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Monetary regimes and macroeconomic stabilization policies within the European economic area : a theoretical and empirical analysis

Romain Legrand

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THÈSE

pour l'obtention du grade de

DOCTEUR DE L'UNIVERSITÉ DE CERGY-PONTOISE

Discipline : Sciences économiques

Présentée et soutenue publiquement

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Romain LEGRAND

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Régimes monétaires et politiques conjoncturelles de stabilisation

dans l'espace économique européen:

une analyse théorique et empirique

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Résumé

La mise en place de l'Euro en 1999 a constitué un événement économique majeur pour les Etats européens. La crise financière de 2007, puis la crise de la dette souveraine en 2010, ont amené à remettre en question la pérennité de la zone Euro, et la capacité de certains de ses membres à respecter leurs engagements vis-à-vis de la monnaie unique. Les mesures d'austérité mises en oeuvre au sein de l'Union Economique et Monétaire dans le contexte actuel de crise peuvent constituer pour certains Etats une tentation supplémentaire pour quitter la monnaie unique et recouvrer leur indépendance monétaire et fiscale. Une sortie de la zone Euro de la Grèce, voire d'autres Etats membres en difficulté (Portugal, Irlande, Italie, et Espagne) n'est aujourd'hui plus un scénario à exclure. Cette thèse se propose de considérer la question du régime monétaire optimal, régime de change flexible ou union monétaire, pour les 17 pays de la zone Euro, dans le cadre des crises financières et de dettes souveraines qui les affectent actuellement. Le premier chapitre est général et vise à démontrer formellement la survenue d'une rupture structurelle due au passage à la monnaie unique en 1999. Il montre qu'une telle rupture s'est bien produite pour les pays de la zone Euro autour de l'année 1992, qui a marqué l'adoption du traité de Maastricht et la mise en place des critères de convergence pour l'adoption de l'Euro. Cette rupture n'est pas partagée par les trois pays européens qui ont préservé leur monnaie (Royaume-Uni, Suède, et Danemark). Le second chapitre constitue le coeur de ce travail. Il présente le modèle de référence utilisé pour mener la comparaison entre les deux régimes monétaires considérés pour la zone Euro. Il s'agit d'un modèle à deux pays intégrant des rigidités financières dans le cadre des transactions interbancaires conclues entre les Etats membres. Le modèle, une fois étalonné pour la zone Euro, suggère que les rigidités financières peuvent jouer un rôle considérable dans la dynamique de ces Etats, les chocs affectant les économies partenaires pouvant contribuer de manière significative à la dynamique nationale. Les simulations numériques préliminaires de crise financières menées sur le modèle ne permettent pas d'apporter de réponse concluante quant aux performances des deux régimes monétaires envisagés, le régime de change flexible semblant amener une stabilité accrue, là où une union monétaire permet une récupération plus rapide suite à la crise initiale. Le dernier chapitre remplit un double objectif. Il propose d'abord un critère de bien-être formel pour l'évaluation des performances respectives des deux régimes considérés. Il développe également un certain nombre d'extensions au modèle de référence, afin d'intégrer la dette souveraine, et les politiques de crédit (Covered Bonds Purchase Programme et Securities Markets Programme) mises en place par la BCE depuis le début de la crise. Les résultats montrent qu'en l'absence de politiques interventionnistes de la part de la Banque Centrale Européenne, une grande majorité des Etats de la zone Euro (15 sur 17) bénéficieraient d'un plus haut niveau de bien-être dans un régime de change flexible. Toutefois, les conclusions s'inversent dans le cadre du Securities Markets Programme, où les Etats membres deviennent alors majoritairement favorables au régime d'union monétaire. Cela suggère que la BCE a un rôle à jouer au sein de l'espace monétaire Européen qui va au-delà de sa fonction première d'instigatrice de la politique monétaire.

Mots clés: Union Economique et Monétaire, monnaie unique, régime de change flexible, rupture structurelle, modèles d'équilibre général en économie ouverte, rigidités financières, crise financière, crise de dette souveraine, bien-être, Banque Centrale Européenne, politiques exceptionnelles de crédit.

Abstract

The introduction of the Euro currency in 1999 represented a major event for the European economies. The 2007 financial crisis and the subsequent 2010 sovereign debt crisis have led to question the sustainability of the Euro area and the capacity of certain member states to fulfil their commitments with respect to the single currency. The numerous austerity plans implemented within the Economic and Monetary Union in the current context of crisis constitute additional arguments for certain states to leave the single currency and retrieve their fiscal and monetary independences. It is not unconceivable anymore for countries such as Greece, Portugal, Ireland, Italy and Spain (the PIIGS) to envisage exiting the Euro area. This thesis considers the issue of determining the optimal monetary regime – flexible exchange rates or monetary union – for the 17 Eurozone countries, accounting for the current financial and sovereign debt crises. Chapter 1 is general and aims at formally establishing the occurrence of a structural break attributable to the 1999 passage to the single currency. It shows that such a break did take place for Euro area countries around 1992, the year which marked the adoption of the Maastricht Treaty and the settlement of the convergence criteria for the Euro. This break is not shared by the three European States which chose to preserve their own currencies (the United Kingdom, Sweden and Denmark). Chapter 2 constitutes the core of this work. It introduces the benchmark model used to perform the comparison between the two monetary regimes considered for the Euro area. It features a two-country open-economy model integrating financial frictions through cross-border interbank markets. Once calibrated for the Euro area, the model suggests that financial rigidities may play a substantial role in the dynamics of Eurozone economies, with a potentially significant impact of shocks affecting the partner economies over national developments. Preliminary financial crisis simulations run on the model prove inconclusive to assess the performances of the two monetary regimes contemplated. On the one hand, the flexible exchange rate regime results in improved stability, but on the other hand the monetary union typically allows for faster recovery following the initial crisis trigger. The third and final chapter meets a double purpose. It first proposes a formal welfare criteria to assess the respective performances of the two monetary regimes under consideration for the Euro area. It then augments the benchmark model with a number of extensions, so as to integrate sovereign debt and the diverse credit policies (Covered Bonds Purchase Programme and Securities Markets Programme) implemented by the ECB since the beginning of the crisis to the basic framework. The results show that absent credit policies, a vast majority of Euro area members (15 out of 17) would enjoy higher welfare levels under a flexible exchange rate regime. These conclusions nevertheless reverse under the Securities Markets Programme, where a majority of member states then favour the monetary union. This suggests that the ECB has a role to play for the Euro area which goes beyond its primary function of monetary policy maker.

Key words: Economic and Monetary Union (EMU), monetary union, flexible exchange rate regime, structural break, open economy general equilibrium models, financial frictions, financial crisis, sovereign debt crisis, welfare, European Central Bank, exceptional credit policies.

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Notations used in chapter 2 and 3

Parameter	Interpretation	First occurrence in equation
α	Capital share of output	(2.44)
β	Household discount rate	(2.15)
γ	Relative size of Home	(2.4)
γ_1	Debt parameter on taxation	(3.9)
γ_2	Output parameter on government expenditure	(3.10)
δ	Depreciation rate of capital	(2.35)
ε	Portfolio adjustment cost/interest premium	(2.18)
ζ	Probability for a firm not to update its price	(2.49)
η	Risk aversion parameter for consumption	(2.16)
θ^h, θ^f	Fraction of assets that may be diverted	(2.57)
θ_1	SMP parameter on sovereign debt	(3.46)
θ_2	SMP parameter on private debt	(3.49)
ϑ^h, ϑ^f	Share of output dedicated to government expenditure	(2.34)
ϖ	Inflation dynamics parameter	See Appendix B.2
ι	Capital adjustment cost parameter	(2.39)
κ	Markup parameter	(2.1)
μ	Home/Foreign bundles elasticity of substitution	(2.3)
ν^h, ν^f	Trade openness parameter	(2.4)
ξ^h, ξ^f	Transfer rates to new bankers	(2.96)
ρ_a	Autoregressive parameter for TFP process	(2.45)
ρ_m	Autoregressive parameter for monetary process	(2.87)
ρ_k	Autoregressive parameter for capital quality process	(2.36)
σ	Survival rate of bankers	(2.56)
σ_a	Standard error of productivity shocks	See calibration
σ_m	Standard error of monetary shocks	See calibration
σ_k	Standard error of capital quality shocks	See calibration
τ^h	Taxation rate	(3.9)
ϕ	Inverse Frisch elasticity of labour supply	(2.16)
$\phi_\pi^h, \phi_\pi^f, \phi_\pi$	Taylor rule parameter on inflation	(2.86)
$\phi_y^h, \phi_y^f, \phi_y$	Taylor rule parameter on output gap	(2.86)
φ	CBPP parameter	(3.34)
φ^h, φ^f	Bias towards domestic goods	(2.3)
χ^h, χ^f	Utility weight of labour	(2.16)
$\bar{\psi}^h, \bar{\psi}^f$	Steady-state value of capital quality process	(2.36)
ω	Fraction of diverted assets that creditors may recover	(2.57)
\bar{A}^h, \bar{A}^f	Steady-state value of TFP	(2.45)
Γ	Welfare loss, shock-specific criterion	(3.4)
Δ	Risk aversion parameter for government expenditure	(3.6)
Φ	Welfare loss, general criterion	(3.1)

Variable	Interpretation	First occurrence in equation
Y_t^h, Y_t^f	Output	(2.43)
C_t^h, C_t^f	Total consumption	(2.3)
$C_{H,t}^h, C_{H,t}^f$	Consumption of Home bundle	(2.1)
$C_{F,t}^h, C_{F,t}^f$	Consumption of Foreign bundle	(2.2)
K_t^h, K_t^f	Productive capital	(2.35)
L_t^h, L_t^f	Labour	(2.15)
I_t^h, I_t^f	Investment	(2.35)
G_t^h, G_t^f	Government expenditure	(2.32)
T_t^h, T_t^f	Taxation	(2.17)
W_t^h, W_t^f	Wage	(2.17)
$\Psi_t^h, \Psi_t^f, MC_t^h, MC_t^f$	Marginal cost	(2.49)
$P_{H,t}, P_{F,t}$	Producer Price Index	(2.7)
$\pi_{H,t}, \pi_{F,t}$	Consumer Price Index	(2.50)
$P_{H,t}^*, P_{F,t}^*$	Optimal reset price	(2.49)
P_t^h, P_t^f	PPI inflation	(2.9)
π_t^h, π_t^f	CPI inflation	(2.23)
ε_t	Exchange rate	(2.9)
$R_{t+1}^h, R_{t+1}^f, R_{t+1}$	Riskless interest rate	(2.17)
R_{t+1}^i	Interbank interest rate	(2.53)
R_{t+1}^d	Interest rate on sovereign debt	(3.8)
Λ_t^h, Λ_t^f	Stochastic discount factor	(2.26)
Q_t^h, Q_t^f	Price of capital	(2.40)
B_t^h, B_t^f	Interbank bonds	(2.52)
B_t^d	Sovereign debt bonds	(3.8)
B_t^c	Covered bonds	(3.34)
S_t^h, S_t^f	Assets	(2.52)
$D_t^h, D_t^f, F_t^h, F_t^f$	Riskless deposits	(2.17)
N_t^h, N_t^f	Net worth	(2.52)
Z_t^h, Z_t^f	Dividend	(2.46)
v_t^h, v_t^f	Shadow price of deposits	(2.60)
$v_{b,t}^h, v_{b,t}^f$	Shadow price of interbank loans	(2.60)
$v_{d,t}^h$	Shadow price of sovereign debt	(3.19)
$v_{c,t}^h$	Shadow price of covered bonds	(3.40)
$v_{s,t}^h, v_{s,t}^f$	Shadow value of assets	(2.60)
ϕ_t^h, ϕ_t^f	Leverage ratio	(2.66)
x_t^h, x_t^f	Interbank funds share in assets	(2.65)
Ω_t^h, Ω_t^f	Stochastic marginal value of net worth	(2.67)
$\theta_{1,t}^h$	SMP sovereign debt intermediation	(3.46)
$\theta_{2,t}^h$	SMP private asset intermediation	(3.49)
A_t^h, A_t^f	Total Factor Productivity	(2.44)
ψ_t^h, ψ_t^f	Capital quality process	(2.35)
$\zeta_t^h, \zeta_t^f, \zeta_t$	Monetary process	(2.86)

Introduction

Fifty years ago, the idea of a universal currency would have seemed better in theory than in practice, and his author would have been deemed to live in the realm of dreams rather than in the one of reality. Nowadays, the European nations are sufficiently aware of their preeminent interests so that such an idea, relying on evident truths, may attract their attentions.

Here are these evident truths: money is nothing but a merchandise. Trade between nations must be eased and enhanced by every possible means, and it would indeed be eased and enhanced, should these nations share similar weights and measures; but it would be even more enhanced if at the same time, a universal currency was introduced.

Dietrich Hermann Hegewish, German historian, jurist and diplomat,
Kleine Schriften, 1786.

The Euro constitutes the greatest achievement of the European integration process. Since its introduction in 1999, it has become the common currency of 18 European States, representing a total of 332 million European citizens using it for their daily transactions. Fourteen years after its creation, a question which may naturally arise is whether this single currency succeeded in improving the life of European citizens. Until recently, the answer may have seemed deceptively simple. The prospect of a unified currency for Europe arose several centuries ago, and its actual fulfilment took place over the last 50 years, perceived as an economical progress as well as a political necessity. Its settlement in 1999 hence constituted the outcome of a long process, resulting in much hope and enthusiasm. Support towards the Euro proved strong during the early years: over the 1999-2005 period, more than two third of Euro users were favourable to a European Monetary Union with one single currency, and more than one half found the Euro advantageous (Deroose et al., 2007). The 2008 financial crisis contributed to qualify this perspective. As the subprime crisis plunged Europe into a recession never seen since the Great Depression of the 1930's, a unified monetary policy appeared more and more as a constraint for the most severely affected economies. With the subsequent sovereign debt crisis, it became clear that the default of a single state could jeopardize the very existence of the single currency, obliging many governments to implement harsh fiscal reforms entailing serious social consequences.

Despite more than 50 years of development of economic theory relative to optimal currency areas, the gradual settlement of a Monetary Union in Europe, and nearly 15 years of coexistence with the single currency, it is still not clear whether the benefits of the Euro outweigh its costs. The issue remains all the more inconclusive since research has to these days mainly focused on monetary unions under regular conditions, largely excluding exceptional circumstances like the current financial and sovereign debt crises. This thesis proposes an answer to this problem. Using structural econometric methods, it estimates the nature of the structural changes brought by the settlement of the single currency. Relying on the tools provided by the DSGE methodology and the recent developments in international and financial macroeconomics, it empirically assesses the performances of the Eurozone countries under the alternative regimes of floating exchange rates or of a monetary union.

I.1. Currency unions in Europe

I.1.1. Early experiments

The idea of a currency union is certainly not new in Europe¹. The first occurrence of a common currency goes back to the Carolingian dynasty, through the economical and monetary reforms initiated by Pepin and pursued by Charlemagne until 801 to counter the lack of a unified set of laws, weights and currencies within the empire. A new currency, the silver penny, was created and served as the unique currency over the entire imperial territory. Its principal objective was to ease trade and tax collection. The governmental authorities re-established the public character of currency issuance, attributing to the State the monopolistic prerogative of minting new coins. Coinage was defined over determined quantities of silver based on a new weighting system, subdividing coins into denarius, solidus and pounds. Furthermore, the values were matched with the values of Muslim coinage to encourage trade. The system steadily declined throughout the feudal period as the gap between the nominal value of coins and the quantity of metal that they effectively contained gradually widened. Eventually, the uncertainty prevailing over the real value of coins was such that only the system of account subsisted.

¹ For useful historical references, the reader is referred e.g. to Bekerman (2001), Patat (2005) or Gerbet (2007).

During the ensuing medieval periods, merchants were greatly penalized by this monetary uncertainty. To guard against those difficulties, some of them started to issue their bills in a specific currency of payment during fairs. Certain cities then generalized the practice, defining a common currency to be used for their trade operations. Thus, the Wendish cities of the Hanseatic League established in 1379 a common type of thaler, closely followed by certain Rhenish princedoms in 1386. These unions, however, remained relatively exceptional initiatives in the context of a European continent torn apart by the Hundred Years' War.

The first attempt of an international monetary union is due to Charles V, ruler of the Holy Roman Empire. The monetary regulation of November 10th, 1524 replaced the local currencies of the German part of the empire by the mark of Cologne. However, the initiative aroused fierce protest, so that the project had to be abandoned. Further promulgations, by Charles V in 1551 and by his successor Ferdinand in 1559 with his Münzedikt ended in a similar failure.

Along these limited realisations, the first intellectual movements towards a universal currency started to emerge. By the end of the 16th century, the mathematician Simon Stevin from Bruges devised a system aiming at harmonizing weights, measures and currencies according to some decimal basis. His project echoed the renewed desire of certain poets of his time to achieve the antique dream of a universal model: Jean Stigel, friend of Luther, declaims: *“Unum pondus, mensura una, moneta sit una, Status et illoesus totius orbis erit.* One weight, one measure, one single currency, and the entire world will be stable.” Similarly, the landgrave Philippe of Hesse composes these verses as a wish: *“Hatten wir alle einen Glauben/Gott und Gerechtigkeit vor augen/Ein Gewicht, Maas, Munz und Geld/Dann stünde es besser in dieser Welt.* Were we all to share only one faith/one justice and God before the eyes/same weights, measures, coins and currencies/ better would be the state of the world”. The first technical project for a universal currency, however, is owed to the Italian economist Gasparo Scaruffi in his treaty “L'Alitinonfo”, published in 1579. The author pleads for a coinage realised only from ingots, with a unique weight and a unique value, in order to eliminate the hazards linked to variability and counterfeiting. A metallic parity between gold, silver and copper would allow to constitute a consistent and unified system of currencies intended to improve the transparency of contracts and ease calculations. Nevertheless, the propositions of the Alitinonfo did not meet much success with the Italian and European

sovereigns, the German princedoms favouring monetary pluralism, while France and the United Kingdom were trying to protect their own currencies rather than opening to new ones.

I.1.2. Modern currency unions

The first step towards actual monetary integration was taken in the 19th century by the numerous kingdoms and territories composing the German confederation. The division of the confederation into 35 princedoms and four independent cities entailed considerable disparities in terms of weights, units of measurement, currencies and tariffs. Because this situation was extremely damageable not only for internal trade but also for international trade, discussions took place as soon as 1820 to establish a unification project within the confederation. The completion of the process took place in March 1833 with the creation of the “Zollverein” or German Customs Union, an agreement taken between Prussia and 25 other kingdoms to set a common market with a unique tariff. Most importantly, article 14 of the convention establishing the Zollverein provided that the contracting entities were to find an immediate agreement concerning the issue of weights, measures and currencies. Following, the Munich convention of 1837 consecrated the creation of a common guilder for the southern states, while the 1838 Dresde convention instituted the Prussian thaler as the unique currency for the other states. A fixed parity between the two currencies was determined. A monetary union was created, but not yet a single currency. This remained all the more true since the joining of Austria in 1856 after the Vienna conference introduced a third currency in the system, the Austrian guilder. The system shortly outlived the 1871 German unification until the eventual creation of the mark in 1873.

Despite the significance of the German experience, the most important episode of a monetary union prior to the Euro remains by far the Latin Monetary Union. The latter was set in the context of the 1860's crisis of the bimetallic system, due to speculative movements over silver after the discovery of gold in Australia and California. The remarkable longevity of this union (1865-1927) contrasts with its relatively modest initial ambitions. Indeed, its primary intent was merely to define a set of technical rules to guarantee the inter circulation of coins between the participating countries (Belgium, France, Greece, Italy and Switzerland), and restore a smooth functioning of bimetallism.

The convention establishing the union was laying the basis of a European bimetallic system based on gold and silver. It provided a set of rules to harmonize coinage across its members: the predominant gold and silver coins of each country were to share the same weight, size, and quantity of metal. In addition, the legal tender of silver coins in each State had to be compliant with the one set in the other countries. Intrinsically, the Latin Union did not constitute a considerable step. First, a *de facto* union existed already prior to it, as the participating countries were all using the French Germinal Franc. Also, because all coins were backed by the same quantity of metal, accepting foreign coins at par value with the national coins was no major concession.

The union met difficulties from its early years. As soon as 1870, the bimetallic system was endangered again, due this time to a relative depreciation of silver following the discovery of new mines in Nebraska and technological innovations rendering extraction easier. France and Belgium first limited minting of silver coins, before a full cessation of silver coining took place in 1878 in all the participating countries. The Union remained a bimetallic system in theory, but became in fact a gold-based monometallic device, silver coins being totally depreciated. The situation could only deteriorate in the following years: gold demand increased sharply as more countries adopted the Gold Standard, especially Germany in 1871 and the United States in 1879. Combined with the continuous rise in silver supply and the economic disruptions caused by World War 1, the Union could not preserve its coherency and was finally disbanded in 1927.

Other experiences of monetary unions are noteworthy. The Scandinavian Monetary Union, directly inspired from the Latin Union, was established in 1873 between Denmark, Norway and Sweden. This Union not only led to the creation of a common coin, the krone, but also allowed conversion at par of bank notes between countries. Because coins of higher values (10 and 20 kronas) were made of gold, while coins of lesser values (2 kronas and below) were made of common metals, the gradual generalisation of the Gold Standard and the rise of the gold value made it more and more difficult to sustain the convertibility of smaller coins. By 1931, the three States had abandoned gold convertibility, disbanding *de facto* the Union.

The Belgium-Luxembourg Economic Union (BLEU), signed by the two countries in 1921 comprised the settlement of a currency union by fixing the parity between the Belgian and Luxembourgian francs. Despite parity revisions in 1935 and 1944, and occasional tensions

such as in 1982 when Belgium decided a unilateral devaluation, the Union carried on until 1999, date of introduction of the Euro.

Finally, it is interesting to mention the small yet successful currency union established by the State of Liechtenstein with Switzerland. By the law of May 26th, 1924, Liechtenstein officially adopted the Swiss franc as its currency. The union was hence decided unilaterally. The partnership with Switzerland nevertheless went smoothly as Liechtenstein adopted a cooperative behaviour, adapting its legislation and banking norms to the Swiss model. In 1980, the currency union was formalized by a treaty between the two countries.

I.1.3. The way to the Euro

World War 2 left the European continent ravaged and disorganised. An urgent need for political and economical reorganisation arose, in Europe but also for the rest of the world. As a first step to support economic reconstruction, the Organisation for European Economic Cooperation (OEEC) was created in 1948. Its objectives were to guarantee an even distribution of the financial aid provided by the United States to European countries through the Marshall plan, foster economic cooperation among these countries, and promote free trade.

Three years later, the French Minister to Foreign Affairs Robert Schuman proposed to institute a supra-national management of the European production of coal and steel. Six countries (France, Germany, Italy, Belgium, Luxembourg and the Netherlands) ratified the treaty, hence creating the European Coal and Steel Community (ECSC). At the economical level, the ECSC settled a common market for coal and steel, neutralising competition between European countries over those resources. At a political level, the ECSC served as a means of unification between European nations.

On the strict monetary side, realisations between European countries over this period have remained limited. The main reason lies in the adoption of the Bretton Woods system in 1944. Each country had to establish a fixed parity in terms of the reserve currency (i.e. the dollar) or gold, and maintain its exchange rate within a band of 1% around that parity, by intervening on foreign exchange markets if necessary. The question of currency convertibility was hence solved de facto. There was, still, some progress towards monetary cooperation. The European

Payments Union, set in 1951, established a multilateral system of international reserve adjustment between 17 OEEC members, a notorious improvement with respect to the former system of bi-lateral agreements. It also created the first common unit of account, the European Currency Unit, based on a gold weight, similarly to the dollar.

The major advance in the European integration process intervened with the Rome Treaty of 1957, which gave birth to the European Economic Community (EEC). The EEC was to bring economic integration among its six founding members, with the objective of constituting a common market over a twelve year transition period. The Treaty also laid specific provisions with respect to the convertibility of European currencies between them. Currency convertibility, however, was already granted by the Bretton Woods system, so that these provisions did not constitute a major adjustment. Hence, in 1958, the European Monetary Agreement successfully replaced the European Payment Union, restricting the band of fluctuation around the dollar to 0.75%. Also, and most importantly, the European Monetary Agreement marked the return to convertibility: currencies could now be freely traded on the exchange market, and benefitted from a guaranteed convertibility. Trade in capital was made significantly easier. By the end of the 1950's, a first form of currency unification was achieved in Europe, the Bretton Woods system acting as a stability warrant.

The situation would gradually deteriorate during the 1960 decade. Persistent current account deficits in the US, partly due to military expenses, contributed to feed the overall dollar supply in the rest of the world. The expansion of the eurodollar market magnified the problem. The anchor currency of the Bretton Woods mechanism was now threatening the stability of the whole system because of its overabundance. Widespread inflationary pressures in Europe, combined with divergences of economic policies and monetary crises (British pound in 1967, French franc in 1968) further contributed to render the fixed parity system less and less sustainable.

A series of initiatives was then taken to consider the transition to a European currency union. The Marjolin Memorandum of 1962 represents the first explicit reference to a monetary union, the memorandum stating that the stabilisation of exchange rates between EEC members should lead to the creation of a single European currency. Under the provisions of the memorandum, a Committee of Governors of the EEC national central banks was created in 1964. The plan proposed in 1968 by Raymond Barre goes one step ahead by proposing new

measures towards economical and monetary integration. It prescribed the definition of objectives for the evolution of the main aggregates of each country (output, unemployment, price levels...), assorted by active cooperation between the ministers of economy to reach these objectives and achieve better economic convergence. Concerning monetary collaboration, the plan established a mechanism of mutual credit assistance, for short and medium term debt. At the La Haye summit of 1969, the Heads of States formally endorsed this orientation, and decided to “undertake everything necessary to achieve economic and monetary integration”. The conclusion of the process was the publishing of the Wermer report in 1970, and that proposed to achieve monetary unification by 1980, through the irrevocable determination of parity between European currencies, and the settlement of a common system of central banks.

The project was short-lived, however. On August 15th 1971, the dramatic shrinking of US gold reserves led President Nixon to suspend the gold to dollar convertibility, marking the demise of the Bretton Woods system. The Baale Agreement concluded in 1972 among EEC members constituted an attempt to retrieve a form of stability. It established the “snake in the tunnel” principle, by which European currencies were allowed to deviate from their dollar parity by 2.25% (the tunnel), with a maximum deviation between intra-European parities of 2.25% (the snake). Further, as fixed parities implied interventionism of national central banks to sell or buy reserves, the European Monetary Cooperation Fund was created in 1973 to manage payment compensations between member states. With the determination of fixed exchange rates and a preliminary form of central bank, much had been accomplished in just a few months to direct Europe towards a monetary union .

Yet, the United States proved unsuccessful in maintaining the dollar at its defined parity. In 1973, the dollar became floating, the snake was henceforth evolving without a tunnel. Simultaneously, sharp movements of capital and strong inflationary pressures exacerbated by the first oil shock were inducing wide exchange rate fluctuations. The snake system was less and less sustainable, causing EEC countries to defect it successively. By the end of 1978, it was reduced to a “Deutschmark area” comprising five countries: Germany, Belgium, the Netherlands, Luxembourg and Denmark. In 1976, the Jamaica Agreement consecrated the floating exchange rate regime.

In 1979, the process of monetary integration was revamped with the founding of the European Monetary System (EMS). Its basic elements were: the definition of a European Currency Unit (ECU) as a basket of currencies; an Exchange Rate Mechanism, acting as an improved snake system allowing for periodic parity adjustments with respect to the ECU; and a formal framework for cooperation between member states, comprising the partial collectivisation of reserves and a system of mutual credits allowing to delay reserve payments to other member states.

The Single European Act (1986) set as an objective the establishment of a Common Market (free circulation of merchandises, services, people and capital) by 1992. The reformed legislative process within the EEC, especially the introduction of the cooperation procedure and the extension of qualified majority voting, allowed for a rapid adaptation of national legislations. Following the success of the Common Market, the monetary union appeared as the natural next step towards integration. The idea was appealing on many respects. A monetary union would consolidate the stability brought by the SME. It would help to maintain budgetary discipline and low inflation rates, and avoid bad economic practices such as competitive devaluations. It would consolidate the common market by improving price transparency. Finally, and not least, it would constitute a factor towards political unification. Consequently, the Delors report of 1989 advocated the settlement of a European Monetary Union (EMU) leading to a single currency which would replace the ECU. The transition would follow a three-stage process. The first stage would entail tighter cooperation between central banks. The second stage would consist in the settlement of a European System of Central Banks (ESCB) and the gradual transfer of decision-making on monetary policy to supranational institutions. Finally, stage 3 would mark the irrevocable fixing of conversion rates of the national currencies before their replacement by the single currency. The signature of the Maastricht Treaty in 1992 created the European Union, and opened the way to the realisation of stage 2. It also laid five convergence criteria for the adoption of the single currency, consisting mainly in minimal performance requirements relative to inflation, public deficits and sovereign debt. On December 31st 1998, the Euro was officially introduced in 11 European Union countries.

I.1.4. The Euro after its introduction

The Euro had now been officially introduced, initiating the transition phase which would result, by 2002, in the replacement of national notes and coins. The question then arose whether the states which had taken an opt-out (Denmark, Sweden and the United Kingdom) should eventually decide to join the Eurozone. The referendums respectively held in 2000 and 2003 by Denmark and Sweden led to the rejection of the single currency. Greece, on the other hand, successfully joined the Euro in 2001.

Meantime, the beginnings of the Euro proved difficult. As the Eurozone enjoyed poorer economic performance than the US, lacked coordinated fiscal policies and suffered from irresolute monetary policy from the ECB, the currency continuously depreciated over the 1999-2002 period. From a parity of 1.17 dollar in January 1999, it depreciated to 0.90 dollars in 2002, with a trough at 0.82 in 2000. The trend yet reversed from 2002 onward. The United States then opted for a weak dollar strategy, cutting interest rates and running budget deficits to enhance their economy. As a result, the Euro steadily appreciated, getting close to 1.3 dollars in 2004. But while Euro sceptics were mocking a weak Euro, a strong Euro was now reproached to harm exports and hinder the recovery of European economies. Over 2002 and 2003, in a context of strong currency and low economic growth, France and Germany were unable to fulfil their obligations with respect to the Stability and Growth Pact and ran deficits over 3%. The two countries benefited from a suspension of the Pact in 2003, a decision that was rejected in 2004 by the Court of Justice of the European Union. A reform of the Stability and Growth Pact in 2005 relaxed the rules of the procedure for excessive deficit. This seemed necessary as more countries were struggling to meet their obligations: by 2006, 12 European Union States were in excessive deficits, including five Euro area members (Germany, France, Italy, Greece and Portugal). Greece and Italy were also displaying alarming levels of sovereign debt, amounting to 110% of GDP for Greece in 2004 and 108% of GDP for Italy in 2006. This was just a flavour, yet, of the incoming crises.

I.1.5. The bank crisis

In 2007, liquidity problems started to surface in both the United States and Europe. U.S Home prices had started to decline, and assets tied to U.S mortgages became questionable in value.

As the U.S. housing market collapsed, banks in the United States and in Europe incurred large losses, and growing uncertainty about the quality of their assets made it increasingly difficult for them to borrow. In the United States, the subprime crisis culminated with the bankruptcy of Lehman Brothers in September 2008. In Europe, the high degree of interdependence between banks led the initial crisis to spread over the whole banking sector. As even wealthy credit institutions were not able anymore to borrow and lend funds on the interbank market, the situation of many banks was getting critical, entailing a serious risk of generalised bankruptcy. National governments and central banks then adopted several alternative approaches to settle the issues of troubled assets and recapitalization needs. Some banks were nationalised, such as Northern Rock in the United Kingdom, or Anglo Irish Bank in Ireland. Others were sold to a third party, like the German Sachsen LB or the financial conglomerate Fortis. But most of them benefitted from large public recapitalization plans and government bailouts: in October 2008, the UK government announced a 50 billion pounds fund for the recapitalization of ailing banks; the same month, Germany established the Financial Market Stabilization Fund (SoFFin), authorized to provide loan guarantees up to 400 billion Euros, while Austria, France, Greece and Portugal respectively announced the allocation of 100, 40, 28 and 20 billion Euros for similar motives. Banks also benefitted from individual bailouts and recapitalization, like the Dutch ING group or the Dexia group.

As for the ECB, it undertook a number of action to try to ensure liquidity in the interbank market. In 2008, the ECB first decided to anticipate liquidity needs and extend the maturity of its loans to Euro area banks. From September onwards, it adopted a fixed rate tender procedure with full allotment for all of its weekly main refinancing operations. It also increased the number and frequency of refinancing operations, and temporarily extended the list of assets eligible for use as collaterals. By 2009, however, it had become clear that these measures were insufficient to renew available liquidities on the interbank market. Hence in May, the ECB announced the activation of the Covered Bond Purchase Programme (CBPP), a 60 billion euro plan of covered bond purchase designed to restore liquidity on this market.

I.1.6. The sovereign debt crisis

The Euro area, along with most of the world, was emerging from recession in 2009, and growth started again. Yet a second crisis was on the way to hit Europe. The introduction of

the Euro had led Eurozone countries of different credit worthiness to receive similar and low interest rates for their bonds and private credit over the 2002-2008 period. Consequently, creditors in countries with originally higher interest rates enjoyed more favourable credit terms, which stimulated private and government spending and created an economic boom. Concurrently, many governments had concealed part of their deficit and debt levels through diverse techniques, including inconsistent accounting, off-balance sheet structures and complex credit derivative structures. In November 2009, Greek Prime Minister Papandreou admitted that Greece's annual budget deficit would be 12.7% of GDP, more than twice the previously announced figure. Following, Greece sovereign credit rating was downgraded. This marked the beginning of the sovereign debt crisis, with causes varying by country. In Greece, the massive increase in indebtedness is mainly connected to fiscal irresponsibility: the government had increased its commitment to public workers with extremely generous pensions and wage benefits, the latter having doubled over ten years. The role of strong trade imbalance has also been stressed, countries like Ireland, Portugal, Spain and Italy having made use of cheap funds to increase consumption and consumptive investments rather than investing them to increase productivity. In Ireland and Spain particularly, the property bubbles are mainly to be blamed, private debts being transferred to sovereign debt as a result of banking system bailouts. It is actually likely that a significant part of the general debt increase observed in the Eurozone is imputable to the large bailout packages provided to the financial sector during the crisis and the low economic growth prevailing since then, rather than to excessive public welfare spending. Still, the Eurozone by the end of 2012 has been characterized by deep recession, with a group of weaker economies (Portugal, Ireland, Italy, Greece and Spain or PIIGS) suffering from high deficits and levels of indebtedness, severe credit rating downgrading and high sovereign debt spreads. Despite a series of austerity packages (2009 for Ireland, 2009 and 2010 for Greece, 2010 for Spain, Portugal and Italy), it became evident that the PIIGS could not be saved from bankruptcy without external intervention. Greece was first to agree on a 110 billion euro bailout plan, jointly set in April 2010 by the Eurozone nations and the IMF. Ireland, Portugal, Spain and Cyprus also requested and received bailout plans over the 2009-2012 period. The European Union also adopted a set of emergency measures, with the settlement of the European Financial Stability Facility (EFSF), a fund providing loans to Eurozone states in difficulty up to a 440 billion dollar capacity jointly guaranteed by the Eurozone governments. Similarly, the European Financial Stabilisation Mechanism (EFSM) provided funds raised on financial markets under the supervision of the Commission, using the Union budget as collateral. The European

Central Bank also multiplied exceptional measures. As the pressing issue was the alarming risk on the sovereign debt of the PIIGS, the ECB initiated in 2010 a Securities Market Program (SMP) to purchase government bonds on secondary markets. Overall, the SMP covered a nominal amount of 218 billion Euros, distributed among Italy (102 billion), Spain (44 billion), Greece (34 billion), Portugal (23 billion) and Ireland (14 billion). In August 2012, the SMP was terminated and replaced by a programme of Outright Monetary Transactions (OMTs), also aimed at acquiring sovereign bonds. In addition, the ECB launched a second CBPP program in 2011 to enhance liquidity on the interbank market.

By the time this work is written (August 2013), great uncertainty remains about the Euro. No one can predict whether Greece will be able to remain within the Single currency, although considerable efforts are made by the central authorities to support its economy. More generally, it is difficult to assess whether the Eurozone will succeed in preserving its cohesion, or if certain members will eventually leave it, by economic necessity or by choice.

I.2. Theory of optimal currency areas

I.2.1. Historical overview

The settlement of the Euro appeared as the natural continuation of the European integration process initiated after World War 2. Concurrently to this process, an economic theory relative to optimal currency areas (OCA) has been progressively developed to determine whether such currency areas could be optimal and under which conditions. This literature is now briefly presented. For a more comprehensive treatment, see Mongelli (2002) and Mongelli (2008).

Adopting the terminology of Mongelli (2002), one can distinguish four main periods, or phases, in the advancement of OCA theory. The first phase, or “pioneering phase”, from the early 1960's to the early 70's, initiated the analysis of the benefits and costs from monetary integration. It enabled to highlight a certain number of characteristics, or prerequisites, that a State had to satisfy to profitably integrate a monetary union. Friedman (1953) highlights the trade-off between domestic price adjustments and exchange rate adjustments. When the domestic prices and wages of partner countries enjoy a high degree of flexibility, the terms of

trade quickly adapt after a shock, which reduces the need of compensation through the exchange rate, or by unemployment if the latter is not possible. Highly flexible prices thus plead in favour of currency unions. In his seminal contribution, Mundell (1961) stresses the importance of factor mobility: if partner countries are characterized by highly integrated markets for factors of production, including labour, then factor movements may reduce the need for an adjustment by the exchange rate or the terms of trade in response to disturbances, making a monetary union more attractive. While labour mobility may be limited in the short run e.g. because of migration costs, it may be higher in the long run, implying better adjustments to permanent shocks (Corden (1972)). McKinnon (1963) proposes an alternative criterion based on trade openness: a higher degree of openness induces a faster transmission of international price changes on the domestic cost of living. This contributes to reduce money and exchange rate illusion for domestic residents. Similarly, as a devaluation is transmitted faster to exported goods, its intended effect will be partly negated. The nominal exchange rate is therefore rendered less useful as an adjustment instrument for highly opened economies. Kenen (1969) deals with the issue of diversification in production and consumption. With a high diversification in production, imports and exports, the effect of a shock on a specific sector is more likely to be diluted if the other sectors remain unaffected. Therefore, diversification provides insulation against a variety of disturbances, diminishing the use of the nominal exchange rate as a factor of adjustment.

Aside these major contributions, a variety of additional criteria have been underlined during the pioneering phase to characterize OCAs. Ingram (1973) argues that financial market integration reduces the need for exchange rate adjustment. Fleming (1971) links the stability of terms of trade to inflation differentials, thus reducing the utility of exchange rates when differentials are low. Kenen (1969) put the emphasis on fiscal integration, while Mintz (1970) and Tower and Willett (1976) highlight the role of political integration. Overall, OCA theory at the end of the pioneering period suffered from a “problem of inconclusiveness” (Tavlas (1994)). Given the multiplicity of criteria, it was likely that a given country would match certain of them but fail to others, and could hence be retained or not as an OCA by referring to different OCA properties.

This issue will be partly tackled during the “reconciliation phase” of the 1970's, characterized by the will of several authors to integrate the different OCA criteria in a joint perspective.

Corden (1972) notes that a currency union induces the relinquishment of monetary policy and exchange rate adjustments. The renunciation to such “expenditure switching policies” will be costly if wages and prices are rigid. In this case, the country would be instead obliged to rely on “expenditure absorption policies” such as fiscal expansion to mitigate the effect of shocks. Therefore, price flexibility may reduce the amount of absorption policy required in a currency union. Tower and Willett (1976) analyse the diverse OCA properties and the trade-offs entailed with respect to different exchange rate regimes. They determine that the cost of joining a monetary union depends on the origin, size and types of shocks, and diminish with trade openness. They conclude to the indeterminacy of the quantitative importance of any OCA property and thus call for further empirical assessment. In the same line, Ishiyama (1975) acknowledges the limitations of an approach restricted to a single OCA property and advances that each country should evaluate the costs and benefits of integrating a currency area through a joint set of criteria, considering its own interest only. Another important factor of optimality for a currency area lies in the degree of similarity of shocks that its members face, along with the country of origin of the shock. This aspect is illustrated by McKinnon (1963) and Corden (1972) in the case of foreign shocks. They conclude that a group of countries with comparable characteristics and reactions to shocks will require less adjustment from the exchange rate, making them more suitable for a monetary union. Mundell (1973) pushes the argument further by stressing the role of financial market integration: if international risk-sharing is high and portfolio sufficiently diversified, a monetary union may be feasible even between countries subject to heterogeneous shocks as long as private financial markets provide mutual insurance. Hence shock similarity, although desirable, is not strictly required.

The reconciliation phase gave more structure to the OCA theory by allowing concomitant consideration of the diverse primary OCA properties. It also integrated the important issue of the similarity of shocks. However, by the end of the 1970's, it had still failed to provide most OCA properties with an empirical content. Most importantly, theoretical developments in different economical fields were progressively leading to revise certain previously established properties of OCAs. This constitutes the third phase, or “reassessment” phase of OCA theory.

One main cost associated with the integration to a monetary union is the loss of monetary policy as a business cycle stabilization device, especially in a situation where partners of the union face asymmetric shocks. Yet, the integration of rational expectations into the traditional

economic framework led to question the validity of former monetary policy considerations. In particular, the traditional short-term Phillips curve, once augmented with expected inflation, implies that perfectly anticipated policy changes have no impact over real activity (McCallum (1989)). Following, policy makers could not hope using monetary policy as an instrument to reach a desired level of unemployment. Consequently, the cost resulting from the loss of control over monetary policy becomes low. Calvo and Rheinart (2002) support this conclusion, advancing that relinquishing monetary policy is costly only to the extent that it is properly used as a stabilization instrument. In the converse case, joining a union might even be beneficial if it allows to gain in credibility: a country with a historical record of breaches over low inflation commitments might gain low-inflation credibility by forsaking monetary sovereignty and delegate it to a third, low-inflation country serving as an anchor for the union (Giavazzi and Giovannoni (1989)). Also, monetary policy can be properly used but inefficient: Erkel-Rousse and Mélitz (1995) and Canzoneri et al. (1996) show that in most E.U countries, monetary policy is powerless to affect real variables such as output and unemployment. Hence, the loss of this instrument implies limited costs. Still, the issue over money neutrality did not reach a consensus. Tavlas (1993) and Melitz (1996) point to the possible necessity of different policy responses even in case of homogenous shocks, if the members of the union display heterogeneous characteristics (tax structure, trade responsiveness, price and wage flexibility). De Grauwe (2000) also argues that different degrees in inflation aversion may render a unified monetary policy suboptimal.

Another issue is whether exchange rate changes are effective altogether. If exchange rate adjustments are not actually effective, the loss resulting from their suppression would be limited. Branson (1989) claims that exchange rates do not constitute an appropriate adjustment tool as changes typically operate with considerable lags. Also, with Ricardian equivalence and perfect foresights, modifications in macroeconomic policy may not even affect the exchange rate (de Grauwe (1989)). Further, movements in exchange rates are primarily determined by monetary and financial shocks, rather than by purposeful macroeconomic policy (Canzoneri et al. (1996)). However, Sachs and Wyploz (1986) and De Grauwe (2000) report episodes of successful exchange rate adjustments, for the cases of France and Belgium in 1982, suggesting that exchange rates may remain an important instrument.

Considerations over the role of labour markets, finally, were also renewed, emphasizing the role of labour market institutions. Supply shocks may result in different macroeconomic effects depending on the degree of centralization in wage bargaining (Bruno and Sachs (1985)). When shocks are asymmetric across countries or regions, centralized wage bargaining, by imposing similar nominal growth rates at the union or region scale, would result in a loss of competitiveness for the countries or regions with low growth in labour productivity. Countries with different degrees of centralization may thus incur a cost from joining a monetary union, even if wages are flexible (De Grauwe (2000)).

By the end of the 1980's, the reassessment phase had led to the emergence of a “new” optimum currency area theory (Tavlas (1993)). In the wake of these advances, new developments in econometrics, theoretical advance and, foremost, the renovated interest towards European economic integration initiated the transition to the final and current phase of OCA theory, the “empirical phase”. It has mainly consisted in empirical studies analysing and comparing certain OCA properties over specific group of countries, often relying on econometric methods, to assess whether they could form a optimal currency area. The emphasis has been essentially set on Europe, as it provides empirical data for the process of monetary integration. This literature has now become quite large, making it difficult to provide a comprehensive treatment (for an exhaustive review of this literature, the reader is referred to Mongelli (2002), Lane (2006) and Ball (2010)). Much attention has been paid on specific issues regarding the Euro area, such as the persistence of price and wage rigidities, labour market integration, financial market integration, trade and economic openness, convergence of inflation rate, fiscal consolidation and the similarity of shocks. Because these issues constitute today the core of the debate over the Single currency, they will be the subject of the incoming and final section.

I.2.2. OCA theory and the Eurozone: persistence of the indeterminacy issue

Considerable progress has been accomplished in the development of OCA theory over the last 60 years. From its initial paradigm, consisting in identifying the properties required to qualify as an OCA, theory has progressively moved to a more varied and weighted analysis of the costs and benefits associated to a monetary union. Further, OCA properties have now been empirically estimated and tested, allowing these costs and benefits to be analyzed in greater

depth. These costs and benefits are now briefly listed, before the discussion turns to their empirical assessment. The main benefits can be classified as follows:

Benefits resulting from the suppression of exchange rate uncertainty. This probably constitutes one of the main gains from a monetary union, and concerns as much consumers and firms (for imports and exports of goods), than banks (for interbank transactions and general capital flows) and public governments (for their sovereign debts). Also, the suppression of exchange rate uncertainty implies the suppression of the hedging costs previously incurred by risk averse agents.

Benefits resulting from the reduction of transaction costs. Changing one currency for another generates the payment of commissions towards the bank or trader executing the operation. Also this cost is not a net loss (it produces a revenue for the bank or trader), it induces undesirable and arbitrary redistribution of wealth among economic entities. In addition, the resources that banks apply for these tasks could be used more usefully in other areas, so that transaction costs still constitute an opportunity cost.

Benefits resulting from increased transparency on goods markets across countries, which discourages price discrimination and decrease market segmentation.

Benefits resulting from deeper financial market integration, characterized by broader and more transparent financial markets, along with an increased degree of international risk sharing.

Benefits resulting from improved overall price stability and convergence of inflation rates. This can be exogenous to the currency union, if for example member states are required to abide by certain inflation objectives to enter the currency area, as is done with the Stability and Growth Pact. Or it can be endogenous to it, if the common monetary policy established by the unique central bank is conducted with a strong emphasis on price stability and enjoys a significant degree of independence vis-à-vis national governments. Gains in credibility can also be obtained for countries with a past reputation of breaching monetary policy commitments.

Benefits from the reduction of fluctuations in output and unemployment caused by asymmetric shocks, such as different fiscal or monetary policies, or unexpected shocks to interest parities impacting the exchange rate.

Benefits resulting from the elimination of non-cooperative behaviours, such as “beggar-thy-neighbour” policies based on unilateral devaluations.

The main costs, on the other hand, are listed below. They are mainly of macroeconomic order:

Costs from the relinquishment of national monetary policy to the central bank of the monetary union. This is probably one of the main costs associated with joining a monetary union. First because the national monetary policy can act as a stabilization tool in case of asymmetric shocks, or even as a revival instrument in case of economic downturn. Also, there can be costs directly arising from the monetary policy implemented by the common central bank. If a particular member state experience significantly asymmetric economic cycles with respect to its currency union partners, and if monetary policy is conducted mainly with respects to the latter, then this state can be made worse off by the monetary union.

Costs from the limitations to use fiscal policy as a stabilization tool. As a general principle, budgetary discipline is necessary to prevent certain governments to conduct possibly unsustainable national fiscal policies. The criteria laid by the Stability and Growth Pact for the Eurozone constitute a typical example of such restrictions. The limitations may be relatively more stringent for States with sustained deficits and/or levels of sovereign debts, which may yet be the States in greater economic difficulty. Also, a monetary union may typically lack a supra-national risk-sharing arrangement that may assist its members with asymmetric shocks.

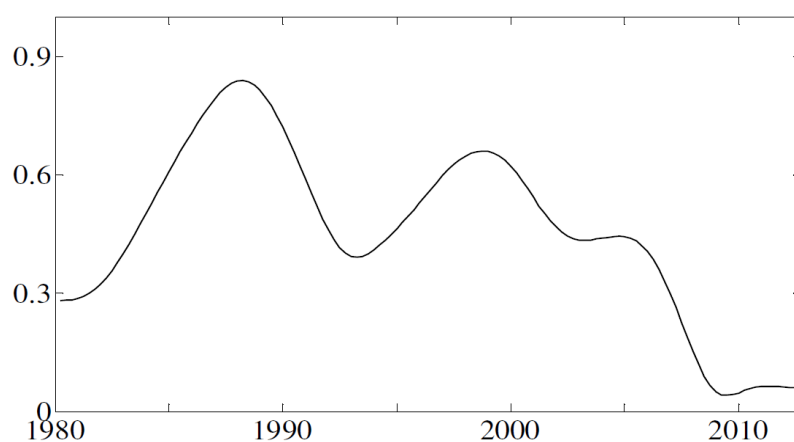
Costs from the loss of the exchange rate as an automatic stabilizer against asymmetric shocks. Asymmetric shocks may result in demand shifts from one country to another. Exchange rate appreciations or depreciations may then contribute to revert the shift. In a monetary union, the absence of exchange rate will require the adjustment to be operated either through shifts in relative prices and wages, or in case of strong nominal rigidities, through adjustments in employment.

Costs from the loss of determination of one own inflation rate. The loss of control over inflation implies forsaking the determination of the inflation tax. Similarly, it also implies the loss of the possibility to “inflate away” the national debt.

Costs from negative external effects. A country joining a monetary union incurs a moral hazard: if one or several other members of the union come to accumulate unsustainable levels of debt, there might be a necessity to eventually monetize these debts. This would in turn affect the interest rate of the union and might eventually ruin international confidence in the common currency.

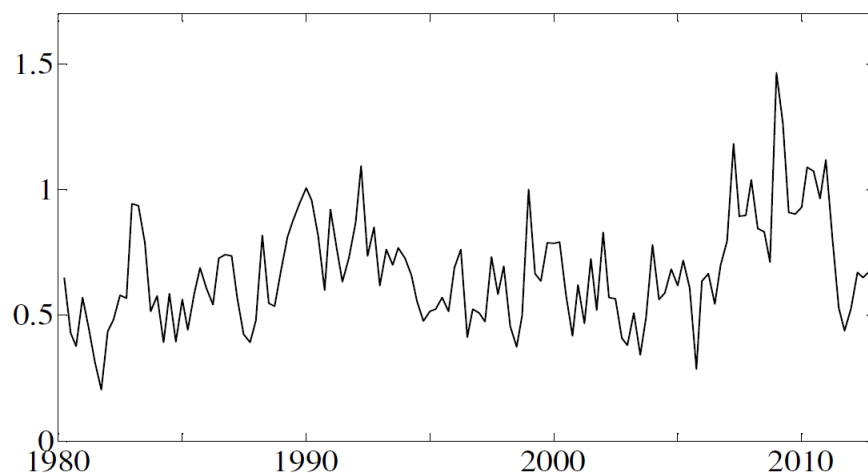
How successful is the Eurozone, from an empirical point of view? The issue is whether the benefits outweigh the costs. Naturally, the higher the benefits, the more likely it is that they outweigh the costs. As a very general and preliminary assessment, one can use output to measure the performance of the Euro area since the introduction of the single currency in 1999. If the benefits from exchange rate stabilization, enhanced trade, improved fiscal discipline, financial market integration and inflation rate convergence are significant, one would expect a positive shift in the growth rate of output resulting from the introduction of the Euro. Also, if the Euro improves overall stability, one should observe a drop in absolute cycle size, along with a reduced volatility of business cycles among countries.

Figure 1
Real GDP, average quarterly growth rate
EMU 11 members



Source: Author's calculations based on Eurostats data

Figure 2
Standard deviation of real GDP cycles
EMU 11 members, per unit of GDP



Source: Author's calculations based on Eurostats data

Table 1
Absolute cycle mean

	<i>1980-1999</i>	<i>1999-2007</i>	<i>Ratio</i>
Austria	1094.5	2553.8	2.3
Belgium	1627.8	2424.6	1.5
Finland	1641.2	2151.4	1.3
France	9080.6	14423.1	1.6
Germany	16315.3	27745.6	1.7
Ireland	927.7	2930.3	3.2
Italy	8347.3	13992.3	1.7
Luxembourg	234.0	526.2	2.2
Netherlands	2842.2	6247.4	2.2
Portugal	1566.1	1635.7	1.0
Spain	5242.8	7919.4	1.5

Source: Author's calculations based on Eurostats data

Figure 1 and 2 along with table 1 display general indicators of output performance. They are derived from real GDP series for the 11 countries which originally joined the Euro in 1999, considered over the 1980-2012 period. Data is first HP-filtered to isolate the trend from the cycles. Figure 1 displays the average quarterly growth rate for output trend², while Figure 2 displays the standard deviation of business cycles among the 11 E.U countries³. Table 1

² Precisely, the series obtains as the weighted average of output growth rates, with the weights given by the respective output share of each country with respect to total output from the 11 Euro countries.

³ Cycles are normalised by real GDP in order to keep estimates comparable between countries.

reports the average value of absolute cycles for each of the 11 countries, before and after the Euro introduction (years beyond 2007 are excluded to avoid exceptional variations imputable to the crisis). Overall, the results are mixed. Concerning output trend, the growth rate over the whole period seems rather downward sloping. Nevertheless, there is a significant increase over the 1992-1999 period, where major economical reforms were undertaken by the E.U States to prepare the introduction of the Euro, and over the 2003-2007 period. Also, it is hard to determine whether the downturns observed over the 2000-2003 and 2007-2011 periods are mainly attributable to the single currency or to the global recessions that were prevailing over these years. Concerning cycles, the Euro clearly contributed to reduce their dispersion over the 1992-2007 period, contrasting with the pre-EMU period of 1980-1992. The renewed increase in dispersion observed after 2007 is likely attributable to the economical crisis. There is hence significant evidence that the Euro contributed to improve cycle coordination within the Euro area. However, when looking at cycle sizes, it becomes apparent that business cycles fluctuations did not shrink with the Euro. On the contrary, fluctuations have become considerably larger, even doubling or tripling in sizes for countries such as Austria, Ireland, Luxembourg or the Netherlands. These findings are consistent with the results found in the empirical literature: although the Euro seems to have initiated a process of convergence of shocks and cycles, especially within a block made of Germany and its immediate neighbours, asymmetries persist within the Euro area, especially with Eastern Europe countries (Bayoumi and Eichengreen (1993,1996), Frenkel and Nickel (2005), Ben Arfa (2009)). Furthermore, shocks in Europe seem to exhibit a higher degree of correlation at the regional scale than at the international scale (Decressin and Fatas (1995), Viñals and Jimeno (1996), Forni and Reichlin (1997), Funke (2000)).

Another indicator of success for the EMU is the extent to which it fosters trade among European countries. Rose (2000) famously estimated that a currency union would increase trade among member countries by 200%. Micco et al. (2003) estimate this gain at around 8-16%, Baldwin (2006) finds an interval of 5-15%, and Frankel (2008) concludes to 10-15%. This improvement in trade is supposed to result from greater price transparency among countries, the single currency allowing to reduce price discrimination for tradables. Overall, price convergence within the Euro area seems to have been limited. Some papers show that tradable prices within Europe are becoming progressively more aligned (Beck and Weber (2001), Allington et al. (2005)), while other suggest that convergence has been achieved

mainly before the Euro introduction (Engel and Rogers (2004)), or has remained modest (Lane (2006)).

Table 2
Trade / GDP ratios

	1992	1998	2004	2010
Austria	0.27	0.33	0.46	0.48
Belgium	n.a	0.73	1.01	1.10
Finland	0.13	0.17	0.18	0.19
France	0.13	0.15	0.20	0.20
Germany	0.15	0.17	0.22	0.26
Greece	0.11	0.12	0.12	0.13
Ireland	0.19	0.27	0.22	0.18
Italy	0.12	0.15	0.18	0.19
Luxembourg	n.a	0.60	0.75	0.74
Netherlands	0.28	0.28	0.57	0.72
Portugal	0.21	0.25	0.32	0.41
Spain	0.12	0.17	0.24	0.24

Source: Author's calculations based on OECD data for real GDP, and International Monetary Fund's Direction of Trade Statistics database for intra-European trade
Note: Trade (exports plus imports) as a ratio to GDP

Table 3
European trade / total trade ratios

	1992	1998	2004	2010
Austria	0.42	0.45	0.48	0.43
Belgium	n.a	0.57	0.70	0.64
Finland	0.30	0.27	0.25	0.22
France	n.a	0.35	0.39	0.32
Germany	0.36	0.31	0.30	0.27
Greece	0.36	0.43	0.27	0.23
Ireland	0.26	0.22	0.15	0.11
Italy	0.40	0.36	0.36	0.32
Luxembourg	n.a	0.33	0.31	0.23
Netherlands	0.38	0.30	0.48	0.49
Portugal	n.a	0.49	0.51	0.52
Spain	0.41	0.41	0.44	0.38

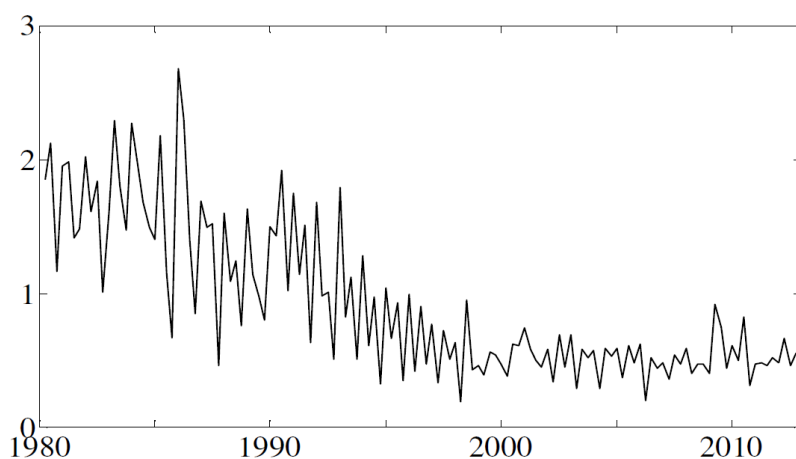
Source: Author's calculations based on OECD data for total trade, and International Monetary Fund's Direction of Trade Statistics database for intra-European trade
Note: Trade (exports plus imports) between E.U members as a ratio to total trade

Table 2 and Table 3 contribute to understand these findings, and, at the same time, to qualify them. Table 2 shows that the share of GDP dedicated to trade with Euro area partners has significantly increased over the 1992-2010 period. This confirms the hypothesis of a rise in

trade subsequently to the settlement of the Euro area. Yet, as noted by Lane (2006), other factors have also increased international trade over this period, including falling communication and transports costs, the expansion of Foreign direct investment, and the increase in trade with China. In this respect, Table 3 is quite revealing. It shows that for many Euro members, the share of European trade over total trade has remained relatively stable over the considered period. In some cases such as Finland, Greece or Italy, it even significantly declined. Also, the financial crisis has hampered the integration process, as all countries faced a drop in the share of European trade between 2004 and 2010. Eventually, it remains unclear whether the observed expansion in trade between Euro area members is attributable to the introduction of the single currency.

Closely related to the questions of cycle convergence and price convergence in tradable goods is the issue of inflation alignment between members of the Euro area. Figure 3 plots the unweighted standard deviation of inflation rates for 12 Euro members (the 11 original members plus Greece) over the period 1980-2012. One may observe an unambiguous decrease in the volatility of inflation across countries over the years 1980-1999. From there on ensues a period of stabilization, with inflation dispersion revolving around 0.5%.

Figure 3
Standard deviation of inflation
EMU 12 members



Source: Author's calculations based on OECD data

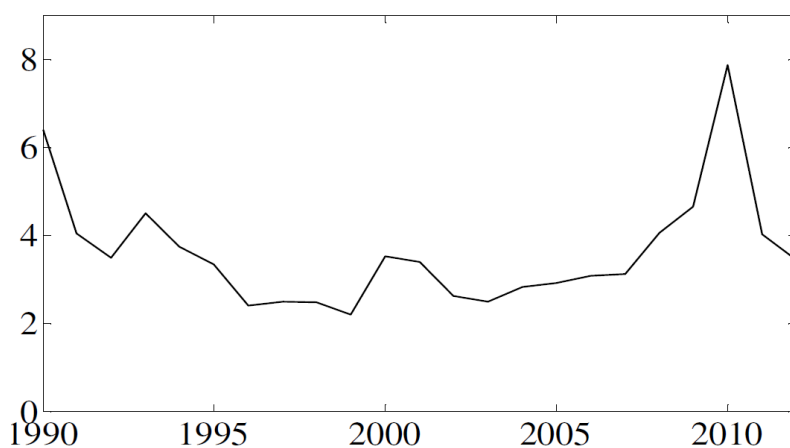
This sharp fall in inflation dispersion is confirmed by Lane (2006) and Mongelli and Wyplosz (2009). Yet several studies also point to the fact that sustained, though not large, differences

in inflation rates persist within the Euro area (ECB (1999), OECD (1999)). As the single currency implies that changes in bilateral real exchange rates take place only through inflation differentials, countries continue to build up real exchange rate misalignments that will have to be reversed some day (Lane (2006), Ball (2010)). Overall, however, the Euro significantly contributed to reduce inflation dispersion within Europe, mostly owing to the inflation criterion set by the Stability and Growth Pact.

Fiscal integration is supposed to be another asset provided by the introduction of the single currency. Figure 4 displays the standard deviation of public deficits across the 12 initial Euro members. As the graph indicates, the settlement of the EMU engendered a sensible fall of deficit dispersion in Europe up to 1999. This convergence in deficits yet reverted from 2000 on, and deficit spreads enlarged again before a peak was reached in 2010, at the height of the crisis. De Bandt and Mongelli (2000) find similar evidence on European data. As a corollary, Artis and Buti (2000) find that Euro area countries would be able to sustain even severe disturbances once they have complied with the deficit criterion of the Stability and Growth Pact.

Figure 4

**Standard deviation of public deficits as a percentage of GDP
EMU 11 members**

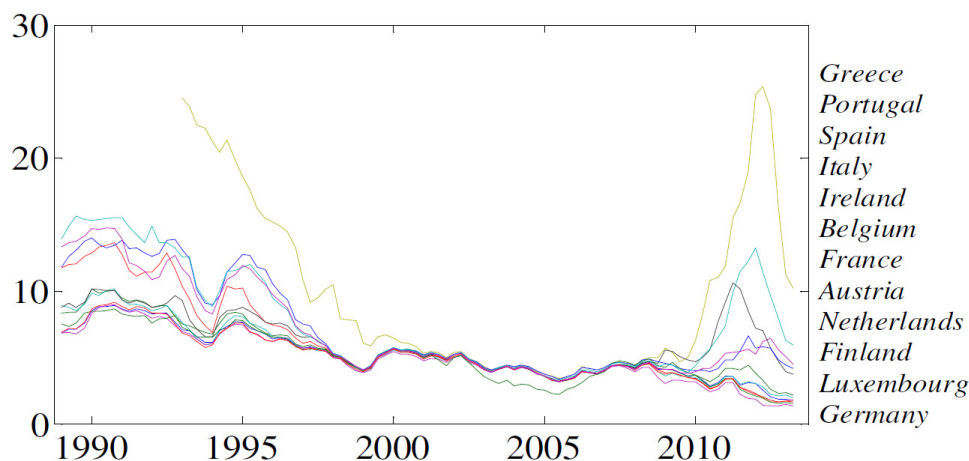


Source: Author's calculations based on Eurostats data

Some authors yet point to the lack of a public risk sharing facility that might provide a supra national or federal budget, and help mitigate adverse shocks on the budget of a particular member (Sachs and Sala-i-Martin (1991), Atkeson and Bayoumi (1993)). In the absence of such mechanism, the interest rate paid by each country on its public debt continues to vary

with its perceived risk of default. This is clearly visible on Figure 5, which represents the variation of the long term government bond yields for the 12 initial Euro area members.

Figure 5
Convergence criterion bond yield
EMU 12 members



Source: Eurostats

Prior to 2000, substantial spreads were still prevailing among Euro countries. Following the introduction of the single currency in 1999, spreads shrunk dramatically up to nearly zero, providing evidence that fiscal consolidation and integration was perceived as largely successful thanks to the Euro. From 2007 on, however, spreads started to soar again, the financial crisis and the sovereign debt crisis shedding light on the large underestimation of the sovereign debt of certain member States. Thanks to the SMP and OMT programmes initiated by the European Central Bank, spreads reverted again in 2010. There is, hence, some evidence of fiscal integration within the Eurozone, but recent developments suggest that consolidation is far from being achieved.

The last major advantage that a monetary union is expected to provide is a deepening of financial market integration. Table 4 reports, for 12 Euro area countries, the share of total cross-border security allocated to its Euro partners.

Table 4
Share of international portfolio allocated to European partners

	<i>Portfolio share</i>								
	<i>Total</i>			<i>Equity</i>			<i>Bonds</i>		
	<i>1997</i>	<i>2006</i>	<i>2011</i>	<i>1997</i>	<i>2006</i>	<i>2011</i>	<i>1997</i>	<i>2006</i>	<i>2011</i>
Austria	47.5	63.2	64.2	50.2	56.5	67.1	46.7	65.4	63.3
Belgium	67.0	77.9	79.9	84.1	79.7	84.9	55.6	76.7	77.9
Finland	29.9	57.4	45.0	34.9	38.8	38.5	28.1	72.4	49.5
France	43.2	61.2	62.5	39.3	52.8	57.2	45.2	64.8	64.0
Germany*	62.3	65.8	66.1	59.7	65.3	71.6	64.7	66.2	64.0
Greece*	36.8	31.8	22.7	50.1	43.0	56.7	34.0	29.8	16.7
Ireland	31.6	36.1	30.6	13.9	30.5	28.4	42.6	38.1	31.6
Italy	31.1	71.8	73.3	55.6	79.2	87.5	21.0	65.3	63.9
Luxembourg*	49.0	45.5	40.8	37.0	33.6	29.4	56.6	56.2	48.0
Netherlands	44.5	49.6	49.7	22.7	26.7	26.8	68.5	66.6	66.6
Portugal	45.2	66.1	75.5	54.0	71.8	63.9	42.3	64.2	78.2
Spain	36.2	62.0	72.9	45.8	77.0	71.6	27.6	56.7	73.1

Source: Author's calculations based on IMF's Coordinated Portfolio Investment Survey database

* 2001 instead of 1997

It appears that this share has substantially increased since 1997. From this point of view, it is undeniable that the Euro has enhanced financial market integration. Asset trade between members of the Euro area now exceeds extra European asset trade by an extent such that Lane and Milesi-Ferretti (2005) actually refer to a “Euro bias” in cross-border equity and bond holdings. One may nevertheless notice that the share of bond holding has declined in certain countries between 2006 and 2011, as a consequence of the sovereign debt crisis affecting Europe.

Empirical studies unanimously conclude to the positive role of the Euro with respect to financial markets, whether the aspect considered is cross-border portfolio holding (Baele et al. (2004)), foreign direct investment between member countries (Barr et al. (2003), De Sousa and Lochard (2011)) or substitutability between domestic and euro-area claims (Spiegel (2004)).

Empirical studies have also paid close attention to other conventional OCA properties, and concluded overall that the Eurozone does still not comply with these properties. Angeloni et al. (2006), for instance, find that the Euro did not trigger any radical change in price rigidities. Also, Calmfors (2001) and Boeri et al. (2001) find that wages remain quite rigid across most

European countries. Duval and Elmeskov (2005) find that the momentum of reform process towards more flexibility has actually slowed down in the Euro area.

As a consequence, labour market integration remains likewise limited. Eichengreen (1990) and Bertola (2000) highlight the low degree of labour mobility in Europe. Bentolilla (1997) shows that unemployment differentials intervene only marginally in the probability of moving across countries. Braunerhjelm et al. (2001) note inefficiencies in the inter-regional job matching process and high mobility costs.

I.3. Analytical framework and objectives of the thesis

The preceding paragraphs provided an overview of the empirical findings established during the empirical phase. Overall, these results reveal a mixed picture of the benefits implied by the settlement of a common currency in Europe. In some sectors, the gains appear significant and unambiguous (financial integration, inflation convergence), while in other sectors, they are more qualified (economic openness, fiscal integration, cycle and shock coordination). Is it nevertheless possible, thanks to these elements, to determine whether the benefits resulting from the introduction of the Euro in 1999 are larger than the costs implied by a currency area? The answer is, unfortunately, no. On the one hand, studies have become comprehensive so that OCA properties can be discussed in more details. It has become possible to determine why, and to what extent certain OCA properties are shared by a country or group thereof. On the other hand, nevertheless, there still lacks a methodology allowing to consider these elements together in order to conclude to the greater performance of a monetary union in comparison with alternative regimes. This is made all the more difficult since empirical analyses suggest that the benefits of the Euro remain, overall, mixed. In 1992, the “One Market, One Money” Report by Emerson et al., commissioned as an input for the Delors report, was pointing out that:

“There is no ready-to-use theory for assessing the costs and benefits of economic and monetary union (EMU)”.

Twenty years later, this remains true, and quoting Mongelli (2002), one can only reach the conclusion that:

“We can gain some important insights by studying the OCA properties in great detail. But it is still difficult to weigh and reconcile all OCA properties: i.e., we might still face a problem of inconclusiveness or of inconsistency”.

The only way out of this issue seems then to depart from studies based on individual OCA properties - as the latter will always result in some inconclusiveness issue - to adopt a broader analytical framework that would allow to consider these properties simultaneously. This thesis aims at fulfilling this objective. In this respect, the framework provided by the new international macroeconomics literature appears appropriate. Initiated with the seminal works of Backus et al. (1992) and Obstfeld and Rogoff (1995), this field has integrated the tools of the real business cycles methodology to issues traditionally treated by the conventional international macroeconomic literature. This framework offers two appealing features. First, its microfounded approach allows to introduce an exhaustive number of salient OCA properties in the modelling process. Hence, the simultaneity issue of OCA properties can be efficiently handled within this framework. Secondly, in the line of the works by Lucas (1987) and Rotemberg and Woodford (1999), this approach has permitted the derivation of explicit welfare criteria. These welfare criteria then allow for a direct comparison of the respective performances under alternative regimes, for example between a floating exchange rate regime and a monetary union.

A suitable methodology hence exists today to assess the respective performance of alternative monetary regimes and eventually overcome the inconclusiveness issue. Yet, and perhaps surprisingly, to these days the literature remains nearly silent on this subject. The New macroeconomic literature has now become wide in scope, dealing with subjects as diverse as Home-bias in preferences (Corsetti and Pesenti (2001)), the impact of pricing-to-market (Betts and Devreux (2000)), the effect of monetary shocks in a staggered price setting (Chari et al. (2002)). Recurring subjects also include the analysis of international business cycles (Kehoe and Perri (2002), Hairault, (2002), Ambler et al. (2004)), of exchange rate volatility (Devreux and Engel (2002), Benigno 2004), and international financial markets (Lane and Milesi-Feretti (2001), Obstfeld (2004)). Naturally, the Euro has given rise to considerable attention on monetary integration and the conduct of optimal policy, either in a regime of

flexible exchange rates (Corsetti and Pesenti (1997), Devereux and Engel (2003), Gali and Monacelli (2004)), or in a monetary union (Benigno (2004), Beetsma and Jensen (2004), Gali and Monacelli (2008), to cite just a few).

Nevertheless, only few authors went one step further and attempted to provide an explicit comparison between the two regimes. Obstfeld and Rogoff (1999) propose an assessment of several monetary regimes, but what they consider is a floating exchange rate regime, as opposed to a fixed exchange rate regime requiring interventionism to maintain fixity, which differs from a monetary union. Alesina and Barro (2002), Clerc et al. (2008), and Ferreira-Lopes (2010) discuss certain characteristics under which a country may profitably join a currency area, but they do not propose a general methodology to compare alternative regime performances. Strobel (2001) tries to determine the optimal schedule to leave a monetary union.

Finally, Monacelli (2000), Carré and Collard (2001), Cooley and Quadrini (2003), Kollmann (2004), and Forlati (2009) do propose an explicit comparison of the performance of a monetary union with respect to a floating exchange rate regime, and their findings are in favour of the monetary union. These papers, however, also raise several difficulties. First, the models that they propose are relatively stylised, making it likely that many important OCA properties are not included in the analysis they develop. Also, these models are not empirically applied to the Euro area, so that it is not certain that their conclusions would remain valid for the Eurozone. Finally, these papers have been written before the 2007 financial crisis. Therefore, they neglect a number of features which have since then proven to become significant, if not predominant, within the Euro area. In particular, financial frictions have played a crucial role during the late period of recession, emphasizing the role of financial market structure. Classical financial macroeconomic literature has largely treated the structure of financial markets as a veil, in accordance with the Modigliani-Miller theorem (1958), or focused on credit constraints faced by non-financial-borrowers (Bernanke and Gertler (1989), Kiyotaki and Moore (1997)) rather than by financial institutions. Yet, the pioneering work by Gertler and Kiyotaki (2010), has recently highlighted the role of financial rigidities faced by credit institutions to raise funds and subsequently finance productive investment. This literature on financial frictions is now developing fast (Gertler and Karadi (2011), Gertler et al. (2012), Gertler and Kiyotaki (2013)). Given the recent developments in the Eurozone, it seems essential to include it in the analytical framework proposed by this

thesis in order to keep in line with the main features characterizing the Euro area. In a similar spirit, it would be difficult to conduct an appropriate comparison of the Euro area with a flexible exchange rate regime if the ongoing sovereign debt crisis was left aside, along with the set of exceptional policies implemented by the ECB to overcome it.

This thesis is hence situated at the crossroads of the new international macroeconomics literature and the recent developments in financial macroeconomics. It is organized as follows:

Chapter 1 is fairly general and aims at formally establishing the occurrence of a structural break due to the introduction of the Euro currency. Indeed, if one admits the classical postulate of money neutrality, money is merely a veil so that switching from one currency to another should be mainly innocuous. In this case, the OCA literature becomes irrelevant and there is no legitimacy in pushing further an empirical analysis of the Euro area. It is then meaningful to endogenously test for the presence of a structural break in the dynamic process of Eurozone economies. As a first step, the procedure developed by Bec and Bastien (2007) for a single break, and the multiple break procedure developed by Qu and Perron (2007) are implemented on a three-variable VAR model for ten European countries, including seven Euro members and three non-Euro members. As a second step, the structural VAR methodology initiated by Blanchard and Quah (1989) is applied to the VAR models, prior and subsequently to the identified breaks, to decompose the dynamics of European economies in terms of supply shocks, real demand shocks and monetary shocks. The estimation process confirms that most Eurozone countries have faced a structural break around 1992, a date that corresponds to the adoption of the Maastricht Treaty and the settlement of the Euro convergence criteria. As the latter had to be implemented by 1994, the break date can be reasonably imputed to the Euro settlement process. As an additional support, the three non-Euro countries display very different and heterogeneous break dates. The second result of interest derives from the forecast error variance decomposition of the SVAR model. It reveals that the role played by supply shocks in Euro countries has increased after the structural break relatively to demand disturbances. Also, the part of monetary disturbances within total demand typically shrunk at short horizons, but raised at longer horizons. This confirms the greater stabilization role played by the ECB in the management of short-run business cycles, but suggests higher long-run volatility implied e.g. by the inadequacy between long-lasting effects of past policies and domestic economic developments.

Chapter 2 introduces the model that constitutes the core of the analytical framework used in this thesis. It relies on the dynamic and stochastic general equilibrium (DSGE) methodology adopted by the new international macroeconomic literature. The model features a region composed of two countries, which may or may not constitute a monetary union. In the floating exchange rate version of the model, the two countries preserve their own currencies, and the conduct of monetary policy by their national central banks. Trade with the economy abroad, whether in terms of goods or of securities, implies currency conversion at the prevailing exchange rate. In the monetary union version of the model, a single currency is introduced, which suppresses the uncertainty linked to exchange rate fluctuations. However, a unique central bank is now in charge of the conduct of the monetary policy at the scale of the union.

The primary objective of the model is to assess the respective performances of the Euro area economies under these two alternative regimes, taking into account the structure of financial markets and the context of the current financial crisis. In this respect, the model owes much to Gertler and Kiyotaki (2010) and proposes a comparable financial accelerator mechanism. This financial accelerator introduces financial rigidities in the model, as banks may be constrained in the quantity of funds they may obtain either from abroad partner banks or from domestic and abroad depositors. This in turn affects the real side of the economy, as only a limited quantity of productive investments may then be financed. This mechanism hence provides an efficient way of incorporating both financial market integration and financial frictions into the model. Other essential OCA properties are also included in the model: in line with the New Keynesian macroeconomic literature, monopolistic competition and nominal price rigidities characterize the behaviour of firms. The degree of wage rigidities and bias towards domestic goods depends on the national-specific preferences of the representative household inhabiting each country. Fiscal policy is implemented discretionary by national public authorities. The regulation of inflation, finally, is determined by the monetary authorities, at the national or union level, depending on the regime.

The model is then calibrated so as to mirror the Euro area: one country (Home) represents a particular country of the Eurozone, and the other country (Foreign) represents the rest of the Euro area. While calibration is common to Home and Foreign for certain parameters, most of them are given country-specific values in order to address the simultaneity issue of OCA theory and produce adequate quantitative conclusions.

Preliminary crisis experiments based on impulse response functions highlight a certain number of features. First, financial rigidities matter. The response of fundamental variables

such as output, consumption or capital is considerably more protracted as financial rigidities increase, and the initial fall due to the trigger of the crisis is larger. Secondly, as one would expect, the recession in Home is more severe when the crisis affects simultaneously the two countries instead of originating from one country only. This highlights the importance of interactions between Eurozone economies in national outcomes, and in particular the role of relations between banking systems. In this respect, the model accounts well for the current European crisis. Concerning the comparison of monetary regimes, finally, the analysis is overall not conclusive. In the event of a financial crisis, a regime of floating exchange rates provides better stabilization performances than a regime of monetary union with respect to certain essential variables (output and labour), but the conclusion is reversed for other variables (consumption and capital). This calls for a more formal welfare analysis.

Chapter 3, eventually, meets a double purpose. First, it derives formal welfare criterion to compare the performances of the Home economy under alternative monetary regimes. The first criterion is general and follows the methodology employed by Monacelli (2000) and Kollmann (2004). A second criterion, proposed by Carré and Collard (2001) in the line of Lucas (1987), is then used to highlight shock-specific results. The general criterion reveals that over 17 Euro area members, 15 would be better-off under a regime of floating exchange rate, Belgium and Slovakia being the two exceptions. This analysis however does not consider the cost of leaving a monetary union, which is likely to be large. The shock-specific criterion then provides more insight and shows that the poorer performance of the monetary union regime is mainly due to the adverse effect of shocks emanating from the abroad economy. These conclusions suggest that the symmetry of shocks remains a primordial issue within the Euro area.

The second objective of this chapter is to introduce a number of extensions to the benchmark model developed in chapter 2, to account for the recent European sovereign debt crisis and the set of exceptional policies employed by the ECB to mitigate it. In the first extension, the assumption of a balanced public budget developed in the benchmark version is relaxed. Taxation becomes proportional to current output, so that a recession implies the creation of a budget deficit and an increase of the public sovereign debt. To reverse the process, public authorities must act in two ways: by increasing taxation, and by cutting public expenditure. The extension also assumes that government spending becomes a component of the household utility function. Following, the fluctuations induced by imbalanced public budgets impact

directly on household welfare. The second extension introduces the Covered Bond Purchase Programme into the benchmark model. The ECB intervenes directly on financial markets by providing a quantity of interbank funds proportional to the spread between the interbank rate and the riskless rate. This allows to partially relax the constraint faced by banks to obtain funds and to speed up the recovery of the economy after a recession. The last extension augments the first extension with the Securities Market Programme/Outright Monetary Transactions. The ECB in this case purchases on secondary markets a fraction of sovereign debt proportional to the deviation from steady-state. It may also invest directly in private securities. This relaxes both the budget constraint of public authorities and the financial pressure over the banking sector as fewer projects have to be financed by the private sector. In terms of welfare, the first extension overall confirms the results obtained with the benchmark version, as only 3 out of 17 countries enjoy higher welfare in a monetary union. There is, however, a strong contrast between the second and the third extension. With the CBPP, the welfare performances of the monetary union are hardly any better than in the benchmark version. Some welfare improvement does occur thanks to the programme, but it remains insufficient with respect to the difference prevailing with the flexible exchange rate regime. Conversely, there is a significant difference in the performance of the monetary union regime when the SMP/OMT programme is included. The best results are obtained when the ECB intervenes both in the sovereign debt market and invests in the private security market. In this case, a majority of Euro area members (9 out of 17) become better off under a monetary union. This suggests that the ECB has an essential role to play for the success of the Euro area. It should not behave as mere institution in charge of the daily conduct of monetary policy, but should provide active fiscal and financial assistance to the Euro area member States, endorsing the functions of a supranational risk-sharing authority.

Chapter 1

Euro implementation: has there been a structural change? A study on 10 European Union countries

Abstract

The implementation of the Euro currency in January 1999 consecrated the achievement of a single currency system within most of the European Union. Despite the dramatic change in the macroeconomic dynamics that this event is likely to have caused, the literature has paid little attention to testing for the existence of such a break and establishing its qualitative characteristics.

This empirical study, based on the Blanchard and Quah (1989) methodology for seven countries having adopted the Euro currency and three members of the European Union which have kept their own currencies reveals:

- i)* very significant breaks for the Euro countries in 1992 – the year of adoption of the Maastricht Treaty – and 2001, not shared by the three non-Euro countries.
- ii)* an increase in the influence of supply shocks on output, unemployment and the interest rate after the breaks for the Euro countries, along with an increase of the part played by monetary disturbances within total demand at long horizons. These conclusions do not generally hold for the three non-Euro countries.

JEL classification: C32, C5

Introduction

The demise of the Bretton Woods system, marked by the end of the dollar's convertibility into gold on August 15th 1971, was followed by a general floating of the world currencies. With the oil crisis of the early 1970's, the European currencies faced great general monetary instability, the cause of serious economic and social difficulties. The member states hence sought to set up a framework which could provide a minimum of stability, at least at the European level, and which could lead to a monetary union.

In 1972 the "snake in the tunnel" system represented the first attempt towards such a union. It narrowed the fluctuation margins between the Community currencies (the snake) in relation to those operating between these currencies and the dollar (the tunnel). However, the oil shocks during the 1970's caused the European economies to react in diverse ways. This led to sharp fluctuations in the exchange rate, such that by the end of 1977, only five of the nine member states (Germany, Belgium, the Netherlands, Luxembourg and Denmark) remained within the mechanism, the others having allowed their currencies to float freely. In 1979, the snake was finally replaced by the European Monetary System. This resulted in the creation of the European Exchange Rate Mechanism (ERM), as a renewed attempt to reduce exchange rate variability and achieve monetary stability in Europe.

The revival of the monetary union was initiated in 1989, with the report of the Delors Committee envisaging the achievement of a European Monetary Union (EMU) in three stages. The first stage would consist in tighter cooperation between central banks. The settlement of a European System of Central Banks (ESCB) and the gradual transfer of decision-making on monetary policy to supranational institutions would be the second stage. Finally, in the third stage, the national currencies would have their exchange rates irrevocably fixed in order to be replaced by the European single currency.

In June 1989, the European Council decided that the first stage towards the EMU would begin in July 1990. The Treaty of Maastricht was agreed by the heads of state of the European Union in December 1991, setting out the framework for stages two and three of progress towards the EMU.

Following, from 1994 on, the states had to abide by five convergence criteria in order to lay safe foundations for the third stage. This mainly consisted in an inflation rate aligned on the best performing states, a budgetary deficit not exceeding 3% of GDP, and a government debt not exceeding 60% of GDP. In June 1997, the European Council in Amsterdam adopted the Stability and Growth Pact, designed to ensure budgetary discipline during the third stage. The Euro was finally implemented in January 1st 1999, in 11 European Union countries. Since then, six additional countries have adopted the Euro currency, and seven other candidate countries should also integrate the system as soon as they abide by the Euro criteria. On the other hand, some countries chose to preserve their monetary independence (Sweden and the United Kingdom), while others such as Denmark decided to adopt the new semi-peg regime implemented by ERM2.

Each of these numerous steps constitutes an opportunity for a structural change in the macroeconomic reaction of the member state economies, especially to monetary policies. This study then focuses on the case of ten European Union countries, seven of them having adopted the Euro and three of them having maintained their own currencies.

The empirical literature dedicated to the study of the macroeconomic dynamics within the Euro framework has become relatively large. But surprisingly enough, there are only few articles taking into account the possibility of a change in macroeconomic dynamics due to the passage to the single currency.

Probably closest to the present work is the article by Boivin et al. (2008). These authors estimate the dynamic response of European Union economies following unexpected monetary shocks, prior to and subsequently to the Euro implementation. They conclude that the passage to the Euro has led to more homogenous transmission mechanisms across E.U countries, and to a global decrease of the impact of monetary shocks. The present work supports these conclusions, but only when one considers short horizons. In addition these authors rely on a FAVAR model used to extract the principle components of the economic activity, while this article uses a structural decomposition in terms of supply and demand shocks. Most importantly, these authors arbitrarily select the date of the structural break - established in 1988 or in 1999 - whereas this work endogenously determines the date of the break and formally tests for its statistical significance.

Other articles in this literature very often include VAR models integrating some monetary factor in order to assess the EMU dynamics (see e.g. Mojon and Peersman (2003), Peersman and Smets (2005), Van Els et al. (2003)). Some papers pay more attention to potential asymmetries in the reaction of EU countries or EU applicants towards supply and demand shocks. Their main aim is to appreciate the suitability of the single currency area, and as such, they pay no attention to a potential break in the reaction mechanism. Such papers include Babetskii et al. (2004), or Frankel and Nickel (2005). Other papers also rely on structural decompositions to analyze the effect of monetary shocks on the economy, such as Funke (1997) or Eickmeier et al. (2007).

Part of the literature on the other hand examines the possibility of a structural break in the dynamic process, such as Weber et al. (2009). These authors use an endogenous bootstrap method to determine the date of the break, found to be 1996. However, the methodology they follow does not actually identify a unique break, but rather a range of breaks all over the period 1984-1996, with an arbitrary choice of the last period. Fratzscher and Stracca (2009) try to determine whether the Euro implementation has altered the transmission of political shocks, from domestic or Union-wide origin. This leads them to postulate the break date to be 1999. Zha et al. (2004) analyze the effects of the change in monetary policy due to the passage to the EMS. They also postulate the date of the break (in 1979) and do not test it explicitly.

Finally, a few papers are explicitly testing for the presence of a structural break in the data, but they are usually dedicated to subjects different from macroeconomic dynamics as regards supply and demand shocks. Marotta (2008) examines the possibility of a structural break in the size and speed of the pass-through of market rates into business lending. Bordes et al. (2007) endogenously test for a potential structural break in equilibrium velocity in the Euro area. Interestingly enough, the two breaks they identify – in 1992/1993 and 2000/2001 – match the results of the present paper. Gregoriou et al. (2006) study potential persisting inflation differentials between European Union members. Fitzenberger et al. (2007) consider the possibility of a break due to the German reunification.

Shortcomings of the current literature are twofold: either the analysis is restricted to pure monetary components, and hence neglects the role played by other factors such as supply or real demand shocks as regards the dynamics; or the analysis does use some decomposition but

neglects the possibility of a break, carrying very limited information on the actual change brought by the Euro. Furthermore, when some break is taken into consideration, it is most of the time determined exogenously.

The present paper thus contributes to the literature in the two ways. Following the methodology proposed by Bec and Bastien (2007), it first formally dates and tests for the occurrence of a break - potentially due to the Euro implementation - in the economy of ten E.U. countries. The study includes seven Euro countries – Germany, France, Italy, Spain, Belgium, Portugal and Ireland – and three non-Euro countries for the sake of comparison: Denmark, Sweden and the United Kingdom. The second contribution of this work is to propose a quantitative analysis of the respective contributions of each kind of disturbances in the dynamics of these economies, prior to and after the estimated breaks. It relies on a simple 3-variables VAR model, using a Blanchard and Quah (1989) decomposition to identify three kinds of shocks: supply shocks, real demand shocks, and monetary shocks. It thus allows to highlight the role played by the sole monetary component as compared to the other shocks, and assess a possible change due to the implementation of the Euro⁴.

The selection of countries aimed at choosing the most representative countries of the Euro zone, and is consequently arbitrary to some extent. Countries that have joined the Eurozone after 1999 have also been excluded in order to get as many time periods including the Euro in the data. Regarding the choice of non-Euro countries, Sweden and the United Kingdom appear as natural candidates since they have kept their own currencies. Denmark, as a member of ERM since 1979 and of ERM II since 1999 represents an interesting case. On the one hand, Denmark preserves its own currency and some liberty in the conduct of its monetary policy, since its exchange rate is allowed to float within a range of 15% with respect to a central rate against the Euro. On the other hand, as a long-lived member of ERM, then ERM II, Denmark's exchange rate has always proven remarkably stable against this central rate, to such an extent that its fluctuation bands have been tightened to 2.25%. In this respect, the passage from ERM to ERM II in 1999 may not have represented such a considerable change for the Danish economy.

⁴ Legrand (2012) follows a similar methodology. This paper however focuses on Germany only, and the structural decomposition which is employed only encompasses supply and demand shocks. The findings of this work nevertheless support the conclusions of the present study.

The other non-Euro EU members, namely Latvia, Lithuania, the Czech Republic, Poland, Hungary, Romania and Bulgaria were also excluded. A first reason is that their joining to the EU is too recent to carry much information about their monetary policy as compared to the Euro. A second reason is that they are either members of ERM II or assumed to join ERM II and the Eurozone; hence it is difficult to consider their monetary policy as independent from the Euro policy. A final reason is that finding data of satisfying quality may prove difficult for these countries.

The remainder of the paper is organized as follows: section 1.1 proposes a simple theoretical model motivating the use of the Blanchard-Quah decomposition; section 1.2 presents the data, then focuses on the testing of a structural break, adapting the Sup test defined in Bai et al. (1998) for a structural shift in the transmission mechanism of a VAR model. Section 1.3 presents the dynamic effect of supply and demand disturbances and section 1.4 evaluates their relative contributions to the fluctuations of output and the interest rates of the different countries. Section 1.5 concludes.

1.1 Theoretical foundations

This empirical study extends the Blanchard and Quah methodology, assuming that the economy is affected by three kinds of disturbances: supply disturbances, real demand disturbances and monetary disturbances, the latter two composing together the total demand side of the economy. This section proposes a simple theoretical model. It is purely illustrative and shows how the dynamics of the three variables included in the incoming VAR setup (real GDP, the unemployment rate and the interest rate) may be represented in terms of supply, real demand and monetary shocks. In addition, for a three-variable VAR model, the identification of this structural decomposition requires the implementation of three restrictions on the dynamics of the variables; the following model also provides a theoretical justification to such restrictions. It is a simple AS/AD setting with a money market, variant of the monetary model of Gray (1978). Following Fisher (1977), it assumes rational expectations and nominal wage rigidities.

The production process characterizing the economy is a Cobb-Douglas technology given by:

$$Y_t = \exp(z_t)L_t^\delta, \quad 0 < \delta < 1 \quad (1.1)$$

where Y_t denotes output, and L_t the quantity of labour used by firms. z_t represents global factor productivity. Expressing this function in logarithm yields:

$$y_t^s = z_t + \delta l_t \quad (1.2)$$

and z_t is assumed to follow a random walk process:

$$z_t = z_{t-1} + \varepsilon_t^s \quad (1.3)$$

with ε_t^s an *i.i.d* supply shock. For the goods market, aggregate demand is given a simple (logarithmic) form:

$$y_t^d = \phi - p_t - r_t + \varepsilon_t^d \quad (1.4)$$

with p_t the price level, and r_t the nominal interest rate. ε_t^d is an *i.i.d* real demand shock, representing e.g. investment, fiscal policy or consumer confidence. On the money market, the central authorities are assumed to follow a simple Taylor rule of the form:

$$r_t = \bar{r} + \beta(p_t - \bar{p}) + \gamma(y_t - \bar{y}) - \varepsilon_t^m \quad (1.5)$$

where \bar{r} , \bar{p} and \bar{y} denote respectively the interest, price and output targets of the central bank. ε_t^m is an *i.i.d* monetary shock, that can be interpreted as the non-systematic component of monetary policy.

Firms are maximizing profit. Hence, their optimal demand for labour is found as being:

$$l_t^d = \frac{1}{1-\delta} [\log(\delta) + z_t - (w_t - p_t)] \quad (1.6)$$

with w_t the nominal wage. The price level adjusts instantaneously to equate aggregate supply and aggregate demand on the goods market.

Combined with the interest rate rule (1.5), the price level is found to be:

$$p_t = \frac{1}{(1+\beta)} [\phi - \bar{r} + \beta \bar{p} + \gamma \bar{y} - (1+\gamma)z_t - (1+\gamma)\delta l_t + \varepsilon_t^m + \varepsilon_t^d] \quad (1.7)$$

Then, (1.6) and (1.7) imply labour demand:

$$l_t^d = \omega_1 - \omega_2 w_t + \omega_3 z_t + \omega_4 (\varepsilon_t^m + \varepsilon_t^d) \quad (1.8)$$

Where ω_1 , ω_2 , ω_3 and ω_4 are constant terms⁵. The wage setting behaviour is introduced through contracts *à la Fisher* (1977): the wage is chosen one period in advance in order to achieve (expected) full employment in the next period:

$$w_t = w | \{ E_{t-1}(l_t^d) = \bar{l} \} \quad (1.9)$$

E_{t-1} denotes the rational expectation as regards labour demand, given the information set available in period $t-1$. \bar{l} denotes the constant total labour supply. Then, from (1.9), the equilibrium nominal wage at every period is:

$$w_t = \frac{\omega_1 - \bar{l}}{\omega_2} + \frac{\omega_3}{\omega_2} z_{t-1} \quad (1.10)$$

With this value, it becomes possible to solve for the equilibrium labour demand l_t^d ; using this value in the aggregate supply equation (1.2), one then finally obtains the equilibrium value for output:

$$y_t = \delta \bar{l} + z_t + \delta \omega_3 \varepsilon_t^s + \delta \omega_4 (\varepsilon_t^m + \varepsilon_t^d) \quad (1.11)$$

But z_t being a random walk process, this can be rewritten as:

$$y_t = \delta \bar{l} + \delta \omega_3 \varepsilon_t^s + \delta \omega_4 (\varepsilon_t^m + \varepsilon_t^d) + \varepsilon_t^s + \varepsilon_{t-1}^s + \varepsilon_{t-2}^s + \varepsilon_{t-3}^s \dots \quad (1.12)$$

⁵ Precisely: $\omega_1 = \frac{\phi - \bar{r} + \beta \bar{p} + \gamma \bar{y} + (1+\beta) \log \delta}{(1+\beta)(1-\delta) + (1+\gamma)\delta}$, $\omega_2 = \frac{(1+\beta)}{(1+\beta)(1-\delta) + (1+\gamma)\delta}$, $\omega_3 = \frac{\beta - \gamma}{(1+\beta)(1-\delta) + (1+\gamma)\delta}$ and

$\omega_4 = \frac{1}{(1+\beta)(1-\delta) + (1+\gamma)\delta}$. While ω_2 and ω_4 are strictly positive, as one would expect, the sign of ω_1 and ω_3 remain ambiguous, depending on policy parameters.

Defining then unemployment for each period as: $u_t = \bar{l} - l_t^d$ and solving for this renders:

$$u_t = -\omega_3 \varepsilon_t^s - \omega_4 (\varepsilon_t^m + \varepsilon_t^d) \quad (1.13)$$

Finally, solving for the equilibrium value of the interest rate renders:

$$r_t = \tilde{r} - \alpha_1 \varepsilon_t^s - \alpha_2 \varepsilon_t^m + \alpha_3 \varepsilon_t^d - \alpha_4 z_t \quad (1.14)$$

with \tilde{r} the long run value of the interest rate, α_1, α_2 and α_3 positive constants, and

$$\alpha_4 = \frac{\beta - \gamma}{1 + \beta} \text{ the coefficient on the productivity process } z_t \text{ }^6.$$

It is clear from (1.12) that supply shocks have a permanent effect on output, which should therefore be integrated of order one. On the other hand, due to nominal wage rigidities, real demand disturbances have only a transitory effect on output, as do monetary shocks. The model hence suggests that the effect of real demand shocks and monetary shocks should be neutral in the long run. This provides for the first two long-run restrictions needed for the incoming structural decomposition.

(1.13) shows that the three disturbances have only a short-run impact on unemployment, that should consequently be a stationary variable. From (1.14) the interest rate is seen to be a linear combination of present and past supply shocks through the productivity term z_t , as well as present real demand and monetary shocks. In the particular case where $\beta = \gamma$, the value of α_4 becomes 0, so that the interest rate is stationary. Otherwise, it is integrated of order one. The model hence implies that depending on policy parameters, the interest rate may be either $I(1)$ or stationary, illustrating the possible reasons for which no consensus has yet been reached in the literature on this issue. In the subsequent parts of this study, the stationary case will be retained⁷.

To identify the model, an additional restriction is required. The transitory effects of all three types of shocks on unemployment and the interest rate (in the case $\beta = \gamma$) provide a range of

⁶ Precisely: $\tilde{r} = \frac{\bar{r} + \beta(\phi - \bar{p}) - \gamma\bar{y} - (\beta - \gamma)\delta\bar{l}}{(1 + \beta)}$, $\alpha_1 = \frac{(\beta - \gamma)\delta\omega_3}{1 + \beta}$, $\alpha_2 = \frac{1 + \delta(\beta - \gamma)\omega_4}{(1 + \beta)}$ and $\alpha_3 = \left(\frac{\beta - \delta(\beta - \gamma)\omega_4}{(1 + \beta)} \right)$.

⁷ Indeed, the case $\beta = \gamma$ does not appear as implausible: see e.g. Hein and Truger (2007) or Clarida et al. (1998) for empirical estimates of β and γ . Most importantly, unit root tests run on the interest rate series used in this study support the stationary hypothesis. The unit root test tables are available from the author upon request.

candidate long-run restrictions. Note however that not every choice of restriction is implementable. In particular, the structural decomposition is not identified if one assumes that the effect of supply shocks on unemployment or the interest rate is neutral in the long run (see Lemma in Appendix A.3). In the following, a conventional restriction is adopted, assuming that monetary shocks may have no long-run effect on the unemployment rate. This merely reflects the usual assumption that monetary policy is neutral in the long-run on real variables⁸.

1.2. Testing for a structural break

This work is based on a VAR setup similar to the one developed in the seminal work by Blanchard and Quah (1989). The number of variables is increased to three in order to be able to decompose the demand shocks into a real demand component and a monetary component. For each country, the vector autoregression includes the index of industrial production⁹, as well as an unemployment rate and an interest rate series. The data is monthly and runs from January 1979 to January 2011. The choice of a starting date in 1979 is made so as to avoid the possibility of multiple breaks, here especially the oil shocks in the 1970's. The series come from Datastream, the OECD or Eurostats¹⁰. Interest rate series are typically 3-month overnight rates. For Euro countries, they first represent the national rates (FIBOR, RIBOR...), and from 1999 on they are replaced by the EURIBOR.

Conclusions from the previous section as well as traditional economic theory suggest that output should be integrated of order one, and hence taken in first difference before its

⁸ One could also mix short-run and long-run restrictions as in Gali (1992); an example of conventional restrictions would then be that real demand and monetary disturbances have no long-run effect on output, while monetary disturbances have no instantaneous effects on unemployment. A replication of the study with the latter set of restrictions produced results essentially similar to the one presented in the study. The main difference with the alternative set of restrictions is that the increase in the influence of monetary shocks within total demand is usually seen at shorter (business cycles) horizons (see section 1.4 for results on Forecast Error Variance Decomposition).

⁹ Precisely, this work follows the common practice consisting in taking the logarithm of the index, multiplied by 100. The reference year for the index is 2005.

¹⁰ The reader interested in the references and sources of each series may refer to Appendix A.1. It must be noted that there is an issue about the German unemployment series. Due to the reunification, the data includes only the Federal Republic of Germany prior to 1992, and reunified Germany as a whole afterwards. This creates a discontinuity in the series at the beginning of the year 1992, and entails the risk of detecting a break at that date that would be a pure statistical artefact. To avoid this, the discontinuity was removed by chaining the series between the last quarter of 1991 and the first quarter of 1992. To our knowledge, there exists unfortunately no dataset providing harmonized data for unemployment prior to and after the year 1992.

inclusion in the VAR. Conventional unit root tests confirm these results¹¹. For unemployment and the interest rate, results from the usual tests are not clear-cut, nor is economic theory: while some authors (see e.g. Gali (1999)) suggest interest rates should be treated as I(1) variables, others like Clarida et al. (1998) or Fuhrer and Moore (1995) argue that such unit-roots do not have very meaningful economic interpretation. There is no ideal solution to this problem. Given that the unit-root tests by Banerjee et al. (1992) suggest stationarity, and since keeping the series in level produces stationary VARs, the following developments will consider unemployment and the interest rates as I(0) variables. Nevertheless, as a check for robustness, for each country the case of a VAR with the unemployment rate and interest rates taken in first difference will also be studied.

Given the previous analysis, the basic VAR setup includes the log of the industrial production index taken in first difference, the unemployment and interest rate series in level, and a drift term:

$$\begin{pmatrix} \Delta Y_t \\ u_t \\ r_t \end{pmatrix} = \begin{pmatrix} a_y \\ a_u \\ a_r \end{pmatrix} + \begin{pmatrix} a_{11,1} & a_{12,1} & a_{13,1} \\ a_{21,1} & a_{22,1} & a_{23,1} \\ a_{31,1} & a_{32,1} & a_{33,1} \end{pmatrix} \begin{pmatrix} \Delta Y_{t-1} \\ u_{t-1} \\ r_{t-1} \end{pmatrix} + \dots + \begin{pmatrix} \varepsilon_{yt} \\ \varepsilon_{ut} \\ \varepsilon_{rt} \end{pmatrix} \quad (1.15)$$

Or, more compactly:

$$X_t = \mu + A(L)X_t + \varepsilon_t \quad (1.16)$$

Where X_t , μ and ε_t represent respectively 3×1 vectors of variables, constants and residuals, and $E(\varepsilon_t \varepsilon_t')$ is assumed to be equal to Σ . $A(L)$ is a lag polynomial whose roots all lie outside the unit circle. For each country, the order of the lag polynomial was set in accordance

¹¹ The unit-root tests initially run on the data are the usual ADF, Phillips-Perron and KPSS tests. However, it is well-known that the power of these tests may be very low, especially if there is a structural break in the data. In this case, the test developed by Banerjee, Lumsdaine and Stock (1992) that specifically aims at allowing for a structural break while testing for the presence of a unit-root is appropriate and hence run on the data. For industrial production, the tests unambiguously fail to reject the null of a unit root. On the other hand, for the unemployment rate and the interest rate, for each of the three statistics (recursive, rolling and sequential), the t_{\max} statistics usually yield mixed results, while the $t_{df}, t_{\min}, t_{diff}$ statistics strongly support the stationary hypothesis. It is hence reasonable to conclude that these two variables are stationary. Results for the conventional unit root tests are not reported in the text but are available from the author upon request. Results for the Banerjee, Lumsdaine and Stock tests are reported in Appendix A.2.

with the Akaike criterion, and as the minimum number of lags required in order to suppress the residual autocorrelation¹².

Before estimating the dynamics of aggregate supply and aggregate demand shocks, the presence of a potential break in the VAR dynamics is formally tested. Following Bec and Bastien (2007), notice that a VAR model allowing for a structural break at time $\tau = \lambda T$, with $\lambda \in (0,1)$ can be written as:

$$X_t = \mu_0 + A(L)X_t + D(t > \tau)[\mu_1 + B(L)X_t] + \varepsilon_t \quad (1.17)$$

The break in the process is allowed for by D , a dummy taking the value of 1 for $t > \tau$, and 0 otherwise.

When compared with (1.16), the VAR model (1.17) differs only to the extent of the term that multiplies the dummy. Hence, assuming a null hypothesis of no break at time τ in (1.17) is equivalent to:

$$H_0 : \mu_1 = 0 \text{ and } B = 0$$

If the date of the break is known, it is possible to use a simple likelihood ratio test relying on a Chi-square distribution. Letting (1.16) denote the restricted model, and (1.17) the unrestricted model, this test can be written as:

$$LR_\tau = 2 \times (\mathcal{L}_\tau - \mathcal{L}_r) = (T - q)(\log \det (\Sigma_\tau) - \log \det (\Sigma_r)) \sim \chi^2(q)$$

Where \mathcal{L}_τ and Σ_τ denote respectively the log-likelihood and the variance-covariance matrix of the unrestricted model for a break at time τ , and \mathcal{L}_r and Σ_r denote the log-likelihood and variance-covariance matrix of the restricted model. q denotes the number of constrained coefficients implied by constraining (1.17) to become (1.16), that is by the null hypothesis.

Nevertheless, given the number of candidates, the break date must be estimated endogenously, so that the test statistics will not be anymore Chi-square distributed. An

¹² The lag orders are as follows: 7 for Germany, 6 for France, 7 for Italy, 11 for Spain, 11 for Portugal, 5 for Ireland, 7 for Belgium, 9 for Denmark, 8 for Sweden and 5 for the United Kingdom.

alternative solution is therefore to follow the practice initiated by Davies (1987) and adopted by Bec and Bastien (2007), which consists in using sup tests of the type:

$$\text{SupLR}(\tau) = \sup_{\tau \in [\tau_{\text{inf}}, \tau_{\text{sup}}]} \text{LR}_T(\tau)$$

Where τ_{inf} corresponds to the initial fraction of the full sample T which is trimmed, usually $0.15T$, and τ_{sup} equals $T - \tau_{\text{inf}}$. The best candidate date for the break is then selected as being:

$$\hat{\tau} = \underset{\tau_{\text{inf}} \leq \tau \leq \tau_{\text{sup}}}{\text{argmax}} \text{LR}_T(\tau) = 2 \times (\mathcal{L}_\tau - \mathcal{L}_r)$$

As shown in Andrews (1993), the asymptotic null distribution of $\text{SupLR}(\lambda)$ is free of nuisance, and hence the critical values of the test could be computed. However, the empirical use of it would involve only a limited number of observations, so that it seems safer to rely on residual bootstrap methods based on simulations, as developed in Bec and Bastien (2007). In addition, Bai et al. (1998) provides for a simple way of computing confidence intervals around $\hat{\tau}$ (see equation 2.21), relying on the limiting distribution of the estimator (see theorem 4 in Bai et al (1998); the explicit function for the limit density comes from Picard (1985)).

During the estimation process, three different null hypotheses are tested. Indeed, if a structural break is detected for the VAR model as a whole, it is possible that it affects in fact only a sub-component of the model, namely the drift term μ_1 or the propagation mechanism B . The following null hypotheses are thus tested:

$$H_0 : \mu_1 = 0 \text{ and } B = 0 \qquad H_0^d : \mu_1 = 0 \qquad \text{and } H_0^p : B = 0$$

Table 1.1 Displays the results of the tests, for 5000 bootstrap replications. The first column displays the point estimate for the date of the break, while column 2 indicates the 95% confidence interval for the break. Columns 3,4 and 5 give the p-value for respectively H_0 , H_0^d and H_0^p . The last column indicates the date of the break for the alternative VARs with variables used in first difference.

Table 1.1
SupLR tests of a structural break

	Break date for the full process	95% confidence interval	p-value (full model)	p-value (drift)	p-value (propagation)	I(1) models
Germany	1989M6	1989M1:1989M11	0.008	0.23	0.009	1989M6
France	2000M11	2000M10:2000M12	0.099	0.537	0.090	1991M6
Italy	1992M7	1992M3:1992M11	0.008	0.359	0.002	1992M7
Spain	1992M2	1991M12:1992M4	0.000	0.000	0.000	1992M2
Portugal	1993M7	1993M4:1993M10	0.333	0.362	0.285	1993M4
Belgium	1990M9	1990M5:1991M1	0.117	0.044	0.067	1992M3
Ireland	1993M1	1992M10:1993M5	0.001	0.004	0.000	1993M1
Denmark	1983M9	1983M8:1983M11	0.000	0.155	0.000	1983M9
Sweden	2005M3	2005M2:2005M4	0.001	0.115	0.052	2005M3
UK	1984M2	1983M12:1984M4	0.000	0.502	0.000	1984M2

The first noteworthy result is that the breaks are overall extremely significant. Over the seven Euro countries, only one displays a clearly non significant break: Portugal. Then four out of six of the remaining Euro states have gone through a statistically significant structural break at the 5% level, while for France and Belgium the break is significant at the 10% level of significance. Looking at columns 4 and 5 of Table 1.1 however reveals that these breaks were mainly - when not only - due to the propagation mechanism in the VAR. While the drift component is often not significant, the propagation mechanism often manages to reject the null of no break, even at the 5% level. For France, Portugal and Belgium, the break remains non significant at the 5% level but the power increases when the sole propagation mechanism is considered. Besides, the dates of the partial breaks¹³ for France and Portugal match the dates for the process as a whole. All these results suggest that the Euro countries have faced a change in the very nature of their transmission mechanism, rather than a simple jump in the level of the variables. This supports the hypothesis of a drastic change in the economy dynamics, possibly due to the passage to a single currency.

The second result is that the break dates present at first sight only moderate homogeneity, suggestive that the results may reflect country-specific changes rather than a simultaneous break process. However, it is in fact possible to distinguish three cases within the ten countries.

¹³ Not reported in Table 1.1. The dates for France and Portugal when only the propagation mechanism is allowed to change are respectively 2000M11 and 1993M7. The complete set of dates for partial breaks is available upon request.

Germany on its own represents a specific case. The break date, 1989, does not overlap with the other six Euro countries. With a confidence interval expanding to the end of year 1989, it seems clear that the obtained date represents the German reunification. The economic consequences of this event were considerable: a massive influx of workers coming from the former Eastern Germany and willing to accept lower wages than their Western Germany compatriots contributed to create simultaneously a major supply shock and a large demand shock. The adoption of the Deutschmark as a single currency as soon as 1990 can also be assimilated to a sharp monetary shock. As such, no break due to the Euro implementation in 1999 is detected. This might be due to the overwhelming consequences of the reunification, but also to the fact that Germany is precisely the state whose monetary policy was used as a benchmark for the setting up of the Euro. As such, the switch from the Deutschmark to the Euro would not be constitutive of a structural break, but on the contrary can be described as a measure of reinforcement of the Bundesbank monetary policy.

The second group is composed by France, Italy, Spain, Belgium, Ireland and Portugal. These countries have gone through their breaks around two central dates: the year 1992, and the end of 2000 for France. There is a clear overlap of confidence intervals over the year 1992 for Italy, Spain and Ireland, despite extremely tight confidence bands around the point estimates. This date corresponds to the adoption and implementation of the Maastricht Treaty, which fixed the Euro convergence criteria. Even though these criteria were strictly speaking only implemented in 1994, governments and financial markets very often anticipate such constraining changes in order to be able to fulfil them in due time. This explains well the spread of the dates over years 1992 and 1993. In this respect, the Belgian break seems somewhat early. It must be noted however that it is not significant for the process as a whole. Allowing for a break in the model subcomponents only (drift term or propagation mechanism), one finds a significant break at the 5% level for the drift term corresponding to the date 1992M3, while the break for the propagation mechanism remains insignificant. The date of the significant break hence aligns with the other Euro countries. However, as the only country displaying a break in the drift term, Belgium remains an exception.

The French break occurring around 2001 is harder to interpret. A possible, although not very likely explanation is that this break corresponds to the introduction of the actual coins and notes within the Euro economies as a replacement of their former national currencies. Given that consumption represents the largest part of GDP, and that consumption transactions were

still realised in national currencies until 2002, the full impact of the passage to the single currency, especially on the demand side of the economies, might not have been felt before 2002.

A more plausible explanation can be found in the global recession that took place in the early 2000's. Many events contributed to this global slowdown, ranging from the dot-com bubble crash to the 9/11 events, as well as a succession of interest rate increases from the FED that sharply affected the US economy. Interestingly enough, in a European perspective, the Euro might still have played a role in this recession. Since its creation on in 1999, the Euro had its value continuously plummet, to reach a final 30% bilateral depreciation with the dollar by the end of 2001. Paired with the depreciation, a fuelling in inflation occurred during the summer 2001. The ECB reaction, which resulted in a rapid rise of the Euro in 2002 (parity with the dollar was reached on July 2002) nevertheless contributed to hurt companies based in Europe, as they suffered an unfavourable exchange rate. As such, these events may be captured by the identified break, in which case the Euro would remain at least part of the explanation.

Finally, it should be noted that the results are extremely robust to specifications in level of in first difference. The models assuming I(1) variables produce dates that are extremely close to their basic model specifications, except in the French case for which a break around 1992 is now identified.

The last group is formed by the three non-Euro countries. The United Kingdom has gone through a break at the beginning of 1984, Denmark went through a break in 1983, while Sweden faced its break at the end of 2004. The confidence bands of the three countries do not overlap with any of the Euro countries. These break dates are, however, not easy to interpret. In the United Kingdom, the economic policy implemented by Margaret Thatcher - mainly tighter monetary policy, tax changes and cuts in government spending as well as reformed industrial relations - had plunged the economy into a full-scale recession by the middle of the 1980's. This early 1980's recession, culminating with the 1984 miner's strike, might be the element identified by the break.

Denmark on the other hand was facing a major recession by the end of 1982, along with flaming inflation and high unemployment levels. The newly elected conservative government gradually implemented in 1983 a set of drastic global stabilization measures. The latter can be

divided in two broad categories: income policy measures on the one hand, such as the suspension of indexation of wages for the public and private sectors, or the profit freeze for goods sold in the country; and fiscal reforms on the other hand, such as the suspension of price indexation of certain social payments, a considerable increase in the contribution to unemployment insurance by workers and employers, or the introduction of taxes on pension funds and life insurances¹⁴. This fiscal consolidation initiated a highly unexpected recovery episode for the Danish economy: the budget deficit improved by 12.5% of GDP over the 1983-1986 period, while over the same period output growth reached a rate of 4% and inflation dropped from 9% to 3%. Such a radical turnout of the economy is likely to be the event identified by the break.

The Swedish break is definitely harder to interpret. The most recent noteworthy economic event took place in the beginning of the 90's, with the burst of the real estate bubble, a date far from the one identified by the break. The withdrawal of the ratification process for the European Constitutional Treaty in June 2005 was perceived as a major political event, but does not appear as a plausible explanation for a break in the dynamics of the economy. An early detection of the 2007 crisis may eventually remain a possible explanation for this date, which, otherwise, offers little meaningful economic interpretation.

The above results for the Euro countries suggest the possibility that the dynamic processes went through multiple breaks, around 1992 and 2001. More generally, to check whether a potential break may have been omitted, it seems relevant to test for potential multiple breaks following the econometric methodology developed by Qu and Perron (2007), that extends the analysis of Bai and Perron (1998) to a system of multivariate equations. Table 1.2 displays the results from the test run for all the countries¹⁵.

The table reports both the Sup test statistics for the null of no break against the alternative of m breaks, and the statistics for the sequential test of a null of ℓ breaks versus the alternative of $\ell + 1$ breaks. The breaks identified by the procedure are extremely similar to the breaks identified with the SupLR procedure used above, confirming the relevance of the previous

¹⁴ For a more comprehensive treatment of the Danish fiscal reforms of 1983, see e.g. Bergman and Hutchinson (2010).

¹⁵ the author wishes to thank Zhongjun Qu and Pierre Perron for kindly providing the Gauss codes used to estimate the dates and test statistics for the breaks, as well as Frédérique Bec for kindly providing the Gauss codes used to derive the exact distribution of the different test statistics.

results. The multiple break tests also suggest that most country have been facing no more than one single break. The SupLR 0/1 rejects the null of no break in favour of the alternative of one break in six out of ten cases. However, the SupLR 0/2 and the Seq 2/1 tests most of the time yield contradictory results regarding the occurrence of a second break: eventually, the two tests agree in only two cases, namely Spain and Ireland.

Table 1.2
Qu and Perron tests for multiple structural breaks

	Break 1	Break 2	SupLR 0/1 break	SupLR 0/2 breaks	Seq Test 2/1
Germany	1989M10	2005M3	146.9/156.9	245.7/333.1	145.8/162.2
France	2000M2	1984M5	106.8/105.7	198.7/215.1	123.3/109.7
Italy	1992M9	1998M5	125.3/123.6	277.1/253.2	101.6/127.8
Spain	1992M4	2004M2	212.8/189.7	496.9/426.5	285.3/197.3
Portugal	1994M4	2001M5	144.3/189.7	236.2/426.5	156.9/197.3
Belgium	1991M6	2001M8	81.3/123.6	123.3/253.2	80.6/127.8
Ireland	1993M1	1998M10	166.3/91.3	376.1/187.4	140.9/95.4
Denmark	1983M10	1998M11	84.9/156.9	266.4/333.1	163.1/162.2
Sweden	2005M3	1995M4	198.4/136.6	296.3/287.7	99.3/141.7
UK	1984M7	2002M4	56.3/91.3	113.7/187.4	58.6/95.4

First number in the last three columns represents the test statistics, while numbers in italic represent the 5% critical value for the test

Interestingly enough, Italy and Ireland prove to have faced a second break around 1999, the date of the official settlement of the Euro currency, while in the cases of Portugal and Belgium the second break is estimated around 2001, a date similar to the initial French break. While these second breaks are not significant (except for Ireland), they provide further evidence of simultaneous shifts in the Euro countries.

Finally, it is to be noted that the second breaks for the United Kingdom and Denmark turn out to be close to the secondary breaks of some Euro countries. For the Danish case, the results of the SupLR and Seq tests remain ambiguous: this implies that the passage to the Euro might after all have affected in some way the dynamics of the economy. On the other hand, for the United Kingdom, the second break would appear as unambiguously insignificant.

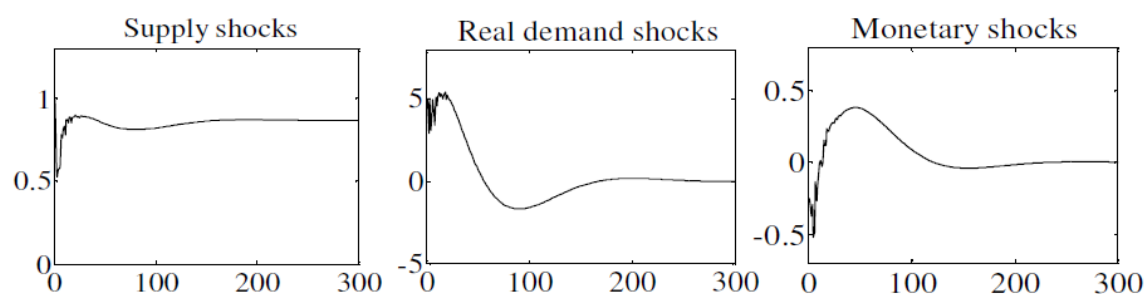
1.3. The Dynamic effect of aggregate supply and aggregate demand shocks

Once the breaks are estimated, it is possible to obtain the impulse response functions to supply, real demand and monetary shock. For each country, the Blanchard and Quah decomposition was applied to both subsamples, prior to the break and after it. For each subsample, the lag number was once again selected following the Akaike Criterion and the minimum lag length succeeding in eliminating the residual autocorrelation. As an illustration, Figure 1.1 displays the simulated impulse response functions for the first subsample of France¹⁶. This figure plots the responses to a unit shock.

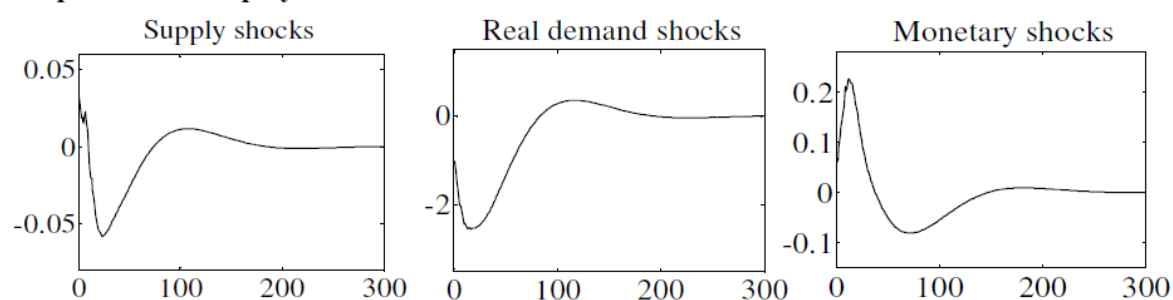
Figure 1.1

Impulse response functions for France, 1979M1-2000M11

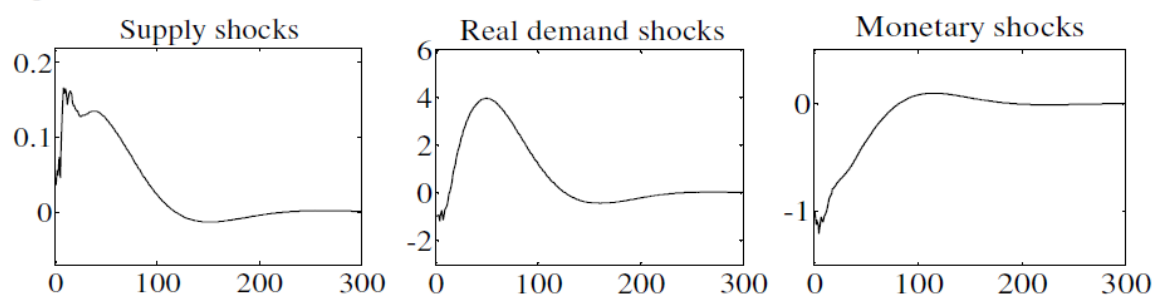
Response of output:



Response of unemployment:



Response of interest rate:



¹⁶ In the present section and the incoming one, the text only displays the results for those countries which are the most characteristic for our purpose. The full set of tables and graphs for the impulse response functions, forecast error variance decomposition and historical decompositions are available from the author upon request.

The results are globally consistent with a traditional view of the dynamic effect of supply and demand shocks. A positive supply shock has a positive, permanent effect on output, stabilizing after approximately 15 years. Real demand shocks have a positive effect on output, stabilizing to zero after roughly 20 years. The same holds for monetary shocks, but the effect of the shock is negative over the first few periods, before it recovers its expected positive sign. A similar phenomenon occurs for the unemployment rate: both real demand and monetary shocks contribute to reduce unemployment temporarily, but again the initial response to monetary shocks turns out to be negative. Supply shocks also trigger an initial increase in unemployment, before the effect reverses to negative after a few months.

Explaining the unexpected sign of the responses to monetary shocks over the first few periods is difficult. However, these results seem to be relatively robust, since they are observed for most countries included in the study. Although surprising, they are consistent with the monetary 'puzzles' identified by e.g. Eichenbaum (1992) or Gordon and Leeper (1994)¹⁷. On the other hand, as emphasized in Blanchard and Quah (1989), the initial positive response of unemployment to supply shocks suggests the presence of rigidities, both real and nominal, within the economy. A positive supply shock contributes to temporarily increase the interest rate, supporting the classical view that capital is remunerated at its marginal productivity. A real demand shock increases the interest rate in the short run, whereas a positive monetary shock drops it temporarily, in accordance with traditional Keynesian theories. Impulse responses for the other countries are most of the times qualitatively similar. Some of them do display unexpected signs, but usually recover a correct sign in the long run¹⁸.

The simulated responses, along with their bootstrap 95% confidence interval, are displayed in Appendix A.4. The responses are qualitatively speaking quite similar, and in majority they are significant at least over a few periods, for all the three variables.

¹⁷ According to these authors, the observed puzzles may be essentially due to imperfect measurements of monetary shocks and monetary policy. In particular, Gordon and Leeper (1994) show that a general equilibrium model separating the Reserve market and the M2 market achieves results consistent with conventional theory. As such is not the case with the present Blanchard and Quah decomposition, this may represent a possible explanation of the observed results.

¹⁸ Finally, it must be noted that models where variables are taken in first difference produce responses in accordance with the aforementioned results. Of course, the unit root in the process implies that the shocks will produce a permanent effect, except for the variables carrying long-run restrictions.

1.4. Relative contributions of supply and demand disturbances

Once the relevance of the dynamics of the innovations has been established from the impulse response functions, the next step consists in assessing the relative contribution of each shock to output, unemployment, and the interest rate. There are two ways to do so. The first way entails the use of the forecast error variance decomposition at various horizons. The second is informal and implies the use of an historical decomposition analysis.

Table 1.3 presents the forecast error variance decomposition for output and the interest rate for Spain, prior to the estimated break and after the break¹⁹. In the table, for each subsample, the first three columns present the respective contributions of supply shocks, real demand shocks and monetary shocks in the variable's fluctuations. The fourth column indicates the part played by monetary shocks as compared to the total demand component, defined as the addition of the contributions of real demand shocks and monetary shocks.

The forecast error variance decomposition indicates that the role played by the supply side of the economy has considerably increased. This is true at short horizons for both output and the interest rate. At very long horizons, supply shocks dominate the decomposition of both variables for the second subsample. This is hardly surprising, since the convergence criteria settled by the Maastricht Treaty constrained the demand side of all the Euro candidates from 1992 on.

The second result of interest is the sharp increase in the part played by monetary shocks when considering the total demand side of the economy. This is true at any horizon for output; for the interest rate, the role of monetary shocks within demand actually shrinks in the short run, but increases in the long run. At long horizons, the monetary component becomes in fact majority within demand for both variables.

¹⁹ The main text only presents selected tables for output and the interest rate; Appendix A.5 provides the reader with additional synthetic tables including the decomposition unemployment, along with other results mentioned later on in this section. The full set of tables is available upon request.

Table 1.3
Forecast Error Variance Decomposition, Spain

Decomposition of Output									
Pre-break : 1979M1-1992M2					Post-break : 1992M3-2011M1				
Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand	Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand
3 months	75.14	19.54	5.31	21.37	3 months	90.34	1.69	7.97	82.47
6 months	69.12	22.07	8.82	28.55	6 months	80.55	3.98	15.47	79.52
9 months	65.87	23.69	10.43	30.57	9 months	67.79	6.13	26.09	80.98
1 year	63.85	25.15	10.99	30.41	1 year	58.01	10.93	31.06	73.97
2 years	64.20	19.91	15.89	44.37	2 years	47.38	25.18	27.44	52.14
3 years	62.55	17.74	19.71	52.63	3 years	45.92	30.80	23.29	43.05
5 years	59.11	18.03	22.86	55.91	5 years	42.40	28.04	29.56	51.33
10 years	53.14	26.34	20.51	43.78	10 years	53.91	16.17	29.92	64.92
15 years	52.04	28.62	19.33	40.32	15 years	64.58	12.74	22.68	64.02

Decomposition of Interest rate									
Pre-break : 1979M1-1992M2					Post-break : 1992M3-2011M1				
Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand	Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand
3 months	5.80	58.09	36.10	38.33	3 months	8.71	84.88	6.41	7.02
6 months	5.46	58.51	36.03	38.11	6 months	18.28	75.58	6.14	7.52
9 months	4.94	57.39	37.67	39.62	9 months	28.66	64.17	7.17	10.05
1 year	4.85	56.85	38.30	40.26	1 year	36.17	51.97	11.86	18.59
2 years	4.89	56.61	38.51	40.49	2 years	47.63	30.87	21.51	41.06
3 years	4.86	55.63	39.51	41.53	3 years	49.50	26.49	24.01	47.54
5 years	5.61	56.03	38.36	40.64	5 years	50.41	21.75	27.85	56.15
10 years	6.11	57.15	36.75	39.13	10 years	51.09	19.83	29.08	59.46
15 years	6.19	57.17	36.64	39.06	15 years	51.24	19.50	29.26	60.00

These features globally hold for the other Euro countries. On the one hand, the rise in the role of the supply component after the break is generally verified. On the other hand, the rise in the role played by monetary shocks as compared to total demand is also verified, but typically in the long run only²⁰. Two noteworthy exceptions are found in the case of Germany and Ireland for which the role of monetary disturbances within total demand actually shrinks at any time horizons after the break. These results are hardly surprising: as Ireland went through a period of fast economic growth and development over the 1990 decade until the 2007 financial crisis, inflationary pressures appeared and the restrictive monetary policy led by the ECB must have

²⁰ Precisely: for a 3-variable VAR model over 7 Euro countries, i.e. a total of 21 variables, the increase – at least in the long run – of the role played by supply shocks after the break is verified for 16 out of 21 variables, while the increase of monetary shocks within total demand is verified in 15 out of 21 variables.

acted as a strong stabilization tool. Besides, Germany is well-known to be the State whose monetary policy largely inspired the policy of the ECB. In this respect, the Euro implementation represents more of a strengthening of the Bundesbank's action than a change in the monetary policy.

For the other Euro countries, the short-run decrease in the role of monetary shocks tends to confirm the stabilizing role of the monetary policy led by the ECB, while the long run increase suggests that such a policy might eventually create larger fluctuations, for example if the ECB restrictive policy differs much from the one that was run by the National authorities prior to the break.

Nevertheless, the comparison with Denmark, Sweden and the United Kingdom shows that these features do not generally hold outside the EMU. Table 1.4 displays the forecast error variance decomposition for Denmark. It should be noted first that the increase in the role played by the supply side of the economy after the break is not verified. It is not verified for output and is not verified either for the interest rate. In addition, the increase in the role played by the monetary component is also qualified. If the role played by monetary disturbances has increased for output, it has shrunk in the case of the interest rate. Results are not so clear-cut for the other two non-Euro countries, but they globally present the same characteristics as Denmark. In the case of Sweden, the increase in the role played by monetary policy is verified neither for output nor for the interest rate; as for the increase in the role played by supply, it is verified, but only in the very long run for output. For the United Kingdom, both the increase in the role played by supply shocks and the increase of the part of monetary shocks within total demand are not verified for output. In addition, for the interest rate, in the very long run, the part played by supply shocks after the break actually shrinks²¹.

These results tend to confirm that the role played by money after the break has more specifically increased in the Euro countries as compared to other Member States of the European Union. Hence the passage to the Euro might well be the main explanation for the break, implying a strengthening of the impact of unexpected monetary policy within the new single-currency area.

²¹ Eventually, for the three non-Euro countries, over their three variables – i.e. 9 variables – the increase in the role of supply shocks is verified in only 5 cases, and the increase in the role played by the monetary component in 5 cases, a result clearly less conclusive than for Euro countries.

Table 1.4
Forecast Error Variance Decomposition, Denmark

Decomposition of Output									
Pre-break : 1979M1-1992M2					Post-break : 1992M3-2011M1				
Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand	Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand
3 months	56.15	30.79	13.07	29.80	3 months	47.79	6.87	45.34	86.83
6 months	43.59	46.91	9.51	16.85	6 months	43.24	10.11	46.64	82.18
9 months	35.37	56.28	8.35	12.92	9 months	38.44	13.00	48.56	78.88
1 year	32.82	59.70	7.48	11.13	1 year	34.18	15.78	50.04	76.03
2 years	35.29	59.73	4.98	7.69	2 years	30.48	24.19	45.33	65.20
3 years	48.76	47.75	3.50	6.82	3 years	38.59	24.73	36.68	59.73
5 years	71.40	26.74	1.85	6.49	5 years	58.23	16.62	25.14	60.20
10 years	88.85	10.43	0.72	6.45	10 years	76.06	9.55	14.39	60.11
15 years	93.25	6.32	0.44	6.45	15 years	83.28	6.69	10.02	59.95

Decomposition of Interest rate									
Pre-break : 1979M1-1992M2					Post-break : 1992M3-2011M1				
Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand	Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand
3 months	9.85	56.38	33.77	37.45	3 months	2.75	76.73	20.51	21.09
6 months	11.66	52.75	35.59	40.29	6 months	3.79	71.49	24.72	25.69
9 months	11.00	52.32	36.68	41.21	9 months	3.48	69.48	27.05	28.02
1 year	11.27	52.08	36.65	41.31	1 year	3.01	67.25	29.75	30.67
2 years	11.31	51.82	36.87	41.57	2 years	2.02	60.11	37.87	38.65
3 years	11.41	51.78	36.81	41.55	3 years	1.74	58.05	40.22	40.93
5 years	11.48	51.76	36.76	41.52	5 years	2.20	59.56	38.24	39.10
10 years	11.49	51.76	36.75	41.52	10 years	2.78	61.44	35.78	36.80
15 years	11.49	51.76	36.75	41.52	15 years	2.83	61.67	35.50	36.53

In section 1.3, it was found that two Euro States, namely Spain and Ireland, had gone through two statistically significant breaks. Hence, as a check for robustness, the forecast error variance decomposition is also estimated for the two subsamples following the main 1992 break, in order to check for possible variations across secondary breaks (see Table A.7 in Appendix A.5). The conclusion is that even when allowing for multiple breaks, the previous conclusions are not altered: the part of the variation due to supply shocks typically increases for both subsamples after the main break. In addition, in the case of Ireland, the part of output variation due to monetary disturbances as compared to total demand drastically shrinks for the two subsamples subsequently to 1992. This result is similar to the findings with only one break, and confirms the strong stabilizing role held by the settlement of the single monetary policy. In the case of Spain, the increase of the role of monetary shocks is more specifically

verified after the second break in 2004. This is consistent with an initial phase of successful stabilization of the activity over the 1992-2004 period, with the action of the by the ECB from 1999 on, and a second phase of larger fluctuations due to monetary shocks after 2004, which probably embeds the 2007 financial crisis. Finally, an analysis of the forecast error variance decomposition for models in first difference reveals essentially similar features. The results, however are not as clear-cut as for the models in level: they either confirm the previous conclusions, or display a relative stability of the role of the diverse components across breaks. They however do not contradict the findings of the models in level.

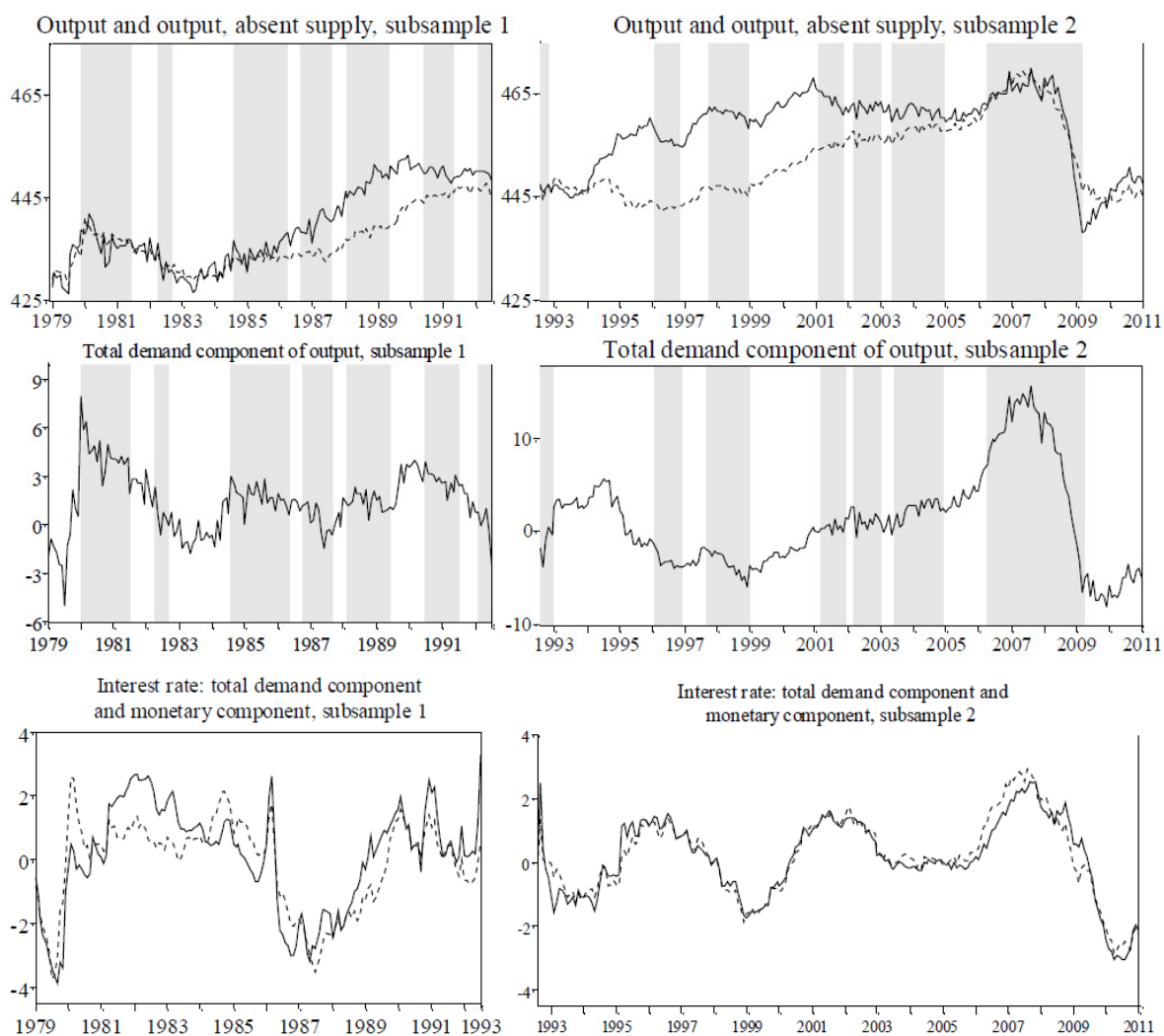
The contributions of each type of shock can also be assessed from a historical analysis of output fluctuations. Figure 1.2 displays the historical decomposition for Italy prior to the break (left panels) and subsequently to the break (right panels). The top graphs represent actual output (plain line) along with its forecast when the supply component is set to zero in the structural decomposition (dotted line). The second row on the other hand represents the total demand contribution to output fluctuation. Superimposed on these graphs are the growth cycles as identified by the ECRI²². Each shaded area starts with a peak and ends up with a trough.

If output is predominantly influenced by demand disturbances, the business cycles and the slope of the demand components must be correlated. Namely, demand components should have high values at peaks and low values at troughs. This is exactly what happens for the first subsample: the business cycles match fairly well the peaks and troughs of the demand components predicted by the model, except for the 1988-1989 cycle.

In addition, the top panel of Figure 1.2 shows that there is little difference between actual output and output, absent supply. The gap widens over the 1987-1992 period but the two components remain relatively close. This suggests a period where the supply component generate little variation in output, while demand components play the major part in explaining the fluctuations.

²² European Cycle Research Institute, New York and London. One should note however that the ECRI does not provide business cycles estimates for every country within the European Union. For estimates of growth cycles of such countries, one must then rely on individual research papers, potentially implementing different methodologies and covering shorter periods. Estimates for Portugal can be found e.g. in Castro (2011), estimates for Belgium in Bodart et al. (2006), some estimates for Ireland in Artis et al. (1997) and some estimates for Denmark in Christoffersen (2000). Recent research papers also propose dates similar to the ones proposed by the ECRI for growth business cycles within the European Union. See e.g. Anas and Ferrara (2004) or Artis, et al. (2003).

Figure 1.2
Historical decomposition of output : Italy



The situation becomes different when the second subsample is considered. The relation between the demand components and the identified business cycles (middle panel) remains verified over the 1996-1999 period, but becomes quite loose otherwise. It also appears that on the top panel, a tremendous gap appears between actual output and its forecast, absent supply, until 2007. This suggests a period where the supply component now dominates output fluctuations. It is however interesting to notice how close actual output and output absent demand become again after 2007. This is no doubt due to the financial crisis which resulted in monetary and fiscal components originating large output fluctuations.

Finally, the closer relation between the monetary component and total demand component following the break is perhaps best illustrated by the interest rate decomposition: prior to the break, some fair variation is allowed between total demand and its monetary component,

while after the break the two components are virtually similar. This suggests a period where most of the fluctuations due to total demand are explained by the monetary component.

1.5. Conclusion

The main objective of this work was to assess the dynamic effect of aggregate supply and aggregate demand disturbances within the European Union, in order to obtain an overview of the monetary consequences of the Euro implementation. This led to identify and test for the presence of a structural break. The main result obtained is the presence of structural breaks in the dynamic process of supply and demand shocks for Euro countries, occurring in 1992 and 2001. These breaks can be attributed to the adoption of the Maastricht Treaty for the year 1992, and possibly to the Euro introduction in 1999 and 2001, along with the consequences of the 2001 global recession. No such breaks are found for non-Euro countries such as Sweden and the United Kingdom. The second noteworthy result is the sharp increase in the importance of supply shocks in the variations of output, unemployment and interest rate in Euro countries after these breaks. Besides, although the dynamic effect of the demand component as a whole declined subsequently to the break, the part played by the monetary component within demand has increased at long horizons for the Euro countries. The results do not generally hold for Denmark, Sweden and the United Kingdom, the three non-Euro countries.

Appendix A.1: Data sources

Germany:

Industrial production index: OECD Stat Extract (<http://stats.oecd.org>): Main Economic Indicators, MEI Original release data and revisions, Index of Industrial Production.

Unemployment rate: OECD Stat Extract (<http://stats.oecd.org>): Registered Unemployment and Job Vacancies (MEI): Registered unemployment rates and levels.

Interest Rate: OECD Stat Extract (<http://stats.oecd.org>): Financial Indicators (MEI): Interest rates, short-term interest rates.

France:

Industrial production index: OECD Stat Extract (<http://stats.oecd.org>): Main Economic Indicators, MEI Original release data and revisions, Index of Industrial Production.

Unemployment rate: Standardized Unemployment Rate; OECD Main economic indicators via Datastream, series: 'FROCSUN%E'.

Interest Rate: Central Government Bond Yield via Eurostats , series 'irt_h_cgby_m'.

Italy:

Industrial production index: OECD Stat Extract (<http://stats.oecd.org>): Main Economic Indicators, MEI Original release data and revisions, Index of Industrial Production.

Unemployment rate: Harmonized Unemployment Rate; OECD Main economic indicators via Datastream, series: 'ITOUN014Q'.

Note: the series starts in December 1979. The few missing months are obtained by chaining on the growth rate of: OECD Economic Outlook via Datastream, series: 'ITOCFUNRQ'.

Interest Rate: Money Market Rate (Federal Fund), IMF International Financial Statistics via Datastream, series: 'ITI60B..'.

Spain:

Industrial production index: Production Index of Total Industry, OECD Main Economic Indicators via Datastream, series 'ESOPRI35H'.

Unemployment rate: OECD Stat Extract (<http://stats.oecd.org>): Main Economic Indicators, MEI Original release data and revisions, Harmonized Unemployment Rates (MEI).

Interest Rate: Central Government Bond Yield via Eurostats, series: 'irt_h_cgby_m'.

Belgium:

Industrial production index: Production Index of Total Industry, OECD Main Economic Indicators via Datastream, series 'BGOPRI35G'.

Unemployment rate: Registered Unemployment rate; OECD Main Economic Indicators via Datastream, series 'BGOUN013Q'.

Interest Rate: Treasury Bill Rate, IMF International Financial Statistics via Datastream, series: 'BGI60C..'.

Ireland:

Industrial production index: OECD Stat Extract (<http://stats.oecd.org>): Main Economic Indicators, MEI Original release data and revisions, Index of Industrial Production.

Unemployment rate: Harmonized Unemployment Rate; OECD Main Economic Indicators via Datastream, series: 'IROUN014Q'.

Note: the series starts in 1983. For the initial periods, the series is chained on the growth rate of the unemployment rate, computed from the Datastream series 'IROUN0080' and 'IROCFEMPO'.

Interest Rate: 3-month rates for Euro area countries via Eurostats, series 'irt_h_mr3_m'.

Portugal:

Industrial production index: Industrial Production Index, IMF International Financial Statistics via Datastream, series: 'PTI66..CE'

Unemployment rate: Harmonized Unemployment Rate, OECD Main Economic Indicators via Datastream, series: 'PTOUN014Q'.

Note: the series starts in 1983. For the initial periods, the series is chained on the growth rate of the unemployment rate, computed from the Datastream series 'PTOUN0080' and 'PTOCFEMPO'

Interest Rate: 3-month rates for Euro area countries via Eurostats, series 'irt_h_mr3_m'.

Denmark:

Industrial production index: Industrial Production Index, IMF International Financial Statistics via Datastream, series: 'DKI66..BH'.

Unemployment rate: Harmonized Unemployment Rate, OECD Main Economic Indicators via Datastream, series: 'DKOUN014Q'.

Note: the series starts in 1982. For the initial periods, the series is chained on the growth rate of the series: Registered Unemployment Rate; OECD Main Economic Indicators via Datastream, series: 'DKOUN013Q'.

Interest Rate: Interbank 3-month rates via Datastream, series: 'DKESSFON'.

Sweden:

Industrial production index: Industrial Production Index, IMF International Financial Statistics via Datastream, series: 'PTI66..CE'.

Unemployment rate: Survey-Based Unemployment Rate; OECD Main Economic Indicators via Datastream, series: 'SDOUN015Q'.

Interest Rate: OECD Stat Extract (<http://stats.oecd.org>): Financial Indicators (MEI): Interest rates, immediate interest rates.

United Kingdom:

Industrial production index: Industrial Production Index, IMF International Financial Statistics via Datastream, series: 'UKI66..IG'.

Unemployment rate: Registered Unemployment Rate; OECD Main Economic Indicators via Datastream, series: 'UKOUN013Q'.

Interest Rate: 3-month LIBOR, IMF International Financial Statistics via Datastream, series: 'UKI60EA'.

Appendix A.2: Banerjee, Lumsdaine and Stock unit root tests

Table A.1

BLS Unit-root tests: Log(output)

	Recursive				Rolling			Sequential			
	t_{DF}	\hat{t}_{DF}^{\max}	\hat{t}_{DF}^{\min}	\hat{t}_{DF}^{diff}	\bar{t}_{DF}^{\max}	\bar{t}_{DF}^{\min}	\bar{t}_{DF}^{diff}	Shift in mean		Shift in trend	
								$\tilde{t}_{DF}(\tilde{\delta})$	\tilde{t}_{DF}^{\min}	$\tilde{t}_{DF}(\tilde{\delta})$	\tilde{t}_{DF}^{\min}
Germany	-4.78	-1.11	-4.95	3.83	-0.87	-4.97	4.10	-2.30	-3.35	-1.05	-3.38
France	-3.33	-1.40	-4.27	2.86	-0.88	-8.77	7.89	-2.92	-2.93	-3.31	-3.55
Italy	-1.31	-0.90	-2.29	1.39	-0.60	-6.98	6.38	-1.31	-1.45	-2.02	-2.04
Spain	-2.07	-1.62	-3.59	1.97	-0.65	-5.33	4.68	-1.50	-1.51	-2.26	-2.26
Belgium	-5.28	-2.36	-6.03	3.67	-1.45	-5.85	4.40	-1.79	-2.39	-2.75	-2.75
Ireland	-0.82	-0.40	-2.32	1.92	-0.43	-3.79	3.37	0.12	-0.68	-0.42	-0.57
Portugal	-1.52	-1.10	-2.30	1.20	-0.70	-2.63	1.93	-0.70	-0.70	-0.88	-0.89
Denmark	-4.74	-0.97	-6.74	5.97	-1.60	-6.01	4.41	-1.44	-1.49	-2.41	-2.41
United Kingdom	-2.92	-0.69	-3.43	2.74	-0.80	-9.59	8.78	-0.14	-0.62	-1.00	-1.03
Sweden	-3.90	-1.63	-5.43	3.80	0.19	-4.99	5.18	-1.32	-2.04	-2.53	-2.53
5%critical value	-3.42	-1.92	-4.18	3.57	-1.47	-4.79	4.65	-4.77	-4.78	-4.39	-4.39

5% critical values from Banerjee et Al. (1992), tables 1 and 2, for T=500. The number of lags included for the computation of each statistics was determined in accordance with the AIC criterion.

Table A.2

BLS Unit-root tests: Unemployment

	Recursive				Rolling			Sequential			
	t_{DF}	\hat{t}_{DF}^{\max}	\hat{t}_{DF}^{\min}	\hat{t}_{DF}^{diff}	\bar{t}_{DF}^{\max}	\bar{t}_{DF}^{\min}	\bar{t}_{DF}^{diff}	Shift in mean		Shift in trend	
								$\tilde{t}_{DF}(\tilde{\delta})$	\tilde{t}_{DF}^{\min}	$\tilde{t}_{DF}(\tilde{\delta})$	\tilde{t}_{DF}^{\min}
Germany	-9.54	-0.66	-10.86	10.20	-0.46	-9.85	9.39	-8.95	-9.63	-8.70	-8.93
France	-10.34	0.29	-10.34	10.64	0.18	-19.05	19.24	-12.49	-12.49	-6.15	-8.04
Italy	-5.01	0.38	-5.03	5.41	1.58	-7.49	9.06	-5.84	-6.73	-3.38	-3.91
Spain	-3.58	0.93	-3.70	4.64	1.19	-4.69	5.88	-3.44	-3.72	-2.68	-2.80
Belgium	-10.58	-0.24	-10.61	10.35	-0.55	-12.51	11.96	-6.32	-10.30	-5.42	-9.85
Ireland	-2.42	0.15	-2.61	2.76	6.73	-9.59	16.33	-2.09	-2.99	-2.20	-2.45
Portugal	-1.71	-0.15	-4.86	4.71	-0.34	-9.33	8.98	-3.40	-3.40	-4.85	-4.86
Denmark	-8.73	-1.67	-9.42	7.75	-0.40	-10.82	10.42	-4.33	-7.79	-7.23	-7.23
United Kingdom	-8.80	-0.45	-8.87	8.42	1.71	-17.96	19.68	-7.96	-7.98	-6.63	-7.12
Sweden	-8.43	1.71	-8.43	10.14	1.63	-10.32	11.95	-4.60	-5.80	-2.89	-5.92
5%critical value	-3.42	-1.92	-4.18	3.57	-1.47	-4.79	4.65	-4.77	-4.78	-4.39	-4.39

5% critical values from Banerjee et Al. (1992), tables 1 and 2, for T=500. The number of lags included for the computation of each statistics was determined in accordance with the AIC criterion.

Table A.3

BLS Unit-root tests: Interest rate

	Recursive				Rolling			Sequential			
	t_{DF}	\hat{t}_{DF}^{\max}	\hat{t}_{DF}^{\min}	\hat{t}_{DF}^{diff}	\bar{t}_{DF}^{\max}	\bar{t}_{DF}^{\min}	\bar{t}_{DF}^{diff}	Shift in mean		Shift in trend	
								$\tilde{t}_{DF}(\tilde{\delta})$	\tilde{t}_{DF}^{\min}	$\tilde{t}_{DF}(\tilde{\delta})$	\tilde{t}_{DF}^{\min}
Germany	-12.37	-1.75	-12.37	10.62	-0.12	-12.06	11.93	-6.75	-8.56	-1.20	-8.41
France	-9.71	-1.04	-9.71	8.67	-1.41	-10.26	8.85	-9.04	-10.12	-6.87	-8.32
Italy	-5.92	-0.43	-5.92	5.49	0.53	-6.78	7.31	-4.78	-5.86	-5.11	-5.62
Spain	-8.07	-0.86	-8.24	7.38	-1.54	-8.79	7.25	-5.05	-7.38	-6.67	-7.46
Belgium	-7.71	-1.33	-7.72	6.39	-0.57	-18.09	17.52	-7.56	-8.64	-5.18	-7.62
Ireland	-8.81	-0.52	-8.81	8.29	0.45	-31.82	32.27	-7.20	-10.40	-7.33	-9.36
Portugal	-3.81	-0.44	-3.81	3.38	-0.15	-8.43	8.28	-3.95	-3.95	-2.39	-3.16
Denmark	-11.17	-1.01	-11.17	10.17	-0.66	-14.48	13.82	-5.68	-10.11	-6.89	-9.29
United Kingdom	-7.11	-0.92	-7.11	6.20	0.09	-12.71	12.80	-5.28	-6.93	-7.20	-7.20
Sweden	-6.51	-0.89	-6.51	5.62	-1.53	-16.32	14.79	-6.01	-6.90	-2.95	-6.34
5%critical value	-3.42	-1.92	-4.18	3.57	-1.47	-4.79	4.65	-4.77	-4.78	-4.39	-4.39

5% critical values from Banerjee et Al. (1992), tables 1 and 2, for T=500. The number of lags included for the computation of each statistics was determined in accordance with the AIC criterion.

Appendix A.3: the Blanchard and Quah decomposition

Retaining the formulation of (1.16), a n -variables VAR model can be written as:

$$X_t = A(L)X_t + \varepsilon_t \quad (\text{A.1})$$

where $A(L)$ denotes a lag polynomial, and where $\Sigma = E(\varepsilon_t \varepsilon_t')$ denotes the variance-covariance matrix of the vector of residuals. Because the roots of the lag polynomial are assumed to lie within the unit circle, the Wold theorem applies and the VAR model can be inverted into an infinite-order moving average:

$$X_t = C(L)\varepsilon_t \quad (\text{A.2})$$

where $C(L)$ denotes a lag polynomial of infinite order in the residuals of the model. From this model, one wants to identify a size n vector of fundamental or structural shocks η_t . These fundamental disturbances are supposed to be uncorrelated, so that their variance-covariance matrix is diagonal. A convenient normalization assumption then allows to assume that this matrix is identity:

$$I = E(\eta_t \eta_t') \quad (\text{A.3})$$

with I the n -dimension identity matrix. It is furthermore assumed that the VAR residuals are linear composites of the structural shocks. Then, there exists a $n \times n$ matrix S so that:

$$\varepsilon_t = S\eta_t \quad (\text{A.4})$$

Combining (A.2) and (A.4) implies that the VAR can be expressed as an infinite-order moving average in the structural disturbances:

$$X_t = C(L)\varepsilon_t = C(L)S\eta_t = \hat{C}(L)\eta_t \quad (\text{A.5})$$

with $\hat{C}(L) = C(L)S$ an infinite-order lag polynomial in the fundamental shocks.

To identify S , one may further note that combining (A.3) and (A.4) yields:

$$\Sigma = E(\varepsilon_t \varepsilon_t') = E(S\eta_t \eta_t' S') = SE(\eta_t \eta_t')S' = SIS' = SS' \quad (\text{A.6})$$

For an n -variable VAR, S has n^2 entries to identify. Because (A.6) imposes only $\frac{n(n+1)}{2}$

restrictions on S , $\frac{n(n-1)}{2}$ additional constraints must be imposed on the VAR to identify S .

The Blanchard and Quah identification methodology consists in imposing long-run neutrality restrictions on the VAR behaviour with respect to the structural shocks. That is, one must impose that $\frac{n(n+1)}{2}$ entries of $\hat{C}(1)$ to be 0. The most convenient case is to impose these constraints so that $\hat{C}(1)$ is lower triangular. S then becomes easy to recover. Indeed, consider the $n \times n$ matrix M defined as:

$$M = C(1)\Sigma C(1)' \quad (\text{A.7})$$

Definition (A.7) implies that M is symmetric with non-zero entries²³. Then from (A.5) and (A.6), one obtains:

$$M = C(1)\Sigma C(1)' = C(1)SS'C(1)' = \hat{C}(1)\hat{C}(1)' \quad (\text{A.8})$$

Because M is symmetric, it has a Choleski factor, that is, there exists a $n \times n$ lower triangular matrix G such that:

$$GG' = M \quad (\text{A.9})$$

Then (A.8) and (A.9) imply that:

$$G = \hat{C}(1) \quad (\text{A.10})$$

S can then be recovered by combining (A.5) and (A.10):

²³ M can actually have one or more 0 entries, but only in degenerate cases, that is if $C(1)$ is zero and/or Σ is a zero matrix (implying that the VAR residuals have variance and covariance equal to zero).

$$\begin{aligned}
\hat{C}(L) &= C(L)S \\
\Rightarrow \hat{C}(1) &= C(1)S \\
\Rightarrow S &= C(1)^{-1}\hat{C}(1) \\
\Rightarrow S &= C(1)^{-1}G
\end{aligned} \tag{A.11}$$

The present paper uses this identification scheme by setting long-run constraints so that $\hat{C}(1)$ is lower triangular. Blanchard and Quah (1989) show that S is always identified when $\hat{C}(1)$ is lower triangular and when $n = 2$. The proof generalizes in a straightforward way for $n > 2$ when $\hat{C}(1)$ is lower triangular, so that identification of the structural decomposition is always guaranteed in this paper.

It is actually possible to get even more. S may be identified even if $\hat{C}(1)$ is not lower triangular. Define the *pseudo Choleski factor* of an $n \times n$ symmetric matrix M as the $n \times n$ matrix G such that $\frac{n(n-1)}{2}$ entries of G are 0, and such that $GG' = M$. In other words, G has as many 0 entries as a normal Choleski factor, but these entries are not necessarily located on the upper triangular parts of G . Assume then that one wants to set the long-run restrictions on $\hat{C}(1)$ so that $\hat{C}(1)$ is not lower triangular. That is, $\hat{C}(1)$ has $\frac{n(n-1)}{2}$ entries which are zero, but these are not necessarily located on the upper triangular part of $\hat{C}(1)$. Then, if one can identify a pseudo Choleski factor of M (as defined in (A.8)), with the 0 entries located in accordance with $\hat{C}(1)$, (A.10) will still hold so that S can be recovered from (A.11). S is not always identified from this scheme, however, when $\hat{C}(1)$ is not lower diagonal. It actually turns out that for certain specific sets of constraints, S is never identified. This is stated in the next lemma.

Lemma: assume a n -variable VAR similar to (A.1) and consider the Blanchard and Quah decomposition for this VAR model. If the set of long-run constraints on $\hat{C}(1)$ is such that for any two rows i and j of $\hat{C}(1)$, each column of $\hat{C}(1)$ comprises a zero entry in either row i or row j , then S is not identified.

Proof:

The proof proceeds by contradiction. Assume that S exists. Then (A.1)-(A.11) hold. In particular, from (A.8), one gets $M = \hat{C}(1)\hat{C}(1)'$. Consider entries $m_{i,j}$ of M . Because $m_{i,j}$ is the vector product of row i of $\hat{C}(1)$ with row j of $\hat{C}(1)$, and because at least one of the two vectors has a 0 for each of the n entries, then this product is zero. Therefore $m_{i,j}$ is equal to zero. But this is a contradiction, as (A.7) implies that M is symmetric with non-zero entries. \square

For this reason, the structural decomposition implemented in the present paper is not identified if one assumes that the effect of supply shocks on unemployment or the interest rate is neutral in the long run. Indeed consider the VAR model (1.15), and the vector of structural

innovations $\eta_t = (\varepsilon_t^s \ \varepsilon_t^d \ \varepsilon_t^m)'$, with the shocks defined as in (1.3), (1.4) and (1.5). Then these restrictions would imply $\hat{C}(1)$ to be defined as either:

$$\hat{C}(1) = \begin{pmatrix} \hat{c}_{11} & 0 & 0 \\ 0 & \hat{c}_{22} & \hat{c}_{23} \\ \hat{c}_{31} & \hat{c}_{32} & \hat{c}_{33} \end{pmatrix} \text{ or } \hat{C}(1) = \begin{pmatrix} \hat{c}_{11} & 0 & 0 \\ \hat{c}_{21} & \hat{c}_{22} & \hat{c}_{23} \\ 0 & \hat{c}_{32} & \hat{c}_{33} \end{pmatrix},$$

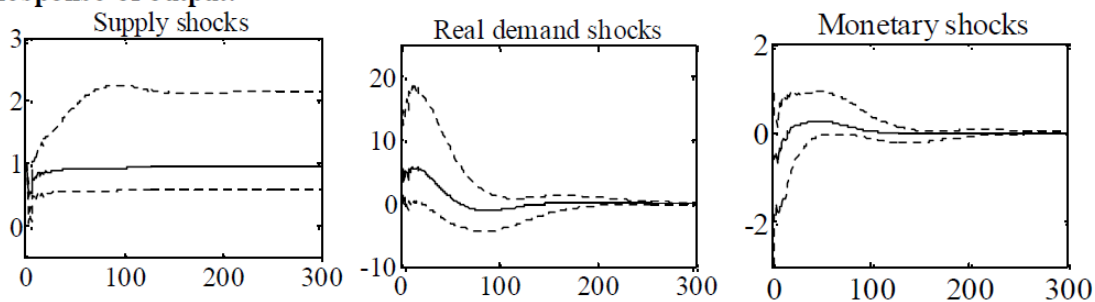
implying either m_{12} or m_{13} to be 0, contradicting (A.7).

Appendix A.4: Impulse response function tables

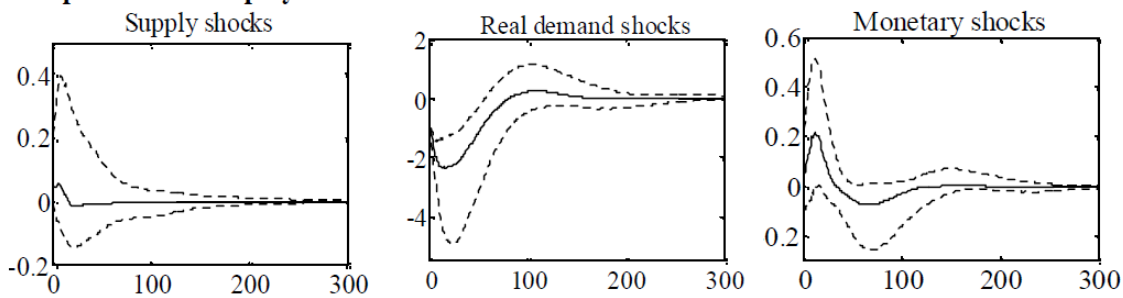
Figure A.1

Simulated impulse response functions and 95% confidence interval
France, 1979M1-2000M11

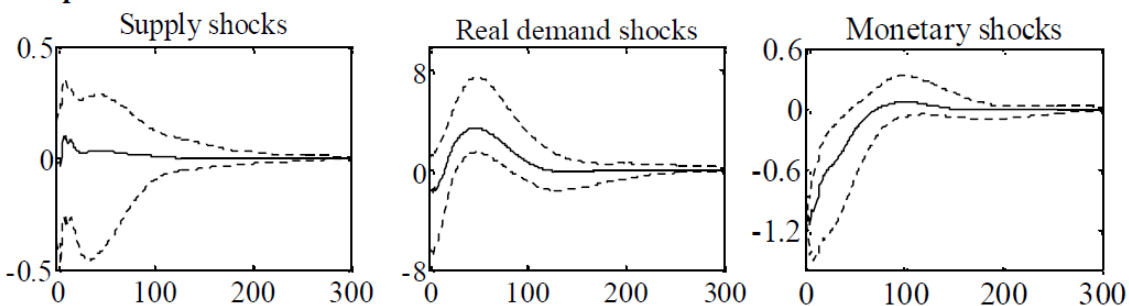
Response of output:



Response of unemployment:



Response of interest rate:



Appendix A.5: Forecast Error Variance Decomposition Tables

Table A.4

Forecast Error Variance Decomposition, Output

Pre-break : 1979M1-1992M2					Post-break : 1992M3-2011M1				
Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand	Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand
Germany : pre-break : 1979M1-1989M6					Germany : post-break : 1989M7-2011M1				
6 Months	72.37	12.63	15.00	54.29	6 Months	0.93	33.64	65.43	66.04
1 year	74.10	14.72	11.18	43.17	1 year	1.46	32.33	66.21	67.19
5 years	80.86	14.93	4.21	22.00	5 years	8.02	16.75	75.23	81.79
15 years	93.73	4.89	1.38	22.02	15 years	38.99	25.70	35.31	57.88
France : pre-break : 1979M1-2000M11					France : post-break : 2000M12-2011M1				
6 Months	73.21	23.39	3.40	12.68	6 Months	5.10	83.13	11.77	12.40
1 year	76.92	21.45	1.63	7.07	1 year	22.23	60.55	17.22	22.14
5 years	86.48	11.84	1.67	12.36	5 years	72.56	15.29	12.15	44.28
15 years	94.12	5.06	0.83	14.04	15 years	88.86	6.20	4.94	44.33
Italy : pre-break : 1979M1-1992M7					Italy : post-break : 1992M8-2011M1				
6 Months	59.38	24.92	15.71	38.66	6 Months	59.80	35.59	4.62	11.48
1 year	58.39	29.10	12.51	30.06	1 year	62.09	35.22	2.68	7.08
5 years	66.72	30.14	3.14	9.44	5 years	70.75	28.14	1.11	3.79
15 years	88.23	10.69	1.08	9.15	15 years	89.26	10.14	0.59	5.53
Spain : pre-break : 1979M1-1992M2					Spain : post-break : 1992M3-2011M1				
6 Months	69.12	22.07	8.82	28.55	6 Months	80.55	3.98	15.47	79.52
1 year	63.85	25.15	10.99	30.41	1 year	58.01	10.93	31.06	73.97
5 years	59.11	18.03	22.86	55.91	5 years	42.40	28.04	29.56	51.33
15 years	52.04	28.62	19.33	40.32	15 years	64.58	12.74	22.68	64.02
Belgium : pre-break : 1979M1-1990M9					Belgium : post-break : 1990M10-2011M1				
6 Months	75.54	12.17	12.29	50.25	6 Months	55.73	9.67	34.61	78.17
1 year	75.70	14.38	9.92	40.82	1 year	60.02	7.60	32.38	80.99
5 years	82.01	13.02	4.98	27.66	5 years	73.77	13.67	12.57	47.91
15 years	93.96	4.27	1.77	29.36	15 years	89.72	5.22	5.06	49.22
Ireland : pre-break : 1979M1-1993M1					Ireland : post-break : 1993M2-2011M1				
6 Months	41.34	41.37	17.29	29.47	6 Months	51.02	48.60	0.39	0.79
1 year	32.48	57.08	10.44	15.46	1 year	34.90	64.77	0.33	0.50
5 years	38.63	58.47	2.91	4.73	5 years	31.04	68.86	0.10	0.14
15 years	76.78	22.23	0.99	4.26	15 years	82.11	17.87	0.03	0.14
Portugal:pre-break : 1979M1-1993M7					Portugal:post-break : 1993M8-2011M1				
6 Months	61.93	15.47	22.60	59.38	6 Months	11.34	73.13	15.53	17.52
1 year	62.98	17.73	19.29	52.10	1 year	12.83	67.79	19.38	22.24
5 years	63.66	28.71	7.63	21.00	5 years	22.56	55.83	21.62	27.91
15 years	84.82	12.05	3.13	20.61	15 years	47.03	37.54	15.43	29.12
Denmark : pre-break : 1979M1-1983M9					Denmark :post-break : 1983M10-2011M1				
6 Months	43.59	46.91	9.51	16.85	6 Months	43.24	10.11	46.64	82.18
1 year	32.82	59.70	7.48	11.13	1 year	34.18	15.78	50.04	76.03
5 years	71.40	26.74	1.85	6.49	5 years	58.23	16.62	25.14	60.20
15 years	93.25	6.32	0.44	6.45	15 years	83.28	6.69	10.02	59.95
Sweden : pre-break : 1979M1-2005M3					Sweden : post-break : 2005M4-2011M1				
6 Months	72.22	7.03	20.75	74.70	6 Months	31.00	61.84	7.16	10.38
1 year	73.76	9.19	17.05	64.97	1 year	26.93	67.63	5.45	7.45
5 years	78.40	16.15	5.45	25.24	5 years	92.19	7.27	0.53	6.83
15 years	90.58	7.16	2.26	24.03	15 years	97.64	2.20	0.16	6.83
United Kingdom : pre-break : 1979M1-1984M2					United Kingdom:post-break : 1984M3-2011M1				
6 Months	39.36	17.49	43.15	71.17	6 Months	16.53	71.38	12.09	14.48
1 year	37.44	19.27	43.29	69.19	1 year	14.18	61.87	23.96	27.91
5 years	68.39	17.70	13.91	44.01	5 years	13.82	69.19	16.99	19.72
15 years	90.06	7.60	2.34	23.56	15 years	47.86	44.40	7.74	14.85

Table A.5

Forecast Error Variance Decomposition, Unemployment

Pre-break : 1979M1-1992M2					Post-break : 1992M3-2011M1				
Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand	Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand
Germany : pre-break : 1979M1-1989M6					Germany : post-break : 1989M7-2011M1				
6 Months	3.17	16.20	80.63	83.27	6 Months	64.40	34.75	0.85	2.39
1 year	3.20	36.03	60.77	62.78	1 year	55.28	42.02	2.69	6.02
5 years	15.03	69.81	15.16	17.85	5 years	55.65	40.25	4.10	9.25
15 years	15.03	69.56	15.42	18.14	15 years	70.69	26.58	2.72	9.29
France : pre-break : 1979M1-2000M11					France : post-break : 2000M12-2011M1				
6 Months	2.33	90.33	7.33	7.51	6 Months	23.47	33.39	43.14	56.37
1 year	1.09	88.58	10.33	10.44	1 year	19.71	45.51	34.79	43.32
5 years	4.30	90.21	5.49	5.73	5 years	13.22	45.26	41.52	47.84
15 years	4.39	88.75	6.86	7.18	15 years	13.07	45.10	41.82	48.11
Italy : pre-break : 1979M1-1992M7					Italy : post-break : 1992M8-2011M1				
6 Months	30.18	24.76	45.06	64.54	6 Months	26.60	15.12	58.28	79.40
1 year	30.95	33.44	35.61	51.57	1 year	22.36	18.94	58.69	75.60
5 years	26.64	62.69	10.68	14.55	5 years	59.09	22.16	18.75	45.83
15 years	25.17	64.92	9.91	13.24	15 years	71.26	18.66	10.07	35.05
Spain : pre-break : 1979M1-1992M2					Spain : post-break : 1992M3-2011M1				
6 Months	14.49	12.15	73.36	85.80	6 Months	6.41	5.35	88.25	94.29
1 year	16.66	21.25	62.09	74.50	1 year	4.97	12.11	82.92	87.26
5 years	17.75	69.81	12.44	15.12	5 years	13.63	30.24	56.12	64.98
15 years	17.07	69.13	13.80	16.64	15 years	15.68	22.25	62.06	73.61
Belgium : pre-break : 1979M1-1990M9					Belgium : post-break : 1990M10-2011M1				
6 Months	7.14	5.41	87.44	94.17	6 Months	12.96	0.67	86.37	99.23
1 year	10.22	14.54	75.24	83.81	1 year	10.81	0.53	88.66	99.41
5 years	19.01	51.75	29.24	36.10	5 years	31.36	9.73	58.91	85.83
15 years	18.29	50.24	31.48	38.52	15 years	34.31	16.10	49.59	75.49
Ireland : pre-break : 1979M1-1993M1					Ireland : post-break : 1993M2-2011M1				
6 Months	27.53	19.91	52.56	72.53	6 Months	41.16	58.71	0.13	0.21
1 year	32.06	44.89	23.04	33.92	1 year	44.89	55.06	0.05	0.09
5 years	31.16	65.81	3.03	4.41	5 years	49.73	50.27	0.01	0.02
15 years	31.07	66.29	2.64	3.83	15 years	49.80	50.19	0.01	0.02
Portugal:pre-break : 1979M1-1993M7					Portugal:post-break : 1993M8-2011M1				
6 Months	41.59	40.24	18.17	31.10	6 Months	0.56	6.86	92.57	93.10
1 year	20.14	69.50	10.37	12.98	1 year	1.30	7.34	91.35	92.56
5 years	7.68	88.85	3.47	3.76	5 years	10.93	11.36	77.71	87.25
15 years	7.73	88.96	3.31	3.59	15 years	43.43	17.10	39.47	69.78
Denmark : pre-break : 1979M1-1983M9					Denmark :post-break : 1983M10-2011M1				
6 Months	57.25	37.43	5.32	12.44	6 Months	32.93	16.73	50.34	75.05
1 year	58.87	39.62	1.51	3.68	1 year	32.26	24.04	43.70	64.51
5 years	61.06	38.40	0.54	1.39	5 years	27.70	41.78	30.52	42.21
15 years	61.07	38.39	0.54	1.38	15 years	26.34	42.13	31.54	42.81
Sweden : pre-break : 1979M1-2005M3					Sweden : post-break : 2005M4-2011M1				
6 Months	16.75	32.01	51.24	61.55	6 Months	62.04	19.01	18.96	49.93
1 year	15.64	38.24	46.11	54.67	1 year	74.12	16.11	9.78	37.77
5 years	9.75	72.43	17.82	19.74	5 years	76.13	15.92	7.95	33.31
15 years	7.67	73.84	18.49	20.03	15 years	76.13	15.92	7.95	33.31
United Kingdom : pre-break : 1979M1-1984M2					United Kingdom:post-break : 1984M3-2011M1				
6 Months	4.22	20.67	75.12	78.42	6 Months	0.25	0.04	99.71	99.96
1 year	2.52	6.15	91.33	93.69	1 year	1.95	0.06	97.99	99.94
5 years	5.78	17.00	77.21	81.95	5 years	51.51	1.78	46.71	96.33
15 years	29.75	18.47	51.77	73.70	15 years	67.70	2.64	29.67	91.83

Table A.6

Forecast Error Variance Decomposition, Interest rate

Pre-break : 1979M1-1992M2					Post-break : 1992M3-2011M1				
Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand	Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand
Germany : pre-break : 1979M1-1989M6					Germany : post-break : 1989M7-2011M1				
6 Months	12.51	56.21	31.27	35.74	6 Months	4.16	92.37	3.47	3.62
1 year	14.00	47.28	38.71	45.02	1 year	2.79	90.51	6.70	6.89
5 years	14.15	52.34	33.51	39.03	5 years	36.15	57.62	6.23	9.76
15 years	14.25	52.41	33.33	38.87	15 years	87.13	11.59	1.28	9.93
France : pre-break : 1979M1-2000M11					France : post-break : 2000M12-2011M1				
6 Months	2.02	3.79	94.19	96.13	6 Months	27.91	68.20	3.89	5.39
1 year	7.71	3.12	89.18	96.62	1 year	27.35	66.32	6.33	8.72
5 years	11.22	41.44	47.33	53.32	5 years	17.58	54.25	28.17	34.18
15 years	10.45	51.67	37.88	42.30	15 years	17.36	54.05	28.58	34.59
Italy : pre-break : 1979M1-1992M7					Italy : post-break : 1992M8-2011M1				
6 Months	0.17	34.09	65.74	65.85	6 Months	32.58	20.82	46.60	69.12
1 year	0.30	29.24	70.46	70.67	1 year	47.98	11.05	40.97	78.76
5 years	6.46	29.88	63.66	68.06	5 years	61.00	7.22	31.78	81.49
15 years	7.93	35.02	57.05	61.96	15 years	63.44	7.87	28.69	78.48
Spain : pre-break : 1979M1-1992M2					Spain : post-break : 1992M3-2011M1				
6 Months	5.46	58.51	36.03	38.11	6 months	18.28	75.58	6.14	7.52
1 year	4.85	56.85	38.30	40.26	1 year	36.17	51.97	11.86	18.59
5 years	5.61	56.03	38.36	40.64	5 years	50.41	21.75	27.85	56.15
15 years	6.19	57.17	36.64	39.06	15 years	51.24	19.50	29.26	60.00
Belgium : pre-break : 1979M1-1990M9					Belgium : post-break : 1990M10-2011M1				
6 Months	17.32	65.04	17.64	21.34	6 months	29.34	45.60	25.06	35.46
1 year	16.05	60.04	23.91	28.48	1 year	26.66	35.56	37.78	51.51
5 years	13.08	46.96	39.96	45.97	5 years	21.66	27.30	51.04	65.15
15 years	13.80	48.60	37.61	43.63	15 years	21.58	27.11	51.30	65.43
Ireland : pre-break : 1979M1-1993M1					Ireland : post-break : 1993M2-2011M1				
6 Months	9.36	52.37	38.27	42.22	6 months	2.25	4.65	93.10	95.24
1 year	10.59	53.46	35.96	40.21	1 year	18.06	26.34	55.60	67.85
5 years	13.80	55.65	30.55	35.44	5 years	33.58	37.60	28.83	43.40
15 years	14.57	56.26	29.17	34.14	15 years	36.60	39.47	23.92	37.73
Portugal:pre-break : 1979M1-1993M7					Portugal:post-break : 1993M8-2011M1				
6 Months	1.87	83.00	15.14	15.42	6 months	84.49	12.33	3.18	20.48
1 year	4.46	77.79	17.75	18.58	1 year	83.79	12.06	4.14	25.57
5 years	7.86	72.40	19.74	21.43	5 years	83.32	11.83	4.85	29.09
15 years	7.88	72.28	19.84	21.54	15 years	83.24	11.79	4.98	29.68
Denmark : pre-break : 1979M1-1983M9					Denmark :post-break : 1983M10-2011M1				
6 Months	11.66	52.75	35.59	40.29	6 months	3.79	71.49	24.72	25.69
1 year	11.27	52.08	36.65	41.31	1 year	3.01	67.25	29.75	30.67
5 years	11.48	51.76	36.76	41.52	5 years	2.20	59.56	38.24	39.10
15 years	11.49	51.76	36.75	41.52	15 years	2.83	61.67	35.50	36.53
Sweden : pre-break : 1979M1-2005M3					Sweden : post-break : 2005M4-2011M1				
6 Months	2.20	44.81	52.99	54.18	6 months	87.34	10.74	1.92	15.19
1 year	3.10	37.70	59.21	61.10	1 year	86.35	12.54	1.11	8.14
5 years	8.93	23.81	67.27	73.86	5 years	85.95	13.12	0.94	6.67
15 years	7.66	46.22	46.12	49.94	15 years	85.95	13.12	0.94	6.67
United Kingdom : pre-break : 1979M1-1984M2					United Kingdom:post-break : 1984M3-2011M1				
6 Months	12.45	41.97	45.58	52.06	6 months	82.10	5.87	12.03	67.20
1 year	18.30	37.63	44.07	53.94	1 year	71.14	4.57	24.29	84.17
5 years	27.57	27.52	44.91	62.01	5 years	53.11	3.13	43.75	93.32
15 years	79.93	9.40	10.68	53.19	15 years	68.17	3.38	28.45	89.39

Table A.7

Forecast Error Variance Decomposition, multiple breaks

Decomposition of Output: Spain									
Post-break, subsample 1: 1992M4-2004M2					Post-break, subsample 2: 2004M2-2011M1				
Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand	Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand
3 months	51.63	4.84	43.53	89.98	3 months	2.68	62.21	35.11	36.08
6 months	54.57	4.74	40.70	89.58	6 months	2.10	43.63	54.27	55.44
9 months	53.64	4.78	41.57	89.68	9 months	1.95	43.32	54.73	55.81
1 year	51.63	9.04	39.32	81.30	1 year	2.08	37.78	60.14	61.42
2 years	39.86	34.00	26.14	43.46	2 years	7.02	36.86	56.13	60.36
3 years	38.45	38.14	23.41	38.04	3 years	13.94	37.95	48.11	55.91
5 years	46.13	31.94	21.93	40.71	5 years	20.96	40.37	38.68	48.93
10 years	64.61	18.62	16.78	47.40	10 years	29.08	44.84	26.08	36.78
15 years	74.32	12.32	13.36	52.01	15 years	32.85	47.37	19.78	29.46

Decomposition of Interest rate: Spain									
Post-break, subsample 1: 1992M4-2004M2					Post-break, subsample 2: 2004M2-2011M1				
Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand	Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand
3 months	8.45	83.47	8.08	8.83	3 months	92.37	2.49	5.14	67.39
6 months	19.15	77.03	3.82	4.72	6 months	64.42	0.76	34.82	97.86
9 months	22.22	75.77	2.01	2.59	9 months	35.02	0.44	64.53	99.32
1 year	21.41	75.70	2.89	3.68	1 year	19.61	0.32	80.07	99.60
2 years	29.63	66.27	4.10	5.83	2 years	9.42	0.23	90.35	99.74
3 years	33.52	62.44	4.04	6.08	3 years	13.93	0.22	85.85	99.74
5 years	33.32	63.01	3.66	5.50	5 years	23.66	0.19	76.15	99.75
10 years	33.80	62.55	3.65	5.51	10 years	52.77	0.12	47.11	99.75
15 years	33.81	62.53	3.66	5.52	15 years	80.99	0.05	18.96	99.75

Decomposition of Output: Ireland									
Post-break, subsample 1: 1993M2-1998M10					Post-break, subsample 2: 1998M11-2011M1				
Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand	Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand
3 months	3.31	92.88	3.81	3.94	3 months	0.36	98.52	1.12	1.12
6 months	7.27	89.04	3.69	3.97	6 months	1.21	97.70	1.10	1.11
9 months	13.12	84.25	2.63	3.03	9 months	2.01	96.81	1.18	1.20
1 year	18.95	79.01	2.04	2.52	1 year	2.68	96.11	1.21	1.25
2 years	32.56	66.44	1.00	1.49	2 years	4.50	94.54	0.96	1.01
3 years	39.98	59.38	0.64	1.06	3 years	5.57	93.72	0.70	0.75
5 years	49.97	49.69	0.34	0.67	5 years	6.81	92.74	0.44	0.47
10 years	67.04	32.84	0.12	0.35	10 years	8.52	91.26	0.23	0.25
15 years	78.53	21.42	0.05	0.24	15 years	9.99	89.86	0.15	0.17

Decomposition of Interest rate: Ireland									
Post-break, subsample 1: 1993M2-1998M10					Post-break, subsample 2: 1998M11-2011M1				
Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand	Horizon	\mathcal{E}^s	\mathcal{E}^d	\mathcal{E}^m	\mathcal{E}^m over total demand
3 months	14.63	5.35	80.02	93.73	3 months	14.82	6.01	79.17	92.94
6 months	13.19	7.37	79.44	91.51	6 months	7.92	4.69	87.40	94.91
9 months	13.93	7.31	78.76	91.51	9 months	4.94	4.18	90.88	95.60
1 year	14.47	7.26	78.26	91.51	1 year	4.24	3.85	91.91	95.98
2 years	14.87	7.24	77.88	91.49	2 years	10.72	3.07	86.21	96.56
3 years	15.87	7.16	76.97	91.49	3 years	22.51	2.56	74.93	96.69
5 years	18.19	6.96	74.85	91.49	5 years	44.48	1.88	53.64	96.61
10 years	25.21	6.36	68.43	91.49	10 years	74.57	1.06	24.37	95.84
15 years	33.87	5.63	60.50	91.49	15 years	87.39	0.71	11.89	94.36

Chapter 2

A canonical model of financial frictions in open economies and monetary unions with an application to the Euro area

Abstract

Recent economic events within the European Monetary Union have led to question the sustainability of a monetary union when confronted to exceptional events such as the 2007 financial crisis. While the literature treating financial frictions comparable to those faced during the 2007 crisis is quickly expanding following the seminal work by Gertler and Kiyotaki (2010), it has not yet been applied to open economies nor to monetary unions. This paper hence aims at filling this gap by proposing a two-country model of the Euro area featuring financial frictions.

The model has two broad objectives: first, provide a quantitative assessment of the reaction of open economies in a context of financial frictions; second, propose a benchmark model allowing to compare monetary regimes (flexible exchange rates or monetary union) and assess their respective performances within the context of “financial crisis” developed in the paper. Preliminary experiments based on numerical simulations for the Euro area reveal that:

- 1) Financial markets matter, and their effects over the economy increase with the degree of financial frictions. Furthermore, financial interactions between countries of the Euro area play a considerable role in domestic developments, so that the situation of financial markets in partner economies may contribute to worsen or mitigate the impact of a crisis.
- 2) The ranking between a floating exchange rate regime and a monetary union is ambiguous: while the floating regime performs better in terms of capacity to stabilize the economy for some variables (output and employment), the conclusion is reversed for other essential variables such as consumption or capital. This calls for a more formal welfare analysis.

JEL classification: E44, F41, F42, F47

Introduction

On August 9th 2007, The Guardian was reporting that central banks had been active on markets. The ECB had pumped 95 billion Euros into the Euro banking system to allay fears of a subprime credit crunch after the announcement by French BNP Paribas of the suspension of three investment funds specialised in US mortgage debt. “Complete evaporation of liquidity”, said BNP representatives, had made it “impossible to value certain assets” of the subprime mortgage sector. The news, deemed hardly worth of any interest, had been buried deep on page 29 of the issue.

Few could imagine that this event would mark the beginning of the worst economic and financial crisis that Europe and the world had faced since the Great depression of the 1930's²⁴. The burst of the U.S housing bubble in 2006 had caused housing prices to plummet, and American banks subject to large exposure to subprime securities had incurred massive losses. The episode culminated with the bankruptcy of Lehman Brothers, on September 15th 2008, and the federal takeover of Fannie Mae and Freddie Mac in September 2008. Contagion to Europe was quick. As almost every bank in Europe had been heavily investing in securities linked to the American housing market, the European banking system also faced tremendous losses²⁵. By 2008, the whole European banking system went under the threat of complete collapse, forcing national governments to adopt emergency measures. These consisted mainly in nationalisations (Fortis Bank, Anglo Irish Bank), State guarantees for depositors and investors, and, most importantly, in large and costly bailout plans at the national levels and for individual banks such as Dexia or ING Group.

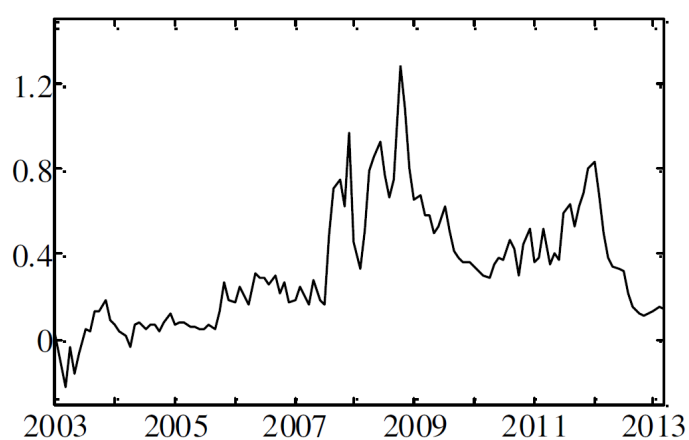
The bank crisis was not settled, however. With the fall of Lehman, it became apparent that the notion of banks being too large to go bankrupt no longer held true. Combined with significant exposure to questionable subprime derivatives, every bank was now considered as potentially risky. Confidence among credit institutions disappeared and banks found it increasingly difficult to borrow funds. Figure 2.1 and 2.2 illustrate this point. Figure 2.1 displays the spread between the unsecured rate that banks charge one another for short-term funds, the EURIBOR, and a secured short-term rate, the EONIA. Figure 2.2 on the other hand indicates

²⁴ Adam Applegarth, Northern Rock's chief executive, would later refer to this as "The day the world changed". For an exhaustive account of the economical crisis which has stricken Europe since 2007, see Shambaugh (2012).

²⁵ IMF estimates in January 2009 amounted to 1.2 trillion dollars.

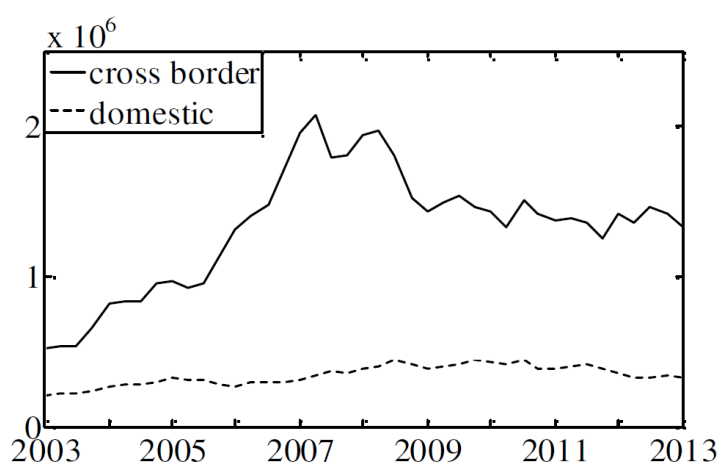
the volume of interbank lending within the Euro area, both for cross-border and domestic loans. One may observe on figure 2.1 that the spread has remained fairly stable over the 2004-2007 period, but started to rise in 2007 and 2008 and then reached a peak in the fall of 2008 after the failure of Lehman. This confirms the increase in risk perceived by the banking system with respect to interbank operations. Concurrently, the volume of cross-border interbank lending, which had been steadily increasing over years 2003-2007, began to decline, to shrink by roughly 30% of its 2007 value in 2009.

Figure 2.1
Spread Euribor/EONIA



Source: author's calculation based on OECD data

Figure 2.2
Euro area interbank lending
billions of US dollars



Source: author's calculation based on Bank for International Settlement (BIS) data

After an initial episode of insolvency, the banking sector was hence facing a liquidity crisis, and this one could be long-lasting in the absence of exterior intervention. Despite some measures implemented at the national level to provide liquidity to domestic banks (France through the Société de Financement de l'Economie Française, Sweden through the National Debt Office), most credit easing has been the doing of the ECB. In October 2008, it first adopted a fixed rate tender procedure with full allotment for all its weekly refinancing operations. It also increased the number and frequency of longer-term refinancing operations, and extended temporarily the list of assets eligible for use as collaterals. In 2009, it implemented the more radical Covered Bond Purchase Programme, a 60 billion Euros worth package of covered bond acquisition destined to provide fresh liquidity to the interbank market. The initial programme was successful, and terminated in June 2010. As Figure 2.1 shows, this coincides with a renewed increase in the interbank rate spread. Following, the programme was reactivated on November 2011 (CBPP2), for an outright purchase of 40 billion Euros, resulting once more in a rapid drop of the spread.

These events stress the importance of financial rigidities in the determination of cyclical activity within the Eurozone over the last few years. One would actually find it difficult to provide an accurate picture of the economic situation currently prevailing in the Euro area without some consideration to such financial mechanisms. Yet to these days, the literature on the Euro area remains relatively silent over these subjects. The Euro area literature developed since the beginning of the 2000's has mainly focused on producing the fittest model for forecasting purposes. To this end, the strategy adopted by many authors has consisted in introducing traditional market imperfections within the model (monopolistic competition, price and wage rigidities), and then estimate the model with the benefit of recent computational methods such as Bayesian estimation. The seminal paper of Smets and Wouters (2003) paved the way for this branch of research, followed by Smets and Wouters (2004, 2005), Ratto et al. (2008), Gerali et al. (2010), Quint and Rabanal (2011).

On the other hand, a literature on endogenous financial frictions has been lately developing at a fast pace; while financial macroeconomics has traditionally paid attention to credit constraints on non-financial borrowers (Bernanke and Gertler (1989), Kiyotaki and Moore (1997)), this new branch acknowledges the idea that capital constraints on financial intermediaries are an important factor in the determination of the performance of these institutions, and, following, of the economy in general. Preliminary works included a role for

financial intermediaries, but no endogenous capital constraint on the banking sector. Examples of such works include Aslam and Santoro (2008) and Curdia and Woodford (2011), who focus on the spread between deposit and lending rates, and Markovic (2006) or Meh and Moran (2008), who concentrate on the role of bank equity and capital in the transmission of macroeconomic shocks. A renewal of the literature has been initiated with the works of Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Gertler et al. (2012). These authors take a step forward by introducing a financial accelerator mechanism in the banking sector. The latter generates an endogenous credit constraint on financial intermediaries as a function of bank capital, which generates financial rigidities. Comparable mechanisms have been exploited in recent papers with more quantitative scopes (Mendoza (2010), He and Krishnamurthy (2012), Dewachter and Wouters (2012)).

The two literatures have not met yet. This paper aims at filling this gap by proposing an open economy model for the Euro area featuring endogenous financial frictions. The main component of the model is the benchmark settings proposed by Gertler and Kiyotaki (2011), adapted to open economies. In addition, the model incorporates a number of conventional new-Keynesian features such as monopolistic competition and price rigidities, which have now become standard in the class of open economy models (see e.g. Galli and Monacelli 2005). In the model, one country (Home) represents a single country of the current Euro area, while the other (Foreign) represents the remaining countries of the Eurozone. While Foreign in itself is assumed to constitute a monetary union, Home may or may not be a Member of this monetary union. That is, it can keep its own currency, in which case it can be considered for example as a Euro candidate or as a former Euro area member which has defected the common currency. Or it can adopt the single currency prevailing in Foreign, in which case the model can be properly considered as being the Eurozone. To this end, this study presents two declinations of the same core model. The first version treats Home and Foreign as countries which have preserved their monetary independence, which implies the presence of an exchange rate and separate central banks, while the second version treats Home and Foreign as the two members of a monetary union sharing the same unique currency and directed by a single central bank.

This work contributes to the literature in three ways. First, it proposes an open economy model of financial rigidities. While a model of closed economy may provide useful insights, it may not be able to represent accurately the main mechanisms behind the dynamics of highly

integrated economies such as the ones composing the Eurozone. This is especially true for economies with a high degree of integration on goods and financial markets. For example, Figure 2.2 shows that a major portion of interbank funds in the Euro area are obtained from cross-border transactions; also, the slowdown of interbank trade since 2008 is mainly imputable to cross-border operations, domestic transactions having remained relatively stable over the whole period. In this respect, it is the interactions with abroad financial markets rather than developments in domestic financial markets that explain the struggle of Euro area economies since the beginning of the crisis in 2007. The model integrates this feature by assuming the existence of a cross-border interbank market between Home and Foreign, rather than a purely domestic interbank channel.

Secondly, the model integrates both financial shocks and more conventional productivity and monetary shocks. Indeed, Gertler and Kiyotaki (2010), along with most of the recent financial macroeconomic literature integrate in their settings a financial crisis shock and a financial accelerator mechanism, but otherwise express their models purely in real terms. While this may be an acceptable assumption for a qualitative analysis, it may become problematic for more precise quantitative assessments. This is particularly true for the Euro area where monetary policy has been and remains an essential tool of stabilization policies since the beginning of the crisis. Gertler and Karadi (2011), on the other hand, do include monetary policy and a financial accelerator in their model, but no financial shocks. This paper hence proposes to reconcile the two approaches by gathering in a unified framework the traditional real and monetary shocks with the more recent financial shocks in order to provide an accurate picture of the Eurozone.

Finally, this paper proposes two declinations of the same model: one in a regime of independent currencies, and one in monetary union. The reason is simple: the financial crisis has shed light on the weaknesses of the Euro area, and the possibility that one or several members may eventually leave it if the burden of the single currency becomes too heavy, or if the sustainability of the Euro comes to be threatened. In this respect, the two versions of the model allow to compare the two monetary regimes and assess their respective performances within the context of 'financial crisis' developed in the paper.

The remainder of this paper consists of four sections. Section 2.1 introduces the floating exchange rate version of the model of financial frictions in open economies. Section 2.2

develops the modifications implied by the monetary union declination of the model. Section 2.3 then details the calibration of the model to fit the 17 countries of the Euro area. Section 2.4 finally proposes a preliminary analysis of the Euro area based on numerical simulations obtained from the model. Section 2.5 concludes.

2.1. A model of financial frictions in open economies

2.1.1. Households

The model considers two countries, Home and Foreign. Each country is inhabited by a representative household of unit mass. Household members consume goods, supply labour, and – as described later on in the model – are entrepreneurs who own capital producing firms and financial intermediaries. At any time, they can be divided into two categories: “workers”, in proportion $1 - f$, and “bankers”, in proportion f . Workers supply labour and earn wage which is returned to the household. Bankers manage financial intermediaries or “banks” that transfer earnings back to the household, subject to the flow of fund constraint. However, as will be developed thereafter, the household members may switch between the two occupations. There is perfect consumption insurance within the household, so that workers and bankers can be considered in a similar way with respect to consumption decisions.

As consumers, household members in both countries consume differentiated varieties of goods, produced both in Home and Foreign. Each country produces a continuum of varieties defined over an interval $[0,1]$ of goods. Let $C_{H,t}(j)$ denote consumption of variety j produced at Home. Then, the composite index for consumption of goods produced in Home $C_{H,t}$ is defined as:

$$C_{H,t} = \left(\int_0^1 C_{H,t}(j)^{\frac{\kappa-1}{\kappa}} dj \right)^{\frac{\kappa}{\kappa-1}} \quad (2.1)$$

Similarly, the composite index for consumption of goods produced in Foreign $C_{F,t}$ is defined as:

$$C_{F,t} = \left(\int_0^1 C_{F,t}(j)^{\frac{\kappa-1}{\kappa}} dj \right)^{\frac{\kappa}{\kappa-1}} \quad (2.2)$$

A superscript h or f is used to denote respective consumption by the household in Home or in Foreign. Hence $C_{H,t}^h(j), C_{H,t}^h, C_{F,t}^h(j)$ and $C_{F,t}^h$ represent consumption of the varieties and consumption indices by the Home Household, while quantities consumed by Foreign are denoted by $C_{H,t}^f(j), C_{H,t}^f, C_{F,t}^f(j)$, and $C_{F,t}^f$. A similar convention will be used in the subsequent parts of the model.

Final consumption is derived from a composite index including the bundles from Home and Foreign. For Home consumers, it is defined as:

$$C_t^h = \left((\varphi^h)^\mu C_{H,t}^h \frac{\mu-1}{\mu} + (1-\varphi^h)^\mu C_{F,t}^h \frac{\mu-1}{\mu} \right)^{\frac{\mu}{\mu-1}} \quad (2.3)$$

where φ^h represents the bias towards Home goods in the composite index, defined as:

$$\varphi^h = 1 - (1-\gamma)v^h \quad (2.4)$$

It comprises γ , the relative size of Home, and the Home-specific parameter v^h which measures trade openness of Home. Similarly, for consumers in Foreign, it is defined as:

$$C_t^f = \left((\varphi^f)^\mu C_{F,t}^f \frac{\mu-1}{\mu} + (1-\varphi^f)^\mu C_{H,t}^f \frac{\mu-1}{\mu} \right)^{\frac{\mu}{\mu-1}} \quad (2.5)$$

$$\text{where } \varphi^f = 1 - \gamma v^f. \quad (2.6)$$

The good of variety j produced at Home is sold at price $P_{H,t}(j)$, expressed in Home currency. The good of variety j produced in Foreign is sold at price $P_{F,t}(j)$, expressed in Foreign currency. The nominal exchange rate, defined as the price of one unit of Foreign currency in terms of Home currency is denoted by ε_t . Assuming that there are no impediments to trade, the Law of One Price holds, which implies that for any variety j produced and sold at price $P_{F,t}(j)$ in Foreign, the retail price in Home will be $\varepsilon_t P_{F,t}(j)$, while a product produced and sold at price $P_{H,t}(j)$ in Home will have a retail price of $\varepsilon_t^{-1} P_{H,t}(j)$ in Foreign.

The price aggregators for Home and Foreign (expressed in their respective currencies) are defined as:

$$P_{H,t} = \left(\int_0^1 P_{H,t}(j)^{1-\kappa} dj \right)^{\frac{1}{1-\kappa}} \quad (2.7)$$

and

$$P_{F,t} = \left(\int_0^1 P_{F,t}(j)^{1-\kappa} dj \right)^{\frac{1}{1-\kappa}} \quad (2.8)$$

Finally, the Law of One price implies that the Consumer Price Indices at Home and Foreign are defined in their respective currencies as:

$$P_t^h = \left(\varphi^h (P_{H,t})^{1-\mu} + (1-\varphi^h)(\varepsilon_t P_{F,t})^{1-\mu} \right)^{\frac{1}{1-\mu}} \quad (2.9)$$

and

$$P_t^f = \left(\varphi^f (P_{F,t})^{1-\mu} + (1-\varphi^f)(\varepsilon_t^{-1} P_{H,t})^{1-\mu} \right)^{\frac{1}{1-\mu}} \quad (2.10)$$

From a cost minimization problem and the structure of consumption and price indices (2.1), (2.2), (2.3), (2.4) and (2.7)-(2.10), optimal demands for Home and Foreign varieties are given by:

$$C_{H,t}^h(j) = \varphi^h \left(\frac{P_t^h}{P_{H,t}} \right)^\mu \left(\frac{P_{H,t}}{P_{H,t}(j)} \right)^\kappa C_t^h = \left(\frac{P_{H,t}}{P_{H,t}(j)} \right)^\kappa C_{H,t}^h \quad (2.11)$$

$$C_{F,t}^h(j) = (1-\varphi^h) \left(\frac{P_t^h}{\varepsilon_t P_{F,t}} \right)^\mu \left(\frac{P_{F,t}}{P_{F,t}(j)} \right)^\kappa C_t^h = \left(\frac{P_{F,t}}{P_{F,t}(j)} \right)^\kappa C_{F,t}^h \quad (2.12)$$

$$C_{F,t}^f(j) = \varphi^f \left(\frac{P_t^f}{P_{F,t}} \right)^\mu \left(\frac{P_{F,t}}{P_{F,t}(j)} \right)^\kappa C_t^f = \left(\frac{P_{F,t}}{P_{F,t}(j)} \right)^\kappa C_{F,t}^f \quad (2.13)$$

$$C_{H,t}^f(j) = (1-\varphi^f) \left(\frac{\varepsilon_t P_t^f}{P_{H,t}} \right)^\mu \left(\frac{P_{H,t}}{P_{H,t}(j)} \right)^\kappa C_t^f = \left(\frac{P_{H,t}}{P_{H,t}(j)} \right)^\kappa C_{H,t}^f \quad (2.14)$$

Consider first the Home household. Household members share their income between consumption and savings, in the form of bank deposits. In this respect, they have access to two different deposit securities: a domestic deposit security proposed by banks in Home, and, thanks to open financial markets, an international deposit security proposed by banks in Foreign. Home deposits take the form of a riskless, one-period bond sold at unit price (in Home currency) and repaying gross nominal interest R_{t+1}^h at maturity, one period later. Similarly, Foreign deposit securities are riskless one period bonds sold for one unit of Foreign currency and repaying (in Foreign currency) gross interest of R_{t+1}^f at maturity. Finally, household members are subject to lump-sum taxation.

Then, let nominal wage paid at Home be denoted by W_t^h , aggregate riskless debt in domestic bonds held from the previous period by D_{t-1}^h , aggregate riskless debt in Foreign bonds held from the previous period by F_{t-1}^h , and aggregate taxation by T_t^h . The household at Home chooses consumption C_t^h , hours worked L_t^h , and deposits D_t^h and F_t^h so as to solve the program:

$$\underset{C_t^h, L_t^h, D_t^h, F_t^h}{Max} E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}^h, L_{t+i}^h) \quad 0 < \beta < 1 \quad (2.15)$$

$$\text{with } U(C_t^h, L_t^h) = \frac{(C_t^h)^{1-\eta}}{1-\eta} - \frac{\chi^h}{1+\phi} (L_t^h)^{1+\phi} \quad (2.16)$$

subject to the budget constraint:

$$P_t^h C_t^h + D_t^h + \varepsilon_t F_t^h + T_t^h + P_t^h A C_t^h = W_t^h L_t^h + R_t^h D_{t-1}^h + R_t^f \varepsilon_t F_{t-1}^h, \quad (2.17)$$

and the appropriate transversality condition²⁶. η, ϕ and χ^h are parameters respectively denoting the risk aversion, inverse Frisch elasticity of labour supply and the (Home-specific) utility weight of labour. In addition, $A C_t^h$ is a convex intermediation cost on Foreign bond holdings, defined as:

$$A C_t^h = \frac{\varepsilon}{2} \left(\frac{\varepsilon_t F_t^h}{P_t^h} - \frac{\varepsilon F^h}{P^h} \right)^2, \quad (2.18)$$

with ε some constant, and where variables without a t subscript denote steady-state values²⁷.

The first-order conditions for this problem are then given by:

$$\chi^h (L_t^h)^\phi (C_t^h)^\eta = \frac{W_t^h}{P_t^h} \quad (2.19)$$

$$1 = R_{t+1}^h \beta E_t \left[\left(\frac{C_t^h}{C_{t+1}^h} \right)^\eta \frac{P_t^h}{P_{t+1}^h} \right] \quad (2.20)$$

and

²⁶ Formally, given the consumption structure, the constraint writes as:

$\int_0^1 P_{H,t}(j) C_{H,t}^h(j) dj + \int_0^1 P_{F,t}(j) C_{F,t}^h(j) dj + D_t^h = W_t^h L_t^h + \Gamma_t^h + R_t^h D_{t-1}^h$. Optimal demands (2.10) and (2.11) and the structure of price and consumption indices (2.1)-(2.8) allow to show that the two formulations are equivalent.

²⁷ Bec and Hairault (1997) show that open economy models with incomplete asset markets are characterized by a unit root. Schmitt-Grohé and Uribe (2006) and Lubik (2007) show that this kind of intermediation costs constitutes one possible solution to stationarize the model.

$$1 + \vartheta \left(\frac{\varepsilon_t F_t^h}{P_t^h} - \frac{\varepsilon F^h}{P^h} \right) = R_{t+1}^f \beta E_t \left[\left(\frac{C_t^h}{C_{t+1}^h} \right)^\eta \frac{P_t^h}{P_{t+1}^h} \frac{\varepsilon_{t+1}}{\varepsilon_t} \right] \quad (2.21)$$

(2.19) is a standard labour/leisure arbitrage condition, while (2.20) and (2.21) constitute instances of conventional Euler equations.

Defining the real wage in Home as $W_{r,t}^h = \frac{W_t^h}{P_t^h}$, Home CPI inflation as $\pi_t^h = \frac{P_t^h}{P_{t-1}^h}$, and real

Foreign bond holdings as $F_{r,t}^h = \frac{\varepsilon_t F_{r,t}^h}{P_t^h}$, the above first-order conditions rewrite in real terms as:

$$\chi^h (L_t^h)^\phi (C_t^h)^\eta = W_{r,t}^h \quad (2.22)$$

$$1 = R_{t+1}^h \beta E_t \left[\left(\frac{C_t^h}{C_{t+1}^h} \right)^\eta \frac{1}{\pi_{t+1}^h} \right] \quad (2.23)$$

and

$$1 + \vartheta (F_{r,t}^h - F_r^h) = R_{t+1}^f \beta E_t \left[\left(\frac{C_t^h}{C_{t+1}^h} \right)^\eta \frac{1}{\pi_{t+1}^h} \frac{\varepsilon_{t+1}}{\varepsilon_t} \right] \quad (2.24)$$

(2.23) and (2.24) can be combined to obtain a modified interest parity condition:

$$R_{t+1}^h \left[1 + \vartheta (F_{r,t}^h - F_r^h) \right] = R_{t+1}^f E_t \left(\frac{\varepsilon_{t+1}}{\varepsilon_t} \right) \quad (2.25)$$

Also, defining respectively the nominal and real stochastic discount factor between t and $t+i$ as:

$$\Lambda_{t,t+i}^h = \beta^i \left(\frac{C_t^h}{C_{t+i}^h} \right)^\eta \frac{P_t^h}{P_{t+i}^h} \quad \text{and} \quad \Lambda_{rt,t+i}^h = \beta^i \left(\frac{C_t^h}{C_{t+i}^h} \right)^\eta, \quad (2.26)$$

the first-order condition (2.23) rewrites as:

$$1 = R_{t+1}^h \Lambda_{t,t+1}^h \quad \text{or} \quad 1 = R_{t+1}^h E_t \left(\frac{\Lambda_{rt,t+1}^h}{\pi_{t+1}^h} \right)$$

Finally, defining real domestic deposit holding as $D_{r,t}^h = \frac{D_t^h}{P_t^h}$ and real taxation as $T_{r,t}^h = \frac{T_t^h}{P_t^h}$, the

budget constraint (2.17) rewrites in real terms as:

$$C_t^h + D_{r,t}^h + F_{r,t}^h + T_{r,t}^h + A C_t^h = W_{r,t}^h L_t^h + \frac{R_t^h}{\pi_t^h} D_{r,t-1}^h + \frac{R_t^f}{\pi_t^h} \frac{\varepsilon_t}{\varepsilon_{t-1}} F_{r,t-1}^h \quad (2.27)$$

The problem is fairly similar for Foreign. One assumes for simplicity that household members in Foreign realize savings only by the way of the international bond proposed by domestic banks. Also, because the bonds are issued domestically, it is assumed that Foreign savers do not incur the intermediation cost on bond holdings. The household members in Foreign hence choose consumption C_t^f , labour L_t^f , and Foreign deposits F_t^f to solve the program:

$$\text{Max}_{C_t^f, L_t^f, F_t^f} E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}^f, L_{t+i}^f) \quad 0 < \beta < 1 \quad (2.28)$$

$$\text{with } U(C_t^f, L_t^f) = \frac{(C_t^f)^{1-\eta}}{1-\eta} - \frac{\chi^f}{1+\phi} (L_t^f)^{1+\phi} \quad (2.29)$$

subject to the budget constraint:

$$P_t^f C_t^f + F_t^f + T_t^f = W_t^f L_t^f + R_t^f F_{t-1}^f,$$

The first-order conditions, in real terms, are given by:

$$\chi^f (L_t^f)^\phi (C_t^f)^\eta = W_{r,t}^f \quad (2.30)$$

and

$$1 = R_{t+1}^f \beta E_t \left[\left(\frac{C_t^f}{C_{t+1}^f} \right)^\eta \frac{1}{\pi_{t+1}^f} \right] \quad (2.31)$$

2.1.2. Governments

In each country, a government is assumed to realize at each period a fixed, exogenous amount of public spending. Expenditures take the form of a CES composite basket over differentiated varieties; however, it is assumed that the government is biased towards domestic goods, so that only domestic varieties enter into its consumption. The CES aggregator is defined as:

$$G_t^h = \left(\int_0^1 G_t^h(j)^{\frac{\kappa-1}{\kappa}} dj \right)^{\frac{\kappa}{\kappa-1}} \quad (2.32)$$

From a cost minimization problem, optimal demand is derived as:

$$G_t^h(j) = \left(\frac{P_{H,t}}{P_{H,t}(j)} \right)^\kappa G_t^h \quad (2.33)$$

and government expenditure is found to be equal to $P_{H,t}G_t^h$. The amount of output spent by the Home governments is assumed to be constant and given by:

$$G_t^h = \vartheta^h Y^h \quad (2.34)$$

where ϑ^h is a Home-specific exogenous constant with $0 < \vartheta^h < 1$, and Y^h denotes the steady-state value of Home output. The problem is similar for Foreign and is hence not developed.

2.1.3. Investment firms

In each country, there exists two types of firms: investment firms which produce new capital; and retail firms that use labour and capital as input to produce differentiated varieties of goods. Consider first investment firms. Capital producers operate in the domestic market only. At each period, capital producers produce and sell new capital to retail firms. Letting then I_t^h denote aggregate investment in Home, δ the capital depreciation rate, and ψ_{t+1}^h a Home-specific shock to the quality of capital, the law of motion of capital in Home is given by:

$$K_{t+1}^h = \psi_{t+1}^h [I_t^h + (1 - \delta)K_t^h] \quad (2.35)$$

(2.35) takes a conventional form, except for the ψ_{t+1}^h term. The latter enables to introduce a simple exogenous source of variation in the value of capital, and will serve as an exogenous trigger of asset price dynamics. As such, it may equivalently be denominated as a financial shock. The stochastic variable ψ_t^h is defined as:

$$\psi_t^h = (\bar{\psi})^{1-\rho_k} (\psi_{t-1}^h)^{\rho_k} e^{\epsilon_{k,t}^h} \quad (2.36)$$

$\bar{\psi}$ is some constant, ρ_k is a capital-specific (autoregressive) parameter with $0 < \rho_k < 1$, and $\epsilon_{k,t}^h$ is an *i.i.d* capital quality shock with a mean of 0. It can be readily verified that the steady-state value of ψ_t^h is $\bar{\psi}$, the latter being set to 1. This formulation also ensures that ψ_t^h cannot go negative. Capital producers build new capital using domestic varieties as sole input. The investment aggregator is then defined as:

$$I_t^h = \left(\int_0^1 I_t^h(j)^{\frac{\kappa-1}{\kappa}} dj \right)^{\frac{\kappa}{\kappa-1}} \quad (2.37)$$

From a cost-minimization problem, optimal demand for variety j produced at Home from capital producers is given by:

$$I_t^h(j) = \left(\frac{P_{H,t}}{P_{H,t}(j)} \right)^\kappa I_t^h \quad (2.38)$$

This also implies that aggregate investment expenditure is equal to $P_{H,t} I_t^h$.

Capital producers in Home sell capital at unit price Q_t^h . On the other hand, capital production involves convex (nominal) adjustment costs given by $P_{H,t} f(I_t^h - I_{t-1}^h)$, with f such that $f(0) = f'(0) = 0$ and $f''(I_t^h - I_{t-1}^h) > 0$. Here, f is chosen as :

$$f(I_t^h - I_{t-1}^h) = \frac{\iota}{2} (I_t^h - I_{t-1}^h)^2 \quad (2.39)$$

with $\iota > 0$ an investment adjustment cost parameter. Following, the objective of the capital producer is to choose I_t^h to solve:

$$\max_{I_t^h} E_t \sum_{i=0}^{\infty} \Lambda_{r,t+i}^h \left\{ Q_{r,t+i}^h I_{t+i}^h - P_{H,t+i} (I_{t+i}^h + f(I_{t+i}^h - I_{t+i-1}^h)) \right\}$$

Defining the real price of capital as $Q_{r,t}^h = \frac{Q_t^h}{P_t^h}$, the objective of the capital producer rewrites in real terms as:

$$\max_{I_t^h} E_t \sum_{i=0}^{\infty} \Lambda_{r,t+i}^h \left\{ Q_{r,t+i}^h I_{t+i}^h - \frac{P_{H,t+i}}{P_{t+i}^h} (I_{t+i}^h + f(I_{t+i}^h - I_{t+i-1}^h)) \right\} \quad (2.40)$$

The first-order condition with respect to I_t^h yields:

$$Q_{r,t}^h = \frac{P_{H,t}}{P_t^h} (1 + f'(I_t^h - I_{t-1}^h)) - E_t \Lambda_{r,t+1}^h \frac{P_{H,t+1}}{P_{t+1}^h} f'(I_{t+1}^h - I_t^h) \quad (2.41)$$

Or, given the postulated form for adjustment costs:

$$Q_{r,t}^h = \frac{P_{H,t}}{P_t^h} [1 + \iota (I_t^h - I_{t-1}^h)] - \iota E_t \Lambda_{r,t+1}^h \frac{P_{H,t+1}}{P_{t+1}^h} (I_{t+1}^h - I_t^h) \quad (2.42)$$

The setting for Foreign is similar, and is hence not developed further.

2.1.4. Retail firms

In both countries, monopolistic retail firms produce a continuum of differentiated good varieties over the interval $[0,1]$. Consider Home. The differentiated varieties of final goods are denoted by $Y_t^h(j)$, for $j \in [0,1]$. The final output composite for Home, denoted by Y_t^h , is a CES composite of the varieties and is given by:

$$Y_t^h = \left(\int_0^1 Y_t^h(j)^{\frac{\kappa-1}{\kappa}} dj \right)^{\frac{\kappa}{\kappa-1}} \quad (2.43)$$

For each retail firm, production is assumed to be realized by the way of a conventional constant return to scale function:

$$Y_t^h(j) = A_t^h K_t^h(j)^\alpha L_t^h(j)^{1-\alpha} \quad \forall j \in [0,1] \quad (2.44)$$

with A_t^h total factor productivity in Home assumed to be common across firms, and $K_t^h(j)$ and $L_t^h(j)$ respectively denoting capital and labour used by retail firm j . A_t^h is defined as:

$$A_t^h = (\bar{A}^h)^{1-\rho_a} (A_{t-1}^h)^{\rho_a} e^{\epsilon_{a,t}^h} \quad (2.45)$$

with \bar{A}^h the steady-state value of A_t^h . ρ_a is a productivity-specific autoregressive parameter such that $0 \leq \rho_a \leq 1$, and $\epsilon_{a,t}^h$ an *i.i.d* productivity shock with 0 mean.

Retail firms own capital, but its acquisition is financed by banks. A bank funding capital acquisition for a retail firm first lends Q_t^h – the price of one unit of capital – to the firm for each unit of capital it acquires. In return, it obtains a claim (or “asset”) over the stream of future returns from one unit of capital:

$$\psi_{t+1}^h Z_{t+1}^h(j), (1-\delta)\psi_{t+2}^h Z_{t+2}^h(j), (1-\delta)^2\psi_{t+3}^h Z_{t+3}^h(j) \dots$$

where $Z_t^h(j)$ is the dividend paid by retail firm j to the bank for each unit of capital acquired, and defined as:

$$Z_t^h(j) = \frac{P_{H,t}(j)Y_t^h(j) - W_t^h L_t^h(j)}{K_t^h(j)}$$

Price rigidities (developed hereafter) imply that future returns are unknown and will vary from one retail firm to another due to different opportunities of price updating. Hence the

return obtained by the different banks may differ according to the particular projects they finance. In order to avoid this situation, and in order to keep the model simple, it is assumed that banks are risk averse and hence decide to fund each retail firm equally. Doing so, each bank obtains a representative portfolio that contains a similar amount of assets from all retail firms and yields a riskless average return Z_t^h per unit of capital funded. Indeed, defining the dividend obtained by a bank over its representative portfolio as:

$$Z_t^h = \int_0^1 Z_t^h(j) dj$$

one can then easily show that this dividend rewrites as:

$$Z_t^h = \frac{P_{H,t} Y_t^h - W_t^h L_t^h}{K_t^h} \quad (2.46)$$

Defining the real dividend as $Z_{r,t}^h = \frac{Z_t^h}{P_t^h}$, (2.46) rewrites in real terms as:

$$Z_{r,t}^h = \frac{\frac{P_{H,t}}{P_t^h} Y_t^h - W_{r,t}^h L_t^h}{K_t^h} \quad (2.47)$$

Under this assumption, one also obtains the convenient result that the aggregate amount of capital will be equalised across retail firms. (2.44) then rewrites:

$$Y_t^h(j) = A_t^h (K_t^h)^\alpha L_t^h(j)^{1-\alpha} \quad (2.48)$$

It is assumed that firms face price rigidities à la Calvo (1983). Namely, at each period, a retail firm may reset its price only with a probability $(1 - \zeta)$. So at each period, a proportion $(1 - \zeta)$ of firms reset their prices while a proportion ζ don't. Firm j in Home reoptimizing its price in period t will choose a price $P_{H,t}^*(j)$ that maximizes the current market value of the profits generated while this price remains effective. Hence its program can be formally stated as:

$$\text{Max}_{P_{H,t}^*(j)} \sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^h \left[P_{H,t}^*(j) Y_{t+i|t}^h(j) - \Psi_{t+i}^h \left(Y_{t+i|t}^h(j) \right) \right] \right\}$$

subject to the stream of final demands: $Y_{t+i|t}^h(j) = C_{H,t+i}^h(j) + C_{H,t+i}^f(j) + G_{t+i}^h(j) + I_{t+i}^h(j)$, where $Y_{t+i|t}^h(j)$ denotes output in period $t+i$ for a firm that last reset its price in period t , and $\Psi_{t+i}^h(j)$ is the total cost function for retail firm j in Home at period $t+i$.

The first-order condition for this program is:

$$P_{H,t}^*(j) = \frac{\kappa}{\kappa-1} \cdot \frac{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^h Y_{t+i|t}^h(j) \Psi_{t+i}^{h^{-1}}(Y_{t+i|t}^h(j)) \right\}}{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^h Y_{t+i|t}^h(j) \right\}} \quad (2.49)$$

where $\Psi_{t+i}^{h^{-1}}(Y_{t+i|t}^h(j))$ is the nominal marginal cost incurred by firm j in period $t+i$ if it last reset its price in period t , and $\frac{\kappa}{\kappa-1}$ represents the firm's markup over the marginal cost it incurs. In the case where $\zeta=0$ (no price rigidity), this collapses to the conventional flexible equilibrium: $P_{H,t}^*(j) = \frac{\kappa}{\kappa-1} \Psi_{t+i}^{h^{-1}}(Y_t^h(j))$, so that $\frac{\kappa}{\kappa-1}$ can be interpreted as the frictionless markup. Also, defining Home inflation as the gross inflation rate for goods produced at Home:

$$\pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}} \quad (2.50)$$

one can obtain an expression for Home aggregate price dynamics as:

$$(\pi_{H,t})^{1-\kappa} = \zeta + (1-\zeta) \left(\frac{P_{H,t}^*}{P_{H,t-1}} \right)^{1-\kappa} \quad (2.51)$$

The setting is similar in Foreign and is not presented.

2.1.5. Banks

In both countries, there exists a banking system which allows to finance capital acquisition by domestic retail firms. Home is considered first. To finance capital, banks in Home obtain funds from two different markets: a retail market where banks raise deposits from household members; and an interbank market, where they may borrow and lend money from banks in Foreign. For simplicity, lending of funds between domestic banks is ignored. On the interbank market, it is assumed that transactions take the form of an international bond purchased at the price of one unit of Foreign currency. This can be justified by the fact that Foreign, as a group of countries gathered in a monetary union, is “large” with respect to Home, and has hence sufficient market power to impose its own currency for international fund trading. This implies that Home will have to bear the exchange rate uncertainty. In addition to the funds obtained from the retail and interbank markets, the bank may use its own net worth,

accumulated from retained earnings. A bank in Home then decides the volume of loans or “assets” s_t^h to make to retail firms to acquire capital. A simple way to think about these assets is to consider them as equity²⁸. Each loan sells at price Q_t^h and, as stated earlier, represents a bank's claim on the future profits from one unit of physical capital of retail firm. Then, for an individual bank, the flow-of-fund constraint implies that at each period, the value of the loans funded by the bank $Q_t^h s_t^h$ must equal the sum of the bank's net worth n_t^h , its borrowing on the interbank market $\varepsilon_t b_t^h$ and total deposits collected from Home d_t^h :

$$Q_t^h s_t^h = n_t^h + \varepsilon_t b_t^h + d_t^h \quad (2.52)$$

Let R_t^i denote the gross international nominal rate applying to loans realized on the international retail market. Then net worth at t is the gross payoff from assets funded at $t-1$, net borrowing costs:

$$n_t^h = [Z_t^h + (1-\delta)Q_t^h] \psi_t^h s_{t-1}^h - \varepsilon_t R_t^i b_{t-1}^h - R_t^h d_{t-1}^h \quad (2.53)$$

Defining real net worth as $n_{r,t}^h = \frac{n_t^h}{P_t^h}$, real interbank bonds as $b_{r,t}^h = \frac{\varepsilon_t b_t^h}{P_t^h}$ and real deposits as

$d_{r,t}^h = \frac{d_t^h}{P_t^h}$, the latter two equations rewrite:

$$Q_{r,t}^h s_t^h = n_{r,t}^h + b_{r,t}^h + d_{r,t}^h \quad (2.54)$$

and

$$n_{r,t}^h = [Z_{r,t}^h + (1-\delta)Q_{r,t}^h] \psi_t^h s_{t-1}^h - \frac{\varepsilon_t}{\varepsilon_{t-1}} \frac{R_t^i}{\pi_t^h} b_{r,t-1}^h - \frac{R_t^h}{\pi_t^h} d_{r,t-1}^h \quad (2.55)$$

As exposed later in this section, banks may be financially constrained. Hence, to overcome this situation, a bank might be willing to accumulate net worth up to the point where the constraint is not binding any more. To avoid the situation where a bank would accumulate net worth without restriction, turnover is allowed between workers and bankers. With *i.i.d* probability $(1-\sigma)$, a banker will cease its banking activity next period and become a worker (implying an average duration of banking activities of $\frac{1}{1-\sigma}$ periods). While becoming a worker, the banker transfers any retained earnings to the household. To keep the number of

²⁸ Although equity appears as the most meaningful interpretation of these assets, in some instances it will be useful to adopt a broader definition which also encompasses bonds. See for example section 3.4.1.

bankers constant, $(1-\sigma)f$ workers become bankers at the same time. Since the bank transfers earnings only when it exits, the bank's problem at the end of period t is to maximize the expected present value of future real net worth flows, as:

$$\text{Max } V_t^h = E_t \sum_{i=1}^{\infty} (1-\sigma)\sigma^{i-1} \Lambda_{r,t+i}^h n_{r,t+i}^h \quad (2.56)$$

Without some limit on the capacity of a bank to collect funds, a bank would find it optimal to expand its assets indefinitely by borrowing additional funds from households or interbank markets. To avoid this, a simple agency problem/moral hazard is introduced, which creates an endogenous limit on the bank's ability to obtain funds. At each period, after a bank obtains funds, the banker may transfer a fraction θ^h of “divertable” assets to the members of his family within the household. Divertable assets consist of total real gross assets $Q_{r,t}^h s_t^h$ net a fraction ω of real interbank borrowing with Foreign banks $b_{r,t}^h$.

That is: divertable assets = $\theta^h (Q_{r,t}^h s_t^h - \omega b_{r,t}^h)$, or using (2.54):

$$\text{divertable assets} = \theta^h (n_{r,t}^h + (1-\omega)b_{r,t}^h + d_{r,t}^h) \quad (2.57)$$

In this respect, the parameter ω can be interpreted as the relative degree of frictions in financial markets. As shown by (2.57), when $\omega=1$ the interbank market is perfectly frictionless as banks cannot divert assets financed by interbank borrowing. On the other hand, when $\omega=0$, the creditors are unable to recover the funds they lent to banks in Home if the bank defaults. In this case, the interbank market faces maximum frictions. In general, ω may take any value between 0 and 1 to represent diverse degrees of market imperfection. The cost for the banker is that upon diverting assets, its creditors can force the bank into bankruptcy and recover the value of non-diverted assets. Because the creditors (both depositors and other banks) are aware of the incentive to divert funds, they will restrict their amounts of lending. Namely, to prevent the bank from diverting assets, creditors must limit lending so that the value of the bank V_t^h remains larger than what the banker can obtain by diverting assets. Hence, the following incentive constraint must hold:

$$V_t^h \geq \theta^h (Q_{r,t}^h s_t^h - \omega b_{r,t}^h) \quad (2.58)$$

In general, the value of the bank (2.56) at the end of period t satisfies the Bellman equation:

$$V_t^h(s_t^h, b_{r,t}^h, d_{r,t}^h) = E_t \Lambda_{r,t+1}^h \left[(1-\sigma)n_{r,t+1}^h + \sigma \text{Max}_{s_{r,t+1}^h, b_{r,t+1}^h} V_{t+1}^h(s_{r,t+1}^h, b_{r,t+1}^h, d_{r,t+1}^h) \right] \quad (2.59)$$

where (2.59) emphasizes the fact that the banker at period t will become a worker at $t+1$ with a probability $(1-\sigma)$, and continues banking activities with probability σ . To solve for the bank's decision problem, one guesses first that the value function is a linear function of the balance sheet arguments, and verifies this guess later:

$$V_t^h(s_t^h, b_{r,t}^h, d_{r,t}^h) = v_{s,t}^h s_t^h - v_{b,t}^h b_{r,t}^h - v_t^h d_{r,t}^h \quad (2.60)$$

$v_{s,t}^h, v_{b,t}^h$ and v_t^h are time-varying parameters representing the shadow values of the different balance sheet components. In this respect, $v_{s,t}^h$ is the marginal value of a unit of assets, while $v_{b,t}^h$ and v_t^h denote respectively the marginal costs of one unit of interbank borrowing and one unit of deposit received by the bank. The maximization of (2.60) subject to the incentive constraint (2.58) yields the Lagrangian:

$$\mathcal{L} = [v_{s,t}^h s_t^h - v_{b,t}^h b_{r,t}^h - v_t^h d_{r,t}^h] (1 + \lambda_t^h) - \lambda_t^h \theta^h [Q_{r,t}^h s_t^h - \omega b_{r,t}^h]$$

where λ_t^h denotes the Lagrange multiplier with respect to the incentive constraint. The necessary first-order conditions with respect to $s_t^h, b_{r,t}^h$ and λ_t^h are respectively given by:

$$\left[\frac{v_{s,t}^h}{Q_{r,t}^h} - v_t^h \right] (1 + \lambda_t^h) = \theta^h \lambda_t^h \quad (2.61)$$

$$[v_{b,t}^h - v_t^h] (1 + \lambda_t^h) = \theta^h \omega \lambda_t^h \quad (2.62)$$

$$v_{s,t}^h s_t^h - v_{b,t}^h b_{r,t}^h - v_t^h d_{r,t}^h = \theta^h [Q_{r,t}^h s_t^h - \omega b_{r,t}^h] \quad (2.63)$$

Equation (2.61) states that at the optimum, the marginal value of assets (expressed in units of goods) exceeds the marginal cost of deposits if and only if the constraint is binding ($\lambda_t^h > 0$), and proportionally to the degree to which the constraint binds along with the bank capacity to divert assets (namely the values of λ_t^h and θ^h). Similarly, (2.62) states that the optimum implies that the excess marginal cost of interbank borrowing with respect to deposits will be positive if the constraint is binding. It is increasing in the tightness of the constraint, the capacity of banks to divert funds, and in the degree of financial friction (namely in the values of λ_t^h, θ^h and ω).

Combining (2.61) and (2.62) in order to get rid of the Lagrange multiplier, one obtains the following optimality condition:

$$\left(\frac{v_{s,t}^h}{Q_{r,t}^h} - v_t^h \right) = \frac{1}{\omega} (v_{b,t}^h - v_t^h) \quad (2.64)$$

(2.64) states that the excess value of assets is related to the excess cost of interbank borrowing in a way that is increasing in the degree of friction on the interbank market: as more frictions reduce the quantity of funds available to banks by the way of interbank borrowing, and hence implies that banks will finance fewer projects, the bank will get rid of the least profitable projects, therefore increasing the marginal value of assets. In the frictionless case $\omega=1$, one obtains the intuitive zero-profit condition that equates the marginal value of assets with the marginal cost of interbank borrowing.

Finally, (2.63) is the incentive constraint. It can be rewritten as:

$$Q_{r,t}^h s_t^h = \left(\frac{v_t^h}{\theta^h (1-\omega x_{r,t}^h) + v_t^h (1-x_{r,t}^h) + v_{b,t}^h x_{r,t}^h - \frac{v_{s,t}^h}{Q_{r,t}^h}} \right) n_{r,t}^h \quad (2.65)$$

or

$$Q_{r,t}^h s_t^h = \phi_t^h n_{r,t}^h \quad (2.66)$$

with $\phi_t^h = \frac{v_t^h}{\theta^h (1-\omega x_{r,t}^h) + v_t^h (1-x_{r,t}^h) + v_{b,t}^h x_{r,t}^h - \frac{v_{s,t}^h}{Q_{r,t}^h}}$ and $x_{r,t}^h = \frac{b_{r,t}^h}{Q_{r,t}^h s_t^h}$ the proportion of assets

financed by interbank borrowing.

(2.66) is an essential equation for the banking sector: it represents the leverage ratio for the bank, stating that when the incentive constraint is binding, the capacity of the bank to finance assets is limited by its net worth. It can be shown that the leverage ratio is an increasing function in the marginal values of assets $v_{s,t}^h$, and is decreasing in the marginal costs of interbank borrowing and deposits $v_{b,t}^h$ and v_t^h . Because an increase in $v_{s,t}^h$ rises the value of the bank V_t^h , it reduces the bank incentive to divert assets and hence relaxes the constraint, making creditors willing to lend more funds to the bank. The converse holds for an increase in $v_{b,t}^h$ or v_t^h . Also, and quite intuitively, the leverage ratio is increasing in ω , the (inverse) market friction parameter, and decreasing in θ^h , the firm capacity to divert asset. As creditors become able to recover a larger fraction of the funds they lent, the amount the bank can obtain by diverting funds shrinks, and creditors can lend more funds before the risk of default arises.

Combining the conjectured solution (2.60) with the Bellman equation (2.59), net worth (2.55) and the leverage ratio (2.66), one obtains:

$$v_{s,t}^h s_t^h - v_{b,t}^h b_{r,t}^h - v_t^h d_{r,t}^h = E_t \left(\Lambda_{r,t+1}^h \Omega_{t+1}^h n_{r,t+1}^h \right)$$

with:

$$\Omega_t^h = (1 - \sigma) + \sigma \left[v_t^h + \phi_t^h \left\{ \left(\frac{v_{s,t}^h}{Q_{r,t}^h} - v_t^h \right) - (v_{b,t}^h - v_t^h) x_{r,t}^h \right\} \right] \quad (2.67)$$

Then, from an undetermined coefficient method, the values for v_t^h , $v_{b,t}^h$ and $v_{s,t}^h$ obtain as:

$$v_t^h = E_t \left(\frac{R_{t+1}^h}{\pi_{t+1}^h} \Lambda_{r,t+1}^h \Omega_{t+1}^h \right) \quad (2.68)$$

$$v_{b,t}^h = v_t^h E_t \left(\frac{R_{t+1}^i}{R_{t+1}^h} \frac{\varepsilon_{t+1}}{\varepsilon_t} \right) \quad (2.69)$$

$$v_{s,t}^h = v_t^h E_t \frac{\pi_{t+1}^h \left([Z_{r,t+1}^h + (1 - \delta) Q_{r,t+1}^h] \psi_{t+1}^h \right)}{R_{t+1}^h} \quad (2.70)$$

Ω_t^h represents the stochastic marginal value of net worth for the banker, who exits next period with probability $(1 - \sigma)$ and remains active with probability σ . In case of exiting, the marginal value is unity. If the banker continues its activity, the value becomes the sum of the marginal cost of deposits held, with the excess value of assets and excess cost of interbank borrowing weighted by the leverage ratio.

(2.68) shows that at the optimum, the marginal cost of deposits must be equal to the expected real return that the bank will have to pay to its depositors at the subsequent period, adjusted for both the stochastic discount and the stochastic event of pursuing or not banking activities. Then (2.69) states that optimality implies that the marginal cost of interbank borrowing is equal to the marginal cost of deposits, corrected for the relative cost of interbank loans over deposits, and for the repayment uncertainty related to the evolution of the exchange rate. Similarly, (2.70) states that the marginal value of assets is equal to the marginal cost of deposits, corrected for the expected relative return of capital over deposits.

The problem for Foreign banks is similar, except that banks in Foreign do not suffer from the exchange rate uncertainty on interbank loans, and collect deposits from both Home and Foreign. Thus, for an individual bank, at each period, the value of the loans funded by the

bank $Q_t^f s_t^f$ must equal the sum of the bank's net worth n_t^f , its borrowing (or lending) on the interbank market b_t^f and total deposits collected from both Home and Foreign $f_t = f_t^f + f_t^h$:

$$Q_t^f s_t^f = n_t^f + b_t^f + f_t \quad (2.71)$$

Then net worth at t is the gross payoff from assets funded at $t-1$, net borrowing costs:

$$n_t^f = [Z_t^f + (1-\delta)Q_t^f] \psi_t^f s_{t-1}^f - R_t^i b_{t-1}^f - R_t^f f_{t-1} \quad (2.72)$$

Defining the real variables $Q_{r,t}^f = \frac{Q_t^f}{P_t^f}$, $n_{r,t}^f = \frac{n_t^f}{P_t^f}$, $b_{r,t}^f = \frac{b_t^f}{P_t^f}$, and $f_{r,t} = \frac{f_t}{P_t^f}$, these two equations

rewrite in real terms as:

$$Q_{r,t}^f s_t^f = n_{r,t}^f + b_{r,t}^f + f_{r,t} \quad (2.73)$$

and

$$n_{r,t}^f = [Z_{r,t}^f + (1-\delta)Q_{r,t}^f] \psi_t^f s_{t-1}^f - \frac{R_t^i}{\pi_t^f} b_{r,t-1}^f - \frac{R_t^f}{\pi_t^f} f_{r,t-1} \quad (2.74)$$

At each period, a banker exits banking activities with probability $(1-\sigma)$. Since the bank transfers earnings only when it exits, the bank's problem at the end of period t is to maximize the expected present value of future real net worth flows, as:

$$\text{Max } V_t^f = E_t \sum_{i=1}^{\infty} (1-\sigma) \sigma^{i-1} \Lambda_{t,t+i}^f n_{r,t+i}^f \quad (2.75)$$

Also, at each period, after an active banker collects funds, he may transfer a fraction θ^f of divertable assets to his family, consisting of total real gross assets $Q_{r,t}^f s_t^f$ net a fraction ω of real interbank borrowing $b_{r,t}^f$. The incentive constraint of the bank then implies that the value of the bank V_t^f must be at least as large as what the banker may earn by diverting assets:

$$V_t^f \geq \theta^f (Q_{r,t}^f s_t^f - \omega b_{r,t}^f) \quad (2.76)$$

In general, the value of the bank (2.75) at the end of period t satisfies the Bellman equation:

$$V_t^f(s_t^f, b_{r,t}^f, f_{r,t}) = E_t \Lambda_{t,t+1}^f \left[(1-\sigma) n_{r,t+1}^f + \sigma \text{Max}_{s_{t+1}^f, b_{r,t+1}^f} V_{t+1}^f(s_{t+1}^f, b_{r,t+1}^f, f_{r,t+1}) \right] \quad (2.77)$$

To solve for the bank's decision problem, one assumes that the value function takes the following linear form:

$$V_t^f(s_t^f, b_{r,t}^f, f_{r,t}) = v_{s,t}^f s_t^f - v_{b,t}^f b_{r,t}^f - v_t^f f_{r,t} \quad (2.78)$$

This implies the following optimality conditions:

$$\left(\frac{v_{s,t}^f}{Q_{r,t}^f} - v_t^f \right) = \frac{1}{\omega} (v_{b,t}^f - v_t^f) \quad (2.79)$$

$$Q_{r,t}^f s_t^f = \phi_t^f n_{r,t}^f \quad (2.80)$$

$$\text{with } \phi_t^f = \frac{v_t^f}{\theta^f (1 - \omega x_{r,t}^f) + v_t^f (1 - x_{r,t}^f) + v_{b,t}^f x_{r,t}^f - \frac{v_{s,t}^f}{Q_{r,t}^f}} \text{ and } x_{r,t}^f = \frac{b_{r,t}^f}{Q_{r,t}^f s_t^f} \quad (2.81)$$

Combining the conjectured solution (2.78) with the Bellman equation (2.77), net worth (2.74) and the leverage ratio (2.80), one obtains:

$$v_{s,t}^f s_t^f - v_{b,t}^f b_{r,t}^f - v_t^f f_{r,t} = E_t \left(\Lambda_{r,t+1}^f \Omega_{t+1}^f n_{r,t+1}^f \right)$$

with:

$$\Omega_t^f = (1 - \sigma) + \sigma \left[v_t^f + \phi_t^f \left\{ \left(\frac{v_{s,t}^f}{Q_{r,t}^f} - v_t^f \right) - (v_{b,t}^f - v_t^f) x_{r,t}^f \right\} \right] \quad (2.82)$$

Then, from an undetermined coefficient method, the values for v_t^f , $v_{b,t}^f$ and $v_{s,t}^f$ obtain as:

$$v_t^f = E_t \left(\frac{R_{t+1}^f}{\pi_{t+1}^f} \Lambda_{r,t+1}^f \Omega_{t+1}^f \right) \quad (2.83)$$

$$v_{b,t}^f = v_t^f \frac{R_{t+1}^f}{R_{t+1}^f} \quad (2.84)$$

$$v_{s,t}^f = v_t^f E_t \frac{\pi_{t+1}^f \left([Z_{r,t+1}^f + (1 - \delta) Q_{r,t+1}^f] \psi_{t+1}^f \right)}{R_{t+1}^f} \quad (2.85)$$

2.1.6. Monetary policy

The central bank at Home is assumed to use the domestic interest rate as its main tool, following a Taylor-type rule:

$$\log(R_{t+1}^h) = \log(R^h) + \phi_\pi^h \log \pi_{H,t} + \phi_y^h \log \left(\frac{Y_t^h}{Y} \right) + \zeta_t^h \quad (2.86)$$

where ϕ_π^h and ϕ_y^h are Home-specific, positive parameters determined by the central bank. The monetary stochastic process is defined as:

$$\zeta_t^h = \rho_m \zeta_{t-1}^h + \epsilon_{m,t}^h \quad (2.87)$$

with ρ_m a monetary autoregressive parameter, and $\epsilon_{m,t}^h$ an *i.i.d* monetary shock with 0 mean. Monetary policy in Foreign is comparable and hence not presented.

2.1.7. Equilibrium and aggregation

This section closes the model by presenting both aggregation procedures and market clearing conditions. As the conditions are similar in Home and Foreign, only Home is detailed. Consider first the goods market. For variety j produced at Home, the market clearing condition is:

$$Y_t^h(j) = C_{H,t}^h(j) + C_{H,t}^f(j) + I_t^h(j) + G_t^h(j) \quad (2.88)$$

From (2.11), (2.14) (2.33) and (2.38), along with the index definitions of the Home final output index and the Home price index (2.7) and (2.43), (2.88) collapses to a conventional resource constraint:

$$Y_t^h = C_{H,t}^h + C_{H,t}^f + I_t^h + G_t^h \quad (2.89)$$

Turn now to the labour market. Given labour demand for each individual retail firm in Home, the market equilibrium condition obtains from the following relation between the economy average real marginal cost (expressed in terms of domestic prices) and the average marginal product of labour:

$$MC_{r,t}^h = \frac{W_t^h}{P_{H,t}} \times \frac{1}{MPL_t^h}$$

This expression rewrites in terms of CPI real wage as:

$$MC_{r,t}^h = W_{r,t}^h \times \frac{P_t^h}{P_{H,t}^h} \times \frac{1}{MPL_t^h} \quad (2.90)$$

Concerning public authorities: in this benchmark version of the model, it is assumed for simplicity that governments always run balanced budget. They finance public expenditure by lump-sum taxation. Equilibrium for the public sector is thus given by:

$$P_{H,t} G_t^h = T_t^h$$

or, in real terms:

$$\frac{P_{H,t}}{P_t^h} G_t^h = T_{r,t}^h \quad (2.91)$$

Turn finally to the banking sector. First, because all banks in Home are similar, one can conveniently sum (2.63) across banks to obtain the equation determining aggregate asset demand in Home:

$$v_{s,t}^h S_t^h - v_{b,t}^h B_{r,t}^h - v_t^h D_{r,t}^h = \theta^h [Q_{r,t}^h S_t^h - \omega B_{r,t}^h] \quad (2.92)$$

with S_t^h , $B_{r,t}^h$ and $N_{r,t}^h$ denoting respectively aggregate asset, aggregate real interbank borrowing and aggregate real net worth in Home. Next, because each unit of asset finances one unit of capital, market clearing requires that the volume of securities issued equals aggregate capital acquired. This condition yields aggregate asset supply as:

$$S_t^h = I_t^h + (1 - \delta) K_t^h \quad (2.93)$$

(2.54) can be used to obtain the balance sheet of the entire banking sector: aggregate deposits equal the difference between total assets, bank net worth and interbank borrowings as follows:

$$D_{r,t}^h = Q_{r,t}^h S_t^h - N_{r,t}^h - B_{r,t}^h \quad (2.94)$$

Market clearing on the international interbank market requires that net aggregate lending in one country corresponds to net aggregate borrowing in the other, so that the following condition obtains:

$$B_t^h = -B_t^f$$

which, in real terms, yields:

$$B_{r,t}^h \frac{P_t^h}{\varepsilon_t} = -B_{r,t}^f P_t^f \quad (2.95)$$

Concerning eventually the evolution of total net worth $N_{r,t}^h$: at each period, a proportion $(1-\sigma)$ of bankers leave banking activities, while the remaining proportion σ continue their activities. At the same time, $(1-\sigma)f$ workers become bankers, so as to keep the number of employees in each occupation constant. Because new bankers will not be able to operate without some initial funds, retiring bankers transfer remaining earnings to the household, except for a small proportion $\frac{\xi^h}{1-\sigma}$ of total assets that is given to entering bankers as a start-up transfer, with ξ^h a small, Home-specific parameter. Hence, aggregate net worth $N_{r,t}^h$ must equal the sum of the net worth of existing bankers $N_{o,t}^h$ (o for old) and of entering entrepreneurs $N_{y,t}^h$ (y for young). Using (2.55):

$$N_{o,t}^h = \sigma \left\{ [Z_{r,t}^h + (1-\delta)Q_{r,t}^h] \psi_t^h S_{t-1}^h \right\} - \frac{\sigma}{\pi_t^h} \left\{ \frac{\varepsilon_t}{\varepsilon_{t-1}} R_t^i B_{r,t-1}^h + R_t^h D_{r,t-1}^h \right\}$$

On the other hand, the transfer in favour of entering bankers is equal to:

$$N_{y,t}^h = \xi^h \left\{ [Z_{r,t}^h + (1-\delta)Q_{r,t}^h] \psi_t^h S_{t-1}^h \right\}$$

This implies that total real net worth is given by:

$$N_{r,t}^h = (\sigma + \xi^h) \left\{ [Z_{r,t}^h + (1-\delta)Q_{r,t}^h] \psi_t^h S_{t-1}^h \right\} - \frac{\sigma}{\pi_t^h} \left\{ \frac{\varepsilon_t}{\varepsilon_{t-1}} R_t^i B_{r,t-1}^h + R_t^h D_{r,t-1}^h \right\} \quad (2.96)$$

This concludes the presentation of the model. Appendix B.1 provides a summary of the equilibrium conditions for the model. Appendix B.2 reports the linearized equilibrium conditions for the model.

2.2. A canonical model of financial frictions in monetary unions

This section introduces the monetary union version of the model. As most of the model is left unchanged when compared to the independent currency version, the incoming paragraphs will be restricted to the parts of the model for which modifications apply.

2.2.1. Households

Domestic consumption and price indices in both countries remain similar to the independent currency version. However, as Home and Foreign now share a single common currency, the exchange rate disappears from all transactions. Following, the consumer price indices for Home and Foreign are now respectively defined as:

$$P_t^h = \left(\varphi^h (P_{H,t})^{1-\mu} + (1-\varphi^h)(P_{F,t})^{1-\mu} \right)^{\frac{1}{1-\mu}} \quad (2.97)$$

and

$$P_t^f = \left(\varphi^f (P_{F,t})^{1-\mu} + (1-\varphi^f)(P_{H,t})^{1-\mu} \right)^{\frac{1}{1-\mu}} \quad (2.98)$$

and optimal demands for abroad varieties are given by:

$$C_{F,t}^h(j) = (1-\varphi^h) \left(\frac{P_t^h}{P_{F,t}} \right)^\mu \left(\frac{P_{F,t}}{P_{F,t}(j)} \right)^\kappa C_t^h = \left(\frac{P_{F,t}}{P_{F,t}(j)} \right)^\kappa C_{F,t}^h \quad (2.99)$$

and

$$C_{H,t}^f(j) = (1-\varphi^f) \left(\frac{P_t^f}{P_{H,t}} \right)^\mu \left(\frac{P_{H,t}}{P_{H,t}(j)} \right)^\kappa C_t^f = \left(\frac{P_{H,t}}{P_{H,t}(j)} \right)^\kappa C_{H,t}^f \quad (2.100)$$

Consider now the Home household. The problem solved by the Household is fairly similar to the one in the independent currency case. The main difference is that as there is now one single monetary authority (the common central bank), household members have only access to one riskless deposit security common to both Home and Foreign. This security takes the form of a riskless, one-period bond sold at unit price and repaying gross nominal interest R_{t+1} at maturity, one period later. Then the household at Home chooses consumption C_t^h , hours worked L_t^h , and deposits D_t^h so as to solve the program:

$$\text{Max}_{C_t^h, L_t^h, D_t^h, F_t^h} E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}^h, L_{t+i}^h) \quad 0 < \beta < 1$$

$$\text{with } U(C_t^h, L_t^h) = \frac{(C_t^h)^{1-\eta}}{1-\eta} - \frac{\chi^h}{1+\phi} (L_t^h)^{1+\phi} \quad (2.101)$$

subject to the budget constraint:

$$P_t^h C_t^h + D_t^h + T_t^h = W_t^h L_t^h + R_t^h D_{t-1}^h + P_t^h A C_t^h, \quad (2.102)$$

and the appropriate transversality condition. AC_t^h is now an interest *premium* paid to the Home household on bond holdings, defined as $AC_t^h = \frac{\vartheta}{2} \left(\frac{D_t^h}{P_t^h} - \frac{D^h}{P^h} \right)^2$, with ϑ some constant, and where variables without a t subscript denote steady-state values²⁹. This can be interpreted e.g. as a small compensation granted to the Home household for the renunciation of the choice of two different bonds (the Home and Foreign bonds in the flexible exchange version of the model) in favour of the unique bond offered by the Monetary Union. The first-order conditions for this problem, expressed in real terms, are given by:

$$\chi^h (L_t^h)^\phi (C_t^h)^\eta = W_{r,t}^h \quad (2.103)$$

and

$$1 - \vartheta (D_{r,t}^h - D_r^h) = R_{t+1} \beta E_t \left[\left(\frac{C_t^h}{C_{t+1}^h} \right)^\eta \frac{1}{\pi_{t+1}^h} \right] \quad (2.104)$$

The problem for Foreign is similar except that Foreign does not benefit from the interest premium. Hence, the first-order conditions for Foreign are standard and given by:

$$\chi^f (L_t^f)^\phi (C_t^f)^\eta = W_{r,t}^f \quad (2.105)$$

and

$$1 = R_{t+1} \beta E_t \left[\left(\frac{C_t^f}{C_{t+1}^f} \right)^\eta \frac{1}{\pi_{t+1}^f} \right] \quad (2.106)$$

2.2.2. Banks

The financial system is globally similar to the one described in section 2.1.5. The only difference is that as there is no more exchange rate, Home banks do not have anymore to bear the uncertainty relative to the reimbursement of interbank loans. Starting from an individual

²⁹ One may notice that the interest premium is the converse of the intermediation cost proposed for the flexible exchange version of the model. The justification is once again to be found in technical issues related to the Blanchard and Kahn conditions for a unique and stable equilibrium of the model dynamics. In the flexible change version of the model, the model would suffer from a unit root without the intermediation cost, i.e. there would be one unstable eigenvalue in excess and no stable equilibrium. The converse problem appears for the monetary union version: without the interest premium, there would be one unstable eigenvalue too few, so that the equilibrium would not be unique; the interest premium hence guarantees unicity. In both cases, as will be detailed in the next section, ϑ is calibrated to be very small (0.0001 for the intermediation cost, 0.00001 for the interest premium) so that the impact on the dynamics is negligible.

bank, the flow-of-fund constraint states that the value of the loans funded by the bank $Q_t^h s_t^h$ must equal the sum of the bank's net worth n_t^h , its borrowing on the interbank market b_t^h and total deposits collected from Home d_t^h :

$$Q_t^h s_t^h = n_t^h + b_t^h + d_t^h \quad (2.107)$$

Net worth is then defined as:

$$n_t^h = [Z_t^h + (1 - \delta)Q_t^h] \psi_t^h s_{t-1}^h - R_t^i b_{t-1}^h - R_t d_{t-1}^h \quad (2.108)$$

In real terms, these two equations rewrite:

$$Q_{r,t}^h s_t^h = n_{r,t}^h + b_{r,t}^h + d_{r,t}^h \quad (2.109)$$

and

$$n_{r,t}^h = [Z_{r,t}^h + (1 - \delta)Q_{r,t}^h] \psi_t^h s_{t-1}^h - \frac{R_t^i}{\pi_t^h} b_{r,t-1}^h - \frac{R_t}{\pi_t^h} d_{r,t-1}^h \quad (2.110)$$

Assuming an agency problem comparable to the one of the independent currency version of the model, one assumes that the value function of the bank takes a linear form given by:

$$V_t^h(s_t^h, b_{r,t}^h, d_{r,t}^h) = v_{s,t}^h s_t^h - v_{b,t}^h b_{r,t}^h - v_t^h d_{r,t}^h \quad (2.111)$$

And from an undetermined coefficient method, the values for v_t^h , $v_{b,t}^h$ and $v_{s,t}^h$ obtain as:

$$v_t^h = E_t \left(\frac{R_{t+1}}{\pi_{t+1}^h} \Lambda_{r,t+1}^h \Omega_{t+1}^h \right) \quad (2.112)$$

$$v_{b,t}^h = v_t^h E_t \left(\frac{R_{t+1}^i}{R_{t+1}} \right) \quad (2.113)$$

$$v_{s,t}^h = v_t^h E_t \frac{\pi_{t+1}^h ([Z_{r,t+1}^h + (1 - \delta)Q_{r,t+1}^h] \psi_{t+1}^h)}{R_{t+1}} \quad (2.114)$$

The problem for Foreign banks is perfectly similar and is hence not developed.

2.2.3. Monetary policy

The monetary policy of the entire monetary union is now determined by a single central bank. The rule followed by the central bank closely follows the Taylor-type rule implemented by the individual central banks of the independent currency version of the model. However, as

the single central authority must now take into account the economic situation of the two countries it governs, it acts according to a weighted average of the respective output gaps and inflation rates of Home and Foreign, the weight being the size of each country:

$$\log(R_{t+1}) = \log(R_t) + \phi_\pi (\gamma \log \pi_{H,t} + (1-\gamma) \log \pi_{F,t}) + \phi_y \left(\gamma \log \left(\frac{Y_t^h}{Y^h} \right) + (1-\gamma) \log \left(\frac{Y_t^f}{Y^f} \right) \right) + \zeta_t \quad (2.115)$$

where ϕ_π and ϕ_y are positive parameters determined by the central bank, and γ is the relative size of Home, as mentioned in section 2.1.1. Hence, in contrast with the independent currency model, the monetary policy led at Union scale and hence effective in Home may be largely independent from Home domestic activity. In the limit case where γ tends to zero, monetary policy is perfectly insulated from Home. Finally, the monetary stochastic process is defined as:

$$\zeta_t = \rho_m \zeta_{t-1} + \epsilon_{m,t} \quad (2.116)$$

with ρ_m a monetary autoregressive parameter, and $\epsilon_{m,t}$ an *i.i.d* monetary shock with 0 mean.

2.2.4. Equilibrium and aggregation

As interbank transactions are now denominated in a single currency, the market clearing condition in real terms omits the exchange rate and rewrites:

$$B_{r,t}^h P_t^h = -B_{r,t}^f P_t^f \quad (2.117)$$

This closes the description of the model. Appendices B.3 and B.4 respectively provide a summary of the original and linearized equilibrium conditions of the model.

2.3. Calibration and steady-state

2.3.1. calibration

Calibration is conducted on the model for the 17 countries currently composing the Eurozone. The setting that is considered goes as follows: one country is considered separately (the Home economy), while the other Eurozone countries constitute the rest of the region (Foreign). The parameters to be calibrated for the Home and Foreign economies can be divided into parameters that are common to both countries, and parameters which are country-specific, the latter attempting to capture the salient features of the economies under consideration.

Common parameters. The model to be calibrated is quarterly. Then, the value of the subjective discount factor β is set to 0.99, a common value in the literature which amounts to a 4% annual real interest rate. There is no real consensus about the value of η , the risk aversion parameter. Much of the macroeconomic literature has adopted a value around 1, in the line of the seminal work by Arrow (1971) and further studies like Mehra and Prescott (1985) who find CRRA's ranging from near 0 to 2. On the other hand, more recent works in financial economics have provided estimates of the value of individuals's CRRA's that exceed earlier suggestions, with a range from 2 to above 10 (see e.g. Blake (1996), Brav et al. (2002), Campbell (1996)). Following Backus et al. (1992), a value of 2 is attributed, in agreement with both fields. The inverse Frisch elasticity of labour supply ϕ is assumed to be 1, in the line of many microeconomic studies (see Ham and Reilly (2006) for a recent example of such studies). The elasticity of substitution parameter κ is set to 6, following the benchmark value supplied in Rotemberg and Woodford's (1992), implying a steady-state 20% markup of prices over marginal cost. In numerous macro studies, the elasticity of substitution between Home and Foreign goods μ is found to lie between 1 and 2 (Deardorff and Stern (1990), Whalley (1985)). On this basis, and following Backus et al. (1993), μ is set to 1.5. To end up with the consumer side, the parameter ε for intermediation cost/interest premium on portfolio holding must be determined. It is set at 0.0001 for the flexible exchange rate version of the model, and at 0.00001 for the monetary union version. The values are purposely very small in order to limit as much as possible the impact of the parameter on the dynamics of the model. Otherwise, substantial gaps in performances could be created between the two regimes, with the risk of attributing them to the monetary regime while it would actually be due to too large a value for ε .

For the supply side, Willman (2002) estimates the capital share of output α to be around 0.3 in the Euro area. Gözic (2007) finds capital depreciation rates in EU countries ranging from 2% to 9%, with an average value over all the countries around 5.5% (implying a quarterly depreciation rate of 0.013). These values are consistent with the ones commonly found in the recent literature (see e.g. Gomme and Ruppert (2007)) and are hence attributed to α and δ . Angeloni et al. (2004) find that European firms change prices once a year on average, a result supported by Dhyne et al. (2005) who estimate a median price duration of roughly 11 months in Europe. This leads to establish the value of ζ at 0.75. Finally, the adjustment cost parameter ι is set to 0.025, following Mendoza (1991).

Regarding the financial part of the model, only one parameter must be calibrated: σ , the survival rate of bankers. Following Gertler and Kiyotaki (2010), the value of σ is established so that bankers remain on average active for 10 years, implying $\sigma = 0.975$.

Finally, there remains to calibrate σ_a , σ_m and σ_k , the standard error of the structural shocks of the model, along with the corresponding autoregressive parameters of the stochastic process ρ_a , ρ_m , and ρ_k . First, the autoregressive parameters ρ_a , ρ_m and ρ_k are attributed respective values of 0.9, 1/2 and 2/3, which corresponds to moderately persistent capital quality shocks and monetary shocks, and to relatively high-inertia productivity shocks.

The standard errors of productivity shocks σ_a is estimated by fitting Eurozone data to equation (2.45), while σ_m is obtained by first fitting the Taylor rule (2.86) to data, then by estimating the autoregressive process (2.87). The resulting values are $\sigma_a = 0.0041$, and $\sigma_m = 0.0028$. These values are in line with the ones estimated in the recent literature for the standard deviation of productivity and monetary shocks in the Eurozone: see e.g. Cecioni and Neri (2011). The literature provides no estimates for σ_k , the standard error of the capital quality shocks, so that one relies entirely on empirical calculation. σ_k is obtained by first computing the capital quality process ψ_t from (2.35), then fitting an AR(1) model for (2.36). σ_k is then computed as the empirical standard error of the residuals $\epsilon_{k,t}^h$. The resulting value is $\sigma_k = 0.0006$. Details of the derivations of σ_a , σ_m and σ_k are provided in Appendix B.5. In addition, Appendix B.6 reports the sources of all the data used for these derivations and subsequent calculations realized in this section.

As a final remark about structural shocks, one may consider the possibility of including covariance terms between Home and Foreign shocks. Symmetry of shocks does matter, especially for a monetary union where a single monetary authority must determine the policy which will simultaneously apply to two countries potentially affected by heterogeneous shocks. This point, initially made by Mundell (1961), has been recently reaffirmed by Forlatti (2009), who shows that the performance of a monetary union may crucially depends on the degree of similarity of shocks experienced by the member economies. In the current state of the literature, however, there is no estimate of the values that such covariance terms could

take. Empirical evidence also suggests that shock asymmetry remains prevalent within the EMU, and that convergence within the Euro area is at best regional or sectorial (Bayoumi and Eichengreen (1993), Frenkel and Nickel (2005)). For this reason, the model assumes that no cross-correlation exists between Home and Foreign shocks.

Country-specific parameters. The general calibration procedure developed in this paragraph consists in computing individual parameter values for each of the 17 Eurozone countries; then the individual value is retained for the country representing Home, while an average over the values of the remaining countries is used to obtain the parameter value for Foreign.

Steady-state values for total factor productivity \bar{A}^h and \bar{A}^f are obtained by using series of real labour productivity per hour worked for each country. As an intermediary step, TFP in Foreign is computed as the weighted average of TFPs for the countries composing Foreign, where the weights are given by the output share of each country over Foreign. As only relative TFP matters, the value of \bar{A}^f is then normalized to 1, while \bar{A}^h obtains as the ratio of Home TFP over Foreign TFP.

ϑ^h and ϑ^f obtain from series of general government expenditure as a percentage of GDP for each country. ϑ^h is simply the value of the specific country treated as Home, while ϑ^f is computed as the weighted average over the country representing Foreign, the weights being once again the output shares in Foreign.

The trade openness parameter ν^h is computed with a similar weighting method. It is derived as the share of imported goods over GDP. Also, from data for the population of each of the 17 Eurozone countries included in the study, the relative size γ of each Home economy is computed as its share over the total population, while Foreign takes size $(1-\gamma)$. Then, from γ , ν^h and ν^f , one can compute φ^h and φ^f .

The disutility weight of labour in Foreign χ^f is normalized to 5, a conventional value in line with the macroeconomic literature (see e.g. Gertler and Kiyotaki (2010), or Villa and Yang (2011)). Using data for average weekly hours worked in each Eurozone country, the ratio of weekly hours in Foreign over weekly hours in Home is then computed. The value of χ^h is

eventually derived as the latter ratio times the value of χ^f , reflecting the idea that fewer weekly hours of work in Home imply a larger disutility from work, hence a larger value for χ^h .

Following Gertler and Kiyotaki (2010), ξ^h , ξ^f , θ^h and θ^f are chosen to hit exogenously calibrated steady-state values for ϕ^h and ϕ^f , the leverage ratios for Home and Foreign banks, and x^h , the share of total asset financed by interbank borrowing for Home banks³⁰. ϕ^h is calibrated for each individual country from Eurostats series for the leverage of the financial sector (debt to equity), while the value for ϕ^f is computed from the usual output weighting process. Values for x^h are derived, for most countries, from the OECD bank profitability report (2000-2009) which provides aggregate data for the ratio of aggregate interbank liabilities over total assets. For the four countries for which data is missing (Cyprus, Malta, Slovenia, Estonia), the ratio is estimated from the average ratio over three major banks of each country (data being obtained from the annual reports of these banks).

A final issue arises from these values, as they represent total interbank operations (hence including both domestic and international interbank loans), while the present model considers international loans only. According to the ECB, international transactions accounted for roughly two third of unsecured overnight interbank lending before 2008³¹. Data from the BIS (see Figure 2.2) suggests that cross-border transactions actually represent around 80% of interbank transactions. Taking an in-between result, x^h is calibrated as three quarters of the observed ratio of interbank liabilities over assets.

The final part of the calibration process concerns the estimation of the Taylor rule parameters for the ECB and the Central banks of individual countries. In the flexible exchange rate version of the model, the aim of the calibration for Home is to reflect the values that the policy parameters would take for this individual country if it were to leave the Eurozone and retrieve its monetary policy independence. In the monetary Union version of the model (and for the monetary union formed by Foreign in the case of the flexible exchange version), the

³⁰ The reader is referred to Appendix B.7 relative to the steady-state derivation for details about the computation of these parameters.

³¹ Speech by Benoit Coeuré, Member of the executive board of the ECB, 17th global securities financing summit, Luxembourg, 16 January 2013.

aim of the calibration is to replicate the parameter values of the Taylor rule followed by the unique central bank of the Union, in our case the European Central Bank.

For the rules followed by individual Eurozone countries, an issue arises from the fact that most countries have relinquished their monetary policies in 1999, so that one cannot rely on data collected since then to evaluate a potential Taylor rule. The most reasonable solution then seems to rely on historical analysis of the policy led by each country before the Euro implementation, and use it as a proxy for the rule that would be implemented today, being aware that the resulting calibration is at best suggestive. In this respect, Vasicek (2007) and Eleftheriou et al. (2006) review the existing literature on Taylor rule estimates for individual Euro countries and propose personal estimates for a number of different rules, prior to and after the Euro implementation in 1999. These studies are relatively large and exhaustive, but as a synthetic conclusion, one may identify three broad categories of policies over the pre-1999 period. Strict inflation targeting constitutes the first category of policy, applied either by countries following an explicit inflation target, such as Germany, Ireland or Finland, or by countries closely connected to the German monetary policy, as in the case of Austria, Belgium or the Netherlands. On the other hand, some countries attributed less importance to contemporary inflation, either because the monetary authorities were applying a strong degree of interest smoothing or exchange rate peg (Estonia, Cyprus, Malta, Slovenia), or because most weight was put on the output gap (Greece). This second category of rules may then be schematically labelled as output targeting. A third category of policy can then be identified for countries in-between, where central authorities policies attributed significant consideration to both domestic inflation, output gap and external variables such as the exchange rate or the German rate. France, Italy, Spain and Portugal belong to this mixed targeting category.

These three categories seem reasonable to represent the monetary policy that a newly independent country could adopt. As a check, the simple Taylor rule of the present model is then estimated for each of the 17 Eurozone countries prior to 1999 on quarterly data. Although the estimated rules differ from those estimated in Vasicek (2007) and Eleftheriou et al. (2006), the results typically validate the above categories of policy rules. The following calibration is hence applied: for countries classified as strictly inflation-targeting, namely Germany, Austria, Belgium, the Netherlands, Finland, Ireland and Luxembourg, a value of 2.5 is attributed to ϕ_π , while ϕ_y is given a value of 0.25. Output-targeting countries (Cyprus,

Estonia, Slovenia, Slovakia, Greece and Malta) are attributed respective values of 1.5 for ϕ_π and 1.25 for ϕ_y . Finally, for mixed rules (France, Italy, Spain and Portugal), values of 2 and 0.75 are given to ϕ_π and ϕ_y . These values apply only in the flexible exchange rate version of the model, for the individual country considered as Home.

Consider now the policy rule followed by the ECB, which in the present model applies either to the single central bank in the monetary Union version of the model, or to Foreign in the flexible exchange rate version. Sauer and Sturm (2007) review the now substantial literature on the topic, and conclude that little consensus emerges relatively to the values of the policy parameters in the ECB Taylor rules. However, despite the variety of estimates pointed out by these authors, there is a wide agreement on the fact that the ECB largely replicates the policy formerly implemented by the Bundesbank. It is thus assumed that the ECB follows a strict inflation targeting policy, in agreement with the calibration for Germany. As a consequence, the parameter values for Foreign are set to 3 for ϕ_π , and 0.25 for ϕ_y .

The calibrated values are summarized in the following tables.

Table 2.1

Calibration: common parameters

β	0.99	Discount rate
η	2	Risk aversion parameter
ϕ	1	Inverse Frisch elasticity of labour supply
κ	6	Markup parameter
μ	1.5	Elasticity of substitution between domestic and Foreign bundles
ε	0.0001	Intermediation cost (flexible exchange rate regime)
ε	0.00001	Interest rate premium (monetary union regime)
α	0.3	Capital share of output
δ	0.013	Depreciation rate
ζ	0.75	Probability for a firm not to update its price
l	0.025	Capital adjustment cost parameter
σ	0.975	Survival rate of bankers
ρ_a	0.9	Autoregressive parameter for TFP process
ρ_m	1/2	Autoregressive parameter for monetary process
ρ_k	2/3	Autoregressive parameter for capital quality process
σ_a	0.0041	Standard error of productivity shocks
σ_m	0.0028	Standard error of monetary shocks
σ_k	0.0006	Standard error of capital quality shocks

Table 2.2

Calibration: country-specific parameters

	Relative Home TFP	Foreign TFP	Share of government expenditure in Home	Share of government expenditure in Foreign	Trade openness in Home	Trade openness in Foreign	Relative size of Home	Bias towards domestic goods in Home
	\bar{A}^h	\bar{A}^f	ϑ^h	ϑ^f	ν^h	ν^f	γ	ϕ^h
Austria	0.97	1	0.51	0.48	0.39	0.29	0.03	0.62
Belgium	1.22	1	0.50	0.48	0.61	0.28	0.03	0.41
Cyprus	0.53	1	0.42	0.48	0.34	0.30	0.01	0.66
Estonia	0.24	1	0.37	0.48	0.64	0.30	0.01	0.36
Finland	1.02	1	0.51	0.48	0.27	0.30	0.02	0.73
France	1.22	1	0.54	0.46	0.22	0.31	0.19	0.82
Germany	1.11	1	0.47	0.48	0.30	0.29	0.25	0.78
Greece	0.54	1	0.47	0.48	0.28	0.30	0.03	0.73
Ireland	1.19	1	0.39	0.48	0.35	0.29	0.01	0.65
Italy	0.83	1	0.48	0.48	0.21	0.31	0.18	0.82
Luxembourg	1.64	1	0.40	0.48	0.47	0.30	0.01	0.53
Malta	0.40	1	0.42	0.48	0.63	0.30	0.01	0.37
Netherlands	1.20	1	0.47	0.48	0.50	0.28	0.05	0.53
Portugal	0.41	1	0.45	0.48	0.33	0.30	0.03	0.68
Slovakia	0.29	1	0.41	0.48	0.73	0.29	0.02	0.28
Slovenia	0.49	1	0.46	0.48	0.54	0.30	0.01	0.46
Spain	0.72	1	0.41	0.49	0.24	0.30	0.13	0.79

	Bias towards domestic goods in Foreign	Utility weight of labour in Home	Utility weight of labour in Foreign	Leverage ratio in Home	Leverage ratio in Foreign	Transfer rate to new bankers in Home	Transfer rate to new bankers in Foreign
	ϕ^f	χ^h	χ^f	ϕ^h	ϕ^f	ξ^h	ξ^f
Austria	0.99	4.83	5.00	3.4	5.21	0.0044	0.0029
Belgium	0.99	5.02	5.00	4.7	5.18	0.0032	0.0029
Cyprus	0.99	4.63	5.00	13.4	5.14	0.0011	0.0029
Estonia	0.99	4.70	5.00	4.2	5.16	0.0036	0.0029
Finland	0.99	4.93	5.00	3.9	5.19	0.0038	0.0029
France	0.94	4.92	5.00	3.6	5.58	0.0042	0.0027
Germany	0.92	5.25	5.00	5.0	5.23	0.0030	0.0029
Greece	0.99	4.33	5.00	12.4	4.99	0.0012	0.0030
Ireland	0.99	5.10	5.00	2.1	5.22	0.0071	0.0029
Italy	0.94	4.80	5.00	6.8	4.82	0.0022	0.0031
Luxembourg	0.99	4.97	5.00	0.6	5.18	0.0250	0.0029
Malta	0.99	4.73	5.00	5.1	5.16	0.0029	0.0029
Netherlands	0.98	6.09	5.00	2.5	5.34	0.0060	0.0028
Portugal	0.99	4.75	5.00	3.9	5.18	0.0038	0.0029
Slovakia	0.99	4.52	5.00	11.1	5.13	0.0014	0.0029
Slovenia	0.99	4.59	5.00	5.6	5.16	0.0027	0.0029
Spain	0.95	4.70	5.00	7.4	4.89	0.0020	0.0031

Note: implied by ϕ^h Note: implied by ϕ^f

	Ratio interbank loan/asset in Home	Fraction of assets divertable in Home	Fraction of assets divertable in Foreign	Home Taylor rule inflation parameter	Home Taylor rule output parameter	Foreign Taylor rule inflation parameter	Foreign Taylor rule output parameter
	x^h	θ^h	θ^f	ϕ_π^h	ϕ_y^h	ϕ_π^f	ϕ_y^f
Austria	0.22	0.33	0.18	2.5	0.25	2.5	0.25
Belgium	0.24	0.23	0.19	2.5	0.25	2.5	0.25
Cyprus	0.04	0.08	0.19	1.5	1.25	2.5	0.25
Estonia	0.10	0.25	0.19	1.5	1.25	2.5	0.25
Finland	0.12	0.28	0.18	2.5	0.25	2.5	0.25
France	0.24	0.31	0.16	2	0.75	2.5	0.25
Germany	0.22	0.22	0.18	2.5	0.25	2.5	0.25
Greece	0.09	0.08	0.20	1.5	1.25	2.5	0.25
Ireland	0.22	0.50	0.18	2.5	0.25	2.5	0.25
Italy	0.08	0.15	0.20	2	0.75	2.5	0.25
Luxembourg	0.34	2.05	0.17	2.5	0.25	2.5	0.25
Malta	0.14	0.21	0.19	1.5	1.25	2.5	0.25
Netherlands	0.14	0.42	0.18	2.5	0.25	2.5	0.25
Portugal	0.20	0.28	0.19	2	0.75	2.5	0.25
Slovakia	0.11	0.09	0.19	1.5	1.25	2.5	0.25
Slovenia	0.15	0.19	0.19	1.5	1.25	2.5	0.25
Spain	0.16	0.14	0.20	2	0.75	2.5	0.25

Note: implied by x^h Note: implied by x^h

2.3.2. Steady-state

Once the parameters of the model have been calibrated, it is possible to estimate the steady-state of the model, for both versions. The steady-state is somewhat cumbersome to derive. The model is large and, as such, does not allow for a convenient analytical nor numerical solution when considered as a whole. To make computations simpler, one separates the model into its financial component and its real component, and solves them recursively. Appendix B.7 delivers some details about the derivation of the steady-state values. As shown in this appendix, the steady-state values are the same for the flexible exchange rate and the monetary union versions of the model. As an illustration, Table 2.3 displays the steady-values for France obtained from the above calibration (France is chosen as its parameter values are representative of the average Euro area values).

Table 2.3

Steady-state values: France

Output	Y^h	3.34	Y^f	2.71
Total consumption	C^h	0.86	C^f	0.72
Consumption (domestic goods)	C_H^h	0.71	C_F^f	0.67
Consumption (imported goods)	C_F^h	0.15	C_H^f	0.04
Capital stock	K^h	60.22	K^f	48.91
Investment	I^h	0.78	I^f	0.64
Public expenditure	G^h	1.80	G^f	1.25
Real taxation	T_r^h	1.80	T_r^f	1.25
Labour	L^h	0.73	L^f	0.78
Real wage	W_r^h	2.68	W_r^f	2.01
Nominal interest rate	R^h	1.01	R^f	1.01
Nominal interbank rate	R^i	1.01		
Real asset price	Q_r^h	1	Q_r^f	1
Real interbank bonds	B_r^h	14.45	B_r^f	-14.45
Volume of assets	S^h	60.22	S^f	48.91
Home bank deposits	D_r^h	29.04		
Foreign bank deposits	F_r^h	42.26	F_r	54.59
Real net worth	N_r^h	16.73	N_r^f	8.76
Real dividend	Z_r^h	0.02	Z_r^f	0.02
Shadow value of deposits	v^h	1	v^f	1
Shadow value of interbank loans	v_b^h	1	v_b^f	1
Shadow value of assets	v_s^h	1	v_s^f	1
Total factor productivity	A^h	1.22	A^f	1
Capital quality process	ψ^h	1	ψ^f	1
Monetary policy process	ζ^h	0	ζ^f	0

2.4. Crisis experiments

This section concludes this chapter by running preliminary numerical experiments, using the model developed in the preceding paragraphs. The first step in the analysis is the simulation of a financial crisis. The experiment starts with a purely domestic crisis, in the context of independent currencies. The case of a union-wide crisis will be the object of section 2.4.2, while section 2.4.3 will be dedicated to the comparison of crisis experiments under alternative monetary regimes. Finally, section 2.4.3 provides a brief covering of conventional productivity and monetary shocks. France is taken as a benchmark example since its parameter values are representative of the Euro area average, but the other Eurozone countries provide very similar qualitative responses.

2.4.1. An example of domestic crisis

Following Gertler and Kiyotaki (2011), a financial crisis is interpreted as an unanticipated 5% decline in capital quality in Home. This simple ad-hoc feature aims at replicating the way an exogenous force may trigger a decline in the value of Home intermediary assets. Following the trigger, and depending on the degree of financial frictions, the initial drop in capital quality results in an enhanced fall in net worth of domestic banks, due to their high degree of leverage. By this means, the experiment crudely captures the dynamics of the financial crisis that has been prevailing since 2008.

Figure 2.3 reports the response of some key variables to this simulated financial crisis. Three different specifications are presented. The solid line displays the responses to the initial shock in the case of maximum financial frictions (that is, when $\omega=0$). On the other hand, the dashed line represents the responses in the case of frictionless financial markets ($\omega=1$). Finally, the dotted line presents the responses in the case of an RBC open-economy model replicating the core features of the present model – including a similar type of capital quality shocks – but devoid of financial markets³².

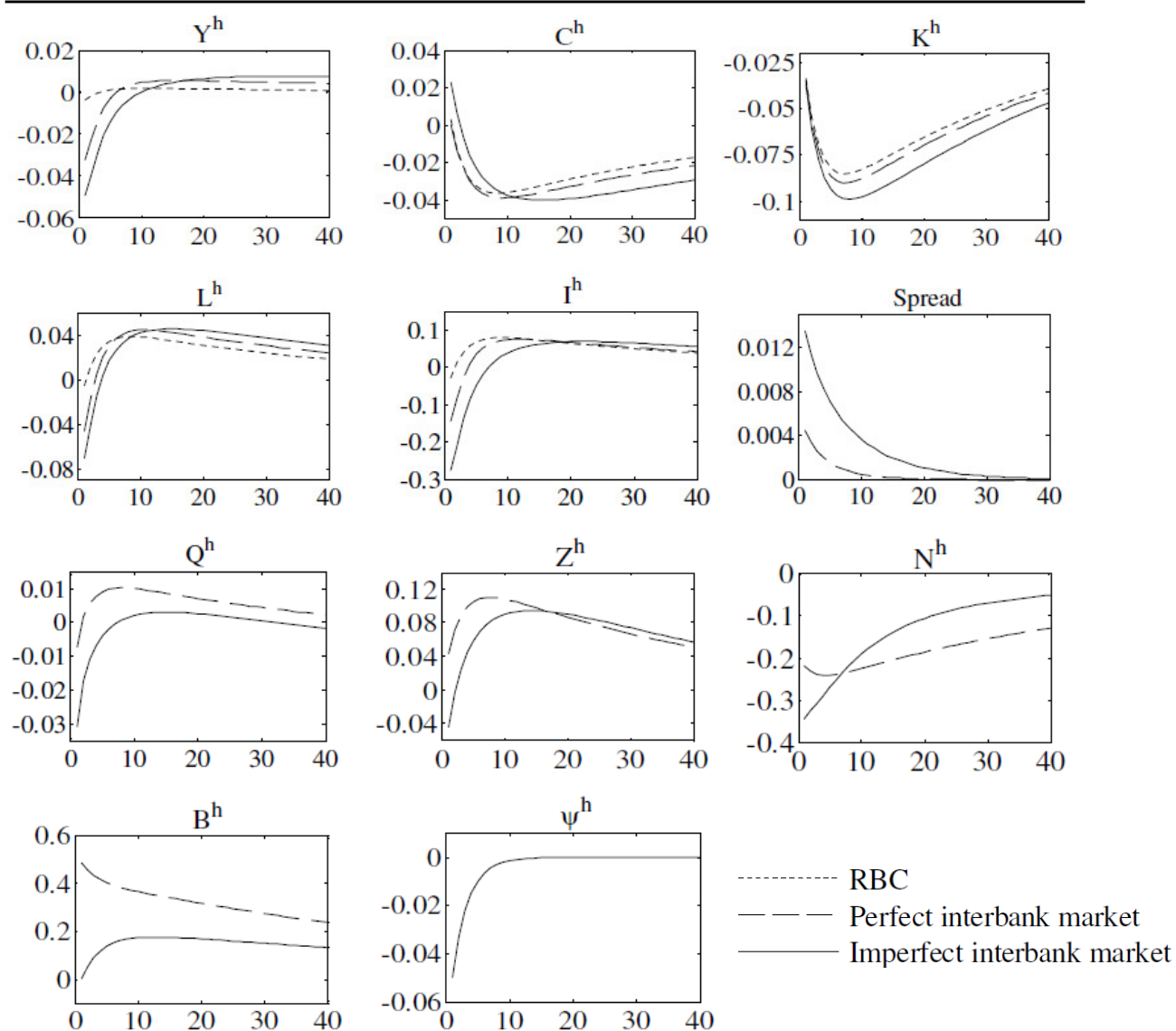
As Figure 2.3 illustrates, in the case of the RBC model, the initial disturbance produces only a slight decline in domestic activity, with a fall in output smaller than 1%. The initial drop in

³² See appendix B.8 for a brief description of this RBC model.

capital calls for a cut in consumption, allowing for increased investment. Coupled with an increase in labour, domestic production rapidly catches up and recovery is fast: output takes hardly more than two quarters to go back to its initial level.

Figure 2.3

Crisis experiment: domestic crisis, independent currencies



By contrast, the model with financial frictions produces a sharp recession. The initial disturbance in capital quality is magnified in two ways. As banks are leveraged, a decline in asset value mechanically results in a drop in their balance sheet net worth, amplified by the leverage ratio. Then, because banks are balance sheet constrained, this first effect is followed by a second effect, namely the sale of assets to meet their balance sheet requirements. As

demand for intermediary assets drops, the price of assets Q_t^h decreases, tightening even more the balance sheet constraint on financial institutions. The effect at work over financial markets is clearly observable through the behaviour of the spread between the expected return on capital and the riskless rate. Also, as a side effect, investment diminishes as the constraint on banks prevents them to finance new projects, delaying effective capital stock rebuilding and recovery of real activity. Following, the crisis feeds itself and recovery is slow. It is noteworthy that during the process, banks are effectively deleveraging, rebuilding net worth at the expense of assets, and it is this transition that prevents a fast recovery of the real part of the economy.

If the logic of the crisis development is the same in the case of maximum frictions and frictionless markets, the speed of recovery varies according to the degree of financial frictions. In the frictionless case, the initial sale of assets is limited due to less stringent balance sheet constraints. This prevents net worth from falling in too much a dramatic way, resulting in an initial drop of a bit more than 20% in the frictionless case, against nearly 35% in the maximum friction case. This contributes to mitigate the initial impact of the crisis. But the crucial difference in terms of recovery is to be accounted for in terms of interbank borrowing. With frictionless interbank markets, the balance sheet constraint on Home banks is looser, which allows for larger borrowing capacities of Home banks from Foreign credit institutions. This is illustrated in a spectacular way by the response of B_t^h following the initial disturbance. In the frictionless case, borrowing of funds from Foreign banks increases by around 50% immediately after the beginning of the crisis: Home banks use the interbank channel to compensate for their own melting in net worth, which can be done without restriction as Foreign banks do not run the risk that domestic banks divert their funds. Following, investment finally recovers and after a few periods even exceeds its steady-state value. As a consequence, demand for assets increases and asset price Q_t^h starts raising again. The gradual recovery of real activity combined to a stock of capital still below its steady-state value generates a sharp rise in dividends, which contributes to improve the banks balance sheet and hence to maintain a high volume of interbank loans. Eventually, the trough in aggregate capital stock reaches its maximum at around 9% after eight quarters and then steadily recovers thanks to sustained investment and gradual recovery of banks net worth. Output also recovers after approximately eight quarters, implying a crisis of limited duration.

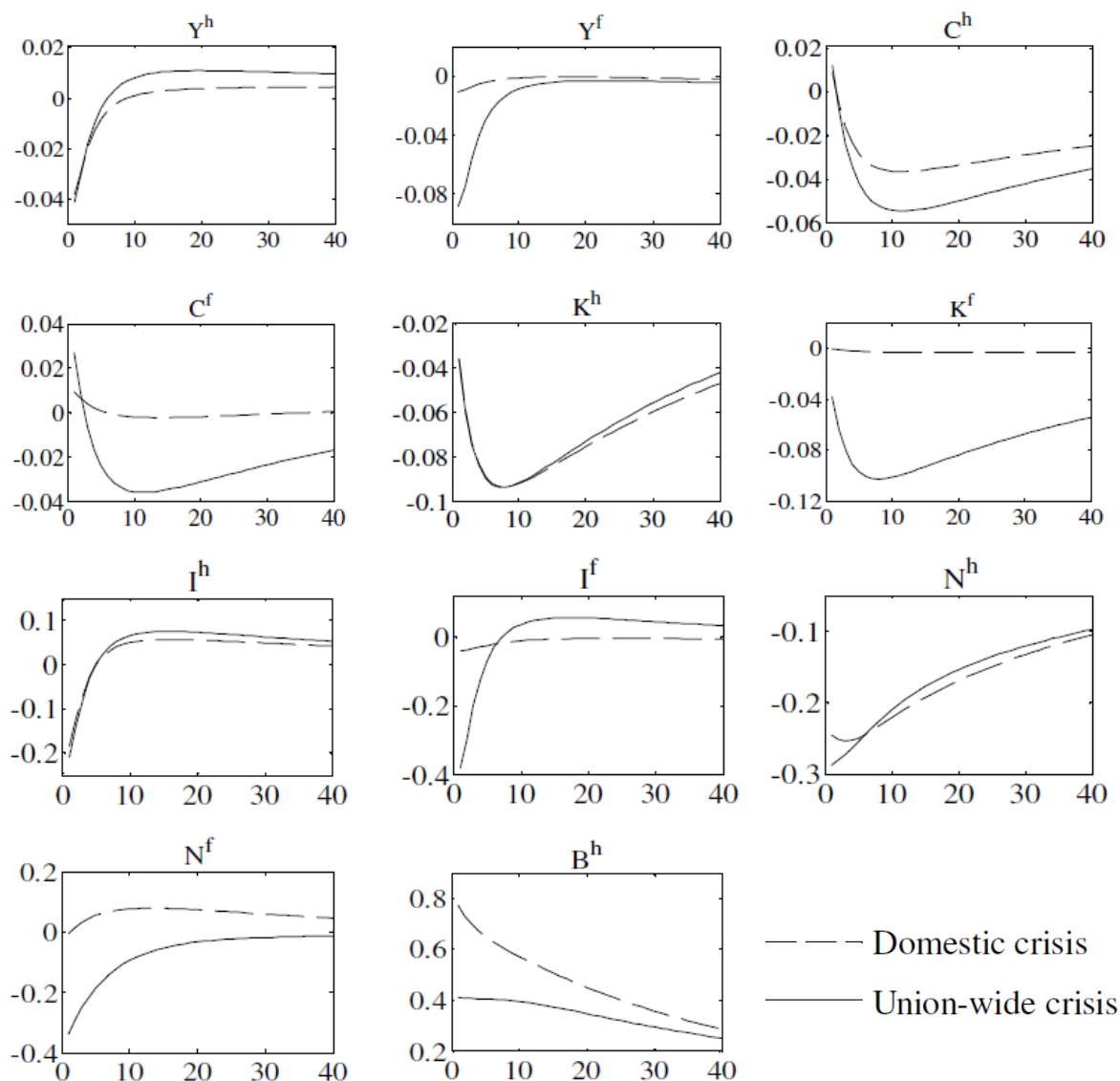
With maximum financial frictions, recovery is significantly slower. Not only is the initial drop in net worth much larger, tightening the balance sheet constraint on Home banks, but also the risk of asset diverting severely limits the volume of interbank loans granted by Foreign banks. Eventually, Home banks only benefit from a 20% increase in interbank loans, after a period of twelve quarters. Banks have then no choice but to sell assets and limit the quantity of projects they finance, which results in a slump in investment and asset prices. Overall, with maximum financial frictions, the variables suffer from both a sharper downturn and more inertia, when compared to the frictionless case. Aggregate capital displays a maximum drop exceeding 10%, and output initially declines by roughly 5%. It also takes around twelve quarters to recover, implying a significantly longer crisis.

2.4.2. Domestic versus union-wide crisis

The analysis now focuses on the differences between a domestic and a union-wide crisis. Figure 2.4 presents the responses of key variables for Home and Foreign. The dashed line reports the responses when the crisis originates from a 5% decline in capital quality in Home only. The plain line displays the responses when the negative shock on capital quality simultaneously occurs in Home and Foreign. Both cases are derived under the assumption of average financial frictions ($\omega = 0.5$).

Note first that a crisis in Home has only a moderate effect over the rest of the union: the recession in Foreign is both short (about four quarters) and limited in magnitude (around 1%). The effect on consumption, capital and investment is nearly nil, while net worth in Foreign actually improves: as the financial crisis in Home triggers a sale of Home assets, Home banks compensate by increasing interbank loans from Foreign, resulting in an increase in Foreign net worth.

Figure 2.4

Crisis experiment: domestic crisis V. union-wide crisis

The second result of interest is that Home is significantly worse off when the crisis occurs at the scale of the union and not just at the domestic level. First, the initial fall in output is larger when the crisis is union-wide, even though the subsequent recovery is faster. Most importantly, consumption drops by nearly 6% with a union-scale crisis, for a bit less than 4% for a purely domestic crisis. With respect to financial markets, the fundamental difference in a union-scale crisis is that as Foreign also enters into recession, Foreign banks suffer from a deleveraging process similar to the one occurring in Home, associated with a cut of nearly 40% in Foreign net worth. Following, as funds become scarce in Foreign banks, the opportunity cost of interbank loans increases, reducing the volume of trade granted to Home:

while with a purely domestic crisis, Home banks manage to increase their loans from Foreign credit institutions by 80%, the rise is limited to 40% in the event of a union-scale crisis. Combined with the sale of assets, banks in Home are more limited in their ability to finance new projects, which explains why the cut in investment is initially larger in the case of a union-wide crisis (around 22% against 18% for a domestic crisis).

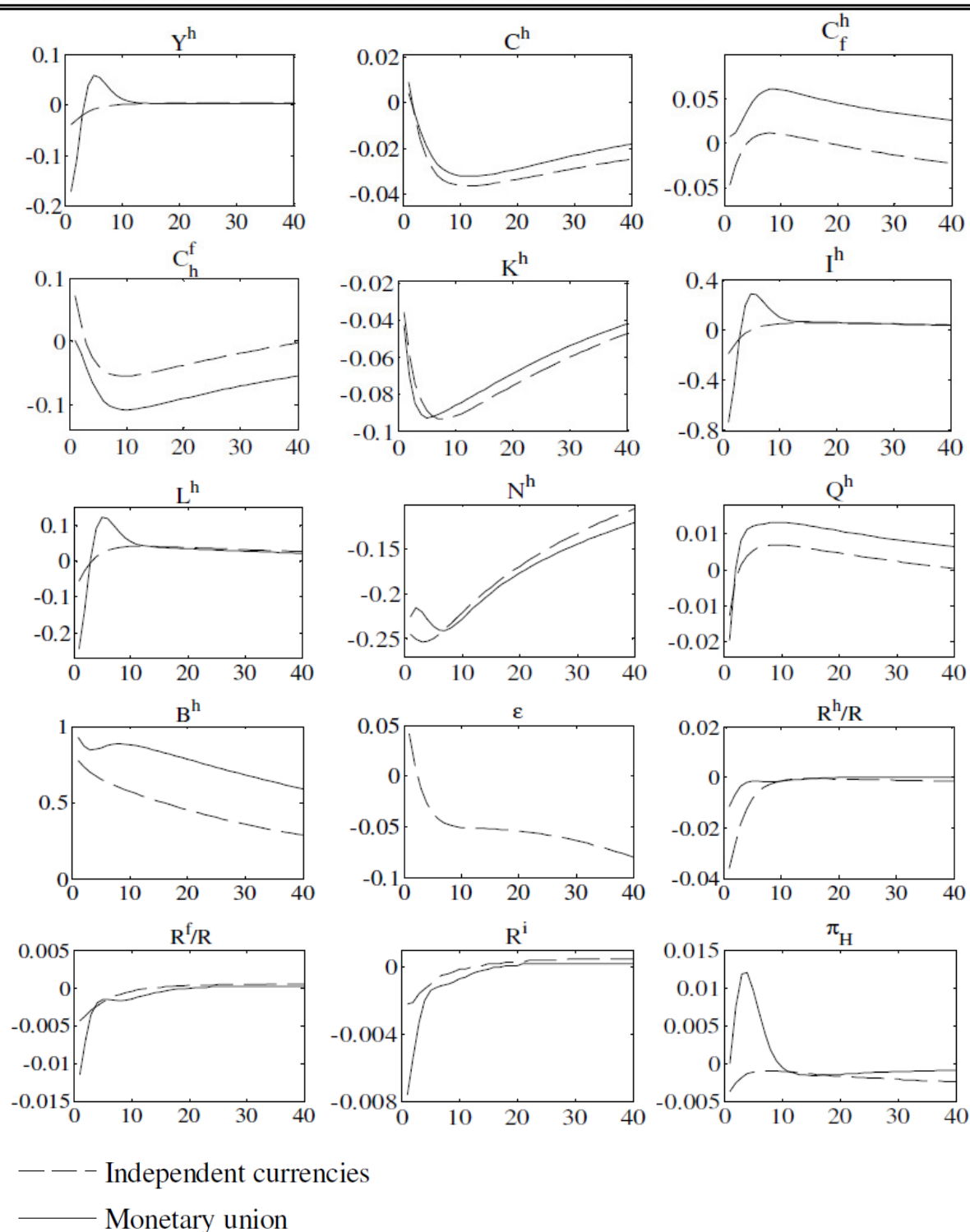
2.4.3. Independent currencies V. monetary union

The incoming section considers a crucial aspect of the analysis: assessing the respective performances of alternative monetary regimes within the framework of the model. Recall that the main two differences that prevail between the two regimes are, on the one hand, the existence or not of an exchange rate, and, on the other hand, the relinquishing of the conduct of monetary policy, in the case of a monetary union. Figure 2.5 displays some responses after a 5% negative shock on capital quality in Home, for an average degree of financial frictions ($\omega = 0.5$).

Overall, results are not clear-cut: for most of the real side of the model, namely for output, consumption, capital, investment and labour, the monetary union typically performs poorly over the first few periods. For instance, the suggested drop in output under a monetary union amounts to no less than 20%, a very strong recession. Nevertheless, this initial slump is followed by a phase of rapid recovery which significantly over performs the independent currency model. A similar pattern can be observed for investment and labour. Also, one may observe that recovery is quicker for consumption and capital under a monetary union, though the difference is not as spectacular as in the case of the three aforementioned variables. Eventually, it seems difficult to draw clear-cut conclusions about the performance of each regime. One may yet infer that in terms of pure stability, the flexible exchange rate regime seems to produce better results, the fluctuations observed for the main variables being typically of smaller amplitude.

Figure 2.5

Crisis experiment: independent currencies V. monetary union



A distinct feature of the model with floating currencies is the behaviour of the exchange rate: as reported in Figure 2.5, the initial shock initially triggers a short depreciation of the exchange rate, rapidly followed by a significant appreciation. The latter originates from the interest rate differential between Home and Foreign. As recession hits Home more severely

than Foreign, the central authorities in Home implement a larger cut in the riskless interest rate than in Foreign. The interest parity condition then implies an expected appreciation of the currency for the next period, which self-fulfils into actual appreciation. For the real side of the economy, this contributes to lower imports of Foreign goods and temporarily increase exports of domestic goods. However, for the monetary union, the sustained expansion that succeeds the initial recession contributes to feed domestic inflation. This increase in the terms of trade causes a substitution effect that turns away consumers from Home goods in favour of Foreign goods. This effect more than compensates for the observed appreciation of the exchange rate occurring in the case of independent currencies, and, as a consequence, the drop in imports of Foreign goods and the increase in exports prove eventually larger in the monetary union.

Not surprisingly, the recovery observed for the real side of the model under the monetary union subsequently to the initial recession has to do with better financial conditions. Notice first that under a monetary union, the initial fall in net worth is more moderate than in the case of floating exchange rates. This contributes to limit the initial sale of assets and the resulting drop in asset price. Consequently, the incentive constraint on Home banks is looser, which explains the higher volume of interbank funds traded in the monetary union (a 100% increase, as compared to 80% in the case of a floating regime), along with the lower interbank interest rate. An interesting feature is that the expected (and actual) appreciation of the exchange rate that occurs under a float should play in favour of this regime, as banks in Home always reimburse interbank loans in Foreign currency. Yet this is not the case: despite the advantage provided by an appreciating exchange rate, the cost of interbank borrowing remains higher because the incentive constraint is much tighter under flexible exchange.

Finally, as one would expect, the riskless nominal interest rate benefits from a larger cut when Home has preserved its monetary independence. As a response to the crisis in progress, the domestic central authorities implement a significant drop in the domestic rate in order to support economic activity. By contrast, when Home is member of a monetary union, the fall in interest rate is only a moderate one. Indeed, as the common central authorities do not focus solely on Home but consider the union as a whole, the response provided to the Home crisis consists in a limited interest rate cut.

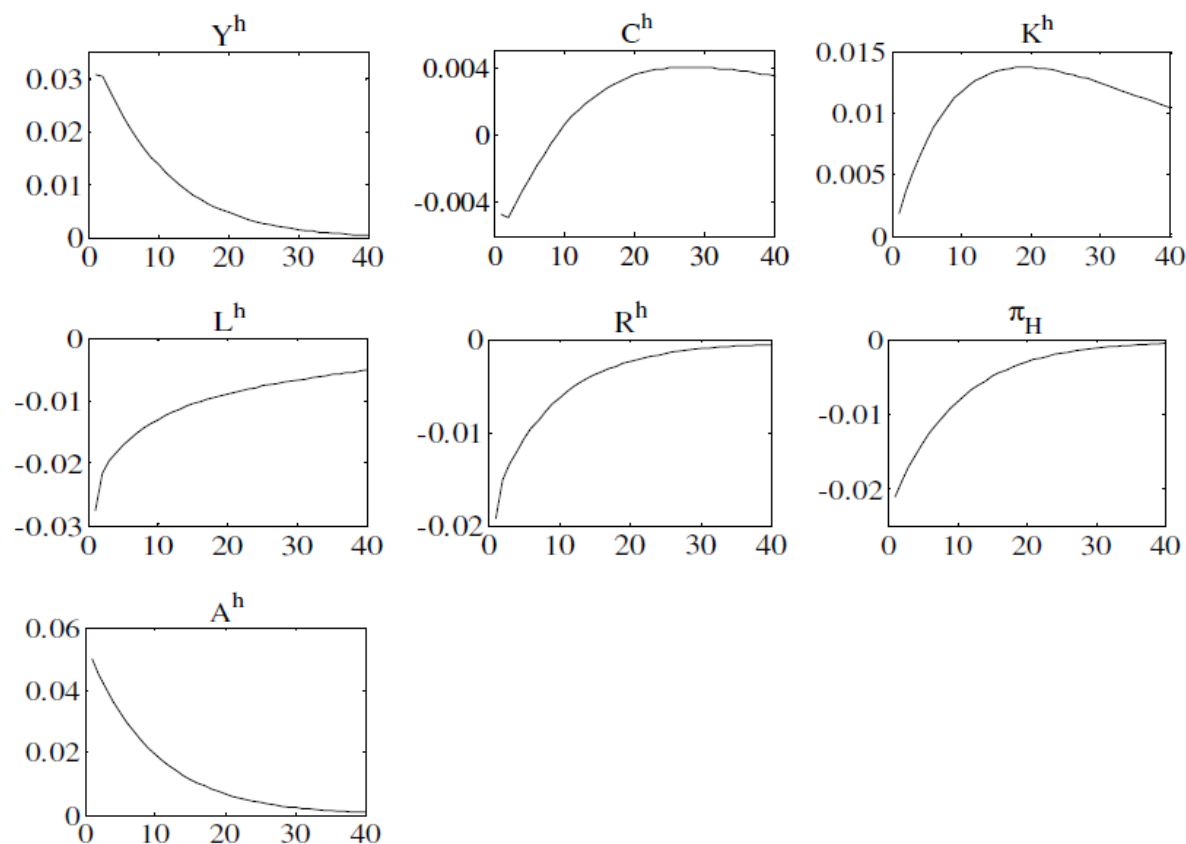
2.4.4. Response to positive productivity shocks and positive monetary shocks

To conclude this section, the case of positive supply shocks and positive monetary shocks (in the context of floating exchange rate) are briefly treated. Figure 2.6 reports the response to a 5% positive productivity innovation in France, while Figure 2.7 illustrates the response to an expansionary 5% monetary innovation.

As an executive summary, the results are in line with the ones typically found in the New Keynesian literature (see e.g. Galí (2008)). A positive productivity shock results in an increase in output, capital, and after a few periods, of consumption. A substitution effect leads employment to temporarily decrease before it steadily goes back to its initial level. Finally, the implied fall in domestic inflation causes the central authorities to partly accommodate the shocks by lowering in turn the nominal rate.

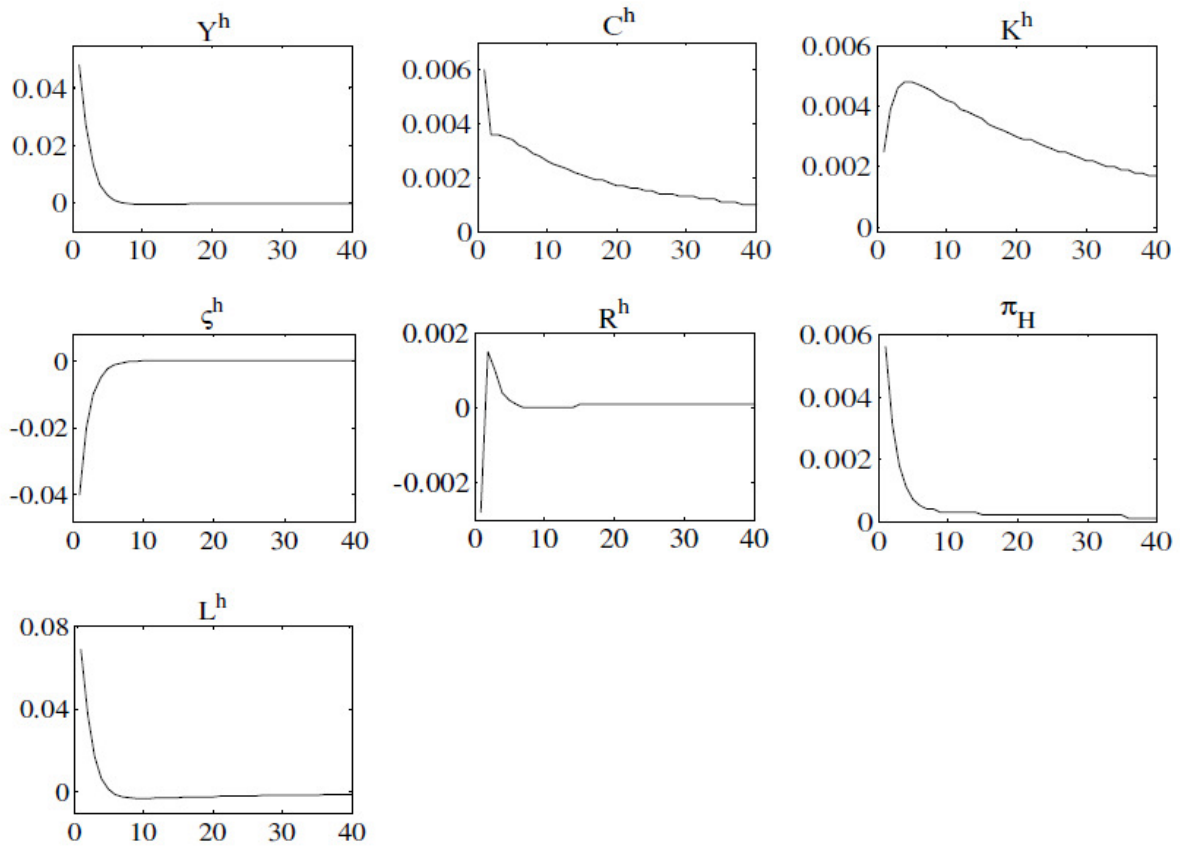
Figure 2.6

Response to a positive productivity shock



On the other hand, a positive monetary shock initially yields a decrease in the nominal interest rate. This interest rate cut initiates an expansion of overall activity illustrated by the rise in output, consumption, capital and labour. The positive output gap also generates an increase in inflation. Then, as both output and inflation exceed their steady-state levels, the central bank reverses its policy and starts to increase the nominal rate. As the shock vanishes and the economy returns to its initial values, the interest rate steadily falls.

Figure 2.7
Response to an expansionary monetary shock



2.5. Conclusion

This paper introduces a model of open economies in a context of financial frictions. Calibrated for the 17 Euro area countries, it constitutes a benchmark framework for the analysis of the current situation of the Eurozone. The analysis, although it provides useful insights relative to the role of financial markets and in particular cross-border transactions, remains up to this point inconclusive. A regime of flexible exchange rates seems to provide overall higher stability than a regime of monetary union, but the latter allows for faster recovery following adverse shocks.

This calls for two possible directions for future research. First, a formal welfare analysis should be carried on the two versions of the model to obtain a definitive assessment of the performance of each regime. And secondly, as the model relies on a conventional calibration procedure, it could be fruitful to investigate further these calibrated values in order to strengthen the validity of the results. In particular, it could be of much interest to formally assess the degree of shock correlation between economies of the Euro area since the beginning of the financial crisis.

Appendix B.1: Summary of equilibrium conditions, flexible exchange rate version of the model

The model comprises 61 variables. The 55 state variables:

$$\{Y_t^h, Y_t^f, C_t^h, C_t^f, C_{H,t}^h, C_{F,t}^h, C_{F,t}^f, C_{H,t}^f, K_{t+1}^h, K_{t+1}^f, L_t^h, L_t^f, I_t^h, I_t^f, G_t^h, G_t^f, T_{r,t}^h, T_{r,t}^f, W_{r,t}^h, W_{r,t}^f, MC_{r,t}^h, MC_{r,t}^f, P_t^h, P_t^f, P_{H,t}, P_{F,t}, P_{H,t}^*, P_{F,t}^*, \pi_t^h, \pi_t^f, \pi_{H,t}, \pi_{F,t}, \varepsilon_t, R_{t+1}^h, R_{t+1}^f, R_{t+1}^i, Q_{r,t}^h, Q_{r,t}^f, B_{r,t}^h, B_{r,t}^f, S_t^h, S_t^f, D_{r,t}^h, F_{r,t}, F_{r,t}^h, N_{r,t}^h, N_{r,t}^f, Z_{r,t}^h, Z_{r,t}^f, v_t^h, v_{b,t}^h, v_{s,t}^h, v_t^f, v_{b,t}^f, v_{s,t}^f\}$$

and the 6 stochastic variables:

$$\{\psi_t^h, \psi_t^f, A_t^h, A_t^f, \zeta_t^h, \zeta_t^f\}$$

are then defined by the following 61 equations:

Consumer Price index:

$$P_t^h = (\varphi^h (P_{H,t})^{1-\mu} + (1-\varphi^h) (\varepsilon_t P_{F,t})^{1-\mu})^{\frac{1}{1-\mu}} \quad (\text{b.1})$$

$$P_t^f = (\varphi^f (P_{F,t})^{1-\mu} + (1-\varphi^f) (\varepsilon_t^{-1} P_{H,t})^{1-\mu})^{\frac{1}{1-\mu}} \quad (\text{b.2})$$

Optimal index demand:

$$C_{H,t}^h = \varphi^h \left(\frac{P_t^h}{P_{H,t}} \right)^\mu C_t^h \quad (\text{b.3})$$

$$C_{F,t}^h = (1-\varphi^h) \left(\frac{P_t^h}{\varepsilon_t P_{F,t}} \right)^\mu C_t^h \quad (\text{b.4})$$

$$C_{F,t}^f = \varphi^f \left(\frac{P_t^f}{P_{F,t}} \right)^\mu C_t^f \quad (\text{b.5})$$

$$C_{H,t}^f = (1-\varphi^f) \left(\frac{\varepsilon_t P_t^f}{P_{H,t}} \right)^\mu C_t^f \quad (\text{b.6})$$

Optimal labour allocation:

$$\chi^h (L_t^h)^\phi (C_t^h)^\eta = W_{r,t}^h \quad (\text{b.7})$$

$$\chi^f (L_t^f)^\phi (C_t^f)^\eta = W_{r,t}^f \quad (\text{b.8})$$

Euler equation:

$$1 = R_{t+1}^h \beta E_t \left[\left(\frac{C_t^h}{C_{t+1}^h} \right)^\eta \frac{1}{\pi_{t+1}^h} \right] \quad (\text{b.9})$$

$$1 = R_{t+1}^f \beta E_t \left[\left(\frac{C_t^f}{C_{t+1}^f} \right)^\eta \frac{1}{\pi_{t+1}^f} \right] \quad (\text{b.10})$$

Budget constraint:

$$C_t^h + D_{r,t}^h + F_{r,t}^h + T_{r,t}^h + A C_t^h = W_{r,t}^h L_t^h + \frac{R_t^h}{\pi_t^h} D_{r,t-1}^h + \frac{R_t^f}{\pi_t^h} \frac{\varepsilon_t}{\varepsilon_{t-1}} F_{r,t-1}^h \quad (\text{b.11})$$

Modified interest parity condition:

$$R_{t+1}^h \left[1 + \vartheta (F_{r,t}^h - F_r^h) \right] = R_{t+1}^f E_t \left(\frac{\varepsilon_{t+1}}{\varepsilon_t} \right) \quad (\text{b.12})$$

Public expenditure:

$$G_t^h = \vartheta^h Y^h \quad (\text{b.13}) \quad G_t^f = \vartheta^f Y^f \quad (\text{b.14})$$

Law of motion of capital:

$$K_{t+1}^h = \psi_{t+1}^h [I_t^h + (1-\delta)K_t^h] \quad (\text{b.15}) \quad K_{t+1}^f = \psi_{t+1}^f [I_t^f + (1-\delta)K_t^f] \quad (\text{b.16})$$

Capital pricing:

$$Q_{r,t}^h = \frac{P_{H,t}}{P_t^h} [1 + \iota(I_t^h - I_{t-1}^h)] - \iota E_t \Lambda_{r,t,t+1}^h \frac{P_{H,t+1}}{P_{t+1}^h} (I_{t+1}^h - I_t^h) \quad (\text{b.17})$$

$$Q_{r,t}^f = \frac{P_{F,t}}{P_t^f} [1 + \iota(I_t^f - I_{t-1}^f)] - \iota E_t \Lambda_{r,t,t+1}^f \frac{P_{F,t+1}}{P_{t+1}^f} (I_{t+1}^f - I_t^f) \quad (\text{b.18})$$

Production technology:

$$Y_t^h(j) = A_t^h (K_t^h)^\alpha L_t^h(j)^{1-\alpha} \quad (\text{b.19}) \quad Y_t^f(j) = A_t^f (K_t^f)^\alpha L_t^f(j)^{1-\alpha} \quad (\text{b.20})$$

Real wage setting:

$$MC_{r,t}^h = W_{r,t}^h \times \frac{P_t^h}{P_{H,t}} \times \frac{1}{MPL_t^h} \quad (\text{b.21}) \quad MC_{r,t}^f = W_{r,t}^f \times \frac{P_t^f}{P_{F,t}} \times \frac{1}{MPL_t^f} \quad (\text{b.22})$$

Dividend:

$$Z_{r,t}^h = \frac{\frac{P_{H,t}}{P_t^h} Y_t^h - W_{r,t}^h L_t^h}{K_t^h} \quad (\text{b.23}) \quad Z_{r,t}^f = \frac{\frac{P_{F,t}}{P_t^f} Y_t^f - W_{r,t}^f L_t^f}{K_t^f} \quad (\text{b.24})$$

Optimal pricing:

$$P_{H,t}^*(j) = \frac{\kappa}{\kappa-1} \cdot \frac{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^h Y_{t+i|t}^h(j) \Psi_{t+i}^h(Y_{t+i|t}^h(j)) \right\}}{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^h Y_{t+i|t}^h(j) \right\}} \quad (\text{b.25})$$

$$P_{F,t}^*(j) = \frac{\kappa}{\kappa-1} \cdot \frac{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^f Y_{t+i|t}^f(j) \Psi_{t+i}^f(Y_{t+i|t}^f(j)) \right\}}{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^f Y_{t+i|t}^f(j) \right\}} \quad (\text{b.26})$$

Law of motion of producer prices:

$$(\pi_{H,t})^{1-\kappa} = \zeta + (1-\zeta) \left(\frac{P_{H,t}^*}{P_{H,t-1}} \right)^{1-\kappa} \quad (\text{b.27}) \quad (\pi_{F,t})^{1-\kappa} = \zeta + (1-\zeta) \left(\frac{P_{F,t}^*}{P_{F,t-1}} \right)^{1-\kappa} \quad (\text{b.28})$$

PPI inflation:

$$\pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}} \quad (\text{b.29}) \quad \pi_{F,t} = \frac{P_{F,t}}{P_{F,t-1}} \quad (\text{b.30})$$

CPI inflation:

$$\pi_t^h = \frac{P_t^h}{P_{t-1}^h} \quad (\text{b.31}) \quad \pi_t^f = \frac{P_t^f}{P_{t-1}^f} \quad (\text{b.32})$$

Optimal security allocation:

$$\left(\frac{v_{s,t}^h}{Q_{r,t}^h} - v_t^h \right) = \frac{1}{\omega} (v_{b,t}^h - v_t^h) \quad (\text{b.33}) \quad \left(\frac{v_{s,t}^f}{Q_{r,t}^f} - v_t^f \right) = \frac{1}{\omega} (v_{b,t}^f - v_t^f) \quad (\text{b.34})$$

Incentive constraint:

$$v_{s,t}^h S_t^h - v_{b,t}^h B_{r,t}^h - v_t^h D_{r,t}^h = \theta^h [Q_{r,t}^h S_t^h - \omega B_{r,t}^h] \quad (\text{b.35})$$

$$v_{s,t}^f S_t^f - v_{b,t}^f B_{r,t}^f - v_t^f F_{r,t} = \theta^f [Q_{r,t}^f S_t^f - \omega B_{r,t}^f] \quad (\text{b.36})$$

Security market clearing:

$$D_{r,t}^h = Q_{r,t}^h S_t^h - N_{r,t}^h - B_{r,t}^h \quad (\text{b.37}) \quad F_{r,t} = Q_{r,t}^f S_t^f - N_{r,t}^f - B_{r,t}^f \quad (\text{b.38})$$

Asset market clearing:

$$B_{r,t}^h \frac{P_t^h}{\varepsilon_t} = -B_{r,t}^f P_t^f \quad (\text{b.39})$$

Evolution of net worth:

$$N_{r,t}^h = (\sigma + \xi^h) \{ [Z_{r,t}^h + (1-\delta)Q_{r,t}^h] \psi_t^h S_{t-1}^h \} - \frac{\sigma}{\pi_t^h} \left\{ \frac{\varepsilon_t}{\varepsilon_{t-1}} R_t^i B_{r,t-1}^h + R_t^h D_{r,t-1}^h \right\} \quad (\text{b.40})$$

$$N_{r,t}^f = (\sigma + \xi^f) \{ [Z_{r,t}^f + (1-\delta)Q_{r,t}^f] \psi_t^f S_{t-1}^f \} - \frac{\sigma}{\pi_t^f} \{ R_t^i B_{r,t-1}^f + R_t^f F_{r,t-1} \} \quad (\text{b.41})$$

Shadow price of deposits:

$$v_t^h = R_{t+1}^h E_t \left(\frac{\Lambda_{r,t+1}^h \Omega_{t+1}^h}{\pi_{t+1}^h} \right) \quad (\text{b.42}) \quad v_t^f = R_{t+1}^f E_t \left(\frac{\Lambda_{r,t+1}^f \Omega_{t+1}^f}{\pi_{t+1}^f} \right) \quad (\text{b.43})$$

Shadow price of interbank borrowing:

$$v_{b,t}^h = v_t^h \frac{R_{t+1}^i}{R_{t+1}^h} E_t \left(\frac{\varepsilon_{t+1}}{\varepsilon_t} \right) \quad (\text{b.44}) \quad v_{b,t}^f = v_t^f \frac{R_{t+1}^i}{R_{t+1}^f} \quad (\text{b.45})$$

Shadow value of assets:

$$v_{s,t}^h = v_t^h E_t \left(\frac{\pi_{t+1}^h [Z_{r,t+1}^h + (1-\delta)Q_{r,t+1}^h] \psi_{t+1}^h}{R_{t+1}^h} \right) \quad (\text{b.46}) \quad v_{s,t}^f = v_t^f E_t \left(\frac{\pi_{t+1}^f [Z_{r,t+1}^f + (1-\delta)Q_{r,t+1}^f] \psi_{t+1}^f}{R_{t+1}^f} \right) \quad (\text{b.47})$$

Final goods market clearing:

$$Y_t^h = C_{H,t}^h + C_{F,t}^h + I_t^h + G_t^h \quad (\text{b.48}) \quad Y_t^f = C_{F,t}^f + C_{H,t}^f + I_t^f + G_t^f \quad (\text{b.49})$$

Government budget constraint:

$$\frac{P_{H,t}}{P_t^h} G_t^h = T_{r,t}^h \quad (\text{b.50}) \quad \frac{P_{F,t}}{P_t^f} G_t^f = T_{r,t}^f \quad (\text{b.51})$$

Concordance of assets with aggregate capital:

$$S_t^h = I_t^h + (1-\delta)K_t^h \quad (\text{b.52}) \quad S_t^f = I_t^f + (1-\delta)K_t^f \quad (\text{b.53})$$

Monetary policy:

$$\log(R_{t+1}^h) = \log(R^h) + \phi_\pi^h \log \pi_t^h + \phi_y^h \log \left(\frac{Y_t^h}{Y} \right) + \zeta_t^h \quad (\text{b.54})$$

$$\log(R_{t+1}^f) = \log(R^f) + \phi_\pi^f \log \pi_t^f + \phi_y^f \log \left(\frac{Y_t^f}{Y} \right) + \zeta_t^f \quad (\text{b.55})$$

Stochastic processes:

$$\zeta_t^h = \rho_m \zeta_{t-1}^h + \epsilon_{m,t}^h \quad (\text{b.56}) \quad \zeta_t^f = \rho_m \zeta_{t-1}^f + \epsilon_{m,t}^f \quad (\text{b.57})$$

$$\psi_t^h = (\bar{\psi})^{1-\rho_k} (\psi_{t-1}^h)^{\rho_k} e^{\epsilon_{k,t}^h} \quad (\text{b.58}) \quad \psi_t^f = (\bar{\psi})^{1-\rho_k} (\psi_{t-1}^f)^{\rho_k} e^{\epsilon_{k,t}^f} \quad (\text{b.59})$$

$$A_t^h = (\bar{A}^h)^{1-\rho_a} (A_{t-1}^h)^{\rho_a} e^{\epsilon_{a,t}^h} \quad (\text{b.60}) \quad A_t^f = (\bar{A}^f)^{1-\rho_a} (A_{t-1}^f)^{\rho_a} e^{\epsilon_{a,t}^f} \quad (\text{b.61})$$

Appendix B.2: Summary of linearized equilibrium conditions, flexible exchange rate version of the model

The model is not linear and does not admit a closed-form solution. It is thus linearized around its steady-state values. The following method, proposed by Uhlig (1999), is used: for any variable considered at period t and denoted by X_t , define:

$$\tilde{X}_t = \log(X_t) - \log(X)$$

where variables without a t subscript denote steady-state values.

Then, using the equality:

$$X_t = X e^{\tilde{X}_t}$$

and taking a first-order Taylor approximation of this equality around $X_t = X$ (that is, around $\tilde{X}_t = 0$), one obtains the following linear approximation:

$$X_t \approx X(1 + \tilde{X}_t)$$

The whole model is then linearized following this method. The full model is made of 57 equations, including 6 stochastic processes. The model is made somewhat smaller than the original, non linearized version, eliminating equations (b.21), (b.22), (b.25) and (b.26) to obtain inflation dynamics in the form of the so-called New-Keynesian Phillips curve. In its final form, the 51 state variables :

$$\{\tilde{Y}_t^h, \tilde{Y}_t^f, \tilde{C}_t^h, \tilde{C}_t^f, \tilde{C}_{H,t}^h, \tilde{C}_{F,t}^h, \tilde{C}_{F,t}^f, \tilde{C}_{H,t}^f, \tilde{K}_{t+1}^h, \tilde{K}_{t+1}^f, \tilde{L}_t^h, \tilde{L}_t^f, \tilde{I}_t^h, \tilde{I}_t^f, \tilde{G}_t^h, \tilde{G}_t^f, \tilde{T}_{r,t}^h, \tilde{T}_{r,t}^f, \tilde{W}_{r,t}^h, \tilde{W}_{r,t}^f, \tilde{P}_t^h, \tilde{P}_t^f, \tilde{P}_{H,t}, \tilde{P}_{F,t}, \tilde{\pi}_t^h, \tilde{\pi}_t^f, \tilde{\pi}_{H,t}, \tilde{\pi}_{F,t}, \tilde{\epsilon}_t, \tilde{R}_{t+1}^h, \tilde{R}_{t+1}^f, \tilde{R}_{t+1}^i, \tilde{Q}_{r,t}^h, \tilde{Q}_{r,t}^f, \tilde{B}_{r,t}^h, \tilde{B}_{r,t}^f, \tilde{S}_t^h, \tilde{S}_t^f, \tilde{D}_{r,t}^h, \tilde{F}_{r,t}, \tilde{F}_{r,t}^h, \tilde{N}_{r,t}^h, \tilde{N}_{r,t}^f, \tilde{Z}_{r,t}^h, \tilde{Z}_{r,t}^f, \tilde{v}_t^h, \tilde{v}_{b,t}^h, \tilde{v}_{s,t}^h, \tilde{v}_t^f, \tilde{v}_{b,t}^f, \tilde{v}_{s,t}^f\}$$

along with the 6 stochastic processes:

$$\{\tilde{\psi}_t^h, \tilde{\psi}_t^f, \tilde{A}_t^h, \tilde{A}_t^f, \tilde{\varsigma}_t^h, \tilde{\varsigma}_t^f\}$$

are determined by the 57 following linearized equations:

$$\begin{aligned}
\tilde{P}_t^h &= \varphi^h \tilde{P}_{H,t} + (1-\varphi^h) \tilde{\varepsilon}_t + (1-\varphi^h) \tilde{P}_{F,t} \\
\tilde{P}_t^f &= \varphi^f \tilde{P}_{F,t} - (1-\varphi^f) \tilde{\varepsilon}_t + (1-\varphi^f) \tilde{P}_{H,t} \\
\tilde{C}_{H,t}^h &= \mu \tilde{P}_t^h - \mu \tilde{P}_{H,t} + \tilde{C}_t^h \\
\tilde{C}_{F,t}^h &= \mu \tilde{P}_t^h - \mu \tilde{\varepsilon}_t - \mu \tilde{P}_{F,t} + \tilde{C}_t^h \\
\tilde{C}_{F,t}^f &= \mu \tilde{P}_t^f - \mu \tilde{P}_{F,t} + \tilde{C}_t^f \\
\tilde{C}_{H,t}^f &= \mu \tilde{\varepsilon}_t + \mu \tilde{P}_t^f - \mu \tilde{P}_{H,t} + \tilde{C}_t^f \\
\tilde{W}_{r,t}^h &= \phi \tilde{L}_t^h - \eta \tilde{C}_t^h \\
\tilde{W}_{r,t}^f &= \phi \tilde{L}_t^f - \eta \tilde{C}_t^f \\
0 &= \tilde{R}_{t+1}^h + \eta \tilde{C}_t^h - \eta \tilde{C}_{t+1}^h - \tilde{\pi}_{t+1}^h \\
0 &= \tilde{R}_{t+1}^f + \eta \tilde{C}_t^f - \eta \tilde{C}_{t+1}^f - \tilde{\pi}_{t+1}^f \\
C^h \tilde{C}_t^h + D_r^h \tilde{D}_{r,t}^h + F_r^h \tilde{F}_{r,t}^h + T_r^h \tilde{T}_{r,t}^h &= W_r^h L^h \tilde{W}_{r,t}^h + W_r^h L^h \tilde{L}_t^h + R^h D_r^h \tilde{R}_t^h - (R^h D_r^h + R^f F_r^h) \tilde{\pi}_t^h + R^h D_r^h \tilde{D}_{r,t-1}^h \\
&+ R^f F_r^h \tilde{R}_t^f + R^f F_r^h \tilde{\varepsilon}_t - R^f F_r^h \tilde{\varepsilon}_{t-1} + R^f F_r^h \tilde{F}_{r,t-1}^h \\
\tilde{R}_{t+1}^h + \vartheta F^h \tilde{F}_{r,t}^h &= \tilde{R}_{t+1}^f + E_t(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_t \\
\tilde{G}_t^h &= 0 \\
\tilde{G}_t^f &= 0 \\
\tilde{K}_{t+1}^h &= \tilde{\psi}_{t+1}^h + \delta \tilde{I}_t^h + (1-\delta) \tilde{K}_t^h \\
\tilde{K}_{t+1}^f &= \tilde{\psi}_{t+1}^f + \delta \tilde{I}_t^f + (1-\delta) \tilde{K}_t^f \\
\tilde{Q}_{r,t}^h &= \tilde{P}_{H,t} - \tilde{P}_t^h + U^h (1+\beta) \tilde{I}_t^h - U^h \tilde{I}_{t-1}^h - \iota \beta I^h E_t \tilde{I}_{t+1}^h \\
\tilde{Q}_{r,t}^f &= \tilde{P}_{F,t} - \tilde{P}_t^f + U^f (1+\beta) \tilde{I}_t^f - U^f \tilde{I}_{t-1}^f - \iota \beta I^f E_t \tilde{I}_{t+1}^f \\
\tilde{Y}_t^h &= \tilde{A}_t^h + \alpha \tilde{K}_t^h + (1-\alpha) \tilde{L}_t^h \\
\tilde{Y}_t^f &= \tilde{A}_t^f + \alpha \tilde{K}_t^f + (1-\alpha) \tilde{L}_t^f \\
Z_r^h K^h \tilde{Z}_{r,t}^h &= Y^h \tilde{P}_{H,t} - Y^h \tilde{P}_t^h + Y^h \tilde{Y}_t^h - W_r^h L^h \tilde{W}_{r,t}^h - W_r^h L^h \tilde{L}_t^h - Z_r^h K^h \tilde{K}_t^h \\
Z_r^f K^f \tilde{Z}_{r,t}^f &= Y^f \tilde{P}_{F,t} - Y^f \tilde{P}_t^f + Y^f \tilde{Y}_t^f - W_r^f L^f \tilde{W}_{r,t}^f - W_r^f L^f \tilde{L}_t^f - Z_r^f K^f \tilde{K}_t^f \\
\tilde{\pi}_{H,t} &= \beta E_t(\tilde{\pi}_{H,t+1}) + \varpi \tilde{W}_{r,t}^h + \varpi \tilde{P}_t^h - \varpi \tilde{P}_{H,t} - \frac{\varpi}{1-\alpha} \tilde{A}_t^h - \frac{\alpha \varpi}{1-\alpha} \tilde{K}_t^h + \frac{\alpha \varpi}{1-\alpha} \tilde{Y}_t^h \\
\tilde{\pi}_{F,t} &= \beta E_t(\tilde{\pi}_{F,t+1}) + \varpi \tilde{W}_{r,t}^f + \varpi \tilde{P}_t^f - \varpi \tilde{P}_{F,t} - \frac{\varpi}{1-\alpha} \tilde{A}_t^f - \frac{\alpha \varpi}{1-\alpha} \tilde{K}_t^f + \frac{\alpha \varpi}{1-\alpha} \tilde{Y}_t^f \\
\text{with : } \varpi &= \frac{(1-\zeta)(1-\zeta\beta)(1-\alpha)}{\zeta(1-\alpha+\alpha\kappa)} \\
\tilde{\pi}_{H,t} &= \tilde{P}_{H,t} - \tilde{P}_{H,t-1} \\
\tilde{\pi}_{F,t} &= \tilde{P}_{F,t} - \tilde{P}_{F,t-1} \\
\tilde{\pi}_t^h &= \tilde{P}_t^h - \tilde{P}_{t-1}^h \\
\tilde{\pi}_t^f &= \tilde{P}_t^f - \tilde{P}_{t-1}^f \\
\tilde{v}_{s,t}^h - \tilde{Q}_{r,t}^h + \left(\frac{1}{\omega} - 1\right) \tilde{v}_t^h &= \frac{1}{\omega} \tilde{v}_{b,t}^h \\
\tilde{v}_{s,t}^f - \tilde{Q}_{r,t}^f + \left(\frac{1}{\omega} - 1\right) \tilde{v}_t^f &= \frac{1}{\omega} \tilde{v}_{b,t}^f \\
S^h \tilde{v}_{s,t}^h + S^h (1-\theta^h) \tilde{S}_t^h - B_r^h \tilde{v}_{b,t}^h - D_r^h \tilde{v}_t^h - D_r^h \tilde{D}_{r,t}^h - B_r^h (1-\theta^h \omega) \tilde{B}_{r,t}^h &= \theta^h S^h \tilde{Q}_{r,t}^h \\
S^f \tilde{v}_{s,t}^f + S^f (1-\theta^f) \tilde{S}_t^f - B_r^f \tilde{v}_{b,t}^f - F_r^f \tilde{v}_t^f - F_r^f \tilde{F}_{r,t}^f - B_r^f (1-\theta^f \omega) \tilde{B}_{r,t}^f &= \theta^f S^f \tilde{Q}_{r,t}^f \\
D_r^h \tilde{D}_{r,t}^h &= S^h \tilde{Q}_{r,t}^h + S^h \tilde{S}_t^h - N_r^h \tilde{N}_{r,t}^h - B_r^h \tilde{B}_{r,t}^h \\
F_r^f \tilde{F}_{r,t}^f &= S^f \tilde{Q}_{r,t}^f + S^f \tilde{S}_t^f - N_r^f \tilde{N}_{r,t}^f - B_r^f \tilde{B}_{r,t}^f
\end{aligned}$$

$$\begin{aligned}
\tilde{B}_{r,t}^h + \tilde{P}_t^h - \tilde{\epsilon}_t &= \tilde{B}_{r,t}^f + \tilde{P}_t^f \\
N_r^h \tilde{N}_{r,t}^h &= (\sigma + \xi^h) Z_r^h S^h \tilde{Z}_{r,t}^h + (\sigma + \xi^h) R^h S^h \tilde{\psi}_t^h + (\sigma + \xi^h) R^h S^h \tilde{S}_{t-1}^h + (\sigma + \xi^h) (1 - \delta) S^h \tilde{Q}_{r,t}^h \\
&+ \sigma (R^i B_r^h + R^h D_r^h) \tilde{\pi}_t^h - \sigma R^i B_r^h \tilde{\epsilon}_t + \sigma R^i B_r^h \tilde{\epsilon}_{t-1} - \sigma R^i B_r^h \tilde{R}_t^i - \sigma R^i B_r^h \tilde{B}_{r,t-1}^h - \sigma R^h D_r^h \tilde{R}_t^h - \sigma R^h D_r^h \tilde{D}_{r,t-1}^h \\
N_{r,t}^f \tilde{N}_{r,t}^f &= (\sigma + \xi^f) Z_r^f S^f \tilde{Z}_{r,t}^f + (\sigma + \xi^f) R^f S^f \tilde{\psi}_t^f + (\sigma + \xi^f) R^f S^f \tilde{S}_{t-1}^f + (\sigma + \xi^f) (1 - \delta) S^f \tilde{Q}_{r,t}^f \\
&+ \sigma (R^i B_r^f + R^f F_r) \tilde{\pi}_t^f - \sigma R^i B_r^f \tilde{R}_t^i - \sigma R^i B_r^f \tilde{B}_{r,t-1}^f - \sigma R^f F_r \tilde{R}_t^f - \sigma R^f F_r \tilde{F}_{r,t-1} \\
\tilde{v}_t^h &= \tilde{R}_{t+1}^h + \eta \tilde{C}_t^h - \eta E_t(\tilde{C}_{t+1}^h) - E_t(\tilde{\pi}_{t+1}^h) + \sigma (1 - \phi^h (1 - x_r^h)) E_t(\tilde{v}_{t+1}^h) \\
&+ \sigma \phi^h E_t(\tilde{v}_{s,t+1}^h) - \sigma \phi^h E_t(\tilde{Q}_{r,t+1}^h) - \sigma \phi^h x_r^h E_t(\tilde{v}_{b,t+1}^h) \\
\tilde{v}_t^f &= \tilde{R}_{t+1}^f + \eta \tilde{C}_t^f - \eta E_t(\tilde{C}_{t+1}^f) - E_t(\tilde{\pi}_{t+1}^f) + \sigma (1 - \phi^f (1 - x_r^f)) E_t(\tilde{v}_{t+1}^f) \\
&+ \sigma \phi^f E_t(\tilde{v}_{s,t+1}^f) - \sigma \phi^f E_t(\tilde{Q}_{r,t+1}^f) - \sigma \phi^f x_r^f E_t(\tilde{v}_{b,t+1}^f) \\
\tilde{v}_{b,t}^h &= \tilde{v}_t^h + \tilde{R}_{t+1}^i - \tilde{R}_{t+1}^h + E_t(\tilde{\epsilon}_{t+1}) - \tilde{\epsilon}_t \\
\tilde{v}_{b,t}^f &= \tilde{v}_t^f + \tilde{R}_{t+1}^i - \tilde{R}_{t+1}^f \\
\tilde{v}_{s,t}^h &= \tilde{v}_t^h + E_t(\tilde{\pi}_{t+1}^h) + \frac{Z_r^h}{R^h} E_t(Z_{r,t+1}^h) + E_t(\tilde{\psi}_{t+1}^h) + \frac{(1 - \delta)}{R^h} E_t(\tilde{Q}_{r,t+1}^h) - \tilde{R}_{t+1}^h \\
\tilde{v}_{s,t}^f &= \tilde{v}_t^f + E_t(\tilde{\pi}_{t+1}^f) + \frac{Z_r^f}{R^f} E_t(Z_{r,t+1}^f) + E_t(\tilde{\psi}_{t+1}^f) + \frac{(1 - \delta)}{R^f} E_t(\tilde{Q}_{r,t+1}^f) - \tilde{R}_{t+1}^f \\
Y^h \tilde{Y}_t^h &= C_H^h \tilde{C}_{H,t}^h + C_H^f \tilde{C}_{H,t}^f + I^h \tilde{I}_t^h + G^h \tilde{G}_t^h \\
Y^f \tilde{Y}_t^f &= C_F^f \tilde{C}_{F,t}^f + C_F^h \tilde{C}_{F,t}^h + I^f \tilde{I}_t^f + G^f \tilde{G}_t^f \\
\tilde{T}_{r,t}^h &= \tilde{P}_{H,t}^h - \tilde{P}_t^h + \tilde{C}_t^h \\
\tilde{T}_{r,t}^f &= \tilde{P}_{F,t}^f - \tilde{P}_t^f + \tilde{C}_t^f \\
\tilde{S}_t^h &= \delta \tilde{I}_t^h + (1 - \delta) \tilde{K}_t^h \\
\tilde{S}_t^f &= \delta \tilde{I}_t^f + (1 - \delta) \tilde{K}_t^f \\
\tilde{R}_{t+1}^h &= \phi_\pi^h \tilde{\pi}_t^h + \phi_y^h \tilde{Y}_t^h + \zeta_t^h \\
\tilde{R}_{t+1}^f &= \phi_\pi^f \tilde{\pi}_t^f + \phi_y^f \tilde{Y}_t^f + \zeta_t^f \\
\zeta_t^h &= \rho_m \zeta_{t-1}^h + \epsilon_{m,t}^h \\
\zeta_t^f &= \rho_m \zeta_{t-1}^f + \epsilon_{m,t}^f \\
\tilde{\psi}_t^h &= \rho_k \tilde{\psi}_{t-1}^h + \epsilon_{k,t}^h \\
\tilde{\psi}_t^f &= \rho_k \tilde{\psi}_{t-1}^f + \epsilon_{k,t}^f \\
\tilde{A}_t^h &= \rho_a \tilde{A}_{t-1}^h + \epsilon_{a,t}^h \\
\tilde{A}_t^f &= \rho_a \tilde{A}_{t-1}^f + \epsilon_{a,t}^f
\end{aligned}$$

Once the model is made linear, the dynamics can be recovered from standard methods (see Blanchard and Kahn (1980)).

Appendix B.3: Summary of equilibrium conditions, monetary union version of the model

The model comprises the 57 variables. The 52 state variables:

$$\{Y_t^h, Y_t^f, C_t^h, C_t^f, C_{H,t}^h, C_{F,t}^h, C_{F,t}^f, C_{H,t}^f, K_{t+1}^h, K_{t+1}^f, L_t^h, L_t^f, I_t^h, I_t^f, G_t^h, G_t^f, T_{r,t}^h, T_{r,t}^f, W_{r,t}^h, W_{r,t}^f, MC_{r,t}^h, MC_{r,t}^f, P_t^h, P_t^f, P_{H,t}, P_{F,t}, P_{H,t}^*, P_{F,t}^*, \pi_t^h, \pi_t^f, \pi_{H,t}, \pi_{F,t}, R_{t+1}, R_{t+1}^i, Q_{r,t}^h, Q_{r,t}^f, B_{r,t}^h, B_{r,t}^f, S_t^h, S_t^f, D_{r,t}^h, D_{r,t}^f, N_{r,t}^h, N_{r,t}^f, Z_{r,t}^h, Z_{r,t}^f, v_t^h, v_{b,t}^h, v_{s,t}^h, v_t^f, v_{b,t}^f, v_{s,t}^f\}$$

and the 5 stochastic variables:

$$\{\psi_t^h, \psi_t^f, A_t^h, A_t^f, \zeta_t\}$$

are then defined by the following 57 equations:

Consumer Price index:

$$P_t^h = \left(\varphi^h (P_{H,t})^{1-\mu} + (1-\varphi^h) (P_{F,t})^{1-\mu} \right)^{\frac{1}{1-\mu}} \quad (\text{b.1}') \quad P_t^f = \left(\varphi^f (P_{F,t})^{1-\mu} + (1-\varphi^f) (P_{H,t})^{1-\mu} \right)^{\frac{1}{1-\mu}} \quad (\text{b.2}')$$

Optimal index demand:

$$C_{H,t}^h = \varphi^h \left(\frac{P_t^h}{P_{H,t}} \right)^\mu C_t^h \quad (\text{b.3}') \quad C_{F,t}^h = (1-\varphi^h) \left(\frac{P_t^h}{P_{F,t}} \right)^\mu C_t^h \quad (\text{b.4}')$$

$$C_{F,t}^f = \varphi^f \left(\frac{P_t^f}{P_{F,t}} \right)^\mu C_t^f \quad (\text{b.5}') \quad C_{H,t}^f = (1-\varphi^f) \left(\frac{P_t^f}{P_{H,t}} \right)^\mu C_t^f \quad (\text{b.6}')$$

Optimal labour allocation:

$$\chi^h (L_t^h)^\phi (C_t^h)^\eta = W_{r,t}^h \quad (\text{b.7}') \quad \chi^f (L_t^f)^\phi (C_t^f)^\eta = W_{r,t}^f \quad (\text{b.8}')$$

Euler equation:

$$1 - \vartheta (D_{r,t}^h - D_r^h) = R_{t+1} \beta E_t \left[\left(\frac{C_t^h}{C_{t+1}^h} \right)^\eta \frac{1}{\pi_{t+1}^h} \right] \quad (\text{b.9}') \quad 1 = R_{t+1} \beta E_t \left[\left(\frac{C_t^f}{C_{t+1}^f} \right)^\eta \frac{1}{\pi_{t+1}^f} \right] \quad (\text{b.10}')$$

Public expenditure:

$$G_t^h = \vartheta^h Y^h \quad (\text{b.11}') \quad G_t^f = \vartheta^f Y^f \quad (\text{b.12}')$$

Law of motion of capital:

$$K_{t+1}^h = \psi_{t+1}^h [I_t^h + (1-\delta)K_t^h] \quad (\text{b.13}') \quad K_{t+1}^f = \psi_{t+1}^f [I_t^f + (1-\delta)K_t^f] \quad (\text{b.14}')$$

Capital pricing:

$$Q_{r,t}^h = \frac{P_{H,t}}{P_t^h} \left[1 + \iota (I_t^h - I_{t-1}^h) \right] - \iota E_t \Lambda_{r,t,t+1}^h \frac{P_{H,t+1}}{P_{t+1}^h} (I_{t+1}^h - I_t^h) \quad (\text{b.15}')$$

$$Q_{r,t}^f = \frac{P_{F,t}}{P_t^f} \left[1 + \iota (I_t^f - I_{t-1}^f) \right] - \iota E_t \Lambda_{r,t,t+1}^f \frac{P_{F,t+1}}{P_{t+1}^f} (I_{t+1}^f - I_t^f) \quad (\text{b.16}')$$

Production technology:

$$Y_t^h(j) = A_t^h (K_t^h)^\alpha L_t^h(j)^{1-\alpha} \quad (\text{b.17}') \quad Y_t^f(j) = A_t^f (K_t^f)^\alpha L_t^f(j)^{1-\alpha} \quad (\text{b.18}')$$

Real wage setting:

$$MC_{r,t}^h = W_{r,t}^h \times \frac{P_t^h}{P_{H,t}^h} \times \frac{1}{MPL_{L_t^h}^h} \quad (\text{b.19}') \quad MC_{r,t}^f = W_{r,t}^f \times \frac{P_t^f}{P_{F,t}^f} \times \frac{1}{MPL_{L_t^f}^f} \quad (\text{b.20}')$$

Dividend:

$$Z_{r,t}^h = \frac{\frac{P_{H,t}^h}{P_t^h} Y_t^h - W_{r,t}^h L_t^h}{K_t^h} \quad (\text{b.21}') \quad Z_{r,t}^f = \frac{\frac{P_{F,t}^f}{P_t^f} Y_t^f - W_{r,t}^f L_t^f}{K_t^f} \quad (\text{b.22}')$$

Optimal pricing:

$$P_{H,t}^*(j) = \frac{\kappa}{\kappa-1} \cdot \frac{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^h Y_{t+i|t}^h(j) \Psi_{t+i}^{h'}(Y_{t+i|t}^h(j)) \right\}}{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^h Y_{t+i|t}^h(j) \right\}} \quad (\text{b.23}')$$

$$P_{F,t}^*(j) = \frac{\kappa}{\kappa-1} \cdot \frac{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^f Y_{t+i|t}^f(j) \Psi_{t+i}^{f'}(Y_{t+i|t}^f(j)) \right\}}{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^f Y_{t+i|t}^f(j) \right\}} \quad (\text{b.24}')$$

Law of motion of producer prices:

$$(\pi_{H,t})^{1-\kappa} = \zeta + (1-\zeta) \left(\frac{P_{H,t}^*}{P_{H,t-1}} \right)^{1-\kappa} \quad (\text{b.25}') \quad (\pi_{F,t})^{1-\kappa} = \zeta + (1-\zeta) \left(\frac{P_{F,t}^*}{P_{F,t-1}} \right)^{1-\kappa} \quad (\text{b.26}')$$

PPI inflation:

$$\pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}} \quad (\text{b.27}') \quad \pi_{F,t} = \frac{P_{F,t}}{P_{F,t-1}} \quad (\text{b.28}')$$

CPI inflation:

$$\pi_t^h = \frac{P_t^h}{P_{t-1}^h} \quad (\text{b.29}') \quad \pi_t^f = \frac{P_t^f}{P_{t-1}^f} \quad (\text{b.30}')$$

Optimal security allocation:

$$\left(\frac{v_{s,t}^h}{Q_{r,t}^h} - v_t^h \right) = \frac{1}{\omega} (v_{b,t}^h - v_t^h) \quad (\text{b.31}') \quad \left(\frac{v_{s,t}^f}{Q_{r,t}^f} - v_t^f \right) = \frac{1}{\omega} (v_{b,t}^f - v_t^f) \quad (\text{b.32}')$$

Incentive constraint:

$$v_{s,t}^h S_t^h - v_{b,t}^h B_{r,t}^h - v_t^h D_{r,t}^h = \theta^h [Q_{r,t}^h S_t^h - \omega B_{r,t}^h] \quad (\text{b.33}')$$

$$v_{s,t}^f S_t^f - v_{b,t}^f B_{r,t}^f - v_t^f D_{r,t}^f = \theta^f [Q_{r,t}^f S_t^f - \omega B_{r,t}^f] \quad (\text{b.34}')$$

Security market clearing:

$$D_{r,t}^h = Q_{r,t}^h S_t^h - N_{r,t}^h - B_{r,t}^h \quad (\text{b.35}') \quad D_{r,t}^f = Q_{r,t}^f S_t^f - N_{r,t}^f - B_{r,t}^f \quad (\text{b.36}')$$

Asset market clearing:

$$B_{r,t}^h P_t^h = -B_{r,t}^f P_t^f \quad (\text{b.37}')$$

Evolution of net worth:

$$N_{r,t}^h = (\sigma + \xi^h) \{ [Z_{r,t}^h + (1-\delta)Q_{r,t}^h] \psi_t^h S_{t-1}^h \} - \frac{\sigma}{\pi_t^h} \{ R_t^i B_{r,t-1}^h + R_t D_{r,t-1}^h \} \quad (\text{b.38}')$$

$$N_{r,t}^f = (\sigma + \xi^f) \{ [Z_{r,t}^f + (1-\delta)Q_{r,t}^f] \psi_t^f S_{t-1}^f \} - \frac{\sigma}{\pi_t^f} \{ R_t^i B_{r,t-1}^f + R_t D_{r,t-1}^f \} \quad (\text{b.39}')$$

Shadow price of deposits:

$$v_t^h = R_{t+1} E_t \left(\frac{\Lambda_{r,t,t+1}^h \Omega_{t+1}^h}{\pi_{t+1}^h} \right) \quad (\text{b.40}') \quad v_t^f = R_{t+1} E_t \left(\frac{\Lambda_{r,t,t+1}^f \Omega_{t+1}^f}{\pi_{t+1}^f} \right) \quad (\text{b.41}')$$

Shadow price of interbank borrowing:

$$v_{b,t}^h = v_t^h \frac{R_{t+1}^i}{R_{t+1}} \quad (\text{b.42}') \quad v_{b,t}^f = v_t^f \frac{R_{t+1}^i}{R_{t+1}} \quad (\text{b.43}')$$

Shadow value of assets:

$$v_{s,t}^h = v_t^h E_t \left(\frac{\pi_{t+1}^h [Z_{r,t+1}^h + (1-\delta)Q_{r,t+1}^h] \psi_{t+1}^h}{R_{t+1}} \right) \quad (\text{b.44}') \quad v_{s,t}^f = v_t^f E_t \left(\frac{\pi_{t+1}^f [Z_{r,t+1}^f + (1-\delta)Q_{r,t+1}^f] \psi_{t+1}^f}{R_{t+1}} \right) \quad (\text{b.45}')$$

Final goods market clearing:

$$Y_t^h = C_{H,t}^h + C_{F,t}^h + I_t^h + G_t^h \quad (\text{b.46}') \quad Y_t^f = C_{F,t}^f + C_{F,t}^h + I_t^f + G_t^f \quad (\text{b.47}')$$

Government budget constraint:

$$\frac{P_{H,t}}{P_t^h} G_t^h = T_{r,t}^h \quad (\text{b.48}') \quad \frac{P_{F,t}}{P_t^f} G_t^f = T_{r,t}^f \quad (\text{b.49}')$$

Concordance of assets with aggregate capital:

$$S_t^h = I_t^h + (1-\delta)K_t^h \quad (\text{b.50}') \quad S_t^f = I_t^f + (1-\delta)K_t^f \quad (\text{b.51}')$$

Monetary policy:

$$\log(R_{t+1}) = \log(R) + \phi_\pi \left(\gamma \log \pi_{H,t} + (1-\gamma) \log \pi_{F,t} \right) + \phi_y \left(\gamma \log \left(\frac{Y_t^h}{Y^h} \right) + (1-\gamma) \log \left(\frac{Y_t^f}{Y^f} \right) \right) + \zeta_t \quad (\text{b.52}')$$

Stochastic processes:

$$\zeta_t = \rho_m \zeta_{t-1} + \epsilon_{m,t} \quad (\text{b.53}')$$

$$\psi_t^h = (\bar{\psi}^h)^{1-\rho_k} (\psi_{t-1}^h)^{\rho_k} e^{\epsilon_{k,t}^h} \quad (\text{b.54}') \quad \psi_t^f = (\bar{\psi}^f)^{1-\rho_k} (\psi_{t-1}^f)^{\rho_k} e^{\epsilon_{k,t}^f} \quad (\text{b.55}')$$

$$A_t^h = (\bar{A}^h)^{1-\rho_a} (A_{t-1}^h)^{\rho_a} e^{\epsilon_{a,t}^h} \quad (\text{b.56}') \quad A_t^f = (\bar{A}^f)^{1-\rho_a} (A_{t-1}^f)^{\rho_a} e^{\epsilon_{a,t}^f} \quad (\text{b.57}')$$

Appendix B.4: Summary of linearized equilibrium conditions, monetary union version of the model

48 state variables:

$$\{\tilde{Y}_t^h, \tilde{Y}_t^f, \tilde{C}_t^h, \tilde{C}_t^f, \tilde{C}_{H,t}^h, \tilde{C}_{F,t}^h, \tilde{C}_{F,t}^f, \tilde{C}_{H,t}^f, \tilde{K}_{t+1}^h, \tilde{K}_{t+1}^f, \tilde{L}_t^h, \tilde{L}_t^f, \tilde{I}_t^h, \tilde{I}_t^f, \tilde{G}_t^h, \tilde{G}_t^f, \tilde{T}_{r,t}^h, \tilde{T}_{r,t}^f, \tilde{W}_{r,t}^h, \tilde{W}_{r,t}^f, \tilde{P}_t^h, \tilde{P}_t^f, \tilde{P}_{H,t}, \tilde{P}_{F,t}, \tilde{\pi}_t^h, \tilde{\pi}_t^f, \tilde{\pi}_{H,t}, \tilde{\pi}_{F,t}, \tilde{R}_{t+1}, \tilde{R}_{t+1}^f, \tilde{Q}_{r,t}^h, \tilde{Q}_{r,t}^f, \tilde{B}_{r,t}^h, \tilde{B}_{r,t}^f, \tilde{S}_t^h, \tilde{S}_t^f, \tilde{D}_{r,t}^h, \tilde{D}_{r,t}^f, \tilde{N}_{r,t}^h, \tilde{N}_{r,t}^f, \tilde{Z}_{r,t}^h, \tilde{Z}_{r,t}^f, \tilde{v}_t^h, \tilde{v}_{b,t}^h, \tilde{v}_{s,t}^h, \tilde{v}_t^f, \tilde{v}_{b,t}^f, \tilde{v}_{s,t}^f\}$$

along with the 5 stochastic processes:

$$\{\tilde{\psi}_t^h, \tilde{\psi}_t^f, \tilde{A}_t^h, \tilde{A}_t^f, \zeta_t\}$$

are determined by the 53 following linearized equations:

$$\begin{aligned} \tilde{P}_t^h &= \phi^h \tilde{P}_{H,t} + (1 - \phi^h) \tilde{P}_{F,t} \\ \tilde{P}_t^f &= \phi^f \tilde{P}_{F,t} + (1 - \phi^f) \tilde{P}_{H,t} \\ \tilde{C}_{H,t}^h &= \mu \tilde{P}_t^h - \mu \tilde{P}_{H,t} + \tilde{C}_t^h \\ \tilde{C}_{F,t}^h &= \mu \tilde{P}_t^h - \mu \tilde{P}_{F,t} + \tilde{C}_t^h \\ \tilde{C}_{F,t}^f &= \mu \tilde{P}_t^f - \mu \tilde{P}_{F,t} + \tilde{C}_t^f \\ \tilde{C}_{H,t}^f &= \mu \tilde{P}_t^f - \mu \tilde{P}_{H,t} + \tilde{C}_t^f \\ \tilde{W}_{r,t}^h &= \phi \tilde{L}_t^h - \eta \tilde{C}_t^h \\ \tilde{W}_{r,t}^f &= \phi \tilde{L}_t^f - \eta \tilde{C}_t^f \\ -\partial D_t \tilde{D}_{r,t}^h &= \tilde{R}_{t+1} + \eta \tilde{C}_t^h - \eta \tilde{C}_{t+1}^h - \tilde{\pi}_{t+1}^h \\ 0 &= \tilde{R}_{t+1} + \eta \tilde{C}_t^f - \eta \tilde{C}_{t+1}^f - \tilde{\pi}_{t+1}^f \\ \tilde{G}_t^h &= 0 \\ \tilde{G}_t^f &= 0 \\ \tilde{K}_{t+1}^h &= \tilde{\psi}_{t+1}^h + \delta \tilde{I}_t^h + (1 - \delta) \tilde{K}_t^h \\ \tilde{K}_{t+1}^f &= \tilde{\psi}_{t+1}^f + \delta \tilde{I}_t^f + (1 - \delta) \tilde{K}_t^f \\ \tilde{Q}_{r,t}^h &= \tilde{P}_{H,t} - \tilde{P}_t^h + U^h(1 + \beta) \tilde{I}_t^h - U^h \tilde{I}_{t-1}^h - \iota \beta I^h E_t \tilde{I}_{t+1}^h \\ \tilde{Q}_{r,t}^f &= \tilde{P}_{F,t} - \tilde{P}_t^f + U^f(1 + \beta) \tilde{I}_t^f - U^f \tilde{I}_{t-1}^f - \iota \beta I^f E_t \tilde{I}_{t+1}^f \\ \tilde{Y}_t^h &= \tilde{A}_t^h + \alpha \tilde{K}_t^h + (1 - \alpha) \tilde{L}_t^h \\ \tilde{Y}_t^f &= \tilde{A}_t^f + \alpha \tilde{K}_t^f + (1 - \alpha) \tilde{L}_t^f \\ Z_r^h K^h \tilde{Z}_{r,t}^h &= Y^h \tilde{P}_{H,t} - Y^h \tilde{P}_t^h + Y^h \tilde{Y}_t^h - W_r^h L^h \tilde{W}_{r,t}^h - W_r^h L^h \tilde{L}_t^h - Z_r^h K^h \tilde{K}_t^h \\ Z_r^f K^f \tilde{Z}_{r,t}^f &= Y^f \tilde{P}_{F,t} - Y^f \tilde{P}_t^f + Y^f \tilde{Y}_t^f - W_r^f L^f \tilde{W}_{r,t}^f - W_r^f L^f \tilde{L}_t^f - Z_r^f K^f \tilde{K}_t^f \\ \tilde{\pi}_{H,t} &= \beta E_t(\tilde{\pi}_{H,t+1}) + \varpi \tilde{W}_{r,t}^h + \varpi \tilde{P}_t^h - \varpi \tilde{P}_{H,t} - \frac{\varpi}{1 - \alpha} \tilde{A}_t^h - \frac{\alpha \varpi}{1 - \alpha} \tilde{K}_t^h + \frac{\alpha \varpi}{1 - \alpha} \tilde{Y}_t^h \\ \tilde{\pi}_{F,t} &= \beta E_t(\tilde{\pi}_{F,t+1}) + \varpi \tilde{W}_{r,t}^f + \varpi \tilde{P}_t^f - \varpi \tilde{P}_{F,t} - \frac{\varpi}{1 - \alpha} \tilde{A}_t^f - \frac{\alpha \varpi}{1 - \alpha} \tilde{K}_t^f + \frac{\alpha \varpi}{1 - \alpha} \tilde{Y}_t^f \\ \text{with : } \varpi &\equiv \frac{(1 - \zeta)(1 - \zeta \beta)(1 - \alpha)}{\zeta(1 - \alpha + \alpha \kappa)} \\ \tilde{\pi}_{H,t} &= \tilde{P}_{H,t} - \tilde{P}_{H,t-1} \\ \tilde{\pi}_{F,t} &= \tilde{P}_{F,t} - \tilde{P}_{F,t-1} \end{aligned}$$

$$\begin{aligned}
\tilde{\pi}_t^h &= \tilde{P}_t^h - \tilde{P}_{t-1}^h \\
\tilde{\pi}_t^f &= \tilde{P}_t^f - \tilde{P}_{t-1}^f \tilde{v}_{s,t}^h - \tilde{Q}_{r,t}^h + \left(\frac{1}{\omega} - 1\right) \tilde{v}_t^h = \frac{1}{\omega} \tilde{v}_{b,t}^h \\
\tilde{v}_{s,t}^f - \tilde{Q}_{r,t}^f + \left(\frac{1}{\omega} - 1\right) \tilde{v}_t^f &= \frac{1}{\omega} \tilde{v}_{b,t}^f \\
S^h \tilde{v}_{s,t}^h + S^h (1 - \theta^h) \tilde{S}_t^h - B_r^h \tilde{v}_{b,t}^h - D_r^h \tilde{v}_t^h - D_r^h \tilde{D}_{r,t}^h - B^h (1 - \theta^h \omega) \tilde{B}_t^h &= \theta^h S^h \tilde{Q}_{r,t}^h \\
S^f \tilde{v}_{s,t}^f + S^f (1 - \theta^f) \tilde{S}_t^f - B_r^f \tilde{v}_{b,t}^f - D_r^f \tilde{v}_t^f - D_r^f \tilde{D}_{r,t}^f - B_r^f (1 - \theta^f \omega) \tilde{B}_{r,t}^f &= \theta^f S^f \tilde{Q}_{r,t}^f \\
D_r^h \tilde{D}_{r,t}^h &= S^h \tilde{Q}_{r,t}^h + S^h \tilde{S}_t^h - N_r^h \tilde{N}_{r,t}^h - B_r^h \tilde{B}_{r,t}^h \\
D_r^f \tilde{D}_{r,t}^f &= S^f \tilde{Q}_{r,t}^f + S^f \tilde{S}_t^f - N_r^f \tilde{N}_{r,t}^f - B_r^f \tilde{B}_{r,t}^f \\
B_{r,t}^h + P_t^h &= B_{r,t}^f + P_t^f \\
N_r^h \tilde{N}_{r,t}^h &= (\sigma + \xi^h) Z_r^h S^h \tilde{Z}_{r,t}^h + (\sigma + \xi^h) R S^h \tilde{\psi}_t^h + (\sigma + \xi^h) R S^h \tilde{S}_{t-1}^h + (\sigma + \xi^h) (1 - \delta) S^h \tilde{Q}_{r,t}^h \\
&\quad + \sigma (R^i B_r^h + R D_r^h) \tilde{\pi}_t^h - \sigma R^i B_r^h \tilde{R}_t^i - \sigma R^i B_r^h \tilde{B}_{r,t-1}^h - \sigma R D_r^h \tilde{R}_t^h - \sigma R D_r^h \tilde{D}_{r,t-1}^h \\
N_r^f \tilde{N}_{r,t}^f &= (\sigma + \xi^f) Z_r^f S^f \tilde{Z}_{r,t}^f + (\sigma + \xi^f) R S^f \tilde{\psi}_t^f + (\sigma + \xi^f) R S^f \tilde{S}_{t-1}^f + (\sigma + \xi^f) (1 - \delta) S^f \tilde{Q}_{r,t}^f \\
&\quad + \sigma (R^i B_r^f + R D_r^f) \tilde{\pi}_t^f - \sigma R^i B_r^f \tilde{R}_t^i - \sigma R^i B_r^f \tilde{B}_{r,t-1}^f - \sigma R D_r^f \tilde{R}_t^f - \sigma R D_r^f \tilde{D}_{r,t-1}^f \\
\tilde{v}_t^h &= \tilde{R}_{t+1}^h + \eta \tilde{C}_t^h - \eta E_t(\tilde{C}_{t+1}^h) - E_t(\tilde{\pi}_{t+1}^h) + \sigma (1 - \phi^h (1 - x_r^h)) E_t(\tilde{v}_{t+1}^h) \\
&\quad + \sigma \phi^h E_t(\tilde{v}_{s,t+1}^h) - \sigma \phi^h E_t(\tilde{Q}_{r,t+1}^h) - \sigma \phi^h x_r^h E_t(\tilde{v}_{b,t+1}^h) \\
\tilde{v}_t^f &= \tilde{R}_{t+1}^f + \eta \tilde{C}_t^f - \eta E_t(\tilde{C}_{t+1}^f) - E_t(\tilde{\pi}_{t+1}^f) + \sigma (1 - \phi^f (1 - x_r^f)) E_t(\tilde{v}_{t+1}^f) \\
&\quad + \sigma \phi^f E_t(\tilde{v}_{s,t+1}^f) - \sigma \phi^f E_t(\tilde{Q}_{r,t+1}^f) - \sigma \phi^f x_r^f E_t(\tilde{v}_{b,t+1}^f) \\
\tilde{v}_{b,t}^h &= \tilde{v}_t^h + \tilde{R}_{t+1}^h - \tilde{R}_{t+1}^h \\
\tilde{v}_{b,t}^f &= \tilde{v}_t^f + \tilde{R}_{t+1}^f - \tilde{R}_{t+1}^f \\
\tilde{v}_{s,t}^h &= \tilde{v}_t^h + E_t(\tilde{\pi}_{t+1}^h) + \frac{Z_r^h}{R} E_t(Z_{r,t+1}^h) + E_t(\tilde{\psi}_{t+1}^h) + \frac{(1 - \delta)}{R} E_t(\tilde{Q}_{r,t+1}^h) - \tilde{R}_{t+1}^h \\
\tilde{v}_{s,t}^f &= \tilde{v}_t^f + E_t(\tilde{\pi}_{t+1}^f) + \frac{Z_r^f}{R} E_t(Z_{r,t+1}^f) + E_t(\tilde{\psi}_{t+1}^f) + \frac{(1 - \delta)}{R} E_t(\tilde{Q}_{r,t+1}^f) - \tilde{R}_{t+1}^f \\
Y^h \tilde{Y}_t^h &= C_H^h \tilde{C}_{H,t}^h + C_F^h \tilde{C}_{F,t}^h + I^h \tilde{I}_t^h + G^h \tilde{G}_t^h \\
Y^f \tilde{Y}_t^f &= C_F^f \tilde{C}_{F,t}^f + C_H^f \tilde{C}_{H,t}^f + I^f \tilde{I}_t^f + G^f \tilde{G}_t^f \\
\tilde{T}_{r,t}^h &= \tilde{P}_{H,t}^h - \tilde{P}_t^h + \tilde{G}_t^h \\
\tilde{T}_{r,t}^f &= \tilde{P}_{F,t}^f - \tilde{P}_t^f + \tilde{G}_t^f \\
\tilde{S}_t^h &= \delta \tilde{I}_t^h + (1 - \delta) \tilde{K}_t^h \\
\tilde{S}_t^f &= \delta \tilde{I}_t^f + (1 - \delta) \tilde{K}_t^f \\
\tilde{R}_{t+1} &= \phi_\pi \gamma \tilde{\pi}_{H,t} + \phi_\pi (1 - \gamma) \tilde{\pi}_{F,t} + \phi_\gamma \gamma \tilde{Y}_t^h + \phi_\gamma (1 - \gamma) \tilde{Y}_t^f + \zeta_t \\
\zeta_t &= \rho_m \zeta_{t-1} + \epsilon_{m,t} \\
\tilde{\psi}_t^h &= \rho_k \tilde{\psi}_{t-1}^h + \epsilon_{k,t}^h \\
\tilde{\psi}_t^f &= \rho_k \tilde{\psi}_{t-1}^f + \epsilon_{k,t}^f \\
\tilde{A}_t^h &= \rho_a \tilde{A}_{t-1}^h + \epsilon_{a,t}^h \\
\tilde{A}_t^f &= \rho_a \tilde{A}_{t-1}^f + \epsilon_{a,t}^f
\end{aligned}$$

Appendix B.5: calibration of σ_a , σ_m and σ_k

Calibration of σ_a :

Start from (2.45):

$$A_t^h = (\bar{A}^h)^{1-\rho_a} (A_{t-1}^h)^{\rho_a} e^{\epsilon_{a,t}^h}$$

Note that this equation rewrites:

$$\tilde{A}_t^h = \rho_a \tilde{A}_{t-1}^h + \epsilon_{a,t}^h \quad (\text{B.1})$$

(B.1) is then fit to a quarterly series of real productivity for the Eurozone considered over the 1995-2012 period. \tilde{A}_t^h is computed as $\tilde{A}_t^h = \log\left(\frac{A_t^h}{A^h}\right)$, the steady-state value A^h being defined as the trend of the real productivity series, obtained by Hodrick-Prescott filtering. σ_a then obtains from the empirical standard error of the series of residuals $\epsilon_{a,t}^h$. The value is $\sigma_a = 0.0041$. Estimates realized on individual Eurozone countries typically yield values of similar amplitude.

Calibration of σ_m

Start from the Taylor rule (2.86):

$$\log(R_{t+1}^h) = \log(R^h) + \phi_\pi^h \log \pi_t^h + \phi_y^h \log\left(\frac{Y_t^h}{Y}\right) + \zeta_t^h$$

As R_{t+1}^h is the gross interest rate prevailing on deposits, one has $R_{t+1}^h = 1 + R_{n,t+1}^h$, where $R_{n,t+1}^h$ denotes the net interest rate. Similarly, $\pi_t^h = 1 + \pi_{n,t}^h$, with $\pi_{n,t}^h$ the net inflation rate. Using the first-order approximation $\log(1 + x_t) \approx x_t$, one may rewrite (2.86) as:

$$R_{n,t+1}^h = R_n^h + \phi_\pi^h \pi_{n,t}^h + \phi_y^h \tilde{Y}_t^h + \zeta_t^h, \quad (\text{B.2})$$

(B.2) takes a convenient form as series for $R_{n,t+1}^h$ and $\pi_{n,t}^h$ can be commonly found in datasets. (B.2) is thus fit by simple OLS, using quarterly series of 3-month money market rate, Harmonized CPI and real GDP for the Eurozone, over the 1999-2008 period. For the computation of \tilde{Y}_t^h , the steady-state value Y^h is defined as the trend of the output series,

obtained by Hodrick-Prescott filtering. The residual series hence yields ζ_t^h , the stochastic monetary process. It then remains to estimate (2.87):

$$\zeta_t^h = \rho_m \zeta_{t-1}^h + \epsilon_{m,t}^h$$

An AR(1) process is thus fit to the series ζ_t^h , and the standard error σ_ζ of the monetary shocks is calculated as the empirical variance of the residual series $\epsilon_{m,t}^h$. The resulting value is $\sigma_m = 0.0028$.

Calibration of σ_k

No estimates exist in the literature for σ_k , the standard error of the capital quality shocks. To obtain an estimate, start from (2.35):

$$K_{t+1}^h = \psi_{t+1}^h [I_t^h + (1 - \delta)K_t^h]$$

The log-linearized form of (2.35) is given by:

$$\tilde{K}_t^h = \tilde{\psi}_t^h + \delta \tilde{I}_{t-1}^h + (1 - \delta) \tilde{K}_{t-1}^h$$

Once rearranged, the equation writes:

$$\tilde{\psi}_t^h = \tilde{K}_t^h - \delta \tilde{I}_{t-1}^h - (1 - \delta) \tilde{K}_{t-1}^h \quad (\text{B.3})$$

Estimation is then run on 11 Eurozone countries: from quarterly series for real capital stock and gross fixed capital formation, one may obtain estimates for \tilde{K}_t^h and \tilde{I}_t^h from

$$\tilde{K}_t^h = \log\left(\frac{K_t^h}{K^h}\right) \text{ and } \tilde{I}_t^h = \log\left(\frac{I_t^h}{I^h}\right), \text{ where the steady-state values are once again defined as the}$$

Hodrick-Prescott trends of the series. Following, direct application of (B.3) yields the series

$$\tilde{\psi}_t^h. \text{ Then, start from (2.36):}$$

$$\psi_t^h = (\bar{\psi})^{1-\rho_k} (\psi_{t-1}^h)^{\rho_k} e^{\epsilon_{k,t}^h}$$

and notice that it rewrites:

$$\tilde{\psi}_t^h = \rho_k \tilde{\psi}_{t-1}^h + \epsilon_{k,t}^h \quad (\text{B.4})$$

So one may simply estimate an AR(1) model for (B.4), and obtain the value of σ_k as the empirical standard error of the residual series $\epsilon_{k,t}^h$. The resulting value is $\sigma_k = 0.0006$, which is obtained as the GDP-weighted average value over the 11 Eurozone countries.

Appendix B.6: source of data used for calibration purposes

Calibration of the standard error of productivity shocks (σ_a):

Series of real labour productivity per hour worked, Eurostats, series namq_aux_lp, considered over the 1995-2012 period.

Calibration of the standard error of monetary shocks (σ_m):

Interest rate series: 3-month money market rates, Eurostats, series irt_h_mr3_q. The period considered is 1980-1998.

Inflation: Harmonized CPI, OECD in its Main Economic Indicator series, (save for Malta obtained from Datastream, series MAI99BIPC, and for Cyprus obtained from Eurostats, series prc_hicp_midx). Inflation is defined as the percentage change of CPI between two periods. The period considered is 1980-1998.

Output: Real GDP, OECD, series VOBARSA (except for Malta and Cyprus obtained from Eurostats (series namq_gdp_k)). The period considered is 1980-1998.

Calibration of the standard error of capital quality shocks (σ_k):

Capital stock: real capital stock, Oxford Economics via Datastream (countries considered are Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Slovakia and Spain³³). The period covered is 1980-2012.

Investment: gross fixed capital formation, Eurostats, series namq_pi6_k. When data is available the period covered is 1980-2012, otherwise reduced to the largest possible period when data is missing.

GDP data: Eurostats, series nama_gdp_k (millions of euro, chain-linked volumes, reference year 2005).

The relative contribution of each of the 17 Eurozone countries to total GDP is computed for each year between 2000 and 2012, then averaged over the period.

³³ The full list of series reference is : OEXKCAP.C, BGXKCAP.C, FNXKCAP.C, FRXKCAP.C, BDXKCAP.C, GRXKCAP.C, IRXKCAP.C, ITXKCAP.C, NLXKCAP.C, PTXKCAP.C, SXXKCAP.C, ESXKCAP.C.

Total factor productivity: Eurostats, series nama_aux_lp (real labour productivity per hour worked, in Euro per hour worked).

Productivity for each of the 17 Eurozone countries is computed as the average over years 2002 to 2009.

Government expenditure: Eurostats, series ei_naga_a (general government expenditure, percentage of GDP).

Government expenditure as a percentage of GDP for each of the 17 Eurozone countries is computed as the average over years 1999-2011.

Imports: Eurostats, series nama_exi_c (imports of goods, percentage of GDP).

The share of imported goods over GDP for each of the 17 Eurozone countries is computed as the average over years 2000-2012.

Relative size: Eurostats, series demo_pjan (total population).

The relative size for each of the 17 Eurozone countries is computed, for each year, as the ratio of the country population over total Eurozone population. Results are then averaged over years 1999-2012.

Daily working time: Eurostats, series lfsa_ewhun2 (average number of usual weekly hours of work in main job).

Daily hours are estimated by assuming a week of five working days, hence dividing weekly time by five. The share of daily time dedicated to work is finally computed by dividing daily working time by 24. Then, for each of the 17 Eurozone countries, an average is taken over years 2000-2011.

Leverage ratio: Eurostats, table tipsfs20 (financial sector leverage, debt to equity).

The leverage ratio for each of the 17 Eurozone country is then computed as the average over years 2006-2011.

Interbank loan share in total bank asset: OECD bank profitability report: 'Bank profitability: financial statements of banks 2010', in the series *OECD Banking Statistics*. The report contains data for 13 Eurozone countries. For these countries, the share is computed as total interbank loans (liabilities) over total asset, averaged over years 2000-2008.

For the four missing countries (Cyprus, Estonia, Malta, Slovenia), an approximation of the ratio is obtained from data of three major domestic banks. Data obtains from the annual reports of these banks. An average is then computed over these banks and over the 2003-2008 period.

The banks under consideration are:

Cyprus: Bank of Cyprus, Cyprus Development Bank, and Cyprus Popular Bank.

Estonia: Krediidipank, SEB Pank, and Tallinna Aripank.

Malta: APS Bank, Bank of Valletta, and Fimbank.

Slovenia: Abanka, Nova Ljubljanska Banka, and Unicredit Banka.

Appendix B.7: derivation of steady-state values

A numerical solution

Consider first the flexible exchange rate version of the model. The set of equations used to derive the steady-state is made of the 61 equations (b.1)-(b.61) detailed in Appendix B.1. Focus is made on a symmetric equilibrium where the all prices are equal and constant at some (unknown) level \bar{P} . That is, in steady-state, prices are such that:

$$P^h = P^f = P_H = P_F = P_H(j) = P_F(j) = P_H^*(j) = P_H^*(j) = \bar{P}$$

where variables without a t subscript denote steady-state values. Because \bar{P} is indeterminate in steady-state, the equilibrium conditions have been expressed in real terms so that \bar{P} can be eliminated from all further steady-state equations (and also from all the linearized equations). This symmetry of the prices implies, from (b.1) and (b.2) that the steady-state exchange rate is set to 1, that is: $\varepsilon = 1$. Also, constant prices imply from (b.27)-(b.32) that inflation variables are set to 1 in steady-state. That is, $\pi_H = \pi_F = \pi^h = \pi^f = 1$.

Several other values are easy to derive. From (b.9) and (b.10), obtain:

$$R^h = R^f = \frac{1}{\beta}$$

From (b.17), (b.18), and the above assumptions on prices:

$$Q_r^h = Q_r^f = 1$$

Finally, from (b.56)-(b.61), the steady-state values of the stochastic processes obtain as:

$$\zeta^h = \zeta^f = 0$$

$$\psi^h = \psi^f = \bar{\psi}, \text{ with } \bar{\psi} \text{ set to 1.}$$

$$A^h = \bar{A}^h \text{ and } A^f = \bar{A}^f$$

These results will be used continuously in the incoming derivations without further reference.

Once these equations are cleared, the steady-state is left with 22 real equations:

(b.3), (b.4), (b.5), (b.6), (b.7), (b.8), (b.13), (b.14), (b.15), (b.16), (b.19), (b.20), (b.21), (b.22), (b.23), (b.24), (b.25), (b.26), (b.48), (b.49), (b.50) and (b.51),

and 20 financial equations:

(b.11), (b.15), (b.16), (b.33), (b.34), (b.35), (b.36), (b.37), (b.38), (b.39), (b.40), (b.41), (b.42), (b.43), (b.44), (b.45), (b.46), (b.47), (b.52) and (b.53).

Start with the real side of the model. Combine first (b.21), (b.25) and (b.7) to obtain:

$$(L^h)^\phi = \frac{(\kappa-1)\bar{A}^h(1-\alpha)}{\chi^h \kappa} \left(\frac{K^h}{L^h} \right)^\alpha (C^h)^{-\eta} \quad (\text{B.5})$$

Similarly, combining (b.22), (b.26) and (b.8)

$$(L^f)^\phi = \frac{(\kappa-1)\bar{A}^f(1-\alpha)}{\chi^f \kappa} \left(\frac{K^f}{L^f} \right)^\alpha (C^f)^{-\eta} \quad (\text{B.6})$$

From (b.13), (b.48), (b.15), (b.19), (b.3) and (b.6):

$$\left((1-\vartheta^h)\bar{A}^h \left(\frac{K^h}{L^h} \right)^{\alpha-1} - \delta \right) K^h = \varphi^h C^h + (1-\varphi^f) C^f \quad (\text{B.7})$$

From (b.14), (b.49), (b.16), (b.20), (b.4) and (b.5):

$$\left((1-\vartheta^f)\bar{A}^f \left(\frac{K^f}{L^f} \right)^{\alpha-1} - \delta \right) K^f = \varphi^f C^f + (1-\varphi^h) C^h \quad (\text{B.8})$$

From (b.21), (b.25) and (b.23):

$$\frac{K^h}{L^h} = (Z_r^h)^{\frac{-1}{1-\alpha}} \left[\bar{A}^h \left(\frac{\kappa - (\kappa-1)(1-\alpha)}{\kappa} \right) \right]^{\frac{1}{1-\alpha}} \quad (\text{B.9})$$

and from (b.22), (b.26) and (b.24):

$$\frac{K^f}{L^f} = (Z_r^f)^{\frac{-1}{1-\alpha}} \left[\bar{A}^f \left(\frac{\kappa - (\kappa-1)(1-\alpha)}{\kappa} \right) \right]^{\frac{1}{1-\alpha}} \quad (\text{B.10})$$

(B.5)-(B-10) can be further used to eliminate C^h, C^f, L^h and L^f until one obtains the single equation:

$$\begin{aligned} & \left(\frac{(Z_r^h)^{\frac{(1-\vartheta^h)\kappa}{\kappa - (\kappa-1)(1-\alpha)} - \delta}}{(Z_r^f)^{\frac{(1-\vartheta^f)\kappa}{\kappa - (\kappa-1)(1-\alpha)} - \delta}} \right) \left(\frac{K^h}{K^f} \right)^{\frac{\eta+\phi}{\eta}} = \left(\frac{Z_r^h}{Z_r^f} \right)^{\frac{-(\alpha+\phi)}{(1-\alpha)\eta}} \times \\ & \left\{ \varphi^h \left[\left(\frac{(\kappa-1)\bar{A}^h(1-\alpha)}{\kappa\chi^h} \right)^{\frac{1}{\eta}} \left[\bar{A}^h \left(\frac{\kappa - (\kappa-1)(1-\alpha)}{\kappa} \right) \right]^{\frac{\alpha+\phi}{(1-\alpha)\eta}} \right] + (1-\varphi^h) \left[\left(\frac{(\kappa-1)\bar{A}^f(1-\alpha)}{\kappa\chi^f} \right)^{\frac{1}{\eta}} \left[\bar{A}^f \left(\frac{\kappa - (\kappa-1)(1-\alpha)}{\kappa} \right) \right]^{\frac{\alpha+\phi}{(1-\alpha)\eta}} \right] \left(\frac{K^h}{K^f} \right)^{\frac{\phi}{\eta}} \right\} \\ & \left\{ \varphi^f \left[\left(\frac{(\kappa-1)\bar{A}^f(1-\alpha)}{\kappa\chi^f} \right)^{\frac{1}{\eta}} \left[\bar{A}^f \left(\frac{\kappa - (\kappa-1)(1-\alpha)}{\kappa} \right) \right]^{\frac{\alpha+\phi}{(1-\alpha)\eta}} \right] + (1-\varphi^h) \left[\left(\frac{(\kappa-1)\bar{A}^h(1-\alpha)}{\kappa\chi^h} \right)^{\frac{1}{\eta}} \left[\bar{A}^h \left(\frac{\kappa - (\kappa-1)(1-\alpha)}{\kappa} \right) \right]^{\frac{\alpha+\phi}{(1-\alpha)\eta}} \right] \left(\frac{K^h}{K^f} \right)^{\frac{\phi}{\eta}} \right\} \end{aligned} \quad (\text{B.11})$$

This equation comprises one single real variable: the ratio $\frac{K^h}{K^f}$. If (B.11) was solvable for this ratio, all the other real values could be derived recursively. However, this is not possible as the equation also comprises the financial variables Z_r^h and Z_r^f , whose values are not yet known. One hence now needs to turn to the financial part of the model.

First note that (b.13), (b.14), (b.50) and (b.51) imply:

$$S^h = K^h \text{ and } S^f = K^f$$

This allows to eliminate S^h and S^f from all the remaining equations, expressing them in terms of ratio with respect to aggregate capital. Use (b.9), (b.10), (b.33), (b.34), (b.37), (b.38), (b.39), (b.46), (b.47), (b.52) and (b.53) to eliminate the variables $R^h, R^f, B_r^f, S^h, S^f, D_r^h, F_r, Z_r^h, Z_r^f, v_s^h$ and v_s^f . This reduces the financial system to 8 equations:

$$\frac{1}{\omega} v_b^h - \frac{1}{\omega} v^h - v_b^h \frac{B_r^h}{K^h} + v^h \frac{N_r^h}{K^h} + v^h \frac{B_r^h}{K^h} = \theta^h - \theta^h \omega \frac{B_r^h}{K^h} \quad (\text{B.12})$$

$$\frac{1}{\omega} v_b^f - \frac{1}{\omega} v^f + v_b^f \left(\frac{B_r^h}{K^h} \right) \left(\frac{K^h}{K^f} \right) + v^h \frac{N_r^f}{K^f} - v^f \left(\frac{B_r^h}{K^h} \right) \left(\frac{K^h}{K^f} \right) = \theta^f + \theta^f \omega \left(\frac{B_r^h}{K^h} \right) \left(\frac{K^h}{K^f} \right) \quad (\text{B.13})$$

$$\left(1 - \frac{\sigma}{\beta} \right) \frac{N_r^h}{K^h} = \left(\frac{\sigma + \xi^h}{\omega} \right) R^i - \left(\frac{\sigma + \xi^h (1 - \omega)}{\beta \omega} \right) - \sigma R^i \frac{B_r^h}{K^h} + \frac{\sigma}{\beta} \frac{B_r^h}{K^h} \quad (\text{B.14})$$

$$\left(1 - \frac{\sigma}{\beta} \right) \frac{N_r^f}{K^f} = \left(\frac{\sigma + \xi^f}{\omega} \right) R^i - \left(\frac{\sigma + \xi^f (1 - \omega)}{\beta \omega} \right) + \sigma R^i \left(\frac{B_r^h}{K^h} \right) \left(\frac{K^h}{K^f} \right) - \frac{\sigma}{\beta} \left(\frac{B_r^h}{K^h} \right) \left(\frac{K^h}{K^f} \right) \quad (\text{B.15})$$

$$\begin{aligned} & (1 - \sigma) \left(\theta^h - \frac{1}{\omega} \right) v^h + \frac{1 - \sigma}{\omega} v_b^h + \frac{1}{\omega} (v^h)^2 + (1 - \sigma) \theta^h \omega \frac{B_r^h}{K^h} + (1 - \sigma) (1 - \theta^h \omega) v^h \frac{B_r^h}{K^h} + v^h v_b^h \frac{B_r^h}{K^h} \\ & = (1 - \sigma) \theta^h + \frac{1}{\omega} v^h v_b^h + (1 - \sigma) v_b^h \frac{B_r^h}{K^h} + (v^h)^2 \frac{B_r^h}{K^h} \end{aligned} \quad (\text{B.16})$$

$$\begin{aligned} & (1 - \sigma) \left(\theta^f - \frac{1}{\omega} \right) v^f + \frac{1 - \sigma}{\omega} v_b^f + \frac{1}{\omega} (v^f)^2 - (1 - \sigma) \theta^f \omega \left(\frac{B_r^h}{K^h} \right) \left(\frac{K^h}{K^f} \right) - (1 - \sigma) (1 - \theta^f \omega) v^f \left(\frac{B_r^h}{K^h} \right) \left(\frac{K^h}{K^f} \right) \\ & - v^f v_b^f \left(\frac{B_r^h}{K^h} \right) \left(\frac{K^h}{K^f} \right) = (1 - \sigma) \theta^f + \frac{1}{\omega} v^f v_b^f - (1 - \sigma) v_b^f \left(\frac{B_r^h}{K^h} \right) \left(\frac{K^h}{K^f} \right) - (v^f)^2 \left(\frac{B_r^h}{K^h} \right) \left(\frac{K^h}{K^f} \right) \end{aligned} \quad (\text{B.17})$$

$$v_b^h = \beta v^h R^i \quad (\text{B.18})$$

$$v_b^f = \beta v^f R^i \quad (\text{B.19})$$

This reduced financial part of the model comprises 8 variables:

$$v^h, v^f, v_b^h, v_b^f, \frac{N_r^h}{K^h}, \frac{N_r^f}{K^f}, \frac{B_r^h}{K^h}, R^i$$

Also, during the simplification process, one obtains the intermediary result that:

$$Z_r^h = Z_r^f = \frac{1}{\omega} R^i - \left(1 - \delta + \frac{1 - \omega}{\beta \omega} \right)$$

This result can be substituted in (B.11) to eliminate Z_r^h and Z_r^f , to obtain:

$$\begin{aligned} & \left(\frac{\frac{1}{\omega} R^i - \left(1 - \delta + \frac{1 - \omega}{\beta \omega} \right)}{\frac{1}{\omega} R^i - \left(1 - \delta + \frac{1 - \omega}{\beta \omega} \right)} \right) \frac{(1 - \vartheta^h) \kappa - \delta}{\kappa - (\kappa - 1)(1 - \alpha)} - \delta \left(\frac{K^h}{K^f} \right)^{\frac{\eta + \phi}{\eta}} = \\ & \left\{ \varphi^h \left[\left(\frac{(\kappa - 1) \bar{A}^h (1 - \alpha)}{\kappa \chi^h} \right)^{\frac{1}{\eta}} \left[\bar{A}^h \left(\frac{\kappa - (\kappa - 1)(1 - \alpha)}{\kappa} \right) \right]^{\frac{\alpha + \phi}{(1 - \alpha)\eta}} \right] + (1 - \varphi^f) \left[\left(\frac{(\kappa - 1) \bar{A}^f (1 - \alpha)}{\kappa \chi^f} \right)^{\frac{1}{\eta}} \left[\bar{A}^f \left(\frac{\kappa - (\kappa - 1)(1 - \alpha)}{\kappa} \right) \right]^{\frac{\alpha + \phi}{(1 - \alpha)\eta}} \right] \left(\frac{K^h}{K^f} \right)^{\frac{\phi}{\eta}} \right\} \\ & \left\{ \varphi^f \left[\left(\frac{(\kappa - 1) \bar{A}^f (1 - \alpha)}{\kappa \chi^f} \right)^{\frac{1}{\eta}} \left[\bar{A}^f \left(\frac{\kappa - (\kappa - 1)(1 - \alpha)}{\kappa} \right) \right]^{\frac{\alpha + \phi}{(1 - \alpha)\eta}} \right] + (1 - \varphi^h) \left[\left(\frac{(\kappa - 1) \bar{A}^h (1 - \alpha)}{\kappa \chi^h} \right)^{\frac{1}{\eta}} \left[\bar{A}^h \left(\frac{\kappa - (\kappa - 1)(1 - \alpha)}{\kappa} \right) \right]^{\frac{\alpha + \phi}{(1 - \alpha)\eta}} \right] \left(\frac{K^h}{K^f} \right)^{\frac{\phi}{\eta}} \right\} \end{aligned} \quad (\text{B.20})$$

Then finally, equations (B.12)-(B.20) constitute a system of 9 equations in the 9 unknown:

$v^h, v^f, v_b^h, v_b^f, \frac{N_r^h}{K^h}, \frac{N_r^f}{K^f}, \frac{B_r^h}{K^h}, R^i, \frac{K^h}{K^f}$. Although the system has no analytical solution, it can be

handled by a numerical solver. The remaining variables are then obtained recursively from these 9 initial values.

A quasi-analytical solution

Although a numerical solution to (B12)-(B.20) does most of the time succeed in finding the steady-state values, an analytical solution is preferable: it yields exact values rather than approximate values, and most importantly, it always manages to solve the system while the numerical algorithm may sometimes fail to converge to the solution.

Preliminary numerical solutions run on (B.12)-(B.20) consistently suggest that $v^h = v_b^h$ and $v^f = v_b^f$, with a value of 1. This indicates that in steady-state, the marginal cost of deposits and the marginal cost of interbank borrowing are equalized, which amounts to the flexible equilibrium situation. This is fairly intuitive as economies typically revert to their flexible values in the long run. From the limited assumptions $v^h = v_b^h$ and $v^f = v_b^f$, a unique, well identified equilibrium obtains then in a mostly analytical way.

First, since $Q^h = 1$ and $Q^f = 1$, the above assumptions allow to conclude from (b.33) and (b.34) that $v_s^h = v^h$ and $v_s^f = v^f$. This states that in steady-state, the marginal return of assets equals the marginal cost of deposit and interbank loans, illustrating once again the reversion of variables to flexible equilibria in the long run. Combining these results with (b.42) and (2.67), one obtains $v^h = 1$. Following, $v^h = v_b^h = v_s^h = 1$. Similarly, from (b.43) and (2.76), one concludes $v^f = v_b^f = v_s^f = 1$. This confirms that the numerical results found were indeed correct. Similarly, the correctness of the results is confirmed by (B.16) and (B.17) which collapse to an identity.

(B.18) and (B.19) then yield: $R^i = \frac{1}{\beta}$, equating the long-run value of the interbank rate with the riskless rate. Using this result, R^i can be eliminated from (B.20), leaving it as a single equation in the single variable $\frac{K^h}{K^f}$:

$$\left(\frac{\frac{1}{\omega\beta} - \left(1 - \delta + \frac{1-\omega}{\beta\omega}\right) \frac{(1-\vartheta^h)\kappa}{\kappa - (\kappa-1)(1-\alpha)} - \delta}{\frac{1}{\omega\beta} - \left(1 - \delta + \frac{1-\omega}{\beta\omega}\right) \frac{(1-\vartheta^f)\kappa}{\kappa - (\kappa-1)(1-\alpha)} - \delta} \right) \left(\frac{K^h}{K^f} \right)^{\frac{\eta+\phi}{\eta}} =$$

$$\frac{\left\{ \varphi^h \left[\left(\frac{(\kappa-1)\bar{A}^h(1-\alpha)}{\kappa\chi^h} \right)^{\frac{1}{\eta}} \left[\bar{A}^h \left(\frac{\kappa - (\kappa-1)(1-\alpha)}{\kappa} \right) \right]^{\frac{\alpha+\phi}{(1-\alpha)\eta}} \right] + (1-\varphi^f) \left[\left(\frac{(\kappa-1)\bar{A}^f(1-\alpha)}{\kappa\chi^f} \right)^{\frac{1}{\eta}} \left[\bar{A}^f \left(\frac{\kappa - (\kappa-1)(1-\alpha)}{\kappa} \right) \right]^{\frac{\alpha+\phi}{(1-\alpha)\eta}} \right] \left(\frac{K^h}{K^f} \right)^{\frac{\phi}{\eta}} \right\}}{\left\{ \varphi^f \left[\left(\frac{(\kappa-1)\bar{A}^f(1-\alpha)}{\kappa\chi^f} \right)^{\frac{1}{\eta}} \left[\bar{A}^f \left(\frac{\kappa - (\kappa-1)(1-\alpha)}{\kappa} \right) \right]^{\frac{\alpha+\phi}{(1-\alpha)\eta}} \right] + (1-\varphi^h) \left[\left(\frac{(\kappa-1)\bar{A}^h(1-\alpha)}{\kappa\chi^h} \right)^{\frac{1}{\eta}} \left[\bar{A}^h \left(\frac{\kappa - (\kappa-1)(1-\alpha)}{\kappa} \right) \right]^{\frac{\alpha+\phi}{(1-\alpha)\eta}} \right] \left(\frac{K^h}{K^f} \right)^{\frac{\phi}{\eta}} \right\}}$$

(B.21)

This equation is the only one that still has to be solved for numerically, but this is easily handled by any numerical solver. $\frac{K^h}{K^f}$ is then identified.

Using $R^i = \frac{1}{\beta}$ in (B.14) gives: $\frac{N_r^h}{K^h} = \frac{\xi^h}{\beta - \sigma}$. But as $\frac{N_r^h}{K^h}$ is the inverse of the steady-state leverage ratio ϕ^h , one can obtain any desired steady-state value ϕ^h for the model by setting the value of ξ^h so that $\xi^h = \left(\frac{\beta - \sigma}{\phi^h} \right)$. Similarly, from (B.15) one can set $\xi^f = \left(\frac{\beta - \sigma}{\phi^f} \right)$.

(B.12) then gives: $\theta^h - \theta^h \omega \frac{B_r^h}{K^h} = \frac{1}{\phi^h}$. As $\frac{B_r^h}{K^h} = \frac{B_r^h}{S^h}$ is the fraction of Home assets financed by interbank borrowing x^h , one can obtain any desired steady-state value x^h in the model by setting the value of θ^h so that: $\theta^h = \frac{1}{\phi^h(1 - \omega x^h)}$.

Finally, from (B.13), one obtains that the value of θ^f must be set as: $\theta^f = \frac{1}{\phi^f \left[1 + \omega x^h \left(\frac{K^h}{K^f} \right) \right]}$.

Then the 9 variables $v^h, v^f, v_b^h, v_b^f, \frac{N_r^h}{K^h}, \frac{N_r^f}{K^f}, \frac{B_r^h}{K^h}, R^i, \frac{K^h}{K^f}$ are identified as :

$$v^h = v^f = v_b^h = v_b^f = 1.$$

$$R^i = \frac{1}{\beta}.$$

ϕ^h, ϕ^f and x^h (that is, $\frac{N_r^h}{K^h}, \frac{N_r^f}{K^f}$ and $\frac{B_r^h}{K^h}$) can be calibrated exogenously, with the values of ξ^h, ξ^f, θ^h and θ^f calculated accordingly.

Finally, $\frac{K^h}{K^f}$ is obtained by numerical resolution of (A1.9). All the other values are obtained recursively from these 9 values.

Turn now to the monetary union version of the model. There are two main differences between the independent currency and the monetary union versions of the model: the existence or not of an exchange rate; and the existence of one or two central banks. Because the exchange rate has a value of 1 in steady-state, its suppression in the monetary union version of the model is of no consequence. As for the Taylor rule, they are not used to derive the steady-state values, whether there is one or two central banks in the model. Following, it is trivial to show that the steady-state values for the model in monetary union are the same than the ones of the independent currency model. Suffice is to notice that total deposit in Foreign $F_r = F_r^h + F_r^f$ is now replaced by D^f , that R^h and R^f are now replaced by the single interest rate value R , and similarly, that ζ^h and ζ^f are replaced by ζ .

Appendix B.8: a RBC model of open economies

The model is an amended version of the open economy RBC model of Backus et al. (1994), adapted to integrate the capital quality shocks introduced in the benchmark model.

Productive setup

The model considers two countries: Home, which has relative size γ , and Foreign, of relative size $1-\gamma$. Each country is inhabited by a Household of mass unity, and by a firm of mass unity operated by the household members. In both countries, this firm produces a single homogenous good from capital and labour. Neither capital nor labour are mobile across countries. Consider Home first. Denoting aggregate output by Y_t^h , capital in Home by K_t^h , aggregate labour by L_t^h , and total productivity by A_t^h , production obtains from the following conventional return-to-scale technology:

$$Y_t^h = A_t^h (K_t^h)^\alpha (L_t^h)^{1-\alpha} \quad (\text{B.22})$$

A_t^h is a stochastic process defined as:

$$A_t^h = (\bar{A}^h)^{1-\rho_a} (A_{t-1}^h)^{\rho_a} e^{\epsilon_{a,t}^h} \quad (\text{B.23})$$

Similarly in Foreign, output obtains from:

$$Y_t^f = A_t^f (K_t^f)^\alpha (L_t^f)^{1-\alpha} \quad (\text{B.24})$$

with:

$$A_t^f = (\bar{A}^f)^{1-\rho_a} (A_{t-1}^f)^{\rho_a} e^{\epsilon_{a,t}^f} \quad (\text{B.25})$$

Aggregate capital held by the Home firm evolves according to the following law of motion :

$$K_{t+1}^h = \psi_{t+1}^h [I_t^h + (1-\delta)K_t^h] \quad (\text{B.26})$$

where I_t^h denotes aggregate investment. ψ_t^h is a shock on the quality of capital, used as an exogenous source of variation for the latter. It is assumed that the shock is defined as a stochastic process defined as:

$$\psi_t^h = (\bar{\psi}^h)^{1-\rho_k} (\psi_{t-1}^h)^{\rho_k} e^{\epsilon_{k,t}^h} \quad (\text{B.27})$$

Similarly in Foreign, capital follows a similar law of motion:

$$K_{t+1}^f = \psi_{t+1}^f \left[I_t^f + (1 - \delta) K_t^f \right] \quad (\text{B.28})$$

with the stochastic process ψ_t^f defined as:

$$\psi_t^f = (\bar{\psi}^f)^{1-\rho_k} (\psi_{t-1}^f)^{\rho_k} e^{\epsilon_{k,t}^f} \quad (\text{B.29})$$

Governments

In both countries, government is assumed to realize at each period a fixed, exogenous amount of public expenditures. The amount of output spent by the Home governments is given by:

$$G_t^h = \vartheta^h Y^h \quad (\text{B.30})$$

where ϑ^h is a fixed exogenous constant with $0 < \vartheta^h < 1$, and Y_H denotes the steady-state value of Home output. Similarly, in Foreign, government consumption is given by:

$$G_t^f = \vartheta^f Y^f \quad (\text{B.31})$$

Households and the social planner problem

Household members in both countries consume goods and supply labour for production purposes. As consumers, they derive consumption from a composite index which includes the goods produced in both countries. Consider Home: let C_t^h denote the composite index, $C_{H,t}^h$ denote consumption of the Home goods by the Home household, and $C_{F,t}^h$ consumption of the Foreign good by the Home household. Then the composite index is defined as:

$$C_t^h = \left((\varphi^h)^{\frac{1}{\mu}} C_{H,t}^h{}^{\frac{\mu-1}{\mu}} + (1-\varphi^h)^{\frac{1}{\mu}} C_{F,t}^h{}^{\frac{\mu-1}{\mu}} \right)^{\frac{\mu}{\mu-1}} \quad (\text{B.32})$$

where $\varphi^h = 1 - (1-\gamma)v^h$ represents the importance of Home goods in the composite index. It comprises γ , the relative size of Home, and v^h which measures trade openness. In Home, a benevolent social planner wants to maximize welfare of the Household. Precisely, it solves the following problem:

$$\underset{\{K_{t+1}^h, L_t^h, C_{H,t}^h, C_{F,t}^h\}}{\text{Max}} E_t \sum_{i=0}^{\infty} \beta^i U^h(C_{t+i}^h, L_{t+i}^h) \quad 0 < \beta < 1 \quad (\text{B.33})$$

with preferences defined as:

$$U^h(C_t^h, L_t^h) = \frac{(C_t^h)^{1-\eta}}{1-\eta} - \frac{\chi^h}{1+\phi} (L_t^h)^{1+\phi}$$

Similarly, for consumers in Foreign, the composite index is defined as:

$$C_t^f = \left((\varphi^f)^{\frac{1}{\mu}} C_{F,t}^{\frac{\mu-1}{\mu}} + (1-\varphi^f)^{\frac{1}{\mu}} C_{H,t}^{\frac{\mu-1}{\mu}} \right)^{\frac{\mu}{\mu-1}} \quad (\text{B.34})$$

The social planner solves:

$$\underset{\{K_{t+1}^f, L_t^f, C_{F,t}^f, C_{H,t}^f\}}{\text{Max}} E_t \sum_{i=0}^{\infty} \beta^i U^f(C_{t+i}^f, L_{t+i}^f) \quad (\text{B.35})$$

and preferences are defined as:

$$U(C_t^f, L_t^f) = \frac{(C_t^f)^{1-\eta}}{1-\eta} - \frac{\chi^f}{1+\phi} (L_t^f)^{1+\phi}$$

The resource constraint for each country is given by:

$$Y_t^h = C_{H,t}^h + C_{F,t}^h + I_t^h + G_t^h \quad (\text{B.36})$$

and

$$Y_t^f = C_{F,t}^f + C_{H,t}^f + I_t^f + G_t^f \quad (\text{B.37})$$

Besides, note from (B.34) that $C_t^f = G(C_{F,t}^f, C_{H,t}^f)$ is homogenous of degree one. Hence, from Euler theorem, we have that in equilibrium:

$$C_t^f = p_t^f C_{F,t}^f + p_t^h C_{H,t}^f \quad (\text{B.38})$$

with p_t^f and p_t^h the prices of the Home and Foreign goods in period t in units of the Foreign composite good³⁴, with:

$$p_t^f \equiv \frac{\partial G(C_{F,t}^f, C_{H,t}^f)}{\partial C_{F,t}^f} \quad (\text{B.39})$$

³⁴ This pricing strategy proposed by Backus et al. (1994) leads to choose the consumption index of one of the two countries to define the relative prices. Here the composite index of Foreign is chosen. As Foreign is the “large” country relative to Home, it makes sense to assume that it is the one that is price maker, as it has the largest market share.

and p_t^h the price of the Home good defined as:

$$p_t^h \equiv \frac{\partial G(C_{F,t}^f, C_{H,t}^f)}{\partial C_{H,t}^f} \quad (\text{B.40})$$

Using (B.38), the resource constraint (B.37) rewrites:

$$Y_t^f = \frac{1}{p_t^f} C_t^f + I_t^f + G_t^f + (C_{F,t}^h - x_t C_{H,t}^f) \quad (\text{B.41})$$

where $x_t \equiv \frac{p_t^h}{p_t^f}$ is the terms of trade. The terms in brackets in the right-hand-side of (B.41)

represent net exports. Assuming balanced budgets in both economies implies that net exports are null. The resource constraint (B.41) then rewrites in a conventional way as:

$$Y_t^f = C_{F,t}^f + x_t C_{H,t}^f + I_t^f + G_t^f \quad (\text{B.42})$$

Similarly the resource constraint in Home rewrites:

$$Y_t^h = C_{H,t}^h + \frac{1}{x_t} C_{F,t}^h + I_t^h + G_t^h \quad (\text{B.43})$$

Consider Home. The social planner determines K_{t+1}^h, L_t^h and $C_{F,t}^h$ to solve (B.33), subject to the set of constraints (B.22), (B.26), (B.30), (B.32), and (B.43). The first-order conditions for this problem are given by:

$$\frac{1}{\psi_{t+1}^h} = \beta E_t \left[\left(\frac{C_{t+1}^h}{C_t^h} \right)^{\frac{1-\eta\mu}{\mu}} \left(\frac{C_{H,t+1}^h}{C_{H,t}^h} \right)^{\frac{1}{\mu}} (\alpha A_{t+1}^h (K_{t+1}^h)^{\alpha-1} (L_{t+1}^h)^{1-\alpha} + (1-\delta)) \right] \quad (\text{B.44})$$

$$(\varphi^h)^{\frac{1}{\mu}} (1-\alpha) (C_t^h)^{\frac{1-\eta\mu}{\mu}} (C_{H,t}^h)^{\frac{-1}{\mu}} A_t^h (K_t^h)^{\alpha} (L_t^h)^{-\alpha} = \chi^h (L_t^h)^{\phi} \quad (\text{B.45})$$

$$\left(\frac{1-\varphi^h}{\varphi^h} \right)^{\frac{1}{\mu}} = \frac{1}{x_t} \left(\frac{C_{F,t}^h}{C_{H,t}^h} \right)^{\frac{1}{\mu}} \quad (\text{B.46})$$

Similarly in Foreign, the social planner solves (B.35), subject to (B.24), (B.28), (B.31), (B.34) and (B.41). The first-order conditions are given by:

$$\left(\frac{1}{\psi_{t+1}^f} \right) = \beta E_t \left(\left(\frac{C_{t+1}^f}{C_t^f} \right)^{\frac{1-\eta\mu}{\mu}} \left(\frac{C_{F,t}^f}{C_{F,t+1}^f} \right)^{\frac{1}{\mu}} \left(\alpha A_{t+1}^f (K_{t+1}^f)^{\alpha-1} (L_{t+1}^f)^{1-\alpha} + (1-\delta) \right) \right) \quad (\text{B.47})$$

$$(\varphi^f)^\mu (1-\alpha) \left(C_t^f \right)^{\frac{1-\eta\mu}{\mu}} \left(C_{F,t}^f \right)^{\frac{-1}{\mu-1}} A_t^f (K_t^f)^\alpha (L_t^f)^{-\alpha} = \chi^f (L_t^f)^\phi \quad (\text{B.48})$$

$$\left(\frac{1-\varphi^f}{\varphi^f} \right)^{\frac{1}{\mu}} = x_t \left(\frac{C_{H,t}^f}{C_{F,t}^f} \right)^{\frac{1}{\mu}} \quad (\text{B.49})$$

Finally, assuming zero net exports, the terms of trade at each period are given by:

$$x_t = \left(\frac{1-\varphi^f}{\varphi^f} \right)^{\frac{1}{\mu-1}} \left(\frac{C_{F,t}^f}{C_{F,t}^h} \right)^{\frac{1}{\mu-1}} \quad (\text{B.50})$$

Then 17 variables:

$$\{Y_t^h, Y_t^f, K_{t+1}^h, K_{t+1}^f, L_t^h, L_t^f, I_t^h, I_t^f, G_t^h, G_t^f, C_t^h, C_t^f, C_{H,t}^h, C_{F,t}^h, C_{F,t}^f, C_{H,t}^f, x_t\}$$

and 4 stochastic processes:

$$\{A_t^h, A_t^f, \psi_t^h, \psi_t^f\}$$

are determined by the 21 equations:

(B.22), (B.23), (B.24), (B.25), (B.26), (B.27), (B.28), (B.29), (B.30), (B.31), (B.32), (A4.34), (B.42), (B.43), (B.44), (B.45), (B.46), (B.47), (B.48), (B.49) and (B.50).

Chapter 3

Of welfare and monetary regimes in the Euro area in a context of financial crisis

Abstract

This paper proposes a formal welfare assessment of two competing monetary regimes, namely a monetary union or a regime of floating exchange rates, for the 17 countries of the Euro area. The analysis relies on the model developed in chapter 2, and proposes two welfare criteria. The first one is a general welfare criterion in the line of the works by Monacelli (2000) and Kollmann (2004). The second adopts a shock-specific approach, following Carré and Collard (2001). The paper also proposes a number of extensions to the benchmark version of the model, integrating in turn to the initial framework sovereign debt and the credit policies run by the ECB, namely the Covered Bond Purchase Programmes, and the Securities Markets Programme.

The result shows that absent credit policies, 15 out of 17 Euro area members would benefit from higher welfare levels under a regime of flexible exchange rates. The main cause of inefficiency of the monetary union seems to lie in its lack of efficiency when dealing with asymmetric shocks originating from abroad partner economies. This shows that shock asymmetry remains a central issue within the Euro area. The welfare performances of the monetary union improve when the ECB operate credit policies. In the case of the SMP, a majority of Euro area countries (9 out of 17) actually become better off under the monetary union regime. This suggests that the European Monetary Union would benefit from the settlement of a supranational risk-sharing facility.

JEL classification: C32, C5

Introduction

Is the Euro beneficial to European countries? The question may seem trivial, as the single currency would not have been introduced in 1999 without sound economic reasons and solid economic studies establishing its implied welfare gains. Reality is somewhat more complex. In 1992, by the time discussions were initiated between E.U. members for the settlement of a monetary union, the matter was not settled. The “One Market, One Money” report by Emerson et al., commissioned as a supplement for the 1992 Delors Report, pointed out that “there is no ready-to-use theory for assessing the costs and benefits of economic and monetary union (EMU)”. Despite this conclusion, the report came out in favour of the establishment of a monetary union for several E.U. countries, arguing that traditional Optimal Currency Area theory was likely to underestimate the gains to be expected from monetary integration.

One can actually consider three main reasons which led to the creation of the EMU. The first one is the great financial instability that followed the demise of the Bretton Woods system in 1971. The diverse attempts to re-establish a structured parity system (snake in the tunnel until 1978, the EMS until 1999) proved mostly unsuccessful in containing the exchange rate fluctuations implied by asymmetric dynamics between E.U. member economies. By the beginning of the 1990's, the EMU was thus facing a dilemma between a regime of pure floating exchange rates, or a monetary union which would de facto terminate parity issues. The settlement of the common market within the E.U. pleaded in favour of the second option. This constitutes the second reason. With the gradual settlement of a common market, characterized by free circulation of goods and free circulation of workers, the solution of flexible exchange rates presented little appeal. Integration could have been hindered if the price of goods purchased abroad and wages earned abroad had been susceptible to vary significantly in domestic currency because of high exchange rate volatility. A monetary union, on the other hand, would guarantee price stability and strengthen market integration. As a third and final reason, the settlement of a common currency would also foster political integration.

The Euro was thus settled for 1999, with yet no guarantee that its advantages would overcome its costs. Despite this uncertainty, academic contributions related to the performance of alternative regimes remained limited during the early years of the single currency. Obstfeld

and Rogoff (1999), Carré and Collard (2001) and Cooley and Quadrini (2003) constitute some examples of such works. Past these initial years however, there seemed to be little reason to push investigations further: despite some economic turmoil and excessive public deficits for Euro area members, the Eurozone had been overall prosperous over the 1999-2007 period. The single currency was steadily appreciating, marking improving confidence of investors towards the process of monetary integration. By 2007, the Euro was appearing as a solid achievement, and it made little sense to consider the possible defection of a member State from the Eurozone.

The European financial and sovereign debt crises of the late 2000's have led to question these conclusions. Certain Euro area member states have been heavily affected by recession (Portugal, Ireland, Italy, Greece and Spain, or PIIGS), and compelled to adopt extremely harsh austerity plans to preserve the credibility and sustainability of the single currency. The cost of remaining within the Euro area may appear high for these countries, so that one cannot exclude anymore the eventuality that one of them may finally leave the Eurozone. Despite these events, nevertheless, there has been little renewal in the debate over alternative monetary regimes for the Euro area (Forlatti (2009) being the only exception).

This paper proposes to address this issue by assessing the respective welfare performances of two possible alternative regimes for the Eurozone: floating exchange rates, and a monetary union, in a context of financial frictions. Naturally, the evaluation of welfare for open economies and currency areas has already been the object of considerable attention. There is now a large number of papers dedicated to the estimation of welfare and optimal economic policies, either under flexible exchange rates (Corsetti and Pesenti (2001), Benigno and Benigno (2003), Devereux and Engel (2003), Gali and Monacelli (2005) and Senay (2008)), or in a monetary union (De Grauwe and Piskorski (2001), Benigno (2004), Lombardo (2002), Beetsma and Jensen (2005), Moons et al. (2007), Brissimis and Skotida (2008), Gali and Moncelli (2008), Ferrero (2008), Adjemian and Devulder (2010)). These approaches present two main limits for an application to the Eurozone. First, they characterize the policy that would optimize welfare under a given regime, but don't exert any kind of cross-regime comparison. Secondly, they aim at characterizing a welfare optimizing policy for a currency area as a whole. Yet, this point of view can be very misleading if one wants to consider the possibility for a particular member to defect the monetary union. Ishiyama (1975) was already advocating an individual cost-benefit analysis, arguing that a correct approach "*tries to*

evaluate costs and benefits of participating in a currency union from the point of view of the self-interest of a particular region or country”, with a recommendation for this country to join the currency union only “*if the pros outweigh the cons from the viewpoint of national self-interest and welfare, rather than from that of global welfare, which might run counter to the former*”. Forty years later, this remains relevant. Greece may care very little that the ECB implements a monetary policy that results in an optimal outcome for the union in general if this policy turns out to be very detrimental to its own economy. It may similarly leave the Euro area with little consideration for its European partners if preserving the Euro implies too high a cost for its own economy in terms of austerity.

For this reason, this work adopts a country-specific, rather than union-wide, approach. It uses the two-country open-economy model for the Euro area developed in chapter 2 to conduct a formal welfare analysis on each of the 17 Euro area countries. It also integrates a number of extensions to the benchmark version of the model to account for the sovereign debt crisis, and the set of exceptional policies adopted by the ECB (the CBPP and SMP programmes) to mitigate the European recession. Running a welfare analysis over these extensions then allows for a straightforward evaluation of the efficiency of such exceptional measures.

The rest of the paper is organised as follows. Section 3.1 details the welfare criteria used in the analysis of the Eurozone and runs a preliminary welfare estimation for the Euro area on the benchmark version of the model. Section 3.2 introduces the first extension, which supplements the benchmark model with sovereign debt. Section 3.3 and 3.4 respectively augment the model with the CBPP and SMP programmes and estimate the welfare gains resulting from these exceptional policies. Section 3.5 concludes.

3.1. Welfare analysis

In this section, the model developed in chapter 2 is used to evaluate the respective performances of two alternative monetary regimes: independent currencies on the one hand, and a monetary union, on the other hand. Before the actual analysis is conducted, it is useful to clearly state the scope and limits of such an analysis. The incoming study will assess the theoretical performance of the two possible monetary regimes under which the model is considered. As such, if the performance is, say, better for the regime of independent

currencies, then there is a theoretical gain for the considered country to leave the Eurozone and retrieve its monetary independence. Conversely, a higher performance of the monetary union implies a theoretical gain from joining the union. There is yet a fundamental limit in such a reasoning: it does not take into consideration the potential costs arising from the switch to an alternative monetary regime. As all the countries under consideration are currently members of the Eurozone, this mainly amounts to ignoring the costs of leaving the Eurozone to recover one's monetary policy independence.

In practice, such costs are however likely to be considerable. In its 2011 study “Euro break-up – the consequences”, UBS delivers a comprehensive analysis of the economic costs associated with the departure of a Eurozone member from the union. A first cost would be a default on domestic debt. Departure from the union is likely to imply a very large depreciation of the new independent currency (around 60% according to UBS). As the sovereign debt would remain Euro denominated, public authorities would have to acquire Euros at a prohibitive price, resulting in an almost certain default. The default is all the more likely if the motivation for exiting the Eurozone was a high level of sovereign indebtedness. Following the default, the long-run cost of capital for the government would increase, generating lasting economic costs.

Also, the default on the sovereign debt would lead to a likely default of the corporate sector: as the government implements the currency change on domestic private companies, the latter will in turn prove unable to reimburse Euro denominated loans because of the massive currency depreciation. In addition, domestic companies earning revenues in the new currency will face problems settling any further debt incurred with overseas banks.

The banking sector would also be facing severe adverse effects: in order to function properly, the seceding government would have forcibly redenominated domestic bank deposits into the new currency. As a first consequence, banks would suffer from the same inability to reimburse debts contracted with overseas banks as the public and corporate sectors. Consequently, their ability to contract new loans from abroad financial institutions would be considerably hindered. But this is not the end of the story: the response of depositors, confronted with the uncertainties surrounding the establishment of a new currency, is likely to be a bank run in order to withdraw Euro-denominated assets before their conversion into the new currency. In addition, the inability of the defaulting public authorities to pay pensions

and salaries would play as a further accelerant to bank runs, providing the citizens with an incentive to withdraw whatever Euro-denominated funds may remain in order to secure some income. The combination of lack of new funds and quick withdrawal of existing capital would most probably drive the banking sector to a general bankruptcy, durably depleting the economy from any kind of financial sourcing.

Finally, the political retaliation of the former Euro partners would entail serious consequences. A possible response to the fierce depreciation experienced by the country leaving the Union would be an equivalent set of tariffs or other trade barriers implemented by the remaining members, which would ruin any potential advantage obtained from the J-curve effect.

Overall, the costs may be very high for the seceding country. For instance, UBS estimates that considering the mass sovereign, corporate and banking default, the increased risk premium for the cost of capital would entail, at best, a 700 basis point premium surge. Eventually, the total cost incurred by a country leaving the union would amount (still, according to UBS) to no less than 50% of GDP for a weak economy such as Greece, and to 25% of GDP for a strong economy like Germany.

Yet, the welfare analysis which is going to be carried out in the incoming section cannot take these costs into consideration. The reason is that it can only assess how well an economy is performing when running under one regime or another. It is not designed to analyze the consequences of switching from one regime to its alternative. In a sense, the analysis is thus restricted to a hypothetical scenario where an economy can switch at no cost from one regime to another, a scenario very unlikely to occur in practice.

Despite this drawback, a welfare analysis may still carry useful information for the purpose of policy analysis. First, if a country may benefit from constant and significant gains by leaving the monetary union, it may still consider enjoying a long-term gain by leaving the union, even if the short-term costs is tremendous. On the other hand, if the gain from seceding is small (or of course, if the union turns out to be the best regime), then clearly a member would have no incentive to defect. It is hence of critical importance to provide a quantitative assessment of the potential gain resulting from a secession. A second argument is that a formal welfare analysis may allow to identify the sources of inefficiency resulting from one regime or the

other. In this respect, the analysis may deliver relevant information to improve the design of economic policy and, consequently, the performance of a particular regime.

The next two paragraphs introduce the formal welfare estimation of the two versions of the model. Following the approaches of Rotemberg and Woodford (1999), Woodford (2003) and Benigno and Woodford (2005), much of the literature has adopted a welfare-based criterion, relying on an approximation of the utility loss experienced by the household with respect to a flexible allocation shown to be efficient. The incoming analysis slightly departs from this methodology for two reasons.

The first reason is that as the two declinations of the model represent two different economic behaviours, their flexible allocations (or “natural levels”) will also differ. Hence, reasoning in terms of deviation from efficient levels would be little meaningful. For example, a monetary union may imply larger deviations from the efficient outcome compared to the floating exchange rate regime. But if the efficient outcome is superior in a monetary union, welfare may still be higher under this regime. As will be exposed in the next section, the methodology retained in this paper avoids this pitfall by estimating welfare with respect to steady-state values, which are similar across the two versions of the model (see Appendix B.7).

The second – and main – reason is that deriving an efficient level would be of little use in the pursuit of the objective of the welfare analysis. The aim of this exercise is to compare how well one monetary regime performs with respect to the other. In this respect, it is the comparison of the regimes that matters, rather than the derivation of their respective efficient values.

The welfare analysis developed in the next two paragraphs is twofold. The first section estimates the general long-term performance of a given regime when the economy is hit by shocks of typical amplitude (that is, whose variance is the one determined by the calibration of the model). The second section makes the analysis more precise by evaluating the respective performances of each regime when the economy is hit by a specific shock of large amplitude.

3.1.1. Welfare analysis: a general assessment

The first measure of welfare is fairly general and evaluates the business cycle cost (or benefit) of relinquishing monetary policy independence. Following Monacelli (2000) and Kollmann (2004), the welfare cost of each regime (independent currency or monetary union) is defined as the fraction of steady-state consumption Φ that the household in Home is willing to forego in order to avoid business cycle fluctuations. Because fluctuations are closely related to the conduct of monetary policy, Φ provides an effective measure of the welfare performance of the economy under the two alternative monetary regimes. For each regime, Φ is thus supposed to satisfy:

$$U^h((1-\Phi)C^h, L^h) = E_t \{U^h(C_t^h, L_t^h)\} \quad (3.1)$$

To evaluate Φ , note first that a second-order Taylor approximation of the Home utility function around steady-state yields³⁵:

$$E_t \{U^h(C_t^h, L_t^h)\} = U^h(C^h, L^h) + \frac{1}{2} U_C^h C^h \left[1 + \frac{U_{CC}^h}{U_C^h} C \right] \text{Var}(\tilde{C}_t^h) + \frac{1}{2} U_L^h L^h \left[1 + \frac{U_{LL}^h}{U_L^h} L^h \right] \text{Var}(\tilde{L}_t^h) \quad (3.2)$$

with U_C^h, U_L^h, U_{CC}^h and U_{LL}^h denoting respectively the steady-state values of the first and second-order derivatives of the utility function with respect to C^h and L^h . Then, using (3.2), and given the functional form of the utility function (2.13), one obtains the value of Φ as:

$$\Phi = 1 - \left[1 + \left(\frac{(\eta-1)^2}{2} \right) \text{Var}(\tilde{C}_t^h) + \left(\frac{\chi^h(1+\phi)(\eta-1)}{2} \right) ((C^h)^{\eta-1} (L^h)^{1+\phi}) \text{Var}(\tilde{L}_t^h) \right]^{-\left(\frac{1}{\eta-1}\right)} \quad (3.3)$$

Φ is comprised between 0 and 1, and is an increasing function of $\text{Var}(\tilde{C}_t^h)$ and $\text{Var}(\tilde{L}_t^h)$. This reflects the intuitive idea that the Home household is adverse to the uncertainty linked to business cycles and hence prefers a policy which provides efficient stabilization of the economy. Therefore, the more successful the policy is in reducing the amplitude of business cycles, the better the household will be. Indeed, in the special case where $\text{Var}(\tilde{C}_t^h) = \text{Var}(\tilde{L}_t^h) = 0$ (perfectly efficient stabilization policy by the central bank), Φ will be equal to 0. In the converse case, when $\text{Var}(\tilde{C}_t^h) \rightarrow \infty$ and/or $\text{Var}(\tilde{L}_t^h) \rightarrow \infty$, Φ will be 1, indicating that the household is ready to give up all its consumption in order to suppress the very strong fluctuations induced by business cycles. Table 3.1 provides the values of Φ for

³⁵ See Appendix C.1 for a detailed derivation of the welfare criterion

the 17 Eurozone countries, for a regime of independent currencies (Φ_{ic}) or monetary union (Φ_{mu}). For each country, $Var(\tilde{C}_t^h)$ and $Var(\tilde{L}_t^h)$ are obtained from empirical estimates resulting from simulations of the model. The first column of the table reports the welfare loss for each country under the regime of independent currencies, while the second column indicates the welfare loss under a monetary union. The last column yields the difference between the two measures, and hence indicates the net gain between the two regimes. When the difference is negative, the country prefers the regime of independent currencies as the loss is larger under a monetary union. Shaded cells are used to highlight this situation.

Table 3.1
Welfare loss estimates

	Independent currencies Φ_{ic}	Monetary Union Φ_{mu}	Difference $\Phi_{ic} - \Phi_{mu}$
Austria	0.029	0.041	-0.012
Belgium	0.038	0.030	0.008
Cyprus	0.044	0.052	-0.009
Estonia	0.040	0.063	-0.022
Finland	0.031	0.044	-0.013
France	0.023	0.039	-0.016
Germany	0.028	0.032	-0.004
Greece	0.045	0.056	-0.012
Ireland	0.020	0.031	-0.011
Italy	0.025	0.042	-0.016
Luxembourg	0.021	0.025	-0.004
Malta	0.041	0.049	-0.008
Netherlands	0.026	0.031	-0.005
Portugal	0.022	0.062	-0.040
Slovakia	0.061	0.051	0.010
Slovenia	0.041	0.050	-0.009
Spain	0.022	0.040	-0.018

note: welfare losses expressed in percentage

The results call for three main comments. First, over 17 Eurozone countries, 15 prefer to be in a regime of independent currencies. This is a clear-cut result in favour of the regime of monetary independence. This may appear as a rather surprising result since monetary unions are precisely meant to grant better stabilization policies than independent currency regimes. As under an independent currency regime, national central banks may suffer from governmental interference and be tempted to support short term activity rather than fulfilling their stabilization role, especially with respect to inflation, a monetary union with independent central authorities is typically considered being fitter for stabilization purposes. The

explanation for this result is perhaps to be found in the seminal work by Mundell (1961). One condition emphasized by Mundell for a common currency area to be optimal is that its members must experience shocks with a strong degree of symmetry. This allows the common central bank to effectively coordinate its monetary policy so as to guarantee that it will be optimal for all the members of the Union as they experience similar periods of growth and recessions. As such is not the case within the EMU, even if the single monetary authority performs very well at stabilizing economic activity at the scale of the union, its policy may be very inefficient at stabilizing the economy of one particular member. This will be especially true if the shocks faced by this member display a significant asymmetry with respect to the shocks experienced by the other members, or if this member receives little consideration from the central bank when it designs its monetary policy. This is what the results reflect for the model: the combination of asymmetric shocks³⁶ with a common monetary policy, primarily designed for the union as a whole rather than according to the specific situation of the Home economy, ends up being less favourable to Home than a regime of monetary independence. Surprisingly, even economies considered as most benefitting from the EMU such as Germany or Luxembourg would turn out to be better off if they were to recover their monetary independence. This may well be due to the fact that their own pre-Euro monetary policy was already very similar to the one currently implemented by the ECB, but was designed solely in function of the domestic economy, eventually granting higher stabilization efficiency. Anyway, these rather unusual results should lead to qualify the current literature on the subject which often concludes to the optimality of monetary union because of their ability to reach union-wide optimum, without taking into consideration the welfare of specific members. As individual countries care about themselves, if a policy successfully stabilizes activity at union-scale but provides to several members an individual incentive to defect the union, the whole union may end up breaking down.

The second comment is that the gains obtained by leaving a union would be most of the time rather small, typically around 0.01% of steady-state consumption. Even in the cases where the theoretical gain is the largest, the benefit remains modest. Portugal, for instance, would enjoy a net gain of 0.04%, while Greece, Estonia and Spain would be better off by around 0.02% of steady-state consumption. Given the tremendous costs that these countries are likely to incur if they secede from the union, these results clearly suggest that it is not currently in the

³⁶ Recall that the calibration of the model implies no cross-correlation between Home and Foreign shocks. For a justification of this assumption, the reader is referred to section 2.3.1.

interest of these countries to part with the EMU. The results imply that they should have considered more seriously their entry before the settlement of the Euro, as they had apparently no profit to expect from joining the monetary union. But as current members, the benefit of such a decision would be perfectly marginal when compared to the costs they would incur. Finally, two results remain to be noted about the amplitude of the welfare gain. First, it is generally admitted that the countries which have most to win by retrieving their monetary independence are the ones which are currently considered as being the weakest within the European Monetary Union (the PIIGS), and are the most likely to leave the EMU if their situation keeps degrading. This is coherent with the results produced by the model, which suggest that Spain, Greece and Portugal are among the countries which are the most structurally inefficient within a monetary union framework. Secondly, the gain enjoyed by the two countries which are actually better off within the EMU (Belgium and Slovakia) turns out to be typically smaller than the gain of the countries preferring monetary independence. This suggests that, overall, the current benefits from the EMU may not be considerable.

The last comment about the results is the heterogeneity of the country profiles in both categories. The two countries which would prefer to be in a monetary union include a strong economy (Belgium) and a weaker economy (Slovakia). The same dissimilarities can be observed among countries which would prefer to recover their monetary independence, with Germany, Finland, and Luxembourg on the one hand, and Slovenia, Estonia, Greece, Spain and Portugal on the other hand. This overall supports the argument relative to the ability of the central bank to effectively stabilize activity: in the event where the unique central bank performs significantly better than a domestic central bank to regulate business cycles, even a weak economy like Slovakia can be better off within the monetary union. Conversely, when a domestic central bank effectively manages to stabilize economic activity, even a strong economy such as Germany can be better off regulating economic activity on its own rather than relying on a common central bank which will only partly consider its specific situation when designing its monetary policy.

3.1.2. Welfare analysis: a shock-specific approach

In the previous paragraph, the approach consisted in analysing the performance of an economy under regular conditions: the economy was allowed to be affected by all the shocks of the model, and the shocks were of small, typical amplitude. The incoming section proposes to refine the analysis by adopting a shock-specific approach, that is, by considering the welfare implications for the economy when it is hit by a single, particular shock of large amplitude. The idea here is that the welfare analysis developed in the previous section provided a general picture of the potential performances of each regime for the EMU countries. However, the performance (or lack thereof) of each regime may be more specifically attributable to a particular subcategory of shocks. For instance, from the point of view of Home, the common central bank of a monetary union might be very efficient at regulating productivity shocks, but performing poorly with monetary shocks. If the consequences of monetary shocks outweigh those of productivity shocks, Home may end up being better off with its own independent monetary policy. Analyzing the welfare implications of each particular shock may therefore help to understand which ones contribute to render a regime preferable to its alternative. This information can then be profitably exploited by policymakers to improve the performance of a specific regime. For example, if Home welfare is relatively lower in a monetary union due to monetary shocks, the common central bank might improve its performance towards Home by deciding to implement unexpected monetary shocks of smaller amplitude. In this sense, such an analysis constitutes a direct tool for policy design.

A second motivation for this kind of analysis is that economic activity is not always characterized by shocks of typical amplitude. Although uncommon, large shocks do occur, and they generally affect economic activity in a substantial way so that the effect of other shocks becomes temporarily marginal. The current financial crisis, for example, constitutes an occurrence of a very large negative financial shock (or, in the terminology of the model, a large negative capital quality shock). When such a large shock hits the economy, a policy maker may be typically interested in knowing whether or not he should expect the present regime to do well, and what could be the appropriate policy to apply to this particular situation.

Following Carré (2001), and in the line of Lucas (1987), the welfare gain (or loss) resulting from joining a monetary union is expressed as the permanent percentage increase of consumption required by the representative household after a specific shock to compensate for relinquishing its monetary policy independence in favour of the single monetary authority of the union. The exercise hence consists in determining the transfer rate Γ required by the Home household, following a particular shock, so that:

$$\sum_{i=0}^{\infty} \beta^i U^{mu} \left((1 + \Gamma) C_{t+i}^{mu}, L_{t+i}^{mu} \right) = \sum_{i=0}^{\infty} \beta^i U^{ic} \left(C_{t+i}^{ic}, L_{t+i}^{ic} \right) \quad (3.4)$$

For the sake of clarity, notation has been slightly modified: as only the Home household is considered, the h superscript may drop without ambiguity; on the other hand, a superscript mu is used when the model is considered in monetary union, while ic designates variables under a regime of independent currencies. If Γ is positive, the household prefers the flexible exchange rate regime, as consumption in monetary union must be permanently raised by a percentage Γ to reach a utility level similar to the one prevailing in flexible exchange rate. Conversely, a negative transfer rate Γ implies that the Household is better-off in a monetary union. Relying on first-order approximations, and considering the functional form of the utility function (2.16), it is straightforward to show that for any shock, Γ obtains as³⁷:

$$\Gamma = \left\{ \frac{\sum_{i=0}^{\infty} \beta^i \left[\frac{(C^{ic} (1 + \tilde{C}_{t+i}^{ic}))^{1-\eta}}{1-\eta} - \frac{\chi^h (L^{ic} (1 + \tilde{L}_{t+i}^{ic}))^{1+\phi}}{1+\phi} \right] + \sum_{i=0}^{\infty} \beta^i \frac{\chi^h (L^{mu} (1 + \tilde{L}_{t+i}^{mu}))^{1+\phi}}{1+\phi}}{\sum_{i=0}^{\infty} \beta^i \frac{(C^{mu} (1 + \tilde{C}_{t+i}^{mu}))^{1-\eta}}{1-\eta}} \right\}^{\frac{1}{1-\eta}} - 1 \quad (3.5)$$

The six shocks of the model are considered in turn (Home and Foreign shocks on capital quality, Home and Foreign productivity shocks, and Home and Foreign monetary shocks). In the case of monetary shocks, the shocks in Home and Foreign are compared to the single monetary shock of the monetary union. In the spirit of the crisis experiment conducted in section 2.4, large shocks are defined as 5% negative deviation from steady-state values. Such shocks have an intuitive interpretation: as stated in section 2.4, a 5% negative shock on capital quality roughly replicates the amplitude of the downturn caused by the current financial crisis. A 5% decline in productivity, on the other hand, can be interpreted as an oil shock similar in magnitude to the ones of 1973 and 1979. Finally, a 5% contractionary shock in monetary

³⁷ See Appendix C.2 for details about the derivation of Γ .

policy amounts, e.g., to a stern disinflationary procedure implemented by the central authorities.

Turn now to the results. Table 3.2 summarizes the results for the 17 Eurozone countries, with respect to the 6 shocks of the model. To ease presentation, cells corresponding to cases where the household prefers the independent currency regime (that is, when Γ is positive) are shaded.

Table 3.2
Welfare loss estimates, 5% negative shocks

	Home shocks			Foreign shocks		
	capital quality shock	productivity shock	monetary shock	capital quality shock	productivity shock	monetary shock
	$\epsilon_{k,t}^h = -0.05$	$\epsilon_{p,t}^h = -0.05$	$\epsilon_{m,t}^h = 0.05$	$\epsilon_{k,t}^f = -0.05$	$\epsilon_{p,t}^f = -0.05$	$\epsilon_{m,t}^f = 0.05$
Austria	-0.92	-0.10	0.02	-1.54	0.15	0.92
Belgium	-1.10	-0.08	0.10	-2.48	0.17	1.60
Cyprus	-1.76	-0.15	0.08	-0.80	0.07	0.79
Estonia	-0.41	-0.16	0.14	-2.45	0.23	1.59
Finland	-1.13	-0.09	0.06	-1.11	0.13	0.61
France	-1.26	0.03	-0.02	-0.94	0.09	0.46
Germany	-1.43	-0.10	0.04	-0.97	0.14	0.50
Greece	-2.34	-0.22	0.04	-0.86	0.16	0.68
Ireland	-0.32	-0.06	-0.05	-1.47	0.12	0.93
Italy	-2.01	-0.18	0.04	-0.73	0.13	0.32
Luxembourg	-0.57	-0.03	-0.08	-1.93	0.11	1.43
Malta	-0.81	-0.14	0.13	-2.45	0.21	1.65
Netherlands	-0.88	-0.08	0.06	-1.90	0.16	1.24
Portugal	-0.55	-0.09	0.02	-1.42	0.17	0.74
Slovakia	-1.12	-0.12	0.16	-2.59	0.22	1.99
Slovenia	-1.27	-0.15	0.10	-2.03	0.20	1.41
Spain	-1.49	-0.14	0.04	-0.81	0.15	0.44

note: welfare losses expressed in percentage

A first striking feature is the extent to which every shock unambiguously implies a preference for one particular regime. While Home capital quality shocks and Home and Foreign productivity shocks unanimously entail a preference for the monetary union regime, Foreign productivity shocks, as well as Home and Foreign monetary shocks nearly unanimously induce a preference for the independent currency regime. This confirms that certain shocks are more specifically responsible for the good or poor performance of each monetary regime.

In addition, a further look at these results supports the hypothesis that the relative inefficiency of the monetary union principally originates from a greater inability to manage Foreign shocks. Save for capital quality shocks, Foreign shocks all result in greater welfare under

independent currencies. By comparison, two about three shocks in Home imply higher welfare in a monetary union, the exception being the monetary shock. This suggests that the inherent restriction of the monetary union to conduct a unique monetary policy for the whole currency area may constitute the main impediment to its success. As this policy is less capable than a domestic central bank to take into account the consequences of Foreign shocks on domestic activity, the Home household prefers to preserve a system of independent currency, notwithstanding the gain resulting from the union, namely the suppression of the fluctuation induced by the variations of the exchange rate.

Useful information can also be obtained by considering shock types. Monetary shocks, whether they originate from Home or Foreign, always plead for the independent currency regime. However, and interestingly enough, the converse holds for capital quality shocks which unanimously imply the household to be better off in a monetary union. As for the technological shocks, results are mixed, Home shocks being in favour of the monetary union, while Foreign shocks indicate a preference for independent currencies. A first conclusion is that one main weakness of the monetary union seems to be the very conduct of its monetary policy, or, to be more precise, of its unexpected component. When confronted to deflationary shocks, whether initiated by Home or Foreign central authorities, the household in Home nearly systematically chooses the independent currency regime. The reason, once again, is probably to be found in the monetary policy design of the monetary union. Even if the common central bank implements a Taylor rule that considers both Home and Foreign, the weight attributed to Home can be low. Hence the disinflation policy, yet successful for union-wide stabilization of the price level, may be at odd with the specific situation of Home, and result in a less efficient conduct of monetary policy than would be realized under a regime of independent currency with a domestic central bank.

It may then seem surprising that the conclusion is reversed in the case of capital quality shocks, where the monetary union performs systematically better. The explanation lies in the behaviour of the interbank market: in an independent currency regime, the downturn of the economy hit by the financial crisis hardly affects its abroad partner, so that abroad central authorities only marginally revise their interest rate. On the other hand, in a monetary union, the negative impact of the crisis urges the common central bank to implement a drop in the single interest rate. This will be all the more true if the crisis occurs in Foreign, as the common central authorities mainly refer to Foreign when they devise their policy, or if Home

is large. As this interest cut applies union-wide, the abroad economy will benefit from a lower riskless rate than would be the case under an independent currency regime. This lower interest rate implies a lower cost of deposits, and, as banks have cheaper access to funds, the opportunity cost of lending interbank funds decreases in turn. Hence, because abroad banks are willing to lend money at a reduced rate, the banks in the economy hit by the crisis will eventually benefit from cheaper interbank loans, resulting in a significant rise in the volume of interbank funds traded. These additional funds help to mitigate the crisis, as banks may continue to finance new projects to support activity. Interestingly enough, this result strongly pleads in favour of monetary unions in times of financial crisis. It suggests that weak countries would not necessarily gain by exiting a monetary area, even if a strong financial crisis prevails. The condition for this however is that the common central bank effectively accommodates for the downturn by significantly cutting the riskless rate.

Overall, these results may seem rather mixed: if monetary shocks advocate for the independent currency regime, capital quality shocks support the monetary union while productivity shocks provide contradictory results depending on the origin of the shock. A central issue is then to determine how such mitigated results can be compatible with the conclusions of the previous section, namely, a consistent preference for the independent currency regime. The answer is that the main benefit of the monetary union is its superior management of financial shocks. Yet, outside periods of financial crisis where a large negative capital quality shock occurs, these shocks are of very small amplitude. The estimated calibration reported in section 2.3.1 indicates especially that their amplitude is considerably smaller than the amplitude of monetary shocks and productivity shocks. Therefore the dynamics of the economies under consideration, both Home and Foreign, is primarily driven by productivity and monetary shocks, for which a monetary union is relatively inefficient in comparison with an independent currency regime. This however suggests that if the amplitude of financial crisis keeps increasing over the incoming years, the European Monetary Union would gain in legitimacy as it would act more effectively than a system of independent currencies.

Two additional remarks can be made to conclude this analysis. First, the welfare implications of shocks, whether in terms of gains or losses, are typically larger when dealing with Foreign shocks than when considering Home shocks. This is true in general and also when one compares a specific shock in Home with its Foreign counterpart. This once again supports the

idea that the main source of inefficiency (or efficiency) of a monetary union lies in its ability to deal with Foreign shocks, Home shocks being rather well handled by the common central bank. Secondly, the larger gains and losses are once again seen to be principally incurred by the countries currently considered as being the weakest within the Eurozone. These include Cyprus, Estonia, Greece, Italy, Malta, Slovakia, Slovenia and Spain. This, again, suggests that these countries are structurally less apt than their partners to be profitably members of a single currency area.

3.2. Extension 1: integrating sovereign debt to the benchmark model

Developments have so far focused on financial rigidities within the Euro area. While the functioning of financial markets has drawn much attention since 2007, it had become clear by 2010 that the crisis was not restricted anymore to its financial component, and was more and more turning into a sovereign debt crisis. This section therefore aims at integrating sovereign debt into the framework laid by the benchmark model.

It could be tempting to model the current phenomenon of increasing indebtedness within the Eurozone. This would be however difficult to implement as an ever-growing debt would imply a non-stationary model which is not compatible with a conventional DSGE design³⁸. In addition, such a representation would make little economic sense: even though many Eurozone countries currently accumulate indebtedness in a seemingly continuous way, such a situation cannot last forever. Greece is a clear illustration that any country will have, at some point, to stop accumulating deficits and start to cleanse its public finances. Nevertheless, it is also clear that Greece will never be able to repay its debt in full. In the end, it is likely that Greece, at least in the medium run, will not be able to achieve more than stabilizing its amount of public indebtedness at its current, critical level. This is the position adopted in this extension. Each Eurozone country is modeled with an initial level of indebtedness, or more precisely, of debt/GDP ratio. In the current context of sovereign debt crisis, one assumes that this initial level constitutes the threshold beyond which the sovereign debt would become unsustainable, so that this level can be exceeded only in a transitory way. Following a negative cycle, the country must hence implement austerity measures to ensure that debt goes

³⁸ Non-stationary model cannot produce well defined theoretical moments for the variables under consideration. Also, approximations around a steady-state become meaningless as any shock would imply a permanent departure from this point, which creates a fundamental flaw in the analysis.

back to its initial, critical level. The dynamics of sovereign debt is thus modeled as a compulsory return to budget balance, characterized by a sequential reduction of deficits implemented through higher taxation and cuts in government spending. To account for the fact that different initial levels of indebtedness imply different degrees of country risk, it is assumed that cuts in government spending during recession are an increasing function of the initial debt/GDP ratio. This can be assimilated to the fact that in order to accept to lend further funds to a highly indebted state, private (and possibly public) credit institutions will demand stronger guaranties of return to a balanced public budget in order to reduce the risk of default. This is exactly the kind of situation which currently characterizes Greece, with private banks, the IMF and the ECB conditioning financial support to significant reforms in the conduct of public expenditure.

The next paragraphs introduce the extension. As the core of the model remains the same, the presentation is restricted to the parts of the model for which modifications apply.

3.2.1. Households

The main modification with respect to the benchmark version of the model is that government expenditure now enters the utility function of the Home household. For the sake of simplicity, and because emphasis is put on Home only, it is assumed that Foreign public authorities continue to run a balanced budget with a fixed amount of public spending which does not enter the household utility function.

When public spending is integrated into Home utility, the program of the household becomes:

$$\begin{aligned} \underset{C_t^h, L_t^h, D_t^h, F_t^h}{\text{Max}} \quad & E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}^h, G_{t+i}^h, L_{t+i}^h) \quad 0 < \beta < 1 \\ \text{with } U(C_t^h, G_t^h, L_t^h) = & \frac{(C_t^h)^{1-\eta}}{1-\eta} + \frac{(G_t^h)^{1-\Delta}}{1-\Delta} - \frac{\chi^h}{1+\phi} (L_t^h)^{1+\phi} \end{aligned} \quad (3.6)$$

subject to the budget constraint:

$$P_t^h C_t^h + D_t^h + \varepsilon_t F_t^h + T_t^h + P_t^h A C_t^h = W_t^h L_t^h + R_t^h D_{t-1}^h + R_t^f \varepsilon_t F_{t-1}^h, \quad (3.7)$$

To keep derivations simple, it is assumed that public spending enters utility in a separable way with respect to consumption. This assumption can be justified by the fact that the goods and services provided by public authorities (education, police, retirement pensions) fundamentally differ in nature from the ones the household may privately purchase. As there is no substitutability between the goods, it makes sense to integrate them separately in the utility function. As utility is separable in consumption and government expenditures, the first-order conditions are similar to the ones of the benchmark model. Therefore, the household problem is not derived further.

3.2.2. Governments

This section formally introduces sovereign debt into the model. The fundamental change with respect to the benchmark version is that the model now allows public authorities to run imbalanced budgets. Precisely, while real taxation and real public expenditure were always balanced in the benchmark version, there may now exist a discrepancy between the two. To ensure payment of public spending, the government may thus have to borrow money from financial institutions, hence accumulating sovereign debt. Therefore, denoting nominal aggregate taxation by T_t^h , government expenditure by G_t^h , nominal aggregate sovereign debt by B_t^d (the superscript h drops without ambiguity as the extension is proper to Home), and the gross nominal interest rate on sovereign debt loans by R_t^d , the budget constraint of the government is given by:

$$T_t^h + B_t^d = P_{H,t} G_t^h + R_t^d B_{t-1}^d \quad (3.8)$$

On the other hand, taxation and public expenditure are given by:

$$T_t^h = P_{H,t} \tau^h Y_t^h + \gamma_1 P_t^h (B_{r,t-1}^d - B_r^d) \quad (3.9)$$

and

$$G_t^h = \vartheta^h Y_t^h + \gamma_2 \vartheta^h (Y_t^h - Y^h) \quad (3.10)$$

where τ^h denotes the taxation rate, and $B_{r,t}^d = \frac{B_t^d}{P_t^h}$ denotes real sovereign debt. In the benchmark version, taxation was always equal to the fixed amount of government expenditure, and would hence guarantee a balanced budget. By contrast, as shown by (3.9),

the model now allows taxation to vary as it is function of current output. Hence, in a situation of crisis where output decreases below its steady-state value, taxation will not be sufficient to finance government expenditure and deficits will start cumulating into a sovereign debt. As an attempt to reverse sovereign accumulation, public authorities will then act in two ways. First, taxation will be increased proportionally to the amount of real debt incurred up to the beginning of the current period (equation (3.9)). Secondly, the government will concurrently reduce public expenditure by a multiple γ_2 of the gap between output and its steady-state counterpart. Hence, when a crisis drives Y_t^h below Y^h , the government will anticipate a drop in the amount of tax collected and cut public expenditure accordingly (equation (3.10)).

γ_2 is the most important parameter of the extension. It is through this parameter that the concept of country risk is introduced in the model. This is done in a simple way by calibrating the value of γ_2 as an increasing function of the country initial debt-to-GDP ratio. Because a high level of sovereign debt implies a higher risk of default, credit institutions may permit additional loans only in exchange of better guarantees. Therefore, in exchange of new loans, financial institutions will constrain the government to decrease the amount of public spending realized in order to limit the deficit and prevent the formation of unsustainable debt³⁹. By how much public authorities will have to reduce expenditure will then depend on γ_2 : a highly indebted country such a Greece will imply a high value for γ_2 , so that borrowing on financial markets will be allowed only at the cost of a substantial fall in the level of public spending. On the other hand, the value of γ_2 for a country only lightly indebted such as Luxembourg will be low, so that the country might accumulate debt and still enjoy a high level of public services. Obviously, as government expenditure enters into the utility function of the household, the higher the value of γ_2 , the larger the welfare cost implied in the event of a recession. In this respect, the model roughly replicates the stylized facts currently observed for the PIIGS. Because bailouts plans have been granted to these countries conditional on

³⁹ Here, “financial institutions” must be considered in a wide sense. For example, in the case of Eurozone countries such as Greece or Portugal, financial institutions include private banking companies, but also public credit institutions such as the IMF, which is the only one explicitly allowed to condition loans to political and economical reforms. One may note however that even private institutions may implicitly state such conditions by proposing interest rates corresponding to the level of risk that they privately estimate. This amounts to proposing affordable loans only if the indebted country accedes to the recommendations of the main financial groups or analysts (S&P for instance). In this respect, the model treats both entities as having similar prerogatives over Eurozone States.

severe austerity plans, public authorities have been led to implement significant cuts in the provision of public services, which considerably impacted the welfare of the populations.

To finish the section, rewrite (3.8) and (3.9) in real terms as:

$$T_{r,t}^h + B_{r,t}^d = \frac{P_{H,t}}{P_t^h} G_t^h + \frac{R_t^d}{\pi_t^h} B_{r,t-1}^d \quad (3.11)$$

and

$$T_{r,t}^h = \frac{P_{H,t}}{P_t^h} \tau^h Y_t^h + \gamma_1 (B_{r,t-1}^d - B_r^d) \quad (3.12)$$

3.2.3. Banks

The mechanisms of the banking market remain essentially the same. Simply, one must now add the amounts borrowed by public authorities. One assumes for the sake of convenience that funds borrowing is realized exclusively towards domestic banks. For an individual bank, the flow-of-fund constraint is given by:

$$Q_t^h s_t^h + b_t^d = n_t^h + \varepsilon_t b_t^h + d_t^h \quad (3.13)$$

Net worth at t is then given by:

$$n_t^h = [Z_t^h + (1 - \delta) Q_t^h] \psi_t^h s_{t-1}^h - \varepsilon_t R_t^i b_{t-1}^h - R_t^h d_{t-1}^h + R_t^d b_{t-1}^d \quad (3.14)$$

Rewrite these conditions in real terms as:

$$Q_{r,t}^h s_t^h + b_{r,t}^d = n_{r,t}^h + b_{r,t}^h + d_{r,t}^h \quad (3.15)$$

and

$$n_{r,t}^h = [Z_{r,t}^h + (1 - \delta) Q_{r,t}^h] \psi_t^h s_{t-1}^h - \frac{\varepsilon_t}{\varepsilon_{t-1}} \frac{R_t^i}{\pi_t^h} b_{r,t-1}^h - \frac{R_t^h}{\pi_t^h} d_{r,t-1}^h + \frac{R_t^d}{\pi_t^h} b_{r,t-1}^d \quad (3.16)$$

At each period, after a bank obtain funds, the banker may transfer a fraction θ^h of divertable assets to the members of his family within the household. Divertable assets consist of total real gross assets $Q_{r,t}^h s_t^h + b_{r,t}^d$ net a fraction ω of real interbank borrowing with Foreign banks $b_{r,t}^h$. That is: divertable assets = $\theta^h (Q_{r,t}^h s_t^h + b_{r,t}^d - \omega b_{r,t}^h)$. From this, the following incentive constraint must hold:

$$V_t^h \geq \theta^h (Q_{r,t}^h s_t^h + b_{r,t}^d - \omega b_{r,t}^h) \quad (3.17)$$

In general, the value of the bank at the end of period t satisfies the Bellman equation:

$$V_t^h(s_t^h, b_{r,t}^d, b_{r,t}^h, d_{r,t}^h) = E_t \Lambda_{t,t+1}^h \left[(1-\sigma)n_{r,t+1}^h + \sigma \underset{s_{t+1}^h, b_{r,t+1}^d, b_{r,t+1}^h}{\text{Max}} V_{t+1}^h(s_{t+1}^h, b_{r,t+1}^d, b_{r,t+1}^h, d_{r,t+1}^h) \right] \quad (3.18)$$

To solve for the bank's decision problem, one guesses first that the value function is a linear function of the balance sheet arguments, and verifies this guess later:

$$V_t^h(s_t^h, b_{r,t}^d, b_{r,t}^h, d_{r,t}^h) = v_{s,t}^h s_t^h + v_{d,t}^h b_{r,t}^d - v_{b,t}^h b_{r,t}^h - v_t^h d_{r,t}^h \quad (3.19)$$

where $v_{d,t}^h$ is the shadow value of one unit of sovereign debt for the bank. The maximization of this function subject to the usual incentive constraint yields the Lagrangian:

$$\mathcal{L} = \left[v_{s,t}^h s_t^h + v_{d,t}^h b_{r,t}^d - v_{b,t}^h b_{r,t}^h - v_t^h d_{r,t}^h \right] (1 + \lambda_t^h) - \lambda_t^h \theta^h \left[Q_{r,t}^h s_t^h + b_{r,t}^d - \omega b_{r,t}^h \right] \quad (3.20)$$

The necessary first-order conditions with respect to $b_{r,t}^d$, s_t^h , $b_{r,t}^h$ and λ_t^h are given by (2.61) and (2.62) from the benchmark model, to which one must add:

$$\left[v_{d,t}^h - v_t^h \right] (1 + \lambda_t^h) = \lambda_t^h \theta^h \quad (3.21)$$

and

$$v_{s,t}^h s_t^h + v_{d,t}^h b_{r,t}^d - v_{b,t}^h b_{r,t}^h - v_t^h d_{r,t}^h = \theta^h \left[Q_{r,t}^h s_t^h + b_{r,t}^d - \omega b_{r,t}^h \right] \quad (3.22)$$

Combine (3.21) and (2.61) to obtain:

$$\frac{v_{s,t}^h}{Q_{r,t}^h} = v_{d,t}^h \quad (3.23)$$

Optimality condition (3.23) states that at the optimum, the bank must acquire quantities of the two types of securities up to the point where their marginal values in terms of goods are equated, a classical microeconomics result. Finally, equation (3.22) is the incentive constraint.

It rewrites as:

$$Q_{r,t}^h s_t^h = \phi_t^h n_{r,t}^h \quad (3.24)$$

with:

$$\phi_t^h = \frac{v_t^h}{\theta^h (1 - \omega x_{1,t}^h + x_{2,t}^h) + v_t^h (1 - x_{1,t}^h + x_{2,t}^h) + v_{b,t}^h x_{1,t}^h - \frac{v_{s,t}^h}{Q_{r,t}^h} - v_{d,t}^h x_{2,t}^h} \quad (3.25)$$

$$\text{and } x_{1,t}^h = \frac{b_{r,t}^h}{Q_{r,t}^h s_t^h}, \quad x_{2,t}^h = \frac{b_{r,t}^d}{Q_{r,t}^h s_t^h}.$$

Combining the conjectured solution with the Bellman equation, net worth, the flow-of-fund constraint and the leverage ratio, one obtains:

$$v_{s,t}^h s_t^h + v_{d,t}^h b_{r,t}^d - v_{b,t}^h b_{r,t}^h - v_t^h d_{r,t}^h = E_t \left(\Lambda_{r,t+1}^h \Omega_{t+1}^h n_{r,t+1}^h \right)$$

with $\Omega_t^h = (1 - \sigma) + \sigma \left[v_t^h + \phi_t^h \left\{ \left[\frac{v_{s,t}^h}{Q_{r,t}^h} - v_t^h \right] + [v_{d,t}^h - v_t^h] x_{2,t}^h + [v_t^h - v_{b,t}^h] x_{1,t}^h \right\} \right]$ (3.26)

Finally, from an undetermined coefficient method, one obtains the usual results for v_t^h , $v_{b,t}^h$ and $v_{s,t}^h$, plus:

$$v_{d,t}^h = v_t^h \frac{R_{t+1}^d}{R_{t+1}^h} \quad (3.27)$$

3.2.4. Market clearing

The aggregate incentive constraint now rewrites as:

$$v_{s,t}^h S_t^h + v_{d,t}^h B_{r,t}^d - v_{b,t}^h B_{r,t}^h - v_t^h D_{r,t}^h = \theta^h [Q_{r,t}^h S_t^h + B_{r,t}^d - \omega B_{r,t}^h] \quad (3.28)$$

Further, the aggregate balance sheet of the banking sector writes:

$$D_{r,t}^h = Q_{r,t}^h S_t^h + B_{r,t}^d - N_{r,t}^h - B_{r,t}^h \quad (3.29)$$

Finally, aggregate net worth is given by:

$$N_{r,t}^h = (\sigma + \zeta^h) [Z_{r,t}^h + (1 - \delta) Q_{r,t}^h] \psi_t^h S_{t-1}^h - \frac{\sigma}{\pi_t^h} \left\{ \frac{\varepsilon_t}{\varepsilon_{t-1}} R_t^i B_{r,t-1}^h + R_t^h D_{r,t-1}^h - R_t^d B_{r,t-1}^d \right\} \quad (3.30)$$

This closes the model. The model in monetary union is essentially the same, save for the usual changes resulting from the suppression of the exchange rates and the modification of the Taylor rule. Appendices C3-C6 provide a summary of the original and linearized equations for both versions of the model.

3.2.5. Calibration and steady-state

Three parameters require to be calibrated for the extension: Δ , γ_1 and γ_2 . All are specific to Home. Δ is given a value of 2, similarly to η . As for γ_1 , it is merely a stabilizing device used to guaranty that sovereign debt will not be explosive and that the model will remain stationary. Its value must hence be sufficiently high to achieve this purpose. It is set at

$\gamma_1 = 0.25$. Finally, the value of γ_2 is set at $\gamma_2 = \frac{2}{3} \times \frac{B_r^d}{Y^h}$, the steady-state value of sovereign debt over GDP. The ratio $\frac{B_r^d}{Y^h}$ is scaled by $\frac{2}{3}$ in order to obtain a stationary dynamics even for highly indebted countries such as Greece. For each Euro area country, the value of $\frac{B_r^d}{Y^h}$ is calibrated from Eurostats data, by taking an average over years 2010-2012. These values are reported in table 3.3:

Table 3.3
Calibration of sovereign debt parameters

	$\frac{Debt}{GDP}$	γ_2
Austria	0.73	0.48
Belgium	0.98	0.65
Cyprus	0.73	0.48
Estonia	0.08	0.05
Finland	0.50	0.33
France	0.86	0.57
Germany	0.82	0.54
Greece	1.59	1.06
Ireland	1.05	0.70
Italy	1.22	0.82
Luxembourg	0.19	0.13
Malta	0.70	0.47
Netherlands	0.67	0.44
Portugal	1.09	0.72
Slovakia	0.45	0.30
Slovenia	0.47	0.31
Spain	0.72	0.48

The steady-state is essentially similar to the one of the benchmark model. See Appendix C.7 for more details.

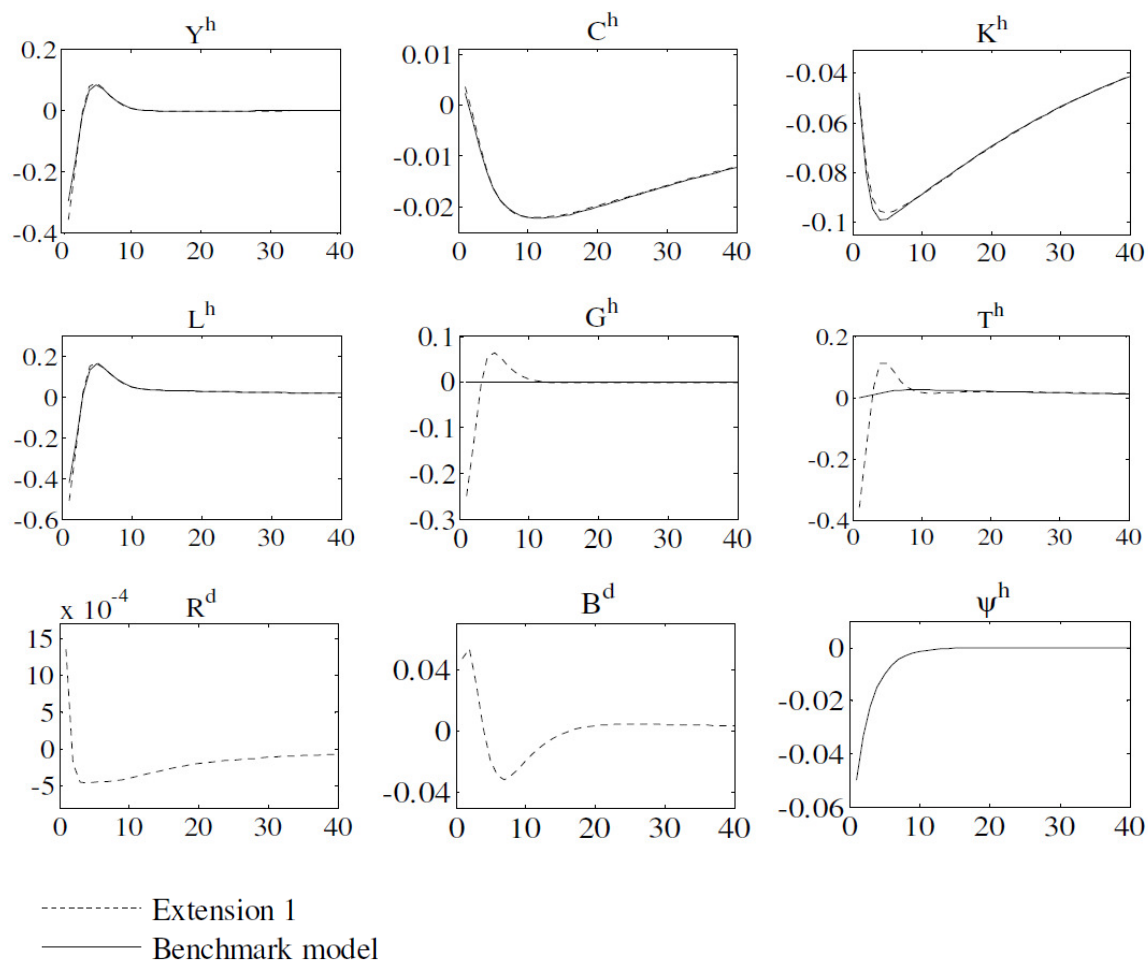
3.2.6. Crisis experiment

Figure 3.1 displays the responses of some key variables within the framework of the first extension, after a 5% decline in capital quality. The country of choice is Ireland, as it has a high level of indebtedness, in the case of a monetary union regime. The responses are comparable to the ones obtained in the benchmark case, except that the recession is more severe in the extension, as can be observed for output and employment. Following the shock

and the initial recession, taxation significantly declines, which results in a concurrent drop in public expenditure. As the latter is smaller than the former, sovereign debt temporarily increases, leading to a temporary increase in the sovereign debt interest rate. As time passes, the recession turns into an expansion, which, combined with increasing indebtedness, leads to an increase in taxation. The economy reverts to its long run values after approximately 20 quarters.

Figure 3.1

Crisis experiment: extension 1



3.2.7. Welfare analysis

A formal welfare analysis is now conducted on the first extension, relying on the general welfare criteria derived in section 3.1.1. Consider the utility function (3.6):

$$U(C_t^h, G_t^h, L_t^h) = \frac{(C_t^h)^{1-\eta}}{1-\eta} + \frac{(G_t^h)^{1-\Delta}}{1-\Delta} - \frac{\chi^h}{1+\phi} (L_t^h)^{1+\phi}$$

The welfare cost of each regime is defined similarly to section 3.1.1 as the fraction of steady-state consumption Φ that the household in Home is willing to forego in order to avoid business cycle fluctuations. Φ is thus supposed to satisfy:

$$U^h((1-\Phi)C^h, G^h, L^h) = E_t \{U^h(C_t^h, G_t^h, L_t^h)\} \quad (3.31)$$

A second-order Taylor approximation of (3.6) around steady-state values yields:

$$E\{U(C_t^h, G_t^h, L_t^h)\} = U(C^h, G^h, L^h) + \frac{(1-\eta)}{2} (C^h)^{1-\eta} \text{Var}(\tilde{C}_t) + \frac{(1-\Delta)}{2} (G^h)^{1-\Delta} \text{Var}(\tilde{G}_t) - \frac{\chi^h(1+\phi)}{2} (L^h)^{1+\phi} \text{Var}(\tilde{L}_t) \quad (3.32)$$

Rearranging, Φ obtains as:

$$\Phi = 1 - \left[1 + \frac{(\eta-1)^2}{2} \text{Var}(\tilde{C}_t) + \frac{(\Delta-1)(\eta-1)}{2} \frac{(C^h)^{\eta-1}}{(G^h)^{\Delta-1}} \text{Var}(\tilde{G}_t) + \frac{\chi^h(1+\phi)(\eta-1)}{2} (C^h)^{\eta-1} (L^h)^{1+\phi} \text{Var}(\tilde{L}_t) \right]^{\frac{-1}{\eta-1}} \quad (3.33)$$

This criterion is then applied to the 17 Euro area countries, for both regimes. The results are displayed in Table 3.4. They are overall consistent with those obtained with the benchmark model. A majority of countries (14 about 17) would enjoy higher welfare levels under a regime of flexible exchange rates. The welfare losses are also of similar amplitude as the ones observed for the benchmark model. This suggests that the results obtained with the benchmark model are robust to specification. Adding the sovereign debt to the model does not fundamentally alter the conclusion, which supports the hypothesis that the monetary union is structurally less efficient than the flexible exchange rate regime.

Table 3.4
Welfare loss estimates, first extension

	Independent currencies Φ_{ic}	Monetary Union Φ_{mu}	Difference $\Phi_{ic} - \Phi_{mu}$
Austria	0.039	0.052	-0.013
Belgium	0.071	0.052	0.020
Cyprus	0.067	0.068	-0.0001
Estonia	0.028	0.064	-0.036
Finland	0.035	0.048	-0.013
France	0.035	0.048	-0.013
Germany	0.038	0.038	-0.001
Greece	0.110	0.079	0.031
Ireland	0.037	0.052	-0.015
Italy	0.052	0.060	-0.008
Luxembourg	0.022	0.026	-0.004
Malta	0.064	0.075	-0.010
Netherlands	0.035	0.041	-0.006
Portugal	0.046	0.099	-0.052
Slovakia	0.073	0.066	0.008
Slovenia	0.035	0.060	-0.025
Spain	0.030	0.049	-0.019

note: welfare losses expressed in percentage

There is yet one significant difference in the case of Greece, which now enjoys higher welfare under a monetary union. This result appears much unexpected, as Greece is among the Euro area countries which have been most suffering from the sovereign debt crisis, and which would be the most likely candidate to exit the Euro. What the results suggest is that the appalling economic situation of Greece is not due mainly to the single currency, but actually stems from its fiscal irresponsibility. Following, switching back to a regime of floating exchange rate would not necessarily make Greece any better off, as it would not change its structural level of indebtedness, nor the harsh reforms required to obtain further funds on financial markets. On the other hand, the monetary union can prove superior to the regime of independent currencies through the higher capability of its monetary policy to stabilize cycles (remember that Greece is one of the country for which the national central bank would implement an output gap targeting policy rather than an inflation targeting policy).

3.3. Extension 2: integrating the Covered Bond Purchase Programme to the model

The Covered Bond Purchase Programme was first publicly announced by Jean-Claude Trichet in his statement of May 7, 2009. The motivation for such a programme, according to the ECB authorities, was the necessity “to revive the (covered bonds) market, which has been very heavily affected”, the ECB being confident that the CBPP could help “with this revival, including the spreads, the depth and the liquidity of the market”. In the statement, the ECB also made it clear that the objective was solely the improvement of the functioning of the market, and not to proceed to any kind of monetary expansion.

The implementation of the CBPP had naturally much to do with the context of the 2008 liquidity crisis. As the crisis was worsening, the moral hazard and lack of confidence between banks was leading to considerable slowdowns in interbank market activity. Spreads in the Euro area covered bond markets kept widening, and liquidity went shrinking. It then appeared indispensable to support this segment of the market, as it was a major source of funding for Euro area banks, representing as much as 20 percent of their financing before the crisis. A more detailed account of the motivations for the settlement of the CBPP can be found in Beirne et Al. (2011).

The programme was operated over two periods. The ECB first intervened between July 2009 and June 2010 (CBPP1), realizing an outright purchase of 60 billion Euros worth of covered bonds. The second programme (CBPP2) was announced in October 2011, and implemented between November 2011 and October 2012. The initially targeted amount of purchases was 40 billion Euros, but the ECB rapidly slowed down the pace of purchases in response both to investors’ increasing demand for Euro area covered bonds and to the decline in their supply. By the time of programme termination, the eventual amount of purchase was not exceeding 16 billion Euros. Overall, the two programmes seem to have been successful. Beirne et Al. (2011) suggest that CBPP1 yielded positive results, as it led to a substitution in the issuance of uncovered bonds by covered ones, and resulted in a sharp decline in spreads. The reactivation of the programme in October 2011 may indicate that the end of CBPP1 was somewhat premature. Nevertheless, the fact that the ECB realized a purchase of covered bond much below the initially intended volume attests that its objective was met prior to the anticipated term.

In the following parts of this section, the CBPP is integrated to the model so as to illustrate how it may facilitate interbank lending during a crisis. Because the CBPP constitutes a set of measures designed by the ECB specifically for the Euro area, the framework is restricted to the monetary union version of the model. A second argument to advocate the fact that the CBPP could not be implemented under a flexible exchange rate regime is that while the ECB holds large reserves allowing to finance such a programme, a single national central bank would typically not possess sufficient funds to do so. As most of the model is left unchanged, the text only presents the modifications applying to the banking sector. Also, because the CBPP is not directly related to the sovereign debt crisis, focus is made on the benchmark version of the model, assuming that governments run balanced budgets at each period.

3.3.1. The model

In the context of the CBPP, the central bank contributes to lower the disruptions on the interbank market by purchasing private bonds issued by financial institutions, therefore channelling funds to the private banking sector. These funds are in turn lent to private firms to acquire new capital. It is assumed that the Central Bank acts as follows: because financial frictions limit the volume of interbank funds traded on the interbank market, an abnormally large credit spread is created between the interbank and the risk-free rate. Therefore, at each period, the central bank will settle the quantity of covered bonds it purchases relatively to the current-period credit spread, in order to expand availability of interbank funds proportionally to the disruption observed on the interbank market. To keep the model simple, and because the primary aim of the exercise lies in the analysis of the Home situation, it is assumed that CBPP bonds are purchased from Home banks only⁴⁰. Denote aggregate covered bonds by B_t^c . Then, the nominal amount of covered bonds purchased at time t is set by the central bank according to the following rule:

$$B_t^c = \varphi B_r^h P_t^h (R_{t+1}^i - R_{t+1}) \quad (3.34)$$

with φ some constant.

⁴⁰ This is without loss of generality. One could assume instead that part of the CBPP funds goes to Home, and part to Foreign. Note that the CBPP still affects Foreign as it increases the overall supply of interbank funds, hence pressing the interbank rate downward.

In the limit case $\varphi = 0$, the extension reduces to the benchmark model. The rule shows that the central authorities determine their purchase of covered bonds both proportionally to the parameter φ and to the steady-state value of real interbank loans B_r^h , so that a country which typically relies on high volumes of interbank funds will receive more support from the central bank. Notice also that in steady-state $R^i = R$, so that $B^c = 0$. This emphasizes the fact that the CBPP constitutes a set of exceptional measures, intended to be employed only outside steady-state during times of crisis, and by nature temporary. Defining real covered bonds as:

$$B_{r,t}^c = \frac{B_t^c}{P_t^h}, \quad (3.34) \text{ rewrites:}$$

$$B_{r,t}^c = \varphi B_r^h (R_{t+1}^i - R_{t+1}) \quad (3.35)$$

It is assumed that the ECB can always enforce contractual obligations with its borrowers, so that banks cannot divert funds obtain from the ECB through CBPP purchase. Apart this, covered bonds are considered as perfect substitutes to interbank loans, and are repaid one period after purchase at a similar interbank market rate R_{t+1}^i . Following, the balance sheet constraint for an individual bank is given by:

$$Q_t^h s_t^h = n_t^h + b_t^h + d_t^h + b_t^c$$

Net worth then obtains as:

$$n_t^h = [Z_t^h + (1 - \delta)Q_t^h] \psi_t^h s_{t-1}^h - R_t^i (b_{t-1}^h + b_{t-1}^c) - R_t d_{t-1}^h$$

The latter two equations rewrite in real terms as:

$$Q_{r,t}^h s_t^h = n_{r,t}^h + b_{r,t}^h + d_{r,t}^h + b_{r,t}^c \quad (3.36)$$

and

$$n_{r,t}^h = [Z_{r,t}^h + (1 - \delta)Q_{r,t}^h] \psi_t^h s_{t-1}^h - \frac{R_t^i}{\pi_t^h} (b_{r,t-1}^h + b_{r,t-1}^c) - \frac{R_t^h}{\pi_t^h} d_{r,t-1}^h \quad (3.37)$$

The bank's problem at the end of period t is to maximize the expected present value of future real net worth flows:

$$\text{Max } V_t^h = E_t \sum_{i=1}^{\infty} (1 - \sigma) \sigma^{i-1} \Lambda_{r,t+i}^h n_{r,t+i}^h$$

The banker may transfer a fraction θ^h of “divertable” assets to the members of his family within the household. Divertable assets consist of total real gross assets $Q_{r,t}^h s_t^h$ net a fraction ω of real interbank borrowing with Foreign banks $b_{r,t}^h$, and the totality of real covered bonds $b_{r,t}^c$.

That is: divertable assets = $\theta^h (Q_{r,t}^h s_t^h - \omega b_{r,t}^h - b_{r,t}^c)$, or using (3.36):

$$\text{divertable assets} = \theta^h (n_{r,t}^h + (1-\omega)b_{r,t}^h + d_{r,t}^h)$$

which clearly shows that covered bonds cannot be diverted. For the bank not to divert assets, the following incentive constraint must hold:

$$V_t^h \geq \theta^h (Q_{r,t}^h s_t^h - \omega b_{r,t}^h - b_{r,t}^c) \quad (3.38)$$

In general, the value of the bank satisfies the Bellman equation:

$$V_t^h (s_t^h, b_{r,t}^h, b_{r,t}^c, d_{r,t}^h) = E_t \Lambda_{r,t,t+1}^h \left[(1-\sigma)n_{r,t+1}^h + \sigma \underset{s_{t+1}^h, b_{r,t+1}^h}{\text{Max}} V_{t+1}^h (s_{t+1}^h, b_{r,t+1}^h, b_{r,t+1}^c, d_{r,t+1}^h) \right] \quad (3.39)$$

To solve for the bank's decision problem, one guesses first that the value function is a linear function of the balance sheet arguments, and verifies this guess later:

$$V_t^h (s_t^h, b_{r,t}^h, b_{r,t}^c, d_{r,t}^h) = v_{s,t}^h s_t^h - v_{b,t}^h b_{r,t}^h - v_{c,t}^h b_{r,t}^c - v_t^h d_{r,t}^h \quad (3.40)$$

The maximization of (3.40) subject to the incentive constraint (3.38) yields the Lagrangian:

$$\mathcal{L} = [v_{s,t}^h s_t^h - v_{b,t}^h b_{r,t}^h - v_{c,t}^h b_{r,t}^c - v_t^h d_{r,t}^h] (1 + \lambda_t^h) - \lambda_t^h \theta^h [Q_{r,t}^h s_t^h - \omega b_{r,t}^h - b_{r,t}^c]$$

where λ_t^h denotes the Lagrange multiplier with respect to the incentive constraint. The necessary first-order conditions with respect to $s_t^h, b_{r,t}^h$ and λ_t^h are respectively given by (2.61), (2.62), and:

$$v_{s,t}^h s_t^h - v_{b,t}^h b_{r,t}^h - v_{c,t}^h b_{r,t}^c - v_t^h d_{r,t}^h = \theta^h [Q_{r,t}^h s_t^h - \omega b_{r,t}^h - b_{r,t}^c] \quad (3.41)$$

Condition (3.41) is the incentive constraint. It rewrites as:

$$Q_{r,t}^h s_t^h = \left(\frac{v_t^h}{\theta^h (1 - \omega x_{1,t}^h) - \theta^h x_{2,t}^h + v_{b,t}^h x_{1,t}^h + v_{c,t}^h x_{2,t}^h + v_t^h (1 - x_{1,t}^h - x_{2,t}^h) - \frac{v_{s,t}^h}{Q_{r,t}^h}} \right) n_{r,t}^h \quad (3.42)$$

or:

$$Q_{r,t}^h s_t^h = \phi_t^h n_{r,t}^h \quad (3.43)$$

$$\text{with } \phi_t^h = \frac{v_t^h}{\theta^h(1-\omega x_{1,t}^h) - \theta^h x_{2,t}^h + v_{b,t}^h x_{1,t}^h + v_{c,t}^h x_{2,t}^h + v_t^h(1-x_{1,t}^h - x_{2,t}^h) - \frac{v_{s,t}^h}{Q_{r,t}^h}},$$

and $x_{1,t}^h = \frac{b_{r,t}^h}{Q_{r,t}^h s_t^h}$, $x_{2,t}^h = \frac{b_{r,t}^c}{Q_{r,t}^h s_t^h}$ the proportion of assets financed by interbank borrowing and covered bonds.

Combining the conjectured solution (3.40) with the Bellman equation (3.39), net worth (3.37) and the leverage ratio (3.43), one obtains:

$$v_{s,t}^h s_t^h - v_{b,t}^h b_{r,t}^h - v_{c,t}^h b_{r,t}^c - v_t^h d_{r,t}^h = E_t \left(\Lambda_{t,t+1}^h \Omega_{t+1}^h n_{r,t+1}^h \right)$$

with:

$$\Omega_t^h = (1-\sigma) + \sigma \left[v_t^h + \phi_t^h \left\{ \left[\frac{v_{s,t}^h}{Q_{r,t}^h} - v_t^h \right] - [v_{b,t}^h - v_t^h] x_{1,t}^h - [v_{c,t}^h - v_t^h] x_{2,t}^h \right\} \right] \quad (3.44)$$

Then, from an undetermined coefficient method, the values for v_t^h , $v_{b,t}^h$, $v_{s,t}^h$ and $v_{c,t}^h$ obtain as (2.68), (2.69), (2.70) and:

$$v_{c,t}^h = v_t^h \frac{R_{t+1}^i}{R_{t+1}^h} \quad (3.45)$$

The rest of the model is left unchanged. Appendices C.8 and C.9 summarize the original and linearized equations for extension 2.

3.3.2. Calibration and steady-state

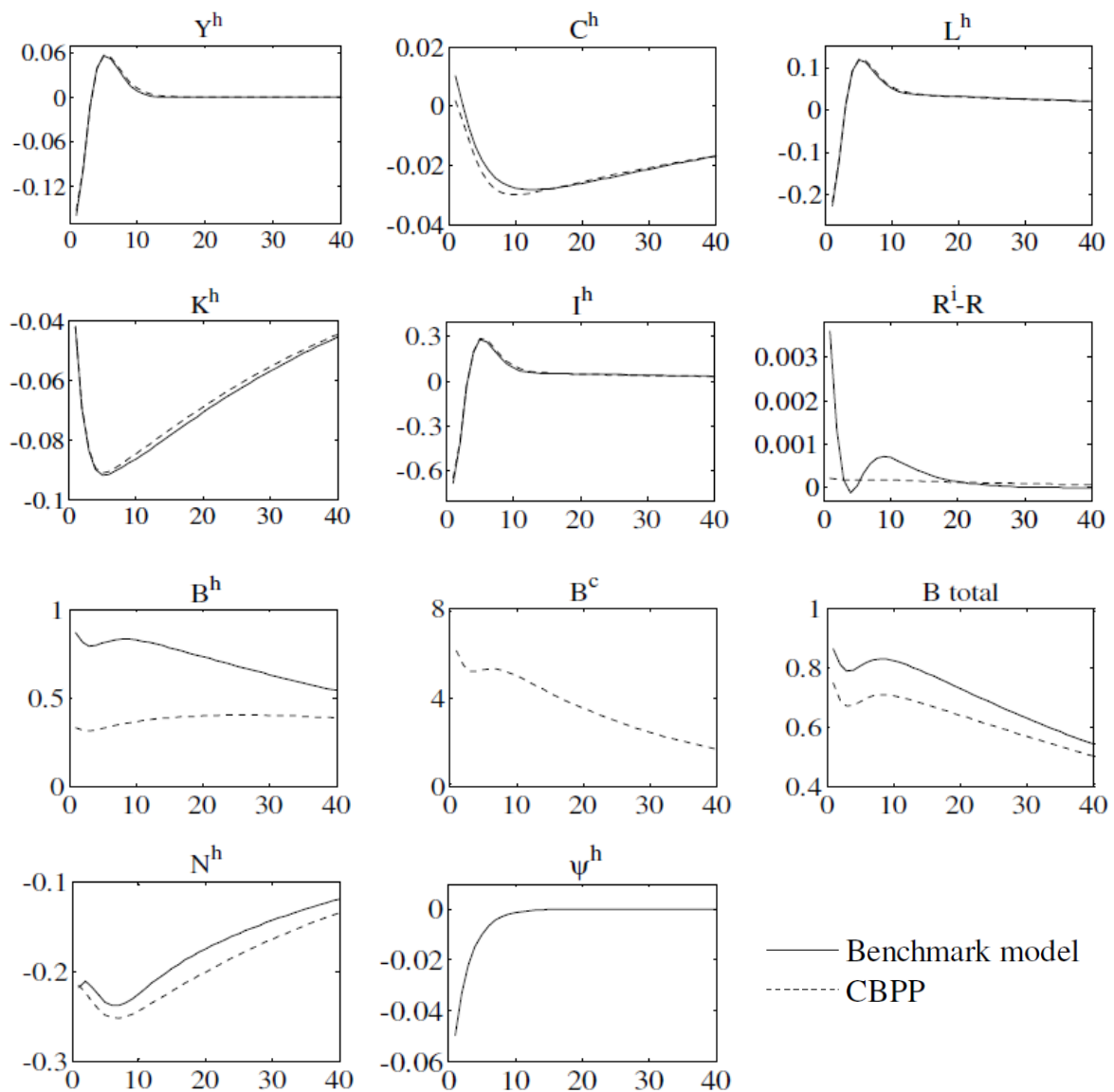
There is only one parameter to calibrate for this extension: φ . Two scenarios are envisaged: a “low interventionism” scenario in which the ECB only provides minimal commitment, and which translates by $\varphi = 20$; and a “high interventionism” profile in which the ECB significantly commits to provide liquidity to the interbank channel, in which case $\varphi = 2000$.

As for the steady-state, it is trivial to show that it is similar to the one of the benchmark version of the model. See appendix C.10 for more details.

3.3.3. Crisis experiment

Figure 3.2 displays the responses of some key variables, after a 5% decline in capital quality. France is chosen for the illustration as the proportion of assets financed by interbank funds is among the highest in Europe. For ease of exposition, the “high interventionism” profile is the only CBPP case presented.

Figure 3.2
Crisis experiment: extension 2



How successful is the CBPP? With respect to fundamental variables, one may observe a very slight improvement imputable to the programme: following the trigger of the crisis, the initial drop in output and employment is smaller (hardly visible on the graphs) than in the benchmark case with no CBPP. Recovery in consumption is also faster with the CBPP. The main reason for this better recovery of the real side of the economy lies in the improved situation of financial markets, which translates into higher investment and faster rebuilding of productive capital. In this respect, it is noteworthy that the primary aim of the CBPP, namely the reduction of the spread between the interbank and riskless rates is fully achieved, the latter becoming nearly null as soon as the crisis begins. Indeed, due to tensions on the spread, the ECB feeds the market with liquidities in order to ensure that Home banks can obtain as much interbank funds as they wish. Doing so, the spread remains close to zero without ever augmenting. It may then seem paradoxical to observe that the total quantity of bonds traded (interbank plus CBPP) is eventually lower in the case of the CBPP. This is actually explained by the strategic behaviour of banks: because covered bonds are not divertable, substituting regular interbank loans in favour of covered bonds allows to relax the incentive constraint of Home banks. Following, credit institutions may limit the quantity of deleveraging, and prefer to draw on their net worth to finance new projects rather than borrowing more funds on the interbank market. In this sense, covered bonds effectively crowd out interbank bonds. Nevertheless, this enhanced access to funding finally enables the banks to finance more projects, which in turn benefits to the real side of the economy.

3.3.4. Welfare analysis

The question which naturally arises is whether the performance of the monetary union exceeds that of the flexible exchange rate regime, once it benefits from the CBPP. Indeed, it may be that the monetary union is structurally less efficient than a regime of independent currency (chapter 2 and previous section), but the ECB and the set of credit policies it may implement certainly represent an asset over a national central bank endowed with too little funds to be able to do so. In this respect, it is not only the structural performance of the two regimes that matters, but also the capacity of central authorities to act as an additional instrument of stabilization. The general welfare criterion (3.3) is thus applied to the CBPP extension to tackle this issue. Table 3.5 reports the welfare loss differences between the two regimes. The first column displays the difference when there is no CBPP (benchmark model),

while the second and third columns present the results for the CBPP in the low and high interventionism scenarios.

Table 3.5
Welfare loss differences, CBPP

	No CBPP	Low interventionism $\varphi = 20$	High interventionism $\varphi = 2000$
Austria	-0.012	-0.005	-0.004
Belgium	0.008	0.011	0.012
Cyprus	-0.009	-0.004	-0.002
Estonia	-0.022	-0.019	-0.015
Finland	-0.013	-0.009	-0.008
France	-0.016	-0.014	-0.013
Germany	-0.004	-0.001	0.000
Greece	-0.012	-0.007	-0.004
Ireland	-0.011	-0.007	-0.007
Italy	-0.016	-0.013	-0.012
Luxembourg	-0.004	-0.001	-0.001
Malta	-0.008	-0.004	-0.002
Netherlands	-0.005	-0.003	-0.002
Portugal	-0.040	-0.035	-0.033
Slovakia	0.010	0.013	0.016
Slovenia	-0.009	-0.005	-0.003
Spain	-0.018	-0.014	-0.012

note: welfare losses expressed in percentage

The results are rather disappointing for the CBPP. On the one hand, it is clear that the program allows for a significant improvement in the stabilization performance of the monetary union. It unambiguously reduces the welfare loss difference with the flexible exchange rate regime, sometimes substantially as in the case of Austria, Cyprus, Greece, Malta and Slovenia. In these cases, the gap between the two regimes becomes very small, rendering the two regimes nearly equivalent. Another asset is that the CBPP seems to benefit most to the weakest economies of the Euro area, indicating that the monetary union may play a strong supportive role for its most fragile members.

On the other hand, however, the impact of the CBPP remains low for many countries, such as Finland, France, Italy or Portugal. Worse, for most countries, a high interventionism profile does not seem to produce much better results than a low interventionism profile, which suggests that even a very strong commitment of the ECB will remain relatively ineffective. Finally, and foremost, the gain in welfare granted by the CBPP remains insufficient to render

the monetary union more attractive than the flexible exchange rate regime, except in the case of Germany for which it becomes marginally better. Eventually, the conclusions from the preceding analyses remain valid: a monetary union provides lower welfare than a regime of independent currencies, even with the support of exceptional policies from its central bank.

3.4. Extension 3: integrating the Securities Market Programme/Outright Monetary Transactions to the model

The last category of measures implemented by the ECB during the financial crisis is constituted by the Securities Market Programme and the Outright Monetary Transactions. The SMP was first introduced in May 2010. The official objectives of the programme were stated as follows⁴¹:

“In view of the current exceptional circumstances in financial markets, characterised by severe tensions in certain market segments which are hampering the monetary policy transmission mechanism and thereby the effective conduct of monetary policy oriented towards price stability in the medium term, a temporary securities markets programme should be initiated. (...) The euro area National Central Banks, (...), and the ECB, in direct contact with counterparties, may conduct outright interventions in the Euro area public and private debt securities markets. The programme’s objective is to address the malfunctioning of securities markets and restore an appropriate monetary policy transmission mechanism.”

“Under the terms of this Decision, Eurosystem central banks may purchase the following: (a) on the secondary market, eligible marketable debt instruments issued by the central governments or public entities of the Member States whose currency is the euro; and (b) on the primary and secondary markets, eligible marketable debt instruments issued by private entities incorporated in the Euro area.”

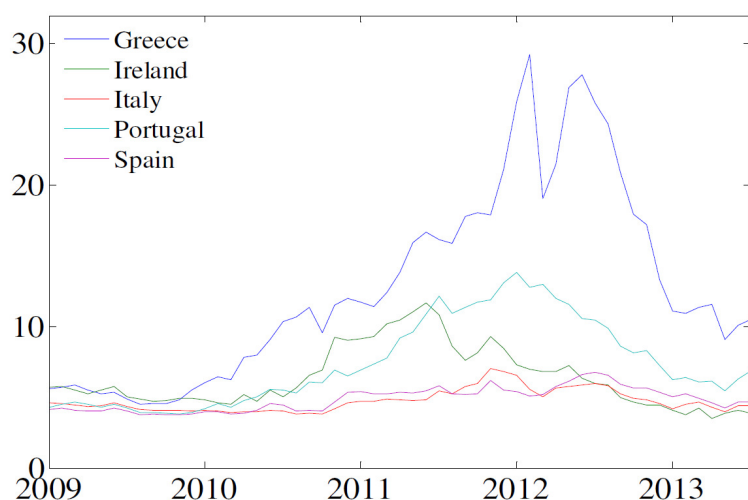
As an executive summary, the SMP constitutes a facility designed to purchase government bonds, in order to provide liquidity to Euro area central governments and alleviate pressures on sovereign debt bond markets. Two facts are noteworthy. First, sovereign debt bonds can only be purchased on secondary markets, in accordance with E.U. legislation. This prevents the central bank to act as a lender of last resort for national governments by purchasing

⁴¹ ECB decision of May 14, 2010.

sovereign debt on primary markets. To prevent debt monetization and abide by its objective of inflation stability, every purchase of debt operated by the ECB has then been sterilized. Secondly, the SMP does not rule out the possibility for the ECB to purchase private debt instruments, along with public sovereign debt. In practice this option has not been exploited by the ECB, the SMP/OMT programmes having exclusively focused on central government debt. Nevertheless, the ECB explicitly acknowledges the role it may endorse in regulating not only the sovereign debt sector, but also potentially the dysfunctioning of the private Euro area sector.

The SMP ran from May 2010 to September 2012, at which time it was replaced by the OMT, which is currently still operating. The sovereign debt titles acquired by the ECB so far concern exclusively the PIIGS, amounting to a total 220 billion Euros. Results from the SMP are globally positive. Figure 3.3 displays the 10-year government bond yield for the PIIGS over the 2009-2013 period.

Figure 3.3
10 year government bond yield, PIIGS



On the one hand, Figure 3.3 makes it clear that the SMP has not yet been completely successful in returning sovereign debt yields to their path prior to the 2009 sovereign debt crisis. On the other hand, yet, it is clear that the SMP contributed to lower sovereign debt interest rates for the PIIGS and initiated a process of convergence to their long-run, pre-crisis value. The yield drop is spectacular in the case of Greece, and must really be attributed to the SMP as there was otherwise no change in the fundamentals of the Greek economy, including

the level of its debt. The convergence in yields also suggests that the action of the ECB has permitted to preserve confidence over the solvency of these highly indebted countries, acting as an implicit insurance over their potential defaults.

The SMP can be integrated to the model in a relatively simple way. As the SMP is tightly linked to the issue of sovereign debt, the first extension of the benchmark model (allowing to integrate public debt) is chosen as the framework of reference. Because the SMP is Euro-specific and requires a large amount of funds that only the ECB is susceptible to possess, it is again assumed that the SMP only concerns the monetary union version of the model. As the core of the model is left unchanged, the presentation restricts to sections for which modifications apply.

3.4.1. Central authorities

In the context of the SMP/OMT, the ECB purchases at each period a fraction of the total amount of debt titles traded on secondary markets⁴². Two types of securities are concerned. First, the ECB acquires sovereign debt titles from Home. Secondly, the extension also considers the possibility for the ECB to acquire private assets although, once again, this prerogative has not been used in practice. Denote by $\theta_{1,t}^h$ the share of total Home sovereign debt purchased under the SMP. Then the latter is set as a function of the amount of sovereign debt accumulated up to the beginning of the current period, according to the rule:

$$\theta_{1,t}^h = \theta_1 \left(\frac{B_{r,t-1}^d - B_r^d}{B_r^d} \right) \quad (3.46)$$

with θ_1 some constant. (3.46) shows that as debt accumulated up to the present period exceeds steady-state debt, the ECB will intervene and start purchase some of this debt on secondary markets. The larger the deviation from its steady-state, critical value, the more the ECB will intervene to ensure that the amount of sovereign debt can eventually return to its steady-state value. Also, (3.46) implies that in steady-state, purchase of sovereign bonds within the SMP will be nil, which emphasizes the characteristics of the SMP as an exceptional policy taking place only during economical downturns.

⁴² For the sake of simplicity, the budget constraint of the ECB is not explicitly included in the extension. This is a mostly harmless assumption, as central banks are not balance sheet constrained, at least not in the sense of private entities. It is then simply assumed that the ECB may always honour its payments.

Concurrently, the ECB also intervenes by purchasing private debt securities. Following Gertler and Kiyotaki (2010), retail firms assets are interpreted in a wide sense so that they are encompassed in the debt securities covered by the SMP. Then, denoting total asset in the Home economy by S_t^h , assets intermediated by private banks by $S_{p,t}^h$ and assets intermediated by the ECB by $S_{g,t}^h$, one obtains:

$$S_t^h = S_{p,t}^h + S_{g,t}^h \quad (3.47)$$

It is assumed that at each period, the ECB intermediates a fraction $\theta_{2,t}^h$ of total assets:

$$S_{g,t}^h = \theta_{2,t}^h S_t^h \quad (3.48)$$

where $\theta_{2,t}^h$ is determined by the ECB as:

$$\theta_{2,t}^h = \theta_2 \left(\frac{Y^h - Y_t^h}{Y^h} \right) \quad (3.49)$$

Hence, the ECB intervenes proportionally to the crisis prevailing in Home: as a recession implies a fire sale of assets and hence the destruction of productive capital, the central authorities try to prevent a severe slump by directly financing private corporate entities.

3.4.2. Government

The government budget constraint and taxation are still given by (3.8) and (3.9). Government expenditure, however, now benefits from the SMP. As the ECB commits to purchase Home sovereign bonds and implicitly acts as an insurance of last resort for Home solvency, financial markets loosen their pressures on Home to cut public expenditure, so that (3.10) is replaced by:

$$G_t^h = \vartheta^h Y^h + \gamma_2 \vartheta^h (Y_t^h - Y^h) + \theta_{1,t}^h G^h \quad (3.50)$$

(3.50) says that public expenditure now comprises fixed expenditure $\vartheta^h Y^h$, minored proportionally to the current recession by an amount $\gamma_2 \vartheta^h (Y_t^h - Y^h)$ to prevent debt to become explosive, but majored by an amount $\theta_{1,t}^h G^h$ thanks to the ECB implicit guarantee.

3.4.3. Banks

The derivations are mostly similar to that of extension 1⁴³. Indeed, the individual behaviour of banks is left unaffected by the SMP. The only changes come at the time of aggregation. First, because the SMP leads the ECB to repurchase a fraction $\theta_{1,t}^h$ of sovereign debt from private banks, the optimality conditions of the private sector are slightly modified. The incentive constraint (c.34') rewrites:

$$Q_{r,t}^h S_{p,t}^h = \phi_t^h N_t^h \quad (3.51)$$

with :

$$\phi_t^h = \left(\frac{v_t^h}{\theta^h (1 - \omega x_{1,t}^h + (1 - \theta_{1,t}^h) x_{2,t}^h) + v_t^h (1 - x_{1,t}^h + (1 - \theta_{1,t}^h) x_{2,t}^h) + v_{b,t}^h x_{1,t}^h - \frac{v_{s,t}^h}{Q_{r,t}^h} - v_{d,t}^h (1 - \theta_{1,t}^h) x_{2,t}^h} \right) \quad (3.52)$$

Also, the balance sheet of the entire sector (c.36') becomes:

$$D_{r,t}^h = Q_{r,t}^h S_t^h + (1 - \theta_{1,t}^h) B_{r,t}^d - N_{r,t}^h - B_{r,t}^h \quad (3.53)$$

Finally, aggregate net worth (c.39') is replaced by:

$$N_{r,t}^h = (\sigma + \xi^h) [Z_{r,t}^h + (1 - \delta) Q_{r,t}^h] \psi_t^h S_{t-1}^h - \frac{\sigma}{\pi_t^h} \{ R_t^i B_{r,t-1}^h + R_t^h D_{r,t-1}^h - R_t^d (1 - \theta_{1,t}^h) B_{r,t-1}^d \} \quad (3.54)$$

Further, as the Central Bank intermediates a fraction $\theta_{2,t}^h$ of assets, asset demand from the private sector as a whole represents only a fraction $(1 - \theta_{2,t}^h)$ of total assets of the economy.

Combining (3.47), (3.48) and (3.51) yields:

$$Q_{r,t}^h S_t^h = \left(\frac{1}{1 - \theta_{2,t}^h} \right) \phi_t^h N_t^h \quad (3.55)$$

Therefore, as long as the Home economy is in recession, $\theta_{2,t}^h$ is positive and the SMP effectively expands the total volume of assets intermediated in the market. This closes the model⁴⁴.

From the developments of the previous and present paragraphs, one can see that the SMP may improve Home welfare from two different channels. First, it contributes to improve in a direct

⁴³ See Appendix C.5 for the set of equations from the first extension to which this section refers.

⁴⁴ See Appendices C.11 and C.12 for a summary of the original and linearized equations for the third extension.

way by restoring confidence over national government's solvency and reducing the cut in public spending. As the provision of public services enters into the household utility, the SMP mitigates the welfare impact of the crisis. But it also fosters welfare indirectly, by allowing banks to bear a lower proportion of project financing, which improves the general functioning of the economy. This results in a faster recovery of the real side of the economy after a crisis, which in turn enhances utility through consumption and labour.

3.4.4. Calibration and steady-state

The two parameters to calibrate for the extension are the constants θ_1 and θ_2 . The value of θ_1 is set at $0.1 \times \frac{B_r^d}{Y^h}$, one tenth of the steady-state level of indebtedness. This value reflects the fact that highly indebted countries may require greater assistance to be able to maintain a sustainable sovereign debt. As for θ_2 , its value is set at 0.2, implying the need of a 5% recession for the ECB to finance 1% of private assets, which represents a relatively modest intervention. These values are only meant to be suggestive.

As for steady-state values, it is straightforward to obtain from (3.46) and (3.49) that $\theta_1^h = 0$ and $\theta_2^h = 0$. Following, it is trivial to show that the steady-state is similar to the one of the first extension.

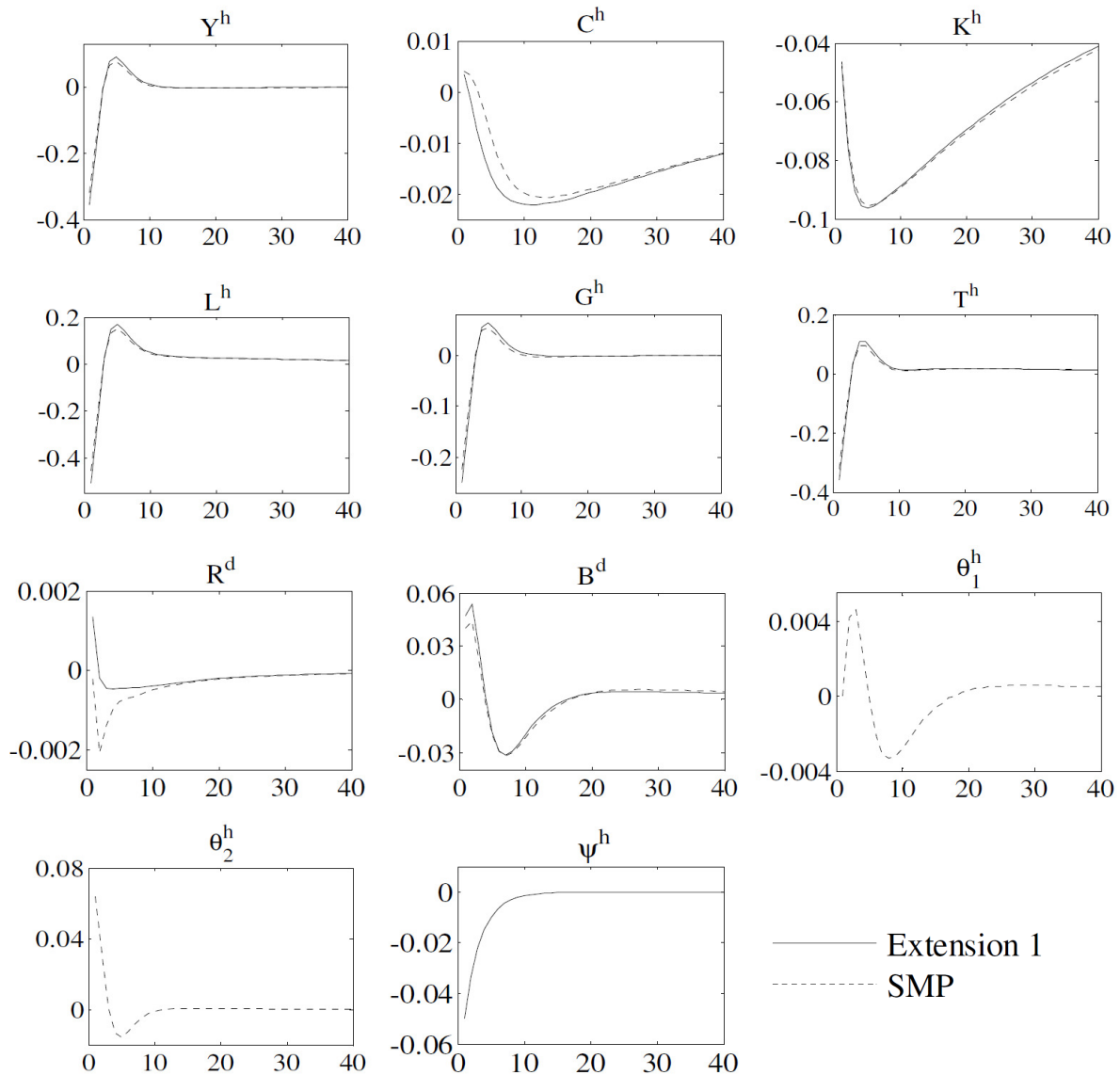
3.4.5. Crisis experiment

Figure 3.4 reports the response of the model to the usual financial crisis experiment, considering again the case of Ireland for the sake of comparison with paragraph 3.2.5. The plain line reports the responses in the context of extension 1 (sovereign debt crisis), while the dotted line gives the responses when the ECB operates the SMP programme. Because the SMP allows to alleviate the burden on banks for both sovereign debt and private, more projects can be financed and the real side of the economy performs better. The initial slump in capital, labour, consumption and output is smaller with the SMP, and the subsequent recovery proves also faster. This indirect effect of the SMP, combined with the direct effect of the SMP as an insurance of last resort both contributes to mitigate the drop in public spending.

Similarly, the lesser fall in output allows taxation to decrease in more moderate proportions, so that the subsequent increase needs not be as dramatic as without the SMP.

Figure 3.4

Crisis experiment: extension 3



The effect of the SMP on sovereign debt is also unambiguously positive. Thanks to enhanced performances of the real side of the economy, higher taxes are raised by governments which limits the creation of deficits and the constitution of the national debt. Hence, not only does the SMP reduce the volume of sovereign debts that the banks must finance, but it also diminish its overall amount. As an additional bonus, the lower demand for debt funding drives the interest rate on public debt downward, which further improves the balance sheet of public authorities.

Finally, it is noteworthy that the duration of the two sections of the programme (sovereign debt and private asset) are quite different. The SMP operates for a rather short time on the private debt market, with a positive purchase of assets over approximately one year, and a resale of the acquired securities over the following year. On the other hand, the action on sovereign debt is longer, taking up the five years before the programme terminates. This is consistent with the way the ECB currently runs the SMP/OMT programmes, the latter having started in 2010 and being still operating today, with no announced termination term.

3.4.6. Welfare analysis

The paragraph concludes the analysis by assessing the welfare performance of the monetary union regime under the SMP. The general welfare criterion (3.33) is used, and two cases are considered. In the first case, the operations conducted under the SMP are restricted to the purchase of sovereign debt, and no private debt is acquired ($\theta_2 = 0$). This case corresponds to the policy currently implemented by the ECB. The second case represents the full SMP, where the ECB also commits in purchasing private assets to support the economy ($\theta_2 = 0.2$). The results for the welfare differences with the floating exchange rate regime are reported in Table 3.6.

Table 3.6
Welfare loss differences, SMP

	No SMP	SMP, no private assets $\theta_2 = 0$	SMP with private assets $\theta_2 = 0.2$
Austria	-0.013	-0.011	-0.003
Belgium	0.020	0.021	0.030
Cyprus	0.000	0.001	0.014
Estonia	-0.036	-0.036	-0.025
Finland	-0.013	-0.013	-0.005
France	-0.013	-0.012	-0.006
Germany	-0.001	0.000	0.005
Greece	0.031	0.036	0.050
Ireland	-0.015	-0.014	-0.005
Italy	-0.008	-0.006	0.004
Luxembourg	-0.004	-0.003	0.000
Malta	-0.010	-0.008	0.006
Netherlands	-0.006	-0.005	0.002
Portugal	-0.052	-0.050	-0.030
Slovakia	0.008	0.010	0.021
Slovenia	-0.025	-0.024	-0.013
Spain	-0.019	-0.018	-0.010

note: welfare losses expressed in percentage

The results prove more conclusive than with the CBPP. The improvement in the performance of the monetary union remains limited when the SMP is restricted to the purchase of public debt, as only two additional countries (Germany and Cyprus) become better off under this regime. In addition, the welfare gain for these two countries is only marginal, although sufficient to reverse preference in favour of the monetary union. On the other hand, when the ECB implements the full SMP, a significant improvement can be observed. Four additional countries (Italy, Luxembourg, Malta and the Netherlands) now prefer to be in a monetary union, and the welfare gain is sometimes especially high, as in the case of Italy and Malta. In the end, with the full SMP, a majority of Euro area countries (9 out of 17) now prefer to live in a monetary union. There is, once again, no typical profile of the country preferring to live under either regime: both categories comprise strong economies (Belgium, Germany and Luxembourg for the monetary union, Austria, Finland and France for the regime of independent currencies), and weaker economies (Cyprus and Slovakia, as opposed to Estonia and Slovenia). This eventually suggests that a monetary union can produce better welfare performances than a regime of floating exchange rates, but only under the sine qua non condition that the central authorities endorse a role beyond the conduct of daily monetary

policy. The ECB must actively commit in intervening to stabilize economic activity, by supporting public governments and private activity. This pleads in favour of the settlement of a supranational risk-sharing authority within the European Monetary Union.

3.5. Conclusion

This paper has proposed a formal welfare analysis of competing regimes (monetary union and floating exchange rates) for the Euro area. Its results suggest that most Eurozone countries would enjoy higher welfare performances under a regime of floating exchange rates, though the analysis does not consider the costs from leaving a currency area. The main inefficiency of the monetary union seems to stem from its reduced ability to handle asymmetric shocks hitting abroad partners. This confirms the fact that shock asymmetry remains a crucial issue within the Euro area.

If the European Central Bank accepts to go beyond its strict role of monetary policy designer and commits in exceptional credit policies, however, welfare gains may be expected. They are only modest in the case of the CBPP, but become significant in the case of the SMP/OMT, leading a majority of members of the Euro area to prefer the monetary union. This calls for the settlement a supranational risk-sharing authority within the European Monetary Union. The European Financial Stability Facility (EFSF) and the European Financial Stabilisation Mechanism (EFSM) constitute preliminary achievements of such devices. The above results hence suggest that the European Monetary Union is currently going into the right direction.

Appendix C.1: derivation of the general welfare measure

Consider Home utility (2.13):

$$U(C_t^h, L_t^h) = \frac{(C_t^h)^{1-\eta}}{1-\eta} - \frac{\chi^h}{1+\phi} (L_t^h)^{1+\phi}$$

A second-order Taylor approximation of (2.13) around steady-state values yields:

$$\begin{aligned} U(C_t^h, L_t^h) = & U(C^h, L^h) + U_C(C_t - C) + U_L(L_t - L) + \frac{1}{2}U_{CC}(C_t - C)^2 + \frac{1}{2}U_{LL}(L_t - L)^2 \\ & + U_{CL}(C_t - C)(L_t - L) + \mathcal{O}(\|\xi\|^3) \end{aligned} \quad (\text{C.1})$$

where U_C^h, U_L^h, U_{CC}^h and U_{LL}^h denote respectively the steady-state values of the first and second-order derivatives of the utility function with respect to C^h and L^h , and $\mathcal{O}(\|\xi\|^3)$ denotes terms of order 3 or above. Ignoring terms beyond order 2, and noticing that utility is separable in consumption and labour, $U_{CL} = 0$ so that (C.1) simplifies to:

$$U(C_t^h, L_t^h) = U(C^h, L^h) + U_C(C_t - C) + U_L(L_t - L) + \frac{1}{2}U_{CC}(C_t - C)^2 + \frac{1}{2}U_{LL}(L_t - L)^2 \quad (\text{C.2})$$

Rearranging:

$$U(C_t^h, L_t^h) = U(C^h, L^h) + U_C C \left(\frac{C_t - C}{C} \right) + U_L L \left(\frac{L_t - L}{L} \right) + \frac{1}{2}U_{CC} C^2 \left(\frac{C_t - C}{C} \right)^2 + \frac{1}{2}U_{LL} L^2 \left(\frac{L_t - L}{L} \right)^2 \quad (\text{C.3})$$

Leave the utility function for now to derive an intermediate step. For any variable X_t and its log deviation $\tilde{X}_t \equiv \log(X_t) - \log(X)$, take a second-order approximation of X_t around $X_t = X$:

$$X_t = X e^{\tilde{X}_t} \approx X e^0 + X e^0 (\tilde{X}_t - 0) + X \frac{1}{2} e^0 (\tilde{X}_t - 0)^2 = X + X \tilde{X}_t + X \frac{1}{2} (\tilde{X}_t)^2 \quad (\text{C.4})$$

Rearranging (C.4), one obtains:

$$\frac{X_t - X}{X} = \tilde{X}_t + \frac{1}{2} (\tilde{X}_t)^2 \quad (\text{C.5})$$

Using (C.5) in (C.3), and rearranging, gives:

$$\begin{aligned} U(C_t^h, L_t^h) = & U(C^h, L^h) + U_C C \tilde{C}_t + U_L L \tilde{L}_t + \frac{1}{2}U_C C (\tilde{C}_t)^2 + \frac{1}{2}U_L L (\tilde{L}_t)^2 \\ & + \frac{1}{2}U_{CC} C^2 (\tilde{C}_t)^2 + \frac{1}{2}U_{LL} L^2 (\tilde{L}_t)^2 + \mathcal{O}(\|\xi\|^3) \end{aligned} \quad (\text{C.6})$$

Ignoring once again terms beyond order 2 and rearranging:

$$U(C_t^h, L_t^h) = U(C^h, L^h) + U_c C \tilde{C}_t + \frac{1}{2} U_{cc} C \left[1 + \frac{U_{cc}}{U_c} C \right] (\tilde{C}_t)^2 + U_L L \tilde{L}_t + \frac{1}{2} U_{LL} L \left[1 + \frac{U_{LL}}{U_L} L \right] (\tilde{L}_t)^2 \quad (\text{C.7})$$

Using the functional form (2.13), (C.7) rewrites:

$$U(C_t^h, L_t^h) = U(C^h, L^h) + (C^h)^{1-\eta} \tilde{C}_t - \chi^h (L^h)^{1+\phi} \tilde{L}_t - \left(\frac{\eta-1}{2} \right) (C^h)^{1-\eta} (\tilde{C}_t)^2 - \left(\frac{\chi^h(1+\phi)}{2} \right) (L^h)^{1+\phi} (\tilde{L}_t)^2 \quad (\text{C.8})$$

Note that for any variable X_t , the unconditional expectation is the steady-state value, that is, $E(X_t^h) = X^h$. This implies $E(\tilde{X}_t^h) = 0$. Following, $E(\tilde{C}_t) = E(\tilde{L}_t) = 0$. Hence, taking unconditional expectation on both sides of (C.8) yields:

$$E\{U(C_t^h, L_t^h)\} = U(C^h, L^h) - \left(\frac{\eta-1}{2} \right) (C^h)^{1-\eta} \text{Var}(\tilde{C}_t) - \left(\frac{\chi^h(1+\phi)}{2} \right) (L^h)^{1+\phi} \text{Var}(\tilde{L}_t) \quad (\text{C.9})$$

Using first (C.9) in (3.1):

$$U((1-\Phi)C^h, L^h) = U(C^h, L^h) - \left(\frac{\eta-1}{2} \right) (C^h)^{1-\eta} \text{Var}(\tilde{C}_t) - \left(\frac{\chi^h(1+\phi)}{2} \right) (L^h)^{1+\phi} \text{Var}(\tilde{L}_t)$$

and then the functional form 2.13, obtain:

$$\begin{aligned} & \frac{((1-\Phi)C^h)^{1-\eta}}{1-\eta} - \frac{\chi^h}{1+\phi} (L^h)^{1+\phi} \\ &= \frac{(C^h)^{1-\eta}}{1-\eta} - \frac{\chi^h}{1+\phi} (L^h)^{1+\phi} - \left(\frac{\eta-1}{2} \right) (C^h)^{1-\eta} \text{Var}(\tilde{C}_t) - \left(\frac{\chi^h(1+\phi)}{2} \right) (L^h)^{1+\phi} \text{Var}(\tilde{L}_t) \end{aligned} \quad (\text{C.10})$$

After some rearrangements, (C.10) finally becomes:

$$\Phi = 1 - \left[1 + \left(\frac{(\eta-1)^2}{2} \right) \text{Var}(\tilde{C}_t) + \left(\frac{\chi^h(1+\phi)(\eta-1)}{2} \right) ((C^h)^{\eta-1} (L^h)^{1+\phi}) \text{Var}(\tilde{L}_t) \right]^{-\left(\frac{1}{\eta-1} \right)} \quad (\text{C.11})$$

which is (3.3) in the text.

Appendix C.2: derivation of the shock-specific welfare measure

Start from (3.4). Because the linearized model is expressed in terms of \tilde{C}_t and \tilde{L}_t rather than C_t and L_t , use is made of the approximations: $C_t = C(1 + \tilde{C}_t)$ and $L_t = L(1 + \tilde{L}_t)$. (3.4) then rewrites:

$$\sum_{i=0}^{\infty} \beta^i U^{mu} \left((1 + \Gamma) C^{mu} (1 + \tilde{C}_{t+i}^{mu}), L^{mu} (1 + \tilde{L}_{t+i}^{mu}) \right) = \sum_{i=0}^{\infty} \beta^i U^{ic} \left(C^{ic} (1 + \tilde{C}_{t+i}^{ic}), L^{ic} (1 + \tilde{L}_{t+i}^{ic}) \right) \quad (C.12)$$

Given (2.13), (C.12) rewrites:

$$\sum_{i=0}^{\infty} \beta^i \left[\frac{\left((1 + \Gamma) C^{mu} (1 + \tilde{C}_{t+i}^{mu}) \right)^{1-\eta}}{1-\eta} - \frac{\chi}{1+\phi} \left(L^{mu} (1 + \tilde{L}_{t+i}^{mu}) \right)^{1+\phi} \right] = \sum_{i=0}^{\infty} \beta^i \left[\frac{\left(C^{ic} (1 + \tilde{C}_{t+i}^{ic}) \right)^{1-\eta}}{1-\eta} - \frac{\chi}{1+\phi} \left(L^{ic} (1 + \tilde{L}_{t+i}^{ic}) \right)^{1+\phi} \right] \quad (C.13)$$

Finally, rearranging (C.13) yields:

$$\Gamma = \left\{ \frac{\sum_{i=0}^{\infty} \beta^i \left[\frac{\left(C^{ic} (1 + \tilde{C}_{t+i}^{ic}) \right)^{1-\eta}}{1-\eta} - \frac{\chi}{1+\phi} \left(L^{ic} (1 + \tilde{L}_{t+i}^{ic}) \right)^{1+\phi} \right] + \sum_{i=0}^{\infty} \beta^i \frac{\chi}{1+\phi} \left(L^{mu} (1 + \tilde{L}_{t+i}^{mu}) \right)^{1+\phi}}{\sum_{i=0}^{\infty} \beta^i \frac{\left(C^{mu} (1 + \tilde{C}_{t+i}^{mu}) \right)^{1-\eta}}{1-\eta}} \right\}^{\frac{1}{1-\eta}} - 1 \quad (C.14)$$

which is (3.5).

Appendix C.3: summary of equilibrium conditions, extension 1, flexible exchange rate version of the model

The model comprises the 64 variables. The 58 state variables:

$$\{Y_t^h, Y_t^f, C_t^h, C_t^f, C_{H,t}^h, C_{F,t}^h, C_{F,t}^f, C_{H,t}^f, K_{t+1}^h, K_{t+1}^f, L_t^h, L_t^f, I_t^h, I_t^f, G_t^h, G_t^f, T_{r,t}^h, T_{r,t}^f, W_{r,t}^h, W_{r,t}^f, MC_{r,t}^h, MC_{r,t}^f, P_t^h, P_t^f, P_{H,t}^h, P_{F,t}^h, P_{H,t}^f, P_{F,t}^f, \pi_t^h, \pi_t^f, \pi_{H,t}^h, \pi_{F,t}^h, \varepsilon_t, R_{t+1}^h, R_{t+1}^f, R_{t+1}^i, R_t^d, Q_{r,t}^h, Q_{r,t}^f, B_{r,t}^h, B_{r,t}^f, B_{r,t}^d, S_t^h, S_t^f, D_{r,t}^h, F_{r,t}^h, F_{r,t}^f, N_{r,t}^h, N_{r,t}^f, Z_{r,t}^h, Z_{r,t}^f, v_t^h, v_{b,t}^h, v_{s,t}^h, v_{d,t}^h, v_t^f, v_{b,t}^f, v_{s,t}^f\}$$

and the 6 stochastic variables:

$$\{\psi_t^h, \psi_t^f, A_t^h, A_t^f, \zeta_t^h, \zeta_t^f\}$$

are then defined by the following 64 equations:

Consumer Price index:

$$P_t^h = \left(\varphi^h (P_{H,t})^{1-\mu} + (1-\varphi^h) (\varepsilon_t P_{F,t})^{1-\mu} \right)^{\frac{1}{1-\mu}} \quad (\text{c.1})$$

$$P_t^f = \left(\varphi^f (P_{F,t})^{1-\mu} + (1-\varphi^f) (\varepsilon_t^{-1} P_{H,t})^{1-\mu} \right)^{\frac{1}{1-\mu}} \quad (\text{c.2})$$

Optimal index demand:

$$C_{H,t}^h = \varphi^h \left(\frac{P_t^h}{P_{H,t}} \right)^\mu C_t^h \quad (\text{c.3})$$

$$C_{F,t}^h = (1-\varphi^h) \left(\frac{P_t^h}{\varepsilon_t P_{F,t}} \right)^\mu C_t^h \quad (\text{c.4})$$

$$C_{F,t}^f = \varphi^f \left(\frac{P_t^f}{P_{F,t}} \right)^\mu C_t^f \quad (\text{c.5})$$

$$C_{H,t}^f = (1-\varphi^f) \left(\frac{\varepsilon_t P_t^f}{P_{H,t}} \right)^\mu C_t^f \quad (\text{c.6})$$

Optimal labour allocation:

$$\chi^h (L_t^h)^\phi (C_t^h)^\eta = W_{r,t}^h \quad (\text{c.7})$$

$$\chi^f (L_t^f)^\phi (C_t^f)^\eta = W_{r,t}^f \quad (\text{c.8})$$

Euler equation:

$$1 = R_{t+1}^h \beta E_t \left[\left(\frac{C_t^h}{C_{t+1}^h} \right)^\eta \frac{1}{\pi_{t+1}^h} \right] \quad (\text{c.9})$$

$$1 = R_{t+1}^f \beta E_t \left[\left(\frac{C_t^f}{C_{t+1}^f} \right)^\eta \frac{1}{\pi_{t+1}^f} \right] \quad (\text{c.10})$$

Budget constraint:

$$C_t^h + D_{r,t}^h + F_{r,t}^h + T_{r,t}^h + A C_t^h = W_{r,t}^h L_t^h + \frac{R_t^h}{\pi_t^h} D_{r,t-1}^h + \frac{R_t^f}{\pi_t^h} \frac{\varepsilon_t}{\varepsilon_{t-1}} F_{r,t-1}^h \quad (\text{c.11})$$

Modified interest parity condition:

$$R_{t+1}^h \left[1 + \vartheta (F_{r,t}^h - F_r^h) \right] = R_{t+1}^f E_t \left(\frac{\varepsilon_{t+1}}{\varepsilon_t} \right) \quad (\text{c.12})$$

Public expenditure:

$$G_t^h = \vartheta^h Y^h + \gamma_2 \vartheta^h (Y_t^h - Y^h) \quad (\text{c.13})$$

$$G_t^f = \vartheta^f Y^f \quad (\text{c.14})$$

Law of motion of capital:

$$K_{t+1}^h = \psi_{t+1}^h [I_t^h + (1-\delta)K_t^h] \quad (c.15) \quad K_{t+1}^f = \psi_{t+1}^f [I_t^f + (1-\delta)K_t^f] \quad (c.16)$$

Capital pricing:

$$Q_{r,t}^h = \frac{P_{H,t}}{P_t^h} [1 + \iota(I_t^h - I_{t-1}^h)] - \iota E_t \Lambda_{r,t+1}^h \frac{P_{H,t+1}}{P_{t+1}^h} (I_{t+1}^h - I_t^h) \quad (c.17)$$

$$Q_{r,t}^f = \frac{P_{F,t}}{P_t^f} [1 + \iota(I_t^f - I_{t-1}^f)] - \iota E_t \Lambda_{r,t+1}^f \frac{P_{F,t+1}}{P_{t+1}^f} (I_{t+1}^f - I_t^f) \quad (c.18)$$

Production technology:

$$Y_t^h(j) = A_t^h (K_t^h)^\alpha L_t^h(j)^{1-\alpha} \quad (c.19) \quad Y_t^f(j) = A_t^f (K_t^f)^\alpha L_t^f(j)^{1-\alpha} \quad (c.20)$$

Real wage setting:

$$MC_{r,t}^h = W_{r,t}^h \times \frac{P_t^h}{P_{H,t}} \times \frac{1}{MPL_t^h} \quad (c.21) \quad MC_{r,t}^f = W_{r,t}^f \times \frac{P_t^f}{P_{F,t}} \times \frac{1}{MPL_t^f} \quad (c.22)$$

Dividend:

$$Z_{r,t}^h = \frac{\frac{P_{H,t}}{P_t^h} Y_t^h - W_{r,t}^h L_t^h}{K_t^h} \quad (c.23) \quad Z_{r,t}^f = \frac{\frac{P_{F,t}}{P_t^f} Y_t^f - W_{r,t}^f L_t^f}{K_t^f} \quad (c.24)$$

Optimal pricing:

$$P_{H,t}^*(j) = \frac{\kappa}{\kappa-1} \frac{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^h Y_{t+i|t}^h(j) \Psi_{t+i}^h(Y_{t+i|t}^h(j)) \right\}}{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^h Y_{t+i|t}^h(j) \right\}} \quad (c.25)$$

$$P_{F,t}^*(j) = \frac{\kappa}{\kappa-1} \frac{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^f Y_{t+i|t}^f(j) \Psi_{t+i}^f(Y_{t+i|t}^f(j)) \right\}}{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^f Y_{t+i|t}^f(j) \right\}} \quad (c.26)$$

Law of motion of producer prices:

$$(\pi_{H,t})^{1-\kappa} = \zeta + (1-\zeta) \left(\frac{P_{H,t}^*}{P_{H,t-1}} \right)^{1-\kappa} \quad (c.27) \quad (\pi_{F,t})^{1-\kappa} = \zeta + (1-\zeta) \left(\frac{P_{F,t}^*}{P_{F,t-1}} \right)^{1-\kappa} \quad (c.28)$$

PPI inflation:

$$\pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}} \quad (c.29) \quad \pi_{F,t} = \frac{P_{F,t}}{P_{F,t-1}} \quad (c.30)$$

CPI inflation:

$$\pi_t^h = \frac{P_t^h}{P_{t-1}^h} \quad (c.31) \quad \pi_t^f = \frac{P_t^f}{P_{t-1}^f} \quad (c.32)$$

Optimal security allocation:

$$\left(\frac{v_{s,t}^h}{Q_{r,t}^h} - v_t^h \right) = \frac{1}{\omega} (v_{b,t}^h - v_t^h) \quad (c.33) \quad \left(\frac{v_{s,t}^f}{Q_{r,t}^f} - v_t^f \right) = \frac{1}{\omega} (v_{b,t}^f - v_t^f) \quad (c.34)$$

Optimal government loans allocation:

$$\frac{v_{s,t}^h}{Q_{r,t}^h} = v_{d,t}^h \quad (c.35)$$

Incentive constraint:

$$v_{s,t}^h S_t^h + v_{d,t}^h B_{r,t}^d - v_{b,t}^h B_{r,t}^h - v_t^h D_{r,t}^h = \theta^h [Q_{r,t}^h S_t^h + B_{r,t}^d - \omega B_{r,t}^h] \quad (c.36)$$

$$v_{s,t}^f S_t^f - v_{b,t}^f B_{r,t}^f - v_t^f F_{r,t} = \theta^f [Q_{r,t}^f S_t^f - \omega B_{r,t}^f] \quad (c.37)$$

Security market clearing:

$$D_{r,t}^h = Q_{r,t}^h S_t^h + B_{r,t}^d - N_{r,t}^h - B_{r,t}^h \quad (c.38) \quad F_{r,t} = Q_{r,t}^f S_t^f - N_{r,t}^f - B_{r,t}^f \quad (c.39)$$

Asset market clearing:

$$B_{r,t}^h \frac{P_t^h}{\varepsilon_t} = -B_{r,t}^f P_t^f \quad (c.40)$$

Evolution of net worth:

$$N_{r,t}^h = (\sigma + \xi^h) \{ [Z_{r,t}^h + (1-\delta)Q_{r,t}^h] \psi_t^h S_{t-1}^h \} - \frac{\sigma}{\pi_t^h} \left\{ \frac{\varepsilon_t}{\varepsilon_{t-1}} R_t^i B_{r,t-1}^h + R_t^h D_{r,t-1}^h - R_t^d B_{r,t-1}^d \right\} \quad (c.41)$$

$$N_{r,t}^f = (\sigma + \xi^f) \{ [Z_{r,t}^f + (1-\delta)Q_{r,t}^f] \psi_t^f S_{t-1}^f \} - \frac{\sigma}{\pi_t^f} \{ R_t^i B_{r,t-1}^f + R_t^f F_{r,t-1} \} \quad (c.42)$$

Shadow price of deposits:

$$v_t^h = R_{t+1}^h E_t \left(\frac{\Lambda_{r,t,t+1}^h}{\pi_{t+1}^h} \Omega_{t+1}^h \right) \quad (c.43) \quad v_t^f = R_{t+1}^f E_t \left(\frac{\Lambda_{r,t,t+1}^f}{\pi_{t+1}^f} \Omega_{t+1}^f \right) \quad (c.44)$$

Shadow price of interbank borrowing:

$$v_{b,t}^h = v_t^h \frac{R_{t+1}^i}{R_{t+1}^h} E_t \left(\frac{\varepsilon_{t+1}}{\varepsilon_t} \right) \quad (c.45) \quad v_{b,t}^f = v_t^f \frac{R_{t+1}^i}{R_{t+1}^f} \quad (c.46)$$

Shadow value of assets:

$$v_{s,t}^h = v_t^h E_t \left(\frac{\pi_{t+1}^h [Z_{r,t+1}^h + (1-\delta)Q_{r,t+1}^h] \psi_{t+1}^h}{R_{t+1}^h} \right) \quad (c.47) \quad v_{s,t}^f = v_t^f E_t \left(\frac{\pi_{t+1}^f [Z_{r,t+1}^f + (1-\delta)Q_{r,t+1}^f] \psi_{t+1}^f}{R_{t+1}^f} \right) \quad (c.48)$$

Shadow value of government loans:

$$v_{d,t}^h = v_t^h \frac{R_{t+1}^d}{R_{t+1}^h} \quad (c.49)$$

Final goods market clearing:

$$Y_t^h = C_{H,t}^h + C_{H,t}^f + I_t^h + G_t^h \quad (c.50) \quad Y_t^f = C_{F,t}^f + C_{F,t}^h + I_t^f + G_t^f \quad (c.51)$$

Taxation:

$$T_{r,t}^h = \frac{P_{H,t}}{P_t^h} \tau^h Y_t^h + \gamma_1 (B_{r,t-1}^d - B_r^d) \quad (\text{c.52}) \quad \frac{P_{F,t}}{P_t^f} G_t^f = T_{r,t}^f \quad (\text{c.53})$$

Government budget constraint:

$$T_{r,t}^h + B_{r,t}^d = \frac{P_{H,t}}{P_t^h} G_t^h + \frac{R_t^d}{\pi_t^h} B_{r,t-1}^d \quad (\text{c.54})$$

Concordance of assets with aggregate capital:

$$S_t^h = I_t^h + (1 - \delta) K_t^h \quad (\text{c.55}) \quad S_t^f = I_t^f + (1 - \delta) K_t^f \quad (\text{c.56})$$

Monetary policy:

$$\log(R_{t+1}^h) = \log(R^h) + \phi_\pi^h \log \pi_t^h + \phi_y^h \log \left(\frac{Y_t^h}{Y} \right) + \zeta_t^h \quad (\text{c.57})$$

$$\log(R_{t+1}^f) = \log(R^f) + \phi_\pi^f \log \pi_t^f + \phi_y^f \log \left(\frac{Y_t^f}{Y} \right) + \zeta_t^f \quad (\text{c.58})$$

Stochastic processes:

$$\zeta_t^h = \rho_m \zeta_{t-1}^h + \epsilon_{m,t}^h \quad (\text{c.59}) \quad \zeta_t^f = \rho_m \zeta_{t-1}^f + \epsilon_{m,t}^f \quad (\text{c.60})$$

$$\psi_t^h = (\bar{\psi})^{1-\rho_k} (\psi_{t-1}^h)^{\rho_k} e^{\epsilon_{k,t}^h} \quad (\text{c.61}) \quad \psi_t^f = (\bar{\psi})^{1-\rho_k} (\psi_{t-1}^f)^{\rho_k} e^{\epsilon_{k,t}^f} \quad (\text{c.62})$$

$$A_t^h = (\bar{A}^h)^{1-\rho_a} (A_{t-1}^h)^{\rho_a} e^{\epsilon_{a,t}^h} \quad (\text{c.63}) \quad A_t^f = (\bar{A}^f)^{1-\rho_a} (A_{t-1}^f)^{\rho_a} e^{\epsilon_{a,t}^f} \quad (\text{c.64})$$

Appendix C.4: summary of linearized equilibrium conditions, extension 1, flexible exchange rate version of the model

The linearized set of equilibrium equations composing the model is presented hereafter. In its final form, the 54 state variables :

$$\{\tilde{Y}_t^h, \tilde{Y}_t^f, \tilde{C}_t^h, \tilde{C}_t^f, \tilde{C}_{H,t}^h, \tilde{C}_{F,t}^h, \tilde{C}_{F,t}^f, \tilde{C}_{H,t}^f, \tilde{K}_{t+1}^h, \tilde{K}_{t+1}^f, \tilde{L}_t^h, \tilde{L}_t^f, \tilde{I}_t^h, \tilde{I}_t^f, \tilde{G}_t^h, \tilde{G}_t^f, \tilde{T}_{r,t}^h, \tilde{T}_{r,t}^f, \tilde{W}_{r,t}^h, \tilde{W}_{r,t}^f, \tilde{P}_t^h, \tilde{P}_t^f, \tilde{P}_{H,t}^h, \tilde{P}_{F,t}^h, \tilde{\pi}_t^h, \tilde{\pi}_t^f, \tilde{\pi}_{H,t}^h, \tilde{\pi}_{F,t}^h, \tilde{\epsilon}_t, \tilde{R}_{t+1}^h, \tilde{R}_{t+1}^f, \tilde{R}_{t+1}^i, \tilde{R}_{t+1}^d, \tilde{Q}_{r,t}^h, \tilde{Q}_{r,t}^f, \tilde{B}_{r,t}^h, \tilde{B}_{r,t}^f, B_{r,t}^d, \tilde{S}_t^h, \tilde{S}_t^f, \tilde{D}_{r,t}^h, \tilde{F}_{r,t}^h, \tilde{F}_{r,t}^f, \tilde{N}_{r,t}^h, \tilde{N}_{r,t}^f, \tilde{Z}_{r,t}^h, \tilde{Z}_{r,t}^f, \tilde{v}_t^h, \tilde{v}_{b,t}^h, \tilde{v}_{s,t}^h, \tilde{v}_{d,t}^h, \tilde{v}_t^f, \tilde{v}_{b,t}^f, \tilde{v}_{s,t}^f\}$$

along with the 6 stochastic processes:

$$\{\tilde{\psi}_t^h, \tilde{\psi}_t^f, \tilde{A}_t^h, \tilde{A}_t^f, \zeta_t^h, \zeta_t^f\}$$

are determined by the 60 following linearized equations:

$$\begin{aligned} \tilde{P}_t^h &= \varphi^h \tilde{P}_{H,t} + (1 - \varphi^h) \tilde{\epsilon}_t + (1 - \varphi^h) \tilde{P}_{F,t} \\ \tilde{P}_t^f &= \varphi^f \tilde{P}_{F,t} - (1 - \varphi^f) \tilde{\epsilon}_t + (1 - \varphi^f) \tilde{P}_{H,t} \\ \tilde{C}_{H,t}^h &= \mu \tilde{P}_t^h - \mu \tilde{P}_{H,t} + \tilde{C}_t^h \\ \tilde{C}_{F,t}^h &= \mu \tilde{P}_t^h - \mu \tilde{\epsilon}_t - \mu \tilde{P}_{F,t} + \tilde{C}_t^h \\ \tilde{C}_{F,t}^f &= \mu \tilde{P}_t^f - \mu \tilde{P}_{F,t} + \tilde{C}_t^f \\ \tilde{C}_{H,t}^f &= \mu \tilde{\epsilon}_t + \mu \tilde{P}_t^f - \mu \tilde{P}_{H,t} + \tilde{C}_t^f \\ \tilde{W}_{r,t}^h &= \phi \tilde{L}_t^h - \eta \tilde{C}_t^h \\ \tilde{W}_{r,t}^f &= \phi \tilde{L}_t^f - \eta \tilde{C}_t^f \\ 0 &= \tilde{R}_{t+1}^h + \eta \tilde{C}_t^h - \eta \tilde{C}_{t+1}^h - \tilde{\pi}_{t+1}^h \\ 0 &= \tilde{R}_{t+1}^f + \eta \tilde{C}_t^f - \eta \tilde{C}_{t+1}^f - \tilde{\pi}_{t+1}^f \\ C^h \tilde{C}_t^h + D_r^h \tilde{D}_{r,t}^h + F_r^h \tilde{F}_{r,t}^h + T_r^h \tilde{T}_{r,t}^h &= W_r^h L^h \tilde{W}_{r,t}^h + W_r^h L^h \tilde{L}_t^h + R^h D_r^h \tilde{R}_t^h - (R^h D_r^h + R^f F_r^h) \tilde{\pi}_t^h + R^h D_r^h \tilde{D}_{r,t-1}^h \\ &+ R^f F_r^h \tilde{R}_t^f + R^f F_r^h \tilde{\epsilon}_t - R^f F_r^h \tilde{\epsilon}_{t-1} + R^f F_r^h \tilde{F}_{r,t-1}^h \\ \tilde{R}_{t+1}^h + \vartheta F^h \tilde{F}_{r,t}^h &= \tilde{R}_{t+1}^f + E_t(\tilde{\epsilon}_{t+1}) - \tilde{\epsilon}_t \\ \tilde{G}_t^h &= \gamma_2 \tilde{Y}_t^h \\ \tilde{G}_t^f &= 0 \\ \tilde{K}_{t+1}^h &= \tilde{\psi}_{t+1}^h + \delta \tilde{I}_t^h + (1 - \delta) \tilde{K}_t^h \\ \tilde{K}_{t+1}^f &= \tilde{\psi}_{t+1}^f + \delta \tilde{I}_t^f + (1 - \delta) \tilde{K}_t^f \\ \tilde{Q}_{r,t}^h &= \tilde{P}_{H,t} - \tilde{P}_t^h + U^h(1 + \beta) \tilde{I}_t^h - U^h \tilde{I}_{t-1}^h - I \beta I^h E_t \tilde{I}_{t+1}^h \\ \tilde{Q}_{r,t}^f &= \tilde{P}_{F,t} - \tilde{P}_t^f + U^f(1 + \beta) \tilde{I}_t^f - U^f \tilde{I}_{t-1}^f - I \beta I^f E_t \tilde{I}_{t+1}^f \\ \tilde{Y}_t^h &= \tilde{A}_t^h + \alpha \tilde{K}_t^h + (1 - \alpha) \tilde{L}_t^h \\ \tilde{Y}_t^f &= \tilde{A}_t^f + \alpha \tilde{K}_t^f + (1 - \alpha) \tilde{L}_t^f \\ Z_r^h K^h \tilde{Z}_{r,t}^h &= Y^h \tilde{P}_{H,t} - Y^h \tilde{P}_t^h + Y^h \tilde{Y}_t^h - W_r^h L^h \tilde{W}_{r,t}^h - W_r^h L^h \tilde{L}_t^h - Z_r^h K^h \tilde{K}_t^h \\ Z_r^f K^f \tilde{Z}_{r,t}^f &= Y^f \tilde{P}_{F,t} - Y^f \tilde{P}_t^f + Y^f \tilde{Y}_t^f - W_r^f L^f \tilde{W}_{r,t}^f - W_r^f L^f \tilde{L}_t^f - Z_r^f K^f \tilde{K}_t^f \end{aligned}$$

$$\tilde{\pi}_{H,t} = \beta E_t(\tilde{\pi}_{H,t+1}) + \omega \tilde{W}_{r,t}^h + \omega \tilde{P}_t^h - \omega \tilde{P}_{H,t} - \frac{\omega}{1-\alpha} \tilde{A}_t^h - \frac{\alpha\omega}{1-\alpha} \tilde{K}_t^h + \frac{\alpha\omega}{1-\alpha} \tilde{Y}_t^h$$

$$\tilde{\pi}_{F,t} = \beta E_t(\tilde{\pi}_{F,t+1}) + \omega \tilde{W}_{r,t}^f + \omega \tilde{P}_t^f - \omega \tilde{P}_{F,t} - \frac{\omega}{1-\alpha} \tilde{A}_t^f - \frac{\alpha\omega}{1-\alpha} \tilde{K}_t^f + \frac{\alpha\omega}{1-\alpha} \tilde{Y}_t^f$$

$$\text{with : } \omega \equiv \frac{(1-\zeta)(1-\zeta\beta)(1-\alpha)}{\zeta(1-\alpha+\alpha\kappa)}$$

$$\tilde{\pi}_{H,t} = \tilde{P}_{H,t} - \tilde{P}_{H,t-1}$$

$$\tilde{\pi}_{F,t} = \tilde{P}_{F,t} - \tilde{P}_{F,t-1}$$

$$\tilde{\pi}_t^h = \tilde{P}_t^h - \tilde{P}_{t-1}^h$$

$$\tilde{\pi}_t^f = \tilde{P}_t^f - \tilde{P}_{t-1}^f$$

$$\tilde{v}_{s,t}^h - \tilde{Q}_{r,t}^h + \left(\frac{1}{\omega} - 1\right) \tilde{v}_t^h = \frac{1}{\omega} \tilde{v}_{b,t}^h$$

$$\tilde{v}_{s,t}^f - \tilde{Q}_{r,t}^f + \left(\frac{1}{\omega} - 1\right) \tilde{v}_t^f = \frac{1}{\omega} \tilde{v}_{b,t}^f$$

$$\tilde{v}_{s,t}^h - \tilde{Q}_{r,t}^h = \tilde{v}_{d,t}^h$$

$$S^h \tilde{v}_{s,t}^h + S^h (1-\theta^h) \tilde{S}_t^h + B_r^d \tilde{v}_{d,t}^h + B_r^d (1-\theta^h) \tilde{B}_{r,t}^d - B_r^h \tilde{v}_{b,t}^h - D_r^h \tilde{v}_t^h - D_r^h \tilde{D}_{r,t}^h - B_r^h (1-\theta^h \omega) \tilde{B}_{r,t}^h = \theta^h S^h \tilde{Q}_{r,t}^h$$

$$S^f \tilde{v}_{s,t}^f + S^f (1-\theta^f) \tilde{S}_t^f - B_r^f \tilde{v}_{b,t}^f - F_r \tilde{v}_t^f - F_r \tilde{F}_{r,t} - B_r^f (1-\theta^f \omega) \tilde{B}_{r,t}^f = \theta^f S^f \tilde{Q}_{r,t}^f$$

$$D_r^h \tilde{D}_{r,t}^h = S^h \tilde{Q}_{r,t}^h + S^h \tilde{S}_t^h + B_r^d \tilde{B}_{r,t}^d - N_r^h \tilde{N}_{r,t}^h - B_r^h \tilde{B}_{r,t}^h$$

$$F_r \tilde{F}_{r,t} = S^f \tilde{Q}_{r,t}^f + S^f \tilde{S}_t^f - N_r^f \tilde{N}_{r,t}^f - B_r^f \tilde{B}_{r,t}^f$$

$$\tilde{B}_{r,t}^h + \tilde{P}_t^h - \tilde{\varepsilon}_t = \tilde{B}_{r,t}^f + \tilde{P}_t^f$$

$$N_r^h \tilde{N}_{r,t}^h = (\sigma + \xi^h) Z_r^h S^h \tilde{Z}_{r,t}^h + (\sigma + \xi^h) R^h S^h \tilde{\psi}_t^h + (\sigma + \xi^h) R^h S^h \tilde{S}_{t-1}^h + (\sigma + \xi^h) (1-\delta) S^h \tilde{Q}_{r,t}^h + \sigma (R^i B_r^h + R^h D_r^h - R^d B^d) \tilde{\pi}_t^h$$

$$- \sigma R^i B_r^h \tilde{\varepsilon}_t + \sigma R^i B_r^h \tilde{\varepsilon}_{t-1} - \sigma R^i B_r^h \tilde{R}_t^i - \sigma R^i B_r^h \tilde{B}_{r,t-1}^h - \sigma R^h D_r^h \tilde{R}_t^h - \sigma R^h D_r^h \tilde{D}_{r,t-1}^h + \sigma R^d B^d \tilde{R}_t^d + \sigma R^d B^d \tilde{B}_{t-1}^d$$

$$N_r^f \tilde{N}_{r,t}^f = (\sigma + \xi^f) Z_r^f S^f \tilde{Z}_{r,t}^f + (\sigma + \xi^f) R^f S^f \tilde{\psi}_t^f + (\sigma + \xi^f) R^f S^f \tilde{S}_{t-1}^f + (\sigma + \xi^f) (1-\delta) S^f \tilde{Q}_{r,t}^f$$

$$+ \sigma (R^i B_r^f + R^f F_r) \tilde{\pi}_t^f - \sigma R^i B_r^f \tilde{R}_t^i - \sigma R^i B_r^f \tilde{B}_{r,t-1}^f - \sigma R^f F_r \tilde{R}_t^f - \sigma R^f F_r \tilde{F}_{r,t-1}$$

$$v_t^h = \tilde{R}_{t+1}^h + \eta \tilde{C}_t^h - \eta E_t(\tilde{C}_{t+1}^h) - E_t(\tilde{\pi}_{t+1}^h) + \sigma (1-\phi^h (1-x_1^h + x_2^h)) E_t(\tilde{v}_{t+1}^h) + \sigma \phi^h E_t(\tilde{v}_{s,t+1}^h) - \sigma \phi^h E_t(\tilde{Q}_{r,t+1}^h)$$

$$- \sigma \phi^h x_1^h E_t(\tilde{v}_{b,t+1}^h) + \sigma \phi^h x_2^h E_t(\tilde{v}_{d,t+1}^h)$$

$$\tilde{v}_t^f = \tilde{R}_{t+1}^f + \eta \tilde{C}_t^f - \eta E_t(\tilde{C}_{t+1}^f) - E_t(\tilde{\pi}_{t+1}^f) + \sigma (1-\phi^f (1-x_r^f)) E_t(\tilde{v}_{t+1}^f)$$

$$+ \sigma \phi^f E_t(\tilde{v}_{s,t+1}^f) - \sigma \phi^f E_t(\tilde{Q}_{r,t+1}^f) - \sigma \phi^f x_r^f E_t(\tilde{v}_{b,t+1}^f)$$

$$\tilde{v}_{b,t}^h = \tilde{v}_t^h + \tilde{R}_{t+1}^i - \tilde{R}_{t+1}^h + E_t(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_t$$

$$\tilde{v}_{b,t}^f = \tilde{v}_t^f + \tilde{R}_{t+1}^i - \tilde{R}_{t+1}^f$$

$$\tilde{v}_{s,t}^h = \tilde{v}_t^h + E_t(\tilde{\pi}_{t+1}^h) + \frac{Z_r^h}{R^h} E_t(Z_{r,t+1}^h) + E_t(\tilde{\psi}_{t+1}^h) + \frac{(1-\delta)}{R^h} E_t(\tilde{Q}_{r,t+1}^h) - \tilde{R}_{t+1}^h$$

$$\tilde{v}_{s,t}^f = \tilde{v}_t^f + E_t(\tilde{\pi}_{t+1}^f) + \frac{Z_r^f}{R^f} E_t(Z_{r,t+1}^f) + E_t(\tilde{\psi}_{t+1}^f) + \frac{(1-\delta)}{R^f} E_t(\tilde{Q}_{r,t+1}^f) - \tilde{R}_{t+1}^f$$

$$\tilde{v}_{d,t}^h = \tilde{v}_t^h + \tilde{R}_{t+1}^d - \tilde{R}_{t+1}^h$$

$$Y^h \tilde{Y}_t^h = C_H^h \tilde{C}_{H,t}^h + C_H^f \tilde{C}_{H,t}^f + I^h \tilde{I}_t^h + G^h \tilde{G}_t^h$$

$$Y^f \tilde{Y}_t^f = C_F^f \tilde{C}_{F,t}^f + C_F^h \tilde{C}_{F,t}^h + I^f \tilde{I}_t^f + G^f \tilde{G}_t^f$$

$$\tilde{T}_{r,t}^h = \tilde{P}_{H,t}^h - \tilde{P}_t^h + \tilde{Y}_t^h + \gamma_1 \frac{B_r^d}{T_r^h} \tilde{B}_{r,t-1}^d$$

$$\tilde{T}_{r,t}^f = \tilde{P}_{F,t}^f - \tilde{P}_t^f + \tilde{G}_t^f$$

$$T_r^h \tilde{T}_{r,t}^h + B_r^d \tilde{B}_{r,t}^d = G^h \tilde{P}_{H,t}^h - G^h \tilde{P}_t^h + G^h \tilde{G}_t^h + R^d B_r^d \tilde{R}_t^d - R^d B_r^d \tilde{\pi}_t^h + R^d B_r^d \tilde{B}_{r,t-1}^d$$

$$\tilde{S}_t^h = \delta \tilde{I}_t^h + (1-\delta) \tilde{K}_t^h$$

$$\tilde{S}_t^f = \delta \tilde{I}_t^f + (1-\delta) \tilde{K}_t^f$$

$$\tilde{R}_{t+1}^h = \phi_\pi^h \tilde{\pi}_t^h + \phi_y^h \tilde{Y}_t^h + \lambda_t^h$$

$$\tilde{R}_{t+1}^f = \phi_\pi^f \tilde{\pi}_t^f + \phi_y^f \tilde{Y}_t^f + \lambda_t^f$$

$$\lambda_t^h = \rho_m \lambda_{t-1}^h + \epsilon_{m,t}^h$$

$$\lambda_t^f = \rho_m \lambda_{t-1}^f + \epsilon_{m,t}^f$$

$$\tilde{\psi}_t^h = \rho_k \tilde{\psi}_{t-1}^h + \epsilon_{k,t}^h$$

$$\tilde{\psi}_t^f = \rho_k \tilde{\psi}_{t-1}^f + \epsilon_{k,t}^f$$

$$\tilde{A}_t^h = \rho_a \tilde{A}_{t-1}^h + \epsilon_{a,t}^h$$

$$\tilde{A}_t^f = \rho_a \tilde{A}_{t-1}^f + \epsilon_{a,t}^f$$

Appendix C.5: summary of equilibrium conditions, extension 1, monetary union version of the model

The model comprises the 60 variables. The 55 state variables:

$$\{Y_t^h, Y_t^f, C_t^h, C_t^f, C_{H,t}^h, C_{F,t}^h, C_{F,t}^f, C_{H,t}^f, K_{t+1}^h, K_{t+1}^f, L_t^h, L_t^f, I_t^h, I_t^f, G_t^h, G_t^f, T_{r,t}^h, T_{r,t}^f, W_{r,t}^h, W_{r,t}^f, MC_{r,t}^h, MC_{r,t}^f, P_t^h, P_t^f, P_{H,t}, P_{F,t}, P_{H,t}^*, P_{F,t}^*, \pi_t^h, \pi_t^f, \pi_{H,t}, \pi_{F,t}, R_{t+1}, R_{t+1}^i, R_t^d, Q_{r,t}^h, Q_{r,t}^f, B_{r,t}^h, B_{r,t}^f, B_{r,t}^d, S_t^h, S_t^f, D_{r,t}^h, D_{r,t}^f, N_{r,t}^h, N_{r,t}^f, Z_{r,t}^h, Z_{r,t}^f, v_t^h, v_{b,t}^h, v_{s,t}^h, v_{d,t}^h, v_t^f, v_{b,t}^f, v_{s,t}^f\}$$

and the 5 stochastic variables:

$$\{\psi_t^h, \psi_t^f, A_t^h, A_t^f, \zeta_t\}$$

are then defined by the following 60 equations:

Consumer Price index:

$$P_t^h = \left(\varphi^h (P_{H,t})^{1-\mu} + (1-\varphi^h) (P_{F,t})^{1-\mu} \right)^{\frac{1}{1-\mu}} \quad (\text{c.1}')$$

$$P_t^f = \left(\varphi^f (P_{F,t})^{1-\mu} + (1-\varphi^f) (P_{H,t})^{1-\mu} \right)^{\frac{1}{1-\mu}} \quad (\text{c.2}')$$

Optimal index demand:

$$C_{H,t}^h = \varphi^h \left(\frac{P_t^h}{P_{H,t}} \right)^\mu C_t^h \quad (\text{c.3}')$$

$$C_{F,t}^h = (1-\varphi^h) \left(\frac{P_t^h}{P_{F,t}} \right)^\mu C_t^h \quad (\text{c.4}')$$

$$C_{F,t}^f = \varphi^f \left(\frac{P_t^f}{P_{F,t}} \right)^\mu C_t^f \quad (\text{c.5}')$$

$$C_{H,t}^f = (1-\varphi^f) \left(\frac{P_t^f}{P_{H,t}} \right)^\mu C_t^f \quad (\text{c.6}')$$

Optimal labour allocation:

$$\chi^h (L_t^h)^\phi (C_t^h)^\eta = W_{r,t}^h \quad (\text{c.7}')$$

$$\chi^f (L_t^f)^\phi (C_t^f)^\eta = W_{r,t}^f \quad (\text{c.8}')$$

Euler equation:

$$1 - \vartheta (D_{r,t}^h - D_r^h) = R_{t+1} \beta E_t \left[\left(\frac{C_t^h}{C_{t+1}^h} \right)^\eta \frac{1}{\pi_{t+1}^h} \right] \quad (\text{c.9}')$$

$$1 = R_{t+1} \beta E_t \left[\left(\frac{C_t^f}{C_{t+1}^f} \right)^\eta \frac{1}{\pi_{t+1}^f} \right] \quad (\text{c.10}')$$

Public expenditure:

$$G_t^h = \vartheta^h Y^h + \gamma_2 \vartheta^h (Y_t^h - Y^h) \quad (\text{c.11}')$$

$$G_t^f = \vartheta^f Y^f \quad (\text{c.12}')$$

Law of motion of capital:

$$K_{t+1}^h = \psi_{t+1}^h [I_t^h + (1-\delta)K_t^h] \quad (\text{c.13}')$$

$$K_{t+1}^f = \psi_{t+1}^f [I_t^f + (1-\delta)K_t^f] \quad (\text{c.14}')$$

Capital pricing:

$$Q_{r,t}^h = \frac{P_{H,t}}{P_t^h} \left[1 + \iota (I_t^h - I_{t-1}^h) \right] - \iota E_t \Lambda_{t,t+1}^h \frac{P_{H,t+1}}{P_{t+1}^h} (I_{t+1}^h - I_t^h) \quad (\text{c.15}')$$

$$Q_{r,t}^f = \frac{P_{F,t}}{P_t^f} \left[1 + \iota (I_t^f - I_{t-1}^f) \right] - \iota E_t \Lambda_{t,t+1}^f \frac{P_{F,t+1}}{P_{t+1}^f} (I_{t+1}^f - I_t^f) \quad (\text{c.16}')$$

Production technology:

$$Y_t^h(j) = A_t^h (K_t^h)^\alpha L_t^h(j)^{1-\alpha} \quad (\text{c.17}') \quad Y_t^f(j) = A_t^f (K_t^f)^\alpha L_t^f(j)^{1-\alpha} \quad (\text{c.18}')$$

Real wage setting:

$$MC_{r,t}^h = W_{r,t}^h \times \frac{P_t^h}{P_{H,t}} \times \frac{1}{MPL_t^h} \quad (\text{c.19}') \quad MC_{r,t}^f = W_{r,t}^f \times \frac{P_t^f}{P_{F,t}} \times \frac{1}{MPL_t^f} \quad (\text{c.20}')$$

Dividend:

$$Z_{r,t}^h = \frac{\frac{P_{H,t}}{P_t^h} Y_t^h - W_{r,t}^h L_t^h}{K_t^h} \quad (\text{c.21}') \quad Z_{r,t}^f = \frac{\frac{P_{F,t}}{P_t^f} Y_t^f - W_{r,t}^f L_t^f}{K_t^f} \quad (\text{c.22}')$$

Optimal pricing:

$$P_{H,t}^*(j) = \frac{\kappa}{\kappa - 1} \cdot \frac{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^h Y_{t+i|t}^h(j) \Psi_{t+i}^{h'}(Y_{t+i|t}^h(j)) \right\}}{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^h Y_{t+i|t}^h(j) \right\}} \quad (\text{c.23}')$$

$$P_{F,t}^*(j) = \frac{\kappa}{\kappa - 1} \cdot \frac{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^f Y_{t+i|t}^f(j) \Psi_{t+i}^{f'}(Y_{t+i|t}^f(j)) \right\}}{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^f Y_{t+i|t}^f(j) \right\}} \quad (\text{c.24}')$$

Law of motion of producer prices:

$$(\pi_{H,t})^{1-\kappa} = \zeta + (1-\zeta) \left(\frac{P_{H,t}^*}{P_{H,t-1}} \right)^{1-\kappa} \quad (\text{c.25}') \quad (\pi_{F,t})^{1-\kappa} = \zeta + (1-\zeta) \left(\frac{P_{F,t}^*}{P_{F,t-1}} \right)^{1-\kappa} \quad (\text{c.26}')$$

PPI inflation:

$$\pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}} \quad (\text{c.27}') \quad \pi_{F,t} = \frac{P_{F,t}}{P_{F,t-1}} \quad (\text{c.28}')$$

CPI inflation:

$$\pi_t^h = \frac{P_t^h}{P_{t-1}^h} \quad (\text{c.29}') \quad \pi_t^f = \frac{P_t^f}{P_{t-1}^f} \quad (\text{c.30}')$$

Optimal security allocation:

$$\left(\frac{v_{s,t}^h}{Q_{r,t}^h} - v_t^h \right) = \frac{1}{\omega} (v_{b,t}^h - v_t^h) \quad (\text{c.31}') \quad \left(\frac{v_{s,t}^f}{Q_{r,t}^f} - v_t^f \right) = \frac{1}{\omega} (v_{b,t}^f - v_t^f) \quad (\text{c.32}')$$

Optimal government loans allocation:

$$\frac{v_{s,t}^h}{Q_{r,t}^h} = v_{d,t}^h \quad (\text{c.33}')$$

Incentive constraint:

$$v_{s,t}^h S_t^h + v_{d,t}^h B_{r,t}^d - v_{b,t}^h B_{r,t}^h - v_t^h D_{r,t}^h = \theta^h [Q_{r,t}^h S_t^h + B_{r,t}^d - \omega B_{r,t}^h] \quad (\text{c.34}')$$

$$v_{s,t}^f S_t^f - v_{b,t}^f B_{r,t}^f - v_t^f D_{r,t}^f = \theta^f [Q_{r,t}^f S_t^f - \omega B_{r,t}^f] \quad (\text{c.35}')$$

Security market clearing:

$$D_{r,t}^h = Q_{r,t}^h S_t^h + B_{r,t}^d - N_{r,t}^h - B_{r,t}^h \quad (\text{c.36}') \quad D_{r,t}^f = Q_{r,t}^f S_t^f - N_{r,t}^f - B_{r,t}^f \quad (\text{c.37}')$$

Asset market clearing:

$$B_{r,t}^h P_t^h = -B_{r,t}^f P_t^f \quad (\text{c.38}')$$

Evolution of net worth:

$$N_{r,t}^h = (\sigma + \xi^h) \{ [Z_{r,t}^h + (1-\delta)Q_{r,t}^h] \psi_t^h S_{t-1}^h \} - \frac{\sigma}{\pi_t^h} \{ R_t^i B_{r,t-1}^h + R_t D_{r,t-1}^h - R_t^d B_{r,t-1}^d \} \quad (\text{c.39}')$$

$$N_{r,t}^f = (\sigma + \xi^f) \{ [Z_{r,t}^f + (1-\delta)Q_{r,t}^f] \psi_t^f S_{t-1}^f \} - \frac{\sigma}{\pi_t^f} \{ R_t^i B_{r,t-1}^f + R_t D_{r,t-1}^f \} \quad (\text{c.40}')$$

Shadow price of deposits:

$$v_t^h = R_{t+1} E_t \left(\frac{\Lambda_{r,t,t+1}^h \Omega_{t+1}^h}{\pi_{t+1}^h} \right) \quad (\text{c.41}') \quad v_t^f = R_{t+1} E_t \left(\frac{\Lambda_{r,t,t+1}^f \Omega_{t+1}^f}{\pi_{t+1}^f} \right) \quad (\text{c.42}')$$

Shadow price of interbank borrowing:

$$v_{b,t}^h = v_t^h \frac{R_{t+1}^i}{R_{t+1}} \quad (\text{c.43}') \quad v_{b,t}^f = v_t^f \frac{R_{t+1}^i}{R_{t+1}} \quad (\text{c.44}')$$

Shadow value of assets:

$$v_{s,t}^h = v_t^h E_t \left(\frac{\pi_{t+1}^h [Z_{r,t+1}^h + (1-\delta)Q_{r,t+1}^h] \psi_{t+1}^h}{R_{t+1}} \right) \quad (\text{c.45}') \quad v_{s,t}^f = v_t^f E_t \left(\frac{\pi_{t+1}^f [Z_{r,t+1}^f + (1-\delta)Q_{r,t+1}^f] \psi_{t+1}^f}{R_{t+1}} \right) \quad (\text{c.46}')$$

Shadow value of government loans:

$$v_{d,t}^h = v_t^h \frac{R_{t+1}^d}{R_{t+1}} \quad (\text{c.47}')$$

Final goods market clearing:

$$Y_t^h = C_{H,t}^h + C_{H,t}^f + I_t^h + G_t^h \quad (\text{c.48}') \quad Y_t^f = C_{F,t}^f + C_{F,t}^h + I_t^f + G_t^f \quad (\text{c.49}')$$

Taxation:

$$T_{r,t}^h = \frac{P_{H,t}}{P_t^h} \tau^h Y_t^h + \gamma_1 (B_{r,t-1}^d - B_{r,t}^d) \quad (\text{c.50}') \quad \frac{P_{F,t}}{P_t^f} G_t^f = T_{r,t}^f \quad (\text{c.51}')$$

Government budget constraint:

$$T_{r,t}^h + B_{r,t}^d = \frac{P_{H,t}}{P_t^h} G_t^h + \frac{R_t^d}{\pi_t^h} B_{r,t-1}^d \quad (\text{c.52}')$$

Concordance of assets with aggregate capital:

$$S_t^h = I_t^h + (1 - \delta)K_t^h \quad (\text{c.53}') \quad S_t^f = I_t^f + (1 - \delta)K_t^f \quad (\text{c.54}')$$

Monetary policy:

$$\log(R_{t+1}) = \log(R_t) + \phi_\pi \left(\gamma \log \pi_{H,t} + (1 - \gamma) \log \pi_{F,t} \right) + \phi_y \left(\gamma \log \left(\frac{Y_t^h}{Y^h} \right) + (1 - \gamma) \log \left(\frac{Y_t^f}{Y^f} \right) \right) + \zeta_t \quad (\text{c.55}')$$

Stochastic processes:

$$\zeta_t = \rho_m \zeta_{t-1} + \epsilon_{m,t} \quad (\text{c.56}')$$

$$\psi_t^h = (\bar{\psi}^h)^{1-\rho_k} (\psi_{t-1}^h)^{\rho_k} e^{\epsilon_{k,t}^h} \quad (\text{c.57}') \quad \psi_t^f = (\bar{\psi}^f)^{1-\rho_k} (\psi_{t-1}^f)^{\rho_k} e^{\epsilon_{k,t}^f} \quad (\text{c.58}')$$

$$A_t^h = (\bar{A}^h)^{1-\rho_a} (A_{t-1}^h)^{\rho_a} e^{\epsilon_{a,t}^h} \quad (\text{c.59}') \quad A_t^f = (\bar{A}^f)^{1-\rho_a} (A_{t-1}^f)^{\rho_a} e^{\epsilon_{a,t}^f} \quad (\text{c.60}')$$

Appendix C.6: summary of linearized equilibrium conditions, extension 1, monetary union version of the model

The model comprises the 56 variables. The 51 state variables:

$$\{\tilde{Y}_t^h, \tilde{Y}_t^f, \tilde{C}_t^h, \tilde{C}_t^f, \tilde{C}_{H,t}^h, \tilde{C}_{F,t}^h, \tilde{C}_{F,t}^f, \tilde{C}_{H,t}^f, \tilde{K}_{t+1}^h, \tilde{K}_{t+1}^f, \tilde{L}_t^h, \tilde{L}_t^f, \tilde{I}_t^h, \tilde{I}_t^f, \tilde{G}_t^h, \tilde{G}_t^f, \tilde{T}_{r,t}^h, \tilde{T}_{r,t}^f, \tilde{W}_{r,t}^h, \tilde{W}_{r,t}^f, \tilde{P}_t^h, \tilde{P}_t^f, \tilde{P}_{H,t}^h, \tilde{P}_{F,t}^h, \tilde{\pi}_t^h, \tilde{\pi}_t^f, \tilde{\pi}_{H,t}^h, \tilde{\pi}_{F,t}^h, \tilde{R}_{t+1}^h, \tilde{R}_{t+1}^f, \tilde{R}_{t+1}^d, \tilde{Q}_{r,t}^h, \tilde{Q}_{r,t}^f, \tilde{B}_{r,t}^h, \tilde{B}_{r,t}^f, \tilde{B}_{r,t}^d, \tilde{S}_t^h, \tilde{S}_t^f, \tilde{D}_{r,t}^h, \tilde{D}_{r,t}^f, \tilde{N}_{r,t}^h, \tilde{N}_{r,t}^f, \tilde{Z}_{r,t}^h, \tilde{Z}_{r,t}^f, \tilde{v}_t^h, \tilde{v}_{b,t}^h, \tilde{v}_{s,t}^h, \tilde{v}_{d,t}^h, \tilde{v}_t^f, \tilde{v}_{b,t}^f, \tilde{v}_{s,t}^f, \tilde{v}_{d,t}^f\}$$

and the 5 stochastic variables:

$$\{\psi_t^h, \psi_t^f, A_t^h, A_t^f, \varsigma_t\}$$

are then defined by the following 56 equations:

$$\begin{aligned} \tilde{P}_t^h &= \varphi^h \tilde{P}_{H,t} + (1 - \varphi^h) \tilde{P}_{F,t} \\ \tilde{P}_t^f &= \varphi^f \tilde{P}_{F,t} + (1 - \varphi^f) \tilde{P}_{H,t} \\ \tilde{C}_{H,t}^h &= \mu \tilde{P}_t^h - \mu \tilde{P}_{H,t} + \tilde{C}_t^h \\ \tilde{C}_{F,t}^h &= \mu \tilde{P}_t^h - \mu \tilde{P}_{F,t} + \tilde{C}_t^h \\ \tilde{C}_{F,t}^f &= \mu \tilde{P}_t^f - \mu \tilde{P}_{F,t} + \tilde{C}_t^f \\ \tilde{C}_{H,t}^f &= \mu \tilde{P}_t^f - \mu \tilde{P}_{H,t} + \tilde{C}_t^f \\ \tilde{W}_{r,t}^h &= \phi \tilde{L}_t^h - \eta \tilde{C}_t^h \\ \tilde{W}_{r,t}^f &= \phi \tilde{L}_t^f - \eta \tilde{C}_t^f \\ -\partial D_r^h \tilde{D}_{r,t}^h &= \tilde{R}_{t+1}^h + \eta \tilde{C}_t^h - \eta \tilde{C}_{t+1}^h - \tilde{\pi}_{t+1}^h \\ 0 &= \tilde{R}_{t+1}^f + \eta \tilde{C}_t^f - \eta \tilde{C}_{t+1}^f - \tilde{\pi}_{t+1}^f \\ \tilde{G}_t^h &= \gamma_2 \tilde{Y}_t^h \\ \tilde{G}_t^f &= 0 \\ \tilde{K}_{t+1}^h &= \tilde{\psi}_{t+1}^h + \delta \tilde{I}_t^h + (1 - \delta) \tilde{K}_t^h \\ \tilde{K}_{t+1}^f &= \tilde{\psi}_{t+1}^f + \delta \tilde{I}_t^f + (1 - \delta) \tilde{K}_t^f \\ \tilde{Q}_{r,t}^h &= \tilde{P}_{H,t}^h - \tilde{P}_t^h + U^h (1 + \beta) \tilde{I}_t^h - U^h \tilde{I}_{t-1}^h - \iota \beta I^h E_t \tilde{I}_{t+1}^h \\ \tilde{Q}_{r,t}^f &= \tilde{P}_{F,t}^f - \tilde{P}_t^f + U^f (1 + \beta) \tilde{I}_t^f - U^f \tilde{I}_{t-1}^f - \iota \beta I^f E_t \tilde{I}_{t+1}^f \\ \tilde{Y}_t^h &= \tilde{A}_t^h + \alpha \tilde{K}_t^h + (1 - \alpha) \tilde{L}_t^h \\ \tilde{Y}_t^f &= \tilde{A}_t^f + \alpha \tilde{K}_t^f + (1 - \alpha) \tilde{L}_t^f \\ Z_r^h K^h \tilde{Z}_{r,t}^h &= Y^h \tilde{P}_{H,t}^h - Y^h \tilde{P}_t^h + Y^h \tilde{Y}_t^h - W_r^h L^h \tilde{W}_{r,t}^h - W_r^h L^h \tilde{L}_t^h - Z_r^h K^h \tilde{K}_t^h \\ Z_r^f K^f \tilde{Z}_{r,t}^f &= Y^f \tilde{P}_{F,t}^f - Y^f \tilde{P}_t^f + Y^f \tilde{Y}_t^f - W_r^f L^f \tilde{W}_{r,t}^f - W_r^f L^f \tilde{L}_t^f - Z_r^f K^f \tilde{K}_t^f \\ \tilde{\pi}_{H,t} &= \beta E_t (\tilde{\pi}_{H,t+1}) + \varpi \tilde{W}_{r,t}^h + \varpi \tilde{P}_t^h - \varpi \tilde{P}_{H,t} - \frac{\varpi}{1 - \alpha} \tilde{A}_t^h - \frac{\alpha \varpi}{1 - \alpha} \tilde{K}_t^h + \frac{\alpha \varpi}{1 - \alpha} \tilde{Y}_t^h \\ \tilde{\pi}_{F,t} &= \beta E_t (\tilde{\pi}_{F,t+1}) + \varpi \tilde{W}_{r,t}^f + \varpi \tilde{P}_t^f - \varpi \tilde{P}_{F,t} - \frac{\varpi}{1 - \alpha} \tilde{A}_t^f - \frac{\alpha \varpi}{1 - \alpha} \tilde{K}_t^f + \frac{\alpha \varpi}{1 - \alpha} \tilde{Y}_t^f \end{aligned}$$

$$\text{with : } \varpi \equiv \frac{(1 - \zeta)(1 - \zeta \beta)(1 - \alpha)}{\zeta(1 - \alpha + \alpha \kappa)}$$

$$\tilde{\pi}_{H,t} = \tilde{P}_{H,t} - \tilde{P}_{H,t-1}$$

$$\tilde{\pi}_{F,t} = \tilde{P}_{F,t} - \tilde{P}_{F,t-1}$$

$$\begin{aligned}
\tilde{\pi}_t^h &= \tilde{P}_t^h - \tilde{P}_{t-1}^h \\
\tilde{\pi}_t^f &= \tilde{P}_t^f - \tilde{P}_{t-1}^f \\
\tilde{v}_{s,t}^h - \tilde{Q}_{r,t}^h + \left(\frac{1}{\omega} - 1\right) \tilde{v}_t^h &= \frac{1}{\omega} \tilde{v}_{b,t}^h \\
\tilde{v}_{s,t}^f - \tilde{Q}_{r,t}^f + \left(\frac{1}{\omega} - 1\right) \tilde{v}_t^f &= \frac{1}{\omega} \tilde{v}_{b,t}^f \\
\tilde{v}_{s,t}^h - \tilde{Q}_{r,t}^h &= \tilde{v}_{d,t}^h \\
S^h \tilde{v}_{s,t}^h + S^h (1 - \theta^h) \tilde{S}_t^h - B_r^h \tilde{v}_{b,t}^h - D_r^h \tilde{v}_t^h - D_r^h \tilde{D}_{r,t}^h - B_r^h (1 - \theta^h \omega) \tilde{B}_{r,t}^h + B_r^d \tilde{v}_{d,t}^h + B_r^d (1 - \theta^h) \tilde{B}_{r,t}^d &= \theta^h S^h \tilde{Q}_{r,t}^h \\
S^f \tilde{v}_{s,t}^f + S^f (1 - \theta^f) \tilde{S}_t^f - B_r^f \tilde{v}_{b,t}^f - D_r^f \tilde{v}_t^f - D_r^f \tilde{D}_{r,t}^f - B_r^f (1 - \theta^f \omega) \tilde{B}_{r,t}^f &= \theta^f S^f \tilde{Q}_{r,t}^f \\
D_r^h \tilde{D}_{r,t}^h &= S^h \tilde{Q}_{r,t}^h + S^h \tilde{S}_t^h + B_r^d \tilde{B}_{r,t}^d - N_r^h \tilde{N}_{r,t}^h - B_r^h \tilde{B}_{r,t}^h \\
D_r^f \tilde{D}_{r,t}^f &= S^f \tilde{Q}_{r,t}^f + S^f \tilde{S}_t^f - N_r^f \tilde{N}_{r,t}^f - B_r^f \tilde{B}_{r,t}^f \\
B_{r,t}^h + P_t^h &= B_{r,t}^f + P_t^f \\
N_r^h \tilde{N}_{r,t}^h &= (\sigma + \xi^h) Z_r^h S^h \tilde{Z}_{r,t}^h + (\sigma + \xi^h) R S^h \tilde{\psi}_t^h + (\sigma + \xi^h) R S^h \tilde{S}_{t-1}^h + (\sigma + \xi^h) (1 - \delta) S^h \tilde{Q}_{r,t}^h \\
&+ \sigma (R^i B_r^h + R D_r^h - R^d B^d) \tilde{\pi}_t^h - \sigma R^i B_r^h \tilde{R}_t^i - \sigma R^i B_r^h \tilde{B}_{r,t-1}^h - \sigma R D_r^h \tilde{R}_t - \sigma R D_r^h \tilde{D}_{r,t-1}^h + \sigma R^d B^d \tilde{R}_t^d + \sigma R^d B^d \tilde{B}_{r,t-1}^d \\
N_r^f \tilde{N}_{r,t}^f &= (\sigma + \xi^f) Z_r^f S^f \tilde{Z}_{r,t}^f + (\sigma + \xi^f) R S^f \tilde{\psi}_t^f + (\sigma + \xi^f) R S^f \tilde{S}_{t-1}^f + (\sigma + \xi^f) (1 - \delta) S^f \tilde{Q}_{r,t}^f \\
&+ \sigma (R^i B_r^f + R D_r^f) \tilde{\pi}_t^f - \sigma R^i B_r^f \tilde{R}_t^i - \sigma R^i B_r^f \tilde{B}_{r,t-1}^f - \sigma R D_r^f \tilde{R}_t - \sigma R D_r^f \tilde{D}_{r,t-1}^f \\
v_t^h &= \tilde{R}_{t+1}^h + \eta \tilde{C}_t^h - \eta E_t(\tilde{C}_{t+1}^h) - E_t(\tilde{\pi}_{t+1}^h) + \sigma (1 - \phi^h (1 - x_1^h + x_2^h)) E_t(\tilde{v}_{t+1}^h) + \sigma \phi^h E_t(\tilde{v}_{s,t+1}^h) \\
&- \sigma \phi^h E_t(\tilde{Q}_{r,t+1}^h) - \sigma \phi^h x_1^h E_t(\tilde{v}_{b,t+1}^h) + \sigma \phi^h x_2^h E_t(\tilde{v}_{d,t+1}^h) \\
\tilde{v}_t^f &= \tilde{R}_{t+1}^f + \eta \tilde{C}_t^f - \eta E_t(\tilde{C}_{t+1}^f) - E_t(\tilde{\pi}_{t+1}^f) + \sigma (1 - \phi^f (1 - x_r^f)) E_t(\tilde{v}_{t+1}^f) \\
&+ \sigma \phi^f E_t(\tilde{v}_{s,t+1}^f) - \sigma \phi^f E_t(\tilde{Q}_{r,t+1}^f) - \sigma \phi^f x_r^f E_t(\tilde{v}_{b,t+1}^f) \\
\tilde{v}_{b,t}^h &= \tilde{v}_t^h + \tilde{R}_{t+1}^h - \tilde{R}_{t+1} \\
\tilde{v}_{b,t}^f &= \tilde{v}_t^f + \tilde{R}_{t+1}^f - \tilde{R}_{t+1} \\
\tilde{v}_{s,t}^h &= \tilde{v}_t^h + E_t(\tilde{\pi}_{t+1}^h) + \frac{Z_r^h}{R} E_t(Z_{r,t+1}^h) + E_t(\tilde{\psi}_{t+1}^h) + \frac{(1 - \delta)}{R} E_t(\tilde{Q}_{r,t+1}^h) - \tilde{R}_{t+1} \\
\tilde{v}_{s,t}^f &= \tilde{v}_t^f + E_t(\tilde{\pi}_{t+1}^f) + \frac{Z_r^f}{R} E_t(Z_{r,t+1}^f) + E_t(\tilde{\psi}_{t+1}^f) + \frac{(1 - \delta)}{R} E_t(\tilde{Q}_{r,t+1}^f) - \tilde{R}_{t+1} \\
\tilde{v}_{d,t}^h &= \tilde{v}_t^h + \tilde{R}_{t+1}^d - \tilde{R}_{t+1} \\
Y^h \tilde{Y}_t^h &= C_H^h \tilde{C}_{H,t}^h + C_H^f \tilde{C}_{H,t}^f + I^h \tilde{I}_t^h + G^h \tilde{G}_t^h \\
Y^f \tilde{Y}_t^f &= C_F^f \tilde{C}_{F,t}^f + C_F^h \tilde{C}_{F,t}^h + I^f \tilde{I}_t^f + G^f \tilde{G}_t^f \\
\tilde{T}_{r,t}^h &= \tilde{P}_{H,t}^h - \tilde{P}_t^h + \tilde{Y}_t^h + \gamma_1 \frac{B_r^d}{T_r^h} \tilde{B}_{r,t-1}^d \\
\tilde{T}_{r,t}^f &= \tilde{P}_{F,t}^f - \tilde{P}_t^f + \tilde{G}_t^f \\
T_r^h \tilde{T}_{r,t}^h + B_r^d \tilde{B}_{r,t}^d &= G^h \tilde{P}_{H,t}^h - G^h \tilde{P}_t^h + G^h \tilde{G}_t^h + R^d B_r^d \tilde{R}_t^d - R^d B_r^d \tilde{\pi}_t^h + R^d B_r^d \tilde{B}_{r,t-1}^d \\
\tilde{S}_t^h &= \delta \tilde{I}_t^h + (1 - \delta) \tilde{K}_t^h \\
\tilde{S}_t^f &= \delta \tilde{I}_t^f + (1 - \delta) \tilde{K}_t^f \\
\tilde{R}_{t+1} &= \phi_\pi \gamma \tilde{\pi}_{H,t} + \phi_\pi (1 - \gamma) \tilde{\pi}_{F,t} + \phi_\gamma \tilde{Y}_t^h + \phi_\gamma (1 - \gamma) \tilde{Y}_t^f + \zeta_t \\
\zeta_t &= \rho_m \zeta_{t-1} + \epsilon_{m,t} \\
\tilde{\psi}_t^h &= \rho_k \tilde{\psi}_{t-1}^h + \epsilon_{k,t}^h \\
\tilde{\psi}_t^f &= \rho_k \tilde{\psi}_{t-1}^f + \epsilon_{k,t}^f \\
\tilde{A}_t^h &= \rho_a \tilde{A}_{t-1}^h + \epsilon_{a,t}^h \\
\tilde{A}_t^f &= \rho_a \tilde{A}_{t-1}^f + \epsilon_{a,t}^f
\end{aligned}$$

Appendix C.7: Steady-state for the first extension

From a procedure similar to the one of the analytical solution for the benchmark model, one deduces that $v^h = v_b^h = v_s^h = 1$. Then, from (c.35), one obtains

$$v_d^h = 1 \quad (\text{C.15})$$

From (C.15) and (c.49), it is straightforward to obtain:

$$R^d = R = \frac{1}{\beta} \quad (\text{C.16})$$

From (c.13), one easily obtains:

$$G^h = \vartheta^h Y^h \quad (\text{C.17})$$

And from (c.52), it easily follows that:

$$T_r^h = \tau^h Y^h \quad (\text{C.18})$$

Then, combining (c.54), (C16), (C.17) and (C.18) yields:

$$B_r^d = \left(\frac{(\tau^h - \vartheta^h)\beta}{1 - \beta} \right) Y^h \quad (\text{C.19})$$

This implies that one can calibrate τ^h to obtain any desired steady-state value for the debt/GDP ratio $\frac{B_r^d}{Y^h}$, by setting:

$$\tau^h = \vartheta^h + \frac{B_r^d}{Y^h} \left(\frac{1 - \beta}{\beta} \right) \quad (\text{C.20})$$

It can be shown that the other steady-state values are the same as in the benchmark model, except for the value of D_r^h , and the calibration of the two parameters θ^h and θ^f .

Appendix C.8: summary of equilibrium conditions, extension 2

The model comprises the 59 variables. The 54 state variables:

$$\{Y_t^h, Y_t^f, C_t^h, C_t^f, C_{H,t}^h, C_{F,t}^h, C_{H,t}^f, C_{F,t}^f, K_{t+1}^h, K_{t+1}^f, L_t^h, L_t^f, I_t^h, I_t^f, G_t^h, G_t^f, T_{r,t}^h, T_{r,t}^f, W_{r,t}^h, W_{r,t}^f, MC_{r,t}^h, MC_{r,t}^f, P_t^h, P_t^f, P_{H,t}^h, P_{F,t}^h, P_{H,t}^f, P_{F,t}^f, \pi_t^h, \pi_t^f, \pi_{H,t}^h, \pi_{F,t}^h, R_{t+1}^h, R_{t+1}^f, Q_{r,t}^h, Q_{r,t}^f, B_{r,t}^h, B_{r,t}^f, B_{r,t}^c, S_t^h, S_t^f, D_{r,t}^h, D_{r,t}^f, N_{r,t}^h, N_{r,t}^f, Z_{r,t}^h, Z_{r,t}^f, v_t^h, v_{b,t}^h, v_{s,t}^h, v_{c,t}^h, v_t^f, v_{b,t}^f, v_{s,t}^f\}$$

and the 5 stochastic variables:

$$\{\psi_t^h, \psi_t^f, A_t^h, A_t^f, \zeta_t\}$$

are then defined by the following 57 equations:

Consumer Price index:

$$P_t^h = (\varphi^h (P_{H,t}^h)^{1-\mu} + (1-\varphi^h)(P_{F,t}^h)^{1-\mu})^{\frac{1}{1-\mu}} \quad (1^*) \quad P_t^f = (\varphi^f (P_{F,t}^f)^{1-\mu} + (1-\varphi^f)(P_{H,t}^f)^{1-\mu})^{\frac{1}{1-\mu}} \quad (2^*)$$

Optimal index demand:

$$C_{H,t}^h = \varphi^h \left(\frac{P_t^h}{P_{H,t}^h} \right)^\mu C_t^h \quad (3^*) \quad C_{F,t}^h = (1-\varphi^h) \left(\frac{P_t^h}{P_{F,t}^h} \right)^\mu C_t^h \quad (4^*)$$

$$C_{F,t}^f = \varphi^f \left(\frac{P_t^f}{P_{F,t}^f} \right)^\mu C_t^f \quad (5^*) \quad C_{H,t}^f = (1-\varphi^f) \left(\frac{P_t^f}{P_{H,t}^f} \right)^\mu C_t^f \quad (6^*)$$

Optimal labour allocation:

$$\chi^h (L_t^h)^\phi (C_t^h)^\eta = W_{r,t}^h \quad (7^*) \quad \chi^f (L_t^f)^\phi (C_t^f)^\eta = W_{r,t}^f \quad (8^*)$$

Euler equation:

$$1 - \vartheta (D_{r,t}^h - D_r^h) = R_{t+1} \beta E_t \left[\left(\frac{C_t^h}{C_{t+1}^h} \right)^\eta \frac{1}{\pi_{t+1}^h} \right] \quad (9^*) \quad 1 = R_{t+1} \beta E_t \left[\left(\frac{C_t^f}{C_{t+1}^f} \right)^\eta \frac{1}{\pi_{t+1}^f} \right] \quad (10^*)$$

Public expenditure:

$$G_t^h = \vartheta^h Y^h \quad (11^*) \quad G_t^f = \vartheta^f Y^f \quad (12^*)$$

Law of motion of capital:

$$K_{t+1}^h = \psi_{t+1}^h [I_t^h + (1-\delta)K_t^h] \quad (13^*) \quad K_{t+1}^f = \psi_{t+1}^f [I_t^f + (1-\delta)K_t^f] \quad (14^*)$$

Capital pricing:

$$Q_{r,t}^h = \frac{P_{H,t}^h}{P_t^h} \left[1 + \iota (I_t^h - I_{t-1}^h) \right] - \iota E_t \Lambda_{r,t,t+1}^h \frac{P_{H,t+1}^h}{P_{t+1}^h} (I_{t+1}^h - I_t^h) \quad (15^*)$$

$$Q_{r,t}^f = \frac{P_{F,t}^f}{P_t^f} \left[1 + \iota (I_t^f - I_{t-1}^f) \right] - \iota E_t \Lambda_{r,t,t+1}^f \frac{P_{F,t+1}^f}{P_{t+1}^f} (I_{t+1}^f - I_t^f) \quad (16^*)$$

Production technology:

$$Y_t^h(j) = A_t^h (K_t^h)^\alpha L_t^h(j)^{1-\alpha} \quad (17^*) \quad Y_t^f(j) = A_t^f (K_t^f)^\alpha L_t^f(j)^{1-\alpha} \quad (18^*)$$

Real wage setting:

$$MC_{r,t}^h = W_{r,t}^h \times \frac{P_t^h}{P_{H,t}^h} \times \frac{1}{MPL_{L_t}^h} \quad (19^*)$$

$$MC_{r,t}^f = W_{r,t}^f \times \frac{P_t^f}{P_{F,t}^f} \times \frac{1}{MPL_{L_t}^f} \quad (20^*)$$

Dividend:

$$Z_{r,t}^h = \frac{\frac{P_{H,t}^h}{P_t^h} Y_t^h - W_{r,t}^h L_t^h}{K_t^h} \quad (21^*)$$

$$Z_{r,t}^f = \frac{\frac{P_{F,t}^f}{P_t^f} Y_t^f - W_{r,t}^f L_t^f}{K_t^f} \quad (22^*)$$

Optimal pricing:

$$P_{H,t}^*(j) = \frac{\kappa}{\kappa-1} \frac{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^h Y_{t+i|t}^h(j) \Psi_{t+i}^h(Y_{t+i|t}^h(j)) \right\}}{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^h Y_{t+i|t}^h(j) \right\}} \quad (23^*)$$

$$P_{F,t}^*(j) = \frac{\kappa}{\kappa-1} \frac{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^f Y_{t+i|t}^f(j) \Psi_{t+i}^f(Y_{t+i|t}^f(j)) \right\}}{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^f Y_{t+i|t}^f(j) \right\}} \quad (24^*)$$

Law of motion of producer prices:

$$(\pi_{H,t})^{1-\kappa} = \zeta + (1-\zeta) \left(\frac{P_{H,t}^*}{P_{H,t-1}} \right)^{1-\kappa} \quad (25^*)$$

$$(\pi_{F,t})^{1-\kappa} = \zeta + (1-\zeta) \left(\frac{P_{F,t}^*}{P_{F,t-1}} \right)^{1-\kappa} \quad (26^*)$$

PPI inflation:

$$\pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}} \quad (27^*)$$

$$\pi_{F,t} = \frac{P_{F,t}}{P_{F,t-1}} \quad (28^*)$$

CPI inflation:

$$\pi_t^h = \frac{P_t^h}{P_{t-1}^h} \quad (29^*)$$

$$\pi_t^f = \frac{P_t^f}{P_{t-1}^f} \quad (30^*)$$

Optimal security allocation:

$$\left(\frac{v_{s,t}^h}{Q_{r,t}^h} - v_t^h \right) = \frac{1}{\omega} (v_{b,t}^h - v_t^h) \quad (31^*)$$

$$\left(\frac{v_{s,t}^f}{Q_{r,t}^f} - v_t^f \right) = \frac{1}{\omega} (v_{b,t}^f - v_t^f) \quad (32^*)$$

Incentive constraint:

$$v_{s,t}^h S_t^h - v_{b,t}^h B_{r,t}^h - v_{c,t}^h B_{r,t}^c - v_t^h D_{r,t}^h = \theta^h \left[Q_{r,t}^h S_t^h - \omega B_{r,t}^h - B_{r,t}^c \right] \quad (33^*)$$

$$v_{s,t}^f S_t^f - v_{b,t}^f B_{r,t}^f - v_t^f D_{r,t}^f = \theta^f \left[Q_{r,t}^f S_t^f - \omega B_{r,t}^f \right] \quad (34^*)$$

Security market clearing:

$$D_{r,t}^h = Q_{r,t}^h S_t^h - N_{r,t}^h - B_{r,t}^h - B_{r,t}^c \quad (35^*)$$

$$D_{r,t}^f = Q_{r,t}^f S_t^f - N_{r,t}^f - B_{r,t}^f \quad (36^*)$$

Asset market clearing:

$$B_{r,t}^h P_t^h = -B_{r,t}^f P_t^f \quad (37^*)$$

Covered bonds determination:

$$B_{r,t}^c = \varphi B_r^h (R_{t+1}^i - R_{t+1}) \quad (38^*)$$

Evolution of net worth:

$$N_{r,t}^h = (\sigma + \xi^h) \{ [Z_{r,t}^h + (1-\delta)Q_{r,t}^h] \psi_t^h S_{t-1}^h \} - \frac{\sigma}{\pi_t^h} \{ R_t^i (B_{r,t-1}^h + B_{r,t-1}^c) + R_t D_{r,t-1}^h \} \quad (39^*)$$

$$N_{r,t}^f = (\sigma + \xi^f) \{ [Z_{r,t}^f + (1-\delta)Q_{r,t}^f] \psi_t^f S_{t-1}^f \} - \frac{\sigma}{\pi_t^f} \{ R_t^i B_{r,t-1}^f + R_t D_{r,t-1}^f \} \quad (40^*)$$

Shadow price of deposits:

$$v_t^h = R_{t+1} E_t \left(\frac{\Lambda_{r,t,t+1}^h \Omega_{t+1}^h}{\pi_{t+1}^h} \right) \quad (41^*) \quad v_t^f = R_{t+1} E_t \left(\frac{\Lambda_{r,t,t+1}^f \Omega_{t+1}^f}{\pi_{t+1}^f} \right) \quad (42^*)$$

Shadow price of interbank borrowing:

$$v_{b,t}^h = v_t^h \frac{R_{t+1}^i}{R_{t+1}} \quad (43^*) \quad v_{b,t}^f = v_t^f \frac{R_{t+1}^i}{R_{t+1}} \quad (44^*)$$

Shadow price of covered bonds:

$$v_{c,t}^h = v_t^h \frac{R_{t+1}^i}{R_{t+1}} \quad (45^*)$$

Shadow value of assets:

$$v_{s,t}^h = v_t^h E_t \left(\frac{\pi_{t+1}^h [Z_{r,t+1}^h + (1-\delta)Q_{r,t+1}^h] \psi_{t+1}^h}{R_{t+1}} \right) \quad (46^*) \quad v_{s,t}^f = v_t^f E_t \left(\frac{\pi_{t+1}^f [Z_{r,t+1}^f + (1-\delta)Q_{r,t+1}^f] \psi_{t+1}^f}{R_{t+1}} \right) \quad (47^*)$$

Final goods market clearing:

$$Y_t^h = C_{H,t}^h + C_{H,t}^f + I_t^h + G_t^h \quad (48^*) \quad Y_t^f = C_{F,t}^f + C_{F,t}^h + I_t^f + G_t^f \quad (49^*)$$

Government budget constraint:

$$\frac{P_{H,t}}{P_t^h} G_t^h = T_{r,t}^h \quad (50^*) \quad \frac{P_{F,t}}{P_t^f} G_t^f = T_{r,t}^f \quad (51^*)$$

Concordance of assets with aggregate capital:

$$S_t^h = I_t^h + (1-\delta)K_t^h \quad (52^*) \quad S_t^f = I_t^f + (1-\delta)K_t^f \quad (53^*)$$

Monetary policy:

$$\log(R_{t+1}) = \log(R) + \phi_\pi (\gamma \log \pi_{H,t} + (1-\gamma) \log \pi_{F,t}) + \phi_y \left(\gamma \log \left(\frac{Y_t^h}{Y^h} \right) + (1-\gamma) \log \left(\frac{Y_t^f}{Y^f} \right) \right) + \zeta_t \quad (54^*)$$

Stochastic processes:

$$\zeta_t = \rho_m \zeta_{t-1} + \epsilon_{m,t} \quad (55^*)$$

$$\psi_t^h = (\bar{\psi}^h)^{1-\rho_k} (\psi_{t-1}^h)^{\rho_k} e^{\epsilon_{k,t}^h} \quad (56^*) \quad \psi_t^f = (\bar{\psi}^f)^{1-\rho_k} (\psi_{t-1}^f)^{\rho_k} e^{\epsilon_{k,t}^f} \quad (57^*)$$

$$A_t^h = (\bar{A}^h)^{1-\rho_a} (A_{t-1}^h)^{\rho_a} e^{\epsilon_{a,t}^h} \quad (58^*) \quad A_t^f = (\bar{A}^f)^{1-\rho_a} (A_{t-1}^f)^{\rho_a} e^{\epsilon_{a,t}^f} \quad (59^*)$$

Appendix C.9: summary of linearized equilibrium conditions, extension 2

The model comprises the 55 variables. The 50 state variables:

$$\{\tilde{Y}_t^h, \tilde{Y}_t^f, \tilde{C}_t^h, \tilde{C}_t^f, \tilde{C}_{H,t}^h, \tilde{C}_{F,t}^h, \tilde{C}_{F,t}^f, \tilde{C}_{H,t}^f, \tilde{K}_{t+1}^h, \tilde{K}_{t+1}^f, \tilde{L}_t^h, \tilde{L}_t^f, \tilde{I}_t^h, \tilde{I}_t^f, \tilde{G}_t^h, \tilde{G}_t^f, \tilde{T}_{r,t}^h, \tilde{T}_{r,t}^f, \tilde{W}_{r,t}^h, \tilde{W}_{r,t}^f, \\ \tilde{P}_t^h, \tilde{P}_t^f, \tilde{P}_{H,t}^h, \tilde{P}_{F,t}^h, \tilde{\pi}_t^h, \tilde{\pi}_t^f, \tilde{\pi}_{H,t}^h, \tilde{\pi}_{F,t}^h, \\ \tilde{R}_{t+1}^h, \tilde{R}_{t+1}^f, \tilde{Q}_{r,t}^h, \tilde{Q}_{r,t}^f, \tilde{B}_{r,t}^h, \tilde{B}_{r,t}^f, B_{r,t}^c, \tilde{S}_t^h, \tilde{S}_t^f, \tilde{D}_{r,t}^h, \tilde{D}_{r,t}^f, \tilde{N}_{r,t}^h, \tilde{N}_{r,t}^f, \tilde{Z}_{r,t}^h, \tilde{Z}_{r,t}^f, \tilde{v}_t^h, \tilde{v}_{b,t}^h, \tilde{v}_{c,t}^h, \tilde{v}_{s,t}^h, \tilde{v}_t^f, \tilde{v}_{b,t}^f, \tilde{v}_{s,t}^f\}$$

and the 5 stochastic variables:

$$\{\tilde{\psi}_t^h, \tilde{\psi}_t^f, \tilde{A}_t^h, \tilde{A}_t^f, \zeta_t\}$$

are then defined by the following 55 equations:

$$\begin{aligned} \tilde{P}_t^h &= \phi^h \tilde{P}_{H,t} + (1 - \phi^h) \tilde{P}_{F,t} \\ \tilde{P}_t^f &= \phi^f \tilde{P}_{F,t} + (1 - \phi^f) \tilde{P}_{H,t} \\ \tilde{C}_{H,t}^h &= \mu \tilde{P}_t^h - \mu \tilde{P}_{H,t} + \tilde{C}_t^h \\ \tilde{C}_{F,t}^h &= \mu \tilde{P}_t^h - \mu \tilde{P}_{F,t} + \tilde{C}_t^h \\ \tilde{C}_{F,t}^f &= \mu \tilde{P}_t^f - \mu \tilde{P}_{F,t} + \tilde{C}_t^f \\ \tilde{C}_{H,t}^f &= \mu \tilde{P}_t^f - \mu \tilde{P}_{H,t} + \tilde{C}_t^f \\ \tilde{W}_{r,t}^h &= \phi \tilde{L}_t^h - \eta \tilde{C}_t^h \\ \tilde{W}_{r,t}^f &= \phi \tilde{L}_t^f - \eta \tilde{C}_t^f \\ -\partial D_r \tilde{D}_{r,t}^h &= \tilde{R}_{t+1}^h + \eta \tilde{C}_t^h - \eta \tilde{C}_{t+1}^h - \tilde{\pi}_{t+1}^h \\ 0 &= \tilde{R}_{t+1}^f + \eta \tilde{C}_t^f - \eta \tilde{C}_{t+1}^f - \tilde{\pi}_{t+1}^f \\ \tilde{G}_t^h &= 0 \\ \tilde{G}_t^f &= 0 \\ \tilde{K}_{t+1}^h &= \tilde{\psi}_{t+1}^h + \delta \tilde{I}_t^h + (1 - \delta) \tilde{K}_t^h \\ \tilde{K}_{t+1}^f &= \tilde{\psi}_{t+1}^f + \delta \tilde{I}_t^f + (1 - \delta) \tilde{K}_t^f \\ \tilde{Q}_{r,t}^h &= \tilde{P}_{H,t}^h - \tilde{P}_t^h + U^h (1 + \beta) \tilde{I}_t^h - U^h \tilde{I}_{t-1}^h - \iota \beta I^h E_t \tilde{I}_{t+1}^h \\ \tilde{Q}_{r,t}^f &= \tilde{P}_{F,t}^f - \tilde{P}_t^f + U^f (1 + \beta) \tilde{I}_t^f - U^f \tilde{I}_{t-1}^f - \iota \beta I^f E_t \tilde{I}_{t+1}^f \\ \tilde{Y}_t^h &= \tilde{A}_t^h + \alpha \tilde{K}_t^h + (1 - \alpha) \tilde{L}_t^h \\ \tilde{Y}_t^f &= \tilde{A}_t^f + \alpha \tilde{K}_t^f + (1 - \alpha) \tilde{L}_t^f \\ Z_r^h K^h \tilde{Z}_{r,t}^h &= Y^h \tilde{P}_{H,t}^h - Y^h \tilde{P}_t^h + Y^h \tilde{Y}_t^h - W_r^h L^h \tilde{W}_{r,t}^h - W_r^h L^h \tilde{L}_t^h - Z_r^h K^h \tilde{K}_t^h \\ Z_r^f K^f \tilde{Z}_{r,t}^f &= Y^f \tilde{P}_{F,t}^f - Y^f \tilde{P}_t^f + Y^f \tilde{Y}_t^f - W_r^f L^f \tilde{W}_{r,t}^f - W_r^f L^f \tilde{L}_t^f - Z_r^f K^f \tilde{K}_t^f \\ \tilde{\pi}_{H,t} &= \beta E_t(\tilde{\pi}_{H,t+1}) + \varpi \tilde{W}_{r,t}^h + \varpi \tilde{P}_t^h - \varpi \tilde{P}_{H,t} - \frac{\varpi}{1 - \alpha} \tilde{A}_t^h - \frac{\alpha \varpi}{1 - \alpha} \tilde{K}_t^h + \frac{\alpha \varpi}{1 - \alpha} \tilde{Y}_t^h \\ \tilde{\pi}_{F,t} &= \beta E_t(\tilde{\pi}_{F,t+1}) + \varpi \tilde{W}_{r,t}^f + \varpi \tilde{P}_t^f - \varpi \tilde{P}_{F,t} - \frac{\varpi}{1 - \alpha} \tilde{A}_t^f - \frac{\alpha \varpi}{1 - \alpha} \tilde{K}_t^f + \frac{\alpha \varpi}{1 - \alpha} \tilde{Y}_t^f \end{aligned}$$

with : $\varpi \equiv \frac{(1 - \zeta)(1 - \zeta \beta)(1 - \alpha)}{\zeta(1 - \alpha + \alpha \kappa)}$

$$\begin{aligned}
\tilde{\pi}_{H,t} &= \tilde{P}_{H,t} - \tilde{P}_{H,t-1} \\
\tilde{\pi}_{F,t} &= \tilde{P}_{F,t} - \tilde{P}_{F,t-1} \\
\tilde{\pi}_t^h &= \tilde{P}_t^h - \tilde{P}_{t-1}^h \\
\tilde{\pi}_t^f &= \tilde{P}_t^f - \tilde{P}_{t-1}^f \\
\tilde{v}_{s,t}^h - \tilde{Q}_{r,t}^h + \left(\frac{1}{\omega} - 1\right) \tilde{v}_t^h &= \frac{1}{\omega} \tilde{v}_{b,t}^h \\
\tilde{v}_{s,t}^f - \tilde{Q}_{r,t}^f + \left(\frac{1}{\omega} - 1\right) \tilde{v}_t^f &= \frac{1}{\omega} \tilde{v}_{b,t}^f \\
S^h \tilde{v}_{s,t}^h + S^h (1 - \theta^h) \tilde{S}_t^h - B_r^h \tilde{v}_{b,t}^h - D_r^h \tilde{v}_t^h - D_r^h \tilde{D}_{r,t}^h - B_r^h (1 - \theta^h \omega) \tilde{B}_{r,t}^h - (1 - \theta^h) B_{r,t}^c &= \theta^h S^h \tilde{Q}_{r,t}^h \\
S^f \tilde{v}_{s,t}^f + S^f (1 - \theta^f) \tilde{S}_t^f - B_r^f \tilde{v}_{b,t}^f - D_r^f \tilde{v}_t^f - D_r^f \tilde{D}_{r,t}^f - B_r^f (1 - \theta^f \omega) \tilde{B}_{r,t}^f &= \theta^f S^f \tilde{Q}_{r,t}^f \\
D_r^h \tilde{D}_{r,t}^h &= S^h \tilde{Q}_{r,t}^h + S^h \tilde{S}_t^h - N_r^h \tilde{N}_{r,t}^h - B_r^h \tilde{B}_{r,t}^h - B_{r,t}^c \\
D_r^f \tilde{D}_{r,t}^f &= S^f \tilde{Q}_{r,t}^f + S^f \tilde{S}_t^f - N_r^f \tilde{N}_{r,t}^f - B_r^f \tilde{B}_{r,t}^f \\
B_{r,t}^h + P_t^h &= B_{r,t}^f + P_t^f \\
B_{r,t}^c &= \phi B_r^h R^i \tilde{R}_{t+1}^i - \phi B_r^h R \tilde{R}_{t+1} \\
N_r^h \tilde{N}_{r,t}^h &= (\sigma + \xi^h) Z_r^h S^h \tilde{Z}_{r,t}^h + (\sigma + \xi^h) R S^h \tilde{\psi}_t^h + (\sigma + \xi^h) R S^h \tilde{S}_{t-1}^h + (\sigma + \xi^h) (1 - \delta) S^h \tilde{Q}_{r,t}^h \\
&+ \sigma (R^i B_r^h + R D_r^h) \tilde{\pi}_t^h - \sigma R^i B_r^h \tilde{R}_t^i - \sigma R^i B_r^h \tilde{B}_{r,t-1}^h - \sigma R D_r^h \tilde{R}_t^i - \sigma R D_r^h \tilde{D}_{r,t-1}^h - \sigma R^i B_{r,t-1}^c \\
N_r^f \tilde{N}_{r,t}^f &= (\sigma + \xi^f) Z_r^f S^f \tilde{Z}_{r,t}^f + (\sigma + \xi^f) R S^f \tilde{\psi}_t^f + (\sigma + \xi^f) R S^f \tilde{S}_{t-1}^f + (\sigma + \xi^f) (1 - \delta) S^f \tilde{Q}_{r,t}^f \\
&+ \sigma (R^i B_r^f + R D_r^f) \tilde{\pi}_t^f - \sigma R^i B_r^f \tilde{R}_t^i - \sigma R^i B_r^f \tilde{B}_{r,t-1}^f - \sigma R D_r^f \tilde{R}_t^i - \sigma R D_r^f \tilde{D}_{r,t-1}^f \\
v_t^h &= \tilde{R}_{t+1}^h + \eta \tilde{C}_t^h - \eta E_t(\tilde{C}_{t+1}^h) - E_t(\tilde{\pi}_{t+1}^h) + \sigma (1 - \phi^h (1 - x_t^h)) E_t(\tilde{v}_{t+1}^h) \\
&+ \sigma \phi^h E_t(\tilde{v}_{s,t+1}^h) - \sigma \phi^h E_t(\tilde{Q}_{r,t+1}^h) - \sigma \phi^h x_t^h E_t(\tilde{v}_{b,t+1}^h) \\
\tilde{v}_t^f &= \tilde{R}_{t+1}^f + \eta \tilde{C}_t^f - \eta E_t(\tilde{C}_{t+1}^f) - E_t(\tilde{\pi}_{t+1}^f) + \sigma (1 - \phi^f (1 - x_t^f)) E_t(\tilde{v}_{t+1}^f) \\
&+ \sigma \phi^f E_t(\tilde{v}_{s,t+1}^f) - \sigma \phi^f E_t(\tilde{Q}_{r,t+1}^f) - \sigma \phi^f x_t^f E_t(\tilde{v}_{b,t+1}^f) \\
\tilde{v}_{b,t}^h &= \tilde{v}_t^h + \tilde{R}_{t+1}^h - \tilde{R}_{t+1} \\
\tilde{v}_{b,t}^f &= \tilde{v}_t^f + \tilde{R}_{t+1}^f - \tilde{R}_{t+1} \\
\tilde{v}_{c,t}^h &= \tilde{v}_t^h + \tilde{R}_{t+1}^h - \tilde{R}_{t+1} \\
\tilde{v}_{s,t}^h &= \tilde{v}_t^h + E_t(\tilde{\pi}_{t+1}^h) + \frac{Z_r^h}{R} E_t(Z_{r,t+1}^h) + E_t(\tilde{\psi}_{t+1}^h) + \frac{(1 - \delta)}{R} E_t(\tilde{Q}_{r,t+1}^h) - \tilde{R}_{t+1} \\
\tilde{v}_{s,t}^f &= \tilde{v}_t^f + E_t(\tilde{\pi}_{t+1}^f) + \frac{Z_r^f}{R} E_t(Z_{r,t+1}^f) + E_t(\tilde{\psi}_{t+1}^f) + \frac{(1 - \delta)}{R} E_t(\tilde{Q}_{r,t+1}^f) - \tilde{R}_{t+1} \\
Y^h \tilde{Y}_t^h &= C_H^h \tilde{C}_{H,t}^h + C_H^f \tilde{C}_{H,t}^f + I^h \tilde{I}_t^h + G^h \tilde{G}_t^h \\
Y^f \tilde{Y}_t^f &= C_F^f \tilde{C}_{F,t}^f + C_F^h \tilde{C}_{F,t}^h + I^f \tilde{I}_t^f + G^f \tilde{G}_t^f \\
\tilde{T}_{r,t}^h &= \tilde{P}_{H,t} - \tilde{P}_t^h + \tilde{G}_t^h \\
\tilde{T}_{r,t}^f &= \tilde{P}_{F,t} - \tilde{P}_t^f + \tilde{G}_t^f \\
\tilde{S}_t^h &= \delta \tilde{I}_t^h + (1 - \delta) \tilde{K}_t^h \\
\tilde{S}_t^f &= \delta \tilde{I}_t^f + (1 - \delta) \tilde{K}_t^f \\
\tilde{R}_{t+1} &= \phi_\pi \gamma \tilde{\pi}_{H,t} + \phi_\pi (1 - \gamma) \tilde{\pi}_{F,t} + \phi_\gamma \gamma \tilde{Y}_t^h + \phi_\gamma (1 - \gamma) \tilde{Y}_t^f + \varsigma_t \\
\varsigma_t &= \rho_m \varsigma_{t-1} + \epsilon_{m,t} \\
\tilde{\psi}_t^h &= \rho_k \tilde{\psi}_{t-1}^h + \epsilon_{k,t}^h \\
\tilde{\psi}_t^f &= \rho_k \tilde{\psi}_{t-1}^f + \epsilon_{k,t}^f \\
\tilde{A}_t^h &= \rho_a \tilde{A}_{t-1}^h + \epsilon_{a,t}^h \\
\tilde{A}_t^f &= \rho_a \tilde{A}_{t-1}^f + \epsilon_{a,t}^f
\end{aligned}$$

Appendix C.10: Steady-state for the second extension

From (43*) and (45*), one concludes that $v_{b,t}^h = v_{c,t}^h$. Following, if one assumes $v^h = v_b^h$ as in the benchmark version, then from (31*), (41*) and (3.44), one obtains again that:

$$v^h = v_b^h = v_s^h = v_c^h = 1$$

Then, the steady-state implies again that $R^i = R$, and this allows to conclude from (38*) that $B_r^c = 0$. Following, it becomes trivial to show that the steady-state is similar to the one of the benchmark model.

Appendix C.11: summary of equilibrium conditions, extension 3

The model comprises the 62 variables. The 57 state variables:

$$\{Y_t^h, Y_t^f, C_t^h, C_t^f, C_{H,t}^h, C_{F,t}^h, C_{F,t}^f, C_{H,t}^f, K_{t+1}^h, K_{t+1}^f, L_t^h, L_t^f, I_t^h, I_t^f, G_t^h, G_t^f, T_{r,t}^h, T_{r,t}^f, W_{r,t}^h, W_{r,t}^f, MC_{r,t}^h, MC_{r,t}^f, P_t^h, P_t^f, P_{H,t}, P_{F,t}, P_{H,t}^*, P_{F,t}^*, \pi_t^h, \pi_t^f, \pi_{H,t}, \pi_{F,t}, R_{t+1}, R_{t+1}^d, R_t^d, Q_{r,t}^h, Q_{r,t}^f, B_{r,t}^h, B_{r,t}^f, B_{r,t}^d, S_t^h, S_t^f, D_{r,t}^h, D_{r,t}^f, N_{r,t}^h, N_{r,t}^f, Z_{r,t}^h, Z_{r,t}^f, v_t^h, v_{b,t}^h, v_{s,t}^h, v_{d,t}^h, v_t^f, v_{b,t}^f, v_{s,t}^f, v_{d,t}^f, \theta_{1,t}^h, \theta_{2,t}^h\}$$

and the 5 stochastic variables:

$$\{\psi_t^h, \psi_t^f, A_t^h, A_t^f, \zeta_t\}$$

are then defined by the following 62 equations:

Consumer Price index:

$$P_t^h = \left(\phi^h (P_{H,t})^{1-\mu} + (1-\phi^h) (P_{F,t})^{1-\mu} \right)^{\frac{1}{1-\mu}} \quad (1^\circ) \quad P_t^f = \left(\phi^f (P_{F,t})^{1-\mu} + (1-\phi^f) (P_{H,t})^{1-\mu} \right)^{\frac{1}{1-\mu}} \quad (2^\circ)$$

Optimal index demand:

$$C_{H,t}^h = \phi^h \left(\frac{P_t^h}{P_{H,t}} \right)^\mu C_t^h \quad (3^\circ) \quad C_{F,t}^h = (1-\phi^h) \left(\frac{P_t^h}{P_{F,t}} \right)^\mu C_t^h \quad (4^\circ)$$

$$C_{F,t}^f = \phi^f \left(\frac{P_t^f}{P_{F,t}} \right)^\mu C_t^f \quad (5^\circ) \quad C_{H,t}^f = (1-\phi^f) \left(\frac{P_t^f}{P_{H,t}} \right)^\mu C_t^f \quad (6^\circ)$$

Optimal labour allocation:

$$\chi^h (L_t^h)^\phi (C_t^h)^\eta = W_{r,t}^h \quad (7^\circ) \quad \chi^f (L_t^f)^\phi (C_t^f)^\eta = W_{r,t}^f \quad (8^\circ)$$

Euler equation:

$$1 - \varepsilon (D_{r,t}^h - D_r^h) = R_{t+1} \beta E_t \left[\left(\frac{C_t^h}{C_{t+1}^h} \right)^\eta \frac{1}{\pi_{t+1}^h} \right] \quad (9^\circ) \quad 1 = R_{t+1} \beta E_t \left[\left(\frac{C_t^f}{C_{t+1}^f} \right)^\eta \frac{1}{\pi_{t+1}^f} \right] \quad (10^\circ)$$

Public expenditure:

$$G_t^h = \vartheta^h Y^h + \gamma_2 \vartheta^h (Y_t^h - Y^h) + \theta_{1,t}^h G^h \quad (11^\circ) \quad G_t^f = \vartheta^f Y^f \quad (12^\circ)$$

Law of motion of capital:

$$K_{t+1}^h = \psi_{t+1}^h \left[I_t^h + (1-\delta) K_t^h \right] \quad (13^\circ) \quad K_{t+1}^f = \psi_{t+1}^f \left[I_t^f + (1-\delta) K_t^f \right] \quad (14^\circ)$$

Capital pricing:

$$Q_{r,t}^h = \frac{P_{H,t}}{P_t^h} \left[1 + \iota (I_t^h - I_{t-1}^h) \right] - \iota E_t \Lambda_{r,t,t+1}^h \frac{P_{H,t+1}}{P_{t+1}^h} (I_{t+1}^h - I_t^h) \quad (15^\circ)$$

$$Q_{r,t}^f = \frac{P_{F,t}}{P_t^f} \left[1 + \iota (I_t^f - I_{t-1}^f) \right] - \iota E_t \Lambda_{r,t,t+1}^f \frac{P_{F,t+1}}{P_{t+1}^f} (I_{t+1}^f - I_t^f) \quad (16^\circ)$$

Production technology:

$$Y_t^h(j) = A_t^h (K_t^h)^\alpha L_t^h(j)^{1-\alpha} \quad (17^\circ) \quad Y_t^f(j) = A_t^f (K_t^f)^\alpha L_t^f(j)^{1-\alpha} \quad (18^\circ)$$

Real wage setting:

$$MC_{r,t}^h = W_{r,t}^h \times \frac{P_t^h}{P_{H,t}^h} \times \frac{1}{MPL_t^h} \quad (19^\circ) \quad MC_{r,t}^f = W_{r,t}^f \times \frac{P_t^f}{P_{F,t}^f} \times \frac{1}{MPL_t^f} \quad (20^\circ)$$

Dividend:

$$Z_{r,t}^h = \frac{\frac{P_{H,t}^h}{P_t^h} Y_t^h - W_{r,t}^h L_t^h}{K_t^h} \quad (21^\circ) \quad Z_{r,t}^f = \frac{\frac{P_{F,t}^f}{P_t^f} Y_t^f - W_{r,t}^f L_t^f}{K_t^f} \quad (22^\circ)$$

Optimal pricing:

$$P_{H,t}^*(j) = \frac{\kappa}{\kappa-1} \cdot \frac{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^h Y_{t+i|t}^h(j) \Psi_{t+i}^{h'}(Y_{t+i|t}^h(j)) \right\}}{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^h Y_{t+i|t}^h(j) \right\}} \quad (23^\circ)$$

$$P_{F,t}^*(j) = \frac{\kappa}{\kappa-1} \cdot \frac{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^f Y_{t+i|t}^f(j) \Psi_{t+i}^{f'}(Y_{t+i|t}^f(j)) \right\}}{\sum_{i=0}^{\infty} \zeta^i E_t \left\{ \Lambda_{t,t+i}^f Y_{t+i|t}^f(j) \right\}} \quad (24^\circ)$$

Law of motion of producer prices:

$$(\pi_{H,t})^{1-\kappa} = \zeta + (1-\zeta) \left(\frac{P_{H,t}^*}{P_{H,t-1}} \right)^{1-\kappa} \quad (25^\circ) \quad (\pi_{F,t})^{1-\kappa} = \zeta + (1-\zeta) \left(\frac{P_{F,t}^*}{P_{F,t-1}} \right)^{1-\kappa} \quad (26^\circ)$$

PPI inflation:

$$\pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}} \quad (27^\circ) \quad \pi_{F,t} = \frac{P_{F,t}}{P_{F,t-1}} \quad (28^\circ)$$

CPI inflation:

$$\pi_t^h = \frac{P_t^h}{P_{t-1}^h} \quad (29^\circ) \quad \pi_t^f = \frac{P_t^f}{P_{t-1}^f} \quad (30^\circ)$$

Optimal security allocation:

$$\left(\frac{v_{s,t}^h}{Q_{r,t}^h} - v_t^h \right) = \frac{1}{\omega} (v_{b,t}^h - v_t^h) \quad (31^\circ) \quad \left(\frac{v_{s,t}^f}{Q_{r,t}^f} - v_t^f \right) = \frac{1}{\omega} (v_{b,t}^f - v_t^f) \quad (32^\circ)$$

Optimal government loans allocation:

$$\frac{v_{s,t}^h}{Q_{r,t}^h} = v_{d,t}^h \quad (33^\circ)$$

Incentive constraint:

$$Q_{r,t}^h S_t^h = \left(\frac{1}{1 - \theta_{2,t}^h} \right) \phi_t^h N_t^h \quad (34^\circ)$$

$$\text{with: } \phi_t^h = \left(\frac{v_t^h}{\theta^h (1 - \omega x_{1,t}^h + (1 - \theta_{1,t}^h) x_{2,t}^h) + v_t^h (1 - x_{1,t}^h + (1 - \theta_{1,t}^h) x_{2,t}^h) + v_{b,t}^h x_{1,t}^h - \frac{v_{s,t}^h}{Q_{r,t}^h} - v_{d,t}^h (1 - \theta_{1,t}^h) x_{2,t}^h} \right)$$

$$v_{s,t}^f S_t^f - v_{b,t}^f B_{r,t}^f - v_t^f D_{r,t}^f = \theta^f [Q_{r,t}^f S_t^f - \omega B_{r,t}^f] \quad (35^\circ)$$

Security market clearing:

$$D_{r,t}^h = Q_{r,t}^h S_t^h + (1 - \theta_{1,t}^h) B_{r,t}^d - N_{r,t}^h - B_{r,t}^h \quad (36^\circ)$$

$$D_{r,t}^f = Q_{r,t}^f S_t^f - N_{r,t}^f - B_{r,t}^f \quad (37^\circ)$$

Asset market clearing:

$$B_{r,t}^h P_t^h = -B_{r,t}^f P_t^f \quad (38^\circ)$$

Evolution of net worth:

$$N_{r,t}^h = (\sigma + \xi^h) \{ [Z_{r,t}^h + (1 - \delta) Q_{r,t}^h] \psi_t^h S_{t-1}^h \} - \frac{\sigma}{\pi_t^h} \{ R_t^i B_{r,t-1}^h + R_t^h D_{r,t-1}^h - R_t^d (1 - \theta_{1,t}^h) B_{r,t-1}^d \} \quad (39^\circ)$$

$$N_{r,t}^f = (\sigma + \xi^f) \{ [Z_{r,t}^f + (1 - \delta) Q_{r,t}^f] \psi_t^f S_{t-1}^f \} - \frac{\sigma}{\pi_t^f} \{ R_t^i B_{r,t-1}^f + R_t^f D_{r,t-1}^f \} \quad (40^\circ)$$

Shadow price of deposits:

$$v_t^h = R_{t+1} E_t \left(\frac{\Lambda_{r,t,t+1}^h \Omega_{t+1}^h}{\pi_{t+1}^h} \right) \quad (41^\circ) \quad v_t^f = R_{t+1} E_t \left(\frac{\Lambda_{r,t,t+1}^f \Omega_{t+1}^f}{\pi_{t+1}^f} \right) \quad (42^\circ)$$

Shadow price of interbank borrowing:

$$v_{b,t}^h = v_t^h \frac{R_{t+1}^i}{R_{t+1}} \quad (43^\circ) \quad v_{b,t}^f = v_t^f \frac{R_{t+1}^i}{R_{t+1}} \quad (44^\circ)$$

Shadow value of assets:

$$v_{s,t}^h = v_t^h E_t \left(\frac{\pi_{t+1}^h [Z_{r,t+1}^h + (1 - \delta) Q_{r,t+1}^h] \psi_{t+1}^h}{R_{t+1}} \right) \quad (45^\circ) \quad v_{s,t}^f = v_t^f E_t \left(\frac{\pi_{t+1}^f [Z_{r,t+1}^f + (1 - \delta) Q_{r,t+1}^f] \psi_{t+1}^f}{R_{t+1}} \right) \quad (46^\circ)$$

Shadow value of government loans:

$$v_{d,t}^h = v_t^h \frac{R_{t+1}^d}{R_{t+1}^h} \quad (47^\circ)$$

Final goods market clearing:

$$Y_t^h = C_{H,t}^h + C_{H,t}^f + I_t^h + G_t^h \quad (48^\circ) \quad Y_t^f = C_{F,t}^f + C_{F,t}^h + I_t^f + G_t^f \quad (49^\circ)$$

Taxation:

$$T_{r,t}^h = \frac{P_{H,t}^d}{P_t^h} \tau^h Y_t^h + \gamma_1 (B_{r,t-1}^d - B_r^d) \quad (50^\circ)$$

$$\frac{P_{F,t}^f}{P_t^f} G_t^f = T_{r,t}^f \quad (51^\circ)$$

Government budget constraint:

$$T_{r,t}^h + B_{r,t}^d = \frac{P_{H,t}^d}{P_t^h} G_t^h + \frac{R_t^d}{\pi_t^h} B_{r,t-1}^d \quad (52^\circ)$$

Concordance of assets with aggregate capital:

$$S_t^h = I_t^h + (1 - \delta) K_t^h \quad (53^\circ) \quad S_t^f = I_t^f + (1 - \delta) K_t^f \quad (54^\circ)$$

SMP rules:

$$\theta_{1,t}^h = \theta_1 \left(\frac{B_{r,t-1}^d - B_r^d}{B_r^d} \right) \quad (55^\circ)$$

$$\theta_{2,t}^h = \theta_2 \left(\frac{Y_t^h - Y_t^f}{Y_t^h} \right) \quad (56^\circ)$$

Monetary policy:

$$\log(R_{t+1}) = \log(R_t) + \phi_\pi \left(\gamma \log \pi_{H,t} + (1 - \gamma) \log \pi_{F,t} \right) + \phi_y \left(\gamma \log \left(\frac{Y_t^h}{Y_t^f} \right) + (1 - \gamma) \log \left(\frac{Y_t^f}{Y_t^f} \right) \right) + \zeta_t \quad (57^\circ)$$

Stochastic processes:

$$\zeta_t = \rho_m \zeta_{t-1} + \epsilon_{m,t} \quad (58^\circ)$$

$$\psi_t^h = (\bar{\psi}^h)^{1-\rho_k} (\psi_{t-1}^h)^{\rho_k} e^{\epsilon_{k,t}^h} \quad (59^\circ) \quad \psi_t^f = (\bar{\psi}^f)^{1-\rho_k} (\psi_{t-1}^f)^{\rho_k} e^{\epsilon_{k,t}^f} \quad (60^\circ)$$

$$A_t^h = (\bar{A}^h)^{1-\rho_a} (A_{t-1}^h)^{\rho_a} e^{\epsilon_{a,t}^h} \quad (61^\circ) \quad A_t^f = (\bar{A}^f)^{1-\rho_a} (A_{t-1}^f)^{\rho_a} e^{\epsilon_{a,t}^f} \quad (62^\circ)$$

Appendix C.12: summary of linearized equilibrium conditions, extension 3

The model comprises the 58 variables. The 53 state variables:

$$\{\tilde{Y}_t^h, \tilde{Y}_t^f, \tilde{C}_t^h, \tilde{C}_t^f, \tilde{C}_{H,t}^h, \tilde{C}_{F,t}^h, \tilde{C}_{H,t}^f, \tilde{C}_{F,t}^f, \tilde{K}_{t+1}^h, \tilde{K}_{t+1}^f, \tilde{L}_t^h, \tilde{L}_t^f, \tilde{I}_t^h, \tilde{I}_t^f, \tilde{G}_t^h, \tilde{G}_t^f, \tilde{T}_{r,t}^h, \tilde{T}_{r,t}^f, \tilde{W}_{r,t}^h, \tilde{W}_{r,t}^f, \\ \tilde{P}_t^h, \tilde{P}_t^f, \tilde{P}_{H,t}^h, \tilde{P}_{F,t}^h, \tilde{\pi}_t^h, \tilde{\pi}_t^f, \tilde{\pi}_{H,t}^h, \tilde{\pi}_{F,t}^h, \\ \tilde{R}_{t+1}^h, \tilde{R}_{t+1}^f, \tilde{R}_{t+1}^d, \tilde{Q}_{r,t}^h, \tilde{Q}_{r,t}^f, \tilde{B}_{r,t}^h, \tilde{B}_{r,t}^f, \tilde{B}_{r,t}^d, \tilde{S}_t^h, \tilde{S}_t^f, \tilde{D}_{r,t}^h, \tilde{D}_{r,t}^f, \tilde{N}_{r,t}^h, \tilde{N}_{r,t}^f, \tilde{Z}_{r,t}^h, \tilde{Z}_{r,t}^f, \theta_{1,t}^h, \theta_{2,t}^h, \tilde{v}_t^h, \tilde{v}_{b,t}^h, \tilde{v}_{s,t}^h, \tilde{v}_{d,t}^h, \tilde{v}_t^f, \tilde{v}_{b,t}^f, \tilde{v}_{s,t}^f\}$$

and the 5 stochastic variables:

$$\{\tilde{\psi}_t^h, \tilde{\psi}_t^f, \tilde{A}_t^h, \tilde{A}_t^f, \varsigma_t\}$$

are then defined by the following 62 equations:

$$\begin{aligned} \tilde{P}_t^h &= \phi^h \tilde{P}_{H,t}^h + (1 - \phi^h) \tilde{P}_{F,t}^h \\ \tilde{P}_t^f &= \phi^f \tilde{P}_{F,t}^f + (1 - \phi^f) \tilde{P}_{H,t}^f \\ \tilde{C}_{H,t}^h &= \mu \tilde{P}_t^h - \mu \tilde{P}_{H,t}^h + \tilde{C}_t^h \\ \tilde{C}_{F,t}^h &= \mu \tilde{P}_t^h - \mu \tilde{P}_{F,t}^h + \tilde{C}_t^h \\ \tilde{C}_{F,t}^f &= \mu \tilde{P}_t^f - \mu \tilde{P}_{F,t}^f + \tilde{C}_t^f \\ \tilde{C}_{H,t}^f &= \mu \tilde{P}_t^f - \mu \tilde{P}_{H,t}^f + \tilde{C}_t^f \\ \tilde{W}_{r,t}^h &= \phi \tilde{L}_t^h - \eta \tilde{C}_t^h \\ \tilde{W}_{r,t}^f &= \phi \tilde{L}_t^f - \eta \tilde{C}_t^f \\ -\partial D_r^h \tilde{D}_{r,t}^h &= \tilde{R}_{t+1}^h + \eta \tilde{C}_t^h - \eta \tilde{C}_{t+1}^h - \tilde{\pi}_{t+1}^h \\ 0 &= \tilde{R}_{t+1}^f + \eta \tilde{C}_t^f - \eta \tilde{C}_{t+1}^f - \tilde{\pi}_{t+1}^f \\ \tilde{G}_t^h &= \gamma_2 \tilde{Y}_t^h + \theta_{1,t}^h \\ \tilde{G}_t^f &= 0 \\ \tilde{K}_{t+1}^h &= \tilde{\psi}_{t+1}^h + \delta \tilde{I}_t^h + (1 - \delta) \tilde{K}_t^h \\ \tilde{K}_{t+1}^f &= \tilde{\psi}_{t+1}^f + \delta \tilde{I}_t^f + (1 - \delta) \tilde{K}_t^f \\ \tilde{Q}_{r,t}^h &= \tilde{P}_{H,t}^h - \tilde{P}_t^h + U^h (1 + \beta) \tilde{I}_t^h - U^h \tilde{I}_{t-1}^h - t\beta I^h E_t \tilde{I}_{t+1}^h \\ \tilde{Q}_{r,t}^f &= \tilde{P}_{F,t}^f - \tilde{P}_t^f + U^f (1 + \beta) \tilde{I}_t^f - U^f \tilde{I}_{t-1}^f - t\beta I^f E_t \tilde{I}_{t+1}^f \\ \tilde{Y}_t^h &= \tilde{A}_t^h + \alpha \tilde{K}_t^h + (1 - \alpha) \tilde{L}_t^h \\ \tilde{Y}_t^f &= \tilde{A}_t^f + \alpha \tilde{K}_t^f + (1 - \alpha) \tilde{L}_t^f \\ Z_r^h K^h \tilde{Z}_{r,t}^h &= Y^h \tilde{P}_{H,t}^h - Y^h \tilde{P}_t^h + Y^h \tilde{Y}_t^h - W_r^h L^h \tilde{W}_{r,t}^h - W_r^h L^h \tilde{L}_t^h - Z_r^h K^h \tilde{K}_t^h \\ Z_r^f K^f \tilde{Z}_{r,t}^f &= Y^f \tilde{P}_{F,t}^f - Y^f \tilde{P}_t^f + Y^f \tilde{Y}_t^f - W_r^f L^f \tilde{W}_{r,t}^f - W_r^f L^f \tilde{L}_t^f - Z_r^f K^f \tilde{K}_t^f \end{aligned}$$

$$\begin{aligned}\tilde{\pi}_{H,t} &= \beta E_t(\tilde{\pi}_{H,t+1}) + \varpi \tilde{W}_{r,t}^h + \varpi \tilde{P}_t^h - \varpi \tilde{P}_{H,t} - \frac{\varpi}{1-\alpha} \tilde{A}_t^h - \frac{\alpha\varpi}{1-\alpha} \tilde{K}_t^h + \frac{\alpha\varpi}{1-\alpha} \tilde{Y}_t^h \\ \tilde{\pi}_{F,t} &= \beta E_t(\tilde{\pi}_{F,t+1}) + \varpi \tilde{W}_{r,t}^f + \varpi \tilde{P}_t^f - \varpi \tilde{P}_{F,t} - \frac{\varpi}{1-\alpha} \tilde{A}_t^f - \frac{\alpha\varpi}{1-\alpha} \tilde{K}_t^f + \frac{\alpha\varpi}{1-\alpha} \tilde{Y}_t^f\end{aligned}$$

$$\text{with : } \varpi \equiv \frac{(1-\zeta)(1-\zeta\beta)(1-\alpha)}{\zeta(1-\alpha+\alpha\kappa)}$$

$$\tilde{\pi}_{H,t} = \tilde{P}_{H,t} - \tilde{P}_{H,t-1}$$

$$\tilde{\pi}_{F,t} = \tilde{P}_{F,t} - \tilde{P}_{F,t-1}$$

$$\tilde{\pi}_t^h = \tilde{P}_t^h - \tilde{P}_{t-1}^h$$

$$\tilde{\pi}_t^f = \tilde{P}_t^f - \tilde{P}_{t-1}^f$$

$$\tilde{v}_{s,t}^h - \tilde{Q}_{r,t}^h + \left(\frac{1}{\omega} - 1\right) \tilde{v}_t^h = \frac{1}{\omega} \tilde{v}_{b,t}^h$$

$$\tilde{v}_{s,t}^f - \tilde{Q}_{r,t}^f + \left(\frac{1}{\omega} - 1\right) \tilde{v}_t^f = \frac{1}{\omega} \tilde{v}_{b,t}^f$$

$$\tilde{v}_{s,t}^h - \tilde{Q}_{r,t}^h = \tilde{v}_{d,t}^h$$

$$S^h \tilde{v}_{s,t}^h + S^h(1-\theta^h) \tilde{S}_t^h - B_r^h \tilde{v}_{b,t}^h - D_r^h \tilde{v}_t^h - D_r^h \tilde{D}_{r,t}^h - B_r^h(1-\theta^h\omega) \tilde{B}_{r,t}^h + B_r^d \tilde{v}_{d,t}^h$$

$$+ B_r^d(1-\theta^h) \tilde{B}_{r,t}^d - B_r^d(1-\theta^h)\theta^h = \theta^h S^h \tilde{Q}_{r,t}^h - N^h \theta_{2,t}^h$$

$$S^f \tilde{v}_{s,t}^f + S^f(1-\theta^f) \tilde{S}_t^f - B_r^f \tilde{v}_{b,t}^f - D_r^f \tilde{v}_t^f - D_r^f \tilde{D}_{r,t}^f - B_r^f(1-\theta^f\omega) \tilde{B}_{r,t}^f = \theta^f S^f \tilde{Q}_{r,t}^f$$

$$D_r^h D_{r,t}^h = S^h \tilde{Q}_{r,t}^h + S^h \tilde{S}_t^h - \tilde{B}_r^d \theta_{1,t}^h + \tilde{B}_r^d \tilde{B}_{r,t}^d - N_r^h \tilde{N}_{r,t}^h - B_r^h \tilde{B}_{r,t}^h$$

$$D_r^f \tilde{D}_{r,t}^f = S^f \tilde{Q}_{r,t}^f + S^f \tilde{S}_t^f - N_r^f \tilde{N}_{r,t}^f - B_r^f \tilde{B}_{r,t}^f$$

$$B_{r,t}^h + P_t^h = B_{r,t}^f + P_t^f$$

$$N_r^h \tilde{N}_{r,t}^h = (\sigma + \xi^h) Z_r^h S^h \tilde{Z}_{r,t}^h + (\sigma + \xi^h) RS^h \tilde{\psi}_t^h + (\sigma + \xi^h) RS^h \tilde{S}_{t-1}^h + (\sigma + \xi^h)(1-\delta) S^h \tilde{Q}_{r,t}^h$$

$$+ \sigma (R^i B_r^h + RD_r^h - R^d B_r^d) \tilde{\pi}_t^h - \sigma R^i B_r^h \tilde{R}_t^i - \sigma R^i B_r^h \tilde{B}_{r,t-1}^h - \sigma RD_r^h \tilde{R}_t^h - \sigma RD_r^h \tilde{D}_{r,t-1}^h + \sigma R^d B_r^d \tilde{R}_t^d$$

$$+ \sigma R^d B_r^d \tilde{B}_{r,t-1}^d - \sigma R^d B_r^d \theta_{1,t}^h$$

$$N_r^f \tilde{N}_{r,t}^f = (\sigma + \xi^f) Z_r^f S^f \tilde{Z}_{r,t}^f + (\sigma + \xi^f) RS^f \tilde{\psi}_t^f + (\sigma + \xi^f) RS^f \tilde{S}_{t-1}^f + (\sigma + \xi^f)(1-\delta) S^f \tilde{Q}_{r,t}^f$$

$$+ \sigma (R^i B_r^f + RD_r^f) \tilde{\pi}_t^f - \sigma R^i B_r^f \tilde{R}_t^i - \sigma R^i B_r^f \tilde{B}_{r,t-1}^f - \sigma RD_r^f \tilde{R}_t^f - \sigma RD_r^f \tilde{D}_{r,t-1}^f$$

$$v_t^h = \tilde{R}_{t+1}^h + \eta \tilde{C}_t^h - \eta E_t(\tilde{C}_{t+1}^h) - E_t(\tilde{\pi}_{t+1}^h) + \sigma(1-\phi^h(1-x_1^h+x_2^h)) E_t(\tilde{v}_{t+1}^h) + \sigma\phi^h E_t(\tilde{v}_{s,t+1}^h) - \sigma\phi^h E_t(\tilde{Q}_{r,t+1}^h)$$

$$- \sigma\phi^h x_1^h E_t(\tilde{v}_{b,t+1}^h) + \sigma\phi^h x_2^h E_t(\tilde{v}_{d,t+1}^h)$$

$$\tilde{v}_t^f = \tilde{R}_{t+1}^f + \eta \tilde{C}_t^f - \eta E_t(\tilde{C}_{t+1}^f) - E_t(\tilde{\pi}_{t+1}^f) + \sigma(1-\phi^f(1-x_r^f)) E_t(\tilde{v}_{t+1}^f)$$

$$+ \sigma\phi^f E_t(\tilde{v}_{s,t+1}^f) - \sigma\phi^f E_t(\tilde{Q}_{r,t+1}^f) - \sigma\phi^f x_r^f E_t(\tilde{v}_{b,t+1}^f)$$

$$\tilde{v}_{b,t}^h = \tilde{v}_t^h + \tilde{R}_{t+1}^h - \tilde{R}_{t+1}$$

$$\tilde{v}_{b,t}^f = \tilde{v}_t^f + \tilde{R}_{t+1}^f - \tilde{R}_{t+1}$$

$$\tilde{v}_{s,t}^h = \tilde{v}_t^h + E_t(\tilde{\pi}_{t+1}^h) + \frac{Z_r^h}{R} E_t(Z_{r,t+1}^h) + E_t(\tilde{\psi}_{t+1}^h) + \frac{(1-\delta)}{R} E_t(\tilde{Q}_{r,t+1}^h) - \tilde{R}_{t+1}$$

$$\tilde{v}_{s,t}^f = \tilde{v}_t^f + E_t(\tilde{\pi}_{t+1}^f) + \frac{Z_r^f}{R} E_t(Z_{r,t+1}^f) + E_t(\tilde{\psi}_{t+1}^f) + \frac{(1-\delta)}{R} E_t(\tilde{Q}_{r,t+1}^f) - \tilde{R}_{t+1}$$

$$\tilde{v}_{d,t}^h = \tilde{v}_t^h + \tilde{R}_{t+1}^d - \tilde{R}_{t+1}$$

$$Y^h \tilde{Y}_t^h = C_H^h \tilde{C}_{H,t}^h + C_H^f \tilde{C}_{H,t}^f + I^h \tilde{I}_t^h + G^h \tilde{G}_t^h$$

$$Y^f \tilde{Y}_t^f = C_F^f \tilde{C}_{F,t}^f + C_F^h \tilde{C}_{F,t}^h + I^f \tilde{I}_t^f + G^f \tilde{G}_t^f$$

$$\tilde{T}_{r,t}^h = \tilde{P}_{H,t} - \tilde{P}_t^h + \tilde{Y}_t^h + \gamma_1 \frac{B_r^d}{T_r^h} \tilde{B}_{r,t-1}^d$$

$$\tilde{T}_{r,t}^f = \tilde{P}_{F,t} - \tilde{P}_t^f + \tilde{G}_t^f$$

$$T_r \tilde{T}_{r,t}^h + B_r^d \tilde{B}_{r,t}^d = G^h \tilde{P}_{H,t} - G^h \tilde{P}_t^h + G^h \tilde{G}_t^h + R^d B_r^d \tilde{R}_t^d - R^d B_r^d \tilde{\pi}_t^h + R^d B_r^d \tilde{B}_{r,t-1}^d$$

$$\tilde{S}_t^h = \delta \tilde{I}_t^h + (1 - \delta) \tilde{K}_t^h$$

$$\tilde{S}_t^f = \delta \tilde{I}_t^f + (1 - \delta) \tilde{K}_t^f$$

$$\tilde{R}_{t+1} = \phi_\pi \gamma \tilde{\pi}_{H,t} + \phi_\pi (1 - \gamma) \tilde{\pi}_{F,t} + \phi_\gamma \gamma \tilde{Y}_t^h + \phi_\gamma (1 - \gamma) \tilde{Y}_t^f + \zeta_t$$

$$\theta_{1,t}^h = \theta_1 \tilde{B}_{r,t-1}^d$$

$$\theta_{2,t}^h = -\theta_2 \tilde{Y}_t^h$$

$$\zeta_t = \rho_m \zeta_{t-1} + \epsilon_{m,t}$$

$$\tilde{\psi}_t^h = \rho_k \tilde{\psi}_{t-1}^h + \epsilon_{k,t}^h$$

$$\tilde{\psi}_t^f = \rho_k \tilde{\psi}_{t-1}^f + \epsilon_{k,t}^f$$

$$\tilde{A}_t^h = \rho_p \tilde{A}_{t-1}^h + \epsilon_{p,t}^h$$

$$\tilde{A}_t^f = \rho_p \tilde{A}_{t-1}^f + \epsilon_{p,t}^f$$

General conclusion

The issue of currency areas started to generate debates among economists more than 50 years ago, and, despite example of the settlement of a currency union among most European nations, it is still not settled today. This thesis proposed to contribute to this debate by proposing a model explicitly adapted to the Euro area, the only yet concrete realisation of such currency area. It is probably not daring to say that this work would not have been possible 20 years ago, for several reasons.

First, the macroeconomic methodology was not yet of sufficient maturity to conduct such a task. In this respect, the democratization of the DSGE methodology, and its application to international economics (Obstfeld and Rogoff (1995)) has constituted a significant step forward for our understanding of monetary unions. In particular, the microfoundations on which it relies allow for the exhaustive and simultaneous inclusion of all OCA properties in a single dynamic setting, through the structural behaviour of individual agents. In this respect, the DSGE methodology represents a first answer to the issue of inconclusiveness (Tavlas (1993)) that has remained prevalent since the early times of OCA theory. From the ad hoc estimations of individual OCA properties, often based on econometric studies, which have characterized the 1980's and 1990's, we have now switched to a rigorous and unified theoretical framework encompassing the whole set of OCA criteria. In addition, the DSGE methodology offers a natural transition from descriptive matters, like the response of open economies in a context of financial frictions, to normative issues such as the determination of the optimal monetary regime for open economies. In this respect, the derivation of formal welfare criteria appears as a particularly valuable instrument for the debate over optimal currency areas.

A second reason lies in the perception of the debate over OCA theory as a relatively abstract matter. For decades, OCA theory has developed without much empirical resonance. Monetary theory hence focused on questions of more direct and immediate consequence for central bankers, such as the short or long-run neutrality of money, and the causality or reverse causality of monetary policy over business cycles (see e.g. Walsh (2010) , chapter 1 for a comprehensive historical treatment of these issues). Without the actual achievement of the

Euro area, it is little likely that OCA economics would have remained more than a field of theoretical discussions presenting little applied attractiveness.

A final and perhaps paradoxical reason can be found in the relative lack of renewed interest over this issue. As the introduction of the Euro in 1999 eventually concretised the creation of a monetary union, the literature switched from one fundamental question, the determination of an optimal monetary regime, to another one, the determination of optimal policy under a given regime. As the Euro proved successful over its early years, there seemed to be little motivation to question further the newly created monetary union. Combined with technical difficulties to exert a cross-regime comparison (data on the Euro was by definition scarce in the immediate aftermath of its introduction, and historical analysis could not be realised with the benefit of hindsight), the matter seemed settled.

With the financial and sovereign debt crises which have hit the Euro area since 2007, and the threat that one or more Euro area member states may ultimately have to leave the currency union, this point of view is no longer possible. The debate over the optimality of currency areas, or even their suitability for their member states, has become lively again. Although recent theoretical developments seem to tip in favour of monetary unions (Mongelli (2002)), the inconclusiveness issue of OCA properties remains. This work has attempted to tackle this issue by proposing a framework that would be as comprehensive as possible with respect to the diverse OCA properties under discussion. By integrating financial frictions, and the credit policies implemented by the ECB, it also ambitioned to account for recent real-life responses brought to the crisis.

Three main conclusions arise from this thesis. The first conclusion, obtained from chapter 1, is that monetary policy and monetary regimes matter. Although there is a relatively wide agreement over the short run impact of money on real activity, there is less consensus over the possible non-neutrality of money in the long run. In addition, even though OCA theory suggests that the delegation of monetary policy to a third party may represent an important aspect of the integration to a currency area, there is no formal evidence that this delegation may trigger a shift in economic dynamics, especially if monetary policy is perfectly anticipated. Chapter 1 formally establishes these two points. It first shows that a structural shift did take place for Euro area countries due to the incoming introduction of the single currency. It also reveals that the part of the monetary component after the shift has typically

decreased for short run fluctuations, but increased for long run fluctuations. This suggests that the single central bank may have enhanced immediate cycle stabilisation, but induces larger long-term volatility, for instance if its long run policy is disconnected from domestic cycles. In this respect, chapter 1 confirms that money is more than a mere unit of account. Money implies monetary policy, and it matters much whether this policy is implemented by a national institution, or at the scale of a monetary union.

The second conclusion, derived from chapters 2 and 3, is that contrarily to the conclusions of the current mainstream literature, the monetary union does not appear, in general, as an optimal regime for European countries. In the absence of any credit policy from the ECB, welfare performance seem consistently higher under a regime of floating exchange rates for a vast majority of countries. This remains true whether one considers or not the question of sovereign debt. Of course, this conclusion cannot pretend to be general: it is the result of an individual analysis, conducted on a specific set of countries, under a particular model which represents a specific crisis conjuncture. Nevertheless, it still produces useful insights for the current situation of the Euro area.

The main explanation found for this relative inefficiency seems to lie in the greater inability of the single central bank to handle asymmetric shocks which may stem from abroad partner economies. Interestingly enough, despite the considerable theoretical and empirical developments that have characterized OCA theory since 50 years, it seems that we are, in the end, back to the initial arguments: the capacity of exchange rates to act as an automatic stabilizer (Friedman (1953)) and the crucial importance of shock asymmetry (Mundell (1961)). On the latter subject, however, lies much potential for further research. The current literature on shock within the EMU remains unsatisfying, being either inconclusive, or restricted to small groups of countries. To these days, this literature has also largely ignored the existence of financial shocks. There still lacks a comprehensive full-scale analysis of shocks within the Euro area, and their possible co-movements. For this reason, the model developed in this work has simply assumed that there was no correlation among the shocks of Euro area countries. It is highly probable that its conclusions could be enriched and strengthened with updated information.

The final conclusion is that there is potential for a monetary union to produce higher welfare than a regime of floating exchange rates. This will be the case if the central authorities

intervene actively in times of recessions. This point was made under the SMP/OMT experiment of chapter 3. With active commitment of the ECB to assist member states with respect to their sovereign debts, but also to support the private sector, the monetary union becomes a superior regime for a majority of Euro area members. These results have yet to be considered with much caution. First, the setting was kept simple, assuming that the ECB had no explicit budget constraint and could always honour any payment. In practice, such assumption is very unlikely to hold, all the more since, as the monetary authority of the Euro area, the ECB must take great care of not being accused of monetizing its policies. Secondly, the results also suggest that public authorities should not blindly intervene to stabilize the economy, as certain types of interventionism produce little welfare improvement, as was illustrated by the CBPP programme. Such policies should remain exceptional, and considered with great care before they can be effectively implemented.

Still, these conclusions represent a significant step forward. Active interventionism is very much against the tradition of the ECB and its predecessor, the Bundesbank. It is also at the radical opposite of the initial ambition to set the ECB as an independent instrument whose exclusive function would be the design of monetary policy. It is not without a reason that E.U. legislation proscribes the purchase of sovereign debt on primary markets, and Chancellor Merkel indeed had much reluctance to accept the settlement of bailout plans by the ECB. There seems yet to be now a consensus that the Euro area could not have survived without such interventionism. As the matter was new, the instruments of these policies have been settled in a rather ad hoc manner, part of them being taken in charge by the ECB (CBPP, SMP/OMT), and part of it directly between the member states (EFSF, EFSM). OCA theory has for long seen the lack of supra-national risk-sharing facilities within currency union as an argument against their success. It turns out the Euro area might well be on its way to implement the first of such mechanisms. It remains yet to be seen whether this evolution only constitutes an emergency response to a temporary crisis, or if it marks the beginning of a new paradigm within the Euro area.

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