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Rémy Delille

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POUR OBTENIR LE GRADE DE

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L'UNIVERSITÉ DE BORDEAUX**

ÉCOLE DOCTORALE ENTREPRISE, ÉCONOMIE, SOCIÉTÉ (ED 42)
SPÉCIALITÉ SCIENCES ÉCONOMIQUES

Par **Rémy DELILLE**

**BILATERAL BARGAINING AND
FARSIGHTEDNESS IN NETWORKS:
ESSAYS IN ECONOMIC THEORY.**

Sous la direction de : Nicolas Carayol, Professeur des Universités
et Jean-Cristophe Perea, Professeur des Universités

Soutenue le 14 Décembre 2015

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Abstract [EN]

The thesis consists in four essays that deal with bargaining and networks in non cooperative game theory. The first chapter introduce river bargaining games in the context of externalities. The first section entitled *The seawall bargaining game* deals with a non cooperative approach of an investment game in a context of positive externalities. In this section, we study the set of bilateral bargaining procedures which can be undertaken among geographically related players. The main result shows that the positioning of the agents impacts their incentives to sit at the bargaining table, leading to a chicken game. Results show that an intermediary player should lead the negotiations to improve the societal welfare. The second section deals with the *River bargaining problem*, that is, more precisely, a non cooperative bargaining on a flowing resource in the presence of negative externalities. The purpose of this study is: (i) to find the bargaining equilibria given an exogenous bilateral bargaining protocol, and (ii) to study the societal desirability of such equilibria in the presence of heterogeneous players. Results show that depending on the instigator of the bargaining sequences, there are analogies between solutions under the Absolute Territorial Sovereignty and the Unlimited Territorial Integrity principles. The results also show that, depending on the protocol, the impasse point reached in the first negotiation can either strengthen or weaken the relative position of the agents in the forthcoming negotiations. The second chapter deals with the formation of networks of manufacturers and retailers in the presence of negative externalities when players are level- K farsighted. The aim of the chapter are, (i) to characterize the level- K farsighted stable sets as the intensity of competition (the intensity of the externality), so as the cost of linking vary; (ii) to formalize a definition of the social optima and to confront these

optima to the stable sets. The results show that, (i) a relatively low level of farsightedness is sufficient to reach the infinite level of farsightedness; (ii) usual definitions of optimality or efficiency find limitations when it comes to be confronted to a set-based definition of stability. (iii) If there is transitive correspondence between the pairwise farsighted stable set and the level- ∞ farsighted stable set, then this set is likely to be strongly efficient. The last chapter is entitled *Allocating value among farsighted players in network formation*. This theoretic chapter proposes the concept of a von Neumann-Morgenstern farsighted stable set with bargaining. Under this solution concept, the stable networks so as the componentwise egalitarian allocation rule emerge endogenously. This chapter provides necessary conditions under which a von Neumann-Morgenstern farsighted stable set with bargaining sustains the strongly efficient networks.

Abstract [FR]

Cette thèse consiste en quatre essais qui traitent de négociation et de réseaux en théorie des jeux non coopérative. Le premier chapitre présente des jeux de négociations dans un contexte d'externalités. La première section intitulée le *jeu de négociation sur la digue* traite d'une approche non coopérative d'un jeu d'investissement dans un contexte d'externalités positives. Dans cette section, nous étudions l'ensemble de procédures bilatérales de négociation qui peuvent être mises en place entre des agents géographiquement liés. Le résultat principal de la section montre que la situation des agents impacte leurs incitations à prendre part aux négociations et se synthétise en un "jeu de la poule mouillée". Du point de vue sociétal, les résultats montrent qu'il est socialement plus efficace qu'un joueur intermédiaire mène les négociations. La seconde section du chapitre traite du *problème de négociation sur la rivière*. Il s'agit d'un jeu de négociation non coopératif sur l'utilisation de la ressource fluviale en présence d'externalités négatives. Le but de cette étude est de (i) déterminer les équilibres de négociation étant donné un protocole de négociation exogène; (ii) d'étudier l'intérêt sociétal de tels équilibres en présence d'acteurs hétérogènes. Les résultats montrent qu'en fonction de l'instigateur des séquences de négociations, il existe des analogies entre les solutions obtenues dans les cas de Souveraineté Territoriale Absolue et d'Intégrité Territoriale Illimitée. Les résultats montrent aussi que, en fonction du protocole retenu, le point d'impasse obtenu lors de la première négociation peut renforcer ou affaiblir la position relative des agents dans les négociations à venir. Le deuxième chapitre traite de la formation de réseaux de producteurs et de détaillants en présence d'externalités négatives lorsque les joueurs sont clairvoyants de degré- K . Le but de ce chapitre est (i) de déterminer les ensembles de réseaux stable

de degré- K lorsque l'intensité de la concurrence (l'intensité de l'externalité) et les coûts de liaison varient; (ii) de donner une définition formelle des optima sociaux afin de les comparer aux ensembles de réseaux stables. Les résultats montrent que (i) un degré de clairvoyance relativement faible est suffisant pour atteindre la clairvoyance absolue ou infinie; (ii) les définitions habituelles de l'optimum ou de l'efficacité ne conviennent pas parfaitement à un concept de stabilité ensembliste. (iii) S'il existe une correspondance transitive entre la stabilité clairvoyante par paires et la stabilité clairvoyante de degré infini, alors l'ensemble stable peut être efficace. Le dernier chapitre est intitulé *attribution de la valeur entre joueurs clairvoyants dans le processus de formation de réseau*. Il s'agit d'un chapitre théorique qui propose le concept de stabilité von Neumann-Morgenstern avec négociation. Dans ce concept de solution, les ensembles de réseaux stables, ainsi qu'une répartition égalitaire au sein des composants du réseau sont déterminés conjointement, et de manière endogène. Ce dernier chapitre met en évidence les conditions nécessaires pour que les réseaux von Neumann-Morgenstern avec négociation soient efficaces.

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Introduction

The thesis consists in four essays in microeconomics. While the internal composition of the document deals with various topics, these four essays have a common point. The thesis studies bilateral relationships in regards to a networked structure. The first chapter addresses the question of the negotiation in an exogenous network structure framework. The second chapter deals with the question of an endogenous network formation with farsighted players. The last chapter proposes an endogenous network formation process that integrates a bargaining procedure and farsighted players. The study remains within the scope of the non cooperative game theory.

The first chapter deals with the management of a natural resource. Economic agents are geographically scattered along a river. In this aligned network structure, the geographic location determines the access to the public good. Indeed, regarding the nature of the valuation of the estate, the strategical advantages depends on the geographic location of the players. Traditionally, the study of public goods has been undertaken using an identical access to the resource and has identified the sources of societal inefficiencies such as free-riding or the tragedy of the commons. In the river setting, the heterogeneous access to the resource lead to reconsider the usual issues and therefore, its solutions. The question of the property rights, or the right to get a priority access quickly arises. Indeed, given the orientation of the stream, an ex-ante property rights division or the fact of having a priority access over the resource induces different forms of inefficiencies and thus, different solutions to decentralize a second-best equilibrium. Results from the cooperative game theory usually lead to the efficient outcome but often fails at explaining the inner process leading to that solution. It appears that a bargaining mechanism leads to the

efficient outcome but the sequences of negotiations need to be fully described. The first chapter aims at opening the "black-box" of the river bargaining using the non cooperative approach of the Rubinstein alternating offers model in the context of externalities. The first section entitled *The seawall bargaining game* is inspired from a paper conjointly written with my PhD. supervisor: Jean Christophe Perea. In this paper, the agents are located from downstream to upstream along an estuary and are exposed to a flooding risk. The players have the ability to invest in facilities to protect themselves from this hazard (for instance a seawall, a dike or a ditch). The agents located upstream the river benefit from the efforts in protection of the players downstream. As the benefits of the local public good increase along the estuary, upstream agents have to bargain for monetary compensation with the most downstream agents in exchange for a greater protection effort. The section analyses different bargaining protocols and determines the conditions under which agents are better off. The results show that no bargaining procedure is obviously preferred by all the players. The results show that the upstream agents are involved in a chicken game when it comes to bargain with the most downstream agent.

The second section entitled *The river bargaining problem* is based on an article conjointly written with Jean Christophe Perea and deals with a river bargaining game in the context of negative externalities. In this model, the players benefit from extracting the resource. The nature of the externality lies in the fact that the resource is scarce and that the sum of the individual optimal extractions exceeds the amount of available water. The section analyses the outcomes of different negotiation procedures between three agents located along a river in a Rubinstein alternating-offers model where agents bargain over transfers and water consumption levels. Two different kind of sequential procedures given the ex-ante division of property rights are investigated. The downstream bargaining procedure is instigated by the most downstream player and the upstream procedure by the most upstream. Results show that under some assumptions on the non cooperative level of extractions and given the Absolute Territorial Sovereignty or the Unlimited Territorial Integrity applies, the procedures yield analogous solutions. Unlike for simultaneous bargaining, agreements in sequential bargaining procedures are not efficient for society, even

if the period between stages becomes infinitely small. This inefficiency results from the player's inside options, which are given by their temporary disagreement payoffs. Results also show that depending on the sequence of moves, inside options and impasse points can strengthen or weaken the relative position of the players involved in the negotiation process.

While the first chapter assumes that the structure of the network is exogenous, driven by geographical constraints, the second and last chapters release the constraint of an exogenous social structure as they deal with endogenous network formation.

The second chapter takes place the context of a vertically related economy. The study of vertical forms of economies has deep historical roots in Industrial Organization. Industrial Organization has raised various forms of questions regarding an exogenous structure of vertical relationships (incentives for the vertical integration, the effects of franchising and exclusive zones...). New theoretic tools such as "the strategic networks analysis" formalized the endogenous formation of networks and thus shed new light on the questioning in Industrial Organization. I investigate the question of the stability of commercial agreements between manufacturers and retailers in regard to their ability to forecast the likely evolution of the network. In this context, I question the typologies of networks that might be formed when the production of a set of differentiated manufacturers has to be distributed through a set of retailers. Thus, the scope of the paper encompasses the bilateral formation of commercial links, *ie.* the consent of a manufacturer and a retailer is needed to build a connection, while distribution channels can be unilaterally severed.

Commercial contracts between firms can be costly to endorse and economic agents have to anticipate the aftermaths induced by their new partnerships and also the consequences of new and/or revoked partnerships of third parties. The decision to discuss a new commercial contract or to abolish an existing relationship is driven by the ability to anticipate. This analysis is therefore based on the depth of reasoning of the players. On the lowest level of reasoning players bilaterally add or unilaterally sever existing links at random, while at the highest level of reasoning, the players anticipate all the deviations that their initial action induce.

Computational tools are used to establish the set of networks that satisfies a pairwise network formation together with an adjustable level of farsightedness for any cost of linking and any intensity of competition between manufacturers.

Beyond being a strategic asset, anticipation is costly in terms of gathering information or in terms of time needed to treat the information. The results show that an intermediary level of anticipation of the player is sufficient to achieve the same outcome as the one obtained when they are fully farsighted. I then confront the stable sets to several optimality and efficiency criteria. It appears that, for low values of products differentiation and high values of linking costs there can be a relation between stability and efficiency.

The question of the endogenous network formation according to a bargaining procedure with farsighted players is the purpose of the third chapter. This last chapter is based on an article conjointly written with my PhD supervisor Nicolas Carayol and Vincent Vannetelbosch. In this chapter, we propose a concept to study the stability of social and economic networks when players are farsighted and allocations are determined endogenously. A set of networks is a von Neumann-Morgenstern farsightedly stable set with bargaining if there exists an allocation rule and a bargaining threat such that (i) there is no farsighted improving path from one network inside the set to another network inside the set, (ii) from any network outside the set there is a farsighted improving path to some network inside the set, (iii) the value of each network is allocated among players so that players suffer or benefit equally from being linked to each other compared to the allocation they would obtain at their respective credible bargaining threat. We show that the set of strongly efficient networks is the unique von Neumann-Morgenstern farsightedly stable set with bargaining if the allocation rule is anonymous and component efficient and the value function is top convex. Moreover, the componentwise egalitarian allocation rule emerges endogenously.

Chapter 1

Bilateral river bargaining with externalities

1.1 Introduction

The first chapter deals with bargaining procedures in the framework of exogenous network and is divided in two main sections. The first section deals with the Seawall bargaining game. In this game, players are ordered along a river from downstream to upstream. The players located upstream benefit from the efforts of the players downstream. The context of positive externalities make coordination harder to achieve and different bargaining protocol are studied. A brief introduction of the motivation for this model are exposed in Section 1.2. The section 1.3 handles the river bargaining game in the presence of negative externalities. In this section, players are ordered from upstream to downstream along a river. Given the principle that prevails (Absolute Territorial Sovereignty or Unlimited Territorial Integrity), two types of sequential bargaining protocols are studied (the downstream and the upstream procedure).

1.2 The Seawall bargaining game

Hirshleifer [26] shows with his “Anarchia Island” fable that citizens have successfully agreed to build seawalls (or dikes) to protect themselves from storms threatening to flood

the coastline despite the weakest-link structure of that local public good. The seawall is known as a particular public good in which the level of effective protection for the whole island depends on the citizen who has constructed the lowest seawall. This chapter analyses a similar problem of flood-protection when agents are located subsequently from downstream to upstream along an estuary and exposed to a flooding hazard. In that case, flood protection does not only consist in building the highest seawall, it also requires the construction of other facilities, such as a first seawall to break the wave, the use of wetland as flood water retention land, a network of ditches to control the flood or a second seawall. This estuarine geography feature implies that when the sea enters, the protection effort implemented by an agent will be a public good for the agents located upstream from his position. At the two ends of the spectrum, the most downstream agent (the closest to the sea) doesn't get additional benefits from the upstream facilities, while the most upstream agent benefits from all the efforts exerted by the players downstream. Thus, the seawall bargaining game is no longer a problem of public good with a weakest-link aggregation technology, but is a public good with positive externalities that increase along the estuary. Since the benefit of the public good depends on the geographical position, it appears that cooperation is hard to achieve. In order to sustain a high level of effort from the most downstream agent, upstream agents have the ability to bargain monetary transfers. The modeling of the negotiations is thus of a great part in the determination of the results. The most obvious approach is to base the analysis on what is known as bargaining theory with the Rubinstein alternating-offers model. Rubinstein [49] describes the process through which negotiating agents can reach an agreement. The agent opening the negotiations makes an offer. The other agent can either accept the offer, in which case the negotiation ends, or reject it and make a counter-offer, which may also be accepted or rejected with a new counter-offer. The interest of the non cooperative approach is that it fully specifies the bargaining sequences. In our framework, an offer will cover two variables related to the effort of sea-flood protection and a monetary transfer. Our framework also assumes a Rubinstein bargaining with 3 players and analyses several bargaining protocols. The chapter determines the likely protocols one could expect to be implemented given the non

cooperative behaviors of the players.

1.2.1 Literature Review

The seawall bargaining game can be modeled as a particular case of global public good in which all agents benefit from the action of the downstream players, whatever their location. In the literature on international environmental agreements, for instance, results show that for identical agents, only a very small number of players will form a coalition. Seminal papers using this approach are Carraro and Siniscalco [11] and Barrett [6], and a survey can be found in Finus [19]. Our framework shares, in common with the “sharing river” model, the downstream/upstream agent structure in which downstream agents create externalities to upstream agents. This literature is based on cooperative game theory and, more precisely, on the core (see Ambec and Sprumont [1] and Beal *et al.* [7] for a survey). The objective is to set up a burden-sharing rule able to favor the cooperation of all. The sharing rule aims at preventing any individual agent, but also any sub-group of agents, from having no incentive to leave the agreement.

However, the coalitional approach often ignores the negotiating process in terms of offers and counter-offers that characterize all negotiations between self-interested agents. The non cooperative solution concept is first introduced by Staahl (1972) [51] and Rubinstein (1982) [49] for a two players bargaining¹. As pointed out by Carraro *et al.* (2007) [10], it is crucial to thoroughly describe the bargaining process that designs the most likely burden-sharing rule so as to implement a non cooperative protocol.

The next subsections present the cooperative and non-cooperative outcomes. The next following subsection is devoted to the analysis of single and double negotiations. In subsection 1.2.4, a specific example shows the main results of the model and the last subsection presents the concluding remarks on the Seawall bargaining game.

¹For an advanced and comprehensive overview of the alternating offers models see Muthoo (1999) [40].

1.2.2 The benchmark

Considering an ordered set of players $N = \{1, 2, \dots, n\}$. Every agent is located from downstream to upstream in a lexicographic ordering². We note e_i , the effort of protection realized by agent i . The respective payoffs are defined as the difference between the concave benefit function (with $B'_i > 0$ and $B''_i < 0$) and the convex cost function ($C'_i > 0$ and $C''_i > 0$). We denote by $i < j$ the fact that agent i is downstream of agent j . Following notation of Ansink and Weikard (2009) [3] $U_i = \{j \in N : j > i\}$ stands for the set of agents located upstream of agent i and reciprocally $D_i = \{k \in N : i > k\}$. The first player only benefits from his own effort, his payoff is $\pi_1(e_1) = B_1(e_1) - C_1(e_1)$. The second agents benefits from his own effort and from the efforts in protection from the previous player. In the end, n benefits from everyone's efforts. The generic payoff function is thus:

$$\pi_i(\{e_j\}_{j \leq i}) = B_i(\{e_j\}_{j \leq i}) - C_i(e_i)$$

The equations show that costs are private while the agents benefit from the efforts of the previous players.

Cooperative outcome

The cooperative solution is given by the program:

$$\max_{e_i} \sum_{i=1}^n \pi_i(\mathbf{e})$$

Effort levels are solution of:

$$\sum_{j \geq i} \partial B_j \left(\sum_{k \leq j} e_k \right) = \partial C_i(e_i) \quad \forall i \in N, k \geq 1, j \leq n.$$

²The opposite of the classical ordering is for convenience worries.

That is in the 3-players setting³:

$$B'_1(e_1) + B'_2(e_1 + e_2) + B'_3(e_1 + e_2 + e_3) = C'_1(e_1) \quad (1.1)$$

$$B'_2(e_1 + e_2) + B'_3(e_1 + e_2 + e_3) = C'_2(e_2) \quad (1.2)$$

$$B'_3(e_1 + e_2 + e_3) = C'_3(e_3) \quad (1.3)$$

Player i takes the impact of his level of effort on the players upstream in his marginal benefit. It returns a unique vector of efforts $e^c = \{e_i^c\}_{i \in N}$. An additional unit of effort by agent i exerts an additional benefit for him and for the agents upstream of his position. At the equilibrium, the marginal cost of that unit equalizes the sum of his marginal benefit and the marginal benefit of the upstream agents. The vector of efforts returns a unique vector of cooperative payoffs $\{\pi_i^c\}_{i \in N}$.

Non-cooperative outcome:

Whenever the players act non cooperatively, the optimality conditions are:

$$\partial B_i \left(\sum_{k \leq i} e_k \right) = \partial C_i(e_i) \quad \forall i \in N, k \geq 1.$$

That is in the 3 players setting:

$$B'_1(e_1) = C'_1(e_1) \quad (1.4)$$

$$B'_2(e_1 + e_2) = C'_2(e_2) \quad (1.5)$$

$$B'_3(e_1 + e_2 + e_3) = C'_3(e_3) \quad (1.6)$$

The previous system returns a unique vector of non-cooperative efforts $e^{nc} = \{e_i^{nc}\}_{i \in N}$. Each agent equalizes his marginal benefit to his marginal cost. The vector of efforts returns a unique vector of payoffs $\{\pi_i^{nc}\}_{i \in N}$. The difference between the cooperative and the non cooperative effort increases as we go downstream. Indeed, downstream players take the positive impact of their action on agents located upstream into account. For

³Considering a three-agent framework is enough to put forward the main results of the paper.

an intermediary player 2, two effects are at stake. The additional effort of agent 1 in the cooperative case reduces the incentives of 2 to do likewise, but due to the positive impact that 3 exerts on 3, the agent 2 tends to increase his effort in the cooperative case. This latter effect dominates the former, since $e_2^{nc} < e_2^c$. However, this is the opposite for the most upstream agent. His effort is reduced in the cooperative case, since he benefits from the effort made by the previous downstream agents, and an additional effort does not exert additional benefit to any player, this leading to $e_3^c < e_3^{nc}$. The aggregate effort is higher in the cooperative case than in the non-cooperative case. In terms of payoffs, agents 2 and 3 are better off in the cooperative case, unlike the most upstream agent, whose payoff only depends on his effort. It turns that agent 1 has to make a greater effort in the non-cooperative case, and this substantially reduces his payoff. It yields $\pi_i^c > \pi_i^{nc}$ for $i = 2, 3$ but $\pi_1^c < \pi_1^{nc}$. This is the main feature of the estuarine geography. The dominant strategy of agent 1 is to implement his non-cooperative effort regardless of the action of the upstream agents. Hence, decentralizing the the non-cooperative case to the cooperative case will never be found profitable for the most downstream agent, even if the aggregate payoff in the cooperative case is higher than in the non-cooperative case.

1.2.3 Negotiation Protocols

If societal gains of cooperation are positive, then the negotiation may improve the payoffs of some agents. In particular, the most downstream agent could find it worth it to increase his effort if the other players can find a mechanism to urge him in doing so. The negotiation can take place between agents over an extra amount of effort that an agent will implement in exchange for a monetary transfer or compensation. However, several protocols must be considered given the multiplicity of pairs of agents. Thus, bargaining between two agents consists in a level of effort and a transfer.

A negotiation between agents i and j has two arguments. A monetary transfers or side-payments in exchange for a modification of the variable that affects both agents. In the estuarine model, the agent i proposes the monetary transfer τ_{ij} to the agent j in exchange for his commitment to increase his effort to a $e_j > e_j^{nc}$. Alternatively, the agent

j could propose the player i to increase his protection level to $e_j > e_j^{nc}$ in exchange for the subvention τ_{ji} . Note that $\tau_{ij} = -\tau_{ji}$ and by convention the first player in the subscript pays the transfer to the second (If i pays $-\tau_{ij}$ then he receives τ_{ji}). An offer or a proposal is a couple denoted by $o_{ij} = (e, \tau_{ij})$ if the offer is made by i to j and is about a variation in $e = e_i$ in exchange for a transfer of value τ_{ji} if $i < j$ and about $e = e_j$ and the transfer τ_{ij} if $j < i$. Bargaining rounds assumes perfect information.

The first and simplest negotiation only concerns two players, either agent 1 with 2 or 3 or between 2 and 3. In this protocol, one player never sits at the bargaining table. In any cases, regardless of the player who doesn't bargain, the player 3 maximizes his individual payoff by setting his effort e_3 . The set of available offers is thus $O_1 = \{o_{12}, o_{13}\}$ and $O_2 = \{o_{23}\}$. The payoffs are denoted by $V_1(e_1) = B_1(e_1) - C_1(e_1)$, $V_2(e_1, e_2) = B_2(e_1 + e_2) - C_2(e_2)$ and $V_3(e_1, e_2, e_3) = B_3(e_1 + e_2 + e_3) - C_3(e_3)$.

In a three-agent case, the general bargaining framework is given by the following net payoff function:

$$\begin{aligned}\pi_1(O_1, t) &= \pi_1^{nc} + \delta_1^t (V_1(e_1) - \pi_1^{nc} - \tau_{12} - \tau_{13}) \\ \pi_2(O_1, O_2; t) &= \pi_2^{nc} + \delta_2^t (V_2(e_1, e_2) - \pi_2^{nc} + \tau_{12} - \tau_{23}) \\ \pi_3(O_1, O_2, e_3; t) &= \pi_3^{nc} + \delta_3^t (V_3(e_1, e_2, e_3) - \pi_3^{nc} + \tau_{13} + \tau_{23})\end{aligned}$$

where $0 < \delta_i < 1$ stands for the discount factor of agent i .

Proposition 1 *When the negotiation is over a single pair (e_i, τ_{ij}) of effort and transfer between agents i (the proposer) and j (the responder) for $i = 1, 2$, $j = 2, 3$, $j \neq i$ and $k \neq j$, the optimal vector of efforts \mathbf{e} satisfies:*

$$\begin{aligned}B'_i(\mathbf{e}) + B'_j(\mathbf{e}) &= C'_i(e_i) \\ B'_j(\mathbf{e}) &= C'_j(e_j) \\ B'_k(\mathbf{e}) &= C'_k(e_k)\end{aligned}$$

and the associated payoffs are:

$$\begin{aligned}\pi_i^*(\mathbf{e}) &= \pi_i^{nc} + \frac{(1 - \delta_j)}{(1 - \delta_i \delta_j)} \Pi \\ \pi_j^*(\mathbf{e}) &= \pi_j^{nc} + \frac{\delta_j(1 - \delta_i)}{(1 - \delta_i \delta_j)} \Pi \\ \pi_k^*(\mathbf{e}) &= \pi_k^{nc} + (V_k^* - \pi_k^{nc})\end{aligned}$$

where $\Pi = (V_i^* - \pi_i^{nc}) + (V_j^* - \pi_j^{nc}) > 0$ stands for the created surplus.

Proof. See the proof in Section 1.4.1. ■

Single negotiations refer to the following cases: 1 bargains with 3 over (e_1, τ_{13}) or 1 bargains with 2 over (e_1, τ_{12}) or 2 bargains with 3 over (e_2, τ_{23}) . In each case, the outsider of the negotiation acts non-cooperatively. When 2 or 3 bargains with 1, the negotiation implies a higher effort for 1, $e_1^c > e_1^* > e_1^{nc}$ and lower efforts for 2 or 3 $e_j^c < e_j^* < e_j^{nc}$, $j = 2, 3$. When the negotiation is between 2 and 3, 2 increases his effort such that $e_2^* > e_2^c > e_2^{nc}$ and $e_3^c < e_3^* < e_3^{nc}$, while the effort of 1 remains unchanged $e_1^* = e_1^{nc}$. When agents bargain, they get a share of the generated surplus while the outsider acts as a free-rider and benefits from the public good. (There is no additional benefits for the most downstream agent, since his payoff equals his non-cooperative outcome). This particular case refers to the maximization of the aggregate payoff under the constraint that the most downstream agent gets his non-cooperative payoff. In the limit when the time between bargaining rounds vanishes $\delta = \delta_i \rightarrow 1 \forall i$, the created surplus is shared equally among the two involved players. In that case, the Rubinstein solution converges to the Nash solution. It turns out that agent 2 is better off when 3 bargains with 1 (the additional efforts of 1 substitute to the effort in protection of 2), and by symmetry, 3 is better off when 2 bargains with 1 (same effect, the additional efforts of 1 and 2 substitute to the effort in protection of 3).

Expecting that the created surplus can be higher when all agents are at the negotiation table, we assume that the three agents negotiate in two bilateral negotiations. This structure ensures the uniqueness of the subgame perfect equilibrium (SPE) ⁴. Two cases

⁴As shown by Shaked and reported by Sutton [52], the use of Rubinstein model in a multilateral

are considered. First, 3 bargains twice over (e_1, τ_{13}) with 1 and over (e_2, τ_{23}) with 2 (setting $\tau_{12} = 0$). Second, 2 bargains twice over (e_1, τ_{12}) with 1 and over (e_2, τ_{23}) with 3 (setting $\tau_{13} = 0$).

The first negotiation yields the following proposition.

Proposition 2 *When 3 bargains as a proposer with 1 over (e_1, τ_{13}) and with 2 over (e_2, τ_{23}) , the Rubinstein bargaining solution shows that:*

1. *The optimal vector of efforts \mathbf{e} satisfies:*

$$\begin{aligned} B'_1(e_1) + B'_3(e_1 + e_2 + e_3) &= C'_1(e_1) \\ B'_2(e_1 + e_2) + B'_3(e_1 + e_2 + e_3) &= C'_2(e_2) \\ B'_3(e_1 + e_2 + e_3) &= C'_3(e_3) \end{aligned}$$

2. *Equilibrium payoffs after transfers are:*

$$\begin{aligned} \pi_i^* &= \pi_i^{nc} + \frac{\delta_i(1-\delta_j)(1-\delta_3)}{\eta} \Pi_3, \quad i = 1, 2 \text{ and } j \neq i \\ \pi_3^* &= \pi_3^{nc} + \frac{(1-\delta_1)(1-\delta_2)}{\eta} \Pi_3 \end{aligned}$$

where $\Pi_3 = \sum_{i=1}^3 (V_i^* - \pi_i^{nc}) > 0$ stands for the created surplus when 3 is involved in two negotiations and $\eta = (1 - \delta_1\delta_3)(1 - \delta_2\delta_3) - \delta_1\delta_2(1 - \delta_3)^2 > 0$.

Proof. See the proof in Section 1.4.2. ■

The second negotiation yields the following proposition.

Proposition 3 *When 2 bargains simultaneously as a proposer with 1 over (e_1, τ_{12}) and with 3 over (e_2, τ_{23}) , the Rubinstein bargaining solution shows that:*

bargaining framework may yield multiple equilibria under the unanimity rule.

1. The optimal vector of efforts \mathbf{e} satisfies:

$$\begin{aligned} B'_1(e_1) + B'_2(e_1 + e_2) &= C'_1(e_1) \\ B'_2(e_1 + e_2) + B'_3(e_1 + e_2 + e_3) &= C'_2(e_2) \\ B'_3(e_1 + e_2 + e_3) &= C'_3(e_3) \end{aligned}$$

2. Equilibrium payoffs (after transfers) are:

$$\begin{aligned} \pi_i^* &= \pi_i^{nc} + \frac{\delta_i(1-\delta_2)(1-\delta_j)}{\phi} \Pi_2, \quad i = 1, 3 \text{ and } j \neq i \\ \pi_2^* &= \pi_2^{nc} + \frac{(1-\delta_1)(1-\delta_3)}{\phi} \Pi_2 \end{aligned}$$

Where $\Pi_2 = \sum_{i=1}^3 (V_i^* - \pi_i^{nc}) > 0$ stands for the created surplus when 2 is involved in two negotiations and $\phi = (1 - \delta_1\delta_2)(1 - \delta_2\delta_3) - \delta_1\delta_3(1 - \delta_2)^2 > 0$ is the additional generated surplus.

Proof. See the proof in Section 1.4.3. ■

In both negotiations, agents 1 and 2 will increase their efforts, such that: $e_1^c > e_1^* > e_1^{nc}$ and $e_2^* > e_2^c > e_2^{nc}$, implying a decrease in the effort for the most upstream agent $e_3^{nc} > e_3^* > e_3^c$ with respect to his non-cooperative effort. Agents get a share of the generated surplus but the proposer still benefit from the first mover advantage (captured through the level of transfers). However, in instantaneous negotiations with discount factors set to the limit $\delta_i = \delta_j = \delta_k = \delta \rightarrow 1$, every player get one third of the surplus. The two negotiations show that agents are better off compared to the non-cooperative outcome $\pi_1^* > \pi_1^{nc} > \pi_1^c$ and $\pi_i^c > \pi_i^* > \pi_i^{nc}$ for $i = 2, 3$. However, in both cases, even if the definition of the generated surplus is identical, Π_3 differs from Π_2 , since equilibrium efforts did not satisfy the same optimality conditions.

1.2.4 Example

We now turn to a comparison between the two bargaining protocols. So as to make the comparison tractable, we assume that players have identical cost and benefit functions

regardless of their location along the estuary:

$$B(z) = az - \frac{b}{2}z^2, \quad a, b > 0$$

$$C(z) = \frac{c}{2}z^2, \quad c > 0$$

It is also assumed that $\delta = \delta_i \rightarrow 1 \forall i$. The outcome for all bargaining protocols under the previous assumptions are summarized in Tables 1.4.4, 1.4.4 and 1.4.4 in Section 1.4.4.

The following assertions can be drawn from the results on the bargaining protocols.

Result 1: The cooperative outcome (highest aggregate effort and payoff) cannot be reached by a particular negotiation protocol. This result comes from the structure of the model. Acting non-cooperatively is a dominant strategy for the most downstream agent. Exerting an effort $e_3 > e_3^{nc}$ always decreases π_3 . The set of bilateral bargaining returns a constrained cooperative solution (an intermediate solution). This solution consists in maximizing the aggregate outcome under the constraint than the most downstream agent always exerts e_3^{nc} . However, the results show that agents are always better off when a set of negotiations over efforts associated to a set of transfers exist.

Result 2: In the single negotiations scheme, if an agreement is reached between agents 1 and 2 (respectively, 1 and 3) the remaining player prefers to free ride and to stand off the bargaining table. It appears that players 2 and 3 both prefer the other player to bargain with 1 since the benefit from the public good. This conflict of interest between agents 2 and 3 comes from the presence of the positive externality. Each agent would rather benefit from the efforts realized by the other at no cost, as shown in Table 1.2.4. The three-agent Seawall bargaining game can be summarized in normal form where the space of strategies of each agent consists in either the acceptance A or the refusal (R) of negotiations. $S_i = \{A, R\}$ for $i = \{1, 2, 3\}$. Notation $i \leftrightarrow j$ means that i negotiates with j . It follows:

If player 1 refuses to bargain ($S_1 = R$) then the best response of 2 and 3 is to play $S_2 = S_3 = A$. however, as seen before, the payoff of player 1 is greater when he find a partner to bargain with compared to his non-cooperative payoff, $\pi_1^{\{1 \leftrightarrow 2\}} > \pi_1^{nc}$ and $\pi_1^{\{1 \leftrightarrow 3\}} > \pi_1^{nc}$.

$$S_1 = A$$

$2 \setminus 3$	$S_3 = A$	$S_3 = R$
$S_2 = A$	$\pi_1^{\{2 \leftrightarrow 1, 3\}}, \pi_2^{\{2 \leftrightarrow 1, 3\}}, \pi_3^{\{2 \leftrightarrow 1, 3\}}$	$\pi_1^{\{1 \leftrightarrow 2\}}, \pi_2^{\{1 \leftrightarrow 2\}}, \pi_3^F$
$S_2 = R$	$\pi_1^{\{1 \leftrightarrow 3\}}, \pi_2^F, \pi_3^{\{1 \leftrightarrow 3\}}$	$\pi_1^{nc}, \pi_2^{nc}, \pi_3^{nc}$

$$S_1 = R$$

$2 \setminus 3$	$S_3 = A$	$S_3 = R$
$S_2 = A$	$\pi_1^{nc}, \pi_2^{\{2 \leftrightarrow 3\}}, \pi_3^{\{2 \leftrightarrow 3\}}$	$\pi_1^{nc}, \pi_2^{nc}, \pi_3^{nc}$
$S_2 = R$	$\pi_1^{nc}, \pi_2^{nc}, \pi_3^{nc}$	$\pi_1^{nc}, \pi_2^{nc}, \pi_3^{nc}$

Table 1.1: The seawall bargaining game.

It appears that $S_1 = A$ is a dominant strategy. The Seawall bargaining game exhibits two Nash equilibria in pure strategies. $(S_1, S_2, S_3) = (A, R, A)$ and (A, A, R) . In both equilibria, agents 2 and 3 are better off when they act as free riders, $\pi_i^F > \pi_i^{\{i \leftrightarrow 1, j\}}$ and $\pi_i^{\{1 \leftrightarrow i\}} > \pi_i^{nc}$ for $i = 2, 3, j \neq i$. The structure of the Seawall bargaining game is a chicken game, as in Carraro and Siniscalco [11]. agent 2 (respectively, 3) prefers that his opponent bargains with 1 and benefits from the outcome of the negotiation without bearing any cost.

In double negotiation, the conflict is over the position of the proposer, but this is a direct consequence of the Rubinstein alternating offer model. When the time between bargaining rounds vanishes, this first mover advantage disappears.

Result 3: It is socially optimal to ask agent 2 to manage the two negotiations with the most downstream and the most upstream agents. The surplus generated in double negotiation is greater than the surplus of single negotiations. The size of the created surplus increases with the number of players at the bargaining table. A negotiation between 2 and 1 yields a greater effort of agent 1 in comparison with a negotiation between 3 and 1 (Player 3 would accept a lower effort from 1 in exchange for a low transfer as he anticipates a positive effort from player 2). This greater effort increases the benefit

of players upstream. A negotiation between 3 and 2 yields a greater effort of agent 2 in comparison with a negotiation between 2 and a 4th player (this 4th player asking less efforts from 2 since he anticipates a positive effort from player 3). This result can be generalized to n agents. A set of geographically restricted bilateral negotiations improves individual payoffs. In this setting, each player directly bargain with his predecessor, n negotiates with $n - 1$, $n - 1$ with $n - 2$, ..., up to the negotiation between 2 with 1.

1.2.5 Conclusions on the Seawall Bargaining game

Hirshleifer [26] shows that cooperation over the building of a seawall can be achieved even if the seawall is known as a weakest-link public good. The seawall example has been revisited for an alternative geographic structure where agents are located from downstream to upstream and have to exert a costly effort to protect themselves from sea floods. This feature implies that the benefit of the public good increases along an estuary. In a simplified three-agent framework, our results show that there does not exist a bargaining protocol that can be preferred by all of the agents. Agents located after the most downstream agent always prefer to free ride rather than entering in single negotiations over an additional effort of the most downstream agent. This case refers to a chicken game. When the negotiation involves all of the agents, our results show that it is more profitable for society to give the right to the agent located in the middle of the estuary to conduct negotiations with both the most downstream and the most upstream agent.

The next section introduces the model with negative externalities.

1.3 The river bargaining problem

Game theoretic analysis of the river sharing problem has been a very active research area over the past two decades (Barret, (1994) [6]; Kilgour and Dinar, (2001) [34] ; Ambec and Sprumont, (2002) [1] and Beal *et al*, (2013) [7] for a recent survey). The main motivation for this research relies on the conflicting nature of water uses between various agents or countries for residential, industrial or agricultural purposes. Analyzing water

allocation rules among agents or countries who are located along a river also raises some interesting questions in terms of efficiency and equity when property rights are not well defined. The sharing problem refers to a situation in which agents have unequal access to the resource depending their location on the river. The most upstream agent have a full access but as the river flow goes down downstream agents get the remaining water left by upstream agents who are located in front of them. Game theoretic models and in particular bargaining models are relevant to deal with such a problem. The literature on water allocation can be classified into two broad approaches.

The first approach is based on cooperative game theory and, more precisely, on the core. The objective is to set up a burden-sharing rule able to favor the cooperation of all, ensuring that the rule prevents that any individual agent, but also any sub-group of agents, from leaving the agreement. Based on a model taking explicitly into account directional flows, Ambec and Sprumont (2002) [1] show that the convexity of the cooperative game ensures a non-empty core. They analyze how a compromise solution between the two international law principles of water sharing in transboundary river basins can be reached in a cooperative game with utility transfers. These two principles are the *absolute territorial sovereignty* (ATS) that prescribes that each agent is free to use all the water he controls on his territory and the *unlimited territorial integrity* (UTI) that states that the amount of available water to an agent cannot be altered by all the agents who are located upstream from his location. An upstream agent is only allowed to consume water if he has the explicit consent of all his downstream agents.

1.3.1 Literature Review

Houba *et al.* (2014) [28] consider a strict interpretation of the UTI rule stating that only the most downstream agent may claim all the water and can restrict all his predecessors to zero extraction as long as no agreement has been reached. Ansink and Weikard (2012a) [4] modeled the river sharing problem as a sequential bankruptcy game in which the sum of the claims of all the agents exceed the availability of the resource (Aumann and Maschler, (1985) [5] and Thomson (2013) [54] for a recent survey). They analyze several sharing

rules specified in terms of amount of water given to each agent. When all the water originates at the head of the river, the sharing rule states that each agent gets the same proportion of his claim and the linear order of the agents does not matter. Otherwise both the distribution of claims and water endowments needed to be considered. Houba *et al.* (2014) [28] show that the most downstream agent always prefers the UTI principle but at least one of the other agent prefers the ATS rule.

The second approach is based on non-cooperative game theory, also with several categories. Ansink *et al.* (2012b) [2] modeled the river sharing problem as a two-stage open-membership cartel game using the concept of internal and external stability (d'Aspremont *et al.*, (1983) [15]). Assuming that a river agreement is a group of agents (a coalition) who have merged and maximize their welfare jointly, their goal is to determine which coalitions are stable, in the sense that no agent wants to leave it or join it. Assuming that agents have identical benefit functions and only differ in their location along the river, results show that a coalition of at least four agents is not stable. Ansink and Weikard (2009) [3] modeled the river sharing problem as a contested game over the property rights in which two agents can decide to bargain or not. In the latter case agents use their outside options by asking a third agent to implement an equitable solution. Results show that agents can end up in an inefficient equilibrium instead of bargaining an efficient outcome. Carraro *et al.* (2007) [10] review several bargaining models to water issues in order to show how an agreement is reached among sectors or countries. They emphasize complex negotiation problems dealing with multilateral and multi-issues features that can only be solved by use of computer simulations. In an alternating-offer Rubinstein model, Houba (2008) [27] interprets the model of Ambec and Sprumont in a bargaining perspective but only in a bilateral case with one upstream agent and one downstream agent.

Most of these approaches and models above mentioned have in common to partly abstract from the negotiating process. This paper uses a simplified Rubinstein model with three agents who can bargain sequentially with endogenous disagreement points. Under the ATS rule, the most downstream agent is constrained in his water consumption while under the UTI rule, the most upstream player can not extract his optimal level

of consumption. Hence negotiation can take place between agents but is not limited to neighboring agents as in Wang (2011) [56]. In modeling terms, the contribution of this paper is to emphasize the role of inside options in a Rubinstein framework with several sequential bargaining procedures. Inside options refer to the payoffs that agents obtain when they temporarily disagree. By assuming in each case that delay between negotiation rounds vanishes, the goals of the paper are to explain the source of possible inefficiencies and the different negotiation outcomes that can result from sequential bargaining. We consider three different negotiation procedures. The first procedure refers to a *simultaneous* negotiation in which the most downstream agent bargains with each upstream agent over water extraction and transfers (side payments). The second procedure assumes that the constrained agent bargains sequentially with both upstream agents. The last procedure considers that the an agent who doesn't suffer from scarcity (the most upstream under ATS, the most downstream under UTI) bargains sequentially with the other agents according to their location. In each case, the outcome in terms of water extraction and transfers is analyzed and compared to the social optimum, showing that with both sequential procedures, and regardless of the property rights division, the outcome is inefficient from the point of view of society. The intuition for this inefficiency comes from the fact that in our game agents bargain over transfers *and* water consumption levels. Thus, not only the distribution of the net surplus is different under different protocols, the water extraction level and, hence, the net surplus itself, is also different. We also show that the inefficiency in the sequential procedures comes from the inside options. Finally, we show that, depending on the sequence of moves in the sequential negotiation, inside options can strengthen or weaken the relative position of the agents involved in the negotiation.

The following subsections present the non cooperative and the cooperative outcomes so as the simultaneous negotiation. Subsection 1.3.3 is devoted to the analysis of sequential negotiations according to the two protocols. In section 1.3.4 a specific example shows the main results of the paper. The last subsection presents the concluding remarks on the River bargaining problem.

1.3.2 The benchmark

Consider a set of $N = \{1, 2, \dots, i, \dots, n\}$ agents located along a river in a lexicographic ordering. Agent 1 is the most upstream agent while agent n is located at the mouth of the river. Each agent extracts an amount of water $x_i \geq 0$ from which he earns $B_i(x_i), \forall i \in N$. The benefit function is assumed to be increasing up to a maximum value equal to \bar{x}_i solution of $B'_i(x_i) = 0$ and decreasing for greater extractions, formally: $\bar{x}_i = \arg \max_i B_i(x_i)$ is the satiation point of the agent i . The players are said to be homogeneous if every benefit functions are identical, that is $\forall \{i_1, i_2\} \in N, B_{i_1}(x_{i_1}) = B_{i_2}(x_{i_2})$. The amount of water available is E . We consider the river merely flows at its own source and the absence of intermediary inflows. We denote by $i_1 < i_2$ the fact that agent i_1 is upstream of agent i_2 . Following notation of Ansink and Weikard (2009), $U_{i_1} = \{i_2 \in N : i_2 < i_1\}$ stands for the set of agents located upstream of the location of player i_1 and reciprocally $D_{i_1} = \{i_3 \in N : i_1 < i_3\}$.

Non cooperative outcome

Under the *Absolute territorial sovereignty* principle, each agent i controls the amount of water $E - \sum_{j \in U_i} x_j$ that hasn't been consumed by the players upstream of the location of i . In the non cooperative case, the most upstream agent chooses how much to extract, under the constraint that this level does not exceed the available amount E . Then, the following agent chooses a level of extraction from the remaining water $E - x_1$. This process goes up to the most downstream agent n . The sub-game perfect equilibria (SPE) of this game shows that each agent i extracts the maximum between his non cooperative level \bar{x}_i and the amount of water he controls: $E - \sum_{j \in U_i} x_j$. The non cooperative set of strategies for $i \in N$ under the ATS principle is the following:

$$S_i = \begin{cases} \bar{x}_i & \text{if } \bar{x}_i \leq E - \sum_{j \in U_i} \bar{x}_j \\ E - \sum_{j \in U_i} \bar{x}_j & \text{if } \bar{x}_i > E - \sum_{j \in U_i} \bar{x}_j \\ 0 & \text{otherwise.} \end{cases}$$

From upstream to downstream, the players extract their best response up to a player who suffers from scarcity. If $\bar{x}_{i_2} \geq E - \sum_{i_1 \in U_{i_2}} \bar{x}_{i_1}$, the subgame perfect equilibrium (SPE) of the game returns a unique vector of extractions:

$$X_{ATS}^{nc} = \begin{cases} \bar{x}_{i_1} & \text{if } i_1 < i_2 \\ E - \sum_{i_1 \in U_{i_2}} \bar{x}_{i_1} & \text{for } i_2 \\ 0 & \text{if } i_3 > i_2 \end{cases}$$

The benefits B_i^{nc} for all $i \in N$ can be derived from the water extractions levels.

Under the *Unlimited Territorial Integrity* principle, the most downstream player can claim to consume an unaltered water. The most downstream agents can claim to consume or extract any level of available water. The most downstream agent chooses his level of extraction, ie $\bar{x}_n = \arg \max_{x_n} B(x_n)$ if $\bar{x}_n < E$. Thus, a part of the resource is clear of property rights. The players upstream can extract a part of the resource provided that the UTI principle still applies, that is: x_n^* is at least flowing on the territory of the most downstream agent. Thus, upstream players apply the following rationale: Player $n - 1$ plays his best response regarding the amount of water his allowed to extract. If $\bar{x}_{n-1} \leq E - \bar{x}_n$, then $x_{n-1}^{nc} = \bar{x}_{n-1}$ is exactly pumped at level $n - 1$. This extraction behavior is enforced by the players all along the river up to the most upstream agent. The non cooperative set of strategies under the UTI principle for $i \in N$ is the following:

$$S_i = \begin{cases} \bar{x}_i & \text{if } \bar{x}_i \leq E - \sum_{j \in D_i} \bar{x}_j \\ E - \sum_{j \in D_i} \bar{x}_j & \text{if } \bar{x}_i > E - \sum_{j \in D_i} \bar{x}_j \\ 0 & \text{otherwise.} \end{cases}$$

From downstream to upstream, the players extract their best response up to a player who suffers from scarcity. If $\bar{x}_{i_2} \geq E - \sum_{i_3 \in D_{i_2}} \bar{x}_{i_3}$, the subgame perfect equilibrium (SPE)

of the game returns a unique vector of extractions:

$$X_{UTI}^{nc} = \begin{cases} 0 & \text{if } i_1 < i_2 \\ E - \sum_{i_3 \in D_{i_2}} \bar{x}_{i_3} & \text{for } \varepsilon_2 \\ \bar{x}_{i_3} & i_3 > i_2 \end{cases}$$

A non cooperative game Γ_P^{nc} under the principle P is a mapping of the ordering of the players onto the individual levels of extraction. The level of extractions (and thus the levels of individual benefits) depends on the ordering of players and the definition of property rights. Let N' be the reversed ordering of N . The following proposition is straightforward and is presented without proof:

Proposition 4 *If $B_1 = B_n$, $\Gamma_{ATS}^{nc}(N) = \Gamma_{UTI}^{nc}(N')$ and $\Gamma_{UTI}^{nc}(N) = \Gamma_{ATS}^{nc}(N')$.*

If the players who suffer from scarcity are identical (player 1 under UTI and player n under ATS), the level of non cooperative extractions are identical for the ordering N under ATS and for the reversed ordering under UTI. If players are heterogeneous, the solution differs conforming to the concavity of the benefit functions.

Cooperative outcome

The cooperative solution is given by the program:

$$\max_{x_1, \dots, x_n} \sum_i B_i(x_i) \quad \text{sc} \quad \sum_i^n x_i \leq E$$

Water extraction levels $X^c = \{x_1^c, \dots, x_n^c\}$ are solution of

$$B'_1(x_1) = \dots = B'_n(x_n)$$

with $x_n = E - \sum_{i \in U_i} x_i$. Efficiency requires the estate E to be shared so as to equalize the marginal benefits among agents. It is straightforward that the cooperative outcome doesn't depend on the principle and that $\Gamma_{ATS}^{nc}(\cdot) = \Gamma_{UTI}^{nc}(\cdot)$ holds for any ordering. Note that $x_{i_1}^{nc} \geq x_{i_1}^c$ for $i_1 \in U_{i_2}$ under the ATS principle and for $i_3 \in D_{i_2}$ under UTI. The

obverse occurs for the remaining players: $x_{i_3}^c \geq x_{i_3}^{nc}$ for $i_3 \in D_{i_2} \cup \{i_2\}$ under ATS and for $i_1 \in U_{i_2} \cup \{i_2\}$ under UTI. This has direct implications in terms of profit, we note that $B_{i_1}^{nc} \geq B_{i_1}^c$ for $i_1 \in U_{i_2}$ under ATS and for $i_3 \in D_{i_2}$ under UTI. The Cooperative outcome has increased the net benefit of players $i_3 \in D_{i_2} \cup \{i_2\}$ under the ATS and $i_1 \in U_{i_2} \cup \{i_2\}$ under the UTI principle.

So as to decentralize the non cooperative outcome toward the cooperative solution under the ATS principle (resp UTI principle), a incentive-based mechanism must be implemented. A monetary compensation is necessary to encourage the most upstream (resp downstream) agents to refrain from overconsuming water. This monetary compensation can be generated by the higher level of benefit associated to a higher water consumption of the most downstream (resp upstream) agents. The strict concavity of the benefit function up to the maximum point ensures that the loss incurred by the upstream (resp downstream) agent will be lower than the gain obtained by the downstream (resp upstream) agent. Thus, there must exist a bargaining scheme that ensures that the agents will individually benefit from decentralizing the non cooperative outcome towards an alternative solution. We now detail in the next section three bargaining protocols.

1.3.3 Negotiation protocols

We restrict our attention to games for which there is a scarcity constraint, that is games for which $\forall j \in \{1, n\}, \sum_{i \in N \setminus \{j\}} \bar{x}_i \leq E \leq \sum_{i \in N} \bar{x}_i$. We assume that negotiations are bilateral. To simplify notation we write: $B_i = B_i(x_i)$. A negotiation between agents i and j has two arguments. A monetary transfers or side-payments in exchange for a reduction of water consumption. The agent j proposes the monetary transfer τ_{ij} to the agent i in exchange for his commitment to reduce his consumption to a level $x_i < x_i^{nc}$. Alternatively, the agent i could propose the player j to reduce his extraction to a level $x_j < x_j^{nc}$ in exchange for the subvention τ_{ji} . Note that $\tau_{ij} = -\tau_{ji}$ and by convention the first player in the subscript pays the transfer to the second (If i pays $-\tau_{ij}$ then he receives τ_{ji}). An

offer or a proposal is a couple denoted by $o_{ij} = (x, \tau)$ with

$$(x, \tau) = \begin{cases} (x_i, \tau_{ji}) & \text{if } -\frac{\partial(B_i+B_j)}{\partial x_i} > 0 \\ (x_j, \tau_{ij}) & \text{if } -\frac{\partial(B_i+B_j)}{\partial x_j} > 0 \end{cases}$$

If the offer is made by i to j and is about an abatement in x_i , then player j increases his consumption in exchange for the transfer τ_{ji} . Bargaining rounds assumes perfect information.

We also assume that the most upstream and the most downstream agents are the only agenda-setters in the negotiation process. Depending on the principle, either the most upstream player or the most downstream player suffers from the scarcity constraint. If not mentioned otherwise, we assume that the player who suffer from the scarcity constraint enters negotiations as the first proposer. That is player 1 and n have the right to make offers as proposer to all the other agents. Players in between only acts as recipients of the proposals. Thus, a 3 players setting is sufficient to put forward the main idea of this paper with 1 or 3 acting as proposers and 2 can thus be seen as representative of players in between.

If player 3 is the proposer (as in the ATS principle), then the general net payoff function (after transfers) for the 3 agents of an agreement reached at period t is:

$$\pi_1(o_{31}; t) = B_1^{nc} + \delta_1^t (B_1 - B_1^{nc} - \tau_{13}) \quad (1.7)$$

$$\pi_2(o_{32}; t) = B_2^{nc} + \delta_2^t (B_2 - B_2^{nc} - \tau_{23}) \quad (1.8)$$

$$\pi_3(o_{31}, o_{32}; t) = B_3^{nc} + \delta_3^t (B_3 - B_3^{nc} + \tau_{13} + \tau_{23}) \quad (1.9)$$

where $0 \leq \delta_i \leq 1$ for $i = \{1, 2, 3\}$ stands for the discount rate, and $t = \{0, 1, \dots\}$ are the periods at which the offers and counter-offers are formulated. A permutation of 3 and 1 in the equations 1.7 to 1.9 returns the payoff under the UTI principle when player 1 is the first proposer. Negotiations can occur simultaneously or in a sequential manner.

Simultaneous negotiation

We set $N = \{i, j, k\}$ without ordering. We assume that the agent who suffers from scarcity initiates the negotiations with the two remaining player. The proposer is identified by i and $x_i = E - x_j - x_k$. Thus i bilaterally negotiates at the same time with k over $o_{ik} = (x_k, \tau_{ik})$ and with j over $o_{ij} = (x_j, \tau_{jk})$. We assume that k is the central player, thus the absence of bargaining between j and k , implies that $\tau_{jk} = 0$.

Let $o_{ij}^{(i)} = (x_j^{(i)}, \tau_{ji}^{(i)})$ be the offer made by i to j . The Rubinstein Bargaining Solution (RBS) that we are looking for is the unique SPE given by the following conditions (Rubinstein, 1982; Muthoo, 1999):

$$\pi_i(o_{ij}^{(j)}, o_{ik}; 0) = \pi_i(o_{ij}^{(i)}, o_{ik}; 1) \text{ and } \pi_j(o_{ij}^{(i)}, o_{ij}; 0) = \pi_j(o_{ij}^{(j)}, o_{ik}; 1).$$

and between i and k :

$$\pi_i(o_{ik}^{(k)}, o_{ij}; 0) = \pi_i(o_{ik}^{(i)}, o_{ij}; 1) \text{ and } \pi_k(o_{ik}^{(i)}, o_{ij}; 0) = \pi_k(o_{ik}^{(k)}, o_{ij}; 1).$$

These two systems of indifference equations states that each agent is indifferent between accepting the current offer of his opponent and making a counteroffer in the next period which will be accepted.

We can now state the following Proposition:

Proposition 5 *The solution of the games $\Gamma_{ATS}^{sim}(j, k, i)$ and $\Gamma_{UTI}^{sim}(i, k, j)$ returns the unique optimal vector of extractions $x^c = \{x_i^c, x_j^c, x_k^c\}$, the highest level of wealth $\Pi = \sum_{i' \in N} (B_{i'}^c - B_{i'}^{nc}) > 0$ and the payoffs after transfers are:*

$$\begin{aligned} \pi_j^* &= B_j^{nc} + \mu_j \Pi \\ \pi_k^* &= B_k^{nc} + \mu_k \Pi \\ \pi_i^* &= B_i^{nc} + (1 - \mu_j - \mu_k) \Pi \end{aligned}$$

with $\mu_j = \frac{(1-\delta_i)(1-\delta_k)\delta_j}{(1-\delta_j\delta_i)(1-\delta_k\delta_i)-\delta_j\delta_k(1-\delta_i)^2}$ and similarly for k .

Proof. see proof 1.4.5. ■

In both cases, the player i who suffers from the scarcity constraints begins the negotiations with players j and k . In the limit case $\delta \rightarrow 1$, each agent gets his impasse point (permanent disagreement payoff) plus an equal share (1/3) of the total created surplus. This simultaneous negotiation accounts for the benchmark case for the sequential bargaining procedures. Under the ATS principle, $\{1, 2, 3\} = \{j, k, i\}$ and player 3 pays a transfer to agents 1 and 2 in exchange for a reduced water extraction. Under the UTI principle, $\{1, 2, 3\} = \{i, k, j\}$ and player 1 pays a transfer to agents 2 and 3 in exchange for an abatement in water consumption.

Once again, if $B_i = B_j$, the Proposition 4 still applies. The solution is sensitive to the concavity of the benefit functions otherwise.

Sequential negotiation

Based on the same set of payoffs (1.7)-(1.9), we analyzed two bargaining procedures that we called the *downstream* and the *upstream procedure*. In the downstream procedure, the most downstream player is assumed to bargain twice (regardless he is the player who suffers from scarcity). In the first round of negotiation⁵, if $j < k < i$, then i and k bargain over (x_k, τ_{ik}) . In the second round, i bargains with j over (x_j, τ_{ij}) . Still, there is no negotiation between j and k . In the upstream procedure, the most upstream player bargains with the two other players in two different rounds. If $j < k < i$, then j bargains with k over $(x_j(x_k), \tau_{jk})$ in the first round and with i in the second round over (x_j, τ_{ij}) . The solutions are found using backward induction in both cases. In both procedures, the second round of negotiations runs knowing that an agreement has been reached in the first stage. The negotiations remain bilateral. We also assume that the discount factor for the intra-negotiations is identical in the two rounds. We set the inter-negotiations discount factor to unity. However as we consider results in the limit case when the delay between negotiation rounds vanishes, this assumption is not restrictive.

⁵It is crucial to distinguish the intra-negotiations between two players (sequences of bilateral offers and counter offer) and the inter-negotiations (sequences of negotiations involving different players). In each round of negotiations, the proposer bargains with a different player.

Asking for water In this first sequential protocol, we assume that the player who suffers from scarcity (i) bargains twice as a proposer. In round two, i bargains with j knowing that an agreement has been reached in round one with k and this agreement will be effective even in the case of a disagreement in this current round. This first round agreement refers to the inside option for agent i . It determines his impasse point.

Proposition 6 *The Rubinstein bargaining solution shows that when i bargains with k in round 1 and with j in round 2,*

1. Water extractions (x_j^*, x_k^*) are solution of:

$$\begin{aligned} B'_j(x_j) &= B'_i(x_i) \\ B'_k(x_k) &= B'_i(x_i) + \frac{\delta_j(1-\delta_i)}{(1-\delta_j\delta_i)} (B'_i(x_i^d) - B'_i(x_i)) \end{aligned}$$

with $x_i^d = E - x_j^{nc} - x_k$

2. Equilibrium payoffs after transfers are:

$$\begin{aligned} \pi_j^* &= B_j^{nc} + \mu_j \Pi' - (1 - \psi_k - \psi_i) [(B_i^d - B_i^{nc}) - (B_k^{nc} - B_k^*)] \\ \pi_k^* &= B_k^{nc} + \mu_k \Pi' + \psi_k [(B_i^d - B_i^{nc}) - (B_k^{nc} - B_k^*)] \\ \pi_i^* &= B_i^{nc} + (1 - \mu_j - \mu_k) \Pi' + \psi_i [(B_i^d - B_i^{nc}) - (B_k^{nc} - B_k^*)] \end{aligned}$$

with $\Pi' = B_i^* - B_i^{nc} + B_j^* - B_j^{nc} + B_k^* - B_k^{nc} > 0$, is the additional surplus generated the downstream procedure, $B_i^d = B_i(x_i^d)$ and the coefficients $\mu_j = \frac{\delta_j(1-\delta_i)}{(1-\delta_j\delta_i)}$, $\mu_k = \frac{\delta_k(1-\delta_i)(1-\delta_j)}{(1-\delta_k\delta_i)(1-\delta_j\delta_i)}$, $\psi_k = \frac{\delta_j\delta_k(1-\delta_i)^2}{(1-\delta_j\delta_i)(1-\delta_k\delta_i)}$ and $\psi_i = \frac{\delta_j(1-\delta_j)(1-\delta_i)}{(1-\delta_k\delta_i)(1-\delta_j\delta_i)}$.

Proof. see proof 1.4.6. ■

Water extractions given by proposition 6 do not maximize social welfare ($x_i^* \neq x_i^c \forall i$). It implies that the net surplus is lower in the sequential case than in the cooperative case. The intuition for this inefficiency comes from the fact that an alternative bargaining protocol does not only modifies the distribution of the net surplus, but also the net surplus itself. Indeed, agents are bargaining over transfers and water extractions, thus the seize of

the net surplus is also determined by the negotiations. The optimality conditions suggest that the player j extracts an intermediary level of water between the cooperative solution and the non cooperative solution: $x_j^{nc} > x_j^* > x_j^c$. The intermediary player reached his lowest level for this protocol: $x_k^* < x_k^c < x_k^{nc}$ while the negotiations allowed i to get his highest level of extractions $x_i^* > x_i^c > x_i^{nc}$.

In the first round, i bargained a low water extraction level or equivalently a high reduction with respect to his initial situation with the player k . The abatement ($x_k^{nc} - x_k^*$) is sufficiently high, the associated transfer is high. Even costly, this outcome is worthy for the player i . Indeed, an agreement with the intermediary player helps securing his position when it comes to enter a bargaining process with the most upstream player. A strengthened position (alternatively, a higher impasse point), would lead to milder consequences if a disagreement with 1 had to occur.

Based on this agreement, i negotiates in the second round with k over a lower reduction effort and a lower transfer. There is not much left to be bargained since most of the exchange gains have been generated in the first round between the players k and i .

Eventually, both k and i are better off at the expense of j , which is only involved in the second round of the negotiation. In terms of payoffs each agent gets a share of the created surplus but even if j gets a higher share ($1/2$ in the limit $\delta \rightarrow 1$ and $1/4$ for k and i), his payoff is reduced by an amount measured by the positive term into brackets ($B_i^d - B_i^{nc}$) $>$ ($B_k^{nc} - B_k^*$). This loss of payoff for j is a gain for k and i . Since the problem is symmetric, a first round between i and j (instead of k) would imply the term ($B_i^d - B_i^{nc}$) $>$ ($B_j^{nc} - B_j^*$). The relative importance of these terms can be used by i to decide whether to bargain first with j or with k , showing that the order of the partners in the sequential process becomes a strategic variable. The first protocol exhibits that being involved in the first round of negotiation is auspicious.

Setting $N = \{j, k, i\}$, Proposition 6 refers to the downstream-sequential protocol under ATS. In this protocol; $i = 3$ suffers from scarcity and bargains with 2 and 1 over an abatement in x_1 x_2 in exchange for monetary transfers. We refer this game to $\Gamma_{ATS}^{down}(j, k, i)$.

Setting $N = \{i, k, j\}$ Proposition 6 refers to the upstream-sequential protocol under UTI. In this protocol; $i = 1$ suffers from scarcity and bargains with 2 and 3 over an abatement in $x_2 x_3$ in exchange for monetary transfers. We refer this game to $\Gamma_{UTI}^{up}(i, k, j)$. These two games are identical for a permutation of players if players are homogeneous while Π' differs in these two games otherwise.

Asking for a transfer Alternatively, we propose the obverse of the previous protocol. In the second protocol, the player who owns property rights over the estate (1 under ATS and 3 under UTI) bargains twice as a proposer. In round two j bargains with i knowing that an agreement has been reached in the first round with k .

Proposition 7 *The Rubinstein bargaining solution shows that when j bargains with k in round 1 and with i in round 2*

1. Water extractions (x_i^*, x_j^*) are solution of

$$\begin{aligned} B'_j(x_j) &= B'_i(x_i) \\ x_k &= x_k^{nc} \end{aligned}$$

2. Equilibrium payoffs after transfers are

$$\begin{aligned} \pi_j^* &= B_j^{nc} + \mu_j \Pi'' \\ \pi_k^* &= B_k^{nc} + \mu_k \Pi'' \\ \pi_i^* &= B_i^{nc} + (1 - \mu_j - \mu_k) \Pi'' \end{aligned}$$

with $\Pi'' = B_i^* - B_i^{nc} + B_j^* - B_j^{nc} > 0$ is the additional surplus from the upstream bargaining protocol compared to the non cooperative outcome. $\mu_i = \frac{(1-\delta_k)(1-\delta_j)}{(1-\delta_k\delta_i)(1-\delta_i\delta_j)}$ and $\mu_k = \frac{\delta_2(1-\delta_i)(1-\delta_j)}{(1-\delta_k\delta_i)(1-\delta_i\delta_j)}$ the sharing parameters.

Proof. see proof 1.4.7. ■

The second protocol also implies that water extractions are not social welfare maximizing. The player k behaves consonantly to the non-cooperative case, indeed: $x_k^* = x_k^{nc}$.

It turns out that $x_i^* < x_i^c$ and $x_j^* < x_j^c$. Analogously to the case of asking for water, each agent gets its impasse point and an additional share of the net surplus. In contrast to the previous protocol, it turns out that being involved in the second round is now opportune. The reason is that the impasse point of j in the second round weakens his relative position in regards to player i . In the event of a perpetual disagreement with i , agent j will consume x_j^{nc} , but will bear the cost of the transfer to k ($\tau_{jk} > 0$). For that reason, j is urged to sign an agreement with i over a low level of water extraction ($x_j^c > x_j^*$) or a high reduction ($x_j^{nc} - x_j^*$), implying a high value ($B_j^{nc} - B_j^*$), which increases the transfer τ_{ij} paid by j and decreases the transfer τ_{jk} .

In the limit $\delta \rightarrow 1$, agents j and k get the same share of the created surplus $\pi_j - B_j^{nc} = \pi_k - B_k^{nc} = \frac{1}{4}\Pi''$, and i is better off $\pi_i = B_i^{nc} + \frac{1}{2}\Pi''$. Asking for a transfer favors the agent who suffers from scarcity and ensures an equal treatment for the remaining agents since j and k get an identical share of the created surplus.

Setting $N = \{j, k, i\}$, Proposition 7 refers to the upstream-sequential protocol under ATS. In this protocol; $j = 1$ bargains with 2 and 3 over an abatement in x_1 and x_2 in exchange for monetary transfers. We refer this game to $\Gamma_{ATS}^{up}(j, k, i)$.

Setting $N = \{i, k, j\}$ Proposition 7 refers to the downstream-sequential protocol under UTI. In this protocol; $i = 1$ suffers from scarcity and $j = 3$ bargains with 1 and 2 over an abatement in x_2 x_3 in exchange for monetary transfers. We refer this game to $\Gamma_{UTI}^{down}(i, k, j)$. Once again, if players are homogeneous, Π'' is identical in both games and $\Gamma_{ATS}^{up}(j, k, i) = \Gamma_{UTI}^{down}(i, k, j)$.

In order to compare the upstream-sequential and the downstream-sequential protocol, we provided an example hereafter.

1.3.4 Example

Let us illustrate the outcomes of the different bargaining protocols discussed above with an example. Assume the following benefit function as in Ambec and Sprumont (2000)

$$B_i = ax_i - \frac{b_i}{2}x_i^2$$

In the non cooperative case, the optimality condition $B'_i = 0$ implies $x_i^{nc} = a/b_i$.

Players are ranked such that $\bar{x}_2 = \lambda\bar{x}_1$ and $\bar{x}_3 = \lambda^2\bar{x}_1$ with $\lambda > 0$. For $\lambda < 1$, $\bar{x}_1 > \bar{x}_2 > \bar{x}_3$ and for $\lambda > 1$, $\bar{x}_1 < \bar{x}_2 < \bar{x}_3$. It implies $\lambda b_2 = b_1$ and $\lambda^2 b_3 = b_1$. Players are homogeneous for $\lambda = 1$.

To ensure negotiations we set $(1 + \lambda + \lambda^2)\bar{x}_1 > E$ (scarcity constraint) and $E > (1 + \lambda)\bar{x}_1$ under ATS. The first player can extract $x_1^{nc} = \bar{x}_1$. The results are given in the Table 1.2 with respect to: $\Pi = b_1 ((1 + \lambda + \lambda^2) x_1^{nc} - E)^2 > 0$ and

$$\varphi = \frac{4\lambda^2(4\lambda^8+24\lambda^7+40\lambda^6+44\lambda^5+40\lambda^4+24\lambda^3+13\lambda^2+4\lambda+1)}{(4\lambda^6+12\lambda^5+16\lambda^4+16\lambda^3+10\lambda^2+4\lambda+1)^2}$$

$$\psi = \frac{(16\lambda^{11}+88\lambda^{10}+240\lambda^9+400\lambda^8+484\lambda^7+452\lambda^6+332\lambda^5+194\lambda^4+88\lambda^3+30\lambda^2+7\lambda+1)}{2(4\lambda^6+12\lambda^5+16\lambda^4+16\lambda^3+10\lambda^2+4\lambda+1)^2}$$

with $\psi > \varphi$

Protocol

	SIM ^(sim-ats)	Downstream ^(D-ats)	Upstream ^(U-ats)
x_1	$\frac{1}{\lambda^2 + \lambda + 1}E$	$\frac{x_1^{nc} + 2\lambda E}{2\lambda^3 + 2\lambda^2 + 2\lambda + 1}$	$d\frac{E - \lambda x_1^{nc}}{\lambda^2 + 1}$
x_2	$\lambda x_1^{sim-ats}$	$\frac{(1 + 2\lambda^2)E - (1 + \lambda^2)x_1^{nc}}{2\lambda^3 + 2\lambda^2 + 2\lambda + 1}$	λx_1^{nc}
x_3	$\lambda^2 x_1^{sim-ats}$	$\lambda^2 x_1^{D-ats}$	$\lambda^2 x_1^{U-ats}$
$\pi_1 - B_1^{nc}$	$\frac{(\lambda + 1)}{6\lambda^2(\lambda^2 + \lambda + 1)}\Pi$	$\frac{\varphi}{2\lambda^2}\Pi$	$\frac{1}{8\lambda^2(\lambda^2 + \lambda + 1)}\Pi$
$\pi_2 - B_2^{nc}$	$\pi_1^{sim-ats} - B_1^{nc}$	$\frac{\psi}{2\lambda^2}\Pi$	$\pi_1^{U-ats} - B_1^{nc}$
$\pi_3 - B_3^{nc}$	$\pi_1^{sim-ats} - B_1^{nc}$	$\pi_2^{D-ats} - B_2^{nc}$	$2(\pi_1^{U-ats} - B_1^{nc})$

Table 1.2: Solutions under ATS

It can be show that in terms of water extraction that $x_1^{D-ats} > x_1^{sim-ats} > x_1^{U-ats}$, $x_2^{U-ats} > x_2^{sim-ats} > x_2^{D-ats}$ and $x_3^{D-ats} > x_3^{sim-ats} > x_3^{U-ats}$ and in terms of payoffs $\pi_1^{sim-ats} > \pi_1^{U-ats} > \pi_1^{D-ats}$, $\pi_2^{D-ats} > \pi_2^{sim-ats} > \pi_2^{U-ats}$ and $\pi_3^{U-ats} > \pi_3^{D-ats} > \pi_3^{sim-ats}$.

This example shows that the most upstream agent prefers the simultaneous negotiation while the most downstream agent prefers the upstream-sequential negotiation and the

Protocol

	SIM ^(sim-uti)	Downstream ^(D-uti)	Upstream ^(U-uti)
x_1	$\frac{1}{\lambda^2} x_3^{sim-uti}$	$\frac{1}{\lambda^2} x_3^{D-uti}$	$\frac{1}{\lambda^2} x_3^{U-uti}$
x_2	$\frac{1}{\lambda} x_3^{sim-uti}$	$\frac{1}{\lambda} x_3^{nc}$	$\lambda \frac{(\lambda^2 + 2)E - (\lambda^2 + 1)x_3^{nc}}{\lambda^3 + 2\lambda^2 + 2\lambda + 2}$
x_3	$\frac{\lambda^2}{\lambda^2 + \lambda + 1} E$	$\frac{\lambda}{\lambda^2 + 1} (\lambda E - x_3^{nc})$	$\lambda^2 \frac{\lambda x_3^{nc} + 2E}{\lambda^3 + 2\lambda^2 + 2\lambda + 2}$
$\pi_1 - B_1^{nc}$	$\pi_3^{sim-uti} - B_3^{nc}$	$\frac{1}{2} (\pi_3^{D-uti} - B_3^{nc})$	$\pi_2^{U-uti} - B_2^{nc}$
$\pi_2 - B_2^{nc}$	$\pi_3^{sim-uti} - B_3^{nc}$	$\pi_3^{D-uti} - B_3^{nc}$	$\frac{\lambda^2 + \lambda + 2}{(4\lambda)(\lambda^3 + 2\lambda^2 + 2\lambda + 2)} \Pi$
$\pi_3 - B_3^{nc}$	$\frac{1}{3} \frac{\lambda^3(\lambda+1)}{(\lambda^2+\lambda+1)} \Pi$	$\frac{1}{8\lambda^2 + 8} \Pi$	$\frac{\lambda^2 + 1}{(\lambda^3 + 2\lambda^2 + 2\lambda + 2)^2} \Pi$

Table 1.3: Solutions under UTI

middle agent the downstream-sequential negotiation.

Under UTI, we set $(\frac{1}{\lambda^2} + \frac{1}{\lambda} + 1) \bar{x}_3 > E$ (scarcity constraint) and $E > (\frac{1}{\lambda} + 1) \bar{x}_3$, the last player can extract $x_3^{nc} = \bar{x}_3$. The results are given in Table 1.3.

We can note that there is a relation between the additional generated surplus under ATS and UTI, indeed, $\lambda^3 S^{sim-ats} = S^{sim-uti}$ are respectively the generated surplus in both cases. Once again, the additional surplus is identical in the presence of homogeneous players. If negotiations are simultaneous, the ATS procedure generates a greater surplus for $\lambda > 1$. In other words, enforcing the ATS principle brings a greater surplus if extraction costs increases as the river goes downstream. This makes sense as the ATS principle tends to favor the players upstream. Conversely, if extraction costs decrease as the river goes downstream, then $\Pi_{UTI}^{sim} > \Pi_{ATS}^{sim}$ since $\lambda < 1$.

It can be show that in terms of water extraction that $x_1^{U-uti} > x_1^{D-uti} > x_1^{sim-uti}$, $x_2^{D-uti} > x_2^{sim-uti} > x_2^{U-uti}$, $x_3^{U-uti} > x_3^{D-uti} > x_3^{sim-uti}$. In terms of payoffs, there is an identical ordering for the the three players $\pi^{sim-uti} > \pi^{D-uti} > \pi^{U-uti}$.

1.3.5 Conclusions on the River bargaining problem

This article has analyzed a bargaining game over two variables, water extractions abatement in exchange for a monetary compensations, between three agents located along a river. Simultaneous and two sequential negotiation procedures have been considered. Results show that simultaneous bargaining leads to a global agreement which is optimal as social welfare is maximized. On the opposite, the upstream and the downstream sequential procedures lead to inefficient agreements for the society. A first contribution of the paper has been to explain the role of inside options in explaining this inefficiency by focusing on the case where the delay between offers and negotiation rounds vanishes. In sequential protocols, the outcome of the agreement in the first round determines the temporary disagreement payoff for the second round.

A second contribution has been to analyze how the the impasse points can strengthen or weaken the relative position of agents, depending on the sequence of moves. With the downstream-sequential procedure in ATS or the upstream-sequential procedure in UTI, the agent suffering from scarcity bargains with the other players in the order of their location. The impasse point of that player in the second round increases his relative bargaining position when facing the remaining agent in the second round. In the upstream-sequential case under ATS (resp. the downstream-sequential procedure under UTI), the most upstream (resp. downstream) agent bargains with the downstream (resp. upstream) agents relative to their position. Results show that the impasse point of the this agent in the second round weakens his relative bargaining position when it comes to bargain with the last player.

1.4 Appendix A

1.4.1 Proof of Proposition 1

We consider the single negotiation between 1 and 2 over the variable e_1 and the transfer τ_{12} before we generalize that result. Agent 1 makes the offer $o_{12}^{(1)} = (e_1^{(1)}, \tau_{12}^{(1)})$ and player 2 makes counteroffers over $o_{21}^{(2)} = (e_1^{(2)}, \tau_{12}^{(2)})$. The subgame perfect equilibrium (SPE)

offers solve the two indifference conditions (Muthoo [40]):

$$\begin{aligned}\pi_1(o_1^{(2)}, 0) &= \pi_1(o_1^{(1)}, 1) \\ \pi_2(o_1^{(1)}, o_2; 0) &= \pi_2(o_1^{(2)}, o_2; 1)\end{aligned}$$

so that each agent is indifferent between accepting the current offer of his opponent and making a counteroffer in the next period. Note that in this case, there is no bargaining between 2 and 3, thus $o_2 = \emptyset$. Specifically, the offers satisfy:

$$V_1(e_1^{(2)}) - \tau_{12}^{(2)} = (1 - \delta_1)\pi_1^{nc} + \delta_1 \left(V_1(e_1^{(1)}) - \tau_{12}^{(1)} \right) \quad (1.10)$$

$$V_2(e_1^{(1)}, e_2) + \tau_{12}^{(1)} = (1 - \delta_2)\pi_2^{nc} + \delta_2 \left(V_2(e_1^{(2)}, e_2) + \tau_{12}^{(2)} \right) \quad (1.11)$$

The optimal offer of agent 1 to agent 2 ($o_{12}^{(1)}$) has to maximize his payoff under the constraint (1.11) for 2. Substitute the expression of the transfer $\tau_{12}^{(1)}$; the offer of 1 satisfies:

$$B'_1(e_1) + B'_2(e_1 + e_2) = C'_1(e_1)$$

It can be shown that the optimal offer of 2 to 1 satisfies the same condition. Assume that 1 is the proposer; the equilibrium transfer is $\tau_{12}^* = \tau_{12}^{(1)}$: As we rewrite 1.10 and 1.11, we get:

$$\tau_{12}^{(2)} = V_1(e_1^{(2)}) - (1 - \delta_1)\pi_1^{nc} - \delta_1 \left(V_1(e_1^{(1)}) - \tau_{12}^{(1)} \right) \quad (1.12)$$

$$\tau_{12}^{(1)} = -V_2(e_1^{(1)}, e_2) + (1 - \delta_2)\pi_2^{nc} + \delta_2 \left(V_2(e_1^{(2)}, e_2) + \tau_{12}^{(2)} \right) \quad (1.13)$$

Substituting $\tau_{12}^{(2)}$ in $\tau_{12}^{(1)}$ and assuming that 1 is the proposer, the equilibrium transfer is $\tau_{12}^* = \tau_{12}^{(1)}$:

$$\tau_{12}^* = \frac{(1 - \delta_1)\delta_2}{(1 - \delta_1\delta_2)} (V_1^* - \pi_1^{nc}) - \frac{(1 - \delta_2)}{(1 - \delta_1\delta_2)} (V_2^* - \pi_2^{nc})$$

and the equilibrium payoffs for player 1 :

$$\begin{aligned}\pi_1^* &= \pi_1^{nc} + (V_1^* - \pi_1^{nc}) - \frac{(1 - \delta_1)\delta_2}{(1 - \delta_1\delta_2)} (V_1^* - \pi_1^{nc}) + \frac{(1 - \delta_2)}{(1 - \delta_1\delta_2)} (V_2^* - \pi_2^{nc}) \\ \pi_1^* &= \pi_1^{nc} + \frac{(1 - \delta_2)}{(1 - \delta_1\delta_2)} ((V_1^* - \pi_1^{nc}) + (V_2^* - \pi_2^{nc}))\end{aligned}$$

And player 2 :

$$\begin{aligned}\pi_2^* &= \pi_2^{nc} + (V_2^* - \pi_2^{nc}) + \frac{(1 - \delta_1)\delta_2}{(1 - \delta_1\delta_2)} (V_1^* - \pi_1^{nc}) - \frac{(1 - \delta_2)}{(1 - \delta_1\delta_2)} (V_2^* - \pi_2^{nc}) \\ \pi_2^* &= \pi_2^{nc} + \frac{(1 - \delta_1)\delta_2}{(1 - \delta_1\delta_2)} ((V_1^* - \pi_1^{nc}) + (V_2^* - \pi_2^{nc}))\end{aligned}$$

Agents 2 and 3 maximize their payoffs after transfers, leading to the optimality conditions:

$$B_2'(e_1 + e_2) = C_2'(e_2)$$

$$B_3'(e_1 + e_2 + e_3) = C_3'(e_3)$$

The bargaining involving 1 and 3 over (e_1, τ_{13}) with $\tau_{12} = \tau_{23} = 0$ is based on the same method. When the negotiation takes place between 2 and 3, the SPE offers solve the two indifference conditions for 2 $\pi_2(o_1, o_2^{(3)}; 0) = \pi_2(o_1, o_2^{(2)}; 1)$ and for 3 $\pi_3(o_1, o_2^{(2)}, e_3; 0) = \pi_3(o_1, o_2^{(3)}, e_3; 1)$. \square

1.4.2 Proof of Proposition 2

Assume that the bargaining takes place between i and j and between i and k (that is $\tau_{jk} = 0$). We set $e_{ij} = \{e_i \text{ if } i < j, e_j \text{ else}\}$.

The two indifference conditions (notations are restricted to the efforts at stake) give respectively:

$$V_j(e_{ij}^{(i)}) + \tau_{ij}^{(i)} = (1 - \delta_j) \pi_j^{nc} + \delta_j (V_j(e_{ij}^{(j)}) + \tau_{ij}^{(j)}), \quad (1.14a)$$

$$V_i(e_{ij}^{(j)}, e_{ik}) - \tau_{ij}^{(j)} - \tau_{ik} = (1 - \delta_i) \pi_i^{nc} + \delta_i (V_i(e_{ij}^{(i)}, e_{ik}) - \tau_{ij}^{(i)} - \tau_{ik}). \quad (1.14b)$$

and

$$V_k \left(e_{ik}^{(i)} \right) + \tau_{ik}^{(i)} = (1 - \delta_k) \pi_k^{nc} + \delta_k \left(V_k \left(e_{ik}^{(k)} \right) + \tau_{ik}^{(k)} \right), \quad (1.15a)$$

$$V_i \left(e_{ij}, e_{ik}^{(k)} \right) - \tau_{ij} - \tau_{ik}^{(k)} = (1 - \delta_i) \pi_i^{nc} + \delta_i \left(V_i \left(e_{ij}, e_{ik}^{(i)} \right) - \tau_{ij} - \tau_{ik}^{(i)} \right). \quad (1.15b)$$

To ensure optimality, the offer made by i to j : $o_{ij}^{(i)} = \left(e_{ij}^{(i)}, \tau_{ij}^{(i)} \right)$ has to maximize the payoff $\pi_i = V_i(e_{ij}^{(i)}, e_{ik}) - \tau_{ij}^{(i)} - \tau_{ik}$ under the constraint given by (1.14a). The optimality conditions depends on the location of players. If the bargaining is about player 3 bargaining twice, then we set $(i, j, k) = (3, 1, 2)$. The optimal offer of 1 to 3 ($o_1^{(1)}$) maximizes his payoff $\pi_1(e_1) = V_1(e_1) + \tau_{13}$ under Constraint (1.14b). Substitute the expression of τ_{13} ; $e_1^{(1)} = e_1$ satisfy $B_1'(e_1) + B_3'(e_1 + e_2 + e_3) = C_1'(e_1)$. By symmetry, the optimal offer of 3 is $e_1^{(3)} = e_1 = e_1^{(1)}$. Also by symmetry, the optimal offer of 2 to 3 ($o_2^{(2)}$) maximizes his payoff under Constraint (1.15b). It yields the optimal offer $e_2^{(2)} = e_2$ that satisfies $B_2'(e_1 + e_2) + B_3'(e_1 + e_2 + e_3) = C_2'(e_2)$. The optimal effort of agent 3 is given by $B_3'(e_1 + e_2 + e_3) = C_3'(e_3)$.

We assume that i is the proposer. As we rewrite 1.14a and 1.14b, we get:

$$\begin{aligned} \tau_{ij}^{(j)} &= \frac{1}{\delta_j} \left((1 - \delta_j) (V_j^c - \pi_j^{nc}) - \tau_{ij}^{(i)} \right) \\ \tau_{ij}^{(i)} &= -\frac{1}{\delta_i} \left((1 - \delta_i) ((V_i^c - \pi_i^{nc}) + \tau_{ik}) + \tau_{ij}^{(j)} \right) \end{aligned}$$

substituting $\tau_{ij}^{(j)}$ of (1.14a) into (1.14b), we get the following transfer when i is assumed to be the proposer:

$$\tau_{ij}^{(i)} = -\frac{(1 - \delta_j)}{(1 - \delta_i \delta_j)} (B_j^c - B_j^{nc}) + \frac{\delta_j (1 - \delta_i)}{(1 - \delta_i \delta_j)} (B_i^c - B_i^{nc} + \tau_{ik}^{(i)})$$

We substitute $\tau_{ik}^{(k)}$ of (1.15a) in (1.15b) to get the second transfer:

$$\tau_{ik}^{(i)} = -\frac{(1 - \delta_k)}{(1 - \delta_i \delta_k)} (B_k^c - B_k^{nc}) + \frac{\delta_k (1 - \delta_i)}{(1 - \delta_i \delta_k)} \left((B_i^c - B_i^{nc}) + \tau_{ij}^{(i)} \right)$$

It returns the equilibrium transfers with $\eta = (1 - \delta_i \delta_j) (1 - \delta_i \delta_k) - \delta_j \delta_k (1 - \delta_i)^2 > 0$ when

i is the initial proposer:

$$\tau_{ij}^* = -\frac{(1-\delta_j)(1-\delta_i\delta_k)}{\eta}(B_j^c - B_j^{nc}) + \frac{(1-\delta_i)(1-\delta_k)\delta_j}{\eta}(B_k^c - B_k^{nc} + B_i^c - B_i^{nc}) \quad (1.16)$$

$$\tau_{ik}^* = -\frac{(1-\delta_k)(1-\delta_i\delta_j)}{\eta}(B_k^c - B_k^{nc}) + \frac{(1-\delta_i)(1-\delta_j)\delta_k}{\eta}(B_j^c - B_j^{nc} + B_i^c - B_i^{nc}) \quad (1.17)$$

That is with $(i, j, k) = (3, 1, 2)$:

$$\begin{aligned} \tau_{13}^* &= -\frac{(1-\delta_1)(1-\delta_2\delta_3)}{\eta}(V_1^* - \pi_1^{nc}) + \frac{\delta_1(1-\delta_2)(1-\delta_3)}{\eta}(V_2^* - \pi_2^{nc} + V_3^* - \pi_3^{nc}) \\ \tau_{23}^* &= -\frac{(1-\delta_2)(1-\delta_1\delta_3)}{\eta}(V_2^* - \pi_2^{nc}) + \frac{\delta_2(1-\delta_1)(1-\delta_3)}{\eta}(V_1^* - \pi_1^{nc} + V_3^* - \pi_3^{nc}) \end{aligned}$$

with $\eta = (1-\delta_1\delta_3)(1-\delta_2\delta_3) - \delta_1\delta_2(1-\delta_3)^2 > 0$. Equilibrium payoffs are given in the proposition. \square

1.4.3 Proof of Proposition 3

Following the proof of Proposition 2, a unique vector of efforts ensures optimality of offers in the case where 2 acts twice as a proposer. From 1.16 and 1.17, we get:

$$\begin{aligned} \tau_{12}^* &= \frac{(1-\delta_1)(1-\delta_2\delta_3)}{\phi}(V_1^* - \pi_1^{nc}) + \left(\frac{\delta_1(1-\delta_2)(1-\delta_3)}{\phi}\right)(V_2^* - \pi_2^{nc} + V_3^* - \pi_3^{nc}) \\ \tau_{23}^* &= -\frac{(1-\delta_1)(1-\delta_2)\delta_3}{\phi}(V_1^* - \pi_1^{nc} + V_2^* - \pi_2^{nc}) + \frac{(1-\delta_3)(1-\delta_1\delta_2)}{\phi}(V_3^* - \pi_3^{nc}) \end{aligned}$$

with $\phi = (1-\delta_1\delta_2)(1-\delta_2\delta_3) - \delta_1\delta_3(1-\delta_2)^2 > 0$. Optimal emissions and payoffs are in the proposition. \square

1.4.4 Summarized Results for the Seawall bargaining game

Results are summarized in the table (1.4).

	Coop	Non-Coop
e_1	$\frac{1 + 4\gamma + 3\gamma^2}{5\gamma + 6\gamma^2 + \gamma^3 + 1}$	$\frac{1}{1 + \gamma}$
e_2	$\frac{\gamma + 2\gamma^2}{5\gamma + 6\gamma^2 + \gamma^3 + 1}$	$\frac{\gamma}{(\gamma + 1)^2}$
e_3	$\frac{\gamma^2}{5\gamma + 6\gamma^2 + \gamma^3 + 1}$	$\frac{\gamma^2}{(\gamma + 1)^3}$
π_1	$\frac{(3\gamma^2 + 4\gamma + 1)(-\gamma^3 + 5\gamma^2 + 5\gamma + 1)}{2(\gamma^3 + 6\gamma^2 + 5\gamma + 1)^2}$	$\frac{1}{2(\gamma + 1)}$
π_2	$\frac{6\gamma^2 + 5\gamma + 1}{2(\gamma^3 + 6\gamma^2 + 5\gamma + 1)}$	$\frac{(3\gamma^3 + 6\gamma^2 + 4\gamma + 1)}{2(\gamma + 1)^4}$
π_3	$\frac{(11\gamma^5 + 46\gamma^4 + 62\gamma^3 + 37\gamma^2 + 10\gamma + 1)}{2(\gamma^3 + 6\gamma^2 + 5\gamma + 1)^2}$	$\frac{(5\gamma^5 + 15\gamma^4 + 20\gamma^3 + 15\gamma^2 + 6\gamma + 1)}{2(\gamma + 1)^6}$

Table 1.4: Efforts (in \bar{e}) and payoffs (in $b\bar{e}^2$) in the cooperative and non-cooperative cases. $\gamma = c/b$ and $\bar{e} = a/b$.

When $\gamma = 0$ (for $c = 0$), $e_1 = \bar{e}$, while $e_i = 0$ for $i = 2, 3$ and $\pi_i = (b/2)\bar{e}^2 \forall i$ in both cooperative and non-cooperative cases. It can be shown that e_1 decreases with γ , while $e_i = 0$ for $i = 2, 3$ first increases, reaches a maximum and then decreases. Payoffs always decrease with γ . A low value of γ means a low marginal cost and/or a high marginal benefit, implying higher efforts and payoffs. On the opposite side, a high value of γ means a high marginal cost (c) and/or a low marginal benefit (b), implying lower efforts and payoffs. These patterns are the same for all of the negotiation protocols.

The negotiation procedures involving only two agents are summarized in Table 1.4.4⁶:

Negotiation protocols consisting in two bilateral negotiations are summarized in Table 1.4.4:

Comparing the different outcomes gives in terms of individuals ⁷ and aggregate efforts:

- $e_1^c > e_1^{\{1\leftrightarrow 2\}} > e_1^{\{2\leftrightarrow 1,3\}} > e_1^{\{1\leftrightarrow 3\}} > e_1^{\{3\leftrightarrow 1,2\}} > e_1^{nc} = e_1^{\{2\leftrightarrow 3\}}$
- $e_2^{\{2\leftrightarrow 3\}} > e_2^{\{3\leftrightarrow 1,2\}} > e_2^{\{2\leftrightarrow 1,3\}} > e_2^c > e_2^{nc} > e_2^{\{1\leftrightarrow 3\}} > e_2^{\{1\leftrightarrow 2\}}$
- $e_3^{nc} > e_3^{\{1\leftrightarrow 3\}} > e_3^{\{2\leftrightarrow 3\}} = e_3^{\{1\leftrightarrow 2\}} > e_3^{\{3\leftrightarrow 1,2\}} > e_3^{\{2\leftrightarrow 1,3\}} > e_3^c$

⁶The created surplus in the negotiation between 1 and 3 is positive for $\gamma > 0.89$.

⁷For agent 2, the rank holds for $\gamma > \gamma^* = \frac{1}{2}(1 + \sqrt{5})$ and for $\gamma \geq 1 : e_2^{\{1,3\leftrightarrow 2\}} \geq e_2^{nc}$, for $\gamma \geq 0.815 : e_2^c \geq e_2^{\{1\leftrightarrow 3\}}$ and for $\gamma \geq 0.52 : e_2^{\{1,3\leftrightarrow 2\}} \geq e_2^{\{1\leftrightarrow 3\}}$

	2 ↔ 3	1 ↔ 3	1 ↔ 2
e_1	$\frac{1}{1 + \gamma}$	$\frac{1 + 2\gamma + 2\gamma^2}{3\gamma + 4\gamma^2 + \gamma^3 + 1}$	$\frac{1 + 2\gamma}{3\gamma + \gamma^2 + 1}$
e_2	$\frac{\gamma + 2\gamma^2}{4\gamma + 4\gamma^2 + \gamma^3 + 1}$	$\frac{\gamma + \gamma^2}{3\gamma + 4\gamma^2 + \gamma^3 + 1}$	$\frac{\gamma}{3\gamma + \gamma^2 + 1}$
e_3	$\frac{\gamma^2}{4\gamma + 4\gamma^2 + \gamma^3 + 1}$	$\frac{\gamma^2}{3\gamma + 4\gamma^2 + \gamma^3 + 1}$	$\frac{\gamma^2}{4\gamma + 4\gamma^2 + \gamma^3 + 1}$
$\pi_1 - \pi_1^{nc}$	0	$\frac{\gamma^6(\gamma^4 + 3\gamma^3 - 2\gamma - 1)}{4(\gamma + 1)^5(\gamma^3 + 4\gamma^2 + 3\gamma + 1)^2}$	$\frac{\gamma^4}{4(\gamma + 1)^3(\gamma^2 + 3\gamma + 1)}$
$\pi_2 - \pi_2^{nc}$	$\frac{\gamma^6}{4(\gamma + 1)^5(\gamma^2 + 3\gamma + 1)}$	$\frac{\gamma^5(2\gamma^3 + 7\gamma^2 + 6\gamma + 2)}{2(\gamma + 1)^3(\gamma^3 + 4\gamma^2 + 3\gamma + 1)^2}$	$\pi_1 - \pi_1^{nc}$
$\pi_3 - \pi_3^{nc}$	$\pi_2 - \pi_2^{nc}$	$\pi_1 - \pi_1^{nc}$	$\frac{(2\gamma^2 + 5\gamma + 2)\gamma^6}{2(\gamma + 1)^5(\gamma^2 + 3\gamma + 1)^2}$

Table 1.5: Efforts (in \bar{e}) and payoffs (in $b\bar{e}^2$) in the two-by-two negotiation process.

	3 ↔ 1, 2	2 ↔ 1, 3
e_1	$\frac{1 + 3\gamma + 2\gamma^2}{4\gamma + 5\gamma^2 + \gamma^3 + 1}$	$\frac{1 + 4\gamma + 2\gamma^2}{5\gamma + 5\gamma^2 + \gamma^3 + 1}$
e_2	$\frac{\gamma + 2\gamma^2}{4\gamma + 5\gamma^2 + \gamma^3 + 1}$	$\frac{\gamma + 2\gamma^2}{5\gamma + 5\gamma^2 + \gamma^3 + 1}$
e_3	$\frac{\gamma^2}{4\gamma + 5\gamma^2 + \gamma^3 + 1}$	$\frac{\gamma^2}{5\gamma + 5\gamma^2 + \gamma^3 + 1}$
$\pi_i - \pi_i^{nc}$	$\frac{(4\gamma^5 + 23\gamma^4 + 38\gamma^3 + 30\gamma^2 + 12\gamma + 2)\gamma^5}{6(\gamma + 1)^5(\gamma^3 + 5\gamma^2 + 4\gamma + 1)^2}$	$\frac{(4\gamma^4 + 17\gamma^3 + 15\gamma^2 + 7\gamma + 1)\gamma^4}{6(\gamma + 1)^5(\gamma^2 + 4\gamma + 1)^2}$

Table 1.6: Efforts (in \bar{e}) and payoffs (in $b\bar{e}^2$) in double negotiation.

- $\sum_i e_i^c > \sum_i e_i^{\{2\leftrightarrow 1,3\}} > \sum_i e_i^{\{3\leftrightarrow 1,2\}} > \sum_i e_i^{\{1\leftrightarrow 2\}} = \sum_i e_i^{\{2\leftrightarrow 3\}} > \sum_i e_i^{\{1\leftrightarrow 3\}} > \sum_i e_i^{nc}$

The individual and aggregate payoffs are:

- $\pi_1^{\{2\leftrightarrow 1,3\}} > \pi_1^{\{3\leftrightarrow 1,2\}} > \pi_1^{\{1\leftrightarrow 2\}} > \pi_1^{\{1\leftrightarrow 3\}} > \pi_1^{nc} = \pi_2^{\{2\leftrightarrow 3\}} > \pi_1^c$
- $\pi_2^c > \pi_2^{\{1\leftrightarrow 3\}} = \pi_2^F > \pi_2^{\{2\leftrightarrow 1,3\}} > \pi_2^{\{3\leftrightarrow 1,2\}} > \pi_2^{\{1\leftrightarrow 2\}} > \pi_2^{\{2\leftrightarrow 3\}} > \pi_2^{nc}$
- $\pi_3^c > \pi_3^{\{1\leftrightarrow 2\}} = \pi_3^F > \pi_3^{\{2\leftrightarrow 1,3\}} > \pi_3^{\{3\leftrightarrow 1,2\}} > \pi_3^{\{2\leftrightarrow 3\}} > \pi_3^{\{1\leftrightarrow 3\}} > \pi_3^{nc}$
- $\sum_i \pi_i^c > \sum_i \pi_i^{\{2\leftrightarrow 1,3\}} > \sum_i \pi_i^{\{3\leftrightarrow 1,2\}} > \sum_i \pi_i^{\{1\leftrightarrow 2\}} > \sum_i \pi_i^{\{1\leftrightarrow 3\}} > \sum_i \pi_i^{\{2\leftrightarrow 3\}} > \sum_i \pi_i^{nc}$

1.4.5 Proof of Proposition 5

This proof is the analog of proof 1. The two indifference conditions give respectively:

$$B_j(x_j^{(i)}) + \tau_{ij}^{(i)} = (1 - \delta_j) B_j^{nc} + \delta_j (B_j(x_j^{(j)}) + \tau_{ij}^{(j)}), \quad (1.18)$$

$$B_i(x_j^{(j)}, x_k) - \tau_{ij}^{(j)} - \tau_{ik} = (1 - \delta_i) B_i^{nc} + \delta_i (B_i(x_j^{(i)}, x_k) - \tau_{ij}^{(i)} - \tau_{ik}). \quad (1.19)$$

and

$$B_k(x_k^{(i)}) + \tau_{ik}^{(i)} = (1 - \delta_k) B_k^{nc} + \delta_k (B_k(x_k^{(k)}) + \tau_{ik}^{(k)}), \quad (1.20)$$

$$B_i(x_j, x_k^{(k)}) - \tau_{ij} - \tau_{ik}^{(k)} = (1 - \delta_i) B_i^{nc} + \delta_i (B_i(x_j, x_k^{(i)}) - \tau_{ij} - \tau_{ik}^{(i)}). \quad (1.21)$$

To ensure optimality, the offer made by i $o_i^{(i)} = (x_j^{(i)}, \tau_{ij}^{(i)})$ has to maximize his payoff $\pi_i = B_i(x_j, x_k) - \tau_{ij} - \tau_{ik}$ under the constraint given by (1.14b). Maximizing returns the optimality condition

$$B'_i(x_j, x_k) = B'_j(x_j). \quad (1.22)$$

(1.22) gives the optimal offer made by i : $x_j^{(i)} = x_j$. By symmetry the optimal offer of j is $x_j^{(j)} = x_j$.

We proceed in the same way for the second negotiation between i and k . The optimal offer made by i : $o_{ik}^{(i)} = (x_k^{(i)}, \tau_{ik}^{(i)})$ maximizes his payoff under (1.15b). It yields the first order condition:

$$B'_i(x_j, x_k) = B'_k(x_k) \quad (1.23)$$

this returns the player i 's optimal offer $x_k^{(i)} = x_k$. By symmetry, k offers $x_k^{(k)} = x_k$. Optimality conditions (1.22) and (1.23) show that the abatement levels are identical to the cooperative abatement. levels of extractions are solution of $(x_j^*, x_k^*) = (x_j^c, x_k^c)$ implying $x_i^* = x_i^c$ and so $B_i^* = B_i^c$.

We assume that i is the proposer. As we rewrite 1.14a and 1.14b, we get:

$$\begin{aligned} \tau_{ij}^{(j)} &= \frac{1}{\delta_j} \left((1 - \delta_j) (B_j^c - B_j^{nc}) - \tau_{ij}^{(i)} \right) \\ \tau_{ij}^{(i)} &= -\frac{1}{\delta_i} \left((1 - \delta_i) ((B_i^c - B_i^{nc}) + \tau_{ik}) + \tau_{ij}^{(j)} \right) \end{aligned}$$

substituting $\tau_{ij}^{(j)}$ of (1.14a) into (1.14b), we get the following transfer when i is assumed to be the proposer:

$$\tau_{ij}^{(i)} = -\frac{(1 - \delta_j)}{(1 - \delta_i \delta_j)} (B_j^c - B_j^{nc}) + \frac{\delta_j (1 - \delta_i)}{(1 - \delta_i \delta_j)} (B_i^c - B_i^{nc} + \tau_{ik}^{(i)})$$

We substitute $\tau_{ik}^{(k)}$ of (1.15a) in (1.15b) to get the second transfer:

$$\tau_{ik}^{(i)} = -\frac{(1 - \delta_k)}{(1 - \delta_i \delta_k)} (B_k^c - B_k^{nc}) + \frac{\delta_k (1 - \delta_i)}{(1 - \delta_i \delta_k)} ((B_i^c - B_i^{nc}) + \tau_{ij}^{(i)})$$

It returns the equilibrium transfers with $\eta = (1 - \delta_i \delta_j) (1 - \delta_i \delta_k) - \delta_j \delta_k (1 - \delta_i)^2 > 0$ and the i as the initial proposer:

$$\begin{aligned} \tau_{ij}^* &= -\frac{(1 - \delta_j) (1 - \delta_i \delta_k)}{\eta} (B_j^c - B_j^{nc}) + \frac{(1 - \delta_i) (1 - \delta_k) \delta_j}{\eta} (B_k^c - B_k^{nc} + B_i^c - B_i^{nc}) \\ \tau_{ik}^* &= -\frac{(1 - \delta_k) (1 - \delta_i \delta_j)}{\eta} (B_k^c - B_k^{nc}) + \frac{(1 - \delta_i) (1 - \delta_j) \delta_k}{\eta} (B_j^c - B_j^{nc} + B_i^c - B_i^{nc}) \end{aligned}$$

note that $\tau_{ij}^* > 0$ and $\tau_{ik}^* > 0$.

The additional gain of simultaneous bargaining for player j is:

$$\begin{aligned}\pi_j^{sim} - B_j^{nc} &= B_j^c + \tau_{ij}^* - B_i^{nc} \\ &= \left(\frac{(\delta_i - 1)(\delta_k - 1)\delta_j}{\eta} \right) \Pi\end{aligned}$$

Where $\Pi = \sum_{\varepsilon=\{i,j,k\}} (B_\varepsilon^c - B_\varepsilon^{nc})$ stands for the sum of the individuals gains of bargaining. Setting $N = \{j, k, i\}$ brings the games $\Gamma_{ATS}^{sim}(N)$ and $\Gamma_{UTI}^{nc}(N')$. \square

1.4.6 Proof of Proposition 6

i first negotiates with k in the first round, then with j in the second round. Using backward induction, the two indifference conditions are:

$$B_j(x_j^{(i)}) - \tau_{ji}^{(i)} = (1 - \delta_j) B_j^{nc} + \delta_j \left(B_j(x_j^{(j)}) - \tau_{ji}^{(j)} \right) \quad (1.24)$$

$$B_i(x_j^{(j)}, x_k^a) + \tau_{ji}^{(j)} + \tau_{ki}^a = (1 - \delta_i) (B_i^d - \tau_k^a) + \delta_i \left(B_i(x_j^{(i)}, x_k^a) + \tau_{ji}^{(i)} + \tau_{ki}^a \right) \quad (1.25)$$

Where x_k^a refers to the agreement reached between i and k in the first round of negotiations. $B_i^d = B_i(E - x_j^{nc} - x_k^a)$ is the payoff of player i in case of disagreement with player j . Assuming that j and i maximizes their payoffs (respectively under constraints (1.24) or (1.25)), it turns out that their optimal offers $x_j^{(j)} = x_j^{(i)} = x_j$ will be solution of:

$$B_j'(x_j) = B_i'(E - x_j - x_k^a) \quad (1.26)$$

Assuming that i is the proposer, the transfer paid by i to j is equal to:

$$\tau_{ij}^{(i)} = \frac{(1 - \delta_j)}{(1 - \delta_j \delta_i)} (B_j - B_j^{nc}) - \frac{\delta_j (1 - \delta_i)}{(1 - \delta_j \delta_i)} (B_i - B_i^{nc}) - \frac{\delta_j (1 - \delta_i)}{(1 - \delta_j \delta_i)} (B_i^{nc} - B_i^d) \quad (1.27)$$

In the first round, the two indifference conditions associated to the negotiation between i and k are

$$B_k(x_k^{(i)}) - \tau_{ki}^{(i)} = (1 - \delta_k) B_k^{nc} + \delta_k \left(B_k(x_k^{(k)}) - \tau_{ki}^{(k)} \right) \quad (1.28)$$

$$B_i(x_j, x_k^{(k)}) + \tau_{ji} + \tau_{ki}^{(k)} = (1 - \delta_i) B_i^{nc} + \delta_i \left(B_i(x_j, x_k^{(i)}) + \tau_{ji} + \tau_{ki}^{(i)} \right) \quad (1.29)$$

The disagreement payoff of i differs from the second round. We assumed that, in case of a failure in the first round, the negotiations are over and all agents get their non cooperative payoffs. Using the implicit relations in the second round $x_j = x_j(x_k)$ and $\tau_{ji} = \tau_{ji}(x_k)$, the optimal water extraction level x_k is solution of the optimal first order condition:

$$\left(1 + \frac{\partial x_j}{\partial x_k} \right) B'_i(E - x_j - x_k) + \frac{\partial \tau_{ji}}{\partial x_k} = B'_k(x_k) \quad (1.30)$$

Optimality conditions (1.26) and (1.30) implies the equilibrium pair (x_j^*, x_k^*) . The transfer associated to that negotiation when i is the proposer is given by:

$$\tau_{ki}^{(i)} = \frac{(1 - \delta_k)}{(1 - \delta_k \delta_i)} (B_k - B_k^{nc}) - \frac{\delta_k (1 - \delta_i)}{(1 - \delta_k \delta_i)} (B_i - B_i^{nc}) - \frac{\delta_k (1 - \delta_i)}{(1 - \delta_k \delta_i)} \tau_{ji} \quad (1.31)$$

Solving the system (1.27)-(1.31) gives $\tau_{ji}^{(i)} = \tau_{ji}^*$ and

$$\begin{aligned} \tau_{ki}^* &= - \frac{\delta_k (1 - \delta_i) (1 - \delta_j)}{(1 - \delta_k \delta_i) (1 - \delta_j \delta_i)} (B_j - B_j^{nc} + B_i - B_i^{nc}) + \frac{(1 - \delta_k)}{(1 - \delta_k \delta_i)} (B_k - B_k^{nc}) \\ &\quad - \frac{\delta_j \delta_k (1 - \delta_i)^2}{(1 - \delta_j \delta_i) (1 - \delta_k \delta_i)} (B_i^d - B_i^{nc}) \end{aligned} \quad (1.32)$$

Substitute the expression $\frac{\partial \tau_{ji}}{\partial x_k}$ derived from (1.32) in (1.27) returns

$$\begin{aligned} B'_j(x_j) &= B'_i(E - x_j - x_k) \\ B'_k(x_k) &= B'_i(E - x_j - x_k) + \frac{\delta_j (1 - \delta_i)}{(1 - \delta_j \delta_i)} (B'_i(E - x_j^{nc} - x_k^a) - B'_i(E - x_j - x_k)) \end{aligned}$$

Note that $x_j^* + x_k^* < x_j^{nc} + x_k^a$ implies that $x_i^* = E - x_j^* - x_k^* > x_i^d = E - x_j^{nc} - x_k^a$. We get that $B'_i(x_i^d) > B'_i(x_i^a)$ since B'_3 is decreasing ($B'' < 0$). Simple manipulations yield the

payoffs for every agent. \square

1.4.7 Proof of Proposition 7

In round 2, j bargains with i based on an agreement in round 1. The two indifference conditions are:

$$\begin{aligned} B_j(x_j^{(i)}) - \tau_{ji}^{(i)} - \tau_{jk} &= (1 - \delta_j) (B_j^{nc} - \tau_{jk}) + \delta_j (B_j(x_j^{(j)}) - \tau_{ji}^{(j)} - \tau_{jk}) \\ B_i(x_j^{(j)}, x_k) + \tau_{ji}^{(j)} &= (1 - \delta_i) B_i^d + \delta_i (B_i(x_j^{(i)}, x_k) + \tau_{ji}^{(i)}) \end{aligned}$$

with $B_i^d = B_3(x_j^{nc}, x_k)$.

In round 1, the two indifference conditions of the negotiation between j and k are:

$$\begin{aligned} B_j(x_j(x_k^{(k)})) - \tau_{ji} - \tau_{jk}^{(k)} &= (1 - \delta_j) B_j^{nc} + \delta_j (B_j(x_j(x_k^{(j)})) - \tau_{ji} - \tau_{jk}^{(j)}) \\ B_k(x_k^{(j)}) + \tau_{jk}^{(j)} &= (1 - \delta_k) B_k^{nc} + \delta_k (B_k(x_k^{(k)}) + \tau_{jk}^{(k)}) \end{aligned}$$

The first negotiation implies:

$$B'_j(x_j) = B'_i(x_i)$$

and

$$\tau_{ji}^{(j)} = \frac{\delta_i (1 - \delta_j)}{(1 - \delta_j \delta_i)} (B_j - B_j^{nc}) - \frac{(1 - \delta_i)}{(1 - \delta_j \delta_i)} (B_i - B_i^d)$$

The second negotiation gives:

$$\left(\frac{\partial x_j}{\partial x_k} \right) B'_j(x_j(x_k)) + \frac{\partial \tau_{ji}}{\partial x_k} + B'_k(x_k) = 0$$

and the transfer:

$$\tau_{jk}^{(j)} = \frac{\delta_k (1 - \delta_j)}{(1 - \delta_k \delta_j)} (B_j - B_j^{nc}) - \frac{(1 - \delta_k)}{(1 - \delta_k \delta_j)} (B_k - B_k^{nc}) + \frac{\delta_k (1 - \delta_j)}{(1 - \delta_k \delta_j)} \tau_{ji}$$

Substitute the derivative $\frac{\partial \tau_{ji}}{\partial x_k}$ in the optimality condition returns:

$$B'_k(x_k) = -\frac{(1 - \delta_i)}{(1 - \delta_j \delta_i)} (B'_i(x_i^d) - B'_i(x_i))$$

Note that, since $x_i^* \geq x_i^d$ and B'_i is decreasing ($B'' < 0$), $B'_i(x_i^d) \geq B'_i(x_i)$ holds, implying $B'_k(x_k) \leq 0$. Since $B'_k(x_k)$ is defined on $[0, +\infty[$ (k will not consume more than his best response \bar{x}_k), it turns that $B'_k(x_k) = 0$ giving the maximum value of $x_k^* = x_k^{nc}$ and as a consequence $B_k^* = B_k^{nc}$ and $B_i^{nc} = B_i^d$

Equilibrium transfers are $\tau_{jk}^* = \tau_{jk}^{(j)}$ and $\tau_{ji}^* = \tau_{ji}^{(j)}$.

$$\begin{aligned} \tau_{ji}^* &= \frac{\delta_i (1 - \delta_j)}{(1 - \delta_j \delta_i)} (B_j^* - B_j^{nc}) - \frac{(1 - \delta_i)}{(1 - \delta_j \delta_i)} (B_i^* - B_i^{nc}) \\ \tau_{jk}^* &= \frac{\delta_k (1 - \delta_j) (1 - \delta_i)}{(1 - \delta_k \delta_j) (1 - \delta_j \delta_i)} (B_j^* - B_j^{nc} + B_i^* - B_i^{nc}) > 0 \end{aligned}$$

Simple manipulations return the proposition. \square

Chapter 2

Level-K Farsightedness in a vertically related economy

2.1 Introduction

This chapter aims at providing a contribution to the endogenous networks formation with farsighted players. A set of differentiated manufacturers have the possibility to be distributed through different channels at different costs, impacting the global structure of interactions between economic agents. The modification of the structure has heavy implications in terms of profits and profits turns out to be of crucial part when firms decide to build new commercial links or decide to abrogate existing contracts. Thus individual incentives and the economic networking influence one another. In this chapter, I investigate how the distribution network can reach a stable state. The positive aspect of stability is determined by the stability criterion we use, this chapter deals with the notion of stability that can be used when we consider the limit of the horizon of the agents. This chapter discusses the way the incentives to change the network depend on the abilities to forecast or to anticipate series of events. I discuss how the available information on the forthcoming states is treated in the present given some limitations in the forecasting. To what extent is the evolution of a distribution network towards a stable state is affected by the knowledge of the information to come. I don't want to discuss the pro and the

cons of myopia against farsightedness when looking at the stability of networks, and that's the reason why the question of the supply network formation is undertaken from the perspective of level- K . The level- K reasoning encompasses the overall spectrum of forecasting from the myopia (level-1) to the infinite farsightedness (level- ∞). If a game is played by level- K farsighted players, then those players are able to anticipate what their own actions induce at $K - 1$ steps ahead. In the level- K reasoning, the concept of time has not to be interpreted in its primary sense, a unit of time corresponding to one alteration of the social structure.

The reason why players are not able to anticipate sequences of shifts in more than a given number of steps ahead results in an either limited rationality, in the presence of some restricted information, or in players discounting the future too heavily (I won't discuss this point). Thus, the present actions are determined by the weight of situations that are likely to emerge within a fixed number of changes of the structure of the network.

With the idea that intermediary states do not account for the determination of deviations and that links among individuals can be unilaterally severed and only bilaterally build with the consent of two, I investigate the role of an adjustable level of farsightedness on the nature of incentives to build a given supply network. Given the numbers of candidate solutions to stability, we substituted an algorithmic solution to the analytic reasoning for the computation of the level- K pairwise farsighted stable sets. This chapter develops three main findings. First, the results established that the theoretical minimum level of farsightedness representative of an infinite farsightedness can be drastically reduced through recursive calculations. I then computed the pairwise farsighted stable set of networks for any level of farsightedness. The results are heavily affected by the level of anticipation. Indeed, the fact that agents are driven by a sum of short-term expectations or by decisions that are the result of expectations rolling away from present alters the stable outcomes. The second result lies in the confrontation between stability and two opposite criteria of social optima that I called *weak* and *strong* correspondence. It turns out that, even if usual definitions of social optima do not perfectly conform to a set-based definition of stability, a tension between stability and social efficiency is observed as the

level of farsightedness increases. In the last section, the comparison between the level- ∞ farsighted stable sets and the pairwise farsighted stable set emphasizes the values of parameters for which there is a transitive correspondence.

2.1.1 Related Literature

Industrial organization deals with various issues such as vertical integration, franchising, exclusive zones, vertical restraints and so forth¹. It generally assumes an exogenous structure of vertical relations without really questioning the consistency of such a networked structure. To my knowledge, the only article that directly addresses the question of network formation in a vertical and competitive environment is the paper by Mauleon, Sempere-Monerris and Vannetelbosch (2001) [38]. The authors investigate the influence of the type of contract signed between manufacturers and retailers on the stability of networks. Their results show holds for the full range of product differentiation and the whole spectrum of linking costs. They show that the three types of competition (inter-brand, intrabrand and in-store rivalry) may fail at being present simultaneously, leading to a conflict between selfish interests and the social optimum. Though this article provide a strong basis for the study of formation of manufacturers and retailers networks, the approach assumes myopic players.

The rather well established literature on networks in economics assumes that, when the individuals are self-interested, the evolution of a network is driven by two processes. The destruction of links is due to an individual deviation while the occurrence of a new link between two individuals needs the consent of these two people (Jackson & Wolinsky (1996) [33], Dutta and Mutuswami (1997) [18]). In this context, when individuals compare their current situation to the directly upcoming situation in order to decide if this deviation could be worth, we talk about pairwise network formation in the presence of myopic players. In this setting, the incentives of an agent or a pair of agents to alter the architecture of the network are based on what the deviation forthwith brings compared to the current situation. The agents may instigate an initial shift that may induce some

¹See Laffont and Tirole [36] for a comprehensive, but nonetheless thorough approach in industrial organization

other players into deviating and, in turn, a initial deviation may trigger an ongoing sequence of further deviation, ultimately leading to a distant situation at the expense of the original defectors (Jackson & Watts (2002) [32]). Considering this argument, a myopic rationale seems like a flawed hypothesis when the model of network formation represents players with forecasting abilities. This main criticism against myopia lies in the fact that, beyond very operative results derived from myopic concepts of stability, a static approach ignores an essential aspect of the rational economic agent and myopia could be seen as representative of a form of limited rationality.

Talking about network formation and forward-looking agents in economics leads us to consider two highly developed fields in economics with a non empty intersection.

The reasoning that embodies the anticipations of the future states in the decision making process is a concept developed by Harsanyi, (1974) [22] as he was questioning the impact of farsightedness in the von-Neumann Morgenstern stable sets [55]. The modification of a structure can induce subsequent amendments, leading in turn to other changes, and so forth. At each step, the structure differs from the preceding by one or several modifications. If the state η_i brings the outcome η_{i+1} which in turn leads to η_{i+2} , then there is a myopic dominance between η_i and η_{i+1} and indirect dominance relation between η_i and η_{i+2} . The solution using indirect dominance returns the set of outcomes one might expect to emerge given the mapping of the dominance relationships. A solution of von Neumann-Morgenstern is reached when a state, or a set of states satisfies external and internal stability. In other words, all states outside the solution are dominated by a state which belongs to the solution while there is no dominance relations between the states within the solution. The main issue with von Neumann-Morgenstern sets including indirect dominance is that such a solution concept is defined by strong requirements and may fail to exist.

The study of individual incentives with farsighted agents has applications when it comes to analyze the formation of groups and when it comes to know the evolution of the intra-group structure, *i.e.* the full architecture of a network. Taking part at explaining the evolution of coalitions, Chwe (1994) [13] defined the farsighted dominance paths by

the preferences of the coalitions over situations and their abilities to reach alternative structures. In his model, Chwe tackles the issue of existence of a stable set by refining the definition of internal stability. He allows dominance cycles within the set and the deviations from an outcome inside the stable set must be deterred by another outcome of the same set. A stable set is then made of the outcomes for which any coalitional deviation that leads outside the set, induce another coalition into returning back to the set at the expense of the initially deviating coalition. The largest consistent set sustaining these requirements has been proved non-empty and unique. Page, Wooders and Kamat (2005) [47] reinterpret the notion of consistency of a set in the context of network games, the coalitional moves being supported by a path in the supernetwork.

Unlike the coalitional approach of farsighted stable set formation, one could consider that links among individuals are added or severed one link at a time. This is how Dutta, Ghosal and Ray (2005) [17] see the formation of farsighted dominance paths. In this paper, farsightedness is captured through a stochastic dynamic process (pairs of players are picked at random at each period of time) and times matters (players take actions considering a discounted stream of payoff). They show that a Markovian network formation process leads to a subgame perfect equilibrium. Discussing the question of coordination failure, they show a tension between stability and efficiency of networks and provided sufficient conditions (Increasing returns to links and the componentwise egalitarian allocation rule) for the efficient network to sustain equilibrium.

Herings, Mauleon and Vannetelbosch [23] developed a model of bilateral network formation for a deterministic mapping of farsighted dominance paths. The farsighted improving paths are designed according to the perspective that a pairwise deviation might induce series of pairwise deviations, each end point of a path being seen as a potential image of the initial deviation. They proved a pairwise farsighted stable set always exists and derived the conditions under which it is unique. So far, these models consider the ability to anticipate infinite series of deviation, this implying a high level of abstraction or tough calculations for large or complex economies. Where the myopic pairwise stability doesn't fit forward-looking agents and may suffer from its simplicity, the pairwise

farsighted stability suffers from its complexity. So as to overcome with a theoretical lack, a larger concept encompassing both approaches as been proposed by Herings Mauleon and Vannetelbosch (2014) [24]. The pairwise level- K farsighted stable set of networks allows any intermediate level of farsightedness, from the myopic players up to the fully (or infinite) farsighted players. They proved a pairwise level- K farsighted stable set in range $[1, \infty]$ exists. Beyond the theoretical concept, Kirchsteiger et al. (2013) [35] empirically revealed that individual players exhibit an intermediate level of farsightedness.

2.1.2 Roadmap

The chapter is organized as follow, Section 2 presents the model of manufacturers and retailers, Section 3 deals with the concepts of pairwise farsighted stability, Section 4 introduces and explains the way the algorithm is running, Section 5 details the results, Section 6 analyses the question of the efficiency. In Section 7, I talk about the reliability of the algorithmic results and the last section concludes.

2.2 Modeling networks of Manufacturers and Retailers

The principle of the game has been borrowed and is inspired from Mauleon et al. (2011) [38]. Consider a set of players \mathbb{N} as the union of two disjoint sets, let $M = \{1, 2\}$ be the set of manufacturers and $R = \{1, 2\}$ the set of retailers, so $\mathbb{N} = M \cup R$ and $M \cap R = \emptyset$. Manufacturers can't directly gain access to the market of consumers, so as to sell on the market of the considered good, they must engage in a commercial relationship with at least one retailer. A manufacturer $i \in M$ can sell part of his production to a a retailer $j \in R$, who, in turn, sells this production on the market of the considered good. The set R is assumed to be homogeneous, the retailers are identical. The set M is heterogeneous and manufacturers sell horizontally differentiated goods according to a parameter $d \in [0, 1]$. The demand is assumed to be linear.

The commercial relationships between manufacturers and retailers are modeled by a

2-mode network with 4 nodes. Whenever a commercial relationship between $i \in M$ and $j \in R$ exists, then i and j are said to be 'linked' in the supply network. We now consider that a network g is a collection of links between the two disjoint sets and we write $ij \in g$ to indicate that i and j are linked in the network g , thus $g = \{ij \mid i \in M, j \in R\}$. Note that the network is undirected and so $ij = ji$. Let $g^{\mathbb{N}}$ be the set of all combination of an element of M and an element of R . The network $g^{\mathbb{N}}$ is referred to as the complete network. The set $\mathbb{G} = \{g \subset g^{\mathbb{N}}\}$ is the set of all possible network and $\#\mathbb{G} = 2^{M \times R}$. $N(g)$ denotes the set of players who have at least one link in the network g , that is:

$$N(g) = \begin{cases} i \in M \mid \exists j \in R \text{ such that } ij \in g \\ j \in R \mid \exists i \in M \text{ such that } ij \in g \end{cases}$$

The manufacturer i is linked to the set of retailers $n_i(g)$ in the network g , that is $n_i(g) = \{j \mid ij \in g\}$. Analogously, the retailer j is linked to a set of manufacturers $n_j(g)$. The network obtained after the addition of the link ij to the network g is denoted by $g+ij$ and the network obtained after the deletion of the existing link ij to g is denoted by $g-ij$. The set of networks $A^g = \{g+ij \mid ij \notin g\} \cup \{g-ik \mid ik \in g\}$ defines the adjacencies to g . That is, the set of networks that can be reached from the network g within a single alteration of the links of g . Figure 2.1 shows the networks that can be formed among the agents of \mathbb{N} .

An alteration of the complete network $g^{\mathbb{N}}$ is a network of mixed distribution (3-links networks) $X = \{x_1, x_2, x_3, x_4\}$. The dyadic networks $ED = \{ed_1, ed_2\}$ refer to the situations of exclusive dealing. Manufacturers monopolies and retailers monopsonies are respectively defined by networks $MM = \{m_1, m_2\}$ and $RM = \{r_1, r_2\}$, eventually, networks made of a single link are defined by $S = \{s_1, s_2, s_3, s_4\}$ and g^\emptyset is the empty network.

Provided a manufacturer i as a link with a retailer, he can sell a part of his production q_{ij} to the retailer j at the price w_i and the demand for the good i is given by $p_i = a - Q_i - dQ_{-i}$ with $Q_i = \sum_{j \in n_i(g)} q_{ij}$ and $-i = M \setminus \{i\}$. Building a link involve a constant cost of $2k$ per link which is equally split among the parties. Given the parameters, the

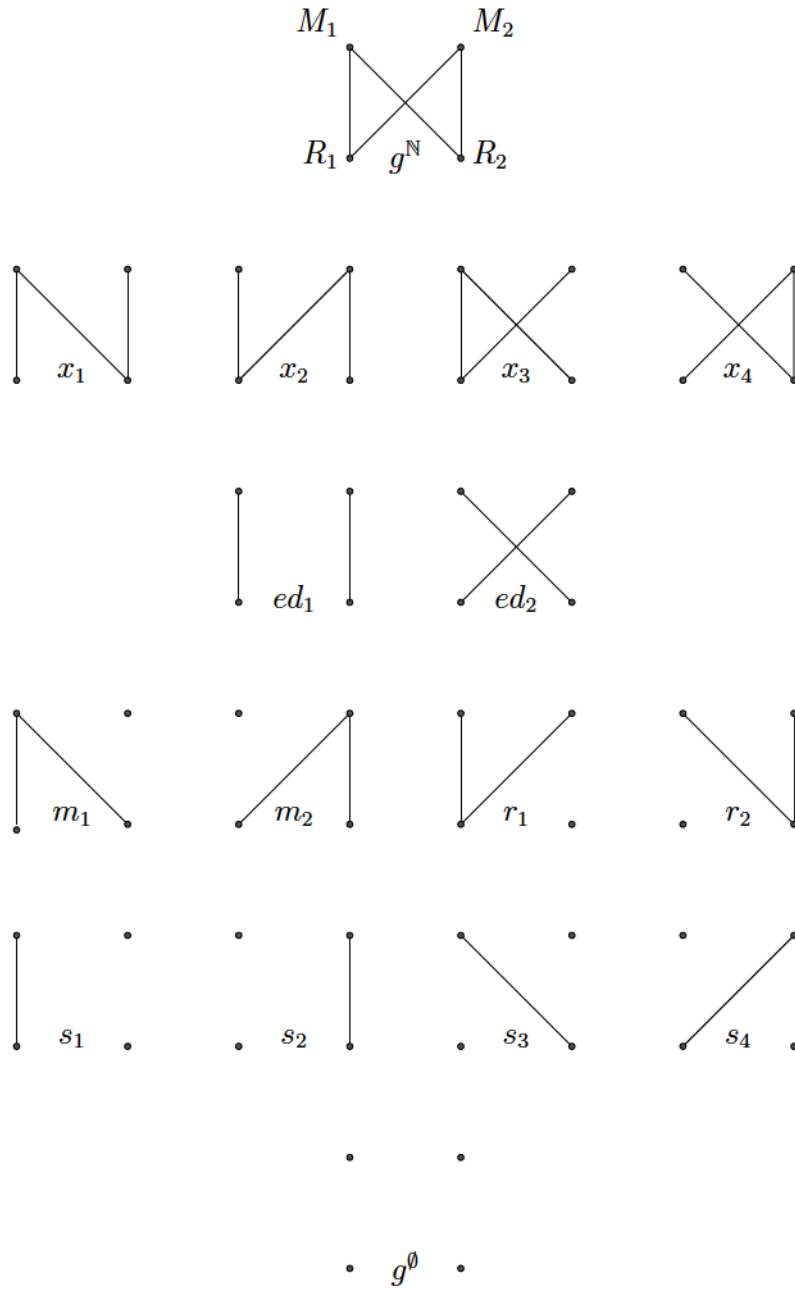


Figure 2.1: Typology of Organizational Structure.

w_i	q_{ij}	$Y_i(g^N)$	$Y_j(g^N)$
$\frac{a+c-ad}{2-d}$	$\frac{a-c}{3(d+1)(2-d)}$	$\frac{2(1-d)(a-c)^2}{3(d+1)(2-d)^2} - 2k$	$\frac{2(a-c)^2}{9(d+1)(2-d)^2} - 2k$

Table 2.1: Equilibrium values for g^N

retailer j is being awarded the following profit:

$$Y_j(g) = \sum_{i \in n_j(g)} (p_i - w_i) q_{ij} - \#n_j(g) k \quad (2.1)$$

The control variables of retailers being the quantities q_{ij} asked to retailers in $n_j(g)$. Manufacturers undergo a constant marginal cost c , thus, when i is linked to the set of retailers $n_i(g)$, he receives the following payoff:

$$Y_i(g) = (w_i - c) Q_i - \#n_i(g) k \quad (2.2)$$

Quantities asked depend on the structure of the network, the following calculations are given for the complete network².

$$\max_{q_{ij}} Y_j(g) \iff q_{ij} = \frac{1}{2} (a - q_{i-j} - dq_{-ij} - dq_{-i-j} - w_i), \quad \forall i \in n_j(g) \quad (2.3)$$

Manufacturers use their structural dominant position to anticipate the best response of each retailer so as to determine their wholesale price w_i . Given that the goods i and $-i$ are imperfect substitutes the best response of manufacturer i is given by the following derivatives:

$$\max_{w_i} Y_i(g) \iff Q_i + (w_i - c) \sum_{j \in n_i(g)} \frac{\partial q_{ij}}{\partial w_i} = 0 \quad (2.4)$$

The upstream duopoly affects the quantities requested by adjusting the wholesale price, this leading to an equilibrium in strategic complements which is summarize in the table 2.1

The payoffs³ for alternative networks can be found in Mauleon et al. [38] and are given

²The best response correspondence function still holds for any network g as we set $q_{ik} = 0, \forall ik \notin g$

³ The payoffs and their division among players are more generally referred to or interpreted as an

in the appendix. The payoffs thus depends on three parameters, d captures the intensity of the rivalry at the upstream level through the global differentiation of the manufactured goods. $2k$, the cost of each link and $(a - c)^2$. The parameter $(a - c)^2$ is an exogenous parameter usually defining the scale of the industry, it has been arbitrarily set to unity so as to easily compare the payoffs according to the parameters d and k exclusively⁴.

Given the profits for each network, the question of the stability is yet to be addressed.

2.3 Farsighted Stability

2.3.1 The pairwise rationale

The agents are able to identify their level of profit for each network of \mathbb{G} . The player λ earns a profit $Y_\lambda(g)$ in the network g . If, for two networks g and g' , we have $Y_\lambda(g) < Y_\lambda(g')$, then, player λ might want to alter the structure of the network g in such a way that g turns into g' . The evolution of the network g into g' is done by cutting her own ties, adding other links or by reassigning links with other players. Nonetheless, the considered player is not necessary able to modify the structure of relations if the alteration of g involves links between third parties. The incentives of a given player doesn't necessarily go hand in hand with the incentives of the other players and the evolution of g into g' can be blocked by other players. In the end, it turns out that the stable networks are the absorbing states at the intersection of the intentions of each players to alter or not, the structure of a given network, one link at a time⁵. This idea is the cornerstone of the concept of pairwise stability by Jackson & Wolinsky (1996) [33]. The idea lying behind pairwise stability is that the alteration of a network is driven by two principles. The first is deviations in deletion. A player decides to cut an existing link if the newly formed network brings her a positive marginal profit. The second principle is deviations in creation. A pair of player is willing to build a link among themselves if they both benefit from doing

imputation or an allocation rule depending on the field of study.

⁴The upper bound on k is set to ensure a non-negative payment to all players for all possible configurations of the network, $\bar{k} = \min \left\{ \frac{1}{36}, \frac{1-d}{3(1+d)(2-d)^2} \right\}$. See Mauleon et al. (2011)

⁵Another strand of the literature addresses this question from a coalitional perspective, see for instance Jackson and Van den Nouweland [31]

so.

Thus, there exists a finite sequence of successive deviations from g to g' if, for a sequence of adjacent networks g_1, g_2, \dots, g_K with $g_1 = g$ and $g_K = g'$ such that for any $\kappa \in \{1, 2, \dots, K-1\}$, either the first principle applies: $g_{\kappa+1} = g_\kappa - ij$ for some ij such that $Y_i(g_\kappa - ij) > Y_i(g_\kappa)$, or the second applies: $g_{\kappa+1} = g_\kappa + ij$ for some ij such that $Y_i(g_\kappa + ij) > Y_i(g_\kappa)$ and $Y_j(g_\kappa + ij) \geq Y_j(g)$.

The sequence from g_1 to g_K is called a myopic improving path. Eventually, a network $g \in \mathbb{G}$ is pairwise stable if:

$$(i) \forall ij \in g, Y_i(g) \geq Y_i(g - ij) \text{ and } Y_j(g) \geq Y_j(g - ij), \text{ and}$$

$$(ii) \forall ij \notin g, \text{ if } Y_i(g) < Y_i(g + ij) \text{ then } Y_j(g) > Y_j(g + ij).$$

More intuitively, a network g is stable if no players no longer wish to alter its structure, or can no longer alter the structure of g at the pairwise level.

A deviation of a player or a pair of player is implemented taking into account what the immediate deviation offers compare to the current network, but as pointed out by Jackson and Watts (2002) [32], the myopic players don't forecast that the deviation they induced may induce other deviations. At the end of the myopic improving path, the initial deviation could be at the expense of its instigator. That's the reason for the introduction of the notion of farsighted improving a path by Herings & al. (2009) [23]

2.3.2 Farsighted improving paths

If players anticipate that an initial deviation may induce a further evolution of the structure of the links, they are said to be farsighted players. The deviations are based on what the ending network offers compared to the current network. The ongoing series of deviation in between is called a farsighted improving path. A farsighted improving path of length $K \geq 0$ from a network g to g' is a sequence of adjacent networks g_1, g_2, \dots, g_K

with $g_1 = g$ and $g_K = g'$ such that for any $\kappa \in \{1, 2, \dots, K - 1\}$, either:

- (i) $g_{\kappa+1} = g_\kappa - ij$, $\exists ij$ such that $Y_i(g_K) > Y_i(g_\kappa)$, or
- (ii) $g_{\kappa+1} = g_\kappa + ij$, $\exists ij$ such that $Y_i(g_K) > Y_i(g_\kappa)$ and $Y_j(g_K) \geq Y_j(g_\kappa)$.

The sequence of networks is a sequence of pairwise deviations taking the final payoffs as reference, this mean that, all along the farsighted improving path, the players don't perceive any stream of payoff. Using notation from Herings, Mauleon & Vannetelbosch (2009) [24], if a farsighted improving path from g to g' of length K exists, then we note $g \longrightarrow_K g'$.

We define the set of networks that can be reached from g by one farsighted improving path of length $K \leq K'$ by:

$$f_{K'}(g) = \{g' \in \mathbb{G} \mid \exists K \leq K' \text{ such that } g \longrightarrow_K g'\}$$

That is, the set $f_{K'}(g)$ is the collection of all the networks that can be reached by the mean of a farsighted improving path in no more than K' alterations of g . Similarly, the set of networks that can be reached by the mean of one farsighted improving path of any length from g is noted $f_\infty(g)$.

2.3.3 Pairwise farsighted stability

Definition 8 *A set of network $G_\infty \subseteq \mathbb{G}$ is a pairwise farsightedly stable set with respect to Y if:*

- (i) $\forall g \in G_\infty$,
 - (ia) $\forall ij \notin g$ such that $g + ij \notin G_\infty^1, \exists g' \in f_\infty(g + ij) \cap G_\infty$ s.t.:
 $(Y_i(g'), Y_j(g')) = (Y_i(g), Y_j(g))$, or $Y_i(g') < Y_i(g)$ or $Y_j(g') < Y_j(g)$.
 - (ib) $\forall ij \in g$ such that $g - ij \notin G_\infty, \exists \{g', g''\} \in f_\infty(g - ij) \cap G_\infty$ s.t.:
 $Y_i(g') \leq Y_i(g)$ and $Y_j(g'') \leq Y_j(g)$.
- (ii) $\forall g' \in \mathbb{G} \setminus G_\infty, f_\infty(g') \cap G_\infty^1 \neq \emptyset$.
- (iii) $\nexists G' \subsetneq G_\infty \mid G' \text{ satisfies conditions (ia), (ib), (ii)}$.

The three criteria respectively correspond to (i) internal stability, (ii) external stability and (iii) Minimality. The condition on external stability is a requirement identical to the one by von-Neumann & Morgenstern in its spirit: the necessary condition for the external stability of a set requires this set to absorb any state outside. In terms of pairwise farsighted deviations, any network outside a stable set has an image in the set through a farsighted improving path. The first condition on the internal stability of a set of networks falls into the deterrence of bilateral deviations (ia) and unilateral deviations (ib). Two players won't be linked in a network of an internal stable set if the addition of their link triggers some series of deviations leading back to the set at their detriment. The existence of a single perspective worse than the initial situation is sufficient to deter the deviation. The unilateral deviation deterrence (ib) works according to the same principle, breaking an existing link is deterred by the perspective that this action could initiate a path bringing back to the set at the expense of the initial defector. The pairwise definition of internal stability allows the dominance paths between adjacent networks inside the set. Thus, unlike the definition of internal stability of a von-Neumann & Morgenstern stable set, the networks of a stable set are not necessarily undominated. To summarize, the networks outside the set lead to the set, there can be inner cycles among networks in the set, but outer cycles are deterred.

A last word about the existence of a pairwise farsighted stable set. Note that \mathbb{G} satisfies both internal and external stability but turns out to be a trivial result. That's the explanation for the last criterion on minimality (iii). If a set that satisfies internal and external stability contains a subset that also satisfies these two requirements, then some networks inside this set are unnecessary to get the two forms of stability. Such a set is evicted from the solution set.

I Now introduce the next paragraph with the following example. Herings et al (2009) proved that, whenever a networks \tilde{g} that strictly pareto dominates all the other networks exists, then a farsighted improving path from any other networks to \tilde{g} exists. Then \tilde{g} is externally stable. A set of cardinality 1 complies with internal stability and is minimal, thus a strictly pareto dominating network is the unique pairwise farsighted stable network.

In this example, converging is of mutual interest and the players manage to get the best solution. This results is achieved by the existence of paths leading to \tilde{g} , that is, by the strong assumption that players with an infinite reasoning get to glimpse all the existing states. Completely relaxing this assumption leads to look at myopic players with presumably contrasting results in terms of the stability of networks. It is thus legitimate to question the role of the level of farsightedness on the stability of the networks. Theoretical tools were developed for players located at the two bounds of farsightedness, the next solution concept filled the gap in between.

2.3.4 Level- K farsighted stability

The limited farsightedness postulates that players anticipate a given number of steps ahead. The decisions to deviate are now based on what a network located at no more than K steps ahead offers compared the current one (see section 2.3.2 for a formal definition). For a $K < \infty$, the limited farsightedness modifies the mapping of farsighted improving paths. Indeed, although the players includes paths of maximum length K in the decision making process, paths of length greater than K may exist. Think for instance that a player i anticipates that the deviation from a network g , to a network in $f_K(g - ij)$ improves his situation. Using the farsighted rationale, if such a deviation is not deterred within K steps, it will occur. A player with limited farsightedness ignores that paths emanating from a network beyond the sequence of deviation may exist. Therefore, the networks of $f_K(g - ij)$ may not be the resting points and some farsighted improving paths may also leave from these networks. Thus when considering limited farsightedness, we need to define the sequences of farsighted improving paths, the compositions of several farsighted improving paths. Let $f_K^m(g)$ be the set of networks that can be reached by the mean of $m \in \mathbb{N}$ compositions of farsighted improving paths initiated in g and of length no greater than K . Formally:

$$f_K^m(g) = f_K(f_K^{m-1}(g)) = \{g'' \in \mathbb{G} \mid \exists g' \in f_K^{m-1}(g) \text{ such that } g'' \in f_K(g')\}$$

By convention, $f_0(g) = g$ and $f_{-1}(g) = \emptyset$ are assumed.

Note that, there can be no more than $\#\mathbb{G} - 1$ sequences of farsighted improving path regardless of the level of farsightedness. Thus the following equality $f_K^m(g) = f_K^\infty(g)$ holds for $m \geq 15$ in the 2×2 networks of manufacturer and retailers. I can now enunciate the definition a pairwise level- K farsighted stability of Herings, Mauleon and Vannetelbosch[24]

Definition 9 For $K \geq 1$, a set of network $G_K^\infty \subseteq \mathbb{G}$ is a pairwise level- K farsightedly stable set with respect to Y if:

$$(i) \forall g \in G_K^\infty,$$

$$(ia) \forall ij \notin g \text{ such that } g + ij \notin G_K^\infty,$$

$$\exists g' \in [f_{K-2}(g + ij) \cap G_K^\infty] \cup [f_{K-1}(g + ij) \setminus f_{K-2}(g + ij)] \text{ s.t.}$$

$$(Y_i(g'), Y_j(g')) = (Y_i(g), Y_j(g)), \text{ or } Y_i(g') < Y_i(g) \text{ or } Y_j(g') < Y_j(g),$$

$$(ib) \forall ij \in g \mid g - ij \notin G_K^\infty,$$

$$\exists \{g', g''\} \in [f_{K-2}(g - ij) \cap G_K^\infty] \cup [f_{K-1}(g - ij) \setminus f_{K-2}(g - ij)] \text{ s.t.}$$

$$Y_i(g') \leq Y_i(g) \text{ and } Y_j(g'') \leq Y_j(g).$$

$$(ii) \forall g' \in \mathbb{G} \setminus G_K^\infty, f_K^\infty(g') \cap G_K \neq \emptyset.$$

$$(iii) \nexists G' \subsetneq G_K^\infty \mid G' \text{ satisfies conditions (ia), (ib), (ii)}$$

Literally, the definition of a level- K farsighted stable set features three criteria. Similarly to the case of fully farsighted players, (i) deals with the "internal stability" of a set G . For any network in the set, any alteration by a player or a pair of players is deterred by the fear of ending worse-off (at least not better-off) at the end of the farsighted improving path. (ia) If two players do not wish to bind in a network in the set, then, either a farsighted improving path of length not greater than $K - 1$ leads back to the set, or a farsighted improving path of length $K - 1$ leads outside the set where at least one of them gets a lower payoff. (ib) For any network in the set, the players do not wish to unilaterally cut an existing links. One of the parties doesn't want to sever the link because a farsighted improving path of length $K - 1$ wouldn't bring him a higher payoff in a network outside the set, or a farsighted improving path of length not greater than

$K - 1$ wouldn't bring him a higher payoff in a network inside the set. The criterion (ii) deals with the "external stability". This second requirement is a condition similar to the definition of an externally quasi stable set by Chvatal and Lovasz (1972) [12]. A set of network is externally quasi stable if a path or a combination of paths from any network outside the externally quasi stable set exists. Finally, (iii) is the "minimality criterion", no subset of a stable set satisfies both internal and external stability.

The definition of a level- K farsighted stability holds for the myopic case up to the level- ∞ of farsightedness. However, since the farsighted improving paths are of limited length (there can't be path of length greater than $\#\mathbb{G}$), the level of farsightedness is also bounded. The lowest minimum level of farsightedness needed to be fully farsighted is theoretically determined. Indeed, whenever $K \geq \#\mathbb{G} - 1$, an increase in the level of farsightedness doesn't change the set of networks that can be reached from an initial deviation ⁶. The definition of the internal stability of a level- K stable set uses deviations that leads back to the set at the level $K - 2$, thus for a $K \geq \#\mathbb{G} + 1$, $G_K^\infty = G_{K+1}^\infty$ holds. Let's denote that level by K^{theo} .

The difference between a pairwise farsighted stable set G_∞ and a level- ∞ farsighted stable⁷ set lies in the definition of the external stability. In the definition of a level- ∞ pairwise farsighted stable, external stability requires that an infinite combination of farsighted improving paths of infinite length from a networks outside the set terminates in the stable set. A level- ∞ farsighted stable G satisfies quasi-external stability: $\forall g' \notin G, f_\infty^\infty(g') \cap G \neq \emptyset$. Unlike a level- ∞ set, a pairwise farsighted stable set G_∞ , requires that a single path of infinite length from a network outside the set leads back to the stable set: $\forall g' \notin G_\infty, f_\infty(g') \cap G_\infty \neq \emptyset$. This has implications in the determination of stable sets, indeed a pairwise farsighted stable set may differ from a level- ∞ stable set.

From the definition of a farsighted improving path, it holds that $f_\infty(g) \subseteq f_\infty^\infty(g)$. The combination of paths from a network outside a level- ∞ stable set makes it easier for a set of networks of low cardinality to sustain external stability and being minimal compared to a stable set G_∞ . This implies that a level- K stable set can not be greater than a G_∞

⁶Since $f_K(g) \subseteq f_{K+1}(g)$, if $K \geq \#\mathbb{G} - 1$, then $f_K(g) = f_{K+1}(g)$ for each g in \mathbb{G} .

⁷Or similarly a level- K^{theo} farsighted stable set.

set. It follows that, for any G_∞ , at least a set $G_K^\infty \subseteq G_\infty$ is a level- K farsighted stable set for any $K \geq K^{theo}$. Indeed a set G_∞ is a set for which the externally quasi stable criterion consists in single paths of infinite length. Since the farsighted stability concept is a subset of the level- ∞ farsighted stability, I will refer to G_∞ as G_∞^1 .

The complexity of the determination of stable sets for each level- K farsightedness first lies in the number of sets to be tested. As seen before, for a $\#M + \#R$ players bipartite network, $\#\mathbb{G} = 2^{\#M\#R}$ different network can be formed. A set of networks is any combination of these $\#\mathbb{G}$ networks, that is, a set of cardinality $\sum_{i=1}^{\#\mathbb{G}} C_{\#\mathbb{G}}^i$. Let's denote by Ω , the set of all feasible sets of networks. For any product differentiation $0 \leq d \leq 1$, any cost of link $0 \leq k \leq \bar{k}$ and any level of farsightedness $0 \leq K \leq K^{theo}$, I now check the level- K farsighted stability of any set of networks in Ω . That's the purpose of the following algorithm.

2.4 The algorithm

The algorithm is quite simple in the sense that it has two main scripts. The first function maps the farsighted improving paths and is twofold. It computes the payoffs of the players for any network $g \in \mathbb{G}$, and a couple of parameters d and k . It then looks at the payoffs to render matrices of farsighted improving paths. Finally, it checks the value of K from which a matrix is identical to the matrix at the level $K - 1$. Observing this value returns the minimum level of farsightedness corresponding to the infinite level of farsightedness \bar{K} .

Once the farsighted improving paths for all $K' \leq \bar{K}$ are found, the program switches to its second part. In the second part of the algorithm, the program uses a recursive dynamic argument to explore the set of feasible. First, the set of all combinations of networks is generated. Starting from the smallest sets (singleton sets), the conditions on internal and external stability are checked. If no singleton set complies with (ia) and (ib) , the program switches to the sets of cardinality 2. Once again, if no set of cardinality 2 complies with conditions (ia) and (ib) it switches to calculations for the upper cardinality, and so forth. Once a set G complying with (ia) and (ib) is found, then it is necessarily

minimal. The program then deletes the super-sets of G which violates minimality. This action shrinks the set of feasible to a $\Omega' \subset \Omega$ and reduces the computing time. The program ends once all the sets of Ω' have been visited or once Ω' is empty. The pseudo code for the identification of the farsighted improving paths, so as the scripts for the following functions can be found in the appendix in section 2.9.1. This section defines the functions used throughout the algorithm:

Each network as been attributed a number, $g^{\mathbb{N}} = 1, x_1 = 2, \dots, g^{\emptyset} = 16$ it will keep. The players are also attributed a number, $M_1 = 1, M_2 = 2, R_1 = 3, R_2 = 4$. The payoffs have been stored in a table with 4 columns (the players) and 16 rows (the networks) according to the following function:

payoff(d, k): For a given couple of parameters d and k returns the table of payoff $Y_i(g)$ for all $i \in N$ and all $g \in \mathbb{G}$. **payoff**(d, k) [1, 1] returns the payoff of the first manufacturer in the complete network.

players(g, g'): This function identifies, for two networks g and $g' \in A^g$, which pair of players could be involved in the alteration of g into g' . Based on the adjacencies matrix of g and g' , **players**(g, g') = (i, j) if $g' = g - ij$ or $g' = g + ij$. That is, **players**(g, g') = **players**(g', g)

compare(g_1, g_2, g_3, d, k): This function is at the heart of the incentives analysis of the players. Based of the payoffs given by d and k , **compare**(.) asks whether or not, a pairwise deviation from g_3 to g_2 will occur knowing that a farsighted improving path leads to g_1 . The function returns 1 in the following cases:

- If $\#g_3 > \#g_2$,

If **payoff**(d, k) [**players**(g_2, g_3) [1], g_1] > **payoff**(d, k) [**players**(g_2, g_3) [1], g_3] or,
if **payoff**(d, k) [**players**(g_2, g_3) [2], g_1] > **payoff**(d, k) [**players**(g_2, g_3) [2], g_3].

- If $\#g_3 < \#g_2$,

If $\text{payoff}(d, k) [\text{players}(g_2, g_3) [1], g_1] \geq \text{payoff}(d, k) [\text{players}(g_2, g_3) [1, g_3]]$, and $\text{payoff}(d, k) [\text{players}(g_2, g_3) [2], g_1] \geq \text{payoff}(d, k) [\text{players}(g_2, g_3) [2], g_3]$ with at least one strict inequality.

The `matfip(.)` function uses the previous function to ask if a level K farsighted improving from g to g' exists. This function returns a set of boolean matrices of farsighted improving path for any level K . Formally $m_K [g, g'] = 1$ if $g \rightarrow_K g'$ exists, otherwise, the matrix m_K is filled with 0. A matrix which columns and rows are the ordering on networks filled of 0 is built. The function begins at $K = 1$ and for any $g \neq g'$, $m_1 [g, g'] = \text{compare}(g', g, g, d, k)$. For higher level of farsightedness, $m_K [g, g'] = \text{compare}(g', g'', g, d, k)$ if $m_{K-1} [g'', g'] = 1$. The matrices m_K can be thought of as the matrices of farsighted adjacencies between networks at the K 's level. The farsighted improving paths of length not greater than K are obtained by the aggregation of the matrices from m_1 to m_K . We note $M_K = \sum_{K'=1}^K m_{K'}$ and $M_K [g, g'] = 1$ if $\sum_{K'=1}^K m_{K'} [g, g'] > 1$. The matrices M_K can be thought of as the matrices of farsighted adjacencies of length not greater than K between networks. Examples of matrices of farsighted adjacencies can be found in the next section 2.5 Note that $m_1 = M_1$ is the adjacency matrix of the supernetwork (see Page et al. [47]). This function exhibits an interesting fact: beyond a given level of farsightedness, the matrices m_K seem to cycle according to the same pattern ($m_K = m_{K+\alpha}$), this impacting the matrices M_K . For a $K \geq K + \alpha$, we can note that $M_K = M_{K+1} = M_{\bar{K}}$. This phenomenon is explained in details below in section 2.5. Starting from the theoretical bound K^{theo} , the matrices M_K identical to their previous matrix M_{K-1} are withdrawn, the process ends when $M_{\bar{K}-1} \neq M_{\bar{K}}$. It's important to note that at the end of this sequence, the value of \bar{K} is known, and a sharp refinement of the maximum farsightedness can be observed (section 2.5). The function returns all the matrices of farsighted improving paths for all $K' \leq \bar{K}$.

`devdet(G', g', g)`: This script works according to the same logic as `compare(.)`. It checks, once all farsighted improving paths for a given K are known, if a deviation is deterred. G' is the set of networks that can be reached by a farsighted improving path emanating from g and a player (or a pair of players) can deviate from g to g' . The function

returns 1 if a player (in case of the severance of a existing link), or at least one of the two players (in case of the creation of a new link) fear ending in a network of G' , returns 0 if the deviation is not deterred.

The $\text{instab}(G, M_K)$ function checks, for a level K , and its matrix of farsighted improving paths M_K , if a set of networks G complies with internal stability. To do this, it checks if a deviation via a network outside the set is deterred by the fear of not ending better-off. For $K = 1$, it's sufficient to check that there is no farsighted improving path emanating from a network within the set. For higher levels, we need to check if any deviation from a network of the set G to a network outside the set is deterred by the fear of ending in a network of G in no more than $K - 1$ steps or ending in any network in exactly $K - 1$ steps. The function returns 1 in the following case: If for any $g \in G$ and any $g' \in A^g$, if $\exists g'' \in G' \cap G$ such that $\text{devdet}(G', g', g) = 1$ according to the matrix M_{K-2} or $\exists g'' \in G'$ such that $\text{devdet}(G', g', g) = 1$ according to the matrix M_{K-1} . If the function returns 0, then G violates internal stability and is withdrawn from Ω . Remind that by convention, $f_{-1}(g) = \emptyset$, so I imposed $M(-1) = \emptyset$. Finally, even though $f_0(g) = g$, I set $M(0) = 0_{[\#G \times \#G]}$ to gain calculation time since the deviation from a network to itself is deterred.

The purpose of the $\text{exstab}(G, M_K)$ script is to check the external stability of the set G given the farsighted improving paths of length not greater than K . The principle is to show that a deviation in any network outside the set to a network inside the set occurs. This deviation leads back to a network of G within an infinity of sequences of farsighted improving paths of length not greater than K . The function runs by successive backward iterations. It first checks if at least a network outside G has a farsighted improving path of length not greater than K leading back to G . If a network g is farsightedly dominated by some network in the tested set, the script uses this information to build sequences of farsighted improving paths of length not greater than K . It first looks at the networks outside the set which are directly dominated by a network in the set, in no more than K steps. Then it looks at the networks outside the set which are dominated by some network found at the first iteration. Then it turns to the networks outside the set for

which 3 sequences of farsighted improving paths lead back to some network inside the set, and so on. If one of the networks outside G have no sequences of paths leading to the tested set, then G fails at being externally stable and the function return 0.

Running the algorithm: The command `matfip(payoff(d, k))` generates the matrices of farsighted improving paths for the payoffs associated to the parameters d and k . There are \bar{K} ordered matrices associated to each level of farsightedness. The program switches to its second part and starts with $K = 1$. Every feasible set in Ω is generated. A function generates the combination of all set of numbers in $[1, 16]$ of size 1 (singletons), then size 2 (every sets made of two numbers out of the sixteen numbers), then size 3 and so forth up to 16 (remind that any network has been attributed a number). Thus Ω is every combination of networks. Once Ω generated, the program begins its exploration with the sets of lowest cardinality. Every sets made of only one networks are tested. Let G be a singleton set, then functions `instab(G, M_1)` and `exstab(G, M_1)` are tested on each $G \in \Omega$. If no set complies with both, the program switches to upper cardinalities. If G complies with the two forms of stability, then its super-sets are withdrawn from Ω . A function applies $\Omega \setminus G'$ whenever $G' \supset G$. The new set is denoted by Ω' . The program ends once every sets of Ω' have been visited, or once Ω' is empty. The results are stored in a table that associates the stable sets to the couple (d, k) . The algorithm runs again for higher levels of farsightedness. Once \bar{K} has been reached, the algorithm starts over for another couple of parameter (d, k) . The program ends once all couples have been visited.

Finally, to end this section, a brief word on the output of the algorithm. After completing calculations, the set of stable sets for each value of K are determined. I chose to represent the output in a Cartesian coordinate system $(\vec{O}d, \vec{O}k)$ for convenience worries. Thus, the first thing to do is to set the parameter of product differentiation d and the cost of the links k . Randomly drawing these values from a uniform distribution found itself of limited interest since some areas of the graph being saturated in points while some others were blank. It was therefore chosen to space the points equidistant so that the results lattice would be sufficiently dense without being overloaded. The algorithm also allows to change the density of the scatter plot. It was also chosen to put points on the bounds

and the corners so as to keep track of threshold effects.

2.5 The results

This section is devoted to look at the stable sets according to the level of farsightedness, the fluctuation of the level of products differentiation (d) and the costs of linking (k). First, the myopic case is studied, then the the level of farsightedness is increased and the second section discuss the evolution of the stable sets for any intermediate level of limited farsightedness. The computations were undertook on a set of 1803 points within the set of feasible for any level of farsightedness in range $[1, 15]$. The choice of this density is a compromise to keep a nice but not overloaded scatter plot together with a reasonable calculation time. Although calculations for a greater number of points have been computed, a higher density (and so greater calculation time) doesn't bring extra information. Each stable set, or a set of stable sets is associated to a point in the space . I choose to depict the results in graphics that partition the space in areas of stability of a given set.

2.5.1 The myopic case G_1^∞

At level $K = 1$, there is a unique farsightedly stable set of networks for a couple of parameters (d, k) . When players are myopic, the stable set is composed of all the networks that belong to a close cycle (see section 2.7.1 for a formal definition).

For expository purposes, an example that gives insights on how the results are obtained is hereafter provided. The payoffs for $(d, k) = (0.7, 0.005)$ are given in Table 2.2 and the associated adjacency matrix is depicted in Table 2.3. In this matrix, a 1 indicates that a deviation from the network in row to the network in column occurs, otherwise, the matrix is filled with 0. In this example, the empty network is dominated by any 1-link network which are in turn dominated by either a monopoly, or monopsony or exclusive dealing network. We can see that $f_1(r_1, r_2, m_1, m_2) \in \{x_1, x_2, x_3, x_4\}$, and more precisely, any monopoly or monopsony is dominated by 2 out of the 4 mixed distribution network.

	M_1	M_2	R_1	R_2
\mathbb{N}	0.05961364	0.05961364	0.06734849	0.06734849
x_1	0.07640787	0.05435935	0.05348933	0.07816900
x_2	0.05435935	0.07640787	0.07816900	0.05348933
x_3	0.07640787	0.05435935	0.07816900	0.05348933
x_4	0.05435935	0.07640787	0.05348933	0.07816900
ed_1	0.08342635	0.08342635	0.04538539	0.04538539
ed_2	0.08342635	0.08342635	0.04538539	0.04538539
r_1	0.04721023	0.04721023	0.16403411	0
r_2	0.04721023	0.04721023	0	0.16403411
m_1	0.15666667	0	0.02277778	0.02277778
m_2	0	0.15666667	0.02277778	0.02277778
s_1	0.12	0	0.0575	0
s_2	0	0.12	0	0.0575
s_3	0.12	0	0	0.0575
s_4	0	0.12	0.0575	0
\emptyset	0	0	0	0

Table 2.2: Payoffs for $(d, k) = (0.7, 0.005)$

Let's note $G = \{g^{\mathbb{N}}, ed_1, ed_2\}$, it turns that $f_1(x_1, x_2, x_3, x_4) \in G$. To summarize, the two exclusive dealing networks, so as the complete network are reached from any other network within a sequence of 4 improving paths of length 1. We can therefore conclude for $g \in \mathbb{N} \setminus G$, $f_1^4(g) = f_1^\infty(g) = G$ which is the definition of the external stability of G . Note that the networks in G are undominated, $\forall g' \in G$, $f_1(g') = \emptyset$, which is a sufficient condition for G to be internally stable. Any subnetwork of G violates external stability, thus G is myopic pairwise stable, *i.e.* a level-1 farsighted stable set and it is unique. In the example, the networks $g^{\mathbb{N}}$, ed_1 and ed_2 are also pairwise stable.

The general results for the level-1 stability in the networks of manufacturers and retailers are summarize in the next proposition and the graphical solution is depicted in Figure (2.2)

Proposition 10 *If $K = 1$, then only four sets of distribution networks may be pairwise stable: $g^{\mathbb{N}}$, $\{X\}$, $\{ED\}$ and $\{g^{\mathbb{N}}, ED\}$. The parameter space (d, k) is partitioned into 4 regions.*

- (i) *Low values of d and k imply that $G_1^\infty = g^{\mathbb{N}}$.*
- (ii) *Low values of d and intermediate values of k imply that $G_1^\infty = \{X\}$.*
- (iii) *Either low values of d and values of k close to the upper bound and large values*

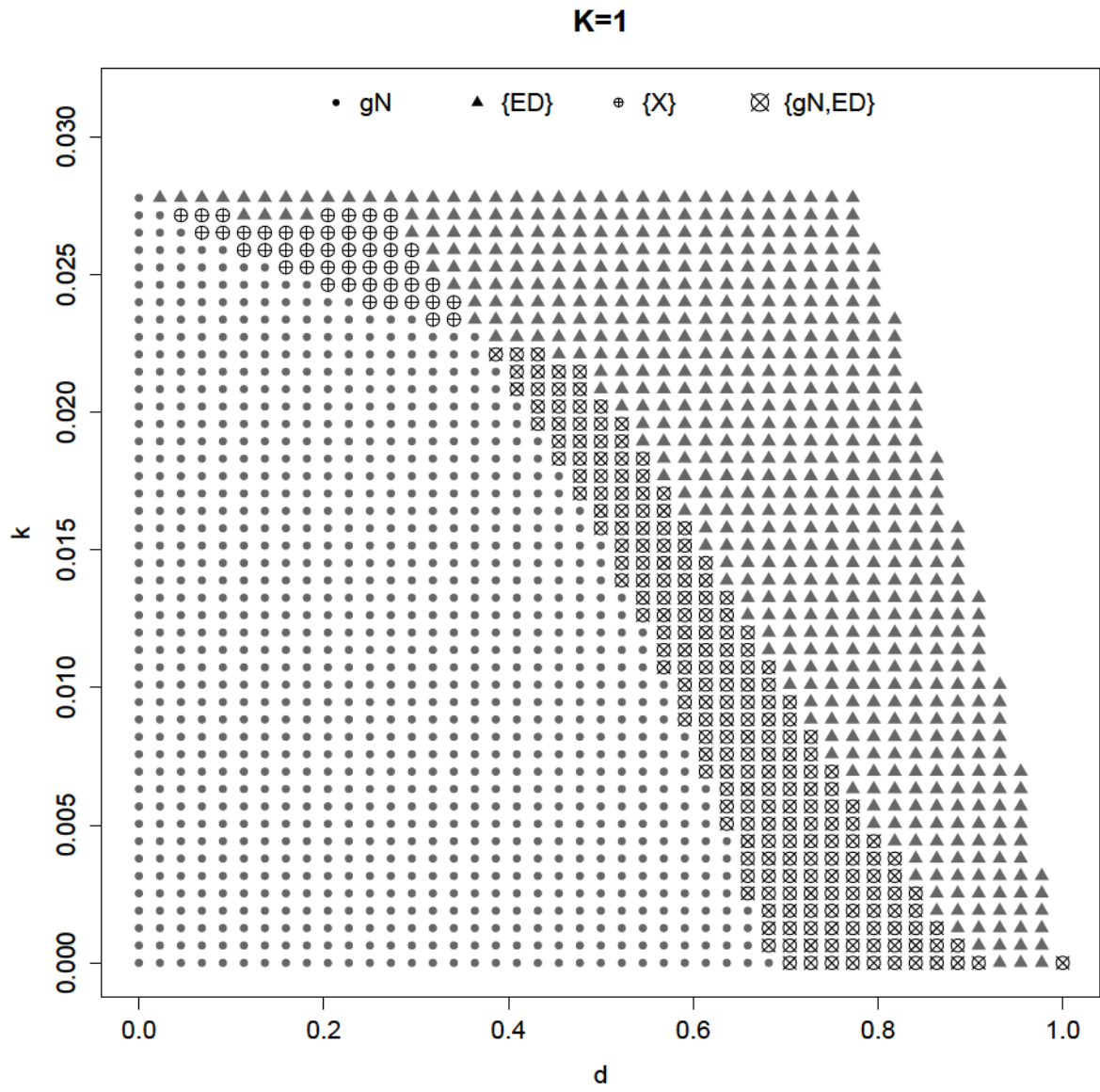


Figure 2.2: Stability for $K = 1$

	\mathbb{N}	x_1	x_2	x_3	x_4	ed_1	ed_2	r_1	r_2	m_1	m_2	s_1	s_2	s_3	s_4	\emptyset
\mathbb{N}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
x_2	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
x_3	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
x_4	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
ed_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ed_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
r_1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
r_2	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0
m_1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
m_2	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
s_1	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0
s_2	0	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0
s_3	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0
s_4	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0
\emptyset	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0

Table 2.3: Adjacencies of the supernetwork.

of d and any k , imply that $G_1^\infty = \{ED\}$.

(iv) Intermediate values of (d, k) along the antidiagonal or $(d, k) = (1, 0)$, imply $G_1^\infty = \{g^N, ED\}$.

There are 4 possible outcomes. For the lowest values of the parameters, the complete network is the unique stable network. This is somehow intuitive. If the cost of building a link is not too high and if the two products are quite different, the agents are attracted by a dense network of exchanges, the increase in the commercial relationships increases the output and so the gross profit. On the opposite, for the highest values of the parameters, the set composed of the two networks of exclusive dealing ed_1 and ed_2 is myopically stable. Once again, this result is intuitive since the products are pretty similar and the cost of links turns out to be an increasing part of the fixed costs of the firms. The solution for the intermediate values of the parameters is also the intermediate solution between the two previous results and $\{g^N, ed_1, ed_2\}$ is stable for the intermediate space of the feasible (The central lower region). For d not too high and the highest values of k , the set of networks $\{x_1, x_2, x_3, x_4\}$ is stable. Once again, the players are attracted by a dense structure but the stability of the 3-links networks has to be explained by two facts. The first thing is that the marginal gain of building the 4th link is offset by its cost. The second is that the players switch from an exclusive dealing network to network of mixed distribution, the products being differentiated enough to make it worth. Eventually, the set $\{g^N, ed_1, ed_2\}$

is also stable when $k = 0$ and $d = 1$, this is due to a threshold effect when the values are set to their extremum. The development of an algorithm for the myopic case is of limited interest. Though the algorithm allowed for the identification of threshold effects as it is the case for $(d, k) = (1, 0)$, this graphical solution doesn't smooth the aliasing and an analytically determined solution for $K = 1$ is more accurate as it is the case in Mauleon et al. 2011 [38].

2.5.2 Greater levels of farsightedness

The computations of the pairwise level- K farsighted stable sets of networks for $K > 1$, so as their economic interpretation in terms of vertically connected economy are provided hereafter. Consider again the example with $(d, k) = (0.7, 0.005)$ and the relative payoffs given in Table 2.2. Remind that $G = \{g^N, ed_1, ed_2\}$ was the stable set at the level-1. The level-2 matrix $M_2(0.7, 0.005)$ is given in Table 2.4. A quick look at the payoffs indicates that the manufacturers get a higher payoff in a network of exclusive dealing compared to their payoff in the complete network. From the level-1 deviations, we know that the two-link manufacturer is better-off in an exclusive network rather than a mixed distribution network. It appears that once a manufacturer sever one of his links in g^N , he anticipates that the other manufacturer will also sever one of his links, in such a way that, in the end, one of the exclusive distribution network has formed. Since $g^N \rightarrow_2 ed_{\{1,2\}}$ exists and is not deterred, a deviation from a network of G to a network within G exists, this set does no longer comply with internal stability at the level-2. The easiest way to determine the level-2 farsighted stable set for $(d, k) = (0.7, 0.005)$ is to notice that for any network outside of $\{ed_1, ed_2\}$ there exists two sequences of farsighted improving paths of length 2 that leads back to the set. Formally: $\forall g \notin \{ed_1, ed_2\}, f_2^2(g) \cap \{ed_1, ed_2\} \neq \emptyset$ (external stability), ed_1, ed_2 are undominated (internal stability) and any subsets violates external stability.

The general results for the level-2 farsighted stability are given in figure 2.3 and summarized in the following proposition:

Proposition 11 *If $K = 2$, then six sets of distribution networks may be pairwise farsight-*

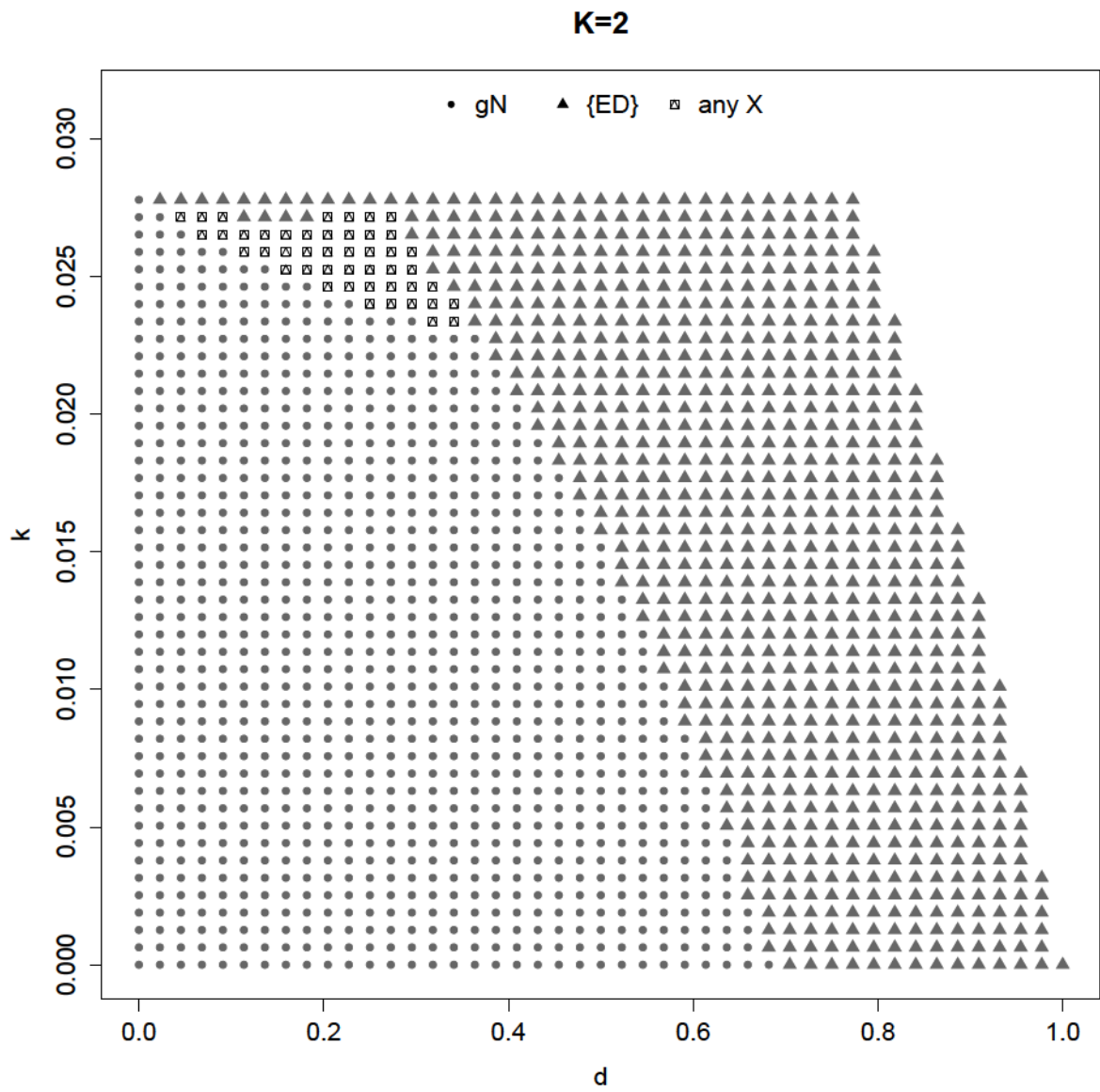


Figure 2.3: Stability for $K = 2$

	\mathbb{N}	x_1	x_2	x_3	x_4	ed_1	ed_2	r_1	r_2	m_1	m_2	s_1	s_2	s_3	s_4	\emptyset
\mathbb{N}	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
x_1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
x_2	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
x_3	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
x_4	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
ed_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ed_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
r_1	1	0	1	1	0	1	1	0	0	0	0	0	0	0	0	0
r_2	1	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0
m_1	1	1	0	1	0	1	1	0	0	0	0	0	0	0	0	0
m_2	1	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0
s_1	0	0	1	1	0	1	0	1	0	1	0	0	0	0	0	0
s_2	0	1	0	0	1	1	0	0	1	0	1	0	0	0	0	0
s_3	0	1	0	0	1	0	1	0	1	1	0	0	0	0	0	0
s_4	0	0	1	1	0	0	1	1	0	0	1	0	0	0	0	0
\emptyset	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0

Table 2.4: Level-2 adjacencies.

edly stable: $g^{\mathbb{N}}$, $\{ED\}$ and any network in X . The parameter space (d, k) is partitioned into 3 regions.

(i) Low values of d and k imply that $G_2^\infty = g^{\mathbb{N}}$.

(ii) Low values of d and intermediate values of k imply that G_2^∞ is either x_1, x_2, x_3 or x_4 .

(iii) Either low values of d and values of k close to the upper bound and large values of d and any k , imply that $G_2^\infty = \{ED\}$.

The transition from the level-1 to the level-2, has no effect on the region for which the complete network is farsightedly stable. The region for which the exclusive dealing networks are stable has increased, absorbing the region for which the intermediate set was level-1 stable. The reason is the one exposed in the previous example. The manufacturers anticipate that an initial deviation of their own (cutting a link) induces the other manufacturer into having an identical behavior. The complete network of distribution is no longer supported. The softer competition upstream leads to an increase in the gross profit of the manufacturers. The area devoted to the mixed distribution networks has undergone some changes. At the level-1, the set $G = \{x_1, x_2, x_3, x_4\}$ used to be stable, but at the level-2, it violates minimality. Indeed, at the level-2, there exist a composition of farsighted improving path from any network outside of G to any of the mixed distribution networks. This implies that any network of X now complies with external stability. This

	\mathbb{N}	x_1	x_2	x_3	x_4	ed_1	ed_2	r_1	r_2	m_1	m_2	s_1	s_2	s_3	s_4	\emptyset
\mathbb{N}	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
x_1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
x_2	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
x_3	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
x_4	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
ed_1	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
ed_2	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
r_1	1	0	1	1	0	1	1	0	0	0	0	0	0	0	0	0
r_2	1	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0
m_1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
m_2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
s_1	1	0	1	1	0	1	0	1	0	1	0	0	0	0	0	0
s_2	1	1	0	0	1	1	0	0	1	0	1	0	0	0	0	0
s_3	1	1	0	0	1	0	1	0	1	1	0	0	0	0	0	0
s_4	1	0	1	1	0	0	1	1	0	0	1	0	0	0	0	0
\emptyset	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

Table 2.5: Level-3 adjacencies.

is sufficient condition for a singleton to be pairwise farsightedly stable.

The general results for the level-3 farsighted stability are given in Figure 2.4 and summarized in the following proposition:

Proposition 12 *If $K = 3$, then eight sets of distribution networks may be pairwise farsightedly stable: $g^{\mathbb{N}}, \{ED\}$, any network in ED and any network in X . The parameter space (d, k) is partitioned into 5 regions.*

- (i) *Low values of d and k imply that $G_3^\infty = g^{\mathbb{N}}$.*
- (ii) *Low values of d and intermediate values of k imply that G_3^∞ is either x_1, x_2, x_3, x_4, ed_1 or ed_2 .*
- (iii) *Either low values of d and values of k close to the upper bound and large values of d and intermediate values of k imply that $G_3^\infty = \{ED\}$.*
- (iv) *Sufficiently high values of d and a low k implies that G_3^∞ is either $g^{\mathbb{N}}, ed_1$ or ed_2 .*
- (v) *The highest values of d implies that G_3^∞ is either ed_1 or ed_2 .*

Although the transition from the level-2 to the level-3 has no effect on the region for which the complete network is farsightedly stable, the increased of the level of farsightedness disrupted the other areas. We can see this through Table 2.5 which refers to the example for $M_3(0.7, 0.005)$.

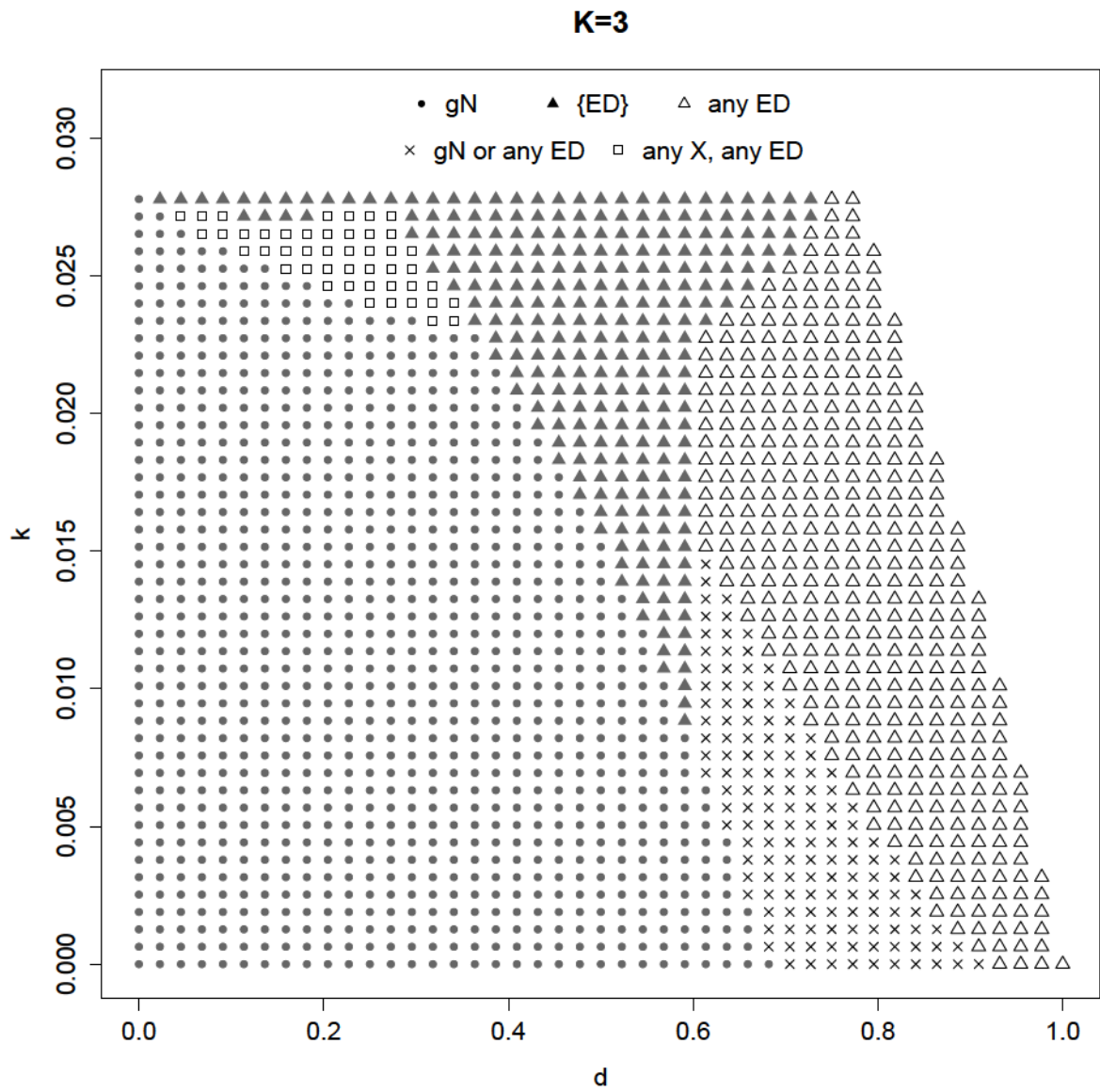


Figure 2.4: Stability for $K = 3$

A cycle within the set $\{g^N, x_1, x_2, x_3, x_4, ed_1, ed_2\}$ can be observed from the adjacencies of matrix 2.5. The mixed distribution networks are dominated by the complete and the two exclusive dealing networks: $f_3(x_1, x_2, x_3, x_4) = \{g^N, ed_1, ed_2\}$, the complete network is dominated the two exclusive dealing networks: $f_3(g^N) = \{ed_1, ed_2\}$ and the two exclusive dealing networks are now farsightedly dominated by any of the mixed distribution networks: $f_3(ed_1, ed_2) = \{x_1, x_2, x_3, x_4\}$. Indeed, the previous levels of farsightedness, informed us that, at the level-1, the monopoly and monopsony networks were dominated by 2 of the 4 mixed distribution networks and that networks with 1 link were dominated by the 2-links networks. At the level-2, the fact that the 1-link networks were dominated by 2 of the 4 mixed distribution networks is a direct consequence of it. Thus, at the level-3 of farsightedness, any of the two retailers will sever his link in any of the two exclusive distribution network, anticipating that this deviation will ultimately lead to a mixed distribution network in which he is better-off (regardless he has 1 or 2 links). This behavioral shift has a great importance in determining stability, contrary to the level-2, the exclusive dealing networks are no longer undominated as we can see in Table 2.5. For $(d, k) = (0.7, 0.005)$, the sets of mixed distribution are externally stable but violates internal stability. Indeed, the deviation of the manufacturer with one link from a network of X is not deterred (For instance, the manufacturer M_1 cuts his single link in x_2 , this leads to the other manufacturer's monopoly m_2 . Since $f_1(m_2) \cap x_2 = \emptyset$ and $f_2(m_2) \setminus f_1(m_2) = \{g^N, ed_1, ed_2\}$ any networks in $\{g^N, ed_1, ed_2\}$ brings him a higher pay-off, the deviation is not deterred). The sets g^N , ed_1 and ed_2 are externally stable. For the complete network, since $g^N \in f_2(X)$, the deviations via a mixed distribution network are deterred, the defecting players fear that a path could go back to the complete network with an identical payoff, thus g^N is internally stable. For the exclusive distribution networks, the deviations consisting in severing a link or building a link are both deterred as a path leads back to where it has originated.

Globally speaking, at the level-3, the region for which the networks of exclusive dealing are stable is now divided in two. The set $\{ed_1, ed_2\}$ is still stable for the highest values of k and the an intermediary d . In this area, the networks in the set are undominated but

	\mathbb{N}	x_1	x_2	x_3	x_4	ed_1	ed_2	r_1	r_2	m_1	m_2	s_1	s_2	s_3	s_4	\emptyset
\mathbb{N}	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
x_1	1	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0
x_2	1	1	0	1	0	1	1	0	0	0	0	0	0	0	0	0
x_3	1	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0
x_4	1	1	0	1	0	1	1	0	0	0	0	0	0	0	0	0
ed_1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
ed_2	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
r_1	1	0	1	1	0	1	1	0	0	0	0	0	0	0	0	0
r_2	1	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0
m_1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
m_2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
s_1	1	1	1	1	1	1	0	1	0	1	0	0	0	0	0	0
s_2	1	1	1	1	1	1	0	0	1	0	1	0	0	0	0	0
s_3	1	1	1	1	1	0	1	0	1	1	0	0	0	0	0	0
s_4	1	1	1	1	1	0	1	1	0	0	1	0	0	0	0	0
\emptyset	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

Table 2.6: Level-4 adjacencies.

doesn't comply with external stability as a singleton. In the other part of the rightmost area, the networks ed_1 and ed_2 dominates each other: $ed_2 \in f_3^2(ed_1)$ and reciprocally. Thus, in this region, any of the two networks complies with external stability and are, as a consequence, pairwise farsightedly stable. Any network in X or ED is stable in the top left remaining zone. To summarize, the space of feasible could split in two, along the antidiagonal. For rather low (d, k) , the complete network is stable, for rather high (d, k) , the exclusive distribution networks are either stable or part of the stable set. For a high k and an intermediary d (Globally, the area of stability of the sets $\{ED\}$, x_1, x_2, x_3, x_4, ed_1 and ed_2 depicted by \blacktriangle and \square) and from the level-3 of farsightedness, an increase in the level of farsightedness won't change the matrices of farsighted improving paths. Indeed, I noted that $M_3(0.5, 0.025) = M_K(0.5, 0.025)$ for $K > 3$. The matrices of farsighted improving paths being identical for a greater $K \geq 3$, the solution concept will return an identical solution in this area for $K \geq 5$. For a high k and an intermediary d , we can state that $\bar{K} = 5$ is enough for the supply network to be fully farsighted and (ii) and (iii) of Proposition 12 still hold for greater level of farsightedness.

The general results for the level-4 farsighted stability are given in figure 2.5 and summarized in the following proposition:

Proposition 13 *If $K = 4$, then nine sets of distribution networks may be pairwise far-*

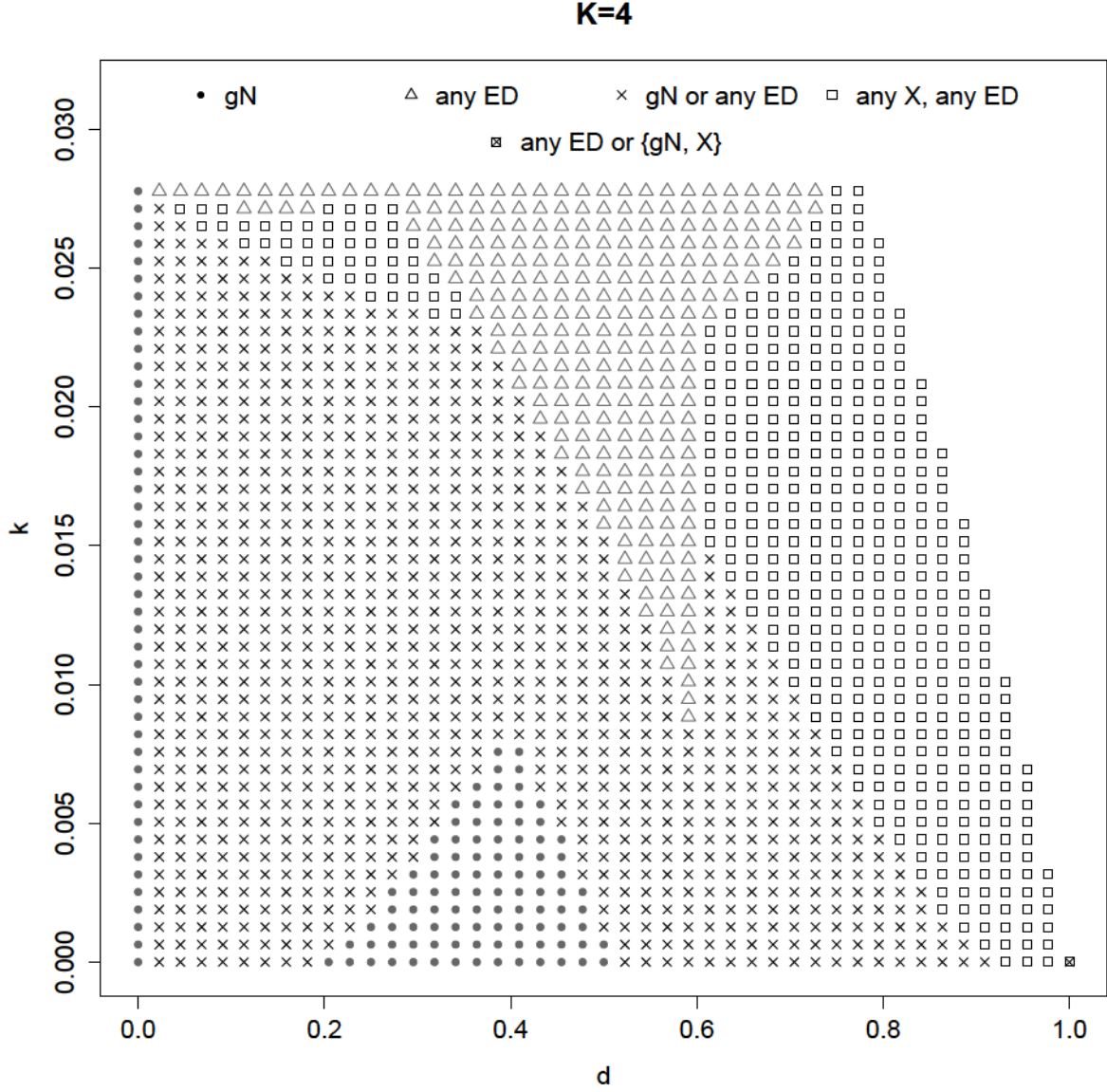


Figure 2.5: Stability for $K = 4$

sightedly stable: $g^N, \{ED\}$, any network in ED, any network in X and $\{g^N, X\}$. The parameter space (d, k) is partitioned into 5 regions.

- (i) *Rather low values of d and k and $d = 0$ imply that $G_4^\infty = g^N$.*
- (ii) *Low values of d and intermediate values of k , so as the highest values of d imply that G_4^∞ is either x_1, x_2, x_3, x_4, ed_1 or ed_2 .*
- (iii) *Either low values of d and values of k close to the upper bound and large values of d and intermediate values of k imply that $G_4^\infty = \{ED\}$.*
- (iv) *The intermediate values of d and k implies that G_4^∞ is either g^N, ed_1 or ed_2 .*

	\mathbb{N}	x_1	x_2	x_3	x_4	ed_1	ed_2	r_1	r_2	m_1	m_2	s_1	s_2	s_3	s_4	\emptyset
\mathbb{N}	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
x_1	1	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0
x_2	1	1	0	1	0	1	1	0	0	0	0	0	0	0	0	0
x_3	1	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0
x_4	1	1	0	1	0	1	1	0	0	0	0	0	0	0	0	0
ed_1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
ed_2	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
r_1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
r_2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
m_1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
m_2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
s_1	1	1	1	1	1	1	0	1	0	1	0	0	0	0	0	0
s_2	1	1	1	1	1	1	0	0	1	0	1	0	0	0	0	0
s_3	1	1	1	1	1	0	1	0	1	1	0	0	0	0	0	0
s_4	1	1	1	1	1	0	1	1	0	0	1	0	0	0	0	0
\emptyset	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

Table 2.7: Level-5 adjacencies.

(v) $(d, k) = (1, 0)$ implies that G_4^∞ is either ed_1 , ed_2 or $\{g^\mathbb{N}, X\}$.

At the 4th level of farsightedness, the complete network and the networks of exclusive dealing are still stable, or part of the stable set for a large part of the spectrum of parameters. The area for which the complete network is the only stable network (for compartmentalized markets $d = 0$ and a low k and d), are the only areas for which the networks of ED are not stable or part of the stable set. The complete network is nonetheless stable under the antidiagonale. The networks of mixed distribution are now stable for a wider spectrum. The area for which the mixed or exclusive distribution networks are stable (\square), so as the points for which $d = 0$ and $(d, k) = (1, 0)$ reached their steady state. In these regions $M_K(d, k) = M_K(d, k)$ for $K \geq 4$.

The general results for the level-5 farsighted stability are given in figure 2.6 and summarized in the following proposition:

Proposition 14 *If $K = 5$, then nine sets of distribution networks may be pairwise farsightedly stable: $g^\mathbb{N}, \{ED\}$, any network in ED , any network in X and $\{g^\mathbb{N}, X\}$. The parameter space (d, k) is partitioned into 6 regions.*

(i) $d = 0$ implies that $G_5^\infty = g^\mathbb{N}$.

(ii) Low values of d and intermediate values of k , so as the highest values of d imply that G_5^∞ is either x_1, x_2, x_3, x_4, ed_1 or ed_2 .

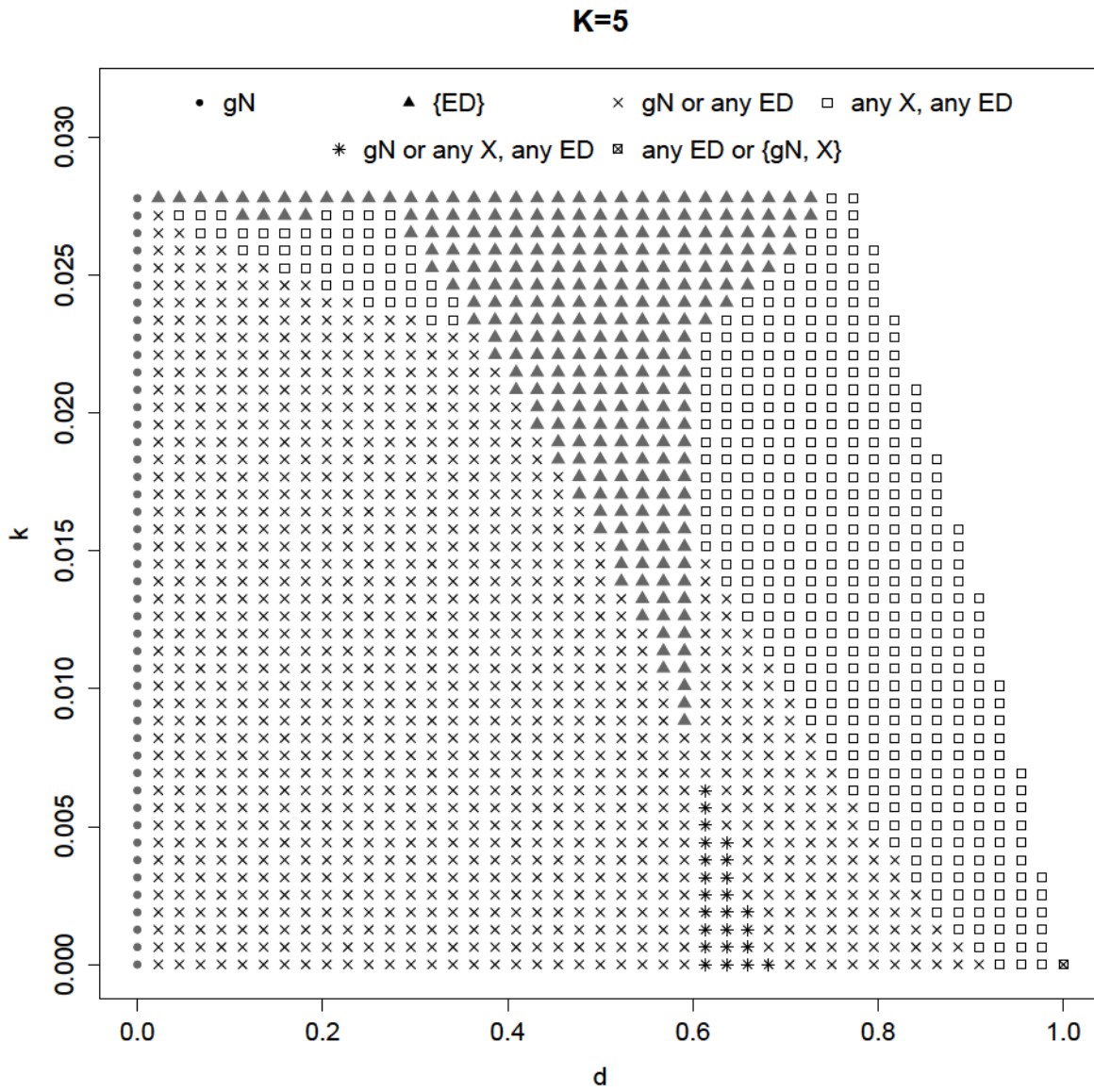


Figure 2.6: Stability for $K = 5$

	\mathbb{N}	x_1	x_2	x_3	x_4	ed_1	ed_2	r_1	r_2	m_1	m_2	s_1	s_2	s_3	s_4	\emptyset
\mathbb{N}	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
x_1	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
x_2	1	1	0	1	1	1	1	0	0	0	0	0	0	0	0	0
x_3	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0
x_4	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0
ed_1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
ed_2	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
r_1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
r_2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
m_1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
m_2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
s_1	1	1	1	1	1	1	0	1	0	1	0	0	0	0	0	0
s_2	1	1	1	1	1	1	0	0	1	0	1	0	0	0	0	0
s_3	1	1	1	1	1	0	1	0	1	1	0	0	0	0	0	0
s_4	1	1	1	1	1	0	1	1	0	0	1	0	0	0	0	0
\emptyset	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

Table 2.8: Level-6 adjacencies and over.

(iii) Either low values of d and values of k close to the upper bound and large values of d and intermediate values of k imply that $G_5^\infty = \{ED\}$.

(iv) The intermediate values of d and k implies that G_5^∞ is either $g^\mathbb{N}$, ed_1 or ed_2 .

(v) $(d, k) = (1, 0)$ implies that G_5^∞ is either ed_1 , ed_2 or $\{g^\mathbb{N}, X\}$.

(vi) Sufficiently high values of d and a low k implies that G_5^∞ is either $g^\mathbb{N}$, x_1 , x_2 , x_3 , x_4 , ed_1 or ed_2 .

The final shape of the different zones is becoming well established for the 5th level of farsightedness. (i) and (vi) still holds for greater level of farsightedness.

The general results for the level-6 farsighted stability are given in Figure 2.7 and summarized in the following proposition:

Proposition 15 *If $K = 6$, then nine sets of distribution networks may be pairwise farsightedly stable: $g^\mathbb{N}$, $\{ED\}$, any network in ED , any network in X and $\{g^\mathbb{N}, X\}$. The parameter space (d, k) is partitioned into 5 regions.*

(i) $d = 0$ implies that $G_6^\infty = g^\mathbb{N}$.

(ii) Low values of d and intermediate values of k , so as the highest values of d imply that G_6^∞ is either x_1, x_2, x_3, x_4, ed_1 or ed_2 .

(iii) [from proposition 14]. Either low values of d and values of k close to the upper bound and large values of d and intermediate values of k imply that $G_6^\infty = \{ED\}$.

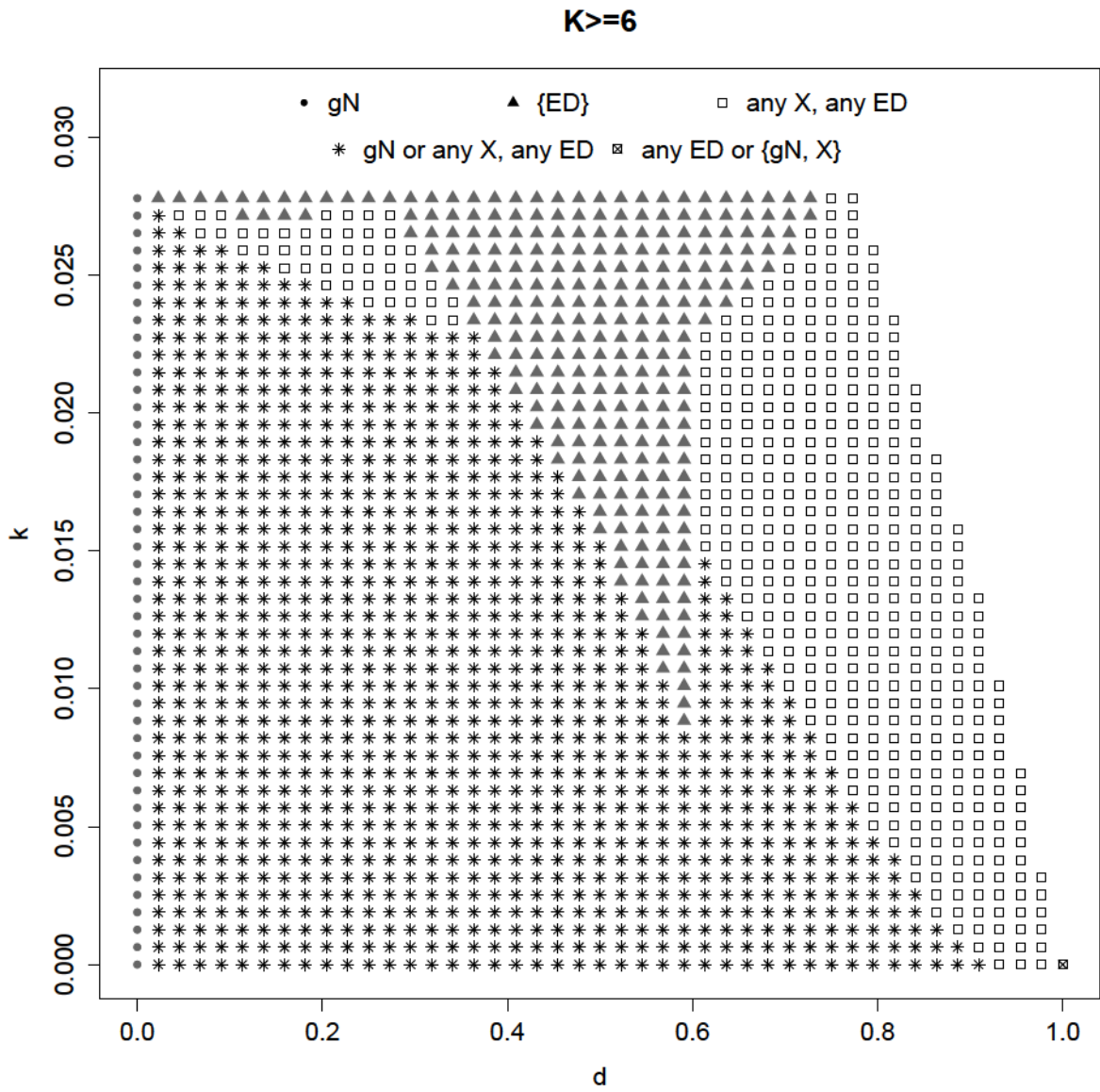


Figure 2.7: Stability for $K \geq 6$

(iv) The intermediate values of d and k implies that G_6^∞ is either g^N , x_1 , x_2 , x_3 , x_4 , ed_1 or ed_2 .

(v) $(d, k) = (1, 0)$ implies that G_6^∞ is either ed_1 , ed_2 or $\{g^N, X\}$.

At the 6th level of farsightedness and for any value of the parameters, the matrices of farsighted improving paths reached their steady state for any couple of parameters. From the Propositions 10 to 15 we can state the following result.

Proposition 16 *In the 2×2 network of manufacturers and retailers, for any (d, k) , any level- K farsighted set is a level- ∞ farsighted set for $K \geq 8$.*

We used a recursive argument on matrices of farsighted improving paths to get this result. For a K sufficiently high, the matrices of farsighted improving paths of length K are identical to the matrices of farsighted improving paths of length $K + \alpha$ for some α , that is: $m_K = m_{K+\alpha}$. The explanation comes from the fact that, for K sufficiently high, every paths are known and increasing the level of farsightedness doesn't bring additional information (eg. the network g is dominated by g' via a path P) but defines longer domination paths (g is dominated by g' via the path P')⁸.

The algorithm aggregates the matrices m_K from m_1 to $m_{K^{theo}}$ and gets the matrix $M_{K^{theo}}$. If $M_{K^{theo}} = M_{K^{theo}-1}$ then $M_{K^{theo}}$ is withdrawn and the process ends once $M_{K^{theo}-\beta-1} \neq M_{K^{theo}-\beta}$. In the model of manufacturers and retailers, the matrices of farsighted improving paths are identical from the 6th degree of farsightedness at most. This directly implies that the pairwise farsightedly stable sets of networks are identical for $K = 8$ and higher levels of farsightedness. More precisely, the calculations revealed that the full farsightedness was reached between the level-5 and the level-8 of farsightedness at most has Propositions 10 to 15 emphasize it. Thus, we can conclude that, $\forall (d, k)$, $G_K^\infty = G_{K+1}^\infty = G_\infty^\infty$ for $K \geq 8$ any couple of parameters. Another interesting fact is that for $K = \{7, 8\}$, even though the matrices of farsighted improving path are different from the level-6, the solution remains unchanged and the plots are identical for $K \geq 6$.

⁸ If the length of P is p , then the length of P' is $p + 2$. Suppose a path emanates from g_1 to g_2 in exactly r_1 deletion of links and r_2 additions, then $g_1 \rightarrow_p g_2$ with $p = r_1 + r_2$. There can be no path of length $K+1$ from g_1 to g_2 since an additional deviation (think for instance, the deletion of a link) ought to be offset by the opposite operation (the creation of a link). however, a path $g_1 \rightarrow_{p+2} g_2$ may exists

This last level of farsightedness exhibits several interesting features. Excepted for isolated markets, the networks of exclusive dealing are always either part of the stable set or pairwise level- K farsighted stable themselves. The same argument can be put forward regarding the networks of mixed distribution. Excepted for the region of stability of the set $\{ED\}$ and a few extreme cases, any network of X is a likely candidate to the pairwise level- K farsighted stability. The last thing to mention is the zone of stability of the complete network, g^N is also a candidate under the antidiagonal. In this region any network in ED, X , or g^N can be stable. This multiplicity of candidate is not surprising. Indeed, a singleton is minimal, internally stable and a high level of farsightedness makes the external stability of a set an more easily satisfied requirement.

The previous proposition has implications in terms of strategies of the firms. The ability of a firm to forecast the aftermath of its actions on a market directly, or indirectly impacts the overall level of benefits. The formulation of a strategy given the strategies of the other players may induce time in calculations and, thus positively impacts the costs of the firm (think for instance of the decision to entry, an adaptive response to a shift in the strategies of a competitor, adaptation to an exogenous shock and so forth...). The fact that a relatively low level of farsightedness is sufficient to reach the highest degree of information implies that the firms can easily be fully farsighted at a relatively low cost in terms of the establishment of a strategy. One can thus expect that the firms seek to be fully farsighted. The difference between \bar{K} and K^{theo} might be due, in great part, to the fact that there is a relatively small number of players, and thus, a relatively small number of available networks. One may expect \bar{K} to increase in the number of players, and thus, in the complexity of the networks.

The study of a level- K farsighted stable set aims at saying which networks, or which sets of networks are the most likely to be stable. If the solution concept doesn't select a unique set among the set of the stable sets, it has the advantage to evict the networks or the sets of networks that are never stable. Out from the results, one can notice that any network including at least one isolated player is never part of a level- K farsightedly stable set or never level- K farsighted stable. Indeed, in the 2×2 manufacturers and retailers

network, the networks having less than two links, monopoly and monopsony never belong to the solution. An isolated manufacturer (resp retailer) systematically finds a retailer (resp manufacturer) who accepts a new partnership, any level- K pairwise farsightedly stable sets of networks exclude no players. A last point is to note that, regardless of the values of the parameters, the stable networks, or the stable sets of networks are made of rather dense networks where no player is excluded g^N , the mixed distribution networks X and the exclusive dealing networks ED . Even if building a link is costly, the equilibrium sustains a high level of upstream competition leading to a significant level of output. A large output being often associated to a high welfare, I turn now to the question of welfare analysis.

2.6 Welfare analysis

The question of the stability of a economic situation often comes along with the question of its efficiency. To state if a given network fit the normative aspect of the economic interactions, we must compare the welfare generated by a stable situation to a neutral and impartial welfare criterion. Beyond the simple comparison of economic values, questioning the reference (*ie.* which concept of efficiency could stand as benchmark) is of interest.

Given that, for some couple of parameters, several sets of networks exists, the attention has been focused on several concepts of efficiency so as to establish a thorough welfare analysis. First, for any couple of parameters we computed the Pareto-optimality and compared the results to the sets of stable networks. Second, we computed the set of strongly stable networks and again, we compared the results to the stable sets. In any case, the conclusions are drawn according to a given level of acceptance that I called *weak* and *strong correspondance*.

For each couple of parameters (d, k) and any network g , we computed the consumers surplus in the following way:

$$Y_C(g) = \frac{1}{2} (Q_1(g) + Q_2(g))^2 + (d-1) Q_1(g) Q_2(g) \quad (2.5)$$

The social welfare is obtained by the aggregation of the payoffs of the 4 players and the consumers surplus:

$$Y(g) = \sum_{\lambda \in \mathbb{N}} Y_{\lambda}(g) + Y_C(g) \quad (2.6)$$

In order to assess the convergence between the stability and the social optimum, we must ensure that, if a stable set G is efficient, it is either unique, or that any other stable set G' is also efficient. If the stability concept allow for several solution, claiming a correspondence between the stable sets and a efficient or optimal outcome requires that any stable set G also belong to an outcome of the efficiency criterion. Alternatively, a lower level of acceptance could be to require that a stable solution which is also efficient or optimal is sufficient to assert the correspondence. Hereafter is defined two concepts of correspondence between a set of stable solutions and the chosen optimal or efficient criterion.

For a given stability concept σ and an optimality or efficiency criterion ψ . let's denote by $\Gamma(\sigma)$, the set made of every stable sets given the stability concept σ . Let $\Lambda(\psi)$, be the set made of all the sets solution of the efficiency or optimality criterion ψ .

Definition 17 *There is a strong correspondence between σ and ψ if*

$$\forall \tau \in \Gamma(\sigma), \exists \alpha \in \Lambda(\psi) \text{ such that } \tau \subseteq \alpha$$

The definition of the strong correspondence is a set-based definition of the correspondence between a solution and a desirable outcome and compels any stable solution to belong to an optimal or efficient set. This definition imposes a strong requirement on the decision to admit a set of solutions as socially desirable. The obverse is defined by the weak correspondence:

Definition 18 *There is a weak correspondence between σ and ψ if*

$$\forall \tau \in \Gamma(\sigma), \exists \alpha \in \Lambda(\psi) \text{ such that } \tau \cap \alpha \neq \emptyset$$

2.6.1 Pareto optimality

This section is devoted to discuss the relation between the stable sets and the Pareto-optimality criterion. Let $\Gamma(K)$ be the set made of every level- K pairwise farsighted stable sets. Let $\Lambda(P)$ be the set of Pareto-optimal networks defined by $\mathbb{G} \setminus g$ such that $\exists g' \in \mathbb{G}$, for which $Y_i(g) \leq Y_i(g'), \forall i \in N$ with at least a strict inequality for one player.

Definition 19 *There is an strong correspondence between $\Gamma(K)$ and P if, for a couple of parameter (d, k) :*

$$G_K^\infty \subseteq P \quad \forall G_K^\infty \in \Gamma(K)$$

The Pareto-optimality is not a set-based criterion, it turns that $\#\Lambda(P) = 1$, so P can be made of several networks but is a unique set. Hereafter are summarized the results for the strong correspondence between the results and the Pareto optimality.

When considering the consumers surplus, the Pareto optimality is a weak criterion. In many cases, too few networks are Pareto dominated. Figure 2.8 depicts the different areas of Pareto optimality. For $d = 0$, the networks $\mathbb{G} \setminus \{g^\emptyset, S, MM\}$ are Pareto-optimal (depicted by \circ). Every networks excepted the empty network and the single-links networks are undominated: $\triangle : \mathbb{G} \setminus \{g^\emptyset, S\}$. Only the empty network is dominated in the area $+$: $\mathbb{G} \setminus g^\emptyset$, the remaining zones are $\times : \mathbb{G} \setminus \{g^\emptyset, S, ED\}$ and $\diamond : \mathbb{G} \setminus \{g^\emptyset, ED\}$. The interpretation of this figure is not easy because of the multiplicity of networks, but several facts can be derived out of this. The empty network is always dominated while networks g^N and X are never dominated. Excepted for the lowest bound on d , the networks of the manufacturer's monopoly MM and the networks of retailer's monopsony RM are undominated. This is somehow intuitive in the sense that those networks maximize the benefit of one player (respectively one manufacturer and one of the retailers) and are, as a matter of fact, undominated. The networks of S (single link networks) can be undominated for a low k and an intermediary d , if the profit associated to the situation of being the only manufacturer and the only retailer is greater than the gross profit of an additional connection. In the same region, the networks of exclusive dealing can be dominated. Indeed, the negative effect of the competition on the profits of the manufacturers is low

Types of Pareto-optimality

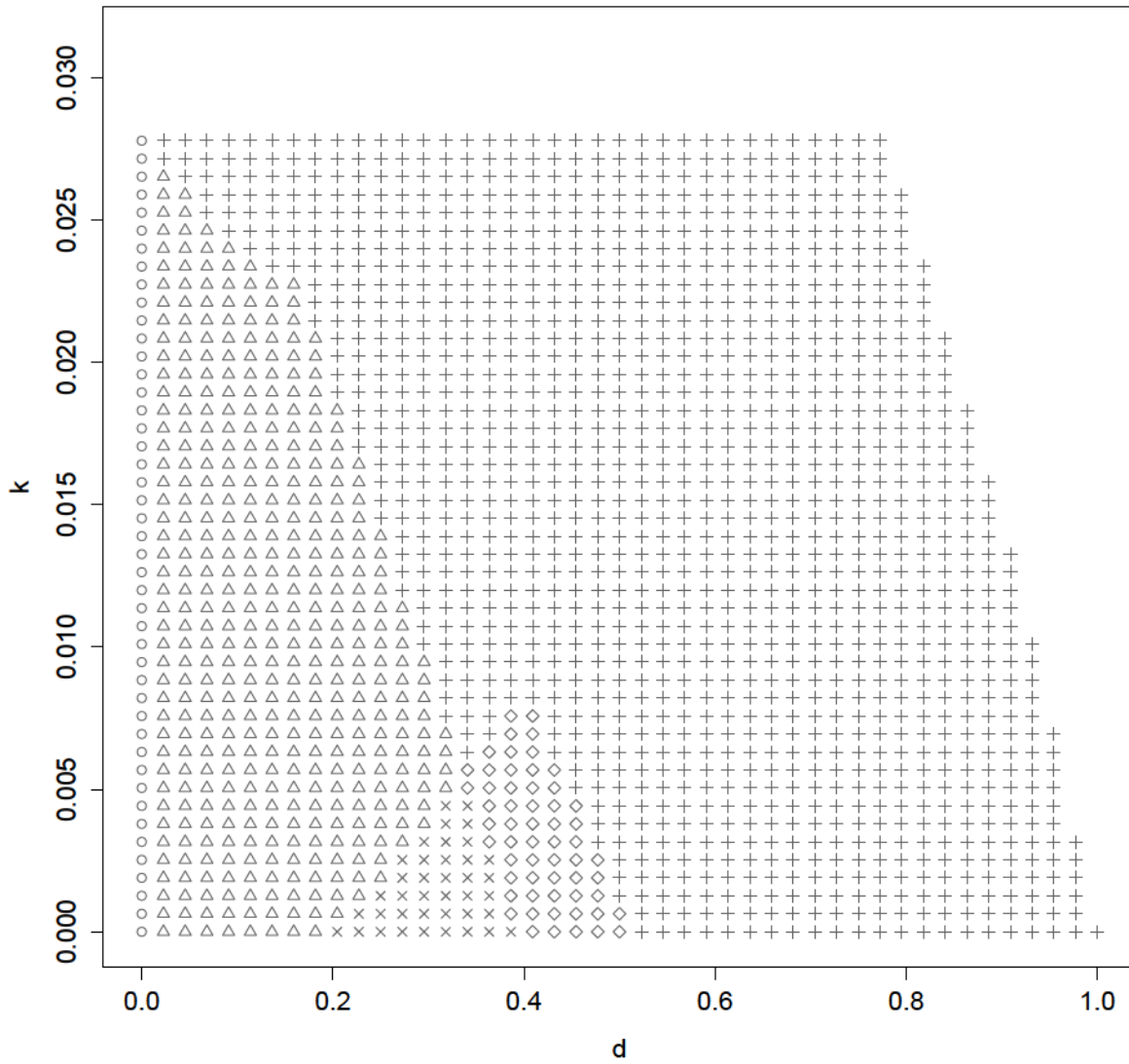


Figure 2.8: Types of Pareto-optimal regions.

enough and the creation of a new link is not too costly in this region. This fact explains that any networks with an additional link weakly increases the overall profits and thus that networks ED are dominated.

It results that the Pareto-optimal network are numerous in many cases and one may expect to often observe a strong correspondence between the stable sets and Pareto-optimal sets. Discussing the strong correspondence between the level- K farsighted stable sets and the Pareto-optimality leads to draw out several facts from the comparison. The strong correspondence between Γ_K and $P(d, k)$ is always ensured for the lowest values of K . For $K \leq 4$, the strong correspondence holds as long as the networks of exclusive dealing remain undominated for a low k and an intermediary d . For $K \geq 5$, in the region for which the networks ED are dominated, the networks of exclusive dealing are also stable. Thus there exists sets that are not included in the Pareto optimal set of networks and the strong correspondence is non longer ensured.

These results may seem ambiguous regarding the relevance of the degree of farsightedness related to a given region and to the strong correspondence. Picking up a couple (d, k) at random, brings a strong correspondence at 100% likely for a level of farsightedness in $[1, 4]$ while the levels 5 and over drops at 94.62% (the difference lies in the lower central region for which the networks ED are dominated)⁹. Regardless of the parameters, when looking at the relationship between the level of farsightedness and the strong correspondence, it can be stated that a stable set for a not too high level of farsightedness is more likely to be optimal¹⁰. These results have to be interpreted in the light of the limitations of the criterion of Pareto-optimality in this vertical setting. Even though the strong correspondence is a very restrictive requirement, the Pareto undominated networks are too numerous and this optimality criterion turns permissive. The weak correspondence requires that at least a network of any stable set is Pareto optimal. Once again, once the networks of exclusive dealing are stable and dominated (the lower central region), the weak correspondence is also unsatisfied. The welfare analysis in terms of strong or weak correspondence returns an identical conclusion.

⁹Graphics don't account for additional information.

¹⁰Though not significant.

2.6.2 Strong efficiency

In order to strengthen the reliability of the correspondence between the stable sets and an optimality criterion, this section switches to a more restrictive concept of optimality and defines the concept of a strongly efficient network (Jackson and Wolinsky [33]). A network g is said to be strongly efficient if, for a couple of parameters, it maximizes the social welfare. $\Lambda(E)$ denotes the set of strongly efficient networks. Formally, for a tuple (G, d, k) : $\Lambda(E) = \arg \max \{Y(g)\}$.

Definition 20 *There is a strong correspondence between $\Gamma(K)$ and $\Lambda(E)$ if, for a couple of parameter (d, k) :*

$$G_K^\infty \subseteq E \quad \forall G_K^\infty \in \Gamma_K$$

Like the Pareto-optimality, the criterion of strong efficiency is not a set-based criteria, $\#\Lambda(E) = 1$, so E can be made of several networks but is a unique set. The Figure 2.9 lists the 3 main areas of strong efficiency. Under the antidiagonal, the complete network is the network that maximizes the social welfare. In this region, a rather high level of differentiation and/or a rather low k implies that a highly connected network generates a high consumers surplus via a high level of competition. In the upper region, the networks of ED are strongly efficient. This comes from the fact that the two goods are still present (high consumers surplus) with a less dense network due to a high cost of connection. In the rightmost part of the space, the networks of mixed distribution are those that maximizes the social welfare. The goods are too similar and the cost of the link is too high to render either an exclusive dealing network or the complete network efficient. Again, there is a threshold effect on the bound $(d, k) = (1, 0)$ for which the complete network so as any network of mixed distribution ensure the maximum of social welfare.

The scatter plots depicted in Figure 2.10 refers to the strong correspondence between the level- K farsightedly stable sets and the strongly efficient networks. For the lowest values of parameters, as long as the complete network is the only stable network ($K \leq 3$), there is a weak correspondence. This result also holds for the weak correspondence and for greater level of farsightedness and compartmentalized markets.

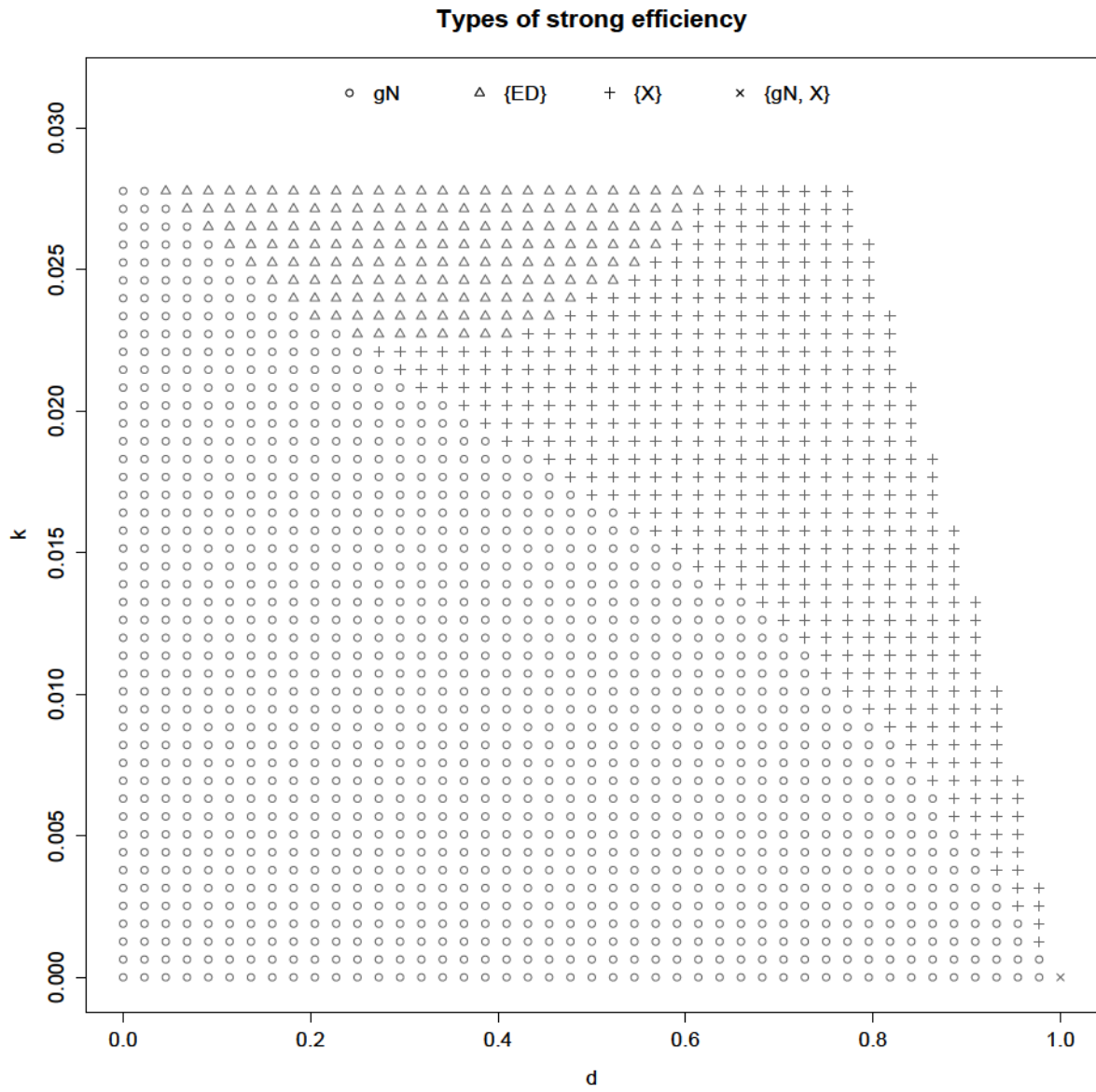


Figure 2.9: The strongly efficient networks.

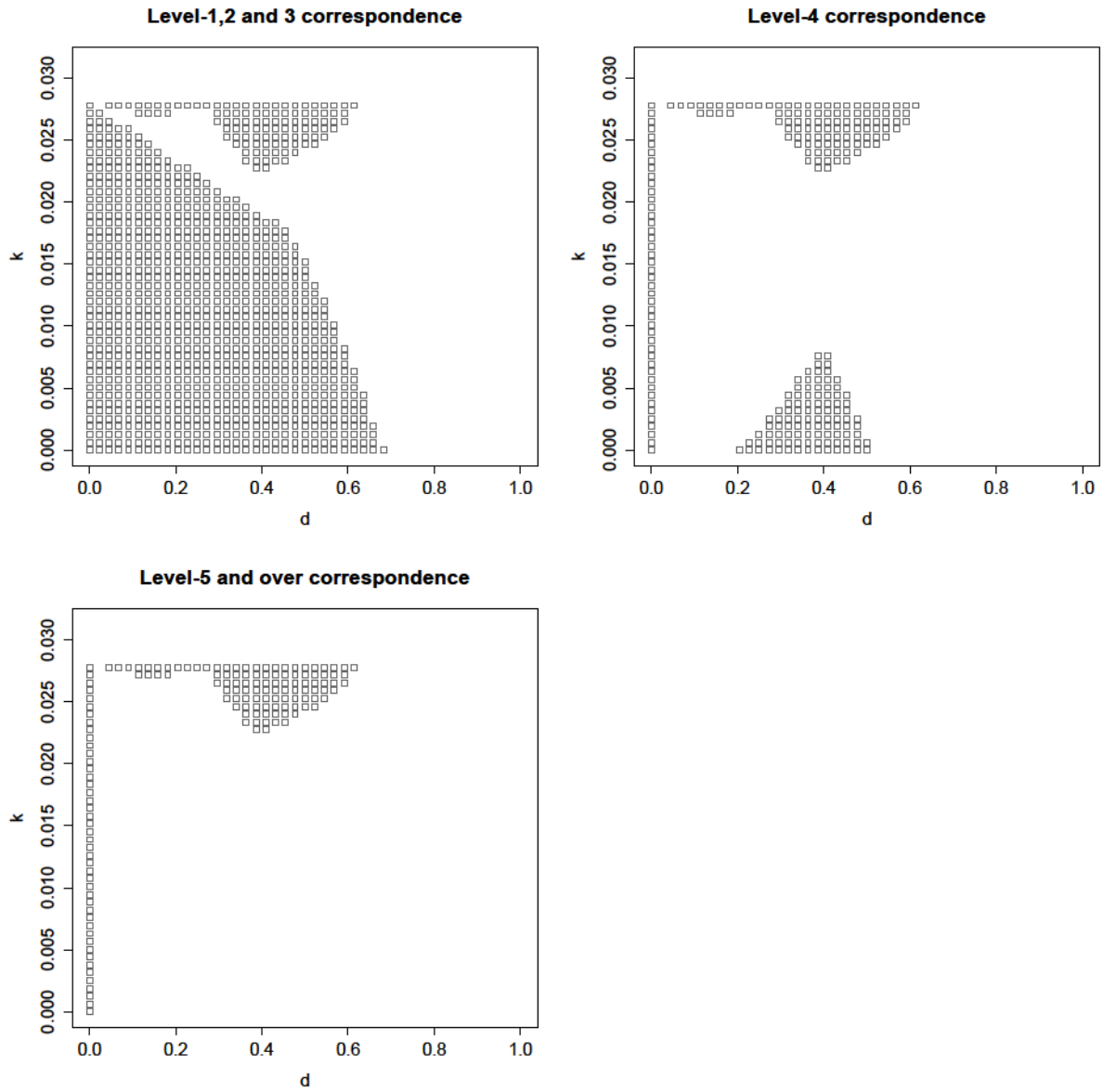


Figure 2.10: Strong correspondence between stable sets and strongly stable networks.

As the level of farsightedness increases, the size the stable sets $\Gamma(K)$ expands for almost each region while the Figure 2.10 obviously reveals that the areas for which the stable sets are also efficient shrinks with the increase of the degree of farsightedness. From 56.74% at the levels 1, 2 and 3, the strong correspondence plummets to 13.59% at the level-4 and to 8.21% at levels 5 and over.

In the networks of manufacturers and retailers, the more the agents are able to anticipate the consequences of their own deviations so as the deviations of the other players, the more the stable sets differs from the social optima regarding the strong correspondence. As seen in the previous section, this result brings wide implications in terms of incentives. The more an agent is informed, or similarly, the more he is able to look forward, the more he is able to formulate decisions that are favorable to him. In this perspective, one can expect the players to be seeking for a high level of farsightedness, this type of behavior directly implying the stable sets to increasingly differ from what is socially desirable. From the the vantage point of strong correspondence, a high level of farsightedness increases the tension between stability and efficiency.

As regards to the weak correspondence between $\Gamma(K)$ and $\Lambda(E)$, there's only a few comments to be addressed. For a high k and an intermediary d , the networks of exclusive dealing can fit the criterion. For the level-1, 66.17% of the space satisfies the weak correspondence, 56.74% for level-2 and 3. Again, the ratio plummets to 13.59% at the level-4 and to 8.21% at levels 5 and over and the regions for which the weak correspondence is satisfied fit the regions for which the strong correspondence is satisfied. Thus, as regards to the strong efficiency, there is a convergence between the strong and weak correspondence for the highest values of parameters.

The correlation between the two concepts of correspondence has also to be seen in the light of the relation between the solution concepts of farsighted stability G_∞^1 and level- ∞ stability G_∞^∞ . Note that the values of parameters for which there is weak and strong correspondence for the level- \bar{K} is included in the set of values of parameters for wich there is a transitive correspondence between G_∞^1 and G_∞^∞ .

A network-based definition of efficiency doesn't harmonize with a set-based notion of stability and it is arduous to define a neutral and fully operative criterion. On one hand, the weak correspondence may seem a too permissive, on the other hand the strong correspondence may look really demanding. Using the Pareto optimality criterion, the two notions are symmetric but the Pareto-optimality seem to be a weak criterion when it comes to be compared to several situations with several players. Even if not fully satisfying, the strong correspondence of the stable networks to strong efficiency appears to be the most tractable method of welfare analysis. Out from the results, it can be concluded that a high level of farsightedness increases the tension between stability and efficiency, moreover, the strong correspondence between stable networks and efficient networks tends to disappear. Once again the convergence between the weak and strong correspondence appears for the highest levels of farsightedness, this convergence is ensured for a subset of the region of parameters for which there is a transitive correspondence between the sets of sets G_∞^1 and G_∞^∞ .

2.7 Extensions

2.7.1 More on the reliability of the algorithmic results

A main aspect of the algorithmic results lies in its reliability. This section covers the extent to which the algorithmic results are consistent with theoretical results one could expect to get and is exclusively descriptive.

The first thing to notice is that the results fits the theoretic expectations at the lowest level of farsightedness. As pointed out by Herings et al. (2009) [23] and [24] (Theorem 3), there is a unique pairwise myopic stable set of networks which is made of all the networks that belong to a closed cycle (Jackson & Watts, 2002 [32]). A set of networks G is a closed cycle if there exists no sequences of farsighted improving paths of length 1 starting in $g \in G$ and ending in $\mathbb{G} \setminus G$. Formally, G is a closed cycle if $f_1^\infty(G) = G$. Regardless of the differentiation of products and the cost of building a link, every solution using the algorithmic determination of stable sets is made of networks that belong to a closed cycle

at the level-1. This can be verified at the level-1 by looking at the adjacencies matrices. In the example given in Section 2.5.1 associated to the matrix given in Table 2.3. We can easily check that g^N , ed_1 and ed_2 are pairwise stable. A pairwise stable network is a closed cycle of size 1, thus $\{g^N, ED\}$ is the level-1 farsightedly stable. The same reasoning can be applied to the other regions at the level-1.

The second thing to heed is that for $d = 0$, the manufacturers sell two absolutely disparate merchandise on two confined markets. The absence of competition exhibits no externalities across components in the networks of exclusive dealing. Furthermore, in this setting, an additional commercial link always improves the payoffs of the agents involved¹¹. Following Herings et al. (2014, Theorem 9), then the complete network should be a level- K farsighted stable set. Once again the algorithm returns an identical solution for any level of farsightedness.

The third thing to note is that the areas of stability of a given set of networks are identical to those in Mauleon et al. 2011 [38] for pairwise stability as pointed out in Section 2.5.1. Ensuring a proper operative algorithm at the lowest level of farsightedness is a prerequisite of the reliability of the programming. Indeed, Herings et al. (2014, Theorem 5) stated that for $K \geq 1$, the unique myopically stable set G_1^∞ contains a level- K farsightedly stable set G_K^∞ . Figures 2.3, 2.4, 2.5, 2.6 and 2.7 compared to Figure 2.2 exhibits this feature, thus the programming seem to operate properly¹²

2.7.2 Relation to the pairwise farsighted stability

This section is dedicated to further explore the relation between the pairwise level- K farsighted stable set G_∞^∞ and the pairwise farsighted stable set G_∞^1 . As mentioned in Section 2.3.4, since $f_\infty(g) \subseteq f_\infty^\infty(g)$ for any $g \in \mathbb{G}$, there is a set $G \subseteq G_\infty^1$ such that G is a G_∞^∞ set. Thus the relation between a pairwise farsighted stable set G_∞^1 and a level- ∞ farsightedly stable set G_∞^1 can be transitive. Figure 2.11 plots the pairwise farsightedly stable sets for any value of the parameters. A quick comparison with Figures 2.3, 2.4, 2.5,

¹¹ This is implicitly the definition of a allocation rule satisfying increasing return to link creation.

¹² Theorem 5 in Herings et al (2014) supposes a generic allocation rule. This results holds for the space (d, k) inside each region, though passing from a region to another supposes a line for which the allocation rule is not generic.

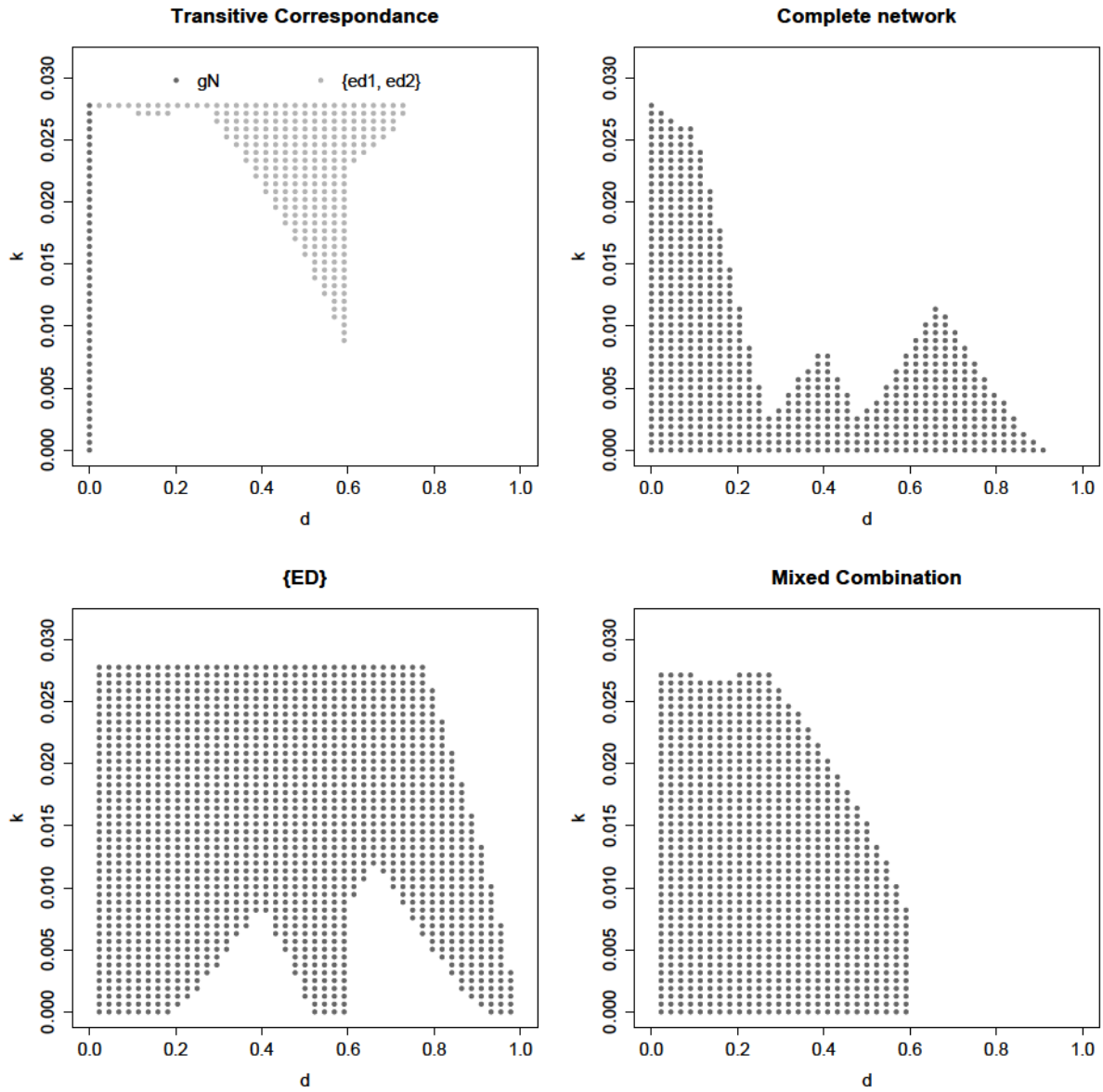


Figure 2.11: Pairwise Frasier stable sets and the transitive correspondence.

2.6 and 2.7 shows that for any se

This fact is clearly the most preeminent assertion advocating for the proper functioning of the algorithm and its thereof results.

The top-right scatter plot details the values of the parameters for which the relation between G_{∞}^1 and G_{∞}^{∞} is transitive, that is, for $K \geq \bar{K}$, the regions for which the level- \bar{K} farsighted set and the level- ∞ are identical. This final result can be stated:

Proposition 21 *For $d = 0$, for $k = 1/36$, and for an intermediary d and a high k , $G_{\infty}^1 = G_{\infty}^{\infty}$.*

This result roughly holds for the region of stability of the set $\{ED\}$. In this sector, the external stability based on the definition of a quasi externally stable set can be replace the definition of an externally stable set without affecting the results nor the conclusions. This observation also displays another interesting fact. The transitive correspondence of the two stability concepts is never optimal regardless of the criterion for the networks of exclusive dealing while this is the opposite for the complete network. This observation has to be related to the low level of farsightedness matching the full farsightedness for these values of the parameters, there can presumably be a relation between the transitive correspondence and a low level of farsightedness matching the full farsightedness (The transitive correspondence being obvious for a network Pareto dominating all the other networks).

2.7.3 Efficiency of the industrial segment.

Another interesting aspect of the the welfare analysis is to focus on the decisive actors, that is the players who can actually interact onto the structure of the exchanges. Unlike the consumer who only undergo the structure, can active actors achieve an optimal of efficient outcome regardless of the externalities that are exerted on third players. The figure 2.12 lists the different zones of Pareto optimality in the exclusion of the consumers.

The legend is identical to the case of Pareto efficiency including the consumers. But the results differs for the upper right region. Regarding the Pareto optimality, the networks $g^{\mathbb{N}}$

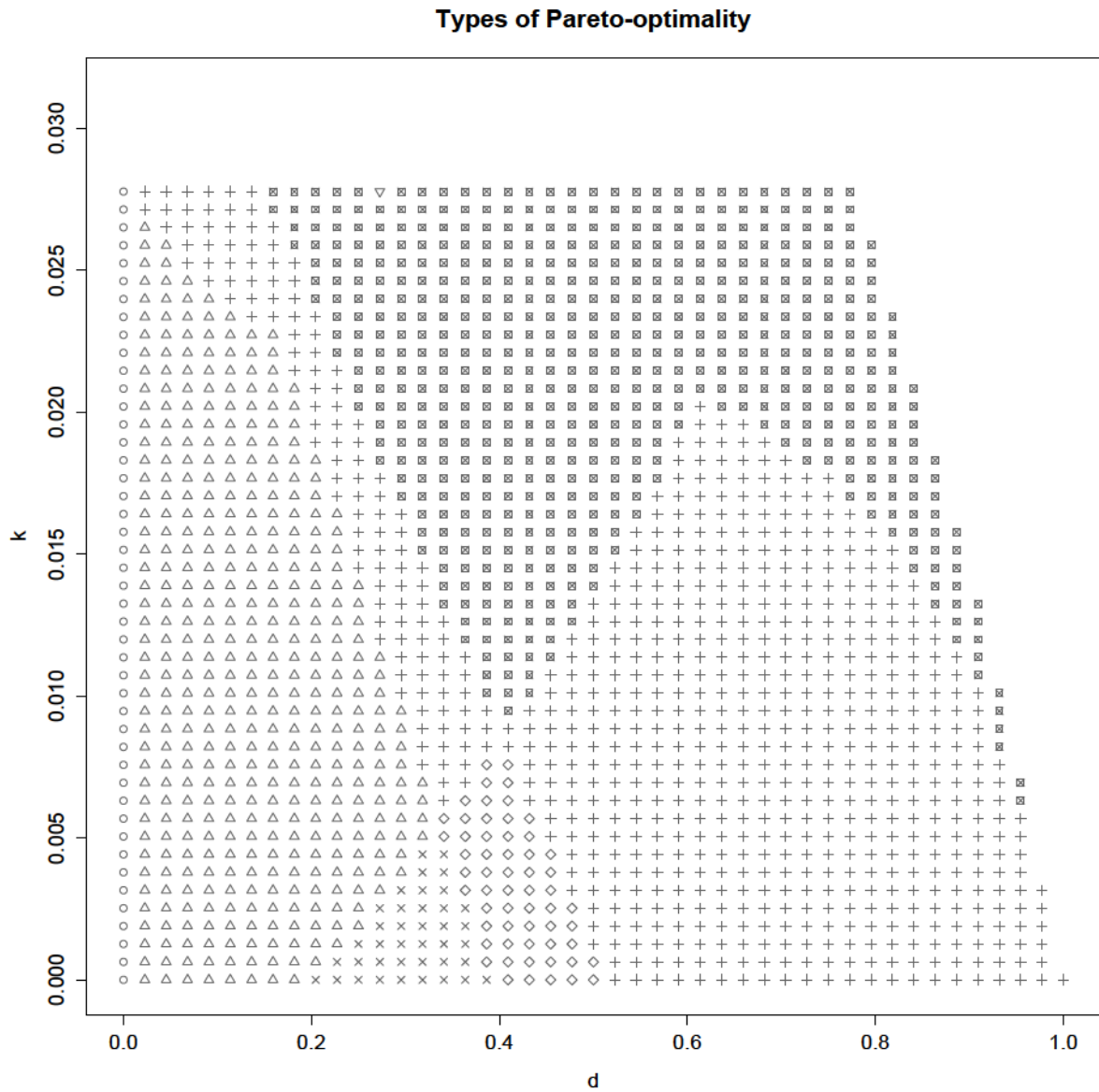


Figure 2.12: Pareto optimality of the industrial segment

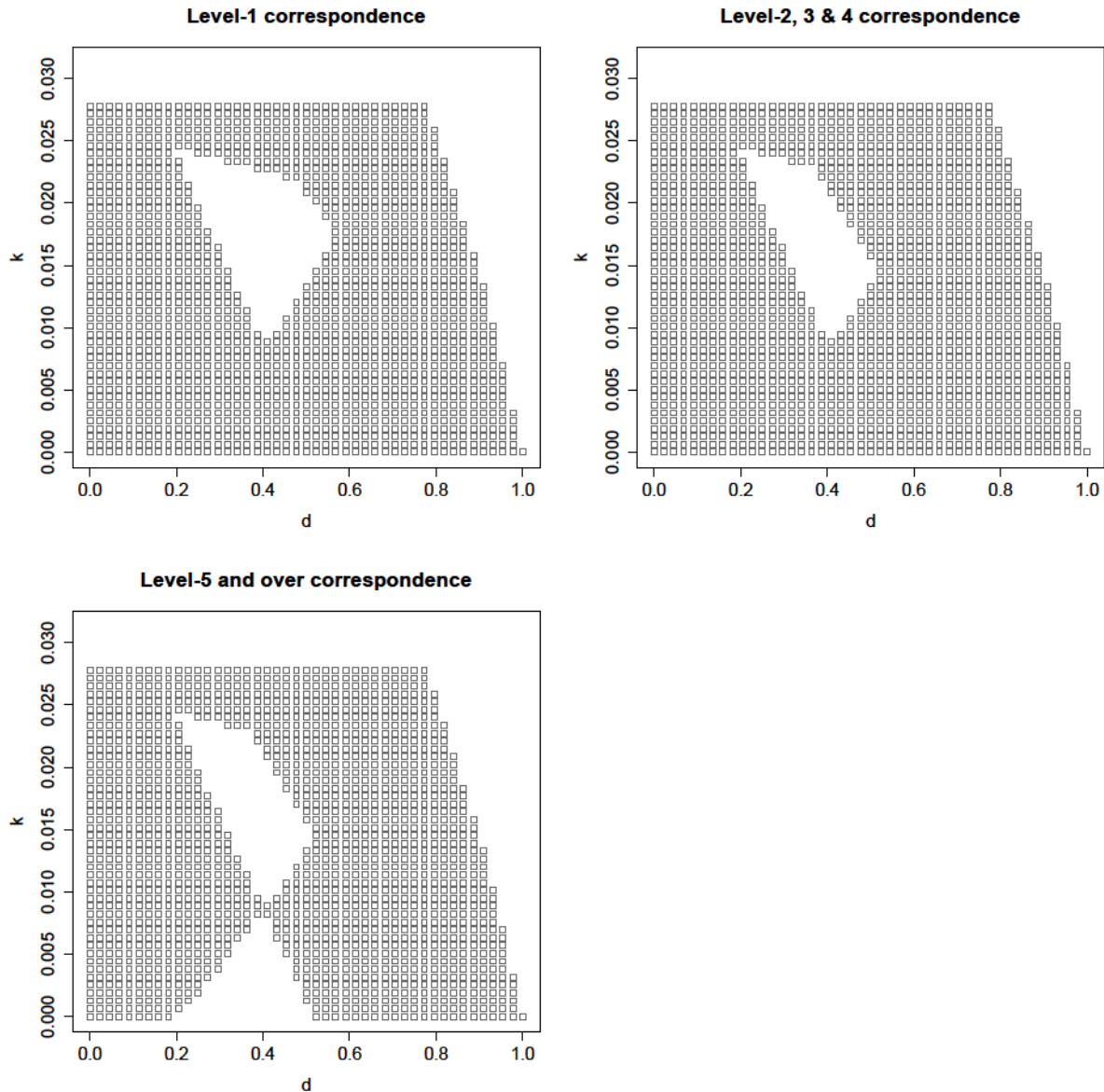


Figure 2.13: Strong Pareto optimality of the industrial segment

and the networks of X are dominated in the area depicted by ticked box. A high level of d , so as a high level of k makes the competition harder and a dense network globally less interesting to achieve. Furthermore, the consumers generate their highest level of surplus in the presence of a high level of competition (interband competition) and a close-knit network (both intra-brand competition and in-store rivalry). This fact no longer accounts for the determination of undominated networks explains that dense networks are now dominated.

Figure 2.13 represents the correspondence between the strong Pareto criterion and

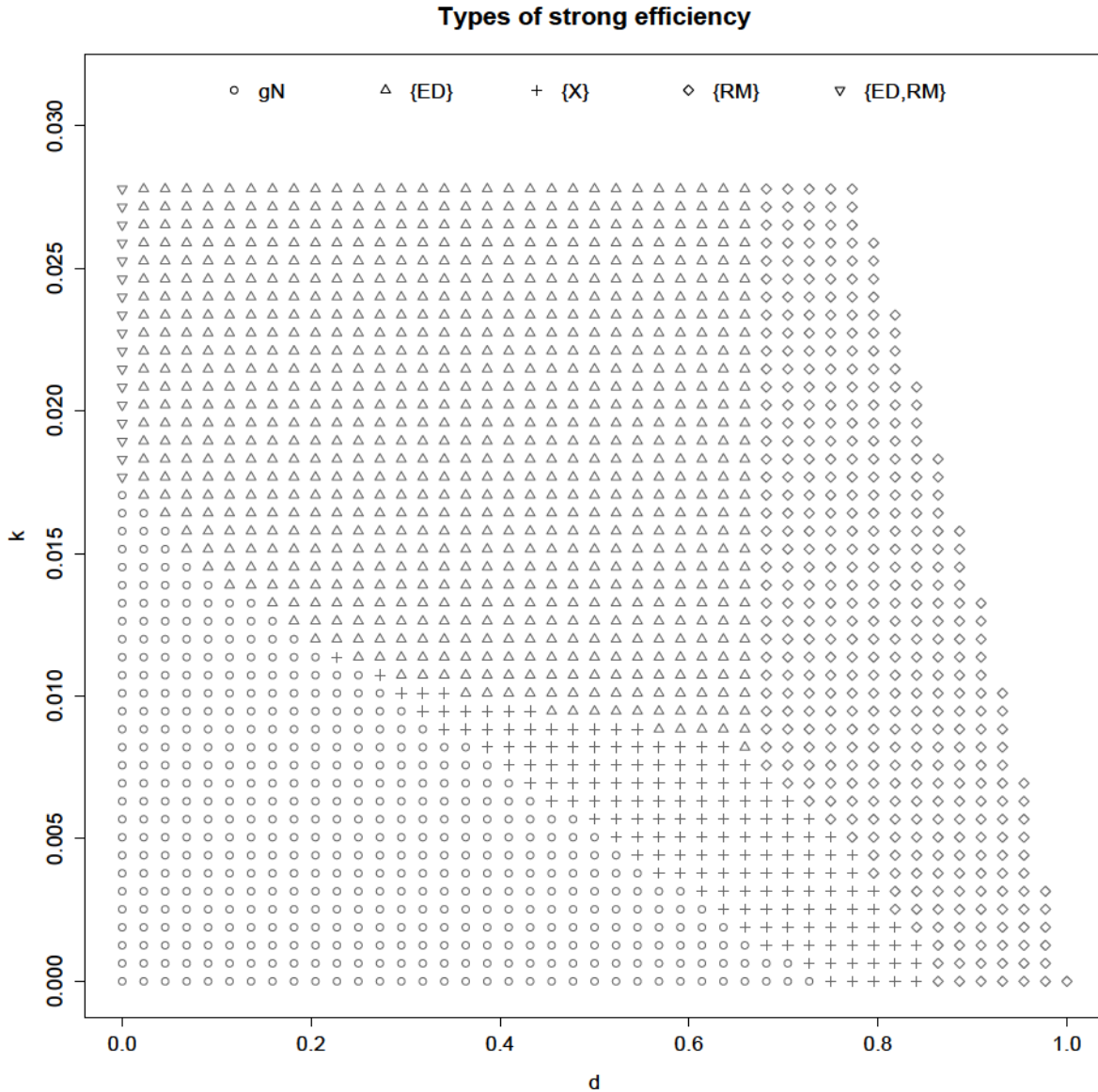


Figure 2.14: Strongly efficient networks of the industrial segment

the stable sets. The same conclusion as for the case with the consumer surplus for the lower central region can be drawn. Shifting from the 4th level of farsightedness to the 5th induce a drop in the correspondence from 91.63% to 86.24%. The correspondence was 89.24% correct at the first level. I thus can not conclude and won't discuss the weak correspondence.

2.14 depicts the strongly efficient networks in the absence of the consumer surplus. The strong correspondence between the stable sets and the strongly efficient networks is ensure for 37.66% of the space of parameter at level-1, 43, 15% for the level-2, 41.76% for

the level-3, 21.08% at the level-4, 15.75% for levels 5 and greater.

The last element to mention is Figure 2.15. As we take a look at the strong correspondence between stable sets of networks and the strongly efficient networks we can note an interesting fact. If, for a couple of parameter (d, k) , there is a transitive correspondence between the pairwise farsighted stable set and the level- ∞ farsightedly stable set, then the stable set is likely to be strongly efficient. This results holds either in the presence of consumers or in their absence.

2.8 Conclusion

Traditionally, the industrial economists have studied vertical relationships using exogenous architectures of networks. Recent researches put forward innovative ways of interpreting the structural organization of economic entities into networked relationships. The strategic networks approach shed new light on the nature of interactions and the determination of the likely layouts of agents. With the perspective of the level- K reasoning, this chapter investigated the question of the stability the networks of manufacturers and retailers for the full range of products differentiation and any cost of linking. The results show that the number of stable sets or stable networks tends to expand sharply with the increase of the level of farsightedness. The computational analysis allow for the determination of the vertically related structures which are likely to emerge. Indeed, the complete network, the mixed distribution so as the exclusive dealing networks are the only stable architectures regardless of the values of parameters or the level of farsightedness. It turns that the minimum level of farsightedness to get to the correspondence between the level- K farsighted stable set and the level- ∞ farsighted set is fairly low in regards to the theoretic expectations. I emphasize a major theoretic failure in the determination of the correspondence between a set-based definition of stability and an even simplistic optimality criterion. This evaluation remains subtle regardless of the level of acceptance. The level of farsightedness has also serious implications when it comes to compare stability to efficiency. Indeed, increasing the level farsightedness doesn't necessarily encourages the involved players in reaching a form of efficiency. Eventually, I identify the values of

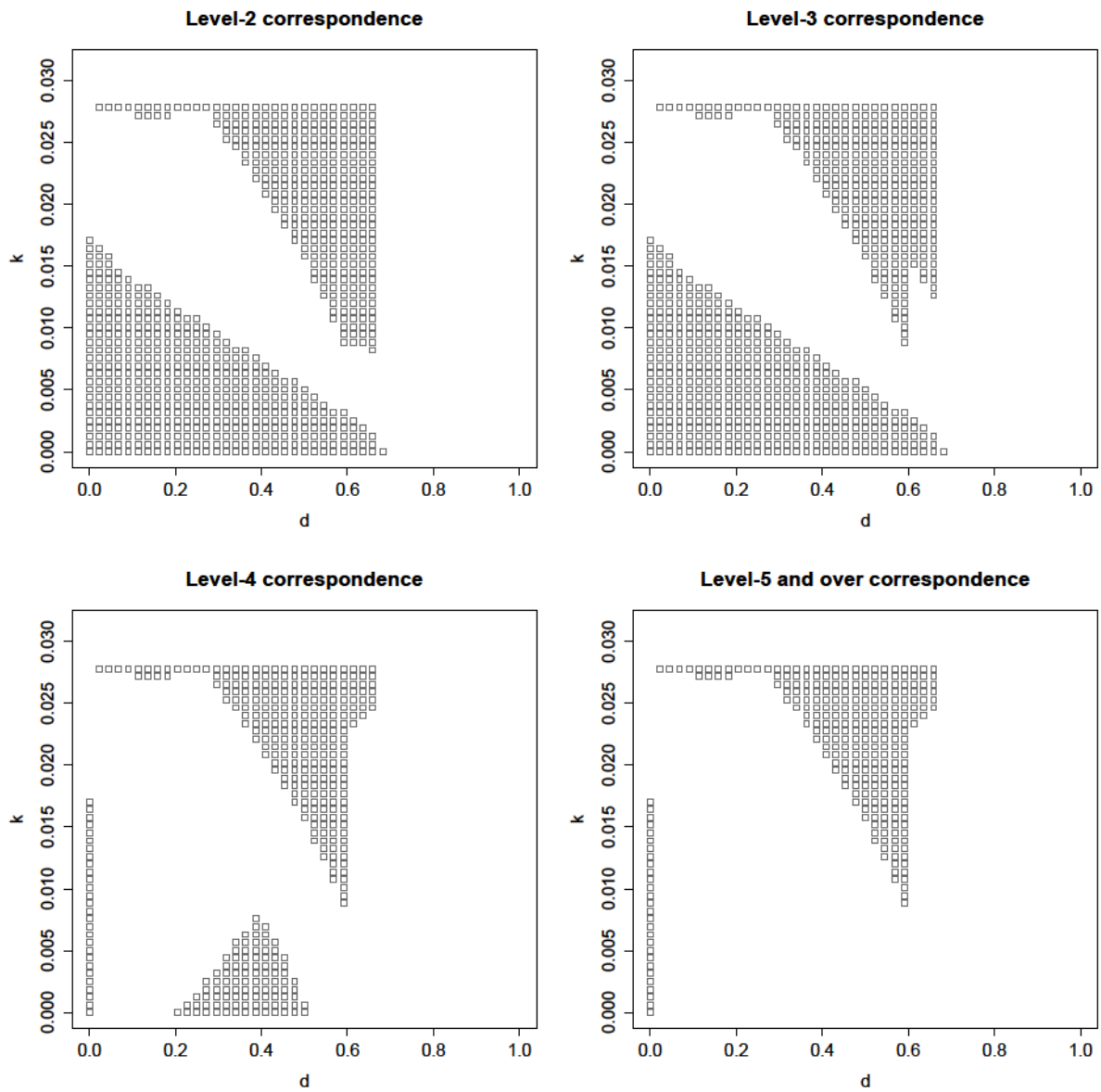


Figure 2.15: Strong correspondence between stable sets and Strongly efficient networks in the industrial segment.

parameters for which there is a transitive correspondence between the pairwise level- ∞ farsighted set G_∞^∞ and the pairwise farsighted set G_∞^1 .

2.9 Appendix B

2.9.1 Payoffs

	g^N	x_1
Y_{M_1}	$\frac{2(1-d)}{3(d-2)^2(d+1)} - 2k$	$\frac{(1-d)(2-d)(2+d)(8+5d)^2}{6(d+1)(7d^2-16)^2} - 2k$
Y_{M_2}	$\frac{2(1-d)}{3(d-2)^2(d+1)} - 2k$	$\frac{(1-d)(d^2-4d-8)^2}{2(7d^2-16)^2(d+1)} - k$
Y_{R_1}	$\frac{2}{9(d+1)(d-2)^2} - 2k$	$\frac{(-3d+2d^2-8)^2}{9(7d^2-16)^2} - k$
Y_{R_2}	$\frac{2}{9(d+1)(d-2)^2} - 2k$	$\frac{52+28d-d^3-7d^2}{36(d+1)(16-7d^2)} - 2k$

Table 2.9: Payoffs for networks g^N and x_1

	ed_1	m_1	r_1	s_1	g^θ
Y_{M_1}	$\frac{2(2-d)}{(d-4)^2(d+2)} - k$	$\frac{1}{6} - 2k$	$\frac{1-d}{2(d-2)^2(d+1)} - k$	$\frac{1}{8} - k$	0
Y_{M_2}	$\frac{2(2-d)}{(d-4)^2(d+2)} - k$	0	$\frac{1-d}{2(d-2)^2(d+1)} - k$	0	0
Y_{R_1}	$\frac{4}{(d+2)^2(d-4)^2} - k$	$\frac{1}{36} - k$	$\frac{1}{2(d+1)(d-2)^2} - 2k$	$\frac{1}{16} - k$	0
Y_{R_2}	$\frac{4}{(d+2)^2(d-4)^2} - k$	$\frac{1}{36} - k$	0	0	0

Table 2.10: Payoffs for networks ed_1 , m_1 , r_1 , s_1 and g^θ

The Tables 2.9 and 2.10 give the payoffs for each network configuration. Remind that the first ratio of the payoffs is expressed in $(a - c)^2 = 1$. The payoffs for alternative networks can be easily obtained by a permutation of players. For instance, the payoffs for the network x_3 are obtained from x_1 by swapping the position of the two retailers and their payments. The payoffs for m_2 are obtained by swapping the the two manufacturers.

2.9.2 Pseudo code

Tables 2.11 to 2.15 hereafter give the pseudo code for the functions used throughout the algorithm are presented. The function **unique**(G) refers to vector where the elements of G appear only once.

```
compare( $g_1, g_2, g_3, d, k$ )


---


1: compare ← function( $g_1, g_2, g_3, d, k$ ) {
2:    $i \leftarrow \mathbf{players}(g_3, g_2)[1]$ 
3:    $j \leftarrow \mathbf{players}(g_3, g_2)[2]$ 
4:   if  $\#g_2 < \#g_3$  {
5:     if  $Y_i(g_1) > Y_i(g_3)$  or  $Y_j(g_1) > Y_j(g_3)$  {
6:       return 1
7:     }
8:   }
9:   if  $\#g_2 > \#g_3$  {
10:    if  $Y_i(g_1) \geq Y_i(g_3)$  and  $Y_j(g_1) > Y_j(g_3)$ 
    or  $Y_i(g_1) > Y_i(g_3)$  and  $Y_j(g_1) \geq Y_j(g_3)$  {
11:      return 1
12:    }
13:  }
14:  else return 0
15: }
```

Table 2.11: The *compare* (g_1, g_2, g_3, d, k) function

matfip(d, k)

```
1: matfip ← function( $d, k$ ) {
2:    $m_0 \leftarrow 0_{[\#\mathbb{G} \times \#\mathbb{G}]}$ 
3:    $m_1 \leftarrow \text{matrix}_{[\#\mathbb{G} \times \#\mathbb{G}]}$ 
4:   for each  $g$  in  $\mathbb{G}$  {
5:     for each  $g'$  in  $A^g$  {
6:        $m_1[g, g'] \leftarrow \text{compare}(g', g', g, d, k)$ 
7:     }
8:   }
9:    $K \leftarrow 2$ 
10:  for each  $K$  in  $\{2 : \#\mathbb{G}\}$  {
11:     $m_K \leftarrow m_0$ 
12:    for each  $g_2$  in  $\mathbb{G}$  {
13:      for each  $g_1$  in  $m_{K-1}[g_2, g_1] = 1$  {
14:        for each  $g_3 \in A^{g_2}$  {
15:          if  $m_K[g_3, g_1] = 0$  {
16:             $m_K[g_3, g_1] \leftarrow \text{compare}(g_1, g_2, g_3, d, k)$ 
17:          }
18:        }
19:      }
20:    }
21:  }
22:   $M_1 \leftarrow m_1$ 
23:  for each  $K$  in  $\{2 : \#\mathbb{G}\}$  {
24:     $M_K \leftarrow M_{K-1} + m_K$ 
25:    if  $M_K[g, g'] > 1$  {
26:       $M_K[g, g'] \leftarrow 1$ 
27:    }
28: }
```

Table 2.12: The *matfip*(d, k) function

$\text{devdet}(G', g', g)$

```
1: devdet ← function( $G', g', g$ )
2:    $i \leftarrow \text{players}(g, g') [1]$ 
3:    $j \leftarrow \text{players}(g, g') [2]$ 
4:   if  $\#g > \#g'$  {
5:     for each  $g_1$  in  $G'$  {
6:       if  $Y_i(g_1) \leq Y_i(g)$  {
7:         for each  $g_2$  in  $G'$  {
8:           if  $Y_j(g_2) \leq Y_j(g)$  {
9:             return 1
10:          }
11:        }
12:      }
13:    }
14:  }
15:  if  $\#g < \#g'$  {
16:    for each  $g_1$  in  $G'$  {
17:      if  $Y_i(g_1) = Y_i(g)$  and  $Y_j(g_1) = Y_j(g)$ 
18:      or  $Y_i(g_1) < Y_i(g)$  or  $Y_j(g_1) < Y_j(g)$  {
19:        return 1
20:      }
21:    }
22:  return 0
```

Table 2.13: The $\text{devdet}(G', g', g)$ function

$\text{instab}(G, M_K)$

```

1: instab ← function( $G, M_K$ ) {
2:    $G' \leftarrow \emptyset$ 
3:   for each  $g$  in  $G$  {
4:      $\tilde{G} \leftarrow A^g \setminus G$ 
5:     if  $\tilde{G} \neq \emptyset$  {
6:       for each  $g_1$  in  $\tilde{G}$  {
7:          $M'_K \leftarrow M_{K-1} - M_{K-2}$ 
8:         for each  $g_2$  in  $M'_K [g_1, g_2] = 1$  {
9:            $G' \leftarrow \{G', g_2\}$ 
10:        }
11:       for each  $g_3$  in  $M_{K-2} [g_1, g_3] = 1$  {
12:         if  $g_3 \in G$  {
13:            $G' \leftarrow \{G', g_3\}$ 
14:         }
15:       }
16:        $G' \leftarrow \text{unique}(G')$ 
17:       if  $\text{devdet}(G', g_1, g) = 0$  {
18:         return 0
19:       }
20:     }
21:   }
22: }
23: return 1
24: }
```

Table 2.14: The $\text{instab}(G, M_K)$ function

$\text{exstab}(G, M_K)$

```
1: exstab ← function( $G, M_K$ ) {
2:    $G_0 \leftarrow G$ 
3:    $G_1 \leftarrow \mathbb{G} \setminus G$ 
4:    $i = 1$ 
5: loop
6:   if  $G_{i-1} = \emptyset$  {
7:     return 1
8:   }
9:   for each  $g$  in  $G_i$  {
10:    for each  $g'$  in  $G_{i-1}$  {
11:      if  $M_K[g, g'] = 1$  {
12:         $G_i \leftarrow G_i \setminus g$ 
13:      }
14:    }
15:  }
16:  if  $G_i = G_{i-1}$  {
17:    return 0
18:  }
19:   $i \leftarrow i + 1$ 
20: }
```

Table 2.15: The $\text{exstab}(G, M_K)$ function

Chapter 3

Allocating value among farsighted players in network formation

3.1 Introduction

The organization of agents into networks and groups plays an important role in the determination of the outcome of many social and economic interactions¹. A simple way to analyze the networks that one might expect to emerge in the long run is to examine the requirement that players do not benefit from altering the structure of the network. An example of such a condition is the pairwise stability notion defined by Jackson and Wolinsky (1996)[33]. A network is pairwise stable if no player benefits from severing one of her links and no two players benefit from adding a link between them. Pairwise stability is a myopic definition. Players are not farsighted in the sense that they do not forecast how others might react to their actions. For instance, the adding or severing of one link might lead to subsequent addition or severing of another link. If players have very good information about how others might react to changes in the network, then these are things one wants to allow for in the definition of the stability concept. For instance, a network could be stable because players might not add a link that appears valuable to them given the current network, as that might in turn lead to the formation of other links

¹See Jackson (2003, 2008) [29] [30], or Goyal (2007) [20] for a comprehensive introduction to the theory of social and economic networks.

and ultimately lower the payoffs of the original players.

Allocation rules keep track of how value is allocated among the players in the network. The allocation rule may simply be the utility that players directly get, accounting for the costs and benefits of being linked to other players in the network. But there are many situations where the allocation rule is the result of some bargaining among linking players. However, most network formation models are such that both the network formation process and the allocation of value among players in a network are separated and the players are not farsighted.

In this chapter we address the question of which networks one might expect to emerge in the long run when the players are farsighted and the allocation of value among players is determined simultaneously with the network formation as players may bargain over their shares of value within their component. Hence, we introduce the notion of von Neumann-Morgenstern farsighted stability with bargaining.

A set of networks is a von Neumann-Morgenstern farsightedly stable set with bargaining if there exists an allocation rule and a bargaining threat such that (i) there is no farsighted improving path from one network inside the set to another network inside the set,² (ii) from any network outside the set there is a farsighted improving path to some network inside the set, (iii) the value of each network is allocated among players so that players suffer or benefit equally from being linked to each other compared to the allocation they would obtain at their respective credible bargaining threat. In contrast to Chwe's (1994) [13] definition of von Neumann-Morgenstern farsighted stability,³ allocations are going to be agreed upon among farsighted players when allocations and links are determined jointly. To capture this idea we request that the allocation rule satisfy the property of equal bargaining power for farsighted players. This property requires that, for each pair of players linked in the network, both players suffer or benefit equally from being linked with respect to their respective bargaining threat. In addition, we request

²A farsighted improving path is a sequence of networks that can emerge when players form or sever links based on the improvement the end network offers relative to the current network. Each network in the sequence differs by one link from the previous one.

³Mauleon, Vannetelbosch and Vergote (2011) [39] have provided the characterization of von Neumann-Morgenstern farsightedly stable sets in one-to-one matching problems and in many-to-one matching problems with substitutable preferences.

the bargaining threat to be credible. Credibility means that the threat can be reached by a farsighted improving path emanating from some network adjacent to the network over which bargaining takes place.

We show that the set of strongly efficient networks is the unique von Neumann-Morgenstern farsightedly stable set with bargaining if the allocation rule is anonymous and component efficient and the value function is anonymous, component additive and top convex. Moreover, the componentwise egalitarian allocation rule emerges endogenously.

We next incorporate the property of equal bargaining power for farsighted players into the original definition of a pairwise farsightedly stable set due to Herings, Mauleon and Vannetelbosch (2009) [23].⁴ We find that if a set of networks is a von Neumann-Morgenstern farsightedly stable set with bargaining, then it is a pairwise farsightedly stable set with bargaining. Hence, if the allocation rule is anonymous and component efficient and the value function is anonymous, component additive and top convex, then the set of strongly efficient networks is a pairwise farsightedly stable set with bargaining where the componentwise egalitarian allocation rule emerges endogenously. However, the set of strongly efficient networks $E(v)$ can also be a pairwise farsightedly stable set with bargaining even for anonymous allocation rules where the value of each component is not shared equally among the members of the component. If the value function is not top convex, then inefficient networks can be farsightedly stable with bargaining. We provide an example where, contrary to pairwise farsighted stability with bargaining, von Neumann-Morgenstern farsighted stability with bargaining only leads to the emergence of inefficient networks.

3.1.1 Literature Review

There are a number of papers that look at the endogenous determination of allocations together with network formation. Currarini and Morelli (2000) [14] have introduced a sequential network formation game, where players propose links and demand allocations.

⁴Other approaches to farsightedness in network formation are Herings, Mauleon, and Vannetelbosch (2004) [25], Page, Wooders and Kamat (2005) [47], Dutta, Ghosal, and Ray (2005) [17], and Page and Wooders (2009) [46].

Given an exogenously given order, each player proposes in turn the links she wants to form and she demands an allocation. Once all proposals have been made, links are formed if both players involved in the link have proposed the link and the demands of the players are compatible (i.e. demands do not exceed the value produced). They have shown that, if the value function satisfies size monotonicity (i.e. if the efficient networks connect all players), then their sequential network formation process with endogenous allocations leads all subgame perfect equilibrium to be efficient.⁵ Mutuswami and Winter (2002) [41] have proposed subscription mechanisms for network formation when the costs from linking are publicly known but the benefits from linking are not known to the social planner. Their mechanism is similar to Currarini and Morelli (2000) [14] sequential network formation game⁶ and leads to the formation of an efficient network.⁷ The payoffs in Currarini and Morelli (2000) [14] and Mutuswami and Winter (2002) [41] are endogenously generated but are highly asymmetric and sensitive to the order in which players make proposals. More recently, Bloch and Jackson (2007) [8] have studied the role played by transfers payments in the formation of networks. They have investigated whether different forms of transfers (direct transfers, indirect transfers or contingent transfers) can solve the conflict between stability and efficiency when there are network externalities that usually lead to the emergence of inefficient networks when transfers are not feasible.⁸ But all these papers have assumed either simultaneous move games (with myopic players) or sequential move games (with finite horizon and specific ordering). We go further by looking at the endogenous determination of payoffs together with network formation in presence of farsighted players.

⁵However, if the network formation process is simultaneous, then there are value functions that satisfy size monotonicity for which inefficient equilibria can arise.

⁶Each player, when it is her turn, proposes the set of links she wants to form and her cost contribution. Once all proposals have been made, the social planner selects the network to be formed and the cost contributions of the players.

⁷Slikker and van den Nouweland (2000) [50] have studied the formation of communication networks with endogenous payoff division but with a strategic form game. Similarly, Matsubayashi and Yamakawa (2004) [37] have proposed a strategic form game to share the cost of building the network in a model where the benefits of the network decays as the distance among players increases.

⁸They have found that indirect transfers together with contingent transfers are needed to guarantee that efficient networks form. Indirect transfers enable players to take care of positive externalities by subsidizing the formation of links by other players; while contingent transfers enable players to overcome negative externalities by preventing the formation of links

The paper most closely related to our work is Navarro (2014) [45] who has studied a dynamic process of network formation that is represented by means of a stationary transition probability matrix. Forward-looking players have a common discount factor and receive payoffs at each moment in time according to a stationary allocation rule. Three properties are imposed on the allocation rule and the transition probability. First, the allocation rule together with the transition probability are expected fair. That is, for each link in the network both players involved in the link benefit or suffer the same stream of discounted expected payoffs from cutting their link at time zero. Second, the allocation rule is component efficient. That is, the value of each component is shared among the members of the component. Third, the expected fair allocation rule and transition probability is a pairwise network formation procedure. That is, the probability that a link is added (or deleted) is positive only if the stream of discounted expected payoffs for the players involved in adding (or deleting) the link is positive. Navarro (2014) [45] has shown that if the common discount factor is small enough (i.e. players are close to be myopic), then there exists an allocation rule together with a transition probability matrix such that the allocation rule is component efficient and the allocation rule together with the transition probability is an expected fair pairwise network formation procedure.⁹

Here, we rather adopt the stability approach because the noncooperative or dynamic approach is much sensitive to the specification of the bargaining game and network formation process, whose fine details (such as how the game ends) can be very important in determining what networks form and how value is allocated.

3.1.2 Roadmap

The chapter is organized as follows. In Section 3.2 we introduce some notations, basic properties and definitions for networks. In Section 3.3 we define the notion of von Neumann-Morgenstern farsighted stability with bargaining and we look at the relationship between von Neumann-Morgenstern farsighted stability with bargaining and efficiency of networks. In Section 3.4 we propose the notion of pairwise farsighted stability with bar-

⁹Navarro (2013) [44] has used her dynamic network formation process and her solution concept to investigate the tension between efficiency and stability.

gaining and we look at its relationship with the von Neumann-Morgenstern farsighted stability with bargaining. In Section 3.5 we discuss some properties. In particular, we address situations where there are externalities across components and we provide a condition such that the set of efficient networks remains the unique von Neumann-Morgenstern farsightedly stable set with bargaining. In Section 3.6 we conclude.

3.2 Allocating value among farsighted players

Networks

Let $N = \{1, \dots, n\}$ be the finite set of players who are connected in some network relationship. The network relationships are reciprocal and the network is thus modeled as a non-directed graph. Players are the nodes in the graph and links indicate bilateral relationships between players. Thus, a network g is simply a list of which pairs of individuals are linked to each other. We write $ij \in g$ to indicate that i and j are linked under the network g . Let g^S be the set of all subsets of $S \subseteq N$ of size 2.¹⁰ So, g^N is the complete network. The set of all possible networks or graphs on N is denoted by \mathbb{G} and consists of all subsets of g^N . The network obtained by adding link ij to an existing network g is denoted $g + ij$ and the network that results from deleting link ij from an existing network g is denoted $g - ij$. Let $g|_S = \{ij \mid ij \in g \text{ and } i \in S, j \in S\}$. Thus, $g|_S$ is the network found deleting all links except those that are between players in S . For any network g , let $N(g) = \{i \mid \exists j \text{ such that } ij \in g\}$ be the set of players who have at least one link in the network g . The neighborhood of player i is the set of players that i is linked to: $N_i(g) = \{j \mid ij \in g\}$. A network g' is adjacent to g if $g' = g + ij$ or $g' = g - ij$ for some ij . Let $A(g)$ be the set of adjacent networks to g .

A path in a network $g \in \mathbb{G}$ between i and j is a sequence of players i_1, \dots, i_K such that $i_k i_{k+1} \in g$ for each $k \in \{1, \dots, K - 1\}$ with $i_1 = i$ and $i_K = j$, and such that each player in the sequence i_1, \dots, i_K is distinct. A non-empty network $h \subseteq g$ is a component of g , if for all $i \in N(h)$ and $j \in N(h) \setminus \{i\}$, there exists a path in h connecting i and j ,

¹⁰Throughout the chapter we use the notation \subseteq for weak inclusion and \subsetneq for strict inclusion. Finally, $\#$ will refer to the notion of cardinality.

and for any $i \in N(h)$ and $j \in N(g)$, $ij \in g$ implies $ij \in h$. The set of components of g is denoted by $C(g)$. Let $\Pi(g)$ denote the partition of N induced by the network g . That is, $S \in \Pi(g)$ if and only if either there exists $h \in C(g)$ such that $S = N(h)$ or there exists $i \notin N(g)$ such that $S = \{i\}$.

Value functions

A value function is a function v that assigns a value $v(S, g)$ to every network g and every coalition $S \in \Pi(g)$. This value $v(S, g)$ can be perfectly distributed among the players in S . Given v , the total value that can be distributed at network g is equal to $v(g) = \sum_{S \in \Pi(g)} v(S, g)$. The set of all possible value functions v is denoted by \mathcal{V} . A value function v is *component additive* (or has no externalities across components) (Jackson and Wolinsky, 1996) if for any $g \in \mathbb{G}$ and $S \in \Pi(g)$, $v(S, g) = v(S, g|_S)$. Component additivity means that the value of a component of the network does not depend on the structure of the network outside the component. Given a permutation of players π and any $g \in \mathbb{G}$, let $g^\pi = \{\pi(i)\pi(j) \mid ij \in g\}$. Thus, g^π is a network that is identical to g up to a permutation of the players. A value function v is *anonymous* (Jackson and Wolinsky, 1996) if for any permutation π , $g \in \mathbb{G}$ and $S \in \Pi(g)$, $v(\{\pi(i) \mid i \in S\}, g^\pi) = v(S, g)$. A network $g \in \mathbb{G}$ is *strongly efficient* relative to v if $v(g) \geq v(g')$ for any $g' \in \mathbb{G}$. Let $E(v)$ be the set of strongly efficient networks. A value function v is *top convex* (Jackson and van den Nouweland, 2005) if some strongly efficient network maximizes the per capita value among players. Let $\rho(v, S) = \max_{g \subseteq g^S} v(g) / \#S$. The value function v is *top convex* if $\rho(v, N) \geq \rho(v, S)$ for any $S \subseteq N$.

Allocation rules

An allocation rule y is a function that assigns a payoff $y_i(g, v)$ to player $i \in N$ from graph g under the value function $v \in \mathcal{V}$. An allocation rule y is *component efficient* (Myerson, 1977) if for any $g \in \mathbb{G}$ and $S \in \Pi(g)$, $\sum_{i \in S} y_i(g, v) = v(S, g)$.¹¹ Given a permutation π , let v^π be defined by $v^\pi(S, g) = v(\{\pi^{-1}(i) \mid i \in S\}, g^{\pi^{-1}})$ for any $g \in \mathbb{G}$. An allocation rule y is *anonymous* (Jackson and Wolinsky, 1996) if for any v , $g \in \mathbb{G}$ and permutation π , $y_{\pi(i)}(g^\pi, v^\pi) = y_i(g, v)$.

¹¹An allocation rule y is *component balanced* (Jackson and Wolinsky, 1996) if for any component additive v , $g \in \mathbb{G}$ and $S \in \Pi(g)$, $\sum_{i \in S} y_i(g, v) = v(S, g|_S)$.

Some prominent allocation rules have been proposed. The egalitarian allocation rule (Jackson and Wolinsky, 1996) y^e is defined by $y_i^e(g, v) = v(g)/n$. For a component additive v and network g , the componentwise egalitarian allocation rule (Jackson and Wolinsky, 1996) y^{ce} is such that for any $S \in \Pi(g)$ and each $i \in S$, $y_i^{ce}(g, v) = v(S, g|_S)/\#S$. For a v that is not component additive, $y^{ce}(g, v) = v(g)/n$ for all g ; thus, y^{ce} splits the value $v(g)$ equally among all players if v is not component additive.

Another allocation rule is the Myerson value:

$$y_i^{MV}(g, v) = \sum_{S \subseteq N \setminus \{i\}} v(g|_{S \cup \{i\}}) - v(g|_S) \left(\frac{\#S!(n - \#S - 1)!}{n!} \right).$$

An allocation rule satisfies *equal bargaining power* if for any component additive v and $g \in \mathbb{G}$ we have $y_i(g, v) - y_i(g - ij, v) = y_j(g, v) - y_j(g - ij, v)$. Equal bargaining power requires that both players equally benefit or suffer from the addition of the link. It does not impose that players split the marginal value of a link. Jackson and Wolinsky (1996) [33] have shown that y satisfies component balance and equal bargaining power if and only if $y(g, v) = y^{MV}(g, v)$ for all $g \in \mathbb{G}$ and component additive v .¹² However, Jackson (2005) [29] has argued that the basic problem with the Myerson value allocation rule is that the value of other possible networks is not properly accounted for in its calculations. For instance, if alternative network structures are taken into account when bargaining over how to allocate value, then values of alternative networks, and not just (adjacent) subnetworks, should be important in determining the allocation. Hence, Jackson (2005) [29] has proposed the player-based flexible network allocation rule:¹³

$$y_i^{PBFN}(g, v) = \frac{v(g)}{\widehat{v}(g^N)} \sum_{S \subseteq N \setminus \{i\}} (\widehat{v}(g^{S \cup \{i\}}) - \widehat{v}(g^S)) \left(\frac{\#S!(n - \#S - 1)!}{n!} \right),$$

where \widehat{v} is the monotonic cover of v defined by $\widehat{v} = \max_{g' \subseteq g} v(g')$.¹⁴ However, the player-

¹²Navarro (2007) [42] has proposed a component efficient and fair allocation rule when the value of the network can exhibit any type of externalities across its components.

¹³Perez-Castrillo and Wettstein (2005) [48] have proposed a sequential mechanism whose subgame perfect equilibria rise to strongly efficient networks and to payoffs that coincide with Jackson's player-based flexible network allocation rule.

¹⁴The monotonic cover of a value function gives the highest value that can be generated by building a network out of a given set of links.

based flexible network allocation rule rule violates both equal bargaining power and component balance.¹⁵ We now provide an example that illustrates the drawbacks of the Myerson value and the player-based flexible network allocation rule. This example also motivates the necessity of determining the allocation rule together with the formation of the network in the long run.

Example 1 *The Myerson value.* Take $N = \{1, 2, 3\}$ and the value function defined by $v(\{1, 2, 3\}, \{12, 13, 23\}) = 0$, $v(\{1, 2, 3\}, \{12, 13\}) = 5$, $v(\{1, 2, 3\}, \{12, 23\}) = 0$, $v(\{1, 2, 3\}, \{13, 23\}) = 0$, $v(\{1, 2\}, \{12\}) = 0$, $v(\{3\}, \{12\}) = 0$, $v(\{1, 3\}, \{13\}) = v(\{2\}, \{13\}) = 0$, $v(\{2, 3\}, \{23\}) = 4$, $v(\{1\}, \{23\}) = 0$, and $v(S, \emptyset) = 0$. We have depicted in Figure 3.1 the different network configurations with their associated allocations. For networks that generate value, the Myerson value leads to equal sharing of the value in $\{12, 13\}$, $y_i^{MV}(\{12, 13\}) = 5/3$ for $i \in N$. Hence, players 2 and 3 obtain less than what they could get in $\{23\}$, namely $y_2^{MV}(\{23\}) = 2 = y_3^{MV}(\{23\})$. Players 2 and 3 have a viable outside option but the Myerson value does not take this option into account because $\{23\}$ is not a subnetwork of $\{12, 13\}$. The player-based flexible network allocation rule provides a more reasonable allocation than the Myerson value for $\{12, 13\}$ by giving higher allocations to players 2 and 3 than player 1: $y_2^{MV}(\{12, 13\}) = 7/3 = y_3^{MV}(\{12, 13\})$ and $y_1^{MV}(\{12, 13\}) = 1/3$. However, it gives $y_2^{PBFN}(\{23\}) = 28/15 = y_3^{PBFN}(\{23\})$ and $y_1^{PBFN}(\{23\}) = 4/15$ violating component balance. Hence, this allocation is unlikely to emerge at $\{23\}$ since the value function is component additive and players 2 and 3 transfer some payoff to player 1 who does not belong to their component. \square

Allocation rules with farsighted players

The equal bargaining power property imposes that, for each link ij in a network g , both players i and j should equally benefit or suffer when bargaining over how to allocate value taking as reference network the adjacent subnetwork $g - ij$. Hence, equal bargaining power presumes that players are myopic, not farsighted, in the sense that they do not forecast how others might react if they break the link ij . For instance, the severing of ij might lead to subsequent severing or addition of another link. Once players are farsighted,

¹⁵Navarro (2010) [43] has proposed three modifications of Jackson's (2005) [29] flexible network axiom when the structure of externalities across components is known.

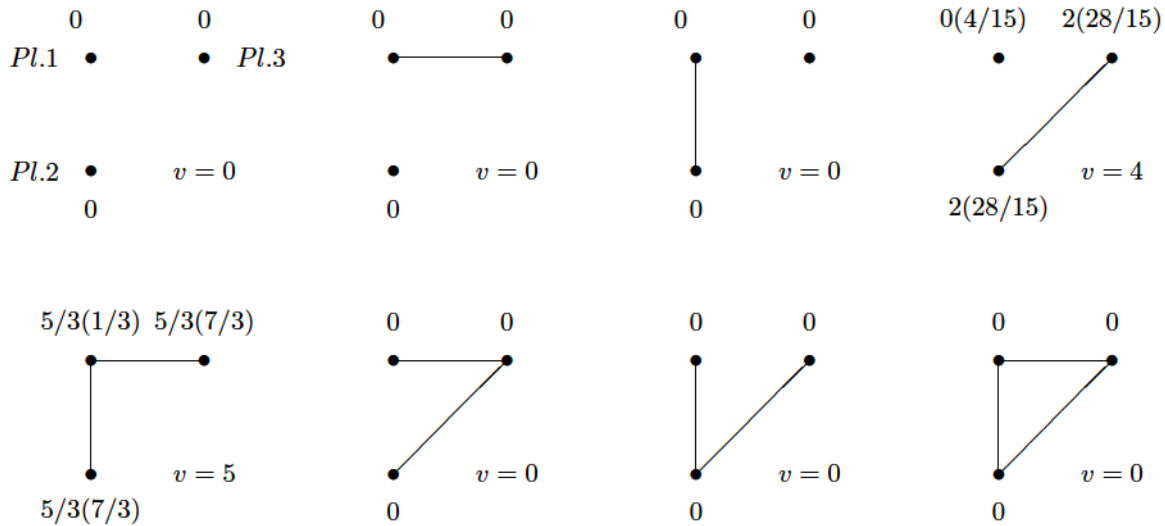


Figure 3.1: The Myerson value (The player-based flexible network allocation rule).

equal bargaining power will impose that players equally benefit or suffer when bargaining over how to allocate value taking as reference network or bargaining threat, not necessarily adjacent subnetworks, but networks that may be reached from adjacent networks through a sequence of networks when players form or delete links based on the improvement the end network offers relative to the current one.

3.3 Von Neumann-Morgenstern farsighted stability with bargaining

We now propose the notion of von Neumann-Morgenstern farsighted stability with bargaining, to predict which networks are likely to emerge and which allocations are going to be agreed upon among farsighted players when allocations and links are determined jointly.

We first introduce the notion of farsighted improving path from Herings, Mauleon and Vannetelbosch (2009) [23] and the notion of bargaining threat. A farsighted improving path is a sequence of networks that can emerge when players form or sever links based on the improvement the end network offers relative to the current network. Each network in the sequence differs by one link from the previous one. If a link is added, then the two

players involved must both prefer the end network to the current network, with at least one of the two strictly preferring the end network. If a link is deleted, then it must be that at least one of the two players involved in the link strictly prefers the end network. Formally, a farsighted improving path from a network g to a network $g' \neq g$ is a finite sequence of graphs g_1, \dots, g_K with $g_1 = g$ and $g_K = g'$ such that for any $k \in \{1, \dots, K-1\}$ either: (i) $g_{k+1} = g_k - ij$ for some ij such that $y_i(g_K, v) > y_i(g_k, v)$ or $y_j(g_K, v) > y_j(g_k, v)$, or (ii) $g_{k+1} = g_k + ij$ for some ij such that $y_i(g_K, v) > y_i(g_k, v)$ and $y_j(g_K, v) \geq y_j(g_k, v)$. For a given network g , let $F(g)$ be the set of networks that can be reached by a farsighted improving path from g . Notice that $F(g)$ may contain many networks and that a network $g' \in F(g)$ might be the endpoint of several farsighted improving paths starting in g .

A bargaining threat z is a function that assigns to each network $g \in \mathbb{G}$ a network $z_i(g) \in \mathbb{G}$ for each player $i \in N$. Intuitively, when player i is negotiating how to share the surplus with other players she is linked to in g , she has in mind the payoff she might obtain at some other network, $z_i(g)$, not necessarily adjacent to g since players are farsighted.

It is also useful to refine the definition of the reachable adjacencies from a player's point of view. Let $A_i^-(g) \cup A_i^+(g)$ be the reachable adjacencies from i 's perspective, with: $A_i^-(g) = \{g - ik \mid k \in N_i(g)\}$, the set of adjacent networks to g that can be reached by i as he deletes one of his links and:

$A_i^+(g) = \{g + ik' \mid k' \notin N_i(g) \text{ s.t. } \forall g' \in F(g + ik'), Y_{k'}(g') > Y_{k'}(g)\}$, the set of adjacent networks to g that can be reached by i as he builds a link with the consent of another player. Note that $F(g + ik) \subset F(g)$ is not requirement here.

A set of networks is a von Neumann-Morgenstern farsightedly stable set with bargaining if there exists an allocation rule and a bargaining threat such that the following conditions hold. First, there is no farsighted improving path from one network inside the set to another network inside the set (internal stability). Second, from any network outside the set there is a farsighted improving path to some network inside the set (external stability). Third, the value of each network is allocated among players so that players suffer or benefit equally from being linked to each other compared to the allocation they would obtain at their respective bargaining threat (equal bargaining power). Fourth,

the bargaining threat at each network is credible. Credibility means that the threat can be reached by a farsighted improving path emanating from some network adjacent to the network over which bargaining takes place. Formally, von Neumann-Morgenstern farsightedly stable sets with bargaining are defined as follows.

Definition 22 *A set of networks $G \subseteq \mathbb{G}$ is a von Neumann-Morgenstern farsightedly stable set with bargaining if there exists an allocation rule y and a bargaining threat z such that*

(i) $\forall g \in G, F(g) \cap G = \emptyset$; (*Internal Stability*)

(ii) $\forall g' \in \mathbb{G} \setminus G, F(g') \cap G \neq \emptyset$; (*External Stability*)

(iii) $\forall g \in \mathbb{G}$ and $ij \in g$,

(a) $y_i(g, v) - y_i(z_i(g), v) = y_j(g, v) - y_j(z_j(g), v)$, (*Equal Bargaining Power*)

(b) $z_i(g) \in (F(g'') \cup \{g''\}) \cap G \neq \emptyset$ for some $g'' \in A_i(g)$ and $z_j(g) \in (F(g''') \cup \{g'''\}) \cap G \neq \emptyset$ for some $g''' \in A_j(g)$. (*Consistency*)

Condition (i) in Definition 22 is the internal stability condition. From any network within G , there is no farsighted improving path leading to some other network in G . Condition (ii) in Definition 22 requires external stability and implies that the networks within the set are robust to perturbations. From any network outside G , there is a farsighted improving path leading to some network in G .¹⁶ Condition (ii) implies that if a set of networks is a von Neumann-Morgenstern farsightedly stable set with bargaining, it is non-empty. Part (a) of condition (iii) in Definition 22 is the equal bargaining power property for farsighted players. It requires that for each pair of players linked in g both players suffer or benefit equally from being linked with respect to their respective bargaining threat. Part (b) of condition (iii) in Definition 22 imposes a consistency requirement on the bargaining threat. When bargaining how to share the value at g , the bargaining threat $z_i(g)$ for each player i has to be such that the threat can be reached by a farsighted

¹⁶There are some random dynamic models of network formation that are based on myopic incentives to form links such as Jackson and Watts (2002) [32] and Tercieux and Vannetelbosch (2006) [53]. These models aim to use the random process to select from the set of pairwise stable networks.

improving path emanating from some adjacent network to g when the adjacent network is not in G . That is, $z_i(g) \in F(g'')$ for some $g'' \in A_i(g)$ when $g'' \notin G$. Moreover, $z_i(g)$ belongs to G , which makes $z_i(g)$ a credible threat.¹⁷

Example 1 (continued). We observe that $E(v) = \{\{12, 13\}\}$ is not a von Neumann-Morgenstern farsightedly stable set with bargaining if y is the Myerson value allocation rule since external stability is violated. There is no farsighted improving path from the network $\{23\}$ to the network $\{12, 13\}$ if y is the Myerson value; players 2 and 3 obtain a higher payoff in $\{23\}$ than in $\{12, 13\}$. Notice that the set $\{\{23\}\}$ is not a von Neumann-Morgenstern farsightedly stable set with bargaining if y is the Myerson value because it violates equal bargaining power for farsighted players at networks where some players are linked to each other and $v = 0$. For instance, players 1 and 2 obtain both 0 at $\{12\}$ but obtain, respectively, 0 and 2 at their consistent bargaining threat ($z_i(\{12\}) = \{23\}$).

If y is the player-based flexible network allocation rule, then $E(v) = \{\{12, 13\}\}$ is not a von Neumann-Morgenstern farsightedly stable set with bargaining even though internal stability, external stability and consistency in Definition 22 are satisfied. But, equal bargaining power for farsighted players is violated at networks where some players are linked to each other and $v = 0$. In general, equal bargaining power for farsighted players may be violated at any network. For instance, take v' such $v'(g) = v(g)$ for all g except for $\{12\}$ where $v'(\{12\}, \{12\}) = 2$ and $v'(\{3\}, \{12\}) = 0$. The player-based flexible networks allocations for the different network configurations are given in Figure 3.2. We observe that $E(v) = \{\{12, 13\}\}$ satisfies internal stability, external stability and consistency, but equal bargaining power is now violated at all networks $g \neq \{12, 13\}$ included networks $\{12\}$ and $\{23\}$.

	\emptyset	$\{12\}$	$\{13\}$	$\{23\}$	$\{12, 13\}$	$\{13, 23\}, \{12, 23\}$ $\{12, 13, 23\}$
y_1	0	$-(1 + 3\varepsilon)/2$	$-(1 + 3\varepsilon)/2$	0	$1 - 2\varepsilon$	$-(2 + 6\varepsilon)/3$
y_2	0	$(1 + 3\varepsilon)/2$	0	2	$2 + \varepsilon$	$(1 + 3\varepsilon)/3$
y_3	0	0	$(1 + 3\varepsilon)/2$	2	$2 + \varepsilon$	$(1 + 3\varepsilon)/3$

Table 3.1: A farsighted allocation rule for value function v

¹⁷Notice that we do not impose that each player chooses her best alternative among her credible threats.

However, $E(v) = \{\{12, 13\}\}$ is a von Neumann-Morgenstern farsightedly stable set with bargaining if y is the allocation rule given in Table 3.1 with $1/2 > \varepsilon > 0$. It can be easily verified that internal stability and external stability are satisfied for $1/2 > \varepsilon > 0$, and that this allocation rule satisfies equal bargaining power for farsighted players and consistency at all networks. This allocation rule also satisfies component balance, a property that is not required by the concept of von Neumann-Morgenstern farsighted stability with bargaining. Component balance makes sense when the value function is component additive, and the value function of our example satisfies component additivity. Notice that the allocation rule given in Table 3.1 is not the unique one that may arise with $E(v) = \{\{12, 13\}\}$ when players are farsighted.

	\emptyset	$\{12\}$	$\{13\}$	$\{23\}$	$\{12, 13\}$	$\{13, 23\}, \{12, 23\}$ $\{12, 13, 23\}$
y_1	0	$-(1 + 4\varepsilon)/2$	$-(1 + 5\varepsilon)/2$	0	$1 - 2\varepsilon$	$-(2 + 9\varepsilon)/3$
y_2	0	$(1 + 4\varepsilon)/2$	0	$(4 - \varepsilon)/2$	$2 + \varepsilon$	$(1 + 3\varepsilon)/3$
y_3	0	0	$(1 + 5\varepsilon)/2$	$(4 + \varepsilon)/2$	$2 + 2\varepsilon$	$(1 + 6\varepsilon)/3$

Table 3.2: Another farsighted allocation rule for value function v

For instance, the set $E(v) = \{\{12, 13\}\}$ is also a von Neumann-Morgenstern farsightedly stable set with bargaining if y is the allocation rule given in Table 3.2 with $1/3 > \varepsilon > 0$. This allocation rule leads to a division of the value of the efficient network $\{12, 23\}$ where player 3 obtains a larger share than player 2 even though players 2 and 3 are symmetric in $\{12, 23\}$. \square

Proposition 23 *Take any bargaining threat z . If y satisfies component efficiency and equal bargaining power for farsighted players then y is such that*

$$y_i(g, v) = y_i(z_i(g), v) + \frac{1}{\#S} \left[v(S, g) - \sum_{j \in S} y_j(z_j(g), v) \right] \quad \forall i \in S, S \in \Pi(g).$$

Proof. See the appendix C. \blacksquare

Proposition 23 tells us that, if the allocation rule satisfies component efficiency and equal bargaining power for farsighted players then this allocation rule will lead to a division

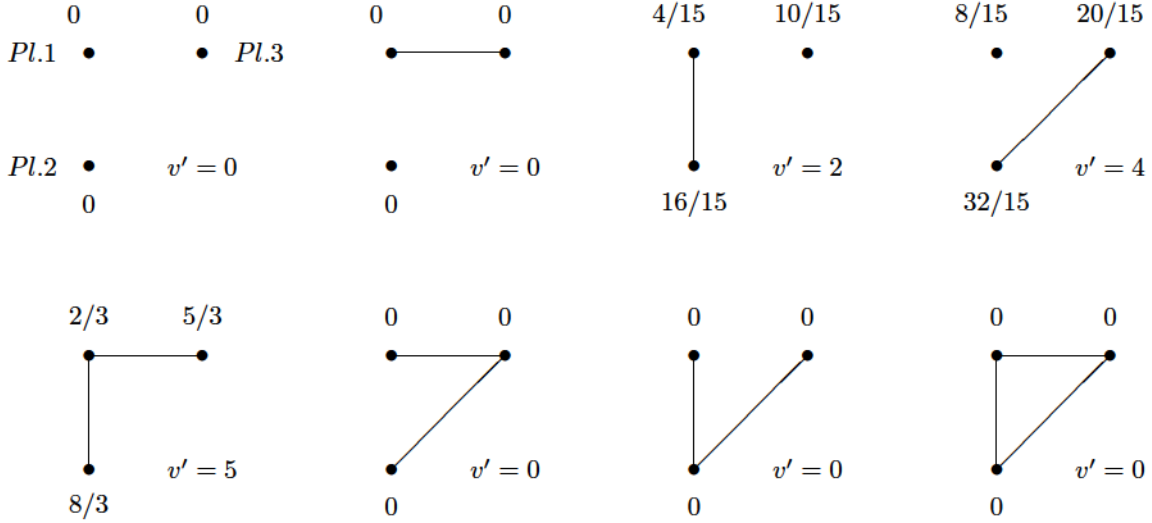


Figure 3.2: The player-based flexible network allocation rule.

of the value of the component where each player gets the payoff at her bargaining threat plus an equal share of the excess between the value and the sum of the threats.

We now turn to the existence of a von Neumann-Morgenstern farsightedly stable set with bargaining. Is the set of strongly efficient networks $E(v)$ always a von Neumann-Morgenstern farsightedly stable set with bargaining?

Proposition 24 *The set of strongly efficient networks $E(v)$ is a von Neumann-Morgenstern farsightedly stable set with bargaining if y is the egalitarian allocation rule.*

Proof.

[Internal stability] We have that $y_i(g, v) = y_j(g, v) = y_i(g', v) = y_j(g', v)$ for all $i, j \in N$, for all $g, g' \in E(v)$ since y is the egalitarian allocation rule. Hence, there is no farsighted improving path from any $g \in E(v)$ to another $g' \in E(v)$, and $E(v)$ satisfies internal stability. [External stability] In addition, $y_i(g, v) > y_i(g', v)$ for all $i \in N$, for all $g \in E(v)$ and $g' \notin E(v)$ since y is the egalitarian allocation rule and g is efficient. Hence, there is a farsighted improving from any $g' \notin E(v)$ to some $g \in E(v)$, and $E(v)$ satisfies external stability. [Equal bargaining power] Since $y_i(g, v) = y_j(g, v) = y_i(g', v) = y_j(g', v)$ for all $i, j \in N$, for all $g, g' \in E(v)$, and $y_i(g'', v) = y_j(g'', v)$ for all $i, j \in N$, for all $g'' \notin E(v)$, we have that equal bargaining power for farsighted players is satisfied for any z such that $z_i(g) \in E(v)$ for $g \in \mathbb{G}$. [Consistency] Since there is a farsighted improving

from any $g' \notin E(v)$ to some $g \in E(v)$, there exists some z such that for all $g \in \mathbb{G}$ and $ij \in g$ we have that $z_i(g) \in (F(g'') \cup \{g''\}) \cap E(v) \neq \emptyset$ for some $g'' \in A_i(g)$ and $z_j(g) \in (F(g''') \cup \{g'''\}) \cap E(v) \neq \emptyset$ for some $g''' \in A_j(g)$. ■

Proposition 24 shows that the egalitarian allocation rule guarantees the existence of a von Neumann-Morgenstern farsightedly stable set with bargaining. However, this allocation rule violates component efficiency and each player's allocation is independent of her position in the network.

Proposition 25 *Consider any anonymous, component additive and top convex value function v . If y is component efficient and anonymous then $E(v)$ is the unique von Neumann-Morgenstern farsightedly stable set with bargaining.*

Proof. See the appendix C. ■

3.4 Pairwise farsighted stability with bargaining

We now incorporate the property of equal bargaining power for farsighted players into the original definition of a pairwise farsightedly stable set due to Herings, Mauleon and Vannetelbosch (2009) [23]. Formally, pairwise farsighted stability with bargaining is defined as follows.

Definition 26 *A set of networks $G \subseteq \mathbb{G}$ is pairwise farsightedly stable with bargaining if there exists an allocation rule y and a bargaining threat z such that*

(i) $\forall g \in G,$

(a) $\forall ij \notin g$ such that $g + ij \notin G, \exists g' \in F(g + ij) \cap G$ such that $(y_i(g', v), y_j(g', v)) = (y_i(g, v), y_j(g, v))$ or $y_i(g', v) < y_i(g, v)$ or $y_j(g', v) < y_j(g, v),$

(b) $\forall ij \in g$ such that $g - ij \notin G, \exists g', g'' \in F(g - ij) \cap G$ such that $y_i(g', v) \leq y_i(g, v)$ and $y_j(g'', v) \leq y_j(g, v),$

(ii) $\forall g' \in \mathbb{G} \setminus G, F(g') \cap G \neq \emptyset.$

(iii) $\forall g \in \mathbb{G}$ and $ij \in g$,

(a) $y_i(g, v) - y_i(z_i(g), v) = y_j(g, v) - y_j(z_j(g), v)$,

(b) $z_i(g) \in (F(g'') \cup \{g''\}) \cap G \neq \emptyset$ for some $g'' \in A_i(g)$ and $z_j(g) \in (F(g''') \cup \{g'''\}) \cap G \neq \emptyset$ for some $g''' \in A_j(g)$.

(iv) $\nexists G' \subsetneq G$ such that G' satisfies Conditions (i), (ii), and (iii).

Condition (ia) in Definition 26 captures that adding a link ij to a network $g \in G$ that leads to a network outside of G , is deterred by the threat of ending in g' . Here g' is such that there is a farsightedly improving path from $g + ij$ to g' . Moreover, g' belongs to G , which makes g' a credible threat. Condition (ib) is a similar requirement, but then for the case where a link is severed. Condition (ii) in Definition 26 requires external stability and implies that the networks within the set are robust to perturbations. From any network outside G there is a farsightedly stable path leading to some network in G . Condition (iiia) in Definition 26 is the equal bargaining power property for farsighted players. Condition (iiib) in Definition 26 imposes a consistency requirement on the bargaining threat. Condition (iv) in Definition 26 is a minimality condition.

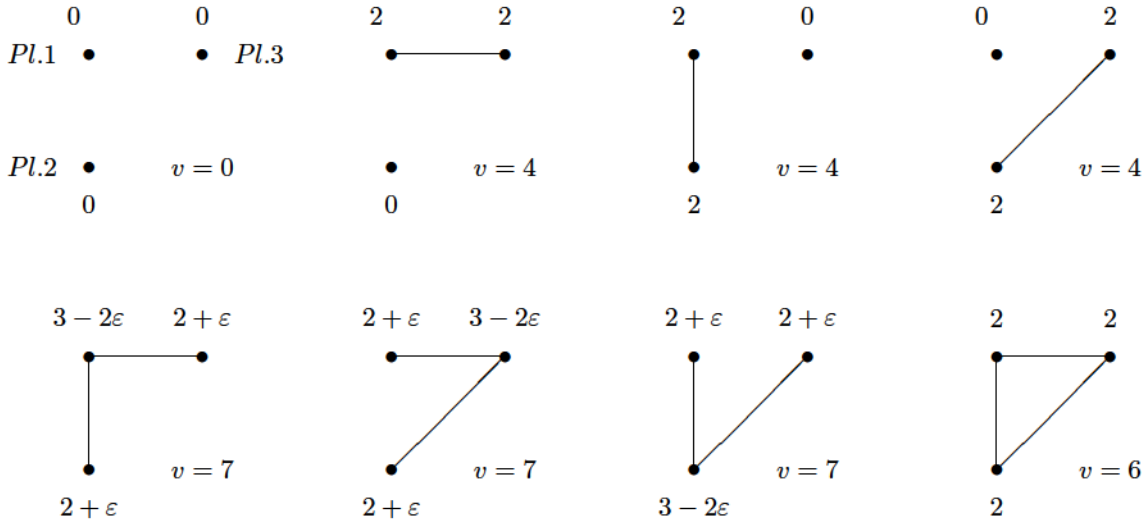


Figure 3.3: Top convexity and farsighted stability with bargaining.

Example 2 *Top convexity and farsighted stability with bargaining.* Take $N = \{1, 2, 3\}$ and the anonymous, component additive, and top convex value function defined by

$v(\{12, 13, 23\}) = 6$, $v(\{12, 13\}) = v(\{12, 23\}) = v(\{13, 23\}) = 7$, $v(\{12\}) = v(\{13\}) = v(\{23\}) = 4$, and $v(\emptyset) = 0$. We have depicted in Figure 3.3 the network configurations with their associated allocations.

First, we show that if y is the componentwise egalitarian allocation rule (i.e. $\varepsilon = 1/3$) then $E(v)$ is the unique von Neumann-Morgenstern farsightedly stable set with bargaining. We have $F(\emptyset) = \mathbb{G} \setminus \{\emptyset, \{12, 13, 23\}\}$; $F(\{13\}) = F(\{12\}) = F(\{23\}) = F(\{12, 13, 23\}) = \{\{12, 13\}, \{12, 23\}, \{13, 23\}\}$; and $F(\{12, 13\}) = F(\{12, 23\}) = F(\{13, 23\}) = \emptyset$. Then, $E(v)$ satisfies internal stability and external stability. The componentwise egalitarian allocation rule also satisfies equal bargaining power and consistency since there is a z such that $z_i(g) \in E(v)$ for $g \in \mathbb{G}$ and $F(g) \cap E(v) \neq \emptyset$ for all $g \notin E(v)$. Hence, $E(v)$ is a von Neumann-Morgenstern farsightedly stable set with bargaining. We now show that $E(v)$ is the unique von Neumann-Morgenstern farsightedly stable set with bargaining. Suppose that G is a von Neumann-Morgenstern farsightedly stable set with bargaining. We have that $E(v) \subseteq G$ since $F(g) = \emptyset$ for all $g \in E(v)$; otherwise, G would violate external stability. In addition, if $E(v) \subsetneq G$ then internal stability is violated because $F(g) \cap E(v) \neq \emptyset$ for all $g \notin E(v)$. Thus, $E(v) = G$.

Second, is $E(v)$ a von Neumann-Morgenstern farsightedly stable set with bargaining if the anonymous and component efficient allocation rule is such that $\varepsilon \neq 1/3$ ($0 < \varepsilon < 1/3$)? Then, equal bargaining power and consistency can still be satisfied as well as external stability but internal stability is violated since now $g \in F(g')$, for any $g, g' \in E(v)$ ($g \neq g'$). Hence, once the allocation rule is determined jointly with the farsighted stability of the network and the value function is anonymous, component additive and top convex, the set of strongly efficient networks is a von Neumann-Morgenstern farsightedly stable set with bargaining only if the sharing of the value follows the componentwise egalitarian allocation rule. \square

The next example is given to give further insights to the reader on the unicity of the stable set.

Example 2 (continued). If y is anonymous then candidate allocations to support a von Neumann-Morgenstern farsightedly stable set with bargaining are given in Figure 3.3.

For $\varepsilon < 0$, then $\{\{12, 13, 23\}\}$ is the unique set to satisfy internal stability and external stability. But, the allocations for $\{ij, ik\}$ violate equal bargaining power because of the consistency requirement. For $\varepsilon = 0$, then $\{\{ij, ik\}, \{12, 13, 23\}\}$ are the sets to satisfy internal stability and external stability. But, the allocations for $\{ij, jk\}$ and $\{ik, jk\}$ violate equal bargaining power because of the consistency requirement. For $0 < \varepsilon < 1/3$ and $1/3 < \varepsilon \leq 1/2$, then $\{\{ij, ik\}\}$ are the sets to satisfy internal stability and external stability. But, the allocations for $\{ij\}$, $\{ik\}$, $\{ij, jk\}$, $\{ik, jk\}$, $\{12, 13, 23\}$ violate equal bargaining power because of the consistency requirement. For $1/2 < \varepsilon$, then $\{\{ij\}, \{12, 13, 23\}\}$ are the sets to satisfy internal stability and external stability. But, the allocations for $\{ij, jk\}$, $\{ij, ik\}$ violate equal bargaining power because of the consistency requirement. For $1/3 = \varepsilon$, the allocation rule reverts to the componentwise egalitarian allocation rule and $E(v) = \{\{12, 13\}, \{12, 23\}, \{13, 23\}\}$ is a von Neumann-Morgenstern farsightedly stable set with bargaining. Hence, $E(v)$ is the unique von Neumann-Morgenstern farsightedly stable set with bargaining and the componentwise egalitarian allocation rule emerges endogenously. \square

Proposition 27 *Consider any anonymous, component additive and top convex value function v . If y is componentwise egalitarian, then $E(v)$ is the unique von Neumann-Morgenstern stable set with bargaining and coincides with the unique pairwise farsightedly farsightedly stable set with bargaining.*

Proof. From Grandjean Mauleon and Vannetelbosch (2011) [21], If the allocation rule is exogenously given and is the componentwise egalitarian allocation rule, then the set of strongly efficient networks is the unique pairwise farsightedly stable set if and only if the value function is top convex. From Herings et al. (2009) [23], If a set G is the unique pairwise farsighted stable set, then it is also the unique von Neumann-Morgenstern farsighted stable set of networks. We only need to show that conditions (iiia) and (iiib) of the von Neumann-Morgenstern farsighted stable set with bargaining and the pairwise farsighted stable set with bargaining are verified for a component additive and top convex value function and the componentwise egalitarian allocation rule. As noticed by Jackson and van den Nouweland (2005) [31], under top convexity, the componentwise egalitarian

allocation rule gives an identical payoff to all players in the efficient networks. The threats are the payoffs the players get in a network of the stable set, here, at a strong efficient network. Thus for any bargaining pair and any network, the equal bargaining power property is verified.

■

Proposition 28 *If G is a von Neumann-Morgenstern farsightedly stable set with bargaining, then G is a pairwise farsightedly stable set with bargaining.*

Proof. Suppose G is a von Neumann-Morgenstern farsightedly stable set with bargaining. Then, conditions (ii) and (iii) in Definition 26 are trivially satisfied for G .

Suppose Condition (i) in Definition 26 is not satisfied. Then there is $g \in G$ and a deviation to $g' \notin G$ such that every $g'' \in F(g') \cap G$ defeats g .¹⁸ In particular, it then follows that $g'' \in F(g)$, a contradiction, since by condition (i) in Definition 22 there is no $g'' \in G$ with that property. Consequently, Condition (i) in Definition 26 holds.

To verify condition (iv) in Definition 26, suppose there is a proper subset $G' \subsetneq G$ that satisfies conditions (i), (ii) and (iii). Let g be in G but not in G' . Then, $F(g) \cap G' \subseteq F(g) \cap G = \emptyset$ since G satisfies condition (i) in Definition 22. It follows that $G' \subsetneq G$ violates condition (ii) in Definition 26, leading to a contradiction. We have shown that G is minimal. ■

From Proposition 28 we have that, in Example 2, $E(v)$ is a pairwise farsightedly stable set with bargaining where the allocation rule is the componentwise egalitarian allocation rule¹⁹. However, the set of strongly efficient networks $E(v)$ is a pairwise farsightedly stable set with bargaining even for anonymous allocation rules where the value of each component is not shared equally among the members of the component.

Example 2 (continued). If y is anonymous then candidate allocations to support a von Neumann-Morgenstern farsightedly stable set with bargaining are given in Figure 3.3.

¹⁸A network g' *defeats* g if either $g' = g - ij$ and $y_i(g', v) > y_i(g, v)$ or $y_j(g', v) > y_j(g, v)$, or if $g' = g + ij$ with $y_i(g', v) \geq y_i(g, v)$ and $y_j(g', v) \geq y_j(g, v)$ with at least one inequality holding strictly.

¹⁹Grandjean, Mauleon and Vannetelbosch (2011) [21] have shown that, if the allocation rule is exogenously given and is the componentwise egalitarian allocation rule, then the set of strongly efficient networks is the unique pairwise farsightedly stable set if and only if the value function is top convex.

For $0 < \varepsilon \leq 1/2$, the set $E(v) = \{\{12, 13\}, \{12, 23\}, \{13, 23\}\}$ is a pairwise farsightedly stable set with bargaining. External stability is satisfied. Notice that pairwise farsighted stability with bargaining does not require internal stability. Equal bargaining power for farsighted players and consistency are satisfied. For instance, in $\{12\}$ the bargaining threat $z_1(\{12\})$ and $z_2(\{12\})$ can be respectively $\{12, 13\}$ and $\{12, 23\}$ (or simply $\{13, 23\}$ for both players). Equal bargaining power is satisfied since $y_1(\{12\}, v) - y_1(\{12, 13\}, v) = 2 - 3 + 2\varepsilon = y_2(\{12\}, v) - y_2(\{12, 23\}, v)$ (or $y_1(\{12\}, v) - y_1(\{13, 23\}, v) = 2 - 2 - \varepsilon = y_2(\{12\}, v) - y_2(\{13, 23\}, v)$) and consistency is satisfied since $z_1(\{12\}), z_2(\{12\}) \in E(v)$ and $z_1(\{12\}), z_2(\{12\}) \in F(\emptyset)$. $E(v)$ is minimal. Any subset of $E(v)$ would violate equal bargaining power for $\varepsilon \neq 1/3$. Take $\{\{12, 13\}, \{12, 23\}\} \subset E(v)$. Then, equal bargaining power is violated at $\{13, 23\}$ because player 3 obtains an allocation smaller or equal than the allocations of players 1 and 2 at $\{12, 13\}$ and $\{12, 23\}$. \square

Proof. See the appendix C. \blacksquare

3.5 Extensions

3.5.1 More on the role of top convexity

We now look at an example where the value function does not satisfy top convexity. We observe that von Neumann-Morgenstern farsighted stability with bargaining is less likely to sustain efficient networks than pairwise farsighted stability with bargaining.

Example 3 *Value function not top convex.* Take $N = \{1, 2, 3\}$ and the anonymous, component additive, and not top convex value function defined by $v(\{12, 13, 23\}) = 1$, $v(\{12, 13\}) = v(\{12, 23\}) = v(\{13, 23\}) = 4/3$, $v(\{12\}) = v(\{13\}) = v(\{23\}) = 1$, and $v(\emptyset) = 0$. We have depicted in Figure 3.4 the network configurations with anonymous and component efficient allocation rules.

a) $0 \leq \varepsilon < 1/36$. There is no von Neumann-Morgenstern farsightedly stable set with bargaining. The set $\{\{12\}, \{13\}, \{23\}\}$ is a candidate but it violates internal stability. In addition, each set $\{\{ij\}\}$ violates equal bargaining power for farsighted players at, for instance, networks $\{ik\}$, $\{jk\}$ and $\{12, 13, 23\}$. But, $\{\{12\}, \{13\}, \{23\}\}$ is a pairwise

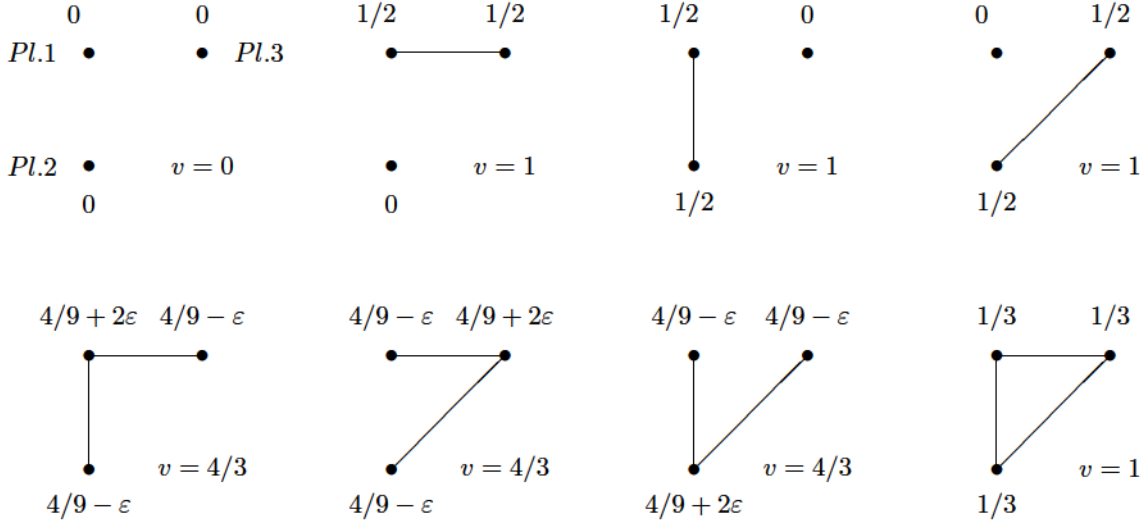


Figure 3.4: Value function not top convex and farsighted stability with bargaining.

farsightedly stable set with bargaining only if $\varepsilon = 0$. It can be easily verified that all conditions are satisfied. It is minimal since any nonempty subset $G \subsetneq \{\{12\}, \{13\}, \{23\}\}$ would satisfy all conditions except that equal bargaining power would be violated at $g \in \{\{12\}, \{13\}, \{23\}\} \setminus G$.

b) $1/36 \leq \varepsilon < 1/9$. The three sets $\{\{12, 13\}, \{12, 23\}\}$, $\{\{12, 13\}, \{13, 23\}\}$ and $\{\{12, 23\}, \{13, 23\}\}$ are pairwise farsightedly stable sets with bargaining. External stability, equal bargaining power, consistency and minimality are satisfied. But, they are not von Neumann-Morgenstern farsightedly stable sets with bargaining because internal stability is violated.

c) $1/9 \leq \varepsilon < 4/9$. There is a von Neumann-Morgenstern farsightedly stable set with bargaining only if $\varepsilon = 1/6$. If $\varepsilon = 1/6$, then $G' = \{\{12\}, \{13\}, \{23\}, \{12, 13, 23\}\}$ is a von Neumann-Morgenstern farsightedly stable set with bargaining. This set G' satisfies internal and external stability, equal bargaining power and consistency. Equal bargaining power requires that at $\{12, 13\}$ we have $y_1(\{12\}, v) - y_1(\{12, 13\}, v) = y_3(\{12\}, v) - y_3(\{12, 13\}, v)$. Since $y_1(\{12\}, v) - y_1(\{12, 13\}, v) = 1/2 - 4/9 - 2\varepsilon$ and $y_3(\{12\}, v) - y_3(\{12, 13\}, v) = 0 - 4/9 + \varepsilon$, equal bargaining power holds only if $\varepsilon = 1/6$. Obviously, G' with $\varepsilon = 1/6$ is also a pairwise farsightedly stable set with bargaining. However, G' is not the unique one. The set $E(v) \cup \{12, 13, 23\}$ is a pairwise farsightedly stable set with

bargaining for $1/36 \leq \varepsilon < 4/9$.

Hence, and contrary to pairwise farsighted stability with bargaining, von Neumann-Morgenstern farsighted stability with bargaining only leads to the emergence of inefficient networks. \square

3.5.2 More on the role of equal bargaining power

We now reconsider Example 2 to show that if the allocation rule y does not satisfy anonymity and/or equal bargaining power, then the componentwise egalitarian allocation does not emerge in the long run. Given a vector $w = (w_1, \dots, w_n) \gg 0$, an allocation rule y satisfies w -weighted bargaining power²⁰ for farsighted players if for all $v \in \mathcal{V}$, for all $g \in \mathbb{G}$, for all $ij \in g$,

$$\frac{1}{w_i} [y_i(g, v) - y_i(z_i(g), v)] = \frac{1}{w_j} [y_j(g, v) - y_j(z_j(g), v)]. \quad (\text{iiia}')$$

Consider the definition of von Neumann-Morgenstern farsighted stability with bargaining where the equal bargaining power condition (iiia) is replaced by the w -weighted bargaining power condition (iiia'). Suppose that (w_1, w_2, w_3) is such that $1 = w_2 = w_3 \leq w_1 = \gamma$. For $\gamma > 1$ the set $E(v)$ is still a von Neumann-Morgenstern farsightedly stable set with bargaining where players share equally the value for each network $g \in E(v)$ while they share unequally the value for each nonempty network $g \notin E(v)$ (the allocations for $\gamma = 2$ are given in Table 3.3). Since each player obtains the same allocation in each efficient star network, the property of weighted bargaining power forces the players to agree on asymmetric allocations at symmetric networks. It can be easily verified that internal stability, external stability and consistency are satisfied too.

	\emptyset	{12}	{13}	{23}	{12, 13}	{12, 23}	{13, 23}	{12, 13, 23}
y_1	0	17/9	17/9	0	7/3	7/3	7/3	22/12
y_2	0	19/9	0	2	7/3	7/3	7/3	25/12
y_3	0	0	19/9	2	7/3	7/3	7/3	25/12

Table 3.3: Allocations satisfying w -weighted bargaining power for $\gamma = 2$

²⁰Such an allocation rule is called w -fairness in Dutta and Mutuswami (1997) [18].

Hence, the property of equal bargaining power for farsighted players is a tight condition for having the componentwise egalitarian allocation rule arising endogenously.

3.5.3 More on the role of component additivity

Component additivity (or no externalities across components) means that the value of a component of the network does not depend on the structure of the network outside the component. We now look at situations where externalities across components can arise and we provide an alternative condition to top convexity such that the set of efficient networks remains the unique von Neumann-Morgenstern farsightedly stable set with bargaining. A value function v is *link monotonic* (Navarro, 2013b) if for any $S \in \Pi(g)$ and any $ij \in g|_S$ we have

(i) $v(S, g) > v(S', g - ij) + v(S'', g - ij)$ if $S' \in \Pi(g - ij)$ with $i \in S'$, $S'' \in \Pi(g - ij)$ with $j \in S''$, and $S' \cap S'' = \emptyset$;

(ii) $v(S, g) > v(S, g - ij)$ if $S \in \Pi(g - ij)$.

A value function v satisfies *strong critical-link monotonicity* (Navarro, 2013b) if v is link monotonic and if for any g , any $S \in \Pi(g)$ and any $ij \in g|_S$ such that $\#\Pi(g) = \#\Pi(g - ij) - 1$ we have $v(S, g)/\#S > \max\{v(S', g - ij)/\#S', v(S'', g - ij)/\#S''\}$ where $S' \in \Pi(g - ij)$ with $i \in S'$ and $S'' \in \Pi(g - ij)$ with $j \in S''$. The link ij is said to be critical. That is, if it is severed, then the component that it was a part of will become two components (or one of the nodes will become disconnected). Strong critical-link monotonicity imposes that if we add a link to the network such that two components become connected then the per-capita value of the new component is greater than the per-capita value of any of the two component before adding the link.

Proposition 29 *Consider any value function that satisfies anonymity and strong critical-link monotonicity. Suppose that y is component efficient and anonymous. The set $\{g^N\}$ is the unique von Neumann-Morgenstern farsightedly stable set with bargaining if and only if y is the componentwise egalitarian allocation rule.*

Proof. Straightforward from the definition of strong critical-link monotonicity. If y is the componentwise egalitarian allocation rule, then adding any link to any network increases the payoffs of the players within the considered component. ■

Thus, if the value function satisfies anonymity and strong critical-link monotonicity and the allocation rule satisfies component efficient and anonymous, then the strongly efficient network is likely to emerge in the long run together with the componentwise egalitarian allocation rule when players are farsighted. In addition, Navarro (2013) [44] has shown that there exists a forward-looking network formation process consisting of an allocation rule and a transition probability matrix such that the allocation rule is component efficient and the complete network is the only absorbing state of the transition probability matrix for any strictly positive common discount factor.

3.6 Conclusions

We have studied the stability of social and economic networks when farsighted players simultaneously form links and bargain over allocations. In particular, we have shown that the set of strongly efficient networks is the unique von Neumann-Morgenstern farsightedly stable set with bargaining if the allocation rule is anonymous and component efficient and the value function is top convex. In addition, the componentwise egalitarian allocation rule emerges endogenously.

3.7 Appendix C

3.7.1 Proof of Proposition 23.

Take any path $\{i_1, i_2, \dots, i_k, i_{k+1}, \dots, i_l\}$ in g between player i and player j with $i_1 = i$ and $i_l = j$. Applying the equal bargaining power property for farsighted players, we have that

$$\begin{aligned} y_{i_1}(g, v) &= y_{i_1}(z_{i_1}(g), v) - y_{i_2}(z_{i_2}(g), v) + y_{i_2}(g, v) \\ &= \Delta_{i_1 i_2}(z_{i_1}(g), z_{i_2}(g), v) + y_{i_2}(g, v), \end{aligned}$$

where $\Delta_{i_1 i_2}(z_{i_1}(g), z_{i_2}(g), v)$ is the difference between player i_1 's allocation at her bargaining threat and player i_2 's allocation at his bargaining threat. Applying the equal bargaining power property for farsighted players, we have also that

$$\begin{aligned} y_{i_2}(g, v) &= y_{i_2}(z_{i_2}(g), v) - y_{i_3}(z_{i_3}(g), v) + y_{i_3}(g, v) \\ &= \Delta_{i_2 i_3}(z_{i_2}(g), z_{i_3}(g), v) + y_{i_3}(g, v). \end{aligned}$$

Hence,

$$y_{i_1}(g, v) = \Delta_{i_1 i_2}(z_{i_1}(g), z_{i_2}(g), v) + \Delta_{i_2 i_3}(z_{i_2}(g), z_{i_3}(g), v) + y_{i_3}(g, v).$$

Notice that $\Delta_{i_k i_{k+1}}(z_{i_k}(g), z_{i_{k+1}}(g), v) = -\Delta_{i_{k+1} i_k}(z_{i_{k+1}}(g), z_{i_k}(g), v)$. Iterating this reasoning, we obtain

$$\begin{aligned} y_{i_1}(g, v) &= \sum_{k=1}^{k=l-1} \Delta_{i_k i_{k+1}}(z_{i_k}(g), z_{i_{k+1}}(g), v) + y_{i_l}(g, v) \\ &= y_{i_1}(z_{i_1}(g), v) - y_{i_l}(z_{i_l}(g), v) + y_{i_l}(g, v). \end{aligned}$$

Hence,

$$y_i(g, v) = y_i(z_i(g), v) - y_j(z_j(g), v) + y_j(g, v).$$

Summing up for all $j \in S$ ($j \neq i$) such that $S \in \Pi(g)$ and $i \in S$, we have

$$(\#S - 1) y_i(g, v) = (\#S - 1) y_i(z_i(g), v) + \sum_{j \in S \in \Pi(g), j \neq i} (y_j(g, v) - y_j(z_j(g), v)).$$

Since y satisfies the component efficiency property, i.e. $\sum_{j \in S \in \Pi(g)} y_j(g, v) = v(S, g)$, we have that

$$(\#S) y_i(g, v) = (\#S - 1) y_i(z_i(g), v) + v(S, g) - \sum_{j \in S \in \Pi(g), j \neq i} y_j(z_j(g), v).$$

Hence,

$$y_i(g, v) = y_i(z_i(g), v) + \frac{1}{\#S} \left(v(S, g) - \sum_{j \in S \in \Pi(g)} y_j(z_j(g), v) \right).$$

□

3.7.2 Proof of Proposition 25.

Suppose that y is anonymous and component efficient and v is anonymous, component additive and top convex. We will show that there is no von Neumann-Morgenstern farsightedly stable set with bargaining $G \neq E(v)$.

(i) Take any G such that $G \cap E(v) = \emptyset$ and G is von Neumann-Morgenstern farsightedly stable set with bargaining.

(ia) Suppose that $G = \{g\}$. Anonymity of v and y imply equal sharing of the value of the complete network. Hence, if g is the complete network, then equal bargaining power and consistency imply that, in any $g' \neq g$, members of each component share equally the value of each component. If g is not the complete network, then equal bargaining power and consistency imply that members of each component of g share equally the value of the component. Then, top convexity of the value function implies that $g \notin F(g'')$ for all $g'' \in E(v)$ (any $g'' \in E(v)$ Pareto dominates all $g' \notin E(v)$). Hence, G fails to satisfy external stability and we have a contradiction.

(ib) Suppose that $\#G > 1$. Internal stability for G implies that players obtain the same allocation in any $g \in G$. Then, equal bargaining power and consistency imply that, in any $g' \notin G$, members of each component share equally the value of each component. Furthermore, top convexity of the value function implies that $g \notin F(g'')$ for all $g'' \in E(v)$ and $g \in G$. Hence, G fails to satisfy external stability and we have a contradiction.

(ii) Take G such that $G \cap E(v) \neq \emptyset$, $G \neq E(v)$ and G is von Neumann-Morgenstern farsightedly stable set with bargaining.

(iia) Suppose that $G = \{g\}$. Notice that g is a strongly efficient network, $g \in E(v)$. Anonymity of v and y imply equal sharing of the value of the complete network. Hence, if g is the complete network, then equal bargaining power and consistency imply that, in any

$g' \neq g$, members of each component share equally the value of each component. If g is not the complete network, then equal bargaining power and consistency imply that members of each component of g share equally the value of the component. As pointed out by Jackson and van den Nouweland (2005) [31], top convexity implies that all components of a strongly efficient network must lead to the same per-capita value (if some component led to a lower per capita value than the average, then another component would have to lead to a higher per capita value than the average which would contradict top convexity). Then, since $G = \{g\} \subsetneq E(v)$, there is $g'' \in E(v)$, $g'' \neq g$, such that $g \notin F(g'')$. Hence, G fails to satisfy external stability and we have a contradiction.

(iib) Suppose that $\#G > 1$. First, consider the case $G \supsetneq E(v)$. Internal stability for G implies that players obtain the same allocation in any $g, g' \in G$, but this is not possible since by top convexity $g \in G \cap E(v)$ Pareto dominates $g' \in G \setminus E(v)$. Hence, G fails to satisfy internal stability and we have a contradiction. Second, consider the case $G \subsetneq E(v)$. Internal stability for G implies that players obtain the same allocation in any $g \in G$, and is satisfied since v is top convex. Then, equal bargaining power and consistency imply that, in any $g' \notin G$, members of each component share equally the value of each component. But there is $g' \notin G$ such that $g' \in E(v)$. Top convexity of the value function implies that there is no $g \in G$ such that $g \in F(g')$ for any $g' \notin G$, $g' \in E(v)$. Hence, G fails to satisfy external stability and we have a contradiction. Third, consider the case $G \not\supseteq E(v)$ and $G \not\subseteq E(v)$. Similar arguments lead to a contradiction.

Thus, if y is anonymous and component efficient and v is anonymous, component additive and top convex, then $E(v)$ is the unique von Neumann-Morgenstern farsighted stable set with bargaining and the endogenously determined allocation rule is the componentwise egalitarian allocation rule. \square

Conclusion

This thesis deals with various aspects of non cooperative game theory but the field of the study is at the intersection of bilateral bargaining, endogenous network formation and farsightedness.

With the idea that a fully specified bargaining procedure that implements the efficient outcome can be found, the second chapter examines few simple candidate procedures. There is still a lot to be learned about the procedures that could or should be implemented in the context sequential bargaining. However, even simplistic bargaining procedures don't necessary match the efficient outcome, and there can't be a consensus on the bargaining protocols. Although this remark could seem negative, it doesn't conclude to its infeasibility and a mechanism that keeps track of the ordering of sequential bilateral negotiations that leads to the efficient outcome can still be found.

The second chapter provides results on the stability of networks of manufacturers and retailers given an adjustable level of depth of reasoning. I emphasize that a rather limited projection is sufficient to reach the highest level of anticipation and that the usual definitions of optimality or efficiency doesn't fit well the level- K solution concept. It is obvious that the level- K solution concept should be studied in different settings to shed new light on usual results regarding economic theory. However, alternative definitions of optimality or efficiency that fit a set-based solution concept should also be of great interest for future research.

The set-based definition of the von Neumann-Morgenstern farsightedly stable set allow the emmergence of an endogenous allocation. Based on standard assumptions on bargaining schemes, the stable sets and the allocation rule are simultaneously determined.

Although further research led to the conclusion that the solution concept is not good at dealing with various forms of externalities, the von Neumann-Morgenstern farsighted stability solution concept remains of interest in the contexts that fits the assumptions.

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**Title: Bilateral Bargaining and Farsightedness in Networks:
Essays in Economic Theory**

Abstract

The thesis consists in four essays that deal with bargaining and networks in non cooperative game theory. The first chapter introduces river bargaining games in the context of externalities. *The seawall bargaining game* deals with a non cooperative approach of an investment game in a context of positive externalities. The main result shows that the positioning of the agents impacts their incentives to sit at the bargaining table, leading to a chicken game. An intermediary player should lead the negotiations to improve the societal welfare. In the *River bargaining problem*, a non cooperative bargaining on a flowing resource in the presence of negative externalities. Results show that depending on the instigator of the bargaining sequences but there are analogies between solutions under the ATS and the UTI principles. The second chapter deals with the formation of networks of manufacturers and retailers in the presence of negative externalities when players are level- K farsighted. The results show that, (i) a relatively low level of farsightedness is sufficient to reach the infinite level of farsightedness; (ii) usual definitions of optimality or efficiency find limitations when it comes to be confronted to a set-based definition of stability. (iii) If there is transitive correspondence between the pairwise farsighted stable set and the level- ∞ farsighted stable set, then this set is likely to be strongly efficient. In *Allocating value among farsighted players in network formation*, we propose the concept of a von Neumann-Morgenstern farsighted stable set with bargaining. Under this solution concept, the stable networks so as the componentwise egalitarian allocation rule emerge endogenously. This chapter provides necessary conditions under which a von Neumann-Morgenstern farsighted stable set with bargaining sustains the strongly efficient networks.

Keywords: Bargaining, Networks, Farsightedness

**Titre: Négociations bilatérales et Clairvoyance dans les Réseaux:
Essais en Théorie Economique.**

Résumé

Cette thèse consiste en quatre essais qui traitent de négociation et de réseaux en théorie des jeux non coopérative. Le premier chapitre présente des jeux de négociations dans un contexte d'externalités. *Le jeu de négociation sur la digue* traite d'une approche non coopérative d'un jeu d'investissement dans un contexte d'externalités positives. Les incitations à prendre part aux négociations se synthétisent en un "jeu de la poule mouillée". Les résultats montrent qu'il est socialement plus efficace qu'un joueur intermédiaire mène les négociations. *Le problème de négociation sur la rivière* est un jeu de négociation non coopératif sur l'utilisation de la ressource fluviale en présence d'externalités négatives. Il existe des analogies entre les solutions obtenues dans les cas de l'ATS et de l'UTI. Le deuxième chapitre traite de la formation de réseaux de producteurs et de détaillants en présence d'externalités négatives lorsque les joueurs sont clairvoyants de degré- K . Les résultats montrent que (i) un degré de clairvoyance relativement faible est suffisant pour atteindre la clairvoyance absolue ou infinie; (ii) les définitions habituelles de l'optimum ou de l'efficacité ne conviennent pas parfaitement à un concept de stabilité ensembliste. (iii) S'il existe une correspondance transitive entre la stabilité clairvoyante par paires et la stabilité clairvoyante de degré infini, alors l'ensemble stable peut être efficace. Dans *Attribution de la valeur entre joueurs clairvoyants dans le processus de formation de réseau*. Il s'agit d'un chapitre théorique qui propose le concept de stabilité von Neumann-Morgenstern avec négociation. Dans ce concept de solution, les ensembles de réseaux stables, ainsi qu'une répartition égalitaire au sein des composants du réseau sont déterminés conjointement, et de manière endogène. Ce dernier chapitre met en évidence les conditions nécessaires pour que les réseaux von Neumann-Morgenstern avec négociation soient efficaces.

Mots-clés: Négociation, réseaux, Clairvoyance

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