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Stability contract for tactical planning in a supply chain : new lot sizing models

Bertrand Hellion

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THÈSE

Pour obtenir le grade de

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Spécialité : **Génie Industriel**

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Présentée par

Bertrand HELLION

Thèse dirigée par **Bernard PENZ**
et codirigée par **Fabien MANGIONE**

préparée au sein du laboratoire **G-SCOP (Grenoble Science pour la Conception et l'Optimisation et la Production)**
et de l'école doctorale **IMEP2 (Ingénierie - Matériaux, Mécanique, Énergétique, Environnement, Procédés, Production)**

Contrats de stabilité pour la planification tactique dans une chaîne logistique : nouveaux modèles de dimensionnement de lots.

Thèse soutenue publiquement le 11 octobre 2013 ,
devant le jury composé de :

Mr Stéphane DAUZÈRE-PÉRÈS

Professeur, École nationale supérieure des mines de Saint-Étienne, Président

Mme Safia KEDAD-SIDHOUM

Maître de conférences, Université Pierre et Marie Curie, Rapporteur

Mr Christian ARTIGUES

Directeur de recherche. CNRS LAAS de Toulouse, Rapporteur

Mme Nadia LEHOUX

Professeur, Université de Laval, Québec, Canada, Examineur

Mr Fabien MANGIONE

Maître de conférences, Institut Polytechnique de Grenoble, Co-Encadrant de thèse

Mr Bernard PENZ

Professeur, Institut Polytechnique de Grenoble, Directeur de thèse



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Même si l'intérêt des travaux présentés dans cette thèse est limité à un domaine très précis de la science, il est indéniable que la prise de recul engendrée par la thèse est utile et valorisable dans toutes les étapes de la vie. Si les études supérieures apprennent l'esprit critique, le doctorat m'a enseigné à cibler ces critiques sur mes propres travaux. Curieuse progression qui m'a animé pendant ces trois ans. La première année était synonyme de pensées entreprenantes et positives ; La deuxième année était porteuse de difficultés et de remises en question ; La troisième année est celle de la prise de recul, avec un bilan mitigé sur mon propre travail, mais extrêmement positif sur les avantages de l'avoir fait.

Introduction Générale

Alors que la fabrication du produit final livré au client est toujours plus complexe, la sous-traitance se massifie et se généralise. Une conséquence est une augmentation du nombre d'acteurs impliqués dans la création, la distribution et la livraison du produit.

Si une gestion centralisée de la chaîne logistique paraît plus efficace ([Arshinder *et al.*, 2008](#)), elle n'est malheureusement pas réaliste ([Giannoccaro and Pontrandolfo, 2004](#)). Le système trouve alors un équilibre dans la gestion séparée de chacun de ces acteurs, tous optimisant leurs propres fonctionnements. Il n'y a donc pas un seul donneur d'ordres, mais plutôt plusieurs, et leurs interactions affectent grandement l'efficacité de la chaîne logistique.

Une bonne collaboration avec ses partenaires permet à l'entreprise d'avoir une vision à long terme, à travers une fiabilisation de ses relations ([Sahin and Robinson, 2002](#)). Il a été démontré qu'une mauvaise collaboration mène à des effets désastreux, l'un d'entre eux étant le célèbre "effet coup de fouet" ([Dejonckheere, 2003](#); [Lee *et al.*, 1997](#)).

L'objet de cette thèse est de présenter un nouveau mécanisme de collaboration permettant de fiabiliser les relations client-fournisseur. Il s'agit de mettre en place des contrats de stabilité dans le but d'assurer les approvisionnements du client tout en garantissant les commandes au fournisseur.

Nous considérons une chaîne logistique livrant un produit unique pour un client unique. La demande du client est définie comme étant la quantité désirée de produits par le client. Dans ce cas, l'efficacité de cette chaîne logistique peut être évaluée. Il s'agit de minimiser les coûts induits par les acteurs, en satisfaisant la demande.

La chaîne logistique considérée contient un unique distributeur, lié à un ou plusieurs fournisseurs. Ces fournisseurs livrent le même produit mais peuvent avoir des caractéristiques différentes, telles que la capacité de production ou le prix demandé.

Nous nous intéressons aux relations à long terme liant ces acteurs. Nous étudions des contrats garantissant des partenariats stables dans le temps, en assurant l'approvisionnement du distributeur tout en minimisant les coûts induits. Ces contrats sont appelés contrats de stabilité et sont développés tout au long de la thèse.

Le chapitre [1](#) introduit la problématique centrale de la thèse. Nous parlons du contexte

environnant, et évoquons brièvement des modèles généraux justifiant nos travaux. Au cours de ce chapitre, les principaux concepts autour de la chaîne logistique sont définis. Ces définitions nous permettent d'introduire un exemple illustrant l'ambition des contrats de stabilité.

Le chapitre 2 est un état de l'art non exhaustif (car la littérature est vaste dans ce domaine) qui décrit une partie des publications reliés directement à notre problématique. Ce chapitre est divisé en deux parties :

- la première partie s'articule autour de la collaboration entre acteurs dans une chaîne logistique à un niveau stratégique ;
- une fois la collaboration décidée, il s'agit d'optimiser la gestion des approvisionnements au niveau tactique, qui peut être modélisée comme un problème du dimensionnement de lots.

Les quatre chapitres qui suivent sont nos contributions directes sur la problématique.

Le chapitre 3 (article soumis en revue) définit les contrats de stabilité. Ceux-ci sont utilisés pour fiabiliser les relations entre fournisseurs et distributeur, sur un horizon stratégique. En particulier, ce chapitre détaille les deux parties des contrats de stabilité :

- l'une permet de contrôler les quantités de commandes passées au fournisseur ;
- l'autre est temporelle et contraint les commandes à s'effectuer dans des fenêtres de temps définies.

Ce chapitre présente également les dépendances entre ces deux parties. Une série d'expériences est présentée, indiquant comment définir les paramètres des contrats, pour aboutir à des relations avantageuses pour tous les partenaires.

Le chapitre 4 est un article publié en 2012 dans *European Journal of Operational Research*. Nous modélisons l'approvisionnement du distributeur par un problème de dimensionnement de lots. Prendre en compte la partie quantitative des contrats de stabilité mène à un problème nommé problème du dimensionnement de lots avec capacité et quantité minimale de commande. Avec des structures de coût de commande et de stockage concaves, ce problème est prouvé polynomial par la présentation d'un algorithme pour le résoudre à l'optimal. Une étude expérimentale est menée pour tester l'exécution de l'algorithme sous différents paramètres.

Le chapitre 5 est un article soumis en revue qui étend le chapitre 4. Considérer la partie temporelle des contrats de stabilité mène à un nouveau problème plus complexe appelé problème du dimensionnement de lots avec capacité minimale et maximale et fenêtres de temps dynamiques. En considérant des structures de coût concaves, cette variante est prouvée polynomiale par la présentation d'un algorithme optimal pour résoudre ce problème, mais de complexité plus grande.

Le chapitre 6 est un article soumis en revue, présentant une chaîne logistique se situant géographiquement dans une région canadienne, appelé la *Côte Nord*. Une papetière a des difficultés d'approvisionnement avec ses fournisseurs (scieries) environnants. Cette chaîne logistique, dont l'activité est de fabriquer de la pâte à papier puis du papier à partir de

copeaux de bois, a besoin de contrats spécifiques pour assurer la pérennité des liens qui la définissent. La solution retenue est de proposer des contrats de stabilité entre les scieries et la papetière pour garantir à cette dernière son approvisionnement.

La conclusion synthétise les contributions qui ont été présentées dans la thèse. Puis, les perspectives à moyen terme et à long terme découlant de ces travaux terminent ce manuscrit.

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Chapitre 1

Présentation du problème

1.1 Généralités et définitions

1.1.1 Chaîne logistique

Les problématiques de la chaîne logistique ont bien évolué lors des 20 dernières années (Daniel and Van Wassenhove, 2009), menant à une multiplication de ses définitions (Mentzer *et al.*, 2001). Parmi elles, d'après Mentzer *et al.* (2001), la chaîne logistique est un ensemble d'au moins trois entités (entreprise ou particulier) directement affectés par les flux physiques, financiers ou d'information circulant du fournisseur jusqu'au client. C'est aussi une succession d'opérations ou d'activités de transformation, stockage ou transport logistique.

La chaîne logistique est donc un ensemble d'acteurs, et peut être représentée par un réseau, dans lequel un lien entre deux acteurs représente un ou plusieurs flux entre ces acteurs (Figure ??).

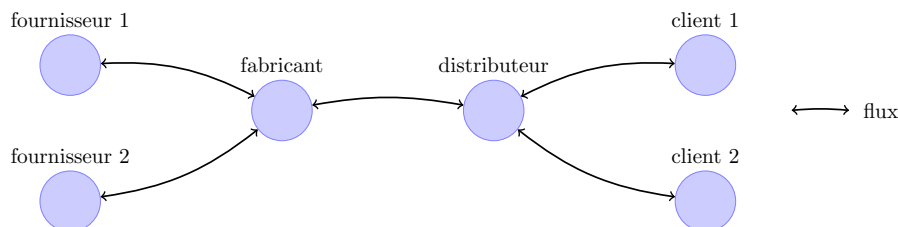


FIGURE 1.1 – Exemple de chaîne logistique à 6 acteurs.

En prenant en compte une seule opération ou activité entre des acteurs, on parle de système à un échelon (Figure ??, ici une activité de stockage). Dans un cas considérant plusieurs opérations ou activités on parle de système multi-échelon (Figure ??).

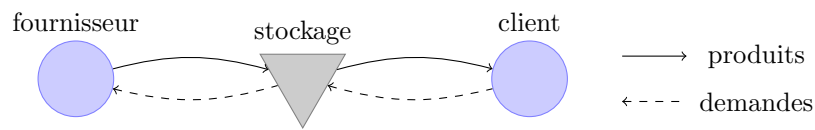


FIGURE 1.2 – Système à un seul échelon.

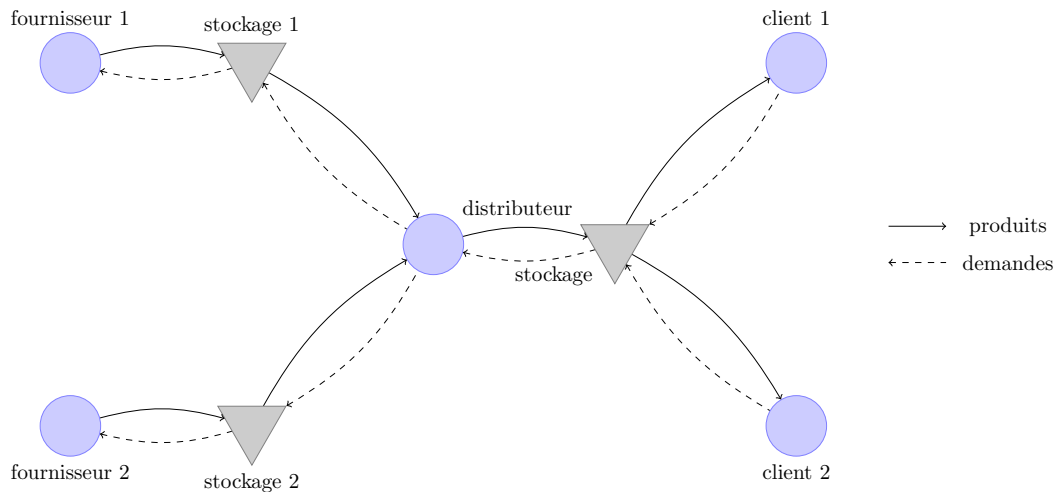


FIGURE 1.3 – Système à plusieurs échelons.

1.1.2 Flux logistiques

Trois types de flux différents circulent dans la chaîne logistique.

- Les flux physiques : ces flux regroupent tous les produits physiques qui traversent la chaîne logistique, en particulier les flux de matières premières et de produits finis. La circulation de ces flux est essentiellement d'amont en aval.
- Les flux financiers : il s'agit des flux monétaires associés aux flux physiques. Ces flux traversent la chaîne essentiellement d'aval en amont.
- Les flux d'information : il s'agit des échanges d'information entre les acteurs de la chaîne. L'information peut concerner l'état du système, le niveau des stocks, ou la demande du client. Ces flux peuvent s'effectuer dans les deux sens.

Ces flux sont représentés sur la figure ??.

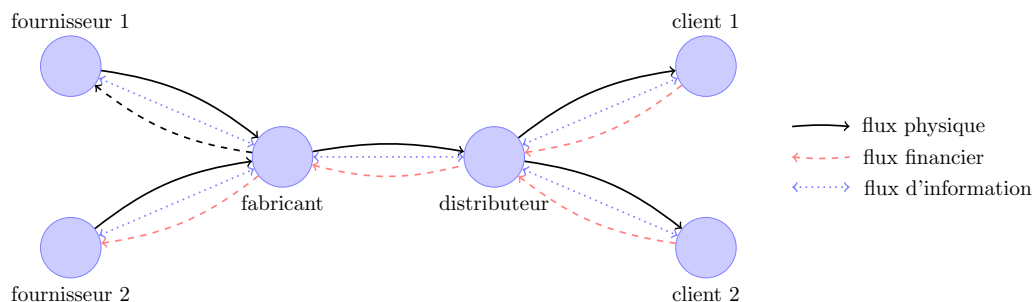


FIGURE 1.4 – Flux logistiques.

1.1.3 Niveaux de décision

Les décisions dans une chaîne logistique peuvent être réparties en trois catégories (Figure ??).

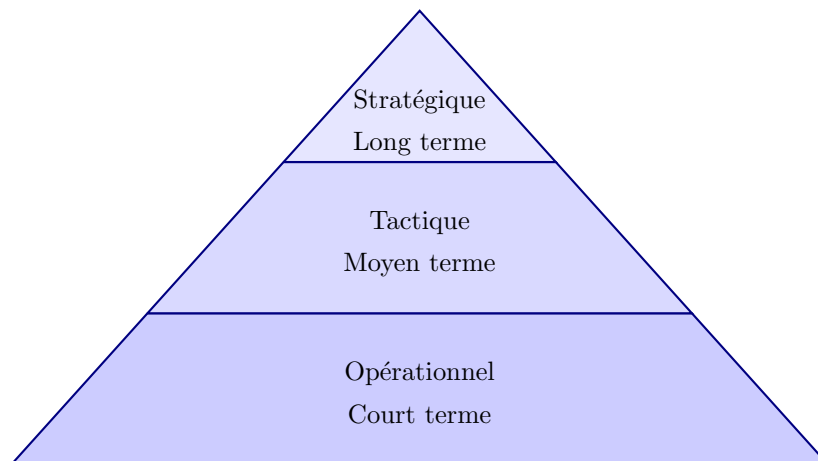


FIGURE 1.5 – Pyramide de décision.

Les définitions suivantes sont tirées du livre de [Giard \(2003\)](#).

“Les décisions stratégiques se traduisent par la formulation de la politique à long terme de l’entreprise (vision à plus de deux ans, en général) [...]. Les ressources stables visées sont aussi bien les machines [...] que les hommes [...], les informations [...] détenues dans des systèmes d’information caractérisés par leur degré d’intégration et de latence et les encours de matières, composants et produits semi-finis [...].

Les décisions tactiques correspondent à un ensemble de décisions à moyen terme. Parmi les décisions tactiques concernant la gestion de la production, on trouve : la planification de la production, qui est une programmation prévisionnelle de la production, agrégée par famille de produits, pour un ensemble de périodes dont l’amplitude varie entre la semaine et le mois [...].

Les décisions opérationnelles assurent la flexibilité quotidienne nécessaire pour faire face aux fluctuations prévues de la demande et des disponibilités de ressources [...], sur un horizon ne dépassant pas quelques dizaines d’heures, dans le cadre d’un découpage temporel généralement de l’ordre de la minute.”

Dans cette thèse, les problématiques étudiées relèvent des décisions stratégiques (définition et création des contrats de stabilité) et des décisions tactiques (optimisation de l’approvisionnement par le problème du dimensionnement de lots).

1.1.4 Problématiques de la chaîne logistique

Les décisions prises par le donneur d'ordres portent sur 3 problématiques distinctes de la chaîne logistique. Il s'agit de la gestion de production, de la gestion de stock et la logistique de transport.

La gestion de production est la principale problématique de la chaîne logistique abordée dans cette thèse, puisqu'elle affecte directement le niveau de stock, celui-ci est aussi considéré.

Gestion de production

D'après [Giard \(2003\)](#), "La gestion de production a pour objet la recherche d'une organisation efficace de la production de biens et de services. En situant la production dans la perspective plus large de la chaîne logistique, la définition de cette organisation doit aussi impérativement prendre en compte la maîtrise des flux entrants (approvisionnements) et celle des flux sortants (distribution) pour assurer le niveau de satisfaction globalement attendu par les clients."

La Figure ?? représente le niveau de stock d'un entrepôt. Chaque baisse d'inventaire correspond à une demande client, tandis que chaque hausse correspond à une production. Sur cette figure, il y a production seulement si l'inventaire est nul.

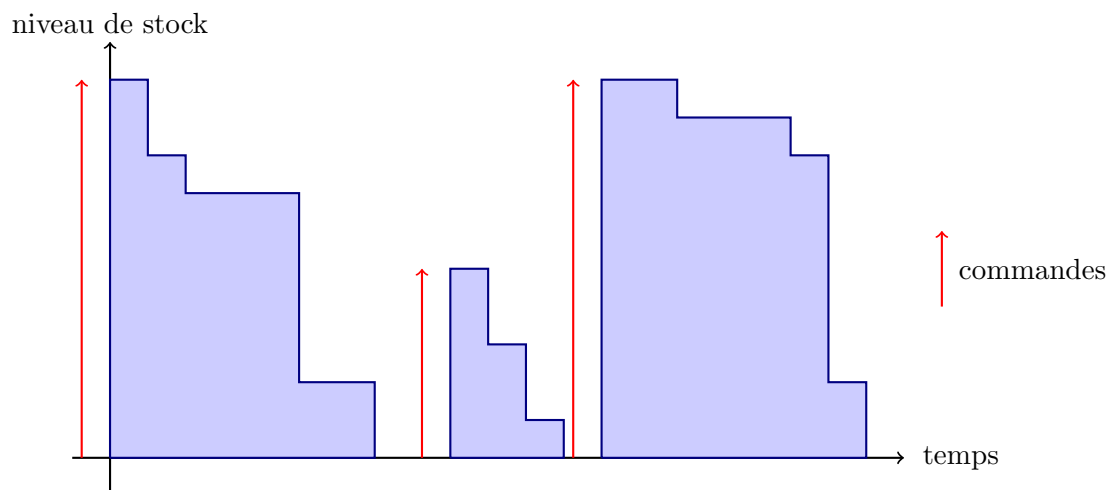


FIGURE 1.6 – Évolution du niveau de stock.

Gestion des stocks

La gestion des stocks est une des priorités de tout preneur de décision d'une chaîne logistique. Si avoir un stock important peut sembler plus confortable, surtout pour satisfaire des demandes incertaines, la tendance actuelle est à la recherche de stock minimaux pour optimiser les coûts (Liker, 2004).

Logistique de transport

La logistique de transport est l'activité engendrée par les ressources (humaines ou matérielles) dans le but de maintenir le flux physique à travers la chaîne logistique. À la différence de la gestion des stocks et la gestion de la production, la logistique de transport peut être gérée séparément.

1.2 Relations entre donneur d'ordres et fournisseur : durabilité et contrat de stabilité

Les relations entre donneurs d'ordre et fournisseurs sont diverses. Elles peuvent être d'importances différentes, et portant à des échelles de temps différentes.

- Au niveau stratégique, la sélection des fournisseurs et la création de contrats sont des étapes majeures des relations entre donneur d'ordres et fournisseur (Aissaoui *et al.*, 2007; Elmaghraby, 2000).
- Au niveau tactique, ces relations regroupent les informations portant sur les commandes journalières, hebdomadaires, ou mensuelles.
- Au niveau opérationnel, elles englobent les changements de dernière minute, retard de livraison, et imprévus.

1.2.1 Durabilité

Dans sa définition la plus générale, le principe de durabilité repose sur trois piliers fondamentaux : économique, social et environnemental (Figure ??). Les contrats de stabilité présentés dans cette thèse portent sur deux des trois dimensions fondamentales :

- la dimension sociale, car les contrats de stabilité tentent de fiabiliser les relations entre donneurs d'ordre et fournisseurs, en permettant ainsi de garantir une certaine stabilité des emplois ;
- la dimension économique, car ces contrats ne sont applicables que si l'augmentation des coûts qu'ils impliquent est faible et maîtrisée.

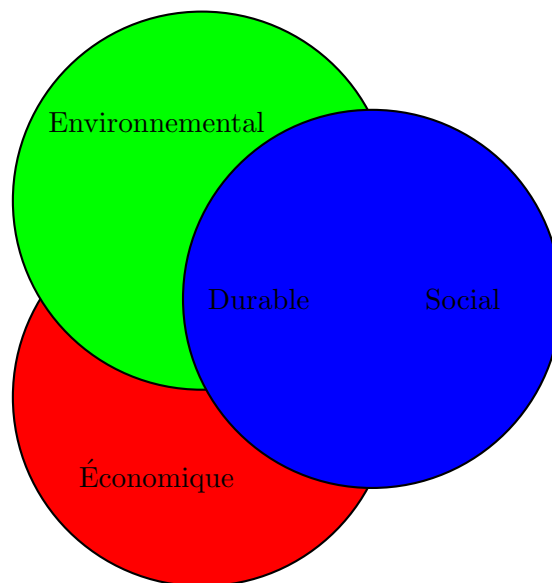


FIGURE 1.7 – Les trois piliers de la durabilité.

1.2.2 Contrat de stabilité

Les contrats de stabilité sont maintenant présentés avec un exemple. Une collaboration réunit deux acteurs : un fournisseur et un distributeur. Sur un horizon tactique, le distributeur a une liste de demandes à satisfaire sur un intervalle de temps, qui est discrétisé en périodes distinctes. Pour cela, il peut soit commander à son fournisseur, soit utiliser l'inventaire présent à la fin de la période précédente.

On suppose que le distributeur ne connaît de son fournisseur que le prix et sa capacité de livraison. Une fois les produits achetés, le fournisseur s'engage à les livrer par camion et dans les délais.

Pour le distributeur, les coûts assumés sont de 4,5 pour le coût fixe de commande et 1 pour le coût unitaire de stockage. Puisque la demande doit être satisfaite, les coûts unitaires de commande ne sont pas pris en compte. En conséquent il n'est rentable de

stocker d'une période sur l'autre que si la quantité d'unité à stocker est inférieur à 4, 5. Le Tableau ?? présente le plan de commande optimal du distributeur, pour un coût de 25.

périodes	1	2	3	4	5	6
demandes	4	6	7	3	2	8
commandes	4	6	12			8
inventaire			5	2		

TABLE 1.1 – Un plan de commande.

Cette solution, bien que satisfaisante pour le distributeur, complique l'établissement du planning de livraison du fournisseur. En effet, les quantités de commande sont variables, et les dates de commandes sont irrégulières. Ce type de planning de commande oblige le fournisseur à prendre des dispositions logistiques particulières (et probablement coûteuses), aux périodes de grosses commandes.

En lissant les commandes, on obtient le plan d'approvisionnement présenté dans la Table ??.

périodes	1	2	3	4	5	6
demandes	4	6	7	3	2	8
commandes	10		10		10	
inventaire	6		3		8	

TABLE 1.2 – Un plan de commande régulier.

Pour le distributeur, ce plan de commande est moins satisfaisant que le premier : son coût est maintenant de 30, 5. En effet, la quantité d'unité à stocker passe de 7 à 17 ; Notez que de manière générale l'augmentation des coûts de stockage est facilement mesurable.

Cependant, du point de vue du fournisseur, l'extrême régularité des commandes facilite l'établissement du planning de livraison :

- la quantité maximum livrée passe de 12 à 10, ce qui peut diminuer le nombre de camions nécessaires ;
- le nombre de livraisons passe de 4 à 3, ce qui allège le planning des transporteurs ;
- les dates de livraison sont mieux réparties sur l'horizon, rendant le planning de production plus constant en diminuant les coûts de stockage du fournisseur.

Tous ces gains sont faciles à imaginer, mais difficiles à mesurer. En résumé, choisir ce plan de commande augmente les coûts de stockage du distributeur de façon mesurable ; alors qu'il diminue les coûts de production, de main d'œuvre et de transport du fournisseur de façon plus difficilement quantifiable.

L'objectif du contrat de stabilité est de compenser cette augmentation du coût de sto-

ckage du distributeur par une réduction de prix de la part du fournisseur. Cette réduction de prix doit être inférieure (ou égale) au gain du fournisseur mais supérieure (ou égale) à la perte du distributeur. Bien évidemment, ce n'est possible que si la perte du distributeur est inférieure (ou égale) aux gains du fournisseur.

De cette manière, les acteurs impliqués pourraient s'engager sur le long terme :

- le distributeur s'engage à commander avec une certaine régularité ;
- le fournisseur s'engage à fournir ces commandes, en garantissant une réduction de prix suffisante.

1.3 Plan et contributions

Ce document est organisé par article. Cependant il est préférable de lire les chapitres 4 à 6 dans l'ordre.

Le chapitre 2 présente un état de l'art sur les relations et la planification des échanges entre distributeurs et fournisseurs. Il est décomposé en deux parties. La première partie détaille les relations de collaboration entre différents acteurs d'une chaîne logistique : de la sélection des fournisseurs à l'établissement de contrats. Elle est suivie d'une autre partie concernant la planification tactique des échanges commerciaux. Une fois le contrat signé et le partenariat commencé, il s'agit d'avoir les outils pour optimiser les flux logistiques circulant à travers la chaîne logistique. Le problème du dimensionnement de lots est adapté à cette optimisation, et sa formulation simple permet l'ajout d'un certain nombre de contraintes industrielles, incluant celles figurant dans le contrat de stabilité.

1.3.1 Motivation et définition des contrats de stabilité : quantité et fenêtres de temps dynamiques

Le chapitre 3 présente les contrats de stabilité en introduisant deux parties distinctes : une partie contraignant les quantités de chaque commande, et une partie sur la régularité temporelle des commandes.

- Les quantités de commandes sont régulées par une borne minimum L et une borne maximum U ;
- deux commandes successives doivent être espacées d'au moins Q périodes et d'au plus R périodes : ces fenêtres de temps dépendent donc chacune du placement de la commande précédente, c'est pourquoi elles sont appelées fenêtres de temps dynamiques ("Dynamic Time Windows", ou DTW : voir Figure ??).

Le problème de dimensionnement de lots résultant, prenant en compte ces deux parties, est original. Un plan d'expérience est établi pour tester l'influence des paramètres (coûts, demandes ou contrats). Les résultats permettent de comprendre pourquoi les coûts fixes

n'ont aucune influence sur le problème. Les coûts induits par les contrats de stabilité sont étudiés sur plusieurs fournisseurs, montrant l'importance de l'augmentation des coûts de stockage.

En se focalisant sur ces derniers, la dernière expérience met en évidence un lien étroit entre les deux parties du contrat. Une partie du contrat est dite serrée quand ses contraintes induites contraignent fortement le problème. Nous montrons que quand une partie est suffisamment serrée, l'autre partie peut être serrée à un niveau équivalent sans payer de coûts de stockage supplémentaires. D'avantageux contrats peuvent être acceptés par des partenaires commerciaux en profitant de cette observation.

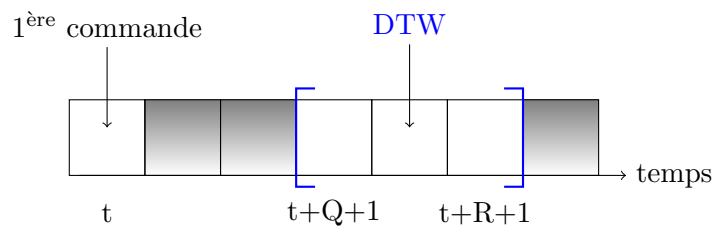


FIGURE 1.8 – Un exemple de fenêtre de temps dynamique pour $Q = 2$ et $R = 4$.

Dans l'exemple, une commande est livrée à la période t . En considérant que $Q = 2$, la commande suivante ne peut pas être passée ni à la période $t + 1$, ni à $t + 2$. Avec $R = 4$, au moins une commande doit être passée dans les cinq périodes suivantes. En conclusion, la prochaine commande doit être passée entre les périodes $t + 3$ ($t + Q + 1$) et $t + 5$ ($t + R + 1$) incluses. Notez que si $Q = R$, une commande doit être passée toutes les $Q + 1$ périodes. En outre, si $Q = R = 0$, une commande doit être passée à toutes les périodes.

1.3.2 Dimensionnement de lots mono-produit avec capacités et quantités minimales d'approvisionnement

Le chapitre 4 présente un problème de dimensionnement de lots avec capacité et quantité minimale de commande. Le problème est polynomial avec des fonctions de coût très simples ($O(T^3)$, Okhrin and Richter (2011), où T représente la longueur de l'horizon de planification, et donc le nombre de périodes considéré), mais sans résultat pour des fonctions de coût plus complexes. Il est connu que sans la quantité minimale de commande, le problème est aussi polynomial ($O(T^4)$, Florian and Klein (1971)). La particularité est que nous considérons à la fois des fonctions de coût concaves (production et stockage) et une quantité minimale de commande.

Avec ces suppositions, nous montrons qu'il existe toujours au moins une solution optimale qui est une succession de sous-plans particuliers. Cette propriété nous permet de décomposer le problème en plusieurs sous-problèmes, qui sont résolus séparément. La meilleure combinaison de sous-plans peut être trouvée en cherchant un plus court che-

min dans un graphe à T^2 arêtes. Cela nous permet de proposer un algorithme exact qui s'exécute en temps polynomial en $O(T^6)$. Une partie expérimentale nous permet de tester le comportement de l'algorithme sur de nombreuses instances.

1.3.3 Dimensionnement de lots mono-produit avec capacités, quantités minimales d'approvisionnement et fenêtres de temps dynamiques

Le chapitre 5 étend l'algorithme du chapitre 4 en prenant en compte les fenêtres de temps dynamiques. Nous montrons que la propriété de dominance des séquences de production est toujours vérifiée, et par conséquent que la décomposition en sous problèmes est encore faisable.

Grâce à une opération de division des sommets, il est possible de prendre en compte les fenêtres de temps dynamiques directement dans la structure du graphe. Ce problème est donc montré polynomial par la présentation d'un algorithme optimal s'exécutant en $O(T^9)$. Cependant, une simple supposition sur la forme des demandes (qu'elles soient à chaque période toutes strictement positives) permet de réduire la taille du graphe et donc la complexité de l'algorithme jusqu'à $O(T^7)$.

1.3.4 Établissement de contrat de stabilité : étude de cas dans l'industrie forestière d'une région du Québec

Le chapitre 6 est une étude de cas d'une chaîne logistique de l'industrie forestière québécoise. Les acteurs considérés sont une papetière et ses trois fournisseurs, qui sont des scieries. Les scieries produisent du bois d'œuvre (utilisé principalement dans la construction de bâtiments) et du copeau issus du sciage du bois. Ces copeaux sont la matière première dont la papetière a besoin pour produire du papier. Le problème est que le copeau n'est pas le principal objectif du sciage, c'est du bois d'œuvre que la scierie tire la majorité de son profit. Cette situation crée une situation de déséquilibre entre les scieries et la papetière.

La méthode retenue est la création de contrats de stabilité spécifiques. Ceux-ci doivent permettre d'assurer à la papetière la quantité de copeau dont elle a besoin, tout en maximisant le profit de la scierie. Deux contraintes industrielles doivent néanmoins être prises en considération lors de la conception des contrats :

- une scierie peut choisir la proportion de copeaux produite avec le bois, néanmoins le processus de sciage nécessite qu'un minimum de copeaux doit être produit ;
- chaque scierie produit un copeau d'une certaine qualité (qui dépend de l'espèce de l'arbre scié). Pour la papetière, une qualité minimum moyenne de copeau est requise pour produire du papier.

Des contrats de stabilité sont créés pour garantir l'approvisionnement en copeau de la papetière, tout en assurant un profit maximum pour les scieries. Ces contrats sont ensuite validés par une série d'expériences.

Chapitre 2

État de l'art

Cet état de l'art est divisé en deux grandes parties. Dans une première partie, la chaîne logistique et la collaboration entre acteurs vont être étudiées. Cela va nous permettre de comprendre les relations entre ces acteurs à un niveau stratégique. La deuxième partie se focalise sur l'optimisation tactique des approvisionnements une fois la collaboration décidée. Le problème d'approvisionnement d'un acteur donné peut se modéliser comme un problème de dimensionnement de lots, et ce dernier est donc approfondi au cours de cette partie.

2.1 Chaîne logistique et collaboration entre acteurs

La chaîne logistique a été étudiée par de nombreux auteurs, son historique a notamment été résumé par [Daniel and Van Wassenhove \(2009\)](#).

2.1.1 Chaîne logistique durable

La chaîne logistique durable est de plus en plus étudiée dans les milieux politiques ou académiques ([Linton *et al.*, 2007](#)). Domaine en forte progression, les états de l'art sur cet aspect de la chaîne logistique sont nombreux ([Tang and Zhou, 2012](#); [Srivastava, 2007](#)).

L'intégration de critères et d'objectifs durables dans les entreprises, poussée notamment par les nouvelles normes ISO ([Corbett and Klassen, 2006](#)), a explosé ces dernières années ([Belin-Munier, 2010](#)). Des chercheurs ont aussi montré que l'incitation pouvait être une motivation d'intégration de critères de durabilité ([Benjaafar *et al.*, 2013](#)).

L'évaluation des performances durables dans une entreprise est un indicateur crucial pour un preneur de décision. Pour ce faire, on peut répartir les enjeux parmi les trois piliers de la durabilité, et ensuite évaluer les bonnes pratiques agissant sur chacun des enjeux ([Baumann, 2011](#)). Cependant, la complexité de l'évaluation vient du grand nombre

d'indicateurs de durabilité d'une chaîne logistique, et de la difficulté de les comparer entre eux (Krajnc and Glavic, 2003).

La prise en compte de ces critères au niveau stratégique se traduit en exigences envers les fournisseurs. Des moyens d'évaluer ces entreprises permettent aux entreprises de bien choisir leurs partenaires (Handfield *et al.*, 2002).

2.1.2 Sélection des fournisseurs

Le choix d'un fournisseur est une décision stratégique importante dans la vie d'une entreprise, et requiert la mise en œuvre d'outils d'analyse spécifiques (Aissaoui *et al.*, 2007). L'évaluation des performances des fournisseurs nécessite une bonne politique de communication, qui parfois se révèle insuffisante (Prahinski and Benton, 2004). Pour aider les entreprises à évaluer et sélectionner leurs fournisseurs, des modèles d'optimisation ont été développés (Feng *et al.*, 2001; Rezaei and Davoodi, 2011).

La sélection des fournisseurs effectuée au niveau stratégique se poursuit au niveau tactique par l'entretien d'une collaboration.

2.1.3 Collaboration entre acteurs

Une bonne collaboration avec les fournisseurs est essentielle au plus haut niveau de décision de l'entreprise (Thomas and Griffin, 1996). Pour être rentable, les activités des entreprises d'une chaîne logistique doivent être communément gérées, par des interactions plus fortes entre acteurs (Chan and Chan, 2010a). Cependant, la collaboration doit être bien guidée pour être rentable pour chacun des acteurs (Lehoux *et al.*, 2013).

Le manque de collaboration s'avère dangereux pour l'ensemble de la chaîne, en impliquant des comportements non-propices au bon fonctionnement du système, et menant à un profit mal réparti entre les acteurs (Tsay, 1999). Le preneur de décision devrait disposer de toutes les informations nécessaires pour mieux optimiser la chaîne logistique. Cependant, c'est le plus souvent opérationnellement irréalisable (Giannoccaro and Pontrandolfo, 2004).

Les facteurs menant à de mauvaises collaborations ont été étudiés. En particulier dans l'industrie automobile, Dyer (1997) met en évidence que les coûts de transaction de Toyota sont moins importants que ceux de General Motors. À la fin de son étude Dyer (1997) montre que Toyota demande beaucoup d'informations à ses partenaires, ce qui lui permet de nouer d'étroites relations symétriques, basées sur la confiance réciproque. Les facteurs de mauvaises collaborations peuvent aussi être mis en évidence grâce à des modèles de collaboration simples (De Souza *et al.*, 2000).

La mauvaise collaboration et le manque d'échange d'information peuvent provoquer l'effet coup de fouet. Il s'agit pour un acteur de répercuter exagérément sa demande sur

ses propres fournisseurs, provoquant une réaction en chaîne. L'effet coup de fouet a été mathématiquement étudié par [Lee et al. \(1997\)](#) et ensuite par [Dejonckheere \(2003\)](#). Tous montrent qu'un minimum de partage d'information est nécessaire à sa prévention.

2.1.4 Partage d'information

Le domaine du partage d'information s'est fortement renforcé depuis quelques années, le nombre de papier a explosé depuis 1996 ([Huang et al., 2003](#)). L'impact favorable du partage d'information commence à être reconnu, notamment grâce à des sondages portant sur certaines grandes entreprises nord-américaines ([Sahin and Robinson, 2002](#)). [Datta and Christopher \(2011\)](#) montrent l'importance du partage d'information entre les membres d'une chaîne logistique faisant face à des incertitudes. Le partage d'information est mathématiquement étudié pour des modèles simples, et à chaque fois leurs auteurs en démontrent l'incidence positive : sur la gestion d'inventaire ([Yu et al., 2001](#)) ou d'information ou de prévision ([Zhao and Xie, 2002](#)). Néanmoins, il peut arriver que l'excès de partage d'information provoque indirectement une perte de profit ([Cachon and Fisher, 2000](#)). La qualité des informations partagées est donc plus importante que leur quantité.

Cette qualité est directement liée à la confiance accordée au partenaire. De nombreuses études portent sur la confiance dans les chaînes logistiques. Certains travaux s'orientent sur la recherche d'informations auprès de nombreuses entreprises, et confirment toute l'importance du partage d'information, en considérant la confiance comme une vraie création de valeur ([Doney and Cannon, 1997](#); [Selnes, 1998](#); [Eckerd and Hill, 2012](#)). Cette confiance dépend aussi de la culture du pays de l'entreprise, les relations entre distributeur et fournisseur sont globalement meilleures au Japon qu'aux États-Unis ou en Corée ([Dyer and Chu, 2003](#)). La différence de poids entre deux entreprises peut aussi conduire à une mauvaise collaboration ([Blomqvist et al., 2005](#); [Audy et al., 2012](#)).

En particulier dans le secteur des pâtes à papier du Canada, la structure de la chaîne logistique pousse les papeteries à être fortement dépendantes des scieries sans réelles contreparties, menant à des situations déséquilibrées ([Lehoux et al., 2009, 2010](#); [Elleuch et al., 2012](#); [Hellion et al., 2013c](#)). De plus, le secteur forestier a récemment subi plusieurs difficultés externes : baisse du prix du bois d'œuvre ([Del Degan and Vincent, 2010](#)), invasion biologique et vieillissement de la forêt ([Bouchard et al., 2005, 2006](#)).

La collaboration et le partage d'information peut parfois se résumer avec efficacité à la gestion conjointe d'inventaire ([Chan and Chan, 2010b](#)), notamment avec une chaîne logistique à deux échelons ([Cachon and Zipkin, 1999](#)). La gestion partagée d'inventaire peut aussi fonctionner dans certains modèles plus complexes, comme par exemple celui de [Hung et al. \(2006\)](#) qui prend en compte deux types de produit et une demande incertaine. Certaines études montrent que la simple méthode du stock de sécurité peut s'avérer

payante (Riddalls *et al.*, 2000).

Rechercher les équilibres dans une relation permet de savoir si chacun des acteurs à intérêt à collaborer. Le nombre de publications dans ce domaine a aussi très largement augmenté (Arshinder *et al.*, 2008). Des modèles simples peuvent être étudiés avec les équilibres de Nash (Çapar *et al.*, 2011), notamment avec demandes stochastique (Cachon, 2003), perturbation des demandes (Qi *et al.*, 2004), demandes dépendantes des prix (Weng, 1999), critères de risque (Cruz, 2008) ou encore coordination d'inventaire (Toktas-Palut and Ülengin, 2011).

Si tous les acteurs trouvent leurs équilibres en se mettant d'accord sur certaines dispositions, alors un contrat peut être ratifié entre eux.

2.1.5 Contrat

Le contrat est le moyen usuel de clarifier une relation de collaborations entre deux entreprises (Elmaghraby, 2000). Cependant, l'engagement à long terme grâce à un contrat exclusif ne doit pas être automatique, et doit être réfléchi (Peleg *et al.*, 2002).

Généralement en contrepartie d'une grosse commande, des remises de prix des fournisseurs peuvent être appliquées, menant à des hausses de profit de chacun des acteurs (Woo *et al.*, 2001; Sirias and Mehra, 2005). Certains modèles sont plus contraignants : Bassok and Anupindi (1997) imposent une quantité minimale de commande, avant d'appliquer la remise de prix.

Certains contrats permettent plus de flexibilité en intégrant des ajustements ou des options. Dans sa forme la plus générale, un ajustement permet à un acheteur d'annoncer à l'avance sa future commande, puis de l'ajuster suivant ses propres prévisions. En contrôlant l'ajustement, le fournisseur obtient une prévision correcte de sa demande (Huang *et al.*, 2005; Milner and Rosenblatt, 2002; Bassok and Anupindi, 2008). Le prix peut aussi en dépendre (Chen, 2003; Wang and Tsao, 2006), tandis que l'ajustement peut être facturé (Cheng *et al.*, 2003).

2.2 Dimensionnement de lots

La collaboration entre les acteurs mise en place au niveau stratégique (sous la forme de contrats) se traduit en contraintes d'approvisionnement au niveau tactique. En y intégrant les contraintes industrielles nécessaires, le problème du dimensionnement de lots est parfaitement adapté pour optimiser la planification de la production, des livraisons ou des commandes.

Le problème du dimensionnement de lots consiste à satisfaire une demande sur un horizon de temps discrétisé en périodes distinctes. La demande est satisfaite soit par une commande à un fournisseur, soit par l'inventaire accumulé sur les périodes précédentes.

L'historique du problème peut être trouvé dans un article écrit par [Wolsey \(1995\)](#). Tandis que le dimensionnement de lots reste très utilisé dans l'industrie ([Robinson et al., 2009](#)), et que des composants spéciaux sont même intégrés à des solveurs commerciaux ([Belvaux and Wolsey, 2000](#)). La variante mono-produit a été largement étudiée les 50 dernières années ([Brahimi et al., 2006b](#); [Akbalik, 2006](#))

2.2.1 Mono fournisseur

La variante mono-produit la plus simple, dite sans capacité et mono-fournisseur, a fait l'objet d'un premier article séminal écrit par [Wagner and Whitin \(1958\)](#). La propriété ZIO (*Zero Inventory Order*) ainsi introduite est qu'il existe toujours une solution optimale telle que le produit de l'inventaire de la période précédente et de la commande est nul, à chaque période. Ce qui revient à énoncer que le donneur d'ordres n'a jamais intérêt à combiner ces deux sources d'approvisionnement (stock et commande) pour satisfaire une demande à une période donnée. L'algorithme (WW) qui en découle s'exécute polynomialement en $O(T^2)$. Cet algorithme a été ensuite indépendamment amélioré jusqu'à une complexité en $O(T \times \log(T))$ ([Federgruen and Tzur, 1991](#); [Wagelmans et al., 1992](#)). Même en considérant des structures de coût concaves, [Wagner \(1960\)](#) montrent que le problème reste polynomial.

Les méthodes pour résoudre le problème sont diverses. La structure et les propriétés du polyèdre engendré par les contraintes du dimensionnement de lots peuvent être étudiées ([Miller, 1999a](#)). Les résolutions sont dépendantes des formulations utilisées. La formulation la plus connue et la plus naturelle est la formulation agrégé : AGG (voir [Florian and Klein \(1971\)](#), par exemple). Cependant, il existe d'autres formulations présentant des propriétés intéressantes. Une formulation à l'origine créée pour le problème du placement d'usine a été adaptée pour le dimensionnement de lots ([Bilde and Krarup, 1977](#)). Cette formulation (FAL pour *Facility Location*) présente une meilleure relaxation linéaire que la formulation classique ([Wolsey, 1995](#)). [Wagelmans et al. \(1992\)](#) améliorent l'algorithme WW ([Wagner and Whitin, 1958](#)) en $O(T \times \log(T))$ en utilisant les propriétés de la formulation duale de FAL.

Toujours sans capacité, des modèles stochastiques ont été étudiés, notamment quand les demandes suivent une loi de poisson ([Lee and Lan, 2013](#)). L'inventaire peut aussi être bornée, ce problème particulier a été montré polynomial par [Liu and Tu \(2008\)](#).

2.2.2 Multi fournisseurs

Dans ce problème, un seul produit est commandé à plusieurs fournisseurs, chacun pouvant avoir des caractéristiques différentes. Des variantes spéciales de ce problème peuvent encore être résolues par la programmation dynamique, que les fournisseurs soient en concurrence (Bai and Xu, 2010) ou placés séquentiellement (van Hoesel *et al.*, 2005). De tels modèles sont aussi abordés avec des approches polyédrales (Zhao and Klabjan, 2012).

2.2.3 Capacité de production

Si le dimensionnement de lots sans capacité a été montré polynomial par Wagner and Whitin (1958), sa version avec capacité a été montrée NP-difficile, même avec certaines simplifications (Florian *et al.*, 1980). Des classifications se basant sur la contrainte de capacité émergent : anciennement (Bitran and Yanasse, 1981) puis plus récemment (Wolsey, 2002). Cependant, en considérant la capacité constante sur l'ensemble des périodes, Florian and Klein (1971) montrent que le problème est polynomial avec une complexité de $O(T^4)$, et ceci en considérant des structures de coût concaves. En considérant des structures de coût de stockage linéaire, cet algorithme peut être optimisé jusqu'à une complexité de $O(T^3)$ (van Hoesel and Wagelmans, 1996).

Diverses structures de coûts ont été étudiées. L'algorithme de Florian and Klein (1971) a été étendu par Jagannathan and Rao (1973) pour l'adapter à des structures de coût générales. Shaw and Wagelmans (1998) résolvent une autre variante en considérant des coûts linéaires par morceau pour la production, et général pour le stockage. Des temps de démarrage peuvent aussi être considérés, menant à une décomposition en plusieurs sous-problèmes, qui peuvent être recombinaés grâce à un algorithme de lissage Trigeiro *et al.* (1989). Certaines particularités industrielles comme les coûts de rupture sur la demande ou les coûts de déficit sur le stock de sécurité ont été étudiés par Absi (2005). Des formulations différentes pour le dimensionnement de lots ont été comparées, il apparaît souvent difficile de décider qu'une est meilleure que l'autre, ceci étant fortement dépendant des paramètres utilisés (Akbalik and Penz, 2009). La notion de capacité peut aussi être étendue à une capacité cumulée de commande, menant dans ce cas à un algorithme d'approximation (Sargut and Romeijn, 2007).

2.2.4 Quantité minimale de commande

Dans la littérature, deux types de quantités minimales de commandes ont été introduites.

À chaque période la quantité de commande doit être au minimum cette quantité minimale : l'absence de commande n'est donc pas autorisée si ce minimum est strictement

positif (Love, 1973). Cette version a été plusieurs fois abordée par des études polyédrales : sans retard (Constantino, 1998) ou avec (Constantino, 2000). Des politiques optimales peuvent être déduites, renforcées par des études expérimentales (Chan and Muckstadt, 1999).

La quantité minimale de commande peut être aussi vue comme un minimum à respecter que si une commande est passée. Cela revient à autoriser l'absence de commandes. Lors d'une étude polyédrale, la comparaison entre ces deux contraintes a été présentée par Van Roy and Wolsey (1986). Autoriser une commande nulle est particulièrement utile dans l'industrie, comme par exemple dans un problème d'assemblage automobile (Lee, 2004), de gestion d'inventaire (Zhou *et al.*, 2007), ou de réapprovisionnement conjoint à plusieurs fournisseurs (Porras and Dekker, 2006). Le problème étant difficile pour de grandes tailles d'instances, des heuristiques basées sur des résolutions successives sur un horizon roulant sont développés (Merce and Fontan, 2003), ou se rapprochant de la politique optimale (Tibben-Lembke, 2004). Des algorithmes optimaux existent en considérant différentes structures de coût, $O(T^3)$ pour des fonctions de production et stockage constantes (Okhrin and Richter, 2011), $O(T^7)$ avec des fonctions de coût concaves (Li *et al.*, 2008).

2.2.5 Fenêtres de temps

Dans la littérature, les fenêtres de temps ont été définies de deux manières différentes.

Contrairement au dimensionnement de lots traditionnel, où toutes les demandes peuvent être satisfaites par une commande équivalente à la première période (sans capacité), ici à chaque demande est associée une date au plus tôt, à partir de laquelle on s'autorise à commander pour satisfaire cette demande. Ces fenêtres de temps peuvent aussi être associées à la périssabilité des produits, pour laquelle à chaque demande est associée une conservation maximum dans l'entrepôt de stockage. Ces fenêtres de temps sont appelées fenêtres de temps sur la production, et sont définies en premier par Dautère-Pérès *et al.* (2002). Ils présentent un algorithme polynomial en $O(T^4)$ pour résoudre le problème. En considérant des structures de coût linéaires, d'autres auteurs exposent un algorithme polynomial en $O(T^3)$, trouvé en montrant l'équivalence entre plusieurs formulations de dimensionnement de lots (Van den Heuvel and Wagelmans, 2008). Ce problème est aussi montré polynomial en $O(T^5)$ dans le cas sans capacité et en considérant des structures de coûts concaves (Hwang, 2007). Le modèle de Dautère-Pérès *et al.* (2002) a été étendu à un problème à plusieurs produits avec capacité, résolu par une décomposition lagrangienne (Brahimi *et al.*, 2006a). On peut considérer qu'enfreindre ces fenêtres de temps est possible, en payant une pénalité. Dans ce cas des équations de récurrence décrivant la politique optimale sont proposées, menant à un algorithme en $O(T^7)$ basé sur la programmation dynamique (Absi *et al.*, 2011).

Lee *et al.* (2001) définissent les fenêtres de temps comme étant une généralisation des demandes. Ces dernières ne sont pas associées à une période : d_t avec $t \in T$, mais plutôt avec un intervalle de période : $d_{tt'}$ avec $t, t' \in T^2$. La demande est satisfaite à n'importe quelle période dans cet intervalle. C'est équivalent à ne pas payer de coûts de stockage ou de retard dans l'intervalle $[t \dots t']$. Ce problème est introduit avec deux algorithmes polynomiaux en $O(T^3)$ et $O(T^2)$, en considérant avec ou sans retard, respectivement (Lee *et al.*, 2001). Ces fenêtres de temps sont déjà connues dans d'autres domaines : par exemple dans un problème hybride d'ordonnancement et dimensionnement de lots (Salomon *et al.*, 1997). Le problème est montré pseudo polynomial par Akbalik and Penz (2011) grâce à la programmation dynamique. La formulation de Lee *et al.* (2001) peut être enrichie d'inégalités valides en utilisant des résultats d'optimisation combinatoire sur les couplages dans des graphes bipartis (Wolsey, 2006).

2.3 Conclusion

De nombreux partenariats entre donneurs d'ordre et fournisseurs ont été étudiés dans la littérature, tous soutenant l'idée que le partage d'information est une clé pour réussir une bonne collaboration. Si des contrats existants partagent notre objectif de fiabiliser les approvisionnements, aucun n'étudie séparément la partie quantitative et la partie temporelle. La définition et l'étude de cette séparation fait l'originalité des contrats de stabilité que nous proposons.

À un niveau tactique, les deux parties distinctes des contrats de stabilité peuvent se traduire en contraintes supplémentaires dans un problème de dimensionnement de lots. Tandis que les contraintes sur les quantités (minimales et maximales) sont bien étudiées et utilisées dans les milieux académiques ou industriels, notre revue de littérature montre qu'il en est différemment pour les contraintes temporelles. D'après nos recherches, les fenêtres de temps dynamiques que nous proposons sont nouvelles et nous conduisent à étudier des problèmes de dimensionnement de lots originaux.

Chapitre 3

Stability contracts between supplier and retailer : a new lot sizing model

Abstract

This work explores the relationship between decision makers in a company and their suppliers, using stability contracts. This relationship can be modeled as a capacitated multi-machine lot sizing problem with minimum order quantity and dynamic time windows, where orders are represented by production levels. Both the amount and the frequency of orders are constrained, the first by upper and lower bounding and the second by dynamic time windows. A mathematical model is provided and an experimental analysis is conducted. A cost study highlights that using proper stability contracts in certain condition can reduce the storage cost of the retailer, improving the whole supply chain efficiency. Conclusions are given leading to insight for decision makers and contract designers.

3.1 Introduction

With the emergence of globalization and international competition, the outsourcing of parts and services has been steadily increasing over the last decades. Consequently, the relationship between decision makers and suppliers becomes one of the most important issues for supply chain design.

The supplier selection phase is the first step in the relationship between decision makers and suppliers. Efficient collaboration is a clear objective in this relationship ([Aissaoui et al., 2007](#)), and therefore supplier selection must be followed by a good management of the relationship. Bad collaboration can have disastrous consequences: the Bullwhip effect

is a good example of what can happen without any information sharing (Dejonckheere, 2003; Lee *et al.*, 1997). In order for companies to have fruitful relationships with their suppliers, pertinent contracts must be established (Elmaghraby, 2000). Contracts strengthen and define the relationship between the partners, with the objective of establishing a long-term, cost-effective relationship (Liker, 2004).

The sizes of each company have to be taken into account when establishing a contract. Considering the example of an asymmetric partnership between 2 actors, Blomqvist *et al.* (2005) bring up the importance of trust in the relationship. They defined 5 requirements to have trust; their case studies show that every time a given partnership comes to end, one of these requirements had been broken (Blomqvist *et al.*, 2005).

Since it is an important part of profit coming from the supply chain, industrial actors and researchers have studied contract design in depth. Elmaghraby provides an overview of contract competition in the manufacturing supply chain, studying supplier selection and additionally giving a detailed a review of contract management (Elmaghraby, 2000). The ethic point of view has been recently studied by Eckerd and Hill, focusing on the relationship between buyer and supplier, within a large review (Eckerd and Hill, 2012).

Some contracts take the price discount directly into account. This is a common business strategy since it allows the reinforcement of the partnership, as well as a way to reward special effort. Bassok and Anupindi modeled the price discount in a simple supply chain of one supplier and one retailer with a minimum commitment. With optimal policy definition, they were able to find the best compromise between minimum commitment and price discount (Bassok and Anupindi, 1997). Quantity discount can be balanced by lead time-dependent discount, with powerful simulation tools that allow the study of hypothetical models as close to the real situations as possible (Sirias and Mehra, 2005).

Contracts have to deal with procurement policy. To ensure that a contract leads to a long-term relationship, it must guarantee regular orders and a certain amount of supplied components. In this paper, we consider that a company has to procure a single product from a set of suppliers. In this case, the problem can be seen as a special case of a multi-machine single-item capacitated lot sizing problem with new constraints. The relationship with each supplier is regulated by a contract in which a principle of regularity of orders must be respected. First, the quantity of items in each order must be within a defined interval. Second, the time interval between two orders must be in a given time window. Finally, all the constraint parameters are different for each supplier.

The single item capacitated lot sizing problem (CLSP) has been widely studied in the last five decades. Bitran and Yanasse (1981) reviewed the whole algorithmic complexity of this problem. In their classification, they clarified whether many sub-problems are NP-hard or not, establishing in some cases polynomial algorithms. Florian and Klein (1971) studied the capacitated version, considering concave cost functions for both production

and storage. They proposed a polynomial time algorithm based on the concept of the optimal sub-sequences. Later, [Shaw and Wagelmans \(1998\)](#) established a two-step procedure to solve the CLSP with linear production costs, setup costs and general holdings costs functions. They extended their work to the case in which the production cost functions are piecewise linear (not necessarily convex or concave). A complete survey of the single-item lot sizing problem is presented by [Brahimi *et al.* \(2006b\)](#).

Each company's supplier can be seen as a single machine in the lot sizing problem, leading to a *multi-machine* CLSP. [Akbalik and Penz \(2009\)](#) studied a multi-machine CLSP with piece-wise linear productions costs. They derived an exact pseudo-polynomial dynamic programming algorithm which makes their problem NP-hard in the ordinary sense. [Bai and Xu \(2010\)](#) studied a single product CLSP in which the retailer may replenish his inventory from several suppliers. Their suppliers present original cost structure characterized by one of two types of order cost structures: incremental quantity discount cost structures and multiple setup cost structures. They derived dominant properties for this problem, and proposed several dynamic programming algorithms. Later, [Toktas-Palut and Ülengin \(2011\)](#) tried to coordinate the inventory policies in a decentralized supply chain with multi-supplier. They developed centralized and decentralized models, and then derived contracts allowing to coordinate the supply chain.

Recently the Minimum Order Quantity (MOQ) constraints have been introduced. These constraints deal with the production level that must be at least equal to the MOQ if production is started. [Constantino \(1998\)](#), from a polyhedral point of view, considered the production level as a continuous variable. He included the MOQ constraint where the capacity is shared for all the machines. He derived strong inequalities which describe the convex hull of the solutions. [Chan and Muckstadt \(1999\)](#) studied a production-inventory system in which the production quantity is constrained by a minimum and a maximum level in each period. They characterized the optimal policy for finite and infinite time horizons, confirmed by an experimental study. The first exact polynomial time algorithm was recently developed by [Okhrin and Richter \(2011\)](#). They solved a special case of the problem in which the unit production cost is constant over the whole horizon, and can therefore be discarded. Furthermore, they assumed that the holding costs are also constant over the T periods. Considering these restrictions, they derived a polynomial time algorithm in $O(T^3)$. [Hellion *et al.* \(2012, 2013a\)](#) developed an optimal $O(T^6)$ polynomial time algorithm to solve the CLSP-MOQ with concave costs functions generalizing [Okhrin and Richter's](#) problem.

The production capacity and MOQ constraints were originally motivated by industrial needs. Considering a retailer ordering from a single supplier, these constraints additionally give the supplier a way to forecast future orders. However, these constraints only affect the

quantity of the orders, and both the supplier and the retailer lack temporal informations. To ensure a long-term partnership, actors must guarantee a certain amount of supplied components and regular orders. The time interval between two orders must in a given time window.

In the existing literature, time windows have been introduced with several definitions. The delivery time window (also called grace period) was first presented by [Lee *et al.* \(2001\)](#). In their model, each demand i must be delivered during a time window. Later, [Akbalik and Penz \(2011\)](#) used a similar definition to compare just in time and time windows policies. The production time windows were introduced by [Brahimi *et al.* \(2006a\)](#). In this problem, items cannot be produced before a defined period. Recently, [Absi *et al.* \(2011\)](#) studied two production time window problems, considering lost sales or backlogs. They used dynamic programming to solve their problems. [Hwang \(2007\)](#) proposed an $O(T^5)$ algorithm for the production time windows and concave production costs. [Van den Heuvel and Wagelmans \(2008\)](#) showed that the formulations with production time windows are equivalent to other models: lot sizing with manufacturing options, lot sizing with cumulative capacities and lot sizing with inventory bounds. These two time window definitions were studied by [Wolsey \(2006\)](#), by proposing valid inequalities and convex hulls.

As seen above, all these time windows definition do not ensure a long-term partnership between actors. The capacities and MOQ only affects the quantity of the orders, and neither of these time windows guarantee regular orders. The time windows considered in this paper are different from those defined in the literature: here, the time window that constrains a supply order is dependent on the period where the last order has been sent. These time windows are denoted dynamic time windows in the following. [Hellion *et al.* \(2013b\)](#) recently presented these time windows in the simple model in which a single supplier and a retailer agree on a minimum and a maximum number of periods between two orders. Considering concave production and holding cost functions, they derived a polynomial exact algorithm.

In this study, we present a lot sizing problem with a set of new constraints. A literature review showed that except the complexity paper defined above ([Hellion *et al.*, 2013b](#)), the dynamic time window constraints have never been studied before. In this paper stability contracts are defined for several supplier, with the introduction of a new contract based on dynamic time windows, leading to a new lot sizing problem. This lot sizing problem is analysed, and an experimental study is made; the resolution efficiency of the multi-supplier case is not investigated, however. The paper is organized as follows: section [3.2](#) describes the problem and introduces mathematical formulation. Section [3.3](#) describes the testing of the model. The design of experiment is detailed, followed by a cost analysis. This section finishes by studying the strong link between the capacity constraints and the time windows. Finally, managerial implications and conclusions are given in section [3.4](#).

3.2 Problem description and mathematical formulation

3.2.1 Description

The single item lot sizing problem consists of satisfying the demands d_t of a product at each period t over T consecutive periods. In our problem, there are S different suppliers (machines in the lot sizing terminology). A demand $d_t \in D$ may be satisfied from the suppliers at period t or from inventory. Backlogs are not allowed. An order cost c_{it} is incurred when a product is ordered from supplier i at period t . The quantity ordered is noted x_{it} . The inventory level at the end of a period t is denoted s_t and the unit storage cost is h_t . A setup cost f_{it} is incurred only if an order is placed to the supplier i at period t . It is assumed that there is no inventory at the beginning of the first period. The problem is to determine the amount x_{it} to be ordered from each supplier, at each period, satisfying the demands and minimizing the total cost.

Each supplier i has a specific contract with the decision maker. For each supplier i , the quantity of products in each order is constrained by a maximum capacity level U_i . The quantity ordered is also constrained by a minimum order quantity L_i . Each subsequent order for supplier i is also constrained by a dynamic time window. There are at least Q_i and at most R_i periods between two consecutive orders.

Figure 3.1 illustrates the dynamic time window for $Q = 2$ and $R = 4$. In the example, an order is placed at period t . Since $Q = 2$, the following order cannot be placed at period $t + 1$ nor $t + 2$. Since $R = 4$, at least one order must be placed in the next five periods. Thereafter, the next order must be placed between $t + 3$ ($t + Q + 1$) and $t + 5$ ($t + R + 1$) included. Consequently, in each interval of length 3 at most one order must be placed. Furthermore, in each interval of length 5, at least one order must be placed. Note that if $Q = R$ an order must be placed at every $Q + 1$ periods. If $Q = R = 0$, an order must be placed at every period.

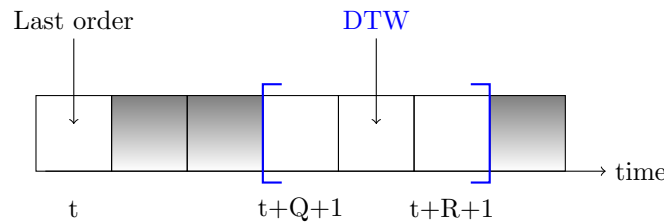


Figure 3.1 – An example of dynamic time window with $Q = 2$ and $R = 4$

The one period sub-problem was proved NP-hard in the ordinary sense by Akbalik and Penz (2009) by a reduction from the knapsack problem. Consequently, the general problem is also NP-hard. However, as seen above the single supplier case considering concave cost functions has been proved polynomial (Hellion *et al.*, 2013b). A simple mixed-integer

linear program approach (MILP) is adopted, since the efficiency of the method is not investigated in this paper. The main objective is to understand the impact of the different parameters and the behaviour of the optimal solutions.

3.2.2 A MILP formulation

A MILP formulation can now be presented. The decision variables are as follows:

- x_{it} : quantity of products ordered from supplier i at period t .
- $y_{it} = \begin{cases} 1 & \text{if an order is placed to the supplier } i \text{ at period } t. \\ 0 & \text{otherwise.} \end{cases}$
- s_t = inventory level at the end of a period t .

Lower and upper bounds on the ordered quantity (MOQ constraints) are integrated into the model, by adding the following constraints:

$$L_i y_{it} \leq x_{it} \leq U_i y_{it}$$

The dynamic time window (DTW) constraints can be written as follows:

$$\sum_{t'=t}^{t+R_i} y_{it'} \geq 1 \qquad \sum_{t'=t}^{t+Q_i} y_{it'} \leq 1$$

Finally, the mathematical formulation is as follows:

$$\text{Min } \sum_{i=1}^S \sum_{t=1}^T c_{it} x_{it} + \sum_{t=1}^T h_t s_t + \sum_{i=1}^S \sum_{t=1}^T f_{it} y_{it} \quad (3.1)$$

$$\sum_{i=1}^S x_{it} + s_{t-1} - s_t = d_t \qquad \forall t \in \{1, \dots, T\} \quad (3.2)$$

$$x_{it} - U_i y_{it} \leq 0 \qquad \forall t \in \{1, \dots, T\}, \forall i \in \{1, \dots, S\} \quad (3.3)$$

$$x_{it} - L_i y_{it} \geq 0 \qquad \forall t \in \{1, \dots, T\}, \forall i \in \{1, \dots, S\} \quad (3.4)$$

$$\sum_{t'=t}^{t+R_i} y_{it'} \geq 1 \qquad \forall t \in \{1, \dots, T - R_i\}, \forall i \in \{1, \dots, S\} \quad (3.5)$$

$$\sum_{t'=t}^{t+Q_i} y_{it'} \leq 1 \qquad \forall t \in \{1, \dots, T - Q_i\}, \forall i \in \{1, \dots, S\} \quad (3.6)$$

$$x_{it} \in \mathbb{R} \qquad \forall t \in \{1, \dots, T\}, \forall i \in \{1, \dots, S\} \quad (3.7)$$

$$y_{it} \in \{0, 1\} \qquad \forall t \in \{1, \dots, T\}, \forall i \in \{1, \dots, S\} \quad (3.8)$$

$$s_t \in \mathbb{R} \qquad \forall t \in \{1, \dots, T\} \quad (3.9)$$

The objective function (3.1) is to minimize the production, holding and setup costs, respectively. Constraint (3.2) is the flow constraint. Constraints (3.3) and (3.4) define the maximum and minimum order quantities. The dynamic time windows are given by (3.5) and (3.6). Constraints (3.7), (3.8) and (3.9) define the validity domain of each variable.

3.3 Computational experiment and analysis

All the following experiments have been performed on a 3 GHz Intel Core 2 Duo Machine with 4GB memory using IBM ILOG CPLEX 12.2.

3.3.1 Design of experiment

To study this problem, we use the *design of experiment* methodology (Kempthorne, 1952) to extract the key factors in the process. The design of experiment can also draw out the interactions of two or more factors. To plan a two-level full factorial design of experiment, all the factor have to be determined. All the costs have to be considered: storage cost, and for each supplier purchasing cost and setup cost. The contract parameters are L_i , U_i , Q_i and R_i for each supplier i . Assuming that the demands are generated by a normal law, the mean μ and the variance σ are also factors of the model.

In this experiment two suppliers are considered, namely A and B . The contract defined by L , U , Q and R is between the supplier A and the retailer. The contract with the supplier B does not constrain the order quantities nor the order frequency, *i.e.* $Q = 0$, $R = T$, $L = 0$ and $U = \infty$, thus B 's purchasing cost is greater. Since the demands must be satisfied, and the total costs are minimized, supplier A 's purchasing costs can be discarded. Both suppliers share the same setup costs. The two-level full factorial design of experiment parameter values are summarized in Table 3.1.

parameter	level 1	level 2
R maximum number of periods between two orders from supplier A	3	T
Q minimum number of periods between two orders from supplier A	0	3
U maximum amount in one order from supplier A	50	$\sum_{\forall t} d_t$
L <i>minimum order quantity</i> for the supplier A	0	30
σ demand variance	2	10
μ demand mean	5	20
f setup production cost for both suppliers	2	10
c supplier B unit production cost	0.1	4
h storage cost	0.1	1

Table 3.1 – *Design of experiment* parameters and their values

Since 9 parameters are studied, 2^9 parameter combinations have to be performed. For

each parameter combination (referred to as sets herein), 25 instances are generated and solved. Solving a single instance takes less than 2 seconds, but considering the number of instances is 12800, the whole experiment takes about 5 hours to compute.

The main effects of each parameter are summarized in Figure 3.2. The largest effect comes from the storage cost, its maximum value strongly affects the value of the solution. Generally speaking, a low storage cost allows the decision maker to deal with contract constraints, by storing more products. An additional experiment based on the inventory level is performed in section 3.3.3 focussing on storage cost.

It is interesting to note that the setup cost (*i.e.*, paying 10.0 for each order instead of 2.0) does not appear to have a strong effect on the solution. In some way, the Q and L constraints play the role of setup cost. This *design of experiment* proves that the setup cost can be discarded from this problem without changing the behavior of the model. Consequently the next experiments in Section 3.3.2 and 3.3.3 will not include the setup cost.

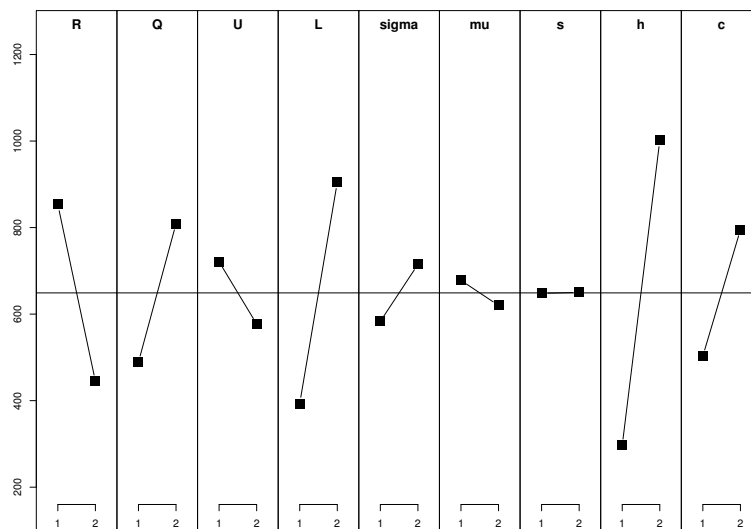


Figure 3.2 – Main effects

We denote as min and max the minimum and maximum amount of products which can be ordered, respectively. Figure 3.3 displays the effect of each parameter R , Q , U and L on min and max . For example, taking the maximum value of Q ($Q_{1 \rightarrow 2}$) decreases max (\searrow), and has no effect on min (-). Each parameter has different effects on min and max , and these effects are summarized in Figure 3.3. For instance, if the decision maker wants to decrease max of the contract, he can choose either to increase Q ($Q_{1 \rightarrow 2}$) or to decrease U ($U_{2 \rightarrow 1}$). All other choices do not decrease max .

parameter	<i>max</i>	<i>min</i>
$Q_{1 \rightarrow 2}$	\searrow	-
$R_{1 \rightarrow 2}$	-	\searrow
$L_{1 \rightarrow 2}$	-	\nearrow
$U_{1 \rightarrow 2}$	\nearrow	-

Figure 3.3 – R , Q , U , L parameter effects on *min* and *max*

3.3.2 Impact of the demand variance on the supply policy

In this experiment, three suppliers are considered with different contracts. We study how the demand variance affects the retailer's supply. More precisely, we compare the amount of product ordered from the 3 suppliers, A , B and C , when the demand variance is increasing.

A contract A dominates a contract B ($A \geq_d B$) if for all parameters, B is more restrictive than A . Formally,

$$A \geq_d B \equiv \begin{cases} L_A \leq L_B \\ U_A \geq U_B \\ Q_A \leq Q_B \\ R_A \geq R_B \end{cases}$$

In our case, $A \geq_d B \geq_d C$. In addition, being more restrictive naturally implies lower cost, consequently $c_A \leq c_B \leq c_C$. The contracts are summarized in Table 3.2.

num	A	B	C
Q	0	2	0
R	0	5	T
L	30	10	0
U	40	100	D
c	1.0	1.1	1.6

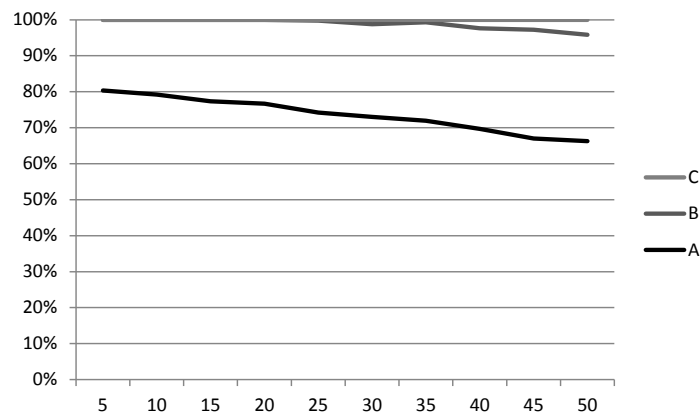
Table 3.2 – Parameters of the contracts

Demands are generated from a normal distribution of mean $\mu = 50$ and variance starting from 5, up to 50 with a step of 5. The number of periods T is 50. Storage costs are fixed to 0.1. For each value of variance, a set of 20 instances is randomly generated. The mean of the total demand is $50 \times 50 = 2500$ products. Results are summarized in Figure 3.4a and 3.4b.

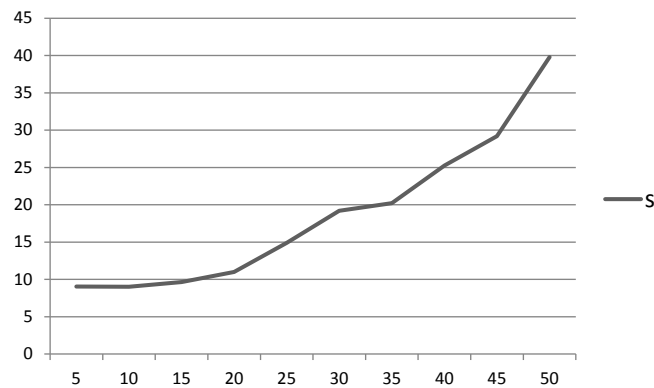
In Figure 3.4a, the stacked proportion of products ordered from each supplier is shown. Each value is the mean of a set of 20 instances. The minimum and maximum proportion of products which can be ordered from supplier A on the whole horizon is 1500 and 2000,

respectively, *i.e.*, 60% and 80% of the total demand. Starting at 80%, the proportion of products ordered from supplier *A* decreases to reach almost 65% of the total demand, which is almost the minimum amount of product which must be ordered (60%). At the same time, the proportion of *B* and *C* varies, increasing from 20% to 30% for *B*, from 0% to 5% for *C*.

A way to deal with high variance is the increase of the storage. This is shown in Figure 3.4b, which displays the inventory level when the variance increases. Starting lower than 10, this average increases to reach 40. Another solution is to order from another supplier, with higher production cost. The Figure 3.4a shows that as demand variance increases, other suppliers are used, despite their higher production cost. This implies a slight cost increase, starting from 2500 (for the lowest variance), up to 2770 (for the greatest variance).



(a) Suppliers allocation (in percentage) as variance increases



(b) Average inventory level in one period (in number of pieces) as variance increases

Figure 3.4 – Impact of the demand variance on the supply policy

3.3.3 Impact of tightening bounds

The next two experiments aim to study the effects of the contracts on the storage cost. Additional costs like transportation costs, production costs, setup costs are not considered.

This allows us to make profound observations of the impacts of these contracts on the storage cost. We only consider a single supplier. The original objective function is as follows:

$$\text{Min} \sum_{i=1}^S \sum_{t=1}^T c_{it}x_{it} + \sum_{t=1}^T h_t s_t + \sum_{i=1}^S \sum_{t=1}^T f_{it}y_{it} \quad (3.10)$$

Assuming that the unit production costs are constant, and that the backlogs are not allowed, the term $c_t x_t$ can be omitted from the objective function (3.10), because it is constant regardless of the ordering. Furthermore, instead of the setup cost, we specify the minimum order quantity, which plays the role of a minor setup cost, as shown in Section 3.3.1. If we additionally assume that the holding costs are also constant, then the objective function is reduced to a single term that minimizes the cumulative inventories (3.11). These assumptions have been recently used in Okhrin and Richter (2011). The objective function 3.11 will be studied in the rest of the paper:

$$\text{Min} \sum_{t=1}^T s_t \quad (3.11)$$

For the supplier, the quantity of products that can be ordered at each period is constrained by the maximum constant capacity level U . The quantity ordered is also constrained by a constant minimum order quantity L .

The contracts can be constrained in two different ways: by tightening the capacity and MOQ constraints, or by tightening the time windows. In the following two experiments, one part of the contract is fixed while the other one varies. Only feasible instances should be solved, and thus a lower and upper bound on the demands are necessary. Uniform distributions are the simplest way to generate the demands.

Cost impact of tightening one contract part

First of all, we will study the quantity part of a contract, *i.e.*, keeping the time windows constant, increasing L and decreasing U . Considering one fixed couple $\{Q, R\}$, and picking 10 different couples $\{L, U\}$ (Table 3.3c), with 3 uniform distributions (Table 3.3a), 100 randomly instances are generated for each set; 3000 instances are generated this way. All these instances are solved for 2 different couples $\{Q, R\}$ (Table 3.3b).

Figures 3.5a and 3.5b show that increasing the uniform distribution variance does not affect the solution. Indeed the three variance-dependant curves are close. Starting with a unconstrained contract: $\{Q, R\} = \{0, 10\}$, tightening the $\{L, U\}$ part makes the graph linear (Figure 3.5a). The differences appear when the fixed part of the contract is already tightened *i.e.* $\{Q, R\} = \{3, 7\}$: see Figure 3.5b. In this case, the graph is almost constant from $\{L, U\} = \{100, 1000\}$ to $\{400, 700\}$, as shown Figure 3.5b. The costs are stable at

				L	U				
				100	1000				
				150	950			Q	R
				200	900			0	10
a	b	Q	R	250	850	L	U	1	9
75	125	0	10	300	800	100	1000	2	8
50	150	3	7	350	750	400	700	3	7
25	175	(b) $\{Q, R\}$ fixed		400	700	(d) $\{L, U\}$ fixed		4	6
(a) Uniform distribution $\mathcal{U}(a, b)$				450	650			5	5
				500	600			(e) $\{Q, R\}$ Tightened	
				550	550				
				(c) $\{L, U\}$ Tightened					

Table 3.3 – Experiment parameters

1.5 values per product (dotted lines).

Inversely to the experiment above, fixed amount constraints (L and U) are considered and time windows (Q and R) are now tightened. Considering one fixed couple $\{L, U\}$, picking 6 different couples $\{Q, R\}$ (Table 3.3e), with 3 uniform distributions (Figure 3.3a), 100 randomly instances are generated for each set, thus 1800 instances are generated. All these instances are solved for 2 different couples $\{L, U\}$ (Table 3.3d).

As in the previous case, Figures 3.5c and 3.5d show that the different variances do not affect the solution. As previously, starting with a unconstrained contract: $\{L, U\} = \{100, 1000\}$ makes the graph linear (Figure 3.5c). Furthermore, if $\{L, U\}$ is tightened *i.e.* $\{L, U\} = \{400, 700\}$, the cost is also constant until a $\{Q, R\}$ is $\{3, 7\}$ (Figure 3.5d).

The two experiments performed above, *i.e.*, tightening $\{L, U\}$ contract part fixing $\{Q, R\}$ and tightening $\{Q, R\}$ fixing $\{L, U\}$ produce similar graphs. If a part of a contract is already tightened, tightening the second part costs nothing for the retailer, to a certain extent: the product cost stays at 1.5. In this situation, if the retailer negotiates a cost variation of 1.5, it will be enough to cover all the storage costs. Mathematically, if a part of a contract is tightened and then fixed, at least one solution among the optimal solutions directly implies a tightened second part of the contract.

Cost impact of tightening both contract parts

The experiment above has showed that the use of a different uniform law does not affect the solution. In this experiment, only one uniform law is considered, the others are discarded.

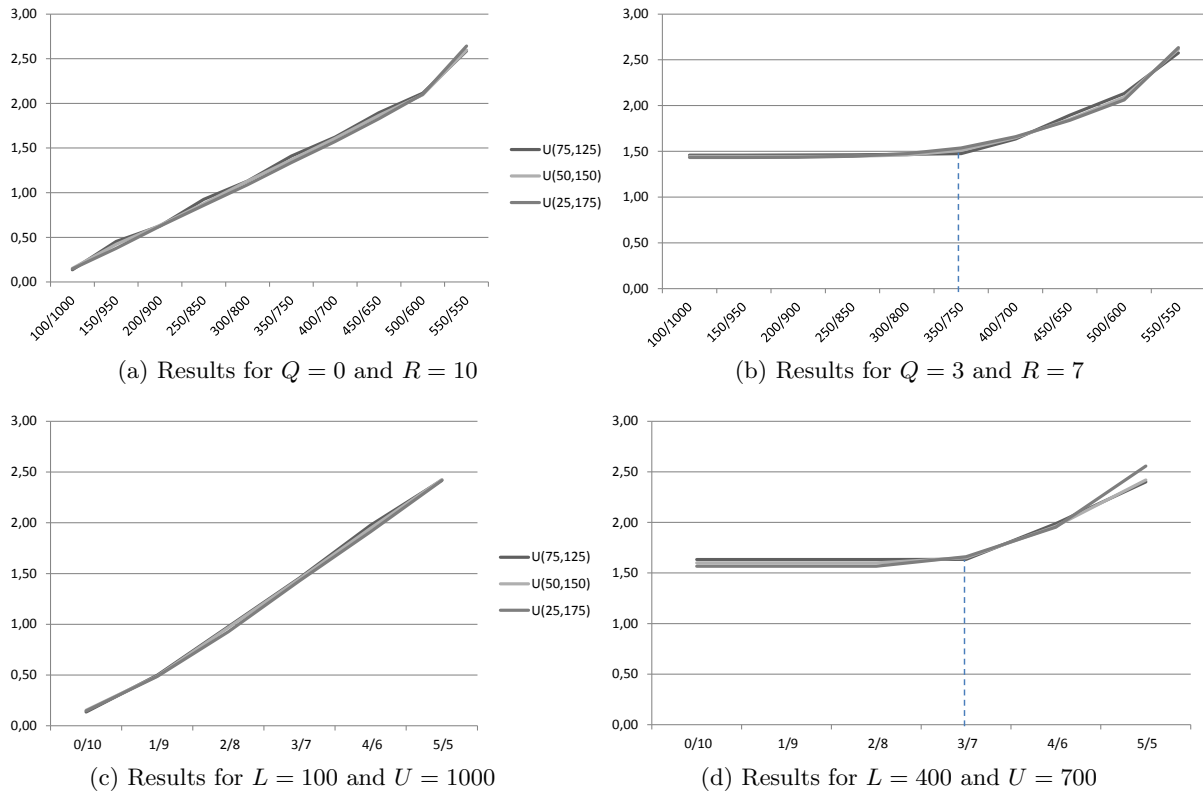


Figure 3.5 – Cost impact of one contract part tightening

The experiment above has highlighted the relationship between the two parts of a contract. The next experiment aims to go further, defining zones where a contract part can be tightened without almost any additional cost. The previous experiment unilaterally tightened a part of a contract, *i.e.* increasing L (or Q) and decreasing U (or R) at the same time. In the following all the combinations respecting $L \leq U$ and $Q \leq R$ are tested.

As previously, one of the contract parts is fixed while the other part varies. Three contracts are fixed for each part, leading to 6 fixed contracts *i.e.* 6 different sets. Each set is presented separately. For each set all the contracts are tested, and the average storage cost for one single product is compared. The percentages are calculated from the least tightened contract of each set (the bottom left corner of each figure). This way, for each cost variation (even if it is zero), a set of equivalent contracts (at same cost for the retailer) can be found.

The 6 fixed contracts are $\{L, U\} = \{300, 800\}$, $\{400, 700\}$, $\{500, 600\}$ (Figures 3.7, 3.8a, 3.8b) and $\{Q, R\} = \{2, 8\}$, $\{3, 7\}$, $\{4, 6\}$ (Figures 3.6, 3.9a, 3.9c). For $\{L, U\}$ fixed contract, Q varies between 0 and 5, whilst R starts at Q , up to 10. For $\{Q, R\}$ fixed contract, L varies between 100 and 500, whilst R starts at L , up to 1000. The demands are generated with a single uniform law $\mathcal{U}(50, 150)$ which implies that the average demand for a period is 100. 50 instances are randomly generated, and these instances are solved for every contract. In the figures, the holes represent unfeasible contracts, either because in

300	49,67%	49,96%	50,88%	53,35%	104%	
350	3,54%	3,86%	4,68%	7,99%	24,45%	85,49%
400	0,41%	0,75%	1,52%	4,85%	19,86%	48,04%
450	0,02%	0,38%	1,12%	4,44%	19,24%	44,37%
500	0%	0,36%	1,1%	4,42%	19,2%	44,07%
550	0%	0,36%	1,1%	4,42%	19,2%	43,99%
600	0%	0,36%	1,1%	4,42%	19,2%	43,99%
650	0%	0,36%	1,1%	4,42%	19,2%	43,99%
700	0%	0,36%	1,1%	4,42%	19,2%	43,99%
750	0%	0,36%	1,1%	4,42%	19,2%	43,99%
800	0%	0,36%	1,1%	4,42%	19,2%	43,99%
850	0%	0,36%	1,1%	4,42%	19,2%	43,99%
900	0%	0,36%	1,1%	4,42%	19,2%	43,99%
950	0%	0,36%	1,1%	4,42%	19,2%	43,99%
1000	0%	0,36%	1,1%	4,42%	19,2%	43,99%
U,L	100	150	200	250	300	350

Figure 3.6 – $\{Q,R\} = \{2,8\}$

2	55,76%	55,76%	56,78%		
3	0,07%	0,07%	1,39%	32,55%	
4	0%	0%	1,31%	28,8%	76,53%
5	0%	0%	1,31%	28,73%	73,16%
6	0%	0%	1,31%	28,73%	73,07%
7	0%	0%	1,31%	28,73%	73,06%
8	0%	0%	1,31%	28,73%	73,06%
9	0%	0%	1,31%	28,73%	73,06%
10	0%	0%	1,31%	28,73%	73,06%
R,Q	0	1	2	3	4

Figure 3.7 – $\{L,U\} = \{300,800\}$

this contract $Q > R$, $L > U$ or because the demands cannot be satisfied: $U \times \lceil \frac{T}{Q+1} \rceil < D$.

Solving a single instance takes less than 2 seconds, however the number of instances is about 30000, therefore the whole experiment takes about 10 hours to compute. The percentages are only displayed for Figures 3.6 and 3.7. The percentages in the other figures are not shown for readability, and are instead replaced by shades of grey, which are detailed in Figures 3.8c and 3.9b.

The experiments show the presence of certain zones of cost variation. For a given cost variation, say 1% in the Figure 3.6, all the contracts satisfying $100 \leq L \leq 150$ and $400 \leq U \leq 1000$ have the same price to the retailer. Therefore fixing $\{Q, R\} = \{2, 8\}$ (the assumption of Figure 3.6) implies that the most constrained contract for this cost variation is $\{L, U\} = \{150, 400\}$. In the following we denote the most constrained contract with a given cost variation as the "best" contract.

Consider a cost variation of 0.01%, in Figure 3.7; all the contracts satisfying $0 \leq L \leq 1$ and $4 \leq R \leq 10$ have the same price to the retailer. Therefore fixing $\{L, U\} = \{300, 800\}$

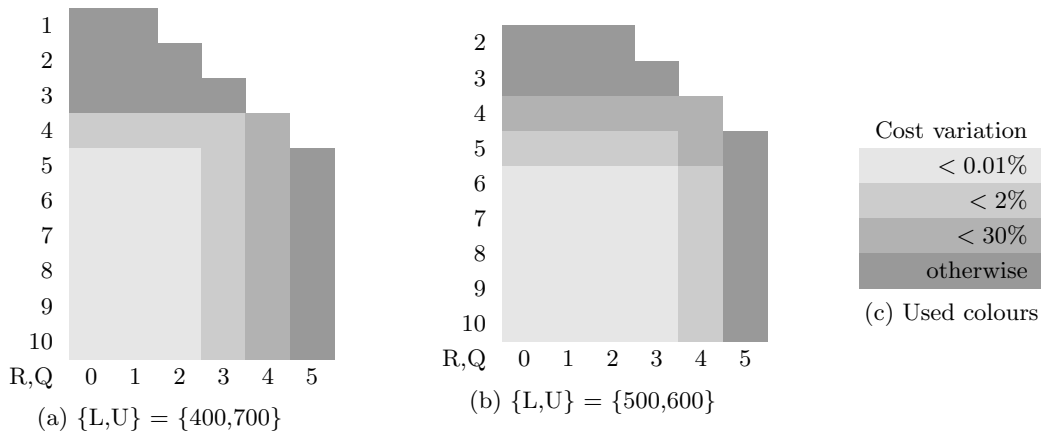


Figure 3.8 – Tightening contracts

(the assumption of Figure 3.7) implies that the best contract for the same cost is $\{Q, R\} = \{1, 4\}$.

Furthermore if a cost variation of 0.01% is considered, in Figures 3.6, 3.9a and 3.9c, the best contracts with this assumption are not the same. Considering $\{Q, R\} = \{2, 8\}$ (Figure 3.6), as seen above the best contract is $\{L, U\} = \{150, 400\}$. Considering $\{Q, R\} = \{3, 7\}$ (Figure 3.9a) the best contract is $\{L, U\} = \{200, 550\}$, whilst for $\{Q, R\} = \{4, 6\}$ (Figure 3.9c) the best contract is $\{L, U\} = \{350, 600\}$.

Consider a cost variation of 2% in Figures 3.7, 3.8a and 3.8b. As seen before, considering $\{L, U\} = \{300, 800\}$ (Figure 3.7) the best contract is $\{Q, R\} = \{2, 3\}$, while considering $\{L, U\} = \{400, 700\}$ (Figure 3.8a) and $\{L, U\} = \{500, 600\}$ (Figure 3.8b), the best contracts are $\{Q, R\} = \{3, 4\}$ and $\{Q, R\} = \{4, 6\}$, respectively.

In conclusion, for any cost variation (even 0%), as one part of a given contract is constrained, the other part corresponding to the *best* combination (as defined above) is naturally constrained.

More formally, for any cost variation, for two fixed contract parts $\{Q_1, R_1\}$ and $\{Q_2, R_2\}$, their best other contract parts are noted $\{L_1^*, U_1^*\}$ and $\{L_2^*, U_2^*\}$.

If $Q_2 \geq Q_1$ and $R_2 \leq R_1$ (*i.e.*, contract 2 is more constrained than contract 1), then $U_2^* \leq U_1^*$ and $L_2^* \geq L_1^*$. The same argument can also be used for any fixed contract part $\{L, U\}$.

3.4 Discussion and conclusion

Contracts can be exploited from several perspectives, which are detailed below.

The retailer establishes a contract to stabilize the relationship with his supplier. This way, the retailer can convince the supplier to apply a price discount in return for his customer loyalty. This price discount for one product has to be at least equal to the cost variation of this product, in respect with the contract constraints.

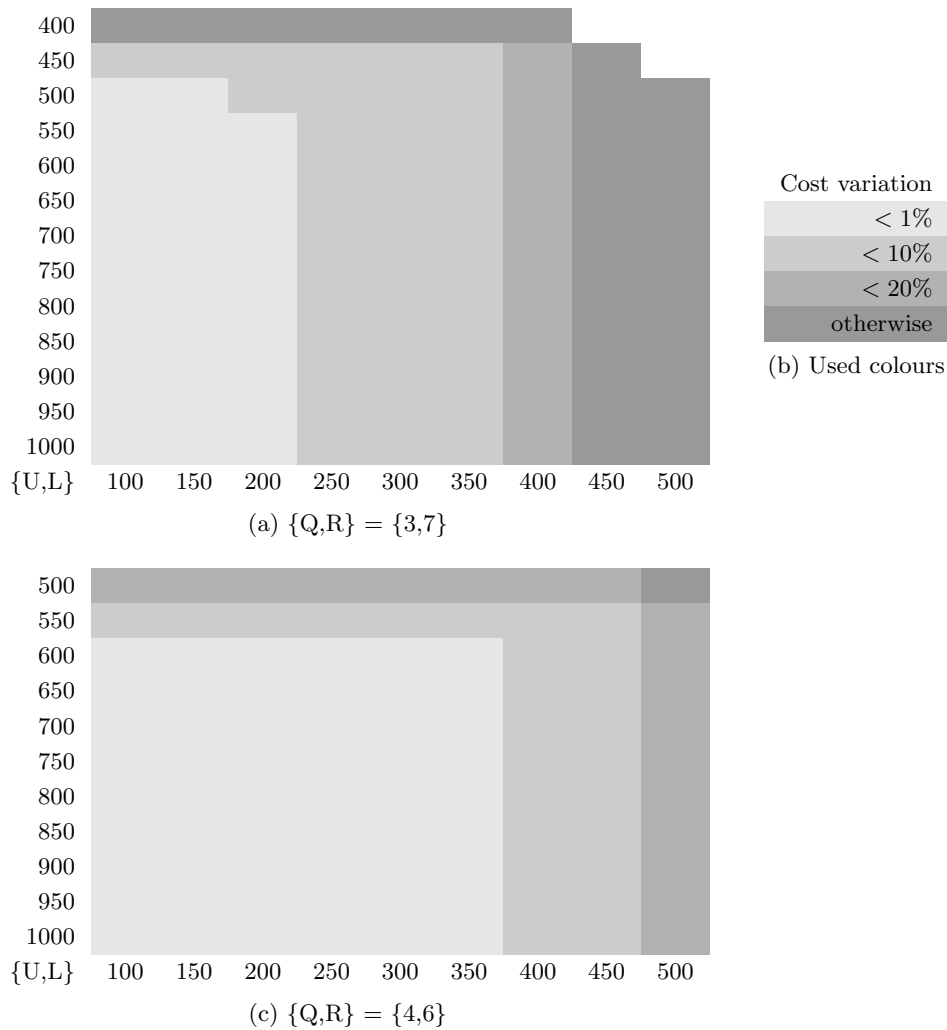


Figure 3.9 – Tightening contracts

The supplier would like to retain its customer, and thus offers the retailer a price discount in return for the respect of the contract, guaranteeing him an upper bound for the additional storage costs.

The supplier and the retailer establish a contract to guarantee each other sustainable relationship. A contract could be established to minimize the total costs of the supply chain and possibly result in a price discount for the final customer.

This paper has focused on a particular set of contracts between suppliers and retailers. Our interest is to strengthen and regularize the relationships between these actors. The contracts studied are divided in two parts: the constraints on order amount and the constraints on order frequency. Constraints on the order amount have been deeply studied and are known in literature as production capacity and minimum order quantity (MOQ). Constraints on the order frequency are introduced, and formally defined. We have called

these constraints *dynamic time windows*, in contrast with both production and delivery time windows, already studied in literature. According to our knowledge, these dynamic time windows and these contracts have never been considered before.

First, a two-level full factorial *design of experiment* helps to highlight the important parameters of the model. It shows the negligible importance of the setup cost in the problem. The next experiment has studied the supplier selection by increasing the demand variance. Finally, the last experiments have studied the effects that a contract part can have on the other part, and help to understand the strong link between these two parts.

The efficiency of the method is not investigated in this paper. Without considering time window, many variants of the problem have been shown to be polynomial (Florian and Klein, 1971; Okhrin and Richter, 2011; Hellion *et al.*, 2012, 2013a). Further work will focus on finding out the theoretical complexity of the studied problem, as well as determining the different assumptions which keep the problem polynomial.

Chapitre 4

A polynomial time algorithm to solve the single-item capacitated lot sizing problem with minimum order quantities and concave costs

Abstract

This paper deals with the single-item capacitated lot sizing problem with concave production and storage costs, and minimum order quantity (CLSP-MOQ). In this problem, a demand must be satisfied at each period t over a planning horizon of T periods. This demand can be satisfied from the stock or by a production at the same period. When a production is made at period t , the produced quantity must be greater to than a minimum order quantity (L) and lesser than the production capacity (U). To solve this problem optimally, a polynomial time algorithm in $O(T^5)$ is proposed and it is computationally tested on various instances.

4.1 Introduction

This paper deals with a generalization of the single-item capacitated lot sizing problem (CLSP) with fixed capacity. The CLSP consists in satisfying a demand at each time period t over a planning horizon T . The demand is satisfied from the stock or by a production. Costs incur for each production and when an item is stored between two consecutive periods. A fixed maximum production capacity (U) must be respected. The problem we consider in this paper contains a minimum order quantity constraint. This constraint imposes that if a production is done at a given period, the quantity must be greater to or equal than a minimum level L . The U and L values are constant for the T periods. This

problem is noted CLSP-MOQ in the following.

The single-item capacitated lot sizing problem is known to be *NP*-Hard (Bitran and Yanasse, 1981). However, some cases are polynomially solvable. This is the case when the capacity is fixed over the T periods. Florian and Klein (1971) considered a case where production and holding cost functions are concave. They proposed an exact method with a time complexity in $O(T^4)$. Later van Hoesel and Wagelmans (1996) improved the complexity of the algorithm in $O(T^3)$ when the holding costs are linear. A complete survey on the single-item lot sizing problem can be found (Brahimi *et al.*, 2006b).

The CLSP-MOQ is relevant in some industrial contexts. Lee (2004) studied an industrial problem where a manufacturer imposes a minimum order quantity to its supplier. The author took an example where the buyer has to choose a supplier among a manufacturer using MOQ constraints and a local dealer. The local dealer supplies the products in just-in-time with a higher cost per unit. The author designed an $O(T^4)$ algorithm which has been tested on industrial data. Porras and Dekker (2006) studied an industrial case where the producer imposes minimum order quantities (MOQ) to produce the items. The company uses containers to ship the products, and set-up costs were not specified explicitly. Consequently, in order to save fixed costs, the producer imposes the MOQ constraint, which plays the role of set-up cost. Zhou *et al.* (2007) analyzed a class of simple heuristic policies to control stochastic inventory systems with MOQ constraints. They also developed insights into the impact of MOQ constraints on repeatedly ordered items to fit in an industrial context.

The first studies on the MOQ constraints were from Constantino (1998) and Miller (1999b). They analyzed these constraints from a polyhedral point of view. Constantino derived strong inequalities which described the convex hull of the solutions, considering the production level as a continuous variable. Miller (1999b) replaced the production level by the amount of product that is produced in excess of the lower bound. Thus he studied the facets of the solutions's convex hull. He focused on multi-item problems and he proved that the single period relaxation is NP-hard. Chan and Muckstadt (1999) studied a production-inventory system in which the production quantity is constrained by a minimum and a maximum level in each period. However, the production level cannot be zero. They characterized the optimal policy for finite and infinite time horizons. The first exact polynomial time algorithm was recently developed by Okhrin and Richter (2011). They solved a special case of the problem in which the unit production cost is constant over the whole horizon and then can be discarded. Furthermore, they assumed that the holding costs are also constant over the T periods. Considering this restriction, they derived a polynomial time algorithm in $O(T^3)$. Li *et al.* (2008) studied the single item lot sizing problem with lower bound and described a polynomial algorithm in $O(T^7)$ to solve the

special case with concave production and storage cost function.

In this study, we extend Okhrin and Richter's result (Okhrin and Richter, 2011) to the problem with concave production and holding costs. We proposed an optimal algorithm called *HMP* with a time complexity in $O(T^5)$. The paper is organized as follows. Section 4.2 describes the problem and introduces the notations. In section 4.3, we give some definitions and present the properties that allow us to solve the problem in polynomial time. The algorithm and its time complexity study are given. In section 4.4, the efficiency of the method is tested on various instances. Finally, concluding remarks and perspectives are given in section 4.5.

4.2 Problem description and notations

The single-item lot sizing problem consists in satisfying the demands over T consecutive periods. At each period t , the demand d_t must be satisfied by production at period t (X_t) and/or from the inventory available at the end of the period $t - 1$ (I_{t-1}).

The production at each period is constrained by a constant capacity U . If a production is done at period t , it must be greater than or equal to a non-zero minimum order quantity L . We also consider production and inventory costs. The production cost is a concave function of the quantity produced ($p_t(X_t)$) and the inventory cost is a concave function of the inventory level ($h_t(I_t)$). Notice that concave cost functions may include set-up costs. The notations are summarized in Table 4.1.

T	number of periods
d_t	demand at period t
X_t	production at period t
I_t	inventory level at the end of period t
U	production capacity
L	minimum order quantity
$p_t(X_t)$	concave production cost function
$h_t(I_t)$	concave storage cost function

Table 4.1 – Notations

The CLSP-MOQ can be easily modeled by a mathematical program. The decision variables are X_t , I_t and a decision variable Y_t defined as follows:

$$Y_t = \begin{cases} 1 & \text{if the } X_t > 0 \\ 0 & \text{otherwise.} \end{cases}$$

The mathematical formulation of the CLSP-MOQ is then:

$$\text{Min } \sum_{t=1}^T p_t(X_t) + \sum_{t=1}^T h_t(I_t) \quad (4.1)$$

$$X_t + I_{t-1} - I_t = d_t \quad \forall t \in T \quad (4.2)$$

$$LY_t \leq X_t \leq UY_t \quad \forall t \in T \quad (4.3)$$

$$X_t, I_t \in \mathbb{R} \quad \forall t \in T \quad (4.4)$$

$$Y_t \in \{0, 1\} \quad \forall t \in T \quad (4.5)$$

The objective function (4.1) minimizes the total production and storage cost. Constraint (4.2) is the flow constraint. Constraint (4.3) insures that the maximum capacity and the minimum order quantity are satisfied. Constraints (4.4) and (4.5) define the validity domain of the variables.

Without loss of generality, we assume that $I_0 = 0$. Unfortunately, I_T can be strictly positive in an optimal solution. These two cases ($I_T = 0$ and $I_T > 0$) are considered in the following section.

4.3 An optimal algorithm

In this section, we introduce some definitions and we prove some properties. Based on these properties, we will be able to derive a polynomial time algorithm to solve the CSLP-MOQ problem.

Definition 1. *Regeneration points*

A period t is called a regeneration point if $I_t = 0$.

Definition 2. *Fractional production periods*

A period t is called a fractional production period if $L < X_t < U$.

Definition 3. *Sequence of production quantities*

The sequence of production quantities from $u + 1$ to v is noted S_{uv} .

Definition 4. *UL-capacity-constrained sequences*

S_{uv} is a UL-capacity-constrained sequence if the following conditions are verified:

- u and v are regeneration points i.e. $I_u = I_v = 0$;
- The demand d_t for $t = \{u + 1, \dots, v\}$ is satisfied;
- For all $t \in \{u + 1, \dots, v - 1\}$, $I_t \neq 0$ i.e. t is not a regeneration point;
- The production X_t for $t \in \{u + 1, \dots, v\}$ is equal to 0, U or L , except for at most one period which can be a fractional production period.

At this time, we consider that $I_T = 0$. The case for which $I_T > 0$ will be considered at the end of this section.

Property 1. *A solution to the CLSP-MOQ problem can be seen as succession of sub-sequences such that both the starting period and the ending period are regeneration points.*

Proof. Assuming that $I_k = 0$ for some $k \in \{1 \dots n - 1\}$. An optimal solution can be found by independently finding solutions to the problems for the first k periods and for the last $T - k$ periods. Consequently, a production plan can be seen as a sequence of consecutive periods in such a way that the stock is empty at the beginning and at the end of each sequence. \square

The problem now is to know if the production plan of each sub-sequence is easy to compute. Fortunately, these sub-sequences have good properties that allow us to find the optimal production plan polynomially.

Property 2. *Let us consider an interval of periods $[u, v]$ such that $I_u = I_v = 0$. UL-capacity-constrained sequences are dominant (i.e. at least one optimal solution is a UL-capacity-constrained sequence).*

Proof. To prove this result, we show that if a solution S_{uv} is not a UL-capacity-constrained sequence, it cannot be an extreme point of the polyhedron, and consequently is dominated by an other solution. In order to prove this result, we show that the solution S_{uv} is a convex combination of two other feasible solutions. Let us consider a solution S_{uv} such that $I_u = I_v = 0$, $I_t \neq 0$ for $t \in \{u + 1, \dots, v - 1\}$ and in such a way that there exists

at least two *fractional production periods* (i.e. i and j are such that $u + 1 \leq i < j \leq v$ and $L < X_i, X_j < U$). Consequently, we can relocate a small value of production between X_i and X_j as follows. Let us define ω as the biggest production quantity we can relocate keeping the solution feasible, and without changing other production levels. Then:

$$\omega = \min\{U - X_j ; U - X_i ; X_i - L ; X_j - L ; \min_{t=i}^{j-1} I_t\}$$

By relocating $\frac{1}{2}\omega$ from i to j , we obtain a solution S'_{uv} . The production plan S'_{uv} is obviously feasible. Symmetrically, by relocating $\frac{1}{2}\omega$ from j to i , we obtain a valid solution S''_{uv} . However, $S_{uv} = \frac{1}{2}S'_{uv} + \frac{1}{2}S''_{uv}$, proving that S_{uv} is not an extreme point. Then S_{uv} is not the unique optimal solution, and it is dominated. \square

In our problem, the fractional period can be anywhere in the production sequence. Okhrin and Richter proved that all maximum size lots must follow all minimum size lots. This is due to their cost function which is linear for the storage and constant for the production. Unfortunately, this property does not hold with concave production and storage cost function.

From now on, we have to verify if finding an optimal UL -capacity-constrained sequence can be done in polynomial time. Let us define α (resp. β) as the number of periods in which the production is equal to U (resp. L). The fractional production is noted ε . Using $D_{uv} = \sum_{t=u+1}^v d_t$, the total demand for the sequence, we can write:

$$\alpha U + \beta L + \varepsilon = D_{uv} \tag{4.6}$$

In some cases, $\alpha U + \beta L = D_{uv}$. This means that there is no fractional production period in the UL -capacity-constrained sequence. In this case we note $\varepsilon = 0$. Note that [Okhrin and Richter \(2011\)](#) define the same parameters, called k and K in their paper.

First of all, we prove that the number of triplets $(\alpha, \beta, \varepsilon)$ is in $O(T)$. Then, we will prove that it is possible to find an optimal plan in $O(T^3)$.

Property 3. *The number of triplets $(\alpha, \beta, \varepsilon)$ is in $O(T)$.*

Proof. Let us consider the maximal interval length with $u = 0$ and $v = T$. Define D as the total demand ($D = \sum_{t=1}^T d_t$). The triplet $(\alpha, \beta, \varepsilon)$ verifies equation (4.6), and obviously the following ones:

$$\alpha + \beta \leq T \tag{4.7}$$

$$L < \varepsilon < U \quad \text{or} \quad \varepsilon = 0 \tag{4.8}$$

The problem now is the enumeration of the distinct feasible triplets $(\alpha, \beta, \varepsilon)$. Let us

define α_{min} (resp. α_{max}) the minimum (resp. the maximum) feasible values for α . In the same way, let us define β_{min} and β_{max} .

Bounds on α : From equation 4.6, we immediately derive:

$$\alpha U + TL + U > D$$

This leads to the following inequality:

$$\alpha > \frac{D - LT - U}{U} \Leftrightarrow \alpha > \frac{D - LT}{U} - 1 \text{ thus } \alpha_{min} = \lceil \frac{D - LT}{U} - 1 \rceil$$

Furthermore we immediately have:

$$\alpha \leq \frac{D}{U} \text{ and then } \alpha_{max} = \lfloor \frac{D}{U} \rfloor$$

And then:

$$\frac{D - LT}{U} - 1 \leq \alpha_{min} \leq \alpha \leq \alpha_{max} \leq \frac{D}{U}$$

The number of possible values for α is at most:

$$\frac{D}{U} - \left(\frac{D - LT}{U} - 1 \right) + 1 = \frac{LT}{U} + 2$$

Bounds on β : Let us consider a feasible α . Using equations (4.6) and (4.8), we have:

$$\beta \geq \frac{D - \alpha U - U}{L} \text{ and then } \beta_{min} = \lceil \frac{D - \alpha U - U}{L} \rceil$$

and

$$\beta \leq \frac{D - \alpha U}{L} \text{ and then } \beta_{max} = \lfloor \frac{D - \alpha U}{L} \rfloor$$

The maximum number of values for β when α is fixed is:

$$\frac{D - \alpha U}{L} - \frac{D - \alpha U - U}{L} + 1 = \frac{U}{L} + 1$$

Maximum number of distinct triplets $(\alpha, \beta, \varepsilon)$: The maximum number of triplets is given by:

$$\left(\frac{LT}{U} + 2 \right) \left(\frac{U}{L} + 1 \right) = \left(1 + \frac{L}{U} \right) T + 2 \frac{U}{L} + 2$$

We must now bound this quantity. We have two cases.

- If $\frac{U}{L} \leq T$, the maximum number of triplets is bounded by $4T + 2$.
- If $\frac{U}{L} > T$ we have $LT < U$. Let us consider $\alpha_{max} = \lfloor \frac{D}{U} \rfloor$. α_{max} is a possible value for α . $\alpha = \alpha_{max} - 2$ is not possible. In this case, at least $2U$ products must be produced on periods for which $X_t = L$ or $X_t = \varepsilon$. As $LT < U$ and $\varepsilon < U$, $LT + \varepsilon < 2U$, the production capacity is not sufficient. Consequently, $\alpha_{max} - 2$

is not a feasible value for α . Then only two values for α can be chosen: $\lfloor \frac{D}{U} \rfloor$ and $\lfloor \frac{D}{U} \rfloor - 1$. In each case, there are at most T possible values for β , and then at most $2T$ distinct triplets.

That concludes the proof. □

The number of triplets $(\alpha, \beta, \varepsilon)$ is in $O(T)$. We must now prove that the best production plan can be found in polynomial time.

Property 4. *A UL-capacity-constrained sequence can be computed in polynomial time.*

Proof. Let us consider u and v , two regeneration points. From property 3, we can compute each valid triplet $(\alpha, \beta, \varepsilon)$. For each α and β , we have a unique ε . At most, we can have K different ε with K in $O(T)$. In the following, we note ε_i for $\{i = 1, \dots, K\}$.

We can now build a directed acyclic graph (\mathcal{UV}) as follows. This graph is divided into levels, for convenience from level u to level v . At level u only one node labeled 0 is put. At level $u + 1$, the nodes are labeled with a feasible cumulative production level $(0, L, \varepsilon_1, \dots, \varepsilon_K, U)$. At each level, several non feasible cumulative production can be discarded (see Section 4.4). The weight of an arc is the cost (production and storage) to have the two cumulative productions at the extremity of the arc. The next levels are built the same way (see Figure 4.1).

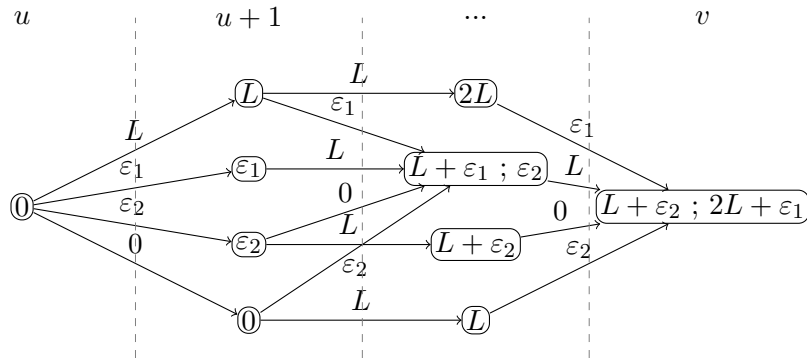


Figure 4.1 – An example of graph \mathcal{UV} with 4 periods

Considering a given triplet $(\alpha, \beta, \varepsilon)$, the number of nodes at each level of the graph is in $O(T)$. The number of triplets is in $O(T)$. Therefore at each level, the number of nodes is thus in $O(T^2)$. We have at most T levels, consequently the number of nodes in the graph is in $O(T^3)$. The shortest path in this graph gives us the optimal production plan. As each node has at most four predecessors, the evaluation of the node can be made in constant time. Consequently, the time complexity for finding an optimal solution is in $O(T^3)$. □

Unfortunately, due to the MOQ constraint, an optimal solution can have items in stock at T . Let us assume that the storage cost $h_{T-1}(I_{T-1})$ for a positive value of I_{T-1} is very high. If the production cost at period T is low and if the demand at period T respects

$0 < d_T < L$, the best strategy could be to produce L at the last period leading to a storage of $I_T = L - d_T$ items. Consequently, we must study the sequences \widehat{S}_{uT} where u is a *regeneration point* and such that $I_T \neq 0$.

Property 5. *An optimal production sequence \widehat{S}_{uT} in such a way that u is a regeneration point, and $I_T \neq 0$ can be computed in polynomial time.*

Proof. First of all, \widehat{S}_{uT} cannot contain a fractional production period. If \widehat{S}_{uT} contains a fractional production period at a period k of value ε , it is possible to decrease the production at this period by $\min\{\varepsilon - L, \min_{t=k}^T I_t\}$.

Furthermore, if \widehat{S}_{uT} contains a period where the production is maximum (U) at a period k , we can easily decrease the production because the storage levels are strictly positive from k to T . Then \widehat{S}_{uT} cannot contain a production level at U .

Consequently, the sequence \widehat{S}_{uT} only has production periods at L and 0 . Furthermore, $I_T < L$, otherwise we can suppress one of the productions, and then we only have to compute the value β as follows:

$$\beta = \lceil \frac{D_{u+1T}}{L} \rceil$$

The best production plan can now be computed in $O(T^2)$ with an algorithm similar to the one presented in Property 4. \square

We can now derive a polynomial time algorithm (called *HMP*) from the previous properties. Let us build a directed acyclic graph (\mathcal{G}) as follows. Let us define $T + 1$ vertices labeled from 0 to T . A vertex t signifies that t is a *regeneration point*. For each pair (i, j) , with $i < j < T$ add an arc. The value on this arc is computed by the algorithm described in Property 4. For each pair (i, T) , we have to add in the graph two arcs, one considering that $I_T = 0$ (Property 4), and one considering that $I_T \neq 0$ (Property 5). A shortest path in the graph leads to an optimal production plan.

Theorem 1. *Algorithm HMP gives an optimal solution for the CLSP-MOQ problem with constant capacity, constant minimum order quantity and concave production and storage costs in time in $O(T^6)$*

Proof. An optimal solution is given by a succession of sub-sequences between two regeneration points (property 1). Then, one of the shortest paths in graph (\mathcal{G}) is an optimal solution. The construction of the graph (\mathcal{G}) is in $O(T^6)$. Indeed, we have $O(T^2)$ arcs in graph (\mathcal{G}) and each arc can be computed in $O(T^4)$ (see properties 4 and 5), leading to a time complexity of $O(T^6)$. Finding one of the shortest paths in (\mathcal{A}) can be made in $O(T^2)$. Finally, we conclude that the time complexity of the algorithm *HMP* is in $O(T^6)$ (Algorithm 1). \square

Algorithm 1 HMP

```

Build the graph ( $\mathcal{G}$ )
for each edge E of ( $\mathcal{G}$ ) do
    Build the corresponding graph ( $\mathcal{UV}$ )
    Perform a shortest path algorithm to find an optimal production sequence and its
    cost
    Set the weight of the edge E with the obtained cost
end for
Perform a shortest path algorithm on graph ( $\mathcal{G}$ ) to find the best production sequence

```

4.4 Computational experiments

4.4.1 Computational settings

To test the efficiency of algorithm *HMP*, we performed intensive experiments. We aim to define different sets in which all the instances share the same parameter values. Then we randomly generate different feasible instances for each set. Demand values are generated from a normal distribution. As in [Okhrin and Richter \(2011\)](#), 4 parameters are selected: the mean and the variance of the demand distribution, the capacity and the MOQ. We choose three different values for the mean μ (low, medium and large) fixed to 40, 200 and 600. For each of these values, we have three different values for the variance σ which correspond to 10%, 20% and 30% of μ . We set three capacity levels: small, medium and large. Each capacity verifies μ/U equal to 0.75, 0.85 and 0.95 as proposed in [Okhrin and Richter \(2011\)](#). The values of these parameters are given in [Table 4.2](#).

Mean demand	Demand variance			Capacity		
	Small	Medium	Large	Small	Medium	Large
40	4	8	12	42	47	53
200	20	40	60	211	235	267
600	60	120	180	632	706	800

Table 4.2 – Mean demand, Variance and Production Capacity Parameters

In addition, we choose the MOQ to be equal to the lower quartile L1, median L2 and upper quartile L3 of the normal distribution as in [Anderson and Cheah \(1993\)](#). The different values for L1, L2 and L3 are given in [Table 4.3](#).

Hence we have 81 possible sets of instances as shown in [Figure 4.2](#), but taking these parameters may lead to infeasible instances. Indeed, the 9 sets with $\mu/U = 0.95$ and L3 are insolvable because of $U < L$. Similarly, in the 3 sets with $\mu/U = 0.85$, L3, and $\sigma = 30\%\mu$, the capacity is smaller than the MOQ. Thus 12 of the 81 sets must be discarded, leaving us with only 69 feasible sets.

To compute the concave costs, both production and storage costs are defined by a fixed

Mean	Variance	L1	L2	L3
40	4	37	40	43
40	8	35	40	45
40	12	32	40	48
200	20	187	200	213
200	40	173	200	227
200	60	160	200	240
600	60	560	600	640
600	120	519	600	681
600	180	479	600	741

Table 4.3 – Minimum Order Quantity Parameters

$$\mu : \begin{pmatrix} 40 \\ 200 \\ 600 \end{pmatrix} \rightarrow U : \begin{pmatrix} \mu/0.95 \\ \mu/0.85 \\ \mu/0.75 \end{pmatrix} \rightarrow \sigma : \begin{pmatrix} 0.1\mu \\ 0.2\mu \\ 0.3\mu \end{pmatrix} \rightarrow L : \begin{pmatrix} \mu - 0.68\sigma \\ \mu \\ \mu + 0.68\sigma \end{pmatrix}$$

Figure 4.2 – Parameter settings

and a linear cost. For each period, these costs are randomly chosen among 3 different values: 10, 20 and 30 for the setup production cost; 1, 2 and 3 for the linear production cost; 1, 2 and 3 for the setup storage cost and 0.1, 0.2 and 0.3 for the linear storage cost.

Finally, we created 69 sets of instances, and for each set, we randomly generated ten feasible instances.

Algorithm *HMP* was implemented in Java 1.6 and has been run on a 3 GHz Intel Core 2 Duo Machine with 4GB memory running Windows 7.

4.4.2 Improvements to Algorithm *HMP*

To increase the algorithm's efficiency, we implemented several improvements. In the construction of graph (\mathcal{G}), if an edge (uv) verifies $D_{u+1,v} > U(u-v)$, the production plan from $u+1$ to v is infeasible and then the vertex u cannot be a regeneration point. The vertex u is removed.

In the construction of graph (\mathcal{UV}), we have to verify that each node created leads to a solution for which the demand is satisfied and the stock level is strictly positive.

These improvements reduce the number of nodes in graphs (\mathcal{G}) and (\mathcal{UV}), and consequently save computational time. They strongly affect the efficiency of Algorithm *HMP*,

as described in section 4.4.3.

4.4.3 Performance evaluation of Algorithm *HMP* on the sets of instances

We now compare the running time of Algorithm *HMP* to test its efficiency. For that, we fix the horizon to 40 periods and launch the algorithm on the 69 instance sets. For each instance, we observe the computational time and the number of nodes generated in graph (\mathcal{UV}) . We denote as *iterations* this number of nodes. Figure 4.3 shows the mean number of iterations, of each sets satisfying $\mu = 200$.

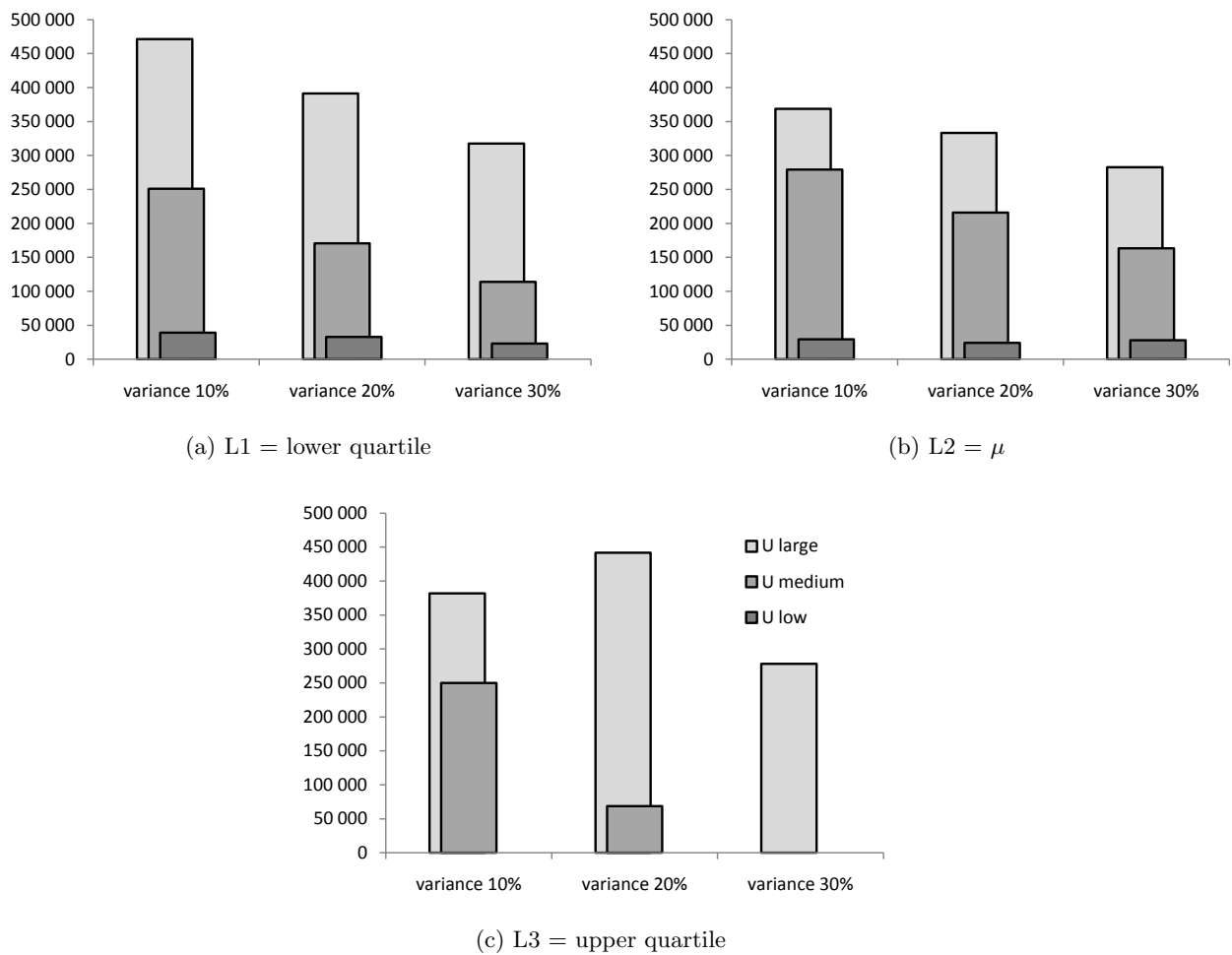


Figure 4.3 – Number of iterations for $\mu = 200$ with *normal* costs

Indeed with another value of μ , the shape of the histograms remains the same.

The value of U hugely affects the number of iterations. For small values of U , the production plan is more constrained and consequently, some periods cannot be regeneration points. Vertices in \mathcal{A} are then removed. For the same reason, large values for σ cause the

deletion of many edges and vertices in graph \mathcal{G} .

Finally, all the instances are solved in less than 30 seconds.

4.4.4 Evaluation of the computational time when T increases

The following experiments have two goals. Firstly, we show that the number of nodes in graph \mathcal{UV} and the running time are strongly dependent. Secondly, we point out the computational complexity of Algorithm *HMP*.

To show this, we choose three sets of instance among the 69 previous ones:

- Instance set for which we have the minimum number of iterations among all the sets (set A);
- Instance set for which we have the mean number of iterations among all the sets (set B);
- Instance set for which we have the maximum number of iterations among all the sets (set C).

The parameters for the 3 instance sets are given in Table 4.4.

sets	μ	σ	L	U
A	40	20%	μ	$\mu/0, 95$
B	40	20%	μ	$\mu/0, 85$
C	200	10%	upper quartile	$\mu/0, 75$

Table 4.4 – Parameters for Instance Sets A, B and C

For each set, 10 instances are generated. The horizon length starts at $T = 40$ and increases with a step of 5 for set C and 10 for sets A and B.

The curves given in Figure 4.4 show the strong dependence of the computational time and the number of nodes in Graph \mathcal{UV} for the 3 instance sets. For the set A, Algorithm *HMP* solves the instances in a few seconds up to 120 periods in the horizon. When $T = 180$, the computational time is less than 1 hour. For instance set B, the instances are more difficult to solve. The algorithm is quick until $T = 70$ but it needs 1 hour to solve instances with $T = 105$ and almost 2 hours when $T = 120$. Finally, for difficult instances (set C) the algorithm is very efficient until $T = 50$, but it needs almost 1 hour when $T = 73$ and almost 4 hours when $T = 80$.

The standard deviations of the number of iterations fluctuate, depending on the instance sets chosen. This is shown in Table 4.5. For easy instances (set A), the standard deviations are large: almost equal to the mean value (69%). Conversely, for set C, the standard deviations are low (7% of the mean value). That means that all the set C instances are difficult to solve. However, the solving difficulty of the set A instances varies

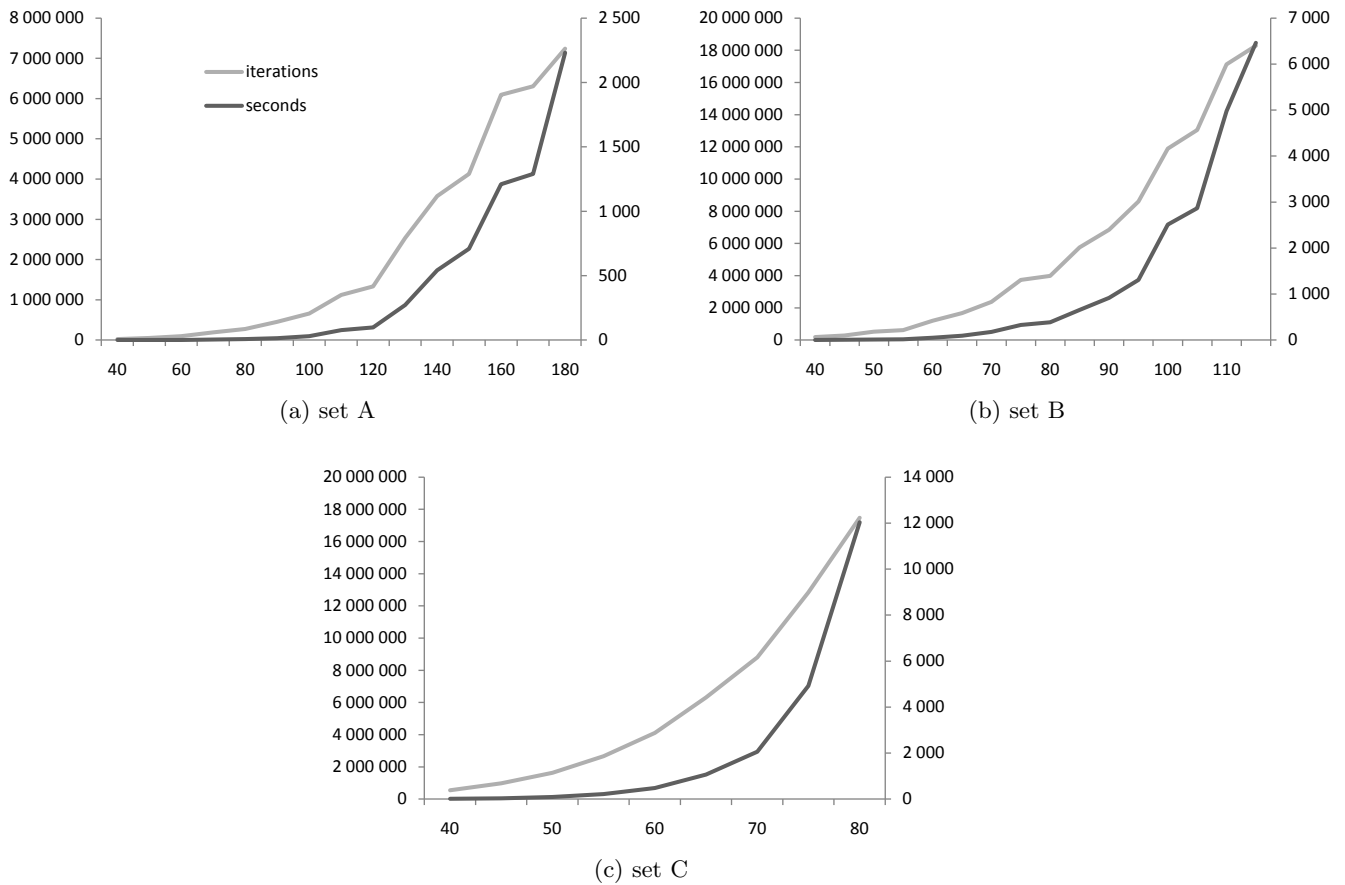


Figure 4.4 – Correlation between iterations (left axis) and time (right axis) increasing the number of periods (horizontal axis)

enormously. This reinforces the idea that the more constrained the instance is, the easier it is to solve.

set A	set B	set C
69%	29%	7%

Table 4.5 – Ratio between standard deviation and the mean of the number of iterations for the 3 sets of instances (on average)

4.5 Conclusion

The focus of this paper is a generalization of the capacitated single item lot sizing problem. In this problem, the production levels must be bounded by a minimum order quantity and a maximum capacity when a production is decided. Both production and storage costs are concave. We proposed an $O(T^5)$ exact algorithm that generalizes Florian

and Klein's (in $O(T^4)$ without the minimum order quantity) and Okhrin and Richter's (in $O(T^3)$ with a simplified cost structure).

The computational experiments show that Algorithm *HMP* solves large instances in reasonable time, even for hard instances. Then, it could be used as a brick to solve more complex lot sizing problems as multi-items ones.

The theoretical complexity of Algorithm *HMP* appears difficult to improve with general concave costs. In future works, it would be interesting to determine if this theoretical complexity can be decreased when the cost structure is limited to a fixed cost plus a linear one like we used in our tests. We could also consider using only linear costs for the storage.

Chapitre 5

A polynomial time algorithm for the single-item lot sizing problem with capacities, minimum order quantities and dynamic time windows

Abstract

This paper deals with the single-item capacitated lot sizing problem with concave production and storage costs, considering minimum order quantity and dynamic time windows. This problem models a lot sizing where the production lots are constrained in amount and frequency. In this problem, a demand must be satisfied at each period t over a planning horizon of T periods. This demand can be satisfied from the stock or by a production at the same period. When a production is made at period t , the produced quantity must be greater than a minimum order quantity (L) and lower than the production capacity (U). The frequency constraints on the production lots are modeled by dynamic time windows. Between two consecutive production lots, there is at least Q periods and at most R periods. An optimal algorithm in $O(T^9)$ is given. The complexity of the algorithm is reduced to $O(T^7)$ when all the demands are strictly positive.

5.1 Introduction

This paper deals with a generalization of the single-item capacitated lot sizing problem (CLSP) with fixed capacity. The CLSP consists in satisfying a demand at each time period t over a planning horizon T . The demand is satisfied from stock or by production. Costs incur for each item produced and also when an item is stored between two consecutive periods. A fixed maximum production capacity (U) must be respected. The problem considered in this paper contains a minimum order quantity constraint (MOQ). This constraint imposes that if an item is produced at a given period, the quantity must be greater than or equal to a minimum level L . The U and L values are constant over the T periods. This problem also includes dynamic time windows (DTW). Between two consecutive production lots, there are at least Q periods and at most R periods. These DTW are useful in a long term partnership between two actors, because they allow the decision makers to stabilize the relationship (Hellion *et al.*, 2013d). This problem is noted CLSP-MOQ-DTW in the following.

The single-item capacitated lot sizing problem is known to be *NP*-Hard (Bitran and Yanasse, 1981). However, some cases are polynomially solvable. This is the case when the capacity is fixed over the T periods. Florian and Klein (1971) considered a case where production and holding cost functions are concave. They proposed an exact method with a time complexity in $O(T^4)$. Later van Hoesel and Wagelmans (1996) improved the complexity of the algorithm in $O(T^3)$ when the holding costs are linear. A complete survey on the single-item lot sizing problem can be found in Brahimi *et al.* (2006a).

Recently minimum order quantity (MOQ) constraints have been developed. These constraints deal with the production level that must be at least the MOQ if the production is to be started. The CLSP-MOQ has been shown relevant in many industrial contexts, for example Lee (2004) has studied an industrial problem where a manufacturer imposes a minimum order quantity to its supplier. Furthermore, Porras and Dekker (2006) have worked on an industrial case where the producer imposes minimum order quantities (MOQ) to produce the items. Zhou *et al.* (2007) have analysed a class of simple heuristic policies to control stochastic inventory systems with MOQ constraints. They also developed insights into the impact of MOQ constraints on repeatedly ordered items to fit in an industrial context. The first exact polynomial time algorithm was developed by Okhrin and Richter (2011). They solved a special case of the problem in which the unit production cost is constant over the whole horizon and then can be discarded. Furthermore, they assumed that the holding costs are also constant over the T periods, with these restrictions they derived a polynomial time algorithm in $O(T^3)$. Li *et al.* (2008) studied the single item lot sizing problem with lower bounds and described a polynomial algorithm in $O(T^7)$ to solve the special case with concave production and storage cost function. Later

Hellion *et al.* (2012, 2013a) developed an optimal $O(T^6)$ polynomial time algorithm to solve the CLSP-MOQ with concave costs functions, improving Li *et al.* (2008) algorithm. Hellion *et al.* (2012) also provide a computational experiment to underline the practical complexity of their algorithm.

The production capacity and MOQ constraints were originally motivated by industrial needs. Considering a retailer ordering from a single supplier, these constraints additionally give the supplier a way to forecast future orders. However, these constraints only affect the quantity of the orders, and both the supplier and the retailer lack temporal informations. To ensure a long-term partnership, actors must guarantee a certain amount of supplied components and regular orders. The time interval between two orders must in a given time window (Hellion *et al.*, 2013d).

In the existing literature, time windows have been introduced with several definitions. The delivery time window (also called grace period) was first presented by Lee *et al.* (2001). In their model, each demand d_t must be delivered during a time window. Later, Akbalik and Penz (2011) used a similar definition to compare just-in-time and time windows policies (introduced by Brahimi *et al.* (2006a)). In this problem, items cannot be produced before a defined period. Recently, Absi *et al.* (2011) studied two production time window problems, considering lost sales or backlogs. They used dynamic programming to solve their problems. Hwang (2007) proposed an $O(T^5)$ algorithm for the production time windows and concave production costs. Van den Heuvel and Wagelmans (2008) showed that the formulations with production time windows are equivalent to other models: lot sizing with manufacturing options, lot sizing with cumulative capacities and lot sizing with inventory bounds. These two time window definitions were studied by Wolsey (2006), he proposed valid inequalities and convex hulls.

However, these time window definitions above do not guarantee regular orders. Hellion *et al.* (2013d) recently presented a new time windows definition in which actors have to agree on a minimum and a maximum number of periods between two orders. Since an order is dependent on the period where the last order occurred, these time windows are dynamic (called DTW as already defined).

In this paper, we extend Hellion *et al.*'s algorithm to the problem with dynamic time windows. The paper is organized as follows: Section 5.2 describes the problem and introduces the notations. Section 5.3 presents the necessary definitions and properties to give a polynomial algorithm. However, under specific assumptions the complexity of the algorithm can be slightly reduced: this case is presented in Section 5.4. Finally, concluding remarks and perspectives are given in Section 5.5.

5.2 Problem description and notations

5.2.1 Description

The single item lot sizing problem consists of satisfying the demands d_t of a product at each period t over T consecutive periods. A demand $d_t \in \mathbb{Z}^+$ may be satisfied by the production of an item at period t (X_t) and/or from inventory (I) available at the end of the period $t - 1$ (I_{t-1}). Backlogs are not allowed. The inventory level at the end of a period t is denoted I_t . It is assumed without loss of generality that there is no inventory at the beginning of the first period. The problem is to determine the amount X_t to be produced at each period, satisfying the demands and minimizing the total cost.

The production at each period is constrained by a constant capacity U . The production level is also constrained by the MOQ: L . Each subsequent production level is also constrained by a dynamic time window (DTW). There are at least Q and at most R periods between two consecutive production lots.

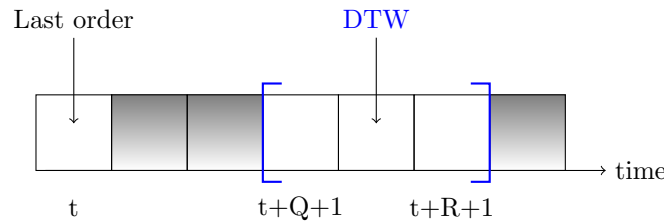


Figure 5.1 – An example of dynamic time window with $Q = 2$ and $R = 4$

Figure 5.1 illustrates the dynamic time window for $Q = 2$ and $R = 4$. In the example, a lot is produced in period t . Since $Q = 2$, the following lot cannot be produced at neither at period $t + 1$ nor at $t + 2$. Since $R = 4$, at least lot must be produced in the next five periods. Thereafter, the next lot must be produced between periods $t + 3$ ($t + Q + 1$) and $t + 5$ ($t + R + 1$) included. Consequently, in each interval of length 3 at most one lot must be produced. Furthermore, in each interval of length 5, at least one lot must be produced. Note that if $Q = R$ one lot must be produced every $Q + 1$ periods. If $Q = R = 0$, one lot must be produced every period.

The production cost is a concave function of the quantity produced $p_t(X_t)$ and the inventory cost is a concave function of the inventory level $h_t(I_t)$. Note that concave cost functions may include set-up costs.

5.2.2 Mathematical formulation

The mathematical formulation is now presented. The decision variables are given as follows:

- X_t : quantity of products ordered at period t .

- $Y_t = \begin{cases} 1 & \text{if an order is placed at period } t. \\ 0 & \text{otherwise.} \end{cases}$
- $I_t =$ inventory level at the end of a period t .

The mathematical formulation of the CLSP-MOQ-DTW is then:

$$\text{Min } \sum_{t=1}^T p_t(X_t) + \sum_{t=1}^T h_t(I_t) \quad (5.1)$$

$$X_t + I_{t-1} - I_t = d_t \quad \forall t \in T \quad (5.2)$$

$$LY_t \leq X_t \leq UY_t \quad \forall t \in T \quad (5.3)$$

$$\sum_{t'=t}^{t+R} Y_{t'} \geq 1 \quad \forall t \in \{1, \dots, T - R\}, \quad (5.4)$$

$$\sum_{t'=t}^{t+Q} Y_{t'} \leq 1 \quad \forall t \in \{1, \dots, T - Q\} \quad (5.5)$$

$$X_t, I_t \in \mathbb{R} \quad \forall t \in T \quad (5.6)$$

$$Y_t \in \{0, 1\} \quad \forall t \in T \quad (5.7)$$

The objective function (5.1) is to minimize the total cost. Constraint (5.2) is the flow constraint. Constraint (5.3) ensures that the maximum capacity and the minimum order quantity are satisfied. The dynamic time windows are given by (5.4) and (5.5). Constraints (5.6) and (5.7) define the domain of validity the variables.

5.3 A polynomial time algorithm

Our work is based on the concept of sub-plan introduced by Florian and Klein (1971), which leads to a polynomial algorithm. We extend the definitions of Hellion *et al.* (2012) to the our problem, *i.e.* including the DTW.

Definition 5. *Regeneration points*

A period t is called a regeneration point if $I_t = 0$.

Definition 6. *Fractional production periods*

A period t is called a fractional production period if $L < X_t < U$.

Definition 7. *Production sequence*

The sequence of production quantities from $u + 1$ to v is noted S_{uv} .

Remark 1. Let S_{uv} and S_{vw} be two production sequences which are separately feasible. The sequence $S_{uv} \cup S_{vw}$ may be unfeasible, due to the DTW incurred by the last period of production of S_{uv} . When computing S_{uv} we cannot know where the first lot of S_{vw} will

be produced. Knowing that $I_v = 0$ is not sufficient because the following demand could be zero. If no assumption on the demand is made, the information given by a production sequence must include the period of the last production. This means that it is possible to know for each feasible S_{uv} where the first lot of S_{uv} can be produced. In order to take this into account, we define a specific production sequence.

The definition below introduces the *DTW-capacity-constrained* sequences $S_{ui,vj}$ (a production sequence between the periods u and v). The additional indexes i and j allow the *DTW* to be respected between two consecutive production sequences. The last lot of the previous production sequence was produced at $u - i$. With R and Q , the *DTW* is known, and the first lot can be produced. The production of the last lot must take place at $v - j$.

Remark 2. Let k be the number of null demands at the beginning of the *DTW-capacity-constrained* sequences. Since $I_u = 0$ the inventory level can be zero at the end of these k periods. In any case $I_t > 0 \forall t \in [u + k + 1, \dots, v - 1]$.

Definition 8. *DTW-capacity-constrained sequences*

$S_{ui,vj}$ is a *DTW-capacity-constrained* sequence if the following conditions are verified:

- u and v are regeneration points i.e. $I_u = I_v = 0$;
- The demand d_t for $t = \{u + 1, \dots, v\}$ is satisfied;
- $X_t \geq L$ for at least one t in $[u - i + Q + 1, \dots, u - i + R + 1]$
- $X_{v-j} \geq L$ and $X_t = 0$ for $t \in [v - j + 1, \dots, v]$
- The production X_t for $t \in \{u + 1, \dots, v\}$ is either equal to 0, L or U , except for at most one period which can be a fractional production period.
- $S_{ui,vj}$ respects the *DTW* constraints.
- $I_t = 0$ before the first production period and $I_t > 0$ after the first production period.

The dominance of the *DTW-capacity-constrained* sequences is proved later.

Property 6. Given two *DTW-capacity-constrained* sequences $S_{ui,vj}$ and $S_{wk,zl}$, the succession of these production sequences is feasible if and only if $v = w$ and $j = k$. $S_{ui,vj}$ and $S_{wk,zl}$ are then said compatible.

Proof. Since a *DTW-capacity-constrained* sequence begins where the previous one ends, $v = w$. As stated above, the last lot of $S_{ui,vj}$ is produced at the period $v - j$. Index wk means that the last lot of the previous sequence was produced at $w - k$. As $v = w$, the *DTW* can be respected if $j = k$. \square

Due to the MOQ constraint, the final storage could be strictly positive. At this time, we consider that $I_T = 0$. The case for which $I_T > 0$ will be considered at the end of this section, in the Property 10.

Property 7. *A solution to the CLSP-MOQ-DTW problem can be seen as succession of compatible DTW-capacity-constrained sequence.*

Proof. Assuming that $I_k = 0$ for some $k \in \{1 \dots n - 1\}$. An optimal solution of CLSP-MOQ-DTW can be found by independently finding solutions to the problems for the first k periods and for the last $T - k$ periods. However the DTW have to be respected, inside and between the production sequences. Consequently, a production plan can be seen as a succession of compatible DTW-capacity-constrained sequences, in accordance to Property 6. \square

Let us build a directed acyclic graph (\mathcal{G}) as follows. Define $T + 1$ vertices labeled from 0 to T . All these vertices are duplicated on $R + 1$ level, and each vertex is labeled (t, i) referring to its period $t \in 0 \dots T$, and its level $i \in 0 \dots R$, respectively. A vertex (t, i) signifies that the period t is a *regeneration point*. For convenience two additional vertices *start* and *end* are added. Each arc $(u, i) \rightarrow (v, j)$ models a DTW-capacity-constrained sequence $S_{ui,vj}$. A number of arcs can be immediately discarded, since the production at periods $u - i$ and $v - j$ is unfeasible considering a given DTW. The vertices *start* and *end* are connected at a null cost with all the nodes $(0, i)$ and $(T, i), \forall i \in [0, \dots, R]$, respectively. For each pair $\{(t, i), (T, j)\}$, we have to add two arcs to the graph, one representing $I_T = 0$ (Property 9 is given below), and one representing $I_T \neq 0$ (Property 10 is given at the end of the section). Figure 5.2 shows an instance of a (\mathcal{G}) considering $T = 3, Q = 1$ and $R = 1$. A shortest path between the nodes *start* and *end* leads to an optimal production plan.

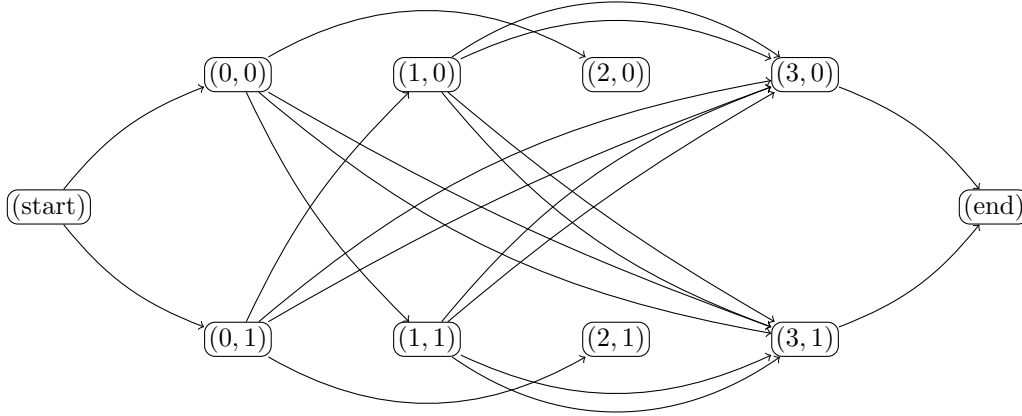


Figure 5.2 – An example of graph \mathcal{G} considering $T = 3$, $Q = 1$ and $R = 1$

Property 8. *Let us consider an interval of periods $[u, v]$ such that $I_u = I_v = 0$. DTW-capacity-constrained sequences are dominant for CLSP-MOQ-DTW (i.e. at least one optimal solution is a DTW-capacity-constrained sequence).*

Proof. To prove this result, we show that if a production sequence $S_{ui,vj}$ is not a DTW-capacity-constrained sequence, it cannot be an extreme point of the polyhedron defined by constraints 5.2 to 5.7, and consequently is dominated by an other solution. In order to prove this result, we show that the solution $S_{ui,vj}$ is a convex combination of two other feasible solutions.

Let us consider a solution $S_{ui,vj}$ such that both u and v are regeneration points. $I_u = I_v = 0$, $I_t \neq 0$ for $t \in \{t', \dots, v-1\}$ where t' is the first lot produced in a sequence (see Remark 2) in such a way that there exist at least two *fractional production periods* a and b such that $t' \leq a < b \leq v$ and $L < X_a, X_b < U$. Since $S_{ui,vj}$ is a feasible sequence for this problem, the DTW are respected. Consequently, we can relocate a small amount of production between X_a and X_b as follows. Let us define ω as the biggest production quantity we can relocate keeping the solution feasible, and without changing other production levels. Then:

$$\omega = \min\{U - X_a ; U - X_b ; X_a - L ; X_b - L ; \min_{t=a}^{b-1} I_t\}$$

By relocating $\frac{1}{2}\omega$ from a to b , we obtain a solution $S'_{ui,vj}$. The production plan $S'_{ui,vj}$ is obviously feasible and the DTW constraints still hold. Symmetrically, by relocating $\frac{1}{2}\omega$ from b to a , we obtain a valid solution $S''_{ui,vj}$. However, $S_{ui,vj} = \frac{1}{2}S'_{ui,vj} + \frac{1}{2}S''_{ui,vj}$, proving that $S_{ui,vj}$ is not an extreme point. Therefore $S_{ui,vj}$ is not the unique optimal solution, and it is dominated. \square

Property 9. *A DTW-capacity-constrained sequence $S_{ui,vj}$ can be computed in $O(T^5)$.*

Proof. Let us define α (resp. β) as the number of periods in which the production is equal to U (resp. L). The fractional production is noted ε with $L < \varepsilon < U$. Using $D_{uv} = \sum_{t=u+1}^v d_t$, the total demand for the sequence, we can write:

$$\alpha U + \beta L + \varepsilon = D_{uv} \tag{5.8}$$

In some cases, $\alpha U + \beta L = D_{uv}$. This means that there is no fractional production period in the *DTW*-capacity-constrained sequence. In this case we note $\varepsilon = 0$. Note also that [Okhrin and Richter \(2011\)](#) define the same parameters, called k and K in their paper.

First of all, we will prove that the number of triplets $(\alpha, \beta, \varepsilon)$ is in $O(T)$. This will be followed by the proof that it is possible to compute a *DTW*-capacity-constrained sequence in $O(T^5)$.

Consider a given β , we have only one feasible value for α , which is $\lfloor \frac{D_{uv} - \beta L}{U} \rfloor$. The magnitude of β is in $O(T)$, thus the number K of triplets $(\alpha, \beta, \varepsilon)$ is in $O(T)$. At most there are K different ε , and they are noted ε_k for $\{k = 1, \dots, K\}$.

We must now prove that the best production plan can be found in polynomial time.

Considering a single *DTW*-capacity-constrained sequence S_{u_i, v_j} , another directed acyclic graph (\mathcal{UV}) can be built (see [Figure 5.3](#)). For each period t such as $u + 1 \leq t \leq v$ we have a node for each feasible cumulative production level. We have an arc labeled x between 2 nodes a and b only if a production $X_b = x$ is feasible, with x being either L , U or ε_k . Each arc is weighted with the associated cost (production and storage). A shortest path allows the minimization of the total cost of this *DTW*-capacity-constrained sequence.

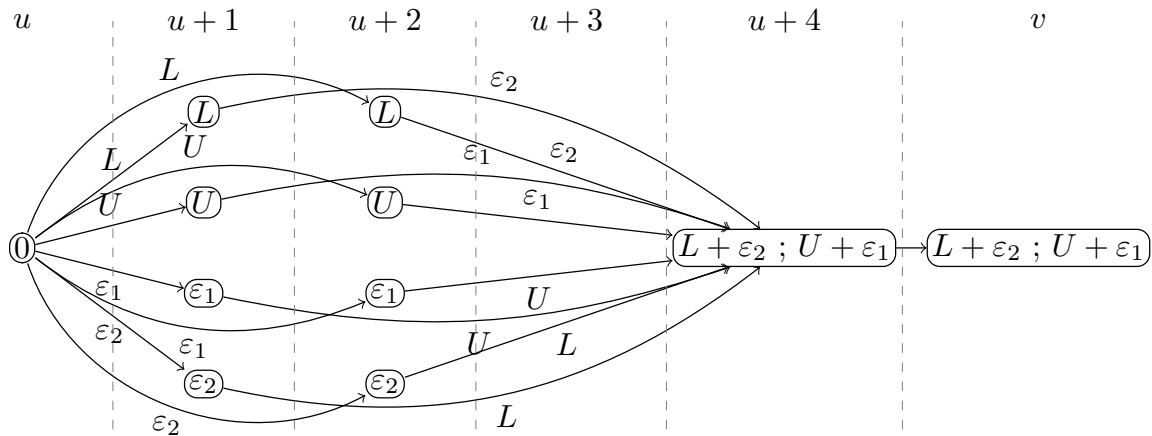


Figure 5.3 – S_{u_1, v_1} : an example of graph \mathcal{UV} with 5 periods, $Q = 1$ and $R = 2$

The [Figure 5.3](#) shows how a given *DTW*-capacity-constrained sequence can be computed. We choose to display S_{u_1, v_1} , considering 5 periods and $Q = 1$ and $R = 2$. In this example, we have to choose between producing at $u + 1$ or $u + 2$, knowing that the last production was at $u - 1$. Lots must also be produced at $u + 4$, because $u + 4 = v - 1$.

Considering a given triplet $k: (\alpha, \beta, \varepsilon_k)$, the number of nodes at each level of the graph is in $O(T^2)$, since the magnitude of α and β are both in $O(T)$. Therefore for this triplet we have $O(T^3)$ nodes. Each node has at most 3 predecessors on each level, leading to $O(T)$ predecessors. The evaluation of a node for the triplet k can be made in $O(T^4)$. The number of triplets is in $O(T)$ and consequently the time complexity for finding an optimal solution is in $O(T^5)$.

This concludes the proof of property 9. \square

Unfortunately, as mentioned above, due to the MOQ constraint an optimal solution can have items in stock at T . Let us assume that the storage cost $h_{T-1}(I_{T-1})$ for a positive value of I_{T-1} is very high. If the production cost at period T is low and if the demand at period T respects $0 < d_T < L$, the best strategy could be to produce L at the last period leading to a storage of $I_T = L - d_T$ items. Consequently, we must study the sequences $\widehat{S}_{ui,Tj}$ where u is a *regeneration point* and such that $I_T \neq 0$.

Property 10. *An optimal production sequence $\widehat{S}_{ui,Tj}$ such that u is a regeneration point, and $I_T \neq 0$ can be computed in polynomial time.*

Proof. First of all, $\widehat{S}_{ui,Tj}$ cannot contain a fractional production period; if $\widehat{S}_{ui,Tj}$ contains a fractional production period at a period t' of value ε , it is possible to decrease the production at this period by $\min\{\varepsilon - L, \min_{t=t'}^T I_t\}$. Furthermore, if $\widehat{S}_{ui,Tj}$ contains a period where the production is maximum (U) at a period t' , we can easily decrease the production because the storage levels are strictly positive from t' to T . Then $\widehat{S}_{ui,Tj}$ cannot contain a production level at U .

Consequently, the sequence $\widehat{S}_{ui,Tj}$ only has production periods at L and 0. Furthermore, $I_T < L$, otherwise we can suppress one of the productions, and then there is only one value for β which is:

$$\beta = \lceil \frac{D_{u+1T}}{L} \rceil$$

Thus there is only one feasible triplet $(\alpha, \beta, \varepsilon)$ in this production sequence. The best production plan can now be computed in $O(T^4)$ (see Property 9) with a graph similar to the one presented in the Figure 5.3. \square

We can now derive a polynomial time algorithm (called *HMP-DTW*) from the previous properties.

Theorem 2. *Algorithm HMP-DTW gives an optimal solution for the CLSP-MOQ-DTW problem with concave production and storage costs in time $O(T^9)$*

Proof. An optimal solution is given by a succession of sub-sequences between two regeneration points (Property 7). One of the shortest paths in graph \mathcal{G} is an optimal solution (see Figure 5.2). The construction of the graph \mathcal{G} is in $O(T^9)$. Furthermore, we have $O(T^4)$

arcs in graph \mathcal{G} and each arc can be computed in $O(T^5)$ (see Properties 9 and 10), leading to a time complexity of $O(T^9)$. Finding one of the shortest paths in \mathcal{G} can be made in $O(T^4)$. We therefore conclude that the time complexity of the algorithm *HMP-DTW* is in $O(T^9)$ (Algorithm 2). \square

Algorithm 2 HMP-DTW

Build the graph (\mathcal{G})
for each edge E of (\mathcal{G}) **do**
 Build the corresponding graph (\mathcal{UV})
 Perform a shortest path algorithm to find an optimal production sequence and its cost
 Set the weight of the edge E with the obtained cost
end for
 Perform a shortest path algorithm on graph (\mathcal{G}) to find the best production sequence

5.4 An $O(T^7)$ algorithm when the demand are strictly positive

In this section it is shown that, with the assumption that $d_t > 0 \forall t \in T$, the complexity of the algorithm can be slightly reduced. Let us called this new problem CLSP-MOQ-DTW with positive demands (CLSP-MOQ-DTW-PD). Consider again two production sequences S_{uv} and S_{vw} such that u , v and w are regeneration points. S_{uv} and S_{vw} are separately feasible, that is to say they both respect all the constraints, including the DTW. The production sequence is defined as $s = S_{uv} \cup S_{vw}$.

Remark 3. Consider that $I_v = 0$ and $d_{v+1} > 0$. When computing S_{vw} a lot must be produced at $v + 1$ to satisfy the demand d_{v+1} . This means that each computed S_{uv} has to allow the production at $v + 1$ for the following production sequence. The information about the last period of production of S_{uv} can therefore be discarded. This decreases the number of DTW-capacity-constrained sequence to compute.

This in turn, leads to another extension of the original definition.

Definition 9. PD-capacity-constrained sequences

S_{uv} is a PD-capacity-constrained sequence if the following conditions are verified:

- u and v are regeneration points i.e. $I_u = I_v = 0$;
- The demand d_t for $t = \{u + 1, \dots, v\}$ is satisfied;
- For all $t \in \{u + 1, \dots, v - 1\}$, $I_t > 0$ i.e. t is not a regeneration point;
- The production X_t for $t \in \{u + 1, \dots, v\}$ is equal to 0, U or L , except for at most one period which can be a fractional production period.
- S_{uv} respects the DTW constraints, considering that $v+1$ is a production period.

We can easily extend the DTW-capacity-constrained sequences properties to the PD-capacity-constrained sequences: two PD-capacity-constrained sequences can follow each other, a succession of them can solve the problem and they are dominant (Properties 6, 7 and 8, respectively). However, as said above, the number of sequences to compute has decreased. The general graph implied (say graph \mathcal{G}_{PD}) is a simplification of the graph \mathcal{G} (Figure 5.2). The only difference is that the $T + 1$ vertices no longer need to be duplicated on $R + 1$ levels.

Property 11. *A PD-capacity-constrained sequence can be computed in $O(T^5)$.*

Proof. Considering a single PD-capacity-constrained sequence $S_{u,v}$, a directed acyclic graph (\mathcal{UV}_{PD}) can be built (see Figure 5.4 for an example). It is similar to the graph (\mathcal{UV}) presented in Figure 5.3.

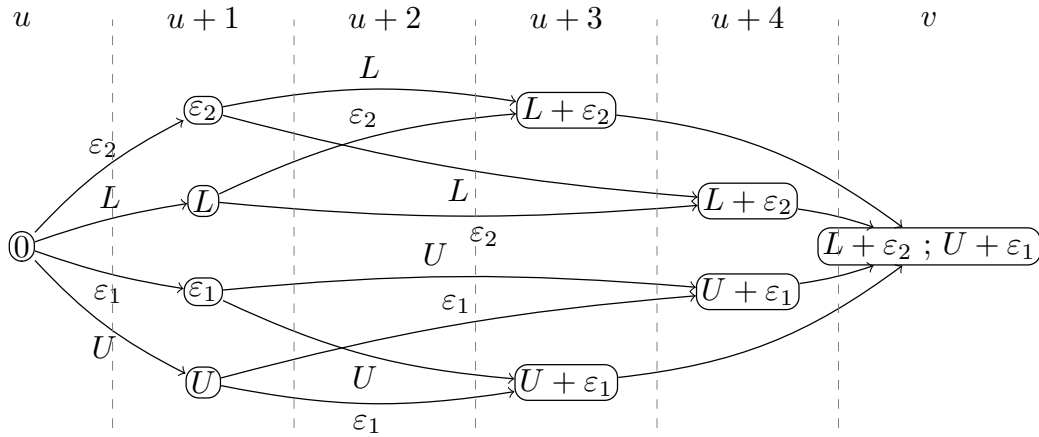


Figure 5.4 – An example of graph \mathcal{UV}_{PD} with 5 periods, $Q = 1$ and $R = 2$

The Figure 5.4 shows how a PD-capacity-constrained sequence S_{uv} can be computed, considering 5 periods and $Q = 1$ and $R = 2$. In the example shown, lots must be produced at $u + 1$. The period $v + 1$ must also be allowed to be in a DTW. Thus there is the choice between producing at $u + 3$ or $u + 4$.

As in Property 9 the time complexity for finding an optimal solution is in $O(T^5)$. \square

Theorem 3. *Assuming that $d_t > 0$ ($\forall t \in T$), the algorithm HMP-DTW-PD gives an optimal solution for the CLSP-MOQ-DTW-PD with concave production and storage costs in time in $O(T^7)$.*

Proof. The proof here is the same as the one presented in Theorem 2. The only difference is that we only have $O(T^2)$ arcs in graph (\mathcal{G}_{PD}), instead of the initial $O(T^4)$ arcs in graph \mathcal{G} . This leads to the new time complexity of $O(T^7)$. \square

5.5 Conclusion

The focus of this paper is a generalization of the capacitated single item lot sizing problem. In this paper, the production levels are bounded by a minimum order quantity and a maximum capacity when a lot is produced. Furthermore, the frequency of the produced lots is modeled by dynamic time windows, recently introduced by [Hellion *et al.* \(2013d\)](#). Between two consecutive production lots, there is at least Q periods and at most R periods. Both production and storage cost functions are concave. We proposed an $O(T^9)$ exact algorithm that generalizes [Hellion *et al.*'s](#) algorithm. A less complex algorithm in $O(T^7)$ is also provided in a case where all the demands are strictly positive. The theoretical complexity of Algorithm HMP-DTW appears difficult to improve with general concave costs. In future work, it would be interesting to determine if this theoretical complexity can be decreased when the cost structure is limited to a fixed cost plus a linear cost. Other assumptions such as the consideration of only linear storage costs ([Okhrin and Richter, 2011](#)) may lead to other improvements in the complexity of the algorithm.

Chapitre 6

Stability contract in the pulp and paper industry : a study case in Canadian forest companies

Abstract

In this paper an industrial case including a papermill and its three suppliers (sawmills) is studied. Sawmills produce lumber and chips from wood, and the paper mill needs these chips to make paper. These sawmills assign a lower priority to the chips market even though the paper mill is their main customer. The focus of the research is therefore on securing the paper mill supply by creating beneficial contracts for both stakeholders. Industrial constraints are taken into account, leading to separate contract designs. Contracts are then tested on various instances and compared to a centralized model that optimizes the total profit of the supply chain. Results show that the decentralized profit with separate contracts is 99.3% the centralized profit, for a fixed demand variance. Difference between centralized and decentralized profit slightly increases with the variance, to reach 3% for a variance of 50%.

6.1 Introduction

Members of the Canadian forest industry agree that the wood market is currently experiencing its worst crisis for a long time. The US is the main final customer for all Canadian wood products. Thus the subprime crisis in 2008 had a disastrous impact on North American investments. In particular, the US lumber demand fell in the following years, causing a lumber price decrease on market from 450\$ (CAD) (near the years 2000) to 298\$ (CAD) (2009) ([Del Degan and Vincent, 2010](#)). The crisis is not the only reason explaining this price decrease. The recent competition of emergent countries such as Chile,

Brazil and China is also highlighted.

In this work, we study an industrial case concerning forest product companies located in the Côte-Nord region in Quebec, Canada. Our interest is focused on the last paper mill of this region, and its three main suppliers, which are sawmills. Sawmills operations and the paper making process are linked together but typically managed independently, leading to a profit waste. Furthermore, the paper mill is the only purchaser for all the wood chip produced by nearby sawmills. The shutting down of this paper mill would have unpredictable consequences.

In order to ensure the collaboration between the main stakeholders, a working group gathering all the forest companies has been created to rethink about their business model (Elleuch *et al.*, 2012). Every week the group meets to discuss about how to satisfy the paper mill demand. Each sawmill must share the informations about the volume of chips to send to the paper mill. These informations include the chips freshness and density, which are related to the tree species. As the sawmills have only one customer, they have to adapt their production planning until the global paper mill needs are satisfied (Elleuch *et al.*, 2012).

Based on this context, we propose the use of beneficial contracts for the stakeholders in order to secure the paper mill supplies. In this way, it becomes possible to deliver the volume of chips needed while better coordinating network operations.

The relationship between decision makers and suppliers becomes one of the most important issues of the supply chain. To be profitable, supply chain activities need to be better coordinated, necessitating stronger interactions between stakeholders (Chan and Chan, 2010a). However, examples of poor collaborations that have disastrous consequences are given by Thomas and Griffin (1996).

The so-called Bullwhip effect is a good example of what can happen without any information sharing. Dejonckheere (2003) studied it from a mathematical and statistical point of view. Lee *et al.* (1997) pointed out the Bullwhip effect in the MIT beer game. De Souza *et al.* (2000) made a large experiment to assess the factors leading to unsuccessful collaborations, highlighting the importance of information sharing.

Collaboration between stakeholders in supply chain is besides a huge subject of interest. Huang *et al.* (2003) presented a review in which they conclude that the number of papers about collaboration exploded between 1996 and 2003.

Camarinha-Matos *et al.* (2009) proposed different classes of collaborative networks reflecting industry's reality. Sahin and Robinson (2002) suggested a review including many industrial references. Many authors have also pointed out the fact that collaborations must be guided in order to be profitable for each supply chain member (Lehoux *et al.*, 2013). For example, Prahinski and Benton (2004) showed that if automotive firms demonstrate

increased willingness to share information, the supplier's commitment to the relationship also increases. The decision maker should have all the stakeholders's information to better optimize a supply chain. However this is usually operationally unrealistic (Giannoccaro and Pontrandolfo, 2004). Consequently, when knowledge is combined, determining the key informations that have to be shared as well as the profit that may occur have been even more studied during the past years. Dyer (1997); Dyer and Chu (2003) studied information sharing in the car industry, and conclude that firms signal their own trustworthiness through a willingness to share information. Datta and Christopher (2011) showed the importance of information sharing between supply chain members to better face uncertainties. Some papers studied the consequences of forecasting error (Zhao and Xie, 2002) and information sharing (Yu *et al.*, 2001) in real case studies.

Since it is an important step in collaboration implementation, practitioners and academics heavily studied the contract design. Elmaghraby (2000) provided an overview of the contract competition in the manufacturing supply chain while Cachon (2003) described different types of contract as coordination mechanisms for the supply chain. More recently, trust has been studied as an important part of the supplier-retailer relationship. Eckerd and Hill (2012) modeled the relation between buyers and suppliers, from an ethical point of view.

Blomqvist *et al.* (2005) found from several case studies that every time the partnership comes to end, a trust rule must have been broken. Trust has also been defined and deeply studied by Doney and Cannon (1997). Researcher as Selnes (1998) demonstrated that enhanced communication contributes significantly to customers satisfaction. The size of the stakeholders may also have an impact on collaboration creation and management, leading to specific leadership and ownership models (Audy *et al.*, 2012).

The context studied in this research concerns the interaction between three sawmills and one paper mill. The paper mill raw material is the chips supplied by the sawmills. In particular, when producing lumber from wood, sawmills generate at the same time chips that can be combined with chemicals to produce pulp and then paper. The paper mill requires a large amount of the wood chips produced by the sawmills, but the latter usually focus on their core business. As a result, sawmills make planning decisions in order to ensure lumber quality rather than chips quality. Chips delivered are therefore variable in terms of volume and quality, leading to higher paper production costs. The purpose of this paper is therefore to secure the paper mill's supply by creating beneficial contracts for both stakeholders.

The paper is organized as follows: Section 6.2 describes the problem and introduces mathematical notations. Section 6.3 exposes the formulations used in the paper, for both

centralized and decentralized cases. Section 6.4 includes the centralized model and a cost analysis. The decentralized model and the contract design are studied in Section 6.5. Contracts are also validated by several experiments on many multi-periods problems with normally distributed demands. Managerial implications and conclusions are given in Section 6.6.

6.2 Problem description

The case studied includes two kinds of stakeholders, a paper mill and three sawmills (see Figure 6.1). All the prices, costs and variables in this paper are based on this industrial context and expressed in m^3 .

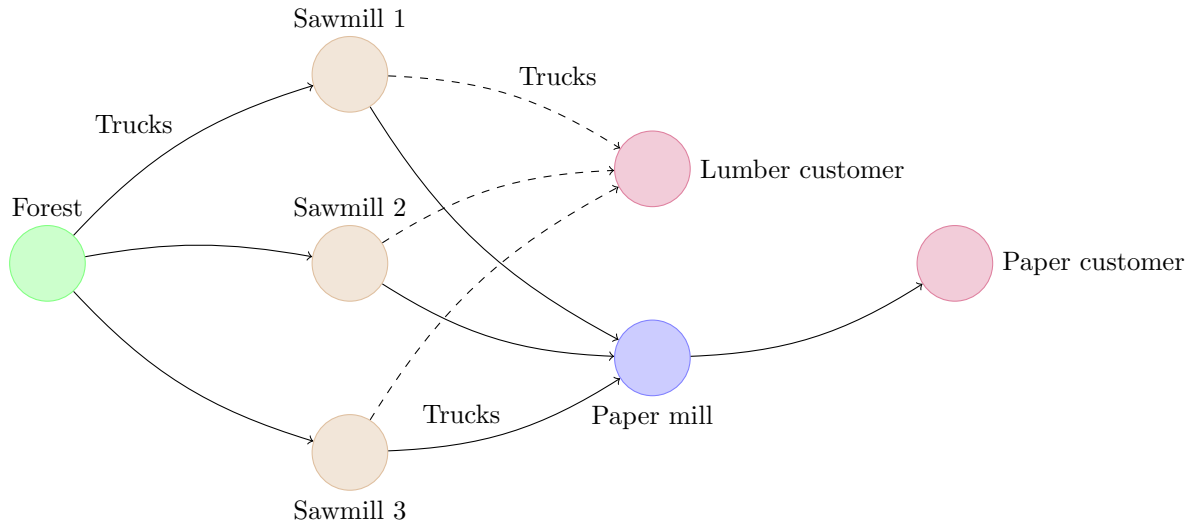


Figure 6.1 – Case study

Sawmill: At each period t , for the sawmill i , wood are delivered by trucks from the forest, at price WP_i . Then the sawmill produces from wood both lumber and chips. The costs for processing wood include harvesting, transportation and sawmilling costs. At each period t , for the sawmill i , lumber are sold to a specific customer at price PB_i , for a maximum demand of $d_{i,t}$, whereas chips are sold to the paper mill at price PC_i . The volume of lumber produced by the sawmill i is constrained by a maximum capacity K_i . The chips produced by the sawmill i has a given quality Q_i . This quality is important for the paper mill and takes value in the real unit interval $[0, 1]$. At the end of each period t , both lumber and chips can be stored at the sawmill i , involving a cost HO_i and HS_i , respectively.

Paper mill: Chips are delivered by trucks from the sawmill to the paper mill. However, these transportation costs are included in the chips price PC_i . At each period t , the paper

mill uses these chips to produce paper at a cost of TC , selling it to a specific customer at price PP , for a maximum demand of dp_t . At the end of each period t , both paper and chips can be stored at the paper mill. The paper mill cost reflects chips purchasing and storage, as well as paper production and storage costs (PC_i , HC_i , TC and HP , respectively).

Sawing capacities: Sawmill capacities are given and known. In particular, the capacity of S_1 and S_3 are the same, and they are both twice the capacity of S_2 . As a result, they can be substituted in the model as K , $0.5K$ and K , as showed in Table 6.3.

Chips quality As said before, the chips produced by the sawmills have a given quality, which is associated with the sawmill. This quality takes value in the real unit interval $[0, 1]$ and depends of the humidity, density, and the tree species. The values exposed here have been given by the paper mill.

Constraint MCP: The sawmill process will conduct to both a certain volume of lumber as main product and a minimum amount of chips as co-product. This minimum is denoted as Minimum Chips Proportion (MCP) in the rest of this paper. We denote as **existing chips** the chips that have been produced by the MCP constraint. The chips that have been produced beyond the MCP constraint are denoted as **additional chips**.

Constraint MCQ: To be effective, the paper production requires a minimum chips quality. More precisely, the average quality of chips must be at least a given value. This value is denoted as Minimum Chips Quality (MCQ) in the rest of this paper.

Variables definition: For convenience, the same variables are used in all the models described in this paper. For each period t :

- Wood arriving at the sawmill is noted $W_{i,t}$. Two products are then produced, lumber and chips.
- At a sawmill i , produced, stored and sold lumber are noted $ZZ_{i,t}$, $IO_{i,t}$ and $Z_{i,t}$, respectively.
- Chips are noted $XP_{i,t}$, afterwards they can be stored ($IS_{i,t}$), and then delivered to the paper mill ($X_{i,t}$). Chips can also be stored at the paper mill ($IC_{i,t}$), and then used in paper production ($XT_{i,t}$).
- At the paper mill, produced, stored and sold paper is noted $YY_{i,t}$, $IP_{i,t}$ and $Y_{i,t}$, respectively.

The general process is summarized in Figure 6.2.

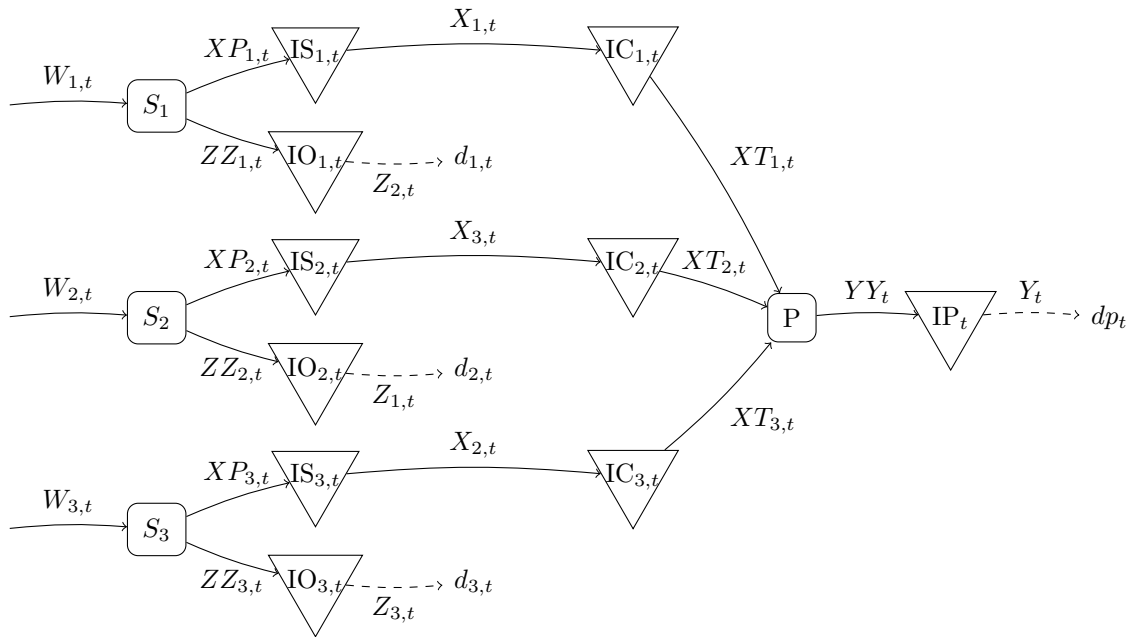


Figure 6.2 – Supply chain modelling

Sawmills, paper mill and customer demands are defined in Table 6.1. Considered costs are presented in Table 6.2. Note that in the models, all costs are constant. Industrial constraints and special notations are listed in Table 6.3. Variables are displayed in Table 6.4.

Table 6.1 – Notation for the mathematical models

P	paper mill
S_1, S_2, S_3	Sawmills
$d_{i,t}$	demand for lumber of the sawmill i at the period t
dp_t	demand for paper of the paper mill at the period t

6.3 Mathematical formulations

In this section, the formulations for both the centralized and the decentralized models are presented. The decentralized formulation is used to test contracts, while the centralized formulation is an upper bound for the profit of the whole supply chain. The latter plays the role of a reference for assessing contracts determined via the decentralized model.

- The centralized model optimizes the supply chain profit, modelling the system as a single decision maker.
- The decentralized model assumes that each stakeholder wants to optimize its own profit *i.e.* each actor is a decision maker. The sum of all stakeholders profit is said to be the profit of the decentralized model.

The profit generated by the centralized and the decentralized model can then be compared.

Table 6.2 – Costs

WP_i	Wood production cost per m^3 of S_i	105\$/ m^3
PC_i	Chip price per m^3 purchased by P at S_i	56\$/ m^3
PB_i	Lumber price per m^3 sold to satisfy $d_{i,t}$ at S_i , at period t	142\$/ m^3
PP	Paper price per m^3 sold to satisfy dp_t at P , at period t	250\$/ m^3
HO_i	Lumber storage cost per m^3 at S_i	0.38\$/ m^3
HC	Chip storage cost per m^3 at P	0.13\$/ m^3
HS_i	Chip storage cost per m^3 at S_i	0.13\$/ m^3
HP	Paper storage cost per m^3 at P	0.58\$/ m^3
TC	Paper production cost per m^3 by P , using chips	156\$/ m^3

Table 6.3 – Industrial constraints

Q_1	Chips quality produced by the sawmill S_i	0.6
Q_2	Chips quality produced by the sawmill S_i	0.75
Q_3	Chips quality produced by the sawmill S_i	0.98
MCQ	Chips quality required to transform chips into paper	0.82
MCP	Minimum chips proportion produced by sawmill.	10%
K_1	Sawing Capacity of the sawmill S_1	K
K_2	Sawing Capacity of the sawmill S_2	0.5K
K_3	Sawing Capacity of the sawmill S_3	K

Table 6.4 – Variables

$W_{i,t}$	Amount of wood produced by S_i , at period t
$XP_{i,t}$	Amount of chips produced by S_i , at period t
$X_{i,t}$	Amount of chips provided by S_i to P , at period t
$XT_{i,t}$	Amount of chips uses to make paper by P , originally provided by S_i , at period t
$Z_{i,t}$	Amount of lumber sold by S_i , at period t , in response to demand $d_{i,t}$
$ZZ_{i,t}$	Amount of lumber produced by S_i , at period t
Y_t	Amount of paper sold by P , at period t , in response to demand dp_t
YY_t	Amount of paper produced by P , at period t
$IO_{i,t}$	Amount of lumber stored by S_i , at the end of the period t
$IS_{i,t}$	Amount of chips stored by S_i , at the end of the period t
$IC_{i,t}$	Amount of chips stored by P , originally provided by S_i , at the end of the period t
IP_t	Amount of paper stored at P , at the end of the period t
$L_{i,t}$	Amount of chips lost for the sawmill i and the paper mill, at period t
Lp_t	Amount of chips lost for the paper mill, at period t

6.3.1 Centralized linear program \mathcal{C}

The centralized model \mathcal{C} aims at maximizing the whole supply chain profit *i.e.*, the sum of all actors profit, for all products (wood, lumber and paper).

Since chips are necessarily produced during the sawmilling process (MCP constraint), sawmills are allowed to throw away a part of them (*e.g.*, if there is no demand for this co-product or if the quality obtained is too poor to be used in other processes). Consequently, chips flow equations include some additional variables : $L_{i,t}$ and Lp_t , which are the chips lost for the sawmill i and the paper mill, respectively.

\mathcal{C} can be defined as follows:

$$\begin{aligned} \max \quad & \underbrace{\sum_t^T \sum_i^S (Z_{i,t} \times PB_i)}_{\text{lumber sale}} + \underbrace{\sum_t^T (Y_t \times PP)}_{\text{paper sale}} - \underbrace{\sum_t^T \sum_i^S (IO_{i,t} \times HO_i + IS_{i,t} \times HS_i)}_{\text{lumber and chips storage at sawmills}} \\ & - \underbrace{\sum_t^T \sum_i^S (W_{i,t} \times WP_i)}_{\text{wood production}} - \underbrace{\sum_t^T \left(\sum_i^S (IC_{i,t} \times HC_i) + IP_t \times HP \right)}_{\text{paper and chips storage at the paper mill}} - \underbrace{\sum_t^T \sum_i^S (XT_{i,t} \times TC)}_{\text{pulp and paper production}} \end{aligned} \quad (6.1)$$

s.t.

$$Z_{i,t} \leq d_{i,t} \quad \forall i \in S, \forall t \in T \quad (6.2a)$$

$$W_{i,t} \leq K_i \quad \forall i \in S, \forall t \in T \quad (6.2b)$$

$$W_{i,t} = XP_{i,t} + ZZ_{i,t} \quad \forall i \in S, \forall t \in T \quad (6.2c)$$

$$W_{i,t} \times MCP \leq XP_{i,t} \quad \forall i \in S, \forall t \in T \quad (6.2d)$$

$$XP_{i,t} + IS_{i,t-1} + L_{i,t} = IS_{i,t} + X_{i,t} \quad \forall i \in S, \forall t \in T \quad (6.2e)$$

$$ZZ_{i,t} + IO_{i,t-1} = IO_{i,t} + Z_{i,t} \quad \forall i \in S, \forall t \in T \quad (6.2f)$$

$$Y_t \leq dp_t \quad \forall t \in T \quad (6.3a)$$

$$\sum_i^S XT_{i,t} = YY_t \quad \forall t \in T \quad (6.3b)$$

$$\sum_i^S (XT_{i,t} \times Q_i) \geq YY_t \times MCQ \quad \forall t \in T \quad (6.3c)$$

$$X_{i,t} + IC_{i,t-1} + Lp_t = IC_{i,t} + XT_{i,t} \quad \forall i \in S, \forall t \in T \quad (6.3d)$$

$$YY_t + IP_{t-1} = IP_t + Y_t \quad \forall t \in T \quad (6.3e)$$

$$X_{i,t}, XP_{i,t}, W_{i,t}, IO_{i,t}, IS_{i,t}, Z_{i,t}, ZZ_{i,t} \in \mathbb{R} \quad \forall i \in S, \forall t \in T \quad (6.4a)$$

$$X_{i,t}, IP_{i,t}, IC_{i,t}, XT_{i,t}, Y_{i,t}, YY_{i,t} \in \mathbb{R} \quad \forall i \in S, \forall t \in T \quad (6.4b)$$

The objective function is defined by (6.1) and aims at maximizing the sales of lumber and paper, while minimizing all production, transformation and storage costs.

Constraints from (6.2a) to (6.2f) are the sawmill constraints. The sawmills cannot sell more lumber than the market demand (6.2a). The sawmilling process is constrained by the sawmill capacity (6.2b). There is no waste nor product creation during the sawmilling process (6.2c). Since it is impossible to only get lumber from trees (*i.e.*, divergent process), a minimum amount of chips have to be produced (6.2d). Equation (6.2e) concerns the sawmill chip flow constraint, after sawmilling. Equation (6.2f) is the sawmill lumber flow constraint, after sawmilling.

Constraints from (6.3a) to (6.3e) reflect the paper mill constraints. The paper mill cannot sell more paper than the market demand (6.3a). One m^3 of chips is used for producing one m^3 of paper (6.3b). Paper quality must be at a minimum given quality (6.3c). Equation (6.3d) is the paper mill “input” chip flow constraint, before the production process. Equation (6.3e) is the paper mill “output” paper flow constraint, after the paper making process.

The range of values for sawmills and paper mill variables are defined by constraints (6.4a) and (6.4b), respectively.

This linear program assumes that there is a single decision maker, aiming at maximizing the total profit. The next section proposes decentralized linear programs, to optimize planning decisions of each stakeholder.

6.3.2 Sawmill decentralized linear program

Each sawmill tries to maximize its own profit generated from both lumber and chips sale. Therefore, from the sawmill point of view, the relevant constraint to consider are constraints (6.2a) to (6.2f), plus the variables definition constraint (6.4a). In order to optimize its profit, a single sawmill i has to solve the following mathematical problem:

$$\begin{aligned}
 \max \quad & \overbrace{\sum_t^T (Z_{i,t} \times PB_i)}^{\text{lumber sale}} + \overbrace{\sum_t^T X_{i,t} \times PC_i}^{\text{chips sale}} \\
 & - \underbrace{\sum_t^T (W_{i,t} \times WP_i)}_{\text{wood production}} - \underbrace{\sum_t^T (IO_{i,t} \times HO_i + IS_{i,t} \times HS_i)}_{\text{lumber and chips storage}} \tag{6.5}
 \end{aligned}$$

s.t.

$$(6.2a), (6.2b), (6.2c), (6.2d), (6.2e), (6.2f), (6.4a)$$

The objective (6.5) tries to maximize the profit *i.e.*, lumber and chips sales, minus wood production and storage costs.

Here it is considered that the paper mill buys all chips produced by the sawmills. In a following section the relevance of such a model is discussed.

6.3.3 paper mill decentralized linear program

In a decentralized supply chain, the paper mill focuses on improving its own profit. In this case, the relevant constraint to take into account are constraint (6.3a) to (6.3e), as well as the variables definition constraint (6.4b). In order to optimize its profit, the paper mill has to solve the following mathematical problem:

$$\begin{aligned} \max \quad & \underbrace{\sum_t^T (Y_t \times PP)}_{\text{paper sale}} - \underbrace{\sum_i^S \sum_t^T o_{i,t} \times PC_i}_{\text{chips purchasing}} \\ & - \underbrace{\sum_t^T \left(\sum_i^S (IC_{i,t} \times HC_i) + IP_t \times HP \right)}_{\text{paper and chips storage}} - \underbrace{\sum_t^T \sum_i^S (XT_{i,t} \times TC)}_{\text{pulp and paper production}} \end{aligned} \quad (6.6)$$

(6.3a), (6.3b), (6.3c), (6.3d), (6.3e), (6.4b)

The objective (6.6) aims at maximizing the profit *i.e.*, paper sales minus costs related to chips purchase and storage costs and paper production.

Here it is considered that the chips ordered by the paper mill have been produced and are available for sale. In a following section the relevance of such a model is discussed.

6.3.4 Discussion on profit

The profitability of the different products is first analysed, using values given in Table 6.2. For that purpose, all the products production profits (lumber, chips and paper) are calculated considering a given stakeholder. The considered stakeholders are the supply chain (if the model is centralized), the sawmill and the paper mill.

When looking at the whole **supply chain**, an interval for the **paper** profit is computed. The lower bound is set as the paper is fully produced from additional chips. The upper bound is computed based on the hypothesis that the paper is fully produced from existing chips.

The **paper** profit generated from additional chips can be calculated as follows :

$$\begin{aligned}\mathbf{profit} &= \text{paper sale} - \text{wood processing} - \text{pulp and paper production} \\ &= 250 - 105 - 156 = \mathbf{-11\$/m^3}\end{aligned}$$

Sawmilling when paper is the only product is not profitable for the supply chain.

The **paper** profit generated from existing chips can be calculated as follows :

$$\begin{aligned}\mathbf{profit} &= \text{paper sale} - \text{pulp paper production} \\ &= 250 - 156 = \mathbf{94\$/m^3}\end{aligned}$$

The MCP constraint forces to produce a minimum amount of chips. Considering that the MCQ is satisfied, producing paper from existing chips is profitable.

We can therefore estimate that profitability for producing paper from one m^3 is between 94\$ and -11\$.

Remark : consider that MCQ is satisfied. To be profitable for the supply chain, the paper must be made with a given minimum proportion of existing chips (*i.e.* produced by the MCP constraint). This value can be calculated. Consider x the minimum proportion of existing chips.

$$\begin{aligned}\mathbf{profit} &= \text{existing chips cost} \times x + \text{additional chips cost} \times (1 - x) \leq \text{paper profit} \\ &= 0 \times x + 105 \times (1 - x) \leq 94 \Leftrightarrow x \geq \frac{11}{105}\end{aligned}$$

The existing chips proportion is $\frac{11}{105} = 10.5\%$. This means that for each $0.105m^3$ of existing chips, it is profitable to produce an additional $0.895m^3$ of chips to make $1m^3$ of paper.

The products profitability for the **sawmills** can be estimated as follows :

Profit for **chips** :

$$\begin{aligned}\mathbf{profit} &= \text{chips sale} - \text{wood processing} \\ &= 56 - 105 = \mathbf{-49\$/m^3}\end{aligned}$$

Wood only used for chips production is not profitable for the sawmills. In a decentralized model, without any incentive, a sawmill has no interest to produce more chips than the MCP constraint.

Profit for **lumber**, considering the MCP constraint

$$\begin{aligned}\mathbf{profit} &= \text{lumber sale} \times (1 - MCP) - \text{wood processing} \\ &= 142 \times 90\% - 105 = \mathbf{22.8\$/m^3}\end{aligned}$$

Focusing on sawmilling to produce the maximum amount of lumber is profitable, even without the chips sales.

If we then look at the **paper mill**, the profit for **paper** can be estimated as follows:

$$\begin{aligned}\mathbf{profit} &= \text{paper sale} - \text{chips purchase} - \text{paper production} \\ &= 250 - 56 - 156 = \mathbf{38\$/m^3}\end{aligned}$$

Considering that the MCQ is satisfied, chips are profitable for the paper mill. In a decentralized model, paper mill has interest to produce and sell as much as paper as possible, while the MCQ holds.

Since in this study, cost are known and constant, the above properties hold in the whole paper.

6.4 Centralized model analysis

This section investigates an unique decision maker using the centralized model. The special case considering constant demand is also studied.

6.4.1 Constant demands

All the demands are now supposed to be constant. Taking this assumption allows to understand the properties that hold in the centralized case. In this context, there is no interest to store any product. In fact, the problem \mathcal{C} can be seen as a succession of identically and separated mono-period problems. Since costs are known and constant, the properties of the Section 6.3.4 still hold. The key decision for the supply chain is to know what optimal quantity of chips should be used for paper, for each quality. To calculate that, a mathematical program can be written, focusing on chips variables. It is a mono-period mathematical program called \mathcal{P}_L .

Mathematical program \mathcal{P}_L

\mathcal{P}_L is found out by simplification of \mathcal{C} . As said previously, there is no interest to store any product, which leads to an elimination of all the inventory variables. Since the resulting mathematical program is mono-period, the flow equations can be simplified in variables equivalences. For instance, the flow constraint 6.3d which is :

$$X_{i,t} + IC_{i,t-1} \geq IC_{i,t} + XT_{i,t} \quad \forall i \in S, \forall t \in T$$

It can be turned into :

$$X_{i,t} \geq XT_{i,t} \quad \forall i \in S$$

In fact, it is a mono-period problem, and therefore the paper mill cannot store chips, it has no interest to buy additional quantity, which are destined to be thrown away. As a result :

$$X_{i,t} = XT_{i,t}$$

All these simplifications lead to keep the following variables, $XP_{i,t}$ and $X_{i,t}$. These variables represent the quantity of chips a sawmill should produce, and the quantity of

chips that should be delivered to the paper mill.

After these variable eliminations, the focus is on the objective function. Discarding all the storage costs, considering mono-period problem, the objective function of \mathcal{C} becomes :

$$\max \quad \overbrace{\sum_i^S (Z_{i,t} \times PB_i)}^{\text{lumber sale}} + \overbrace{Y_t \times PP}^{\text{paper sale}} - \overbrace{\sum_i^S (W_{i,t} \times WP_i)}^{\text{wood production}} - \overbrace{\sum_i^S (XT_{i,t} \times TC)}^{\text{pulp paper production}}$$

in which $Y_{i,t}$ and $XT_{i,t}$ can be turned into $X_{i,t}$ (for the reason explained above). Furthermore $W_{i,t}$ can be replaced by $XP_{i,t}$, because WP_i is the cost for producing any additional chips (beyond the MCP). Also, the term $\sum_i^S (Z_{i,t} \times PB_i)$ can be discarded because lumber demand is not linked to chips flows. This leads to another objective function, which is :

$$\max \quad \overbrace{\sum_i^S X_{i,t}(PP - TC)}^{\text{profit with existing chips}} - \overbrace{\sum_i^S XP_{i,t} \times WP_i}^{\text{cost to produce additional chips}}$$

The constraints to consider are therefore the MCP, the MCQ, the sawmill capacity, and the paper demand.

For a single period t , the mono-period mathematical program \mathcal{P}_L is defined below:

$$\max \quad \sum_i^S X_{i,t}(PP - TC) - \sum_i^S XP_{i,t} \times WP_i \quad (6.7)$$

s.t.

$$XP_{i,t} \geq \min \left\{ \frac{\text{MCP}}{1 - \text{MCP}} \times d_{i,t}, \text{MCP} \times K_i \right\} \quad \forall i \in S \quad (6.8a)$$

$$\sum_i^S (X_{i,t} \times Q_i) \geq \sum_i^S (X_{i,t} \times \text{MCQ}) \quad (6.8b)$$

$$XP_{i,t} \leq K_i \quad \forall i \in S \quad (6.8c)$$

$$\sum_i^S X_{i,t} \leq dp_t \quad (6.8d)$$

$$X_{i,t} \leq XP_{i,t} \quad \forall i \in S \quad (6.8e)$$

$$X_{i,t}, XP_{i,t} \in \mathbb{R}^+ \quad \forall i \in S \quad (6.8f)$$

The amount of lumber produced by a sawmill is bounded either by its capacity, or by the lumber demand. Considering the MCQ, the minimum amount of chips produced by a sawmill is a fraction of either its capacity or the lumber demand. This minimum amount is

given by the constraint (6.8a). Even if this constraint is not linear, it can be transformed into a linear constraint, adding an additional variable. Thus this mathematical program can be written as a linear program.

Constraint (6.8b) (MCQ) verifies that the quality of the chips used is at least the minimum paper quality. The chips produced are also bounded by the capacity of the sawmills (Constraint (6.8c)). The constraint (6.8d) bounds the amount of chips used to the paper demand. Finally, constraints (6.8e) and (6.8f) ensure that the variables are well defined and linked.

The mathematical program \mathcal{P}_L gives the optimal amount of chips produced and used for a single period. Even in a multi-period problem, in the case where demands are constant, the optimal amount of chips produced and used can be provided either by \mathcal{P}_L or \mathcal{C} .

6.5 Decentralized model with contracts

In this section, different decentralized scenarios are investigated. Demands are not constant anymore and may vary. All the problems considered are therefore multi-periods. However, the mono-period results above are used to design contracts at the end of this section.

This decentralized problem \mathcal{D} aims at maximizing each stakeholder's own profit. In this purpose, at each period t , each stakeholder successively optimizes its own planning on a rolling horizon of H periods, *i.e.* from t to $t + H - 1$.

6.5.1 Imbalance of extreme decentralized cases

The first two scenarios show that in a decentralized case, the lack of regulation or contract leads to a profit loss for the supply chain. A new variable $o_{i,t}$ is introduced to reflect the amount of chips ordered by the paper mill at the sawmill i , at period t .

The two scenarios are :

- A decentralized supply chain where the paper mill is dominant, *i.e.* the paper mill chooses the quantity of chips to buy from the sawmills (Algorithm 3).
- A decentralized supply chain where the sawmills are dominant, *i.e.* the sawmills choose the quantity of chips to produce, and then the paper mill chooses to buy an amount lesser or equal than the available amount of chips (Algorithm 4).

Let the paper mill be the dominant (Algorithm 3). At each period, the paper mill orders a certain amount of chips $o_{i,t}$ that the sawmill i has to satisfy (the capacity of the sawmill is respected, *i.e.* $\sum_{t'=1}^t o_{i,t'} \leq t \times K_i$). A unique chips production is not profitable for the sawmills, so any chips quantity ordered beyond the MCP constraint can be a profit waste. The paper is profitable for the paper mill, so depending on the paper demand, a

Algorithm 3 A decentralized algorithm with dominant paper mill

for each period t : **do**
 The paper mill optimizes its planning on the rolling horizon H .
 Then it sends H orders $o_{i,t}$ for the next H periods for each sawmill i , $\sum_{t'=1}^t o_{i,t'} \leq t \times K_i$.
for each sawmill i : **do**
 The sawmill i optimizes its own planning regarding the amount of chips $X_i = o_{i,t}$ to provide to the paper mill.
end for
end for

large quantity of chips could be ordered. Moreover, since the paper profit for the whole supply chain is also negative (see Section 6.3.4), the supply chain profit overall decreases.

Algorithm 4 A decentralized algorithm with dominant sawmills

for each period t : **do**
for each sawmill i : **do**
 On a rolling horizon of H periods, the sawmill i optimizes its planning, producing an amount of chips $XP_{i,t}$. The paper mill buys a quantity of chips $X_{i,t}$ to the paper mill, $X_{i,t} \leq XP_{i,t}$.
end for
 On a rolling horizon of H periods, the paper mill optimizes its planning based on the $X_{i,t}$ provided by the sawmills.
end for

If the sawmills are dominant (Algorithm 4), they produced a given amount of chips. Afterwards, the paper mill chooses what quantity to buy. As said previously, sawmills must respect the MCP constraint, forcing them to produce a minimum quantity of chips. Also, any quantity of chips produced beyond the MCP is a profit waste for a sawmill. Thus the sawmills will only produced the minimum quantity of chips mandatory. Indeed there is no incentive for the sawmills to produce more than the MCP.

Remind that the MCQ has to be respected when paper is produced, involving that certain quantity of chips may be thrown away. However, it would be profitable for the whole supply chain to produce additional high quality chips to produce more paper, using existing chips (see Section 6.3.4). This situation does not allow additional chips production, leading to a non-optimal global solution.

To conclude, both these extreme decentralized situations are not convenient. In the next section, a decentralized scenario with contract is investigated.

6.5.2 Contract design

The next scenario proposed a more balanced and flexible decentralized supply chain dynamic. The previous mathematical program \mathcal{P}_L defines the optimal quantity of chips to order, but only in the case where demands are known and constant. Considering a multi-period problem with varying demands, contracts have to be fixed and constant on the whole horizon in order to guide the different stakeholders in the decisions they make.

Hellion *et al.* (2013d) developed the stability contracts for securing a retailer's supply while ensuring a beneficial relationship for the stakeholders (retailer and suppliers). These stability contracts initially constrained the retailer order in two ways :

- by defining minimum and maximum bounds on orders amounts (denoted L and U , respectively);
- by defining dynamic time windows for each orders.

Based on Hellion *et al.*'s work, stability contracts are used for better satisfying paper mill needs. However, since each period corresponds to a possible delivery, the time discretization does not allow to define any dynamic time windows.

Consequently, the stability contract must define L_i and U_i for each sawmill i . Then, for each period t , $L_i \leq o_{i,t} \leq U_i$. L_i and U_i values are computed according to the sawmills capacity and lumber demand. Sawmills must also provide a quantity of chips $X_{i,t}$ such as $X_{i,t} = o_{i,t}$. Algorithm 5 presents how the supply chain works in the multi-period decentralized context.

Algorithm 5 The decentralized procedure \mathcal{D}

for each sawmill i : **do**

The paper mill and the sawmill i agree on both L_i and U_i bounds.

end for

for each period t : **do**

The paper mill optimizes its planning on the rolling horizon H , calculating the values $o_{i,t'}$, $t' \in \{t \dots t + H - 1\}$, such as $L_i \leq o_{i,t'} \leq U_i$.

The paper mill sends H orders $o_{i,t'}$ at each sawmill i .

for each sawmill i : **do**

The sawmill i optimizes its own planning taking into account the paper mill's orders.

The sawmill i provides an amount of chips $X_{i,t}$ to the paper mill, such as $X_{i,t} = o_{i,t}$, for every H next periods.

end for

end for

Remark : Since demands are not constant in this section, the average demand is used to calculate the contract parameters. The average demand of paper is noted dp . Furthermore, the average demand for lumber at each sawmill i is noted d_i .

The output values of \mathcal{P}_L are the optimal quantities of chips to order so as to satisfy the demand for a single period, a multi-period problem when demands are constant. However, in a decentralized multi-period problem with varying demands, this can be seen as a lower bound of chips for the paper mill at each period, without decreasing the profit of the supply chain. Then the output values X_i of \mathcal{P}_L can be assigned to L_i .

Definition : \min_i is the minimum of chips produced by the sawmill i , according to its lumber market and its capacity.

$$\min_i = \min \left\{ \frac{\text{MCP}}{1 - \text{MCP}} \times d_{i,t}, \text{MCP} \times K_i \right\}$$

Remind that if low quality chips are available, it is profitable for the supply chain to produce additional high quality chips to make paper (see Section 6.3.4). Thus the upper bound U_i should include all chips produced by the sawmill i , according to the MCP constraint. The likely insufficient quality of this mix of chips must be improved by producing additional high quality chips, in order to reach the required quality (MCQ). Formally, U_i defines for each sawmill i the minimum quantity of chips satisfying :

- $U_i \geq \min_i \forall i \in S$;
- $\sum_i^S U_i Q_i \geq \sum_i^S U_i \text{MCQ}$.

This can be calculated by a similar mathematical program than \mathcal{P}_L . Since the demands are no longer constant, it would be profitable to order more chips to later satisfy a large paper demand. Furthermore, U_i values can be computed using \mathcal{P}_L , but the constraint (6.8d) ensuring that the amount of chips provided does not exceed the paper demand for one period, must be discarded. This new mathematical program is called \mathcal{P}_U and is defined as follows :

$$\max \sum_i^S X_{i,t} (PP - TC) - \sum_i^S X P_{i,t} \times W P_i$$

s.t. (6.8a) (6.8b) (6.8c) (6.8e) (6.8f)

The values for U_i are computed based on the output values X_i of this mathematical program.

Remark : Since the only difference between \mathcal{P}_U and \mathcal{P}_L is the constraint (6.8d), if dp is large, the output from \mathcal{P}_U and \mathcal{P}_L are the same. Formally, if $dp \geq \sum_{i \in S} U_i^*$, the constraint (6.8d) has no impact on the formulation and consequently $L_i^* = U_i^*, \forall i \in S$.

To summarize, for each sawmill i , the values for L_i and U_i can be computed using \mathcal{P}_L and \mathcal{P}_U , respectively. Moreover, the solutions of the mathematical programs \mathcal{P}_L and \mathcal{P}_U present some properties :

Property 12. *Optimal solution of \mathcal{P}_U determines the value U_i to its \min_i , except for a set of chips S (which only include the chips with the best quality).*

The solution follows the form below. Sort all the sawmills i by Q_i in descending order :

- $\forall i \in \{1; \dots; k-1\}, U_i = K_i$
- $U_k = \frac{\sum_{i=1}^{k-1} U_i(MCQ - Q_i)}{Q_k - MCQ}$
- $\forall i \in \{k+1; \dots; S\}, U_i = \min_i$

Remark : *the proof explains how to find k .*

Proof. The proof is given in the appendix. □

The property 12 and its proof leads to a simple procedure \mathcal{R}_U (Algorithm 6) which computes the optimal solution of \mathcal{P}_U . For each sawmill i the value of U_i can be assigned via the output of the procedure \mathcal{R}_U .

Algorithm 6 procedure \mathcal{R}_U computing the optimal solution of \mathcal{P}_U

```

for each  $i$  from 1 to  $S$  do
  calculate  $U_k$  as if  $i = k$ 
   $k$  is the first that satisfies  $U_k \leq K_k$ 
end for
calculate  $U_k^*$ .

```

\mathcal{P}_L presents some properties as well. The MCQ is a constraint in the formulation, and since all the chips have the same price, an infinity of optimal solutions exists. However these solutions do not have the same final quality of chips, because MCQ is a constraint, not an objective. The purpose is to find, among the optimal solutions, the solution with the best quality of paper.

The following property (property 13) defines the structure of an optimal solution for \mathcal{P}_L , the one with the best quality of paper.

Property 13. *Sort all sawmills i by Q_i in descending order. The optimal solution of \mathcal{P}_L which maximizes the average paper quality is the following form:*

- $\forall i \in \{1; \dots; k-1\}, L_i = K_i$
- $L_k = \frac{dp(MCQ - Q_m) + (Q_m - Q_i)(\sum_{i=1}^{k-1} K_i + \sum_{i=k+1}^{m-1} \min_i)}{Q_k - Q_m}$
- $\forall i \in \{k+1; \dots; m-1\}, L_i = \min_i$
- $L_m = \{dp - \sum_{j \neq m}^{j \in S} L_j\}$
- $\forall i \in \{m+1; \dots; S\}, L_i = 0$

Remark : *the proof explains how to find k and m .*

Proof. The proof is given in the appendix. \square

Property 13 and its proof leads to a simple procedure \mathcal{R}_L (Algorithm 7) which computes the optimal solution of \mathcal{P}_L . For each sawmill i , the value of L_i can be assigned to the output of the procedure \mathcal{R}_L .

For each sawmill i , the contract is created by assigning at L_i and U_i the outputs of \mathcal{R}_L and \mathcal{R}_U , respectively.

Algorithm 7 procedure \mathcal{R}_L computing the solution with best paper quality among the optimal solutions of \mathcal{P}_L

```

for each  $i$  from 1 to  $S$  do
  calculate  $L_k$  as if  $i = k$ 
   $k$  is the first that satisfy  $L_k \leq K_k$ 
end for
for each  $i$  from  $k + 1$  to  $S$  do
  with the value of  $L_k$ , calculate the final paper quality
end for
keep the  $m$  that maximize the paper quality.
calculate  $L_k^*$ .
calculate  $L_m^*$ .

```

6.5.3 Experiments

In this section, decentralized procedure \mathcal{D} with stability contracts is compared to the centralized procedure \mathcal{C} , using instances with demands following a normal distribution. For each sawmill i , stability contracts are based on parameters L_i and U_i computed by procedures \mathcal{R}_L and \mathcal{R}_U

Different groups of instances are generated, and they differ by lumber and paper average demands. The capacities of the sawmills, which are K , $0.5K$ and K , are fixed to 90, 45 and 90, respectively. The lumber demand can be much larger than the paper demand, and inversely. Also, the capacity of the sawmills can be significant or not. That leads to 4 groups, as shown in Table 6.5, that encompass the average demand for each stakeholder or product. In each group, 20 different instances are generated.

These instances are tested in two experiments. First, the profit of each stakeholders is evaluated using a given fixed variance (20%) (Section 6.5.3). The global profit is then investigated for a variance varying, from 5% to 50%, with a step of 5 (section 6.5.3).

Table 6.5 – Instance parameters

Group	$d_{1,t}$	$d_{2,t}$	$d_{3,t}$	dp_t
G1	60	30	60	10
G2	30	15	30	50
G3	120	60	120	20
G4	120	60	120	150

Comparison of each stakeholder's profit using fixed variance

For each group, the variance is fixed at 20% of the average demand. Results are displayed in Table 6.6.

Table 6.6 – Supply chain member's profit with fixed variance

Group	Scenario	Average profit for the whole group					Chips proportion		
		P	S_1	S_2	S_3	Total	S_1	S_2	S_3
G1	\mathcal{C}	46 450	76 287	38 078	75 864	236 679	0.10	0.10	0.10
	\mathcal{D}	46 436	76 170	37 774	73 599	233 980	0.10	0.10	0.11
G2	\mathcal{C}	47 956	38 195	19 041	27 577	132 769	0.10	0.10	0.16
	\mathcal{D}	47 953	37 775	18 815	27 586	132 129	0.10	0.10	0.16
G3	\mathcal{C}	92 225	102 912	51 513	97 277	343 926	0.10	0.10	0.11
	\mathcal{D}	92 244	102 856	51 516	93 329	339 945	0.10	0.10	0.12
G4	\mathcal{C}	129 462	103 037	51 504	65 380	349 383	0.10	0.10	0.16
	\mathcal{D}	129 473	103 038	51 497	65 319	349 327	0.10	0.10	0.16

Results shows that the profit obtained with the proposed method are close to the ones generated using the centralized method (lower than 1%). Certain values are even better in the proposed method than with the centralized approach, which can be explained by a different profit distribution. However, all values are close to their optimum. The overall profit of the decentralized supply chain is 99.3% the centralized profit. In Table 6.6, poor local results (more than 1% lower than the centralized profit) are displayed in bold. Those results correspond to the cases where the paper demand is low for sawmills having the best chips quality. Decentralized decision making for groups G2 and G4 is very close to the optimal. Considering that these two groups face a large paper demand, contracts seems to be particularly effective in that case.

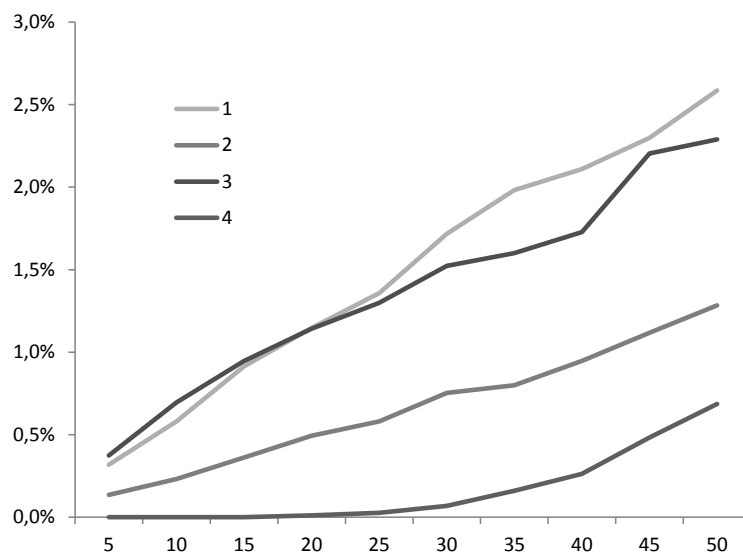
The last three columns of the table display the proportion of chips produced for each sawmill. When the value is greater than 0.1, additional chips have to be produced. The chips proportion of sawmill S_3 is 0.16 for G2 and G4, which are the two groups facing a

large demand for paper.

Comparison of the whole supply chain profit when the variance is increased

The next experiment aims at comparing the profit of the whole supply chain generated from the centralized and decentralized model using an increasing variance. In particular, the variance starts at 5, up to 50, and increases by a step of 5. For each variance and each group, 100 instances are solved using \mathcal{C} and \mathcal{D} . In total, 800 instances are solved.

Figure 6.3 – Comparison of the whole supply chain profit, for each group of instances



For each group of instances, the profit difference increases with the variance. Concerning the decentralized model, contracts are generated based on the average demand for paper and lumber. However, the average demand are less and less indicative as variance increases. For a variance of 50%, the profit difference almost reach 3% for the worst groups G1 and G3. On the other hand, considering the group G2 and G4, even with the largest variance, the profit difference between the centralized and the decentralized system is at 1% and 0.5%, respectively.

In summary, the experiments based in stakeholder's profit show that the supply chain profit is correctly distributed among the stakeholders because their profits are close between the centralized and the decentralized model. The experiment using an increasing variance demonstrates that stability contracts can optimize the supply chain profit, even with large variances on the demands. However, to be effective, these contracts need a good assessment of the current average demand of the market. Furthermore, each change in the average demand should involve a modification of the contract terms to ensure fair distribution of the supply chain profit. By using stability contracts, forest product companies

of the Côte-Nord region could therefore both better respond to the paper mill demand while improving coordination between supply chain operations.

6.6 Conclusion

In this paper, we investigate the case of a paper mill and its three suppliers, which are sawmills. All these companies are located in the Côte-Nord region, in Quebec, Canada. The purpose is on securing the paper mill supplies by creating beneficial contracts for all stakeholders. The paper mill and the sawmills are modeled, with their costs, capacities, and chips quality. Two industrial constraints are considered, reflecting the divergent production process and the quality requirements for paper production, leading to a specific problem. These constraints are then used to design particular contracts between the paper mill and each sawmill.

Two contexts are presented and compared : the centralized environment and the decentralized decision making process. Lumber and chips market are considered in the models to take into account industry's reality. Algorithm used to create the specific contracts are provided, leading to a practical solution. An experimental study shows that the profit of the decentralized model, managed by the contracts, is 99.3% the centralized profit. Furthermore, each individual profit under contract is close to the centralized optimal solution. Another experiment shows the efficiency of the stability contracts, even for largest demand variances.

The experiments showed that if a good assessment of the average demand (for both paper and lumber market) is conducted, the stability contracts could be effective for the whole supply chain, as well as for each stakeholder. The key of the problem is therefore to get a good assessment of the future average demand while determining and negotiating the contracts terms efficiently. In that purpose, the Côte-Nord stakeholders should share the necessary informations to get the best possible demand forecast.

This case study has served well to propose a contract design methodology. Generalization of the methodology to divergent process industries such as those found in refinery or agricultural industry could be done.

6.7 Appendix

Proof. of property 12.

This proof is presented in three parts. First, we demonstrate that there always exists a solution that contains only one index k , such as U_k^* is a fractional value, *i.e.* $U_k^* > \min_k$

and $U_k^* < K_k$. Then, we show how to calculate U_k^* . Finally (with the sawmills sorted by Q_i in descending order), we prove that this formula applied on each i such as $i < k$ leads to $U_i > K_i$.

Preliminary remark : $Q_k > \text{MCQ}$, because there is no reason to produce chips below the MCQ.

Say that it exists an optimal solution such as there is two fractional value, say U_a^* and U_b^* , with $a < b$. In this case we can reallocate the value $\alpha = U_b^* - \min_b$ from b to a . If $U_a^* + \alpha \geq K_a$, assign at U_a^* the value K_a , reassign the rest in U_b^* and thus there is only one fractional value, which is U_b^* . Otherwise, if $U_a^* + \alpha < K_a$, there is only one fractional value, which is U_a^* , and the chips quality of the new solution is better.

The value U_k^* is found below :

$$\sum_{i=1}^S U_i Q_i = \sum_{i=1}^S U_i \text{MCQ}$$

$$\sum_{\substack{i \in S \\ i \neq k}} U_i Q_i + U_k Q_k = \sum_{\substack{i \in S \\ i \neq k}} U_i \text{MCQ} + U_k \text{MCQ}$$

$$U_k = \frac{\sum_{\substack{i \in S \\ i \neq k}} U_i (\text{MCQ} - Q_i)}{Q_k - \text{MCQ}}$$

If this formula is used on a given i , such as $i < k$, this means that $U_k = \min_k$, and $U_i < K_i$. In this case, there is a chips quality loss and the only way to satisfy the MCQ is to have a $U_i > K_i$, which is a contradiction. Thus, all i have to be tried, from 1 to S , and the first i satisfying $U_i \leq K_i$ means $i = k$. \square

Proof. of property 13.

This proof is presented in three parts. First, we show that the optimal solution of \mathcal{P}_L which maximizes the average paper quality admits at most one m and one k , such as $0 < L_m^* < \min_m$ and $\min_k < L_k^* < K_k$. Then, we demonstrate how to compute L_m^* and L_k^* . Finally (with the sawmills sorted by Q_i in descending order), we show how to find m and k .

Say that it exists an optimal solution which maximizes the average paper quality such as there are two indexes, say k and k' , with $k < k'$. These indexes both satisfy $\min_k < L_k^* < K_k$ and $\min_{k'} < L_{k'}^* < K_{k'}$. By reallocating a small value from k' to k , the average paper quality increases, which is a contradiction. Similarly, there are two other indexes, say m and m' , with $m < m'$, $0 < L_m^* < \min_m$ and $0 < L_{m'}^* < \min_{m'}$. By reallocating a small value from m' to m , the average paper quality increases, which is a contradiction.

The purpose of the value of L_m^* is to meet the paper demand dp . In the case where m exists (see the third part of the proof):

$$L_m^* = dp - \sum_{\substack{j \in S \\ j \neq m}} L_j$$

The purpose of the value of U_k^* is to maximize the paper quality :

$$\sum_i^S L_i Q_i = \sum_i^S L_i \text{MCQ} \quad (6.9)$$

Note that $\forall i \in [1; \dots; k-1]$, $L_i = K_i$ and $\forall i \in [k+1; \dots; m-1]$, $L_i = \min_i$ and $L_m = dp - \sum_{i \neq m}^S L_i$ and $\forall i \in [m+1; \dots; S]$, $L_i = 0$.

$$\begin{aligned} (6.9) &\Leftrightarrow \sum_{i=1}^{k-1} K_i Q_i + L_k Q_k + \sum_{i=k+1}^{m-1} \min_i Q_i + Q_m (dp - \sum_{i=1}^{k-1} K_i - L_k - \sum_{i=k+1}^{m-1} \min_i) \\ &= \sum_{i=1}^{k-1} K_i \text{MCQ} + L_k \text{MCQ} + \sum_{i=k+1}^{m-1} \min_i \text{MCQ} + \text{MCQ} (dp - \sum_{i=1}^{k-1} K_i - L_k - \sum_{i=k+1}^{m-1} \min_i) \end{aligned}$$

After development and simplifications:

$$\begin{aligned} (6.9) &\Leftrightarrow L_k Q_k - L_k Q_m \\ &= - \sum_{i=1}^{k-1} K_i Q_i - \sum_{i=k+1}^{m-1} \min_i Q_i - dp Q_m + \sum_{i=1}^{k-1} K_i Q_m + \sum_{i=k+1}^{m-1} \min_i Q_m + dp \text{MCQ} \end{aligned}$$

After rearrangement and factoring:

$$(6.9) \Leftrightarrow L_k = \frac{dp(\text{MCQ} - Q_m) + (Q_m - Q_i)(\sum_{i=1}^{k-1} K_i + \sum_{i=k+1}^{m-1} \min_i)}{Q_k - Q_m}$$

The third part of the proof follows :

If this formula is used on a given i , such as $i < k$, this means that $U_k = \min_k$, and $L_i < K_i$. In this case, there is a chips quality loss and the only way to satisfy the MCQ is to have a $L_i > K_i$, which is a contradiction. Thus, all i have to be tried, from 1 to S , and the first i satisfying $L_i \leq K_i$ means $i = k$.

The value for L_k is dependent of the m chosen. The formula for L_k guarantees that considering a given m , L_k maximizes the quality of paper. Thus for each possible m (between $k+1$ and S) L_k has to be calculated. The final m is the one that maximizes the quality, *i.e.* $\sum_{i=1}^S L_i Q_i$. \square

Conclusion

Contributions

Décisions stratégiques

Dans cette thèse, nous avons étudié les relations entre donneur d'ordres et fournisseurs, à différents niveaux de décision. La problématique centrale de cette thèse est d'établir des contrats de stabilité menant à une collaboration durable entre ces acteurs, en garantissant leurs profits et leurs approvisionnements.

Au niveau stratégique, ces contrats spécifient un ensemble de règles de commande entre un donneur d'ordres et un de ses fournisseurs, en définissant à la fois un minimum et un maximum à respecter sur la quantité de chaque commande, et un intervalle de temps minimum et maximum entre chaque commande.

La prise en compte des contrats de stabilité pour un donneur d'ordres a des répercussions différentes suivant les coûts considérés. En particulier, les coûts fixes de commande sont très peu influents sur le problème, alors que les coûts de stockage ont un fort impact sur l'augmentation des coûts d'application des contrats de stabilité (Section 3.3.1).

Le niveau de stock est également très dépendant de la variance de la demande (Section 3.3.2).

Le travail s'est donc naturellement orienté vers l'augmentation des coûts de stockage. En outre, des expériences ont été menées sur l'interdépendance entre les deux parties des contrats de stabilité (quantitative et temporelle). L'augmentation des coûts de stockage est linéaire d'une partie du contrat, à condition que la deuxième ne soit pas serrée (Section 3.3.3). Pourtant, si une partie est déjà serrée, alors l'augmentation des coûts de stockage n'est plus linéaire mais constante jusqu'à un certain point, puis redevient linéaire (Section 3.3.3). Ces résultats suggèrent que quand le donneur d'ordres est contraint par une partie d'un contrat, ses commandes respecteront naturellement la deuxième partie du contrat jusqu'à un certain point.

Un effet de palier peut être mis en évidence (Section 3.3.3). Quand un donneur d'ordres

accepte d'être contraint par une partie d'un contrat de stabilité, il peut donc respecter l'autre partie du contrat sans payer de coût additionnels.

Ces contrats peuvent être utilisés de différentes manières :

Un donneur d'ordres veut stabiliser la relation avec un de ses fournisseurs. Il peut convaincre ce fournisseur d'appliquer une réduction de prix sur ses commandes, en échange de la régularité de ses commandes. Cette réduction de prix doit être équivalente à l'augmentation des coûts de stockage induits par cette régularité.

Un fournisseur réalise qu'une augmentation de profit est possible en régularisant son planning de production. Également, il aimerait être sûr de la fidélité de ses clients. Il propose donc à chacun de ses clients de contraindre leurs commandes, tout en leur garantissant une réduction de prix qui compense une augmentation maîtrisée de leurs coûts de stockage.

Un donneur d'ordres et son fournisseur désirent établir une relation durable entre eux, en diminuant leurs coûts et augmenter leurs profits. Ils choisissent le contrat qui propose le meilleur compromis entre simplification du planning de production du fournisseur et augmentation des coûts de stockage du donneur d'ordres. La réduction du prix des commandes est ensuite utilisée pour partager le profit ainsi généré.

Par leur simplicité et leur généricité, ces contrats peuvent être applicables entre les donneurs d'ordre et fournisseurs qui rencontrent des problèmes d'approvisionnement. Dans la chaîne logistique forestière canadienne, toutes les forêts sont distribuées par le gouvernement aux scieries. Cependant, elles ne sont pas les seules à profiter des produits du bois : les papetières utilisent comme matière première les copeaux. Si on considère les deux produits du sciage, le bois d'œuvre est plus rentable que le copeau. Cela crée un déséquilibre dans les relations de collaboration entre ces deux types d'acteurs.

Les contrats de stabilité peuvent assurer la garantie de l'approvisionnement de la papetière. Cependant, pour être accepté, ces contrats doivent aussi assurer une stabilisation du profit des scieries. Dans ce cas d'étude, les scénarios décentralisés extrêmes sont inefficaces (Section 6.5.1).

Les contrats de stabilité sont aux acteurs considérés : seule la partie quantitative des contrats est conservée, car la discrétisation du temps ne permet pas l'implantation des fenêtres de temps. Les bornes maximales et minimales sur les commandes sont calculées en prenant en compte diverses contraintes industrielles, ainsi que les demandes des scieries. Une méthode pour calculer les bornes des contrats de stabilité est également fournie (Section 6.5.2). Le scénario décentralisé avec contrat de stabilité est comparé à un scénario centralisé dans lequel un seul donneur d'ordres prend toutes les décisions pour l'ensemble de la chaîne logistique. Les résultats montrent que l'utilisation des contrats permet d'at-

tendre en moyenne 99.3% le profit généré par le modèle centralisé, et cela en équilibrant le profit sur chacun des acteurs (Section 6.5.3).

Décisions tactiques

En considérant un donneur d'ordres et un fournisseur, la collaboration stratégique se formalise par la signature d'un contrat. Dans le cas d'un contrat de stabilité, les bornes quantitatives et temporelles sont décidés. Du point de vue du donneur d'ordres, l'établissement d'un planning d'approvisionnement peut se modéliser comme un problème de dimensionnement de lots.

En considérant des fonctions de coût concaves (production et stockage), Le problème de dimensionnement de lots avec quantité maximales et minimales de commande est un problème original dont la complexité n'avait jamais été spécifiquement étudiée. Une propriété dominante est utilisée pour séparer le problème en plusieurs sous-problèmes simplifiés (Section 4.3). Cette approche par sous-plans permet la création d'un algorithme optimal s'exécutant en temps polynomial en $O(T^6)$. L'algorithme est testé sur un ensemble d'instances aléatoires en faisant varier les paramètres (Section 4.4).

Les fenêtres de temps dynamiques peuvent se traduire en contraintes linéaires, qui sont intégrées dans une formulation de dimensionnement de lots. D'après nos connaissances, le problème de dimensionnement de lots avec fenêtre de temps dynamique n'a jamais été étudié dans la littérature. La même propriété dominante est utilisée pour séparer le problème en plusieurs sous-problèmes simplifiés, menant à un algorithme polynomial en $O(T^9)$ (Section 5.3). Cependant, une simple supposition sur la forme des demandes (qu'elles soient, à chaque période, toutes strictement positives) permet de réduire la taille du graphe et donc la complexité de l'algorithme jusqu'à $O(T^7)$.

Des méthodes efficaces sont donc proposés pour optimiser les plans d'approvisionnements induits par l'établissement des contrats de stabilité.

Perspectives

Comme nous l'avons vu dans de cette thèse, l'efficacité des contrats au niveau stratégique dépend de la quantité et de la valeur des informations échangés entre les acteurs. La mise en place de tels contrats nécessite donc un processus de négociation bien formalisé, de façon à guider les acteurs dans la mise au point. Un axe de recherche serait donc de formaliser ce processus, ses recherches relevant plus de la gestion que de la recherche opérationnelle.

Les contrats de stabilité présentés dans cette thèse sont simples à comprendre. Des pistes pour les améliorer peuvent être suivies. Par exemple, accorder au fournisseur le

droit de modifier une quantité de livraison de façon contrôlée pourrait être intéressante, c'est à dire donner la possibilité au fournisseur de livrer plus ou moins un certain pourcentage de la commande passée. Cette piste a déjà été exploré pour notre étude de cas dans l'industrie forestière, mais elle a été abandonnée parce qu'elle semblait trop compliqué à mettre en place dans ce contexte. Mais elle pourrait être reprise pour d'autres contextes industriels.

Au niveau tactique, les perspectives portent sur les possibles extensions, généralisations ou simplifications des modèles de dimensionnement de lots présentés.

Dans la plupart de nos modèles (Chapitres 1 à 5), la demande est une contrainte rigide du problème : elle doit absolument être satisfaite à la bonne période. Une possibilité d'extension serait d'autoriser les retards de livraison (backlogs), en payant un coût de pénalité.

Dans les modèles traités dans la thèse, nous considérons des structures de coût concaves (Chapitre 4 à 5). Si ces structures de coût ont l'avantage d'être relativement générales, notamment par la prise en compte de coûts fixes ou coûts linéaires par morceaux, elles ont l'inconvénient de compliquer la résolution. Il serait donc intéressant de poursuivre l'étude de ces modèles avec des structures de coût plus simple, de façon à voir le gain que l'on pourrait obtenir sur les actuels algorithmes dont la complexité est élevée ($O(T^7)$ et $O(T^9)$). Considérer des structures de coût simples : de commande linéaire, de stockage linéaire et un coût fixe de commande pourraient apporter de sensibles améliorations à nos algorithmes en terme de complexité.

Plus généralement, ces modèles pourraient aussi être élargis pour considérer des problèmes multi-produits, plus proche des problèmes industriels.

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