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Par

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## Structure des noyaux les plus lourds: spectroscopie du

 noyau ${ }^{251} \mathrm{Fm}$ et développement pour des traitements
## numériques du signal

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## Résumé substantiel de la thèse

## Titre: Structure des noyaux les plus lourds: spectroscopie du noyau ${ }^{251} \mathrm{Fm}$ et développement pour des traitements numériques du signal

L'un des principaux défis de la physique nucléaire moderne est de comprendre la structure nucléaire des éléments les plus lourds. Les barrières de fission calculées dans le modèle de la goutte liquide macroscopique ne parviennent pas à expliquer la stabilité des noyaux avec un nombre de protons $\mathrm{Z} \geq 90$. Cette barrière disparaît pour les éléments transfermium $(Z \geq 100)$ qui ne sont donc stabilisés que par des effets quantiques de couche. Les noyaux lourds sont un laboratoire unique pour étudier l'évolution de la structure nucléaire dans des conditions extrêmes de masse et de champ Coulombien. Bien que de nombreuses théories s'accordent sur l'existence d'un « îlot de stabilité », les prédictions sur son emplacement exact en terme de nombre de protons et neutrons varient grandement. Les études expérimentales des noyaux transfermium s'avèrent donc essentielles pour contraindre les modèles théoriques et mieux comprendre l'évolution des couches nucléaires.

L'interaction entre la particule indépendante et les degrés de liberté collectifs dans le noyau ${ }^{251} \mathrm{Fm}$ a été étudiée par la combinaison de la spectroscopie d'électrons de conversion interne (ECI) et spectroscopie du rayonnement $\gamma$. Les états excités du ${ }^{251} \mathrm{Fm}$ ont été peuplés dans la décroissance $\alpha \mathrm{du}{ }^{255} \mathrm{No}$, produit dans deux réactions de fusionévaporation suivantes: ${ }^{208} \mathrm{~Pb}\left({ }^{48} \mathrm{Ca}, 1 \mathrm{n}\right){ }^{255} \mathrm{No}$ et ${ }^{209} \mathrm{Bi}\left({ }^{48} \mathrm{Ca}, 2 \mathrm{n}\right){ }^{255} \mathrm{Lr}$. Les expériences ont été réalisées au JINR, FLNR, Dubna. Les faisceaux intenses ont été délivrés par le cyclotron U-400, et les séparateurs VASSILISSA ou SHELS ont été utilisés pour sélectionner les résidus de fusion-évaporation. Le spectromètre GABRIELA a été utilisé pour effectuer des mesures des propriétés de décroissance caractéristique corrélées en temps et en position pour isoler les noyaux d'intérêt. La spectroscopie d'électrons de conversion interne $d u{ }^{251} \mathrm{Fm}$ a été réalisée pour la première fois. Ces mesures ont permis d'établir les multipolarités de plusieurs transitions et de quantifier le rapport de mélange $\mathrm{M} 2 / \mathrm{E} 3$ dans la désintégration de l'isomère $5 / 2+$. Le $B(E 3)$ valeur extraite est comparée à celles des autres membres de la chaîne isotonique $\mathrm{N}=151$ et à des calculs théoriques.

Au cours de ce travail, une nouvelle méthode graphique d'extraction des rapports de mélange de transitions nucléaires a été développé. Cette méthode intuitive et illustrative et ses limites d'application, ainsi que certains aspects du calcul des rapports de mélange au-delà de ces limites, sont décrites et discutées.

Les détecteurs silicium double-face à strips (DSDS) sont largement utilisés en spectrométrie nucléaire, en particulier au plan focal de séparateurs pour détecter l'implantation et la désintégration ultérieure des noyaux les plus lourds. Il a été constaté que la présence de strips mécaniquement déconnectés sur une face du DSDS peut conduire à l'apparition de pics d'énergie abaissée sur la face opposée en raison de la variation de la capacité totale. Cet effet, ainsi que les méthodes de correction du spectre, ont été étudiés et discutés. L'utilisation de simulations GEANT4 pour résoudre les effets de sommation $\alpha$-ECI dans le DSDS et pour contraindre les coefficients de conversion interne des transitions impliquées dans la désexcitation du noyau d'intérêt est présentée à l'aide de l'exemple du ${ }^{221} \mathrm{Th}$.

Une bonne partie des travaux ont été consacrés à la R\&D pour un nouveau système électronique numérique pour le spectromètre GABRIELA et aux tests comparatifs de plusieurs cartes d'acquisition numériques. Les résultats de ces tests, ainsi que les algorithmes de traitement numérique du signal mis en œuvre pour une analyse non biaisée hors ligne sont présentés.

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Tiger got to hunt, bird got to fly;
Man got to sit and wonder 'why, why, why?'
Tiger got to sleep, bird got to land;
Man got to tell himselfhe understand.
"Cat's Cradle", Kurt Vonnegut

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The White Rabbit put on his spectacles. "Where shall I begin, please your Majesty?" he asked. "Begin at the beginning," the King said gravely, "and go on till you come to the end: then stop."
"Alice’s Adventures in Wonderland", Lewis Carroll

## 1

## Introduction

### 1.1 Motivation for the research

A major challenge of modern nuclear physics is to extend the frontiers of the nuclear chart. Many theories agree on the existence of a so-called "Island of Stability" - a cluster of spherical, stable or very long-lived super-heavy elements (SHE). These nuclei would be stabilized by quantum shell effects and be grouped around the next shell closures. However, the predictions for the next magic numbers vary over a wide range: $\mathrm{Z}=114,120$ or 126 for protons and $\mathrm{N}=172$ or 184 for neutrons (see, e.g., [1]). It is extremely difficult to check these predictions, since the production cross-sections of the SHE are very low (giving a few events per month). Luckily, the quantum states responsible for the enhanced stability for spherical SHE are also active for the lighter, deformed (rugby ball shaped) transfermium ( $\mathrm{Z}=100-105$ ) nuclei. The study of these nuclei, which also owe their existence to quantum shell effects, may shed much light on the nuclear structure and nuclear dynamics under the influence of large Coulomb forces and large mass. A systematic investigation of transfermium nuclei is thus essential to constrain theoretical models and to obtain a better understanding of the evolution of nuclear shells.

### 1.2 Nuclear models and theoretical approaches

### 1.2.1 Liquid drop model

The first nuclear model able to predict binding energies was the liquid drop model. It considers an atomic nucleus as a droplet of an incompressible quantum fluid consisting of nucleons: protons and neutrons. The nucleons are held together with the residual strong force. The liquid drop model takes into account the fact that the forces on the nucleons at the surface of a nucleus are different to those that are completely surrounded by other attracting nucleons. This is somewhat similar to a droplet of liquid held together by the surface tension. The volume of the liquid drop is proportional to the mass number $A$, and the surface is proportional to $A^{2 / 3}$. Another force that has to be taken into account is the Coulomb repulsion of the protons in a nucleus which makes the nucleus less tightly bound. Since each proton is repelled by every other proton, this term is proportional to $Z(Z-1)$. Assuming a uniformly charged sphere, the binding energy must proportional to $-\frac{Z(Z-1)}{A^{1 / 3}}$. A reasonable approximation of the variation of the nuclear binding energy $E_{b}$ is therefore obtained using the mass number:

$$
\begin{equation*}
E_{b} \approx a_{V} A-a_{S} A^{2 / 3}-a_{C} \frac{Z(Z-1)}{A^{1 / 3}}, \tag{1.1}
\end{equation*}
$$

where $a_{V}=15.5 \mathrm{MeV}, a_{S}=16.8 \mathrm{MeV}$ and $a_{C}=0.72 \mathrm{MeV}$ are empirical coefficients [2].
It may be said that the first attempt to apply nuclear models to understand the systematic behaviour of nuclear properties was made by Bethe and Weizsaecker who, in 1935, wrote the famous semi-empirical equation for the binding energy of nuclei depending on their atomic number $A$ and charge $Z$.

The first three terms of the Weizsaecker equation arise from the liquid drop approach (see eq. 1.1). As it was found that the most stable nuclei have equal or similar numbers of protons and neutrons, an empirical term describing the symmetry of a nucleus in terms of number of protons and neutrons $a_{\text {sym }} \frac{(A-2 Z)^{2}}{A}$ was added to the equation, where the constant $a_{\text {sym }}=23 \mathrm{MeV}$. This term becomes zero at $Z \simeq A / 2$, thus implying an enhanced stability, and grows larger as $N$ and $Z$ become more different. It was also known that nuclei with even N and Z tend to be more stable than the odd-valued configurations. Hence, the last empirical term $a_{P} A^{-3 / 4}$ was added, where $a_{P}=34 \mathrm{MeV}$ when
$Z$ and $N$ are even, $a_{P}=-34 \mathrm{MeV}$ when $Z$ and $N$ are odd and $a_{P}=0$ for an odd $A$.
Thus, the semi-empirical formula [2] describing the binding energies of nuclei becomes:

$$
\begin{equation*}
E_{b}=a_{V} A-a_{S} A^{2 / 3}-a_{C} \frac{Z(Z-1)}{A^{1 / 3}}-a_{\text {sym }} \frac{(A-2 Z)^{2}}{A}+a_{P} A^{-3 / 4} \tag{1.2}
\end{equation*}
$$

Using this expression, a semi-empirical mass formula may also be written:

$$
\begin{equation*}
M(Z, A)=Z \cdot m_{p}+N \cdot m_{n}-E_{b} / c^{2} \tag{1.3}
\end{equation*}
$$

where $M(Z, A)$ is a mass of a nucleus with charge $Z$ and mass number $\mathrm{A}, m_{p}$ and $m_{n}$ are the masses of a proton and of a neutron respectively, $c=3 \cdot 10^{8} \mathrm{~m} / \mathrm{s}$ is the speed of light. The neutron and proton separation energies, i.e. the energies required to remove a nucleon from the core, are

$$
\begin{gather*}
S_{p}=M(Z-1, A-1) c^{2}-M(Z, A) c^{2}+M\left({ }^{1} \mathrm{H}\right) c^{2}  \tag{1.4}\\
S_{n}=M(Z, A-1) c^{2}-M(Z, A) c^{2}+m_{n} c^{2}, \tag{1.5}
\end{gather*}
$$

where $m_{n}$ is the neutron mass and $M\left({ }^{1} H\right)$ is the mass of ${ }^{1} \mathrm{H}$.
Although the applied approach is rather crude, the solutions of eq. 1.2-1.3 compare reasonably well to many experimentally-defined masses and binding energies. However, there are numerous phenomena which cannot be explained by the liquid drop model, e.g. that specific "magic" numbers of nucleons result in an enhanced stability compared to that given by eq. 1.2 or the fact that $\gamma$ transitions have discrete energies. Furthermore, the classical liquid drop model cannot account for non-spherical nuclei.

The limits of stability given by the liquid drop model are more restricted than the experimentally observed nuclear landscape. In fig. 1.1 the green lines trace the limit values of neutron $S_{n}=0$ and proton $S_{p}=0$ separation energies, and the blue lines indicate the decrease and disappearance of the fission barrier $B_{f}$. The liquid drop fission barrier does not explain the lifetimes of the transuranium elements, and completely disappears around $Z \sim 100$. However, transfermium elements are known to exist and have long lifetimes due to the additional stability arising from the quantum shell effects.


Figure 1.1: The limits of nuclear stability according to the liquid drop model (figure by C. Theisen).

### 1.2.2 Shell model

The nuclear shell model describes a nucleus in terms of individual nucleons arranged in shells. It was first proposed by D. Ivanenko and E. Gapon as early as 1932 [3]. The model is partly analogous to Niels Bohr's atomic shell model describing the arrangement of electrons in an atom, which was a great success of that time. Likewise, a filled nuclear shell results in an enhanced stability. The nuclear shell model was inspired by the experimentally observed fact that the nuclear mass formula (see eq. 1.3), which describes the nuclear masses quite well on average, fails for certain "magic numbers", i.e., for $N=20,28,50,82,126$ and $Z=20,28,50,82$. Such nuclei, especially the doublymagic ones, demonstrate much stronger binding energies than predicted by eq. 1.2. Moreover, discreteness of $\gamma$-ray lines in nuclear spectra indicates a limited number of allowed energy states and thus suggests that a nucleon moves in an effective potential well created by all the other nucleons (mean field). This potential is proportional to the nuclear density and can be expressed as follows:

$$
\begin{equation*}
V(r)=\frac{-V_{0}}{1+\exp \left(\frac{r-R}{a}\right)}, \tag{1.6}
\end{equation*}
$$

where $R=R_{0} A^{1 / 3}$ is the mean nuclear radius with $R_{0}=1.25 \mathrm{fm}, a=0.524 \mathrm{fm}$ is the skin


Figure 1.2: A realistic form for the shell-model Wood-Saxon potential.
thickness, i.e. the distance over which charge density falls from $90 \%$ to $10 \%$ and $V_{0}$ is the depth of the potential well [2] (see fig. 1.2). This potential is called Wood-Saxon (WS) potential.

The spectroscopic notation of shells ( $1 s, 1 p, 1 d, 2 s$, etc.) is inherited from the atomic shell model. Each quantum shell may contain $2(2 l+1)$ particles, where $l$ is the orbital angular momentum of the nucleons, the factor ( $2 l+1$ ) comes from the projection of angular momentum $m_{l}$ degeneracy and the factor of 2 comes from the spin projection $m_{s}$ degeneracy. The resulting energy levels are shown in fig. 1.3.

However, the potential 1.6 alone only explained the first three magic numbers: 2 , 8 and 20. For a while this posed a problem, until in 1948 M. Goeppert-Mayer and independently H. Jensen (who shared a Noble prize in 1963) proposed the inclusion of a spin-orbit interaction to the potential. The spin-orbit force is an interaction of a particle's spin with its motion, therefore it is appropriate to define the total angular momentum $j=|l+s|$, where $s= \pm 1 / 2$ is the projection of the spin of a nucleon. The total angular momentum may take values $j=l-1 / 2$ and $j=l+1 / 2$ when $l>0$ and $j=1 / 2$ when $l=0$. Additional splitting of levels over $j$ reduces the degeneracy (see fig. 1.3) and allows all magic numbers to be reproduced. The shell model also predicts and explains to a greater extent other properties of nuclei, such as the spin and parity of nuclear ground states (g.s.) and excited states, as well as their energy spacing.


Figure 1.3: A schematic representation of the shell structure in nuclei. The energy levels calculated with the WS potential without the spin-orbit interaction are given in the left; in the right the effect of the spin-orbit interaction is added.

### 1.2.3 Nuclear shapes, vibrations and rotations

The concept of deformed nuclei was first introduced by J. Rainwater (Noble Prize 1975). It is convenient to define an instantaneous coordinate $R(t)$ (where $t$ is time) of a point on a nuclear surface described by spherical coordinates $(\theta, \phi)$. The deformation can then be expressed in terms of spherical harmonics $Y_{\lambda, \mu}$, where $\lambda$ and $\mu$ are the multipole and the order of deformation respectively, as follows:

$$
\begin{equation*}
R(t)=R_{\circ}\left(1+\sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda, \mu}(t) Y_{\lambda, \mu}(\theta, \phi)\right), \tag{1.7}
\end{equation*}
$$

where $\alpha_{\lambda, \mu}(t)$ is the amplitude of the harmonics. $\lambda=0$ and 1 do not contribute to the deformation, as $\lambda=0$ is a contraction or an expansion of the sphere and $\lambda=1$ is a translation. $\lambda=2$ corresponds to quadrupole vibration, yielding an ellipsoid-shaped nucleus. There are several formalisms to quantify the deformation, one of which is the $\beta$ parameter defined as follows:

$$
\begin{equation*}
\beta=\frac{4}{3} \sqrt{\frac{\pi}{5}} \frac{\Delta R}{R_{\circ} A^{1 / 3}}, \tag{1.8}
\end{equation*}
$$



Figure 1.4: Schematic visualisation of an axially symmetric deformed nucleus.
where $\Delta R$ is the difference between the semiminor and semimajor axes of the ellipse [2]. Nuclei with $\beta>0(\beta<0)$ have a prolate (oblate) ellipsoid shape (see fig. 1.4). The deformation can be stable (independent of $t$ ) as well as vibrational. The quantum of vibrational energy is called a phonon. Thus a $\lambda=2$ vibration is a quadrupole phonon, $\lambda=3$ - an octupole phonon etc..

An additional degree of freedom available to statically deformed nuclei is that of rotation about an axis different to that of the symmetry axis. The kinetic energy of a rotating nucleus is

$$
\begin{equation*}
E_{\text {rot }}=\frac{\hbar^{2}}{2 \mathscr{I}} I(I+1) \tag{1.9}
\end{equation*}
$$

where $I$ is the angular momentum quantum number and $\mathscr{I}$ is the moment of inertia, $\hbar=6.6 \cdot 10^{-34} \mathrm{~m}^{2} \mathrm{~kg} / \mathrm{s}$ is the Planck constant. Increasing $I$ is equivalent to adding rotational energy to the nucleus. The nuclear excited states from such a sequence form a rotational band. Rotational bands can be built on top of ground or excited states. The latter excitations occur when one or more nucleons are excited to higher single-particle levels. These excitations may change the nuclear shape, spin and parity.

The projection of the total angular momentum $K$ of the even-even nuclei is equal to zero. This is not the case for odd-A nuclei, where the expression 1.9 for $K \neq 1 / 2$ and for
rotation around the axis perpendicular to the symmetry axis becomes

$$
\begin{equation*}
E_{\text {rot }}=\frac{\hbar^{2}}{2 \mathscr{I}_{\perp}}\left(I(I+1)-K^{2}\right), \tag{1.10}
\end{equation*}
$$

where $\mathscr{I}_{\perp}$ is the projection of $\mathscr{I}$ on the axis perpendicular to the symmetry axis. When $K=1 / 2$, the rotation energy is also proportional to $\left(I+\left(1+(-1)^{I+1 / 2} a\right) / 2\right)^{2}$, where $a$ is the decoupling parameter [4].

Vibration and rotation were added to the single-particle picture by A. Bohr and B. Mottelson, who formulated a unified model of a deformed nucleus, for which together with J. Rainwater they received the Noble prize in 1975. S.G. Nilsson added the term $\left(l^{2}-<l^{2}>_{N}\right)$, where $<l^{2}>_{N}=1 / 2 N(N+3)$ and $N$ is the principal quantum number, to the Hamiltonian and formulated the Nilsson model. He also introduced a graphical representation called Nilsson diagrams (see e.g. fig. 1.5) [5], where the single particle energy is traced as a function of deformation. In such diagrams each energy level is labelled by the asymptotic quantum numbers as follows:

$$
K^{\pi}\left[N n_{Z} \Lambda\right]
$$

where $N$ is the principal quantum number, $n_{Z}$ is the symmetry axis component of $N$, $\Lambda$ is the projection of $l$ on the $z$ axis, $K=n_{Z}+\Lambda$ is the projection of the total angular momentum on the $z$ axis and $\pi=(-1)^{l}$ is the parity of the state.

### 1.2.4 Mean field approaches

Most generally, a Hamiltonian describing N interacting particles may be written as follows:

$$
\begin{equation*}
H=\sum_{i=1}^{N} T(i)+\sum_{i<j=1}^{N} V^{(2)}(i, j)+\sum_{i<j<k=1}^{N} V^{(3)}(i, j, k)+\ldots, \tag{1.11}
\end{equation*}
$$

where the first term corresponds to the sum of the kinetic energies of the $N$ particles, the second term denotes the two-body, third - three-body etc. interactions [7]. Resolving this Hamiltonian equation results in an N -body problem. The simplest approach to obtain a solution is to assume that the particles do not interact with one another except through an average mean field. In this manner an N -body problem is reduced to


Figure 1.5: Nilsson diagram showing the single-neutron energy levels for heavy nuclei as a function of axial deformation (from [6]); the orbitals active in ${ }^{251} \mathrm{Fm}$ excitations (see chapter 6) are highlighted.
a 1-body problem.
A practical difficulty encountered by all mean field theories is to define the potential of the mean field itself. There are two main courses of attack for this problem: 1) phenomenological, whereby the nuclear potential is parametrised by an appropriate mathematical function, and 2) self-consistent or Hartree-Fock (HF), which aims to deduce the potential from an effective nucleon-nucleon interaction. Though the latter approach seems to be more fundamental, the nucleon-nucleon interaction within the
nucleus is, alas, not known analytically. Thus the concept of an effective interaction was introduced, as a mathematical function with the parameters adjusted to agree with experimental data. The three most prominent schemes are the Skyrme (zero-range interaction), Gogny (finite-range interaction) energy functionals, and the relativistic mean field parameter set [8].

In HF theory the many-body ground state wave function is assumed to be a single Slater determinant $\mid \Phi>$ [7], i.e. an anti-symmetrised product of single-particle wave functions. The energy expectation value then is

$$
\begin{equation*}
E_{H F}=\frac{\langle\Phi| H|\Phi\rangle}{\langle\Phi \mid \Phi\rangle} \tag{1.12}
\end{equation*}
$$

The HF ground state is obtained by minimizing the energy in the space of Slater determinant

$$
\begin{equation*}
\delta\left[<\Phi|H| \Phi>-E_{H F}<\Phi \mid \Phi>\right]=0, \tag{1.13}
\end{equation*}
$$

where the variation is made on all possible variations of single-particle components.
In HF theory particles and holes are uncorrelated. The Random Phase Approximation (RPA) which is based on the HF approach includes particle-hole correlations (see fig. 1.6), replacing the simple HF particle-hole vacuum by a correlated ground state involving many-particle - many-hole excitations [9]. RPA theory was introduced in 1953 by D. Bohm and D. Pines to describe the oscillations in plasma. The mathematical formalism was then applied to nuclear structure physics. In addition to single-particle excitations built on the HF ground state the RPA also describes the collective vibrations representing a "coherent" motion of many nucleons [9].

However, the RPA theory does not include pairing effects. In quasi-particle RPA (QRPA) the hamiltonian is expressed in the Hartree-Fock-Bogoliubov (HFB) representation. HFB theory is a generalisation of the HF equations that includes pairing interaction [10].

### 1.3 Radioactive decays

A radioactive decay is a stochastic process when an unstable nucleus loses energy by emitting radiation, including $\alpha$ particles, $\beta$ particles, $\gamma$-rays, internal conversion elec-


Figure 1.6: Schematic visualisation of proton or neutron energy levels. $\epsilon_{f}$ is the fermi surface.
trons or undergoes spontaneous fission.
The probability of the decay depends exponentially on time $t$. Thus the number of nuclei at time $t$ equals $N(t)=N_{\circ} \exp -t / \tau$, where $N_{\circ}$ is the initial number of nuclei and $\tau$ is the lifetime of a nucleus. The half-life $T_{1 / 2}=\tau \ln (2)$ is the period of time such that $N\left(T_{1 / 2}\right)=1 / 2 N_{0}$.

### 1.3.1 Alpha decay

An $\alpha$ particle is a bound system of two protons and two neutrons. It has a particularly high binding energy $E_{b}(\alpha)=28.3 \mathrm{MeV}$. This makes it energetically more advantageous for many nuclei to decay by emitting an $\alpha$ particle rather than single nucleons. The mother nucleus that undergoes $\alpha$ decay transforms as follows:

$$
{ }_{Z}^{A} X \rightarrow{ }_{Z-2}^{A-4} Y+\alpha
$$

The $Q$-value of $\alpha$ decay is

$$
\begin{equation*}
Q_{\alpha}=M\left({ }_{Z}^{A} X\right) c^{2}-M\left({ }_{Z-2}^{A-4} Y\right) c^{2}-m_{\alpha} c^{2}-2 m_{e} c^{2}, \tag{1.14}
\end{equation*}
$$

where $m_{\alpha}$ and $m_{e}$ are the masses of an $\alpha$ particle and of an atomic electron. Thus, from the conservation laws of energy and momentum, and neglecting the binding energies of an $\alpha$ particle and of the daughter nucleus, the kinetic energy of an $\alpha$ particle for a g.s. $\rightarrow$ g.s. transition is

$$
\begin{equation*}
\mathrm{T}_{\alpha}=\frac{A-4}{A} \mathrm{Q}_{\alpha} . \tag{1.15}
\end{equation*}
$$

$\alpha$-decay can be characterised by a hindrance factor (HF) defined as a ratio of the measured $T_{1 / 2}$ for a given $\alpha$-transition to the one calculated for the g.s. $\rightarrow$ g.s. decay of an even-even nucleus (the calculation procedure can be found e.g. in [11]). For odd nuclei, the closest even-even nucleus (with one nucleon less) is taken. The value of the HF allows an empirical assessment of the properties of a transition. For the transitions between similar or identical initial and final single particle states the HF is lower than 10; for different single particle states without parity change, $\mathrm{HF}=10-100$; for different parity but same spin projection, $H F=100-1000$; for different single particle states with change of parity and a spin flip, HF>1000 [12].

### 1.3.2 Beta decay and electron capture

Beta decay is governed by the weak force. Nucleons are composed of up (u) or down (d) quarks: $p=u u d, n=u d d$. The weak force allows quarks to change type by the exchange of a $\mathrm{W}^{ \pm}$boson and the creation of positron + neutrino or electron + anti-neutrino pair respectively (see fig. 1.7). Thus the nuclei transform by the following laws:

$$
\begin{array}{ll}
\beta^{-}: & { }_{Z}^{A} X \rightarrow \rightarrow_{Z+1}^{A} Y_{N-1}+\mathrm{e}^{-}+\overline{\nu_{e}} \\
\beta^{+}: & { }_{Z}^{A} X \rightarrow{ }_{Z-1}^{A} Y_{N+1}+\mathrm{e}^{+}+\nu_{e}
\end{array}
$$

As the mass of a neutron is greater than that of a proton, a free neutron may $\beta^{-}$decay into a proton, but a free proton does not possess enough energy for $\beta^{+}$decay. In the nucleus the binding energy is "spent" on the $\beta^{+}$decay.

In the case of electron capture (EC), an atomic electron from either the K- or Lorbital participates to the same process, and thus gets transformed into a neutrino:

$$
E C: \quad{ }_{Z}^{A} X+\mathrm{e}^{-} \rightarrow{ }_{Z-1}^{A} Y_{N+1}+\nu_{e}
$$

Such decay is always energetically possible if $\beta^{+}$decay is possible as it requires slightly less energy.


Figure 1.7: The Feynman diagram for $\beta^{-} / \beta^{+} /$EC decay.

### 1.3.3 Spontaneous fission

In spontaneous fission (SF) a nucleus disintegrates into at least two fragments (nuclei of lighter elements) and liberates a large amount of energy in form of neutrons and $\gamma$-rays, the overall kinetic energy being of a few hundreds MeV. SF mostly occurs to the heavy elements. It was discovered in 1940 by G. Flerov and K. Pertzhak in their observations of ${ }^{238} \mathrm{U}$ in the Moscow metro station "Dinamo", 60 metres underground [13].

SF is energetically beneficial for nuclei with low binding energies, when the sum of the binding energies of the two SF fragments is higher than the binding energy of the mother nucleus.

### 1.3.4 Isomers

The half-lives of excited states are usually within the range of few fs. However, some states may have significant lifetimes. An isomer is a metastable excited state of an atomic nucleus. The limit at which a state is called isomeric changes with the advancement of spectroscopic techniques, as shorter and shorter lifetimes can be measured (today the limit is of $\sim \mathrm{few} \mathrm{fs}$ ). The longest-living isomers have lifetimes of several years. Certain isomeric states may even have lifetimes longer than that of the ground state of the same nucleus, ${ }^{180 \mathrm{~m}} \mathrm{Ta}$ being a famous example.

The lifetime of an isomeric state is determined by the transition energy, change of spin and parity and the wave functions of the initial and final state. Isomeric states also may gain their lifetimes for other reasons, e.g. due to a large rearrangement of particles between the initial and final configuration, or high $K$ quantum number.

### 1.4 Decay of excited states

### 1.4.1 Electromagnetic transitions

An excited state of a nucleus may decay by emitting a $\gamma$-ray. The energy of a $\gamma$-ray $E_{\gamma}$ is equal the difference of the excitation energies of the initial $\left(E_{i}\right)$ and final $\left(E_{f}\right)$ states of a transition minus the kinetic energy of the recoiling nucleus $E_{R}$ :

$$
\begin{equation*}
E_{\gamma}=E_{i}-E_{f}-E_{R} . \tag{1.16}
\end{equation*}
$$

As the kinetic energy of the recoiling nucleus is very small (e.g. for a 200 keV transition in ${ }^{251} \mathrm{Fm} E_{R} \sim 0.17 \mathrm{eV}$ ), usually it can be neglected.

An excited nucleus may also electromagnetically interact with one of the atomic electrons. This causes the electrons to be ejected from the atom. Such electrons are called internal conversion electrons (ICE). The kinetic energy of the ICE equals

$$
\begin{equation*}
E_{I C E}=\left(E_{i}-E_{f}\right)-E_{b}, \tag{1.17}
\end{equation*}
$$

where $E_{b}$ is the binding energy of the electron on its atomic shell ( $\mathrm{K}, \mathrm{L}, \mathrm{M}$ etc.).

### 1.4.2 Atomic processes

The ICE emission is always accompanied by atomic radiation. The ejected atomic electron leaves a vacancy in one of the inner atomic shells. This vacancy is filled by one of the electrons from a higher shell. The excess of electron binding energy is released either in a radiative process through X-ray emission, or in the following non-radiative process: Auger, by releasing one of the electrons from higher shells, or Coster-Kröning (which is a special case of Auger) when the emitted electron also belonged to the same shell.

X-ray emission is characterised by the fluorescent yields $\omega_{K}, \omega_{L}$ etc., which represents the probability of the emission of a K-, L- etc. X-ray when filling the vacancy. $\omega_{K}$ is low for low-Z nuclei and high ( $\omega_{K}>90 \%$ ) for medium-heavy and heavy elements. $\omega_{K}$ is always much larger than $\omega_{L}$, which in turn is superior to $\omega_{M}$ etc. [14]. The Auger processes are characterised by Auger yields $\alpha_{K}, \alpha_{L}$ etc. such that $\omega_{K}+\alpha_{K}=1, \omega_{L}+\alpha_{L}=1$ etc..

### 1.4.3 Multipolarities

Each electromagnetic transition is characterised by the multipole order $L$ and the change of parity $\Delta \pi$. For the angular momentum, a selection rule applies $\left|I_{i}-I_{f}\right| \leqslant L \leqslant I_{i}+I_{f}$, where $I_{i}$ and $I_{f}$ are the angular momenta of the initial and final states. The multipolarity ( $\sigma \mathrm{L}$ ) of a transition may be electric ( E ) or magnetic (M) depending whether the radiation is due to a shift in the charge distribution or to a shift in the current distribution

$$
\begin{gather*}
\Delta \pi(E L)=(-1)^{L}  \tag{1.18}\\
\Delta \pi(M L)=(-1)^{L+1} \tag{1.19}
\end{gather*}
$$

The electric monopole (EO) corresponds to a static distribution of charge in the nucleus and can only proceed via internal conversion. As there is no magnetic charge, Mo transitions do not exist.

| $\sigma L$ | Name | $\Delta I$ | Change of $\pi$ |
| :---: | :---: | :---: | :---: |
| Eo | electric monopole | 0 | no |
| E1 | electric dipole | 1 | yes |
| M1 | magnetic dipole | 1 | no |
| E2 | electric quadrupole | 2 | no |
| M2 | magnetic quadrupole | 2 | yes |
| E3 | electric octupole | 3 | yes |
| M3 | magnetic octupole | 3 | no |
| E4 | electric hexadecapole | 4 | no |
| M4 | magnetic hexadecapole | 4 | yes |

Table 1.1: Selection rules and multipolarities.

For electromagnetic transitions the lowest possible multipolarity is, in general, most favoured.

### 1.4.4 Conversion coefficients

As mentioned above, electromagnetic transitions occur via two competing processes: $\gamma$-emission with a probability $P_{\gamma}$ and ICE-emission with a probability $P_{\text {ICE }}$. The conver-
sion coefficient $\alpha$ is defined as the ratio of these probabilities:

$$
\begin{equation*}
\alpha=\frac{P_{\gamma}}{P_{I C E}} ; \tag{1.20}
\end{equation*}
$$

As $P_{\gamma}+P_{\text {ICE }}=1$, the probability to decay via $\gamma$-emission is $P_{\gamma}=\alpha /(1+\alpha)$, and for the ICE $P_{\text {ICE }}=1 /(1+\alpha)$.

As the conversion coefficients can be calculated for various multipolarities of the transition [15], they provide important selection criteria on the spin and parity combinations of the initial and final states when compared to data. The rough approximate formulae for the internal conversion coefficients are:

$$
\begin{gather*}
\alpha(E L)=\frac{Z^{3} \alpha_{\text {f.s. }}^{4}}{n^{3}}\left(\frac{L}{L+1}\right)\left(\frac{2 m_{e} c^{2}}{E_{\gamma}}\right)^{L+5 / 2}  \tag{1.21}\\
\alpha(M L)=\frac{Z^{3} \alpha_{f . s .}^{4}}{n^{3}}\left(\frac{2 m_{e} c^{2}}{E_{\gamma}}\right)^{L+3 / 2} \tag{1.22}
\end{gather*}
$$

where $n$ is the principal quantum number of the ejected electron and $\alpha_{f . s .}=1 / 137$ is the fine structure constant [2].

Conversion coefficients can be derived by measuring the ratio of the $\gamma$-ray and internal conversion electrons (ICE) intensities ( $I_{\gamma}$ and $I_{I C E}$ respectively):

$$
\begin{equation*}
\alpha_{t o t}=\frac{I_{I C E}}{I_{\gamma}}=\alpha_{K}+\alpha_{L}+\alpha_{M}+\ldots \tag{1.23}
\end{equation*}
$$

where $\alpha_{\text {tot }}$ is the total conversion coefficient, and $\alpha_{K}, \alpha_{L}$ etc. are the partial coefficients for K -, L - etc. ICE. When the ICE measurement is missing or incomplete, the coefficients may also be determined through the X-rays that are emitted by the atom replacing the ICE in the shells:

$$
\begin{equation*}
\alpha=\frac{I_{X}}{I_{\gamma} \cdot \omega}, \tag{1.24}
\end{equation*}
$$

where $I_{X}$ is the measured intensity of the X-rays and $\omega$ is the fluorescent yield. This method only allows to quantify K - and sometimes L-conversion, as the X-ray energies of $M$ and higher electron shells are usually too low to be detected.

For mixed multipolarities the experimental conversion coefficients relate to the theoretical ones through the mixing ratio $\delta$ which is defined as the ratio of the reduced
matrix elements:

$$
\begin{equation*}
\delta=\frac{\left\langle I_{f}\right| O\left(\sigma^{\prime} L^{\prime}\right)\left|I_{i}\right\rangle}{\left\langle I_{f}\right| O(\sigma L)\left|I_{i}\right\rangle} \tag{1.25}
\end{equation*}
$$

where $\sigma L$ and $\sigma^{\prime} L^{\prime}$ are the two mixed multipolarities, $L^{\prime}>L ; O(\sigma L)$ and $O\left(\sigma^{\prime} L^{\prime}\right)$ are the electromagnetic mulitpole operators. For $\delta^{2}$ one can write

$$
\begin{equation*}
\delta^{2}=\frac{P_{\gamma}^{\prime}\left(\sigma^{\prime} L^{\prime}\right)}{P_{\gamma}(\sigma L)} \tag{1.26}
\end{equation*}
$$

where $P_{\gamma}(\sigma L)$ and $P_{\gamma}^{\prime}\left(\sigma^{\prime} L^{\prime}\right)$ are the probabilities to decay via each of the multipolarities.

Then the experimental conversion coefficient $\alpha_{\text {exp }}$ becomes

$$
\begin{equation*}
\alpha_{\text {exp }}=\frac{\alpha(\sigma L)+\delta^{2} \cdot \alpha\left(\sigma^{\prime} L^{\prime}\right)}{1+\delta^{2}} \tag{1.27}
\end{equation*}
$$

where $\alpha(\sigma L)$ and $\alpha\left(\sigma^{\prime} L^{\prime}\right)$ are the theoretical [15] ones. For the ratios, e.g. K and L:

$$
\begin{equation*}
\left(\alpha_{K} / \alpha_{L}\right)_{\exp }=\frac{\alpha_{K}(\sigma L)+\delta^{2} \cdot \alpha_{K}\left(\sigma^{\prime} L^{\prime}\right)}{\alpha_{L}(\sigma L)+\delta^{2} \cdot \alpha_{L}\left(\sigma^{\prime} L^{\prime}\right)} . \tag{1.28}
\end{equation*}
$$

The mixing ratio is crucial for the calculation of the transition strength $B(\sigma L)$ which, as it will be discussed in the following subsection, is an important link between theory and experiment in nuclear structure physics. Thus, the technique of determining the mixing ratios from the measured conversion coefficient is of interest to experimentalists. The equations for the calculation of the experimental transition strengths in W . $u$. for the different mixed multipolarities will be given in the next section.

Each of the theoretical values $\alpha(\sigma L), \alpha\left(\sigma^{\prime} L^{\prime}\right)$ etc. has an uncertainty $\Delta \alpha$ associated to it. This uncertainty is of the order of $1-2 \%$ and arises from two factors: the accuracy of the theoretical calculations and the accuracy of interpolation for the non-tabulated values [15]. However, because of the shape of the $\delta(\alpha)$ function this small $\Delta \alpha$ may result in much more significant and, in a general case, asymmetric uncertainties on $\delta$. Certain aspects of the mixing ratio determination will be discussed in chap. 5 .

### 1.4.5 Electromagnetic transition rates

The $\gamma$-decay rate $\lambda(\sigma L)$ i.e. the probability of decay per unit of time, can be expressed as follows:

$$
\begin{equation*}
\lambda(\sigma L)=\frac{2(L+1)}{L[(2 L+1)!!]^{2}}\left(\frac{\omega}{c}\right)^{2 L+1} B(\sigma L), \tag{1.29}
\end{equation*}
$$

where $\omega=2 \pi E_{\gamma}$ is the $\gamma$-ray frequency and $B(\sigma L)=\left.\frac{1}{2 L_{i}+1}\left|<I_{f}\right| O(\sigma L)\left|I_{i}\right\rangle\right|^{2}$ is the reduced transition probability [2]. The decay of an excited state may occur via different decay modes. The total decay rate $\lambda=\ln (2) / T_{1 / 2}$ is a sum of the decay rates of all modes. For an electromagnetic transition $\gamma$ emission often competes with ICE emission. To take this into account a $1 /(1+\alpha)$ factor must be inserted, where $\alpha$ is the total conversion coefficient. Thus, from the eq. 1.29, the transition strength can be expresses as follows:

$$
\begin{equation*}
B(\sigma L)=\frac{L[(2 L+1)!!]^{2}}{2(L+1)}\left(\frac{\hbar c}{E_{\gamma}}\right)^{2 L+1} \frac{\ln (2)}{T_{1 / 2}(1+\alpha)}, \tag{1.30}
\end{equation*}
$$

If the transition is of a mixed multipolarity $L /(L+1)$, then a factor of $1 /\left(1+\delta^{2}\right)$ must be inserted for the $L$ strength and $\delta^{2} /\left(1+\delta^{2}\right)$ for the $L+1$ strength.
V. Weisskopf derived general expressions for the $\gamma$-transition strength with the assumption that the transition results from a change of a single particle (s. p.) inside of a nucleus with a uniform density and with a radius $R=R_{0} A^{1 / 3}$. Such expressions are called Weisskopf single-particle strengths:

$$
\begin{gather*}
B_{s . p .}(E L)=\frac{1}{4 \pi b^{L}}\left(\frac{3}{3+L}\right)^{2} R^{2 L} \cdot e^{2} f m^{2 L}  \tag{1.31}\\
B_{s . p .}(M L)=\frac{10}{\pi b^{L-1}}\left(\frac{3}{3+L}\right)^{2} R^{2 L-2} \cdot \mu^{2} f m^{2 L-2}, \tag{1.32}
\end{gather*}
$$

where $b=10^{-24} \mathrm{~cm}^{2}, e^{2}=1.44 \cdot 10^{-10} \mathrm{keV} \mathrm{cm}$ and $\mu^{2}=1.59 \cdot 10^{-38} \mathrm{keV} \mathrm{cm}^{3}$ [16]. Then, from eq. 1.29-1.32 the partial $\gamma$-ray half-life is

$$
\begin{align*}
T_{1 / 2 s, p}^{\gamma}(E L) & =\frac{\ln (2) L[(2 L+1)]^{2} \hbar}{2(L+1) e^{2} R^{2 L}}\left(\frac{3+L}{3}\right)^{2}\left(\frac{\hbar c}{E_{\gamma}}\right)^{2 L+1}  \tag{1.33}\\
T_{1 / 2 s, p}^{\gamma}(M L) & =\frac{\ln (2) L[(2 L+1)]^{2} \hbar}{80(L+1) \mu^{2} R^{2 L-2}}\left(\frac{3+L}{3}\right)^{2}\left(\frac{\hbar c}{E_{\gamma}}\right)^{2 L+1} \tag{1.34}
\end{align*}
$$

These values are called Weisskopf single particle estimates. The comparison of the experimental transition strengths to the Weisskopf estimates measured in Weisskopf units (W. u.) gives an idea as to how valid the s.p. hypothesis is for a particular transition, and thus to quantify the role of collective effects, such as rotation or vibration or structural effects such as spin-flips causing more retarded decays. Thus, the transition strengths provide an important link between experiment and theory in nuclear structure physics.

## Description of the experimental setup

### 2.1 Fusion-evaporation reactions

Over the last 40 years, fusion evaporation reactions have been the most promising method to produce heavy elements. These reactions require heavy ion beams, such as $\mathrm{Ne}, \mathrm{Ca}$ or Ti and a heavy targets, e.g. $\mathrm{Pb}, \mathrm{U}$ or Pu . The projectile and target nuclei fuse, creating a compound nucleus in a very excited state (see fig. 2.1). It de-excites by evaporating neutrons, protons and/or $\alpha$-particles. The evaporation mode depends on the reacting nuclei as well as on the kinetic energy of the beam.

The beam nucleus should posses a sufficient kinetic energy $E_{b}$ to penetrate the Coulomb


Figure 2.1: A schematic view of experiments with the fusion-evaporation reactions
barrier of the target nucleus. The required beam energy, thus, depends on the interacting nuclei. The excitation energy of the compound nucleus can be expressed as follows:

$$
\begin{equation*}
E^{*}=Q+\Delta E_{K}, \tag{2.1}
\end{equation*}
$$

where $Q=M c^{2}+m c^{2}-M^{\prime} c^{2}$ is the $Q$-value of the reaction ( $M, m$ and $M^{\prime}$ are the masses of the target, beam and compound nuclei respectively) and $\Delta E_{K}$ is the difference between the kinetic energy of the beam particle and of the compound nucleus. In a nonrelativistic approach (which is well applicable to the velocities in question) $\Delta E_{K}$ can be expressed as

$$
\begin{equation*}
\Delta E_{K}=E_{b}-\frac{M^{\prime} V^{\prime 2}}{2}=E_{b} \cdot \frac{M^{\prime}-m}{M^{\prime}} \tag{2.2}
\end{equation*}
$$

where $V^{\prime}$ is the velocity of the compound nucleus. Thus the excitation energy becomes:

$$
\begin{equation*}
E^{*}=Q+E_{b} \cdot \frac{M^{\prime}-m}{M^{\prime}} . \tag{2.3}
\end{equation*}
$$

Thus, as can be seen from eq. 2.1, the excitation energy defines the evaporation mode of the compound nucleus, i.e. the number of particles ( $n, p$ and/or $\alpha$ ) that are emitted by the compound nucleus. This dependence is described by the excitation functions (see, e.g., fig. 2.3 of this thesis), which represent the dependence of the cross-section of a given evaporation mode on the excitation energy of the compound nucleus. The beam energy required for the selected excitation energy can be expressed as follows:

$$
\begin{equation*}
E_{b}=\left(E^{*}-Q\right) \frac{M^{\prime}}{M^{\prime}-m} . \tag{2.4}
\end{equation*}
$$

The same evaporation residue (ER) can be created with various combinations of beam and target particles: for example, to synthesize ${ }^{255} \mathrm{No}$, one can use ${ }^{208} \mathrm{~Pb}\left({ }^{48} \mathrm{Ca}, \mathrm{n}\right)^{255} \mathrm{No}$ or ${ }^{238} \mathrm{U}\left({ }^{22} \mathrm{Ne}, 5 \mathrm{n}\right){ }^{255} \mathrm{No}$ reactions. In the first case, the interacting nuclei are strongly bound: both ${ }^{48} \mathrm{Ca}$ and ${ }^{208} \mathrm{~Pb}$ are doubly-magic, meaning that both their proton and neutron shells are fully occupied. Thus the excitation energy of the compound nucleus is low, and lower evaporation modes $(1-2 n)$ are available. Such reactions are often referred to as cold fusion reactions. In the more asymmetric (in terms of the number of the masses of the beam and target nuclei) reaction of neon on uranium, the interacting nuclei are less bound, and thus the excitation energy of the compound is higher,
which constrains the reaction to higher (3-6n) evaporation modes. Such reactions with highly excited compound nuclei are called hot fusion reactions. The cross-sections are 200 nb for the ${ }^{238} \mathrm{U}\left({ }^{22} \mathrm{Ne}, 5 \mathrm{n}\right){ }^{255} \mathrm{No}$ reaction [17], and 260 nb for ${ }^{208} \mathrm{~Pb}\left({ }^{48} \mathrm{Ca}, \mathrm{n}\right){ }^{255} \mathrm{No}$ [18]. Although the cross-section of the second reaction is larger it does not necessarily mean that producing ${ }^{255}$ No via this reaction would be the most beneficial.

Another factor that has to be taken into account is the thickness of the target material. Typically the target would either be pure metallic (preferred) or a compound if an increased melting point is required (e.g. PbS or $\mathrm{Bi}_{2} \mathrm{O}_{3}$ ). It is often deposited on some sort of backing (e.g. carbon or Al, Ti). The thickness of the target (and backing) material is typically of the order of $0.2-0.5 \mathrm{mg} / \mathrm{cm}^{2}$. This thickness results in an energy loss for the beam particle in the target, as well as energy loss and angular scattering of the ER. The kinetic energy of an ER is reaction-dependent and can be expressed as follows:

$$
\begin{equation*}
E_{K}^{\prime}=\frac{m}{M^{\prime}} \cdot E_{b} \tag{2.5}
\end{equation*}
$$

The ${ }^{22} \mathrm{Ne}$ beam energy for the $5 n$ reaction is 117 MeV [17], thus $E_{K}^{\prime}\left({ }^{22} \mathrm{Ne}\right)=10 \mathrm{MeV}$. For the ${ }^{48} \mathrm{Ca}$ in reaction, the optimal beam energy is 212 MeV [18], and the $E_{K}^{\prime}\left({ }^{48} \mathrm{Ca}\right)=40 \mathrm{MeV}$. This means that the nobelium ER created in ${ }^{238} \mathrm{U}\left({ }^{22} \mathrm{Ne}, 5 n\right){ }^{255} \mathrm{No}$ leaves the target with a smaller energy and a wider angular distribution. This adversely affects the transmission of these nuclei to the focal plane of the separator and makes ${ }^{208} \mathrm{~Pb}\left({ }^{48} \mathrm{Ca}, \mathrm{n}\right){ }^{255} \mathrm{No} \mathrm{a}$ better choice.

### 2.2 SHELS

A fusion-evaporation reaction is by far not the most probable mode of interaction of the beam with the target. A major part of the incident particles traverse the target without interacting or scatters off the target atoms. The incident nucleus and the target may also exchange several nuclides in so-called transfer reactions. This is an important background for our experiments as most of these transfer products $\alpha$ decay. Finally, in some cases the compound nucleus may immediately fission. Thus, ER need to be separated from the background of other reaction products before they are transported to the detection system, where the decay of the nuclei of interest will be studied (see
fig. 2.1).
In order to achieve this goal separators are used. The general principle is to deflect the background products via electromagnetic fields and thus transport only the ERs to the detector at the focal plane of the separator. There are gas-filled separators (such as, e.g., RITU at JYFL), which are filled with inert gas, and vacuum separators such as e.g. SHIP at GSI. The performance of a separator can be estimated through its transmission coefficient. The transmission efficiency is a relation of the ER created at the target to the number of those transported to the focal plane. It can be written as

$$
\begin{equation*}
T=\frac{N_{\text {transmitted }} \cdot \epsilon_{\text {det }}}{N_{\text {incident }} \cdot n_{\text {target }} \cdot \sigma}, \tag{2.6}
\end{equation*}
$$

where $N_{\text {incident }}$ is the number of incident beam particles, $\epsilon_{\text {det }}$ is the detection efficiency, $n_{\text {target }}$ is the number of target particles per $\mathrm{cm}^{2}, \sigma$ is the reaction cross-section and $N_{\text {transmitted }}$ is the number of ER that are detected the focal plane. Another significant parameter to estimate the performance of a separator is its rejection, which denotes the purity of the nuclei that are selected. As mentioned in the previous section, the angular distribution of the ERs as well as their kinetic energy (and hence velocity) is reaction-dependetyytnt. If the angle between the optical axes and the ER trajectory is too large the particle may not enter the aperture of the separator. This is implicitly taken into account by $N_{\text {transmitted }}$. Another important design consideration is the size of the implantation detector at the focal plane of the separator. If its area is too small, it may be that some of the particles transported to the focal plane are not being detected, thus reducing the value of $\epsilon_{\text {det }}$ and hence the transmission.

We use the Separator for Heavy ELement Spectroscopy (SHELS) to study transfermium nuclei. SHELS was developed within a collaboration between the $\mathrm{IN}_{2} \mathrm{P} 3$ (France) and the JINR (Russian Federation) [19]. The cyclotron U-400 of the FLNR provides pulsed beams of heavy ions, such as ${ }^{16} \mathrm{O},{ }^{22} \mathrm{Ne},{ }^{40} \mathrm{Ar},{ }^{48} \mathrm{Ca}$ and ${ }^{50} \mathrm{Ti}$. As the beam intensities are quite high ( $10^{12}-10^{13}$ ions $\cdot \mathrm{s}^{-1}$ ), the targets have to be mounted on a rotating wheel to avoid melting them. The beam is pulsed to avoid irradiating the spokes of the target wheel to reduce the background of scattered beam.

SHELS is an upgrade of the previous vacuum separator called VASSILISSA. While VASSILISSA selected the ERs on the basis of their kinetic energy, SHELS in its turn is


Figure 2.2: The VASSILISSA (bottom) and SHELS(top) separators
a velocity filter. VASSILISSA [20] used to have a configuration QQQEEEQQQM ( $\mathrm{Q}=$ magnetic quadrupole, $\mathrm{E}=$ electric deflector, $\mathrm{M}=$ magnetic dipole). The scheme of the separator may be found in fig. 2.2 (bottom drawing).

The main objective of the upgrade was not only to increase the transmission efficiency for very asymmetric reactions (where the projectile nucleus is significantly lighter than the target nucleus), such as ${ }^{22} \mathrm{Ne}+{ }^{238} \mathrm{U}$ or ${ }^{16} \mathrm{O}+{ }^{244} \mathrm{Pu}$, but also to achieve better transmissions with symmetric reactions (where the projectile and target nuclei are closer in mass range). The configuration for SHELS is QQQEMMEEQQQM (see fig. 2.2, top drawing). One of the ways the acceptance (and hence transmission) of the separator has been improved was by moving the target closer to the first quadrupole, and thus covering a larger solid angle for the ERs. Moreover, the plates of the electric dipoles of SHELS are movable, and thus can be put closer to achieve higher fields, which is useful for symmetric reactions, like $\mathrm{Xe}+\mathrm{Xe}$.

The first commissioning of the new separator took place in May 2013. This was also when I joined the team for my Master's internship. The commissioning reactions were selected so that the reaction cross-sections would be sufficiently high $\sim 10 \mathrm{nb}-1 \mathrm{mb}$ so

| Reaction | VASSILISSA | SHELS |  |
| :---: | :---: | :---: | ---: |
| ${ }^{22} \overline{\mathrm{Ne}^{197}(\mathrm{Au}, 5 \mathrm{nn}}{ }^{214} \mathrm{Ac}$ | $3 \%$ | $6.5 \%$ | $\left(58 \times 58 \mathrm{~mm}^{2}\right)$ |
| ${ }^{40} \mathrm{Ar}\left({ }^{(288} \mathrm{Pb}, 3 \mathrm{n}\right)^{245} \mathrm{Fm}$ | $20 \%$ | $40 \%$ | $\left(58 \times 58 \mathrm{~mm}^{2}\right)$ |
| $\left.{ }^{50} \mathrm{Ti}\left({ }^{164} \mathrm{Dy}, 2 \mathrm{n}\right)\right)^{209} \mathrm{Ra}$ |  | $40 \%$ | $\left(100 \times 100 \mathrm{~mm}^{2}\right)$ |

Table 2.1: Some results for the transmission tests of SHELS compared to those of VASSILISSA. First two measurements were performed with a smaller ( $58 \times 58 \mathrm{~mm}^{2}$ ) focal plane detector. The last result was obtained after the upgrade of the focal plane detector ( $100 \times 100 \mathrm{~mm}^{2}$ )
that a number optical regimes could be scanned in a reasonable period of beam time. The transmission increase for the asymmetrical reactions should be a factor of five with a $100 \times 100 \mathrm{~mm}^{2}$ focal plane detector.

An example of such test reaction was ${ }^{208} \mathrm{~Pb}\left({ }^{40} \mathrm{Ar}, 2-3 \mathrm{n}\right){ }^{245-6} \mathrm{Fm} .{ }^{245} \mathrm{Fm} \alpha$-decays, and ${ }^{246} \mathrm{Fm}$ has a $93.20 \% \alpha$-branch and $6.80 \%$ spontaneous fission (SF) branch [23]. The excitation functions for this reaction are given in fig. 2.3(a). Fig. 2.3(b) contains the spectra of $\alpha$ decay of both ${ }^{245} \mathrm{Fm}$ and ${ }^{246} \mathrm{Fm}$. The production ratio of these two nuclei was used for the accurate definition of the beam energy ( 190 MeV mid-target).

From the number of observed decay events, and given a measured beam dose on target, a value of $40 \%$ was obtained for the transmission of ERs. This indicates a factor of two improvement of the transmission after the upgrade compared to the transmissions obtained with VASSILISSA. Additional comparisons from the commissioning of SHELS are given in table 2.1.

### 2.3 GABRIELA

The detection system at the focal plane of SHELS is called GABRIELA - Gamma Alpha Beta Recoil Investigations with the Electromagnetic Analyser [24]. GABRIELA consists of an implantation silicon detector, situated at the focal plane of the separator SHELS, a box of silicon strip detectors surrounding it (which we often call "the tunnel" due to its shape), an array of germanium detectors equipped with bismuth germanate crystal (chemical formula $\mathrm{Bi}_{4} \mathrm{Ge}_{3} \mathrm{O}_{12}$, hence denoted BGO) shields and a time-of-flight (ToF) detector. The entire setup, as it was during the experiment in January-February 2016, can be seen on fig. 2.4. It is designed to detect the arrival of the reaction products, as well as their subsequent decay via $\alpha$ - or $\beta$-emission, internal transitions emitting $\gamma$

b


Figure 2.3: (a) Excitation functions for the ${ }^{208} \mathrm{~Pb}\left({ }^{40} \mathrm{Ar}, \mathrm{xn}\right)$ reaction, taken from [21]; (b) fit to the $\alpha$ decay lines of ${ }^{245} \mathrm{Fm}$ and ${ }^{246} \mathrm{Fm}$ and corresponding energies [22]
rays, X -rays or internal conversion electrons, as well as spontaneous fission [25]. Thus GABRIELA is sensitive practically to any decay mode that may occur.

### 2.3.1 Time-of-flight detector

The nucleus of interest passes through the SHELS separator, goes through the bending magnet at the entrance to the GABRIELA experimental hall, passes through the time of flight (ToF) detector and gets implanted into the focal plane detector. The ToF detector consists of two emissive foils made of $30-40 \mu \mathrm{~g} / \mathrm{cm}^{2}$ of gold deposited on $40-50 \mu \mathrm{~g} / \mathrm{cm}^{2}$


Figure 2.4: The GABRIELA setup; picture by A. Popeko, taken in February 2016
of polypropylene. The foils in the current design are 102 mm wide and 90 mm high, placed at a distance of 240 mm from one another and 335 mm from the downstream foil to the focal plane.

The foils are equipped with two micro-channel plates (MCP) each. MCPs are plates of highly resistive material with a regular array of holes, $\sim 5-10 \mu \mathrm{~m}$ in diameter and $\sim 15 \mu \mathrm{~m}$ from one another, distributed over the surface. The charged recoil ion passing an emissive foil induces the emission of secondary electrons, which are then accelerated in an electromagnetic field and bent to the MCP plates, which are positioned perpendicular to the foils. Primary electrons trigger cascades of $\sim 10^{6}$ secondary electrons/event in the MCPs. The precise value of the amplification gain depends on the value of the high voltage applied as well as on the charge of the ion passing through the foil.

The time difference between the signals from the two foils is, naturally, proportional to the velocity of the recoil. The emissive foils of the ToF detector also provide a "flag" to distinguish the recoil implantations from the decays that occur in the implantation detector. This principle can be seen in fig. 2.11 and will be further discussed in
section 3.1 of this chapter.

### 2.3.2 The implantation detector

After passing the ToF foils, the recoil impinges on the implantation silicon detector. The depth of the implantation varies between $\sim 1-7 \mu \mathrm{~m}$ [26] and depends on the mass and the velocity of the recoil. The implantation detector is position-sensitive in order to allow position correlations. It is also used to detect $\alpha$ particles, internal conversion electrons and fission fragments. As the decay radiation is isotropic in space, the efficiency of the implantation detector for $\alpha$-particles slightly varies depending on the implantation depths and is between $50-55 \%$ due to geometrical reasons.

The early version of the GABRIELA setup coupled to the VASSILISSA separator was equipped with a position-sensitive silicon detector (PSD) that was $58 \times 58 \mathrm{~mm}^{2}$, $\sim 300 \mu \mathrm{~m}$ thick and was segmented in 16 resistive strips (see fig. 2.5 (a)). The x position of the implantation was given by the strip number. The y position was reconstructed through the following relation:

$$
\begin{equation*}
y=\frac{Q_{\text {top }}}{Q_{\text {top }}+Q_{\text {bottom }}}, \tag{2.7}
\end{equation*}
$$

where $Q_{\text {top }}$ and $Q_{\text {bottom }}$ are the amounts of charge collected from the top and from the bottom of the strips respectively. If the signal comes in coincidence with one of the MCPs (meaning that this particle passed through the foil at the exit of the separator), it is considered to be a recoil, if not - a decay event ( $\alpha$, electron, fission etc.). The principle scheme of the electronic readout from a resistive strip is given in fig. 2.6. The data from 2004-2005 which are presented in chap. 6 of this work were taken with this detector.

A disadvantage of the PSD is that the signals collected from the top and bottom of a strip have to be split in two in order to deduce both the energy and the position. This makes it more difficult to obtain the sufficiently low thresholds ( $\sim 100-200 \mathrm{keV}$ ) to detect ICE signals especially since at that time additional linear amplifiers were not available. In the process of the upgrade the PSD was replaced by a double-sided silicon strip detector (DSSD). A DSSD has orthogonal strips on the front and on the back sides, which efficiently makes pixels. This allows the interaction point to be determined within a pixel without splitting the signal (unlike the PSD construction), and


Figure 2.5: (a): The silicon detectors of the old GABRIELA: the PSD and one face of the tunnel mounted on the support with the cooling system; (b): the 48x48 strip DSSD; (c): the silicon detectors of GABRIELA: the $128 \times 128$ DSSD and $16 \times 16$ tunnel detectors (2 per each side) mounted on the copper support


Figure 2.6: Readout scheme of a PSD resistive strip
thus to achieve significantly lower thresholds [27].
The DSSD used in 2013-2014 had 48 strips on each side and had an area of $6 \times 6 \mathrm{~cm}^{2}$ (see 2.5 (b)). The DSSD used in the experiments in 2015-2016 had $128 \times 128$ strips and was $10 x 10 \mathrm{~cm}^{2}$ (see 2.5 (c)). Thus, not only was the implantation detector upgraded with a larger granularity (and hence fewer random correlations), but also a larger area. The latter improved the acceptance of the system and hence the effective transmission of SHELS.

### 2.3.3 The tunnel detector

The "tunnel" consists of silicon strip detectors located upstream and mounted perpendicular to the implantation detector (see fig. 2.5). It is designed to detect the ICE, the $\alpha$ particles that escape from the implantation detector and the recoiling SF fragments from the decays occurring in the focal detector. In the early version of GABRIELA, the tunnel consisted of four silicon strip detectors (one on each side), each segmented into 4 strips (see fig. 2.5 (a)). In 2013-2014 $6 \times 6 \mathrm{~cm}^{2}$ detectors with 8 strips per side were used. In 2015-2016 the tunnel was upgraded to have two 16x16 strip DSSD detectors on each side (see fig. 2.5 (c)). Though in the experiments discussed in this thesis, only one side of these DSSDs was used, this upgrade allows to use the position information in the future. In particular, it will be possible to reconstruct the energy of the escaped $\alpha$ particles taking into account the angles and thus the dead layer thickness. This may be valuable for studying rare events. The new tunnel detectors are also thicker ( $700 \mu \mathrm{~m}$ instead of $500 \mu \mathrm{~m}$ ) which extends the plateau region of electron efficiency to higher


Figure 2.7: Efficiency of the tunnel detector for the ICE in $6 \times 6 \mathrm{~cm}^{2}$ focal plane configuration (courtesy of Karl Hauschild).
energies.
The efficiency of the tunnel for the $6 \times 6 \mathrm{~cm}$ focal plane setup can be seen on fig. 2.7. The energy thresholds are typically at the order of 50 keV , which is a significant advantage of GABRIELA as whole, because it allows us to measure the low-energy conversion electrons from many transitions. This permits us to directly measure the conversion coefficients, which are (as discussed in chap. 1) the keys to the spin and parity of the initial and final states.

### 2.3.4 The Ge detectors and the BGO shields

The $\gamma$ rays and X -rays are registered by an array of germanium detectors surrounding the focal plane of the separator. Coupled to the germanium detectors, the BGO shields are used in an anti-coincidence scheme. This allows certain backgrounds to be eliminated, such as the Compton scattering, when the $\gamma$ ray scatters out of the volume of the germanium crystal and thus does not deposit the full energy inside the volume. It also allows the rejection of the background $\gamma$ rays that come from outside of the reaction chamber, e.g. from the natural radioactivity of the walls. The use of the BGO shields therefore improves the peak-to-total ratio of the acquired spectra.

In the early GABRIELA, 7 Ge detectors were used (see fig. 2.8): one directed up-


Figure 2.8: The earlier GABRIELA setup with 7 germanium detectors, photo taken in 2004
stream with respect to the beam; the other 6 placed in a ring around the implantation detector. The efficiency of this setup is given in fig. 2.9. The BGO shields in this setup extended beyond the end-cap of the Ge detector. This put a certain limit at the minimal distance between the germanium and implantation detectors.

This system was upgraded in 2015. The upstream detector was replaced by a clover: four square Ge detectors in one cryostat (see fig. 2.10). The granularity reduces the summing and thus to enhance the detection efficiency. In the upgraded GABRIELA there are 4 side detectors. The geometry was made more compact in order to achieve a better efficiency. The efficiency of this system is given in fig. 2.9. The energy resolutions achieved with these detectors are $\sim 1.8-2.1 \mathrm{keV}$ for the clover and $\sim 1.8-2.3 \mathrm{keV}$ for the side detectors. The new vacuum chamber of GABRIELA has very thin 1 mm duraluminium inserts, which allows energy thresholds of $\lesssim 20 \mathrm{keV}$ for the $\gamma$ - and X-rays. In particular, this allows us to detect the L X-rays of many heavy elements.

The new germanium array also has new BGO shields. They are 15 mm thick and


Figure 2.9: Top: Efficiency of an array of 7 Ge detectors (taken from [25]) dashed line: Geant4 simulated curve for a point source positioned at the centre of the stop detector; dotted line: Geant4 simulated curve for a distributed source; square points were measured using ${ }^{133} \mathrm{Ba},{ }^{152} \mathrm{Eu}$ and ${ }^{241} \mathrm{Am}$ sources, triangles - using $\gamma$-e coincidences from the decay of an isomeric state in ${ }^{207} \mathrm{Rn}$ implanted into the PSD using the reaction ${ }^{164} \mathrm{Dy}\left({ }^{48} \mathrm{Ca} ; 5 \mathrm{n}\right){ }^{207} \mathrm{Rn}$, star - measured using $\alpha-\gamma$ coincidences from the finestructure decay of ${ }^{211} \mathrm{Bi}$. Bottom: Geant4-simulated efficiencies of the 2016 germanium setup using the measured ${ }^{255} \mathrm{Lr}$ recoil distribution in 2016 experiment (red solid line) and a point-source distribution with the add-back for the four Clover crystals (blue dashed line) (courtesy of Karl Hauschild).


Figure 2.10: LEFT:The clover germanium detector; RIGHT: a side germanium detector of the upgraded GABRIELA in the BGO shield

15 cm long, and since the clover end-cap is level with the front of the shields it allows the Ge to approach the implantation detector as close as possible. A BGO shield of one of the side detectors can be seen on the right of fig. 2.10.

### 2.4 Readout electronics

Figure 2.11 depicts the electronics scheme of GABRIELA. The general principle for each type of detectors is the same: the signal passes through a charge sensitive pre-amplifier (PA) situated right next to the detector. Then the pre-amplified signal is transported to a spectroscopy amplifier (SA), where the signal is shaped and (in the case of silicon detectors) multiplexed (MUX). At this stage the signal from the germanium detectors passes through an anti-coincidence block with the corresponding BGO. If there is a coincidence, the signal is rejected. After this the accepted signal (or any signal for the silicon detectors) is transported to an analogue-to-digital converter (ADC). In the ADC the energy is determined from the pulse height of the signal and sent to the data acquisition system (DAQ). In the ADC each event is also assigned a time-stamp. The system is triggerless, meaning that all registered events are written to disk.

Due to noise problems in the ToF electronics, the ToF signal was only read out in coincidence with a gate generated by the DSSD (see fig. 2.11). The condition for the ToF is that at least one of the two MCP plates of each emissive foil gives a signal. The ToF


Figure 2.11: A scheme of the electronics of GABRIELA, the January-February 2016 setup
is then proportional to the time difference of these signals. The ToF detector also provides a flag for the implantation events. If a signal of at least one of the MCPs coincides with a signal from the DSSD, the event is marked as a recoil.

The number of strips in the silicon detectors is very large - 256 in the DSSD and 128 in the tunnel. In order to reduce the number of channels of back-end electronics, the signals from multiple strips are processed in the same ADCs. In order to be able to measure the events in neighbouring strips of the DSSD (e.g. inter-strip events), the odd and even strips of the implantation detector are chained separately: 16 odd or 16 even successive strips go to the same ADC channel. For the tunnel, in the current set-up each detector ( 16 strips) goes to one ADC.

The SAs for the silicon detectors work in two gains simultaneously. For the implantation detector it is $\sim 100 \mathrm{keV}$ to $\sim 20 \mathrm{MeV}$ which we denote as " $\alpha$-gain" and $\sim 2 \mathrm{MeV}$ to $\sim 200 \mathrm{MeV}$ which we denote as "fission gain" [28]. As one of the main objectives for the silicon detectors is to detect electrons which have low energies ( $\lesssim 50 \mathrm{keV}$ ), it is important to have low thresholds on them. Thus the tunnel signal is amplified in a fast linear amplifier (LA) before it is shaped in the SA. The spectroscopy amplifier of the tunnel thus has two ranges: " $\beta$-gain" - $\sim 50 \mathrm{keV}-\sim_{1} \mathrm{MeV}$ and the " $\alpha$-gain". A " $\beta$-gain" with the additional LAs is sometimes also used in the DSSD.

### 2.5 Data format

Each crate contains a time module with a lower-level clock, "timeL", distributed to all ADCs (see section 2.6). This clock is a counter that increments every $\mu \mathrm{s}$. It has a 16 bits memory stack associated to it therefore every $65536 \mu$ s the clock resets and a higherlevel PC-generated 16-bit counter "timeH" is incremented which has a length of 1.19 h . A highest-level clock, " $n$ Loops" counts the number of times this timeH passed through zero. Thus the absolute time-stamp may be restored through the formula

$$
\begin{equation*}
t=\left(n \text { Loops } \cdot 2^{32}\right)+\left(\text { timeH } \cdot 2^{16}\right)+\text { timeL. } \tag{2.8}
\end{equation*}
$$

As the ADCs of the silicon detectors acquire signals in two gains, and as they are the most numerous, all the ADC readouts follow the same pattern. Each dataset has a .pro
file associated to it, describing the format in which data is coded. Each ADC event is written to disk in 4 or 5 (depending on the experimental campaign) 16-bit words which, using the information from the .pro file, may be read to the following variables:

- groupID - the number of the ADC; ADC numbers are grouped by the detector types, e.g. $n$ to $n+8$ are the germanium detectors, and $m$ to $m+8$ are the tunnel;
- strip - this word denotes the strip number for the multiplexed detectors;
- timeL- low level timestamp (turns to o every $2^{16} \mu \mathrm{~s}$ );
- timeH - high level timestamp (turns to o every $2^{32} \mu \mathrm{~s}$ );
- dataL - low-gain data (if any);
- dataH - high-gain data.

The data are recorded to disk in this structure in binary format. The format slightly varies from one experiment to another depending on the detectors in use. For example in 2004-2005 data discussed in chapter 6 of this work, a PSD detector was used. The SAs then only had one gain, but the position of the implantation was recorded in three coordinates: strip, $Q_{\text {top }}$ and $Q_{\text {bottom }}$. Thus the format of the data was adapted accordingly, but the philosophy of the data format is always the same.

The data is then sorted into tree structures and analysed in the ROOT data analysis framework [29] and C++ codes.

### 2.6 Acquisition system

All ADCs are positioned in two CAMAC crates [30]. When one of the ADCs gets a signal or when the timing module passes the reset, a "look-at-me" (LAM) flag is incremented and a signal is sent to the crate controller. Putting up the LAM flag takes $3 \mu$ s for the ADCs of the silicon detectors and $5 \mu \mathrm{~s}$ for the ADCs of the germanium detectors. This is related to the integration time in the SA and processing time in the ADC. The crate then transmits the LAM flag to the controller common for both crates. While the LAM of an ADC is on, it "waits" to be read out. No data can be recorded to this ADC in this period of time, so this is the dead time for the given ADC . If at least one ADC in at least


Figure 2.12: A schematic view of the GABRIELA acquisition system.
one crate has a LAM flag, this triggers a readout system driven by the PC. The action of putting up a LAM for a crate takes 30ns. The further actions will be calculated in socalled CAMAC cycles, which is a cycle of the communication between the CAMAC crate and the PC. One CAMAC cycle takes $C=1.2 \mu \mathrm{~s}$.

1. Ask all the ADCs in crate 1 if they have a LAM flag. As each crate has 24 stations, and the cycles are 16 -bit, this action takes 2 cycles per crate ( $2 \cdot C=2.4 \mu \mathrm{~s}$ ). If crate 1 has the LAM flag(s), proceed to steps 2 and 3. If crate 1 has no LAM flags, proceed to step 3.
2. For each ADC in the crate that has a LAM flag ask the following: dataL, dataH and timeL from the timing module. This action takes $3 \cdot C=3.6 \mu$ s per ADC with a LAM flag.
3. Ask all the ADCs in crate 2 if they have a LAM flag. ( $2 \cdot C=2.4 \mu \mathrm{~s}$ ). If yes, perform step 2 for the crate 2.

Thus, the minimal dead-time (in case of only one event in one of the tunnel detectors) is $8.4 \mu \mathrm{~s}$. If there is one DSSD decay event (and thus both front and back strips fire), the dead-time will be $18 \mu \mathrm{~s}$. For a single implantation event the dead-time reaches
almost $26 \mu \mathrm{~s}$. In case of multiple coincident events the dead-times for the individual ADCs reach much larger values. This is among the key motivations for the future upgrade of GABRIELA data acquisition system from analogue to digital design. The reasoning for the upgrade, as well as some of the evaluation tests made with several digital electronics solutions are reviewed in chapter 4 of this thesis.

### 2.7 Data analysis

As mentioned before, the DAQ of GABRIELA is triggerless, i.e. all decay events are written to disk. The entries in the data are mostly time-ordered, although they may be swapped within few $\mu$ s due to CAMAC readout mechanism described above.

Each DSSD event is characterised by two data entries from the front-face and backface strips. These entries follow one another within $\sim 0-2 \mu \mathrm{~s}$. During the analysis they are combined into "pixels", where front and back strip numbers constitute the $x$ and $y$ coordinates of an event. When more than one front and/or back strip readout occurs ( e.g. due to the charge sharing, see chap. 3), the pixel is assembled from the highest energy signals. The events with a recoil flag also have a corresponding ToF entry. The ToF information allows to distinguish between different types of recoil events: ER, transfer products and scattered beam. This is demonstrated in fig. 2.13. Placing conditions on the ToF allows to exclude the scattered beam events, as well as to distinguish between the transfer products and the ER of interest. Although the ER cluster usually overlaps with the banana-shaped region containing the transfer products, many transfer products and ERs may still be identified as such.

The implanted nucleus may then be correlated to its subsequent decay (be it either $\alpha$ or SF) by searching for an event in the same pixel. SF events also yield many $\gamma$ rays and neutrons. As the probability to detect at least one of the $\gamma$ rays from a fission event is close to $100 \%$, the detection of a $\gamma$ ray coincident to a SF fragment can be used as an additional tag to confirm a recoil-SF correlation. As a nucleus is often created in an excited state, it may de-excite by emitting $\gamma$ rays or ICE. Thus, recoil- $\gamma$ correlations may be found between the Ge detectors and the DSSD, and recoil-ICE correlation events occurring in the tunnel or in the same pixel of the DSSD as the recoil implantation.

In such correlations the lifetimes of the nuclear states are also measured. It is often


Figure 2.13: Recoil energy vs time of flight for the ${ }^{48} \mathrm{Ca}\left({ }^{(209} \mathrm{Bi}, 2 \mathrm{n}\right){ }^{255} \mathrm{Lr}$ reaction of the 2004 experimental campaign. The filled green region corresponds to the ERs, the region between the red curves - to the transfer products and the area below the black dashed line - to the scattered beam.
convenient to plot the lifetime in a logarithmic scale (see e.g. fig. 2.14). In this thesis, the correlation plots are put in $\log _{2}$ scale, thus the lifetime equals $\tau=2^{\log _{2}(\Delta T)}$, where $\Delta T$ is the time at which the distribution of time differences between correlated events peaks.


Figure 2.14: Recoil-decay correlations from the ${ }^{48} \mathrm{Ca}\left({ }^{209} \mathrm{Bi}, 2 \mathrm{n}\right){ }^{255} \mathrm{Lr}$ reaction performed in the 2016 experimental campaign.

Figure 2.14 displays recoil-decay correlations measured in the DSSD from the ${ }^{48} \mathrm{Ca}\left({ }^{209} \mathrm{Bi}, 2 \mathrm{n}\right){ }^{255} \mathrm{Lr}$ reaction performed in the 2016 experimental campaign. The recoil-

ICE and recoil- $\alpha$ correlations for ${ }^{255} \mathrm{Lr}$ are marked with red rectangles. The white dashed line indicates the time of random correlations, i.e. when another unrelated decay happens in the same pixel. Some of the long-lived transfer products as well as the daughters and granddaughters from the current and previously made reactions appear with the lifetime of random correlations. The value of this time depends on the event rate in a detector segment, and thus becomes larger when the event rate is lower and when the granularity of the detectors is higher.


Figure 2.15: A scheme of the recoil-decay and decay-decay correlations.

In order to optimise the performance of a correlation code, it is practical to start the search from the event type that occurs more rarely. For example, in recoil- $\alpha$ correlations it is better to start with an $\alpha$ decay event and then "go back" in time to search for a recoil in the same pixel (see fig. 2.15).

It is also possible to reconstruct longer event chains and to reverse-engineer various decay patterns. For example, a nucleus may be created in an isomeric state and then be implanted to the DSSD. It would then de-excite by emitting $\gamma$ rays or ICEs and $\alpha$ decay from the ground state. If the $\alpha$ decay properties of the nucleus are known, then building recoil- $\gamma-\alpha$ or recoil-ICE- $\alpha$ correlation chains (see fig. 2.15) allows to prove that the isomeric decay belongs to the nucleus in question. If several isomeric nuclides are created, an $\alpha$ condition may allow to separate them from one another. This technique also has numerous other applications, like $\gamma-\gamma, \alpha-\gamma, \alpha$-ICE correlations etc..

Some aspects of the operation of double-sided silicon strip detectors

### 3.1 Basic principles of semiconductor detectors

Solid state semiconducting detectors ( Ge and Si ) are widely used in nuclear spectroscopy for radiation detection. They consist of solid crystals of valence-4 atoms forming four covalent bonds with neighbouring atoms. In a perfect crystal the vast majority of electrons are bound in the valence band. At room temperature few electrons ( $\sim_{1}$ in a million) get thermally excited to the conduction band across the energy gap $\Delta E$ of about a few eV, leaving a "hole": a valence-band vacancy. When an electron from a neighbouring atom fills the vacancy a hole may migrate across the crystal.

To increase the number of charge carriers the crystals are doped with small amounts of valence-3 or valence-5 atoms. With valence-3 dopants (e.g. B, Al, Ga, In) one valence bond remains open, thus creating additional holes. Such materials are called p-type semiconductors, where " p " stands for "positive" - the prevailing sign of charge-carriers. Valence-5 dopants (e.g. P, As, Sb ), on the other hand, have the fifth valence electron that is weakly bound and thus can be easily released to the conduction band. Such materials
are called n-type semiconductors, with " n " standing for "negative".


Figure 3.1: A schematic illustration of the principle of work of semiconductor detectors.

As the p-type and n-type materials are brought in contact, the charges from both materials diffuse creating a depletion region in the vicinity of which the charges are neutralised. The space-charge creates an electric field that prevents further diffusion. When radiation enters the depletion region electron-hole pairs are created. The charges migrate in opposite directions and the full collected charge is proportional to the energy of the incident particle. In order to increase the sensitive volume of the detector, as well as to make the charge collection more efficient a reverse bias $V$ is applied (see fig. 3.1). Following the application of a voltage, V, the potential barrier between the valence and conductive bands becomes higher by $e \cdot V$ which suppresses the diffusion across the junction. The leakage current across the junction, which mainly originates from thermally generated electron-hole pairs becomes very small.

As the mean free path of a charged particle in silicon is of the order of $\mu \mathrm{m}$, the detectors used for charged particle application usually consist of an extremely thin sub$\mu \mathrm{m}$ highly doped p -type layer on top of a very low doped n -type bulk. The signals at the detector appear already before the arrival of the charges to the electrodes. During the process of separation electrons and holes induce unequal charges on the electrodes
as the distance to the electrode is different (see fig. 3.2). Holes induce charge $q \frac{x_{n}}{d}$ on the p -side electrodes and $-q \frac{x_{h}}{d}$ on the n -side electrodes, where $d$ is the thickness of the fully-depleted detector, $x_{h}$ is the distance from the occurred event to the p side [31]. Electrons induce charge $q \frac{x_{e}}{d}$ on the $p$-side and $-q \frac{x_{e}}{d}$ on the $n$-side electrodes, where $x_{e}$ is the distance of the occurred event to the n side. Thus the total charge induced on $p$-side electrodes is $q$, while on the $n$-side electrodes it is $-q$.


Figure 3.2: Signal formation in a DSSD by the separation of electron-hole pairs due to the electric field in the space-charge region of the detector.

In DSSDs the electrodes on both faces of the detector are segmented into orthogonal strips insulated from one another by thin strips of silicon oxide (see fig. 3.2). The strip numbers from both sides provide $x$ and $y$ coordinates of the occurred event, efficiently making "pixels". In order to increase the efficiency of the recoil-decay correlations, as well as to enlarge the available time-span before another implantation at the same position (to see the decay of the longer-lived nuclei), it is sufficient to increase the granularity of these detectors. A larger and larger number of strips are thus becoming ever more common. However, as the number of strips increases the chance to have a malfunctioning or disconnected strip bonding also grows. The latter effect results in a change of capacitive couplings and may lead to certain spectroscopic problems. This will be further discussed in section 3.3.

### 3.2 Energy spectra acquired with silicon detectors

In general, the $\alpha$ lines in $\alpha$-decay spectra taken with silicon detectors have shapes described by a Gaussian distribution:

$$
\begin{equation*}
G(x \mid \mu, \sigma)=\frac{1}{\sqrt{2 \sigma^{2} \pi}} e^{-\frac{(x-\mu)^{2}}{2 \sigma^{2}}} \tag{3.1}
\end{equation*}
$$

where $\mu$ is the mean value of the distribution and $\sigma$ is standard deviation.


Figure 3.3: A schematic image of the DSSD and tunnel detectors and the amounts of material crossed by a particle escaping the DSSD. The dead layers are represented in grey.

The nuclei of interest are implanted to the DSSD with an implantation depth of $\sim 2$ $5 \mu \mathrm{~m}$. As the decay may occur isotropically, emitting the particle in any direction, some particles may escape from the DSSD. As an electron has a significantly longer mean free-path in silicon, the energy losses of an escaped electron are much smaller compared to those of an $\alpha$-particle. Nevertheless, electrons detected in the tunnel detector have to cross the dead layers of both the DSSD and the tunnel. Both layers are crossed at an angle which depends on the position of the implantation (see fig. 3.3). Because of these losses, the ICE lines acquired in the tunnel detector have the shape of a Gaussian distribution skewed on low energy side:

$$
f(x \mid \mu, \sigma, \alpha, n)=\frac{1}{\sqrt{2 \sigma^{2} \pi}}\left\{\begin{array}{ll}
e^{-\frac{(x-\mu)^{2}}{2 \sigma^{2}}} & , x>-|\alpha|  \tag{3.2}\\
\frac{\left(\frac{n}{\mid \alpha \alpha}\right)^{2} e^{-\frac{a^{2}}{2}}}{\left(\frac{n}{|\alpha|}-|\alpha|-x\right)^{n}} & , x<-|\alpha|
\end{array},\right.
$$

where $\alpha$ is a parameter which indicates where the skewed edge begins and $n$ is related to the magnitude of the skewed tail. The tail is also followed by a constant uniform background on the lower-energy side of the peak. This background comes from the back-scattering of the electrons out of the sensitive volume of silicon. An example of such spectrum fitted with RooFit libraries [32] can be found in fig. 3.12(a).

The $\alpha$ particles may also escape from the DSSD detector and be detected in the tunnel. As the energy losses of $\alpha$ particles are significantly higher than those of the ICE, the few- $\mathrm{MeV} \alpha$ s lose most of their energy by the time they get to the sensitive volume of the tunnel detector and appear with kinetic energies of the same order of magnitude as those of the ICE. Thus they cause an important background for the ICE observed in coincidence with an $\alpha$ decay. In fig. 6.3 the ICE energies coincident with $\alpha$ decays are plotted as a function of the $\alpha$ energies. The diagonal stripes correspond to the escaped $\alpha$ events: an $\alpha$-particle deposits most of its kinetic energy in the DSSD detector and then hits the tunnel detector with the remaining few hundreds of keV. A possible solution to remove such background would be to apply pulse-shape analysis of the preamplifier responses from the tunnel detector in order to distinguish between the different particles.

While the Ge detectors can be calibrated with standard calibration sources $\left({ }^{60} \mathrm{Co}\right.$, ${ }^{133} \mathrm{Ba},{ }^{137} \mathrm{Cs}$ etc.), the silicon detectors need an in-beam calibration to correct for the energy losses described above. Basically, when an external $\alpha$ or electron source is used, particles have to cross different amounts of the material to get to the sensitive volumes of the sililcon detectors than they do during an experiment. In order to calibrate the DSSD and the tunnel taking into account these energy losses, we perform calibration reactions, like e.g. ${ }^{48} \mathrm{Ca}\left({ }^{164} \mathrm{Dy}, 2-4 \mathrm{n}\right){ }^{208-10} \mathrm{Rn}$. Such reactions produce $\alpha$-decaying nuclei which allow calibration the DSSD with the $\alpha$ particles emitted inside the detector. They also produce ICE-decaying isomers which allow calibration of the tunnel detector with all the dead layers an electron has to pass, as well as to determine the tunnel efficiency from the ratio of the observed ICE and $\gamma$ rays.

If both ICE and $\alpha$ are simultaneously emitted in the direction of the bulk of the detector (which happens roughly in 1 case out of 4), the deposited energies from both particles get summed. This effect, as well as the ways to untangle this summing with the Geant 4 simulations, will be discussed in section 3.5.

|  | $\mathrm{E}, \mathrm{keV}$ | BR | $\mathrm{T}_{1 / 2}, \mathrm{~s}$ |
| :---: | :---: | :---: | :---: |
| ${ }^{209} \mathrm{Ra}$ | $7003(10)$ | $99.3 \%$ | $4.8(2)$ |
| ${ }^{210} \mathrm{Ra}$ | $7016(4)$ | $92.7 \%$ | $3.7(2)$ |
| ${ }^{211} \mathrm{Ra}$ | $6909(4)$ | $99.97 \%$ | $23.6(2)$ |

Table 3.1: The most intense $\alpha$-decay lines in ${ }^{209-11} \mathrm{Ra}$, from [34-36].

### 3.3 The effect of missing strip connections on the total energy spectra

One of the commissioning reactions for the transmission tests of the separator SHELS was ${ }^{50} \mathrm{Ti}+{ }^{164} \mathrm{Dy}$, producing ${ }^{209} \mathrm{Ra},{ }^{210} \mathrm{Ra}$ and ${ }^{211} \mathrm{Ra}$. In this commissioning run the newly developed ${ }^{50} \mathrm{Ti}$ beam was first used at the JINR [33]. The radiums undergo $\alpha$-decay to ${ }^{205-7} \mathrm{Rn}$ (see tab. 3.1). The recoil-decay correlations from this reaction can be seen in fig. 3.4. In this commissioning run, a $300 \mu$ m thick $48 \times 48$ strip $6 \times 6 \mathrm{~cm}^{2}$ DSSD was used at the focal plane of SHELS.


Figure 3.4: Recoil- $\alpha$ correlations spectrum for the ${ }^{50} \mathrm{Ti}\left({ }^{164} \mathrm{Dy}, 3-5 \mathrm{n}\right)^{209-11}$ Ra reaction. The alpha-decay energy is given on the $x$ axis, the time difference between the recoil and decay events in the same pixel is given on the $y$ axis.
${ }^{210} \mathrm{Ra}$ is known [34] to have a short-lived (2.1(1) $\mu \mathrm{s}$ ) 8+ isomeric state which decays through a cascade of $97,578,602,604,752$ and 775 keV transitions (see fig. 3.5).

In the recoil- $\alpha$ correlation plot an unidentified $\alpha$ line with the same lifetime as ${ }^{210} \mathrm{Ra}$ was found. This line also appeared in $\alpha-\gamma$ correlations with $\gamma$ rays from the decay of an isomer in ${ }^{210} \mathrm{Ra}$ (see fig. 3.6).


Figure 3.5: Decay scheme of the isomer in ${ }^{210} \mathrm{Ra}$ [34].


Figure 3.6: $\alpha-\gamma$ correlations for the isomeric $\gamma$-ray transitions in ${ }^{210} \mathrm{Ra}$ (following a recoil implantation within $27 \mu \mathrm{~s}$ ) vs the $\alpha$-decay correlated to the implanted recoil

The energy of this apparent $\alpha$-decay branch is 6927 keV , which is 89 keV lower than the main $7016 \mathrm{keV}{ }^{210} \mathrm{Ra} \alpha$ line. It looked like we had observed fine structure in ${ }^{210} \mathrm{Ra}$ $\alpha$-decay with a $\sim 5 \%$ branching. It could be attributed to the decay to an excited state in ${ }^{206} \mathrm{Rn}$, and could have hypothetically been masked in the previous experiments if ${ }^{210} \mathrm{Ra}$ was always produced along with ${ }^{211} \mathrm{Ra}$, as the $\alpha$ energies are similar and the branching is relatively low. On a closer inspection it was found that a similar fine structure was present in the decay of ${ }^{206} \mathrm{Rn}$, also with a branching of around $5 \%$, also roughly 90 keV below the main line, and also with a right lifetime (from the $\alpha-\alpha$ correlations) to be ${ }^{206} \mathrm{Rn}$. This was starting to look suspicious but could still be explained with similar arguments as for ${ }^{210} \mathrm{Ra}$. Looking even more closely revealed similar smaller lines

|  | Strip, $\mu \mathrm{m}$ | Strip, \% | Inter-strip, $\mu \mathrm{m}$ | Inter-strip, \% |
| :---: | :---: | :---: | :---: | :---: |
| Front face | 1750 | 96.6 | 60 | 3.4 |
| Back face | 1570 | 85 | 235 | 15 |

Table 3.2: The dimensions of the $48 \times 48$ strip DSSD: front and back strip and insulation silicon oxide widths.
associated with every large $\alpha$ line in the spectrum, including the transfer products and the grand-daughters from the previously made reactions which resided in the DSSD. Certainly, this could not result from nuclear structure, but only from the experimental conditions.

As the energies acquired with the front strips have better quality and efficiency (less charge sharing), we have chosen the front face of the DSSD to calibrate and to build the spectra. Thus these are the energies taken from the front side of the DSSD that appear in fig. 3.4 and 3.6. The dimensions of the DSSD used for this commissioning experiment are given in tab.3.2.

Front- and back-strip energies traced on the same histogram are given in fig. 3.7. The main diagonal indicates the full charge collection on both front and back faces of the DSSD. This diagonal contains $\sim 80 \%$ of the total number of events. The vertical bands that descend from each $\alpha$-peak correspond to the charge-sharing in the back side of the DSSD, i.e. when the space-charge from the decay event is spread between two neighbouring back electrodes. In these cases the full energy is still collected in the front-side strips. These bands contain $\sim 15.8 \%$ of all events, which agrees with the percentage of the back face occupied by the inter-strip area (see tab. 3.2).

As the implantations occur at rather shallow depths of a few $\mu \mathrm{m}$ the nuclei implanted in the inter-strip space of the front side of the DSSD are in the vicinity of the oxide insulation layer. During irradiation of the detector this layer builds up a trapped positive charge (see fig. 3.2). This charge attracts and traps the negatively-charged electrons moving away to the back side electrodes. This results in a decrease of charge collection in both front and back sides of the DSSD. The fraction of such events is $\sim 3.6 \%$ of the total amount, which also agrees well with the fraction of the front-face area taken up by the silicon oxide given in table 3.2. These events give rise to a slanted band demonstrated in fig. 3.7.


Figure 3.7: DSSD front VS back energy

The back side of the DSSD had two strips (strip \#10 and strip \#11) with broken bondings. They were thus disconnected from the acquisition system and not grounded. In fig. 3.7 the two lines with $E_{\text {back }} \lesssim 0.1 \cdot E_{\text {front }}$ (labelled "Missing strip effect") correspond to decays that occurred in front of these disconnected strips. These lines indicate that a small signal is detected in the neighbouring back strips (\#9 and \#12). When there is a signal in a back strip, there is a ghost signal induced in its neighbours through capacitive coupling. In a normal situation this ghost peak is small and, in general, comes below the threshold. However, when the back strip is disconnected (see fig. 3.8), the relative difference of potentials between this strip and its neighbours $V_{b b}$ grows, which results in a bigger ghost signal which appears above the threshold and allows to make a "pixel" for such an event. As the back strip is no longer connected to the preamplifier which would immediately compensate for the induced negative charge on the electrode, the difference of potentials between the "missing" strip and the corresponding front strip $V_{f b}$ is somewhat decreased, which is equivalent to having a smaller capacitance between these strips and leads to the occurrence of a smaller signal in the frontside electrode. This is why the corresponding events in ${ }^{210} \mathrm{Ra}$ experiment appeared $\sim 90 \mathrm{keV}$ lower than the normal ones. This can be clearly seen in fig. 3.9, where the front-strip energy is plotted with respect to the associated back-face strip numbers.

The simplest and most brutal solution to clean the spectra from such lower-energy


Figure 3.8: A schematic view of a DSSD with a disconnected back strip. The signal on the corresponding front electrode is decreased: the signal on the neighbouring back electrode is increased.
peaks is to reject all events coming with the strips \#9 and \#12 adjacent to the "missing" ones. However, there is no need for such a loss of statistics. As it can be clearly seen from fig. 3.7, the described events can easily be isolated. When the cuts indicated by the dashed lines are applied, the effect of those "missing strips" is fully eliminated. This is demonstrated in fig. 3.10. Moreover, a separate calibration can be made for the events falling in the "missing strip" cone. Thus the decays of the nuclei implanted in front of the disconnected strips may be fully restored.

### 3.4 Conversion coefficients in ${ }^{210} \mathrm{Ra}$

The commissioning reaction described in the previous section allowed the conversion coefficients for the transitions after the decay of an isomer in ${ }^{210} \mathrm{Ra}$ to be determined. These conversion coefficients were measured for the first time via combined $\gamma$ and ICE spectroscopy. The recoil- $\gamma$ correlation plot from this reaction can be seen in fig. 3.11.

In order to obtain the pure $\gamma$-ray and ICE spectra from the isomer decay with no contamination from the other reaction products, recoil- $\gamma-\alpha$ and recoil-electron- $\alpha$ correlations were performed. Thus, it was required that after an internal transition there is an $\alpha$-decay happening in the same "pixel" as the recoil implantation with the energy and lifetime of ${ }^{210}$ Ra. The $\alpha$-energy gate was chosen between 7014 keV and 7034 keV ,


Figure 3.9: Energy taken from the front strips of the DSSD vs the corresponding back strip numbers. Strips \#10 and \#11 are missing. The spurious peaks (indicated by red rectangles) $\sim 90 \mathrm{keV}$ lower in energy than the real $\alpha$ lines are observed with the strips \#9 and \#12


Figure 3.10: Recoil- $\alpha$ correlation spectrum of ${ }^{211} \mathrm{Ra}$ and ${ }^{210} \mathrm{Ra}$ before (left) and after (right) the correction
the recoil- $\alpha$ time - between 0.3 s and 16 s and the recoil- $\gamma$ and recoil-ICE (with the ICE detected in the tunnel) time-gate was o to $27 \mu \mathrm{~s}$. The resulting $\gamma$-ray and ICE spectra are given in fig. 3.12.

Figure 3.12(a) displays a simultaneous multicomponent fit of the ICE spectrum corresponding to the decay of the isomer. All ICE components, except for the 97 keV peak, were fitted with a skewed Gaussian combined with an error-function. All free parameters, except for the norms, are identical for each transition. The 97 keV peak has multiple components (L-, M- conversion electrons) which have very similar energies and thus result in a wider Gaussian distribution. The integral of this latter peak may be underestimated due to the threshold effects, thus the observed intensity constitutes a


Figure 3.11: Recoil- $\gamma$ correlation spectrum from the ${ }^{50} \mathrm{Ti}+{ }^{164} \mathrm{Dy}$ reaction.

| $E_{\gamma}, \mathrm{keV}$ | 97 | 578 | 602 | 604 | 751 | 775 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $N_{\gamma}$ | $9296 \pm 372$ | $63153 \pm 1336$ | $34104 \pm 1207$ | $97766 \pm 1459$ | $40302 \pm 1147$ | $67032 \pm 1705$ |
| $N_{\text {tot ICE }}$ | $93035 \pm 1087$ |  |  |  |  |  |
| $N_{\text {K ICE }}$ |  | $844 \pm 169$ | $445 \pm 59$ | $1183 \pm 158$ | $84 \pm 106$ | $781 \pm 169$ |
| $N_{\text {LICE }}$ |  | $114 \pm 379$ | $240 \pm 46$ | $638 \pm 123$ |  | $380 \pm 99$ |
| $\alpha_{\text {tot }}$ | $10.0(4)$ |  |  |  |  |  |
| $\alpha_{K}$ |  | $0.013(3)$ | $0.013(2)$ | $0.012(2)$ | $0.002(3)$ | $0.012(3)$ |
| $\alpha_{L}$ |  | $0.002(6)$ | $0.007(1)$ | $0.007(1)$ |  | $0.006(1)$ |
| Mult. | E2 | E2 | E2 | E2 | (E2) | E2 |

Table 3.3: The measure ICE and $\gamma$ intensities, obtained values of the conversion coefficients and the resulting multipolarity assignments. All the intensities are multiplied by the corresponding detector efficiencies. The $\gamma$-intensities take into account the summing lines.
lower limit. As the absolute $\gamma$-ray efficiency and multiplicity were high, the $\gamma$ spectrum in fig. 3.12(b) contains peaks resulting from summing of the most intense $\gamma$-lines in the cascade.

The measured ICE and $\gamma$ intensities multiplied by the corresponding detector efficiencies and (for the $\gamma$ rays) taking into account the summing are given in tab. 3.3. The obtained values of the internal conversion coefficients allow confirmation of the previously assumed [34] E2 nature of these transitions.


Figure 3.12: (a) ICE and (b) $\gamma$-ray spectra of the decay of an isomer in ${ }^{210} \mathrm{Ra}$.

### 3.5 Summing of internal conversion electrons and $\alpha$ particles with the example of ${ }^{221} \mathrm{Th}$

Another commissioning reaction for the separator SHELS, with the goal of measuring the transmission for a more asymmetric reaction, was ${ }^{22} \mathrm{Ne}+{ }^{206} \mathrm{~Pb}$ [37], which produced ${ }^{221} \mathrm{Th}$ in the $\alpha 3 \mathrm{n}$ channel. In this commissioning run, the $10 \times 10 \mathrm{~cm}^{2} 128 \times 128$ strip DSSD was used for the first time. ${ }^{221} \mathrm{Th} \alpha$ decays to ${ }^{217} \mathrm{Ra}$. The $\alpha$ spectra obtained in this experiment can be seen in fig. 3.13. Some of the branches of the $\alpha$-decay are followed by an immediate de-excitation via the emission of an ICE. As both decays occur in the body of the DSSD, all or part of the energy of the ICE (and accompanying X-rays and/or Auger electrons) may be summed with the energy of the $\alpha$-particle. Such summing may produce peaks in the $\alpha$ spectrum above the energy of a true $\alpha$-decay line.


Figure 3.13: Top: the recoil- $\alpha$ correlation plot for the ${ }^{22} \mathrm{Ne}+{ }^{206} \mathrm{~Pb}$ reaction; bottom: the $\alpha-\gamma$ coincidences, courtesy of Araceli Lopez-Martens.

There are five apparent $\alpha$ lines observed in ${ }^{221} \mathrm{Th}: 7735,8146,8243,8379$ and 8468 keV . These $\alpha$ decays are in coincidence with the $\gamma$ transitions of 226, 331, 422 and 752 keV (see fig. 3.13).

In order to interpret the acquired spectra, these processes were simulated with the Geant4 toolkit [38]. There is a specific class in Geant4 called G4RadioactiveDecay which allows to simulate the internal transitions happening in a given nucleus and its subsequent nuclear decay on the basis of a level scheme that can be conditioned accordingly. Such simulations allow us to test potential decay schemes. In particular they allow new $\alpha$-decay branches from the atomic- and ICE- $\alpha$ summing to be distinguished.

The Geant4-simulated spectrum of the decay of ${ }^{221}$ Th nuclei implanted into the DSSD


Figure 3.14: The experimental (left) and Geant4-simulated (right) spectra of the $\alpha$-decay spectrum of ${ }^{221} \mathrm{Th}$.

| $E_{\alpha}, \mathrm{keV}$ | $\mathrm{BR}, \%$ | HF |
| :---: | :---: | :---: |
| 7735 | 4.8 | 0.80 |
| 8146 | 56.8 | 1.23 |
| 8243 | 1.1 | 122.1 |
| 8468 | 37.2 | 15.61 |

Table 3.4: The $\alpha$-decay branches of ${ }^{221} \mathrm{Th}$.
is given in fig. 3.14. The ${ }^{221} \mathrm{Th}$ ions were "created" with a kinetic energy of $6.9 \pm 1.7 \mathrm{MeV}$. The silicon detector was simulated as a $10 \mathrm{~cm} \times 10 \mathrm{~cm} \times 300 \mu \mathrm{~m}$ volume of silicon covered by $0.7 \mu \mathrm{~m}$ of aluminium and segmented in $128^{2}$ sensitive pixels. The resulting implantation depth in the simulations was $\sim 2.6 \mu \mathrm{~m}$. The input level scheme (see fig. 3.15) contains four $\alpha$-decay branches: the $8468,8146,8243$ and 7735 keV lines (see tab. 3.4). The intensities and conversion coefficients of the internal transitions are given in tab. 3.5.

Kuusiniemi et al. [39] observed 8250 keV and and 8375 keV lines in the $\alpha$ spectra of ${ }^{221} \mathrm{Th}$, both of which were attributed to the summing of the $\alpha$ particles and ICEs. As it can be clearly seen from fig. 3.14, the 8379 keV peak (corresponding to the 8375 keV

| $E_{\gamma}, \mathrm{keV}$ | $I_{\gamma}, \%$ | multipolarity | $\alpha_{\text {tot }}$ |
| :---: | :---: | :---: | :---: |
| 226 | 0.94 | M1 | 1.554 |
| 331 | 64.8 | M1 | 0.543 |
| 422 | 19.1 | E2 | 0.059 |
| 752 | 76.1 | M1 | 0.060 |

Table 3.5: The proposed conversion coefficients and multipolarities for the internal transitions in ${ }^{217}$ Ra.


Figure 3.15: The proposed level scheme of ${ }^{221} \mathrm{Th}$.
line in [39]) appears in the simulated spectrum without being explicitly put in the decay scheme. We therefore conclude that this peak indeed comes from summing effects in the DSSD rather than from a new $\alpha$ transition. In fact, it arises from the summing of the $8146 \mathrm{keV} \alpha$ line with the ICE from the 331 keV internal transition. The simulations also allowed us to tentatively attribute the multipolarities indicated in tab. 3.5. The work on the evaluation of the final decay scheme including new $\gamma$ cascades and setting the final limits on conversion coefficients is currently ongoing. However, as the summing between the $\alpha$ particles and ICE cannot explain the 8243 keV line (corresponding to the 8250 keV line in [39]), we conclude that it is indeed a new $\alpha$ branch.

### 3.5.1 On the simulation of $\alpha$-decay with the G4RadioactiveDecay class

During this work we encountered a problem with Geant4 which might be worth mentioning. When I first started with the simulations it became apparent that the kinetic energies of the $\alpha$ particles generated by the G4RadioactiveDecay class did not agree with any experimental values. As this was noticed for a handful of nuclei, we begun to suspect that there could be some conceptual problem.

In search of the explanation we had to investigate how the $\alpha$ energies are defined
in Geant4. It appeared that they are calculated from the masses of the mother and daughter atoms. As an example, we have taken the decay of ${ }^{221} \mathrm{Th}$ to the ground state of ${ }^{217} \mathrm{Ra}$, where the $\alpha$ energy $E_{\alpha}$ was previously measured to be 8470 (5) keV [40], which also agrees with our measurement. To check this number, we took the atomic masses from AME 2003 [41]:

$$
\begin{gathered}
M_{a ~ T h}=205877.112 \mathrm{MeV} \\
M_{a \mathrm{Ra}}=202140.085 \mathrm{MeV} \\
m_{\alpha}=3727.379 \mathrm{MeV} \\
m_{e}=511 \mathrm{keV} \\
Q=M_{a \text { Th }}-M_{a R a}-m_{\alpha}-2 m_{e}=8626 \mathrm{keV} .
\end{gathered}
$$

Neglecting the binding energies of an $\alpha$ particle and of the daughter nucleus (see eq.1.15), this gives

$$
\begin{equation*}
E_{\alpha}=8626 \cdot \frac{217}{221}=8470 \mathrm{keV} \tag{3.3}
\end{equation*}
$$

which agrees with the measured $\alpha$ energy. However the Geant4 output was 8509 keV .

It also appeared that whenever Geant4 was demanded to return the total energy of a nucleus at rest, it was not giving the AME atomic masses, but the masses of fully stripped atoms, i.e. bare nuclei (atomic mass - electrons + binding energy). In an attempt to reverse-engineer the Geant4 calculations, we tried to put these numbers in the calculation instead. The nuclear masses are obtained with the following formula:

$$
\begin{equation*}
M_{n}(A, Z)=M_{A}(A, Z)-Z \cdot m_{e}+B_{e}(Z), \tag{3.4}
\end{equation*}
$$

where $B_{e}(Z)$ is an approximation [42] of the total binding energy of all removed electrons. As a result we obtained:

$$
\begin{gathered}
M_{n \text { Th }}=202095.797 \mathrm{MeV} ; \\
M_{n \mathrm{Ra}}=205831.843 \mathrm{MeV} ; \\
" Q "=8667 \mathrm{keV} ; \\
" E_{\alpha} \text { " }=8510 \mathrm{keV},
\end{gathered}
$$

which is equivalent to what Geant4 returned. Similar calculations were applied to various nuclides with always the same result.

This conceptual mistake was reported on the Geant4 forum, and we got proposed a solution of using the reaction $Q$ values instead of the masses. However, this solution requires to recompile the Geant 4 installation.

One should remark, that use of the atomic masses for the determination of $E_{\alpha}$ is not an optimal approach for heavy nuclei or for many other nuclei far from stability. Few of the masses in the transuranium region have been measured in a direct way. Most of them are derived from the $Q$ values obtained from the $\alpha$-decay energies. As the new $\alpha$ decay measurements may be more precise, it is thus advisable to use the $Q$ values as an input for the simulations. Moreover, as the fully-stripped nuclei of $\mathrm{Z} \sim 90$ are extremely hard to obtain, it is more practical to use the atomic masses by default rather than the nuclear ones.

## 4

# Digital electronics tests for the upgrade of GABRIELA acquisition system 

### 4.1 Common digital signal processing algorithms

### 4.1.1 Introduction

Over the last 10-15 years digital electronics has gained an increasing popularity in nuclear spectroscopy experiments. With conventional electronics the signal is amplified, shaped and integrated in analogue electronics devices. Only the final result (e.g. the energy or the time from a time-to-amplitude converter) is recorded via an ADC. The concept of digital electronics is to record the signal directly from the preamplifier and then perform the signal processing digitally: either within a Field-Programmable Gate Array (FPGA) on board within a specialised digital signal processing (DSP) modules, or off-line on a PC, e.g. using a C++ code. An FPGA is a reprogrammable silicon chip. It consists of programmable logic blocks and a hierarchy of reconfigurable interconnects that can be programmed to perform logical operations or complex mathematical functions. Most FPGAs also include memory stacks (Block-RAM) to store data in order to perform on-board calculations. Generally, the FPGA configuration is performed us-
ing the Hardware Description Language (HDL), but there also exist more user-friendly interfaces, such as, e.g., LabVIEW, that allow programme development in a graphical environment and the generation of an HDL code to be implemented on the FPGA.


Figure 4.1: A simplistic principal scheme of an AC coupled charge integrating preamplifier.

The general scheme of a simple charge integrating preamplifier can be found on fig. 4.1. The shaped output voltage is depicted in more detail in fig. 4.2. The PA output signal has a fast rising step caused by the charge collection. The duration of this step (rise-time) depends on the physical processes happening in the detector, as well as on the bandwidth $B W$ of the preamplifier and can be expressed as $R T(\mathrm{~ns})=0.35 / B W(\mathrm{GHz})$. The rising step is followed by an exponential decay due to the discharge of the capacitors over the resistor. The decay time can be expressed as follows:

$$
\begin{equation*}
\tau=R \cdot C, \tag{4.1}
\end{equation*}
$$

where $R$ and $C$ are feedback resistance and capacitance of the preamp (see fig. 4.1). The signal shape can be approximated with the following expression:

$$
f(t)=\left\{\begin{array}{l}
A \cdot\left[\exp (-t / \tau)-\exp \left(-(t / R T)^{2}\right)\right]+B, t>T_{0}  \tag{4.2}\\
B, t<T_{0}
\end{array}\right.
$$

where $t$ is time, $A$ is the signal amplitude, $B$ is the baseline shift (can be both positive or negative) and $T_{0}$ is the start of the signal (see fig. 4.2) [43]. The digitizer records the traces in a form equivalent to a one-dimensional histogram, attributing the signal magnitude value to each sampling point $X[n]$ equivalent to $f(t)$.


Figure 4.2: A typical charge integrating preamplifier output signal. Digitised signal values, $X[n]$, are represented by the dots.

### 4.1.2 Baseline correction

The PA output signal may have a positive or negative offset B. For the DC-coupled devices changing leakage current of the detector changes the offset. Moreover, in some noisy environments or with some temperature-unstable preamplifiers the baseline may shift during data taking, which may significantly reduce (if not fully destroy) the energy resolution of the acquired spectra. In order to avoid this problem a baseline restoration may be applied. It consists of subtracting the average (over $K$ points) baseline level of a particular signal from the function:

$$
\begin{equation*}
X_{b}[n]=\frac{\sum_{i=0}^{K-1} X[n]}{K} . \tag{4.3}
\end{equation*}
$$

### 4.1.3 Digital simulation of CR- $(R C)^{N}$ circuits

Generally, the signal to noise of the PA output is not sufficient for high precision spectroscopy. In addition, the sharp pointed top (see fig. 4.2) complicates the subsequent pulse-height analysis as the maximum pulse amplitude is maintained only for a short period of time and is largely affected by the high-frequency components of any noise making the energy resolutions obtained from the pulse-height analysis of the PA signal very poor. Thus in classical analogue electronics setups the PA signal is usually shaped in CR- $(\mathrm{RC})^{N}$ integro-differentiating circuits of a SA [43]. The output of the SA is given
by

$$
\begin{equation*}
f(t)=\frac{A}{N!}\left(\frac{t}{\tau}\right)^{N} \cdot \exp (t / \tau) \tag{4.4}
\end{equation*}
$$

which more closely approximates a Gaussian shape the higher the order of $N$. This is why this method is also referred to as Semi-Gaussian shaping. The time required for the shaped pulse to reach maximal amplitude is equal to $N \tau$, thus $N$ is typically chosen between 4 and 7.

The same technique may also be reproduced in DSP algorithms. A numeric CR$(\mathrm{RC})^{4}$ circuit may be implemented as follows:

$$
\begin{align*}
\operatorname{CRRC}^{4}[n]=G \cdot\left(1-e^{-\Delta T / \tau_{s}}\right)^{4} \cdot(1+ & \left.e^{-\Delta T / \tau_{s}}\right) \cdot(X[n]-X[n-S]) \\
& +5 \cdot e^{-\Delta T / \tau_{s}} \cdot \operatorname{CRRC}^{4}[n-1] \\
& -10 \cdot e^{-2 \Delta T / \tau_{s}} \cdot \operatorname{CRRC}^{4}[n-2] \\
& +10 \cdot e^{-3 \Delta T / \tau_{s}} \cdot \operatorname{CRRC}^{4}[n-3] \\
& -5 \cdot e^{-4 \Delta T / \tau_{s}} \cdot \operatorname{CRRC}^{4}[n-4] \\
& +1 \cdot e^{-\Delta 5 T / \tau_{s}} \cdot \operatorname{CRRC}^{4}[n-5], \tag{4.5}
\end{align*}
$$

where $G$ is the gain constant introduced in order to scale the output, $X[n]$ is the preamplified signal, $\tau_{s}$ is the shaping time, $\Delta T$ is the sampling time, $S$ is the step. $\left(1+e^{-\Delta T / \tau}\right)$ is the CR differentiation term, and $\left(1-e^{-\Delta T / \tau_{s}}\right)^{4}$ is the $\mathrm{RC}^{4}$ integration term. The CRRC ${ }^{4}[n]$ reaches a maximum at the peaking time $t_{p}=4 \cdot \tau$. The energy corresponding to an event is proportional to the amplitude of the signal $C R R C_{M A X}^{4}$ at the peaking time.


Figure 4.3: The pole-zero effect on the output of the CR- $(\mathrm{RC})^{\mathrm{N}}$ function.
As the preamp signal is not a step function, but has a long exponential decay tail,
there is a small amplitude undershoot in the CR-(RC) ${ }^{N}$ output (see fig. 4.3). Most shaping amplifiers incorporate a pole-zero cancellation circuit to eliminate this undershoot. The benefit of pole-zero cancellation is the improvement of peak shapes and of the resulting energy resolution. In particular, when the counting rate is high the next signal may be "riding" on the pole-zero undershoot of the previous signal, and thus the energy obtained for this signal would appear significantly smaller. In analogue electronics, an additional CR differentiation circuit with a variable resistance is used in order to adjust the correction for each preamplifier. However, when over-corrected ( $R$ is too high) this CR circuit produces an overshoot after the signal, which also disagrees the energy resolution for similar reasons. Thus, the idea is to make the signal return to the baseline when the shaping is over.

In digital signal processing this undershoot may be corrected for by adding a compensation term proportional to $e^{-\Delta T / \tau}$ to the CR- $(\mathrm{RC})^{\mathrm{N}}$ expression:

$$
\begin{equation*}
\operatorname{CRRC}_{p 0}^{N}[n]=\operatorname{CRRC}_{p 0}^{N}[n-1]+\operatorname{CRRC}^{N}[n]-e^{-\Delta T / \tau} \cdot \operatorname{CRR} C^{N}[n-1] . \tag{4.6}
\end{equation*}
$$

### 4.1.4 Moving Window Deconvolution

The Moving Window Deconvolution (MWD) [44] is one of the most commonly used algorithms to derive the energy spectra from the preamplified signal traces. The principle idea is to make two moving differences $d[n]$ between the points of the trace:

$$
\begin{equation*}
d[n]=(X[n]-X[n-k])-(X[n-l]-X[n-l-k]), \tag{4.7}
\end{equation*}
$$

where $X[n]$ is an array containing the signal digitised points, $l$ and $k$ are the parameters of the MWD. In a similar manner to the CR- $(R C)^{N}$ pole-zero correction, the expression 4.7 has to be corrected for the exponential decay:

$$
\begin{equation*}
r[n]=r[n-1]+d[n]-\exp (-\Delta T / \tau) \cdot d[n-1] . \tag{4.8}
\end{equation*}
$$

Both $d[n]$ and $r[n]$ are depicted in fig. 4.4.
Thus the MWD is a running sum of these two decay-corrected moving differences


Figure 4.4: The moving difference $d[n]$ before (green) and $r[n]$ after (blue) the correction for the decay.
$r[n]:$

$$
\begin{equation*}
M W D[n]=M W D[n-1]+r[n] . \tag{4.9}
\end{equation*}
$$

The MWD algorithm converts an exponentially decaying signal into a step with a flat top of the length $m$. The shape of the MWD trapezoid can be seen on fig. 4.5. The amplitude of the signal at the flat top is proportional to the energy. Hence the energy spectrum can be obtained by taking $M W D[N]$ at a point $N$ on the flat top for each trapezoid. In some cases it may be useful to average over a range of points in the flat top in order to reduce the electronics noise.


Figure 4.5: The Moving Window Deconvolution trapezoid MWD[n].

A clear advantage of this method is it's intuitive simplicity. Also, it requires only one floating point operation (multiplication by $e^{-\Delta T / \tau}$, see eq.4.8). Given that $\Delta T$ is usually « $\tau$ the multiplication term can be approximated as ( $1-\Delta T / \tau$ ) and can be easily converted to an integer operation with sufficient precision by bit-shifting. This makes the MWD
code for an FPGA less demanding on resources and easier to implement than the other algorithms. This method does not require the baseline correction, as such subtraction is already contained in the moving difference (see eq. 4.7).

### 4.1.5 Time-over-overflow

If the signal amplitude exceeds the range of the digitizer, it overflows (see fig. 4.6), in which case the digitized trace no longer resembles the shape described in eq.(4.2) and the algorithms described above are not applicable. However, the amplitude of such signal can still be restored with digital signal processing.


Figure 4.6: The time-over-overflow energy determination.

When the rise-time of the PA output signal may be considered negligible, the expression 4.2 becomes

$$
f(t)=\left\{\begin{array}{l}
B, t<T_{0}  \tag{4.10}\\
A \cdot \exp (-t / \tau)+B, t \geq T_{0}
\end{array}\right.
$$

Then if the the decay constant and the start and stop timestamps of an overflow are known, the initial amplitude of the signal can be restored from geometrical considerations (see fig. 4.6. Thus for the signal amplitude we get:

$$
\begin{equation*}
A=M-B+\exp \left(\frac{T_{O F}}{\tau}\right)=M-B+\exp \left(\Delta T \frac{\frac{n_{4}-n_{3}}{2}-\frac{n_{2}-n_{1}}{2}}{\tau}\right) \tag{4.11}
\end{equation*}
$$

where $M$ is the signal size at the overflow, $T_{O F}$ is the duration of an overflow, $\Delta T$ is the sampling time; $n_{1}$ is the point before the start of an overflow, $n_{2}$ is the first point of the overflow, $n_{3}$ is the last point of the overflow and $n_{4}$ is the first point after the overflow as illustrated in fig. 4.6.

As this method yields rather crude energy resolutions of $\sim 1 \%$, it is not applicable of $\alpha$, ICE or $\gamma$-ray spectroscopy. However it proves to be very handy with high-energy signals where the high precision in energy is not needed, e.g. for the recoil or fission fragment tagging, when a precise measurement of energy is not needed.

### 4.2 Comparative tests of different digitizers

As discussed in chapter 2 of this manuscript, the back-end electronics of GABRIELA is fully based on analogue ADCs. This system has a number of significant disadvantages, among which we count the excessive dead times of the acquisition system and limited temporal resolution. A digital acquisition setup would resolve these problems. It would also permit the application of pulse-shape analysis (PSA) algorithms for particle discrimination. An essential part of the work of this thesis was dedicated to the tests of different digitizers in order to find the best solution for the upgrade of the GABRIELA DAQ system.

### 4.2.1 Characteristics of digitizers

The sampling frequency is one of the key characteristic of a digitizer, as it is related to the precision at which the shape of the signal is described. Thus it is important to have an adequate sampling frequency $\left(f_{s}\right)$ in order to have a good temporal and energy resolution. From the Nyquist-Shannon theorem [45] sinusoidal signals with frequency greater than $0.5 \cdot f_{s}$ can not be distinguished. It is therefore common to limit the signal frequencies using a low-pass filter just before the digitisation stage. Typically the cutoff frequency ( -3 dB or $70.7 \%$ point) of the low-pass filter is set at around $0.5 \cdot f_{s}$ in order to suppress high frequency noise.

As signal shape differs for different particles (i.e. for an $\alpha, e$ or a charged ion in the silicon detector), it is useful to well reproduce the rise-time of the signal in order
to be able to perform the PSA and try to distinguish different types of particles. The typical value of a rise-time in nuclear spectroscopy experiments varies from $\sim 100 \mathrm{ps}$ to few $\mu \mathrm{s}$. Thus, in order to be able to describe the shape of the rising signal, the sampling frequency has to be in the $100 \mathrm{MHz}-10 \mathrm{GHz}$ range, depending on the specific experimental setup.

Another important characteristics of a digitizer is the level of the distortion. Even for an ideal (no noise) digitizer the quantisation noise arises from the discretization of the digitized signal. The digital signal reproduces the analogue signal as a "staircase" (see fig. 4.7) within the precision of $\pm q / 2$ (if no other noise dominates), where $q$ is the minimal peak-to-peak amplitude of the two neighbouring samples. The standard deviation of the digitization noise is $\sigma_{D}=q / \sqrt{12}$ [46].


Figure 4.7: Ideal N -bit ADC Quantization Noise (taken from [46]).

Other sources of noise in the ADC include thermal noise, variations in the voltage supply and reference voltage, clock jitter, non-linear response of the digitizer, gain and offset errors. For obvious reasons, this influences both energy and timing resolutions of the setup, and thus such noise has to be minimized.

The digitizer resolution $r$ is the number of bits returned by the ADC. It is chosen so that $r \geq q / 2$; then for an ideal digitizer $r$ equals $1 L S B$ (Least Significant Bit). When digitized, the signal is divided in $2^{r}$ discrete levels, thus for a given full-scale (FS) in-
put range the minimal voltage that is detectable by the digitizer is $F S / 2^{r}$, where $F S$ is expressed in mV . As different preamplifiers in the setup may have different gains, it is important that digitizer range is sufficient to acquire all signals, but also that the range is fully used, as $F S / 2^{r}$ ratio is proportional to the lower limit of the achievable energy resolution. Typical digitizer resolutions are $r=10-16$ significant bits.

There are several methods of quantifying the performance of a digitizer. The most simple is to measure the fluctuations around a DC input voltage from which one obtains the signal to noise ratio (SNR) relates the signal amplitude to the background noise:

$$
\begin{array}{r}
S N R=\frac{\mu}{\sigma} ; \\
S N R(\mathrm{~dB})=20 \cdot \log \left(\frac{\mu}{\sigma}\right), \tag{4.13}
\end{array}
$$

where $\mu$ is the signal mean value and $\sigma$ is the standard deviation of the noise.
The effective number of bits (ENOB) is a measure of the dynamic performance of an ADC, i.e. the number of bits available above the noise. Usually it is determined through an Fast Fourier or Laplace transform analysis, but it can be determined through the standard deviation of the noise as follows:

$$
\begin{equation*}
E N O B=\log _{2}\left(\frac{R}{\sigma \sqrt{12}}\right)=\log _{2}\left(\frac{2^{r}}{\sigma \sqrt{12}}\right)=r-\log _{2}(\sigma \sqrt{12}), \tag{4.14}
\end{equation*}
$$

where $R$ is the input range; in the last expression $r$ and $\sigma$ must be both expressed in LSB [47].

In nuclear physics it is common to evaluate the noise performance of the whole data acquisition chain through the energy resolution estimated through the full-width half maximum (FWHM) of the spectral lines. For a Gaussian distribution $F W H M=2.35 \sigma_{E}$, where $\sigma_{E}$ is the standard deviation of the energy.

It is important to remember that the DAQ is a system consisting of a preamplifier and a digitizer, and thus much also depends on the PA quality: it should not add additional noise or distort the signal. For PSA it is especially important that the bandwidth of the PA does not stretch the rise-time of the signal, otherwise the differences of the detector responses to various particles may be washed out.

Modern nuclear spectroscopy experiments require an ever increasing number of detector channels, as the granularity and the solid angle coverage tend to grow. Thus, for the sake of reducing both the bulk and the cost of the acquisition systems, it is useful to have more input channels per digitizer card. However there is an optimum, as an FPGA has a limited amount of memory and computation power and can handle only a certain number of operations per cycle.

Digitizers from different manufacturers have a variety of solutions both for the FPGA programming and for the board control. Certain manufactures provide cards with HDL codes already implemented that allow the acquisition of signal traces or energy spectra. In some cases these are "black box" solutions, meaning that the user cannot access or modify the implemented codes. In other cases the custom modification and adjustment of the FPGA program are available, meaning that the specific algorithms may be developed and implemented. The latter is especially attractive for nuclear spectroscopy needs as it allows to adjust the algorithms to particular tasks, e.g. to record the energy only for the events with no pile-up and the full traces to the events with the pile-up in order to perform further off-line treatment. It also allows the implementation of custom PSA algorithms for particle discrimination.

### 4.2.2 First preliminary tests of the Nutaq digitizer

One of the solutions considered for the digital DAQ for GABRIELA was the MI-125 from Nutaq [48]. It provides a 14-bit digitizer resolution at a 125 MHz sampling frequency and has up to 64 input channels per card when placed in a double-width microTCA mother board. The implementation of the FPGA programme for this device is performed in a MatLab-Simulink environment, using Xilinx blocks [49]. The MatLab model may be then compiled into a bit-stream and recorded to the FPGA via 1xPCIe-4x remote host interface. In 2014 we performed the first preliminary tests with the standalone desktop version of the PicoDigitizer 125.

The preamplifier signal traces were recorded to disc and processed off-line on a PC to find the optimal parameters of the MWD and CR- $(R C)^{N}$ algorithms described above. An absolute minimal requirement to be able to test the performance of the digitizer is that the on-board FPGA of a digitizer is programmed to trigger on the preamplifier


Figure 4.8: Principal scheme of the MatLab-Simulink model for the tests of Nutaq PicoDigitizer.
signal pulse and to record a given number of pre-trigger and post-trigger points for each event to disk. These data can then be processed off-line with the C++ codes. As the Nutaq digitizer sent for the tests came with no such programme, we had to develop a MatLab-Simulink model, then convert it to a bit-stream and implemented it to the FPGA.

The simplified scheme of the developed programme is given in fig. 4.8 (a more detailed graphical representation is given in the Appendix A). There are 7 registers used for the following input parameters (see fig. A.1):
threshold the trigger level in ADC counts;
pre-trigger points number of data points before the trigger to be recorded;
number of samples total number of data points to be recorded per event;
number of triggers the demanded number of events to be recorded;
polarity set to " +1 " for positive or " -1 " for negative input signal polarity;
$\mathbf{s}$ trigger sensitivity level in ADC counts;
run enable when set "true", the data taking is activated.
The preamplifier signal acquired by the digitizer is converted to digital form in an onboard ADC, each ADC cycle is 8 ns long. The digitized signal is then pipelined to the trigger block and to the delay memory buffer. In the trigger block (see fig. A.2) the signal first passes through a median filter (see fig. A.3) and then through either the rising-edge detector block if the polarity is set to positive, or through the falling-edge detector if negative polarity is selected. The MaLab-Simulink scheme for the fallingedge detector can be found in fig. A.4. The trigger level $T L$ and trigger sensitivity $s$
are defined by the corresponding input register values. In the falling edge mode, the triggering occurs when all of the following conditions are true:

- $X[n] \leqslant T L-2 \cdot s$
- $X[n-2] \leqslant T L-s$
- $X[n-4] \geqslant T L+s$
- $X[n-6] \geqslant T L+2 \cdot s$
- "run enable" register is set to 1
where $X[n]$ is the current value of the signal in ADC counts and $X[n-2], X[n-4]$ and $X[n-6]$ are the values acquired 2,4 and 6 ADC cycles prior respectively (see fig. 4.9). For the rising edge detector the trigger logic is the same, but the signal polarity is reversed. The median filter block serves to smooth the signal before it reaches to the rising- or falling-edge detector, thus allowing lower trigger levels. When the trigger level is low (e.g. for the ICE detection), smaller $s$ values must be selected. The trigger block also contains a trigger counter which increments each time a trigger occurs. The time-stamp assigned to an event is that of the $X[n]$ point.


Figure 4.9: The falling edge trigger algorithm.

The memory buffer (see fig. A.5) serves to delay the signal by the number of ADC cycles specified in the "pre-trigger points" register. When triggered, the data from this memory buffer are formatted in the formatting block and sent to the PC. The formatting block (see fig. A.6) adds a header before each event trace. The data format is as follows:
channel ID the digitizer channel number (16 bits);
trigger counter the number of the current event ( $2 \times 16$ bits);
time-stamp the time-stamp of the current event ( $3 \times 16$ bits);
number of samples number of sampling points $N$ in the event trace ( 16 bits);
$\mathbf{X}[1]$... $\mathbf{X}[\mathbf{N}]$ the digitized input signal trace consisting of $N$ points, including the demanded number of pre-trigger points. Each sample is 14 bits long, written in 16-bit format for simplicity.


Figure 4.10: Comparison of the ${ }^{241} \mathrm{Am}$ and ${ }^{244} \mathrm{Cm} \alpha$ spectra acquired with the (a),(c) Nutaq MI-125 digitizer and (b),(d) with the TNT digitizer. The ranges were (a) 0-28 MeV, (b) $0-8 \mathrm{MeV}$, (c) $0-125 \mathrm{MeV}$ and (d) 0-40 MeV, thus (a) and (d) are directly comparable.

The tests were carried out on the S3 test-bench at IPHC, Strasbourg. The data were taken with the $100 \times 100 \mathrm{~mm}^{2} 8 \times 8$ pixels S3 prototype 298-12-R52 silicon tunnel detector and two different custom-modified CREMAT preamplifiers with the $62 \mathrm{mV} / \mathrm{MeV}(\mathrm{Si})$ and $173 \mathrm{mV} / \mathrm{MeV}(\mathrm{Si})$ gains and $\tau \simeq 50 \mu \mathrm{~s}$. The detector was cooled to -30C. The TNT digitizer with 100 MHz sampling frequency [50] was used to take data with the same setup for comparison. As a control an analogue DAQ was also used to check the results at all times. Data were taken with a standard triple- $\alpha\left({ }^{239} \mathrm{Pu},{ }^{241} \mathrm{Am},{ }^{244} \mathrm{Cm}\right)$ and ${ }^{207} \mathrm{Bi}$ sources. The particle energies can be found in table 4.1.

The resulting spectra for the $\alpha$ sources are given in fig. 4.10, while those for the ${ }^{207} \mathrm{Bi}$

| Source | Mode | Energy, keV | Intensity |
| :---: | :---: | :---: | :---: |
| ${ }^{241} \mathrm{Am}$ | $\alpha$ | 5338 | $1.4 \%$ |
|  | $\alpha$ | 5443 | $12.8 \%$ |
|  | $\alpha$ | 5486 | $85.2 \%$ |
| ${ }^{244} \mathrm{Cm}$ | $\alpha$ | 5763 | $23.3 \%$ |
|  | $\alpha$ | 5803 | $76.7 \%$ |
| ${ }^{239} \mathrm{Pu}$ | $\alpha$ | 5144 | $17.1 \%$ |
|  | $\alpha$ | 5157 | $70.8 \%$ |
| ${ }^{238} \mathrm{Pu}$ | $\alpha$ | 5456 | $29.0 \%$ |
|  | $\alpha$ | 5499 | $70.9 \%$ |
| ${ }^{207} \mathrm{Bi}$ | K ICE | 482 | $1.52 \%$ |
|  | L ICE | 554 | $0.15 \%$ |
|  | M ICE | 567 | $0.08 \%$ |
|  | K ICE | 976 | $7.00 \%$ |
|  | L ICE | 1048 | $1.84 \%$ |
|  | M ICE | 1060 | $0.54 \%$ |
| ${ }^{133} \mathrm{Ba}$ | K ICE | 240 | $0.34 \%$ |
|  | K ICE | 267 | $0.68 \%$ |
|  | L ICE | 297 | $0.10 \%$ |
|  | K ICE | 320 | $1.31 \%$ |
|  | K ICE | 348 | $0.15 \%$ |

Table 4.1: The energies of the main $\alpha$ and ICE lines of the calibration sources used in the tests.
source are shown in fig. 4.11. All spectra were produced off-line on a PC using PA signal traces of equal durations and with the equivalent MWD parameters for TNT and for MI-125. The resolutions obtained for the $\alpha$ particles are $\sim 19 \mathrm{keV}$ with the TNT within the $0-40 \mathrm{MeV}$ range and over 28 keV with $\mathrm{MI}-125$ within a smaller $0-28 \mathrm{MeV}$ range. For the ICE, the resolution with the TNT was $\sim 14 \mathrm{keV}$ with the TNT and $\sim 21 \mathrm{keV}$ with MI-125 in the same ranges, clearly displaying the disadvantage of the Nutaq digitizer performance.

This dramatic difference in resolutions measured with the TNT and Nutaq digitizers came as a great surprise to us, as we knew that both Jurogam and GREAT at JYFL are instrumented with the 14 -bit 100 MHz Lyrtech digitizers (which are the previous generation of the Nutaq PicoDigitizer), and their performance is very satisfactory. This promoted further tests using a signal generator to determine a baseline noise (fluctuations about a DC input voltage). The reason for the bad performance of the Nutaq device is probably due to a grounding issue. Unfortunately the digitizer we were sent


Figure 4.11: Comparison of the ${ }^{207}$ Bi ICE spectra acquired with the (a),(c) Nutaq MI-125 digitizer and (b),(d) with the TNT digitizer. The ranges were (a) $0-28 \mathrm{MeV}$, (b) $0-8 \mathrm{MeV}$, (c) $0-125 \mathrm{MeV}$ and (d) $\mathrm{O}-$ 40 MeV , thus (a) and (d) are directly comparable.

|  | range | $\sigma$, LSB | $\sigma, \mathrm{mV}$ | $E N O B$ |
| :---: | :---: | :---: | :---: | :---: |
| TNT | $\pm 0.33 \mathrm{~V}$ | 1.92 | 0.07 | 11.3 |
| (card 1008) | $\pm 1.33 \mathrm{~V}$ | 1.45 | 0.21 | 11.8 |
| NUTAQ | $\pm 1 \mathrm{~V}$ | 2.2 | 0.27 | 11.1 |

Table 4.2: The ENOB obtained on the baseline of the TNT and MI-125 digitizers in the 2014 tests.
for these tests was the desktop edition, not the crate version. As it can be seen in table 4.2 the baseline noise of the Nutaq digitizer is considerably worse than that of TNT: the noise in mV is $12.5 \%$ larger. Therefore it was decided to repeat these tests once the possible grounding problem was solved, and also to search for the alternative solutions.

### 4.2.3 Nutaq and NI comparative tests

The comparative tests between a Nutaq MI-125 digitizer (described in the previous subsection) and a National Instruments (NI) NI-5170R digitizer were performed in 2015 using the CSNSM test-bench. As for the preliminary tests, both analogue electronics

| Manufacturer | Digitizer resolution | Frequency | Channels | Range |
| :---: | :---: | :---: | :---: | :---: |
| Nutaq | 14 bits | 125 MHz | up to 64 | $\pm 1 \mathrm{~V}$ |
| NI | 14 bits | 250 MHz | 8 | $\pm 0.1 \mathrm{~V}$ |
|  |  | 125 MHz |  | $\pm 0.2 \mathrm{~V}$ |
|  |  |  |  | $\pm 0.5 \mathrm{~V}$ |
|  |  |  |  | $\pm 1 \mathrm{~V}$ |
|  |  |  |  | $\pm 2.5 \mathrm{~V}$ |
| TNT | 14 bits | 100 MHz | 4 | $\pm 0.33 \mathrm{~V}$ |
|  |  |  |  | $\pm 1.33 \mathrm{~V}$ |

Table 4.3: The characteristics of different digitizers.
and a TNT digitizer were used for the reference measurements. The characteristics of the digitizers are given in tab. 4.3. The NI module has 4 ns sampling rate. After the digitization, the data is divided in two parallel streams of even and odd samples. This is done to achieve a better performance of the FPGA card. The even and odd samples may also optionally be summed in order to reduce the noise, a technique known as over-sampling. This results in an effective sampling rate of 125 MHz allowing a more direct comparison with the MI-125 digitizer from Nutaq.

For the Nutaq digitizer, the programme described in subsection 4.2.2 was used. The NI digitizer had a similar programme, developed within the LabVIEW framework implemented on it as standard (see fig. 4.12).


Figure 4.12: The LabVIEW interface of the control programme of the NI digitizer. The trace (in red) comes from one of the crystals of the clover germanium detector.


Figure 4.13: The scheme of the data acquisition for the measurement of the silicon detector resolution.

|  | FWHM in keV with RC $=300-1000 \mathrm{~ns}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 |
| TNT | 25 | 23.7 | $\mathbf{2 3 . 2}$ | 23.4 | 23.4 | 23.4 | 23.6 | 23.9 |
| Nutaq | 24.7 | 23.8 | $\mathbf{2 3 . 7}$ | 24 | 24.3 | 24.9 | 25.5 | 26 |
| NI (250 MHz) | 28.8 | 27.5 | 26.9 | 26.3 | $\mathbf{2 6}$ | $\mathbf{2 6}$ | 26.5 | 26.3 |
| NI (125 MHz) | 28.9 | 25.9 | 26 | 25.9 | $\mathbf{2 5 . 3}$ | 25.6 | 25.6 | 25.8 |

Table 4.4: Comparison of the $\alpha$ resolutions obtained of the CR-RC ${ }^{4}$ algorithm with different digitizers and with varying $R C$ parameter. The optimal results for each digitizer are highlighted in bold text.

To compare the energy resolutions of the spectra acquired with the three digitizers, a surface-barrier silicon detector with a triple- $\alpha$ source was used with a Mesytec MPR-1 single-channel preamplifier (see fig. 4.13). In order to make a direct comparison, $\pm 1 \mathrm{~V}$ and $\pm 1.3 \mathrm{~V}$ ranges were selected on NI-5170R and TNT respectively, resulting in $\sim 45 \mathrm{MeV}$ full-scale ranges on all three digitizers. The acquired traces were postprocessed with the $\mathrm{CR}-\mathrm{RC}^{4}$ algorithm. The shaping time $\tau=R C$ was varied in order to find the optimal parameters for each digitizer. For comparison, the resolution with the same setup obtained using an analogue Ortec-672 spectroscopy amplifier with $1 \mu \mathrm{~s}$ shaping time and recorded with an Amptek MCA-8000A device in $\sim 16 \mathrm{MeV}$ range was 24.8 keV .

The results of these tests are given in table 4.4. The best energy resolution was obtained with the TNT. The Nutaq digitizer showed comparable though slightly worse result. The tests with the NI-5170R digitizer were performed with the exact same setup but a few weeks later. The obtained resolutions proved to be $\sim 2 \mathrm{keV}$ worse than those of Nutaq and TNT. However, we have a suspicion (proved by the further results) that either the grounding of the Mesytec PA was insufficient for the NI tests, or the detector itself was deteriorating. In fact, the detector became unusable a few weeks later due to high leakage current.


Figure 4.14: The scheme of the data acquisition with a signal generator.

|  | range | $\sigma$, LSB | $\sigma, \mathrm{mV}$ | ENOB |
| :---: | :---: | :---: | :---: | :---: |
| NI | $\pm 0.5 \mathrm{~V}$ | 1.99 | 0.12 | 11.2 |
|  | $\pm 1 \mathrm{~V}$ | 2.17 | 0.26 | 11.1 |
| NI (even+odd) | $\pm 0.5 \mathrm{~V}$ | 1.48 | 0.09 | 11.6 |
|  | $\pm 1 \mathrm{~V}$ | 1.61 | 0.20 | 11.5 |
| TNT | $\pm 0.33 \mathrm{~V}$ | 1.92 | 0.07 | 11.3 |
| (card 1008) | $\pm 1.33 \mathrm{~V}$ | 1.45 | 0.21 | 11.8 |
| NUTAQ (2014) | $\pm 1 \mathrm{~V}$ | 2.2 | 0.27 | 11.1 |
| NUTAQ (2015) | $\pm 1 \mathrm{~V}$ | 1.87 | 0.23 | 11.3 |

Table 4.5: The ENOB obtained on the baseline of different digitizers.

As it can be concluded from the presented results, the energy resolution provided by the surface-barrier detector was far from perfect. To eliminate this effect, the ENOB measurements on the baseline of each digitizer were performed. For this measurements, a signal generator and an attenuator were used (see fig. 4.14). In order to measure the $E N O B$ value at the baseline for each digitizer, the noise variation on the baseline $\sigma$ was estimated by averaging over $2 \cdot 10^{7}$ pre-trigger points in each range (2000 points $\cdot 10000$ traces) for the NI digitizer and $3 \cdot 10^{7}$ pre-trigger points ( 900 points 35000 traces) for the Nutaq digitizer. The results for the $E N O B$ measured on the baseline are presented in tab. 4.5. One may conclude that Nutaq digitizer sent in 2015 performed better on the test bench than that sent in 2014, but its performance is still worse than that of TNT and of the NI digitizer. Yet, the reason may be still in the grounding, as again we only received the desktop edition of the digitizer for the tests despite warning Nutaq of this issue and requesting a micro-TCA crate version. Though the ENOB values for all digitizers are more or less compatible, the energy resolutions are still different. This is due to the fact that the $E N O B$ measurements were only performed at the baseline, and not through the full range of the digitizers.

Finally, tests on the energy resolution were performed with Ge detectors and a ${ }^{60} \mathrm{Co}$ calibration source. The TNT and Nutaq digitizers were compared on the IPNO test-
bench with a phase-1 Ortec detector. The FWHM was 2.07 keV with analogue electronics, 2.1 keV with the TNT and no better than $2.3-2.4 \mathrm{keV}$ with the Nutaq digitizer. Moreover, the Nutaq digitizer spectra with the optimal CR-RC ${ }^{4}$ parameters had low-energy tails that we do not quite understand (see fig. 4.15). No such structure was found with TNT or analogue spectra.


Figure 4.15: 1173 keV and 1332 keV lines ${ }^{60} \mathrm{Co}$ spectrum acquired with the Nutaq digitizer and a phase-1 germanium detector on the IPN test-bench. The smaller line is 1462 keV line is from ${ }^{40} \mathrm{~K}$ in the background.

As we did not have the NI digitizer at the time of the IPNO test, it was tested in Dubna with the GABRIELA clover detector installed. The resolution obtained with CAMAC analogue ADC was 2.1 keV with the clover mounted on the GABRIELA frame and liquid nitrogen autofill. Figure 4.12 shows the LabVIEW control interface for the NI digitizer; the trace on the screen is from crystal 1 of the clover detector. The tests with the NI-5170 digitizer borrowed from NI-Russia for these tests was $1.9-2 \mathrm{keV}$, an improvement over the CAMAC analogue resolutions*.

A general remark on the Nutaq digitizer is that during these tests we had multiple problems with corrupted files when the number of events recorded was greater than $\sim 35000$ and each event had $\sim 10000$ sample points, though that did not occur all the time. The engineers from Nutaq support could not help us find the explanation for this problem.

[^0]To conclude, given the poor resolutions of the Nutaq digitizer from the first tests and with the germanium detector and taking into account the complications with the MatLab/Xilinx framework compared to LabView, we are inclined to favour the NI solution for the upgrade of the GABRIELA DAQ system.

### 4.2.4 Tests on the Dubna test-bench

As purchasing the full set of digital electronics ( $\sim 520$ independent channels) at once is complicate from a budget point of view, an intermediate solution involving the 16channel spectroscopy amplifiers equipped with 16-to-1 channel multiplexers used in GABRIELA DAQ was suggested. As a SA is also shaping the output signal of a PA, it would not be possible to use the full power of the digital electronics, i.e. the PSA or the MWD or CR- $(R C)^{N}$ algorithms, but only the pulse-hight determination of the shaped signals. Thus we performed a set of tests on the FLNR test-bench in order to prove the feasibility of such a solution.


Figure 4.16: The setup of the data acquisition for the tests on the FLNR test-bench.

For the first tests we used the $128 \times 128$ strip DSSD of the GABRIELA focal plane. The PA used was a 16-channel Tekinvest preamplifier with $20 \mathrm{mV} / \mathrm{MeV}(\mathrm{Si})$ gain. The multiplexed GABRIELA SA has six output channels, two of which provide an energy signal in two ranges (" $\alpha$ " and "fission" ranges if no linear amplifier is used) and the other four provide the binary signals ( $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D ) in which the strip number is coded. We used a 16 -channel fast linear amplifier of the GABRIELA DAQ which multiplies the PA signal amplitude by 10 without shaping it to obtain " $\beta$ " and " $\alpha$ " ranges at the SA output (see fig. 4.16). The strip number may be decoded the following way:

$$
\text { strip }=(A+1 / 2 \cdot B+1 / 4 \cdot C+1 / 8 \cdot D) \cdot 8
$$

which gives strip numbers 0 to 15 for the 16 multiplexed channels. As the TNT digitizer used for these tests only has four inputs, the above operation had to be performed in an
analogue electronics block, which summed the four signals, three of which had additional resistances at the input in order to divide the signals (see fig. 4.17). The resulting 16-level analogue strip-ID signal is recorded in TNT along with the two analogue signals from the MUX-SA. The histogram for 16 channels of the DSSD is given in fig. 4.18. This test proved that the spectroscopy amplifiers with multiplexers can be used in combination with digital electronics as a temporary solution to reduce the number of channels.


Figure 4.17: The scheme of the data acquisition with $\mathrm{SA}+\mathrm{MUX}$ and TNT.


Figure 4.18: A 2D histogram of 16 multiplexed DSSD strips.


Figure 4.19: Energy spectrum acquired with ${ }^{238} \mathrm{Pu},{ }^{244} \mathrm{Cm}$ and ${ }^{133} \mathrm{Ba}$ sources in " $\alpha$-range".

As there were certain noise issues at the test-bench for the $128 \times 128$ strip detector, we used a single-channel silicon pin detector to acquire the energy spectra and to demonstrate that the resolutions obtained using TNT as a peak sensing ADC were no worse than those obtained from the CAMAC ADC. We used a ${ }^{133} \mathrm{Ba}$ ICE source and ${ }^{238} \mathrm{Pu}$ and ${ }^{244} \mathrm{Cm} \alpha$-sources to measure the energy resolutions both for both $\alpha$-particles and ICE. The spectrum acquired in " $\alpha$-range" is given in fig. 4.19. The SA output signal traces were acquired with the TNT and treated to find the pulse hight for each trace. The resulting FWHM is 21.4 keV for the $\alpha$ s and 17.7 keV for the ICE.

We also made a similar measurement in " $\beta$-range" with these sources. The ICE spectrum is given in fig. 4.20, the FWHM is 14.3 keV . The $\alpha$ particles in this range are over the range limits resulting in saturation of the SA. An example of an overflow trace is given in fig. 4.21. The time-over-overflow algorithm described in subsection 4.1.5 was used to restore the energies of the $\alpha$ particles. The resulting spectrum is presented in fig. 4.22. The fine structure of ${ }^{238} \mathrm{Pu}$ and ${ }^{244} \mathrm{Cm} \alpha$-decay is not visible, the FWHM is $\sim 65 \mathrm{keV}$. Though such resolution would not be sufficient for $\alpha$ spectroscopy, such FWHM would clearly be satisfactory for the ER energies or for the fission fragments, which cause a similar overflow in "fission-range". Thus, it would be sufficient to use the two ranges " $\beta$ " and " $\alpha$ ", in an experiment with such semi-digital setup.


Figure 4.20: ${ }^{133} \mathrm{Ba}$ energy spectrum acquired in " $\beta$-range".


Figure 4.21: A digitized trace of a saturated SA signal.

### 4.2.5 Conclusions

The decision on the model of the digitizers to be purchased is yet to be taken. From the tests presented in this chapter, we tend to prefer the NI digitizer over the NUTAQ one. Our colleagues from IPHC Strasbourg have conducted similar tests with a CAEN digitizer, which is also one of the probable a candidates for the upgrade of the GABRIELA DAQ system. The final decision is to be taken in the coming months.

As mentioned above, purchasing all $\sim 520$ independent channels of readout elec-


Figure 4.22: An $\alpha$-spectrum obtained with the ${ }^{238} \mathrm{Pu}$ and ${ }^{244} \mathrm{Cm}$ sources in " $\beta$-range" with the time-over-overflow method.
tronics at once may be complicated from a budget point of view. Thus the following intermediate solution, which proves to be feasible and provides satisfactory energy resolutions, is proposed:

- DSSD front side: 128 channels fully digital, which allows to measure fast decays and to test particle discrimination ( 128 channels);
- DSSD back side: analogue SA+MUX to digital (8 channels);
- Tunnel: 1 detector fully digital, 7 detectors analogue SA+MUX to digital (32+14 channels);
- Ge+BGO: Clover: 4 Ge crystals and 1 BGO shield; 4 coaxial Ges and 4 BGOs (13 channels);
- ToF: standard 2-foil configuration - digitisation of the output of a TAC; 1-foil configuration for the very asymmetric reactions - digitization of the logics of the MCPs of one foil (1 or 2 channels).

Such setup requires 192 digitizer channels instead of 520 , meaning that 248 -channel NI digitizers are required.

# Determination of the multipole mixing 

## ratios

### 5.1 Probability density function of the mixing ratio

As discussed in chapter 1 , conversion coefficients may provide valuable information on the spin and parity of the initial and final states of an electromagnetic transition, as well as to measure the admixture coefficients for the mixed transitions. The expression connecting the theoretical conversion coefficients for the two mixed multipolarities $\alpha_{1}$ and $\alpha_{2}$ to the measured value $\alpha_{\text {exp }}$ may be written as follows:

$$
\begin{equation*}
\alpha_{\text {exp }}=\frac{\alpha_{1}+\delta^{2} \alpha_{2}}{1+\delta^{2}} \tag{5.1}
\end{equation*}
$$

where $\delta$ is the mixing ratio. The uncertainty of $\alpha_{\text {exp }}$ comes from the experiment and mostly depends on the acquired statistics. As mentioned in chapter 1 , uncertainty of $\alpha_{1}$ and $\alpha_{2}$ is of the order of $2 \%$ and arises from two factors: the accuracy of the theoretical calculations and the accuracy of interpolation for the non-tabulated values [15]. However, because of the shape of the $\delta(\alpha)$ function, this small $\Delta \alpha$ may result in much more significant and, in a general case, asymmetric uncertainties on the mixing ratio.

The most intuitive thing to do in order to determine the mean value of $\delta$ is to invert the expression 5.1 in the following way:

$$
\begin{equation*}
\delta=\sqrt{\frac{\alpha_{1}-\alpha_{\text {exp }}}{\alpha_{\text {exp }}-\alpha_{2}}} . \tag{5.2}
\end{equation*}
$$

However, in a general case this is wrong, as $\langle y(x)>\neq y(\langle x\rangle)$. We will discuss the applicability of this assumption in section 5.3.

The theoretical and experimental internal conversion coefficients have Gaussian probability density functions (PDFs) associated to them. Each of these probability distributions will contribute to the PDF of the mixing ratio $P(\delta)$. The PDF components for the mixing ratio arising from these three variables (under the assumption that they are uncorrelated) can be determined in the following way:

$$
\begin{equation*}
\int_{0}^{+\infty} P(\delta) d \delta=\int_{0}^{+\infty} G(\alpha) d \alpha=\int_{0}^{+\infty} G(\alpha(\delta)) \frac{\partial \alpha}{\partial \delta} d \delta \tag{5.3}
\end{equation*}
$$

where $\alpha$ stands for $\alpha_{1}, \alpha_{2}$ or $\alpha_{\text {exp }}$ for each partial PDF $P_{1}(\delta), P_{2}(\delta)$ or $P_{\exp }(\delta)$ respectively, and $G(\alpha)$ is the Gaussian distribution. $P(\delta)$ can be retrieved from this expression as follows:

$$
\begin{equation*}
P(\delta)=G(\alpha(\delta)) \frac{\partial \alpha}{\partial \delta} \tag{5.4}
\end{equation*}
$$

For $P_{1}$ and $P_{2}$ we express $\alpha_{1}$ and $\alpha_{2}$ as functions of $\delta$ :

$$
\begin{gather*}
\alpha_{1}(\delta)=\alpha_{\text {exp }} \cdot\left(1+\delta^{2}\right)-\alpha_{2} \cdot \delta^{2} ;  \tag{5.5}\\
\alpha_{2}(\delta)=\frac{1+\delta^{2}}{\delta^{2}} \cdot \alpha_{\exp }-\frac{\alpha_{1}}{\delta^{2}} . \tag{5.6}
\end{gather*}
$$

To calculate the partial PDF of $\alpha_{1}$ the other two parameters must be fixed to their mean values. Then the expression 5.4 becomes:

$$
\begin{equation*}
P_{1}(\delta)=\delta \cdot \exp \left(-\frac{\left(\delta^{2}\left(\alpha_{e x p}-\alpha_{2}\right)+\alpha_{e x p}-\mu\right)^{2}}{2 \sigma^{2}}\right) \tag{5.7}
\end{equation*}
$$

with $\mu=\alpha_{1}, \sigma=\Delta \alpha_{1}$.

In a similar fashion, the PDF for $\alpha_{2}$ is:

$$
\begin{equation*}
P_{2}(\delta)=\frac{1}{\delta^{3}} \cdot \exp \left(-\frac{\left(\frac{1+\delta^{2}}{\delta^{2}} \cdot \alpha_{\exp }-\frac{\alpha_{1}}{\delta^{2}}-\mu\right)^{2}}{2 \sigma^{2}}\right) \tag{5.8}
\end{equation*}
$$

with $\mu=\alpha_{2}, \sigma=\Delta \alpha_{2}$. For $\alpha_{\text {exp }}$ the expression is:

$$
P_{\exp }(\delta)=\frac{\delta}{\left(\delta^{2}+1\right)^{2}} \cdot \exp \left(-\frac{\left(\frac{\alpha_{1}+\delta^{2} \alpha_{2}}{1+\delta^{2}}-\mu\right)^{2}}{2 \sigma^{2}}\right)
$$

with $\mu=\alpha_{\text {exp }}$ and $\sigma=\Delta \alpha_{\text {exp }}$.

The total PDF of $\delta$ is then a convolution of these partial PDFs and therefore, in general, is no longer Gaussian:

$$
\begin{equation*}
P(\delta)=P_{1}(\delta) \otimes P_{2}(\delta) \otimes P_{\exp }(\delta) \tag{5.9}
\end{equation*}
$$

In the following sections the method will be illustrated with a mixed M2/E35/2+ $\rightarrow$ $9 / 2^{-} 200 \mathrm{keV}$ transition in ${ }^{251} \mathrm{Fm}$ [51]. For this particular example $\delta$ will be obtained from the K -conversion coefficient $\alpha_{K}$ measured using the GABRIELA setup (see chapter 2). Plots of the relevant PDFs calculated using RooFit [32] classes are shown in fig. 5.1 and the corresponding parameters $\alpha_{\text {exp }}, \alpha_{1}$ and $\alpha_{2}$ are given in table 5.1.

The mean value of $P(\delta)$ can be derived through the first order moment of the distribution

$$
\begin{equation*}
<\delta>=\int_{0}^{+\infty} \delta P(\delta) d \delta \tag{5.10}
\end{equation*}
$$

The central value (from eq. 5.10) with the associated uncertainties within one standard deviation ( $68 \%$ of the PDF integral calculated such that $34 \%$ are on either side of the mean value) is found to be $\delta=0.92_{-0.37}^{+0.42}$.


Figure 5.1: An example of the PDFs for the K-conversion of 200 keV transition (M2 and E3 admixture) in ${ }^{251} \mathrm{Fm}$. The dashed lines show the partial PDFs $P_{1}$ (red), $P_{2}$ (green) and $P_{\text {exp }}$ (blue). The solid magenta line is the total PDF of $\delta . P_{1}$ and $P_{2}$ are normalised to 1 , while $P_{\exp }$ and $P$ have been normalised to 10 for purely visual reasons. The shaded cyan region represents the $68 \%$ confidence interval around the graphically-obtained mean; the magenta shaded area is the the $68 \%$ confidence interval around the analytically-obtained mean.

### 5.2 Graphical method

We propose to trace the experimental internal conversion coefficient $\alpha$ as a function of $\delta$ to determine both the admixture and the corresponding uncertainty. Again, the mixed $200 \mathrm{keV} \mathrm{M} 2 / E 3$ transition in ${ }^{251} \mathrm{Fm}$ is used as the example. For this case the expression of the experimental K -conversion coefficient as a function of $\delta$ becomes:

$$
\begin{equation*}
\alpha_{K}(\delta)=\frac{\alpha_{K}(M 2)+\delta^{2} \cdot \alpha_{K}(E 3)}{1+\delta^{2}} \tag{5.11}
\end{equation*}
$$

The upper and lower uncertainty limits $\alpha_{K}^{ \pm}(\delta)$ are given by

$$
\begin{gather*}
\alpha_{K}^{ \pm}(\delta)=\frac{\alpha_{K}(M 2)+\delta^{2} \cdot \alpha_{K}(E 3)}{1+\delta^{2}} \pm \\
\frac{\sqrt{\Delta \alpha_{K}(M 2)^{2}+\left(\delta^{2} \Delta \alpha_{K}(E 3)\right)^{2}}}{1+\delta^{2}} \tag{5.12}
\end{gather*}
$$

|  | Mean value | Uncertainty |
| :---: | :---: | :---: |
| $\alpha_{\text {Kexp }}$ | 8.8 | 3.1 |
| $\alpha_{K}(M 2)[52]$ | 14.49 | 0.21 |
| $\alpha_{K}(E 3)[52]$ | 0.227 | 0.004 |
| $\delta_{a}$ | 0.92 | $\substack{-0.42 \\ \hline 0.03 \\ \hline 0.0 .42}$ |
| $\delta_{g}$ | 0.81 | $\substack{+0.32}$ |

Table 5.1: The $\alpha_{K}$ parameters for $5 / 2^{+} \rightarrow 9 / 2^{-} 200 \mathrm{keV}$ transition in ${ }^{251} \mathrm{Fm}$ used for the demonstration and the obtained analytical $\delta_{a}$ and graphical $\delta_{g}$ mixing ratio values.

These theoretical curves are presented in fig. 5.2.


Figure 5.2: In red: K-conversion coefficient $\alpha_{K}$ as a function of $\delta$ with its uncertainties; in magenta: measured value of the K-conversion coefficient $\alpha_{\exp }$ with its uncertainties; in blue: the deduced value of $\delta$ with the associated asymmetric uncertainties

The measured value of the conversion coefficient is $\alpha_{\text {exp }}$ with the upper and lower limits $\alpha_{\text {exp }}^{+}$and $\alpha_{\text {exp }}^{-}$defining the confidence interval. The central value of $\delta$ is then the solution of equation 5.11 with $\alpha_{K}(\delta)=\alpha_{\text {exp }}$.

One should bear in mind, that the function $\alpha(\delta)$ may be decreasing as a function of $\delta$ (as is the case for $\alpha_{K}$ used in this example) as well as increasing (e.g. $\alpha_{L}$ for this same transition). Assuming the maximal error approach, the minimal value of $\delta$ is determined by the lowest intersection of the uncertainties of $\alpha_{K}$ and $\alpha_{\text {exp }}$, and the upper limit is the value of $\delta$ at the highest intersection.

The graphical method gives $\delta=0.81_{-0.36}^{+0.47}$ for this example. This graphical result is
compared to the analytical one in fig. 5.1.

### 5.3 Discussion

As the theoretical uncertainties are restrained to $\sim 2 \%$, the error bars on $\alpha_{\text {exp }}$ are the main source of the uncertainty on $\delta$. It is important to note that the mean value of $P(\delta)$ (see eq. 5.10 ) is, in general, not equal to the solution of eq. 5.2. For example, for the 200 keV transition in ${ }^{251} \mathrm{Fm}$, the means $\left\langle\delta>_{a}=0.92\right.$ and $\left\langle\delta>_{g}=0.81\right.$ are obtained using the analytical and graphical methods respectively. In this case, the solution to equation 5.2 underestimates the mean value by $12 \%$.

When the uncertainty on $\alpha_{\text {exp }}$ is small, both formulae give practically the same result. This is illustrated in fig. 5.3 which shows how the expression $\langle\delta\rangle_{a} /\langle\delta\rangle_{g}-1$ varies as a function of the relative experimental error $\Delta \alpha_{K \text { exp }} / \alpha_{K \text { exp }}$. When the relative error on $\alpha$ exceeds $25 \%$ the mean value obtained from eq. 5.2 begins to deviate from the true mean value. This mismatch reaches $36 \%$ for a relative error of $60 \%$.


Figure 5.3: The dependence of the relative difference of $\delta_{a}$ and $\delta_{g}$ as a function of $\Delta \alpha_{\text {Kexp }} / \alpha_{\text {Kexp }}$. The confidence interval for the of the 200 keV transition in ${ }^{251} \mathrm{Fm}$ obtained with the
graphical method is $6 \%$ larger than the one obtained analytically. The comparison of the two intervals is given in fig. 5.2. $P(\delta)$ provides a precise measure of the uncertainty on $\delta$. The confidence intervals derived through the graphical method are always superior or equal to the true confidence intervals extracted from $P(\delta)$, as the edge values $\alpha^{-}(\delta)=\alpha_{\text {exp }}^{+}$and $\alpha^{+}(\delta)=\alpha_{\text {exp }}^{-}$are beyond the one standard deviation region.

### 5.4 Conclusions

The graphical method of propagation of the uncertainties allows the derivation of the central value of the admixture coefficient with its errorbars in a simple and illustrative manner. It also enables the asymmetric confidence interval of $\delta$ to be calculated. While it requires much less computational power than the analytical estimate, it is important to understand the limitations of this method which can result in significant deviations from the full analytical solution.

For comparison, when the value of $\delta$ is calculated in a "classic" linear approach with $<\delta>$ from eq. 5.2 and $\Delta \delta=\sqrt{\Sigma_{i}\left(\frac{\partial \delta}{\partial \alpha_{i}} \Delta \alpha_{i}\right)^{2}}$, the resulting confidence interval becomes $\delta_{\text {lin }}=0.81 \pm 0.37$, which is an underestimate of both the central value and the confidence interval, and also does not take the asymmetry of the PDF into account.

It is important to notice that the graphical method only gives an upper limit for the confidence interval, and may underestimate the central value of $\delta$ when the uncertainties on the experimental conversion coefficients are high. In the cases when the graphical method is not applicable, the convolution has to be applied.

The graphical method also helps to better understand the influence of the different parameter values and their PDFs on the final result. The demonstrated method may also be applied to the $\alpha_{K} / \alpha_{L}, \alpha_{L} / \alpha_{M}$ and similar measurements, which lead to even bulkier calculations if developed analytically.
"Before I came here I was confused about this subject. Having listened to your lecture I am still confused. But on a higher level."

Enrico Fermi

## 6

## ${ }^{251} \mathrm{Fm}$

### 6.1 Discovery of fermium

The element 100, fermium, was first discovered in 1952, in the fallout from the explosion of the 10-megaton nuclear bomb "Ivy Mike", the first successful hydrogen bomb [53]. The tests by the United States took place on the atoll Enewetak in the Pacific Ocean. "Mike" was the first nuclear test in which part of the explosive yield came from nuclear fusion. The immense flux of neutrons allowed 15-18 neutron capture on ${ }^{238} \mathrm{U}$ contained in the explosive material combined with up to $7 \beta^{-}$decays. Einsteinium (Z=99) was also discovered in "Mike" fallout practically immediately after the test: it was found in the dust that was collected in the filters carried by an aeroplane in the zone of the nuclear explosion. In a couple of months, when more material was brought to the Berkeley laboratory (namely - the corals from an island near the atoll), an observation of an $\alpha$-decaying activity with a half-life of about a day was confirmed. Due to its short lifetime, it could only come form $\beta^{-}$decay of ${ }^{255} \mathrm{Es}$, and thus it was identified as ${ }^{255} \mathrm{Fm}$. The discoveries were declassified and published [54] in 1955.

### 6.2 Previous measurements on ${ }^{251} \mathrm{Fm}$

The first tentative level scheme of ${ }^{251} \mathrm{Fm}$ was built by Eskola et al. [55] through the observation of the fine structure of the ${ }^{255} \mathrm{No} \alpha$ decay. The spin and parity assignments (see fig. 6.1(a)) were based on the deduced hindrance factors and the analogy with ${ }^{253} \mathrm{Fm}$.

In addition to $\alpha$ spectroscopy Bemis et al. [56] also used $\gamma$ detectors in order to have the K X-ray information for the unequivocal identification of transfermium elements. These detectors, though not intended for the studies of nuclear structure, allowed them to make the first $\gamma$-ray observations for ${ }^{251} \mathrm{Fm}$. In their paper, they describe a number of alpha-decay lines from ${ }^{255}$ No populating excited states in ${ }^{251} \mathrm{Fm}$ and two gamma-ray transitions of 187 keV * and 191 keV depopulating these states. The 187 keV transition observed in prompt $\alpha-\gamma$ coincidences corresponds to the $192 \mathrm{keV} \mathrm{I} / 2^{+} \rightarrow 5 / 2^{+}$transition, which is the most intense $\gamma$ line in such spectra. They also made an estimate of the corresponding conversion coefficients $\alpha_{\mathrm{K}}=1.4_{-0.6}^{+2.1}$ and $\alpha_{\text {TOT }}=1.9_{-2.1}^{+4.9}$, which allowed them to conclude that the transition should have either E2 or M1-E2 multipolarity. The delayed K X-rays observed with a half-life of $15.2(23) \mu$ s were attributed to the $5 / 2^{+} \rightarrow 9 / 2^{-}$transition (see fig. 6.1(b)). The energy of the transition of about 191 keV was deduced from the difference between the $\alpha$ branches and corresponds to the 200 keV transition in [51,57] and this work.

The next observation of ${ }^{251} \mathrm{Fm}$ was made by Hessberger et al. [57], who built a much more consistent level scheme (see fig. 6.1(c)), largely adopted from [55]. They observed many prompt $\gamma$-ray transitions: 163 keV and 167 keV , $192 \mathrm{keV}, 195 \mathrm{keV}, 354 \mathrm{keV}$ and 358 keV which have also been observed by Asai et al. [51] and in the present work. Hessberger observed an energy of 200 keV for the isomeric transition, measured the halflife to be $21(3) \mu$ and extracted the K -conversion coefficient from the ratio of $\gamma$ - and K X-ray intensities to be $\alpha_{K}=8.3(29)$, indicating $\mathrm{M} 2+\mathrm{E} 3$ mixing.

The most recent paper on ${ }^{251} \mathrm{Fm}$ by Asai et al. [51] confirmed the level scheme proposed by Hessberger et al. in [57]. Moreover, in the experiment performed by Asai et al. the $\alpha$ spectra were free from the effects of summing with ICEs. This allowed to

[^1]

Figure 6.1: Plots (a) to (d) correspond to the chronological evolution of the ${ }^{251} \mathrm{Fm}$ level scheme. The proposed level schemes of ${ }^{251} \mathrm{Fm}$ from Eskola et al. [55] (a), Bemis et al. [56] (b), Hessberger et al. [57] (c) and Asai et al. [51] (d). In (d) the transitions feeding the 192 keV line are marked in blue, the transitions feeding 200 keV line are marked in green.
more precisely determine the $\alpha$-feeding branches and to assign several new levels (see fig. 6.1(d)). From the delayed K X-rays $\alpha_{K}=16.7(27)$ for the 200 keV isomeric transition is measured. They also report the observation of a 64 keV transition in the ground state
band.
The $\mathbf{N}=151$ isotones are characterised by prompt $\alpha-\gamma$ and $\alpha$ - delayed $-\gamma$ coincidences. The decay of the $5 / 2^{+}$level is hindered, as this level can only decay to the 9/2 ${ }^{-}$ground state band via an M2 or E3 transition. Such isomers have been observed in ${ }^{247} \mathrm{Cm}$ [58], ${ }^{249} \mathrm{Cf}[59],{ }^{251} \mathrm{Fm}$ [51, 57], ${ }^{253} \mathrm{No}$ [60]. There is also recent experimental evidence of such state in ${ }^{255} \mathrm{Rf}$ [61].

The presence of several converted transitions in prompt coincidence makes it impossible to clearly determine the internal conversion coefficients from the $\gamma$ - and X-ray intensities only. To untie this knot, the information on the conversion electrons is required. In the present work, the ICE spectroscopy of the exited states in ${ }^{251} \mathrm{Fm}$ was performed for the first time. This allowed to measure the internal conversion coefficients for the 192 keV and 200 keV transitions, to deduce the M2+E3 mixing ratio of the $5 / 2^{+}$ $\rightarrow 9 / 2^{-}$isomer decay and to set limits on the intensities and conversion coefficients of certain other transitions.

### 6.3 Experimental details

The excited states in ${ }^{251} \mathrm{Fm}$ were populated via the $\alpha$-decay of ${ }^{255}$ No produced in the two following reactions:

$$
\begin{gather*}
{ }^{48} \mathrm{Ca}+{ }^{209} \mathrm{Bi} \rightarrow{ }^{257} \mathrm{Lr}^{*} \rightarrow{ }^{255} \mathrm{Lr}+2 \mathrm{n} \rightarrow{ }^{255} \mathrm{No}  \tag{6.1}\\
{ }^{48} \mathrm{Ca}+{ }^{208} \mathrm{~Pb} \rightarrow{ }^{256} \mathrm{No}^{*} \rightarrow{ }^{255} \mathrm{No}+\mathrm{n} \tag{6.2}
\end{gather*}
$$

The cross-sections of these reactions are 440 nb and 260 nb at beam energies of 220 MeV and 214 MeV mid-target [18] respectively. In 2004 and 2005 experiments with both bismuth and lead targets were performed using the VASSILISSA separator (before the upgrade). For these experiments GABRIELA consisted of a $6 \times 6 \mathrm{~cm}^{2}$ PSD at the focal plane, 4 silicon strip detectors with 4 strips each as the tunnel and 7 phase-1 germanium detectors from the France-UK loan-pool: 1 behind the focal detector, coaxial to the beam and 6 in a ring around the focal plane. In the 2005 experiments, one of the germanium detectors from the ring was missing, leading to the decrease of the total efficiency by a factor of $\sim 1.08$. In January-February 2016 the reaction 6.1 was used in an
experiment with the SHELS separator and the GABRIELA spectrometer as described in chapter 2. However, in the 2016 experiment nearly half of the tunnel strips were not functional due to an electronics problem. Also, the threshold energies varied from 50 to 110 keV depending on the electronics chain (linear+spectroscopy amplifier). These technical problems lead to a significant decrease of the efficiency of the tunnel detector, especially below 100 keV . Nevertheless, some valuable spectroscopic data for the conversion electrons was extracted, as discussed in the following sections.

As the experimental campaigns of 2004 and 2005 were performed in practically identical conditions, the data from both years was treated as one data set. However, the 2016 set-up is different from 2004+2005 in many aspects (thresholds, efficiency, number of strips and detectors etc.), so the dataset from 2016 was treated separately.

In these experiments, several other transfermium nuclei, such as ${ }^{256} \mathrm{Lr}$ and ${ }^{251} \mathrm{Md}$ were also populated. Examples of $\alpha-\gamma$ and $\alpha$-ICE energy correlation plots are given in fig. 6.2 and 6.3 respectively. These nuclei, as well as ${ }^{255} \mathrm{No}, \alpha$ decay to excited states in their daughters, which then de-excite via $\gamma$ emission or internal conversion. As it can be clearly seen from fig. 6.3, the $\alpha$ particles that escape from the DSSD and are detected in the tunnel constitute an important background for the ICE spectroscopic measurements. In order to avoid this contamination in the ${ }^{251} \mathrm{Fm}$ prompt ICE spectra, a cut on the $\alpha$-particle energies indicated by the blue area was applied.

### 6.4 Electromagnetic transitions observed in ${ }^{251} \mathrm{Fm}$

The $\alpha-\gamma$ and $\alpha$-ICE correlation plots gated on ${ }^{255}$ No $\alpha$ decays between 7700 keV and 8160 keV are given in fig. 6.4. The x axis of the histograms is the energy of the particle ( $\gamma$ - or X-ray for the top image and ICE for the bottom image). The time difference between the detection of an $\alpha$ particle and the transition depopulating excited states in the daughter is given on the $y$ axis. A number of prompt and delayed transitions are observed. The spread in time of the prompt coincident events of $\sim 2 \mu \mathrm{~s}$ is due to the limited time-resolution of the DAQ. The delayed ones are from the decay of the $5 / 2^{+}$ isomeric state in ${ }^{251} \mathrm{Fm}$.

The half-life of the isomeric state was determined from the ICE correlations of the $2004+2005$ data. The ICE distribution was chosen as it is cleaner than the $\alpha-\gamma$ distri-


Figure 6.2: $\alpha-\gamma$ coincidence plot for the 2016 data. The red band indicates the coincidences with ${ }^{251} \mathrm{Md}$ $\alpha$ decay, cyan band ${ }^{-256} \mathrm{Lr} \alpha$ decay. The green band indicates the coincidence of ${ }^{251} \mathrm{Fm} \gamma$ rays to ${ }^{255} \mathrm{No}$ $\alpha \mathrm{s}$.


Figure 6.3: $\alpha$-ICE coincidence plot for the 2004+2005 data. The red dashed lines indicate the escaped $\alpha$-particles seen both in the PSD and in the tunnel. The green band indicates the coincidence of ${ }^{251} \mathrm{Fm}$ ICE to ${ }^{255} \mathrm{No} \alpha$ s. The blue band indicates the cut used for the prompt $\alpha$-ICE measurements.
bution, which inevitably contains background events from scattered photons. Using the ICE, the lifetime of the isomer can be fitted with a single component exponential. The result of such time distribution fit is given in fig. 6.5. We measured the lifetime of this isomer $T_{1 / 2}=23.7 \pm 1.1 \mu \mathrm{~s}$. This is in a good agreement with the $T_{1 / 2}=21.1 \pm 1.9 \mu \mathrm{~s}$


Figure 6.4: Top: the $\alpha-\gamma$ correlation plot gated on the ${ }^{255}$ No $\alpha$-decay from the 2016 data. Bottom: $\alpha-e$ correlation plot gated on the ${ }^{255}$ No from the $2004+2005$ data.

## given in [51].

The binding energies of the atomic electrons in fermium are 142 keV for the K -shell, $\sim 27 \mathrm{keV}$ for the L-shells and $\sim 7 \mathrm{keV}$ for the M -shells. Thus, the converted transitions below 142 keV are K-forbidden (do not result in the K-ICE emission), the transitions below 27 keV are L-forbidden and so forth.

### 6.5 The prompt transitions

The ICEs seen in coincidence with the ${ }^{255} \mathrm{No} \alpha$ decay mainly arise from the $192 \mathrm{keV} \mathrm{I} / 2^{+}$ $\rightarrow 5 / 2^{+}$transition (see fig. 6.1(d)). According to [51], this transition has a lifetime of 22 ns . With our $1 \mu$ s event timing resolution we cannot resolve the lifetime of this isomer and consider it a prompt transition. The $1 / 2^{+}$level is populated via direct feeding


Figure 6.5: The lifetime of the ICE from the decay of the $5 / 2^{+}$isomeric state in ${ }^{251} \mathrm{Fm}$ (from 2004+2005 data).
through an $\alpha$ branch, as well as from another $1 / 2^{+}$rotational band above it. In order to measure the conversion coefficients for this transition, a gate on the $\alpha$-energies between 7700 keV and 7920 keV , which incorporates all the transitions feeding the $1 / 2^{+}$ level, was applied. The projections of the ICE correlation spectra between $4 \mu \mathrm{~s}$ and $50 \mu \mathrm{~s}$ from the 2004+2005 and 2016 experiments are given in fig. 6.6.

The ICEs from the 192 keV transition have the following energies: $50 \mathrm{keV}, 167 \mathrm{keV}$ and 186 keV for K -, L- and M-conversion respectively. The K-conversion line mostly appears below the threshold in the 2016 data, hence no data on K -conversion electrons can be extracted from these data. Two internal transitions of 163 keV and 167 keV also populate the $1 / 2^{+}$level, and also contribute to the ICE spectra. The K-ICE of these transitions have energies of 21 keV and 25 keV respectively, so they appear below the threshold and cannot be detected. The L-ICE of both transitions appear in a peak of $\sim 140 \mathrm{keV}$.

The contribution of the 358 keV transition to the prompt ICE spectrum is very small, which means that it most likely has an E2 multipolarity. The 354 keV feeding transition is mostly excluded by our $\alpha$-energy cuts. Moreover, as the $7 / 2^{+}$state is not very populated in this experiment, and as the lowest ICE energy originating from this transition is 217 keV , it does not constitute any significant background to the rest of the observed transitions.

As the applied $\alpha$-energy cut allows for the population of a number of excited states

| $E_{\gamma}$ | N | $\epsilon$ | multipolarity | $N_{\mathrm{XK}}$ |
| :---: | :---: | :---: | :---: | :---: |
| 163 | $12 \pm 3$ | $24 \%$ | M 1 | $412 \pm 113$ |
| 167 | $9 \pm 3$ | $24 \%$ | M 1 | $288 \pm 96$ |
| 192 | $121 \pm 11$ | $22 \%$ | E 2 | $75 \pm 7$ |
| 195 | $20 \pm 4$ | $22 \%$ | M 1 | $469 \pm 103$ |
| 358 | $23 \pm 5$ | $15.8 \%$ | E 2 | $10 \pm 2$ |
| TOT |  |  |  | $\mathbf{1 2 5 4} \pm \mathbf{1 8 1}$ |
| $\mathrm{K}_{\alpha}$ | $247 \pm 16$ | $26.3 \%$ |  | $939 \pm 60$ |
| $\mathrm{~K}_{\beta}$ | $81 \pm 9$ | $25.5 \%$ |  | $318 \pm 35$ |
| TOT |  |  |  | $\mathbf{1 2 4 8} \pm \mathbf{6 9}$ |

Table 6.1: The number of K X-rays expected from the observed $\gamma$-ray intensities from 2016 data compared to the observed $\gamma$-ray intensities.
in ${ }^{251} \mathrm{Fm}$, there might be some highly converted transitions contributing to the prompt ICE spectra that have not been seen in $\gamma$ rays. In order to exclude such contributions, we have compared the observed KX-ray intensities to the number of KX-rays expected from the observed $\gamma$ ray intensities. All transitions observed in $\gamma$-rays were taken with the most likely (lowest possible) multipolarities. The results of this comparison are presented in table 6.1. As the expected number of K X -rays agrees well with the observations we may conclude that there is no other important K -conversion contribution to the ICE spectra that has not been taken into account.

In order to deduce the intensity of each ICE line, multicomponent fits were performed using the RooFit libraries [32]. Each peak is approximated with a Gaussian skewed to low energy (RooCBShape class in RooFit, see chapter 3.2). There is also a flat tail to the left of each peak, which is due to the partial energy deposition of the particle in the detector, and which constitutes a background for every other peak lower in energy. These tails are approximated with error functions starting from the mean values of the Gaussians. The ratio of the integrals of an error function and a Gaussian is related to the physical properties of the detector (thickness of the dead layer) and to the implantation depth of the ER for a given reaction, and thus remains the same for each ICE line. The results of the fits are given in fig. 6.6. The intensities of the ICE lines are given in table 6.2.

The $\gamma$ - and X-ray spectra seen in a prompt coincidence with the ${ }^{255} \mathrm{No} \alpha$-decay with the same $E_{\alpha}$ and time gates as for the ICE are given in fig. 6.7. In the 2004+2005 data,

|  | $E_{\text {ICE }}$ | $2004+2005$ |  | 2016 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | N | $\epsilon$ | N |  |
| $\mathrm{~K}(192)$ | 50 keV | $15.5 \%$ | $50 \pm 8$ | - | - |
| $\mathrm{L}(192)$ | 167 keV | $17.5 \%$ | $79 \pm 12$ | $10.4 \%$ | $76 \pm 11$ |
| $\mathrm{M}+(192)$ | 186 keV | $17.5 \%$ | $40 \pm 8$ | $10.4 \%$ | $34 \pm 8$ |
| $\mathrm{~L}(163+167)$ | $\sim 140 \mathrm{keV}$ | $17 \%$ | $21 \pm 6$ | $10.4 \%$ | $20 \pm 7$ |

Table 6.2: The energies, detection efficiencies and intensities of the prompt ICE transitions in ${ }^{251} \mathrm{Fm}$ from the experiments performed in 2004+2005 and 2016

|  | 2004+2005 | 2016 | E2[52] |
| :---: | :---: | :---: | :---: |
| $\alpha_{K}$ | $0.76(18)$ | - | $0.139(2)$ |
| $\alpha_{L}$ | $1.06(25)$ | $1.23(21)$ | $1.03(2)$ |
| $\alpha_{M}$ | $0.54(15)$ | $0.55(14)$ | $0.293(5)$ |
| $\alpha_{\text {tot }}$ | $2.36(35)$ | - | $1.57(2)$ |

Table 6.3: The comparison of the experimental internal conversion coefficients for the 192 keV transition from 2004+2005 and 2016 data compared to the theoretical E2 conversion coefficients
$31 \pm 6192 \mathrm{keV} \gamma$-rays are observed with a detection efficiency of $7.3 \%$. In the 2016 data, there are $126 \pm 11 \gamma$-rays in the 192 keV peak with a $21.2 \%$ detection efficiency.

The 64 keV line present in the 2016 data is from the contaminant ${ }^{212 \mathrm{~m}}$ At which is a transfer product in this reaction. It can be seen from fig. 6.2 that the $64 \mathrm{keV} \gamma$-rays are coincident with the wrong $\alpha$ energy to be the $11 / 2^{-} \rightarrow 9 / 2^{-}$transition.

The internal conversion coefficients for the 192 keV transition deduced from the numbers of detected $\gamma$ rays and ICE are given in table 6.3. The comparison of the experimental values to the theoretical ones [52] confirms the E2 nature [51] of this transition.

As the ICE of the 163 keV and 167 keV transitions cannot be separated, and given the relatively low statistics of these transitions, it is not possible to calculate their conversion coefficients from these data.

### 6.6 The $5 / 2^{+}$isomer

In order to select the $\alpha$ decay branches contributing to the feeding of the $5 / 2^{+}$isomeric state in ${ }^{251} \mathrm{Fm}$, the correlations with $\alpha$-decays of 7700 keV to 8160 keV were selected.


Figure 6.6: The ICE coincident to the $\alpha$-decay of ${ }^{255} \mathrm{No}\left(E_{\alpha}=7700 . .7920 \mathrm{keV}\right.$ ) from 2004+2005 data (top) and from 2016 data (bottom).

The time window was selected to be $4 \mu \mathrm{~s}$ to $50 \mu \mathrm{~s}$ in order to avoid random correlation background in the $\gamma$-ray spectra. The $\gamma$-and X-ray spectra of the isomer from the $2004+2005$ data are given in fig. 6.8 and from 2016 in fig. 6.9. In order to deduce the intensities of the lines multicomponent fits were applied. In the 2016 data, each line is fitted with a combination of a Gaussian representing the shape of the peak and an error function starting at the mean value of the Gaussian representing the Compton tail. In the 2004+2005 data, a flat background component was also added to describe the random background. The intensities obtained from the fits are given in table 6.4.

The ICE spectra of the isomer are given in fig. 6.10. As for the prompt electrons, each peak was fitted with a combination of a skewed Gaussian and an error function. In the 2016 data the thresholds were too high for the detection of the K-ICE of this transition. The determined ICE intensities from both datasets are given in table 6.5.

The internal conversion coefficients calculated from these numbers are given in


Figure 6.7: The $\gamma$-and X -rays coincident to the $\alpha$-decay of ${ }^{255}$ No ( $E_{\alpha}=7700-7920 \mathrm{keV}$ ) from 2004+2005 data (top) and from 2016 data (bottom).

|  |  | $2004+2005$ |  | 2016 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\epsilon$ | N | $\epsilon$ | N |
| $\mathrm{K} \alpha_{2}$ | 115 | $9.6 \%$ | $44 \pm 9$ | $26.5 \%$ | $115 \pm 13$ |
| $\mathrm{~K} \alpha_{1}$ | 121 | $9.4 \%$ | $98 \pm 10$ | $26.2 \%$ | $224 \pm 16$ |
| $\mathrm{~K} \beta_{1}$ | 136 | $8.8 \%$ | $20 \pm 6$ | $25.4 \%$ | $90 \pm 10$ |
| $\mathrm{~K} \beta_{2}$ | 141 | $8.6 \%$ | $23 \pm 6$ | $24.9 \%$ | $33 \pm 7$ |
| $\gamma$ | 200 | $7.1 \%$ | $14 \pm 5$ | $20.7 \%$ | $33 \pm 6$ |

Table 6.4: The energies, detection efficiencies and intensities of the $\gamma$ - and X-rays from the isomeric transition in ${ }^{251} \mathrm{Fm}$ from the experiments performed in $2004+2005$ and 2016
table 6.6. The values of these coefficients clearly indicate the mixed $\mathrm{M} 2 / \mathrm{E} 3$ nature of the 200 keV transition. The mixing ratios deduced from individual conversion coefficients are also given in table 6.6. As the error-bars on most of the measurements are


Figure 6.8: The isomeric $\gamma$-transition (left) and X-rays (right) following the $\alpha$-decay of ${ }^{255}$ No $\left(E_{\alpha}=7700-8160 \mathrm{keV}\right)$ in the 2004+2005 data.


Figure 6.9: The isomeric $\gamma$ - and X-rays following the $\alpha$-decay of ${ }^{255} \mathrm{No}\left(E_{\alpha}=7700-8160 \mathrm{keV}\right)$ in the 2016 data.

|  | $2004+2005$ | 2016 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\epsilon$ | N | $\epsilon$ | N |
| K | 58 keV | $15.5 \%$ | $266 \pm 20$ | - | - |
| L | 173 keV | $17.5 \%$ | $280 \pm 31$ | $10.4 \%$ | $134 \pm 19$ |
| M | 193 keV | $17.5 \%$ | $129 \pm 20$ | $10.4 \%$ | $61 \pm 16$ |
| $\mathrm{~N}+$ | 200 keV | $17.5 \%$ | $29 \pm 10$ | - | - |

Table 6.5: The energies, detection efficiencies and intensities of the ICE from the isomeric transition in in ${ }^{251} \mathrm{Fm}$ from the experiments performed in $2004+2005$ and 2016.
significant, as discussed in chapter (4), the mean value of the mixing ratio $<\delta>$ cannot be assumed to be equal to $\delta(<\alpha>)$. Thus the graphical method to determine $\delta$ is not applicable and the mixing ratios were calculated numerically through the convolutions of the corresponding PDFs.


Figure 6.10: The isomeric ICE following the $\alpha$-decay of ${ }^{255} \mathrm{No}\left(E_{\alpha}=7700-8160 \mathrm{keV}\right)$ 2004+2005 data (top) and 2016 data (bottom).

The mean value of the mixing ratio was calculated via the maximum likelihood approach. Each $\delta$ value was weighted by the size of the corresponding confidence interval:

$$
\begin{equation*}
<\delta>=\frac{\sum_{i} \omega_{i} \delta_{i}}{\sum_{i} \omega_{i}}=\frac{\Sigma_{i} \delta_{i} /\left(\sigma_{L i}+\sigma_{R i}\right)^{2}}{\Sigma_{i} 1 /\left(\sigma_{L i}+\sigma_{R i}\right)^{2}}, \tag{6.3}
\end{equation*}
$$

where $i=1 . .7, \omega_{i}$ is the weight of each measurement point, $\sigma_{L i}$ and $\sigma_{R i}$ are the left and right uncertainties and $\delta_{i}$ is the mean value of the mixing ratio for each measurement.

In order to determine the mean confidence interval, the right and left uncertainties of the mixing ratios were treated separately:

$$
\begin{equation*}
<\sigma_{L}>=\sqrt{\frac{1}{\Sigma_{i} \sigma_{L i}^{2}}} ; \quad<\sigma_{R}>=\sqrt{\frac{1}{\Sigma_{i} \sigma_{R i}^{2}}} . \tag{6.4}
\end{equation*}
$$

|  | $2004+2005$ | 2016 | M2 [52] | E3 [52] | $\delta(2004+2005)$ | $\delta(2016)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha_{K}$ | $8.8(31)$ | - | $14.49(21)$ | $0.227(4)$ | $0.92_{-0.37}^{+0.43}$ | - |
| $\alpha_{K X}$ | $10.2(37)$ | $11.5(23)$ |  |  | $0.81_{-0.34}^{+0.44}$ | $0.56_{-0.23}^{+0.23}$ |
| $\alpha_{L}$ | $7.9(29)$ | $8.1(19)$ | $6.75(10)$ | $11.05(16)$ | $1.20_{-0.72}^{+1.11}$ | $1.10_{-0.61}^{+0.86}$ |
| $\alpha_{\text {tot }}$ | $21.2(75)$ | - | $23.8(4)$ | $15.93(23)$ | $0.71_{-0.16}^{+0.16}$ | - |
| $\alpha_{K} / \alpha_{L}$ | $1.07(16)$ | - | $2.15(5)$ | $0.0205(4)$ | $0.80_{-0.12}^{+0.12}$ | - |

Table 6.6: The conversion coefficients and deduced mixing ratios of the 200 keV isomeric transition in ${ }^{251} \mathrm{Fm}$.

The resulting mean value of the mixing ratio is $\delta=0.76_{-0.19}^{+0.20}$ (see fig. 6.11).


Figure 6.11: The mixing ratios obtained through the different conversion coefficients measurements. The dashed blue area represents the mean confidence interval of $\delta$.

### 6.6.1 Other isomeric decays

In both the 2004+2005 and the 2016 data we have observed hints for another isomeric ICE transition. This transition could be the decay of the $5 / 2^{+}$isomeric state to the $11 / 2^{-}$ member of the ground state rotational band. From the energy of the $7 / 2^{-}$state observed in the direct $\alpha$ feeding [51], the energy of this transition should be 136 keV . Such transition would have an E3 multipolarity, and thus be highly converted. The theoretical conversion coefficients for this transition are $\alpha_{\text {tot }}=118.4(17)$ and $\alpha_{L}=82.3$ (12) [52]. The K-conversion for such a transition is energetically forbidden. The L-ICE, M-ICE and NICE would have energies of $111 \mathrm{keV}, 130 \mathrm{keV}$ and 135 keV respectively. No corresponding
$\gamma$ rays were observed in either of these data, which is natural, as the transition in question is highly converted.


Figure 6.12: The isomeric ICE following the $\alpha$-decay of ${ }^{255} \mathrm{No}\left(E_{\alpha}=7700-8160 \mathrm{keV}\right)$ from the 2004+2005 data (top) and from the 2016 data (bottom), fitted with a supplementary 110 keV component for the supposed 136 keV transition.

In order to estimate the upper limit of the branching ratio of such a transition, a 110 keV component was added to the multicomponent fits of the isomeric ICE. The energies for the L and M ICE lines were allowed to vary between 100 keV and 120 keV and between 125 keV and 140 keV respectively. The fitted maxima appear at the energies of 115 keV and 132 keV respectively in the 2004+2005 data, and 108 keV and 134 keV respectively in the 2016 data. The results of the fits are given in fig. 6.12. The intensities of the evident peaks are given in table 6.7.

From these intensities, an upper limit of the branching ratio for the $5 / 2^{+} \rightarrow 11 / 2^{-}$ transition is determined to be $\sim_{1} \%$. Given the small branching ratio, more statistics would be required to confirm the existence of this 136 keV transition.

|  | $2004+2005$ | 2016 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\epsilon$ | N | $\epsilon$ | N |
| L | $\sim 111 \mathrm{keV}$ | $17.5 \%$ | $15 \pm 3$ | $10.4 \%$ | $15 \pm 5$ |
| M | $\sim 130 \mathrm{keV}$ | $17.5 \%$ | $13 \pm 7$ | $10.4 \%$ | $9 \pm 3$ |

Table 6.7: The detection efficiencies and intensities of the supposed 136 keV isomeric ICE transitions in ${ }^{251} \mathrm{Fm}$ from the experiments performed in 2004+2005 and 2016

### 6.7 Physical interpretation

### 6.7.1 The low-lying $5 / 2^{+}$level

The assignment of the $9 / 2^{-}$spin and parity to the ground state in ${ }^{251} \mathrm{Fm}$ comes from the systematics for the $\mathrm{N}=151$ isotones. This ground state configuration was identified in ${ }^{247} \mathrm{Cm}$ [62] and ${ }^{253} \mathrm{No}$ [63], and also indirectly in ${ }^{249} \mathrm{Cf} \alpha$-decaying to ${ }^{245} \mathrm{Cm}$ which is known [62] to have a $7 / 2^{+}$[624] ground state. The Nilsson diagram of the neutron orbitals involved in this region is given in fig. 6.13.

For the $N=149$ isotones in this region the ground state is $7 / 2^{+}[624]$, and the first excited state is $5 / 2^{+}$[622] [65]. $N=147$ isotones have a $5 / 2^{+}$[622] [66] ground state. Thereby the first excited single-particle state in $\mathrm{N}=151$ isotones, which have one more occupied level, should be $7 / 2^{+}$[624]. However, in ${ }^{251} \mathrm{Fm}$ a low-lying $5 / 2^{+}$level is observed below the $7 / 2^{+}[624]$ at an excitation energy of 200 keV . The spin and parity of this level are deduced from the decay properties of the isomer: an M2/E3 multipolarity implies that the level should be of opposite parity and be $\pm 2$ units of angular momentum. Thus it can be either $5 / 2^{+}$or $13 / 2^{+}$. As there is no $13 / 2^{+}$orbital present in the region, and also as the $\alpha$ decay from $1 / 2^{+}$ground state in ${ }^{255}$ No to a $13 / 2^{+}$state would be very hindered, the low-lying state has to involve the $5 / 2^{+}[622]$ orbital. Similar behaviour can be traced in the ${ }^{247} \mathrm{Cm}$ to ${ }^{253} \mathrm{No} \mathrm{N}=151$ isotones (see fig 6.17). There is also recent experimental evidence of a $5 / 2^{+}$isomer in ${ }^{255} \mathrm{Rf}$ [61], although a firm assignment is yet to be established. The obtained value of the M2/E3 mixing ratio and the measured lifetime of the 200 keV isomeric transition allow the corresponding Weisskopf single-particle


Figure 6.13: Single-particle spectra of ${ }^{250} \mathrm{Fm}$ for protons (top) and neutrons (bottom) obtained with SLy4 interaction, taken from [64]. The vertical grey bar indicates the range of ground-state deformations predicted for this and neighboring nuclei; the coloured orbitals are the ones involved in the $2^{-}$ octupole vibration.
estimates for the transition strengths in ${ }^{251} \mathrm{Fm}$ to be determined:

$$
\begin{gather*}
B(M 2)=3.0 \cdot 10^{-3} \pm 0.6 \cdot 10^{-3} \text { W.u.; }  \tag{6.5}\\
B(E 3)=17.9 \pm 6.0 \text { W.u.. } \tag{6.6}
\end{gather*}
$$

As the measured $B\left(E_{3}\right)$ strength is very large, it is likely that the low excitation energy of the $5 / 2^{+}$state is related to the presence of octupole correlations.


Figure 6.14: Systematics of the excited states in $\mathrm{N}=151$ isotones, taken from [58-61, 67] and this work.

### 6.7.2 $\mathrm{N}=150$ isotonic chain

The existence of this collective effect was first suggested by Yates et al. [68] who observed a $K^{\pi}=2^{-}$phonon coupled to the ground states in ${ }^{248} \mathrm{Cf}$ and ${ }^{249} \mathrm{Cf}$ via transfer reactions. The octupole vibration mainly arises from the interplay of the levels coming from the $g 9 / 2$ and $j 15 / 2$ neutron shells having $\Delta j=\Delta l=3$ (see fig. 6.16). The $5 / 2^{+}$[622] and $9 / 2^{-}$[734] orbitals stemming from these shells give rise to an octupole phonon. There is a similar occurrence on the proton side with $7 / 2^{+}[633]$ and $3 / 2^{-}$[521] orbitals stemming from the proton shells $f 7 / 2$ and $i 13 / 2$ respectively. It turns out that the lowest octupole phonon has $\mathrm{K}^{\pi}=2^{-}$. This phonon yields a low-lying $2^{-}$collective state in the even-even $\mathrm{N}=150$ isotonic chain from ${ }^{246} \mathrm{Cm}$ to ${ }^{252} \mathrm{No}$. The systematics of the experimental 2- energies is given in fig. 6.15.

The $2^{-}$phonon has not yet been observed in ${ }^{254} \mathrm{Rf}$. However, the observation of an $893 \mathrm{keV} \gamma$-ray in coincidence with a cascade of converted transitions reported by David et al. [70] may be the signature of the presence of this collective excitation. They interpret it as a high-K isomer decay followed by one or several converted transitions of $\sim 450 \mathrm{keV}$, however, the K-isomer may as well be decaying to the $2^{-}$state emitting the ICE. The 893 keV transition would then correspond to the 799 keV line in ${ }^{246} \mathrm{Cm}$ [72], the 593 keV line in ${ }^{248} \mathrm{Cf}$ [68], the 834 keV line in ${ }^{250} \mathrm{Fm}$ [73] and 883 keV line in ${ }^{252} \mathrm{No}$ [69]. If the transition is $2^{-} \rightarrow 2^{+}$, the excitation energy of the $2^{-}$state would be $\sim 940 \mathrm{keV}$, as the $2^{+}$must be $\sim 45 \mathrm{keV}$ above the ground state. Such estimate well agree with the systematics (see fig. 6.15). If (like e.g. in ${ }^{252} \mathrm{No}$ ) the most intense transition is between


Figure 6.15: Systematics of the $K^{\pi}=2$ - collective excited state in $N=150$ isotones. Experimental values (blue) taken from [69, 70]; the QPM calculations with Nilsson potential [69] are given in red; selfconsistent QRPA calculations with DıM parametrization of Gogny interaction by I. Deloncle and S. Peru are given in violet; QPM calculations with the Wood-Saxon (WS) potential from [71] are given in green.
the higher members of the $2^{-}$and ground state bands, the excitation energy is not easy to estimate.


Figure 6.16: A schematic diagram of the proton and neutron orbitals active around ${ }^{251} \mathrm{Fm}$. The $\mathrm{Z}=100$ and $\mathrm{N}=152$ gaps come from the Wood-Saxon calculations (see e.g. [6]). The orbitals and asymptotic Nilsson labels are indicated on the right in black, the spherical shell model labels - on the left in blue.

In fig. 6.15 the experimental measurements of the excitation energy of the $2^{-}$state are compared to theoretical calculations. From the experimental values (traced in blue) it is clearly visible that the excitation energy of the $2^{-}$level is almost constant for $\mathrm{Z}=96$ 102 with the exception of a noticeable kink in ${ }^{248} \mathrm{Cf}$. In californium the proton single-

|  |  | Nilsson | Gogny |  |  | Wood-Saxon |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{E}^{*}, \mathrm{keV}$ | $\mathrm{E}^{*}, \mathrm{keV}$ | $\pi, \%$ | $\nu, \%$ | $\mathrm{E}^{*}, \mathrm{keV}$ |
| ${ }^{246} \mathrm{Cm}$ | 842 | 1000 | 1030 | 28 | 72 | 949 |
| ${ }^{248} \mathrm{Cf}$ | 593 | 800 | 920 | 34 | 66 | 612 |
| ${ }^{250} \mathrm{Fm}$ | 881 | 890 | 1000 | 28 | 72 | 1061 |
| ${ }^{252} \mathrm{No}$ | 930 | 1150 | 1115 | 18 | 82 | 998 |

Table 6.8: Theoretical calculations for the $2^{-}$vibrational state in $\mathrm{N}=150$ isotones. QRPA calculations from [69] with Nilsson potential; self-consistent QRPA calculations by I. Deloncle and S. Peru [77] with the D1M parametrization of the Gogny interaction, $\pi$ and $\nu$ are the proton and neutron content of the phonon respectively; QPM calculations with the Wood-Saxon potential from [71].
particle states $7 / 2^{+}$[633] and $3 / 2^{-}$[521] are near the Fermi surface, and in ${ }^{249} \mathrm{Bk}$ [74] it is known that they are nearly degenerate. Thus the proton collective component is more pronounced in ${ }^{248} \mathrm{Cf}$ than in other members of the $\mathrm{N}=150$ isotonic chain. This is clear evidence that the shell gap occurs at $Z=100$ as given e.g. in [6] and not at $Z=98$, as it is with the Gogny DiM parametrization (see e.g. fig. 6.13). The same may also be concluded from the masses and from the energies of the $2^{+}$states in this region. Different theoretical calculations for the $2^{-}$collective state are given in table 6.8. The QRPA calculations by Robinson et al. [69] are performed with the Nilsson potential within the theoretical framework described in [75]. The calculations performed by Jolos et al. [71] were carried out with the Wood-Saxon potential using the quasiparticle-phonon model (QPM). The QRPA calculations by I. Deloncle and S. Peru were performed with the HFB approximation with the Gogny DiM parametrization [76]. Since the shell gap appears at $Z=98$ with the Gogny interaction, unlike Nilsson and Wood-Saxon potentials where it appears at $Z=100$, the dip in the excitation energy of the $2^{-}$level at $Z=98$ is not as pronounced in these calculations. In the QPM calculations with the shell gap at $\mathrm{Z}=100$, the 2qp-proton component in the $2^{-}$state is $62 \%$ and the $2 q p$-neutron component is 16\% [7]].

It is obvious from fig. 6.15 that none of the theoretical calculations reproduce the experimental excitation energies. However, the absolute value of the excitation energy highly depends on the parametrization of the nucleon-nucleon interaction and a small shift in the parameters of the order of $2 \%$ may result in an energy shift of a few hundred keV [69]. Thus, it is more important for the calculations to follow the general trend of the experimental values. In this respect the QPM calculations reproduce the behaviour
of the $2^{-}$level more closely, despite the energy shift by $\sim 200 \mathrm{keV}$.

### 6.7.3 $\mathrm{N}=151$ isotonic chain

In the $N=151$ isotonic chain, the coupling of the $2^{-}$phonon to the single-particle $9 / 2^{-}$[734] ground state yields a vibrational $5 / 2^{+}$state, which in turn interacts with the $5 / 2^{+}$[622] single-particle level yielding two mixed states one of which carries more of a singleparticle component and the other larger phonon component. Thus the $5 / 2^{+}[622]$ level acquires a collective octupole admixture (hence the strong octupole component in the decay) and gets pushed down in energy, becoming the first excited state in the $\mathrm{N}=151$ isotones (see fig. 6.17). It is also suggested in [71] that the $5 / 2^{+}$[622] single-particle level should lie closer to the $7 / 2^{+}[624]$ level than it appears from the Wood-Saxon potential.


Figure 6.17: Systematics of the $5 / 2^{+}$level in $\mathrm{N}=151$ isotones. Experimental values (blue) taken from $[58-60,67]$ and this work; QRPA calculations by I. Deloncle and S. Peru with DiM parametrization of the Gogny interaction [77] are plotted in red; QPM calculations with Wood-Saxon (WS) potential from [78] are given in green.

The same pattern is observed in doubly-magic ${ }^{208} \mathrm{~Pb}$ region, where a similar configuration of $g 9 / 2$ and $j 15 / 2$ neutron shells gives rise to existence of $\mathfrak{3} 3^{-}$octupole phonon yielding a $3^{-}$first excited state in ${ }^{208} \mathrm{~Pb}$ [79]. In ${ }^{209} \mathrm{~Pb}$ the phonon couples to the 9/2+ ground state yielding a $15 / 2^{-}$level which mixes and repels with the single-particle 15/2(see fig. 6.18). The higher-lying $5 / 2^{+}$level carrying a larger fraction of the $9 / 2^{-}[734] \otimes 2^{-}$ component was not observed in $\mathrm{N}=151$ isotones, probably due to the low $\alpha$ branching.


Figure 6.18: a) the $3^{-}$phonon in ${ }^{208} \mathrm{~Pb}$ and ${ }^{209} \mathrm{~Pb}$, taken from [79]; b) $2^{-}$phonon in ${ }^{250} \mathrm{Fm}$ and ${ }^{251} \mathrm{Fm}$.

|  | $\mathrm{E}_{\text {exp }}^{*}, \mathrm{keV}$ | $\mathrm{B}(\mathrm{E} 3)_{\exp }$ | $\mathrm{E}_{\text {theor }}^{*}, \mathrm{keV}$ | $\pi, \%$ | $\nu, \%$ | $\mathrm{~B}(\mathrm{E} 3)_{\text {theor, }}$ W.u. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{247} \mathrm{Cm}$ | 227 | 8 W.u. | 611 | 15 | 88 | 9.5 W.u. |
| ${ }^{249} \mathrm{Cf}$ | 145 | 11 W.u. | 534 | 18 | 92 | 11 W.u. |
| ${ }^{251} \mathrm{Fm}$ | 200 | 18 W.u. | 590 | 13 | 87 | 9 W.u. |
| ${ }^{253} \mathrm{No}$ | 168 | 14 W.u. | $(1029)$ | $?$ | $?$ | $?$ |

Table 6.9: Preliminary results of self-consistent calculations with DiM parametrization of Gogny interaction by I. Deloncle and S. Peru [77] for the $5 / 2^{+}$state in $\mathrm{N}=151$ isotones. $\pi$ and $\nu$ are the proton and neutron content of the phonon respectively. The excitation energy $E_{\text {theor }}^{*}$ is given for the $9 / 2^{-}[734] \otimes 2^{-}$level without taking into account mixing with the quasiparticle levels. The excitation energies and strengths are from [58-60] and this work.

Figure 6.17 shows the comparison between the experimental $5 / 2^{+}$excitation energy and theoretical calculations. The QRPA predictions by I. Deloncle and S. Peru based of the HFB calculations with the Gogny DiM interaction do not take into account the interaction of the $9 / 2^{-} \otimes 2^{-}$collective $5 / 2^{+}$state with the single-particle $5 / 2^{+}$[622] level and thus only give the excitation of the phonon-induced state. The behaviour of the phonon according to these calculations seems to follow the trend of the experimental values for $\mathrm{Z}=96-100$, but fails at $\mathrm{Z}=102$. One should remark that these the preliminary results for the $\mathrm{N}=151$ isotones are the current "state of the art", as this is the first time QRPA calculations with a Gogny interaction have been performed in heavy odd-A nuclei. Transition strengths were also calculated, though they also do not account for mixing with quasiparticle states, and thus only provide an upper limit for the strengths that can be measured experimentally. Nevertheless the predicted $B(E 3)_{\text {theor }}$ (see table 6.9) are of the same order or smaller than the experimental values and which therefore requires further investigation.

|  | $\mathrm{E}_{\text {exp }}^{*}, \mathrm{keV}$ | $\mathrm{E}^{*}, \mathrm{keV}$ | Structure |
| :---: | :---: | :---: | :---: |
| ${ }^{247} \mathrm{Cm}$ | 227 | 647 | $5 / 2^{+}[622] 85 \%+9 / 2^{-}[734] \otimes 2^{-} 10 \%$ |
|  | $?$ | 1096 | $9 / 2^{-}[734] \otimes 2^{-} 86 \%+5 / 2^{+}[622] 10 \%$ |
| ${ }^{249} \mathrm{Cf}$ | 145 | 613 | $5 / 2^{+}[622] 77 \%+9 / 2^{-}[734] \otimes 2^{-} 18 \%$ |
|  | $?$ | 982 | $9 / 2^{-}[734] \otimes 2^{-} 79 \%+5 / 2^{+}[622] 18 \%$ |
| ${ }^{251} \mathrm{Fm}$ | 200 | 630 | $5 / 2^{+}[622] 88 \%+9 / 2^{-}[734] \otimes 2^{-} 8 \%$ |
|  | $?$ | 1099 | $9 / 2^{-}[734] \otimes 2^{-} 87 \%+5 / 2^{+}[622] 9 \%$ |
| ${ }^{253} \mathrm{No}$ | 168 | 597 | $5 / 2^{+}[622] 89 \%+9 / 2^{-}[734] \otimes 2^{-} 7 \%$ |
|  | $?$ | 1099 | $9 / 2^{-}[734] \otimes 2^{-} 86 \%+5 / 2^{+}[622] 8 \%$ |

Table 6.10: Results of QPM calculations with the Wood-Saxon potential [78] for the $5 / 2^{+}$states in $\mathrm{N}=151$ isotones.

The QPM calculations (see tab. 6.10) take the interaction of all quasiparticle states into account. According to these calculations, the excitation energies for the $5 / 2^{+}$states consisting mostly of the phonon component are $\sim 400 \mathrm{keV}$ above the $5 / 2^{+}$levels with $\sim 10 \%$ of the phonon component. The maximum of the phonon mixing is at ${ }^{249} \mathrm{Cf}$, where, as for ${ }^{248} \mathrm{Cf}$, the vibration is most pronounced. As it was mentioned before, though the predicted excitation energies are $\sim 300 \mathrm{keV}$ higher then the experimental ones, a small change of the parameters may result in a significant variation of these values, and thus the general trend is more important. These calculations reproduce the general tendency of the behaviour of the $5 / 2^{+}$level observed experimentally (see fig. 6.17), though the predicted variation of the excitation energy for ${ }^{249} \mathrm{Cf}$ is less strong than the experimentally observed one. One of the possible explanations is that in the odd nuclei the relative positioning of the quasiparticle levels plays a bigger role than in the evens, and thus a slight change in level placing, e.g. closer relative location of $5 / 2^{+}$[622] and $7 / 2^{+}$[624] [71], might solve this problem.

### 6.7.4 Summary

To conclude, the particle-phonon interaction plays an important role in the structure of nuclei in the region near the $\mathrm{Z}=100$ and $\mathrm{N}=152$ shell gaps. The $\mathrm{M} 2 / \mathrm{E} 3$ nature of the transition de-exciting the isomer in ${ }^{251} \mathrm{Fm}$ was firmly established via $\gamma$ - and ICEspectroscopy. The preliminary results of the self-consistent calculations for the $\mathrm{N}=151$ isotonic chain were compared to the Wood-Saxon calculations in order to better un-
derstand the underlying structure of the $5 / 2^{+}$isomer. More data is needed in order to explore the full range of the region where the octupole correlations play a role. In particular, a firm assignement for the $5 / 2^{+}$isomer in ${ }^{255} \mathrm{Rf}$ is needed. The mixing ratio of the $5 / 2^{+}$isomer decay needs to be measured. It would also be interesting to search for the higher-lying $5 / 2^{+}$levels which contain a larger fraction of the collective component that are predicted for this isotonic chain. On the theoretical side, the interaction of the phonon with the quasiparticle state has to be included in the self-consistent calculations calculations. It would also be interesting to compare the experimental $B\left(E_{3}\right)$ values to the QPM calculations. In fact, these calculations are currently being performed by our Russian colleagues.

For the $\mathrm{N}=150$ isotones, the calculations with the Gogny interaction are compared to the Nilsson and Wood-Saxon based ones. All three calculations reproduce the general trend for these isotones to an extent, though the location of the shell gap at $\mathrm{Z}=100$ proves to be important to fully reproduce excitation energy dip in ${ }^{248} \mathrm{Cf}$. As discussed in subsection 6.7.2, there is vague experimental evidence of the $2^{-}$collective state in ${ }^{254} \mathrm{Rf}$. More data is needed to check this assumption. Theoretical calculations for both ${ }^{254} \mathrm{Rf}$ and ${ }^{255} \mathrm{Rf}$ would also be of great value.

This octupole vibration should also be active in the odd-Z berkelium, einsteinium and mendelevium nuclei in this region, though no such observations have been made so far.

## 7

## Conclusions and perspectives

A large fraction of the work of this thesis was dedicated to the commissioning of the SHELS separator and GABRIELA spectrometer. Some of these test reactions allowed to not only determine the values for the transmission of SHELS, but also to obtain certain interesting results in spectroscopy and spectroscopic techniques.

In particular, the effect of the presence of rear-face DSSD strips with broken bondings has been investigated with the example of the $\alpha$ decay of ${ }^{209-211}$ Ra produced in the ${ }^{50} \mathrm{Ti}+{ }^{164} \mathrm{Dy}$ reaction. A method to restore the front-face spectra without any loss of statistics by applying special conditions on the front-face VS back-face correlations is proposed. The conversion coefficients for the $96.5 \mathrm{keV}, 578 \mathrm{keV}, 602 \mathrm{keV}, 604 \mathrm{keV}$ and 751 keV transitions following the decay of a $8+2.1 \mu$ s isomeric state in ${ }^{210} \mathrm{Ra}$ were measured with the $\gamma$ and ICE intensity ratios for the first time. The obtained values allowed us to confirm the previously proposed E2 multipolarities of these transitions.

The fine structure of the $\alpha$ decay of ${ }^{221} \mathrm{Th}$ produced in ${ }^{22} \mathrm{Ne}+{ }^{206} \mathrm{~Pb}$ reaction and the summing in the DSSD of the $\alpha$ particles with the ICE from internal transitions in ${ }^{217} \mathrm{Ra}$ was studied by comparing the experimental spectra to Geant 4 simulations. The 8243 keV line was proved to be a new $\alpha$ decay branch, tentatively assigned to the decay of the ground state in ${ }^{221} \mathrm{Th}$ to the $7 / 2^{+}$excited state in ${ }^{217} \mathrm{Ra}$. The conversion coefficients for the $226 \mathrm{keV}, 331 \mathrm{keV}, 422 \mathrm{keV}$ and 751 keV electromagnetic transitions in ${ }^{217} \mathrm{Ra}$ are
suggested. This analysis is about to be finalized and prepared for publication. During this work, a significant bug in the Geant 4 class simulating radioactive decays, namely in the calculation of $\alpha$ energies, was found and reported to the Geant 4 community.

A novel graphical method for the extraction of mixing ratios for the mixed nuclear transitions was developed, and its limits of applicability, as well as some other aspects of the calculation of mixing ratios from the experimental conversion coefficients were carefully studied. This work has been submitted to the Nuclear Instruments and Methods A journal.

A significant fraction of the work for this thesis was dedicated to the tests of the digital electronics for the upgrade of GABRIELA DAQ system. The performance of the digital ADCs from Nutaq and NI manufacturers was evaluated on the basis of the ENOB on the baseline of each digitizer and the energy resolutions for the data acquired with the semiconductor detectors and standard calibration sources. The performance of these digitizers was also compared to that of the TNT digitizer and to the results from analogue electronics, both of which were used as a reference. Currently, the tender for the purchase of new digital electronics is closed and the final evaluation tests are being conducted. An intermediate semi-digital design of the DAQ setup for the first stage of the upgrade was also proposed, and its feasibility has been proved.
$\gamma$ ray and ICE spectroscopy of the excited states in ${ }^{251} \mathrm{Fm}$ populated by $\alpha$ decay of ${ }^{255}$ No allowed the measurement of conversion coefficients for the 192 keV and 200 keV electromagnetic transitions. The evidence of a new $5 / 2^{+} \rightarrow 11 / 2^{-}$transition in the decay of the $5 / 2^{+}$isomer has been observed in the ICE spectra, and the limit on the branching for this decay has been set. The M2/E3 mixing ratio for the decay of the $5 / 2^{+}$isomer to the ground state was measured to be $\delta=0.76_{-0.19}^{+0.20}$. The transitions strengths $B(E 3)=17.9(60) \mathrm{W} . u$. and $B(M 2)=3.0(6) \cdot 10^{-3} \mathrm{~W}$. u. have also been extracted. The particlephonon mixing in the $5 / 2^{+}$state in $N=151$ isotopes is responsible for its enhanced $B(E 3)$ strength as well as for the lowering of its excitation energy. This phonon is traced in the experimental data in the other members of $\mathrm{N}=151$ isotonic chain as well as in the $\mathrm{N}=150$ isotones. The experimental data are compared to preliminary results from self-consistent QRPA calculations with the Gogny interaction, a QPM model with the Woods-Saxon potential and QRPA based on the Nilsson potential.

This work may be continued by acquiring more experimental data for other nuclei
in this region, namely ${ }^{254} \mathrm{Rf}$ and ${ }^{255} \mathrm{Rf}$ to continue the $\mathrm{N}=150$ and 151 isotonic chains and the odd-Z ${ }^{247} \mathrm{Bk},{ }^{249} \mathrm{Es}$ and ${ }^{253} \mathrm{Md}$. It is also possible that the presence of octupole correlations is responsible for other phenomena, such as for example the lowering of first 2+ energies and masses in $\mathrm{Z}=98$ isotopes [80]. Further advance on the theory side, namely the QPM calculations for the transition strengths in the $\mathrm{N}=151$ isotones and the full self-consistent QRPA calculations are expected shortly.

## A

## MatLab-Simulink model for the NUTAQ <br> digitizer

In this appendix the data acquisition developed for the tests of the Nutaq MI-125 digitizer is presented. The software was developed within MatLab-Simulink framework making use of Xilinx blocks [49]. The details of the DSP tests may be found in chapter 4 of this thesis. The green blocks are input and output registers ans board control blocks. The blue-grey boxes are Xilinx blocks. The white boxes contain custom-made sub-programs folded into them.

The programme main control screen contains the input registers and the board control units, the memory buffer block, the trigger block and the data formatting block and MatLab signal generator. The trigger block contains the median filter for smoothing the signal before triggering and rising- and falling-edge detectors for triggering with positive- and negative-polarity signals respectively. The memory buffer block serves to delay the data by the requested number of ADC cycles. The formatting block adds a header and formats the data for writing to disk.






Figure A.6: The data formatting block of the MatLab-Simulink programme for the data acquisition with NUTAQ PicoDigitizer.

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Titre: Structure des noyaux les plus lourds: spectroscopie du noyau ${ }^{251} \mathrm{Fm}$ et developpement pour des traitements numériques du signal
Mots clés: physique nucléaire, structure nucléaire, éléments lourds, spectroscopie, états excitées, traitement numérique du signal
Résumé: L'un des principaux défis de la physique nucléaire moderne est de comprendre la structure nucléaire des éléments les plus lourds. Les barrières de fission calculées dans le modèle de la goutte liquide macroscopique ne parviennent pas à expliquer la stabilité des noyaux avec un nombre de protons $Z \geq 90$. Cette barrière disparaît pour les éléments transfermium ( $\mathrm{Z} \geq 100$ ) qui ne sont donc stabilisés que par des effets quantiques de couche. Les noyaux lourds sont un laboratoire unique pour étudier l'évolution de la structure nucléaire dans des conditions extrêmes de masse et de champ Coulombien. Bien que de nombreuses théories s'accordent sur l'existence d'un «îlot de stabilité », les prédictions sur son emplacement exact en terme de nombre de protons et neutrons varient grandement. Les études expérimentales des noyaux transfermium s'avèrent donc essentielles pour contraindre les modèles théoriques et mieux comprendre l'évolution des couches nucléaires.

L'interaction entre la particule indépendante et les degrés de liberté collectifs dans le noyau ${ }^{251} \mathrm{Fm}$ a été étudiée par la combinaison de la spectroscopie d'électrons de conversion interne (ECI) et spectroscopie du rayonnement $\gamma$. Les états excités du ${ }^{251} \mathrm{Fm}$ ont été peuplés dans la décroissance $\alpha \mathrm{du}{ }^{255} \mathrm{No}$, produit dans deux réactions de fusion-évaporation suivantes: ${ }^{208} \mathrm{~Pb}\left({ }^{48} \mathrm{Ca}, 1 \mathrm{n}\right){ }^{255} \mathrm{No} \mathrm{et}{ }^{209} \mathrm{Bi}^{48}{ }^{48} \mathrm{Ca}$, $2 \mathrm{n})^{255}$ Lr. Les expériences ont été réalisées au JINR, FLNR, Dubna. Les faisceaux intenses ont été délivrés par le cyclotron U-400, et les séparateurs VASSILISSA ou SHELS ont été utilisés pour sélectionner les résidus de fusion-évaporation. Le spectromètre GABRIELA a été utilisé pour effectuer des mesures des propriétés de décroissance caractéristique corrélées en temps et en position pour isoler les noyaux d'intérêt. La spectroscopie d'électrons de conversion interne $\mathrm{du}^{251} \mathrm{Fm}$ a été réalisée pour la première fois. Ces mesures ont permis d'établir les multipolarités de plusieurs transitions et de quantifier le rapport de mélange M2/E3 dans la désintégration de l'isomère $5 / 2+$. Le $B\left(E_{3}\right)$ valeur extraite est comparée à celles des autres membres de la chaîne isotonique $N=151$ et à des calculs théoriques.

Au cours de ce travail, une nouvelle méthode graphique d'extraction des rapports de mélange de transitions nucléaires a été développé. Cette méthode intuitive et illustrative et ses limites d'application, ainsi que certains aspects du calcul des rapports de mélange au-delà de ces limites, sont décrites et discutées.

Les détecteurs silicium double-face à strips (DSDS) sont largement utilisés en spectrométrie nucléaire, en particulier au plan focal de séparateurs pour détecter l'implantation et la désintégration ultérieure des noyaux les plus lourds. Il a été constaté que la présence de strips mécaniquement déconnectés sur une face du DSDS peut conduire à l'apparition de pics d'énergie abaissée sur la face opposée en raison de la variation de la capacité totale. Cet effet, ainsi que les méthodes de correction du spectre, ont été étudiés et discutés. L'utilisation de simulations GEANT4 pour résoudre les effets de sommation $\alpha$-ECI dans le DSDS et pour contraindre les coefficients de conversion interne des transitions impliquées dans la désexcitation du noyau d'intérêt est présentée à l'aide de l'exemple du ${ }^{221} \mathrm{Th}$.

Une bonne partie des travaux ont été consacrés à la $R \& D$ pour un nouveau système électronique numérique pour le spectromètre GABRIELA et aux tests comparatifs de plusieurs cartes d'acquisition numériques. Les résultats de ces tests, ainsi que les algorithmes de traitement numérique du signal mis en œuvre pour une analyse non biaisée hors ligne sont présentés.

Title: Structure of the heaviest nuclei: spectroscopy of ${ }^{251} \mathrm{Fm}$ and digital signal processing development
Key words: nuclear physics, nuclear structure, heavy elements, spectroscopy, excited states, digital signal processing
Abstract: One of the major challenges of modern nuclear physics is to understand the nuclear structure of the heaviest elements. Fission barriers calculated within the macroscopic liquid drop model fail to explain the stability of nuclei with a number of protons $\mathrm{Z} \geq 90$. Transfermium elements $(Z \geq 100)$ have a vanishing liquid-drop barrier and are solely stabilized by quantum shell effects. They provide a unique laboratory to study the evolution of nuclear structure under the extreme conditions of large mass and strong Coulomb force. Although many theories agree on the existence of an "Island of Stability", the predictions on its exact location in terms of number of protons and neutrons vary greatly. Hence a systematic study of transfermium nuclei is essential to constrain theoretical models and to obtain a better understanding of the evolution of nuclear shells.

The interplay between single-particle and collective degrees of freedom in ${ }^{251} \mathrm{Fm}$ was investigated by means combined internal conversion electron (ICE) and $\gamma$-ray spectroscopy. Excited states in ${ }^{251} \mathrm{Fm}$ were populated via the $\alpha$-decay of ${ }^{255}$ No produced in the two following fusion-evaporation reactions: ${ }^{208} \mathrm{~Pb}\left({ }^{48} \mathrm{Ca}, 1 \mathrm{n}\right){ }^{255} \mathrm{No}$ and $\left.{ }^{209} \mathrm{Bi}^{48} \mathrm{Ca}, 2 \mathrm{n}\right){ }^{255} \mathrm{Lr}$. The experiments were performed at the FLNR, JINR, Dubna. The intense beams were delivered by the U-400 cyclotron and the separators VASSILISSA or SHELS were used to select fusion evaporation residues. At the focal planes of these separators the GABRIELA spectrometer was used to perform a time and position correlated measurement of the characteristic decay properties to further isolate the nuclei of interest. The ICE spectroscopy of ${ }^{251} \mathrm{Fm}$ was performed for the first time. These measurements allowed to establish the multipolarities of several transitions in ${ }^{251} \mathrm{Fm}$ and to quantify the $\mathrm{M}_{2} / \mathrm{E}_{3}$ mixing ratio in the decay of the low-lying $5 / 2+$ isomer. The extracted $\mathrm{B}\left(\mathrm{E}_{3}\right)$ value is compared to those found in other members of the $\mathrm{N}=151$ isotonic chain and to theoretical calculations.

During this work a novel graphical method of extracting mixing ratios for nuclear transitions was developed. This intuitive and illustrative method and it's limits of applicability, as well as certain aspects of the calculation of mixing ratios beyond these limits, are described and discussed.

Double-sided silicon strip detectors (DSSD) are widely used in nuclear spectrometry, in particular at the focal plane of separators to detect the implantation and subsequent decay of the heaviest nuclei. It was found that the presence of mechanically disconnected strips on one face of the DSSD leads to the occurrence of lower energy peaks on the opposite face due to the change of the total capacitance. This effect, along with the methods of restoring the correct spectra, has been studied and discussed. The use of GEANT4 simulations for resolving $\alpha$-ICE summing in the DSSD and for constraining the internal conversion coefficients of the transitions involved in the decay of the nucleus of interest is presented with the example of ${ }^{221} \mathrm{Th}$.

A significant part of the thesis work was dedicated to the R\&D for a new digital electronics system for the GABRIELA spectrometer and to the comparative tests of several digital acquisition cards. The results of these tests, as well as the digital signal processing algorithms implemented for an unbiased off-line analysis are presented.


[^0]:    *The resolutions quoted by the manufacturer were $1.8-1.95 \mathrm{keV}$ on a test bench

[^1]:    *It is inconceivable that the calibration was incorrect by 5 keV at energies so close to those of the X-rays. In the ORNL annual report the gamma-ray spectrum does not extend high enough to show the " 187 keV " line. Since the text is ambiguous, it is highly likely that the transition energy was assigned from alpha energy differences.

