Vehicle Sharing Systems Pricing Optimization

Optimisation des systèmes de véhicules en libre service par la tarification

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One-way Vehicle Sharing Systems (VSS)

Bike Sharing Systems *e.g.* Vélib' Paris (2007)

Protocol

1. Take a bike at a station
2. Use it
3. Return it to the chosen station

In more than 400 cities !

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Car Sharing Systems – Same protocol

- Car2Go (2008) $>$ 15 cities

- Autolib' Paris (dec. 2011)

Is it really freedom?

Frequent and uncontrolled dissatisfaction

- Taking impossible (no vehicle available)
- Returning impossible (no free parking spot)

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Causes

- *Gravitation* (Topography – Montmartre hill, Vélib' Paris)

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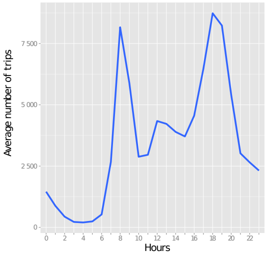
- *Tides* (Home $\leftrightarrow$ Work)

Source Côme (2012) on Vélib', Paris



A day

Spatial distribution of morning tides

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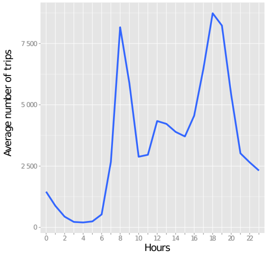
Current optimization

- Fleet/station sizing — Bikes ✓, Cars ✓
- Truck redistribution — Bikes ✓, ~~Cars~~



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  - Chemla, Meunier, and Wolfler Calvo (2012)
  - Raviv, Tzur, and Forma (2013)
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Our approach - An alternative

$\Rightarrow$ Self regulation through incentives (pricing) Bikes ✓, Cars ✓

On models' metaphysics

Mental abstraction

Is pricing a relevant leverage for VSS optimization?

Decide

On models' metaphysics

Truth

Interpret ↓ ↑ Experiment

Empirical knowledge

- Intuition
- Experience
- Analyses

Insights ↓ ↑ Observe

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+ Tradition
+ Implicit context
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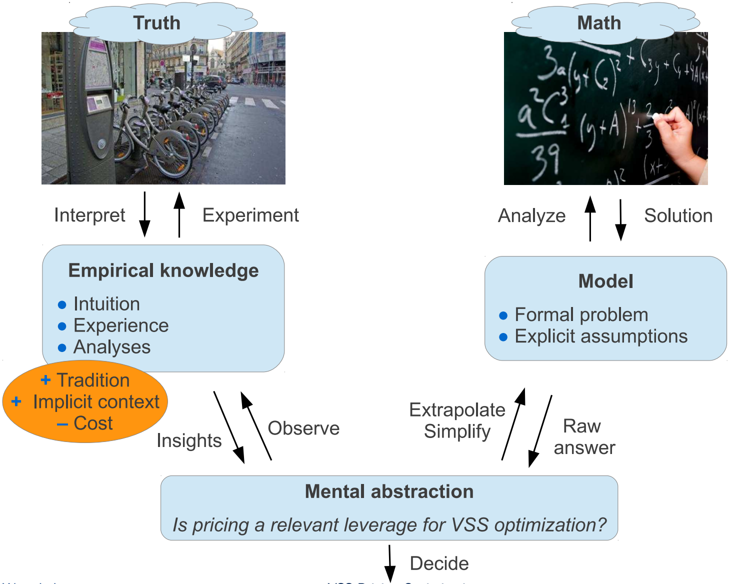

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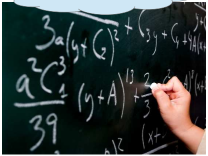
Math



Analyze ↑ ↓ Solution

Model

- Formal problem
- Explicit assumptions

+ Low cost
+ "Speed"
– Assumptions

Extrapolate Simplify ↑ ↓ Raw answer

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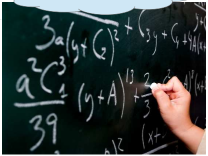
On models' metaphysics

Truth

= under assumptions

Math



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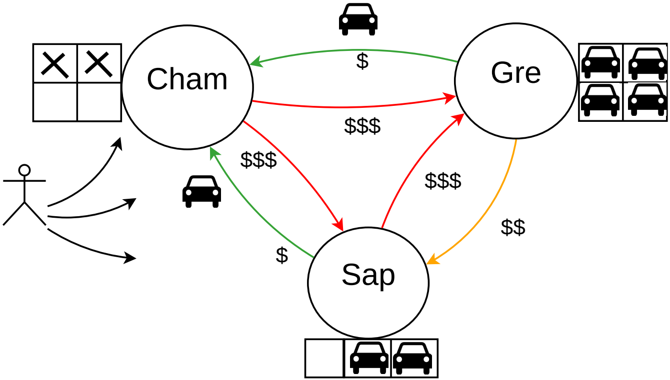
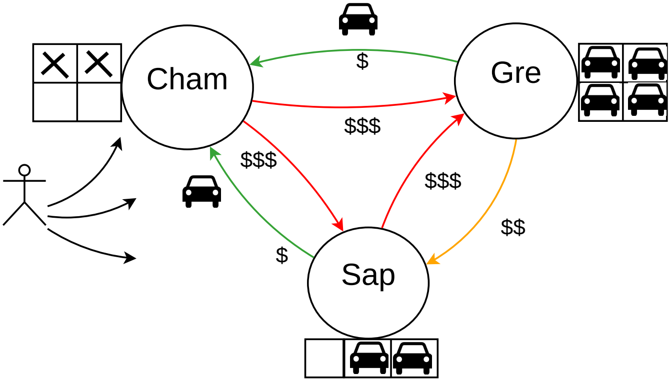
Study assumptions



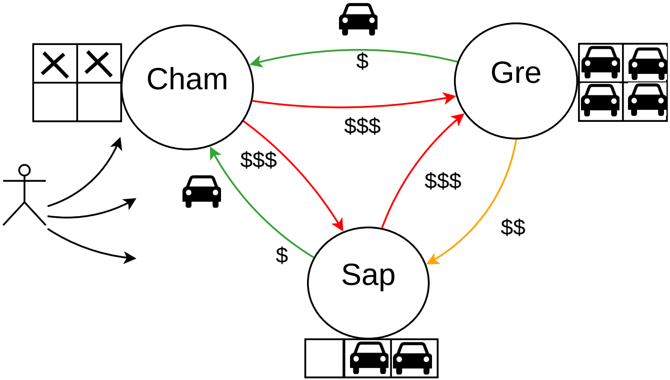
- Stochastic demand

Study assumptions



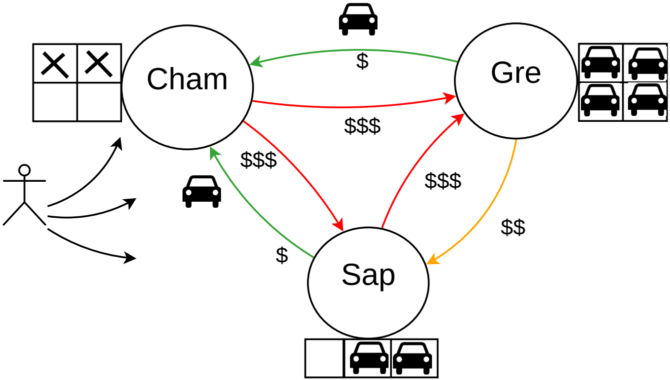
- Stochastic demand
- For a station to station trip

Study assumptions



- Stochastic demand
- For a station to station trip
- In real-time

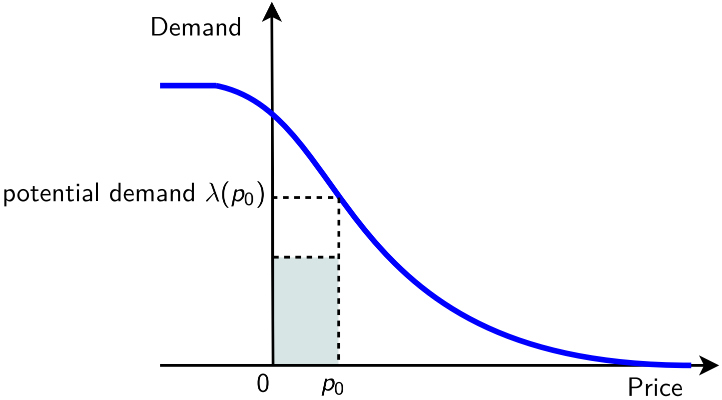
Study assumptions



- Stochastic demand
- For a station to station trip
- In real-time
- With reservation of parking spot at destination

Study assumptions

An elastic demand



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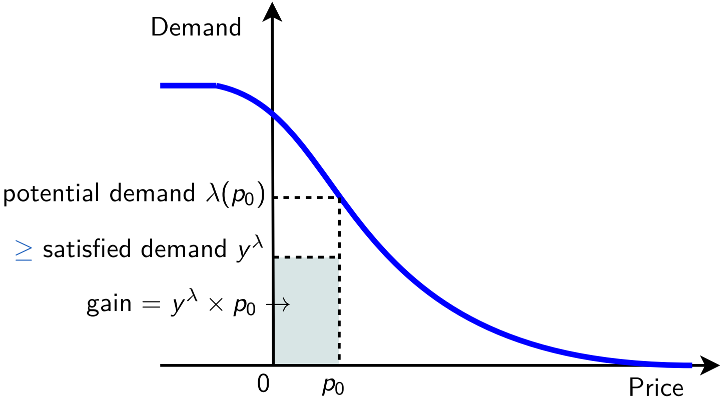


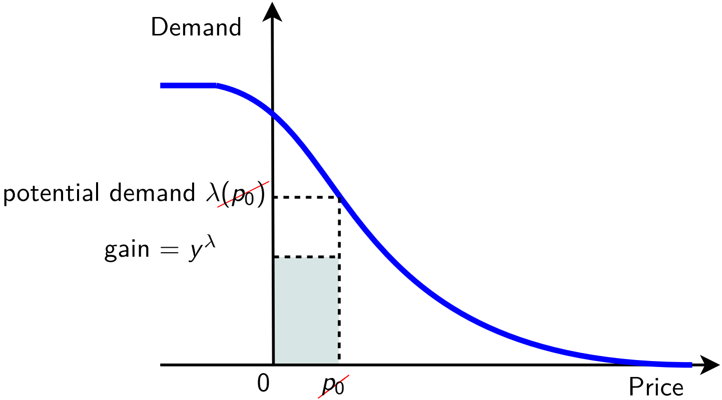
Study assumptions

An elastic demand

Objective: Maximize transit

$\rightarrow$ Implicit pricing/incentive

$\Rightarrow$ Set demand rate λ



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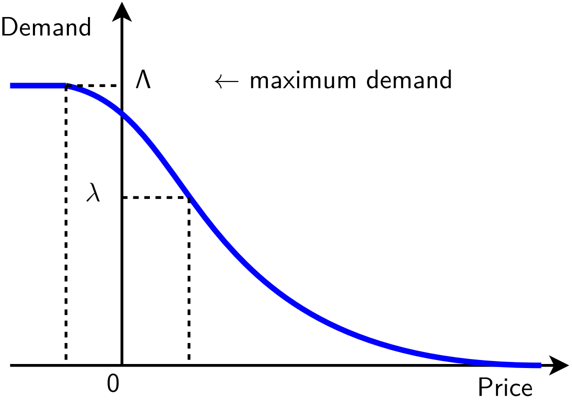
Continuous demand

- Maximum demand Λ

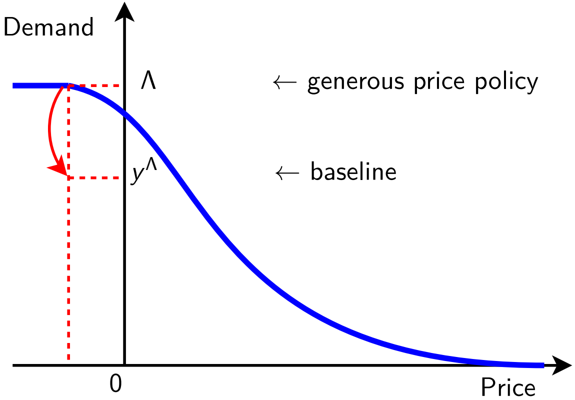
$\Rightarrow$ Any demand $\lambda \in [0, \Lambda]$ reachable



Research question

Can pricing improve on the transit of the generous policy?

$$\sum_{a,b} y^{\Lambda}_{a,b}$$



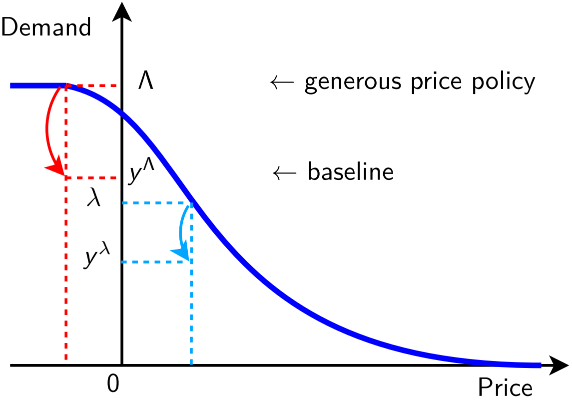
$\leftarrow$ generous price policy

$\leftarrow$ baseline

Research question

Can pricing improve on the transit of the generous policy?

$$\Leftrightarrow \ \exists? \text{ pricing policy } \lambda \text{ such that } \sum_{a,b} y^{\lambda}_{a,b} > \sum_{a,b} y^{\Lambda}_{a,b}$$



VSS stochastic optimization problem

Input

- **Time-dependent** continuous stochastic demand bounded by Λ^t
- A fleet of N vehicles
- A set of M stations with capacity $\mathcal{K}_a$

Output Set the demand (= price) on each trip (a, b) at each instant t

- $\lambda^t_{a,b} \in [0, \Lambda^t_{a,b}]$

Objective

$\Rightarrow$ Maximize the number of trips sold

VSS stochastic evaluation model

Closed queuing network – Finite capacities – Time-varying rates λ^t

- M stations of size $\mathcal{K}_a$
- N vehicles

VSS stochastic evaluation model

Closed queuing network – Finite capacities – Time-varying rates λ^t

- M stations of size $\mathcal{K}_a$ (servers) (example with $M = 2$)
- N vehicles (jobs) $(\sum_{a \in \mathcal{M}} n_a = N)$

VSS stochastic evaluation model

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- M stations of size $\mathcal{K}_a$ (servers) (example with $M = 2$)
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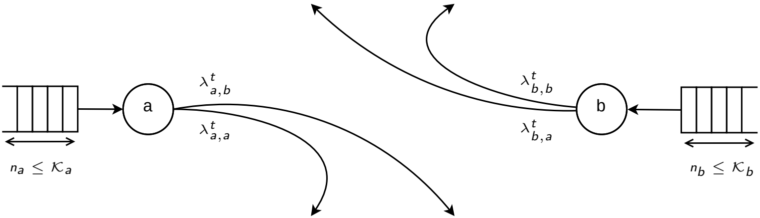
- Users arrivals following a time-dependent Poisson process
  - $\rightsquigarrow$ $\lambda^t_{a,b}$ for trips from a to b at time-step t

VSS stochastic evaluation model

Closed queuing network – Finite capacities – Time-varying rates λ^t



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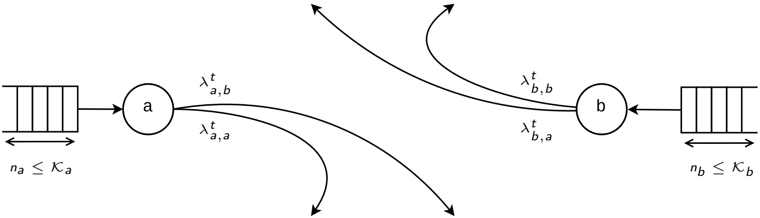

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- Exponential transportation time of mean ${\mu^t_{a,b}}^{-1}$

VSS stochastic evaluation model

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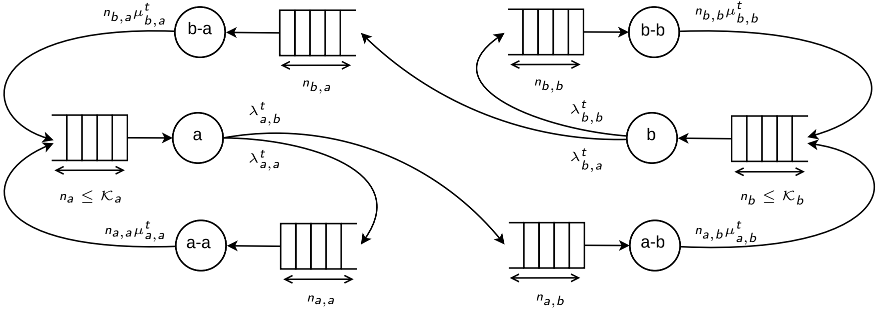
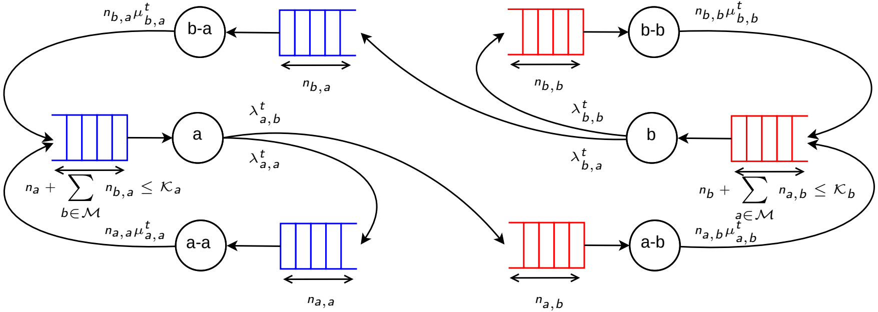


- M stations of size $\mathcal{K}_a$ (servers) (example with $M = 2$)
- N vehicles (jobs) $\left(\sum_{a\in\mathcal{M}} n_a + \sum_{b\in\mathcal{M}} n_{a,b} = N\right)$
- Users arrivals following a time-dependent Poisson process
  - $\rightsquigarrow$ $\lambda^t_{a,b}$ for trips from a to b at time-step t (service time and routing)
- Exponential transportation time of mean ${\mu^t_{a,b}}^{-1}$ (infinite server a-b)

VSS stochastic evaluation model

Closed queuing network – Finite capacities – Time-varying rates λ^t



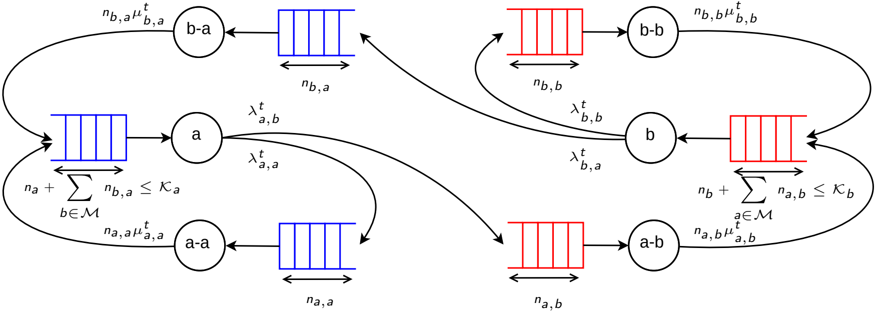
Blocking issues

- Parking spot reservation at destination
  - *Blocking Before Service* type

$\rightarrow$ Joint constraint on "station" and "transport" queue sizes

VSS stochastic evaluation model

Closed queuing network – Finite capacities – Time-varying rates λ^t

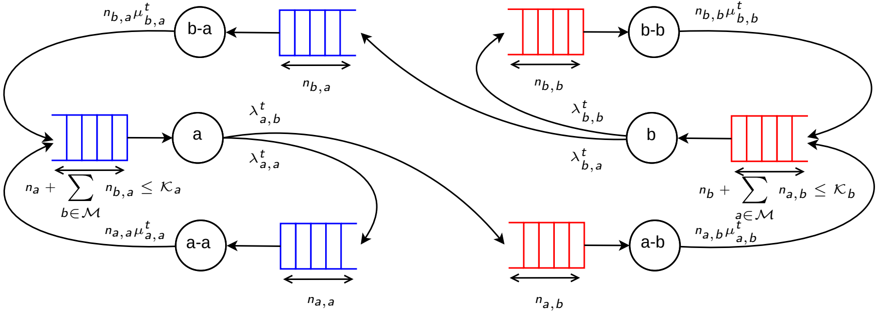


State of the art – Another optimization: Fleet sizing

- Only fixed stationary demand $\lambda^t = \lambda$ (**NOT** pricing)
- George and Xia (2011)
  $\rightarrow$ Infinite station capacities
- Fricker and Gast (2012)
  $\rightarrow$ Perfect cities $\lambda^t_{a,b} = \lambda$ and $\mu^t_{a,b} = \mu$

VSS stochastic evaluation model

Closed queuing network – Finite capacities – Time-varying rates λ^t



An intractable model

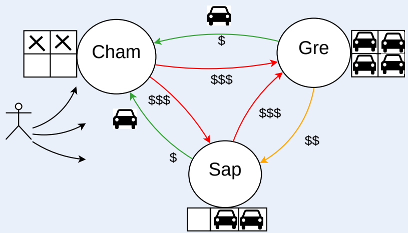
With all our assumptions

- Exact evaluation of the transit for a given demand "hard"
  - Curse of dimensionality

$\Rightarrow$ Easy to evaluate by simulation

VSS pricing optimization

An “intractable” stochastic model

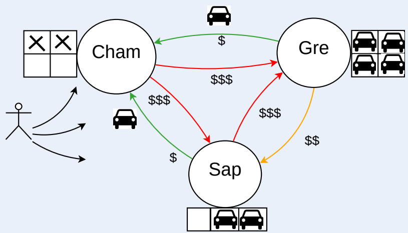


Simplified stochastic model already hard to evaluate (exactly)

“Keep it as simple as possible but not simpler” (A. Einstein)

VSS pricing optimization

An “intractable” stochastic model



Optimization on approximations $\Downarrow$

“Tractable” models

	Heuristic	Upper bound	
• Simplified stoch. models	✓	✓	W. and Jost (2013a)
• Scenario-based approach	APX-hard	✓	W., Jost, and Brauner (2013b)
• Fluid approximation	✓	✓	W. and Jost (2013b)

VSS pricing optimization

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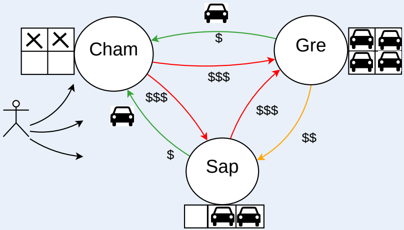

⇒ Preliminary answer

Optimization on approximations ⇓ ⇑ Evaluation by simulation

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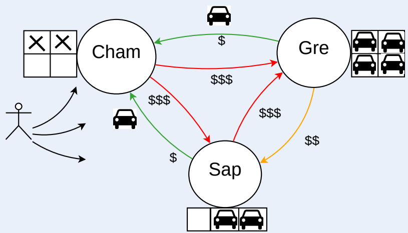
VSS pricing optimization

(1) An "intractable" stochastic model



$\Rightarrow$ (6) Preliminary answer

Optimization on approximations $\Downarrow$ $\Uparrow$ (5) Evaluation by simulation

"Tractable" models

	Heuristic	Upper bound	
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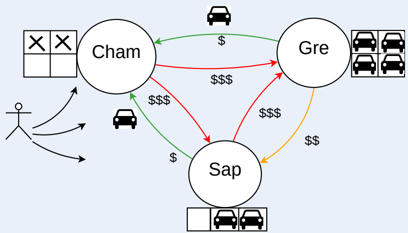
VSS pricing optimization

An "intractable" stochastic model



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Optimization on approximations ⇓ ⇑ (5) Evaluation by simulation

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- Decomposable MDP Exact Solution W., Gayon, and Jost (2013a)

Looking for "tractable" solution methods

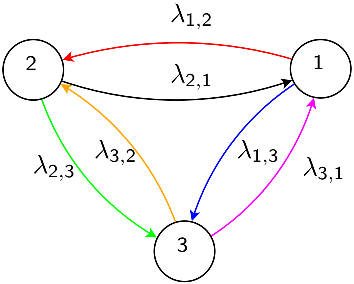
1. Simplified stochastic model
   - No station capacity and no time-varying demand as in George and Xia (2011) $+$ no transportation times

   $\rightarrow$ Evaluate exactly a pricing policy

   $\Rightarrow$ "Feel" stochastic optimization
2. Scenario based approach
3. Fluid approximation

Stochastic optimization of a simplified model

Null transportation times, stationary demand ($\lambda^t = \lambda$), infinite station capacity ($\mathcal{K} = \infty$)



Demand graph, $M = 3$ stations

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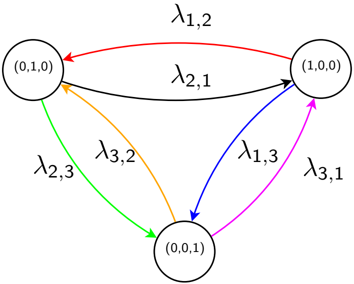

State graph, $M = 3$, $N = 1$ vehicle

Evaluation: a Continuous-Time Markov Chain (CTMC)

- State: $(n_1, \ldots, n_M)$, $\sum n_a = N$
- n_a: number of vehicles in station a

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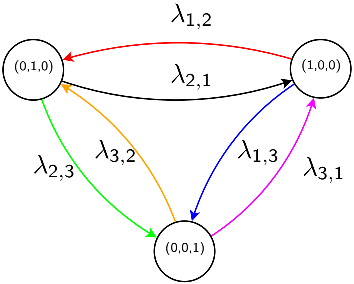

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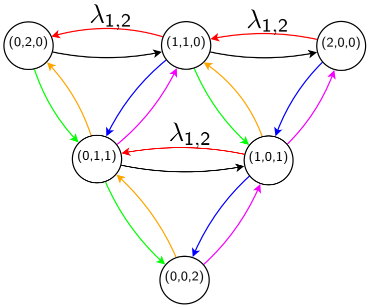

State graph, $M = 3$ stations, $N = 2$ vehicles

Evaluation: a Continuous-Time Markov Chain (CTMC)

- State: $(n_1, \ldots, n_M)$, $\sum n_a = N$
- n_a: number of vehicles in station a

$\rightarrow$ State graph of exponential size: $|\mathcal{S}| = \binom{N+M-1}{N}$ states

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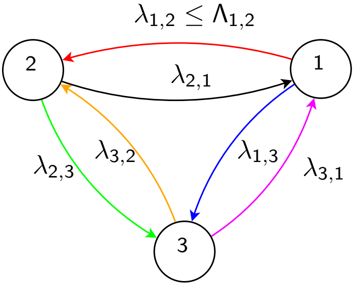

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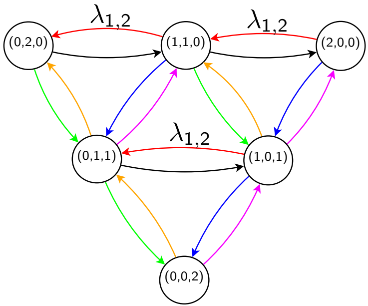
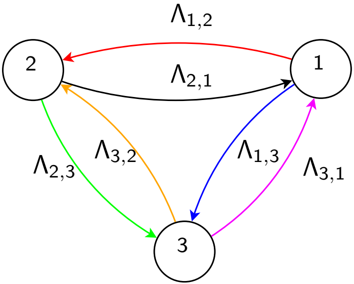


State graph, $M = 3$ stations, $N = 2$ vehicles

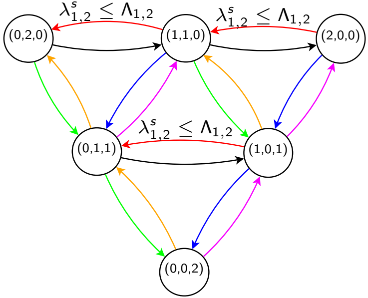
- Static policy
  - $=$ Not state dependent
  - $\rightarrow$ Decisions on the demand graph

Stochastic optimization of a simplified model

Null transportation times, stationary demand ($\lambda^t = \lambda$), infinite station capacity ($\mathcal{K} = \infty$)



Demand graph, $M = 3$ stations



State graph, $M = 3$ stations, $N = 2$ vehicles

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- Dynamic policy
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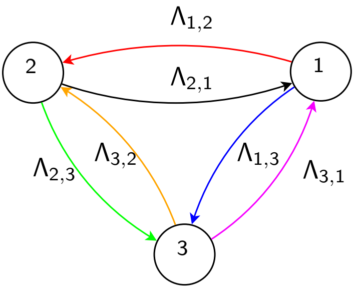

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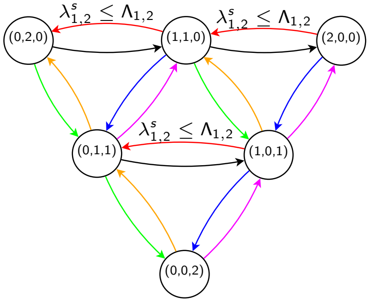

State graph, $M = 3$ stations, $N = 2$ vehicles

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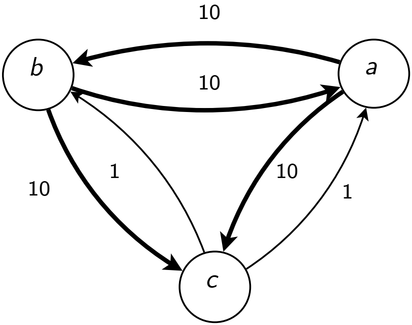
Can static policies improve on the generous policy?

$N = 1$ vehicle



Demand graph

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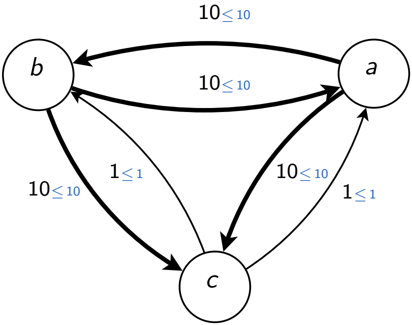
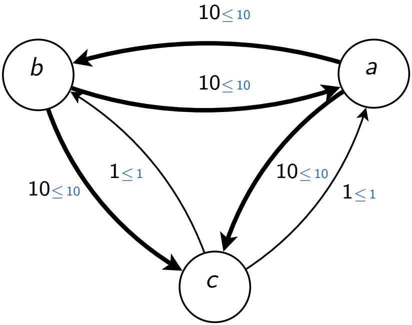
$N = 1$ vehicle



Generous policy ($\lambda_{\leq \Lambda}$)

Can static policies improve on the generous policy?

$N = 1$ vehicle



Generous policy ($\lambda_{\leq \Lambda}$)

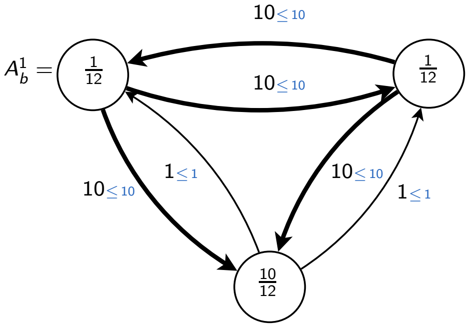
Availability A_a^N: probability to have a vehicle in station a

Transit on trip $y_{a,b} = A_a^N \lambda_{a,b}$: expected transit for trip (a, b)

Total transit $\sum_{(a,b)\in\mathcal{D}} y_{a,b}$

Can static policies improve on the generous policy?

$N = 1$ vehicle



Generous policy ($\lambda_{\leq \Lambda}$)

- Generous policy
  - 1 vehicle $\rightarrow$ 5 trips/hour

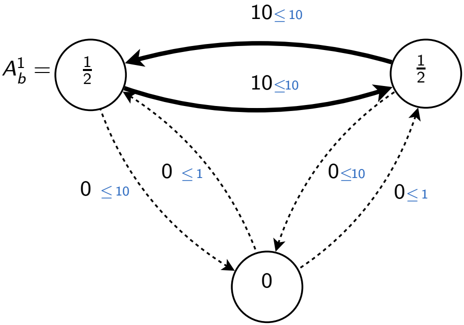
Availability A_a^N: probability to have a vehicle in station a

Transit on trip $y_{a,b} = A_a^N \lambda_{a,b}$: expected transit for trip (a, b)

Total transit $\sum_{(a,b)\in\mathcal{D}} y_{a,b}$

Can static policies improve on the generous policy?

$N = 1$ vehicle



Policy closing station c ($\lambda_{\leq \Lambda}$)

- Generous policy
  - 1 vehicle $\rightarrow$ 5 trips/hour
- Policy closing station c
  - 1 vehicle $\rightarrow$ 10 trips/hour

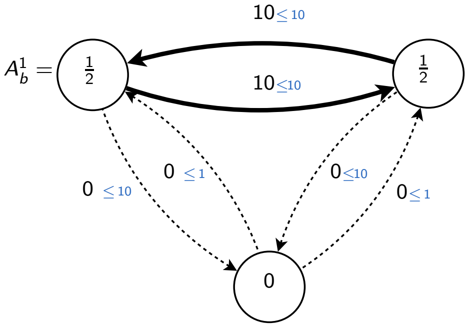
Availability A_a^N: probability to have a vehicle in station a

Transit on trip $y_{a,b} = A_a^N \lambda_{a,b}$: expected transit for trip (a, b)

Total transit $\sum_{(a,b)\in\mathcal{D}} y_{a,b}$

Can static policies improve on the generous policy?

$N = 1$ vehicle



Policy closing station c ($\lambda_{\leq \Lambda}$)

- Generous policy
  - 1 vehicle $\rightarrow$ 5 trips/hour

  $\Rightarrow$ ∞ vehicles $\rightarrow$ dominant?
- Policy closing station c
  - 1 vehicle $\rightarrow$ 10 trips/hour

  $\Rightarrow$ Optimal policy $\forall N$?

Availability A_a^N: probability to have a vehicle in station a

Transit on trip $y_{a,b} = A_a^N \lambda_{a,b}$: expected transit for trip (a, b)

Total transit $\sum_{(a,b)\in\mathcal{D}} y_{a,b}$

Optimizing static policies

Exact optimization for N vehicles

A_a^N: probability to have a vehicle in station a

$y_{a,b}$: expected transit for trip (a,b) with demand $\lambda_{a,b}$

Maximize $$\sum_{(a,b)\in\mathcal{D}} y_{a,b} \qquad \text{(Expected flow)}$$

Optimizing static policies

Exact optimization for N vehicles

A_a^N: probability to have a vehicle in station a
$y_{a,b}$: expected transit for trip (a,b) with demand $\lambda_{a,b}$

$$\text{Maximize} \quad \sum_{(a,b)\in\mathcal{D}} y_{a,b} \qquad \text{(Expected flow)}$$

$$\text{s.t.} \quad \sum_{(a,b)\in\mathcal{D}} y_{a,b} = \sum_{(b,a)\in\mathcal{D}} y_{b,a} \qquad \forall a \in \mathcal{M} \qquad \text{(Flow conservation)}$$

$$y_{a,b} = A_a^N \lambda_{a,b} \qquad \forall (a,b) \in \mathcal{D} \qquad \text{(Satisfied demand)}$$

Optimizing static policies

Exact optimization for N vehicles

A_a^N: probability to have a vehicle in station a

$y_{a,b}$: expected transit for trip (a,b) with demand $\lambda_{a,b}$

$$
\begin{aligned}
\text{Maximize} \quad & \sum_{(a,b)\in\mathcal{D}} y_{a,b} && && \text{(Expected flow)}\\
\text{s.t.} \quad & \sum_{(a,b)\in\mathcal{D}} y_{a,b} = \sum_{(b,a)\in\mathcal{D}} y_{b,a} && \forall a \in \mathcal{M} && \text{(Flow conservation)}\\
& y_{a,b} = A_a^N \lambda_{a,b} && \forall (a,b) \in \mathcal{D} && \text{(Satisfied demand)}\\
& 0 \leq A_a^N \leq 1 && \forall a \in \mathcal{M} && \text{(Probability)}\\
& {\color{red} A^N \in \mathcal{A}^N} && && \text{(Admissible Proba)}
\end{aligned}
$$

Optimizing static policies

Exact optimization for N vehicles

A_a^N: probability to have a vehicle in station a

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& A^N \in \mathcal{A}^N && && \text{(Admissible Proba)}\\
& 0 \le \lambda_{a,b} \le \Lambda_{a,b} && \forall (a,b)\in\mathcal{D} && \text{(Max Demand)}
\end{aligned}
$$

Optimizing static policies

Exact optimization for N vehicles

A_a^N: probability to have a vehicle in station a
$y_{a,b}$: expected transit for trip (a,b) with demand $\lambda_{a,b}$

$$
\begin{array}{llrr}
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\text{s.t.} & \displaystyle\sum_{(a,b)\in\mathcal{D}} y_{a,b} = \sum_{(b,a)\in\mathcal{D}} y_{b,a} & \forall a \in \mathcal{M} & \text{(Flow conservation)} \\
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\end{array}
$$

- Evaluation of a policy λ polynomial in N and M George and Xia (2011)
  $\rightarrow$ Optimization problem $\in$ NP ... exact complexity remains open

Optimizing static policies

Relaxation for N vehicles

$A_a^N = 1$: always a vehicle available

$y_{a,b}$: expected transit for trip (a, b) with demand $\lambda_{a,b}$

$$
\begin{aligned}
\text{Maximize} \quad & \sum_{(a,b)\in\mathcal{D}} y_{a,b} & & \text{(Expected flow)} \\
\text{s.t.} \quad & \sum_{(a,b)\in\mathcal{D}} y_{a,b} = \sum_{(b,a)\in\mathcal{D}} y_{b,a} & \forall a \in \mathcal{M} & \text{(Flow conservation)} \\
& y_{a,b} = \cancel{A_a^N}\lambda_{a,b} & \forall (a,b) \in \mathcal{D} & \text{(Satisfied demand)} \\
& A_a^N = 1 & \forall a \in \mathcal{M} & \text{(Probability)} \\
& \cancel{A^N \in \mathcal{A}^N} & & \text{(Admissible Proba)} \\
& 0 \le \lambda_{a,b} \le \Lambda_{a,b} & \forall (a,b) \in \mathcal{D} & \text{(Max Demand)}
\end{aligned}
$$

Optimizing static policies

Relaxation for N vehicles

$A_a^N = 1$: always a vehicle available

$y_{a,b}$: expected transit for trip (a, b) with demand $\lambda_{a,b}$

$$\text{Maximize} \quad \sum_{(a,b)\in\mathcal{D}} y_{a,b} \qquad \text{(Flow)}$$

$$\text{s.t.} \quad \sum_{(a,b)\in\mathcal{D}} y_{a,b} = \sum_{(b,a)\in\mathcal{D}} y_{b,a} \qquad \forall a \in \mathcal{M} \qquad \text{(Flow conservation)}$$

$$y_{a,b} = \lambda_{a,b} \qquad \forall (a,b) \in \mathcal{D} \qquad \text{(Satisfied demand)}$$

$$0 \leq \lambda_{a,b} \leq \Lambda_{a,b} \qquad \forall (a,b) \in \mathcal{D} \qquad \text{(Max Demand)}$$

Optimizing static policies

Maximum Circulation

$A_a^N = 1$: always a vehicle available

$\lambda_{a,b}$: expected transit for trip (a, b)

$$\text{Maximize} \quad \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b} \qquad \text{(Flow)}$$

$$\text{s.t.} \quad \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b} = \sum_{(b,a)\in\mathcal{D}} \lambda_{b,a} \qquad \forall a \in \mathcal{M} \qquad \text{(Flow conservation)}$$

$$0 \leq \lambda_{a,b} \leq \Lambda_{a,b} \qquad \forall (a, b) \in \mathcal{D} \qquad \text{(Max. demand)}$$

Relaxation

$\Rightarrow$ Maximum Circulation is an upper bound on static policies

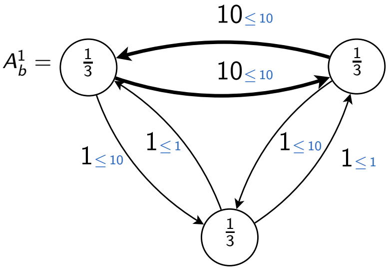
Optimizing static policies

Maximum Circulation policy

$$\text{Maximize} \quad \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b} \qquad \text{(Flow)}$$

$$\text{s.t.} \quad \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b} = \sum_{(b,a)\in\mathcal{D}} \lambda_{b,a} \qquad \forall a \in \mathcal{M} \qquad \text{(Flow conservation)}$$

$$0 \le \lambda_{a,b} \le \Lambda_{a,b} \qquad \forall (a,b) \in \mathcal{D} \qquad \text{(Max. demand)}$$



Circulation policy ($\lambda_{\le \Lambda}$)

- Generous policy
  - 1 vehicle → 5 trips/hour
- Policy closing station c
  - 1 vehicle → 10 trips/hour
- Circulation policy
  - 1 vehicle → 8 trips/hour

Optimizing static policies

Maximum Circulation policy

$$\text{Maximize} \quad \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b} \qquad \text{(Flow)}$$

$$\text{s.t.} \quad \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b} = \sum_{(b,a)\in\mathcal{D}} \lambda_{b,a} \qquad \forall a \in \mathcal{M} \qquad \text{(Flow conservation)}$$

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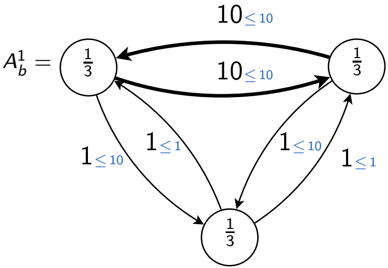

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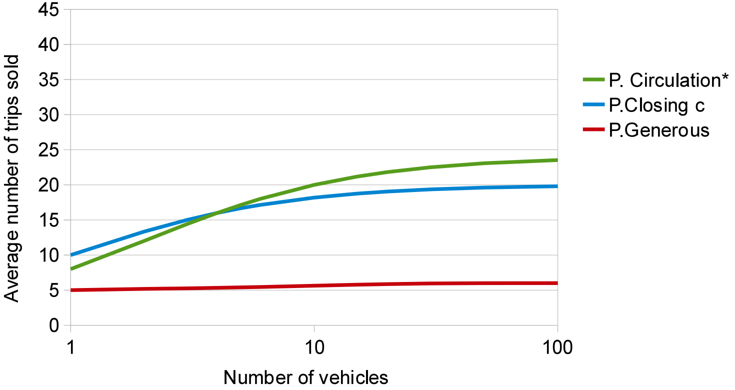
$\Rightarrow$ N vehicles?

Policies performance for N vehicles

Quantifying policies quality $\rightarrow$ Upper Bound (UB)



Policies performance for N vehicles

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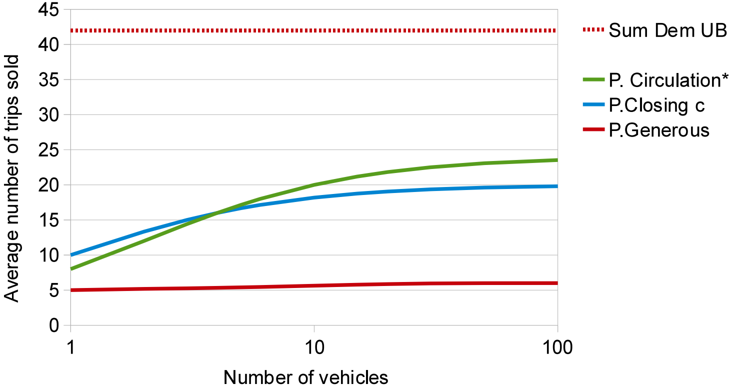
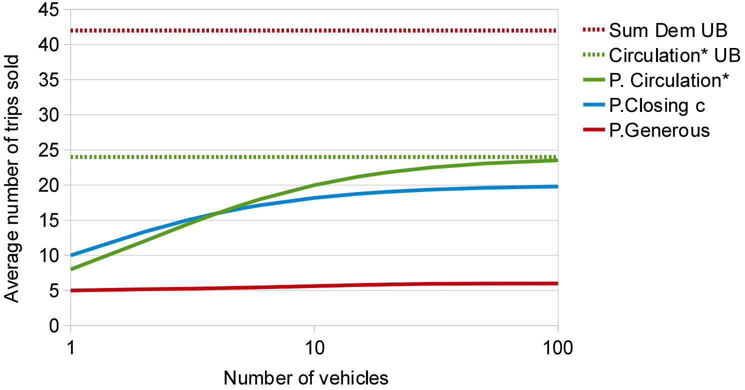


Upper bounds on optimal dynamic policy P_{dyn^*}

- (Trivial) Satisfying all demands $\quad P_{dyn^*} \leq \sum \Lambda_{a,b} = 42$

Policies performance for N vehicles

Quantifying policies quality $\rightarrow$ Upper Bound (UB)

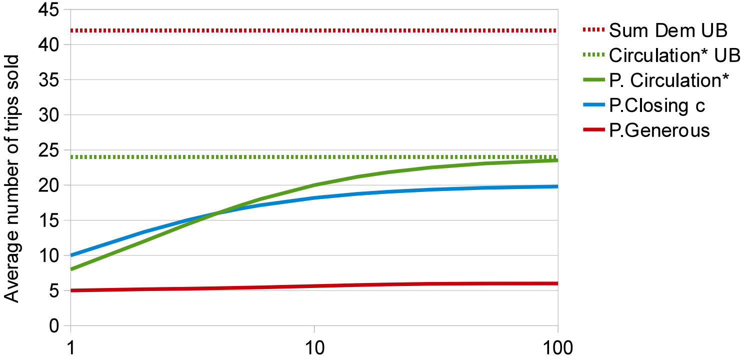


Upper bounds on optimal dynamic policy P_{dyn^*}

- (Trivial) Satisfying all demands $\qquad P_{dyn^*} \leq \sum \Lambda_{a,b} = 42$
- MAXIMUM CIRCULATION value $\qquad P_{dyn^*} \leq \sum \lambda_{a,b}^{\text{P.Circ}^*} = 24$

Policies performance for N vehicles

Quantifying policies quality → Upper Bound (UB)

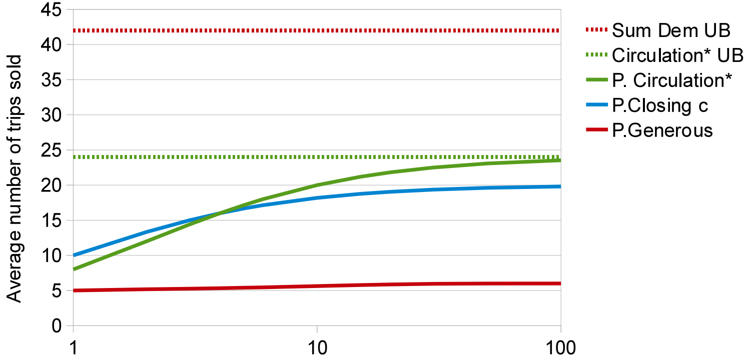


Theorem (For M stations and N vehicles)

MAXIMUM CIRCULATION policy is a $\frac{N}{N+M-1}$-approximation on P_{dyn^*}

Policies performance for N vehicles

Quantifying policies quality → Upper Bound (UB)



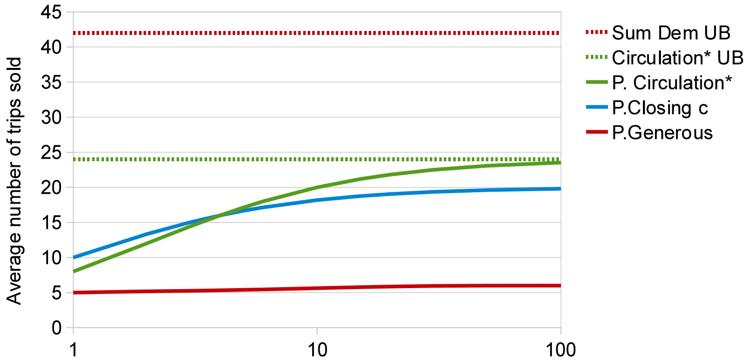
Theorem (For M stations and N vehicles)

MAXIMUM CIRCULATION policy is a $\frac{N}{N+M-1}$-approximation on P_{dyn^*}

- For 9 vehicles per station ($N = 9M$) $\Rightarrow$ $\frac{9}{10}$-approximation

Policies performance for N vehicles

Quantifying policies quality → Upper Bound (UB)



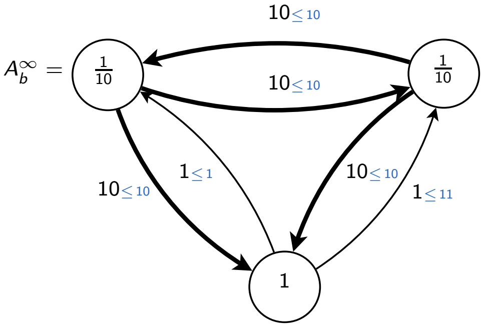
Theorem (For M stations and N vehicles)

MAXIMUM CIRCULATION policy is a $\frac{N}{N+M-1}$-approximation on P_{dyn^*}

- For 9 vehicles per station ($N = 9M$) $\Rightarrow$ $\frac{9}{10}$-approximation
- Sketch of proof

Availability when number of vehicle $N \to \infty$

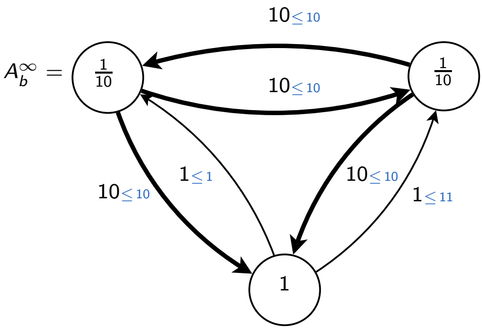
Why generous so bad?



Generous policy ($\lambda_{\leq \Lambda}$)

Availability when number of vehicle $N \to \infty$

Why generous so bad?



Generous policy ($\lambda_{\leq \Lambda}$)

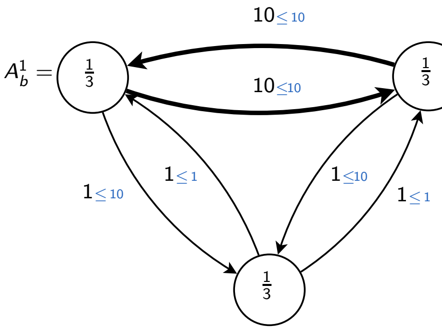
$$\lim_{N\to\infty} A_a^N = \frac{A_a^1}{\max_{b\in\mathcal{M}} A_b^1}$$

George and Xia (2011)

$$\lim_{N\to\infty} \text{Transit} = 6$$

Availability when number of vehicle $N \to \infty$

Why circulation so good?



Circulation policy ($\lambda_{\leq \Lambda}$)

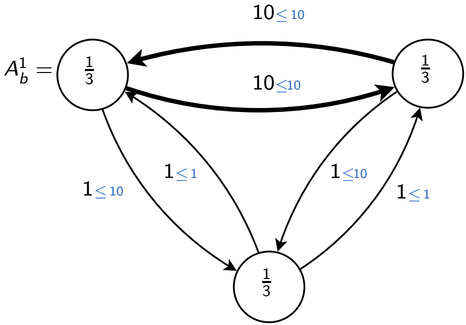
Circulation "balances" demand

$$\forall a \quad \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b} = \sum_{(b,a)\in\mathcal{D}} \lambda_{b,a}$$

$\to$ Availabilities A is the same for all stations

Availability when number of vehicle $N \to \infty$

Why circulation so good?



Circulation policy ($\lambda_{\leq \Lambda}$)

Circulation "balances" demand

$$\forall a \quad \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b} = \sum_{(b,a)\in\mathcal{D}} \lambda_{b,a}$$

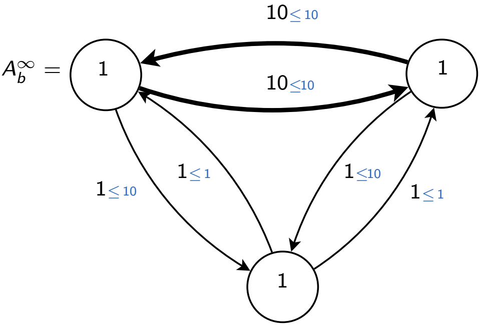
$\rightarrow$ Availabilities A is the same for all stations

Availabilities for N vehicles and M stations

$$\forall a \in \mathcal{M}, \quad A_a^N = A^N = \frac{N}{N+M-1}$$

Availability when number of vehicle $N \to \infty$

Why circulation so good?



Circulation policy ($\lambda \leq \Lambda$)

$$\lim_{N\to\infty} A_a^N = \frac{A_a^1}{\max_{b\in\mathcal{M}} A_b^1}$$

George and Xia (2011)

$$\lim_{N\to\infty} \text{Transit} = 24$$

Availabilities for N vehicles and M stations

$$\forall a \in \mathcal{M}, \quad A_a^N = A^N = \frac{N}{N+M-1}$$

Circulation policy approximation

Analytic transit evaluation

$Circ^* =$ value of Maximum Circulation
$P_{Circ^*} =$ value of the static circulation policy
$A^N = \frac{N}{N+M-1} =$ Availability at any station

Circulation policy approximation

Analytic transit evaluation

$Circ^*$ = value of MAXIMUM CIRCULATION
P_{Circ^*} = value of the static circulation policy
$A^N = \frac{N}{N+M-1}$ = Availability at any station

Analytic transit of circulation policy

$$P_{Circ^*} = \sum_{(a,b)\in\mathcal{D}} A_a^N \lambda_{a,b}^{Circ^*}$$

Circulation policy approximation

Analytic transit evaluation

$Circ^*$ = value of MAXIMUM CIRCULATION
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$$P_{Circ^*} = \sum_{(a,b)\in\mathcal{D}} A_a^N \lambda_{a,b}^{Circ^*} = A^N \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b}^{Circ^*}$$

Circulation policy approximation

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$$P_{Circ^*} = \sum_{(a,b)\in\mathcal{D}} A_a^N \lambda_{a,b}^{Circ^*} = A^N \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b}^{Circ^*} = \frac{N}{N+M-1} Circ^*$$

Circulation policy approximation

Analytic transit evaluation

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Claim $Circ^*$ is an UB on optimal dynamic policy P_{Dyn^*}

$$P_{Dyn^*} \leq Circ^*$$

Circulation policy approximation

Analytic transit evaluation

$Circ^*$ = value of MAXIMUM CIRCULATION
P_{Circ^*} = value of the static circulation policy
$A^N = \frac{N}{N+M-1}$ = Availability at any station

Analytic transit of circulation policy

$$P_{Circ^*} = \sum_{(a,b)\in\mathcal{D}} A_a^N \lambda_{a,b}^{Circ^*} = A^N \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b}^{Circ^*} = \frac{N}{N+M-1} Circ^*$$

Claim $Circ^*$ is an UB on optimal dynamic policy P_{Dyn^*}

$$P_{Dyn^*} \leq Circ^*$$

$$\Leftrightarrow \frac{N}{N+M-1} P_{Dyn^*} \leq \frac{N}{N+M-1} Circ^* = P_{Circ^*}$$

Circulation policy approximation

Analytic transit evaluation

$Circ^*$ = value of MAXIMUM CIRCULATION
P_{Circ^*} = value of the static circulation policy
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Analytic transit of circulation policy

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Claim $Circ^*$ is an UB on optimal dynamic policy P_{Dyn^*}

$$P_{Dyn^*} \leq Circ^*$$
$$\Leftrightarrow \frac{N}{N+M-1} P_{Dyn^*} \leq \frac{N}{N+M-1} Circ^* = P_{Circ^*}$$

P_{Circ^*} cannot be worse than $\frac{N}{N+M-1} P_{Dyn^*} \Rightarrow \frac{N}{N+M-1}$-approximation $\square$

Looking for "tractable" solution methods

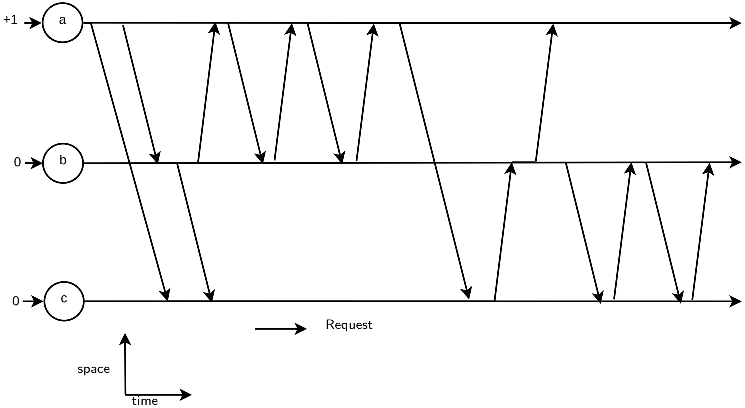
1. Simplified stochastic model
   - ▲ Good approximation algorithm
   - ▼ No transportation times, No time-varying demand, No station capacity
2. Scenario based approach
3. Fluid approximation

Looking for “tractable” solution methods

1. Simplified stochastic model
   - ▲ Good approximation algorithm
   - ▼ No transportation times, No time-varying demand, No station capacity
2. Scenario based approach
   - Deterministic problem
   - Optimize on a scenario → off line optimization problem
3. Fluid approximation

Scenario approach

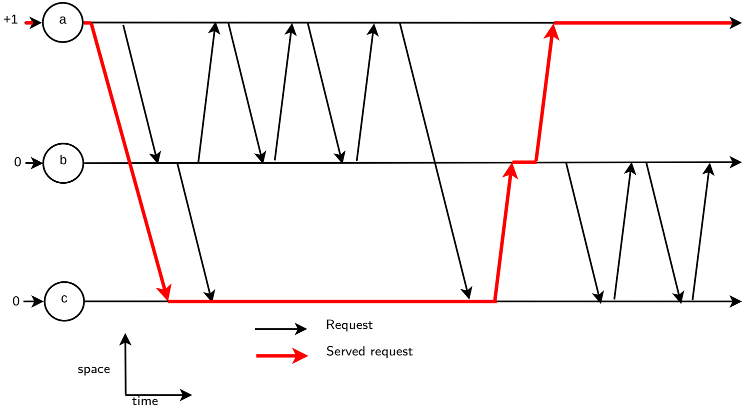
First Come First Served Flow (FCFS)



15 requests

Scenario approach

First Come First Served Flow (FCFS)

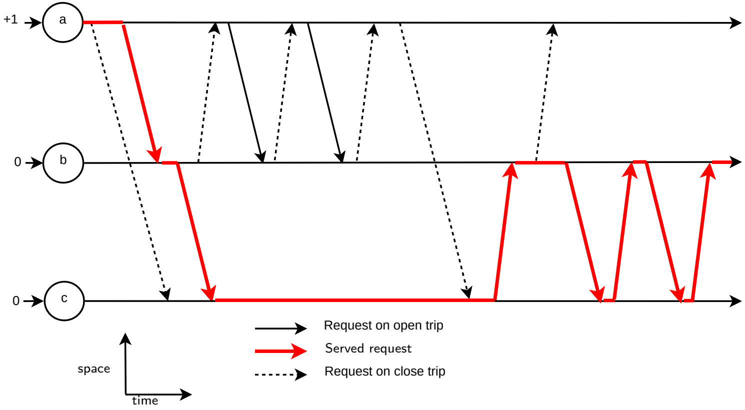


15 requests

⇒ 3 trips sold with Generous policy

Scenario approach

First Come First Served Flow (FCFS)



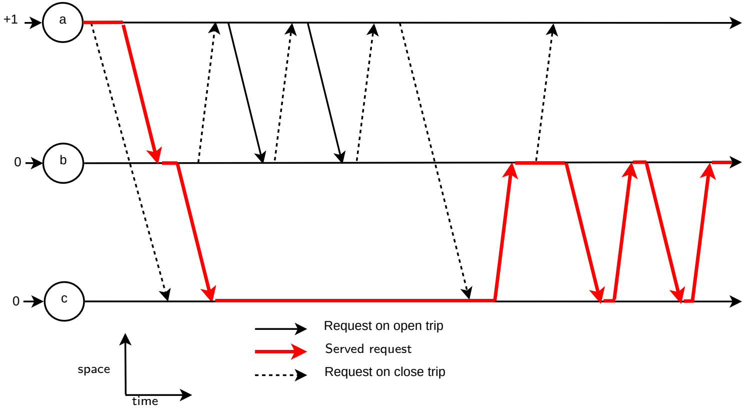
15 requests

⇒ 7 trips sold with FCFS "{Open,Close}" trip pricing policy

– Closing always trips (a, c) and (b, a)

Scenario approach

First Come First Served Flow (FCFS)

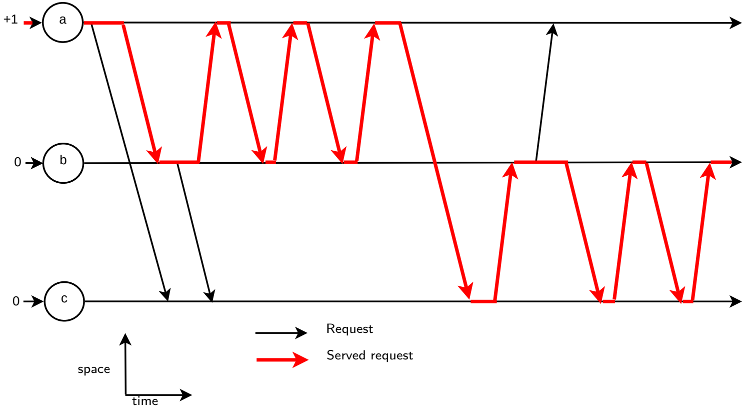


Complexity of computing the best static policy?

$\Rightarrow$ FCFS FLOW TRIP PRICING is APX-Hard

Scenario approach

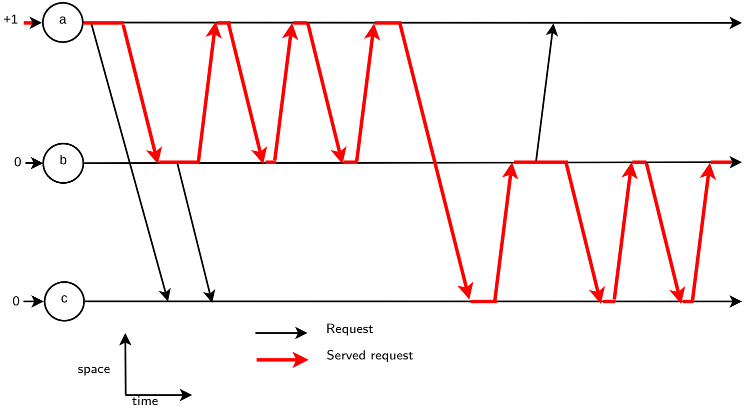
MAX FLOW UB



MAX FLOW serves 12 trips $>>$ 7 sold in optimal FCFS policy

Scenario approach

Max Flow UB



Max Flow serves 12 trips >> 7 sold in optimal FCFS policy

- UB theoretical guaranty in $[2^M - M - 1,\ (M+2)!]$

$\Rightarrow$ Still... Max Flow UB competitive in practice

Looking for "tractable" solution methods

1. Simplified stochastic model
   - ▲ Good approximation algorithm
   - ▼ No realistic assumptions
2. Scenario based approach
   - ▲ Upper bound considering all our constraints
   - ▼ No good heuristic policy
3. Fluid approximation

Looking for "tractable" solution methods

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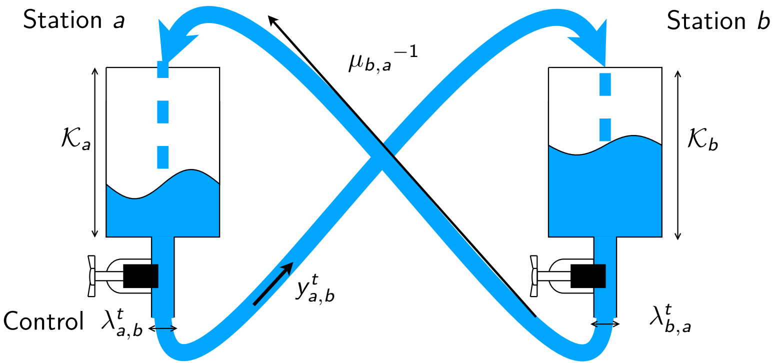
3. Fluid approximation
   - Another deterministic approach
   - → A plumbing problem

Fluid approximation

Known technique but not directly usable

- Discrete stochastic demand $\rightarrow$ deterministic continuous
- Stations $\rightarrow$ tanks linked by pipes
- Vehicles $\rightarrow$ fluid evolving deterministically
- Pricing control $\rightarrow$ pipe sizing (tap) $\lambda^t \in [0, \Lambda^t]$



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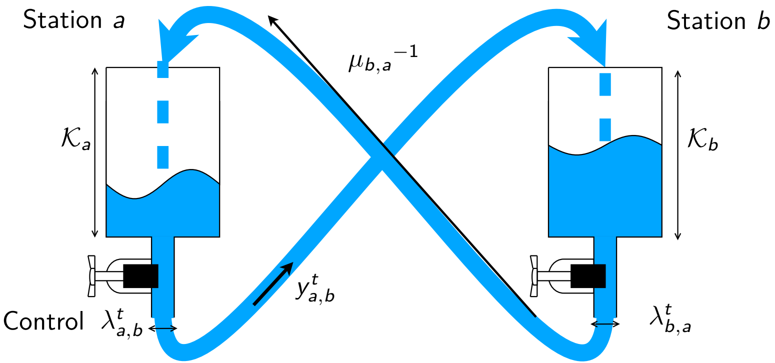
$\Rightarrow$ Static policy & Upper Bound(?)



Fluid model

Continuous Linear Program (CLP)

$$\max \int_0^T \sum_{(a,b)\in D} y^t_{a,b} dt \qquad (Flow)$$

s.t. (Continuous periodic conservation flow)

(Number of vehicles)

(Reservation & Station capacities)

$$0 \leq y^t_{a,b} \leq \lambda^t_{a,b} \leq \Lambda^t_{a,b} \quad \forall (a,b) \qquad \text{(Max demand)}$$

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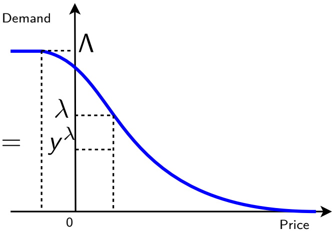
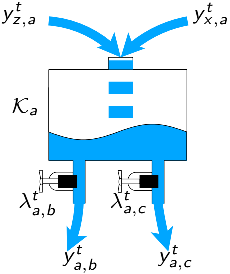
flow =





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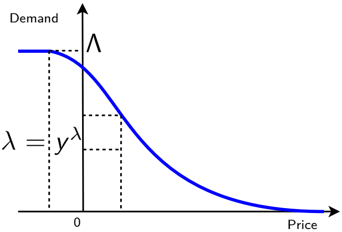

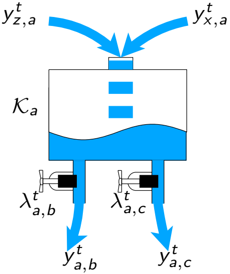

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$$0 \leq \lambda_{a,b}^t \leq \Lambda_{a,b}^t \qquad \forall (a,b) \qquad \text{(Max demand)}$$

Generalization of flow constraints

s_a^t: stock of vehicle at instant t in station a

$$s_a^t = s_a^0 + \int_0^t \sum_{(b,a)\in\mathcal{D}} \lambda_{b,a}^{\theta-\mu_{b,a}^{-1}} - \lambda_{a,b}^{\theta}\, d\theta \qquad \forall a \in \mathcal{M}, \ \forall t \in [0, T]$$

Fluid model

State Constrained Separated Continuous Linear Program (SCSCLP)

$$\max \int_0^T \sum_{(a,b)\in D} \lambda_{a,b}^t dt \qquad (Flow)$$

$$\begin{aligned} \text{s.t. } & \text{(Continuous periodic circulation flow)} \\ & \text{(Number of vehicles)} \\ & \text{(Reservation \& Station capacities)} \\ & \text{(Maximum demand)} \end{aligned}$$

- CLP $\in$ SCSC**LP** class, $\exists$ efficient algorithms (Luo and Bertsimas (1999))

→ Static heuristic policy

→ CLP value conjectured to be an UB on dynamic policies

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$\rightarrow$ Static heuristic policy

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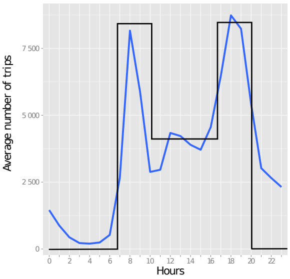
SCSCLP still complicated to compute...
Interest of considering time-dependent demand?

S-Fluid PSA

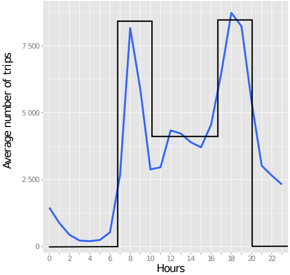
Pointwise Stationnary Approximation (Green and Kolesar, 1991)



S-Fluid PSA

Pointwise Stationnary Approximation (Green and Kolesar, 1991)

1 LP for each time-step t



$$\max \sum_{(a,b)\in\mathcal{D}} \lambda^t_{a,b} \qquad \text{(Flow)}$$

$$\text{s.t.} \sum_{(a,b)} \lambda^t_{a,b} = \sum_{(b,a)} \lambda^t_{b,a} \quad \forall a \qquad \text{(Circulation)}$$

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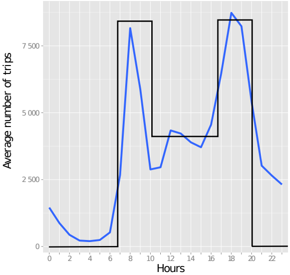

$$
\begin{aligned}
\max \quad & \sum_{(a,b)\in\mathcal{D}} \lambda^t_{a,b} && && \text{(Flow)}\\
\text{s.t.} \quad & \sum_{(a,b)} \lambda^t_{a,b} = \sum_{(b,a)} \lambda^t_{b,a} && \forall a && \text{(Circulation)}\\
& 0 \le \lambda^t_{a,b} \le \Lambda^t_{a,b} && \forall (a,b) && \text{(Max. demand)}\\
& \sum_{(a,b)} \frac{1}{\mu^t_{a,b}} \lambda^t_{a,b} \le N && && \text{(Nb. vehicles)}\\
& \sum_{b} \frac{1}{\mu^t_{a,b}} \lambda^t_{a,b} \le \mathcal{K}_a && \forall a && \text{(Reservation)}
\end{aligned}
$$

S-Fluid PSA

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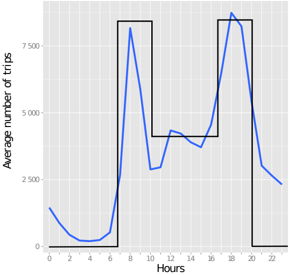

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- Concatenate the solution of each independent LP
  $\Rightarrow$ Static heuristic policy

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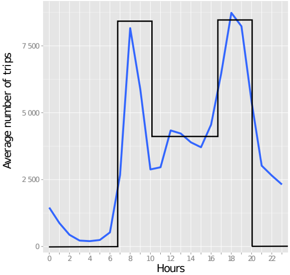

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Theorem LP value is an UB on dynamic policies on each time step

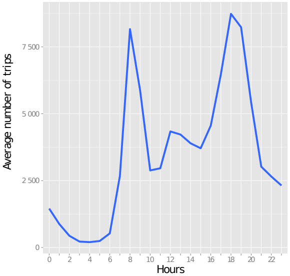
(Not the case when concatenated)

Looking for "tractable" solution methods

1. Simplified stochastic model
   - ▲ Good approximation algorithm
   - ▼ No realistic assumptions
2. Scenario based approach
   - ▲ Upper bound
   - ▼ No heuristic policy
3. Fluid approximation
   - ▲ Heuristic policy considering time-dependent demand
   - ▼ No proved upper bound

   → Interest of a time-dependent model?

Evaluation on simple instances

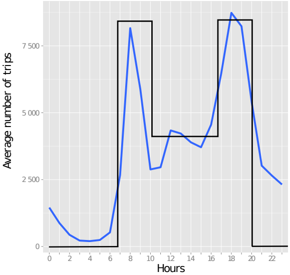


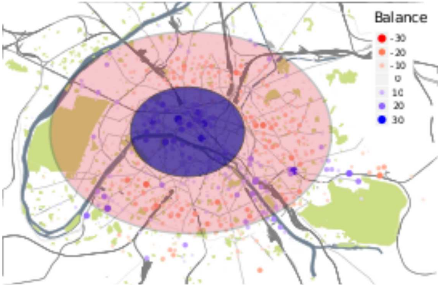
A day

Spatial distribution of morning tides

Source Côme (2012) on Vélib', Paris

Evaluation on simple instances





Trips → Demand

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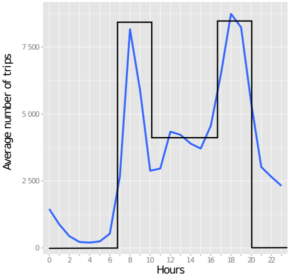

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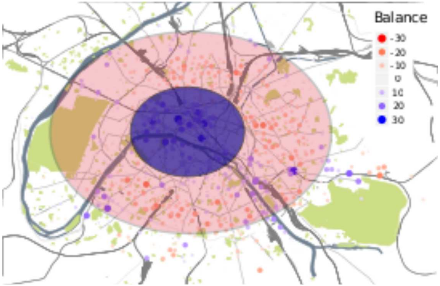


Spatial distribution of morning tides

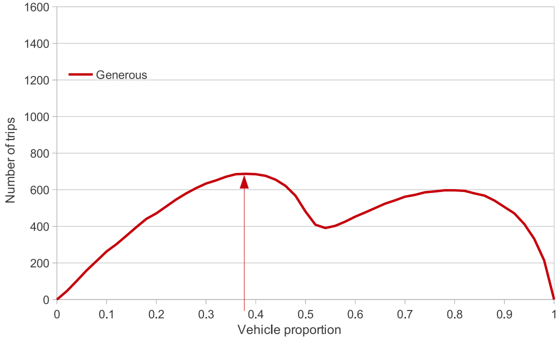
Reproducible benchmark

- Start with uniform demand
- \+ Tides
- \+ Gravitation

- Stations on a grid
- Manhattan distances
- Stations of size $\mathcal{K} = 10$

Simulation results

24 stations – Tide – Demand $\Lambda = 18$ users/hour/station



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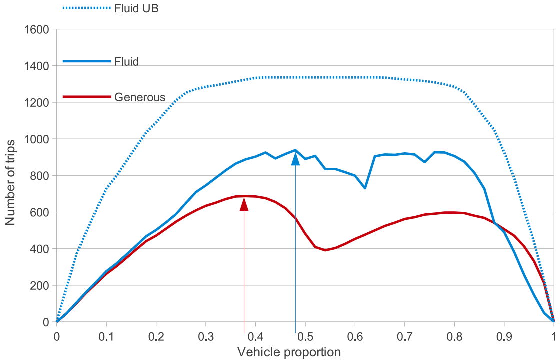

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	Heuristic	Upper Bound
• Fluid Approximation	$\checkmark$	$\checkmark^?$

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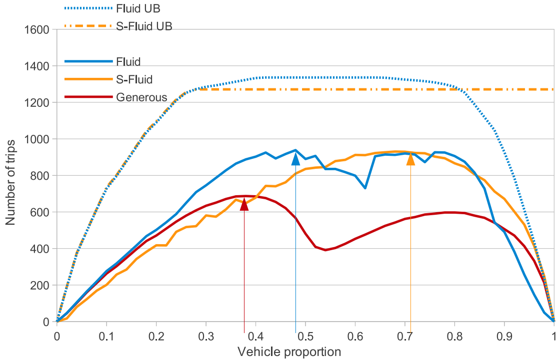

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• Fluid Approximation	✓	$\checkmark^?$
• Stable Fluid PSA	✓	$\checkmark^{\lambda^t=\lambda}$

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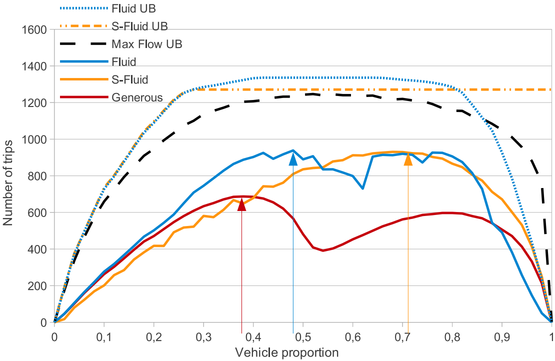

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• **Max-Flow** on a scenario		✓

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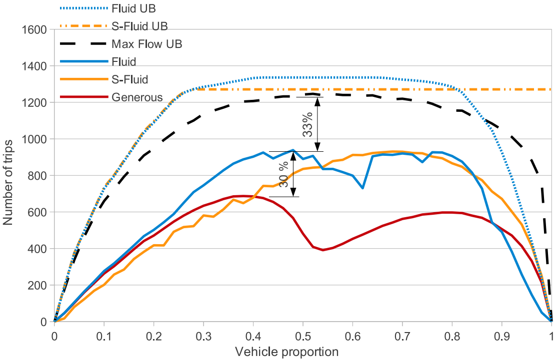

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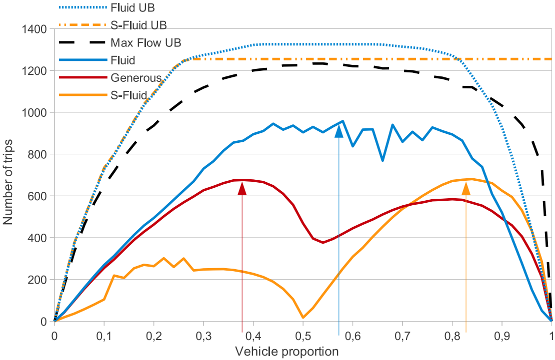
Simulation results

Another tide type – S-Fluid PSA blindness



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• **Max-Flow** on a scenario		$\checkmark$

Conclusion

1. A pioneer study on a real-practical problem
    - Development of a methodology
    - Dissection into sub-problems

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$\Rightarrow$ YES pricing can improve Vehicle Sharing Systems performance

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$\Rightarrow$ YES pricing can improve Vehicle Sharing Systems performance

- Under assumptions...

Perspectives

- Optimization
  - Extend MAX CIRCULATION approximation to consider transportation times
  - Develop heuristics for scenario approach
  - Incorporate availabilities in the fluid approximation
  - Optimization by simulation (*e.g.* dynamic threshold policies)

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- More realistic models (utility models / economics)
  - Spatio-temporal flexibilities
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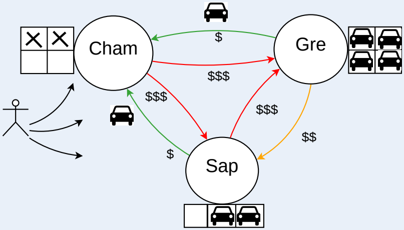
  - Optimization by simulation (*e.g.* dynamic threshold policies)
- More realistic models (utility models / economics)
  - Spatio-temporal flexibilities
  - Demand elasticity
- Improving the benchmark (statistics / data mining)
  - Estimate uncensored demand ($\lambda \neq y$ trips sold)

VSS pricing optimization

An "intractable" stochastic model



⟹ Preliminary answer

Optimization on approximation ⇓ ⇑ Evaluation by simulation

"Tractable" models

	Heuristic	Upper bound	
• Simplified stoch. models	✓	✓	W. and Jost (2013a)
• Scenario-based approach	APX-hard	✓	W., Jost, and Brauner (2013b)
• Fluid approximation	✓	✓	W. and Jost (2013b)

• Decomposable MDP	Exact Solution	W., Gayon, and Jost (2013a)

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Circulation policy approximation

Theorem – For M stations and N vehicles

MAXIMUM CIRCULATION policy is a $\frac{N}{N+M-1}$-approximation on optimal dynamic policy.

Circulation policy approximation

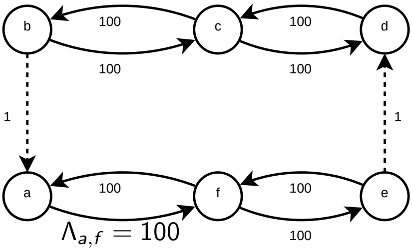
Theorem – For M stations and N vehicles

Maximum Circulation policy is a $\frac{N}{N+M-1}$-approximation on optimal dynamic policy.

Sketch of proof

- We assume Maximum Circulation policy is strongly connected

$\rightarrow$ Otherwise need to spread vehicles in the clustered city



Demand graph

Circulation policy approximation

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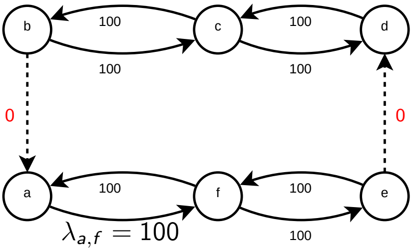


Maximum Circulation policy

Circulation policy approximation

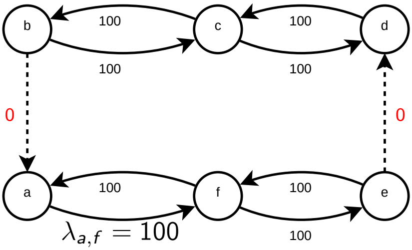
Theorem – For M stations and N vehicles

MAXIMUM CIRCULATION policy (together with its optimal vehicle distribution) is a $\frac{N}{N+M-1}$-approximation on optimal dynamic policy.

Sketch of proof

- We assume MAXIMUM CIRCULATION policy is strongly connected

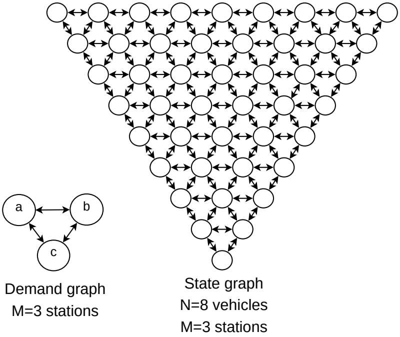
$\rightarrow$ Otherwise need to spread vehicles in the clustered city



Maximum Circulation policy

Circulation policy approximation (1/3)

Circulation policy $\leftrightarrow$ uniform stationary distribution



π_s: probability to be in state $s \in \mathcal{S}$

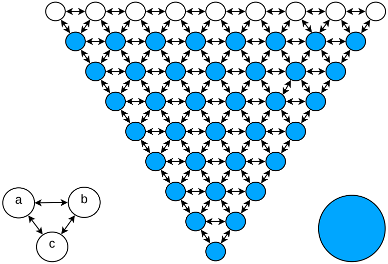
Circulation policies have a uniform stationary distribution

$$\rightarrow \ \forall s \in \mathcal{S}, \ \pi_s = \frac{1}{|\mathcal{S}|}$$

Circulation policy approximation (2/3)

Availability $\Leftrightarrow$ number of states

A_a^N: probability to find a vehicle available in station a



Demand graph
M=3 stations

State graph
N=8 vehicles
M=3 stations

State with at least
1 vehicle in c
$(n_a, n_b, n_c \geq 1)$

Circulation policy approximation (2/3)

Availability ⇔ number of states

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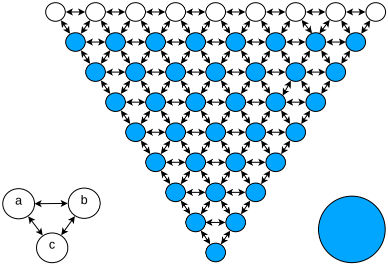

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M=3 stations

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N=8 vehicles
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State with at least
1 vehicle in c
$(n_a, n_b, n_c \geq 1)$

For M stations & N vehicles

$$|\mathcal{S}| = |\mathcal{S}(N, M)| = \binom{N + M - 1}{N}$$

Here

$$|\mathcal{S}(7,3)| = 36$$

$$|\mathcal{S}(8,3)| = 45$$

$$\rightarrow \quad A_c^8 = \frac{36}{45} = \frac{8}{10}$$

Circulation policy approximation (2/3)

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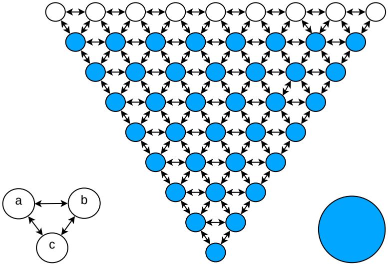

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Availability for N vehicles and M stations

$$A^N = \frac{|\mathcal{S}(N-1, M)|}{|\mathcal{S}(N, M)|} = \frac{N}{N+M-1}$$

Circulation policy approximation (3/3)

Analytic transit evaluation

$Circ^*$ = value of MAXIMUM CIRCULATION
P_{Circ^*} = value of the static circulation policy
$A_a^N = A^N = \frac{N}{N+M-1}$ = Availability at station a

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Analytic transit of circulation policy

$$P_{Circ^*} = \sum_{(a,b)\in\mathcal{D}} A_a^N \lambda_{a,b}^{Circ^*}$$

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Claim $Circ^*$ is an UB on optimal dynamic policy P_{Dyn^*}

$$P_{Dyn^*} \leq Circ^*$$

Circulation policy approximation (3/3)

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$$P_{Dyn^*} \leq Circ^*$$

$$\Leftrightarrow \frac{N}{N+M-1} P_{Dyn^*} \leq \frac{N}{N+M-1} Circ^* = P_{Circ^*}$$

Circulation policy approximation (3/3)

Analytic transit evaluation

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$$P_{Dyn^*} \leq Circ^*$$

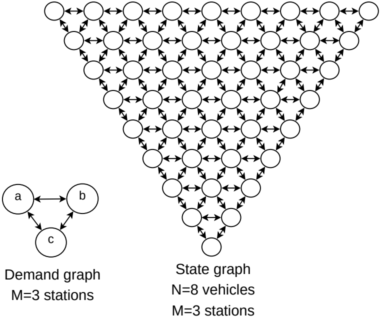
$$\Leftrightarrow \frac{N}{N+M-1} P_{Dyn^*} \leq \frac{N}{N+M-1} Circ^* = P_{Circ^*}$$

P_{Circ^*} cannot be worse than $\frac{N}{N+M-1} P_{Dyn^*} \Rightarrow \frac{N}{N+M-1}$-approximation □

Dynamic policies optimization

Decomposable CTMDP – (W., Gayon, and Jost (2013a))

Continuous-Time Markov Decision Process (CTMDP) $\rightarrow$ Dynamic policy



- M stations, 2 prices per trip
  $\rightarrow$ $\lambda^s_{a,b} \in \{0, \Lambda_{a,b}\}$
- "Classic" CTMDP
  $\rightarrow$ 2^{M^2} decisions per state

State graph
$M = 3$ stations, $N = 8$ vehicles

Dynamic policies optimization

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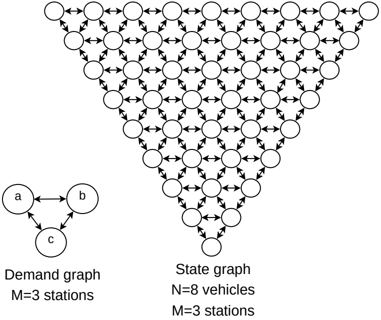

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  $\rightarrow$ Reduced to $2 \times M^2$ decisions

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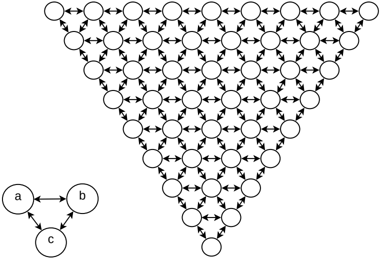

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State graph
N=8 vehicles
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- Action Decomposable CTMDP
  → Reduced to $2 \times M^2$ decisions

State graph
$M = 3$ stations, $N = 8$ vehicles

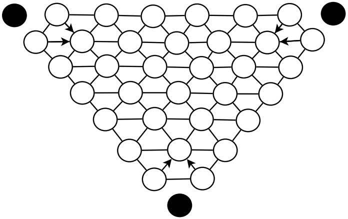
Still exponential number of states
... Work only for toy systems

→ Need compact representation of dynamic policies

Dynamic policies optimization

Optimal dynamic policies characterization?

In homogeneous cities $\rightarrow \Lambda^t_{a,b} = 1, \; \forall (a,b) \in D$



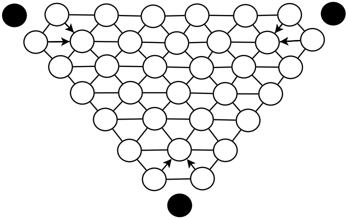
State graph for 8 vehicles

- Refusing 8 vehicles in a station
- Refusing trip if passing from states (6,1,1) $\rightarrow$ (7,1,0)

Dynamic policies optimization

Optimal dynamic policies characterization?

In homogeneous cities $\rightarrow \Lambda^t_{a,b} = 1, \ \forall (a,b) \in D$



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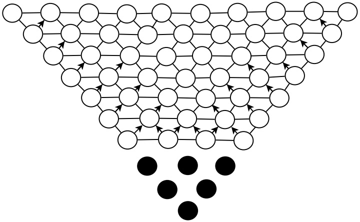

“Spike” for 30 vehicles

- Refusing 28, 29 or 30 vehicles in a station
- Refusing trip if . . .

Dynamic policies optimization

Optimal dynamic policies characterization?

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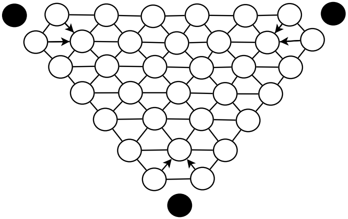

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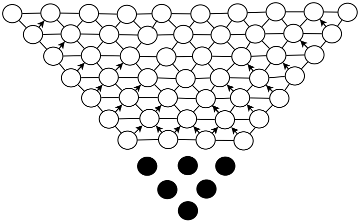


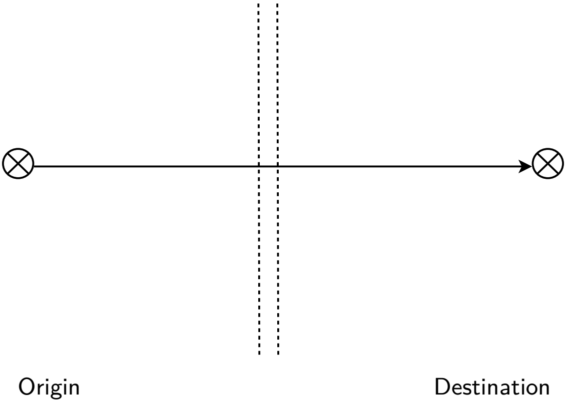
"Spike" for 30 vehicles

"Simple" threshold policies sub-optimal...
Representation of optimal policies?

Dynamic policies optimization problem $\in$ NP?

Study assumptions

A station to station demand



Study assumptions

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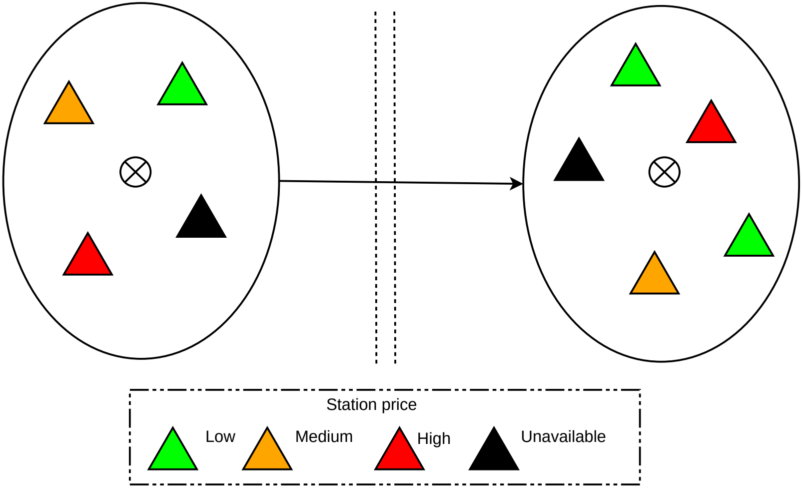

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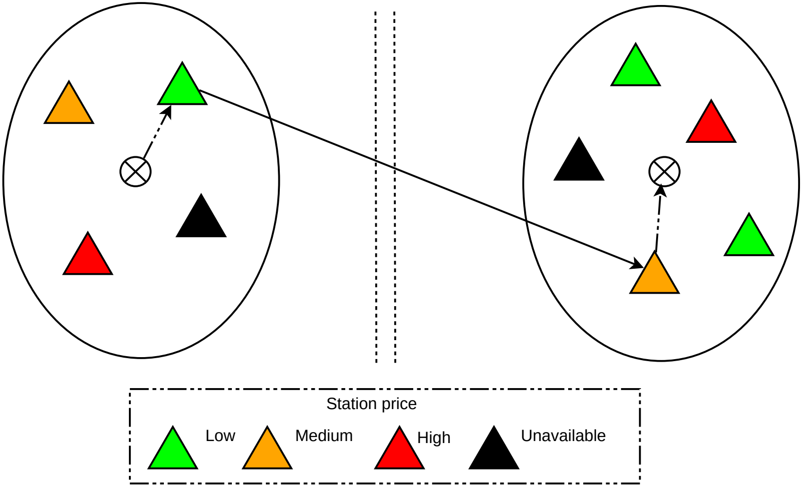

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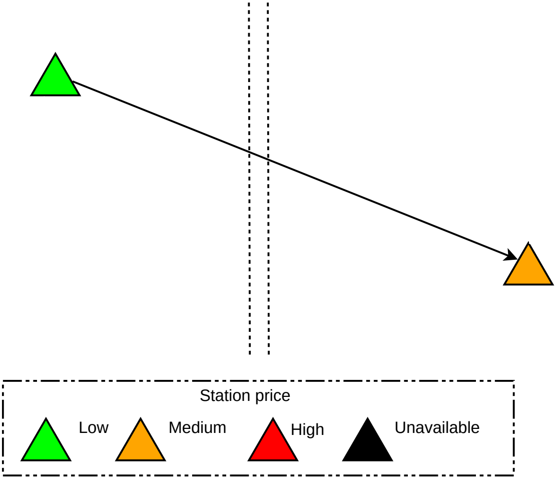
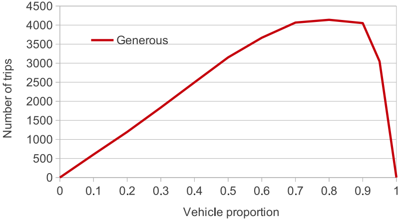


A real case analysis

Capital bikeshare, Washington DC

Simulation results



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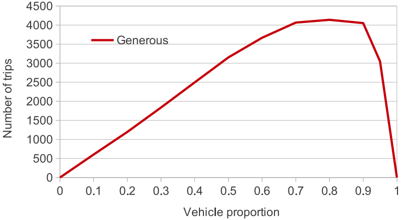
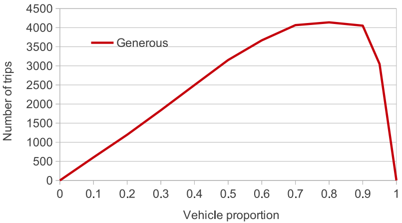


- 30 000 trips sold per week in real-life... 4000 in the simulation

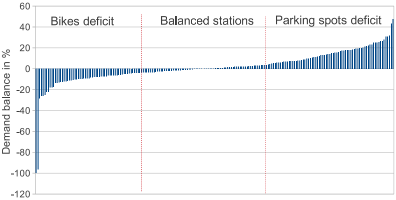
A real case analysis

Capital bikeshare, Washington DC

Simulation results



Stations average balance



- 30 000 trips sold per week in real-life... 4000 in the simulation
- Use of truck

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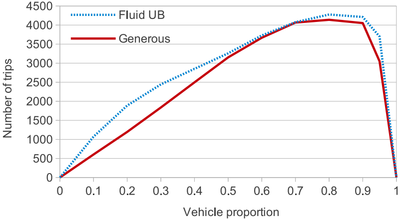
A ≈null optimization gap



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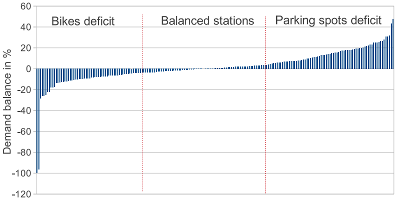

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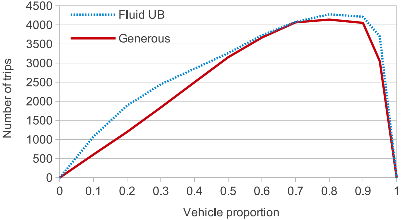

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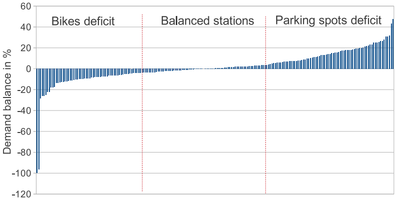


- 30 000 trips sold per week in real-life... 4000 in the simulation
- Use of truck
- Fluid UB information: no optimization gap for these data
  - $\rightarrow$ Corrupted data, only the trips sold
  - Need to isolate problems
  - $\Rightarrow$ Work on toy instances to provide information

Fluid approximation $=?$ ∞-scaled problem

Modèle fluide – espace d'état continu

$$\mathcal{S}^F = \Big\{ \Big(n_a \in \mathbb{R}: \ a \in \mathcal{M}, \ n_{a,b} \in \mathbb{R}: \ (a,b) \in \mathcal{D}, \ t \in [0,T] \Big) \ / \sum_{i \in \mathcal{M} \cup \mathcal{D}} n_i = N \ \& \ n_a + \sum_{b \in \mathcal{M}} n_{b,a} \leq \mathcal{K}_a, \ \forall a \in \mathcal{M}, \ \forall t \in [0,T] \Big\}$$

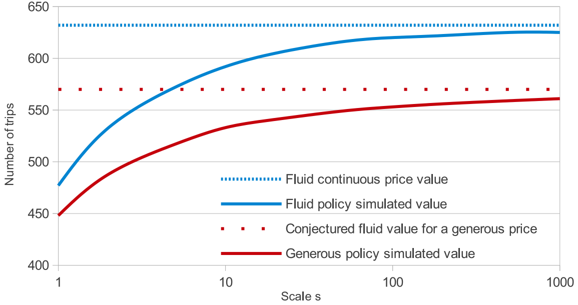
s-scaled problème à prix continus – espace d'état discret ($R = \{1, \dots, s\}$)

$$\mathcal{S}(s) = \Big\{ \Big(s.n_a \in \mathbb{N}: \ a \in \mathcal{M}, \ n^r_{a,b} \in \mathbb{N}: \ ((a,b),r) \in \mathcal{D} \times R, \ s.t \in \mathcal{T} \Big) \ / \sum_{i \in \mathcal{M} \cup \mathcal{D} \times R} n_i = N \ \& \ n_a + \sum_{r \in R} \sum_{b \in \mathcal{M}} n^r_{b,a} \leq \mathcal{K}_a, \ \forall a \in \mathcal{M}, \ \forall s.t \in \mathcal{T} \Big\}$$

- Espace d'état rescalé, unité entier $\rightarrow$ unité fraction $1/s$
- Chaque pas de temps divisé en s parties $\rightarrow$ durée $(s\mathcal{T})^{-1}$
- Temps de transport $\rightarrow$ s serveurs en séries avec taux $s\mu^t_{a,b}$
- Transitions accélérées par un facteur s $\rightarrow$ $\Lambda^t_{a,b}(s) = s\Lambda^t_{a,b}$
- Contrôle continu sur les prix

  $\rightarrow$ demande $\lambda^t_{a,b}(s) \in [0, \Lambda^t_{a,b}(s)]$ obtenue au prix $\frac{1}{s} p^t_{a,b}(\frac{1}{s}\lambda^t_{a,b}(s))$.

Fluid approximation $=$? ∞-scaled problem



Conjecture

SCSCLP policies

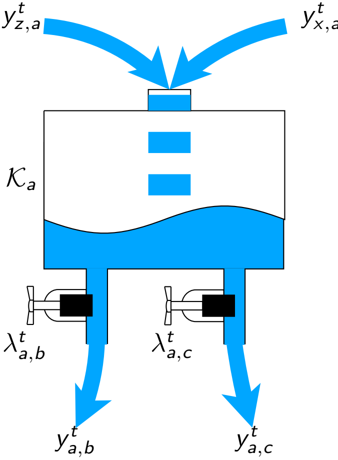
$=$ *asymptotic limit of s-scaled problem*

- *Upper Bound on dynamic policies*

A plumbing problem with *equity* (FCFS rule)

Flow evaluation y for fixed demand λ

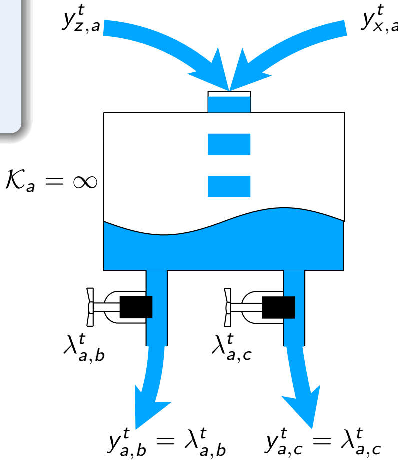
- Departure equity $\rightleftharpoons$ Arrival equity?



A plumbing problem with *equity* (FCFS rule)

Flow evaluation y for fixed demand λ

- Departure equity $\rightleftharpoons$ Arrival equity?
- Infinite size – Only departure equity



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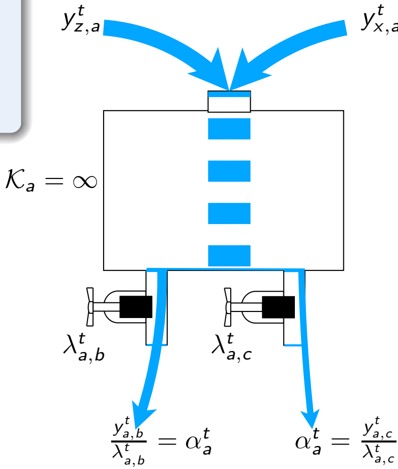


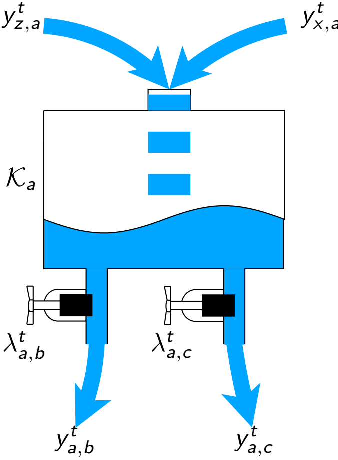
A plumbing problem with *equity* (FCFS rule)

Flow evaluation y for fixed demand λ

- Departure equity $\rightleftharpoons$ Arrival equity?
- Infinite size – Only departure equity
- Finite size – Non linear dynamic!

$\rightarrow$ Steady state evaluation "hard"

... Optimization "hard" with discrete prices ...



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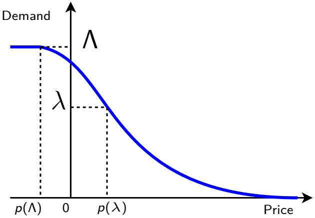
$\rightarrow$ Steady state evaluation "hard"

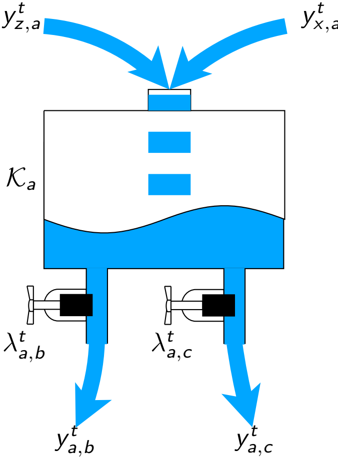
... Optimization "hard" with discrete prices ...

$\Rightarrow$ Use of continuous prices

Always fill the pipes: $y^t_{a,b} = \lambda^t_{a,b}$



Elastic demand $\lambda^t_{a,b} \in [0, \Lambda^t_{a,b}]$



Fluid approximation – Continuous control

Continuous **Non Linear** Program

$$\max \sum_{(a,b)\in D} \int_0^T \lambda_{a,b}(\theta) p(\lambda_{a,b}(\theta)) d\theta \qquad \text{(Gain)}$$

$$\text{s.t.} \sum_{a\in\mathcal{M}} s_a(0) = N \qquad \text{(Nb. vehicles)}$$

$$s_a(0) = s_a(T) \qquad \forall a \qquad \text{(Flow stabilization)}$$

$$s_a(t) = s_a(0) + \int_0^t \sum_{(b,a)\in\mathcal{D}} \lambda_{b,a}(\theta - \mu_{b,a}^{-1}) - \lambda_{a,b}(\theta)\ d\theta \qquad \forall a, t \qquad \text{(Flow conservation)}$$

$$0 \le \lambda_{a,b}(t) \le \Lambda_{a,b}^t \qquad \forall a, b, t \qquad \text{(Max demand)}$$

$$r_a(t) = \sum_{b\in\mathcal{M}} \int_0^{\mu_{b,a}^{-1}} \lambda_{b,a}(t-\theta)\ d\theta \qquad \forall a, t \qquad \text{(Reservation)}$$

$$0 \le s_a(t) + r_a(t) \le \mathcal{K}_a \qquad \forall a, t \qquad \text{(Station capacity)}$$

$$\lambda_{a,b}^t = y_{a,b}^t$$

Fluid approximation – Continuous control

State-Constrained Separated Continuous **Linear** Program (SCSCLP)

$$\max \sum_{(a,b)\in D} \int_0^T \lambda_{a,b}(\theta)\cancel{p(\lambda_{a,b}(\theta))}d\theta \qquad \text{(Flow)}$$

$$\text{s.t.} \sum_{a\in\mathcal{M}} s_a(0) = N \qquad \text{(Nb. vehicles)}$$

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Fluid approximation – Continuous control

State-Constrained Separated Continuous **Linear** Program (SCSCLP)

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$$0 \leq s_a(t) + r_a(t) \leq \mathcal{K}_a \qquad \forall a, t \qquad \text{(Station capacity)}$$

- $\in$ SCSC**LP** class, $\exists$ efficient algorithms (Luo and Bertsimas (1999))

$\rightarrow$ Static heuristic policy

Fluid approximation – Continuous control

State-Constrained Separated Continuous **Linear** Program (SCSCLP)

$$\max \sum_{(a,b)\in D} \int_0^T \lambda_{a,b}(\theta) d\theta \qquad \text{(Flow)}$$

$$\text{s.t.} \sum_{a\in\mathcal{M}} s_a(0) = N \qquad \text{(Nb. vehicles)}$$

$$s_a(0) = s_a(T) \qquad \forall a \qquad \text{(Flow stabilization)}$$

$$s_a(t) = s_a(0) + \int_0^t \sum_{(b,a)\in\mathcal{D}} \lambda_{b,a}(\theta - \mu_{b,a}^{-1}) - \lambda_{a,b}(\theta)\ d\theta \qquad \forall a, t \qquad \text{(Flow conservation)}$$

$$0 \leq \lambda_{a,b}(t) \leq \Lambda_{a,b}^t \qquad \forall a, b, t \qquad \text{(Max demand)}$$

$$r_a(t) = \sum_{b\in\mathcal{M}} \int_0^{\mu_{b,a}^{-1}} \lambda_{b,a}(t-\theta)\ d\theta \qquad \forall a, t \qquad \text{(Reservation)}$$

$$0 \leq s_a(t) + r_a(t) \leq \mathcal{K}_a \qquad \forall a, t \qquad \text{(Station capacity)}$$

¿ Upper bound on dynamic policies ?

Fluid approximation – Continuous control

State-Constrained Separated Continuous **Linear** Program (SCSCLP)

$$\max \sum_{(a,b)\in D} \int_0^T \lambda_{a,b}(\theta)d\theta \qquad \text{(Flow)}$$

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$$0 \le s_a(t) + r_a(t) \le \mathcal{K}_a \qquad \forall a,t \qquad \text{(Station capacity)}$$

¿ Upper bound on dynamic policies ?

¿ Interest of considering time dependant demand ?

Fluid approximation – Stationary demand

Stable Fluid Linear Program

$$\max \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b} \qquad \text{(Flow)}$$

$$\text{s.t.} \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b} = \sum_{(b,a)\in\mathcal{D}} \lambda_{b,a} \qquad \forall a \in \mathcal{M} \qquad \text{(Flow conservation)}$$

$$0 \leq \lambda_{a,b} \leq \Lambda_{a,b} \qquad \forall (a,b) \in \mathcal{D} \qquad \text{(Max. demand)}$$

$$\sum_{(a,b)\in\mathcal{D}} \frac{1}{\mu_{a,b}} \lambda_{a,b} + \sum_{a\in M} s_a = N \qquad \text{(Nb. vehicles)}$$

$$\sum_{b\in\mathcal{M}} \frac{1}{\mu_{a,b}} \lambda_{a,b} + s_a \leq \mathcal{K}_a \qquad \forall a \in \mathcal{M} \qquad \text{(Reservation)}$$

- $\lambda_{a,b} = y_{a,b}$

Fluid approximation – Stationary demand

Stable Fluid Linear Program

$$\max \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b} \qquad \text{(Flow)}$$

$$\text{s.t.} \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b} = \sum_{(b,a)\in\mathcal{D}} \lambda_{b,a} \qquad \forall a \in \mathcal{M} \qquad \text{(Flow conservation)}$$

$$0 \leq \lambda_{a,b} \leq \Lambda_{a,b} \qquad \forall (a,b) \in \mathcal{D} \qquad \text{(Max. demand)}$$

$$\sum_{(a,b)\in\mathcal{D}} \frac{1}{\mu_{a,b}} \lambda_{a,b} + \cancel{\sum_{a\in M} s_a} = N \qquad \text{(Nb. vehicles)}$$

$$\sum_{b\in\mathcal{M}} \frac{1}{\mu_{a,b}} \lambda_{a,b} + \cancel{s_a} \leq \mathcal{K}_a \qquad \forall a \in \mathcal{M} \qquad \text{(Reservation)}$$

- $\lambda_{a,b} = y_{a,b}$
- If $N \leq \sum_{a\in M} \mathcal{K}_a$

Fluid approximation – Stationary demand

Stable Fluid Linear Program

$$
\begin{aligned}
\max \quad & \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b} & & \text{(Flow)}\\
\text{s.t.} \quad & \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b} = \sum_{(b,a)\in\mathcal{D}} \lambda_{b,a} & \forall a \in \mathcal{M} & \text{(Flow conservation)}\\
& 0 \leq \lambda_{a,b} \leq \Lambda_{a,b} & \forall (a,b) \in \mathcal{D} & \text{(Max. demand)}\\
& \sum_{(a,b)\in\mathcal{D}} \frac{1}{\mu_{a,b}} \lambda_{a,b} \leq N & & \text{(Nb. vehicles)}\\
& \sum_{b\in\mathcal{M}} \frac{1}{\mu_{a,b}} \lambda_{a,b} \leq \mathcal{K}_a & \forall a \in \mathcal{M} & \text{(Reservation)}
\end{aligned}
$$

Theorem (W. and Jost (2013b))

Stable fluid LP value is an upper bound on dynamic policies.

Sketch of proof

- Any dynamic policy is giving a solution of stable fluid with same value

Fluid approximation – Stationary demand

Stable Fluid Linear Program

$$
\begin{aligned}
\max \quad & \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b} && && \text{(Flow)}\\
\text{s.t.} \quad & \sum_{(a,b)\in\mathcal{D}} \lambda_{a,b} = \sum_{(b,a)\in\mathcal{D}} \lambda_{b,a} && \forall a \in \mathcal{M} && \text{(Flow conservation)}\\
& 0 \le \lambda_{a,b} \le \Lambda_{a,b} && \forall (a,b) \in \mathcal{D} && \text{(Max. demand)}\\
& \sum_{(a,b)\in\mathcal{D}} \frac{1}{\mu_{a,b}} \lambda_{a,b} \le N && && \text{(Nb. vehicles)}\\
& \sum_{b\in\mathcal{M}} \frac{1}{\mu_{a,b}} \lambda_{a,b} \le \mathcal{K}_a && \forall a \in \mathcal{M} && \text{(Reservation)}
\end{aligned}
$$

Adaptation to time dependent demands

$\Rightarrow$ Pointwise Stationnary Approximation (PSA) (Green and Kolesar, 1991)