Regime switching in bond yield and spread dynamics

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Regime Switching in Bond Yield and Spread Dynamics

Changements de régimes dans la dynamique des taux et écarts de taux obligataires

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Résumé

Cette thèse développe des modèles de la structure par terme de taux d'intérêt –et des écarts de taux d'intérêt– dans lesquels les changements de régimes occupent une place centrale. On montre notamment que ce type de modélisation est particulièrement adapté pour rendre compte du comportement non-linéaire de ce type de variables financières. Ces modèles sont exploités afin de répondre à des questions clés pour les décideurs économiques et/ou les participants de marché, notamment: quels sont les effets de la crise financière récente sur la structure par terme des taux d'Etat ? Comment modéliser l'influence du risque systémique sur la courbe des taux ? Comment l'illiquidité d'une obligation ou le risque de défaut de son émetteur influencent la valorisation de ce titre ? Quelles sont les compensations demandées par les investisseurs pour supporter le risque de taux associé aux décisions de politique monétaire ?

Les modèles présentés sont fondés sur l'hypothèse d'absence d'opportunité d'arbitrage. Ce type d'approche a acquis une importance croissante pour les décideurs comme pour les participants de marché au cours de la dernière décennie. En effet, tout en permettant un ajustement fin des données, les modèles vérifiant cette hypothèse fournissent un cadre cohérent pour l'analyse des fluctuations des taux obligataires et des primes de risque qu'elles incluent.

Dans la spécification de nos modèles, une grande importance est accordée à la simplicité des formules de valorisation d'actifs. Ceci est crucial pour l'estimation des paramètres des modèles et pour faciliter l'inférence statistique. La simplicité des calculs repose sur l'utilisation des propriétés des processus composés auto-

regressifs (Car) pour obtenir des formules quasi-explicites de prix obligataires[1]. Dans notre approche, les dynamiques physique et risque-neutre des processus sont explicites. Disposer de la dynamique historique rend notamment possibles les exercices de prévision et, de manière générale, est important pour les besoins de gestion des risques financiers (notamment pour le calcul des *Values-at-Risk*, VaR).

Le premier chapitre est une revue de la littérature liée aux différents sujets étudiés dans cette thèse. Dans le deuxième chapitre, nous développons un cadre général de modélisation des fluctuations de courbes de taux associées à différents émetteurs. Les probabilités de défaut des emprunteurs sont fonctions de facteurs observables ou non, à valeurs discrètes ou réelles. Alors que l'accent est mis sur la modélisation du risque de défaut, on montre comment ce cadre permet également de valoriser les titres obligataires illiquides. Une version simple du modèle est estimée pour reproduire la dynamique des écarts de taux entre les obligations émises par des entreprises américaines d'une part et celles émises par le Trésor américain (*Treasuries*) d'autre part. Ce second chapitre montre par ailleurs comment les changements de régimes peuvent être utilisés pour reproduire des phénomènes de contagion sectorielle. Enfin, le cadre initial est étendu pour modéliser les transitions de notations de crédit et l'influence de celles-ci sur les structures par terme de taux d'intérêt.

Le chapitre 3 présente une analyse des fluctuations jointes de courbes de taux d'Etat de dix pays de la zone euro entre 1999 et 2012. Deux régimes interviennent dans le modèle, l'un de ceux-ci correspondant aux périodes de crise financière. Ces régimes conditionnent la dynamique de cinq facteurs observables. Le taux d'intérêt sans risque de court-terme, ainsi que les intensités de défaut et d'illiquidité, dépendent linéairement de ces cinq facteurs. Ces spécifications permettent d'expliquer la majeure partie des variations des taux d'intérêt inclus dans l'échantillon –pour les différents pays et les différentes maturités considérées. L'estimation suggère en outre que l'introduction du régime de crise est importante pour expliquer l'accroissement de la volatilité des écarts de taux sur la période récente. Cette étude propose également un moyen d'identifier la partie des taux

[1] Pour une analyse exhaustive de ces processus, voir Darolles, Gourieroux et Jasiak (2006).

d'intérêt liée à la valorisation de l'illiquidité relative des obligations souveraines. A cet égard, les résultats indiquent que bien que la liquidité soit un facteur important pour expliquer les écarts de taux d'Etat sur les cinq dernières années, les inquiétudes relatives à la qualité de crédit des Etats de la zone euro constituent le principal motif de leurs variations sur la période 2010-2012.

Le chapitre 4 complète l'analyse du chapitre précédent en se concentrant sur les cinq dernières années, i.e. la période de crise financière 2007-2012. La modélisation repose sur des facteurs de risque latents (non observables). La modélisation des périodes de crise est plus précise que dans le chapitre précédent. En effet, nous distinguons ici deux types de tensions : celles liées à des motifs de liquidité et celles liées à des motifs de crédit. Plus précisément, nous introduisons deux chaînes, l'une dite de liquidité et l'autre dite de crédit. Pour la chaine de liquidité, deux états sont possibles: «faibles tensions» et «périodes de turbulences». Pour la chaîne de crédit, un troisième niveau de tension («crise aigue») s'ajoute aux deux premiers. L'estimation met en évidence un lien de causalité entre les deux types de tensions : la probabilité d'entrer dans une période de turbulences liées à des modifs de crédit est plus forte lorsque la situtation de liquidité est déjà détériorée.

Le chapitre 5 examine l'influence de la politique monétaire sur la courbe des taux d'intérêt. Bien que les taux directeurs fixés par la banque centrale jouent un rôle central dans la dynamique de la structure par terme des taux d'intérêt, peu de modèles sont cohérents avec les spécificités des trajectoires de taux directeurs. Ce chapitre vise à pallier ce manque en présentant un cadre dans lequel le taux d'intérêt (de court terme) auquel les banques se refinancent auprès de la banque centrale est à valeurs discrètes (ce sont des multiples de 0.25%) et positives. En particulier, contrairement à la plupart des modèles de la structure par terme, celui-ci est conforme à l'existence d'une borne inférieure (en zéro) pour les taux courts. Ces propriétés découlent d'une utilisation innovante (et intensive) des changements de régimes. En dépit d'un très grand nombre de régimes (246), le modèle reste maniable, ce qui est illustré par son estimation sur données quotidiennes relatives à la zone euro, l'échantillon couvrant les 13 dernières années. Les résultats suggèrent que la partie courte de la courbe des taux intègre des primes de risque, celles-ci correspondant aux compensations demandées par les investisseurs pour supporter le risque de taux associé aux décisions de politique monétaire. Ce modèle est également utilisé afin d'évaluer l'influence sur la courbe des taux d'engagements de la banque centrale sur une trajectoire future de son principal taux directeur.

Summary

This doctoral thesis studies the potential of regime-switching models to capture salient features of the dynamics of interest rates. It is notably shown that these techniques can be used in several ways to reproduce various forms of yield and spread non-linearities. Different innovative frameworks, combining flexibility and tractability, are proposed. They are brought to data so as to tackle questions that are key for both policy-makers and practitioners alike. These questions include the following: What are the effects of the ongoing financial crisis on the term-structure of sovereign spreads? How to model yield-curve reactions to increases in systemic risk? What are the effects of market liquidity on the term structure of interest rates? How are priced the probabilities of default (PDs) in defaultable-bond yields of different maturities? What are the compensations required by investors to hedge against uncertain monetary-policy decisions?

The models that are presented throughout this thesis rule out arbitrage opportunities. Such models are becoming increasingly important to policy makers and practitioners. Indeed, beyond being able to provide a good fit of interest rates along the whole maturity spectrum, these models allow to study the driving factors behind the term structure of interest rates and the risk premia within a consistent framework.

Particular attention is paid to the tractability of the proposed models. Tractability is notably obtained through an extensive use of Car's –Compound autoregressive processes– properties,[2] which leads to quasi-explicit formulas for bond prices. Both historical and risk-neutral dynamics are explicitly modeled, which is helpful for

[2]For an in-depth analysis of Car processes, see Darolles, Gourieroux and Jasiak (2006).

choosing appropriate specifications under the physical –or real-world– measure, for dealing simultaneously with pricing and forecasting or also for risk-management purposes (e.g. Value-at-Risk calculations).

The first chapter of this dissertation goes through the different topics that are studied in the thesis and reviews the connected literature. The second chapter develops a general framework aimed at modelling the joint dynamics of yield curves associated with different issuers. In this reduced-form framework, the default probabilities are modeled directly as functions of observable or latent factors, the latter being discrete or real-valued. Regime-switching features lie at the heart of this framework. While the focus is on default modelling, the specifications can also account for the pricing of some liquidity premia using the same machinery (as in Duffie and Singleton, 1999). A basic form of the model is fitted on the term structure of spreads between U.S. corporate BBB-rated bonds and risk-free (Treasury) yields. Some extensions are proposed, including a sector-contagion model as well as the explicit modelling of credit-rating transitions.

In Chapter 3, the framework is applied to model the joint fluctuations of ten euro-area sovereign yield curves over the period 1999-2012. In the model, there are two regimes: a "tranquil" regime and a crisis one. These regimes affect the dynamics of five euro-area wide observable factors. These factors affect the riskfree short-term rate as well as the default and illiquidity intensities associated with the different issuers. This framework is able to capture most of the fluctuations of the various interest rates (over the different countries and different maturities) of the estimation sample. Further, the setup makes it possible to account for the dramatic rise in spreads that have been observed for some countries over the last few years. Also, this study proposes a way to identify liquidity-related components in sovereign bond yields. Regarding the latter point, the estimation results suggest that while liquidity is an important driver of euro-area sovereign spreads, most of the 2010-2012 spreads' fluctuations correspond to concerns regarding the credit quality of sovereign issuers.

Chapter 4 builds on the previous chapter by focusing on the last five years (instead of the last thirteen years hereinbefore). Contrary to Chapter 3, the pricing factors are unobservable. In addition, the crisis modelling is more precise. Specifically, we distinguish between two kinds of crises: liquidity-related ones and credit-related ones. For the credit chain, there are three possible states: “calmer periods”, “turmoil periods” and “severe-crisis periods”; for the liquidity chain, there are two possible states: “calmer periods” and “turmoil periods”. The empirical part of Chapter 4 provides evidence of causality between the two types of crisis, the probability of switching from the calm credit state to the credit-crisis state being higher when the liquidity situation is already deteriorated.

Chapter 5 investigates the influence of monetary policy on the yield curve. In this study, a key role is given to the central-bank policy rate. In the model, the policy rate follows a realistic step-like path (with values that are multiples of 0.25%) and can not turn negative. Therefore, by contrast with most of the existing term-structure models, this one is consistent with the zero-lower bound (ZLB). These appealing features are obtained thanks to an extensive and innovative use of regime shifts. In spite of a very large number of regimes (246), the model remains tractable and is easily brought to data. This is illustrated by estimating the model on euro-area daily data covering the last 13 years. The results notably point to the existence of monetary-policy-related risk premia at the short-end of the yield curve. Furthermore, this model is used in order to assess the influence of forward-guidance measures –defined as commitments of the central bank regarding the future paths of the policy rate– on the yield curve.

1. Survey of the literature

Abstract: This first chapter reviews the literature connected to the present thesis. Section 1.1 surveys the contributions that have highlighted the ability of Markovian regime-switching techniques to model nonlinear dynamics in a tractable way. Section 1.2 illustrates the fact that these techniques have been employed in many studies exploring the dynamics of economic and financial variables. Section 1.3 focuses on the use of regime switching in term-structure models: Subsection 1.3.1 deals with risk-free yields and Subsection 1.3.2 considers defaultable-bond pricing. The subsequent Sections deal with the additional topics that are covered by this thesis: the simultaneous modelling of different yield curves (Section 1.4), systemic risk and contagion (Section 1.5), credit-rating migrations (Section 1.6), monetary-policy and the yield curve (Section 1.7) and the decomposition in spreads into liquidity and credit components (Section 1.8).

Résumé

Il existe une importante littérature sur l'existence de non-linéarités dans la dynamique des taux d'intérêt (voir par exemple Aït-Sahalia, 1996, Stanton, 1997 ou Boudoukh et al., 1999). Plus précisément, plusieurs études montrent l'existence de différents régimes conditionnant la dynamique des taux d'intérêt (voir notamment Hamilton, 1988 ou Ang et Bekaert, 2002).

Alors que le comportement récent des taux d'intérêt illustre de manière édifiante la notion de changement de régimes, l'utilisation de modèles à changements de régimes pour l'analyse de la structure par terme des taux d'intérêt est encore relativement limitée. Alors que différentes études présentent des modèles dans lesquels les déformations d'une unique courbe de taux dépendent de l'état d'une variable aléatoire à valeurs discrètes (Monfort et Pégoraro, 2007, Ang, Bekaert et Wei, 2008, Dai, Singleton et Yang, 2007, ou Pérignon and Smith, 2007), un nombre très restreint de contributions considèrent la modélisation jointe de différentes courbes de taux affectées par des changements de régimes (Dionne et al., 2011 et Siu, Erlwein et Mamon, 2008).

Ce premier chapitre propose une revue de la littérature concernant la modélisation des changements de régimes d'une manière générale et leurs applications à la modélisation des variables financières et des taux d'intérêt en particulier. Cette revue de la littérature couvre également divers champs d'études auxquels les travaux présentés dans cette thèse sont liés, ceux-ci ont trait à:

- la modélisation du risque systémique et des phénomènes de contagion;
- la modélisation des changements de notations de crédit;
- l'influence de la politique monétaire sur la structure par terme des taux d'intérêt;
- l'influence de la valorisation de la qualité de crédit et de la liquidité sur la structure par terme des taux d'intérêt.

1.1. Regime switching: A tool to model non-linear dynamics

Linear models –such as autoregressive (AR) models, moving average (MA) models, and mixed ARMA models– are extensively used to model the dynamics of economic or financial variables. These models, that are extremely popular among academics, practitioners and policy makers, are quite successful in numerous applications. However, it has often been found that simple linear time series models usually leave certain aspects of economic and financial data unexplained. By definition, they are unable to capture nonlinear dynamic patterns such as asymmetry, extreme events or volatility clustering. Typically, the properties of output growth in recessions are, in various ways, different from expansion time (see e.g. Hamilton, 1989, Lo and Piger, 2005 or Sichel, 1994 among innumerable others). Inflation also presents different kinds of nonlinearities, notably in crises periods (see e.g. Stock and Watson, 2010). Therefore, in many cases, linear models are not sufficient and non-linear approaches have to be resorted to. Accordingly, over the last two to three decades, we have witnessed a rapid growth of the development of nonlinear time series models (see e.g. Granger and Teřasvirta, 1993).

Regime-switching models, closely linked to the seminal work of Hamilton (1988, 1989 and 1990), are among the most popular nonlinear time series models in the literature. The fact that the regimes can switch over time makes it possible to account for various non-linear behavior of the modeled variables. In the standard regime-switching framework, the change in the regimes is controlled by an unobservable state variable that follows a Markov chain, that is, the current value of the state variable depends on its immediate past value. A given regime can be persistent or not, depending on the probabilities of switching to alternative regimes. The Markov switching model is therefore suitable for describing correlated data that exhibit distinct dynamic patterns during different time periods. The standard framework has notably been extended by Filardo (1994) to allow for time-varying transition probabilities (implying that regime-switching models encompass threshold auto-regressive models of Tong and Lim, 1980) or, from an econometric point of view, by Kim (1994) that integrates the regime-switching features within a state-space framework including unobserved factors affected by Gaussian shocks.

Of course, regime-switching models are not the only models that can handle non-linear behavior of random variables. There exist models that can handle more general forms of non-linearity. In particular, the so-called artificial neural network models, due to their "universal approximation" property, are capable of characterizing any nonlinear pattern in data (see e.g. Kuan and White, 1994). Unfortunately, these models suffer from identification-related problems and are far less tractable than regime-switching models. As will be illustrated throughout the present thesis, the latter still allow for a substantial degree of flexibility, making them appropriate to study a wide range of phenomena.

By appropriately mixing conditional normal (or other types of) distributions, large amounts of non-linear effects can be generated within regime-switching frameworks. Regime switching models can provide a good approximation for more complicated processes driving security returns. Regime switching models also nest as a special case jump models, since a jump is a regime which is immediately exited next period and, when the number of regimes is large, the dynamics of a regime switching model approximates the behavior of time-varying parameter models where the continuous state space of the parameter is appropriately discretized.

1.2. Regime switching in economics and finance

Abrupt changes are a prevalent feature of economic systems and financial markets. These changes are of different natures: some are transitory (jump-like) and some tend to persist for protracted periods. As mentioned above, both types of changes can be captured by regime-switching models (see Ang and Timmermann, 2011 and Guidolin, 2011); this is going to be illustrated in the present thesis.[1] Regime-switching models parsimoniously capture stylized behavior of many financial series

[1] Specifically, in the model developed in Chapter 3, the crisis regime may last for several years. By contrast, the stress episodes introduced in Chapter 4 have a life expectancy of a few weeks.

including asymmetries, fat tails, skewness, persistently occurring periods of turbulence followed by periods of low volatility, volatility clustering, time-varying correlations.

The use of the regime-switching method for modelling dynamics and asymmetries in stock prices has become very popular and various adaptations of the basic setup have been proposed (e.g. Perez-Quiros and Timmermann, 2001 or Ang and Chen, 2002). Regime-switching setups have also been estimated to analyze the dynamics of exchange rates (Ang, 2011, Kanas, 2006, Engel, 1994, Bollen, Gray and Whaley, 2000 and Dewachter, 2001), and of various alternative prices such as electricity prices (e.g. Haldrup and Nielsen, 2006) or commodity prices (e.g. Chen and Insley, 2012).

Regime-switching models have proven useful in building coincident indicators (Kim and Yoo, 1995 and Kim and Nelson, 1998) or in developing forecasting tools (Chauvet and Potter, 2000) or optimal portfolio choice (Guidolin and Timmermann, 2007).

The ongoing financial crisis is strengthening the case for including regime-switching features in financial models (see Christensen, Lopez and Rudebusch, 2008). This is notably illustrated by Chapters 3 and 4 of the present thesis that study the dynamics of government-bond interest rates amid the so-called euro-area sovereign debt crisis (exploiting the general framework presented in Chapter 2). The crisis period itself can be seen as a succession of different regimes or phases; this idea is omnipresent: it can be found in academic work (e.g. Bech and Lengwiler, 2011), in official speeches (Stark, 2009) or in the medias (The Guardian, 2011).

The fact that the idea of regime changes is natural and intuitive has contributed to its popularity. Economic explanations for these types of time-variation in a series' dynamics point into main three directions. According to the first, regimes identified by econometric methods can be associated with different periods in regulation, policy, and other secular changes (see e.g. Hamilton, 1988, Sims and Zha, 2006, Davig, 2004). The second strand of economic explanations relates market-price movements to macroeconomic fundamental influences. In particular, numerous studies confirm that the conditional moments of stock returns are business cycle dependent (Cecchetti, Lam and Mark, 1990, Hamilton and Lin, 1996, Schwert, 1989, Campbell et al., 2001 or Perez-Quiros and Timmermann, 2001). The third type of explanation attributes nonlinearities to particular behavior of market participants (e.g. noise traders). There is a large literature that reports that speculative trading may cause fads, bubbles or even market crashes (Funke, Hall and Sola, 1994, van Norden and Vigfusson, 1998 or Jeanne and Rose, 2002).

1.3. Yield-curve dynamics and regime switching

1.3.1. Regime shifts in default-free yield-curve dynamics

Strong evidence points to the existence of regime switching in the dynamics of the term structure of interest rates. Thus, Hamilton (1988) finds that changes in the Federal reserve operating procedures leads to regime-switching in the dynamics of the term structure of interest rates. In addition to such a shift, Cai (1994) finds that the 1974 oil shock resulted in a regime shift in the asymptotic volatility of the three-month Treasury bill. Gray (1996) shows that the assumption of a single regime is a source of misspecification in models of the short rate. Garcia and Perron (1996) use the Hamilton filter to characterize the time series behavior of the ex-post U.S. real interest rate during the period 1961 to 1986 and show that the real interest rate series during this time period would be best characterized by three states. Adding term spread in their estimation, Ang and Bekaert (2002) identify regimes that are closely linked to business cycles, suggesting that large periodic shifts in interest rates across distinct regimes present a systematic risk to investors (see also Wu and Zeng, 2005 or Bansal and Zhou, 2002). The same authors (2002) show that regime switching is efficient in capturing nonlinear dynamics of the short-term interest rate exhibited by Aït-Sahalia (1996). Christiansen (2004) estimates a two-state Markov-switching model for the short-rate and the slope of the yield curve: his estimated regimes turn out to depict low and high variances regimes for short-rate changes. The economy appears to have been in the high-variance state during unusual economic periods such as oil or stock-market crises, or more generally during the official recession periods.

Monfort and Pegoraro (2007) show that the introduction of regime switching in term-structure models leads to term-structure models that are well-specified under the historical probability and that are able to explain the expectation-hypothesis puzzle (why the long and short term interest rate differential does not predict the future interest rate changes), over short and long horizons. Following Veronesi and Yared (1999) and Evans (2003), Ang Bekaert and Wei (2008) develop term structure models with regime shifts to investigate the joint dynamics of real and nominal yields. They identify inflation and real factor sources behind regime shifts and analyze how they contribute to nominal interest-rate variations. Dai, Singleton and Yang (2007) develop a model with regime-shift risks that are priced by investors. Allowing for state-dependent transition probabilities, their model makes it possible to conveniently capture asymmetry in the cyclical behavior of interest rates. Pérignon and Smith (2007) show that allowing for regime shifts in the pricing factor volatilities dramatically improves the model's fit.

In Chapters 2 to 4 of the present thesis, the emphasis is put on defaultable-bond pricing. By contrast, in Chapter 5, an innovative use of regime-switching features is proposed to model the term structure of riskfree yields. Contrary to the above-mentioned studies, the number of regimes involved in the model introduced in Chapter 5 can be very large (tens or hundreds). In spite of that, the model remains tractable and makes it possible to model the specific dynamics of the central-bank policy rate in a satisfying way. The latter point implies that this model can be exploited to investigate the effects of monetary-policy on the yield curve.[2]

1.3.2. Regime shifts in spreads' dynamics

While the previous subsection puts forward the importance of modelling regime switching in yield-curve models, a few has been done to integrate such a feature in term-structure models of defaultable bonds. However, empirical studies point

[2]This is also discussed below in 1.7.

to the existence of different regimes in the default risk valuation. Davies (2004 and 2008) uses Markov-Switching Vector Auto-Regression (MS-VAR) estimation techniques and finds that credit spreads exhibit distinct high- and low-volatility regimes. Alexander and Kaeck (2008) detect a pronounced regime-specific behavior of Credit default swap (CDS) spreads. Cenesizoglu and Essid (2010) or Bruche and Gonzales-Aquado (2010) find switching behavior in default rates and recovery-rate distributions. Hackbarth, Miao and Morellec (2006) build a theoretical model to explain the dependence of credit spread on business-cycle regimes. In the same vein, Bhamra, Kuehn and Strebulaev (2007), Chen (2008) and David (2008) adopt a Merton structural model including regime switching to assess the influence of different states of the economic cycles on the credit-risk premia. This can be related to the analysis of Bangia et al. (2002) who illustrate the importance of distinguishing between expansion and contraction phases for the assessment of loss distribution of credit portfolios. Without deriving a complete model of the credit-spread term structure, Maalaoui, Dionne and François (2009) estimate Markov-switching specifications to investigate the links between credit spreads and their determinants. Their results suggest that the failure of single-regime models to find significant links between potential determinants (see e.g. Collin-Dufresne, Goldstein and Martin, 2001) may stem from the fact that these determinants have opposite average effects in the two regimes they identify. Dionne et al. (2011) propose a model of the term-structure of interest rates associated with defaultable bonds. Regime switching affects the dynamics of the risk factors, that are observable macroeconomic variables. Siu, Erlwein and Mamon (2008) present a framework to price credit default swaps in the presence of regime-switching in the default intensities processes.

The potential of regime-switching features to account for the fluctuations of the term-structure of (credit-)risky yields in a no-arbitrage framework is explored in Chapters 2 to 4 of this thesis. In the proposed setups, the probabilities of default of the debtors depend on the different regimes and on factors that can be observed or latent. Therefore, the whole term structure of interest rates is affected by the regimes. While this framework is highly flexible, it remains particularly tractable, bond yields being given by quasi-explicit formula. This property stems from the fact that the processes involved are Compound auto-regressive (Car), implying that multi-horizon Laplace transforms of these processes are obtained by recursive formulas.

1.4. Jointly modelling the physical and risk-neutral dynamics of different yield curves

Motivated by derivative-pricing or credit-risk-management objectives, a large strand of the recent literature related to fixed-income securities has focused on the joint modelling of several yield curves. In this context, Jarrow, Lando, Turnbull (1997), Lando (1998) or Duffie and Singleton (1999) have highlighted the potential of affine term-structure models (ATSM) to describe the joint dynamics of yield curves associated with various obligors subject to default risk. Their intensity-based –or reduced-form– approaches used to model defaults differ from the more structural approaches originating in Black and Scholes (1973) and Merton (1974). In the latter, the default of a firm is modeled in terms of the relationship between its assets and liabilities. The asset value process is modeled as a geometric Brownian motion and default occurs when the asset value at maturity is lower than the liabilities. Important industry models like KMV's Portfolio Manager or the JP Morgan's CreditMetrics model are based on this approach (see Crouhy, Glai and Mark, 2000for a comparative analysis of industry credit-risk models). Cathcart and El-Jahel, 2006) have shown that the two approaches (reduced-form and structural) are somewhat reconcilable. As shown by Duffie and Singleton (1999), in an intensity-based framework, the modelling of defaultable claims is based on the standard affine term-structure machinery readily available for default risk modelling and estimation. Since then, numerous further developments have illustrated the flexibility and tractability of affine-term structure models to jointly model different yield curves (see e.g. Duffee, 1999, Collin-Dufresne and Solnik, 2001, Dai and Singleton, 2003, Collin-Dufresne, Goldstein and Hugonnier, 2004 and Gourieroux, Monfort and Polimenis, 2006).

Despite the importance of sovereign credit risk in the financial markets, relatively little research proposing models of the joint dynamics of sovereign yields has appeared in the literature. Notable recent exceptions include Pan and Singleton (2008) and Longstaff et al. (2011). These two contributions point to an important degree of commonality across sovereign credit risk. More precisely, they show that the risk premia included in sovereign credit spreads are substantial and covary importantly with economic measures of global event risk. According to Longstaff et al., an important source of commonality in sovereign credit spreads may be their sensitivity to the funding needs of major investors in the sovereign credit markets.

Chapters 3 and 4 propose models that depict the joint dynamics of different euro-area sovereign yield curves. In these models, the dynamics of the stochastic discount factor implies that the physical and the risk-neutral dynamics of the pricing factors –and notably the default process– do not coincide. The risk-neutral dynamics is the dynamics of the pricing factors that would be consistent with observed prices under the (potentially false) assumption that investors are risk-neutral. In our framework, we can assess the size of the (potential) errors that are implied by assuming that the historical and the risk-neutral dynamics coincide. A typical example lies in the computation of market-based probabilities of default (PDs). To get these, the vast majority of practitioners or market analysts resort to approaches ending up with risk-neutral PDs.[3] While risk-neutral PDs are relevant for pricing purposes, historical ones are needed (a) if one wants to extract real-world investors' perception of the credit quality of the issuer, (b) for the sake of forecasting or more generally (c) for risk management purposes.[4]

[3] Most of these methodologies build on Litterman and Iben (1991), see e.g. (amongst many others) Bank of England (2012), CMA (2011) and O'Kane and Turnbull (2003). Studies resorting to these methods are usually silent about this caveat. Notable recent exceptions include Blundell-Wignall and Slovik (2010), in an OECD study, who note: "In the real world, actual defaults are fewer than market-driven default probability calculations would indicate. That is because market participants demand a risk premium – an excess return – compared to the risk-neutral rate, and that premium cannot be observed. This makes it difficult to use the above measure [the risk-neutral PDs] to imply the likelihood of actual defaults in the periphery of Europe or anywhere else."

[4] Regarding the latter point, note for instance that Value-at-Risk measures (VaR) should be based on the real-world measure and not on the risk-neutral one (see Gourieroux and Jasiak,

The results of Chapters 3 and 4 suggest that the sources of common fluctuations across euro-area countries' yields command such credit-risk premia. This is consistent with the fact that sovereign risk cannot be diversified away. The analysis shows that, because of these premia, the physical probabilities of default of euro-area countries are substantially lower than their risk-neutral counterparts.

1.5. Systemic risk, default clustering and contagion

While there is no strong consensus on the definition of systemic risk, the general view is that this kind of risk would differ from the systematic ones in terms of the severity and frequency of the associated shocks. More precisely, systematic shocks are frequent and not extreme while systemic shocks would be infrequent and extreme (see e.g. Das and Uppal, 2004 or Baur and Schulze, 2009). For de Bandt and Hartmann (2000), a systemic event is an event where the release of bad news about a financial institution, or even its failure, or the crash of a financial market leads in a sequential fashion to considerable adverse effects on one or several other financial institutions or markets, e.g. their failure or crash. Obviously, disentangling systematic from systemic risks may not be a trivial task. In particular, difficulties arise from the fact that systematic shocks can turn into systemic ones. For instance, in some contexts –notably when the level of uncertainty is high–, temporary systematic shocks can lead to defaults and generate significant negative aftershocks, including liquidity spirals.[5]

In a model accommodating regime shifts, it is natural to associate systematic and systemic risk with the Gaussian shocks and the regime shifts, respectively (see e.g. Gonzales-Hermosillo and Hesse, 2009 or Abdymomunov, 2012). Billio et al. (2012) propose an other use of regime-switching features to investigate systemic risks; in their approach, the regimes are key to model of the interconnectedness of the financial system.

2009).

[5]See Brunnermeier and Pedersen, 2009 for a structural analysis of this and e.g. Hesse and Gonzalo-Hermosillo (2009) for empirical evidence.

The contagion literature focuses on the interdependencies between the defaults of different debtors.[6] In the so-called contagion models, if one of the debtor defaults, it affects the default probability of the other debtors. Contagion effects, whose consequences are cascades of subsequent spread changes, are explained by the existence of close ties between firms. These ties may be of legal (e.g. parent-subsidiary), financial (e.g. trade credit), or business nature (e.g. buyer-supplier). Through these channels, economic distress of one firm can have an immediate adverse effect on the financial health of that firm's business partners (Giesecke, 2004, Egloff, Leippold and Vanini, 2005). Jarrow and Yu (2001) develop a *primary-secondary* approach: in case a primary entity defaults, the spreads of other debtors jump upwards; meanwhile, default of secondary firms do not have any impact on other debtors in the portfolio. In the *infectious-default* model developed by Davies and Lo (2001), the default of a debtor triggers a regime shift: in the high-risk regime, the default intensities of all debtors are increased.[7]

Das et al. (2007) test whether default events can reasonably be modeled as dependent solely on exogenous observable factors.[8] As Duffie et al. (2009) and Giesecke and Kim (2010), they find that doubly-stochastic settings perform badly if no latent covariates –also called frailty components– enter the intensity specifications. Duffie et al. (2009) further argue that including frailty covariates in the hazard-rate specifications is necessary to accommodate default clustering.[9] Collin-Dufresne, Goldstein and Helwege (2008), Bai et al. (2012) and Jorion and Zhang (2007) also find that default events are associated with significant increases in the credit spreads of other firms, consistent with default clustering in excess of that suggested by the standard doubly stochastic models. Azizpour and Giesecke (2008) find that contagion effects represent a significant additional source of default clus-

[6]For an extensive survey of the contagion literature, see e.g. Lütkebohmert (2009).

[7]Other contagion mechanisms based on the same kinds of approaches are proposed by Frey and Backhaus (2003) or Yu (2007).

[8]Nevertheless, using a different specification of the default intensity, Lando and Nielsen (2008) cannot reject the assumption of conditional independence for default histories recorder by Moody's between 1982 and 2006. Lando and Nielsen conclude that the test proposed by Das et al. (2007) is mainly a misspecification test.

[9]Frailty models come from the biostatistics literature. In these models, the intensity of a point process is proportional to an unobservable variable, the frailty parameter. For a survey of frailty models, see Hougaard (2000) [155].

tering (over and beyond the effect due to firms' exposure to observable and frailty risk factors). Koopman, Lucas and Schwaab (2012) show that modelling frailty contributes to obtain a proper modelling of default rates during crisis.

Two Subsections of Chapter 3 specifically deal with systemic risk and contagion. Subsection 2.8.2 shows that the general framework introduced in Chapter 3 can accommodate the specific contagion case where one entity –or, for the sake of tractability, a small number of them– affects the default probability of the others: it suffices to make one of the regimes corresponds to the default state of this entity. Further, Subsection 2.8.3 explains how the regime-switching feature can be exploited in order to capture "sector-contagion" phenomena. The sectors can represent different industries or different geographical areas. Each sector can be "infected" or not, and when a sector gets infected, the default intensities of its constituents (the debtors) shift upwards. In this context, sector contagion stems from the parameterization of the matrix of regime-transition probabilities. For instance, it is easy to model infection probabilities that depend positively on the number of sectors already infected.

1.6. Credit-migration modelling

The default of a debtor is the most basic credit event. More generally, credit events include changes in credit ratings like these attributed by agencies like Moody's, Standard & Poor's or Fitch. There are several reasons why it may be desirable to model not only default events but also rating transitions (see Cantor, 2004 or Gagliardini and Gourieroux, 2001). Several of the main credit models currently being used in the industry, such as J.P. Morgan's CreditMetrics (1997), draw on the credit-migration approach. For presentation, comparison and evaluation of these models, one can refer to Crouhy, Glai and Mark (2000), Gordy (2000) or Lopez and Saidenberg (2000). First, because of the importance of ratings in terms of risk management, modelling credit migration is key for practitioners. For instance, the VaR or capital adequacy numbers may be based on a portfolio rating's distribution (see Saidenberg and Schuermann, 2003). In addition, some portfolio managers are constrained by limits based on the ratings of the bond they held. Second, such models are obviously required to price credit-event options. Third, when complete default historical data sets are not available (or do not go back far in time), exploiting credit-migration matrices may allow to extrapolate long-term default predictions from short-term credit risk dynamics. Similarly, to the extent that rating classes are seen as approximately homogenous, having a rating-based term structure model at one's disposal makes it quick to get a rough estimate of the fair value of a bond (given the rating of the issuer).[10]

In their seminal study of credit spread, Jarrow, Lando and Turnbull (1997) model rating transitions as a time-homogenous Markov chain. That is, in their model, whether a firm's rating will change in the next period depends on its current rating only and the probability of changing from one rating to the other remains the same over time. In addition, in their setting, the market risk and the credit risk are assumed to be independent. Different studies suggest however that –per-period– transition probabilities are time-varying (see e.g. Lucas and Lonski, 1992, Belkin, Suchower and Wagner, 1998, Farnsworth and Li, 2007 or Feng, Gourieroux and Jasiak, 2008). In addition to time-variability, Nickell, Perraudin and Varotto (2000) show that conditioning a transition matrix on the industry (to which the company belongs) is desirable.

Lando (1998) extends the framework developed by Jarrow, Lando and Turnbull (1997) by allowing for dependence between the market risk and the credit risk[11] and by making the rating-transition probabilities depend on the state variables. Other examples of term-structure models allowing for time-varying probabilities of rating migrations include Bielecki and Rutkowski (2000) and Wei (2003). In Subsection 2.8.4 of the present thesis, it is shown how the general framework proposed in Chapter 3 can be extended in order to model credit-rating migrations. In that model, the probabilities of migrating from one rating to another is time-varying and can, in particular, depends on regimes. In such a context, bond prices

[10] This assumption is for instance made in J.P. Morgan's CreditMetrics (1997). It is also made, e.g., by Feldhütter and Lando (2008).

[11] Amongst the earliest studies suggesting that such a feature is required, see Longstaff and Schwartz (1995) or Duffee (1998).

are still given by closed-form recursive formulas.

1.7. Monetary-policy and the yield curve

While there is a strong empirical support for the assertion that monetary policy is a major driver of the yield-curve fluctuations (see e.g. Cochrane and Piazzesi, 2002 or Rigobon and Sack, 2004), the quantitative aspects regarding the transmission mechanism along the yield curve –from the overnight interbank market to longer-term interest rates– are less clear.[12] Among the vast number of interest-rate term-structure models, only a very few deal explicitly with monetary-policy decisions. This lack, which is particularly pronounced at a time when policymakers have to consider all possible options to deal with the crisis, partly reflects the specificities of the process followed by the policy rate –or central-bank target– and the technical difficulties associated with incorporating such a process in a no-arbitrage framework.[13] Piazzesi (2005) and Fontaine (2009) propose term-structure models in which changes in the target rate have (realistic) discrete supports. They estimate their models on U.S. data covering respectively the periods 1994-1998 (weekly) and 1994-2007 (daily). However, their models technically imply non-zero probabilities of negative interest rates for all maturities on the term structure. While this caveat may be tenable when the short-term interest rate is far enough from zero –the conditional probabilities of having negative interest in the subsequent periods being negligible–, it is more problematic when the zero-lower bound (ZLB) is binding.

Actually, most of the tractable yield-curve models are not consistent with this zero lower bound (See Dai and Singleton (2003) or Piazzesi (2010)). Hamilton and Wu (2012) propose a way to adapt the standard Gaussian framework to account for an extended period of constant short-term rate. However, they implicitly assume that when this phase ends, (a) such a phenomenon cannot happen again and (b),

[12] This is the so-called interest-rate channel of monetary-policy decisions.

[13] See e.g. Rudebusch (1995), Hamilton and Jorda (2002), Balduzzi, Bertola and Foresi (1997) and Balduzzi et al. (1998) for models of the U.S. Federal Funds rate target (the Fed funds rate is the U.S. overnight interbank rate).

the short-term rate can turn negative again. Andreasen and Meldrum (2011) or Kim and Singleton (2011) show that the quadratic Gaussian framework can be used to preclude negative interest rates. Indeed, in these models, the short-term rate is a quadratic function of underlying factors; this quadratic function can be such that the short-rate –and therefore longer-term rates– is always positive. Nevertheless, to ensure the tractability of this approach, the underlying factors are affected by homoskedastic Gaussian shocks. Hence, the probability that a quadratic combination of these factors remains very close to zero for a protracted period of time is extremely low. The latter point implies that these models are not consistent with prolonged periods of very low interest rates, limiting their relevance in the current context. By contrast, as is illustrated in Chapter 5 of the present thesis, regime switching features make it possible to satisfyingly account for long periods of time of very low and/or constant policy rates.

A notable feature of the monetary policy behavior is that changes in the policy rate tend to be followed by changes of the same direction, giving rise to easing/tightening monetary-policy phases (see e.g. Mooreand Richard, 2002, Heinemann and Ullrich, 2007 or the speech by Smaghi, 2009). These phases are usually very persistent and typically last for a few quarters or years. For Bikbov and Chernov (2008), shifts in the overall monetary policy stance (from accommodative to tightening or vice versa) may have more important effects on interest rates than a single interest rate change does. Bikbov and Chernov show that a model with regime shifts is the most convenient tool to capture such policy behavior. Davig and Gerlach (2006) identify states that imply different responses of the yield curve to unexpected changes in the federal funds target.

The model introduced in the fifth chapter of this thesis addresses these different issues. This innovative model builds on an extensive use of regime-switching features. In this model, the short end of the yield curve is explicitly influenced by the central-bank policy rate, the latter being a multiple of 25 basis points. Occurrences of target moves depend on a hidden monetary-policy regime and on the level of the current target rate. An appealing feature of this model is that it is consistent with positive policy rates, making it appropriate to deal with the zero-lower-bound restriction. To illustrate the flexibility and tractability of this model, it is estimated on daily euro-area data. The results suggest that the dynamics of the term structure of riskfree (OIS) rates is closely related to monetary-policy expectations. The estimation also reveals the existence of sizable risk premia at the short-end of the yield curve, which suggests that the widespread market practice that consists in using money-market forwards to proxy market forecasts of future target moves is biased.

1.8. Decomposing the term structure of spreads

There is compelling evidence that yields and spreads are affected by liquidity concerns[14]. In particular, using euro-area data, Beber, Brandt and Kavajecz (2009) provide evidence of a nontrivial role in the dynamics of sovereign bond spreads, especially for low credit risk countries and during times of heightened market uncertainty.[15] In recent studies, some authors develop affine term-structure models to breakdown several kinds of spreads into different components, including liquidity-related ones. These approaches are based on the assumption that there exists commonality amongst the liquidity components of asset prices and bond in particular.[16] For instance, Liu, Longstaff and Mandell (2006) use a five-factor affine framework to jointly model Treasury, repo and swap term structures. One of their factors is related to the pricing of the Treasury-securities liquidity and another factor reflects default risk.[17] Feldhütter and Lando (2008) develop a six-factor model for Treasury bonds, corporate bonds and swap rates that makes it possible to decompose swap spreads into three components: a convenience yield from hold-

[14]See, e.g., Longstaff (2004), Landschoot (2004), Chen, Lesmond and Wei (2007), Covitz and Downing (2007) or Acharya and Pedersen (2005).

[15]Such a behaviour is captured in a theoretical framework by Vayanos (2004).

[16]See e.g. Chordia and Subrahmanyam (2000), Brockman, Chung and Pérignon (2009), Fontaine and Garcia (2012), Feldhütter and Lando, (2008), Longstaff, Mithal and Neis (2005), Liu, Longstaff and Mandell (2006) or Dick-Nielsen, Feldhütter and Lando (2011).

[17]As noted by Feldhütter and Lando (2008), the identification of the liquidity and credit risk factors in Liu et al. relies critically on the use of the 3-month general-collateral repo rate (GC repo) as a short-term risk-free rate and of the 3-month LIBOR as a credit-risky rate. Liu et al. define the liquidity factor as the spread between the 3-month GC repo and the 3-month Treasury-bill yield (and is therefore observable). In each yield, their liquidity component is the share of the yield that is explained by this factor.

ing Treasuries, a credit-element associated with the underlying LIBOR rate, and a factor specific to the swap market. They find that the convenience yield is by far the largest component of spreads. Longstaff, Mithal and Neis (2005) use information in credit default swaps –in addition to bond prices– to obtain measures of the nondefault components in corporate spreads. They find that the nondefault component is time-varying and strongly related to measures of bond-specific illiquidity as well as to macroeconomic measures of bond-market liquidity.

In recent studies, some authors rely on the affine-term structure framework to model yield curves associated not only with different obligors but also with different fixed-income instruments (e.g. bonds, repos, swaps). Further, the authors exploit this modelling to breakdown credit spreads or swap spreads into different components. Specifically, Liu, Longstaff and Mandell (2006) use a five-factor affine framework to jointly model Treasury, repo and swap term structures. One of their factors is related to the pricing of the Treasury-securities liquidity and another factor reflects default risk.[18] Feldhütter and Lando (2009) develop a six-factor model for Treasury bonds, corporate bonds and swap rates that makes it possible to decompose swap spreads into three components: a convenience yield from holding Treasuries, a credit-element associated with the underlying LIBOR rate, and a factor specific to the swap market. They find that the convenience yield is by far the largest component of spreads. Longstaff, Mithal and Neis (2005) use information in credit default swaps –in addition to bond prices– to obtain measures of the nondefault components in corporate spreads. They find that the nondefault component is time-varying and strongly related to measures of bond-specific illiquidity as well as to macroeconomic measures of bond-market liquidity.

Chapter 3 and 4 present no-arbitrage affine term-structure model (ATSM) of the dynamics of euro-area sovereign yields and spreads, respectively. In addition to the term structures of sovereign entities, the dataset includes yields associated with

[18]As noted by Feldhütter and Lando (2009), the identification of the liquidity and credit risk factors in Liu et al. relies critically on the use of the 3-month general-collateral repo rate (GC repo) as a short-term risk-free rate and of the 3-month LIBOR as a credit-risky rate. Liu et al. define the liquidity factor as the spread between the 3-month GC repo and the 3-month Treasury-bill yield (and is therefore observable). In each yield, their liquidity component is the share of the yield that is explained by this factor.

KfW (*Kreditanstalt für Wiederaufbau*), a German agency. A liquidity-related pricing factor is then identified by exploiting the term structure of the the KfW-*Bund* spreads. Indeed, the bonds issued by KfW, guaranteed by the Federal Republic of Germany, benefit from the same credit quality than their sovereign counterparts –the *Bunds*– but are less liquid. Therefore, the KfW-*Bund* spread should be essentially liquidity-driven.[19] It is demonstrated that liquidity-related factors significantly contribute to the dynamics of intra-euro spreads, supporting recent findings by Favero et al. (2010) or Manganelli and Wolswijk (2009).

[19] See Schwarz (2009).

2. Default, liquidity and crises: An econometric framework[1]

Abstract: In this Chapter, we present a general discrete-time affine framework aimed at jointly modelling yield curves associated with different debtors. The underlying fixed-income securities may differ in terms of credit quality and/or in terms of liquidity. The risk factors follow conditionally Gaussian processes, with drifts and variance-covariance matrices that are subject to regime shifts described by a Markov chain with (historical) non-homogenous transition probabilities. Importantly, bond prices are given by quasi-explicit formulas, ensuring the tractability of the framework. This tractability is illustrated by the estimation of a term-structure model of the spreads between U.S. BBB-rated corporate bonds and Treasuries. Alternative applications are proposed, including a sector-contagion model as well as the explicit modelling of credit-rating transitions.

[1]This Chapter is based on an article featuring the same title, published in the *Journal of Financial Econometrics* and co-authored with Alain Monfort. We are grateful to Christian Gourieroux, Damiano Brigo, Olesya Grishchenko, Wolfgang Lemke, Andrew Siegel, Simon Dubecq and Hans Dewachter for helpful discussions and comments on previous versions of this paper. We are also grateful to participants at the Banque de France internal seminar, at the C.R.E.D.I.T. conference (Venice) 2010, at CREST seminar 2010, at the Paris finance international meeting 2010, at CORE Econometrics Seminar 2011, at SoFiE annual meeting (Chicago) 2011, at Erasmus University (Rotterdam) 2011 and at Financial Risk Forum (Paris) 2011. We thank Béatrice Saes-Escorbiac and Aurélie Touchais for excellent research assistance. Any remaining errors are ours. The views expressed in this Chapter are ours and do not necessarily reflect the views of the Banque de France.

Résumé

Ce chapitre présente un cadre économétrique général visant à modéliser de manière jointe les fluctuations de courbes de taux associées à différents émetteurs obligataires.

Les titres sous-jacents à ces courbes peuvent différer en termes de qualité de crédit de l'émetteur et/ou en termes de liquidité.

- Les émetteurs des obligations peuvent faire défaut (risque de crédit), impliquant une perte pour les détenteurs des obligations qu'ils ont émises. Le fait que la probabilité de défaut d'un émetteur peut varier dans le temps implique que la valorisation des obligations varie également.
- Le cadre présenté dans ce chapitre permet également de modéliser l'influence des différences de liquidité –cette dernière étant définie par la facilité avec laquelle il est possible de trouver une contrepartie pour acheter/vendre un titre– sur les prix obligataires.

Les risques de crédit et de liquidité sont respectivement modélisés par le biais d'intensités de défaut et d'illiquidité. Dans ce modèle de forme réduite, les intensités et le taux court sans risque dépendent de trois types de variables: des facteurs «macroéconomiques», des facteurs «microéconomiques» et une variable de régimes. Les facteurs dits «macroéconomiques» peuvent affecter les intensités (de défaut et d'illiquidité) caractérisant toutes les entités de l'économie considérée; les facteurs «microéconomiques» sont spécifiques aux différentes entités. Tous ces facteurs suivent des processus auto-regressifs multi-variés et sont affectés par des chocs gaussiens dont les covariances dépendent du régime qui prévaut au moment du choc. Les tendances (*drifts*) de ces processus dépendent également des régimes. La dynamique des régimes est définie par une chaîne de Markov dont les probabilités de transitions peuvent être non-homogènes sous la mesure historique (elles peuvent dépendre des valeurs retardées des facteurs). Conditionnellement aux facteurs et aux régimes, les défaut des différentes entités de l'économie sont indépendants.

L'introduction d'un facteur d'escompte stochastique permet la valorisation d'actifs

et notamment d'obligations dans cette économie. Le facteur d'escompte stochastique dépend des facteurs et des régimes. Nous en déduisons la dynamique risque-neutre du vecteur regroupant les facteurs et les régimes et montrons que celle-ci est composée auto-régressive (voir Darolles, Gourieroux et Jasiak, 2006 pour une étude de ces processus «Car») lorsque les probabilités de transition des régimes sont homogènes –i.e. ne dépendent pas du temps– sous la mesure risque-neutre. Dans ce cas, la valorisation d'obligations émises par des entités risquées (pouvant faire défaut) est obtenue à partir de formules quasi-explicites reposant sur un algorithme récursif. Ce résultat découle des propriétés des processus composés auto-régressifs, et plus précisément du calcul de leurs transformée de Laplace multi-horizon (qui est exponentielle affine). Ainsi, bien que riche, ce modèle bénéficie de l'existence de méthodes simples pour valoriser les obligations de maturité longue émise par les entités risquées.

L'estimation de ce type de modèle est ensuite étudiée. La méthode d'estimation dépend du caractère observable ou non des différents facteurs et régimes. Les différents cas (facteurs observables ou non, régimes observables ou non, soient quatre possibilités) sont considérés tour à tour. En particulier, dans le cas où tout ou partie des facteurs et des régimes sont inobservables, on présente une méthode de calcul de la vraisemblance reposant à la fois sur le filtrage de Kitagawa-Hamilton et sur les techniques d'inversion à la Chen et Scott (1993).

La simplicité de ces calculs est illustrée en estimant une des nombreuses spécifications envisageables sur données américaines. Dans cet exemple, on modélise la dynamique de la structure par terme des écarts de taux d'intérêt entre (a) les obligations émises par des entreprises américaines présentant la notation de crédit BBB d'une part et (b) les obligations du Trésor américain d'autre part. Les données d'estimation couvre la période allant de mars 1995 à juillet 2011, à fréquence hebdomadaire. Cet exemple montre notamment que l'utilisation des régimes permet de reproduire de manière satisfaisante les moments d'ordre trois et quatre de la distribution des écarts de taux sur la période considérée (contrairement aux modèles dans lesquels les facteurs suivent des processus gaussiens).

Ce chapitre propose également une utilisation innovante des changements de régime pour modéliser des phénomènes de contagion. Dans ce modèle, on considère N secteurs (qui peuvent correspondre à différentes régions ou encore différents secteurs industriels). Chaque secteur est constitué de différentes entités (pouvant faire défaut) qui émettent des obligations. A chaque période, chacun des secteurs peut être «infecté» ou non. Le modèle comprend 2^N régimes, chaque régime décrivant l'état de chacun des secteurs (infecté ou non). Les probabilités de défaut des entités d'un secteur dépendent de l'état de leur secteur respectif. Dans ce cadre, il est aisé de faire dépendre la probabilité d'infection d'un secteur des états des autres secteurs, ce qui permet de modéliser des effets de contagion entre secteurs. Par exemple, on peut faire dépendre la probabilité d'infection d'un secteur du nombre de secteurs qui sont infectés à la date précédente. Les formules de valorisation d'obligations sont toujours valables dans ce cadre.

Ce chapitre présente également une extension du cadre décrit précédemment visant à permettre la modélisation des transitions de notations de crédit et l'influence de celles-ci sur les courbes de taux. Dans cette extension, on augmente le nombre d'états de crédit, de deux (défaut ou non défaut) à K classes de qualité de crédit: la $K^{ième}$ correspond à l'état de défaut et la première à la meilleure qualité de crédit (par exemple AAA dans le système de notations de Standard&Poor's). Les probabilités de transition d'une notation à une autre varient dans le temps ; la matrice de probabilités de transition dépendant notamment des régimes. On montre que des formules quasi-explicites pour les prix obligataires sont encore disponibles dans ce cadre. Un exemple numérique illustre le fonctionnement de ce type de modèle.

2.1. Introduction

There is strong evidence of regime switching in the dynamics of interest rates (see, e.g., Hamilton, 1988 or Cai, 1994). Regime shifts have been successfully introduced in term-structure models of risk-free interest rates by, amongst others, Bansal and Zhou (2002), Monfort and Pegoraro (2007), Dai, Singleton and Yang (2007) or Ang Bekaert and Wei (2008). Whereas these contributions put forward the importance of modelling regime switching in yield-curve models, a few has been done to integrate such a feature in term-structure models of defaultable bonds. However, empirical studies point to the existence of different regimes in the default risk valuation (see, e.g., Davies, 2004 or Alexander and Kaeck, 2008). From a theoretical point of view, Hackbarth, Miao and Morellec (2006) provide a theoretical model to explain the dependence of credit spreads on business-cycle regimes. In the same vein, Bhamra, Kuehn and Strebulaev (2007) and David (2008) adopt structural models including regime switching to assess the influence of different states of the economic cycles on the credit-risk premia.

The main aim of the present Chapter is to propose a general multi-issuer dynamic framework including switching regimes, both in the historical and the risk-neutral worlds. Particular attention is paid to the tractability of the model and its estimation. Tractability is notably obtained through an extensive use of Car's –Compound autoregressive processes– properties (see, e.g. Darolles, Gourieroux and Jasiak, 2006), which leads to quasi-explicit formulas for riskless and defaultable bond prices. Both historical and risk-neutral dynamics are explicitly modelled, which is helpful for choosing appropriate specifications under the historical measure, for dealing simultaneously with pricing and forecasting, for Value-at-Risk calculations or for Sharpe-ratio computations.[2]

In our modelling of defaults, correlations between default events arise through dependence on some common underlying stochastic factors –also called "risk factors"–

[2]Regarding the latter point, see Duffee (2010). The fact that our framework is defined in discrete time makes it easier (compared with continuous-time models) to properly specify the dynamics of the observable risk factors under the historical probability measure (see e.g. Duffie and Singleton, 1999) or Gourieroux, Monfort and Polimenis, 2006).

which influence the default probabilities of every single loans. Some of the factors may be unobserved, so, in this sense, our model accommodates frailty. This feature is advocated by recent papers suggesting that including only observable covariates in default-intensity specifications results in poorly-estimated conditional probabilities of default (see e.g. Lando and Nielsen, 2008 or Duffie et al., 2009).

In our approach, regime shifts may affect pricing through several channels: (i) regimes affect the historical and risk-neutral dynamics of the risk factors, (ii) regimes appear in the stochastic discount factor (s.d.f.) –which implies that regime-transition risk is priced– and (iii) regimes appear in the default-intensity functions. This results in a large degree of flexibility in the model specifications, which is illustrated by several numerical examples in the Chapter. In particular, since default intensities can be affected by the regime variable, our model is appropriate to capture default clustering.

In order to show some of the framework advantages and to illustrate its tractability, we estimate a simple model of the term structure of the spreads between U.S. BBB-rated corporate bonds and Treasuries. In particular, a comparison of this model with purely Gaussian model highlights the potential of regime switching to capture salient features of the spread distributions.

Beyond the enrichment of the specifications of the risk factors and those of the default intensities by introducing nonlinearities, the regime-switching feature can be further exploited to handle specific forms of contagions. Contagion effects, whose consequences are cascades of subsequent spread changes, is explained by the existence of close ties between firms (see, e.g., Jarrow and Yu, 2001, Davies and Lo, 2001 or Giesecke, 2004). Contagion takes place when the default probability of any debtor can be affected by the default event of another one. Given that our baseline model relies on the conditional-independence assumption –which states that, conditional on the underlying factors and regimes, the default events of the firms in a portfolio are independent– direct contagion effects is not captured. Nevertheless, we can model specific contagion effects in two distinct ways. First, our framework can accommodate the specific contagion case where one entity –or, for the sake of tractability, a small number of them– affects the default probability of the others: it suffices to make one of the regimes corresponds to the default state of this entity. Second, the regime-switching feature can be exploited in order to capture "sector-contagion" phenomena. The sectors can represent different industries or different geographical areas. Each sector can be "infected" or not, and when a sector gets infected, the default intensities of its constituents (the debtors) shift upwards. In this context, sector contagion stems from the parameterization of the matrix of regime-transition probabilities. For instance, it is easy to model infection probabilities that depend positively on the number of sectors already infected.

Our baseline model considers only one credit event: the default of the debtor. However, credit events include more generally the changes in credit ratings like those attributed by agencies like Moody's, Standard & Poor's or Fitch.[3] It turns out that our framework can be adapted to accommodate time-varying credit-rating migration probabilities along the lines of Lando (1998) while keeping quasi-explicit bond-pricing formulas.[4]

The remainder of the Chapter is organised as follows. Sections 2.2 and 2.3 respectively present the historical and risk-neutral dynamics of the variables. Section 2.4 gives the bond-pricing formulas with zero or non-zero recovery rates. Section 2.5 deals with internal-consistency restrictions that arise when yields or asset returns are included amongst the risk factors. In Section 2.6, we discuss the estimation of such models, which is illustrated by an estimation of a term-structure model of spreads between U.S. BBB-rated corporate bonds and Treasuries. Section 2.7 shows how the model accommodates the pricing of liquidity. Section 2.8 investigates possible extensions of the framework: Subsection 2.8.1 deals with multi-lag dynamics of the risk factors; Subsection 2.8.2 deals with the specific case where one of the Markov chains coincides with the default state of a given entity;

[3] Several of the main credit models currently being used in the industry draw on the credit-migration approach. For presentation, comparison and evaluation of these models, see e.g. Gordy (2000), Cantor (2004) or Gagliardini and Gourieroux (2001).

[4] Other examples of term-structure models allowing for time-varying rating-migration probabilities include Bielecki and Rutkowski (2000) and Wei (2003).

Subsection 2.8.3 presents a sector-contagion model and Subsection 2.8.4 shows how to introduce rating-migration modelling in the framework. Section 2.9 concludes.

2.2. Information and historical dynamics

2.2.1. Information

The new information of the investors at date t is $w_t' = (z_t', y_t', x_t', d_t')$ where z_t is a regime variable that can take a finite number J of values, y_t is a multivariate macroeconomic factor, $x_t' = (x_{1,t}', \ldots, x_{N,t}')$ is a set of specific multivariate factors $x_{n,t}$ associated with debtor n, and $d_t' = (d_{1,t}, \ldots, d_{N,t})$ is a set of binary variables indicating the default ($d_{n,t} = 1$) or the non-default ($d_{n,t} = 0$) state of entity n. The whole information set of the investors at date t is $\underline{w}_t' = (w_1', \ldots, w_t')$. At this stage, we do not make any assumption about the observability of these variables by the econometrician (this is done below in Section 2.6). The regimes influence bond pricing through different channels (they will appear in the dynamics of the risk factors y_t and $x_{n,t}$'s, in the stochastic discount factor and in the default-intensity functions). In the baseline framework, the regimes are viewed as transitory: none of these regimes is absorbing but this restriction is relaxed in a specific case presented in Subsection 2.8.2.

2.2.2. Historical dynamics

It is convenient to make the regime variable z_t valued in $\{e_1, \ldots, e_J\}$, the set of column vectors of the identity matrix I_J.[5] The conditional distribution of z_t given $\underline{w}_{t-1}$ is characterised by the probabilities:

$$p\left(z_t \mid \underline{w}_{t-1}\right) = \pi\left(z_t \mid z_{t-1}, y_{t-1}\right). \tag{2.1}$$

[5]Indeed, this implies that any function of the regimes taking the value f_j in the j^{th} regime, say, is the linear function of z_t: $f' z_t$ with $f' = (f_1 \ldots f_J)$.

The probability $\pi(e_j \mid e_i, y_{t-1})$ that z_t shifts from regime i to regime j between period $t-1$ and t, conditional on y_{t-1}, is also denoted by $\pi_{ij,t-1}$. These specifications allow for state-dependent transition probabilities, as in Ang and Bekaert (2002) or Dai, Singleton and Yang (2007).

The conditional distribution of y_t given z_t and $\underline{w}_{t-1}$ is Gaussian and given by:[6]

$$y_t = \mu\left(z_t, z_{t-1}\right) + \Phi y_{t-1} + \Omega\left(z_t, z_{t-1}\right)\varepsilon_t \tag{2.2}$$

where the ε_t are independently and identically $N(0, I)$ distributed. Specifications (2.1) and (2.2) imply that, in the universe (z_t, y_t), z_t Granger-causes y_t, y_t causes z_t and there is instantaneous causality between z_t and y_t. Moreover, in the universe $w_t = (z_t, y_t, x_t, d_t)$, (x_t, d_t) does not cause (z_t, y_t). As noted by Ang, Bekaert and Wei (2008), instantaneous causality between z_t and y_t implies that the variances of the factors y_t, conditional on $\underline{w}_{t-1}$, embed a jump term reflecting the difference in drifts μ across regimes. Such a feature, that allows for conditional heteroskedasticity, is absent from the Dai, Singleton and Yang (2007) setting. It should be noted that our framework nests the case where there is no instantaneous causality between z_t and y_t in the historical dynamics.[7] Contrary to Bansal and Zhou (2002), matrix Φ is not regime-dependent: this is for the sake of tractability when it comes to bond pricing.[8]

The $x_{n,t}$'s, $n = 1, \ldots, N$ are assumed to be independent conditionally to $(z_t, y_t, \underline{w}_{t-1})$. The conditional distribution of $x_{n,t}$ is Gaussian and defined by:

$$x_{n,t} = q_{1n}\left(z_t, z_{t-1}\right) + Q_{2n} y_t + Q_{3n} y_{t-1} + Q_{4n} x_{n,t-1} + Q_{5n}\left(z_t, z_{t-1}\right)\eta_{n,t} \tag{2.3}$$

where the shocks $\eta_{n,t}$ are $IIN(0, I)$. Specifications (2.1), (2.2) and (2.3) imply that, in the universe $(z_t, y_t, x_{n,t})$, (z_t, y_t) causes $x_{n,t}$, $x_{n,t}$ does not cause (z_t, y_t) and there is instantaneous causality between (z_t, y_t) and $x_{n,t}$. Moreover, denoting by

[6] These specifications allow for various and rich dynamics of the risk factors y_t such as, notably, threshold auto-regressive dynamics (TAR) or self-exciting TAR (SETAR).

[7] Formally, this corresponds to $\mu\left(z_t, z_{t-1}\right) = \mu\left(z_{t-1}\right)$ and $\Omega\left(z_t, z_{t-1}\right) = \Omega\left(z_{t-1}\right)$.

[8] Indeed, the model of Bansal and Zhou (2002) [21] does not admit a closed-form exponential affine solution (they proceed by linearizing the discrete-time Euler equations and by solving the resulting linear relations for prices).

$\overline{x}_{n,t}$ the vector x_t excluding $x_{n,t}$, $(\overline{x}_{n,t}, d_t)$ does not cause $(z_t, y_t, x_{n,t})$ in the whole universe w_t.

Finally, the $d_{n,t}$'s, $n = 1, \ldots, N$, are independent conditionally to $(z_t, y_t, x_t, \underline{w}_{t-1})$ and the conditional distribution of $d_{n,t}$ is such that:

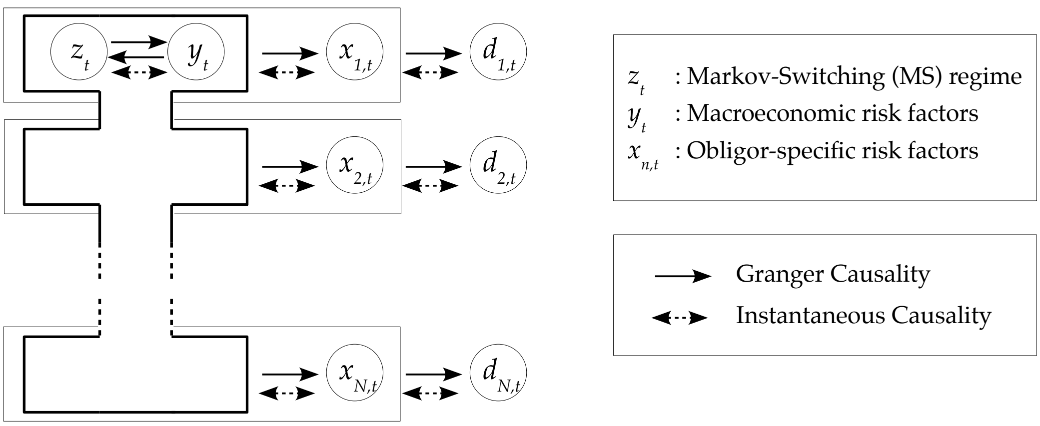
$$p\left(d_{n,t} = 1 \mid z_t, y_t, x_t, \underline{w}_{t-1}\right) = \begin{cases} 1 & \text{if } d_{n,t-1} = 1, \\ 1 - \exp\left(-\lambda_{n,t}\right) & \text{if } d_{n,t-1} = 0, \end{cases} \tag{2.4}$$

with $\lambda_{n,t} = \alpha_n^{'} z_t + \beta_n^{'} y_t + \gamma_n^{'} x_{n,t}$.

In other words, state 1 of $d_{n,t}$ is an absorbing state and $\exp\left(-\lambda_{n,t}\right)$ is the survival probability. Since the default probability $1 - \exp\left(-\lambda_{n,t}\right)$ is close to $\lambda_{n,t}$ if $\lambda_{n,t}$ is small, $\lambda_{n,t}$ is called the default intensity. The default intensity is expected to be positive, which is not necessarily the case since the ε_t's are Gaussian. However, the parameterization of the model may make this extremely unfrequent.

In the universe $(z_t, y_t, x_{n,t}, d_{n,t})$, $(z_t, y_t, x_{n,t})$ causes $d_{n,t}$ whereas $d_{n,t}$ does not causes $(z_t, y_t, x_{n,t})$ and there is instantaneous causality. In the whole universe w_t, $(\overline{x}_{n,t}, \overline{d}_{n,t})$ does not cause $(z_t, y_t, x_{n,t}, d_{n,t})$. The causality scheme is summarised in Figure 2.1.

Figure 2.1.: Causality scheme



In the proposition below, we consider the historical conditional Laplace transform of the distribution of (z_t, y_t) given $\underline{w}_{t-1}$, that is $\varphi_{t-1}(u, v) = E_{t-1}\left[\exp\left(u' z_t + v' y_t\right)\right]$.

Proposition 1. *The historical conditional Laplace transform of* (z_t, y_t) *given* $\underline{w}_{t-1}$

is:

$$\varphi_{t-1}(u,v) = \exp\left(v'\Phi y_{t-1} + [l_1, \ldots, l_J]\, z_{t-1}\right), \tag{2.5}$$

where $l_i = \log \sum_{j=1}^{J} \pi_{ij,t-1} \exp\left\{u_j + v'\mu(e_j, e_i) + \frac{1}{2} v'\Omega(e_j, e_i)\,\Omega'(e_j, e_i)\, v\right\}$.

Proof. See Appendix 2.A.1. □

This Laplace transform is not, in general, exponential affine in (z_{t-1}, y_{t-1}), since y_{t-1} appears in the $\pi_{ij,t}$'s. This is the case if the $\pi_{ij,t}$'s do not depend on y_{t-1} and then, by definition, (z_t, y_t) is Car(1).[9] As detailed in Darolles, Gourieroux and Jasiak (2006) or Bertholon, Monfort and Pegoraro (2008), Car processes are omnipresent in asset-pricing literature due to the fact that, when the dynamics is Car, there exist recursive algorithms to compute multi-horizon Laplace transforms, which is key for term-structure modelling. As will be illustrated below, we will rely heavily on the properties of the Car processes in the risk-neutral world to ensure tractability in bond pricing (section 2.4), but, as mentioned above the historical dynamics will not be Car in general.

2.3. Stochastic discount factor and risk-neutral dynamics

2.3.1. Stochastic discount factor

We assume that the riskless short-term rate between $t-1$ and t is:

$$r_{t-1} = a_1^{'} z_{t-1} + b_1^{'} y_{t-1}. \tag{2.6}$$

This includes the case where r_{t-1} is the first component of y_{t-1} ($a_1 = 0$ and $b_1 = e_1$). Then, we define the stochastic discount factor $M_{t-1,t}$ between $t-1$ and

[9] Recall that a random process Λ_t is Car(1) if its conditional Laplace transform (given information available up to date $t-1$) is exponential affine in Λ_{t-1}.

t by:

$$\begin{aligned} M_{t-1,t} &= \exp\Big[-a_1^{'} z_{t-1} - b_1^{'} y_{t-1} - \frac{1}{2}\nu^{'}\left(z_t, z_{t-1}, y_{t-1}\right)\nu\left(z_t, z_{t-1}, y_{t-1}\right) + \\ &\quad + \nu^{'}\left(z_t, z_{t-1}, y_{t-1}\right)\varepsilon_t + \delta^{'}\left(z_{t-1}, y_{t-1}\right) z_t\Big], \end{aligned} \tag{2.7}$$

The ν and δ vectors can be seen respectively as the prices of risk associated with the (standardised) innovations of the process y_t and the regimes z_t. Regarding the latter, the fact that we must have $E_{t-1}(M_{t-1,t}) = \exp(-a_1^{'} z_{t-1} - b_1^{'} y_{t-1})$ implies that $E_{t-1} \exp\left(\delta^{'}\left(z_{t-1}, y_{t-1}\right) z_t\right) = 1$, which is equivalent to:

$$\sum_{j=1}^{J} \pi_{ij,t-1} \exp\left[\delta_j\left(e_i, y_{t-1}\right)\right] = 1, \; \forall i, y_{t-1}, \tag{2.8}$$

where δ_j is the j^{th} component of δ.

In our framework, the variables $(x_{n,t}, d_{n,t})$, specific to entity n, do not appear in the stochastic discount factor, which reflects the fact that these entities have no impact at the macroeconomic level (in Subsection 2.8.2, we discuss the case where one entity has a "systemic" status).

2.3.2. Risk-neutral dynamics

2.3.2.1. The conditional risk-neutral distribution of (z_t, y_t) given $\underline{w}_{t-1}$

Let us now consider the conditional risk-neutral Laplace transform of (z_t, y_t) given $\underline{w}_{t-1}$, $\varphi_{t-1}^{\mathbb{Q}}(u, v) := E_{t-1}^{\mathbb{Q}}\left(\exp\left[u' z_t + v' y_t\right]\right)$, and let us introduce the simplified notations:

$$\begin{aligned} \mu_t &= \mu\left(z_t, z_{t-1}\right) \\ \Omega_t &= \Omega\left(z_t, z_{t-1}\right), \; \Sigma(z_t, z_{t-1}) = \Omega_t \Omega_t' = \Sigma_t \\ \nu_t &= \nu\left(z_t, z_{t-1}, y_{t-1}\right) \\ \delta_{t-1} &= \delta\left(z_{t-1}, y_{t-1}\right). \end{aligned}$$

Proposition 2. *The conditional risk-neutral Laplace transform of* (z_t, y_t) *given* $\underline{w}_{t-1}$ *is:*

$$\varphi_{t-1}^{\mathbb{Q}}(u,v) = \exp\left[v'\Phi y_{t-1} + \begin{pmatrix} A_{1,t-1}(u,v) & \ldots & A_{J,t-1}(u,v) \end{pmatrix} z_{t-1}\right], \quad (2.9)$$

where

$$\begin{aligned} A_{i,t-1}(u,v) \quad = \quad & \log(\sum_{j=1}^{J} \pi_{ij,t-1} \exp\left\{v'\Omega\left(e_j, e_i\right)\nu\left(e_j, e_i, y_{t-1}\right) + \frac{1}{2}v'\Sigma\left(e_j, e_i\right)v + \right. \\ & v'\mu\left(e_j, e_i\right) + u_j + \delta_j\left(e_i, y_{t-1}\right)\}) \,. \end{aligned}$$

Proof. See Appendix 2.A.2. □

As mentioned above, Car processes are particularly convenient because the computation of their multi-horizon Laplace transforms is straightforward, as will be shown below. This motivates the next Corollary.

Corollary 1. *The risk-neutral dynamics of* (z_t, y_t) *is Car(1) if the risk sensitivities* δ *and* ν*, appearing in the s.d.f., satisfies the constraints (for any* i*,* j *and* t*):*

$$\begin{cases} \delta_j\left(e_i, y_{t-1}\right) & = \log\left[\pi_{ij}^{*}/\pi\left(e_j \mid e_i, y_{t-1}\right)\right] \\ \nu\left(e_j, e_i, y_{t-1}\right) & = \Omega\left(e_j, e_i\right)^{-1}\left[\Phi^{*} y_{t-1} + \mu^{*}\left(e_j, e_i\right)\right], \end{cases} \quad (2.10)$$

for any transition matrix $\pi_{ij}^{*} = \pi^{*}(e_j \mid e_i)$*, any matrix* Φ^{*} *and any function* μ^{*}*.*

It is important to note that these constraints still allow for a large number of degrees of freedom in the specification of the s.d.f., since the transition matrix $\{\pi_{ij}^{*}\}$, the matrix Φ^{*} and the vectors $\mu^{*}\left(e_j, e_i\right)$ are arbitrary. If the constraints (2.10) are satisfied, the risk-neutral conditional Laplace transform becomes:

$$\varphi_{t-1}^{\mathbb{Q}}(u,v) = \exp\left[v'\left(\Phi + \Phi^{*}\right) y_{t-1} + \begin{pmatrix} A_1^{*}(u,v) & \ldots & A_J^{*}(u,v) \end{pmatrix} z_{t-1}\right], \quad (2.11)$$

where, for any i,

$$A_i^*(u,v) = \log\left(\sum_{j=1}^{J}\pi_{ij}^*\exp\left\{u_j + v'\left[\mu\left(e_j,e_i\right)+\mu^*\left(e_j,e_i\right)\right]+\frac{1}{2}v'\Sigma\left(e_j,e_i\right)v\right\}\right).$$

Comparing with equation (2.5), we deduce that the risk-neutral dynamics of (z_t, y_t) is then defined by:

$$y_t = \mu\left(z_t,z_{t-1}\right)+\mu^*\left(z_t,z_{t-1}\right)+\left(\Phi+\Phi^*\right)y_{t-1}+\Omega\left(z_t,z_{t-1}\right)\varepsilon_t^*, \tag{2.12}$$

where, under $\mathbb{Q}$, z_t is an homogenous Markov chain defined by the transition matrix $\{\pi_{ij}^*\}$, and ε_t^* –defined by $\varepsilon_t^* = \varepsilon_t - \Omega^{-1}\left(z_t,z_{t-1}\right)\left[\mu^*\left(z_t,z_{t-1}\right)+\Phi^* y_{t-1}\right]$– is $IIN^{\mathbb{Q}}(0,I)$. Note that$\tilde{\mu} = \mu + \mu^*$ and $\tilde{\Phi} = \Phi + \Phi^*$ are arbitrary and that the Ω function is the same in the historical and risk-neutral worlds.

2.3.2.2. The risk-neutral distribution of (x_t, d_t) given $(z_t, y_t, \underline{w}_{t-1})$

Lemma 1. *Let us consider a partition of* $w_t = \left(w_{1,t}', w_{2,t}'\right)'$. *If* $M_{t-1,t}$ *is a function of* $(w_{1,t}, \underline{w}_{t-1})$, *the risk-neutral probability density function, or p.d.f. , of* $w_{1,t}$ *given* $\underline{w}_{t-1}$ *is:*

$$f_{\mathbb{Q}}\left(w_{1,t} \mid \underline{w}_{t-1}\right) = f\left(w_{1,t} \mid \underline{w}_{t-1}\right) M_{t-1,t}\exp\left(r_{t-1}\right)$$

(where f *is the historical conditional p.d.f. of* $w_{1,t}$ *given* $\underline{w}_{t-1}$*) and the conditional risk-neutral distribution of* $w_{2,t}$ *given* $(w_{1,t}, \underline{w}_{t-1})$ *is the same as the corresponding historical distribution.*

Proof. See Appendix 2.A.3. □

Since $M_{t-1,t}$ is a function of (z_t, y_t) but not of (x_t, d_t), the previous lemma shows that the risk-neutral distribution of (x_t, d_t) given $(z_t, y_t, \underline{w}_{t-1})$ is the same as the historical one and it is given by equations (2.3) and (2.4). In particular, the functional forms of the default intensities $\lambda_{n,t}$ are the same as in the historical world. Of course, since the dynamics of (z_t, y_t) are different in the two worlds, the same is true for the $x_{n,t}$'s and the $\lambda_{n,t}$'s.

In addition, it can be shown that $(z_t, y_t, x_{n,t})$ is Car(1) under the risk-neutral measure (see Appendix 2.A.4). However, it is not the case for $(z_t, y_t, x_{n,t}, d_{n,t})$.

It is also clear that the causality structure of the risk-neutral dynamics is similar to the historical one, the only difference being the non-causality from y_t to z_t implied by the homogeneity of the matrix $\{\pi^*_{ij}\}$.

2.4. Pricing

2.4.1. Defaultable bond pricing with zero recovery rate

The price at t of a riskless zero-coupon bond with residual maturity h is given by:

$$B(t,h) = E_t^{\mathbb{Q}}\left[\exp\left(-r_t - \ldots - r_{t+h-1}\right)\right], \tag{2.13}$$

where $r_{t+i-1} = a_1' z_{t+i-1} + b_1' y_{t+i-1}$, $i = 1, \ldots, h$. The following proposition shows that, thanks to the risk-neutral causality structure of our model, there exists an analogous formula for the price of defaultable bonds with zero recovery rates. Naturally, the case of risk-free bond pricing is nested within the more general defaultable-bond pricing case (with a zero default intensity).

Proposition 3. *The price of a zero-recovery-rate zero-coupon defaultable bond issued by debtor n is given by:*

$$B_n^D(t,h) = E_t^{\mathbb{Q}}\left[\exp\left(-(r_t + \lambda_{n,t+1}) - \ldots - (r_{t+h-1} + \lambda_{n,t+h})\right)\right], \tag{2.14}$$

which is exponential linear in $(z_t, y_t, x_{n,t})$:

$$B_n^D(t,h) = \exp\left(-c_{n,h}' z_t - f_{n,h}' y_t - g_{n,h}' x_{n,t}\right) \tag{2.15}$$

The defaultable yields are therefore linear in $(z_t, y_t, x_{n,t})$:

$$R_n^D(t,h) = \frac{1}{h}\left(c'_{n,h} z_t + f'_{n,h} y_t + g'_{n,h} x_{n,t}\right), \tag{2.16}$$

In Equations (2.15) and (2.16), $(c'_{n,h}, f'_{n,h}, g'_{n,h})$ is computed recursively by:

$$\left(c'_{n,h}, f'_{n,h}, g'_{n,h}\right) = (a'_1, b'_1, 0) - \overline{a}\left(\omega_{H-h+1} - \left(c'_{n,h-1} - a'_1, f'_{n,h-1} - b'_1, -g'_{n,h-1}\right)'\right)$$

where

- *the sequence ω_h, $h = 1, \ldots, H$ is defined by $\omega_H = (-\alpha'_n, -\beta'_n, -\gamma'_n)$ and $\omega_h = (-\alpha'_n - a'_1, -\beta'_n - b'_1, -\gamma'_n)$ for $h = 1, \ldots, H-1$, with $c_{n,0} = a_1$, $f_{n,0} = b_1$, $g_{n,0} = 0$,*
- *The function $\bar{a}$ is defined by $\overline{a}(u,v,w) = [(\tilde{A}_1, \ldots, \tilde{A}_J), (v' + w'Q_{2n})(\Phi + \Phi^*) + w'Q_{3n}, w'Q_{4n}]$, where*

$$\begin{aligned}
\tilde{A}_i(u,v,w) \quad = \quad & \log(\sum_{j=1}^{J} \pi^*_{ij} \exp\{u_j + (v' + w'Q_{2n})\left[\mu\left(e_j, e_i\right) + \mu^*\left(e_j, e_i\right)\right] + \\
& w' q_{1n}\left(e_j, e_i\right) + \frac{1}{2}(v' + w'Q_{2n})\Sigma\left(e_j, e_i\right)\left(v + Q'_{2n} w\right) + \\
& \frac{1}{2} w' Q_{5n}\left(e_j, e_i\right) Q'_{5n}\left(e_j, e_i\right) w\}).
\end{aligned}$$

Proof. The price of a zero-coupon bond providing one money unit at $t+h$ if entity n is still alive at $t+h$ and zero otherwise is:

$$\begin{aligned}
B_n^D(t,h) \quad &= \quad E_t^{\mathbb{Q}}\left[\exp\left(-r_t - \ldots - r_{t+h-1}\right) \mathbb{I}_{\left\{d_{n,t+h}=0\right\}}\right] \\
&= \quad E_t^{\mathbb{Q}}\left[E^{\mathbb{Q}}\left(\exp\left(-r_t - \ldots - r_{t+h-1}\right) \mathbb{I}_{\left\{d_{n,t+h}=0\right\}} \mid \underline{z}_{t+h}, \underline{y}_{t+h}, \underline{x}_{n,t+h}, d_{n,t} = 0\right)\right] \\
& \quad\; E_t^{\mathbb{Q}}\left[\exp\left(-r_t - \ldots - r_{t+h-1}\right) \mathbb{Q}\left(d_{n,t+h} = 0 \mid \underline{z}_{t+h}, \underline{y}_{t+h}, \underline{x}_{n,t+h}, d_{n,t} = 0\right)\right].
\end{aligned}$$

Moreover,

$$\begin{aligned}
& \mathbb{Q}\left(d_{n,t+h} = 0 \mid \underline{z}_{t+h}, \underline{y}_{t+h}, \underline{x}_{n,t+h}, d_{n,t} = 0\right) \\
= \quad & \textstyle\prod_{i=1}^{h} \mathbb{Q}\left(d_{n,t+i} = 0 \mid \underline{z}_{t+h}, \underline{y}_{t+h}, \underline{x}_{n,t+h}, d_{n,t+i-1} = 0\right)
\end{aligned}$$

and, since $d_{n,t}$ does not $\mathbb{Q}$-cause $(z_t, y_t, x_{n,t})$ in the Granger's or Sims' sense, we have:[10]

$$\begin{aligned} & \mathbb{Q}\left(d_{n,t+i} = 0 \mid \underline{z}_{t+h}, \underline{y}_{t+h}, \underline{x}_{n,t+h}, d_{n,t+i-1} = 0\right) \\ = & \mathbb{Q}\left(d_{n,t+i} = 0 \mid \underline{z}_{t+i}, \underline{y}_{t+i}, \underline{x}_{n,t+i}, d_{n,t+i-1} = 0\right) \\ = & \exp\left(-\lambda_{n,t+i}\right). \end{aligned}$$

where the last equality comes from the fact that the conditional historical and risk-neutral distributions of $d_{n,t}$ are the same (see Subsectionsec. 2.3.2.2). Hence, Equation(2.14) holds. The latter suggests that since the r_{t+i}'s and the $\lambda_{n,t+i}$'s are linear in the $(z_{t+i}, y_{t+i}, x_{n,t+i})$'s, the bond prices are multi-horizon Laplace transforms.

Besides, it can be shown (see Appendix 2.A.4) that $(z_t, y_t, x_{n,t})$ is Car(1) under $\mathbb{Q}$, with a conditional Laplace transform of the type $\exp[\overline{a}'(u, v, w).(z_t', y_t', x_{n,t}')']$. The recursive formulas presented in Proposition 3 directly stem from Appendix 2.A.5, where it is explained how to exploit the Car(1) property of $(z_t, y_t, x_{n,t})$ to compute its multi-horizon Laplace transforms. □

2.4.2. Defaultable bond pricing with non-zero recovery rate

In the next Proposition, we present conditions under which quasi-explicit formulas are still available in the case of non-zero recovery rates.

Proposition 4. *If, for any bond issued by debtor n before t, the recovery payoff –that is assumed to be paid at time t in case of default between $t-1$ and t of debtor n– is equal to the product of a function $\zeta_{n,t}$ (with $0 \leq \zeta_{n,t} \leq 1$) of the information available at time t by the survival-contingent market value of the bond at t, the price at t of a bond with residual maturity h is:*

$$B_n^D(t,h) = E_t^{\mathbb{Q}}\left[\exp(-r_t - \ldots - r_{t+h-1} - \tilde{\lambda}_{n,t+1} - \ldots - \tilde{\lambda}_{n,t+h})\right], \tag{2.17}$$

[10] A process X_t does not cause Y_t in Granger's sense if and only if, for any t, Y_t is independent of $(X_{t-1}, \ldots, X_1)$ conditionally on $(Y_{t-1}, \ldots, Y_1)$. This is equivalent to the non-causality in Sims' sense (X_t does not cause the stochastic process Y_t in Sims' sense iff X_t is independent from$(Y_{t+1}, Y_{t+2}, \ldots, Y_T)$ conditionally on $(Y_t, X_{t-1,}Y_{t-1}, \ldots, X_1, Y_1)$).

where $\tilde{\lambda}_{n,s}$ *is defined by (for any s):*

$$\exp(-\tilde{\lambda}_{n,s}) = \exp(-\lambda_{n,s}) + (1 - \exp(-\lambda_{n,s}))\,\zeta_{n,s}.$$

Proof. The proof of this proposition is a special case of Appendix 4.A, associated with Chapter 4 (where a slightlty more general framework is presented). □

The assumption of Proposition 4 is similar to the "Recovery of Market Value" assumption made by Duffie and Singleton (1999) except that, in their discrete-time approach, they assume that ζ_t is known at time $t-1$, and that conditionally to the information at $t-1$, $d_{n,t}$ is independent of the recovery payoff at t.

2.5. Internal consistency (IC) conditions

2.5.1. IC conditions based on riskless yields

If the short rate r_t is a component of y_t, for instance the first one, we have to impose an internal consistency condition implying that $r_t = a_1^{'} z_t + b_1^{'} y_t$ is equal to the first component of y_t, that is:

$$a_1 = 0,\, b_1 = \tilde{e}_1,$$

where $\tilde{e}_i$ is the vector selecting the i^{th} component of y_t.

Moreover, if another component of y_t, for instance the second one, is equal to a riskless yield of maturity h_0 –i.e. $R(t, h_0)$– we have to impose that $(1/h_0)\left(a_{h_0}^{'} z_t + b_{h_0}^{'} y_t\right)$ is equal to the second component of y_t, that is

$$\begin{cases} a_{h_0} & = 0 \\ b_{h_0} & = h_0 \tilde{e}_2. \end{cases}$$

2.5.2. IC conditions based on defaultable yields

Similarly, if the first component of $x_{n,t}$ is a defaultable yield with residual maturity h_0, equation (2.15) implies that we have to impose:

$$\begin{cases} c_{n,h_0} & = 0 \\ f_{n,h_0} & = 0 \\ g_{n,h_0} & = h_0 \hat{e}_1. \end{cases}$$

where $\hat{e}_i$ denotes the vector selecting the i^{th} component of $x_{n,t}$.

2.5.3. IC conditions based on asset returns

If the first component of y_t is the geometric return of a market index, we have to impose

$$\exp\left(-r_t\right) E_t^{\mathbb{Q}}\left(\exp\left(y_{1,t+1}\right)\right) = 1.$$

Using equation (2.11), this gives

$$\begin{pmatrix} A_{1,0}^* & \dots & A_{J,0}^* \end{pmatrix} z_t + \left(\Phi_1 + \Phi_1^*\right) y_t = a_1^{'} z_t + b_1^{'} y_t,$$

with $A_{i,0}^* = \log\left\{\sum_{j=1}^{J} \pi_{ij}^* \exp\left[\mu_1\left(e_j, e_i\right) + \mu_1^*\left(e_j, e_i\right) + \frac{1}{2}\sigma_1^2\left(e_j, e_i\right)\right]\right\}$, μ_1 and μ_1^* being the first components of μ and μ^* respectively, σ_1^2 being the $(1,1)$ entry of Σ and Φ_1 and Φ_1^* the first rows of Φ and Φ^* respectively. Then we get

$$\begin{cases} a_1 & = \begin{pmatrix} A_{1,0}^* & \dots & A_{J,0}^* \end{pmatrix}^{'} \\ b_1 & = \left(\Phi_1 + \Phi_1^*\right)^{'}. \end{cases}$$

Similarly, if the first component of $x_{n,t}$ is the return of a stock attached to entity n, we must have:

$$\exp\left(-r_{t+1}\right) E_t^{\mathbb{Q}}\left(\exp\left(x_{1,n,t+1}\right)\right) = 1$$

or

$$r_t = \log\left[E_t^{\mathbb{Q}}\left(\exp\left(x_{1,n,t+1}\right)\right)\right].$$

Using the fact that $(z_t, y_t, x_{n,t})$ is Car(1) under $\mathbb{Q}$ (see Appendix 2.A.4), it is readily seen that $\log\left[E_t^{\mathbb{Q}}\left(\exp\left(x_{1,n,t+1}\right)\right)\right]$ is linear in $z_t, y_t, x_{n,t}$ and the IC constraint follows.

2.6. Inference

2.6.1. Observability

We assume that y_t and the $x_{n,t}$'s are partitioned into $y_t = (y_{1t}^{'}, y_{2t}^{'})'$ and $x_{n,t} = (x_{1,n,t}^{'}, x_{2,n,t}^{'})'$, that y_{1t} and $x_{1,n,t}$ are observed by the econometrician and that y_{2t} and $x_{2,n,t}$ are not. Moreover, we assume that the regime variable z_t is equal to $z_{1,t} \otimes z_{2,t}$ where $z_{1,t}$ and $z_{2,t}$ are valued respectively in $E_1 = \{e_1, \ldots, e_{J_1}\}$ and $E_2 = \{e_1, \ldots, e_{J_2}\}$, where $\otimes$ denotes the Kronecker product operator. We assume further that $z_{1,t}$ is observed by the econometrician whereas $z_{2,t}$ is not. Besides, we observe at each date t a vector of risk-free yields denoted by R_t and, for each obligor n, a vector of defaultable yields denoted by $R_{n,t}^D$. Note that if some yields are included in the vectors y_t or $x_{n,t}$, they do not enter the vectors R_t and $R_{n,t}^D$.

2.6.2. Estimation methods

Regarding estimation, it is convenient to distinguish two main kinds of equations. While the first kind of equations defines the dynamics of the factors (i.e., equations 2.2 and 2.3), the second kind is concerned with the fit of observed yields. If the number of unobserved factors is lower than the number of yields to fit, some pricing errors arise. Obviously, if one wants to compute the log-likelihood of the model, one has to specify a distribution type for these pricing errors. Usually, these are supposed to be (i.i.d.) normally distributed in the affine term-structure literature.

In the absence of latent factors or regimes, the computation of the likelihood of the model is straightforward. On the contrary, specific techniques are required as soon as some factors and/or some Markov chains are unobserved. Table 2.1 proposes techniques that can be implemented in the different possible cases. For instance, when the model includes latent variables but no unobserved Markov chains, the log-likelihood can be computed by means of the Kalman filter or the so-called inversion techniques (see Chen and Scott, 1993) may be resorted to. Absent unobserved factors $y_{2,t}$, the Kitagawa-Hamilton filter can be used if some regimes are unobserved. Appendix 2.B shows how to adapt the standard Hamilton filter in order to deal with partially-hidden Markov chains. Finally, if there are both unobserved regimes and factors, two techniques can be implemented. First, one can use Kim's (1994) filter that allows to approximate the log-likelihood in the presence of both kinds of unobserved processes. Second, inversion techniques *à la* Chen and Scott (1993) may still be used; the implied adjustments to deal with unobserved regimes being detailed in Appendix 2.C.

Table 2.1.: Estimation methods

Notes: This Tablesums up the different estimation procedures that can be implemented depending on the observability of the regimes (z_t) and of the factors (y_t). The unobserved regimes and factors (if any) are respectively denoted by $z_{2,t}$ and $y_{2,t}$. In the Table, the notation $y_{2,t} = \oslash$ (respectively $z_{2,t} = \oslash$) corresponds to those models in which there are no latent factors (respectively no latent regimes).

	$y_{2,t} = \oslash$	$y_{2,t} \neq \oslash$
$z_{2,t} = \oslash$	"Standard techniques"	Kalman filter / Inversion techniques
$z_{2,t} \neq \oslash$	Kitagawa-Hamilton filter	Kim filter / Kitagawa-Hamilton filter + Inv. techniques

2.6.3. Estimation example: a simple model of the BBB-Treasury spreads

In this subsection, we illustrate the flexibility and the tractability of the framework by estimating a model using real data. Note that this example reflects only one, out of many, possible uses of the framework. (the multiplicity of its applications is addressed in Section 2.8.)

We consider one defaultable entity whose funding costs are representative of those of BBB-rated corporates.[11] As commonly assumed, bonds issued by the U.S. Treasury are supposed to be riskfree (i.e., the U.S.-Treasury default intensity is zero). Dropping the debtor index in that subsection(since we consider only one risk entity), the BBB-rated-firms' default intensity is defined as:

$$\lambda_t = y_{1,t} + y_{2,t}$$

where the $y_{i,t}$'s are some risk factors following:

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} \mu_1 & \mu_1 & \mu_1 \\ 0 & 0 & \mu_2 \end{bmatrix} z_t + \begin{bmatrix} \varphi_1 & 0 \\ 0 & \varphi_2 \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \omega\varepsilon_t \\ 0 \end{bmatrix} \tag{2.18}$$

with $\varepsilon_t \sim$ i.i.d. $\mathcal{N}(0,1)$ and where z_t is a three-state Markov chain which is independent from ε_t and has a matrix of transition probabilities of the form:

$$P = \begin{bmatrix} p_{11} & (1-p_{11}) & 0 \\ (1-p_{22}-p_{23}) & p_{22} & p_{23} \\ 0 & (1-p_{33}) & p_{33} \end{bmatrix} \tag{2.19}$$

While the first regime is conceived as being a "tranquil" regime, the third is supposed to correspond to a "crisis" regime. The second acts as an intermediary regime: under this regime, the risk factors $y_{1,t}$ and $y_{2,t}$ have the same dynamics as under the tranquil regime, but with the threat of switching to the third regime (such a threat does not exist under the first regime since the probability of switching from the first to the third regime is null). The "crisis" nature of the third regime stems from the fact that the drift associated with the process $y_{2,t}$ (i.e. $[0, 0, \mu_2]z_t$) is strictly positive only under this last regime, assuming that $\mu_2 > 0$.

It is important to note that conditionally on the information available at time t, the means and variances of future hazard rates λ_{t+k} depend on the current regimes. For instance, whereas the one-period-ahead variance of the intensity is ω^2 under

[11] Among others, Feldhütter and Lando (2008) also consider firms that are representative of some credit-rating classes.

the first regime, it is $\mu_2^2 p_{23}(1-p_{23})+\omega^2$ ($>\omega^2$) under the second regime. This illustrates in particular the fact that the model is able to generate some forms of stochastic volatility.

The risk-neutral dynamics of $y_{1,t}$ and $y_{2,t}$ is assumed to be similar to its historical counterparts (equations 2.18 and 2.19), except that parameters φ_1, φ_2, μ_1, μ_2 and the p_{ij}'s are respectively replaced by φ_1^*, φ_2^*, μ_1^*, μ_2^* and by some p_{ij}^*'s. Besides, the ε_t's in equation (2.18) are replaced by some ε_t^*'s that are normal in the risk-neutral world.

For the sake of simplicity, we assume that $y_{1,t}$ and $y_{2,t}$ are independent from the factors driving the short-term risk-free rate under both historical and the risk-neutral measure. This implies that we can estimate the dynamics of $y_{1,t}$ and $y_{2,t}$ without defining a process for the short rate and that the estimation requires only spreads data.[12]

The data are weekly and cover the period from 17 March 1995 to 1 July 2011. The spreads are computed by subtracting from the corporate yields the Treasury zero-coupon rates of the same maturities.[13] We consider four maturities: 1, 2, 3 and 5 years. The spreads are assumed to be observed with i.i.d. measurement errors. The model can be seen as a state-space model with (a) four measurement equations (relating the observed spreads to the modelled ones, the discrepancy being the measurement –or pricing– errors) and (b) transition equations defined by (2.18) and (2.19). The parameters are estimated by maximising the log-likelihood, using the approximation proposed by Kim's (1994). Additional details regarding the estimation –including the developed state-space version of the model and the parameter estimates– are presented in Appendix 2.D.

The upper panel of Figure 2.2 displays the estimated components of the default intensity (i.e. factors $y_{1,t}$ and $y_{2,t}$). The second panel shows the (smoothed) probabilities of being in each of the three regimes. As expected, the failure of LTCM

[12]Such an assumption is for instance made by Pan and Singleton (2008) or Longstaff et al. (2011).

[13]Zero-coupon yield curves have been obtained by applying bootstrap techniques on the BBB (coupon) yield curve provided by Bloomberg (tickers C009). The risk-free yields are US STRIPS yields extracted from Bloomberg (tickers C079).

(Fall 1998), the bursting out of the internet bubble (2001) or the recent financial crisis (2007-2009) are associated with the crisis-regime periods (see the black areas of this second panel).[14] The lowest two panels of Figure 2.2 display respectively the 2-year and 5-year observed spreads together with their model-implied counterparts, showing that the model captures most of the spread fluctuations (close to 99% of the spread fluctuations are accounted for by the model).

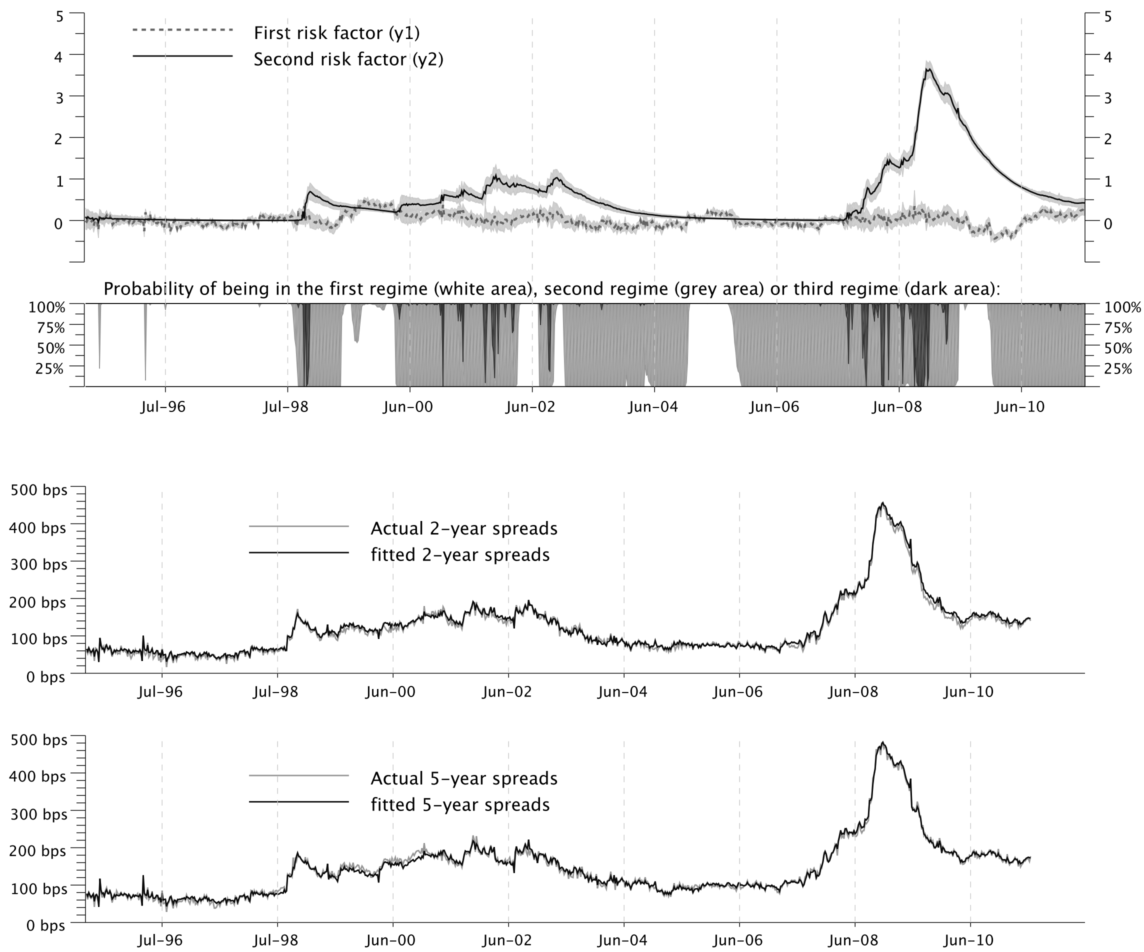
An important feature of the model is that it is not only capable of fitting the data, but it is also relevant to simulate realistic ones. Obviously, this is key if one wants to use the model to compute Values-at-Risk, for instance. In order to illustrate this, we have compared our estimated regime-switching model (RS model hereinafter) with two alternative (purely) Gaussian models. In the first alternative model, the default intensity is a simple AR(1); in the second model, the default intensity is a sum of two independent Gaussian AR(1) processes. The standard deviations of the spread pricing errors obtained with the RS model, the 1-factor Gaussian model and the 2-factor Gaussian model are respectively of 8, 10 and 7 basis points. However, the quality of the data resulting from Monte-Carlo simulations of the Gaussian models is poor in comparison with the RS model. This is illustrated in Figure 2.3. The first row of charts shows, for the 2-year and the 5-year maturities, the unconditional distributions of spreads simulated by the three models mentioned above. These distributions are compared with the sample-data ones. The charts show that the Gaussian models are inappropriate to capture the tails' shapes: in particular, while the one-factor Gaussian model often generates negative spreads, the two-factor Gaussian model fails to generate high spreads. By contrast, the RS model is able to keep the number of simulated negative spreads at a minimum while allowing for frequent high (crisis) spreads. The skewness and kurtosis of the sample data are impressively well reproduced by the RS model: considering the 5-year maturity, the skewness of the simulated spread is of 1.78 vs. 1.83 for the sample data and the kurtosis are respectively of 7.45 vs. 7.46. The superiority of the RS model in terms of simulation of plausible spreads is further

[14] Looking both at the first and second panel in Figure 2.2, one can check that the second factor $y_{2,t}$ is pushed upwards during crisis periods.

highlighted by the lower plots in Figure 2.3: according to these charts, the RS model performs well in terms of fitting the 5th, 50th and 95th quantiles of the spreads (as well as their mean).

Figure 2.2.: BBB vs. Treasury Spreads, Estimation results

Notes: The upper panel presents the smoothed (using Kim's (1994) filter) estimates of the two factors $y_{1,t}$ and $y_{2,t}$ that are such that the default intensities λ_t of BBB-rated corporates is given by $\lambda_t = y_{1,t} + y_{2,t}$. Grey-shaded areas correspond to 95% confidence intervals. The second panel reports the (smoothed) probabilities of being in the "tranquil-times" regime 1 (white), the "intermediary" regime 2 (in grey) or the "crisis" regime 3 (in black). For each date, the three vertical bars (white, grey and black) sums to one. The lowest two panels display model-implied spreads together with observed ones for two respective maturities: 2 years and 5 years.



2.7. Liquidity risk

There is compelling evidence that yields and spreads contain components that are closely linked to liquidity.[15] The estimation of the liquidity premium is of concern for several reasons. First, gauging the liquidity-risk premium provides policy makers –central bankers in particular– with insights on the valuation of liquidity by the markets (see Taylor and Williams, 2008 or Michaud and Upper, 2008). Second, if one wants to extract default probabilities from market data, one has to distinguish between what is related to default and what is caused by the liquidity of the considered bonds.

However, the identification of the liquidity premium, that is, distinguishing between the default-related and the liquidity-related components of yield spreads, remains a challenging task. Empirical evidence points to the existence of commonality amongst the liquidity components of prices of different bonds (see e.g. Fontaine and Garcia, 2012). Therefore, the identification of the liquidity component relies on the ability to exhibit risk factors that reflects liquidity valuation. Liu, Longstaff and Mandell (2006) and Feldhütter and Lando (2008) develop affine term-structure models where a liquidity factor is latent and the identification is based on assumptions regarding the relative liquidity of different interest-rate instruments.[16] In the euro area context, Chapters 3and 4 identify a liquidity latent factor by exploiting the term-structure of the KfW-Bund spreads. KfW is a German public agency whose issuances are fully and explicitly guaranteed by the Federal Republic of Germany. Accordingly, the spreads between the yields of bonds issued by KfW and those issued by the German government (called “Bunds”) mainly reflect liquidity-pricing effects. Alternatively, the liquidity factor could be proxied by observable factors.[17] One may resort to intermediate –or mixed– ap-

[15] The influence of liquidity effects on bond pricing has been investigated, amongst others, by Longstaff (2004) [193], Chen, Lesmond and Wei (2007) [63], Covitz and Downing (2007) [76].

[16] In both studies, the liquidity factor that is estimated corresponds to the so-called “convenience yield”, that can be seen as a premium that one is willing to pay when holding Treasuries. This premium stems from various features of Treasury securities, such as repo specialness (see Feldhütter and Lando, 2008).

[17] Among which: bid-ask spreads, market-depth measures, bond supply, spread between bonds of the same maturity but with different ages or spread between off-the-run and on-the -run Treasuries (see, e.g., Longstaff, 2004 or Beber, Brandt and Kavajecz, 2009).

proach, where part of the liquidity-factor dynamics is observable (through observed proxies) and part of it is latent.

Let us come back to our modelling framework. We have seen above (Section 2.4) that incorporating default risk in the pricing methodology implies to replace the short rate r_t by a "default-adjusted" short-rate $r_t + \lambda_{n,t+1}$. Besides, in order to take into account recovery-rate effects, $\lambda_{n,t+1}$ can be seen as a "recovery adjusted" default intensity between t and $t+1$ (Subsection 2.4.2). So the price at t of a defaultable asset providing the payoff $g\left(\underline{w}_{t+h}\right)$ at $t+h$ in case of absence of default, is:

$$E_t^{\mathbb{Q}}\left[\exp\left(-r_t - \lambda_{n,t+1} - \ldots - r_{t+h-1} - \lambda_{n,t+h}\right) g\left(\underline{w}_{t+h}\right)\right].$$

As suggested by Duffie and Singleton (1999), intensity-based model can also account for liquidity effects by introducing a stochastic process that is interpreted as the carrying cost of non-liquid defaultable securities. This process then appears alongside the default intensity in the spread between the "pure" –i.e. default and liquidity-adjusted– short rate and the short rate associated with a defaultable bond. The affine term-structure literature is relatively silent on the interpretation or the micro-foundations of the illiquidity intensity. In a theoretical paper analysing interactions between credit and liquidity risks, He and Xiong (2012) show that such an illiquidity intensity may reflect the probability of occurrence of a liquidity shock; upon arrival of this shock, the bond investor has to exit by selling his bond at a fractional cost (i.e. the selling price is equal to a fraction of the price that would have prevailed in the absence of the liquidity shock); the fractional cost is the analogous to the fractional loss $(1-\zeta)$ in the default case (see also Ericsson and Renault, 2006 for a similar interpretation). Let us introduce an "illiquidity intensity" between t and $t+1$, denoted with $\lambda^L_{n,t+1}$.[18] If $\lambda_{n,t+1}$ and

[18] Chapter 4 (see 4.2.1) explicitly relates the illiquidity intensity to "portfolio liquidation" processes as in He and Xiong (2012) and Ericsson and Renault (2006).

$\lambda_{n,t+1}^{L}$ are specified in an affine way,

$$\begin{cases} \lambda_{n,t+1} &= \alpha_n^{'} z_{t+1} + \beta_n^{'} y_{t+1} + \gamma_n^{'} x_{n,t+1} \\ \lambda_{n,t+1}^{L} &= \alpha_n^{L'} z_{t+1} + \beta_n^{L'} y_{t+1} + \gamma_n^{L'} x_{n,t+1}, \end{cases}$$

we could price not only riskless bonds $B_n\left(t,h\right)$ and defaultable bonds $B_n^D\left(t,h\right)$ as above, but also bonds facing liquidity risk $B_n^L\left(t,h\right)$ and bonds facing both default and liquidity risk $B_n^{DL}\left(t,h\right)$. We would have:

$$\begin{cases} B\left(t,h\right) &= E_t^{\mathbb{Q}}\left[\exp\left(-r_t - \ldots - r_{t+h-1}\right)\right] \\ B_n^D\left(t,h\right) &= E_t^{\mathbb{Q}}\left[\exp\left(-r_t - \lambda_{n,t+1} - \ldots - r_{t+h-1} - \lambda_{n,t+h}\right)\right] \\ B_n^L\left(t,h\right) &= E_t^{\mathbb{Q}}\left[\exp\left(-r_t - \lambda_{n,t+1}^L - \ldots - r_{t+h-1} - \lambda_{n,t+h}^L\right)\right] \\ B_n^{DL}\left(t,h\right) &= E_t^{\mathbb{Q}}\left[\exp\left(-r_t - \lambda_{n,t+1} - \lambda_{n,t+1}^L - \ldots - r_{t+h-1} - \lambda_{n,t+h} - \lambda_{n,t+h}^L\right)\right]. \end{cases}$$

In the context of a Car(1) risk-neutral dynamics of $(z_t, y_t, x_{n,t})$, these prices are exponential linear in $(z_t, y_t, x_{n,t})$ and the corresponding yields are linear in $(z_t, y_t, x_{n,t})$.

2.8. Model extensions

2.8.1. Multi-lag dynamics for y_t and $x_{n,t}$ processes

The model can easily be extended to allow for y_t and $x_{n,t}$ dynamics that include several lags. In particular, when observed data are used in the estimation process –the $y_{1,t}$ and $x_{1,n,t}$ defined in Section 2.6–, preliminary analysis of the data could point to the need of taking different lags into account to model the historical dynamics of these variables. The flexibility in the choice of the lag structure constitutes an advantage of working in discrete-time over most continuous-time models (see, e.g., Monfort and Pegoraro, 2007 or Gourieroux, Monfort and Polimenis, 2006).

Equations (2.2) and (2.3) imply that the multivariate factors y_t and x_t follow autoregressive process of order one. However, to the extent that a VAR(p) amounts to a

VAR(1) once the last p lags of the endogenous variable are stacked in the same vector, the pricing techniques of the bonds –namely equation (2.16)– are not affected if y_t and x_t follow VAR(p). However, in order to make the estimation strategy presented in Section 2.6 still effective –in particular regarding inversion techniques–, the unobserved vector variables $y_{2,t}$ and $x_{2,n,t}$ should not enter equations (2.2) and (2.3) with lags larger than one. To the extent that this restriction only applies to the unobserved factors –for which insights on the appropriate distributions are a priori not readily available– such a constraint is not really restrictive.

2.8.2. Interpretation of a regime as the default state of an entity

In this subsection, we consider the specific case where the regime variable z_t is the Kronecker product of several basic regime variables, one of them corresponding to the default or non-default state of a given entity (indexed by zero). The specificity of that situation lies in the fact that the default of this entity then enters the s.d.f.. Therefore, we leave the framework described in Subsection 2.3.1 where all defaultable entities were small enough to have no impact at the macroeconomic level. As a consequence, the "zero" entity may represent a whole industry or a very big institution. This could be extended to a few major entities but one has to bear in mind that increasing their number results in an exponential growth in the dimension of z_t.

The fact that this default enters the s.d.f. results in new components in bond prices. As pointed out by Yu (2002) and Jarrow, Lando and Yu (2005), such components arise only when the default-event risk is not diversifiable.

As mentioned in the introduction, this interpretation is also linked with previous studies attempting to introduce contagion effects in affine term-structure models. Indeed, the default of entity zero may lead to a simultaneous increase in the default intensities of any other debtor (through the regime variable z_t that may enter all default intensities).

For sake of simplicity, let us assume that such a crisis variable is the only regime captured by z_t, which can be observable or not. In this case, assuming that the state $e_2 = (0,1)'$ is the absorbing crisis state, we have:

$$\begin{aligned}\pi\left(e_2 \mid e_2, y_{t-1}\right) &= 1 \\ \pi\left(e_1 \mid e_2, y_{t-1}\right) &= 0.\end{aligned}$$

Moreover, we could specify:

$$\pi\left(e_1 \mid e_1, y_{t-1}\right) = \exp\left(-\lambda_{0,t-1}\right),$$

with $\lambda_{0,t-1} = \alpha_0 + \beta_0' y_{t-1}$. In this case, $\lambda_{0,t-1}$ can be interpreted as a systemic-risk intensity. Conditions (2.10) $\{\pi\left(e_j \mid e_i, y_{t-1}\right) \exp\left[\delta_j\left(e_i, y_{t-1}\right)\right] = \pi^*_{ij}\}$ imply the followings:

- $\pi^*_{21} = 0$, π^*_{22}=1, $\delta_1\left(e_2, y_{t-1}\right)$ is undefined, $\delta_2\left(e_2, y_{t-1}\right) = 0$ and, therefore, $\delta'\left(e_2, y_{t-1}\right) z_t = 0$.
- $\exp\left[\delta_1\left(e_1, y_{t-1}\right)\right] = \pi^*_{11} \exp\left(\lambda_{0,t-1}\right)$ or $\delta_1\left(e_1, y_{t-1}\right) = \log \pi^*_{11} + \alpha_0 + \beta_0' y_{t-1}$.
- $\exp\left[\delta_2\left(e_1, y_{t-1}\right)\right] = \left(1 - \pi^*_{11}\right)\left[1 - \exp\left(-\lambda_{0,t-1}\right)\right]^{-1}$, or $\delta_2\left(e_1, y_{t-1}\right) = \log\left(1 - \pi^*_{11}\right) - \log\left[1 - \exp\left(-\alpha_0 - \beta_0' y_{t-1}\right)\right]$.

Denoting $\pi^*_{11} = \exp\left(-\lambda^*_0\right)$, λ^*_0 being the systemic-risk intensity in the risk-neutral world, we get:

$$\begin{aligned}\delta_1\left(e_1, y_{t-1}\right) &= \lambda_{0,t-1} - \lambda^*_0 \\ \delta_2\left(e_1, y_{t-1}\right) &= \log\left[1 - \exp\left(-\lambda^*_0\right)\right] - \log\left[1 - \exp\left(-\lambda_{0,t-1}\right)\right] \\ &\simeq \log\left(\lambda^*_0\right) - \log\left(\lambda_{0,t-1}\right) \text{ if } \lambda^*_0, \lambda_{0,t-1} \text{ are small.}\end{aligned}$$

In particular, the risk-neutral intensity λ^*_0 and the historical intensity$\lambda_{0,t-1}$ are different functions, contrary to what happened in the previous Sections. Both the riskless yields:

$$R(t,h) = \frac{1}{h}\left(a_h' z_t + b_h' y_t\right)$$

and the defaultable yields:

$$R_n^D(t,h) = \frac{1}{h}\left(c_{n,h}' z_t + f_{n,h}' y_t + g_{n,h}' x_{n,t}\right)$$

will be different functions of y_t (and of x_{nt} for $R_n^D(t,h)$) before and after the systemic crisis. The term structure of the impact of the systemic crisis will be:

$$\begin{cases} a_{2,h} - a_{1,h} & \text{for the riskless yield of residual maturity } h, \\ c_{2,n,h} - c_{1,n,h} & \text{for the defaultable yield of residual maturity } h, \text{ for the } n^{th}\text{entity.} \end{cases}$$

2.8.3. A sector-contagion model

2.8.3.1. General approach

In this subsection, we propose another specific use of the regimes that makes it possible to model sector-contagion phenomena. As explained in the introduction, our assumptions prevent us from making the default intensity of any entity depend on the default event of other entities. In other words, the baseline framework does not allow us to account for contagion at the debtor level (except in the specific case presented in 2.8.2). Nevertheless, as shown here, this can be done at a sector level, the sectors representing for instance different industries or different geographical areas.

Specifically, in this model, each debtor belongs to one of the sectors. At each period, a sector is either "infected" or not infected. When a sector is infected, the default intensities of its constituent entities tend to be higher. Let us denote by $S_{i,t}$ the state the i^{th} sector at time t: $S_{i,t}$ is equal to $[0,1]'$ if the i^{th} sector is infected at time t, and is equal to $[1,0]'$ otherwise. If we have N_S sectors, then we have to consider 2^{N_S} regimes, the regime variable z_t being given by:

$$z_t = S_{1,t} \otimes S_{2,t} \otimes \ldots \otimes S_{N_S,t}$$

where $\otimes$ denotes the Kronecker product. In such a model, one can make the default intensity of any firm depend on the state of the sectors (and, in particular, on the state of its own sector). Further, the sector-contagion phenomena can be obtained through the specification of the regime-transition matrix. Indeed, this matrix contains the probabilities that any sector gets infected (or cured) given the states of the other sectors.

2.8.3.2. Numerical example

In this example, we make use of processes y_t and z_t whose dynamics are defined in Tables 2.2 and 2.3. We consider three homogeneous sectors. The probability that a sector gets cured/infected at time t depends on the number of infected sectors at the previous period. In that case, the regime-transition matrix is defined by a set of probabilities like the one reported in Table2. In our example, the probability of getting infected is far higher when at least one sector is already infected than when none of them is infected. The default intensities of sector-i firms are given by:

$$\lambda_{i,t} = 0.01 + 0.02 \times \mathbb{I}_{\left\{S_t^i=1\right\}} + 0.02 \times \mathbb{I}_{\left\{S_t^1=1\right\}}\mathbb{I}_{\left\{S_t^2=1\right\}}\mathbb{I}_{\left\{S_t^3=1\right\}} + 0.002 y_t,$$

where $S_t^i = [0,1]S_{i,t}$. This implies that the default intensity of a Sector-i entity increases by two percentage points when Sector i gets infected and increases by an additional two percentage points if all sectors become infected simultaneously.

Let us now consider a portfolio of 600 debtors, with 200 debtors in each sector. Figure 2.2 shows a simulation of the timing of defaults for this portfolio. Each panel corresponds to one of the three sectors. At one point, Sector 1 gets infected (see the grey area in the first panel of Figure 2.2). While the default intensities of Sector-2 and Sector-3 firms are not contemporaneously impacted by the infection

Table 2.2.: Calibration of the sector-contagion model

Notes: The second (respectively the third) lines reports the probabilities, for any sector, of getting infected (resp. cured), depending on the number of infected sectors during the previous period.

Numb. of infected sect. ($\sum_i [0,1].S_{i,t}$)	0	1	2	3
Proba. of getting infected (in $t+1$)	0.25%	10%	10%	–
Proba. of getting cured (in $t+1$)	–	10%	10%	10%

of the first sector, 5-year default probabilities of Sector-2 and Sector-3 firms shift upwards. This is a consequence of the fact that once Sector 1 is infected, the probability that Sector 2 and Sector 3 get infected over the next periods is higher. A few periods later, Sector 3 and then Sector 2 get infected.

2.8.4. modelling credit-rating transitions

In their seminal study of credit spread, Jarrow, Lando and Turnbull (1997) model rating transitions as a time-homogenous Markov chain. That is, in their model, whether a firm's rating will change in the next period depends on its current rating only and the probability of changing from one rating to the other remains the same over time. Different studies suggest however that –per-period– transition probabilities are time-varying and that simple Markov processes are not appropriate to model credit migrations (see e.g. Lucas and Lonski, 1992 or Feng, Gourieroux and Jasiak, 2008 or Bangia et al., 2002).

In the present subsection, we show how our framework can be adapted in order to account explicitly for rating migration. Building on Lando's (1998) approach (see also Feldhütter and Lando, 2008), the structure accommodates a time-varying rating-migration matrix while allowing different ratings to respond in a correlated yet different fashion to the same change in the general economic conditions. The time variability of the rating-migration probabilities results from Gaussian shocks as well as from regime shifts. Note that the model that we propose here is very general and may be suited to address various features of empirical evidences regarding credit-rating transitions. In particular, this framework is such that the marginal dynamics of the credit ratings (once the regime variable and the factors have been integrated out) depends on the whole history of the past ratings and therefore is not Markovian.

2.8.4.1. Adaptation of the framework

While most of the previous framework is still valid, some changes regard the modelling of the default intensity. Specifically, the historical dynamics of $(z_t, y_t, x_{n,t})$, as well as the s.d.f. specifications are still given by equations (2.1), (2.2), (2.3) and (2.7). However, in this adapted framework, each firm n is also characterised by a credit-rating process, denoted by $\tau_{n,t}$. For any firm n and period t, $\tau_{n,t}$ can take one of K values: the first $K-1$ values correspond to credit ratings and the K^{th} corresponds to the default state. For instance, rating 1 can be the highest (Aaa in Moody's rankings) and $K-1$ can be the lowest (C in Moody's rankings). In addition, we have, $d_{n,t} = \mathbb{I}\left(\tau_{n,t} = K\right)$. Like the $d_{n,t}$'s, the $\tau_{n,t}$'s, $n = 1, \dots, N$, are independent conditionally to$(z_t, y_t, x_t, \underline{w}_{t-1})$. In addition, we assume that the rating transition probabilities, for firm n and from period $t-1$ to period t, is a function of $(z_t, y_t, x_{n,t})$. Accordingly, this transition matrix is denoted with $\Pi(z_t, y_t, x_{n,t})$ and we have:

$$P(\tau_{n,t} = j \mid \tau_{n,t-1} = i, \underline{z}_t, \underline{y}_t, \underline{x}_t) = \Pi_{i,j}(z_t, y_t, x_{n,t}),$$

where $\Pi_{i,j}(z_t, y_t, x_{n,t})$, the (i,j) entry of the transition matrix $\Pi(z_t, y_t, x_{n,t})$, represents the actual probability of going from state i to state j in one time step. Each of these entries must be in $[0,1]$ and for each row, the sum of the entries must sum to one. In other words, $\left[\begin{array}{ccc} 1 & \cdots & 1 \end{array}\right]'$ is an eigenvector of $\Pi(z_t, y_t, x_{n,t})$ associated with the eigenvalue 1. In addition, the default state being absorbing, the bottom row of $\Pi(z_t, y_t, x_{n,t})$ is equal to $\left[\begin{array}{cccc} 0 & \cdots & 0 & 1 \end{array}\right]$. Importantly, the entries of Π are the same function of $(z_t, y_t, x_{n,t})$ under both measures (as the default intensities in the baseline model).[19]

In this context, a defaultable zero-coupon bond providing one money unit at $t+h$ if entity n is still alive in $t+h$ and zero otherwise has a price, in period t, that is

[19] Note however that this does not imply that the distributions of these entries are the same under both measures (since the distributions of $(z_t, y_t, x_{n,t})$ differ under $\mathbb{Q}$ and $\mathbb{P}$).

given by (assuming that entity n has not defaulted before t):

$$B_n^D(t,h) = E_t^{\mathbb{Q}}\left[\exp\left(-r_t - \ldots - r_{t+h-1}\right)\mathbb{I}_{\left\{\tau_{n,t+h}<K\right\}}\right]. \tag{2.20}$$

In order to keep a quasi-explicit formula for defaultable zero-coupon bonds, we assume that $\Pi(z_t, y_t, x_{n,t})$ admits the diagonal representation:

$$\Pi(z_t, y_t, x_{n,t}) = V.\Psi(z_t, y_t, x_{n,t}).V^{-1},$$

where the columns of V are the eigenvectors of $\Pi(z_t, y_t, x_{n,t})$ and constitute a basis in $\mathbb{R}^K$ and $\Psi(z_t, y_t, x_{n,t})$ is a diagonal matrix of real eigenvalues that are positive and smaller than one. Given that 1 is an eigenvalue of $\Pi(z_t, y_t, x_{n,t})$, $\Psi(z_t, y_t, x_{n,t})$ can be written in the following manner:

$$\Psi(z_t, y_t, x_{n,t}) = \begin{bmatrix} \exp\left[-\psi_1\left(w_t\right)\right] & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \exp\left[-\psi_{K-1}\left(w_t\right)\right] & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix},$$

with, for any $i < K$, $\psi_i\left(w_t\right) \geq 0$. Then, it is easily seen that, conditionally on $\left(\underline{z}_{t+h}, \underline{y}_{t+h}, \ \underline{x}_{n,t+h}, \tau_{n,t} = i\right)$ the probability of defaulting before $t+h$ corresponds to the entry (i, K) of the matrix that is given by:

$$V.\Psi(z_{t+1}, y_{t+1}, x_{n,t+1}) \ldots \Psi(z_{t+h}, y_{t+h}, x_{n,t+h}).V^{-1}.$$

This probability is therefore:

$$P(\tau_{n,t+h} = K \mid \underline{z}_{t+h}, \underline{y}_{t+h}, \underline{x}_{n,t+h}, \tau_{n,t} = i) = \sum_{j=1}^{K} V_{i,j} V_{j,K}^{-1} \exp\left[-\sum_{p=1}^{h} \psi_j\left(w_{t+p}\right)\right],$$

where $V_{i,j}$ and $V_{i,j}^{-1}$ are the entries (i, j) of , respectively,V and V^{-1}. Since

$V_{i,K}V_{K,K}^{-1} = 1$ (see Appendix 2.E) using $\psi_K \equiv 0$, we get:

$$P(\tau_{n,t+h} < K \mid \underline{z}_{t+h}, \underline{y}_{t+h}, \underline{x}_{n,t+h}, \tau_{n,t} = i) = -\sum_{j=1}^{K-1} V_{i,j}V_{j,K}^{-1} \exp\left[-\sum_{p=1}^{h} \psi_j\left(w_{t+p}\right)\right]. \tag{2.21}$$

If the ψ_j's are some linear combinations of $(z_t, y_t, x_{n,t})$, equations (2.20) and (2.21) imply that the price of a bond is a sum of $K-1$ multi-horizon Laplace transforms. As a consequence, the bond prices can be obtained using the algorithm presented in Lemma 2. However, it should be noted that in this context, the yields are no longer affine in the factors, which implies in particular that the Kalman filter has to be adapted so as to accommodate the nonlinearity of the state-space measurement equations. In such a context, Feldhütter and Lando (2008) use the extended Kalman filter. As an alternative, the unscented Kalman filter can be implemented.

2.8.4.2. Numerical example

Let us consider the processes r_t and z_t whose dynamics are specified in Table 2.3. In the present model, the credit-migration matrices are of the form:

$$\Pi(z_t, y_t, x_{n,t}) = V.\begin{bmatrix} \exp\left[-\alpha_1 z_t - \beta_1 y_t\right] & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \exp\left[-\alpha_{K-1} z_t - \beta_{K-1} y_t\right] & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix}.V^{-1}$$

In order to get plausible plausible matrices, the first-regime calibration –that involves the $\alpha_{i,1}$'s– is based on the one-year-average rating-migration matrix for European corporates provided by Moody's (Moody's, 2010). This matrix is reported in Table 2.4. The spectral decomposition of this matrix provides us with the matrix of eigenvectors V. The eigenvalues are real and comprised between 0 and 1. Accordingly, they are of the form $\exp(-\alpha_{i,1})$. The $\alpha_{i,1}$ are reported in Table 2.5. The definition of the second regime requires a second set of α_i's, denoted by $\{\alpha_{i,2}\}_{i=1\ldots K-1}$. We calibrate the latter in order to have 5-year default probabilities that are higher than those obtained with the first-regime transition matrix (see Table5). Finally, the β_i's are given by $(\alpha_{i,1} - \alpha_{i,2})/5$.

Table 2.3.: Dynamics of risk factors under both measures

Notes: The shocks ε_t and ε_t^* are i.i.d., $\varepsilon_t \sim \mathcal{N}^{\mathbb{P}}(0,1)$ and $\varepsilon_t^* \sim \mathcal{N}^{\mathbb{Q}}(0,1)$. The risk-free short-term rate is $r_{t+1} = 4\% + y_t/100$.

	Under $\mathbb{P}$	Under $\mathbb{Q}$
Dynamics of	$y_t = 0.6y_{t-1} + \varepsilon_t$	$y_t = 0.3 + 0.8y_{t-1} + \varepsilon_t^*$
Transition proba.	$\{\pi_{i,j}\} = \begin{bmatrix} 0.98 & 0.02 \\ 0.25 & 0.75 \end{bmatrix}$	$\{\pi_{i,j}^*\} = \begin{bmatrix} 0.98 & 0.02 \\ 0.01 & 0.99 \end{bmatrix}$

Table 2.4.: Baseline matrix of rating-migration probabilities

Notes: This matrix is based on Moody's (2010) (Exhibit 12: One-year average ratings-transition for European corporates 1985-2009). According to the industry standard, the probability of transitions to the "not rated" state is distributed among all states in proportion to their values (see Bangia et al., 2002).

	Aaa	Aa	A	Baa	Ba	B	Caa-C	Default
Aaa	0.911	0.084	0.004	0.000	0.001	0.000	0.000	0.000
Aa	0.009	0.902	0.083	0.005	0.000	0.000	0.000	0.000
A	0.000	0.042	0.898	0.055	0.003	0.000	0.000	0.001
Baa	0.000	0.004	0.072	0.868	0.041	0.009	0.003	0.001
Ba	0.000	0.000	0.007	0.074	0.788	0.107	0.012	0.011
B	0.000	0.000	0.004	0.004	0.073	0.794	0.092	0.033
Caa-C	0.000	0.003	0.001	0.000	0.007	0.106	0.706	0.177

Table 2.5.: Eigenvalues of the transition matrix under both regimes

Notes: "Regime 1" is consistent with the transition matrix reported in Table3. Regime 2 is intended to depict a "crisis" regime. The $\alpha_{i,j}$'s ($i = 1, \dots, 7$, $j = 1, 2$) are such that the $\exp(-\alpha_{i,j})$'s are the eigenvalues –those different from 1– of the rating-transition matrix obtained under regime j (when $y_{r,t} = 0$). The 5-year default probabilities are computed conditionally on the absence of regime switching (i.e. as if the current regime is to last 5 years).

5-yr default prob.	Aaa	Aa	A	Baa	Ba	B	Caa-C
Regime 1	0.057%	0.24%	0.80%	1.91 %	8.72%	21.8%	52.0%
Regime 2	0.774 %	1.79 %	3.01%	6.40%	16.74%	32.6%	63.2%
-log(eigenvalues)	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}
$\alpha_{i,1}$ $(i = 1, \dots, K-1)$	0.009	0.069	0.097	0.143	0.213	0.311	0.464
$\alpha_{i,2}$ $(i = 1, \dots, K-1)$	0.017	0.110	0.146	0.205	0.294	0.463	0.807

Figure 2.5 displays yield curves for selected ratings under both regimes (for $y_t = 0$, i.e. its unconditional value). Figure 2.6 presents some simulation results. The upper panel shows the time fluctuations of downgrade probabilities for two different ratings. The lower panel displays yield spreads between 10-year zero-coupon bonds

issued by A-rated or Baa-rated firms and 10-year zero-coupon bonds issued by Aaa-rated firms.

2.9. Conclusion

In this Chapter, we have proposed an econometric framework aimed at jointly modelling yield curves associated with different defaultable issuers. Default intensities and yields are affine functions of a multivariate process which is Compound autoregressive (Car) in the risk-neutral world and thus provides us with quasi-explicit (recursive) formulas for both risk-free and defaultable bond prices.

The risk factors follow discrete-time conditionally Gaussian processes, with drifts and variance-covariance matrices that are subject to regime shifts described by a Markov chain with (historical) non-homogenous transition probabilities. The regime-switching feature is relevant for credit models in several respects. First, it makes it possible to capture non-linear behaviours of yields and spreads, which is consistent with empirical evidence. Second, it is appropriate to capture default clusters. Third, it offers some ways of dealing with specific forms of contagion. In this respect, we show how the framework can be used to capture sector-contagion phenomena. An other extension accommodates credit-rating migrations. While flexible, the model remains tracTableand amenable to empirical estimation. To that end, a sequential estimation strategy is proposed in the Chapter.

2.A. Proofs of Sections 2.3 and 2.4

2.A.1. Proof of Proposition 1

We have

$$\begin{aligned}
\varphi_{t-1}(u,v) &= E_{t-1}\left(\exp\left[u'z_t + v'y_t\right]\right) \\
&= E_{t-1}\left(\exp\left[u'z_t + v'\mu\left(z_t, z_{t-1}\right) + v'\Phi y_{t-1} + v'\Omega\left(z_t, z_{t-1}\right)\varepsilon_t\right]\right) \\
&= E\left(E\{\exp\left[u'z_t + v'\mu\left(z_t, z_{t-1}\right) + v'\Phi y_{t-1} + \right.\right. \\
&\quad \left.\left. v'\Omega\left(z_t, z_{t-1}\right)\varepsilon_t\right] \mid \underline{w}_{t-1}, z_t\} \mid \underline{w}_{t-1}\right) \\
&= \exp(v'\Phi y_{t-1})E\left(\exp\left\{u'z_t + v'\mu\left(z_t, z_{t-1}\right)\right\} \times\right. \\
&\quad \left. E\left(\exp\left\{v'\Omega\left(z_t, z_{t-1}\right)\varepsilon_t \mid \underline{w}_{t-1}, z_t\right\}\right) \mid \underline{w}_{t-1}\right) \\
&= \exp(v'\Phi y_{t-1})E\left(\exp\left\{u'z_t + v'\mu\left(z_t, z_{t-1}\right)\right\} \times\right. \\
&\quad \left. \frac{1}{2}v'\Omega\left(z_t, z_{t-1}\right)\Omega'\left(z_t, z_{t-1}\right)v \mid \underline{w}_{t-1}\right) \\
&= \exp\left(v'\Phi y_{t-1} + \left[l_1, \ldots, l_J\right] z_{t-1}\right).
\end{aligned}$$

Using the expression given for the l_i's leads to the result.

2.A.2. Proof of Proposition 2

$$\begin{aligned}
\varphi^{\mathbb{Q}}_{t-1}(u,v) &= E^{\mathbb{Q}}_{t-1}\left(\exp\left[u'z_t + v'y_t\right]\right) \\
&= E_{t-1}\left(\exp\left[-\frac{1}{2}\nu_t'\nu_t + \nu_t'\varepsilon_t + \delta_{t-1}'z_t + u'z_t + v'y_t\right]\right) \\
&= \exp\left(v'\Phi y_{t-1}\right) \times \\
&\quad E_{t-1}\left(\exp\left[-\frac{1}{2}\nu_t'\nu_t + \nu_t'\varepsilon_t + \delta_{t-1}'z_t + u'z_t + v'\mu_t + v'\Omega_t\varepsilon_t\right]\right) \\
&= \exp\left(v'\Phi y_{t-1}\right) \times \\
&\quad E_{t-1}\left(\exp\left[-\frac{1}{2}\nu_t'\nu_t + \frac{1}{2}\left(\nu_t' + v'\Omega_t\right)\left(\nu_t' + v'\Omega_t\right)' + v'\mu_t + u'z_t + \delta_{t-1}'z_t\right]\right) \\
&= \exp\left(v'\Phi y_{t-1}\right)E_{t-1}\left(\exp\left[v'\Omega_t\nu_t + \frac{1}{2}v'\Sigma_t v + v'\mu_t + u'z_t + \delta_{t-1}'z_t\right]\right).
\end{aligned}$$

Using the expression given for $A_{i,t-1}(u,v)$ in sec. 2.3.2.1 leads to the result.

2.A.3. Pdf under the $\mathbb{Q}$ world

By definition,

$$f_{\mathbb{Q}}(w_{1,t}|\,\underline{w}_{t-1})f_{\mathbb{Q}}(w_{2,t}|\,w_{1,t},\underline{w}_{t-1}) = f(w_{1,t}|\,\underline{w}_{t-1})f(w_{2,t}|\,w_{1,t},\underline{w}_{t-1})M_{t-1,t}\exp(r_{t-1}).$$

Integrating both sides w.r.t. $w_{2,t}$ and using the fact that $M_{t-1,t}$ does not depend on $w_{2,t}$, we get the expression of $f_{\mathbb{Q}}(w_{1,t}|\,\underline{w}_{t-1})$ and, as a consequence,

$$f(w_{2,t}|\,w_{1,t},\underline{w}_{t-1}) = f_{\mathbb{Q}}(w_{2,t}|\,w_{1,t},\underline{w}_{t-1}).$$

2.A.4. The risk-neutral Laplace transform of $(z_t, y_t, x_{n,t})$

In this appendix, we compute $E^{\mathbb{Q}}_{t-1}\left(\exp\left[u'z_t + v'y_t + w'x_{n,t}\right]\right)$ and show that it is exponential affine in $(z_{t-1}, y_{t-1}, x_{n,t-1})$, that is, we show that $(z_t, y_t, x_{n,t})$ is Car(1) (see Darolles, Gourieroux and Jasiak, 2006).

$$
\begin{aligned}
& E^{\mathbb{Q}}_{t-1}\left(\exp\left[u'z_t + v'y_t + w'x_{n,t}\right]\right) \\
= \;& E^{\mathbb{Q}}_{t-1}\left(\exp\left[u'z_t + v'y_t + w'\left(q_{1n}\left(z_t, z_{t-1}\right) + \right.\right.\right. \\
& \left.\left.\left. Q_{2n}y_t + Q_{3n}y_{t-1} + Q_{4n}x_{n,t-1} + Q_{5n}\left(z_t, z_{t-1}\right)\eta_{n,t}\right)\right]\right) \\
= \;& \exp\left(w'Q_{3n}y_{t-1} + w'Q_{4n}x_{n,t-1}\right) \times \\
& E^{\mathbb{Q}}_{t-1}\left(\exp\left[u'z_t + (v' + w'Q_{2n})y_t + \right.\right. \\
& \left.\left. w'q_{1n}\left(z_t, z_{t-1}\right) + w'Q_{5n}\left(z_t, z_{t-1}\right)\eta_{n,t}\right]\right) \\
= \;& \exp\left(w'Q_{3n}y_{t-1} + w'Q_{4n}x_{n,t-1}\right) \times \\
& E^{\mathbb{Q}}_{t-1}\left(\exp\left[u'z_t + w'q_{1n}\left(z_t, z_{t-1}\right) + w'Q_{5n}\left(z_t, z_{t-1}\right)\eta_{n,t} + \right.\right. \\
& \left.\left. (v' + w'Q_{2n})\left((\mu_t + \mu^*_t) + (\Phi + \Phi^*)\,y_{t-1} + \Omega_t\varepsilon^*_t\right)\right]\right) \\
= \;& \exp\left[\left\{(v' + w'Q_{2n})\left(\Phi + \Phi^*\right) + w'Q_{3n}\right\}y_{t-1} + \right. \\
& \left. w'Q_{4n}x_{n,t-1} + \begin{pmatrix} \tilde{A}_1(u,v,w) & \ldots & \tilde{A}_J(u,v,w) \end{pmatrix} z_{t-1}\right]
\end{aligned}
$$

Using the expression given for $\tilde{A}_i(u,v,w)$ in Proposition 3 leads to the result.

2.A.5. Multi-horizon Laplace transform of a Car(1) process

Let us consider a multivariate Car(1) process Z_t and its conditional Laplace transform given by $\exp\left[a'(s)Z_t + b(s)\right]$. Let us further denote by $L_{t,h}(\omega)$ its multi-horizon Laplace transform given by:

$$
L_{t,h}(\omega) = E_t\left[\exp\left(\omega'_{H-h+1}Z_{t+1} + \ldots + \omega'_H Z_{t+h}\right)\right],\; t = 1,\ldots,T,\; h = 1,\ldots,H,
$$

where $\omega = (\omega'_1, \ldots, \omega'_H)$ is a given sequence of vectors. We have, for any t,

$$
L_{t,h}(\omega) = \exp\left(A'_h Z_t + B_h\right),\; h = 1,\ldots,H,
$$

where the sequences A_h, B_h, $h = 1, \ldots, H$ are obtained recursively by:

$$\begin{aligned} A_h &= a(\omega_{H-h+1} + A_{h-1}) \\ B_h &= b(\omega_{H-h+1} + A_{h-1}) + B_{h-1}, \end{aligned}$$

with the initial conditions $A_0 = 0$ and $B_0 = 0$.

Proof. The formula is true for $h = 1$ since:

$$L_{t,1}(\omega) = E_t\left(\omega'_H Z_{t+1}\right) = \exp\left[a'(\omega_H) Z_t + b(\omega_H)\right]$$

and therefore $A_1 = a(\omega_H)$ and $B_1 = b(\omega_H)$.

If it is true for $h-1$, we get:

$$\begin{aligned} L_{t,h}(\omega) &= E_t\left[\exp\left(\omega'_{H-h+1} Z_{t+1}\right) E_{t+1}\left(\exp\left(\omega'_{H-h+2} Z_{t+2} + \ldots + \omega'_H Z_{t+H}\right)\right)\right] \\ &= E_t\left[\exp\left(\omega'_{H-h+1} Z_{t+1}\right) L_{t+1,h-1}(\omega)\right] \\ &= \exp\left[a(\omega'_{H-h+1} + A_{h-1}) Z_t + b(\omega'_{H-h+1} + A_{h-1}) + B_{h-1}\right] \end{aligned}$$

and the result follows. □

2.B. Kitagawa-Hamilton algorithm for partially-hidden Markov chains

In this appendix, we describe how to use the Hamilton's (1990) algorithm within the estimation strategy presented in Section 2.6, when the Markov chain is partially observed. As noted by Hamilton (1994), while the algorithm was originally presented in a model with fixed transition probabilities, it readily generalizes to processes in which transition probabilities depend on a vector of observed variables.[20]

[20]See e.g. Filardo (1994) ord Diebold, Lee and Weinbach (1993) for implementation examples of Hamilton's algorithm in models with time-varying transition probabilities. For introductions to regime-switching models, see Hamilton (1994) or Kim and Nelson (1999).

Let us denote with $\hat{y}_t$ the vector of observed variables $(\tilde{y}_t', R_{1t}, z_{1t}')'$. The Hamilton's algorithm consists in computing recursively the probabilities $p(z_{2t} \mid \underline{\hat{y}}_t)$. As a by product, the algorithm provides the conditional densities $f(\hat{y}_t \mid \underline{\hat{y}}_{t-1})$, which makes it possible to estimate the model parameters by maximization of the log-likelihood. The algorithm is based on the iterative implementation of the following steps (the input being $p(z_{2t-1} \mid \underline{\hat{y}}_{t-1})$):

1. The joint probability $p\left(z_{2t}, z_{2t-1} \mid \hat{y}_{t-1}\right)$ is computed using:

$$p\left(z_{2t}, z_{2t-1} \mid \underline{\hat{y}}_{t-1}\right) = p\left(z_{2t} \mid z_{2t-1}, \underline{\hat{y}}_{t-1}\right) \times p\left(z_{2t-1} \mid \underline{\hat{y}}_{t-1}\right)$$

   where the first term of the right-hand side is a sum of entries of the transition matrix $\{\pi_{ij,t-1}\}$ and the second term is the input.

2. The joint conditional density $f(\hat{y}_t, z_{2t}, z_{2t-1} \mid \underline{\hat{y}}_{t-1})$ is then given by:

$$f(\hat{y}_t, z_{2t}, z_{2t-1} \mid \underline{\hat{y}}_{t-1}) = f(\hat{y}_t \mid z_{2t}, z_{2t-1}, \underline{\hat{y}}_{t-1}) \times p\left(z_{2t}, z_{2t-1} \mid \underline{\hat{y}}_{t-1}\right)$$

   where

$$\begin{aligned} f(\hat{y}_t \mid z_{2t}, z_{2t-1}, \underline{\hat{y}}_{t-1}) &= f(\tilde{y}_t, R_{1t}, z_{1t} \mid z_{2t}, z_{2t-1}, \underline{\hat{y}}_{t-1}) \\ &= f(\tilde{y}_t, R_{1t} \mid z_{1t}, z_{2t}, z_{2t-1}, \underline{\hat{y}}_{t-1}) \times p(z_{1t} \mid z_{2t}, z_{2t-1}, \underline{\hat{y}}_{t-1}) \end{aligned}$$

   with

$$p(z_{1t} \mid z_{2t}, z_{2t-1}, \underline{\hat{y}}_{t-1}) = \frac{p(z_{1t}, z_{2t} \mid z_{2t-1}, \underline{\hat{y}}_{t-1})}{p(z_{2t} \mid z_{2t-1}, \underline{\hat{y}}_{t-1})}$$

   and all the terms can be computed.

3. The conditional density$f(\hat{y}_t \mid \underline{\hat{y}}_{t-1})$ is given by:

$$f(\hat{y}_t \mid \underline{\hat{y}}_{t-1}) = \sum_{z_{2,t}} \sum_{z_{2,t-1}} f(\hat{y}_t, z_{2t}, z_{2t-1} \mid \underline{\hat{y}}_{t-1}).$$

4. The joint density $p\left(z_{2t}, z_{2t-1} \mid \underline{\hat{y}}_t\right)$ comes from:

$$p\left(z_{2t}, z_{2t-1} \mid \underline{\hat{y}}_t\right) = \frac{f(\hat{y}_t, z_{2t}, z_{2t-1} \mid \underline{\hat{y}}_{t-1})}{f(\hat{y}_t \mid \underline{\hat{y}}_{t-1})}.$$

5. And eventually:

$$p\left(z_{2t} \mid \underline{\hat{y}}_t\right) = \sum_{z_{2,t-1}} p\left(z_{2t}, z_{2t-1} \mid \underline{\hat{y}}_t\right).$$

2.C. Inversion techniques in the presence of unobserved regimes

In this appendix, we detail an approach using jointly the Kitagawa-Hamilton filter and the so-called inversion techniques *à la* Chen and Scott (1993). Such an approach is aimed at estimating models in which there are both latent factors ($y_{2,t}$) and latent regimes ($z_{2,t}$) (see Section 2.6 for notations). Note that the implementation of the following estimation strategy requires that the transition probabilities do not depend on the unobserved vectors $y_{2,t-1}$. The period of observation is $\{1, \ldots, T\}$.

2.C.1. Decomposition of the joint p.d.f. and estimation strategy

Let us denote by θ^{zy} the vector of parameters defining the historical dynamics of (z_t, y_t), by θ^x_n the vector of parameters defining the conditional p.d.f. of $x_{n,t}$ given $\underline{z}_t, \underline{y}_t, \underline{x}_{n,t-1}$ and by θ^d_n the vector of parameters defining the conditional p.d.f. of $d_{n,t}$ given $\underline{z}_t, \underline{y}_t, \underline{x}_{n,t}, \underline{d}_{n,t-1}$.

The joint p.d.f. of $\underline{w}_T$ is:

$$\begin{aligned} f(\underline{w}_T, \theta) \quad = \quad & \prod_{t=1}^{T} f\left(z_t, y_t \mid \underline{z}_{t-1}, \underline{y}_{t-1}; \theta^{zy}\right) \\ & \times \prod_{n=1}^{N} \prod_{t=1}^{T} f\left(x_{n,t} \mid \underline{z}_t, \underline{y}_t, \underline{x}_{n,t-1}; \theta_n^x\right) \\ & \times \prod_{n=1}^{N} \prod_{t=1}^{T} f\left(d_{n,t} \mid \underline{z}_t, \underline{y}_t, \underline{x}_{n,t}, \underline{d}_{n,t-1}; \theta_n^d\right). \end{aligned}$$

The parameters appearing in $M_{t-1,t}$ are denoted by θ^*. The theoretical values of R_t and R_{tn}^D given by the model are denoted by $R_t(\theta^{zy}, \theta^*)$ and $R_{nt}^D\left(\theta^{zy}, \theta_n^x, \theta_n^d, \theta^*\right)$ respectively. A sequential strategy of estimation is the following:

1. Estimate θ^{zy} and θ^* from the observations of y_{1t}, z_{1t}, R_t, $t = 1, \ldots, T$.
2. Estimate the θ_n^x's and the θ_n^d's from the observations of $x_{1n,t}$ and $R_{n,t}^D$, $t = 1, \ldots, T$, taking as given the values of θ^{zy} and θ^*, and the values of $y_{2,t}$ and $z_{2,t}$ being fixed at the approximated values obtained from step 1.

The remaining of the current section details these two steps. The methodology that is proposed builds on the so-called inversion technique developed by Chen and Scott (1993). This technique is adapted in order to accommodate regime switching.

2.C.2. Estimation of the parameters (θ^{zy}, θ^*)

Using equation (2.16), we have, with obvious notations:

$$R_t(\theta^{zy}, \theta^*) = Az_t + B_1 y_{1,t} + B_2 y_{2,t}.$$

If m is the dimension of y_{2t}, let us partition R_t in $\left(R_{1,t}', R_{2,t}'\right)'$ where $R_{2,t}$ is of dimension m. With obvious notations, we get:

$$R_{2,t}(\theta^{zy}, \theta^*) = A_2 z_t + B_{21} y_{1,t} + B_{22} y_{2,t},$$

and denoting $\left(y_{1,t}^{'}, R_{2,t}^{'}\right)^{'}$ by $\tilde{y}_t$ we get:

$$\tilde{y}_t = \begin{pmatrix} I & 0 \\ B_{21} & B_{22} \end{pmatrix} y_t + \begin{pmatrix} 0 \\ A_2 \end{pmatrix} z_t$$

or

$$\tilde{y}_t = \tilde{B} y_t + \tilde{A} z_t$$

and

$$y_t = \tilde{B}^{-1} \left(\tilde{y}_t - \tilde{A} z_t\right)$$

and from equation (2.2) we get:

$$\tilde{B}^{-1} \left(\tilde{y}_t - \tilde{A} z_t\right) = \mu\left(z_t, z_{t-1}\right) + \Phi \left[\tilde{B}^{-1}\left(\tilde{y}_{t-1} - \tilde{A} z_{t-1}\right)\right] + \Omega\left(z_t, z_{t-1}\right) \varepsilon_t$$

or

$$\tilde{y}_t = \tilde{A} z_t + \tilde{B} \mu\left(z_t, z_{t-1}\right) + \tilde{B} \Phi \left[\tilde{B}^{-1}\left(\tilde{y}_{t-1} - \tilde{A} z_{t-1}\right)\right] + \tilde{B} \Omega\left(z_t, z_{t-1}\right) \varepsilon_t$$

or

$$\tilde{y}_t = \tilde{\mu}\left(z_t, z_{t-1}\right) + \tilde{\Phi} \tilde{y}_{t-1} + \tilde{\Omega}\left(z_t, z_{t-1}\right) \varepsilon_t, \tag{2.22}$$

with

$$\begin{cases} \tilde{\mu}\left(z_t, z_{t-1}\right) & = \tilde{A} z_t + \tilde{B} \mu\left(z_t, z_{t-1}\right) - \tilde{B} \Phi \tilde{B}^{-1} \tilde{A} z_{t-1} \\ \tilde{\Phi} & = \tilde{B} \Phi \tilde{B}^{-1} \\ \tilde{\Omega}\left(z_t, z_{t-1}\right) & = \tilde{B} \Omega\left(z_t, z_{t-1}\right). \end{cases}$$

The conditional distribution of $\tilde{y}_t$ given $\underline{z}_t, \underline{\tilde{y}}_{t-1}$, is similar to that of y_t given $\underline{z}_t, \tilde{y}_{t-1}$, and in particular is Gaussian, the difference being that $\tilde{y}_t$ is fully observable.

Assuming moreover that the $R_{1,t}$ are observed with Gaussian errors we get, with obvious notations:

$$\begin{aligned} R_{1,t} &= A_1 z_t + B_{11} y_{1,t} + B_{12} y_{2,t} + \xi_t \\ &= A_1 z_t + B_{11} y_{1,t} \\ &\quad + B_{12} B_{22}^{-1} \left(R_{2t} - A_2 z_t - B_{21} y_{1,t} \right) + \xi_t, \end{aligned} \tag{2.23}$$

with $\xi_t \sim IIN\left(0, \sigma^2 I\right)$.

Putting equations (2.22),(2.23) and (2.1) together, we have a dynamic model in which the only latent variables are $z_{2,t}$ and which can be estimated by the maximum likelihood methods using Hamilton's approach. At this stage, IC constraints on (θ^{zy}, θ^*) must be taken into account.

2.C.3. Estimation of $\left(\theta_n^x, \theta_n^d\right)$

From the inversion method of 2.C.2, we can get approximations of the $y_{2,t}$'s and smoothing algorithms provide approximations of the $z_{2,t}$'s (see Kim, 1994) the z_{2t} are replaced by those states presenting the highest smoothed probabilities. Then using equation (2.16), we get:

$$R_{t,n}^D = C_1^n z_{1,t} + C_2^n z_{2,t} + D_1^n y_{1,t} + D_2^n y_{2,t} + F_1^n x_{1,n,t} + F_2^n x_{2,n,t}. \tag{2.24}$$

and using equations (2.2), (2.3) and (2.24) and replacing $y_{2,t}$ and $z_{2,t}$ by their approximations, we get a system in which the only latent variables are the $x_{2,n,t}$. Taking θ^{zy} and θ^* as given, the parameters θ_n^x and θ_n^d can be estimated either by an inversion technique or by Kalman filtering, taking into account IC conditions.

Note that in this strategy, the observable variables $d_{n,t}$'s have not been used. If the recovery rate was effectively zero, $\lambda_{n,t}$ would be the default intensity and the conditional p.d.f. of $d_{n,t}$ given $\underline{z}_t, \underline{y}_t, \underline{x}_{n,t}, d_{n,t-1}$ would be:

$$d_{n,t} d_{n,t-1} + \left(1 - d_{n,t-1}\right) \exp\left[-\left(1 - d_{n,t}\right) \lambda_{n,t}\right] \times \left[1 - \exp\left(-\lambda_{n,t}\right)\right]^{d_{n,t}}.$$

This p.d.f. could be incorporated in the likelihood function. However, in the more realistic case of non-zero recovery rate, we have seen that (see Subsection 2.4.2) the $\lambda_{n,t}$'s must be interpreted as risk-neutral "recovery adjusted" default intensities and, therefore, they cannot be used for describing the historical dynamics of the $d_{n,t}$'s.

2.D. Estimation example: U.S. BBB-AAA corporate spreads

2.D.1. State-space model

The model introduced in 2.6.3 can be written as a state-space system for the purpose of estimation. Let denote by s_t the 4×1 vector containing the BBB-Treasury spreads with respective maturities of 1, 2, 3 and 5 years. Using a matrix representation, the measurement equations of the state-space model are given by:

$$s_t = cz_t + fy_t + \varepsilon_{err,t},$$

where the $\varepsilon_{err,t}$ are some i.i.d. pricing (measurement) errors and where the matrices c and f, that are respectively of dimension 4×3 (because there are three regimes) and 4×2 (because there are two factors $y_{i,t}$), are computed by applying the recursive pricing formulas introduced in Proposition 3. Because both $y_{1,t}$ and $y_{2,t}$ are unobserved, the transition equations read:

$$y_t = \mu z_t + \Phi y_{t-1} + \Omega \varepsilon_t,$$

where μ, Φ and Ω are constrained along the lines presented in 2.6.3. The state-space model is completed by the specification of the matrix of regime-switching probabilities $\{\pi_{i,j}\}$.

2.D.2. Estimation results

The estimation is conducted by maximizing the log-likelihood (approximated by the filter proposed by Kim, 1994). Some of the parameters are calibrated. First, the unconditional variance of the first (purely Gaussian) factor $y_{1,t}$ is constrained to be relatively small with comparison to the overall standard deviation of the spreads, so as to make sure that most of the spread fluctuations are to be explained by the second factor $y_{2,t}$. Specifically, the standard deviation of $y_{1,t}$ is set to 10 bp. Alternative estimations have shown that the results are fairly robust to this first choice. Second, the matrix of probabilities of regime shifts is parameterized so as to be consistent with the regimes' interpretation. The "tranquil" regime is supposed to be persistent and to prevail 50% of the time. By contrast, the crisis regime is supposed to be relatively short-lived (with an average length of 4 weeks) and to prevail only 5% of the time. Formally, these constraints mean that (a) the ergodic distribution of the Markov chain is [50%,45%,5%]' and that (b) the third diagonal entry of the matrix of transition probabilities (i.e. p_{33}) is such that $4 = 1/(1 - p_{33})$ (4 weeks = average length of the third regime). The resulting matrix of transition probabilities (under the historical measure) is:

$$\begin{bmatrix} 0.976 & 0.024 & 0 \\ 0.027 & 0.945 & 0.028 \\ 0 & 0.250 & 0.750 \end{bmatrix}.$$

The estimated dynamics of y_t under the historical and the risk-neutral measures are, respectively (standard errors, based on the outer-product estimate of the

Fisher information matrix, are reported in parentheses below the point estimates):

$$y_t = \begin{bmatrix} \underset{(2111)}{0.023} & \underset{(2111)}{0.023} & \underset{(2111)}{0.023} \\ 0 & 0 & \underset{(0.007)}{0.219} \end{bmatrix} z_t + \begin{bmatrix} \underset{(0.010)}{0.841} & 0 \\ 0 & \underset{(0.001)}{0.981} \end{bmatrix} y_{t-1} + \begin{bmatrix} \underset{(0.001)}{0.053} & 0 \\ 0 & 0 \end{bmatrix} \varepsilon_t$$

$$y_t = \begin{bmatrix} \underset{(0.0001)}{-0.0029} & \underset{(0.0001)}{-0.0029} & \underset{(0.0001)}{-0.0029} \\ 0 & 0 & \underset{(0.0001)}{0.0063} \end{bmatrix} z_t +$$
$$\begin{bmatrix} \underset{(0.000)}{1} & 0 \\ 0 & \underset{(0.000)}{1} \end{bmatrix} y_{t-1} + \begin{bmatrix} \underset{(0.001)}{0.053} & 0 \\ 0 & 0 \end{bmatrix} \varepsilon_t^*,$$

where ε_t and ε_t^* are i.i.d. normally distributed shocks under the historical and the risk-neutral measures, respectively. Besides, the hazard rate is

$$\lambda_t = \underset{(0.024)}{0.622} + y_{1,t} + y_{2,t}.$$

The risk-neutral probabilities of transition (the $\pi^*_{i,j}$'s) are estimated via MLE (together with the parameterization of the dynamics of y_t):

$$\begin{bmatrix} \underset{(0.001)}{0.988} & \underset{(0.001)}{0.012} & 0 \\ \underset{(0.014)}{0.000} & \underset{(0.534)}{0.511} & \underset{(0.524)}{0.488} \\ 0 & \underset{(0.007)}{0.000} & \underset{(0.007)}{1.000} \end{bmatrix}.$$

Finally, the pricing-error standard-deviation estimate (i.e. the standard deviation of the $\varepsilon_{err,t}$'s defined in 2.D.1) is 0.08%, or 8 basis points (the standard deviation of the parameter estimate is 0.001%, or 0.1 bp).

2.E. About the eigenvectors of the rating-migration matrix Π

In this appendix, using the notations presented in Subsection 2.8.4, we outline some properties of matrices Π and V. For notational simplicity, we drop arguments and time subscripts associated with these matrices.

- As the sum of the entries of each line of Π is equal to 1, the vector $\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}'$ is an eigenvector of Π associated with the eigenvalue 1. Consequently, since this eigenvalue is supposed to be the last one appearing in Ψ, the last column of V –that collects the eigenvectors of Π– is proportional to $\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}'$.
- The fact that default is an absorbing state implies that the last row of Π is $\begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}$. Since we have $\Pi V = V\Psi$, it comes (considering the last line of this equation):

  $$\forall j \quad V_{K,j} = V_{K,j} \exp\left(-\psi_j\right),$$

  which implies: $\forall j < K,\ V_{K,j} = 0$.
- The two previous points imply that the matrix V admits the following form:

  $$V = \begin{bmatrix} V_{1,1} & \cdots & V_{1,K-1} & \gamma \\ \vdots & \ddots & \vdots & \vdots \\ V_{K-1,1} & \cdots & V_{K-1,K-1} & \gamma \\ 0 & \cdots & 0 & \gamma \end{bmatrix}$$

  Since $VV^{-1} = I$, we have (considering the last line and using the notation $V_{i,j}^{-1}$ for the entry (i,j) of V^{-1})

  $$\begin{bmatrix} V_{K,1}^{-1} & \cdots & V_{K,,K-1}^{-1} & V_{K,K}^{-1} \end{bmatrix} = \begin{bmatrix} 0 & \cdots & 0 & \frac{1}{\gamma} \end{bmatrix}$$

  and, therefore, for $i = 1, \ldots, K$, we have $V_{i,K} V_{K,K}^{-1} = 1$.

Figure 2.3.: BBB vs. Treasury Spreads, Simulations

Notes: This Figurecompares the distributions of spreads simulated by different models (with the sample distributions of spreads, the sample covering the period from March 1995 to July 2011). Three alternative models are used: the regime-switching one (presented in 2.D) and two "purely Gaussian" models (involving respectively one and two AR(1) factors). Simulations are based on 50.000 replications of each models. The lower row of panels present the term-structures of the spreads (observed for the left plot and implied by the models for the other plots); for each panel, the grey shaded area is delimited by the 5th and the 95th percentiles of the spreads at each considered maturity. In addition, the lower-row plots present the term structures of medians and means of the spreads.

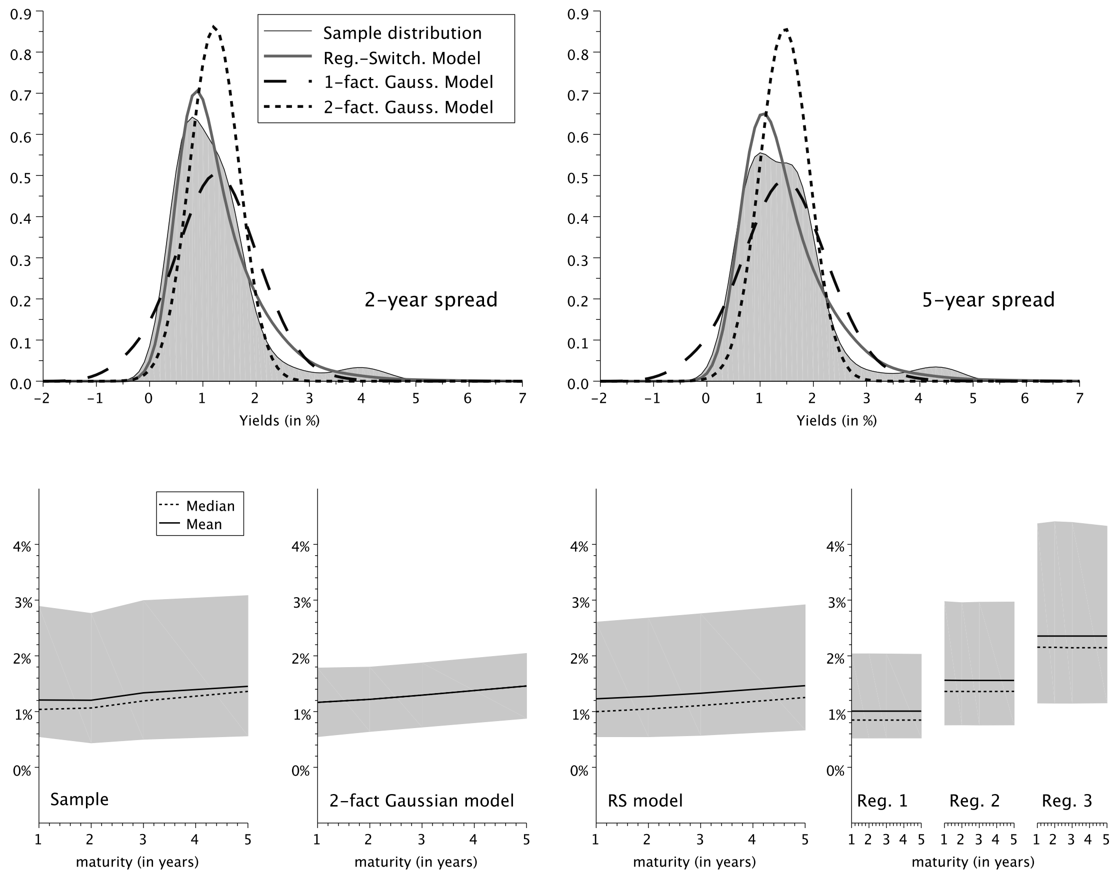


Figure 2.4.: Simulated sample of the sector-contagion model

Notes: Each panel corresponds to one sector. There are 600 debtors in the portfolio (200 per sector). The vertical bars represent the number of firms that have defaulted during the considered period. At the end of each period, defaulted firms are replaced by new ones (of the same sector). Grey-shaded areas indicate periods during which the considered sector is in distress. Darker areas indicate periods when all three sectors are in distress.

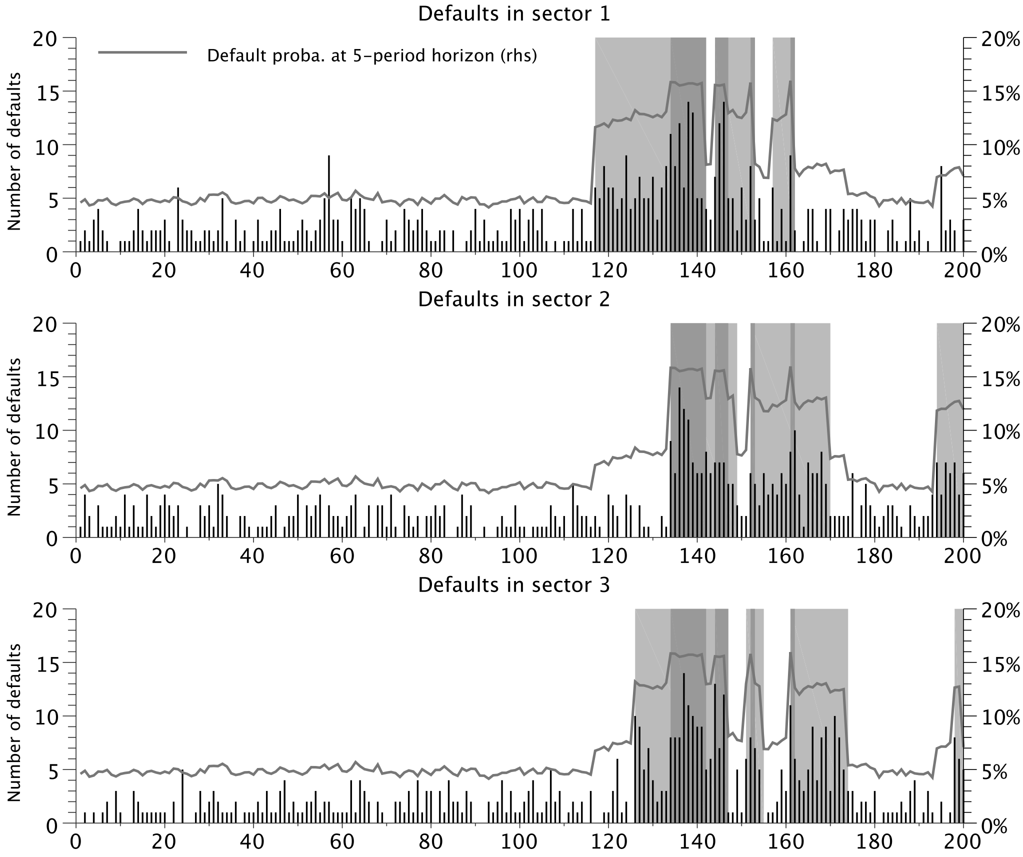


Figure 2.5.: Yield curves for selected ratings (with impact of regimes)

Notes: The left plot shows yield curves for selected ratings, with $y_t = 0$ and $z_t = [1, 0]'$ (solid lines) or $z_t = [0, 1]'$ (dashed lines). The right plot shows the term structure of spreads vs. Aaa-rated bonds.

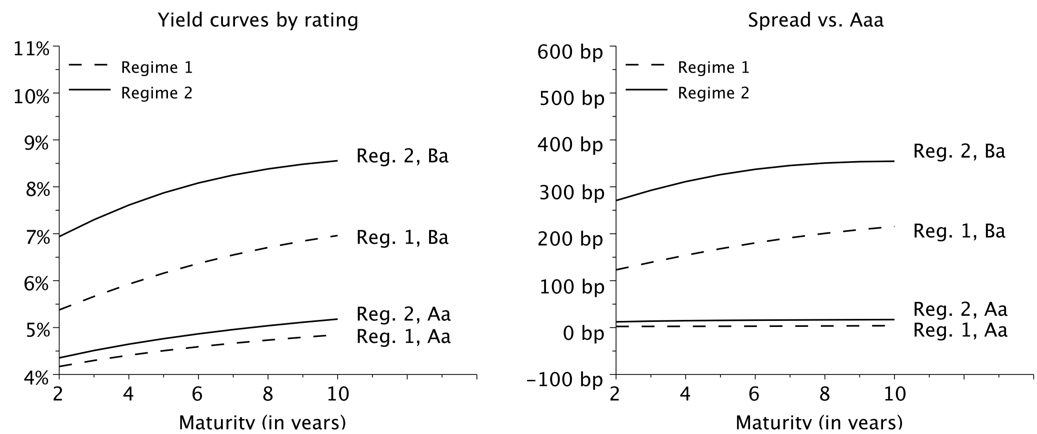
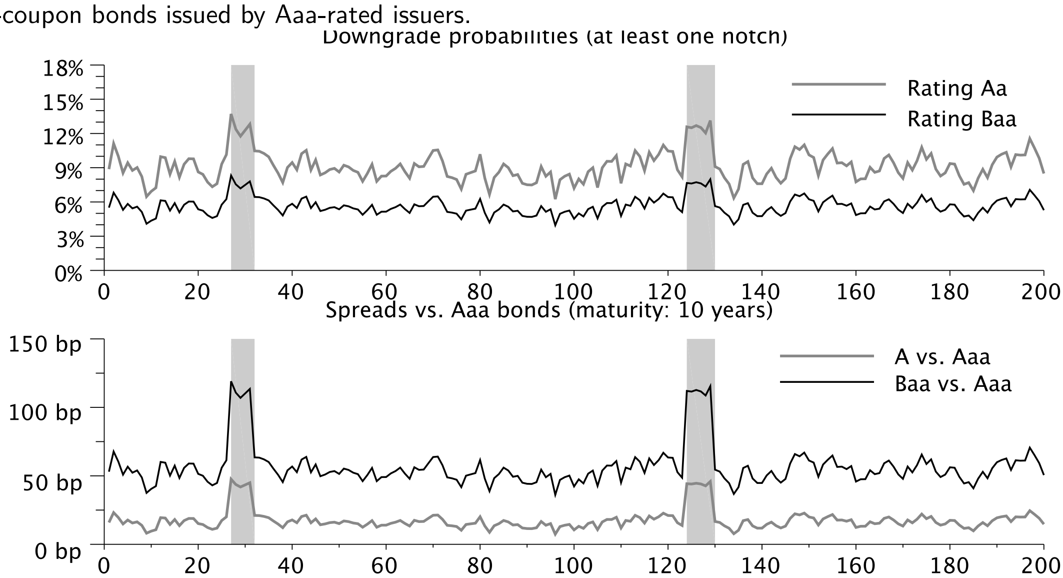


Figure 2.6.: Simulated downgrade probabilities and spreads

Notes: The upper plot shows simulated downgrade probabilities for two ratings (the downgrade can be of one or more notches). Formally, for rating j, the upper panel plots $P(\tau_{n,t} > \tau_{n,t-1} \mid \underline{z}_t, \underline{y}_t, \underline{x}_{n,t}, \tau_{n,t-1} = j)$. The grey-shaded areas indicate "crisis" periods. The lower plot shows the yield spreads between 10-year zero-coupon bonds issued by A-rated or Baa-rated debtors and zero-coupon bonds issued by Aaa-rated issuers.



3. Credit and liquidity risks in euro-area sovereign yield curves[1]

Abstract: In this Chapter, we propose a model of the joint dynamics of euro-area sovereign yield curves. The arbitrage-free valuation framework involves five factors and two regimes, one of the latter being interpreted as a crisis regime. Each country is characterized by a hazard rate, specified as a linear combination of the factors and the regimes. The hazard rates incorporate both liquidity and credit components, that we aim at disentangling. We find that a substantial share of the changes in euro-area yield differentials is liquidity-driven.

[1]This Chapter is based on an article co-authored with Alain Monfort and entitled “Credit and liquidity risks in euro-area sovereign yield curves”. We are grateful to Christian Gourieroux, Glenn Rudebusch, Damiano Brigo, Olesya Grishchenko, Vladimir Borgy, Valère Fourel, Wolfgang Lemke, Simon Gilchrist, Kristoffer Nimark, Tao Zha, Christian Hellwig, Jean-Sébastien Fontaine and Adrien Verdelhan for helpful discussions and comments. We are also grateful to participants at the Banque de France internal seminar, at the C.R.E.D.I.T. conference 2010, at CREST seminar 2010, at the Paris finance international meeting 2010, at CORE Econometrics Seminar 2011, at 2011 ESEM meeting, at 2011 IESEG-University of Cambridge conference on yield-curve modelling, at AFSE annual meeting, at the Bank of England seminar, at CDC seminar and at the ECB Workshop on Asset pricing models in the aftermath of the financial crisis, at Computational and Financial Econometrics conference (London 2011), at Bank of Canada seminar. We thank Beatrice Saes-Escorbiac and Aurélie Touchais for excellent research assistance. Any remaining errors are ours. The views expressed in this Chapter are ours and do not necessarily reflect the views of the Banque de France.

Résumé

Ce chapitre propose une modélisation jointe des dynamiques de courbes de taux de dix Etats de la zone euro sur la période 1999-2012. Ce modèle, qui exclut les opportunités d'arbitrage, est un cas particulier du cadre général proposé dans le chapitre précédent. Chaque pays est caractérisé par une intensité de crédit –reflétant son risque de défaut– et une intensité de liquidité –reflétant la valorisation de la relative illiquidité des titres émis par ces Etats. Chacune de ces intensités est une combinaison linéaire de cinq facteurs ainsi que d'un vecteur de sélection de dimension 2×1 indiquant quel régime prévaut à chaque date, l'un de ces deux régimes étant interprété comme un régime de crise. Les facteurs et les régimes sont communs aux dix pays.

Les cinq facteurs sont eux-mêmes des combinaisons linéaires de taux d'intérêt observés. Les trois premiers sont tirés de taux d'Etat allemand, qui sont considérés comme des taux sans risque : le premier correspond au taux à 10 ans, le second est un facteur de pente (différence entre un taux long et taux court) et le troisième est un facteur de convexité (position d'un taux de maturité intermédiaire par rapport à la moyenne d'un taux long et d'un taux court). Le quatrième et le cinquième facteurs sont les deux premières composantes principales d'un échantillon de quatre séries temporelles d'écarts de taux; ce sont les écarts entre les taux à 10 ans français, espagnols, italiens et hollandais d'une part et le taux à 10 ans allemand d'autre part.

Dans ce modèle, les taux sont des combinaisons linéaires des facteurs et des régimes. Les facteurs sont donc des fonctions de taux d'intérêt qui dépendent eux-mêmes des facteurs. Aussi, une procédure d'estimation spécifique doit-elle être définie pour respecter la cohérence interne du modèle. Cette procédure est mise en oeuvre lors de l'estimation de la dynamique risque-neutre des facteurs. Une méthode spécifique, fondée sur une utilisation de la méthode des moments généralisés, est proposée. Le calcul des écarts-types des paramètres estimés via cette méthode est détaillé.

La dynamique historique des facteurs est estimée séparément de la dynamique

risque-neutre. Il a été montré que l'on tend à sous-estimer la persistence des processus en échantillon fini (voir notamment Jardet, Monfort et Pegoraro, 2013). Afin de traiter ce problème, qui a d'importantes conséquences pour l'estimation des primes de risque, nous mettons en oeuvre la méthode proposée par Kim et Orphanides (2012). Cette méthode consiste à pénaliser, lors de l'estimation du modèle, ces combinaisons de paramètres qui impliquent que les prévisions de taux fondées sur le modèle sont éloignées de celles réalisées par les prévisionnistes. Nous utilisons trois séries de prévisions: l'une pour le premier facteur (taux long allemand) et les deux autres pour le quatrième et le cinquième facteur (composantes principales d'écarts de taux). Ces séries temporelles de prévisions sont issues du *Consensus Forecast*.

L'estimation suggère que l'existence du régime de crise est clé pour expliquer l'accroissement de la volatilité des taux sur la période récente. L'ajustement des données est de bonne qualité. En moyenne à travers les pays et les maturités, l'écart-type des erreurs de mesure de taux d'intérêt est de 18 points de base (0.18%).

A ce stade, on dispose d'un modèle dans lequel chaque taux d'intérêt est une combinaison linéaire des cinq facteurs et de la variable de régime. Il reste ensuite à décomposer chaque taux d'intérêt en une composante crédit et une composante liquidité. Cela revient à décomposer en deux parties l'intensité globale associée à chaque pays. Notre stratégie d'identification de la partie liquidité de chaque intensité repose sur l'interprétation de l'écart de taux *KfW-Bund*. *KfW* est une banque publique allemande dont les titres sont complètement et explicitement garantis par l'état fédéral allemand. Aussi, les *Bunds*, qui sont les obligatons émises par l'Etat fédéral allemand, et les obligations émises par *KfW* bénéficient-elles de la même qualité de crédit. En conséquence, l'écart de taux entre ces titres reflète essentiellement la valorisation de la différence de liquidité entre les deux types de titres. Nous vérifions que les écarts de taux *KfW-Bund* sont très corrélés avec le même type d'écarts de taux relatifs à d'autres pays de la zone euro. Nous en déduisons que la valorisation de la liquidité obligataire en zone euro repose sur un unique facteur de liquidité. L'identification de celui-ci est réalisée en incluant la structure par terme associée aux obligations émises par *KfW* dans notre estimation. Il en résulte que le facteur de liquidité est lui aussi une combinaison des cinq facteurs et de la variable de régime. Pour chacun des autres pays, l'intensité d'illiquidité est obtenue comme une transformation affine du facteur de liquidité précédemment identifié. Cette transformation est estimée en maximisant la part des fluctuations de chaque intensité nationale (incluant crédit et liquidité) pouvant être expliquées par les fluctuations du facteur de liquidité, tout en intégrant des objectifs supplémentaires concernant la positivité des écarts de taux et des intensités de crédit (qui correspondent à des probabilités de défaut, qui sont donc positives).

Nos résultats suggèrent que la liquidité est déterminante pour expliquer les variations des prix obligataires. En particulier, jusqu'en 2009, les différences de liquidité expliquent la majeure partie des écarts de taux d'intérêt entre les pays les mieux notés par les agences de crédit (Allemagne, Autriche, Finlande, France et Pays-Bas). En revanche, depuis lors, c'est surtout la dégradation de la qualité de crédit vis-à-vis de celle de l'Allemagne –du moins telle que perçue par les marchés– qui explique le creusement des écarts de taux.

3.1. Introduction

One of the most spectacular symptoms of the crisis that began in mid-2007 is the dramatic rise in intra euro-area government-bond yield spreads. Whereas all euro-area sovereign 10-year bond yields were contained in a range of 50 bp between 2002 and 2007, the average spreads over Germany of only two countries were lower than 50 basis points in 2011, the debt-weighted mean being of about 250 bp. Since the inception of the euro in 1999 and the resulting elimination of exchange-rate risk, intra-euro-area spreads reflect the fluctuations of compensations demanded by investors for holding essentially two kinds of risks: credit and liquidity risks.[2] The credit risk is linked to the issuer's probability of default (PD). If investors assess that the PD of some indebted country is higher than in the past, the prices of the bonds issued by this country fall because the expected loss increases. Liquidity risk arises from the potential difficulty that one may have in selling the asset before its redemption (for instance if one is required to do so in distressed market conditions, where it is difficult to find a counterpart for trade relatively quickly). In many ways, the ongoing financial crisis has illustrated why, along with credit risk, liquidity risks matter and should not be underestimated (see Brunnermeier, 2009).

Disentangling credit and liquidity effects in bond prices is important in several respects. For instance, appropriate policy actions that may be needed to address a sharp rise in spreads depend on the source of the movement: if the rise in spreads reflects poor liquidity, policy actions should aim at improving market functioning. But if it is linked to credit concerns, the solvency of the debtors should be enhanced (see Codogno, Favero and Missale, 2003). Furthermore, optimal investment decisions would benefit from such a decomposition. In particular, those medium to long-term investors who buy bonds to hold them until redemption seek to buy bonds whose price is low because of poor liquidity, since it provides them

[2]Indeed, an overwhelming share of the euro-area sovereign debt is denominated in euros (see Eurostat, 2011). Note however that over the recent period, i.e. since Spring 2012 onward, there is evidence that there have been fears on the part of investors of the reversibility of the euro (see notably the speech by Draghi on 6 September 2012). This period is not covered by the empirical studies of this Chapter and the following.

with higher long-run returns than more liquid bonds with the same credit quality (see Longstaff, 2009).

In this chapter, we present a no-arbitrage affine term-structure model (ATSM) of the dynamics of ten euro-area sovereign yield curves. Jointly modelling these different yield curves allows us to identify and price credit and liquidity risk factors that are common to euro-area countries. Being euro area-wide, these risks can not be diversified away by the investors, who demand risk premia to be compensated for carrying them. The size and dynamics of such risk premia will be studied more in depth in the next Chapter, the present one focusing on the credit/liquidity decomposition.

The framework allows for transitions between tranquil and crisis periods, which is obviously well-suited to account for the fluctuations of yields and spreads over the last three years. In this reduced-form framework, the default probabilities are modeled directly instead of defining a stochastic process for the obligor's asset value that triggers default when the process reaches some threshold (as in Merton, 1974).[3] While the focus is on default modelling, the specifications account for the pricing of some liquidity premia, as originally proposed by Duffie and Singleton (1999) . The state variables, also named "risk factors", follow discrete-time inter-related Gaussian processes. Exploiting the framework developed in the previous Chapter, the Gaussian processes present drifts and variance-covariance matrices that are subject to regime shifts. The latter are described by a two-state Markov chain. The model is estimated using yield data covering the last twelve years. The five-factor and two-regime model accounts for more than 98% of the variances of yields driving eleven term structures of interest rates. The fact that a small set of factors is able to account for most of the fluctuations of sovereign spreads is consistent with findings by Geyer, Kossmeier and Pichler (2004) and, more recently, by Longstaff et al. (2011) .

[3] After having developed criteria to measure the performances of credit models in terms of default discrimination and relative value analysis, Arora, Bohn and Zhu (2005) compare structural (e.g. Merton's) and reduced-form models. Their results suggest that the reduced-form model outperforms the others when the issuer has many bonds in the market, which is typically the case for sovereign issuers.

In addition to the yield curves of ten euro-area countries, we model the yield curve of KfW (*Kreditanstalt für Wiederaufbau*), a German agency. We identify a liquidity-related pricing factor by exploiting the term structure of the the KfW-*Bund* spreads. Indeed, the bonds issued by KfW, guaranteed by the Federal Republic of Germany, benefit from the same credit quality than their sovereign counterparts –the *Bunds*– but are less liquid.[4] Therefore, the KfW-*Bund* spread should be essentially liquidity-driven.[5] The resulting liquidity-related factor contributes significantly to the dynamics of intra-euro spreads, supporting recent findings by Favero et al. (2010) or Manganelli and Wolswijk (2009).

We propose an efficient estimation method to bring the model to the data. The risk factors are some linear combinations of observed yields. Being observed, the estimation of the (historical) risk-factor dynamics boils down to the estimation of a Markov-switching vector-autoregression model. The regime-switching feature of the model turns out to be particularly relevant to account for the rise in volatility experienced by fixed-income markets over the last years.[6] The fact that the factors are observed yield combinations raises internal consistency issues when it comes to estimating their risk-neutral dynamics: the model has indeed to correctly price the bond portfolios that are reflected by these yield combinations. These internal-consistency restrictions are taken into account by our estimation procedure.

Our estimation dataset is supplemented with survey-based forecasts. As evidenced by Kim and Orphanides (2005), this alleviates the downward small-sample bias in the persitence of the yields obtained with conventional estimation.[7] Such biases typically result in too sTablelong-horizon expectations of yields and, as a consequence, overstate the variability of term premia. Generating reliable expectations is key if one wants to use the model to recover probabilities of default from bond

[4]By abuse of language, we use here the term *Bunds* for the German sovereign bonds of any maturity although this name is usually used for ten-year bonds only.

[5]See Schwarz (2009).

[6]The pricing framework allows for risk premiums demanded by the investors to be compensated for the systematic nature of the regime shifts. Regime shifts represent a systematic risk in the sense that this risk can not be diversified away.

[7]This way of reducing the bias is not the only one. In particular, Jardet, Monfort and Pegoraro (2009) use a "near-cointegrated framework" specification of the factors (averaging a stationary and a cointegrated specification).

prices (as will be done in the next Chapter).

This study contributes to the term-structure modelling literature in three main directions. First, we estimate an ATSM explicitly incorporating liquidity and credit aspects on European data, in a multi-country set up.[8] Second, we investigate the potential of the regime-switching feature in credit ATSM. Third, we propose an efficient estimation methodology, conveniently dealing with internal consistency problems and incorporating survey-based forecasts data.

The remaining of this Chapter is organized as follows. Section 3.2 presents the model and details how bonds are priced in this framework. Section 3.3 deals with the choice and the construction of the data. Section 3.4 presents the estimation of the model and Section 3.5 examines the implication of the model in terms of liquidity and credit pricing. Section 3.6 summarizes the results and makes concluding remarks.

3.2. The model

In this Section, we present the dynamics of the pricing factors and regimes. We consider three types of variables: five macroeconomic factors gathered in a vector $y_t = [y_{1,t}, y_{2,t}, y_{3,t}, y_{4,t}, y_{5,t}]'$, a regime variable z_t that can take two values: $[1, 0]'$ and $[0, 1]'$ and $d_t^{'} = (d_{1,t}, \ldots, d_{N,t})$, a set of binary variables indicating the default ($d_{n,t} = 1$) or the non-default ($d_{n,t} = 0$) states of the countries indexed by n. The next two subsections respectively describe the dynamics under the historical measure and under the risk-neutral measure. Then Subsection 3.2.3 deals with the hazard rates and, in particular, introduces the modelling of liquidity pricing.

3.2.1. Historical dynamics of factors (y_t) and regimes (z_t)

The conditional distribution of y_t given z_t is Gaussian and is given by:

[8] Geyer, Kossmeier and Pichler (2004) have also presented a multi-country ATSM. However, their model only accounts for the spreads' dynamics (which are supposed to be driven by factors that are independent from the the riskfree rates) and it does not explicitly accomodate liquidity-pricing effects.

$$\begin{bmatrix} y_{1,t} \\ \vdots \\ y_{p,t} \end{bmatrix} = \begin{bmatrix} \mu_{1,1} & \mu_{1,2} \\ \vdots & \vdots \\ \mu_{p,1} & \mu_{p,2} \end{bmatrix} z_t + \Phi \begin{bmatrix} y_{1,t-1} \\ \vdots \\ y_{p,t-1} \end{bmatrix} + \begin{bmatrix} \sigma'_{11} z_t & 0 & 0 \\ \vdots & \ddots & 0 \\ \sigma'_{p1} z_t & \cdots & \sigma'_{pp} z_t \end{bmatrix} \varepsilon_t$$
$$= \mu z_t + \Phi y_{t-1} + \Omega(z_t)\varepsilon_t, \tag{3.1}$$

where the ε_t's are independently and identically $N(0, I)$ distributed. It is a vector autoregressive model where the drift and the covariance matrix of the innovations are subject to regime shifts. The regime variable z_t follows a two-state Markov chain whose probabilities of transition are denoted with $\pi_{i,j}$. Formally:

$$P(z_t = j | z_{t-1} = i) = \pi_{i,j}. \tag{3.2}$$

Equation (3.1) implies that there is instantaneous causality between z_t and y_t, as in Ang, Bekaert and Wei (2008).[9] If country n has not defaulted before t, the conditional probability that country n defaults in time t is given by $1 - \exp(-\lambda^d_{n,t})$ where the default intensity $\lambda^d_{n,t}$ is a function of (z_t, y_t). Our framework builds on the "doubly stochastic" assumption, under which the default times of the different countries are correlated only as implied by the correlation of their default intensities. The default state is absorbing, in the sense that $d_{n,t} = 1$ implies $d_{n,t+h} = 1$ for any positive h.

The risk-free one-period rate r_{t+1}, that is the return of a one-period risk-free investment between t and $t+1$ (known in t) is a linear combination of y_t and z_t:

$$r_{t+1} = a_1 z_t + b_1 y_t.$$

[9] Ang et al. (2008) remark that instantaneous causality between z_t and y_t implies that the variances of the factors y_t, conditional on past values of (z_t, y_t), embed a jump term reflecting the difference in drifts μ accross regimes. This feature is absent from the Dai, Singleton and Yang (2007) setting.

3.2.2. The risk-neutral dynamics

Under the risk-neutral measrure $\mathbb{Q}$, the dynamics of y_t is given by:

$$\begin{aligned}
\begin{bmatrix} y_{1,t} \\ \vdots \\ y_{p,t} \end{bmatrix} &= \begin{bmatrix} \mu^*_{1,1} & \mu^*_{1,2} \\ \vdots & \vdots \\ \mu^*_{p,1} & \mu^*_{p,2} \end{bmatrix} z_t + \Phi^* \begin{bmatrix} y_{1,t-1} \\ \vdots \\ y_{p,t-1} \end{bmatrix} + \begin{bmatrix} \sigma'_{11} z_t & 0 & 0 \\ \vdots & \ddots & 0 \\ \sigma'_{p1} z_t & \cdots & \sigma'_{pp} z_t \end{bmatrix} \varepsilon^*_t \quad (3.3) \\
&= \mu^* z_t + \Phi^* y_{t-1} + \Omega(z_t)\varepsilon^*_t \quad (3.4)
\end{aligned}$$

where, under $\mathbb{Q}$, z_t is an homogenous Markov chain defined by a transition matrix $\{\pi^*_{ij}\}$, and where ε^*_t is $IIN\,(0, I)$.

Given the historical and the risk-neutral dynamics, it can be shown that the stochastic discount factor (s.d.f.) is exponential affine in (z_t, y_t). More precisely, in this context, the s.d.f. $M_{t-1,t}$ between $t-1$ and t is of the form (see Chapter 2):

$$\begin{aligned}
M_{t-1,t} &= \exp\Big[-a_1' z_{t-1} - b_1' y_{t-1} - \frac{1}{2}\nu'\left(z_t, z_{t-1}, y_{t-1}\right)\nu\left(z_t, z_{t-1}, y_{t-1}\right) + \\
&\quad +\nu'\left(z_t, z_{t-1}, y_{t-1}\right)\varepsilon_t + [\delta' z_{t-1}]' z_t\Big], \quad (3.5)
\end{aligned}$$

where δ is a 2×2 matrix whose (i, j) entry is $\ln(\pi^*_{ij}/\pi_{ij})$ and where $\Omega\left(z_t\right)\nu\left(z_t, y_{t-1}\right) = (\Phi^* - \Phi)y_{t-1} + \left(\mu^*\left(z_t\right) - \mu\left(z_t\right)\right)$. The risk-sensitivity matrix δ and function ν respectively price the (standardized) innovations ε_t of y_t and the regimes z_t.

3.2.3. Hazard rates

As explained in Section 2.7, in such a framework, the pricing of defaultable bonds boils down to the pricing of risk-free bonds if the risk-free short rate is replaced with a short rate embedding credit and liquidity risks. The differential between the latter and the risk-free short rate is termed with hazard rate and is denoted by $\lambda_{n,t}$ (for country n). Intuitively, in the absence of liquidity pricing and with a zero recovery rate, the hazard rate would simply be the default intensity $\lambda^d_{n,t}$. In the presence of non-zero recovery rate, the pure default intensity has to be replaced by the a loss-adjusted credit intensity$\lambda^c_{n,t}$ (see Subsection 2.4.2). Assuming that the recovery payoff is equal to a constant fraction ζ of the bond price that would have prevailed in the absence of default, the credit intensity $\lambda^c_{n,t}$ is given by:[10]

$$\exp\left(-\lambda^c_{n,t}\right) = \exp\left(-\lambda^d_{n,t}\right) + \zeta\left[1 - \exp\left(-\lambda^d_{n,t}\right)\right].$$

Liquidity-pricing effects are introduced through an illiquidity intensity denoted by $\lambda^\ell_{n,t}$.[11] We assume further that credit and illiquidity intensities are affine in (z_t, y_t). More precisely, under both measures, the hazard rate of the bonds issued by country n reads:

$$\lambda_{n,t} = \underbrace{(\alpha^c_n)' z_t + (\beta^c_n)' y_t}_{\text{credit-related } (\lambda^c_{n,t})} + \underbrace{\left(\alpha^\ell_n\right)' z_t + \left(\beta^\ell_n\right)' y_t}_{\text{liquidity-related } (\lambda^\ell_{n,t})}. \tag{3.6}$$

Further, we assume that the country-specific illiquidity intensities $\lambda^\ell_{n,t}$ are driven by a unique factor denoted by λ^ℓ_t, the latter being a linear combination of (z_t, y_t). Formally, for all countries n, we have:

$$\lambda^\ell_{n,t} = \gamma^0_{\ell,n} + \gamma^1_{\ell,n} \times \lambda^\ell_t = \gamma^0_{\ell,n} + \gamma^1_{\ell,n} \times \left(\alpha'_\ell z_t + \beta'_\ell y_t\right). \tag{3.7}$$

3.2.4. Pricing

It is well-known that the existence of a positive stochastic discount factor is equivalent to the absence of arbitrage opportunities (see Hansen and Richard, 1987

[10] Of course, when ζ is equal to zero,$\lambda^c_{n,t} = \lambda^d_{n,t}$, and when$\zeta$ is equal to one, the bond is equivalent to a risk-free bond.

[11] See Section 2.7. The affine term-structure literature is relatively silent on the interpretation or the microfoundations of the illiquidity intensity. In a theoretical paper analyzing interactions between credit and liquidity risks, He and Xiong (2012) show that such an illiquidity intensity may reflect the probability of occurence of a liquidity shock; upon the arrival of ths shock, the bond investor has to exit by selling his bond at a fractional cost (i.e. the selling price is equal to a fraction of the price that would have prevailed in the absence of the liquidity shock); the fractional cost is the analogous to the fractional loss $(1 - \zeta)$ in the default case (see also Ericsson and Renault, 2006 for a similar interpretation).

and Berholon, Monfort and Pegoraro, 2008) and that the price at t of a risk-free zero-coupon bond with residual maturity h, denoted by $B_{0,t,h}$, is given by:

$$B_{0,t,h} = E_t^{\mathbb{Q}}\left[\exp\left(-r_{t+1} - \ldots - r_{t+h}\right)\right], \tag{3.8}$$

where $r_{t+i} = a_1^{'} z_{t+i-1} + b_1^{'} y_{t+i-1}$, $i = 1, \ldots, h$.[12] Under our recovery assumptions, the price of a defaultable and illiquid zero-coupon bond issued by country n and with a residual maturity of h has a price at time t that is given by (if debtor n has not defaulted before time t; see Appendix 4.A for a proof in a more general context):

$$B_{n,t,h} = E_t^{\mathbb{Q}}\left[\exp\left(-r_{t+1} - \ldots - r_{t+h} - \lambda_{n,t+1} - \ldots - \lambda_{n,t+h}\right)\right]. \tag{3.9}$$

Since both the r_{t+i}'s and the $\lambda_{n,t+i}$'s are affine in (z_t, y_t), and since(z_t, y_t) is compound auto-regressive of order one under $\mathbb{Q}$, the prices of bonds are exponential affine in (z_t, y_t):[13]

$$B_{n,t,h} = \exp\left(-c_{n,h}^{'} z_t - f_{n,h}^{'} y_t\right) \tag{3.10}$$

and the associated yields are:

$$R_{n,t,h} = \frac{1}{h}\left(c_{n,h}^{'} z_t + f_{n,h}^{'} y_t\right), \tag{3.11}$$

where $(c'_{n,h}, f'_{n,h})$ are computed recursively.[14]

[12] As for the hazard rates (see equation 3.6), the risk-free short-term rate is the same function of (z_t, y_t) under both measures.

[13] Appendix 3.A.1 derives the Laplace transform of (z_t, y_t) and shows that (z_t, y_t) is Compound auto-regressive of order one. Appendix 2.A.5 shows how to compute the multi-horizon Laplace transform of compound auto-regressive processes. (See Darolles, Gourieroux and Jasiak, 2006 or Bertholon, Monfort and Pegoraro, 2008 for in-depth presentations of compound auto-regressive –or Car– processes.)

[14] The general recursive formulas are presented in Appendix 2.A.55.15. To apply these in the current case, one has (a) to use the Laplace tansform of (z_t, y_t) presented in Appendix 2.A.5 and (b) take a sequence ω_h, $h = 1, \ldots, H$ defined by $\omega_H = (-\alpha'_n, -\beta'_n)$ and $\omega_h = (-\alpha'_n - a'_1, -\beta'_n - b'_1, -\gamma'_n)$ for $h = 1, \ldots, H-1$, with $c_{n,0} = a_1$ and $f_{n,0} = b_1$.

3.3. Data

The data are monthly and cover the period from July 1999 to March 2011. We exclude the first 6 months of 1999 so as to avoid potential effects linked to the euro introduction. The estimation involves end-of-month yields as well as survey-based yield forecasts. We consider the yield curves of ten euro-area countries: Austria, Belgium, Finland, France, Germany, Ireland, Italy, the Netherlands, Portugal and Spain. Greece data are excluded from the analysis because appropriate euro-denominated bond yields are not available before 2001, when Greece joined the euro area. Consistently with the fact that, among sovereign euro-area bonds, the German *Bunds* are perceived to be the "safest haven" both in terms of credit quality and liquidity, we consider the German bonds as risk-free.[15]

Appendix 3.B details the sources of the data and the preliminary computations performed to get end-of-month zero-coupon yields. The following subsection(3.3.1) introduces the KfW-*Bund* spreads that will be exploited to identify the liquidity-related latent factor λ_t^ℓ. In 3.3.2, we provide a preliminary analysis of euro-area yield differentials and in 3.3.3, we detail the computation of the factors $y_{1,t}, \dots, y_{5,t}$.

3.3.1. The KfW-*Bund* spread

Our identification of a liquidity-related latent factor is based on the yield spreads between German federal bonds and KfW agency bonds. The latter are less liquid than the sovereign counterparts, the so-called *Bunds*, but are explicitly and fully guaranteed against default by the German federal government.[16] Consequently,

[15]In particular, the German bond market is the only one in Europe that has a liquid futures market, which boosts demand for the German Bund compared to other euro area debt and bolsters its liquidity (see e.g. Pagano and von Thadden, 2004, Ejsing and Sihoven, 2009 or Barrios et al., 2009).

[16]An understanding between the European Commission and the German Federal Ministry of Finance (1 March 2002) stated that the guarantee of the Federal Republic of Germany will continue to be available to KfW. The three main rating agencies –Fitch, Standard and Poor's and Moody's– have assigned a triple-A rating to KfW (see KfW website http://www.kfw.de/kfw/en/KfW_Group/Investor_Relations/index.jsp). In addition, as the German federal bonds, KfW's bonds are zero-weighted under the Basel capital rules. The

the spread between these two kinds of bonds can be seen as a measure of the German government bond-market liquidity premium demanded by investors. In the same spirit, Longstaff (2004) computes liquidity premia based on the spread between U.S. Treasuries and bonds issued by Refcorp, that are guaranteed by the U.S. Treasury.

Panel A of Figure 3.1 shows that the KfW-Bund spreads of different maturities are highly correlated. This suggests that a single factor may be adequate to model the term structure of these spreads. Here, it is important to check that this liquidity-pricing measure is not purely specific to Germany. To that purpose, we look at comparable liquidity-driven spreads –between government-guaranteed bonds and their sovereign counterparts– in alternative countries.[17] In France for instance, the CADES (Caisse d'amortissement de la dette sociale) issues bonds that are guaranteed by the French government. Panel B compares one of the KfW-Bund spreads with a CADES-OAT spread (OATs are French government-issued bonds) and displays spreads of government-guaranteed bank bonds –issued by the Dutch NIBC bank and the Austrian Raiffeisen Zentalbank– over their respective sovereign counterparts. This exercise points to a substantial degree of correlation among liquidity-driven spreads from different European countries.

3.3.2. Euro-area government yields

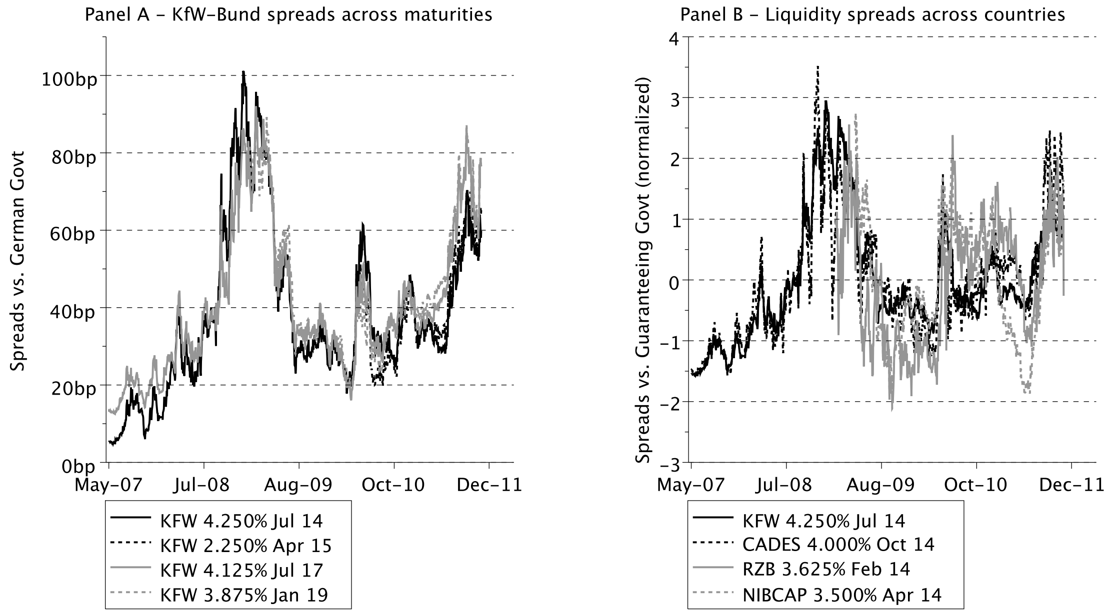
Table 3.1 suggests that euro-area government yields are highly correlated across countries and across maturities (see also Favero, Pagano and von Thadden, 2010 [115]). Table 3.2 reports the correlations between the spreads vs. Germany for different countries over the sample periods and presents a principal-component analysis of these spreads across countries. The correlations suggest that spreads largely comove across countries. The principal-component analysis (see lower part of Table 3.2) indicates that, for different maturities (2, 5 and 10 years), the first two

relevance of the KfW-*Bund* spread as a liquidity proxy is also pointed out by McCauley (1999), the ECB, 2009 and is exploited by Schwarz (2009).

[17] Note that such alternative (term structures of) spreads are not available on our whole estimation period, that is why we use essentially KfW-Bund spreads to identify our liquidity factor within our econometric approach.

Figure 3.1.: Differentials between government and government-guaranteed bonds

Notes: The first plot shows the spreads between KfW bond yields and their sovereign counterparts. The second plot compares the spread between a KfW bond maturing in 2014 and its sovereign counterpart with other spreads between government-guaranteed European bonds and their respective sovereign counterparts: CADES', NIBCAP's and RZB's bonds are respectively guaranteed by the French, Dutch and Austrian governments (the spreads are demeaned and standardized). The yields come from Barclays Capital.



principal components roughly explain 90% of the spread variances across countries. This suggests that a model with a limited number of common factors may be able to explain the bulk of euro-area yield-differential fluctuations. The estimation is based on four benchmark maturities per country: 1, 2, 5 and 10 years. The short end of the risk-free yield curve is augmented by the 1-month EONIA swap.[18]

3.3.3. Construction of the factors y_t

As explained in Section 3.2.4, our framework implies that (modeled) bond yields end up being some linear combinations of the regime variables z_t and of the factors y_t. Therefore, appropriate factors have to capture a large share of the common fluctuations of yields. Natural candidates for the y_t's are the principal components

[18] Data providers such as Bloomberg do not propose 1-month sovereign German yields. We decide to replace it with the 1-month EONIA swap rates as swap yields are often considered as risk-free yields, see e.g. Collin-Dufresne, Goldstein and Martin (2001).

Table 3.1.: Descriptive statistics of selected yields

Notes: The Tablereports summary statistics for selected yields. The data are monthly and cover the period from July 1999 to March 2011. Two auto-correlations are shown (the 1-month and the 1-year auto-correlations). The yields are continuously compounded and are in percentage annual terms (see Appendix 3.B for details about their construction). The lower panel of the Tablepresents the covariances and the correlations (in italics) of the yields. The 1-month rate is the 1-month EONIA swap.

		German yds		Italian yds		Portuguese yds		Irish yds	
	1-mth	2-year	10-year	2-year	10-year	2-year	10-year	2-year	10-year
Mean	2.761	2.961	4.086	3.288	4.517	3.428	4.609	3.537	4.672
Median	2.832	3.091	4.084	3.32	4.459	3.474	4.45	3.51	4.568
Standard dev.	1.371	1.18	0.718	1.046	0.581	1.046	0.79	1.125	1.048
Skewness	-0.243	-0.303	-0.076	0.175	0.17	0.488	0.952	1.285	1.974
Kurtosis	2.09	2.131	2.323	2.059	2.189	3.664	4.967	6.398	9.232
Auto-cor. (lag 1)	0.998	0.988	0.973	0.98	0.962	0.896	0.962	0.936	0.963
Auto-cor. (lag 12)	0.475	0.53	0.586	0.491	0.571	0.132	0.29	0.037	0.279
Correlations \ **Covariances**									
1-mth EONIA swap	1.867	1.521	0.73	1.306	0.432	0.835	0.114	0.636	-0.276
German 2-yr yd	*0.946*	1.385	0.713	1.149	0.423	0.728	0.144	0.577	-0.18
German 10-yr yd	*0.744*	*0.843*	0.516	0.588	0.345	0.349	0.201	0.283	0.037
Italian 2-yr yd	*0.917*	*0.937*	*0.785*	1.086	0.451	0.84	0.316	0.747	0.12
Italian 10-yr yd	*0.545*	*0.619*	*0.827*	*0.746*	0.337	0.406	0.343	0.415	0.325
Portug. 2-yr yd	*0.586*	*0.593*	*0.466*	*0.773*	*0.671*	1.086	0.605	1.095	0.61
Portug. 10-yr yd	*0.106*	*0.155*	*0.355*	*0.384*	*0.75*	*0.737*	0.622	0.721	0.753
Irish 2-yr yd	*0.415*	*0.438*	*0.351*	*0.639*	*0.638*	*0.938*	*0.815*	1.256	0.855
Irish 10-yr yd	*-0.193*	*-0.146*	*0.049*	*0.11*	*0.536*	*0.561*	*0.913*	*0.73*	1.092

of the set of yields time series. However, since we do not have survey-based forecasts of all the yields that we consider in the estimation –there are 40 of them–, doing so would deprive us of survey-based forecasts of the factors. If, as in Kim and Orphanides (2012), we want to incorporate such data in the estimation of the historical dynamics of the factors, these need to be based on variables for which some forecasts are available. To that respect, the *Consensus Forecasts* provide us with 3-month-ahead and 12-month-ahead forecasts of the 10-year sovereign yields of 5 countries: France, Germany, Italy, the Netherlands and Spain. As a consequence, if we construct some factors that are given by combinations of these yields, 3-month and 12-month ahead survey-based forecasts of these factors can be included in the estimation procedure. (The advantages of using survey forecasts in the estimation of the historical dynamics of the factor are outlined in Section 3.1.)

The *Consensus Forecasts* are produced monthly by *Consensus Economics*, which surveys financial and economics forecasters. The survey is released around the middle of the month.[19] Note that the survey implicitly targets yields-to-maturity of coupon bonds and not zero-coupon bonds. However, our zero-coupon yields remain very close to coupon yields over the estimation sample. The remaining discrepancy will be attributed to the deviation between the survey-based forecasts and the model-based ones (the $\varepsilon_{j,h,t}$'s introduced in equation 3.12 below).

Nevertheless, all of our factors can not be based on 10-year yields since we would then miss the drivers of the deformation of the term structure of interest rates. In other words, we also have to consider factors that will be able to capture the changes in the slope and the curvature of the yield curves.[20]

Taking all these remarks into account, we use the following factors: the first three are the level, the slope and the curvature of the German yield curve;[21] the last two factors are the first two principal components of the 10-year-maturity spreads (vs. Germany) of France, Italy, the Netherlands and Spain. Eventually, survey-based forecasts are available for three out of five factors (the first factor, i.e. the 10-year German yield, and the last two factors, associated with 10-year spreads vs. Germany).

The factors $y_{1,t}$, ..., $y_{5,t}$ that result from this procedure are plotted in the upper two panels in Figure 3.2.

[19] The number of respondents varies across time and countries. One average over the estimation period, while more than 20 forecasters contribute to the German forecasts, around 10 take part to the Italian ones. For each yield, we use the mean of the forecasts produced by the different survey contributors.

[20] The importance of such factors has been investigated by various empirical studies in the wake of Litterman and Scheinkman (1991).

[21] The first (level) factor is the 10-year German rate, the second (slope) factor is the difference between the spread between the 10-year and the 1-year rates, the third (curvature) factor is computed as the difference between (a) the 3-to-10 year and (b) the 1-to-3 year slope of the yield curve (that is, 2 times the 3-yr yield minus the sum of the 1-yr and the 10-yr yields).

3.4. Estimation

3.4.1. Main lines of the estimation strategy

As Ang, Piazzesi and Wei (2006) or Moench (2008), our estimation procedure involves two steps. In the first one, we estimate the historical dynamics of factors y_t and regimes z_t by maximizing the log-likelihood using the Kitagawa-Hamilton algorithm. At the end of this first step, the Kitagaw-Hamilton smoother is used to estimate the regime variables z_t and these are taken as fixed in the next step. The latter concerns the joint estimation of the risk-neutral dynamics of (z_t, y_t) and of the specifications of the hazard rates $\lambda_{n,t}$. This second step is based on non-linear-least-squares techniques, taking into account the internal-consistency issue. Then, it remains to perform the decomposition of the hazard rates into credit and liquidity components. This final operation will be detailed in Section 3.5.

3.4.2. Historical dynamics of (z_t, y_t)

The historical dynamics of (z_t, y_t) is defined by a Markov-switching VAR (see equations 3.1 and 3.2). This set of five equations is augmented with equations linking survey-based forecasts to their model-based equivalent. These six additional equation read:

$$E^{CF}_{j,h,t} = E_t(y_{j,t+h}) + \varepsilon_{j,h,t}, \quad j \in \{1,4,5\},\ h \in \{3,12\}, \tag{3.12}$$

where $E^{CF}_{j,h,t}$ is the h-period ahead survey-based forecast, $E_t(y_{j,t+h})$ is its equivalent model-based forecast, and the $\varepsilon_{j,h,t}$'s are the measurement errors, assumed to be normally i.i.d.. The model-based forecasts stem from:

$$E_t(y_{t+h}) = \left[\mu P^h + \Phi\mu P^{h-1} + \ldots + \Phi^{h-1}\mu P\right] z_t + \Phi^h y_t. \tag{3.13}$$

The parameters are estimated by maximizing the associated log-likelihood. Two kinds of constraints are imposed in the estimation. First, we impose some constraints on the matrix of regime-switching probabilities. The probability of remaining in the crisis regime is then calibrated so as to imply an average length of the crisis of 2 years; this length being consistent with the findings of Cecchetti, Kohler and Upper (2009) who investigate worldwide banking crises over the ast 30 years.[22] Second, we constrain the unconditional means of the factors. Except for the first factor, the unconditional means of the factors are set to their sample means. The mean of the first factor (10-year German yield) is set to 4.75%. Indeed, its sample mean, which is of 4.10%, is low compared to the average of the long-term forecasts for this yield, the latter being expected to be less affected by short-sample biases.[23] Finally, as in Kim and Orphanides (2012), we let the estimation to decide the standard deviations of the measurement errors $\varepsilon_{j,h,t}$ in equations (3.12).

Parameter estimates are reported in Table 3.3 and Table 3.4. The second regime, that we identify as a "crisis" regime, is characterized by particularly high standard deviations of the innovations ε_t, especially for the shocks affecting $y_{4,t}$ and $y_{5,t}$ (see Table 3.4).

The grey-shaded areas in Figure 3.2 indicate the crisis periods. These periods are estimated as those for which the smoothed probabilities of being in the crisis regime are larger than 50%. Three crisis periods are estimated: a first between September 2008 and August 2009, a second between December 2009 and January 2010 and a last that starts in April 2010 and that lasts till the end of the sample (March 2011).

Figure 3.3 displays survey-based forecasts of three of the factors ($y_{1,t}$, $y_{4,t}$ and $y_{5,t}$) together with their model-based equivalent, computed using equation (3.13). Except for the 12-month ahead forecasts of the fifth factor (bottom right panel in Figure 3.3), the model is able to reproduce most of the survey-based forecasts'

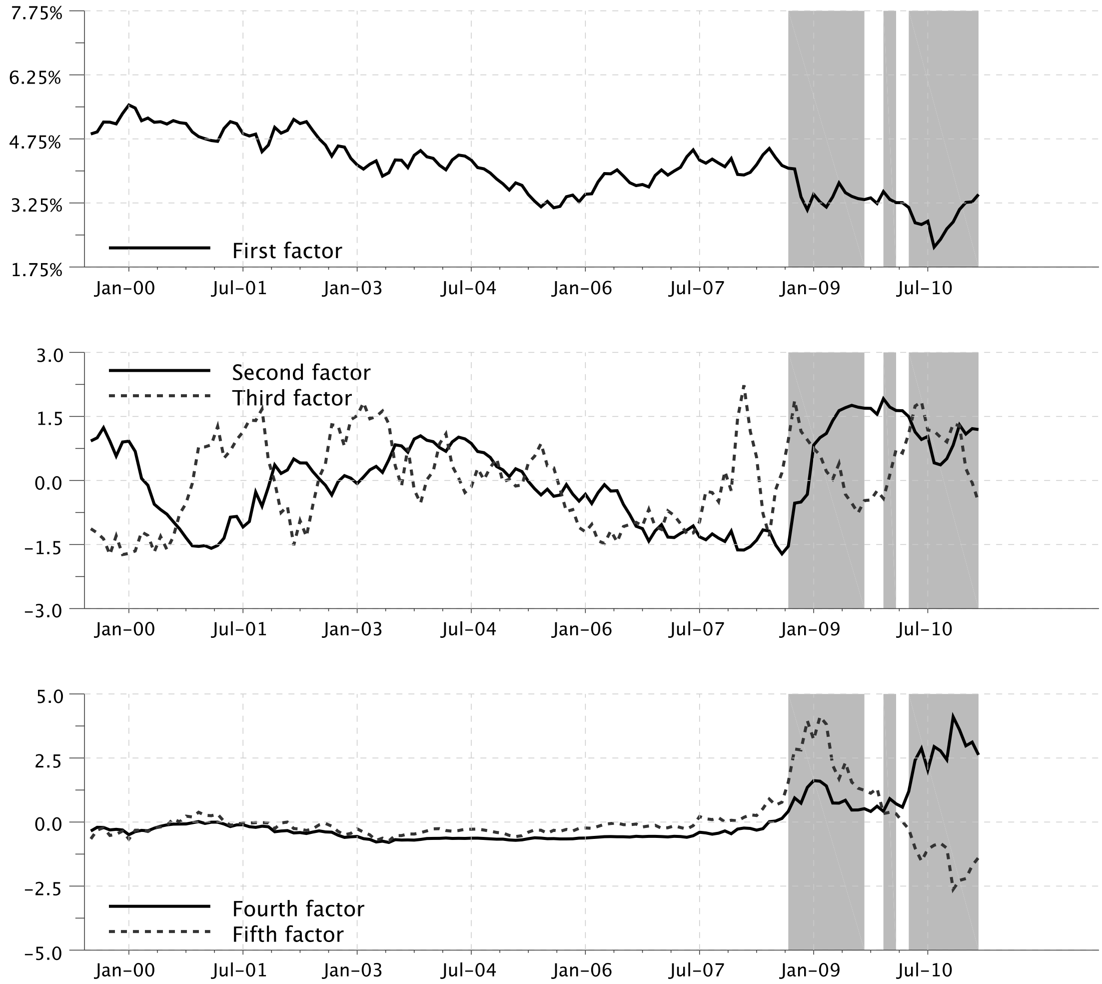
[22] Which translates into $\pi_{C,C} = 95\%$. Cecchetti et al. study 40 systemic banking crises since 1980. This constraint is imposed because preliminary unconstrained estimations resulted in probabilities of remaining in each of the regimes that was implausibly high.

[23] For comparison, the average of the 10-year-Bund yield over the last 20 years is approximately 5%. Twice a year, in April and October, the *Consensus Forecasts* present long-term forecasts of macroecononmic variables (up to 10 years ahead). Over the last 10 years, the average of the long-term forecasts of the 10-year German yield is of 4.78%.

fluctuations.

Figure 3.2.: The five factors y_t and the estimated regime variable z_t

Notes: These plots show the factors $y_{1,t}, \ldots, y_{5,t}$ that are used in the analysis. The first factor is the 10-year zero-coupon German yield (minus 4.75%). The second factor is a proxy of the yield-curve slope (difference between the 10-year German yield and the 1-month yield). The third is a proxy of the yield-curve curvature (10-year German yield + 1-month yield − 2 times the 3-year German yield). The fourth and fifth factors are the two first PCs of a set of four 10-year spreads vs. Germany (France, Italy, the Netherlands and Spain). The shaded areas correspond to periods for which the smoothed probability of being in the crisis regime is above 50% (using Kim's algorithm, 1993).

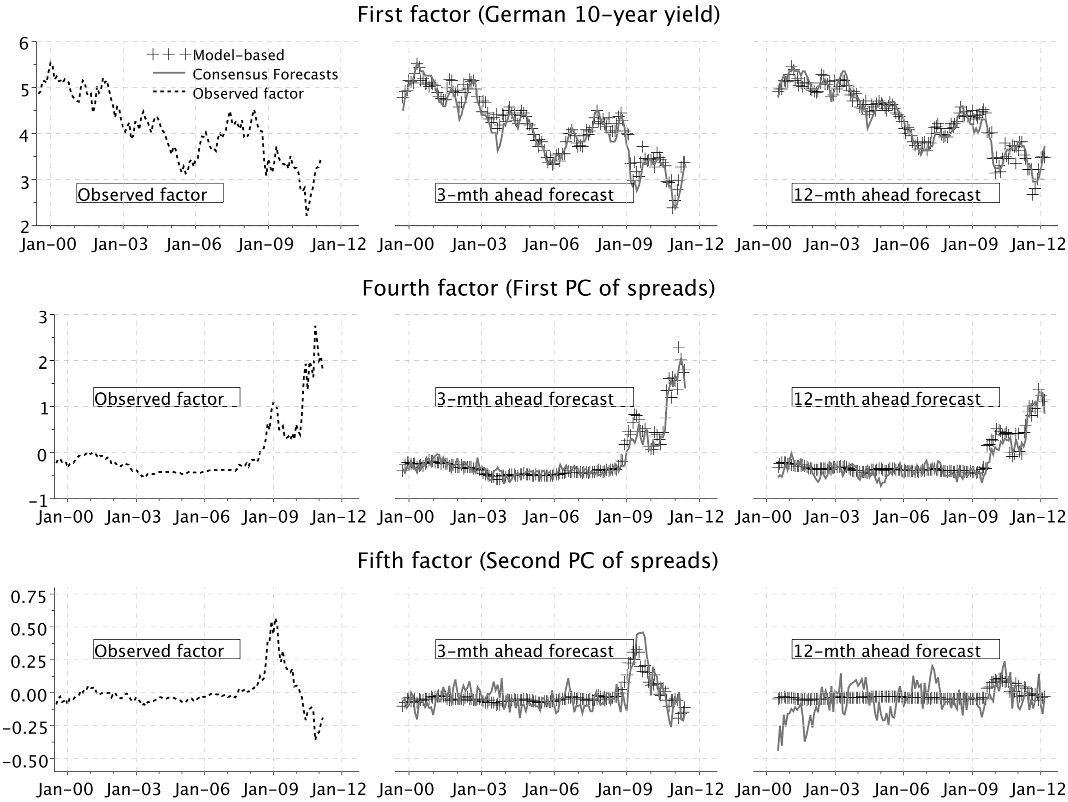


3.4.3. Risk-neutral dynamics

The vector θ of parameters defining the risk-neutral dynamics –that is, matrices μ^*, Φ^*, $\left\{\pi^*_{i,j}\right\}$– and those defining the default intensities –the α's and the β's– is

Figure 3.3.: Model-based vs. survey-based forecasts

Notes: The Figures compare survey-based forcasts of the factors (derived from the *Consensus forecasts*) with model-based forecasts. The charts of the left column display the three factors for which some survey-based forecasts are available, namely $y_{1,t}$, $y_{4,t}$, and $y_{5,t}$. The first factor is the German 10-year yield (minus 4.75 percentage points). The fourth and fifth factors are the first two principal components of a set of 10-year spreads vs. Germany for 4 countries (France, Italy, Spain and the Netherlands).



estimated by means of non-linear least squares. Basically, we aim at minimizing the sum of squared measurement errors, or SSME, across countries and maturities (1, 2, 5 and 10 years).[24] In addition, we have to deal with internal consistency conditions. These conditions arise from the fact that our pricing factors $y_{1,t}, \dots, y_{5,t}$ are known linear combinations of the yields; the latter being in turn some combinations of the factors (see equation 3.11). To maintain internal consistency, the model has to correctly "price" the factors (that reflect observed bond-portfolios' prices). The internal-consistency restrictions involve highly non-linear transformations of the parameters. As a consequence, numerically minimizing the SSME under the consistency constraints would considerably slow down the optimization

[24]The measurement errors are defined as the deviations between modeled and actual yields. In addition to sovereign yields, KfW's yields are also used in the estimation.

procedure.[25] We therefore resort to an alternative solution that consists in augmenting the SSME with a term penalizing deviations from internal-consistency restrictions. More precisely, denoting observed yields by $\tilde{R}_{n,t,h}$, modeled yields by $R_{n,t,h}(\theta)$, observed factors by $\tilde{y}_{i,t}$ and modeled factors by $y_{i,t}(\theta)$, the estimator $\hat{\theta}$ results from:

$$\hat{\theta} = \arg\min_{\theta} \sum_{n,t,h} \left(\tilde{R}_{n,t,h} - R_{n,t,h}(\theta)\right)^2 + \chi \sum_{t,i} \left(\tilde{y}_{i,t} - y_{i,t}(\theta)\right)^2 . \tag{3.14}$$

where χ is a parameter defining the relative penalization of the deviations between modeled ($\tilde{y}_t$) and observed (y_t) factors.

The loss function that we aim at minimizing (see equation3.14) being highly nonlinear in the underlying model parameters, it is necessary to find good starting values so as to achieve convergence in a reasonable computing time.[26] We proceed as follows: (a) we consider only the risk-free rates in (3.14) and we assume that their term-structure depends on the first three factors ($y_{1,t}$, $y_{2,t}$ and $y_{3,t}$) only, (b) we incorporate the risky yields of a subset of debtors (namely Germany, KfW and Portugal) and we (re-)estimate the parameterization of the risk-neutral dynamics (for the five factors y_t) as well the hazard rates of these three entities, (c) we estimate the hazard rates of the remaining entities, one by one, taking the other parameters as given. In the final stage, all the parameters are (re)estimated jointly.[27]

Table 3.5 and Table 3.3 present the parameter estimates. The standard deviation of these estimates are based on a Newey-West (1987) heteroskedasticity and autocorrelation consistent (HAC) covariance matrix estimator (see Appendix 3.C).

The parameterizations of the hazard rates, presented in Table 3.5, stem from the decomposition of the hazard rates between liquidity-related and credit-related com-

[25] See e.g. Duffie and Kan (1996) for a simple example. Considering only one debtor and no regime-switching, Joslin, Singleton and Zhu (2011) find a parameterization of their Gaussian model that automatically satisfies internal consistency restrictions.

[26] Optimizations are based on iterative uses of quasi-Newton and Nelder-Mead algorithms (as provided by the Scilab software).

[27] The final stage is itself decomposed into several sub-steps: first, the penalty factor χ (for the internal-consistency restrictions) is set to zero. Then, it is progressively increased, till 1, level at which deviations between modeled and actual factors y_t become neglectible.

ponents, that will be discussed in the next Section. Indeed, the minimization of the loss function specified in (3.14) leads to estimates of the α_n's and β_n's, with $\alpha_n = \alpha_n^c + \alpha_n^\ell$ and $\beta_n = \beta_n^c + \beta_n^\ell$ (α_n's and β_n's estimates are not reported). A first look at Table 3.5 suggests that the estimation results in significant impacts of the factors on the hazard rates.

3.5. Results and interpretation

To begin with, the approach results in a satisfiying fit of the data. Modeled versus observed spreads are displayed in Figure 3.4 (grey lines for observed spreads, dotted lines for modeled spreads). On average across countries and maturities (i.e. across 45 series), the ratios of the measurement-error variances over those of the yields are lower than 2%: the average (across countries and maturities) measurement-error standard deviation is around 18 basis points. In the sequel of this Section, we focus on two specific issues: liquidity pricing and extraction of default probabilties from bond yields.

3.5.1. The illiquidity intensity

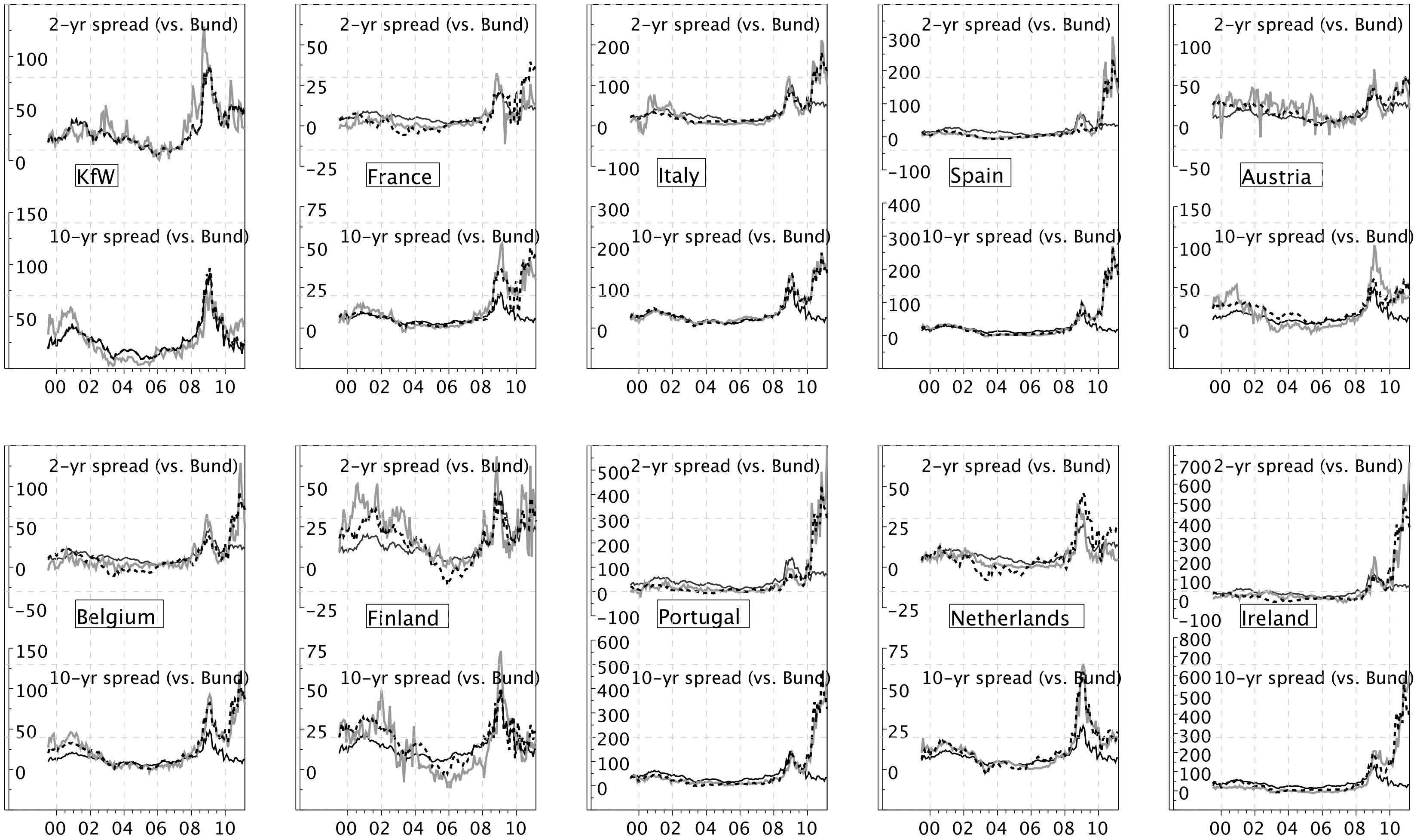
In our model, we assume that there is a single factor that drives the liquidity pricing in euro-area bond yields. As documented in 3.3.1, the bonds issued by KfW and those issued by the German government embed the same credit risks – assumed to be nil here– but are not equally exposed to the liquidity-related factor. Accordingly, we simply have:

$$\lambda_t^\ell = \lambda_{KfW,t}. \tag{3.15}$$

The left part of Table 3.5 presents the estimated specification of λ_t^ℓ. According to the Student-t ratio, the liquidity factor is significatively linked to the five factors, especially the fifth one (which is the second PC of a set of four 10-year spreads vs. Germany). In addition, the α_ℓ estimates indicate that the liquidity factor

Figure 3.4.: Actual vs. model-implied spreads vs. Germany

Notes: These plots compare observed (light-grey solid lines) and model-implied (dotted lines) spreads between the yields of 9 countries (+ KfW, a German agency) and their German counterparts. Two maturities are considered: 2 years (upper plot) and 10 years (lower plot). The black solid line is the model-implied contribution to the spreads of the liquidity factor λ_t^ℓ (these contributions are computed as the spread that would prevail if the credit parts $\lambda_{n,t}^c$ of the debtor intensities were equal to zero). For KfW (upper-left plot), the fact that the dotted line and the black solid line are confounded results from the identiification of the liquidity factor λ_t^ℓ.



jumps upwards in crisis periods. The resulting estimate of the liquidity factor is displayed in the upper plot in Figure 3.5, together with a 90% confidence interval.[28] It turns out that this European factor has some comovements with other proxies of liquidity pricing. Two such measures are displayed in Figure 3.5 (middle and lower plot). A first proxy, inspired by Manganelli and Wolswijk (2009), consists of a dispersion measure of the bond yields of Aaa-rated countries. This proxy is based on the assumption according to which a significant share of the spreads between Aaa-rated countries should reflect liquidity differences since they are all supposed to have a very high credit quality.[29] The second liquidity proxy is the bid-ask spread on the 10-year French benchmark bond (lower plot in Figure 3.5). In addition to concomitant rises in the three proxies in early 2008, one can observe a common decreasing trend between the early 2000 and 2005.

The liquidity-related factor λ_t^ℓ presents three main humps: in the early 2000s, in 2008 and in 2010. The rise in liquidity premia in the early 2000s –concomitant with the collapse of the Internet bubble– is also found in U.S. data by Fontaine and Garcia (2012), Longstaff (2004) or Feldhütter and Lando (2008). The fact that the liquidity factor is particularly high during crises periods (burst of the dotcom bubble and post-Lehman periods) is consistent with the findings of Beber, Brandt and Kavajecz (2009) who pinpoint that investors primarily chase liquidity during market-stress periods.[30]

Given the liquidity-related factor λ_t^ℓ, it remains to perform the default/liquidity decompositions of the country-specific hazard rate (see equations 3.6 and 3.7). Specifically, we have to estimate the pair of parameters $(\gamma_{\ell,n}^0, \gamma_{\ell,n}^1)$ for each country n (recall that $\lambda_{n,t}^\ell = \gamma_{\ell,n}^0 + \gamma_{\ell,n}^1 \lambda_t^\ell$). Intuitively, we look for parameters $\gamma_{\ell,n}^0$'s and $\gamma_{\ell,n}^1$'s that are such that (a) an important share of the spread *fluctuations* is explained by the liquidity intensity $\lambda_{n,t}^\ell$ under the constraints that (b) the implied risk-neutral

[28] The computation of this confidence interval is based on the delta method, exploiting the fact that at each point in time, the estimate of λ_t^ℓ is a function of the parameter estimates and of y_t and z_t ($\lambda_t^\ell = \alpha_\ell' z_t + \beta_\ell' y_t$).

[29] To compute this proxy, we use sovereign yield data (the same as in the rest of the analysis) of Austria, Finland, France, Germany and the Netherlands, which are the five countries that remain Aaa-rated over the whole period.

[30] Such a behaviour is captured in a theoretical framework by Vayanos (2004).

and historical PDs are positive and that (c) the liquidity-related parts of the spreads are positive. In order to achieve this for each country n, we construct a loss function $\mathcal{L}_n$ that quantifies the previous objectives and we look for parameters $(\gamma^0_{\ell,n}, \gamma^1_{\ell,n})$ that minimize this function. This procedure is detailed in Appendix 3.D.

The estimated $\gamma^0_{\ell,n}$ and $\gamma^1_{\ell,n}$ are shown in the lower panel of Table 3.5. Note that these parameters are non-linear combinations of the parameters that were estimated in two steps of the estimation procedure. In particular, each $\gamma_{\ell,n}$ is largely dependent on the estimation of α_{KfW} and β_{KfW} that define the liquidity-related factorλ^ℓ_t. The standard deviations of the estimated $\gamma_{\ell,n}$'s (reported in Table 3.5) result from the delta method, taking all these dependencies into account.[31]

Figure 3.6.: Sensitivity to the liquidity factor versus debt outstanding

Notes: The coordinates of the countries correspond to (x) the sensitivities $\gamma^1_{\ell,n}$ of their hazard rates $\lambda^\ell_{n,t}$ to the liquidity factor λ^ℓ_t (these sensitivities are reported in the lowest row of Table 3.5) and (y) their total markeTablesovereign debt (as of the end of 2009, Source: Eurostat).

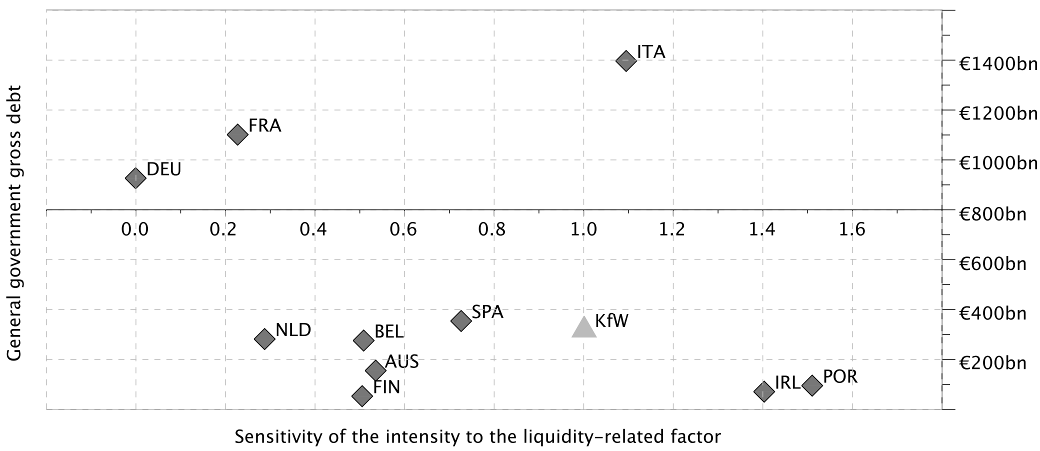


Figure 3.6 shows a scatter plot where the coordinates of the countries are the sensitivities $\gamma^1_{\ell,n}$ to the liquidity-related factor and the total markeTabledebt of the different countries. Leaving Italy aside, there seems to be a negative relationship between these sensitivities and the debt outstanding. In spite of the large size of

[31] We assume that the large covariance matrix of the parameter estimates obtained in the first step and in the second step of the estimation is block diagonal. This would be exact if both steps of the estimations were independent. This is not rigorously the case since the covariance matrices of the factor innovations $(\Omega(z_t)\Omega(z_t)')$– are the same under both measures.

the tradable debt issued by the Italian government, Italy's hazard rate appears to be particularly sensitive to the liquidity factor (among the countries considered in our subset, only Ireland and Portugal are more exposed than Italy to the liquidity factor).[32]

Moreover, in order to gauge the relative importance of the liquidity-related part of the spreads, we have computed the spreads (versus German yields) that would prevail if the credit part of the countries' hazard rates were equal to zero. Figure 3.4 presents the resulting spreads (black solid lines). While, for most countries, the liquidity-related part of the spread is less important than the credit-related one (as in Codogno, Favero and Missale, 2003), it turns out to account for a substantial part of the changes in spreads, especially over the earlier part of the estimation sample.

3.6. Conclusion

In this Chapter, we present a no-arbitrage model of the joint dynamics of euro-area sovereign yield curves. In addition to five Gaussian shocks, the model includes a regime-switching feature that makes it possible to distinguish between tranquil and crisis periods. Such a regime-switching feature is well suited to account for the recent/current economic and financial market stress times. As a source of systematic risk, the regime shifts are priced by investors. Quasi-explicit formulas are available, which makes the model tracTableand the estimation feasible. The model is estimated over the last twelve years. The resulting fit is satisfying since the standard deviation of the yields pricing errors –across countries and maturities– is of 18 basis points. Our estimation suggests that the regimes are key in explaining the fluctuations of yields over the last three years.[33] Further, some credit and liquidity intensities are estimated for each European country included in our dataset. The

[32] To some extent, such a finding is consistent with the results of Chung-Cheung, de Jong and Rindi (2004) according to which transitory costs would be more important in the Italian market, dominated by local traders.

[33] Counterfactual experiments –whose results are not reported here– have been conducted to gauge the impact of the crisis regime on model-implied yields: when the crisis periods are replaced by no-crisis ones, simulated (counterfactual) spreads remain flat from 2008 onwards.

liquidity intensities are driven by a single European factor whose identification is based on the KfW-*Bund* spreads. Indeed, the bonds issued by KfW, guaranteed by the Federal Republic of Germany, benefit from the same credit quality than their sovereign counterparts –the *Bunds*– but are less liquid. Therefore, the KfW-*Bund* spread should be essentially liquidity-driven. Our results indicate that a substantial part of intra-euro spreads is liquidity-driven. The remaining parts of the spreads reflect credit-risk pricing. In the next Chapter, we focus on the financial-crisis period. Further, we extend the analysis by deriving probabilities of default under the physical measure.

3.A. Proofs

3.A.1. Laplace transform of (z_t, y_t)

The risk-neutral conditional Laplace transform of (z_t, y_t) the information available in time $t-1$ is:

$$\varphi_{t-1}^{\mathbb{Q}}(u, v) = \exp\left(v'\Phi^* y_{t-1} + [l_1, \ldots, l_J]\, z_{t-1}\right), \tag{3.16}$$

where $l_i = \log \sum_{j=1}^{J} \pi_{ij}^* \exp\left\{u_i + v'\mu^* e_j + \frac{1}{2} v'\Omega(e_j)\,\Omega'(e_j)\, v\right\}$ and where e_j is the j^{th} column of the identity matrix. Therefore, (z_t, y_t) is compound auto-regressive of order one –denoted by Car(1)– under the risk-neutral measure.

Proof. We have

$$\begin{aligned}
\varphi_{t-1}^{\mathbb{Q}}(u,v) &= E_{t-1}^{\mathbb{Q}}\left(\exp\left[u'z_t + v'y_t\right]\right) \\
&= E_{t-1}^{\mathbb{Q}}\left(\exp\left[u'z_t + v'\mu^* z_t + v'\Phi^* y_{t-1} + v'\Omega(z_t)\varepsilon_t\right]\right) \\
&= E_{t-1}^{\mathbb{Q}}\Big(E_{t-1}^{\mathbb{Q}}\{\exp\left[u'z_t + v'\mu^* z_t + v'\Phi^* y_{t-1} + \right. \\
&\quad \left. v'\Omega(z_t)\varepsilon_t\right] \mid z_t\}) \\
&= \exp(v'\Phi^* y_{t-1}) E_{t-1}^{\mathbb{Q}}\left(\exp\left\{u'z_t + v'\mu^* z_t\right\} \times \right. \\
&\quad E_{t-1}^{\mathbb{Q}}\left(\exp\left\{v'\Omega(z_t)\varepsilon_t \mid z_t\right\}\right)\Big) \\
&= \exp(v'\Phi^* y_{t-1}) E_{t-1}^{\mathbb{Q}}\left(\exp\left\{u'z_t + v'\mu^* z_t\right\} \times \right. \\
&\quad \frac{1}{2}v'\Omega(z_t)\Omega'(z_t)v\Big) \\
&= \exp\left(v'\Phi^* y_{t-1} + \left[l_1,\ldots,l_J\right]z_{t-1}\right).
\end{aligned}$$

Using the expression given above for the l_i's leads to the result. □

3.B. Sovereign yield data

The estimation of the model requires zero-coupon yields. However, governments usually issue coupon-bearing bonds. For Germany, France, Spain and Netherlands, we bootstrap constant-maturity coupon yield curves provided by Barclays Capital.[34] For Belgium, we use zero-coupon yields computed by the National Bank of Belgium and made available by the BIS. For remaining countries, we resort to a parametric approach (see BIS, 2005, for an overview of zero-coupon estimation methods). The yield curves are derived from bond pricing data on regularly replenished populations of sovereign bonds. We choose the parametric form originally proposed by Nelson and Siegel (1987). Specifically, the yield of a zero-coupon bond with a time to maturity m for a point in time t is given by:[35]

[34]For details about bootstrapping methods, see e.g. Martellini, Priaulet and Priaulet (2003)

[35]We use the Nelson-Siegel form rather than the extended version of Svensson (1994) because the latter requires more data to be estimated properly (and for some countries and some dates, we have too small a number of coupon-bond prices).

$$
\begin{aligned}
R_t^m(\theta) &= \beta_0 + \beta_1 \left(-\frac{\tau_1}{m}\right)\left(1 - \exp(-\frac{m}{\tau_1})\right) + \\
&\quad \beta_2 \left[\left(\frac{\tau_1}{m}\right)\left(1 - \exp(-\frac{m}{\tau_1})\right) - \exp(-\frac{m}{\tau_1})\right]
\end{aligned}
$$

where Θ is the vector of parameters $[\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2]'$. Assume that, for a given country and a given date t, we dispose of observed prices of N coupon-bearing bonds (with fixed coupon), denoted by $P_{1,t}, P_{2,t}, \ldots, P_{N,t}$. Let us denote by $CF_{k,i,t}$ the i^{th} (on n_k) cash flows that will be paid by the k^{th} bond at the date $\tau_{k,i}$. We can use the zero-coupon yields $\{R_t^m(\Theta)\}_{m\geq 0}$ to compute a modeled (dirty) price $\hat{P}_{k,t}$ for this k^{th} bond:

$$
\hat{P}_{k,t}(\Theta) = \sum_{i=1}^{n_k} CF_{k,i,t} \exp\left(-\tau_{k,i} R_t^{\tau_{k,i}-t}(\Theta)\right).
$$

The approach then consists in looking for the vector Θ that minimizes the distance between the N observed prices and modeled bond prices. Specifically, the vector Θ_t is given by:

$$
\Theta_t = \arg\min_{\Theta} \sum_{k=1}^{N} \omega_k (P_{k,t} - \hat{P}_{k,t}(\Theta))^2
$$

where the ω_k's are some weights that are chosen with respect to the preferences that one may have regarding the fit of different parts of the yield curve. Intuitively, taking the same value for all the ω_k's would lead to large yield errors for financial instruments with relatively short remaining time to maturity. This is linked to the concept of duration (i.e. the elasticity of the price with respect to one plus the yield): a given change in the yield corresponds to a small/large change in the price of a bond with a short/long term to maturity or duration. Since we do not want to favour a particular segment of the yield-curve fit, we weight the price error of each bond by the inverse of the remaining time to maturity.[36]

[36] Using remaining time to maturity instead of duration has not a large effect on estimated yields as long as we are not concerned with the very long end of the yield curve.

Coupon-bond prices come from Datastream.[37] In the same spirit as Gurkaynak et al. (2005), different filters are applied in order to remove those prices that would obviously bias the obtained yields. In particular, the prices of bonds that were issued before 1990 or that have atypical coupons (below 1% or above 10%) are excluded. In addition, the prices of bonds that have a time to maturity lower than 1 month are excluded.[38]

3.C. Computation of the covariance matrix of the parameter estimates

The second step of the estimation deals with the parameters defining the risk-neutral dynamics of (z_t, y_t) and the parameterization of the hazard rates. In this appendix, we detail how the covariance matrix of these estimates is derived. The non-linear least square estimator $\hat{\theta}$ is defined by (this is equation 3.14):

$$\hat{\theta} = \arg\min_{\theta} \sum_{n,t,h} \left(\tilde{R}_{n,t,h} - R_{n,t,h}(\theta)\right)^2 + \chi \sum_{t,i} \left(\tilde{y}_{i,t} - y_{i,t}(\theta)\right)^2$$

where $y_{i,t}(\theta)$ is the i^{th} entry of the vector of "theoretical" factors, in the sense that it is a linear combination of the "theoretical" yields $R_{n,t,h}(\theta)$, that are themselves a combination of observed factors $\tilde{y}_t$.

This estimator must satisfy the first-order conditons:

$$\sum_{n,t,h} \frac{\partial R_{n,t,h}(\theta)}{\partial \theta} (\tilde{R}_{n,t,h} - R_{n,t,h}(\theta)) + \chi \sum_{t,i} \frac{\partial y_{i,t}(\theta)}{\partial \theta} (\tilde{y}_{i,t} - y_{i,t}(\theta)) = 0,$$

where the left-hand side of the previous equation is of dimension $k \times 1$ (the dimension of vector θ). The Taylor expansion of the previous equation in a neighborood

[37] Naturally, the number of bonds used differ among the countries (from 19 bonds for the Netherlands to 175 bonds for Germany).

[38] The trading volume of a bond usually decreases considerably when it approaches its maturity date.

of the limit value θ_0 leads to (after multiplication by $1/\sqrt{T}$):

$$\begin{aligned} 0 \ \simeq \ & \frac{1}{\sqrt{T}}\left[\sum_{n,t,h}\frac{\partial R_{n,t,h}(\theta_0)}{\partial\theta}(\tilde{R}_{n,t,h}-R_{n,t,h}(\theta_0))+\chi\sum_{t,i}\frac{\partial y_{i,t}(\theta_0)}{\partial\theta}\left(\tilde{y}_{i,t}-y_{i,t}(\theta_0)\right)\right]+ \\ & \sqrt{T}\left(\hat{\theta}-\theta_0\right)\left[\frac{1}{T}\sum_{n,t,h}\left[\frac{\partial^2 R_{n,t,h}(\theta_0)}{\partial\theta\partial\theta'}(\tilde{R}_{n,t,h}-R_{n,t,h}(\theta_0))-\frac{\partial R_{n,t,h}(\theta_0)}{\partial\theta}\left(\frac{\partial R_{n,t,h}(\theta_0)}{\partial\theta}\right)'\right]+\right. \\ & \left.\frac{1}{T}\chi\sum_{t,i}\left[\frac{\partial^2 y_{i,t}(\theta_0)}{\partial\theta\partial\theta'}(\tilde{y}_{i,t}-y_{i,t}(\theta_0))-\frac{\partial y_{i,t}(\theta_0)}{\partial\theta}\left(\frac{\partial y_{i,t}(\theta_0)}{\partial\theta}\right)'\right]\right]. \end{aligned}$$

Since $E(\tilde{R}_{n,t,h}-R_{n,t,h}(\theta_0))=0$ and $E(\tilde{y}_{i,t}-y_{i,t}(\theta_0))=0$ (for any i), we have

$$\frac{1}{T}\sum_{n,t,h}\frac{\partial^2 R_{n,t,h}(\theta_0)}{\partial\theta\partial\theta'}(\tilde{R}_{n,t,h}-R_{n,t,h}(\theta_0))\overset{a.s.}{\to}0,$$

$$\frac{1}{T}\sum_{t,i}\frac{\partial^2 y_{i,t}(\theta_0)}{\partial\theta\partial\theta'}(\tilde{y}_{i,t}-y_{i,t}(\theta_0))\overset{a.s.}{\to}0.$$

Therefore:

$$\begin{aligned} \sqrt{T}\left(\hat{\theta}-\theta_0\right) \ \simeq \ & \left[\frac{1}{T}\sum_{n,t,h}\frac{\partial R_{n,t,h}(\theta_0)}{\partial\theta}\left(\frac{\partial R_{n,t,h}(\theta_0)}{\partial\theta}\right)'+\frac{1}{T}\chi\sum_{t,i}\frac{\partial y_{i,t}(\theta_0)}{\partial\theta}\left(\frac{\partial y_{i,t}(\theta_0)}{\partial\theta}\right)'\right]^{-1}\times \\ & \frac{1}{\sqrt{T}}\left[\sum_{n,t,h}\frac{\partial R_{n,t,h}(\theta_0)}{\partial\theta}(\tilde{R}_{n,t,h}-R_{n,t,h}(\theta_0))+\chi\sum_{t,i}\frac{\partial y_{i,t}(\theta_0)}{\partial\theta}\left(\tilde{y}_{i,t}-y_{i,t}(\theta_0)\right)\right]. \end{aligned}$$

Hence, the asymptotic distribution of $\sqrt{T}\left(\hat{\theta}-\theta_0\right)$ is estimated by $\hat{\mathcal{J}}^{-1}\hat{\mathcal{I}}\hat{\mathcal{J}}^{-1}$ where:

$$\hat{\mathcal{J}}^{-1}=\left[\frac{1}{T}\sum_{n,t,h}\frac{\partial R_{n,t,h}(\hat{\theta})}{\partial\theta}\left(\frac{\partial R_{n,t,h}(\hat{\theta})}{\partial\theta}\right)'+\frac{1}{T}\chi\sum_{t,i}\frac{\partial y_{i,t}(\hat{\theta})}{\partial\theta}\left(\frac{\partial y_{i,t}(\hat{\theta})}{\partial\theta}\right)'\right]^{-1}.$$

The second matrix, denoted by $\hat{\mathcal{I}}$, is the estimate of the covariance matrix of $1/\sqrt{T}\sum_t \gamma_t(\theta_0)$ where

$$\gamma_t = \sum_{n,h} \frac{\partial R_{n,t,h}(\theta_0)}{\partial \theta}(\tilde{R}_{n,t,h} - R_{n,t,h}(\theta_0)) + \chi \sum_{i} \frac{\partial y_{i,t}(\theta_0)}{\partial \theta}\left(\tilde{y}_{i,t} - y_{i,t}(\theta_0)\right).$$

To compute$\hat{\mathcal{I}}$, we use the Newey-West (1987) HAC estimator. This estimate is given by:

$$\hat{\mathcal{I}} = \sum_{i=-(T-m+1)}^{i=T-m-1} \kappa\left(\frac{i}{m}\right) \hat{cov}(\hat{\gamma}_t, \hat{\gamma}_{t+i})$$

where $\hat{\gamma}_t = \gamma_t(\hat{\theta})$ and where $\hat{cov}$ denotes the sample covariance operator. In practice, we use the Bartlett kernel $\kappa(x) = 1 - |x|$ and a bandwidth of 5.

3.D. Disentangling credit from liquidity risks: the loss function

In that appendix, we details the loss function introduced in 3.5.1. This function is aimed at being minimized in order tofind pairs of $(\gamma^0_{\ell,n}, \gamma^1_{\ell,n})$ that are such that (a) an important share of the spread *fluctuations* is explained by the liquidity intensity $\lambda^{\ell}_{n,t}$ under the constraints that (b) the implied risk-neutral and historical PDs are positive and that (c) the liquidity-related parts of the spreads are positive. Actually, an additional "shadow" parameter is introduced in the loss function to account for the fact that objective (a) focuses on the fluctuations and not on the level the spread (this will be clarified below). We consider linearized versions of the spreads in order to facilitate the optimization. This considerably fasten the optimization to the extent that (1) it avoids computations of multi-horizon Laplace transforms defined by (3.18) at each evaluation of the loss function and (2), it implies that analytical derivatives of the loss functions are available (which is particularly welcome when implementing the delta method to get standard deviations of the estimated $\gamma^0_{\ell,n}$ and $\gamma^1_{\ell,n}$). Formally, we define the following loss function $\mathcal{L}_n$

for each country n:

$$\begin{aligned}\mathcal{L}_n(\delta_0,\delta_1,\delta_2) &= \sum_t \Bigg\{\left(\overline{\lambda_{n,t}}^{\mathbb{Q}} - \left\{\delta_0 + \delta_1\overline{\lambda^{\ell}_{n,t}}^{\mathbb{Q}}\right\} + \delta_2\right)^2 \\ &\quad +\chi_1\left[\left(\left[\overline{\lambda_{n,t}}^{\mathbb{P}} - \left\{\delta_0 + \delta_1\overline{\lambda^{\ell}_{n,t}}^{\mathbb{P}}\right\}\right]_-\right)^2 + \left(\left[\overline{\lambda_{n,t}}^{\mathbb{Q}} - \left\{\delta_0 + \delta_1\overline{\lambda^{\ell}_{n,t}}^{\mathbb{Q}}\right\}\right]_-\right)^2\right] \\ &\quad +\chi_2\left(\left[\delta_0 + \delta_1\overline{\lambda^{\ell}_{n,t}}^{\mathbb{Q}}\right]_-\right)^2\Bigg\} \end{aligned} \tag{3.17}$$

where $[x]_-$ is equal to x if $x < 0$ and 0 otherwise, and where the operator $\overline{\bullet}^{\mathbb{Q}}$ is defined by (for any time series x):

$$\overline{x_t}^{\mathbb{Q}} = \frac{1}{h}E_t^{\mathbb{Q}}\left(x_{t+1} + \ldots + x_{t+h}\right). \tag{3.18}$$

When x is replaced by the hazard rate λ_n, we get a linearized approximation of the spread vs. Germany at maturity h. The operator $\overline{\bullet}^{\mathbb{P}}$ is the equivalent expectation computed under the historical measure.[39] The maturity h is supposed to be a benchmark maturity that is priviledged regarding objectives (a) to (c). We use $h = 60$ months.

Using this loss function, the estimation of the $\gamma^0_{\ell,n}$'s and the $\gamma^1_{\ell,n}$'s is based on the following optimization:

$$(\gamma^0_{\ell,n},\gamma^1_{\ell,n},\gamma^2_n) = \underset{\delta_0,\delta_1,\delta_2}{\arg\min}\,\mathcal{L}_n(\delta_0,\delta_1,\delta_2).$$

The three parts of the loss function (the second part including two terms) reflect the three criteria (a), (b) and (c) mentionned above. (a) The more the fluctuations of $\overline{\lambda_{n,t}}^{\mathbb{Q}}$ can be tracked by those of $\overline{\lambda^{\ell}_{n,t}}^{\mathbb{Q}}$, the lower the first part of the loss function is. In this first term, the shadow parameter δ_2 is introduced because we want this first part of $\mathcal{L}_n$ to focus on the *fluctuations* and not on the *level* of the intensities. Without the shadow parameter δ_2, we would arbitrarily favour those specifications

[39]If the relationship between spreads and intensities were linear, then $\gamma^0_{\ell,n} + \gamma^1_{\ell,n}\overline{\lambda^{\ell}_{n,t}}^{\mathbb{Q}}$ would be the part of the h-period spread (country n vs. Germany) corresponding to liquidity effects. Though the linearity assumption does not strictly hold, the approximation is reasonable as long as the λ's remain small.

of the liquidity intensity that imply close-to-zero-mean default-related spreads. (b) The second part of the loss function penalizes the specifications of the liquidity intensity that generate negative default compensations (under both measures). (c) The third term implies an additional cost when the liquidity-related part of the spread is negative.

Generating positive PDs is arguably a more important objective than getting positive liquidity compensations. As a consequence, χ_1 is taken higher than χ_2. We use $\chi_1 = 4$ and $\chi_2 = 1$ (see equation 3.17) for all countries except for Finland, for which we set these parameters to zero. With $\chi_1 = 4$ and $\chi_2 = 1$, we get positive and statistically significant Finnish PDs in the early 2000s. It may be due to the fact that the liquidity of Finnish bonds has increased over the last decade; but in our framework, we can not increase the liquidity spreads in the early 2000s without producing deeply negative PDs in the late 2000s (penalized when $\chi_1 = 4$).

Table 3.2.: Correlations and preliminary analysis of euro-area yield differentials

Notes: Panel A reports the covariances and correlations (in italics) of 10-year spreads (vs. Germany) across nine euro-area countries. Panel B presents results of principal-component analyses carried out on the spreads. There are three analyses that correspond respectively to three maturities: 2 years, 5 years and 10 years. For each analysis, Panel B reports the eigenvalues of the covariance matrices and the propotions of variance explained by the corresponding component (denoted by "Prop. of var." in Panel B).

Panel A: Covariance and *correlations* of 10-year spreads vs. Germany									
	Fr.	It.	Sp.	Au.	Be.	Fi.	Po.	Ne.	Ir.
France	0.015	0.045	0.052	0.023	0.027	0.012	0.077	0.013	0.117
Italy	*0.915*	0.163	0.202	0.062	0.088	0.023	0.313	0.032	0.459
Spain	*0.818*	*0.951*	0.277	0.069	0.113	0.025	0.434	0.032	0.623
Austria	*0.867*	*0.684*	*0.585*	0.05	0.047	0.028	0.093	0.024	0.15
Belgium	*0.922*	*0.887*	*0.871*	*0.854*	0.061	0.024	0.169	0.023	0.253
Finland	*0.599*	*0.358*	*0.293*	*0.767*	*0.615*	0.026	0.028	0.015	0.053
Portugal	*0.738*	*0.904*	*0.962*	*0.486*	*0.8*	*0.204*	0.736	0.042	1.03
Netherlands	*0.878*	*0.67*	*0.514*	*0.911*	*0.794*	*0.785*	*0.41*	0.014	0.074
Ireland	*0.783*	*0.918*	*0.956*	*0.545*	*0.83*	*0.263*	*0.97*	*0.497*	1.534
Panel B: Principal components									
Component	1	2	3	4	5	6	7	8	9
2-year spread									
Eigenvalue	6.07	1.46	0.71	0.33	0.16	0.12	0.08	0.05	0.02
Prop. of var.	0.67	0.16	0.08	0.04	0.02	0.01	0.01	0.01	0
Cumul. prop.	0.67	0.84	0.92	0.95	0.97	0.98	0.99	1	1
5-year spread									
Eigenvalue	6.68	1.56	0.38	0.13	0.09	0.07	0.05	0.02	0.01
Prop. of var.	0.74	0.17	0.04	0.01	0.01	0.01	0.01	0	0
Cumul. prop.	0.74	0.92	0.96	0.97	0.98	0.99	1	1	1
10-year spread									
Eigenvalue	6.83	1.62	0.27	0.12	0.06	0.05	0.02	0.02	0.01
Prop. of var.	0.76	0.18	0.03	0.01	0.01	0.01	0	0	0
Cumul. prop.	0.76	0.94	0.97	0.98	0.99	0.99	1	1	1

Table 3.3.: Parameters defining the historical and risk-neutral dynamics (Part 1/2)

Notes: The table reports the estimates of the parameters defining the dynamics of the factor under historical and risk-neutral measures. The estimation data are monthly and span the period from April 1999 to March 2011. Standard errors and Student-t are reported, respectively, in parenthesis and in square brackets below the coefficient estimates. ***, ** and * respectively denote significance at the 1%, 5% and 10% significance level.
The historical-dynamics parameterization is estimated by maximizing the log-likelihood (equation 3.3). The covariance matrix of the parameter estimates is based on the Hessian of the log-likelihood function. The risk-neutral dynamics of the factors is estimated together with the hazard-rate specifications reported in Table 3.5 using non-linear least squares. For the latter, the covariance matrix of the parameter estimates is computed using the Newey-West (1987 adjustment (see Appendix 3.C).

	Non-Crisis	Crisis		$\Phi_{i,1}$	$\Phi_{i,2}$	$\Phi_{i,3}$	$\Phi_{i,4}$	$\Phi_{i,5}$
μ_1	0.0054	-0.0052	$\Phi_{1,i}$	0.98***	0.0089***	0.017***	-0.015*	-0.12***
	(0.0086)	-		(0.0032)	(0.0024)	(0.0053)	(0.0081)	(0.033)
	[0.63]	-		[310]	[3.7]	[3.3]	[-1.9]	[-3.6]
μ_2	0.003	-0.0028	$\Phi_{2,i}$	-0.012	1.02***	0.17***	-0.049	0.25**
	(0.019)	-		(0.013)	(0.0108)	(0.017)	(0.031)	(0.12)
	[0.15]	-		[-0.94]	[94]	[10.4]	[-1.6]	[2]
μ_3	0.086*	-0.082*	$\Phi_{3,i}$	0.029	-0.054***	0.88***	0.091*	0.28
	(0.048)	-		(0.024)	(0.02)	(0.034)	(0.054)	(0.17)
	[1.8]	-		[1.2]	[-2.7]	[26]	[1.7]	[1.6]
μ_4	-0.057***	0.054***	$\Phi_{4,i}$	0.0023	0.00092	-0.019***	0.93***	-0.083*
	(0.0054)	-		(0.0033)	(0.0025)	(0.0046)	(0.0076)	(0.045)
	[-10.7]	-		[0.72]	[0.37]	[-4.2]	[124]	[-1.8]
μ_5	-0.016***	0.015***	$\Phi_{5,i}$	0.00054	0.0012	-0.0033	-0.0043	0.85***
	(0.0025)	-		(0.0015)	(0.0013)	(0.0024)	(0.0033)	(0.016)
	[-6.5]	-		[0.36]	[0.96]	[-1.4]	[-1.3]	[54]

	Non-Crisis	Crisis		$\Phi^*_{i,1}$	$\Phi^*_{i,2}$	$\Phi^*_{i,3}$	$\Phi^*_{i,4}$	$\Phi^*_{i,5}$
μ^*_1	0.017***	0.016***	$\Phi^*_{1,i}$	1***	0.011***	0.0026***	0.00015	0.00027
	(0.00047)	(0.0022)		(0.0004)	(0.00026)	(0.00063)	(-0.00038)	(-0.00025)
	[37]	[7.3]		[2511]	[42]	[4.1]	[0.17]	[0.058]
μ^*_2	0.044***	0.069***	$\Phi^*_{2,i}$	0.013***	0.98***	0.13***	-0.00042	-0.0017
	(0.0054)	(0.019)		(0.003)	(0.0038)	(0.0097)	(-0.0028)	(-0.0036)
	[8.2]	[3.7]		[4.4]	[256]	[14]	[-0.061]	[-0.051]
μ^*_3	-0.074***	-0.109***	$\Phi^*_{3,i}$	-0.021***	-0.013**	0.88***	-0.0006	-0.0016
	(0.0059)	(0.025)		(0.0043)	(0.005)	(0.011)	(-0.0041)	(-0.0047)
	[-13]	[-4.3]		[-4.8]	[-2.6]	[79]	[-0.056]	[-0.031]
μ^*_4	0.0034***	-0.00106	$\Phi^*_{4,i}$	0.00028	-0.00016	-0.00103	1***	0.023***
	(0.00103)	(0.0027)		(-0.00045)	(-0.0056)	(-0.018)	(0.0015)	(0.0051)
	[3.3]	[-0.39]		[0.54]	[-0.45]	[-0.83]	[689]	[4.6]
μ^*_5	-0.00025	-0.004**	$\Phi^*_{5,i}$	0.000093	-0.00108***	-0.000058	0	1***
	(0.00087)	(0.002)		(-0.0051)	(-0.0021)	(-0.024)	(0.00067)	(0.0033)
	[-0.29]	[-2]		[0.21]	[-3.4]	[-0.04]	[0.0086]	[308]

Markov-switching probabilities			
$\pi_{NC,NC}$	0.96***	$\pi^*_{NC,NC}$	1***
$\pi_{C,C}$	*0.96*	$\pi^*_{C,C}$	1***

Table 3.4.: Parameters defining the historical and risk-neutral dynamics (part 2/2)

Notes: See previous table. This table presents the estimated covariance matrices $\Sigma(z_t)$ of the Gaussian shocks $\Omega(z_t)\varepsilon_t$ in equation (3.1) (we have $\Sigma(z_t) = \Omega(z_t)\Omega(z_t)'$). The upper (respectively lower) part of the table reports the covariance matrix associated with the non-crisis (respectively crisis) regime.

	$\Sigma_{i,1}$	$\Sigma_{i,2}$	$\Sigma_{i,3}$	$\Sigma_{i,4}$	$\Sigma_{i,5}$
Non-crisis regime					
$\Sigma_{1,i}$	0.027***				
	(0.0036)				
	[7.5]				
$\Sigma_{2,i}$	0.026***	0.036***			
	(0.0038)	(0.0048)			
	[6.9]	[7.5]			
$\Sigma_{3,i}$	-0.031***	-0.027***	0.079***		
	(0.0053)	(0.0057)	(0.011)		
	[-5.8]	[-4.7]	[7.1]		
$\Sigma_{4,i}$	-0.0031***	-0.0032**	-0.00104	0.0038***	
	(0.00105)	(0.0013)	(0.0022)	(0.00064)	
	[-3]	[-2.5]	[-0.48]	[6]	
$\Sigma_{5,i}$	-0.00069*	-0.00057	-0.0011	0.0012***	0.00055***
	(0.00038)	(0.00047)	(0.00082)	(0.00022)	(0.000092)
	[-1.8]	[-1.2]	[-1.4]	[5.5]	[5.9]
Crisis regime					
$\Sigma_{1,i}$	0.069***				
	(0.019)				
	[3.6]				
$\Sigma_{2,i}$	0.058***	0.11***			
	(0.021)	(0.032)			
	[2.8]	[3.5]			
$\Sigma_{3,i}$	-0.0073	0.0013	0.084***		
	(0.015)	(0.02)	(0.025)		
	[-0.48]	[0.064]	[3.3]		
$\Sigma_{4,i}$	-0.041*	-0.0103	0.073***	0.15***	
	(0.022)	(0.026)	(0.027)	(0.042)	
	[-1.9]	[-0.4]	[2.7]	[3.6]	
$\Sigma_{5,i}$	-0.0081	0.004	-0.0049	-0.0061	0.0106***
	(0.0055)	(0.0069)	(0.0061)	(0.008)	(0.003)
	[-1.5]	[0.59]	[-0.81]	[-0.77]	[3.6]

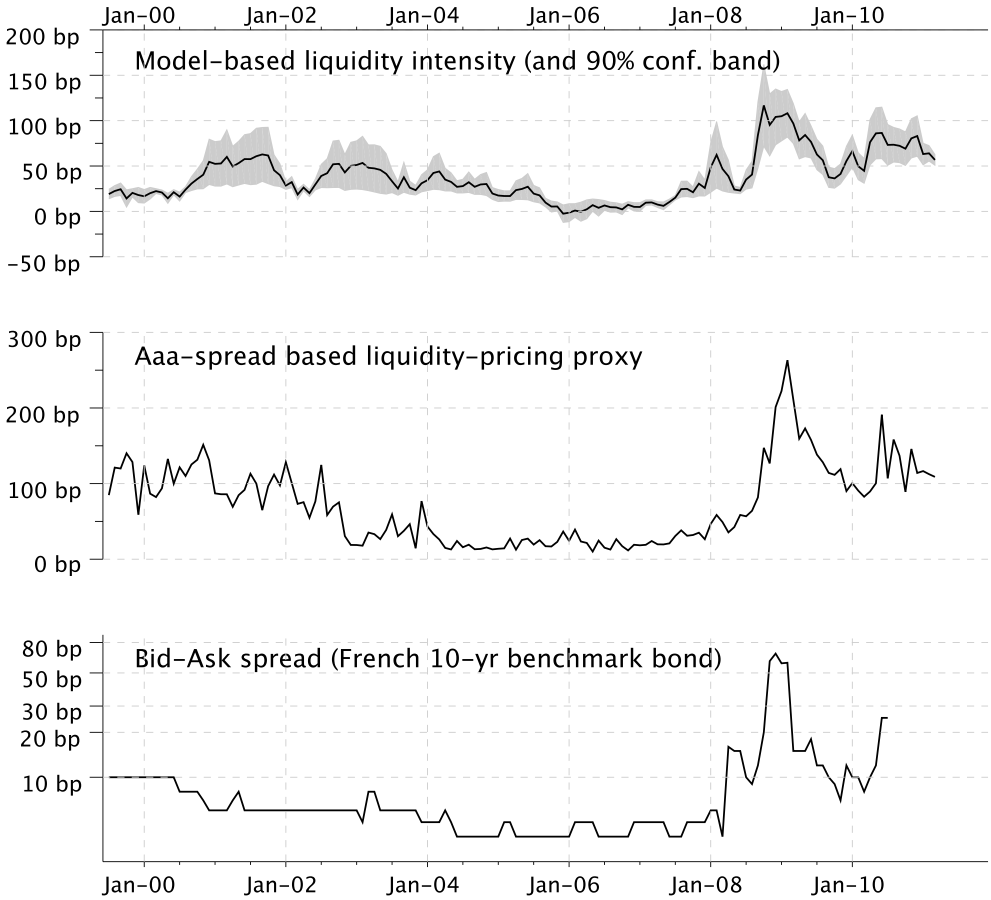
Table 3.5.: Estimation of the hazard-rate ($\lambda_{n,t}$) parameterizations

Notes: The Table reports the estimates of the hazard-rate parameterizations: $\lambda_{n,t} = (\alpha_n^c)' z_t + (\beta_n^c)' y_t + \lambda_{n,t}^\ell$, where α_n^c and β_n^c define the credit-related part of the hazard rate and $\lambda_{n,t}^\ell = \gamma_{\ell,n}^0 + \gamma_{\ell,n}^1 \left(\alpha_\ell' z_t + \beta_\ell' y_t\right)$ is the liquidity-related part (see Section 3.2.3). The first column of the Tablereports the parameterization of the illiquidity intensity λ_t^ℓ (with $\lambda_t^\ell = \alpha_\ell' z_t + \beta_\ell' y_t$) that drives all country-specific illiquidity intensites$\lambda_{n,t}^\ell$. The estimation data are monthly and span the period from April 1999 to March 2011. Standard errors and Student-t are reported, respectively, in parenthesis and in square brackets below the coefficient estimates. ***, ** and * respectively denote significance at the 1%, 5% and 10% significance level. The estimates of $\gamma_{\ell,n}^0$ and $\gamma_{\ell,n}^1$ are based on the following penalty factors (see equation 3.17): $\chi_1 = 4$ and $\chi_2 = 1$.

	KfW (λ_ℓ)		France	Austria	Netherlands	Finland	Belgium	Italy	Spain	Portugal	Ireland
$\alpha_{\ell,NC}$	0.41***	$\alpha_{c,NC}$	-0.037**	0.084**	-0.027	0.23***	0.014	0.088**	0.071	0.06	0.014
	(0.073)		(0.017)	(0.04)	(0.028)	(0.04)	(0.022)	(0.04)	(0.052)	(0.12)	(0.11)
	[5.7]		[-2.3]	[2.1]	[-1]	[5.8]	[0.7]	[2.2]	[1.4]	[0.5]	[0.1]
$\alpha_{\ell,C}$	0.64***	$\alpha_{c,C}$	0.052*	0.0049	0.045	0.18**	0.0072	-0.006	0.0046	0.102	0.19*
	(0.16)		(0.028)	(0.046)	(0.033)	(0.082)	(0.027)	(0.042)	(0.057)	(0.102)	(0.106)
	[3.9]		[1.9]	[0.1]	[1.4]	[2.2]	[0.3]	[-0.1]	[0.1]	[1]	[1.8]
$\beta_{\ell,1}$	0.11***	$\beta_{c,1}$	-0.024*	0.035	-0.025	0.13***	-0.006	-0.12***	-0.097***	-0.23***	-0.3***
	(0.035)		(0.013)	(0.023)	(0.017)	(0.021)	(0.021)	(0.025)	(0.025)	(0.044)	(0.07)
	[3.2]		[-1.9]	[1.5]	[-1.5]	[6.3]	[-0.3]	[-4.7]	[-3.9]	[-5.1]	[-4.2]
$\beta_{\ell,2}$	0.036*	$\beta_{c,2}$	-0.041***	0.018	-0.031	0.027*	-0.089***	-0.058*	-0.093***	-0.13***	-0.092
	(0.02)		(0.0108)	(0.03)	(0.023)	(0.015)	(0.019)	(0.03)	(0.016)	(0.038)	(0.065)
	[1.8]		[-3.8]	[0.6]	[-1.3]	[1.9]	[-4.6]	[-1.9]	[-5.7]	[-3.5]	[-1.4]
$\beta_{\ell,3}$	0.28**	$\beta_{c,3}$	-0.14***	-0.22***	-0.22***	0.16*	-0.22***	-0.24***	-0.13***	-0.26	-0.78***
	(0.13)		(0.037)	(0.074)	(0.048)	(0.083)	(0.054)	(0.081)	(0.048)	(0.18)	(0.22)
	[2.2]		[-3.9]	[-2.9]	[-4.5]	[1.9]	[-4.1]	[-3]	[-2.7]	[-1.4]	[-3.6]
$\beta_{\ell,4}$	0.14***	$\beta_{c,4}$	0.054	0.094**	0.044	0.082***	0.21***	0.3***	0.47***	0.75***	0.98***
	(0.043)		(0.04)	(0.047)	(0.049)	(0.03)	(0.044)	(0.065)	(0.066)	(0.13)	(0.2)
	[3.1]		[1.3]	[2]	[0.9]	[2.8]	[4.8]	[4.7]	[7.2]	[5.6]	[4.9]
$\beta_{\ell,5}$	0.61***	$\beta_{c,5}$	-0.23**	-0.28*	0.16	-0.098	-0.52***	-0.97***	-1.7***	-3.4***	-3.5***
	(0.093)		(0.105)	(0.16)	(0.16)	(0.106)	(0.15)	(0.22)	(0.32)	(0.58)	(0.83)
	[6.5]		[-2.2]	[-1.7]	[1]	[-0.9]	[-3.4]	[-4.3]	[-5.4]	[-5.7]	[-4.2]
		γ_ℓ^0	-0.067***	-0.096***	-0.093***	-0.076***	-0.14***	-0.22***	-0.2***	-0.37***	-0.49***
			(0.023)	(0.031)	(0.025)	(0.012)	(0.023)	(0.027)	(0.034)	(0.058)	(0.14)
			[-2.9]	[-3.1]	[-3.7]	[-6.5]	[-6]	[-8.2]	[-5.9]	[-6.4]	[-3.4]
		γ_ℓ^1	0.23**	0.53***	0.29*	0.51***	0.51***	1.09***	0.73***	1.5***	1.4**
			(0.094)	(0.14)	(0.17)	(0.061)	(0.12)	(0.23)	(0.2)	(0.4)	(0.68)
			[2.4]	[3.7]	[1.7]	[8.3]	[4.4]	[4.9]	[3.7]	[3.8]	[2.1]

Figure 3.5.: Liquidity intensity λ_t^ℓ and liquidity-pricing proxies

Notes: The upper panel presents the estimate of λ_t^ℓ, which is the factor driving the country-specific illiquidity intensities $\lambda_{n,t}^\ell$ ($\lambda_{n,t}^\ell = \gamma_{\ell,n}^0 + \gamma_{\ell,n}^1 \lambda_t^\ell$, see Section 3.2.3). The shaded area corresponds to the 90% confidence band based on the covariance matrix of the parameter estimates presented in Table 3.5 (the delta method is employed, using the fact that at each point in time, the estimate of λ_t^ℓ is a function of the parameter estimates and of y_t and z_t: $\lambda_t^\ell = \alpha_\ell' z_t + \beta_\ell' y_t$). The confidence band does not take into account the uncertainty stemming from the estimation of the regime variable z_t. The middle plot presents a liquidity-pricing measure inspired by Manganelli and Wolswijk (2009): for each period t, it is the mean of the absolute values of the spreads between the 10-year Aaa-rated-country yields and their average. (The underlying assumption being that most of the spreads between Aaa countries should be liquidity-driven.) The lower plot shows the bid-ask spreads on the 10-year French benchmark bond (computed as the monthly medians of high-frequency trade data provided by Thomson Reuters Tick History).



4. Credit and liquidity pricing within the financial crisis[1]

Abstract: In this chapter, we model the joint dynamics of euro-area sovereign bond spreads during the crisis period 2007-2012. It is therefore closely related to the previous one but departs from it in several ways. First, thanks to a more intensive use of regime-switching features, we identify credit and/or liquidity stress periods and explore the causality between these two types of stresses. Second, while the pricing factors were observable in the previous chapter, they are latent in the present one. Third, the analysis is extended by deriving market-perceived default probabilities. The latter are obtained by filtering liquidity-pricing effects and risk premia out of the spreads. We find that the actual –or physical, or real-world– probabilities of default are significantly lower than their risk-neutral counterparts. This is consistent with the existence of a non-diversifiable euro-area sovereign credit risk.

[1]This Chapter is based on an article entitled "Decomposing euro-area sovereign spreads: credit and liquidity risks", coauthored with Alain Monfort. We are grateful to Christian Gourieroux, Glenn Rudebusch, Thomas Sargent, Vladimir Borgy, Valère Fourel, Wolfgang Lemke, Simon Gilchrist, Kristoffer Nimark, Tao Zha, Christian Hellwig, Jean-Sébastien Fontaine and Adrien Verdelhan for helpful discussions and comments. We are also grateful to seminar participants at the Banque de France, CREST, the Paris finance international meeting 2010, CORE Econometrics Seminar 2011, ESEM annual meeting 2011, IESEG-University of Cambridge conference on yield-curve modeling, AFSE annual meeting, the Bank of England, CDC, the ECB Workshop on Asset pricing models in the aftermath of the financial crisis, Computational and Financial Econometrics conference (London 2011), the Bank of Canada, the Bundesbank. We thank Beatrice Saes-Escorbiac and Aurélie Touchais for excellent research assistance. Any remaining errors are ours. The views expressed in this Chapter are ours and do not necessarily reflect the views of the Banque de France.

Résumé

Comme le précédent, ce chapitre présente une analyse empirique des écarts de taux d'Etat de la zone euro. La période d'étude est plus courte que précédemment (2007-2012 dans le présent chapitre versus 1999-2012 dans le précédent) et la fréquence d'échantillonage est plus élevée (hebdomadaire versus mensuelle). Par ailleurs, une utilisation différente des régimes est faite dans le présent chapitre. Enfin, l'analyse est complétée par le calcul de probabilités de défaut des différents Etat considérés (telles que perçues par les participants de marché).

Comme dans les chapitres précédents, chaque émetteur (Etat) est caractérisé par une intensité de crédit, reflétant son risque de défaut, et une intensité d'illiquidité, reflétant l'illiquidité relative des titres qu'il émet. Les intensités de crédit et d'illiquidité sont des processus auto-régressifs à innovations gaussiennes dont les constantes dépendent respectivement de régimes de crédit et de liquidité. Les premiers sont au nombre de trois (état relativement calme, état de stress intermédiaire ou état de stress élevé) et les seconds au nombre de deux (en état de stress ou non). Ces deux chaînes sont communes à tous les pays considérés. Le modèle comporte donc six régimes.

Ce type de spécifications permet d'introduire des relations de causalité entre les deux chaînes de manière simple. Par exemple, la probabilité de passer en régime de crise de liquidité peut dépendre de l'état, en période précédente, de la chaîne définissant le stress lié au risque de crédit.

Les facteurs, comme les régimes, ne sont pas directement observables. L'estimation du modèle repose sur l'utilisation de l'algorithme de Kim (1994), celui-ci permettant de traiter simultanément le caractère latent des deux types de variables. L'estimation des paramètres du modèle repose sur la maximisation de la fonction de vraisemblance. Une fois les paramètres estimés, on peut calculer, pour chaque date, les probabilités d'être dans chacun des six régimes possibles (conditionnellement aux variables observées que sont les écarts de taux). Ces probabilités permettent de découper la période 2007-2012 en différentes phases se distinguant par la prégnance des problèmes de liquidité et/ou de de perception de la qualité

de crédit des Etats de la zone euro. L'estimation met par ailleurs en évidence l'existence de relations de causalité entre les deux chaînes (crédit et liquidité).

Comme dans le cadre du chapitre précédent, l'identification des intensité d'illiquidité des différents pays repose sur l'inclusion de taux relatifs à la banque publique allemande *KfW* dans les données d'estimation. Plus précisément, notre stratégie d'identification des intensités d'illiquidité des différents pays repose sur deux hypothèses: (a) l'écart de taux *KfW-Bund* est intégralement expliqué par un facteur de liquidité et (b) les intensitiés d'illiquidité des différents émetteurs souverains sont identiques à une transformation affine près. Malgré une modélisation différente, cette nouvelle décomposition des écarts de taux en une composante crédit et une composante liquidité confirme les résultats du chapitre précédent.

Dans ce chapitre, nous montrons également comment calculer les probabilités de défaut des États (perçues par les participants de marché) à partir du modèle estimé. Si les investisseurs étaient neutres au risque (et si l'illiquidité relative des obligations n'était pas valorisée), alors l'écart entre le taux d'une obligation émise par une entité pouvant faire défaut et celui d'une référence sans risque correspondrait à la perte moyenne (ou "espérée") en cas de défaut de l'émetteur. Dans ce contexte, moyennant une hypothèse de taux de recouvrement (qui est le ratio entre le montant recouvré en cas de défaut et la valeur faciale de l'obligation), il serait aisé de déduire les probabilités de défaut (perçues par les participants de marché) à partir de prix d'obligations.

Cette approche, largement utilisée par les analystes de marché, est pourtant erronée car les investisseurs ne sont pas neutres aur risque. Autrement-dit, les probabilités résultant de l'approche précédente sont des probabilités risque-neutres et non physiques. L'aversion au risque –reflétée dans notre cadre par le facteur d'escompte stochastique sous-jacent au modèle– explique par exemple pourquoi nous sommes prêts à acquérir des polices d'assurance coûtant en moyenne plus cher que les remboursements espérés de la part de l'assureur.

Les différences entre les probabilités de défaut calculées sous l'hypothèse de neutralité des investisseurs vis-à-vis du risque et celles issues de notre estimation sont très importantes. A titre d'exemple, alors que certaines analyses de marché, comme celle de CMA (2012), aboutissaient fin 2011 à des probabilités de défaut à cinq ans de près de 20% pour la France, et près de 30% pour l'Espagne et l'Italie, notre approche fournit des probabilités de défaut perçues par les marchés qui seraient respectivement 4%, 12% et 15%. Les différences enre les probabilités de défault risque-neutre et les probabilités de défaut dites physiques correspondent à des primes de risque de crédit. L'existence de telles primes de risque découle de l'aspect non-diversifiable du risque souverain en zone euro (les contributions récentes étudiant ce risque souverain incluent Borri and Verdehlan, 2012 and Longstaff et al., 2011).

4.1. Introduction

In this Chapter, we develop a multi-issuer no-arbitrage affine term-structure framework to model the dynamics of bond spreads, with a twofold objective: to disentangle credit and liquidity components in euro-area sovereign spreads and to identify the part of these spreads corresponding to risk premia, defined as the part that would not be present if agents were risk-neutral. Risk premia are demanded by risk-averse investors to be compensated for non-diversifiable –or systematic– risk, and our results are supportive of the findings of Pan and Singleton (2008) and Longstaff et al. (2011) who point to the systematic nature of sovereign risk.[2] The resulting risk premia associated with sovereign credit quality implies that physical, or real-world, probabilities of default differ from their risk-neutral counterparts. Yet, the latter, derived from basic models like Litterman and Iben (1991), are extensively used by market practitioners, who refer to them as *implied default probabilities*.[3] Our approach makes it possible to assess the deviations between the two kinds of PDs and we show that these can be substantial. In particular, these results are of significant interest in the current context where regulators want banks to model the *actual* default risk of even high-rated government bonds.[4]

In our framework, each country is characterized by a risk intensity which is the sum of a credit intensity and an illiquidity one. We propose an original use of regime-switching features to account for the joint dynamics of credit- and liquidity-related crises, the aim being to make the model consistent with theoretical approaches highlighting the potentital interactions between these two kinds of risks.[5] Credit- and liquidity-crisis regimes are key drivers of the countries' intensities, the latter

[2]Borri and Verdelhan (2011) propose a theoretical framework to investigate the implications of the investors' inability to hedge against correlated sovereign risks.

[3]See e.g. Hull, Predescu and White (2005), Berd, Mashal and Wang (2003), Caceres, Guzzo and Segoviano (2010) or Berg (2009).

[4]In early 2012, the European Union introduced new rules on trading-book capital, known as Basel 2.5. This package notably requires the banks to model the default risk of all sovereign entities for the first time. This contrasts with the special status that government bonds have enjoyed since the Basel Committee for Banking Supervision (BCBS) first proposed rules on the capital treatment of market risks in 1993. As stressed by Carver (*Risk Magazine*, 2012), these changes in regulation reveal the practitioners' lack of tools to extract actual default probabilities from market prices.

[5]See e.g. Brunnermeier and Pedersen (2009)[48] or Garleanu and Pedersen (2007)[126].

being also affected by Gaussian shocks. In this framework, the spreads turn out to be linear combinations of the regime variables and of latent factors that follow Gaussian auto-regressive processes whose drifts depend on the regimes. Therefore, the model can be seen as a linear state-space model with regime switching.[6] The countries' illiquidity intensities are driven by a single European liquidity-related factor. The identification of this factor is based on the exploitation of the term structure of the spreads between KfW (*Kreditanstalt für Wiederaufbau*), a German agency, and the *Bunds*, which are the bonds issued by the Federal Republic of Germany. Indeed, the bonds issued by KfW, guaranteed by the Federal Republic of Germany, benefit from the same credit quality than the *Bunds* but are less liquid.[7] Therefore, the KfW-*Bund* spread should be essentially liquidity-driven.[8] The resulting liquidity-related factor significantly contributes to the dynamics of intra-euro spreads, supporting findings by Favero et al. (2010) or Manganelli and Wolswijk (2009).

The model is estimated on weekly data covering the last five years. These data consist of sovereign-bond yields associated with eight euro-area countries. Our estimation dataset is supplemented with survey-based forecasts. As evidenced by Kim and Orphanides (2012), this alleviates the downward small-sample bias in the persitence of the yields obtained with conventional estimation.[9] Such biases typically result in too sTablelong-horizon expectations of yields and, as a consequence, overstate the variability of term premia. Generating reliable expectations is crucial given our goal of recovering historical –or actual, or real-world– probabilities of default from bond prices.

The remaining of this Chapter is organized as follows. Section 4.2 presents the model and details how bonds are priced in this framework. Section 4.3 deals with the choice and the construction of the data. The estimation of the model

[6] Accordingly, we use Kim's (1994) algorithm to estimate the model parameters by maxmizing the likelihood.

[7] By abuse of language, we use here the term *Bunds* for the German sovereign bonds of any maturity although this name is usually used for ten-year bonds only.

[8] See Schwarz (2009). This is also discussed in Subsection 3.3.1.

[9] This way of reducing the bias is not the only one. In particular, Jardet, Monfort and Pegoraro (2009) use a "near-cointegrated framework" specification of the factors (averaging a stationary and a cointegrated specification).

is detailed in Section 4.4 and Section 4.5 examines the implications of the model in terms of liquidity and credit pricing. In particular, this Sectionpresents model-implied physical probabilities of default. Section 4.6 summarizes the results and makes concluding remarks.

4.2. The model

We consider zero-coupon bonds issued by N debtors. These entities may default and their bonds are not perfectly liquid, both aspects having an impact on the bonds prices. Heuristically, a bondholder fears about the default of the bond's issuer –that would result in a early and reduced repayment of the bond– and about the risk of being hit by a liquidity shock.[10] In the latter case, the bondholder is forced to precipitately liquidate her bond holdings and, in such circumstances, illiquid bonds are sold at a discount.

Subsection 4.2.1 presents the notations and introduces default and liquidity intensities. The historical (respectively risk-neutral) dynamics of the model's variables is developed in Subsection 4.2.2 (4.2.3). The implications in terms of bond pricing are developed in Subsection 4.2.4.

4.2.1. Default events, liquidity shocks and associated intensities

At date t, each investor is provided with the new information $\tilde{w}_t = (r_t,\, z_t',\, \tilde{\lambda}_{d,t}',\, \tilde{\lambda}_{\ell,t}',\, d_t',\, \ell_t)'$ where r_t is the risk-free short-term rate, z_t is a crisis-regime variable, $\tilde{\lambda}_{d,t}$ is a N-dimensional vector containing the default intensities associated with the respective N debtors, d_t is a N-dimensional vector of binary variables $d_t^{(n)}$ indicating whether debtor n is in default at date t ($d_t^{(n)} = 1$, which is an absorbing state) or not ($d_t^{(n)} = 0$),[11] $\tilde{\lambda}_{\ell,t}$ is the liquidity-shock intensity and ℓ_t is a binary

[10] The liquidity shock may occur e.g. as a result of unexpected cash shortages, the need to rebalance a portfolio in order to maintain a hedging or diversification strategy, or a change in capital requirements (see He and Xiong, 2012).

[11] We use parenthesis to distinguish country from exponentiation in the superscript.

variable indicating if the bondholder is affected by the liquidity shock at date t ($\ell_t = 1$) or not ($\ell_t = 0$).

Denoting by w_t the vector $w_t = (r_t, z_t', \tilde{\lambda}_{d,t}', \tilde{\lambda}_{\ell,t}')'$ and by $\underline{\tilde{w}}_t$ the cumulated information available at date t, i.e. $\underline{\tilde{w}}_t = (\tilde{w}_t, \tilde{w}_{t-1}, \ldots, \tilde{w}_1)$, the conditional probability of default of debtor n is given by:

$$\mathbb{P}\left(d_t^{(n)} = 1 \middle| w_t, d_{t-1}^{(n)} = 0, \underline{\tilde{w}}_{t-1}\right) = 1 - \exp\left(-\tilde{\lambda}_{d,t}^{(n)}\right),$$

which is close to $\tilde{\lambda}_{d,t}^{(n)}$ when this intensity is small. Let us consider a bond issued by debtor n with a residual maturity of h at date t. We denote by $B_{t,h}^{(n)}$ the non-default price of this bond. If debtor n defaults between date $t-1$ and date t, the bondholder is assumed to receive –from the borrower– a fraction ζ of the price that would have prevailed otherwise at date t. In other words, in the case of default, the recovery pay-off is $\zeta B_{t,h}^{(n)}$.

The conditional probability, for an investor, of being hit by the liquidity shock is:

$$\mathbb{P}\left(\ell_t = 1 \middle| d_t, w_t, \underline{\tilde{w}}_{t-1}\right) = 1 - \exp\left(-\tilde{\lambda}_{\ell,t}\right).$$

In particular, this probability does not depend on $\underline{\ell}_{t-1}$ and $\underline{d}_t$. Upon the arrival of the liquidity shock ($\ell_t = 1$), the bond investor has to exit by selling her bond holdings at a fractional cost $1-\theta^{(n)}$, that is, the proceed of the sale is then $\theta^{(n)} B_{t,h}^{(n)}$. A theoretical basis for such a fractional cost can be found in Ericsson and Renault (2006)[12]

Conditionally on $(w_t, \underline{\tilde{w}}_{t-1})$, the $d_t^{(n)}$'s and ℓ_t are independent. However, conditionally on the past information $\underline{\tilde{w}}_{t-1}$, the default events and the liquidity shocks are not independent because the associated intensities are correlated with each other.[13]

[12] In their model, an investor hit by the liquidity shock must liquidate her bond holdings in a limited time (between t and t^+, say). Then, she obtains a Poisson-distributed number K of offers from traders ($K \sim \mathcal{P}(\gamma^{(n)})$) and retains the best one, each offer being a random fraction ω_i ($i \in [1,,K]$) of $B_{t,h}^{(n)}$, which can then be seen as the price she would get if $\gamma^{(n)}$ was infinite. Therefore the higher $\gamma^{(n)}$, the more liquid the bonds issued by n.

[13] This assumption appears in the "doubly stochastic" framework (see e.g. Duffie et al., 2005, Pan and Singleton, 2008 or Longstaff et al., 2011).

For the sake of bond pricing, it will prove convenient to introduce the fractional-loss intensities $\lambda_{d,t}^{(n)}$ and $\lambda_{\ell,t}^{(n)}$ (see Appendix 4.A). Intuitively, they correspond to the expected losses, conditional to w_t, associated associated with, respectively, the default of debtor n and the arrival of a liquidity shock (expressed as fractions of the price that would have prevailed, absent the default and/or the liquidity shock). Appendix 4.A shows that these intensities are defined through:

$$\begin{cases} \exp\left(-\lambda_{d,t}^{(n)}\right) & = \exp\left(-\tilde{\lambda}_{d,t}^{(n)}\right) + \zeta\left(1 - \exp\left(-\tilde{\lambda}_{d,t}^{(n)}\right)\right) \\ \exp\left(-\lambda_{\ell,t}^{(n)}\right) & = \exp\left(-\tilde{\lambda}_{\ell,t}\right) + \theta^{(n)}\left(1 - \exp\left(-\tilde{\lambda}_{\ell,t}\right)\right). \end{cases} \tag{4.1}$$

When the $\tilde{\lambda}_{d,t}^{(n)}$'s and $\tilde{\lambda}_{\ell,t}$ are small, these equations are approximately $\lambda_{d,t}^{(n)} = (1-\zeta)\tilde{\lambda}_{d,t}^{(n)}$ and $\lambda_{\ell,t}^{(n)} = (1-\theta^{(n)})\tilde{\lambda}_{\ell,t}$.

Naturally, when the fractional recovery pay-offs (ζ and $\theta^{(n)}$) are equal to one, the fractional-loss intensities are null. Hence, when both kinds of losses are ruled out ($\zeta = \theta^{(n)} = 1$), the bonds issued by debtor n turn out to be risk-free bond. By contrast, when the recovery pay-offs are null, the fractional-loss intensities correspond to the conditional probabilities of default and to the probability of being hit by the liquidity shock, respectively.

4.2.2. Historical dynamics of w_t

4.2.2.1. Short rate, credit- and liquidity-related Markov chains

As in Pan and Singleton (2008) or Longstaff et al. (2011), we assume that the short-term risk-free interest rate is exogenous. Hence, we work conditionnally to observed values of the r_t's.

The joint dynamics of the recovery-adjusted default intensities ($\lambda_{d,t}^{(n)}$) and of the liquidity intensities ($\lambda_{\ell,t}^{(n)}$) crucially depends on an exogenous Markov chain z_t. The regime variable z_t is obtained by crossing two regime variables. A first regime variable $z_{\ell,t}$ defines the liquidity situation, which can be distressed ($z_{\ell,t} = [0,1]'$) or not ($z_{\ell,t} = [1,0]'$). A second regime variable $z_{c,t}$ represents the credit situation, the latter being either non-stressed ($z_{c,t} = [1,0,0]'$), distressed ($z_{c,t} = [0,1,0]'$) or severely distressed ($z_{c,t} = [0,0,1]'$).[14] The credit/liquidity regime of the economy at date t is then summarized by the six-dimensional selection vector z_t, which is the Kronecker product of $z_{\ell,t}$ and $z_{c,t}$:

$$z_t = z_{\ell,t} \otimes z_{c,t}, \tag{4.2}$$

The vector z_t is valued in $\{e_1^{[6]}, \ldots, e_6^{[6]}\}$, where $e_i^{[M]}$ denotes the M-dimensional vector whose all entries are equal to 0, except the i^{th} that is equal to 1.

Importantly, there may be causal relationships between $z_{\ell,t}$ and $z_{c,t}$. For instance, we allow for the probability of a change in the liquidity state to depend on the credit regime (and vice-versa). Formally, let us denote by Π the matrix of transition probabilities, whose (i,j) entry, denoted by $\pi_{i,j}$, corresponds to $p(z_{t+1} = e_j^{[6]} | z_t = e_i^{[6]})$. The entries of the row of this matrix summing to one, 30 parameters are required to specifiy this matrix. In order to keep the model parismonious, some constraints are introduced. With these constraints, which are detailed in Appendix 4.B, 11 parameters are required to specifiy the matrix Π.

4.2.2.2. Historical ($\mathbb{P}$) dynamics of the $\lambda_{d,t}^{(n)}$'s and the $\lambda_{\ell,t}^{(n)}$'s

The dynamics of the intensities $\lambda_{d,t}^{(n)}$ and $\lambda_{\ell,t}^{(n)}$ are connected through the regime variables. Consistently with the liquidity shock interpretation introduced in Subsection 4.2.1, we assume that the illiquidity intensities are driven by a single factor denoted by $\lambda_{\ell,t}$.[15] This factor, as well as the credit-related ones, follow

[14]Preliminary modelling with a unique level of credit-distress regime led to a less satisfying fit of the data. That is why this additional level of credit distress (compared with the unique liquidity-distress regime) has been introduced in the framework.

[15]However, this factor is not rigorously equal to the liquidity-shock intensity $\tilde{\lambda}_{\ell,t}$. Indeed, for this to be the case in a context where the $\theta^{(n)}$ are not time-varying, the $\lambda_{\ell,t}^{(n)}$'s should be the same up to a multiplicative factor. In other words, the $\alpha_{0,\ell}^{(n)}$'s in equation (4.4) should be equal to zero. That being so, we use these additional degrees of freedom to improve the model fit.

auto-regressive processes with drifts depending on the regime variables. Formally:

$$\begin{cases} \lambda_{d,t}^{(n)} &= \mu_d^{(n)\prime} z_{d,t} + \rho_d \lambda_{d,t-1}^{(n)} + \sigma_d^{(n)} \varepsilon_{d,t}^{(n)} \\ \lambda_{\ell,t}^{(n)} &= \alpha_{0,\ell}^{(n)} + \alpha_{1,\ell}^{(n)} \lambda_{\ell,t} \end{cases} \quad \forall n \tag{4.3}$$

$$\lambda_{\ell,t} = \mu_\ell' z_{\ell,t} + \rho_\ell \lambda_{\ell,t-1} + \sigma_\ell \varepsilon_{\ell,t} \tag{4.4}$$

where the $\varepsilon_{d,t}^{(n)}$'s –some country-specific credit shocks– and the $\varepsilon_{\ell,t}$'s –some liquidity-related shocks– are i.i.d. $\mathcal{N}(0,1)$. We denote by λ_t the $(N+1)\times 1$ vector containing the recovery-adjusted default intensities and the liquidity-related factor, i.e. $\lambda_t = [\lambda_{d,t}^{(1)}, \dots, \lambda_{d,t}^{(N)}, \lambda_{\ell,t}]'$, and by ε_t the associated innovations, i.e. $\varepsilon_t = [\varepsilon_{d,t}^{(1)}, \dots, \varepsilon_{d,t}^{(N)}, \varepsilon_{\ell,t}]'$. By abuse of notation, we may denote the entries of λ_t by $\lambda_{i,t}$ in the following.[16] Then, denoting by μ the $6 \times (N+1)$ matrix of drifts,[17] by Φ the matrix whose diagonal entries are ρ_d (N times) and ρ_ℓ, and by Σ the matrix whose diagonal entries are the $\sigma_d^{(n)}$'s and σ_ℓ, the dynamics of λ_t reads:

$$\lambda_t = \mu' z_t + \Phi \lambda_{t-1} + \Sigma \varepsilon_t \tag{4.5}$$

Equation (4.5) means that the conditional distribution of λ_t given $(r_t, z_t, \underline{\tilde{w}}_{t-1})$ is $\mathcal{N}(\mu' z_t + \Phi\lambda_{t-1}, \Sigma^2)$, implying in particular that this distribution depends on $\underline{\tilde{w}}_{t-1}$ through λ_{t-1} only. Moreover, since r_t and z_t are exogenous, this implies that the distribution of w_t given $\underline{\tilde{w}}_{t-1}$ does not depend on $(\underline{d}_{t-1}, \underline{\ell}_{t-1})$, that is, (d_t, ℓ_t) does not Granger-cause w_t.

It can be seen that the $\lambda_{i,t}$'s are positively marginally skewed as soon as the μ vectors contain only positive entries. Moreover, the lower the standard deviations σ of the Gaussian shocks (in comparison with the drifts μ), the more often the $\lambda_{i,t}$'s are positive, which is important given their interpretations in terms of probabilities.

Furthermore, the instantaneous causality between z_t and λ_t implies that the variances of the $\lambda_{i,t}$'s, conditionally on $\underline{\tilde{w}}_{t-1}$, depend on the regime variable z_{t-1}. More precisely, conditionally to $\underline{\tilde{w}}_{t-1}$, the distributions of the $\lambda_{i,t}$'s are some mixtures

[16] $\lambda_{i,t} = \lambda_{d,t}^{(i)}$ for $i \leq N$ and $\lambda_{N+1,t} = \lambda_{\ell,t}$.

[17] The columns of this matrix are $\mu_d^{(1)}, \dots \mu_d^{(N)}$, and μ_ℓ.

of Gaussian distributions, thereby involving a form of heteroskedasticity in the innovations.[18]

4.2.3. Stochastic discount factor and risk-neutral ($\mathbb{Q}$) dynamics

We assume that the stochastic discount factor (s.d.f.) has the following expression:

$$M_{t-1,t} = \exp\left[-r_{t-1} - \frac{1}{2}\nu_t'\nu_t + \nu_t'\varepsilon_t + (\delta z_{t-1})' z_t\right] \tag{4.6}$$

where δ is a 6×6 matrix and where the entries of ν_t are affine in z_t and in the corresponding entries of λ_{t-1}, that is $\nu_{i,t} = \nu_{\lambda,i}\lambda_{i,t-1} + \nu_{z,i}' z_t$, say. ($\nu_{\lambda,i}$ is a scalar and $\nu_{z,i}$ is a vector.) The risk-sensitivity matrix δ and the vectors ν_t respectively price the regimes z_t and the (standardized) Gaussian innovations ε_t of λ_t. The fact that we must have $E_t(M_{t,t+1}) = \exp(-r_t)$ implies that the entries of δ are of the form $\ln(\pi_{ij}^*/\pi_{ij})$ where the π_{ij}^* are such that $\Sigma_j \pi_{ij}^* = 1$ for any i. Note that the variables (d_t, ℓ_t) do not appear in $M_{t-1,t}$, in other words, we assume that the risk aversion is completely captured by the pricing of the innovation process ε_t and the regime process z_t. It can be shown that, in such framework, the risk-neutral ($\mathbb{Q}$) dynamics of (z_t, λ_t) is of the same form as its historical counterpart.[19] More precisely, under $\mathbb{Q}$, z_t follows a time-homogenous Markovian chain whose dynamics is described by the matrix Π^* of transition probabilities $\{\pi_{ij}^*\}$ and, denoting by $\lambda_{i,t}$ the i^{th} entry of λ_t, we have:

$$\lambda_{i,t} = \mu_i^{*'} z_t + \rho_i^* \lambda_{i,t-1} + \sigma_i \varepsilon_t^* \tag{4.7}$$

where $\varepsilon_{i,t}^* \sim \mathcal{N}^{\mathbb{Q}}(0,1)$, $\mu_i^* = \mu_i + \sigma_i \nu_{z,i}'$ and $\rho_i^* = \rho_i + \sigma_i \nu_{\lambda,i}$.

Let us turn to the risk-neutral dynamics of d_t and ℓ_t. As shown in Appendix 4.C, the conditional distributions –given $(w_t, \underline{\tilde{w}}_{t-1})$– of these binary variables are the same functions of w_t under $\mathbb{P}$ and $\mathbb{Q}$. In other words, for any n, $\tilde{\lambda}_{d,t}^{(n)}$ is the same

[18]Such a feature is discussed in Ang, Bekaert and Wei (2008).
[19]This results stems from Lemma 1.

process in both worlds, and the same is true for $\tilde{\lambda}_{\ell,t}$. This stems from the fact that the variables d_t and ℓ_t do not enter the s.d.f. (that depends on w_t only).[20] However, it is important to stress that while the intensities are the same processes under both measures, their $\mathbb{Q}$- and $\mathbb{P}$-dynamics are different (because the $\mathbb{Q}$- and $\mathbb{P}$-dynamics of (z_t, λ_t) differ). As a consequence, the probabilites of default are different under $\mathbb{P}$ and $\mathbb{Q}$.

4.2.4. Bond pricing

In this framework, the price of a defaultable and illiquid zero-coupon bond issued by country n (not in default at date t) and with residual maturity h has a price at time t that is given by (see Appendix 4.A):

$$B_{t,h}^{(n)} = E_t^{\mathbb{Q}}\left[\exp\left(-r_t - \ldots - r_{t+h-1} - \lambda_{d,t+1}^{(n)} - \ldots - \lambda_{d,t+h}^{(n)} - \lambda_{\ell,t+1}^{(n)} - \ldots - \lambda_{\ell,t+h}^{(n)}\right)\right]. \tag{4.8}$$

where r_t is the return of a risk-free investment between t and $t+1$ and where $E_t^{\mathbb{Q}}$ is the conditional expectation given $\underline{\tilde{w}}_{t-1}$ in the risk-neutral world.

The short-term risk-free interest rate being exogenous, we have:

$$\begin{aligned} B_{t,h}^{(n)} &= E_t^{\mathbb{Q}}\left[\exp\left(-r_t - \ldots - r_{t+h-1}\right)\right] \times \\ & \quad E_t^{\mathbb{Q}}\left[\exp\left(-\lambda_{d,t+1}^{(n)} - \ldots - \lambda_{d,t+h}^{(n)} - \lambda_{\ell,t+1}^{(n)} - \ldots - \lambda_{\ell,t+h}^{(n)}\right)\right]. \end{aligned} \tag{4.9}$$

Denoting by $y_{t,h}^{(n)}$ the yield-to-maturity of this bond, we obtain:

$$\begin{aligned} y_{t,h}^{(n)} &= -\frac{1}{h}\ln(B_{t,h}^{(n)}) \\ &= y_{t,h}^{(0)} - \frac{1}{h}\ln\left(E_t^{\mathbb{Q}}\left[\exp\left(-\lambda_{d,t+1}^{(n)} - \ldots - \lambda_{d,t+h}^{(n)} - \lambda_{\ell,t+1}^{(n)} - \ldots - \lambda_{\ell,t+h}^{(n)}\right)\right]\right) \end{aligned} \tag{4.10}$$

[20] Appendix 4.C also shows that in that context, the fact that the distribution of w_t given $\underline{\tilde{w}}_{t-1}$ does not depend on $(\underline{d}_{t-1}, \underline{\ell}_{t-1})$ –i.e. that (d_t, ℓ_t) does not cause w_t– is true under both measures.

where $y_{t,h}^{(0)}$ denotes the yield to maturity of a risk-free zero-coupon bond of residual maturity h at date t. The vector (z_t, λ_t) being compound auto-regressive of order one under $\mathbb{Q}$, the second term on the right-hand side of (4.10) is linear in (z_t, λ_t).[21] Therefore, the spread between the yield associated with the defaultable bond and the risk-free bond of the same maturity is of the form:

$$y_{t,h}^{(n)} - y_{t,h}^{(0)} = a_h^{(n)\prime} z_t + b_h^{(n)\prime} \lambda_t \tag{4.11}$$

where the $(a_h^{(n)\prime}, b_h^{(n)\prime})$ vectors are computed recursively.[22]

4.3. Data

4.3.1. Overview

The data are weekly (end of weeks), and cover the period from 1 June 2007 to 13 April 2012 (255 dates), encompassing the ongoing financial crisis. We consider the yield curves of eight euro-area countries: Austria, Belgium, Finland, France, Germany, Italy, the Netherlands and Spain. We exclude from the analysis those countries that were placed under EU-IMF programs during that period, namely Greece, Ireland and Portugal (in April 2010 for Greece, in November 2010 for Ireland and in May 2011 for Portugal). The choice of removing these countries from the analysis stems from the facts that (a) the three EU-IMF programs cover important shares of the total estimation period and that (b) these programs coincide with severe impairments of associated sovereign-debt markets, notably illustrated by a fall in primary-market activity.[23]

[21] Appendix 3.A.1 derives the Laplace transform of (z_t, y_t) and shows that (z_t, y_t) is Compound auto-regressive of order one. Appendix 2.A.5 further shows how to compute the multi-horizon Laplace transform of compound auto-regressive processes. See Darolles, Gourieroux and Jasiak (2006) or Bertholon, Monfort and Pegoraro (2008) for in-depth presentations of compound auto-regressive –or Car– processes.

[22] The general recursive formulas are presented in Appendix 2.A.5.

[23] These impairments are illustrated by bid-ask spreads on government bonds. Based on bond prices extracted from the Thomson Reuters tick history database, the bid-ask spreads on 10-year bond issued by Greece, Ireland and Portugal were on average above 200 bp in 2011 (i.e. 2% of the face value, or 3% to 4% of the bond value) while they were lower than 40 bps for other euro-area countries.

Sovereign zero-coupon yields are extracted from Bloomberg. As will be detailed below, our estimation strategy also involves yields of bonds issued by KfW, a public German agency. The latter (zero-coupon) yields come from the Thomson Reuters Tick History database. The estimation dataset is completed by 12-month-ahead forecasts of 10-year sovereign yields for France, Germany, Italy, Spain and the Netherlands. These forecasts are the mean values of the respondents' forecasts by the Consensus Economics' expert panel. The survey is released around the middle of the month. Note that the survey implicitly targets yields-to-maturity of coupon bonds and not zero-coupon bonds. However, our zero-coupon yields remain very close to coupon yields over the estimation sample. The remaining discrepancy, of a few basis points, will be attributed to the deviation between the survey-based forecasts and the model-based ones (the $\xi_t^{(n)}$'s introduced in equation 4.13 below). This monthly series is converted into a weekly one using a cubic spline.

The risk-free rates are proxied by the Overnight Index Swap (OIS) rates. An OIS is a fixed-for-floating interest rate swap with a floating rate leg tied to the index of overnight interbank rates, that is the EONIA in the euro-area case.[24] OIS have become especially popular hedging and positioning vehicles in euro financial markets and grew significantly in importance during the financial turmoil of the last few years. The OIS curve is more and more seen by market participants as a proxy of the risk-free yield curve (see e.g. Joyce et al., 2011).[25]

4.3.2. Euro-area government yields

Table 4.1 reports the correlations between the spreads vs. Germany for different countries over the sample period.[26] The results suggest that euro-area sovereign

[24]For maturties higher than 12 months, OIS rates are homogenous to constant-maturity coupon yields. Therefore, we first have to convert Bloomberg-extracted OIS rates into zero-coupon yields. This is done using standard bootstrapping methods.

[25]While OIS rates reflect the credit risk of an overnight rate, this may be regarded as negligible in most situations. Besides, even during financial-markets turmoils, the counterparty risk is limited in the case of a swap contract, due to netting and credit enhancement, including call margins (see Bomfin, 2003).

[26]German debt is often considered as a safe haven in terms of credit and liquidity quality. The German bond market is the only one in Europe that has a liquid futures market, which boosts demand for the German Bund compared to other euro area debt and bolsters its liquidity

spreads are highly correlated across countries and across maturities (see also Favero, Pagano and von Thadden, 2010). According to these descriptive statistics, spreads' distributions are positively skewed and often leptokurtic. Table 4.2 presents a principal-component analysis of these spreads across countries. This analysis indicates that, for different maturities (2, 5 and 10 years), the first two principal components explain more than 95% of the spread variances across countries (75% for the first principal component alone). This highlights the importance of common sources of risk in euro-area sovereign spreads.

Table 4.1.: Descriptive statistics of selected spreads

Notes: The Table reports summary statistics for selected spreads (versus Germany). Two auto-correlations are shown (the 1-month and the 1-year auto-correlations). The underlying yields are continuously compounded and are in percentage annual terms. The lower panel of the Tablepresents the covariances and the correlations (in italics) of the spreads. The data are weekly and cover the period from 1 June 2007 to 13 April 2012.

	France		Italy		Netherlands		Spain	
	2-year	10-year	2-year	10-year	2-year	10-year	2-year	10-year
Mean	0.198	0.345	1.188	1.388	0.127	0.237	1.073	1.286
Median	0.126	0.255	0.723	0.915	0.092	0.175	0.517	0.751
Standard dev.	0.216	0.321	1.319	1.391	0.109	0.173	1.161	1.238
Skewness	2.736	2.023	2.101	1.756	1.669	1.135	1.169	0.896
Kurtosis	12.245	6.944	7.331	5.432	5.511	3.546	3.73	2.69
Auto-cor. (lag 1)	0.957	0.979	0.984	0.99	0.922	0.963	0.979	0.987
Auto-cor. (lag 12)	0.838	0.894	0.916	0.936	0.782	0.876	0.908	0.926
Correlations \ Covariances								
France 2-yr yd	0.047	0.065	0.261	0.271	0.014	0.026	0.194	0.198
France 10-yr yd	*0.938*	0.103	0.392	0.425	0.018	0.043	0.307	0.336
Italy 2-yr yd	*0.915*	*0.926*	1.738	1.808	0.055	0.143	1.421	1.479
Italy 10-yr yd	*0.899*	*0.952*	*0.986*	1.933	0.059	0.162	1.486	1.597
Netherlands 2-yr yd	*0.604*	*0.521*	*0.382*	*0.388*	0.012	0.015	0.03	0.032
Netherlands 10-yr yd	*0.706*	*0.775*	*0.626*	*0.675*	*0.787*	0.03	0.101	0.119
Spain 2-yr yd	*0.771*	*0.824*	*0.929*	*0.921*	*0.237*	*0.506*	1.346	1.406
Spain 10-yr yd	*0.741*	*0.845*	*0.906*	*0.928*	*0.235*	*0.559*	*0.979*	1.532

(see e.g. Pagano and von Thadden, 2004).

4.4. Estimation

4.4.1. State-space form of the model

Our estimation is conducted by the maximium-likelihood method, in a single step. The likelihood function is approximated by means of the Kim's (1994) filter, that handles state-space models with Markov-switching.[27] The measurement equations of the model are of two kinds: a first set of equations relates observed spreads – stacked in a vector denoted by S_t– to modeled ones; a second one relates observed survey-based spreads' forecasts –stacked in a vector denoted by CF_t– to model-implied ones. Let us make these two sets of equations equations more precise.

Consistently with the fact that spreads versus Germany are the most scrutinized spreads in the euro-area sovereign bond market, the set of observed spreads consists of spreads versus German sovereign-bond yields. As for the German spreads included in the vector S_t, we take the yield differentials between Bunds' yields and (zero-coupon) OIS rates with comparable maturities (2, 5 and 10 years). These first measurement equations read:

$$S_t = Az_t + B\lambda_t + \xi_{S,t} \tag{4.12}$$

where the entries of the matrices A and B are respectively based on the $a_h^{(n)}$'s and the $b_h^{(n)}$'s appearing in equation (4.11). More precisely, consistently with the choice of the observed spreads, and replacing the German index (1) (say) by GER, the entries of A and B corresponding to German yields are respectively the a_h^{GER}'s and the b_h^{GER} with the appropriate maturities h.[28] Those entries of A and B corresponding to other debtors ($n > 1$) are of the form $a_h^{(n)} - a_h^{GER}$ and $b_h^{(n)} - b_h^{GER}$. The vector $\xi_{S,t}$ contains i.i.d. normally-distributed pricing errors.

[27] See also Kim and Nelson (1999). The algorithm has been slightly adapted for this application. In particular, for each iteration of the algorithm, in the updating step, we prevent the algorithm from resulting in values of the ith unobserved variables $\lambda_{i,t}$ that would be below $-2\sqrt{\sigma_i^2/(1-\rho_i^2)}$. Note that $\sigma_i^2/(1-\rho_i^2)$ would be the unconditional variance of the ith unobserved factor $\lambda_{i,t}$ if μ_i was null (since the vector μ_i is positive, the unconditional mean of the ith factor is higher than zero, implying that the unconditional probability of having $\lambda_{i,t} < 0$ is lower than 2.5%).

[28] More rigorously, the vector $b_h^{(1)'}$ defines one line of the B matrix.

This set of measurement equations is augmented with equations linking survey-based 12-month-ahead forecasts of spreads to their model-based equivalent. Four spreads are considered: the yield differentials between Dutch, French, Italian and Spanish 10-year bonds and their German counterparts. These equations read:

$$CF_t^{(n)} = E_t^{\mathbb{P}}\left(y_{t+h,H}^{(n)} - y_{t+h,H}^{GER}\right) + \xi_t^{(n)}, \quad n \in \{2,3,4,5\} \tag{4.13}$$

where $H = 52 \times 10$, $h = 52$, the $\xi_t^{(n)}$'s are i.i.d. normally-distributed measurement errors and where the model-based forecasts $E_t(y_{H,t+h}^{(n)} - y_{H,t+h}^{GER})$ are easily derived using equation (4.11) and:

$$\begin{cases} E_t^{\mathbb{P}}(\lambda_{t+h}) & = \left[\mu\Pi^h + \Phi\mu\Pi^{h-1} + \ldots + \Phi^{h-1}\mu\Pi\right] z_t + \Phi^h\lambda_t \\ E_t^{\mathbb{P}}(z_{t+h}) & = \Pi^h z_t. \end{cases} \tag{4.14}$$

Appendix 4.B presents and discusses different constraints that are imposed on the parameter estimates. In particular, it details the relationship between λ_t^{KfW} and λ_t^{GER} that is aimed at identifying the liquidity-related factor $\lambda_{\ell,t}$.

4.4.2. Estimation procedure and results

The log-likelihood function is highly non-linear in the underlying model parameters. Therefore, good starting values ar required to achieve convergence in a reasonable computing time.[29] In a first step, we estimate the model using data associated with a subset of debtors, namely Germany, KfW, Italy and France. In a second step, the parameters defining the dynamics of the risk intensities of the remaining countries are estimated successively, one country after the other, taking the other parameters as given. In the final stage, all the parameters are (re)estimated jointly.

The approach results in a satisfiying fit of the data. Modeled spreads versus observed ones are displayed in Figure 4.1 (grey lines for observed spreads, dotted lines

[29] Optimizations are based on iterative uses of quasi-Newton and Nelder-Mead algorithms (as provided by the Scilab software).

for modeled spreads). The average of the measurement-error standard deviations is around 18 basis points (across 27 time series: 3 maturities for 9 entities).

Table 4.3 and Table 4.4 present the parameter estimates. The standard deviations of these estimates are based on the outer product of the first derivative of the likelihood function. Important differences arise in the parameters across countries. Naturally, those countries that have been characterized by the highest rises in spreads are more affected by the crises regimes. Notably, in an intense credit crisis regime, the drift of the the Italian credit-related factor is 70 times larger than in a less intense credit crisis (this is obtained by comparing the entries of the line "μ_c" with those of the line "μ_{cc}" in Table 4.3). It can also be noted that the auto-regressive coefficients (the ρ's) of the different factors are higher under the risk-neutral measure than under the historical one. This suggests that credit and liquidity intensities factors are more persistent under the risk-neutral measure than under the historical one. Note that another source of persistence originates from the regime-switching features: indeed, low switching probabilities generate persistence in the processes that depend on these regimes (the $\lambda_{i,t}$'s here). These transition probabilities are discussed in the next Section.

Table 4.3.: Parameter estimates (1/2)

Notes: This Tablereports the estimates of the parameters defining the dynamics of the intensities under the historical and the risk-neutral measures. It is completed by an additional Table(Table 4.4) that presents the matrices of transition probabilities under both measures. The estimation data are weekly and span the period from 1 June 2007 to 13 April 2012. Standard errors are reported in parentheses below the coefficient estimates. ***, ** and * respectively denote significance at the 1%, 5% and 10% significance level. The standard deviations of these estimates are based on the outer product of the first derivative of the likelihood function. The entries of the $\mu_c^{(n)}$ vectors (see equation 4.3) that correspond to the two crises regimes are reported in the lines "μ_c" and "μ_{cc}" of the table. The entries of the $\mu_c^{(n)}$ vectors (see equation 4.3) that correspond to the non-credit-crisis regime is assumed to be zero (consistently with the existence of periods of very low spreads). it can be checked that $\alpha_{1,\ell}^{KfW} = \alpha_{1,\ell}^{GER} + 1$; this constraint is imposed in order to identify the liquidity-related factor (see 4.B.1).

	KfW	GER	SPA	ITA	FRA	NET	BEL	FIN	AUS
Specification of the liquidity intensities									
$\alpha_{0,\ell}$	-0.058*	-0.24***	-0.025	-0.24***	-0.24***	-0.24***	-0.24***	-0.16***	-0.24***
	(0.03)	(0.027)	(0.28)	(0.027)	(0.027)	(0.027)	(0.027)	(0.047)	(0.027)
$\alpha_{1,\ell}$	0.73***	-0.27***	0.52	1.1*	0.084	0.41***	0.53*	0.43***	0.63***
	(0.045)	(0.045)	(0.45)	(0.6)	(0.104)	(0.051)	(0.27)	(0.075)	(0.091)
Historical dynamics of the intensities λ									
μ_c	0.0101***	0.0101***	0.0052***	0.0051***	0.0051***	0.0103***	0.0051***	0.0085***	0.0105***
	(0.00087)	(0.00087)	(0.00044)	(0.00044)	(0.00044)	(0.00052)	(0.00044)	(0.0015)	(0.0018)
μ_{cc}	0.0102***	0.0102***	0.27***	0.38***	0.092***	0.0104***	0.24***	0.013***	0.071***
	(0.00087)	(0.00087)	(0.032)	(0.05)	(0.007)	(0.00052)	(0.022)	(0.0041)	(0.0078)
ρ_c	0.99***	-	-	-	-	-	-	-	-
	(0.00031)	-	-	-	-	-	-	-	-
σ_c	0.0025***	0.0025***	0.02***	0.022***	0.0051***	0.0028***	0.012***	0.0031***	0.0057***
	(0.000027)	(0.000027)	(0.00022)	(0.00024)	(0.000056)	(0.000031)	(0.00013)	(0.000034)	(0.000063)
μ_ℓ	0.076***	-	-	-	-	-	-	-	-
	(0.0027)	-	-	-	-	-	-	-	-
ρ_ℓ	0.9***	-	-	-	-	-	-	-	-
	(0.0043)	-	-	-	-	-	-	-	-
σ_ℓ	0.0101***	-	-	-	-	-	-	-	-
	(0.0002)	-	-	-	-	-	-	-	-
Risk-neutral dynamics of the intensities λ									
μ_c^*	0.0043*	0.0043*	0.0022*	0.0022*	0.0022*	0.0024**	0.0024**	0.0022*	0.0025*
	(0.0023)	(0.0023)	(0.0012)	(0.0012)	(0.0012)	(0.0012)	(0.0012)	(0.0013)	(0.0012)
μ_{cc}^*	0.0044*	0.0044*	0.009	0.0083	0.0047	0.0025**	0.0023**	0.0026**	0.0048
	(0.0023)	(0.0023)	(0.012)	(0.013)	(0.0043)	(0.0012)	(0.0012)	(0.0013)	(0.0044)
ρ_c^*	1***	1***	1***	1***	1***	1***	1***	1***	1***
	(0.00012)	(0.00012)	(0)	(0)	(0)	(0)	(0.00054)	(0)	(0)
μ_ℓ^*	0.00059	-	-	-	-	-	-	-	-
	(0.00037)	-	-	-	-	-	-	-	-
ρ_ℓ^*	1***	-	-	-	-	-	-	-	-
	(0)	-	-	-	-	-	-	-	-
Measurement errors									
σ_{perr}	0.093***	0.06***	0.49***	0.53***	0.12***	0.068***	0.28***	0.075***	0.14***
	(0.00051)	(0.00033)	(0.0027)	(0.0029)	(0.00068)	(0.00037)	(0.0016)	(0.00041)	(0.00076)

Table 4.4.: Parameter estimates (2/2)

Notes: The Tablereports the estimates of the regime-transition probabilities, i.e. the matrices Π and Π^*. Note that only 11 parameters are used to define the 36 entries of each of these two matrices (see Subsectionsec. 4.2.2.1 and Appendix 4.B). Standard errors of the estimates, based on the outer-product approximation of the Information matrix, are reported in parentheses below the coefficient estimates. ***, ** and * respectively denote significance at the 1%, 5% and 10% significance level. NL: no liquidity crisis, L: liquidity crisis, NC: no credit crisis, C: (non intense) credit crisis, CC: intense credit crisis. Each line of the Tableindicates the probabilities of switching from one regime (defined by the first column) to another (defined by the second line of the table).

				Historical dynamics			
		at date t:	NL-NC	NL-C	NL-CC	L-NC	L-C	L-CC
at date $t-1$	NL-NC		0.9994***	0.00041	0	0.0002***	0	0
			(0.00097)	(0.00097)	-	(0)	(0)	-
	NL-C		0.00014***	0.98***	0.000045***	0***	0.022***	0***
			(0)	(0.0081)	(0)	(0)	(0.0081)	(0)
	NL-CC		0	0.59***	0.39***	0	0.013**	0.0087*
			-	(0.12)	(0.12)	-	(0.0052)	(0.0046)
	L-NC		0.19**	0.032	0	0.66***	0.11***	0
			(0.09)	(0.03)	-	(0.14)	(0.038)	-
	L-C		0.065**	0.22***	0.076**	0.11***	0.39***	0.13**
			(0.028)	(0.063)	(0.036)	(0.032)	(0.093)	(0.056)
	L-CC		0	0.22***	0.14**	0	0.38***	0.25***
			-	(0.063)	(0.064)	-	(0.106)	(0.081)
				Risk-neutral dynamics				
		at date t:	NL-NC	NL-C	NL-CC	L-NC	L-C	L-CC
at date $t-1$	NL-NC		0.94***	0.025	0	0.036	0.00094	0
			(0.045)	(0.016)	-	(0.045)	(0.0012)	-
	NL-C		0.000059***	0.91***	0.000042***	0	0.092	0
			(0)	(0.091)	(0)	(0)	(0.091)	(0)
	NL-CC		0	0.103	0.8***	0	0.0105	0.082
			-	(0.12)	(0.19)	-	(0.021)	(0.073)
	L-NC		0.0089	0.00036	0	0.95***	0.039	0
			(0.055)	(0.0023)	-	(0.069)	(0.034)	-
	L-C		0	0.000052***	0	0.094	0.87***	0.036
			(0)	(0)	(0)	(0.103)	(0.13)	(0.04)
	L-CC		0	0	0.000053***	0	0.11	0.89***
			-	(0)	(0)	-	(0.14)	(0.14)

Table 4.5.: Conditional probabilities of transition

Notes: The Tablereports selected probabilities of transition. These probabilities are some combinations of those presented in Table 4.4. "Credit crisis" refers to one of the two credit-crisis regimes (intense or not). For instance, the probability of staying in a credit-crisis regime (intense or not) when in a liquidity crisis (at $t-1$) is 82% under the historical measure. $\mathbb{P}$ (resp. $\mathbb{Q}$) correspond to the historical (resp. risk-neutral) measure. Standard errors are reported in parentheses below the coefficient estimates. ***, ** and * respectively denote significance at the 1%, 5% and 10% significance level.

	at date t	at date $t-1$			
		no liquidity crisis		liquidity crisis	
		no credit crisis	credit crisis	no credit crisis	credit crisis
under $\mathbb{P}$	liq. crisis	0.0002***	0.022***	0.78***	0.64***
		(0)	(0.0081)	(0.12)	(0.09)
under $\mathbb{Q}$	liq. crisis	0.036	0.092	0.99***	1***
		(0.046)	(0.091)	(0.057)	(0)
under $\mathbb{P}$	cred. crisis	0.00041	0.99986***	0.14**	0.82***
		(0.00097)	(0)	(0.064)	(0.051)
under $\mathbb{Q}$	cred. crisis	0.026	0.99994***	0.039	0.91***
		(0.016)	(0)	(0.034)	(0.103)

4.5. Interpretation

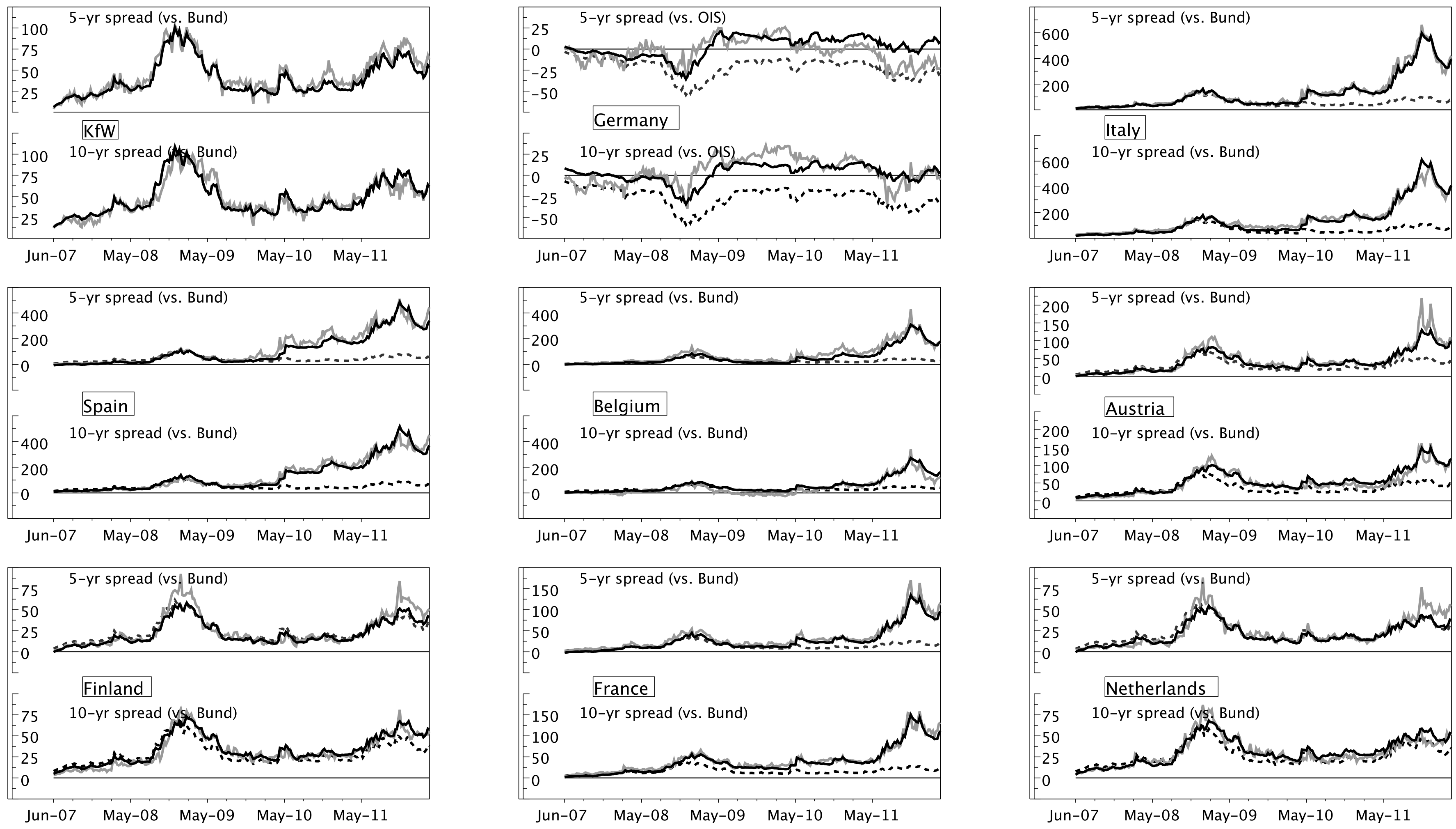
4.5.1. Credit and liquidity crises

The upper two panels in Figure 4.2 present the smoothed probabilities of being in the different crisis regimes.[30] The first period of liquidity crisis begins with the collapse of Bear Sterns in March 2008. This period is relatively short (a few weeks). By contrast, the second liquidity-crisis period, that begins with the Lehman Brothers' bankruptcy in Spetember 2008 lasted about six months. The premise of the so-called euro-area crisis (April 2010) and the latest development of the same crisis (starting in mid-2011) are also identified as liquidity-crisis periods. Turning to the credit crises, one can distinguish two long stress periods: November

[30] The smoothed probabilities are obtained by applying Kim's (1994) filter. While filtered probabilities, as of date t, use only information available up to date t, smoothed probabilities exploit all sample information.

Figure 4.1.: Actual vs. model-implied spreads

Notes: These plots compare observed (light-grey solid lines) and model-implied (dotted lines) spreads. For all entities, except Germany, the spreads are yield differentials between zero-coupon 5- and 10-year zero-coupon yields and their German counterparts. For Germany, the yield differentials are against (zero-coupon) overnight-index-swap rates. The black solid line is the model-implied contribution of the liquidity intensity $\lambda_{\ell,t}^{(n)}$ to the spreads (these contributions are computed as the spread that would prevail if the credit parts $\lambda_{c,t}^{(n)}$ of the debtor intensities were equal to zero). For KfW (upper-left plot), the fact that the dotted line and the black solid line are confounded results from the identiification of the liquidity factor $\lambda_{\ell,t}$.



2008 to mid-2009 and April 2010 to the end of the sample (April 2012). Within these credit-crisis phases, several peak periods of severe market stress –indicated by black-shaded areas in the second panel of Figure 4.2– are observed, notably in Autumn 2011. The lower panels in Figure 4.2 display the (smoothed) estimates of the unobserved factors λ_t. For instance, looking at the first and the third panel, one can observe the influence of the occurrence of liquidity crises on the liquidity-related factor $\lambda_{\ell,t}$.

Besides, the estimated specifications of the regimes' dynamics are meaningful. The historical (risk-neutral) dynamics is described by the matrix Π (the matrix Π^*) reported in Table 4.4. It appears that the adverse states of the world are more long-lived under the risk-neutral measure than under the historical one, which tends to give rise to risk premia associated with those bad states of the world. To set an example, while the probability of remaining in the most adverse regime (liquidity crisis and severe credit crisis) is of 25% under the historical measure, it is of 89% under the risk-neutral one. So as to facilitate the interpretation of the transition probabilities, Table 4.5 presents selected combinations of these. More precisely, it gives the probabilities of switching to liquidity- or credit-crisis regimes conditional on the existence of a crisis at the previous period, ruling out the distinction between the intense and the less intense credit crises. These probabilities illustrate the causality between the two kinds of crises. Indeed, the probability of switching to a credit (resp. liquidity) crisis is significantly higher when there is a liquidity (resp. credit) crisis at the previous period. For instance, the probability of switching to a credit crisis between date $t-1$ and date t is of 14% (resp. 0.04%) if there is (resp. not) a liquidity crisis at date $t-1$.

4.5.2. Liquidity intensity and pricing

In our model, a single factor ($\lambda_{\ell,t}$) drives liquidity pricing in euro-area bond yields. The first panel in Figure 4.3 illustrates the striking comovements between our estimated liquidity factor and another proxy of liquidity pricing, the bid-ask spreads associated with French 10-year benchmark bonds.

The lower part Figure 4.3 is aimed at relating the countries' sensitivities to the liquidity-related factor ($\alpha_{1,\ell}^{(n)}$, see equation 4.3) to national marketable-debt characteristics: (a) the countries' debt outstanding and (b) the average bid-ask spreads of the countries 10-year benchmark bonds. The first scatter plot of Figure 4.3 shows that, leaving Italy aside, there seems to be a negative relationship between these sensitivities and the debt outstanding. In spite of the large size of the tradable debt issued by the Italian government, Italy's intensity appears to be particularly sensitive to the liquidity factor.[31] The second scatter plot (bottom-right panel in Figure 4.3) points to a positive relationship between the countries' sensitivities and the bid-ask spreads.

In order to gauge the relative importance of the liquidity-related part of the spreads, we have computed the spreads (versus German yields) that would prevail if the default intensities were equal to zero. Figure 4.1 presents the resulting spreads (black solid lines).[32] The liquidity-related parts of the spreads turn out to account for a substantial part of the changes in spreads, especially for the less indebted countries (the Netherlands and Finland). The German plot reveals that the high liquidity of the German Bunds translates into negative spreads versus swap rates. Such negative spreads can be attributed to the so-called convenience yield of holding government-issued securities and/or to flight-to-liquidity phenomena taking place amid the financial crisis.[33] While the liquidity factor was explaining the main part of the spreads' fluctuations for most of the countries in the post-Lehman period, the part of the spreads explained by credit-related factors became predominant for several countries (Austria, Belgium, France, Italy and Spain) over the last year of the sample.

[31] To some extent, such a finding is consistent with the results of Chung-Cheung, de Jong and Rindi (2004) according to which transitory costs would be more important in the Italian market, dominated by local traders.

[32] Due to non-linearity effects, the sum of this counterfactual spreads and those that would be obtained, alternatively, by switching off the liquidity intensities are not strictly equal to the complete modeled spreads. However, the differences are visually imperceptible.

[33] See Feldhütter and Lando, 2008 or Liu, Longstaff and Mandell 2006 for empirical studies and discussions of convenience yield on U.S. data. Flight-to-liquidiyty effects in the euro area sovereign bond market are investigated by Beber, Brandt and Kavajecz 2009.

4.5.3. Default probabilities

In the remaining of the Chapter, we show how our results can be exploited to compute the default probabilities implied by the yield data. In the spirit of Litterman and Iben (1991), various methodologies that are widely used by practitioners or market analysts end up with risk-neutral PDs (see, e.g. Chan-Lau, 2006). Our framework makes it possible to investigate the potential differences that exist between the latter and their historical, or real-world, counterparts. As stated above (see Subsection 4.2.3), while the intensities of default are the same processes under both measures, the $\mathbb{P}$- and $\mathbb{Q}$-probabilities of default are not the same because the $\mathbb{P}$- and $\mathbb{Q}$-dynamics of these processes differ.

In our framework, the actual PD between time t and time $t+h$ is given by

$$\begin{aligned}\mathbb{P}\left(d_{t+h}^{(n)}=1\middle|\,\underline{\tilde{w}}_t, d_t^{(n)}=0\right) &= E_t^{\mathbb{P}}\left(\mathbb{I}_{\left\{d_{t+h}^{(n)}=1\right\}}\middle|\,d_t^{(n)}=0\right)\\ &= 1-E_t^{\mathbb{P}}\left(\mathbb{I}_{\left\{d_{t+h}^{(n)}=0\right\}}\middle|\,d_t^{(n)}=0\right)\\ &= 1-E_t^{\mathbb{P}}\left(\exp(-\tilde{\lambda}_{d,t+1}^{(n)}-\ldots-\tilde{\lambda}_{d,t+h}^{(n)})\right). \quad (4.15)\end{aligned}$$

We are then left with the computation of the survival probability $E_t^{\mathbb{P}}(\exp(-\tilde{\lambda}_{d,t+1}^{(n)}-\ldots-\tilde{\lambda}_{d,t+h}^{(n)}))$. Recall that $\exp(-\lambda_{d,t}^{(n)})=\exp(-\tilde{\lambda}_{d,t}^{(n)})+\zeta[1-\exp(-\tilde{\lambda}_{d,t}^{(n)})]$. When $\lambda_{d,t}^{(n)}$ is small, the first order approximation leads to:

$$\tilde{\lambda}_{d,t}^{(n)} \simeq \frac{1}{1-\zeta}\lambda_{d,t}^{(n)}. \quad (4.16)$$

Up to this approximation, the survival probability is a multi-horizon Laplace transform of a compound auto-regressive process of order one. In the same way as for the yields, the recursive algorithm detailed in Appendix 2.A.5 can be used in order to compute these probabilities. In the computation, we use a constant recovery rate of 50%, which corresponds to the average of the recovery rates observed for sovereign defaults over the last decade (see Moody's, 2010).

Figure 4.4 shows the model-based 5-year probabilites of default (i.e. the probabilities that the considered countries will default during the next 5 years). One-standard-deviation bands are also reported. These standard deviations take two kinds of uncertainty into account: (1) the smoothing errors that are associated with the Kim's (1994) smoothing algorithm used to estimate the intensities λ_t and (2) the uncertainty stemming from the parameters' estimation (MLE).[34]

Figure 4.5 presents the model-implied term-structure of PDs as of 9 May 2008 and 30 December 2011. This Figureillustrates the dramatic changes in the term-structure of PDs that took place over these 3 years. For all countries and especially for the more indebted ones, the term-structure of the PDs is much higher and steeper in late 2011 than in Spring 2008.

Finally, it is worth noting that even when taking into account the uncertainty regarding the estimated real-world PDs, the gap between these and their risk-neutral counterparts is significant in many cases, particularly for the most recent years (see Figure 4.4). Nevertheless, as stated above, risk-neutral PDs are extensively used by market practitioners and analysts. This mainly stems from the fact that risk-neutral PDs are relatively easy to compute, using basic methods inspired by the one proposed by Litterman and Iben (1991) [35]. To illustrate, Figure 4.6 compares the PDs estimates derived from our model with alternative estimates, as of the end of 2011. Two kinds of alternative estimates are considered: (a) PDs that are based on the Moody's credit ratings and the associated matrix of long-run credit-rating-migration probabilities and (b) risk-neutral probabilities computed by CMA Datavision (2011). Figure 4.6 shows that our estimates lie somewhere between the two others.[36] In addition, it appears that our risk-neutral PDs (the triangles) are relatively close to the risk-neutral CDS-based ones computed by CMA.[37]

[34] The computation of these standard errors is inspired from Hamilton (1986). It relies on the assumption that the two kinds of errors (smoothing and MLE) are independent from each other.

[35] In particular, these methods do not care about liquidity-pricing effects.

[36] Credit-rating-based PDs are extremely small (for instance: 6.10^{-6} for a AAA-rated countries, 6.10^{-4} for a A-rated countries). This reflects the fact that transition probabilities are based on past 25-year history of rating changes, during which quick sovereign downgrades were infrequent (contrary to during the current crisis period).

[37] The remaining differences between the latter two risk-neutral estimates may be attributed to

4.6. Conclusion

In this Chapter, we present a multi-country no-arbitrage model of the joint dynamics of euro-area sovereign spreads. At the heart of our model is an innovative approach capturing the intertwined dynamics of credit- and liquidity-related crises by a joint modelling of two kinds of switching regimes. These crises are key drivers of, respectively, credit and illiquidity intensities associated with the different issuers.

Using euro-area spread data covering the last five years, we estimate such intensities for eight euro-area countries. The resulting fit is satisfying, the standard deviations of the yields pricing errors –across countries and maturities– being of 18 basis points. Interestingly, we provide evidence of causal relationships between credit and liquidity crises periods.

Our approach makes it possible to exhibit the part of the spreads reflecting liquidity-pricing effects. A key assumption is that the country-specific illiquidity intensities perfectly comoves, that is, that there exists a single European liquidity-pricing factor. The identification of the latter is based on the term sturcture of the yield differentials between the bonds issued by KfW (a German agency) and the German sovereign bonds (the *Bunds*). Indeed, KfW's liabilities are explicitly and unconditionally guaranteed by the Federal Republic of Germany. Therefore, the KfW-*Bund* spread should be essentially liquidity-driven. Our results indicate that a substantial part of intra-euro spreads is liquidity-driven.

Given some assumptions regarding the recovery process, our framework makes it possible to decompose the credit part of the spreads between actual, or real-world, probabilities of default on the one hand and risk premiums on the other hand. To that respect, our results suggest that actual PDs are often significantly lower than their risk-neutral counterparts. According to these results, relying on risk-neutral PDs to assess the market participants expectations regarding future

(i) the fact that we consider spreads w.r.t. Germany in our methods while the CMA's method involves "absolute" CDS, (ii) the absence of treatment of liquidity-pricing effects in the CMA methodology (while empirical evidence suggests that CDS contain liquidity premia, see Buhler and Trapp, 2008) or also to (iii) the measurement errors of our approach (see Figure 4.1).

sovereign defaults would be misleading.

4.A. Pricing of defaultable bonds

For the sake of notational convenience, we drop the issuer subscript n in this Appendix.

Let us consider the price $B_{T-1,1}$, at date $T-1$, of a one-period bond issued by the debtor (before $T-1$). If the debtor is not in default at $T-1$, then:

$$B_{T-1,1} = \exp(-r_{T-1})E^{\mathbb{Q}}\left[(1-d_T+\zeta d_T)(1-\ell_T+\theta\ell_T) \mid \underline{\tilde{w}}_{T-1}\right]$$

($\underline{\tilde{w}}_{T-1}$ containing the information $d_{T-1}=0$)

$$\begin{aligned}
&= \exp(-r_{T-1})E^{\mathbb{Q}}\left[E^{\mathbb{Q}}\left((1-d_T+\zeta d_T)(1-\ell_T+\theta\ell_T) \mid w_T, \underline{\tilde{w}}_{T-1}\right)\Big|\, \underline{\tilde{w}}_{T-1}\right] \\
&= \exp(-r_{T-1})E^{\mathbb{Q}}\Big[\Big\{\exp\left(-\tilde{\lambda}^{(n)}_{d,T}\right) + \zeta\left(1-\exp\left(-\tilde{\lambda}^{(n)}_{d,T}\right)\right)\Big\} \times \\
&\qquad \Big\{\exp\left(-\tilde{\lambda}_{\ell,T}\right) + \theta\left(1-\exp\left(-\tilde{\lambda}_{\ell,T}\right)\right)\Big\}\Big)\Big|\, \underline{\tilde{w}}_{T-1}\Big].
\end{aligned}$$

The last equality is obtained by using the conditional independence of d_t and ℓ_t and the expressions of the conditional $\mathbb{Q}$-distributions of d_t and ℓ_t (that are the same as their historical counterparts, as shown in Appendix 4.C). From that, using the definitions of $\lambda_{d,T}$ and $\lambda_{\ell,T}$ given in formula (4.1), it follows that:

$$\begin{aligned}
B_{T-1,1} &= \exp(-r_{T-1})E^{\mathbb{Q}}\left[\exp(-\lambda_{d,T}-\lambda_{\ell,T}) \mid \underline{\tilde{w}}_{T-1}\right] \\
&= \mathcal{B}(r_{T-1}, z_{T-1}, \lambda_{T-1}) \quad \text{(say)}
\end{aligned}$$

The fact that $E^{\mathbb{Q}}\left[\exp(-\lambda_{d,T}-\lambda_{\ell,T}) \mid \underline{\tilde{w}}_{T-1}\right]$ is a function of (z_{T-1}, λ_{T-1}) originates from the assumptions on the distribution of (z_T, λ_T) given $\underline{\tilde{w}}_{T-1}$, in particular the non-causality from (d_t, ℓ_t) to (z_t, λ_t) under $\mathbb{Q}$ (see Appendix 4.C).

Let us then consider $B_{T-2,2}$, we have:

$$B_{T-2,2} = \exp(-r_{T-2})E^{\mathbb{Q}}\left[(1-d_{T-1}+\zeta d_{T-1})(1-\ell_{T-1}+\theta\ell_{T-1})\mathcal{B}(r_{T-1},z_{T-1},\lambda_{T-1})|\,\underline{\tilde{w}}_{T-2}\right],$$

$\underline{\tilde{w}}_{T-2}$ containing the information $d_{T-2}=0$.

Conditioning first by $(w_{T-1},\underline{\tilde{w}}_{T-2})$ and using the fact that $\mathcal{B}_{T-1}(z_{T-1},\lambda_{T-1})$ only depends on w_{T-1}, we get:

$$B_{T-2,2} = E^{\mathbb{Q}}\left[\exp(-r_{T-2}-\lambda_{d,T-1}-\lambda_{\ell,T-1})\mathcal{B}(r_{T-1},z_{T-1},\lambda_{T-1})|\,\underline{\tilde{w}}_{T-2}\right].$$

Replacing $\mathcal{B}(r_{T-1},z_{T-1},\lambda_{T-1})$ by $E^{\mathbb{Q}}\left[\exp(-r_{T-1}-\lambda_{d,T}-\lambda_{\ell,T})\mid\underline{\tilde{w}}_{T-1}\right]$ and using the fact that $\exp(-r_{T-2}-\lambda_{d,T-1}-\lambda_{\ell,T-1})$ is function of $\underline{\tilde{w}}_{T-1}$, we get:

$$\begin{aligned} B_{T-2,2} &= E^{\mathbb{Q}}\left[E^{\mathbb{Q}}\left(\exp(-r_{T-2}-\lambda_{d,T-1}-\lambda_{\ell,T-1}-r_{T-1}-\lambda_{d,T}-\lambda_{\ell,T})\mid\underline{\tilde{w}}_{T-1}\right)\Big|\,\underline{\tilde{w}}_{T-2}\right] \\ &= E^{\mathbb{Q}}\left[\exp(-r_{T-2}-\lambda_{d,T-1}-\lambda_{\ell,T-1}-r_{T-1}-\lambda_{d,T}-\lambda_{\ell,T})|\,\underline{\tilde{w}}_{T-2}\right]. \end{aligned}$$

Applying this methodology recursively leads to equation (4.8).

4.B. Parameter constraints

4.B.1. Econometric identification of the liquidity factor $\lambda_{\ell,t}$

As documented in 3.3.1, the bonds issued by KfW and those issued by the German government embed the same credit risks but are not equally exposed to the liquidity-related factor. Therefore, the sum of the recovery-adjusted default intensity and the liquidity intensity of KfW is given by:[38]

$$\begin{aligned} \lambda_t^{KfW} &= \lambda_{d,t}^{GER} + \lambda_{\ell,t}^{KfW} \\ &= \lambda_{d,t}^{GER} + \alpha_{0,\ell}^{KfW} + \alpha_{1,\ell}^{KfW} \lambda_{\ell,t}, \end{aligned} \tag{4.17}$$

that is, the risk intensities of KfW and the Federal Republic of Germany differ only through α_ℓ^{KfW}. We impose $\alpha_{1,\ell}^{KfW} = \alpha_{1,\ell}^{GER} + 1$ so as to identify the liquidity-related factor $\lambda_{\ell,t}$ (without loss of generality).

4.B.2. Specification of the matrix of transition probabilities Π

Here, we present the specification of the matrix Π that defines the dynamics of z_t, which is the Kronecker product of the liquidity-crisis variable $z_{\ell,t}$ and of the credit-crisis variable $z_{c,t}$. First, we assume that there is no instantaneous causality between $z_{c,t}$ and $z_{\ell,t}$, meaning that conditionally to z_{t-1}, $z_{c,t}$ and $z_{\ell,t}$ are independent. Second, whereas the switching probabilities of the liquidity-regime variable $z_{\ell,t}$ between date $t-1$ and date t may be influenced by the existence of a credit crisis at date $t-1$, it does not depend on the *distinction* between the two credit-crisis levels.[39] Third, the probabilities of switching from the severe credit-crisis state ($z_{c,t-1} = e_3^{[3]}$) to the no-credit-stress regime ($z_{c,t} = e_1^{[3]}$) is zero, as well as the opposite. That is, the first credit-crisis level ($z_{c,t} = e_2^{[3]}$) acts as an intermediary regime between the two others. Fourth, the probability of remaining in the severe-credit-crisis state does not depend on $z_{\ell,t}$. With these restrictions, 11 parameters are required to define the matrix Π.

4.B.3. The size of the Gaussian shocks

The standard deviations of the Gaussian shocks entering equations (4.3) and (4.4) are constrained to make sure that the regime variables z_t are the main sources of

[38] For the sake of clarity, we slightly modify the notations by replacing the (n) subcripts by "KfW" and "GER".

[39] Formally, $p(z_{\ell,t}| z_{\ell,t-1}, z_{c,t-1} = e_2^{[3]}) = p(z_{\ell,t}| z_{\ell,t-1}, z_{c,t-1} = e_3^{[3]})$.

the spreads fluctuations. If such constraints are not imposed, most of the spread fluctuations tend to be accounted for by the Gaussian shocks. This phenomenon, that reflects that Gaussian shocks are more flexible than the discrete-numbered regimes to fit the spreads, has two undesirable implications within our framework. First and foremost, the higher the standard deviation of the Gaussian shocks, the higher the frequency of generating/estimating negative intensities. Second, the lower the importance of the regime variables, the less information about the relationships between liquidity- and credit-crises the estimation is brought to reveal. Accordingly, we constrain the parameters to be such that a limited part of the (unconditional) fluctuations of the intensities is accounted for by Gaussian shocks. Practically, we impose the following constraints on the parameter estimates: $\sigma_i/\sqrt{1-\rho_i^2} \leq 10\%\tilde{\sigma}_i$, where $\tilde{\sigma}_i$ is the sample standard deviation of the (observed) spreads associated with entity i (and where σ_i is expressed in the same unit as the spreads). This calibration implies unconditional distribution of the intensities that is consistent with mainly positive intensities. Alternative estimation (with ratios of 5% and 20%) suggest that the qualitative results presented above are fairly robust to changes in the 10% ratio.

4.B.4. The auto-regressive coefficient ρ_c

Under the historical measure, the auto-regressive coefficient ρ_c is assumed to be constant across countries. This choice is related to our use of survey-based forecasts to address the downward bias in the estimated persistency of the factors. Kim and Orphanides (2012) have shown that using survey-based forecasts of yields makes it possible to overcome this bias. However, we have survey-based forecasts of spreads vs. Germany for only four countries (France, Italy, Spain and the Netherlands). Under the assumption that the persistence of the credit factor is common across countries, the information content of available survey-based forecasts benefits the parameterizations of all debtors' intensities.

4.B.5. The standard deviations of the pricing errors

For a given debtor, the standard deviations of the pricing errors (gathered in the vector $\xi_{S,t}$, see equation 4.12) are assumed to be the same across maturities. However, they differ across countries, proportionally to the standard deviations of the observed spreads (the proportionality coefficient being estimated by the MLE). There are two exceptions: First, given the crucial role of the KfW-Bund spreads in the identification of the liquidity factor $\lambda_{\ell,t}$, the proportionality coefficient associated with the pricing errors of the KfW-Bund spreads is twice lower than the others (to make sure that the liquidity factor properly fits the KfW-Bund spread). Second, given its different nature, the standard deviation of the Bund-OIS pricing-errors is twice larger than the others. (What we want to fit in the first place are the highly scrutinized spreads versus Germany).

4.C. Relationship between the risk-neutral and historical intensities

In this Appendix, we show that the default intensities are the same processes under both measures ($\mathbb{P}$ and $\mathbb{Q}$) when the stochastic discount factor depends on w_t only.

Proof. Recalling that $\tilde{w}_t = (w_t', d_t', \ell_t)'$, we have:

$$f_{d,\ell}^{\mathbb{Q}}(d_t, \ell_t | \, w_t, \underline{\tilde{w}}_{t-1}) = \frac{f^{\mathbb{Q}}(\tilde{w}_t | \, \underline{\tilde{w}}_{t-1})}{f_w^{\mathbb{Q}}(w_t | \, \underline{\tilde{w}}_{t-1})}. \tag{4.18}$$

Using the definition of the s.d.f., the numerator of (4.18) can be expressed as:

$$\begin{aligned} f^{\mathbb{Q}}(\tilde{w}_t | \, \underline{\tilde{w}}_{t-1}) &= M_{t-1,t} \exp(r_{t-1}) f^{\mathbb{P}}(\tilde{w}_t | \, \underline{\tilde{w}}_{t-1}) \\ &= M_{t-1,t} \exp(r_{t-1}) f^{\mathbb{P}}(d_t, \ell_t | \, w_t, \underline{\tilde{w}}_{t-1}) f_w^{\mathbb{P}}(w_t | \, \underline{\tilde{w}}_{t-1}) \end{aligned} \tag{4.19}$$

Since the s.d.f. $M_{t-1,t}$ depends on w_t only, the integration of both sides of (4.19) w.r.t. (d_t, ℓ_t) leads to:

$$f_w^{\mathbb{Q}}(w_t|\,\underline{\tilde{w}}_{t-1}) = M_{t-1,t}\exp(r_{t-1}) f_w^{\mathbb{P}}(w_t|\,\underline{\tilde{w}}_{t-1}). \tag{4.20}$$

Using (4.19) and (4.20) to compute the r.h.s. of (4.18), we obtain $f_{d,\ell}^{\mathbb{Q}}(d_t, \ell_t|\,w_t, \underline{\tilde{w}}_{t-1}) = f_{d,\ell}^{\mathbb{P}}(d_t, \ell_t|\,w_t, \underline{\tilde{w}}_{t-1})$.$\square$

Moreover, since we have assumed that $f_w^{\mathbb{P}}(w_t|\,\underline{\tilde{w}}_{t-1}) = f_w^{\mathbb{P}}(w_t|\,\underline{w}_{t-1})$ and since $M_{t-1,t}\exp(r_{t-1})$ does not depend on $(\underline{d}_{t-1}, \underline{\ell}_{t-1})$, equation (4.20) implies that the same is true for $f_w^{\mathbb{Q}}(w_t|\,\underline{\tilde{w}}_{t-1})$. In other words, since we assume that if (d_t, ℓ_t) does not Granger cause w_t under $\mathbb{P}$, the same is true in the risk-neutral world.

Table 4.2.: Principal component analysis of euro-area yield differentials

Notes: This Tablepresents results of principal-component analyses carried out on the spreads versus Germany. There are three analyses that correspond respectively to three maturities: 2 years, 5 years and 10 years. For each PC analysis, the Tablereports the eigenvalues of the covariance matrices and the proportions of variance explained by the corresponding component (designated by "Prop. of var."). The data are weekly and cover the period from 1 June 2007 to 13 April 2012. the spred (versus Germany) of seven countries are included in the analysis (Austria, Belgium, Finland, France, Italy, Netherlands, Spain).

Component	1	2	3	4	5	6	7
2-year spread							
Eigenvalue	5.26	1.34	0.17	0.12	0.06	0.03	0.02
Prop. of var.	75%	19%	2%	2%	1%	0%	0%
Cumul. prop.	75%	94%	97%	98%	99%	100%	100%
5-year spread							
Eigenvalue	5.87	0.88	0.12	0.06	0.04	0.02	0.01
Prop. of var.	84%	13%	2%	1%	1%	0%	0%
Cumul. prop.	84%	96%	98%	99%	100%	100%	100%
10-year spread							
Eigenvalue	5.56	1.11	0.14	0.12	0.03	0.02	0.01
Prop. of var.	79%	16%	2%	2%	0%	0%	0%
Cumul. prop.	79%	95%	97%	99%	99%	100%	100%

Figure 4.2.: Estimated regimes and intensities

Notes: The first two panels display the (smoothed) probabilities of being in the crises regimes (Kim's (1994) algorithm). More precisely, there are two liquidity regimes (normal and crisis) and three credit regimes (normal, distress and severe distress). The last three panels show the nine unobserved factors (λ_t). The dynamics of the liquidity factor $\lambda_{\ell,t}$ (resp. credit factors $\lambda_{c,t}^{(n)}$) depends on the liquidity regime (resp. the credit regime) but not on the credit regime (resp. the liquidity regime).

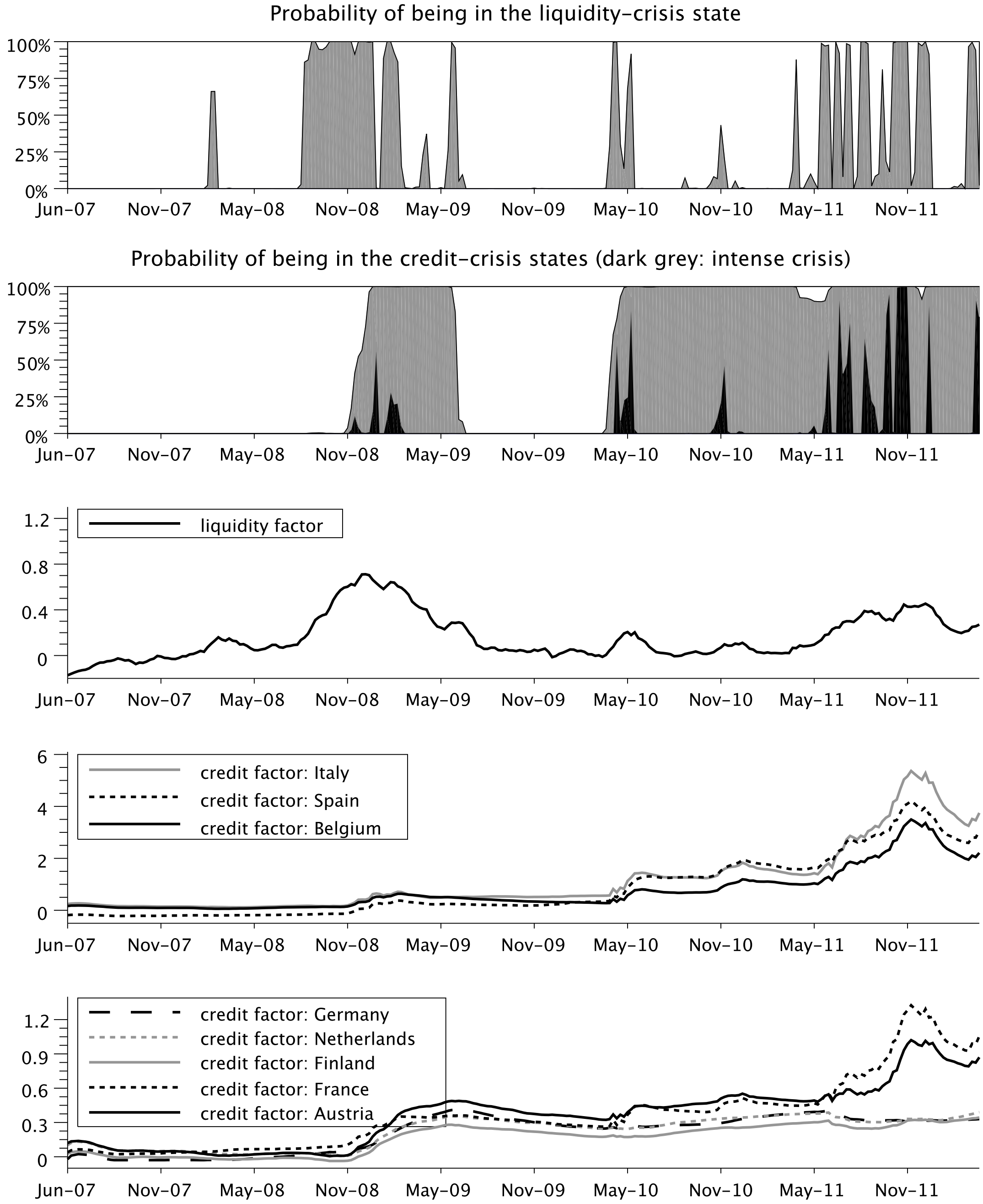


Figure 4.3.: Sensitivity to the liquidity factor versus debt outstanding

Notes: The upper panel plots the estimated liquidity-related factor $\lambda_{\ell,t}$ together with the bid-ask spread on French 10-year benchmark bonds (source: Thomason Reuters Tick History database). In the first scatter plot, the coordinates of the countries correspond to (x-coordinates) the sensitivities $\mu_{1,\ell}^{(n)}$ of their risk intensities to the European liquidity factor $\lambda_{\ell,t}$ (these sensitivities are reported in the upper part of Table 4.3) and (y-coordinates) their total markeTablesovereign debt (as of the end of 2009, Source: Eurostat). In the first scatter plot, the abscissa of the countries are the same than for the previous plot, and the y-coordinates are the average (2010-2012) of the bid-ask spreads of 10-year sovereign bonds (source: Thomson Reuters Tick History database).

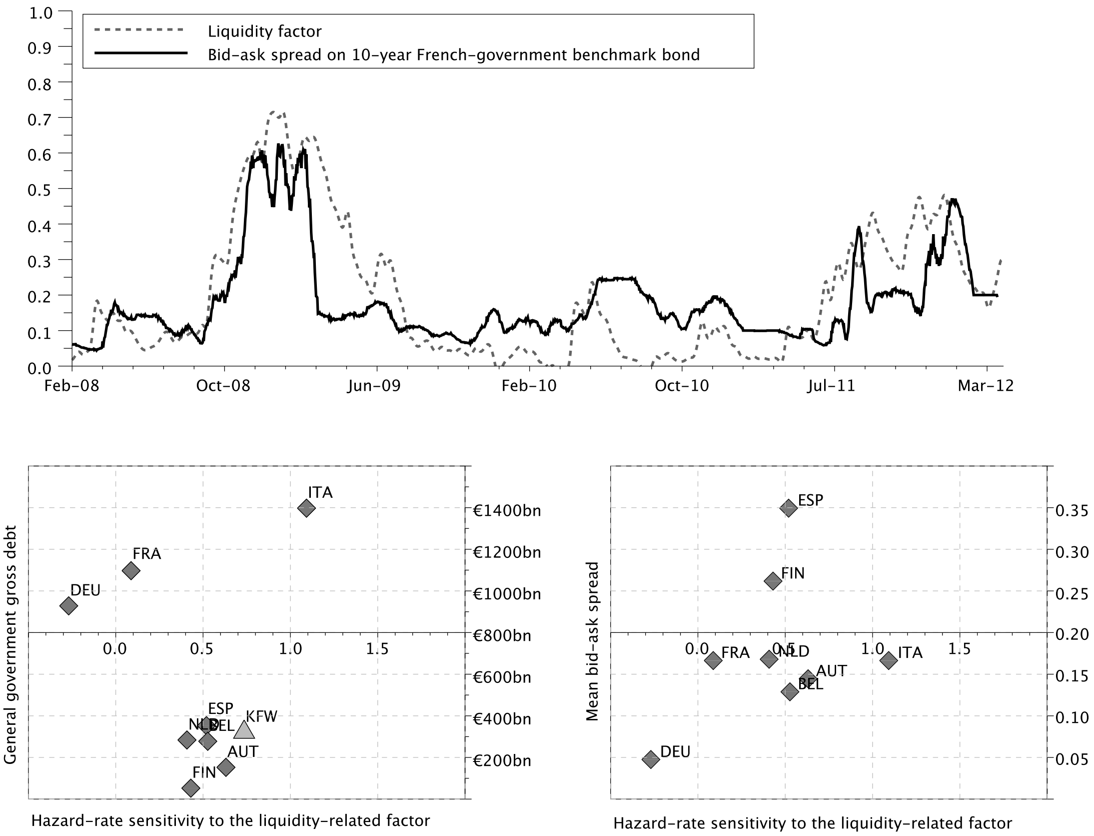


Figure 4.4.: Default probabilities estimates (5-year horizon)

Notes: Notes: These plots display the model-implied 5-year default probabilities computed under both measures (risk-neutral: dotted line, historical: solid line). Formally, the dotted line corresponds to the time series of $E_t^{\mathbb{Q}}(\mathbb{I}\{d_{t+5yrs}^{(n)} = 1\}|d_t^{(n)} = 0)$, where E_t denotes the expectation (under the historical measure) conditonal to the information available at time t (see Section 4.5.3 for the computation of these default probabilities). The black solid line represents$E_t^{\mathbb{P}}(\mathbb{I}\{d_{t+5yrs}^{(n)} = 1\}|d_t^{(n)} = 0)$. One-standard-devation bands are reported. These standard devations account for smoothing errors (associated to Kim's smoothing algorithm, 1994) as well as uncertainty related to the parameter estimates, following Hamilton's (1986) approach. The y-axis scales differ across countries.

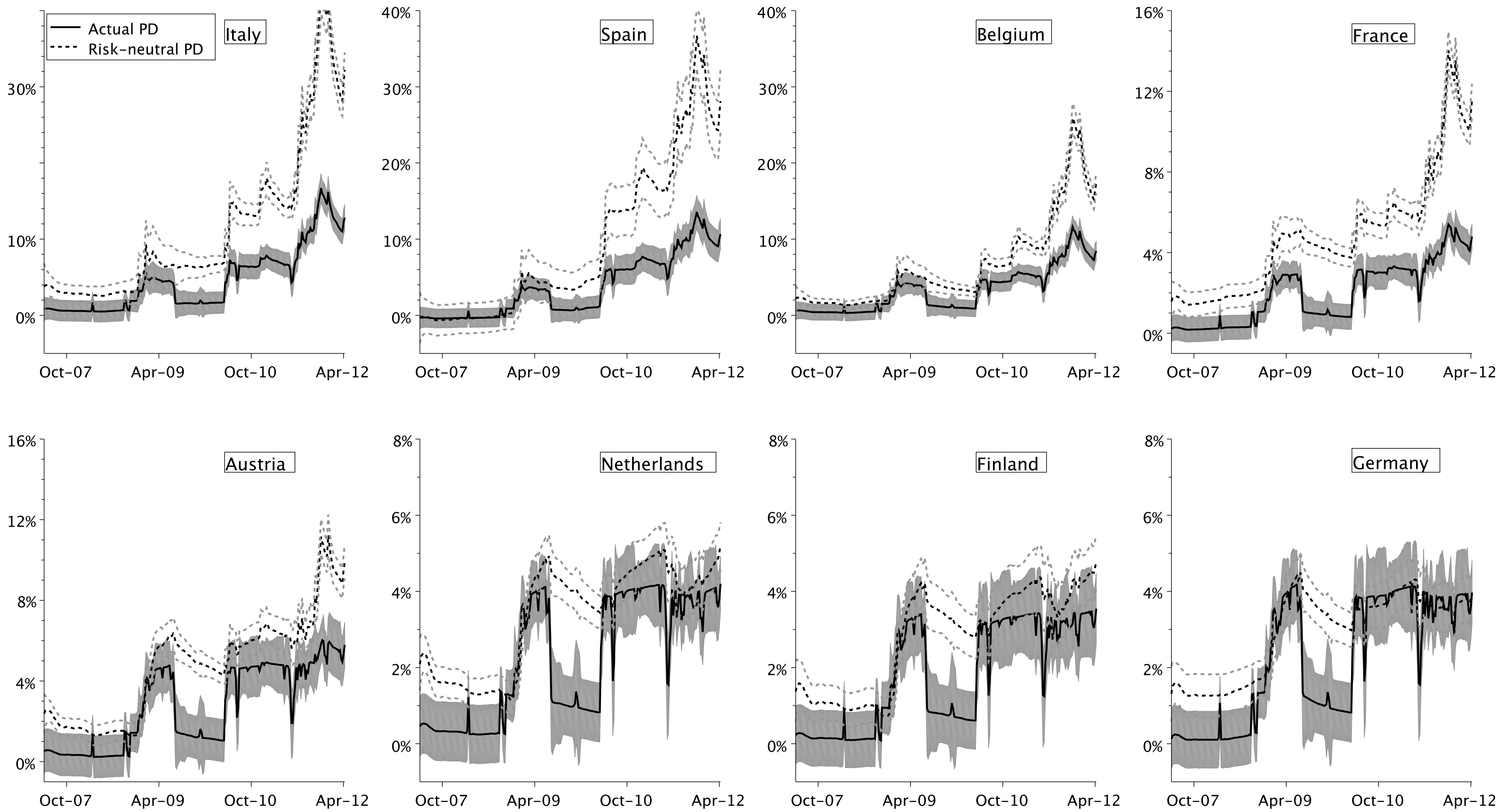


Figure 4.5.: Term structure of default probabilities

Notes: These plot display the term structure of the default probabilities for two distinct dates. Formally, for date t and debtor n, the plot shows $E_t^{\mathbb{P}}(\mathbb{I}\{d_{t+h}^{(n)} = 1\}|d_t^{(n)} = 0)$ for h between 1 month and 10 years (where $E_t^{\mathbb{P}}$ denotes the expectation –under the historical measure– conditional to the information available at time t). The grey lines delimit the ± 1 standard deviation area. These standard devations account for smoothing errors (associated to Kim's smoothing algorithm, 1994) as well as uncertainty related to the parameter estimates, following Hamilton's (1986) approach. The y-axis scales differ across countries.

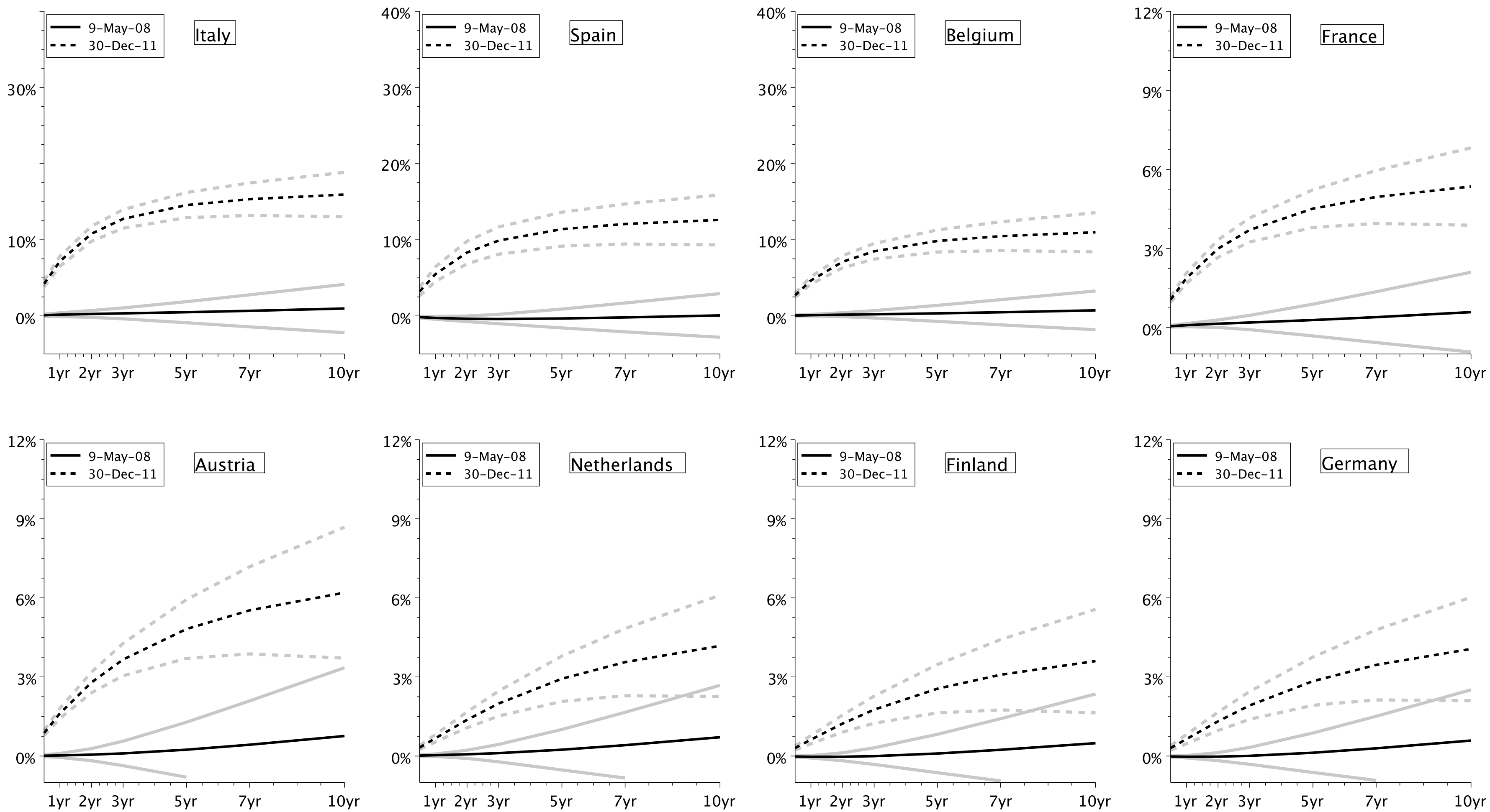
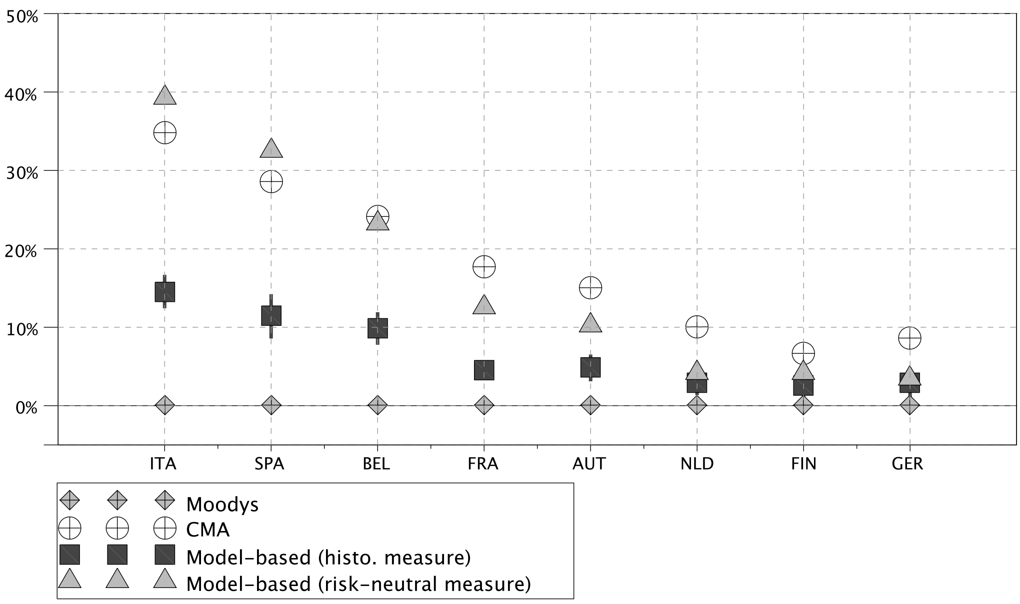


Figure 4.6.: Default probabilities estimates (5-year horizon)

Notes: This plot displays different estimates of probabilities of default (PD) of 8 euro-area governments (as of 30 December 2011). The squares and the triangles correspond to outputs of our model. While the squares indicate "real-world" PDs (i.e. the default proabilities obtained under the physical, or historical, measure), the triangles are risk-neutral PDs. The vertical black bars associated with squares delineate the delimit the ± 2 standard-deviation area. These standard devations account for smoothing errors (associated to Kim's smoothing algorithm, 1994) as well as uncertainty related to the parameter estimates, following Hamilton's (1986) approach. The circles indicate the PDs computed by CMA, using an industry standard model and proprietary CDS data from CMA Datavision (2011). The diamonds correspond to PDs that derive from (a) the Moody's' ratings of the countries (as of 2011Q4) and (b) the matrice of credit-rating-migration probabilities given by Moody's (2010).



5. A model of the euro-area yield curve with discrete policy rates[1]

Abstract: This Chapter presents a no-arbitrage model of the yield curve that explicitly incorporates the central-bank policy rate. After having estimated the model using daily euro-area data, I explore the behaviour of risk premia at the short end of the yield curve. These risk premia are neglected by the widely-used practice that consists in backing out market forecasts of future policy-rate moves from money-market forward rates. The results suggest that this practice is valid in terms of sign of the expected target moves, but that it tends to overestimate their size. As an additional contribution, the model is exploited to simulate forward-guidance measures. A credible commitment of the central bank to keep its policy rate unchanged for a given period of time can result in substantial declines in yields. For instance, a central-bank commitment to keep the policy rate at 1% over the next 2 years would imply a decline in the 5-year rate of about 25 basis points.

[1]When writing this Chapter, I have benefited from discussions with Narayan Bulusu, Hans Dewachter, Simon Dubecq, Jean-Sébastien Fontaine, Rodrigo Guimaraes, Imen Ghattassi, Wolfgang Lemke, Andrew Meldrum, Jean-Stéphane Mésonnier, Emmanuel Moench, Benoît Mojon, Fulvio Pegoraro, Francisco Rivadeynera Sanchez, Thomas Sargent, Andrew Siegel and Paul Whelan. I thank participants at Banque de France seminar, at Bank of England seminar, at Canadian Economic Association annual meeting (2012), at AFSE annual meeting (2012) and at ESEM annual meeting (2012), at the ECB workshop "Excess liquidity and money-market functioning" (2012), at AFFI Paris finance meeting 2012. I thank Béatrice Saes-Escorbiac and Aurélie Touchais for excellent research assistance. Any remaining errors are mine. The views expressed in this Chapter are mine and do not necessarily reflect the views of the Banque de France.

Résumé

Les évolutions récentes des taux d'intérêt ont mis en lumière une limite de la majorité des modèles de taux d'intérêt, à savoir leur incapacité à assurer une probabilité nulle à l'occurrence de taux d'intérêt négatifs. En effet, dans un contexte de taux d'intérêt courts très bas, les modèles dans lesquels les taux courts sont constamment affectés par des chocs de moyenne nulle attribuent une probabilité strictement positive à l'occurrence de taux courts négatifs dans le futur (voir notamment Andreasen et Meldrum, 2012). Ce chapitre montre comment l'utilisation des changements de régimes permet de construire un modèle dans lequel les taux d'intérêt sont positifs. De plus, il y est montré comment ce type de modèle, dans lequel la politique monétaire occupe une place centrale, peut être exploité afin d'étudier l'influence de cette forme de politique économique sur la structure par terme des taux d'intérêt.

La partie empirique de ce chapitre poprose une modélisation de la structure par terme des taux OIS (Overnight indexed swaps). Ces instruments ont pris une importance grandissante au cours de la dernière décennie sur les marchés de taux d'intérêt. Le taux sous-jacent à cet instrument dérivé est le taux interbancaire au jour-le-jour, appelé EONIA en zone euro. L'OIS de maturité m mois est un contrat dans lequel deux contreparties se mettent d'accord pour échanger deux flux à la maturité de ce swap (i.e. dans m mois). A cette date, l'une des contreparties donnera à l'autre l'EONIA capitalisé sur cette période (cette somme n'est pas connue au moment de la négociation du contrat); inversement, la seconde paie un montant indexé sur un taux fixé à la date de négaociation (c'est le taux du swap). Il peut être montré que le taux du swap est homogène à un taux obligataire. Pour modéliser la structure par terme des taux OIS, il convient (a) de définir le processus suivi par le taux court (l'EONIA) et (b) de spécifier le facteur d'escompte stochastique.

Une des principales composantes de l'EONIA est le taux des opérations principales de refinancement des banques auprès de la banque centrale européenne. Ce dernier, appelé «taux directeur» dans ce qui suit, est fixé par le conseil des gou-

verneurs de l'Eurosystème ayant lieu, approximativement, à fréquence mensuelle. Nous modélisons l'EONIA comme la somme de ce taux directeur et d'une composante résiduelle que l'on nomme «écart de taux EONIA». Une particularité du taux directeur réside dans l'aspect discret de son support. En effet, les principales banques centrales ont pour pratique de fixer ce taux d'intérêt à des niveaux qui sont des multiples de 0,25%. La modélisation du taux directeur repose sur une utilisation intensive des changements de régime. Chaque régime est en effet défini (a) par un niveau du taux directeur (élément de l'ensemble {0% ; 0,25% ; 0,50% ;... ; 10%}), (b) par une «phase» de politique monétaire (durcissement, assouplissement ou statu quo) et, (c) par l'existence, ou non, d'une situation de sur-liquidité des banques. Cette dernière caractéritique est introduite afin de reproduire les chutes persistantes de l'écart de taux EONIA observées depuis la mise en place par la BCE d'injections de liquidités à taux fixe en 2008. Au total, le modèle compte 246 régimes ($41 \times 3 \times 2$). Pour chaque niveau de taux et chaque phase de politique monétaire, on définit une probabilité de décision de hausse ou de baisse de taux lors des conseils des Gouverneurs. Afin de ne pas faire face à un explosion du nombre de paramètres requis pour spécifier ce modèle, les probabilités de transition d'un régime à l'autre sont données par des fonctions paramétriques du niveau de taux directeur.

Pour l'écart de taux EONIA, nous spécifions une dynamique mêlant processus auto-régressif gaussien et bruit blanc distribué suivant des lois Beta. Ces spécifications permettent notamment de reproduire l'épaisseur des queues de la distribution empirique de cet écart de taux.

La spécification du facteur d'escompte stochastique est telle que la dynamique du taux court sous la mesure risque-neutre est du même type que la dynamique physique définie précédemment. En revanche, la paramétrisation de cette dynamique risque-neutre est différente. Dans ce contexte, nous montrons comment calculer les taux des OIS. Nous présentons un algorithme permettant de calculer rapidement ces taux sans avoir recours à des formules récursives pouvant ralentir le calcul lorsque la fréquence d'échantillonnage du modèle est élevée.

Le modèle est estimé sur données quotidiennes couvrant la période allant de janvier 1999 à février 2012. Les données d'estimation incluent l'EONIA ainsi que des taux OIS de maturités 1, 3, 6, 12, 24 et 48 mois. Comme dans les chapitres 3 et 4 de cette thèse, des données issues d'enquêtes auprès de prévisionnistes sont également utilisées afin d'améliorer l'estimation de la dynamique historique des processus. Le modèle est estimé par maximisation de la vraisemblance. La modélisation inclut des régimes inobservables ainsi que des variables latentes (car l'écart de taux EONIA comporte plusieurs composantes). Pour traiter simultanément ces deux types de latence, le calcul de la vraisemblance repose sur une utilisation jointe du filtre de Kitagawa-Hamilton et de la technique d'inversion à la Chen et Scott (1993). Une mise en oeuvre innovante de ces dernières est toutefois proposée afin de réduire l'ampleur d'un problème lié à cette méthode; ce problème est celui du caractère arbitraire du choix de la maturité des taux supposés être modélisés sans erreur. La méthode proposée permet d'éviter ce choix et de répartir l'erreur de mesure sur les différents maturités considérées. Toujours à propos du calcul de la vraisemblance, il est important de remarquer que du point de vue de l'économètre, les 246 régimes ne sont que «partiellement» inobservables : à chaque période, l'économètre observe le niveau du taux directeur mais il n'observe pas (a) la phase de politique monétaire (durcissement, assouplissement ou statu quo) et (b) la situation de liquidité bancaire (en excès de liquidité ou non). Autrement dit, du point de vue de l'économètre, le modèle est un modèle à six régimes inobservables (3×2) avec probabilités de transition variant dans le temps (en fonction du niveau observable du taux directeur).

Le modèle obtenu permet l'étude du comportement de primes de risque présentes dans la partie courte de la courbe des taux. Ces primes de risques sont généralement négligées par la pratique considérant que les taux courts *forward* calculés à partir des taux OIS peuvent être interprétés comme les anticipations de marché des taux de politique monétaire futurs. Les résultats suggèrent que cette pratique est valide en termes de direction des mouvements anticipés du taux directeur mais qu'elle tend à surestimer l'amplitude de ces variations. De plus, nous montrons que le biais résultant de l'absence de prise en compte des primes de risque est partciulièrement grand durant les phases de durcissement monétaire.

Le modèle est également utilisé pour simuler l'impact sur la courbe des taux d'un engagement de la banque centrale concernant la trajectoire future de son taux directeur. Ce type de mesure peut avoir une influence substantielle : un engagement crédible de la banque centrale à maintenir son principal taux directeur à un niveau de 1% au cours des deux prochaines années entraînerait une baisse du taux à 5 ans de 25 points de base.

5.1. Introduction

The standard view of the monetary policy transmission mechanisms suggests that central banks' actions are mainly transmitted to the economy through their effect on market interest rates. According to this standard view, a restrictive monetary policy pushes up both short-term and long-term interest rates, leading to less spending by interest-sensitive sectors of the economy, and vice versa. While there is a strong empirical support for the assertion that monetary policy is a major driver of the yield-curve fluctuations, only a very few term-structure models explicitly incorporate the "policy rate", that is the main central bank's instrument. Arguably, this reflects the technical difficulties associated with accommodating the dynamics of this discrete-valued process.[2]

This Chapter proposes a novel and tracTableno-arbitrage term-structure model where changes in the monetary-policy rate are explicit and central. This model is particularly adapted to depict the dynamics of the short-end of the yield curve, where the influence of monetary policy decisions is the most evident (see Cochrane and Piazzesi, 2002). The estimation, carried out on euro-area daily data covering the last 13 years, sheds light on the influence of the ECB monetary policy on the term-structure of interest rates. Notably, the results show the key effect of the monetary-policy phases –tightening, easing or status quo– on the shape of the yield curve. Besides, the analysis provides evidence of the existence of substantial risk premia at the short- to medium-end of the term structure of interest rates.[3] This implies in particular that the common market practice that consists in backing out market forecasts of next policy-rate moves from money-market forward rates is biased.[4] More precisely, the results suggest that while this practice is valid in terms of sign of the expected target moves, it tends to overestimate their size. Besides, these risk premia turn out to be the most important when the monetary

[2]See e.g. Rudebusch (1995), Hamilton and Jorda (2002), Balduzzi, Bertola and Foresi (1997) and Balduzzi et al. (1998) for models of the U.S. Federal Funds rate target.

[3]The existence of such risk premia in the short end of the euro-area yield curve has notably been evidenced by Durré, Evjen and Pilegaard (2003).

[4]This common market practice implicitly assumes that the expectation hypothesis holds at the short-end of the yield curve.

policy is in a tightening phase, the deviation between the 12-month-ahead risk-neutral forecast of the policy rate (this forecast is approximately a forward rate) and its physical counterpart being of about 50 basis points.

As an additional contribution, this model is exploited to assess the potential effects of so-called *forward policy guidance* measures. These measures, that consist of commitments of the central bank regarding the future path of its policy rate, are expected to provide more accommodation at the zero lower bound (ZLB).[5] Indeed, the objective of these measures is to provide a stimulus to the economy by making market participants revising down their expectations of future short-term interest rates, thereby pushing down medium- to long-term interest rates. The effectiveness of such measures is the subject of substantial debate (Williams, 2011). Using new-Keynesian general equilibrium models, Eggertsson and Woodford (2003), Campbell et al. (2012) or Levin et al. (2010), among others, investigate the impacts of forward policy guidance. While the former two studies find that forward guidance can be effective in terms of macroeconomic stabilisation, the latter shows that such measures may be insufficient to deal a “Great Recession”-style shock. As in Gagnon et al. (2011), Kool and Thornton (2012), Rudebusch and Bauer (2011) or Jardet, Monfort and Pegoraro (2010), the present Chapter focuses on the effects of unconventional monetary policies on the term structure of interest rates. Specifically, as in the latter paper, the model is used to simulate the effects of commitments of the central bank to keep its policy rate at its current level for (at least) a deterministic period of time. In the present framework, where the policy rate is explicit, such a simulation is carried out in a straightforward and consistent manner. According to the results, forward-guidance measures could lead to a substantial downward shift in the yield curve. The lower the policy rate, the larger the effect: for instance, in a context characterised by a policy rate of 1% (respectively 3.5%), the model predicts that the announcement of a commitment to keep the target rate unchanged for at least 2 years would be followed by a 25 bp (resp. a 5 bp) decline in the 5-year yield.

[5]See Bernanke and Reinhart (2004) for a list and discussion of the potential policy options available to monetary-policy authorities when the zero bound is binding.

The papers that are the closest to the present one are those by Piazzesi (2005) and Fontaine (2009). In both papers, the authors propose term-structure models that explicitly involve the target for the policy rate. They estimate their models on U.S. data covering respectively the periods 1994-1998 (weekly) and 1994-2007 (daily). A common drawback of these frameworks is that they do not preclude negative policy rates. While this caveat may be tenable when the short-term interest rate is far enough from zero –the conditional probabilities of having negative interest in the subsequent periods being negligible–, it is more problematic in the current context of very low interest rates. More generally, many of the tracTableyield-curve models are not consistent with the ZLB restriction.[6] This limitation is addressed in the present framework.

In my model, changes in the policy rate rate take place on pre-determined monetary-policy meeting dates and are multiples of 25 basis points (or 0.0025). The model is consistent with the fact that target-rate changes occur infrequently, on a daily time scale, and with policy inertia (i.e. that target changes are often followed by additional changes in the same direction). These appealing features stem from an original use of regime-switching techniques, each regime being characterised by a given tick of the policy rate and a given monetary-policy phase: tightening, easing or status quo. The definition of these phases is consistent with observed central banks' target-setting behaviour and communication (see Smaghi, 2009). The probabilities of occurrences of target moves depend on the monetary-policy phase and on the level of the target rate. In particular, the probability of a cut in the policy rate is zero when this rate is at the ZLB, thereby precluding negative rates.

The shortest-term rate of the yield curve that is considered here is the interbank overnight interest rate, which most central banks aim at stabilising to a level close

[6]See Dai and Singleton (2003) or Piazzesi (2010). Hamilton and Wu (2012) propose a way to adapt the standard Gaussian framework to account for an extended period of constant short-term rate. However, they implicitly assume that when this phase ends, (a) such a phenomenon cannot happen again and (b), the short-term rate can turn negative again. Andreasen and Meldrum (2011) or Kim and Singleton (2011) show that the quadratic Gaussian framework can be used to preclude negative interest rates. However, these latter models can not accommodate long periods of unchanged interest rates.

to the policy (or target) rate. Therefore, after having specified the dynamics of the latter, the model is completed by specifying the dynamics of the so-called EONIA spread, that is the yield differential between the Euro Over-Night Index Average and the main policy rate. While this spread was mostly transitory before 2007, persistent deviations appeared in October 2008, following changes in the monetary-policy implementation in the euro area in response to the financial crisis. To capture that change in the behaviour of the EONIA spread, an additional two-state Markov-switching process is introduced, one of these two states corresponding to a situation in which banks' excess liquidity translates into a drop of the interbank rate with respect to the target (see Soares and Rodrigues, 2011).

Consistently with the choice of the EONIA as the shortest-term rate, the empirical exercise uses Overnight Index Swap (OIS) rates as longer-term yields.[7] An OIS is a fixed-for-floating interest rate swap with a floating rate leg tied to the index of daily interbank rates, that is the EONIA in the euro-area case. OIS have become especially popular hedging and positioning vehicles in euro financial markets and grew significantly in importance during the financial turmoil of the last few years.[8] The OIS curve is closely watched by practitioners to gauge what policy-rate changes the market has already priced in.

The model involves a lot of Markovian regimes –more than 200–, obtained by crossing the regimes describing the policy rate, the monetary policy phases and the liquidity states. This distinguishes the present framework from earlier term-structure models involving regime switching.[9] In spite of this unusual feature, the approach remains tracTableboth in terms of bond pricing and estimation. The yields of different maturities turn out to be equal to linear combinations of the factors (including the regime variable), the factors loadings being given by

[7]This is done only for the second part of my sample, i.e. 2005-2011. Indeed, long-term OIS are not available before then. In the first part of the sample, I use EURIBOR swaps (see Subsection 5.2.2).

[8]While the United States has a liquid Fed Funds future contract (Gurkaynak, 2005 or Gurkaynak, Sack and Swanson, 2007), markets in most other countries rely exclusively on their local-currency-denominated OIS market for hedging central bank policy (Lang, 2010).

[9]See, e.g., Bansal and Zhou (2002), Dai, Singleton and Yang (2007), Ang, Bekaert and Wei (2008) or Lemke and Archontachis (2008).

simple formulas involving a limited number of matrix products.[10] The model can generate the usual shapes of the yield curve (steep, flat, inverse, humped, inverse-humped) and accommodates heteroskedasticity in the yield dynamics. As regards the estimation, a key point is that regimes are only partially hidden: a characteristic of the regimes, namely the central-bank policy rate, is observed by the econometrician.[11] Therefore, the econometric model can be seen as a six-hidden-state (three monetary-policy regimes and two liquidity regimes) Markov-switching model with heterogenous probabilities of transition, the latter depending on the observed target rate.

The model is estimated by maximum likelihood techniques. The computation of the log-likelihood is based on an innovative joint use of the Kitagawa-Hamilton's filter and so-called inversion techniques introduced by Chen and Scott (1993). The fit of the model is satisfying, the standard deviations of the pricing errors being of 8 basis points (from 1 month to 4 years). An important output of the approach are the probabilities of being in the different hidden Markovian states. To that respect, this approach is an illustration of the results of Bikbov and Chernov (2008) who underline the importance of using yield-curve information to identify monetary-policy regimes.

The remainder of this chapter proceeds as follows. Section 5.2 presents the data and emphasises stylised facts. Section 5.3 develops the model. Section 5.4 presents the estimation strategy and results. Section 5.5 documents the behaviour of policy-rate-related risk premia. Section 5.6 derives some implications of the model regarding the commitment of the central bank to keep the target rate fixed for a given period of time. Section 5.7 concludes.

[10] In particular, the derivation of the term-structure of yields does not rely on the recursive algorithms usually used to solve discrete-time term structure models (as in Ang and Piazzesi, 2003). This point is crucial to make the model easily amenable to estimation using high-frequency data.

[11] I assume that market participants observe latent regimes and factors, as in most yield-curve studies involving latent factors.

5.2. Data and stylised facts

5.2.1. The EONIA and the Eurosystem's framework

Contrary to the Fed or the Bank of England, the ECB does not have an explicit interest-rate target. However, its aim is explicitly to *"influence money market conditions and steer short-term interest rates"* (ECB, 2011). This is done by using primarily the official interest rates: *"The (long) chain of causes and effect linking monetary policy decisions with the price level starts with a change in the official interest rates by the central bank on its own operations."*

In order to influence short-term money-market rates, a shortage of liquidity is created by imposing mandatory reserves on banks within the euro area. Specifically, credit institutions are required to hold compulsory cash deposits on accounts with the Eurosystem. The reserve requirements are based on the amount and profile of liabilities on a bank's balance sheet as of every month end. The banks can refinance themselves through the ECB's weekly Main Refinancing Operations (MROs). In these weekly refinancing operations, the ECB returns liquidity to the market by allowing banks to tender for cash (against collateral). By abuse of language, the rate at which liquidity is supplied in the regular weekly monetary policy operations is referred to as the "policy rate" (or the "target rate") in this chapter. However, there are two additional policy rates in the Eurosystem framework. Indeed, the latter is completed by a symmetric corridor bracketing the main policy rate.[12] The lower bound of the corridor, called the deposit-facility rate, is the rate at which counterparties can deposit cash overnight with the Eurosystem. The upper bound is the lending-facility rate, at which counterparties can borrow funds overnight from the Eurosystem. The target rate and the corridor is displayed in Panel A of Figure 5.1.

After having been fixed till June 2000, the MROs' rate then became variable.[13] In October 2008, in a context of worldwide financial stress, the Eurosystem adopted

[12] See Kahn (2010) for a comprehensive description and an international comparison of "corridor" systems.

[13] At that time, the target, or refi rate, acted as the minimum bid rate at the MRO.

a fixed-rate full allotment (FRFA) tender procedure: since then, the ECB accommodates any demand for liquidity its bank counterparties might express at the policy rate –against eligible collateral– in unlimited amounts.

While the policy rate defines the rate at which banks can refinance themselves through the ECB against collateral, EONIA (Euro OverNight Index Average) fixings reflect rates at which banks refinance themselves on the interbank market on an unsecured basis.[14] In "normal" circumstances, EONIA rates trade in close relation to ECB marginal rates but can also include a premium related to the unsecured nature of the lending.

Panel A of Figure 5.1 compares the fluctuations of the target with these of the EONIA. Changes in the policy rate are decided during the first of the bimonthly meetings of the ECB's Governing Council. On a daily scale, this implies a step-like behaviour for the target rate. Over the estimation sample (January 1999 – February 2012), there were 18 rises in the target rate (16 of 25 bp and 2 of 50 bp) and 16 cuts in target rates (7 of 25 bp, 8 of 50 bp and one of 75 bp). Panel A of Figure 5.1 also suggests that the EONIA is closely linked to the target rate. However, by displaying the EONIA spread –i.e. the yield differential between the EONIA and the policy rate–, Panel B highlights the break in the relationships between these two rates that occurred in 2008. This break can be related to non-standard monetary-policy measures that were taken in response to the financial crisis. A particularly important decision was the one to move from variable rate tender procedures in liquidity providing operations to FRFA. Together with the expansion of the spectrum of maturities at which liquidity was being offered to the market, this measure generated a steady excess of liquidity balances in the overnight market, as banks began supplying in the interbank market the precautionary cash buffers that they were securing at the ECB.[15] An excess supply of

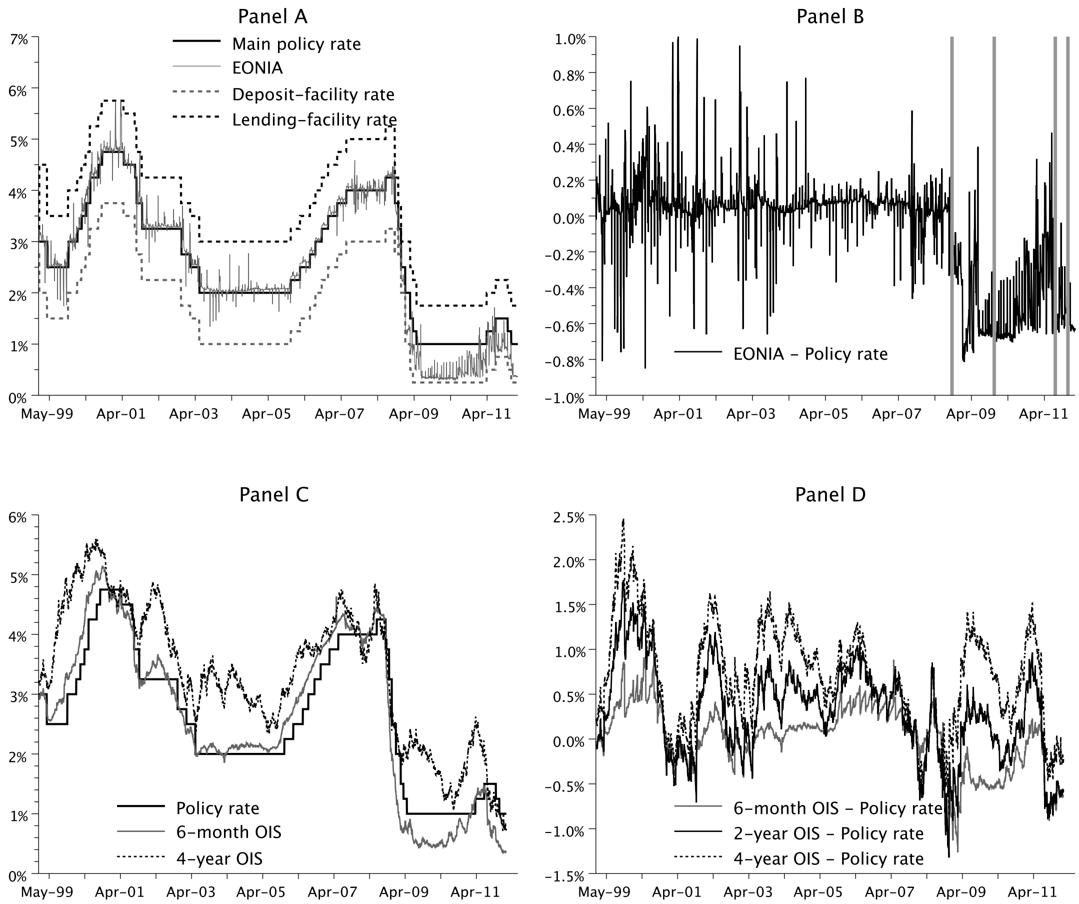
[14]The EONIA is computed as a weighted average of all overnight unsecured lending transactions undertaken in the interbank market, initiated within the euro area by the contributing banks. It is computed by the ECB at the end of every TARGET day (since January, 4 1999). The banks contributing to the EONIA are the same first class market standing banks as the panel banks quoting for Euribor. See www.euribor-ebf.eu for more details.

[15]A large share of the cash buffers is held with the ECB, the banks using massively the marginal deposit facility.

liquidity in overnight trades put downward pressure on the overnight interest rate, which drifted toward the lower limit of the monetary policy corridor (see Beirne, 2012 and Fahr et al., 2010).

Figure 5.1.: Target rate, EONIA and OIS

Notes: Panel A shows the target rate together with the overnight interbank interest rate (EONIA). Panel B displays the EONIA spread, i.e. the spread between the EONIA and the target. The four vertical bars in Panel B indicate the four following dates, respectively: 8 October 2008 (introduction of Fixed-Rate Full Allotment procedures), 3 December 2009 (announcement of the phasing out of the very long-term refinancing operations), 4 August 2011 (given the renewed financial-market tensions, announcement of supplementary 6-month LTRO), 8 December 2011 (3-year VLTRO). Panel C presents the target rate and two OIS yields, the spreads between the latter and the target being reported in Panel D.



5.2.2. The Overnight Index Swaps

An overnight index swap (OIS) is an interest rate swap whose floating leg is tied to an overnight rate (the EONIA in the euro-area case), compounded over a specified term. OIS contracts involve the exchange of only the interest payments, the principal amount being notional. That is, the two parties agree to exchange, on the agreed notional amount, the difference between interest accrued at the fixed rate and interest accrued through daily compounding (or geometric averaging) of the floating overnight index rate. While the tenor of these swaps was usually below 2 years before 2005, the OIS maturities were extended afterwards to more than 10 years (see Barclays, 2008). The OIS curve is more and more seen by market participants as a proxy of the risk-free yield curve (see e.g. Joyce et al., 2011).[16] In spite of that, OIS have failed to attract significant attention from academics for the time being.

As an interest-rate swap, an OIS can be used to manage interest-rate risks. In particular, the OIS are structured in such a way that if a bank (a) has some money available for investment, (b) has access to the overnight interbank market and (c) can enter OIS contracts, then this bank can synthetically create a fixed-income instrument that is equivalent to a maturity-h bond paying a coupon equal to the maturity-h OIS rate.

An important point that is going to be investigated below relates to the use of OIS curves to back out market expectations of future policy rate's moves. Heuristically, under the expectation hypothesis, the forward rates based on the OIS term structure should reflect the market expectations of the interbank rate, that is supposed to be close to the target rate. This principle is widely used by market analysts, investors or central banks themselves.[17]

[16]While OIS rates reflect the credit risk of an overnight rate, this may be regarded as negligible in most situations. Besides, even during financial-markets turmoil, the counterparty risk is limited in the case of a swap contract, due to netting and credit enhancement, including call margins (see Bomfin, 2003). To that respect, one can note that German sovereign bonds, usually perceived as being the European "safest haven" both in terms of credit quality and liquidity, trade at levels that have remained close to the OIS yield curve over the last years.

[17]See e.g. Barclays, 2008, Joyce, Relleen and Sorensen, 2008, Joyce and Meldrum 2008, Bank of England, 2005 or Lang (2010).

5.2.3. Data sources and treatments

The sample period is January 15, 1999 to February 17, 2012 (3416 dates). While the target rate and the EONIA series come from the ECB, the OIS yields are taken from Bloomberg. All yields are translated on a continuously compounded basis, and market holidays are filled with observations from the previous trading days' rates.[18] In the estimation, we consider six maturities (in addition to the overnight one): 1 month, 3 months, 6 months, 12 months, 2 years and 4 years.

As said above, OIS yields are not available for longer-than-one-year maturities before 2005. Before that date, we use EURIBOR swaps data in place of the 2-year and 4-year OIS yields. This appear to be a reasonable assumption given that the short-term EONIA swaps and maturity-matching EURIBORs had extremely close variations before 2007.[19] Swap yields are homogenous to coupon-bond yields. Since the pricing formula presented below (Subsection 5.3.2) are consistent with zero-coupon yields, zero-coupon yields are computed using classic bootstrapping methods.[20]

The estimation procedure involves survey-based forecasts of short-term yields (as in Kim and Orphanides, 2012; this is discussed in Section 5.4). Specifically, 12-month-ahead forecasts provided by the *Consensus Forecasts* are used. Forecasts of the ECB's policy rate are available since July 2009 only; before that, I use 3-month EURIBOR forecasts.[21] Since EURIBOR and OIS were closely linked until summer 2007, using EURIBOR forecasts instead of OIS forecasts is appropriate till then. In mid-2007 however, the widening in the EURIBOR-OIS spread is likely

[18] Let r denote a market-quoted interest rate (the OIS, say). Using the fact that money-market rate are based on the ACT/360 day-count basis, the corresponding continuously compounded rate is computed as $\ln(1 + d \times r/360) \times 365/d$, where d is the residual maturity of the instrument.

[19] During summer 2007, credit and liquidity risks affected unsecured interbank lending rates (IBOR), leading to a sudden widening of the IBOR-OIS spreads. Before that, this spread was small and steady. For each maturity (2-year and 4-year), I subtract the 2005-2006 IBOR-OIS average spread from the EURIBOR swap series used in the estimation before 2005, which is about 10 basis points (standard deviation below 3 basis points).

[20] OIS rates with a maturities lower than one year are already homogenous to zero-coupon instruments. The bootstrapping methods are applied only for longer-than-one-year maturities. See Barclays (2008) for more information about EONIA swaps.

[21] Naturally, the fact that the nature of the forecasted rate changes in mid-2009 is taken into account in the estimation procedure.

to induce a bias in the forecasts. This is addressed by subtracting from the EURIBOR forecasts –from August 2007 to June 2009– the 1-year-ahead forward spread between the 3-month EURIBOR and OIS rates (averaged over the same period). All these survey-based expectations are available at the monthly frequency only and are released about mid-month. Using a cubic spline, this series is converted into a daily one. The discrepancies that arise from these approximations are expected to be captured by measurement errors of the state-space model that will be presented below.[22]

5.2.4. Preliminary analysis of the yields

Table 5.1 reports descriptive statistics for the different yields used in the analysis. These statistics suggest that yields are highly persistent. While the daily autocorrelation is nearly one, the correlations between the yields and their 1-year lags is still substantial (higher than 50%). The correlation across maturities is also extremely high, with near-unit correlations for adjacent maturities. Mean and median statistics show that the term structure is positively sloped on average.

The lowest Panel in Table 5.1 shows the results of a principal component analysis carried out on the set of seven spreads between OIS yields –with maturities of 1 day to 4 years– versus the policy rate. The three principal components are sufficient to explain most of the fluctuations of these spreads. Notably, the first principal component explains more than 90% of the variances of the spreads associated with yields of maturities comprised between 3 months and 1 year. This is graphically illustrated in Panel D of Figure 5.1, that highlights the common fluctuations in some of these spreads. Half of the variance of the EONIA spread and of the spread between the 4-year rate and the target rate is accounted for by this first factor, indicating that there are important correlations between the EONIA spread and longer-term spreads. However, further investigations mitigate this finding. Specifi-

[22] Anticipating on the estimation results presented in Section 5.4, the standard deviation of the measurement errors associated to the forecasts is slightly larger than 20 bps (σ_{fcst} in Table 5.2), which is of the same order of magnitude as the errors expected from the previous points.

cally, the same kind of analysis has been carried out on a shorter sample, excluding the crisis period: 1999-2008 (bottom of Table 5.1). On that period, the EONIA spread turns out to be almost orthogonal to the first principal component. Therefore, the apparent comovement between the EONIA spread and the other spreads on the whole sample seems to be related to the fall in the EONIA spread that took place in mid-2008 (see Subsection 5.2.1 for a description of this phenomenon).

5.3. The model

This Sectionformulates a model of the daily dynamics of the overnight interbank interest rate.[23] Two dynamics are considered: the historical (or physical, or real-world) one and the risk-neutral (or pricing) one. The knowledge of the risk-neutral dynamics of the interbank rate makes it possible to price financial instruments – such as the OIS contracts– whose cash flows depend on the overnight interbank rate. The simultaneous knowledge of the two dynamics allows to study term premiums' behaviour, as will be done in Section 5.5. The historical ($\mathbb{P}$) and the risk-neutral ($\mathbb{Q}$) dynamics of the different processes are of the same kind, but their respective parameterizations differ. These differences and the implied stochastic discount factor (s.d.f.) are detailed in Subsection 5.3.2, that also deals with the derivation of the term-structure of OIS rates. Before that, the next subsectionpresents the different components of the overnight interest rate.

5.3.1. The components of the overnight interest rate

The target rate prevailing at date t is denoted by $\bar{r}_t$. As is the case in most currency areas, the target rate is assumed to be a multiple of 0.25%. I proceed under the assumption that the target rate is lower than a maximal rate denoted by r_{max} and equal to $0.25\% \times N$, say. Therefore:

$$\bar{r}_t = \Delta' z_{r,t}$$

[23]The extension to a lower frequency is straightforward.

where $z_{r,t}$ is a selection vector, i.e. one of the column of I_{N+1}, the identity matrix of dimension $(N+1) \times (N+1)$ and where the entries of the vector Δ are the continuously-compounded possible policy rates. Specifically, using the money-market day-count convention, the ith entry of Δ is given by $\log(1 + (i - 1)0.25\%/360)$. Note that at the daily frequency, many of the successive $\bar{r}_t$'s are equal. In particular, $\bar{r}_{t-1} = \bar{r}_t$ as soon as there is no policy meeting at date t. This results in a step-like process for the policy rate (as seen in Panel A of Figure 5.1).

The interbank overnight interest rate is denoted by r_t. Its deviations from the target rate are accounted for by two components: ξ_t and s_t:

$$r_t = \bar{r}_t + s_t + \xi_t \tag{5.1}$$

I assume that $\bar{r}_t$, s_t and ξ_t are independent of each others.[24] The variables s_t and ξ_t are unobservable but can be inferred from yields through the bond-pricing model. The historical dynamics of these factors are presented in the following. The risk-neutral dynamics are of the same kind, but their parameterizations is different from their physical counterparts. These differences are made explicit in Subsection 5.3.2.

5.3.1.1. The dynamics of the target rate $\bar{r}_t$

Central bankers can decide to change the target rate at their regular meetings. On these dates, the target can be raised or cut if the the tightening regime or the easing regime respectively prevail, but the target remains necessarily unchanged under the status quo regime. Formally, the monetary regime is represented by a 3-dimensional selection vector $z_{m,t}$ that is valued in the set of the three columns of the identity matrix I_3, corresponding respectively to the tightening, the status quo and the easing regimes. Contrary to the econometrician, market participants observe the regime, this knowledge being based on a variety of detailed policy-relevant information that is not modelled here.

[24]Such independence assumptions are common in that literature (see Balduzzi et al., 1997 and 1998, or Piazzesi (2005)).

The Kronecker product of the selection vectors $z_{r,t}$ and $z_{m,t}$, denoted by $\bar{z}_t$, is also a selection vector that is valued in the set of the columns of $I_{3(N+1)}$ (recall that $N+1$ is the number of possible values of the target rate, between 0% and $r_{max} = N \times 0.25\%$). The dynamics of $\bar{z}_t$ is described by a Markov chain. The matrix of transition probabilities of $\bar{z}_t$ is denoted by $\bar{\Pi}_t$. These matrices are time-inhomogenous, but in a deterministic way. Indeed, the matrices $\bar{\Pi}_t$ can take two values, one of them being specific to those days at which a monetary-policy meeting are scheduled.[25] The number of entries of these $\bar{\Pi}$ matrices is considerable: for $r_{max} = 10\%$, there are 15.129 of them. However, owing to the following assumptions, most of these entries are zero:

1. Conditionally on being in an easing, a status quo or a tightening regime, the target moves are respectively valued in $\{-0.50\%, -0.25\%, 0\}$, $\{0\}$ and $\{0, +0.25\%, +0.50\%\}$.
2. Easing or tightening phases are necessarily followed by status quo phases.

Even with these restrictions, many of $\bar{\Pi}_t$'s entries still require to be parameterised. Eight sets of probabilities needs to be defined: two of them contain the probabilities of switching to the status quo regime (the probability of exiting the easing and the tightening regimes are respectively denoted by p_{ES} and p_{TS}), two others are the probabilities of exiting the status quo regime (p_{SE} and p_{ST}), two of them contain the probabilities of 25-bp changes in the target rate (rise: p_{r25}; cut: p_{c25}) and the last two are the probabilities of 50-bp moves (rise: p_{r50}; cut: p_{c50}). These probabilities may vary with the policy rate. For instance, the probability of switching from the tightening to the status quo regime could be larger for higher target rates, say. In order to keep the model parsimonious, the probabilities are based on logit-based parametric functions of the target rate $\bar{r}$. Formally, let me

[25] Contrary to the policy rate ($z_{r,t}$), that can change only following a monetary-policy meeting, the monetary-policy regime ($z_{m,t}$) can switch at any date. For instance, such changes could be triggered by ECB officials' speeches or the release of macroeconomic news or figures.

define the function f by:

$$f(\bar{r}, [a_1, a_2]') = [1 + \exp(a_1 + a_2\bar{r})]^{-1} . \tag{5.2}$$

For $i \in \{TS, ES, SE\}$, the probabilities p_i are characterised by some 2×1 vectors of parameters α_i and are given by $f(\bar{r}, \alpha_i)$. Further, so as to have $p_{ST} + p_{SE} < 1$, the probabilities p_{ST} are defined by $(1-p_{SE}(\bar{r}))f(\bar{r}, \alpha_{ST})$. Moreover, α_i vectors are not defined for each of the four kinds of target moves, but only for two: one for the rises in the policy rate (α_r) and one for the cuts (α_c). Two additional parameters, k_r and k_c, are then introduced to share the rise and cut probabilities into those of 25-bp and 50-bp moves. Formally, the conditional probabilities of target-rate changes (i.e. p_{c25}, p_{r25}, p_{c50} and p_{r50}) are defined through:

$$\begin{cases} p_{r25}(\bar{r}) = k_r f(\bar{r}, \alpha_r) \quad \text{and} \quad p_{r50}(\bar{r}) = (1 - k_r) f(\bar{r}, \alpha_r) \\ p_{c25}(\bar{r}) = k_c f(\bar{r}, \alpha_c) \quad \text{and} \quad p_{c50}(\bar{r}) = (1 - k_c) f(\bar{r}, \alpha_c) \end{cases}$$

where k_c and k_r are valued in $[0, 1]$.[26] Eventually, the 15.129 entries of matrix $\bar{\Pi}$ are defined by 16 parameters only.

5.3.1.2. The dynamics of ξ_t

The factor ξ_t is aimed at capturing the volatile and short-lived (noise) fluctuations of the EONIA spread. However, as clearly appears on Panel B of Figure 5.1, the distribution of this component is related to the level of this spread. Typically, the noise distribution became strongly positively skewed after the drop in the EONIA spread, in late 2008. As discussed in Subsection 5.2.1, this drop follows the implementation of non-standard monetary-policy measures that gave rise to a *banks' excess liquidity regime.* Hence, both the distribution of the noise component of

[26]Before November 2001, possible changes in the policy rate were discussed in each of the bi-weekly meetings of the ECB Governing Council. Since then, they are considered during the first of these two bi-weekly meetings only. Accordingly, for the first part of the sample (up to November 2001), the target-moves probabilities are divided by two so as to result in (approximately) the same probabilities of target moves over a month.

the EONIA spread as well as its (conditional) mean have to be conditioned on the excess liquidity regime. The latter is modelled by an additional Markovian regime process $z_{exc,t}$. This process can take two values $[1, 0]'$ (no excess-liquidity conditions) or $[0, 1]'$ (excess liquidity conditions). The matrix of transition probabilities associated with this process is time-homogenous and is denoted by Π_{exc}.[27] Formally, ξ_t is given by:

$$\xi_t = \left[\begin{array}{cc} (w_{norm} + \xi_{norm,t}) & (w_{exc} + \xi_{exc,t}) \end{array} \right] z_{exc,t}$$

where the w_i's are scalar parameters and the $\xi_{i,t}$'s follow taylor-made distributions, denoted by $\mathcal{L}$, that allows for non-zero skewness and fat tails. The definition and features of this distribution are detailed in Appendix 5.A. The support of this distribution is the compact $[-1, 1]$ (in annualised terms), which is consistent with the fact that the EONIA is bounded by the corridor set by the ECB's standing facilities.[28]

5.3.1.3. The dynamics of s_t

The variable s_t is aimed at contributing to persistent fluctuations of yields that can not be accounted for by the regime variables ($z_{r,t}$, $z_{m,t}$ and $z_{exc,t}$). Combined with ξ_t, the latter are expected to account for most of EONIA's fluctuations. Therefore, the variable s_t is expected to have a far lower impact on the overnight rate than on longer-term yields. To obtain such a feature (without resorting to an explosive dynamics for s_t), s_t is decomposed into two components denoted by $s_{1,t}$ and $s_{2,t}$,

[27] The columns of Π_{exc} sum to one. Note that, as $z_{m,t}$, the regime variable $z_{exc,t}$ is assumed to be observed by market participants but not by the econometrician ($z_{r,t}$ is observed by everybody).

[28] Note that the width of the corridor has changed over time (between 150 and 200 bp). However, taking into account such a variability would induce severe complexity in the framework.

that is, $s_t = s_{1,t} + s_{2,t}$. The dynamics of $[s_{1,t}, s_{2,t}]'$ is given by:

$$\begin{bmatrix} s_{1,t} \\ s_{2,t} \end{bmatrix} = \Phi \begin{bmatrix} s_{1,t-1} \\ s_{2,t-1} \end{bmatrix} + \Sigma\varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d. } \mathcal{N}(0, I) \tag{5.3}$$

$$\text{where} \qquad \Phi = \begin{bmatrix} \rho_1 & \beta \\ 0 & \rho_2 \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} 0 & 0 \\ 0 & \sigma \end{bmatrix}$$

The smaller β, the less variable $s_{1,t}$ is. In the limit, if β is equal to zero and if $s_{1,t}$ was zero at some point in the past, then $s_t = s_{2,t}$. I assume this is the case under the physical measure, but not under the risk-neutral one. Under the latter measure, if the ρ_i^*'s –the risk-neutral counterpart of the ρ_i's– are close to one, a shock on $s_{2,t}$ can have a very persistent impact on s_t. In addition, if β^* is large enough, these effects are multiplied by feeding through $s_{1,t}$. Therefore, s_t's innovations may have a far more long-lasting impact under the risk-neutral measure than under the physical measure. This implies that s_t may account for a far larger variance of long-term yields than of short-term yields.[29]

5.3.1.4. Definition of the single regime variable z_t

Defining a single regime variable will prove convenient for notational reasons in the remaining of this chapter. Accordingly, I introduce the selection vector z_t, defined as the Kronecker product of $\bar{z}_t$ and $z_{exc,t}$. Since $\bar{z}_t$ is itself the Kronecker product of $z_{r,t}$ and $z_{m,t}$, I have:

$$z_t = z_{r,t} \otimes z_{m,t} \otimes z_{exc,t}.$$

Hence, z_t is valued in the set of the columns of $I_{6(N+1)}$, each of the $6(N+1)$ different regimes being characterised by the policy rate (there are $N+1$ of them), a monetary-policy stance (there are three of them) and the situation of Eurosystem's liquidity (the situation being "normal" or "in surplus"). Recall that z_t is observed by market participants but not by the econometrician (who observes $z_{r,t}$ but not

[29] The choice of this dynamics builds on Dubecq and Gourieroux (2011).

$z_{m,t}$ and $z_{exc,t}$).

Given the assumption of independence between $\bar{r}_t$ and (w_t, ξ_t), the matrix of transition probabilities of z_t, denoted by Π_t, is equal to the Kronecker product of $\bar{\Pi}_t$ and Π_{exc}.

5.3.1.5. About the seasonality of the EONIA spread

This framework do not account for potential seasonality in the EONIA spread. While this could bias the pricing of short-term yields (with maturities of one week, say), this simplification has a limited impact for longer maturities. As noted by Balduzzi et al. (1998), only little seasonal variability of the overnight interest rate should be transmitted to longer-term rates, since seasonal variability is "averaged out" in the expectation process (especially if one considers maturities that are multiple of the reserve maintenance period, which is the case in that study).

5.3.2. Pricing

5.3.2.1. The stochastic discount factor (s.d.f.)

I assume that the risk-neutral dynamics of z_t and s_t are of the same kinds as their historical counterparts except that the Π_t's and Φ are respectively replaced by Π_t^*'s and Φ^* matrices, that depend on the same number of free parameters.[30] In this context, it can be shown that the stochastic discount factor (s.d.f.), or pricing kernel, is explicit.[31] Specifically, the s.d.f. $M_{t-1,t}$ between $t-1$ and t is given by:

$$M_{t-1,t} = \exp\left(-\Delta_m^{'} z_{t-1} - s_{t-1} - \xi_{t-1} - \frac{1}{2}\nu_{t-1}^{'}\nu_{t-1} + \nu_{t-1}^{'}\varepsilon_t + (z_{t-1}^{\prime}\delta_t)z_t\right)$$

where Δ_m is the concatenation of six vectors Δ, that is $\Delta_m = 1_{6\times1} \otimes \Delta$, which reflects the fact that there are three monetary regimes ($z_{m,t}$) and two Eurosystem-liquidity situations ($z_{exc,t}$), and where the risk sensitivities δ_t and ν_t –that price

[30] In particular, the p_{ES}^*, p_{SE}^*, p_{ST}^*, p_{TS}^*, p_{r25}^*, p_{c25}^*, p_{r50}^* and p_{c50}^*, that define the Π_t^*'s matrices, are based on functions $f(\bar{r}, \bullet)$. Still using the superscript $*$ to denote risk-neutral parameters, these probabilities depend on some vectors α_i^* (see end of Subsectionsec. 5.3.1.1).

[31] See Chapter 2.

respectively the risks associated to the regime shifts and to the Gaussian shocks ε_t– are defined by:

$$\begin{cases} \delta_{ij,t} & = \log\left(\Pi^*_{t,ij}/\Pi_{t,ij}\right) \\ \nu_t & = \Sigma^{-1}(\Phi^* - \Phi)\left[\begin{array}{cc} s_{1,t} & s_{2,t} \end{array}\right]' \end{cases} \quad \forall\ i,j,t. \tag{5.4}$$

5.3.2.2. Bond prices

It is well-known that the existence of a positive stochastic discount factor is equivalent to the absence of arbitrage opportunities (see Hansen and Richard, 1987) and that the price at t of a zero-coupon bond with residual maturity h, denoted by $P\left(t,h\right)$ is given by:

$$\begin{aligned} P\left(t,h\right) &= E_t\left(M_{t,t+1} \times \ldots \times M_{t+h-1,t+h}\right) \\ &= E_t^{\mathbb{Q}}\left(\exp\left[-r_t - \ldots - r_{t+h-1}\right]\right). \end{aligned} \tag{5.5}$$

Substituting equation (5.1) into equation (5.5) leads to:

$$P(t,h) = E_t^{\mathbb{Q}}\left(\exp\left[-\sum_{i=0}^{h-1}\left(\bar{r}_{t+i} + s_{t+i} + \xi_{t+i}\right)\right]\right) \tag{5.6}$$

Under the assumption that $\bar{r}_t$, s_t and (w_t, ξ_t) are independent processes, it comes:

$$\begin{aligned} P(t,h) &= \left(E_t^{\mathbb{Q}} e^{-\sum_{i=0}^{h-1}\bar{r}_{t+i}}\right)\left(E_t^{\mathbb{Q}} e^{-\sum_{i=0}^{h-1}\xi_{t+i}}\right)\left(E_t^{\mathbb{Q}} e^{-\sum_{i=0}^{h-1}s_{t+i}}\right) \\ &= P_1(t,h) \times P_2(t,h) \times P_3(t,h) \qquad \text{(say)}. \end{aligned}$$

The computations of $P_1(t,h)$, $P_2(t,h)$ and $P_3(t,h)$ are detailed in Appendix 5.C. It is important to stress that explicit formulas are available to compute each of these three terms, each of them turning out to be exponential affine in $(z_t', s_t)'$. Accordingly, the yields associated with zero-coupon bonds of maturity h, denoted by $y(t,h)$, are of the form:

$$y(t,h) = -\frac{1}{h}\left[G(t,h)z_t + A_h + B_h s_t\right] \tag{5.7}$$

Note that $G(t,h)$ is deterministic (i.e., the only stochastic components of the yields are z_t and s_t).

5.4. Estimation

5.4.1. The state-space form of the model

Kim and Orphanides (2012) have shown that the estimation of dynamic no-arbitrage term structure models with a flexible specification of the market price of risk is beset by a severe small-sample problem arising from the highly persistent nature of interest rates. They show that using survey-based forecasts of a short-term interest rate as an additional input to the estimation can overcome this problem. Following their approach, I enlarge the state-space model to make the estimated model consistent with 12-month-ahead forecasts of short-term rates provided by the *Consensus Forecasts*.[32]

Let me denote by R_t a vector of M observed yields of maturities $h_1, \ldots, h_M$, that is $R_t = [y(t,h_1), \ldots, y(t,h_M)]'$. Equation (5.7) shows that the these yields are affine in (z_t, s_t). It is straightforward to show that it is also the case for the 12-month-ahead forecasts included in the estimation. These forecasts are denoted by CF_t. Introducing some vectors of –supposedly i.i.d. normal– measurement errors denoted by ξ, we can write:

$$\begin{cases} R_t & = \Lambda_{z,R} z_t + \Lambda_{s,R} s_t + \xi_t^R \\ CF_t & = \Lambda_{z,C} z_t + \Lambda_{s,C} s_t + \xi_t^C \end{cases} \tag{5.8}$$

where the Λ matrices are functions of the model parameters (see Subsection 5.3.2).

[32] Other methodologies have been proposed to address this problem, see e.g. Jardet, Monfort and Pegoraro (2009).

The model admits a Markov-switching state-space representation whose measurement equations are given by (5.8). The dynamics of the state vectors s_t and z_t are respectively defined by equation (5.3) and by the matrices of transition probabilities Π_t.

5.4.2. Computation of the log-likelihood

Whereas the Markov chain $z_{r,t}$ is observed, the remaining state variables (s_t, $z_{m,t}$ and $z_{exc,t}$) are not. This latency is handled by using an estimation strategy building on the one mentioned in Section 2.6. The approach consists in applying inversion techniques *à la* Chen and Scott (1993) together with the Kitagawa-Hamilton filter to address the hidden nature of the switching regimes. The idea of the inversion technique is the following: assuming that a combination of the yields –gathered in the vector R_t– is observed without error, one can recover the latent variable s_t as a function of R_t and z_t. Further, one can compute the likelihood function based on the specified dynamics of the latent factor as well as on the distribution of the (remaining) pricing errors. Usually, one uses trivial perfectly-priced combinations of yields: specifically, if there are m latent factors with continuous support in the model, one assumes that m yields are priced without error. However, as noted for instance by Piazzesi (2010), the choice of this maturity is arbitrary. Therefore, I resort to an original alternative approach and choose s_t in order to minimise the average squared pricing errors across the different maturities.[33] In that case, the latent factor s_t (as a function of R_t and z_t) is simply obtained by using the OLS formula:[34]

$$s_t = (\Lambda'_{s,R}\Lambda_{s,R})^{-1}\Lambda'_{s,R}(R_t - \Lambda_{z,R}z_t). \tag{5.9}$$

Details of the exact computation of the likelihood are provided in Appendix 5.D.

[33] I am grateful to Simon Dubecq for providing me with this procedure. To our knowledge, though particularly efficient compared to classic inversion techniques, it has not been used in the existing literature.

[34] Note that this procedure results in one s_t conditionally to each of the different hidden regimes.

5.4.3. Estimation results

Table 5.2 reports the maximim-likelihood parameter estimates.[35] The computation of the estimates' standard errors are based on the outer product of the first derivative of the likelihood function. The standard deviation of the pricing error –i.e. the deviation between modelled and observed yields) is equal to eight basis points–, which is comparable to Piazzesi's (2005) fit of the U.S. yield curve.[36] Panels B, C and D of Figure 5.2 respectively show the fit of the 3-month, the 2-year and the 4-year yields. These plots also show the part of those yields that is explained by the regime variable z_t. It appears that most of the yields' fluctuations can be accounted for by z_t: more than 95% of the sample variances of yields with maturities lower than 2 years are captured by the term $G(t,h)z_t$ appearing in equation (5.7).[37]

Panel A of Figure 5.3 illustrates the ability of the model to reproduce survey-based forecasts of the target rate. Panel B and Panel C respectively present the estimated (smoothed) probabilities of being in the different monetary-policy regimes ($z_{m,t}$) and in the liquidity-surplus regime ($z_{exc,t}$) characterised by the disconnection of the EONIA from the main ECB policy rate.[38] According to the estimation, the first period of the liquidity-surplus regime is October 17, 2008, i.e. a few days after the announcement of the fixed-rate full-allotment procedure by the ECB. This regime was interrupted three times since then. The last interruption ended on August 2, 2011, two days before the ECB announced supplementary 6-month long-term refinancing operations (LTRO) in a context of renewed financial tensions.[39]

Searching for potential explanations of each change in regime is beyond the scope of this chapter. For the sake of illustration, though, let me highlight an episode

[35]In order to avoid that the factor s_t, thanks to its flexible Gaussian dynamics, explains too large a share of the yield fluctuations, I limit the size of its unconditional variance in the estimation. Specifically, I impose that the unconditional standard deviation of the s_t-related component of the one-year yield is lower than 10 basis points. Eventually, fifty one parameters remain to be estimated.

[36]Note however that the sample period used by Piazzesi (2005) is shorter (4 years against 13 years here) and the frequency is higher here (daily vs. weekly).

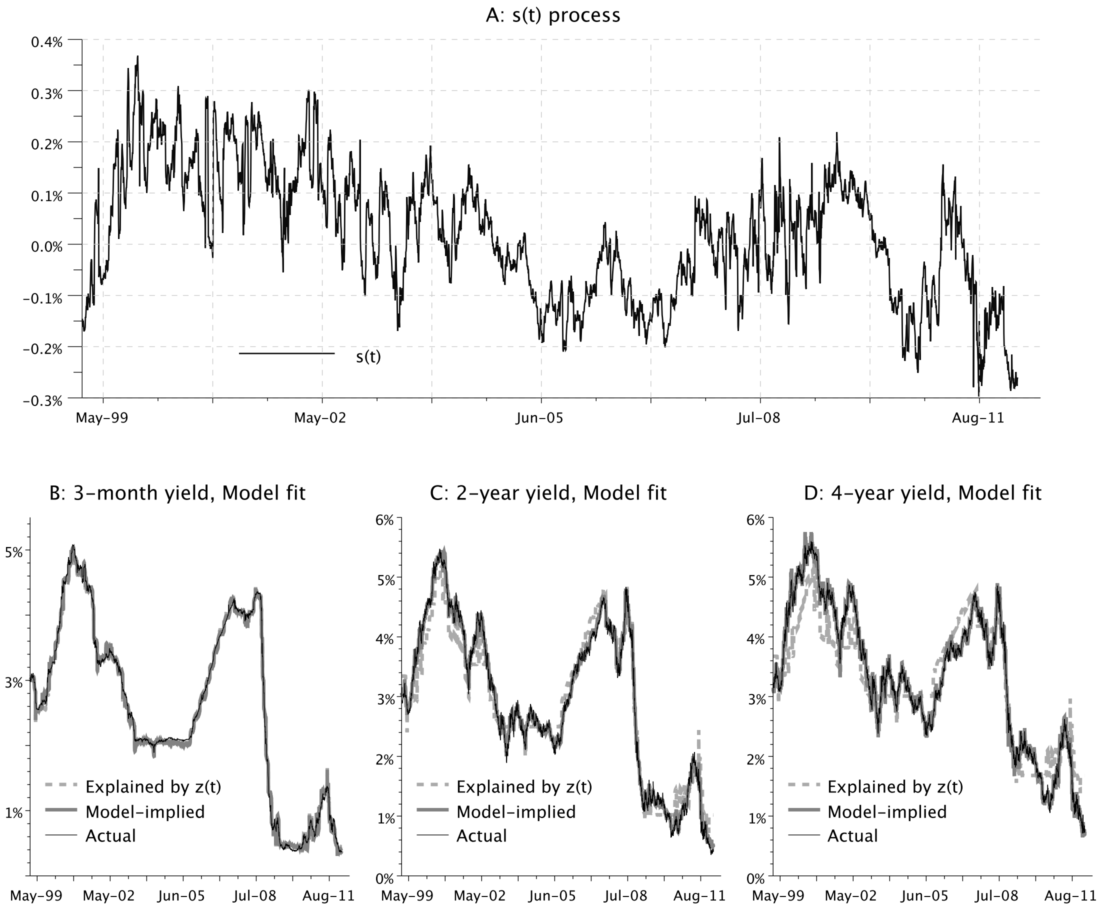
[37]85% of the variance of the 4-year yield is accounted for by $G(t,h)z_t$.

[38]Smoothing is based on Kim's (1993) algorithm.

[39]See the press release at http://ecb.int/press/pr/date/2011/html/pr110804_1.en.html.

Figure 5.2.: Estimated s_t process and model fit

Notes: Panel A displays the estimated s_t process (see equation (5.9)). Panels B, C and D compare model-implied yields with their data (actual) counterparts. The latter panels also display (grey dashed line) the part of the model-implied yields that is accounted for by the regime variable z_t (that is $-\frac{1}{h}\left[G(t,h)z_t + A_h\right]$ in equation 5.7).



where monetary-policy-regime shifts can be directly related to central bankers' announcements.[40] During the press conference following the ECB Governing Council that took place on 5 June 2008, J.-C. Trichet said: "we could decide to move our rates [by] a small amount in our next meeting". As is shown in Figure 5.4, this triggered a change in the monetary-policy regime, from status quo to tightening. A rate hike was then decided by the Governing Council in the next meeting, on 3 July 2008. The latter meeting was however followed by a more dovish press conference by Trichet, which induced a return to the status-quo regime in the next

[40]Naturally, central bankers' speeches are key events that are subject to indicate changes in monetary-policy regimes (see e.g. Rosa and Verga, 2008 in the euro-area case).

few days.

Figure 5.5 illustrates the influence of the monetary-policy regimes on the term structure of interest rates. For three dates, the modelled yields are compared with the observed ones. For each date, three additional yield curves are displayed, each of them corresponding to one of the three monetary-policy regimes. The modelled yield curve corresponds to one of these three curves, the attribution being based on the smoothed probabilities associated with the Markov chain $z_{m,t}$.[41] The two remaining curves are the answers to the question: what if the monetary-policy stance were different on that date? These plots show that monetary-policy regimes are key to shape the yield curve. Furthermore, this Figureillustrates the ability of the model to reproduce various shapes of the yield curve (steep, flat, humped, inverse-humped).

Figure 5.6 displays the 30-day-ahead probabilities of change in the monetary-policy regime as well as in the policy rate as functions of the latter.[42] Both historical and risk-neutral probabilities are reported. Interestingly, all three monetary-policy regimes are more persistent under the risk-neutral measure than under the physical one, which can be seen from the fact that the risk-neutral probabilities of exiting a given monetary-policy phase are lower than their historical counterparts. The implications of the differences between the two dynamics (historical vs. risk-neutral) are explored in Section 5.5. Overall, the probabilities of monetary-policy changes substantially depend on the target rate: This appears on the plots of Figure 5.6 and is also reflected by the statistical significance of the parameters a_2 (equation 5.2) that relate the probabilities of changes in the policy rate or in the monetary-policy regime to the level of the policy rate (see Table 5.2).

In this model, the volatility of the policy rate, and hence of the whole term structure of interest rates is not trivial. This is illustrated in Figure 5.7, that displays the standard deviation associated with the model-implied 3-month-ahead forecasts

[41] In the present case, the smoothing algorithm results in a clear-cut identification of the hidden monetary-policy regime: Most of the time, the smoothed probabilities are either 1 or 0.

[42] These probabilities are based on the matrix product $\bar{\Pi}_{MP}\bar{\Pi}_{NMP}^{29}$, where $\bar{\Pi}_{MP}$ and $\bar{\Pi}_{NMP}$ are the two possible matrices of transition probabilities for $\bar{z}_t$ $(= z_{r,t} \otimes z_{m,t})$: $\bar{\Pi}_{MP}$ (respectively $\bar{\Pi}_{NMP}$) is the matrix that corresponds to a monetary-policy-meeting day (resp. a day without meeting).

of the policy rate. The left-hand (right-hand) side plot regards the historical (risk-neutral) measure. The volatility of the policy rate turns out to strongly depend on the level of the rate itself as well as with the monetary-policy phase. Notably, these results echo those of Fontaine (2009) who finds –using U.S. data– that the uncertainty is lowest (highest) in tightening (loosening) cycles.

5.5. Term premia associated with target changes

The fact that the historical ($\mathbb{P}$) and the risk-neutral ($\mathbb{Q}$) dynamics of $\bar{r}_t$ differ gives rise to target-related risk premia.[43] The existence of such term premia is important in several respects. Let me mention two of them. First, if these risk premia are sizeable, OIS forward rates should not be interpreted as the market perceptions of future target rates, though this is the basis of a widespread market practice (see Subsection 5.2.2). Second, the existence of risk premia at the short-end of the yield curve implies that excess returns associated with a long position in money-market instruments may be partially predicTableor, alternatively said, that the expectation hypothesis does not hold at the short-end of the yield curve. While there is strong evidence against the expectation hypothesis for long-term yields, the evidence is weaker for short-term ones (see Longstaff, 2000).

In order to assess the size of target-related risk premia, policy-rate forecasts are computed under the two different measures. Conceptually, under the risk-neutral measure $\mathbb{Q}$, the forecasted paths of the policy rate are very close to the term structure of forward annualised rates (up to small Jensen-inequality correction terms). Here, emphasis is put on the risk premia associated with policy-rate changes, those associated with the s_t process having a straightforward and orthogonal influence.[44] Figure 5.8 displays the term structure of the policy-rate forecasts. Nine pairs of plots are reported. Each pair of plot corresponds to a given policy rate (1%, 2.5%

[43]These target-related premia contribute to the total term premia, that also include risk premia associated with the s_t component of the EONIA.

[44]The mean reversion of s_t being far larger under the historical measure than under the risk-neutral measure, the risk premia associated with this factor are almost $-B_h s_t/h$ (see Subsection 5.3.2 and equation (5.7) for details regarding the latter expression).

or 4%) and a given monetary-policy phase (tightening, status quo or easing). For each pair of charts, an upper plot presents the forecasts of the policy rate (w.r.t. the horizon forecast, on the x axis) and a lower one displays the associated risk premia, i.e. the spread between the $\mathbb{Q}$ and $\mathbb{P}$ forecasts. 90% confidence intervals for the risk premia are reported in the lower charts.[45] These premia are discussed in the following.

First, it appears that the risk premia can be substantial, even at the short end of the yield curve. In particular, under the tightening regime (see the first column of charts in Figure 5.8), the risk premia are higher than 50 basis points for maturities higher than 12 months. Furthermore, for policy rates that are higher than the sample average (of about 2.5%), the risk premia associated with tightening and easing monetary-policy regimes turn out to have opposite signs at the short- to medium-end of the yield curve (see the second and third rows of pairs of charts, corresponding respectively to a 2.5% and a 4% policy rates). This stems from the fact that the probabilities of remaining in the tightening and easing regimes are higher under the risk-neutral measure than under the historical one (as shown in Figure 5.6), implying higher life expectancies for these regimes and, thereby, a higher probability –compared with the physical measure $\mathbb{P}$– of having several policy-rate moves in the next months or quarters. This translates into positive (negative) risk premia at the short end of the yield curve when the tightening (easing) regime prevails. Therefore, the estimation results suggest that under the risk-neutral measure, the central bank is more "aggressive", in the sense that the yield curve reflects the behaviour of a central bank that tends to rise (respectively cut) the policy rate in a more rapid way than under the real-world measure when in the tightening (resp. easing) regime.[46] This supports the findings of Balduzzi et al. (1997), who observe that the target-change predictions that may be obtained from the short-end of the yield curve –under the expectation hypothesis– are correct in

[45]The confidence intervals are based on bootstrap techniques, the parameter estimates being drawn from their asymptotic distribution, see Figure 5.8's caption for more details.

[46]Regarding the rise in rate, this is not any more the case for high target rates, since the risk-neutral probability of a rise in the target is lower than its historical counterpart when the policy rate is above 4%. However, note that the unconditional probability of being in the targeting regime when the target rate is higher than 4% is low (see lowest Panel of Figure 5.6).

terms of sign, but tend to overestimate the size of realised target moves.

5.6. Estimated impact of forward policy guidance

In the present framework, the behaviour of the central bank is modelled through a set of probabilities: some of them correspond to probabilities of switching from one regime to the other (tightening, easing and status quo), some of them correspond to probabilities of rises or cuts in the target rate (the latter being conditional to the monetary-policy regime). If a change in these probabilities is made public, it may have an impact on the whole yield curve because the pricing of financial assets depend in part on the entire expected future path of short-term interest rates. This expectation channel of monetary policy transmission is at the heart of the rationale for *forward policy guidance* measures. In the current context in which the zero bound is binding for the overnight nominal interest rate, these measures are aimed to provide additional stimulus to the economy by pushing down medium- to long-term interest rates and, thereby, to support other asset prices (see e.g. Bernanke and Reinhart, 2004).

My framework makes it possible to assess the impacts of such announcements in a straightforward and consistent manner. In the following, I consider a basic form of forward guidance in which a central bank commits itself to maintaining its target rate constant for (at least) a deterministic period of time. The recent decision by the U.S. Federal Reserve to release federal funds rate forecasts and to extend its pledge to keep rates near zero at least through late 2014 is of that kind.[47] In the past, other central banks have signalled future policy intentions through official communication. For instance, the Bank of Canada announced on April 21, 2009 its conditional commitment to *"hold current policy rate [close to zero] until the end of the second quarter of 2010."*[48]

[47] See the Fed press release at http://www.federalreserve.gov/monetarypolicy/files/fomcprojtabl20120125.pdf

[48] There exist older cases of forward policy guidance: the Reserve Bank of New Zealand announced a path for its 3-month bank bill rate in 1997, it was followed by the Norges Bank and the Riskbank in 2005 and 2007, respectively.

As in nearly all of the existing literature, the following simulations abstract from issues that could arise under imperfect credibility of the central bank and focus on the case where the monetary authorities benefit from a perfect commitment technology.

Let me assume that the central bank has announced at date t that it will keep its policy rate unchanged for the next p periods. Then, equation (5.7) can be used to compute the yields of different maturities, up to a few parameters' adjustments: the matrix $G(t,h)$ has simply to be replaced by $\tilde{G}(t,h)$, the latter being computed in the same way as the former (i.e. using the formulas presented in Appendix 5.C) after having modified the matrices of Π^*_{t+i}, $i \leq p$ by setting the probabilities of policy-rate moves to zero.[49]

Figure 5.9 displays the results of four simulations. These simulations are based on two different target rates (1% and 3.5%) and two commitment durations (12 months and 24 months). Consistently with the fact that the policy rate is fixed for several months, the monetary-policy regime is set to the status-quo one (in the baseline as well as in the counterfactual case). The results suggest that such measures would have a statistically significant downward impact on the yield curve (90% confidence intervals of the downward effects are reported for each of the four cases presented in Figure 5.9). The impact appears to be far larger when the current target rate is low. For instance, a commitment to keep the target rate unchanged for the next 24 months leads to a decrease in the 5-year yield of about 25 bp when the target rate is of 1% and of about 5 bp when the target rate is of 3.5%.

5.7. Conclusion

While central banks' decisions are obvious drivers of the fluctuations of the term structure of interest rates, only few of the available yield-curve models feature a

[49]The fact that the probabilities of having policy-rate moves over the next p periods are equal to zero implies that the same is true under the risk-neutral measure because $\mathbb{P}$ and $\mathbb{Q}$ are equivalent measures. If this was not the case, it would imply the existence of infinitely large Sharpe ratios associated with policy-rate changes.

realistic modelling of the policy rate. This chapter proposes a framework that captures simultaneously the dynamics of the policy rate and the yields of longer maturities. Importantly, this model is consistent with the existence of the zero-lower-bound restriction, making it appealing in the current context of extremely low interest rates.

A key ingredient of the model is an extensive and innovative use of switching-regime features. Each regime is characterised by (a) a target level, (b) a monetary-policy regime (easing, tightening or status quo) and (c) the Eurosystem aggregate liquidity situation (normal or “in surplus”). The latter is introduced so as to accommodate the recent situation in which banks resort massively to the ECB deposit facility, which has an impact on the overnight interbank rate –the shortest-maturity yield considered in the model.

In order to illustrate the flexibility of the model, it is estimated using daily data covering the last thirteen years. Consistently with the choice of the interbank rate (EONIA) as the shortest yield, the overnight index swap (OIS) curve is fitted. Being impressively tractable, the model is estimated by standard maximum likelihood techniques. In order to alleviate potential small-sample bias and, hence, to properly estimate the physical dynamics of the processes, the estimation data set includes survey-based forecasts of short-term rates.

Various by-products are available, including the estimation of the market-perceived monetary-policy regime (at the daily frequency). In addition, the model is used in order to explore the size and influence of risk premia at the short end of the yield curve, the approach making it possible to exhibit monetary-policy-related risk premia. My analysis suggests that market yields reflect the behaviour of a central bank that would tend to rise (respectively cut) the target rate more rapidly than is physically observed when in a tightening (resp. easing) phase. This has implications regarding the common practice that consists in inverting the OIS yield curve to extract market-based short-term forecasts of the policy-rate path. Specifically, it means that such a practice –that assumes that the expectation hypothesis holds at the short-end of the yield curve– is valid in terms of sign of next target changes, but tend to overestimate their size.

Finally, the model is exploited to predict the potential effects of a forward policy guidance measure that consists of a commitment of the central bank to keep its rate unchanged for (at least) a given period of time. The simulations show that, in the current context of low short-term rates and with a commitment duration of 2 years, such an (unanticipated) announcement would be followed by a decrease of about 25 basis points of the 5-year rate.

5.A. The $\mathcal{L}$ distribution

The $\mathcal{L}$ distribution accommodates non-zero skewness and fat tails. A random variable follows the distribution $\mathcal{L}(p, \alpha_P, \beta_P, \alpha_N, \beta_N)$ if it is equal to $\mathbb{I}_{\{u=0\}} v_P - \mathbb{I}_{\{u=1\}} v_N$, where u is Bernoulli distributed with success probability p, and where v_P and v_N follow beta distributions with respective parameters (α_P, β_P) and (α_N, β_N) [“P” and “N” respectively stand for “positive” and “negative”].

The bond-pricing formula (Subsection 5.3.2 and Appendix 5.C.2) require the computation of $E(\exp[\xi])$, where $\xi \sim \mathcal{L}(p, \alpha_P, \beta_P, \alpha_N, \beta_N)$:

$$
\begin{aligned}
E(\exp[\xi]) &= pE(\exp \xi_P) + (1-p)E(\exp \xi_N) \\
&= p.f(\alpha_P, \beta_P) + (1-p).f(\alpha_N, \beta_N)
\end{aligned}
$$

where $f(\alpha, \beta)$ is given by:

$$
f(\alpha, \beta) = 1 + \sum_{k=0}^{\infty} \frac{1}{k!} \left(\prod_{i=0}^{k-1} \frac{\alpha + i}{\alpha + \beta + i} \right).
$$

5.B. Multi-horizon Laplace transform of a (homogenous) Markov-switching process

In the following, I consider a n-state Markov process z_t, valued in $\{e_1, \dots, e_n\}$, the set of columns of I_n, the identity matrix of dimension $n \times n$. I assume that the matrix of transition probabilities is deterministic and denoted by P_t (the columns sum to one). We have: $\mathbb{P}(z_{t+1} = e_i | z_t) = e_i' P_{t+1} z_t$.

Computation of $E_t(\exp(\alpha' z_{t+1}))$

$$
\begin{aligned}
E_t(\exp(\alpha' z_{t+1})) &= \sum_{i=1}^{n} \exp(\alpha_i) e_i' P_{t+1} z_t \\
&= \left(\sum_{i=1}^{M} \exp(\alpha_i) e_i' \right) P_{t+1} z_t \\
&= \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} D(\exp \alpha) P_{t+1} z_t
\end{aligned}
$$

where $\exp \alpha$ is the vector whose entries are the $\exp(\alpha_i)$'s and where $D(x)$ is a diagonal matrix whose diagonal entries are the elements of the vector x.

Computation of $E_t(\exp[\alpha_1' z_{t+1} + \alpha_2' z_{t+2}])$

The law of iterated expectations leads to:

$$
\begin{aligned}
E_t(\exp[\alpha_1' z_{t+1} + \alpha_2' z_{t+2}]) &= E_t(E_t[\exp[\alpha_1' z_{t+1} + \alpha_2' z_{t+2}] | z_{t+1}]) \\
&= E_t(\exp[\alpha_1' z_{t+1}] E_t[\exp[\alpha_2' z_{t+2}] | z_{t+1}])
\end{aligned}
$$

Then, using the previous case:

$$\begin{aligned}
& E_t(\exp[\alpha_1' z_{t+1} + \alpha_2' z_{t+2}]) \\
= \; & E_t\left(\exp[\alpha_1' z_{t+1}] \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} D(\exp \alpha_2) P_{t+2} z_{t+1}\right) \\
= \; & E_t\left(\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} D(\exp \alpha_2) P_{t+2} z_{t+1} \exp[\alpha_1' z_{t+1}]\right) \\
= \; & E_t\left(\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} D(\exp \alpha_2) P_{t+2} z_{t+1} z_{t+1}' D(\exp \alpha_1) \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}'\right).
\end{aligned}$$

Using the facts that $z_{t+1} z_{t+1}'$ commutes with any matrix and that

$$z_{t+1} z_{t+1}' \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}' = z_{t+1},$$

we get:

$$\begin{aligned}
E_t(\exp[\alpha_1' z_{t+1} + \alpha_2' z_{t+2}]) &= E_t\left(\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} D(\exp \alpha_2) P_{t+2} D(\exp \alpha_1) z_{t+1}\right) \\
&= \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} [D(\exp \alpha_2) P_{t+2}] [D(\exp \alpha_1) P_{t+1}] z_t.
\end{aligned}$$

Generalisation

It is straightforward to generalise and to show that:

$$\begin{aligned}
E_t(\exp[\alpha_1' z_{t+1} + \ldots + \alpha_h' z_{t+h}]) &= \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} [D(\exp \alpha_h) P_{t+h}] \times \ldots \\
& \quad \ldots \times [D(\exp \alpha_1) P_{t+1}] z_t.
\end{aligned}$$

5.C. Pricing formulas

In this appendix, I detail the computation of the three multiplicative components of $P(t,h)$ (the price at date t of a bond with residual maturity h), namely $P_1(t,h)$, $P_2(t,h)$ and $P_3(t,h)$. More precisely, this appendix propose a way to compute

$G_1(t,h)$, $G_2(t,h)$, A_h and B_h that are such that:

$$\begin{cases} P_1(t,h) & = G_1(t,h)z_t \\ P_2(t,h) & = G_2(t,h)z_t \\ P_3(t,h) & = \exp(A_h + B_h s_t) \end{cases}$$

These formulas leads to equation (5.7).[50]

5.C.1. Computation of $P_1(t,h)$

The targets $\bar{r}_t$ are the only stochastic variables involved in the computation of $P_{1,}(t,h)$. The previous Appendix shows that the expectation of an exponential-affine combination of a variable that follows a Markov-switching process is available in closed form. This leads to the following formula:

$$\begin{aligned} P_1(t,h) &= E_t^{\mathbb{Q}}\left(\exp\left[-\sum_{i=0}^{h-1}\bar{r}_{t+i}\right]\right) = G_1(t,h)z_t \qquad (5.10) \\ \text{with } G_1(t,h) &= \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}\left[\prod_{i=h-1}^{1} D(\exp[-\Delta_m])\Pi^*_{t+i}\right] D(\exp[-\Delta_m]) \end{aligned}$$

and where

- $D(x)$ is a diagonal matrix whose diagonal entries are those of the vector x.
- The matrices Π^*_t, which are of dimension $6(N+1)\times 6(N+1)$, contain the risk-neutral probabilities of switching from one regime –defined by a policy rate, a monetary-policy regime and a bank's liquidity situation– to another. As their physical-measure counterparts, these matrices can take two values, depending on whether a monetary-policy meeting is scheduled at date t or not.
- The product operator $\prod$ works in a backward direction: if X_1 and X_2 are some square matrices, $\prod_{i=2}^{1} X_i = X_2 X_1$

[50]In equation (5.7), the i^{th} entry of $G(t,h)$ is the logarithm of the i^{th} entry of $G_1(t,h)+G_2(t,h)$.

It is important to stress that this formula does not require the use of time-demanding recursive algorithms used by most alternative discrete-time affine term-structure models. Since policy meetings do not take place at a fully regular frequency, the matrices G_t should be computed for every date. As in Piazzesi (2005), I resort however to an intermediate approach where I consider only the exact number of days until the next decision meeting whereas subsequent meetings are assumed to be equally spread (every 30 days). The latter approximation, that leads to the computation of (only) 31 matrices G_i (instead of one per day), results in negligible pricing errors.

5.C.2. Computation of $P_2(t,h)$

The computation of $E^{\mathbb{Q}}\left(\exp\left[-\sum_{i=0}^{h-1}\xi_{t+i}\right]\right)$ is very close to this of $P_1(t,h)$. Indeed, using the law of iterated expectations, it comes:

$$P_2(t,h) = E_t^{\mathbb{Q}}\left(E^{\mathbb{Q}}\left[\exp\left(-\sum_{i=0}^{h-1}\xi_{t+i}\right)\right]\Bigg| z_{exc,t+1},\dots,z_{exc,t+h-1}\right). \tag{5.11}$$

Then remark that $w_{t+i}+\xi_{t+i}=\left[\begin{array}{cc}(w_{norm}+\xi_{norm,t+i}) & (w_{exc}+\xi_{exc,t+i})\end{array}\right]z_{exc,t+i}$ and recall that the ξ's follow $\mathcal{L}$ distributions based on beta distributions. Appendix 5.A gives the Laplace transform of a variable drawn from a $\mathcal{L}$ distribution, which provides us with $E(\exp(-\xi_{j,t}))$ for $j\in\{norm, exc\}$. This leads to:

$$E_t^{\mathbb{Q}}\left(\exp\left[-\sum_{i=0}^{h-1}\xi_{t+i}\right]\right) = E_t^{\mathbb{Q}}\left(\exp\left[\sum_{i=0}^{h-1}\left[\begin{array}{cc}\vartheta_{norm} & \vartheta_{exc}\end{array}\right]z_{exc,t}\right]\right)$$

where $\exp\vartheta_j = E(\exp(-w_j-\xi_{j,t}))$. Then, using Appendix 5.B again, one obtains:

$$\begin{aligned} P_2(t,h) &= G_2(t,h)z_t \qquad (5.12)\\ \text{with } G_2(t,h) &= \left[\begin{array}{cc}1 & 1\end{array}\right]\left\{D\left(\exp\left[\begin{array}{c}\vartheta_{norm}\\ \vartheta_{exc}\end{array}\right]\right)\Pi_{exc}\right\}^{h-1} D\left(\exp\left[\begin{array}{c}\vartheta_{norm}\\ \vartheta_{exc}\end{array}\right]\right)H_{exc}\end{aligned}$$

where H_{exc} is the selection matrix (whose entries are 0 or 1) that is such that $z_{exc,t} = H_{exc} z_t$.

5.C.3. Computation of $P_3(t,h)$

We have

$$\begin{array}{rcl} P_3(t,h) & = & E_t^{\mathbb{Q}} e^{-\sum_{i=0}^{h-1} s_{t+i}} = E_t^{\mathbb{Q}} e^{-\sum_{i=0}^{h-1} s_{1,t+i}+s_{2,t+i}} \\ \text{where} \left[\begin{array}{c} s_{1,t} \\ s_{2,t} \end{array}\right] & = & \Phi^* \left[\begin{array}{c} s_{1,t-1} \\ s_{2,t-1} \end{array}\right] + \Sigma\varepsilon_t^*, \quad \varepsilon_t^* \sim \text{i.i.d. } \mathcal{N}^{\mathbb{Q}}(0,I). \end{array}$$

In that Appendix, I describe an algorithm originally presented by Borgy et al. (2011). This algorithm results in the same matrices than the recursive formula given in the seminal paper by Ang and Piazzesi (2003). However, this latter approach turns out to be time-demanding for high-frequency (weekly or daily) processes. As shown by Borgy et al. (2011), the algorithm described below is substantially quicker when h is large.

Let me denote by X_t the vector $[s_{1,t}, s_{2,t}, s_{1,t-1}, s_{2,t-1}]'$. X_t follows:

$$X_t = \tilde{\mu}^* + \tilde{\Phi}^* X_{t-1} + \tilde{\Sigma}\varepsilon_t^*, \quad \varepsilon_t^* \sim \mathcal{N}^{\mathbb{Q}}(0,I),$$

where $\tilde{\mu}^*$, $\tilde{\Phi}^*$ and Σ^* are easily deduced from μ^*, Φ^* and Σ. In the following, I show how to compute the vectors A_h and C_h that are such that

$$P_3(t,h) = E_t^{\mathbb{Q}}(\exp(\delta' X_{t+1} + \ldots + \delta' X_{t+h})) = \exp\left(A_h + C_h X_t\right)$$

where $\delta = [0,0,1,1]'$. Denoting by $F_{t,t+h}$ the random variable $X_{t+1} + \ldots X_{t+h}$, we get:

$$P_3(t,h) = E_t^{\mathbb{Q}}(\exp(\delta' F_{t,t+h})$$

Note that$F_{t+h,h}$ is a Gaussian random variable. We have

$$
\begin{aligned}
F_{t+h,h} &= \left(hI + (h-1)\Phi^* + \ldots + \Phi^{*h-1}\right)\mu^* + \\
&\quad \left(\Phi^* + \Phi^{*2} \ldots + \Phi^{*h}\right) X_t + \\
&\quad \left(I + \ldots + \Phi^{*h-1}\right)\varepsilon^*_{t+1} + \left(I + \ldots + \Phi^{*h-2}\right)\varepsilon^*_{t+2} + \ldots + \varepsilon^*_{t+h}.
\end{aligned}
$$

Therefore $F_{t+h,h} \sim \mathcal{N}^{\mathbb{Q}}(\Lambda_{0,h} + \Lambda_h X_t, \Omega_h)$ with

$$
\begin{cases}
\Lambda_h &= \Phi^*\left(\Phi^{*h} - I\right)\left(\Phi^* - I\right)^{-1} \\
\Lambda_{0,h} &= \left[\chi_{1,h} - hI\right]\left(\Phi^* - I\right)^{-1}\mu^*
\end{cases}
$$

and with

$$
\begin{aligned}
\Omega_h &= \mathrm{Var}\Big(\left(I + \ldots + \Phi^{*h-1}\right)\varepsilon^*_{t+1} + \left(I + \ldots + \Phi^{*h-2}\right)\varepsilon^*_{t+2} + \ldots + \varepsilon^*_{t+h}\Big) \\
&= \left(\Phi^* - I\right)^{-1}\Big[\left(\Phi^{*h} - I\right)\Sigma\Sigma'\left(\Phi^{*h} - I\right)' + \ldots \\
&\quad + \left(\Phi^* - I\right)\Sigma\Sigma'\left(\Phi^* - I\right)'\Big]\left(\Phi^* - I\right)'^{-1} \\
&= \left(\Phi^* - I\right)^{-1}\left[(h-1)\Sigma\Sigma' - \Lambda_h\Sigma\Sigma' - \Sigma\Sigma'\Lambda_h' + \Pi(h, \Phi^*, \Sigma)\right]\left(\Phi^* - I\right)'^{-1}
\end{aligned}
$$

where $\Pi : (h, \Phi^*, \Sigma) \rightarrow \left(\Phi^{*h}\right)\Sigma\Sigma'\left(\Phi^{*h}\right)' + \ldots + \left(\Phi^*\right)\Sigma\Sigma'\left(\Phi^*\right)' + \Sigma\Sigma'$. Instead of using a brute-force approach (based on h loops) to compute $\Pi(h, \Phi^*, \Sigma)$, we exploit the fact that $\Pi(kp, \Phi^*, \Sigma) = \Pi(k, \Phi^{*p}, \Pi(p, \Phi^*, \Sigma) - \Sigma\Sigma') + \Sigma\Sigma'$. This can substantially reduce the computation time to compute. It suffices to apply the latter formula a few times, based on an integer factorization of h. Finally

$$
\begin{cases}
A_h &= \delta'\Lambda_{0,h} + \frac{1}{2}\delta'\Omega_h\delta \\
C_h &= \delta'\Lambda_h.
\end{cases}
$$

Finally, since $s_{1,t} \equiv 0$ under $\mathbb{P}$, denoting by B_h the second column of C_h, we get:

$$P_3(t,h) = \exp\left(A_h + B_h s_t\right). \tag{5.13}$$

5.D. Computation of the likelihood

$$\begin{cases} R_t - \Lambda_{z,R} z_t - \Lambda_{s,R}\Lambda_s(R_t - \Lambda_{z,R} z_t) & = \xi_t^R \\ CF_t - \Lambda_{z,C} z_t - \Lambda_{s,C}\Lambda_s(R_t - \Lambda_{z,R} z_t) & = +\xi_t^C \end{cases} \tag{5.14}$$

$$\begin{cases} (I - \Lambda_{s,R}\Lambda_s)R_t - \Lambda_{z,R} z_t + & \Lambda_{s,R}\Lambda_s\Lambda_{z,R} z_t = \xi_t^R \\ CF_t & = \Lambda_{z,C} z_t + \Lambda_{s,C} s_t + \xi_t^C \end{cases} \tag{5.15}$$

This Appendix complements Subsection 5.4.2. For a given vector of observed yields R_t and a regime vector z_t, the latent factor s_t is given by $s_t = \Lambda_s(R_t - \Lambda_{z,R} z_t)$ with $\Lambda_s = (\Lambda'_{s,R}\Lambda_{s,R})^{-1}\Lambda'_{s,R}$ (this is eq. 5.9). For a given regime vector z_t, there is the same information in R_t as in $\{s_t, \tilde{R}_t\}$, where $\tilde{R}_t$ is any subvector of R_t containing $M-1$ yields of distinct maturities. Without loss of generality, I assume that $\tilde{R}_t = [y(t,h_2), \dots, y(t,h_M)]'$. As a consequence, from the econometrician point of view, the model reads:

$$\begin{cases} \tilde{\Gamma}\xi_t^R & = \tilde{\Gamma}\left\{\left(I - \Lambda_R\right) R_t - \left(I - \Lambda_R\right)\Lambda_{z,R} z_t\right\} \\ \xi_t^C & = CF_t - \left(\Lambda_{z,C} - \Lambda_C\Lambda_{z,R}\right) z_t - \Lambda_C R_t \\ \varepsilon_t & = \frac{1}{\sigma}\Lambda_s\left[\left(R_t - \rho_2 R_{t-1}\right) - \Lambda_{z,R}\left(z_t - \rho_2 z_{t-1}\right)\right] \end{cases}$$

where $\Lambda_R = \Lambda_{s,R}\Lambda_s$, $\Lambda_C = \Lambda_{s,C}\Lambda_s$ and where $\tilde{\Gamma}$ is the $(M-1)\times M$ matrix that selects the last $M-1$ entries of an $M \times 1$ vector.

Assuming that the ε_t's, the ξ_t^R's and the ξ_t^{cf}'s are i.i.d. normal, the computation of the log-likelihood associated with the previous model is easily obtained by applying the Kitagawa-Hamilton filter. However, this likelihood is the one associated with the vector $\{s_t, \tilde{R}_t, CF_t\}_{t=1,\ldots,T}$, while we need to maximise the one associated with actually observed data $\{R_t, CF_t\}_{t=1,\ldots,T}$. The latter is obtained by multiplying the former by the determinant of the Jocobian resulting from this change in variables, that is $\left|\partial \left[s_t, \tilde{R}'_t\right] / \partial R_t\right| = \frac{1}{\Lambda'_{s,R}\Lambda_{s,R}} \Lambda_{s,R,1}$ where $\Lambda_{s,R,1}$ is the first entry of $\Lambda_{s,R}$.

Table 5.1.: Descriptive statistics of yields

Notes: The Tablereports summary statistics for selected yields. The data are monthly and cover the period from January 1999 to February 2012. Two auto-correlations are shown (the 1-day and the 1-year auto-correlations). The yields are continuously compounded and are in percentage annual terms. Panel B presents the covariances and the correlations of the yields. The EONIA spread is the yield differential between the (annualized) EONIA and the target rate. Panel C reports some results of a principal-component analysis carried out on the spreads between the yields and the target rate. More precisely, it shows the share of the variances of the different spreads that are explained by the first three principal components. Two samples are cnosidered: January 1999 to February 2012 and January 1999 to August 2008.

Panel A - Descriptive statistics 1999-2012								
	Target	EONIA	1-mth	3-mth	6-mth	12-mth	2-yr	4-yr
Mean	2,64	2,56	2,58	2,60	2,64	2,74	2,94	3,35
Median	2,50	2,57	2,60	2,64	2,67	2,77	2,91	3,37
Standard dev.	1,15	1,34	1,33	1,34	1,34	1,34	1,28	1,16
Skewness	0,14	0,14	0,14	0,14	0,13	0,13	0,17	0,22
Kurtosis	1,91	2,04	2,01	1,95	1,93	1,92	2,02	2,23
Auto-cor. (1 day)	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
Auto-cor. (1 year)	0,52	0,56	0,57	0,57	0,57	0,58	0,60	0,62
Panel B – *Correlations* \ **Covariances**								
Target	1,31	1,51	1,50	1,49	1,48	1,44	1,34	1,16
EONIA	*0,98*	1,80	1,77	1,77	1,75	1,71	1,60	1,39
1-mth OIS	*0,98*	*0,99*	1,77	1,77	1,76	1,73	1,62	1,41
3-mth OIS	*0,98*	*0,99*	*1,00*	1,78	1,79	1,76	1,65	1,43
6-mth OIS	*0,96*	*0,98*	*0,99*	*1,00*	1,80	1,78	1,68	1,46
12-mth OIS	*0,94*	*0,96*	*0,97*	*0,99*	*1,00*	1,79	1,70	1,48
2-yr OIS	*0,91*	*0,93*	*0,95*	*0,96*	*0,98*	*0,99*	1,65	1,46
4-yr OIS	*0,87*	*0,90*	*0,91*	*0,93*	*0,94*	*0,96*	*0,99*	1,34
Panel C – Principal component analysis of spreads vs. target								
1999-2012								
	Eonia	1-mth	3-mth	6-mth	12-mth	2-yr	4-yr	Total
1st PC	0,51	0,80	0,93	0,96	0,93	0,80	0,48	0,77
2d PC	0,86	0,96	0,97	0,96	0,96	0,99	0,91	0,94
3rd PC	0,99	0,96	0,99	1,00	0,98	0,99	0,99	0,98
1999-2008								
1st PC	0,03	0,58	0,89	0,93	0,95	0,89	0,63	0,70
2d PC	0,84	0,74	0,90	0,93	0,96	0,93	0,71	0,86
3rd PC	0,99	0,85	0,97	0,96	0,96	0,99	0,95	0,95

Figure 5.3.: Regimes' estimates

Notes: Panel A compares the model-implied forecasts with the survey-based ones (*Consensus Forecasts*). Panel B displays the (smoothed) probabilities of being in the different monetary-policy regimes. Panel C shows the smoothed probabilities of being in the excess-liquidity regime. The four vertical lines reported in Panel C indicate the following dates: 8 October 2008 (introduction of Fixed-Rate Full Allotment procedures), 3 December 2009 (announcement of the phasing out of the very long-term refinancing operations), 4 August 2011 (given the renewed financial-market tensions, announcement of supplementary 6-month LTRO), 8 December 2011 (3-year VLTRO).

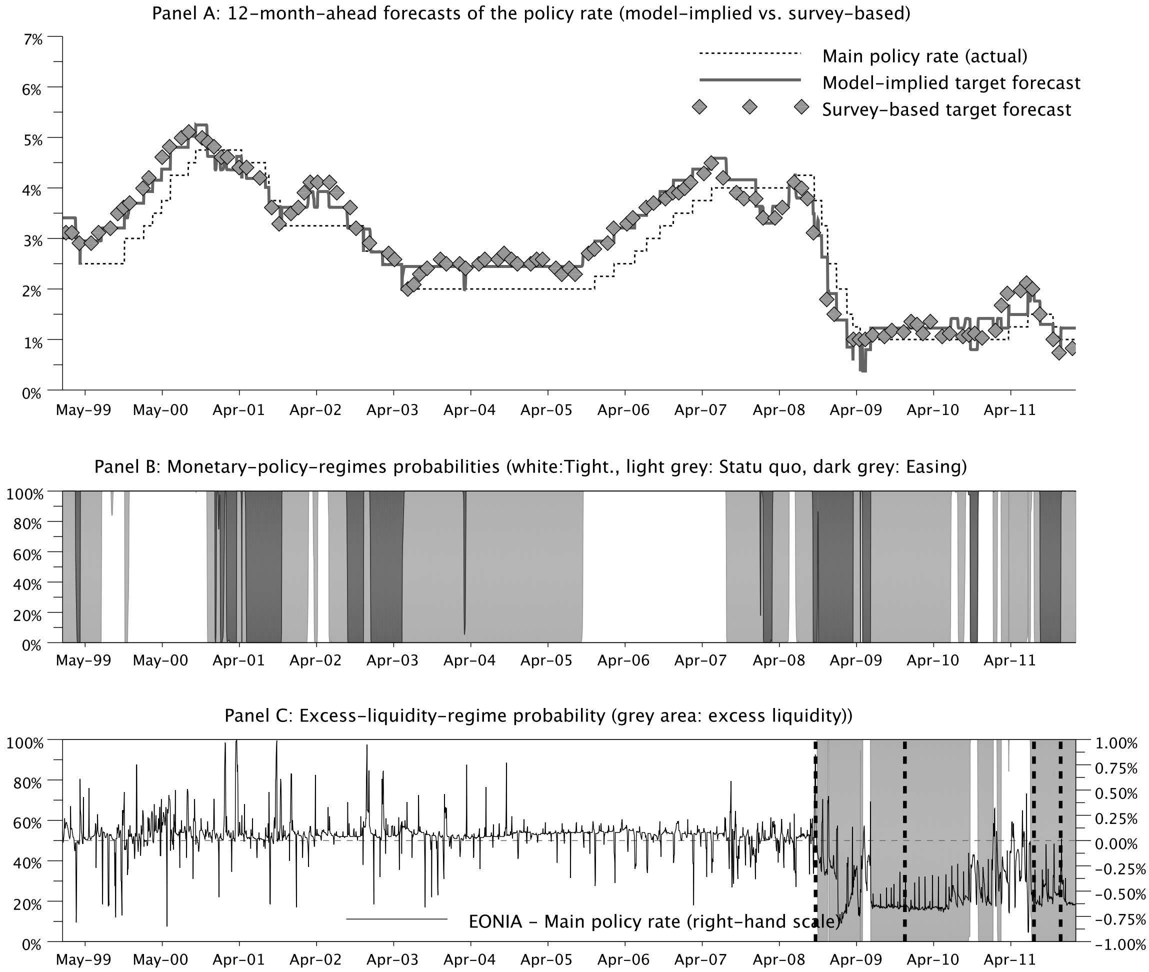


Figure 5.4.: Changes in monetary-policy regimes and central-bankers' announcements

Notes: This Figurerelates some specific (estimated) changes in the monetary-policy regimes to specific central bankers speeches (summer 2008). The grey shaded area corresponds to the (smoothed) probability of being in the tightening monetary-policy regime. The two vertical bars indicate the dates of two subsequent ECB governing councils (5 June 2008 and 3 July 2008). The quotation from J.-C. Trichet comes from the "Questions & Answers" part of the Press Conference held at the ECB on 5 June 2008. The extract from the *Financial Times* comes from www.ft.com (article entitled "ECB raises interest rates to 4.25%", by Ralph Atkins).

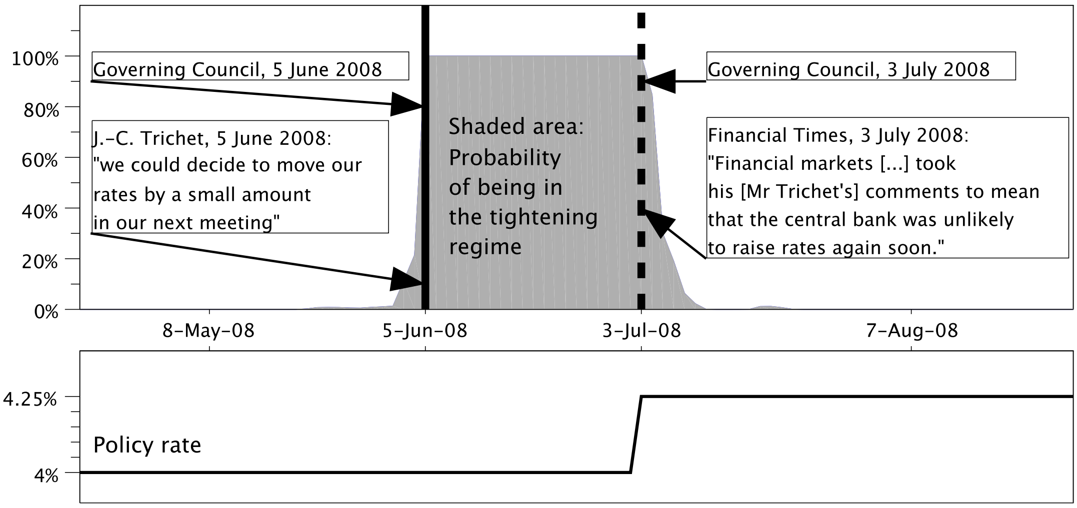


Figure 5.5.: Fitted yield curves and influence of monetary-policy regimes

Notes: These plots compare model-implied (diamonds) with observed (black circles) yield curves at different dates. In addition, each plot reports the (model-implied) yield curves that would have been obtained if other monetary-policy regimes had prevailed. The seven circles (and diamonds) correspond respectively to the following maturities: 1 day, 1, 3, 6 months, 1, 2 and 4 years.

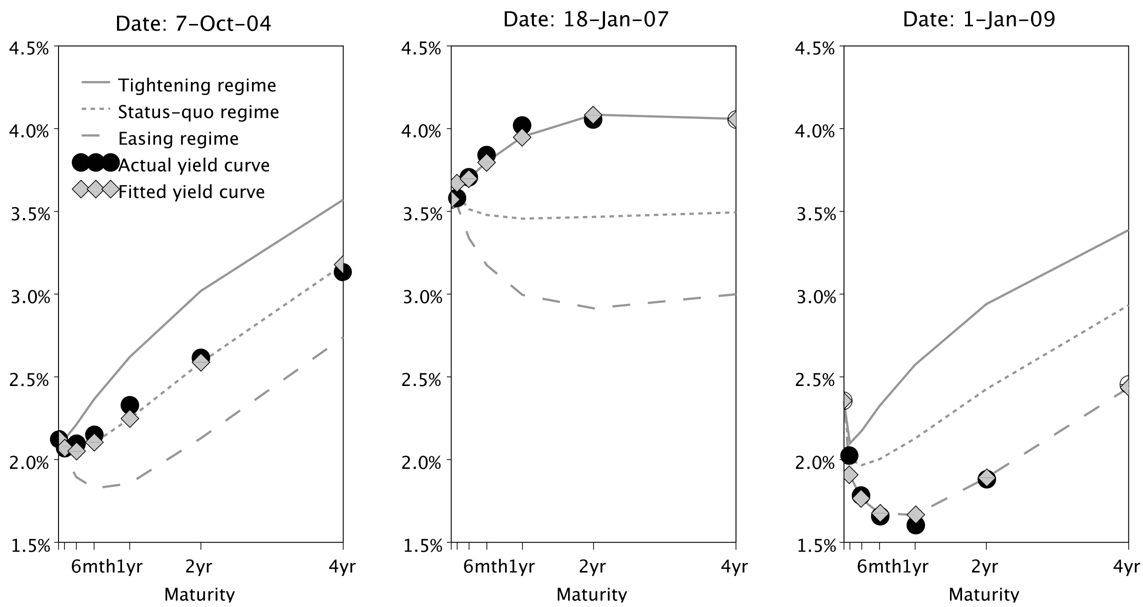


Figure 5.6.: Estimated probabilities of regime changes

Notes: These plots show the estimated probabilities of regime change over the next 30 days (period which includes only one monetary-policy meeting). As detailed in Subsection 5.3.1, these curves are based on some parametric forms of the target rate $\bar{r}_t$. Each plot displays the historical, or physical, probabilities as well as the risk-neutral ones. The upper four panels define the probabilities of monetary-policy-regime changes, the lower two show the target-change probabilities. Altogether, these probabilities define the matrices Π_t and Π_t^* describing respectively the dynamics of the Markov chain z_t (indicating the current target rate and the monetary-policy regime) under the physical and the risk-neutral measures.

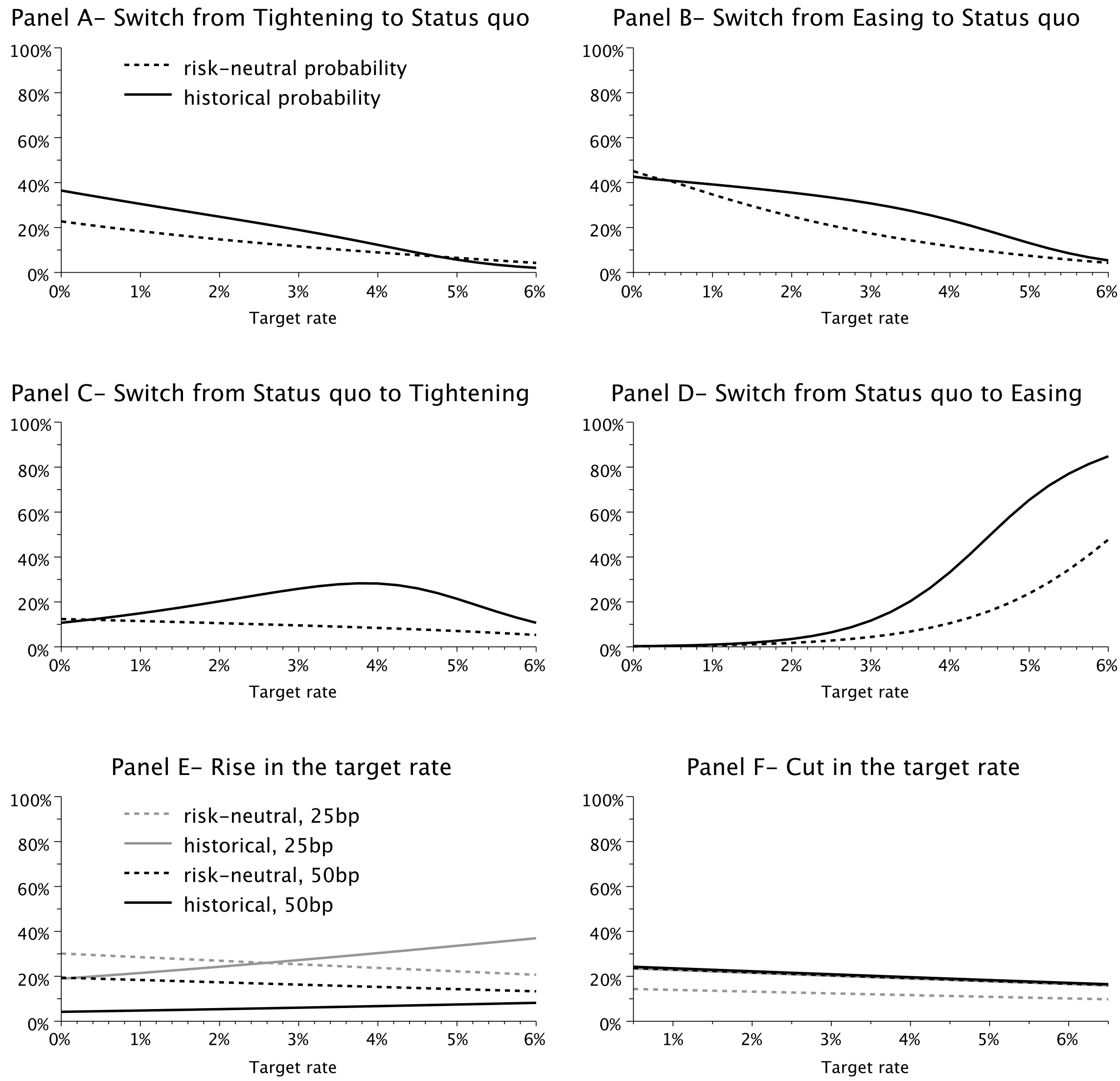


Figure 5.7.: Standard deviations associated with the 3-month-ahead forecasts of the policy rate

Notes: These plots present the standard deviations (reported in basis points) associated with the 3-month-ahead forecasts of the policy rate. These standard deviations depend on the target rate and on monetary-policy phase, which illustrates the heteroskedasticity of the policy rate in the model.

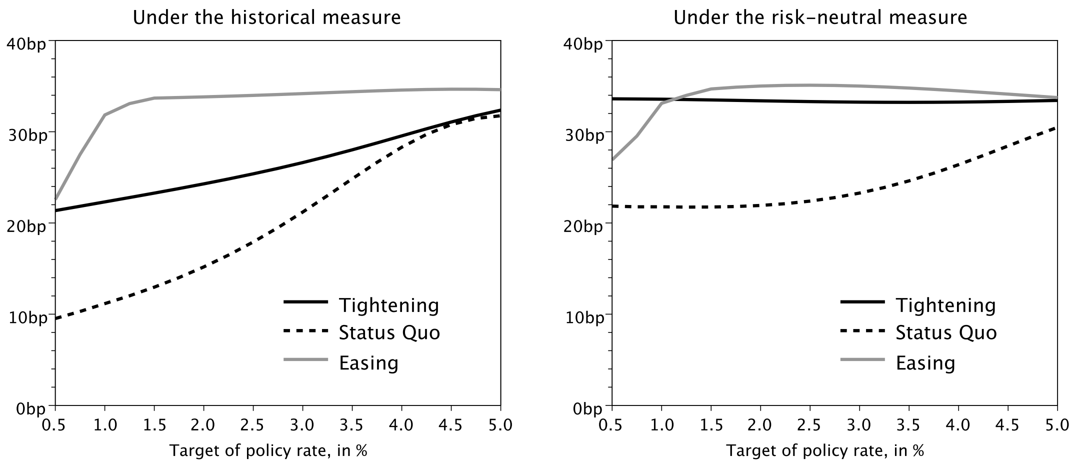


Table 5.2.: Parameter estimates

Notes: The Tablereports the estimates of the parameters defining the dynamics of the factor under historical and risk-neutral measures. The estimation data are daily and span the period from January 1999 to February 2012. Standard errors are reported in parenthesis. The sign "*" (after a number) denotes significance at the 5% level. The parameters a_i relate the probabilities of changes in the policy rate or in the monetary-policy regime to the level of the policy rate (see Subsectionsec. 5.3.1.1 and notably equation 5.2). The parameters that define the risk-neutral dynamics are indicated by *. The dynamics of the Markov chain $z_{exc,t}$ is defined by $p_{exc,exc}$ and $p_{norm,norm}$ which are, respectively, the probabilities of remaining in the excess-liquidity regime and the non-excess-liquidty regime. σ_{fcst} and σ_{pric} are, respectively, the standard deviations of the measurement errors ξ_t^C and of the pricing errors ξ_t^R (see equation 5.8).

	a_1^*	a_2^*	k^*	a_1	a_2	k
rise in	-0	0.11*	0.61	1.2*	-0.17*	0.82*
the target	(0.81)	(0.036)	(0.58)	(0.23)	(0.014)	(0.105)
cut in	0.41	0.098*	0.38	-0.00011	0.11*	0.49*
the target	(0.46)	(0.02)	(0.44)	(0.31)	(0.0104)	(0.14)
ES	3.8*	0.4*		3.9*	0.065*	
	(0.065)	(0.022)		(0.22)	(0.032)	
SE	9*	-0.87*		9*	-1.3*	
	(0.49)	(0.091)		(0.32)	(0.072)	
TS	4.7*	0.25*		4.1*	0.19*	
	(0.068)	(0.018)		(0.37)	(0.058)	
ST	5.3*	0.109		5.4*	-0.33*	
	(0.081)	(0.058)		(0.16)	(0.09)	

$\alpha_{P,norm}$	$\alpha_{N,norm}$	$\beta_{P,norm}$	$\beta_{N,norm}$	μ_{norm}	p_{norm}
0.76*	0.22*	8.3*	0.86*	0.04*	0.48*
(0.035)	(0.0056)	(0.13)	(0.064)	(0.00012)	(0.0105)
$\alpha_{P,exc}$	$\alpha_{N,exc}$	$\beta_{P,exc}$	$\beta_{N,exc}$	μ_{exc}	p_{exc}
0.75*	0.5*	4.6*	9*	-0.65*	0.55*
(0.05)	(0.038)	(0.25)	(1.6)	(0.0012)	(0.021)

ρ_1^*	ρ_2^*	β^*	ρ_2	σ
0.9999*	0.9999*	16*	0.94*	0.00007*
(0)	(0)	(6.1)	(0.0012)	(0.00003)

$p_{norm,norm}^*$	$p_{exc,exc}^*$	$p_{norm,norm}$	$p_{exc,exc}$	σ_{fcst}	σ_{pric}
0.9999*	0.9999*	0.99*	0.9999*	0.23*	0.085*
(0.0001)	(0.00014)	(0.00093)	(0)	(0.0029)	(0.00021)

Figure 5.8.: Risk-neutral vs. physical policy-rate forecasts, and associated risk premia

Notes: These plots show the term structures of the forecasts of the policy rate under the physical (grey circles) and the risk-neutral (black circles) measures. Up to the Jensen inequality, these curves can be considered as forward rates of the policy rate (as regards the risk-neutral measure). The three columns of plots correspond to the current (period 0) monetary-policy regime (either tightening, status quo or easing). The three rows of plots correspond to different (current) policy rates (1%, 2.5% and 3.5%). Each of the 9 plots presenting the policy-rate forecasts is completed by a plot (placed below the first plot) of the corresponding risk premia, i.e. the spread between the two forecast curves (in basis points). 90% confidence intervals are reported. These confidence intervals are based on bootstrap techniques: the asymptotic distribution of the parameter estimates is used to draw 1000 alternative sets of parameter estimates that, in turn, are used to compute 1000 sets of alternative risk premia; the dashed lines correspond to the 5 and 95 percentiles of the obtained risk-premia distributions.

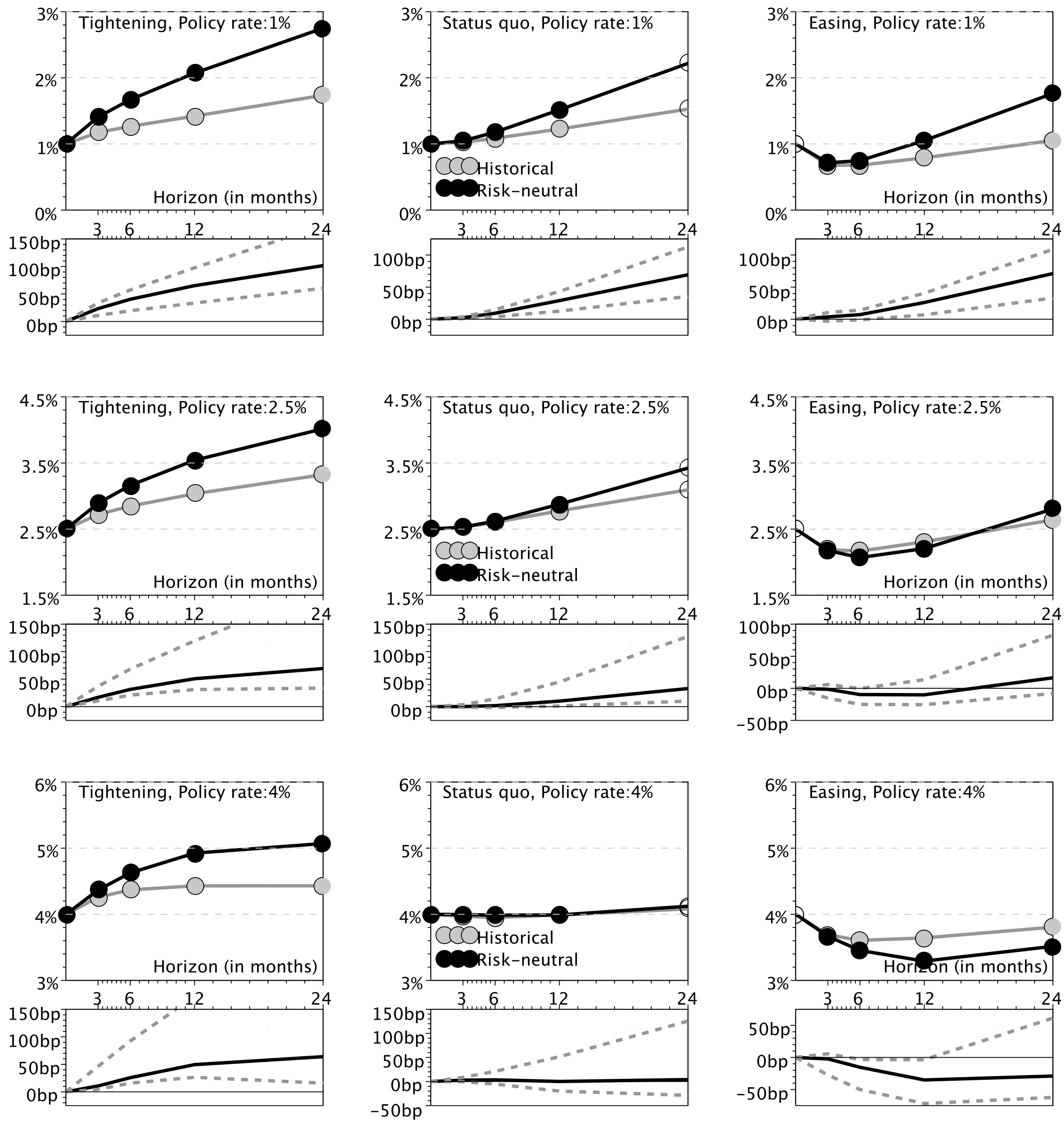
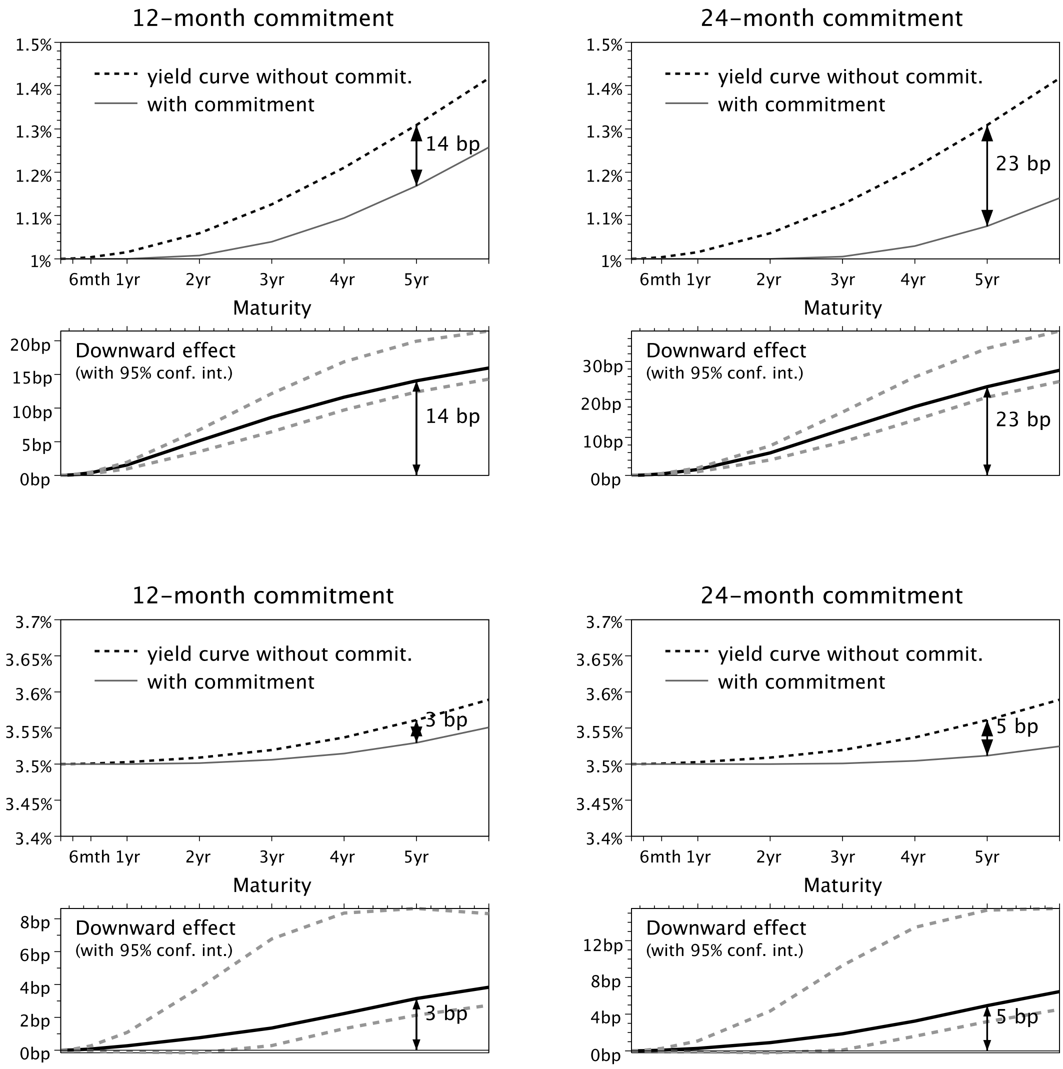


Figure 5.9.: Simulation of forward-guidance measures

Notes: These plots show the term-structure impact of a central bank's commitment to keep the target rate at its current level for 12 or 24 months. Two different policy rates are considered (1% and 3.5%). For each policy rate (1% or 3.5%) and each commitment durations (12 or 24 month), two plots are reported: the upper one displays yield curves with/without commitment of the central bank, the lower plot present the associated downward effect of the forward-guidance measure (that is the spread between the two curves plotted in the upper plot, in basis points). Note that here, I abstract from the effects of the excess-liquidity regime (w_t) and s_t is set at 0, its unconditional level (the rationale behind this is that in my framework, these two latter factors are independent from the policy rate, which is the only factor affected by the measure). In the baseline as well as in the counterfactual case, the monetary-policy regime is set to the status-quo regime. Regarding the downward effect of the measure (lower plots of each pair of charts), 90% confidence intervals are reported. These confidence intervals are based on bootstrap techniques: the asymptotic distribution of the parameter estimates is used to draw 1000 alternative sets of parameter estimates that, in turn, are used to compute 1000 sets of alternative effects of the measure; the dashed lines correspond to the 5 and 95 percentiles of the obtained downward-effects distributions.



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Nomenclature

AR	Auto-Regressive
ARMA	Auto-Regressive Mobile Average model
ATSM	Affine Term-Structure Model
Car	Compound auto-regressive
CDS	Credit Default Swap
MA	Mobile Average
MS-VAR	Markov Switching Vector Auto-Regressive
PD	Probability of default
RS	Regime Switching
VAR	Vector Auto-Regressive model
VaR	Value-at-Risk
ZLB	Zero lower bound