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Yu Cao

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Long-Distance Procurement Planning in Global Sourcing

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devant un jury composé de :

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Long-Distance Procurement Planning in Global Sourcing

Yu CAO

March 26, 2015

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Abstract

This research discusses procurement planning problems engaged in global sourcing. The main difficulty is caused by the geographically long distance between buyer and supplier, which results in long lead times when maritime transport is used. Customer demands of finished-products usually evolve during the shipment, thus extra costs will be produced due to unpredictable overstocks or stockouts. This thesis presents adaptive planning approaches to make adequate long-distance procurement plans in a cost-efficient manner.

Firstly, an adaptive procurement planning framework is presented. The framework deploys demand forecasting and optimal planning in a rolling horizon scheme. In each sub-horizon, demands are assumed to follow some known distribution patterns, while the distribution parameters will be estimated based on up-to-date demand forecasts and forecast accuracy. Then a portable processing module is presented to transform the sub-horizon planning problem into an equivalent standard lot-sizing problem with stochastic demands.

Secondly, optimal or near-optimal procurement planning methods are developed to minimize expected total costs including setup, inventory holding and stockout penalty in sub-horizons. Two extreme stockout assumptions are considered: backorder and lost sale (or outsourcing). The proposed methods can serve as benchmarks to evaluate other methods. Numerical tests have validated the high efficiency and effectiveness of both sub-horizon planning methods and the overall adaptive planning approaches.

Keywords: global sourcing, procurement planning, uncertainty, stochastic demand, rolling horizon, lot sizing, lead time, long distance, cost optimization

Résumé

Cette thèse porte sur l'optimisation de l'approvisionnement dans les zones géographiquement lointaines. Au moment de planifier des approvisionnements de matières premières ou de composants dans des pays lointains, la longue distance géographique entre l'acheteur et le fournisseur devient un enjeu essentiel à prendre en compte. Puisque le transport se fait souvent par la voie maritime, le délai d'approvisionnement est si long que les besoins peuvent évoluer pendant la longue période de livraison, ce qui peut engendrer un risque de rupture élevé. Cette thèse présente des approches adaptatives afin d'élaborer des plans d'approvisionnements lointains d'une manière rentable.

Tout d'abord, nous proposons un cadre d'adaptation de la planification des approvisionnements lointains. Il déploie des techniques de prévision de la demande et des méthodes d'optimisation d'approvisionnements à horizon glissant. En utilisant ce cadre, nous transformons le problème de la planification sur l'horizon globale en plusieurs problèmes standards de lotissement avec demandes stochastiques sur des sous-horizons. Ce cadre permet aussi d'évaluer la performance sur une longue période des méthodes utilisées.

Nous considérons ensuite la planification optimale d'approvisionnement sur les sous-horizons. Deux hypothèses de ruptures de stocks sont considérées: livraison tardive et vente perdue (ou sous-traitance). Nous développons des approches optimales ou quasi-optimales pour faire des plans d'approvisionnement tout en minimisant les coûts totaux prévus de commande, de stockage et de rupture sur les sous-horizons. Les méthodes proposées peuvent servir de repères pour évaluer d'autres méthodes. Pour chaque hypothèse, nous menons des expériences numériques pour évaluer les algorithmes développés et les approches adaptatives de planification globales. Les résultats expérimentaux montrent bien leur efficacité.

Mots clés: approvisionnement lointain, planification optimale, incertitude, demande stochastique, horizon glissant, lotissement, longue période de livraison, minimisation des coûts

Contents

Acknowledgement	i
Abstract	iii
Résumé	v
List of Figures	x
List of Tables	xii
1 Introduction	1
1.1 Background	2
1.2 Objective	4
1.3 Methodology	4
1.4 Contributions	5
1.5 Thesis Outline	6
2 Procurement Planning in Global Sourcing: State-of-the-Art	7
2.1 Overview	8
2.2 Planning in Supply Chain Management	8
2.2.1 Supply Chain Management (SCM)	8
2.2.2 Planning in SCM	10
2.2.3 The Cost Structure	11
2.3 Procurement Planning	14
2.3.1 Hierarchical Procurement Planning	14

2.3.2	State-of-the-Art on Procurement Planning	15
2.4	Global Sourcing	19
2.4.1	Context and Definition	19
2.4.2	Main Features	20
2.4.3	State-of-the-Art on Global Sourcing	21
2.5	Procurement Planning in Global sourcing: An Illustration	24
2.6	Conclusion	26
3	Adaptive Procurement Planning: A Rolling Horizon Forecasting Approach	27
3.1	Overview	28
3.2	Literature Review	28
3.2.1	Model Identification	28
3.2.2	Rolling Horizon Planning (State-of-the-Art)	30
3.3	Problem Description	34
3.3.1	Assumptions	34
3.3.2	Notation	37
3.4	Mathematical Formulation	39
3.5	An Adaptive Optimization Framework	39
3.5.1	The Rolling Horizon Scheme	40
3.5.2	Sub-Horizon Planning	41
3.5.3	Ex-Post-Facto Experimental Evaluation	48
3.6	Conclusion	49
4	Procurement Planning with Backorders	51
4.1	Overview	52
4.2	Literature Review	52
4.2.1	The Backorder Systems	52
4.2.2	State-of-the-Art on Backorder Models	53
4.3	Problem Description	62
4.3.1	Assumptions	62
4.3.2	Notation	63

4.4	Mathematical Formulation	64
4.4.1	The Cumulative Inventory Property (CIP) of Backorder Models	64
4.4.2	Mathematical Formulation	65
4.5	Optimization Algorithm	69
4.5.1	An Optimal Solution Procedure	71
4.5.2	A Simplified 2-Stage Heuristic Method (S2S Method)	73
4.5.3	The Normal Distribution Case	78
4.6	Numerical Examples	79
4.6.1	A 5-Period Planning Example and the Optimal Solution	79
4.6.2	Results of Simplified 2-Stage Heuristic Method (S2S Method)	81
4.6.3	A 200-Period Planning Example and Results of Proposed Adaptive Planning Approach	82
4.7	Conclusion	86
5	Procurement Planning with Lost Sales or Outsourcing	89
5.1	Overview	90
5.2	Literature Review	90
5.2.1	The Lost-Sale Systems	90
5.2.2	State-of-the-Art on Lost-Sale Models	92
5.3	Problem Description	96
5.3.1	Assumptions	96
5.3.2	Notation	98
5.4	Mathematical Formulation	100
5.4.1	The Backward Inventory Property (BIP) of Lost-Sale Models	100
5.4.2	Mathematical Formulation	103
5.5	A Forward Procedure	107
5.5.1	The Normal Distribution Case	111
5.5.2	The Programming Structure	113
5.6	Numerical Examples	114
5.6.1	A 5-Period Planning Example and Results of Proposed Heuristic Method	114

5.6.2	A 200-Period Planning Example and Results of Proposed Adaptive Planning Approach	118
5.7	Conclusion	118
6	Conclusion and Future Research	121
6.1	Conclusion	122
6.2	Future Research	123
	Bibliography	124

List of Figures

1.1	How to improve procurement plans	3
1.2	Research methodology	4
1.3	Dissertation structure	6
2.1	A supply chain example	9
2.2	Supply chain planning matrix (Rohde et al., 2000)	10
2.3	Procurement planning problem in global sourcing	25
3.1	Demand variation and uncertainty	32
3.2	Procurement planning problem engaged in global sourcing	36
3.3	Observation period and period of interest	38
3.4	The rolling horizon scheme	41
3.5	Sub-horizon planning module	41
3.6	Time series decomposition analysis	43
3.7	Box-Jenkins methodology on time series model building	44
4.1	Illustration of quantiles	57
4.2	A simple illustration of scenario tree	59
4.3	Illustration of fill rate and ready rate	61
4.4	The dynamic programming procedure	73
4.5	A randomly generated demand scenario	83
5.1	Backorder and lost-sale inventory systems	91
5.2	Inventory states of a lost-sale system	101
5.3	The Backward Inventory Property of lost-sale models	102

5.4	Recursive calculated inventory level distributions	105
5.5	Cost $C_{3,4}(x_3)$	117

List of Tables

2.1	Practical applications of procurement planning	16
2.2	Synthetic analysis of global sourcing	22
4.1	Meta-heuristic examples (literature)	58
4.2	Three service level definitions	60
4.3	Demand data set	80
4.4	Optimal accumulative order quantity Y_{st}	80
4.5	Cost \mathcal{G}_{st}^* associated with optimal order cycles	81
4.6	Network optimization, minimum expected costs Γ_{st}	81
4.7	An actual demand scenario (extracted portion)	84
4.8	Demand forecasts made at period -6	84
4.9	Net requirements	84
4.10	Updated demand forecasts (Observed at Period -5)	85
4.11	Procurement plan to be executed	85
4.12	Adaptive procurement planning	87
5.1	5-period planning example data set	114
5.2	Cumulative demand data set	115
5.3	Optimal procurement order quantity x_s^*	115
5.4	Cost $C_{s,t}^*(x_s^*)$ associated with optimal order cycles	116
5.5	Network optimization, minimum expected costs Γ_{st}	117

List of Notations

Decision Variable:

- x_i : The delivery quantity arriving at the beginning of period i .
- δ_i : A binary decision variable. $\delta_i = 1$ if $x_i > 0$, or 0 otherwise.
- X_i : The cumulative delivery quantity arriving from period 1 up to and including period i . $X_i = \sum_{t=1}^i x_t$.
- $\mathbf{x}_{i,j}$: The vector of order quantities arriving in the periods $i, i+1, \dots, j$. $\mathbf{x}_{i,j} = (x_i, x_{i+1}, \dots, x_j)^T$.
- $X_{i,j}$: The cumulative order quantity arriving in the periods $i, i+1, \dots, j$. $X_{i,j} = \sum_{t=i}^j x_t$.

Random Variable:

- d_i : The actual demand quantity of period i . The exact value of d_i cannot be acquired until the end of period i .
- D_i : The cumulative actual demand quantity counting from period 1 up to and including period i . $D_i = \sum_{t=1}^i d_t$.
- $D_{i,j}$: The cumulative actual demand quantity counting from the beginning of period i to the end of period j ($i \leq j$). $D_{i,j} = \sum_{t=i}^j d_t$.
- I_i^o : The inventory level at the beginning of period i .

- I_i^N : The calculated inventory level at the end of period i . $I_i^N = I_i^o + x_i - d_i$.
- I_i : The inventory holding level at the end of period i .
- $\hat{\square}_{i,k}$: The forecast on the random variable \square of period k , which is made at the end of period i ($i < k$). Here \square could be d , I^o and I . Note that i is the observation period number, and k indexes the period of interest in which the demand d_k occurs.

Cost Parameter:

- h_i : The inventory holding cost (per unit held) which is charged at the end of period i .
- b_i : The backorder cost (per unit unfulfilled) which is charged at the end of period i .
- c_i : The purchase cost (per unit purchased) of the order that is placed in period i .
- p_i : The lost-sale (or outsourcing) cost (per unit unfulfilled) in period i .
- K_i : The mandatory setup cost for each procurement order that is placed in period i .

Sub-Horizon Symbol:

- d_k^i : The actual net demand quantity of the k^{th} period in sub-horizon η_i .
- μ_k^i : The mean value of d_k^i .
- σ_k^i : The standard deviation of d_k^i .
- y_k^i : The interim order decision which is "supposed" to arrive at the beginning of the k^{th} period in sub-horizon η_i .

Probability Symbol:

- $U_i(\mathbf{x}_{1,i})$: The probability that a lost sale occurs at the end of period i ($i \geq 1$), knowing the supply vector $\mathbf{x}_{1,i}$. For initial state, $U_0 = 1$.
- $W_{i,j}^-(\mathbf{x}_{i,j})$: The probability that a lost sale occurs at the end of period j , while no lost sale occurs in the periods $i, i+1, \dots, j-1$, knowing that the previous lost sale occurs at the end of period $i-1$.
- $W_{i,j}^+(\mathbf{x}_{i,j})$: The probability that no lost sale occurs in the periods $i, i+1, \dots, j$, knowing that a lost sale occurs at the end of period $i-1$.
- $Q_{i,j}(\mathbf{x}_{i,j}, h)$: The probability density function of the calculated inventory level h at the end of period j , while no lost sale occurs in the periods $i, i+1, \dots, j-1$, knowing that a lost sale occurs at the end of period $i-1$.

Chapter 1

Introduction

1.1 Background

Global sourcing is becoming a common practice in industrial activities. It offers firms opportunities to enhance its competitiveness by procuring raw materials and/or components from supply sources all around the world, usually with lower prices and/or improved quality. Meanwhile, firms that participate in global supply chains are also exposed to increased complexity and uncertainty compared to those that operate domestically. Global sourcing thus gives rise to a wide range of issues and impacts different levels of decision making. To address such a problem, we focus on tactical and operational decision making.

Among many efficiencies brought by global sourcing, cost reduction is probably the most marked driver. However, this efficiency can be achieved only when global sourcing activities are well operated, since global sourcing has its key disadvantages including long lead times, unexpected incidents interrupting supply (weather interference, pirates, etc.) and so on. In this thesis, we attempt to answer the following questions: How to make procurement plans for global sourcing activities in a cost efficient manner? Are classical domestic procurement planning policies also efficient in global sourcing? Two issues should be specified: when to buy, and how much should buy. The main objective is to minimize expected long-run per-period total costs. In this thesis, we consider three types of procurement cost: setup, inventory holding and stockout penalty.

A major concern of global sourcing is the geographically long distance between buyer and supplier. While maritime transport is the most common transportation mode engaged in global sourcing, the procurement lead time is often so long that the customer requirements for finished-products usually evolve during the shipment. Then a large demand uncertainty should be considered in corresponding procurement planning models.

This thesis follows up on the research work of (Hubert, 2013). In order to improve the procurement plans, one should, on the one hand, improve the demand forecasting so that it is close to actual demand, and on the other hand, optimize the procurement planning based on forecasts. As it is widely admitted, demand forecast is never 100% accurate. This requires to know how accurate the obtained forecast is. In this way, we can anticipate the demand uncertainty when establishing the procurement plans. That is why forecast accuracy assessment is an essential part of the process. The overall process is shown in Figure 1.1. For step ①, Hubert proposed a methodology to select an appropriate forecasting method and to update it dynamically. For step ②, Hubert proposed a detailed model of forecast accuracy and its evolu-

tion in function of time horizon involved. This forecast accuracy model is applied to a real-life flow management problem in global sourcing. The method has been validated by its application on real-life data from a world-leading automobile corporation, which has demonstrated a clear superiority compared to existing ones in terms of both service level and inventory level.

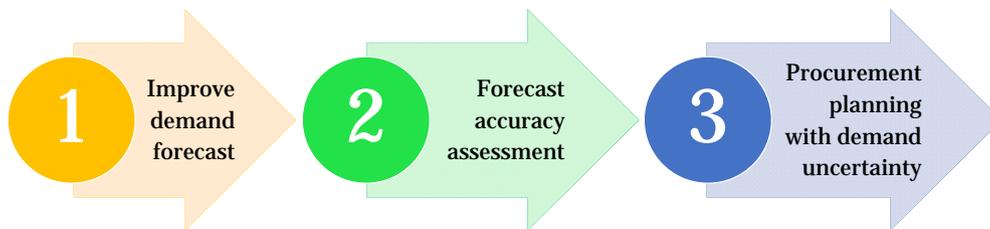


Figure 1.1: How to improve procurement plans

The thesis of Hubert contributed very much to the two first steps of the process. This current thesis focuses on step ③. Based on Hubert's study, we consider developing an adaptive planning approach, which can make cost-efficient procurement plans for global sourcing activities for the long run. We intend to deploy demand forecasting and procurement planning techniques in a rolling horizon procedure. The rolling horizon procedure can help in observing up-to-date demand forecasts and forecast accuracy, using which we can well estimate demand distributions over a certain horizon in the near future. Then the procurement planning problem over the above horizon can be modeled as a stochastic lot-sizing problem. Note that stochastic lot-sizing problems are NP-hard (Halman et al., 2012), therefore, we should make effort to develop efficient and effective methods to find optimal or near-optimal solutions.

In practical procurement planning engaged in global sourcing, stockout often takes place due to demand uncertainty. In this thesis, we discuss two extreme assumptions for stockouts: backorder and lost sale. For each assumption, we will develop respective solution procedure to determine adequate procurement plans that implements global sourcing in a cost efficient manner for the long run. In real-life situations, in case of stockout, part of demand is backordered while the remaining part is lost. Such situations can be addressed by combining the results of this thesis.

1.2 Objective

This work aims at supporting industrial multinationals in determining tactical and operational procurement plans for raw materials and/or components in distant lands.

The main objective is:

Research Objective:

develop optimal procurement planning policies for global sourcing activities, which minimize expected long-run per-period total costs including setup, inventory holding and stockout penalty.

1.3 Methodology

This research is carried out by the following five main steps: surveying existing literature, specifying the problem under study, formulating the problem, proposing optimization algorithms, and finally evaluating the proposed solution procedure, as shown in Figure 1.2.

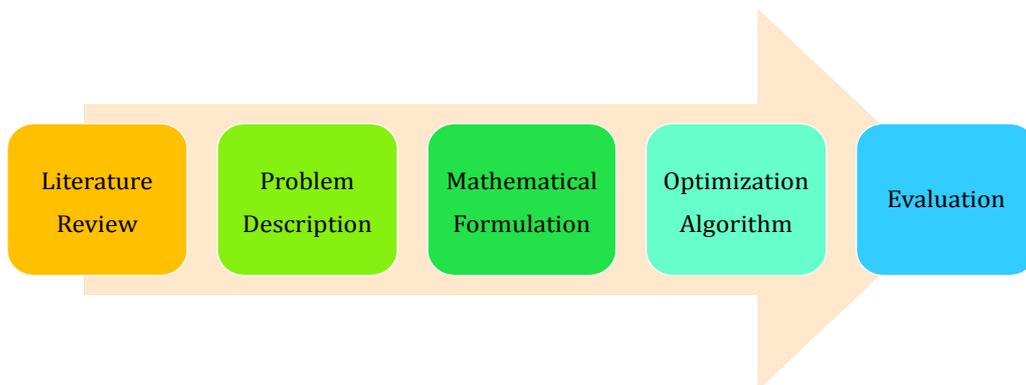


Figure 1.2: Research methodology

Literature review can help in specifying the problem under study. For a big research topic, a general survey of existing published works will provide valuable assistance in addressing central research issues and pointing out the gaps. In addition, the state-of-the-art on a specific research issue can also help in developing better solution procedures which improve certain performances (accuracy, efficiency, effectiveness, etc.) of existing procedures.

Based on literature review, the problem under study is then clearly defined. The following step is to formulate the problem mathematically. Good mathematical formulations can even boost development of good solution procedures.

The next step is to develop effective procedures which can find optimal or near-optimal solutions efficiently. This is the key step of the overall methodology.

Finally, performances of the proposed solution procedures will be evaluated by numerical tests.

1.4 Contributions

The contributions of this dissertation mainly include two parts.

Firstly, we present an adaptive optimization framework for procurement planning problems engaged in global sourcing. The framework deploys demand forecasting and short-term procurement planning in a rolling horizon scheme. By employing the proposed framework, the procurement planning problem engaged in global sourcing can be split into sub-horizon stochastic procurement planning problems, which reduces greatly the overall computational complexity. The proposed adaptive framework can also be used to evaluate the long-run performances of other methods.

Secondly, we develop detailed sub-horizon optimal or near-optimal planning methods for two extreme cases against stockouts: backorder and lost sale (or outsourcing). The proposed methods can help in determining optimal or near-optimal procurement plans that minimize expected total costs including set-up, inventory holding and stockout penalty in sub-horizons. When implemented with the aforementioned adaptive optimization framework, we can make adequate procurement plans for global sourcing activities.

To the best of our knowledge, existing literature mainly focuses on qualitative analysis of procurement planning and global sourcing policies. Due to economic globalization, global sourcing has become a key cost-control step for many companies. This research work provides an effective and efficient planning procedure to determine adequate operational procurement plans for global sourcing activities, which fills a gap of academic research in this field.

1.5 Thesis Outline

This dissertation is organized as shown in Figure 1.3.

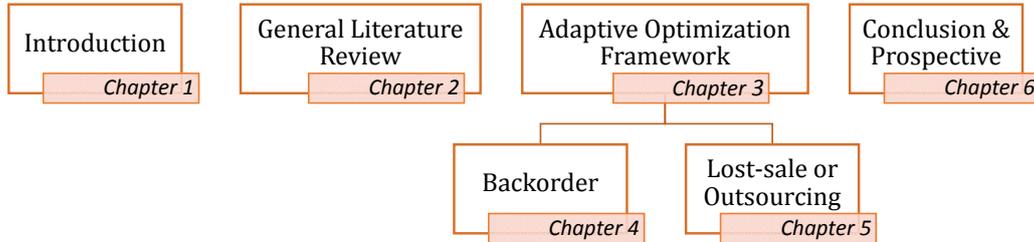


Figure 1.3: Dissertation structure

Chapter 2 provides a general literature review of research on procurement planning and global sourcing. The gaps are pointed out and distinguish this research with existing works.

Chapter 3 develops an adaptive optimization framework to deal with procurement planning problems engaged in global sourcing. The costs of set-up, inventory holding and stockout penalty are considered.

Chapter 4 presents optimal and near-optimal sub-horizon procurement planning approaches for the case that all the unfulfilled demands are backordered.

Chapter 5 presents an effective near-optimal sub-horizon procurement planning approach for the case that all the unfulfilled demands are lost or outsourced.

Chapter 6 draws conclusions of this research and discusses some potential research directions.

Chapter 2

Procurement Planning in Global Sourcing: State-of-the-Art

2.1 Overview

Supply chain management is one of the hottest economic topics in the world today. It spans the movement and storage of raw materials, work-in-process inventory, and finished products. Along a supply chain, numerous dependent or independent tasks need to be coordinated, while a large number of corresponding decisions of different importance should be made. The preparation work supporting aforementioned decision making activities is planning. Supply chain planning tasks can be divided into four stages along a product's life cycle: procurement, production, distribution and sales (Rohde et al., 2000). As the starting planning process of a supply chain, procurement planning is the first cost-control step of overall supply chain planning. Nowadays, supply chain management is inevitably linked to global sourcing due to economic globalization. Global sourcing is usually associated with a centralized procurement planning strategy, which seeks to minimize total procurement costs while achieving a required service level. In this context, procurement planning in global sourcing has become a key cost-control step and is worthy of serious consideration.

This chapter gives a general literature review of related issues about procurement planning in global sourcing. Section 2.2 presents a systematical structure of supply chain planning, and specifies the importance of procurement planning in overall supply chain planning process. Section 2.3 investigates the state-of-the-art on procurement planning. When procurement planning is engaged in global sourcing, the problem will be more complicated. Section 2.4 reviews the state-of-the-art on global sourcing, and specifies the main features of procurement planning problems engaged in global sourcing. Moreover, Section 2.5 gives an illustration of procurement planning problem engaged in global sourcing, which will serve for problem description and formulation in the following chapters. The conclusion is given in Section 2.6.

2.2 Planning in Supply Chain Management

2.2.1 Supply Chain Management (SCM)

A *supply chain* can be defined as the whole system involved in moving a product or service from (initial) supplier to (ultimate) customer. It consists of all the organizations, people, activities, information and resources related to fulfilling a(n) (ultimate) customer request, that is, not only suppliers and manufacturers, but also warehouses, transporters, retailers and customers

themselves. In addition, a supply chain also includes all corresponding functions such as new product development, marketing, finance, operations, distribution and customer service.

In short, supply chain activities transform natural resources (raw materials) and components into a finished product which is delivered to the (ultimate) customer (Nagurney, 2006).

Figure 2.1 illustrates a simplified supply chain network of a manufacturing firm.

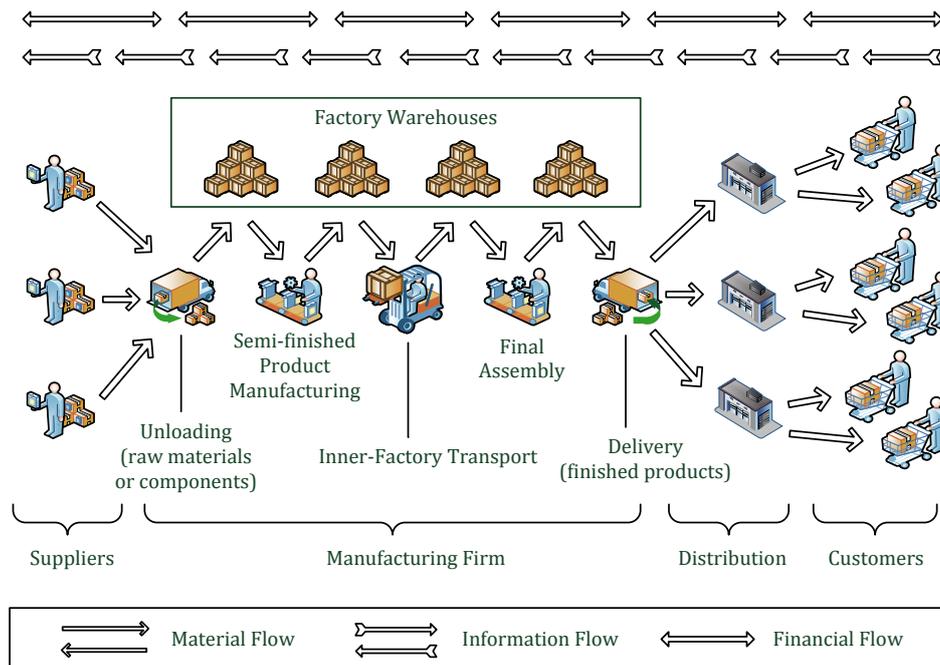


Figure 2.1: A supply chain example

Usually, *supply chain management* (SCM) not only focuses on divergent or convergent flows within a single chain, but also has to deal with a large number of flows within a complex network due to different customer orders to be handled in parallel. In order to ease complexity, a related organization should be established to concentrate on different portions of the overall supply chain serving to centralized strategic decision making.

In a word, SCM covers the tasks of integrating organizational units along a supply chain, and coordinating material, information and financial flows in order to fulfill (ultimate) customer demands with the aim of improving the competitiveness of a supply chain as a whole (Stadtler and Kilger, 2008).

2.2.2 Planning in SCM

Along a supply chain, numerous dependent or independent tasks need to be coordinated, while a large amount of corresponding decisions of different importance should be made. The preparation work supporting such decision making activities is *planning*. Planning can be subdivided into five phases (Meyr and Günther, 2009):

- problem recognition and analysis,
- objective definition,
- future development forecasting,
- feasible solution identification and evaluation,
- good (or best) solution selection.

The planning tasks along a supply chain can be classified according to different rules. (Rohde et al., 2000) proposed a *supply chain planning matrix* (SCP-Matrix) which distinguishes the planning tasks in two dimensions: planning-horizon dimension, and supply-chain-process dimension.

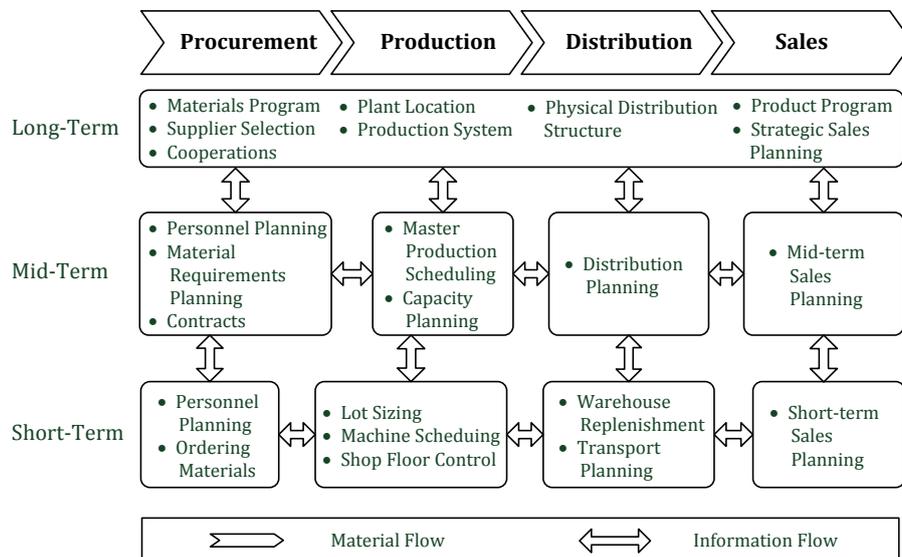


Figure 2.2: Supply chain planning matrix (Rohde et al., 2000)

The SCP-Matrix (see Figure 2.2) shows typical tasks occurred in most common supply chain types, following a hierarchical structure. Long-term planning decisions are also called strategic decisions, which usually concern the structural design of a supply chain and have long-term effects. Mid-term and short-term planning decisions are called operational ones, yet some

existing literature distinguishes the mid-term planning decisions as tactical ones. In particular, mid-term planning determines a "contour line" of regular operations, while short-term planning has to specify all the activities as detailed instructions for immediate execution and control (Anthony, 1965; Silver et al., 1998).

As shown in Figure 2.2, for each partner along a generalized supply chain network, the internal supply chain usually consists of four main supply chain processes with substantially different planning tasks:

- Procurement,
- Production,
- Distribution,
- Sales.

Procurement process includes all the activities that provide necessary resources for production. Production process is conducted under the aforementioned resource capacity limitation. Distribution process bridges the distance between production sites and retailers or downstream processing firms. And all the above planning decisions are driven by order/demand forecasts determined by the sales process.

2.2.3 The Cost Structure

A large part of planning problems aim at minimizing total costs incurred in the corresponding supply chain process, so as to maximize the profit gained. Costs are being charged along a product's life cycle (Stevenson and Hojati, 2007). When a replenishment activity is launched, a replenishment cost (including setup and purchasing) will be assessed. For the inventory management of stocks, the holding costs are assessed. When the inventory can satisfy a demand, a profit will be gained; otherwise, an additional penalty cost will be charged.

(1) Replenishment Cost

When an order is placed, the following two costs will be assessed (Freimer et al., 2006).

The first is setup cost. It is the initial cost related to replenishment activities, and can appear in different forms according to various industries, since the supplier could be manufacturer, packager, distributor, and so on. For example, if the supplier engages in manufacturing process, then a series of preparatory work (such as equipment installation and commissioning) should

be done as soon as an order arrives. This type of expense is just-for-once and fixed, which is independent of the production quantity. The set-up cost is usually signified by symbol K .

The second is variable cost. It increases proportionally with the replenishment quantity. The symbol c is often used to denote the production or purchase cost of product (per unit).

Let x denote the replenishment quantity, the total cost of a replenishment activity is calculated as:

$$R(x) = \begin{cases} 0, & x = 0; \\ K + cx, & x > 0. \end{cases}$$

In real commercial activities, the setup cost is rarely explicitly discussed, since people actually focus much more on the order quantity and the unit price of product. However, it is usual that the supplier is not willing to start a production or distribution activity for just a small order quantity, which reflects implicitly the effect of setup cost. Generally speaking, once a production or distribution activity is set up, people always want to produce or distribute more in order to achieve an economy of scale. Moreover, the setup cost is sometimes hard to quantify. Many suppliers prefer to set the unit price of a product according to the order quantity. In this situation, the setup cost is actually split onto each unit of product (Allahverdi and Soroush, 2008).

In this dissertation, the setup cost and the variable cost are assumed to be independent and can be calculated respectively. This assumption is widely used in inventory analysis.

(2) Holding Cost

A holding cost is defined as the expense incurred to maintain a stock of goods, which may be materials/components awaiting use in production or finished products to be traded and shipped.

The first is capital opportunity cost (Rajan et al., 2000). In general, the capital invested (to purchase goods, rent warehouses, etc.) will be held until the goods are sold. If the given capital was invested in other potential investment activities, a certain rate of return could be expected. The value of the best alternative forgone is defined as the capital opportunity cost. Let I denote the expected rate of return, and c denote the order cost of unit goods, then the capital opportunity cost can be given as Ic (per unit goods, per unit time).

Other holding costs comprise: the resource consumption during inventory holding, such as water, electricity and space; the deterioration in the quality of goods, such as the damage caused by handling, weather, etc.; the loss of goods through mishandling, poor record keeping or theft; the management of storage, such as the equipment and labor to operate the warehouses; the insurances, taxes and security, and so on (Raman and Kim, 2002). Let I' denote the related cost for per unit value of goods in per unit time, then the related cost is given as $I'c$ (per unit goods, per unit time).

For many real inventory systems, the capital opportunity cost is much higher than other holding costs. Moreover, I' can be mathematically converted into I . As a result, the holding cost for unit goods in unit time can be given as: $h = Ic$. Let y denote the average inventory level, then the holding cost per unit time is: $H = hy$.

(3) Penalty Cost

The principal function of stock is to satisfy the customers' requirements in time. When demand cannot be satisfied, a penalty cost will be charged.

In real life, two basic assumptions have been made to deal with unsatisfied demand: backorder and lost sale (Graves et al., 1993). For many consumable commodities, customers can often easily find the alternatives when their demand cannot be satisfied in time. In this case, the lost-sale assumption is usually used. However, when some customers insist on certain specific products or brands, they are more likely to wait until the replenishment. In this case, the backorder assumption is used.

For lost-sale cases, the expected profit that could have gained from the corresponding item if it was not in shortage is lost. While for backorder cases, although the customer's order has been retained, an additional cost concerning order management will be assessed. In both cases, there is an indirect cost for loss of goodwill. Let p denote the penalty cost due to each stockout, and z denote the unsatisfied requirement quantity per unit time, then the penalty cost in per unit time is $P = pz$.

The penalty cost structure is determined by the stockout assumption. However, the stockout assumption decides not only the penalty cost. It is no exaggeration that the change of stockout assumption will influence the overall cost structure of the inventory system, and will result in totally different solution procedures.

2.3 Procurement Planning

As the starting planning process of a supply chain, an adequate procurement planning may satisfy the raw material or component requirements for production process in a cost efficient manner, thus procurement planning is the first cost-control step of overall supply chain planning, and is worthy of serious consideration.

2.3.1 Hierarchical Procurement Planning

According to the length of planning horizon, procurement planning can be divided into long-term, mid-term and short-term.

Long-term procurement planning comprises materials program, supplier selection and co-operations (see Figure 2.2). In particular, materials program is often linked to product program (in long-term sales planning) for supplying raw materials and component parts. The product program conceives the architecture of whole product range, based on the prospects of existing product lines, future product developments and potential new sales regions. In materials program, price, quality and service (such as availability and reliability) are three key criteria used in material/component selection. Moreover, suppliers will be evaluated equivalently. Further reduction of procurement costs may be achieved by strategic cooperations with suppliers, such as simultaneous reduction of inventories and backorders using ideas like JIT (*just-in-time*) supply and VMI (*vendor managed inventory*), see (Magad and Amos, 1995; Fry et al., 2010).

In mid-term planning, the potential sale of a product in a specific region is forecasted at first. Then the master production scheduling (MPS) is applied to decide how to use the available production capacity of one or more facilities efficiently. The material requirement planning (MRP) should be implemented to calculate the order quantities of raw materials or component parts. Besides MRP, mid-term procurement planning also includes the personnel planning, which calculates the personnel capacity for procurement activities, by considering the availability of specific personnel groups according to their labor contracts. If there are not enough available employees to fulfill the work load, personnel planning should give the necessary amount of additional part-time employees.

Short-term procurement planning accomplishes mid-term planning in a detailed manner. For example, the short-term personnel planning determines the detailed schedule of each staff considering the employment agreement and labor costs. Besides, the short-term order planning

specifies the purchasing activity of every day in order to fulfill the following material/component requirements in a cost efficient manner.

2.3.2 State-of-the-Art on Procurement Planning

In general, existing research work on procurement planning can be divided in the following three stages.

General Scheme and Strategic Analysis

In the early work of (Farmer and Taylor, 1975), the idea of mastering resources in a systematic way instead of taking resources for granted to meet production requirements is proposed. The authors introduce a new concept of *resource management*, which is an efficient and effective deployment of resources to meet requirements. This paper mainly concerns benefits of corporate procurement planning in the long term.

(Spekman, 1981, 1985) specifies the importance of procurement planning to industry, and attempts to bridge the gap between resource availability and production decisions. The author presents the notion of *strategic procurement planning*, and proposes a general framework for better integration of procurement within a firm's strategic plans.

Based on analysis and comparison of different strategic procurement models, (Rink and Fox, 1999) study the relationship between procurement planning and other functions (such as production planning) across a product's life cycle, and further clarify the top-management stature of strategic procurement planning.

Real-Life Case Study

Due to its complexity and variety, many practical applications of procurement planning for real-life industrial cases have been studied, see Table 2.1. These papers concern different realistic problems such as the fluctuating prices of raw materials and finished products, long lead times, capacity limits, and so on, which are introduced as follows.

(Bonser and Wu, 2001; Sun et al., 2010, 2011) study the fuel procurement planning problem in electrical utilities. In particular, (Bonser and Wu, 2001) reduce the problem complexity by using practical management insights, and propose an efficient heuristic solution procedure in two stages: in the first stage, a priori plan is made according to long-term interests; in the sec-

Table 2.1: Practical applications of procurement planning

Industry	Paper	Main concerns
Electrical utility	(Bonser and Wu, 2001) (Sun et al., 2010, 2011)	fluctuating fuel price
Knockdown production system	(Lim and Kim, 2001)	plant location; capacity limits
Wafer fabrication facility	(Swaminathan, 2002)	long lead times
Biorefinery	(Yun et al., 2009a,b)	fluctuating prices of raw materials & finished products
Petrochemical industry	(Chu et al., 2012)	crude oil transportation; capacity limits
Oil refinery	(Oddsdottir et al., 2013)	nonstop crude oil supply; capacity limits

ond stage, a short-term detailed procurement plan is made by applying a linear programming technique. On the other side, (Sun et al., 2010, 2011) solve the procurement planning problem with a two-stage fuzzy programming technique.

(Lim and Kim, 2001) consider an integrated problem of plant location and component procurement planning in knockdown production systems. The knockdown production system is a global manufacturing system which is widely used in automobile and electronics industries. It usually consists of a home production site and several globally dispersed local production sites. In particular, the home production site produces both the final products and the knocked-down components required to assemble final products; while the local production site performs only the final assembly operations, using the knocked-down components supplied by home production site or other suppliers. In this paper, the authors formulate the procurement planning problem as a mixed integer program, and develop a two-stage solution procedure. In the solution procedure, the problem is decomposed into two tractable sub-problems (respectively a dynamic plant location problem and a multi-period capacity planning problem) which are solved sequentially.

(Swaminathan, 2002) addresses the issue of tool procurement planning at a semiconductor wafer fabrication facility which makes specific integrated circuits. The fabrication process involves building interconnected layers of metal lines and insulation material to produce a required circuitry on a wafer. The processing time is fairly long that the manufacturer has to forecast the customer demands in advance. In order to meet the forecasted demand, the manu-

facurer also needs to procure additional tools and equipment, which are quite costly and usually have long delivery lead times, for the tools are highly customized and made-to-order. In this paper, the author presents a stochastic planning model for tool procurement by developing a strategy that plans for a set of demand scenarios. The problem is formulated as a large-scale mixed integer program for each demand scenario. Efficient heuristics have been developed to solve the industrial size problem.

Besides, (Yun et al., 2009a,b) discuss the application of raw material procurement planning within a biorefinery. The profit of a biorefinery is highly affected by the prices of its raw materials and the margins of its products. The prices of raw materials change for a variety of reasons such as seasonal effects, states of harvest and policy changes; while the margins of products fluctuate due to changing market conditions. In this paper, the authors curtail the risks by purchasing diversified raw materials and their future contracts. An operational procurement planning model is proposed to decrease the profit variability of the refinery, by flexibly operating an integrated production process for multiple products. The optimal procurement plan for an integrated biorefinery process is determined based on different price scenarios and product requirements.

Moreover, (Chu et al., 2012; Oddsdottir et al., 2013) investigate crude oil supply problems. In particular, (Chu et al., 2012) formulate a crude oil transportation problem into a single-item lot sizing problem with limited production and inventory capacities, and then develop a strongly polynomial dynamic programming algorithm to solve it. On the other hand, (Oddsdottir et al., 2013) investigate the crude oil procurement problem in oil refineries. An oil refinery is designed to process a wide range of crude oil types into finished products, such as gasoline, kerosene and diesel oil. According to fluctuant market conditions, the refinery should have the flexibility to shift between crude oils and process various crude blends into required products. Oil refineries operate 24h a day, thus a shut down is extremely costly and results in major material loss, as well as extreme cleaning and security activities. The procurement plan should make sure that: (1) there is always enough supply of crude blends to avoid shutdowns; (2) the supply should not exceed the storage capacity of refinery; (3) the quality of the supply has to be feasible for the downstream processing units. In this paper, the authors introduce a mixed integer nonlinear programming model, and develop an efficient two-stage solution approach which can generate a feasible procurement plan within acceptable computational time.

To sum up, studies on real-life procurement planning problems are inevitably linked to physical characteristics of their systems, such as the fuel supply, refining and wafer fabrication

processes. In particular, a large part of above papers mainly focus on procurement planning for raw materials whose prices fluctuate widely, such as fuels and agricultural products, see (Bonser and Wu, 2001; Sun et al., 2010, 2011; Yun et al., 2009a,b; Oddsdottir et al., 2013). The authors usually solve the problem in two stages: firstly, make a priori plan according to long-term interest; secondly, based on spot market trends, develop adequate planning methods to determine cost-efficient short-term procurement plans. On the other hand, for manufacturing industries such as OEMs (*original equipment manufacturers*) and ODMs (*original design manufacturers*), logistics and inventory management have become the main concerns (Lim and Kim, 2001; Swaminathan, 2002). Moreover, due to economic globalization, distance and long lead time are worthy of particular attention when global sourcing is involved.

Academic Operational Analysis

Instead of studies on real-life procurement planning cases, researchers have also published a few academic papers about operational procurement planning for a specific class of model settings.

(Chauhan et al., 2009) investigate a stochastic-lead-time procurement planning model involved in a customized product assembly scenario. Due to high price fluctuations and technological advances, some key components for assembling finished customized products are not suitable for being stocked in advance. Usually, a delivery date of finished products is given, and a procurement plan to order needed components from different suppliers should be made to assure assembly of finished products. The authors have developed an effective approach to determine the ordering time for each component so as to minimize the expected inventory costs including holding and backorders.

(Geunes et al., 2009) consider a procurement planning problem with price-sensitive demands. An integrated model involving pricing and procurement planning under capacity limits and scale economy is used. The revenue functions are assumed to be concave. The authors seek to maximize total profit (revenue minus procurement and inventory holding costs), and develop polynomial-time solution methods for both the dynamically varying price case and the constant price case.

In addition, (Balakrishnan and Natarajan, 2013) discuss the coordinated procurement planning problem for large multi-division firms. Based on firm-wide purchasing power, coordinating procurement policies across multiple divisions to leverage volume discounts from suppliers can yield great cost savings. The authors propose an integrated optimization model that considers firm-wide volume discounts as well as divisional ordering and inventory costs. An effective

solution procedure is developed. Numerical results show that the proposed method generates near-optimal solutions within reasonable computational time.

To summarize, for domestic procurement planning, prices of raw materials or components are of special importance in total cost optimization. The above papers mainly discussed how to maximize profits when considering impacts of price changes on order decisions. Global sourcing can be an effective way to procure raw materials or components in lower prices, but will also bring significant difficulties, such as long lead time, and the underlying demand uncertainty.

Conclusion

To conclude, early studies on procurement planning mainly address general definitions, framework structuring and strategic analysis. From the new century, more research work has been done on real-life industrial cases. As the first cost-control step of supply chain planning, procurement planning has attracted increasing research interest in recent years. In existing literature, most attention was paid to price fluctuations that impact strongly decision making. However, due to economic globalization, the number of possible material/component suppliers has been increasing immensely. Therefore, global sourcing makes effects on material/component price reduction. But other difficulties will be produced, such as the long lead time and large demand uncertainty caused by the long period separating the date of actual consumption from the date where procurement decision is made, which are the main focuses of this thesis.

2.4 Global Sourcing

2.4.1 Context and Definition

Global sourcing is the practice of sourcing from the global market for goods or services under certain geopolitical constraints (Antràs and Helpman, 2004). It is a natural product of supply chain globalization, which is the outcome of today's ever-expanding global trade markets.

In order to concentrate on *core competencies*, which signify the unique ability that a firm inherits or develops and that cannot be easily imitated (Prahalad and Hamel, 1990), many firms decide to outsource other activities as far as possible. Consequently, the features (functions, price, quality, etc.) of a product/service sold to customers largely depend on multiple firms involved in its creation.

For the purpose of cost control, a lot of developing countries have been brought into consideration for their lower labor and production costs. This brings about new challenges for the integrated cooperation of legally separated and geographically far-away firms and the coordination of material, information and financial flows on a global scale.

Thanks to rapid improvements in transport and communication technologies, firms struggling to meet dynamic needs of growing markets and new consumer segments may learn that suppliers located in the whole planet are achievable. In such an environment, firms are usually faced with two possible sourcing choices:

- (1) choose a local supplier, usually with higher price and short lead time,
- (2) choose a distant supplier, usually with lower price and long lead time.

It is noticeable that, despite the net financial profits due to difference of material/component prices, the latter choice might lose its advantage rapidly owing to heavy import duty, high shipment cost or severe stockout penalty, and so on. Therefore, how to achieve global sourcing in a cost efficient manner has become an important issue.

In practice, global sourcing touches upon all multinational firms, while the suppliers usually cluster in China, India, the Middle East, Russia, and so on. When developing sourcing strategies on a global scale, companies have to consider not only the manufacturing cost and the fluctuation of exchange rates but also the availability of infrastructures such as transportation and energy (Kotabe and Murray, 2004). In addition, the complex nature of global sourcing introduces many constraints to its successful execution. In particular, logistics, inventory management and distance have become several major concerns for multinational firms engaged in global sourcing.

2.4.2 Main Features

Compared to local supply chain management and local sourcing, global supply chain management and global sourcing have their own constraints (Cohen and Huchzermeier, 1999; Meixell and Gargeya, 2005; Dornier et al., 2008).

The most common transportation mode engaged in global sourcing is maritime transport. However, the corresponding procurement activities can be profitable only when the order quantity exceeds a certain threshold to achieve the economy of scale, that is, the product should fill the containers as full as possible.

Consequently, many problems come along. On the one hand, an extra-large quantity of procurement implies a significant capital requirement and required storage space. On the other hand, the vagaries during the long-distance maritime transport (such as customs inspection, weather interference, labor strike, route security, etc.) will bring a great uncertainty in the lead time, leading to stockout risk. In addition, the cultural and geopolitical differences will also add to the difficulties. And most importantly, the finished-product demand (determined by sales process) is hard to forecast resulting from the long lead time of material/component delivery.

The previous features might all influence, directly or indirectly, the procurement order decisions. Consider the essential factors for planning, the first step should be specifying the required material/component quantity of each period. In this thesis, we focus on the large uncertainty of future finished-product demand due to long distance.

2.4.3 State-of-the-Art on Global Sourcing

Outsourcing labor-intensive products to localities with lower labor costs is an implicit success key for nearly all businesses. This phenomenon usually appeared domestically in early times, while after World War II, countries of the whole planet seek cooperation in multiple aspects and national borders are no longer formidable barriers for international trades (Hickman and Hickman, 1992).

Early study on global sourcing can be dated back to (Barnet and Müller, 1974; Moxon, 1974; Leroy, 1976), in which the authors advocate the prospective future of offshore production in less developed countries and the power of multinational corporation. Besides, (Levitt, 1983) reviews the globalization of markets by illustration of multinationals in Japan, Europe and the United States. (Kotabe and Omura, 1986) analyze the typology of global sourcing strategies. Further, (Kotabe and Omura, 1989; Kotabe and Murray, 1990; Kotabe, 1992) compare various sourcing patterns adopted by European and Japanese multinational manufacturing firms. Their work empirically confirms that competitive advantages can be gained through global sourcing in precondition of skillful execution.

Synthetic Analysis of Global Sourcing

Since the 1990s, studies on global sourcing have been blooming. The main research work has been focused on synthetic (organizational, structural) analysis of long-term global sourcing strategies, see Table 2.2.

Table 2.2: Synthetic analysis of global sourcing

Class	Paper	Description
Knowledge building	(Cavusgil et al., 1993)	motives, location/item selection, sourcing configuration
	(Murray et al., 1995)	effects of sourcing-related factors
	(Samli et al., 1998)	opportunistic sourcing v.s. strategic sourcing
	(Petersen et al., 2000)	critical factors impacting global sourcing effectiveness
Relationship	(Palaniswami and Lingaraj, 1994)	supplier partnering
	(Munson and Rosenblatt, 1997)	governmental regulation
	(Bozarth et al., 1998)	buyer-supplier relationship
Framework & Decision-aide model	(Meredith Smith, 1999)	an item-selection model
	(Arnold, 1999)	3 models for global sourcing organization: central purchasing model, coordination model, outsourcing model
	(Kim et al., 2002)	a global manufacturing system
	(Quintens et al., 2006)	a Global-Purchasing-Strategy (GPS) system

Firstly, the major issues engaged in global sourcing have been addressed.

(Cavusgil et al., 1993) specify the decision variables involved in global sourcing, and describe how they are interrelated with each other, by addressing three critical issues: motives of global sourcing activities, selection of locations and materials/components for offshore sourcing, and optimal design on sourcing configuration for a firm's global operations. (Murray et al., 1995) investigate the effects of three sourcing-related factors (bargaining power of supplier, proprietary technology, elements of transaction costs) on the relationship between global sourcing strategy and a product's market performance. (Samli et al., 1998) indicate the conceptual distinction between opportunistic and strategic sourcing. The authors appeal that, other than being considered as a casual gainful opportunity, global sourcing should be taken as a key element in a firm's overall corporate strategic plan. (Petersen et al., 2000) find that global sourcing structures/processes, global sourcing business capabilities, international language capabilities, and top management commitment to global sourcing are four critical factors that impact global sourcing strategy effectiveness.

Secondly, relationships (both external and internal) engaged in global sourcing have been discussed.

(Palaniswami and Lingaraj, 1994) introduce the idea of *supplier partnering*, in which the

buyer and supplier form a closer relationship where they mutually participate in advertising, marketing, branding, product development, and other business functions. The authors discuss the role of proposed global supplier partnering system in manufacturing firms which plan to revitalize their operations through strategies such as just-in-time, flexible manufacturing and total quality management. (Munson and Rosenblatt, 1997) investigate local content purchasing rules which force firms to purchase a certain amount of components from suppliers located in the country where they wish to operate. Through case studies, (Bozarth et al., 1998) indicate that managing the buyer-supplier relationship is important for global manufacturing firms. The authors have also analyzed the interrelationships between international sourcing decisions, sourcing strategies, and supplier performance.

Finally, some decision-aide frameworks and models have been proposed.

(Meredith Smith, 1999) proposes a decision-matrix based model which can provide an initial scheme for selecting items that may benefit from global sourcing. (Arnold, 1999) develops three global sourcing organization models: central purchasing model, coordination model, and outsourcing model. These models can be used to give suggestions for different types of firms on how to organize global sourcing. (Kim et al., 2002) construct a global manufacturing system for a shoe-making firm. The system helps in allocating production tasks between remote places (headquarters and manufacturing plants), and generating corresponding procurement plans. (Quintens et al., 2006) develop a global purchasing strategy (GPS) system to execute organizational alignment of purchasing functions inside the multinational firms engaged in global sourcing.

Planning in Global Sourcing

In recent years, there are several published works studying the planning activities engaged in global sourcing.

Above all, (Hartmann et al., 2008) explain the organizational implications of different control mechanisms in global sourcing, and indicate that planning is a central mechanism for coordinating activities of organizational units. Besides, (Golini and Kalchschmidt, 2011) investigate the impact of global sourcing on inventory levels. The authors show that the negative impact brought by complex nature of global sourcing on inventory performance can be partially reduced via proper supply chain planning practices.

Moreover, (Holweg et al., 2011; Hu and Motwani, 2013) contribute on detailed model

building and algorithm development. In particular, (Holweg et al., 2011) propose an analytical total cost model for global sourcing. The model has been validated through case studies. And (Hu and Motwani, 2013) develop a methodology to minimize downside risks in global sourcing under price-sensitive stochastic demand, exchange rate uncertainties, and supplier capacity constraints.

There are also some researchers who are vigilant to the disadvantages of global sourcing. (Schaibly, 2004; Kotabe and Murray, 2004; Matthyssens et al., 2006; Steinle and Schiele, 2008; Kotabe and Mudambi, 2009) explore the potential limitations and negative consequences of sourcing strategies on a global scale, and propose remedial approaches.

Conclusion

To conclude, the majority of published works have been focused on synthetic analysis of global sourcing, which can be generally divided into three groups: knowledge building, (external/internal) relationship, and structural framework/model. However, to the best of our knowledge, there is little literature discussing how to make procurement plans in a global sourcing context.

Due to economic globalization, optimal planning for global sourcing activities has become a crucial step in a firm's cost-control strategy. For real-life applications, tactical and operational planning of procurement activities on a global scale is fairly important. Therefore, it will be interesting and meaningful to develop an optimal long-distance procurement planning approach, which may achieve global sourcing in a cost efficient way. The proposal is based on the gap of research work about tactical and operational procurement planning engaged in global sourcing.

For convenience, we use "procurement planning" to signify "tactical and operational procurement planning" in the remainder of the dissertation.

2.5 Procurement Planning in Global sourcing: An Illustration

For nearly every multinational company engaged in global sourcing, a central procurement organization is established to seek an optimal procurement plan which minimizes total costs.

When cost structure is specified, we use the following example to figure the procurement

planning problem engaged in global sourcing. See Figure 2.3 for illustration.

Example:

Firm A (in France) plans to procure component X from Supplier B (in China) in order to produce Product Y. The transport from B to A takes about 1.5-2 months. Once a procurement order is placed, a mandatory set-up cost occurs. When there is a stockout of component X in A’s production process, the production for final-product Y will be interrupted, therefore the customer demand for Y cannot be fully satisfied. Consequently, a penalty cost is charged to firm A for unfulfilled demand.

The objective is to draw up an optimal procurement planning, which minimizes the expected long-run per-period total procurement costs including set-up, inventory holding and stockout penalty, by specifying two issues: when a procurement order should be placed, and how many component X should be ordered.

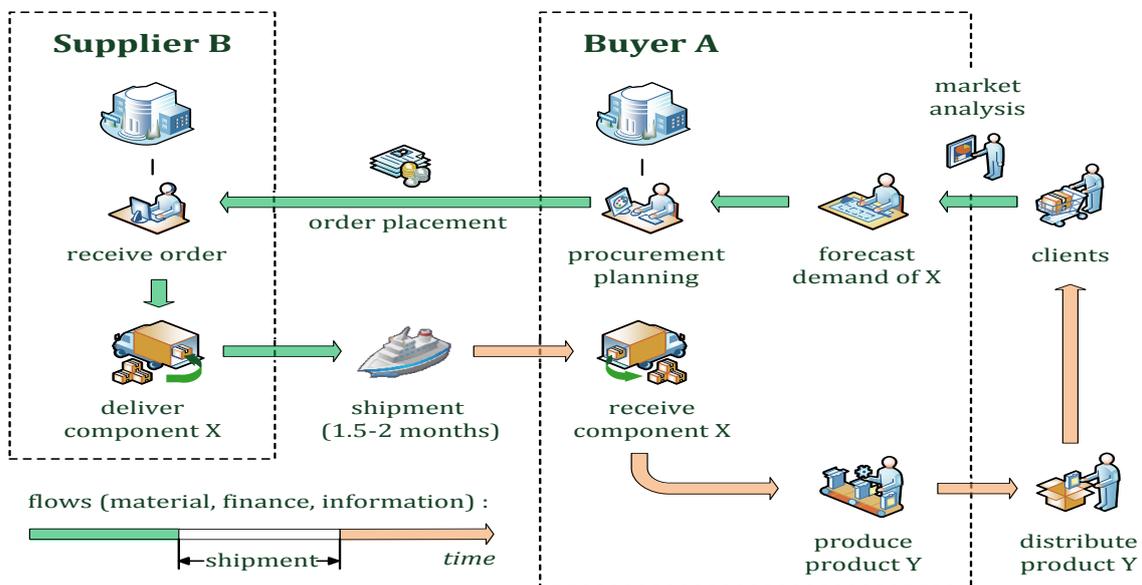


Figure 2.3: Procurement planning problem in global sourcing

To the best of our knowledge, there are few published works discussing long-distance procurement planning problem in global sourcing. In order to make up for the gap of literature, we tend to expand our literature research to similar operations research issues, such as production planning.

Procurement planning is, to some extent, similar to production planning. The major difficulty of procurement planning in global sourcing comes from the long distance between buyer and supplier. The long distance and the maritime transport mode result in long delivery lead time, thus a large uncertainty of future customer demand. In recent years, uncertain demand is more and more considered in production planning problems. Though the scales of uncertainties are different, production planning problem with uncertain demand can provide some reference to procurement planning problem engaged in global sourcing which is studied in this dissertation.

Since there are enormous academic papers on production planning, we will survey related literature according to specific issues in the following chapters.

2.6 Conclusion

Procurement planning in global sourcing has attracted increasing attention. This chapter presents a general literature review of related research issues. To the best of our knowledge, the published papers on procurement planning and global sourcing mainly discussed in a qualitative manner. There exists a little literature that investigates procurement planning and global sourcing respectively, by employing analytical models. However, research on procurement planning problems engaged in global sourcing is still a gap in existing literature. Due to its extensive application in real life, it is important and meaningful to develop effective and efficient approaches to make adequate procurement plans for global sourcing activities. This chapter also illustrates a standard procurement planning problem engaged in global sourcing, which serves as a base for specifying problem description and formulation in the following chapters.

Chapter 3

Adaptive Procurement Planning: A Rolling Horizon Forecasting Approach

3.1 Overview

A major problem of procurement planning in global sourcing is the long lead time caused by the geographically long distance between supplier and buyer. This requires that an order be placed long time before it is actually consumed. As a consequence, the demand forecast should be made long time before. This inevitably induces a large uncertainty. This chapter presents an adaptive optimization framework, which deploys demand forecasting and optimal procurement planning in a rolling horizon scheme. By employing this framework, the long-distance procurement planning problem engaged in global sourcing will be split into optimal procurement planning problems in sub-horizons, which reduces largely the overall computational complexity, and also allows to cope with the demand uncertainty with updated information. Moreover, this adaptive framework can also be used to evaluate long-run performances of other methods.

The chapter is organized as follows. Firstly, the key factors determining inventory models are identified for the specific problem under study (Section 3.2.1). The assumptions and notation are then addressed in Section 3.3. The problem is mathematically formulated in Section 3.4. Based on literature review (Section 3.2.2), an adaptive optimization framework is developed to cope with the large demand uncertainty caused by long distance (Section 3.5). The proposed adaptive optimization framework is developed to deploy the short-term optimal planning methods presented in Chapter 4 and Chapter 5. The conclusion is drawn in Section 3.6.

3.2 Literature Review

3.2.1 Model Identification

In this thesis, the long-distance procurement planning problem engaged in global sourcing is studied. As presented in Chapter 2, we find little direct reference in published works on this specific issue. Since procurement planning is, in some measure, similar to production planning, we turn to literature on production planning with uncertain demand for some reference. However, although we focus our problem on single item and single location, there are still numerous applicable inventory models due to various parameters which determine the models' structure. For clear model identification, (Graves et al., 1993) addressed three key factors that influence the essential structure of inventory models, which are summarized as follows.

Key Factor (1): Demand

Since virtually all the procurement plans are made to satisfy certain customer demand, the assumptions that we make about demand are, in most cases, the most important in determining the problem complexity. On the basis of existing literature, the following assumptions are commonly defined:

- (a) *Deterministic and stationary.* Demand is constant and known. This assumption is usually made based on the fact that either the demand is expected to stay unchangeable, or the demand can be predicted in advance. A famous application is the classical *economic order quantity* (EOQ) model, which is developed by (Harris, 1913) and popularized by (Wilson, 1934).
- (b) *Deterministic and time varying.* In this case, the changes in demand are supposed to be predictable. This demand pattern is widely considered for lot sizing problems, in the context of manufacturing final products from components and raw materials.
- (c) *Uncertain.* The term "uncertainty" refers particularly to the case that demand distribution is known, while the exact demand quantities cannot be predicted in advance. Usually, the demand distribution can be estimated from historical demand data. This demand pattern is used in majority of the planning problems which consider indeterminate demand.
- (d) *Unknown.* This demand pattern is resulting from the practical condition that demand uncertainty is so large that even the demand distribution is unknown. A feasible approach is to assume that demand follows certain distribution form, and update the distribution parameters according to up-to-date demand observations.

In this thesis, we consider the unknown demand.

Key Factor (2): Costs

The main objective of most planning problems is to minimize total costs. Thus the assumptions that one makes about costs also play an important role in defining the problem. Section 2.2.3 has addressed the commonly-used cost assumptions. In this thesis, we consider the costs of setup, inventory holding and stockout penalty.

Key Factor (3): System's physical characteristics.

The physical aspects of inventory system will also influence the problem complexity. The following physical features are often considered:

- (a) *The review process.* Two types of review processes are usually considered: continuous and periodic. Continuous review process assumes that inventory level is known at all times, and replenishment order can be placed at any time. It is widely used in modern supermarkets with scanning devices connected to central inventory management system. On the other side, periodic review process assumes that inventory level is known only at discrete time points, and order decisions can only be made at these points. Note that when period length is extremely tiny, periodic review process can be approximately regarded as continuous review process.
- (b) *Lead time assumptions.* Lead time is defined as the delay between the placement and the arrival of a replenishment order. It reports the system response time. A simplest assumption is instantaneous replenishment, in this case, lead time is assumed to be zero. This assumption is made when lead time is short enough compared to the time between reorder decision in practice. In periodic review process, a more general assumption is that lead time equals some integral multiple of period length.
- (c) *Stockout assumptions.* When demand cannot be fulfilled, a stockout assumption should be made. The simplest assumption is that all the unsatisfied demand is backordered. The backordered demand is mathematically described as a negative inventory level. Another extreme is that the unfulfilled demand is totally lost, which often occurs in retail environment.

In this thesis, we consider a periodic review process. Lead time is assumed to be some integral multiple of period length. When a stockout occurs, the backorder and lost-sale assumptions are respectively used in Chapter 4 and Chapter 5.

3.2.2 Rolling Horizon Planning (State-of-the-Art)

In procurement planning problems, the information (on actual demands, inventory states, etc.) cannot be well observed unless within an appropriate forecast lead time. In order to acquire up-to-date forecasts, a rolling horizon planning procedure is commonly used.

The explicit academic research on rolling horizon planning can be dated back to the early work of (Baker, 1977). The author has designed an experimental study to investigate the efficiency of rolling horizon planning with the classical Wagner-Whitin model (Wagner and Whitin, 1958). In his paper, the standard procedure of rolling horizon planning is described as follows:

"solve the model and implement only the first period's decisions; for the following period, update the model to reflect information collected in the interim, re-solve the model, and again implement only the imminent decision pending subsequent model runs."

Baker has tested the rolling horizon procedure with different demand patterns. He indicates that for constant or trend demand without seasonal effects, the optimal demand forecast horizon is the natural cycle derived by EOQ model (Harris, 1913; Wilson, 1934). With the seasonal demand, the most appropriate demand forecast horizon is the integral multiple of the seasonal cycle. When demand forecast horizon is properly determined, the effectiveness of a rolling horizon procedure can be independent of the demand profile. The author has also shown that the longest demand forecast horizon is not necessarily optimal.

Efficiency Comparison on Exact and Heuristic Algorithms Under Rolling Horizon Policy

(Blackburn and Millen, 1980) implemented the rolling horizon procedure with three well-known lot sizing methods: Part Period Balancing (PPB), Silver Meal (SM), and Wagner-Whitin (WW) algorithm. Numerical experiments show that rolling WW method does not always perform the best. As a matter of fact, the WW decisions are optimal only in a fixed-horizon problem. Extending the planning horizon by one period, the optimal decisions might change. This instability is named "nervousness" in MRP. However, for the heuristic methods such as PPB and SM, the nervousness degree is probably lower since not all the future information is needed to make current order decision. Thus under certain conditions, rolling SM approach can produce lower total costs than rolling WW approach, which has been validated by numerical examples in the paper.

Nevertheless, (Chand, 1982) presented a modified version of rolling WW approach, which gives better cost performance than rolling WW approach and rolling SM approach. The improvement on rolling WW approach requires very few additional computations.

The argument whether exact algorithms (such as Wagner-Whitin method) or heuristic algorithms (such as Silver-Meal method) perform better in a rolling horizon environment goes through the following decade, see (Carlson et al., 1982; Wemmerlöv and Whybark, 1984; Evans,

1985; Bookbinder and H'ng, 1986; Saydam and McKnew, 1987; Bahl and Taj, 1991). In these papers, lot sizing under demand uncertainty in a rolling horizon environment is studied. The uncertainty is usually described in the form of forecast error. The authors distinguish the terminology "demand uncertainty" from "demand variation". As illustrated by Figure 3.1, demand variation specifies the known or predictable trend that demand follows, which is expressed by the difference between demand expectations. Otherwise, the actual demand is not exactly equal to the expectation, but usually in a predictable range deviated from the demand expectation. This deviation is corresponding to demand uncertainty.

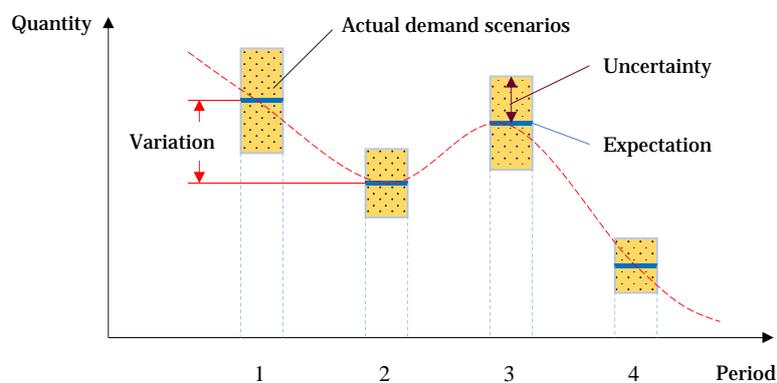


Figure 3.1: Demand variation and uncertainty

The authors indicate that demand uncertainty impairs rolling WW approach more than rolling SM approach, while demand variation impairs rolling SM approach more than rolling WW approach. Another important conclusion is that when demand uncertainty is involved, more information on demand variation and uncertainty is better than less for making order decisions. This result is different from Baker's conclusion that "less is better than more" for deterministic models, see (Baker, 1977).

Based on previous works, (Russell and Urban, 1993) developed an improving solution for rolling WW approach under large demand variation condition. The authors summarize that rolling SM approach often outperforms rolling WW approach with very short demand forecast windows. However, when demand variation is large, rolling SM approach is no longer well applicable. The authors also demonstrate that the performance of rolling WW approach can be improved significantly even with quite small forecast window extensions (i.e., extending the forecast window by only a small number of periods). Numerical examples are presented to confirm the effectiveness of using rolling extensive Wagner-Whitin (EWW) approach to make order decisions under demand uncertainty.

Besides, (Stadtler, 2000) introduces the idea of "time between orders (TBO)", which can be calculated by the heuristic algorithm presented in (Groff, 1979). In a rolling horizon procedure, let N denote the entire planning horizon length, n denote the myopic planning horizon length. In most cases, $N \gg n$, and the demands in the n -period horizon have already been acquired at the planning time. Let τ_i ($1 \leq i \leq n$) denote the TBO for period i in which an order is placed. The author employs modified cost coefficients for the period i such that $i + \tau_i - 1 > n$. For the period beyond n , a demand forecast is used. Then the same recursion procedure of Wagner-Whitin is implemented. Numerical tests show that in a rolling horizon environment, Stadtler's (ST) approach performs at least as well as well-known heuristics (such as Silver Meal method) and is fairly insensitive to the planning horizon length.

In a rolling horizon procedure, the optimal planning for the n -period problem is not necessarily optimal for the N -period problem. For example, it is always optimal for the n -period problem to leave no stock at the end of period n , while this might be not optimal for the N -period problem. This phenomenon is named "end-effect" (Grinold, 1983) or "truncated horizon effect" (Federgruen and Tzur, 1994). (Fisher et al., 2001) present an ending inventory valuation (EIV) method, which includes a valuation term about the end-of-horizon inventory in the objective function of the n -period problem. Numerical tests show that under several demand patterns, rolling EIV approach surpasses both rolling WW approach and rolling SM approach.

However, rolling EIV approach assumes the availability of quite accurate future demand estimates, while rolling WW and SM approaches do not use any demand information beyond the n -period horizon. (van den Heuvel and Wagelmans, 2005) argue that the superior performance of rolling EIV approach as illustrated in (Fisher et al., 2001) is, to a large extent, due to the aforementioned unfairness. The authors have compared rolling EIV, EWW and ST approaches, and conclude that rolling EWW approach outperforms the other approaches under multiple different demand patterns and horizon lengths.

Involving Demand Forecasting Techniques in Rolling Horizon Procedure

The above literature mainly discusses about the efficiency of rolling planning procedure. Since demand uncertainty is much more considered in recent years, the inventory model that explicitly includes demand forecasting is taken into attention.

(Iida and Zipkin, 2006) published a significant paper describing how the optimal procurement policy depends on current forecasts of future demands. The procurement planning problem is formulated as a dynamic program with multidimensional state space. The authors

propose a forecast-corrected base-stock policy which is combined with a variant of MMFE (martingale model of forecast evolution, see (Heath and Jackson, 1994)). The authors indicate that near-term forecasts contain more useful information than long-term ones. Based on this result, a novel myopic policy is developed and is proven to be optimal under certain circumstances. Note that setup cost is not considered in this paper.

MMFE is essentially a framework that represents the dynamics of demand forecasts. An alternative is to employ a demand model based on a specific forecasting technique, such as time series forecasting techniques. (De Gooijer and Hyndman, 2005) provide a comprehensive review on multiple time series forecasting techniques. Moreover, (Poler and Mula, 2011) propose an automatic selection method among these techniques for better adapting different work settings and reducing forecast errors.

Conclusion

To conclude, rolling horizon procedures are effective and widely used in practice, for the situation that future demand information cannot be completely acquired. In addition, forecasting techniques can be involved in the procedure so as to better deal with demand uncertainties, since forecast errors can be continuously revised in a rolling horizon procedure.

In the above papers, the authors use exact planning methods to make myopic plans based on demand forecasts. However, when demand uncertainty is so large that even demands in the myopic planning horizon cannot be considered "determinate" as the demand forecasts (see "unknown demand" defined in Section 3.2.1), the above exact myopic planning methods are no longer applicable. In this chapter, we investigate the rolling horizon planning procedure with above-mentioned "unknown demand".

3.3 Problem Description

3.3.1 Assumptions

In order to specify the features of long-distance procurement planning problem engaged in global sourcing, the following example is used.

Example:

Firm A (in France) plans to procure component X from Supplier B (in China) in order to produce Product Y. The transport from B to A takes about 1.5-2 months. Once a procurement order is placed, a mandatory set-up cost occurs. When there is a stockout of component X in A's production process, the production for final-product Y will be interrupted, therefore the customer demand for Y cannot be fully satisfied. Consequently, a penalty cost is charged to firm A for unfulfilled demand.

The objective is to draw up an optimal procurement planning, which minimizes the expected long-run per-period total procurement costs including set-up, inventory holding and stockout penalty, by specifying two issues: when a procurement order should be placed, and how many component X should be ordered.

The following assumptions can be extracted from the above example.

Assumptions:

- (a) The planning problem is considered under a periodic-review policy.
- (b) The demand quantity in each period is non-negative, independent and unknown. It might follow certain trend and seasonality which could be forecasted adaptively from the historical data.
- (c) The procurement capacity is unlimited.
- (d) Unsatisfied demand is either backordered or lost, according to stockout assumptions. In any case, a penalty cost will be charged.
- (e) The inventory cost is assessed at the end of each period, including holding or stockout penalty. No disposal of inventory is allowed, and the item considered is not perishable.
- (f) For each order placed, the lead time L is assumed to be fixed. L is a positive integer that cannot be neglected. Moreover, demand forecasts will evolve during the delivery of order.
- (g) A mandatory setup cost occurs when a procurement order is placed.

The mission is to make an optimal procurement plan which can minimize the long-run per-period total costs including setup, inventory holding and stockout penalty. Two principal issues should be specified in the solution: (1) in which period should a procurement order be placed; (2) how much should be ordered.

Figure 3.2 illustrates the specific problem under study.

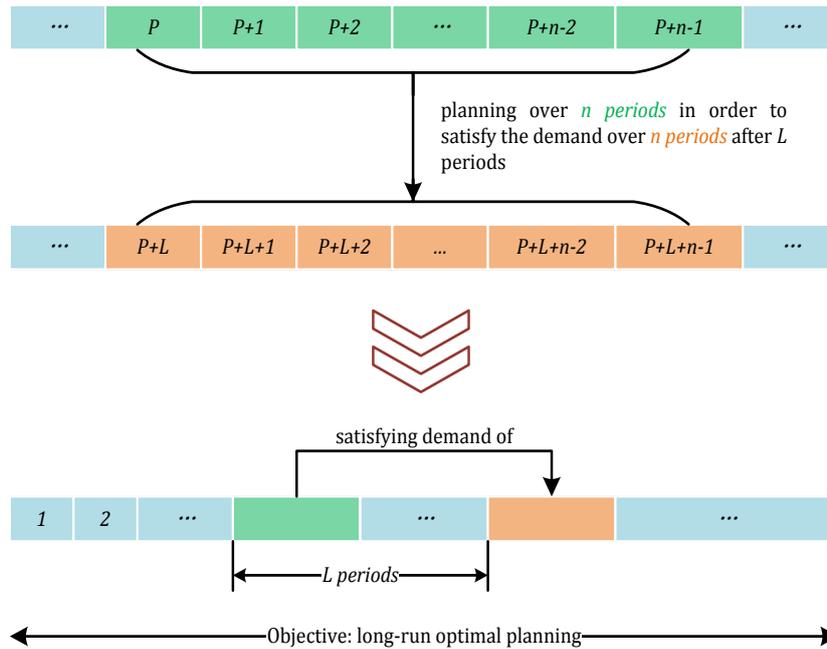


Figure 3.2: Procurement planning problem engaged in global sourcing

In order to satisfy the demand in a certain period i , the corresponding procurement order should be placed at least L periods before. In this thesis, we use a rolling horizon procedure to acquire up-to-date demand forecasts.

Let n denote the planning horizon. The procurement orders made in periods $P, P+1, \dots, P+n-1$ are placed, very likely, to satisfy the demands in periods $P+L, P+L+1, \dots, P+L+n-1$. The eventual objective is to make an adequate procurement plan which minimizes expected long-run per-period total costs.

As presented in Chapter 2, a major difficulty in making procurement plans in a global sourcing environment is the long lead time caused by the geographically long distance between supplier and buyer. This requires that an order be placed long time before it is actually consumed. As a consequence, the demand forecast should be made long time before. This in-

duces a very large demand uncertainty in inventory models, which implies that even the demand distribution is unknown.

In most existing literature on rolling horizon planning, the authors deploy demand forecasting techniques in a rolling horizon procedure, and make interim optimal plans for each sub-horizon using up-to-date demand forecasts as "determinate demands". While in this thesis, the demand uncertainty is so large that the above assumption of "determinate demands" in the sub-horizon is no longer valid. In this thesis, we assume that actual demands in the sub-horizon follow some known distribution pattern, and estimate the distribution parameters using up-to-date demand forecasts and forecast accuracy.

3.3.2 Notation

In this chapter, the following notation is used.

Decision Variable:

- x_i : The delivery quantity arriving at the beginning of period i .
- δ_i : A binary decision variable. $\delta_i = 1$ if $x_i > 0$, or 0 otherwise.

Random Variable:

- d_i : The actual demand quantity of period i . The exact value of d_i cannot be acquired until the end of period i , thus we define d_i as a random variable at the decision-making phase.
- I_i^o : The actual inventory level at the beginning of period i .
- I_i : The actual inventory level at the end of period i . Note that $I_i = I_{i+1}^o$.
- $\hat{\square}_{i,k}$: The forecast on the random variable \square of period k , which is made at the end of period i ($i < k$). Here \square could be d , I^o and I . Note that i is the observation period number, and k indexes the period of interest in which demand d_k occurs.

An *observation period* is defined as the period at the *end* of which that the forecast is made. Besides, the *period of interest* specifies the period the demand of which is to be forecasted. Figure 3.3 gives an illustration of the foresaid time concept.

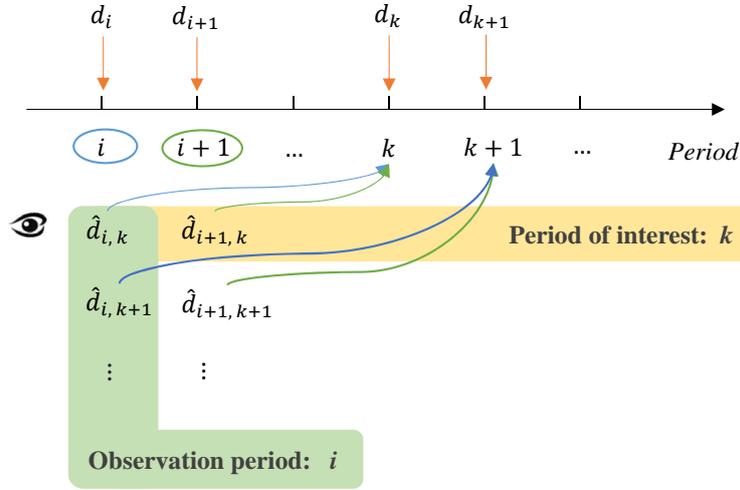


Figure 3.3: Observation period and period of interest

Other Parameter:

- $\epsilon_{i,k}^{\square}$: The estimated standard deviation of $\hat{\square}_{i,k}$. $\epsilon_{i,k}^{\square}$ depends on forecast errors. In this assumption, $\square_k \sim \mathcal{D}(\hat{\square}_{i,k}, \epsilon_{i,k}^{\square})$, where \mathcal{D} denotes some distribution form. We make the realistic assumption that $\epsilon_{i,k}^{\square} \geq \epsilon_{i+1,k}^{\square} \geq \dots \geq \epsilon_{k,k}^{\square} = 0$.
- h_i : The inventory holding cost (per unit held) which is charged at the end of period i .
- p_i : The penalty cost (per unit unfulfilled) which is charged at the end of period i .
- c_i : The purchase cost (per unit purchased) of the order that is placed in period i .
- K_i : The mandatory setup cost for each procurement order that is placed in period i .

In order to distinguish the points that an order is placed and that it arrives, the subscript " $i-L$ " is used to denote the period in which an order is *placed* to arrive at period i . Period i is called an "*order period*" because an order quantity x_i *arrives*. Note that the decision of x_i is made at period $i-L$, thus the corresponding setup cost and the purchase cost coefficient should be respectively K_{i-L} and c_{i-L} .

3.4 Mathematical Formulation

The objective of procurement planning engaged in global sourcing is to minimize the expected long-run per-period total costs. For mathematical formulation, we need to specify the order quantities x_i , $i = 1, 2, \dots, n$, that produce the minimum total costs (including setup, inventory holding and stockout penalty), that is:

$$\text{Minimize } \sum_{i=1}^n C_i, \quad (3.1)$$

where C_i denotes the cost incurred in period i .

Without loss of generality, the structure of C_i is defined as

$$C_i = K_{i-L} \cdot \delta_i + c_{i-L} \cdot x_i + h_i \cdot I_i^+ + p_i \cdot I_i^-, \quad (3.2)$$

where I_i^+ and I_i^- respectively denote the overplus and shortage quantities at the end of period i . Note that I_i^+ and I_i^- are both non-negative, and only one of them can be positive. The detailed definition of I_i^+ and I_i^- will be presented in the subsequent chapters, which regards the stockout assumptions about unsatisfied demands. For stationary purchase cost case, the term " $c_{i-L} \cdot x_i$ " can be eliminated from C_i .

3.5 An Adaptive Optimization Framework

In a long-distance procurement planning problem engaged in global sourcing, the main objective is to determine the order periods and the corresponding order quantities.

The planning is based on data about future developments. The data may be estimated by forecast models, while in most cases, the forecast errors cannot be neglected. The forecast errors are likely to reduce product availability, and thus impair the customer satisfaction targets that a firm seeks. In order to improve the service level, safety stocks are usually used as buffers against stockouts. Whereas, safety stock is not the only way to confront uncertainty.

In the very great majority of cases, the actual demands will deviate from the forecasts. Thus the procurement plans should be revised to reduce avoidable economic loss caused by too many overstocks or stockouts.

Although it is virtually impossible to predict future demand accurately, good predictions can bring a great help in the subsequent procurement planning. Empirically, the forecast of demand in a certain period k is gradually revised along the time, which can be expressed as

$$\epsilon_{i,k}^d \geq \epsilon_{i+1,k}^d \geq \dots \geq \epsilon_{k-1,k}^d \geq \epsilon_{k,k}^d = 0, \quad \forall i \leq k \quad (3.3)$$

A rolling horizon scheme is proposed in order to acquire the *up-to-date* forecasts as far as possible.

3.5.1 The Rolling Horizon Scheme

Let us start with a straightforward example. The ultimate objective is to determine an adequate procurement plan over one year. The planning horizon is divided into periods (months). In order to get the up-to-date forecast data as far as possible, a rolling horizon policy is used. At the beginning of January, a procurement plan is made, which has considered the probable demands from January to June. Only the *order for January* is actually *released* in practice. Here, January is named "*the frozen period*". At the beginning of February, a new procurement plan is made, which covers February to July, using the updated demand information. Note that the frozen period is February at this moment, and only the updated order decision for February is actually executed. Thus it goes on.

In the above example, the sub-horizon "February-July" overlaps the sub-horizon "January-June", but reaches one period (month) further. At the beginning of February, the actual demand of January is known, then the demand forecast error of January can be estimated in order to revise the subsequent procurement plan. Moreover, demand forecasts of the following periods will be updated.

Without loss of generality, the principle of rolling horizon scheme is demonstrated in Fig.3.4. In a rolling horizon scheme, the entire planning horizon is cut into *multiple partly overlapping* sub-horizons along the time axis. In each step, only one sub-horizon of n periods is considered.

Let η_i denote the sub-horizon including the periods from i to $i + n - 1$, and ρ_i denote the planning problem corresponding to the sub-horizon η_i . In a rolling horizon scheme, the planning problems $\rho_1, \rho_2, \rho_3, \dots$ are successively solved. Note that the solution to the planning problem ρ_i will give an interim optimal plan over the sub-horizon η_i , but only the order decision

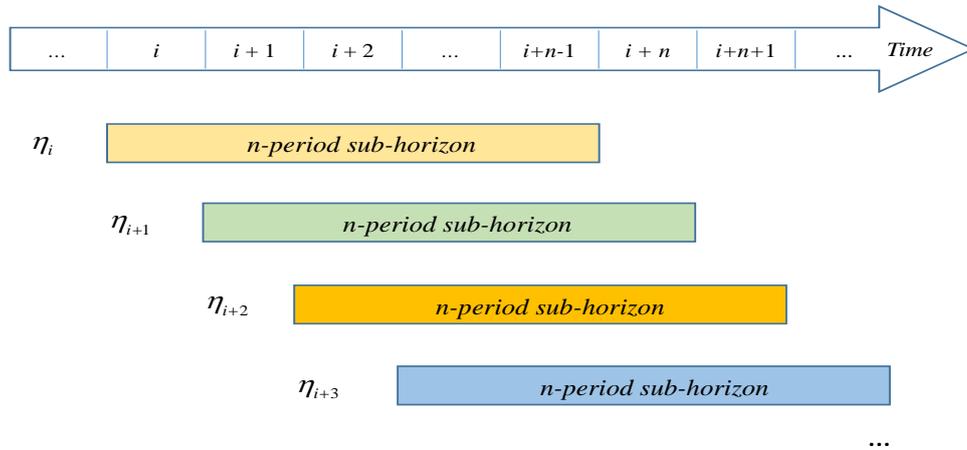


Figure 3.4: The rolling horizon scheme

for the corresponding frozen period (the first period of η_i) is actually executed.

Since the demand forecasts keep updated over time, and the order decisions for the subsequent periods are continuously revised as the planning horizon rolls, the rolling horizon scheme is naturally an "adaptive" one.

3.5.2 Sub-Horizon Planning

Now consider the procurement planning problem ρ_i in the sub-horizon η_i . Since the sub-horizon planning will be implemented repeatedly in a rolling horizon scheme, it would better be a portable module.

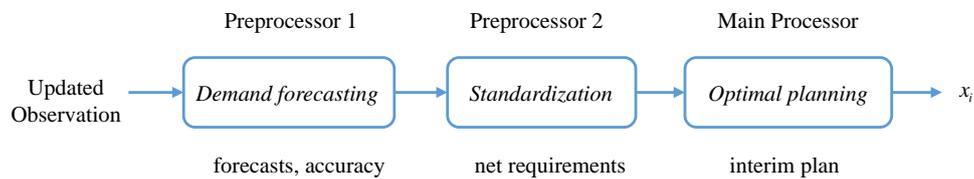


Figure 3.5: Sub-horizon planning module

Figure 3.5 illustrates a standard portable planning module for sub-horizon η_i . This module will not be launched until the following two conditions are satisfied: (1) the order decision x_i in order to satisfy the demand in period i is urged to be determined (considering the long lead

time), and (2) the up-to-date demand observations have been acquired.

Basically, the sub-horizon planning module consists of two preprocessors and one main processor. Firstly, the demand forecasting preprocessor makes demand forecasts and estimates forecast accuracy, which will help in estimating demand distributions. Then the initial stock of η_i will be estimated. Afterwards, the *net* demand quantities are calculated in the standardization procedure. The main processor generates interim optimal procurement plan over the sub-horizon, and only the order decision for the frozen period (the first period of η_i) will be extracted and actually executed in practice.

Preprocess 1: Demand Forecasting

Among the various methods for forecasting, the technique of time-series forecasting holds an important part and is strongly applicable in procurement planning.

A *time series* is defined as a sequence of data (such as the demands in this dissertation), measured at successive time instants spaced at uniform time intervals. The *time-series forecasting* is the use of time-series models to predict future values based on previously observed ones.

The time-series models can have diverse forms which represent different stochastic processes. Stationarity, linearity, trend and seasonality are the four major characteristics that determine time-series models (Palit and Popovic, 2005).

A stationary time series implies that the mean value and variance of observation data remain constant over time. The demands in our problem are non-stationary. Nevertheless, most non-stationary time series can be still transformed into the stationary time series by certain differentiations, such as the removal of trend and seasonality.

A linear time series can be represented as a linear function of its current and past states. The famous examples include the AR (autoregressive), MA (moving average), ARMA (autoregressive moving average) and ARIMA (autoregressive integrated moving average) models. Similarly, the nonlinear time series can be described by the corresponding nonlinear models.

The trend component of a times series expresses its long-term feature that is affected by the global evolvement to which the time series data relates. It is usually represented by a detectable disturbance with upward or downward trend.

The seasonal times series is widely used in economic applications. The seasonality is

often explicitly described by the periodical fluctuation, since the data pattern repeats hourly, daily, weekly, monthly or even yearly in real life.

Step (1): Decomposition analysis

In order to make proper demand forecasts, the decomposition analysis of a time series should be employed at first.

As above mentioned, a time series $X(t)$ is probably constituted by the random component $R(t)$, the trend component $T(t)$, and the seasonality component $S(t)$. Generally, two types of model are used: the additive model

$$X(t) = R(t) + T(t) + S(t), \quad (3.4)$$

and the multiplicative model

$$X(t) = R(t) \times T(t) \times S(t). \quad (3.5)$$

Both models are useful in practice, which manifest the corresponding effects that the trend and seasonality act on customer's requirements.

The decomposition analysis identifies and extracts the partial data that is superimposed to the main time series data, such as the trend and seasonal components as shown in Figure 3.6.

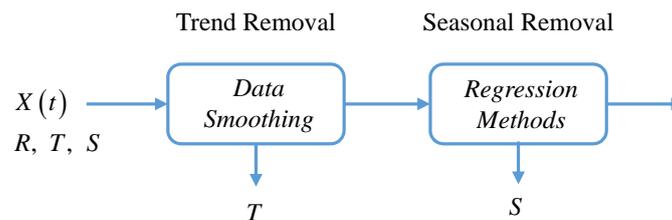


Figure 3.6: Time series decomposition analysis

Multiple mature tools have been developed to implement the time series decomposition analysis, such as the X-12-ARIMA developed by the United States Census Bureau, and the TRAMO/SEATS developed by the Bank of Spain.

Step (2): Model building

The decomposition analysis helps in obtaining the residual time series data deprived of

the trend and seasonal components. In most cases, the residual time series can be considered as stationary. (Box et al., 2013) developed a systematic methodology to build the stationary time series models, as presented in Figure 3.7.

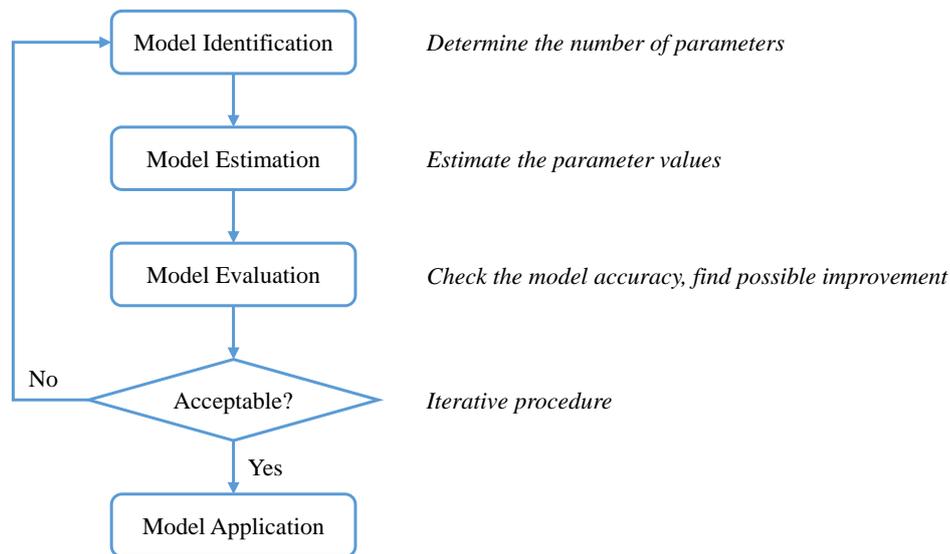


Figure 3.7: Box-Jenkins methodology on time series model building

The model identification phase is a rough procedure to profile the collected observation data, and then define the initial model structure. Usually, an existing model pattern is employed, and the autocorrelation approach is used to check if the employed model pattern matches the observation data well enough.

Next, the specific parameter values of the preliminary model will be determined. Some special statistical techniques are required, such as the maximum-likelihood estimation (MLE) and the least-squares estimation (LSE).

The model evaluation procedure is to verify if the built model fits well the collected observation data, and to check the model sensitivity to the input data.

Step (3): Forecasting methods

Once the time series model has been built, an adequate forecasting method should be selected to acquire as accurate as possible the future demand information.

There exist many mathematical tools that help in making forecasts, such as the trend analysis (based on trend line fitting of time series data, using a linear, quadratic, or exponential function), the regression approaches, the Box-Jenkins methods (forecasting using AR, MA, ARMA and ARIMA models), and the data smoothing techniques.

For our specific demand forecasting problem involved in procurement planning in a global sourcing environment, the main difficulty is the likely large forecast errors due to the long forecast lead times. (Hubert, 2013) proposed a methodology to select an appropriate demand forecasting method in this context, and developed a detailed model of the forecast accuracy and its evolution with time. The method has been validated by its application on real-life data from a world-leading automobile corporation, which has demonstrated a clear superiority compared to existing ones in terms of both service level and inventory level.

Hubert's study has laid a solid foundation on demand forecasting for our adaptive procurement planning approach. In his dissertation, the real-life case study has shown that the relative forecast errors are reduced to 3%. This promising result is due to the dynamically updating forecasting method that he has developed, which is also compatible with our adaptive planning framework. Furthermore, he proposed models to estimate the forecast accuracy, which have been proven to be reliable based on tests of real life data.

Based on Hubert's work, we can assume that the demand in each period of the myopic sub-horizon can be considered to follow some distribution pattern. The demand forecast can be regarded as the mean value of demand distribution, while the forecast accuracy can be regarded as the standard deviation.

Preprocess 2.1: Initial Stock Estimation

For the sub-horizon η_i , the initial stock is give as

$$I_i^o = I_{i-L}^o + \sum_{j=i-L}^{i-1} (x_j - d_j). \quad (3.6)$$

The interim procurement plan for η_i is made at period $i-L$, thus the initial stock I_{i-L}^o and the decisions x_j , $j = i-L, \dots, i-1$ are known. The distributions of d_j , $j = i-L, \dots, i-1$ have been estimated in the previous preprocess, then the distribution of I_i^o can be derived.

According to Equation (3.6) and the expectation properties, the mean value of I_i^o can be

given as

$$\hat{I}_{i-L, i}^o = I_{i-L}^o + \sum_{j=i-L}^{i-1} (x_j - \hat{d}_{i-L, j}), \quad (3.7)$$

and the standard deviation is

$$\epsilon_{i-L, i}^{I^o} = \sqrt{\sum_{j=i-L}^{i-1} (\epsilon_{i-L, j}^d)^2}. \quad (3.8)$$

Preprocess 2.2: Net Requirements Determination

The above preprocesses have estimated the equivalent distributions of the initial stock and demands for the sub-horizon η_i . Then a standardization procedure is employed to obtain the net demand quantities.

Similar to the *Bill of Material* (BOM) process used in *Material Requirement Planning* (MRP), the distributions of net demands can be estimated as follows.

For simplification, the following notation is used:

Notation:

- d_k^i The *net* actual demand of the k^{th} period in the sub-horizon η_i .
- μ_k^i The mean value of d_k^i .
- σ_k^i The standard deviation of d_k^i .
- y_k^i The interim order decision which is supposed to arrive at the beginning of the k^{th} period in the sub-horizon η_i .

Let r_k^i denote the expected remaining stock at the end of the k^{th} period in the sub-horizon η_i , if no order arrives. Then we have

$$r_k^i = \max\{0, r_{k-1}^i - \hat{d}_{i-L, i+k-1}\}. \quad (3.9)$$

The mean values μ_k^i , $k = 1, \dots, n$ can be calculated as

$$\mu_k^i = \begin{cases} 0, & r_k^i > 0; \\ \hat{d}_{i-L, i+k-1} - r_{k-1}^i, & r_k^i = 0. \end{cases} \quad (3.10)$$

Assume the first positive net demand expectation occurs in the m^{th} ($1 \leq m \leq n$) period of the sub-horizon η_i . The standard deviation can be calculated according to the *square root law*:

$$\sigma_k^i = \begin{cases} \sqrt{(\epsilon_{i-L, i}^{I^o})^2 + \sum_{t=1}^k (\epsilon_{i-L, i+t-1}^d)^2}, & k \leq m; \\ \epsilon_{i-L, i+k-1}^d, & k > m. \end{cases} \quad (3.11)$$

The actual net demand quantities d_k^i , $k = 1, \dots, n$ can be regarded to follow probabilistic distributions with the mean value μ_k^i and the standard deviation σ_k^i . Then the planning problem in sub-horizon η_i can be transformed into a standard lot-sizing problem with stochastic demands.

Interim Optimal Planning

The interim optimal planning problem in the sub-horizon is equivalently a standard lot-sizing problem with stochastic demands. The demand in each period follows some probabilistic distribution pattern with different parameters.

The procurement order decisions are made before the sub-horizon begins, thus it is probable that a stockout happens during the sub-horizon. Usually, a penalty cost will be charged for the unsatisfied demand.

The assumptions made for dealing with stockouts influence dominantly the procurement decisions. Chapter 4 and Chapter 5 will discuss how to make interim optimal procurement plans for the equivalent stochastic lot sizing problems in the sub-horizon, respectively under the assumptions of backorders and lost sales (or outsourcing).

Conclusion

To summarize, when the required information (up-to-date demand observation, sub-horizon definition, etc.) is prepared under a rolling horizon scheme, the interim procurement planning over the corresponding sub-horizon can be made in the following steps: demand forecasting,

initial stock estimation, net requirements standardization, optimal planning, and frozen-period decision extraction (to be executed in practice). The sub-horizon planning procedure will be repeatedly employed, thus is strongly suggested to be designed as an independent portable programming module in real-life application.

3.5.3 Ex-Post-Facto Experimental Evaluation

In the above sections, an adaptive optimization approach is developed to make adequate procurement plans in a global sourcing environment. The main difficulty comes from the long distance between supplier and buyer, due to which the large demand uncertainties will significantly augment the risk level. In order to acquire as accurate as possible demand forecasts, a rolling horizon scheme is employed. For the sub-horizon planning problem, a systematical solution module has been developed.

This section presents an evaluation method to estimate the performance of proposed adaptive optimization approach. Since the actual demand of each period can be all known only after the end of the entire planning horizon, an *ex-post-facto* experiment is designed to evaluate the proposed approach. The ex-post-facto evaluation is widely used in forecasting models.

The proposed optimization approach is developed to minimize *expected long-run* per-period total costs, therefore, we should generate randomly sufficient demand scenarios with considerably long planning horizon.

To launch the experiments, a large amount of demand scenarios should be randomly generated at first. Since trend and seasonality are common cases in practical procurement process, the following formula is used to generate actual demand scenarios:

$$d_t = aT(t) + (b + c \times randn) \left[d + \sin\left(\frac{2\pi}{e} \times \left(t + \frac{f}{4}\right)\right) \right] \quad (3.12)$$

where t is the time period, $T(t)$ is a polynomial function of t , a, b, c, d, e, f are constant integer parameters, and $randn$ is a random number that follows standard normal distribution.

For each demand scenario, an ex-post-facto optimal solution can be determined at the end of the scenario (when the demand quantities of all the periods in this scenario are known), by employing any existing deterministic dynamic lot-sizing algorithm.

Let C^* denote the ex-post-facto minimum total cost figured out as aforementioned for

some demand scenario. Let C denote the actual total cost applying the proposed adaptive optimization approach to the same demand scenario. Regarding the randomness of the problem, the final evaluation of the proposed approach should be a statistical result and not decided by a single demand scenario. Letting κ denotes the number of demand scenarios that are randomly generated following the same parameter setting, we use two measures to evaluate the proposed approach, expressed respectively as

$$\mathcal{R}_1^\kappa = \overline{C}/\overline{C^*} - 1, \quad \text{and} \quad \mathcal{R}_2^\kappa = \overline{C/C^*} - 1, \quad (3.13)$$

where a symbol with an overline bar represents the mean value of the quantity represented by the same symbol without the overline bar. Note that \mathcal{R}_1^κ and \mathcal{R}_2^κ especially represent the ratio of the mean values, and the mean value of the ratios.

3.6 Conclusion

This chapter has presented an adaptive optimization framework for procurement planning problems engaged in global sourcing. The feasibility and high efficiency of rolling horizon planning procedure to deal with uncertain demand have been proven in existing literature. However, when demand uncertainty is very large, existing rolling horizon planning methods are no longer applicable. In order to deal with the large demand uncertainties due to the geographically long distance between buyer and supplier in a global sourcing environment, we estimate the demand distributions instead of using directly the demand forecasts as "determinate demands" to make myopic plans in the near future. This chapter established a structural framework that deploys demand forecasting and optimal procurement planning in a rolling horizon procedure. The detailed optimal sub-horizon procurement planning methods will be presented in Chapter 4 and Chapter 5. Besides, the proposed adaptive framework can also be used to evaluate long-run performances of other methods.

Chapter 4

Procurement Planning with Backorders

4.1 Overview

For planning problems with demand uncertainty, a penalty rule is usually predefined in case of unsatisfied demand. In this chapter, the unfulfilled demand is assumed to be all backordered, with a time-varying expense charged for each backordered item per unit time. As a matter of fact, backorder is regarded as the most common consideration when demand exceeds supply, while the other extreme is that all excess demand is lost (see Chapter 5).

This chapter is organized as follows. Section 4.2 surveys existing publications on planning problems with demand uncertainty and backorders. The problem is described in Section 4.3, and mathematically formulated in Section 4.4. Section 4.5 presents optimization algorithms to solve sub-horizon procurement planning problems, which can be deployed with the rolling-horizon framework presented in Chapter 3. Numerical results are presented in Section 4.6, validating the high efficiency and effectiveness of both the proposed sub-horizon planning methods, and the overall adaptive procurement planning procedure. Conclusion is given in Section 4.7.

4.2 Literature Review

4.2.1 The Backorder Systems

When a customer places an order for an item that the supplier does not have enough in stock, typically, the supplier will inform the customer of the probable stockout, and the estimated waiting time for the replenishment of the required item. The customer can choose to cancel the order immediately or wait for the item until it is available again. When the item is irreplaceable and hard to find an adequate substitute, the customer is usually willing to wait.

A *backorder* is defined as a customer order that cannot be currently filled or shipped, but for which the customer is prepared to wait until the item becomes available again. The percentage of the order postponed and the backorder time are important measures of a company's customer service quality and inventory management effectiveness (Silver et al., 1998).

When a customer demand is backordered, a penalty cost will be charged. The cost includes the management operations for unfulfilled orders, the possible negative customer relations, the interest expenses, and so on. Sometimes the items are backordered longer than

expected, then the suppliers may waive the shipping costs or offer other compensations to encourage customers' patience of waiting, in order to retain the customer loyalty.

Although customers can be willing to wait for the unsatisfied order in certain situations, they generally want their orders to be filled immediately. Thus the suppliers are always trying to avoid backorders. However, due to the uncertain nature of the demand, backorder cannot be completely avoided, unless with a huge amount of safety stock, which leads obviously to a huge inventory holding cost.

Companies are always trying to balance their stock so as to minimize total costs including inventory holding and backorder penalty.

4.2.2 State-of-the-Art on Backorder Models

For multi-period planning problems, lot sizing is a mature and effective technique to find optimal solutions.

The Standard Lot-Sizing Model with Backorders

The study of backorder models can be dated back to the early work of (Zangwill, 1966), in which a deterministic multi-facility, multi-period production planning model is analyzed. The author assumes that each facility can backlog the total demand of its product for a certain integral number of periods. By determining the general form of minimum cost production schedule, a 2-facility 3-period planning case is specifically studied.

Zangwill accomplished his study on backorders in (Zangwill, 1969). In this paper, the author extended the famous Wagner-Whitin model (Wagner and Whitin, 1958) with backorder permission. The objective is to find a production schedule that minimizes total production and inventory costs (including holding and backorders). Based on reformulated recursion equations, an efficient dynamic programming algorithm for calculating the optimal production schedule is developed. Note that in this paper, the marginal production cost is assumed to be independent of the amount produced in each period, and the fixed charges, the inventory holding and shortage costs can differ from period to period.

With the backward $O(n^2)$ (n is the number of periods involved in the planning horizon) algorithm presented in (Zangwill, 1969), the optimal decision of period 1 can be specified provided that all information about demands and costs over the entire planning horizon is given.

(Blackburn and Kunreuther, 1974) improved this algorithm by developing *planning horizon theorems*. The planning horizon here refers in particular the length t that the optimal production quantities in periods 1, 2, \dots , t remain optimal for the problems with length $t + j$ for all $j > 0$. If such planning horizons can be found, the optimal decision for some periods can be determined by limited demand and cost information, which is rather computation-reducing. After developing several planning horizon properties, the author developed a forward $O(n^2)$ algorithm to find the optimal production quantities. The proposed algorithm has been applied to the same example used in (Zangwill, 1969), and yields an alternative optimal solution which is different from the one obtained by Zangwill.

Important contribution on reducing the computational complexity is published by (Federgruen and Tzur, 1993). The authors developed a simple $O(n \log n)$ algorithm for the standard Wagner-Whitin model with backorders. The fixed set-up cost is considered, and the unit holding and backorder costs are supposed to be linear. In some special situations, the method could even be $O(n)$.

Some Variants for Real Application

The standard lot-sizing model assumes that demand of a given period should be fulfilled in the period. If backorder is not allowed, the demand of a given period should be delivered at a determinate time. If backorder is allowed, the demand can be delivered later at the expense of backorder cost. The Wagner-Whitin model with backorders is just a simplified paraphrase of what might actually happen in real life. In the following years, many more realistic applications have been studied.

(Lee et al., 2001) studied the demand time window case. In daily lives, customers usually offer a grace period during which a specific demand can be fulfilled without penalty. This grace period is named "*demand time window*". In other words, for each demand, the customer specifies acceptable earliest and latest delivery dates. The authors developed a polynomial time algorithm to compute the optimal solution for the dynamic lot sizing problem with demand time windows. The complexity of the proposed method is $O(n^3)$ when backorder is allowed.

(Li et al., 2004) studied two important variants which are applicable to a wide range of real-world situations: batch ordering and delivery truckload discount. In the model with batch ordering, production in each period is restricted to a multiple number of a constant batch size Q . This model is applicable to situations when the items are produced in a batch process with a fixed capacity Q , or the production is performed to fill up multiple truckloads of finished prod-

ucts. The authors have developed an $O(n \log n)$ algorithm to solve the problem with backorders. In the model with truckload discount, the authors consider a replenishment cost structure that includes a fixed setup cost, a variable unit production or purchase cost, and a delivery cost with a truckload discount scheme. Under the truckload discount scheme, the total transportation cost for a truck will no longer increase when a specific proportion of the truck is filled. The authors developed an $O(n^3 \log n)$ method to solve the problem with backorders. In addition, (Grunder, 2010) considered a single-item batch scheduling problem with batch-size-dependant setup times. The author developed an efficient dynamic programming method to determine production and transportation plans in order to meet customer demands at minimum total costs.

(Chu and Chu, 2007, 2008; Chu et al., 2013) addressed production planning problems, which can be formulated as single-item dynamic lot-sizing problems with backlogging, outsourcing and inventory capacity. The costs of set-up, inventory holding, backlogging and outsourcing are considered. The backlogging level at each period is also limited. The goal is to satisfy all demands in the planning horizon at minimal total cost. The authors developed polynomial time algorithms respectively for backlogging, outsourcing and combined cases.

Lot-Sizing Models with Demand Uncertainties and Backorder Permissions

The above papers concentrate on deterministic lot-sizing problems with backorders. In these problems, the demand of each period is given, and the backorders are tactically permitted in order to find better balance between storage and shortage costs. In other words, the supplier can always satisfy the customers' punctual delivery requirements if he likes.

However, in real life, the customer demand is unpredictable. Since the procurement planning should be determined before the actual demand is known, the situation that supply cannot satisfy demand often occurs. In this case, the backorder is not alike that in the deterministic problems, but is a specific measure to counter the unavoidable stockout. Moreover, the objective of optimal planning turns into minimizing the *expected* total costs, which is a statistical result instead of a specific scenario.

In Section 3.2.1, the customer demand has been divided into four classes: (1) deterministic and stationary, (2) deterministic and time varying, (3) uncertain, (4) unknown. Demand uncertainty is inevitable in real life. Practically, the assumptions (3) and (4) have described different magnitudes of demand uncertainty.

In recent years, research on lot-sizing problems with demand uncertainties has become

a new trend. In particular, assumption (3) is widely used in the literature, of which the two following branches are mainly studied.

(1) Probabilistically distributed demand

In many published papers concerning uncertain demand, a common hypothesis is that the customer demand follows some widely used continuous or discrete probability distributions, such as the famous uniform distribution and normal distribution (Gaussian distribution).

Lot sizing with uncertain demand is recognized to be a very difficult problem to solve. (Porteus, 1990) provides a good review of the early work done in this area. The common practice is to separate the problem into simpler sub-problems. In each sub-problem, the demand of each period is set equal to the forecasted value, and a buffering mechanism (such as a safety stock) is used to provide a certain protection against the shortage risk. Then the problem is solved as a deterministic lot sizing problem, and usually deployed with a rolling horizon scheme. Other prevailing approaches suppose that the actual demand in the sub-problem follow the same probability distribution, which leads to considerable simplicity of solving procedures.

The approaches cited by (Porteus, 1990) all require appropriate forecasts of customer demand. *Quantile regression* is widely used to estimate the demand quantities. This type of regression introduces on purpose a bias in the result, instead of seeking simply the average value of the variable to be predicted. Quantile regression can give the median and any other quantiles (or percentiles) of a stochastic variable. Figure 4.1 illustrates three distinct forecasts: a 25% quantile forecast in violet, a 50% quantile (mean) forecast in green, and a 75% quantile forecast in red.

(Gélinas et al., 1995) reviewed and compared three available methods to compute the demand quantiles for planning problems with uncertain demand. The authors also developed a new forecasting procedure based on smoothed order statistics. In this paper, the concept of *cumulative quantity* is firstly proposed. When backorder is allowed, the inventory level at the end of a certain period equals the cumulative delivered order quantity minus the cumulative demand quantity since the first period, provided that the initial stock is zero.

(Sox, 1997) reformulated the dynamic lot sizing problem with random demand and non-stationary costs. By using the cumulative quantities and assuming that the distribution of the cumulative demand is given, the problem can be mathematically modeled as a mixed integer

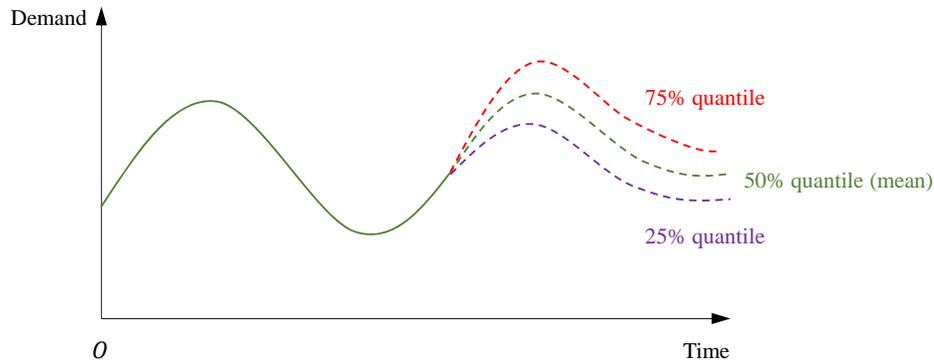


Figure 4.1: Illustration of quantiles

nonlinear problem. The author developed a forward dynamic programming (DP) method to find the optimal order sequence for the reformulated problem. The proposed method is, in some respect, similar to the Wagner-Whitin algorithm for deterministic planning problem, but with some additional feasibility constraints. The author also demonstrated that the lot sizes determined with a rolling-horizon scheme of the proposed method will be bounded below the optimal lot sizes determined directly with a stochastic dynamic programming formulation.

When the holding cost and backorder cost of each period is proportional, (Vargas, 2009) proved that the cumulative order quantity is nondecreasing with the period number. The result implies that the corresponding order quantity of each period is nonnegative, which confirms the feasibility in practice. When demand follows normal distribution, the author proposed a DP procedure to find the optimal solution. This paper provides a means to calculate the optimal objective function value for related planning problems with uncertain demand, thus can serve as a basis to evaluate the new and existing heuristics.

Moreover, (Babai et al., 2011) analyzed a single-item inventory system where the demand and the lead time are stochastic. Demand is modeled as a compound Poisson process. The stock is controlled according to a continuous time order-up-to (OUT) level policy. The objective is to minimize total inventory costs including holding and backorders. The authors proposed a method for determining the optimal OUT level, and validated its high efficiency by numerical tests.

Besides, some meta-heuristic methods have been applied on lot sizing problems with uncertain demand, as shown in Table 4.1.

Table 4.1: Meta-heuristic examples (literature)

Publication	Meta-Heuristic Method
(Piperagkas et al., 2012)	Particle Swarm Optimization (PSO) Differential Evolution (DE) Harmony Search (HS)
(Wong et al., 2012)	Artificial Neural Network (ANN) modified Ant Colony Optimization (ACO)
(Şenyigit et al., 2013)	Artificial Neural Network (ANN) Genetic Algorithm (GA) Bee Algorithm (BA)

(Piperagkas et al., 2012) are the first to tackle the specific problem by applying three popular meta-heuristic methods descending from evolutionary computation and swarm intelligence, namely particle swarm optimization (PSO), differential evolution (DE) and harmony search (HS). The authors manipulated the three methods properly to fit the problem, and investigated their performance (in terms of run-time and solution accuracy) on test cases. The method developed by (Vargas, 2009) was used to evaluate the performance of the previous meta-heuristics, and the planning horizon are varying from 12 up to 48 periods. Computational results show that the used meta-heuristics can give promising solutions in both efficiency and accuracy, if they are properly configured.

(Wong et al., 2012) studied a stochastic dynamic lot sizing problem with asymmetric deteriorating commodity. The objective is to determine the optimal unit cost of material and unit holding cost. The problem comprises a sub-problem of replenishment planning which is NP-hard. The authors use a decision system based on an artificial neural network (ANN) and modified ant colony optimization (ACO) to solve the problem. In particular, ANN is used to learn the simulation results, and the modified ACO is used to find the optimal decision variables. Computational results show that the proposed solving procedure outperforms the classical response surface methodology.

(Şenyigit et al., 2013) attempted to solve the stochastic single-item lot sizing problem by using a variety of artificial neural networks (such as the feed-forward neural network, FF-NN) trained with meta-heuristics (such as the genetic algorithm, GA; and the bee algorithm, BA). The combined approaches were compared with three classical heuristics, respectively the re-

vised Silver Meal (RSM), the revised least unit cost (RLUC), and the cost benefit (CB). Computational results show that RLUC and FF-NN trained with BA outperform the other approaches.

(2) Scenario-based demand

Besides the probabilistically distributed demand assumption, another part of existing literature uses the scenario-based demand assumption, among which some key work has been contributed by (Guan et al., 2006; Guan and Miller, 2008; Guan and Liu, 2010; Guan, 2011).

In Guan's papers, the problem parameters (such as costs, demands and lead times) are assumed to evolve as discrete-time stochastic processes with finite probability space. Let n denote the number of periods in the problem horizon, $1 \leq i \leq t \leq n$. Then the whole information structure can be interpreted as a scenario tree with n stages.

Figure 4.2 illustrates a simple example of scenario tree (also called decision tree). The nodes represent the states of system at a particular instant, while the arcs represent different realizations of random variables (such as demands), and each arc is associated with a realization of stochastic process and a probability of occurrence. For example, as shown in Figure 4.2, the node i in stage t of the tree gives the state of the system that can be distinguished by the information available up to time stage t .

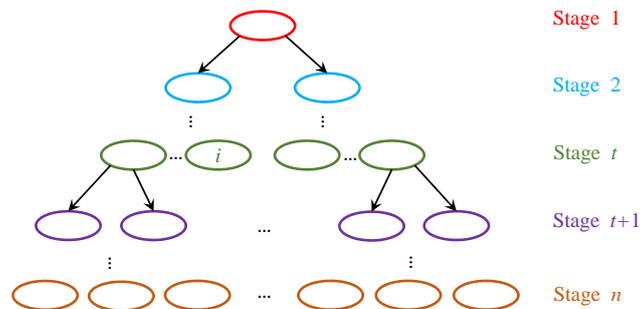


Figure 4.2: A simple illustration of scenario tree

By introducing the architecture of scenario tree, the authors proved a named production-path property which is analogous to the Wagner-Whitin optimality property for the deterministic case. Based on this property, the authors developed a polynomial-time algorithm to find the minimum expected total costs for the single-item uncapacitated lot-sizing problem with scenario-based uncertain demand. The algorithm could be more efficient if the setup cost is not considered.

Service-Level Constraint

In the above papers, the authors suppose that the customer demand should be fulfilled in time, otherwise a penalty cost will be charged. In real-life application, there is an alternative hypothesis that a prescribed service level should be achieved.

In some production systems, the stoppage due to the shortage in material or component supply is quite costly. Moreover, the stockout penalty on the side of the unfulfilled customer demand is expensive. In this case, people often use a service-level constraint instead of the charges of penalty cost which is difficult to evaluate in practice.

Table 4.2 gives the definition of three widely used service levels. In real-life situations, it often happens that on-hand stock cannot fulfill demand and the expected order doesn't arrive in time, then a stockout occurs. Let γ denote the probability of stockout at a certain period, then the service level $SL_1 = 1 - \gamma$. Besides, Figure 4.3 gives a simple illustration to SL_2 and SL_3 . When a service level is considered, it is important to clarify its definition, and follow up the real customer service according to the required level.

Table 4.2: Three service level definitions

Symbol	Alias	Description
SL_1	-	probability of "no stockout" per order cycle, or probability that an order arrives in time
SL_2	fill rate	fraction of demand that can be satisfied immediately from stock on hand
SL_3	ready rate	fraction of time with positive stock on hand

To the best of our knowledge, SL_1 (see Table 4.2) is mostly studied in the literature, and is usually denoted by the Greek letter α .

(Bookbinder and Tan, 1988) remark that for high service-level constraints ($\alpha \geq 0.9$), the probability of observing stockouts at the end of a period is negligible, therefore, the shortage cost can be ignored in the model. On the other side, (Silver et al., 1998) show that high α is a realistic assumption when the stockout penalty is high and independent of the stockout duration.

With the introduction of adequate α , the backorder cost term could be eliminated from the objective function. (Tarim and Kingsman, 2004; Tarim et al., 2011; Tempelmeier, 2011) developed efficient computational solutions on this issue. In addition, (Rekik and Sahin, 2012)

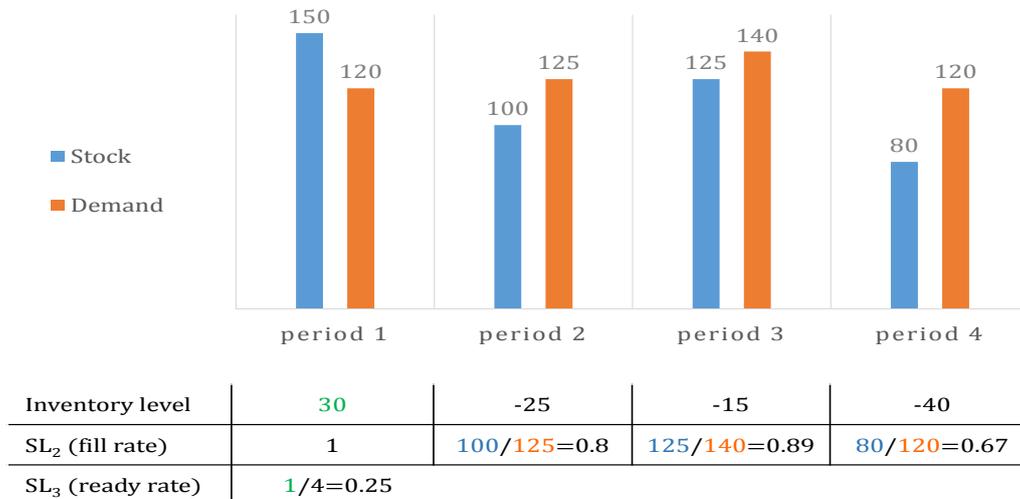


Figure 4.3: Illustration of fill rate and ready rate

investigated the real-life situation that the inventory records in the IT system are inaccurate. The authors compared four scenarios of inventory management systems and provided an optimal joint ordering and inspection policy for each scenario under a service-level constraint.

Conclusion

To conclude, the stochastic lot sizing models with backorders are really different from the deterministic ones. For deterministic models, the backorder may be allowed if it proves cost-saving. When backorder is not allowed, we need to find a good balance between the setup cost and the inventory holding cost. However, when backorder is allowed, the balance should be found among the setup cost, the holding cost and the backorder cost. It is possible to find better reorder points when backorder is allowed, as well as a lower total cost. While for stochastic models, the backorder is more considered as a "defense mechanism" against the unavoidable stockouts caused by unpredictable actual customer requirements. But some natural properties are the same, such as the cost structure and the calculation formula of inventory position.

In particular, there are a few papers investigating the stochastic lot-sizing models (see above sections). When demand is uncertain but with known distributions, some effective solving procedures were proposed in the literature, which provide a good basis for the problem with unknown demand studied in the current thesis. Besides, some authors use the service-level constraint to eliminate the backorder cost term in the objective function, which might also be useful to simplify the stochastic model. Note that the objective of stochastic lot-sizing prob-

lems is to minimize the *expected* total costs, which is a statistical result instead of a specific demand scenario.

4.3 Problem Description

4.3.1 Assumptions

In order to describe the procurement planning problem with backorder permissions, the similar example as presented in Chapter 2 is used.

Example:

Firm A (in France) plans to procure component X from Supplier B (in China) in order to produce Product Y. The transport from B to A takes about 1.5-2 months. Once a procurement order is placed, a mandatory set-up cost occurs. When there is a stockout of component X in A's production process, the production for final-product Y will be interrupted, therefore the customer demand for Y cannot be fully satisfied. The unsatisfied demand is totally backordered and a penalty cost is charged per unit.

The objective is to draw up an optimal procurement planning, which minimizes the expected long-run per-period total procurement costs including set-up, inventory holding and backorders, by specifying two issues: when a procurement order should be placed, and how many component X should be ordered.

The following assumptions are extracted from the example.

Assumptions:

- (a) The planning problem is considered under a periodic-review policy.
- (b) The demand quantity in each period is non-negative, independent and unknown. It might follow certain trend and seasonality which could be forecasted adaptively from the historical data.
- (c) The procurement capacity is unlimited.

- (d) Unsatisfied demand is totally backordered. A penalty cost will be charged for each item backordered per period.
- (e) The inventory cost is assessed at the end of each period, including holding cost or backorder cost. No disposal of inventory is allowed, and the item considered is not perishable.
- (f) For each order placed, the lead time L is assumed to be fixed. L is a positive integer that cannot be neglected. Moreover, the demand forecasts will evolve during the delivery of order.
- (g) A mandatory setup cost occurs when a procurement order is placed.

The mission is to make an optimal procurement plan, with the objective of minimizing the *expected* long-run per-period total costs including setup, inventory holding and backorders. Two principal issues should be specified in the solution: (1) in which period should a procurement order be placed; and (2) how much should be ordered.

4.3.2 Notation

In this chapter, the following notation is used.

Decision Variable:

- x_i : The delivery quantity arriving during period i .
- δ_i : A binary decision variable. $\delta_i = 1$ if $x_i > 0$, or 0 otherwise.
- X_i : The cumulative delivery quantity arriving from period 1 up to and including period i . $X_i = \sum_{t=1}^i x_t$.

Random Variable:

- d_i : The *actual* demand of period i . The exact value of d_i cannot be acquired until the end of period i , thus we define d_i as a random variable at the decision-making phase.

- D_i : The actual cumulative demand quantity counting from period 1 up to and including period i . $D_i = \sum_{t=1}^i d_t$.
- I_i^o : The actual inventory level at the beginning of period i .
- I_i : The actual inventory level at the end of period i . Note that $I_i = I_{i+1}^o$.

Other Parameter:

- h_i : The inventory holding cost (per unit held) which is charged at the end of period i .
- b_i : The backorder cost (per unit unfulfilled) which is charged at the end of period i .
- c_i : The purchase cost (per unit purchased) of the order that is placed in period i .
- K_i : The mandatory setup cost for each procurement order that is placed in period i .

In order to distinguish the points that an order is placed and that it arrives, the subscript " $i - L$ " is used to denote the period in which an order is *placed* to arrive at period i . Period i is called an "*order period*" because an order quantity x_i *arrives*. Note that the decision of x_i is made at period $i - L$, thus the corresponding setup cost and the purchase cost coefficient should be respectively K_{i-L} and c_{i-L} .

4.4 Mathematical Formulation

4.4.1 The Cumulative Inventory Property (CIP) of Backorder Models

As cited in Section 4.2.2, (Gélinas et al., 1995) are the first to apply the idea of *cumulative quantities* in the reformulation of planning problems with backorder permissions.

In this dissertation, we use the capital letters to denote the cumulative quantities. The

cumulative delivery quantity arriving from period 1 to period i is

$$X_i = \sum_{t=1}^i x_t.$$

The actual cumulative demand quantity counting from period 1 to period i is

$$D_i = \sum_{t=1}^i d_t.$$

Using the cumulative quantities, the inventory level at the end of period i can be simplified as

$$I_i = I_1^o + \sum_{t=1}^i (x_t - d_t) = I_1^o + X_i - D_i,$$

where I_1^o is the inventory level at the beginning of period 1, named the initial stock.

The objective of classical lot sizing problems is often to specify the order quantities x_i that minimize the total costs. By introducing the cumulative quantities, the problem can be reformulated and the objective turns into finding the optimal sequence of X_i .

4.4.2 Mathematical Formulation

When the backorder assumption is used, the procurement planning problem with unknown demand can be formulated as follows:

P

$$\min \sum_{i=1}^n [K_{i-L}\delta_i + E(h_i \cdot I_i^+ + b_i \cdot I_i^-) + c_{i-L} \cdot x_i] \quad (4.1)$$

$$s.t. \quad I_i = I_{i-1} + x_i - d_i, \quad \forall i \quad (4.2)$$

$$I_i^+ = \max\{I_i, 0\}, \quad \forall i \quad (4.3)$$

$$I_i^- = \max\{-I_i, 0\}, \quad \forall i \quad (4.4)$$

$$0 \leq x_i \leq M\delta_i, \quad \forall i \quad (4.5)$$

$$\delta_i \in \{0, 1\}, \quad \forall i \quad (4.6)$$

where $E(t)$ denotes the expectation of the random variable t .

As presented in the above section, by using the cumulative quantities, the recursion formula (4.2) can be rewritten as

$$I_i = I_1^o + X_i - D_i. \quad (4.7)$$

Then the inventory levels become

$$I_i^+ = \max\{I_1^o + X_i - D_i, 0\}, \quad (4.8)$$

and

$$I_i^- = \max\{D_i - I_1^o - X_i, 0\}. \quad (4.9)$$

Therefore, the inventory variables I_i , I_i^+ and I_i^- can be eliminated from the formulation **P**.

Let L_i denote the expected inventory cost (including holding and backorder) incurred in period i , we have

$$\begin{aligned} L_i(X_i) &= E\left(h_i \cdot I_i^+ + b_i \cdot I_i^-\right) \\ &= E\left(h_i \cdot \max\{I_1^o + X_i - D_i, 0\} + b_i \cdot \max\{D_i - I_1^o - X_i, 0\}\right) \\ &= h_i \cdot \int_{-\infty}^{I_1^o + X_i} (I_1^o + X_i - D_i) f_i(D_i) dD_i + b_i \cdot \int_{I_1^o + X_i}^{+\infty} (D_i - I_1^o - X_i) f_i(D_i) dD_i, \end{aligned} \quad (4.10)$$

where $f_i(D_i)$ denotes the probability density function (PDF) of D_i .

In addition, the total purchase cost can be rewritten as

$$\sum_{i=1}^n c_{i-L} \cdot x_i = \sum_{i=1}^n c_{i-L} \cdot (X_i - X_{i-1}) = \sum_{i=1}^n (c_{i-L} - c_{i-L+1}) \cdot X_i, \quad (4.11)$$

where we define $c_{n-L+1} = c_{-L} = 0$.

Then the problem **P** can be reformulated as:

RP

$$\min \quad \sum_{i=1}^n [K_{i-L}\delta_i + L_i(X_i) + (c_{i-L} - c_{i-L+1}) \cdot X_i] \quad (4.12)$$

$$\begin{aligned} s.t. \quad & 0 \leq X_i - X_{i-1} \leq M\delta_i, \quad \forall i \quad (4.13) \\ & (4.6), (4.10) \end{aligned}$$

For procurement planning problems in global sourcing, the most distinctive feature is the significant demand uncertainty due to long distance. Although it is virtually impossible to forecast the future demand accurately, good demand forecasts can bring a great help in the subsequent procurement planning.

Section 3.5 has presented an adaptive optimization framework to solve the long-distance procurement planning problem. The proposed framework is based on the fact that the demand forecasts are being regularly updated. In particular, Section 3.5.1 has described the rolling-horizon scheme applied to the overall procurement planning problem, and Section 3.5.2 has specified the concrete processing steps to solving each sub-problem.

As presented in Section 3.5.2, after the preprocessing of demand forecasting and standardization (see Figure 3.5), the planning problem in the sub-horizon η_i can be transformed into a standard single-item stochastic lot-sizing problem.

In order to formulate the sub-problem, the following notation is used.

Notation:

- d_k^i : The actual *net* demand quantity of the k^{th} period in sub-horizon η_i .
- D_k^i : The actual cumulative *net* demand quantity counting from the 1st to the k^{th} period in sub-horizon η_i .
- μ_k^i : The mean value of random variable d_k^i .
- σ_k^i : The standard deviation of d_k^i .
- y_k^i : The interim order decision which is supposed to arrive during the k^{th} period in sub-horizon η_i .
- Y_k^i : The interim cumulative order decision counting from the 1st to the k^{th} period

in sub-horizon η_i .

z_k : A binary variable. $z_k = 1$, if $y_k^i > 0$; 0 otherwise.

Besides, the cost parameters K, h, b, c are still used to denote respectively the setup cost, the unit inventory holding cost, the unit backorder cost and the unit purchase cost. In order to distinguish them from the cost parameters corresponding to the overall planning problem, we use a superscript i to indicate sub-horizon η_i , and a subscript k to denote the k^{th} period in the sub-horizon. As a result,

$$h_k^i = h_{i+k-1}, \quad b_k^i = b_{i+k-1}.$$

And

$$K_k^i = K_{i+k-1-L}, \quad c_k^i = c_{i+k-1-L},$$

here for simplification, we eliminate the term " $-L$ " from the subscript by adequate mathematical transformation.

The planning problem in the sub-horizon η_i can be formulated as:

Sub-P

$$\min \quad \sum_{k=1}^n \left[K_k^i z_k + L_k^i(Y_k^i) + (c_k^i - c_{k+1}^i) \cdot Y_k^i \right] \quad (4.14)$$

$$s.t. \quad 0 \leq Y_k^i - Y_{k-1}^i \leq M z_k, \quad \forall k \quad (4.15)$$

$$z_k \in \{0, 1\}, \quad \forall k \quad (4.16)$$

$$L_k^i(Y_k^i) = h_k^i \cdot \int_{-\infty}^{Y_k^i} (Y_k^i - D_k^i) f_k^i(D_k^i) dD_k^i + b_k^i \cdot \int_{Y_k^i}^{+\infty} (D_k^i - Y_k^i) f_k^i(D_k^i) dD_k^i, \quad \forall k \quad (4.17)$$

where f_k^i denotes the PDF of D_k^i . Note that in the transformed standard stochastic lot-sizing problem in the sub-horizon, the initial stock equals to zero.

Equation (4.17) can be simplified:

$$\begin{aligned} L_k^i(Y_k^i) &= h_k^i \cdot \int_{-\infty}^{Y_k^i} (Y_k^i - D_k^i) f_k^i(D_k^i) dD_k^i + b_k^i \cdot \left[\int_{-\infty}^{+\infty} (D_k^i - Y_k^i) f_k^i(D_k^i) dD_k^i - \int_{-\infty}^{Y_k^i} (D_k^i - Y_k^i) f_k^i(D_k^i) dD_k^i \right] \\ &= (h_k^i + b_k^i) \cdot \int_{-\infty}^{Y_k^i} (Y_k^i - D_k^i) f_k^i(D_k^i) dD_k^i - b_k^i \cdot (Y_k^i - U_k^i), \end{aligned} \quad (4.18)$$

where U_k^i denotes the mean of D_k^i , $U_k^i = \int_{-\infty}^{+\infty} D_k^i f_k^i(D_k^i) dD_k^i$.

Assuming that s and $t+1$ ($1 \leq s \leq t \leq n$) are two consecutive periods in which a procurement order lot arrives, we have $Y_{s-1}^i < Y_s^i = Y_{s+1}^i = \dots = Y_t^i < Y_{t+1}^i$. For simplification, we use Y to substitute Y_s^i . The expected total cost incurred in the periods $\{s, s+1, \dots, t\}$ of the sub-horizon η_i is given as

$$\mathcal{G}(Y, s, t) = K_s^i + \sum_{k=s}^t L_k^i(Y) + (c_s^i - c_{t+1}^i) \cdot Y. \quad (4.19)$$

Let Y_{st} denote the cumulative order quantity from the 1^{st} period up to and including the t^{th} period in the sub-horizon, while s is the last order period before period $t+1$. Suppose k_1, k_2, \dots, k_l are the order periods that $z_k = 1$, where $1 \leq k_1 \leq k_2 \leq \dots \leq k_l \leq n$. The objective function (4.14) can be rewritten as

$$\min \sum_{\tau=1}^l \mathcal{G}(Y_{k_\tau, (k_{\tau+1})-1}, k_\tau, (k_{\tau+1})-1). \quad (4.20)$$

Note that $k_0 = 0$, and $k_{l+1} = n+1$.

4.5 Optimization Algorithm

By substituting Equation (4.18) into Equation (4.19), we have

$$\begin{aligned} \mathcal{G}(Y, s, t) &= K_s^i + \sum_{k=s}^t \left[(h_k^i + b_k^i) \cdot \int_{-\infty}^Y (Y - D_k^i) f_k^i(D_k^i) dD_k^i - b_k^i \cdot (Y - U_k^i) \right] + (c_s^i - c_{t+1}^i) \cdot Y \\ &= \left[K_s^i + \sum_{k=s}^t (b_k^i \cdot U_k^i) \right] + \left(c_s^i - c_{t+1}^i - \sum_{k=s}^t b_k^i \right) \cdot Y + \sum_{k=s}^t \left[(h_k^i + b_k^i) \cdot \int_{-\infty}^Y (Y - D_k^i) f_k^i(D_k^i) dD_k^i \right]. \end{aligned} \quad (4.21)$$

$L_k^i(Y_k^i)$ is smooth and convex, then $\mathcal{G}(Y, s, t)$ is smooth and convex. Therefore, $\mathcal{G}(Y, s, t)$ is derivable:

$$\begin{aligned} \frac{d\mathcal{G}(Y, s, t)}{dY} &= \left(c_s^i - c_{t+1}^i - \sum_{k=s}^t b_k^i \right) + \sum_{k=s}^t (h_k^i + b_k^i) \cdot \int_{-\infty}^Y f_k^i(D_k^i) dD_k^i \\ &= \left(c_s^i - c_{t+1}^i - \sum_{k=s}^t b_k^i \right) + \sum_{k=s}^t (h_k^i + b_k^i) \cdot F_k^i(Y), \end{aligned} \quad (4.22)$$

where F_k^i denotes the cumulative distribution function (CDF) of D_k^i , $F_k^i(Y) = \int_{-\infty}^Y f_k^i(D_k^i) dD_k^i$.

We define $A(Y, s, t) = \sum_{k=s}^t (h_k^i + b_k^i) \cdot F_k^i(Y)$, and $B(s, t) = \sum_{k=s}^t b_k^i + c_{t+1}^i - c_s^i$. Then

$$\frac{d\mathcal{G}(Y, s, t)}{dY} = A(Y, s, t) - B(s, t) \triangleq \mathcal{G}'(Y, s, t).$$

Note that the CDF of a distribution is always nondecreasing, and $F_k^i(Y) \in (0, 1)$, $F_k^i(-\infty) = 0$, $F_k^i(+\infty) = 1$. Thus $A(Y, s, t)$ is nondecreasing, and $A(Y, s, t) \in \left(0, \sum_{k=s}^t (h_k^i + b_k^i)\right)$.

Proposition 4.1 *Suppose $s, t+1$ ($1 \leq s \leq t \leq n$) are the two consecutive order periods, $z_s = z_{t+1} = 1$, $z_{s+1} = \dots = z_t = 0$. Let $(\mathbf{Y}^*, \mathbf{z}^*)$ be the optimal solution of Sub-P. $\mathbf{Y}^* = (Y_1^*, Y_2^*, \dots, Y_n^*)$, $\mathbf{z}^* = (z_1^*, z_2^*, \dots, z_n^*)$.*

(1) *If $B(s, t) \leq 0$, there does not exist an optimal solution $(\mathbf{Y}^*, \mathbf{z}^*)$ such that $z_s^* = z_{t+1}^* = 1$ and $z_{s+1}^* = \dots = z_t^* = 0$;*

(2) *If $B(s, t) \in \left(0, \sum_{k=s}^t (h_k^i + b_k^i)\right)$, and if there exists an optimal solution $(\mathbf{Y}^*, \mathbf{z}^*)$ such that $z_s^* = z_{t+1}^* = 1$ and $z_{s+1}^* = \dots = z_t^* = 0$, then $\mathcal{G}'(Y_s^*, s, t) = 0$;*

(3) *If $B(s, t) \geq \sum_{k=s}^t (h_k^i + b_k^i)$, there does not exist an optimal solution $(\mathbf{Y}^*, \mathbf{z}^*)$ such that $z_s^* = z_{t+1}^* = 1$ and $z_{s+1}^* = \dots = z_t^* = 0$.*

Proof. Suppose there exists an optimal solution $(\mathbf{Y}^*, \mathbf{z}^*)$ for Sub-P such that $z_s^* = z_{t+1}^* = 1$ and $z_{s+1}^* = \dots = z_t^* = 0$.

(1) If $B(s, t) \leq 0$, then $\mathcal{G}'(Y, s, t) > 0$, therefore $\mathcal{G}(Y, s, t)$ is monotonically increasing. The mathematically optimal solution to minimize $\mathcal{G}(Y, s, t)$ is $Y_s^* = -\infty$. However, it is more convenient to set $Y_s^* = Y_{s-1}^*$ according to Equation (4.15), then s is not an order period, $z_s^* = 0$, which is a contradiction.

(2) If $B(s, t) \in (0, \sum_{k=s}^t (h_k^i + b_k^i))$, then $\mathcal{G}'(Y, s, t) = 0$ is solvable. Since $\mathcal{G}'(Y, s, t)$ is nondecreasing with Y , let Y_{st}^* be its smallest root, then $\mathcal{G}(Y, s, t)$ gets its mathematical minimum at Y_{st}^* . If $Y_{st}^* > Y_{s-1}^*$, then $Y_s^* = Y_{st}^*$; otherwise, we can prove that the solution (\mathbf{Y}, \mathbf{z}) such that $z_s = z_{t+1} = 1$ and $z_{s+1} = \dots = z_t = 0$ is not optimal for Sub-P, see proof in Section 4.5.1.

(3) If $B(s, t) \geq \sum_{k=s}^t (h_k^i + b_k^i)$, then $\mathcal{G}'(Y, s, t) < 0$, therefore $\mathcal{G}(Y, s, t)$ is monotonically decreasing. The mathematically optimal solution to minimize $\mathcal{G}(Y, s, t)$ is $Y_s^* = +\infty$. However, with the following transformation:

$$B(s, t) \geq \sum_{k=s}^t (h_k^i + b_k^i) \iff \sum_{k=s}^t b_k^i + c_{t+1}^i - c_s^i \geq \sum_{k=s}^t (h_k^i + b_k^i) \iff c_s^i + \sum_{k=s}^t h_k^i \leq c_{t+1}^i,$$

we find that, it is more economic to buy the item at unit price c_s^i and hold the inventory until the end of period t (i.e. the beginning of period $t+1$) than to buy it directly at unit price c_{t+1}^i . In other words, period $t+1$ cannot be an optimal order period. Let q denote the supposed order quantity to arrive at period $t+1$, then we can save a cost of $c_{t+1}^i - (c_s^i + \sum_{k=s}^t h_k^i) > 0$ per unit if we add a quantity q to the order period s , besides, the setup cost corresponding to period $t+1$ can also be saved. Therefore, we have $z_{t+1}^* = 0$, which is a contradiction. \square

Lemma 4.1 *If $Y_{r, s-1}^* > Y_{st}^*$ for some r, s, t ($1 \leq r < s \leq t \leq n$), then there does not exist an optimal solution $(\mathbf{Y}^*, \mathbf{z}^*)$ for Sub-P such that $z_r^* = z_s^* = z_{t+1}^* = 1$, $z_{r+1}^* = z_{r+2}^* = \dots = z_{s-1}^* = z_{s+1}^* = z_{s+2}^* = \dots = z_t^* = 0$.*

Proof. Suppose there exists an optimal solution $(\mathbf{Y}^*, \mathbf{z}^*)$ such that for some r, s, t ($1 \leq r < s \leq t \leq n$), $z_r^* = z_s^* = z_{t+1}^* = 1$, $z_{r+1}^* = z_{r+2}^* = \dots = z_{s-1}^* = z_{s+1}^* = z_{s+2}^* = \dots = z_t^* = 0$, and $Y_{r, s-1}^* > Y_{st}^*$. According to Proposition 4.1(2), we have $Y_{s-1}^* = Y_{r, s-1}^* > Y_{st}^* = Y_s^*$, which does not satisfy the constraint equation (4.15) of Sub-P. \square

4.5.1 An Optimal Solution Procedure

Proposition 4.1 and Lemma 4.1 give the instructions to find the interim optimal cumulative order quantity Y_{st}^* , provided that s and $t+1$ are consecutive order periods. As a result, a forward dynamic programming procedure can be used to solve Sub-P.

Let Γ_t denote the minimum expected total cost incurring from the 1st to the t^{th} period in

the sub-horizon. We have:

$$\begin{aligned} \Gamma_0 &= 0; \\ \Gamma_t &= \min_{\substack{1 \leq s \leq t, \\ Y_{s-1}^* < Y_{st}^*}} \{\Gamma_{s-1} + \mathcal{G}^*(s, t)\}, \end{aligned} \quad (4.23)$$

where $\mathcal{G}^*(s, t)$ is the minimal value of $\mathcal{G}(Y, s, t)$, $\mathcal{G}^*(s, t) = G(Y_{st}^*, s, t)$.

Now we will prove that when $Y_{st}^* \leq Y_{s-1}^*$, the solution (\mathbf{Y}, \mathbf{z}) such that $z_s = z_{t+1} = 1$ and $z_{s+1} = \dots = z_t = 0$ can not be optimal for Sub-P, which is a complementary proof for Proposition 4.1(2).

Proof. Suppose there exists an optimal solution $(\mathbf{Y}^*, \mathbf{z}^*)$ for Sub-P such that $z_s^* = z_{t+1}^* = 1$ and $z_{s+1}^* = \dots = z_t^* = 0$. Let Y_{s-1} and Y_{st} denote respectively the cumulative order quantity at period $s-1$ and periods s, \dots, t . We define $\Gamma_{s,t} = \Gamma_{s-1}(Y_{s-1}) + \mathcal{G}(Y_{st}, s, t)$. $\Gamma_{s-1}(Y_{s-1})$ is convex and has its minimum at Y_{s-1}^* . $\mathcal{G}(Y_{st}, s, t)$ is also convex and has its minimum at Y_{st}^* . If $Y_{st}^* > Y_{s-1}^*$, let $Y_{s-1} = Y_{s-1}^*$ and $Y_{st} = Y_{st}^*$, then $\Gamma_{s,t}$ gets its minimum. If $Y_{st}^* \leq Y_{s-1}^*$, there exists a Y^{**} at which $\Gamma_{s,t}$ gets its minimum, and $Y^{**} \in (Y_{st}^*, Y_{s-1}^*)$. Therefore, let $Y_{s-1} = Y_{st} = Y^{**}$, $\Gamma_{s,t}$ will get its minimum. In this case, $z_s = 0$, which is a contradiction. \square

Figure 4.4 describes the detailed computing procedure for an optimal solution $(\mathbf{Y}^*, \mathbf{z}^*)$. In particular, the flow chart (1) is used to calculate the minimum expected total cost incurred in the sub-horizon, and the flow chart (2) is used to find the optimal order periods and corresponding order quantities.

As shown in Figure 4.4, the key step is to find Y_{st}^* at which the step expected cost $\mathcal{G}(Y, s, t)$ is minimized. Here s and $t+1$ are assumed to be consecutive order periods. Since we use a forward solution procedure, the interim optimal cumulative order quantity Y_{jk}^* is known, for any $1 \leq j \leq k < s$.

For each $s \leq t$, $B(s, t)$ can be calculated. Proposition 4.1(1)(3) and Lemma 4.1 can be used to eliminate a part of unfeasible (s, t) pairs including which the overall order sequence cannot be optimal.

For the remaining (s, t) pairs, $A(s, t)$ can be specified. Proposition 4.1(2) can be used to find Y_{st}^* , if any. For a given (s, t) , since we use a forward solution procedure, the interim

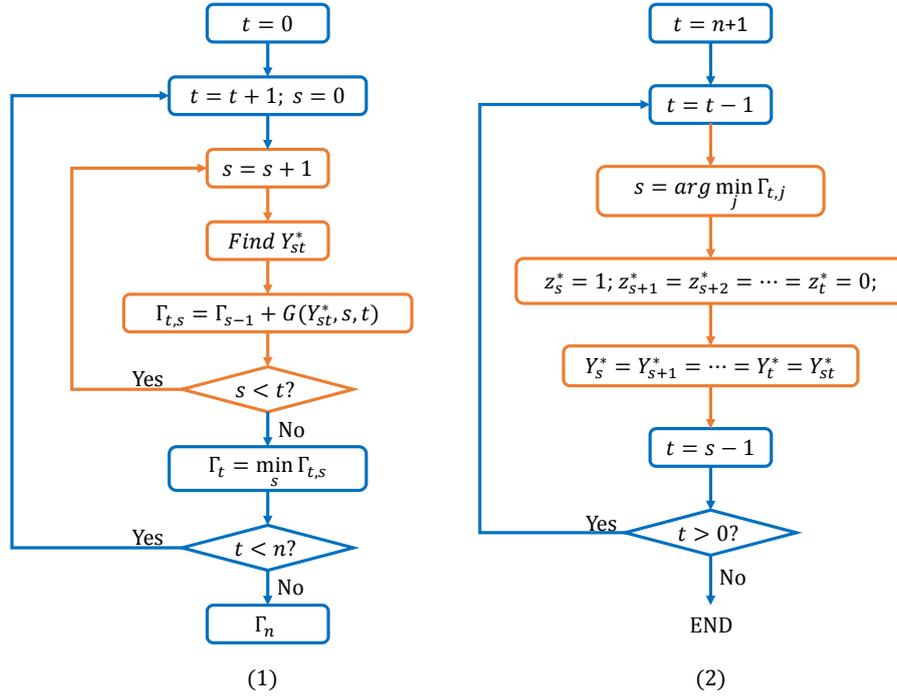


Figure 4.4: The dynamic programming procedure

optimal solution considering only the first $s - 1$ periods of the sub-horizon has been determined. Let Y_{s-1}^* denote the interim optimal cumulative order quantity of period $s - 1$. If $B(s, t) \in (0, \sum_{k=s}^t (h_k^i + b_k^i))$ and $Y_{st}^* > Y_{s-1}^*$, let $Y_s^* = Y_{st}^*$; the other cases are unnecessary to consider, since the order sequence such that $z_s = z_{t+1} = 1$ and $z_{s+1} = \dots = z_t = 0$ cannot be optimal.

For the feasible (s, t) pairs, the recursion formula (4.23) is used to find the minimum expected total cost incurred in the sub-horizon, that is, Γ_n . Afterwards, using a back-tracing procedure, the optimal order periods and the corresponding order quantities can be specified.

4.5.2 A Simplified 2-Stage Heuristic Method (S2S Method)

The above section has presented an optimal solution procedure for Sub-P. It can be regarded as a benchmark to evaluate other near-optimal methods.

The most time-consuming step is to solve the nonlinear equation $\mathcal{G}'(Y, s, t) = 0$. Note that $1 \leq s \leq t \leq n$, the number of such nonlinear equations is on the order of n^2 . Moreover, the solution procedure for Sub-P will be practically deployed with a rolling horizon scheme as presented in Chapter 3. In order to improve computational efficiency, we have developed a

simplified heuristic algorithm as follows.

The core idea of the proposed heuristic is to decouple the deterministic part (i.e. means) and the stochastic part (i.e. standard deviations) in Sub-P, and determine the optimal procurement plan of the sub-horizon in two stages:

- (1) determine the interim optimal procurement plan $y_1^i, y_2^i, \dots, y_n^i$ while considering the "determinate" demands $\mu_1^i, \mu_2^i, \dots, \mu_n^i$;
- (2) find an adequate safety stock to cope with the probable stockout caused by demand uncertainties.

The interim optimal plan $y_1^i, y_2^i, \dots, y_n^i$ can be easily determined by employing any classical deterministic dynamic lot sizing algorithm. However, the selection of safety stock is not really explicit.

The Safety Stock (SS) Policy

A *safety stock* (SS), or buffer stock, is a level of extra inventory held as a buffer against the mismatch between the forecast and actual demand due to uncertainties. It can also be interpreted as an additional stock that can be used as a protection against demand variations (Axsäter, 2006).

In many practical applications, companies often use an identical safety stock through the entire planning horizon. In recent years, dynamic safety stocks have been considered. (Kanet et al., 2009) performed an empirical study in 190 U.S. producers and retailers, among which most companies think that improving service level is the primary goal of inventory management. The study also demonstrates that significant savings can be achieved by employing dynamically planned safety stocks.

Adequate safety stocks permit business operations to proceed according to their plans, serving as an insurance against stockouts (Monk and Wagner, 2008). Too much safety stock can result in high inventory holding cost, while too little safety stock may result in stockout penalty. Therefore, finding the adequate safety stock is essential.

Existing Safety Stock Determining Approaches

A large amount of research has focused on determining safety stocks, of which the majority is based on statistical analysis and simplifications. The safety stocks can also be approximately

calculated, which has sufficient accuracy and ensures a slightly higher stock availability.

In many applications, the safety stock is determined under a prescribed *service level* (SL) constraint. The commonly used service level definitions can be found in Table 4.2. For different definitions of service level, the safety stock is differently determined.

For the service level defined as SL_1 in Table 4.2, a frequently used rule for calculating the safety stock is

$$SS_1 = SF(SL_1) \cdot \sigma_d, \quad (4.24)$$

where the demand is supposed to be normally distributed with average μ_d and standard deviation σ_d , and SF denotes the service factor that depends on the service level SL_1 (Graves et al., 1993).

In Equation (4.24), the safety stock SS_1 is defined as a proportional function of the service level SL_1 which can be mathematically expressed as the percentage of total demands that is served punctually. The service factor SF can be determined by the inverse standard normal distribution:

$$SF(x) = \Phi^{-1}(x). \quad (4.25)$$

Approximately, the service factor can be calculated as $SF(\alpha) \approx (2\alpha - 1)/(1 - \alpha)^{0.2}$ for $\alpha > 50\%$.

For the fill rate and ready rate constraints (respectively defined as SL_2 and SL_3 in Table 4.2), when a (R, Q) policy is used, we can approximately calculate the service level as follows:

$$SL_2 = SL_3 \approx 1 - \frac{\sigma_d}{Q} G\left(\frac{R - \mu_d}{\sigma_d}\right), \quad (4.26)$$

where Q denotes the order quantity, R denotes the reorder point, and the loss function $G(x) = \int_x^{+\infty} (t - x)\varphi(t)dt$. $\varphi(t)$ is the probability density function of standard normal distribution. See (Axsäter, 2006) for detailed discussion. In this case, the reorder point R can be considered as a safety stock.

Among the three service levels as described in Table 4.2, SL_1 is also called " α -availability", interpreted as the ability to deliver during the replenishment time, while SL_2 is as well called " β -availability", interpreted as the stock availability.

(Alicke, 2005) extended the method (4.24) by introducing the lead time L :

$$SS_1^{(1)} = SF(SL_1) \cdot \sigma_d \cdot \sqrt{L}. \quad (4.27)$$

He also provided a similar calculation rule using forecast errors:

$$SS_1^{(2)} = SF(SL_1) \cdot \sigma_f \cdot \sqrt{L}, \quad (4.28)$$

where σ_f denotes the standard deviation of the demand forecast errors during the lead time L . σ_f is calculated statistically based on historical demand data, thus is independent of a specific demand scenario.

(Axsäter, 2006) considered both stochastic demands and stochastic lead times. The author proved that when demand quantities and lead times are independent, an optimal safety stock policy can be given as:

$$SS_1^{(3)} = SF(SL_1) \cdot \sqrt{\sigma_d^2 \cdot \mu_L + \mu_d^2 \cdot \sigma_L^2}, \quad (4.29)$$

where μ_L and σ_L denote respectively the average and standard deviation of lead time L . The method expressed by (4.29) is derived from the *theorem of large numbers*, as well as the *law of error propagation* and the *fluctuation law*. Further discussions could be referred to (Song, 1994; Zipkin, 2000).

(Gudehus and Kotzab, 2012) proposed an approach to calculate the service factor SF_β for the required β -availability situation:

$$SF_\beta(SL_2) = \Phi^{-1} \left(1 - \frac{(1 - SL_2) \cdot Q}{L \cdot \mu_d} \right). \quad (4.30)$$

As a result, when demands and lead times are both stochastic, the safety stock can be calculated by:

$$SS_2 = SF_\beta(SL_2) \cdot \sqrt{\sigma_d^2 \cdot \mu_L + \mu_d^2 \cdot \sigma_L^2}. \quad (4.31)$$

The alternative to a service level is the stockout cost, such as the backorder cost per unit per period in this chapter. (Axsäter, 2006) proved that the relation between service levels and

backlogging cost is:

$$SL_2 = SL_3 = \frac{b}{h+b}, \quad (4.32)$$

where h and b denote respectively the inventory holding and backorder cost per unit per period.

Formula (4.32) is very useful in practice, when the service level constraints or stockout costs are not explicit in practical inventory management. Besides, formula (4.32) can also be used to evaluate whether a certain shortage cost is reasonable or not.

An Optimal Approach to Determine Safety Stock for Sub-P

The above section has surveyed the existing widely-used methods to determine safety stocks. In the Sub-P studied in this chapter, a time-varying backorder cost is charged instead of using a service-level constraint, under a periodic-review policy. Due to the lack of applicable solution in the literature, we develop the following method to determine the optimal safety stock.

As the proposed S2S heuristic method is applied to Sub-P, the interim optimal plan $y_1^i, y_2^i, \dots, y_n^i$ is firstly determined. Since the backorder is allowed, it is possible that $y_1^i = 0$.

Proposition 4.2 *If $y_1^i = 0$ in the interim optimal plan determined by the first stage, it is unnecessary to find an optimal safety stock for the sub-horizon η_i .*

Proof. Consider Sub-P for the sub-horizon η_i . If $y_1^i = 0$, then the demand of the 1st period is fully backordered. The expected backorder cost assessed at the end of this period is $b_1^i \mu_1^i$.

If we start a compulsory order for the safety stock, then at least a setup cost K_1^i is charged. Since y_1^i, \dots, y_n^i is the interim optimal plan considering the "determinate" demands μ_1^i, \dots, μ_n^i , it is obvious that $b_1^i \mu_1^i < K_1^i$. Note that if $b_1^i \mu_1^i \geq K_1^i$, it will be more economic to order μ_1^i units at the 1st period, that is, $y_1^i > 0$, which is a contradiction.

Therefore, it is not optimal to place an order specifically for the safety stock at the 1st period. Since only the order decision for the 1st period of the sub-horizon will be actually executed in a rolling horizon scheme, there is no need to find safety stocks applicable to the subsequent periods of the sub-horizon. \square

When y_1^i, \dots, y_n^i have been determined, the objective function (4.14) can be rewritten as

$$\min \sum_{k=1}^n L_k^i(SS), \quad (4.33)$$

where SS denotes the safety stock, and

$$L_k^i(SS) = (h_k^i + b_k^i) \cdot \int_{-\infty}^{SS + Y_k^i} (SS + Y_k^i - D_k^i) f_k^i(D_k^i) dD_k^i - b_k^i \cdot (SS + Y_k^i - U_k^i). \quad (4.34)$$

Note that $Y_k^i = \sum_{j=1}^k y_j^i$, $L_k^i(SS)$ is smooth and convex, then $\sum_{k=1}^n L_k^i(SS)$ is also smooth and convex. We have

$$\frac{d \sum_{k=1}^n L_k^i(SS)}{d SS} = \sum_{k=1}^n (h_k^i + b_k^i) \cdot F_k^i(SS + Y_k^i) - \sum_{k=1}^n b_k^i. \quad (4.35)$$

The optimal safety stock can be determined by solving the following equation:

$$\sum_{k=1}^n (h_k^i + b_k^i) \cdot F_k^i(SS + Y_k^i) = \sum_{k=1}^n b_k^i. \quad (4.36)$$

4.5.3 The Normal Distribution Case

When the demands d_1^i, \dots, d_n^i follow independent normal distributions, the cumulative demand $D_k^i = \sum_{j=1}^k d_j^i$ also follows normal distribution, with the mean:

$$U_k^i = \sum_{j=1}^k \mu_j^i,$$

and the standard deviation:

$$\rho_k^i = \sqrt{\sum_{j=1}^k (\sigma_j^i)^2}.$$

Let φ and Φ respectively denote the PDF and CDF of the standard normal distribution, we

have

$$\varphi(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2},$$

and

$$\Phi(x) = \int_{-\infty}^x \phi(t)dt.$$

Then the PDF and CDF of D_k^i can be described as

$$f_k^i(t) = \frac{1}{\rho_k^i} \varphi\left(\frac{t - U_k^i}{\rho_k^i}\right),$$

and

$$F_k^i(x) = \Phi\left(\frac{x - U_k^i}{\rho_k^i}\right).$$

Note that for normal distribution, if $x \geq U_k^i + 3\rho_k^i$, $F_k^i(x)$ is very nearly equal to 1. Then Equation (4.36) will be fairly simple to solve.

4.6 Numerical Examples

This section illustrates numerical examples of the proposed solution methods.

4.6.1 A 5-Period Planning Example and the Optimal Solution

Consider a 5-period planning problem as follows. For simplification, the holding cost h_i is taken to be 1 (per unit per period) in all periods, while the backorder cost b_i is taken to be 9 (per unit per period). We suppose that the demand in each period is normally distributed. Take the demand forecast as the mean demand, and the forecast accuracy as the standard deviation. Table 4.3 presents a sample set of data. Note that the cumulative demand $D_i = \sum_{t=1}^i d_t$ also follows a normal distribution, whose average and standard deviation are also shown in Table 4.3.

Table 4.4 presents the optimal cumulative order quantity Y_{st} that minimizes the expected

Table 4.3: Demand data set

Period i	Set-up Cost K_{i-L}	Period Demand		Cumulative Demand	
		Mean (μ_i)	Stand.Dev. (σ_i)	Mean ^a (U_i)	Stand.Dev. ^b (ρ_i)
1	85	69	7.7	69	7.7
2	102	29	3.2	98	8.3
3	102	36	4.0	134	9.2
4	101	61	6.8	195	11.5
5	98	61	6.8	256	13.3

^aThis is the cumulative sum of mean demand.

^bThe standard deviation of a sum follows a square root law

total costs $\mathcal{G}(Y, s, t)$ incurred in periods $s, s+1, \dots, t$, knowing that the consecutive procurement orders are placed in period s and $t+1$, and y_1, y_2, \dots, y_{s-1} have already been determined to minimize the expected total costs Γ_{s-1} incurred in periods $1, 2, \dots, s-1$.

Table 4.4: Optimal accumulative order quantity Y_{st}

$s \backslash t$	1	2	3	4	5
1	79	105	139	198	256
2		109	142	201	259
3			146	205	263
4				210	267
5					273

Table 4.5 presents the minimum expected total cost \mathcal{G}_{st}^* corresponding to the optimal cumulative order quantity Y_{st} as listed in Table 4.4.

Then the following expected costs can be calculated: $\Gamma_{st} = \Gamma_{s-1} + \mathcal{G}_{st}^*$.

Using recursion formula (4.23):

$$\Gamma_0 = 0,$$

$$\Gamma_t = \min_{1 \leq s \leq t} \Gamma_{st},$$

we find the optimal order sequence is: $z_1^* = z_2^* = z_4^* = 1, z_3^* = z_5^* = 0$ (see Table 4.6). The order

quantities are respectively:

$$y_4^* = Y_{45} - Y_{24} = 66,$$

$$y_2^* = Y_{24} - Y_{11} = 122,$$

$$y_1^* = Y_{11} = 79.$$

The minimum expected total cost of planning horizon is $C^* = 417.6$.

Table 4.5: Cost \mathcal{G}_{st}^* associated with optimal order cycles

$s \backslash t$	1	2	3	4	5
1	98.5	137.3	218.1	413.3	666.1
2		116.6	164.0	299.9	494.7
3			118.2	195.1	331.5
4				121.1	199.4
5					121.5

Table 4.6: Network optimization, minimum expected costs Γ_{st}

$s \backslash t$	1	2	3	4	5
1	98.5	137.3	218.1	413.3	666.1
2		215.1	262.5	298.4	593.2
3			255.5	332.5	468.8
4				339.2	417.6
5					453.9
Predecessor period s	1	1	1	2	4
Minimum cost up to t	98.5	137.3	218.1	298.4	417.6

4.6.2 Results of Simplified 2-Stage Heuristic Method (S2S Method)

The same demand data set in the above section is used.

By employing the famous Wagner-Whitin algorithm, we find the interim optimal procurement plan as: $y_1 = 134$, $y_4 = 122$, $y_2 = y_3 = y_5 = 0$. Then we have $Y_1 = Y_2 = Y_3 = 134$, $Y_4 = Y_5 = 256$.

According to Equation 4.36, we have

$$\Phi\left(\frac{SS+65}{7.7}\right) + \Phi\left(\frac{SS+36}{8.3}\right) + \Phi\left(\frac{SS}{9.2}\right) + \Phi\left(\frac{SS+61}{11.5}\right) + \Phi\left(\frac{SS}{13.3}\right) = 4.5.$$

Due to its physical meaning, $SS \geq 0$. For standard normal distribution, we have $\Phi(\tau) \approx 1$ for $\tau > 3$. Thus we have the approximate equation as follows:

$$\Phi\left(\frac{SS}{9.2}\right) + \Phi\left(\frac{SS}{13.3}\right) = 1.5. \quad (4.37)$$

By solving the approximate equation, we obtain $SS = 7$. The the final procurement plan is: $y_1 = 141, y_4 = 122, y_2 = y_3 = y_5 = 0$.

Using this procurement plan, we can calculate the expected inventory cost of each period: $C_1 = 72, C_2 = 43, C_3 = 18.8, C_4 = 68, C_5 = 32.2$.

The expected total cost of the planning horizon is: $C = K_1 + K_4 + \sum_{t=1}^5 C_t = 420$.

Compared to the optimal planning algorithm presented in Section 4.5.1, the relative error of simplified heuristic is:

$$\left| \frac{C - C^*}{C^*} \right| \approx 0.6\%.$$

Note that in the optimal planning method, we need to solve $\mathcal{O}(n^2)$ nonlinear equations (n is the number of periods involved in the planning horizon). While in the simplified heuristic method, only one nonlinear equation like (4.37) needs to be solved. Therefore, the proposed simplified heuristic can give a near-optimal procurement plan in a much more efficient manner, by sacrificing a tiny profit.

4.6.3 A 200-Period Planning Example and Results of Proposed Adaptive Planning Approach

For procurement planning problems engaged in global sourcing, the proposed optimization method will be deployed with an adaptive planning framework as presented in Chapter 3.

Section 3.5 has specified detailed solution procedures of both the rolling horizon scheme

and sub-horizon planning. By employing rolling horizon scheme, the overall planning problem is split into stochastic procurement planning problems in each sub-horizon. In particular, for sub-horizon planning, the sub-problem is processed by three successive steps: demand forecasting, standardization and interim optimal planning (see Figure 3.5).

(Hubert, 2013) proposed a methodology to select an appropriate demand forecasting method, and developed a detailed model of the forecast accuracy and its evolution with time. The method has been validated by a real-life application on a set of data involving different products from a world-leading automobile corporation (with a forecast lead time of 48 days), whose results show that the relative forecast errors are reduced to 3%.

Moreover, the standardization step has been described in Section 3.5, and the interim optimal planning methods for sub-horizon problems are presented in this chapter and Chapter 5, respectively for backorder and lost-sale case.

We use the following example to illustrate the overall adaptive planning procedure for a long-distance procurement planning problem engaged in global sourcing.

The actual demand scenarios are generated by Equation (3.12) (c.f. Section 3.5.3):

$$d_t = aT(t) + (b + c \times randn) \left[d + \sin\left(\frac{2\pi}{e} \times \left(t + \frac{f}{4}\right)\right) \right],$$

in which we set $a = 0.02$, $T(t) = -0.001t^3 + t^2 - 160t + 9000$, $b = 25$, $c = 30$, $d = 0.3$, $e = 8$, $f = -20$. Figure 3.5.3 illustrates a demand scenario generated by this setting.

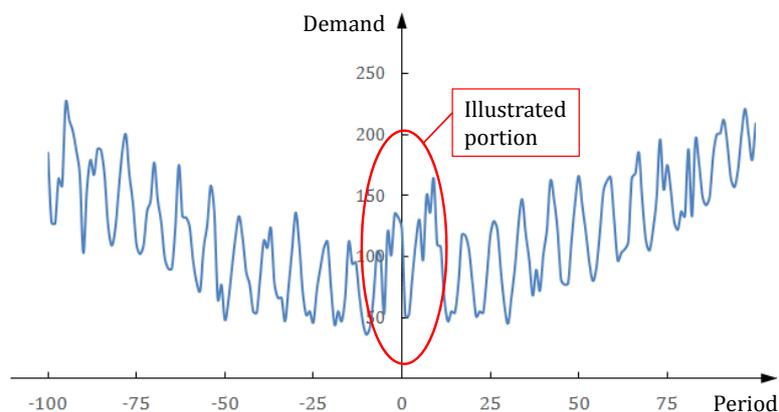


Figure 4.5: A randomly generated demand scenario

Due to page size limit, we only illustrate an extracted portion of 12-period planning in

detail. Here we assume that a period signifies a week in real life, and the lead time $L = 7$. For simplification, the holding cost h_i is taken to be 1 (per unit per period) in all periods, while the backorder cost b_i is taken to be 9 (per unit per period). Table 4.7 shows actual demand and setup cost of each period in the portion.

Table 4.7: An actual demand scenario (extracted portion)

Period i	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12
Setup cost K_{i-L}	140	192	120	87	154	87	93	185	193	115	195	180	199	115	86	220	188	200	255
actual demand d_i	100	53	120	101	135	132	123	51	52	84	113	130	97	150	136	164	110	108	66

Set the sub-horizon length $n = 5$. Consider the planning problem in sub-horizon 1 (periods 1-5). At period -6, we forecast the demand in sub-horizon 1 ($=-6+7$) as in Table 4.8. Note that in the table, the number in green means the actual demand.

Table 4.8: Demand forecasts made at period -6

period number	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5
demand forecast	100	54	121	102	134	131	121	54	55	83	111	133
abs. forecast error	-	0.5	1.2	0.7	1.1	1.3	2.2	2.6	3.4	1.5	2.2	2.9

From previous planning, the inventory level at the end of period 0 is estimated to be 5. Its standard deviation σ_0^1 can be calculated following a square root law:

$$\sigma_0^1 = \sqrt{0 + 0.5^2 + 1.2^2 + 0.7^2 + 1.1^2 + 1.3^2 + 2.2^2} = 3.1.$$

Then the net requirements can be estimated as in Table 4.9.

Table 4.9: Net requirements

period number	1	2	3	4	5
net.req., mean	$54 - 5 = 49$	55	83	111	133
net.req., stand.dev.	$\sqrt{3.1^2 + 2.6^2} = 4.0$	3.4	1.5	2.2	2.9
cumul.net.req., mean	49	104	187	298	431
cumul.net.req., stand.dev.	4.0	5.3	5.5	5.9	6.6

Consider the mean of net requirements as "determinate" demands, by applying Wagner-Whitin algorithm, the interim optimal plan is: $y_1^1 = 104$, $y_3^1 = 194$, $y_5^1 = 133$, $y_2^1 = y_4^1 = 0$. Using

the proposed method, we find an optimal safety stock of 6. Therefore, at period -6, we should place an order of 110 units.

At period -5, the demand forecasts are updated as in Table 4.10. Note that the actual demand of period -5 has been acquired, and the demand forecast of period 1 has been revised.

Table 4.10: Updated demand forecasts (Observed at Period -5)

period number	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5
demand forecast	100	53	121	102	134	131	121	53	55	83	111	133
abs.forecast error	-	-	0.6	0.5	0.8	1.1	1.8	2.1	2.9	1.3	2.0	2.6

Based on updated demand forecasts, the inventory level at the end of period 1 can be estimated: $I_1^o = 5 + (54 - 53) + 110 - 53 = 63$. Knowing that the updated demand forecast of period 2 is 55. We do not need to place an order at period -5.

Table 4.11: Procurement plan to be executed

period number	1	2	3	4	5	6	7	8	9	10	11	12
adaptive planning	110	0	188	0	232	0	149	298	0	293	0	0
ex-post-facto optimal planning	103	0	197	0	227	0	150	300	0	284	0	0

Table 4.12 describes the overall adaptive procurement planning procedure for the portion of 12-period planning horizon.

Table 4.11 shows the procurement plan by using proposed adaptive planning procedure, and the ex-post-facto optimal plan when the actual demand of each period has been known. The total costs including setup, inventory holding and backorders are respectively 1600 and 1535. By applying the proposed adaptive planning procedure, the total cost of finally executed procurement plan is slightly higher than the ex-post-facto optimal plan by a relative ratio of 4.2%.

By implementing a numerical test of 500 actual 200-period demand scenarios which are generated with the same parameter setting, we obtain:

$$\mathcal{R}_1^{500} = \overline{C}/\overline{C^*} - 1 = 4.35\% , \quad \text{and} \quad \mathcal{R}_2^{500} = \overline{C}/\overline{C^*} - 1 = 4.35\%.$$

Here C denotes the total cost incurred by finally executed procurement plan determined with proposed adaptive planning procedure, and C^* denotes the total cost incurred by ex-post-facto

optimal procurement plan for the same demand scenario. \mathcal{R}_1^{500} and \mathcal{R}_2^{500} especially represent the ratio of the mean values, and the mean value of the ratios, which are both statistical results of 500 randomly generated demand scenarios.

Note that the results can be considered promising, since the "absolutely" optimal procurement plan to a certain demand scenario can only be obtained ex post facto. In addition, the ex-post-facto optimal plan for one demand scenario will be, in most cases, no longer optimal to another demand scenario.

4.7 Conclusion

For procurement planning problems engaged in global sourcing, the shortage situation that demand exceeds stock often occurs. In this chapter, the unfulfilled demand is assumed to be fully backordered. An optimal planning method is developed to minimize expected total costs including setup, inventory holding and backorders in sub-horizons. This method can be considered as a benchmark for other planning methods. In addition, for efficiency improvement, a near-optimal 2-stage heuristic method is proposed with a safety stock policy. The presented methods can be deployed with the adaptive planning framework described in Chapter 3. Numerical results have shown the high efficiency and effectiveness of both the proposed sub-horizon planning methods and the overall adaptive procurement planning framework.

Chapter 5

Procurement Planning with Lost Sales or Outsourcing

5.1 Overview

In Chapter 4, the unsatisfied demands are assumed to be fully backordered (i.e., the customers keep waiting for a new delivery to arrive), and a time-varying backorder cost is assessed per unit per period as a stockout penalty. The backorder assumption is widely used in the literature, especially for applications in manufacturing industry.

However, in some circumstances, lost-sale assumption is more relevant. Besides immediate lost sales, suppliers can also recover the deficit by purchasing the stockout item from an external source at a higher price, and then supply it to the customer. As a result, the problem modeled with outsourcing assumption can be comprised into that with lost-sale assumption.

This chapter is organized as follows. Section 5.2 investigates the existing literature on lost-sale inventory models. The specific research problem is described in Section 5.3, and mathematically formulated in Section 5.4. An heuristic planning method is developed in Section 5.5, which minimizes expected total costs including setup, inventory holding and lost sales. Numerical examples are given in Section 5.6. Research work of this chapter is concluded in Section 5.7.

5.2 Literature Review

5.2.1 The Lost-Sale Systems

Lost sale is a common phenomena in real life, especially in retail industry.

A famous survey funded by Procter & Gamble shows that during customers' shopping trips in the supermarkets, one of every 13 items on the shopper's list tends to be unavailable, and 21% of a shopper's time is wasted on looking for an OOS (out-of-stock) item (Gruen and Corsten, 2007). Gruen's report also reveals that only 15% of the customers faced stockouts will wait for the items until replenishment, while the remaining 85% will either choose an alternative (45%), or visit another store (31%), or move the item out from the shopping list (9%), thus a lost sale occurs. (Verhoef and Sloot, 2006) provides similar conclusions, confirming that most of the unsatisfied demands could be considered as lost sales in many practical settings, especially in retail environment.

(Aastrup and Kotzab, 2010) surveys the recent 40-year research on stockouts, from both

demand side and supply side, with the objective of analyzing the customer responses to OOS items and the causes of OOS situations. The report indicates that the OOS rates have fallen into an average level at about 7-8% as a result of 40-year research, but seem to be at a standstill for further improvement.

Therefore, lost sales are inevitable and considerable phenomena that need to be taken into account. In real life, the supplier can recover the deficit by purchasing the stockout item from an external source at a certain (usually higher) price, and then supply it to the customer. Therefore, the outsourcing case can be comprised into the lost-sale case.

To the best of our knowledge, in spite of the early starting of academic research on lost-sale inventory systems around the 1960s (Hadley and Whitin, 1963; Fabrycky and Banks, 1967), there are fewer publications considering lost sales than that consider backorders.

The lost-sale inventory models perform differently from the backorder models, and lead to much more modeling difficulties. In an inventory model considering lost sales, the inventory level cannot be negative. Figure 5.1 illustrates the difference between backorder and lost-sale inventory systems under (R, Q) policy.

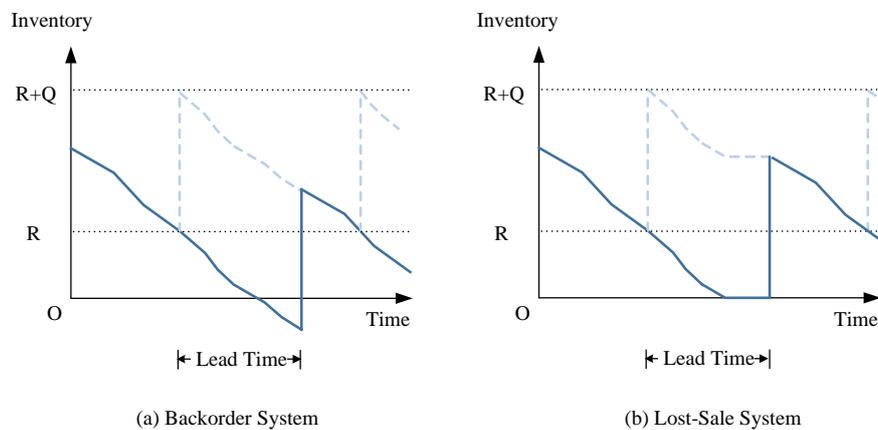


Figure 5.1: Backorder and lost-sale inventory systems

Figure 5.1(a) illustrates a simple backorder inventory system. According to Chapter 4, in a backorder system, the inventory level equals the cumulative order quantities that have arrived since the beginning of planning horizon, plus the initial stock, and minus the cumulative demand

quantities, i.e.,

$$I_i = I_1^o + X_i - D_i.$$

This feature helps in eliminating the variables denoting different inventory states (such as I_i^+ for overstock, and I_i^- for stockout) in problem formulation, and simplifies the solution procedure.

However, in a lost-sale inventory system as shown in Figure 5.1(b), the unsatisfied demands are supposed to be totally lost instead of being backordered. In other words, the inventory level will not decrease when a stockout occurs but remain zero until a new delivery, which is likely to bring about singularities in inventory costs calculation. In such an inventory system, the inventory level is calculated as

$$\begin{aligned} I_1 &= \max\{0, I_1^o + x_1 - d_1\}, \\ I_t &= \max\{0, I_{t-1} + x_t - d_t\} \quad (t > 1), \end{aligned} \tag{5.1}$$

where I_1^o denotes the initial stock of period 1, I_t denotes the inventory holding level at the end of period t , x_t denotes the order quantity that arrives at the beginning of period t , and d_t denotes the demand quantity in period t .

Equations (5.1) implies that it is virtually impossible to preview the future inventory states merely with on-hand inventory and demand information. Consequently, in order to calculate the future inventory costs, an information vector with the same length of lead time should be necessarily required. Moreover, the state space that describes the inventory system will grow exponentially with the vector's length.

Therefore, the lost-sale inventory models are usually much more complicated than the backorder ones.

5.2.2 State-of-the-Art on Lost-Sale Models

Optimal Approaches

The academic research on lost-sale inventory systems can be dated back to the early work of (Bellman et al., 1955). They formulated the lost-sale inventory management problem under periodic-review policy, in which demands are assumed to follow the same distribution, while setup and holding costs are not considered. Complete solutions are obtained for some special

cases, such as the 1-period-lead-time case, with the objective to minimize the expected total costs including purchasing and lost-sale penalty. Bellman's model is less applicable in practice, due to its limits on cost structure and restrictions on lead times.

(Arrow, 1958) has extended Bellman's model with holding costs and released the restrictions on lead times. In Arrow's model, the lead time is assumed to be a fixed integral multiple of review periods. An exact dynamic programming method is developed. Moreover, for the 1-period-lead-time case, the cost function and optimal order quantities are proven to be well-defined and bounded. (Yaspan, 1961) publishes similar results about the boundedness.

Furthermore, (Morton, 1969) extends Arrow's conclusion inductively to a more general case, in which the lead time is any integral multiple of review period length. The assumptions of linear and proportional holding, purchasing and penalty costs are made, while no fixed setup cost is considered.

Based on Arrow and Morton's work, (Zipkin, 2008) has reformulated the problem with transformed state variables, and demonstrated that the newly-defined optimal cost function possesses L-natural-convexity (i.e. the function is convex and submodular, plus an additional property related to diagonal dominance). The author indicates that the optimal order quantities are monotone in the transformed state variables, with limited sensitivity. Additional bounds on the optimal policy are derived, and the conclusion has been extended with introduction of capacity constraints, stochastic lead times and so on. Note again that this result prohibits fixed setup costs.

(Yaspan, 1961) compares the optimal replenishment policies between backorder and lost-sale systems. The author indicates that for the same inventory status observed, the optimal order quantity of lost-sale system is smaller. Besides, (Janakiraman et al., 2007) sets the same cost parameters (inventory holding and shortage penalty) to an average-cost model considering respectively backorders and lost sales, and demonstrates that the lost-sale model dominates the backorder model for any start state in infinite-horizon planning.

Near-Optimal Policies

Among the above-mentioned literature, the main research work has been performed in deriving properties and bounds of the optimal order quantities for lost-sale systems under periodic review policy. Since it requires quite some computational effort to find the optimal solution (especially for large inventory systems), many near-optimal replenishment policies have been developed.

(Morton, 1971) has proposed a myopic policy based on his early work. The upper bound of the optimal order quantity derived in (Morton, 1969) is executed as the actual order quantity. In this policy, the order quantity needs to be sufficient to satisfy the demand until the next delivery. Thus the holding costs tend to be considerable. A recursive procedure is developed to calculate the solutions.

The base-stock policy is commonly used in practice. In a base-stock policy, we make a replenishment order once the inventory position is below the base-stock level R . Based on the early work of (Gaver, 1959) and (Morse and Teichmann, 1958), (Pressman, 1977) extends their 1-period-lead-time model to the general case where the lead time is any integral multiple of review period. (Downs et al., 2001) have demonstrated that when fixed lead times are considered, the expected total cost function is convex with the order-up-to level. In order to find the optimal order-up-to level, a bisection method could be used.

The performances of base-stock policies in both lost-sale and backorder systems have been evaluated by (Janakiraman et al., 2007) and (Huh et al., 2009). The authors indicate that the optimal order-up-to level for the backorder model is asymptotically optimal for the lost-sale model. This implies that when the lost-sale penalty cost is high enough, the backorder model can be used as approximation for the lost-sale model. However, (Reiman, 2004) has already shown that the base-stock policies perform worse than a constant order quantity policy in the aforementioned case.

Besides, (Johansen, 2001) proposes a modified base-stock policy, in which the pair (S, t) is specified as decisions. S is the base stock, and t is a minimum number of review periods between two consecutive orders. Numerical examples show that the modified base-stock policy may realize the major part of cost reduction that would be achieved by using the pure base-stock policy.

The objective of all the policies as mentioned so far is to minimize the sum of expected holding and penalty costs. Holding costs are assessed due to the risk of ordering too much, while penalty costs are charged due to the risk of ordering too few units. (Levi et al., 2008) have developed a dual-balancing policy, which balances the two above-mentioned risks. The authors indicate that the expected total cost is, at the worst, twice of the cost using optimal policy.

Other Models: Service, Lead Times

In addition to the cost model, the service model is also used for lost-sale inventory systems. (van Donselaar et al., 1996) apply base-stock policies to satisfy a target fill rate. In this paper, a dynamic procedure is developed to set the order-up-to levels.

Moreover, stochastic lead times are considered in lost-sale inventory systems.

Generally, the lead times are assumed to be independent random variables, and the orders are assumed to be received in the same sequence that they are placed. (Nahmias, 1979) has developed an approximation policy based on dynamic programming equations. (Janakiraman and Roundy, 2004) have established some sample-path properties. They have proven the convexity of expected on-hand inventory level at the beginning of each period in function of the order-up-to level, which justifies the use of common search techniques to find the optimal order-up-to levels.

In the above-mentioned papers, lead time is assumed to be integral multiple of review period. However, there still exists some literature studying fractional lead times (i.e., the lead time is smaller than the length of one review period). Markov chain is usually used to model such inventory systems, see (Chiang, 2006; Sezen, 2006).

Setup Costs

Setup costs are not considered in the above literature. When delivery is instantaneous, the (s, S) policy has been proven to be optimal for lost-sale inventory systems, while a fixed order cost is charged for each order placed, see (Veinott and Wagner, 1965; Veinott, 1966; Shreve, 1976; Cheng and Sethi, 1999; Xu et al., 2010).

However, for positive lead-time cases, there is no simple optimal order policy. Based on numerical examples, (Hill and Johansen, 2006) show that neither an (R, Q) nor an (s, S) policy is optimal for a lost-sale inventory system which considers both setup costs and positive lead times.

Nevertheless, the (R, Q) and (s, S) policies are near-optimal and easy-running in real-world practice.

(Johansen and Hill, 2000) applies an (R, s, Q) policy in a periodic-review continuous-demand lost-sale model with a fixed lead time, assuming that at most one order is excess at

any time. The optimal s and Q are determined by a policy iteration algorithm. Numerical examples show that the approach is close to optimal in certain practical applications.

(Bijvank and Vis, 2012) establish a service model for a periodic-review lost-sale system, with no restrictions in lead times or the number of excess orders. The optimal policies such as base stock and (R, s, S) are used, and new heuristic procedures are developed to set the reorder level and base-stock level.

Conclusion

To conclude, there are two major factors that influence the searching procedure of optimal solutions for periodic-review lost-sale inventory models: lead time and setup cost.

For the cases that no setup cost is considered, much research work has been performed on deriving properties and bounds on the optimal order quantities. Usually, the lead time is assumed to be integral multiple of review period length. Since it takes quite some computational effort to find the optimal order quantities, many alternative and efficient replenishment policies have been developed.

For the cases that zero lead time and fixed order costs are considered, the (R, s, S) policy has been proven to be optimal.

However, for the cases considering both positive lead times and setup costs, no explicit optimal solution has been proposed. This is the main focus in this chapter.

5.3 Problem Description

5.3.1 Assumptions

In order to describe the long-distance procurement planning problem with lost sales (or outsourcing) engaged in global sourcing, the similar example as presented in Chapter 2 is used.

Example:

Firm A (in France) plans to procure component X from Supplier B (in China) in order to produce Product Y. The transport from B to A takes about 1.5-2 months. Once a procurement order is placed, a mandatory set-up cost occurs. When there is a stockout of component X in A's production process, the production for final-product Y will be interrupted, therefore the customer demand for Y cannot be fully satisfied. In this case, all the unsatisfied demands are lost (or outsourced) and a penalty cost is charged.

The objective is to draw up an optimal procurement planning, which minimizes the expected long-run per-period total procurement costs including set-up, inventory holding and penalty, by specifying two issues: when a procurement order should be placed, and how many component X should be ordered.

The following assumptions are extracted from the example.

Assumptions:

- (a) The planning problem is considered under a periodic-review policy.
- (b) The demand quantity in each period is non-negative, independent and unknown. It might follow certain trend and seasonality which could be forecasted adaptively from the historical data.
- (c) The procurement capacity is unlimited.
- (d) Unsatisfied demand is totally lost (or outsourced), and a penalty cost will be charged for each unfulfilled item.
- (e) The inventory cost is assessed at the end of each period, including holding cost or penalty cost. No disposal of inventory is allowed, and the item is not perishable.
- (f) For each order placed, the lead time L is assumed to be fixed. L is a positive integer that cannot be neglected. Moreover, demand forecasts will evolve during the delivery of order.
- (g) A mandatory setup cost occurs when a procurement order is placed.

The mission is to make an optimal procurement plan over the entire planning horizon, with the objective of minimizing the expected long-run per-period total costs including setup, inventory holding and lost sales (or outsourcing). Two principal issues should be specified in the solution: (1) in which period should a procurement order be placed, (2) how much should be ordered.

5.3.2 Notation

In this chapter, the following notation is used.

Decision Variable:

- x_i : The delivery quantity arriving during period i .
- δ_i : A binary decision variable. $\delta_i = 1$ if $x_i > 0$, or 0 otherwise.
- $\mathbf{x}_{i,j}$: The vector of order quantities arriving in the periods $i, i+1, \dots, j$.
 $\mathbf{x}_{i,j} = (x_i, x_{i+1}, \dots, x_j)^T$.
- $X_{i,j}$: The cumulative order quantity arriving in the periods $i, i+1, \dots, j$.
 $X_{i,j} = \sum_{t=i}^j x_t$.

Random Variable:

- d_i : The actual demand quantity of period i . The exact value of d_i cannot be known until the end of period i , therefore, we consider d_i as a random variable at the decision-making phase.
- $D_{i,j}$: The cumulative actual demand quantity counting from the beginning of period i to the end of period j ($i \leq j$). $D_{i,j} = \sum_{t=i}^j d_t$.
- I_i^o : The inventory level at the beginning of period i .
- I_i^y : The calculated inventory level at the end of period i . $I_i^y = I_i^o + x_i - d_i$.
- I_i : The inventory holding level at the end of period i . $I_i = \max\{I_i^y, 0\}$, and $I_i = I_{i+1}^o$.

Probability Symbol:

- $U_i(\mathbf{x}_{1,i})$: The probability that a lost sale occurs at the end of period $i \geq 1$, knowing the supply vector $\mathbf{x}_{1,i}$. For initial state, $U_0 = 1$.
- $W_{i,j}^-(\mathbf{x}_{i,j})$: The probability that a lost sale occurs at the end of period j , while no lost sale occurs in any of the periods $i, i+1, \dots, j-1$, knowing that the previous lost sale occurs at the end of period $i-1$.
- $W_{i,j}^+(\mathbf{x}_{i,j})$: The probability that no lost sale occurs in any of the periods $i, i+1, \dots, j$, knowing that a lost sale occurs at the end of period $i-1$.
- $Q_{i,j}(\mathbf{x}_{i,j}, h)$: The probability density function (PDF) of the calculated inventory level h at the end of period j , while no lost sale occurs in the periods $i, i+1, \dots, j-1$, knowing that a lost sale occurs at the end of period $i-1$.

Other Parameter:

- h_i : The inventory holding cost (per unit held) charged at the end of period i .
- p_i : The per-unit lost-sale (or outsourcing) cost at period i .
- K_j : The mandatory setup cost for each order that is placed in period i .
- M : A sufficiently large positive number.

In order to distinguish the points that an order is placed and that it arrives, the subscript " $i-L$ " is used to denote the period in which an order is *placed* to arrive at period i . Period i is called an "*order period*" because an order quantity x_i *arrives*. Note that the decision about x_i is made at period $i-L$, thus the corresponding setup cost should be K_{i-L} .

5.4 Mathematical Formulation

The procurement planning problem with lost sales (or outsourcing) can be formulated as follows:

Q

$$\min \sum_{i=1}^n [K_{i-L}\delta_i + E(h_i \cdot I_i + p_i \cdot I_i^-)] \quad (5.2)$$

$$s.t. \quad I_i^y = I_{i-1} + x_i - d_i, \quad \forall i \quad (5.3)$$

$$I_i = \max\{I_i^y, 0\}, \quad \forall i \quad (5.4)$$

$$I_i^- = \max\{-I_i^y, 0\}, \quad \forall i \quad (5.5)$$

$$0 \leq x_i \leq M\delta_i, \quad \forall i \quad (5.6)$$

$$\delta_i \in \{0, 1\}, \quad \forall i \quad (5.7)$$

where L denotes the lead time, $E(t)$ denotes the expected value of random variable t . I_i^y is named *calculated inventory level*, which equals the algebraic result of $(I_{i-1} + x_i - d_i)$. I_i^- is the lost-sale quantity.

5.4.1 The Backward Inventory Property (BIP) of Lost-Sale Models

The lost-sale inventory model is much more complex than the backorder model. The recursive maximum functions in Equation (5.1):

$$I_1 = \max\{0, I_1^o + x_1 - d_1\},$$

$$I_t = \max\{0, I_{t-1} + x_t - d_t\} \quad (t > 1)$$

have explained how the inventory system reacts to excess demands. We are not able to describe the inventory level of each period in such an explicit way as

$$I_i = I_1^o + X_i - D_i$$

which describes the inventory level for a backorder system.

Figure 5.2 enumerates the possible states of inventory level at the end of each period. In the figure, the node 0 denotes initial state, while each node (i, j) denotes the j^{th} possible inventory state at the end of period i .

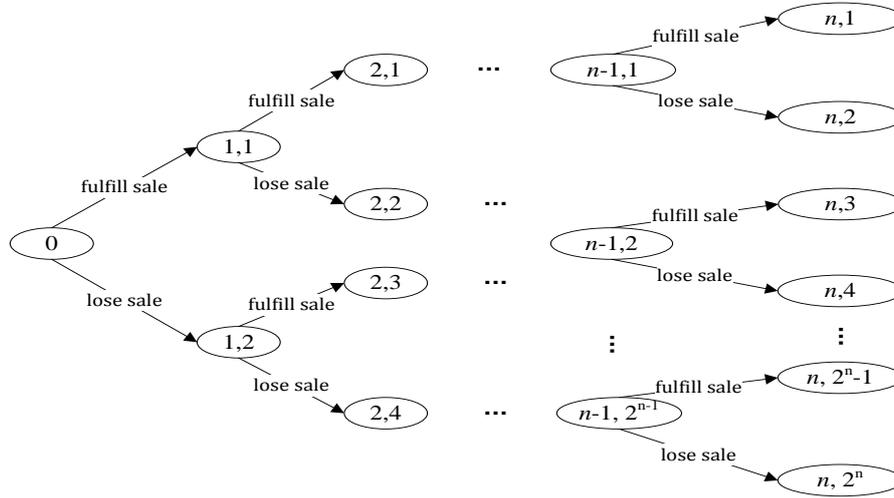


Figure 5.2: Inventory states of a lost-sale system

According to Equations (5.1), for each node (i, j) which denotes one inventory state, the subsequent states could have only two possibilities, respectively signifying whether a lost sale occurs or not.

Let I_0 denote the initial inventory level of the planning horizon. The inventory level at the end of period 1 is $I_1 = \max\{0, I_0 + x_1 - d_1\}$, thus I_1 has $2(= 2^1)$ possible states. Similarly, I_2 has $4(= 2^2)$ possible states, I_3 has $8(= 2^3)$ possible states, and so on. As a result, the number of possible inventory states at the end of period n can be given as 2^n . Thus the state space grows exponentially with the length of planning horizon, which implies huge computational burden for further solution procedure.

In order to avoid exponentially-growing calculations, we reconsider the problem in a backward direction.

Suppose the latest lost sale occurring before period i has taken place at the end of period $\chi(i)$. It is natural that $0 \leq \chi(i) < i$. Since the demand exceeds supply in period $\chi(i)$, and the unsatisfied demand is totally lost, the initial stock of period $\chi(i) + 1$ is zero. By definition of $\chi(i)$, no sale will be lost from the beginning of period $\chi(i) + 1$ until the end of period $i - 1$, thus

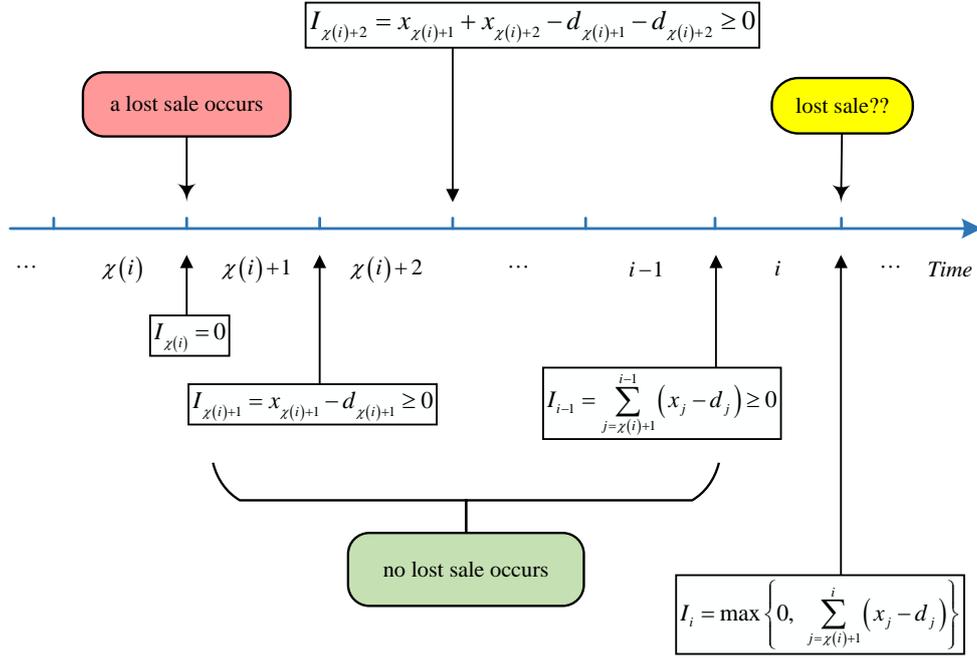


Figure 5.3: The Backward Inventory Property of lost-sale models

the inventory level at the end of period i is given as

$$I_i = \max\left\{0, \sum_{t=\chi(i)+1}^i (x_t - d_t)\right\}, \quad (5.8)$$

where $\sum_{k=\chi(i)+1}^t (x_k - d_k) \geq 0$ for any $k \in [\chi(i)+1, i-1]$.

Since $\chi(i) \in \{0, 1, 2, \dots, i-1\}$, the inventory *holding* level at the end of period i has $i+1$ possible states: $\sum_{t=1}^i (x_t - d_t)$, $\sum_{t=2}^i (x_t - d_t)$, \dots , $(x_{i-1} + x_i) - (d_{i-1} + d_i)$, $x_i - d_i$, 0 . Here 0 signifies that a lost sale occurs at the end of period i .

This property is named "*Backward Inventory Property (BIP)*", for the possible inventory states are counted from an inverse time path. BIP has reduced the growth of inventory state space from exponential ($O(2^n)$) to polynomial ($O(n)$), thus implies a possible optimal solution for our problem.

5.4.2 Mathematical Formulation

According to the *Backward Inventory Property* (BIP), the inventory level at the end of period j is

$$I_j = \max \left\{ 0, \sum_{t=\chi(j)+1}^j (x_t - d_t) \right\},$$

where $\chi(j) \in \{0, 1, \dots, j-1\}$.

Whether a lost sale will occur at the end of period j depends on the location of $\chi(j)$, the demand quantities in periods $\chi(j) + 1, \dots, j$, and the orders arriving during these periods, that is,

$$\mathcal{P} \left(I_j^y = \sum_{t=i}^j (x_t - d_t) \right) = \mathcal{P}(i = \chi(j) + 1),$$

or equivalently,

$$\mathcal{P} \left(I_j^y = \sum_{t=i}^j (x_t - d_t) \right) = \mathcal{P} \left(I_{i-1}^y < 0 \cap x_i - d_i \geq 0 \cap \dots \cap \sum_{t=i}^{j-1} (x_t - d_t) \geq 0 \right).$$

In Chapter 3, an adaptive planning framework has been presented to acquire the approximate demand distributions. In the following sections of this chapter, we focus on developing optimal procurement planning method for a sub-horizon of n periods. For concise writing, the notation used after here signifies the same physical feature (demand, inventory state, etc.) in the sub-horizon. We suppose that the procurement planning problem in the sub-horizon has already been standardized in previous processes.

Let f_i denote the PDF (probability density function) of d_i , and $f_{i,j}$ denote the PDF of the cumulative demand quantity $D_{i,j}$, $D_{i,j} = \sum_{t=i}^j d_t$ ($i \leq j$).

Suppose $\chi(j) = i - 1$, then the expected total inventory cost incurred in period j can be calculated as follows:

$$C_j^i(\mathbf{x}_{i,j}) \Big|_{\chi(j)=i-1} = h_j \cdot \int_{-\infty}^{X_{i,j}} (X_{i,j} - q) f_{i,j}(q) dq + p_j \cdot \int_{X_{i,j}}^{+\infty} (q - X_{i,j}) f_{i,j}(q) dq, \quad (5.9)$$

where $X_{i,j}$ is the cumulative order quantities arriving during the periods $i, i+1, \dots, j$, $X_{i,j} = \sum_{t=i}^j x_t$.

Note that C_j^i signifies that the inventory cost of period j is calculated provided that the initial stock of period i is zero, and no lost sale occurs in the periods $i, i+1, \dots, j$, thus the *calculated* inventory level at the end of period j can be written as $I_j^Y = X_{i,j} - D_{i,j}$. For concise writing, the symbol q is used to replace $D_{i,j}$ in Equation (5.9).

Therefore, the expected total cost incurred in period j is

$$C_j(\mathbf{x}_{1,j}) = K_{j-L}\delta_j + \sum_{i=1}^j [\mathcal{P}(i = \chi(j) + 1) \cdot C_j^i(\mathbf{x}_{i,j})]. \quad (5.10)$$

The probability factor $\mathcal{P}(i = \chi(j) + 1)$ is the key component in Equation (5.10). In order to calculate $\mathcal{P}(i = \chi(j) + 1)$, a recursive method is developed as follows.

Suppose a lost sale occurs at the end of period $i-1$.

Let $Q_{i,j}(\mathbf{x}_{i,j}, h)$ denote the PDF of the calculated inventory level h at the end of period j , provided that no lost sale occurs during the periods $i, i+1, \dots, j-1$. Let g denote the calculated inventory level at the end of period $j-1$. Since no lost sale occurs at period $j-1$ ($g \geq 0$), we have

$$h = g + x_j - d_j,$$

or equivalently,

$$d_j = g + x_j - h. \quad (5.11)$$

By definition, $Q_{i,j-1}(\mathbf{x}_{i,j-1}, g)$ denotes the PDF of the calculated inventory level g at the end of period $j-1$, provided that no lost sale occurs during the periods $i, i+1, \dots, j-2$. Then the PDF of h can be derived as

$$Q_{i,j}(\mathbf{x}_{i,j}, h) = \int_0^{+\infty} Q_{i,j-1}(\mathbf{x}_{i,j-1}, g) f_j(g + x_j - h) dg. \quad (5.12)$$

Figure 5.4 illustrates the recursive feature of calculated inventory level distributions.

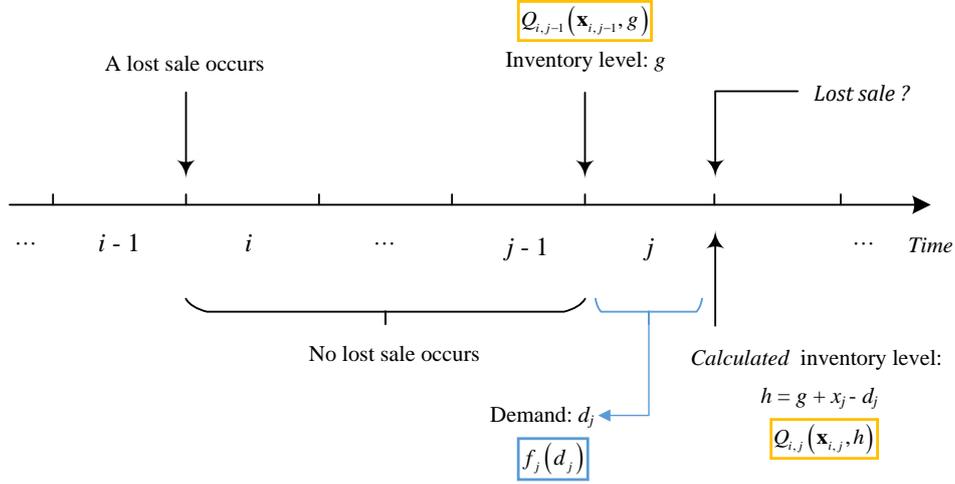


Figure 5.4: Recursive calculated inventory level distributions

The initial state $Q_{i,i}(x_i, h)$ can be derived as follows.

Let F_i^V denote the CDF (cumulative distribution function) of the calculated inventory level I_i^V at the end of period i . By definition of $Q_{i,i}(x_i, h)$, a lost sale occurs at the end of period $i-1$, thus $I_i^V = x_i - d_i$, and

$$F_i^V(h) = \mathcal{P}(I_i^V \leq h) = \mathcal{P}(x_i - d_i \leq h) = \mathcal{P}(d_i \geq x_i - h) = \int_{x_i - h}^{+\infty} f_i(t) dt. \quad (5.13)$$

$Q_{i,i}(x_i, h)$ is defined as the PDF of I_i^V , provided that $I_{i-1}^V < 0$, we have

$$Q_{i,i}(x_i, h) = \frac{dF_i^V(h)}{dh} = -f_i(x_i - h) \cdot \frac{d(x_i - h)}{dh} = f_i(x_i - h). \quad (5.14)$$

Still suppose a lost sale occurs at the end of period $i-1$.

Let $W_{i,j}^-(\mathbf{x}_{i,j})$ denote the probability that the next lost sale occurs at the end of period j , while no lost sale occurs during the periods $i, i+1, \dots, j-1$, knowing the supply vector $\mathbf{x}_{i,j}$.

Thus $W_{i,j}^-(\mathbf{x}_{i,j})$ is equivalently an integral of $Q_{i,j}(\mathbf{x}_{i,j}, h)$ over the interval $(-\infty, 0)$, we have

$$W_{i,j}^-(\mathbf{x}_{i,j}) = \int_{-\infty}^0 Q_{i,j}(\mathbf{x}_{i,j}, h) dh, \quad (i \leq j). \quad (5.15)$$

Correspondingly, let $W_{i,j}^+(\mathbf{x}_{i,j})$ denote the probability that no lost sales occurs during the periods $i, i+1, \dots, j$. Then we have

$$W_{i,j}^+(\mathbf{x}_{i,j}) = \int_0^{+\infty} Q_{i,j}(\mathbf{x}_{i,j}, h) dh, \quad (i \leq j). \quad (5.16)$$

Note that the sum of $W_{i,j}^-(\mathbf{x}_{i,j})$ and $W_{i,j}^+(\mathbf{x}_{i,j})$ equals the probability that no lost sales occurs during the periods $i, i+1, \dots, j-1$, that is,

$$W_{i,j-1}^+(\mathbf{x}_{i,j-1}) = W_{i,j}^-(\mathbf{x}_{i,j}) + W_{i,j}^+(\mathbf{x}_{i,j}), \quad \text{if } i < j; \quad (5.17)$$

and

$$W_{i,i}^-(\mathbf{x}_{i,i}) + W_{i,i}^+(\mathbf{x}_{i,i}) = 1. \quad (5.18)$$

Therefore, the probability that a lost sale occurs at the end of period j is given as:

$$U_j(\mathbf{x}_{1,j}) = \mathcal{P}(I_j^v < 0) = \sum_{i=1}^j U_{i-1}(\mathbf{x}_{1,i-1}) \cdot W_{i,j}^-(\mathbf{x}_{i,j}), \quad (5.19)$$

and for initial condition, $U_0 = 1$.

Let \mathcal{S}^v denote the state vector of the calculated inventory level I_i^v at the end of period i in a lost-sale inventory system. According to the BIP (backorder inverse property, see Section 5.4.1) of lost-sale models, we have

$$\mathcal{S}^v = \begin{pmatrix} \sum_{t=1}^i (x_t - d_t) \\ \sum_{t=2}^i (x_t - d_t) \\ \vdots \\ (x_i + x_{i-1}) - (d_i + d_{i-1}) \\ x_i - d_i \end{pmatrix}.$$

Let \mathcal{P}^v denote the corresponding probability state vector of I_i^v , we have

$$\mathcal{P}^v = \begin{pmatrix} W_{1,i-1}^+(\mathbf{x}_{1,i-1}) \\ U_1(\mathbf{x}_{1,1}) \cdot W_{2,i-1}^+(\mathbf{x}_{2,i-1}) \\ \vdots \\ U_{i-2}(\mathbf{x}_{1,i-2}) \cdot W_{i-1,i-1}^+(\mathbf{x}_{i-1,i-1}) \\ U_{i-1}(\mathbf{x}_{1,i-1}) \end{pmatrix}.$$

Note that $\mathcal{P}^v(k)$ ($k = 1, 2, \dots, i$) denotes the probability of $I_i^v = \mathcal{S}^v(k)$. Compared to Equation (5.10), the expected total cost incurred in period j can be rewritten as

$$C_j(\mathbf{x}_{1,j}) = K_{j-L}\delta_j + \sum_{i=1}^j \{U_{i-1}(\mathbf{x}_{1,i-1}) \cdot W_{i,j-1}^+(\mathbf{x}_{i,j-1}) \cdot C_j^i(\mathbf{x}_{i,j})\}, \quad (5.20)$$

and for concise writing, let $W_{j,j-1}^+(\mathbf{x}_{i,j}) = 1$.

Therefore, the sub-horizon planning problem can be formulated as:

Sub-Q

$$\min \quad \sum_{j=1}^n C_j(\mathbf{x}_{1,j})$$

$$s.t. \quad (5.6), (5.7), (5.9), (5.12), (5.14), (5.15), (5.16), (5.19), (5.20).$$

5.5 A Forward Procedure

Let Γ_j denote the minimum expected cumulative total costs incurred in periods $1, 2, \dots, j$. By the definition, $\Gamma_0 = 0$.

Consider the two consecutive order periods s and $t+1$ ($s \leq t$), that is, an order arrives at the beginning of period s , and no order arrives in the periods $s+1, s+2, \dots, t$. The expected total costs incurred in periods $s, s+1, \dots, t$ is:

$$C_{s,t}(\mathbf{x}_{1,t}) = K_{s-L} + \sum_{j=s}^t \sum_{i=1}^j \{U_{i-1}(\mathbf{x}_{1,i-1}) \cdot W_{i,j-1}^+(\mathbf{x}_{i,j-1}) \cdot C_j^i(\mathbf{x}_{i,j})\}. \quad (5.21)$$

$C_{s,t}(\mathbf{x}_{1,t})$ is defined for $1 \leq s \leq t \leq n$. When $t = n$, no replenishment subsequent to period s occurs during the horizon.

The minimum expected cumulative costs incurred in periods $1, 2, \dots, t$ can be determined by

$$\Gamma_t = \min_{\substack{1 \leq s \leq t, \\ x_1, \dots, x_s \geq 0 \\ x_{s+1} = \dots = x_t = 0}} \{ \Gamma_{s-1}(\mathbf{x}_{1,s-1}) + C_{s,t}(\mathbf{x}_{1,s}) \} \quad (5.22)$$

Our objective is to specify $x_i, i = 1, \dots, n$, in order to minimize the expected total costs over the sub-horizon, in other words, to find Γ_n and the corresponding order vector $\mathbf{x}_{1,n}$.

We develop a forward heuristic method to find near-optimal solutions, using the relaxed recursion formula as follows:

$$\Gamma_0 = 0;$$

$$\Gamma_t = \min_{\substack{1 \leq s \leq t, \\ \mathbf{x}_{1,s-1} \text{ is given}}} \left\{ \Gamma_{s-1} + \min_{\substack{x_s \geq 0 \\ x_{s+1} = \dots = x_t = 0}} C_{s,t}(x_s) \right\}. \quad (5.23)$$

In the recursion formula (5.23), x_s is the unique decision variable, since x_1, x_2, \dots, x_{s-1} have already been derived from the searching procedure of Γ_{s-1} , and $x_{s+1} = x_{s+2} = \dots = x_t = 0$. As a result, the key problem is to find the optimal x_s^* which minimizes $C_{s,t}$. $C_{s,t}(x_s)$ is continuous on the closed interval $[0, M]$, by *Extreme Value Theorem*, the global minimum $C_{s,t}^* = C_{s,t}(x_s^*)$ exists.

Proposition 5.1 For any $s < i < j \leq t$, $W_{i,j}^+ = W_{i,j}^- = 0$.

Proof. Note that x_1, x_2, \dots, x_{s-1} are known at this stage, and $x_{s+1} = x_{s+2} = \dots = x_t = 0$, thus x_s is the unique variable in $W_{i,j}^+(\mathbf{x}_{i,j-1})$ and $W_{i,j}^-(\mathbf{x}_{i,j-1})$. By definition,

$$W_{i,j}^+(x_s) = \mathcal{P} \left(x_i - d_i \geq 0 \bigcap (x_i + x_{i+1}) - (d_i + d_{i+1}) \geq 0 \bigcap \dots \bigcap \sum_{t=i}^j (x_t - d_t) \geq 0 \right)$$

$$\begin{aligned}
&= \mathcal{P} \left(d_i \leq x_i \cap d_{i+1} \leq x_i + x_{i+1} - d_i \cap \cdots \cap d_j \leq \sum_{t=i}^j x_t - \sum_{t=i}^{j-1} d_t \right) \\
&= \int_0^{x_i} f_i(d_i) dd_i \int_0^{x_i + x_{i+1} - d_i} f_{i+1}(d_{i+1}) dd_{i+1} \cdots \int_0^{\sum_{t=i}^j x_t - \sum_{t=i}^{j-1} d_t} f_j(d_j) dd_j \\
&\triangleq \int_0^{x_i} f_i(\tau_i) d\tau_i \int_0^{x_i + x_{i+1} - \tau_i} f_{i+1}(\tau_{i+1}) d\tau_{i+1} \cdots \int_0^{\sum_{t=i}^j x_t - \sum_{t=i}^{j-1} \tau_t} f_j(\tau_j) d\tau_j. \quad (5.24)
\end{aligned}$$

For $s < i < j \leq t$, we have $x_i = \cdots = x_j = 0$, thus $W_{i,j}^+ = \int_0^0 f_i(\tau_i) d\tau_i \int \cdots = 0$. Similarly,

$$W_{i,j}^-(x_s) = \int_0^{x_i} f_i(\tau_i) d\tau_i \int_0^{x_i + x_{i+1} - \tau_i} f_{i+1}(\tau_{i+1}) d\tau_{i+1} \cdots \int_{\sum_{t=i}^j x_t - \sum_{t=i}^{j-1} \tau_t}^{+\infty} f_j(\tau_j) d\tau_j, \quad (5.25)$$

and $W_{i,j}^- = 0$ for $s < i < j \leq t$.

As a matter of fact, since the initial stock of period i is zero according to $W_{i,j}$'s definition, and no order will arrive during the periods $i, i+1, \dots, j$, the demands in periods $i, i+1, \dots, j-1$ are impossible to be fulfilled and will be totally lost. Therefore, neither $W_{i,j}^+$ nor $W_{i,j}^-$ makes practical sense, due to their presupposition that "no lost sale occurs during the periods $i, i+1, \dots, j-1$ ". \square

Lemma 5.1 For any $s < i \leq t$, $W_{i,i}^+ = 0$, and $W_{i,i}^- = 1$.

Proof. If $s < i \leq t$, the initial stock of period i is zero, and no order will arrive in the period i . Then the demand of period i will be certainly lost, in other words, $W_{i,i}^+ = 0$, and $W_{i,i}^- = 1$. \square

From the above results, we have the following lemma:

Lemma 5.2

$$C_{s,t}(x_s) = K_{s-L} + \sum_{i=1}^s [U_{i-1} \cdot W_{i,s-1}^+ \cdot C_s^i(x_s)] + \sum_{j=s+1}^t \left\{ \sum_{i=1}^s [U_{i-1} \cdot W_{i,j-1}^+(x_s) \cdot C_j^i(x_s)] + U_{j-1}(x_s) \cdot C_j^j \right\}. \quad (5.26)$$

Lemma 5.3 For $s < j \leq t$, $C_j^j = p_j \cdot \mu_j$.

Proof. Similar to the proof of Lemma 5.1, if $s < j \leq t$, the demand of period j will be certainly lost. Consequently, the expected total cost incurred in period j is the totally lost-sale penalty cost. Since μ_j denotes the average demand of period j , we have $C_j^j = p_j \cdot \mu_j$. \square

Proposition 5.2 For any $i \leq s \leq j \leq t$, $W_{i,j}^+(x_s)$ is monotone nondecreasing.

Proof. Equation (5.24) implies that $W_{i,j}^+(x_s)$ is continuous and derivable over the whole real domain.

For $i \leq s < j \leq t$,

$$\frac{dW_{i,j}^+(x_s)}{dx_s} = \int_0^{x_i} f_i(\tau_i) d\tau_i \int \cdots \int_0^{\sum_{k=i}^s x_k - \sum_{k=i}^{j-2} \tau_k} f_{j-1}(\tau_{j-1}) \cdot f_j \left(\sum_{k=i}^s x_k - \sum_{k=i}^{j-1} \tau_k \right) d\tau_{j-1}.$$

For $i \leq s = j \leq t$,

$$\frac{dW_{i,s}^+(x_s)}{dx_s} = \int_0^{x_i} f_i(t_i) dt_i \int \cdots \int_0^{\sum_{k=i}^{s-1} x_k - \sum_{k=i}^{s-2} t_k} f_{s-1}(t_{s-1}) \cdot f_s \left(\sum_{k=i}^s x_k - \sum_{k=i}^{s-1} \tau_k \right) d\tau_{s-1}.$$

Note that the PDF f_i is nonnegative over the whole real domain, thus for any $i \leq s \leq j \leq t$, we have $\frac{dW_{i,j}^+(x_s)}{dx_s} \geq 0$, that is, $W_{i,j}^+(x_s)$ is monotone nondecreasing.

As a matter of fact, increasing x_s will certainly raise the possibility that demands in periods $s, s+1, \dots, j$ are satisfied rather than lost, knowing that period s is the last period that an order arrives before period $t+1$. \square

Proposition 5.3 For any $s \leq j \leq t$, $U_j(x_s)$ is monotone nonincreasing.

Proof. According to Equation (5.19), we have

$$\begin{aligned} U_j(x_s) &= \sum_{i=1}^s U_{i-1} \cdot W_{i,j}^-(x_s) + U_{j-1}(x_s) \\ &= \sum_{i=1}^s U_{i-1} \cdot W_{i,j}^-(x_s) + \sum_{i=1}^s U_{i-1} \cdot W_{i,j-1}^-(x_s) + U_{j-2}(x_s) \\ &= \dots \end{aligned}$$

$$= \sum_{i=1}^s U_{i-1} \cdot [W_{i,j}^-(x_s) + W_{i,j-1}^-(x_s) + \cdots + W_{i,s}^-(x_s)] + U_{s-1}.$$

Besides, Equations (5.17) and (5.18) imply that:

$$\begin{aligned} & W_{i,j}^-(x_s) + W_{i,j-1}^-(x_s) + \cdots + W_{i,s}^-(x_s) \\ &= [\cancel{W_{i,j-1}^+(x_s)} - W_{i,j}^+(x_s)] + [\cancel{W_{i,j-2}^+(x_s)} - \cancel{W_{i,j-1}^+(x_s)}] + \cdots + [\cancel{W_{i,s}^+(x_s)} - \cancel{W_{i,s+1}^+(x_s)}] + [W_{i,s-1}^+ - \cancel{W_{i,s}^+(x_s)}] \\ &= W_{i,s-1}^+ - W_{i,j}^+(x_s). \end{aligned}$$

Therefore, the expression of $U_j(x_s)$ can be simplified as:

$$U_j(x_s) = \sum_{i=1}^s U_{i-1} \cdot [W_{i,s-1}^+ - W_{i,j}^+(x_s)] + U_{s-1}. \quad (5.27)$$

Note that U_k is positive and known for $k < s$, since x_1, \dots, x_k have been determined in the previous processes. Besides, $W_{i,s-1}^+$ is nonnegative and independent with x_s , $W_{i,j}^+(x_s)$ is nondecreasing for any $i \leq s \leq j \leq t$. Then

$$\frac{dU_j(x_s)}{dx_s} = - \sum_{i=1}^s U_{i-1} \cdot \frac{dW_{i,j}^+(x_s)}{dx_s} \leq 0.$$

Thus $U_j(x_s)$ is monotone nonincreasing, if $s \leq j \leq t$. \square

5.5.1 The Normal Distribution Case

When demands follow normal distributions, the following proposition is developed.

Proposition 5.4 *There exists some x_s^β that for any $x_s \geq x_s^\beta$, $C_{s,t}(x_s) \geq C_{s,t}(x_s^\beta)$.*

Proof. According to Lemma 5.2 and Equation (5.27), $C_{s,t}(x_s)$ can be rewritten as

$$\begin{aligned} & C_{s,t}(x_s) \\ &= K_{s-L} + \sum_{i=1}^s [U_{i-1} \cdot W_{i,s-1}^+ \cdot C_s^i(x_s)] + \sum_{j=s+1}^t \left\{ \sum_{i=1}^s [U_{i-1} \cdot W_{i,j-1}^+(x_s) \cdot C_j^i(x_s)] + \left[\sum_{i=1}^s U_{i-1} \cdot [W_{i,s-1}^+ - W_{i,j-1}^+(x_s)] + U_{s-1} \right] \cdot C_j^j \right\} \\ &= K_{s-L} + \sum_{j=s+1}^t \left(U_{s-1} + \sum_{i=1}^s U_{i-1} \cdot W_{i,s-1}^+ \right) \cdot C_j^j + \sum_{i=1}^s [U_{i-1} \cdot W_{i,s-1}^+ \cdot C_s^i(x_s)] + \sum_{j=s+1}^t \sum_{i=1}^s [U_{i-1} \cdot W_{i,j-1}^+(x_s) \cdot (C_j^i(x_s) - C_j^j)]. \end{aligned}$$

We define $\mathcal{A}_{i,j,s}(x_s) \triangleq W_{i,j-1}^+(x_s) \cdot (C_j^i(x_s) - C_j^j)$, where $1 \leq i \leq s < j \leq t$. The derivative of $\mathcal{A}_{i,j,s}(x_s)$ is:

$$\frac{d\mathcal{A}_{i,j,s}(x_s)}{dx_s} = \frac{dW_{i,j-1}^+(x_s)}{dx_s} \cdot (C_j^i(x_s) - C_j^j) + W_{i,j-1}^+(x_s) \cdot \frac{dC_j^i(x_s)}{dx_s},$$

where $\Phi(x)$ is the cumulative distribution function (CDF) of standard normal distribution, and

$$\frac{dC_j^i(x_s)}{dx_s} = (h_j + p_j) \Phi\left(\frac{X_{i,j} - \mu_{i,j}}{\sigma_{i,j}}\right) - p_j. \quad (5.28)$$

The function $\Phi\left(\frac{X_{i,j} - \mu_{i,j}}{\sigma_{i,j}}\right)$ is monotone increasing with x_s (x_s is implicitly involved in $X_{i,j}$, $X_{i,j} = \sum_{m=i}^{s-1} x_m + x_s$). Let x_s^r denote the unique root of the equation $\frac{dC_j^i(x_s)}{dx_s} = 0$. Then $\frac{dC_j^i(x_s)}{dx_s} < 0$ for $x_s < x_s^r$, and $\frac{dC_j^i(x_s)}{dx_s} > 0$ for $x_s > x_s^r$. Therefore, the nonnegative function $C_j^i(x_s)$ is convex and has its minimum at x_s^r . Moreover, $C_j^i(x_s)$ is monotone increasing on the interval $[x_s^r, +\infty)$.

Let x_s^u denote the greater root of equation $C_j^i(x_s) - C_j^j = 0$, if any. It is obvious that $C_j^i(x_s) - C_j^j$ is nonnegative and increasing on the interval $[x_s^u, +\infty)$.

Let $x_s^{i,j} = \max\{x_s^r, x_s^u\}$ ($1 \leq i \leq s < j \leq t$). Note that $W_{i,j-1}^+(x_s) \geq 0$ and $\frac{dW_{i,j-1}^+(x_s)}{dx_s} \geq 0$. Then $\frac{d\mathcal{A}_{i,j,s}(x_s)}{dx_s} \geq 0$ on the interval $[x_s^{i,j}, +\infty)$, thus $\mathcal{A}_{i,j,s}(x_s)$ is nondecreasing on the interval.

Similarly, $C_s^i(x_s)$ ($1 \leq i \leq s$) is increasing on the interval $[x_s^{i,s}, +\infty)$, where $x_s^{i,s}$ is the root of equation $\frac{dC_s^i(x_s)}{dx_s} = 0$.

Let $x_s^\beta = \max_{\substack{1 \leq i \leq s \\ s < j \leq t}} \{x_s^{i,j}\}$. Since the probability factors $U \geq 0, W^+ \geq 0$, we have $\frac{dC_{s,t}(x_s)}{dx_s} \geq 0$.

Thus $C_{s,t}(x_s)$ is nondecreasing on the interval $[x_s^\beta, +\infty)$. \square

Proposition 5.4 implies that the minimum $C_{s,t}(x_s)$ locates on the interval $[0, x_s^\beta]$. In order to find the minimum $C_{s,t}(x_s)$, we can use the necessary condition $\frac{dC_{s,t}(x_s)}{dx_s} = 0$, where

$$\frac{dC_{s,t}(x_s)}{dx_s} = \sum_{i=1}^s \left[U_{i-1} \cdot W_{i,s-1}^+ \cdot \frac{dC_s^i(x_s)}{dx_s} \right] + \sum_{j=s+1}^t \sum_{i=1}^s U_{i-1} \cdot \left[\frac{dW_{i,j-1}^+(x_s)}{dx_s} \cdot (C_j^i(x_s) - C_j^j) + W_{i,j-1}^+(x_s) \cdot \frac{dC_j^i(x_s)}{dx_s} \right]. \quad (5.29)$$

Since $\frac{dC_{s,t}(x_s)}{dx_s}$ is continuous on the interval $[0, x_s^\beta]$, the equation $\frac{dC_{s,t}(x_s)}{dx_s} = 0$ has a finite number of roots. By comparing the corresponding local minimum values of $C_{s,t}(x_s)$, we can obtain the optimal x_s^* and the global minimum $C_{s,t}^*(x_s^*)$.

In real application, many numerical computing softwares (such as Mathematica and Matlab) can help to find the optimal x_s^* rapidly.

5.5.2 The Programming Structure

The heuristic solution procedure can be programmed as follows:

Stage 1:

$\Gamma_0 = 0$.

For $t = 1, 2, \dots, N$

For $s = 1, 2, \dots, t$

Compute $C_{s,t}^*(x_s^*) := \min_{\substack{x_s \geq 0 \\ x_{s+1} = \dots = x_t = 0}} C_{s,t}(x_s)$.

$\Gamma_{s,t} := \Gamma_{s-1} + C_{s,t}^*(x_s^*)$.

$\Gamma_t := \min_{1 \leq s \leq t} \{\Gamma_{s,t}\}$.

Stage 2:

$t := N$.

While $t > 1$

$s := \arg \min_{1 \leq \kappa \leq t} \{\Gamma_{\kappa,t}\}$.

$z_s := 1$.

For $\kappa = s + 1, \dots, t$, $z_\kappa := 0$.

For $\kappa = s + 1, \dots, t$, $x_\kappa := 0$.

$t := s - 1$.

5.6 Numerical Examples

This section illustrates numerical examples of the proposed solution method.

5.6.1 A 5-Period Planning Example and Results of Proposed Heuristic Method

Consider a 5-period planning problem as follows. For simplification, the holding cost h_i is taken to be 1 in all periods, while the lost-sale penalty p_i is taken to be 20. We suppose that the demand in each period is normally distributed. Take the demand forecast as the mean demand, and the forecast accuracy as the standard deviation. Table 5.1 presents a sample set of data.

Table 5.1: 5-period planning example data set

Period (i)	Setup Cost (K_{i-L})	Demand	
		Mean (μ_i)	Stand.Dev.(σ_i)
1	85	69	7.7
2	102	29	3.2
3	102	36	4.0
4	101	61	6.8
5	98	61	6.8

The cumulative demand $d_{i,j} = \sum_{t=i}^j d_t$ also follows a normal distribution, whose average and standard deviation are shown in Table 5.2.

Table 5.3 presents the optimal order quantity x_s^* that minimizes the expected total costs $C_{s,t}(x_s)$ incurred in periods $s, s+1, \dots, t$, knowing that the consecutive procurement orders are placed in period s and $t+1$, and x_1, x_2, \dots, x_{s-1} have already been determined to minimize the expected total costs Γ_{s-1} incurred in periods $1, 2, \dots, s-1$.

Table 5.4 presents the minimum expected total cost $C_{s,t}^*(x_s^*)$ corresponding to the optimal order quantities as listed in Table 5.3.

The following example is presented to illustrate the detailed calculation process of x_s^* and $C_{s,t}^*(x_s^*)$.

Suppose $s = 3, t = 4$. The order quantities of period 1 and 2 have been determined in

Table 5.2: Cumulative demand data set

Mean ^a ($\mu_{i,j}$)					
$i \backslash j$	1	2	3	4	5
1	69	98	134	195	256
2		29	65	126	187
3			36	97	158
4				61	122
5					61

Standard Deviation ^b ($\sigma_{i,j}$)					
$i \backslash j$	1	2	3	4	5
1	7.7	8.3	9.2	11.5	13.3
2		3.2	5.1	8.5	10.9
3			4.0	7.9	10.4
4				6.8	9.6
5					6.8

^aThis is the cumulative sum of mean demand.

^bThe standard deviation of a sum follows a square root law.

Table 5.3: Optimal procurement order quantity x_s^*

$s \backslash t$	1	2	3	4	5
1	82	109	144	205	265
2		31	65	129	189
3			41	102	155
4				71	125
5					69

the previous procedures: $x_1 = 109$, $x_2 = 0$. Note that $x_4 = 0$. Thus x_3 is the unique decision variable to be determined here.

Let $\Phi(x)$ denote the cumulative probability function of standard normal distribution, for the normal distribution with mean μ and standard deviation σ , the cumulative probability $F(x) =$

Table 5.4: Cost $C_{s,t}^*(x_s^*)$ associated with optimal order cycles

$s \backslash t$	1	2	3	4	5
1	101.0	143.5	229.6	434.6	699.5
2		119.2	171.3	314.4	517.5
3			120.3	203.6	247.6
4				123.7	181.4
5					124.9

$\Phi\left(\frac{x-\mu}{\sigma}\right)$. According to the *Empirical Rule* of normal distribution, for $x < \mu - 3\sigma$, $F(x) \approx 0$; for $x > \mu + 3\sigma$, $F(x) \approx 1$. Thus $U_1 = 0$. Since $x_2 = 0$, by definition, $W_{2,2}^+ = 0$, $W_{2,3}^+ = 0$, $W_{2,3}^- = 0$. Besides, it is easy to calculate that $W_{1,2}^+ = 0.906$, $U_2 = 0.094$.

Let $\phi(x)$ denote the probability density function of standard normal distribution, $\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$. For the normal distribution with mean μ and standard deviation σ , the density $f(x) = \frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right)$. According to Equations (5.24) and (5.25), we have

$$W_{1,3}^+(x_3) = \int_0^{x_1} f_1(\tau_1)d\tau_1 \int_0^{x_1+x_2-\tau_1} f_2(\tau_2)d\tau_2 \int_0^{x_1+x_2+x_3-\tau_1-\tau_2} f_3(\tau_3)d\tau_3,$$

$$W_{1,3}^-(x_3) = \int_0^{x_1} f_1(\tau_1)d\tau_1 \int_0^{x_1+x_2-\tau_1} f_2(\tau_2)d\tau_2 \int_{x_1+x_2+x_3-\tau_1-\tau_2}^{+\infty} f_3(\tau_3)d\tau_3,$$

$$W_{3,3}^+(x_3) = \Phi\left(\frac{x_3 - \mu_3}{\sigma_3}\right),$$

$$W_{3,3}^-(x_3) = 1 - \Phi\left(\frac{x_3 - \mu_3}{\sigma_3}\right),$$

where $f_i(x) = \frac{1}{\sigma_i}\phi\left(\frac{x-\mu_i}{\sigma_i}\right)$.

According to Equation (5.19), $U_3(x_3) = W_{1,3}^-(x_3) + U_2 \cdot W_{3,3}^-(x_3)$. According to Lemma 5.2, we have

$$C_{3,4}(x_3) = K_{3-L} + W_{1,2}^+ \cdot C_3^1(x_3) + U_2 \cdot C_3^3(x_3) + W_{1,3}^+(x_3) \cdot C_4^1(x_3) + U_2 \cdot W_{3,3}^+(x_3) \cdot C_4^3(x_3) + U_3(x_3) \cdot C_4^4.$$

Figure 5.5 shows the evolution of function $C_{3,4}(x_3)$. It is obvious that the minimum cost occurs at $x_3^* = 102$, and $C_{3,4}^*(x_3^*) = 203.6$.

Table 5.5 shows the details of shortest path optimization through this network. The optimal order sequence is determined in a backward direction. It is easy to find that $\Gamma_5 = \min_{1 \leq s \leq 5} \{\Gamma_{s,5}\} = 391.1$, where the predecessor order period is 3. Then we find $\Gamma_2 = \min_{1 \leq s \leq 2} \{\Gamma_{s,2}\} = 143.5$, where the predecessor order period is 1. Thus the optimal order policy is to procure 109 units at period 1-L, and 155 units at period 3-L. Therefore, the minimum expected total cost over the whole planning horizon is 391.1.

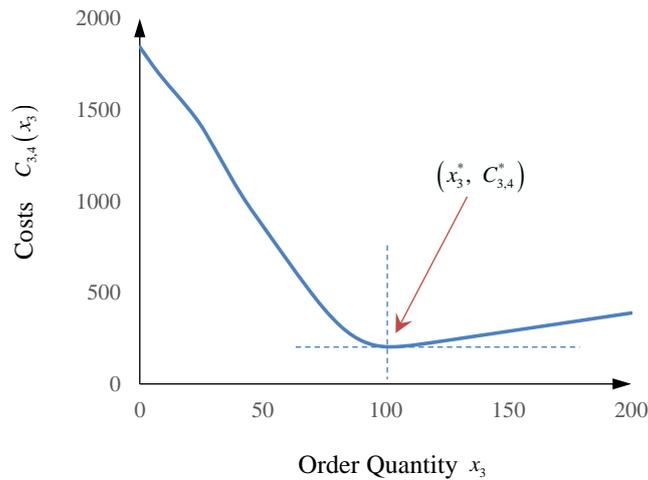


Figure 5.5: Cost $C_{3,4}(x_3)$

Table 5.5: Network optimization, minimum expected costs Γ_{st}

$s \backslash t$	1	2	3	4	5
1	101.0	143.5	229.6	434.6	699.5
2		220.3	272.4	415.5	618.6
3			263.8	347.1	391.1
4				353.3	411.0
5					472.0
Predecessor period s	1	1	1	3	3
Minimum cost up to t	101.0	143.5	229.6	347.1	391.1

5.6.2 A 200-Period Planning Example and Results of Proposed Adaptive Planning Approach

For procurement planning problems engaged in global sourcing, the proposed heuristic method will be deployed with the adaptive planning framework presented in Chapter 3.

In Section 4.6.3, we have illustrated how to implement the proposed adaptive planning procedure for a long-distance procurement planning problem engaged in global sourcing, provided that all unfulfilled demand is assumed to be backordered.

In this section, we study the same problem with lost-sale assumption. For data setting, we suppose that a penalty of 20 will be charged for each lost sale (or outsourced item) instead of the backorder cost used in Section 4.6.3.

The overall adaptive planning framework is the same as presented in Section 4.6.3. The only difference is that, in this section, we use the forward planning approach presented in Section 5.5 for sub-horizon interim optimal planning.

By implementing a numerical test of 500 actual 200-period demand scenarios which are generated with the same parameter setting as in Section 4.6.3, we obtain:

$$\mathcal{R}_1^{500} = \overline{C}/\overline{C^*} - 1 = 4.11\% , \quad \text{and} \quad \mathcal{R}_2^{500} = \overline{C}/\overline{C^*} - 1 = 4.11\% .$$

Here C denotes the total cost incurred by finally executed procurement plan determined with proposed adaptive planning procedure, and C^* denotes the total cost incurred by ex-post-facto optimal procurement plan for the same demand scenario. \mathcal{R}_1^k and \mathcal{R}_2^k especially represent the ratio of the mean values, and the mean value of the ratios, which are both statistic results of 500 randomly generated demand scenarios.

5.7 Conclusion

In this chapter, the procurement planning problem in global sourcing is considered. For unfulfilled demand, the lost-sale (or outsourcing) assumption is used. In this chapter, we developed a forward heuristic method based on a relaxed recursion formula, which can provide near-optimal solutions for sub-horizon planning problems. The proposed approach can serve as a benchmark for other planning heuristics. When deployed with the adaptive planning frame-

work in Chapter 3, the proposed solution procedure can help in making adequate procurement plans for global sourcing activities, whose effectiveness has been validated by numerical tests.

Chapter 6

Conclusion and Future Research

6.1 Conclusion

Due to economic globalization, global sourcing has become one of the most important cost-control measures in virtually every industry. Businesses have learnt that they are now all part of some extended enterprises with trading partners from all around the world. Companies of all scales may find that some part along their supply chain contains suppliers in distant locations, where the costs of primary products or services are considerably low. In such an environment, global sourcing has become a major issue that is worthy of attention.

Global sourcing is often associated with a centralized procurement planning system that seeks a balance among costs of setup, inventory holding and stockout penalty. How to make adequate procurement plans for global sourcing activities has attracted our attention for its extensive applications in real life.

Maritime transport is the most used transportation mode in global sourcing. The long delivery lead time due to geographically long distance between the buyer and the supplier has become our main concern. Long lead times lead to great difficulties in demand forecasting, thus companies might probably face unavoidable shortages and subsequent negative effects (such as the drop in customer loyalty and the damage on a firm's reputation).

In Chapter 2, a general literature review of research on procurement planning and global sourcing is presented. The majority of published papers in this field focus on qualitative analysis and strategic planning. For operational analysis that makes concrete procurement plans, little research work has been published.

In order to fill the above-mentioned gap of literature, we propose an adaptive procurement framework in Chapter 3. In this framework, demand forecasting and optimal planning techniques are deployed with a rolling horizon procedure. By employing the framework, the procurement planning problem in global sourcing can be split into optimal procurement planning problems in sub-horizons. For each sub-horizon, a systematical processing procedure is presented to transform the sub-horizon planning problem into an equivalent standard lot-sizing problem with stochastic demands. Besides, an evaluation method is proposed to test the proposed framework through numerical experiments.

Shortages are inevitable for practical procurement planning in global sourcing. In this dissertation, we consider a penalty cost for each stockout item. Chapter 4 and Chapter 5 have discussed respectively the backorder and lost-sale assumptions for shortages. For each

case, optimal or near-optimal planning methods minimize expected total costs (including setup, inventory holding and shortage penalty) are developed. When implemented with the framework presented in Chapter 3, we achieve in determining adequate procurement plans for global sourcing activities in a cost efficient manner. Numerical results show that the overall solution procedure is effective and efficient.

6.2 Future Research

Procurement planning in global sourcing is receiving increasing attention due to its extensive application in real life. This dissertation contributes a general adaptive planning framework and detailed sub-horizon optimal or near-optimal planning methods for backorder and lost-sale cases, which can help in determining adequate procurement plans for global sourcing activities in a cost efficient manner. However, there is still enough room for conducting further research. In what follows, we discuss some potential research questions that are interesting from both theoretical and practical perspectives.

In Chapter 4, we present an optimal sub-horizon procurement planning method that can minimize expected total costs for the backorder case. For efficiency improvement, we also develop a near-optimal heuristic method that can reduce computational time effectively. In Chapter 5, we have developed an effective near-optimal sub-horizon procurement planning method for the lost-sale case. However, due to problem complexity of lost sales, we haven't found an efficient alternative for computational time reduction. It is worthwhile to continue the development of efficient heuristics that can find near-optimal solutions rapidly. As a matter of fact, the planning methods presented in this dissertation can serve as benchmarks to evaluate other methods.

In addition, we evaluate the proposed approaches by numerical tests. Future research can be conducted in analytical analysis on performances of proposed approaches, such as the boundedness of the proposed adaptive framework, the worst-case performance, and so on.

Moreover, this thesis focuses on large demand uncertainties due to long distance, while lead time is considered to be fixed and known. In future research, it will be interesting to consider random lead time, which is more close to real life.

Besides, other shortage assumptions can be considered, such as a joint model with partial backorders and partial lost sales (or outsourcing).

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Résumé :

Cette thèse porte sur l'optimisation de l'approvisionnement dans les zones géographiquement lointaines. Au moment de planifier des approvisionnements de matières premières ou de composants dans des pays lointains, la longue distance géographique entre l'acheteur et le fournisseur devient un enjeu essentiel à prendre en compte. Puisque le transport se fait souvent par la voie maritime, le délai d'approvisionnement est si long que les besoins peuvent évoluer pendant la longue période de livraison, ce qui peut engendrer un risque de rupture élevé. Cette thèse présente des approches adaptatives afin d'élaborer des plans d'approvisionnements lointains d'une manière rentable.

Tout d'abord, nous proposons un cadre d'adaptation de la planification des approvisionnements lointains. Il déploie des techniques de prévision de la demande et des méthodes d'optimisation d'approvisionnements à horizon glissant. En utilisant ce cadre, nous transformons le problème de la planification sur l'horizon globale en plusieurs problèmes standards de lotissement avec demandes stochastiques sur des sous-horizons. Ce cadre permet aussi d'évaluer la performance sur une longue période des méthodes utilisées.

Nous considérons ensuite la planification optimale d'approvisionnement sur les sous-horizons. Deux hypothèses de ruptures de stocks sont considérées: livraison tardive et vente perdue (ou sous-traitance). Nous développons des approches optimales ou quasi-optimales pour faire des plans d'approvisionnement tout en minimisant les coûts totaux prévus de commande, de stockage et de rupture sur les sous-horizons. Les méthodes proposées peuvent servir de repères pour évaluer d'autres méthodes. Pour chaque hypothèse, nous menons des expériences numériques pour évaluer les algorithmes développés et les approches adaptatives de planification globales. Les résultats expérimentaux montrent bien leur efficacité.

Mots clés : approvisionnement lointain, planification optimale, incertitude, demande stochastique, horizon glissant, lotissement, longue période de livraison, minimisation des coûts

Abstract:

This research discusses procurement planning problems engaged in global sourcing. The main difficulty is caused by the geographically long distance between buyer and supplier, which results in long lead times when maritime transport is used. Customer demands of finished-products usually evolve during the shipment, thus extra costs will be produced due to unpredictable overstocks or stockouts. This thesis presents adaptive planning approaches to make adequate long-distance procurement plans in a cost-efficient manner.

Firstly, an adaptive procurement planning framework is presented. The framework deploys demand forecasting and optimal planning in a rolling horizon scheme. In each sub-horizon, demands are assumed to follow some known distribution patterns, while the distribution parameters will be estimated based on up-to-date demand forecasts and forecast accuracy. Then a portable processing module is presented to transform the sub-horizon planning problem into an equivalent standard lot-sizing problem with stochastic demands.

Secondly, optimal or near-optimal procurement planning methods are developed to minimize expected total costs including setup, inventory holding and stockout penalty in sub-horizons. Two extreme stockout assumptions are considered: backorder and lost sale (or outsourcing). The proposed methods can serve as benchmarks to evaluate other methods. Numerical tests have validated the high efficiency and effectiveness of both sub-horizon planning methods and the overall adaptive planning approaches.

Keywords: global sourcing, procurement planning, uncertainty, stochastic demand, rolling horizon, lot sizing, long distance, cost optimization