Errata

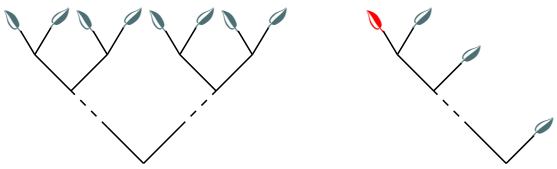
Some minor editions have been made since the defence of this thesis (improvements in the bibliography, fixing a few margins and some typos): they won't be listed here, as those errors were not altering the meaning of this work.[1]

However, while writing an abstract for TERMGRAPH 2014, I realized that the simulation of Proof Circuits by an Alternating Turing Machine, in the Section 2.4 of Chapter 2 was not correct.

More precisely, $\mathbf{bPCC^i}$ is not an object to be considered (as it is trivially equal to $\mathbf{PCC^i}$) and Theorem 2.4.3, p. 51, that states that "For all $i > 0$, $\mathbf{bPCC^i} \subseteq \mathbf{STA}(\log, *, \log^i)$", is false. The ATM constructed do normalize the proof circuit given in input, but not within the given bounds. This flaw comes from a mismatch between the *logical depth* of a proof net (Definition 2.1.10) and the *height of a piece* (Definition 2.2.6), which is close to the notion of *depth of a Boolean circuit* (Definition 2.1.2).

First, remark that the class $\mathbf{bPCC^i}$ (Definition 2.2.8) is ill-defined: recall that (Lemma 2.3.1) in the case of $\mathscr{D}^k_{\text{isj}}$ and $\mathscr{C}^k_{\text{onj}}$ pieces, one entry is at the same logical depth than the output, and all the other entries are at depth 3 plus the depth of the output.

In the process of decomposing a single n-ary operation in $n-1$ binary operations, the "depth-efficient way" is not the "*logical* depth-efficient way". Let us consider a tree with n leafs (🍃) and two ways to organize it:



Let us take the *distance* to be the number of edges between a leaf and the root of the tree. Then, on the left tree, every leaf is at the same distance $\log(n)$, and on the right tree, the greatest distance is n.

Everything differs if one consider that every binary branching is a $\mathscr{D}^2_{\text{isj}}$ or $\mathscr{C}^2_{\text{onj}}$ piece and consider the logical depth rather than the distance. In that case, on the left-hand tree, the ith leaf (starting from the left) is at logical depth 3 times the number of 1 in the binary encoding of i.[2] On the right-hand tree, every leaf is at logical depth 3, except for the red leaf (the left-most one), which is at logical depth 0.

So the logical depth is in fact independent of the fan-in of the pieces, and it has the same "logical cost" in terms of depth to compute $\mathscr{D}^i_{\text{isj}}$ or $\mathscr{C}^i_{\text{onj}}$ for any $i \geqslant 2$. A proof circuit of $\mathbf{PCC^i}$ *is* a proof circuit of $\mathbf{bPCC^i}$: every piece of $\mathscr{P}_u$ can be obtained from pieces of $\mathscr{P}_b$ *without increasing the logical depth*. So we have that for all $i \in \mathbb{N}$,

$$\mathbf{PCC^i} = \mathbf{bPCC^i}.$$

Secondly, the proof of Theorem 2.4.3 was an attempt to adapt the function value(n, g, p) [135, p. 65] that prove that an ATM can normalize a Boolean circuit with suitable bounds. The algorithm is correct,

[1] Except maybe for a silly mistake in Proposition 4.3.1, spotted by Marc Bagnol.
[2] Sic. This sequence is known as A000120 and starts with $0, 1, 1, 2, 1, 2, 2, 3, 1, 2, 2, 3, 2, 3, 3, 4, \ldots$.

but not the bounds: it is stated that "Each call to f uses a constant number of alternations, and $\mathcal{O}(d(\mathsf{P}_n))$ calls are made. As P_n is of depth $\log^i(n)$, $\mathcal{O}(\log^i(n))$ calls are made.", and this argument is wrong.

Consider that $\mathsf{P}_n \in \mathbf{PCC}^{\mathsf{i}}$ (this is harmful taking the first remark into account), when the ATM calls f to evaluate the output of a $\mathscr{D}^k_{\text{isj}}$ or $\mathscr{C}^k_{\text{onj}}$ piece, it makes $k-1$ calls to f at a lesser depth, and one call at the same depth. The only bound on the fan-in of the pieces, k, is the size of the proof circuit, that is to say a polynomial in the size of the input: this is a disaster.

tl;dr: bPCC$^{\mathsf{i}}$ should not be considered as a pertinent object of study and the correspondence between the proof circuits and the Alternating Turing Machines is partially wrong. This does not affect anyhow the rest of this work and this part has never been published nor submitted.

Créteil, 7 November 2014.