

**Enseignement et apprentissage des mathématiques
en anglais langue seconde**

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ANNEXES

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ANNEXE 1 : Situation des *Nombres Triangulaires*

Liste des documents

Document 1 : Différence de deux carrés

Document distribué aux élèves à l'issue d'une séance située en amont de la séquence sur les nombres figurés. Ce document concerne les manipulations spatio-visuelles portant sur des configurations géométriques.

Document 2

Transcription du document Powerpoint utilisé en classe lors de la séance 1 de la séquence sur les nombres figurés.

Document 3

Document sur les nombres polygonaux (commenté et distribué aux élèves)

Document 4

Lexique joint au document précédent et distribué aux élèves.

Document 5

Transcription du document Powerpoint utilisé en classe lors de la Séance 1 bis de la séquence sur les nombres figurés. Il s'agit de la séance précédent celle relative à la Situation des Nombres Triangulaires proprement dite.

Document 6

Transcriptions de la Séance 1 bis et commentaires

Document 7 (fourni aux élèves)

Lexique constitué des termes et expressions rencontrées à l'occasion de la séance 1 bis (précédent celle relative à la Situation des Nombres Triangulaires).

Document 8

Consignes données lors de la Situation des Nombres Triangulaires (Séance 2).

Document 9

Test donné aux élèves de Première européenne à l'issue de la séquence des Nombres Triangulaires

Posters

Posters réalisés en phase adidactique par la classe de première européenne

Document 1

Difference of two squares

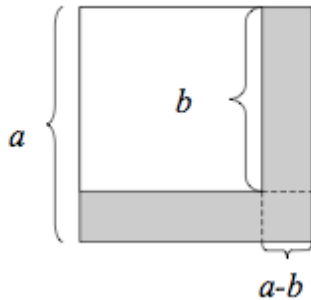
The difference of two squares can also be illustrated geometrically as the difference of two square areas in a [plane](#).

In the diagram, the shaded part represents the difference between the areas of the two squares, i.e. $a^2 - b^2$.

The area of the shaded part can be found by adding the areas of the two rectangles:

$$a(a - b) + b(a - b), \text{ which can be factorized to } (a + b)(a - b).$$

$$\text{Therefore } a^2 - b^2 = (a + b)(a - b)$$



Another geometric proof proceeds as follows:

We start with the figure shown in the first diagram below, a large square with a smaller square removed from it. The side of the entire square is a , and the side of the small removed square is b . The area of the shaded region is $a^2 - b^2$.

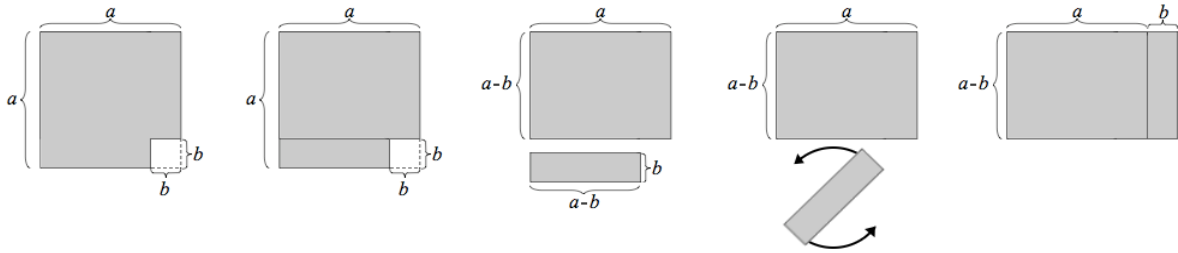
A cut is made, splitting the region into two rectangular pieces, as shown in the second diagram.

The larger piece, at the top, has width a and height $a-b$. The smaller piece, at the bottom, has width $a-b$ and height b . Now the smaller piece can be detached, rotated, and placed to the right of the larger piece.

In this new arrangement, shown in the last diagram below, the two pieces together form a rectangle, whose width is $a + b$ and whose height is $a - b$.

This rectangle's area is $(a + b)(a - b)$. Since this rectangle came from rearranging the original figure, it must have the same area as the original figure.

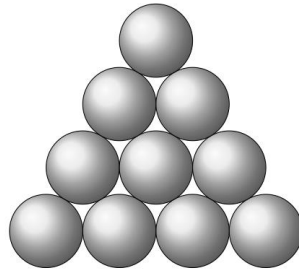
Therefore, $a^2 - b^2 = (a + b)(a - b)$.



Document 2 (transcription du document Powerpoint utilisé en classe lors de la Séance 1)

Diapositive n°1

Triangular number



Diapositive n°2

Numbers for the Ancient Greeks

A breakthrough in mathematical understanding occurred when mathematicians realized that, in addition to being useful as tools for calculation, numbers are also interesting objects of study in their own right.

Diapositive n°3

Some of the first people to study numbers as objects were the Pythagoreans, who were obsessed with the mystical properties of numbers. One of the most important properties to the Pythagoreans was a number's shape.

Diapositive n°4

Figurate number

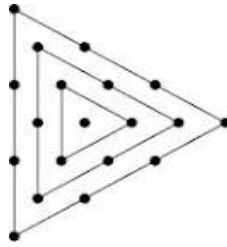
A figurate number is a group of dots.

dot : point

figurate : figuré

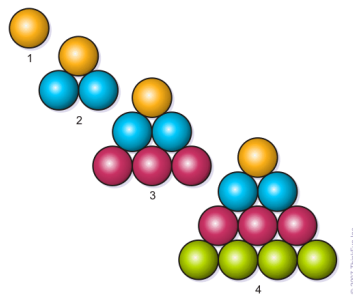
Diapositive n°5

Centered triangular number



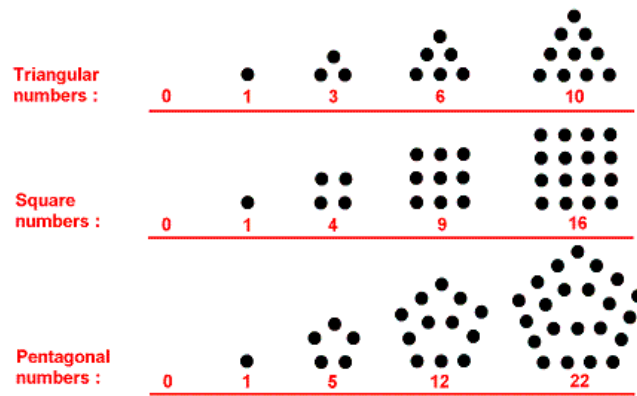
Diapositive n°6

Triangular numbers



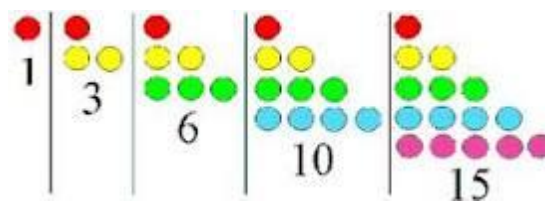
Diapositive n°7

Polygonal numbers



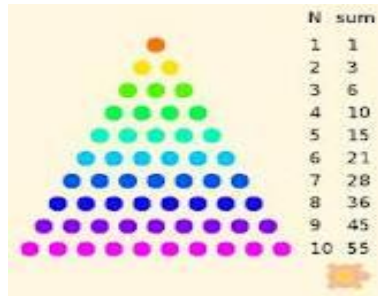
Diapositive n°8

other arrangements



Diapositive n°9

several triangular numbers simultaneously



Triangular number

Numbers for the Ancient Greeks

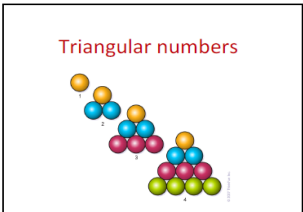
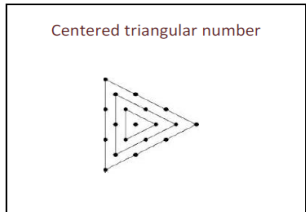
A breakthrough in mathematical understanding occurred when mathematicians realized that, in addition to being useful as tools for calculation, numbers are also interesting objects of study in their own right.

Some of the first people to study numbers as objects were the Pythagoreans, who were obsessed with the mystical properties of numbers. One of the most important properties to the Pythagoreans was a number's shape.

Figurate number

A figurate number is a group of dots.

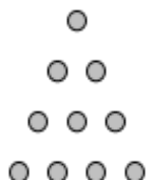
dot : point
figurate : figuré



In mathematics, a **polygonal number** is a number represented as dots or pebbles arranged in the shape of a regular polygon. The dots were thought of as alphas (units). These are one type of 2-dimensional figurate numbers.



The number 10, for example, can be arranged as a triangle :



But 10 cannot be arranged as a square.

The number 9, on the other hand, can be:



Some numbers, like 36, can be arranged both as a square and as a triangle (square triangular number):



By convention, 1 is the first polygonal number for any number of sides.

The rule for enlarging the polygon to the next size consists in

extending two adjacent arms by one point

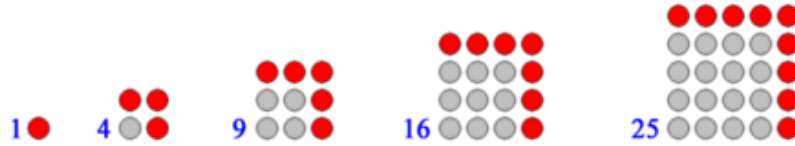
and in then adding the required extra sides between those points.

In the following diagrams, each extra layer is shown as in red:

Triangular numbers

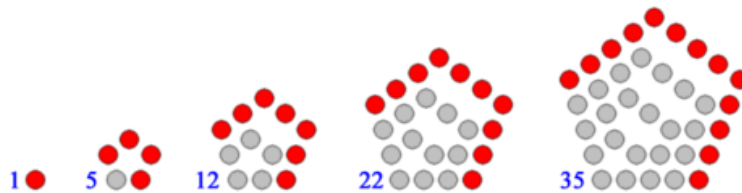


Square numbers

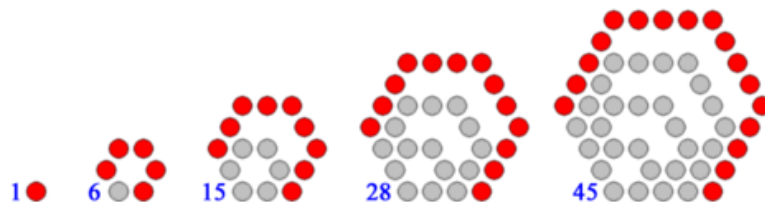


Polygons with higher numbers of sides, such as pentagons and hexagons, can also be constructed according to this rule, although the dots will no longer form a perfectly regular lattice like above.

Pentagonal numbers



Hexagonal numbers



A breakthrough in mathematical understanding occurred when mathematicians realized that, in addition to being useful as tools for calculation, numbers are also interesting objects of study in their own right.

Some of the first people to study numbers as objects were the Pythagoreans, who were obsessed with the mystical properties of numbers.

One of the most important properties to the Pythagoreans was a number's shape.

Document 4 (fourni aux élèves)

to enlarge : agrandir, élargir

pebble : caillou, galet

required : requis, exigé

extra layer : couche supplémentaire

constructed according to a rule : construit selon une règle

lattice : treillis

breakthrough : découverte capitale, percée

to occur : se produire

in one's own right : par son seul talent, pour soi, en soi

mystical : mystique

mysticisme : doctrine, croyance fondée sur une union intime de l'homme et de la divinité, union rendue possible dans la contemplation.
doctrine religieuse essentiellement fondé sur le sentiment de la divinité plus que sur une conception rationnelle de celle-ci.

A figurate number is a group of dots.

to bring together : regrouper

to reverse, to turn round : retourner (un objet)

to put upside down : mettre à l'envers

layout : disposition, agencement

arrangement : disposition, arrangement

to move, to shift : bouger, déplacer

calculation : calcul [étymologie / calcul : du latin *calculus*, *caillou*]

the method's right but the calculations are wrong : le raisonnement est bon mais le calcul est faux

**Document 5 (transcription du document Powerpoint effectivement utilisé
en classe pour la Séance 1 bis)**

diapositive n° 1

Warm-up

diapositive n° 2

In mathematics,
a polygonal number
is a number
represented as dots or ??????? arranged in the shape
of a ??????? polygon.

diapositive n° 3

In mathematics,
a polygonal number
is a number
represented as dots or “**cherries**” arranged in the shape
of a “**red**” polygon.

diapositive n° 4

Remember:
Calculus is a Latin word
meaning “*pebble*” or *stone*
used for counting.
Definition: a “*regular*” polygon is
a polygon that has *all sides equal*
and *all interior angles equal*.

diapositive n° 5

In mathematics,
a polygonal number
is a number
represented as dots or **pebbles** arranged in the shape
of a **regular** polygon.

diapositive n° 6



What are the next **3** triangular numbers?

What **rule**

do you apply to get the next numbers?

diapositive n° 7

We add 6 to the 5th number , then 7 to the 6th , 8 to the 7th.

We add « 1 more unit » to the rank and add the obtained number to the previous triangular number.

diapositive n° 8

One among the first 8

triangular numbers

(i.e. 1, 3, 6, 10, 15, 21, 28, 36)

is « particular ».

What is this number,

and *in what respect*

is it « particular »?

diapositive n° 9

Some numbers, like **36**,

can be arranged both as a **??????**

and as a **????????**

diapositive n° 10

Some numbers, like 36,

can be arranged both as a **square** and as a **triangle**.

diapositive n° 11

Isn't it funny

to say that something

is *triangular* and *square*

at the same time?

diapositive n° 12

Try to explain

in what respect

we can say that

a figurate number

(such as 36)

is square and triangular.

diapositive n° 13

Each figurate number has a certain numerical value.

Each figurate number corresponds

to a certain amount of dots.

We say that the number

(corresponding to a numerical value)

is triangular

if the dots can be arranged

so as to form a triangular pattern.

But in some cases, the same number of dots

can be arranged as a square.

diapositive n° 14

The possibility for some numbers (of dots)

to be arranged in different ways

justifies the fact

that a number is sometimes

either triangular or square or else...

diapositive n° 15

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$$

$$6 \times 6 = 36$$

A triangular number

is the ??? of ?????????? integers

while a square number

is the ?????? of an integer

by ?????? .

diapositive n° 16

A triangular number

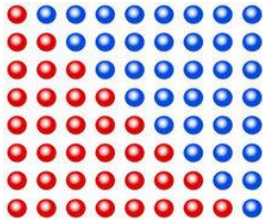
is the **sum** of **consecutive** integers
 while a square number
 is the **product** of an integer
 by **itself**.

diapositive n° 17

As the saying goes:

“Strength lies in unity”

diapositive n° 18



A rectangular number is made up of two identical triangular numbers.


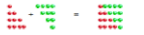


We can count the total number of dots easily.


diapositive n° 19

By bringing together
2 identical triangular numbers
we get a rectangular number




Le document conçu initialement est un ensemble de diapositives pour lesquelles il était envisagé que toutes ne soient pas nécessairement utilisées comme support d’interactions orales (voir précédemment pour le support effectivement utilisé). Par conséquent, certaines d’entre elles ont été réservées à la constitution d’un document de synthèse (voir ci-dessous), obtenu une fois l’ensemble imprimé en pdf.

<p>Warm-up</p>	<p>In mathematics, a polygonal number is a number represented as dots or ??????? arranged in the shape of a ??????? polygon.</p>	<p>We add 6 to the 5th number, then 7 to the 6th, 8 to the 7th.</p> <p><i>We add « 1 more unit » to the rank and add the obtained number to the previous triangular number.</i></p>	<p>One among the first 8 triangular numbers (i.e. 1, 3, 6, 10, 15, 21, 28, 36) is « particular ».</p> <p>What is this number, and in what respect is it « particular »?</p>
<p>In mathematics, a polygonal number is a number represented as dots or « cherries » arranged in the shape of a « red » polygon.</p>	<p>Remember:</p> <p><i>Calculus</i> is a Latin word meaning «pebble» or stone used for counting.</p> <p>Definition: a «<i>regular</i>» polygon is a polygon that has <i>all sides equal</i> and <i>all interior angles equal</i>.</p>	<p>Some numbers, like 36, can be arranged both as a ?????? and as a ????????</p>	<p>Some numbers, like 36, can be arranged both as a square and as a triangle.</p>
<p>In mathematics, a polygonal number is a number represented as dots or pebbles arranged in the shape of a regular polygon.</p>	 <p>What are the next 3 triangular numbers?</p> <p>What rule do you apply to get the next numbers?</p>	<p>Isn't it funny to say that something is triangular and square at the same time?</p>	<p><i>Try to explain</i> in what respect we can say that a figurate number (such as 36) is square and triangular.</p>
<p>Each figurate number has a certain numerical value. Each figurate number corresponds to a certain amount of dots. We say that the number (corresponding to a numerical value) is triangular if the dots can be arranged so as to form a triangular pattern. But in some cases, the same number of dots can be arranged as a square.</p>	<p>The possibility for some numbers (of dots) to be arranged in different ways justifies the fact that a number is sometimes either triangular or square or else...</p>	<p>By bringing together 2 identical triangular numbers we get a rectangular number.</p>  $2T_4 = 4(4+1)$ $T_4 = \frac{4(4+1)}{2}$	<p>Try to generalize the previous observation.</p> <p>We intend to express T_n in terms of n.</p>
<p>$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$</p> <p>$6 \times 6 = 36$</p> <p>A triangular number is the ??? of ???????????? integers while a square number is the ????????? of an integer by ?????????.</p>	<p>A triangular number is the sum of consecutive integers while a square number is the product of an integer by itself.</p>	<p>Generalization</p> $2T_n = n(n+1)$ $T_n = \frac{n(n+1)}{2}$	<p>Question:</p> <p>Is there a relation between a square number and triangular numbers?</p>
<p>As the saying goes:</p> <p>« strength lies in unity »</p>	 <p>total = 9×8 dots</p> <p>Hence $T_9 = \dots$</p>	<p>Here is a saying that may be of help to you: « divide and rule »</p> <p><i>We have to divide in order to rule.</i></p> <p>to rule = to command a saying : un dicton</p>	<p><i>As is said in the saying start by considering the following square number and divide!</i></p>  <p>Then write : $S_5 = \dots + \dots$</p>



$$S_5 = T_5 + T_4$$



2 ² =	1 + 3	= 4
3 ² =	3 + 6	= 9
4 ² =	6 + 10	= 16

The purpose of what follows is to make you *perceive* the relation between the *geometrical manipulations* performed on, or with, the figurate numbers and their *algebraic transcription*.

Generalize
in order to express the *nth square number* as a sum of *2 triangular numbers*.

$$S_n = T_n + T_{n-1}$$

$$S_5 = T_5 + T_4$$

Check that the formula works for $n=6$ for example (by considering *numerical values*)

Fill in the gaps:

$$S_6 = \dots\dots\dots$$

$$T_6 + T_5 = \dots\dots\dots + \dots\dots\dots$$

We now intend to compare the « *visual proof* » with a « *standard* » (i.e. modern) proof.

Document 6 Transcriptions de la Séance 1 bis et commentaires

Phase interactive d'introduction avec support Powerpoint

1. P So, let's start.
I want you to take an active part in this lesson.
(quelques explications supplémentaires sur le thème/ non retranscrites)
2. P (s'adressant à E1)
Can you tell me what we did last time ?
3. E1 We saw a picture with balls
4. P What were the terms I used?
5. E2 Rows and columns
6. E3 In a rectangle
7. P Just a rectangle, with only the outlines?

8. P Let me switch on the Interactive Board
(affiche la première diapositive)
This is the first slide
9. P We spoke of imaginary lines...
Did we see a real triangle?
What is a real triangle?

10. E1 A real triangle is defined by three points

11. P A triangle is made up of sides, of vertices ...

Does a point exist?

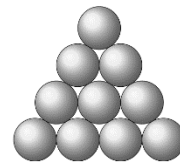
12. E5 No because... (hésite)

13. P (commentaires non retranscrits)

What are the missing words?
Can you match the question marks with a word?

diapositive n° 1

Warm-up



diapositive n° 2

In mathematics,
a polygonal number
is a number
represented as dots or ???????
arranged in the shape
of a ??????? polygon.

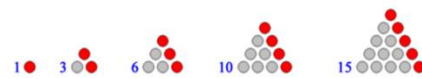
14. Try to guess the missing words
What do you suggest?
- A dot is already an abstraction
Something not visible
- To you, a dot is like a little circle
Do you accept dots as objects?
15. What sort of polygon do we actually consider?
16. E3 regular
17. P Let's check diapositive n° 3
(P affiche la diapositive suivante)
(P lit le contenu de la diapositive)
- In mathematics,
a polygonal number
is a number
represented as dots or **"cherries"**
arranged in the shape
of a **"red"** polygon.
18. P Can you tell me the origin of the
word calculus?
Nobody knows?
(P lit le contenu de la diapositive)
- Take a look at the next slide
- diapositive n° 4
Remember:
Calculus is a Latin word
meaning *"pebble"* or *stone*
used for counting.
Definition: a *"regular"* polygon is
a polygon that has *all sides equal*
and *all interior angles equal*.
19. P [...] diapositive n° 5
Arranged in the shape of a regular
polygon.
What does the adjective *regular*
mean?
- Can you rephrase?
- In mathematics,
a polygonal number
is a number
represented as dots or **pebbles**
arranged in the shape
of a **regular** polygon.
20. E6 Same dimensions [...]
21. He has...

22. P It's not a person, it's a polygon so, say "it" and not "he"
23. P You could describe similar things in three dimensions?
In this case we'd talk about a regular **polyhedron**.
24. P Give me an example of a polygon
25. E7 A square
26. We see that it can be inscribed in a circle
27. Remind me of what we said about the Ancient Greeks.

(s'adresse à un autre élève)
What did they do? concerning numbers

28. E8 They made pictures for numbers
29. P What sort of numbers?
30. E1 triangular
31. P Here are the first 5 triangular numbers.
By the way, we say the first three or the first five. "Les cinq premiers"
In English, the order is reversed.
What are the next three. Take a look at the slide.
What rule do you apply to get the next numbers?

diapositive n° 6



What are the next **3** triangular numbers?

What **rule**

do you apply to get the next numbers?

32. E9 21
33. P Right
(s'adresse à E10)
And the following ones ?
34. E10 28 and ...36
35. P This is mental calculation.
You're right.
So, you seem to understand the rule.
Who can explain the rule?
36. E3 You have to add a number...you can divide...
37. P No
38. E1 If you take ... [...] you add the precedent number of **the** pebbles...
39. P To explain things, it's better to denote things
40. E1 To calculate the sixth, you take the fifth. T_5 and you add 6.

41. P What is 6 with respect to the previous number.
You add one more unit to the ...?
To the what...?
(attente)...
(personne ne répond)
To the rank. "Le rang"
You recognize the rank when you say
 T_5
(P insiste sur "five")
42. P We add one more unit to the rank and add it to the previous number
- diapositive n° 7
We add 6 to the 5th number , then 7 to the 6th , 8 to the 7th.
- We add « 1 more unit » to the rank and add the obtained number to the previous triangular number.***
43. P (P lit le contenu de la diapositive)
- diapositive n° 8
One among the first 8 triangular numbers
(i.e. 1, 3, 6, 10, 15, 21, 28, 36)
is « particular ».
Which one?
- One among the first 8 triangular numbers
(i.e. 1, 3, 6, 10, 15, 21, 28, 36)
is « particular ».
What is this number,
and in what respect
is it « particular »?
44. E1 36
Because it's a square
45. P (P montre la diapositive suivante)
- diapositive n° 9
Some numbers, like 36,
can be arranged both as a ??????
and as a ????????
46. E3
- As a square and a triangle
47. P (P montre la diapositive suivante)
Right
- diapositive n° 10
Some numbers, like 36,

- can be arranged both as a **square** and
as a **triangle**.
48. Isn't it funny to say that something is
triangular and *square* at the same
time?

In maths, it makes sense.
- diapositive n° 11
Isn't it funny
to say that something
is triangular and square
at the same time?
49. E1 Can we represent this phenomenon?
50. P Look at the next slide
(P affiche la diapositive)

Try to explain in what respect you
can say that a figurate number is
square and triangular.
It's just a matter of description.
Remember
A number was just a group of ...
Of what?
- diapositive n° 12

Try to explain
in what respect
we can say that
a figurate number
(such as 36)
is square and triangular.
51. P Dots
52. P What can you do with objects?
53. E10 We can play...
54. P What kind of game?
55. E11 Basket ball
56. P No
57. E5 Make a picture
58. p A number, seen as a group of dots,
can be arranged as...
59. P (P montre la diapositive suivante)
- diapositive n° 13
Each figurate number has a certain
numerical value.
Each figurate number corresponds
to a certain amount of dots.
We say that the number
(corresponding to a numerical value)

is triangular
if the dots can be arranged
so as to form a triangular pattern.
But in some cases, the same number of
dots
can be arranged as a square.

60. (commentaires non retranscrits)

diapositive n° 14
 The possibility for some numbers (of
 dots)
 to be arranged in different ways
 justifies the fact
 that a number is sometimes
 either triangular or square or else...

61. idem

diapositive n° 15
 $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$
 $6 \times 6 = 36$
 A triangular number
 is the ??? of ?????????? integers
 while a square number
 is the ??????? of an integer
 by ?????? .

62. Idem ...

...

Voir Document 5 pour les dernières diapositives et la transcription de leur contenu

Document 7 (fourni aux élèves)

- Some numbers, like 36, can be arranged both as a square and as a triangle.

Certains nombres, comme 36, peuvent être représentés/(mis sous la forme) à la fois d'un carré ou d'un triangle.

- in what respect : *dans quelle mesure.*
- amount of dots : *quantité de points.*
- so as to form a triangular pattern : *afin de former un motif triangulaire.*
- dots arranged in different ways : *points placé(s)/ disposé(s) de diverses manières.*
- A triangular number is the sum of consecutive integers while a square number is the product of an integer by itself.

Un nombre triangulaire est la somme d'entiers consécutifs tandis qu'un carré est le produit d'un entier par lui-même.

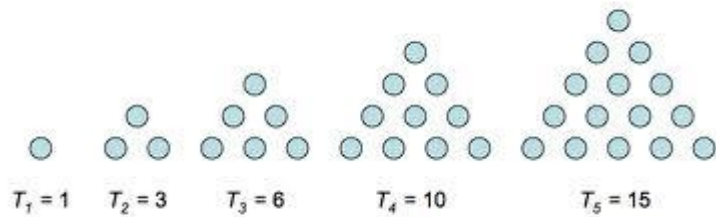
- a saying : *un dicton*
- as the saying goes : *comme dit le proverbe.*
- “Strength lies in unity”: *“l'union fait la force”.*
- « divide and rule » : *« diviser pour régner ».*
- to rule = to command
- the n-th square number : *le n-ième nombre carré.*
- We now intend to ... : *nous avons maintenant l'intention de ...*
- The purpose of what follows is to make you perceive the relation between the geometrical manipulations performed on, or with, the figurate numbers and their algebraic transcription.

L'objet de ce qui suit est de vous faire percevoir la relation entre les manipulations géométriques réalisées sur, ou avec, les nombres figurés et leur transcription algébrique.

- to move sthg closer to sthg : *rapprocher qqch de qqch.*
- to get closer (to) : *se rapprocher (de) .*

Document 8 **Consignes données lors de la Situation des Nombres**
Triangulaires

GROUP WORK
INSTRUCTIONS



- 1) Respect the work of others by *keeping your voices down*.
- 2) *Listen to and respect* your team mates.
- 3) *No French* is to be spoken in the class.
- 4) Inside the team some works / tasks can be divided up.
Remember that there is a time when you have to coordinate your results.
The motto of a team is: “*all for one, one for all*”
- 5) At the end of your research, the *spokesperson will present* the results of the work.

Part I

Denote by T_n the n th triangular number.

Recall : $T_1 = 1$

The purpose is to *calculate the numerical value of T_{100} directly, i.e. without calculating all the previous numbers.*

Hint: *use pictures, arrangements, bring triangular numbers together.*

Present *on a poster* the different steps of your research and try to justify your calculations.

Never forget that “strength lies in unity”.

Part II

Can we obtain 5^2 from triangular numbers?

What about other square numbers?

Present *on a poster* the different steps of your research and try to justify your calculations.

Document 9 Test donné aux élèves de Première européenne à l'issue de la séquence des Nombres Triangulaires

Durée : 1 heure

- 1) What is a *figurate number* (a polygonal number)?
Give an example. Explain in what way a number can be said to be “triangular” for example.
Is the notion of figurate number a modern one?...
- 2) Did you enjoy working in group and if so, why?
- 3) Imagine yourself as a teacher (but only momentarily!).
What *instructions* would you give to your pupils to let them work in proper conditions?
- 4) During a group-work session, you showed that a square number is made up of two triangular numbers.
Describe in a few words the manipulations you then performed on figurate numbers.
- 5) Here is the *algebraic* way of proving that the expression of the n th triangular number T_n in

terms of n is $\frac{n(n+1)}{2}$:

$$Tn = 1 + 2 + 3 + \dots + n$$

$$\cdot \quad Tn = n + (n-1) + (n-2) + \dots + 1 \quad (\text{reversed order})$$

hence, by adding side
to side and by columns

we get:
$$2Tn = (n+1) + (n-1+2) + (n-2+3) + \dots + (n+1)$$

notice that there are n terms in the sum,
each of them equal to $(n+1)$

that is :
$$2Tn = n(n+1)$$

and therefore :
$$Tn = \frac{n(n+1)}{2}$$

Try to establish a *correspondence* between this proof and the *manipulations* you performed on triangular numbers in order to get the general expression in terms of n .

Key-words : adding / rectangular number / number of dots / reversing the order / bringing together / putting upside down ...

Posters réalisés par la classe de première européenne

PART 1

$T_n + T_n = n \times (n+1)$
 $2T_n = n \times (n+1)$
 $T_n = \frac{n \times (n+1)}{2}$


We have to find a formula

example:

$2T_3 = 3 \times 4$
 $T_3 = 6$

We apply the formula for T_{100} :

$2T_{100} = 100 \times (100+1)$
 $2T_{100} = 100 \times 101$
 $2T_{100} = 10 \cdot 100$
 $T_{100} = 5 \ 050$

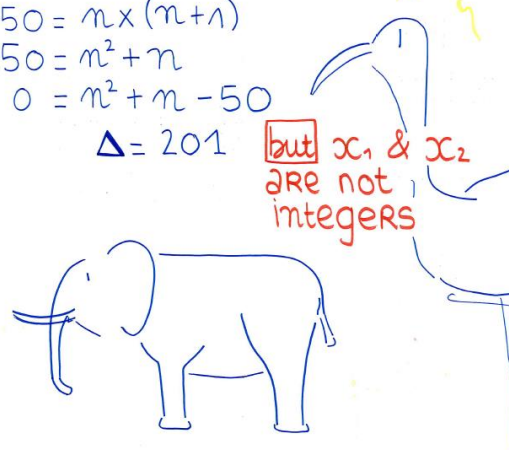


Beauty Queen

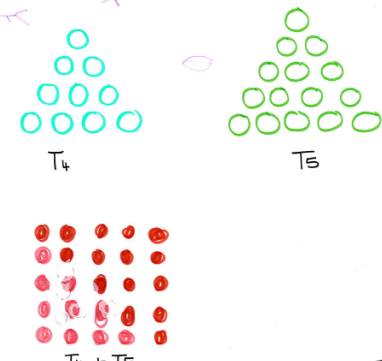
PART 2

$5^2 = 25$
 $T_n = 25$
 $25 = \frac{n \times (n+1)}{2}$
 $50 = n \times (n+1)$
 $50 = n^2 + n$
 $0 = n^2 + n - 50$
 $\Delta = 201$

but x_1 & x_2 are not integers

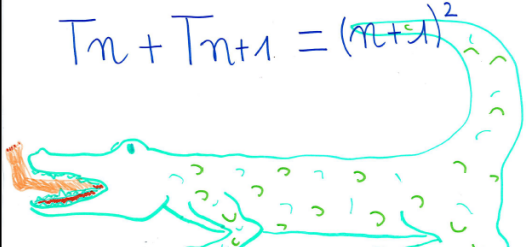


PART 2




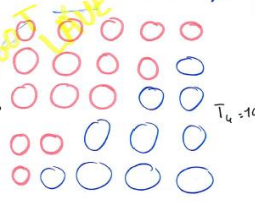
$T_4 + T_5 = 5^2 = 25$
 $10 + 15 = 5^2 = 25$

$T_n + T_{n+1} = (n+1)^2$



Part 2





$T_n + T_{n+1} = (n+1)^2$
 $T_4 + T_5 = (4+1)^2$
 $T_4 + T_5 = 25$

MARGOT LARVE.

oudart - Godeluck
Laruelle - BMS - Larue



$5^2 = 25$

$5^2 = T_5 + T_4$

This same process may also be applied in a general case:

$n^2 = x$

$T_n + T_{n-1} = x$

TEAM SUSHI!

* $1 + 2 + 3 + \dots + (n-1) + n = T_n$

+ $n + (n-1) + (n-2) + \dots + 2 + 1 = T_n$

$(1+n) + (1+n) + (1+n) \dots (1+n) + (1+n) = 2(T_n)$

$= n \times (1+n) = 2 T_n$

$T_n = \frac{n \times (1+n)}{2}$

* For example, we will use T_3 to demonstrate the formula:

$T_3 = \frac{3 \times (1+3)}{2}$

$T_3 = 6$

* To explain further, we have drawn the images that represent this formula:

Almasione Sophia, Diarra Emma, Gaultier Mathieu, Teubard Lucas
Souillard Guillaume

The tetris team

PART 1

Example:

$T_3 = \frac{3 \times (3+1)}{2} = 6^2$

The formula

$T_n = \frac{n \times (n+1)}{2}$

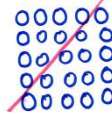
$T_{100} = \frac{100 \times (100+1)}{2} = 5050$

PART 2
First Reflexion

$$5^2 = \left(\frac{n(n+1)}{2} \right) - n$$

$$= \left(\frac{5(5+1)}{2} \right) - 5$$

$$= 30 - 5$$

$$= 25$$


$\implies n(n+1) - n = 5^2$

Second Reflexion

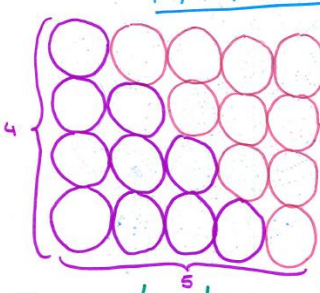
$$T_n + (T_n - 1) = \frac{n(n+1)}{2} + \frac{(n-1)(n-1+1)}{2}$$

$$= \frac{n(n+1) + n(n-1)}{2}$$

$$= \frac{n^2 + n + n^2 - n}{2}$$

$$= \frac{2n^2}{2} = n^2$$

PART I



Let's explain what we've done with the example of T_4

$$2T_4 = 5 \times 4$$

$$\Leftrightarrow T_4 = \frac{5 \times 4}{2}$$

$$\Leftrightarrow T_4 = 10$$


Generalisation:

$$T_n = \frac{n \times (n+1)}{2}$$


Let's do this with T_{100} :

$$T_{100} = \frac{100 \times (100+1)}{2}$$

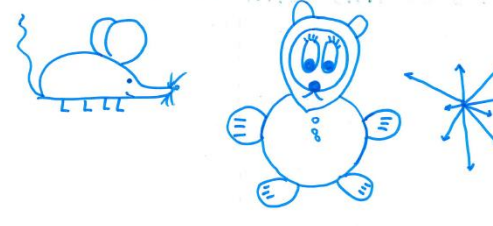
$$= \frac{10100}{2}$$

$$= 5050$$


3-10 BACANI
 2-10 BACANI
 1-10 BACANI
 0-10 BACANI



Quantum dot



PART II

$$\frac{n \times (n+1)}{2} = 5^2$$





$$\Leftrightarrow n^2 + n = 50$$

$$\Leftrightarrow n^2 + n - 50 = 0$$

$$\Delta = b^2 - 4ac = 1 + 200 = 201$$


$$x_1 = \frac{-1 - \sqrt{201}}{2}$$

(negative so excluded)

$$x_2 = \frac{-1 + \sqrt{201}}{2}$$






PART II

We have



T_4 T_5

$T_4 + T_5 = 25$



$$T_{(n-1)} + T_n = \frac{n(n-1)}{2} + \frac{n(n+1)}{2}$$

$$= \frac{n(n-1) + n(n+1)}{2}$$

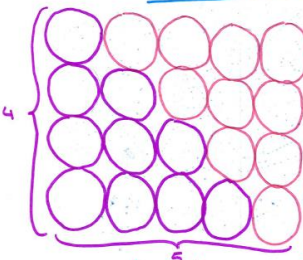
$$= \frac{n^2 - n + n^2 + n}{2}$$

$$= \frac{2n^2}{2}$$

$$= n^2$$

$T_{n-1} + T_n = n^2$

PART I



Let's explain what we've done with the example of T_4

$$2T_4 = 5 \times 4$$

$$\Leftrightarrow T_4 = \frac{5 \times 4}{2}$$

$$\Leftrightarrow T_4 = 10$$

Generalisation:

$$T_n = \frac{n \times (n+1)}{2}$$

Let's do this with T_{100} :

$$T_{100} = \frac{100 \times (100+1)}{2}$$

$$= \frac{10100}{2}$$

$$= 5050$$

ANNEXE 2 : Cartes mentales / Mind maps

Thème : signe des expressions algébriques (classe de première européenne)

1) Définition et caractéristiques d'une carte mentale

Inventée par le psychologue Tony Buzan, la carte mentale (aussi appelée carte heuristique) permet d'organiser ses connaissances sur un sujet.

Elle repose sur un principe de visualisation de liens entre les idées ou les concepts et peut facilement être réalisée, que ce soit sur papier, à l'aide de crayons ou de feutres, ou bien à l'aide d'un logiciel dédié (de type Freemind ou Xmind par exemple).

Un de ses avantages, et non le moindre, est de permettre une vision à la fois détaillée et globale d'un sujet.

Pour le propos de notre thèse, nous retiendrons cependant l'intérêt didactique qui réside dans les processus-mêmes de création ou d'analyse d'une telle carte.

Nous laissons de côté, en revanche, d'autres types d'avantages qu'elle peut procurer, à savoir la facilitation de la prise de notes par exemple, ou encore la mémorisation ou la restitution d'informations dans une perspective de révision avant le passage d'un examen.

2) Utilisation dans une perspective didactique orientée-tâche.

D'un point de vue didactique, l'élaboration de cartes mentales peut faire l'objet d'un travail en groupe, à propos d'un thème d'étude préalablement fixé.

Inversement, la donnée d'une carte mentale peut donner lieu à une discussion quant à la nature des relations impliquées, qui sont certes visualisées, mais le plus souvent ne sont pas explicitées.

Ces relations peuvent être de types très différents et leur explicitation peut constituer, elle aussi, un objectif didactique dans le cadre d'une présentation orale par exemple.

A cet effet, il semble nécessaire que les élèves aient été auparavant, une ou plusieurs fois, confrontés à ce type de démarche et qu'ils disposent d'un répertoire suffisant en ce qui concerne la description des différents types de relations ou de catégories.

Néanmoins, il ne faut pas perdre de vue qu'une relation entre deux concepts d'une carte mentale, lorsqu'elle n'est pas explicitée, peut tout-à-fait être verbalisée de façon non univoque.

On pourra donc dans un premier temps se focaliser sur des relations élémentaires, de type causal puis, progressivement, dans une démarche métacognitive, interpréter les relations (matérialisées par des traits) comme des consignes éventuelles, voire des stratégies, permettant par exemple de résoudre tel ou tel type de problèmes, ou encore d'accomplir telle ou telle tâche.

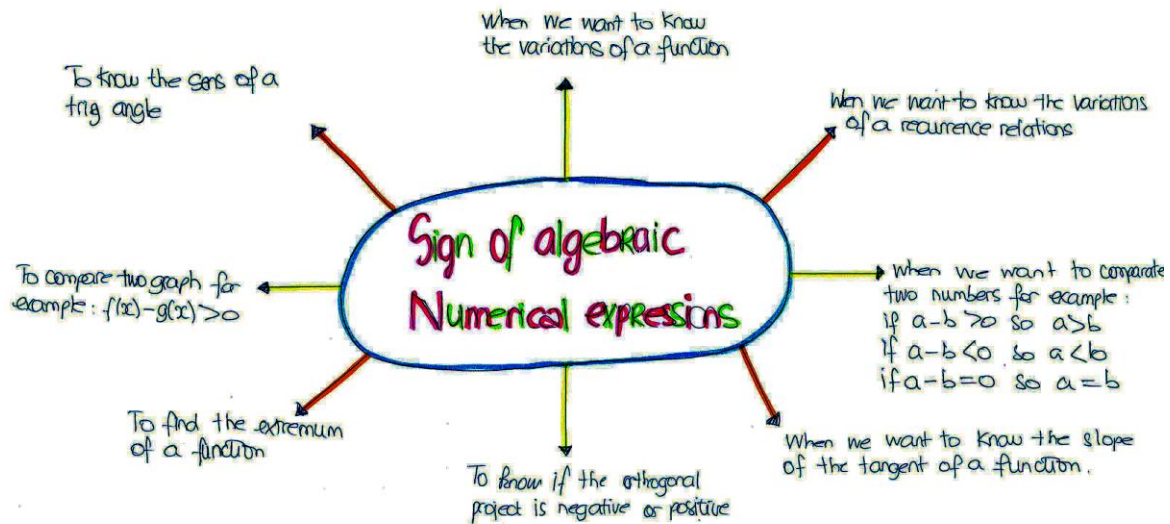
Cela sera sans doute aussi l'occasion de faire prendre conscience, par les élèves, de la structuration cognitive non linéaire de nos connaissances mémorisées. On pourra d'ailleurs contraster celle-ci avec la linéarité du processus sémiotique de la pensée lorsque celle-ci se traduit par une verbalisation intériorisée ou oralisée.

3) Production d'une carte mentale en contexte CLIL

Le document ci-dessous est le résultat d'un travail en groupes.

Un responsable de chaque groupe devait présenter le résultat de sa recherche devant la classe.

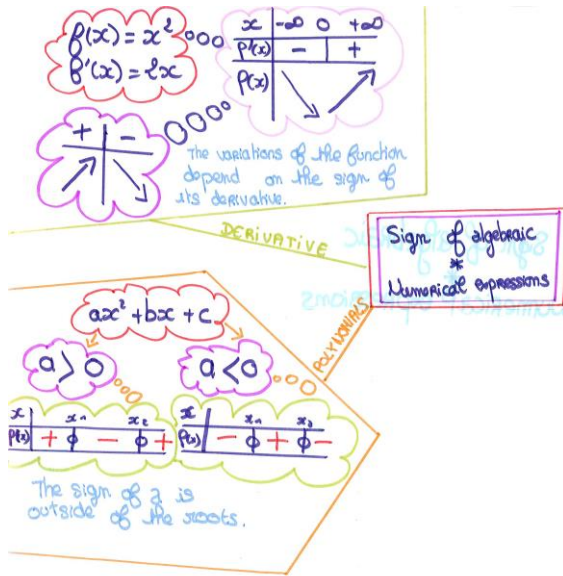
Groupe Work: Amoziane Sophia / Banse Paul / Diarra Emma / Larue Margot / Oudart Antoine / Pauly Julie : 609 and 613



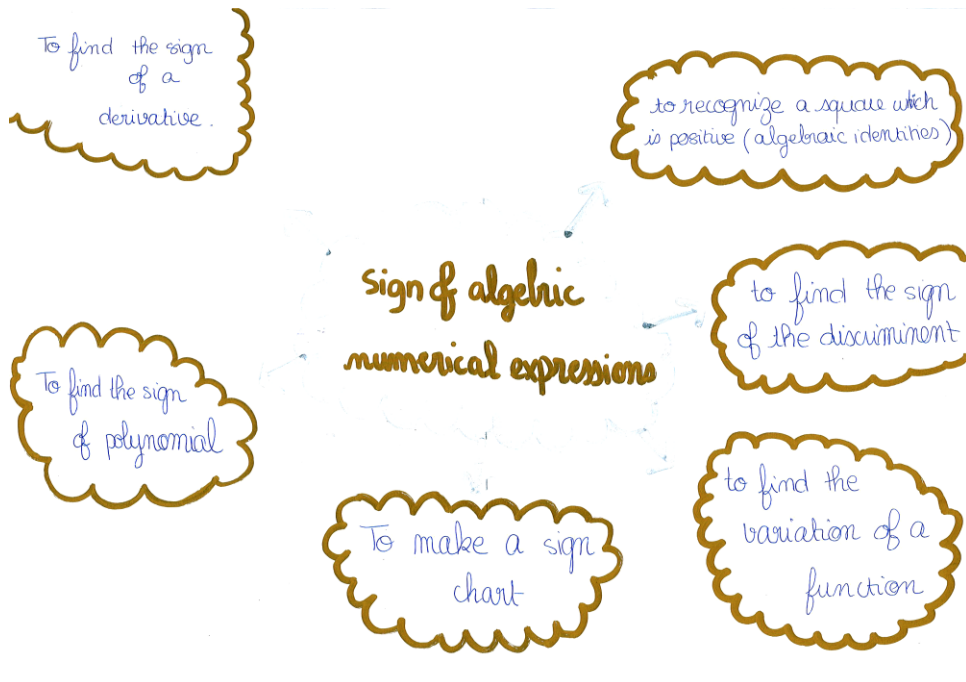
A l'issue de l'activité les élèves ont reçu un document présentant ce qui avait été évoqué lors de la phase d'institutionnalisation. Les erreurs de nature lexicale ou syntaxique apparaissant sur les cartes produites par les élèves ont bien évidemment été rectifiées ! Les élèves n'avaient pas l'habitude de constituer de telles cartes et n'ont pas eu l'occasion de réfléchir à l'avance sur le thème proposé. Ce n'est donc pas une carte dont l'objectif serait de dresser un bilan des connaissances en fin de chapitre. En examinant les éléments dans le sens inverse des aiguilles d'une montre (« anticlockwise ») et en commençant par le haut, on peut relever :

- ❖ *to know the variations of a function* est incorrect / on dira *to study or investigate the behaviour of a function*.
- ❖ *sense of a right triangle* est calqué sur le français : on dira plutôt : *a positively oriented right triangle*
- ❖ il manqué un *s* à *graph*.
- ❖ l'élève utilise *orthogonal project (ion)* en pensant au produit scalaire (*dot product*).
- ❖ *compare* est erroné et résulte d'un transfert de la forme *-ate* (comme dans *enumerate*) sur la forme correcte *compare*
- ❖ *a recurrence relations* / mis à part le pluriel inapproprié, on dira plutôt *recurring pattern* ou encore *recursive pattern* / de plus, parler de variations s'appliquerait plutôt aux suites (sequences) : *in order to know if a sequence is increasing or decreasing*.

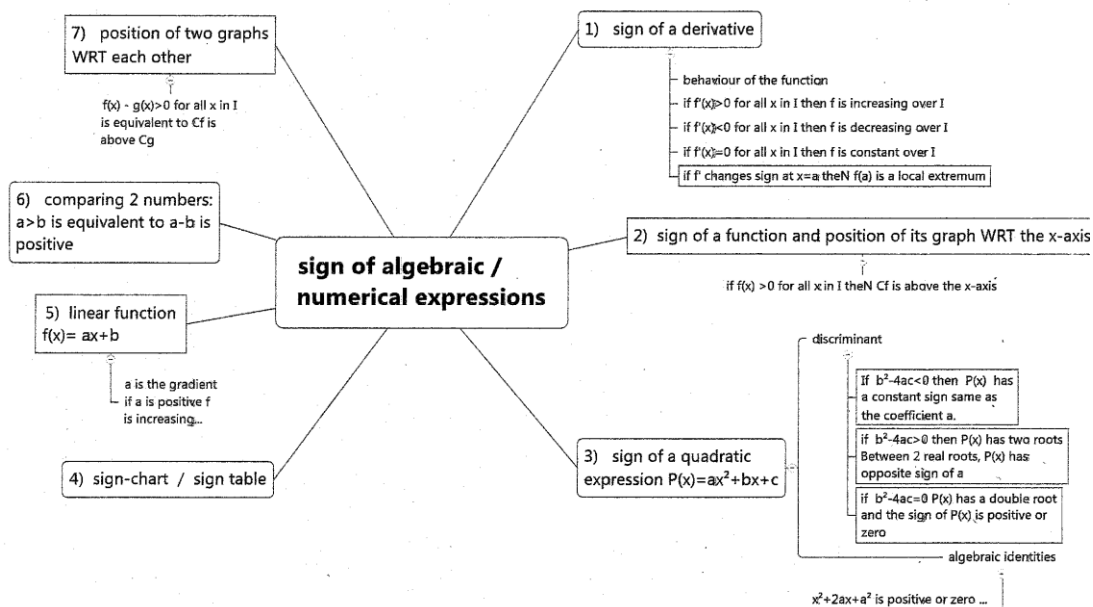
Autres cartes mentales (réalisées à la main)



Alain LAURELLE
 Claire BERTELLA
 Guillaume SCHLIEDT
 Camille ESTILCON



Carte mentale réalisée avec freemind pour l'institutionnalisation

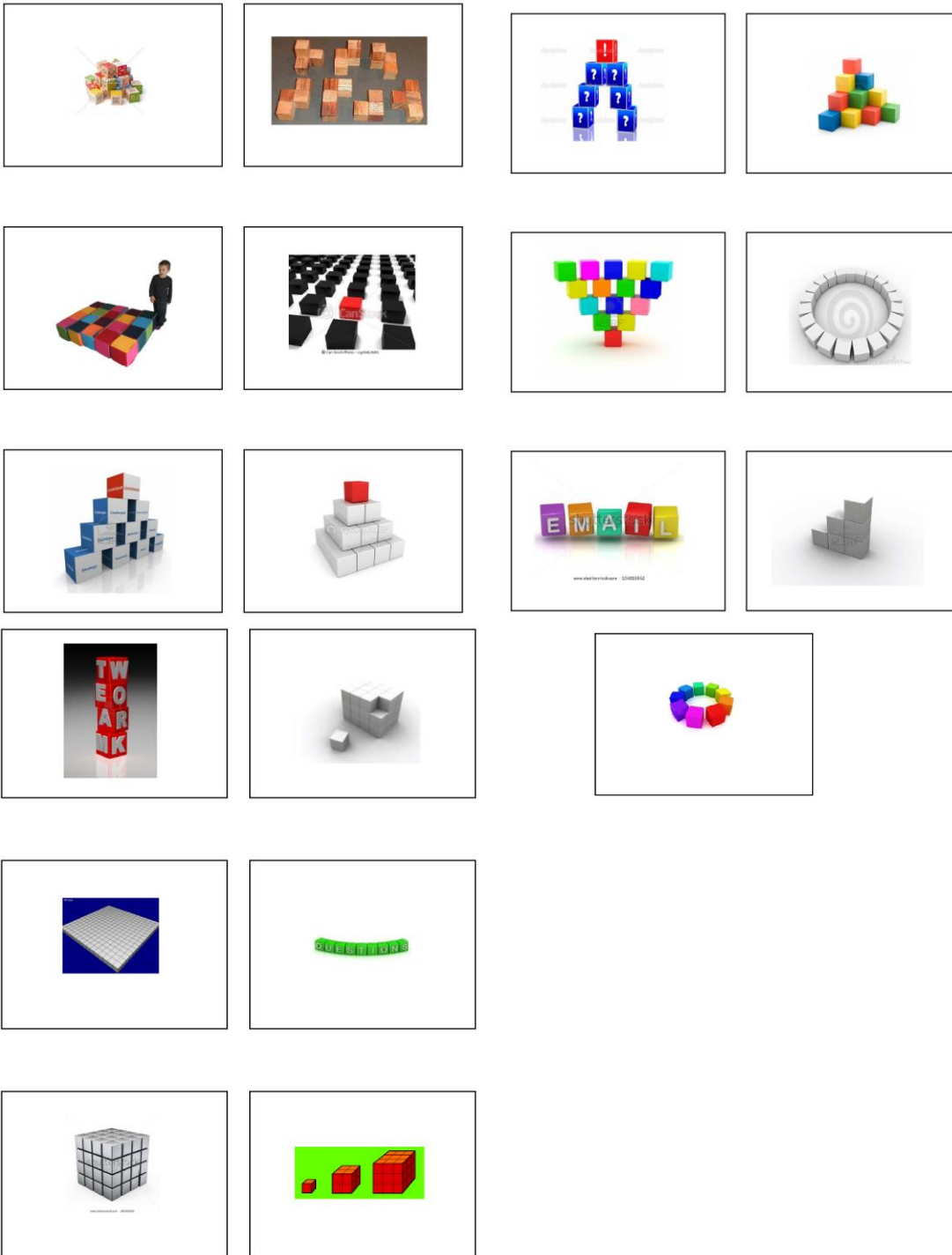


ANNEXE 3 : Situation intitulée *Somme des Carrés*

Préparation pour la séance Somme des cubes

Powerpoint basé sur l'idée de *scaffolding* (suivi d'un lexique distribué aux élèves) dans le but de familiariser les élèves avec les termes tels que :

stack, pile up, arrange etc.



Lexique phraséologique

a heap : un tas

to assemble small identical cubes into a single cube

to be oriented, to be placed in a position

to manipulate

arrangement : arrangement disposition /

to arrange as a square: disposer sous forme de carré

so as to form ... : de manière à former

to line up = to align

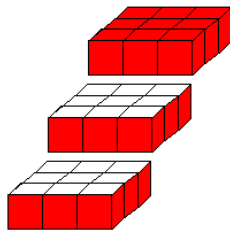
the screen displays n lined-up cubes.

to stack (up) : ranger en hauteur

to heap up : empiler (en tas)

a gift boxed set of 10 stacking cubes (largest measures 16 x 16cm)

a pyramid made of stacked up cubes



cube layers

a layer: une couche

level : niveau

to be arranged in a vertical plane

5 red cubes and 4 blue cubes are placed at random in a row.

find the probability that both end cubes are red.



to place cubes in a row

to remove : enlever / to shift : déplacer

concentric : concentrique

each sugar cube will be stacked one on top of the other in a single, vertical column

the basic piece consists of a single cube

its design also allows it to support four or more **vertically stacked cubes** in required configurations

Document fourni aux élèves

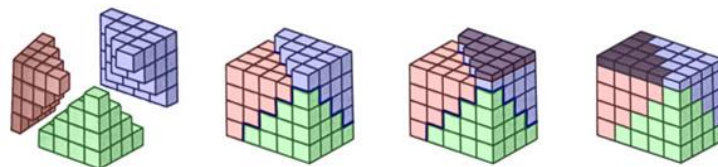
Sum of consecutive square numbers : second Visual Proof

We have **already proved** that the following property is true :

$$(1^2 + 2^2 + \dots + n^2) = \frac{1}{6}n(n+1)(2n+1) \quad (E1)$$

We did it by making a **Proof by Induction**.

We also investigated a **3D-Visual Proof** based on the grouping together of three pyramids.



Let's consider the following equality, denoted by (E2) :

$$3(1^2 + 2^2 + \dots + n^2) = (2n+1) \frac{n(n+1)}{2}$$

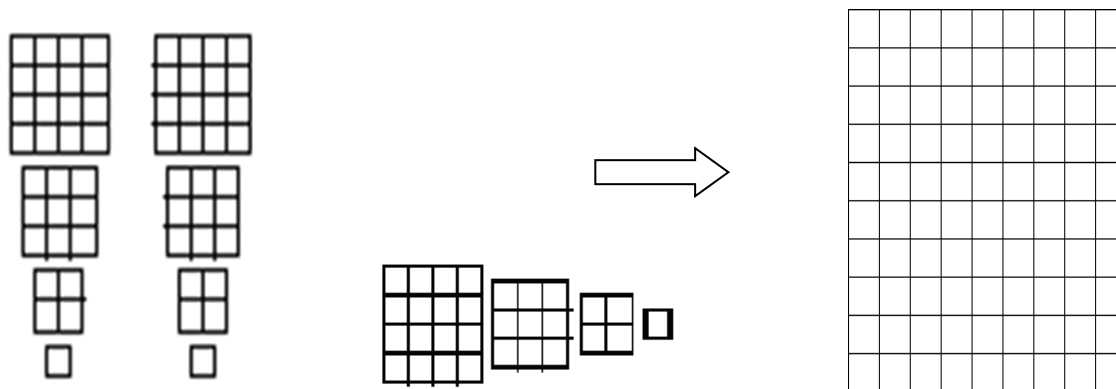
- 1) Prove that : (E1) is equivalent to (E2).
- 2) Then deduce that $3(1^2 + 2^2 + \dots + n^2) = (2n+1)(1+2+\dots+n)$.

The product $(2n+1)(1+2+\dots+n)$ can be interpreted either as the area of a rectangle or as the number of squares in a rectangle whose dimensions are $(2n+1)$ and $(1+2+\dots+n)$ and $3(1^2 + 2^2 + \dots + n^2)$ can be seen as 3 copies of a pyramid.

The purpose is now to elaborate a **second Visual Proof** based on the previous observations.

The arrangements of squares are no longer in 3D-space. (the former 3 pyramids are supposed to have been laid flat)

- 3) Considering **three copies** of the sum of square numbers (for the case $n=4$), Show how to rearrange the squares so as to fill in the rectangle.



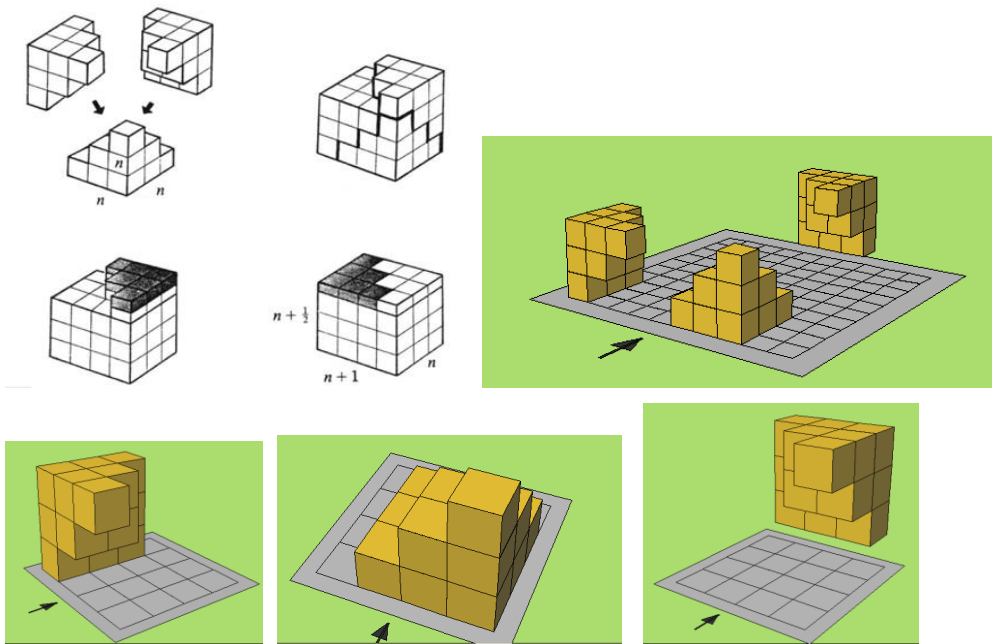
Hints:

*Think of **gnomons!** (gnomons are hidden) Arrange **methodically!** Introduce **order!***

*Establish **relationships** (between corresponding rows, columns, gnomons, squares)*

Use colour

Première Preuve visuelle pour la Somme des Carrés preuve en 3 D



$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{3}n(n+1)\left(n + \frac{1}{2}\right)$$

This equality is equivalent to :

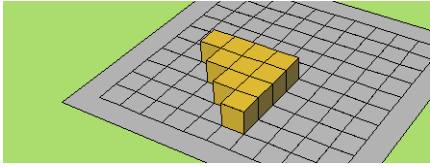
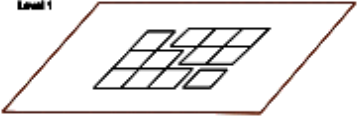
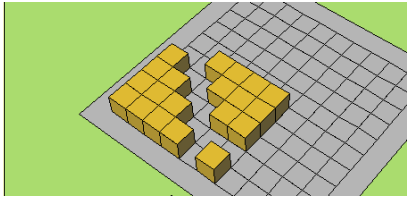
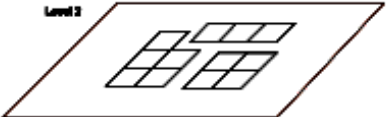
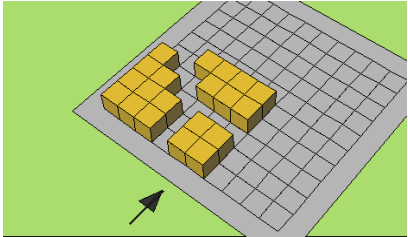
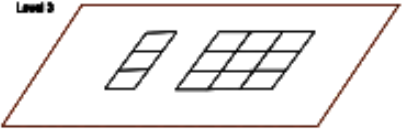
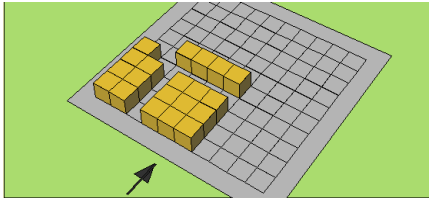
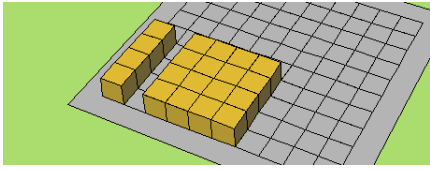
$$3(1^2 + 2^2 + 3^2 + \dots + n^2) = n(n+1)\left(n + \frac{1}{2}\right)$$

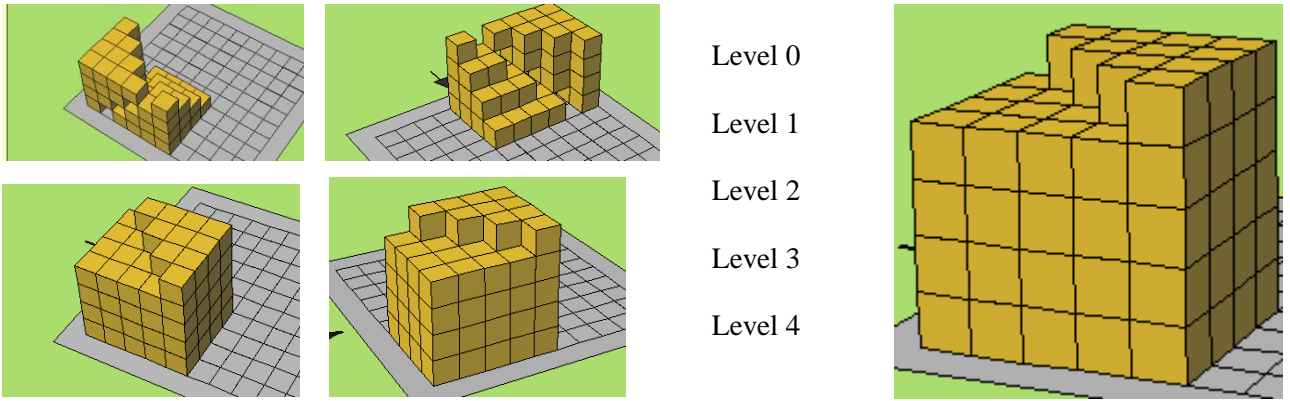
$n(n+1)\left(n + \frac{1}{2}\right)$ corresponds to the volume of a parallelepiped

Its dimensions are : n , $n+1$ and $n + 1/2$

And $3(1^2 + 2^2 + 3^2 + \dots + n^2)$ corresponds to the volume of 3 identical pyramids

Schematization

		<p>Level 0 (top of the pyramid)</p>
<p>Level 1</p> 		<p>Level 1</p>
<p>Level 2</p> 		<p>Level 2</p>
<p>Level 3</p> 		<p>Level 3</p>
<p>etc</p>		<p>Level 4 (bottom of the pyramid)</p>
<p>Les figures de cette colonne correspondent au rang n=4 alors que celles de la colonne de gauche correspondent au rang n=3</p>		



Powerpoint pour la séance Somme des Carrés (2ème preuve visuelle)
Classe de terminale européenne

Visual Proofs

Triangular Numbers Revisited
 Representing T_n with dots
 Let's try a visual representation:

 T_n blue dots and T_n red dots for a grand total of $2T_n$ dots

Sum of Squares
 Definition: Let S_n be the sum of the first n perfect squares. So
 $S_n = 1^2 + 2^2 + 3^2 + \dots + n^2$
 Theorem:
 $S_n = \frac{n(n+1)(2n+1)}{6}$

Triangular Numbers Revisited
 Proof:

 $2T_n = \text{Total Dots} = n(n+1)$

Sum of Squares
 Elegant Proof:

 $2S_n = \text{Black Dots} + S_n = \text{Color Dots} = 3S_n, \text{Total Dots} = \frac{n(n+1)(2n+1)}{2}$

Sum of Squares
 $3n^2 + 2^2 + \dots + n^2 = (2n+1)(2n+1)$

 $3S_n = (2n+1)(2n+1)$

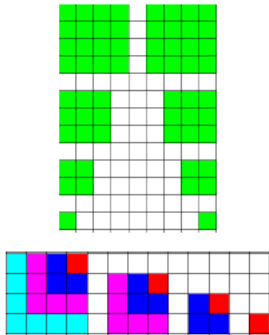
Sum of Squares
 Hints:
 Think of gnomons!
 (gnomons are hidden)
 Arrange methodically!
 Establish relationships
 (between corresponding rows, columns, gnomons, squares)
 Use colour to convince.

Sum of Squares
 We need three copies of the sum of squares.

 S_n, S_n, S_n
 $(n+1)^2$ in the picture
 Sum of the five consecutive squares

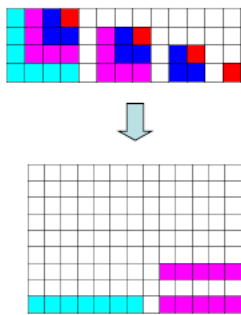
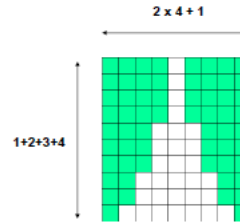
Deuxième preuve visuelle pour la Somme des Carrés

institutionnalisation après la séance adidactique



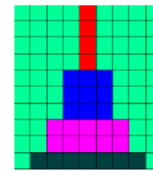
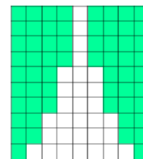
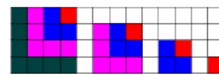
Start with
3 copies of
the sum of
the first four
square
numbers

Place the 4
consecutive
squares
along the edges
of
a rectangle whose
dimensions
are : $1+2+3+4$
and $(2 \times 4 + 1)$

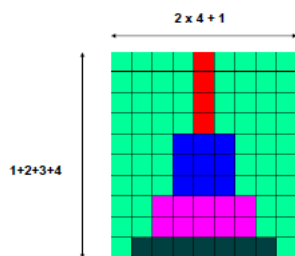


Make the
Gnomons
horizontal

fit the
Gnomons
into the
empty
spaces



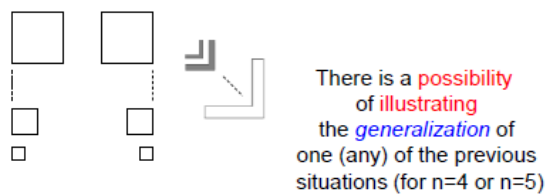
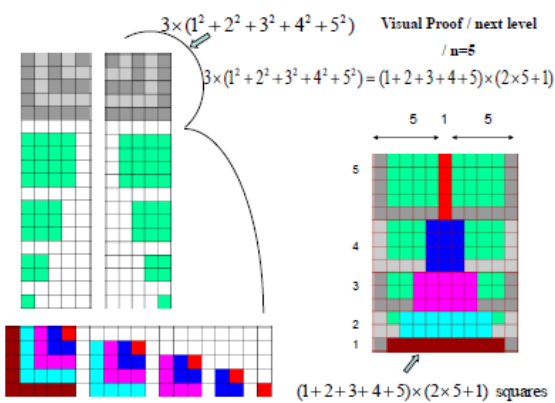
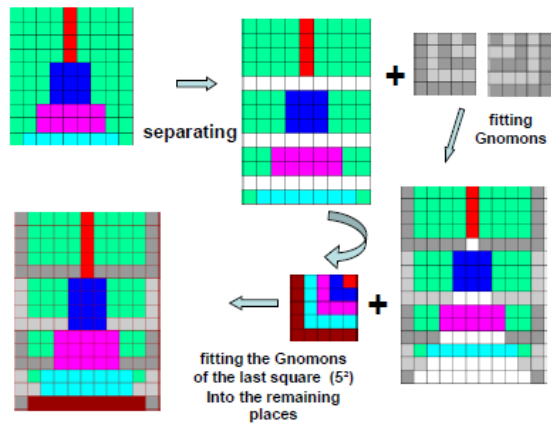
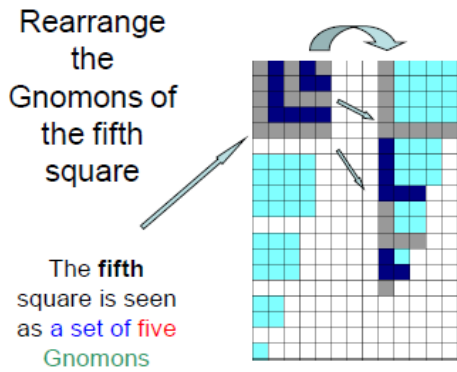
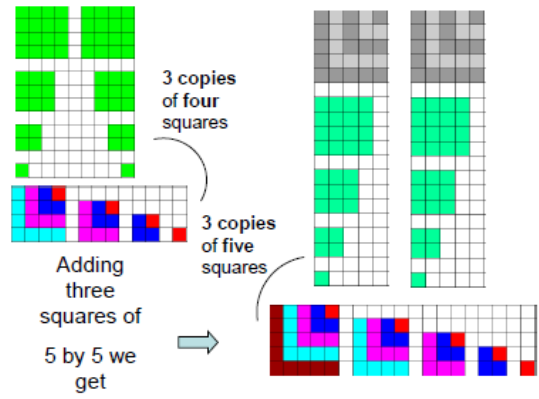
This is the
Visual proof
with $n=4$



$$3 \times (1^2 + 2^2 + 3^2 + 4^2) = (1+2+3+4) \times (2 \times 4 + 1)$$

Establish a
relationship with
the
next pattern
(3 copies of the
sum of the first **five**
square numbers)

The next pattern corresponds to a visual proof with 5 squares instead of 4 instead of 4




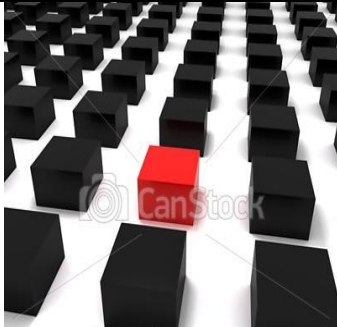

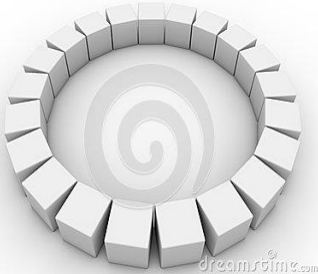



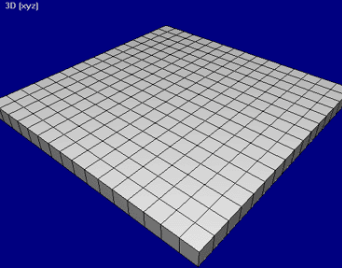

by using dotted lines or suspension dots

And labels such as :

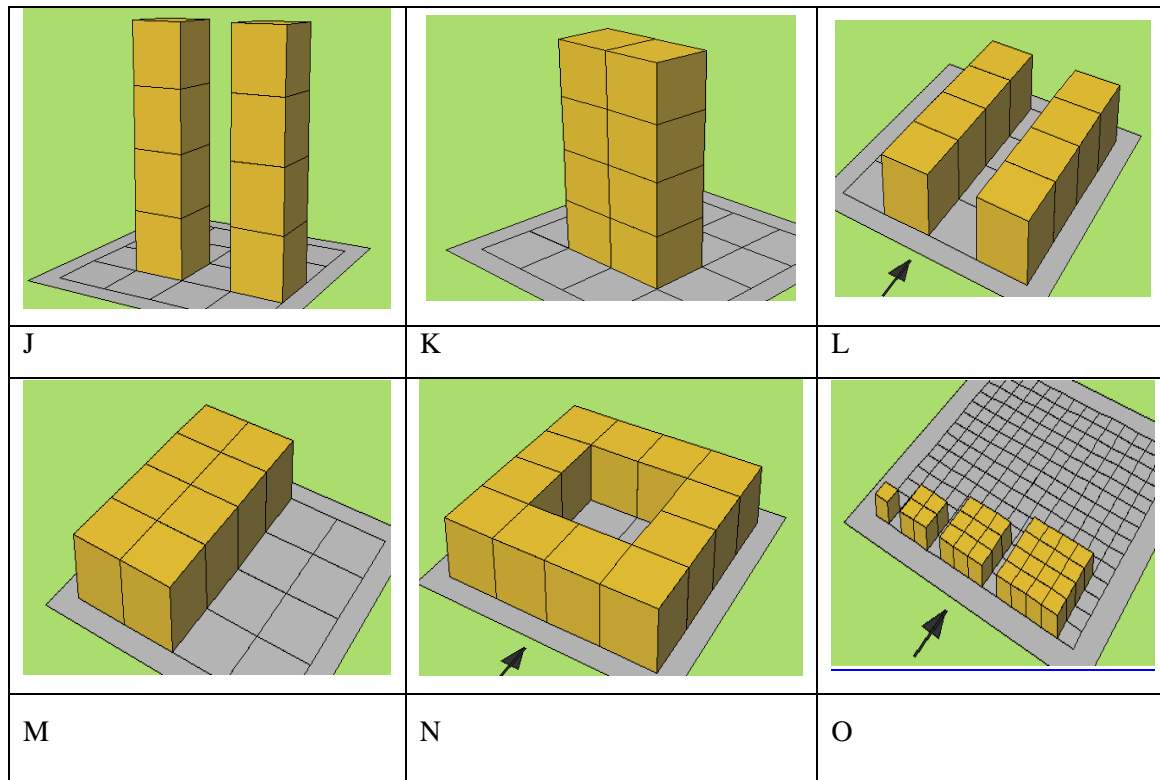
$$n, n+1, n^2 \text{ or } (n+1)^2$$

Exercice linguistique de type *consolidating*

Match the following pictures with the corresponding number(s)

		
A	B	C
		
D	E	F
		
G	H	I

- The cubes are arranged as a circle.
- It's a square of 16 by 16 cubes.
- Each level corresponds to a triangular number.
- Two parallel columns of cubes.
- All the cubes have been laid flat so as to form a square.
- The cubes are arranged around a circle.
- This is a stack of decorated cubes.

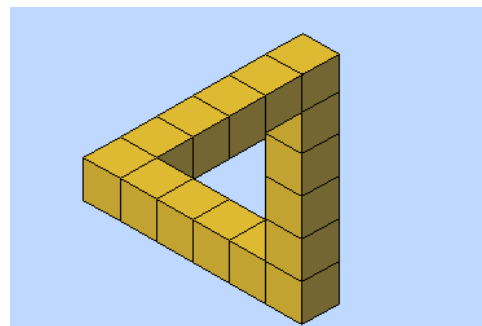


8. The cubes form a flat pyramid that makes us think of a triangular number.
9. The number of cubes at each level is a square number.
10. All cubes in this pyramid are located in a vertical plane.
11. The cubes are piled up so as to create words.
12. The picture depicts two columns in contact.
13. Cubes are placed on an imaginary grid.
14. There is a sort of passage-way. Cubes form an arch.
15. This pyramid consists of four layers of cubes.
16. The columns or the rows are apart from each other.
17. There are two rows close to each other.
18. The arrangement reveals a square-shaped hole.
19. All the cubes of the staged pyramid have been laid flat.

optical illusion

Comment upon the picture opposite:

What do you think?



Gnomons

document distribué aux élèves après une phase adidactique de découverte

Convention :

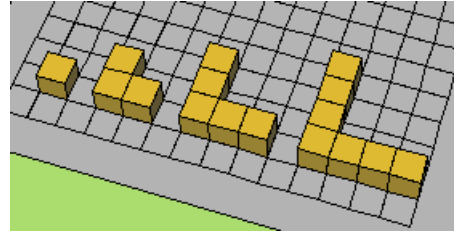
We agree to denote by G_n the n th Gnomon

$$G_1 = 1$$

$$G_2 = 3$$

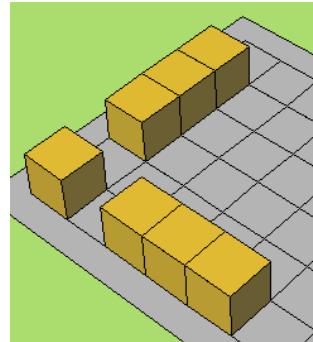
$$G_3 = 5$$

$$G_n = 2n - 1$$



check that $G_4 = 1 + 2 \times 3$

but we also have : $G_4 = 2 \times 4 - 1$



$$G_n = 1 + 2(n - 1) \quad \text{hence } G_n = 2n - 1$$

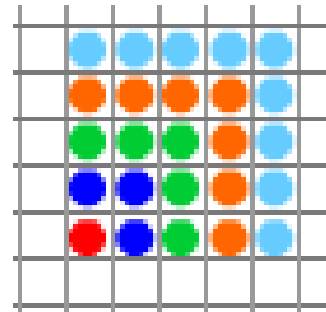
Visual proof of the following property:

The sum of consecutive odd numbers is a square (number).

On the picture opposite we have represented the case of the sum of the first five odd numbers.

$$1 + 3 + 5 + 7 + 9 + 11 = 5^2.$$

$$G_1 + G_2 + G_3 + G_4 + G_5 = 5^2$$



The principle of extension from the particular case (corresponding to the value $n=5$ for the previous picture) is natural.

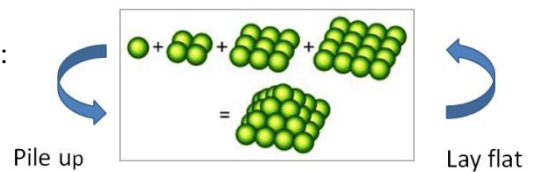
It is based on the fact that we can get a new square (the next one) by adding a Gnomon to a given square.

In the activity, our **purpose** was to :

Establish a second visual proof for the following equality:

$$3(1^2 + 2^2 + \dots + n^2) = (2n+1)(1+2+\dots+n).$$

It's no longer necessary to consider pyramids.



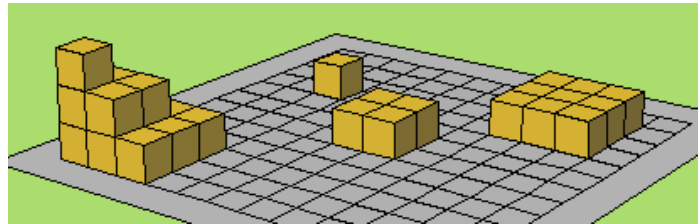
The **new visual proof** will be based on a **plane arrangement**.

We have indeed seen that the expression in the right side of the equality can be interpreted as **the product of the dimensions of a rectangle**: $(2n+1)$ and $(1+2+\dots+n)$

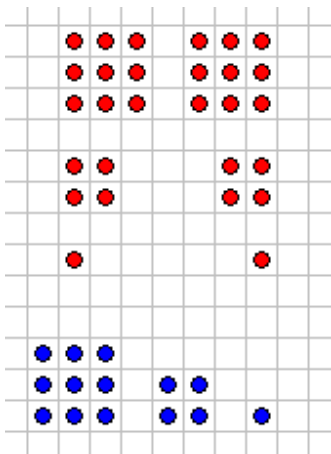
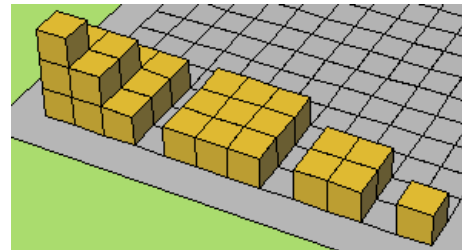
(this is the way we compute areas and it amounts to the same as counting the objects arranged as a rectangle)

Hence, we'll keep in mind a rectangle of $(2n+1)$ by $(1+2+3+\dots+n)$.

We can lay flat the different layers of the pyramid. On the picture opposite, this has been done at random.



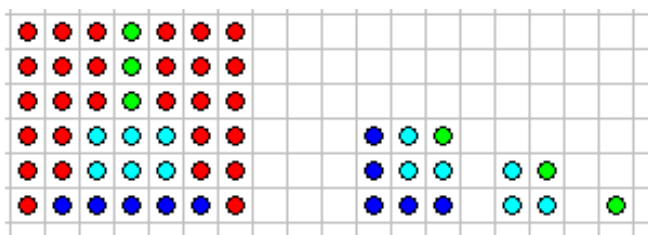
But it's more natural to lay flat the layers of cubes along the edge of the grid.



Then, we can go on, using either cubes that have been laid flat or more simply, arrangements (in a plane) of small squares (or any other objects like on the picture opposite). On this picture, we have represented **three copies of the sum** $(1^2 + 2^2 + 3^2)$.

Thinking of Gnomons leads us to the following arrangement :

In this case (corresponding to $n=3$), the rectangle has dimensions $2 \times 3 + 1$ and $1 + 2 + 3$.



Nevertheless, the **principle of extension** (possibility of extending from the particular diagram) **cannot be so easily perceived and stated**.

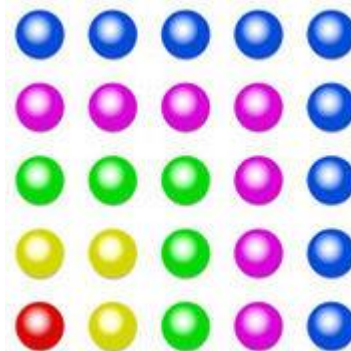
It is **not based on the mere adding of 1 single Gnomon** to the whole picture/ shape (which, in this case, is a rectangle). Hence we might find this proof is less convincing than the proof of the sum of consecutive odd numbers but more convincing than that with the 3 pyramids.

Implicite et explicite schématique / principe d'extension / gnomons

Document distribué aux élèves

Our purpose is **to make** the usual Visual Proof concerning the sum of consecutive integers **more convincing**.

We consider the visual Visual Proof concerning the sum of consecutive integers. It is based on pictures like the one opposite. It is not accompanied by any description or any comments.



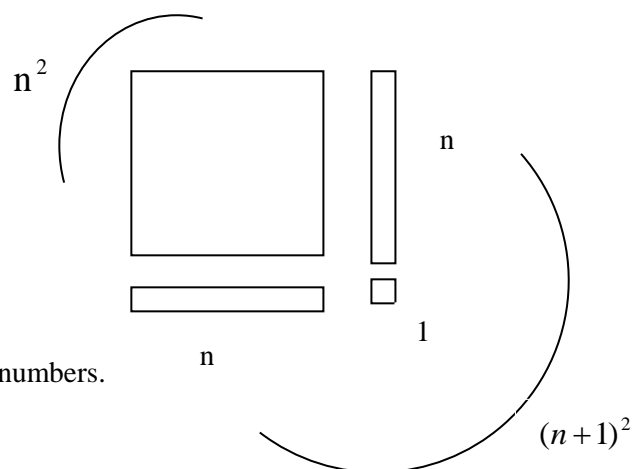
Anyway, it contains an **implicit** principle of extension.

That is: we can (and sometimes quite easily, depending on the individuals) **perceive** or **just feel** (sometimes using mental representations) the possibility of extending the picture to the next square by adding the next gnomon.

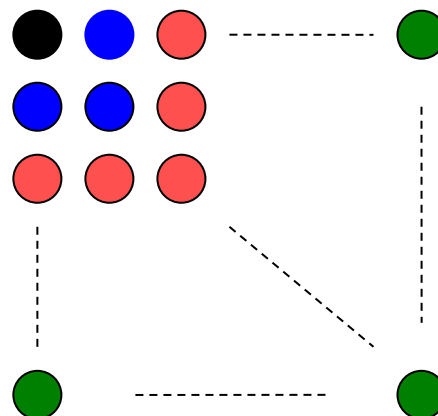
We consider that **it would be better** to add other diagrams so that the visual proof can keep closer to the standard proof by induction.

Diagram corresponding to the algebraic equality $(n+1)^2 = n^2 + 2n + 1$.

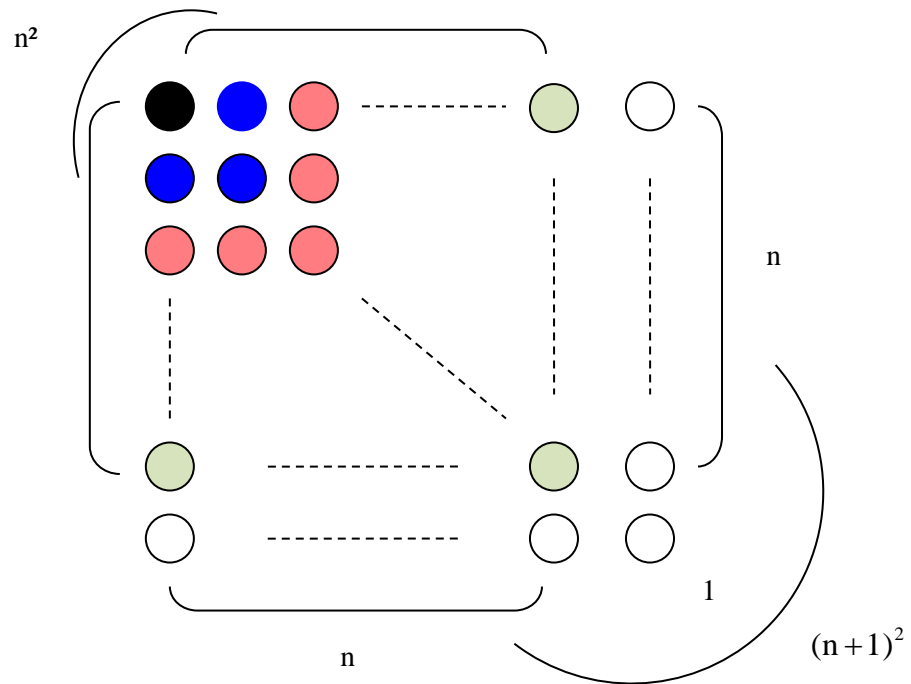
This illustrates the fact that adding a Gnomon to a square gives the next square. But this diagram **says nothing about the sum** of odd numbers.



The picture opposite is necessary to illustrate the fact that a sum of consecutive odd numbers is a square (number). Moreover, it contains an **explicit** principle of extension. The possibility of extension is **materialised** by a dotted line (which plays a similar role to suspension dots in the sum $1+3+5+ \dots + (2n+1)$). The dotted line prompts you to extend (mentally) the diagram, to extend the pattern.



We can also combine the last two diagrams:



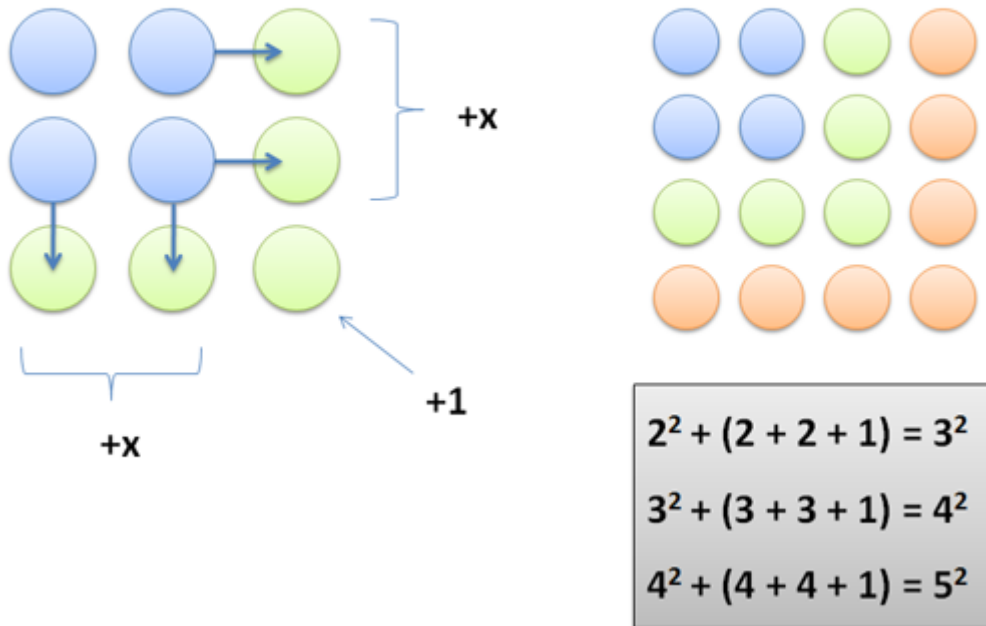
According to me, **this** diagram would deserve to be called a visual proof.

What do you think?

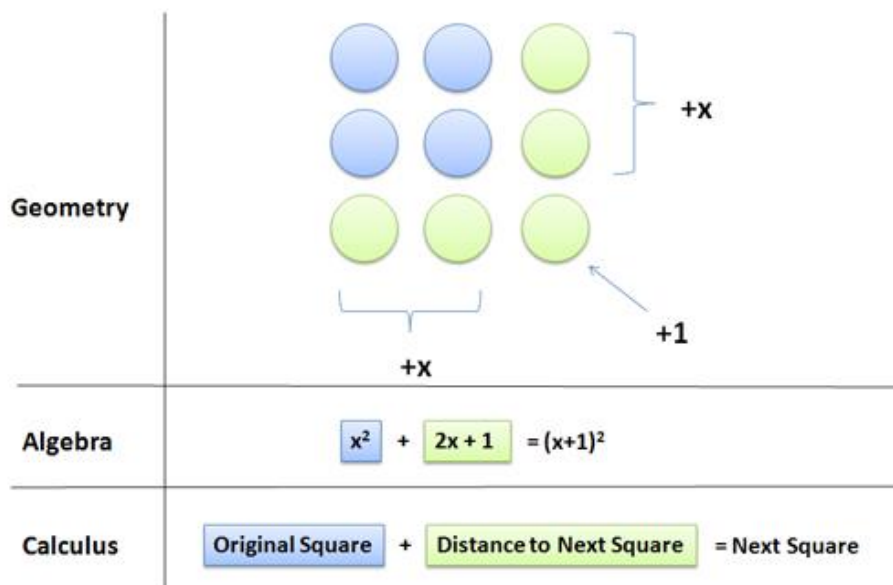
Arguments are explicit, not from a linguistic but from a visual point of view.

Document trouvés sur le net et portant sur le thème des *growing patterns*

Growing the Squares



Related Interpretations



ANNEXE 4 : Situation intitulée *Somme des Cubes*

Powerpoint utilisé lors de la séance *Somme des Cubes*

Rappel : les élèves doivent élaborer une preuve visuelle

Sum of the consecutive cubes

The formula for the *sum of the consecutive cubes of integers* is one of the **most elegant** in elementary mathematics.

The sum of the cubes **seems to be a square number**

question: it's the square of « what »?

What number is it the square of?

$n=1 \quad 1^3 = 1 = 1^2 = (\ ?)^2$
 $n=2 \quad 1^3 + 2^3 = 9 = 3^2 = (\ ?)^2$
 $n=3 \quad 1^3 + 2^3 + 3^3 = 36 = 6^2 = (\ ?)^2$
 $n=4 \quad 1^3 + 2^3 + 3^3 + 4^3 = 100 = 10^2 = (\ ?)^2$

what sort of number is in the bracket?

Purpose of the introduction:

establish a **conjecture** for the formula concerning the sum of the cubes.

Denote by n the number of cube numbers in the sum.

What do the numbers **1, 3, 6, 10 ...** remind you of? Do you recognize the numbers?

Hence we have:
 $n=1 \quad 1^3 = 1 = 1^2 = (1)^2$
 $n=2 \quad 1^3 + 2^3 = 9 = 3^2 = (1+2)^2$
 $n=3 \quad 1^3 + 2^3 + 3^3 = 36 = 6^2 = (1+2+3)^2$
 $n=4 \quad 1^3 + 2^3 + 3^3 + 4^3 = 100 = 10^2 = (1+2+3+4)^2$

Consider the following sums:

$n=1 \quad 1^3 = ?$
 $n=2 \quad 1^3 + 2^3 = ?$
 $n=3 \quad 1^3 + 2^3 + 3^3 = ?$
 $n=4 \quad 1^3 + 2^3 + 3^3 + 4^3 = ?$

$n=1 \quad 1^3 = 1$
 $n=2 \quad 1^3 + 2^3 = 9$
 $n=3 \quad 1^3 + 2^3 + 3^3 = 36$
 $n=4 \quad 1^3 + 2^3 + 3^3 + 4^3 = 100$

what do we notice?

We can now establish the **conjecture:**

$1^3 + 2^3 + \dots + n^3$ seems to be equal to $(1+2+3+\dots+n)^2$

We can notice that the expression $1+2+3+\dots+n$ can be replaced by $\frac{n(n+1)}{2}$

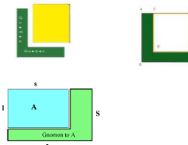
Purpose of the next part of the activity (groupwork): elaborate a **Visual Proof** of the previous property

The **Visual Proof**, as a demonstration, must be both **practical** (using real cubes) and **schematic** (making diagrams)

you must be able to explain or demonstrate

why, if something works at **some level** (for some particular value of n),

it will **necessarily** work at the **next level** (for $n+1$).



explain or demonstrate **why**, if something works at **some level** (for some particular value of n), it will **necessarily** work at the **next level** (for $n+1$).

All manipulations will have to be **accompanied** by a **verbal (oral) description** and, if possible, a **schematization**

Make a **poster** for **explanations** and, if possible, a **generalization**

The diagram opposite illustrates the fact that adding a **Gnomon** to a square gives the next square. But this diagram says **nothing** about the **sum of odd numbers**.

The picture opposite contains an **explicit principle of extension**

The possibility of extension is **materialised** by a **dotted line**

Anticipate what you are going to say (during the *physical* demonstration)

Recall a **Gnomon is usually L-shaped** (has the form of the letter L) its dimensions are **not necessarily** fixed

Cubes available : (until $n=3$)

Next cube (for $n=4$): not available!

Introduce order while arranging the (small) cubes

arrange **methodically**

think of a **generalization** of the way you arrange the small cubes.

Document-support distribué pendant la séance

(après la phase d'établissement de la conjecture et au début de la phase adidactique)

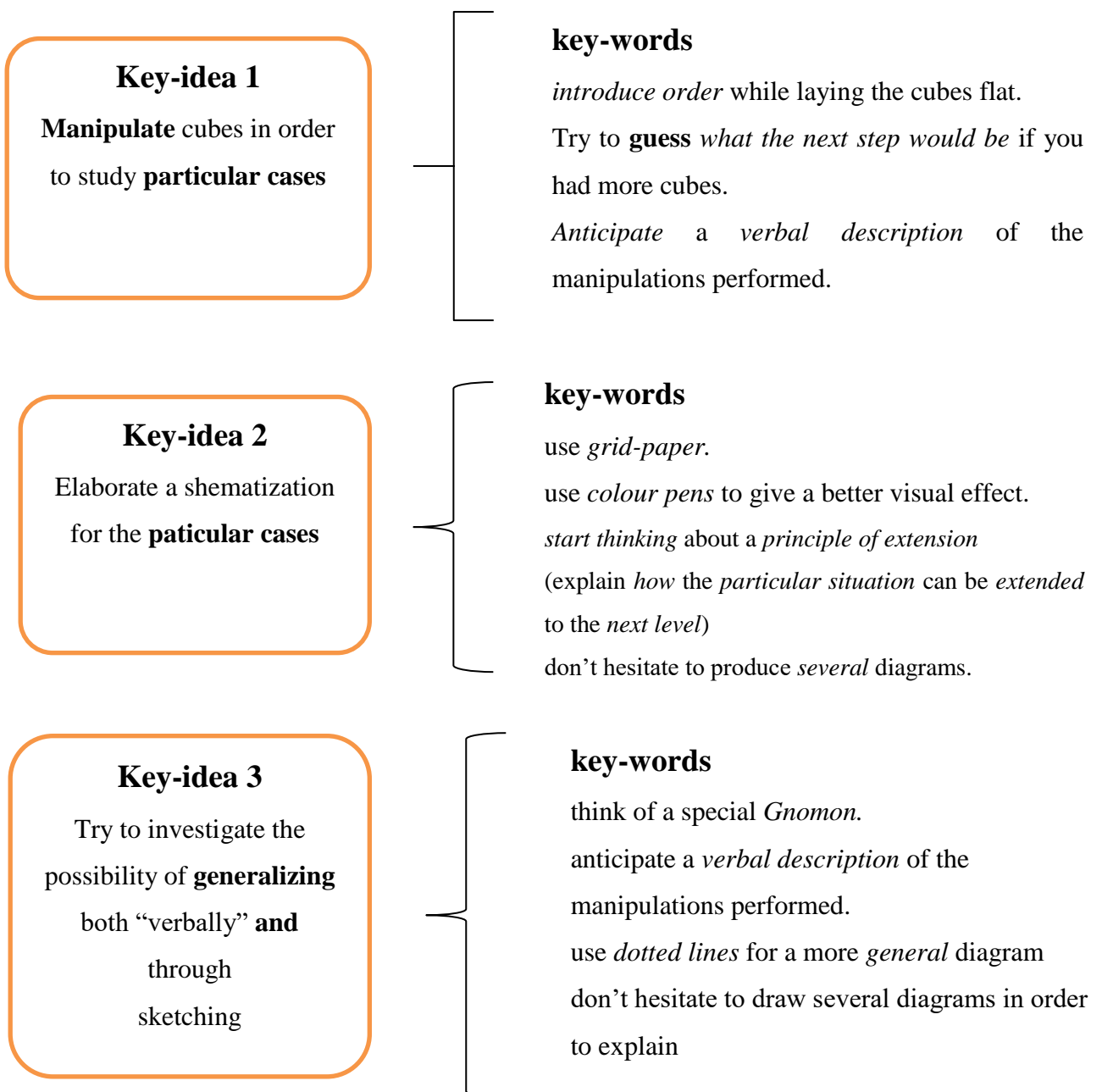
Sum of consecutive cubes

$$1^3 + 2^3 + \dots + n^3 = (1 + 2 + 3 + \dots + n)^2$$

this property has been established as a conjecture

Purpose of the activity : **elaboration of a Visual Proof** after manipulating real cubes.

We expect both a **concrete proof** , “using real cubes” , **and** a proof based on a **diagram**.



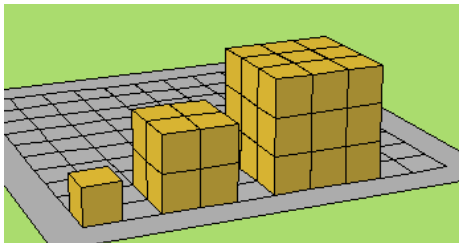
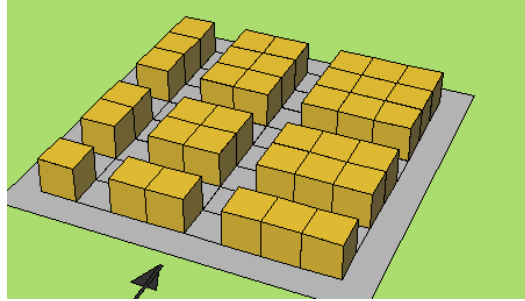
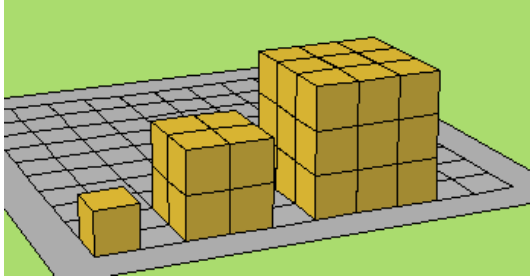
Convincingness of the proof

It is *based on* the following fact:

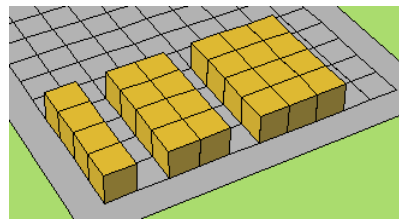
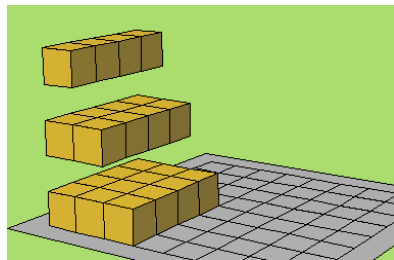
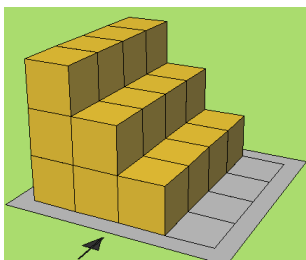
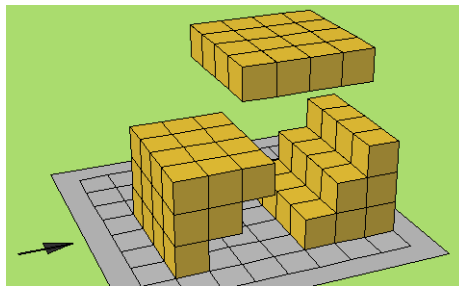
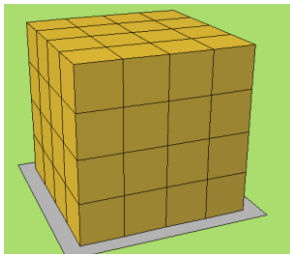
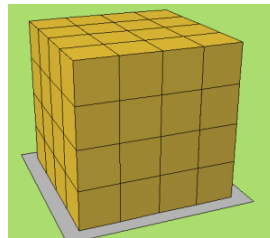
you must *be able to explain or demonstrate* (“making both a **physical and a schematic demonstration**”) why, if something works at some level (for some particular value of n) it will **necessarily** work at the next level.

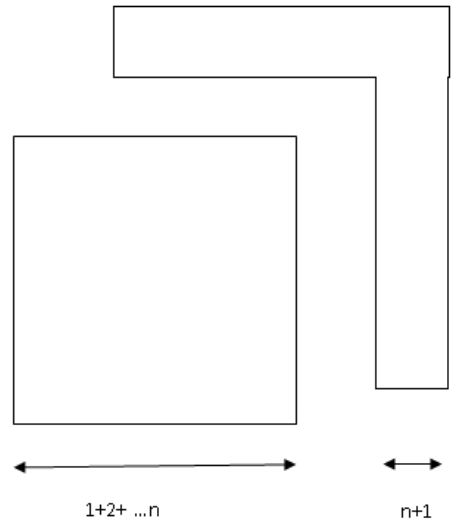
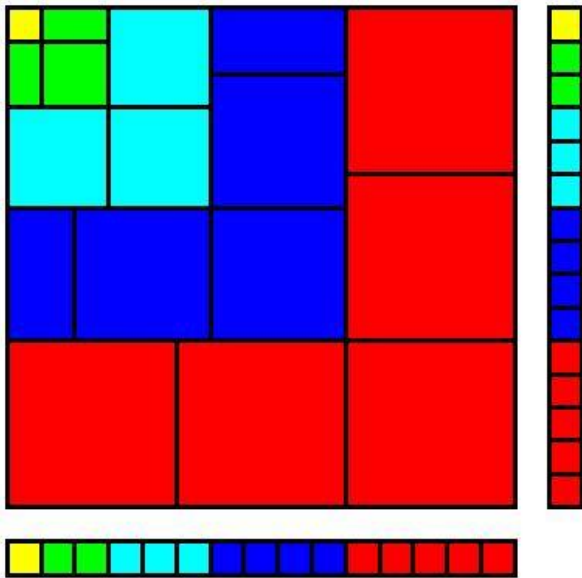
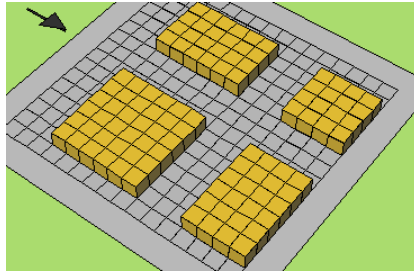
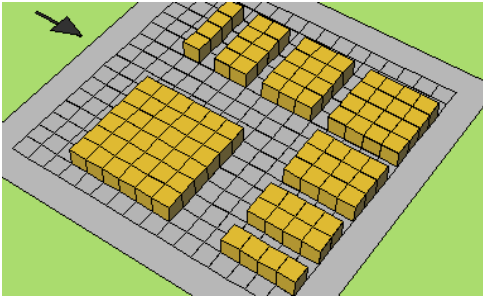
Montage pour la phase d'institutionnalisation de la séance Somme des Cubes

Montage réalisé à l'aide d'un logiciel du site Wisweb



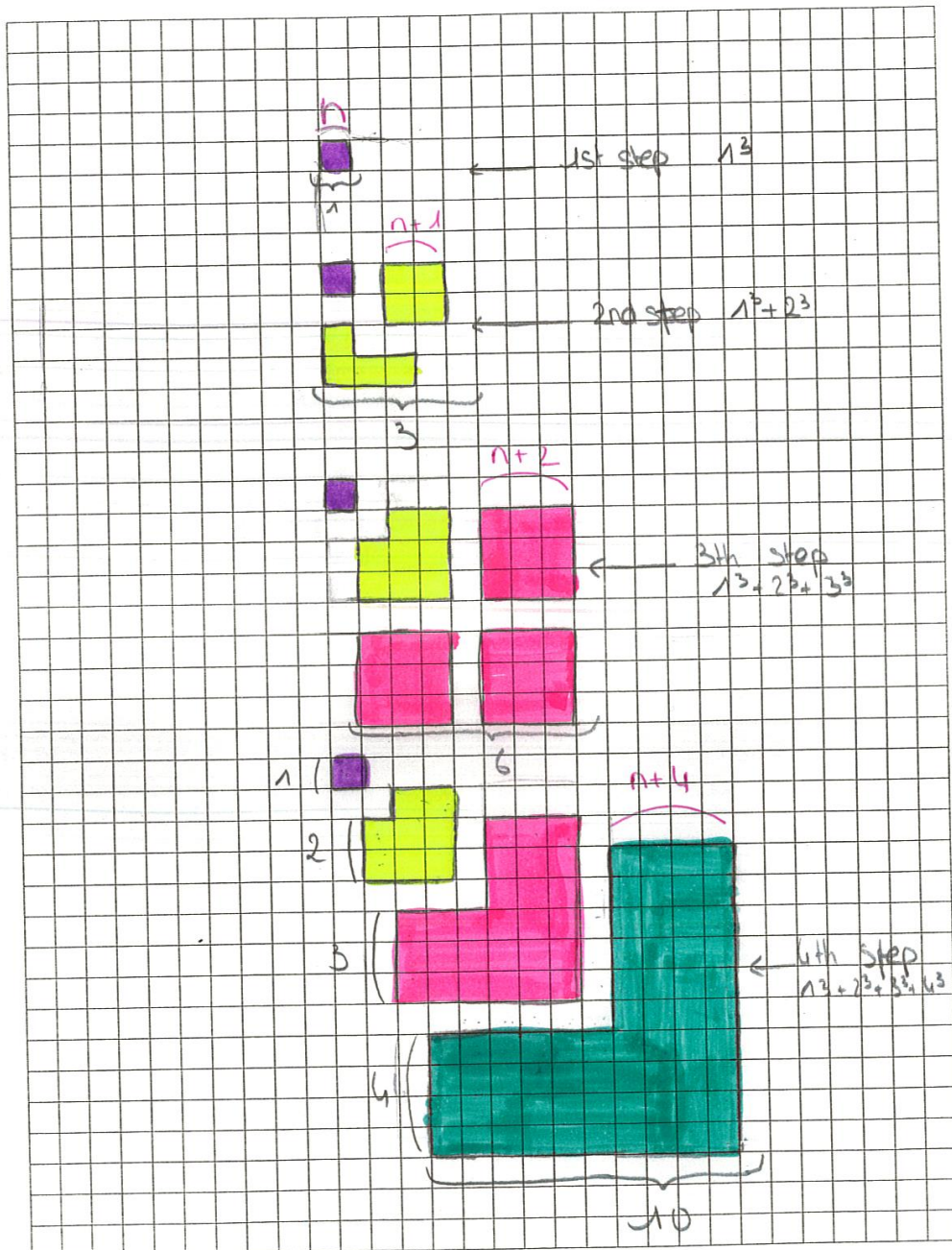
+



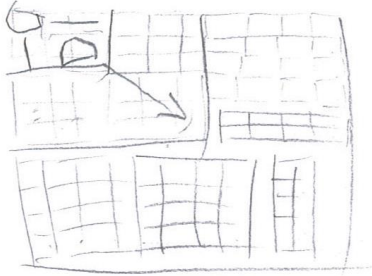
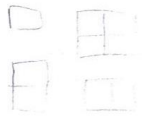


Posters réalisés lors de la séance Somme des Cubes

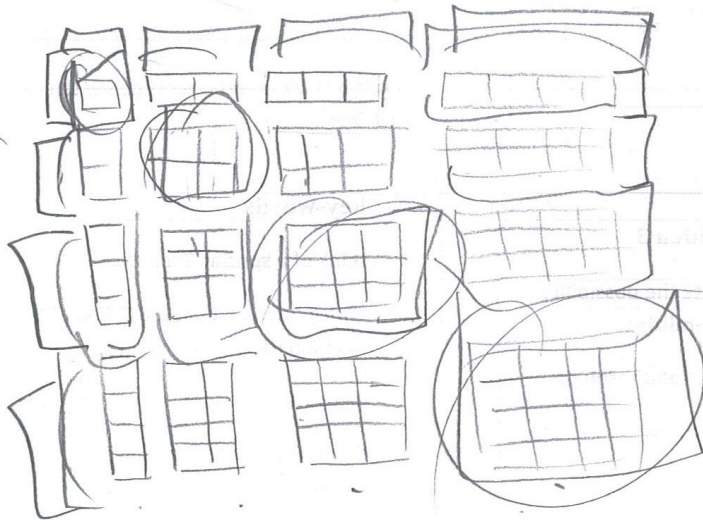
Documents produits par les élèves (classe de terminale européenne)



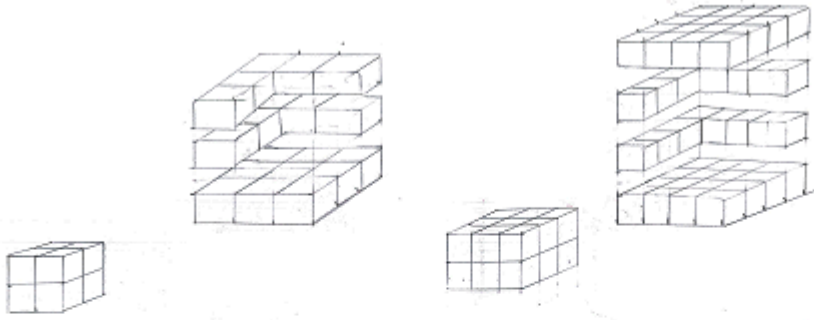
Lola, Faustine, Sophie, Emma



XXXX

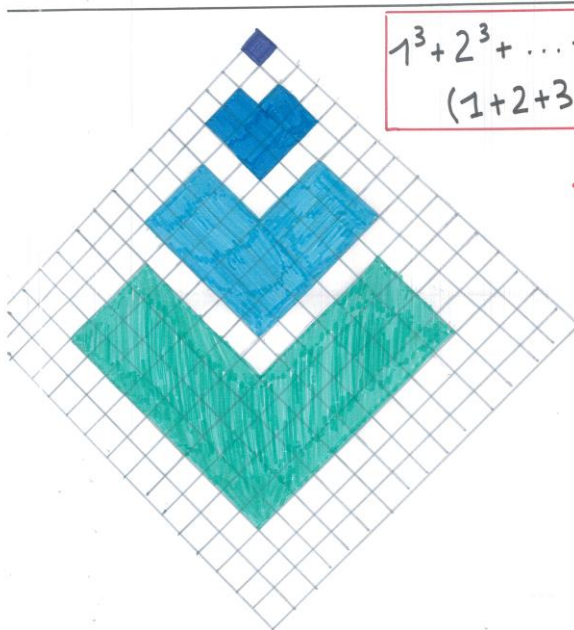


Why does it work?
 Key-idea 3

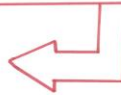


LAVIELLE Claire
 LARUE Margot
 BANSE Paul
 TERRADE Gauthier
 SOUILLARD Guillaume

LAVIELLE Claire, SOUILLARD Guillaume, LARUE Margot, BANSE Paul, TERRADE Gauthier

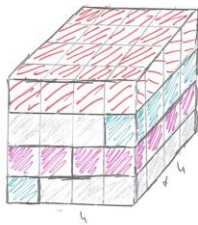
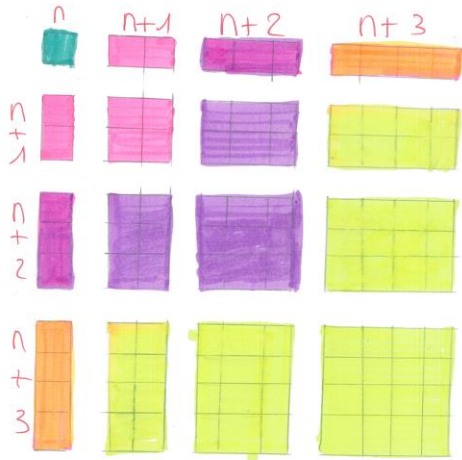


$$1^3 + 2^3 + \dots + n^3 = (1 + 2 + 3 + \dots + n)^2$$



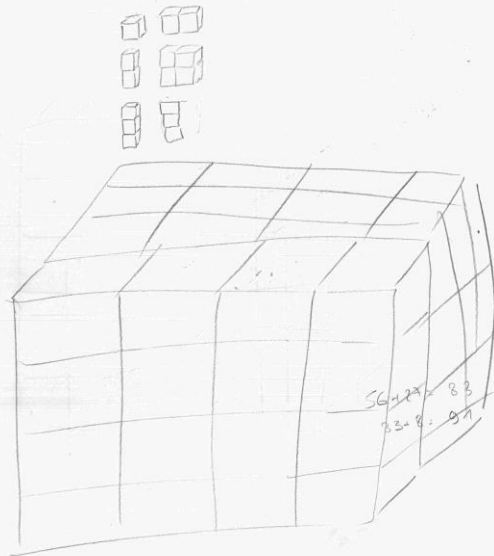
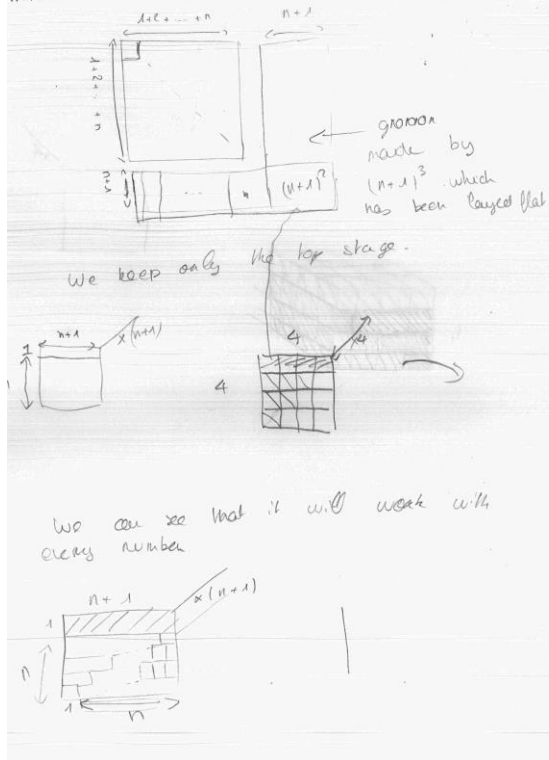
HATRY, TRUCHETET, RENARD, DIARRA

SUM OF CONSECUTIVE CUBES

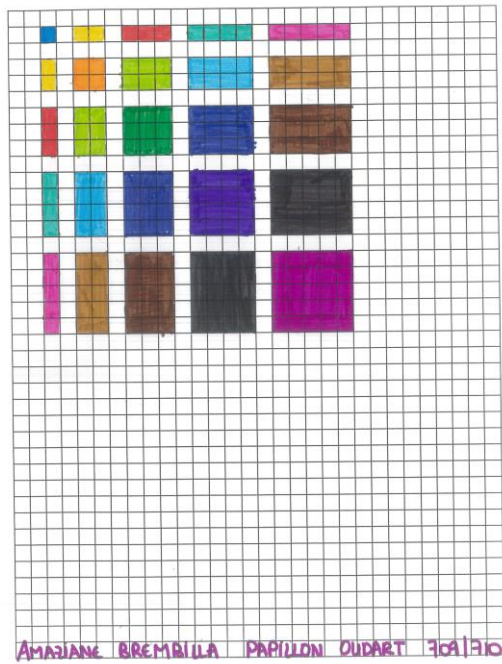
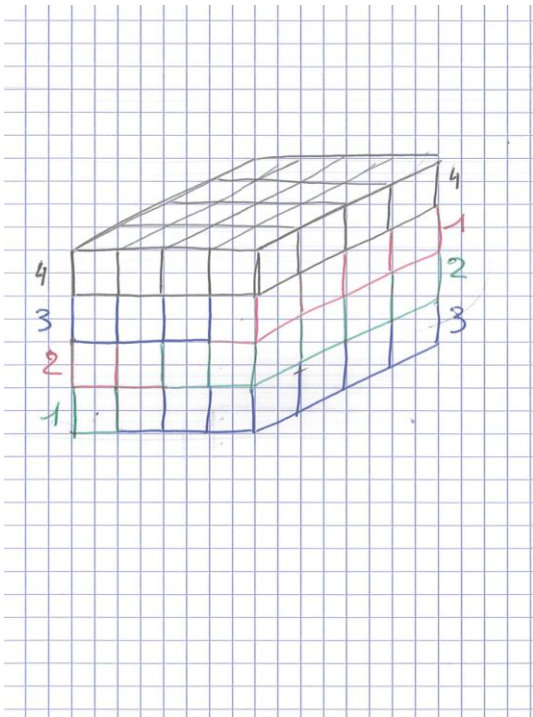
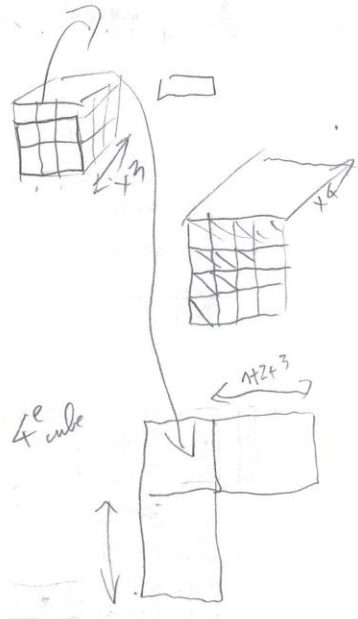


We have layered flat the cube 4×4 to make the gnomon.

HATRY
TRUCHETET
RENARD
DIARRA

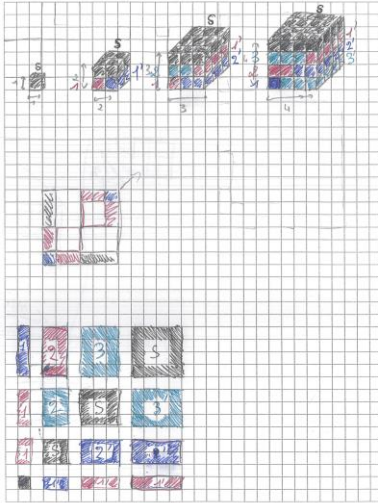


$(n+2)^2 = n^2 + 6n + 4$
 $m^3 + (n+1)^3 = m^3 + n^3 + 3n^2 + 3n + 1$
 $1^3 + 2^3 = 9$
 $1^3 + 2^3 + 3^3 = 36$
 $1^3 + 2^3 + 3^3 + 4^3 = 100$
 $(m+n)^3 = (m+n)^2(m+n)$
 $(m+n)^2 = (m^2 + 2mn + n^2)$
 $(m+n)^3 = (m^2 + 2mn + n^2)(m+n)$
 $(n+1)^3 + m^3 = 2n^3 + 3n^2 + 3n + 1$
 $(m+n)^3 = m^3 + 3m^2 + 3m + 1$
 $m^3 + 3m^2 + 3m + 1 = m(m^2 + 3m + 3) + 1$
 $= m^3 + 3m^2 + 3m + 1$
 $= 2n^3 + 3n^2 + 3n + 1$
 $= m(2n^3 + 3n^2 + 3n + 1)$



GAUTIER MATHIEU
 RENOIR MALCOLM

711

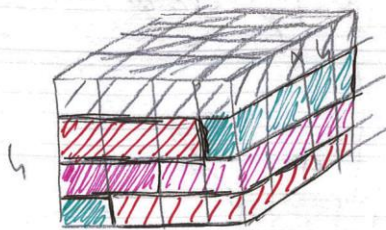
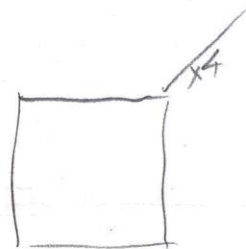
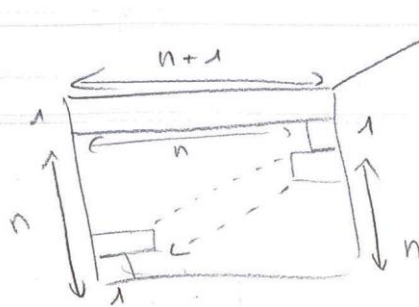
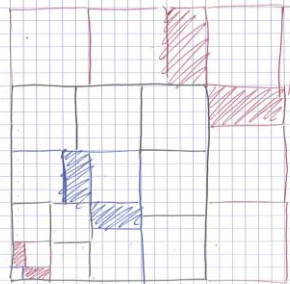
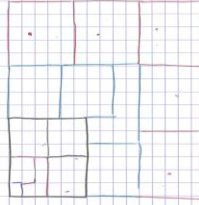


GAUTIER M.
 RENOIR M.

711

$$u_4 = 1$$

$$u_{min} = (n+1)!$$



CBM 26

FINDAIR

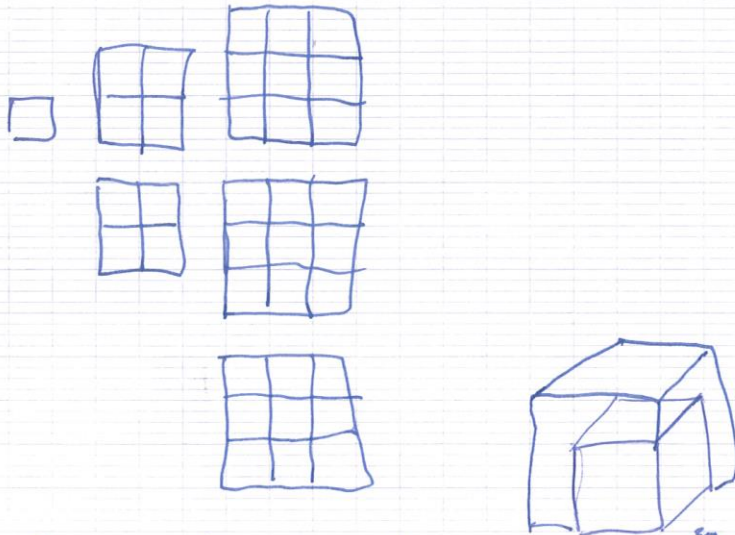
Clare
Margot
Guillaume
Gauthier
Paul

$1^3 = 1$	1^2	
$1^3 + 2^3 = 9$	3^2	+2
$+ 3^3 = 36$	6^2	+3
$+ 4^3 = 100$	10^2	+4

$$\sum_{k=1}^n k^3 = (n-1)^2 + n)^2$$

$$= (1+2+3+\dots)^2$$

$$= \left(\sum_{k=1}^n k\right)^2 = \left(\frac{n(n+1)}{2}\right)^2$$



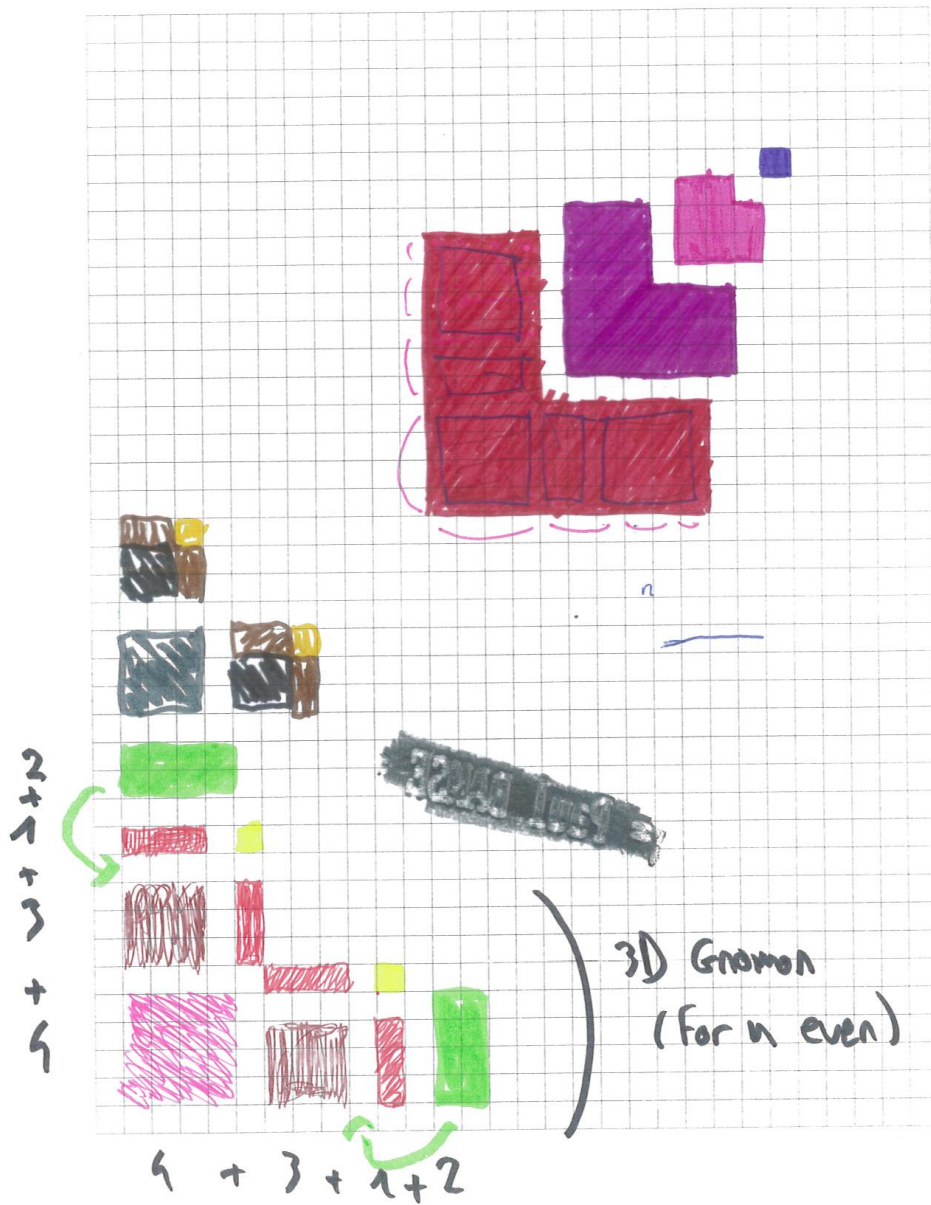
$$n^3 = n^2 + 2(n-1)^2 + (n-1) + (n-1)^{3/2}$$

~~$1(n) + 3 \times (n-1+n)$~~
 ~~$1(n+n-1+n-2+n-3 \dots 0)$~~
 ~~$3(n+n-1+n-2+n-3 \dots 0)$~~

$$\frac{n^3}{3} = \underbrace{(n-1)^3} + \underbrace{(2n+1) \times n^2}$$

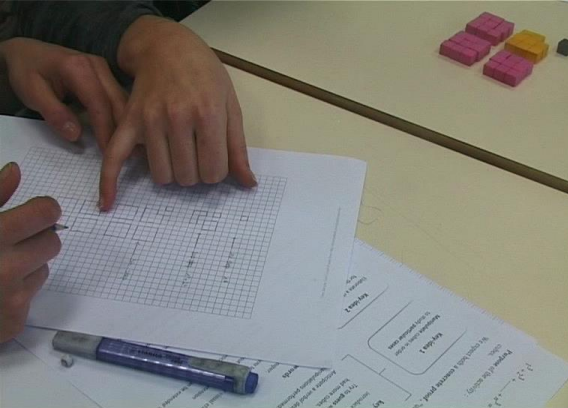


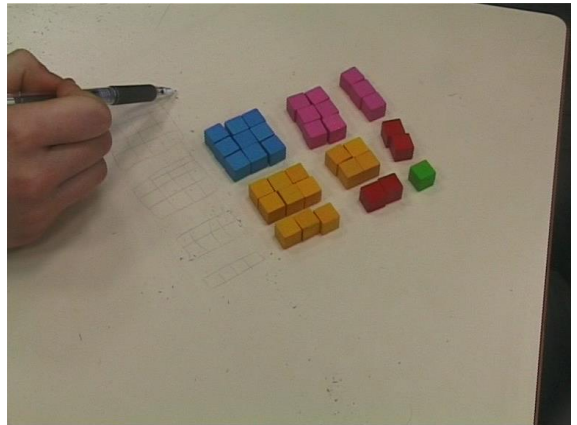
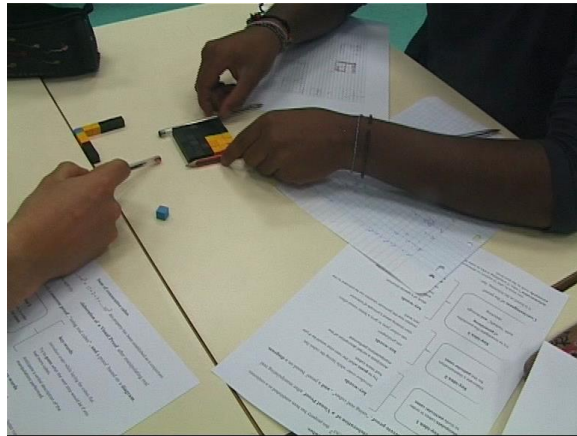
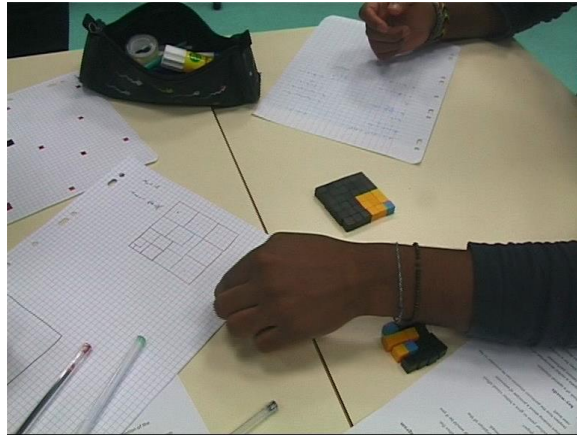
Claire
Margot
Guillaume
Gauthier
Paul

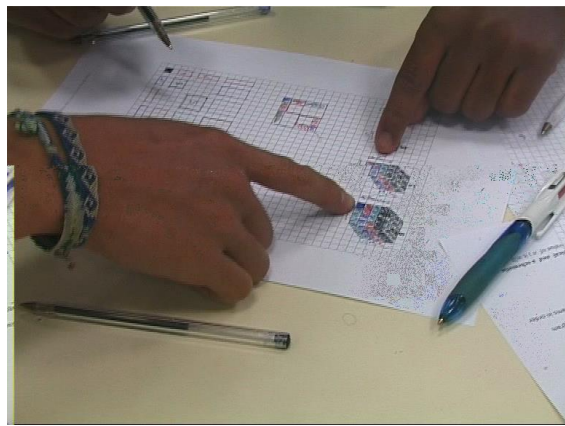
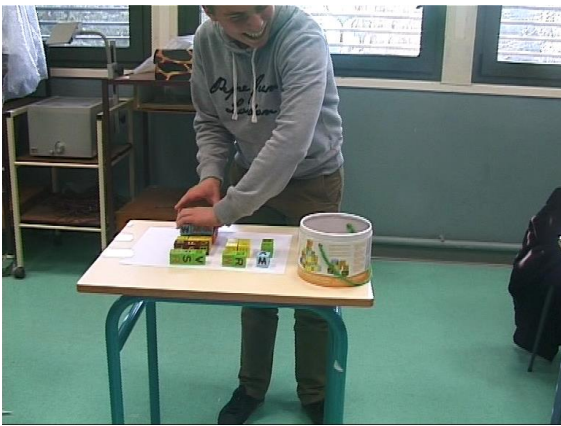
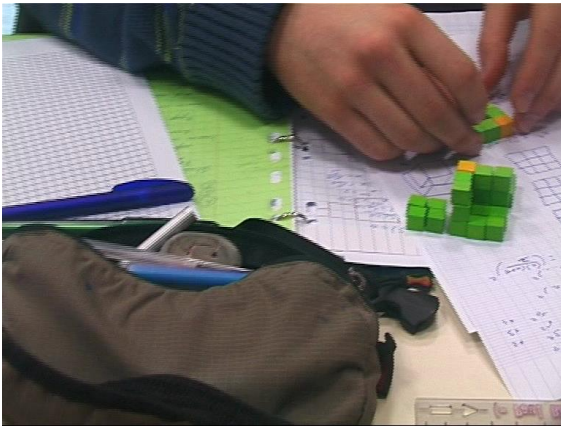
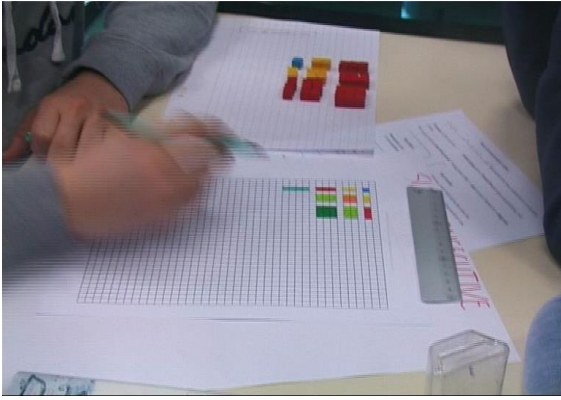


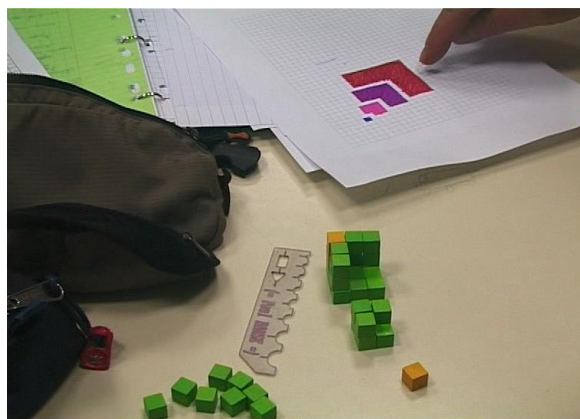
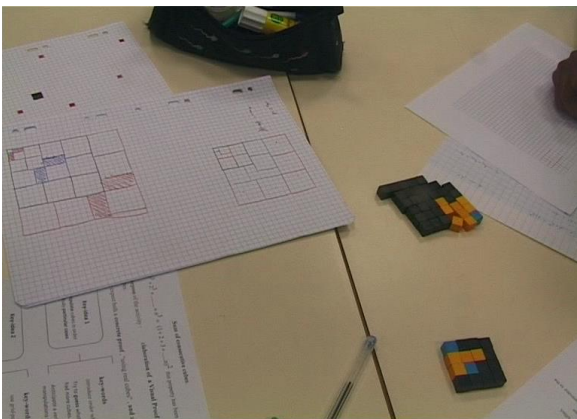
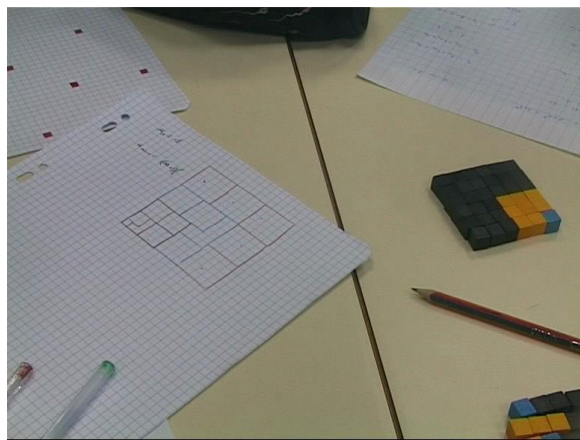
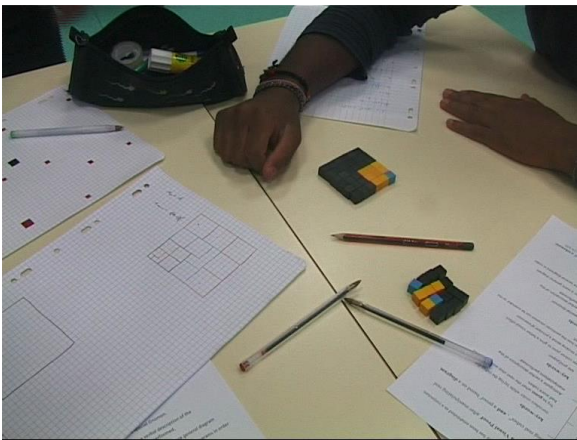
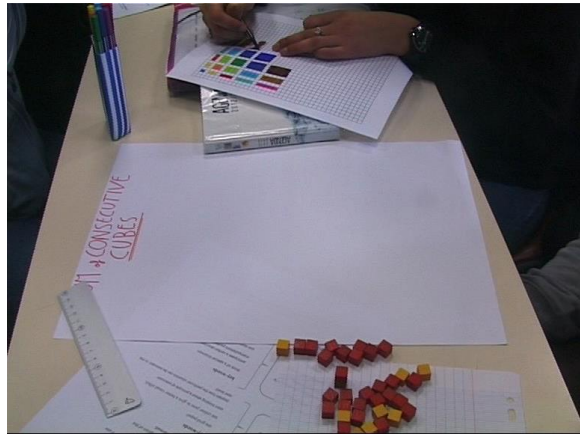
Photos

Séance expérimentale Somme des cubes et Preuve visuelle









Transcriptions de la séance et commentaires

I) Phase interactive d'introduction avec support Powerpoint

La première phase est un « warming-up »

1. P The topic is about cube numbers
About the sum of consecutive cube numbers
Is that clear to you what a cube number is?
(s'adressant à E1)
Give me an example of cube number

Premières diapositives du PowerPoint ayant servi de support à cette phase interactive

Warming-up et établissement de la conjecture

2. E1 8
3. P Why is it a cube?
4. E1 It's 2 times 2 times 2
5. P [?] sum of consecutive cubes
What is the first cube?

Sum of the consecutive cubes

6. E1, E2 ensemble
1

The formula
for the sum of the consecutive
cubes of integers
is one of the most elegant
in elementary mathematics.

7. T Justify why
8. E2 It's 1 times 1 times 1
9. T 1 cubed is equal to 1

(répète en insistant sur la dernière syllabe)
1 "cubed"

The purpose of the first part of the activity is to establish a conjecture

Purpose of the introduction:

That's what we are going to do now
Establish a property but only at the level of a conjecture

establish a conjecture

(s'adressant à toute la classe)

for the formula concerning the sum of the cubes.

Does anybody know the formula giving the sum of consecutive cubes?

10. E3 You should multiply the sum of the consecutive squares and ...heu and [?] by the ...
(inaudible)

11. P There is square behind this but it's the sum of the consecutive numbers

[légère pause]

all “squared”¹

You know the property but you cannot state it properly

You’ve got some memory of the formula anyway

[E3 n’a pas suivi le même cursus scolaire]

Normally you’re not supposed to know this formula

Denote by n ...

1 cubed, 2 cubed ...

You understand

That’s the sum

Look

Take a look at this slide

Voir désormais page 48

pour le détail des diapositives
utilisées dans la suite de la phase
interactive d’établissement de la
conjecture

[P indique les égalités à compléter ; elles figurent sur la 5^{ème} diapositive p48]

For 1, one cubed is...

E4 [spontanément]

One

P One cubed plus eight is ...

E5 nine

P plus 27

3 times 3 times 3

3 times 9

27

[P interroge E6 du doigt]

E6 36

P What sort of number is it?

E1 + E2 +....

A square

P that’s a square

Take a look at that

[P indique en même temps la 6^{ème} diapositive]

¹ all squared : *le tout au carré*

One cubed plus... plus four cubed that is 64.

It's a square number

You see

It works

So what is the conjecture?

[P s'adresse à E7 en particulier, en amorçant par la formulation...]

It looks like or it seems that ...

E7 the square of the sum of the four...

[E7 montre en même temps au tableau, à distance, la formule concernée]

P the question is

And this will be a bit colloquial

C'est familier [alternance codique volontaire]

"That's the square of what"

If you want to be more correct

"what number is it the square of?"

You understand this question??"

[P attend l'assentiment des élèves]

E1 + E2 + ...

Yes

P It's difficult to state it in English

[P s'adresse à E8]

So, what is in the bracket?

E1 + E2+... [spontanément]

It's one

P that's the square of 1

And then 3

Here, it's 6

What sort of number is in the bracket?

[P s'attend à ce que les élèves reconnaissent des « nombres triangulaires]

So ?

E6 Triangular number(s)

P Triangular numbers

Exactly

What is a triangular number?

I mean, as a number

You remember the picture.

- Forget it.
- As a number
- What is a triangular number?
- If the first one is 1
- How do you get the next one?
- [P s'adresse à E9]
- You have one, then you add 2, that's 3
- The you ...
- It's the sum of ...
- E1 + E2 + ...
- Consecutive numbers
- P Integers
- Is that the conjecture?
- Can you formulate the conjecture as a property now?
- E1 + E2 + ...
- Yes
- [P s'adresse à E8]
- Try to formulate the property with "any" integer n
- E8 The sum of the consecutive cube numbers is equal to ... [hésite]
- P to the
- [P indique l'exposant 2 sur la diapositive correspondante/ P attend]
- E5 [rebondit]
- Is equal to the sum of the square of the consecutive... [pause] integers
- P exactly
- That's exact
- Now
- Do you agree with what she said?
- E1 + E2 + ...
- Yes
- P do you want to make a traditional proof by induction?
- E1 + E2 + ...
- No, no
- P no, that's not what you're supposed to do now
- You're going to explain things through manipulations
- ... try to find a principle of extension
- You're going to make diagrams

And when you feel confident enough, you will show me you have understood using gestures.

[P affiche la diapositive suivante afin de récapituler : « we can now establish a conjecture, etc... »]

You remember the formula

$E1 + E2 + \dots$

Yes

P But we don't need it (really)

Forget it

That's not algebraic

It's $1 + 2 + 3$ squared

That is, it's a square

What does it make you think of?

What does it call to mind?

[à ce moment, E9 lève déjà la main]

That's physical

Because cubes are something physical

We are going to “**play**” with **real** cubes. [insistance]

[à ce moment, P aperçoit E9 qui lève déjà la main]

Yes ?

E9 It makes me think of a “stack” of cubes.

[E9 vient de réinvestir un terme rencontré lors de la séquence précédente ; celle-ci incluait une phase de consolidation²]

P A stack of cubes or, more generally, an arrangement

It's 3D geometry

It's (in) space

You have to do something in space

But a square is not really in space

How could you symbolize a square in a diagram?

We can symbolize cubes by some small squares.

But you remember all that, ok?

You must keep it in mind

You are going to establish a visual proof

I don't expect a traditional proof

² Voir Annexes 3

I want you to schematize

You are going to demonstrate with diagrams and you will have to manipulate real cubes

You will have to explain why, if something works at some level, it will necessarily work at the next level

This is the principle of a proof by induction

This is the **logical** principle

But we are not going to make a proof by induction

We are going to make something concrete

We are going to make manipulations

All the manipulations will have to be accompanied – that's the linguistic aspect- will be accompanied by a verbal description

I want you to speak and describe what you do

Is that clear?

E1 + E2 + ...

Yes

P

I expect you to describe and comment upon your schematization

You will have to make a poster

You know how to generalise

So, if possible, make a generalisation of the property

Use dotted lines

On diagrams

Anticipate what you are going to say

You prepare your explanations to come

[P présente une diapositive, en 3D, élaborée avec Wisweb]

Think of introducing order

Arrange methodically

Think of the way you arrange small cubes at a given level and describe how to generalise

And don't forget Gnomons

Remember that they are usually L-shaped

Ensuite les élèves didtribuent le document-support (voir p 48).



Ils se mettent rapidement en groupes de trois ou quatre pour travailler.

	<p>Courts extraits :</p> <p>Groupe 1</p> <p>E1 « après on a une longueur de 2 ici, donc c'est $n!$ »</p> <p>E2 « non, c'est $n-1$, parce que ton n c'est 3 »</p>
--	---

	<p>Groupe 2</p> <p>E3 « Bon, juste une question. Est-ce que c'est notre figure initiale, Est-ce qu'on en a besoin pour notre poster ? »</p>
--	---

Les échanges sont nombreux. Ils concernent par exemple les propositions d'agencement des cubes, la manière de les nommer, la manière de les schématiser, etc...

Le paragraphe suivant est une transcription de la phase adidactique.

II) Phase adidactique

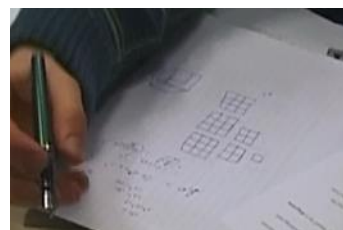
La phase adidactique débute par la distribution du document-support et des cubes.

Les élèves se placent en groupes de trois ou quatre. Au sein des groupes, les élèves sont invariablement notés E1, E2, E3 et éventuellement E4.

La caméra passe de groupe en groupe.

Groupe 1

1. E1 [...] On peut faire des gnomons 3D
Un élève dessine une figure en perspective
Deux élèves manipulent les cubes (E2, E3)
2. E2 Non, c'est pas comme ça le gnomon 3D !
3. E3 Non, c'est comme ça...
En fait tu rajoutes 1 en bas
Et ... [pause] sur les côtés...
4. E2 Oui
Faut que ce soit en biais, oui
E2 rajoute un cube en prolongeant la disposition amorcée par E1
5. E1 Attends. Après il va falloir qu'on cale ça avec des stylos pour que ça fasse bien droit.
E2 place des stylos
E3 place des cubes de son côté
6. E2 Comme ça...
Il va falloir peut-être des cubes en couleur.
Je me répète, là, mais, heu...
7. E3 « English » !
8. E1 Non, on a le droit de parler en anglais, non , heu..., en français.
[pause]
Après, avec le 3D, à la base, là, on a un cube normal...
enfin, à la base, on a un carré.
Donc c'est n^2 ...
9. E2 Hum [acquiesce] ...
E2 suit le raisonnement de E1
10. E1 Ici, on a $n-1$ carrés et $n-1$ carrés



Après on a une longueur de 2 ici, donc c'est n
Ici c'est n

11. E2 Parce qu'il a dit qu'il fallait décomposer [?]
... une formule [?]

12. E3 Non, c'est n-1.
Parce que ton n c'est 3

13. E2 Parce que $(n-1)^2$ c'est ça et ça...

14. E3 C'est les petits [?]

*E2 prend deux carrés (2x2) dans les mains à
partir de l'empilement 3D*

E2 fait tomber les cubes

15. E2 On l'a détruit !
De toute façon, fallait le refaire...

[...]

Groupe 2

[...]

16. E1 Non, c'est pas ça !

17. E2 C'est parce qu'on a pas fait de carré(s).

18. E1 Non, mais genre comme ça...

Moi, j'aurais fait... et hop... un petit carré là

E3 prend la fiche avec les consignes et relit

Groupe 3

19. E1 On avait 100 tout à l'heure...

20. E2 Ah non ... mais c'est n... $n \times 3$
Quatre couches.

21. E1 Tout à l'heure, les nombres qu'on avait...

La somme des nombres positifs,

On avait 1, 8... on avait 1, 3, 6...

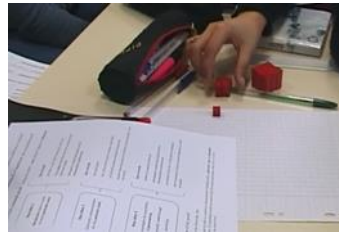
Groupe 4

22. E1 Tu peux me passer ton stylo.

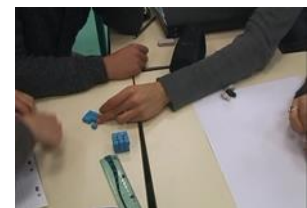
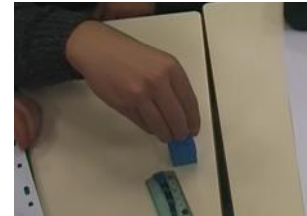
23. P Vous vous arrêtez à 3.

Moi, je veux l'étape suivante.

24. E2 Mais oui ça fait 6



25. E3 Parce que là, si on met sur le côté...
26. E4 Vas-y, décompose...
27. E3 Non, non...
28. E4 Regarde, tu laisses la base en bas, hop...
Tu mets sur le côté... Et là
« Plus one »
 1^2 , ça c'est 2^2
29. E3 Si tu les ajoutes, les deux, ça fait 3^2
 $1+2$ au carré.
30. E2 Ça c'est 2 au cube
E1 Oui mais si tu les ajoutes les deux ça fait quoi...
Ça fait plus un carré, ça fait un rectangle
Si je le mets là ça fait un rectangle
*E1 pense au parallépipède « rectangle »
obtenu ou qu'il va faire apparaître en ajoutant 3^2
sur le haut*
*E1 place la couche carrée 3x3 au-dessus, la
maintient et la repose*
31. E2 Mais t'es bête ou quoi
*E1 persiste dans sa manière de voir et de
faire*
32. E2,E3 Non, arrête... mais arrête
C'est pas ça qu'il faut faire
E2, E3 font tomber les cubes E1 rigole
33. E2 Regarde
34. E3 C'était très bien ce qu'on a fait.
Là, regarde
A la limite on peut faire pareil ici
*E2, E3 ont vu que E1 partait sur une mauvaise
voie*
Et ça comblera parfaitement...
35. E2 Et celui-là, là
E2 montre 3 au cube
36. E1 Je vais le refaire, t'inquiète
E1 s'apprête à mettre à plat 3 au cube
37. E3 Attention les cubes vont tomber entre les tables.



E1 dispose les cubes en forme de gnomon, en hésitant

38. E3 Mais pourquoi tu fais ça ?
Tu pars de la base autour... 1, 2, 3 hop 1, 2, 3...

39. E2 Attends... 1, 2, 3

40. E3 Fais autour

La redistribution de 3 au cube est celle d'un carré et non pas celle d'un gnomon !

41. E2 Ah non, mais c'est pas ça qu'il fallait faire.

42. E1 Il faut mettre comme ça

E1 fait un geste le long de 3^2

43. E2 Attends

44. E1 Il faut que tu le rajoutes comme ça pour que ça fasse un gros carré.

45. E2 Comme ça...

46. E1 Non, il faut que tu le rajoutes là et là...

47. E3 Ça fait 27... 27 + 9 ...

Groupe 5

48. E1 [...] Et après, ça fait [hésite]

49. E2 Non, il en manque là.

50. E1 Ça c'est un L ...non...

Ah non, c'est pas un L.

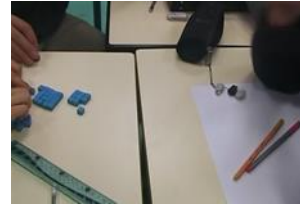
51. E3 Il aurait fallu [?]

52. E1 Tu veux une autre couleur ?

53. E2 Non ,non..

Groupe 3

54. P Il faut faire toute la somme



Groupe 5

55. E1 Non, tu le mets là ça.
56. P Ça commence a être sympa ça !
57. E2 Là, tu le mets comme ça.
58. E3 C'est bon là t'es sûre ?
59. E2 1,2, 3 pour 4, 5, 6 ;
7, 8, 9 ... 10, 11, 12 ... 26,27
C'est bon
60. E3 Et donc ?
61. E2 Ben oui, 3 au cube ça fait 27
62. E3 C'est la fatigue
63. E2 En fait, ça, ce serait ? ...
64. E3 C'est ça
 3^2 est également présent sur le côté
65. E2 On a 2 au cube et 3 au cube
66. E3 Là c'est bon
E1, E2, E3 relisent les consignes
67. E1 Allez, je le dessine
68. E2,E1 Nous, on fait « algébrique »
E3 prend la feuille quadrillée
69. E3 Donc on va faire le 1 comme ça
Ensuite j'en fais quatre



Groupe 1

70. P L'idée des gnomons, c'est pas forcément au départ
Eh oui !
P ne veut pas révéler la solution
C'est peut-être pour passer d'une somme à la
somme $n=4$, d'accord ?
Là je veux un seul carré de façon ...
D'abord vous commencez avec 1 ensuite vous
rajoutez le cube suivant...
71. E2 [?]
72. E1 Moi je sais !...
73. E3 Là, ça fait 3^2
74. E4 Oui mais faut le coupler au visuel
75. E1 Il faut le coupler avec des chiffres.+3 au cube, ça



76. E2 fait 3 puissance 3,... 9
 77. E3 Ça fait 27 !
 78. E2 [rigole] oui, ça fait 27
 79. E1 On en a 27, là...

Groupe 2

80. P C'est quoi les dimensions du grand carré ?
 81. E1 C'est ça, là ...
 82. P Et le suivant, vous voyez ce que c'est ?
 Je ne vous dis pas plus...
 Par contre, il faut me dire pourquoi ça marche
 Il faut me dire pourquoi ça va venir pile...
 C'est ça l'idée, hein ?



83. E2 Le nombre d'après, c'est $(n+1)$ au cube
 84. E4 4 au cube, c'est 4 fois, heu...
 Ça fait 16×4 , 24, 64...voilà
 Ça fera $36 + 64$, ça fera 100
 Ah, ben je sais !

[perçoit la possibilité de généraliser]

85. E1 Ça rajoute le nombre de côtés chaque fois...
 Ce qu'il y a à rajouter, c'est le nombre
 E2 et E4 discutent à part sur le cas algébrique
 86. E2 n , déjà, ça va être sur le côté [...]

Groupe 5

E1 dessine

87. E1 Je fais mes trois carrés
 88. E2 Tu les fais pas ensemble maintenant
 89. E1 Non, non, non
 Tu fais ça et après je changerai de couleur pour le quatrième...
 90. E2 Regarde
 Parce que là, on les a rajoutés
 Donc ceux-là, il faudrait...
 Il faut que tu les rajoutes



E1 efface la dernière partie et reprends en tenant compte de la remarque de E2

E1 passe du dessin à la disposition à plat

91. E1 Colle-moi les trois là ! [...]



Groupe 2

92. P Là, vous avez réussi
 Vous avez assez de cubes pour pouvoir faire la
 somme jusqu'à trois
 Et après ça fait un carré
 Et après je veux l'étape suivante...
 Mais vous n'avez plus assez de cubes
 Va falloir que vous m'expliquiez...
 Vous essayez de me donner une méthode... de me
 dire pourquoi ça marche.
 Ok ?
 Vous pouvez faire des dessins à côté, d'accord ?

93. E2 Non, faut mettre...

94. E3 C'est « chiant » ces petits trucs, là...

E4 stabilise l'agencement avec un double-décimètre



Groupe 3

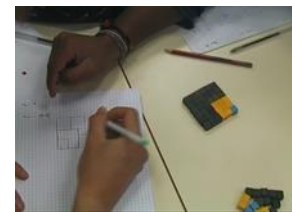
E1 suit attentivement ce que E2 dessine

95. E2 Le dernier, il est comme ça
 Un gnomon avec, heu ...

E2 fait des gestes dans la zone concernée

*E2 parcourt le dessin au-dessus d'un gnomon
 imaginaire*

96. E1 Après faudra rajouter 5, 5
 « tac, tac » 6, 6 ... 7, 7
 Après faut trouver « la propriété »
 [*l'extension !*]



Groupe 1

97. P Faites un petit dessin
Le groupe est bloqué après avoir disposé convenablement les cubes en carré avec des gnomons détachés



98. E1 Ben, j'attends ça, hein ?

99. P Vous prenez du brouillon

100. E1 Bon, juste, question...

Est-ce que c'est notre figure initiale

E1 indique les cubes agencés

Est-ce qu'on en a besoin dans notre poster ?

Est-ce que c'est une figure essentielle, le poster ?...

[les élèves demandent ici des précisions sur la consigne]

Oui

101. E3 Ensuite, est-ce que je dessine les couleurs comme

102. E1 ça, là [*E1 indique les cubes*]

Non, tous de la même couleur

103. E3 Non, fais juste des ronds vides

104. E4 On les remplira

C'est des carrés

105. E2 Bon, bon, des carrés [...]

106. E4



Groupe 2

107. E1 Là, tu en as 1, 2, 3...

Là, tu vas en avoir 4

E1 fait une référence au gnomon suivant

108. E2 T'en auras 4 ici aussi



Groupe 3

109. P Ah, vous avez mis les cubes comme ça, c'est curieux ...oui, on peut...

Et on aurait pas pu avoir une disposition plus symétrique ?

C'est pas plus « joli » le symétrique » ?...

Vous voyez la symétrie par rapport à...

P fait un geste le long de la diagonale



principale

Bon, vous avez pensé au gnomon à l'intérieur du « truc » [relâchement de P !]

Ça c'est pas mal

Tout ça c'est un gnomon et à l'intérieur vous avez mis un autre gnomon

En fait, moi je veux savoir pourquoi ça marche « **tout le temps** » ...

Quand je vais rajouter le cube, je veux savoir exactement ce qui va se passer

On rajoute la longueur qui est donnée

110. E1 Par exemple, là ... 1, 2, 3, 4, 5

Si on veut rajouter 6

On va rajouter 6 ici, 6 là

Ouais, ouais...

111. P Là, ça va pas « rentrer pile »

112. E1 E1 *pense aux carrés par couches*

Il y en a deux qui vont rentrer pile de chaque côté et il y aura un gnomon en plus

C'est parce que tu as remarqué que le phénomène

113. P est alterné ... Ça, je suis d'accord

Maintenant, il y a juste un truc...

Pourquoi, quand tu prends un cube, ça fait soit un truc comme ça, quand tu les mets à plat, ou soit ça fait ça... Ok ?

Il vous manque juste ça, ok ?

Pourquoi, avec le cube en 3D, on peut le mettre à plat comme ça...

Groupe 5

Le dessin est bien avancé (coloriage effectué)

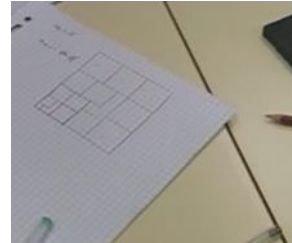
114. E1 En fait tu mets 4 sur les côtés et tu complètes le tour

115. E2 En fait le 4 c'est « tout con » !

Groupe 2

116. P C'est curieux d'avoir mis les gnomons à l'intérieur

Groupe 5



P s'adresse à E1

117. P L'extension, c'est vraiment ça
118. E2 Ah, je ne comprends pas, j'ai vraiment du mal...
119. E1 Faut savoir que sur les côtés tu rajoutes toujours 4
 E2 *tente de suivre le processus de généralisation*
 Là, il y a le 2, là, le 3, là le 4
120. E2 Regarde
121. P Pourquoi tu as rajouté ça dedans
 Regarde, je peux te montrer
 P est contraint d'apporter une aide
122. E2 Ah, je pensais que...

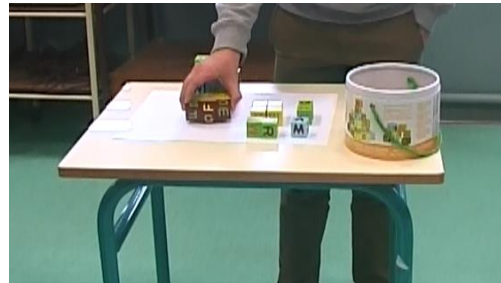
III) Phase multimodale

1. P Are you ready?
2. E Yes.
3. P Let's go
Show me something
4. E At first we have three cubes
One big, one medium and one little

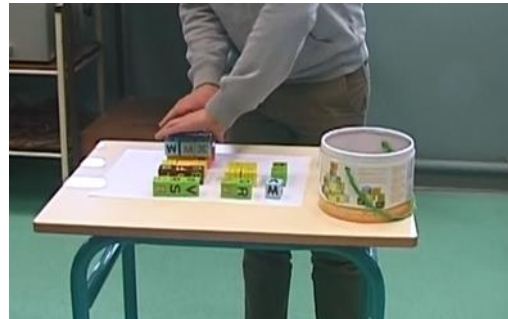


So, we can see that the cube can be decomposed of a square and, heu, two little row(s)... Yes?

5. P Yes.
6. E And then the second one...



7. P You may use both hands!
8. E OK.



9. E When we compose the three cubes we have the big square.
10. P So where is the big square?
11. E Here.
12. P Yes.
13. P So what would be the next step?
14. E [hésite]
The next step would be ... four...



	The row of four cubes...	
	[Hesite]	
15. P	And then? Go on. [ne saisit pas où P veut en venir]	
16. P	If we had a cube of side 4. 4 by 4 by 4. Where would you place the rows?	
17. E	Here.	
18. P	It's just one (row)	
	[E s'apprête à prendre d'autres cubes dans la boîte mais P intervient car la consigne initiale était d'expliquer verbalement]	
19. P	No ! Don't use...	
20. E	Ah, ok.	
21. P	Just explain without any cubes. That' the principle. How should we lay the cubes flat?	
22. E	We have a (cube) like this (row?) Then (next row?) Then three And then 4 and 4	
23. P	Ok. Can we generalise that?	
24. E	Yes.	
25. P	Is it easy to explain without cubes?	
26. E	No	
27. P	So what should we do?	
28. E	[hésite]	
29. P	Generalise	

We could do something instead of
manipulating cubes.


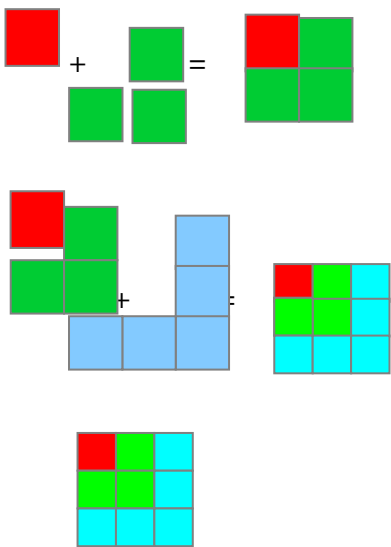
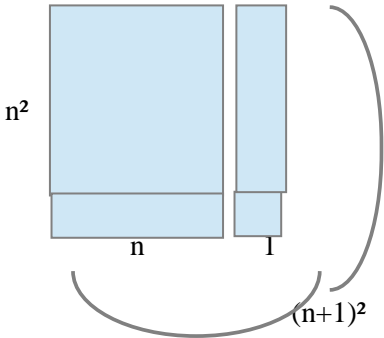
What have we got to do?

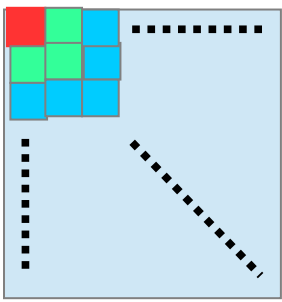
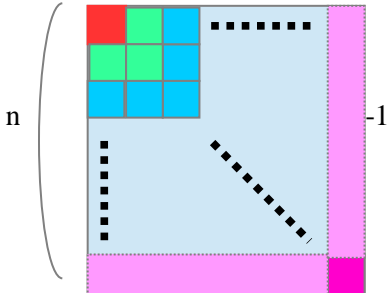
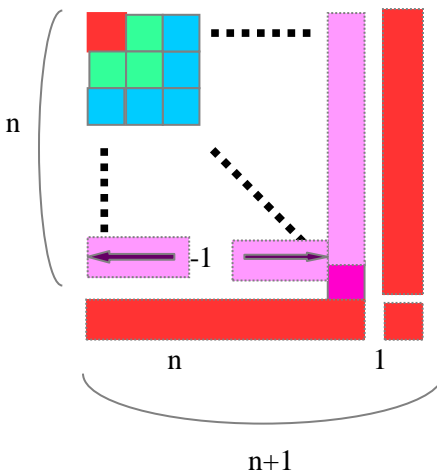
- 30. E A drawing.
- 31. P Thank you.
- 32. E Ok...



ANNEXE 5 : Les identités algébriques et les preuves en L1 et en L2

Parallèle entre preuve schématique et preuve par induction

schematic / visual elements	intermediate		algebraic transcription
	an object a	a number a	1
	particular configurations		<p> $1 + 3 = 2^2$ </p> <p> $2^2 + 5 = 3^2$ </p> <p> $1 + 3 + 5 = 3^2$ </p>
	properties		$(n + 1)^2 = n^2 + 2n + 1$
	physical Visual identity Adding a to a square (of side n) yields the next	algebraic Algebraic identity (squares of ntegers) “(n+1) squared is equal to plus the product plus	

	<p>The sum of gnomons is a square</p>	<p>The sum of consecutive numbers is a square number</p>	$1 + 3 + 5 + \dots = ()^2$
	<p>More precision</p>		$1+3+ \dots + (2n-1) = n^2$
	<p>The last is recognizable : its value is $(2n-1)$</p>	<p>The last in the sum is no longer implicit : $(2n-1)$</p>	
	<p>Surprisingly, it's almost more difficult to guess that the last term in the sum is $(2n-1)$ (right column) than to notice that the inner gnomon is $2n-1$ (left column)</p>		
<p>Schematic proof / visual proof</p> 	<p>Schematic induction</p> <p>1) start by looking at the corner we observe that the first squares are made up of gnomons</p> <p>2) By the gnomon of value $2n+1$ to the with side n we get the next square (i.e. the square with side $n+1$)</p> <p>3) Conclusion : A square is made up of consecutive gnomons</p>	<p>Comments</p> <p>the first step is a matter of the property with the first value(s) of n or along the main diagonal, starting at the top left corner</p> <p>By adding a gnomon, or by adding $2n+1$, we get the next physical (left column) or the next square number (right column)</p> <p>the conclusion is standard</p>	<p>Proof by induction</p> <p>1) initialization With $n = 1$: 1 is simultaneously the first odd and a square (1^2) Or with $n = 2$: 1+3 is a square (2^2) 2) step Assuming that, for some n, the following holds : $1+3+ \dots + (2n-1) = n^2$ and the identity : $n^2 + 2n + 1 = (n+1)^2$ we get : $1+3+ \dots + (2n+1) = (n+1)^2$</p> <p>3) conclusion : the sum of numbers is always a square number</p>

Dimension-outil des identités algébriques

- **Activité en classe**

Part I

Here is a logical pattern whose rule is to be conjectured:

R U

R U R U U U

R U R U U U R U U U U U

- 1) Can you determine the next pattern ?

We now consider a path along a coordinate grid so that **R** stands for a *Shift of 1 unit to the Right* (Right Shift) and that **U** stands for a *Shift of 1 unit Up* (Upward Shift).

The point attained after a succession of one **R** and some number of **U**'s is denoted by A_i .

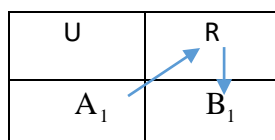
The next point, attained from A_i after a *Shift of 1 unit to the Right* is denoted by B_i .

The point A_0 coincides with the origin O of the axes.

Hence, the summarization of the succession of shifts in the following table:

Points attained after a succession of unit-Shifts (R or UUU...U)								
Shift		R	U	R	UUU	R	UUUUU	
Point	$O = A_0$	B_0	A_1	B_1	A_2	B_2	A_3	

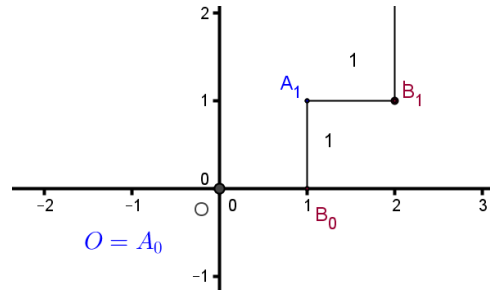
The next picture (taken from the previous table) shows that, starting at A_1 , if we shift *1 unit to the Right* we reach B_1



- 2) Plot points A_i and B_i on the coordinate grid for $i = 1$ to 3.

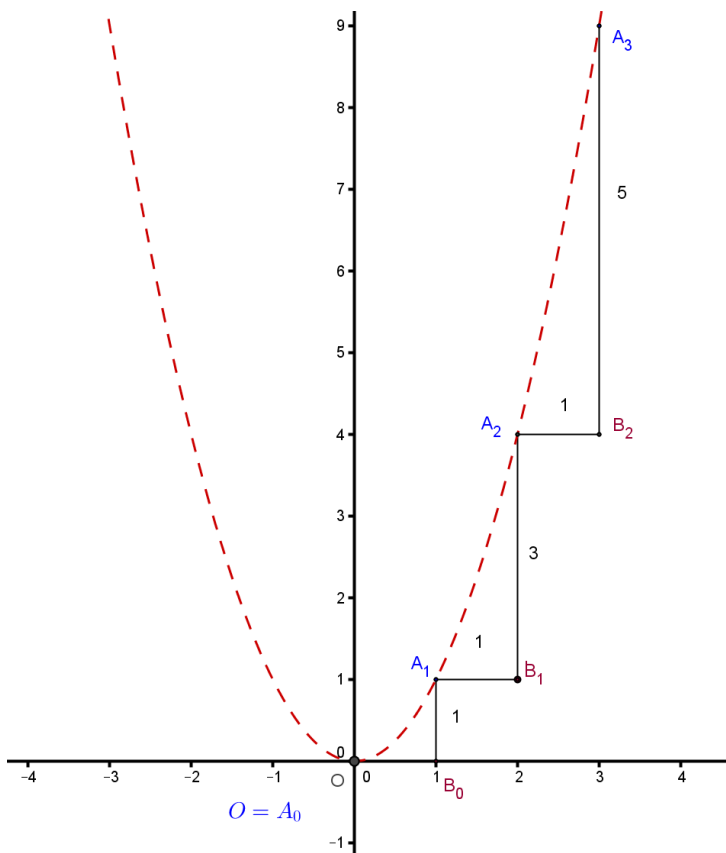
The picture opposite represents the first points:

O , B_0 , A_1 and B_1



The points A_i seem to lie on a well-known curve.

- 3) Do you recognize this curve? Justify your answer and give an equation of the curve.



- 4) What do you think as to the location of points B_i ?
Justify.

Part II

Write an algorithm that returns the “straight” distance $A_{n-1} A_n$ if we input n .

- **Homework**

- ❖ More about Gnomons

A Gnomon is a kind of arrangement of a certain amount of objects according to a specific shape. (so far, we have only met L-shaped gnomons).

(a physical Gnomon is made up of real, physical objects: balls, cubes, etc...)

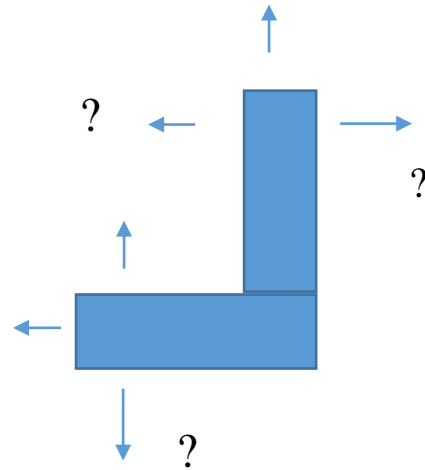
A physical Gnomon can be symbolized on a diagram by an arrangement of unit-squares.

(in this case we get a schematized Gnomon)

As an arrangement, a Gnomon has a shape and can be enlarged or reduced.

This enlargement can be performed according to a rule.

The length and/or the width can be enlarged. The order in which we add unit-squares to get the new Gnomon doesn't really matter.



As a number of objects, arranged according to a particular shape, a Gnomon has a specific value. This value can be increased according to a rule. (the first gnomon you encountered was L-shaped and its value was $2n-1$, expressed in terms of the rank n of the design).

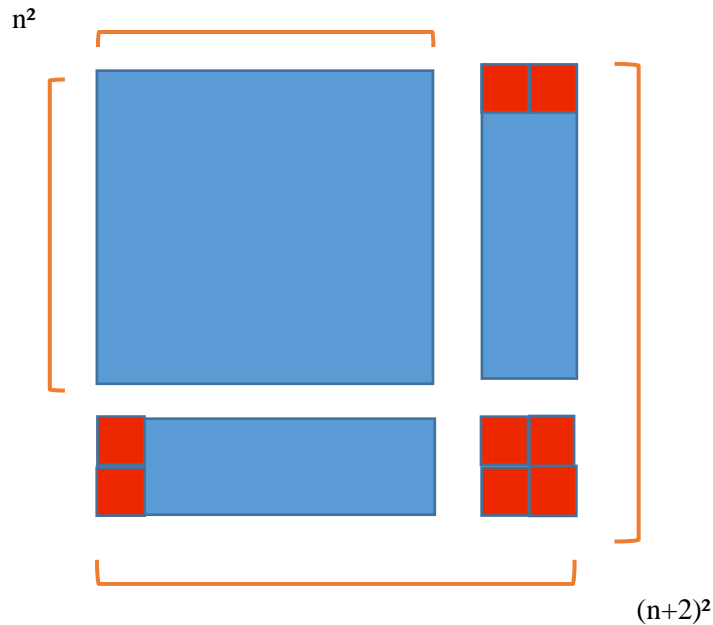
(This rule consists in determining how many new objects are added and in the way of counting these objects)

To sum up:

- ✓ A Gnomon has a specific shape.
- ✓ A Gnomon is a particular arrangement of physical or symbolized objects.
- ✓ A Gnomon has a numerical value which is the number of objects.
- ✓ A Gnomon can be enlarged by following a rule.

❖ New type of L-shaped Gnomons

The next diagram shows a larger Gnomon and establishes a visual, schematic relationship between n^2 and $(n+2)^2$.

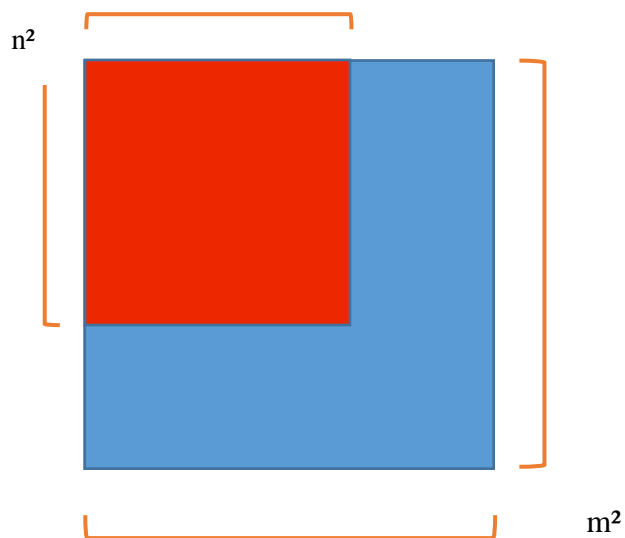


- 1) Closely examine this diagram and put it into algebraic terms.
(establish an algebraic transcription)

There are several different ways of counting the numbers of objects in the arrangement of the Gnomon.

❖ General situation

The next diagram shows an even larger Gnomon and establishes a visual, schematic relationship between two squares, that is between n^2 and m^2 where m and n are **any** integers.



- 2) Closely examine this diagram and put it into algebraic terms.

Part II

Here is a logical pattern whose rule is to be conjectured:

R U

R U R U U

R U R U U R U U U

1) Can you determine the next pattern ?

We now consider a path along a coordinate grid so that **R** stands for a *Shift of 1 unit to the Right*

(Right Shift) and that **U** stands for a *Shift of 1 unit Up* (Upward Shift).

The point attained after a succession of one **R** and some number of **U**'s is denoted by A_i .

The next point, attained from A_i after a *Shift of 1 unit to the Right* is denoted by B_i

The point A_0 coincides with the origin O of the axes.

Hence, the summarization of the succession of shifts in the following table:

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Point	$O = A_0$	B_0	A_1	B_1	A_2	B_2	A_3	

The next picture (taken from the previous table) shows that, starting at A_1 , if we shift *1 unit to the*

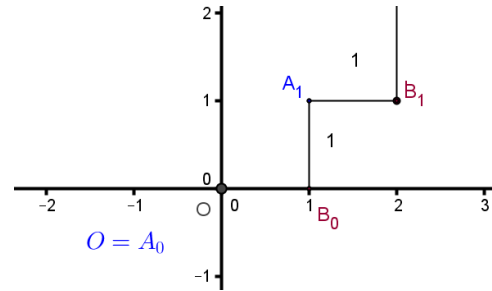
Right we reach B_1

U	R
A_1	B_1

- 2) Plot points A_i and B_i on the coordinate grid for $i = 1$ to 3.

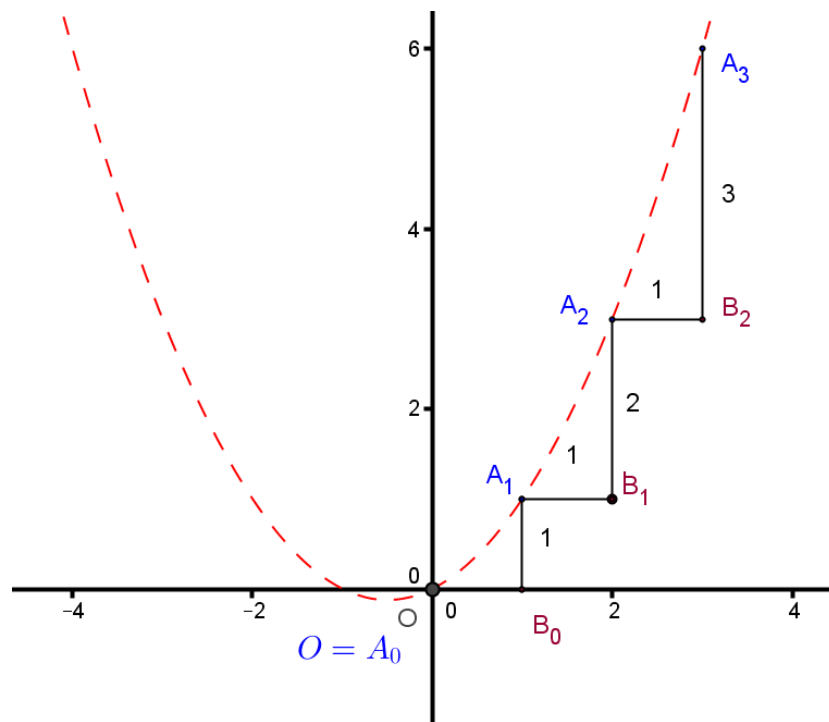
The picture opposite represents the first points:

O , B_0 , A_1 and B_1



The points A_i seem to lie on a parabola.

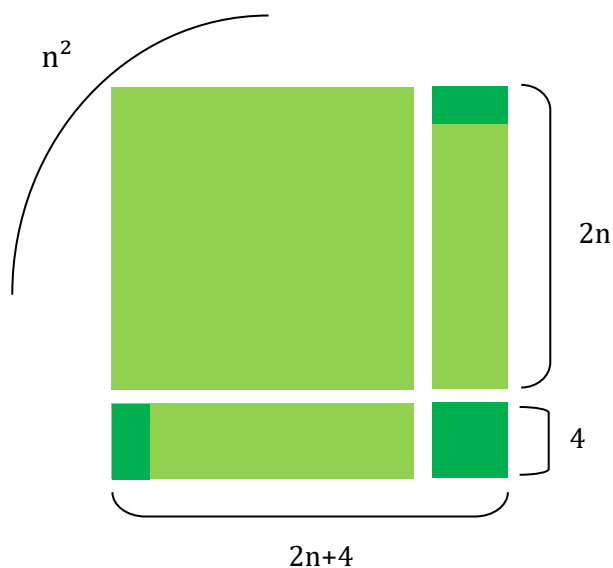
- 3) Give an equation of the curve. Justify your answer.



- 4) Determine the coordinates $(\alpha; \beta)$ of the vertex of the parabola and write the equation in vertex form (that is, in canonical form: $y = a(x - \alpha)^2 + \beta$).

Nous présentons ci-dessous une production d'élève relative au sujet précédent.

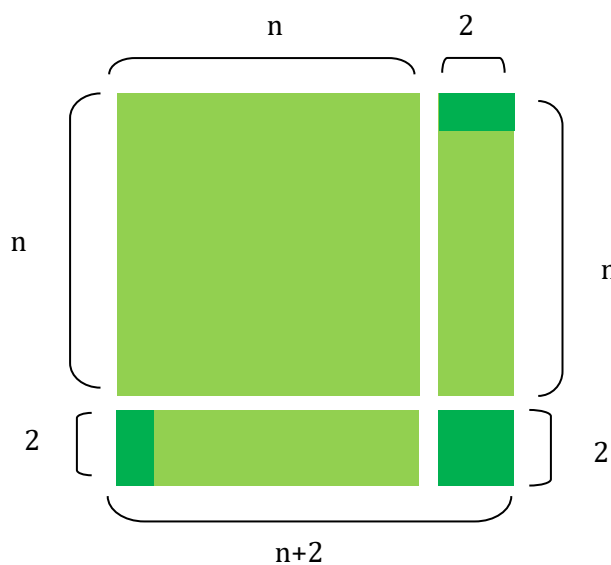
❖ In terms of type of objects :



First, we can expand the algebraic identity in terms of name of objects that is that the big light green square is “named” n^2 , the two rectangles are “called” $2n$ and the small dark green square is 4 . By adding the objects, we get:

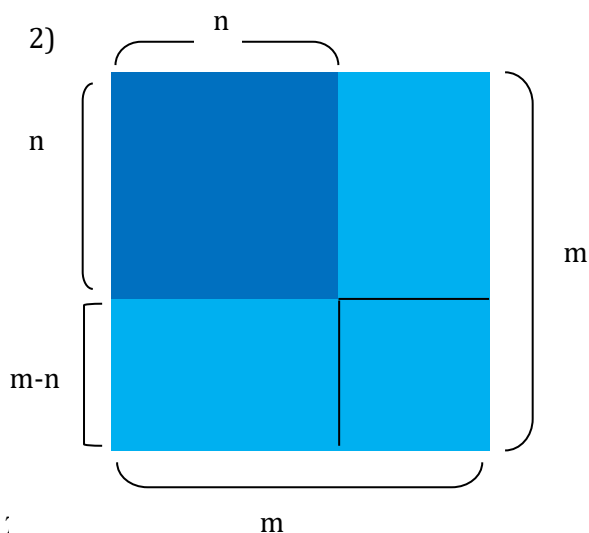
$$n^2 + 2 \times 2n + 4 = n^2 + 4n + 4 = \boxed{(n+2)^2}$$

❖ In terms of areas of objects:



We can also see the algebraic identity in terms of area of the objects. That’s why we consider that the small dark green square is composed of 4 unit squares. This way, its side is 2, the length of the two rectangles is n and its width is 2, and the side of the big light green square is n . By expressing and calculating the areas, we get:

$$n \times n + 2(2 \times n) + 2 \times 2 = n^2 + 4n + 4 = \boxed{(n+2)^2}$$



We based our examination on our previous answer. Indeed, the big light blue square has been obtained by adding the small dark blue one and a L-shaped gnomon which is composed of two rectangles and a square. This way, the side of the small light blue square is $m-n$, the length of the two rectangles is n and its width is $m-n$, and the side of the dark blue square is n . By expressing and calculating the areas, we get:

$$n^2 + 2 \times [n(m-n)] + (m-n)^2 = n^2 + 2(nm-n^2) + m^2 - 2mn + n^2 = 2n^2 - 2n^2 + 2mn - 2mn + m^2 = \boxed{m^2}$$

Part II:

Write an algorithm that returns the “straight” distance $A_{n-1}A_n$ if we input n .

Input: Enter n

Treatment: x_a takes value $n-1$

y_a takes value $(n-1)^2$

x_b takes value n

y_b takes value n^2

d takes value $\sqrt{(x_b - x_a)^2 + (y_b - y_a)^2}$

Output: Display d

1) R U

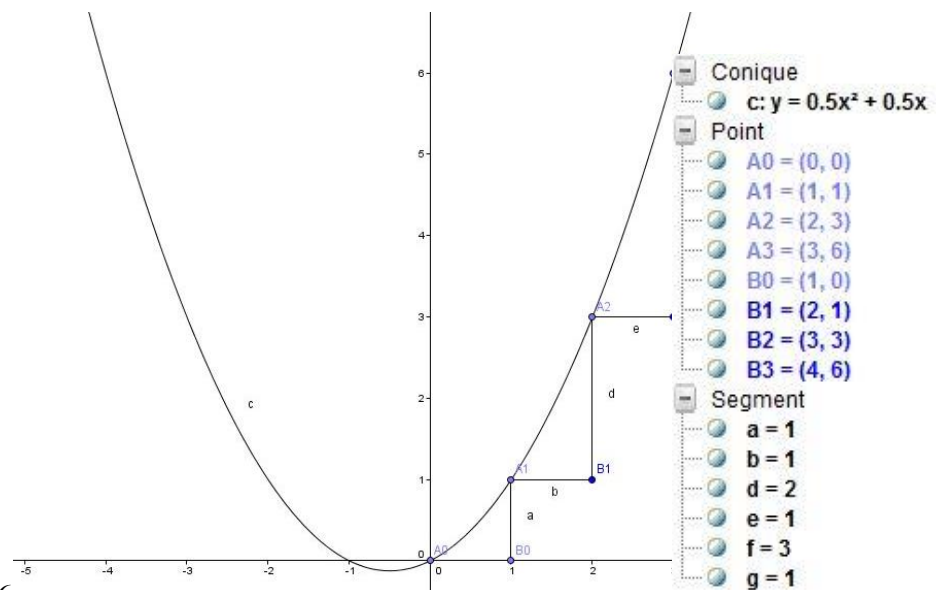
R U R U U

R U R U U R U U U

We can determine the next pattern which is: R U R U U R U U U R U U U U

2) Then, we plot points A_i and B_i on a coordinate grid for $i = 1$ to 3 and whose coordinates are given in the following table:

Points	A_1	B_1	A_2	B_2	A_3	B_3
x-value	1	2	2	3	3	4
y-value	1	1	3	3	6	6



Thèse Larue 14/04/2016

3) The curve is a parabola, so its equation is in the form $y = ax^2 + bx +$

c . The y-value of point A_0

$(0;0)$ is the y-intercept of the parabola so $c = 0$. Using the coordinates of point $A_1 (1;1)$, and

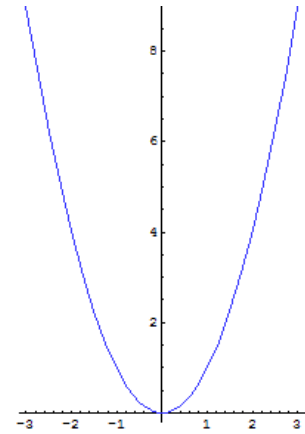
$A_2 (2;4)$, we get the following system:

ANNEXE 6 : Compléments

Exemple de formulation utilisant le ton humoristique³

Le premier exemple est tiré du paragraphe [Images and metaphors](#) :

- We think of a function that has a positive derivative as “going up”. But the function *isn't really going up*, it just sits there like your pet rock.
- We think of the parabola $y = x^2$ as like a bowl whose sides go up forever. But “forever” is a metaphor. *There is no time involved.*
- Besides, we may forget that *the sides go out in the positive and negative x direction “forever”*, too (every vertical line cuts the graph!), even though the picture looks like *they are too steep to do that.*
- As these examples illustrate, we think about math objects in terms of *images and metaphors* that we have developed out of our experience with them. These are valuable insights *but they generally cannot be used to prove theorems about them.*



Le second exemple porte sur des *exemples d'images* :

Children don't always sort these out correctly. Father: "We are all going to fly to Saint Paul to see our cousin Petunia." Little child: "But Dad, I don't know how to fly!"

We know what a **pyramid** looks like. But when we refer to the government's food pyramid *we are not talking about actual food constituting a pyramidal pile.* We are talking about a **visual image** of the pyramid. In the study of literature, the word “image” is used in a more general way, to refer to an evocation of a sensory experience.

Examples of metaphors

We know by direct physical experience what it means to be **warm** or **cold**. We use these words as [metaphors](#) in many ways:

We refer to a person as having a warm or cold personality. *This has nothing to do with their body temperature.*

³ Extrait du site abstractmath.org

When someone is on a treasure hunt we may tell them they are “getting warm”, even if they are hunting outside in the snow.

Le troisième exemple concerne la distinction kind / property:

The ideas of kind and property are not really different.

You could define a kind of mathematical object called **ent** to denote an even integer and another kind **ont** to denote odd integers. Then you could say “42 is an ent” using the same grammar as when you say “42 is an integer”. So although we think of “integer” as a *kind* of thing and “even” as a *property* of things, calling even integers ents allows us to think of “even integer” as a kind.

Exemples de Collocations (en anglais) fréquemment utilisées en Mathématiques

this is evident from
 suppose that $_$, then
 it follows that
 it now follows that
 since $_$, it follows that $_$
 since $_$, we see that $_$
 as $_$, we have $_$
 as $_$, then $_$
 if we combine this with lemma 1,
 now if we recall (2.1),
 we arrive at the conclusion that
 we come to the conclusion that
 under the assumption that
 if either $_$ or $_$, then $_$
 and, in addition,
 and, additionally,
 for all $_$ such that $_$, we have $_$
 for any $_$ such that $_$, it follows that $_$
 there exist no such
 there does not exist
 turning now to $_$, we see that
 we begin by considering
 take an arbitrary $n > 3$
 consider an arbitrary $n > 3$
 the function f defined on $_$ by $f(x) = _$
 we want to
 it will be proved that
 it will appear later that
 when dealing with
 to a large extent
 to a great extent
 we are now in a position to define $_$
 as was noted before,

this proves
 this amounts
 this entails
 this provides
 this is only true for $_$
 by the suppositions on $_$,
 by the above, we have
 as a result, we obtain
 according to the definition,
 according to definition,
 because of $_$,
 the following theorem holds.
 the object of this work is not only to find $_$ but
 also to use $_$
 not only is it finite, but it even consists of
 we show that if $_$, then
 each $_$ may be written as $_$
 for all integers $n > 3$
 for every integer $n > 3$
 for any $_$, there exists a $_$
 in addition, suppose that
 furthermore, assume that
 suppose $_$ satisfies the assumption
 let us show that
 we now prove that
 we also show how
 we claim that $_$. indeed,
 one of these cases is realized
 we are interested in whether $_$
 also, as is shown in $_$,
 as was shown above,
 as it was shown above,
 as was shown before,

as it was noted before,
 is of interest in itself
 as may seem at first glance
 the previous two cases
 the next three cases
 the last three functions
 the question as to for which _
 the question of whether _
 we remind that
 recall that
 remind that
 we recall the results of
 takes the form
 can be viewed as
 has a representation of the form

we require that _ be _
 three such elements
 all three expressions
 is of great importance to us
 depending on whether
 both functions are
 for verification of whether _ it is sufficient to
 and vice versa
 instead of _ it would have sufficed to take _
 in one sense or another
 provides a way of describing
 a way of constructing
 may be regarded as
 a graph each of whose components is a tree
 a graph whose each component is a tree

Extrait de lexique monolingue (anglais) non phraséologique

Verbes d'action

	To level off
to count	To top
to solve	To fall off
to compute	To recover
to work out	
to determine	Noms
to look for	Calculation
to search for	Reasoning
to raise to the square	proof
to assume	Deduction
to deduce from	theorem
to conclude	Axiom
to prove	Hypothesis
to show	algorithm
to assert	instruction
to recap	definition
to rank	property

to estimate	proposition
to evaluate	relation
to be equal to	condition
to equal	comparison
to forecast	opposite
to foresee	reciprocal
to figure	converse
grouping	contrapositive
to transpose	counterexample
to extract	a premise, conjecture
to simplify	paradox
to factorize	unit
to expand= to develop	unity
to substitute	array
to replace	matrix
to eliminate	row
to transform, to change into	column
to change the subject	line
to rearrange the formula	symbol
to vanish	variable
to cancel	random number
to satisfy	random variable
to increase by	calculator
to increase to	computer
to decrease by	Adjectifs
to decrease to	included
to remain constant	excluded
to rise	finite
to raise	infinite
to vary (by)	implicit
The level rises to a peak	explicit
To pick up	symmetric (en soi)
To reach a peak	symmetrical (par rapport à)
To plummet	unique

Exemple de lexique simple (non phraséologique) avec phonétique

Extrait du site Emilangues

Mathematics vocabulary with phonetics and sound (English to French)		
<i>(Pour accéder au son, cliquer sur la phonétique)</i>		
angle bisector	æŋɡl̩, baɪ'sektəʃ	bissectrice
Archimedes	ˌɑːki'miːdiz	Archimède
circumcentre	ˌsɜːkəm'sentəʃ	centre du ...
circumcircle	ˌsɜːkəm'sɜːkl̩	cercle circonscrit
converse	'kɒnvɜːs	réciproque (nom)
diameter	daɪ'æmɪtəʃ	diamètre
divisible	dɪ'vɪzəbl̩	divisible
division	dɪ'vɪʒn̩	division
divisor	dɪ'vaɪzəʃ	diviseur
equation	ɪ'kwɛɪʃn̩	équation
equidistant from	ˌiːkwɪ'dɪstənt, ˌekwɪ'dɪstənt frɒm	équidistant de
equilateral	ˌiːkwɪ'læt(ə)r(ə)l̩	équilatéral
frequency	friːkwəns	effectif
finite	'fɪnɪt	fini
geometry	dʒɪ'ɒmɪtri	géométrie
histogram	'hɪstəgræm	histogramme
hypotenuse	haɪ'pɒtənjuːz	hypoténuse
hypothesis plur. -ses	haɪ'pɒθɪsɪs -sɪz	hypothèse
image	'ɪmɪdʒ	image
infinite	ɪn'fɪnɪt	infini
integer	ɪn'tɪdʒəʃ	entier relatif
interval	ɪn'təvl̩	intervalle
irrational	ɪ'ræʃənl̩	irrationnel
isosceles	aɪ'sɒsɪliːz	isocèle
mathematician	ˌmæθmə'tɪʃn̩	mathématicien
mathematics	ˌmæθ'mætiːks	mathématiques
measure	'meʒəʃ	mesure
measurement	'meʒə'mənt	mesure
median	'miːdiən	médiane
multiple	'mʌltɪpl̩	multiple
negative	'negətɪv	strictement négatif
numerator	njuː'merətəʃ	numérateur
orthogonal	ɔːθəg(ə)n(ə)l̩	orthogonal
parallel	'pærəlel̩	parallèle (droites)
parallel	'pærəlel̩	collinéaire (vecteurs)
parallelogram	ˌpærə'leləgræm	parallélogramme
parenthesis plur. -ses	pə'renθɪsɪs -sɪz	parenthèse
per cent (GB)	pə'sent	pour cent %
percent (US)	pə'sent	pour cent %
percentage	pə'sentɪdʒ	pourcentage
perpendicular bisector	ˌpɜːpen'dɪkjʊləʃ, baɪ'sektəʃ	médiatrice
polygon	'pɒlɪɡɒn	polygone
polyhedron plur. -dra	ˌpɒlɪ'hɪdrən -drə	polyèdre

LEXIQUE ANGLAIS-FRANCAIS DE MATHÉMATIQUES AVEC PHONÉTIQUE ET SON
Alain Gauchet - Emilangues - 2011

Exemples de résultats fournis avec des dictionnaires phraséologiques

Dictionnaire : Linguee

À propos de Linguee | Linguee in English | Participer | Connexion | Outils | Contact | Aide

français ↔ anglais à á é è é é i i ô ú ü ç œ æ

Linguee Recherche

Désormais sur Linguee: La paire français- allemand et beaucoup d'autres langues

Dictionnaire rédactionnel

pattern *nom*

- motif *m*
- modèle *m*
- schéma *m*
- dessin *m*
- patron *m*
- scénario *m*
- prototype *m*
- formule *f*

pattern *verbe*

- modeler *v* · ouvrir *v*

Exemples :

checked pattern *nom*

- quadrillage *m*

Exemples de traduction provenant de sources externes pour 'pattern'

anglais	français
This potential will vary depending on the exposure pattern to the substance. <small>↳ guidance.echa.europa.eu</small>	Ce potentiel varie en fonction du schéma d'exposition à la substance. <small>↳ guidance.echa.europa.eu</small>
In order to evaluate whether injurious dumping would be likely to recur if the measure currently in force against Russia were to be removed, reference must be made to the pattern of trade between Russia and the Community in the investigation period and the period leading up to it. <small>↳ eur-lex.europa.eu</small>	Pour évaluer le risque de réapparition du dumping préjudiciable en cas d'abrogation des mesures actuellement en vigueur pour la Russie, il faut tenir compte de la structure des échanges entre la Russie et la Communauté au cours de l'enquête et de la période qui l'a précédée. <small>↳ eur-lex.europa.eu</small>
This pattern of evasion is unworthy of a government that walks around claiming that it is clean. <small>↳ www2.parl.gc.ca</small>	Cette pratique de l'esquive est indigne d'un gouvernement qui se vante d'être intègre. <small>↳ www2.parl.gc.ca</small>
This also helps to explain the somewhat volatile pattern of inflation perceptions. <small>↳ ecb.europa.eu</small>	Il permet également d'expliquer le caractère assez volatil de la perception de l'inflation. <small>↳ ecb.europa.eu</small>
This pattern of restructuring and consolidation continued in 1999. <small>↳ ecb.europa.eu</small>	Ces opérations de restructuration et de consolidation du secteur bancaire se sont poursuivies en 1999. <small>↳ ecb.europa.eu</small>

Dictionnaire : Oxford collocations dictionary for students of English

<http://oxforddictionary.so8848.com/search1?word=pattern>

pattern *noun*

¹ arrangement of lines, shapes, etc.

ADJ. intricate | geometric | floral

VERB + PATTERN have The jumper has a geometric pattern on it. | design, make, print, produce, weave

PREP. in a/the ~ He had arranged the glasses in a pattern on the table. | ~ on the pattern on the carpet

² usual manner

ADJ. basic, existing, familiar, normal, predictable, regular, set, traditional There is no set pattern for these meetings. | changing, ever-changing | complex | main | overall The overall pattern of our life changes little. | behaviour | employment | weather

VERB + PATTERN establish, set | follow Their actions follow a very predictable pattern. | fall into, fit into ideas that do not fit neatly into his patterns of thought | show 67% of patients showed a similar pattern of improvement.

PATTERN + VERB develop, emerge Similar patterns are emerging all over Eastern Europe. | change

PREP. ~ for the normal pattern for a boy/girl relationship | ~ in the main patterns in English spelling | ~ of patterns of behaviour

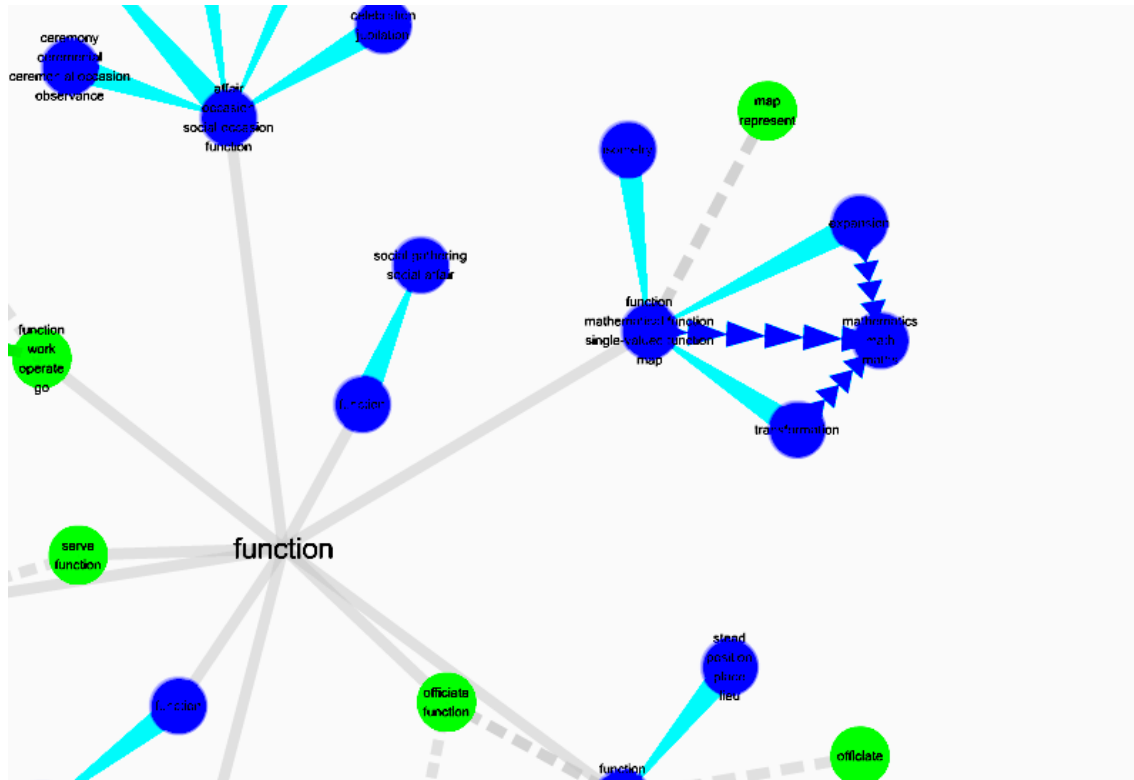
You can also check Google Dictionary: [pattern \(English, 中文解释\)](#)

Exemples de résultats fournis par les dictionnaires visuels (visual dictionaries)

Dictionnaire : visuwords

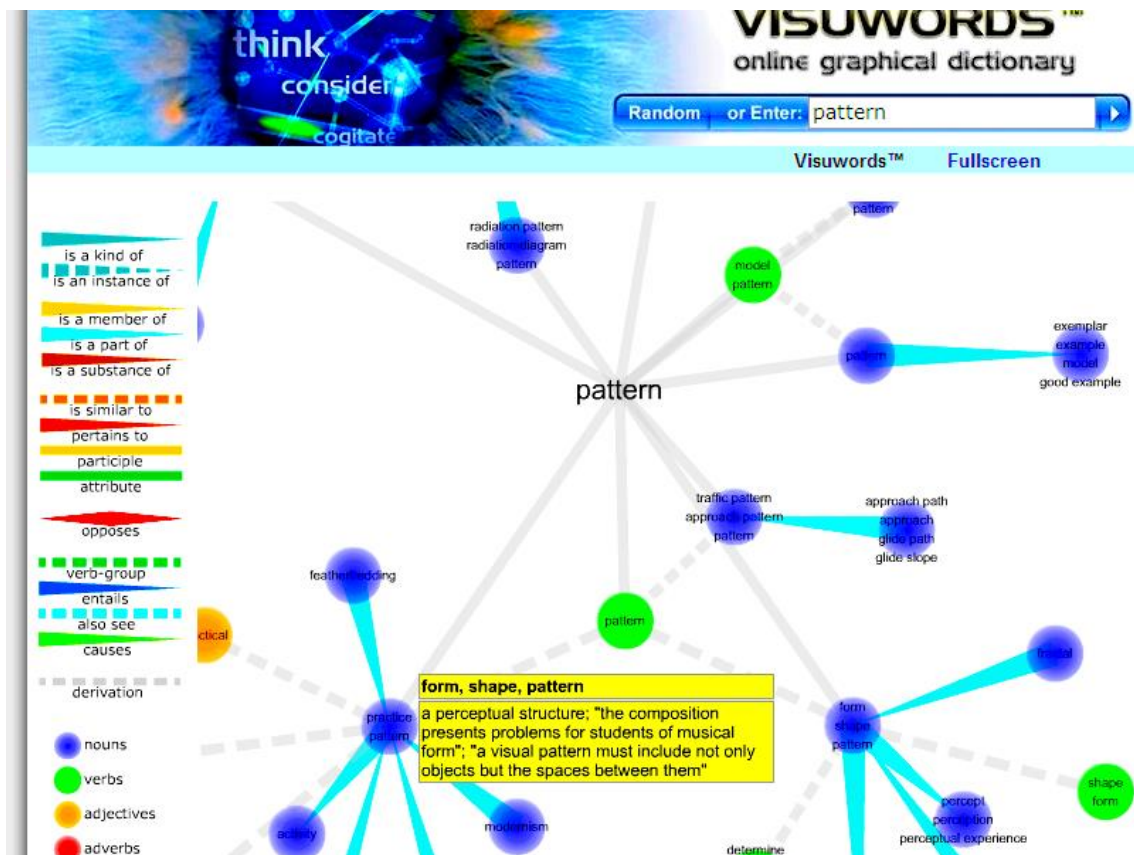
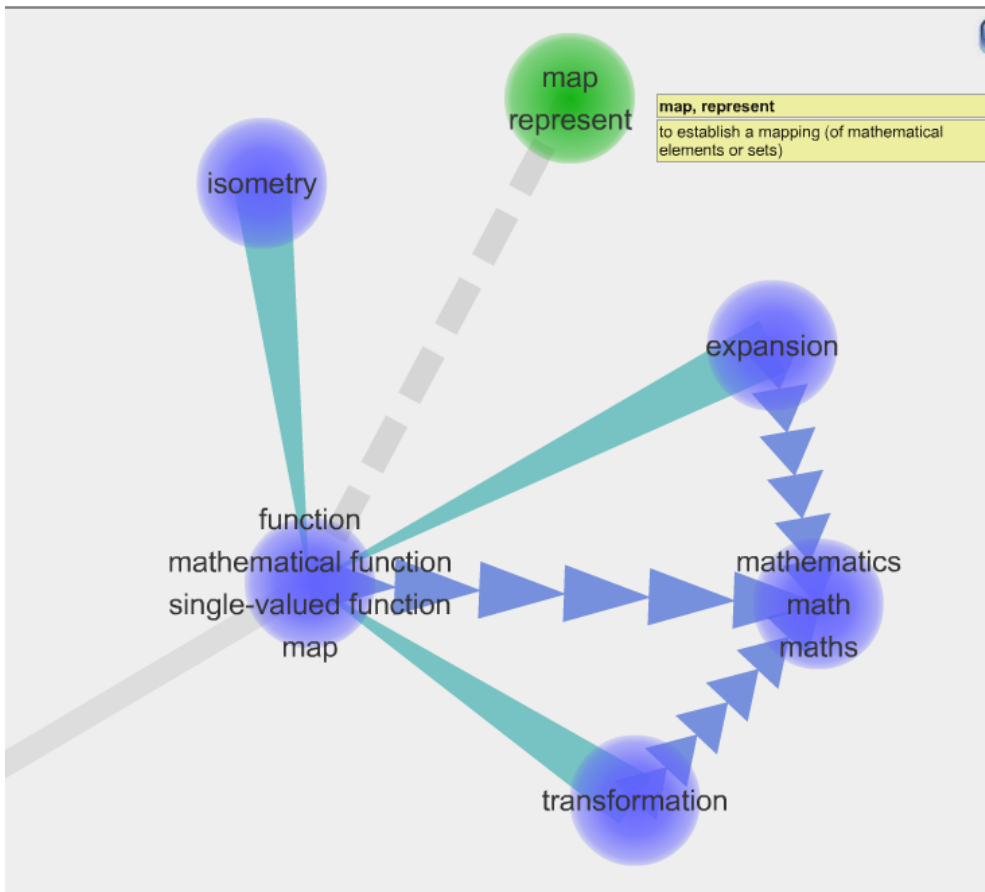
<http://www.visuwords.com>

autour des termes *function*, *pattern*, *map*

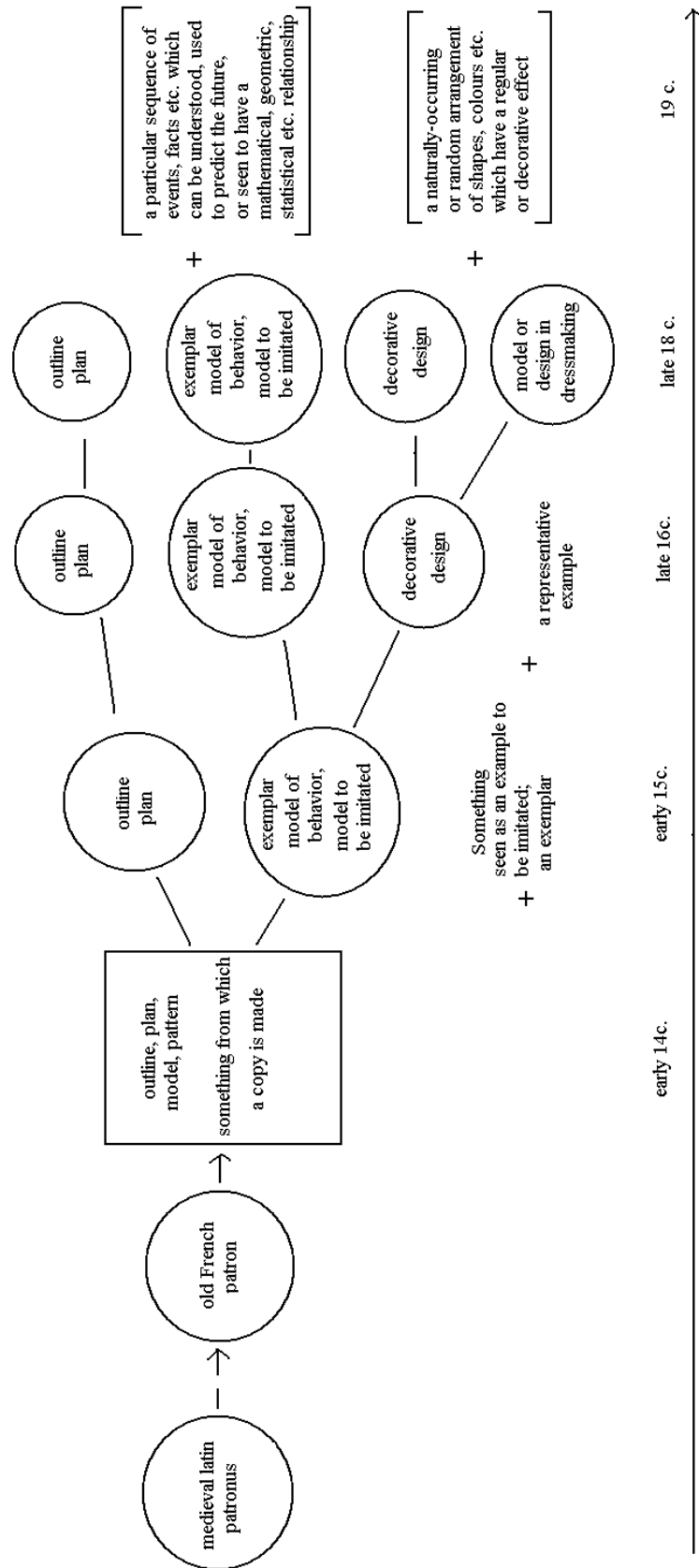


Dans les figures qui suivent, des renseignements d'ordre définitoire ou phraséologique apparaissent dans les encadrés (en jaune).

Au-delà de son caractère convivial et original, le dictionnaire *visuwords* constitue une ressource intéressante pour la phraséologie. Les structures phraséologiques (combinaisons de mots du type collocations) apparaissent en lien direct avec les nœuds sémantiques du réseau (voir ci-dessous).



Evolution diachronique de *pattern*



Définitions lexicales (*raisonnement, preuve, démonstration, etc...*)

Les définitions proviennent des sites [http://www.larousse.fr/dictionnaires/francais/...](http://www.larousse.fr/dictionnaires/francais/)
suivants : [http://www.cnrtl.fr/definition/...](http://www.cnrtl.fr/definition/)

- Raisonement 1 (Larousse)

1.a. Activité, exercice de la raison, de la pensée : *comprendre les choses par intuition plutôt qu'en faisant appel au raisonnement.*

1.b. Suite d'arguments, de propositions liés les uns aux autres, en particulier selon des principes logiques, et organisés de manière à aboutir à une conclusion : *suivez bien mon raisonnement.*

- Raisonement 2 (CNRTL)

2.a. Faculté d'analyser le réel, de percevoir les relations entre les êtres, les rapports entre les objets, présents ou non, de comprendre les faits; exercice de cette faculté, activité de la raison discursive.

2.b. Capacité de connaître, de juger, de convaincre; aptitude à comprendre, envisagée du point de vue de son développement plus ou moins grand selon les personnes; qualité d'une personne, disposition d'un esprit qui juge avec discernement, avec sagesse.

2.c. *Notamment en logique et dans le langage Scientifique.* Opération qui consiste à lier deux propositions pour en former une troisième (ou conclusion), au moyen de règles logiques.

2.d. Ensemble des arguments issus d'une réflexion, mis en œuvre dans une discussion, et qui ont pour but de convaincre quelqu'un ou de démontrer, de prouver quelque chose.

2.e. *Notamment en logique et dans le langage Scientifique.* Suite logique de propositions aboutissant à une conclusion. *Mettre en forme, construire un raisonnement.*

- Preuve 1 (Larousse)

1.a. Élément matériel (exemple document contractuel, attestation) qui démontre, établit, prouve la vérité ou la réalité d'une situation de fait ou de droit : *la preuve d'un crime.*

1.b. Opération par laquelle on contrôle l'exactitude d'un calcul ou la justesse de la solution d'un problème.

- Preuve 2 (CNRTL)

2.a. Fait, témoignage, raisonnement susceptible d'établir de manière irréfutable la vérité ou la réalité de (quelque chose).

- Démonstration 1 (Larousse)

1.a. Action de démontrer, de rendre évidente la vérité d'une loi scientifique, d'un raisonnement, d'une donnée objective : *démonstration d'une théorie.*

1.b. Ensemble de formules qui s'enchaînent à l'intérieur d'une théorie (théorie de la démonstration) et dont la dernière constitue un théorème, appelé également *thèse.*

1.c. *Chez Descartes,* raisonnement mathématique qui, procédant du plus simple au plus complexe, parvient à montrer la vérité de propositions déduites d'autres propositions déjà admises comme vraies.

- Démonstration 2 (CNRTL)

2.a. Action de montrer, d'expliquer par des expériences faites sous les yeux de l'assistance la vérité d'une donnée scientifique.

2.b. Action de démontrer par le raisonnement.

2.c. *Logique*. Raisonnement qui établit la vérité d'une proposition déductivement, c'est-à-dire en la rattachant par un lien nécessaire à d'autres propositions admises comme vraies ou antérieurement démontrées.

- Argumentation 1 (Larousse)

1.a. Ensemble de techniques discursives destinées à provoquer ou à accroître l'adhésion de l'interlocuteur aux thèses qui lui sont présentées.

- Argumentation 2 (CNRTL)

1.b. Action d'argumenter; ensemble des raisonnements par lesquels on déduit les conséquences logiques d'un principe, d'une cause ou d'un fait, en vue de prouver le bien-fondé d'une affirmation, et de convaincre.

- Logique 1 (Larousse)

1.a. Science du raisonnement en lui-même, abstraction faite de la matière à laquelle il s'applique et de tout processus psychologique.

1.b. Caractère logique, rationnel de quelque chose : *admirez la logique de son raisonnement*.

1.c. Manière de raisonner, d'agir cohérente, rationnelle : *travailler avec logique*.

1.d. Manière dont les faits s'enchaînent, découlent les uns des autres : *c'est dans la logique des choses*.

- Logique 2 (CNRTL)

2.a. Science relative aux processus de la pensée rationnelle (induction, déduction, hypothèse p. ex.) et à la formulation discursive des vérités.

2.b. *Logique formelle*. Étude des concepts, jugements et raisonnements considérés abstraitement et sans considération des objets qu'ils désignent. *Logique propositionnelle, modale, bivalente, plurivalente, binaire; logique des prédicats; logique des propositions*.

2.c. *Logique naturelle*. Capacité naturelle en vertu de laquelle l'intelligence discerne dans les phénomènes de la nature des rapports de causalité : *C'est de l'extension d'une certaine géométrie naturelle, suggérée par les propriétés générales et immédiatement aperçues des solides, que la logique naturelle est sortie. C'est de cette logique naturelle, à son tour, qu'est sortie la géométrie scientifique*. Bergson, *Évol. créatr.*, 1907, p. 162.

- Vérité 1 (Larousse)

1.a. Adéquation entre la réalité et l'homme qui la pense.

1.b. Idée, proposition qui emporte l'assentiment général ou s'accorde avec le sentiment que quelqu'un a de la réalité : *les vérités éternelles*.

1.c. Connaissance ou expression d'une connaissance conforme à la réalité, aux faits tels qu'ils se sont déroulés ; les faits réels eux-mêmes : *le témoin a caché la vérité*.

1.d. *En logique*. Position d'une proposition par rapport aux conditions imposées par un langage donné et par rapport à un état de fait.

- Vérité 2 (CNRTL)

2.a. Connaissance reconnue comme juste, comme conforme à son objet et possédant à ce titre une valeur absolue, ultime.

2.b. *En logique*. Conformité de la pensée ou de son expression avec son objet. Rapport de non-contradiction entre une proposition et un ensemble de propositions servant de référence.

Formes d'enseignement et d'apprentissage des mathématiques⁴

(en anglais)

The Coding of communication activities is based on the CEFR: R = reception; P = production; I = interaction; M = mediation; O = oral; W = written.

Presentation by the teacher (teacher-learner interaction as monologic instruction) using visual aids (maps, diagrams, data tables, graphs, computer animations, applets dynamic geometry software etc.) (AuR, WR and WP note-taking)	teacher-learner interaction as monologic instruction / frontal education
Teacher-learner interaction as dialogic instruction (OI)	teacher-learner interaction as dialogic instruction / pedagogical dialogue / IRE model ⁵
Learners presenting the results of their homework using visual aids (OWP), comparing the results (AuR, WR), asking and answering questions using visual aids (OWI)	learner-learner interaction as monologic instruction / presentations by students
Learners explaining a mathematical conception, an assertion, a rule, a procedure, a proof etc. to others (OWI)	learner-learner interaction as dialogic instruction / instruction by peers
Learners read the textbook (WR) and solve problems (WP) or work individually within a learning environment (WR) and (WP)	Individual work (problem solving)
Learners working on mathematical exercises individually (WP)	Individual work (practising)
Learners exploring individually a mathematical state of affairs or testing conjectures systematically (WR, WP)	Individual work (exploring)

⁴ LINNEWEBER-LAMMERSKITTEN H. (2012) Items for a description of linguistic competence in the language of schooling necessary for learning/ teaching mathematics (*in secondary education*). An approach with reference points, *in language and school subjects linguistic dimensions of knowledge building in school curricula, n° 4, Language Policy Unit Directorate of Education, DGII*, Platform of resources and references for plurilingual and intercultural education Council of Europe.

⁵ The IRE model (Initiation by the teacher – Response by the learners – Evaluation by the teacher) was presented by Sinclair & Coulthard (1975).

Gathering information (WR and WP: note-taking)	Individual work (gathering information)
Activities run as projects (linking different competences) of individual research; e.g. inventing new mathematical problems (WP) departing from solved problems (WR)	Individual work (working on an individual project / creative work)
Learners writing a study diary (WP)	Individual work (metacognition)
Learners interact with fellow pupils in group work (WR, WP, OI, OWI) solving a problem or working together with a learning environment	Group work (problem solving)
Learners exploring individually a mathematical state of affairs or testing conjectures systematically (OWI, WP)	Group work (exploring)
Searching information: planning the search (OI), gathering information (WR and WP; note-taking); sharing the results (OWI)	Group work (planning the search for information), individual work (gathering information)
Presentations of the results of group work by learners (OP) based on notes, using PowerPoint (O&WP), Blackboard (O&WP), etc.; answering questions (OWI)	Group work (preparing a presentation), individual work (presenting)
Activities run as projects (linking different competences) as group research (OWI, O&WP)	Group work (working on a common project)
Teamwork: Developing new mathematical problems (OI, WP)	Group work (creative work)
Controlling and reflecting results (WR, OI)	Group work (reflective work)

Niveaux de discours

Level	Type of Discourse	Description
1	Answering, stating, or sharing	A student gives a short right or wrong answer to a direct question, or a student makes a simple statement or shares his or her results in a way that does not involve an explanation of how or why.
2	Explaining	A student explains a mathematical idea or procedure by describing how or what he or she did but does not explain why.
3	Questioning or challenging	A student asks a question to clarify his or her understanding of a mathematical idea or procedure, or a student makes a statement or asks a question in a way that challenges the validity of an idea or procedure.
4	Relating, predicting, or conjecturing	A student makes a statement indicating that he or she has made a connection or sees a relationship to some prior knowledge or experience, or a student makes a prediction or a conjecture based on his or her understanding of the mathematics behind the problem.
5	Justifying or generalizing	A student provides a justification for the validity of a mathematical idea or procedure, or makes a statement that is evidence of a shift from a specific example to the general case.

Extrait de :

Oregon Mathematics Leadership Institute Spring 2007 Evaluation Report

by D. Weaver, 2007, Portland, OR, RMC Research Corporation, p. 26.

Le projet que nous mentionnons, et qui était focalisé sur le discours mathématique des élèves, a fait l'objet d'une analyse et d'un compte-rendu (Fredericks et Van Cleave)

Exemples de questions incitant au retour réflexif sur la tâche ou facilitant l'anticipation ou l'imagination

<u>Questions invitant à un retour réflexif sur le travail ou l'activité :</u>	<u>Questions facilitant la l'anticipation, l'imagination ou la résolution de problème :</u>
<p>What mathematics were you investigating?</p> <p>What questions arose as you worked?</p> <p>What were you thinking when you made decisions?</p> <p>What strategy did you select to solve the problem?</p> <p>What changes did you have to make to solve the problem?</p> <p>What was the most challenging part of the task?</p> <p>What were the steps involved?</p> <p>What was the most important thing you learned in this activity today?</p> <p>What else could you have done?</p>	<p>What would happen if?</p> <p>What decisions can you make from the pattern that you discovered?</p> <p>How else might you have solved the problem?</p> <p>Will it be the same if we use different numbers?</p> <p>How is this pattern like ?</p> <p>How can you prove that there is only one possible answer?</p> <p>Can you tell me what is the same and what is different?</p> <p>How can you prove your estimate?</p>

Exemple de séance à focalisation linguistique

Nous insistons auprès du lecteur sur le fait que nous avons dû faire des choix en ce qui concerne les thèmes abordés au niveau des séances expérimentales. Pour des raisons de place limitée, mais aussi du fait que nous considérons avoir choisi jusqu'à présent un moyen d'illustrer convenablement nos propos théoriques en concentrant nos efforts sur le thème des preuves visuelles, et de manière plus générale sur celui de l'arithmétique, nous ne présenterons dans cette partie que quelques exemples issus de séances que nous avons effectuées avec les élèves de première ou de terminale européenne. Nous joignons également quelques documents complémentaires.

Cette séance a été effectuée avec l'aide de dictionnaires en ligne :

<http://www.larousse.fr/dictionnaires/anglais-français>

<http://www.thesaurus.com>

<http://www.etymonline.com>

<http://www.prefixsuffix.com>

Elle est inspirée de l'article « Wort.Schatz » de Tanja Tajmel (Tajmel 2011).

Keyword	To differentiate
Examples of use Everyday language	he cannot differentiate between mother and daughter / a policy which is differentiated by sector and sub-sector / to identify what differentiates the two categories /
Other languages	Spanish : diferenciare German : unterscheiden French : différencier
Meanings in mathematical context	to calculate or get the derivative of (a function)
Examples in mathematical context	to differentiate the function x^2+2x+3 WRT x
Collocations	unable to differentiate between/ differentiate between right and wrong hard to differentiate
Prepositional phrases	differentiate X from Y / differentiate between X and Y/ differentiate X and Y
Synonyms	distinguish / separate
Antonyms	link / group math: to integrate
Idioms / proverbs / rhymes	
Similar words / words that look similar	dissociate / disclose / disrupt / detach / dismiss / distract referentiate
Word root	difference / differ, infer radical: "fer"
Etymology	1816, from Medieval Latin <i>differentiatus</i> , past participle of <i>differentiare</i> , from Latin <i>differentia</i> originally a mathematical term
Prefix / suffix	dis- / -iate or -tiate

Exemples de sujets donnés à l'épreuve spécifique de terminale

Rappel :

L'élève dispose de 20 minutes de préparation et présente sa solution pendant 10 minutes.

L'épreuve se prolonge par 10 minutes supplémentaires consacrées à un entretien sur un des thèmes d'une liste proposée par l'élève.

Les sujets 2 et 3 sont des sujets plus propices à un élargissement du discours. Ils sont très récents (épreuve de mai 2015, Académie de Bordeaux). Le sujet 1 n'intègre pas les recommandations récentes selon lesquelles le questionnement doit permettre à l'élève de ne pas cantonner son discours et sa réflexion au seul formalisme mathématique.

Sujet 1⁶ :

Exercise 1:

Let g be the function defined on $[0; +\infty[$ by : $g(x) = 2xe^{-x^2}$.

- 1) For any number a in $[0; +\infty[$, let $I(a)$ be defined as:

$$I(a) = \int_0^a g(x) dx$$

Express $I(a)$ in terms of a and then deduce the limit of $I(a)$ as x tends to positive infinity.

- 2) Justify that g is a density of probability.
 3) Let X be a random variable following a distribution with g as density of probability. Determine the real number m in such a way that $P(X \leq m) = 0.5$.

What does m represent?

⁶ Nous n'avons fait figurer qu'un seul exercice, le sujet comportant en fait un deuxième exercice. Le candidat choisit un exercice parmi les deux proposés.

Sujet 2

ARROWHEAD

The arrowhead on the picture opposite was found on an archaeological digging site near Stonehenge.



Figure 1

This gave a math teacher the insight to simplify its shape and make a word-problem out of it.

(see figure 2).

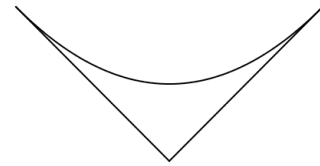


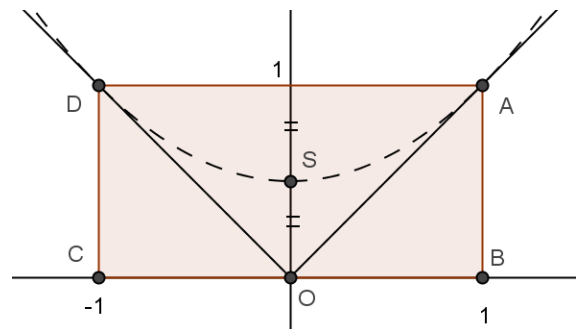
Figure 2

He then asked the whole class if they could solve the following problem:

Let ABCD be a rectangle drawn on a coordinate plane and symmetrical with respect to the y-axis.

Is it possible to draw a parabola

- whose vertex is on the y-axis, halfway between 0 and 1 and
- which is tangent both to line (OA) at point A and to line (OD) at point D ?



Paul claimed that this is mathematically impossible, saying that the only possibility would be to consider an arc of a circle.

Lucy on the contrary claimed that she could determine a solution to the problem.

She even added that there exists one and only one such parabola.

Where do you stand on this?

Justify your point of view.

Vocabulary:

arrowhead = tip of arrow

Sujet 3

Tangent to a parabola

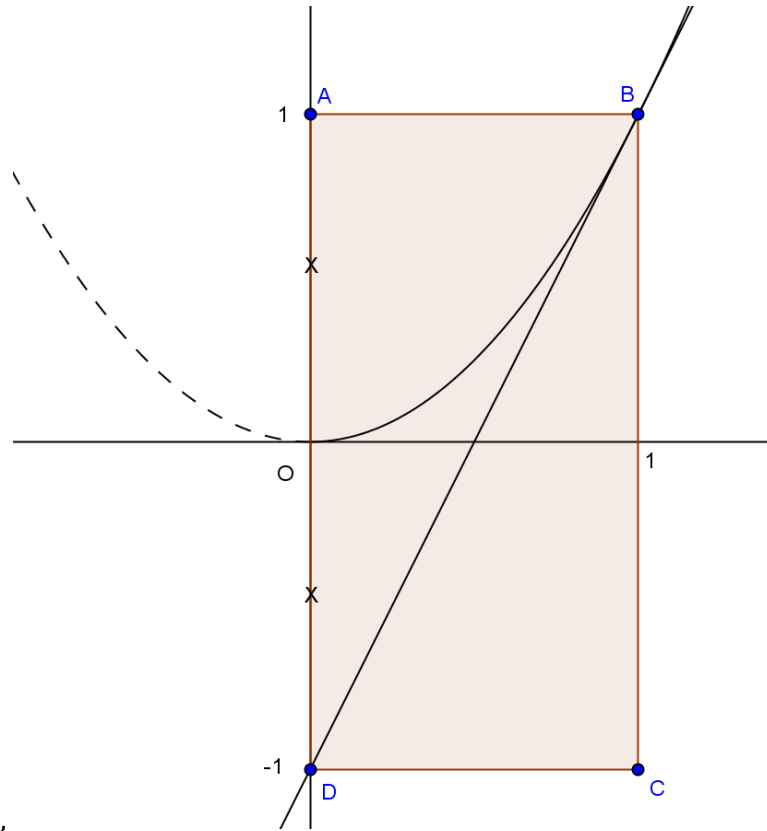
The diagram below shows the parabola $y=x^2$ in the coordinate plane.

The rectangle ABCD is made up of two unit-squares, as shown on the picture.

The origin of the axes is halfway from points A and D, as symbolized through the two crosses on the vertical axis.

At first sight, the diagonal of the rectangle seems to be tangent to the parabola.

- 1) Where do you stand on this rather immediate and natural conjecture?
Justify your answer.



By reasons of symmetry, and provided that question 1 receives a favourable answer, there should exist another parabola that would open down and also be tangent to line (BD).

- 2) What do you think?
Would you go so far as giving an equation of this parabola?