



# Optimised space vector modulation for variable speed drives

Hamid Khan

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**T h è s e**

Présentée par

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pour obtenir le grade de

**DOCTEUR D'UNIVERSITÉ**

**SPECIALITE : ÉLECTRONIQUE DE PUISSANCE**

**Optimised Space Vector Modulation for Variable Speed Drives**

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# Résumé

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Le travail effectué au cours de cette thèse consiste à étudier et développer des techniques innovantes de modulation de largeurs d'impulsions (MLI) qui visent à optimiser les chaînes de traction électriques embarquées dans des véhicules hybrides ou électriques. La MLI joue un rôle stratégique au cœur des variateurs de vitesse, elle influe sur le comportement général de la chaîne de traction et sur sa performance. La MLI présente des degrés de liberté qui peuvent contribuer avantageusement à redimensionner les composants du variateur tels que le circuit de refroidissement, le filtre EMI et le condensateur du bus continu.

Les véhicules hybrides constituent une étape naturelle dans la transition énergétique entre les véhicules thermiques et les véhicules électriques.

Notre étude contribue à l'optimisation des variateurs de vitesse en général et ceux au cœur des véhicules hybrides ou électriques en particulier. Notre apport consiste à proposer une MLI performante afin de rendre le variateur plus léger et plus compacte tout en garantissant les fonctionnalités traditionnelles. La compétitivité de ces variateurs et par conséquent des véhicules hybrides ou électriques devient alors accessible.

Les véhicules hybrides ou électriques utilisent généralement une machine de traction à courant alternatif en raison de nombreux avantages que celle-ci présente par rapport à une machine à courant continu. La source d'alimentation au bord d'un véhicule est une batterie, il est donc nécessaire d'utiliser un onduleur pour transformer la tension continue en tension alternative à amplitude et fréquence variables. Le contrôle de cet onduleur est réalisé par des techniques de modulation de largeurs d'impulsions (MLI) ce qui permet ainsi de réguler le couple de la machine. Les techniques MLI produisent une composante basse fréquence, le fondamental qui est le signal désiré et des composantes hautes fréquences appelées harmoniques de commutation qui sont indésirables.

Dans les véhicules modernes, il y a de plus en plus de charges mécaniques pilotées par des machines électriques et des systèmes électroniques. Il est impératif d'éliminer le risque d'interférences électromagnétiques entre ces différents systèmes pour éviter le dysfonctionnement ou la défaillance. Il faut donc filtrer ces harmoniques indésirables pour qu'elles ne perturbent pas les calculateurs et autres circuits électroniques de faibles niveaux de tensions. Il existe des techniques de modulation aléatoire (RPWM) qui permettent d'étaler les harmoniques à la fréquence de commutation et ses multiples. Dans cette étude, notre choix s'est porté sur la technique de modulation vectorielle aléatoire (RSVM) qui présente plusieurs avantages par rapport à la MLI intersective.

Les machines pilotées par une MLI produisent des tensions de mode commun dites « shaft voltage », qui peuvent provoquer des courants à travers les roulements de la machine, ces derniers pouvant être destructifs. Nous avons pu développer une technique MLI vectorielle basée sur un choix judicieux des vecteurs nuls pour réduire cette tension de mode commun.

La chaleur produite par les pertes dans les convertisseurs à commutation dure lors de l'ouverture et de la fermeture des interrupteurs doit être évacuée rapidement, ce qui réduit le stress thermique, évite la défaillance et augmente la durée de vie des interrupteurs. Une technique utilisée pour réduire ces pertes par commutation est la modulation discontinue (DPWM); une amélioration est apportée à cette technique dans ce travail. Cette amélioration est présentée sous forme d'une technique discontinue évolutive (EDSVM) qui s'adapte au régime du moteur pour minimiser les pertes. Grâce à cette technique une meilleure distribution du stress thermique sur les différents bras de l'onduleur est rendue possible et permet ainsi d'augmenter la durée de vie de l'onduleur.

Une autre variante de modulation est abordée dans ce travail; cette technique utilise des vecteurs non adjacents et un placement dynamique des pulses permettant ainsi de réduire le stress électrique sur le condensateur du bus continu et de réduire le nombre de capteurs de courants requis pour la régulation du couple de la machine.

Les effets indésirables de la MLI cités ci-dessus ont été abordés séparément et des techniques de modulation dédiées ont été développées telles que : la modulation aléatoire, la modulation discontinue et la modulation discontinue évolutive. Ces techniques permettent de réduire le filtrage passif souvent encombrant et d'utiliser des condensateurs du bus continu moins volumineux. Elles permettent également de réduire les interférences électromagnétiques (EMI) et l'effort de refroidissement.

Un banc de test complet associant l'électronique de puissance à un système de contrôle performant à base de DSP a été réalisé. Toutes les validations expérimentales sont précédées par une étude théorique et validées par simulation.

**Mots clés:** Electric drives, Interférence Electromagnétique, Véhicules Hybride-Electrique, Pertes par Commutation, Modulation par largeur d'impulsion, MLI Discontinue, MLI Aléatoire, MLI Vectorielle.

# Abstract

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The dissertation documents research work carried out on Pulse Width Modulation (PWM) strategies for hard switched Voltage Source Inverters (VSI) for variable speed electric drives. This research is aimed at Hybrid Electric Vehicles (HEV). PWM is at the heart of all variable speed electric drives; they have a huge influence on the overall performance of the system and may also help eventually give us an extra degree of freedom in the possibility to rethink the inverter design including the re-dimensioning of the inverter components.

HEVs tend to cost more than conventional internal combustion engine (ICE) vehicles as they have to incorporate two traction systems, which is the major discouraging factor for consumers and in turn for manufacturers. The two traction system increases the maintenance cost of the car as well. In addition the electric drives not only cost extra money but space too, which is already scarce with an ICE under the hood. An all-electric car is not yet a viable idea as the batteries have very low energy density compared with petrol or diesel and take considerable time to charge. One solution could be to use bigger battery packs but these add substantially to the price and weight of the vehicle and are not economically viable. To avoid raising the cost of such vehicles to unreasonably high amounts, autonomy has to be compromised. However hybrid vehicles are an important step forward in the transition toward all-electric cars while research on better batteries evolves. The objective of this research is to make electric drives suitable for HEVs i.e. lighter, more compact and more efficient -- requiring less maintenance and eventually at lower cost so that the advantages, such as low emissions and better fuel efficiency, would out-weigh a little extra cost for these cars.

The electrical energy source in a vehicle is a battery, a DC Voltage source, and the traction motor is generally an AC motor owing to the various advantages it offers over a DC motor. Hence the need for a VSI, which is used to transform the DC voltage into AC voltage of desired amplitude and frequency. Pulse width modulation techniques are used to control VSI to ensure that the required/calculated voltage is fed to the machine, to produce the desired torque/speed. PWM techniques are essentially open loop systems where no feedback is used and the instantaneous values differ from the required voltage, however the same average values are obtained.

Pulse width modulated techniques produce a low frequency signal (desired average value of the switched voltage) also called the fundamental component, along with unwanted high frequency harmonics linked to the carrier signal frequency or the PWM period. In modern cars we see more and more mechanical loads driven by electricity

through digital processors. It is very important to eliminate the risk of electromagnetic interference between these systems to avoid failure or malfunction. Hence these unwanted harmonics have to be filtered so that they do not affect the electronic control unit or other susceptible components placed in the vicinity. Randomised modulation techniques (RPWM) are used to dither these harmonics at the switching frequency and its multiple. In this thesis a random modulator based on space vector modulation is presented which has additional advantages of SVM.

Another EMI problem linked to PWM techniques is that they produce common mode voltages in the load. For electric machines, common mode voltage produces shaft voltage which in turn provokes dielectric stress on the motor bearings, its lubricant and hence the possibility of generating bearing currents in the machine that can be fatal for the machine. To reduce the common mode voltage a space vector modulation strategy is developed based on intelligent placement of zero vectors.

For hard switched converters, commutations or the switching of the power switches produce losses that heat up the switches and have to be evacuated rapidly as thermal stress reduces the component life and makes it prone to failure. The higher the switching losses the higher the thermal stress that the switch undergoes. The heat sink dimensions are proportional to the energy lost in the form of heat to be dissipated. So higher switching losses result in a bigger heat sink. Discontinuous modulators (DPWM) are used to reduce the switching losses. Here we have developed an improved discontinuous modulator which can adapt itself to the changing machine speed and load to minimise the switching losses. It also offers the possibility to regulate the thermal stress between the inverter legs to increase the inverter life.

A PWM technique to reduce the electric stress on the DC-Link capacitors and reduce the number of current sensors required for torque regulation is presented as well. This technique makes use of non-adjacent active vectors and dynamic pulse placement.

Each of the aforementioned side effects and its alleviation is dealt with separately and dedicated modulation strategies namely Randomized, Discontinuous Space Vector Modulation and Optimised PWM in terms of reduced ripple content of the inverter input current are developed to achieve it. These techniques will eventually result in inverters with a smaller EMI filter, a smaller heat sink, smaller DC-link capacitor i.e. a compact and cheaper inverter.

A befitting test bench is realised to calculate the real gains and check the practical feasibility of these techniques in terms of execution on embedded processor. All experimental work is systematically preceded by theoretical study where analytical expressions are developed to prove the claims made and validation by simulation tools.

**Keywords:** Electric drives, Electromagnetic Interference, Hybrid Electric Vehicles, Commutation Losses, Pulse Width Modulation, Discontinuous-PWM, Random PWM, Space Vector Modulation.

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# List of Abbreviations

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DPWM	Discontinuous Pulse Width Modulation
DFT	Discrete Fourier Transformation
DSP	Digital Signal Processor
DSVM	Discontinuous Space Vector Modulation
DTC	Direct Torque Control
ECU	Electronic Control Unit
EMC	Electromagnetic Compatibility
EMF	Electromotive Force
EMI	Electromagnetic Interference
FFT	Fast Fourier Transformation
FOC	Field Oriented Control
HEV	Hybrid Electric Vehicle
ICEV	Internal Combustion Engine Vehicle
IGBT	Insulated Gate Bipolar Transistor
MOSFET	Metal Oxide Field Effect Transistor
PMSM	Permanent Magnet Synchronous Motor
PSD	Power Spectral Density
PWM	Pulse Width Modulation
RCF	Random Carrier Frequency
RPP	Random Pulse Position
RPWM	Random Pulse Width Modulation
RSVM	Random Space Vector Modulation
SVM	Space Vector Modulation
THIPWM	Third Harmonic Injection PWM
THD	Total Harmonic Distortion
VSI	Voltage Source Inverter
ZSS	Zero sequence signal

# List of Notations

---

$g_a$	Upper switch Gating signal for phase 'a'
$\langle V_a \rangle_T$	Mean phase voltage over a modulation period
$C_a$	Clamping duration phase 'a'
$C_{NK}$	Fourier coefficients
$C_{ws}$	Parasitic capacitance between the stator windings and stator frame
$E$	Back EMF of the machine
$f(m_i)$	Harmonic distortion factor
$f_{PWM}$	Carrier frequency
$i_h$	Harmonic current
$L$	Inductance
$m_i$	Modulation index
$P_{sw}$	Switching losses
$S(f)$	Power density spectrum
$u_0$	Zero sequence voltage
$V_a^*$	Voltage reference, phase 'a'
$V_{a0}$	Phase 'a' voltage with respect to DC mid-point
$V_{an}$	Phase voltage with respect to the load neutral
$V_{CM}$	Common mode voltage
$V_{dc}$	Inverter input voltage
$V_i$	Space vectors ( $i=0,1,2,3,4,5,6,7$ )
$V_{max}$	Maximum instantaneous value of a 3 phase system
$V_{n0}$	Potential difference between load neutral and the DC mid-point
$V_\alpha, V_\beta$	Voltages in $\alpha\beta$ -plane
$\alpha_i$	Duty cycle of space vectors
$\lambda_h$	Harmonic flux
$\varphi$	Phase angle between the phase voltage and current

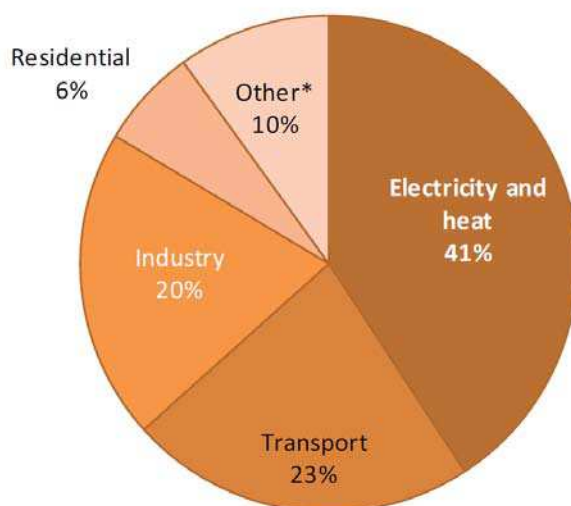


# Introduction

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Vehicles contribute enormously to atmospheric pollution, about 20%-35% of total atmospheric pollution [1]. An average European car produces about 4 tonnes of CO<sub>2</sub> every year [2], [3]. These emissions can be classified further as exhaust emissions including dangerous gases such as carbon monoxide, oxides of nitrogen, hydrocarbons and particulates and evaporative emissions vapours of fuel which are released into the atmosphere without being burnt. Some of these gases contribute to the greenhouse effect which is a threat to the planet.

EVs or HEV can help to considerably reduce these emissions. Depending on the way the electricity is produced and on the type and extent of electrification of the vehicle (e.g. micro hybrid, mild hybrid, Plug in HEVs range extenders, pure electric etc.) the emissions can be reduced from 5% to 100%. Statistics for some HEVs are given in Table I. Toyota Prius is most sold HEV.



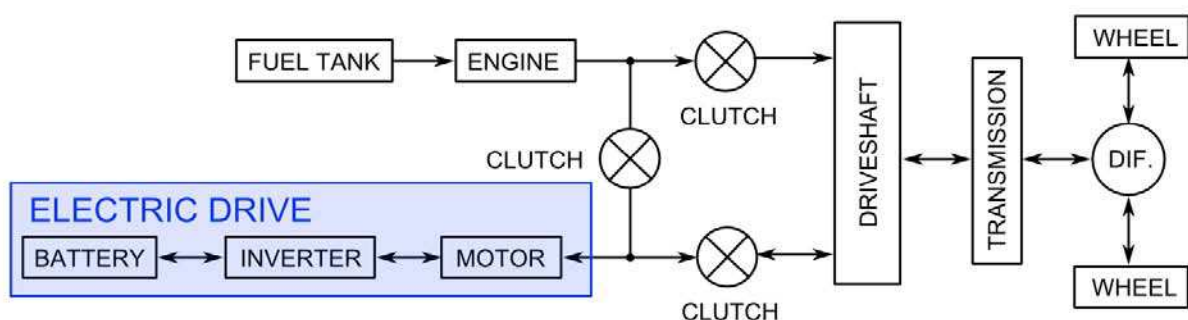
**Figure 1. World CO<sub>2</sub> emissions by sector in 2009 [3]**

Figure 1 shows the carbon dioxide emissions by sector for the year 2009. It is clear that transport represents the second largest chunk on the pie chart, however electricity and heat remain the biggest contributors of this greenhouse gas. Hence all the more reason to concentrate on Hybrids rather than on all-electrics, until we get this percentage down. However this doesn't apply to countries which do not use fossil fuel to generate electricity. The 'Other' on the chart includes commercial/public services, agriculture/forestry, fishing, energy industries other than electricity and heat generation, and other emissions not specified elsewhere.

	CO2 Reduction (%)	Fuel consumption Reduction (%)
Toyota Prius	42	38
Ford Fusion Hybrid	36	31
Lincoln MKZ Hybrid	36	31
Honda Civic Hybrid	31	28
Lexus HS 250h	30	28
Mazda Tribute Hybrid	26	23
Nissan Altima Hybrid	21	19

**Table I. HEV pollution reduction**

ICEVs give good performance and autonomy, taking advantage of the rich petroleum fuels. The efficiency and pollution of such vehicles is a threat to the environment and limited energy reserves. Whereas battery powered EVs have high efficiency and zero emission but very low operating range per battery charge. HEVs have the advantages of an ICEV and an EV while alleviating their drawbacks [1], [6]. Since HEVs have two energy sources and converters it could considerably increase the cost and space requirements as can be seen in Figure 2. Many types of hybrid structures are possible like series, parallel, series-parallel and complex hybrid [7]. Parallel hybrid best meets the objective of increased efficiency and low emissions. There are serious drawbacks of the series hybrid drivetrain, such as the energy conversion takes place twice and the electric traction motor needs to be rated for maximum power. Whereas for a parallel drivetrain, a complex mechanical coupling design is required with the additional complexity of regulating/blending two parallel power sources [8], [1].



**Figure 2. Parallel HEV Drivetrain**

Regenerative braking, or energy recuperation, is the principal means through which the kinetic energy of the vehicle is returned to electric energy storage rather than burned off as heat in the brake pads. But there are practical limits to how much and how fast

regenerative braking can be applied. The machine runs in the generator mode and the inverter in rectifier mode, hence a reversible converter is required. Smooth braking is the result of a fine balance between electric motor energy recuperation and the vehicle's foundation brakes. The best brake system for a hybrid is what is known as series regenerative braking system (RBS). With series RBS the electric motor extracts braking energy without application of the service brakes, then when higher braking forces are required, or if the brake pedal is depressed faster than a prescribed threshold, the service brakes are engaged so that total braking effort is delivered. Not only are such cars energy efficient they have better performance owing to very dynamic torque response, particularly under Field Oriented Control (FOC), it is quicker than ICE response. During gear-shifting the electric motor can add torque to the driveline, thus filling in for lack of engine-supplied torque and give a better driving experience. Hybrid electric power trains require a large investment in electric motor and power electronics technology. Package space is extremely restricted so that even with a ground-up design for a hybrid there is little space to put 20 to 100 kW electric machines and the power electronics to drive them. Such machines must not only have the highest power density but they must also be robust and efficient. The power electronics must be of the highest power density both gravimetrically and volumetrically [1].

Nonetheless EVs have already penetrated the off-road vehicles where the required autonomy is not the limiting factor and is known beforehand and where low acoustic noise and clean air are a priority. Such applications include airport vehicles for passenger and ground support; recreational vehicles like golf carts and for theme parks, plant operation vehicles like forklifts and loader trucks. All of this is possible because of the technological advancements in power electronics and digital signal processors. The recent developments in the field of power solid state devices or Power Electronics has changed completely the form of electric propulsion system, it has made the use of AC machines possible. Here we'll discuss electric drives with a portable energy source, the battery for automotives. A typical modern electric drive is shown in Figure I-1. The AC motor is fed by a battery via a Voltage Source Inverter (VSI), it can be seen as the interface between the DC voltage source (battery) and the AC Motor. The inverter can convert DC Voltage in a poly phase AC voltage source of variable amplitude and frequency. They are made up of power switches that can be electronically switched on and off. These electronic signals are voltage pulses for IGBTs, which have been used in this work as they meet our power and frequency requirements. An Electronic Control Unit (ECU) calculates the duty cycle of the pulses using the information fed to it by an acquisition circuit, this is known as Pulse Width Modulation.

The research revolves around the development of modulation techniques for hard switched three phase two level inverters for variable speed drives destined for HEV to alleviate the drawbacks of the hybrid drivetrains mentioned above. Random and discontinuous modulation techniques have been developed to address the issues of EMI

interference and switching losses respectively and their digital implementation has been given equal importance. Techniques to reduce DC-link capacitors are also developed.

Hard switched PWM converters have the following drawbacks:

1) Electromagnetic pollution caused by the switching harmonics and switching transients [11] may hamper the proper functioning of digital electronic circuitry used extensively in modern cars. This is normally dealt by adding bulky and voluminous L-C filters and shielding of the power converter.

2) Switching Losses [12] are not only a waste of energy but give rise to another very important concern -- evacuating this energy dissipated in the form of heat in the power switches. Thermal stress can lead to poor functioning and in extreme cases complete failure of the switch.

3) Shaft voltages may cause an electric discharge through the lubricant around the ball bearing and the stator called bearing current and destroy the motor [13].

4) Acoustic noise in power converters [14] for switching frequencies in the audible range till 22kHz can be very annoying.

To alleviate these drawbacks this work has the objective of reducing the cost and volume of the electric drivetrain by developing innovative PWM techniques to reduce the need of auxiliary components required to suppress the side effects of such systems namely EMI, DC link fluctuations and heating of the power switches and at the same time increasing the efficiency and hence an improved autonomy on battery. These auxiliaries are namely the passive filters to absorb the switching harmonics, DC stabilizing capacitors and voluminous cooling circuit.

A very important aspect of this research is the integration of EMI mitigation techniques and meeting Electromagnetic Compatibility (EMC) standards at the development stages of the electric drive rather than troubleshooting at the end, which is a costly and time-consuming process. The motivation behind the work is to reduce the cost to market of HEVs which is significantly higher than conventional cars. Innovative techniques based on the classical PWM techniques such as RPWM and DPWM exist to address the issue of unwanted harmonics linked to the switching frequency and the switching losses. These techniques are explained in the chapters to follow. Space Vector Modulation (SVM) is a relatively new PWM technique based on mathematical transformations and has some advantages over conventional techniques. Since more and more sophisticated techniques are used, such as FOC, DTC, the digital signal processors have become indispensable and this means SVM can be implemented at no extra cost. SVM has been taken as the basic modulation technique and its derivatives are developed to address the issues mentioned above.

In such electric drives, PWM methods influence heavily the behaviour of the drive. A meticulously programmed technique cannot only give improved performance but also reduce many of the unwanted secondary effects of modulated power supply. The work

presented here is on developing such PWM techniques to alleviate the problems in the drives mentioned in the previous section.

The thesis is divided into four parts, the first part puts into perspective the need for this study and an assessment of the state of the art of the field, explaining briefly the major problems that need to be addressed. Introduction to EMI is given and then an overview of some performance indicators of Pulse Width Modulators. The second part gives details of the PWM techniques developed during this PhD. The third part gives the details of the experimental setup and the experimental validations of the techniques developed in the second part. The fourth part is the conclusion and few suggestions for future works.

# **PART I**

# **PRELIMINARIES**

# I. Preliminaries

The research documented in this thesis relates to pulse-width modulation (PWM) techniques for hard-switched three phase two level power electronic inverters for variable speed drives destined for vehicle propulsion. Focus has been on two different types of modulators that introduce randomness and discontinuity to the system for reasons to be described shortly. Such modulators are generally designated as random PWM techniques in the literature to emphasize their non-deterministic properties.

## I.1. Introduction

An electric drive comprises of an inverter which interfaces the energy source to the motor/generator. In the context of vehicle propulsion system the inverter helps feeding the motor as required by the driver but also recharging the batteries during deceleration and braking. The inverter is comprised of electronically controllable switches. The PWM schemes control the state of the switches whether conducting or not. The maximum DC side voltage is about 600V hence 2 level IGBT inverters are sufficient. However to improve the signal quality one can imagine the use of multilevel inverters but for automotive applications it is not practical owing to the extra cost it will add to the overall system, i.e. extra gate drivers, extra PWM peripherals, extra processing power, extra space.

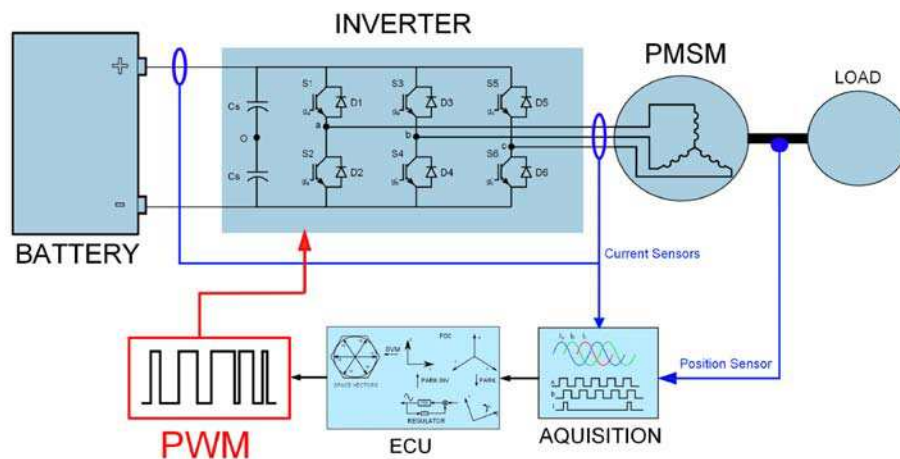


Figure I-1: Schematic diagram of an Electric Drive

Permanent Magnet Synchronous Motor (PMSM) are preferred as they are superior to the DC and induction motors in servo applications due to their high power density, efficiency, moment to current ratio and their low moment of inertia [9]. Permanent Magnet Synchronous Motors (PMSM) have been unanimously declared to be the most suitable for

HEVs. Table I-1 recapitulates how different electrical machines fair on grounds mentioned above.

	DC	IM	PMSM	SRM
<b>Power Density</b>	--	+	++	0
<b>Efficiency</b>	--	+	++	0
<b>Cost</b>	+	+	0	+

**Table I-1 Comparison of Electrical Machines**

The two most pertinent control schemes for AC motors are Field Oriented Control (FOC) and Direct Torque Control (DTC). The former was chosen because DTC is a method based on hysteresis comparators which require the controller to work at a very high frequency in order to confine the error in the hysteresis band which means introducing error to the system and hence torque ripple. FOC unlike DTC is based on regulators which calculate the exact value of the phase voltage to be fed to the motor. PWM techniques are used to calculate the duty cycle of the pulse to be applied to the switch to produce the required output voltage to be applied to the motor [10]. PWM techniques are at the heart of such drives, they have a huge influence on various aspects apart from the quality of the voltage produced, like the losses and electromagnetic interference.

## **I.2. Literature review**

Pulse Width Modulation is an interface between the control circuit and the inverter, where the modulator is given a value that is required at the converter terminal and it has to produce it for a given period and a given DC voltage. Modulation technique was developed by communication engineers to transmit a baseband signal by transforming it into a pass-band signal. The use of PWM for electric drives dates as back as the early 1960s [15].

The concept is to achieve a variable voltage from a fixed DC voltage source while regulating the duty cycle of the power converter control signal. Half bridge or a converter leg configuration is shown in Figure I-2. The control signals  $g_a$  and  $g_a'$  of the two switches S1 and S2 are complementary in nature. The voltage output ( $V_{a0}$ ) for a sinusoidal reference signal ( $V_a^*$ ) is the modulated signal generated by comparing a high frequency carrier wave with the reference signal, the switch changes state each intersection of the these two signals The voltage produced is known as Pulse Width Modulated Voltage. The average value of the modulated voltage over a carrier period is equal to the reference voltage for that carrier period this is also known as Volt-Sec match. For good results the carrier frequency should be at least 20 times higher than the fundamental frequency.



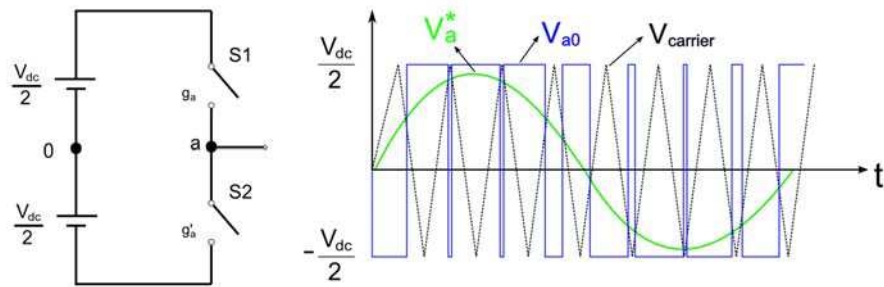


Figure I-2: Half Bridge

### I.2.1. Fundamentals of PWM

Different types of carrier signal and the frequency of the reference voltage updates schemes can be envisioned and some of the most common methods are discussed here. The two predominant carrier signals used are sawtooth, triangular and the reference voltage updates usually employed are naturally sampled, regularly sampled (symmetrical and asymmetrical).

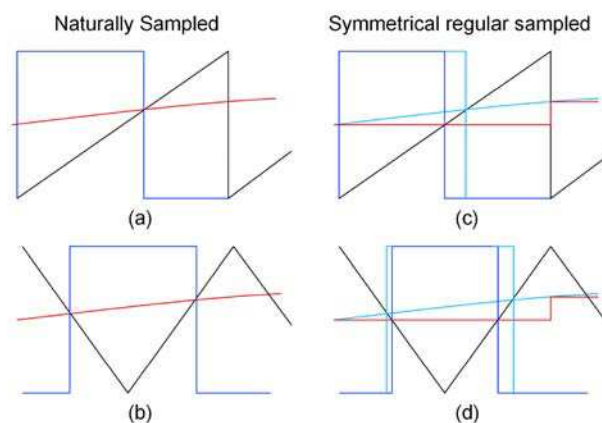


Figure I-3: (a), (b): Naturally sampled, (c), (d) regularly sampled

The naturally sampled scheme is realized by an analog circuit and therefore the comparator is updated continuously and thus the most accurate. Whereas other techniques such as symmetrical, asymmetrical, multi-sampled schemes are digitally implemented using DSP/FPGA. In Figure I-3 and Figure I-4 the evolution the reference signal is highly exaggerated as the carrier is very high compared to the reference signal frequency for a carrier period the reference signal can be considered constant.

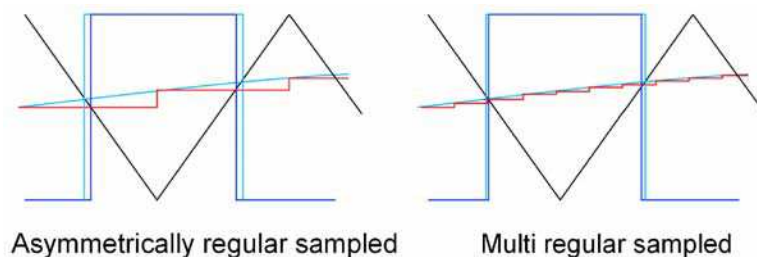


Figure I-4: Asymmetrically sampled

### 1.2.2. Classical Sinusoidal PWM

Three half bridges in parallel form a three phase inverter. For a 3-phase system given by (1.1) the voltage reference of each leg is phase delayed by  $120^\circ$ .

$$\begin{bmatrix} v_a^* \\ v_b^* \\ v_c^* \end{bmatrix} = A \begin{bmatrix} \sin(\theta) \\ \sin(\theta - 2\frac{\pi}{3}) \\ \sin(\theta - 4\frac{\pi}{3}) \end{bmatrix} \quad (1.1)$$

The most basic and straight forward PWM strategy is the Sinusoidal PWM. This method is used specially for loads with neutral tied to the ground or the DC mid-point.

$$d_i = \frac{T_{PWM}}{2} \left(1 + \frac{2v_i^*}{v_{dc}}\right) \times 100 \quad (1.2)$$

where  $i \in \{a, b, c\}$

Equation (1.2) gives the duty cycle in percentage of the modulation period ' $T_{PWM}$ ' for symmetrically sampled PWM. Since the duty cycle cannot be greater than 100, again from equation (1.3) one can deduce that the amplitude of the reference is limited to half of  $V_{dc}$ . Hence the maximum value of A is  $A_{max} = V_{dc}/2$ .

Modulation Index ( $m_i$ ) is given by (1.3) to evaluate the extent to which the DC input voltage is used.

$$m_i = \frac{V_{fundamental-PWM}}{V_{fundamental-six\ step}} \quad (1.3)$$

Six step refers to square wave phase voltage where for the positive half cycle the phase is clamped to the positive terminal of the DC source and for the negative half to the negative terminal hence the voltage is not modulated and produces the maximum output voltage. This definition of  $m_i$  is chosen so because it makes sure that  $0 \leq m_i \leq 1$ . The fundamental component of a square wave is  $2 V_{dc} / \pi$ . So the modulation index for SPWM is  $\frac{\pi}{4}$  or 0.785.

Frequency domain analysis of the modulated signal helps visualizing the presence of the reference signal in the square pulse train. The switching harmonics are the by-product of the PWM, frequency domain analysis of the PWM signals are elaborated later in the thesis. These unwanted high frequency voltage causes conducted and radiated electromagnetic emissions. Passive filters are used to send back these voltages back to the source and absorb some of these unwanted signals however to reduce the filtering effort one can use Random PWM methods explained in the next section.

### 1.2.3. Hysteresis Band control

Before we go further I'd like to mention a slightly different type of controller, called the hysteresis controller or a current regulator. As the name current regulator suggest this technique directly controls the current in the inverter. What distinguishes it from the other PWM techniques is that it is technique is a closed loop technique i.e. it requires a feedback. This is the most basic current control method that doesn't require current regulators. The switches are controlled to maintain the current around the desired value defined by the hysteresis band (HB). As indicated in Figure I-5, if the actual current exceeds the HB, the upper device of the half-bridge is turned off and the lower device is turned on. As the current decays and crosses the lower band, the lower device is turned off and the upper device is turned on. If the HB is reduced, the harmonic quality of the wave will improve, but the switching frequency will increase, which will in turn cause higher switching losses.

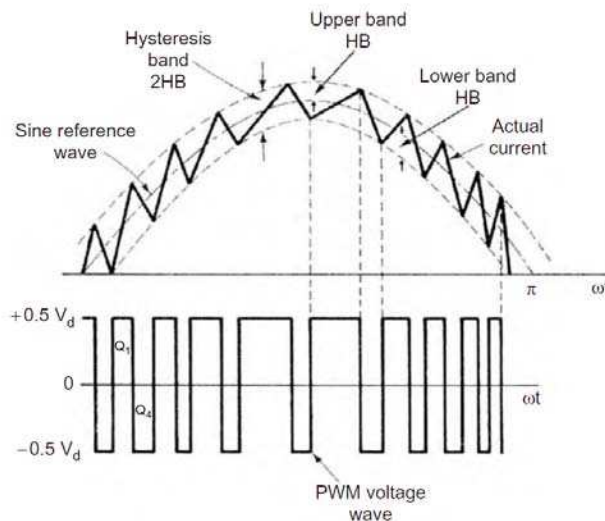


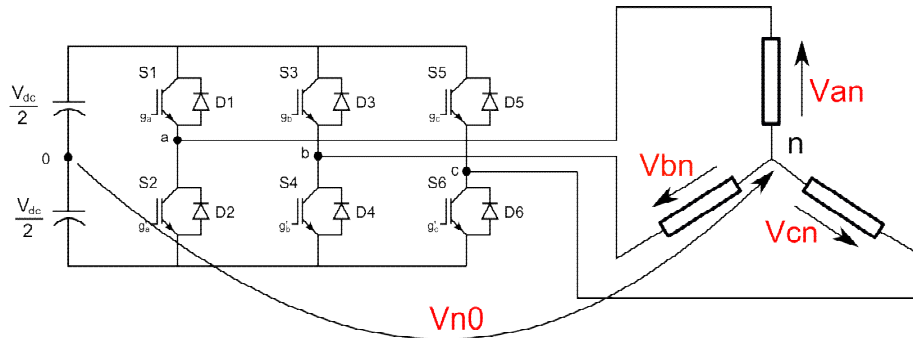
Figure I-5: Hysteresis Band

Basically, the current loop error signal generates the PWM voltage wave through a comparator with a hysteresis band. Although the technique is simple, control is very fast, and device current is directly limited, the disadvantages are a harmonically non-optimum waveform and the phase lag is frequency dependent, increases with the increase in frequency.

### 1.2.4. Zero Sequence voltage injection

If the neutral point on the poly-phase load has a floating neutral i.e. not connected to the DC side midpoint 0 or the ground, the zero sequence signals (ZSS) in the phase voltages disappear from the line voltages and thus the phase currents depend only on the potential difference between the phases and the phase to neutral voltages are perfectly sinusoidal and in turn the phase currents.

A 3-phase 3-wire system, i.e. neutral free to float with respect to the ground or the DC mid-point is shown in Figure I-6.

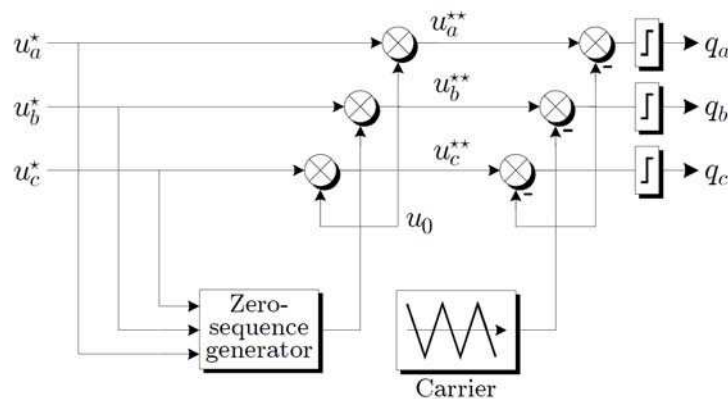


**Figure I-6: Inverter fed floating neutral 3-Phase load**

However certain rules must be followed while inserting ZSS into the reference voltages. In equation (1.4) the  $u_0$  is the ZSS voltage added and  $u^*$  is the original reference voltage generally sinusoidal and  $u^{**}$  is the final voltage seldom a sinusoid of different amplitude.

$$u^{**} = u^* + u_0 \quad (1.4)$$

Figure I-7 shows a generalised modulator commonly abbreviated as GPWM.



**Figure I-7: Generalized modulator: ZSS injection**

Figure I-8 helps visualize the concept of floating neutral and how one can achieve higher phase voltages injecting the right zero sequence component without creating an imbalance, in the next section we'll see what ZSS should be injected if the goal is to achieve higher phase voltages. Whatever be the zero sequence voltage injected into the system it should not make the phase voltages go beyond the maximum calculated value, beyond this value it will introduce imbalance in the system. Another very important aspect about ZSS component is the possibility of adding discontinuity to the modulator.

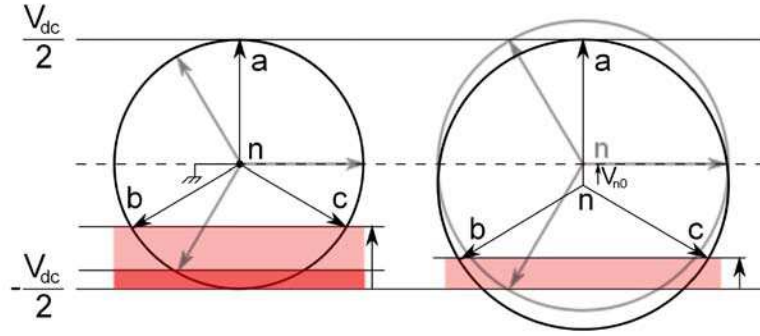


Figure I-8: Floating neutral

From the above figure the maximum value of the phase to neutral voltage can be calculated.

$$|V_{an} - V_{bn}| \leq V_{DC}$$

$$V_{\max} \angle 240^\circ - V_{\max} \angle 0^\circ = V_{dc}$$

$$V_{\max} \cos 30^\circ + V_{\max} \cos 30^\circ + V_{\max} \cos 60^\circ - V_{\max} \cos 60^\circ = V_{dc}$$

$$V_{\max} = \frac{V_{dc}}{2 \cos 30^\circ} = \frac{V_{dc}}{\sqrt{3}}$$

$$V_{\max} = \frac{V_{dc}}{\sqrt{3}} \quad (1.5)$$

From the above derivation it is evident that there are limits to the ZSS that can be added without creating an imbalance in the poly-phase system.

#### I.2.4.1. Third Harmonic Injection PWM

Third harmonic injection PWM (THIPWM) can increase the value of the fundamental component of the phase voltage, Figure I-9 shows that after the addition of the zero sequence component the region which would have been over-modulated or un-modulated is now under linear modulation region. The optimum injection value can be calculated as following: [16].

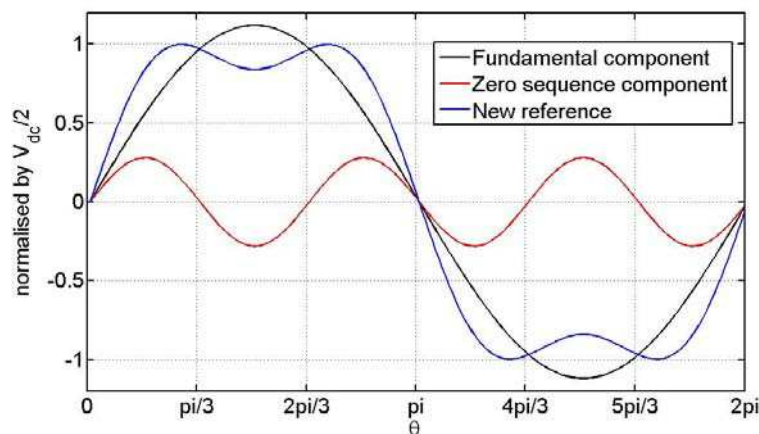


Figure I-9: 1/4th THIPWM

The most prominent third harmonic injection schemes are THIPWM1/4 and THIPWM1/6 where the zero sequence voltage to be injected is given by (1.6).

$$\begin{aligned} u_0 &= A_3 \cos(3\omega t) \\ A_3 &= A_1/4 \text{ or } A_3 = A_1/6 \end{aligned} \quad (1.6)$$

Where  $A_1$  and  $A_3$  are the amplitudes of the fundamental and the third harmonic components respectively. Injecting  $1/4$  of the fundamental is theoretically superior in terms of harmonic distortion [17] while injecting  $1/6$  increases the fundamental to the maximum possible value, this can be shown in the following manner: Let Eq.(1.7) give the resultant voltage after third-harmonic injection.

$$V_a = \frac{V_{dc}}{2} (M_1 \cos \theta + M_3 \cos 3\theta) \quad (1.7)$$

Dividing by  $M_1 V_{dc}/2$  we get:

$$v = \cos \theta + \gamma \cos 3\theta$$

$$\text{Where } \gamma = \frac{M_3}{M_1}.$$

$$\frac{dv}{dt} = \sin \theta + 3\gamma \sin 3\theta = 0 \quad (1.8)$$

The maxima or minima of (1.7) would lie on the values of  $\theta$  and  $\gamma$  that satisfy equation (1.8) and  $\sin 3\theta$  can be written as:

$$\sin 3\theta = (4\cos^2 \theta - 1)\sin \theta$$

Substituting it in equation (1.8) gives

$$\sin \theta + 3\gamma(4\cos^2 \theta - 1)\sin \theta = 0 \text{ from where the value of } \cos \theta \text{ can be calculated as:}$$

$$\cos \theta = \sqrt{\frac{3\gamma - 1}{12\gamma}} \quad (1.9)$$

$\cos 3\theta$  can similarly be expanded to give as  $\cos 3\theta = (4\cos^2 \theta - 3)\cos \theta$  and therefore

$$\cos 3\theta = -\frac{6\gamma + 1}{6\gamma} \sqrt{\frac{3\gamma - 1}{12\gamma}}.$$

Substituting these values back in (1.8) give:

$$v_{\max} = -\frac{3\gamma - 1}{3} \sqrt{\frac{3\gamma - 1}{3\gamma}} \quad (1.10)$$

Differentiating this time with respect to  $\gamma$  and equating to 0.

$$\frac{dv_{\max}}{d\gamma} = \sqrt{1 - 1 - \frac{1}{3}\gamma} - \frac{1}{6} \frac{3\gamma - 1}{\sqrt{1 - \frac{1}{3}\gamma}} \frac{1}{3\gamma^2}$$

$$0 = \sqrt{1 - \frac{1}{3}\gamma} \left(1 + \frac{1}{6\gamma}\right)$$

We get two values of  $\gamma$ .

$$\gamma = \frac{1}{3} \text{ \& } \gamma = -\frac{1}{6} \quad (1.11)$$

The first one can be neglected and clearly produces 0 when substituted in (1.10). Hence a minimum.

With the second value of  $\gamma$  the maximum attainable fundamental component is found.

$$A_{1,\max} = \frac{V_{dc}}{\sqrt{3}} \quad (1.12)$$

It should be noted that the amplitude is one-sixth of the  $A_{1,\max}$  given by equation (1.12). It is the same value that was calculated earlier in this chapter hence it confirms that injecting this ZSS the maximum possible phase voltage can be achieved.

#### I.2.4.2. Discontinuous PWM

ZSS can be injected in the system not only to make better use of the DC link voltage, it can also reduce the effective switching frequency over a fundamental period avoiding commutations during a part of the fundamental period. The idea behind such techniques is clamping a phase to either side of the DC rail. To clamp phase 'i' the following condition must be met:

$$v_i^* = v_{\max} \vee v_{\min} \quad (1.13)$$

This condition basically assures that the line voltages will not be distorted and since line voltages dictate the current in the phases for loads with floating neutral. Respecting this condition one can imagine quite a few discontinuous techniques. The commonly found techniques in the literature are briefly covered here. All the figures in this section show three waveforms: the zero sequence signal in red, the initial voltage reference signal in black and the new resultant signal (blue)  $u^{**}$ . In this method the voltage clamping is symmetrical for the positive and negative half cycles.

### 1.2.4.2.1. DPWM1

DPWM1 is one of the most basic types of modulation scheme where the phase with the maximum absolute amplitude is clamped. The expression of the voltage to be injected in this case is given by (1.14).

$$u_0 = \frac{V_{dc}}{2} \text{sign}(v_{\max}) - v_{\max} \quad (1.14)$$

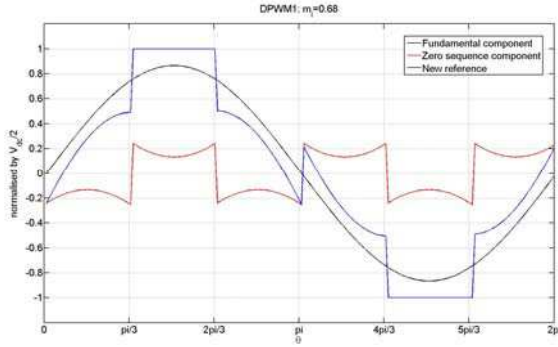


Figure I-10: DPWM1, mi=0.68

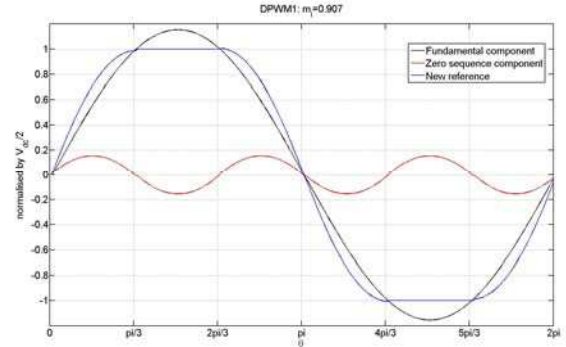


Figure I-11: DPWM1, mi=0.907

Where  $V_{\max} = \max(\text{abs}(V_a, V_b, V_c))$ . Figure I-10 and Figure I-11 show the DPWM1 method for two different modulation index.

### 1.2.4.2.2. DPWMMIN & DPWMMAX

Voltage clamping for these methods is done either at maximum voltage or minimum. This method unlike DPWM1 is an unsymmetrical method where clamping is done only during one of the two half cycles for  $120^\circ$ . The harmonic injection law for DPWMMIN is given by (1.15):

$$u_0 = -\frac{V_{dc}}{2} - v_{\min} \quad (1.15)$$

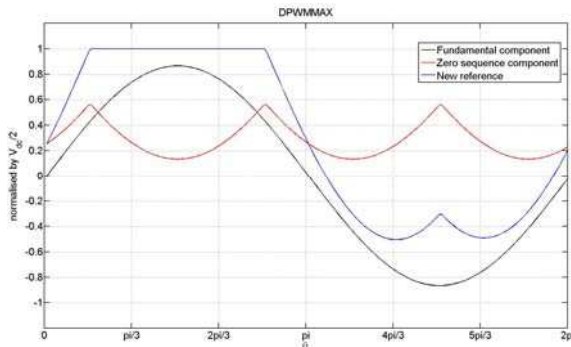


Figure I-12: DPWMMAX

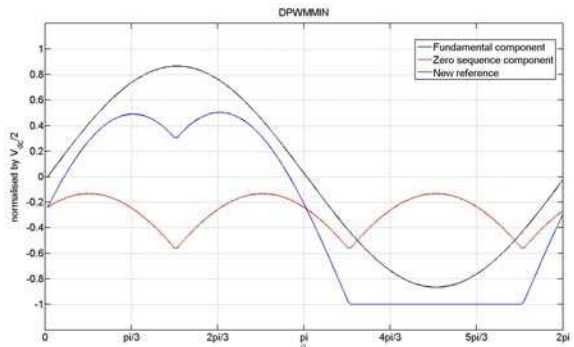


Figure I-13: DPWMMIN

Similarly the zero sequence component to be inserted is given by equation (1.16). Figure I-12 and Figure I-13 show the modulation functions for DPWMMAX and DPWMMIN respectively.



$$u_0 = \frac{V_{dc}}{2} - v_{\max} \quad (1.16)$$

### 1.2.4.2.3. DPWM3

DPWM3 has four 30° discontinuous segments as can be seen in Figure I-14 and Figure I-15, the zero sequence voltage to be injected is given by:

$$u_0 = \frac{\text{sign} V_{dc} - \{(\text{sign} + 1)V_{\max} - (\text{sign} + 1)V_{\min}\}}{2} \quad (1.17)$$

If  $|V_a| < |V_b| < |V_c|$ , then  $\text{sign}=1$  if  $V_a > 0$  and  $-1$  otherwise.

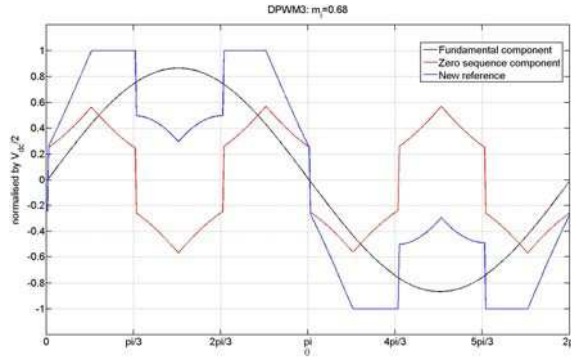


Figure I-14: DPWM3,  $m_i=0.68$

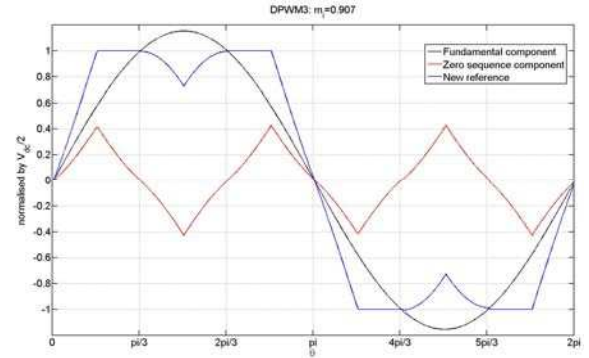


Figure I-15: DPWM3,  $m_i=0.907$

The reference signal with the intermediate magnitude defines the zero sequence signal. This method has low harmonic distortion characteristics [20].

$$u_0 = \text{sign}(v_{\max}) \frac{V_{dc}}{2} - v_{\max}$$

where,

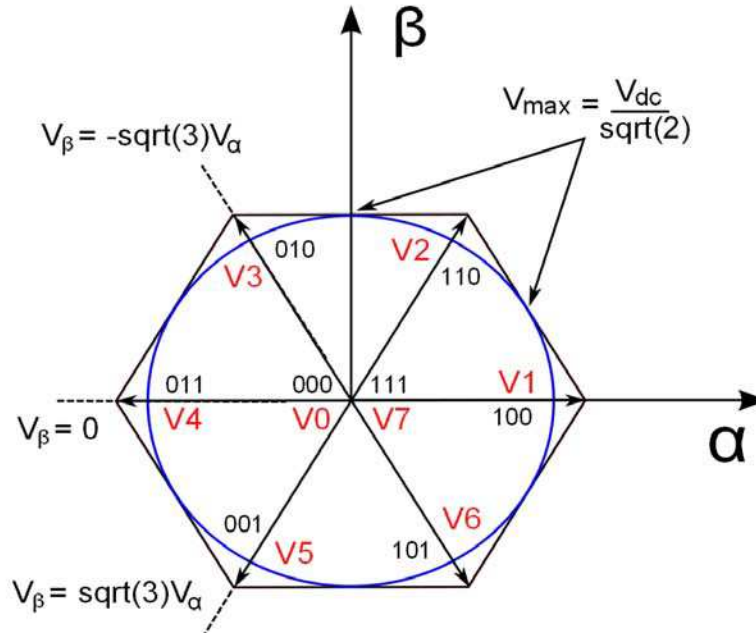
$$v_{\max} = \max(|v_a^*(\omega t + \phi)|, |v_b^*(\omega t + \phi)|, |v_c^*(\omega t + \phi)|)$$

$$\phi \in \{0, 30^\circ, 60^\circ\} \quad (1.18)$$

There are plenty of methods to achieve voltage clamping but they are not all discussed here as the only the concept was to be introduced and only most of common of these discontinuous modulators discussed. Such modulators are called generalized discontinuous modulator where a generic type of zero sequence can be added [19].

### 1.2.5. Space Vector Modulation

Space vector modulation (SVM) is a digital modulation technique based on mathematical transformation. A three phase electrical system can be represented on the  $\alpha\beta$ -plane using the well-known Clark transform (1.20). as shown in Figure I-16. to calculate the duration of the active vectors to applied [16], the modulation period is completed by the zero vectors ( $V_0$  and  $V_7$ ). Figure I-16 depicts the vectors representing all possible inverter states that form the vertex of a hexagon on the  $\alpha\beta$ -plane.



**Figure I-16: Voltage Space Vectors**

The three bit binary subscript denotes the state of upper switch of the inverter leg corresponding the three phases 'a, b and c' in the same order. '0' and '1' represent the OFF and ON state respectively. The upper and lower switch states of a leg are complimentary to avoid short circuiting the voltage source. The other vectors are to be interpreted in the same way. SVM is the generation of a voltage vector using two adjacent vectors among the six active vectors (explained later in document) and the two zero vectors.

The phase voltages ' $v_{in}$ ' are given by (1.19).

$$\begin{pmatrix} v_{an} \\ v_{bn} \\ v_{cn} \end{pmatrix} = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{pmatrix} v_{ao} \\ v_{bo} \\ v_{co} \end{pmatrix} \quad (1.19)$$

$$\text{where } v_{io} = V_{dc} \left( g_i - \frac{1}{2} \right) \& i \in \{a, b, c\}$$

A 3-phase system can be projected on the  $\alpha\beta$ -plane using the following transformation.

$$\begin{bmatrix} x_{\alpha} \\ x_{\beta} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \quad (1.20)$$

Applying (1.20) and (1.19) on (1.1) we get the projections in the  $\alpha\beta$ -plane as shown in the Table I-2.

$g_a$	$g_b$	$g_c$	$v_\alpha$	$v_\beta$	Vector
0	0	0	0	0	V0
0	0	1	$-\frac{V_{dc}}{\sqrt{6}}$	$-\frac{V_{dc}}{\sqrt{2}}$	V5
0	1	0	$-\frac{V_{dc}}{\sqrt{6}}$	$\frac{V_{dc}}{\sqrt{2}}$	V3
0	1	1	$-\frac{2V_{dc}}{\sqrt{6}}$	0	V4
1	0	0	$\frac{2V_{dc}}{\sqrt{6}}$	0	V1
1	0	1	$\frac{V_{dc}}{\sqrt{6}}$	$-\frac{V_{dc}}{\sqrt{2}}$	V6
1	1	0	$\frac{V_{dc}}{\sqrt{6}}$	$\frac{V_{dc}}{\sqrt{2}}$	V2
1	1	1	0	0	V7

**Table I-2 Vector magnitudes in  $\alpha\beta$ -plane**

The magnitudes of all the active vectors can be given by (1.21).

$$|V_i| = \sqrt{\frac{2}{3}} V_{dc} \quad (1.21)$$

where  $i \in \{1, 2, 3, 4, 5, 6\}$

Using (1.20) a balanced three phase system expressed by (1.1) can be represented by a circle whose radius is given by A.

$$v_\alpha = \sqrt{\frac{3}{2}} A \sin(\theta), \quad (1.22)$$

$$v_\beta = -\sqrt{\frac{3}{2}} A \cos(\theta)$$

$$\sqrt{v_\alpha^2 + v_\beta^2} = \sqrt{\frac{3}{2}} A \quad (1.23)$$

Let  $A_{\max}$  be the maximum amplitude of the 3-phase Voltage that can be generated using the space vectors shown in Figure I-16. Then the radius of such system is given by (1.24).

$$r_{\max} = \sqrt{\frac{3}{2}} A_{\max} \quad (1.24)$$

The circle inscribed in the hexagon represents a circle whose radius is ' $r_{\max}$ '. Hence this defines the boundary limit of the attainable voltage. From the (1.24) we get (1.25).

$$r_{\max} = \sqrt{\frac{2}{3}} V_{dc} \cos\left(\frac{\pi}{6}\right) \quad (1.25)$$

Comparing (1.24) and (1.25) we get  $A_{\max}$  as expressed by (1.26).

$$A_{\max} = \frac{V_{dc}}{\sqrt{3}} \quad (1.26)$$

### 1.2.5.1. Sector and time calculation

The Space vector hexagon can be divided into six parts according to the six active vectors. They are called sectors. A vector in any sector can be produced as the resultant of the two vectors defining the sector. Active vector application duration is calculated with the help of Figure I-16 in the following manner.

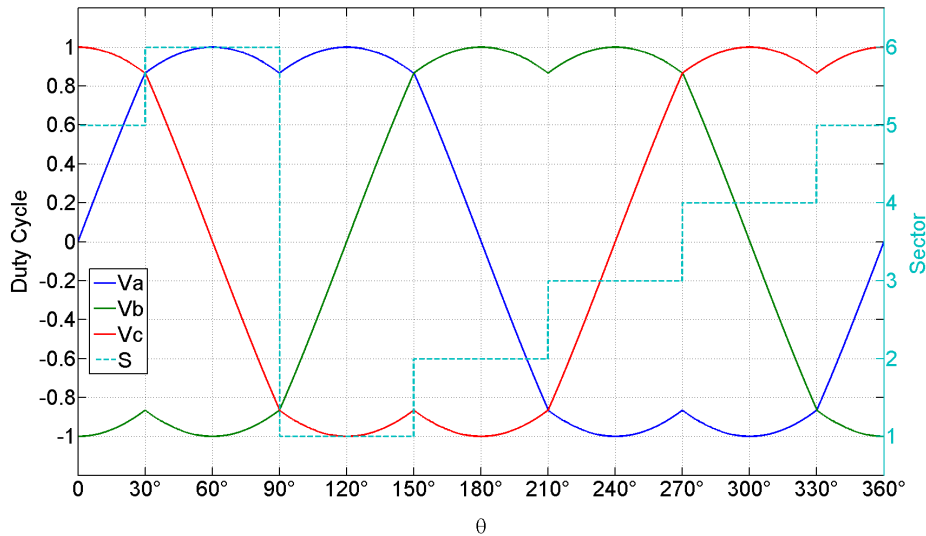
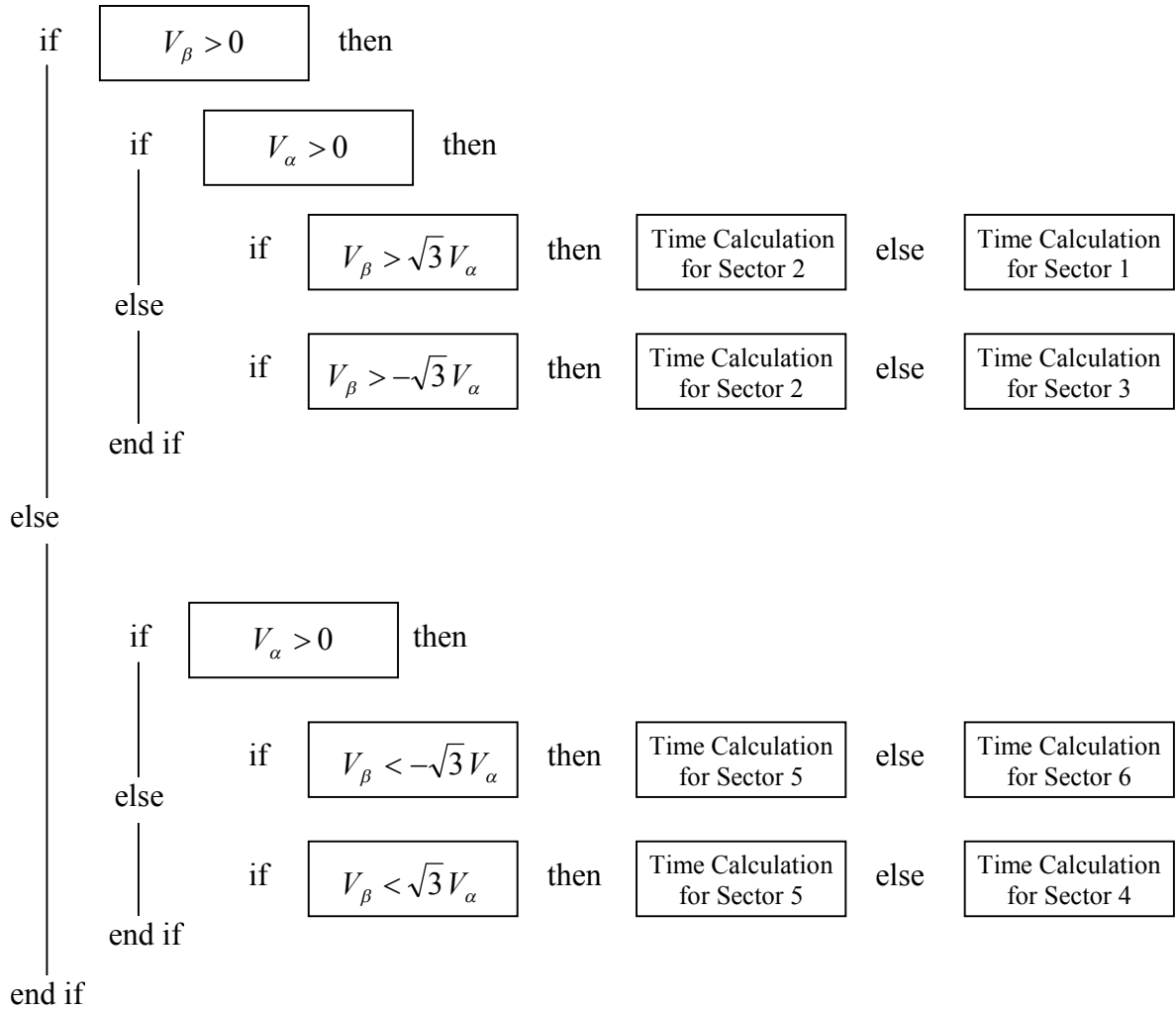


Figure I-17: SVM; modulation function and corresponding sectors

For a balanced three phase system the space vector modulation function and sectors in the time domain are shown in Figure I-17. Each sector is  $60^\circ$  long i.e. one-sixth of the fundamental period.

The  $\alpha\beta$ -plane is divided by lines;  $V_\beta = \sqrt{3} V_\alpha$ ,  $V_\beta = 0$  and  $V_\beta = -\sqrt{3} V_\alpha$  in 6 sectors to make the calculations simple. Any arbitrary vector in a sector would be a resultant of the two adjacent active vectors named  $V_i$  and  $V_{i+1}$ . Figure I-18 gives the algorithm to identify the sector.

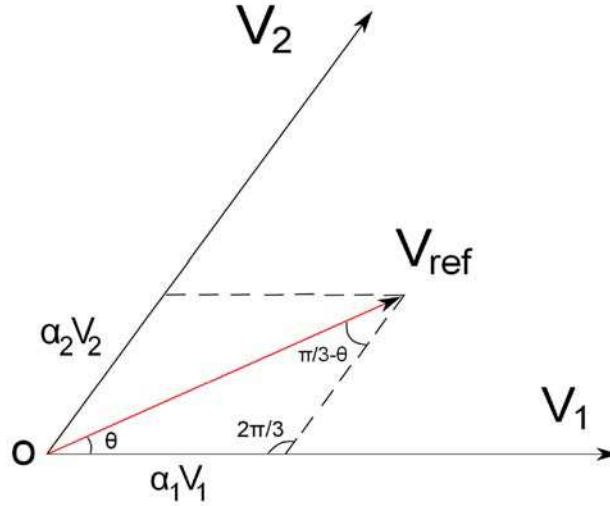


**Figure I-18: SVM Sector identification**

Once the sector has been identified the calculation of the duty cycles of the corresponding active vectors are calculated. A reference vector lying in the first sector is shown in Figure I-19, i.e.  $0 \leq \theta \leq 60^\circ$ .

$$\alpha_i = \frac{T_i}{T_{svm}} \quad (1.27)$$

The duty cycle calculations are done using simple geometric identities and rules, for e.g. the calculations for the duty cycle ' $\alpha$ ' for the two active vectors for the first sector is shown here.



**Figure I-19: Time calculation sector 1**

Applying the sine law to the triangle of Figure I-19 we get:

$$\frac{\alpha_2 V_2}{\sin \theta} = \frac{\alpha_1 V_1}{\sin\left(\frac{\pi}{3} - \theta\right)} = \frac{V_{ref}}{\sin\left(2\frac{\pi}{3}\right)} \quad (1.28)$$

From where,

$$\alpha_1 = \frac{V_{ref}}{V_1} \left( \cos \theta - \frac{\sin \theta}{\sqrt{3}} \right) \text{ and } \alpha_2 = \frac{V_{ref}}{V_2} \left( \frac{2 \sin \theta}{\sqrt{3}} \right) \quad (1.29)$$

Substituting the values of  $\sin \theta = \frac{V_{\beta\_ref}}{V_{ref}}$ ,  $\cos \theta = \frac{V_{\alpha\_ref}}{V_{ref}}$  and  $V_1 = V_2 = \sqrt{\frac{2}{3}} V_{dc}$  in

equation (1.29) leads to the following result for the duty cycle of each of the two vectors used.

$$\alpha_i = \frac{1}{V_{dc}} \left( \sqrt{\frac{3}{2}} \cdot v_{\alpha_{ref}} - \sqrt{\frac{1}{2}} \cdot v_{\beta_{ref}} \right) \text{ \& } \alpha_{i+1} = \frac{1}{V_{dc}} \sqrt{2} \cdot v_{\beta_{ref}} \quad (1.30)$$

In a similar way we calculate the duration of the vector to be applied for other sectors, the time expressions are given below. For a given sector k,  $\theta = \theta - (k-1)\frac{\pi}{3}$  in equation (1.29).

Sector 2:

$$\alpha_i = \frac{1}{V_{dc}} \left( \sqrt{\frac{3}{2}} \cdot v_{\alpha_{ref}} + \sqrt{\frac{1}{2}} \cdot v_{\beta_{ref}} \right) \text{ \& } \alpha_{i+1} = \frac{1}{V_{dc}} \left( -\sqrt{\frac{3}{2}} \cdot v_{\alpha_{ref}} + \sqrt{\frac{1}{2}} \cdot v_{\beta_{ref}} \right) \quad (1.31)$$

Sector 3:

$$\alpha_i = \frac{1}{V_{dc}} \sqrt{2} \cdot v_{\beta_{ref}} \quad \& \quad \alpha_{i+1} = \frac{1}{V_{dc}} \left( -\sqrt{\frac{3}{2}} \cdot v_{\alpha_{ref}} - \sqrt{\frac{1}{2}} \cdot v_{\beta_{ref}} \right) \quad (1.32)$$

Sector 4:

$$\alpha_i = \frac{1}{V_{dc}} \left( -\sqrt{\frac{3}{2}} \cdot v_{\alpha_{ref}} + \sqrt{\frac{1}{2}} \cdot v_{\beta_{ref}} \right) \quad \& \quad \alpha_{i+1} = -\frac{1}{V_{dc}} \sqrt{2} \cdot v_{\beta_{ref}} \quad (1.33)$$

Sector 5:

$$\alpha_i = \frac{1}{V_{dc}} \left( -\sqrt{\frac{3}{2}} \cdot v_{\alpha_{ref}} - \sqrt{\frac{1}{2}} \cdot v_{\beta_{ref}} \right) \quad \& \quad \alpha_{i+1} = \frac{1}{V_{dc}} \left( \sqrt{\frac{3}{2}} \cdot v_{\alpha_{ref}} - \sqrt{\frac{1}{2}} \cdot v_{\beta_{ref}} \right) \quad (1.34)$$

Sector 6:

$$\alpha_i = \frac{1}{V_{dc}} (0 \cdot v_{\alpha_{ref}} - \sqrt{2} \cdot v_{\beta_{ref}}) \quad \& \quad \alpha_{i+1} = \frac{1}{V_{dc}} \left( \sqrt{\frac{3}{2}} \cdot v_{\alpha_{ref}} + \sqrt{\frac{1}{2}} \cdot v_{\beta_{ref}} \right) \quad (1.35)$$

Once the active vector duty cycles are calculated the time durations can be calculated by the following expression:

$$T_i = \alpha_i T_{svm} \quad \& \quad T_{i+1} = \alpha_{i+1} T_{svm} \quad (1.36)$$

The remaining time i.e.  $T_{zero} = T_{svm} - (T_i + T_{i+1})$  is distributed equally to the two zero vectors,  $V_0$  and  $V_7$ . Hence  $T_0 = T_7 = T_{zero} / 2$ .

### 1.2.5.2. Space Vector PWM

Space Vector PWM (SVPWM) is the analog method of achieving SVM characteristics, this is done by injecting zero sequence voltages in the reference voltages. The ZSS to be injected can be given by (1.37). This voltage is the median voltage, for e.g. if  $V_a > V_b > V_c$  then  $u_0 = -V_b/2$ .

$$u_0 = -\frac{v_{\min} + v_{\max}}{2} \quad (1.37)$$

In the Figure I-20 the zero sequence component is shown in red and resultant modulation function in blue. This is a triangular signal and has all the multiple of 3 of the fundamental frequency.

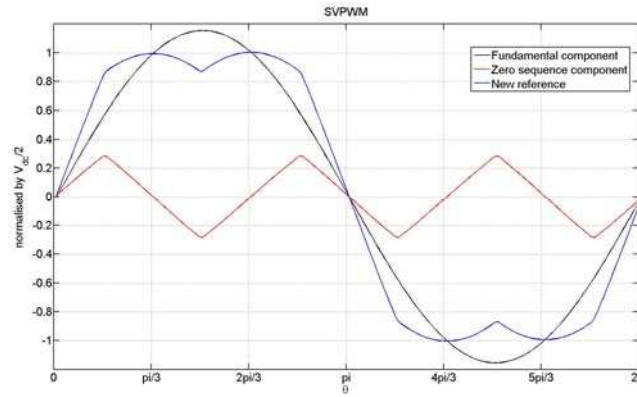


Figure I-20: SVPWM

Figure I-21 shows the spectrum of a phase voltage.  $f_3$ ,  $f_6$ ,  $f_9$ , are the zero sequence harmonics that appear in the phase voltage however these are cancelled out for line voltages.

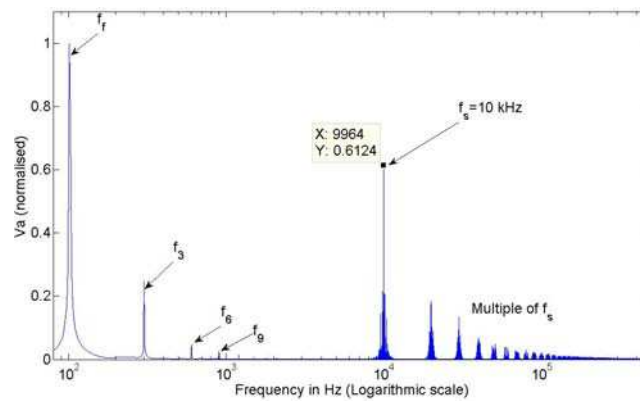


Figure I-21: Frequency Spectrum - Phase Voltage  $V_{a0}$

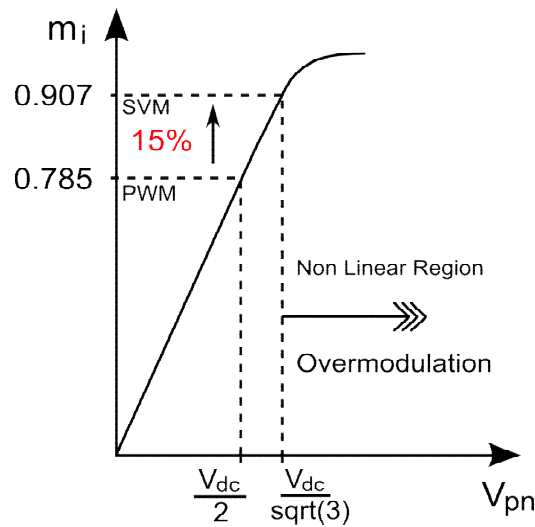
## WHY SVM?

Why do we think that SVM is best suited to meet our objectives. It might not be very evident now but the fact that SVM is a mathematical method which makes it possible to calculate which vectors to be applied and gives their duty cycle too hence one gets an extra degree of freedom to apply these vectors at one's will. In later chapter we'd see how this feature unique to SVM can be exploited to one's advantage in more than one way, which cannot be done in PWM which is a simple method of comparing the reference values with a carrier wave or if digitally implemented with a counter where one does not control the vector sequencing.

The other important reason is that SVM intrinsically increases the inverter's capability to optimally use the input DC voltage  $V_{dc}$ . We saw that  $A_{max}$  for PWM and SVM are  $V_{dc}/2$  and  $V_{dc}/\sqrt{3}$  respectively. This means an increase in the  $m_i$  by 15.47% as seen in Figure I-22, which means a better DC voltage utilization. Third harmonic injection techniques as seen earlier can also increase the output voltage of inverters [29]. However these techniques are not suitable for dynamic systems where the amplitude and the frequency of the fundamental component vary randomly and abruptly with time. Furthermore in vector

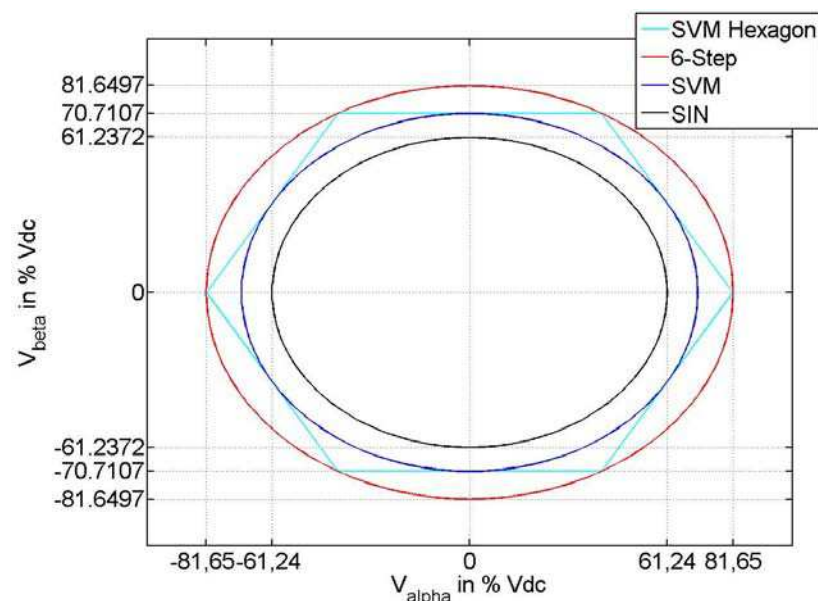


control, instantaneous values are fed to the controller and the complete form of the reference signal has no significance. Since it is automatically done using SVM one can worry about other issues without having to worry about this aspect.



**Figure I-22: Increased linearity of Modulation Index**

The table below shows the effect of injecting different signals into the reference signal, on the magnitude of the fundamental component. It is possible to get comparable results with SVM by injecting  $V_{med}$  (SPWM) which is basically third harmonic and its multiples. However in SVM one doesn't explicitly inject these harmonics, SVM is based on mathematical calculations and the graphical representation helps show the boundary values very vividly, since it is a purely mathematical method that unlike SPWM depends only on instantaneous values of the reference signal and not a specific waveform to calculate the signal to be injected.



**Figure I-23: Amplitudes of different modulation schemes in  $\alpha\beta$ -plane**

Figure I-23 shows the amplitudes of the different modulation techniques namely, sinusoidal, SVM and six-step modulation in the  $\alpha\beta$ -plane. All the values are given in percentage of the inverter input voltage ' $V_{dc}$ '. This gives us an idea of the voltage amplitudes with respect to the space vector representation, where each vertex of the hexagon represents an active vector. However since we are more used to representing a three phase system by their phase voltage amplitudes, below these values are given for some of the common modulation techniques.

Modulation technique	Maximum amplitude of the fundamental of $V_{pn}$
Conventional (sinusoidal)	$\frac{1}{2}V_{dc}$
THIPWM4 $\left(A_3 = \frac{A_1}{4}\right)$	$\frac{1.12}{2}V_{dc}$
THIPWM6 $\left(A_3 = \frac{A_1}{6}\right)$	$\frac{V_{dc}}{\sqrt{3}} \approx \frac{1.15}{2}V_{dc}$
SVM & SVPWM with Injection of $V_{med}$ (Linearity Limit)	$\frac{V_{dc}}{\sqrt{3}} \approx \frac{1.15}{2}V_{dc}$
Six-step (Over-modulation Limit)	$\frac{4}{\pi} \left(\frac{V_{dc}}{2}\right) \approx \frac{1.27}{2}V_{dc}$

**Table I-3 Attainable Voltage using different techniques**

Table I-3 shows the maximum phase to neutral voltage  $V_{pn}$  that can be produced by the PWM techniques seen earlier in this chapter without going into overmodulation, the fundamental value for a six-step wave or an un-modulated output is given to give an how the others compare to the maximum possible output voltage.

### ***1.2.6. Random PWM***

The concept is to randomize the modulation function parameters, like the pulse position, the switching frequency to spread the frequency spectrum around the switching frequency and hence reduce Electromagnetic Interference (EMI) and in turn reduce the filtering effort [20]. The analog way of doing it is to generate a random carrier wave. In Figure I-24 the red dots represent the parameters to be varied randomly within a given range with a specific effective value to get the desired harmonic dithering. Which means one can randomize the lead/lag pattern of the pulses and modulation period.

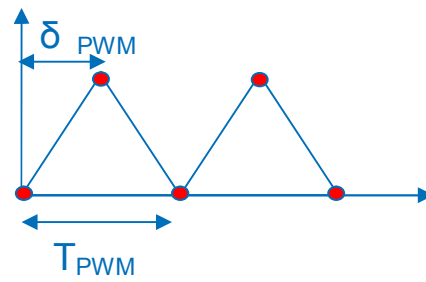


Figure I-24:Carrier signal randomization

### I.2.6.1. Random Carrier Frequency-PWM

Random Carrier Frequency PWM (RCF-PWM) refers to randomizing the carrier signal frequency is the most effective method of dithering the switching harmonics in the frequency domain [22]. The idea is to have many carrier frequency that are randomly chosen to decrease the power density at the switching frequency while distributing it to its surrounding values. This means that modulated signal would contain more harmonics but of smaller amplitude hence less effective. However the duty cycles remain constant, i.e. the on-time to the period ratio remains constant. Figure I-24 shows the principle of RCFM and conservation of V-seconds per modulation period.

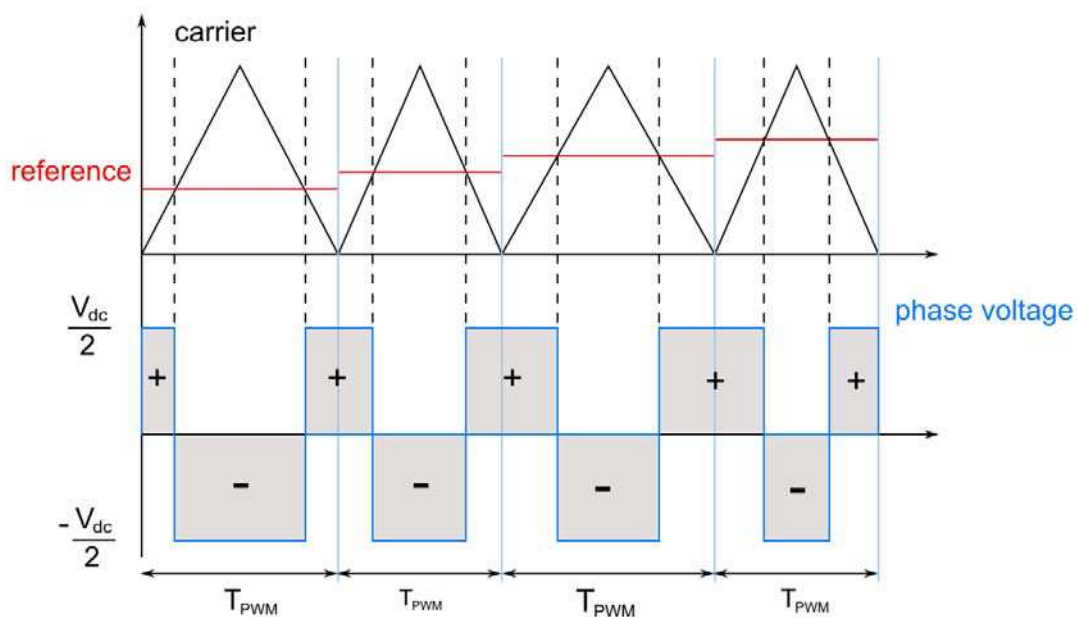
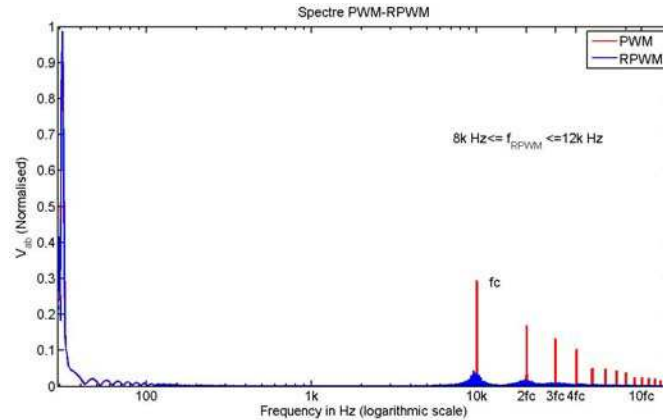


Figure I-25:RCFM

The frequency limits need to be carefully chosen. Lower frequencies increase the current ripple for inductive loads. (1.38) shows the evolution of current of a machine, winding resistance is neglected and  $E$  is the back-emf while  $V$  is the applied voltage. The change in the stator current is proportional to  $dt$ , hence higher the switching frequency lower the current ripple.

$$di = \frac{V - E}{L} dt \quad (1.38)$$

Some power switches are faster than others e.g. Metal Oxide Semiconductor Field Effect Transistor (MOSFET) can operate for switching frequencies as high as 1MHz whereas Insulated Gate Bipolar Transistor (IGBT) are normally limited to 100kHz. Another factor limiting the switching frequency is the permissible switching losses. The higher the switching frequency higher the losses.

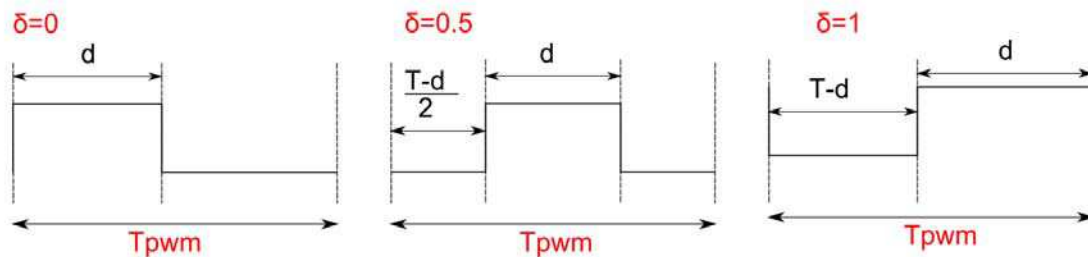


**Figure I-26:Frequency Spectrum PWM-RPWM**

Frequency Spectrum of the modulated signals for fixed and random carrier frequency are shown in Figure I-24. The curve in red and blue are for fixed and random PWM respectively. It can be clearly seen that switching harmonic peaks are considerably reduced and distributed to the surroundings represented by small humps.

### I.2.6.2. Random Pulse Position PWM

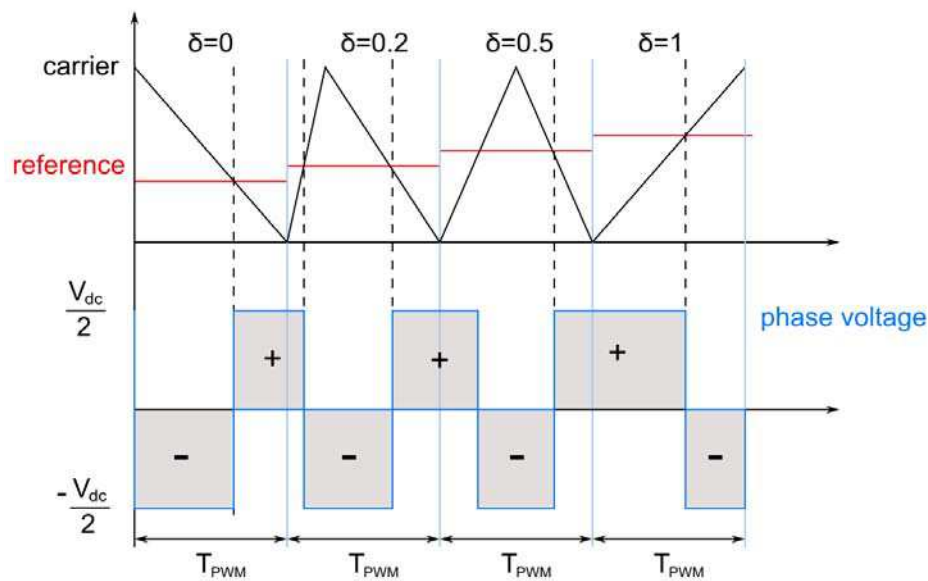
Random Pulse Position PWM (RPP-PWM) as the name suggests is randomizing the placement of the pulse in the modulation period. The pulse position is denoted by  $\delta$  as shown in Figure I-27.



**Figure I-27: Pulse positioning**

To randomize the pulse position the top vertex of the triangular carrier wave is randomly placed along the breadth of the period. Randomizing the pulse position does not

dither the switching harmonics to the same extent as RCFM and hence is not given much importance [23].



**Figure I-28:RPPM**

Figure I-28 shows how different types of carrier signals upon comparison with the reference signal can lead to different types of pulse placements leading, lagging centred, other intermediate cases can be imagined as well.

### **I.2.6.3. Dual Randomization**

Combining RCF-PWM and RPP-PWM is called dual randomization. As the name suggests here both the switching frequency and pulse position vary randomly from one modulation period to another which gives a slightly better voltage spectrum compared to RCFM modulation as shown in [24]. This technique would not be developed further in the work owing to some of the advantages of the centred pulses. It is shown later in the chapter that centred pulses have much lower harmonic current ripple amplitude, half to be precise, compared to completing lagging or leading pulses.

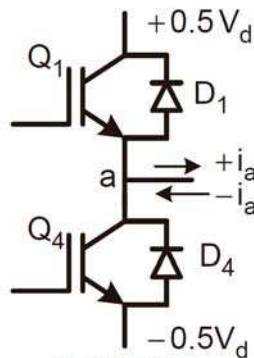
### **I.2.7. Practical aspects of PWM**

Some precautions must be taken while implementing PWM techniques. In this section some of the practical aspects are introduced like avoiding short-circuiting the voltage source, switching losses, and some EMI aspects related to hard switched converter fed loads.

#### **I.2.7.1. Dead time**

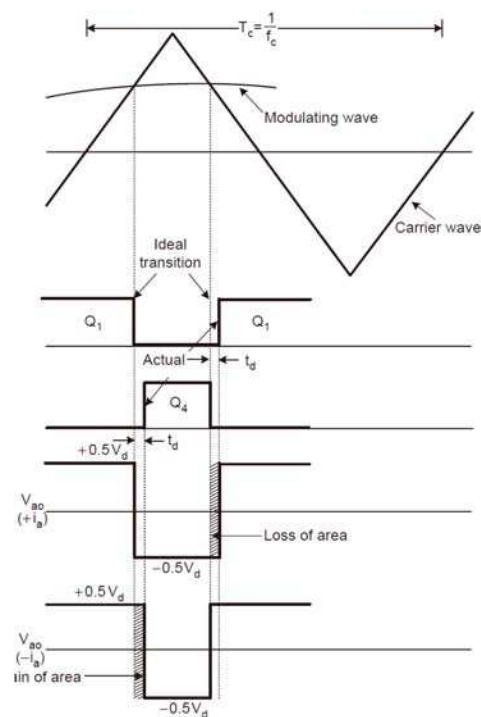
Theoretically the two switches of the same inverter leg are controlled by complementary signals however dead time must be inserted in the gating signals to avoid

short-circuit faults. An inverter leg is drawn in Figure I-29 with current conventions. To understand the phenomenon the commutations have to be studied closely.



**Figure I-29: Half bridge**

In Figure I-30 the theoretical instances of commutations are shown by dotted lines, and it can be seen that there is a slight delay when the signal changes from 0 to 1. Two cases arise depending of the polarity of the current. In the first case where the current is positive during the first commutation the dead time doesn't have any effect on the voltage output at the terminal as the phase is still clamped to the negative rail through  $D_4$ , whereas for the second commutation the phase again gets clamped to the negative rail which causes loss of volt-seconds. In the second case where the current is negative the opposite phenomenon takes place and volt-seconds are gained.



**Figure I-30: Dead time**

Dead time effect adds baseband deformations and affects the quality of the output voltage. Figure I-31 shows the effect of dead time in time domain.

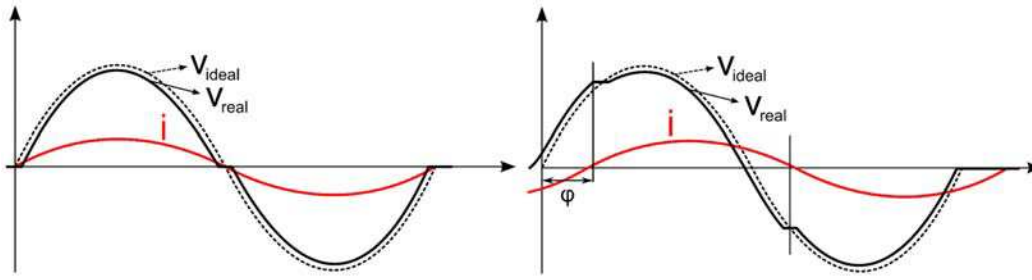


Figure I-31: Dead time; Voltage distortion

Dead time effects can be corrected by modifying the duty cycles, since the current polarity decides the dead time effect, to compensate for it current information is needed, which makes it a feedback system.

### I.2.7.2. Switching losses

In VSI the phase voltage generated has only two levels  $\pm V_{dc}/2$  with respect to the DC mid-point and since they both have the same absolute value it becomes a constant while calculating the switching losses hence the only variable then is the phase current [12] which leads to the expression (1.39) for switching losses.

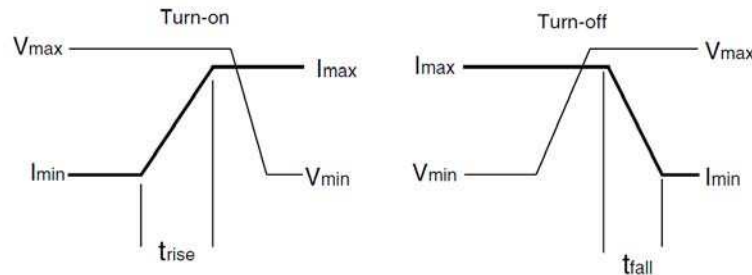


Figure I-32: Hard Switching

$$\begin{aligned}
 P_{sw}^{on} &= \frac{f_{sw} V_{dc} t_{rise}}{2} \frac{1}{2\pi} \int_0^{2\pi} |i(\theta)| d\theta \\
 P_{sw}^{off} &= \frac{f_{sw} V_{dc} t_{fall}}{2} \frac{1}{2\pi} \int_0^{2\pi} |i(\theta)| d\theta \\
 P_{sw} &= P_{sw}^{on} + P_{sw}^{off}
 \end{aligned} \tag{1.39}$$

This means the faster the switching lesser the losses and the higher the frequency the greater the losses. Some stray elements in the switches may alter the switching transient of the switch and hence alter the switching losses.

### I.2.7.3. Leakage Currents

Pulse width modulated voltages have a very high rate of change of voltage often called dv/dt ratio. This is getting worse by the day with the switching speeds constantly

increasing. On the other hand faster switches have lower switching losses. The leakage current depends a lot on the high frequency model of the load because switching transients are extremely short of about a few Nano-Seconds and hence a very high  $dv/dt$  of about a few GV/s.

High frequency model of an electrical machine, Figure I-33 shows a capacitive coupling between the stator end windings and the stator body [41]. Since the stator is connected to the protective earth the current leaks into the earth therefore also called common mode current. These are the main cause of the radiated noise [42]. The expression for the leakage current is given by (1.40) where  $C_{ws}$  is about a few (1-8) Nano-Farads for well-constructed machine while ' $dv/dt$ ' is about 5G V/s which can induce currents through these stray capacitors for which the instantaneous values can be as high as 10-15 amperes. The occurrence of the peaks is equal to the number of commutations.

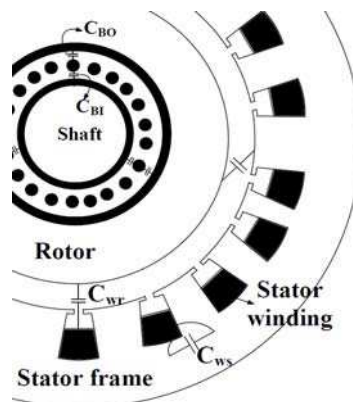


Figure I-33: High frequency model of an electric motor

$$I_{leakage} = c_{ws} \frac{dv}{dt} \quad (1.40)$$

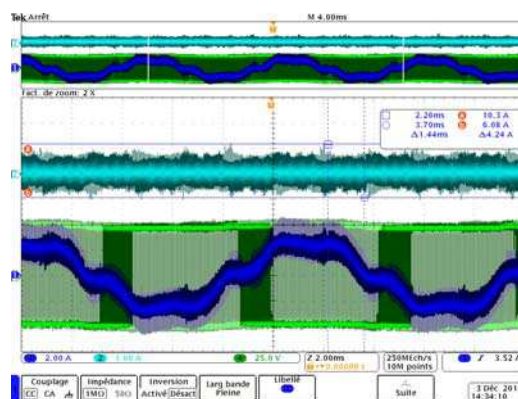


Figure I-34: Leakage current (top),  $I_a$  and  $V_{a0}$

In Figure I-34, the curves in green and navy blue are the phase voltage and current respectively while the turquoise curve on top is the leakage current as observed at our test bench. The density of the leakage current directly depends on the modulation frequency  $f_{PWM}$  and the modulation technique used. The modulation frequency determines how



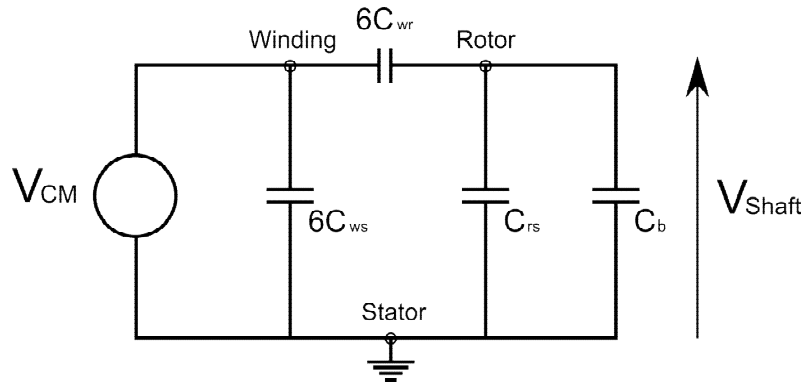
frequently the voltage commutations occur whereas the type of modulation technique could alter the effective modulation frequency such as in the case of DPWM techniques.

Another type of leakage current commonly known as the bearing current is also found in electric machines again owing to the stray capacitive coupling being manifested by high  $dv/dt$  but unlike in the previous case here the common mode voltage is responsible for this phenomenon.

For three phase, two level inverter of Figure I-8 the common mode voltage if the point '0' is connected to the ground is given by equation (1.41):

$$V_{n0} = \frac{V_{a0} + V_{c0} + V_{c0}}{3} \quad (1.41)$$

$V_{n0}$  is either  $V_{dc}/6$  for active vectors or is  $V_{dc}/2$  for zero vectors. This means the lower the modulation index ' $m_i$ ' the higher value of the common mode voltage is applied most of the time. Shaft voltage is proportional to the common mode voltage. Analysing the high frequency circuit the following representative circuit can be drawn [40].

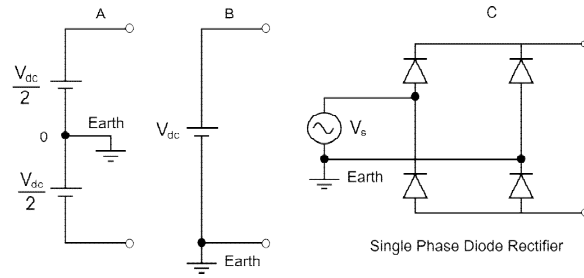


**Figure I-35: High frequency equivalent model**

From Figure I-35 we get (1.42) which shows that linear and proportional relationship of shaft voltage and  $V_{CM}$ .

$$V_{Shaft} = \frac{C_{wr}}{C_{wr} + C_{rs} + C_b} V_{CM} \quad (1.42)$$

The suffix w, r and s correspond to stator-winding, rotor and stator-frame respectively. This shaft voltage can induce currents between the stator connected to earth and the rotor through the ball bearings. The electric drive topology is a very important factor that needs to be taken into consideration to understand how it affects the generation of  $V_{CM}$ .



**Figure I-36: Inverter feeding topologies**

Common mode voltage is given for topology 'A' by (1.41). However this topology is used only for academic purposes to measure phase voltages creating this artificial mid point where the neutral is inaccessible. For topology 'B'  $V_{CM}$  for active vectors is the same as for the topology 'A' but for zero vectors its completely different, its 0 Volts and  $V_{dc}$  for  $V_0$  and  $V_7$  respectively. For the topology 'C' of Figure I-36 the common mode voltage calculation would get a little complicated. There are two cases to consider now according to the polarity of the supply voltage ' $V_s$ '. Table I-1 summarizes the magnitude of  $V_{CM}$  according to the vector generated and the voltage source polarity.

VECTOR	$V_{CM} (V_s > 0)$	$V_{CM} (V_s < 0)$
$V_0$	0	$V_{dc}/3 + V_s$
$V_1$	$V_{dc}/3$	$2V_{dc}/3 + V_s$
$V_2$	$2V_{dc}/3$	$V_{dc}/3 + V_s$
$V_3$	$V_{dc}/3$	$2V_{dc}/3 + V_s$
$V_4$	$2V_{dc}/3$	$V_{dc}/3 + V_s$
$V_5$	$V_{dc}/3$	$2V_{dc}/3 + V_s$
$V_6$	$2V_{dc}/3$	$V_{dc} + V_s$
$V_7$	$V_{dc}$	$V_s$

**Table I-4:  $V_{CM}$  for a single phase full wave rectifier**

Similarly other topologies can be imagined like a three phase rectifier where the  $V_{CM}$  would be still different. Normally for HEVs the topology 'B' is used and hence a strategy to minimize common mode voltage for this case is discussed in the coming chapters.

## I.3. Electromagnetic interference

Electromagnetic Interference (EMI) is the effect of Electromagnetic signals on other equipment electrically connected or otherwise. Institute of Electrical and Electronics Engineers (IEEE) has defined the concept Electromagnetic Compatibility (EMC) in the following way:

“Electromagnetic Compatibility, EMC, is the ability of a device, equipment or system to function satisfactorily in its electromagnetic environment without introducing intolerable electromagnetic disturbances to anything in that environment. ” Here we'll see some basics about EMI, how it propagates from one device to another, how it can be measured and some standards that help assure electromagnetic compatibility among different electrical equipment operating at proximity.

### I.3.1. Introduction

The propagation of the signal from one device to another is the most important aspect of understanding EMI. There are 3 types of coupling (paths) possible for the transmission of these signals and these are; electrical, magnetic and capacitive.



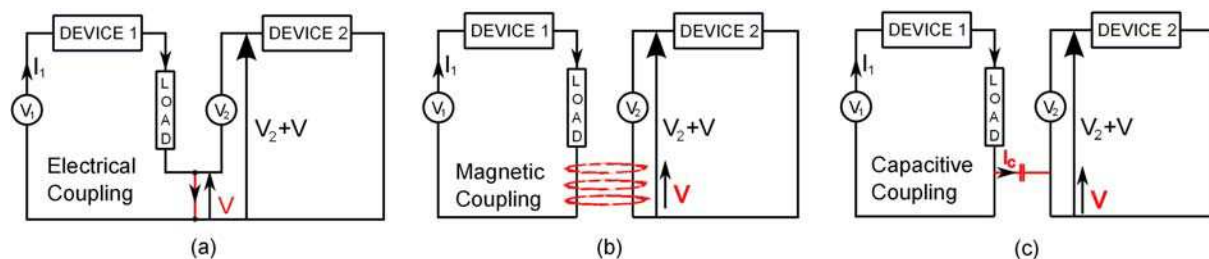
Figure I-37: EMI propagation

Looking at Figure I-37 there are three possibilities of reducing the EMI:

- Controlling the emissions at the source
- Removing/reducing the coupling
- Increasing the immunity of the victim

Different coupling types are illustrated in Figure I-38. In Figure I-38-(a) electric coupling is shown, which arises because a common cable is used, ideally cables have negligible impedance for low frequency signals, as the frequency goes up they can be modelled by lossy inductances, and hence the flow of current produced a voltage drop across the cable and this voltage in turn would be seen by DEVICE 2.

The Magnetic coupling doesn't need an electrical connection as can be seen in Figure I-38 (b), current carrying wires induce magnetic fields and changing magnetic field induces a voltage in the nearby conductor, the voltage induced is directly proportional to the magnetic coupling between the conductors and the rate of change of current in the first conductor.



**Figure I-38: Modes of coupling**

The third type of coupling Figure I-38-(c) manifest itself in the presence of high rate of change of voltage. A changing electric field will induce a voltage in the nearby conductor. In this case the induced voltage is proportional to the strength of the stray capacitance.

In this research work we'd only focus on EM noise emission as we'd be studying the side effects of PWM methods. The working PWM frequencies generally do not exceed 40kHz for electric traction applications therefore we'd focus rather on conducted emissions normally measured between 150kHz to 30MHz whereas the radiated emissions are measured from 30MHz to 1GHz. The reason behind is that very big antennas would be required to transmit low frequency signals. Since the size of the equipment is very small compared to the frequency radiated noise can be neglected.

EMI can be classified into two categories, common mode (CM) and the differential mode (DM) noise. Common mode refers to a voltage impressed on all the lines. This voltage is with respect to the ground. A current flows in the same direction in all the lines and the return is ground. Differential mode means the normal transfer of energy down the line. A voltage across the line with a current flowing in one direction in one wire and the opposite direction in the other wire is differential mode. For two wire system, the differential mode and common mode are the same; between line and ground.

### ***1.3.2. EMI Standards and measurements***

EMI standards for emissions and immunity are put in place for harmonious cohabitation of different electronic and electrical systems, ensuring good power quality, compatibility between different equipment, electromagnetic spectrum protection.

Some common EMI standards are DOD (Dept. of Defense, USA) military standards for Emissions and immunity, IEC (International Electrochemical commission) for immunity standards. CISPR (Comité International Spécial des Perturbations Radioélectriques) or better known as International Special Committee on Radio Interference. FCC Federal Communication Commission USA emission standards. EN are the European standards similar to IEC/CISPR Immunity and emission standards. There are standards for Electrostatic discharge, radiated radio frequency, surge immunity, voltage dips immunity. There are also emission standards for industrial, scientific and medical equipment, automobiles, household equipment, IT, radio frequency devices, limits of harmonic current emission, etc. CISPR 12 Automobile (emission), IEC 61800-3 electric drives (Wind and PV).

LISN (Line Impedance Stabilization Network) and an antenna are the transducers for CE and RE measurements respectively. They both are terminated across a 50 Ohm resistor and measured by an EMI receiver or a spectrum analyser. The EM signals are measured in dB $\mu$ V, dB $\mu$ A, dB $\mu$ V/m, dB $\mu$ A/m, e.g. -20dB $\mu$ V = 0.1 $\mu$ V  $\sim$  0.0002pW.

LISN isolates noise from source to Device Under Test (DUT) and provides a defined impedance at Radio Frequency across the 50 Ohm termination at the measuring receiver. The measurements are done in a well defined set-up in order to make the measurements repeatable. AC power is routed through the LISN to the device under test. The LISN standardizes the measurement impedance to 50  $\Omega$  and provides an isolated RF output to a spectrum analyser, which provides a plot of the conducted emissions coming from the device.

### I.3.3. EMI Filters

EMI filtering is quite different compared to dissipative nature of the signal filters where most of the noise is sent back to the source. Generally the EMI filters are low pass filters. A typical EMI filter is depicted in Figure I-39.

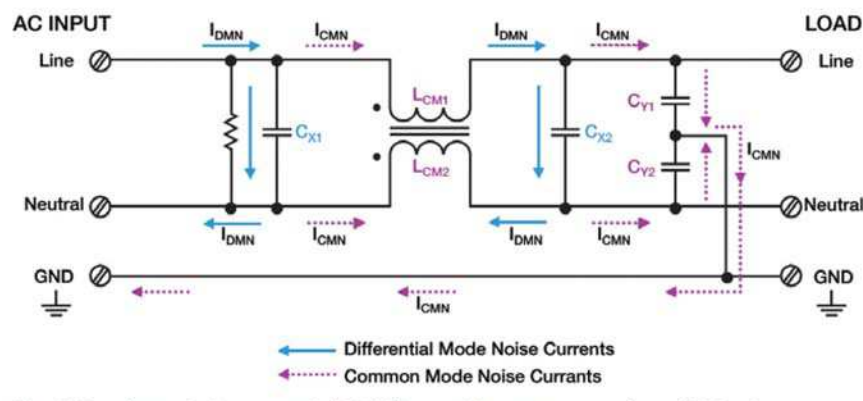


Figure I-39: Conducted noise filter

Common Mode Noise is suppressed by using dual-wound toroid type inductors (LCM1 and LCM2). These inductors are wound on a single core in such a way that they present a high impedance to the in-phase common-mode noise on each ac conductor. In addition, the Y-capacitors (CY1 and CY2) shunt or bypass the high-frequency common mode noise to ground. Differential Mode Noise on each AC conductor is suppressed by the two X-capacitors (CX1 and CX2), which tend to neutralize the out-of-phase high-frequency. The input resistor discharges these capacitors when the power is turned off.

## I.4. Analytical analysis of PWM schemes

In this section frequency and type domain analysis of PWM signal is discussed to better understand and quantify the switching harmonic component present in them. Some

evaluation tools are needed to compare the performance of different modulation schemes. Assuming that the PWM strategies are well programmed and the switching frequency is much superior to the fundamental frequency, the subcarrier harmonic content can be neglected.

#### **1.4.1. Frequency domain analysis**

Before getting into the details a simple Fourier series expansion of classical sinusoidal PWM is shown to give an idea how the switching frequency and the modulation index affect the harmonic amplitudes. To calculate the amplitude of the switching harmonics Fourier series expansion. For a three phase inverter the phase voltage can be broken into the Fourier series as shown in (1.43).

$$\begin{aligned}
 V_a = & V_f \cos(2\pi f_f t) \\
 & + C_{10} \cos(2\pi f_s t) + \sum_{K=1}^{\infty} C_{1K} \{ \cos(2\pi(f_s + 2Kf_f)t) + \cos(2\pi(f_s - 2Kf_f)t) \} \\
 & + 0 + \sum_{K=1}^{\infty} C_{2K} \{ \cos(2\pi(2f_s + (2K-1)f_f)t) + \cos(2\pi(2f_s - (2K-1)f_f)t) \} \\
 & + C_{30} \cos(2\pi f_s t) + \sum_{K=1}^{\infty} C_{3K} \{ \cos(2\pi(3f_s + 2Kf_f)t) + \cos(2\pi(3f_s - 2Kf_f)t) \} \\
 & + 0 + \sum_{K=1}^{\infty} C_{4K} \{ \cos(2\pi(4f_s + (2K-1)f_f)t) + \cos(2\pi(4f_s - (2K-1)f_f)t) \} \\
 & + \dots
 \end{aligned} \tag{1.43}$$

$$V_f = \frac{mV_{dc}}{2} \tag{1.44}$$

The Fourier series coefficients can be approximated rather accurately by Bessel functions, [44].

For odd n:

$$\begin{aligned}
 |C_{N0}| &= \frac{4}{N\pi} \left[ \frac{V_{dc}}{2} \right] \left| J_0 \left( \frac{\pi}{2} Nm \right) \right| \\
 |C_{Nk}| &= \frac{4}{N\pi} \left[ \frac{V_{dc}}{2} \right] \left| J_{2K} \left( \frac{\pi}{2} Nm \right) \right|
 \end{aligned} \tag{1.45}$$

For even n:

$$|C_{NK}| = \frac{4}{N\pi} \left[ \frac{V_{dc}}{2} \right] \left| J_{2K-1} \left( \frac{\pi}{2} Nm \right) \right| \tag{1.46}$$

Figure 1-40 shows the analytical frequency spectrum of pulse width modulated voltage.

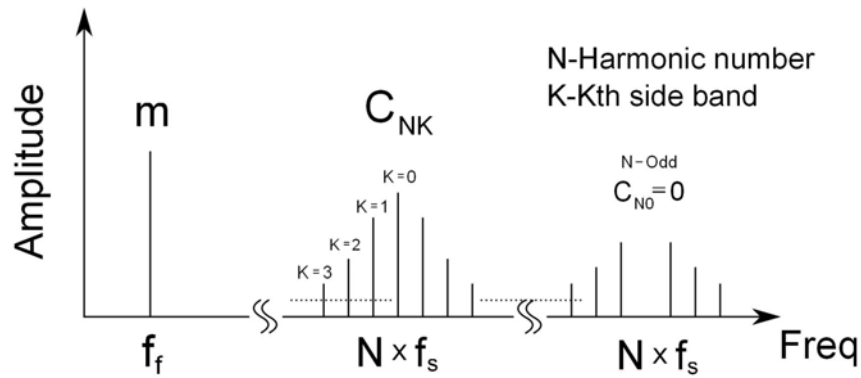


Figure I-40: Frequency Spectrum: PWM

Figure I-41 shows the carrier first five harmonic amplitudes with respect to the modulation depth, it can be observed that for higher modulation index the amplitudes decrease considerably specially for the first carrier harmonic.

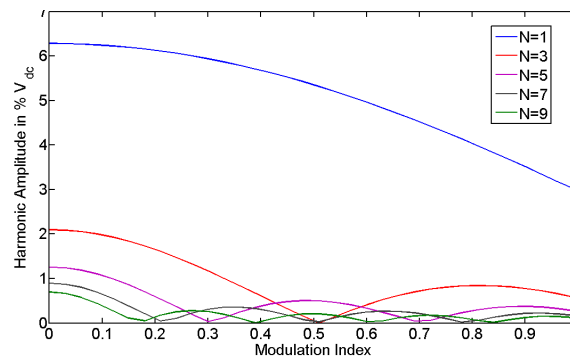


Figure I-41: Carrier Harmonic Amplitudes

Figure I-42 shows the how the sideband harmonic dependence on the harmonic number and the modulation index. It is clear from the graphic that for higher harmonic number and high modulation index the amplitudes decrease.

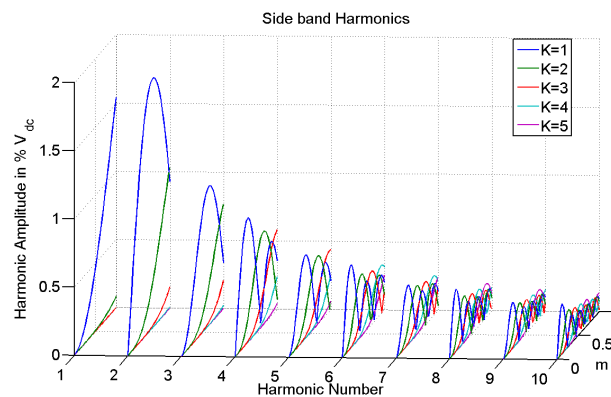


Figure I-42: Sideband Harmonic Amplitudes

#### **1.4.1.1. Power Spectrum Density**

The theoretical methods for analysing harmonic spectrum like Fourier series expansion seen previously are not suitable for randomly varying carrier frequency or position. Power Spectrum Density (PSD) of such signals is an easier alternative to analyse such signals.

The power density spectrum for a periodic function can be expressed as the Fourier transform of the autocorrelation of the signal.

$$S(f) = \int_{-\infty}^{\infty} R(\tau) e^{-j2\pi f\tau} d\tau \quad (1.47)$$

For wide-sense stationary (WSS) random signals the autocorrelation function is given by the expression below where E is the statistical expectation.

$$R(\tau) = \lim_{T_0 \rightarrow \infty} \frac{1}{2T_0} \int_{-T_0}^{T_0} E\{x(t)x(t-\tau)\} dt \quad (1.48)$$

The derivation of the expression for PSD for three phase DC/AC modulators is out of the scope of this research work, however the work from M. Bech's thesis [26] on the theoretical analysis of RPWM techniques has been taken as reference where he compared the theoretical results with experimental results. The calculated spectrums are comparable to the measured spectrums which lays down a theoretical background for random modulation techniques.

More over it was shown that the most simple random distribution law, Uniform distribution is as good as any other complex law like Gaussian etc. for either the switching harmonics [27].

#### **1.4.2. Waveform quality**

To evaluate the quality of PWM signals one of the method is to compare the unwanted harmonics to the desired fundamental component, an ideal sine wave. The most common performance indicators are discussed here.

##### **1.4.2.1. Harmonic Voltage Distortion**

The output voltage is generally sinusoidal with a given periodicity, so one method is to compare the total RMS value of the output voltage to the RMS value of the fundamental component.

The output voltage can be expressed as:

$$v(t) = V_0 + V_1 \cos(\omega_1 t) + V_2 \cos(2\omega_1 t) + V_3 \cos(\omega_3 t) + \dots \quad (1.49)$$

The RMS value of this voltage is given by:



$$\begin{aligned}
 V_{RMS} &= \sqrt{\frac{1}{T} \int_0^T v(t)^2 dt} \\
 &= \sqrt{\frac{1}{T} \int_0^T \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} V_n V_k \cos(n\omega_1 t) \cos(k\omega_1 t) dt}
 \end{aligned} \tag{1.50}$$

From (1.51) it can be readily seen that for  $n \neq k$  the integral over a complete period would be zero and for  $n=k$  the integral comes out to be  $\frac{1}{2}$ .

$$\cos(n\omega t) \cos(k\omega t) = \frac{1}{2} \{ \cos[(n+k)\omega t] + \cos[(n-k)\omega t] \} \tag{1.51}$$

Normally such voltages don't have a DC and even harmonic components and can be reduced to:

$$V_{RMS} = \sqrt{\sum_{n=1,3,5,\dots}^{\infty} \frac{V_n^2}{2}} \tag{1.52}$$

From which:

$$V_{RMS} = \sqrt{\sum_{n=1,3,5,\dots}^{\infty} V_{n,RMS}^2} \tag{1.53}$$

The total RMS value can be rewritten factoring the desired term, usually the fundamental component as:

$$\begin{aligned}
 V_{RMS} &= V_{1,RMS} \sqrt{1 + \sum_{n=3,5,7,\dots}^{\infty} \frac{V_{n,RMS}^2}{V_{1,RMS}^2}} \\
 &= V_{1,RMS} \sqrt{1 + THD^2}
 \end{aligned} \tag{1.54}$$

From where the unwanted components termed as total harmonic distortion (THD) is:

$$\begin{aligned}
 THD &= \sqrt{\sum_{n=3,5,7,\dots}^{\infty} \frac{V_{n,RMS}^2}{V_{1,RMS}^2}} \\
 &= \sqrt{\sum_{n=3,5,7,\dots}^{\infty} \frac{V_n^2}{V_1^2}}
 \end{aligned} \tag{1.55}$$

Or the THD can be expressed as:

$$THD = \sqrt{\left(\frac{V_{RMS}}{V_{1,RMS}}\right)^2 - 1} \tag{1.56}$$

A square wave voltage output can be expanded as:

$$v_{sqw}(t) = \frac{2V_{dc}}{\pi} \left( \cos \omega t - \frac{1}{3} \cos 3\omega t + \frac{1}{5} \cos 5\omega t - \frac{1}{7} \cos 7\omega t + \dots \right) \quad (1.57)$$

It should be noted that the third order harmonics disappear for three phase inverters, so the output voltage is less distorted.

$$\begin{aligned} THD_{sqw} &= \sqrt{\sum_{n=3,5,7,\dots}^{\infty} \frac{1}{n^2}} \\ &= \sqrt{\frac{\pi^2}{8} - 1} \end{aligned} \quad (1.58)$$

Till now we discussed the output voltage quality of the inverter which makes sense as it is the quantity which is controlled by the PWM. However it is the current that is of more interest for loss, torque ripple, output power calculations and hence calculating the current harmonic distortion  $THD_i$  may be more useful. Current depends on the load characteristics and therefore cannot be known before hand, however in many applications the load can be modelled by a lossy inductance, e.g. electric motors. Then the current harmonics can be given by:

$$I_n \cong \frac{V_n}{n\omega L} \quad (1.59)$$

$$\begin{aligned} THD_i &= \sqrt{\sum_{n=2}^{\infty} \left( \frac{V_n}{n\omega L} \right)^2} \\ &= \frac{1}{\omega L} \sqrt{\sum_{n=2}^{\infty} \left( \frac{V_n}{n} \right)^2} \end{aligned} \quad (1.60)$$

To get rid of the inductance this  $THD_i$  can be normalised with respect to  $I_1$ , the fundamental component of the current, this quantity is known as weighted total harmonic distortion (WTHD).

$$WTHD = \frac{\sqrt{\sum_{n=2}^{\infty} \left( \frac{V_n}{n} \right)^2}}{V_1} \quad (1.61)$$

The WTHD for the square wave of equation (1.57) can be calculated in a similar manner and is given by (1.62).

$$\begin{aligned} WTHD_{sqw} &= \sqrt{\sum_{n=3,5,7,\dots}^{\infty} \frac{1}{n^4}} \\ &= 0.1212 \end{aligned} \quad (1.62)$$

There is a serious limitation to THD as a yardstick to compare different PWM methods as the RMS value used for the calculation is the same for all being equal to  $V_{dc}$ .

Only the fundamental component differs from one technique to another with causes a change in THD values. In the next section another figure of merit is introduced which will be used to compare the modulation techniques developed during the course of this thesis. However WTHD is a better figure of merit, in the next section we'll calculate the harmonic distortion factor which give the harmonic losses in the load and is quite similar to WTHD.

#### I.4.2.2. Harmonic Distortion Factor

The current quality is sometimes a better criterion to compare PWM methods as they give an idea about the Harmonic losses, Harmonic distortion factor. The RMS copper losses due to switching over the interval  $T$  are proportional to the average of the square of the current during this time, which can be integrated over the fundamental period to get the cumulative losses over a complete cycle.

Harmonic currents are calculated under the assumption that the average internal electromotive force (EMF) of the load over a switching period is constant and the losses be in the load can be neglected for calculating the ripple current. That would eventually mean that load EMF and the average applied voltage can be considered equal in magnitude and in quadrature because of the inductive nature of the load.

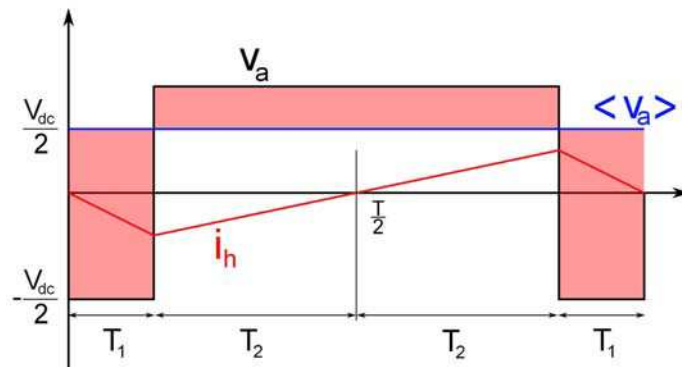


Figure I-43: Harmonic current

The harmonic current is the result of the harmonic voltage represented by the shaded regions. From the figure it can be noticed that the average value of the harmonic current is zero for a given modulation period, i.e. voltage harmonics do not have any impact on the average value of the current which is nothing but the consequence of zero average voltage ripple over a modulation period which is the basic condition to meet to satisfy the volt-sec balance given by:

$$\langle V_a \rangle_T = \frac{1}{T} \int_0^T v_a(t) dt \quad (1.63)$$

The harmonic power dissipated will be proportional to the square of current times load resistance. For single phase half bridge and full bridge inverters the harmonic current calculation is quite straightforward. For centre placed pulses are symmetrical about the half period mark, hence calculating the harmonic current for the first half is enough.

$$i_h(t) = \frac{v_a(t) - \langle v_a \rangle}{L} t$$

$$\text{Where : } v_a(t) = -V_{dc}/2 \quad \text{for } 0 \leq t \leq T_1$$

$$\text{and } v_a(t) = V_{dc}/2 \quad \text{for } T_1 \leq t \leq T_2$$
(1.64)

From the figure it can be seen that at  $T/2$  the harmonic becomes zero:

$$i_h(T_2) = 0$$
(1.65)

With the help of equation (1.64) and (1.65):

$$i_h(T_1) = \frac{V_{dc}}{2L} (-1-u)T_1$$

$$i_h(T_2) = \frac{V_{dc}}{2L} [(-1-u)T_1 + (1-u)T_2]$$

$$\text{Where : } u = \frac{2v_a}{V_{dc}}$$
(1.66)

From equation (1.65) and (1.66)  $T_1$  and  $T_2$  can be expressed in terms of

$$T_1 + T_2 = T/2$$

$$T_1 = \frac{(1-u)T}{4} \text{ and } T_2 = \frac{(1+u)T}{4}$$
(1.67)

The average square value of the ripple current can be found out by integrating the squared value over the entire period.

$$\langle i_h^2 \rangle_T = \left( \frac{V_{dc}}{2L} \right)^2 \frac{2}{T} \left\{ \int_0^{T_1} (1+u)^2 t^2 dt + \int_{T_1}^{T_1+T_2} [(-1-u)T_1 + (1-u)(t-T_1)]^2 dt \right\}$$
(1.68)

Simplifying it gives the following expression:

$$\langle i_h^2 \rangle_T = \left( \frac{V_{dc}}{2L} \right)^2 \frac{T^2(1-u^2)^2}{48}$$
(1.69)

The term ' $u$ ' can also be written as:

$$u = \frac{2v_a}{V_{dc}} = \frac{2A_1 \cos \theta}{V_{dc}}$$

$$= \frac{4}{\pi} m_i \cos \theta$$
(1.70)

So the average harmonic loss over a fundamental period can be calculated as follows:

$$\langle i_h^2 \rangle = \left( \frac{V_{dc}}{2L} \right)^2 \frac{T^2}{48} \int_0^{2\pi} \left( 1 - \frac{4}{\pi} m_i^2 \cos^2 \theta \right)^2 d\theta \quad (1.71)$$

That upon solving reduces to:

$$\langle i_h^2 \rangle = \left( \frac{V_{dc}}{2L} \right)^2 \frac{T^2}{48} \left( 1 - \frac{4}{\pi} m_i^2 + \frac{3}{2\pi} m_i^4 \right) \quad (1.72)$$

To compare different topologies irrespective of the switching frequency and the DC link voltage, harmonic distortion function can be introduced as:

$$f(m_i) = \left( 1 - \frac{4}{\pi} m_i^2 + \frac{3}{2\pi} m_i^4 \right) \quad (1.73)$$

Similarly the harmonic distortion function can be calculated for other inverter topologies. For full phase single bridge configuration is given by:

$$f(m_i) = \left( \frac{8}{\pi} m_i^2 - \frac{128}{3\pi^2} m_i^3 + \frac{6}{\pi} m_i^4 \right) \quad (1.74)$$

Under similar hypothesis HDF can be calculated for three phase PWM techniques, the expression for the harmonic current becomes (1.75).

$$\langle i_h^2 \rangle_T = \left( \frac{V_{dc}}{L} \right)^2 \frac{T^2}{48} \left\{ (u_2 - u_1)^2 + (u_2 - u_1)^3 + (u_2 - u_1)(u_2^3 - u_1^3) \right\} \quad (1.75)$$

The results for some of the techniques mentioned in the previous section are shown here, [16]. Equation (1.76) gives the HDF for the most basic modulation scheme, i.e. the sinusoidal modulation scheme where  $m$  varies from 0 to 0.785.

$$f_{SPWM}(m_i) = \frac{4}{\pi} \left( \frac{3}{2} m_i^2 - \frac{4\sqrt{3}}{\pi} m_i^3 + \frac{9}{8} m_i^4 \right) \quad (1.76)$$

Similarly for one-sixth and one-quarter third harmonic injection THIPWM1/6 and THIPWM1/4 are given by (1.77) and (1.78) respectively. The maximum value of the modulation index for THIPWM1/4 is slightly less than 0.907 which is the maximum possible value.

$$f_{THIPWM1/6}(m_i) = \frac{4}{\pi} \left( \frac{3}{2} m_i^2 - \frac{4\sqrt{3}}{\pi} m_i^3 + m_i^4 \right) \quad (1.77)$$

$$f_{THIPWM1/4}(m_i) = \frac{4}{\pi} \left\{ \frac{3}{2} m_i^2 - \frac{4\sqrt{3}}{\pi} m_i^3 + \frac{63}{64} m_i^4 \right\} \quad (1.78)$$

HDF for SVM and SVPWM are given by (1.79).

$$f_{SVPWM}(m_i) = \frac{4}{\pi} \left\{ \frac{3}{2} m_i^2 - \frac{4\sqrt{3}}{\pi} m_i^3 + \frac{9}{8} \left( \frac{3}{2} - \frac{9\sqrt{3}}{8\pi} \right) m_i^4 \right\} \quad (1.79)$$

For DPWMMIN, DPWMMAX, DPWM0, DPWM2 the HDF is the same and is given by equation (1.80).

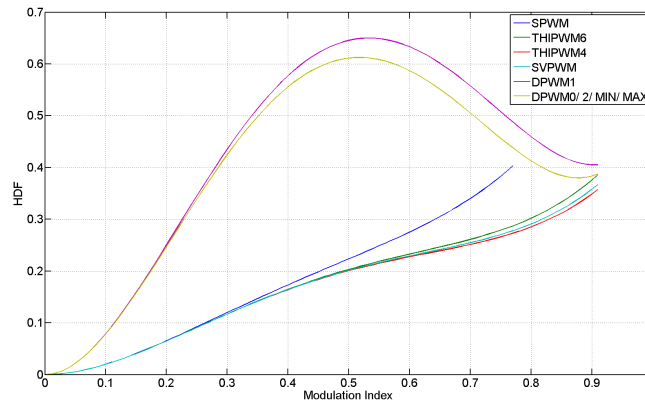
$$f_{DPWMMIN}(m_i) = \frac{4}{\pi} \left\{ 6m_i^2 - \frac{35\sqrt{3}}{2\pi} m_i^3 + \left( \frac{27}{8} - \frac{81\sqrt{3}}{64\pi} \right) m_i^4 \right\} \quad (1.80)$$

For DPWM1 and DPWM3 are given by equation (1.81) and (1.82) respectively.

$$f_{DPWM1}(m_i) = \frac{4}{\pi} \left\{ 6m_i^2 - \left( \frac{45}{2\pi} + \frac{4\sqrt{3}}{\pi} \right) m_i^3 + \left( \frac{27}{8} + \frac{27\sqrt{3}}{32\pi} \right) m_i^4 \right\} \quad (1.81)$$

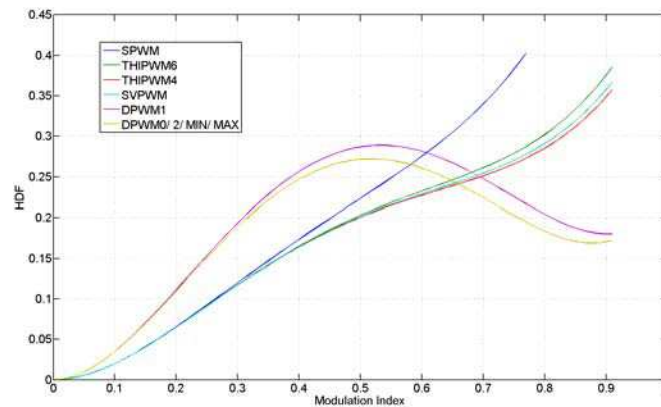
$$f_{DPWM3}(m_i) = \frac{4}{\pi} \left\{ 6m_i^2 - \left( \frac{45}{2\pi} - \frac{31\sqrt{3}}{\pi} \right) m_i^3 + \left( \frac{27}{8} + \frac{27\sqrt{3}}{16\pi} \right) m_i^4 \right\} \quad (1.82)$$

These functions can be plotted and compared. It can be seen that for low modulation index the DPWM techniques are have very high harmonic distortion. It can be noticed on Figure I-44 that THIPWM1/4 has the least HDF.



**Figure I-44: HDF**

The discontinuous techniques have lower effective frequency, the techniques mentioned here have 2/3rds the frequency of the continuous methods. It can be unfair to compare two techniques with different switching frequencies hence a factor of 2/3 can be added to make them comparable.



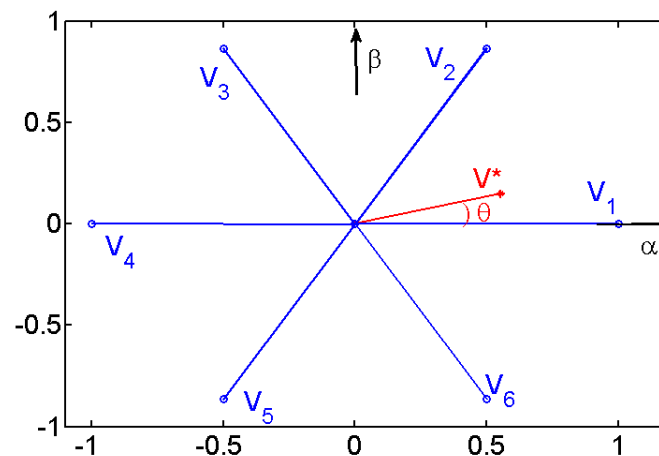
**Figure I-45: HDF; same effective frequency**

Now the HDF for the continuous and discontinuous techniques are more or less comparable with continuous methods perform better for low modulation index and the discontinuous method for high modulation index.

### I.4.2.3. Subcarrier harmonic analysis

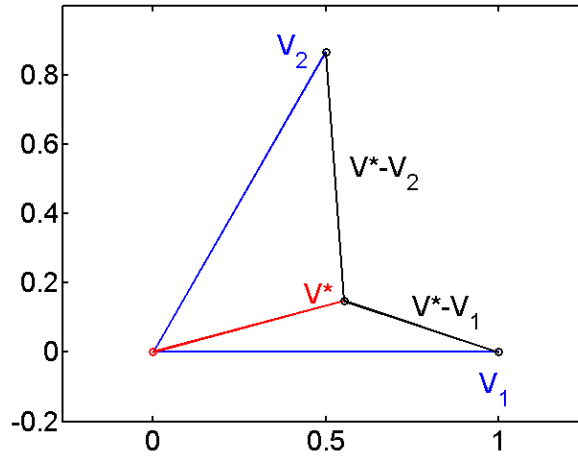
Till now we discussed how different modulation schemes can be evaluated in terms of voltage and current harmonic distortion and more importantly in terms of harmonic losses in the load. In this section a finer study is carried out to find the impact at the modulation period level. It is also called microscopic or subcarrier time scale analysis.

The most straightforward method is the space vector method. Let us consider an arbitrary reference vector  $V^*$ , let it be in the first sector as shown in the Figure I-46.



**Figure I-46: Space vector, arbitrary reference  $V^*$**

According to the standard SVM theory the reference voltage vector is formed by the two adjacent space vectors and the two zero vectors. Hence the harmonic space vector voltages are shown by Figure I-47 in black.

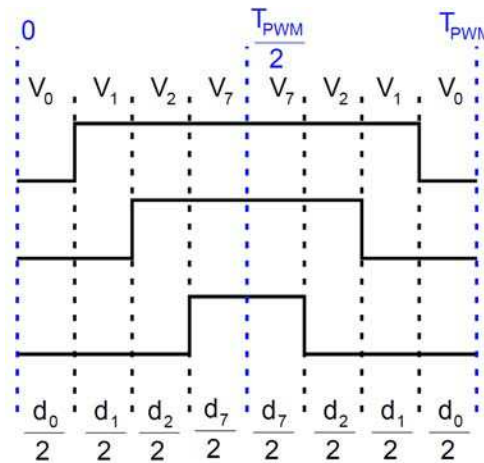


**Figure I-47: Harmonic voltage vectors**

Under the same assumptions as earlier i.e. considering the load as purely inductive. The harmonic currents can be calculated by the following expression (1.83), where  $V_j$  is the adjacent space vector.

$$i_h = \frac{1}{L} \int_0^T (V_j - V^*) dt \quad (1.83)$$

From the above expression it is obvious that the harmonic current depends on the reference vector, hence on  $m_i$  and  $\theta$ , as well as the space vectors. For standard SVM implementation the sequence of vector application is 0127-7210 as shown in Figure I-48.



**Figure I-48: SVM switching sequence**

So a total of four vectors are used and for sector one these are  $V_0$ ,  $V_1$ ,  $V_2$ , and  $V_7$ . To get rid of the load inductance value harmonic flux can be calculated instead by multiplying equation (1.83) by  $L$ . Harmonic flux for all of these vectors is given by (1.84).



$$\lambda_h = \begin{cases} -(m_i e^{j\theta}) \frac{d_0}{2} \\ \left( \frac{\pi}{3} - m_i e^{j\theta} \right) \frac{d_1}{2} \\ \left( \frac{\pi}{3} e^{\frac{j\pi}{3}} - m_i e^{j\theta} \right) \frac{d_2}{2} \\ -(m_i e^{j\theta}) \frac{d_7}{2} \end{cases} \quad (1.84)$$

The pulses are symmetrical about the half period, the flux trajectory for each inverter state is shown below for the first half of the modulation period. It can be seen that the harmonic flux forms a closed triangle and the resultant harmonic flux vector being zero at the end of the half period.

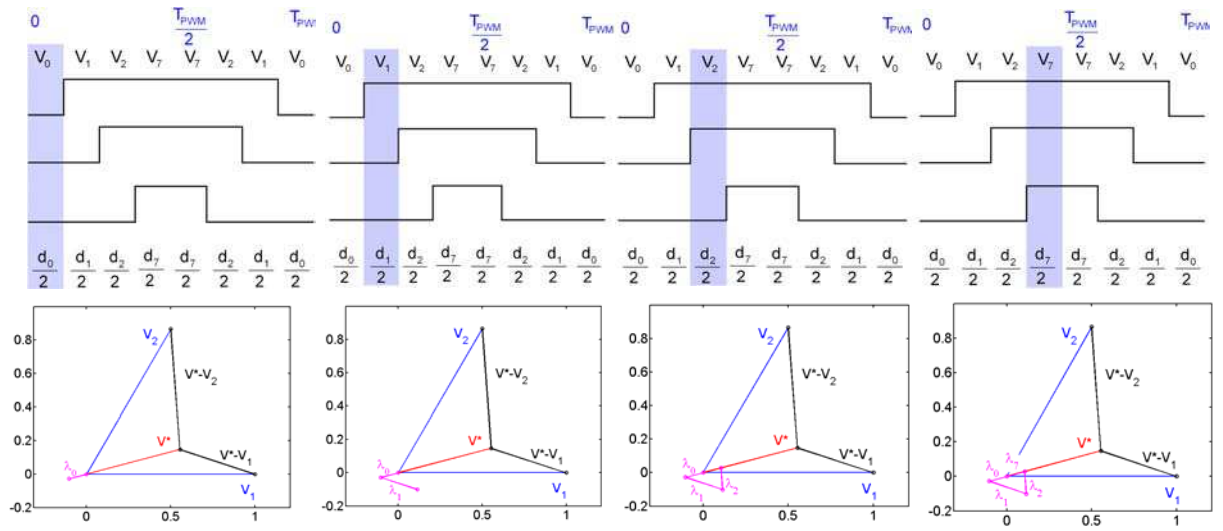


Figure I-49: SVM, Harmonic flux trajectories

Similarly the harmonic flux trajectory can be calculated for the next half period because of the symmetry they both result in similar triangular trajectory, the complete flux trajectory is shown in Figure I-50.

It is clear that the manner in which the vectors are applied can influence the flux trajectory. It is interesting to check out the different carrier types like, the sawtooth and RPP as well as other PWM with atypical zero vector placement.

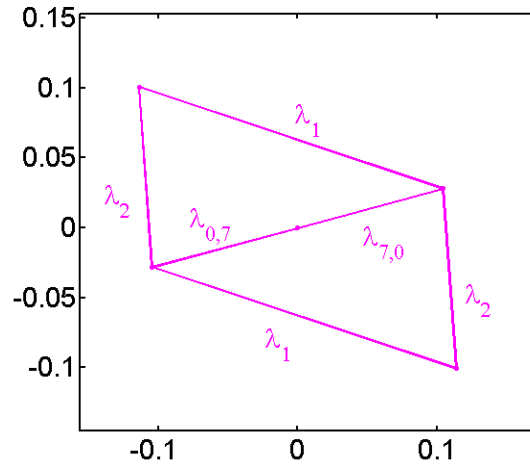


Figure I-50: SVM, Harmonic flux

Let's study the sawtooth carrier case. There are two types of sawtooth carrier leading or lagging edge. We'll discuss only one case here as the other has similar effect on the harmonic currents. The same of Figure I-46 is studied, from the Figure I-51 it can be seen that the sequence and duration of vector application is completely different.

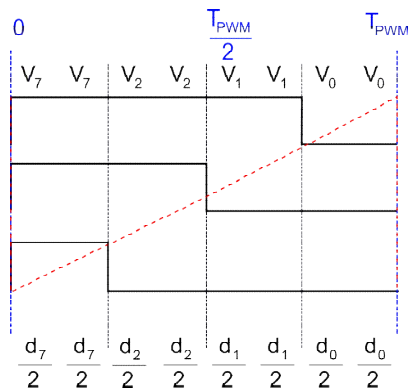


Figure I-51: Vector application Sawtooth carrier

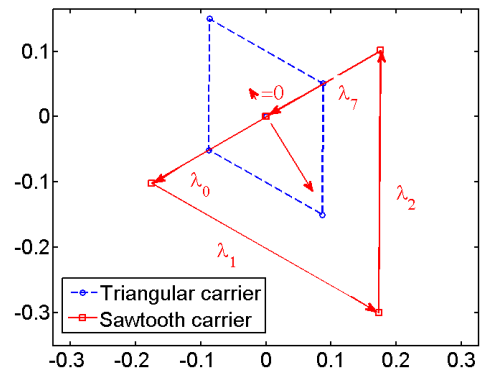


Figure I-52:  $\lambda_h$ ; triangular and sawtooth carrier

The harmonic flux trajectories for a triangular case as seen earlier and for the sawtooth case are plotted on the Figure I-52. For the sawtooth only one triangle is formed which is a lot bigger compared to the two triangles for a triangular carrier. This means there is less distortion with centred pulse and that current sampling can be done at the beginning, middle and at the end of the period to get the accurate average current.

It is interesting to see the effect of different zero vector partitioning. Let us take the example of discontinuous modulation scheme. As seen earlier on of the inverter leg stays idle, consider that the pulses are centred, Figure I-53.

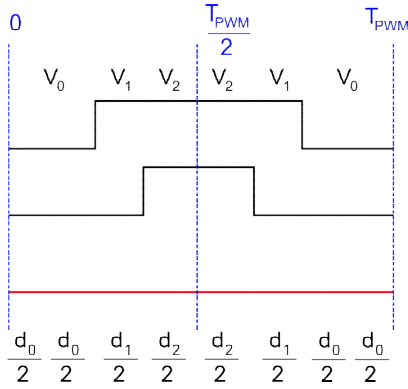
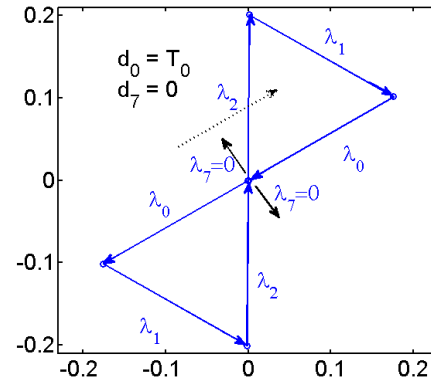


Figure I-53: Vector application DPWM

Figure I-54:  $\lambda_h$  DPWM triangular carrier

As can be seen in Figure I-54 due to symmetry about the half period like in the case of standard SVM two triangles are formed with the same dimensions but are displaced along the reference vector because of uneven zero vector distribution. Similarly other intermediate cases can be observed due to uneven use of vectors  $V_0$  and  $V_7$ .

To summarise this section it is observed that centred pulses produce lower harmonic currents whereas any type of PWM would produce zero average harmonic current over a complete switching period.

## I.5. Summary

In this first part of the thesis fundamentals of PWM methods are laid down; different types of carrier waves, different types of sampling updates their advantages and drawbacks. Emphasis was given on three phase inverters. Different types of switching functions, duty ratios and space vectors, principles were reviewed. The concept of a floating neutral was introduced as well which was followed by zero sequence voltage injection methods to increase the linearity of the pulsed output voltage without going into overmodulation and achieving discontinuities in the modulation function without creating any imbalance in the system.

Randomised PWM techniques were discussed as well, one that randomises the carrier frequency and the other that randomises the pulse position. Some practical aspects were discussed such as dead times, leakage current, current harmonics. This was followed by some performance evaluation tools and indices for PWM voltages. Different types of modulation schemes were compared. It was seen that at high modulation index DPWM techniques fared better than continuous PWM strategies. This framework is used as the developing grounds for the remaining parts of the thesis. This would conclude the first part of the thesis

## **PART II**

# **DEVELOPMENT OF PWM SCHEMES**

## II. Development of PWM Schemes

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In the previous chapter the fundamentals of PWM, the state of art was discussed to prepare ground for further development of PWM techniques. In this chapter some new PWM techniques developed for the thesis are presented and a comparison with the existing methods is made, these techniques can be classified into discontinuous, randomized modulators and two other modulation techniques that can optimise the number of current sensors and the DC-link capacitor. All of these techniques are based on space vector representation and calculation.

### II.1. Introduction

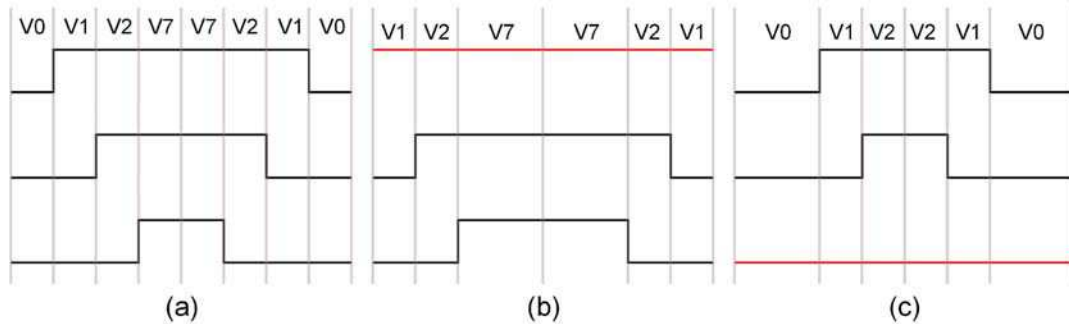
In order to meet the objectives of this research several different issues are dealt separately, one at a time. The issue of switching losses is dealt by introducing an innovative discontinuous modulator. To reduce the filtering effort and hence avoid the use of bulky EMI filters a random modulator is proposed. We know that the DC-link capacitors can amount to up to 50% of the inverter volume. A technique to reduce it has been put forward too.

In the first section of this chapter we'd do an in depth analysis of the degree of freedom related to the floating neutral loads to be able to develop a PWM strategy to minimise the switching losses. A generalised technique is presented which can adapt itself to the different types of load characteristics to optimise losses in the inverter. In the second part randomisation of PWM techniques looked into details and a randomised technique best suited under given conditions is proposed which is analogous to a RCF modulator. This is followed by techniques to reduce the number of current sensors required to perform a vector control on AC machines and a technique to reduce the inverter input current component which is directly related to the DC-link capacitor size.

### II.2. Discontinuous Space Vector Modulation

We saw we could achieve discontinuous modulation strategies upon injecting a zero sequence component in the voltage. On the other hand using space vector calculations it becomes easier, the following example will make it clearer. For SVM we calculate the active vector ( $V_i$ ,  $V_j$ ) duty cycles ( $T_i$ ,  $T_j$ ) and then complete it with the two zero vectors ( $V_0$ ,  $V_7$ ) with equal participation, which gives the following sequence  $V_0 \rightarrow V_i \rightarrow V_j \rightarrow V_7 \rightarrow V_7 \rightarrow V_j \rightarrow V_i \rightarrow V_0$ . Now if  $T_0 \neq T_7$  and we consider the two extreme cases where  $T_0=0$  and  $T_7=T_{\text{zero}}$  or  $T_7=0$  or  $T_0=T_{\text{zero}}$ , where  $T_{\text{zero}} = T_{\text{PWM}} - (T_i + T_j)$ . These two case

will lead to two different sequences;  $V_i \rightarrow V_j \rightarrow V_7 \rightarrow V_7 \rightarrow V_j \rightarrow V_i$  and  $V_0 \rightarrow V_i \rightarrow V_j \rightarrow V_j \rightarrow V_i \rightarrow V_0$  respectively. Of course the average output voltage will be the same in the two cases. However there will be a difference in the inverter states used for a modulation period, this will be clear from the following illustrations. Let us assume that the reference vector be in the first sector, this will give the standard SVM sequence as shown in Figure II-1-(a) shows 2 commutations for each inverter leg hence a total of 6 commutations per period. Whereas in the second case, Figure II-1-(b) only  $V_7$  is applied it can be seen that phase 'c' stays high throughout the period similarly for the third case Figure II-1-(c) only  $V_0$  is used phase 'a' stays low throughout. To generalise its the phase with the biggest duty that stays high through out and its the phase with the smallest value of duty cycle that stays low throughout for the two cases respectively. Hence only 4 commutations per modulation period for these two methods compared to 6 for the standard modulation technique.



**Figure II-1: SVM, DSVMMAX, DSVMMIN**

These two methods are analogous to the DPWMMAX and DPWMMIN hence they both produce  $120^\circ$  discontinuities in the positive and the negative half cycles respectively of the fundamental phase voltages. They can be referred to as DSVMMAX and DSVMMIN. It can be seen as an intrinsically or naturally discontinuous PWM scheme. This is one of the major advantages of SVM that one doesn't have to explicitly inject ZSS to achieve better linearity and discontinuity in the modulation function. In the next section it can be seen how this extra degree of freedom that we get in using the zero vectors.

### **II.2.1. Zero Vector Placement**

All the DPWM techniques that we saw in the previous chapter can be reproduced using space vector calculations by careful placement of the zero vectors. Some of the basic PWM schemes earlier as DPWMMAX, DPWM0, DPWM1, DPWM2, DPWM3 are shown in Figure II-2. For e.g. for DPWMMAX only  $V_7$  is used throughout and for the other techniques where there is a symmetrical distribution of the discontinuity between the positive and negative half cycles of the fundamental voltage  $V_7$  and  $V_0$  are used alternately.

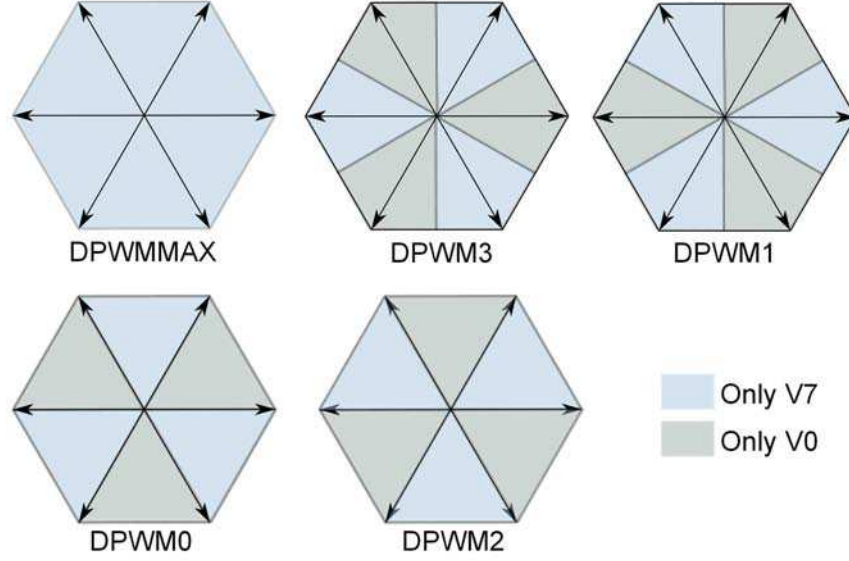


Figure II-2: Zero Vector Placement

### II.2.2. *Evolutionary Discontinuous SVM*

Evolutionary discontinuous SVM (EDSVM), developed during this PhD work was published in [30] and [31]. This modulation technique reduces the switching losses for a load with a varying voltage-current phase lag, for e.g. a variable speed drive where according to the speed and charge the load characteristics evolve in time. EDSVM is a very generic method that makes full use of the entire degree of freedom for phase clamping, i.e. depending on the load characteristics this method can lead to different types of clamping strategies, symmetrical or unsymmetrical about a phase or can have either equal or unequal clamping durations for different phases;

We saw earlier that the switching losses are proportional to the current commutated by the switch, the rest of the parameters remain more or less constant hence the current becomes the only variable in the equation (1.39). This means that a phase with maximum instantaneous current should be clamped. To do so the limits or boundary conditions and different possibilities should be studied.

#### II.2.2.1. *Clamping Study*

Considering the balanced three phase system given by (1.1), the following conditions must be met in order to clamp a phase without creating an unbalance in the system. To clamp a phase 'k' (K=a, b or c) the following condition (2.1) must be met:

$$v_k^* = v_{\max} \vee v_{\min} \quad (2.1)$$

Where  $v_{\max} = \max(v_a^*, v_b^*, v_c^*)$  and  $v_{\min} = \min(v_a^*, v_b^*, v_c^*)$  is the maximum and minimum instantaneous value of the voltage amongst the three phases respectively, or the

same function with a phase lag of  $\pi$  radians. Which means the permissible clamping zone for each phase can be given by the expressions (2.2) & (2.3).

$$v_{\max} = \begin{cases} v_a^*, & \text{for } \frac{\pi}{6} \leq \omega t < \frac{2\pi}{3} + \frac{\pi}{6} \\ v_b^*, & \text{for } \frac{2\pi}{3} + \frac{\pi}{6} \leq \omega t < \frac{4\pi}{3} + \frac{\pi}{6} \\ v_c^*, & \text{for } \frac{3\pi}{2} \leq \omega t < 2\pi + \frac{\pi}{6} \end{cases} \quad (2.2)$$

This means for the positive fundamental half cycle each phase can be clamped for a maximum of  $2\pi/3$ . This is the case for DPWMMAX.

$$v_{\min} = \begin{cases} v_a^*, & \text{for } \frac{\pi}{6} + \pi \leq \omega t < (\frac{2\pi}{3} + \frac{\pi}{6}) + \pi \\ v_b^*, & \text{for } (\frac{2\pi}{3} + \frac{\pi}{6}) + \pi \leq \omega t < (\frac{4\pi}{3} + \frac{\pi}{6}) + \pi \\ v_c^*, & \text{for } \frac{\pi}{2} \leq \omega t < \frac{\pi}{6} + \pi \end{cases} \quad (2.3)$$

Similarly for the negative fundamental half cycle each phase can be clamped for a maximum of  $2\pi/3$  so this would lead to DPWM technique we know as DPWMMIN. From where the maximum clamping duration for a leg per fundamental period is given by (2.4)

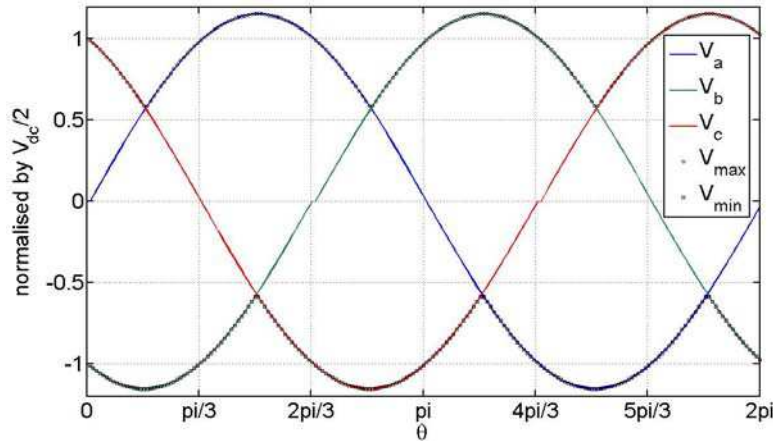


Figure II-3: Clamping zones

$$T_{c_{\max}} = \frac{2\pi}{3} \quad (2.4)$$

Other possibilities can be imagined as well where a phase can be clamped in both the fundamental half cycles, i.e. to the positive and negative rail successively in the same period. Let us consider the clamping possibilities for phase 'a'. Expression (2.5) regroups the positive and negative clamping regions, where C is the clamping durations, the first



suffix is the phase and the second represents the positive or negative clamping, and T the total duration.

$$T_{a\_clamping} = \begin{cases} C_{a\_+ve}, & \text{for } \frac{\pi}{6} \leq \omega t < \frac{2\pi}{3} + \frac{\pi}{6}, \\ & \text{or} \\ C_{a\_ -ve}, & \text{for } \frac{\pi}{6} + \pi \leq \omega t < \frac{2\pi}{3} + \frac{\pi}{6} + \pi \end{cases} \quad (2.5)$$

From where we can say that they both are comprised in the same interval given by (2.6).

$$\text{where } C_{a\_+ve}, C_{a\_ -ve} \in \left[0, \frac{2\pi}{3}\right] \quad (2.6)$$

All the existing DPWM techniques follow the condition given by equation (2.7). Where 'k' is any arbitrary phase, which basically means that all the phases will have the same total clamping duration.

$$C_{k\_+ve} + C_{k\_ -ve} = \frac{2\pi}{3} \quad (2.7)$$

In addition to that DPWM0, DPWM1, DPWM2, DPWM3 equally distribute this time, given by (2.8).

$$C_{a\_+ve} = C_{a\_ -ve} = \frac{\pi}{3} \quad (2.8)$$

Whereas for DPWMMAX is given by (2.9) and for DPWMMIN it's given by (2.10).

$$C_{a\_+ve} = \frac{\pi}{3}, C_{a\_ -ve} = 0 \quad (2.9)$$

$$C_{a\_+ve} = 0, C_{a\_ -ve} = \frac{\pi}{3} \quad (2.10)$$

This makes these two techniques unsymmetrical about a phase. In the next section we'd try to exploit all the freedom to develop a technique to reduce the switching losses.

### II.2.2.2. Switching loss reduction

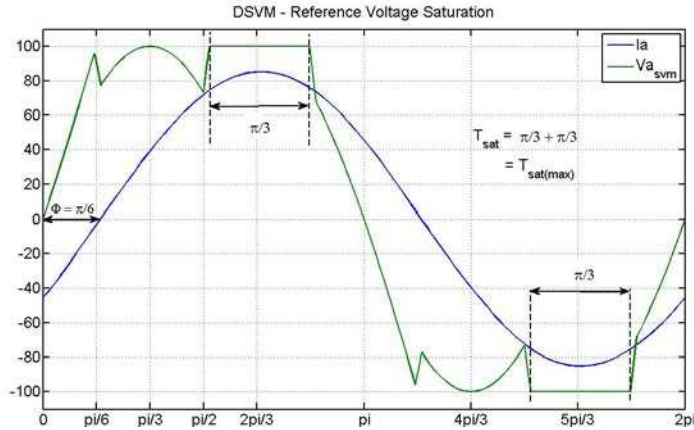
The proposed DSVM algorithm allows minimizing the switching losses . The simplest discontinuous SVM technique, referred to above as DSVMMIN, as the basic technique to complement the generalized modulator in terms of the load angle  $\phi$ . The DSVM Algorithm is very simple and intelligent with almost no extra burden on the processor. First of all the duty cycles are calculated for the given reference voltages (generated by the torque control algorithm for electric drives) using the algorithm

DSVMMIN i.e. using V0. Now to optimally clamp the inverter legs we use the measured phase currents, this makes it a modulation technique with feed-back but it's not the output voltage that is fed back hence it remains an atypical closed loop system. DSVMMIN was chosen over other DSVM techniques because in our case (the inverter is fed by a battery) the zero vector 'V0' does not generate any common mode voltage whereas vector 'V7' generates the maximum common mode voltage equal to DC bus voltage,  $V_{dc}$ . Hence lower chances of a discharge through the motor bearings as seen earlier. However it is obvious that clamping to the positive DC rail will make it impossible to avoid the space vector V7.

This technique follows equation (2.7) that means that all the inverter legs will have the same degree of discontinuity. However it is neither bounded by (2.8) or (2.9) and (2.10), the repartition of the clamping durations 'x' for both the half cycles are completely flexible. With x belonging to the closed interval  $[0 \ 2\pi/3]$ .

$$T_{a\_clamping} = \begin{cases} x, & \text{clamped to positive DC rail,} \\ \frac{2\pi}{3} - x, & \text{clamped to negative DC rail} \end{cases} \quad (2.11)$$

The variable 'x' depends on the phase difference between the voltage and the current,  $\phi$ . For example for  $0 \leq \phi \leq 30^\circ$ , in the Figure II-4 the modulation function is shown for the limiting value of  $\phi = 30^\circ$  where  $x = \pi/3$ , till this value of  $\phi$  there is equal repartition of the clamping zones between the two half cycles.



**Figure II-4: Intelligent Switching Function –  $\phi = 30^\circ$**

For  $\phi > 30^\circ$  x becomes less than  $\pi/3$  and leads to an unsymmetrical voltage clamping as can be seen in Figure II-5. This technique modifies the modulation function to adapt to the load power factor angle to track the maximum power transfer through all the phases. For variable speed drive the power factor angle varies with the speed and this technique uses a current feedback to keep adapting the switching function to minimize the switching losses and hence is named here as Evolutive Discontinuous Space Vector Modulation (EDSVM).

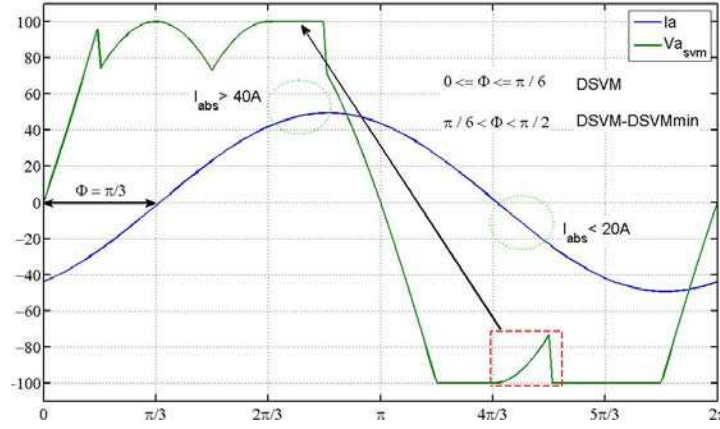


Figure II-5: Intelligent Switching Function -  $\phi > 30^\circ$

The flowchart shown in Figure II-6 depicts the EDSVM algorithm. It can be noted that not many instructions and conditions are needed to alter the initially calculated duty cycle values to optimize the switching losses.

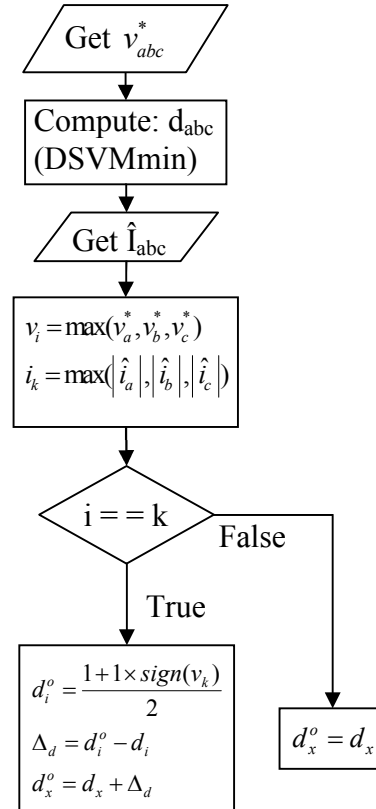
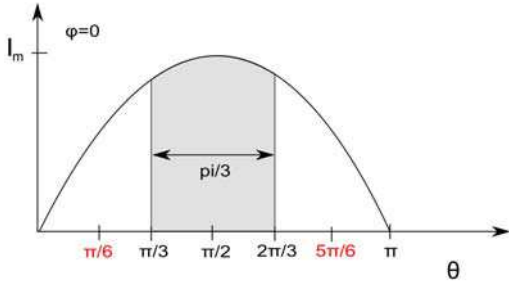


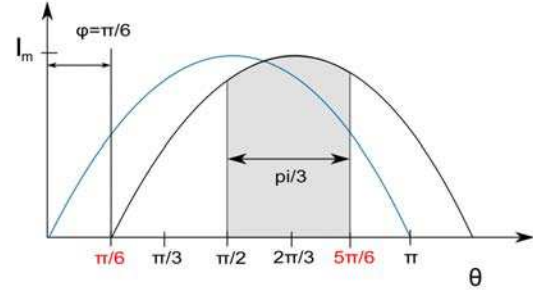
Figure II-6: Algorithm EDSVM

The switching losses for the modulation technique proposed in this section are calculated using (1.39), which assumes linear relationship between switching losses and the switched currents. Figure II-10 shows the theoretical switching loss reduction while feeding a load with a variable power factor angle with the proposed method compared to

the standard continuous SVM. It should be noted that the curve is symmetric about  $0^\circ$  and  $90^\circ$  and hence could be extended to  $-180^\circ$  to  $180^\circ$  for motor/generator mode.

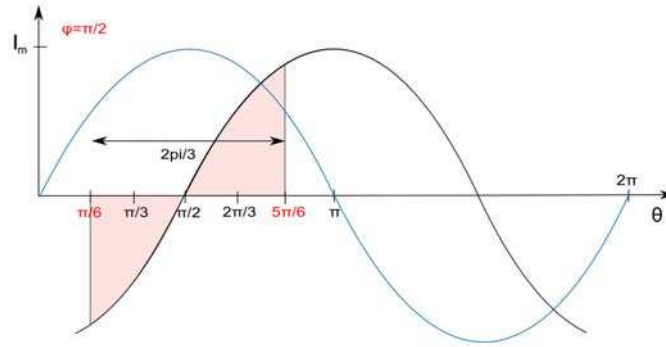


**Figure II-7: Switching loss reduction  $\varphi=0^\circ$**



**Figure II-8: Switching loss reduction  $\varphi=30^\circ$**

The reduction in switching losses using EDSVM can be calculated with help of Figure II-9 and Figure II-10 for  $\varphi < 30^\circ$  only one half cycle of the voltage and current are shown here. the other half being identical to the first one due to symmetrical clamping. The reduction in the switching loss is the shaded area which amounts to be half of the total area under the curve.



**Figure II-9: Switching loss reduction  $\varphi=90^\circ$**

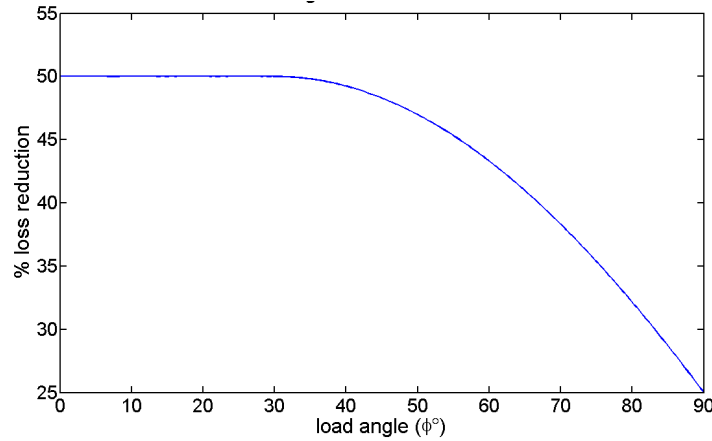
For  $\varphi > 30^\circ$  the phase is asymmetrically clamped to the positive and negative terminal, in the Figure II-9,  $\varphi=90^\circ$  an extreme case where the clamping is done only in one half period. Similarly the shaded area is the reduction of switching losses.

$$\cos \theta \Big|_{\pi/3}^{2\pi/3} \quad \text{for } 0 \leq \varphi < \frac{\pi}{6} \quad (2.12)$$

EDSVM can be divided in to two category; symmetric and unsymmetrical. The reduction in switching losses for symmetrical clamping can be given by equation (2.12), which represents the shaded region of Figure II-9 and Figure II-10.

$$\cos \theta \Big|_{\phi - 5\pi/6}^{\phi - \pi/2} \quad \text{for } \frac{\pi}{6} \leq \varphi < \frac{\pi}{2} \quad (2.13)$$

For the second case the reduction in switching losses can be given by equation (2.13). These two expressions are traced in Figure II-10.



**Figure II-10: Switching loss reduction-DSVM**

Theoretically the switching losses using the proposed method can be reduced by 50% for  $0 \leq \phi \leq 30$  and afterwards it continues to decrease till hits its lowest point at 25%. This means no matter what the load power factor this technique can reduce the switching losses by at least 25%.

### II.2.2.3. Unbalanced load condition

All three phase systems are prone to imbalance, in this section we consider the possibility of an unbalanced three phase system, i.e. the impedance of all the phases is not identical. This would mean that different the RMS value of the currents induced in the phases would not be the same. This would mean that the switching losses too will be different for each phase. In order to optimize the losses this imbalance needs to be taken into account. In electrical machines turn-turn short can occur due to weak isolation in the stator windings which can create an imbalance. Unlike seen in the previous section we'd increase the degree of freedom by not following equation (2.7). This condition is replaced by (2.14).

$$C_{a_{+ve}}, C_{a_{-ve}} \in \left[0, \frac{2\pi}{3}\right] \& C_{a_{+ve}} + C_{a_{-ve}} \in \left[0, \frac{4\pi}{3}\right] \quad (2.14)$$

where  $C_a + C_b + C_c \in [0, 2\pi]$

Let us imagine that phase 'a' carries the highest RMS current and phase 'c' the least then the clamping durations allotted to each phase can be given by (2.15).

$$C_{a_{+ve}} + C_{a_{-ve}} \geq \frac{2\pi}{3} \quad (2.15)$$

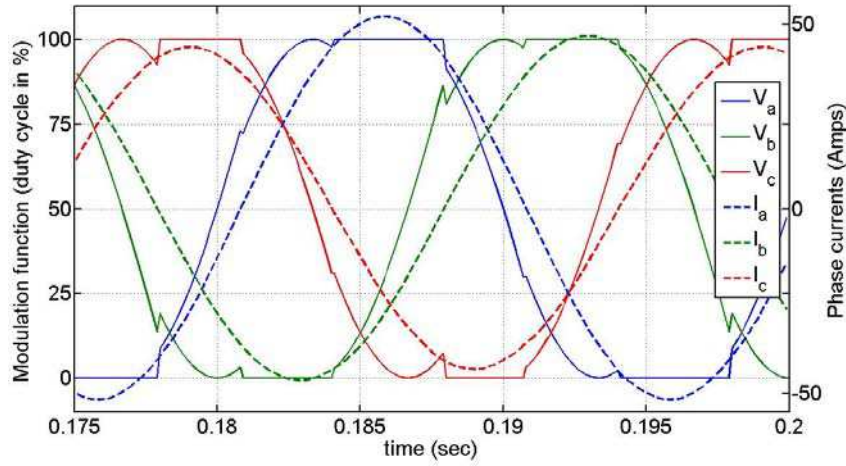
$$\& C_a > C_b > C_c$$

For any arbitrary phase 'k', the boundary conditions for a phase, for both positive and negative half cycles over a complete fundamental period is  $2\pi/3$  which can be given by (2.16).

$$C_{k_{+ve}} + C_{k_{-ve}} = \frac{4\pi}{3} \quad (2.16)$$

$$\& C_{k_{+ve}} = C_{k_{-ve}} = \frac{2\pi}{3}$$

This would in turn mean that only two-thirds of the fundamental time period is left for the other two phases which shows the unsymmetrical nature of the proposed method. The maximum clamping duration for a three phase system is  $C_{total} = 2\pi$ , hence the effective switching frequency would be reduced to two-thirds. An exaggerated case can be seen in Figure II-11 it can be noticed that all the phases have different clamping durations, phase 'a' being allocated the most and phase 'c' the least. The proposed method can extend the life of the inverter before the fault could be repaired by allocating the concerned phase the appropriate clamping to avoid complete failure of the corresponding inverter leg.



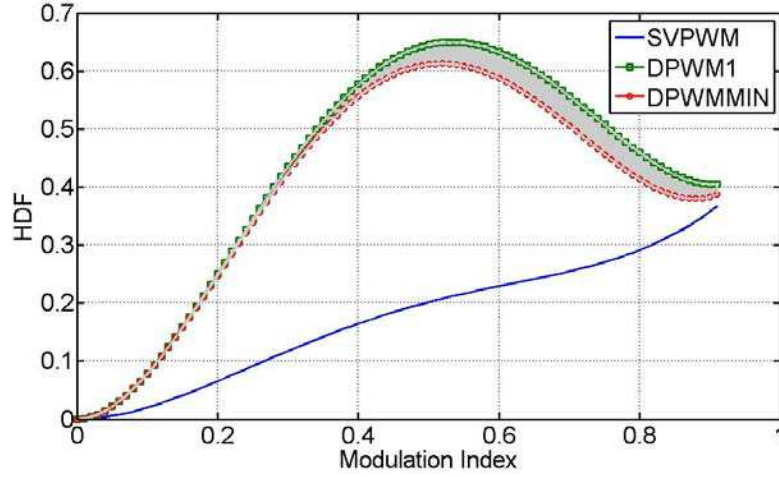
**Figure II-11: DSVM: unsymmetrical clamping**

No two power switches are exactly identical in behaviour, the switching transients may vary along with the rest of the parameters like stray capacitances between different terminals and which can cause the switches to commute at different speeds which in turn means different switching losses. Another possibility can be observing the IGBT temperature, normally for high voltage high power applications IGBTs are used. The temperatures of the inverter legs can be regulated by providing the hottest leg with the least number of commutations. Due to uneven heating of the different switches the switch suffering the highest thermal stress becomes the weakest link and would reduce the life of the inverter. This can be avoided by regulating the IGBT temperature as proposed here.

#### **II.2.2.4. Waveform quality**

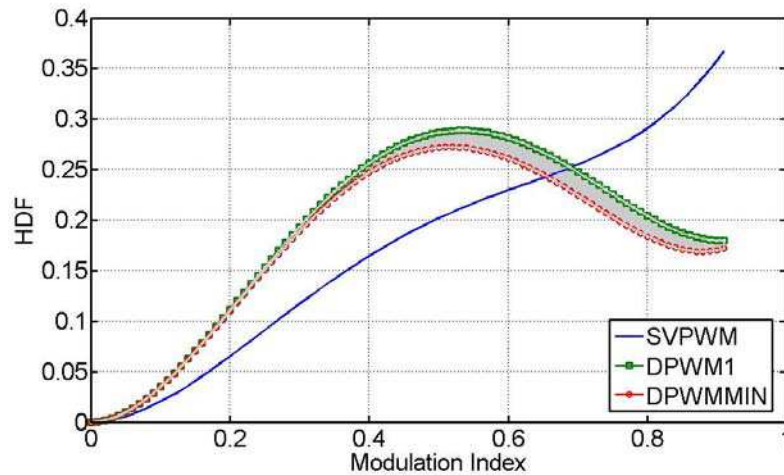
The effectiveness of this method in terms of reduction of switching losses and the possibility to regulate the switch temperature has been seen so far in this section now an analysis in terms of harmonic current quality as seen the previous chapter. So harmonic

distortion factor (HDF) will be calculated for the EDSVM technique. The EDSVM technique has different type of discontinuities depending upon load angle. However it varies between two well-known techniques the DPWMMIN and DPWM1, so the HDF for EDSVM will be confined between these two values given by (1.80) and (1.81).



**Figure II-12: HDF EDSVM**

The shaded area of the Figure II-12 represents the HDF for EDSVM which is the area between the curve for DPWM1 and DPWMMIN. It can be seen the HDF for EDSVM is greater for low modulation index values and becomes somewhat comparable at higher modulation index. As the discontinuous techniques have lower effective frequency compared to continuously modulated voltage. The switching frequency for this continuous modulator can be increased to get same effective frequencies for both discontinuous and continuous modulators.

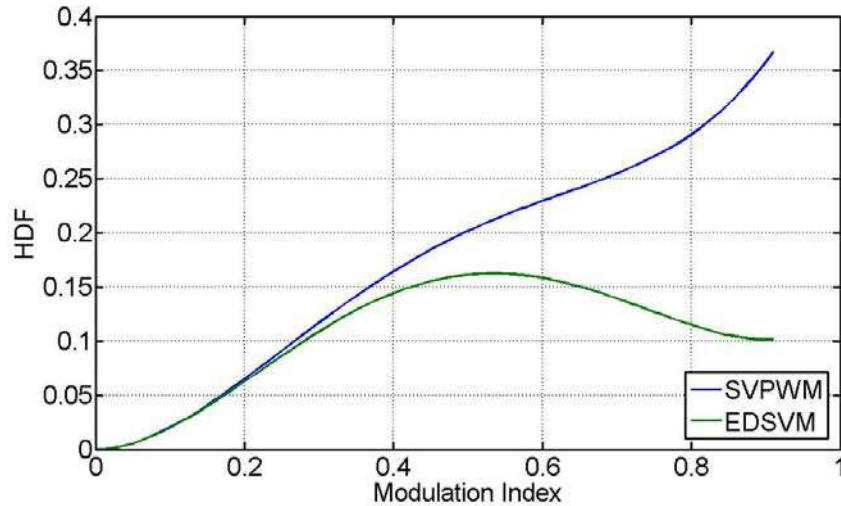


**Figure II-13: HDF EDSVM; same effective frequency**

With complete  $2\pi$  clamping the effective frequency is reduced to two-thirds of the real the switching frequency. Figure II-13 shows the HDF for an increased frequency of the discontinuous modulator by one and half times. It can be observed that the HDF for



EDSVM is still greater for low modulation index values but becomes smaller for modulation index higher than 0.7. In this cases the switching losses for EDSVM gets increased by the same factor as the increase in the switching frequency. As the discontinuous techniques have lower effective frequency compared to continuously modulated voltage.



**Figure II-14: HDF EDSVM; equal switching losses**

Another case can be imagined where the load angle  $\varphi$  is  $\leq 30^\circ$ , and the switching losses for EDSVM are reduced by 50% hence to have same switching losses for both the cases the switching frequency for EDSVM should be doubled. Figure II-14 shows that with twice the switching frequency the HDF factor for EDSVM is well below that of SVM for all values of modulation index. Hence according to the requirement higher switching frequency with lower switching losses and better quality voltage signals can be attained.

#### **II.2.2.5. Simulations**

PWM is basically an open loop technique but since current feedback is required for EDSVM though not for voltage control but for clamping with current tracking, hence closed loop simulations are required to validate the algorithm. All simulations are done on Matlab/Simulink. The inverter and PMSM are taken from the Simulink library 'SimPowerSystems'.



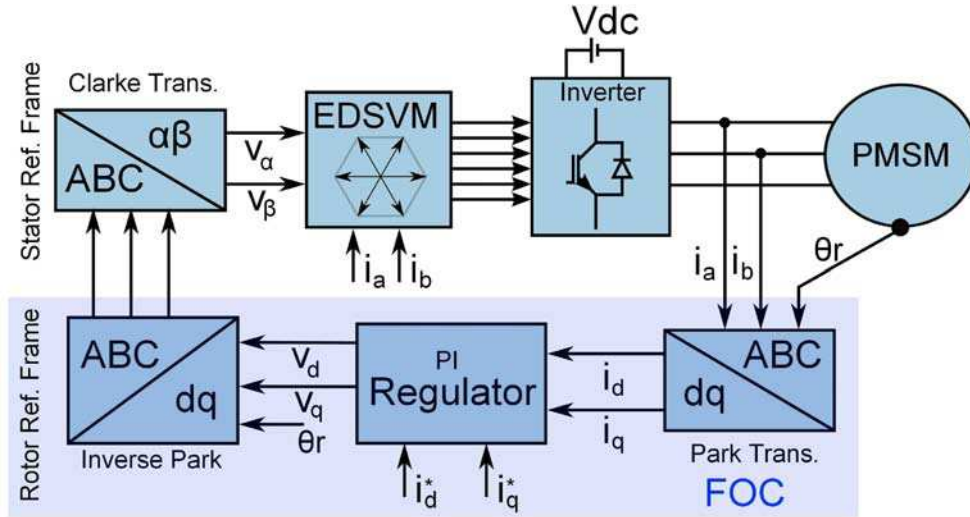


Figure II-15: FOC with DSVM

A PMSM electric drive Figure II-15 is simulated using Field Oriented Control (FOC) which decouples the stator currents into two components viz. a torque producing ' $i_q$ ' and a magnetizing ' $i_d$ ' component as can be seen in the phasor diagram, Figure II-16 This leads to a linear relationship between the torque and the current's q component and for maximum torque the magnetizing component is kept zero, (2.17).

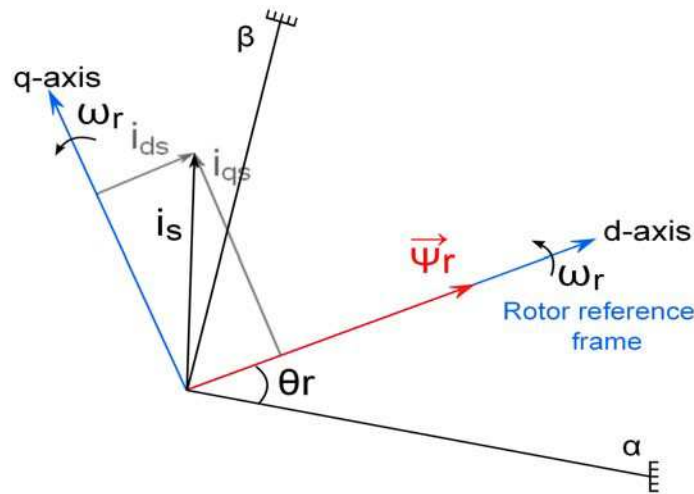
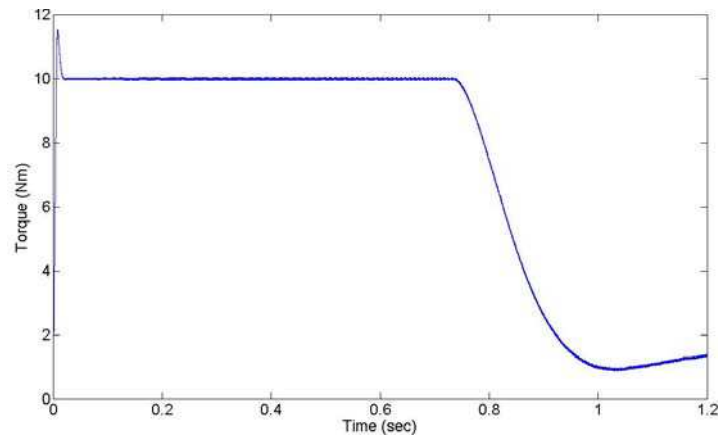


Figure II-16: FOC Phasor diagram

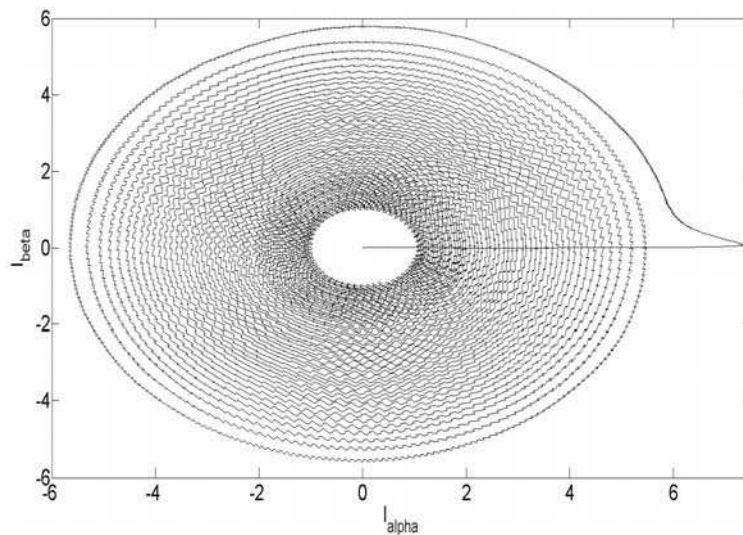
$$i_q^* = \frac{2}{3P\psi_{PM}\tau^*} \quad (2.17)$$

$$i_d^* = 0$$



**Figure II-17: EDSVM torque response**

Figure II-17 shows the torque response of the machine for a desired torque of 10Nm at stand still at no load conditions, the drive is simulated till it reaches its steady state speed at around 0.8 seconds where the torque starts to die down.



**Figure II-18: Stator Current Evolution**

Figure II-18 shows the stator current evolution in the  $\alpha$ - $\beta$  plane. We get anticipated results. The outer circle represents the starting of the motor, when it draws in the maximum current and the inner most circles represent low currents when the machine reaches its steady state and draws lower current.

Figure II-19 shows the stator current ' $i_a$ ' and the evolving switching function ' $d_a$ '. To facilitate the comparison between the duty cycle and the current, the current is multiplied by a constant gain. It can be observed that the duty cycle is saturated to 100 and 0 with positive and negative current peaks adapting itself with the changing fundamental frequency and load angle. As we saw previously switching losses are directly proportional

to the switching losses hence placing the discontinuous with current peaks minimises the losses.

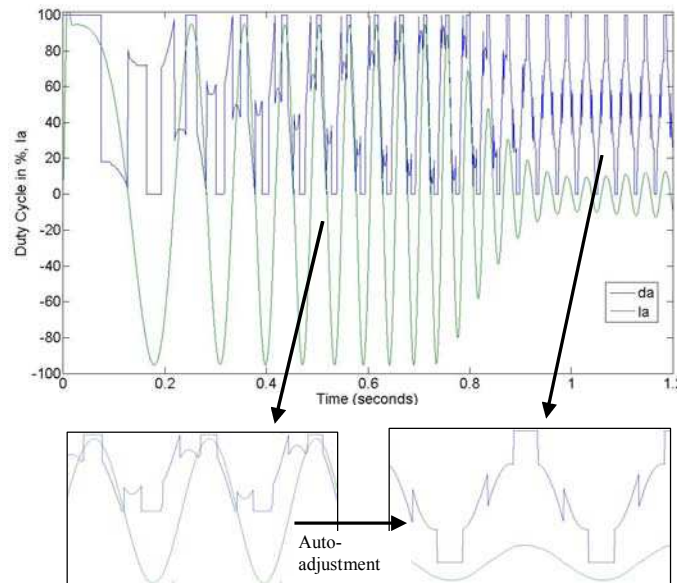


Figure II-19: EDSVM- Ia, da

### II.2.3. Summary

A self-adapting discontinuous SVM is introduced which can drastically reduce the switching losses and can also adapt itself to any imbalance in the load as well as regulate the power switch temperatures which in turn would increase the inverter life expectancy. Theoretical calculations show that with EDSVM it is possible to reduce the switching losses till 50%. The simulation results validate the theory with comparable results for SVM and DSVM. Now the inverter efficiency of this new techniques has to be practically calculated which is done in the next part of the thesis.

## II.3. Randomised Space Vector Modulation

In this section the randomization of space vector modulator will be studied. Random pulse width modulation techniques that were discussed in the first part will be used to compare the possibilities and analogies between the two techniques. First of all, the different parameters would be identified which will be followed by an appropriate analysis of randomizing that particular parameter and its influence on the output voltage.

### II.3.1. Randomisation parameters

The different parameters of the space vector modulator are first to be identified and ways of randomising these parameters are developed. In this chapter five parameters

have been put forth with example and there relevance towards achieving the goal is discussed as well. Each subsection introduces each one of these 5 parameters.

### II.3.1.1. Sequence of vector application

SVM has a specific sequence in which the vectors applied within a modulation period, as seen earlier the standard sequence for SVM is  $V_0 \rightarrow V_i \rightarrow V_j \rightarrow V_7 \rightarrow V_7 \rightarrow V_j \rightarrow V_i \rightarrow V_0$ , where the  $j=i+1$  for  $i=1, \dots, 5$  and  $j=1$  for  $i=6$ . This sequence of application of vectors may be randomized. Vectors  $V_0$ ,  $V_i$ ,  $V_j$  and  $V_7$  can be applied in any random order. Let us consider first the case of pulses symmetrical about the half period ' $T_{PWM}/2$ '. Let us consider the desired voltage vector lies in the first sector, then the sequence would be  $V_0 \rightarrow V_1 \rightarrow V_2 \rightarrow V_7 \rightarrow V_7 \rightarrow V_2 \rightarrow V_1 \rightarrow V_0$  as shown in the Figure II-20 on the left hand side. It can be noticed that there are total of 6 commutations required to realise this sequence.

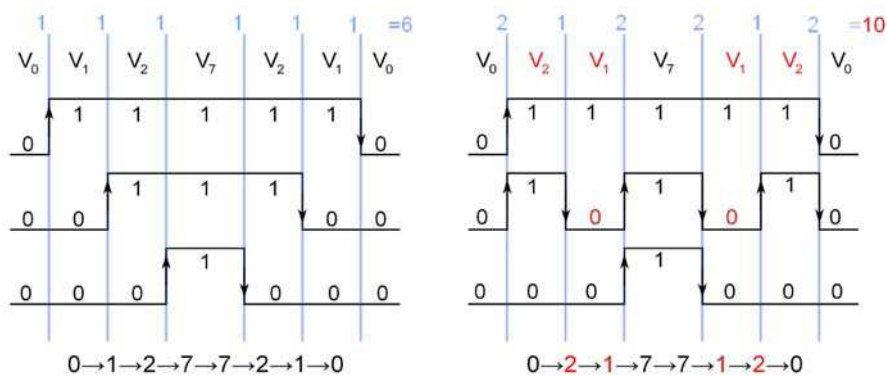
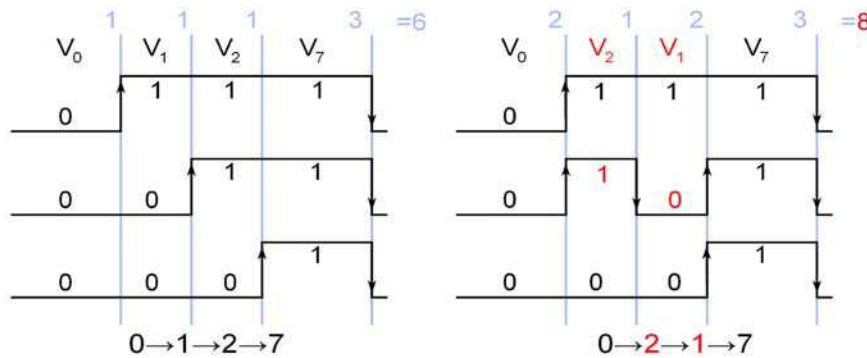


Figure II-20: Random Vector Sequence, triangular carrier

On the other hand on the right a different sequence is imagined which is  $V_0 \rightarrow V_2 \rightarrow V_1 \rightarrow V_7 \rightarrow V_7 \rightarrow V_1 \rightarrow V_2 \rightarrow V_0$  as can be seen in the Figure II-20 on the right hand side. This adds extra commutations, there are a total of 10 commutations instead of 6 as for the standard SVM which increases the number of commutations by two-thirds this will increase the switching losses in similar proportions. Apart for the increased switching losses another serious drawback of this method that it is increases the complexity of generating the pulses by adding several other conditions whereas with the standard technique a mere comparison of the duty cycle and the counter is required to produce the pulses.

The second possibility if considering the lesser common form of SVM with a leading/trailing edge counter. Let us consider the trailing edge case, the sequence would be  $V_0 \rightarrow V_i \rightarrow V_j \rightarrow V_7$ . Many combinations may be imagined, let us again take the example of the reference voltage vector lying in the first sector, hence the vector sequence for standard sawtooth PWM would be  $V_0 \rightarrow V_1 \rightarrow V_2 \rightarrow V_7$  as can be seen in Figure II-21 on the left hand side.



**Figure II-21: Random Vector Sequence, sawtooth carrier**

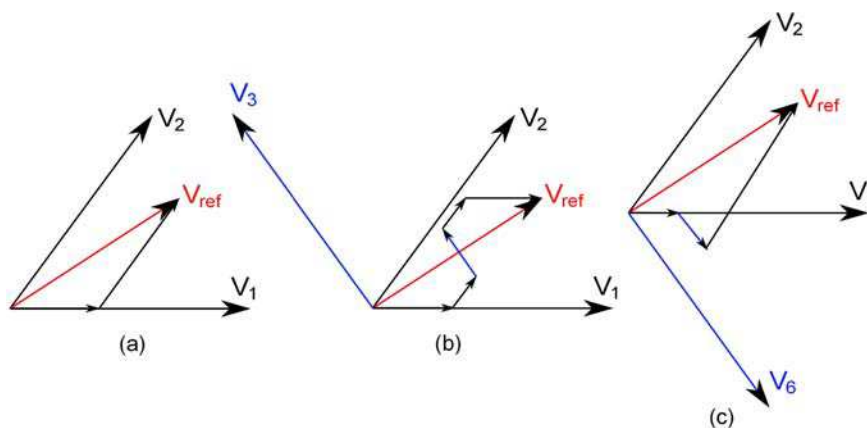
As we did in the previous example just changing the sequence of active vector  $V_1$  and  $V_2$  change the commutation pattern of the leg 'b' of the inverter, adding two extra commutations. Similar conclusions can be drawn for this case as well.

Randomizing the vector application sequence increases the effective switching frequency which may push the switching harmonics a little further in the spectrum but they'd still remain cluttered at the same frequency hence the power associated with these switching harmonics would not be distributed as we would have liked which makes this technique less attractive, moreover the pulse generation will become a little complex compared to one simple comparison between the duty cycle and the modulator counter for standard cases. Hence this method is of no interest to us and no further effort would be made to explore it any further.

### II.3.1.2. Choice of active vectors

Another possible parameter that can be randomised is the choice of the inverter state to eventually achieve the required volt-seconds. In the standard SVM technique only adjacent vectors are used. In this section other possibilities will be studied.

Let us again take an example where the desired output voltage vector lies in the first sector. The standard way to produce a voltage vector lying in the first sector is using the vectors  $V_1$  and  $V_2$  as shown in the Figure II-22-(a).

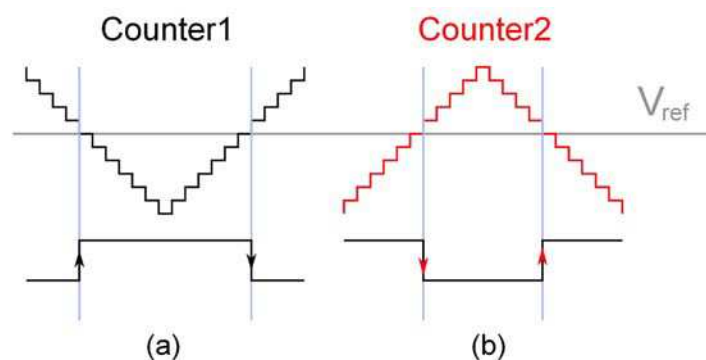


**Figure II-22: Random active vector selection**

However other vectors may be used as well two other possibilities are shown in the Figure II-22-(b) and (c); where in addition to vector  $V_1$  and  $V_2$  vector  $V_3$  and  $V_6$  are used respectively. Other possibilities can be imagined as well like using only  $(V_1+V_3)$ ,  $(V_2+V_6)$ , even combinations formed by vectors  $V_4$  and  $V_5$  can be imagined. This sure would depend on the magnitude of the desired vector or the modulation index at high  $m_i$  there is lesser margin to use wider vectors.

Of course it is important to find out the influence of applying this method at the modulated voltage signal. Let us see now the repercussions it will have on the commutation signals. The first thought that comes to mind is that more than two commutations will be required at least for a phase to be able to produce three different active vectors in one modulation period, of course this is the most simple of going about it. However increasing the number of commutations leads to increased switching losses and increases the complexity in of the pulse generator. Hence this would be an undesirable way of going about it.

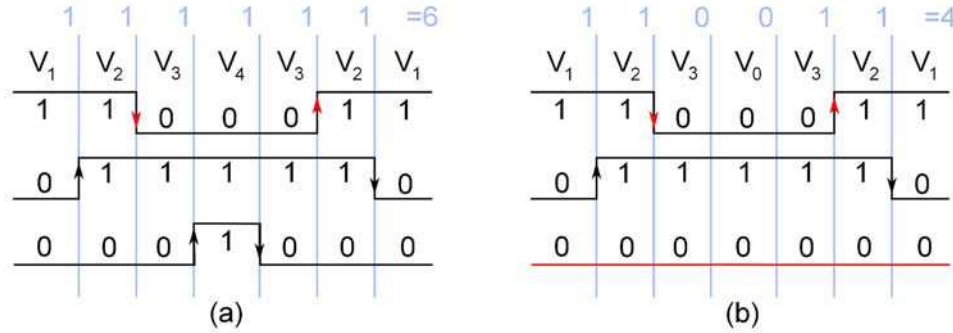
Another approach to this problem is shown in this section, only the symmetrical case will be discussed here however other ways of generating the commutation signal for cases where more than two active vectors are used or when two non-adjacent active vectors are used will be discussed in more detail. Let us suppose we have to triangular counters as shown below, Figure II-23. The comparison of the reference with these two complementary counters leads to two different forms of pulses. For the first counter, 'Counter1' Figure II-23-(a), the first commutation is from low to high whereas for the second the counter, 'Counter2' Figure II-23-(b) it is from low to high. Of course they both have equal volt-seconds. The triangular counter assures symmetry of the pulses. They both have two commutations each. Hence there is no side effect of this technique in terms of switching losses. Other types of counters can be imagined as well like the sawtooth carrier.



**Figure II-23: Complementary Counters**

Now using two counters at time may solve the problem of using vectors other than the standard adjacent vectors. Again sticking by the same example of the reference vector in the first sector. Let us imagine that the PWM signal for leg 'a' is generated through

comparison by 'Counter2' and for the other two legs using 'counter'. This case is shown in Figure II-24.



**Figure II-24: Random active vector selection; commutation signals**

For Figure II-24-(a) vectors V3 and V4 appear in the sequence and it can be noticed no zero vectors are required. Figure II-24-(b) shows that this technique is applicable to discontinuous PWM strategies. In this case V3 is the extra active vector used. The random selection of the vectors can be done by using both counters and randomly assigning them to any of the three phases.

Observing the commutation signal for this type of technique where non adjacent vectors are equally considered, there is not much difference in terms of the voltage commutation and randomizing the space vector this way would only randomize the harmonic quality of the waveform, it is obvious the greater the distance between the reference voltage vector and the voltage vector used to produce it the greater would be the harmonic content.

### II.3.1.3. Zero vector distribution

There are two zero vectors though they don't produce any voltage but are very important for the volt-second balance. Here we'll explore the possibility of randomizing the distribution of these two zero vectors at sub-carrier time scale. In standard SVM implementation the time  $T_{zero}$  given by (2.18), is equally divided between the two zero vectors V0 and V7.

$$T_{zero} = T_{PWM} - (T_i + T_j) \quad (2.18)$$

The time of application of these vectors is given by expression (2.19).

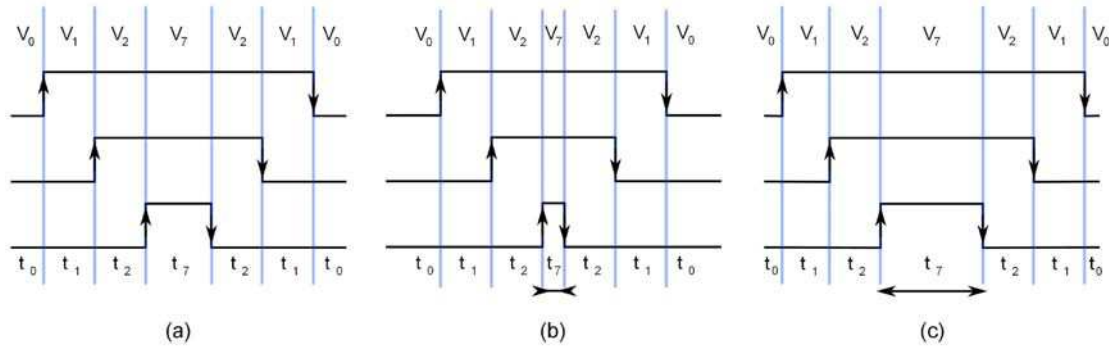
$$T_0 = T_7 = \frac{T_{zero}}{2} \quad (2.19)$$

We saw in the case of DPWM that one of these vectors would be assigned the totality of this time to avoid commutations, however another scenario can be imagined where this time can be randomly distributed amongst these two vectors.



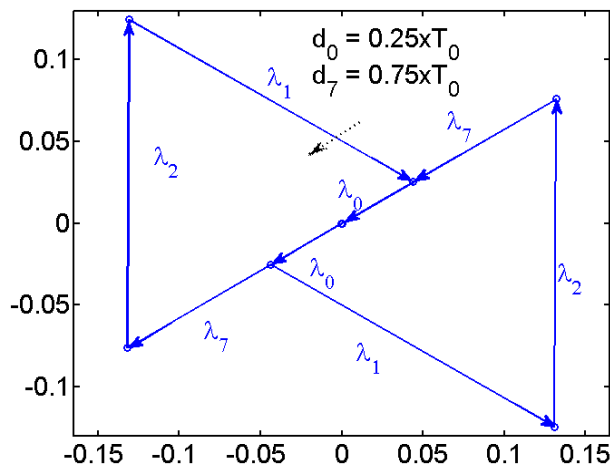
$$\begin{aligned} T_0 &= rT_{zero} \\ T_7 &= (1-r)T_{zero} \end{aligned} \quad (2.20)$$

The random distribution can be given by (2.20) where 'r' is a random variable comprised between 0 and 1, it can be a closed or open interval depending on if discontinuity is accepted or not. It is interesting to find out how this would alter the commutation signals.



**Figure II-25: Random zero vector distribution**

As can be seen from the Figure II-25, the breadth of the pulses get altered while the rest remains the same, the number commutations the symmetry etc.



**Figure II-26: Harmonic flux,  $\lambda_h$  for  $d_0 \neq d_7$**

In the previous chapter the harmonic flux trajectory for different PWM strategies were shown. An example is shown where the random variable 'r' is taken to be 0.25. It can be seen compared to the other techniques considered in that section shown by Figure I-52 and Figure I-54. In this particular case the two triangles will be free to move in either direction along the axis of the reference vector.

Randomising the zero vector distribution does not seem to be of much interest, since it doesn't change switching pattern, the switching harmonics will remain the way they were. This method though easy to implement would not be considered any further.



### II.3.1.4. Counter profile

The counter has two variables that can be randomised, the counter frequency and the counter vertex. This method is analogous to the RCF and RPP-PWM methods seen in the previous chapter for RPWM techniques. Unlike the analog carrier signals the digital counter have a fixed discrete smallest step depending on the clock frequency and the length of the counter variable is fixed as well. This means that the counter will have only discrete values of time period and vertex position. Let us consider a generic counter and an arbitrary pulse as shown in the Figure II-27, three basic parameters can be identified viz. the counter time period ' $T_{PWM}$ ', the duty cycle ' $d$ ' and the delay ' $\delta$ '.

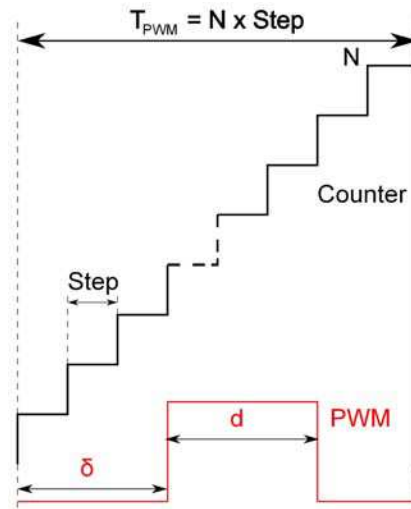


Figure II-27: Digital Counter

The smallest step is the  $1/f_{clock}$ , where  $f_{clock}$  is the clock frequency. This will give the highest resolution pulses. 'N' is the maximum value after which the counter returns to zero and restarts the count, this variable decides the switching frequency,  $f_s = f_{clock}/N$ .

This counter resembles a sawtooth carrier and therefore we are inclined to think that it will produce a trailing edge PWM when the duty cycle is compared to the counter. However Different pulse position can be obtained with such counters adding a little intelligence can get the other two most common PWM, the centred and the leading edge PWM. Let us assume that all the parameters are normalised with respect to the time period ' $T_{PWM}$ ' hence The maximum value of the counter is 1 and the duty cycle lies between  $[0,1]$ .

For trailing edge PWM it is obvious, a simple comparison given by (2.21) is needed.

$$d > counter \quad (2.21)$$

While for leading edge the expression is given by (2.22).

$$1 - d < counter \quad (2.22)$$

To place a pulse at the centre, two conditions are required, they both represent the first and the second commutation, (2.23).

$$\left\{ \frac{1-d}{2} \right\} < \text{counter}, \text{ First commutation}$$

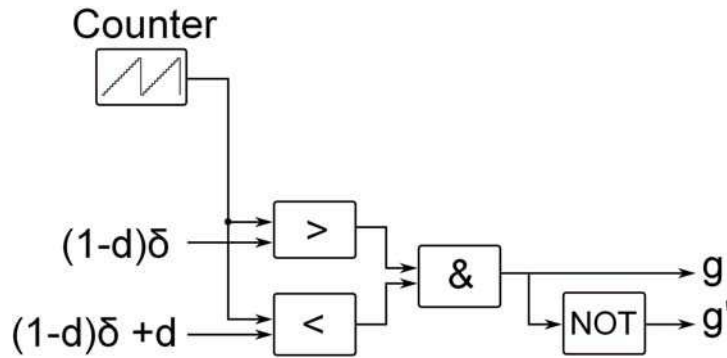
&

$$\left\{ \frac{1+d}{2} \right\} > \text{counter}, \text{ Second commutation}$$
(2.23)

It would be interesting to find a general condition to get whatever pulse position is required. Let us define the variable  $\delta$  that lies between [0,1].  $\delta=0$  for trailing edge pulses,  $\delta=0.5$  for centred pulses and  $\delta=1$  for leading edge pulses. The two necessary conditions can be given by (2.24).

$$\{(1-d)\delta\} < \text{counter} \quad \& \quad \{(1-d)\delta + d\} > \text{counter} \quad (2.24)$$

This flexible pulse generator can be modelled by the following block diagram Figure II-28, where  $g$  and  $g'$  are the gating signals for the upper and the lower switch of an inverter leg.



**Figure II-28: Pulse generator**

Any type of pulse positioning can be achieved from trailing edge to leading edge by varying  $\delta$  from 0 to 1 respectively in the expression (2.24). This pulse generator developed in this thesis was published in [28].

Randomising ' $\delta$ ' would be analogous to RPP-PWM, however as seen earlier this method is not of much interest compared to RCF-PWM for switching harmonic dithering, nonetheless it was important to show how such system can be designed digitally, with just an extra condition any type of pulse positioning can be obtained.

It is obvious that Randomising 'N' would give us different counter period and therefore will lead to the random modulator that would be analogous RCF-PWM. This particular case of randomising the counter period for space vector modulation would be studied in further detail later on in the chapter where a new randomised SVM is developed.

In this section we saw all the space vector modulator parameters that could be randomised and their effect on the output voltage harmonics, before going any further with the development of a randomised modulator we should think of how are we going to provide this the required randomness to the system. The next section will give us some insight into random numbers and their generation.

### II.3.2. Random number generation

A random number source is a prerequisite for any RPWM technique. Normally random number are generated by physical process for example analog electric circuits can be used. A physical random number generator can be based on an essentially random atomic or subatomic physical phenomenon whose unpredictability can be traced to the laws of quantum mechanics. Hence such things cannot be programmed and need external information from physical systems. For this reason algorithms are developed to be able to produce a sequence which appears random with a suitable period before which it starts to repeat itself, hence these are called Pseudorandom numbers. professor D. H. Lehmer defined random numbers as:

*A random sequence is a vague notion ... in which each term is unpredictable to the uninitiated and whose digits pass a certain number of tests traditional with statisticians ...*

For most applications pseudorandom numbers generated by software may be acceptable. A lot of research has been done on pseudorandom number generators. Just to get an idea we'll see a couple of famous methods vastly used. The most common random number generator is the multiplicative congruential algorithm also introduced by Prof. Lehmer given by (2.25).

$$x_{k+1} = (ax_k + c) \bmod m \quad (2.25)$$

Many of today's random number generators use this algorithm. Lehmer's generators involve three integer parameters, a, c, and m, and an initial value,  $x_0$ , called the seed. The period of the sequence is m-1.

The Matlab function 'rand' till 1995 used this algorithm with the following values given by (2.26) and recommended by [43] which make the arithmetic mod and multiplication quicker.

$$\begin{aligned} a &= 7^5, \\ c &= 0, \\ m &= 2^{31} - 1 \end{aligned} \quad (2.26)$$

With these parameters the sequence of the multiplicative congruential is  $2^{31}-2$  which is 2147483646 which is a huge period. To give an idea, a modulator with an expected mean frequency of 20kHz can run for 4 days without repeating the same frequencies.

However Matlab has switched to another algorithm which is based on the Marsaglia-Zaman random number generator [45]. Unlike Lehmer's multiplicative congruential algorithm Marsaglia-Zaman's doesn't involve any multiplications and divisions. It is specifically designed to produce floating-point values. The results are not just scaled integers. In place of a single seed, the new generator has 35 words of internal memory or state. Thirty-two of these words form a cache of floating-point numbers,  $z$ , between 0 and 1. The remaining three words contain an integer index  $i$ , which varies between 0 and 31, a single random integer  $j$ , and a "borrow" flag  $b$ . This entire state vector is built up a bit at a time during an initialization phase. Different values of  $j$  yield different initial states. The generation of the  $i$ th floating-point number in the sequence involves a "subtract-with-borrow" step, where one number in the cache is replaced with the difference of two others, as expressed by (2.27).

$$z_i = z_{i+20} - z_{i+5} - b \quad (2.27)$$

The three indices,  $i$ ,  $i+20$ , and  $i+5$ , are all interpreted mod 32 (by using just their last five bits). The quantity  $b$  is left over from the previous step; it is either zero or a small positive value. If the computed  $z_i$  is positive,  $b$  is set to zero for the next step. But if the computed  $z_i$  is negative, it is made positive by adding 1 before it is saved and  $b$  is set to  $2^{-53}$  for the next step. This method has a huge period of about  $2^{1430}$ . However Matlab's modified version of Marsaglia-Zaman would generate uniformly spaced  $2^{1492}$  random numbers comprised between 0 and 1 before repeating itself.

We have used Matlab's modified version of Marsaglia-Zaman random number generator algorithm throughout the work, for simulations as well as for experimental tests.

### ***II.3.3. Random Space Vector Modulation***

Random Space Vector Modulation (RSVM), a new solution is proposed to conserve the advantages of SVM while adding those of the RCF-PWM. The proposed method is to vary the switching frequency randomly and calculating the corresponding duty cycles. This technique is analogous to RCF-PWM, here it is implemented digitally i.e. no carrier signals. There are about half a dozen papers on RSVM; reading them thoroughly we find some incoherence with the basic principles of SVM. [32] Proposes a 3-level inverter with RSVM to reduce acoustic noise, however the generation of gate signals is done through comparison of the reference signal by a triangular carrier signal, and this technique is commonly known as SVPWM. [33] Proposes a random SVM technique, which is not really random, only two switching frequencies are used with scalar control, however the pulse generation remains ambiguous. Similarly there are other papers such as [34], [35] that treat the problem in a similar way, which in our opinion requires to be treated further.

In the previous chapter it was seen that depending on how the inverter is fed different common mode voltage is produced by different inverter state. For electric drives fed through a battery whose negative terminal is grounded doesn't produce any common

mode voltage on the application of the vector  $V_0$  whereas it produces a common mode voltage of  $V_{dc}$  on the application of  $V_7$ . Therefore the RSVM technique developed in this section will only use one zero vector to complete the modulation period.

Randomised space vector modulation function is shown in Figure II-29, at first glance the randomness may seem to be uneven and unsymmetrical, one would expect similar envelope all along the modulation function, but if you see the line voltage it appears to be perfectly symmetrical. This is because the degree of randomness is through out the same hence for smaller duty cycles the absolute deviation from the statistical mean is less than that of higher values.

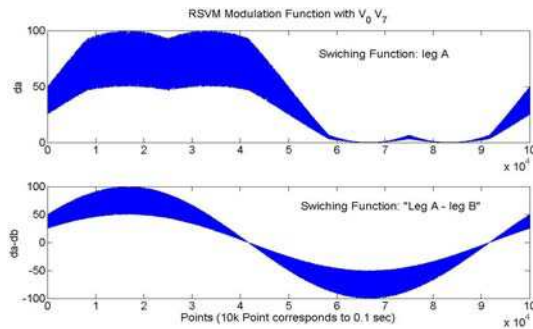


Figure II-29: RSVM continuous

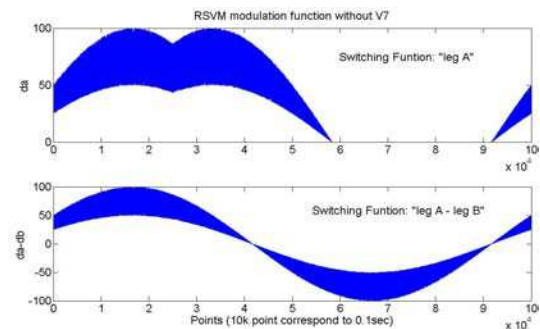


Figure II-30: RSVM: (No V7)

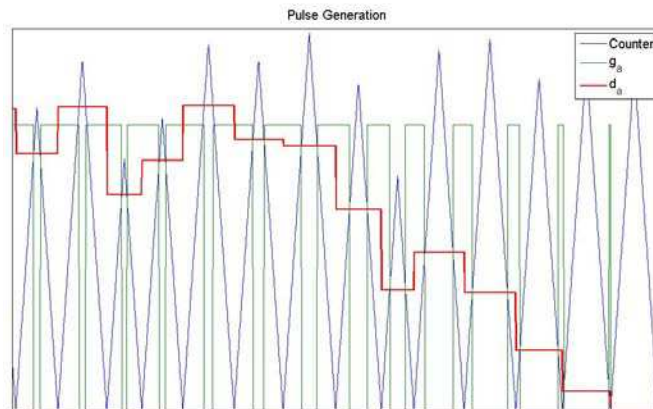
Figure II-30 shows the randomised modulation function from DSVMMIN where only  $V_0$  is used. The variation of the modulation function is done according to the randomly generated SVM-period. For digital control system the switching (or modulating) function should correspond to the randomly generated frequency at every cycle to update the comparator registers with the corresponding new duty cycles. In the following sub-sections some practical aspects from the point of view of digital implementation are discussed.

### II.3.3.1. Pulse Generation

As seen earlier in the chapter the pulses are generated by comparing the duty cycle with a counter. We saw a generic counter that could easily adapt to any type of pulse position but required one extra comparison compared to the classical up-down counter. Since we don't plan to implement RPP-PWM type of modulator we can avoid using one extra condition by using an up-down counter to get centred pulse.

So here an up-down counter is employed for the pulse generation, the counter needs to be adjustable in terms of counter periodicity. A flexible counter that counts till a given number for a given period i.e. flexible in width and amplitude.

Figure II-31 shows the flexible counter (blue) that adapts itself with the random modulation frequency, the duty cycle in red corresponding to the randomly changing time period and the gate signals in green are centred around the modulation period. The maximum tolerance of the pulse is kept 1% to keep the execution time of the model small. In practice it can be kept as 0.1%.



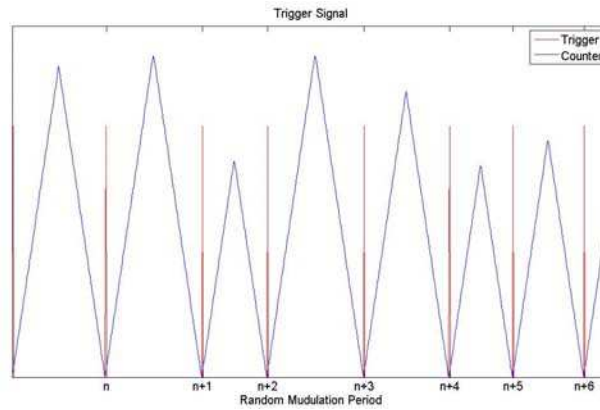
**Figure II-31: Pulse Generation**

### **II.3.3.2. Acquisition and control sequence**

For deterministic PWM the duty cycles are updated at regular interval depending upon the type of PWM the most common duty cycle updating techniques are the symmetrical and asymmetrical regular sampling. In symmetrical sampling the duty cycle is updated once every modulation period whereas for the asymmetrical sampling the duty cycle is updated twice in a period once at the beginning of the period and once half through the period. For non-deterministic modulators the comparators cannot be updated at regular intervals, they have to be synchronised with the random time period of the randomised modulator.

For applications where current regulation is required like speed drives with vector control, the current acquisition should be done carefully. As seen in the previously in the microscopic analysis of harmonic currents, the average value of the harmonic current over the entire period is zero, however instantaneous values are not expect on two occasions once at the beginning (or the end) of the period and at the middle for centred pulse and at the beginning for leading or trailing edge PWM. The points with highest instantaneous value of harmonic currents are at  $\frac{1}{4}$  and  $\frac{3}{4}$  of the period for centred PWM and at the half period for leading or trailing edge PWM. This means that if the acquisition is not done carefully erroneous values of current will be sampled. For changing switching frequency the acquisition system should be adapted accordingly to get the values closest to the average current in the phase.

To address these issues of duty cycle updates and current sampling the carrier has to be studied to identify the point of interest such as the half time period and the end of the period. When the counter reaches its maximum count 'N', this even can be used to update the duty cycles or get the sampled values of current to be used by the current regulators.

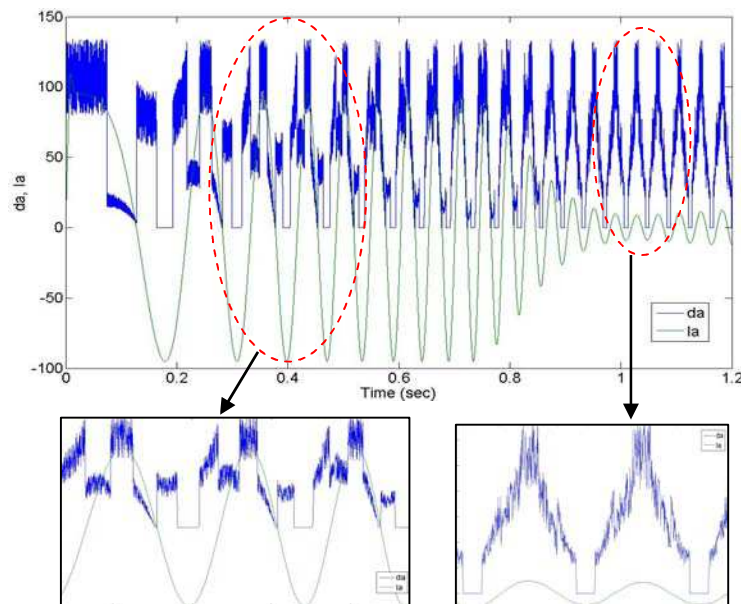


**Figure II-32: Triggered Iterations**

Figure II-32 shows the generation of the trigger pulse at the end of each period, which updates the pulse generator comparators with the new values of the duty cycle and get a new current samples to be used for the calculation of the voltage references for the next iteration.

#### ***II.3.4. Random discontinuous modulation***

In the previous chapter a new type of discontinuous modulator was introduced, the evolutive discontinuous space vector modulation (EDSVM). This modulator was developed to reduce the switching losses, it is preferable to have a single modulator which could reduce the switching harmonic peaks as well as reduce the losses. Hence these two different modulators will be fused together to form a random discontinuous modulator that will have the advantages of both, reduced switching losses and dithered switching harmonics. This research work has been published in [37].



**Figure II-33: RDSVM – phase current and modulation function**

Figure II-33 shows the stator current  $I_a$  and the randomised switching function  $d_a$ , evolving in time to track the phase current. To facilitate the comparison between the duty cycle and the current, the current is multiplied by a constant gain. It can be observed that the duty cycle is saturated to 100 and 0 with positive and negative current peaks adapting itself with the changing frequency and load angle,  $\phi$ .

### II.3.5. Simulation

Although PWM is an open loop voltage control technique most applications of hard switched converters have feed-back loops like AC motor control. Hence both an open loop and loop cases will be dealt with in this section. All simulations are done on Matlab/Simulink. The inverter and PMSM are taken from the Simulink library 'SimPowerSystems'.

#### II.3.5.1. Open loop

A balanced three phase voltage is produced by an ideal DC source and a three phase two level IGBT inverter. Three sine-waves of 20 Hz out of phase to form a balanced 3-phase voltage system are taken as reference signals. The open loop results are shown in this section. The switching frequency for SVM is kept constant at 10 kHz, whereas for RSVM it varies randomly between 7.5 and 12.5 kHz and therefore has an expected mean frequency of 10 kHz. The degree of randomness may be defined as (2.28).

$$r = \frac{\Delta f}{f_{mean}} \quad (2.28)$$

Where  $\Delta f$  is  $f_{max} - f_{min}$  and  $f_{mean}$  is the statistical mean of the random frequency. For this case the degree of randomness is kept to be 0.5. For the determinist case the reference sine wave is sampled at 10kHz and synchronised single updates with the switching frequency. For the randomised modulator the sine wave is sampled at 12.5kHz but the updates are synchronised with the randomly changing switching frequency.

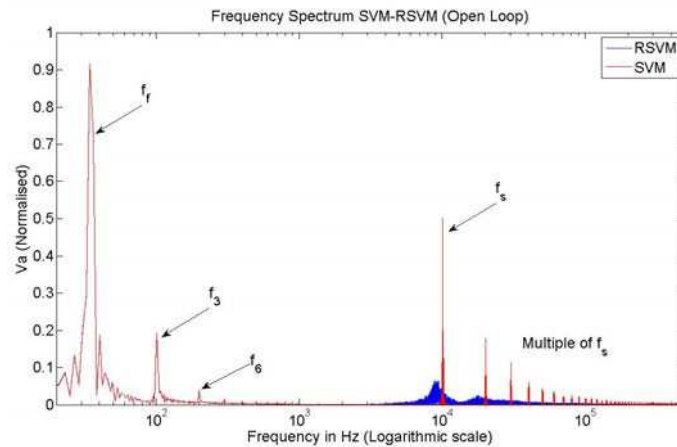


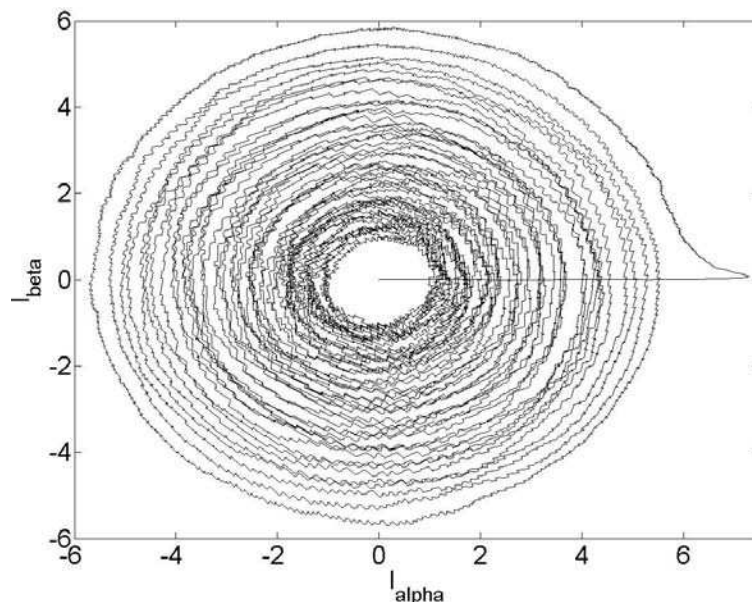
Figure II-34: SVM, RSVM; Frequency Spectrum Contrast



Frequency spectrum of the output phase voltages with respect to the DC mid-point at no load. for the determinist and random SVM are shown in Figure II-34. We can see a peak at 10 kHz for SVM and for RSVM we see a small hump around 10 kHz while the rest remains the same. The peak value of the switching harmonics is 10 times lower than the peak value for an SVM at a constant frequency. As we have already seen that SVM intrinsically adds third-order harmonics in the phase voltages, this can be seen in the graph marked by  $f_3$ ,  $f_6$ . Higher multiples are not marked but can be seen on the chart. These are zero sequence harmonics and therefore disappear from the line voltage. The subcarrier frequency behaviour of both the modulators is identical and completely superimpose each other. The simulation result validates the concept in open loop as we can see the low frequency spectrums perfectly superimpose each other.

### II.3.5.2. close loop

The sampling frequency decides the closed loop bandwidth of the controller. The switching frequency and the sampling frequency of the current controller may not necessarily be equal, they can be a multiple of the regulator frequency. This may look absurd to have a higher switching frequency than the controller frequency, but there are applications where the permissible harmonic distortion may impose a certain switching frequency and the digital processor may not be fast enough to be able to execute the controller instructions at the switching frequency. However to avoid erroneous data acquisition the current sampling must be in sync with the PWM unit. For random carrier frequency PWM the controller has to be synchronised with the changing PWM frequency, this would mean a variable frequency discrete regulator would be required.

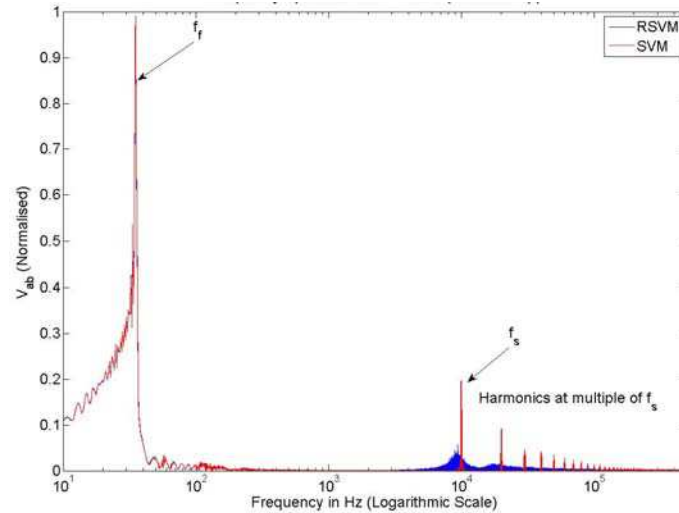


**Figure II-35: RSVM Stator Current Evolution**

Here we have used a linear PI regulator, since the updating of the PI current controller is synchronized to the random carrier frequency which implies a non-uniform

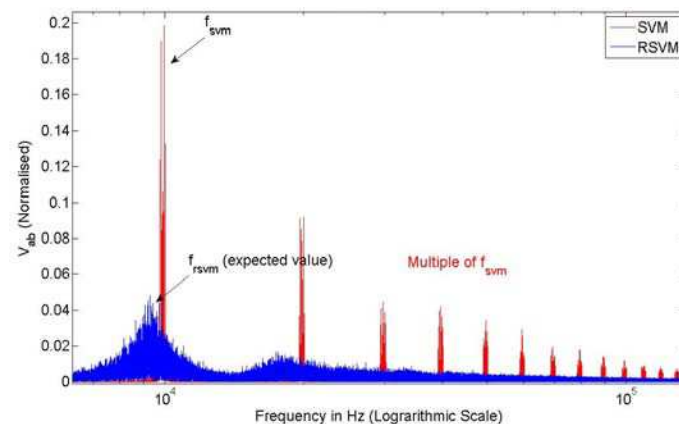
sampling rate of the controller. This requires that the coefficients of the difference equation to be adapted at every sampling period which is also the randomly varying switching frequency to in order to better approximate the response of the discrete digital implementation to the response of the underlying continuous analog PI-controller, independent of the instantaneous sampling rate.

Figure II-35 shows the stator current evolution in the machine in the  $\alpha\beta$ -plane, in amperes. It can be observed that compared to Figure II-18 is a little blurry due to the randomness added to the system.



**Figure II-36: Frequency Spectrum Contrast (Closed Loop)**

The frequency spectrum of the line voltage ' $V_{ab}$ ' is shown in Figure II-36, SVM in red and RSVM in blue. The first peak represents the evolution of the fundamental frequency till it reaches its steady state at 35 Hz, which corresponds to the steady-state speed of 1050 rpm for a 4 pole electric machine. The peaks in red and the humps in blue represent the harmonics at the switching frequency and its multiples.



**Figure II-37: Switching Harmonics (RSVM and SVM)**

Figure II-37 is a zoom around  $f_s$  (10 kHz) and its multiple. We can see the harmonic peak for RSVM is five times smaller for a frequency range of 5 kHz.

### ***II.3.6. Summary***

In this chapter we saw different types of randomisation possibilities for a space vector modulator, their physical interpretation and their digital implementation. Amongst all of the methods introduced only randomising carrier frequency was chosen because it was the only modulator which alters the switching function to dither the switching harmonics and alleviate the problem of acoustic noise annoyance in hard switched converters.

Different practical implementation aspects of the random carrier frequency modulators such as system performance, acquisition of feedback signals and the design considerations for the digital controller were discussed. Finally the validity of such technique was established through simulations.

## **II.4. PWM Optimized embedded electric drives**

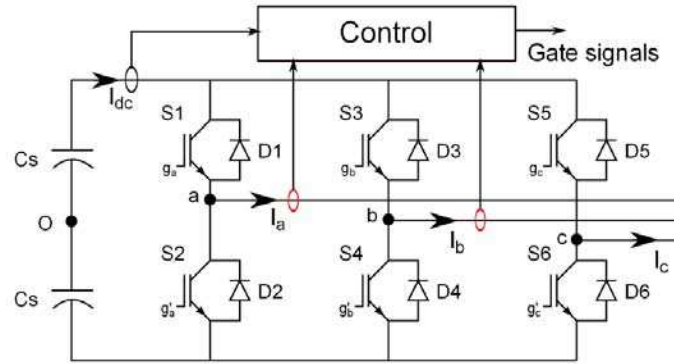
The focus of this chapter is how a pulse width modulated drive be optimised in terms of cost and volume but unlike in the previous chapters the focus would not be entirely on the output voltage and the switching losses here even the DC current fed to the inverter is studied at subcarrier period scale in correlation with the inverter discrete state. Studying the inverter states and the DC input current, it is observed that the phase currents can be deduced and the DC link current fluctuations may be reduced to which in turn would mean a small stabilising capacitor at the inverter input. Different PWM techniques would be introduced here, that can potentially reduce the cost and size of the inverter drive by altering the PWM techniques a little.

### ***II.4.1. Introduction***

Embedded electric drives have certain design constraints which are the volume, weight and robustness as they are more prone to varying working environment, harsh temperatures, and possible mechanical shocks. Not to mention cost minimization is a very important engineering aspect and is true for any application. In this section we'd develop two modulation techniques which would reduce the design cost with decreased volume and weight linked to the DC-link capacitor and a decrease in the number of current sensors. Talk briefly about volume and the cost of stabilising capacitors add to the system. similarly for the current sensors.

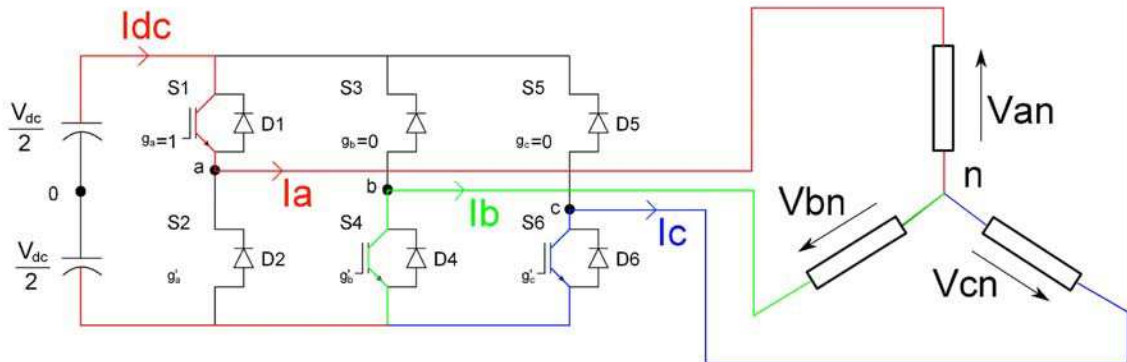
### ***II.4.2. Phase current reconstruction***

In this section we'll study the possibility of deducing the two phase currents necessary for the feed-back control of PWM speed drives. Modulation strategy will be developed allowing the possibility of removing the phase current sensors.



**Figure II-38: Three phase Inverter and current sensors**

Three current sensors are required in a feedback electric drive, Figure II-38 the DC current sensor for protective reasons and the two phase current sensors for the speed/torque controller. All the three inverter legs are in parallel, the switches within each leg are controlled in a complementary manner which means that for every active inverter state all the current  $I_{dc}$  flows through one leg. The two zero states of the inverter, when all the upper switches (S1, S2, S3) are open or closed disconnect the inverter from the voltage source and hence no current flows into the inverter and load is short-circuited hence no information on the phase current can be deduced from these two states.



**Figure II-39: Current channelling**

Let us consider the space vector approach, where each vector represents a discrete inverter state. Table II-1 shows the relationship between the inverter input current  $I_{dc}$  and the inverter phase currents  $I_a$ ,  $I_b$  and  $I_c$ . 'NI' is no information.

Vector ( $g_a g_b g_c$ )	V0 (000)	V1 (100)	V2 (110)	V3 (010)	V4 (011)	V5 (001)	V6 (101)	V7 (111)
$I_{dc}$	NI	$I_a$	$-I_c$	$I_b$	$-I_a$	$I_c$	$-I_b$	NI

**Table II-1: Phase current and space vectors**

The controller needs two phase currents to perform the torque regulation the third can be calculated using (2.29) for loads with floating neutral point.

$$I_a + I_b + I_c = 0 \quad (2.29)$$

One active vector gives information about only one phase current, hence for every modulation period two active inverter states are required to successfully deduce the phase currents and one zero vector for ground fault detection.

#### II.4.2.1. Limitations and Boundary conditions

Apart from the requirement of two active vectors per PWM period another limiting factor is the absolute duration of vector application, the dead-time and the reverse recovery time of the diode, this time should be sufficiently large for a stable analog to digital conversion.

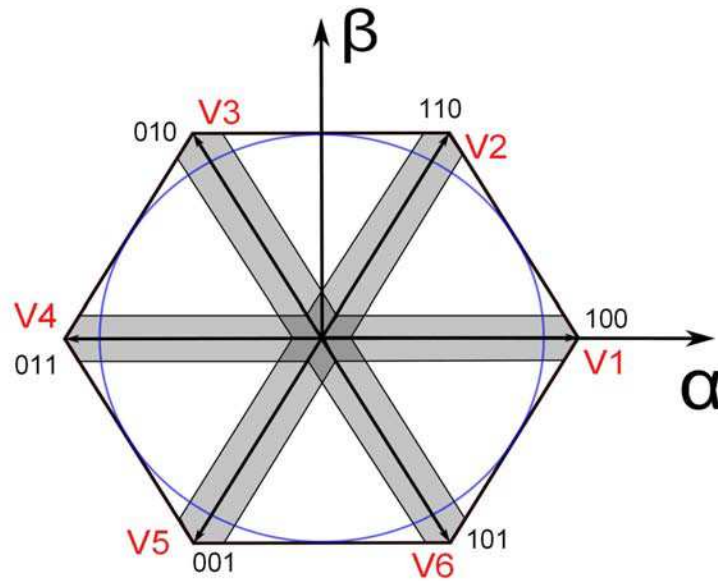


Figure II-40. Space vector representation

For small modulation index the inverter stays in the active states for a very short duration and makes it impossible to get a stable current acquisition, hence for weak modulation index standard modulation techniques won't work and to increase the application time of active vectors non-adjacent vectors can be used. There are six limiting cases where only one active vector or one inverter state is needed to produce the desired average output voltage, the condition is given by (2.30) which essentially define the active vector axis.

$$v_{\beta_{ref}} = 0 \text{ OR } v_{\beta_{ref}} = \pm \sqrt{3} v_{\alpha_{ref}} \quad (2.30)$$

In the vicinity of these axis represented by the shaded area of the Figure II-40 represents the regions where the length of one of the active vector or the inverter active state duration is not long enough to extract the complete phase current information from the  $I_{dc}$  measurement.

### II.4.2.2. State of the art

A number of papers have been written to overcome these problems. [48] does a review of the papers published on phase current reconstruction from the inverter input current. The common techniques used are adding measurement vectors [46] where a set of three active vectors V1, V3 and V5 are applied as shown in Figure II-41, each with a duration long enough for accurate current measurement these vectors have zero average value hence don't add to the average to desired voltage. The advantage of this method is that all the three phase currents are reconstructed separately whereas the drawbacks are high HDF and switching losses owing to the 6 extra commutations.

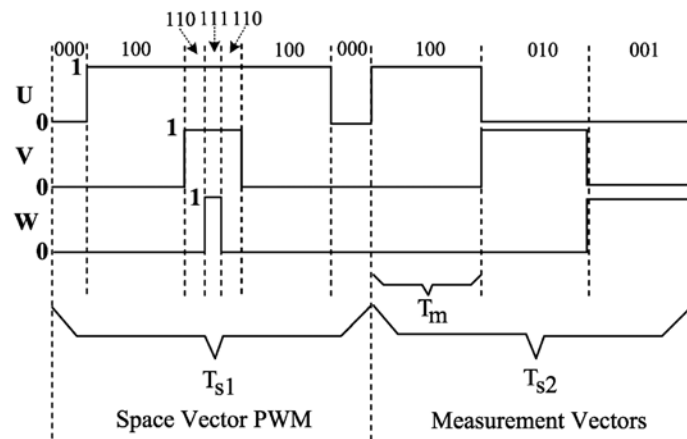


Figure II-41: Measurement vectors, [46]

Another way is to add an extra active vector in the by displacing the pulse in the modulation period [48].

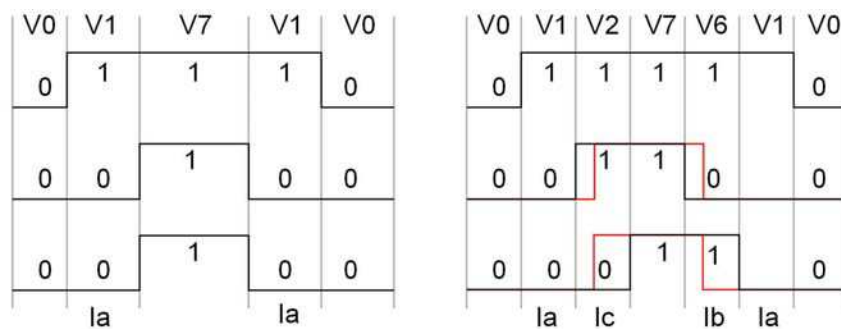


Figure II-42: Pulse displacement, [48]

Figure II-42 shows a case where  $db=dc$  and only the current in phase can be detected whereas shifting the two pulses all the three phase currents can be reconstructed from the inverter input current. This can equally be used where the duty cycles are nearly equal.

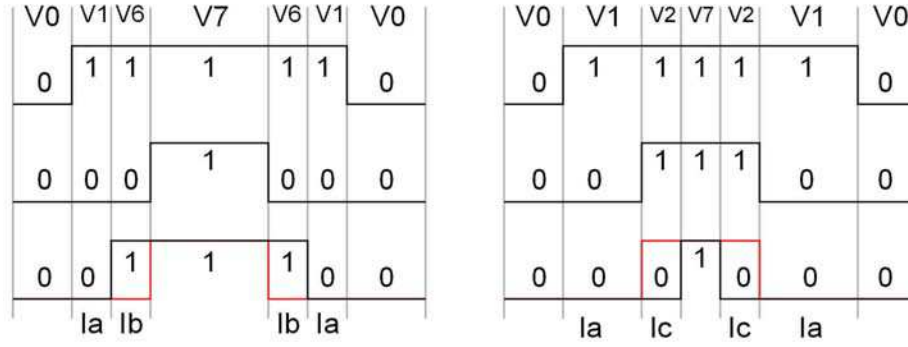


Figure II-43: duty cycle alteration, [49]

Another way of dealing with this problem is proposed in [49] where the duty cycle is altered for current reconstruction and is compensated in the next switching period, the Figure II-43 shows two consecutive switching period with modified duty cycles (in red the original duty cycle).

#### II.4.2.2.1. Reconstruction error

In pulse width modulated loads, the load current has two components the average or the desired component and the ripple component introduced by the harmonic voltage, though the average value of this current over a modulation period is zero but not the instantaneous value, Figure II-44 shows the average (over a modulation period) and instantaneous values of the voltage and current within a given modulation period in solid and dashed lines respectively for a phase for a triangular PWM, i.e. the pulses are centred.

$$i_{ripple} = \frac{1}{L_a} \int (\langle V_{a0} \rangle_T - V_{a0}) dt \quad (2.31)$$

The analytical expression for this ripple current for an inductive load can be approximated by (2.31).

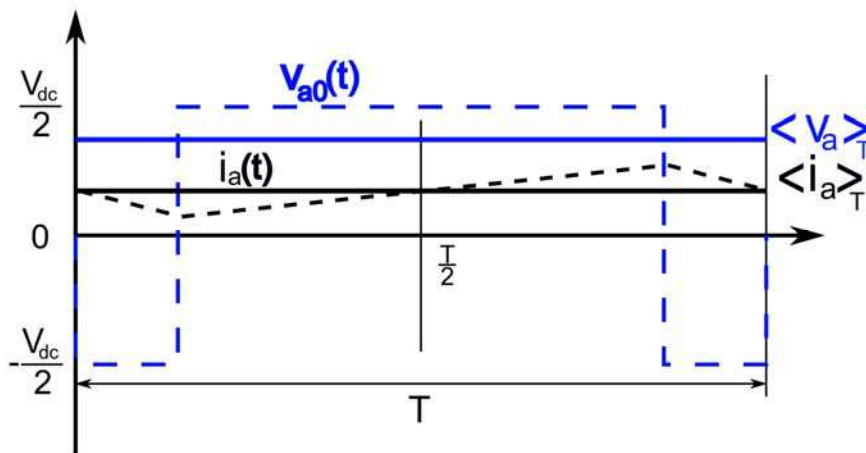


Figure II-44: Average current and harmonic current

For current controlled pulse width modulated loads, the current regulators should be fed the average value. From the Figure II-44 it can be observed that the ripple current is zero on three occasions viz. at the beginning of the period, at half-period and at the end of the period. The current acquisition should be done in a way to get the ripple free current samples hence the current sampling has to be synchronised with the PWM.

However for phase current reconstruction from the DC link current sensing the phase currents have to be synchronised with the inverter state and hence the values have to be sampled somewhere within the switching period.

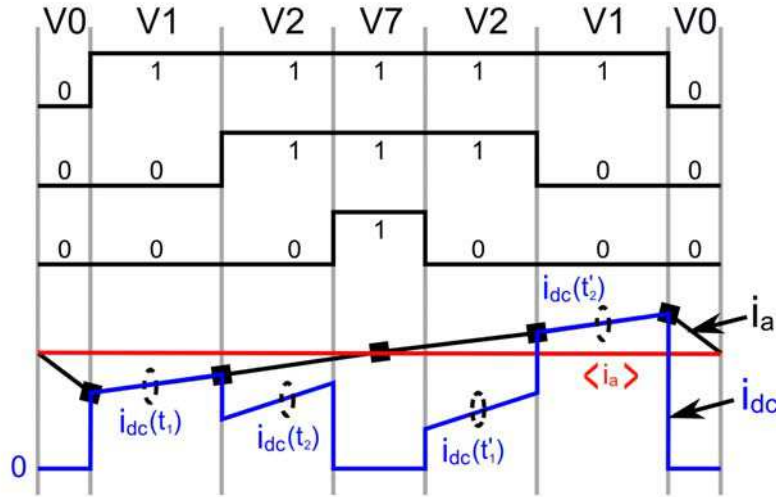


Figure II-45: Current reconstruction error elimination

Figure II-45 shows an example of current sampling to remove the ripple component from the current sensed somewhere between the period. The dotted circles show the sampling instances, the DC link current is sampled twice for each phase. (2.32) and (2.33) should be used to get the desired average values of the phase currents.

$$\langle i_a \rangle_T = \frac{i_{dc}(t_1) + i_{dc}(t_2)}{2} \quad (2.32)$$

$$\langle i_c \rangle_T = -\frac{i_{dc}(t_2) + i_{dc}(t_1)}{2} \quad (2.33)$$

This technique requires the pulses to be symmetrical about the half-period and that the reference voltage vector to be in the comfortable zone to be able to get a suitably long durations for accurate analog to digital conversions.

#### II.4.2.2.2. Fault Detection

An electric drive is vulnerable to different types of faults that can be destructive to the inverter and the load if not detected and isolated quickly. So in general an electric drive should be protected against:

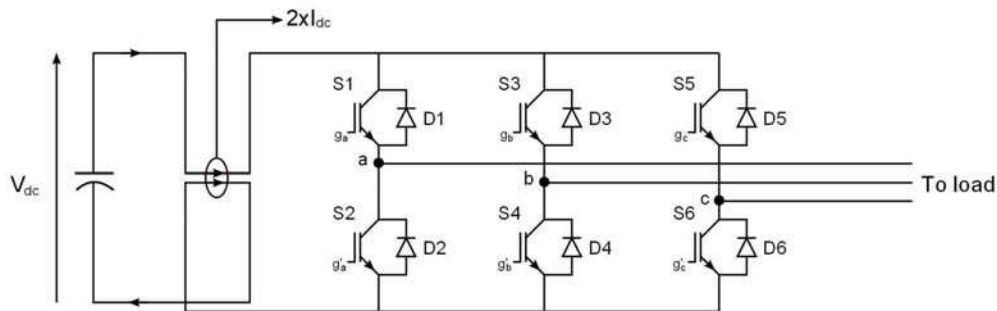
- Ground faults



- overloading
- Short-circuiting the DC link

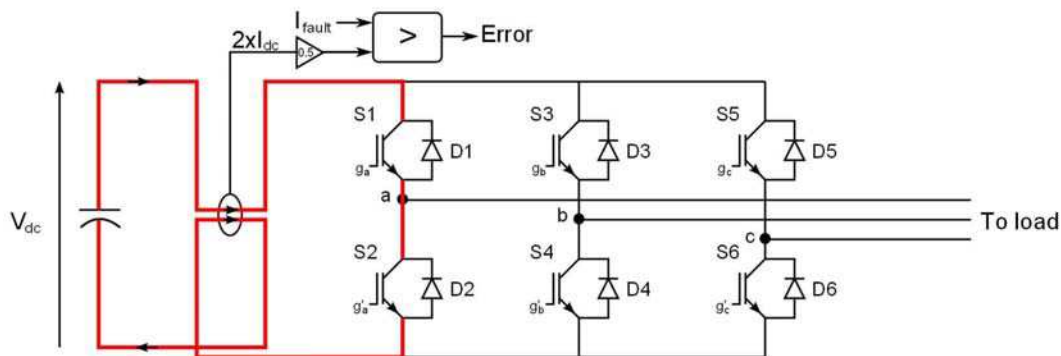
To detect these faults normally four current sensors are required, 3 sensors to detect the ground faults, one for each phase. A DC link current sensor for overloading and short circuit detection. Electric drives with only one current sensors should be able to detect these faults as well to provide same safety levels.

In [51] an interesting solution is proposed that is capable of providing the protection required using only one current sensor as shown in Figure II-46 which measures the DC link current at both negative and positive rails.



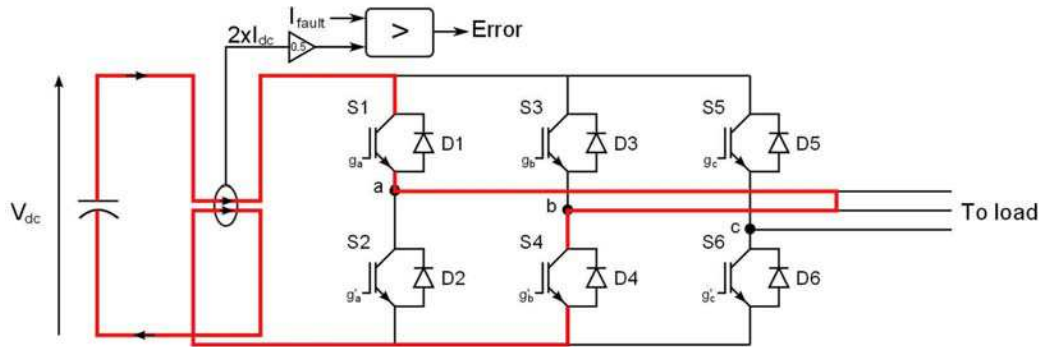
**Figure II-46: Fault detection with one current sensor, [51]**

With this special arrangement the current sensor will give twice the value of inverter current under normal operating conditions and can be used for phase current reconstruction. This configuration coupled with the gating signals can detect all the errors.



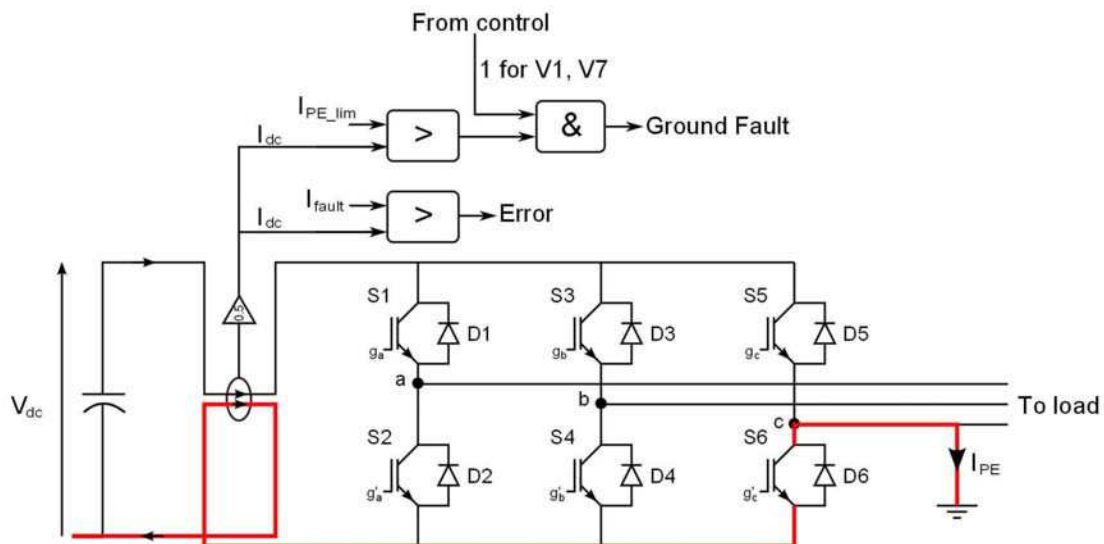
**Figure II-47: DC link short-circuit**

A shoot-through fault is shown in Figure II-47, which occurs when both switches of an inverter leg are in the conducting state at the same time, this may be caused by malfunctioning of the electronic circuitry producing the gating signals or the controller sending the wrong signals. This type fault can be detected simply employing a comparator, digital or electronic to compare the measured value to the maximum permissible inverter input current.



**Figure II-48: Phase short-circuit**

A short circuit fault is shown in the Figure II-48 where any two phases of the load get shorted generally due to wearing off the insulation. Like in the previous case this fault will induce high DC currents and can be deducted in exactly the same way as the other one.



**Figure II-49: Ground Fault**

When the zero vectors are used the inverter is disconnected from the DC link and no current should be flowing in either of the DC rails, Figure II-49 shows a ground fault condition which can be detected through a simple comparison between the current measured by the sensor and the maximum tolerable earth current while the inverter in the zero state. Hence in this way the inverter can be protected from all types of fault using only one current sensor. This method has two drawbacks the first is that the current sensor to be used has to be rated twice the maximum input current which will makes it more expensive and the second disadvantage is that the measurement resolution gets affected provided the ADC on the controller remaining the same. The conversion resolution is given by :

$$res = \frac{V_{\max} - V_{\min}}{2^N} \quad (2.34)$$

Where  $N$  is the number of bits on which the digital value is stored,  $V_{\max}$  and  $V_{\min}$  are the maximum and minimum values to be read by the ADC. For this topology since the current measured twice the amplitude of the DC-link current, the measurement resolution decreases by the same factor hence a poorer resolution. However the same authors in [50] propose another method to remove these drawbacks.

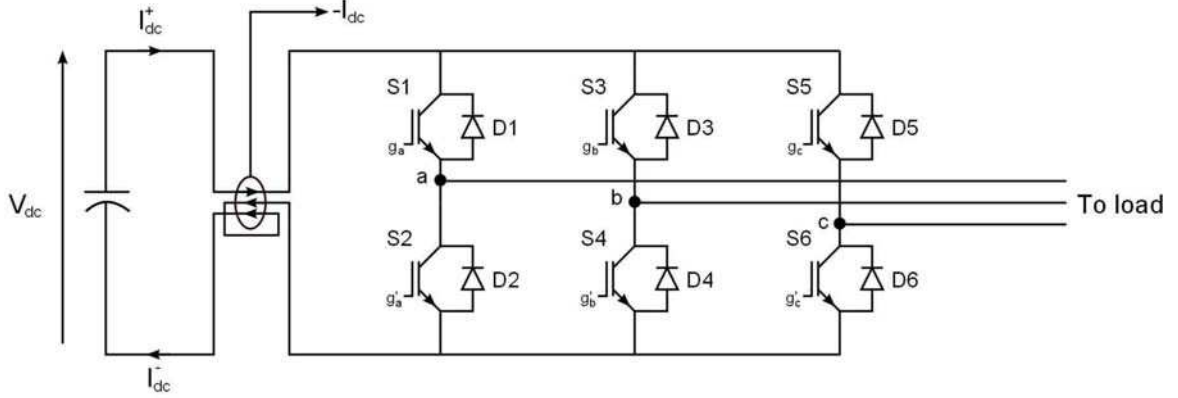


Figure II-50: Fault detection using one current sensor, [50]

As can be seen in Figure II-50 an additional winding is introduced in the sensor so now the current measured by the sensor is given by (2.35).

$$I_{\text{sensor}} = I_{dc}^+ - 2I_{dc}^- \quad (2.35)$$

Under normal operation the currents following in the positive and negative DC rails are equal so the sensor current equals  $-I_{dc}$ , in this way the drawbacks of the previous topology can be removed. For the ground fault detection the comparison of the sensor current will have to be done with twice the acceptable limit tolerable for ground current since the sensor under this fault will output twice the current flowing through the ground loop.

#### II.4.2.3. Proposed solution

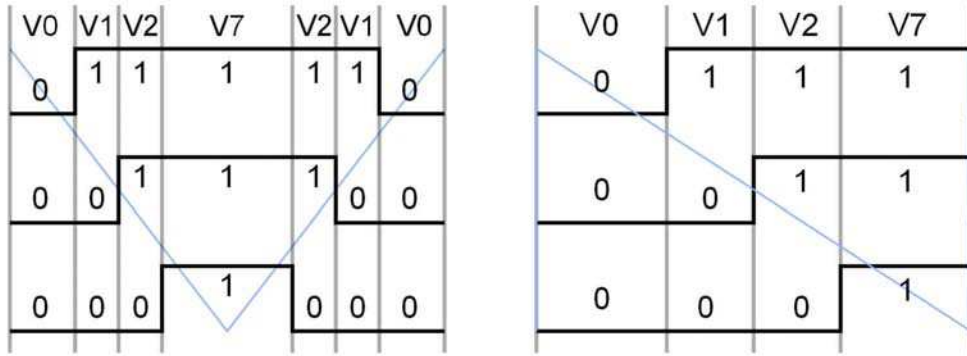
A method based on using non adjacent vectors in low modulation region and the six limiting cases shown by the shaded region in Figure II-40. The proposed method is a generic method for all sectors, this method assures that at least one zero vector is used in every modulation period for ground fault detection and use the closest possible active vectors to minimise the voltage harmonic distortion. This work was published in [28].

Analytical expressions are used to define pulse placements in a period, the duration for the ADC is also compared with standard methods like the triangular carrier or the centred pulses or the sawtooth carrier with leading or trailing edge PWM techniques. In each sector the duty cycles follow a certain pattern that is easily identified, no matter what

modulation technique is used, sine wave, harmonic injection, or space vector they can all be given by (2.36), where  $d$  is the duty cycle and  $S$  the sector defined by the space vectors in the  $\alpha\beta$ -plane.

$$\begin{aligned}
 d_a &\geq d_b \geq d_c & \text{for } S = 1 \\
 d_b &\geq d_a \geq d_c & \text{for } S = 2 \\
 d_b &\geq d_c \geq d_a & \text{for } S = 3 \\
 d_c &\geq d_b \geq d_a & \text{for } S = 4 \\
 d_c &\geq d_a \geq d_b & \text{for } S = 5 \\
 d_a &\geq d_c \geq d_b & \text{for } S = 6
 \end{aligned} \tag{2.36}$$

Let us consider the first sector using the two standard techniques namely triangular and sawtooth carrier which eliminate the possibility of the inverter being in any other active state other than  $V_1$  and  $V_2$  as can be seen in Figure II-51.



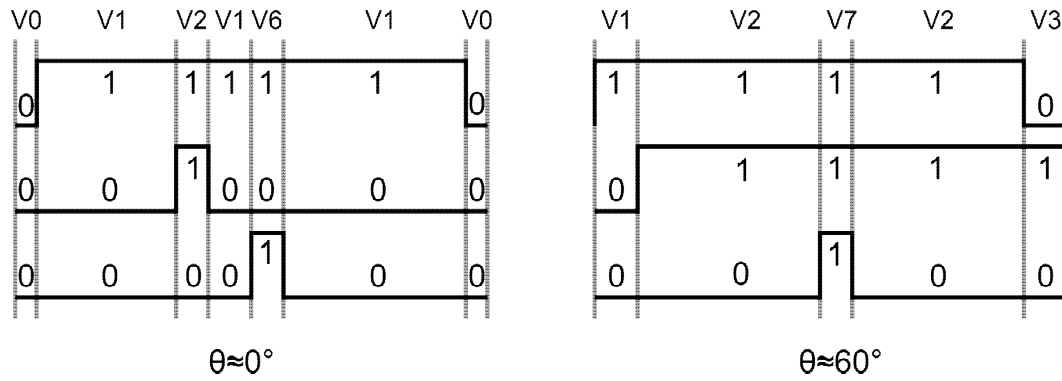
**Figure II-51: Triangle and sawtooth modulation**

It can be seen depending upon the type of modulation the duration of active vectors change, for triangle modulation the same active vectors appear twice whereas for the triangular modulation each inverter state appears only once in a period. The active vector duration can be given by (2.37) for triangle and by (2.38) for sawtooth.

$$\begin{aligned}
 T_{tri\_V1} &= \frac{d_a - d_b}{2} \\
 T_{tri\_V2} &= \frac{d_b - d_c}{2}
 \end{aligned} \tag{2.37}$$

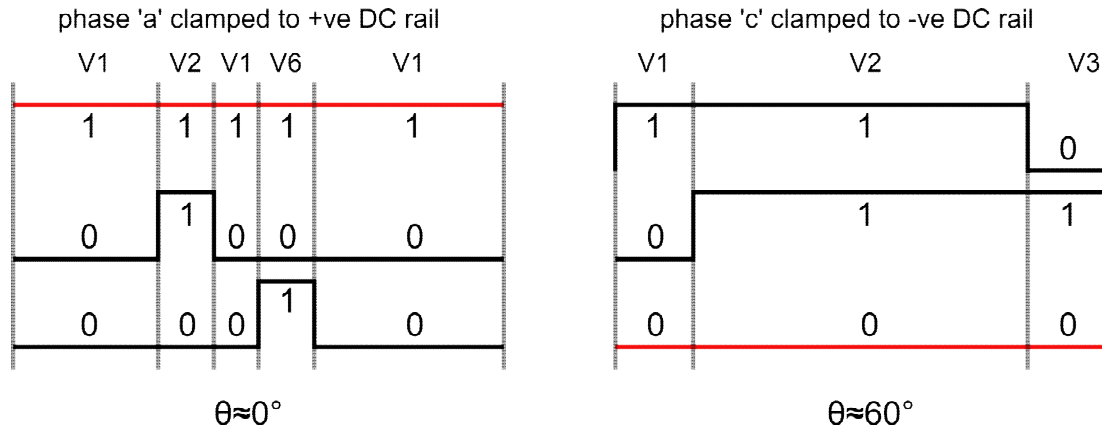
$$\begin{aligned}
 T_{saw\_V1} &= d_a - d_b \\
 T_{saw\_V2} &= d_b - d_c
 \end{aligned} \tag{2.38}$$

It is clear that for sawtooth modulation we have twice as much time for which the input current is channelized through phase 'a' and 'c'.



**Figure II-52: Phase current deduction; high  $m_i$  and sector extremities**

Whereas for extreme cases when any of the two duty cycles are nearly the same,  $d_b \approx d_c \approx 10\%$  and  $d_a \approx d_b \approx 90\%$  at the reference vector angles  $\theta \approx 0^\circ$  and  $\theta \approx 60^\circ$  respectively in the  $\alpha\beta$ -plane for a modulation index of 0.907, Figure I-17, which needs to be dealt with. The methods found in literature use pulse displacement techniques to the legs with nearly equal duty cycles, which can increase the application time of the active vectors, Figure II-52. However this falls short for high switching frequencies, to further increase the applicability of phase current reconstruction from the input current measurement a discontinuous PWM scheme is proposed, Figure II-53.



**Figure II-53: Discontinuous PWM for phase current deduction**

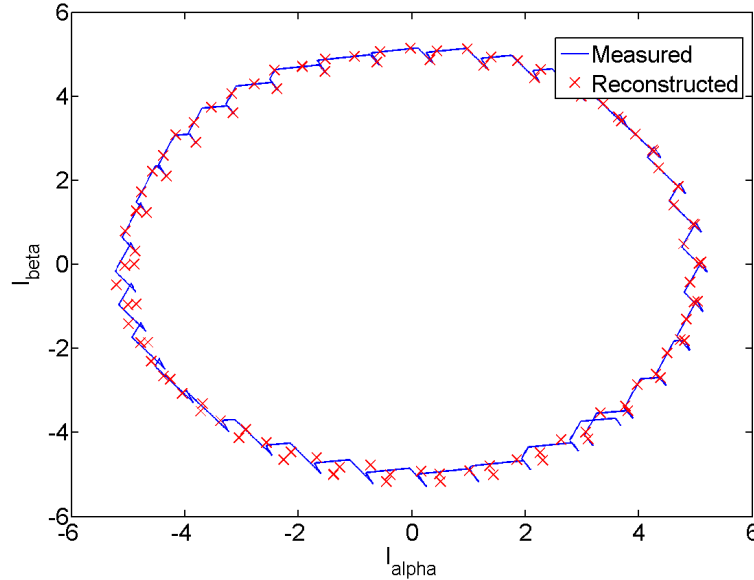
The proposed method can increase the active vector duration by an extra 10% by clamping the phase 'a' and 'c' to the positive DC rail for reference vector angle of about  $0^\circ$  and to the negative DC rail for vector angle of about  $60^\circ$ . The increase in the duration can be given by (2.39).

$$\begin{aligned} T_{increase\_V2} &= 1 - d_a \\ T_{increase\_V1} &= d_c \end{aligned} \quad (2.39)$$

#### II.4.2.4. Simulation

The simulations are done on MATLAB/SIMULINK. The inverter, the three phase RL load (PMSM) and the DC source are taken from the “*SimPowerSystems*” library. The

simulation models are completely discretized, with the simulation step being  $10\mu\text{s}$ . To avoid the use of multiple counters the pulse placement technique shown in section II.3.1.4 is used and hence only one digital counter for pulse generation is used with a precision of 1% to accelerate the SIMULINK model execution.



**Figure II-54: Phase current in the  $\alpha\beta$ -plane**

The phase currents and DC link current are measured. With the gate signals the applied vector is deduced and the inverter input current is associated to a phase current value. The simulation results, Figure II-54 shows coherence between measured and reconstructed values of current for a high modulation index of 0.907. The currents are shown in the  $\alpha\beta$  plan which makes it easier to visualize all the three phases in a single graphic. However no closed loop control has been performed yet nevertheless we plan to perform Field Oriented Control for a Permanent Magnet Synchronous Machine.

#### **II.4.2.5. Summary**

The reconstructed phase currents contain ripple currents present at the sub-carrier level cannot be removed as the symmetry of the voltage pulse is lost under extreme conditions where pulse displacement is used in order to extract information during the dead intervals. This adds to the error in the feedback controller. However it remains a good bargain as it considerably increases the operating region of such drives to maximum modulation index, with the introduction of discontinuities in the modulation function. Whereas the use of non-adjacent active vectors increases the overall harmonic content of the voltage and hence has a poor performance compared to schemes where only adjacent vectors are used.

### II.4.3. DC link capacitor reduction

An intermediate circuit capacitor called the DC-Link capacitor is used in converters of different kinds. Here we'd be studying the DC link capacitor used in inverter drives fed through a battery. The role of these capacitors is to provide transient power and high frequency current. To fulfil these purposes this high capacitance is required. In general aluminium electrolytic capacitors are used in power electronic circuits due to their very high power density. In spite the use of these high density electrolytic capacitors they account up to 40% of the total volume of the inverter module. In this section some methods will be explored that would help reduce the high frequency current demands of the inverter which would mean a capacitor with a lower capacitance could be used or the life of the capacitor could be prolonged.

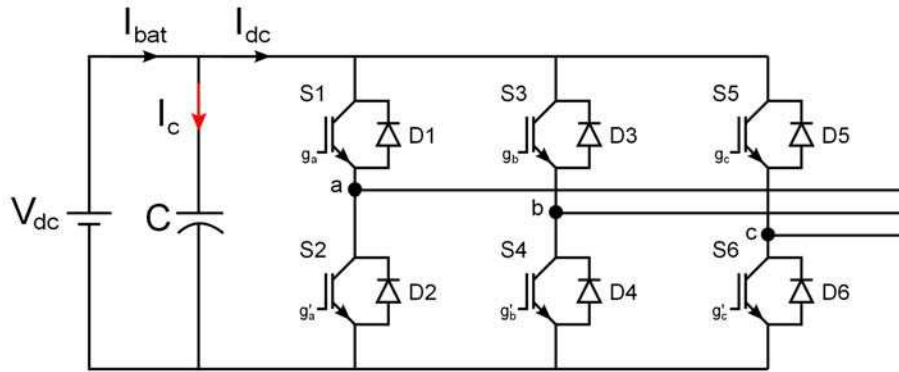


Figure II-55: Current in the DC-link

The battery feeds the inverter with an average input current  $I_{bat}$  whereas the current  $I_c$  is the fluctuating or ripple current [53].

$$I_c = I_{bat} - I_{dc} \quad (2.40)$$

#### II.4.3.1. Ripple component of the inverter input current

Since  $I_{bat}$  supplies a steady DC current the current circulating in the capacitor depends directly on the current absorbed by the inverter. The inverter input current  $I_{dc}$  whose average value to equal to the average value of the DC current if the losses in the DC-link capacitor and the inductive cables are neglected. If the transient value of this current is brought closer to the average value less current will circulate in the DC-Link capacitor. So here we'd see how this can be done by wisely choosing the inverter states.

$$I_{c,rms}^2 = \overline{I_{dc,rms}^2} - \overline{I_{dc}^2} \quad (2.41)$$

From the balance of power between the DC supply and inverter input gives us the average value of the DC current as follows:

$$V_{dc} \bar{I}_{dc} = \frac{3}{2} I_p V_p \cos \varphi \quad (2.42)$$

Where  $V_p = \frac{2m_i V_{dc}}{\pi}$

Therefore the average value of the inverter input current can be given by (2.43).

$$\bar{I}_{dc} = \frac{3}{\pi} I_p m_i \cos \varphi \quad (2.43)$$

The RMS values of the inverter input current can be calculated as:

$$I_{dc,rms}^2 = \frac{2}{T} \int_0^{\frac{T}{2}} I_{dc}^2 dt \quad (2.44)$$

From where the average value of the input RMS current can be calculated as shown by (2.45), [54]. Because of the symmetry the calculation can be done for only one sector.

$$\bar{I}_{dc,rms}^2 = \frac{3}{\pi} \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} I_{dc,rms}^2 dt \quad (2.45)$$

Let us calculate the average RMS value of the inverter input current for SVM.

$$I_{dc,rms}^2 = \alpha_1 I_a^2 + \alpha_2 I_c^2 \quad (2.46)$$

Substituting the values of  $\alpha_1$  and  $\alpha_2$  from section I.2.5.1, equation (1.30) which can also be expressed by (2.47).

$$\alpha_1 = \frac{2\sqrt{3}}{\pi} V_{dc} m_i \cos\left(\omega t + \frac{\pi}{3}\right) \text{ and } \alpha_2 = \frac{2\sqrt{3}}{\pi} V_{dc} m_i \sin(\omega t) \quad (2.47)$$

$I_a$  and  $I_c$  are the phase currents which are given by (2.48).

$$\begin{aligned} I_a &= I \sin(\omega t - \varphi), \\ I_b &= I \sin\left(\omega t - \frac{2\pi}{3} - \varphi\right) \text{ and} \\ I_c &= I \sin\left(\omega t + \frac{2\pi}{3} - \varphi\right) \end{aligned} \quad (2.48)$$

The average value of the inverter input current for SVM is given by (2.49).

$$\bar{I}_{dc,rms\_SVM}^2 = \frac{I^2}{\pi^2} m_i \left\{ 2\sqrt{3} + 5 \left( \frac{1}{2} + \cos^2 \varphi \right) - 5 \right\} \quad (2.49)$$



Substituting this value and  $\bar{I}_{dc}$  from equation (2.43) in equation (2.41) gives the ripple current in the DC link capacitor.

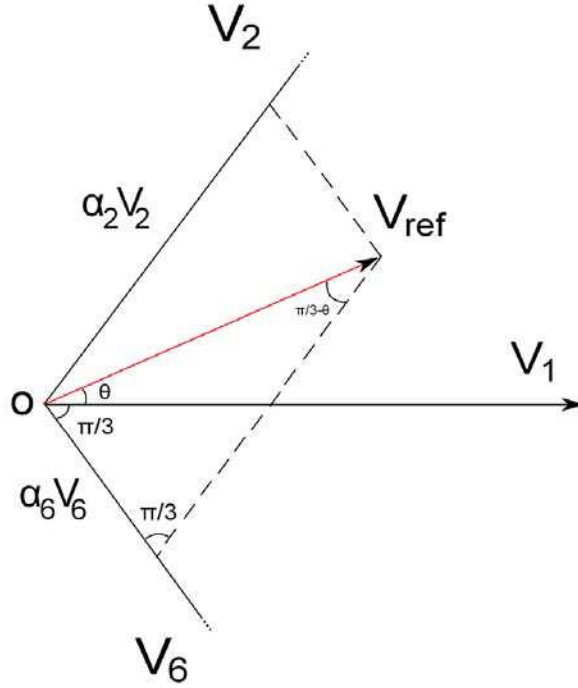
$$I_{c,rms\_SVM} = \frac{I}{\pi} \sqrt{m_i \frac{(6-5\sqrt{3})}{2} + m_i (5\sqrt{3} + 6 - 3m_i) \cos^2 \varphi} \quad (2.50)$$

In order to reduce this ripple current in the DC link capacitor, the inverter input current should be brought as closer to the average input current. This can be done avoiding the jumps to zero inverter input current each time a zero vector is applied. It was seen in the previous section that every active inverter state the inverter input current can be associated to one of the phase currents while for zero vectors the inverter input current is zero. It can be observed that the zero states of the inverter contribute heavily to the ripple component of the input current. In order to reduce the ripple component the zero vectors should be avoided.

#### **II.4.3.1.1. Reduction of input RMS ripple current**

To increase the active vector application time non adjacent vectors may be used as shown in Figure II-56. However for reference voltage smaller than a given value would still need a zero vector to complete the switching period, this method is presented in [55] but no analytical expressions for the input ripple current are presented to justify the claim. So the contribution of this section for this particular modulation strategy is to derive the mathematical expressions to quantify the ripple current flowing in the DC-link capacitor. The objective behind this work is to provide a faire ground to compare the magnitude of the ripple component of this current with the standard space vector technique, for which an analytical expression was derived earlier in this section.

The smallest value of  $V_{ref}$  for which  $\alpha_2 + \alpha_6 = 1$  can be calculated through simple trigonometric manipulations of the figure below.  $V_{ref,min} = \frac{V_{dc}}{\sqrt{6}}$  which corresponds to a maximum phase voltage of  $V_{max} = \frac{V_{dc}}{3}$ , which in turn corresponds to a modulation index of  $m_i = 0.6$ .



**Figure II-56: Two non-adjacent vectors – case1**

The RMS value of the inverter input current for this particular strategy for modulation index,  $m_i < 0.6$  can be calculated as follows.

$$I_{dc,rms}^2 = \alpha_2 I_c^2 + \alpha_6 I_b^2 \quad (2.51)$$

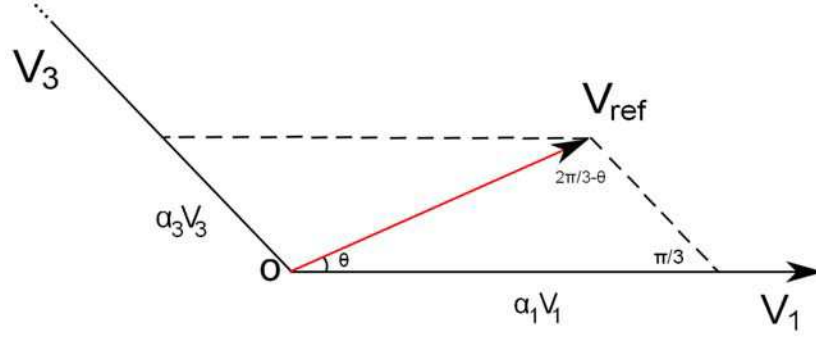
From Figure II-56 applying the law of sines we get:

$$\frac{\alpha_2 V_2}{\sin\left(\theta + \frac{\pi}{3}\right)} = \frac{\alpha_6 V_6}{\sin\left(\frac{\pi}{3} - \theta\right)} = \frac{V_{ref}}{\sin\left(\frac{\pi}{3}\right)} \quad (2.52)$$

From where:

$$\alpha_2 = \frac{\sqrt{\frac{3}{2}}V_\alpha + \sqrt{\frac{1}{2}}V_\beta}{V_{dc}} \text{ and } \alpha_6 = \frac{\sqrt{\frac{3}{2}}V_\alpha - \sqrt{\frac{1}{2}}V_\beta}{V_{dc}} \quad (2.53)$$

The second possibility using two non-adjacent vectors for the reference vector in the first is using vectors V1 and V3 as shown in Figure II-57



**Figure II-57: Two non-adjacent vectors – case2**

The duty cycles can be calculated in the same manner. The duty cycle expressions for the first sector is given by (2.54).

$$\alpha_1 = \frac{\sqrt{\frac{3}{2}}V_\alpha + \sqrt{\frac{1}{2}}V_\beta}{V_{dc}} \text{ and } \alpha_3 = \frac{\sqrt{2}V_\beta}{V_{dc}} \quad (2.54)$$

The choice between the two cases should be made wisely depending on the polarity of the currents. For example for sector one if  $i_b$  and  $i_c$  have the same polarity then V2 and V6 or else V1 and V3 if  $i_a$  and  $i_b$  have the same polarity. This makes sure that  $I_{dc}$  does not make jumps from a positive to a negative value which would increase input RMS ripple content of the input current. The same train of thought can be extended to the remaining five sectors.

Now substituting the values of  $V_\alpha = \frac{\sqrt{6}}{\pi}m_iV_{dc}\cos\omega t$  and  $V_\beta = \frac{\sqrt{6}}{\pi}m_iV_{dc}\sin\omega t$  in (2.53) we get:

$$\alpha_2 = \frac{2\sqrt{3}}{\pi}m_i\sin\left(\omega t + \frac{\pi}{6}\right) \text{ and } \alpha_6 = \frac{2\sqrt{3}}{\pi}m_i\sin\left(-\omega t + \frac{\pi}{3}\right) \quad (2.55)$$

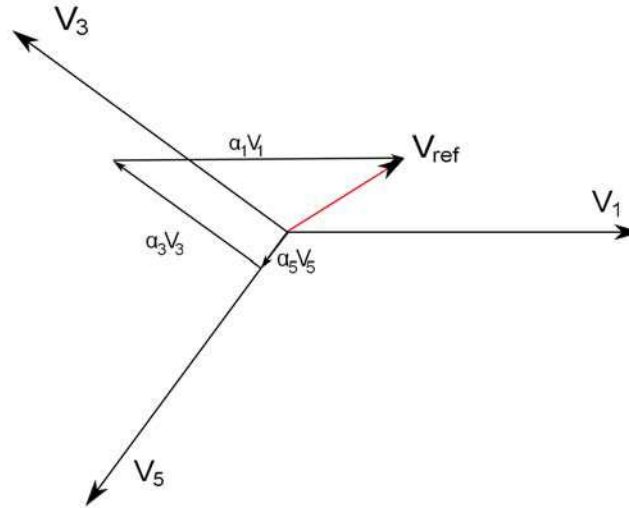
After some tedious calculations we get

$$\bar{I}_{dc,rms}^2 = \frac{\sqrt{3}}{2\pi^2}I^2m_i\left\{2\left(3\sqrt{3}-10\right)\cos^2\varphi-1\right\} \quad (2.56)$$

From where the ripple current into the capacitor can be given by (2.57).

$$I_{c,rms} = \frac{I}{\pi}\sqrt{\frac{\sqrt{3}}{2}m_i\left(3\sqrt{3}-10\right)\cos 2\varphi + \left(5-6\sqrt{3}\right)\sin 2\varphi - 9m_i^2\cos^2\varphi} \quad (2.57)$$

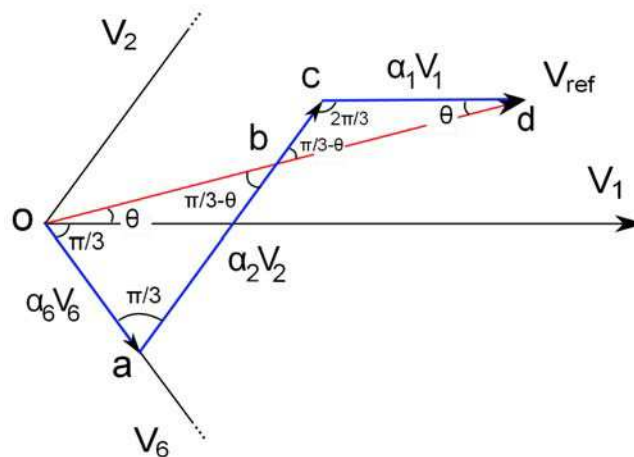
It should be noted that using a set of three non-adjacent vectors V1, V3 and V5 (shown in Figure II-58) or V2, V4 and V6 using zero vectors be avoided for very low modulation index.



**Figure II-58: Three non-adjacent vectors**

However this will not solve the problem of reducing the inverter input current ripple as we know that the sum of all the currents is zero,  $I_a + I_b + I_c = 0$  which implies that only two of the phase currents have the same polarity, which would mean that the inverter current would jump from a negative value to a positive value which is worse than jumping to zero current as in the case of using zero vectors. Hence this method will not be explored any further.

Now for  $m_i > 0.5$  zero vectors can be avoided if an extra adjacent vector is used, i.e. three adjacent vectors can be used to construct a given reference voltage vector, Figure II-59. In the example vectors  $V_6$ ,  $V_1$  and  $V_2$  are shown however using the same logic as in the case of two non-adjacent vectors the other option of using vectors  $V_1$ ,  $V_2$  and  $V_3$  can be used as well. For example  $V_1$ ,  $V_2$ ,  $V_3$  should be used while  $i_a$  and  $i_b$  have the same polarity else  $V_6$ ,  $V_1$  and  $V_2$  should be used if  $i_b$  and  $i_c$  have the same polarity. Again using some geometrical properties of triangles and some trigonometric identities we can calculate the values of the duty cycle for each of the three active vectors.



**Figure II-59 Three adjacent vectors**

Applying sine rule to triangles,  $\Delta oab$  and  $\Delta bdc$  we get (2.58) and (2.59) respectively.

$$\frac{ab}{\sin\left(\theta + \frac{\pi}{3}\right)} = \frac{ob}{\sin\left(\frac{\pi}{3}\right)} = \frac{\alpha_6 V_6}{\sin\left(\frac{\pi}{3} - \theta\right)} \quad (2.58)$$

$$\frac{\alpha_1 V_1}{\sin\left(\frac{\pi}{3} - \theta\right)} = \frac{bd}{\sin\left(\frac{2\pi}{3}\right)} = \frac{bc}{\sin(\theta)} \quad (2.59)$$

Where  $\alpha_2 V_2 = ab + bc$  and  $V_{ref} = ob + bd$ . Imposing  $\alpha_1 + \alpha_2 + \alpha_6 = 1$  the expressions for the duty cycles are given by the following expression, (2.60).

$$\alpha_1 = \frac{2\sqrt{3}-3}{\pi} m \sin \omega t, \alpha_2 = 1 - \frac{3}{\pi} m \left\{ \cos \omega t - \frac{1}{\sqrt{3}} \sin \omega t \right\} \quad (2.60)$$

$$\text{and } \alpha_6 = \frac{3}{\pi} m \left\{ \cos \omega t - (1 - \sqrt{3}) \sin \omega t \right\}$$

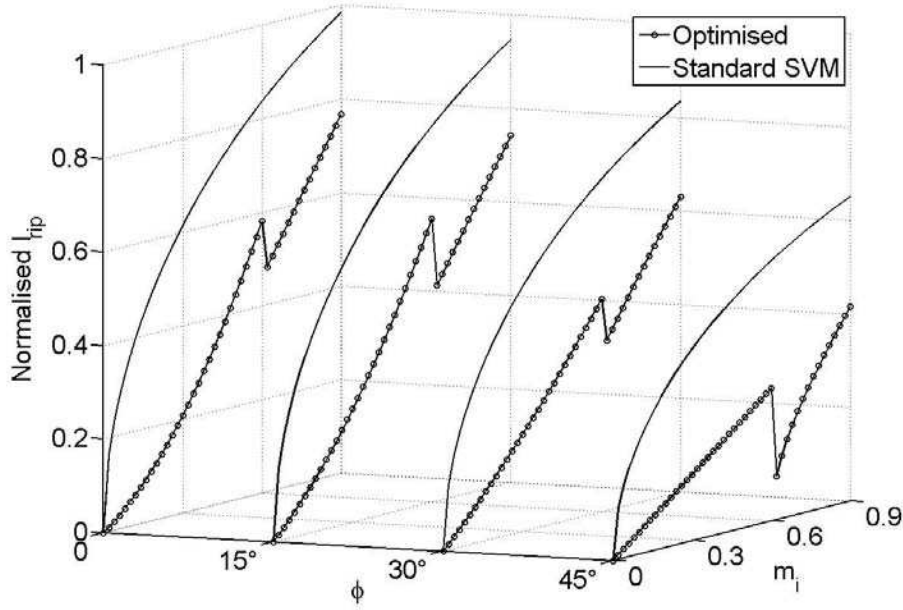
For a given sector k,  $\theta = \theta - (k-1)\frac{\pi}{3}$  in equations (2.58) and (2.59).

Equation (2.61) gives the expression for the average RMS current flowing in to the inverter.

$$\bar{I}_{dc,rms} = \frac{I}{\pi} \sqrt{1 + \frac{9\sqrt{3}}{\pi^2} m_i - \left\{ \frac{3}{2\pi} + \frac{(3\sqrt{3}+15)}{4\pi^2} m_i \right\} \cos 2\varphi + \frac{(5\sqrt{3}-30)}{4\pi^2} m_i \sin 2\varphi} \quad (2.61)$$

From where the ripple current  $I_{c,rms}$  into the capacitor can be given by calculated subtracting the average input current,  $\bar{I}_{dc}$  given by equation (2.43) from the average input RMS current,  $\bar{I}_{dc,rms}$  given by (2.61).

To compare the new technique of reduction of current stress on the capacitor with the standard space vector method, In the Figure II-60 we've plotted the RMS ripple content of the capacitor current, normalised with respect to the RMS values of the phase current ' $I$ ', for the two cases for different values of the load angle and for the complete linear range (0 to 0.907) of the modulation index ' $m_i$ '.



**Figure II-60: RMS input current ripple**

An abrupt decrease in the ripple component can be observed for the optimised technique for  $m_i = 0.6$  which represents the transition from the technique where two non-adjacent active vectors along with 1 zero vector are used to the other technique where only three adjacent active vectors are used.

In this section we successfully showed how the inverter input ripple current can be reduced by modifying the PWM strategy. Now we'll study how it affects the input voltage.

#### **II.4.3.2. Ripple component of the inverter input Voltage**

Based on the same hypothesis that all the ripple component of the inverter input current is absorbed by the DC link capacitor, the voltage across the capacitor can be given by:

$$\tilde{V}_c = \frac{1}{C} \int (\bar{I}_{dc} - I_{dc}) dt \quad (2.62)$$

The instantaneous value of the inverter input current depends on the state of the inverter, we'll study the case where the reference vector falls in the first sector. The calculations are made for the standard SVM where vectors V1 and V2 are used along with the zero vectors, V0 and V7.

$$\tilde{V}_c = \frac{1}{C} \left\{ \int_0^{T_1} (\bar{I}_{dc} - i_a) dt + \int_0^{T_2} (\bar{I}_{dc} + i_c) dt + \int_0^{T_0} \bar{I}_{dc} dt \right\} \quad (2.63)$$

Where  $T_1 = \alpha_1 T$ ,  $T_2 = \alpha_2 T$  and  $T_0 = T - (T_1 + T_2)$  and the vector duty cycles are given by (2.47) for SVM. Now the mean square value of the ripple voltage over a PWM period can be calculated as:

$$\tilde{V}_{c,T}^2 = \frac{1}{TC^2} \left\{ (\bar{I}_{dc} - i_a)^2 \int_0^{T_1} t^2 dt + (\bar{I}_{dc} + i_c)^2 \int_0^{T_2} t^2 dt + \bar{I}_{dc}^2 \int_0^{T_0} t^2 dt \right\} \quad (2.64)$$

Which leads to the following expression for the macroscopic mean ripple voltage:

$$\tilde{V}_{c,T}^2 = \frac{T^2}{3C^2} \left\{ \bar{I}_{dc}^2 (1 - \alpha_1 - \alpha_2)^3 + (\bar{I}_{dc} - i_a)^2 \alpha_1^3 + (\bar{I}_{dc} + i_c)^2 \alpha_2^3 \right\} \quad (2.65)$$

Now the average value of this voltage over a fundamental period can be calculated as:

$$\tilde{V}_{c,rms}^2 = \frac{3}{\pi} \int_{\pi/6}^{\pi/2} \tilde{V}_{c,T}^2 d\omega t \quad (2.66)$$

Upon some simplifying we obtain the final expression as:

$$\tilde{V}_{c,rms}^2 = \frac{T^2 I^2}{\pi C^2} \left\{ \frac{\bar{I}_{dc}^2}{I^2} \left( \frac{\pi}{3} + \frac{3k^2}{16} (4\pi + 2\sqrt{3}) \right) + \frac{\bar{I}_{dc} k^3}{16I} \left( (2\sqrt{3} - 2\pi - 11) \cos \varphi + (2\pi\sqrt{3} + 4\pi - 5\sqrt{3} + 11) \sin \varphi \right) + k^3 \left( 3 + 3\sqrt{3} + \frac{(5 - 21\sqrt{3})}{10} \cos 2\varphi + \frac{(15\sqrt{3} - 24)}{10} \sin 2\varphi \right) \right\} \quad (2.67)$$

Where  $k = \frac{2\sqrt{3}}{\pi} m_i$ .

Similarly for the optimised technique developed earlier in this section and we'd get two separate expressions for  $m < 0.6$  and  $m > 0.6$ , given by (2.68) and (2.69) respectively.

$$\tilde{V}_c = \frac{1}{C} \left\{ \int_0^{T_2} (\bar{I}_{dc} + i_c) dt + \int_0^{T_6} (\bar{I}_{dc} + i_b) dt + \int_0^{T_0} \bar{I}_{dc} dt \right\} \quad (2.68)$$

$$\tilde{V}_c = \frac{1}{C} \left\{ \int_0^{T_1} (\bar{I}_{dc} - i_a) dt + \int_0^{T_2} (\bar{I}_{dc} + i_c) dt + \int_0^{T_6} (\bar{I}_{dc} + i_b) dt \right\} \quad (2.69)$$

Where  $T_2 = \alpha_2 T$ ,  $T_6 = \alpha_6 T$  and  $T_0 = T - (T_2 + T_6)$  and

$T_1 = \alpha_1 T$ ,  $T_2 = \alpha_2 T$  and  $T_6 = \alpha_6 T$  given by equation (2.55) and (2.60) respectively.

The mean square value of the ripple voltage across the DC-link capacitor for  $m < 0.6$  is given by (2.70).

$$\tilde{V}_{c,rms}^2 = \frac{T^2 I^2}{\pi C^2} \left\{ \begin{aligned} & \frac{\bar{I}_{dc}^2}{I^2} \left( \frac{\pi}{3} + \frac{k^2}{2} (4\pi - 32\sqrt{3}) + \frac{5k^3}{12} \right) + \\ & \frac{\bar{I}_{dc} k^3}{24I} \left( (3\sqrt{3} + 2\pi) \cos \varphi + (2\pi(\sqrt{3} + 6) + 21\sqrt{3} + 6) \sin \varphi \right) \\ & + k^3 \left( \frac{8 - 15\sqrt{3}}{10} + 2\sqrt{3} \cos^2 \varphi - 3 \cos \varphi \sin \varphi \dots \right. \\ & \left. - \frac{(3\sqrt{3} + 16)}{5} \cos 2\varphi + \frac{(5\sqrt{3} + 4)}{5} \sin 2\varphi \right) \end{aligned} \right\} \quad (2.70)$$

The mean square value of the ripple voltage across the DC-link capacitor for  $m > 0.6$  is given by (2.71).

$$\tilde{V}_{c,rms}^2 = \frac{T^2 I^2}{\pi C^2} \left\{ \begin{aligned} & \frac{\bar{I}_{dc}^2}{I^2} \left( \frac{\pi}{3} - \frac{11\sqrt{3}}{2} + \frac{139}{15} + m \left( \pi - \frac{3\sqrt{3}}{\pi} \right) \right) + \\ & \frac{\bar{I}_{dc}}{I} \left( \frac{(2\sqrt{3} - 3)}{\pi} m \left( \left( \frac{\pi}{8} + \frac{7\sqrt{3}}{64} \right) \cos \varphi - \frac{15}{64} \sin \varphi \right) + \sin \varphi + k \left( \left( \frac{27}{4} - \frac{4\sqrt{3}\pi}{3} \right) \cos \varphi - \frac{1}{3} \sin \varphi \right) \dots \right. \\ & \left. + \frac{k}{64} \left( (-342 + 2\pi + 179\sqrt{3}) \cos \varphi + (147 + (36\sqrt{3} - 64)\pi) \sin \varphi \right) \right) \\ & + \frac{(2\sqrt{3} - 3)}{\pi} m \left( \frac{19\sqrt{3}}{80} \cos^2 \varphi + \frac{1}{180} \cos \varphi \sin \varphi - \frac{1}{5} \sin(2\varphi) + \frac{11\sqrt{3}}{160} \right) + \frac{\pi}{6} + \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{2} \cos^2 \varphi \dots \\ & + k \left( \frac{\pi}{6} - \frac{6\sqrt{3}}{5} + \frac{88}{24} + \left( \frac{11}{2} + \frac{\pi}{3} - \frac{159\sqrt{3}}{40} \right) \cos^2 \varphi + \left( \frac{93\sqrt{3} - 164}{20} \right) \cos \varphi \sin \varphi \right) \end{aligned} \right\} \quad (2.71)$$

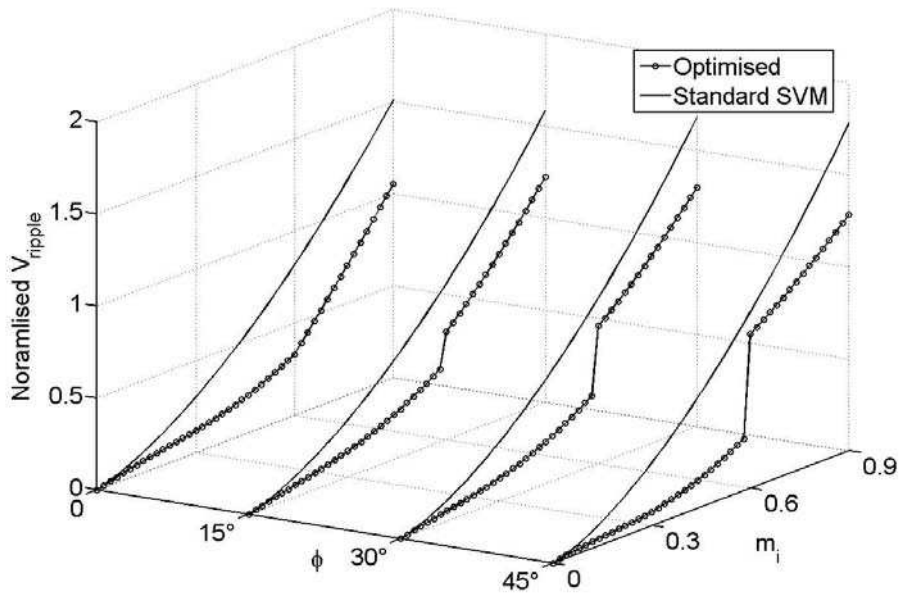


Figure II-61: RMS input voltage ripple



The RMS value of the input voltage ripple would be the square root of the expressions given by (2.70) and (2.71). Normalising these expressions with  $\frac{TI}{C}$  to free it from any dependence on the switching frequency, the load current or the capacitor value. These normalised values as a function of ' $m_i$ ' are traced in Figure II-61 for different values of the load angle. Lesser input voltage can be expected for the case where the input ripple current is minimised. Again an abrupt change can be noticed for the optimised technique for  $m_i = 0.6$  which represents the transition from the technique where two non-adjacent active vectors along with 1 zero vector are used to the other technique where only three adjacent active vectors are used.

### II.4.3.3. Simulation

Simulation are done on Simulink the inverter and three phase RL load are taken from the SimPowerSystem library. Here we observe the inverter input current rest remaining for the standard SVM technique and for the one proposed here to reduce the ripple content of this current. Results are shown for static RL load.

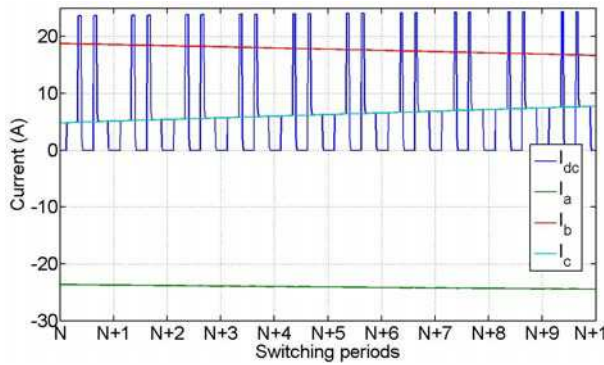


Figure II-62:  $I_{dc}$  and  $I_{abc}$ , SVM. E.g.-1

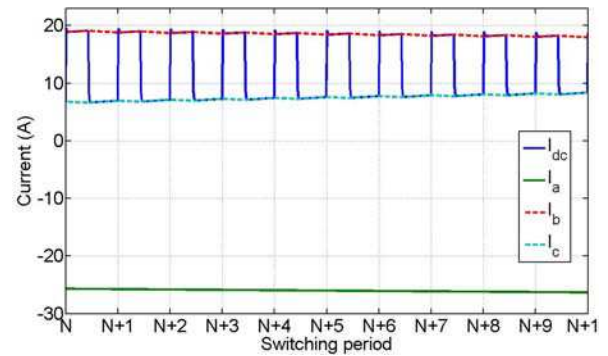


Figure II-63:  $I_{dc}$  and  $I_{abc}$ , Opt. E.g.-2

Figure II-62 and Figure II-63 trace the inverter currents for standard SVM technique and the optimised technique respectively. The curve in blue represents the inverter input current and the other three are the phase currents. While phase currents are more or less similar for the two cases, a striking difference can be observed while comparing the inverter input current. For the standard case we see that the input current takes up three values,  $I_c$ ,  $-I_a$  and zero. Whereas for the optimised technique the input current takes up only two values  $I_b$  and  $I_c$ . It is evident that the input current varies less in the second case which implies lesser RMS input ripple current.

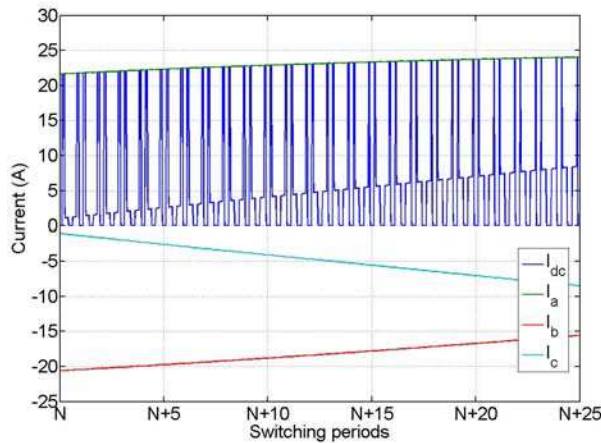


Figure II-64:  $I_{dc}$  and  $I_{abc}$ , SVM. E.g.-2

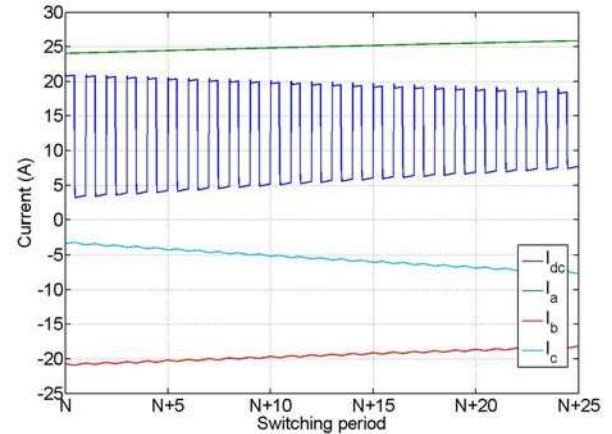


Figure II-65:  $I_{dc}$  and  $I_{abc}$ , Opt. E.g.-2

Similarly another example is shown in Figure II-64, Figure II-65. While the phase currents remain similar we can again notice stark difference in the inverter input currents. For the standard case the input current again takes up three different values,  $I_a$ ,  $-I_c$  and zero. Whereas for the optimised technique it again takes up only values;  $-I_b$  and  $-I_c$ . It can be easily said looking up at these two traces that there is less variation in the value of the input current for the optimised current and hence a lower value of RMS ripple current.

#### II.4.3.4. Summary

In this section we studied the correlation between the PWM techniques and the inverter input current. The objective of this study was to modify the PWM technique in so that a smaller DC-link capacitor can be used. We saw that it is the DC-link capacitor which feeds the inverter with the transient power and high frequency pulse currents whereas the battery feeds the average inverter input. This pulse current is represented by the RMS input current ripple.

Here we proposed some changes in the modulation strategy which attenuates the demand of the inverter in terms of pulsed current. Performance wise there is one drawback of this technique which is the increase of harmonic content in the output voltage. This increase is due to the use of non-adjacent vectors. The effectiveness of these techniques depends on the modulation index and the load angle. It is analytically shown that the proposed techniques can decrease the input RMS ripple current by at least 20% under all operating zones. Finally its repercussions on the ripple voltage are studied and lower ripple content is observed for the proposed modulation strategy. These techniques are complicated to implement. They require precise pulse placement. These techniques are feedback techniques which require the current information or the load angle information to make intelligent choices between different sets of vectors to be used.

# **PART III**

## **EXPERIMENTAL VALIDATION**

# III. Experimental Validation

In order to validate the modulation strategies developed in earlier chapters they are to be tested for closed loop electric drives for which they are destined. A versatile test bench has to be developed in order to properly carry out the tests needed for validation not only from a purely technical point of view but also evaluate its feasibility as an embeddable solution. For this reason a new test platform with a digital signal processor (DSP) is developed. A careful choice of DSP is made to be able to fulfil the needs of a digitally controlled electric drive. Interfacing the Control Unit (DSP) with the Power Unit (electric machine and inverter) is done through an electronic circuit which would condition and scale the sensor output to levels acceptable to the ADC or binary ports of the DSP. Similarly gate signals to control the inverter switches had to be adapted to levels acceptable to the IGBT drivers. The experimental evaluation of the PWM schemes are done on the data acquired through an oscilloscope to get a fine resolution, which are then processed in Matlab.

## III.1. Introduction

Test bench consists of a 20kW inverter, a PMSM and a DSP. The DSP is interfaced with the inverter through a PCB. Standard vector controlled torque regulation strategy, the Field Oriented Control (FOC) method is used to run the machine.

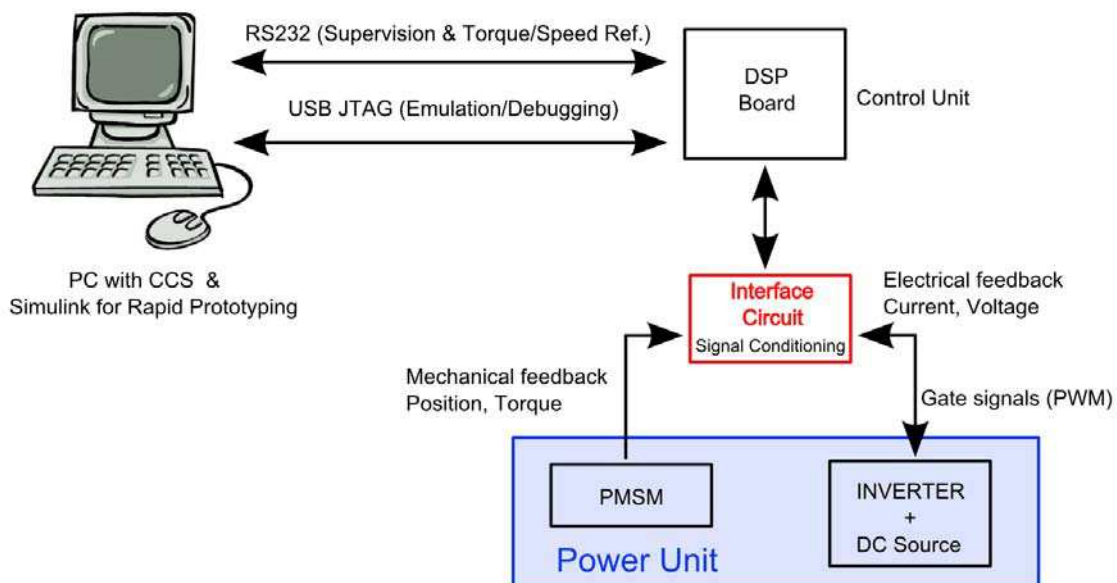


Figure III-1: Test bench, schematic diagram

## III.2. Test bench

The experimental setup consists of a 20 kW, 3-phase 2-level IGBT inverter designed for a nominal DC input voltage of 600 V with current and voltage sensors directly mounted on the legs for stator current and line voltage measurements, a 3 kW PMSM with an incremental encoder (4096 points), a 150 MHz Floating-Point DSP “TMS32F8335”, high speed acquisition system, a buffer card for gate signal conditioning with incorporated dead time. The test bench consists of two identical PM synchronous machines mechanically coupled, one that is part of the system under observation is controlled and the other acts as a load. In this section a detailed description of the test bench is given. The test setup is divided into 3 parts the power unit, the control unit and an electronic unit to interface the power and control unit.

### III.2.1. Power Unit

This unit comprises of a three phase IGBT inverter, a *Magna-PowerElectronics*' three phase full wave rectifier and a '*Leroy Somer*' electric motor, 6 Pole surface mounted permanent magnet sinusoidal synchronous motor (PMSM), with a nominal speed of 6000rpm and a maximum torque of 4.5Nm.

#### III.2.1.1. Power Converter -- VSI

The converter is a DC to AC three phase voltage source inverter. The maximum inverter input voltage and current are 600V and 100Amps respectively. The maximum rated power of the inverter is 20kW. The inverter can be switched till 30kHz.

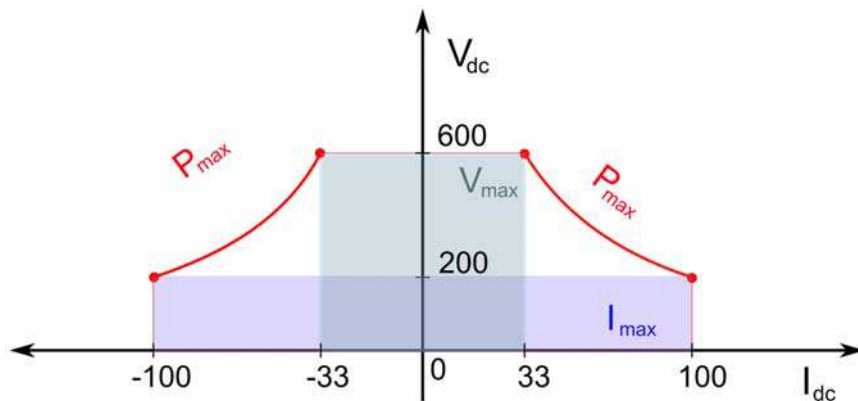
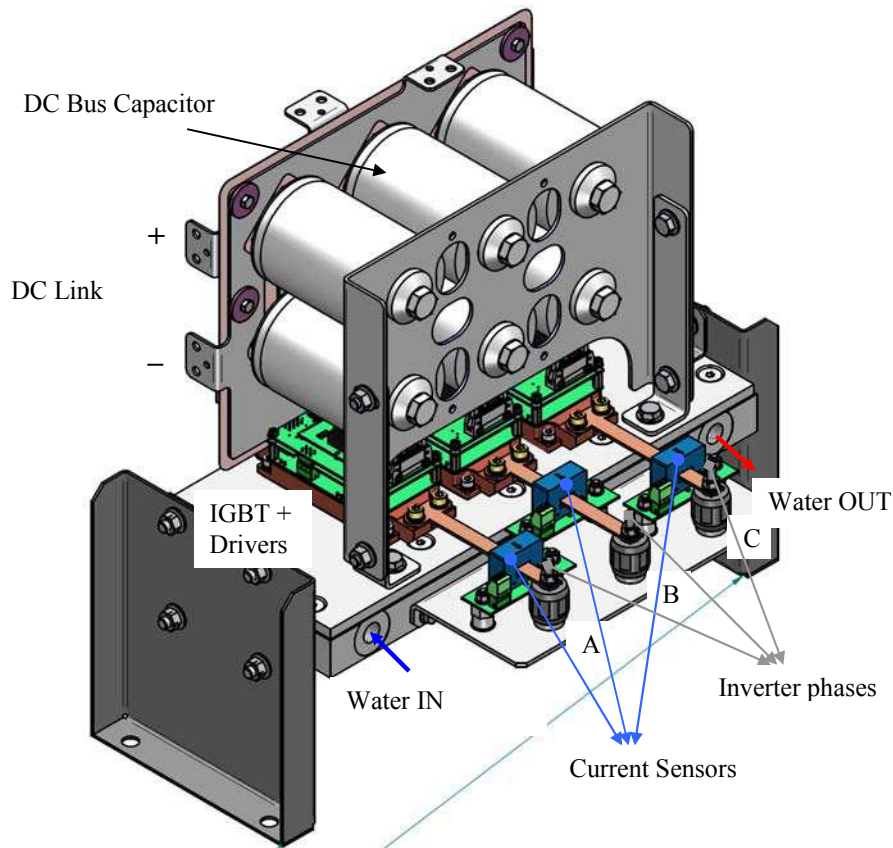


Figure III-2: Inverter V-I characteristics

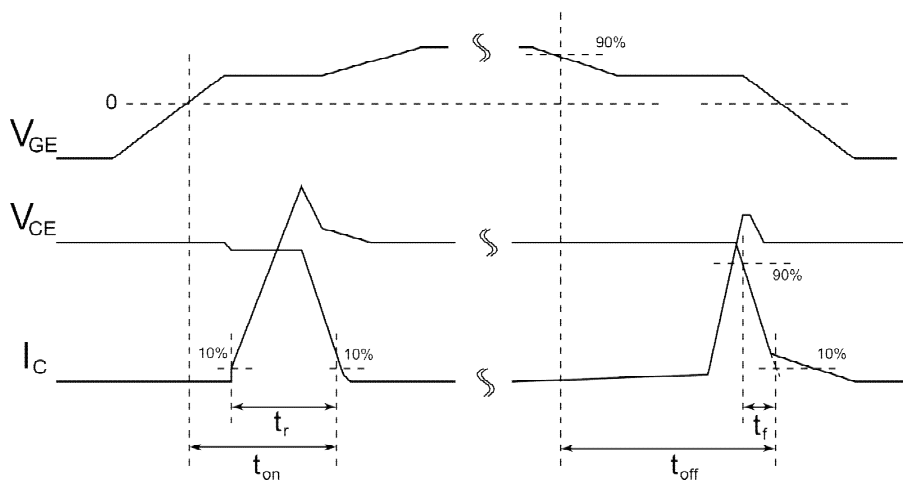
For these power ratings and switching frequencies IGBTs are better suited. *Fuji Electric Device and Technology* IGBT modules are used, ref. no. 2MBI225U4N-120-50. The different operating regions are graphically depicted in the Figure III-2, which shows basically three boundary conditions where either the input voltage or the input current or the input power reaches the maximum rated values. For e.g. for maximum rated voltage of

600V the input current should not exceed 33Amps to respect the inverter maximum power ratings. The redline on the figure shows the different set of inverter voltage and current values while respecting the inverter power ratings.



**Figure III-3: Inverter module**

The maximum RMS phase voltage without pulse dropping is ~244V hence for maximum power an RMS current of ~27Amps. The inverter is water cooled and each inverter leg is equipped with a temperature sensor. The drivers are capable of detecting short-circuits in case of inverter malfunction.



**Figure III-4: IGBT Switching characteristics**



The switching characteristics of IGBTs are shown in Figure III-4. The switching characteristics for an input voltage of 600V conducting about 50Amps of current at 25°C are  $t_{on}=250ns$ ,  $t_r=60ns$ ,  $t_{off}=500ns$ ,  $t_f=90ns$ .

### III.2.1.2. DC source

The DC source is TSa600-32 by *Magna-Power Electronics* 20kW, 3 phase full wave rectifier, supplies filtered DC voltage with an output voltage ripple component of 250mV RMS and an efficiency of 88% .



Figure III-5: DC source

### III.2.1.3. Electric machine

The electric machine used is a three phase *Leroy Somer* permanent magnet synchronous machine reference number *095E2C600BACAA100190*. Has a 4096 point incremental encoder mounted on the motor shaft. The machine characteristics are given in the Table III-1. Where Drive  $V_{PWM}$  is the rated input voltage of the DC link feeding the inverter which makes the nominal RMS phase voltage for the machine at a modulation index of 0.907 equal to 98V.

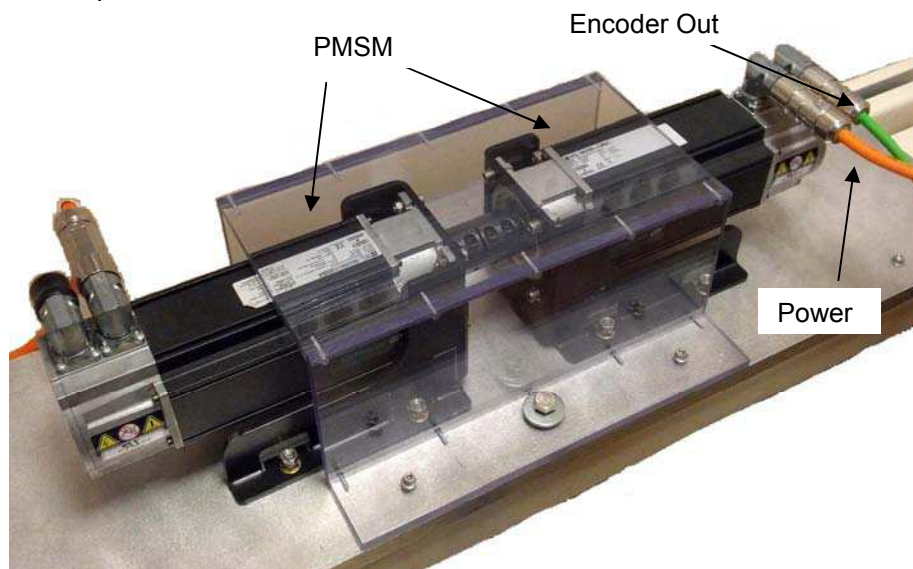


Figure III-6: Electric Machine

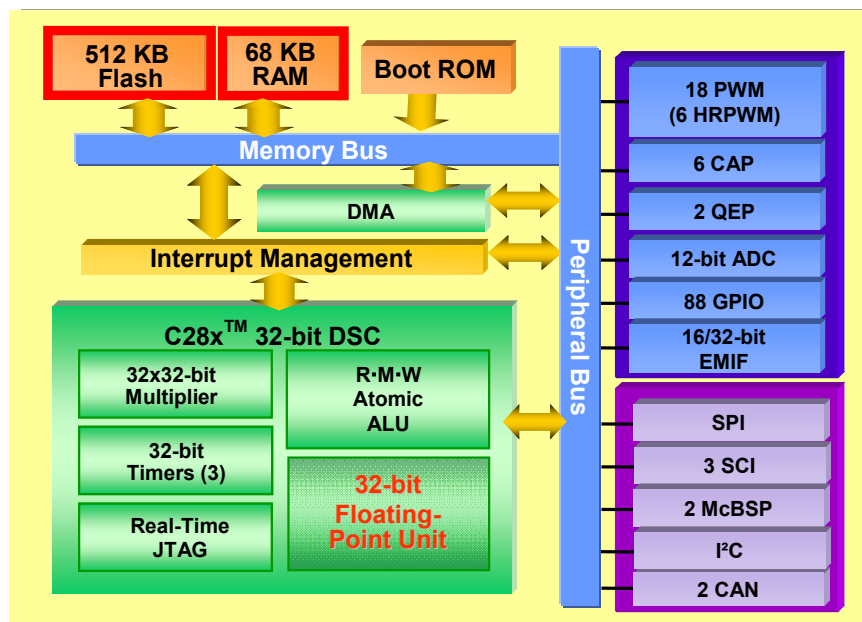
<b>Rated Power</b>	1.76kW
<b>Drive <math>V_{PWM}</math></b>	220-240V
<b>Rated Torque</b>	2.8Nm
<b>Stall torque</b>	5.9Nm@12.7Amps
<b>Kt : 1.6 Nm/A</b>	0.47Nm/Amp
<b>Rated speed</b>	6000rpm
<b>Max Speed</b>	63000rpm
<b>Ke : 98V/Krpm</b>	28.5V/Krpm

**Table III-1: Machine characteristics**

The load connected to the power supply is comprised of two AC machines connected to the same shaft. These permanent magnet synchronous machines are rated at 90Vrms/ 3.2Arms; the rated speed is 6000 rpm. Again, the machine operating as a generator is loaded by a variable resistance capable of dissipating the required power.

### ***III.2.2. Control unit***

The test platform should be able to implement FOC control for a PMSM this requires the stator currents and the rotor position information, the position information can be either pulses in the case of a encoder or analog signals for resolvers.



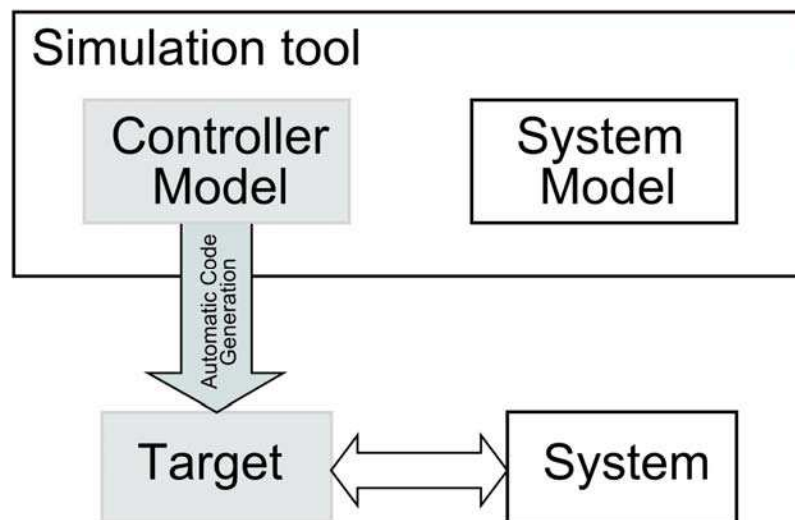
**Figure III-7: F28x architecture**



A part from this it should be capable of generating high frequency pulses for PWM inverter control and should also be able to read high frequency quadrature pulses from optical encoders required for electric machine control. This can be done using a standard DSP used for signal processing and CPLD/ FPGA for high frequency high resolution pulse generation. Now a days digital motor control is being widely implemented and dedicated signal processors with PWM and Quadrature pulse reading peripherals are easily available on a number of DSPs dedicated for electric motor control.

Figure III-7 shows the hardware architecture of DSP family F26x, with all the peripherals on the right hand side. To shorten the development cycle *Spectrum Digital's* eZdsp™ with TMS32F8335 which is a ready to use DSP board specially designed for electric motor control and has the following hardware features:

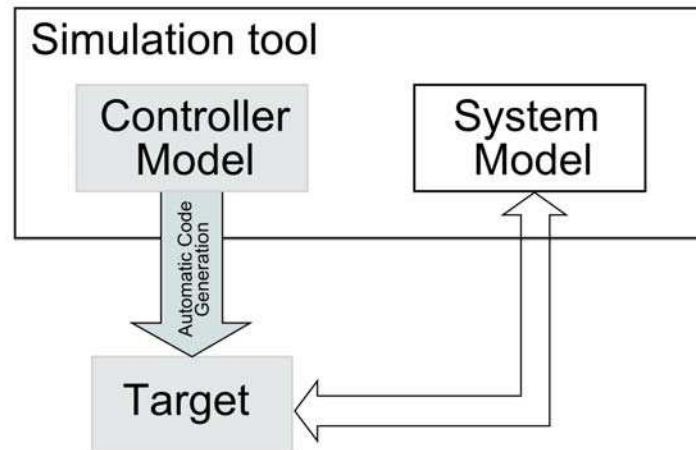
- 150 MHz. operating speed
- 32 x 32 MAC Operations
- On chip 12 bit Analog to Digital (A/D) converter with 16 input channels
- 68K bytes on-chip RAM, 512K bytes on-chip Flash memory
- on board RS-232 connector with line driver and CAN 2.0 interface



**Figure III-8: Rapid prototyping**

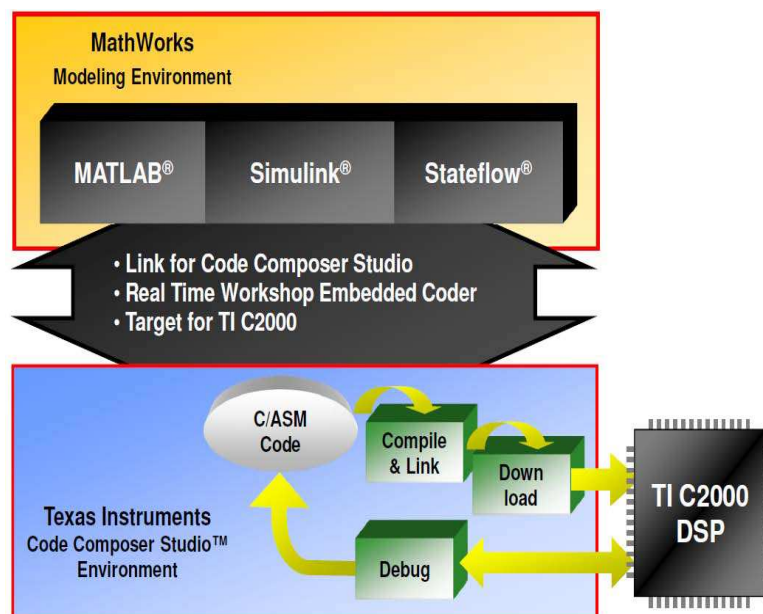
Some of the interesting software features are: Automatic code generation (Rapid prototyping) from Matlab/Simulink models, Figure III-8. Hardware drivers for Simulink, Code Composer Studio™ Integrated Development Environment, Texas Instruments' Flash APIs to support the F28335, Texas Instruments' F28335 header files and example softwares.

The simulation tool here is Simulink, the controller and system model are prepared first. The system model should be as closely represented by the model as possible. Before actually controlling the and then simulation in the loop (SiL) as shown in Figure III-9.



**Figure III-9: Simulation in the Loop**

Pure simulation tests are performed where the entire system; the control and the plant model are modelled in a simulation environment before actually programming the DSP. The Figure III-10 shows how the rapid prototyping works for a *Simulink* and *TI* DSPs.



**Figure III-10: Automatic code generation**

The *Simulink* model is translated into a C program which is then compiled by code composer studio and the target, DSP in the case is flashed with the executable via JTAG liaison.

### **III.2.3. Power and control Interface**

Interface circuitry is required to make the exchange of the electrical signals between the DSP and the inverter coherent as they work at different voltage levels. The ADCs used in the given DSP accept a voltage level of 0-3 volts. All information coming

from the sensors to the ECU need to be processed to perform a feedback control or the fault are to be brought to an acceptable form and level. The voltage and current sensors output the image of the alternating current and voltages representing the real voltage and current in the inverter. So any mechanical or electrical information is to be transformed into a voltage signal of 0-3V. The IGBT drivers are controlled by binary logic 0 and 15V whereas the ECU provides binary signals of 0 and 3.3V.

The circuits were designed and the components used were chosen, the layout and the size of the board defined the fabrication of the Printed Circuit Board (PCB) was outsourced.

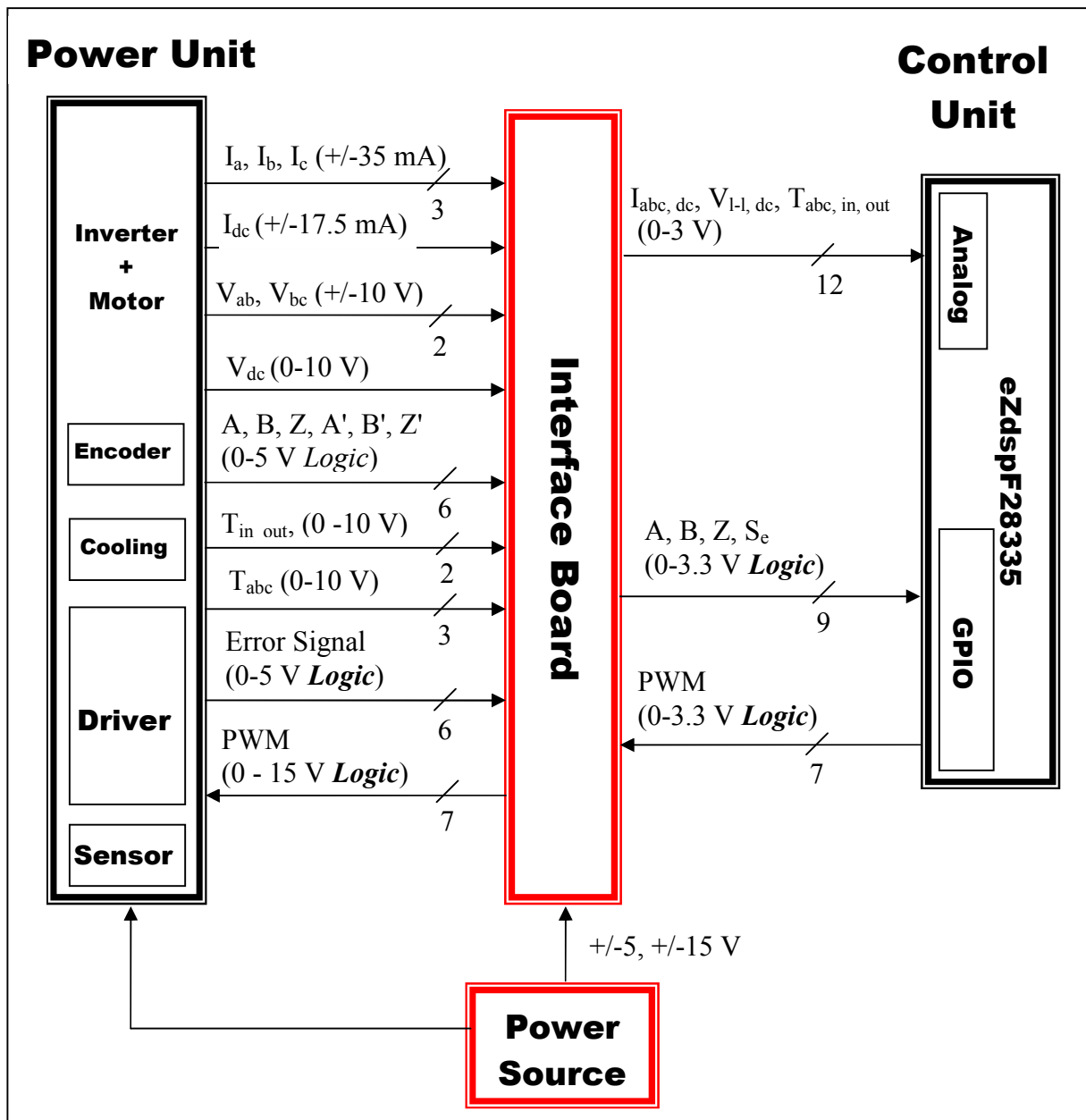


Figure III-11: Schematic diagram of the interface circuit

The schematic diagram of the interface is shown in Figure III-11. Here it can be seen that the signals to be interfaced can be divided into two groups analog and binary. Different approaches are undertaken while designing the interface board to assure immunity from electromagnetic noise present in such environments, have high precision and general ruggedness of the electrical circuit.

### III.2.3.1. Analog signal interfacing

The use of readymade circuits for analog signal conditioning is avoided and custom-made circuits are designed to get maximum precision on the measurements by using the complete input range of the Analog to Digital Converter input.

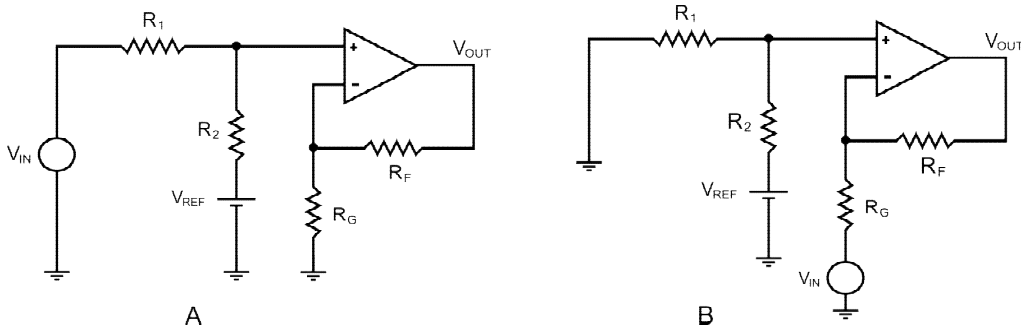


Figure III-12: Analog Signal Scaling

$$V_{OUT} = V_{IN} \left( \frac{R_2}{R_1 + R_2} \right) \left( \frac{R_F + R_G}{R_G} \right) + V_{REF} \left( \frac{R_1}{R_1 + R_2} \right) \left( \frac{R_F + R_G}{R_G} \right) \quad (3.1)$$

$$V_{OUT} = -V_{IN} \frac{R_F}{R_G} + V_{REF} \left( \frac{R_1}{R_1 + R_2} \right) \left( \frac{R_F + R_G}{R_G} \right) \quad (3.2)$$

The two basic topologies used are shown in Figure III-12. The circuit can be mathematically represented by a straight line, as shown in (3.1) and (3.2) for topology A and B respectively. A pair of points would help determine the slope and a suitable offset value can be chosen if needed.

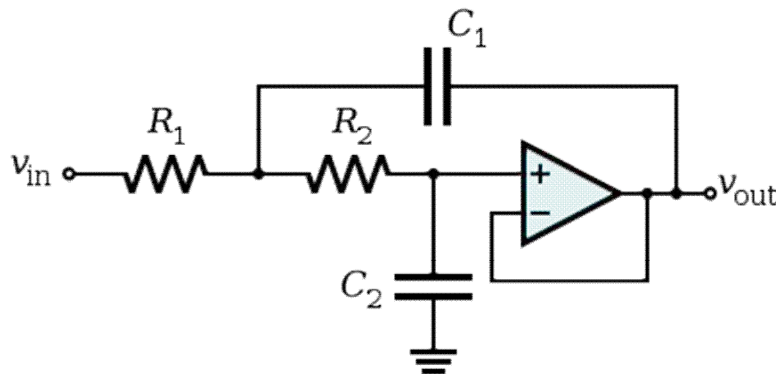


Figure III-13: Anti-aliasing filter

Now that these information signals have been put to scale, anti-aliasing filters have to be used before they are fed to the ADC. We have used second order Butterworth sallen-key topology with unity gain.

$$\omega_0^2 = \frac{1}{R_1 R_2 C_1 C_2} \quad (3.3)$$

where  $\omega_0 = 2\pi f_c$

The filter can be calculated using (3.3) and the behaviour can be controlled using (3.4)  $f_c$  is the cut-off frequency of the filter and  $Q$  is the quality factor which determines the behaviour of the filter and can be chosen to satisfy the requirements for a given application.  $Q$  basically determines the filters step response.

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 C_1 + R_2 C_1 - (K-1)R_1 C_2} \quad (3.4)$$

A low  $Q$  value should be chosen if overshoot and ringing is undesirable and fast response is not required where as high  $Q$  values result in shorter rise times, bigger overshoots and longer ringing. We used  $Q=1/\sqrt{2}$  which gives a nice compromise between the rise time and the settling time.

### III.2.3.2. Digital signal interfacing

The DSP used is a 3.3V I/O CMOS technology device which has to be interfaced with circuits operating at higher voltages or/ and driving high current loads. The optical incremental encoder, the IGBT drivers and the inverter error signal are the binary signals that need to be interfaced.

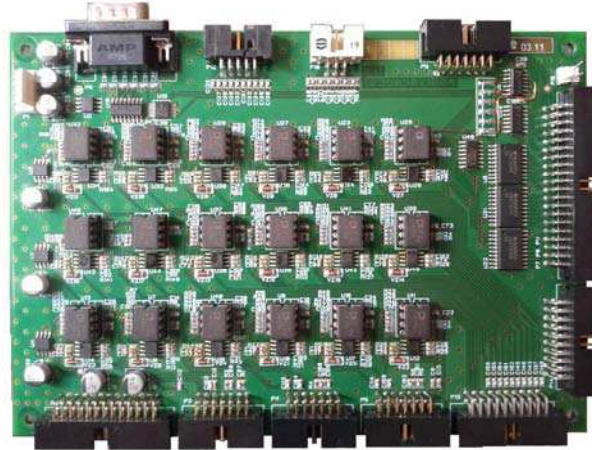


Figure III-14: Interface board

The IGBT driver require 15V input which is interfaced with TTL buffers with high voltage open-collector outputs.  $Tl$  component, reference number, 'SN5417' is used. For the encoder input line receivers are used for better Electromagnetic immunity as the electric

drives create high electromagnetic pollution, electrostatic discharge immunity with a high data rate, the component used is SN 65175 from TI. Figure III-14 shows the resulting printed circuit board for the interface circuit.

### III.3. Test bench validation

Now that every aspect of the development of the test bench has been explained in this chapter. To get the complete system all the three units namely; the Power Unit, the Control Unit and the Interface Unit are put together as shown in Figure III-15. It is very important to ensure the proper functioning at the system level, the power links and the communication links binding the different units together have to be verified.

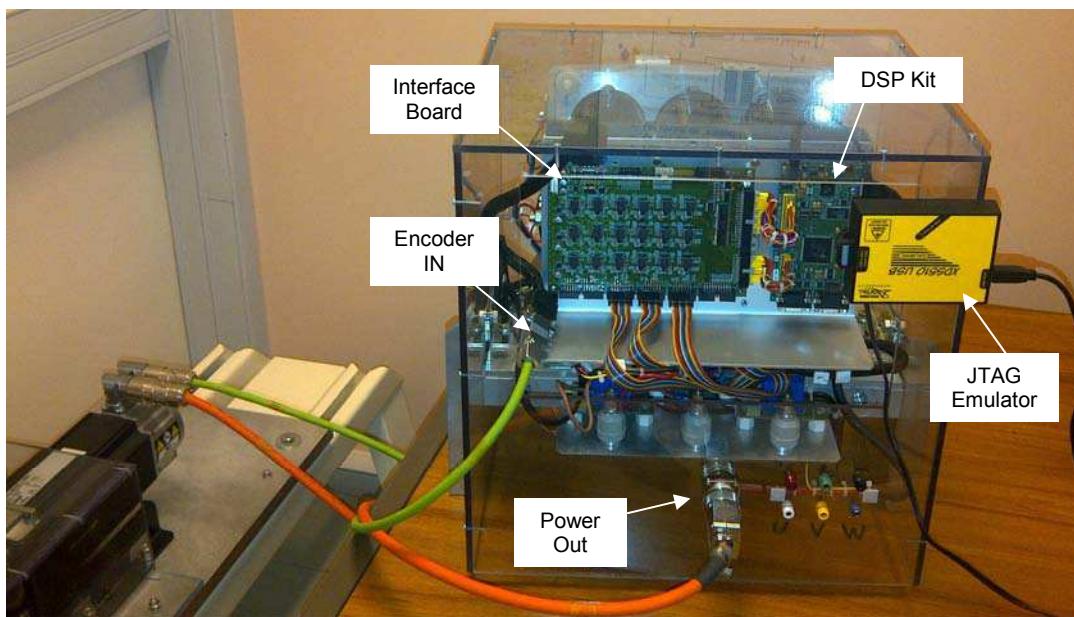


Figure III-15: Test bench

Before getting started with the experiments a step by step validation of each function was undertaken, such as calibrating the ADCs for current and voltage sensors, verifying the incremental encoder, checking the PWM signals for right voltage levels and dead-time insertion.

#### III.3.1. ADC Modules and sensors

The sensors on the inverter were applied with known voltages and currents and the sensor output was measured using an oscilloscope which helped calculate the offsets and confirm the polarity of measured signals. Similarly the ADC conversion is also verified comparing the actual current and ADC reading to accurately calculate the ADC gains for each sensor. With 3 to 4 different values the gains and offset are calculated.



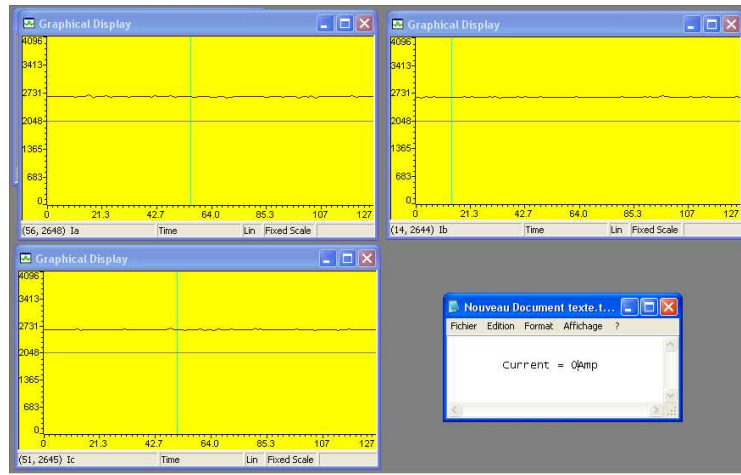


Figure III-16: ADC calibration

Figure III-16 shows ADC calibration with a DC current through with the DC offset and the correct current polarity.

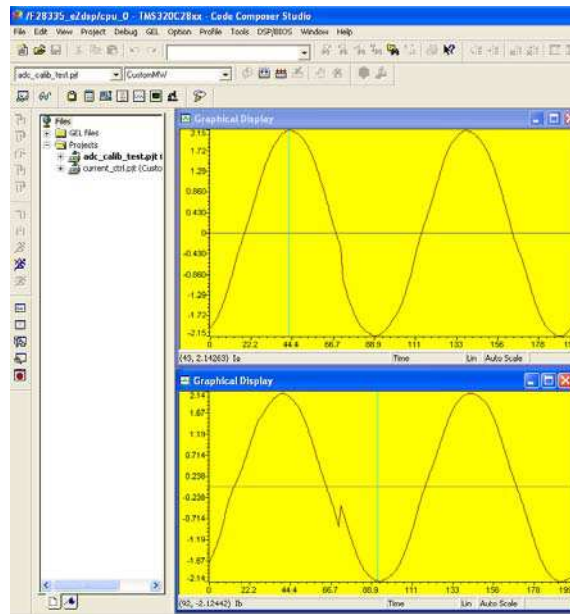


Figure III-17: Current calibration

Once the ADCs are calibrated, the AC values can be read accurately. Figure III-17 shows the reading of the ADC registers for an AC current in the load.

### III.3.2. Incremental encoder

The machine is run at a given speed and the pulses coming from the encoder, *A* and *B* are counted by the DSP to reconstruct the position of the rotor and calculate the speed. The Figure III-18 shows the reconstruction of the rotor position and speed.

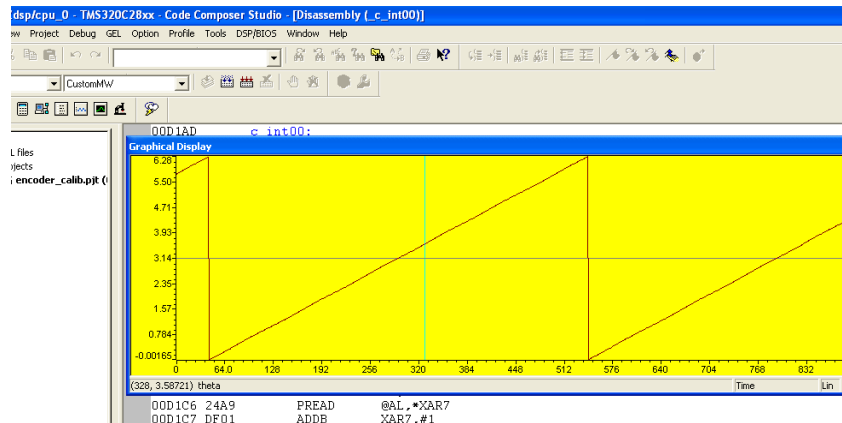


Figure III-18: Encoder verification

### III.3.3. PWM Module

The PWM signals are verified at the DSP output and at the driver input with a constant duty cycle and a given dead-time.

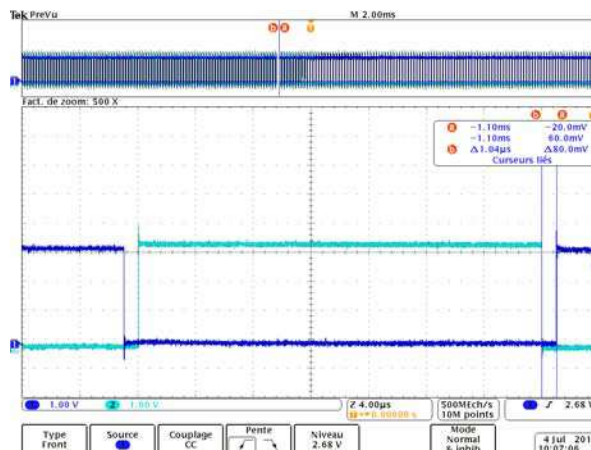


Figure III-19: PWM signals from DSP

These tests are very helpful to the test bench and validate the interfacing of the power unit and control unit. This confirms good functioning of the bench.

## III.4. Performance indicators

The test bench is fully functional and before we start comparing the different modulation strategies we need to decide upon the performance indicators that should be used. We'd like to compare these techniques on three grounds, the harmonic content of the modulated voltage, the frequency spectrum and the switching losses.

The waveform quality or the harmonic content is compared in terms of Harmonic Distortion Factor (HDF) /Weighted Total Harmonic Distortion (WTHD), whereas HDF is better suited for comparing the performance analytically whereas WTHD is better suited



when comparing experimental waveforms. The relationship between the two indicators can be established as follows. In section 1.4.2.2 expressions for HDF for different three phase PWM schemes can be generalised by (3.5).

$$\langle i_h^2 \rangle_T = \left( \frac{V_{dc}}{L} \right)^2 \frac{T^2}{48} f(m) \quad (3.5)$$

Where  $f(m)$  for different modulation techniques is given by equations (1.76) to (1.82). From equation (1.61) the expression for WTHD for a modulation index of  $m=0.907$  can be given by (3.6).

$$WTHD0|_{m=0.907} = \frac{\sqrt{\sum_{n=2}^{\infty} \left( \frac{V_n}{n} \right)^2}}{2V_{dc}} \quad (3.6)$$

$V_1$  at  $m=0.907$  has a line to line amplitude of  $2V_{dc}$ . The harmonic current induced by these harmonic voltages in their peak values is given by equation (3.7)

$$I_n = \frac{V_n}{n\omega_f L} \quad (3.7)$$

The total RMS value of the harmonic current content can be given by equation (3.8)

$$I_{h,l-l,RMS} = \sum_{n=2}^{\infty} \frac{1}{2} I_n^2 \quad (3.8)$$

Substituting equation (3.7) in equation (3.8) gives the following expression for harmonic line to line RMS current content of the load.

$$I_{h,l-l,RMS} = \sum_{n=2}^{\infty} \frac{1}{2} \left( \frac{V_n}{n\omega_f L} \right)^2 \quad (3.9)$$

Rearranging equation (3.9) as in equation (3.10).

$$\sum_{n=2}^{\infty} \frac{V_n^2}{n^2} = 2 \left( \omega_f L I_{h,l-l,RMS} \right)^2 \quad (3.10)$$

Substituting this result in equation (3.6) we get the WTHD in terms of line to line harmonic current content.

$$WTHD0|_{m=0.907} = \frac{\omega_f L I_{h,l-l,RMS}}{\sqrt{2}V_{dc}} \quad (3.11)$$

Now again substituting value of harmonic current from equation (3.5) in equation (3.11) gives.

$$\begin{aligned}
 WTHD0|_{m=0.907} &= \frac{\omega_f L \sqrt{\left(\frac{V_{dc}}{L}\right)^2 \frac{T^2}{48} f(m)}}{\sqrt{2} V_{dc}} \\
 &= \frac{\omega_f T \sqrt{f(m)}}{4\sqrt{6}}
 \end{aligned} \tag{3.12}$$

Since T is the modulation period equation (3.12) simplifies to

$$WTHD0 = \frac{\pi f_f}{f_s} \frac{\sqrt{HDF}}{2\sqrt{6}} \tag{3.13}$$

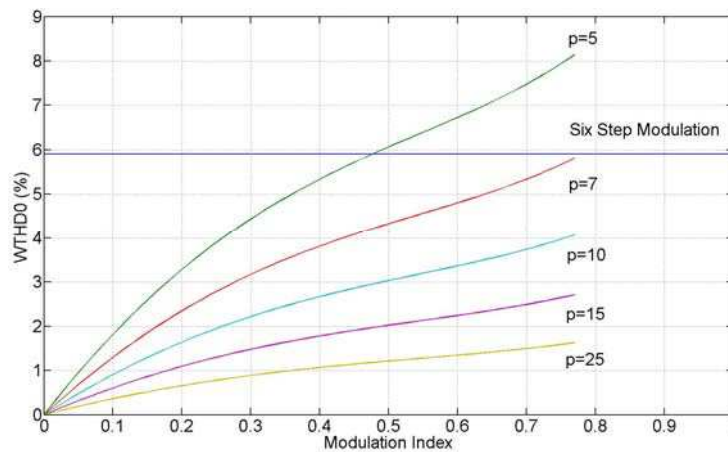
This can be further written in terms of pulse ratio  $p$ , defined as the ratio of the PWM switching frequency and the fundamental frequency,  $f_s/f_f$ .

$$WTHD0 = \frac{\pi}{p} \frac{\sqrt{HDF}}{2\sqrt{6}} \tag{3.14}$$

While deriving the expression for HDF it was assumed that the load EMF was constant over each half modulation period as the pulse ratio was considered sufficiently high. It is interesting to see how the pulse ratio affects the performance of a modulator, for this purpose we'll compare it with a six-step voltage wave i.e. no modulation. The WTHD for a six-step can be calculated in a similar way as was done for a single phase inverter, equation (1.62) as below.

$$WTHD0|_{six-step} = 5.91\% \tag{3.15}$$

It is interesting to see how the pulse ratio affects the waveform quality. The Figure III-20 traces WTHD0 for a sinusoidal modulator for different pulse ratios.



**Figure III-20: WTHD0 for SPWM for different pulse ratios**

The WTHD0 for a six-step modulator is shown by a straight line, which helps compare the waveform quality of modulated voltages for different pulse ratios to the un-

modulated six-step voltage. It can be seen that for pulse ratios inferior to 7 six-step has superior performance, which means that the pulse ratio should kept higher than 7.

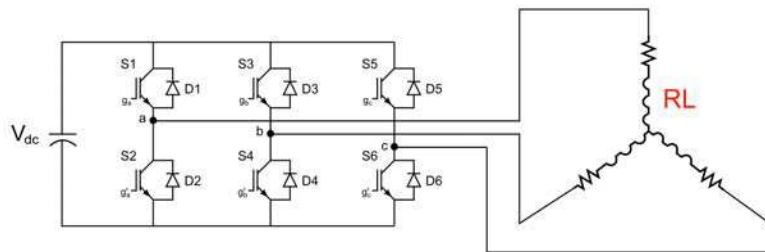
This gives us good platform to analyse the experimental results. In the next section different modulation strategies will be implemented experimentally and with compared against each other for waveform quality.

## III.5. Experimental Results

Now that the test bench is functional the previously developed techniques can be implemented experimentally. In this section we'll compare these new techniques to the existing standard techniques, in terms of the spectral behaviour of the pulse width modulated voltage, the harmonic component and inverter switching losses. First of all we test these new techniques on a stable resistive-inductive load and once these are validated we proceed with testing them on a dynamic load; a PMSM.

### III.5.1. Static Load

In this section all the major types of modulators seen in sections I.2.2, I.2.4, II.2 and II.3 are implemented experimentally on a static RL load before actually using it on an electric machine. These different techniques will be evaluated according to two indicators, the harmonic content of the modulated wave and its frequency spectrum. These two indicators will help give an idea about the waveform quality and the electromagnetic interference that the output voltage can generate. These tests are divided into three sub categories depending upon the type of modulator namely continuous, discontinuous and randomised modulators.



**Figure III-21: Inverter feeding a static load**

The test configuration is shown in Figure III-21 where the inverter feeds an inductive load. The test conditions are given below:

- DC link voltage ' $V_{dc}$ '=200V
- Modulation index ' $m_i$ '=0.7
- Fundamental frequency ' $f_i$ '=200Hz
- switching frequency ' $f_s$ '=10kHz, which makes the pulse ratio ' $p$ '=50

### III.5.1.1. Continuous Modulators

In this section two of the most common continuous modulation strategies, the sinusoidal and the SVM strategies are implemented and analysed experimentally. SVM being a continuous modulator with the presence of zero sequence harmonics. In order explicitly show the presence of these harmonics phase voltages are considered for the harmonic and spectral analysis.

#### III.5.1.1.1. Sinusoidal Pulse Width Modulator

First the sinusoidal modulator commonly known as SPWM is implemented with the parameters specified earlier.

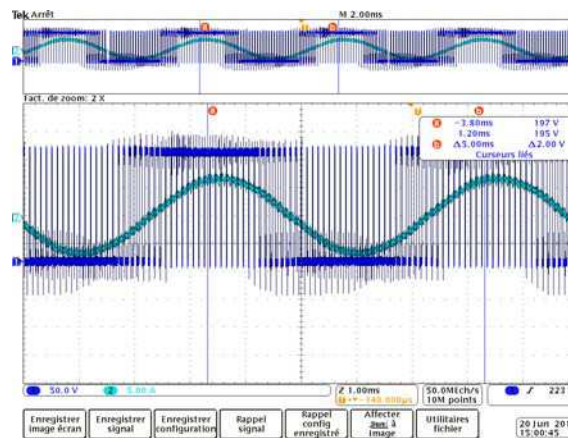


Figure III-22: SPWM: Phase voltage and current

Figure III-22 is the oscillogram of the phase voltage measured with respect to the negative terminal of the DC rail and the corresponding phase current. The blue curve is the modulated voltage and the turquoise curve is the current. The first verifications are done on the oscilloscope by checking the switching frequency and the fundamental frequency.

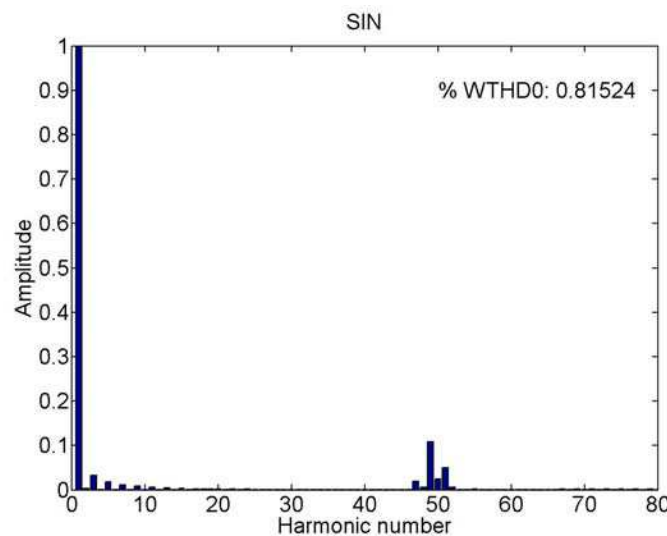
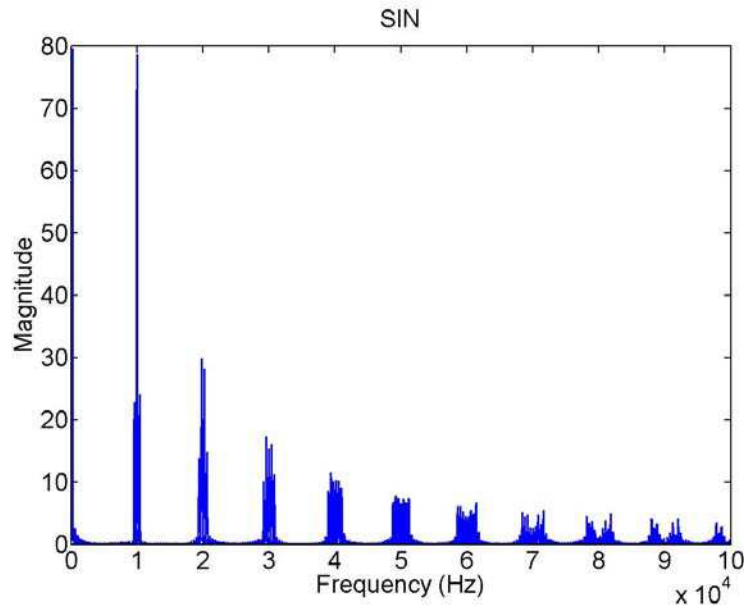


Figure III-23: SPWM: Harmonic spectrum

The waveform quality of the modulated signal is measured in terms of WTHD0 for the reasons mention at the beginning of the chapter. Figure III-23 shows the harmonic spectrum of the phase voltage, only the first 80 harmonics are shown on the chart, where the 50<sup>th</sup> harmonic represents the switching harmonics. The harmonic amplitudes are normalised with respect to the amplitude of the fundamental component. The WTHD0 for SPWM under for the given parameters is of about 0.81%. This is calculated for the first 45 harmonics.



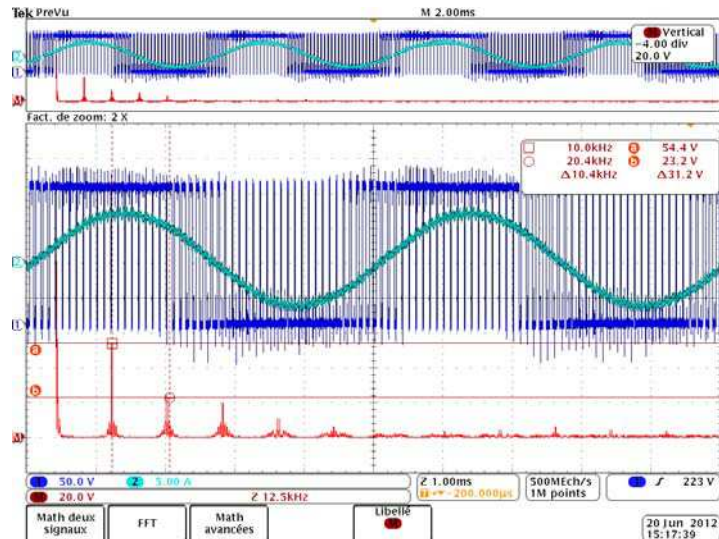
**Figure III-24: SPWM: frequency spectrum**

The frequency spectrum of the modulated voltage is shown in Figure III-24. The magnitude of the harmonics are adjusted to correspond to the real amplitude of the sinusoidal signals and is given in volts.

The experimental results are in perfect coherence with the theoretically calculated values for the switching harmonic amplitudes for the given modulation index with a relatively high pulse ratio, i.e. the first switching harmonic of almost the same amplitude with the side bands being about 30% and the second harmonic being about 40%.

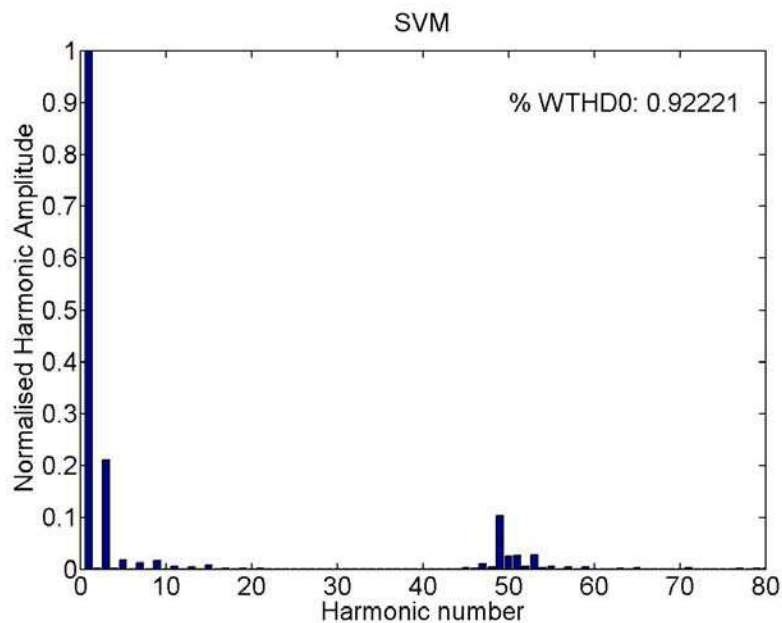
#### **III.5.1.1.2. Space Vector Modulator**

The space vector modulator is implemented here with the same parameters. Similarly Figure III-25 is the oscillogram of the phase voltage measured with respect to the negative terminal of the DC rail and the corresponding phase current.



**Figure III-25: SVM: Phase voltage and current**

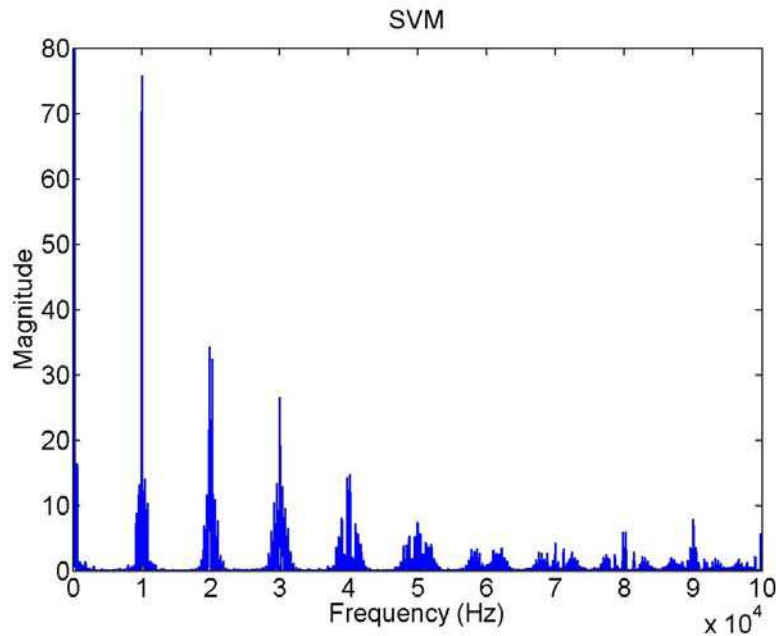
The blue curve is the modulated voltage and the turquoise curve is the current. Whereas the red curve is the discrete Fourier transform (DFT) of the voltage signal, where a DC component can be noticed, which can be explained by the fact that we are measuring the phase voltage with respect to the negative terminal of the DC link and not with respect to the DC mid-point.



**Figure III-26: SVM: Harmonic spectrum**

Figure III-26 shows the harmonic spectrum of the phase voltage, only the first 80 harmonics are shown on the chart, where the 50<sup>th</sup> harmonic represents the switching harmonics, the presence of zero sequence component can be seen by the non-negligible harmonics present at low harmonic numbers, these are the 3<sup>rd</sup> harmonic and its multiples.

The WTHD0 for SVM under for the given parameters is of about 0.92%, it should be noted that for this case harmonic component at  $N=3, 6$  was not included while calculating the THD as this is the desired behaviour of the modulator. This is calculated for the first 45 harmonics.



**Figure III-27: SVM: frequency spectrum**

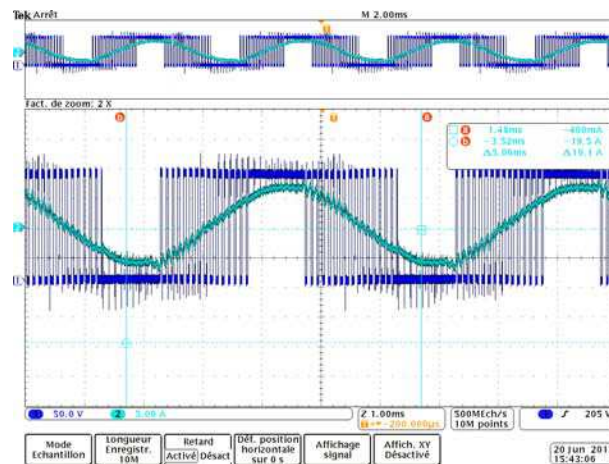
The frequency spectrum of the modulated voltage is shown in Figure III-27. The magnitude of the harmonics are adjusted to correspond to the real amplitude of the sinusoidal signals. Some low frequency components can be seen in the figure which again are due to the presence of the zero sequence voltages.

### **III.5.1.2. Discontinuous Modulator**

In this section three of the most common discontinuous modulation strategies, the DPWM1, DPWMMIN and DPWMMAX are implemented and analysed experimentally. Since these types of modulators require a zero sequence voltage to be injected. In order explicitly show the presence of these harmonics phase voltages are considered for the harmonic and spectral analysis.

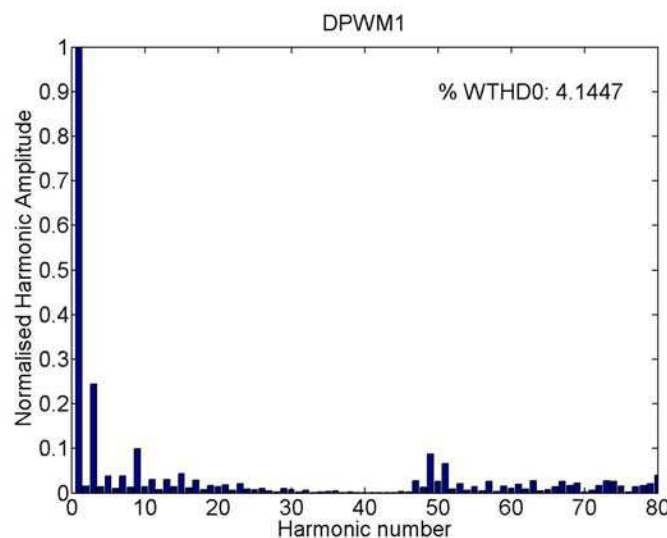
#### **III.5.1.2.1. DPWM1**

DPWM1 as seen in section I.2.4.2.1 is discontinuous method where the discontinuities occur at the maximum absolute value of the desired voltage for one sixth of the fundamental time period.



**Figure III-28: DPWM1: Phase voltage and current**

Figure III-28 is the oscillogram of the phase voltage measured with respect to the negative terminal of the DC rail and the corresponding phase current. The discontinuities are clearly visible on the blue curve which is the modulated voltage and the turquoise curve is the current which remains sinusoidal. However higher current ripples can be observed due the reduced effective switching frequency.



**Figure III-29: DPWM1: Harmonic spectrum**

Figure III-29 shows the harmonic spectrum of the phase voltage, only the first 80 harmonics are shown on the chart, where the 50<sup>th</sup> harmonic represents the switching harmonics, the presence of zero sequence component can be seen by the non-negligible harmonics present at low harmonic numbers. The WTHD0 for DPWM1 under for the given parameters is of about 4%. For this case too WTHD is calculated for the first 45 harmonics. The effective frequency being two-thirds of the switching frequency the WTHD0 should also be scaled down by the same factor which comes out to be around 2.7.



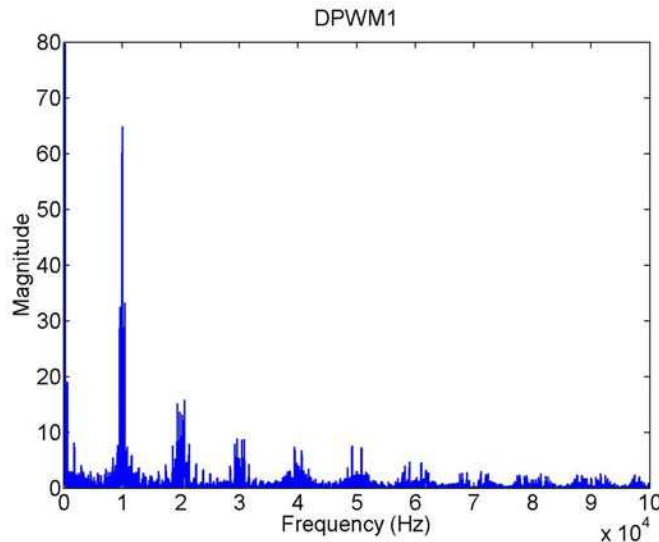


Figure III-30: DPWM1 SPWM: frequency spectrum

The frequency spectrum of the DPWM1 modulated voltage is shown in Figure III-30. Here too the magnitude of the harmonics are adjusted to correspond to the real amplitude of the different frequency components present in the pulse width modulated voltage. Some low frequency components can be seen in the figure which again are due to the presence of the zero sequence voltages, Whereas it can be noticed that the switching harmonic are smaller in amplitude which again is linked to the fact that this modulator has a lower effective frequency.

#### III.5.1.2.2. DPWMMIN & DPWMMAX

DPWMMIN and DPWMMAX as seen in section I.2.4.2.2 are two discontinuous modulators which are similar to each other. For these two methods the discontinuities occur at the maximum and minimum value of the desired voltage for one-third of the fundamental time period respectively. Here the discontinuity appears only in half-cycle of the fundamental period and is twice as long compared to DPWM1 or other common discontinuous modulators.

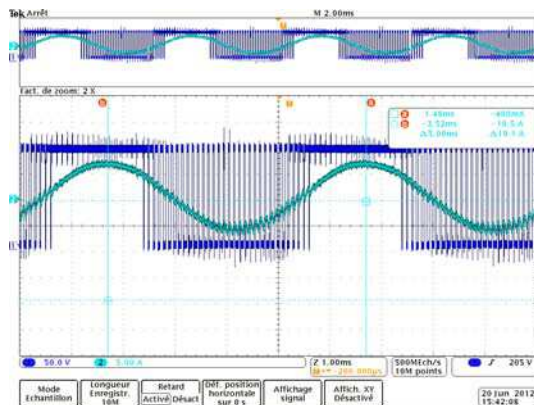


Figure III-31: DPWMMAX: Phase voltage and current

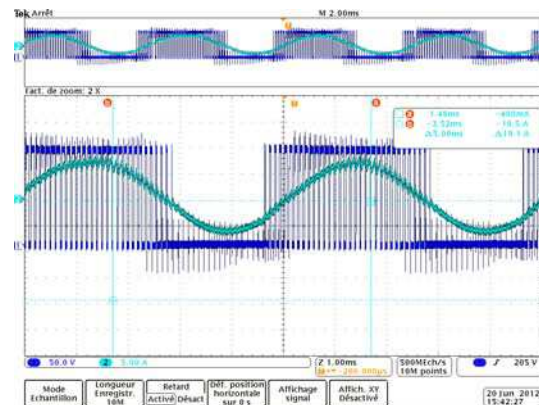
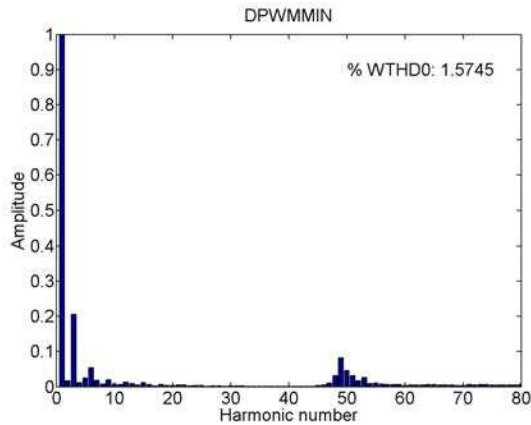


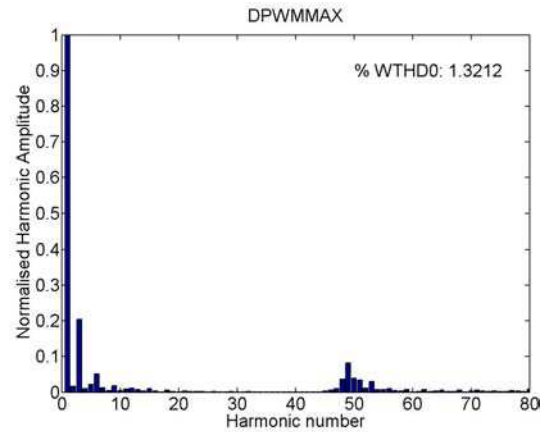
Figure III-32: DPWMMIN: Phase voltage and current

Figure III-31 and Figure III-32 are the oscillograms of the phase voltage measured with respect to the negative terminal of the DC rail and the corresponding phase current for DPWMMAX and DPWMMIN respectively. The discontinuities are clearly visible on the blue curves which are the modulated voltage, the turquoise curves are the phase currents which again remain sinusoidal.

However these have higher current ripples, the current curve is a little thicker which can be easily observed on the figures. This is due to the reduced effective switching frequency, which increases the harmonic content of the phase currents.

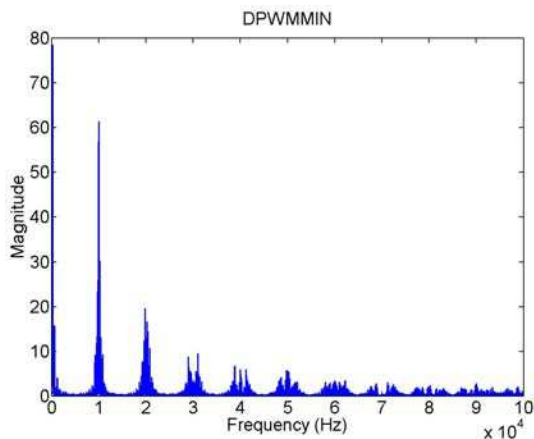


**Figure III-33: DPWMMIN: Harmonic spectrum**

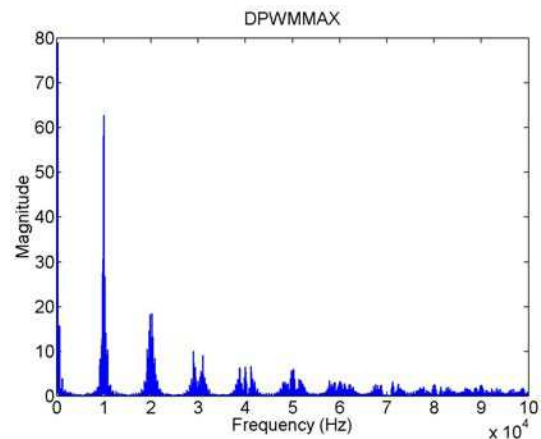


**Figure III-34: DPWMMAX: Harmonic spectrum**

The Harmonic spectrum of the DPWMMIN and DPWMMAX modulated voltages are shown in Figure III-33 and Figure III-34 respectively. The WTHD0 for both these the techniques are about 1.5%. The effective frequency for these modulators again being two-thirds of the actual switching frequency, the WTHD0 should also be scaled down by the same factor which comes out to be around 1%.



**Figure III-35: DPWMMIN: frequency spectrum**

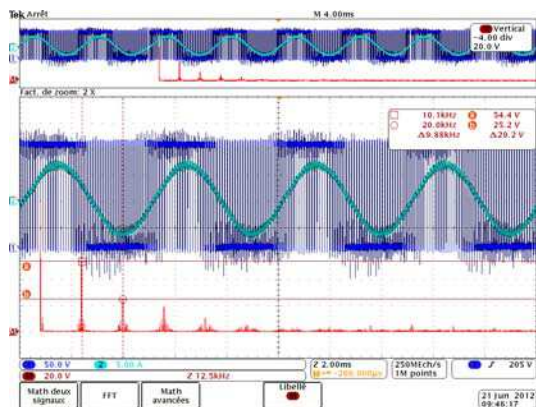


**Figure III-36: DPWMMAX: frequency spectrum**

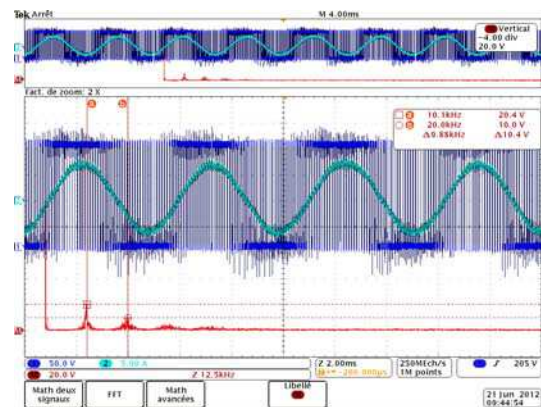
The frequency spectrum of the DPWMMIN and DPWMMAX modulated voltages are shown in Figure III-35 and Figure III-36 respectively. Here too the magnitude of the harmonics are adjusted to correspond to the real amplitude of the different frequency components present in the pulse width modulated voltage. Some low frequency components can be seen in the figure which again are due to the presence of the zero sequence voltages, whereas it can again be noticed that the switching harmonics are smaller in amplitude which again is linked to the fact that this modulator has a lower effective frequency. Hence we obtain expected results.

### III.5.1.3. Random modulator

A random space vector modulator of section II.3.3 which is basically a random carrier frequency type modulator, is implemented in this section and is compared to the standard deterministic modulator. The randomisation factor of 0.2 was taken with a statistically expected value of the switching frequency being 10kHz, this will make the switching frequency vary randomly between 9 and 11kHz.

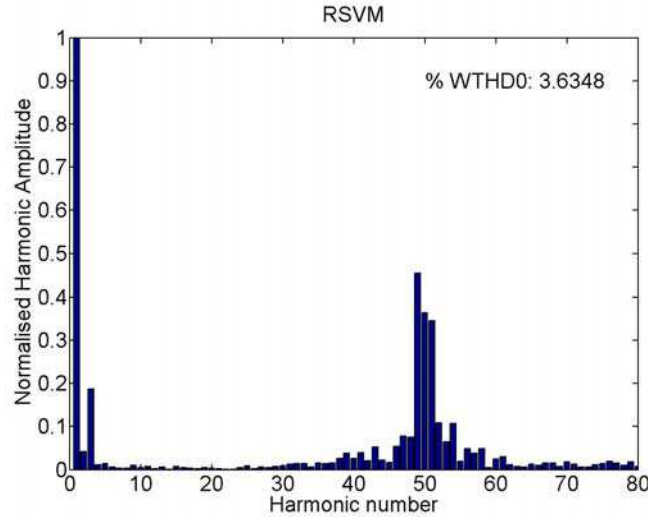


**Figure III-37: SVM: Phase voltage and current**



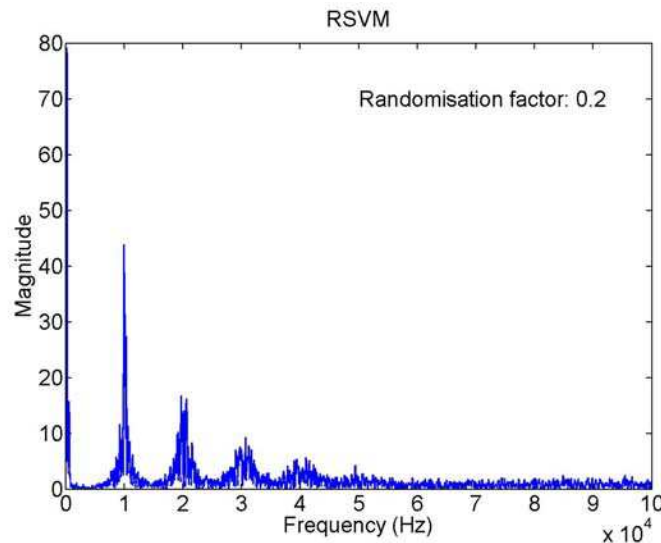
**Figure III-38: RSVM: Phase voltage and current**

The oscillograms, Figure III-37 and Figure III-38 for deterministic SVM and randomised SVM are put side by side to so that their frequency spectrum can be compared easily. The blue curve is the modulated voltage and the turquoise curve is the current. Whereas the red curve is the discrete Fourier transform (DFT) of the voltage signal, where a DC component can be noticed, which can be explained by the fact that we are measuring the phase voltage with respect to the negative terminal of the DC link and not with respect to the DC mid-point. The first switching harmonic for deterministic SVM has an amplitude of 54V whereas for the randomised SVM the amplitude is of 20V. This a significant reduction in the peak harmonic values.



**Figure III-39: RSVM: Harmonic spectrum**

The Harmonic spectrum of the RSVM is shown in Figure III-39, many peaks can be noticed around the harmonic number 50, which is due to the randomised switching frequency. The WTHD0 for the randomised SVM technique comes out to be about 3.6% which is relatively high for a continuous modulator, but this is normal as the pulse ratio varies between 45 and 55 and hence while calculating the WTHD these add up to increase the total harmonic content of the voltage signal.



**Figure III-40: DPWMMAX: frequency spectrum**

The frequency spectrum of RSVM voltage waveform is shown in Figure III-40. Again the harmonic magnitudes correspond to the actual amplitudes contained in the modulated voltage. It can be noticed that first switching harmonic peak is of just about 45V which is almost half compared to the first harmonic amplitude of the deterministic SVM. This peak can be further reduced by increasing the randomisation factor ' $RF$ ', however higher the  $RF$  higher the harmonic distortion content of the voltage waveform.

PWM Strategy	%WTHD0	Scaled %WTHD0	Mag. of first switching harmonic
SPWM	0.81	-	80V
SVM	0.92	-	75V
DPWM1	4.14	2.76	65V
DPWMMIN	1.57	1.04	61V
DPWMMAX	1.32	0.88	62V
RSVM (RF=0.2)	3.63	-	45V

**Table III-2: Experimental performance comparison of different PWM strategies**

Table III-2 summarises the different results obtained for the experiments performed on a static load.

### ***III.5.2. Dynamic Load***

Now that the different modulators were tested on static loads with satisfactory results in this section we proceed with similar test but this time in a closed loop control system for a PMSM as shown in Figure II-15.

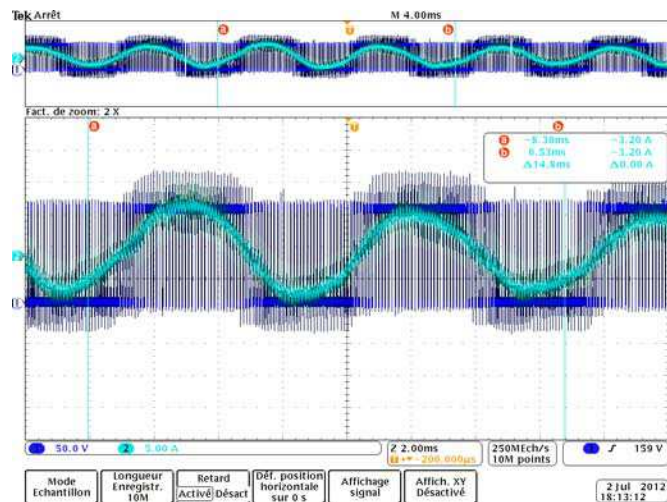
We'll proceed in a similar as we did for previous case, i.e. we've again divided the tests in three categories viz. continuous, discontinuous and randomised modulators. These strategies are implemented along with a vector control of the PMSM and the inverter output voltage waveforms are analysed experimentally. The electric machine running at 2700rpm which corresponds to an electrical frequency of about 135Hz for a machine with 3 pole pairs.

#### **III.5.2.1. Continuous Modulator**

In this section two of the most common continuous modulation strategies, the sinusoidal PWM and the SVM.

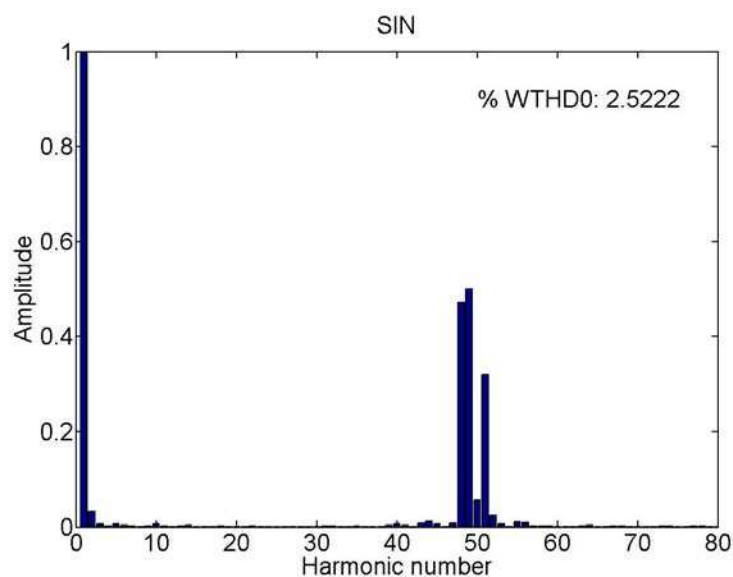
##### ***III.5.2.1.1. Sinusoidal Pulse Width Modulator***

In the same way as we proceeded with tests for a static load, implementing different type of modulators we start by showing results for the sinusoidal modulator commonly known as SPWM is implemented with the parameters specified earlier.



**Figure III-41: SPWM: Phase voltage and current**

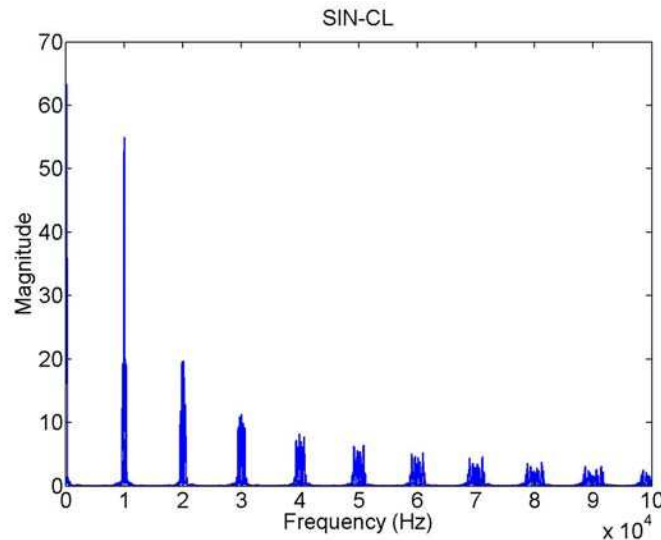
Figure III-41 is the oscillogram of the phase voltage measured with respect to the negative terminal of the DC rail and the corresponding phase current. The blue curve is the modulated voltage and the turquoise curve is the current. The first verifications are done on the oscilloscope by checking the switching frequency and the fundamental frequency.



**Figure III-42: Vector controlled SPWM: Harmonic spectrum**

The waveform quality of the modulated signal is measured in terms of WTHD0 for the reasons mention at the beginning of the chapter. Figure III-42 shows the harmonic spectrum of the phase voltage, only the first 80 harmonics are shown on the chart, where the 50<sup>th</sup> harmonic represents the switching harmonics. The WTHD0 for SPWM under for the given parameters is of about 2.5%. This is calculated for the first 45 harmonics.



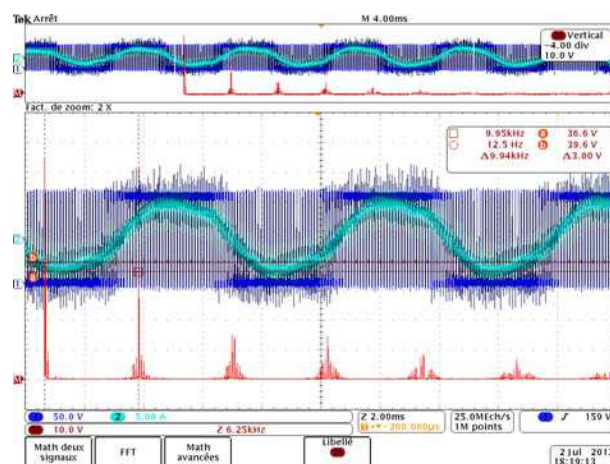


**Figure III-43: Vector controlled SPWM: frequency spectrum**

The frequency spectrum of the modulated voltage is shown in Figure III-43. The magnitude of the harmonics are adjusted to correspond to the real amplitude of the sinusoidal signals. The experimental results are in perfect coherence with the theoretically calculated values for the switching harmonic amplitudes for the given modulation index with a relatively high pulse ratio, i.e. the first switching harmonic of almost the same amplitude with the side bands being about 30% and the second harmonic being about 40%.

#### **III.5.2.1.2. Space Vector Modulator**

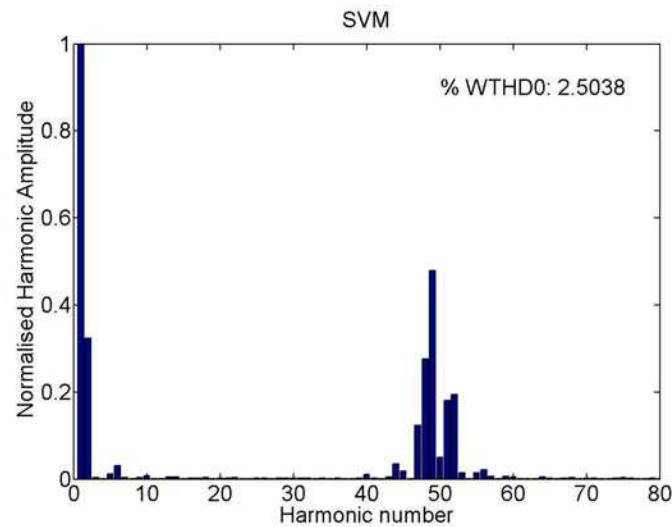
The space vector modulator is implemented here with the same parameters. Similarly Figure III-44 is the oscillogram of the phase voltage measured with respect to the negative terminal of the DC rail and the corresponding phase current.



**Figure III-44: Vector controlled SVM: Phase voltage and current**

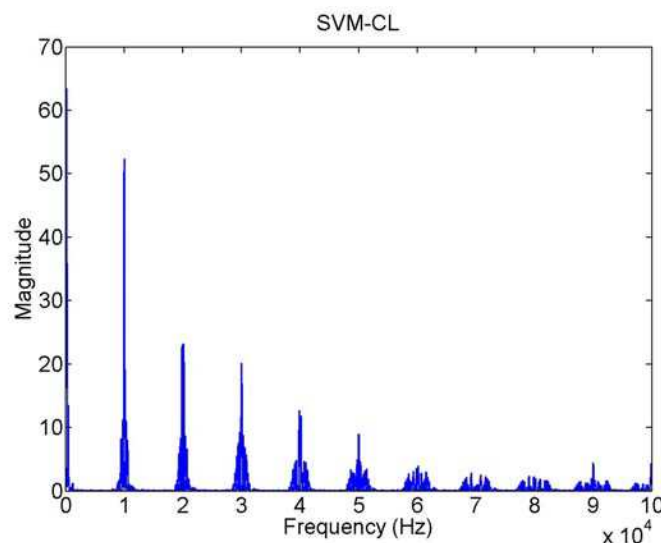
The blue curve is the modulated voltage and the turquoise curve is the current. Whereas the red curve is the discrete Fourier transform (DFT) of the voltage signal, where

a DC component can be noticed, which can be explained by the fact that we are measuring the phase voltage with respect to the negative terminal of the DC link and not with respect to the DC mid-point.



**Figure III-45: Vector controlled SVM: Harmonic spectrum**

Figure III-45 shows the harmonic spectrum of the phase voltage, only the first 80 harmonics are shown on the chart, where the 50<sup>th</sup> harmonic represents the switching harmonics, the presence of zero sequence component can be seen by the non-negligible harmonics present at low harmonic numbers, these are the 3<sup>rd</sup> harmonic and its multiples. The WTHD0 for SVM under for the given parameters is of about 2.5%, it should be noted that for this case harmonic component at N=3, 6 was not included while calculating the THD as this is the desired behaviour of the modulator. This is calculated for the first 45 harmonics.



**Figure III-46: Vector controlled SVM: frequency spectrum**



The frequency spectrum of the modulated voltage is shown in Figure III-46. The magnitude of the harmonics are adjusted to correspond to the real amplitude of the sinusoidal signals. Some low frequency components can be seen in the figure which again are due to the presence of the zero sequence voltages.

### III.5.2.2. Discontinuous modulator

In this section three discontinuous modulation strategies DPWMMIN and DPWMMAX and EDSVM are implemented and analysed experimentally. Evolutive DSVM is chosen in this case as we have a dynamic load, we can do the current tracking. Since these types of modulators require a zero sequence voltage to be injected. In order explicitly show the presence of these harmonics phase voltages are considered for the harmonic and spectral analysis.

#### III.5.2.2.1. DPWMMIN & DPWMMAX

DPWMMIN and DPWMMAX as seen in section I.2.4.2.2 are two discontinuous modulators which are similar to each other.

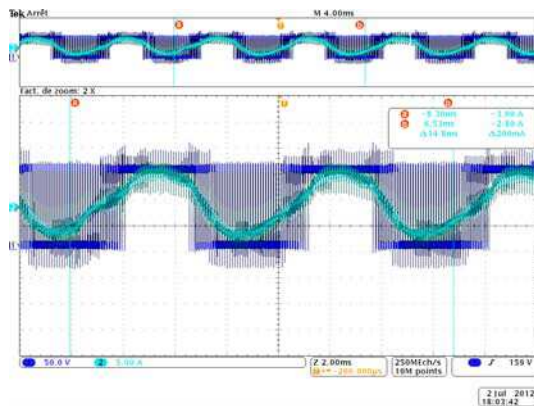


Figure III-47: Vector controlled DSVMAX: Phase voltage and current

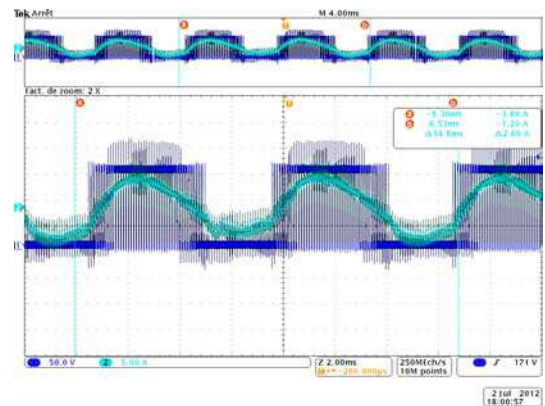
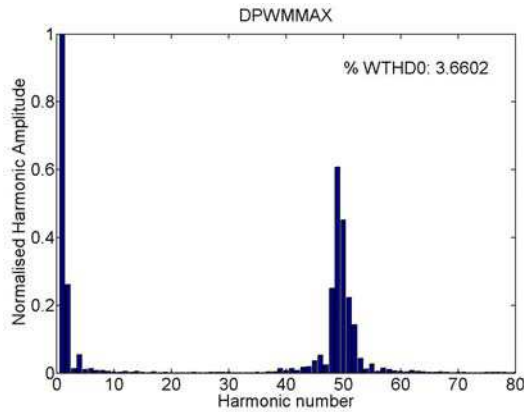
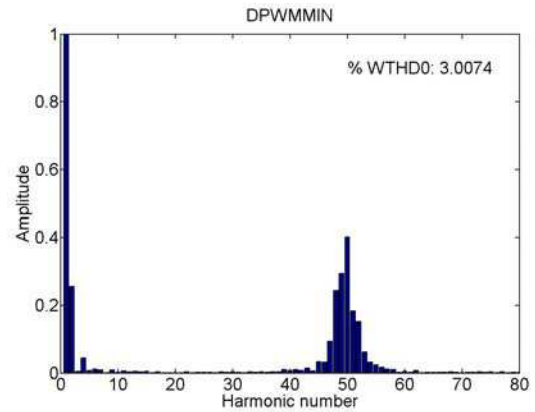


Figure III-48: Vector controlled DSVMIN: Phase voltage and current

Figure III-47 and Figure III-48 are the oscillograms of the phase voltage measured with respect to the negative terminal of the DC rail and the corresponding phase current for DPWMMAX and DPWMMIN respectively. The discontinuities are clearly visible on the blue curves which are the modulated voltage, the turquoise curves are the phase currents which again remain sinusoidal. However higher current ripples, the current curve is a little thicker which can be easily observed on the figures this again is due to the reduced effective switching frequency.

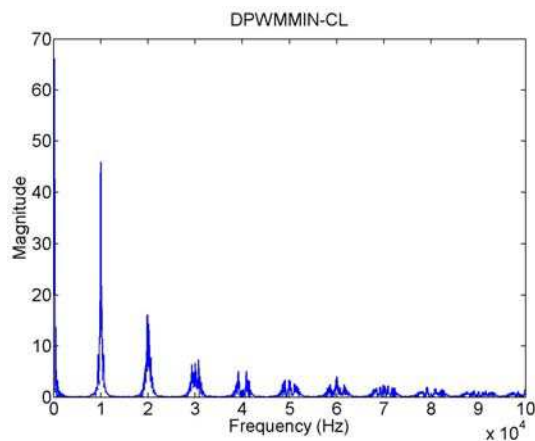


**Figure III-49: Vector controlled DPWMMAX: Harmonic spectrum**

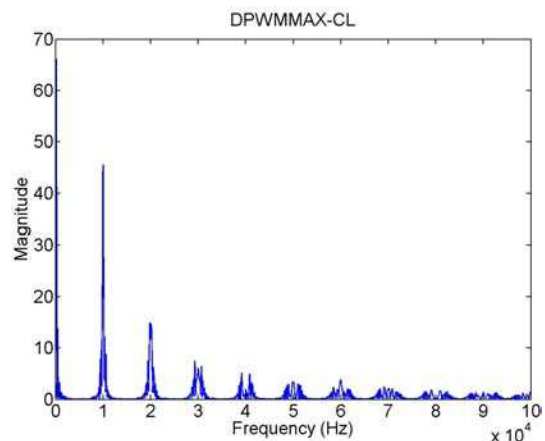


**Figure III-50: Vector controlled DPWMMIN: Harmonic spectrum**

The Harmonic spectrum of the DPWMMIN and DPWMMAX modulated voltages are shown in Figure III-49 and Figure III-50 respectively. The WTHD0 for both these the techniques are about 1.5%. The effective frequency for these modulators again being two-thirds of the actual switching frequency, the WTHD0 should also be scaled down by the same factor which comes out to be around 1%.



**Figure III-51: Vector controlled DPWMMIN: frequency spectrum**

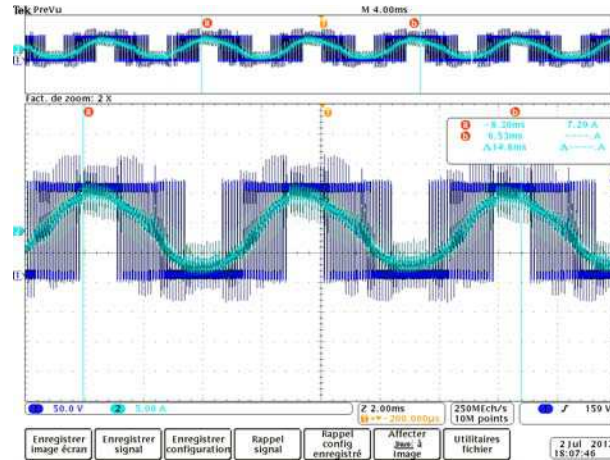


**Figure III-52: Vector controlled DPWMMAX: frequency spectrum**

The frequency spectrum of the DPWMMIN and DPWMMAX modulated voltages are shown in Figure III-51 and Figure III-52 respectively. Here too the magnitude of the harmonics are adjusted to correspond to the real amplitude of the different frequency components present in the pulse width modulated voltage. Some low frequency components can be seen in the figure, which again are due to the presence of the zero sequence voltages, whereas it can again be noticed that the switching harmonics are smaller in amplitude which again is linked to the fact that this modulator has a lower effective frequency. Hence we obtain expected results.

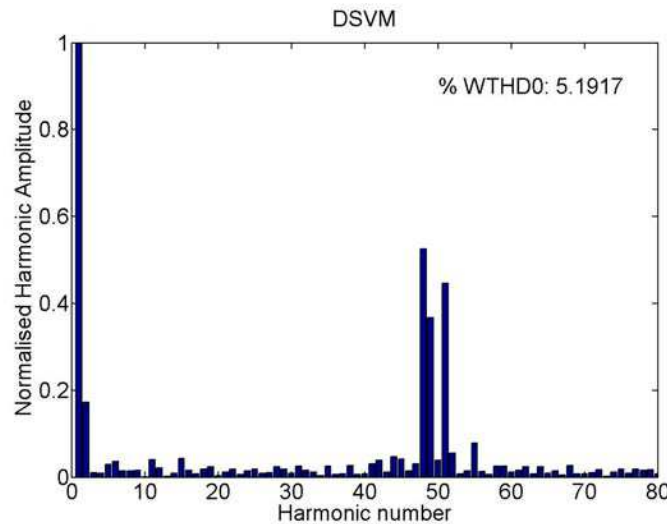
### III.5.2.2.2. EDSVM

The Evolutive discontinuous space vector modulator as described in section II.2.2 is implemented here to optimise the switching losses.



**Figure III-53: Vector controlled EDSVM: Phase voltage and current**

Figure III-53 is the oscillogram of the phase voltage measured with respect to the negative terminal of the DC rail and the corresponding phase current. The discontinuities on the blue curve which is the modulated voltage correspond to peak values of the turquoise curve is the current which remains sinusoidal. As seen earlier higher the current commuted through higher are the switching losses in the switch. Here the phase with highest absolute value of current is clamped to either the positive or the negative DC rail.

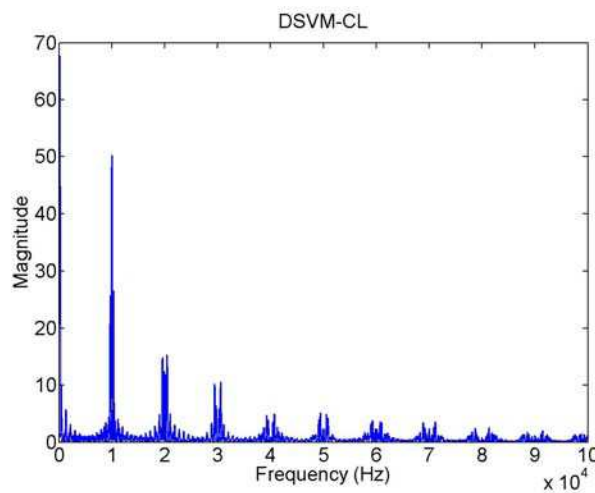


**Figure III-54: Vector controlled EDSVM: Harmonic spectrum**

Figure III-54 shows the harmonic spectrum of the phase voltage for EDSVM, only the first 80 harmonics are shown on the chart, where the 50<sup>th</sup> harmonic represents the switching harmonics, the presence of zero sequence component can be seen by the non-negligible harmonics present at low harmonic numbers.

The WTHD0 for EDSVM under for the given parameters is of about 5%. The effective frequency being two-thirds of the switching frequency the WTHD0 should also be scaled down by the same factor which comes out to be around 3.3. The experimental results show a decrease in the global losses from about 6.8% to 5.6% which translates to an overall reduction of a little more than 17%. However comparing only the switching losses accurately is practically impossible for fast switching transients ( $dv/dt \sim 300M \text{ v/s}$ ).

The analysis is done over two complete fundamental electrical period with, high frequency data acquisition is done through an oscilloscope to get accurate. Global losses are calculated measuring the power supplied by the DC link and comparing it with the power at the inverter legs for the proposed and the standard techniques. The analysis takes into account the DC link capacitor losses as well. However no tests under unbalanced conditions could be undertaken.

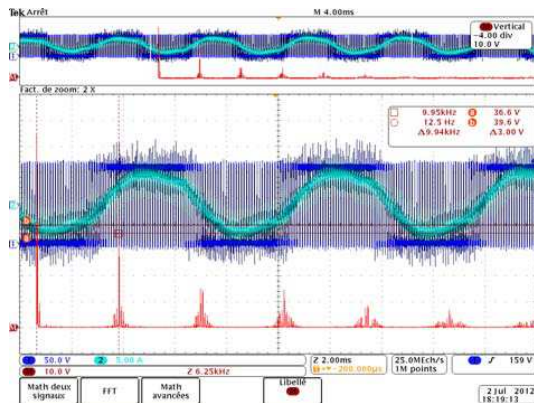


**Figure III-55: Vector controlled EDSVM: frequency spectrum**

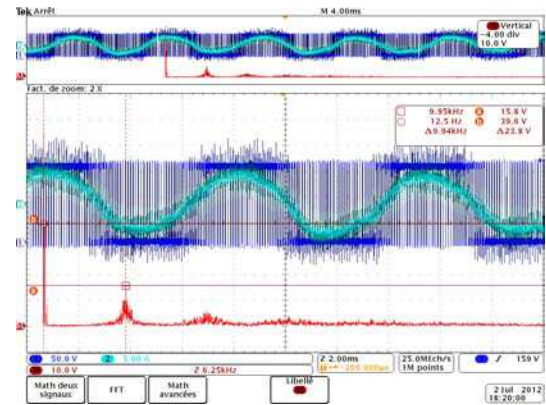
The frequency spectrum of the DPWM1 modulated voltage is shown in Figure III-55. Here too the magnitude of the harmonics are adjusted to correspond to the real amplitude of the different frequency components present in the pulse width modulated voltage. Some low frequency components can be seen in the figure which again are due to the presence of the zero sequence voltages, Whereas it can be noticed that the switching harmonic are smaller in amplitude which again is linked to the fact that this modulator has a lower effective frequency.

### **III.5.2.3. Randomised modulator**

A random space vector modulator of section II.3.3 which is basically a random carrier frequency type modulator, is implemented in this section and is compared to the standard deterministic modulator. The randomisation factor of 0.2 was taken with a statistically expected value of the switching frequency being 10kHz, this will make the switching frequency vary randomly between 9 and 11kHz.

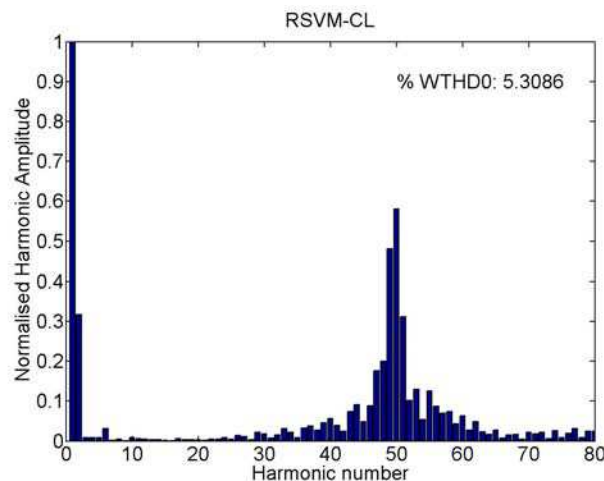


**Figure III-56: Vector controlled SVM:  
Phase voltage and current**



**Figure III-57: Vector controlled RSVM: Phase voltage and current**

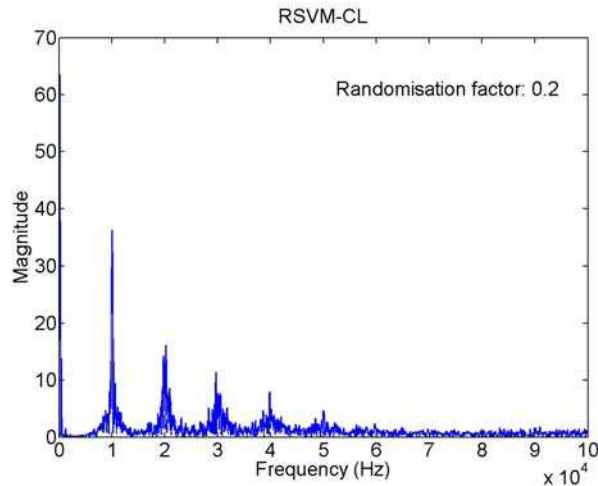
The oscillograms, Figure III-56 and Figure III-57 for deterministic SVM and randomised SVM are put side by side so that their frequency spectrum can be compared easily. The blue curve is the modulated voltage and the turquoise curve is the current. Whereas the red curve is the discrete Fourier transform (DFT) of the voltage signal, where a DC component can be noticed, which can be explained by the fact that we are measuring the phase voltage with respect to the negative terminal of the DC link and not with respect to the DC mid-point. The first switching harmonic for deterministic SVM has an amplitude of 36.6V whereas for the randomised SVM the amplitude is of 15.8V. This is a significant reduction in the peak harmonic values.



**Figure III-58: Vector controlled RSVM: Harmonic spectrum**

The Harmonic spectrum of the RSVM is shown in Figure III-58, many peaks can be noticed around the harmonic number 50, which is due to the randomised switching frequency. The WTHD0 for the randomised SVM technique comes out to be about 3.6% which is relatively high for a continuous modulator, but this is normal as the pulse ratio varies between 45 and 55 and hence while calculating the WTHD these add up to increase the total harmonic content of the voltage signal.



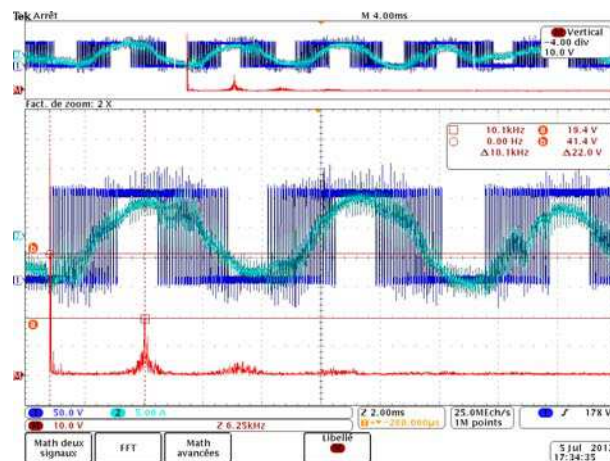


**Figure III-59: Vector controlled RSVM: frequency spectrum**

The frequency spectrum RSVM voltage waveform is shown in Figure III-59. Again the harmonic magnitudes correspond to the actual amplitudes contained in the modulated voltage. It can be noticed that first switching harmonic peak is of just about 35V which is almost half compared to the first harmonic amplitude of 55V for the deterministic SVM, shown in Figure III-46. This peak can be further reduced by increasing the randomisation factor 'RF', however higher the RF higher the harmonic distortion content of the voltage waveform.

#### III.5.2.4. Random discontinuous modulator

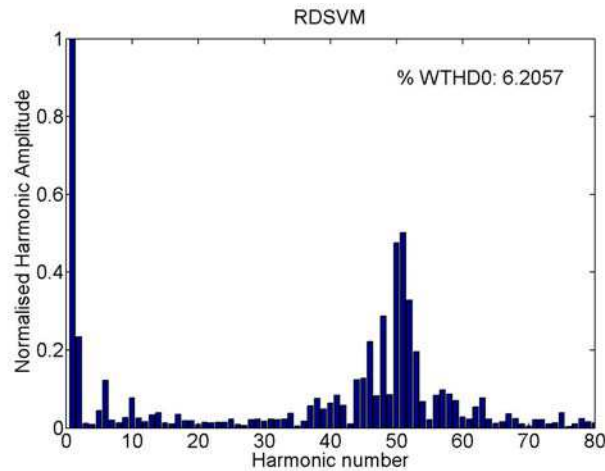
The random discontinuous space vector modulator as described in section II.3.4 is implemented here to optimise the switching losses.



**Figure III-60: Vector controlled RDSVM: Phase voltage and current**

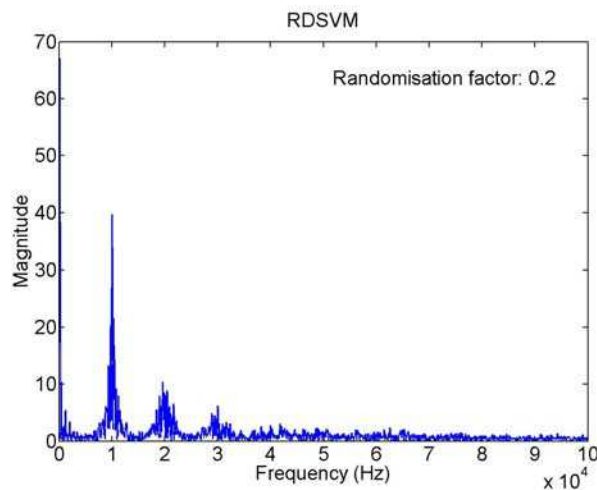
Figure III-60 is the oscillogram of the phase voltage measured with respect to the negative terminal of the DC rail and the corresponding phase current. The discontinuities on the blue curve which is the modulated voltage correspond to peak values of the turquoise curve is the current which remains sinusoidal. As seen earlier higher the current

commuted through higher are the switching losses in the switch. Here the phase with highest absolute value of current is clamped to either the positive or the negative DC rail.



**Figure III-61: Vector controlled RSVM: Harmonic spectrum**

The Harmonic spectrum of the RSVM is shown in Figure III-61, many peaks can be noticed around the harmonic number 50, which is due to the randomised switching frequency. The WTHD0 for the randomised discontinuous SVM technique comes out to be about 6.2% which is relatively high but if scaled down to its effective frequency it is acceptable at about 4.1%, but this is normal as the pulse ratio varies between 45 and 55 and hence while calculating the WTHD these add up to increase the total harmonic content of the voltage signal.



**Figure III-62: Vector controlled RSVM: frequency spectrum**

The frequency spectrum RSVM voltage waveform is shown in Figure III-62. Again the harmonic magnitudes correspond to the actual amplitudes contained in the modulated voltage. It can be noticed that first switching harmonic peak is of just about 40V which is comparable to the RSVM technique where the peak was about 35V, shown in Figure

III-59. This peak can be further reduced by increasing the randomisation factor ' $RF$ ', however higher the  $RF$  higher the harmonic distortion content of the voltage waveform.

PWM Strategy	%WTHD0	Scaled %WTHD0	Mag. of first switching harmonic
SPWM	2.52	-	55V
SVM	2.50	-	52V
DPWMMIN	3.66	2.44	45V
DPWMMAX	3.00	2.00	45V
EDSVM	5.20	3.46	50V
RSVM (RF=0.2)	5.30	-	36V
RDSVM (RF=0.2)	6.20	4.13	40V

**Table III-3: Experimental performance comparison of PWM strategies in close-loop**

Table III-3 summarises the different results obtained for the experiments performed on a dynamic load in closed with a vector controlled torque regulation of an electric machine. From the oscillograms we can notice low frequency distortion for the current waveforms which are not perfect sinusoids. This is due to the imperfections related to torque regulation of the PMSM. Hence these results should be interpreted and used with caution.

## III.6. Summary

This part of the thesis was dedicated to the experimental validation of the PWM strategies developed in Part-II (pg. 68). Every aspect of the experimental analysis is considered, i.e. from the test bench setup to the experiments and the analysis of the experimental data. We saw the different aspects of the test bench setup, choices of the power converter, electrical machine, DSP and interfacing all of them together and finally the calibration of the current and voltage sensors, extraction of the rotor position and speed from an incremental encoder.

Finally the modulation strategies were implemented and compared against standard techniques according to three basic criteria, the weighted harmonic content of the resultant voltage, its frequency spectrum and the switching losses for cases wherever it was applicable. The experimental results obtained were in line with the theoretical and simulation results.



## **PART IV**

# **CONCLUSION & FUTURE WORK**

# IV. Conclusion & Future Work

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## IV.1. Conclusion

This thesis addressed optimisation issues of electric drives by innovating the PWM techniques. These techniques are at the heart of the electric drive train: they act directly on the conditioning of the DC voltage from the battery through an inverter to feed the electric machine with a fully controllable AC voltage. The objective of this research is to make HEVs economically a practical and reasonable solution to regular cars and to reduce the CO<sub>2</sub> emissions by making them more energy efficient. A nicely designed HEV emits 40% less CO<sub>2</sub> and has lower fuel consumption.

Various aspects of the PWM techniques are thoroughly studied by carrying out time and frequency domain analysis on the different voltage and currents in the inverter. This provided an overview on how PWM strategies influence an electric drive. These analyses are then used to develop a new PWM technique that would alter the inverter architecture to make it cheaper, more compact and more efficient.

In the first part of the thesis, standard PWM techniques, such as SPWM, SVM and THIPWM, some basic discontinuous modulation techniques such as DPWM1, DPWMMIN as well as the common randomised modulation techniques are studied. Their unique features, advantages and shortcomings are discussed. Before moving to the next part where new PWM techniques are developed, tools to evaluate the performance of the output waveform are discussed. The choice of a performance indicator is established to compare the different modulation strategies and get an idea about the factors influencing the quality of the output signal. This is done on a macroscopic scale, i.e. at the scale of the time period of the fundamental voltage time period as well as at a microscopic scale i.e. on the scale of switching period. Some conclusions drawn from this analysis are that continuous PWM techniques have a lower harmonic distortion factor compared with DPWM for the same switching frequency. For the same effective switching frequency, the DPWM techniques produce higher quality output for high modulation index, while the macroscopic analysis on the waveform quality showed that centred voltage pulses have lower harmonic component than lagging or leading pulses.

In the second part, different modulation techniques developed with the different objectives in mind are presented. First of all a discontinuous modulation scheme is presented that optimises switching losses for variable speed drives. The advantages of this technique is the simple algorithm able to adapt the modulation technique to the changing load factor of the machine, while balancing the switching losses and/or

temperature difference, if any, between the different inverter legs. This not only increases the efficiency of the inverter, leading to a smaller heat sink and hence reduced volume but also ensures a longer life. Owing to its intelligence, this modulation technique can introduce non-uniformity in terms of the switching patterns among the three phases. This lets the weakest phase idle longer than the other two phases, extending the life cycle of the inverter. A random carrier frequency modulator is proposed and the choice is theoretically justified. Frequency domain analysis shows considerable reduction in the switching harmonics. This will lead to reduced EM emissions and hence lower possibility of these high frequency voltages interfering with other electronic components in the vicinity. However no emission measurement could be undertaken due to lack of time and resources.

A technique where phase currents can be deduced by measuring only the inverter input current is also presented. This technique is made applicable through out the hexagon using pulse placement techniques. This technique avoids the need of two phase current sensors hence reducing the cost of the inverter module. The last technique presented in this part is the reduction of the inverter input current ripple. This is done by using non-adjacent active vectors. From the implementation point of view, these techniques are a little complicated and require the right set of vectors to be used. The proposed technique can reduce the RMS value of the inverter input current ripple by 20%. This decrease in the ripple component means a smaller DC-link capacitor in volume by the same factor. However the use of non-adjacent vectors decreases the waveform quality of the output voltage. These techniques could not be experimentally verified due to the architecture of the power module, where the current sensor is placed before the DC-Link capacitor and hence gives filtered or average values.

After laying down the theoretical foundations that were tested through computer simulations, the discontinuous and randomised techniques are experimentally tested with PMSM as the load. A test platform was developed for this purpose. The basic work done was to interface the DSP with the IGBT module while conditioning the signals. One of the main aspects of the test bench is that it supports rapid prototyping through Simulink. One of the requirements relates to the controllability of the fundamental component of the voltage and its quality and the other is its harmonic spectrum. The proposed techniques are successfully implemented. The results obtained are very satisfactory. Reduction of inverter losses for the proposed discontinuous modulator and harmonic dithering for the randomised modulator were confirmed experimentally.

## **IV.2. Suggestions for future work**

The field of science knows no limits; one can work all one's life in a specific domain and discover new things everyday, the possibilities of improving and to reaching perfection never end. A Ph.D. offers a limited time to do research and document it; I was given three years. In this thesis I've presented my finding during these past three years as a Ph.D. candidate. The field of PWM is vast, a lot has been done and still remain to be accomplished. I hope I will be able to work some more on some of the issues.

The objective of my thesis here was to reduce the cost, volume and weight of electric drives through innovative PWM techniques. Results were shown that have a direct influence on these aforementioned parameters. EMI measurements have to be carried out to quantify the real gain in terms of electromagnetic emissions. This would help the dimensioning of the passive filters to meet the EMI standards for the new techniques which can then be compared against EMI filters required for standard deterministic PWM techniques. Similarly the heat sink needs to be re-dimensioned according to the reduced losses for the discontinuous modulator.

Experimental validation of the technique proposed to reduce the inverter input current ripple content has to be done, followed by re-dimensioning of the DC-link capacitors. A major task that should be looked into is the fusion of these techniques, this sure is a challenge, when there is more than one parameter to minimize. This will turn into an optimisation problem with a rather complex cost function. Only after performing these tasks can the final impact on the cost, volume and weight of the electric drive be calculated.

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