrium with the following properties:*

- *The uniqueness is up to a free choice of  $p^d$ .*
- *At equilibrium, we have:*

$$p^s = \frac{D_0 + \alpha p^a}{3\alpha} - p^d, \quad (6.2)$$

$$p^c = \frac{D_0 - 2\alpha p^a}{3\alpha} + p^d. \quad (6.3)$$

- *The net user payment per demand  $p^s + p^c$  is unique and is given by*

$$p^s + p^c = \frac{2D_0 - \alpha p^a}{3\alpha}.$$

*Any  $p^d$  paid by the CP is collected from the user and further returned back to the user by the ISP.*



- The demand is unique and is given by  $(D_0 + \alpha p^a)/3 > 0$ .
- The utilities of the ISP and CP are equal and given by

$$U_{ISP} = U_{CP} = \frac{(D_0 + \alpha p^a)^2}{9\alpha}.$$

(b) For each choice of  $p^d$ , a strategy profile  $(p^s, p^c)$  constitutes a Nash equilibrium with zero demand if and only if the following two inequalities hold:

$$p^s \geq D_0/\alpha + p^a - p^d, \quad (6.4)$$

$$p^c \geq D_0/\alpha + p^d. \quad (6.5)$$

*Proof.* We first observe that at equilibrium,  $U_{ISP}$  and  $U_{CP}$  are both nonnegative. If not, the ISP (resp. CP) has strictly negative utility. He can raise the price  $p^s$  (resp.  $p^c$ ) to a sufficiently high value so that demand becomes zero, and therefore  $U_{ISP} = 0$  (resp.  $U_{CP} = 0$ ). Thus a deviation yields a strict increase in utility and therefore cannot be an equilibrium. It follows that at equilibrium, we may take the revenues per demand for the ISP and CP to be nonnegative, i.e.,  $p^s + p^d \geq 0$ , and  $p^c + p^a - p^d \geq 0$ .

We next deduce (b), which is a characterization of all the pure strategy NE with zero demand. Consider a fixed  $p^d$ . If a pair  $(p^s, p^c)$  were an equilibrium with zero demand, then clearly

$$D_0 \leq \alpha(p^s + p^c),$$

and

$$U_{ISP} = d(p^s, p^c) \times (p^s + p^d) = 0.$$

Moreover, the ISP should not be able to make his utility positive, i.e., any  $p^s$  that makes demand strictly positive,  $p^s < D_0/\alpha - p^c$ , must also render price per unit demand zero or negative,  $p^s + p^d \leq 0$ . This can happen only if  $(D_0/\alpha - p^c) + p^d \leq 0$  which is the same as (6.5). Similarly, the CP should not be able to make his utility positive, i.e., any  $p^c$  that makes demand strictly positive,  $p^c < D_0/\alpha - p^s$ , must render CP price per unit demand nonpositive,  $p^c + p^a - p^d \leq 0$ . This can happen only if  $(D_0/\alpha - p^s) + p^a - p^d \leq 0$  which is the same as (6.4). This proves the necessity of (6.4) and (6.5). We now show sufficiency. Let (6.4) and (6.5) hold. Then addition of  $p^c$  to both sides of (6.4) and some rearrangement yields

$$p^c + p^a - p^d \leq p^s + p^c - D_0/\alpha. \quad (6.6)$$

Since the left side is the revenue per unit demand for the CP, it must be nonnegative, and hence  $p^s + p^c - D_0/\alpha \geq 0$ , which upon rearrangement yields  $D_0 - \alpha(p^s + p^c) \leq 0$ . The demand  $d(p^s, p^c)$  is therefore zero. Let us now consider a deviation by the CP for a fixed ISP price  $p^s$  that satisfies (6.4). We will show that the least deviation (decrease in price) that sets the demand at the threshold of positivity results in a negative revenue per demand for the CP. Indeed, this critical price  $q^c$  that sets the demand at the threshold of positivity satisfies the equation

$$D_0 - \alpha(q^c + p^s) = 0.$$

Again, addition of  $q^c$  to both sides of (6.4) yields, by the same steps above that led to (6.6),

$$q^c + p^a - p^d \leq p^s + q^c - D_0/\alpha = 0.$$

Further reduction in price to make demand strictly positive only results in negative revenue and negative utility. Consequently, the CP does not have a deviation that yields a higher revenue. A similar argument shows that, under (6.5), the ISP can make demand strictly positive only if its revenue is negative. It too does not have a deviation with a strictly greater utility. Thus (6.4) and (6.5) constitute zero demand equilibrium prices.

Let us now search for an equilibrium with a strictly positive demand. Such a  $(p^s, p^c)$  must lie in the interior of the set of all pairs satisfying  $D_0 \geq \alpha(p^s + p^c)$ . As  $U_{\text{ISP}}$  is concave in  $p^s$  for a fixed  $p^c$  and  $p^d$ , whenever the utility is strictly positive, we must have a local maximum at equilibrium, i.e.,

$$\begin{aligned} \frac{\partial U_{\text{ISP}}}{\partial p^s} &= \frac{\partial}{\partial p^s} (D_0 - \alpha(p^s + p^c))(p^s + p^d) \\ &= D_0 - \alpha p^s - \alpha p^c - \alpha(p^s + p^d) \\ &= 0, \end{aligned}$$

which yields

$$p^s = \frac{D_0 - \alpha p^c - \alpha p^d}{2\alpha}.$$

Analogously,  $U_{\text{CP}}$  is concave in  $p^c$  for fixed  $p^s$  and  $p^d$  wherever the function is positive, and so the equilibrium  $p^c$  should be a local maximum, i.e.,

$$\begin{aligned} \frac{\partial U_{\text{CP}}}{\partial p^c} &= \frac{\partial}{\partial p^c} (D_0 - \alpha(p^s + p^c))(p^c + p^a - p^d) \\ &= D_0 - \alpha p^s - \alpha p^c - \alpha(p^c + p^a - p^d) \\ &= 0, \end{aligned}$$

which yields

$$p^c = \frac{D_0 - \alpha p^s - \alpha(p^a - p^d)}{2\alpha}.$$

Solving these two simultaneous equations in the variables  $p^s$  and  $p^c$ , we see that  $p^s$  and  $p^c$  are given by (6.2) and (6.3), respectively. Note that the choice of  $p^d$  is free. Once this is chosen, the choice fixes both  $p^s$  and  $p^c$ . This proves the second bullet. We shall return to prove the first bullet after proving the others.

Adding these two, we see that  $p^s + p^c$  is a constant for any such equilibrium. Choice of  $p^d$  fixes both  $p^s$  and  $p^c$ . This is true for any Nash equilibrium with a strictly positive demand. Furthermore, any  $p^d$  that is paid reduces  $p^s$  by that amount and increases  $p^c$  by the same amount. This proves the third bullet.

The last two bullets follow by direct substitutions into  $d(p^s, p^c)$ ,  $U_{\text{ISP}}$ , and  $U_{\text{CP}}$ .

As a consequence of the observation that  $U_{\text{ISP}} = U_{\text{CP}}$  at any equilibrium regardless of the value of  $p^d$ , we have

$$U_{\text{ISP}}^\gamma \times U_{\text{CP}}^{1-\gamma}$$

is independent of  $p^d$  at any equilibrium, for any fixed bargaining power  $\gamma \in (0, 1)$ . The arbitrator may thus pick any  $p^d$ . This proves the first bullet. (This observation holds even for zero-demand equilibria). The proof is now complete.  $\square$

*Remarks:* 1) Every choice of  $p^d$  can also result in the undesirable zero-demand equilibria, and not just the desirable equilibrium with strictly positive demand.

2) For this strictly positive demand equilibrium, the natural choices of  $p^d$  are those that make  $p^d = 0$ , i.e., there is no payment from CP to ISP, or  $p^c = 0$ , there is no payment from the user to the CP, or  $p^s = 0$ , there is no payment from the user to the ISP.

3) If one places the additional restriction that  $p^s \geq 0$ , the only effect of this constraint is that the choice of  $p^d$  is restricted to  $p^d \leq (D_0 + \alpha p^a)/(3\alpha)$ , and the above theorem continues to hold.

4) It is easily seen that if  $p^d$  is controlled by either agent, the only equilibria fall amongst the deadlocking zero demand points. Thus bargaining beforehand induces a good equilibrium point.

### 6.2.2 Bargaining after actions

We next consider the case when the CP and ISP decide on their respective prices first, knowing that they will subsequently bargain over  $p^d$ , say in the presence of the arbitrator.

**Theorem 6.2.2.1.** *When the CP and the ISP set prices simultaneously before agreeing on  $p^d$  and then bargain in the presence of an arbitrator, there is a unique pure strategy Nash equilibrium with the following properties:*

- The uniqueness is up to a free choice of either  $p^s$  or  $p^c$ . Without loss of generality, we may assume a free  $p^s$ .
- At equilibrium, the net user payment per demand is uniquely given by

$$p^s + p^c = \frac{D_0 - \alpha p^a}{2\alpha}.$$

- The demand is unique and is given by  $(D_0 + \alpha p^a)/2 > 0$ .
- The arbitrator will set  $p^d$  so that the net revenue per demand  $p^s + p^c + p^a = \frac{D_0 + \alpha p^a}{2\alpha}$  is shared in the proportion  $\gamma$  and  $1 - \gamma$  by the ISP and the CP, respectively.

*Proof.* As in the previous section, it is clear that the revenues per demand and the utilities for both agents are nonnegative. If this is not the case, the aggrieved CP or the ISP

guarantees himself a strictly larger zero utility by raising the price under his control so that demand reduces to 0.

Let us now perform a search for equilibria with strictly positive demand. Such a  $(p^s, p^c)$  is an interior point among all those pairs that satisfy  $D_0 - \alpha(p^s + p^c) \geq 0$ . Consider a fixed interior point  $(p^s, p^s)$ . The arbitrator sets  $p^d$  to

$$\begin{aligned} & \arg \max_{p^d} U_{\text{ISP}}^\gamma \times U_{\text{CP}}^{1-\gamma} \\ &= \arg \max_{p^d} \left[ \gamma \log(p^s + p^d) + (1 - \gamma) \log(p^c + p^a - p^d) \right], \end{aligned}$$

where the equality follows because the demand can be pulled out of the optimization. The optimization is over the set of  $p^d$  that ensure that the arguments inside the logarithm remain strictly positive. It is easy to see that the latter function is concave in  $p^d$ , and thus the maximizing  $p^d$  satisfies

$$\frac{\gamma}{p^s + p^d} - \frac{1 - \gamma}{p^c + p^a - p^d} = 0,$$

which yields  $p^d = \gamma(p^c + p^a) - (1 - \gamma)p^s$ .

Substitution of this  $p^d$  yields

$$\begin{aligned} p^s + p^d &= \gamma(p^s + p^c + p^a) \\ p^c + p^a - p^d &= (1 - \gamma)(p^s + p^c + p^a). \end{aligned}$$

Clearly,  $p^s + p^c + p^a$  is the net revenue per demand for both ISP and CP put together, and the ISP and the CP share this booty in the fraction of their bargaining powers.

Knowing this action of the arbitrator, the ISP will respond optimally to a CP's  $p^c$  by maximizing

$$U_{\text{ISP}} = d(p^s, p^c)(p^s + p^d) = (D_0 - \alpha(p^s + p^c)) \times \gamma(p^s + p^c + p^a).$$

This is a concave function of  $p^s$ , and the maximum is at

$$p^s = \frac{D_0 - \alpha p^a}{2\alpha} - p^c. \quad (6.7)$$

Similarly, for an ISP's  $p^s$ , the CP's best response is

$$p^c = \frac{D_0 - \alpha p^a}{2\alpha} - p^s,$$

which is the same equation as (6.7).

At equilibrium, we thus have  $p^s + p^c$  uniquely determined and given by the second bullet. A substitution yields that the demand is given by

$$d(p^s, p^c) = D_0 - \alpha(p^s + p^c) = \frac{D_0 + \alpha p^a}{2},$$

which proves the third bullet.

The revenue per demand is easily seen to be  $(D_0 + \alpha p^a)/(2\alpha)$ . Further substitution yields that net revenue is

$$d(p^s, p^c)(p^s + p^c + p^a) = (D_0 + \alpha p^a)^2/(4\alpha),$$

a strictly positive quantity shared in proportion of the bargaining powers by the ISP and CP. This proves the last bullet.

Finally, for any  $p^s$ , the arbitrator will set  $p^d$  to ensure this proportion, and thus  $p^s$  may be taken as a free variable. Each  $p^s$  and  $p^c$  satisfying the above conditions is a Nash equilibrium. This proves the first bullet.

Finally, it still remains to prove that there is no zero-demand equilibrium. Suppose that  $(p^s, p^c)$  is such that we get a zero-demand, i.e.,  $D_0 \leq \alpha(p^s + p^c)$ . With  $\varepsilon = (D_0 + \alpha p^a)/2 > 0$ , the ISP can set his new price to

$$q^s = D_0/\alpha - p^c - \varepsilon/\alpha$$

yielding a demand  $D_0 - \alpha(q^s + p^c) = \varepsilon > 0$  and a revenue

$$\gamma(q^s + p^c + p^a) = \gamma(D_0/\alpha - \varepsilon/\alpha + p^a) = \gamma\varepsilon/\alpha > 0,$$

and therefore a strictly positive utility. A unilateral deviation yields the ISP a strict increase in his utility. Thus a  $(p^s, p^c)$  with zero demand cannot be a pure-strategy equilibrium. This concludes the proof.  $\square$

*Remarks:* 1) The equilibrium utility for the ISP under post-bargaining is easily seen to be  $9\gamma/4$  fraction of that under pre-bargaining. Clearly then, post-bargaining is favourable if  $\gamma \geq 4/9$ .

2) Similarly, the equilibrium utility for the CP under post-bargaining is  $9(1 - \gamma)/4$  fraction of that under pre-bargaining. The CP prefers post-bargaining if  $1 - \gamma \geq 4/9$  or  $\gamma \leq 5/9$ .

3) Thus, if  $\gamma \in \left[\frac{4}{9}, \frac{5}{9}\right]$ , both will prefer post-bargaining. For  $\gamma > 5/9$ , ISP prefers post-bargaining while CP prefers pre-bargaining. Opposite is the case when  $\gamma < 4/9$ .

### 6.3 The case of multiple CPs

We now consider the case when there are several content providers. Internauts connect to each of the content providers through the single ISP. See Figure 7.1. The parameters of this game are given in the table that follows.

Parameter	Description
$n$	Number of content providers.
$p_i^s$	Price per unit demand paid by the users to the ISP for connection to CP $i$ . This can be positive or negative.
$p_i^c$	Price per unit demand paid by the users to CP $i$ . This too can be positive or negative.
$p_i^a$	Advertising revenue per unit demand, earned by the CP. This satisfies $p_i^a \geq 0$ .
$p_i^d$	Price per demand paid by the CP to the ISP. This can be either positive or negative.
$p^x$	Vectors of aforementioned prices, where $x$ is one of $s, c, a, d$ .
$d_i(p^s, p^c)$	Demand for CP $i$ as a function of the prices. See (7.1) below and the following discussion.
$r_{CP,i}$	The revenue per unit demand of CP $i$ , given by $p_i^c + p_i^a - p_i^d$ .
$r_{ISP,i}$	The revenue per unit demand of ISP coming from content provided by CP $i$ , given by $p_i^s + p_i^d$ .
$U_{ISP}$	The revenue or utility of the ISP, given by $\sum_i d_i(p^s, p^c)(p_i^s + p_i^d)$ .
$U_{CP,i}$	The revenue or utility of the CP, given by $d_i(p^s, p^c)(p_i^c + p_i^a - p_i^d)$ .
$\gamma_i$	Bargaining power of the ISP with respect to the CP.

Table 6.2: Multiple CP and single ISP net neutrality game

### 6.3.1 Demand function: Strictly positive demands

The demand function for content from CP  $i$  is such that it depends on  $p^s$  and  $p^c$  only through the sum  $p^s + p^c$ , the vector of net payment per unit demand from the inter-nauts. An interesting feature we wish to model is a *positive correlation in demand with respect to others' prices*. If CP  $i$  and ISP increase their prices for content from CP  $i$ , demand for this content naturally goes down. On the other hand, when the price for CP  $j$  content increases, where  $j \neq i$ , the decrease in demand for content from CP  $j$  frees up some capacity. This provides a marginally better delay experience for the inter-nauts of other CPs, and particularly inter-nauts of CP  $i$ . This positive effect creates a marginal increase in the demand for content from the other CPs, and in particular, an increase in the demand for content from CP  $i$ . We model this correlation effect by setting the

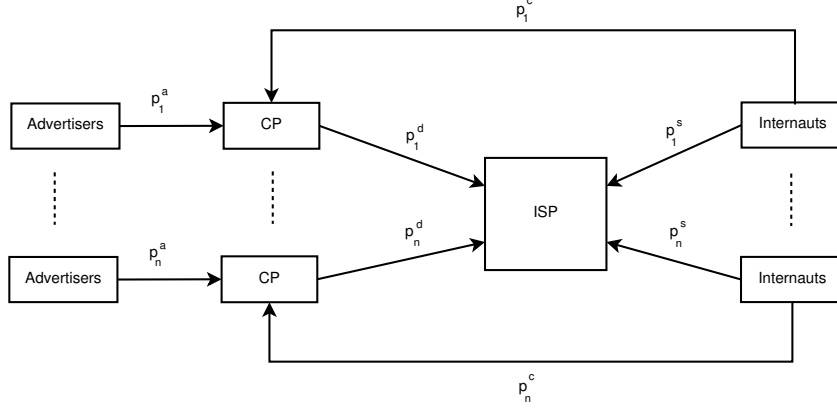


Figure 6.3: Monetary flow in a nonneutral network with multiple CPs.

demand functions to be

$$d_i(p^s, p^c) = \left[ D_0 - \alpha(p_i^s + p_i^c) + \beta \sum_{j:j \neq i} (p_j^s + p_j^c) \right] \quad (6.8)$$

provided each of the demands are strictly positive. Here  $\beta$  is the sensitivity parameter for the increase in demand for CP  $i$  content per unit increase in price of CP  $j$  content, when  $j \neq i$ .

While (7.1) is justifiable when all demands are positive, further thought suggests that it must be refined a little to account for the following. When the price  $p_i^s + p_i^c$  charged to CP  $i$  internauts is such that it forces demand  $d_i$  to be zero, then any additional increase in  $p_i^s + p_i^c$  simply continues to hold this demand  $d_i$  at zero. The capacity freeing and the consequent phenomenon of increase in demand for other CPs' contents no longer occurs, and additional price rise for CP  $i$  content will have no further tangible effect on other internauts' behavior. We shall return to this refinement shortly after addressing some points on the positive demand case.

Let the evaluations in (7.1) be strictly positive for each  $i$ . If this is placed as a requirement, one could view it as a joint constraint on the actions of the ISP and CPs: given the other prices, CP  $i$  will not set too high a price that makes  $d_i$  zero; neither will the ISP. We may write  $d_i(p^s, p^c) > 0$  for every  $i$  as

$$D_0 - \alpha(p_i^s + p_i^c) + \beta \sum_{j:j \neq i} (p_j^s + p_j^c) > 0, \quad i = 1, \dots, n, \quad (6.9)$$

which is compactly summarized as follows. Define the matrix  $A = (\alpha + \beta)I_n - \beta J_n$  where  $I_n$  is the identity matrix of size  $n \times n$ , and  $J_n$  is the square matrix with all-one

entries of size  $n \times n$ . The matrix  $A$  has diagonal entries  $\alpha$  and all off-diagonal entries  $-\beta$ . Also define  $E_n$  to be the all-one vector of size  $n \times 1$ . Then the constraint (6.9) in matrix notation is

$$D_0 E_n - A(p^s + p^c) > 0. \quad (6.10)$$

Sum up the components in (6.9) over all  $i$  and set the sum price  $P = \sum_i (p_i^s + p_i^c)$ , and we see that the total demand is

$$nD_0 - (\alpha - (n-1)\beta)P$$

under the assumption that each  $d_i$  is strictly positive. For this total demand to be negatively correlated with the average price per unit demand  $P/n$ , we must have that

$$(n-1)\beta \leq \alpha, \quad (6.11)$$

an assumption that we make from now on. As before we assume that  $p_i^s$  and  $p_i^c$  can be negative, i.e., the ISP and CP can pay the interanauts for their usage, with a consequent increase in demand.

It is a simple matter to verify that  $U_{\text{ISP}}$  given by

$$U_{\text{ISP}} = \sum_{i=1}^n d_i(p^s, p^c)(p_i^s + p_i^c)$$

is a concave quadratic function of the vector of service provider prices  $p^s$ , under the constraint (6.9). Indeed, simple calculations show that the Hessian matrix is  $-2A$ . Matrices of this form arise quite often in the sequel. To see that this Hessian matrix is negative semidefinite, observe that

$$-2A = -2\alpha \times [(1-\rho)I_n + \rho J_n]$$

where  $\rho = -\beta/\alpha$ . The matrix  $(1-\rho)I_n + \rho J_n$  has  $1-\rho$  as an eigenvalue repeated  $n-1$  times and  $1+\rho(n-1)$  once, and is therefore positive semidefinite by our assumption (6.11). (It is positive definite if there is strict inequality in (6.11)). Consequently, the Hessian  $-2A$  is negative semidefinite, and  $U_{\text{ISP}}$  is a concave function of  $p^s$ .

### 6.3.2 General demand function

As alluded to above, the demands in (7.1) have to be refined to account for the lack of further positive correlation after a demand reaches zero. See the discussion in the paragraph following the one containing (7.1). With a suitable reindexing, we may assume that the vector  $p = p^s + p^c$  has components in the increasing order, i.e.,  $p_1 \leq p_2 \leq \dots \leq p_n$ , where  $p_i = p_i^s + p_i^c$ . For brevity, we shall abuse notation and refer to  $d_i(p^s, p^c)$  as  $d_i(p)$ . Common sense suggests that if demand for CP  $i$  content is zero, then demand for CP  $j$  content for a  $j \geq i$  must also be zero since its price is higher. It will be illuminating to study the evolution of the demand function as the price vector increases from the



all-zero vector to  $p$  via  $\min\{xE_n, p\}$ , where  $x$  is a scalar parameter that increases from 0 to  $+\infty$  and the min operation is taken component-wise.

For  $x \in [0, p_1]$ , we have  $\min\{xE_n, p\} = xE_n$ ; all internauts are charged the same (net) price of  $x$  per unit demand. It is then immediate that all demands are equal, and from (7.1), this value is strictly positive if and only if

$$x < \frac{D_0}{\alpha - (n-1)\beta}.$$

In particular, demand for CP 1 is strictly positive at  $x = p_1$  if and only if

$$p_1 < \frac{D_0}{\alpha - (n-1)\beta} =: T(1). \quad (6.12)$$

If (6.12) does not hold, the demand for the cheapest content is zero, and our common sense conclusion suggests that all other demands are also zero. If (6.12) holds, then at  $x = p_1$ , demand for CP 1 is strictly positive. For  $x \in [0, p_1)$ , the demand  $d_1$  for content from CP 1 decreased with  $x$ . But further increase in  $x$  leaves the price for CP 1 content unchanged at  $p_1$ , and our observations about positive correlation with respect to others' prices indicates that  $d_1$  must now begin to linearly increase with  $x$  for  $x > p_1$ . This is illustrated in Figure 6.4. Thus for  $x \in [p_1, p_2]$ , we see

$$\begin{aligned} d_1 &= D_0 - \alpha p_1 + (n-1)\beta x, \text{ for CP 1} \\ d_i &= D_0 - (\alpha - (n-2)\beta)x + \beta p_1, \text{ for CP } i \geq 2. \end{aligned} \quad (6.13)$$

At  $x = p_2$ , the demand from CP 2, given by (6.13) for  $i = 2$ , is positive if and only if

$$p_2 < \frac{D_0 + \beta p_1}{\alpha - (n-2)\beta} =: T(2). \quad (6.14)$$

When (6.14) holds,  $d_1$  is linear in  $x$  with positive slope  $(n-1)\beta$  for  $x$  up to  $p_2$ , and all other  $d_i$  are linear and decreasing in  $x$  with negative slope  $-(\alpha - (n-2)\beta)$ . Again see Figure 6.4. If (6.14) does not hold,  $d_i = 0$  for  $i \geq 2$ , but  $d_1$  is set up to the value  $D_0 - \alpha p_1 + (n-1)\beta x^*$  where  $x^* = T(2)$ . All demands are thus set in this latter case. If (6.14) holds, the former case, then one proceeds further in a similar fashion until  $x^* = p_n$  and all demands are set, or until  $x^* \in (p_{k^*}, p_{k^*+1}]$  for some  $k^*$ , when demands  $d_j = 0$  for all  $j \geq k^*$ , and demands  $d_i$  are set with prices  $\min\{x^*E_n, p\}$ . To get an explicit expression for the demands, let us define

$$T(k) := \frac{D_0 + \beta \sum_{j:k < j} p_j}{\alpha - (n-k)\beta}, \quad k = 1, 2, \dots, n. \quad (6.15)$$

Let  $k^*$  be the smallest index among  $k = 0, 1, \dots, n$  for which

$$\begin{aligned} p_i &< T(i), \quad i = 1, \dots, k \\ p_{k+1} &\geq T(k+1). \end{aligned} \quad (6.16)$$

To further clarify (6.16), if  $p_1 \geq T(1)$  then  $k^* = 0$ ; if  $p_i < T(i)$  for  $i = 1, \dots, n$ , then  $k^* = n$ . In all other cases, the definition in (6.16) is unambiguous. Straightforward manipulations show that

$$T(k) > T(k+1) \text{ if and only if } p_k < T(k), \quad k = 1, \dots, n-1.$$

It follows from the definition of  $k^*$  that

$$T(1) > T(2) > \dots > T(k^*) > T(k^* + 1) \leq T(k^* + 2), \quad (6.17)$$

where the last two inequalities hold if the corresponding indices are between 1 and  $n$ . Let us now get back to identifying the demands. Given  $k^*$ , we set  $x^*$  such that

$$D_0 - \alpha x^* + \beta \sum_{j < k^* + 1} p_j + \beta(n - k^* - 1)x^* = 0;$$

the solution is  $x^* = T(k^* + 1)$ . The demands are now specified by

$$d_i(p) = \begin{cases} D_0 - \alpha p_i + \beta \sum_{j < k^* + 1, j \neq i} p_j + (n - k^*)\beta T(k^* + 1) & i = 1, \dots, k^* \\ 0, & i > k^*. \end{cases} \quad (6.18)$$

This describes the behavior of the internauts for any given price vector  $p = p^s + p^c$  and models the positive correlation of demand with other internauts' prices. Figure 6.4 depicts the procedure outlined above to evaluate the demands when there are  $n = 3$  CPs. The other parameters are  $D_0 = 100$ ,  $\alpha = 10$ ,  $\beta = 2$ , and the price vector  $p = (5, 10, 20)$ . The slope of demand functions in different intervals are also marked. Here  $k^* = 2$ . The demand of each CP is obtained by noting the respective value of their demand curve at  $x^* = T(3)$ .

Let us now summarize the demands as a function of internaut prices ( $p = p^s + p^c = (p_1, p_2)$ ) for the case when  $n = 2$ . See Figure 6.5. As shown, we can divide the set of prices into four regions. A description of the regions is given below.

- Denote the vector of net prices by  $p = (p_1, p_2)$ . If it lies in the interior of the region bounded by lines  $AO$  and  $BO$ , denoted as Region 1, demands for contents from both the CPs are strictly positive.
- In the rectangular region enclosed between lines  $OC$  and  $OD$ , denoted as Region 2, the demands for contents from both CPs are zero.
- In the region enclosed between the lines  $AO$  and  $OC$ , denoted as Region 3, demand of CP 1 content is zero and that of CP 2 content is positive. Any point  $p$  that lies on the line  $AO$  is such that  $p_1 = (D_0 + \beta p_2)/\alpha$  with  $p_2 < D_0/(\alpha - \beta)$ .
- In the region enclosed between the lines  $BO$  and  $OD$ , denoted as Region 4, demand of CP 2 content is zero and that of CP 1 is positive. Any point  $p$  that lies on line  $BO$  is such that  $p_2 = (D_0 + \beta p_1)/\alpha$  with  $p_1 < D_0/(\alpha - \beta)$ .

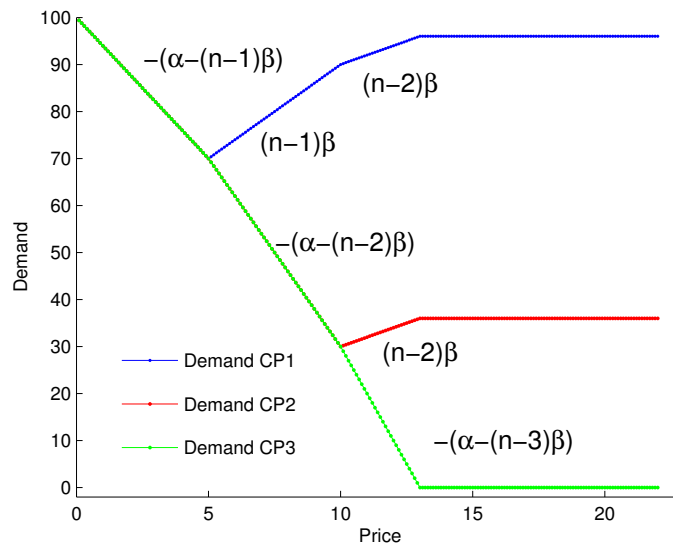


Figure 6.4: Demand functions

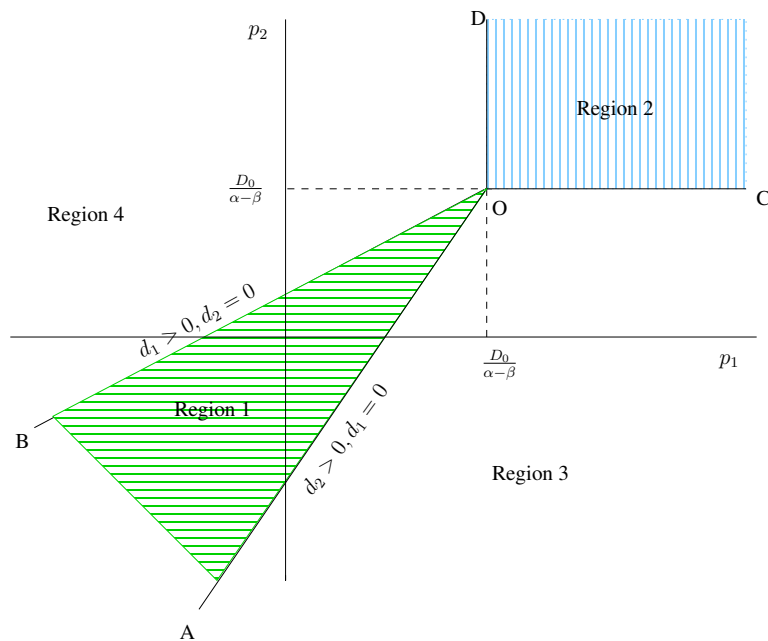


Figure 6.5: Characterization of the demand region

### 6.3.3 Timing of actions

The timing of actions for the games are indicated as follows. For bargaining over Nash equilibria, the following is the action sequence.

- The ISP bargains with each of the CPs, separately and simultaneously, over the payment  $p^d$  from the CP to the ISP. This can be positive or negative. In bargaining with CP  $i$ , the ISP shall bring only that revenue into consideration which is generated by internauts connected to CP  $i$ .
- All the CPs choose their price  $p_i^c$ . The ISP chooses the vector  $p^s$ . All these actions are taken simultaneously.
- The internauts react to the prices and set their demands as per the discussion in the previous subsection.

As before, bargaining comes later in the second game, and the new sequence of actions is as follows.

- The CP and the ISP set their respective access prices  $p_i^c$  and  $p^s$  simultaneously.
- The ISP and each of the CPs bargain over the payment  $p_i^d$  from the CP to the ISP. This can be positive or negative. Yet again, the ISP shall be able to bring only that revenue into consideration which is generated by internauts connected to CP  $i$ .
- The internauts react to the set prices and set their demands.

The case when  $\beta = 0$  is easily handled in either scenario. The actions of the various CPs (prices) do not influence each other. Though the ISP's utility is the sum over all revenues accrued from access to each CP, in bargaining with CP  $i$ , only the revenue generated by accesses to content of CP  $i$  matters. The ISP's utility is thus separable, and the problem separates into  $n$  single-CP single-ISP problems. The results of Theorems 6.2.1.1 and 6.2.2.1 immediately extend to this case. We shall henceforth assume that  $\beta > 0$ .

### 6.3.4 Bargaining over Nash equilibria

In this subsection, the CPs and the ISP bargain over Nash equilibria. We divide the presentation in this subsection into three parts. In the first part, we characterize all equilibria with all demands being strictly positive. In the second part, we characterize all equilibria with all demands being zero. In the third part, due to combinatorial complexity reasons, we restrict attention to two CPs ( $n = 2$ ) and characterize all equilibria where demand for one CP's content is strictly positive and demand for the other CP's content is zero.

### Equilibria with all demands strictly positive

We now characterize all equilibria where all demands take strictly positive values. Recall the definition of the matrix  $A$  and the vector  $E_n$ , given after (6.9). The matrix  $A$  has diagonal entries  $\alpha$  and off-diagonal entries  $-\beta$ .  $E_n$  is the  $n \times 1$  vector of all 1s.

**Theorem 6.3.4.1.** *Assume  $\alpha > (n - 1)\beta > 0$  and consider the case when the CPs and the ISP bargain beforehand. Among profiles with strictly positive demand, a strictly positive pure strategy Nash equilibrium exists if and only if the matrix  $(A + 2\alpha I_n)^{-1}[D_0 E_n + A p^d]$  is made of strictly positive entries. When this condition holds, the pure strategy Nash equilibria have the following properties.*

- The uniqueness is up to a free choice of the vector  $p^d$ .
- At equilibrium, for each  $i$ , there exist constants  $g_i$  and  $h_i$  that depend only on  $p^d$ ,  $D_0$ ,  $\alpha$ ,  $\beta$  such that

$$\begin{aligned} p_i^s &= g_i - p_i^d \\ p_i^c &= h_i + p_i^d. \end{aligned}$$

- For each CP  $i$ , the net internaut payment per unit demand is unique and is given by  $p_i^s + p_i^c = g_i + h_i$ . Any payment  $p_i^d$  paid by CP  $i$  is collected from the internaut, and this in turn is returned to the internaut by the ISP.
- The demand vector is unique and does not depend on  $p^d$ .
- The revenues per unit demand, and therefore the total revenues collected by the CPs and the ISP, does not depend on  $p^d$ .

*Proof.* The recipe for the proof is identical to that of Theorem 6.2.2.1, only with some matrix algebra. See Appendix 6.6.1. □

*Remarks:* 1) Yet again we notice that the actual choice of  $p^d$  does not affect the net cost per unit demand to the internauts; neither does it affect the equilibrium demand. It merely affects the way in which the payment by internaut is split between CP  $i$  and ISP. The mere fact that they agreed on an *arbitrary*  $p^d$  suffices to get an equilibrium more favorable than the case when  $p^d$  is under the control of one of the players.

2) For concreteness, we give the specific results for the case when  $n = 2$ ; see (6.36) in Appendix 6.6.1. Let  $\tau = \beta/\alpha$ . The negative definiteness condition is then  $\tau < 1$ , and

thus  $\tau \in (0, 1)$ . The equilibrium prices turn out to be

$$\begin{aligned} p^s &= -p^d + \frac{1}{3(1 - \tau^2/9)} \begin{bmatrix} 1 & \tau/3 \\ \tau/3 & 1 \end{bmatrix} p^a \\ &\quad + \frac{D_0}{3\alpha(1 - \tau)(1 - \tau/3)} E_2, \end{aligned} \quad (6.19)$$

$$\begin{aligned} p^c &= p^d - \frac{2}{3(1 - \tau^2/9)} \begin{bmatrix} 1 & \tau/3 \\ \tau/3 & 1 \end{bmatrix} p^a \\ &\quad + \frac{D_0}{3\alpha(1 - \tau/3)} E_2. \end{aligned} \quad (6.20)$$

An interesting observation from (6.19) is that when  $\tau \lesssim 1$ , any increase in CP 2 price causes a reduction in demand for that content, but results in nearly similar in magnitude increase in demand for content 1, and vice-versa. The ISP resources thus remain nearly fully utilized which encourages the ISP to charge a high price, as evidenced by the appearance of  $1 - \tau$  in the denominators for  $p^s$ . The prices charged by the CPs in (6.20) remain bounded.

### Equilibria with all demands being zero

We now study the case of equilibria with all demands being zero. Obviously (6.12) must not hold; additional conditions are also needed.

**Theorem 6.3.4.2.** *A price vector  $(p^s, p^c)$  is an equilibrium with all demands being zero if and only if the following conditions hold for all  $i = 1, 2, \dots, n$ :*

$$p_i^s \geq \frac{D_0}{\alpha - (n-1)\beta} + p_i^a - p_i^d \quad (6.21)$$

$$p_i^c \geq \frac{D_0}{\alpha - (n-1)\beta} + p_i^d. \quad (6.22)$$

*Proof.* See Appendix 6.6.2. □

*Remarks:* 1. Equations (6.22) and (6.21) are the same as saying that revenues per unit demand due to CP  $i$  content, to CP  $i$  and to the ISP, are at least  $\frac{D_0}{\alpha - (n-1)\beta} + p_i^a = T(1) + p_i^a$ , and this holds for each  $i$ . In such a case, all the CPs and the ISP are charging too high a price resulting in a deadlock equilibrium with all demands zero.

2. When  $n = 1$ , (6.21) and (6.22) reduce to (6.4) and (6.5), as they should.

### Equilibria with mixed demands with $n = 2$

In order to avoid combinatorial complexities, and for ease of exposition, we focus on the case when  $n = 2$  and now characterize all equilibria where demand for one content is strictly positive and demand for the other is zero.

**Theorem 6.3.4.3.** (a) A price profile  $((p_1^s, p_2^s), p_1^c, p_2^c)$  is an equilibrium with  $d_1 > 0$  and  $d_2 = 0$  if and only if

$$p_1^s = \frac{D'_0 + \alpha' p_1^a}{3\alpha'} - p_1^d \quad (6.23)$$

$$p_1^c = \frac{D'_0 - 2\alpha' p_1^a}{3\alpha'} + p_1^d \quad (6.24)$$

$$p_2^s \geq \frac{D_0 + \beta(p_1^s + p_1^c)}{\alpha} - p_2^d + p_2^a \quad (6.25)$$

$$p_2^c \geq \frac{D_0 + \beta(p_1^s + p_1^c)}{\alpha} - p_2^s, \quad (6.26)$$

where  $D'_0 = D_0(\alpha + \beta)/\alpha$  and  $\alpha' = (\alpha^2 - \beta^2)/\alpha$ .

(b) A price profile  $((p_1^s, p_2^s), p_1^c, p_2^c)$  is an equilibrium with  $d_2 > 0$  and  $d_1 = 0$  if and only if the same conditions as above hold with indices 1 and 2 interchanged.

*Proof.* See Appendix 6.6.3. □

*Remarks:* 1) Region 1 equilibria are characterized in Theorem 6.3.4.1. Region 2 equilibria are characterized in Theorem 6.3.4.2. Equilibria in Regions 3 and 4 are characterized in Theorem 6.3.4.3. We have therefore characterized all equilibria in the  $n = 2$  case.

2) Conditions (6.23) and (6.24) together constitute an equilibrium in case of a single CP with  $D_0$  and  $\alpha$  replaced by  $D'_0$  and  $\alpha'$ , respectively.

3) Conditions (6.25) and (6.26) may be interpreted as

$$r_{\text{ISP},2} \geq T(2) + p_2^a \text{ and } p_2 \geq T(2)$$

where  $r_{\text{ISP},2} = p_2^s + p_2^d$  is the revenue to the ISP from CP 2 content.

### 6.3.5 Bargaining after setting prices

As done previously, the ISP and the CPs will choose their respective prices knowing that the revenue they will get is the outcome of bargaining. We shall present our results for  $n = 2$ , due to combinatorial complexity reasons.

As in the  $n = 1$  case, the ISP and CP  $i$  will share  $p_i^s + p_i^c + p_i^a$ , the revenue coming from internauts accessing content from CP  $i$ , in the proportion  $\gamma_i$  and  $1 - \gamma_i$ . One immediate observation is that at equilibrium, this revenue should be nonnegative if demand is strictly positive because otherwise CP  $i$  can raise price and force demand to be zero, change his loss to zero, and strictly improve. Another observation is that all utilities and the constraints depend on  $p_i^s$  and  $p_i^c$  only through the sum  $p_i^s + p_i^c$ . While this sum is bounded if the demand vector is to be strictly positive, neither  $p_i^s$  nor  $p_i^c$  need be bounded, and so the action sets for each of the agents is unbounded. We shall present

our main result for bargaining after setting prices for  $n = 2$  and under a condition on the bargaining powers, namely, the matrix  $H$  with entries

$$H_{ij} = \begin{cases} \gamma_i & i = j, \\ -(\frac{\beta}{\alpha}) \left( \frac{\gamma_i + \gamma_j}{2} \right) & i \neq j, \end{cases} \quad (6.27)$$

is positive definite. This condition arises to keep the utility of the ISP a concave function of  $p^s$  in Region 1.

**Proposition 6.3.6.** *Let  $\tau = \beta/\alpha$  and  $n = 2$ . The matrix  $H$  is positive definite if and only if*

$$\sqrt{\max \left\{ \frac{\gamma_1}{\gamma_2}, \frac{\gamma_2}{\gamma_1} \right\}} \leq \frac{1 + \sqrt{1 - \tau^2}}{\tau}.$$

*Under this condition, the Hessian of  $U_{ISP}$  in Region 1, given by  $-2\alpha H$ , is negative definite, and so  $U_{ISP}$  is a concave function of  $(p_1^s, p_2^s)$  in Region 1.*

*Proof.*  $H$  is a  $2 \times 2$  matrix and the statement is straightforward to verify by direct evaluation of eigenvalues and requiring that they be positive. The expression for  $U_{ISP}$  immediately yields that the Hessian is  $-2\alpha H$ . We omit the details.  $\square$

This condition holds, for example, when the  $\gamma_i$ 's are equal and  $\alpha > \beta$ .

Our main result of this section is the following mixed bag. Recall that the case  $\beta = 0$  was already considered and disposed; so we shall consider only  $\beta > 0$ .

**Theorem 6.3.6.1.** *Consider  $n = 2$ . Let the matrix  $H$  given by (6.27) be positive definite. Also let  $\alpha > \beta > 0$ . Without loss of generality, assume  $p_1^a \geq p_2^a$ . For the case when bargaining is done after setting the prices, the following hold.*

- *If  $p_1^a$  is large enough so that*

$$p_1^a \geq (2\alpha/\beta)p_2^a + (2\alpha/\beta - 1)D_0, \quad (6.28)$$

*then there exists a pure strategy Nash equilibrium with  $d_1 > 0$  and  $d_2 = 0$ . Such an equilibrium satisfies all the properties of a single-CP and single-ISP equilibrium given in Theorem 6.2.2.1 with  $D_0$  and  $\alpha$  replaced by  $D'_0 = D_0(\alpha + \beta)/\alpha$  and  $\alpha' = (\alpha^2 - \beta^2)/\alpha$ . There is no other pure strategy Nash equilibrium.*

- *If (6.28) does not hold, there exists no pure strategy Nash equilibrium.*

*Proof.* See Appendix 6.6.4.  $\square$

Thus even though post-arbitration in the single-CP single ISP case always gave a unique Nash equilibrium with the desirable strictly positive demand, the desirable feature disappears when there are multiple CPs,  $\alpha > \beta > 0$ , and  $p_1^a$  is not high enough to satisfy (6.28). In particular, when  $p_i^a$  are equal, there is no post-arbitration equilibrium. Pre-arbitration continues to yield a unique Nash equilibrium among those profiles with strictly positive demand vectors. In the case when there is indeed a post-arbitration equilibrium, under (6.28), CP 2 is shut out by CP 1. The above result corrects an error in Theorem 4 in [125] where the equilibrium under (6.28) was missed.



## 6.4 Dynamics

In this section we constrain prices to remain in Region 1 of Figure 6.5. This yields a coupled constraint which is a significant difference with respect to the unconstrained model in Section 7.2. For this new setting, we discuss two dynamic models with multiple content providers. Again, for ease of exposition, we restrict to the case of two CPs.

### 6.4.1 Continuous dynamics

Let us assume that the players set their prices such that the demand from each CP is nonnegative, i.e., (7.1) is greater than or equal to zero for both CPs. This imposes coupled constraints on the set of prices  $(p^s, p^c) \in \mathbb{R}^4$  given by

$$\begin{aligned} d_1(p^s, p^c) &\geq 0, & d_2(p^s, p^c) &\geq 0, \\ p_1^s + p_1^d &\geq 0, & p_2^s + p_2^d &\geq 0, \\ p_1^c - p_1^d + p_1^a &\geq 0, & p_2^c - p_2^d + p_2^a &\geq 0. \end{aligned}$$

Let  $R$  denote the set of prices that satisfy these constraints. It is easy to verify that the above constraints also result in the following upper bounds on the prices:

$$\begin{aligned} p_1^s &\leq \frac{D_0}{\alpha - \beta} + p_1^a - p_1^d, & p_2^s &\leq \frac{D_0}{\alpha - \beta} + p_2^a - p_2^d \\ p_1^c &\leq \frac{D_0}{\alpha - \beta} + p_1^d, & p_2^c &\leq \frac{D_0}{\alpha - \beta} + p_2^d, \end{aligned}$$

and thus the set  $R$  is compact. Furthermore, due to the linearity of the constraints in the prices,  $R$  is convex. As argued in Section 7.2, for any price vector  $p = (p^s, p^c) \in R$ , the mappings  $U_{\text{ISP}}(\cdot, p^c)$ ,  $U_{\text{CP},1}(p^s, \cdot, p_2^s)$  and  $U_{\text{CP},1}(p^s, p_1^c, \cdot)$  are concave functions in the “ $\cdot$ ” variables.

Given the concave utility functions defined on the coupled constraint set  $R$ , we are in the setting of  $n$ -person concave games studied by Rosen [49]. We can then directly use the dynamic model proposed by Rosen in Section 4 in [49]. In our game setting the system of differential equations for the strategies  $p_1^s, p_2^s, p_1^c, p_2^c$  is:

$$\begin{aligned} \frac{dp_1^s}{dt} &= \frac{\partial U_{\text{ISP}}(p^s, p^c)}{\partial p_1^s} + u_1(p) \frac{\partial d_1(p)}{\partial p_1^s} + u_2(p) \frac{\partial d_2(p)}{\partial p_1^s} + u_3(p) \\ \frac{dp_2^s}{dt} &= \frac{\partial U_{\text{ISP}}(p^s, p^c)}{\partial p_2^s} + u_1(p) \frac{\partial d_1(p)}{\partial p_2^s} + u_2(p) \frac{\partial d_2(p)}{\partial p_2^s} + u_4(p) \\ \frac{dp_1^c}{dt} &= \frac{\partial U_{\text{CP},1}(p^s, p^c)}{\partial p_1^c} + u_1(p) \frac{\partial d_1(p)}{\partial p_1^c} + u_2(p) \frac{\partial d_2(p)}{\partial p_1^c} + u_5(p) \\ \frac{dp_2^c}{dt} &= \frac{\partial U_{\text{CP},2}(p^s, p^c)}{\partial p_2^c} + u_1(p) \frac{\partial d_1(p)}{\partial p_2^c} + u_2(p) \frac{\partial d_2(p)}{\partial p_2^c} + u_6(p). \end{aligned}$$

In the above dynamics it is assumed that a central agent computes  $u(p) = (u_1(p), \dots, u_6(p))$  as in Equation 4.5 in [49] and communicates the values to the players. The above dynamics tend to an equilibrium as is established next.

**Theorem 6.4.1.1.** *Let  $\alpha > \beta$ . Starting from any point  $p \in R$ , the continuous solution  $p(t)$  to the above system of differential equations remains in  $R$  for all  $t$  and converges to the unique equilibrium point.*

*Proof.* The first claim follows directly from Rosen's Theorem 7 in [49]. To prove the second part, we verify the so-called *diagonal strict concavity* property of

$$\sigma(p) = U_{\text{ISP}}(p) + U_{\text{CP},1}(p) + U_{\text{CP},2}(p), \quad p \in R.$$

Let  $g(p)$  denote the gradient of  $\sigma(p)$  given by

$$g(p) = \begin{bmatrix} \partial U_{\text{ISP}}(p)/\partial p_1^s \\ \partial U_{\text{ISP}}(p)/\partial p_2^s \\ \partial U_{\text{CP},1}(p)/\partial p_1^c \\ \partial U_{\text{CP},2}(p)/\partial p_2^c \end{bmatrix}.$$

With  $\tau := \beta/\alpha$ , the Jacobian  $G(p)$  of the above matrix can be verified to be the symmetric matrix

$$G(p) = -\alpha \begin{bmatrix} 2 & -2\tau & 1 & -\tau \\ -2\tau & 2 & -\tau & 1 \\ 1 & -\tau & 2 & -\tau \\ -\tau & 1 & -\tau & 2 \end{bmatrix}.$$

It is easy to see that the eigenvalues of  $-G(p)/\alpha$  are

$$\begin{aligned} & ((3\tau + 4) \pm \sqrt{(3\tau + 4)^2 - 4(\tau^2 + 4\tau + 3)})/2 \\ & ((4 - 3\tau) \pm \sqrt{(4 - 3\tau)^2 - 4(\tau^2 - 4\tau + 3)})/2, \end{aligned}$$

and that these eigenvalues are strictly positive for  $\tau \in [0, 1)$ .  $G(p)$  is therefore negative definite. By Theorem 6 and 9 in [49],  $\sigma(p)$  is diagonally strictly concave and the equilibrium point is globally asymptotically stable for the system of differential equations, respectively; this establishes convergence.  $\square$

## 6.4.2 Discrete dynamics

In this subsection we study discrete dynamics motivated by the best response dynamics. We assume the providers set their price, say, at the beginning of each day, as the best response to prices set by the other players on the previous day.

Let  $p_t = ((p_{1t}^s, p_{2t}^s), p_{1t}^c, p_{2t}^c)$  denote the price set by the players on day  $t$ . Recalling the concavity properties of the utility functions, the price set by the players on day

$m = t + 1$  are obtained by setting

$$\partial U_{\text{ISP}}((p_{1m}^s, p_{2m}^s), p_{1t}^c, p_{2t}^c) / \partial p_1^s = 0 \quad (6.29)$$

$$\partial U_{\text{ISP}}((p_{1m}^s, p_{2m}^s), p_{1t}^c, p_{2t}^c) / \partial p_2^s = 0 \quad (6.30)$$

$$\partial U_{\text{CP},1}((p_{1t}^s, p_{2t}^s), p_{1m}^c, p_{2m}^c) / \partial p_1^c = 0 \quad (6.31)$$

$$\partial U_{\text{CP},2}((p_{1t}^s, p_{2t}^s), p_{1m}^c, p_{2m}^c) / \partial p_2^c = 0. \quad (6.32)$$

The ISP controls the price  $(p_{1m}^s, p_{2m}^s)$  and sets them so that both (6.29) and (6.30) are simultaneously satisfied. The above conditions straightforwardly result in the following equations:

$$\begin{aligned} 2\alpha p_{1m}^s - 2\beta p_{2m}^s &= D_0 - \alpha p_{1t}^c + \beta p_{2t}^c - \alpha p_1^d + \beta p_2^d \\ 2\alpha p_{2m}^s - 2\beta p_{1m}^s &= D_0 - \alpha p_{2t}^c + \beta p_{1t}^c - \alpha p_2^d + \beta p_1^d \\ 2\alpha p_{1m}^c &= D_0 - \alpha p_{1t}^s + \beta p_{2t}^s + \beta p_{2t}^c - \alpha(p_1^a - p_1^d) \\ 2\alpha p_{2m}^c &= D_0 - \alpha p_{2t}^s + \beta p_{1t}^s + \beta p_{1t}^c - \alpha(p_2^a - p_2^d). \end{aligned}$$

This is a linear mapping that can be compactly written in the matrix form as

$$p_{t+1}^T = X p_t^T + Y \quad (6.33)$$

where, with  $\tau = \beta/\alpha$ , we take

$$X = \frac{1}{2} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & \tau & 0 & \tau \\ \tau & -1 & \tau & 0 \end{bmatrix}, Y = \begin{bmatrix} \frac{D_0 - (\alpha - \beta)p_1^d}{2(\alpha - \beta)} \\ \frac{D_0 - (\alpha - \beta)p_2^d}{2(\alpha - \beta)} \\ \frac{D_0 - \alpha(p_1^a - p_1^d)}{2\alpha} \\ \frac{D_0 - \alpha(p_2^a - p_2^d)}{2\alpha} \end{bmatrix}.$$

An easy guess of the fixed point to the iteration in (6.33) is

$$p_{\text{opt}}^T = (I - X)^{-1}Y. \quad (6.34)$$

Under the assumptions of Theorem 6.3.4.1 for two CPs ( $n = 2$ ), it can be verified that  $p_{\text{opt}}^T$  is the solution of that theorem given in (6.36). Under the same assumptions, the dynamics converge to that solution, as guaranteed next.

**Theorem 6.4.2.1.** *For  $\tau \in [0, 1)$ , the dynamics given in (6.33) converges to the fixed point  $p_{\text{opt}}^T = (I - X)^{-1}Y$ .*

*Proof.* The eigenvalues of the matrix  $X$  can be straightforwardly evaluated; they are

$$\frac{\frac{\tau}{2} \pm \sqrt{(\frac{\tau}{2})^2 + 1 - \tau}}{2} \quad \text{and} \quad \frac{-\frac{\tau}{2} \pm \sqrt{(\frac{\tau}{2})^2 - 1 - \tau}}{2}.$$

For  $\tau \in [0, 1)$ , these eigenvalues are nonzero, of magnitudes strictly smaller than 1, distinct, and hence  $X$  is diagonalizable in the form  $X = UDU^{-1}$ , where  $D$  is the diagonal

matrix of eigenvalues and  $U$  is an invertible matrix.  $X$  is also invertible. Consequently,  $p_{\text{opt}}^T$  in (6.34) is well-defined and satisfies

$$p_{\text{opt}}^T = X p_{\text{opt}}^T + Y.$$

Using this and (6.33), with  $p_0^T$  as the initial iterate, the norm of the error at iteration  $t + 1$  telescopes as

$$\begin{aligned} \|p_{t+1}^T - p_{\text{opt}}^T\| &= \|X(p_t^T - p_{\text{opt}}^T)\| \\ &= \|X^{t+1}(p_0^T - p_{\text{opt}}^T)\| \\ &= \|(UDU^{-1})^{t+1}(p_0^T - p_{\text{opt}}^T)\| \\ &= \|UD^{t+1}U^{-1}(p_0^T - p_{\text{opt}}^T)\|. \end{aligned}$$

Since the magnitudes of the eigenvalues are strictly less than 1, the error vector converges to 0 exponentially quickly in the number of iterations.  $\square$

*Remarks:* 1) The iterates converge if  $\tau = \beta/\alpha < 1$ . However, to guarantee that the solution is in  $R$ , we need the other necessary and sufficient condition  $(A + 2\alpha I_n)^{-1}[D_0 E_n + A p^a]$  to be made of strictly positive entries.

2) Even if these hold, the iterates may not remain in  $R$  due to the coupled nature of the constraints. Strictly speaking then, the dynamics is not the best response dynamics. Indeed, with  $D_0 = 200, \alpha = 6, \beta = 3, p_1^d = 10, p_2^d = 25, p_1^a = 45, p_2^a = 10$ , it can be seen that with  $p_0 = (19, 2, 25, 28)$  when demand for both contents is positive, we get  $p_1 = (15.8333, 6.8333, -2.8333, 34.1667)$  where demand for CP 1 content alone is positive. Nevertheless, the iterates converge to the unique equilibrium with strictly positive demands.

## 6.5 Discussion

In this paper, we studied a model of a nonneutral network and investigated the role of bargaining power in identifying side payments. We began with the simple case of a single CP and a single ISP and studied the equilibria when they bargained with each other over the side payment. The bargaining could happen either before they set their prices or afterward. The relative bargaining power of the ISP with respect to the CP was captured by a single real number between 0 and 1. We highlighted several interesting features of the equilibria. If the agents bargain beforehand, there can be zero-demand equilibria, but there is one unique desirable equilibrium with strictly positive demand. The actual amount of the payment that they agree upon is inconsequential to the users, but the mere fact that they agree upon it before-hand (via bargaining) is beneficial to all agents. If the agents bargain afterward, the agents share the revenue in proportion of their bargaining powers. There is a unique equilibrium and it has the desirable feature of a strictly positive demand.

When there are multiple CPs and demand for content  $i$  depends only on the content  $i$ 's price (sum of CP price and ISP price), then the problem reduces to  $n$  separable single-CP single-ISP problems. If the demand for content  $i$  is positively correlated with pricing of other contents, then for pre-bargaining, the results are qualitatively similar to the single-CP single-ISP case. We demonstrated that there is exactly one unique equilibrium that has strictly positive demand for all contents, and this holds if and only if a certain condition on the payments holds. For post-bargaining, however, no equilibrium exists unless the advertisement revenues widely differ. This is in contrast to the single-CP single-ISP case.

Our model is, needless to say, a mere caricature that captures certain types of interactions between ISPs, CPs, and internauts. The biggest benefit is that it is tractable, as evidenced by the obtained expressions in this paper. The litmus test of its usefulness will be its ability, or otherwise, to explain some observed behavior, even if only qualitatively. Studies in this direction are ongoing. Finally, aspects of (1) investment in infrastructure by the CPs and the ISPs and (2) collusion between some CPs and the ISP should be brought in to enrich the model. The latter was briefly considered in [125]. We hope to pursue some of these in future works.

## 6.6 Appendices

### 6.6.1 Proof of Theorem 6.3.4.1

Consider a fixed  $p^d$ . We shall only focus on strategies jointly constrained so that  $d_i > 0$  for all  $i$ . The joint constraint on  $p^s$  and  $p^c$  is given by (6.9), and the demands are given by (7.1). Let us look at  $U_{\text{ISP}}$  as a function of  $p^s$  and  $U_{\text{CP},i}$  as a function  $p_i^c$ . We already saw that the former is concave since  $\alpha > (n-1)\beta$ . Inspection of the expression for  $U_{\text{CP},i}$  shows that it is also quadratic and strictly concave in  $p_i^c$ . Since we seek equilibria with strictly positive demand, such equilibria are interior points of, for example in case of  $n = 2$ , Region 1 in Figure 6.5. It is therefore necessary that first order optimality conditions hold for such equilibria. So, setting the gradient of  $U_{\text{ISP}}$  with respect to  $p^s$  to zero, we get

$$\begin{aligned} \frac{\partial U_{\text{ISP}}}{\partial p_k^s} &= \sum_{j:j \neq k} \beta(p_j^s + p_j^d) - \alpha(p_k^s + p_k^d) \\ &\quad + D_0 - \alpha(p_k^s + p_k^c) + \beta \sum_{j:j \neq k} (p_j^s + p_j^c) = 0 \end{aligned}$$

for each  $k$ . Similarly, setting each  $\partial U_{\text{CP},k} / \partial p_k^c = 0$  yields

$$D_0 - \alpha(p_k^s + p_k^c) + \beta \sum_{j:j \neq k} (p_j^s + p_j^c) - \alpha(p_k^c + p_k^a - p_k^d) = 0$$

for each  $k$ . We next write these  $2n$  equations in matrix notation. For this purpose recall that the matrix  $A = (\alpha + \beta)I_n - \beta J_n$ , where all diagonal elements are  $\alpha$  and all off-diagonal elements are  $-\beta$ , and define  $B = (\alpha + \beta/2)I_n - (\beta/2)J_n$ , where all diagonal

elements are  $\alpha$  and all off-diagonal elements are  $-\beta/2$ . Also recall that  $E_n$  is the vector of size  $n \times 1$  with all-one entries. Then the above equations become:

$$\begin{bmatrix} 2A & A \\ A & 2B \end{bmatrix} \begin{bmatrix} p^s \\ p^c \end{bmatrix} = \begin{bmatrix} -A & \bigcirc \\ \alpha I_n & -\alpha I_n \end{bmatrix} \begin{bmatrix} p^d \\ p^a \end{bmatrix} + D_0 E_{2n}. \quad (6.35)$$

The matrices  $A$  and  $B$  commute because both are linear combinations of the commuting matrices  $I_n$  and  $J_n$ . Moreover, the determinant of the matrix on the left side is

$$\begin{aligned} \det(4AB - A^2) &= \det(A(A + 2\alpha I_n)) \\ &= \det(A) \det(A + 2\alpha I_n) \\ &= (\alpha + \beta)^{n-1} (\alpha - (n-1)\beta) \\ &\quad \cdot (3\alpha + \beta)^{n-1} (3\alpha - (n-1)\beta) \\ &> 0. \end{aligned}$$

This follows because the eigenvalues of the matrix

$$M(\rho) = (1 - \rho)I_n + \rho J_n$$

are  $1 - \rho$  repeated  $n - 1$  times and  $1 + (n - 1)\rho$  occurring once. The matrices  $A$  and  $A + 2\alpha I_n$  are scaled versions of  $M(\rho)$  with appropriate choices for  $\rho$ . Thus the matrix on the left side of (6.35) is invertible. From the fact that  $A$  and  $B$  commute, the fact that  $4AB - A^2 = A(A + 2\alpha I_n)$ , and the formula for the inverse of two-by-two block matrices with commutable entries, one writes by inspection that

$$\begin{bmatrix} 2A & A \\ A & 2B \end{bmatrix}^{-1} = (A(A + 2\alpha I_n))^{-1} \circ \begin{bmatrix} 2B & -A \\ -A & 2A \end{bmatrix},$$

where the symbol “ $\circ$ ” implies that the matrix before it left-multiplies all the elements of the bigger matrix following it. Multiplying (6.35) by the above inverse, and observing that  $2B + \alpha I_n = A + 2\alpha I_n$ , we get

$$\begin{aligned} \begin{bmatrix} p^s \\ p^c \end{bmatrix} &= \begin{bmatrix} -I_n & \alpha(A + 2\alpha I_n)^{-1} \\ I_n & -2\alpha(A + 2\alpha I_n)^{-1} \end{bmatrix} \begin{bmatrix} p^d \\ p^a \end{bmatrix} \\ &\quad + D_0 \begin{bmatrix} \alpha(A(A + 2\alpha I_n))^{-1} E_n \\ (A + 2\alpha I_n)^{-1} E_n \end{bmatrix}. \end{aligned} \quad (6.36)$$

Let us now verify that the revenues to each of CPs and the ISP are nonnegative. First we handle the ISP. Observe that the components of  $p^s + p^d$  constitute revenues from each family of internauts. From (6.36) we gather that

$$\begin{aligned} p^s + p^d &= \alpha(A + 2\alpha I_n)^{-1} p^a + \alpha D_0 (A + 2\alpha I_n)^{-1} A^{-1} E_n \\ &= \alpha A^{-1} (A + 2\alpha I_n)^{-1} (A p^a + D_0 E_n). \end{aligned} \quad (6.37)$$

Next, consider the CPs. Again from (6.36) we gather that

$$\begin{aligned} p^c - p^d + p^a &= (I_n - 2\alpha(A + 2\alpha I_n)^{-1}) p^a \\ &\quad + D_0 (A + 2\alpha I_n)^{-1} E_n \\ &= (A + 2\alpha I_n)^{-1} (A p^a + D_0 E_n). \end{aligned} \quad (6.38)$$

From (6.36), it also follows that

$$p^s + p^c = (A + 2\alpha I_n)^{-1}(-\alpha p^a + D_0(I_n + \alpha A^{-1})E_n)$$

so that the demand vector  $d = D_0 E_n - A(p^s + p^c)$  can be written (after observing that all involved matrices commute) as

$$d = \alpha(A + 2\alpha I_n)^{-1}(Ap^a + D_0 E_n). \quad (6.39)$$

Using this in (6.37), we see that  $p^s + p^d = A^{-1}d$  so that

$$U_{\text{ISP}} = d^T A^{-1}d. \quad (6.40)$$

Necessity of

$$(A + 2\alpha I_n)^{-1}(Ap^a + D_0 E_n) > 0 \quad (6.41)$$

is then clear from (6.38) and (6.39). Indeed, if any component on the left-hand side of (6.41) is nonpositive, the corresponding CP derives a nonpositive revenue per unit demand, and the demand for this CP's content truncates to 0. Such a point is either not an equilibrium, or if so, not all demands are strictly positive.

Sufficiency of (6.41) is obtained as follows. If (6.41) holds, then (6.38), (6.39), and (6.40) yield a point with strictly positive revenue for all agents and strictly positive demand. Indeed, from (6.38), revenue per unit demand is strictly positive for all CPs; from (6.39), all demands are strictly positive and consequently, all CPs' utilities are strictly positive; from (6.40) and the fact that  $A^{-1}$  has strictly positive eigenvalues, the ISP revenue is also positive. Furthermore, this point satisfies first-order optimality conditions. Given the concavity of the utility functions, it is a Nash equilibrium.

We have thus established that (6.41) is necessary and sufficient for a pure strategy Nash equilibrium to exist. When this holds, the pure strategy Nash equilibria are such that (6.37)-(6.40) hold, for a given  $p^d$ .

Let us now bring bargaining into the picture. Since the choice of  $p^d$  does not affect the demands  $d_i(p^s + p^d)$  as in (6.39), and the collections per unit demand by each of the CPs and the ISP are as in (6.38) and (6.37), respectively, the optimal solution  $p^d$  to the bargaining problem can be taken as any vector.

It then follows that the unique demand is given by (6.39) which establishes the fourth bullet. The form of the solution for  $p^s$  and  $p^c$  in (6.36) shows that  $p_i^s = g_i - p_i^d$  and  $p_i^c = h_i + p_i^d$  which verifies the second and the third bullets. Notice that  $p^d$  can be any vector, and so the solution is unique up to a free choice of  $p^d$ , and the statement of the first bullet is verified. The last bullet follows from the observation that the demand vector, the price charged by the CPs in (6.38), and the the revenue of the ISP in (6.40) do not depend on  $p^d$ . This concludes the proof.

### 6.6.2 Proof of Theorem 6.3.4.2

We first prove the necessity of these conditions. Let  $(p^s, p^c)$  be an equilibrium with all demands being zero; it must be the case that (6.12) is violated, and so

$$p^s + p^c \geq \left( \frac{D_0}{\alpha - (n-1)\beta} \right) E_n.$$

CP  $i$  should not be able to reduce his price, increase demand  $d_i$  to a strictly positive value, and derive a strictly positive utility. It must therefore be the case that even the least reduced price  $q_i^c$  that keeps the demand  $d_i$  on the threshold of positivity is too low a price bringing him a negative revenue. More precisely, a price

$$q_i^c + p_i^s = \frac{D_0}{\alpha - (n-1)\beta} \quad (6.42)$$

when demand for CP  $i$  content is on the threshold of positivity should imply

$$q_i^c - p_i^d + p_i^a \leq 0, \quad (6.43)$$

a negative revenue per unit demand for CP  $i$ . Substitution of (6.42) in (6.43) yields necessity of (6.21).

We next verify necessity of (6.22) by contraposition. Let  $i$  be a content index for which (6.22) does not hold, and so

$$p_i^c - p_i^d < D_0 / (\alpha - (n-1)\beta). \quad (6.44)$$

Take

$$\varepsilon = \frac{1}{2} \left( \frac{D_0}{\alpha - (n-1)\beta} - (p_i^c - p_i^d) \right) > 0, \quad (6.45)$$

and set  $q_i^s$  so that  $q_i^s + p_i^d = \varepsilon > 0$ , i.e., the ISP revenue from CP  $i$  content is  $\varepsilon > 0$ . Also set all other  $p_j^s$  for  $j \neq i$  to high values so that demand for these other contents is zero. Demand  $d_i$  for CP  $i$  content is however strictly positive because, by using  $\varepsilon = q_i^s + p_i^d$ , (6.44) and (6.45), we get

$$\begin{aligned} q_i^s + p_i^c &= (q_i^s + p_i^d) - p_i^d \\ &= \varepsilon + p_i^c - p_i^d \\ &= \frac{1}{2} \left( \frac{D_0}{\alpha - (n-1)\beta} - (p_i^c - p_i^d) \right) + (p_i^c - p_i^d) \\ &< \frac{D_0}{\alpha - (n-1)\beta}. \end{aligned}$$

Thus (6.12) holds, and so  $d_i > 0$ . (All other demands are zero). The ISP now has a strictly positive utility, and the profile cannot be a Nash equilibrium. By contraposition, necessity of (6.22) is established.

We now argue sufficiency of (6.21) and (6.22). Take a profile that satisfies these conditions. A glance at the proof of necessity of (6.21) indicates that there is no deviation



for CP  $i$  to derive a positive utility. To see that there is no deviation for the ISP that will yield a positive revenue, let  $q^s$  be any vector of ISP prices. Without loss of generality, reorder the prices  $q_i = q_i^s + p_i^c$ ,  $i = 1, 2, \dots, n$ , so that  $q_1 \leq q_2 \leq \dots \leq q_n$ . If ISP revenue for content  $j$  were strictly positive, then

$$0 < q_j^s + p_j^d \leq q_j^s + p_j^c - \frac{D_0}{\alpha - (n-1)\beta}$$

where the second inequality follows from (6.22), and so

$$q_j = q_j^s + p_j^c > \frac{D_0}{\alpha - (n-1)\beta} = T(1) \quad (6.46)$$

for any such  $j$  with strictly positive ISP revenue for content  $j$ . ( $T(1)$  is defined in (6.12)). However, from (6.17) and (6.16), any index with strictly positive demand satisfies  $q_i < T(i) \leq T(1)$ . Comparing this with (6.46), we deduce that indices with strictly positive demand have nonpositive revenue per unit demand. The ISP revenue is therefore nonpositive, and there is no deviation that will yield a better revenue. This proves sufficiency of the stated conditions, and the characterization of all Nash equilibria with all demands being zero is complete.

### 6.6.3 Proof of Theorem 6.3.4.3

We shall prove only (a). Proof of (b) is similar and is omitted.

We first prove necessity of the stated conditions. Let  $((p_1^s, p_2^s), p_1^c, p_2^c)$  be an equilibrium with  $d_1 > 0$  and  $d_2 = 0$ . Then, from the discussion on demands, we must have

$$\begin{aligned} p_1 &= p_1^s + p_1^c < \frac{D_0}{\alpha - \beta} \\ p_2 &= p_2^s + p_2^c \geq \frac{D_0 + \beta p_1}{\alpha}. \end{aligned} \quad (6.47)$$

Necessity of (6.26) is immediate from (6.47).

We next prove the necessity of (6.25). Since  $d_2 = 0$ , the current utility for CP 2 is zero. No unilateral deviation of CP 2 should yield him a strictly positive utility. For a strictly positive utility, he must reduce his price to make the demand for his content strictly positive. But even the least reduction in his price that puts the demand for his content on the threshold of positivity, a price  $q_2^c$  such that  $q_2^c + p_2^s = (D_0 + \beta p_1)/\alpha$  should already yield him a net nonpositive revenue  $q_2^c + p_2^a - p_2^d \leq 0$ . Substitution of the former equality in the latter inequality yields (6.25) as a necessary condition.

We now consider deviations of the ISP. We first observe that ISP's utility must be nonnegative. Next, given that the price profile falls in Region 4 of Figure 6.5, the ISP can reduce the price of  $p_2^s$  to  $q_2^s$  such that

$$q_2 = q_2^s + p_2^c = T(2) = \frac{D_0 + \beta p_1}{\alpha} \quad (6.48)$$

without affecting the demand  $d_1$  and keeping the demand  $d_2 = 0$ . His revenue does not change, and the price profile  $(p_1, q_2)$  is now on the line BO in Figure 6.5. The ISP's utility is thus

$$\begin{aligned}
 U_{\text{ISP}}(p_1, p_2) &= U_{\text{ISP}}(p_1, q_2) \\
 &= (D_0 - \alpha p_1 + \beta q_2)(p_1^s + p_1^d) \\
 &\quad + (D_0 - \alpha q_2 + \beta p_1)(q_2^s + p_2^d) \\
 &= (D_0 - \alpha p_1 + \beta q_2)(p_1 - p_1^c + p_1^d) \\
 &\quad + (D_0 - \alpha q_2 + \beta p_1)(q_2 - p_2^c + p_2^d).
 \end{aligned} \tag{6.49}$$

Let us now consider infinitesimal deviations either into Region 1 or along the line BO, and prove necessity of (6.23) and (6.24). The ISP can clearly change  $p_1^s$  and  $q_2^s$  simultaneously to place the price vector in a neighborhood of  $(p_1, q_2)$  inside Region 1 or on the line BO. Such deviations are given by increments  $u = (u_1, u_2)$  that satisfy  $u_2 \leq (\beta/\alpha)u_1$ . Since  $U_{\text{ISP}}(p_1, q_2)$  given by (6.49) is differentiable in this region, and there must be no direction pointing into Region 1 in which  $U_{\text{ISP}}$  increases, we must have the dot-product

$$\nabla U_{\text{ISP}}(p_1, q_2)^T u \leq 0 \quad \forall u \text{ with } u_2 \leq (\beta/\alpha)u_1.$$

It follows that the direction of steepest ascent for  $U_{\text{ISP}}$  at  $(p_1, q_2)$  which is  $\nabla U_{\text{ISP}}(p_1, q_2)$  must be normal to the line defined by  $u_2 = (\beta/\alpha)u_1$  and pointing away from Region 1, i.e.,

$$\frac{\partial U_{\text{ISP}}}{\partial q_2} = -\frac{\alpha}{\beta} \frac{\partial U_{\text{ISP}}}{\partial p_1}. \tag{6.50}$$

From (6.49), and after noting that  $r_{\text{ISP},1} := (p_1 - p_1^c + p_1^d)$  and  $r_{\text{ISP},2} := (q_2 - p_2^c + p_2^d)$  are the ISP revenues per unit demand arising from contents of CP 1 and CP 2, respectively, we get

$$\frac{\partial U_{\text{ISP}}}{\partial q_2} = \beta r_{\text{ISP},1} + d_2 - \alpha r_{\text{ISP},2} = \beta r_{\text{ISP},1} - \alpha r_{\text{ISP},2}$$

and

$$\frac{\partial U_{\text{ISP}}}{\partial p_1} = \beta r_{\text{ISP},2} + d_1 - \alpha r_{\text{ISP},1}.$$

Substitution of these in (6.50) yields that the condition

$$d_1 = \alpha' r_{\text{ISP},1} \tag{6.51}$$

is a necessary condition for direction of increase for the ISP's utility. We now use  $d_1 = D_0 - \alpha p_1 + \beta q_2$ , the expression for  $q_2$  given in (6.48), and the definition of  $r_{\text{ISP},1}$  and rewrite (6.51) as

$$D_0' - \alpha'(p_1^s + p_1^c) = \alpha'(p_1^s + p_1^d). \tag{6.52}$$

Note that this equation fixes  $p_1^s$  given a  $p_1^c$ :

$$p_1^s = \frac{D_0' - \alpha'(p_1^c + p_1^d)}{2\alpha'}. \tag{6.53}$$

Furthermore, if we can establish the necessity of (6.24) which fixes  $p_1^c$ , then (6.52) implies the necessity of (6.24) as well, as can be verified by direct substitution.

We now establish the necessity of (6.24). Consider first an interior point of Region 4. Small deviation by CP 1 move the point along the abscissa, and if small enough the deviation keeps the resulting point inside the interior of Region 4. Then  $d_2$  continues to be 0 and  $d_1 > 0$ . As a consequence, it follows that  $d_1 = D'_0 - \alpha' p_1$ , where  $p_1 = p_1^s + p_1^c$  and the variation here is in  $p_1^c$ . The revenue for CP 1 is  $p_1^c + p_1^a - p_1^d$  so that

$$U_{CP,1} = (D'_0 - \alpha'(p_1^s + p_1^c))(p_1^c + p_1^a - p_1^d).$$

It is thus necessary that the first order optimality condition hold, and so

$$\frac{\partial U_{CP,1}}{\partial p_1^c} = (D'_0 - \alpha'(p_1^s + p_1^c)) - \alpha'(p_1^c + p_1^a - p_1^d) = 0 \quad (6.54)$$

so that

$$(D'_0 - \alpha'(p_1^s + p_1^c)) = \alpha'(p_1^c + p_1^a - p_1^d). \quad (6.55)$$

Solving the simultaneous equations (6.52) and (6.55), we get the necessity of (6.23) and (6.24) among interior points of Region 4.

Now consider points on the line BO. Let us denote the right-hand sides of (6.23) and (6.24) as  $p_{1,opt}^s$  and  $p_{1,opt}^c$ , respectively, i.e.,

$$p_{1,opt}^s = \frac{D'_0 + \alpha' p_1^a}{3\alpha'} - p_1^d \quad (6.56)$$

$$p_{1,opt}^c = \frac{D'_0 - 2\alpha' p_1^a}{3\alpha'} + p_1^d \quad (6.57)$$

If  $p_1^c \geq p_{1,opt}^c$ , consider an infinitesimal decrease in  $p_1^c$  which puts the point in the interior of Region 4. The left partial derivative is

$$\frac{\partial^- U_{CP,1}}{\partial p_1^c} = (D'_0 - \alpha'(p_1^s + p_1^c)) - \alpha'(p_1^c + p_1^a - p_1^d), \quad (6.58)$$

the right-hand side of (6.54). We then have the following chain of equalities:

$$\begin{aligned} & \frac{\partial^- U_{CP,1}}{\partial p_1^c} \\ &= (D'_0 - \alpha'(p_1^s + p_1^c)) - \alpha'(p_1^c + p_1^a - p_1^d) \\ &= (D'_0 - \alpha'(p_{1,opt}^s + p_{1,opt}^c)) - \alpha'(p_{1,opt}^c + p_1^a - p_1^d) \\ & \quad + \alpha'(p_{1,opt}^s - p_1^s) + 2\alpha'(p_{1,opt}^c - p_1^c) \end{aligned} \quad (6.59)$$

$$= 0 - (1/2)\alpha'(p_{1,opt}^c - p_1^c) + 2\alpha'(p_{1,opt}^c - p_1^c) \quad (6.60)$$

$$= (3/2)\alpha'(p_{1,opt}^c - p_1^c), \quad (6.61)$$

where (6.59) follows by adding and subtracting

$$(D'_0 - \alpha'(p_{1,opt}^s + p_{1,opt}^c)) - \alpha'(p_{1,opt}^c + p_1^a - p_1^d).$$

Equation (6.60) follows because (6.55) and (6.52) hold for the pair  $(p_{1,\text{opt}}^c, p_{1,\text{opt}}^s)$ , and from (6.53) we see that

$$p_1^s - p_{1,\text{opt}}^s = -(1/2)(p_1^c - p_{1,\text{opt}}^c).$$

From (6.61),  $p_1^c > p_{1,\text{opt}}^c$  implies that an infinitesimal decrease results in a strict increase for CP 1. It must therefore be that  $p_1^c \leq p_{1,\text{opt}}^c$  for the profile under consideration to be an equilibrium.

When  $p_1^c \leq p_{1,\text{opt}}^c$ , consider an infinitesimal increase in  $p_1^c$  which puts the point in the interior of Region 1, i.e., both  $d_1$  and  $d_2$  become positive. As a consequence, the right-derivative is now

$$\frac{\partial^+ U_{\text{CP},1}}{\partial p_1^c} = (D'_0 - \alpha'(p_1^s + p_1^c)) - \alpha(p_1^c + p_1^a - p_1^d); \quad (6.62)$$

observe that the difference with (6.58) is that the second term is multiplied only by  $\alpha$  instead of  $\alpha'$  as now both CPs have positive demand upon deviation in Region 1. Following the same steps leading to (6.61), we now get

$$\begin{aligned} \frac{\partial^+ U_{\text{CP},1}}{\partial p_1^c} &= 0 - (1/2)\alpha'(p_{1,\text{opt}}^c - p_1^c) + (\alpha' + \alpha)(p_{1,\text{opt}}^c - p_1^c) \\ &= (\alpha + \alpha'/2)(p_{1,\text{opt}}^c - p_1^c) \end{aligned}$$

and now the right-hand side has a different scale when compared with (6.61). When  $p_1^c < p_{1,\text{opt}}^c$ , we have  $\frac{\partial^+ U_{\text{CP},1}}{\partial p_1^c} > 0$  yielding a strict increase in CP 1 utility. It follows that we must have  $p_1^c = p_{1,\text{opt}}^c$ . This establishes the necessity of (6.24), and the proof of necessity is complete.

Next, to address sufficiency of the stated conditions, consider a profile satisfying them. Our necessity argument for (6.25) also shows that CP 2 has no deviation yielding him a strictly positive utility. For the ISP, the necessity argument considered an equivalent point on the line BO, and showed that there are no infinitesimal deviations around this point that will yield a better utility. But on account of concavity of the utility functions, no other point in Region 1 (including the boundary AO) will yield a strictly better utility. Since the boundary AO has also been considered, and the any point in Region 3 yields him the same utility as the point on the line AO with the same ordinate, no point in Region 3 will yield a better utility. Similarly, on account of concavity, CP 1 too as no deviation (infinitesimal or otherwise) that will yield him a strictly better utility. This concludes the proof of sufficiency.

#### 6.6.4 Proof of Theorem 6.3.6.1

The system has two CPs,  $n = 2$ . When  $\beta = 0$ , the problem separates into two smaller problems each with one CP and one ISP, and Theorem 6.2.2.1 applies. We now assume  $\alpha > \beta > 0$ . It will be useful to recall Figure 6.5 which has four regions.

1. We now argue there are no pure strategy equilibria in Region 2. This is the region with both demands zero. Let the ISP prices  $p_1^s, p_2^s$  and the CP prices  $p_1^c$  and  $p_2^c$  be such that

$$\begin{aligned} p_1 &= p_1^s + p_1^c \geq D_0/(\alpha - \beta) \\ p_2 &= p_2^s + p_2^c \geq D_0/(\alpha - \beta). \end{aligned}$$

Consider the point O in Figure 6.5 given by  $(D_0/(\alpha - \beta), D_0/(\alpha - \beta))$ . ISP can bring down both his prices to move the price point to O, and demand and revenue collected remain zero. Now consider further deviation along the line BO. To realise this, ISP reduces both prices so that the net price denoted  $(q_1, q_2)$  satisfies the equation  $q_2 = (D_0 + \beta q_1)/\alpha$ . Along this line  $d_1 = D'_0 - \alpha' q_1 > 0$  and  $d_2 = 0$ , where  $D'_0$  and  $\alpha'$  are given in the statement of the theorem. But this puts us in a single-CP single-ISP case. By the last part of the proof of Theorem 6.2.2.1, we see that the ISP has a deviation that yields a strictly positive revenue for itself. So no point in Region 2 can be a pure strategy equilibrium.

2. We now argue that no point in the interior of Region 1 can be an equilibrium. Let the prices be such that the total prices on the internauts is  $(p_1, p_2)$ , a point in Region 1. In this case

$$\begin{aligned} d_1(p_1, p_2) &= D_0 - \alpha p_1 + \beta p_2 > 0 \\ d_2(p_1, p_2) &= D_0 - \alpha p_2 + \beta p_1 > 0. \end{aligned} \tag{6.63}$$

Clearly, the net revenue coming from internauts  $i$  is  $p_i + p_i^a$ , and so

$$\begin{aligned} U_{\text{ISP}} &= d_1(p_1, p_2)\gamma_1(p_1 + p_1^a) + d_2(p_1, p_2)\gamma_2(p_2 + p_2^a) \\ U_{\text{CP},i} &= d_i(p_1, p_2)(1 - \gamma_i)(p_i + p_i^a), \quad i = 1, 2. \end{aligned}$$

Since the utilities depend on  $p_i^s$  and  $p_i^c$  only through  $p_i = p_i^s + p_i^c$ , partial derivatives with respect to  $p_i^s$  and  $p_i^c$  may be obtained by considering partial derivatives with respect to  $p_i$ . These are (in Region 1)

$$\begin{aligned} \frac{\partial U_{\text{CP},i}}{\partial p_i} &= (1 - \gamma_i)(d_i(p_1, p_2) - \alpha(p_i + p_i^a)), \quad i = 1, 2 \\ \frac{\partial U_{\text{ISP}}}{\partial p_1} &= \gamma_1(d_1(p_1, p_2) - \alpha(p_1 + p_1^a)) + \gamma_2\beta(p_2 + p_2^a) \\ &= \frac{\gamma_1}{1 - \gamma_1} \frac{\partial U_{\text{CP}_1}}{\partial p_1} + \gamma_2\beta(p_2 + p_2^a) \\ \frac{\partial U_{\text{ISP}}}{\partial p_2} &= \gamma_2(d_2(p_1, p_2) - \alpha(p_2 + p_2^a)) + \gamma_1\beta(p_1 + p_1^a) \\ &= \frac{\gamma_2}{1 - \gamma_2} \frac{\partial U_{\text{CP}_2}}{\partial p_2} + \gamma_1\beta(p_1 + p_1^a). \end{aligned}$$

(In passing, we note that from here, it is but a short step to verify that the Hessian for  $U_{\text{ISP}}$  with respect to  $(p_1^s, p_2^s)$  is  $-2\alpha H$ ). The first order necessary conditions imply that the above partial derivatives are zero, and we immediately deduce that  $p_i + p_i^a = 0$  for

both  $i = 1, 2$ , i.e., the revenue for each CP's content is zero. Substitution of these in  $\frac{\partial U_{CP_i}}{\partial p_i} = 0$  above yields  $d_i = 0$  for both  $i = 1, 2$ . But this is contrary to the assumption that the point is on the interior of Region 1. So no point in the interior of Region 1 can be an equilibrium.

3. Let us now consider a candidate equilibrium in Region 4, with  $p_1 < D_0/(\alpha - \beta)$  and  $p_2 \geq (D_0 + \beta p_1)/\alpha$ .

Let us consider deviations by the ISP. First, he may reduce  $p_2^s$  to  $q_2^s$  so that  $p_2$  reduces to  $q_2 = (D_0 + \beta p_1)/\alpha$  so that the resulting point  $(p_1, q_2)$  is on the line BO, and  $d_2$  is on the threshold of positivity, but revenue of CP 1, revenue of CP 2 (which is zero), and revenue of the ISP still remain unaffected. ISP can now consider deviations from  $(p_1, q_2)$  along the line BO or into Region 1, i.e., along the vector  $(u_1, u_2)$  where  $u_2 \leq (\beta/\alpha)u_1$ . For such deviations to be fruitless,  $\nabla U_{ISP}(p_1, q_2)$  must point into Region 4, and must in particular be normal to the line BO, and so (6.50) should hold, which in the present case yields

$$\begin{aligned} \gamma_2 d_2(p_1, q_2) - \alpha \gamma_2(p_2 + p_2^a) + \gamma_1 \beta(p_1 + p_1^a) \\ = -(\alpha/\beta)(\gamma_1 d_1(p_1, q_2) - \alpha \gamma_1(p_1 + p_1^a) + \gamma_2 \beta(p_2 + p_2^a)). \end{aligned}$$

After cancelations and after using the fact that  $d_2(p_1, q_2) = 0$ , the above equality simplifies to

$$\begin{aligned} p_1 + p_1^a &= \alpha d_1(p_1, q_2)/(\alpha^2 - \beta^2) \\ &= d_1(p_1, q_2)/\alpha' \\ &= (D'_0 - \alpha' p_1)/\alpha', \end{aligned} \tag{6.64}$$

solving which we get

$$p_1 = \frac{D'_0 - \alpha' p_1^a}{2\alpha'}, \tag{6.65}$$

the solution for the single-CP and single-ISP case. It is easily verified that the net revenue is  $p_1 + p_1^a = (D'_0 + \alpha' p_1^a)/(2\alpha') > 0$  and further, from (6.64),

$$d_1(p_1, q_2) = \alpha'(p_1 + p_1^a) = (D'_0 + \alpha' p_1^a)/2 > 0,$$

as in the single-CP and single-ISP case.

Let us next consider local deviations by CP 1 who can increase or decrease  $p_1^c$  and therefore perturb  $p_1$ . From the above argument,  $p_1$  must satisfy (6.65). If  $(p_1, p_2)$  is an interior point of Region 4, any deviation by CP 1 effectively moves the point  $(p_1, q_2)$ , a point that is effectively equivalent to the original  $(p_1, p_2)$ , along the line BO. It is easy to see, using  $q_2 = (D_0 + \beta p_1)/\alpha$ , that

$$\begin{aligned} \frac{\partial}{\partial p_1} U_{CP,1} &= \frac{\partial}{\partial p_1} (d_1(p_1, q_2)(p_1 + p_1^a)(1 - \gamma_1)) \\ &= \frac{\partial}{\partial p_1} ((D'_0 - \alpha' p_1)(p_1 + p_1^a)(1 - \gamma_1)) \\ &= (d_1(p_1, q_2) - \alpha'(p_1 + p_1^a))(1 - \gamma_1) \\ &= 0 \end{aligned} \tag{6.66}$$

where the last equality comes from (6.64). Thus, when  $(p_1, p_2)$  is an interior point of Region 4, CP 1 does not benefit from a local deviation. When  $(p_1, p_2)$  is on the line BO, it is just  $(p_1, q_2)$ . A decrease in  $p_1$  moves the point to the interior of Region 4, and the equivalent point moves lower and left along the line BO. Then the argument leading to (6.66) holds for the left partial derivative  $\frac{\partial^-}{\partial p_1} U_{CP,1}$ , and decrease in  $p_1^c$  does not yield a gain. On the other hand, an increase in  $p_1^c$  increases  $p_1$  and puts the system in the interior of Region 1, and we then have

$$\begin{aligned}
 \frac{\partial^+}{\partial p_1} U_{CP,1} &= \frac{\partial}{\partial p_1} (d_1(p_1, q_2)(p_1 + p_1^a)(1 - \gamma_1)) \\
 &= \frac{\partial}{\partial p_1} ((D_0 - \alpha p_1 + \beta q_2)(p_1 + p_1^a)(1 - \gamma_1)) \\
 &= (d_1(p_1, q_2) - \alpha(p_1 + p_1^a))(1 - \gamma_1) \\
 &= (d_1(p_1, q_2) - \alpha'(p_1 + p_1^a))(1 - \gamma_1) \\
 &\quad + (\alpha' - \alpha)(p_1 + p_1^a)(1 - \gamma_1) \\
 &= (\alpha' - \alpha)(p_1 + p_1^a)(1 - \gamma_1) \\
 &< 0
 \end{aligned}$$

where the penultimate equality follows because of (6.64), and the last inequality follows because  $\alpha' < \alpha$ , but the other two factors are strictly positive. But this implies an infinitesimal increase in  $p_1^c$  yields a strict decrease in his utility. There are thus no utility increasing infinitesimal deviations for CP 1.

Lastly, we consider infinitesimal deviations by CP 2. If  $p_2 > q_2$ , then CP 2 can bring down his price so that  $p_2$  reduces to  $q_2$  without a change in his revenue or without a change in demand for his content. Any further decrease moves the operating point into the interior of Region 1, and renders  $d_2$  strictly positive. For such a deviation to be fruitless, the revenue for CP 2 at the operating point  $(p_1, q_2)$  should be nonpositive, i.e.,

$$\begin{aligned}
 0 &\geq q_2 + p_2^a \\
 &= \frac{D_0 + \beta p_1}{\alpha} + p_2^a.
 \end{aligned}$$

Substitution of (6.65) and rearrangement yields (6.28) as a necessary condition for equilibrium. If (6.28) does not hold, there is no pure strategy Nash equilibrium in Region 4.

4. Consider points in Region 3. An argument analogous to above yields that an analogue of (6.28), with indices 1 and 2 interchanged, is a necessary condition. But as  $p_1^a \geq p_2^a$ , such a condition cannot hold, and there is no pure strategy Nash equilibrium in Region 3

It is thus clear that if (6.28) does not hold, there exists no pure strategy Nash equilibrium. This proves the second statement. If (6.28) does hold, we saw above that the only possible equilibria, if any, are in Region 4 with  $p_1$  as in (6.65), and  $p_2 \geq (D_0 + \beta p_1)/\alpha$ . From the first order conditions, no infinitesimal deviation yields a better revenue for any of the agents. From the facts that

- $U_{\text{ISP}}$  is concave in Region 1 by the assumption that  $H$  is positive definite,
- $U_{\text{CP},1}$  and  $U_{\text{CP},2}$  are strictly concave in Region 1,
- they extend continuously to the boundaries AO and BO from Region 1,
- for each point in Region 4, the utilities are determined by the utilities on an equivalent point on the line BO, and similarly,
- for each point in Region 3, the utilities are determined by the utilities on an equivalent point on the line AO, and finally,
- the utilities earned in Region 2 are zero,

it follows that no deviation, infinitesimal or otherwise, will yield a better revenue for any of the agents. So  $p_1$  given in (6.65) and  $p_2 \geq (D_0 + \beta p_1)/\alpha$  characterize the pure strategy Nash equilibrium. This concludes the proof of the theorem.





# Chapter 7

## Game Theoretic Analysis of Collusions in Nonneutral Networks

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### 7.1 Introduction

Recently there were several debates to allow internet service providers (ISP) to charge content providers (CPs) for providing the last mile connectivity. The proponents of network neutrality are in favor of the current neutral network where CPs do not pay ISP for connectivity to the end users. This neutrality principle also meant that packets could not be discriminated according to their origin or destination, the application, or the protocol they use.

Those opposing the neutrality argue that (i) some applications (such as peer-to-peer (P2P) streaming applications) require costly resources, and (ii) if a neutral policy is pursued, there would be no incentive for investing in the infrastructure of the network in order to upgrade it. In a nonneutral network, discrimination of packets can mean a selective blocking of packets, selective throttling of flows (allocating less throughput to some flows), preferential treatment coming from exclusive agreements between the access provider and some content or service providers, and discriminatory of charging. For detailed discussion on the network neutrality debate see [123], [122] and the references therein.

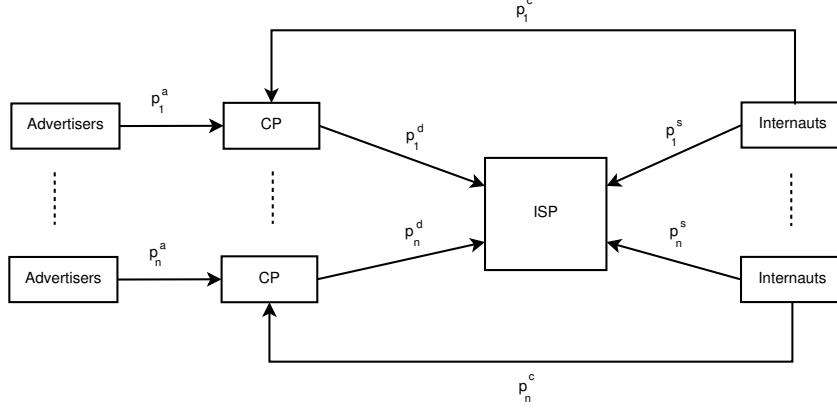


Figure 7.1: Monetary flow in a nonneutral network with multiple CPs.

In this paper we consider the nonneutral regime where ISP can charge the CPs. This regime can prompt the CPs to make an exclusive agreement with ISP to get a preferential treatment for their content. However, a CP and an ISP will be willing to consider an agreement only if it leads to improvement in their revenues. Such collusions were studied in the context of routing games [127]-[128]. It is often observed in these networks that the subset of players that collude may end up with a worse performance than without collusion, and can also degrade the social performance. To capture this degradation or improvement in social performance a metric called *price of collusion* is proposed in [127]. Extending this line of thought several other performance metrics related to collusion were proposed in [128] to quantify the effect of collusion on those involved in the collusion and those are not involved. We use these performance metrics to study the effect of collusion in nonneutral network.

The chapter is organized as follows. In Section 7.2 we introduce the model for the nonneutral network with multiple CPs. In Section 7.3 we allow an exclusive contract between a CP and ISP and establish existence of equilibrium. Section 7.4 quantifies the impact of collusion on the colluding and the noncolluding players. Finally, we end with concluding remarks and a brief discussion on ongoing work in Section 8.9. The recipe of the proofs is same as those in [126].

## 7.2 Model and prior work

In [125] we proposed a simple model to capture interaction between a single ISP and a CP and extended it to include several CP's. The model with multiple CPs is as follows. Several groups of *internauts* wish to access content from CPs. There is one CP per in-

ternaut group. All their access requests go through the ISP. The CPs and the ISP charge the interauts for access. In addition, the CPs get additional revenue from advertisers. The ISP has a known bargaining power when negotiating payment with a CP. See Fig. 7.1 for a payment flow diagram. The parameters of this game are given in the following table.

Parameter	Description
$n$	Number of content providers.
$p_i^s$	Price per unit demand paid by the users to the ISP for connection to CP $i$ . This can be positive or negative.
$p_i^c$	Price per unit demand paid by the users to CP $i$ . This too can be positive or negative.
$p_i^a$	Advertising revenue per unit demand, earned by the CP. This satisfies $p_i^a \geq 0$ .
$p_i^d$	Price per demand paid by the CP to the ISP. This can be either positive or negative.
$p^x$	Vectors of aforementioned prices, where $x$ is one of $s, c, a, d$ .
$d_i(p^s, p^c)$	Demand for CP $i$ as a function of the prices. See (7.1) below and the following discussion.
$r_{CP,i}$	The revenue per unit demand of CP $i$ , given by $p_i^c + p_i^a - p_i^d$ .
$r_{ISP,i}$	The revenue per unit demand of ISP coming from content provided by CP $i$ , given by $p_i^s + p_i^d$ .
$U_{ISP}$	The revenue or utility of the ISP, given by $\sum_i d_i(p^s, p^c)(p_i^s + p_i^d)$ .
$U_{CP,i}$	The revenue or utility of the CP, given by $d_i(p^s, p^c)(p_i^c + p_i^a - p_i^d)$ .

**Table 7.1:** Multiple CP and single ISP net neutrality game (collusion)

The case of a single-CP and single-ISP was quite easy [125]. The demand was a function of the sum of the CP and ISP prices, and decreased linearly with the sum. In the multiple-CP single-ISP case, there was a possibility of positive correlation in demand for a particular CP's content with respect to other contents' prices. See [126] for details. The demand can be summarised as follows.

As in [125]-[126], we write  $p^s = (p_1^s, p_2^s, \dots, p_n^s)$  and  $p^c = (p_1^c, p_2^c, \dots, p_n^c)$  to denote the price set by the ISP and CPs respectively. The vector  $p = p^s + p^c$  denotes the total

price charged to the internauts. The positive correlation in demand with respect to others' prices was modeled via

$$d_i(p^s, p^c) = \left[ D_0 - \alpha(p_i^s + p_i^c) + \beta \sum_{j:j \neq i} (p_j^s + p_j^c) \right] \quad (7.1)$$

provided each of the demands are strictly positive. Here  $\alpha > 0$ , and  $\beta > 0$  is the sensitivity parameter for the increase in demand for CP  $i$  content per unit increase in price of CP  $j$ 's content, for  $j \neq i$ . When the prices  $p^s$  and  $p^c$  are constrained to lie in the region of strictly positive demand for all contents, i.e.,  $d_i(p^s, p^c) > 0$  for all  $i = 1, 2, \dots, n$ , we can write the demand vector compactly as

$$D_0 E_n - A_n(p^s + p^c) > 0, \quad (7.2)$$

where  $A_n$  is a matrix of size  $n \times n$  with diagonal elements as  $\alpha$  and off-diagonal elements as  $-\beta$ , and  $E_n$  is a column matrix of size  $n \times 1$  with all elements equal to 1. We further argued in [126] that for the total demand to be negatively correlated with the average price per unit demand  $\sum_{i=1}^n (p_i^s + p_i^c)/n$ , we must have  $(n-1)\beta \leq \alpha$ . This assumption also resulted in the concavity of the utility of the ISP in  $p^s$ . We continue to make this assumption in this paper.

As before we assume that  $p_i^s$  and  $p_i^c$  can be negative, i.e., the ISP and CP can pay the internauts for their patronage, with a consequent increase in demand. While (7.1) was good so long as all the demands were strictly positive, the positive correlation in  $d_i$  with respect to  $p_j$  disappears as soon as  $d_j$  equals zero. Further increase in the price of CP  $j$  content keeps  $d_j$  at zero, there is no freeing of ISP capacity, and consequently no increase in demand  $d_i$ , i.e., positive correlation disappears. This subtlety required a refinement of the demand function which was studied [126].

In [125] we considered a game between ISP and CP with the following sequence of actions.

- The ISP bargains with each of the CPs, separately and simultaneously, over the payment  $p^d$  from the CP to the ISP. This can be positive or negative. In bargaining with CP  $i$ , the ISP shall bring only that revenue into consideration which is generated by internauts connected to CP  $i$ .
- All the CPs choose their price  $p_i^c$ . The ISP chooses the vector  $p^s$ . All these actions are taken simultaneously.
- The internauts react to the prices and set their demands as per the discussion in the previous subsection.

We analyzed this game of  $n + 1$  players, derived the equilibrium prices and the equilibrium demand. Our aim in the current work is to extend the analysis to the case when some of the players collude by means of an exclusive contract.

### 7.3 Exclusive Contract

Suppose now that one of the CP makes an exclusive contract with the ISP. The ISP and the colluding CP make a joint decision on the price charged to the internauts. This situation also arises when the ISP himself provides content. Without loss of generality assume that CP 1 makes an exclusive contract with the ISP, i.e., and together they decide the price  $p_1^s + p_1^c$  to charge from user group 1. The total revenue obtained by CP 1 together with ISP is  $p_1^s + p_1^c + p_1^a$  which they share among themselves in some mutually agreed fashion. We denote the colluding pair as  $\overline{\text{ISP}}$  and their utility  $U_{\overline{\text{ISP}}}$ . Recall that  $p_1^d$  denotes the price that CP 1 will pay to the ISP. Because of exclusive contract this price does not affect  $U_{\overline{\text{ISP}}}$ . The utility of each of the  $n$  players is as follows:

$$\begin{aligned} U_{\overline{\text{ISP}}}(p_1^s + p_1^c, p_2^s, \dots, p_n^s, p_2^c, \dots, p_n^c) \\ = [D_0 - \alpha(p_1^s + p_1^c) + \beta \sum_{j \neq 1} (p_j^s + p_j^c)](p_1^s + p_1^c + p_1^a) \\ + \sum_{i \neq 1} [D_0 - \alpha(p_i^s + p_i^c) + \beta \sum_{j \neq i} (p_j^s + p_j^c)](p_i^s + p_i^d), \end{aligned} \quad (7.3)$$

and for  $i = 2, 3, \dots, n$ ,

$$\begin{aligned} U_{\text{CP},i}(p_1^s + p_1^c, p_2^s, \dots, p_n^s, p_2^c, \dots, p_n^c) \\ = [D_0 - \alpha(p_i^s + p_i^c) + \beta \sum_{j \neq i} (p_j^s + p_j^c)](p_i^c + p_i^a - p_i^d). \end{aligned} \quad (7.4)$$

It is easy to verify that  $U_{\overline{\text{ISP}}}$  is a concave function of  $\bar{p}^s := (p_1^s + p_1^c, p_2^s, \dots, p_n^s)$  for a given  $p^a := (p_1^a, p_2^a, \dots, p_n^a)$  and  $\bar{p}^d := (p_2^d, p_3^d, \dots, p_n^d)$ , and for each for  $i = 2, 3, \dots, n$ ,  $U_{\text{CP},i}$  is a concave function  $p_i^c$ . Indeed, Hessian matrix of  $U_{\overline{\text{ISP}}}$  is  $-2A_n$  which is negative definite. The following theorem establish the existence of equilibrium prices and some of its properties.

**Theorem 7.3.0.1.** *Assume  $\alpha > (n-1)\beta$  and consider the case when CP 1 enters into an exclusive contract with the ISP. Among the profiles with strictly positive demand, a strictly pure strategy Nash equilibrium exists. Further, the equilibrium prices satisfy the following properties*

- The equilibrium is unique upto free choice of  $p^d$ ,
- The equilibrium prices are

$$p_1^s + p_1^c = \frac{-p_1^a}{2} + \frac{D_0}{2(\alpha - (n-1)\beta)}, \quad (7.5)$$

for  $i = 2, 3, \dots, n$ ,

$$p_i^s = g_i - p_i^d \quad \text{and} \quad p_i^c = h_i + p_i^d, \quad (7.6)$$

where constants  $g_i$  and  $h_i$  depends only on  $p^a, D_0, \alpha, \beta$

- The demand vector, the revenue per unit demand and therefore the total revenues collected by ISP and CP does not depend on  $p^d$ .

*Remarks:* 1) From (7.5), the equilibrium price of the colluding pair depends on only its advertisement revenues and is independent of other price quantities. As the number of CPs increase, the colluding pair charges a higher price from its internauts at equilibrium.

2)  $p^d$  has no influence on internaut's per unit demand price. We may have anticipated this given our previous observation of influence of  $p^d$  on internaut's price when there was no exclusive contract (See [126][Thm. 3]).

With the expressions for the equilibrium prices we proceed to analyze how a collusion between the ISP and a CP affects their own payoff and that of the noncolluding CP's. For simplicity we restrict attention to two CP's.

## 7.4 Price of Collusion

In this section we study the impact of a single collusion formed between a CP 1 and ISP. We may study this as game with two players, the super ISP denoted  $\overline{\text{ISP}}$  consisting of CP 1 and the ISP as player 1 and CP 2 as player 2. Let  $U_{\overline{\text{ISP}}}$  and  $U_{\text{CP},2}$  denote the utility of the colluding pair and that of the noncolluding player respectively in the game with a single coalition.

In the case when there is indeed a collusion between CP 1 and the ISP, straightforward simplification of the equilibrium prices evaluated in Theorem 7.3.0.1 can be shown to yield the following prices for the two CP case [125].

$$\begin{bmatrix} p_1^s + p_1^c \\ p_2^s \\ p_2^c \end{bmatrix} = \begin{bmatrix} 0 & -1/2 & 0 \\ -1 & \tau/6 & 1/3 \\ 1 & -\tau/3 & -2/3 \end{bmatrix} \cdot \begin{bmatrix} p_2^d \\ p_1^a \\ p_2^a \end{bmatrix} + \frac{D_0}{6\alpha} \begin{bmatrix} 3/(1-\tau) \\ (2+\tau)/(1-\tau) \\ 2 \end{bmatrix}, \quad (7.7)$$

where  $\tau = \beta/\alpha$  and  $\tau \in [0, 1)$ . Then the net price per unit demand on the internauts is given by

$$\begin{bmatrix} p_1^s + p_1^c \\ p_2^s + p_2^c \end{bmatrix} = \begin{bmatrix} -1/2 & 0 \\ -\tau/6 & -1/3 \end{bmatrix} \cdot \begin{bmatrix} p_1^a \\ p_2^a \end{bmatrix} + \frac{D_0}{6\alpha(1-\tau)} \begin{bmatrix} 3 \\ (4-\tau) \end{bmatrix}, \quad (7.8)$$

and thus  $p_2^d$  has no influence on the internauts' equilibrium price per unit demand.

For the case of no exclusive contract we computed the equilibrium prices for the posi-

tive demand in [125], which we recall below

$$p^s = -p^d + \frac{1}{3(1-\tau^2/9)} \begin{bmatrix} 1 & \tau/3 \\ \tau/3 & 1 \end{bmatrix} p^a + \frac{D_0}{3\alpha(1-\tau)(1-\tau/3)} E_2, \quad (7.9)$$

$$p^c = p^d - \frac{2}{3(1-\tau^2/9)} \begin{bmatrix} 1 & \tau/3 \\ \tau/3 & 1 \end{bmatrix} p^a + \frac{D_0}{3\alpha(1-\tau/3)} E_2. \quad (7.10)$$

This resulted in the following internauts prices

$$\begin{bmatrix} p_1^s + p_1^c \\ p_2^s + p_2^c \end{bmatrix} = -\frac{1}{3(1-\tau^2/9)} \begin{bmatrix} 1 & \tau/3 \\ \tau/3 & 1 \end{bmatrix} \cdot \begin{bmatrix} p_1^a \\ p_2^a \end{bmatrix} + \frac{2D_0(1-\tau/2)}{3\alpha(1-\tau)(1-\tau/3)} E_2. \quad (7.11)$$

With the explicit equilibrium prices for the case of exclusive contract and no exclusive contract we can compare utilities in both cases. In particular, we are interested in the following questions:

- Does the colluding pair improve their total revenue by agreeing to cooperate?
- how does this collusion affect the total revenues of the noncolluding player.

In this regard we use two relevant definitions proposed in [128], namely, individual single collusion price (ISCP) and single collusion externality price (SCEP). When there is only one coalition formation ISCP is defined as the ratio of the total utility of the colluding players before and after collusion at equilibrium. Similarly, for a given single collusion SCEP is defined as the ratio of the total utility of the noncolluding players before and after collusion at equilibrium. Let  $(p^s, p^c)$  and  $(\bar{p}^s, \bar{p}^c)$  denote the equilibrium prices for the cases of no exclusive contract and exclusive contract respectively. In the present case with single coalition denoted by  $\bar{\text{ISP}} = \{\text{ISP}, \text{CP1}\}$ , the performance metrics ISCP and SCEP are given by

$$\text{ISCP}(\bar{\text{ISP}}) = \frac{U_{\text{ISP}}(p^s, p^c) + U_{\text{CP},1}(p^s, p^c)}{U_{\bar{\text{ISP}}}(\bar{p}^s, \bar{p}^c)}, \quad (7.12)$$

$$\text{SCEP}(\bar{\text{ISP}}) = \frac{U_{\text{CP},2}(p^s, p^c)}{U_{\text{CP},2}(\bar{p}^s, \bar{p}^c)}. \quad (7.13)$$

Using equilibrium prices in (7.7)-(7.8) the utilities in the case of exclusive contract can be directly computed. After simple manipulation they can be written as

$$\begin{aligned} \bar{U}_{\text{ISP}} = & \left( \frac{D_0(3+\tau)}{6} + \frac{\alpha p_1^a(3-\tau^2)}{6} - \frac{\beta p_2^a}{3} \right) \cdot \left( \frac{p_1^a}{2} + \frac{D_0}{2\alpha(1-\tau)} \right) \\ & + \left( \frac{D_0}{3} - \frac{\alpha p_1^a \tau}{3} + \alpha \frac{p_2^a}{3} \right) \cdot \left( \frac{p_1^a \tau}{6} + \frac{p_2^a}{3} + \frac{D_0(2+\tau)}{6\alpha(1-\tau)} \right), \end{aligned} \quad (7.14)$$



$$U_{CP,2} = \alpha \left( \frac{D_0}{3\alpha} + \frac{p_2^a}{3} - p_1^a(\tau/3) \right)^2. \quad (7.15)$$

Similarly, using equilibrium prices in (7.9)-(7.11) the utilities in the case of no exclusive contract can be computed as

$$\begin{aligned} U_{ISP} + U_{CP} = & \left( \frac{D_0}{3-\tau} + \frac{\alpha}{(9-\tau^2)} (-2p_1^a\tau + (3-\tau^2)p_2^a) \right) \\ & \cdot \left( \frac{p_1^a\tau + 3p_2^a}{(9-\tau^2)} + \frac{D_0}{\alpha(1-\tau)(3-\tau)} \right) + \\ & \left( \frac{D_0}{3-\tau} + \frac{\alpha}{(9-\tau^2)} (-2p_2^a\tau + (3-\tau^2)p_1^a) \right) \\ & \cdot \left( \frac{p_1^a(6-\tau^2) - p_2^a(\tau)}{(9-\tau^2)} + \frac{2D_0(2-\tau)}{\alpha(1-\tau)(3-\tau)} \right), \end{aligned} \quad (7.16)$$

$$U_{CP,2} = \alpha \left( \frac{D_0}{\alpha(3-\tau)} + \frac{1}{(9-\tau^2)} ((3-\tau^2)p_2^a - 2\tau p_1^a) \right)^2. \quad (7.17)$$

We first evaluate the impact of collusion between ISP and CP 1 on the noncolluding pair. By comparing (7.15) and (7.17) the following observation is immediate.

**Proposition 7.4.1.**  $SCEP(\overline{ISP}) \geq 1$  if and only if

$$p_2^a \leq \frac{3-\tau^2}{2\tau} p_1^a + \frac{D_0(3+\tau)}{\alpha(2\tau)}. \quad (7.18)$$

Thus if the advertisement revenues  $(p_1^a, p_2^a)$  satisfies the relation (7.18) CP 1 going for an exclusive contract will lead the noncolluding CP to incur loss.

*Remarks:* 1) In [125], we observed that when (7.18) holds both the user group pay less per unit demand. Hence though the collusion between ISP and CP 1 hurts the noncolluding CP it benefits the interanauts.

2) It is a simple to verify that under the condition (7.18) collusion results in the reduction of demand from CP 2.

To compare the performance of the colliding pair we evaluate  $ISCP(\overline{ISP})$  by dividing (7.16) by (7.14).  $ISCP(\overline{ISP})$  is shown in Figure 7.2. In this figure we observe that colluding pair benefits if (7.18) holds. However, if the condition (7.18) is not satisfied, then the colluding pair are not gaining compared to the case of no collusion. Thus making an exclusive contract is not always beneficial. This observation is made in several earlier work in the context of routing games, for example see [128]. Further, collusion always hurts one of the player, colluding pair or noncolluding player unless equality holds in (7.18).

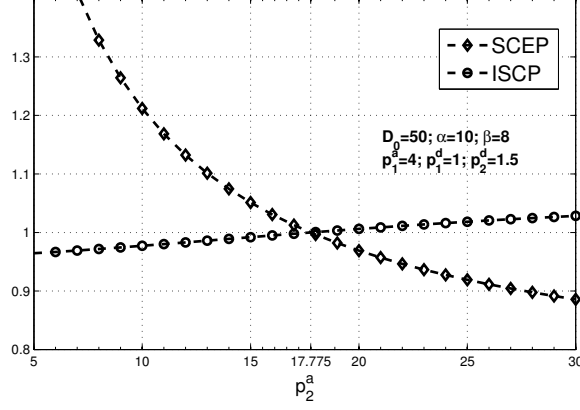


Figure 7.2: Individual Single collusion price for ISP

## 7.5 Conclusion and Future Work

We considered a nonneutral network in which ISP can charge the CPs for connecting them to the end users. In such networks we studied the case where a CP can collude with the ISP by making an exclusive contract. In this contract the colluding pair maximize the sum of their utility. We showed that such collusion may not be beneficial to the colluding pair. They will benefit if the advertisement revenue of the noncolluding ISP is low. Otherwise, they will not benefit but the noncolluding will improve its revenues.

In future, we plan to consider other forms of cooperation between a CP and the ISP. An example is the Nash bargaining solution in which the colluding pair try to maximize the product of their utilities or the  $\alpha$ -fair sum of their utilities [124].

## 7.6 Appendices

### 7.6.1 Proof of Theorem 7.3.0.1

Recall our argument that  $U_{\text{ISP}}$  is a concave function in  $\bar{p}^s = (p_1, p_2^s, \dots, p_n^s)$ , and for each  $k = 1, 2, \dots, n$ ,  $U_{\text{CP},k}$  is concave in  $p_k^c$ . Then, the equilibrium prices must satisfy

the following first order optimality conditions.

$$\begin{aligned}
 \frac{\partial U_{\overline{ISP}}}{\partial p_1} &= D_0 - 2\alpha p_1 + 2\beta \sum_{j \neq 1} p_j^s + \beta \sum_{j \neq 1} p_j^c \\
 &- \alpha p_1^a + \beta \sum_{j \neq 1} p_j^d = 0, \\
 &\text{and for } k = 2, 3, \dots, n, \\
 \frac{\partial U_{\overline{ISP}}}{\partial p_k^s} &= D_0 + 2\beta p_1 - 2\alpha p_k^s + 2\beta \sum_{j \neq k, 1} p_j^s - \alpha p_k^c \\
 &+ \beta \sum_{j \neq k, 1} p_j^c + \beta p_1^a - \alpha p_k^d + \beta \sum_{j \neq k, 1} p_j^d = 0 \\
 \frac{\partial U_{CP,k}}{\partial p_k^c} &= D_0 + \beta p_1 - \alpha p_k^c + \beta \sum_{j \neq k, 1} (p_j^s + p_j^c) \\
 &- 2\alpha p_k^c - \alpha(p_k^a - p_k^d) = 0.
 \end{aligned}$$

Let  $\bar{p}^c = (p_2^c, p_3^c, \dots, p_n^c)$  denote the CP price vector. The above set of  $2n - 1$  equations can be compactly written in matrix form as follows

$$\begin{bmatrix} 2\alpha & -b^T \\ -b & C \end{bmatrix} \begin{bmatrix} \bar{p}^s \\ \bar{p}^c \end{bmatrix} = \begin{bmatrix} -\alpha & c^T \\ a & D \end{bmatrix} \begin{bmatrix} p^a \\ \bar{p}^d \end{bmatrix} + D_0 E_{2n-1}, \quad (7.19)$$

where  $b^T$  and  $c^T$  are row vectors of size  $1 \times (2n - 2)$  given by  $b^T = [2\beta E_{n-1}^T \ \beta E_{n-1}^T]$  and  $c^T = [0 \cdot E_{n-1}^T \ \beta E_{n-1}^T]$ .  $a$  denotes a column vector of size  $(2n - 2) \times 1$  given by  $a^T = [\beta \cdot E_{n-1}^T \ 0 \cdot E_{n-1}^T]$ .  $C$  and  $D$  are  $2 \times 2$  block matrix given by

$$C = \begin{bmatrix} 2A_{n-1} & A_{n-1} \\ A_{n-1} & 2B_{n-1} \end{bmatrix} \quad D = \begin{bmatrix} \bigcirc & -A_{n-1} \\ -\alpha I_{n-1} & \alpha I_{n-1} \end{bmatrix}.$$

The solution to system of equation in (7.19) exists if the matrix

$$\begin{bmatrix} 2\alpha & -b^T \\ -b & C \end{bmatrix}$$

is invertible. By inspection we can write its inverse as

$$\frac{1}{\mu} \begin{bmatrix} 1 & b^T C^{-1} \\ C^{-1} b & \mu C^{-1} + C^{-1} b b^T C^{-1} \end{bmatrix}, \quad (7.20)$$

where  $\mu = (2\alpha - b^T C^{-1} b)$  and  $C^{-1}$  denotes the inverse of matrix  $C$ . For the above inverse to exist the following must hold.

- $C$  is invertible and
- $b^T C^{-1} b \neq 2\alpha$ .

We next verify these conditions. Invertibility of  $C$  is guaranteed by its definition. Indeed,

$$\begin{aligned}\det(C) &= \det(A_{n-1}(A_{n-1} + 2\alpha I_{n-1})) \\ &= (\alpha + \beta)^{(n-2)}(\alpha - \beta(n-2))(3\alpha + \beta)^{(n-2)} \\ &\quad \times (3\alpha - \beta(n-2)) > 0,\end{aligned}$$

and it can be computed as

$$C^{-1} = (A_{n-1} + 2\alpha I_{n-1})^{-1} \circ \begin{bmatrix} 2B_{n-1}A_{n-1}^{-1} & -I_{n-1} \\ -I_{n-1} & 2I_{n-1} \end{bmatrix}.$$

Further, all the terms in matrix (7.20) can be expressed in term of inverse of matrix  $A_{n-1}$  as follows:

$$\begin{aligned}C^{-1}b &= \beta \begin{bmatrix} A_{n-1}^{-1}E_{n-1} \\ 0 \end{bmatrix}, \\ b^TC^{-1} &= \beta \begin{bmatrix} E_{n-1}^TA_{n-1}^{-1} & 0 \end{bmatrix}, \\ C^{-1}bb^TC^{-1} &= \begin{bmatrix} \beta^2 A_{n-1}^{-1}E_{n-1}E_{n-1}^TA_{n-1}^{-1} & \bigcirc \\ \bigcirc & \bigcirc \end{bmatrix}.\end{aligned}\tag{7.21}$$

Left multiplying matrix (7.21) by  $b^T$  we have

$$\begin{aligned}b^TC^{-1}b &= 2\beta^2 E_{n-1}^TA_{n-1}^{-1}E_{n-1} \\ &= \frac{2\beta^2(n-1)}{\alpha - (n-2)\beta}.\end{aligned}\tag{7.22}$$

The above relation follows by noting that the sum of elements in each row of the adjacent matrix of  $A_{n-1}$  and its determinant are given by  $(\alpha + \beta)^{n-2}$  and  $(\alpha + \beta)^{n-2}(\alpha - (n-2)\beta)$ , respectively. The left hand side in (7.22) is equal to  $2\alpha$  only when  $\alpha = \beta$  and  $n = 2$ . But, this contradicts our assumption  $\alpha > (n-1)\beta$ . This completes the proof of existence of equilibrium.

We next compute the equilibrium prices and the corresponding demand. Rearranging (7.19), equilibrium prices can be written as

$$\begin{aligned}\begin{bmatrix} \bar{p}^s \\ \bar{p}^c \end{bmatrix} &= \frac{1}{\mu} \begin{bmatrix} 1 & b^TC^{-1} \\ C^{-1}b & Y \end{bmatrix} \begin{bmatrix} -\alpha & c^T \\ a & D \end{bmatrix} \begin{bmatrix} p^a \\ \bar{p}^d \end{bmatrix} \\ &\quad + \frac{D_0}{\mu} \begin{bmatrix} 1 & b^TC^{-1} \\ C^{-1}b & Y \end{bmatrix} E_{2n-1},\end{aligned}\tag{7.23}$$

where  $Y = \mu C^{-1} + C^{-1}bb^TC^{-1}$ .

To further simplify the expression for equilibrium prices, we use the following relations

- $2b^T C^{-1}a = b^T C^{-1}b$  and
- $c^T + b^T C^{-1}D = 0$ ,

which are easy to verify and we skip the details. Using these relations we get

$$\begin{aligned}
 \begin{bmatrix} \bar{p}^s \\ \bar{p}^c \end{bmatrix} &= \begin{bmatrix} -1/2 & \bigcirc \\ C^{-1}(a - b/2) & C^{-1}D \end{bmatrix} \begin{bmatrix} p^a \\ \bar{p}^d \end{bmatrix} \\
 &+ \frac{D_0}{\mu} \begin{bmatrix} 1 & b^T C^{-1} \\ C^{-1}b & Y \end{bmatrix} E_{2n-1} \\
 &= \begin{bmatrix} -1/2 & \bigcirc \\ C^{-1}(a - b/2) & C^{-1}D \end{bmatrix} \begin{bmatrix} p^a \\ \bar{p}^d \end{bmatrix} \\
 &+ D_0 \begin{bmatrix} -1/2\beta + (\beta + \alpha)/\beta\mu \\ C^{-1}b(\frac{-1}{2\beta} + \frac{\beta + \alpha}{\beta\mu}) + C^{-1}E_{2n-2} \end{bmatrix} \\
 &= \begin{bmatrix} -1/2 & \bigcirc \\ C^{-1}(a - b/2) & C^{-1}D \end{bmatrix} \begin{bmatrix} p^a \\ \bar{p}^d \end{bmatrix} \\
 &+ D_0 \begin{bmatrix} -1/2\beta + (\beta + \alpha)/\beta\mu \\ \frac{X^{-1}}{2} \circ \begin{bmatrix} -E \\ 2E \end{bmatrix} + \frac{\beta + \alpha}{\mu} \begin{bmatrix} A^{-1}E \\ \bigcirc \end{bmatrix} \end{bmatrix}, \tag{7.24}
 \end{aligned}$$

where  $X = (A_{n-1} + 2\alpha I_{n-1})^{-1}$ , and we used notation  $A := A_{n-1}$  and  $E := E_{n-1}$  for ease of presentation.

The product of matrix  $C^{-1}$  and  $D$  in the above expression can be computed as

$$C^{-1}D = \begin{bmatrix} \alpha X^{-1} & I_{n-1} \\ -2\alpha X^{-1} & -I_{n-1} \end{bmatrix} \tag{7.25}$$

Substituting this relation in (7.24), it is easy to see that equilibrium prices  $p_i^s$  and  $p_i^c$  depend only on  $p_i^d$  all  $i = 2, 3, \dots, n$ . This verifies the claims in second bullet. Further, using (7.24), it follows that at equilibrium prices paid by each user group can be computed as

$$\begin{aligned}
 \begin{bmatrix} p_1 \\ \bar{p}_{-1}^s + \bar{p}^c \end{bmatrix} &= \begin{bmatrix} -1/2 & \bigcirc \\ -(\beta/2)X^{-1}E & -\alpha X^{-1} \end{bmatrix} p^a \\
 &+ D_0 \begin{bmatrix} -1/2\beta + (\beta + \alpha)/\beta\mu \\ X^{-1}E/2 + (\beta + \alpha)/\mu A^{-1}E \end{bmatrix},
 \end{aligned}$$

where  $\bar{p}_{-1}^s$  denotes the ISP price vector without the component  $p_1$ . The corresponding equilibrium demand can be computed as

$$\begin{aligned}
 D_0 E_n - \begin{bmatrix} -\frac{\alpha}{2} + \frac{\beta^2}{2E^T X E} + \beta \alpha E^T X \\ \frac{\beta}{2}E - \frac{\beta}{2}AE - \alpha AX \end{bmatrix} - \\
 D_0 \begin{bmatrix} \frac{\alpha}{2(\alpha - (n-1)\beta)} - \beta E^T X E - \frac{\beta(\beta + \alpha)}{\mu} E^T A^{-1}E \\ \frac{-\beta}{2(\alpha - (n-1)\beta)}E + \frac{(\beta + \alpha)}{\mu}E + AX E/2 \end{bmatrix}.
 \end{aligned}$$

Note that both equilibrium prices and the corresponding demand do not depend on  $p_d$ , verifying the claim in the last bullet. Also, notice that  $p_d$  can be any vector, and so the solution is unique up to a free choice of  $p_d$ , and the statement of the first bullet is verified.



## Chapter 8

# Network Non-Neutrality through Preferential Signaling

### Contents

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### 8.1 Introduction

The past few years have seen much public debate and legislation initiatives concerning access to the global Internet. Some of the central issues concerned the possibility of discrimination of packets by service providers according to their source or destination, or the protocol used. A discrimination of a packet can occur when preferential treatment is offered to it either in terms of the quality of service it receives, or in terms of the cost to transfer it. Much of this debate took part in anticipation of the legislation over "Net Neutrality", and several public consultations were launched in 2010 (e.g. in the USA, in France and in the E.U.). Network neutrality asserts that packets should not be discriminated. Two of the important issues concerning discrimination of traffic are whether (i) an internet service provider (ISP) may or may not request payment from a content provider (CP) in order to allow it reach its end users, and (ii) whether



or not an ISP can have an exclusive agreement with a given CP resulting in a vertical monopoly. Indeed, for Hahn and Wallsten [143], net neutrality “usually means that broadband service providers charge consumers only once for internet access, do not favor one content provider over another, and do not charge content providers for sending information over broadband lines to end users”.

The network neutrality legislation will determine much of the socio-economic role of the internet in the future. The internet has already had a huge impact not only on economy, but also on the exercise of socio-cultural freedom. Directive 2002/22/EC of the European Union, as amended by the Directive 2009/136/EC, established internet access as a universal service<sup>1</sup>. However, internet is a conglomeration of several profit making entities. Interaction among these entities is largely governed by economic interests, and their decisions can adversely impact the socio-economic role of the internet. Thus, it is necessary to understand the interplay between various agents involved, and the knowledge gained can be used in enabling laws that benefits society and its economic development.

This paper pursues a line of research that we have been carrying on for modeling exclusive agreements between service and content providers and study their economic impact. Such agreements are often called “vertical monopolies”. In some branches of industry, steps have been taken against vertical monopolies. As a result, several railway companies in Europe had to split the railway infrastructure activity from the transportation activity. However, in the telecommunication industry impact of vertical monopolies is not yet clear. The international community is still debating the laws to regulate, or not to regulate, interaction between various agents in the internet.

In this paper we study another form of nonneutrality<sup>2</sup> resulting from vertical monopolies that arises when a CP provides private information to an ISP. The private information could be popularity of its content, profiles of users interested in different types of content, traffic characterization, usage pattern, etc. We assume that the CP’s private information is related to the demand generated through the ISPs. If CPs can share this private information with an ISP, then that ISP can adopt a more efficient pricing policy than its competitors. For example, recent acquisition of Dailymotion by France Télécom (an ISP) enables it to have exclusive information about demand for its video content. We derive game theoretic models that enable to compute the impact of such discrimination on the utility of the ISPs. We model the interaction between the ISPs and a CP as a game, where the CP can share its private information through signals. We also look at the possibility of ISPs paying CP for access to its private information and study mechanism to decide these payments.

**Related Work:** We have used in the past game theoretical models to study two aspects of vertical monopolies. In [125] and [139], we studied the impact of collusion between an ISP and a CP by jointly determining the price each one charges. We evaluated the

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<sup>1</sup>A universal service has been defined by the EU, as a service guaranteed by the government to all end users, regardless of their geographical location, at reasonable quality and reliability, and at affordable prices that do not depend on the location.

<sup>2</sup>The traditional net-neutrality discussion is about ISPs discriminating CPs by giving them preferential treatment.

impact of such collusion, both on the colliding companies as well as on the benefits of other ISPs and CPs. In [142] we studied the impact that an ISP can have by proposing preferential quality of service or cheaper prices for accessing a CP with which it has an exclusive agreement. We refer the reader to [134] for a survey on net neutrality debate. In [131], the authors study a signaling game between high quality and low quality firms in a Bertrand oligopoly [133]. The quality of each firm is a private information which is signaled to others by the price set on their products. In [137], the authors propose a metric called *price of collusion* to study impact of collusion. In [138], similar definitions are proposed to consider several other scenarios. The authors in [110] study cooperation in routing games using Nash bargaining solution concept. They study degradation in network performance by introducing a metric called *price of selfishness*. Nash bargaining solution is also used in [100] to study contracts in nonneutral networks.

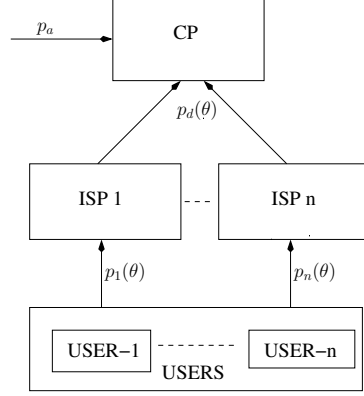
**Our contributions:** In this paper, we propose a simple model with one CP and several ISPs to analyze a network with vertical monopolies.

- We first consider the neutral network where the CP shares private information with all the ISPs or none of them. We compare this case with a nonneutral network where the CP colludes with one of the ISPs, say  $ISP_1$ , and provides signal only to  $ISP_1$ . We show that an ISP receiving signal improves its monetary gains, while CP may not.
- We then consider a case where the CP charges the ISPs for sharing private information. We show that the colluding pair, i.e., the CP and  $ISP_1$  that obtains signal on payment, may not always gain. We characterize the price, that colluding ISP pays to the CP, that results in collusion beneficial to the colluding pair.
- We then propose mechanisms based on weighted proportional fairness criteria for deciding payments that the colluding ISP makes to the CP to obtain signal. We compare the social utilities induced by these mechanisms with the optimal social utility by introducing a metric termed as *price of partial bargaining*.

The paper is organized as follows: In Section 8.2, we introduce the model and set up the notations. In Section 8.3, we consider the neutral network in which the CP provides signals either to all the ISPs or to none of them. In section 8.5, we study the competition assuming the demand generated through ISPs is linear in the user price. Section 8.4 studies a nonneutral behavior in which the CP colludes with one of the ISPs. In Section 8.6, we allow the CP to charge the colluding ISP for providing the signals. In Section 8.7, we consider two mechanisms to determine the payment between the colluding pair. In Section 8.8, we propose a new metric to compare social utilities induced by these mechanism. Finally, we end with conclusions in Section 8.9.

## 8.2 Model

Consider  $n$  competing internet service providers (ISPs), namely  $ISP_i$ ,  $i = 1, 2, \dots, n$ , that provide access to a common content provider (CP). Each ISP determines the price (per unit of content) that it charges its subscribers. In our model we consider single



**Figure 8.1:** Monetary flow in a nonneutral network.

CP as few players, like YouTube, Netflix, account for a significant amount of traffic generated in the internet. The demand generated by the subscribers of  $\text{ISP}_i$  depends on the price of all ISPs as well as on some parameter  $\theta$  reflecting private information of the CP. We assume that  $\theta$  takes values in some discrete space  $\Theta$ . The model is shown in Figure 8.1. We summarize the parameters of the model in Table 8.1.

Let the vector  $\mathbf{p}(\theta) := (p_1(\theta), p_2(\theta), \dots, p_n(\theta))$  denote the price set by all the ISPs when the signal is  $\theta$ . We write the demand generated by the subscribers of  $\text{ISP}_i$  as

$$d_i(\theta, \mathbf{p}(\theta)) = d_i(\theta, p_i(\theta), \mathbf{p}_{-i}(\theta)) \quad i = 1, 2, \dots, n$$

where

$$\mathbf{p}_{-i}(\theta) = (p_1(\theta), p_2(\theta), \dots, p_{i-1}(\theta), p_{i+1}(\theta), \dots, p_n(\theta)).$$

We shall assume that for each  $\theta$  and  $i$  the demand functions are twice differentiable and satisfy the following monotonicity properties

$$\frac{\partial d_i(\theta, \mathbf{p}(\theta))}{\partial p_i} < 0 \text{ and } \frac{\partial d_i(\theta, \mathbf{p}(\theta))}{\partial p_j} > 0 \text{ for } j \neq i. \quad (8.1)$$

These conditions imply that if  $\text{ISP}_i$  increases the access price  $p_i$  then subscribers of  $\text{ISP}_i$  can shift to other ISPs, decreasing the demand generated through  $\text{ISP}_i$  while increasing that generated from the other ISPs. Above conditions are common in modeling demand functions in a price competition [136]. The CP is assumed to have knowledge on the exact value of  $\theta$ . The probability  $P(\theta)$  that the private information is of type  $\theta$  is a common knowledge to all ISPs.

The utility of  $\text{ISP}_i$  is assumed to have the form

$$U_{\text{ISP}_i}(\theta, p_i(\theta), \mathbf{p}_{-i}(\theta)) = p_i(\theta) d_i(\theta, p_i(\theta), \mathbf{p}_{-i}(\theta)).$$

Parameter	Description
$\theta$	indicator of private information of the CP (signal).
$p_i(\theta)$	Price per unit demand charged by $\text{ISP}_i$ to its users; this can be a function of $\theta$ .
$d_i(\cdot)$	Demand generated by $\text{ISP}_i$ . It is a function of the price set by all the ISPs and $\theta$ .
$p_a$	Advertising revenue per unit demand, earned by the CP. This satisfies $p_a \geq 0$ .
$p_d$	Price per unit demand paid by the ISPs to the CP for providing signals.
$U_{\text{ISP}_i}$	The revenue or utility of the $\text{ISP}_i$ .
$U_{\text{CP}}$	The revenue or utility of the CP.
$\gamma$	Bargaining power of the ISPs with respect to the CP. This satisfies $0 < \gamma < 1$ .
$n$	Number of ISPs.

*Table 8.1: Single CP and multiple ISP net neutrality game*

The CP earns a fixed advertisement revenue of  $p_a$  per unit demand. The total revenue earned by the CP depends on the effective demand generated by all the ISPs. Utility of the CP is given by

$$U_{\text{CP}}(\theta, \mathbf{p}(\theta)) = \sum_{i=1}^n d_i(\theta, \mathbf{p}(\theta)) p_a.$$

If  $\text{ISP}_i$  does not know the actual signal  $\theta$  then it can set the price knowing only the distribution of  $\theta$ . In this case we denote the price by simply  $p_i$  (doesn't depend on particular realization of  $\Theta$ ). With some abuse of notation, we denote the utilities of  $\text{ISP}_i$  as  $U_{\text{ISP}_i}(\cdot)$  in both the cases. It should be clear from the context if an ISP obtains signals or not. In the current setting CP acts as a passive player. It can only provide signals to the ISPs, but does not control any prices. Its revenue is influenced by the prices set by the ISPs. Again, with some abuse of notation we denote the utility of CP as  $U_{\text{CP}}$  in all the cases.

The demand functions defined above are quite general. To study the price competition between the ISPs we further assume that the demand function  $d_i(\theta, \mathbf{p}(\theta))$  is supermodular for each  $i$  and  $\theta$ , and satisfy 'dominant diagonal' property. For a twice differentiable function supermodularity property is equivalent to the condition **Assumption 8.2.1** (Supermodularity, [129]).

$$\frac{\partial^2 d_i(\theta, \mathbf{p})}{\partial p_i \partial p_j} \geq 0 \quad j \neq i.$$

The dominant diagonal property is defined for all  $\gamma, \theta \in \Theta$  as

**Assumption 8.2.2** (dominant diagonal).

$$\sum_{j=1}^n d_i(\theta, \mathbf{p}) \frac{\partial^2 d_i(\gamma, \mathbf{p})}{\partial p_i \partial p_j} - \frac{\partial d_i(\theta, \mathbf{p})}{\partial p_i} \frac{\partial d_i(\gamma, \mathbf{p})}{\partial p_j} \leq 0.$$

For simplicity, we also assume that the price charged by  $\text{ISP}_i$  is bounded, say by  $p_i^{\max}$  for all  $\theta$ , such that demand from for all the ISPs is positive. Also, the price sensitivity of the subscribers is the same for all the ISPs. If  $\text{ISP}_i$  increases access charges while the others maintain their price, then a fraction of the subscribers move from  $\text{ISP}_i$  to other ISPs without assigning preference to any particular ISP. Thus demand function of all the ISPs is symmetric.

In the next two sections we study price competition in neutral and nonneutral networks. We define utility and objectives of all the players and compare their revenues in each cases.

### 8.3 Neutral behavior

In this section we study price competition in a neutral network. In the neutral regime the CP does not discriminate between the ISPs: It shares private information about its content with all the ISPs or none of them. We study these two cases separately, and analyze the impact of having the information on the expected utility of each ISP and the CP at equilibrium.

#### 8.3.1 No information

We first consider the neutral behavior in which no information is shared with the ISPs. The ISPs set their prices knowing only distribution  $P(\theta)$ . Recall that in this case we denoted the price charged by  $\text{ISP}_i$  as  $p_i$ . The objective of  $\text{ISP}_i$  is to set  $p_i$  that maximizes its expected utility, i.e.,

$$\begin{aligned} E[U_{\text{ISP}_i}] &:= E_\theta[U_{\text{ISP}_i}(\theta, p_i, \mathbf{p}_{-i})] \\ &= \int P(d\theta) p_i d_i(\theta, p_i, \mathbf{p}_{-i}). \end{aligned}$$

where the operator  $E_\theta[\cdot]$  denotes expectation with respect to the random signal  $\theta$ .

#### 8.3.2 Full information

Let us consider the case where the CP gives signals to all the ISPs, i.e., all the ISPs are given  $\theta$ . We also assume that the signal is sent to all the ISPs simultaneously. Note that the CP providing signals to all the ISPs is a non-discriminatory act. Hence we consider this case under neutral regime.

ISPs can use knowledge of  $\theta$  to set the price charged form their users. The objective of  $\text{ISP}_i$  is to maximize its expected utility given by

$$\begin{aligned} E[U_{\text{ISP}_i}] &= E[U_{\text{ISP}_i}(\theta, p_i(\theta), \mathbf{p}_{-i}(\theta))] \\ &= \int P(d\theta) U_i(\theta, p_i(\theta), \mathbf{p}_{-i}(\theta)). \end{aligned}$$

Note that if any vector  $(p_i(\theta))_{\theta \in \Theta}$  maximizes the expected utility for a given  $\{p_{-i}(\theta) : \theta \in \Theta\}$ , then  $p_i(\theta)$  also maximizes  $U_{\text{ISP}_i}(\theta, \mathbf{p}(\theta))$  for each  $\theta$ . Thus the objective of each player is to maximize  $U_{\text{ISP}_i}(\theta, p_i(\theta), \mathbf{p}_{-i}(\theta))$  for each value of  $\theta$ . In this case strategy of each  $\text{ISP}_i$  is to choose a pricing function on  $\Theta$ , i.e.,  $p_i : \Theta \rightarrow [0, p_i^{\max}]$ .

**Theorem 8.3.2.1.** *Assume that the demand function are supermodular and satisfy the dominant diagonal property for all the ISPs and  $\theta$ . Then the price competition in the neutral regime has the following properties:*

- When all the ISPs obtain the signals, equilibrium exists and unique.
- When none of the ISPs obtain the signals, equilibrium exists and unique.

## 8.4 Nonneutral behavior

In a nonneutral network the CP can discriminate between the ISPs by giving preferential treatment to, or making an exclusive agreement with, one of the ISPs. In this subsection we assume that CP shares information with one of the ISPs. Without loss of generality we assume that the CP shares information with  $\text{ISP}_1$  through signals. Then  $\text{ISP}_1$  can set access price knowing the signal, whereas the other ISPs does so knowing only the distribution. In this case we say that CP and  $\text{ISP}_1$  are in collusion, and refer to them as colluding pair. The utilities of  $\text{ISP}_1$  and other ISPs are given, respectively, as follows:

$$\begin{aligned} U_{\text{ISP}_1}(\theta, p_1(\theta), p_{-1}) &= d_1(\theta, p_1(\theta), p_{-1}) p_1(\theta) \\ U_{\text{ISP}_j}(\theta, p_j, p_{-j}) &= d_j(\theta, p_j, p_{-j}) p_j \quad j = 2, \dots, n. \end{aligned}$$

In the above utilities we write  $p_1(\cdot)$  as a function of the  $\theta$ , whereas  $p_j$ ,  $j = 2, 3, \dots, n$  are constants chosen knowing only distribution of  $\theta$ . The objective of  $\text{ISP}_1$  and  $\text{ISP}_j$ ,  $j = 2, 3, \dots, n$  is to maximize their expected utilities given, respectively, as follow

$$\begin{aligned} E[U_{\text{ISP}_1}] &= \int P(d\theta) U_1(\theta, p_1(\theta), p_{-1}), \\ E[U_{\text{ISP}_j}] &= \int P(d\theta) U_j(\theta, p_j, p_{-j}). \end{aligned}$$

**Theorem 8.4.0.2.** *Assume that the demand functions are supermodular and satisfy diagonal dominance property for each ISP and  $\theta$ . Assume that the CP provides information only to  $\text{ISP}_1$ . Then equilibrium exists in the nonneutral regime and is unique.*

After establishing existence of equilibrium prices in both neutral and nonneutral regimes it is interesting to compare the utilities in both the regimes to see the impact of sharing private information on revenues of the ISPs. Though it appears that an ISP receiving signal should obtain higher revenue compared to an ISP without signal, this observation is not true in general: One can construct simple examples where a player with more information can gain or loose even when equilibrium is unique. For example, see [130]. Also, the authors in the same paper give conditions on the feasible payoff sets that ensures higher utilities at equilibrium for a more informed player. However, these conditions are not easy to verify in a game with continuous strategy space.

There are several demand functions which are log-supermodular and satisfy monotonicity and diagonal property. For example, Linear, Logit, Constant Elasticity of Substitution (CES), Cobb-Douglas, etc [136]. By appropriately choosing the term that depends on the signal  $\theta$ , we can use any of them to model demand functions. In the following we restrict our attention to linear demand function to study the impact of signaling. Linear demand functions are often used to model demand functions in economic literatures [136] due to simplicity of analysis.

## 8.5 Linear demand function

For given price vector  $\mathbf{p}(\theta)$ , assume that demand generated through ISP<sub>*i*</sub> is given by

$$d_i(\theta, \mathbf{p}(\theta)) = D_i(\theta) - \alpha p_i(\theta) + \beta \sum_{j \neq i} p_j. \quad (8.2)$$

where  $\alpha > 0$  and  $\beta > 0$ , and  $D_i(\theta)$  denotes the demand generated through ISP<sub>*i*</sub> when private information of the CP corresponds to  $\theta$  and all the ISPs give free access to the CP. For simplicity, we assume that all the ISPs are equally competitive and set  $D_i(\theta) = D(\theta)$ , i.e., if all the ISPs offer free subscription then the demand generated through each ISP is the same. Also, the users are assumed to be equally sensitive to the price set by each ISP, thus we take  $\alpha$  and  $\beta$  to be the same in the demand function of each ISP. Note that linear demand function is supermodular and satisfies the dominant diagonal property if  $\alpha > (n - 1)\beta$ . We assume this relation holds in the sequel.

Due to simple structure of linear demand function one can compute the equilibrium prices and equilibrium utilities in both the neutral regime and nonneutral regime explicitly and compare.

**Theorem 8.5.0.3.** *Assume that demand function is (8.2) for each ISP and  $\alpha > (n - 1)\beta$ . Then, in the neutral regime, ISPs obtain higher revenue when all of them receive signals, compared to the case where none of them receive signals. In the nonneutral regime,*

1. *the colluding ISP obtains higher revenue at equilibrium than the noncolluding ISPs.*
2. *the revenue of the colluding ISP further improves if the noncolluding ISPs also receive signals.*
3. *the revenue of the noncolluding ISPs remains the same as in the neutral regime where none of the ISPs receive signals (no information).*

Finally, the revenue of the CP at equilibrium is the same in all the cases.

As we note from the above theorem, if ISPs receive signal they only earn higher revenues. However, this increase in ISPs revenue is not because of increased demand, but due to the optimal choice of subscription prices. In particular, as shown in the proof, the demand generated through each ISP remains the same at equilibrium, irrespective of whether it receives signal or not. Thus, the CP does not gain anything by sharing its private information as its revenue depends only on the total demand generated. Hence the CP has an incentive to charge the ISPs for sharing its private information.

In the rest of the paper we restrict our attention to the case with just two ISPs for ease of exposition. However, it is not very restrictive as end users often face a duopoly ISP market.

## 8.6 Signaling with Side Payments

In this section we assume that CP charges ISPs for providing signaling information. We assume that if an ISP receives signal from the CP it pays a fixed price of  $p_d$  per unit demand generated by its subscribers. We refer to  $p_d$  as side payment. We consider the nonneutral regime where  $ISP_1$  gets preferential treatment from the CP. Our aim is to study the impact of side payment on the equilibrium utilities of the colluding pair ( $ISP_1$  and CP) and on the non-colluding  $ISP_2$ . In particular, we will be interested in characterizing the values of side payment that makes the collusion with the CP beneficial to  $ISP_1$  and vice versa.

### 8.6.1 Nonneutral network with pricing

We define the utility of the  $ISP_1$ ,  $ISP_2$  and CP respectively as follows:

$$\begin{aligned} U_{ISP_1}(\theta, p_1(\theta), p_2) &= (D_0(\theta) - \alpha p_1(\theta) + \beta p_2)(p_1(\theta) - p_d) \\ U_{ISP_2}(\theta, p_1(\theta), p_2) &= (D_0(\theta) - \alpha p_2 + \beta p_1(\theta))p_2 \\ U_{CP}(\theta, p_1(\theta), p_2) &= (D_0(\theta) - \alpha p_1(\theta) + \beta p_2)(p_a + p_d) \\ &\quad + (D_0(\theta) - \alpha p_2 + \beta p_1(\theta))p_a \end{aligned}$$

The CP informs the value of  $p_d$  to  $ISP_1$  while they enter in to the agreement. Thus CP acts as a passive player, providing signals, which in turn affects the demand generated by the subscribers of the ISPs. We proceed to analyze the game between  $ISP_1$  and  $ISP_2$ . The objective of each ISP is to maximize its expected utility.

**Proposition 8.6.2.** *In the collusion assume that the CP imposes price on  $ISP_1$  for sharing private information rather than giving it for free. Then, equilibrium revenue of  $ISP_1$  decreases, whereas that of the noncolluding ISP increases.*

Thus pricing in the nonneutral network have a positive externality on the non-colluding ISP. This behavior can be explained as follows: When  $ISP_1$  is charged, it



too charges its subscribers higher to compensate for the extra payment it makes to the CP (see proof of Prop. 8.6.2). Whereas ISP<sub>2</sub> does not need to increase its access fee, and also, some of the users of ISP<sub>1</sub> shift to ISP<sub>2</sub>. This increases demand generated through ISP<sub>2</sub>, thus improving the revenue of the non-colluding ISP. Unlike for the non-colluding ISP<sub>2</sub>, revenue of the colluding pair may or may not improve. It depends on the value of  $p_d$ . The following theorem characterizes its range.

**Theorem 8.6.2.1.** *Assume that in a collusion CP provides signal to ISP<sub>1</sub> by charging a price  $p_d$  per unit demand. Also assume that the distribution of  $\theta$  is such that  $\text{Var}(D(\theta)) \leq E[D(\theta)]$ . Then,*

1) *ISP<sub>1</sub> has an incentive to collude with the CP if and only if*

$$p_d \leq \frac{(2\alpha + \beta)E[D(\theta)]}{2\alpha^2 - \beta^2} \left\{ 1 - \sqrt{1 - \frac{(2\alpha - \beta)^2 \text{Var}(D(\theta))}{4\alpha^2 E^2[D(\theta)]}} \right\}.$$

2) *The CP has an incentive to collude if and only if*

$$p_d \leq \frac{E[D(\theta)](4\alpha^2 - \beta^2)}{(2\alpha - \beta)(2\alpha^2 - \beta^2)} - \frac{(2\alpha^2 - \beta^2 - \alpha\beta)}{2\alpha^2 - \beta^2} p_a \quad (8.3)$$

*Further, the colluding ISP obtains higher revenue than the non-colluding ISP if and only if*

$$p_d \leq \frac{(2\alpha + \beta)E[D(\theta)]}{2\alpha^2 - \beta^2 + \alpha\beta} \times \left\{ 1 - \sqrt{1 - \frac{(2\alpha - \beta)^2 \text{Var}(D(\theta))(2\alpha^2 - \beta^2 + \alpha\beta)}{4\alpha^2 E^2[D(\theta)](2\alpha^2 - \beta^2 - \alpha\beta)}} \right\}.$$

In Figure 8.2, we plot the utilities of colluding ISP and CP as a function of  $p_d$ . In generating the plot we used the following parameters. The signal  $\theta$  takes three values: high (H), medium (M) and low (L), which corresponds to demands  $D(H) = 200, D(M) = 50, D(L) = 20$ . The distribution of  $\Theta$  is taken as  $\Pr(\Theta = H) = 0.1, \Pr(\Theta = M) = 0.6, \Pr(\Theta = L) = 0.3$ . The other parameters are  $\alpha = 2, \beta = 1$  and  $p_a = 5$ .

As shown in the figure, if the side payment lies in the region marked A, then the colluding ISP obtains higher revenue at equilibrium. If it is charged a price outside the region A, then collusion with the CP is not beneficial to any ISP. For the CP, it appears that higher side payment will increase its revenues. But this is not the case. If ISP has to pay higher price to CP its demand goes down, which in turn reduces the revenue of the CP. Thus it is not beneficial for the CP to charge high prices from ISPs. Indeed, if CP charges price beyond region B, governed by (8.3), it will not improve its revenue. The collusion between CP and ISP is profitable to both if and only if side payment lies in the region A. Also, Note that in the region A, though both ISP and CP benefit, CP can obtain higher revenues by increasing the side payment but at the cost of reducing the revenue of ISP<sub>1</sub>. Thus it becomes important to decide how the side payments should be set so that all the players remain satisfied. In the next section we look for mechanisms to address this issue.

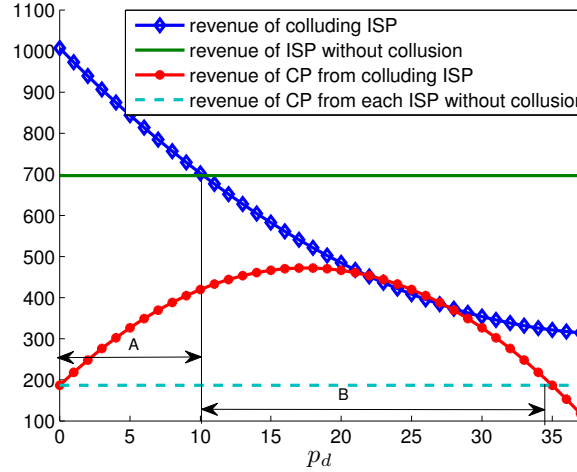


Figure 8.2: Utilities of colluding ISP and CP.

## 8.7 Mechanisms for setting side payments

In this section we look for mechanisms that take into account the bargaining power (weight) of each player. As in the previous section,  $ISP_1$  is in collusion with CP, in which CP shares private information with  $ISP_1$  on payment. We assume that both  $ISP_1$  and CP decide payment in presence of an arbitrator, and refer to the process as *bargaining*. Arbitrator can be a regulating authority, or a disinterested third party who aims to set a side payment that maximizes, in a sense made precise below, the revenues earned by the colluding pair. We consider the following two game models.

The timing for the first game is as follows.

- $ISP_1$  and the CP bargain over the payment  $p^d$ .
- $ISP_1$  and  $ISP_2$  set the access price. The prices are set simultaneously .
- The subscribers react to the prices and set the demand generated through each ISP.

In the second game, timing is as follows:

- $ISP_1$  and  $ISP_2$  set their access price simultaneously.
- $ISP_1$  and the CP then bargain over the payment  $p_d$ .
- The subscribers react to the prices and set the demand generated through each

ISP.

The first game arises when the private information of the CP changes over a slower time-scale making the agreement between  $\text{ISP}_1$ -CP last for longer duration, whereas the ISPs vary access fee over a comparatively faster time-scale. The second game arises in cases where private information of the CP varies over a faster time-scale making the  $\text{ISP}_1$ -CP to renegotiate the side payments often, whereas the ISPs price to its subscribers varies over a slower time-scale. We analyze both models via backward induction and identify the equilibria. In the sequel, we refer to the first game as *pre bargaining* game, and the second one as *post bargaining* game.

In deciding side payment the arbitrator takes into account only revenue of CP earned by traffic generated through  $\text{ISP}_1$ . For a given  $\{p_1(\theta), \theta \in \Theta\}$  and  $p_2$ , the arbitrator decides side payment  $p_d$  from  $\text{ISP}_1$  to the CP based on weighted proportionally fair allocation criteria given as follows:

$$p_d^* \in \arg \max_{p_d} E[U_{\text{ISP}_1}]^\gamma E[U_{\text{CP}}]^{1-\gamma}. \quad (8.4)$$

The parameter  $\gamma \in [0, 1]$  determines the bargaining power of  $\text{ISP}_1$  with respect to the CP.

If we take  $\gamma = 1/2$ , then the above maximization is equivalent to that of product of the utilities of  $\text{ISP}_1$  and the CP. This is then the standard proportional fair allocation [135] and is based on *Nash bargaining solution* which is known to satisfy set of four axioms [104]. We note the method discussed in this section is a modified version of the standard Nash bargaining solution, abusing terminology we continue to refer to it as bargaining. We may imagine that the bargaining is done by another player, the regulator, whose (log) utility equals

$$\bar{U}_{\text{regulator}} := \gamma \bar{U}_{\text{ISP}_1} + (1 - \gamma) \bar{U}_{\text{CP}}, \quad (8.5)$$

where  $\bar{U}_{\text{ISP}_1} = \log E[U_{\text{ISP}_1}]$  and  $\bar{U}_{\text{CP}} = \log E[U_{\text{CP}}]$ .

Recall that the strategy of  $\text{ISP}_1$  is to choose price vector  $\{p_1(\theta), \theta \in \Theta\}$  and that of  $\text{ISP}_2$  is to choose  $p_2$  knowing only distribution of  $\Theta$ , and utilities of  $\text{ISP}_1$ ,  $\text{ISP}_2$  and CP are defined respectively as follows:

$$\begin{aligned} E[U_{\text{ISP}_1}] &= E[(D_0(\theta) - \alpha p_1(\theta) + \beta p_2)(p_1(\theta) - p_d)] \\ E[U_{\text{ISP}_2}] &= E[(D_0(\theta) - \alpha p_2 + \beta p_1(\theta))] p_2 \\ E[U_{\text{CP}}] &= E[(D_0(\theta) - \alpha p_1(\theta) + \beta p_2)](p_a + p_d) \\ &\quad + E[(D_0(\theta) - \alpha p_2 + \beta p_1(\theta))] p_a. \end{aligned}$$

We will now return to our game model where the colluding pair decides side payment in presence of the arbitrator.. In both games the CP is a passive player. In the first game,  $\text{ISP}_1$  and the CP bargain over side payment and then the ISPs set their price competitively. In the second,  $\text{ISP}_1$  choose price knowing that he will bargain with the CP subsequently. Our aim is to compare the expected utilities of each player as a function of  $\gamma$ , and study how the bargaining power influences the players' preference for the bargaining modes, i.e, pre bargaining or post bargaining.

### 8.7.1 Pre bargaining

At the beginning, ISP<sub>1</sub> bargain with the CP and decide side payments. In this bargaining process we take into account the bargaining power of each player. Once the side payment is set, the game is between the two ISPs alone who set their prices competitively to maximize their revenue.

We computed the equilibrium utilities of players in the proof of Proposition 8.6.2 given as follows:

$$E[U_{ISP_1}] = \alpha E \left[ \left( \frac{D(\theta)}{2\alpha} + \frac{\beta E[D(\theta)]}{2\alpha(2\alpha - \beta)} - \frac{p_d(2\alpha^2 - \beta^2)}{4\alpha^2 - \beta^2} \right)^2 \right] \quad (8.6)$$

$$E[U_{ISP_2}] = \alpha \left( \frac{E[D(\theta)]}{(2\alpha - \beta)} + \frac{\alpha\beta}{4\alpha^2 - \beta^2} p_d \right)^2 \quad (8.7)$$

$$\begin{aligned} E[U_{CP}] &= \frac{2\alpha E[D(\theta)] p_a}{2\alpha - \beta} - \alpha \frac{(2\alpha^2 - \beta^2)}{4\alpha^2 - \beta^2} p_d^2 \\ &+ \left( \frac{\alpha E[D(\theta)]}{2\alpha - \beta} - \frac{\alpha(2\alpha^2 - \beta^2 - \alpha\beta) p_a}{4\alpha^2 - \beta^2} \right) p_d. \end{aligned} \quad (8.8)$$

Utility of the CP given above consists of revenue generated from traffic of both the ISPs. The portion that comes from ISP<sub>1</sub> is given by

$$\alpha E \left[ \left( \frac{D(\theta)}{2\alpha} + \frac{\beta E[D(\theta)]}{2\alpha(2\alpha - \beta)} - \frac{p_d(2\alpha^2 - \beta^2)}{4\alpha^2 - \beta^2} \right) \right]$$

Then the optimization problem of the arbitrator is given by

$$\begin{aligned} &\arg \max_{p_d} \alpha E \left[ \left( \frac{D(\theta)}{2\alpha} + \frac{\beta E[D(\theta)]}{2\alpha(2\alpha - \beta)} - \frac{p_d(2\alpha^2 - \beta^2)}{4\alpha^2 - \beta^2} \right)^2 \right]^\gamma \times \\ &E \left[ \left( \frac{D(\theta)}{2\alpha} + \frac{\beta E[D(\theta)]}{2\alpha(2\alpha - \beta)} - \frac{p_d(2\alpha^2 - \beta^2)}{4\alpha^2 - \beta^2} \right) \right]^{1-\gamma} (p_a + p_d)^{1-\gamma}. \end{aligned} \quad (8.9)$$

It is easy to verify that the above optimization problem has unique solution.

### 8.7.2 Post Bargaining

In the second game, ISPs set price competitively knowing that the arbitrator will decide side payment between ISP<sub>1</sub> and CP according to (8.4).

As in the pre bargaining case, we analyze this game in the reverse order, i.e., we first look at the side payment set by the arbitrator as a function of ISP prices, and then study the competition between the ISPs. Note that the demand function  $d_2(\theta, p_1(\theta), p_2)$  does

not depend on  $p_d$ . Then the side payment set by the arbitrator for a given  $\{p_1(\theta), \theta \in \Theta\}$  and  $p_2$  is such that

$$p_d^* \in \arg \max_{p_d} E[d_1(\theta, p_1(\theta), p_2)(p_1(\theta) - p_d)]^\gamma \times E[d_1(\theta, p_1(\theta), p_2)(p_a + p_d)]^{1-\gamma}.$$

**Lemma 8.7.3.** For a given  $\{p_1(\theta), \theta \in \Theta\}$  and  $p_2$  the arbitrator sets side payment as

$$p_d^* = \frac{E[d_1(\theta, p_1(\theta), p_2)(p_1(\theta))]}{E[d_1(\theta, p_1(\theta), p_2)]} - \frac{E[d_1(\theta, p_1(\theta), p_2)p_a]}{E[d_1(\theta, p_1(\theta), p_2)]}.$$

Substituting this expression of  $p_d^*$  in the utility of ISP<sub>1</sub> and CP, the modified utilities are as follows:

$$\begin{aligned} \bar{U}_{ISP_1} &= \gamma E[d_1(\theta, p_1(\theta), p_2)(p_1(\theta) + p_a)] \\ \bar{U}_{CP_1} &= (1 - \gamma) E[d_1(\theta, p_1(\theta), p_2)(p_1(\theta) + p_a)] \end{aligned}$$

Note that in the above expressions  $E[d_1(\theta, p_1(\theta), p_2)(p_1(\theta) + p_a)]$  is total revenue earned by both ISP<sub>1</sub> and CP from the traffic generated through ISP<sub>1</sub>. In the post bargaining they shares this total revenue in proportion to their bargaining power. We proceed to analyze the game between ISPs with the modified utility of ISP<sub>1</sub>.

**Proposition 8.7.4.** In the post bargaining game with the modified utilities, the equilibrium utilities are as follows:

$$\begin{aligned} E[U_{ISP_1}] &= \gamma \alpha E \left[ \left( \frac{D(\theta)}{2\alpha} + \frac{\beta E[D(\theta)]}{2\alpha(2\alpha - \beta)} + \frac{(2\alpha^2 - \beta^2)p_a}{4\alpha^2 - \beta^2} \right)^2 \right] \\ E[U_{ISP_2}] &= \alpha \left( \frac{E[D(\theta)]}{2\alpha - \beta} - \frac{\alpha \beta p_a}{4\alpha^2 - \beta^2} \right)^2 \\ E[U_{CP}] &= \alpha \left( \frac{E[D(\theta)]}{2\alpha - \beta} - \frac{\alpha \beta p_a}{4\alpha^2 - \beta^2} \right) p_a + \\ & (1 - \gamma) \alpha E \left[ \left( \frac{D(\theta)}{2\alpha} + \frac{\beta E[D(\theta)]}{2\alpha(2\alpha - \beta)} + \frac{(2\alpha^2 - \beta^2)p_a}{4\alpha^2 - \beta^2} \right)^2 \right]. \end{aligned}$$

We can now compare the expected utilities of the players in the two games obtained in Proposition 8.7.4 and equations (8.6)-(8.8). First note that the expression for the utility of ISP<sub>1</sub> and the CP in the pre bargaining game is similar to that in the post bargaining game with  $p_d$  replaced by  $-p_a$ . Also, as seen from (8.9) the side payment set in the pre bargaining game satisfies  $p_d > -p_a$ . This gives the impression that ISP<sub>1</sub> prefers the post bargaining mechanism in setting side payment. However, note the multiplicative factor  $\gamma$  in the utility of ISP<sub>1</sub> in the post bargaining game. When the bargaining power of ISP<sub>1</sub> is small, it gets only a small fraction of the joint revenue it earns with the CP. Thus ISP<sub>1</sub> prefers pre bargaining to decide side payment, whereas CP<sub>1</sub> prefers post bargaining. As the bargaining power of the ISP<sub>1</sub> increases, CP gets only a smaller fraction of the total revenue earned, and it prefers post bargaining. We plot the expected utility of

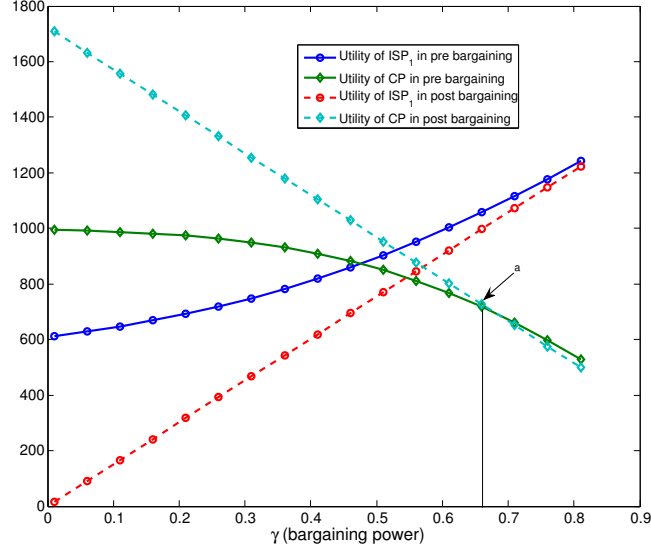


Figure 8.3: ISP and CP utility in two game models  $\alpha = 2, \beta = 1.5$

ISP<sub>1</sub> and CP in both the game models as a function of  $\gamma$  in Figure 8.3. In generating the plots we used the same parameters as in Figure 8.2 with  $\beta = 1.5$ . As seen, there is a threshold on the bargaining power, marked as point *a* in the figure, below which CP<sub>1</sub> prefers pre bargaining and above post bargaining.

## 8.8 Price of Partial Bargaining

In the previous section we studied mechanisms to decide the payment based on weighted proportionally fair criteria. Another natural choice is to set a side payment such that the sum of the utility of all the players is maximized at equilibrium. Let  $\tilde{p}_d$  denote this side payment, i.e.,

$$\tilde{p}_d \in \arg \max_{p_d} E[U_{ISP_1}] + E[U_{ISP_2}] + E[U_{CP}],$$

where the utilities are computed at the equilibrium prices of the players. We denote the expected sum of equilibrium utilities calculated at  $\tilde{p}_d$  as  $\tilde{U}$ . Recall that we denoted the side payment obtained in weighted proportional fairness solution as  $p_d^*$  in (8.4). Let the expected sum of the equilibrium utilities calculated at the weighted proportional

fairness solution be denoted as  $\bar{U}$ . We will be interested in studying how good  $\bar{U}$  is compared to  $\tilde{U}$ .

In this section we do not take into account the bargaining power of each player. We shall be interested in simply the product of utilities (i.e., without the exponents in (8.4)). A more interesting analysis would be to compare optimal  $\alpha$ -fair social equilibrium utility, interpreting fairness factor as the bargaining power. However, we will not pursue this thought in this work.

In [110], the authors proposed a new measure called *Price of Selfishness* (PoS) to compare the optimal social utility with the social utility obtained at the Nash bargaining solution. However, their definition of PoS is not suitable in our setting to compare  $\tilde{U}$  and  $\bar{U}$ . This is because, in [110] the problem is defined in a cooperative context in which the regulator determines the actions taken by all the players, i.e,  $p_1(\theta)$  and  $p_2$ , and also the value of  $p_d$  that maximizes the product of the utilities of all the players. In our case the problem is not fully cooperative. Bargaining is restricted to the parameter  $p_d$  alone. The other parameters are set through competition. Thus in our model bargaining is over a subset of the parameters. We therefore propose an alternative metric called *Price of Partial Bargaining* (PoPB), which we define as

$$PoPB = \frac{\tilde{U}}{\bar{U}}.$$

We will next compute the PoPB in the nonneutral regime analyzed in the previous section where CP shares private information with ISP<sub>1</sub> on payment. In the pre bargaining game, side payment  $\tilde{p}_d$  is the maximizer of sum of utilities given by (8.6)-(8.8) and  $p_d^*$  is obtained from (8.9). The resulting optimal values and the corresponding utilities are cumbersome to manipulate. We plot the PoPB in Figure 8.4 as a function of  $\tau = \beta/\alpha$  fixing  $\alpha = 2$ . From the figure we note that when  $\tau$  is close to 1, the PoPB is large. When  $\tau$  is close to 1, the demand generated from each ISP is equally sensitive to price set by the competing ISPs, in this case pre bargaining leads to poor social utility. However, when  $\tau$  is close to 1/2, PoPB is close to one. This implies that when demand generated from an ISP half as much sensitive as to its own price then the resulting social utility in pre bargaining is close to optimal. When  $\tau$  is close to zero, again the pre bargaining results in poor equilibrium social utility.

We note that our definition of PoPB is not appropriate for the post bargaining game. In this game, first optimal side payment is evaluated for a given price of ISPs, and then the equilibrium prices are computed. But the similar process of evaluating social utility becomes independent of side payment.

## 8.9 Conclusions

In this paper we studied preferential treatment of ISPs by CPs through collusions. We modeled a nonneutral behavior in which a CP shares private about its content through signals. We showed that the CP may not benefit sharing its private information, whereas ISPs always benefit receiving signals. If the CP charges the ISPs to share

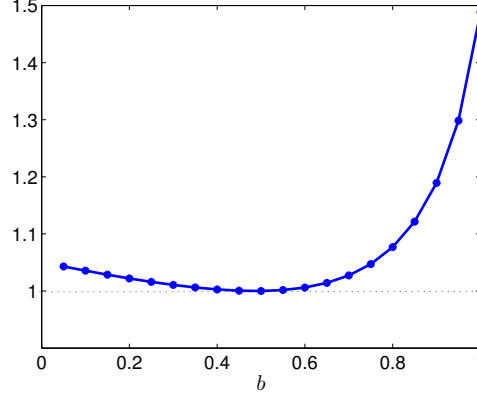


Figure 8.4: Price of partial Bargaining  $\alpha = 2$

its private information, both the CP and the ISP in collusion may lose, whereas the ISPs which do not receive signals may gain.

We also studied two mechanisms based on weighted proportional fairness criteria to set the price (side payments) that ISP pays to the CP for providing signals. In deciding this side payments we took into account the bargaining power of the players. We noted that the bargaining power influences players preference for the mechanisms. We also introduced a new performance measure to compare the social utility at equilibrium with the optimal social utility when some parameters are agreed through bargaining and others are set competitively.

## 8.10 Appendices

### 8.10.1 Proof of Theorem 8.3.2.1

*Proof.* We begin with the neutral regime with no information. Taking the logarithm of the utility of  $\text{ISP}_i$  we get

$$\log E[U_{\text{ISP}_i}] = \log E[d_i(\theta, p_i, \mathbf{p}_{-i})] + \log p_i$$

Using Assumption 8.2.1 and monotonicity properties of  $d_i(\theta, \mathbf{p})$  given in (8.1), it is easy to verify that  $\log E[d_i(\theta, p_i, \mathbf{p}_{-i})]$  satisfies supermodular property, i.e.,

$$\frac{\partial^2 \log E[d_i(\theta, p_i, \mathbf{p}_{-i})]}{\partial p_i \partial p_j} \geq 0 \text{ for } j \neq i.$$

Then existence of equilibrium follows from Topkis's theorem [129].



Using the dominant diagonal property it is easy to verify that for all  $i = 1, 2, \dots, n$

$$\sum_{j=1}^n \frac{\partial^2 \log Ed_i(\theta, \mathbf{p})}{\partial p_i \partial p_j} \leq 0.$$

Then uniqueness of equilibria follows from [129].

Now consider the case with full information. We first note that the demand function for each ISP is separable in  $\theta$ . Thus, given  $\theta$ ,  $\text{ISP}_i$  sets a price that maximizes  $U_{\text{ISP}}(\theta, p_i(\theta), \mathbf{p}_{-i}(\theta))$  independent of what other ISPs set when the signal is different from  $\theta$ . Hence we can restrict the study of price competition between the ISP for a given  $\theta$ . It can be easily verified that  $d_i(\theta, p(\theta))$  is log-supermodular. Also, by setting  $\theta = \gamma$  in Assumption 8.2 the condition

$$\sum_{j=1}^n \frac{\partial^2 \log d_i(\theta, p(\theta))}{\partial p_i \partial p_j} \leq 0$$

holds for all  $\theta$  and  $i$ . Then existence and uniqueness follows from Topkis's theorem [129].

□

### 8.10.2 Proof of Theorem 8.5.0.3

*Proof.* For a given  $\theta$  the utility of each ISP is quadratic in price. We compute the equilibrium prices by simply solving the best response. A straight forward calculations results in the following equilibrium utilities when the CP does not give signal to any of the ISPs:

$$E[U_{\text{ISP}_1}] = E[U_{\text{ISP}_2}] = \frac{\alpha(E[D(\theta)])^2}{(2\alpha - \beta)^2}, \quad (8.10)$$

$$E[U_{\text{CP}}(\theta, p_1^*, p_2^*)] = \frac{2\alpha E[D(\theta)]}{2\alpha - \beta} p_a. \quad (8.11)$$

Similar calculation results in the following utilities of when the CP gives signal to both the ISPs.

$$E[U_{\text{ISP}_1}] = E[U_{\text{ISP}_2}] = \frac{\alpha}{(2\alpha - \beta)^2} E[D^2(\theta)], \quad (8.12)$$

$$E[U_{\text{CP}}(\theta, p_1^*(\theta), p_2^*(\theta))] = \frac{2\alpha E[D(\theta)]}{2\alpha - \beta} p_a. \quad (8.13)$$

Subtracting the expected utility of the  $\text{ISP}_1$  in (8.10) from (8.12) we have

$$\begin{aligned} & \frac{\alpha}{(2\alpha - \beta)^2} (E[D^2(\theta)] - E^2[(\theta)]) \\ &= \frac{\alpha}{(2\alpha - \beta)^2} \text{Var}(D(\theta)) \geq 0. \end{aligned}$$

Where  $\text{Var}(D(\theta))$  denotes the variance of the random variable  $D(\theta)$ . Now assume that CP colludes with  $\text{ISP}_1$  and shares private information only it. Then, the expected utility of  $\text{ISP}_2$  and CP at equilibrium can be computed, respectively, as follows:

$$E[U_{\text{ISP}_1}] = \frac{E[D^2(\theta)]}{4\alpha} + \frac{\beta E^2[D(\theta)](4\alpha - \beta)}{4\alpha(2\alpha - \beta)^2}. \quad (8.14)$$

$$E[U_{\text{ISP}_2}] = \frac{\alpha E^2[D(\theta)]}{(2\alpha - \beta)^2}, \quad (8.15)$$

$$E[U_{\text{CP}}] = 2\alpha \frac{E[D(\theta)]}{(2\alpha - \beta)}. \quad (8.16)$$

We now compare the performance of ISP that receives the signaling information with the ISP which do not have this information. To prove the first claim in the nonneutral regime we compare the expected utility in (8.14) with the expected utility in (8.10) obtained when both the ISPs do not get signaling information. A simple manipulations yields that (8.14) is larger than (8.10) if and only if  $E[D^2(\theta)] \geq E^2[D(\theta)]$ , which always holds.

To prove the second claim we compare the expected utility in (8.14) with the expected utility in (8.12) obtained when both the ISPs get signaling information. Again, a simple manipulation shows that (8.12) is larger than (8.14) if and only if  $E[D^2(\theta)] \geq E^2[D(\theta)]$  which holds always.

The third claim holds by comparing utility of the non-colluding ISP in (8.12) and (8.15). Finally, the last claim follows by noting that expected utility of the CP given by in the three cases reo(8.15) and (8.10) are the same.  $\square$

### 8.10.3 Proof of Proposition 8.6.2

*Proof.* Best response of  $\text{ISP}_1$  for a given value of  $\theta$  and  $p_2$  is

$$p_1(\theta) = \frac{D(\theta) + \beta p_2 + \alpha p_d}{2\alpha}. \quad (8.17)$$

Similarly, the best response of  $\text{ISP}_2$  for a given strategy profile  $\{p_1(\theta) : \theta \in \Theta\}$

$$p_2 = \frac{E[D(\theta + \beta p_1(\theta))]}{2\alpha}.$$

Solving the above best response equations simultaneously, the equilibrium prices are given by

$$\begin{aligned} p_1^*(\theta) &= \frac{D(\theta)}{2\alpha} + \frac{\beta E[D(\theta)]}{2\alpha(2\alpha - \beta)} + \frac{2\alpha^2 p_d}{4\alpha^2 - \beta^2}, \\ p_2 &= \frac{E[D(\theta)]}{2\alpha - \beta} + \frac{\alpha\beta}{4\alpha^2 - \beta^2}. \end{aligned}$$

Substituting these prices in the utility functions and taking expectation we obtain the equilibrium utility for each player as follows:

$$\begin{aligned}
 E[U_{ISP_1}] &= \alpha E \left[ \left( \frac{D(\theta)}{2\alpha} + \frac{\beta E[D(\theta)]}{2\alpha(2\alpha - \beta)} - \frac{p_d(2\alpha^2 - \beta^2)}{4\alpha^2 - \beta^2} \right)^2 \right] \\
 E[U_{ISP_2}] &= \alpha \left( \frac{E[D(\theta)]}{(2\alpha - \beta)} + \frac{\alpha\beta}{4\alpha^2 - \beta^2} p_d \right)^2 \\
 E[U_{CP}] &= \frac{2\alpha E[D(\theta)]}{2\alpha - \beta} - \alpha \frac{(2\alpha^2 - \beta^2)}{4\alpha^2 - \beta^2} p_d^2 \\
 &\quad + \left( \frac{\alpha E[D(\theta)]}{2\alpha - \beta} - \frac{\alpha(2\alpha^2 - \beta^2 - \alpha\beta)p_d}{4\alpha^2 - \beta^2} \right) p_d.
 \end{aligned}$$

The claim follows by comparing the above utilities of the ISPs with the utilities given in (8.14)-(8.15) which corresponds to the case when both the ISPs receive signals.  $\square$

#### 8.10.4 Proof of Theorem 8.6.2.1

*Proof.* Collusion with the CP is beneficial to  $ISP_1$  if it can get higher expected utility compared to the case when it does not enter into any agreement. This happens if the expected utility, given in (8.6), is larger than that given in (8.10). Subtracting (8.10) from (8.6) and simplifying, we get the following quadratic equation in  $p_d$ .

$$f(p_d) := p_d^2 - \frac{2E[D](2\alpha + \beta)p_d}{2\alpha^2 - \beta^2} + \frac{(4\alpha^2 - \beta^2)^2 \text{Var}(D(\theta))}{(2\alpha^2 - \beta^2)4\alpha^2}.$$

The roots of this quadratic equation are

$$\frac{(2\alpha + \beta)E[D(\theta)]}{2\alpha^2 - \beta^2} \left\{ 1 \pm \sqrt{1 - \frac{(2\alpha - \beta)^2 \text{Var}(D(\theta))}{4\alpha^2 E^2[D(\theta)]}} \right\}.$$

Let  $x_1$  and  $x_2$  denote the smaller and larger root respectively. Note that  $f(p_d)$  is a concave function in  $p_d$ . It takes non negative values outside the interval  $[x_1, x_2]$ . It is easy to verify that for  $p_d \geq x_2$  revenue obtained by the colluding ISP is negative. Thus the claim follows by noting that  $f(p_d)$  is nonnegative for  $p_d \leq x_1$ .

Similarly, collusion with  $ISP_1$  is beneficial to CP if it can get higher expected utility compared to the case when it does not enter into any agreement. This happens if the expected utility for CP in collusion, given in (8.8), is larger than that given in (8.11). Subtracting (8.11) from (8.8) and simplifying, we get the following quadratic equation in  $p_d$ .

$$g(p_d) := p_d \left( \frac{E[D(\theta)]}{2\alpha - \beta} - \frac{(2\alpha^2 - \beta^2 - \alpha\beta)p_d}{4\alpha^2 - \beta^2} - \frac{2\alpha^2 - \beta^2}{4\alpha^2 - \beta^2} p_d \right).$$

Now the claim follows by noting that  $g(p_d)$  is positive if and only if  $p_d$  satisfies the relation (8.3).

The last claim can be verified in a similar way by comparing expected utility of  $ISP_1$  and  $ISP_2$  given in (8.6) and (8.7) respectively.  $\square$

## Chapter 9

# Conclusion

In this thesis we analyzed performance of communication networks using game-theoretic models. We considered two aspects that affects the performance. In the first part, we considered the spatial effects on the performance of mobile ad hoc networks and cellular networks. In the second part we considered the economical aspects.

Geometric considerations play a central role in wireless communications, since the attenuation of wireless channels strongly depends on the distance between transmitter and receiver. Models that take into account the exact location of mobiles are often too complex to analyze or to optimize. Our objective in the first two chapters was to analyze the performance of mobile ad hoc networks (MANET) where the location of the nodes is modeled as a realization of homogeneous Poisson point process.

In Chapter 2, we analyzed the case where an ad hoc network operates on a licensed spectrum and each node can be charged for the spectrum usage. When the nodes are selfish, we showed that the spectrum owner can mitigate the selfish behaviour of the nodes by appropriately charging the nodes for the spectrum usage. In particular, we considered average throughput and average delay as the performance metric that the selfish nodes aim to optimize, and in both cases we derived optimal pricing factors that resulted in global optimal performance at equilibrium. Further, we considered the case where the nodes operate on an unlicensed band and analyzed the spatial performance taking into consideration transmission costs.

Decentralized nature of the MANETs leads to many security threats. Some nodes can behave maliciously and aim to degrade the performance of the network. In Chapter 3, we studied a jamming game in MANETs, where the jamming nodes form a realization of Poisson point process and degrade the aim to degrade the performance of the network. We modeled the interaction between the legitimate nodes and the jamming nodes taking into consideration their transmission costs. We showed that if the legitimate nodes transmit at a power level higher than certain threshold, while the jamming nodes operate at a fixed power level, then the jamming nodes will cease to operate at equilibrium.

Energy consumption in cellular networks is a major operational cost incurred by the

network operators. The operators can reduce the energy consumption by turning OFF base stations (BSs) in the off-peak period. However, the switching OFF BSs requires the mobile nodes to re-establish the connection with the active BSs that could be far away. This leads to increased uplink power transmissions by the mobile modes, and expose humans to more radiations. In Chapter 4, we studied the effect of switching OFF BSs in cellular networks where the BSs form a realization of Poisson point process. We computed the amount of radiation that human body can be exposed to which is proportional to the uplink power transmitted by the mobiles. Our analysis showed that switching OFF base stations leads to significant increase in uplink power when density of the BSs is low.

In all the above problems we have extensively used the thinning property of the Poisson point process, which leads to closed form expressions for the performance metric of interest. It is interesting to extend the results using more structured point processes, for instance exhibiting attraction (hot spots) or repulsion (more elaborate medium access control than Aloha like e.g. CSMA). The new models developed in [148] are useful for more tractable analysis of medium access control protocols like CSMA.

In the second part of the thesis we studied the issues discussed in the ongoing net neutrality debate. One of the central issues in the ongoing net neutrality debate is the impact of the last-mile internet service providers (ISPs) charging the content providers (CPs). Mostly, ISPs are in favour of nonneutral regime, whereas small sized CPs are in favour of neutral regime which does not allow such price to be imposed on them. We proposed models to study monetary interaction between ISPs and the CPs in the nonneutral regime and analyzed its impact on the various players in the Internet, i.e., ISPs, CPs and the end users.

In Chapter 6, we considered a nonneutral network where the last-mile ISPs can charge the CPs. We studied two mechanisms to decide payment between a monopoly ISP and a CP based on the Nash bargaining solution. In this mechanism, we took into account the relative bargaining power the ISP with respect to the CP. We showed that if the payment is set by a regulator, instead of it being decided by one of the players, both players and the end users stand to gain. We then extended the model to include several CPs.

In the nonneutral regime, a CP can make an exclusive contract with the ISP to obtain preferential treatment. In Chapter 7, we studied a scenario where one of the CP enters into an exclusive contract with the ISP, and in this contract the colluding ISP and the CP makes joint decisions on the price charged from the end users. We showed that such contracts may not always benefit the CP. Whether entering into such contracts help CP depends on its advertisement revenues relative to the noncolluding CPs. In particular, we showed that if the colluding CP's advertisement revenue is smaller compared to the noncolluding CP by a certain factor, exclusive contract will not improve the revenues of the colluding pair.

A significant portion of the traffic on the Internet is due to few CPs, and these CPs have information about the demand pattern for their content. If this information is made available to the ISPs, the ISPs can better price the users and increase their rev-

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enues. In Chapter 8, we analyzed a scenario where a CP can share such private information with the ISPs. We showed that the ISPs can improve their revenues with the private information of the CP. We further considered the case where the CP shares its private information with only an ISP who pays for it. We showed that demanding high prices from the ISP will not increase the revenues of the CP. We derived mechanisms to decide the payment between the ISP and CP based on Nash bargaining mechanism.

In all the above problems we considered a demand function that is linear in price charged by the ISPs and CPs. This assumption is partly due to the lack of knowledge on how the pricing affects the demand in the Internet. It is interesting to consider more realistic demand models and see if the results derived in this thesis continue to hold. To compare the influence of bargaining mechanism on the total revenues of the colluding ISP and CP, we introduced a metric called price of partial bargaining (PoPB). In this thesis, we restricted the analysis of PoPB only to linear demand functions, it is interesting to study this performance metric for other general demand functions.



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