



The estimate of the underground economy : the contributions of the fuzzy modeling

Mostafa Tahmasebi

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THÈSE

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Présentée par

Mostafa TAHMASEBI

Thèse dirigée par **Michel ROCCA**

préparée au sein du **Centre de Recherche en Économie de Grenoble**
dans l'**École Doctorale de Sciences Économiques**

L'estimation de l'économie souterraine : les apports d'une modélisation floue

Thèse soutenue publiquement le **28 mai 2015**
devant le jury composé de :

Mme Sophie DABO

Professeure à l'Université Charles de Gaulle, Lille 3 (Rapporteure)

M. Philippe ADAIR

Maitre de Conférences HDR à l'Université Paris-Est Créteil Val de Marne
(Rapporteur)

Mme Véronique SIMONNET

Professeure à l'Université Pierre Mendès-France, Grenoble 2
(Présidente du Jury)

M. Michel ROCCA

Maitre de Conférences HDR à l'Université Pierre Mendès-France, Grenoble 2
(Directeur)



This document is dedicated to

My Parents

and **My wife**

for supporting me all the way!

Acknowledgment

It must be admitted that writing a good economic thesis is not a facile work. It requires extensive studies on previous literature, being equipped with various economic and statistical knowledge, proposing models, gathering the data, testing the models and comparing the results with those of previous methods and models. Hence, it could not be accomplished individually or solely by ourselves.

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Abstract

Much attention has been paid in recent years to the study of the underground economy in many developed and developing countries. The consequences and the policy implications linked with this ambiguous part of the economy have raised concerns among economists and governments that led to proposing various measures and estimation methods. It is not however easy to estimate accurately and precisely the size and trend of the underground economy due to its hidden nature. But, some techniques have been used by economists to directly and indirectly estimate the size of the underground economy.

In this thesis, we have focused on the underground economy as a universal phenomenon having its unique and inescapable manifestation in the form of both legal and illegal activities. The main purpose of this thesis is to propose fuzzy methods to measure the size and the amount of the underground economy.

We first constructed a conceptual framework that allowed us to study the specifications of the underground economy. The common numerous estimation methods of the underground economy, their weaknesses and strengths were reviewed then. The basic conditions of applying the fuzzy concept and the initial conditions of the underground economy were investigated to see if they match.

Since fuzzy modeling allows for rapid modeling even with imprecise and incomplete data and lets us to model non-linear functions of arbitrary complexity and achieve simplicity and flexibility, we were encouraged to apply fuzzy method.

Three fuzzy methods were proposed to estimate the underground economy based on initial investigations during the period 1985-2010, including: the fuzzy modeling applying mean and variance, the fuzzy modeling applying fuzzy clustering, and Multiple Indicators and Multiple Causes (Structural Equation Modeling) with fuzzy data.

Finally, the size of the underground economy was measured for France, Germany, Italy, USA and Canada and the results of these fuzzy methods were compared with other conventional

methods. It can be claimed that the proposed methods in this work are qualitatively comparable to the common methods used to estimate the underground economy.

Keywords: Underground Economy, Structural Equation Modeling, MIMIC, Fuzzy Modeling, Fuzzy Linear Regression, Estimation Methods.

JEL Classification: O17, H26, C22, C51, E26

Résumé

Une grande attention a été accordée au cours des années récentes à l'étude de l'économie souterraine dans de nombreux pays développés et en voie de développement. Les conséquences et les implications politiques associées à cette partie ambiguë de l'économie ont suscité des inquiétudes parmi les économistes et les gouvernements qui ont été amenés à proposer diverses mesures et méthodes d'estimation. Il n'est cependant pas facile d'évaluer avec exactitude et précision la taille (l'ampleur) et la tendance de l'économie souterraine à cause de sa nature cachée (dissimulée, discrète). Néanmoins, certaines techniques ont été utilisées par les économistes pour estimer directement ou indirectement la taille de l'économie souterraine.

Dans ce manuscrit de thèse, nous nous intéressons à l'économie souterraine comme un phénomène universel ayant une manifestation unique et incontournable sous forme d'activités à la fois légales et illégales. Le but principal de cette thèse est de proposer des "méthodes floues" (méthode de la logique floue) pour en mesurer la taille et la quantité.

Dans un premier temps, nous avons construit un cadre conceptuel nous permettant d'étudier les spécificités de l'économie souterraine. Ensuite, de nombreuses (plusieurs, différentes...) méthodes communes d'estimation (méthodes en vigueur pour l'estimation) de l'économie souterraine ont été examinées. Les conditions de base de l'application du concept flou (de la logique floue) ainsi que les conditions initiales de l'économie souterraine ont été explorées (interrogées) pour voir si elles correspondent les unes aux autres (si elles font la paire, si elles sont cohérentes).

Dès lors que la logique floue permet une modélisation rapide même avec des données imprécises et incomplètes et des fonctions non-linéaires d'une complexité arbitraire et d'atteindre la simplicité et la flexibilité, nous avons été encouragés à appliquer cette méthode.

Trois méthodes floues ont été proposées pour évaluer l'économie souterraine sur la base des enquêtes initiales réalisées pendant la période 1985-2010, à savoir : la modélisation floue appliquant la moyenne et la variance, la modélisation floue appliquant le regroupement flou, la modélisation floue utilisant de multiples indicateurs et causes avec des données floues.

En dernier lieu, la taille de l'économie souterraine a été mesurée pour la France, l'Allemagne, l'Italie, les Etats-Unis et le Canada. Les résultats issus de ces différentes méthodes floues

ont ensuite été comparés avec ceux obtenus par d'autres méthodes conventionnelles. Il peut être affirmé que les méthodes proposées dans ce travail de recherche sont qualitativement comparables avec les méthodes couramment utilisées pour évaluer l'économie souterraine.

Mots clés : économie souterraine, modélisation par équation structurelle, MIMIC, modélisation floue, régression linéaire floue, méthodes d'estimation,

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List of Abbreviations

- A: Average (Linguistic term of the underground economy)
- AAEP: Average Absolute Error Percentage
- B: Big (Linguistic term of the underground economy)
- DTR: Direct Tax Rate
- FCM: Fuzzy C-Means (Clustering method)
- FLSR: Fuzzy Least-Squares Regression
- FM: Fuzzy Modeling
- GDP: Gross Domestic Product
- GDPC: Real GDP per Capita
- GLS: Generalized least squares
- GNI: Gross National Income
- GNE: Gross National Expenditure
- GR: Government Regulation
- H: High (Linguistic term of the causes)
- HSR: Household Net Saving Rate
- IC: Index of Confidence
- IR: Inflation Rate
- ITR: Indirect Tax Rate
- L: Low (Linguistic term of the causes)
- LIML: Limited Information Maximum Likelihood
- LT: Linguistic Term
- LTUR: Long Term Unemployment/Labor Force
- LV: Linguistic Variable
- MF: Membership Function
- MIMIC: Multiple Indicators Multiple Causes
- MLE: Maximum Likelihood Estimation
- N: Normal (Linguistic term of the causes)
- NDP: National Domestic Product

- OECD: Organization for Economic Cooperation and Development
- OLS: Ordinary Least Squares
- PRL: Participation Ratio of the Labor force
- S: Small (Linguistic term of the underground economy)
- SBU: Small Business Unit
- SD: Standard Deviation
- SEM: Structural Equation Modeling
- SER: Self-employment/Labor Force
- SNA: System of National Accounts
- SSE: Sum of Squares for Error term
- SST: Sum of Squares for Total of error term
- STFNN: Symmetrical Triangular Fuzzy Numbers
- TFN: Triangular Fuzzy Numbers
- UE: Underground Economy
- UR: Unemployment Rate
- USA: United States of America
- VB: Very Big (Linguistic term of the underground economy)
- VH: Very High (Linguistic term of the causes)
- VL: Very Low (Linguistic term of the causes)
- VOF: Values of Objective Function
- VS: Very Small (Linguistic term of the underground economy)

Chapter 1

General Introduction

"Black holes are thought to exist where nothing seems to exist where everything that enters vanishes. Gravitational forces are so strong inside a black hole that not even light can get out - hence, the name. Since no one can devise a way to visit a black hole to report on what they find there, what happens in a black hole has to be deduced from what cannot be seen."

Lester Thurow, 1992

1.1 Problem Statement

Many countries have made efforts for more than two decades to transform their political economies. The complexity of this transformation has appeared in different outcomes throughout these countries. Some countries (especially, central and Southeastern Europe countries) have succeeded more than others in engineering economic growth and controlling poverty and inequality [Svejnar, 2006]. Despite this, there is still economic inequality in many societies which manifests itself in some indicators that affect the economic growth, such as the underground economy.

There is no doubt that a considerable and increasing deal of economic activities take place outside the official economy which is known as the underground economy. From a long time ago, many trades, productions and consumptions are performed outside the legal system,

frequently in violation of the laws in effect, thus, the underground economy has existed in different economies like developing, transition and high income economies. Prostitution, sale of stolen goods or bribing the government officials for privileges are examples of such activities. Many usual economic activities which are supposed to be measured and taxed take place in the underground economy. These activities escape being detected in the official estimates gross domestic product (GDP) or taxation, thus, they cannot be captured in the official statistics [Tanzi, 1999].

The development and growth of the underground economy have led to many concerns in recent years, especially in developing countries and economies in transition where there are not enough legal and political institutions to support efficient market activities [De Soto, 1989]. Some researchers claim that the underground activities are increasing at a rapid rate [Feige, 2007]. Schneider [2002] has demonstrated that the underground economy varies considerably among transition countries. His research about 25 transition countries revealed that 38 % of GNP on average was produced in this sector between 1999 and 2000. In his recent study on 116 developing countries between 1999 and 2006/2007, Schneider et al. [2010] found that the underground economy has reached an average value of 28% of official GDP in those countries which is quite remarkable (see Figure (1.1)).

The increasing share of the underground activities has drawn many attention in the last few years since many observers have come to believe that the growth of this hidden sector of the economy is much faster than the official economy and it might become the most important sector of the advanced socio-economies. It should be noted that these observations have been mostly made in the countries with more immigrants [Paquet et al., 1996]. Although there is a universal knowledge about the existence and importance of the underground economy, the role of this sector in market performance and its share in allocation of resources is not fully understood. Consequently, there is no consensus on the nature and type of the underground activities which help to recognize them and categorize them under the name of the underground economy. However, one observer has commented that they range "from relatively legal to totally criminal, that somehow escape official attention and may distort official statistics and lead to erroneous policies" [Tanzi, 1983].

On the whole, the underground economy -also known as the unrecorded economy- consists of three main types of activities: non-market economic activities such as home productions

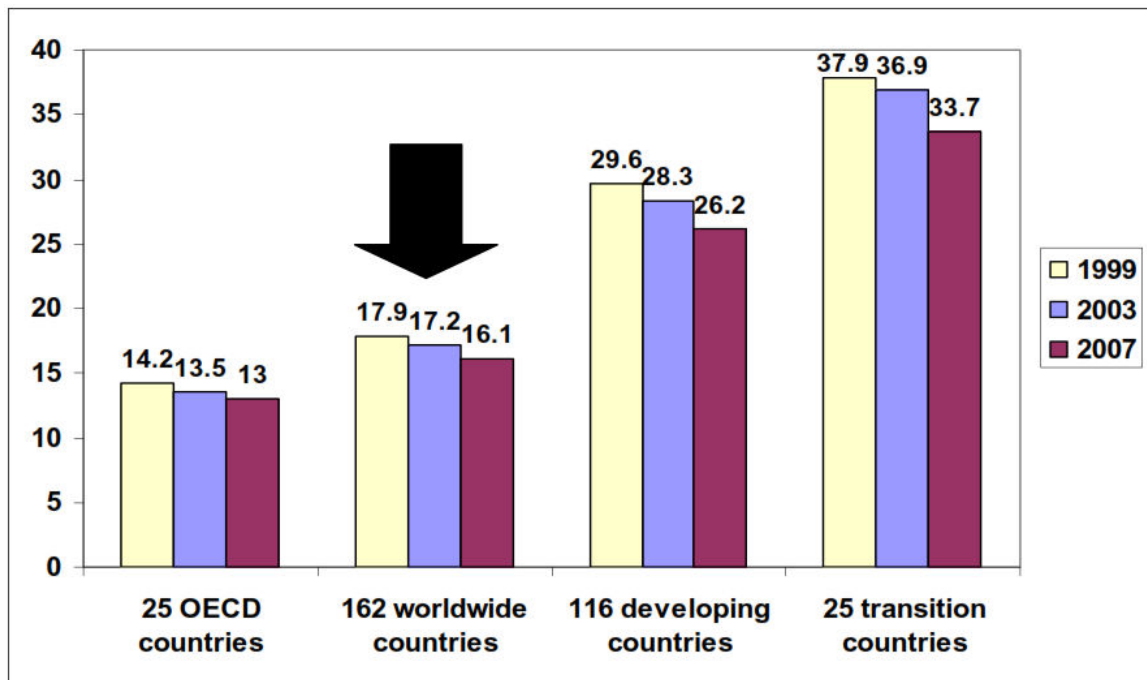


Figure 1.1: Size and development of the underground economy of various countries Groups [Schneider et al., 2010].

which are consumed by members of the family, illegal market activities such as prohibited production and distribution of proscribed substances, and legal market activities which are hidden due to reasons such as tax-evasion. Among these activities, the latter group are more important to economists [Jie et al., 2011].

The existence of the underground activities makes the official and social indicators inaccurate and unreliable and confuse the policy makers in a country. For example, the official number of unemployed people may hide that an (unknown) share of them actually work and receive income. Thus, the true unemployment rate might be overestimated and the economic and social conditions of individuals, household and countries might be evaluated in a biased way by relying on the official statistics. On the other hand, if the underground economy plays a greater role and influence positively the economy of a developing country, the government should pay more attention and support further the underground activities.

It is also known that the underground markets are usually cheaper than official ones because of their reduced costs of productions and services. The existence of the underground economy therefore distorts the market competition since the production situations are not the same in the official economy and in the underground economy.

Since the policy makers use the official statistics to design economic policies, the efficiency of the policies depends to a large extent on the accuracy of these statistics. As the underground economy distort the official statistics due to its hidden nature, not including the outcomes of the underground economy in statistics may result in underestimates of the true economic growth rate and the national income and the outcome of the economic policies tend to be inefficient then. For instance, effective monetary and fiscal policy require a level of precision in the estimates of key statistics (such as: income, consumption, unemployment, etc.), but the existence of the underground economy might distort these measures [Albu, 1995].

Every change in tax brings about its consequences and complications and that is why the tax policy is known as the most complex and sensitive area of the economic policy. To discuss the impacts of taxation, one cannot only rely on economic theories, but empirical estimates may help, too.

In theoretical tax models, it is almost always assumed that everybody follows the rules, while the reality says that it is not true. The behavioral response to tax changes has a wider range of variation, sometimes beyond the choice between labor and leisure. For example, entrepreneurs prefer to work underground in order to circumvent government-induced distortions such as excessive taxes, regulation, and graft. Friedman et al. [2000] explains that entrepreneurs go underground not to escape official taxes but to reduce the burden of bureaucracy and corruption. On the other hand, sometimes honest citizens decide to break the law in order to escape taxes. So, the loss of tax revenue due to the existence of the underground activities which escape taxation is another problem and concern for economists and policy makers. Consequently, in the discussions of tax policy, the way it affects the underground economy and vice versa should be essentially considered.

Most of economists agree upon the existence of a strong bidirectional causal relationship between effective tax rate and the size of underground economy, while some disagree. The formers believe that if the underground economy grows due to a rise in tax rates beyond maximum level, it may eventually lead to an erosion of tax revenue and social security base [Feige, 2007]. Both tax rates and enforcement affect the tax evasion, therefore, tax policy must be chosen regarding the type of enforcement that accompanies it. Taxation aims to raise revenue to pay for public goods, but it also have some effects on economic growth and income distribution which are often undesirable, and sometimes difficult to predict.

The underground economy can also manifest an unhealthy relation between the citizens and the government. When the taxpayers are not satisfied with public services they get for their contributions, they seek to redress the balance by evading to the underground economy and a vicious equilibrium comes up as a consequence of this unhealthy state. Individuals tend to work underground to escape taxation and social security burdens which results in the erosion of tax and social security bases and consequently increases the budget deficit and the growth of the underground economy. This, therefore, forces the government to raise tax rates on the official sector which, in turn, incentivize to participate in the underground economy. The consequence of this equilibrium will be gradual weakening of the economic and social basis of the collective arrangement [Feige, 2007].

The causes and impacts of the underground economy differ significantly between developed and developing countries. In developed countries, the inappropriate perception of government or high tax rates may influence the incentives to perform the underground activities. Due to the hidden nature of the underground economy, people can avoid and evade the taxes. They consequently tend to move to the underground sector. Over-regulation in the official economy also affects the size of the underground economy.

In developing countries with high unemployment rates, working in the underground economy is more of a survivalist strategy. Domestic firms in such countries cannot compete with multinational corporations that have entered their markets, so, these firms are forced to seek for alternatives. Since these employees are not usually skilled workers, they cannot readily find another job in the official sector. Also, the unemployed people and new entrants into the labor market cannot participate in the official labor market and in the absence of a strong social welfare system, there will be no choice other than employment or self-employment in the underground economy. It can be said that in developing countries, the underground economy is crucial as a source of income for low income families to sustain themselves until they are employed officially. But, the aim of participation in the underground activities is providing more saving in developed countries. Briefly, providing the survival needs and more saving are the two different reasons of working in the underground economy in developing and developed countries, respectively.

From other side, the reason that people go underground in developing countries is being unaware of the rules and laws, whereas, people consciously disobey the government regulations

in developed countries.

Small and family enterprises that mostly participate in the underground economy exist more in developing countries but less in developed countries. Immigration can also affect the rate of participation in the underground economy. It seems that the participation of the immigrants in the underground activities is more in developed countries in comparison with the developing countries [Gerxhani, 2004].

Another important factor is that different regulations are applied in different countries. As an example, drug and prostitution are legal in Netherland while these activities are forbidden in many other countries.

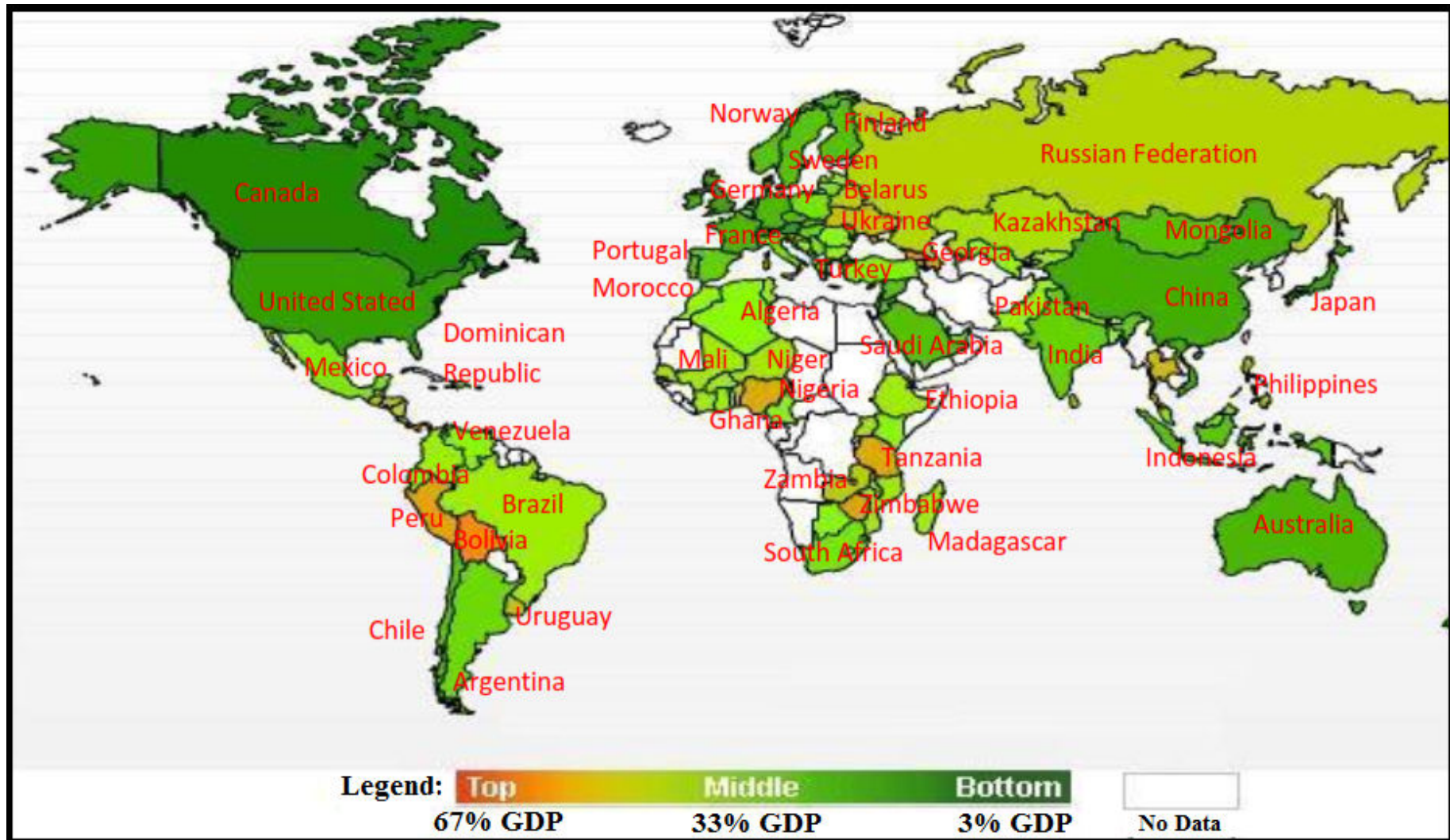


Figure 1.2: Average size of the shadow economy of 162 countries over 1999-2007 [Schneider et al., 2010].

Until here, some negative impacts of the underground economy were discussed while positive effects of this hidden sector of economy must be also considered. In this regard, a very first impact of the underground economy is providing some revenues for low-income families of the society and improving their welfare [Tahmasebi and Rocca, 2015]. Unemployed people, especially in developing countries, have the opportunity to work in the underground economy. The underground economy can substitute the official economy in certain types of economic activities where evasion is easier such as in the relatively low productive areas of the economy. More human resources are therefore involved in underground sector in comparison to the official sector in the developing countries.

On the other hand, a large underground enterprise may be regarded as an opportunity for future economic growth within the official economy. This is particularly important from a development perspective in poor and emerging economies with larger shares of the underground activities than developed countries.

De Soto [1989] believes that a key institutional weakness is the cause of underdevelopment in developing and less developing countries which restrict the productive capacity of the economy; that is, the most of the potentially productive capital is outside the system of formal property rights. In other words, in these countries, it is rather difficult to establish clear property rights in the first instance and afford protections by a formal property rights system. In contrast, in countries with mature property rights systems, capital can be leveraged extensively for productive activity. Wider participation in the formal economy is therefore hindered in developing and less developing countries; it favors the few and leaves the many out.

To sum up the positive aspects of the underground economy, Spiro [2005] summarizes the following positive consequences for the underground economy:

- Government spending is sometimes excessive and dispensable. The underground economy can be regarded as a kind of tax protest that forces the governments to control, or maybe limit, their spending.
- Government regulations on economic activities are sometimes too strict or unnecessary. In such situations, a willing seller and a willing buyer who cannot make a legal exchange prefer to go underground. In other words, the underground economy is a useful

outlet that increases economic welfare in such cases. It should be noted that this is a situation where the activity which has gone underground does not aim to evade taxation, otherwise, this activity usually takes place in a less efficient manner than it would be in the legal economy and there would be welfare costs.

Some positive and negative aspects of the underground economy that affect different policies were discussed. But, without reliable estimations that enable us to analyze and measure the underground economy, no further decisions or actions can be made. Despite many attempts made by economists and statisticians to measure the size of the underground economy in various countries, there is not a single reliable method for this purpose and the resultant estimates obtained from different methods are not always comparable. But it is still essential to measure the contribution of the underground sector in terms of income generation and employment creation due to the socioeconomic roles of the underground economy and its influences on the official sector.

The methods that have been proposed for the estimation of the underground economy can be categorized in three groups: direct methods, indirect methods and modeling methods. Direct or microeconomic methods generate the estimates for a specific point in time regarded as low-end estimates. Indirect, indicator or macroeconomic methods employ several indicators which can provide some information about the development of the underground economy. Multiple Indicators Multiple Causes (MIMIC) is known as a modeling method and benefits the use of multiple indicators that demonstrate the effects of the underground economy over time as well as multiple causes of the growth and development of the underground economy. Various methods examine different aspects of the underground economy based on the causes and/or indicators they use and their primary conditions. As an instance, a macroeconomic approach proposed by Elgin and Oztunali [2012] and a MIMIC model approach proposed by Schneider and Buehn [2012], both estimated the same trend and a similar average value for the magnitude of the underground economy in Canada and Italy. But these two models gave completely different estimates for Germany; Elgin-Oztunali model demonstrated a nearly constant trend unlike Schneider-Buehn's.

On the whole, the evidence presented has failed to achieve a consensus on a reliable method to measure the size of the underground economy without any strong assumptions. Breusch [2005] has discussed in his study the reason that the MIMIC estimations give different results

for the size of the underground economy in a specific country, although the MIMIC method is an agreed method among researchers.

Based on what was discussed and according to the literature, some of the limitations encountered when investigating the underground economy are:

- There is not a single comprehensive definition for the underground economy;
- The causes of the underground economy varies among countries or in the course of time;
- Lack of information and difficulty in finding the relation between the underground economy and its causes and indicators;
- The data sets of the causes and indicators of the underground economy are not sufficient;
- The uncertainty of the initial data (causes and indicators of the underground economy) affects the result of the estimation method,
- Strong assumptions and other weaknesses of the estimation methods also affects the results,
- There is not a single reliable method to estimate the underground economy.

1.2 Research Questions

Until now, some positive and negative aspects of the underground economy were discussed. The problems that the policy makers and economists encounter due to the existence of the underground economy were explained. Different categories of estimation methods proposed for the underground economy were expressed. It was explained that there is not an agreed method for the estimation of the underground economy and the present methods have some shortcomings which make their results not much reliable. Regarding these points, we answer the following questions in this research:

1. What is a comprehensive definition for the underground economy which clarifies and classifies different underground activities?

2. What are the common methods used to estimate the size of the underground economy?
What are the weak and strong points of each method?
3. Is it possible to investigate the underground economy applying the fuzzy concept? Is it possible to estimate a possible interval for the size of underground economy in each year?
4. Is the fuzzy method enough reliable for this purpose? How much reliable are the obtained results from the proposed fuzzy methods?
5. How large is the size of the underground economy in France? Can we use the proposed methods for other countries?
6. What was/is the impact of economic crisis on the underground economy in developed countries?
7. What is the impact of GDP growth on the extension of the underground economy after economic crisis?
8. What are the positive consequences of the underground economy in developed economy?

1.3 Hypothesis of the Research

It was mentioned that direct methods, indirect methods and modeling methods are the three main categories of estimation methods of the underground economy. In direct methods, surveys and questionnaires are used to collect the required data and the estimations are based on the obtained data. Difficulties associated with selecting appropriate samples, the respondent's willingness to cooperate and their honesty when responding are some defects of the direct methods.

Indirect approaches, also called indicator methods, are mainly macroeconomic methods that employ various indicators which can provide some information about the development of the underground economy. In other words, these methods focus on the discrepancy between the official and actual data such as the differences between national expenditure and income. These methods are however criticized for using some simplifying and limiting assumptions such as focusing on one specific indicator and neglecting other indicators which could also affect the development of the underground economy.

Applying the multiple indicators and multiple causes and making a structural equation model with a latent variable (here, the underground economy) is a modeling method which is known as MIMIC.

All these methods have some shortcoming which affect their final results such as applying only one indicator in indirect methods or possibility of selecting improper sample size in direct methods. The most agreed method used by the economists, MIMIC method, is also quite sensitive to sample size and the quality of the data [Breusch, 2005].

Regarding these constraints, it is crucial to choose or propose a proper method which covers these limitations best.

It is however difficult to get the exact numerical data about complex systems such as the underground economy. The complexity of systems themselves, the vagueness in people's thinking and judgment and the influence of various uncertain factors existing in the boundary environment around the systems might affect the information. These specifications and constraints together with the facilities and accuracy that the fuzzy concept offers, encourage to apply the fuzzy concept to study the underground economy. Unlike other methods, quality of the data and sample size have a minimum effect on the result of the fuzzy method which

makes it an advantageous method.

Applying fuzzy logic together with structural equation modeling, which is the basis of the MIMIC, makes the results even more reliable. In other words, we combine the strengths of the fuzzy concept and structural equation modeling to boost our method.

In this research, we also hypothesize that the effects of direct and indirect tax rate, government regulation, unemployment and some other causes are significant on the size of the underground economy. According to the literature, we also consider that the GDP is an indicator that underground economy affects directly.

The underground economy has a distorting effect on the accuracy of statistics and affects both fiscal and monetary policies implemented by regulatory authority. Therefore, the government and policy makers at all levels of state administration should seek for a solution to the underground economy problem through making decisions and taking measures that result in the long-run.

1.4 Significance of the Research

There is a widespread feeling that fighting the underground activities by police pugnacity or regulatory powers will restrict these activities, but this actually brings about adverse effects. In other words, fighting against the underground economy will yield nothing but encouraging the undergrounders to become more sophisticated in their activities.

A variety of reasons motivate the labor force to participate in the underground economy the three main of which are [Paquet et al., 1996]:

1. If they work in the official economy, their income will be overtaxed;
2. Their activities may be regarded as dehumanizing in the official market; and,
3. Undergrounders fail to see the taxes they should pay as the prices they pay for public goods.

There is a common belief that the high income tax rate is the main reason to go underground. Comparing the size of the underground economy in France (as a country with high income tax rates) and other European countries in different years demonstrates that this belief is not true and the tax reform is not the main cause. Apart from tax rates, another possible reason can be a dual effort to redesign the private and public organizations to improve dramatically accountability and very serious attempts at civic education to ensure that the society is fully conscious of the links between taxes and public goods and services. For instance, the social services of France encourage people to participate in the official economy in order to benefit from these social services in the future. All these efforts and attempts are a reframing of governance and a change in the organizational culture which cannot be accomplished overnight but are bound to take years, decades maybe. These long-run modifications of the socio-economic policies are clearly more important and fundamental than tax tinkering. Tax reform may be necessary but it will be never sufficient.

Based on these reasons and socio-economic considerations, the estimation of the underground economy is crucial. In fact, having an estimation for the size of the underground economy is indispensable in policy making process. Knowing that the existing methods used to estimate the underground economy include some strong assumptions and shortcomings in certain conditions encouraged us to propose more accurate methods which fit best the initial

conditions of the underground economy.

From other side, it is known that the administrative officials in many countries emphasize less on regulatory control and there is a large loophole in regulating the irregular activities which might eventually destroy the tax bases and social security burden. This is another reason that emphasizes the importance of this research.

This research can serve as a cautionary reminder to the government to seriously consider and take into account the consequences of the underground economy that may hamper or stimulate the economic growth and social welfare. We also hope that we have encouraged enough in further research in the area of fuzzy estimation of the economy, especially the underground economy.

1.5 Research Objectives

1.5.1 General Objectives

The importance of the underground economy and its impacts on the economy of each country cannot be denied. Ignoring the existence of the underground economy in the development process of countries, especially in developing countries, can cause serious damages which cannot be easily compensated. The underground economy exists in different countries, either developing or developed, and appears in different forms. The continuous changes in the structure of the underground economy shows the importance of the periodical studies about this hidden sector of the economy. These changes may be the reason of the absence of a comprehensive definition for the underground economy which can cover different activities of this sector. The main purposes of our research are therefore:

1. To present a comprehensive theoretical framework as the basis for a study on the nature of and reasons behind the underground economy;
2. To recognize various forms of the underground economy each of which has its specific causes and implications;
3. To demonstrate the importance of the underground economy for public finance and provision of public goods;
4. To give a descriptive and critical comparative overview of common methods used to determine the amount of the underground economy;
5. To present some shortcomings of the MIMIC method as a universally accepted method which is used to estimate the underground economy;
6. To give the case study analysis of some developed countries explaining the nature and relative size of the underground economy, using the fuzzy methods.

1.5.2 Specific Objectives

And more specially, this research intends to:

-
- I Introduce the underground economy as a complex system and explain the need for fuzzy concept to model and measure the size of the underground economy;
 - II Propose new fuzzy methods to estimate a fuzzy number for the size of the underground economy as a possible interval in a period of time;
 - III Estimate the size of the underground economy in some developed countries;
 - IV Outline the consequences of economic crisis on growth of the underground economy, and;
 - V Discuss the impacts of the underground economy on official statistics, policy making process and economic growth.

1.6 Organization of the Research

The importance of the study of the underground economy, its positive and negative consequences, the effects it has on policy making, the importance of estimating the underground economy and different methods for this purpose were discussed until here. These topics as well as the findings of this work will be described more in the following chapters structured as follows:

1. Chapter 2 presents an overview on the underground economy, its definitions, causes, indicators and consequences. A complete schema of the underground economy is provided in this chapter.
2. Chapter 3 reviews various estimation methods of the underground economy and more especially discusses the MIMIC method as the most common method.
3. Chapter 4, provides the necessary definitions of the fuzzy concept and describes different steps of fuzzy modeling.
4. Chapter 5 outlines the methodologies proposed in this research for measuring the size of the underground economy.
5. Chapter 6 presents and discusses the estimation results of the annual size of the underground economy using the proposed fuzzy methods during the period 1989-2010.
6. Chapter 7 concludes the research.

Part I

Literature Review

Chapter 2

Underground Economy: An economic question

Introduction

In all countries, there is a parallel or unofficial market behind the formal market (which is under the government's supervision). This unofficial market is created with the aim of escaping from the government regulations or breaking them, which take place in two conditions [Tahmasebi and Rocca, 2015]:

- Production and trade of illegal products: in this case, the products are completely illegal and they are against the government regulations like, the production and trade of drugs and prostitution ¹.
- Production and trade of legal products while they take place in this market in order to escape taxation and some supportive regulations of human resources.

This unofficial market makes the second part of the economy and it is named the underground economy as it is against the official or legal economy. In the literature, the underground economy is also called by many other terms such as black, irregular, unofficial, non-observed,

¹ It must be considered that this item varies among different countries and it depends on the country under study. For example, drug trade and consumption in the Netherlands and prostitution in Germany are legal while they are forbidden in France.

hidden, unobserved and shadow economy [Gyllys, 2005; Pommerehne and Frey, 1984]. This variation of terms may reveal the fact that the underground economy is an ambiguous concept. This can be concluded also from a wide variety of definitions for the underground economy presented in the literature [Gayraud et al., 2012; Ögünç and Yılmaz, 2000] and also from the fact that the activities it encompasses are mobile and dynamic. Moreover, the structure of the underground economy has displayed several variations as the time goes by. Consequently, the confusion in terminology and concept in this ambiguous socio-economic phenomenon could bring about huge cognitive losses [Gyllys, 2005].

The tax rate is a reason that encourages people to participate in the underground activities [Buehn and Schneider, 2008]. The high tax rate increases the incentives of people to work and trade in the underground market as the underground market is an untaxed market. An increase in the tax evasion decreases the tax revenues of the government and makes it difficult to provide the budget for the public services. From other side, the existence of the underground market may also spoil the official statistics and render them unreliable. So, estimating the tax gap, the distribution of income, real economic growth and increasing the impact of financial and economic policies are crucial and can be achieved through studying the underground economy [Schneider and Buehn, 2012; Schneider and Enste, 2013; Tahmasebi and Rocca, 2015].

The importance and necessity of studying and determining the size and development of the underground economy is clear now as they let us to estimate the tax evasion, purify the economic criteria and increase the impact of financial and economic policies [Fortin et al., 2009]. The present chapter aims at presenting:

- An introduction to the the underground economy, its definitions, segmentation and its actors;
- The positive and negative consequences of the underground economy;
- Different causes of the underground economy; and,
- Different indicators such as monetary, labour market and state of the official economy.

The theoretical considerations of the underground economy including the definitions, activities and the actors are presented in the next section. Then, the macroeconomic and microeconomic

views to the underground economy are discussed. A macroeconomic model of an economy with the underground economy is proposed and further explained in four sub-models. Next, the positive and negative effects of the underground economy and its consequences on the official economy are provided. Afterwards, the causes and indicators of the underground economy are introduced and the selected causes and indicators used in the models proposed in this research are determined. Finally, different sectors of the underground economy are presented.

2.1 Underground Economy: Definitions and actors

Despite the government regulations in developing and developed countries, there are large numbers of market transactions (mostly in cash) in various activities and services which are not reported and declared to the government and make a phenomenon known as the underground economy [Fortin et al., 2009]. Most governments restrict these activities through prosecuting, punishing and educating culprits, but the changes in the form of such activities have been considerable over the course of time [Jie et al., 2011].

A quick review on economic situation of OECD countries reveals the considerable impacts of this section of economy which is called underground economy, shadow economy, non-observed economy and etc.

Opposite to the common belief that the underground economy is a recent phenomenon in economic literature claimed by several influential publications in the late 70s, first economic inquiries about the underground economy date back at least to the early 40s².

The economy of every country is divided into two main sections, formal and informal. In different resources, other terms such as observed and non-observed, regular and irregular, official and unofficial, legal and illegal economy and etc. are used as well to name these sections [Blades and Roberts, 2003; Gylys, 2005]. The recording and reporting of economic activities are two main criteria to make this differentiation. Underground activities exert some adverse effects on the official economy and they may affect government policies, planning and legislation.

Some of the undesirable and negative effects of the underground economy underlined in the

² See, for example Jostock [1943]

literature are: the official statistics may become unreliable; tax revenue may reduce due to the tax evasion in the underground markets; financial and political policies may become inefficient; and the development process of developing countries may become depressed [Frey and Schneider, 2000; Tahmasebi and Rocca, 2015].

On the contrary, some researchers believe that the underground economy has positive desirable effects on the development process in different countries [Dell'Anno, 2007; Dell'Anno et al., 2007], as it may cause market competition and boost GDP [Zhuge et al., 2009]. Moreover, legal underground activities can provide earnings for low-income families in both developing and developed countries.

Some researchers have demonstrated the relation and correlation between the size of the underground economy and corruption (Dreher and Schneider [2010], Schneider [2004b] and Lafay [1990]), tax evasion and avoidance (Schneider and Enste [2000], Elgin and Garcia [2010] and Elgin [2010]) and income (Dreher and Schneider [2010])³. However, Schneider, Fery and some other researchers believe that the underground economy is not only associated with illegality and corruption, but also it comprises the legal activities which are not reported and recorded [Amir et al., 2010; Feige, 1997a; Frey and Schneider, 2000; Schneider and Enste, 2000; Zhuge et al., 2009].

All these effects and concerns have led to a widespread attempt and share of economic research for better recognition of the underground economy concept. There is no agreement on a single definition for the underground economy among researchers [Öğünç and Yılmaz, 2000] but there has been always a real interest to define the underground economy clearly and figure out its key concepts.

Knowledge about the concept and specifications of the underground economy provides information about the size of the tax gap, the distribution of income, real economic growth and on the whole, it helps to obtain reliable economic criteria. In addition, fiscal and monetary policies are not much efficacious without considering the impact of the underground economy, which means that the underground economy has a significant effect on economic policies and regulations [Buehn and Schneider, 2013; Schneider and Buehn, 2012]. All these effects and concerns necessitate the recognition and study of the underground economy in greater detail,

³ The effect of these items on the underground economy will be completely discussed in the Causes section of this research

in particular its size and trends.

Defining the underground economy is an important initial step towards the understanding of its characteristics and how this "invisible hand" works. A comprehensive definition can also help to have simple and reliable method which does not include any strong assumptions.

Researchers and economists have studied the underground economy from different perspectives, either positively or negatively, with high degree of diversity. It is important to examine the characteristics and determinants of the underground economy to:

1. Standardize a clear definition which facilitates the understanding of the underground economy;
2. Identify areas of focus for better controlling the growth of the underground economy;
3. Have a comprehensive classification to recognize different activities for further researches (estimation, ...);
4. find main actors and their roles to control underground economy much more better; and,
5. Have the simple and reliable method to estimate the size and trend of the underground economy.

2.1.1 Denominations and Definitions

It was not until the early 80s that the unobserved economy emerged as an international phenomenon. In 1983, the first international conference on the underground economy took place in Bielefeld, Germany ⁴. One of its primary goals was to reach common grounds on definitional issues. But, the participants failed to reach a consensus and were using widely diverging definitions. However, there were some unifying themes, in particular:

- the underground economy captures a significant part of the world economy;
- for the sake of completeness of the SNA, the underground economy should be taken into account; and
- two principal types of the underground activities can be distinguished as:

⁴ See Gaertner and Wenig [1985], for the proceedings of the conference

First the activities that are legal according to a particular country's legislation but the income earned from them is either understated or hidden from state authorities (the legal underground economy);

Second the activities that are prohibited by law of a country (the criminal underground economy).

In spite of tremendous efforts to give the best definition for this section of economy, there is not a unique definition for it. Most researchers who try to measure and estimate the underground economy face the difficulty of how to define it. There are several names to call the underground economy such as: irregular economy (Ferman and Ferman, 1973), subterranean economy (Gutmann, 1977), underground economy (Simon and Witte, 1982; Houston, 1987), black economy (Dilnot and Morris, 1981), shadow economy (Frey, Weck and Pommerehne, 1982; Cassel and Cichy, 1986), and informal economy (McCrohan and Smith, 1986), unofficial, parallel second underground, non-observed, hidden, invisible, unrecorded or moonlighting but the term most often used is black economy (le travail au noir, Schwarzarbeit, svarta sektor). Also, the popular media uses terms such as invisible, hidden, submerged, shadow, irregular, non-official, unrecorded, or clandestine (U.S. Department of Labor, 1992).

Fleming et al. [2000] explains two different approaches which can be recognized in various definitions of the underground economy:

First Definitional approach which considers underground activities as simply unrecorded economic activities. This approach can be seen in the definitions proposed by Thomas [1999]: *"...those activities which...are not recorded in the national income accounts"* and Schneider and Enste [2000] : *"all economic activities which contribute to the officially calculated (or observed) gross national product"*⁵. There is another public definition by Smith which is cited widely in literature; he defines the underground economy as: *"...market-based production of goods and services, whether legal or illegal, which escapes detection in the official estimates of GDP"* [Smith, 1985]⁶. Feige [1997a] defines the underground economy as *"those economic activities transactions which are*

⁵ Similar definitions are used in another publications, (Feige, 1994, 1989), Schneider (1994a, 1998, 2000, 2002, 2004, 2005, 2007, 2008, 2009 and 2010), Frey and Pommerehne (1984), and Lubell (1991).

⁶ This definition is also mostly used by Schneider in his papers, Schneider (1994a, 1998, 2000, 2002, 2004, 2005, 2007, 2008, 2009, 2010) and by Smith (1985, 1994).

unreported or are unmeasured by society's current techniques for observing economic activity".

Second Behavioral approach which emphasizes the relevance of institutional rules legislated by government and governmental agencies and regards the underground economy as a change in economic agents behaviors being a reaction to institutional constraints and legislations. This approach can be noticed in definitions proposed by ?: "...*economic activities include conscious efforts to avoid official detection*", Feige [1990]: "The characteristics of each distinct informal economy are determined by the particular set of institutional rules that its members circumvent" and Loayza [1996] : "*Informal economy is unregulated by the institutions of society, in a legal and social environment in which similar activities are regulated*".

There is another classification for the definitions of the underground economy based on the purpose of definitions. In this classification, the most precise and frequently used definitions seek to relate the underground economy to officially measured national income; e.g., it comprises all presently not recorded productive (i.e., value-adding) activities which should have been included in the Gross National Product (GNP) [Frey and Schneider, 2000]. However, Frey and Schneider [2000] believe that this definition excludes two major set of activities:

1. The productions which are not a part of GNP by convention; in particular, private household activities. Depending on the approach and measurement technique, the household sector comprises between 30% and 50% of GNP.
2. The tax evasion which is not value adding but is redistributive, thus, is not included in the definition above. However, generally, no tax is paid on the underground activities which is value-adding. Therefore, the underground activities and tax evasion are related but certainly not identical.

There are also another definitions usually referred to in literature such as: "...those economic activities and the income derived from them that circumvent or otherwise avoid government regulation, taxation or observation" [Dell'Anno, 2007; Dell'Anno et al., 2007]⁷ or another one

⁷ See also Thomas (1999), Fleming, Roman and Farrell (2000) or Feld and Larsen (2005, p.25), Dell'Anno and Schneider (2003) and Feige (1989)

by Tanzi [1982a] that defines the underground economy as the activities which are unrecorded or unreported in the national accounts as a result of tax evasion. He completes his definition as: "The first term is about the production (or income) missed in the official statistics and the other term refers to the revenue not reported and not recognized by the tax authorities". In another definition, he mentions the tax evasion as a possible government revenue missed due to underground activities, "...the first measure implies that the country is richer than the official statistics show, the second (namely, tax evasion) indicates that the tax administration draws less revenue than it should." [Tanzi, 1999].

Draeseke and Giles [2002] state that the underground economy involves (legal or illegal) transactions, activities and services which are not measured as they are not reported in order to evade tax liabilities. Ihrig and Moe [2004] point out a similar specification of the underground economy, escaping from reporting and recording and define the underground economy as a sector which produces legal goods but does not comply with government regulations. In another definition, the underground economy includes economic activities carried out outside the framework of bureaucratic public and private sector establishments [Hart, 2008].

"The value-added activities that the official statistics do not register although they should" is another definition suggested by the System of National Accounts (SNA) as a good benchmark [Bovi and Dell'Anno, 2010]. Another definition refers to the concept that the underground activities are a part of market and economy which can be moved to official market according to legality or illegality: *"the secondary market with trade deals that would be possible in official economy but were taxed or would not be possible at all because of law"* [Startienė and Trimonis, 2010]. Also, Thomas [1999] presents a definition which considers legal and illegal market transactions and non-market transactions, separation of underground economic sections from each other and their encountering manner to tax.

A Romanian dictionary of economic terms, published in 1999, defines the underground economy as *"the totality of economic activities that are not declared to the institutions that calculate taxes and social securities and that escape statistical recording and national accountancy"* [Dobrotă, 1999; Radulescu and Stimmelmayer, 2010]. Also, in the American Heritage Dictionary ⁸ of Business, the underground economy is defined as business or trade

⁸ The American Heritage Dictionary of Business Terms in <http://business.yourdictionary.com/underground-economy>

that avoids notice or observation by the government agencies.

A more narrow and complete definition is also cited in many studies which defines the underground economy as: *"All market-based, lawful trade in goods and services deliberately concealed from public authorities in order to evade:*

1. *Payment of income, value added or other taxes;*
2. *Payment of social security taxes;*
3. *Certain legal labor market standards, such as minimum wages, maximum working hours, safety standards and etc.; and,*
4. *Compliance with administrative procedures, such as filling out paperwork"* [Buehn and Schneider, 2009].

On the whole, three main characteristics can be noticed from a variety of the definitions presented for the underground economy including: the underground economy consists of the transactions, activities and services; the components of the underground economy are either legal or illegal; and, these activities are not reported or recorded in spite of the government regulations due to unconsciousness, evasion or avoidance from the government regulations or shortcomings of the government agents. It is considerable that the participators of this sector trade underground goods and services in cash to prevent transactions being traced.

On the other hand, Gerxhani [2004] underlines that a universal definition for the underground economy is not necessary; she explains that: "As long as researchers share a definition for each separate aspect/activity of the informal sector, there will be, first, a basis of comparison between studies (for each activity) and second, the sum of all activities will provide a picture of the informal sector as a whole". She emphasizes that the recognition of the activities which form this part of economy can help to provide a schema of the underground economy. Thus, activity categorization can be a reliable method which helps to define the underground economy. Defining underground economy based on activity categorization is the most popular and preferable method. The first definition based on this method was presented by Feige and was completed by Schneider and colleague [Feige, 1997b; Frey and Schneider, 2000].

Briefly, we agree with Gerxhani (2004)'s opinion about the necessity of the universal definition for the underground economy and consider the three main specifications which are common

in various definitions of the underground economy: *”the underground economy includes transactions, activities and services; these activities are either legal or illegal; and finally, they are not reported or recorded in spite of government regulations due to unconsciousness, evasion or avoidance of government regulations, or the shortcomings of government agents”* (Tahmasebi and Rocca, 2015).

Considering these concepts, Table 2.1 ⁹ summarizes the types of the underground activities which are commonly considered in various studies. This table serves as a complement of the definitions above and is also used in this research.

Table 2.1: A taxonomy of types of the underground activities [Schneider and Enste, 2000].

	Monetary Transaction	Nonmonetary Transactions		
Illegal Activities	Trade in stolen goods; drug dealing and manufacturing; prostitution; gambling; smuggling and fraud.	Barter: drugs, stolen goods, smuggling etc. Produce or growing drugs for own use. Theft for own use.		
	Tax Evasion	Tax Avoidance	Tax Avoidance	Tax Avoidance
Legal Activities	Unreported income from self-employment; wages, salaries and assets from unreported work related to legal services and goods	Employee discounts and fringe benefits	Barter of legal services and goods	All do-it-yourself work and neighbor help

2.1.2 SNA, Production Boundary and underground economy

Despite the major efforts to present a unified theoretical definition for the underground economy, there is no agreement among researchers. Researchers sometimes use quite vague definitions with only a few of them referring to the System of National Accounts(SNA). Although SNA terminology is not entirely devoid of ambiguity and overlapping either, but it can be argued that it is the strict.

In each country, the level of real economic activity can be measured more precisely with the help of accounting (or in other words, SNA). However, as noted in Bloem and Shrestha [2000], accounting can also be uncertain as some activities of this type cannot be given a

⁹ This Table is presented in most of publications about the underground economy: (Schneider, 1998), (Schneider and Enste, 2002), (Buehn and Schneider, 2009) and etc. The structure of the table is taken from Lippert and Walker [1997] with additional remarks; also from Mirus and Smith [1997] and with additional remarks by Schneider and Enste (2000) and Edwards and Flaim [2008].

solid monetary measure (e.g., activities of households, some activities of charities and other non-profit organizations, etc.).

In spite of this empirical obstacle, a number of researchers believe that the underground economy should be also identified with all informal activities, at least theoretically ¹⁰. Based on this belief, the underground economy also includes such activities that are not supposed to be a part of the GDP ¹¹.

The legal activities which are not reported or recorded in related organizations are a part of the SNA as the first component of the underground economy; but, there has been considerable argument about including the second element of the underground economy, the illegal activities, into the SNA. There is a point of view that the criminal activities can be regarded as producing "bads" rather than goods and contributed nothing to a nation's welfare, thus, they are not necessarily meant to be a part of the SNA. But, the SNA principally measures the level of economic activity as well as other important aggregates such as expenditure or saving, rather than the welfare. It is therefore necessary to consider both legal and illegal activities as parts of SNA.

Following these considerations, a revision of the SNA was recommended by the United Nations in 1993 and 2008 ¹² to ensure the consistency and international comparability of SNA in terms of the underground economy components. According to SNA93, "all illegal actions that fit the characteristics of transactions - notably the characteristic that there is mutual agreement between the parties- are treated the same way as legal actions." [Commission et al., 1993]. Hence, criminal and legal transactions are equally important for the SNA. However, a criminal act has little importance for SNA purposes, but, it is the occurrence of a transaction and the value that is being added or redistributed that matters. To make it clear, burglary should not be directly included into the SNA as there is no mutual compliance between parties, but a contract murder (a service provided with consent of the customer and the murderer) or sale of stolen goods or drug production and distribution (sales transactions) should be regarded just like any other legal transaction which involves goods and services that are resold or new. There are some considerations associated with this treatment of the underground transactions.

¹⁰ See Thomas (1992).

¹¹ The empirical problem associated with this view can be dealt with, at least to a certain extent, by using such measurement methods that go beyond the realm of monetary transactions, such as the labor force participation method as an indirect method which will be discussed in next sections.

¹² These documents are called SNA93 and SNA2008 in our work in [Commission et al., 1993, 2008].

First, in some cases, the industry of crime is indirectly interrelated with the SNA in which, the occurrence of a transaction may not be vital. For example, existence of criminals necessitates protection devices or security personnel. Thus, to better demonstrate the interdependence of a country's legal and illegal industries, the mutual agreement condition may need to be removed. Second, it is hard for some elements of the illegal underground economy to say if a transaction actually took place, as in the case of the legal underground economy. As Lotspeich [1997] notes, underground protection or law enforcement services, for example, often imply a transaction with no consent. As pointed out by Friedman and Friedman [1984], the official equivalents of such types of activities establish the state functions of "tyranny" that do not assume consent between parties either but are deeply involved in the SNA.

An advantage of treating illegal and legal transactions alike is that such treatment does not restrict the criminal underground economy by a nation's criminal law, but, allows for international comparison of the underground economy. It is noticeable that some activities are legal within one country but illegal within another (for example, distribution of certain drugs, prostitution). On the other hand, the legal status of certain activities can change with time (for example, legalization of arms or alcohol). Thus, it is not simple to differentiate between legal and illegal activities unless we define illegal underground economy as illegal transactions rather than illegal actions.

Without national statistics or SNA, defining the underground economy would be pointless. Existence of the SNA implies that there is a state without which the entire economy would be unobserved simply because there would be nobody to observe (i.e., record) it. Consequently, the underground economy can be defined as any economic activity, regardless of being legal or social, that is not included into the actual SNA of countries.

If we define the underground economy as an economic reality that is not recorded -for various reasons- in the aggregated SNA of all countries, the components of the underground economy are determined based on the definition of economic reality. Thus, the most accurate terminology for the unobserved economic reality should be defined in compliance with the SNA93 concept of "production boundary".

In order to ensure thorough GDP estimates (i.e., what should be considered the economic reality), this concept introduces a general production boundary and the SNA production boundary to determine what should be included in national accounts. The general production boundary

differentiates between non-economic and economic production. On the other hand, the SNA production boundary includes all "human controlled activities resulting in outputs capable of being exchanged." Consequently, purely natural processes with no human involvement (e.g., unmanaged growth of fish stocks) are excluded while natural processes with human control (e.g., fish farming) are not. The production for own use is also included since it is an activity "resulting in outputs capable of being exchanged", even if it is not necessarily exchanged.

Based on marketability, an activity can be economically productive only if it is performed by someone other than the one benefiting from it. Basic human activities such as eating, sleeping, etc. are therefore not included into the general production boundary, whereas activities like washing, preparing meals and babysitting are included.

The SNA production boundary is more confined; it defines a range of productive activities that should be included in GDP estimates. The SNA93 summarized this boundary as follows:

1. the production of all individual or collective goods or services that are supplied to units other than their producers, or intended to be so supplied, including the production of goods and services used up in the process of producing such goods or services;
2. the own-account production of all goods that are retained by their producers for their own final consumption or gross capital formation;
3. the own-account production of housing services by owner- occupiers and of domestic and personal services produced by employing paid domestic staff.

This definition of the economic reality that was supposed to be contained in the SNA seems rather blurred and conditional, at least on the final destination of goods or services which is not always known, particularly in household production. Par. 6.24 of the SNA93 lists the most common types of production that were included irrespective of whether or not the final destination was own consumption: production of agricultural products and their subsequent storage (including for example, gathering berries and mushrooms, wood-cutting for firewood, hunting etc.); production of other primary products (for example, mining salt, cutting peat etc.); processing of agricultural products (milling flour, threshing grain); and other kinds of processing (dress making, pottery, furnishing, etc.).

According to first and second items above, major repairs of dwelling were the only cases

when the SNA production boundary includes own-account services. On the contrary, unpaid domestic or personal services provided by household members, which are very significant in many developing and transitional economies as well as volunteer services rendered for the community were generally not included in the SNA production boundary.

The SNA93 also contained definitions of informal, illegal and the underground activities which will be presented in the following sections. Informal, illegal and underground activities were all overlapping terms but remained within the production boundary and thus represented the economic reality that should be measured by the SNA.

In the strict SNA terminology, the underground economy consisted of the informal, illegal and underground activities as defined in the SNA93 which are all unobserved. Therefore, the underground economy can be defined as a number of activities that fall within the SNA production boundary but are not recorded in the SNA of countries for several reasons. In principle, a particular country's system of criminal law and administrative rules should not matter once the activities regulated by these rules are within the boundary. In practice, however, most countries have not compiled their national accounts including the illegal activities due to associated technical difficulties.

The illegal underground economy ¹³ is defined to include globally economic activities that fall within the SNA production boundary but are not included in the SNA for various reasons; the legal underground economy includes legal productive activities that are not included in the SNA although included in the SNA production boundary.

The illegal activities of the underground economy include transactions such as: drug trafficking, exploitation and abatement of prostitution, production of branded counterfeits, forgery of banknotes, intellectual property rights infringements, smuggling of goods, people, livestock, unlicensed profession practicing, unlicensed production of goods, rent seeking and the legal activities of the underground economy includes: income under reporting, overstatement of tax deductible expenses, "off-the-books" production of goods and services, non- market production of goods, small-scale agriculture ¹⁴, paid domestic services.

Although the two sector of the underground economy obviously coexist, they may have nothing in common. For example, illegal sector of the underground economy is largely driven

¹³ In some references is also called criminal section of the underground economy.

¹⁴ In SNA, it means an agriculture with less than 5 workers

by regulations (in particular, by criminal law) while, the legal sector is framed by tax evasion which means that they exhibit different degrees of responsiveness to both fiscal and restrictive aspects of state regulation.

A brief history of production boundary and the role of SNA was presented. OECD ¹⁵ believes in the production boundary as one of the key concepts in national accounts which helps to define the underground economy. Briefly, the production boundary is composed of the rules that determine what should be included as production and what should be excluded and demonstrate the scope of most current and capital transactions in national accounts. It was mentioned that the last "general production boundary" draws the line between economic and non-economic production. Economic production includes all activities "carried out under the control and responsibility of an institutional unit that uses inputs of labour, capital, goods and services to produce outputs of goods or services ... of a kind that can be delivered or provided to other institutional units". The System of National Accounts production boundary is more restricted than the general boundary. It describes the range of productive economic activities that should be included in GDP estimates and is thus the boundary relevant to considerations of exhaustiveness [Giles, 1999; ILO, 2002a].

The groups of activities most likely to be non-observed are those that are underground, illegal, informal or undertaken by households for their own final use. However, some activities may be missed because of deficiencies in the basic data collection program. These five mentioned groups of activities are referred to as non-observed economy problem areas. OECD has presented a set of useful definitions for these five groups of activities:

1. Underground production: Certain activities may be both economically productive and rather legal (in compliance with certain standards or regulations) but intentionally concealed from public authorities due to reasons mentioned above [Schneider, 2005]. Some examples of this type of activities are:

- The enterprises that do not declare a share or all of their income in order to evade or avoid direct or indirect taxation;
- The enterprises that ignore employment and social protection regulations or immi-

¹⁵ Organization for Economic Cooperation and Development, is an organization that acts as a meeting ground for 30 countries which believe strongly in the free market system, The OECD provides a forum for discussing issues and reaching agreements, some of which are legally binding.

gration laws by hiring labor "off the books";

- The enterprises that work unofficially to circumvent long and costly bureaucratic procedures, and;
- Self-employed workers who declare fraudulently that they are unemployed in order to receive unemployment benefits.

2. Illegal production: is a genuine process whose outputs consist of goods and services the sale, distribution or possession of which are prohibited by law but there is an effective market demand for them, e.g., production and distribution of illegal drugs, the legal activities carried out by unauthorized actors, hence, become illegal such as unlicensed practice of medicine. The SNA93 states explicitly that illegal activities should be included in the system of national accounts, noting that "despite the obvious practical difficulties in obtaining data on illegal production, it is included within the production boundary" and "All illegal actions that fit the characteristics of transactions notably the characteristic that there is mutual agreement between the parties are treated in the same way as legal actions".

3. Informal production: Most of enterprises, which operate in informal sector provide goods and services the production and distribution of which are perfectly legal and this characteristic differentiate informal production from illegal production. Another distinction between informal and underground activities is that the informal activities are not necessarily performed with the aim of evading the payment of taxes or social security contributions, or infringing labor legislation or other regulations. However, there exist some overlaps, like some informal enterprises may prefer to remain unregistered or unlicensed in order to avoid compliance with regulations and thereby reduce production costs.

Informal production may be mostly known as a number of units engaged in the production of goods or services with the primary goal of providing employment and incomes to the people concerned. More precisely, it refers to productive institutional units which are characterized by:

- low level of organization;

- little or no division between labour and capital, and;
- Work relations based on occasional jobs, kinship, or personal relations.

Examples of this type of activity are the activities of craftsmen, peddlers without licenses, farm workers, home workers and the unregistered activities of small sellers.

Table 2.2 presents the structure of informal work. As can be seen, informal employment is comprised of both self-employment in informal enterprises (i.e., small and/or unregistered such as Home-based workers and street vendors) and wage employment in informal jobs (i.e., without secure contracts, worker benefits, or social protection). It is notable that self-employment comprises a greater share of informal employment (outside of agriculture) in developing countries than wage employment [ILO, 2002b].

Table 2.2: Structure of Informal Work (ILO, 2002)

	Primary work for company	Extra work for current employer
Person works for someone else	<ul style="list-style-type: none"> • Works off-the-books • Works under-the-table • Paid in cash and not taxed 	<ul style="list-style-type: none"> • Works extra hours on weekends or evenings (paid in cash and not taxed)
	Person operates own small business	Does sporadic odd jobs
Person self-employed	<ul style="list-style-type: none"> • Cash-only exchanges • On-going, regular • Business is seen as primary income • May invest income into business 	<ul style="list-style-type: none"> • Seasonal work • Helps keep household expenses down by bartering for services or offering low-cost products • Not as formalized as a "small business"

Loayza [1996] explains that people who work for someone else may be doing informal jobs as their primary job or as extra work for their primary employer in addition to their primary work. For example, a building cleaning contractor who "employs" people off-the-books and pays them in cash, or someone who is employed by his/her current employer to come at the weekend, stay the evenings or to take work home to receive more money.

In self-employed type, people may run their own businesses, either as a primary source of income, mostly in developing countries, or as a way to increase their income beside what they receive from their primary job, which is more common in developed countries.

For instance, a person owns a lawn care business as the primary source of income or another person has a household repair business to earn additional income other than what he gets from the full-time employment. In another case, the person is still self-employed but the way he works may be less formal such as someone who makes craft boxes around the holidays and sells them to friends.

4. Household production for own final use: Production undertaken by household unincorporated enterprises solely for their final use is not a part of informal sector. The examples are: production of crops and livestock, production of other goods for their own final use, construction of own houses and other own-account fixed capital formation, imputed rents of owners-occupiers and services of paid domestic servants.
5. Production missed due to deficiencies in data collection program: It comprises all productive activities that were supposed to be accounted by the basic data collection program but are missed as a result of statistical deficiencies. It is sometimes referred to as the statistical underground similar to the economic underground that includes the activities that have been concealed by producing units for economic reasons. From production approach to GDP compilation point of view, the reasons why activities may escape direct measurement by the basic data collection system can be summarized in three main categories [ILO, 2002a]:
 - Under coverage of enterprises: Enterprises and every parts of them should be included in the data collection program, but sometimes they are not. The reasons may be that an enterprise is new and has not yet been included in the survey frames, or it falls below the size cut-off for surveys, or it has been incorrectly classified by kind of activity or by region and thus improperly excluded from the survey frame.
 - Non-response by enterprises: Enterprises are included in the sample but no data are collected from them (for example, because the survey questionnaire was wrongly addressed or the enterprise, or part of it, did not return the questionnaire) and no one takes the responsibility of the missing observations.
 - Under reporting by enterprises: Data are obtained from enterprises, but are mis-reported by the respondent in such a way to under report value added, or correct

data are received but are inappropriately edited or weighted.

More explanations and information are presented in Table 2.3.

Table 2.3: Unit by activity by cause classification of underground activities (OECD, 2002)

Type of activity	Type of cause	Type of Unit		
		Formal	informal	Household Production
Illegal and Underground	Not registered	Statistical underground, units not registered	Illegal production, units not registered	Informal sector, units not registered
	Not registered		Economic Underground, units not registered	
	Underreporting		Economic Underground, underreporting	
legal and not Underground	Not registered		Informal sector, units not registered	
	Not Updated		Statistical underground, units not updated	
	Non-response		Statistical underground, non-response	

2.1.3 Activities of the underground economy

Not all the activities which are performed in the underground economy are illegal. The concept of the underground economy includes both illegal activities (trafficking, corruption and etc.) and some legal activities which are unrecorded at the level of national accountancy (working home, volunteering) or which are undeclared to social and fiscal administrations (fiscal fraud, working on the black market). Moreover, the forms of the underground activities vary according to economic systems, economic regulations and other local specifications.

The different types of activities which are performed in the underground economy are demonstrated in **bird's eye view** model in Figure 2.1. In this model, economic activities are divided into two main sections: "Productive activities" and "Unproductive activities"¹⁶. Underground economy is placed among productive activities and consists of two sections. The first section

¹⁶ It is clear that we consider both "Goods" and "Services" in this division.

is called black economy and includes unrecorded companies which work without any registrations and companies that carry dissimulated activities. The second section is illegal activities, such as drug trade, weapons trade, human trafficking and etc. Based on another segmentation,

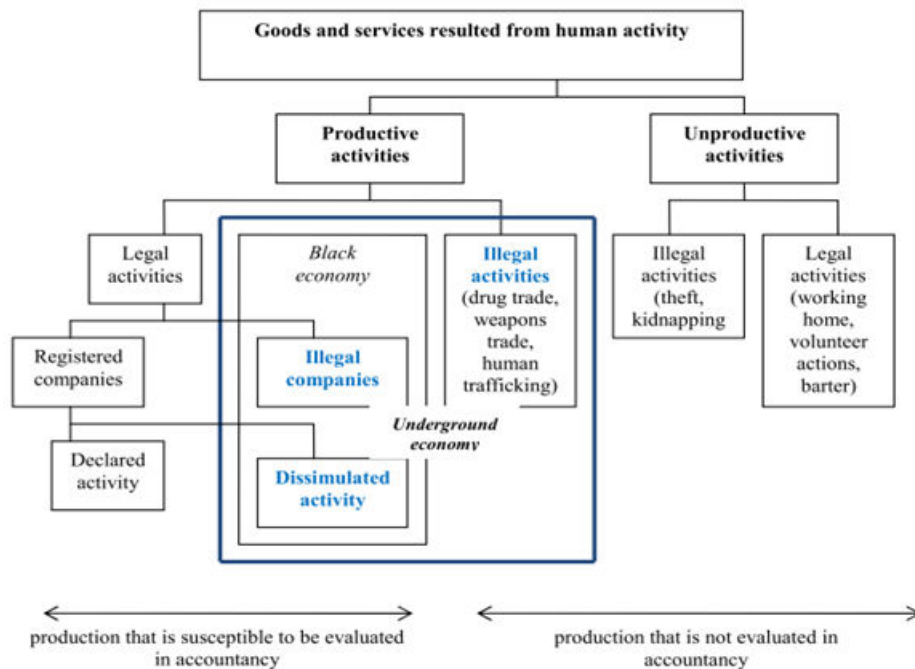


Figure 2.1: The place of underground economy [Cazenave, 1993]

all economic activities can be divided into legal and illegal sections. Each section is then split into reported or unreported activities. As depicted in Figure 2.1, the underground economy consists of:

1. Reported illegal activities
2. Unreported illegal activities
3. Unreported legal activities

The legal activities which are reported to official national accounting systems belong to the formal economy.

Based on another division (Figure 2.2), the underground economy is composed of black economy and unorganized economy. Unorganized activities include household sector and informal sector, none of which are illegal. On the contrary, black economic activities are all illegal with the characteristics of classical crimes like burglary, robbery, drug, smuggling tax

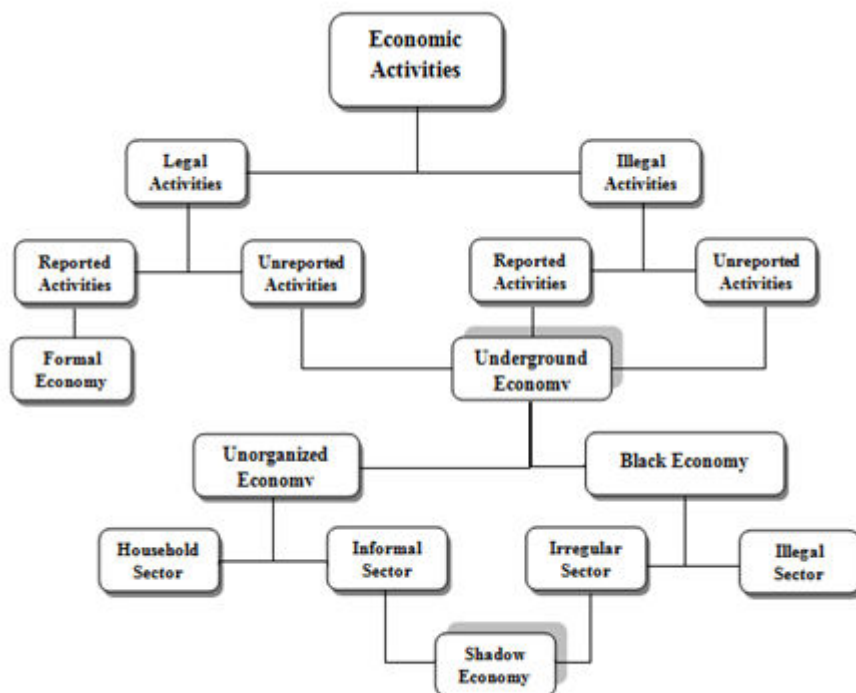


Figure 2.2: Underground and Shadow economy in economic activities framework [Razmi et al., 2013]

evasion, and etc. [Razmi et al., 2013]. Moreover, black economy is ambiguous, so, it is more complicated to calculate and measure it. OECD explains that areas of productive activities in

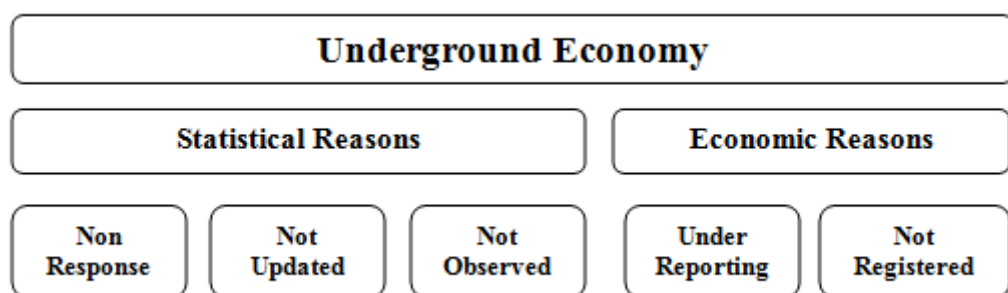


Figure 2.3: Production Boundary Segmentation of underground economy [ILO, 2002a]

the underground economy (Figure 2.3) are not directly observed due to:

- Economic reasons; for example, in case of the activities carried out deliberately to avoid taxes, social contributions in favor of employees or to avoid observing the law provisions concerning minimum wages, the number of work hours, job safety, etc.

- Statistical reasons; in this case production activities are not registered due to:
 - The failure to fill out the administrative forms or statistics questionnaires properly because the people in charge of filling forms are not familiar with statistics and/or due to shortcomings in the statistics system;
 - The difficulty in perceiving the changes of a rapidly evolving productive system containing small productive activities which are often not detectable with the traditional survey techniques [ILO, 2002a].

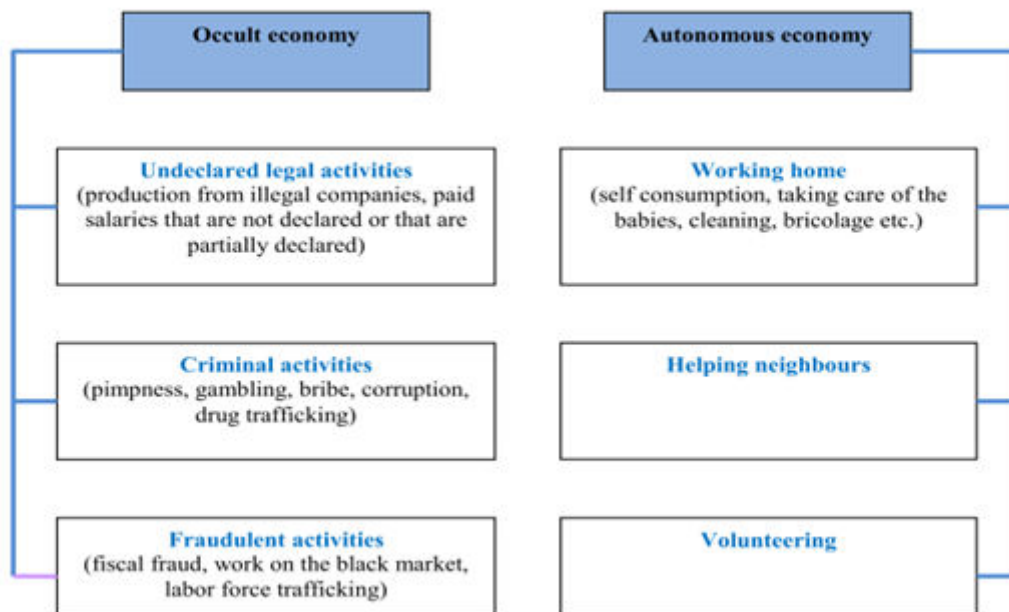


Figure 2.4: Occult economy and Autonomous economy [Debare, 1992]

Radulescu and Stimmelmayer [2010] explain that the activities of the underground economy exceeds the area of domestic activities. Some activities in the underground economy are not dealt on market; they are considered "Occult". They say further that some goods and services may be produced more economically from a social point of view if they are made in a limited area: individual actions, family, volunteering (Figure 2.4).

2.1.4 Actors of the underground economy and their motivations

The underground economy serves as an indication of the situation of people's lives, economy and etc. Some important points are the bases of the discussion about the underground economy, such as: the main actors, their motivations, the concerns about the underground economy and etc. These points are gained by answering the following questions:

1. Who takes part in the underground activities?
2. What are the roles of these actors in the underground economy?
3. What is the motivation to participate in the underground activities?

In every system, there is a boundary between internal factors/actors and external factors/actors. This boundary, which reflects the complexity of the system, is defined by the designer of the system or the one who studies the system. Considering too many actors in a system makes it complicated to study the whole system properly and it may affect the target of the study. Thus, the system bounds and actors must be determined carefully. When defining the system bound,:

1. We must consider the whole system including the details required;
2. We should not complicate the system by choosing too many actors. Still, the main actors, their roles and a simple model of the system based on these actors must be clarified;
3. The system bounds and actors must be clearly explained and noticed;
4. And, we have to bear in mind the targets of study.

In each society, the government legislates for many things such as traffic, trade, finance, market and so on to govern the society, and consequently, it affects the underground economy by changing the tax policies, regulation, punishment, etc. On the other hand, because of the restrictions imposed by government and other related institutions, society tends to follow the underground activities to violate the government regulations. Therefore, the people's incentive plays the main role in extending the underground economy.

Researchers have always studied the economic event, have analyzed the advantages and

defects of the government regulations and their effect on economic phenomena specially on the underground economy. So, researchers affect government and society indirectly and the underground economy directly.

After a brief acquaintance with the main actors in the underground economy, a question arises: What motivates of these actors?

Before answering this question, we need to know why underground activities are demanded. Schneider carried out extensive research on the demand of underground activities in Germany in 2003. He investigated whether people regularly work in the underground economy or not. According to his survey, 25% of German respondents work in the underground economy, while 46% of them regularly demand the underground activities. In his survey, he asked about the reasons that the underground activities are on demand and found out money saving as the most important reason. The respondents explained that the underground products and services are much cheaper than the official ones. According to 73% of the respondents, high tax and social security burden are the second most important reasons. Other reasons are listed in the Table (2.4) [Schneider, 2005]. It must be noted that these reasons are a part of the motivations

Table 2.4: Reasons for participation in the underground activities - Survey results for Germany [Schneider, 2005].

Reasons for why the underground activities are demanded	Values (%)
One saves money or they are much cheaper than in the official one	90
The tax and social security burden is much too high	73
Due to the high labour costs in the official economy one would not demand these activities (extreme assumption no shadow economy: 23 demand these activities in the official economy, 9% make do-it-yourself)	68
The firms offer them from themselves	52
It is so easy to become quick and reliable persons	31

in developed countries. Gerxhani [2004] also has the same opinion for developed countries but she explains that the motivations and reasons are different in developing countries and the survival demands are the main motivations for participation in the underground economy in developing countries. Chen [2007] also explains that:

- Despite predictions of its eventual demise, the underground economy has not only grown in many countries but also emerged in new guises and unexpected places;
- And, despite continuing debates about its defining features, supporting underground en-

terprises and improving underground jobs are increasingly recognized as key pathways to promoting growth and reducing poverty.

Due to reasons explained above, the underground economy has continued to prove an important concept to many policymakers and researchers as the reality it captures is so large and significant. It demonstrates that a large share of economic units and workers remain outside the world of regulated economic activities and protected employment relationships.

Up to now, we found out the motivations of society and researchers in relation with the underground economy. Next, we discuss the roles of government in the underground economy. Politicians have effect on the underground economy through legislating and establishing the rules (such as, work conditions, tax rate, punishments, social security and etc.) which lead to limiting or extending the underground economy. Radulescu and Stimmelmayer [2010] explain why the politicians should notice this section of economy:

- Increase of fiscal burden may cause the growth of the underground economy and implicitly, decreases the basis for taxes and social security [Olson, 2008]. The result may be a defective cycle with a rise of budgetary deficit or taxes, and this also causes a further augmentation of the underground economy which eventually may lead to attenuation of the economy and weaken the social basis of collective commitments;
- With high measure of underground economy, the economic policy will be based on fake "official" indicators (such as unemployment, official labor force, income, consumption) or on indicators with inaccurate magnitude;
- The effects of the rising underground economy over the official economy must be noted. Due to more benefits of underground activities, an increased underground economy attracts local and foreign workers outside the official economy and make more competitors for such companies. On the other hand, at least two thirds of the income from the Underground Economy is spent immediately in the official sector and has a positive effect on the Official Economy [Radulescu and Stimmelmayer, 2010; Schneider, 1998].

Frey and Schneider [2000] point out how the underground economy may affect the interaction of government and society. They explain that the underground economy indicates the satisfaction of society from government or in other words, it is an indicator of an unhealthy state

between society and government. If taxpayers are not satisfied with public services they get for their contributions, they will seek to sustain the balance by performing the underground activities.

2.2 Modelization of the underground economy

2.2.1 Macro and Microeconomics view to underground economy

The underground economy can be investigated by two different approaches: Macroeconomics view and Microeconomics view. A brief discussion about the underground economy regarding these two views are presented in this section.

From **macroeconomics view**, a theoretical macro model of business cycle as well as tax and monetary policy linkages with the underground economy (see Figure 2.5) reveals a positive relationship between the growth of the underground economy and the official economy. This positive relation is established under certain assumptions such as very low entry costs of underground economy due to low probability of enforcement [Frey and Schneider, 2000].

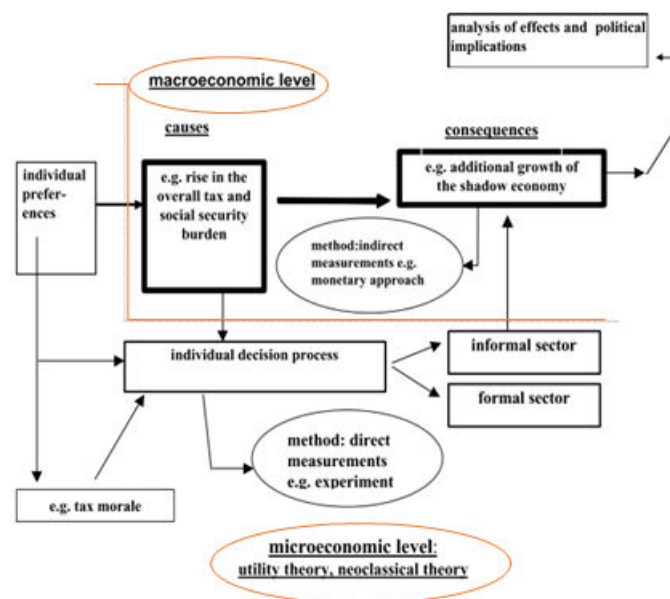


Figure 2.5: Macro and Microeconomic approaches of the interaction of underground and official Economy [Schneider and Enste, 1998]

Different studies imply that an expansionary fiscal policy has a positive stimulus on both official and underground economy. Empirical findings of Schneider show that over 66 % of the earnings in this section of economy are rather immediately spent in the official part [Schneider and Enste, 1998]. Hence, the positive effects of these expenditures on economic growth and on (mostly indirect) tax revenues must be taken into account as well. About

the United Kingdom , evidences suggest that the underground economy has a significant stimulative effect on consumer expenditure [Frey and Schneider, 2000].

In **Micro economic view**, the underground economy causes an unfair competitive condition among firms. The firms that participate in the underground economy -Unregistered firms- have no legal regulations and they can implement and sell their services and products at lower price than general market price ,thus, they can increase their sales volume and profit.

The underground economy may also deteriorate financial power of social security institutions; the increase in the underground economy decreases government tax revenue which may restrict consequently the social services provided for people on low-income. These limitations make living harder for people on low-income and that may increase social tensions in society. From other side, the underground economy has some positive effects on Micro economic indicators; e.g., it creates jobs in a country. It was considered that unregistered firms which operate in the underground economy have a lower cost structures (lower wage, non tax, etc.) than registered firms, so their labor demand and employment capacity can be higher than the firms in the official economy. Society welfare level may also increase as a result of the underground economy. Briefly, the underground firms may sell their goods and services at a lower price than general market price, thus, lower prices may increase purchasing power of society and increase general welfare level of the public.

2.2.2 Macroeconomic model of the underground economy

As it was discussed earlier, we can consider the society, companies, financial markets, government and etc. as the main actors of a comprehensive model of macroeconomics. The question then arises: How these actors whether increase or restrict the motivation to participate in this part of the economy?

The object of macroeconomics is the entire economy or in other words, the survey of some of the main variables or issues in the level of the society. The macroeconomic results can be obtained from study and analysis of the demand and supply of the goods and services, human resources, saving and etc. which help to realize the effects of the main actors in the underground economy.

In this section, we introduce a model which consists of the main actors of macroeconomics by considering the underground economy phenomena. Furthermore, we analyze the effects of these actors on this part of economy.

First model: Families and firms

In the first possible segmentation, the firms are divided into the official and underground firms. The criteria of this segmentation will be completely presented in the definition of underground economy. The main question here is that the firms which perform a part of their activities (manufacturing and services) as underground activities belong to official or underground sector?

It is noticed that each part of a firm is a Small Business Unit (SBU) based on the value it adds by its activities. Thus, each SBU can be considered as an economic firm whether official or underground based on the definitions and other criteria.

The families or society and firms are the two main parts of this model. The families supply and provide "Production Factors" of the manufacturing firms and receive "Income" for renting their production factors. The production factors are the inputs which the firms use and apply in the production process. as an instance, the "Human Resource" is a main production factor. Also, land, machinery and investment are other production factors provided by families. The families rent these factors and receive an revenue; rent and profit make the income of the families.

In this model, the families spend their incomes to buy goods and services from firms and firms produce goods and services applying the production factors and sell them to families. There is a conceptual difference between families and firms based on their functions. As an example, the manager or owner of a firm, indeed when he manages his firm, he is also a member of a family who gives its work force to this firm and he receives and spends the income he gets as a member of a family and not as the owner of the firm.

The firms which provide goods and services participate and work either formal or underground market. Regarding all these concepts, in an economically stable condition (Economic growth or economic recession), abrupt changes in the incomes of the families can represent the changes in the size of the underground activities. In other words, sudden changes in the consumption and expenditure of the families can be an indication of gaining new income

which are not registered in the official statistics. Moreover, the changes in the official statistics of the production factors (specially, human resources), demand and supply of final and intermediate goods, product and services and etc. can be a further proof for the changes in the underground market.

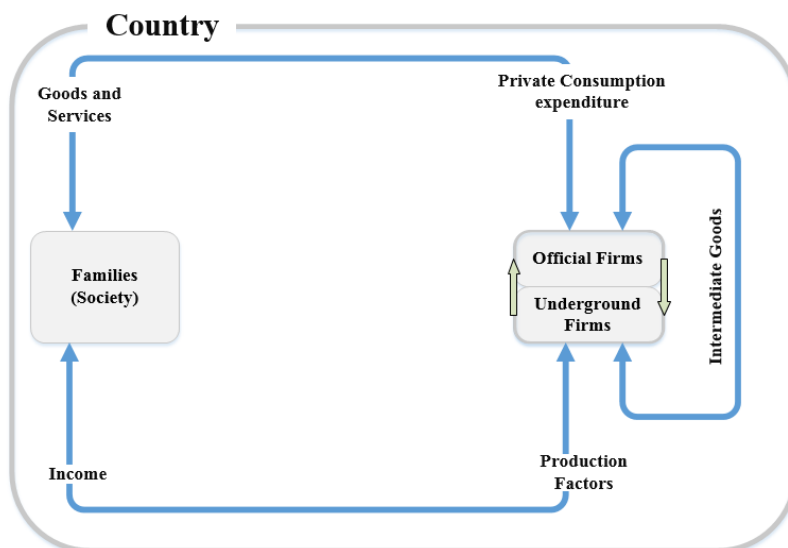


Figure 2.6: First macroeconomic model of the underground economy: simple macroeconomic model of the underground economy.

The process mentioned above can be shown in a product and income cycle as shown in Figure (2.6)¹⁷. As depicted in Figure (2.6), the firms process the raw materials and intermediate goods bought from other firms applying production factors in the production procedure. The output of the production line is sold to families. So, there are two bidirectional flows in the model as indicated in Figure (2.6).

The first flow is the flow of goods and services provided by firms for families and against, the cost of the goods and services is paid by families to firms. This flow indicates that the expenditure of goods and services is measured as a variable named: "consumption expenditure of private sector".

The second flow is related to the supply of production factors by families side the income received from firms which is known as "firms paid to production factors". From this model, the total amount of the final product equals with consumption expenditure of private sector as well as the income gained from the payments for the production factors.

¹⁷ The four models presented in this sub section are developed by the author.

Second model: First model with the financial market

Financial or investment market is a market that families can invest a portion of their income in and receive a loan loan for their investment purposes.

The first model is completed by adding the financial market and the new model is shown in Figure (2.7). In this model, family income paid by firms can be invested in financial market in addition to consumption mentioned in the first model. In this case, the income is not only spent only for consumption but, a portion of it is invested in financial and investment markets.

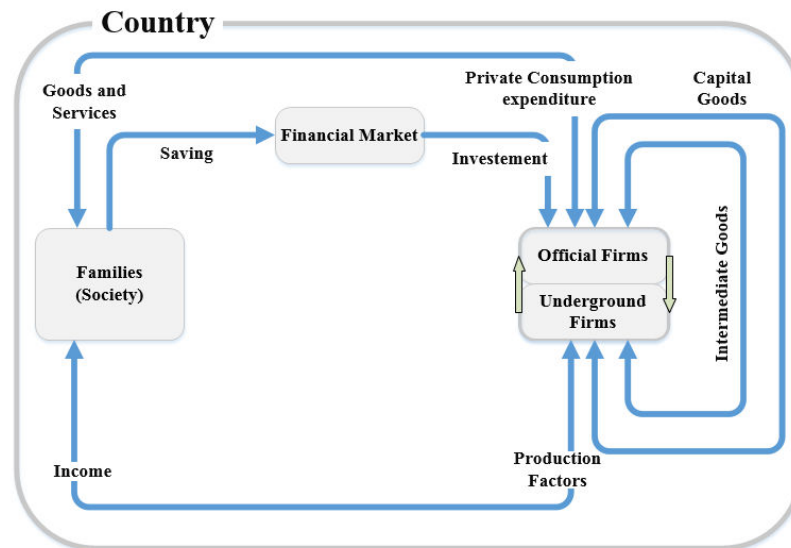


Figure 2.7: First model with the financial market (Second model).

On the other hand, the firms buy capital goods from other firms and consume them to produce more goods using the investments of the financial markets. In other words, a part of the final products will be invested and they are not completely consumed.

Based on this explanation, the amount of saving and investment of families in the financial market can illustrate the situation of the underground activities. Also, considering the economic situation and the official statistics can improve the result of this estimation.

The flow of the capital goods in Figure (2.7) started from the firms and ended to them indicates that a part of the final production in firms come back to the firms as an investment.

Third model: Second model with the Government

The government is the fourth element which adds to the model. The government deals with families, firms and financial markets. As depicted in Figure (2.8), government receives a part of the family income as the direct and indirect tax and spends it to buy goods and services from the firms. These goods and services are distributed as the public consumption of the society and other consumption. So, it can be stated that the family income will be spent on the private consumption, saving and investment and the tax paid to government.

Based on these items, the amount of the direct and indirect tax, government spending and consumption, government policies and legislation and etc. encourage families to participate and work in underground activities. Moreover, the satisfaction of families from public services which is related to the government revenue sources, affects on the extension of underground market or moves underground market to official market and vice versa.

The firms continuously move from underground market to official market and vice versa based on the economic situation, policies and legislation including supportive legislation and etc.; thus, the official and underground firms are shown in one box in the models.

Also, "the government consumption expenditure"¹⁸ can affect the production factors decision on how to continue their activities in the underground market. In other words, the government consumption expenditure may cause the extension of the underground economy and affect the satisfaction of families from the government.

About the relation of the government and the financial market, the government is an institution which affect the financial market, also, it can directly legislate for the financial market. In a very simple case, it can be mentioned that the government can compensate for budget deficit from financial market which indicates a flow from financial market to government.

Fourth model: Final model

Various elements of the final model were introduced step by step in previous models to illustrate perfectly the role of each element (e.g. government, financial market and etc.) on the firms and specially, the underground or informal firms.

¹⁸ It can be considered that the some of the final goods and services produced in the firms can be dedicated for the government consumption and these are known as "government consumption expenditure"

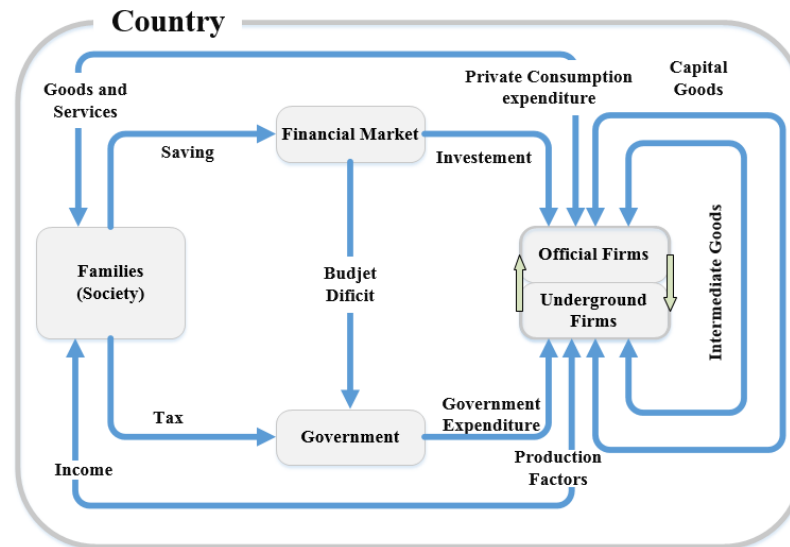


Figure 2.8: Second model with the Government (Third model).

To complete the model, the international trade and capital trade are added to the model as the final element. Each country has some international trade and capital trade with other countries, which means that some of the final goods and services, which are produced by the firms are exported to other countries instead of consumption and investment inside the country. On the other hand, a part of goods needed inside the country may be provided by other countries as shown in Figure (2.9).

By dividing the economy of a country to the official economy (consists of financial market, government and formal or official firms) and underground economy (consists of underground firms), there are four significant linkages between the underground economy and the official economy which are depicted in Figure (2.9):

- **Income and expenditure flow:** Firstly, the underground economy earns income from the goods market by selling capital and intermediate goods and services to the official economy.
- **Unofficial or informal wage flows from official sector:** The second flow, originating from the labour market, is related to wages earned by workers who are employed unofficially in the official sector.
- **Financial flows:** Underground firms and households are able to attract the capital from official financial market to cover the cost of their purchases from the official economy.

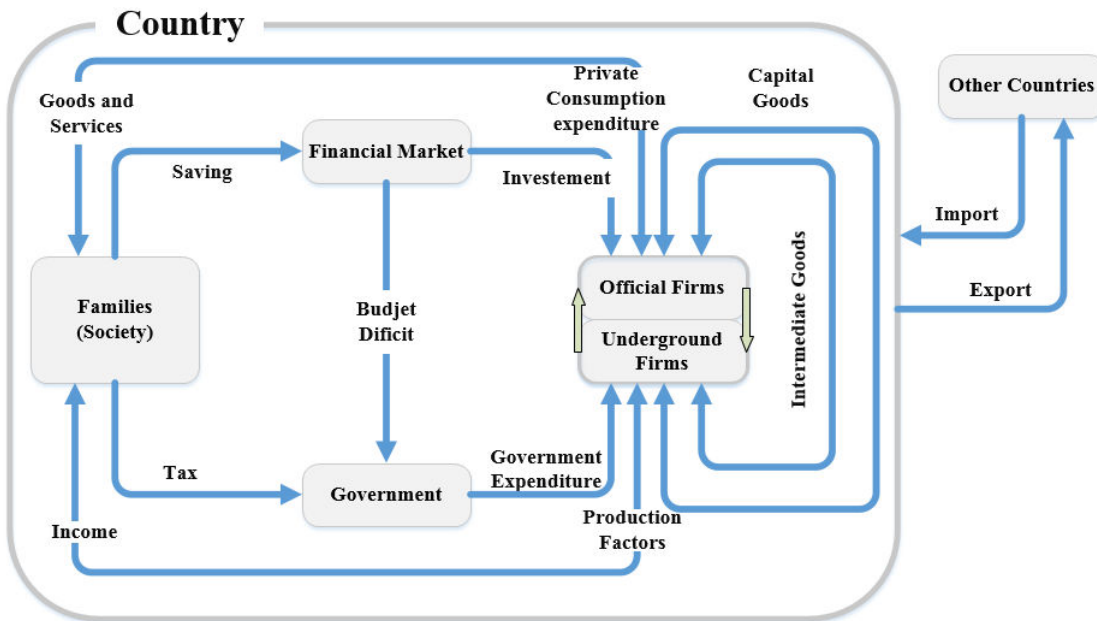


Figure 2.9: Final macroeconomic model of the underground economy (Developed by the author).

- Social transfers: The final flow refers to social transfers (old-age pensions, child support and disability grants) from government to households in the underground economy.

It can be stated that all products within the country can be exported to other countries, be invested in the firms or be consumed by families and government in addition to the goods and services which are imported from other countries. On the whole, domestic products are used for private consumption, investment, governmental consumption and net exports ¹⁹.

To summarize the models presented, the effects of the underground market are significant in three main macroeconomic areas: economic growth or GDP, unemployment and inflation. The economic growth is a long term issues while inflation and unemployment are short term issues ²⁰. Naturally, a favorable economic situation is the one with max rate of economic growth and minimum short term fluctuations. Thus, minimizing the inflation and unemployment and maximizing the economic growth are the objects of policy making in macroeconomic. It must be considered that the effective policies are the ones which are established by considering the underground economy specifications, size and trend. The three areas mentioned above, economic growth or GDP, unemployment and inflation, are discussed later as the causes and

¹⁹ The net exports is defined as the difference of import and export

²⁰ Short term indicates a period of time during which one of the production factors are unchanged at least.

indicators of the underground economy.

2.3 Consequences of the underground economy

As it was mentioned, there are several definitions for the underground economy and it is not simple to select a definition as the most complete and accepted. But no matter how it is defined, it is obvious that underground economy is an important concept as it is impossible to see the complete and big picture of economy without considering the underground economy and due to significant financial losses caused by tax evasion and tax avoidance.

However, it is unfair to say that the underground economy is completely destructive or in other words, it is not wise to consider the whole unofficial sector as a negative phenomenon from economic point of view. It was discussed that some of the underground activities have beneficial influence on social welfare.

The underground economy has also some positive effects on economy such as, adding competitive power to economy, creating jobs and providing economic activity, several economic effects including cost reduction, providing resource to the official economy, bringing dynamism to economy and being the assurance of socioeconomic system and social balancing [Dell'Anno, 2007; Dell'Anno et al., 2007; Frey and Schneider, 2000; Schneider and Enste, 1998; Tahmasebi and Rocca, 2015].

Nevertheless, we should not forget the negative effects of underground economy on economy like levying more tax from the very same elaborates. On the whole, it is clear that the governments prefer the individuals participate in the official activities rather than in the activities of this sector of economy. Since there are various types of the underground activities and it is hard to generalize the influence, but it is probable that only illegal activities have unfavorable impacts in total. Here, some of the positive and negative effects of the underground economy are demonstrated and discussed.

2.3.1 Negative effects and influences of the underground economy

It is clear that the underground economy has some negative impacts which can be discussed from different economic aspects. The first important issue is about inequity. More clearly, the fact that some individuals buy goods and services at prices affected by the tax imposed by government while others may buy similar but cheaper goods or services produced by the underground companies, makes the economic system less fair. It becomes more significant

when we know that all these individuals benefit from government services and those who operate in the official economy pay taxes while those who operate in the underground economy do not.

Eilat and Zinnes [2000] introduce several negative impacts for the existence of the underground economy which can be considered as macroeconomic, microeconomic or social problems.

One of the main macroeconomic consequences is the reduction in tax revenue. It is clear that growth of the underground economy means shrinkage of official economy. In a country, when a considerable share of the economy goes into the underground, it causes tax losses and the tax revenue will eventually start to fall.

From other side, the underground companies which do not meet their tax obligations also use government services. Thus, to provide these services, the government is forced to raise the tax rates (while, it also has the reduction in tax revenue) on those who pay the taxes and this action make it even more difficult for the companies to work in the official economy. From a pessimistic view, this "vicious spiral" which is made from increasing tax rate for providing more tax revenue in one hand and the tax evasion of companies due to the increase in tax rate on the other hand, can lead to a collapse of the whole system and government. On the contrary, it is also probable that government end up with less tax revenue needed to prevent macroeconomic difficulties.

A large underground economy may also discourage foreign direct investment into a country because foreign direct investment generally is often associated with better controlled and more taxed activities.

In this case, government can take two different approaches: One is going into debt and the other is coinage. After going into debt, government attracts efficient private sector investments to itself which would affect private sector investments negatively due to crowding out effect. Besides, this strategy will increase the bond prices and the interest rates. The inflation rates will also increase and country will face more problems. The effect of the second approach, coinage, is the same. Anyhow, the lack of revenue is a serious cause of problems in financing all government activities such as wages, public services and goods, pensions etc.

Another negative impact of the underground economy is distortion of official statistics (distortion of their current situation, size and distributions). In other words, the large size of the underground activities completely disrupt the economic indicators and cause inaccuracy in

the measurements of several important economic criteria including: national income statistics, growth rates, tax burden, unemployment rate, productivity, and so on.

The governments pursue the changes in some of these criteria to determine the policies, but, the inaccuracy in their size caused by the underground economy may spoil macro-policy decisions. As an example, the national income may be growing faster than the statistics indicate or the tax burden may be different from the official estimates. Consequently, the measured GDP per capita would not be any more a reliable measure for the standard of living. Moreover, estimates of growth rates are doubtful and the disappointing news of decline in productivity (i.e. in the GDP per worker ratio) may or may not correctly reflect the "true" state of the economy. It is probable that measured productivity has declined whilst the "true" not-fully measured productivity is either stable or increasing.

In another example, unemployment rate increase may be due to either a decrease in overall employment or the movement of human resources from official activities to underground activities. But, as the size of the underground economy is not properly captured by the national statistical office, the policies that the government establishes may be wrong.

If the macro-policies are inefficient, the information about the economic situation won't be reliable any more.

Also, it can be considered that the disintegration of social norms is a macro-social consequence of the underground economy. Disrespect of official institutions, norms and law supported by corruption can lead to a significant deficit of legitimacy of the current social order. However, it is questionable whether this "bad moral" is a consequence or cause of the underground economy.

The inefficiency in resource allocation can be a significant unfavorable microeconomic impact of the underground economy. As an example, the underground companies, as they do not pay taxes and do not obey official regulations, may offer higher wages which is an unfair advantage over the labour market. Similarly, their advantages in goods (or services) market are due to lower costs (no taxes for purchaser). On the other side, these companies would not be registered and so they would seem very unreliable. Hence, the investments would be most likely distracted from such firms. This means that the underground companies are either small firms (without the need of high investments) or large firms (which are registered, but misreport output).

It must be considered that the costs of running these companies decrease due to avoiding official regulations, taxes, licensing, etc., which is in fact another microeconomic problem related to the underground companies.

2.3.2 Positive effects and influences of the underground economy

The positive consequences and impacts of the underground economy are not discussed so completely and perfectly as the negative ones. As it was mentioned earlier, the underground activities may be either legal or illegal, but, it is obvious that only the legal activities of the underground economy should be considered when discussing about the positive effects of the underground economy.

Some economists and researchers believe that the underground activities respond to the demand of economic environment for small services and manufacturing. This hypothesis implies that the underground activities create an entrepreneurial atmosphere and more competition which lead to higher efficiency and lower prices in the official sector. From other side, the underground revenues provide financial resources and make it possible to extent the market and create new jobs [Asea, 1996]. There is also another idea that the possibility of choosing between the official and the underground economy can contribute to a higher potential for economic growth.

A considerable positive effect of the underground economy is related to the employment. The underground economy has a significant influence on employment, especially during recession or in the early stages of transformation (turning from a developing country to a developed one). It is believed that during this period, the majority of people working in the underground sector are the ones who could not get a job in the official economy [Eilat and Zinnes, 2000]. From other side, human resources can be trained and acquire primary skills necessary for their jobs in the underground sector.

Another positive impact is that the major part of the money earned from the underground activities is spent in the official sector. Thus, a significant part (approximately 66 percent according to Schneider [2002, 2007]; Schneider and Enste [2002]) of tax revenue loss will return to the official economy through consumption and etc. Moreover, the revenues of the underground market can improve the life standards in the low-income families which may

consequently decrease the corruption and the incentives to participate in the illegal activities of the underground economy. In this regard, Schneider [2007] explains that why a government may not be interested to limit the underground activities. He states that the income earned in the underground economy increases the standard of living of at least $\frac{1}{3}$ of the working population. Besides, the people working in the underground economy have less time for other things like going to demonstrations, etc. Moreover, between 40% and 50% of the underground activities have a complementary character, which means that additional value added is created, which consequently increases the official (overall) GDP.

From other side, most of the immigrants to the developed countries cannot easily find a job in the formal markets, but, the underground market can give them a chance to integrate with the society and consequently find a job in the formal market. These are a few reasons stating the positive consequences of the underground economy which encourage the governments to do not have serious struggle and debate with this market.

2.3.3 Effects on the official economy

The underground economy can have positive and negative impacts on the official or registered economy. Schneider and Enste [2000] believe that theoretical and empirical studies do not conclusively explain how an increase in the underground economy affects economic growth. In one hand, Adam and Ginsburgh (1985), Tedds (1998), Giles [2000], Tedds and Giles [2002], Chaudhuri and Schneider (2003), Alan and Gmez-Antonio (2003) estimate a positive relationship between official and unofficial economy; on the other hand, Frey and Weck-Hannemann (1984), Loayza [1996], Kaufmann and Kaliberda (1996), Eilat and Zinnes [2000], Dell'Anno [2003], Dell'Anno and Schneider (2004), Dell'Anno, Antonio and Pardoand (2007) and Ene and Stefanescu [2011] consider an inverse relationship between them.

The impacts of the underground economy become highly considerable in policy-making process. A high underground economy results in unreliable official macroeconomic criteria such as unemployment rate and income level. Economic policy decisions made based on these inexact official macroeconomics data are likely to be ineffective. On the contrary, it is probable that the underground economy can provide an entrepreneurial environment, establish new markets, increase efficiency and eventually, stimulate growth in official economy [Buehn

and Schneider, 2009; Ögünç and Yılmaz, 2000].

According to the **Neoclassical view** presented earlier, the underground sector provides a more dynamic economy with entrepreneurial spirit which increases the competition on the market, the efficiency and the financial resources. These, consequently, increase the economic growth of the official economy [Asea, 1996].

On the contrary, Loayza [1996] states that in economies where the statutory tax burden is higher than the optimal tax burden and where the enforcement of compliance is too weak, the increase in the relative size of the underground economy reduces the economic growth of the official sector. He also believes that increasing the underground activities might decrease tax revenue of government which may consequently diminish public infrastructure investments that are the basic elements of the economic growth. On the whole, there is no consensus on the relationship between the growth of the underground economy and the growth of the official economy Schneider [2007].

It is considerable that decreasing the tax rate or tax burden can not singly increase the tax revenues. Because there are more other factors (such as punishment, enforcement of legislation and etc.) which may increase or decrease the motivations to stay in the underground activities or not and consequently, the tax revenues will be decreased or increased.

2.4 Causes of the underground economy

It is not reasonable to attribute the accelerating growth of the underground economy in the recent past to a single factor while several factors, known as the causes, affect it. In different countries, a confluence of forces has been at work that would appear to be the main causes of this phenomenon [Kanniainen et al., 2004].

In general, any government intervention into market structure (taxes, fees, regulations etc.) can be considered as a primary cause of the second part of economy. These interventions affect the decision of certain group of people on how to work on the market (e.g., transforming a part of their taxable income into cash transactions).

Marinov [2008] confirms the effects of government intervention and explains that the socio-economic reasons for people to choose the "quit" option (moving to underground) is generally influenced by the government policy in terms of tax and regulation measures. There are several concrete factors that directly or indirectly contribute to increase in the size of the underground economy such as:

- the growth of self-employment;
- the loss of confidence in the government and its impact on tax morale;
- the communities generated in a country by immigration waves;
- the higher level of unemployment and the survival strategies it has triggered;
- the possibility of interests and dividends being paid beyond the borders;
- the failure of mega organizations in mobilizing the loyalty of their employees.

The burden of direct and indirect taxation and social security system, intensity of the regulation in the official sector, social welfare system, public sector services, forced reduction in weekly working hours; unemployment rate, earlier retirement, prohibitions and corruption are additional causes which are mostly used in empirical researchers [Jie et al., 2011; Schneider, 2002, 2005; Schneider and Buehn, 2012; Tanzi, 2002] and will be discussed next.

2.4.1 Direct and indirect taxation

The tax burden is the most influencing factor among others causes [Schneider, 2005; Startienė and Trimonis, 2010; Torgler and Schneider, 2009]. Tanzi [2012a,b] demonstrates that many years before, when there were no taxes, or there existed poll taxes or they were just related to "presumptive" and highly visible activities, no underground economic activities existed because of tax. However, as modern states developed, taxes became heavier and they were no longer "presumptive". Thus, taxpayers started to avoid them by doing a part of their activities "underground" or "in the shadow" [Andrei, 2011].

In almost all studies, direct and indirect taxation are among the most recommended causes and is known as the burden of direct and indirect taxation, the effective direct and indirect tax rate or the ratio of total tax revenue to GDP.

Higher tax rates cause the taxpayers become more inclined to evade taxation by switching to non-official and unreported transactions. The benefit of these attempts may go either to those providing goods and services (the sellers) or those buying them; but often both sides benefit. In this condition, penalties ²¹ may act as a deterrent for those who evade taxes; the higher the penalties imposed on tax evaders, the less tax evasions. However, it is important that an efficient justice system mete out these penalties, unless otherwise they would be useless. Penalties, however, play a lesser role in practice than what is assumed.

Another important element related to taxation is known as the cost of compliance. Compliance costs refers to the costs that taxpayers bear as a consequence of complying with all the obligations (additional to the payment of the tax). These costs include keeping accounts, reporting the information to tax authorities, keeping updated about the changes in tax laws, hiring tax advisers, obtaining tax forms and filling them out, standing in line to make tax payments and etc. High compliance costs are likely to contribute to the development of the underground economic activities because potential taxpayers who find these costs burdensome, try to avoid them.

Taxation also affects labor/leisure trade off and labor supply in the underground economy which is known as an untaxed market. Different amount of total revenue and cost of labor in the official economy and underground economy give the workers more incentives to participate

²¹ For a fundamental theoretical discussion on the role of penalties, see Allingham and Sandmo [1972] or Becker [1968].

in the underground activities. Since the mentioned differences depend broadly on the social security system and the overall tax burden, they play a key role in the existence and rise of the underground activities.

It is not easy to define a measure for taxation as it differs among countries. However, total direct tax burden (including social security contributions), total indirect tax burden and tax system complexity are all known as effective factors on the size of the underground economy²². In several empirical analysis, the estimated coefficients attribute the most significant influence to the burden of direct taxation, followed by the complexity of the tax system. The share of total tax revenues and the government expenditures (as percentage of GDP) can be used as main proxies for direct and indirect taxes to circumvent the complexity of tax system. There are several types of tax which operating in the underground activities makes it possible to avoid them, including [Tanzi, 2012a,b]:

1. income taxes;
2. value added taxes;
3. excises;
4. social security taxes;
5. foreign trade taxes; and,
6. taxes on capital transfers.

Tanzi [2012a,b] believes that both the tax rates and the capacity of tax administration to uncover underground activities affect the extent of the underground activities. It must be considered that well-motivated, well-organized and well-funded tax administration will be able to limit migration from the official economy to the underground economy.

There are some factors other than what was mentioned above which could play a role in encouraging tax-induced underground activities such as: corrupted tax inspectors and taxpayers' morale.

Obviously if tax inspectors are corrupt and can be bought at a price lower than the cost of

²² See Allingham and Sandmo (1972), Schneider (1986), Neck et al. (1989), Schneider and Neck (1992) and Loayza (1997).

tax, some taxpayers will choose the cheaper option, bribing the corrupted inspectors. This however requires the corrupt inspector to have a monopoly over the decision of whether the taxpayer can avoid paying taxes and it depends on the internal policies of the tax administration. Various controls on tax administrators and other administrative policies can resolve this problem, but, these controls are not always successful or may not exist in many countries.

Tax payer's morale refers to as the willingness of individual to pay the right tax at the right time [Maurin et al., 2006] has often been mentioned in the literature as a potential factor. According to Maurin et al. [2006], perceived fairness in taxation regulations, public's attitude toward the government, and the public religious and cultural characteristics determine the tax morality either positively or negatively. According to Torgler and Schneider [2009], governments that respect the preferences of the publics or democracy would receive more supports from its citizens in tax payment compared to a "bad" government. Some believe that because of different tax morality, the taxpayers of different countries have more or less propensity to avoid paying taxes.

Others dismiss this sociological explanation arguing that most taxpayers respond more or less in similar ways to similar sets of economic incentives. Given the incentives, most taxpayers would react in the same way, regardless of their nationalities. A declining tax morality tends to increase the size of the underground economy [Torgler and Schneider, 2009].

2.4.2 Intensity of Regulation

Regulations are important instruments of governmental policy. Economic regulations often substitute for taxing and public spending to achieve specific governmental roles [Tanzi, 2002]. They are applied to many areas connected with the economic activities of individuals and enterprises. The more the regulations against the underground economy or regulation in the official economy (the more the regulations related to underground economy and official economy), the less the individuals turn to it [Andrei, 2011; Fortin, 2002]. Entrepreneurs and large businesses may face outrageous demand from the government bureaucracy or suffer in a hostile environment [Friedman et al., 2000] ²³, consequently, they move to the underground

²³ Friedman et al. [2000] illustrate the logic behind the corruption view as: anybody from the State authorize (as an example, fire inspector, zoning committee member, mayor and etc.) can come and request extortionate

market. Moreover, increases in the pressure of regulation give a strong incentive to go underground market [Johnson et al., 1997; Pickhardt and Shinnick, 2008]. Besides, governments or official institutions cannot guarantee absolute fair regulations for business communities, so, people might search other options that include the practice of the underground economy [Risteski, 2009].

Intensity of the regulations or the index of the degree of regulations is measured by the number of laws and requirements such as licenses and rules related to the labor force. It includes supportive rules from labor forces in hiring and firing, labor restrictions for foreigners, minimum wage, minimum age, pension contributions, vacation time, trade barriers and some rules which refers to safety conditions related to fire, health or environmental hazards and etc. [Schneider et al., 2010]. These regulations often lead to an increase in labor costs in some activities of official economy (most of which must be paid by employees) or even make it impossible or difficult to operate legally in them. So, people are encouraged to work in the underground economy to escape these costs. Empirical evidences confirms this hypothesis; as an example, Johnson et al. [1997] explains that countries with more general regulation usually confront a bigger size of the underground economy (as a share of GDP).

Briefly, over regulation and labor costs in the official labor market are driving forces for the firms and individuals to participate in the underground activities and consequently, a higher share of the underground economy in total GDP will be resulted (increased).

However, measuring the intensity of government regulations (number of people working in the underground economy because of the restrictions affecting business and labor market) is a challenging task and few indexes such as Heritage Foundation's business or economic freedom and World Bank's regulatory quality index can be applied as the intensity of government regulations.

2.4.3 The social welfare system

The social welfare system may act as a negative incentive for those who work in the official economy. Rowley and Schneider [2008] note that the welfare beneficiaries have high disin-

and arbitrary demands one by one.

centives to work in the official economy. To clarify the effects of social welfare system, it is enough to consider a system which provides welfare for individuals but demotivates them from even searching for work in the official economy because the overall income will be higher if they receive these welfare payments and work in the underground economy at the same time [Finlayson and Peacock, 2002; Lippert and Walker, 1997].

From other side, the social welfare programs which are provided from the tax revenues and other revenues of the government can affect the incentives of the immigrants to work in the underground economy. That is, some immigrants pay a high tax to receive these services in return, but, they never benefit from them as they work in the society in a short period. So, they may prefer to work in the underground market which is an untaxed market. It must be considered that this case happens more in developed countries.

The social welfare system, as one of the causes of the underground economy, can be measured by a few variables such as the total expenditure for the social welfare (as a share of GDP) and the size of marginal tax rates.

2.4.4 The Public sector services

The quality of public services is another important factor. Taxes are costs paid by citizens in return for the public services they receive from the government. Therefore, the better the services are, the greater will be the willingness of the taxpayers to comply with the tax obligations. The decrease in the State revenues and consequently, the reduction in the quality of public goods and services can be a consequence of the bigger share of the underground economy on the overall economy. Also, an increase in the tax burden or combination of them (i.e., high tax rates and low quality or insufficient quantity of public goods and services) can be alternatively another possible reason. And again and again, it leads to stronger incentives to participate in the underground economy in this cycle.

According to Ott [2002], government's over spending or spending on unnecessary expenditure also result in the increase of the underground economy. In a country where public spending is inefficient, public officials are believed to be corrupt, or the spending is not equitably distributed, it is hard to encourage taxpayers to stay in the official economy and fully satisfy their tax obligations. Irrational government budget can add financial burden to tax payers thus

directly contributes to the growth of the underground economy [Ott, 2002]. In such a country, the underground economy is likely to become larger. It is difficult to determine how much of a role this factor plays in reality, but, it is believed to play some roles.

It is a hard work to capture the quality of public sector services, but the World Bank's Government effectiveness criterion, which measures the quality of public and civil services and the degree of the independence from politics, can be used for this purpose. It also captures the quality of policy formulation and reliability of government in this matter.

2.4.5 Inflation and CPI rate

Inflation shows the rate of the change of prices in the economy as a whole [Schneider et al., 2010] and is measured by the annual growth rate of the GDP implicit deflator.

The consumer price index measures the general price level of goods and services in the official market, but it does not include the price level in the underground economy which are lower due to the lower cost and the tax evasion in the underground economy. Therefore, official inflation rates overestimate the price level in the economy which causes a distortion in certain economic indicators such as real economic growth, real interest rates, and real effective exchange rate. On the whole, the inflation rate can be regarded as a cause which motivates people to buy goods and services of the underground firms as they are cheaper, and consequently, the increase in the demands of underground outputs make incentives to expand the underground firms.

2.4.6 Official economy

The condition of the official economy can affect the choice of individuals for working in the underground sector or not. In a healthy thriving economy, where people have the same and more opportunities to get a high quality work with sufficient salary, the motivation to participate in the underground economy is quite low. In contrast, the people are motivated more to participate in the underground activities in an economy with recession, high rate of unemployment, high inflation rate and etc. The unemployment rate, inflation rate (GDP deflator), openness (international trade as percentage of GDP) and some other parameters can be considered as the criteria which describe the situation of the official economy.

2.4.7 Unemployment rate and official working hours

Two other factors are also broadly discussed in literature as the causes of the underground economy: the forced reduction of official working hours and the unemployment rate. In most OECD countries, unemployment rate was to a large extent caused by economic crisis in 2008 and high labor costs in hiring and firing. The reduction in working hours in the official economy was consequently introduced by governments (e.g., France) and/or labor unions (e.g., Germany) in order to reduce the unemployment.

The main idea behind this policy is the fact that there is a limited quantity of work. But, this policy neglects that a forced reduction of working hours contrary to employee preferences increases the potential hours they can work in the underground economy [Schneider et al., 2010]. Other factors such as early retirement and part time work also provide opportunities for the labor force to get involved in the untaxed unregulated activities.

2.4.8 Prohibitions

Prohibitions can be considered as special forms of regulations so they could also be classified together with the category of regulations. Prohibitions are related to the destination countries; some activities are prohibited in certain countries while they may be allowed in others. For instance, Germany allows prostitution or Netherlands allows the sales and consumption of some drugs that are illegal elsewhere. Thus, the prohibitions can play a major role in pushing the labor force to participate in the underground activities.

Some banned activities are highly profitable as there is a great demand for them. So, those who are engaged in them are in fact involved in illegal or even criminal activities. In a private cost benefit evaluation, the tendency of people for participating in these activities demonstrates that the high potential profits of these works exceed the charges of being caught or punished. Major forms of these activities are production and distribution of illegal drugs, illegal gambling, lending money at usury rates, production and sale of weapons, prohibited biological substances, various services including prostitution, and etc. Still, we can add more explicit criminal activities such as robberies, murders for hire, kidnapping, extortions, etc. to these activities. Moreover, the reported estimates of the earnings from the mentioned activities are enormous. Therefore, large resources, in terms of real and financial inputs, go into these

activities.

There are some controversies among statisticians and other experts on whether the proceeds of these activities, or at least of some of them, ought to be included in the estimates of national incomes. Some argue that as long as people demand these "services" and are willing to pay for them, they ought to be included. One difficulty is that some countries allow at least some of these activities while others forbid them. As it was mentioned, Germany allows prostitution, thus, the German tax authorities expect prostitutes to report fully their "incomes" and pay taxes on them. Important questions are therefore raised by prohibited and criminal activities for definition and measurement of the underground activities. Because of the large estimates of these activities, their inclusion or exclusion can make a significant difference to the measurements.

2.4.9 Corruption

Corruption, either political or bureaucratic, is one of the possible causes of the underground economy and can generate high incomes to those engaged in corrupt activities. Many researchers have associated bribery or corruption with underground economy especially in the government sector [Çiçek and Elgin, 2011; Dell'Anno and Schneider, 2009; Friedman et al., 2000; Lafay, 1990; Ott, 2002]. The question is whether the proceeds from corruption should be taken into consideration in the estimates of the underground economy. In some countries, these proceeds can be very large and corruption can change the official prices of some goods sold in the market.

There is either a substitution or complementary relationship between the underground economy and corruption. The substitute model explains that corruption and underground economy are substitutes since the existence of the underground economy reduces the propensity of officials to demand bribes [Choi and Thum, 2004, 2005; Dreher et al., 2005, 2007, 2009]. We know that the labor force can be either employed in the official sector or in the underground economy.

The complement models [Hendricks, 2002; Johnson et al., 1998] explain that corruption can be regarded as one particular form of taxation and regulation. Thus, the increase in corruption drives the entrepreneurs underground and increases the underground economy.

2.4.10 Other causes and some remarks

There are some other causes besides the ones mentioned above. Torgler and Schneider [2009] propounded another factor: the local autonomy and explain that the tendency to go underground can be counteracted by high level of autonomy that allows expression of public preferences and enhancement of identification with the governments . They also believe that high urbanization increases the underground transactions because urbanization would maintain the anonymity and reduces the loyalty to government. Urbanization characterizes majority of people and business premise clusters in a highly populated area. The possibility and potential of having the underground economy in urbanization is high, especially when the government cannot fully satisfy public expectations and preferences [Torgler and Schneider, 2009].

Unequal income and poverty also increases the underground activities in the society. But, there are arguments that if it is true for developing or transition economy or not. Some believe that the underground economy increases the standard of living in the society, but it also leads to income inequality. Moreover, quality of public goods and services are affected by the size of the underground economy; increased underground economy will reduce the quality and quantity of publicly provided goods and services. In countries where underground activities are relatively high, revenue generation will be low and the quality and quantity of publicly provided goods and services will be lower than average.

The discussions presented about the causes of the underground economy can be summarized in four items:

- Government is one of the predominant factors that contribute to the underground economy, particularly great attention must be paid to the efficiency of the bureaucracy, trust crisis between people and government, government transparency, quality of public sector service, education expenditure, social transfer, government budget, and bribery of government agents.
- Regulation is another factor that affects the size of the underground economy, specifically, the intensity or corrupted regulation, slow and closed legislative system, equality under the law and labour market regulation.
- Economic conditions such as economic recession, unemployment, high public debt and

inadequate and slow economic growth also play important roles in the underground economy.

- Social mentality including the break in market traditional, cultures characteristics, public religious, survival strategies, public attitude, bribery, urbanization and growth in self-employment are another factors that one should take into account when examining the underground economy.

2.4.11 Selection of the Causes

The best way to comprehend the causes of the underground economy is to analyze the motivation of the firms or individuals for moving from the official market to the underground. As it was mentioned earlier, the most popular reason for turning to the underground activities is the tax rate (both marginal and total). In other words, the most important determinant of the increase of the underground economy is the rise in tax and social security burdens ²⁴.

Schneider and Enste [2000] state that the bigger the difference between the total cost of labor in the official economy and after tax earnings (from work), the greater is the incentive to avoid this difference and work in the underground economy.

From other side, Johnson et al. [1997] and Friedman et al. [2000] believe that the institutional aspects are of greater importance in increasing the underground activities compared to tax. In cross country studies that included less-developed, transition and OECD countries, Johnson et al. [1998] and Friedman et al. [2000] recognized the extent of regulatory discretion as the key determinant of the underground activities. When regulations are lax and rule of law is weak, bureaucrats decide freely about individual cases without any supervision. And this leads to corruption, which causes the firms to go underground.

Aigner et al. [1986] believe that a rise in the size of the public sector and/or the degree of regulations is a motivation to enter the underground economy. Johnson et al. [1997] also believes that the government regulation is a key factor in measuring the unofficial economy. Giles and Tedds [2002] empirically confirm that the government regulation is positively correlated with the underground economy. In another study, Prokhorov [2001] demonstrated a significant and positive relationship between the proxy of regulation burden and the underground economy.

²⁴ see Tanzi (1982, 1999); Frey and Pommerehne (1984); Feige (1989); Pozo (1996) and Johnson, Kaufmann, and Zoido-Lobato (1998)

Fugazza and Jacques [2004] used a continuous time series model and showed that the magnitude of the underground economy expands with the increase of tax rate and degree of government regulations. Ihrig and Moe [2004] found that the tax rate and the government's regulatory policies could both affect the size of the underground economy. Furthermore, they found that the tax rate effects are stronger than the government regulation and policies. Therefore, they suggested that the decrease in the tax rate along with the enhancement of government regulation and punishments in the form of fines, would reduce the magnitude of the underground economy.

Table 2.5: Researches and different causes

	Effective tax rate or Tax Burden	Share of direct taxation in GDP	Share of indirect taxation in GDP	Unemployment rate	Public employment	Self-employment	Index or intensity of regulation	Corruption	Efficacy of Judicial System	Small companies	Working Hours	Household net saving rate	Index of Illegality	Inflation Rate	Social security contributions
Ene and Stefanescu [2011]	*	*	*	*				*							
Klarić [2011]		*	*	*											*
ENE [2010]		*	*	*				*				*			
Buehn and Schneider [2008]	*			*			*				*				
Dell'Anno et al. [2007]		*	*	*	*	*									*
Dell'Anno [2007]	*			*	*	*	*								*
Yu et al. [2006]	*						*								
Fugazza and Jacques [2004]	*			*			*								
Ihrig and Moe [2004]	*						*								
Dell'Anno [2003]	*			*		*	*		*				*		
Draeseke and Giles [2002]	*						*								
Prokhorov [2001]	*			*	*		*	*						*	
Bordignon and Zanardi [1997]				*		*				*					
Johnson et al. [1997]							*								

About unemployment, Giles and Tedds [2002] explain that two opposite forces determine the relationship between the unemployment rate and the underground economy which means that this relationship is ambiguous. From one hand, an increase in unemployment implies a

decrease in the official economy and due to the positive relationship between the underground economy and the growth rate of GDP, the unemployment is negatively correlated to the underground economy. On the other hand, a share of official unemployed spends a part of their time working in the underground economy, thus, there is a positive correlation. In another research, Tanzi [1999] confirmed the ambiguous relation of unemployment and the underground economy explaining that *"...for OECD countries there seems to be a broad relation between the panel data of the size of the underground economy and the official unemployment rates"*.

From another point of view, the rate of self-employment as a percentage of the labor force is considered as a determinant of the underground economy. Bordinon and Zanardi [1997] believe that the development of small firms as well as the large proportion of professionals and self-employed with respect to the total workforce, has main effects on Italian productive system. They explain that *"...a large proportion of professionals and self-employed implies greater possibilities for transferring expenses from consumption to production (to be deducted from taxes), simplified accounting and easier path collusion with customers"*.

Some of the causes frequently used in different studies or their effects on the underground economies are more analyzed are listed in Table ²⁵ (2.5). According to Table (2.5), the tax system (consists of direct and indirect tax rate and tax burden), unemployment rate and intensity of the regulations are the most widely used causes in the estimation of the underground economy. It means that most of the researchers believe that these factors affects significantly the extension of the underground economy and the decision of people to work in the underground economy.

For the proposed methods in this work, seven causes ²⁶ are used as input variables. It is notable that the variables must be selected in a way to cover the effects of other ignored causes. According to the descriptions above, the selected causes are:

1. The Taxes on incomes and Profits as The Effective Direct Tax Rate or the share of direct taxation in GDP (DTR);
2. The Taxes on goods and services as The Effective Indirect Tax Rate or the share of indirect taxation in GDP (ITR);

²⁵ This Table is developed by the author

²⁶ The six causes are only used in the proposed models based on availability of the data for all countries.

3. Proxy of Government Regulation (GR), defined as the ratio of Government consumption and expenditure to GDP [Yu et al., 2006];
4. Unemployment Rate (UR);
5. Inflation Rate (IR);
6. And, Self-employment/Labor Force(SER).

2.5 Indicators of the underground economy

Indicators are dependent variables which capture the changes in the size of the underground economy and indicate the situation of the underground economy. Schneider and Enste [2000] believe that the changes in the size of the underground economy may be reflected as a change in some official economy criteria such as:

- Monetary indicators: the underground transactions are mostly in cash. Thus, an increasing demand for currency may indicate the increase of the underground activities [Laflèche, 1994].
- Labor market participation rates and working hours: the participation rates in the official economy may fall as the number of people who work in hidden sector expands. However, it must be considered that this participation can be as a second job besides the primary job. Thus, the participation rate may fall or not in this case.
- Output statistics: as the underground economy develops, the production inputs, especially labor forces, move at least partly out of the official economy. This displacement may decrease the official growth rate of economy.

Also, they suggest that all indicators can be classified in these three groups which are discussed next.

2.5.1 Monetary indicators

It was mentioned that People trade underground goods and services in cash to prevent transactions being traced, while the effects of cash transactions are reflected in overuse of cash or currency. The M_0 over M_1 (M_0/M_1) are used as the indicator to consider these effects of the underground economy . Also, the M_0 over M_2 (M_0/M_2), where M_2 corresponds to M_1 plus term and other deposits is also applied as a monetary indicator ²⁷.

²⁷ M_0 corresponds to the currency outside the banks and the usual definition for M_1 is M_0 plus deposits (Schneider, et al., 2010).

2.5.2 Labour market

The labour force is a market which can be strongly influenced by changes in the underground economy. The people who are heavily involved in the underground activities and do not work in the official sector are excluded from official labor force statistics. The indicators of the labor market can therefore reflect the situation of the underground activities and can be used as the main indicators in the estimation of the underground economy.

The labor market indicators are:

1. labour force participation rate, is the ratio of the population that is economically active to population in the working age (usually 15-64 years). The economically active population includes all people who supply labor for the production of goods and services during a specified period; and,
2. growth rate of total labour force, which is the growth of the economically active population supplying labour force [Schneider et al., 2010].

2.5.3 State of the official economy

As it was explained before, the underground activities strongly affect the official economic criteria. As an example, the increase in the size of the underground economy can reduce the tax revenues and GDP, and consequently reduces the government expenditures. The Real GDP per capita and its growth are considered as the indicators which reflect the state of the official economy.

2.5.4 Selection of the Indicators

Dell'Anno et al ²⁸ used real GDP, participation ratio of the labor force and currency in circulation outside of banks as the indicators to estimate the underground economy in three Mediterranean countries using MIMIC approach.

In another study, Real GDP index, Population Activity Rate and M_1 share in M_2 were chosen as the indicators for estimating the underground economy of Romania [Ene and Stefanescu,

²⁸ [Dell'Anno, 2003, 2007; Dell'Anno et al., 2007; Dell'Anno and Schneider, 2009; Schneider and Dell'Anno, 2003]

2011]. Buehn and Schneider selected the GDP volume index and monetary aggregate M_1 as indicators for estimating the underground economy in France [Buehn and Schneider, 2008]. Gerxhani [2004] explains that there are great differences between the significance of underground activities in developed and developing countries. In developing countries these activities are identified with low income, lacking any capacity for accumulation, whereas in developed countries the possibilities for accumulation and incomes are often comparable to formal activities. She further explains that the reasons for participating in the underground economy differ between these countries; survival demands and savings are the main reasons for participating in the underground economy in developing and developed countries, respectively. The amount of household saving can consequently also be considered as an indicator of participation in the underground economy of developed countries.

According to what was discussed above about the most predominant indicators, the following indicators were selected as the best represents of the amount of the underground economy in this study:

1. Real GDP per Capita (GDPC) which is the ration between the real GDP and the population in working age (15-64 years old):

$$GDPC = \frac{\text{GDP market price value/Deflator of GDP}}{\text{Population in working age}}$$

2. Participation Ratio of the Labor force (PRL) which is the ratio between the total labor force and population in working age ,
3. and the Household Net Saving Rate (HSR)

It is notable that the data for all these causes and indicators are obtained from OECD statistical compendium database [OECD, 2014].

2.6 Sectors of the underground economy

It was mentioned that there are several names for the underground economy such as irregular economy, subterranean economy, underground economy, black economy and etc. The most frequently used name is the underground economy. The significance and characteristics of the underground economy differs from one country to another, but, there are also some similarities. For example, among OECD members "black practices" and their motivations (avoidance of direct and indirect taxes and social security) are common concerns.

In eastern countries before the reforms towards a market economy, they used to have their own private sector which was the same as the underground economy. Until the fall of communist regime and planned economy, these countries witnessed a parallel economy, totally illegal called "secondary economy". Secondary economy is the transfer of productions from planned sector to unofficial sector. The underground economy is thus developed by means of turning into a parasite of the official economy. This type of economy covers the deficiencies and the rigidity of planned economy and answers particularly to poverty, standardization and poor quality of products. If secondary economy is meant for the production of goods that will be consumed or exchanged on the parallel markets, the underground activities in planned economies have the same varied forms as the ones in the industrialized countries [Kureshi et al., 2009].

The general economic condition is heterogeneous among developing countries. In these countries, the unofficial economy is the dominant way of production and determines the level of development. It is often said that under-developed economies are thought of as being "dual". In these countries, one may distinguish between two fundamental forms of the underground economy:

1. A primitive form that includes activities that come directly after domestic activities like agricultural self-production and everything it involves.
2. The other form includes handicraft, trade or small scale activities. Their multiplication rate is related to rural exodus and demographic boom which further leads to urban increase.

Proliferation of these activities is carried out within the framework of the law and escape state control. Local public authorities tolerate these activities because they absorb the labor force

and reduce poverty [Radulescu and Stimmelmayer, 2010].

From what has been discussed above, it is clear that giving a precise definition for the underground economy is difficult. This is mainly due to the complexity of this phenomenon and to the fact that the underground economy is like "running water": it easily adapts to the changes within an economic, fiscal, judicial or moral system [Mogensen et al., 1995]. However, it is essential to identify the complete definitions of the underground economy in order to determine the causes for its growth and to analyze its implications for a broad range of social and political issues. Major efforts have been undertaken to differentiate between various types of the underground economy to identify them and create operational methods for estimating their size, characteristics, composition and growth.

Different authors addressing different questions have employed a variety of alternative definitions of the underground economy and this has led to the criticism that it is "an exceedingly fuzzy concept" which "obscures analysis of central issues and is counterproductive" [Peattie, 1987]. As a classification, we can distinguish four distinct sectors in the underground economy which are introduced next.

2.6.1 Illegal economy

The illegal economy includes the income produced by those economic activities pursued in violation of legal statutes that define the scope of legitimate forms of commerce. Illegal economy participants are engaged in production and distribution of inhibited goods and services. From economic development perspective, the most significant illegal activities are illegal production of forbidden substances such as drugs and illegal services such as black market currency exchange or black market medicine. The illegal production, provides a profitable source of income that competes directly with the production of other cash crops. On the contrary, these illegal productions lack the stability and responsibility of political, legal and economic institutions that might otherwise serve to facilitate the development process. From other hand, the existence of illegal black market currency transactions affect the development process by reducing the transaction costs of obtaining foreign currencies and providing a hedge against the fluctuations of domestic currency value.

2.6.2 Unreported economy

The unreported economy consists of those economic activities that circumvent or evade the institutionally established fiscal rules as codified in the tax code. A summary measure of the unreported economy is the amount of income that should be reported to the tax authority but is not reported [?]. These unreported activities cause the "Tax Gap" which is the difference between the amount of tax revenues the fiscal authorities expect and amount of tax revenue actually collected.

The size and growth of unreported income and tax gap affect the size of budget deficits, the government debt and tax reform policies as a consequence of which, the government may change its policies and regulations. This may be a reason that the government increases the tax rate or punishments of tax evasion. Without empirical knowledge of the size and growth of unreported income, it is difficult to ascertain who are the winners and losers from policy changes. This section of underground economy is a real problem in developing countries whose already weakened fiscal authority must now shift a greater burden of revenue collection to monetary policy.

The consequences of unreported economy are clear at microeconomic level. ? believes that the participants in the unreported economy face a different configuration of relative prices for both inputs and outputs, hence, they face different transformation and transaction costs than participants in the reported economy. Consequently, allocation and distribution will be distinct as well. He mentions that in developed nations, research on unreported income has been a major stimulus to tax reform and has stimulated the simplification of tax laws, lowering of tax rates and broadening of tax base to reduce rates of non compliance and lower the efficiency and administrative costs of tax system. In summary, the unreported economy affects the regulations and tax rate which should be considered together in every policy making process by governments and related associations [?].

2.6.3 Unrecorded economy

The unrecorded economy is the amount of unrecorded income that should be recorded in national accounting systems under existing rules and conventions but is not recorded. Un-

recorded income represents a discrepancy between total income/output and the actual amount of income/output captured or recorded by the statistical accounting system designed to measure economic activity and economic growth.

As mentioned earlier, the unrecorded economy involves activities and services which should be recorded but are not so. For unrecorded activities, one particularly important component in developing nations is the production which takes place within household for their consumption. Also, the unrecorded services take place frequently in developing countries when the service providers do not want to record them.

A large and rapidly changing unrecorded economy can systematically bias estimates of key economic indicators such as unemployment rates, savings rates, productivity and price levels [McGee and Feige, 1989]. Since the macroeconomic policy is based on systematically biased information, well intentioned policies are likely to produce unexpected and counterproductive outcomes. Also, empirical research in economy relies almost exclusively on macroeconomic indicators that are derived from official national accounting or survey information systems. Thus, any systematic discrepancies in underlying data bases arising from the unrecorded activities will distort the research results.

It is noticeable that the activities and services which are produced and provided in the unrecorded economy are neither illegal nor informal.

2.6.4 Informal economy

There are two opinions about the first use of the term "informal sector"; one, in early 1970s, suggested by Hart in a study about Ghana [Hart, 2008], the other one, in 1972, by the International Labor Organization in a widely read study on Kenya. However, the common thread is that these activities which may be either legal or illegal are not recorded or imperfectly reflected in official national accounting systems [Adair, 1985; Losby et al., 2002].

In the dual world of economic activities in developing countries, the formal sector is the result of the government encouragement of entrepreneurs to apply new methods and technologies of production for mass production and gaining more benefits in big markets. On the other hand, the informal sector represents the continuation of traditional production methods which

develops due to self-employment to sustain poor population of developing countries and to create job for them.

About informal sector in many developing countries, it must be considered that business activities and particularly the non-productive ones are prominent activities within employment structure. Multiple price system, which is established to support low-income groups in society, plays also an important role in development of the informal sector. It is obvious that many governments have approved the informal sector as an inevitable reality to decrease public pressure on them. Furthermore, some of them have included the provision of financial and non-financial supports for the informal sector in their short-term policies to reduce the unemployment rate in both developing and some developed countries.

In the foregoing definition, Amin [1987] explains that: "the informal sector may be defined as including enterprises that have one major and dominant attribute in common: the absence of official status. In other words, enterprises and individuals in the informal sector operate outside the incentive or social security system offered by the State and its institutions."²⁹

In another definition, the informal economy comprises those economic activities that circumvent the costs and are excluded from the benefits and rights incorporated in the laws and administrative rules covering property relationships, commercial licensing, labor contracts, torts, financial credit and social security systems. It can be also said that informal economy is the income generated by economic agents that operate informally [?].

Ferry and Gray [1987] believe that because the informal sector is defined in terms of income earning activities which escape official regulation and the constraints of labor legislation, an analysis of the role of the State is crucial to any understanding of its development.

The distinction between a formal and informal sector is based on the organizational characteristics of exchange relationships and the position of economic activity in relation to the State. Basically, the formal sector includes government activity itself and those enterprises in the private sector which are officially recognized, fostered, nurtured and regulated by the State, whilst the operations in the informal sector are characterized by the absence of such benefits [Weeks, 1975].

The situation of many countries may become clearer beneath institutional governance and

²⁹ Also, refer to: [Portes and Sassen-Koob, 1987], ([Portes; Blitzer and Curtis, 1986], [Feige, 1989,1990] and [Adair, 2004, 2009, 2012].

political situations. In developed and developing countries, the applicable Institutional governance may increase the formal GDP while at the same time reducing informal GDP. However, the informal sector plays an important role in transition countries. Knowing the shadow economy causes is a tendency to control illegal activities through measures such as punishment, prosecution, economic growth or education [Schneider and Enste, 2002].

Different factors cause the extension of informal economy such as: population growth, urbanization growth, immigration, economic and social laws and regulations, supervision and management of productive sections, efficiency and its importance, salary and working hours. The mentioned four sectors of the underground economy presented above are summarized in Table 2.6. Some of the causes frequently used in different studies or their effects on the underground economies are more analyzed are listed in Table ³⁰ (2.5). According to Table (2.5), the tax system (consists of direct and indirect tax rate and tax burden), unemployment rate and intensity of the regulations are the most widely used causes in the estimation of the underground economy. It means that most of the researchers believe that these factors affects significantly the extension of the underground economy and the decision of people to work in the underground economy.

³⁰ This Table is developed by the author

Table 2.6: The four sections of the underground economy (Feige, 1997; Rdulescu, et al., 2010)

		Define	Broken institutional rules	Activities	Examples and consequences
Underground Economy	Illegal Economy	Totality of the revenues that are generated by those economic activities that violate the legal status of legitimate forms of trade	Production and distribution of illegal goods and services	Illegal activities such as illegal drugs dealing, black market of currency exchange, money laundering, unlicensed money lending, illegal gambling prostitution and pornography.	Drug-related criminal activities submination of the stability institutions
	Unreported Economy	Totality of economic activities that escape or avoid fiscal rules as they are defined in fiscal codes	Fiscal status	Unreported revenue either from legal sources or illegal sources.	Tax evasion
	Unrecorded Economy	Activities that avoid institutional conventions that define the necessary requirements for the report to governmental agencies for statistics	Accounting convention	-	Distorted information Wrong macroeconomics policies
	Informal Economy	Economic activities that avoid costs and excluded from the rights and benefits that come along with leasing, work contracts, loan and social security	Legal and administrative norms	Revenue that generated by economic agents that operate informally.	Lack of rights and benefits

2.7 Summary

In this chapter,;

1. Various definitions proposed for the underground economy in the literature were presented. As it was stated, there is not a single definition for this sector of economy [Öğünç and Yılmaz, 2000], however, considering the variety of definitions focusing on separate aspects and activities of the underground economy can provide a comprehensive picture of this ambiguous economic phenomenon as a whole.

Different actors of the underground economy, their motivations to participate in this sector and different activities of the underground economy were also completely discussed. It was explained that three main concepts are common in different definitions presented for the underground economy. The underground economy not only consists of activities, but also includes the transactions and services. Different components of the underground economy can be either legal or illegal. Also, the underground activities are not reported or recorded, in spite of the government regulations, in order to avoid or evade the regulations or due to the shortcomings of the government agents [Tahmasebi and Rocca, 2015].

Categorizing the underground activities provides a global vision to this economic phenomenon, thus, Table (2.1) that serves as a complement of the definitions was reproduced from Schneider and Enste [2000]. This Table will be used in this research.

2. The macro and micro economic views to the underground economy were discussed. A macroeconomic model of the underground economy that illustrated the roles of the underground firms was proposed firstly and was completed further by adding different actors of the underground economy. A more complete model consisting of the society, the government, the financial market, the official and the underground firms, their interactions and the role of other countries in an economy was finally developed. This model (Figure (2.9)) helps to better understand the causes and the indicators of the underground economy
3. The positive and negative impacts of the underground economy were explained. From one side, the underground activities have beneficial influences on the social welfare, the

economic growth, the employment rate and etc. From other side, they may bring about financial losses and affect the social services and the government programs because of the tax evasion, tax avoidance and etc. Also, the underground economy might render the official statistics unreliable [Jie et al., 2011; Tahmasebi and Rocca, 2015].

4. Different causes of the underground economy such as taxation, regulation, inflation, etc. and the indicators of the underground economy such as monetary, labour market and state of the official economy which are mostly considered and used in various estimation methods were introduced. It was mentioned that the direct and indirect tax rate, unemployment rate and intensity of regulations are the most frequently used causes in the analysis and estimation of the underground economy. Also, as it was discussed, the taxes on incomes and profits known as the Effective Direct Tax Rate or the share of direct taxation in GDP (DTR); the taxes on goods and services known as the Effective Indirect Tax Rate or the share of indirect taxation in GDP (ITR); Long Term Unemployment/Labor Force(LTUR); Proxy of Government Regulation (GR) defined as the ratio of Government consumption and expenditure to GDP [Yu et al., 2006]; Unemployment Rate (UR); Inflation Rate (IR); and Self-employment/Labor Force(SER) are the causes that will be used in our proposed models. Real GDP per Capita (GDPC) which is the ratio between the real GDP and the population in working age (15-64 years old); Participation Ratio of the Labor force (PRL) which is the ratio between the total labor force and population in working age; and, the Household Net Saving Rate (HSR) will be used as the indicators.

Chapter 3

Estimation Methods of the Underground Economy

Introduction

In previous chapter, an overall introduction to the underground economy including the definitions, causes and indicators of the underground economy, its consequences and actors was provided. A macroeconomic model was proposed based on these facts in order to illustrate more this economic phenomenon.

The two main consequences of the underground economy more frequently referenced and discussed are [Jie et al., 2011; Schneider and Buehn, 2012; Tahmasebi and Rocca, 2015]:

- Tax evasion and avoidance in the underground economy which cause budget deficit; and,
- The effect of the underground economy on the official statistics and economic criteria, such as economic growth, unemployment and GDP, and making them unreliable.

These two main effects of the underground economy and other effects reflect the necessity of an estimation method for the size of the underground economy in order to determine the tax gap and obtain reliable criteria [Pommerehne and Frey, 1984]. Economists and statisticians have therefore sought for some proper methods with the aim of measuring the size of the

underground economy in various countries.

This chapter introduces various methods used to determine the size and growth of the underground economy. The aim is to illustrate the nature and the extent of the problems that official institutions and international researchers face when dealing with such a variety of methods. The methods and approaches proposed for the estimation of the underground economy can be classified into three main categories: direct, indirect and model approaches with some subcategories under each [Fortin, 2002]. Direct approach includes two methods: survey and tax audit. There are numerous indirect methods to determine the size and trend of the underground economy, but, we focus more on five indirect methods including: the discrepancy between national expenditure and income approach, the discrepancy between actual and observed labor, the transaction approach, the currency demand approach and the physical input method [Barthélemy, 1997; Schneider et al., 2010].

The most famous method applied more to estimate the size of the underground economy is MIMIC (Multiple Indicators Multiple Causes). This method, known as a Model approach, will be completely discussed in this chapter [Schneider et al., 2010].

3.1 Direct approach

Direct approaches, also known as micro approaches, apply a survey- based technique to quantify the underground economy. Two different direct approaches called the survey and tax auditing methods are introduced next.

3.1.1 Survey method

Survey method exploits a well-designed survey and sample base to measure the size of the underground employment [Schneider, 2003]. To get the essential and accurate information from the respondents of the survey, it is important to structure the survey carefully. In other words, the survey should be expressed in a way that the information obtained fulfills what the researcher needs to prove his/her argument. *"It includes prompts describing different types of work and payment and allows a relatively precise identification of work that is in legal terms taxable"* [OECD, 2004].

The analyzer cannot interview all citizens of a country as its costly and time consuming. Thus, only a sample that represents the country as a whole will be analyzed. This sample includes the individuals who participate in the underground sector, tax audits or etc. On the whole, three phases can be noticed in a survey approach, known as the 1-2-3 survey [Saunders, 2008]:

Phase 1 : Collecting the information pertaining to the labor force;

Phase 2 : Identifying underground and official workers; and,

Phase 3 : Collecting the information about income and expenditure.

This approach provides valuable information about the nature of the underground activities, also about the structure of illegal employment in form of either self-employed or employees in the underground sector. However, the survey approach does not give any value for the size of the underground economy.

In survey approach, there are three frequently used methods to identify the workers of the underground sector:

1. Enterprise-based: An individual is recognized as an underground worker as long as he works for an underground or unregistered enterprise;
2. Employment relationship: no matter the company is registered or not, an individual is recognized as an underground or illegal worker if he is deprived of certain employment benefits or if he works in inferior working conditions such as: lack of medical aid, pension fund, paid leave, permanent employment or retirement benefits. The individual is classified as an underground or illegal employee irrespective of the company's registration [Heintz and Posel, 2008].
3. Worker characteristics: This approach is mostly used for Latin American countries [Gasparini and Tornarolli, 2007; Henley et al., 2009]. In this case, different characteristics of the workers are considered to identify them as underground workers, such as: educational attainment, skills level of occupation, remuneration, as well as public or private sector status.

The results obtained from the survey approach not only provide a deep insight into the structure of the underground economy, but also into its demography, geographical patterns and work

profile of the illegal workers, such as: race, gender, skill level in the province that they reside, etc. However, Schneider and Enste [2002] believe that these results are extremely sensitive to the way in which the survey has been designed.

The main shortcoming of this approach is a common issue among all surveys: the individuals are not very inclined to cooperate fully. Moreover, it is nearly impossible to measure the undeclared work with the survey approach. In other words, the respondents hesitate to reveal their participation in the underground activities as they fear getting caught. Consequently, the responses would be mostly inappropriate and unreliable. The reason for their fears relates mainly to tax evasion or illegal or unlicensed activities [Buehn and Schneider, 2013; Schneider and Enste, 2013].

Also, Saunders [2008] believes that complete coverage of the underground markets and their workers, without omissions and duplications, is difficult or impossible in this method due to the cost implications and limited resources.

To sum up, this approach cannot provide any real estimate for the underground economy in monetary terms [Schneider and Enste, 2002]. The survey method has been extensively used to estimate the underground economy in different countries such as Canada, Britain, Germany, USA, Brazil, South Africa, etc.¹. In South Africa, it has been the most commonly used approach over the years.

3.1.2 Tax auditing method

Tax auditing method provides an estimation on tax evasion and gives the information about the strongest tax evaders, specially the self-employed who have more opportunities for secrecy [Frey and Schneider, 2000]. The main objective of this method is to quantify the amount of undeclared taxable income to make tax collection more efficient but it can also be applied to measure the size of the underground economy.

In other words, this method seeks the discrepancy between the amount of income reported for tax purposes and the tax audits conducted by the Internal Revenue Service (IRS) called

¹ e.g., in Norway by Isachsen et al. (1982), and Isachsen and Strom (1985). For Denmark this method is used by Mogensen et. al. (1995) and Van Eck and Kazemier (1988) and Kazemier (2006) in the Netherlands and other newer studies like Feld and Larsen (2005, 2008, 2009) , Haigner et al. (2011) and Enste and Schneider (2006) for Germany.

selective checks². The difference between the income declared for tax purposes and the income calculated by selective checks let us estimate the size of the underground activities [Frey and Weck-Hanneman, 1984]. Thus, this approach demonstrates the importance of fiscal auditing programs [Schneider, 2003] and how they may affect the tax evasion to decrease.

However, the sample population is a bottle neck. To avoid biased results, the sample population should be selected carefully in order to match the target population.

According to Schneider [2003], the tax payers are not totally selected at random; instead, they are chosen based on submission of (tax) returns. The authorities pick up tax evasion by the random checks in the parts in which the tax evasion is rife. Also, this method does not give a comprehensive insight into the income earned in the underground economy.

Another shortcoming of this method is that the income discovered by tax authorities during selected checks, is only a portion of the income earned in the underground economy [Saunders, 2008]. Moreover, this method deals with taxable activities and give no information about the extent of tax evasion [Frey and Schneider, 2000]. Consequently, this approach just provides a lower bound estimate for the size of the underground, therefore, the results are likely biased. However, the trend can be significant. [Greenidge et al., 2009]. This approach is used in a number of countries such as Canada, Italy and the USA, etc.

3.1.3 A conclusion on direct method

Direct or microeconomic methods estimate the size of the underground economy based on the results of surveys, interviews and tax audits of people. These methods only yield estimates for a specific point in time which can be regarded as low-end estimates. Difficulty selecting appropriate samples, the respondent's willingness to cooperate, their honest responses and measurement errors regarding the surveys are some defects of direct methods [Albu, 1995; Elgin and Oztunali, 2012]. It must be also considered that the results from these kinds of surveys are very sensitive to the way the questionnaire is formulated.

Thus, it is difficult to assess the amount of undeclared work from a direct questionnaire. It is clear that most respondents hesitate to confess to their fraudulent behavior and responses are

² The IRS is the USA tax institution

of uncertain reliability which makes it difficult to calculate a true estimate of the size of the underground economy.

About tax auditing approach, there are some main difficulties which may affect the result of this method. First, finding an appropriate sample of the population is equivalent to using tax compliance data. In this method, the selection of individuals by tax audits is not random but based on the properties of submitted tax returns that indicate a certain likelihood of tax fraud. So, such a sample does not stand for the whole population and the estimates based upon this biased sample may be inaccurate. Second, the estimates based on tax audits demonstrate only a part of the underground income discovered by authorities and it is likely to fail to reveal the whole hidden income.

Detailed information about the underground activities, knowing about the underground markets which are attractive for underground workers, finding the structure and composition of these activities as well as the socio-economic characteristics and motives of those who work in these markets are of the main advantages of direct methods.

3.2 Indirect approach

The indirect methods also known as "indicator" approaches [Pommerehne and Frey, 1984] are mostly macroeconomic methods. Various macroeconomic indicators are used in these methods to determine the size, trend and development of the underground economy over a period of time.

3.2.1 Discrepancy between national expenditure and income

Gross Domestic Product (GDP) can be estimated via three different methods : the income, the expenditure and production methods. As all these methods measure the same thing, they should Presumably give the same result in spite of different points in the circular flow [Mohr et al., 2008]. This indirect method only focuses on discrepancy of the results of two methods: the income and expenditure methods which are applied to estimate the GDP. Thus, having an independent estimate of the expenditure side of the national accounts, the difference with the income side of the national accounts can indicate the extent of the underground economy. In income approach, the incomes that productive factors earn should equal the value of their products, thus, GDP can be determined by finding the sum of all producers' incomes. In other words, the income approach considers the incomes earned by various productive factors: wage (labor), rent (land), interest (capital), royalties (intellectual property) and profit (entrepreneurs) [Mohr et al., 2008].

The basis of the expenditure approach is that people buy all products; therefore, the value of the total products should equals people's total expenditures in buying things. In fact, the expenditure approach is an output accounting method that focuses on the expenditures within an economy. This approach provides a detailed analysis of total spending by focusing on real growth rates:

$$GDP = C + I + G + (X - M)$$

where:

- C = household consumption expenditures or personal consumption expenditures;
- I = gross private domestic investment;
- G = government consumption and gross investment expenditures;

- X = gross exports of goods and services, and;
- M = gross imports of goods and services.

The use of two independent data sets which share the same concept for measuring the GDP, make this approach efficient. The difference between expenditure and income methods demonstrates the net effect of the underground [Schneider, 2003].

Moreover, this approach is effective for the estimation of the particular components of the underground economy. It is, however, only applicable if there are two independent data sources measuring the same thing which may not always be the case [Saunders, 2008].

In expenditure method, the estimations are at market prices, whereas in income approach, they are at factor cost [Mohr et al., 2008]. Thus, before performing any analysis, the results of the income approach should be converted to market prices. To do so, these two approaches should be converted to Gross National Income (GNI) and Gross National Expenditure (GNE). As the informal expenditures are mostly paid for in cash, the discrepancies arise. Moreover, this income cannot be observed and it results in an under-estimation of income in the economy which consequently leads to the discrepancy between income and expenditure. If the discrepancies are large, the data must be normally revised. Saunders [2008] states that the final reported difference, called the residual item, is not useful for the estimation of the underground economy. So, many statisticians are more inclined to lessen it as much as possible. During the calculation process of GDP, many factors initially included in the calculation are removed later, such as seasonality. This is the reason that the first estimate should be used to determine the size of the underground economy, instead of the published results which are mostly manipulated and give a debatable estimate [Schneider, 2003]. This approach has been used in countries such as Germany, Italy, Canada and USA.

3.2.2 Discrepancy between official and actual labour force

This approach benefits the survey data to estimate the size of the economically active labour force involved in the underground economy [Saunders, 2008]. The total labour force participation rate is assumed to be constant, thus, if the participation rate of the labour force in the official economy declines, it probably implies a consequent increase in the activities of the underground economy and vice versa [Tanzi, 1982b].

On the other hand, other factors also affect the labour force including demographical reasons (fertility rates, mortality, etc.), the impact of sicknesses such as HIV/AIDS on the supply of labour, immigration and emigration [Barker, 2007]. There is also another case where the labour force are actively participating in both the underground and official economies. These are the shortcomings which make this approach less reliable.

3.2.3 Transaction approach

Transaction approach was firstly explained by Feige in 1979. He developed this method using the information on the overall volume of transactions in the total economy [Greenidge et al., 2009]. The constant relationship over time between the volume of transactions and the official GNP is the main assumption of the transaction approach that can be used to estimate the value of the unrecorded income. This approach applies the Fisherian quantity equation to estimate the size of the underground economy and the equation is [Feige, 1997a; Schneider, 2003]:

$$M \times V = P \times T$$

where:

- M = Money;
- V = Velocity;
- P = Price and
- T = Total transactions.

If the total value of the transactions i.e. both the official and the underground transactions are unknown, then it is possible to estimate the total value of payments (MV).

$$(py)^* = (py)_r + (py)_u$$

and

$$PT = C \times V_c + D \times V_d$$

where:

- $(py)^*$: Total income;
- $(py)_r$: Recorded income;

- $(py)_u$: Unrecorded income;
- C : Currency;
- V_c : Currency velocity;
- D : Checkable deposits;
- V_d : Checkable deposits velocity and
- PT : Total transactions

If we assume that the total transactions are proportional to total income, then, the equation can be rewritten:

$$PT/(py)^* = k^* \times (C \times V_c + D \times V_d)/(py)^*$$

and

$$(py)_u = (py)^* - (py)_r = [(C \times V_c + D \times V_d)/k^*] - (py)_r$$

where, k^* is the benchmark parameter.

In this approach, it is assumed that the benchmark year (also called the base year) is determined and there was no undeclared income for this year. Consequently, the size of the underground economy would be zero in the benchmark year.

As a result of the benchmark parameter (k^*), the amount of the unrecorded income can be estimated. Moreover, to establish the benchmark, we assume that accurate data exists for a particular period of time where all income is properly recorded [Saunders, 2008].

The later assumption is one of the weaknesses of this approach. It refers to the velocity of money and the relationships between the value of total transactions and total (unofficial + official) nominal GDP. By relating total nominal GDP to total transactions, the size of the underground economy can be calculated by subtracting the official GDP from the total nominal GDP. Another weakness of the transactional approach is the assumption that implies all variations between total value transactions and the official GNP is due to underground activities. Due to the large amount of assumptions and data required in this model, Schneider [2003] states that the results of this approach are extremely questionable.

This approach has been used in countries such as the USA, Germany, Britain, Canada, Italy and Cyprus.

3.2.4 Currency demand approach

This approach was first used by Cagan in 1958 to investigate the correlation between the demand for currency and the tax burden for the United States over the periods 1919 to 1955 [Saunders, 2008; Schneider, 2003]³.

Later in 1977, Gutmann applied this approach, without using any advanced statistical procedures like econometric modeling, to find out the correlation between the currency demand and tax policy. He decomposed the US money supply, M_1 , into currency and demand deposits [Georgiou, 2007]. The main assumption of this approach is that all the payments in the underground economy are in cash because payments made in cash cannot be traced while other payments are more easily traced. Consequently, an increase in the demand for cash may indicate the growth of the underground economy [Greenidge et al., 2009].

Based on the assumptions of this method, if one can estimate the amount of cash being specifically used for the underground transactions, and if the rate of using that cash is the same as that in the official economy, this estimate can provide an indirect measure of the underground economy.

Tanzi further developed this approach in 1980 and 1983 by using econometric estimates of the currency demand approach. He estimated the amount of underground activities in the US between 1929 and 1980. He also assumed in his approach that the people and companies who participate in the underground activities, trade on a cash basis to avoid being tracked by authorities. Thus an increase in the size of the underground economy would be an increase in the demand for currency.

An equation for the currency demand has been estimated econometrically over time which was a tax-related method developed exclusively for the US economy. This equation estimated the demand for cash linked to attempts to evade paying US income taxes. It incorporated several independent variables such as the direct and indirect tax burden, government regulation as well as the complexity of the tax system, which altogether reflect why people participate in the underground activities. Other possible conventional factors such as the generation of income, payment habits and interest rates are incorporated [Schneider, 2003].

The basic regression equation for the currency demand is [Tanzi, 1983]:

$$\ln(C/M_2)_t = \beta_0 + \beta_1 \ln(1 + T_w)_t + \beta_2 \ln(W_s/Y)_t + \beta_3 \ln(R_t) + \beta_4 \ln(Y/N)_t + \mu_t$$

³ See Tanzi [1982a, 1983]

with considering $\beta_1, \beta_2, \beta_3 > 0$ and \ln denoting natural Logarithms.

- C/M_2 is the ratio of cash holdings to current and deposit accounts;
- T_w is a weighted average tax rate;
- W_s/Y is a proportion of wages and salaries in the national income (to capture changing payment and money holding patterns);
- R is the interest paid on saving deposits (to capture the opportunity cost of holding cash); and,
- Y/N is the national income per capita.

Therefore, if any large increase in currency demand cannot be explained by conventional factors mentioned above, it is probably associated with the tax burden and other reasons that encourage the people to participate in the underground economy. The assumed income rate for currency used in the underground economy is the same as for the M_1 in the official economy [Caridi and Passerini, 2001].

Over the last decades, the original equation has been modified and new variables were added. The outcome has been applied for the estimation of the underground economy in different countries, including Brazil. There are, however, a few comments about these measurements. The first comment implies that the underground economy may exist even without any tax evasion, although tax evasion and the underground economy often go together. On the other side, tax evasion may exist even in the absence of the underground economy; e.g., when somebody overstates the deductions in the payment of the income tax or does not declare the whole income earned. This briefly means that the estimates of tax evasion are not necessarily the same as the estimates of the underground economy linked to tax evasion. On the other hand, tax related estimates of the underground economy may be different from those that are not tax related.

Many researchers assume incorrectly that national income statistics are estimated using only the information of tax authorities. They conclude that the tax evasion leads directly to underestimation of national income, thus, the estimation of the underground economy implies that the national income is underestimated. But in reality, most of the national income statistics are

not derived from tax information. As an example, the information on the agricultural outputs are mostly obtained through direct estimation of observed output together with the prices of that output estimated to be produced. Thus, the estimation of an output may be incorrect but not due to the existence of the underground economy. It implies that there may be just little relation between the estimations of the underground economy and those of national income or gross national product.

According to Tanzi [1982a,b], an estimation of the underground economy around 40 percent does not mean that the GDP is underestimated by 40 percent or the country is in fact 40 percent richer than estimated.

However, Schneider [2003] explains that this approach is widely used, in spite of its shortcomings. In one of the surveys of the underground economy in Norway carried out by Isachen and Storm (1985), it was found that only 80% of the underground activities were paid for in cash. This finding indicates that the economical activities in this part of economy could have been larger than estimated and the estimated value is only a possible minimum estimate or a lower limit for the underground economy. Another criticism Schneider [2003] is that the tax evasion is not the only cause of the underground economy as it is considered in many studies. But, some other factors should be considered as well, such as: tax payer attitude, the impact of regulation, etc.

As it was mentioned earlier, the currency demand approach implies that any increase in the currency demand demonstrates the expansion of the underground economy, but, this is not always the case. For example, in 1996, Feige recognized that the increase in currency demand deposits was largely due to the slowdown in the demand deposits and not because of an increase in the underground activities.

Also, the assumption that there is no underground economy or tax evasion in a base year is largely disputed. This assumption would imply an upward adjustment in the size of the underground economy [Schneider, 2003]. In addition, econometric results are sensitive to selection of period which means that the results could have been influenced with eventual exogenous shocks in this period.

This approach has been used in a number of countries such as the USA, Germany, Britain, Canada, Italy, Cyprus, Belgium, Denmark, Finland, France, Switzerland, Netherlands, Norway, Australia, New Zealand and South Africa.

3.2.5 Physical input method: electricity consumption

In physical input method, it is assumed that the ratio of electric power consumption to the GDP can be estimated via econometric methods, then, any difference with the expected levels can be ascribed to underground activities [Fleming et al., 2000]. Two methods are proposed based on this approach by Kauffmann- Kaliberda and Lack:

Kaufmann Kaliberda Method:

In this method, it is assumed that the electricity consumption is the best physical indicator for economic activity. This method estimates the overall economic activity in both the official and underground economies and it considers a positive relation between the growth in the total electricity consumption and that of overall GDP. Briefly, the growth in the electricity consumption reflects the extent of economic activities [Saunders, 2008]. The consumption of electricity can be a proxy measurement as it may act as a reason for the growth in both economies. Considering this proxy in the overall economy, the discrepancy between the growth rate of the official economy and the electricity consumption points out the size of underground activities.

However, Schneider [2003] has criticized this approach and explained that not all underground activities require large electricity supply, e.g., fishermen, taxi drivers and vendors who sell things in the street. So, this approach includes only a part of the underground activities. Moreover, various technological improvements over the time, which lead to more efficient electricity consumption in both the official and the underground activities, are ignored in this method. Finally, the price elasticity of electricity consumption may differ among countries.

Lackó method:

In Lackó method, it is assumed that a definite part of the underground economy is linked with household electricity consumption. It is further considered that in countries with high consumption of household electricity, the rest of the underground activities are also high, as many underground goods and services are offered by home participants.

To determine the household electricity consumption, Lackó method employs regression analysis in order to express the impact of these factors on the underground economy. The

related equation is [Saunders, 2008]:

$$\ln(ER_i) = \alpha_1 \ln(C_i) + \alpha_2 \ln(AG_i) + \alpha_3 \ln(G_i) + \alpha_4 \ln(Q_i) + \alpha_5 \ln(PR_i) + \alpha_6 \ln(H_i) + \mu_i$$

with $\alpha_1, \alpha_2, \alpha_3, \alpha_4 > 0$

and,

$$H_i = \beta_1(T_i) + \beta_2(S - T)_i + \beta_3(D_i) \quad \beta_1, \beta_2, \beta_3 > 0$$

where:

- i = index which indicates each country;
- ER = household electricity consumption (kWh) per capita;
- C = real consumption of households per capita (at purchasing power);
- AG = agricultural ratio in total official GDP;
- G = index for the weather differences;
- Q = the total ratio of other energy sources other than electricity;
- PR = real price of consumption of one kWh of residential electricity;
- H = output of the unofficial economy per capita;
- T = the ratio of the sum of paid personal income, corporate profit and taxes paid on goods and services to GDP;
- S = the ratio of public social welfare expenditure to GDP;
- D = the sum of dependents over 13 years and inactive earners per 100 active earners.

To obtain the per capita output of the underground sector, we substitute the second equation into the first one:

$$\ln(ER_i) = \alpha_1 \ln(C_i) + \alpha_2 \ln(AG_i) + \alpha_3 \ln(G_i) + \alpha_4 \ln(Q_i) + \alpha_5 \ln(PR_i) + \alpha_6 \ln(\beta_1(T_i) + \beta_2(S - T)_i + \beta_3(D_i)) + \mu_i$$

Now, to determine the contribution of the underground economy to GDP, the contribution of one unit of electricity to GDP has to be known. Then, multiplying the contribution per unit by the number of units, the size of the underground economy and its contribution to GDP can be determined.

Schneider [2003] criticizes that certain underground activities requires energy sources other than electricity. Moreover, the underground activities not necessarily take place in household environments. Also, the base value used to calculate the size of the underground economy for all countries, especially, for transitional and developing countries must be argued.

The electricity consumption method was used in countries like Russia, Ukraine, Uzbekistan, Slovenia, Slovakia, Poland, Romania, Bulgaria, Canada, Germany, Italy, Britain and the USA.

3.2.6 A conclusion on indirect method

Indirect, macroeconomic or indicator methods focus more on the discrepancy between the official and actual criteria. These methods are generally criticized for employing various simplifying and limiting assumptions, such as focusing on a specific aspect or indicator and neglecting the others which may also have a predominant effect on the underground economy [Albu, 1995; Elgin and Oztunali, 2012]. Also, some of the shortcomings of currency demand approach are:

- All transactions in the underground market are not paid in cash;
- The role of the international currency -such as US dollar- in underground transactions of other countries is not regarded;
- Increases in currency demand deposits are largely due to a slowdown in demand deposits rather than to an increase in currency caused by underground activities;
- The tax burden is not the only cause of the underground economy;
- Assuming the same velocity of the currency in both the official and the underground economy; and,
- The main criticism is that there is no underground economy in a base year.

There are also some criticisms about the physical input method:

- All parts of the underground economy do not necessarily require a considerable amount of electricity and other energy sources can be used as well;
- The underground activities do not take place only in the household sector;

- The technical progresses in the production procedure and use of electricity over time are ignored;
- Using the ratio of social welfare expenditures as the explanatory factor of the underground economy, especially in transition and developing countries; and,
- The differences or changes in the elasticity of the electricity/GDP across countries and over time are ignored.

On the whole, these methods are simpler than direct methods for the estimation of the underground economy; they do not need people to provide data through questionnaire and etc., however, they are very sensitive to the quality and quantity of the raw data. The underground economy can be estimated more quickly by indirect methods compared to direct methods. Besides, the outcome of indirect methods is a time series for the size of the underground economy while the outcome of direct methods is a number as the size of the underground economy in a specific year.

Direct methods depend on the people willingness to cooperate and their honest responses. Another problem with direct methods is that they cannot be applied to more than last two or three years since the initial data should be obtained from people and they may forget some answers and cannot answer the questions properly even if they want to do it correctly. On the contrary, the indirect methods do not deal with these limitations and can be used to estimate the time series of the underground economy more readily.

3.3 Model Approach (MIMIC Model Approach)

All methods described up to now consider just one cause or indicator that to indicate or capture all effects of the underground economy. However, underground economy effects show up simultaneously in many markets such as the production, labor, and money markets.

An even more important critique is that the causes that determine the size of the underground economy are taken into account only in some of the monetary approach studies that usually consider one cause, the tax burden.

The empirical method used is quite different from those used so far. The Model approach is based on the statistical theory of latent or unobserved variables. The model approach explicitly considers multiple causes of the existence and growth of the underground economy as well as the multiple effects of the underground economy over time for its estimation.

The MIMIC model approach employs the unobserved or latent variables in econometrics [Georgiou, 2007]. It can be divided into two parts: the measurement model which associates the unobserved variables to the observed variables, and the structural model which determines the causal relationships among the latent variables. In the underground economy estimation, the size of the underground economy is considered as the only the only unknown variable.

According to recent researches, the MIMIC is a promising model for the estimation of underground economy. In this model, it is assumed that the causes and the output (or income) of underground economy are observable. Moreover, there are two types of observed variables called "causal" variables and "indicator" variables, which are connected by a single unobserved index: the underground Economy. The two sets of equations must be specified in MIMIC model: one set of equation between the causes and the underground economy and the other one between the underground economy and outputs.

The MIMIC model is generally composed of two parts: unobserved (or latent) and observed variables [Schneider, 2003]. Values of the latent or unobserved variable over time are estimated via the statistical model using the data of causes and indicators and by predicting the coefficients of the equations. The fitted coefficients are then interpreted as a time-series estimate of the size of underground economy.

The main statistical idea of this approach is to compare a sample covariance matrix. This covariance matrix is related to the observable variables consist of the causes and indicators,

with the parametric structure which imposed by a hypothesized model.

Also, there is an economic theory which is tested based on the structure of the model and data. The economic theory is tested examining the consistency of actual data with the hypothesized relationships between the unobservable (or latent) variable or factor and the observable (or measurable) variables.

Frey and Weck-Hanneman [1984] firstly tried to measure the underground economy using MIMIC method in 1984. They applied four causes for the underground activities including the burden of the state on economic agents (e.g. the actual and perceived tax burden; the share of public employees in total labour force), the unemployment rate, tax morality and the level of economic development. They also used three indicator variables in their model called: male labour force participation rate, hours worked and real GDP growth. They however did not gave a clear definition for the underground economy and the model failed to define the underground economy. Furthermore, they used the public employees as share of the total labour force as an indicator of the burden of regulation while the number of regulations seems to be a more obvious and direct measure.

Georgiou [2007] criticizes the Frey and Weck-Hanneman approach and explains that the complexity of morality which cannot simply be measured by an index. He states that the attitudes towards tax are governed by historical, social and cultural factors.

In general, there are two main goals in solving the MIMIC model:

1. to estimate parameters such as coefficients and variances of the model; and,
2. to assess the fit of the model.

For the analysis of the underground activities these two goals mean:

1. to estimate the relationships between a set of observable variables (causes and indicators) and unobservable variable (underground economy); and,
2. to test if the researcher's theory or the derived hypotheses as a whole fit the data.

3.4 Discussion on MIMIC Method

There is no doubt that the underground economy cannot be directly observed, so, various methods have been proposed to estimate its trend and magnitude. "Household incomes and expenditures" is a measure used by national statistical agencies to examine the discrepancies that might indicate unreported incomes. "Tax audits" are valuable and informative, but they usually target suspected offenders and hence are biased estimates of aggregate behavior. Other frequent surveys question about cash payments or below the counter payments (or incomes) directly, thus, the unreliable answers or no response to the questions are always concerns. Generally, deductions appear from imbalances between the expenditure, income and product data that are collected from various sources for national accounting purposes. The most popular estimation methods of the underground economy are based on macroeconomic models of either the demand for currency holdings (perhaps in comparison to bank account balances) or the consumption of some standard commodity such as electricity [Breusch, 2005].

As it was mentioned, the MIMIC model can be considered as the most famous method used to estimate the underground economy in recent years. This model which is also called the structural equation, is established on "multiple indicators, multiple causes". The origin of MIMIC comes from the factor analysis literature of psychometrics and its exposure in economics is via the latent variable models of Zellner [1970] and Goldberger [1972].

In the first application of MIMIC to estimate the underground economy, Frey and Weck-Hanneman [1984] examined a pooled data set of 17 OECD countries. The idea was further developed by Aigner, Schneider and Ghosh, who made some lagged adjustments in a dynamic MIMIC model and applied it to the United States [Aigner et al., 1986].

Another modification in the approach was introduced in the estimation of underground economy in New Zealand. Giles [2000] modified the model in his study by incorporating the developments in time-series methods, especially unit roots and co-integration analysis. Later, Giles and Tedds [2002] thoroughly described the approach and applied it to Canada, the results of which was entitled : "state of the art of dynamic MIMIC modeling".

Bajada and Schneider [2005] adopted the method after Giles and Tedds to study Australia and other Pacific nations. Also, Schneider and Dell'Anno [2003] estimated the underground economy in Italy and reported the results for other OECD countries.

Researchers describe the underground incomes in two ways: some researchers measure it as a percentage of recorded GDP while some others consider the "tax gap" between the actual and the potential revenue when all taxable incomes are reported.

All the estimated values explained above are presented in three or four digits of precision without any confidence interval, consequently the results are not reliable enough. On the other hand, these values are large and cause major concerns, thus, the estimated size of the underground economy is shown to be growing strongly. In other words, these results are serious alarms in the areas of tax administration, national income accounting, stabilization policy and social fairness and cohesion.

In some researches, the causal and indicator variables are not sufficient and convincing for the estimation of the underground economy ⁴. Another shortcoming discussed by Helberger and Knepel is related to the relevance of the variables. The direct relevance of the causal and indicator variables complicates the model and increases the number of coefficients which must be calculated. Even it may exclude the model from the structural equation form.

Smith (2002) and Hill (2002) also criticized the method proposed by Tedds and Giles, especially the absence of economic theory to indicate the specifications and the complexity of the estimation strategy. The specifications and results of Tedds and Giles were examined in detail by Breusch [2005]. It revealed that the time path of their estimation for Canada has little to do with any underground activities, but mostly reflects price inflation and the real growth in the observed economy. Moreover, the estimated values they obtained were incidentally meaningful with no connection to any evidence in the data.

Despite all this, the MIMIC method is still a reliable approach among researchers for the estimation of the underground economy. Before describing the principles of the MIMIC method, two simple regression models with one latent variable which are the basis of the MIMIC are presented and discussed.

⁴ Helberger and Knepel (1988)'s opinion about the results of Frey and Weck-Hannemann

3.4.1 Latent variable model

Simple linear regression model with one latent or unobserved variable is defined as equation (3.1):

$$y = \lambda\eta^* + \varepsilon \quad (3.1)$$

where y is an observed scalar, η^* is the latent variable and ε is the error term. It must be considered that η^* and ε are independent. It is assumed that the latent variable is in standard units which means the mean and variance of the latent variable are respectively zero and one [Weeks, 2000]. we can calculate the variance and mean of y using the following equation (3.2),

$$\text{var}(y) = \text{var}(\lambda\eta^* + \varepsilon) = \lambda^2 * \text{var}(\eta^*) + \text{var}(\varepsilon) = \lambda^2 + \sigma_\varepsilon^2 \quad (3.2)$$

λ should be also determined.

To determine the unknown parameters, the following variable change (equations (3.3) and (3.4)) is suggested [Weeks, 2000]:

$$\eta = \eta^* + u \quad (3.3)$$

$$\eta^* = \beta'X \quad (3.4)$$

where, $X_{(s \times 1)} = (x_1, x_2, \dots, x_s)'$ is the vector of causes and β is the same vector of unknown parameters. Also, $E(\varepsilon \times u)$ is zero and η^* and u are independent.

From above equations, the structural equations can be written as equations (3.5) and (3.6):

$$y = \lambda(\beta'X) + \varepsilon \quad (3.5)$$

$$\eta = \beta'X + u \quad (3.6)$$

The reduced form of equations (3.5) and (3.6) are equation (3.7):

$$\begin{pmatrix} y \\ \eta \end{pmatrix} = \begin{pmatrix} X & 0 \\ 0 & X \end{pmatrix} \begin{pmatrix} \pi \\ \beta' \end{pmatrix} + \begin{pmatrix} \varepsilon \\ u \end{pmatrix} \quad (3.7)$$

where, $\pi = \lambda\beta$ and the covariance matrix of ε and u are diagonal. A simple application of the above equations was illustrated in Income model by Friedman [1957]. He defined

the variable Y as Consumption, the variable η^* as permanent income and the variable η as measured income [Weeks, 2000].

An alternative approach is based on the method of instrumental variables. As an instance, in the context of the above problem, it is possible that a variable q exists which is correlated with η^* and y but not with u . It is quite favorable, if the model contains a number of fully observed exogenous variables and one explanatory variable which is measured with error.

One strategy would be to simply drop the unobservable variable and incur the normal omitted variable bias. Alternately, it is possible that it includes a proxy or indicator variable. Since this variable will contain measurement error, the estimation of parameters will also be inconsistent. It is noticeable that the equation (3.6) satisfies all the requirements of Ordinary Least Squares (OLS). Thus, β and the asymptotic covariance matrix for β can be estimated.

Also, for the structural equation (3.5), by substituting $\widehat{\eta^*} = \widehat{\beta}'x$, the estimation of λ can be possible.

Pagan [1984] demonstrated that this procedure is consistent as asymptotically efficient as full Maximum-Likelihood Estimation (MLE) and even much easier to implement.

3.4.2 Latent variable model with multiple indicators

It was mentioned that additional information is needed to determine the unknown parameters. To obtain the unknown parameters, the latent variable must be substituted with a linear equation of its causes as it was explained and done.

Here, we discuss that how the information in the form of multiple indicators of η^* can also be applied to identify the parameters of the model. Another assumption is that the latent variable is in standard units. Consider the equation (3.8):

$$y_j = \lambda_j \eta^* + \varepsilon_j, \quad j = 1, 2, \dots, n \quad (3.8)$$

where, $Y_{(n \times 1)} = (y_1, y_2, \dots, y_n)'$ is the vector of observed indicators, λ and ε are the same as Y in dimension, λ is the vector of the unknown parameters of the latent variable and ε denotes the vector of error terms of the equations. It is assumed that σ_j^2 is the variance of ε_j and also, $cov(\varepsilon_i, \varepsilon_j), \forall i \neq j$ are zero. It must also be considered that η^* and ε_j are independent.

Based on equation (3.8), there are " $2n$ " unknown parameters that must be calculated including:

" n " parameters as the coefficients of the equation, $(\lambda_1, \lambda_2, \dots, \lambda_n)$ and " n " parameters as variance of error terms, $(\sigma_1, \sigma_2, \dots, \sigma_n)$. The " $n \times n$ " covariance matrix of Y can be easily obtained. The diagonal elements (S_{jj}) in this matrix represent the variance of y_i as equation (3.9):

$$\text{var}(y) = S_{jj} = \lambda^2 + \sigma^2 \quad (3.9)$$

The other elements in this matrix (S_{ij}) depicts the $(\text{cov}(y_i, y_j), \forall i \neq j)$ and correspond to $\lambda_i \lambda_j$. The related equation is (3.10):

$$\begin{aligned} \text{cov}(y_i, y_j) &= \text{cov}(\lambda_i \eta^* + \varepsilon_i, \lambda_j \eta^* + \varepsilon_j) \\ &= \lambda_i \lambda_j \times \text{var}(\eta^*) + \text{cov}(\varepsilon_i, \varepsilon_j) = \lambda_i \lambda_j \end{aligned} \quad (3.10)$$

It should be noted that $\text{cov}(y_i, y_j)$ and $\text{cov}(y_j, y_i), \forall i \neq j$ are equal. So, the covariance matrix has " n " unique elements in diagonal and " $(n^2 - n)/2$ " elements in off-diagonal⁵. Totally, the covariance matrix has " $(n \times (n+1))/2$ " unique elements, as follow:

$$\underbrace{\frac{(n^2 - n)}{2}}_{\text{unique elements in off-diagonal}} + \underbrace{n}_{\text{unique elements in diagonal}} = \underbrace{\frac{(n \times (n+1))}{2}}_{\text{unique elements in covariance matrix}}$$

From the above results, the coefficients can be calculated using the following equation (3.11):

$$\lambda_m = (S_{mj} \times S_{mi})^{0.5} / S_{ij}, \forall i \neq j \quad (3.11)$$

The variance of error terms can be also calculated as equation (3.12):

$$\sigma_j^2 = S_{jj} - \lambda_j^2 \quad (3.12)$$

Based on the equations ((3.11) and (3.12)), the elements of regression are completely identified. But, a major issue arises from the value of the " n " or the number of the equations in equation (3.8).

⁵ n^2 is the number of all elements in covariance matrix and $(n^2 - n)$ is the number of elements in off-diagonal

Let " $n = 3$ "; the coefficients would be:

$$\lambda_1 = (S_{12} \times S_{13})^{0.5} / S_{23}, \quad \lambda_2 = (S_{21} \times S_{23})^{0.5} / S_{13}, \quad \lambda_3 = (S_{31} \times S_{32})^{0.5} / S_{12}$$

It is shown that there are the " $2n$ " elements which must be identified and for a " $n \times n$ " covariance matrix of Y , there are $(n \times (n+1)) / 2$ unique elements. To identify the regression elements, the elements of covariance matrix must be equal or larger than the unknown elements:

$$\frac{(n \times (n + 1))}{2} \geq 2n \Rightarrow n \geq 3$$

If " $n \geq 3$ ", the number of unique sample variance and covariance terms in covariance matrix of Y would be more than the " $2n$ " unknown parameters. Otherwise, the model is undefined and the unknown parameters cannot be determined.

3.4.3 Multiple Indicators and Multiple Causes (MIMIC)

The Multiple Indicators and Multiple Causes or MIMIC is represented in Giles and Tedds [2002, chapter 6] work as a relation between three sets of variables:

Set 1. A vector consisting of Indicator Variables called $Y(p \times 1)$

Set 2. A vector consisting of Causal Variables called $X(q \times 1)$

Set 3. And an unobserved latent variable η (scalar).

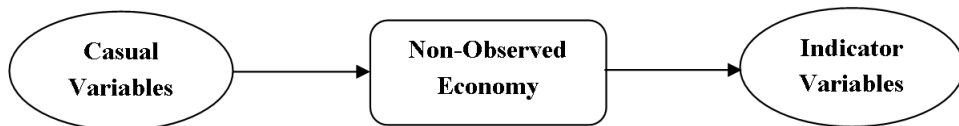


Figure 3.1: Relations between the variables in MIMIC Method

Their relations can be shown as figure (3.1). As it is shown in figure (3.1), the causal and indicator variables are connected by an unobserved latent variable η which is scalar via the

equations (3.13) and (3.14):

$$y_t = \lambda \eta_t + \varepsilon_t \quad (3.13)$$

$$\eta_t = \gamma' x_t + \xi_t \quad (3.14)$$

where, $\gamma(q \times 1)$ and $\lambda(p \times 1)$ are vectors of unknown parameters ⁶. As it can be seen in equations (3.13) and (3.14), two error terms are used in these equations:

1. The error term $\varepsilon_t(p \times 1)$ is assumed to have a normal distribution with zero mean and a variance equal to $\Theta = \text{diag}(\theta_1, \dots, \theta_p)$; and,
2. The error term $\xi_t(\text{scalar})$ is assumed to have a normal distribution with zero mean and a variance equal to Ψ

It is also assumed that the error terms are not correlated with each other. It is clear that with the model consisting of equations (3.13) and (3.14) we cannot specify the scale of all of the parameters, so, a normalization condition is required.

There are many possibilities for a normalization condition. For instance, Giles and Tedds [2002] set the convention of setting the first element of λ to unit, as $\lambda_1 = 1$. It is considerable that there is not any restriction in selection of the λ_i for initializing ($\lambda_i = 1$) and other elements of the vector λ can be also selected for this purpose. Moreover, the normalization can be carried out by initializing one of the elements of the vector γ (the coefficient of the causal variables) knowing that any other element of γ_j can be initialized to 1 ($\gamma_j = 1$).

The data are a time series of observations in different years where $t = 1, \dots, T$. Estimation of the parameters by using MLE Method (Maximum Likelihood Estimation) needs some additional assumptions such as the error terms ε and ξ have normal distributions and they are independent over time.

In MIMIC method, causal variables are independent variables that affect the model without being affected by the model. In other words, causal variables (A vector $X(q \times 1)$) are exogenous and all of the other variables depend on them directly or indirectly as it was shown in figure (3.1). So, the model forms particular structures based on the conditional mean and variance of

⁶ The vector γ' is transpose of the vector γ and therefore, $\gamma'(1 \times q)$.

the causal variables (equations (3.15) and (3.16)):

$$E[y_t|x_t] = E[\lambda(\gamma'x_t + \xi_t) + \varepsilon_t|x_t] = \lambda\gamma'x_t \quad (3.15)$$

$$\text{var}[y_t|x_t] = \text{var}[\lambda(\gamma'x_t + \xi_t) + \varepsilon_t|x_t] = \text{var}[\lambda\xi_t + \varepsilon_t|x_t] = \lambda\lambda'\Psi + \Theta \quad (3.16)$$

As a reduced form regression equation, the expressions (3.13) and (3.14) can be written as equation(3.17):

$$y_t = \Pi x_t + v_t \quad (3.17)$$

where, $\Pi = \lambda\gamma'$.

Also, it can be proved that variance of y_t is the same as variance of v_t and can be written as $\Omega = \lambda\lambda'\Psi + \Theta$.

From other side, based on means of the error terms (ε_t and ξ_t), the mean of v_t will be zero and can be shown as $v_t = (0, \Omega)$ and $\Omega = \lambda\lambda'\Psi + \Theta$.

Generally, the structure of the MIMIC model will explain the restrictions on the reduced parameters, Π and Ω .

3.4.4 MIMIC method and econometrics models

In this section, more details about MIMIC and its relation to econometrics topics are provided. As it was explained earlier, two main relations, one between "the causal variables and the latent variable" and the other between "the latent variable and the indicator variables" form the two sets of equations and are defined in MIMIC method as equations (3.13) and (3.14):

$$y_t = \lambda\eta_t + \varepsilon_t \quad (3.13)$$

$$\eta_t = \gamma'x_t + \xi_t \quad (3.14)$$

where $y_t(p \times 1)$ is the vector of indicator variables, $x_t(q \times 1)$ is the vector of causal variables, η_t (scalar) is the latent variable, the error terms are $\varepsilon_t(p \times 1)$ and ξ_t (scalar) with a normal distribution (with zero means and their variances are respectively $\Theta = \text{diag}(\theta_1, \dots, \theta_p)$ and Ω) and $\gamma(q \times 1)$ and $\lambda(p \times 1)$ are the vectors of unknown parameters.

Furthermore, the equation (3.17) was written as a reduced form regression of the MIMIC

equations (equations (3.13) and (3.14)):

$$y_t = \Pi x_t + v_t \quad (3.17)$$

where, $\Pi = \lambda \gamma'$, $v_t = (0, \Omega)$, and $\Omega = \lambda \lambda' \Psi + \Theta$.

As an assumption, the coefficient λ will be a vector ($p \times 1$) with the first element initialized to one ($\lambda_1 = 1$) based on Giles and Tedds [2002] convention. So, about λ , $\Pi = (\lambda \times \gamma')$ and the variance of the error term $\varepsilon(p \times 1)$ we can write (equation (3.18)):

$$\lambda = \begin{bmatrix} 1 \\ \lambda_2 \\ \vdots \\ \lambda_{p-1} \\ \lambda_p \end{bmatrix} \Rightarrow \Pi = \lambda \times \gamma' = \begin{bmatrix} \gamma' \\ \lambda_2 \gamma' \\ \vdots \\ \lambda_{p-1} \gamma' \\ \lambda_p \gamma' \end{bmatrix}, \Theta = \begin{bmatrix} \theta_1 & 0 & \dots & 0 & 0 \\ 0 & \theta_2 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & \theta_{p-1} & 0 \\ 0 & 0 & \dots & 0 & \theta_p \end{bmatrix} \quad (3.18)$$

Representing the reduced form equations in full, we will have equations (3.19) and (3.20):

$$y_{1t} = \gamma' x_t + v_{1t} \quad (3.19)$$

$$\left\{ \begin{array}{l} y_{2t} = \lambda_2 \gamma' x_t + v_{2t} \\ y_{3t} = \lambda_3 \gamma' x_t + v_{3t} \\ \vdots \\ y_{pt} = \lambda_p \gamma' x_t + v_{pt} \end{array} \right. \quad (3.20)$$

As can be seen, the coefficients of the variable x_t in the equations (3.19) and (3.20) have a constant ratio (This constant ratio is λ_i). According to the definitions above, the variance

matrix of the v_t (error term) can be written as equation (3.21):

$$\begin{aligned}
 \text{var} \begin{bmatrix} v_{1t} \\ v_{2t} \\ \vdots \\ v_{pt} \end{bmatrix} &= \lambda \lambda' \Psi + \Theta = \begin{bmatrix} 1 \\ \lambda_2 \\ \vdots \\ \lambda_{p-1} \\ \lambda_p \end{bmatrix} \times \begin{bmatrix} 1 & \lambda_2 & \cdots & \lambda_{p-1} & \lambda_p \end{bmatrix} \times \Psi + \begin{bmatrix} \theta_1 & 0 & \cdots & 0 & 0 \\ 0 & \theta_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \theta_{p-1} & 0 \\ 0 & 0 & \cdots & 0 & \theta_p \end{bmatrix} \\
 &= \begin{bmatrix} \Psi + \theta_1 & \lambda_2 \Psi & \cdots & \lambda_{p-1} \Psi & \lambda_p \Psi \\ \lambda_2 \Psi & \lambda_2^2 \Psi + \theta_2 & \cdots & \lambda_2 \lambda_{p-1} \Psi & \lambda_2 \lambda_p \Psi \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_{p-1} \Psi & \lambda_{p-1} \lambda_p \Psi & \cdots & \lambda_{p-1}^2 \Psi + \theta_{p-1} & \lambda_{p-1} \lambda_p \Psi \\ \lambda_p \Psi & \lambda_2 \lambda_p \Psi & \cdots & \lambda_{p-1} \lambda_p \Psi & \lambda_p^2 \Psi + \theta_p \end{bmatrix} \quad (3.21)
 \end{aligned}$$

The variance value of the v_t (error term) can be also calculated as equation (3.22):

$$\begin{aligned}
 \text{var}(v_{it}|x_t) &= \text{var}(y_{it} - \lambda_i \gamma' x_t | x_t) = \text{var}(y_{it} | x_t) = \text{var}(\lambda_i (\gamma' x_t + \xi_t) + \varepsilon_t | x_t) \\
 &= \text{var}(\lambda_i \xi_t + \varepsilon_t | x_t) = \lambda_i^2 \text{var}(\xi_t | x_t) + \text{var}(\varepsilon_t | x_t) = \lambda_i^2 \Psi + \theta_i \quad (3.22)
 \end{aligned}$$

It should be considered that the elements $\lambda_i^2 \Psi + \theta_i$ are the diagonal elements of the variance matrix of the v_t (error term) in the final matrix (equation (3.21)). Also, the off-diagonal elements of the final matrix in equation (3.21) are covariance of the term errors $\text{Cov}(v_{it}, v_{jt})$. On the whole, about the variance and covariance of the term errors, these equations (3.23) can be written:

$$\begin{aligned}
 \text{var}(v_{it}|x_t) &= \lambda_i^2 \Psi + \theta_i \\
 \text{Cov}(v_{it}, v_{jt}) &= \lambda_i \lambda_j \Psi, \quad \forall i \neq j \quad (3.23)
 \end{aligned}$$

By solving the regression of the reduced form in equation (3.17), the vectors of unknown parameters $\gamma(q \times 1)$ and $\lambda(p \times 1)$ are determined easily.

About the elements of the covariance matrix, there are P diagonal elements and $P^2 - P$ non-diagonal elements. It is considerable that the non-diagonal elements above and below the diagonal elements are similar. Thus, there are $(P^2 - P)/2$ distinct non-diagonal elements. On

the whole, there are $(p^2+p)/2$ distinct elements in the covariance matrix:

$$\text{Distinct elements in covariance matrix:} = \frac{(p^2 - p)}{2} + p = \frac{(p^2 + p)}{2}$$

Knowing the vector λ , no additional restriction exist on the variances and there will be $(p^2+p)/2$ distinct elements as it was mentioned above and $(p+1)$ parameters $(\Psi, \theta_1, \theta_2, \dots, \theta_p)$ in the covariance matrix. So, it can be written:

$$\frac{(p^2 + p)}{2} \geq (p + 1) \Rightarrow p^2 - p - 2 \geq 0 \Rightarrow p \geq 2$$

It was demonstrated that the number of the indicators in the restricted reduced form must be more than two. It should be noticed that when there are more than two indicator variables in the model, there are restrictions on the covariance matrix as well as on the coefficient vectors of the restricted reduced form. It can be seen that the $p \times q$ elements for Π must be obtained and $(p^2+p)/2$ additional elements in variance of the error term (v_t) in the reduced form equation. On the whole, $p(\frac{2q+p+1}{2})$ elements must be determined. However, the reduced form equation has $2p + q$ parameters as shown in the Table (3.1). By considering the

Table 3.1: Parameters of MIMIC method in reduced form equation

coefficient γ	(q) parameters
coefficient λ	(p-1) parameters
variance (ξ) = Ψ	(1) parameter
variance (ε) = Θ	(p) parameters
Total	(2p+q) parameters

standard econometrics form of the Simultaneous Equations Model, the equation (3.24) can be written by multiplying the equation (3.19) by λ_i and then subtracting the result from the equation (3.20) while u_{it} is defined as equation (3.24):

$$y_{1t} = \gamma' x_t + v_{1t} \quad (3.19)$$

$$y_{it} = \lambda_i \gamma' x_t + v_{it} \quad (3.20)$$

$$\begin{array}{ccc}
 \left\{ \begin{array}{l} y_{2t} = \lambda_2 \gamma' x_t + v_{2t} \\ y_{3t} = \lambda_3 \gamma' x_t + v_{3t} \\ \vdots \\ y_{pt} = \lambda_p \gamma' x_t + v_{pt} \end{array} \right. & \xrightarrow[\lambda_i y_{1t} = \lambda_i \gamma' x_t + \lambda_i v_{1t}]{\text{Are subtracted from}} & \left\{ \begin{array}{l} y_{2t} - \lambda_2 y_{1t} = v_{2t} - \lambda_2 v_{1t} = u_{2t} \\ y_{3t} - \lambda_3 y_{1t} = v_{3t} - \lambda_3 v_{1t} = u_{3t} \\ \vdots \\ y_{pt} - \lambda_p y_{1t} = v_{pt} - \lambda_p v_{1t} = u_{pt} \end{array} \right. \\
 & & \hline
 & & y_{it} - \lambda_i y_{1t} = v_{it} - \lambda_i v_{1t} = u_{it} \quad (3.24)
 \end{array}$$

With this variable change, the equation (3.24) is converted to the equation (3.25):

$$\begin{array}{ccc}
 y_{1t} = \gamma' x_t + v_{1t} & & (3.19) \\
 \left\{ \begin{array}{l} y_{2t} = \lambda_2 y_{1t} + u_{2t} \\ y_{3t} = \lambda_3 y_{1t} + u_{3t} \\ \vdots \\ y_{pt} = \lambda_p y_{1t} + u_{pt} \end{array} \right. & & \\
 & & \hline
 y_{it} = \lambda_i y_{1t} + u_{it} & & (3.25)
 \end{array}$$

Now, the equations (3.25) and (3.19) make a "P"-equation linear simultaneous model, with p endogenous variables and q exogenous variables.

The error term variance of the above equations or in other words, the covariance matrix between u_{it} and v_{1t} , is unrestricted and it can be calculated as follows:

$$\begin{aligned}
 cov(u_{it}, v_{1t}) &= cov(v_{it} - \lambda_i v_{1t}, v_{1t}) = cov(v_{it}, v_{1t}) + cov(-\lambda_i v_{1t}, v_{1t}) \\
 &= cov(v_{it}, v_{1t}) + (-\lambda_i) \times cov(v_{1t}, v_{1t}) \\
 &= cov(v_{it}, v_{1t}) + (-\lambda_i) \times Var(v_{1t}) \quad (3.26)
 \end{aligned}$$

Based on the covariance of v_{it} and v_{1t} and variance of v_{it} which were calculated in equation (3.23), the final result of the covariance of u_{it} and v_{1t} in equation (3.26) can be written as

equation (3.27):

$$\begin{aligned}
 cov(u_{it}, v_{1t}) &= cov(v_{it}, v_{1t}) + (-\lambda_i) \times Var(v_{1t}) \\
 &= \lambda_i \Psi + (-\lambda_i) \times (\Psi + \theta_1) \\
 &= -\lambda_i \theta_1
 \end{aligned} \tag{3.27}$$

Also, the variance of u_{it} can be calculated from equation (3.28):

$$\begin{aligned}
 var(u_{it}) &= var(v_{it} - \lambda_i v_{1t}) = var(v_{it}) + var(-\lambda_i v_{1t}) + 2cov(v_{it}, -\lambda_i v_{1t}) \\
 &= var(v_{it}) + \lambda_i^2 \times var(v_{1t}) - 2\lambda_i \times cov(v_{it}, v_{1t})
 \end{aligned} \tag{3.28}$$

Based on the variance of v_{it} which was calculated in equation (3.23), the final result for the variance of u_{it} can be obtained from equation (3.29):

$$\begin{aligned}
 var(u_{it}) &= var(v_{it}) + \lambda_i^2 \times var(v_{1t}) - 2\lambda_i \times cov(v_{it}, v_{1t}) \\
 &= (\lambda_i^2 \Psi + \theta_i) + \lambda_i^2 \times (\Psi + \theta_1) - 2\lambda_i \times (\lambda_i \Psi) = \lambda_i^2 \theta_1 + \theta_i
 \end{aligned} \tag{3.29}$$

For giving more explanations and details, the case of two indicators ($p = 2$) will be discussed next. Based on the assumptions, the coefficient λ is a (2×1) vector with the first element set to one ($\lambda_1 = 1$) according to Giles and Tedds [2002] convention. As it was mentioned, any other element either from the vector λ or the vector γ (the coefficients of the causal variables) can be initialized to 1 ($\lambda_i = 1$ or $\gamma_j = 1$).

The restricted reduced form equations for two indicators and "q" causes, with their coefficients and term errors can be written as equations (3.30) and (3.31):

$$y_t = \Pi x_t + v_t \tag{3.30}$$

$$\lambda = \begin{bmatrix} 1 \\ \lambda_2 \end{bmatrix} \Rightarrow \Pi = \lambda \times \gamma' = \begin{bmatrix} \gamma' \\ \lambda_2 \gamma' \end{bmatrix}; \quad \Theta = \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix} \tag{3.31}$$

With the condition $q \geq 2$, the reduced form will be restricted by the model. Writing the restricted reduced form equations in full gives (equations (3.32) and (3.33)):

$$y_{1t} = \gamma' x_t + v_{1t} \quad (3.32)$$

$$y_{2t} = \lambda_2 \gamma' x_t + v_{2t} \quad (3.33)$$

The variance of the term error v_t would be (equation (3.34)):

$$\text{var} \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix} = \begin{bmatrix} \Psi + \theta_1 & \lambda_2 \Psi \\ \lambda_2 \Psi & \lambda_2^2 \Psi + \theta_2 \end{bmatrix} \quad (3.34)$$

It is clear that " $2q$ " elements in " Π " must be obtained as well as " 3 " more elements from the variance of the term error " v_t " in the reduced form equation. Totally, " $2q + 3$ " elements must be obtained. However, the reduced form equation has " $q + 4$ " parameters ($\gamma, \lambda_2, \Psi, \theta_1, \theta_2$).

As can be seen, the coefficients of the variable x_t in equations (3.32) and (3.33) have a constant ratio. Since λ_2 is obtained by solving the reduced form regression in equation (3.30), there is no additional restriction on the variances, so there are three distinct elements and three parameters (Ψ, θ_1 and θ_2) in the variance matrix.

Regarding the standard econometrics form of a Simultaneous Equations Model, the equation (3.35) is obtained by multiplying the equation (3.32) by λ_2 and subtracting the result from equation (3.33):

$$\begin{cases} y_{2t} = \lambda_2 \gamma' x_t + v_{2t} \\ (y_{1t} = \gamma' x_t + v_{1t}) \times \lambda_2 \end{cases} \Rightarrow \begin{cases} y_{2t} = \lambda_2 \gamma' x_t + v_{2t} \\ \lambda_2 y_{1t} = \lambda_2 \gamma' x_t + \lambda_2 v_{1t} \end{cases} \xrightarrow{\text{Subtracting}} \quad (3.35)$$

$$y_{2t} - \lambda_2 y_{1t} = v_{2t} - \lambda_2 v_{1t} = u_t(\text{say})$$

The equation (3.35) can be rewritten as (equation (3.37)):

$$y_{2t} = \lambda_2 y_{1t} + u_t \quad (3.36)$$

The two equations (3.37) and (3.32) form a simultaneous set of two linear equations with two endogenous variables and q exogenous variables:

$$y_{2t} = \lambda_2 y_{1t} + u_t \quad (3.37)$$

$$y_{1t} = \gamma' x_t + v_{1t} \quad (3.32)$$

The covariance matrix between u_t and v_{1t} is unrestricted and can be written as (equation (3.38)):

$$\text{var} \begin{bmatrix} u_t \\ v_{1t} \end{bmatrix} = \text{var} \begin{bmatrix} y_{2t} - \lambda_2 v_{1t} \\ v_{1t} \end{bmatrix} = \begin{bmatrix} \lambda_2^2 \theta_1 + \theta_2 & -\lambda_2 \theta_1 \\ -\lambda_2 \theta_1 & \Psi + \theta_1 \end{bmatrix} \quad (3.38)$$

This matrix has three free elements and determines three parameters $(\Psi, \theta_1, \theta_2)$ while λ_2 is determined as a coefficient in the main reduced form regression. The equations (3.37) and (3.32) are a two-equation linear simultaneous model, with two endogenous variables and q exogenous variables. In fact, maximum likelihood defines the standard econometric procedure of LIML on equation (3.37), because the equation (3.32) is already in reduced form, and the covariance matrix between u_t and v_{1t} is unrestricted⁷.

Thus, the indicator and causal variables of the MIMIC model match exactly the endogenous and exogenous variables of econometrics. The measurement equations in the MIMIC model define the structural relationship by which the endogenous variables are jointly determined in the model. Also, the variance parameters in the MIMIC model can be solved from the following econometrics model equations (3.39)-(3.41):

$$\theta_1 = \frac{-\text{cov}(u_{it}, v_{1t})}{\lambda_i} \quad (3.39)$$

$$\theta_i = \text{var}(u_{it}) - \lambda_i^2 \theta_1 \quad (3.40)$$

$$\Psi = \text{var}(v_{1t}) - \theta_1 \quad (3.41)$$

⁷ Limited Information Maximum Likelihood (LIML) recognizes the structure of one equation of a system, while treating the other equations in their reduced form and ignoring any covariance matrix restrictions. It therefore uses the same information about the structure of the model as two-stage least squares, to which it is asymptotically equivalent. With more than two indicator variables in the MIMIC model, maximum likelihood estimation is not simply LIML, because the restrictions on the covariance matrix would be ignored in LIML [Breusch, 2005].

3.4.5 MIMIC Conclusion

For an acceptable estimation using MIMIC model, the variances expressed above must be positive. On the other hand, LIML estimation does not impose non-negativity condition on the solution, and there are possible examples with implied negative variances. In case of having two indicator variables, at most one of the implied variances can be negative.

It is noticeable that LISREL or Amos ⁸ packages do not restrict economists because of negative variance. However, a warning message appears when the estimated variance matrix is not positive.

Explaining the MIMIC model as simultaneous equations, there is no need to use the famous software in econometrics for estimations. The easiest and most perceptive way to apply LIML estimation is to identify its equivalence to iterated Generalized Least Squares ⁹ (or Aitken) estimation for apparently unrelated regressions (see, Pagan [1984] and Breusch [2005]).

The GLS procedure is a well-known available module (or commend) with several applications in econometrics, but the implementation of the GLS procedure may be problematic. The iterated GLS estimates the coefficients of the two main equations in (3.37) and (3.32) and maybe the variance matrix in equation (3.38). The estimation of other parameters in MIMIC model can be done easily; by calculating the variances v_t and u_t in equation (3.38), the variances (Ψ, θ_1 and θ_2) can be computed by substituting the GLS variances and coefficient estimates into equations (3.39)-(3.41).

The main application of the MIMIC model in this work is to extract the latent variable η_t . Then, the estimated size of the underground economy will be related to the predicted value of the indicator variable (the one which is normalized to have unit coefficient in λ) expressed as equation (3.32):

$$E(\eta_t|x_t) = E(y_{1t}|x_t) = \gamma'x_t \quad (3.42)$$

The estimation will be performed via LIML or GLS procedures using the restricted reduced form. The MIMIC method establishes a proportionality relationship between the vectors of coefficients in the two reduced form equations. Thus, the coefficients of indicator variable are

⁸ Two packages which are used to determine the elements of the structural equation modeling

⁹ Generalized least squares (GLS) is a statistic method used to estimate the unknown parameters of a linear regression model. The GLS method can be applied either when there is a definite degree of correlation between the observations or when the variances of the observations are not equal. In these conditions, using ordinary least squares may be statistically faulty, or even give inaccurate conclusions.

related together. In other words, by rescaling the prediction of the indicator variable on which the normalization is made, the indicator variable is estimated and the proportionality factor is the estimate of λ_2 . Apart from the change in scale, as the same estimates are defined, it does not matter principally which indicator variable is chosen for normalization in the variance of MLE ¹⁰. But, the two main consequences of the normalization should be considered: the practical issue and its effect on the interpretation of the results [Breusch, 2005].

When estimated by OLS, one unrestricted reduced form equation may fit the data markedly better than the other. It means that in the language of instrumental variables, the exogenous variables may be much practical for one of the endogenous variables than the other. In that case, the restricted LIML estimates of the reduced form coefficients will more closely resemble the unrestricted OLS estimates of the equation with the higher R-squared. Then the estimated latent variable will be similar to the unrestricted OLS prediction from the reduced form equation which best fits, perhaps scaled by λ_2 if it is necessary to normalize on the other indicator variable. It is noticeable that if the reduced form equations have very different fits by OLS, the iterations will converge faster and more reliably if the model is normalized on the indicator (endogenous) variable with the highest R-squared [Hamilton and Waggoner, 2005]. In MIMIC model, the normalization of the model may affect the interpretation of the results. In the standard assumption of $\lambda_1 = 1$ for the model of equations (3.13) and (3.14), the latent variable is linked to the first-listed indicator variable by the normalization. Reordering the variables will switch another variable to become the normalizing indicator and hence it will rescale the latent variable. So, there is a degree of indeterminacy in scale, which needs to be resolved if the latent variable is to be interpreted as an estimate of the underground economy [Hamilton and Waggoner, 2005].

¹⁰ Maximum Likelihood Estimation

3.5 Summary

In this chapter,:

1. A comprehensive review on three types of methods developed over time for measuring the growth and size of the underground economy was provided. The conditions and constraints of each method as well as their strengths and weaknesses were emphasized. It is obvious that these specifications must be taken into account when interpreting and comparing the final results.
2. Direct methods were discussed: these methods build up the estimates of the size of the underground economy based on the results of surveys, questionnaires, interviews and tax auditions of firms and/or households. These methods only yield estimates for a specific point in time, they do not provide a time series and can be regarded as low-end estimates. Also, it seems that they cannot be applied to more than two or three years before, since the people who participate or answer the questionnaires cannot answer correctly even if they want to do well as it is possible that they forget the exact answers. From other side, the difficulty in selecting the appropriate samples, the possibility of selection bias and measurement errors regarding the surveys are among the defects of direct methods [Albu, 1995; Elgin and Oztunali, 2012].
3. Indirect or macroeconomic methods as more appropriate ones compared to direct methods, were discussed: these methods focus on the discrepancy between the official and actual criteria such as differences between the national income and expenditure, transactions and national income, and electricity consumption and GDP. These methods are generally criticized for employing various simplifying and limiting assumptions, such as focusing on a specific aspect or indicator and neglecting the other causes which may also have a predominant effect on the underground economy [Elgin and Oztunali, 2012; Tahmasebi and Rocca, 2015]. The outcome of the indirect methods is a time series of the underground economy while the outcome of the direct methods is an estimation of the underground economy in a specific year which is considered as the minimum possible size of the underground economy [Tahmasebi and Rocca, 2015].
4. The most popular method used to estimate the underground economy, Multiple In-

dicators Multiple Causes (MIMIC) method, was presented next. MIMIC approach as a complex structural equation model, considers some causes and indicators of the underground economy (for which the data are available) and estimates the dependencies among these variables. The results will be further used to determine the size of the underground sector as a time series. Similar to two other methods, this approach is based on the use of simplified econometric conditions, making it vulnerable to statistical errors. Another deficiency of this approach is that it does not rely on any micro-foundations [Breusch, 2005]. Schneider [2003] considers some of the shortcomings of the MIMIC model and states that:

Firstly, the estimated coefficients of the two sets of the equations (the equations between the latent variable (the underground economy) and the causes, and the equations between the latent variable and the indicators) may become unstable if the sample size changes ¹¹. Also, by changing the causal variables and/or the period of estimation, the final results may change as well.

Secondly, inferring the accurate data for causal variables, except for the tax burden, is quite challenging.

It means that although the MIMIC method can be considered as a comprehensive model for the estimation of the underground economy, it requires a considerable amount of data which is not easily available [Greenidge et al., 2009]. Saunders [2008] also agrees with Schneider and admits that any small changes in the specifications of this model may affect significantly the final results. Thus, it is not surprising if for a specific country different estimates are obtained by two distinct researchers [Breusch, 2005]. However, it can be claimed that MIMIC is still the most used method to estimate the underground economy.

It is now evident that we deal with an economic phenomenon with vague initial conditions, some unclear or unknown activities, not enough clear causes and ambiguous relation between the causes and this economic phenomenon. Besides, as it was explained in this chapter, the estimation methods commonly used to measure this economic phenomenon are not free from

¹¹ It must be considered that the best sample size to obtain reliable results is not clear even if we consider that it is the only shortcoming of this method

some errors, shortcomings and weaknesses. So, the need for another methods which cover these defects is evident. An eventual method for this purpose should satisfy the following criteria :

1. The method should address the vagueness of the causes and the economic phenomenon;
2. The uncertainty of the initial data must be considered and addressed in the method;
3. The method should not be sensitive to the sample size;
4. The method should not be sensitive to the data specifications;
5. The method should be free of strong assumptions as much as possible;
6. The obtained results should be reliable; and,
7. The method should be simple, smart and easy to understand.

These are some specifications which are determined according to the weaknesses and strengths of the existing estimation methods. It may seem that some of them are so idealistically considered, but they help to choose the best method among the possible methods.

In the following chapters, we can see how these specifications are consistent with the features of the fuzzy methods.

Chapter 4

Fuzzy Concepts and Logic

Introduction

A general introduction to the underground economy including the definitions, causes and indicators of the underground economy, its segmentations and actors as well as a macroeconomic model was provided earlier. The main issue concerning the underground economy is that the participants in the underground markets try to not to be recognized. Another issue is that the causes and the indicators of the underground economy and some other specifications which specially belong to it are not clear enough.

In chapter 3, three main categories of methods used to estimate the underground economy were fully introduced. Direct or microeconomic methods determine the size of the underground economy in a specific year (the result is not a time series) and are completely influenced by the honesty of the people who are selected to get the data from. But, since some people do not want to be recognized, they may conceal some facts (the main issue stated above). Indirect or macroeconomic methods determine the size of the underground economy applying only one indicator. But it seems that applying just one indicator is not so effective. Another critique about this method is using some simplifying and strong assumptions.[Albu, 1995; Elgin and Oztunali, 2012].

The MIMIC method, the most famous method for this estimation, uses both the causes and the indicators to determine the size of the underground economy as a latent variable. Quantity

and quality of the sample size, distribution assumptions and data dimension are some of the critiques of this method [Breusch, 2005; Schneider, 2003].

Many systems are naturally fuzzy due to the uncertainty in their initial conditions and parameters [Ratiu et al., 2009]. The concept of fuzzy and its method can be applied in many cases such as an insufficient number of observations or samples; an unclear relationship between response and explanatory variables; difficulties verifying distribution assumptions; the ambiguity of events or the degree to which they occur; and inaccuracy and distortion due to linearity assumptions [Ross, 2009]. The criteria cited here correspond very closely to the initial conditions of the underground economy, offering further encouragement for using fuzzy logic to model and estimate the underground economy.

Before applying the fuzzy concept to estimate and model the underground economy, in this chapter, we review some main definitions and concepts which form the basis of the proposed methods in this thesis.

4.1 Fuzzy Concept

"It is the mark of an instructed mind to rest satisfied with that degree of precision which the nature of the subject admits, and not to seek exactness where only an approximation of the truth is possible."

Aristotle, 384-322 BC

"Precision is not truth."

Henri Matisse, 1869-1954

"All traditional logic habitually assumes that precise symbols are being employed. It is therefore not applicable to this terrestrial life but only to an imagined celestial existence."

Bertrand Russell, 1923

"We must exploit our tolerance for imprecision."

Lotfi Zadeh

Professor, Systems Engineering, UC Berkeley, 1973

All these legendary quotes above share a common thread that illustrates the relationship between precision and uncertainty. The more complex or uncertain a system is, the less precise or exact is the information provided to characterize it. It can be claimed, then, that the information, complexity and precision are interrelated in the problems that an eventual solution is required. To better face this challenge, Professor Lotfi Zadeh suggests to accept some level of imprecision for most of the problems which exist and he introduced the Fuzzy logic. In other words, a balance must be maintained between the degree of precision in a problem and its associated uncertainty.

Fuzzy logic is a reasoning approach based on "degrees of truth" rather than the usual "true or false" (1 or 0) Boolean logic. This logic not only includes 0 and 1 as the extreme cases of truth, but also includes various states of truth from 'almost certain' to 'very unlikely'. Dr. Lotfi Zadeh of the University of California at Berkeley come up with the idea of fuzzy logic for the first time in the 1965s when working on the problem of computer understanding of natural language. To make it clear, the statement, today is sunny, might be 100% true if there are no clouds, 80% true if there are a few clouds, 50% true if it's hazy and 0% true if it rains all day. Fuzzy logic is believed also to be closer to the way the brains work.

4.1.1 Why use fuzzy logic?

When conventional logic fails to find a solution, fuzzy logic comes in. Fuzzy concept with its fuzzy rules and membership functions estimates any continuous function to a degree of precision. The idea behind fuzzy logic is to resemble human decision making use of natural language terms instead of quantitative terms. In other words, this approach exploits linguistic variables which are the variables whose values are words or sentences in natural language. Any relation between two linguistic variables can be expressed in terms of fuzzy if-then rules which will be discussed later. Moreover, fuzzy logic provides a better alternative solutions to simulate or control nonlinear systems including most real-life physical systems.

Fuzzy logic offers several unique features which individualize this method from other method: [Ross, 2009]

- Fuzzy logic is based on natural language which the basis for human communication;
- The concept of fuzzy logic is easy to understand; fuzzy reasoning applies simple

mathematical concepts. Moreover, since fuzzy logic applies the qualitative description used in everyday language, it is easy to use.

- Fuzzy logic manipulates imprecise data; it can handle and model those problems in which imprecise and incomplete data must be used.
- Fuzzy logic has been shown to be able to approximate or model any nonlinear functions of arbitrary complexity; it is possible to create a fuzzy system to match any set of input-output data.
- Fuzzy logic can be used in many cases with conventional control techniques to augment them and simplify their implementation.

4.1.2 Why fuzzy economy?

It was stated that many dynamic economic systems naturally become fuzzy due to the uncertain initial conditions and parameters [Ratiu et al., 2009]; so, fuzzy sets and fuzzy logic and their combination with conventional methods offer a useful approach in different domains like engineering sector, economic events, bioinformatics and computational biology and other fields of science.

The study of some economic systems with fuzzy-logic based representation of the economic agent's behavior has presented a field named fuzzy economics. The fuzzy economics is defined as human-centric and is closer to the reality [Ratiu et al., 2009].

Most of economic events are unobservable; their causes are often vague and there is a very limited knowledge about the nature of the relationships between these causes and the economic event. Collecting reliable raw data is also challenging and poses some problems in modeling these economic events. From other side, fuzzy logic, specifically fuzzy modeling or fuzzy system, allows for rapid modeling, even with imprecise and incomplete data as it is not necessary to know all the details about the relations before starting. Using linguistic terms to relate input and output variables make the fuzzy rules easy to understand. Also, it can model nonlinear functions of arbitrary complexity and provide simplicity and flexibility [Yu et al., 2006].

Chang [1977] believes that fuzzy is a useful concept to be applied in economy and can be used in three different areas: economic modeling, economic forecasting and economic policy. He explains that *"In modeling it gives expression to the fact that after structural equations are obtained from theoretical considerations, the parameters cannot be estimated accurately. In forecasting a solution is given to the multiple model problem. In economic policy, it offers a range of optimum policies depending on a risk parameter and leaves the final policy choice to the decision maker"*.

Yu et al. [2006] considers two advantages for applying fuzzy logic to model economic events: fuzzy logic is free of complex calculations in conventional econometric models and fuzzy rules with linguistic terms are easy for human to understand.

Ponsard [1995] confirms the above reasons and explains why economists should consider the economic events from a fuzzy view:

1. *"economic spaces where men live (regions, market areas, urban influence zones, etc.) are imprecise; they possess "more or less" constituent characteristics and they are not partitioned into very homogeneous and well delineated sub-spaces."*
2. *"economic agents pursue vague and sometimes incompatible objectives ¹; the constraints which limit their means are elastic, either because they are imperfectly appraised, or because they are objectively fuzzy."*

[Aliev, 2009] states also that an economic system can be considered as a *"human centric and imperfect information based realistic multi-agent system with fuzzy-logic-based representation of the economic agents' behavior and with imprecise constraints."* and explains that the best way to model the human (or agent) behavior is using the fuzzy concept.

On the whole, the initial conditions of economic events (specially the underground economy in this research) are very close to initial conditions of applying fuzzy logic and the simplicity it offers encourages the researchers to apply fuzzy logic to model the economic events.

¹ Barthélemy [1997] also considers the same problem in economic modeling in his research.

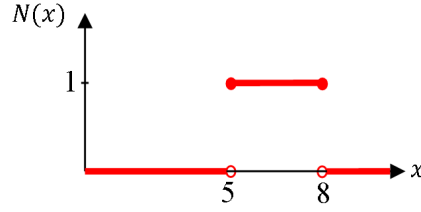


Figure 4.1: The classical set C in which the degree of membership of any object is either 0 or 1.

4.2 Fuzzy Logic

4.2.1 Crisp and fuzzy set

Crisp or classical bivalent set C is a set wherein the degree of the membership function of any object in the set is either 0 or 1. For instance, if N maps the elements of the reference set M to the set 0, 1 as follows:

$$N(x) = \begin{cases} 1 & \text{if } x \text{ is an element of set } M \\ 0 & \text{if } x \text{ is not an element of set } M \end{cases}$$

Then, the classical set C will be: $C = \{(x, N(x)) | x \in M\}$ which is depicted in Figure 4.1.

A fuzzy set F is characterized by a membership function μ_F ² which takes values in the interval $[0, 1]$ namely $\mu_F : M \rightarrow [0, 1]$. On the whole, a fuzzy set F in a continuous universe M may be represented as a pair (M, μ_F) . Let $x \in M$, then:

$$F = \{(x, \mu_F(x)) | x \in M\}$$

Membership function $\mu_F(x)$ can take any value between 0 and 1; the closer the value of $\mu_F(x)$ is to unity, x has the higher grade of membership in fuzzy set F .

x is called a fuzzy member if $0 \leq \mu_F(x) \leq 1$; x is not included or is not a member in the fuzzy set if $\mu_F(x) = 0.0$, x is called fully included or full member if $\mu_F(x) = 1$ and x is called a graded member if $\mu_F(x) = 0.5$.

² The membership function can be represented with various symbols ($\mu_F(x)$, $\mu(x)$ and μ_F), but all of them share the same meaning.

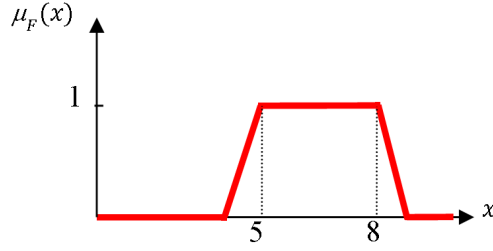


Figure 4.2: The fuzzy set F whose membership function can take a value in the interval $[0, 1]$.

It is good to know that a fuzzy set has two properties, it is normal and convex:

$$\text{Normal} : \max_{m \in M} \mu_F(m) = 1$$

$$\text{Convex} : \mu_F(\lambda m_1 + (1 - \lambda)m_2) \geq \min(\mu_F(m_1), \mu_F(m_2)) \quad m_1, m_2 \in M \quad \lambda \in [0, 1]$$

These properties will be discussed completely later. In short, in the classical set theory, when M is a set in the ordinary sense of the term, its membership function only takes on two values 0, 1, but about fuzzy sets, the membership function can take a value in the interval $[0, 1]$ as depicted in Figure (4.2).

4.2.2 Linguistic Variables

A fuzzy set is concerned with linguistic concepts and variables. As an example, price is a concept and expensive is a linguistic variable. Consider that the price of a personal computer is 25\$ (The compute which is named Raspberry Pi). When it is asked whether it is expensive? A yes or no answer is a Boolean dependency (0 or 1, yes or no and etc) while another possible answer is "it is not expensive and really cheap". In this case, a fuzzy set called "expensive computer" can be defined which its dependency can be mapped to $[0, 1]$. The dependency is 0 when a personal computer costs less than 5\$ and it is 1 when it costs for example more than 1, 500\$. The mentioned dependency is called the membership function of "expensive computer" as a fuzzy set which is illustrated in Figure (4.3).

Note that the membership function should not be confused with probability. Rather than a probability, the magnitude of membership indicates the level of association of an entity with the given concept and precisely, with the given fuzzy set. Moreover, the degree of membership of a set does not necessarily sum to unity.

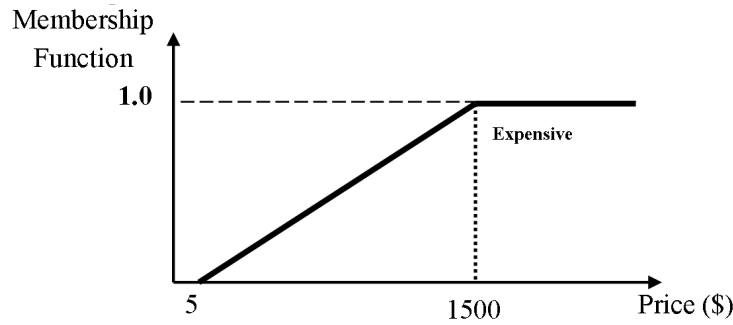


Figure 4.3: The membership function of "expensive computer" example.

The use of fuzzy sets provides a systematic way to manipulate ambiguous and imprecise concepts. In particular, fuzzy sets are applied to define linguistic variables. A linguistic variable can be considered as a variable whose values are defined in linguistic terms.

Briefly, a linguistic variable (X) is presented by a quintuple $X = (x, T(x), M, G, H)$ where:

X : name of the linguistic variable;

x : represents the input variable;

$T(x)$: set of linguistic terms for input variables.

For every input variable x , the general form to represent $T(x)$ with k linguistic terms can be described as:

$$T(x) = \{T_x^1, T_x^2, T_x^3, \dots, T_x^k\}$$

Where T_x^i represents the i^{th} linguistic term for a range of input variable x . For example, "Fuzzy Speed" is defined as a linguistic variable and its term set $T(x)$ could be:

$T(x) = \{veryslow, slow, moderate, fast, veryfast\}$. Each term in $T(x)$ is characterized with a fuzzy set in a universe of discourse $M = [0, 100]$.

G : a syntactic rule for generating the linguistic terms for each x ;

H : a semantic rule for associating each term with its meaning.

It is possible that slow is defined as the speed below $40Km/h$, moderate as the speed close to $55Km/h$ and fast as the speed above $70Km/h$. These linguistic terms and their membership functions are shown in Figure (4.4) [Lee, 1990].

A complete discussion about membership function is provided in next sections.

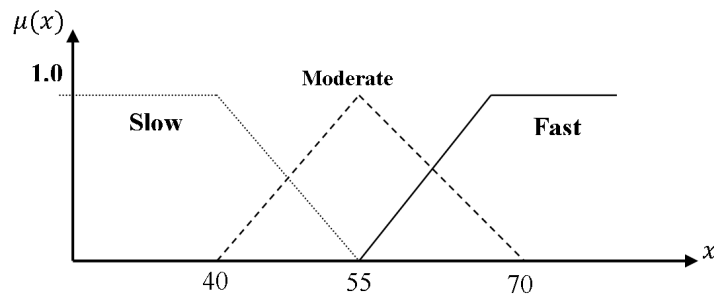


Figure 4.4: Definition of a linguistic variable called "fuzzy speed".

4.2.3 Membership function

The membership function of a fuzzy set contains all the information of that set. Figure (4.5) depicts the regions in the universe making the membership function of a typical fuzzy set. As it can be seen in Figure (4.5), for a typical fuzzy set A, the *core* of the membership function is that region of the universe which is characterized by full membership in the set A. That is the core covers those elements x of the universe such that $\mu_A(x) = 1$.

For the same fuzzy set A, the *support* of the membership function is that region of the universe which is characterized by nonzero membership in the set A. That is the support covers those elements x of the universe such that $\mu_A(x) > 0$.

For the same fuzzy set A, the boundaries of the membership function are that region of the

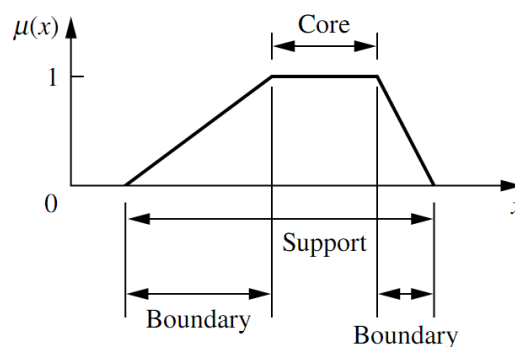


Figure 4.5: Core, support, and boundaries of a typical fuzzy set.

universe containing elements that have a nonzero membership but not complete membership. That is the boundaries covers those elements x of the universe such that $0 < \mu_A(x) < 1$. These elements of the universe are those with some degree of fuzziness, or only partial membership in the fuzzy set A.

A normal fuzzy set has a membership function with at least one element x in the universe

whose membership equals to one. If in a fuzzy set, the membership of one and only one element is unity, this element is commonly referred to as the prototype of the set or the prototypical element. Figure (4.6) depicts typical normal and subnormal fuzzy sets. A

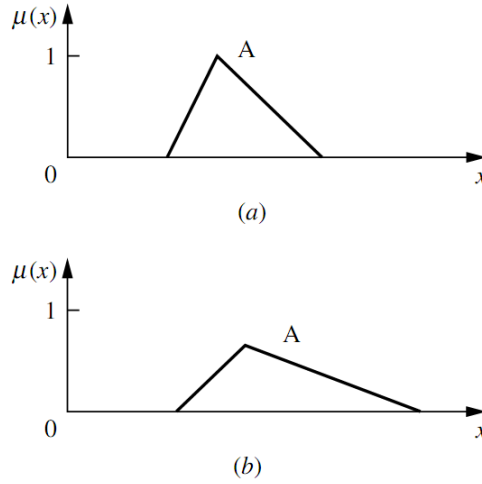


Figure 4.6: Fuzzy sets that are normal (a) and subnormal (b).

convex fuzzy set is described by a membership function whose membership values are strictly monotonically increasing or monotonically decreasing or initially monotonically increasing then monotonically decreasing with the increase in the values of the elements of that particular fuzzy set. It is notable that this definition of convexity is not the same as some definitions of this term in mathematics. Equation (4.1) summarizes the considered definition of convexity. It implies that for any elements x, y , and z in a fuzzy set A, if the relation $x < y < z$ implies that:

$$\mu_A(y) \geq \min[\mu_A(x), \mu_A(z)] \quad (4.1)$$

then A would be a convex fuzzy set [Ross, 1995]. Figure (4.7) illustrates a typical convex fuzzy set and a typical non convex fuzzy set. The intersection of two typical convex fuzzy sets, say A and B, is also a convex fuzzy set, as shown in Figure (4.8). That is, for A and B, which are both convex, $A \cap B$ is also convex.

The *crossover points* of a membership function are defined as the elements of a particular fuzzy set A in the universe for which: $\mu_A(x) = 0.5$.

The height of a fuzzy set A is the maximum value of the membership function, i.e., $hgt(A) = \max(\mu_A(x))$. If the $hgt(A) < 1$, the fuzzy set would be subnormal. The $hgt(A)$ can be

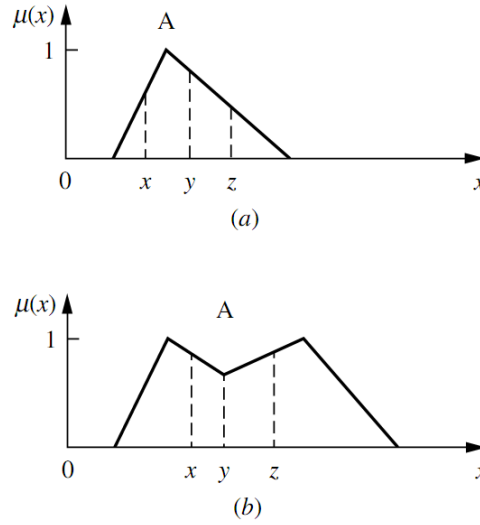


Figure 4.7: Convex, normal fuzzy set (a) and non convex, normal fuzzy set (b).

considered as the degree of validity of information [Klir and Wierman, 1999].

If A is a convex single-point normal fuzzy set defined on the real line, then A is said to be a fuzzy number.

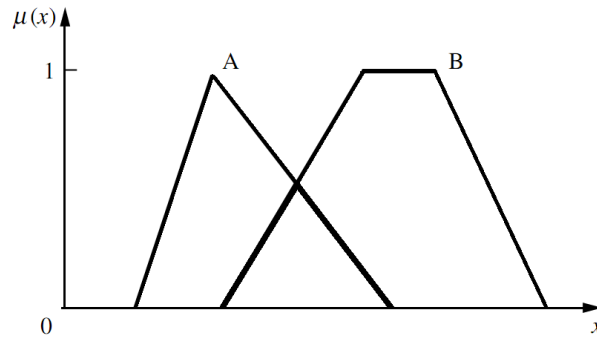


Figure 4.8: The intersection of two convex fuzzy sets produces a convex fuzzy set.

4.2.4 Various form of membership function

Membership functions are mostly normal and convex in form. However, subnormal and non convex fuzzy sets exist also due to many operations on fuzzy sets, hence on membership functions. For example, the extension principle and the union operator both can result in subnormal or non convex fuzzy sets.

Membership functions are either symmetrical or asymmetrical. They are normally de-

defined on one-dimensional universes, but also can be described on multidimensional (or n -dimensional) universes. For example, the membership functions presented in this section are one-dimensional curves, they become surfaces in two dimensions and hyper surfaces for three or more dimensions. These hyper surfaces, or curves, are simple mappings from combinations of the parameters in the n -dimensional space to a membership value on the interval $[0, 1]$. This membership value still expresses the degree of membership that the specific combination of parameters in the n -dimensional space has in a particular fuzzy set defined on the n -dimensional universe of discourse.

Although normal and subnormal fuzzy sets (Figure (4.6)) that are described by ordinary membership functions are the most common cases in practice, another types of fuzzy membership functions have been also proposed by Klir and Wierman [1999] as generalized membership functions. The idea is that the values used in developing ordinary membership functions are often excessively precise. That is, a specific membership value, $\mu(x)$, must be assigned to each element of the universe x on which the fuzzy set A is defined. The level of information is, hence, not adequate to specify membership functions with this precision.

For example, for a typical fuzzy set, only the upper and lower bounds of membership grades may be specified for each element of the universe. Such a fuzzy set can be described by an interval-valued membership function as illustrated in Figure (4.9). As can be seen in this Figure, the membership of a particular element, $x = z$, in the fuzzy set A , i.e., $\mu(z)$, can be expressed by the membership interval $[\alpha_1, \alpha_2]$. Interval-valued fuzzy sets can be generalized further by allowing their intervals to become fuzzy. Each membership interval then becomes an ordinary fuzzy set. This type of membership function is referred to in the literature as a type-2 fuzzy set. Other generalizations of the fuzzy membership functions are also available Klir and Wierman [1999].

4.2.5 Fuzzy intersection and union

Fuzzy intersection and union can be specified by three commonly-used groups of rules named Standard, Algebraic and Bounded [Klir and Yuan, 1995], as shown in Table 4.1. These rules are acceptable as they satisfy certain essential requirements such as the boundary conditions.

It is observed that the largest (the most loose or the weakest) intersection is provided by

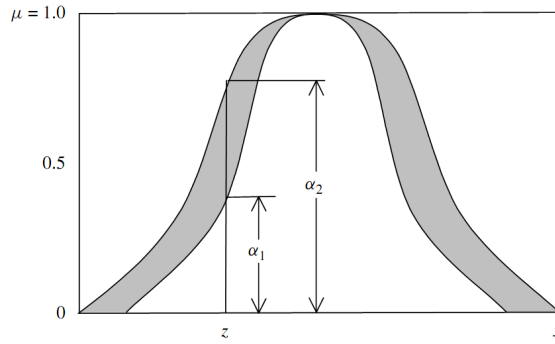


Figure 4.9: An interval-valued membership function.

Table 4.1: Some basic forms of fuzzy operations

Type of operation	Intersection $i(a, b) = a \cap b$	Union $u(a, b) = a \cup b$
Standard	$\min(a, b) = i_{\max}$	$\max(a, b) = u_{\min}$
Algebraic	$a * b$	$a + b - a * b$
Bounded	$\max(0, a + b - 1)$	$\min(1, a + b)$

Standard fuzzy operations among all the permitted forms; i.e., all other forms give a smaller, or at least no larger, value for the intersection. By contrast, the Standard form gives the smallest (the most tight or the strongest) union among these forms. That is the reason for labeling these operations as i_{\max} and u_{\min} in Table 4.1. It is notable that among all permitted forms, the Standard fuzzy intersection and union are the only ones which satisfy the intuitively and substantively desirable condition of "idempotency", namely: $i(a, a) = a$ and $u(a, a) = a$. Consequently, it can be considered that the operator *AND* is the intersection of the sets and the operator *OR* is the union of them. Let A and B be two fuzzy sets in M with membership functions $\mu_A(x)$ and $\mu_B(x)$, respectively. The Standard set theoretic operations of union and intersection for fuzzy sets are defined via their membership functions as follows:

- The membership function for the standard fuzzy *OR*-operator ($\mu_{A \cup B}$, which is called the membership function of " $A \text{ OR } B$ ") is defined as the maximum membership function of the common elements of A and B ($\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$).
- The membership function for the standard fuzzy *AND*-operator ($\mu_{A \cap B}$, which is called the membership function of " $A \text{ AND } B$ ") is defined as the minimum membership function of the common elements of A and B ($\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$).

4.3 Fuzzy modeling

A fuzzy model commonly is comprised of three general steps: fuzzification, fuzzy inference unit (decision making logic), and defuzzification [Von Altrock, 1996]. Figure (4.10) illustrates how these units are related.

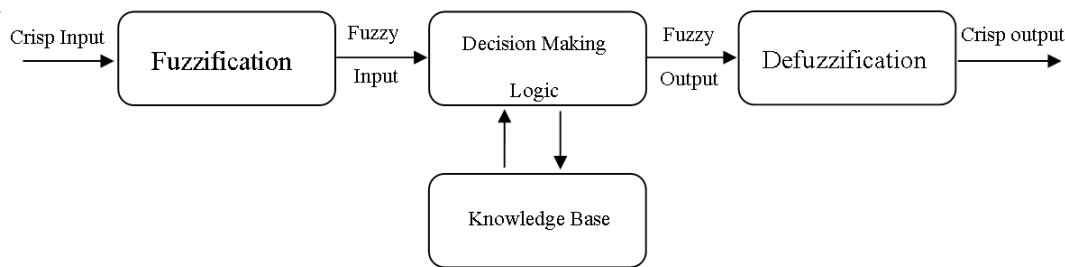


Figure 4.10: General steps in a typical fuzzy model.

- In the fuzzification unit, a fuzzifier receives the current crisp values of input variables and convert them into relevant membership functions using linguistic terms;
- Decision making unit, also called fuzzy rule-base unit, performs inference operation on the fuzzy data based on fuzzy rules;
- In defuzzification unit, a defuzzifier converts fuzzy results of inference unit into a crisp output using a suitable transformation method.

Ross [2004] demonstrates that the knowledge base (also called fuzzy inference engine) consists of two subsections called data base and rule base:

- A data base contains the information about the membership functions of fuzzy sets used in fuzzy rules, the domains of variables and types of normalization;
- A rule base contains a number of fuzzy If-Then rules.

These three steps of fuzzy method are discussed thoroughly next.

4.3.1 Fuzzification

Fuzzification is the process of transforming crisp values into fuzzy terms quantified by fuzzy membership function. It can be done based on the recognition that many quantities that are considered to be crisp and deterministic are not deterministic at all, but they are uncertain to some extents. If the uncertainty is a result of imprecision, ambiguity, or vagueness, then it can be concluded that the variable is probably fuzzy. In the real world, any hardware such as a digital thermometer generates crisp data, but these data are subject to experimental error. Figure (4.11) demonstrates a possible range of errors for a typical temperature reading and the associated membership function that might represent such imprecision.

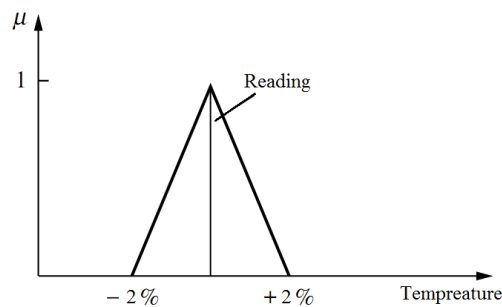


Figure 4.11: Membership function representing imprecision in "crisp temperature reading".

When using those data in fuzzy systems, it is useful to represent them as fuzzy sets but not mandatory. This idea is depicted in Figure (4.12): the data are considered to be crisp in Figure (4.12a), or fuzzy in Figure (4.12b).

In Figure (4.12a), it is shown that the intersection of the fuzzy set "low temperature" and the crisp temperature reading occurs at a membership of 0.35 which means that the fuzzy set and crisp reading agree at a membership value of 0.35. In Figure (4.12b), the fuzzied reading intersects the fuzzy set "medium temperature" at a membership of 0.5. Moreover, in Figure (4.12b), the set intersection of the two fuzzy sets is a small triangle, whose largest membership occurs at the membership value of 0.5.

4.3.2 Fuzzy Rule and Fuzzy Inference

Applying algebraic expressions to merge data provides better visualization, understanding and interpretation of data relations. Data modeling is a good option to merge the data. It is

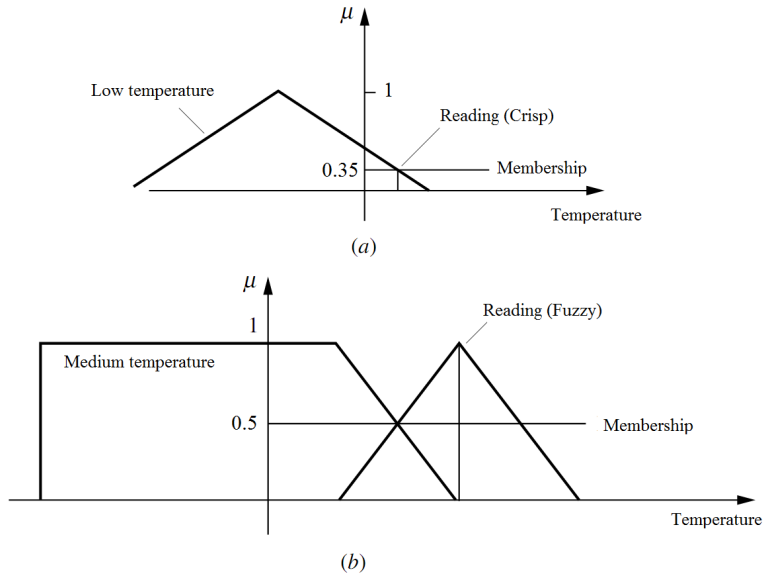


Figure 4.12: Comparisons of fuzzy sets and crisp or fuzzy readings: (a) fuzzy set and crisp reading; (b) fuzzy set and fuzzy reading.

defined as an attempt to represent the available information from a set of data in a clear way [Lambert-Torres and da Silva, 2003]. Data modeling is a very first stage for generating a fuzzy statement, mainly in large databases, and helps the operator in a decision-making process. A fuzzy statement can be defined as an attribution of a linguistic term, either single term or a composed term (with two or more single terms that are combined by linguistic conjunctions), to a variable. The general form of a fuzzy statement is: " x_i is T_{x_i} ", where T_{x_i} represents the linguistic term of the variable x_i [Lambert-Torres and da Silva, 2003].

In a fuzzy conditional statement, the premise and/or consequence of the *If – Then* rule is represented by fuzzy statements [Lambert-Torres et al., 1992]. A fuzzy rule can be defined as a conditional statement in the form: *If x is A , Then y is B* , where x is the input variable. A and B are the linguistic terms determined by fuzzy sets on the universe of discourses M and U , respectively.

A typical structure of a decision system in fuzzy conditional statement has multiple input and output variables (MIMO systems) [Lambert-Torres and da Silva, 2003]. Consider a system with p input variables x in the premise of the rules and m output variables y in the consequence. The general form of a fuzzy conditional statement will be:

$$\text{If } \{x_1 \text{ is } T_{x_1}, x_2 \text{ is } T_{x_2}, \dots, x_p \text{ is } T_{x_p}\} \text{ Then } \{y_1 \text{ is } T_{y_1}, y_2 \text{ is } T_{y_2}, \dots, y_m \text{ is } T_{y_m}\}$$

It must be noted that the condition and consequence of *If – Then* rules can be either fuzzy or

crisp numbers, but at least one of them must be fuzzy. Thus, three types of *If – Then* rules can be distinguished:

- Condition is fuzzy and consequence is crisp;
- Condition is crisp and consequence is fuzzy;
- Condition is fuzzy and consequence is fuzzy.

In this research, the latter (If-Then form with fuzzy numbers in condition and consequence) is used.

4.3.3 Defuzzification

There are situations which necessitates the output of a fuzzy process to be a single scalar quantity. Thus, another stage is required to generate specific responses. Defuzzication is the conversion of fuzzy outputs into precise crisp outputs. The output of a fuzzy process can be the logical union of two or more fuzzy membership functions defined on the universe of discourse of the output variable. For example, a fuzzy output may be comprised of two parts: C_1 , a triangle shape shown in Figure (4.13 a), and C_2 , another triangular membership shape shown in Figure (4.13 b). The union of these two membership functions ($C = C_1 \cup C_2$) involves the max operator shown in Figure (4.13 c). This is graphically the outer envelope of these two shapes shown in Figure (4.13 a) and (4.13 b). A general fuzzy output process may involve more than two parts and their membership functions can have shapes other than triangles. Moreover, the membership functions may not always be normal as illustrated in Figure (4.13 a). The fuzzy output can be generally shown as:

$$C_k = C_1 \cup C_2 \cup \dots \cup C_k = C \quad (4.2)$$

Seven common methods proposed for defuzzifying fuzzy output functions (membership functions) in the literature are described next [Hellendoorn and Thomas, 1993].

1. *Max membership principle*: Also known as the height method, is limited to peaked

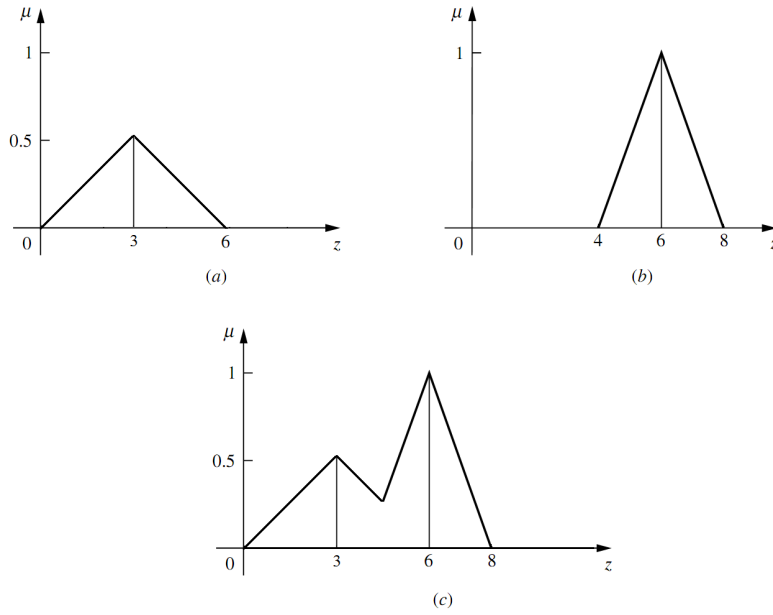


Figure 4.13: Typical fuzzy process output: (a) first part of fuzzy output; (b) second part of fuzzy output; (c) union of both parts.

output functions. This method can be represented by the algebraic expression:

$$\mu_C(z^*) \geq \mu_C(z) \quad \text{for all } z \in Z \quad (4.3)$$

where z^* is the defuzzified value, and is shown graphically in Figure (4.14).

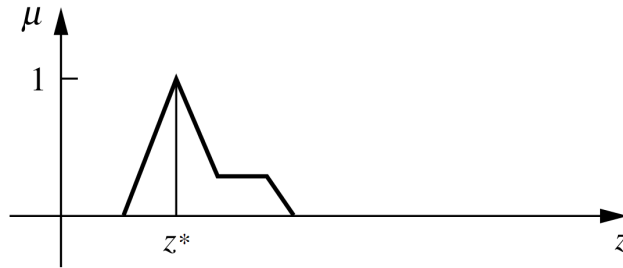


Figure 4.14: Max membership defuzzification method.

2. *Centroid method:* Also called center of area or center of gravity, this method is the most prevalent and physically appealing among all defuzzification methods [Ross, 2004]. It is represented by the algebraic expression:

$$z^* = \frac{\int \mu_C(z) \cdot z \, dz}{\int \mu_C(z) \, dz} \quad (4.4)$$

where \int denotes an algebraic integration. This method is shown in Figure (4.15).

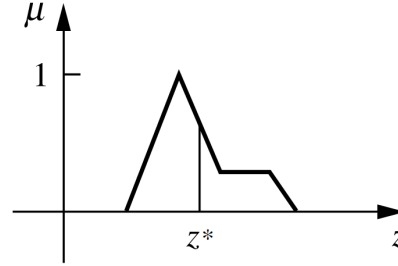


Figure 4.15: Centroid defuzzification method.

3. *Weighted average method*: since this method is one of the most computationally efficient methods, it is frequently used in fuzzy applications. Being limited to symmetrical output membership functions is the only restriction of this method. Instead of using a composite output membership function, this method uses the individual clipped or scaled output fuzzy sets. It is represented by the algebraic expression:

$$z^* = \frac{\sum \mu_C(\bar{z}) \cdot \bar{z}}{\sum \mu_C(\bar{z})} \quad (4.5)$$

where \sum denotes the algebraic sum and z is the centroid of each symmetric membership function. This method is shown in Figure (4.16). The weighted average method is formed by weighting each membership function in the output by its respective maximum membership value. As an example, the two functions shown in Figure (4.16) would result in the following general form for the defuzzied value:

$$z^* = \frac{a(0.5) + b(0.9)}{(0.5) + (0.9)}$$

Since the method is limited to symmetrical membership functions, the values a and b are the means (centroids) of their respective shapes.

4. *Mean max membership*: this method, also called middle-of-maxima, is close to the first method except that the locations of the maximum membership can be non-unique (i.e., the maximum membership can be a plateau rather than a single point). It is represented by the algebraic expression [Ross, 2004, 2009]:

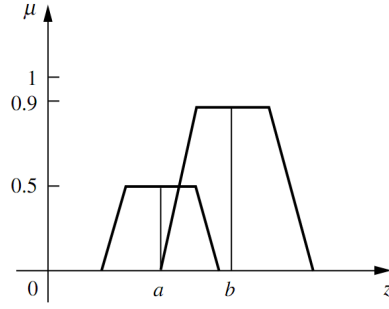


Figure 4.16: Weighted average method of defuzzification.

$$z^* = \frac{a + b}{2} \quad (4.6)$$

where a and b are as defined in Figure (4.17).

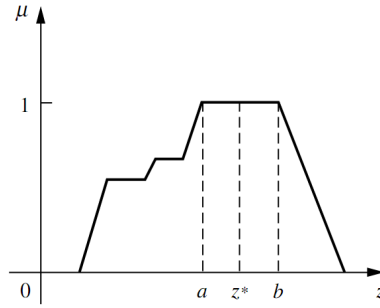


Figure 4.17: Mean max membership defuzzification method.

5. *Center of sums*: This method is faster than many common defuzzification methods and is not restricted to symmetric membership functions. Instead of using the algebraic union of outputs, this method involves the algebraic sum of individual output fuzzy sets: C_1 and C_2 . There are two drawbacks in this method: the intersecting areas are added twice, and the method involves finding the centroids of the individual membership functions. The defuzzied value z^* is given by the following equation:

$$z^* = \frac{\int_Z \bar{z} \sum_{k=1}^n \mu_C(z) dz}{\int_Z \sum_{k=1}^n \mu_C(z) dz} \quad (4.7)$$

where the symbol \bar{z} is the distance to the centroid of each of the respective membership functions. This method is very similar to weighted average method (Equation (4.5))

except in the center of sums method the weights are the areas of the respective membership functions whereas in the weighted average method the weights are individual membership values. Figure (4.18) illustrates the center of sums method.

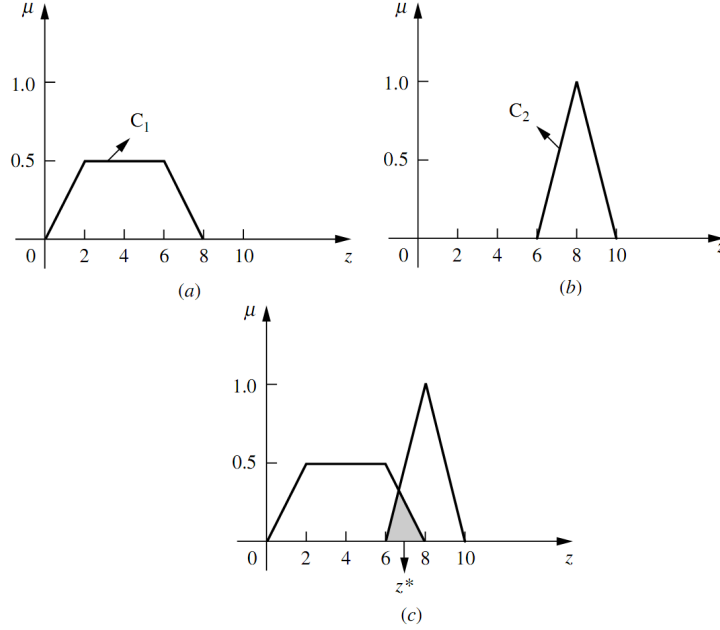


Figure 4.18: Center of sums method: (a) first membership function; (b) second membership function; and (c) defuzzification step.

6. *Center of largest area:* If the output fuzzy set has at least two convex sub regions, then the center of gravity (i.e., z^* is calculated using the centroid method, Equation (4.8)) of the convex fuzzy sub region with the largest area is used to obtain the defuzzified value z of the output. It is illustrated in Figure (4.19), and the algebraic expression is:

$$z^* = \frac{\int \mu_{C_m}(z) \cdot z \, dz}{\int \mu_{C_m}(z) \, dz} \quad (4.8)$$

where C_m is the convex sub region that has the largest area making up C_k . This condition applies in the case when the overall output C_k is non convex; and in the case when C_k is convex, z^* is the same quantity as determined by the centroid method or the center of largest area method (because then there is only one convex region).

7. *First (or last) of maxima:* the overall output or union of all individual output fuzzy sets C_k is used in this method to determine the smallest value of the domain with maximized

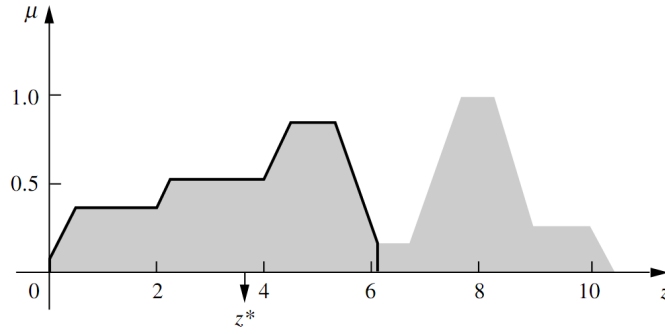


Figure 4.19: Center of largest area method (outlined with bold lines), shown for a non convex C_k .

membership degree in C_k . The equations for z^* are as follow:

First, the largest height in the union (denoted $\text{hgt}(C_k)$) is determined,

$$\text{hgt}(C_k) = \sup_{z \in Z} \mu_{C_k}(z) \quad (4.9)$$

Then, the first of the maxima is found,

$$z^* = \inf_{z \in Z} \{z \in Z \mid \mu_{C_k}(z) = \text{hgt}(C_k)\} \quad (4.10)$$

An alternative to this method is called the last of maxima, and represented as:

$$z^* = \sup_{z \in Z} \{z \in Z \mid \mu_{C_k}(z) = \text{hgt}(C_k)\} \quad (4.11)$$

In Equations (4.9)-(4.11), the supremum (sup) is the least upper bound and the innum (inf) is the greatest lower bound. It is illustrated in Figure (4.20). In the case illustrated in this Figure, the first max is also the last max and as it is a distinct max, it is also the mean max. Hence, the methods presented in Equation (4.3)(max or height), (4.6) (mean max), (4.10) (first max), and (4.11) (last max) all provide the same defuzzied value, z^* , for the particular situation illustrated in Figure (4.20).

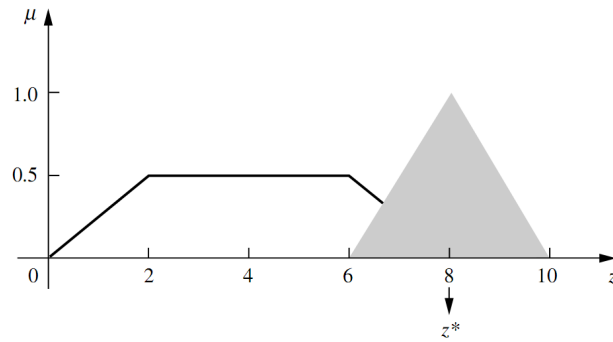


Figure 4.20: First of max (and last of max) method.

4.4 Summary

In this chapter,:

1. Two main questions have been answered: what is the advantage of fuzzy logic and why we should benefit from this concept in economic researches. Having an insufficient sample size, an unclear relationship between response and explanatory variables, difficulties verifying distribution assumptions, etc. are of the cases where the fuzzy logic can offer a solution with much less complication compared to other conventional methods. Possibility of modeling the agent's behavior and modeling or measuring the economic phenomena with fuzzy logic are of the main reasons that encourage to apply the fuzzy economy.
2. Some primary definitions or principles of the fuzzy logic including: fuzzy set, linguistic term and variable, membership functions and mathematical operations on the fuzzy sets (fuzzy intersection and union) were explained.
3. Fuzzy modeling and the steps (fuzzification, fuzzy inference or fuzzy decision making and defuzzification) were fully introduced. In fuzzification, the crisp data are converted to fuzz data; in fuzzy inference, the decision making process on fuzz data is done and in defuzzification, the resulted fuzzy data are converted to crisp data to make the possibility of comparing the obtained results with the outcomes of other methods [Ross, 2004].

The concepts and definitions presented in this chapter are crucial to understand the proposed fuzzy models in this research. The fuzzy models developed for the estimation of the

underground economy in this work and their results will be presented in the following chapters.

Summary of Part I

Until here we discussed the following topics:

- In chapter 2, the underground economy was introduced. Firstly, various definitions of the underground economy were presented. It was mentioned that the underground economy consists of legal and illegal transactions, activities and services which are not reported or recorded in spite of government regulations due to unconsciousness of people, evasion or avoidance from government regulations, or the shortcomings of the government agents (Tahmasebi and Rocca, 2015). Table (2.1) was provided to complete the definitions.

Different actors of the underground economy, their roles and motivations from participating in the underground market were discussed. A complete macro model of the underground economy consisting of the society, the government, the financial market, the official and the underground firms, their interactions and the role of other countries in an economy was proposed.

The positive and negative impacts of the underground economy were discussed. Different causes (the factors that motivate people to work in the underground markets) and indicators (the factors that represent the condition of the underground economy) of the underground economy and finally its segmentations were presented.

- In chapter 3, three categories of estimation methods of the underground economy with their strengths and weaknesses were introduced. It was explained that the direct methods are based on the results of surveys and questionnaires and only yield estimates for a specific point in time. They do not provide a time series and can be regarded as low-end estimations. Selecting appropriate samples, the possibility of selection bias, measurement errors regarding the surveys, the respondent's willingness to cooperate

and their honest responses are among the defects of direct methods [Albu, 1995; Elgin and Oztunali, 2012].

Indirect or macroeconomic methods focus on the discrepancy between the official and actual criteria. As an example, if the total labour force participation is assumed to be constant, a decrease in the official rate of participation can indicate an increase in the participants of the underground activities. These methods are generally criticized for employing various simplifying and limiting assumptions, such as focusing on a specific aspect or indicator and neglecting the other causes which may also have a predominant effect on the underground economy [Elgin and Oztunali, 2012; Tahmasebi and Rocca, 2015].

The modeling method or Multiple Indicators Multiple Causes (MIMIC) simultaneously considers some causes and indicators of the underground economy. It is composed of two sets of equations (first set is the equations between the latent variable (the underground economy) and the causes and second set is the equations between the latent variable and the indicators) to determine a time series for the underground economy. This approach is very sensitive to sample size, data specifications, data distributions etc. [Breusch, 2005; Schneider, 2003].

- In chapter 4, the required fuzzy concepts to better understand the models were introduced. It was explained that having an insufficient sample size, an unclear relationship between response and explanatory variables, difficulties verifying distribution assumptions, possibility of modeling the agent's behavior and modeling or measuring the economic phenomena were of the cases where the fuzzy logic can be used in economic researches. The concept of fuzzy set, linguistic term and variable, membership functions and mathematical operations on fuzzy sets were fully explained. Fuzzy modeling and its steps including fuzzification: converting the crisp data to fuzzy data; fuzzy inference: the decision making process on fuzz data; and defuzzification: converting the fuzzy data to crisp data; were discussed [Ross, 2009].

In the next part, we are going to introduce the proposed fuzzy methods in this thesis which aim to estimate a time series for the underground economy.

Part II

Methods and Results

Chapter 5

Proposed Fuzzy Models

Introduction

A complete schema of the underground economy was already provided and the necessity of an estimation method is evident. Various estimation methods including the direct, indirect and modeling (or MIMIC) methods have been proposed for this purpose. Although MIMIC is a widely used method to estimate the underground economy, there is not yet any consensus on a specific method among researchers [Buehn and Goethel, 2010; Tahmasebi and Rocca, 2015]. In previous chapter, we explained about the advantages of the fuzzy logic and discussed that why the fuzzy logic and methods are promising for the estimation of the underground economy. In this chapter, the three fuzzy methods proposed in this research to estimate the size of the underground economy as a time series will be introduced. These methods are: fuzzy modeling applying mean and variance, fuzzy modeling applying fuzzy clustering, and structural equation modeling with fuzzy data.

The first two methods are composed of three steps previously discussed in Chapter 4: Fuzzification, Fuzzy inference and Defuzzification. The latter involves two main steps: changing the structural equation model to a reduced form, then making a non-linear model from the reduced-form equations applying fuzzy linear regression concept and solutions.

5.1 Fuzzy Modeling

It was mentioned that a fuzzy model, also called a fuzzy system, is a way of mapping an input space to an output space using fuzzy logic. Fuzzy model uses a collection of fuzzy membership functions and rules, instead of Boolean logic, to reason about data. The functional operations in fuzzy modeling proceed in the following steps:

1. Fuzzification;
2. Fuzzy Inferencing (apply implication method) and;
3. Aggregation of all outputs and Defuzzification

The rules which are used in fuzzy inference are fuzzy rules of the form:

- if p then q , where p and q are fuzzy statements.

As an instance, consider the fuzzy rule:

- if x is medium and y is low then z is high.

Here x is *medium*; y is *low*; z is *high* are fuzzy statements; x and y are input variables; z is an output variable. *Medium*, *low* and *high* are fuzzy sets. All these concepts are completely discussed in previous chapter. After a short review of fuzzy concepts, the main steps of the proposed fuzzy models are discussed next.

5.1.1 Fuzzification

It was explained earlier that the first step in fuzzy modeling is to take the crisp inputs (for example, x_1 and y_1 as project funding and project staffing) and determine the degree or the membership function to which these inputs belong to each of the appropriate fuzzy sets (Figure (5.1)).

As illustrated in Figure (5.1), the two main parts of the fuzzification step are defining the fuzzy sets and their membership functions.

In the proposed models, the causes that are believed to affect the size of underground economy are determined and used as the input variables. The required data of the causes were obtained

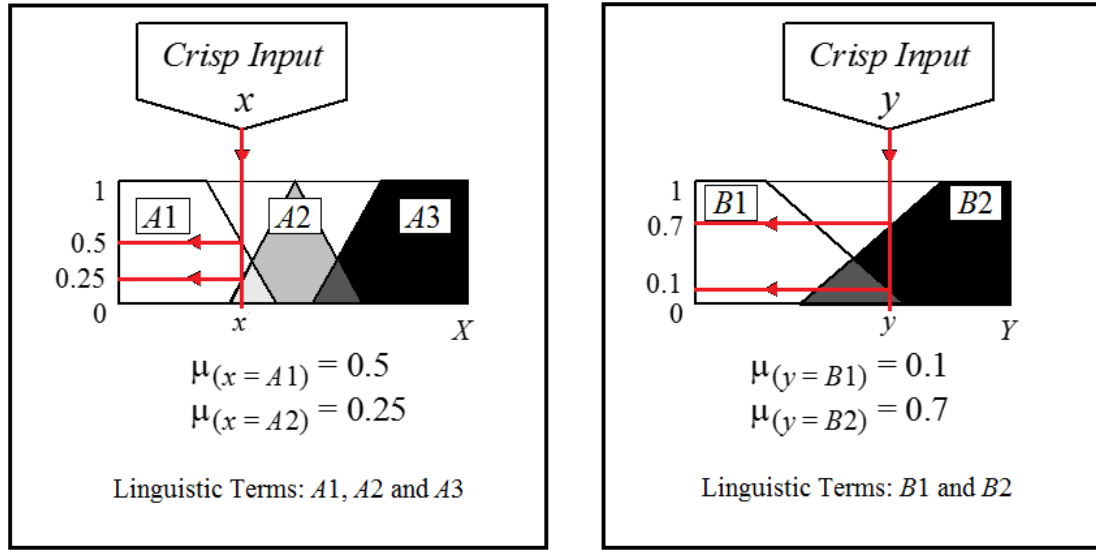


Figure 5.1: The operations of the fuzzification.

from OECD statistics database [OECD, 2014]. There are three different reasons to use this database:

- Five countries including France, Germany, United Kingdom, Canada and the United States are selected to determine the size of the underground economy in. It is better then to extract all necessary data from one database instead of using the different data bases and the OECD statistics database provides all necessary data for all of these countries which distinguishes it from other data bases.
- Another advantage of the OECD database is that all the required data are also available as weekly and monthly data, although we do not use this characteristic.
- We have compared our results with the findings of Professor Schneider, Dr. Buehn, Dr. Dell'Anno, Ceyhun Elgin and Oguz Oztunali. To have a reasonable comparison it is essential to use the same raw data. As these researchers had used the OECD database in their works, we also preferred to use the same database in order to be able to compare the results.

The applied data for France are provided in Table (A.1) ¹.

It was explained that in the fuzzification step, the crisp data are converted to the fuzzy data.

¹ The applied data for other countries are provided in Appendix

Now, two different models are proposed based on different fuzzification methods including:

1. Fuzzification using Mean and Variance and;
2. Fuzzification using Fuzzy Clustering.

These methods are completely discussed next.

Table 5.1: The initial data of France (OECD statistics Data Base)

	Causes						Indicators		
	DTR	ITR	UR	GR	SER	IR	GDPC	HSR	PRL
1985	15.985	29.680	8.93	51.88	15.02	5.83	20.428	10.3	65.678
1986	16.525	29.279	9.08	51.28	14.74	2.54	15.545	9.24	65.709
1987	16.674	29.256	9.26	50.71	14.51	3.29	13.737	7.19	65.641
1988	16.082	29.388	8.83	50.06	14.18	2.7	14.213	7.55	65.603
1989	16.086	28.735	8.34	48.93	13.71	3.5	15.851	8.1	65.705
1990	16.048	28.405	8	49.58	13.36	3.23	13.940	9.04	68.251
1991	16.860	27.202	8.46	50.69	12.86	3.22	14.615	10.06	68.286
1992	15.996	26.470	9.32	52	12.33	2.41	13.901	10.84	68.463
1993	15.833	26.204	10.55	55.07	11.89	2.08	14.803	11.98	68.154
1994	16.104	27.056	11.12	54.07	11.44	1.67	14.937	11.33	68.505
1995	16.274	27.675	10.53	54.39	10.97	1.79	13.775	12.47	68.491
1996	16.863	27.954	10.97	54.5	10.61	2	14.319	11.44	68.875
1997	18.192	27.434	11.14	54.18	10.29	1.19	16.899	12.46	69.040
1998	23.120	26.959	10.74	52.76	9.91	0.64	17.731	11.86	69.637
1999	23.937	26.594	10.38	52.6	9.62	0.54	19.161	11.26	70.422
2000	24.932	25.811	9.02	51.69	9.31	1.69	23.347	11.01	70.702
2001	25.363	25.272	8.18	51.66	9.01	1.63	24.832	11.66	70.898
2002	23.920	25.735	8.31	52.87	8.91	1.92	23.987	12.97	71.028
2003	23.189	25.565	8.89	53.4	8.9	2.11	19.356	11.87	71.057
2004	23.365	25.591	9.28	53.26	8.97	2.13	17.868	12.22	70.930
2005	23.537	25.374	9.28	53.57	9.05	1.74	18.387	11.08	70.854
2006	24.240	24.916	9.25	52.98	9.06	1.68	18.971	11.21	70.966
2007	23.884	24.845	8.38	52.61	9.01	1.49	17.846	11.72	70.905
2008	24.155	24.592	7.78	53.28	9.03	2.81	17.051	11.7	70.550
2009	20.727	25.094	9.53	56.77	9.19	0.09	17.507	12.55	70.844
2010	21.885	25.035	9.77	56.55	9.21	1.53	18.634	12.06	70.927

Fuzzification using Mean and Variance

The fuzzification step itself is composed of two more phases: first, defining the fuzzy sets; second, defining the membership function.

Defining the fuzzy sets

The very first thing to do when applying fuzzy logic is to define the fuzzy sets including the linguistic terms and membership functions. Five linguistic terms are used to describe the causes in this work: *Very Low (VL)*, *Low (L)*, *Normal (N)*, *High (H)* and *Very High (VH)*. The related linguistic terms of underground economy are: *Very Small (VS)*, *Small (S)*, *Average (A)*, *Big (B)* and *Very Big (VB)*.

The average value of each cause for the five past years up to the desired date is considered as the Mean value for the series of that cause. Furthermore, the standard deviation of the data in these years is calculated using the relevant Mean value. For example, the Mean value related to a variable for 1985 is the average value of the period from 1981 to 1985. By assigning *Normal* as the linguistic term to the Mean value, the linguistic terms obtained around the Mean value will be:

⇒ *Very high: Mean value + 2 × Standard Deviation;*

⇒ *High: Mean value + Standard Deviation;*

⇒ *Normal: Mean value;*

⇒ *Low: Mean value - Standard Deviation and;*

⇒ *Very low: Mean value - 2 × Standard Deviation.*

Table (5.2) represents the linguistic terms used for the causes and the underground economy.

Table 5.2: The linguistic terms used for the causes and the underground economy.

Linguistic terms for Causes				
Very Low(VL)	Low(L)	Normal(N)	High(H)	Very High(VH)
Mean*-2×SD**	Mean-SD	Mean	Mean+SD	Mean+2×SD
Linguistic terms for Underground Economy (UE)				
Very Small (VS)	Small (S)	Average (A)	Big (B)	Very Big (VB)
* <i>Mean value (Mean)</i>			** <i>Standard Deviation (SD)</i>	

Defining the membership function

Each input variable in a fuzzy set is associated with one or two linguistic terms and their membership functions. To determine the membership functions for each input variable, firstly,

the value (m^*) associated with the peak of the membership function ($\mu(x) = 1.0$) in (*Normal* linguistic term should be obtained (see Figure (5.2)). It is done by calculating the average of the value of that cause in every five consecutive years as it was explained above. Afterwards, the relevant values associated with the peaks of the membership functions in remaining linguistic terms (*Very Low*, *Low*, *High* and *Very High*) are calculated by adding or subtracting one or two standard deviations (SD) (see Table (5.2)).

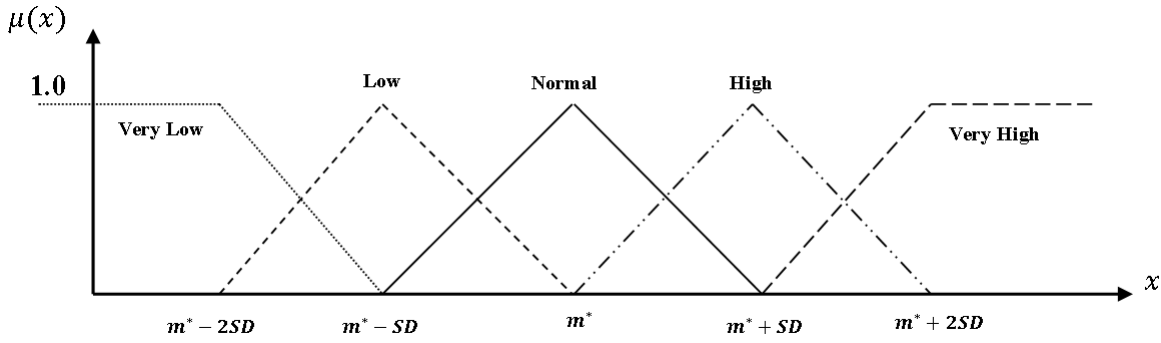


Figure 5.2: A schema of the membership function.

To depict the membership functions, the inclined lines are drawn from the peak of a linguistic term with $\mu(x) = 1.0$ to the associated value of the peaks of its neighboring linguistic terms with $\mu(x) = 0.0$ as it is shown in Figure (5.2) [Yu et al., 2006]. For example, to depict the linguistic term N (*Normal*), the first inclined line is drawn from *Mean value* (M_N^*) with $\mu(x) = 1.0$, to $(M_N^* - SD)$ with $\mu(x) = 0.0$ and the other line is similarly drawn from *Mean value* (M_N^*) with $\mu(x) = 1.0$, to $(M_N^* + SD)$ with $\mu(x) = 0.0$.

It should be noted that the value of membership functions for input variables lower than the relevant value of *Very Low* peak ($M_N^* - 2 \times SD$) as well as higher than the relevant value of *Very High* peak ($M_N^* + 2 \times SD$) equal unity (see Figure (5.2)).

To associate the membership functions to each linguistic term, Table (5.3) will be used. According to the linguistic term(s) of an input variable (x) and by finding its related condition from the second column, the related mathematical form of the membership function can be gained from the third column. Consequently, the value of the membership function associated with each linguistic term can be calculated. For the linguistic terms *Low*(L), *Normal*(N) and *High*(H), (m_i^*) which is a point on x axis with maximum membership function ($\mu(x) = 1.0$), Lower Bound ($L.B_i = m_i^* - SD$) and Upper Bound ($U.B_i = m_i^* + SD$) are illustrated in Figure (5.2).

It is noticeable that in most cases, two linguistic terms are assigned to each input variable (x). On the other hand, each linguistic term is essentially associated with one membership function. Therefore, most of input variables have two different linguistic terms with their membership functions. Until here, we realized how crisp inputs are converted into fuzzy

Table 5.3: The linguistic terms and their membership functions. m_i^* of each linguistic term is the point with the highest membership function $\mu(m_i^*) = 1.0$.

Linguistic terms	Related Conditions	Membership Function
Very Low (VL)	$if\ x \leq m_N^* - 2SD$	$\mu_{VL}(x) = 1.0$
	$if\ m_N^* - 2SD \leq x \leq m_N^* - SD$	$\mu_{VL}(x) = \frac{(m_N^* - SD) - x}{SD}$
$i=(L), (N) \text{ and } (H)$	$if\ L.B_i \leq x \leq m_i^*$	$\mu_i(x) = \frac{x - (L.B_i)}{SD}$
	$if\ m_i^* \leq x \leq U.B_i$	$\mu_i(x) = \frac{(U.B_i) - x}{SD}$
Very High (VH)	$if\ m_N^* + SD \leq x \leq m_N^* + 2SD$	$\mu_{VH}(x) = \frac{x - (m_N^* + SD)}{SD}$
	$if\ x \geq m_N^* + 2SD$	$\mu_{VH}(x) = 1.0$

values comprised of linguistic terms and membership functions. Briefly, the linguistic terms are defined for the causes and the underground economy, then, one (or two) linguistic term(s) with related membership function(s) is(are) associated with each of the causes in each year. The fuzzification process can also be carried out using Fuzzy clustering. This process will be discussed next.

Fuzzification using Fuzzy Clustering

Clustering

Clustering methods are used to classify data into categories or groups based on similarities among individual data items; e.g., groups of schools or students with similar properties, groups of buildings based on their heights, an aggregation of stars or galaxies that appear close together in the sky and are gravitationally associated. It is considerable that the quantitative (numerical) data, qualitative (categorical) data or a mixture of both can be used as initial data in clustering techniques.

Clustering algorithms are useful in situations where little prior knowledge exists, because most of them do not rely on assumptions common to conventional statistical methods such as the underlying statistical distribution of data.

Table 5.4: Technical descriptions of some vehicles.

Vehicle	Top speed (Km/h)	Colour	Air resistance	weight (Kg)
V1	220	red	0.3	1300
V2	230	black	0.32	1400
V3	260	red	0.29	1500
V4	140	gray	0.35	800
V5	155	blue	0.33	950
V6	130	white	0.4	600
V7	100	black	0.5	3000
V8	105	red	0.6	2500
V9	110	gray	0.55	3500

To illustrate more the concept of clustering, technical descriptions of some vehicles are provided in Table (5.4). Then, in Figure (5.3), these vehicles are divided to three main clusters based on their similarities: their top speeds and weights.

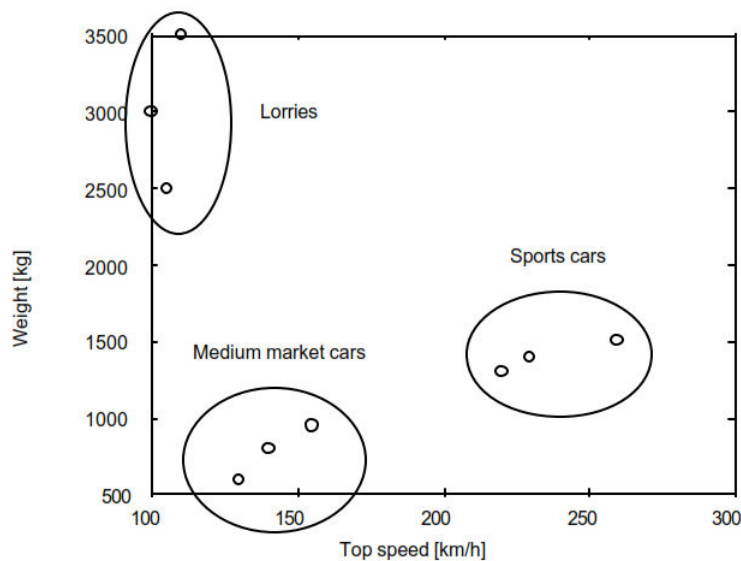


Figure 5.3: Clustering some vehicles based on top speed and weight.

As it can be seen in Figure (5.3), each cluster is composed of some objects or data points and is named by a label. These objects or data points are classified based on some similarities or features which are top speed and weight of the vehicles in this example. It is also possible to make other clusters based on other similar features if any exists. The terminology of the clusters are demonstrated in Figure (5.4). There are various definitions for a cluster based on the objective of clustering. Generally, a cluster is a group of objects that are more similar to

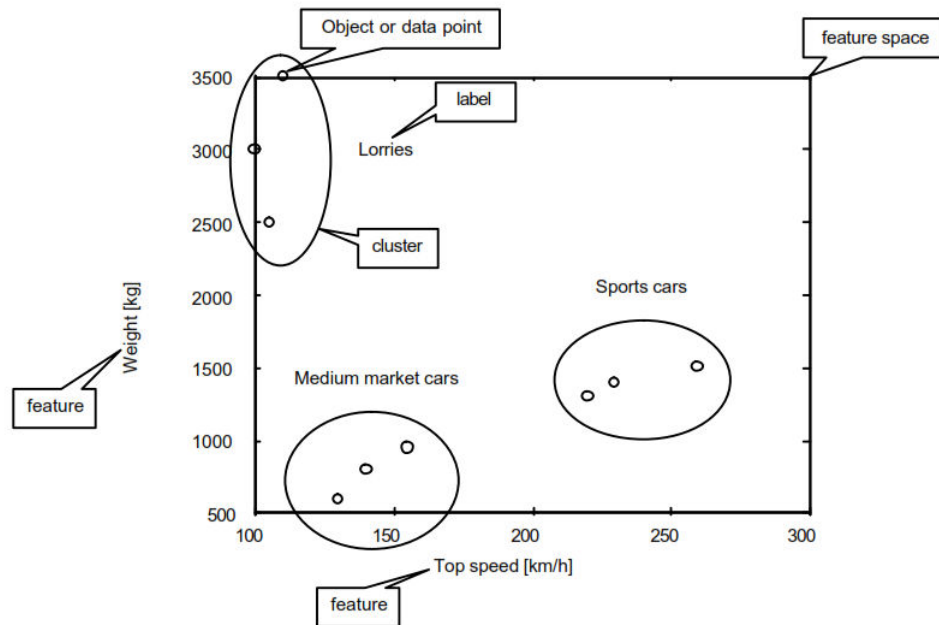


Figure 5.4: The terminology of the clusters.

one another than to members of other clusters [Bezdek et al., 1984; Jain and Dubes, 1988]. The term "similarity" should be clarified as mathematical similarity, measured in some well-defined sense.

A distance norm is often used to indicate the similarity when the problem is proposed in metric spaces. The Distance can be determined among the data vectors themselves or be defined as a distance from a data vector to some prototypical objects (prototype) of the cluster. Usually, the prototypes are not known before and will be specified by the clustering algorithms together with the partitioning of the data. The prototypes may be vectors of the same dimension as the data objects, but they can also be defined as "higher-level" geometrical objects, such as linear or nonlinear sub-spaces or functions. The geometrical shapes and densities of the individual clusters together with the spatial relations and distances among the clusters influence the performance of most clustering algorithms. Clusters can be well-separated, continuously connected to each other or overlapping each other. On the whole, clusters can formally be seen as subsets of the data set.

Fuzzy C-means Clustering

Clustering methods can be classified as to whether the subsets are fuzzy or crisp (hard). Hard

clustering methods are based on classical set theory and require an object to either belong or not to a cluster(see Figure (5.3)). In hard clustering, the data are divided into a specified number of mutually exclusive subsets.

However, in fuzzy clustering methods, the objects on the boundaries between several classes are not forced to fully belong to one of the classes; so, the objects may belong to several clusters simultaneously with different degrees of membership. In many situations, fuzzy clustering is more favorable than hard clustering. Moreover, the discrete nature of the hard partitioning causes difficulties with algorithms based on analytic functionals, since these functionals are not differentiable.

The optimization of the basic c-means objective function or its modification is the basis for the most analytical fuzzy clustering algorithms. A large family of fuzzy clustering algorithms is based on minimization of the fuzzy c-means functional formulated as Equation (5.1a)[Bezdek et al., 1984; Dunn, 1974].

$$J(\mathbf{Z}; \mathbf{U}, \mathbf{V}) = \sum_{i=1}^c \sum_{k=1}^N (\mu_{ik})^m \|\mathbf{z}_k - \mathbf{v}_i\|_A^2 \quad (5.1a)$$

where

$$\mathbf{U} = [\mu_{ik}] \in M_{fc} \quad (5.1b)$$

is known as the matrix of membership functions or a fuzzy partition matrix of \mathbf{Z} and

$$\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_c], \quad \mathbf{v}_i \in \mathbb{R}^n \quad (5.1c)$$

is a vector of cluster centers or cluster prototypes, which have to be determined,

$$D_{ikA}^2 = \|\mathbf{z}_k - \mathbf{v}_i\|_A^2 = (\mathbf{z}_k - \mathbf{v}_i)^T \mathbf{A} (\mathbf{z}_k - \mathbf{v}_i) \quad (5.1d)$$

is a squared inner-product distance norm and $m \in [1, \infty)$ is a parameter which determines the fuzziness of the resulting clustering.

Also, \mathbf{Z} is the given data set, \mathbf{A} is the norm-inducing matrix, m is the weighting exponent ($m > 1$) and c is the number of clusters ($1 < c < \mathbf{N}$) which must be firstly determined.

The minimization of the c-means functional generates a nonlinear optimization problem

that can be solved by methods like, iterative minimization, simulated annealing or genetic algorithms. The most popular method is a simple Picard iteration through the first-order conditions for stationary points of Equations (5.1a)-(5.1d), known as the fuzzy c-means (FCM) algorithm [Szczepaniak et al., 2000].

The fuzzy c-means algorithm used in this thesis consists of three steps:

Step 0: Initializing the membership function matrix (partition matrix) randomly, such that

$$U(0) \in MF_c$$

Repeat for $l = 1, 2, \dots$

Step 1: Computing the cluster means (cluster prototype):

$$V_i^{(l)} = \frac{\sum_{k=1}^N (\mu_{ik}^{(l-1)})^m \mathbf{z}_k}{\sum_{k=1}^N (\mu_{ik}^{(l-1)})^m}, \quad 1 \leq i \leq c.$$

Step 2: Computing the distance:

$$D_{ikA}^2 = (\mathbf{z}_k - \mathbf{v}_i^{(l)})^T \mathbf{A} (\mathbf{z}_k - \mathbf{v}_i^{(l)}), \quad 1 \leq i \leq c, \quad 1 \leq k \leq N.$$

Step 3: Updating the membership function matrix:

for $1 \leq k \leq N$

if $D_{ikA} > 0$ for all $i = 1, 2, \dots, c$

$$\mu_{ik}^{(l)} = \frac{1}{\sum_{j=1}^c (D_{ikA} / D_{jkA})^{2/(m-1)}},$$

otherwise: $\mu_{ik}^{(l)} = 0$ if $D_{ikA} > 0$, and $\mu_{ik}^{(l)} \in [0, 1]$ with $\sum_{i=1}^c \mu_{ik}^{(l)} = 1$.

until $\|U^{(l)} - U^{(l-1)}\| < \varepsilon$.

where \mathbf{Z} is the given data set, c is the number of clusters ($1 < c < N$) which must be determined² and must be lower than the number of observations (N), m is the weighting exponent ($m > 1$), ε is the termination tolerance $\varepsilon > 0$ and \mathbf{A} is the norm-inducing matrix.

The termination condition of the algorithm expressed as "until condition", means that the

² Number of clusters in the proposed method is equal to the number of the linguistic terms and will be fixed in the different steps of the proposed method

difference of the consecutive objective functions (membership functions) must be less than the considered difference (ε).

Fuzzification using Fuzzy Clustering

It was mentioned that the fuzzification step is itself composed of two main parts: defining the fuzzy linguistic terms of causes and that of underground economy as well as the membership functions.

As it was explained earlier, the causes are transformed to five linguistic terms: *Very Low (VL)*, *Low (L)*, *Normal (N)*, *High (H)* and *Very High (VH)*. The related linguistic terms of the underground economy are *Very Small (VS)*, *Small (S)*, *Average (A)*, *Big (B)* and *Very Big (VB)*. The fuzzy c-means (FCM) algorithm presented above is used to assign the data to different clusters defined. The number of clusters is equal to the number of linguistic terms. Each element of the final matrix U indicates the membership function of the related cluster (or linguistic term) in the related year. At the end of the iteration, the initial data are converted to fuzzy numbers consisting of the linguistic terms and their membership functions. Each value of the membership function indicates the degree of dependency of an input value or initial value of the causes to the related cluster or linguistic term.

5.1.2 Fuzzy Inference

Fuzzy Inference Methods

There are two well-known methods which are used in the fuzzy inference: Mamadani and Sugeno. Fuzzy rules, more precisely If-then rule statements, are used to formulate the conditional statements in fuzzy inference step.

Fuzzy rules are briefly reviewed in this section and different inference methods are presented and compared in the following.

It was mentioned earlier that a single fuzzy if-then rule has the general form:

$$\text{if } x \text{ is } A \text{ then } y \text{ is } B$$

where A and B are linguistic values defined by fuzzy sets on the ranges X and Y, respectively. It was mentioned that the if-part or conditional-part of the rule, " x is A", is called the antecedent or premise, while the then-part or response-part of the rule, " y is B", is called the consequent or conclusion (see Figure (5.5)).

To make it more clear, consider a rule as: *If apartment is big then charge is high.*

Here, *big* is a linguistic value for apartment which takes a number between $80m^2$ and $100m^2$.

high is a linguistic value for charge which takes a number between 100 euros and 120 euros.

The consequent then is an assignment that assigns a linguistic term from fuzzy set B to the output variable y according to premise of the rule (Figure (5.5)). In another example depicted

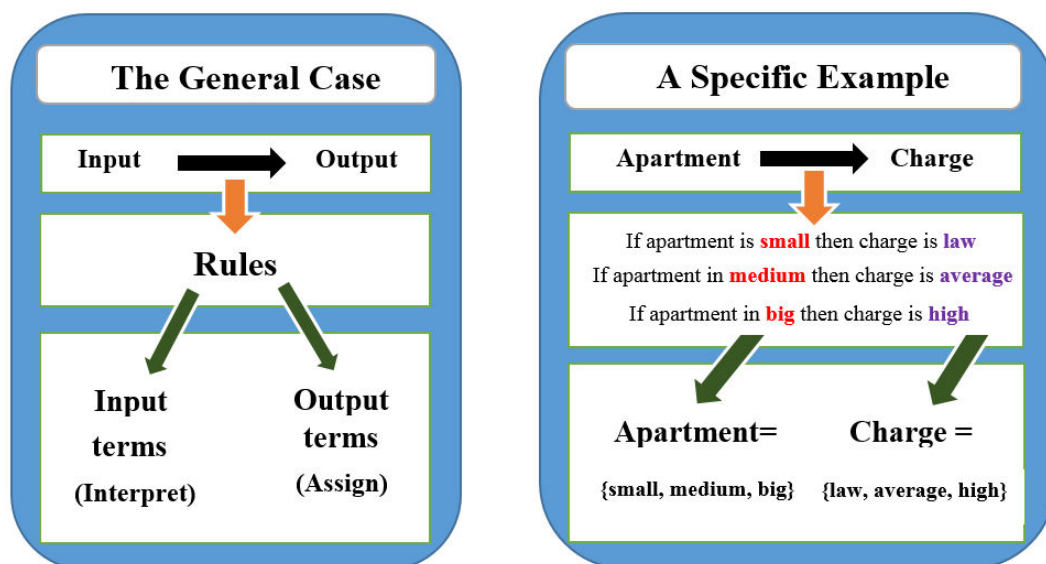


Figure 5.5: Illustration of fuzzy rule with an example.

in Figure (5.6), the main part of a rule is completely presented. It must be noted that the consequent or conclusion consists of two parts: the linguistic value and its membership function. The output of each rule is a fuzzy set. But generally, one rule alone is not effective. Thus, two or more rules that can play off one another are required. The output fuzzy sets obtained from various rules are then aggregated into a single output fuzzy set. Later on, the resulting set will be defuzzified, or resolved to a single number which is defined as defuzzification.

Mamdani-Type Fuzzy Inference

Mamdani method is the most frequently used fuzzy inference technique developed by Profes-

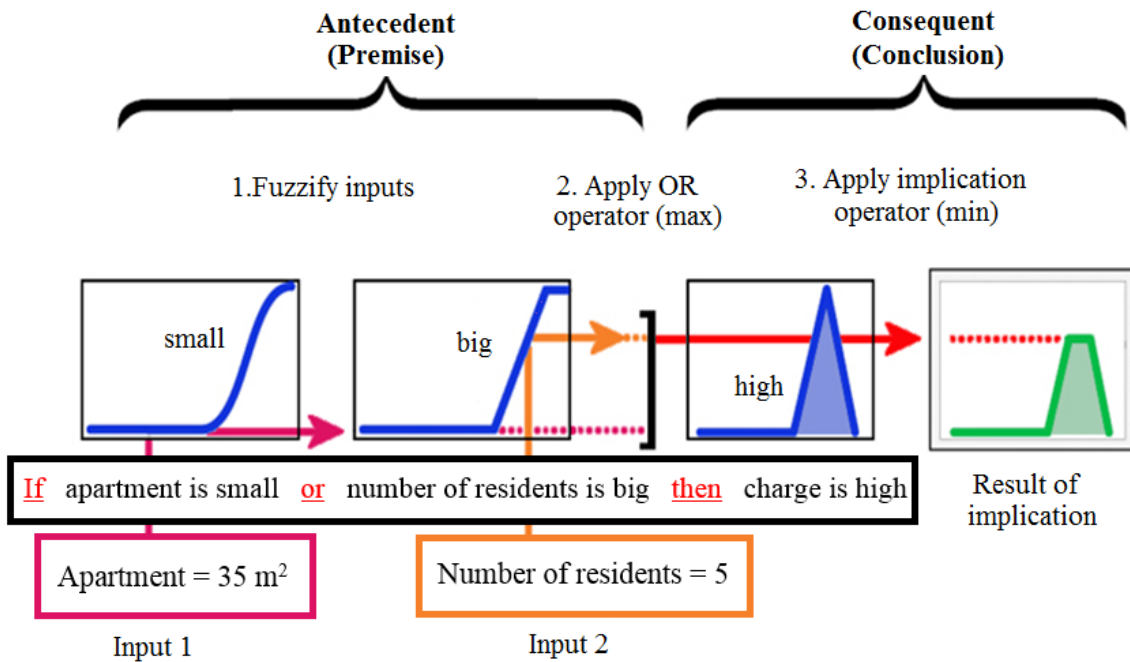


Figure 5.6: Different parts of the If-Then rule.

Dr. Ebrahim Mamdani of London University in 1975. He built one of the first fuzzy systems to control a steam engine and boiler combination. According to Lotfi Zadeh's 1973 paper on fuzzy algorithms for complex systems and decision processes [Zadeh, 1973], he applied a set of fuzzy rules supplied by experienced human operators [Mamdani and Assilian, 1975].

In Mamdani-type inference, the output membership functions are expected to be fuzzy sets. The aggregation process then produces a fuzzy set for each output variable that needs defuzzification. The important feature of this method is that the consequent of the rule ("y is B") is a fuzzy expression (unlike the Sugeno-Type).

Sugeno-Type Fuzzy Inference

Another fuzzy inference technique, which is similar to the Mamdani method in many respects, is Sugeno or Takagi-Sugeno-Kang method introduced in 1985 [Sugeno, 1985]. The main difference between Mamdani and Sugeno is that the Sugeno output membership functions are either linear or constant. In fact, there is no output membership function at all in the Sugeno method, instead, the output is a crisp number computed by multiplying each input by a constant and then adding up the results.

A typical rule in a Sugeno fuzzy model has the form:

$$\text{if } Input1 = x \text{ and } Input2 = y \text{ then output is } z = ax + by + c.$$

For a zero-order Sugeno model, the output level z is a constant ($a = b = 0$).

A serious shortcoming of Sugeno method is that there is no good method for determining the coefficients a , b and c . Furthermore, the outputs of this method are only crisp which may not be what is desired. But as there are algorithms for optimizing Sugeno method, this method can be an alternative in fuzzy inference step.

Comparison of Sugeno and Mamdani

It is known that Sugeno method is computationally more efficient than Mamdani system. Sugeno makes use of optimisation and adaptive techniques for constructing fuzzy models which let to customize the membership functions so that the fuzzy system best models the data. But Mamadani method is still more conventional in the fuzzy inference step for capturing expert knowledge. It allows to describe the expertise in more human-like manner. However, Mamdani-type fuzzy inference encompasses considerable computational burden.

Fuzzy Inference (Decision making logic)

Defining the fuzzy rules

Until here, fuzzy sets were determined including the related value of the membership functions and linguistic terms for each input value of causes. Now, fuzzy inference should be carried out. Fuzzy inference table can be constructed for both inference methods, Mamdani and Sugeno. In this work, the Mamdani-Type fuzzy inference method is applied to construct fuzzy inference table.

Fuzzy rules in form of conditional statements were defined as presented in Table (5.5) [Lindström, 1998]. As it was explained earlier, the conditional statements used in fuzzy rules are in *If-Then* form.

As an instance, if the *Cause 1* and *Cause 2* are respectively supposed as government regulation

Table 5.5: Table of fuzzy rules and relevant degrees (written next to each linguistic term). Rule (i,j) represents the rule in row i and column j .

			Cause 2				
			1	2	3	4	5
			VH	H	N	L	VL
Cause 1	1	VH	VB, 1	VB, 1	B, 1	B, 0.8	A, 0.5
	2	H	VB, 0.95	B, 1	B, 0.8	A, 1	S, 0.8
	3	N	B, 0.95	B, 0.75	A, 1	S, 0.8	S, 1
	4	L	B, 0.75	A, 0.95	S, 0.75	S, 1	VS, 1
	5	VL	A, 0.45	S, 0.75	S, 0.95	VS, 1	VS, 1

(GR) and the share of taxation in GDP (TR), the first rule in Table (5.5) will be:

$$\underline{\text{If}} (GR = VH \text{ and } TR = VH) \underline{\text{Then}} (UE = VB).$$

And can be read as:

If (government regulation is Very High and the share of taxation in GDP is Very High) Then (the underground economy is Very Big).

The resulted rule means that when the government regulation is very high, tax payers tend to participate in underground activities to avoid the regulations. Also, when the share of tax in GDP or in other words tax burden is very high, tax payers would like to participate in underground works to avoid paying taxes. Hence, when both conditions are matched, the underground economy is considered as very big.

The fuzzy rules presented in Table (5.5) are based on *If-Then* decision rule; they are easy to handle and understand. The basic rules in the main diagonal (including the rules $(1, 1)$, $(2, 2)$, $(3, 3)$, $(4, 4)$ and $(5, 5)$) are directly resulted and the other rules are symmetrically gained from Lindstrom method. As an example, the underground economy of a state characterized by *Very High* for *Cause 1* and *Normal* for *Cause 2* equals the underground economy of a state characterized by *Normal* for *Cause 1* and *Very High* for *Cause 2*. Furthermore, a situation marked by *Normal* for both *Cause 1* and *Cause 2* is equally distanced from *Very High* for both *Cause 1* and *Cause 2* and a situation marked by *Very Low* for both *Cause 1* and *Cause 2* [Lindström, 1998]. The term degree in Table (5.5) is used to correct the membership function of the resultant linguistic term from each rule. As an example, in Rule $(3, 2)$, the degree

0.8 means that the magnitude of the underground economy in the mentioned year is not completely associated to Big and it needs this correction value to use the Rule (3, 2) for the mentioned year.

Table (5.5) can be customized for every two causes that affect the underground economy sufficiently but not for two causes with low effects on the underground economy. The reason is that with selection of two weak causes, the magnitude of the underground economy becomes meaningless and unreliable.

Application of fuzzy rules (linguistic terms and membership function)

The initial data for each cause were converted to linguistic terms with corresponding membership functions. Now, having the linguistic terms and their membership functions of the causes, the linguistic term of each pair of causes with related degree can be obtained using Table (5.5). For each rule of Table (5.5), the *AND-operator* is used to connect two causes (*Cause1* and *Cause2*). Lindström [1998] considers that the outcome of the *AND-operator* weights the membership function to a pair of causes. This implies that the membership function of a specific pair of causes composed of *Cause 1* and *Cause 2* is calculated as the minimum value of the membership functions associated with both *Cause 1* and *Cause 2* for that particular rule. Hence, the Min-operator is used to calculate the membership function:

$$\mu_{Cause\ 1\ and\ cause\ 2} = Min\{\mu_{Cause\ 1}, \mu_{Cause\ 2}\} \times d$$

where, d is the relevant degree in Table (5.5).

In some cases, the results of the fuzzy rules might be the same linguistic terms with different membership functions. Then, the *maximum membership function* for the same linguistic term must be considered as the resultant membership function.

5.1.3 Defuzzification

After applying the fuzzy inference, the fuzzy results of each year including linguistic terms and their memberships are obtained, but, they are not numeric values. To use these values for comparing or ranking, they should be converted into numerical values in a defuzzification procedure.

Before defuzzification step, the aggregation process should be done. By passing the fuzzy inference for each pair of causes, there is no linguistic term with multiple membership functions but by considering the results of two pairs of causes all together, there might be some linguistic terms with two different membership functions. Now, the main questions arises: if a linguistic term has two different membership functions, which of them must be picked up and applied in the defuzzification process?

The answer to this questions is in aggregation step. The aggregation is the process where the outputs of each rule are combined into a single fuzzy set. The input of the aggregation process is a list of truncated output functions returned by the implication process for each rule and its output is one fuzzy set for each output variable.

In other words, all fuzzy sets assigned to each output variable are combined together to form a single fuzzy set for each output variable using a fuzzy aggregation operator. Some of the most commonly used aggregation operators are:

- Maximum : pointwise maximum over all of the fuzzy sets;
- Sum : pointwise sum over all of the fuzzy sets; and,
- Probabilistic sum or probabilistic OR.

Afterwards, The Average of First Maxima and the Center Of Area (COA) [Ross, 2004; Yen and Langari, 1998] are used for defuzzification. The Average of First Maxima is indicated in equation (6.4):

$$UE \text{ percent} = \frac{\sum_{i=1}^n (m_i^* \pm (1 - \mu_i) \times SD_i)}{n} \quad \begin{cases} + & VS \text{ Linguistic term} \\ - & Other \text{ Linguistic terms} \end{cases} \quad (5.2)$$

where, m_i^* is the value of each related linguistic term of the underground economy with maximum membership function $\mu(m_i^*) = 1.0$, μ_i is the associated membership function to the linguistic term, SD_i is the related standard deviation which is related to each linguistic term and n represents the number of linguistic terms in this year.

The Center Of Area (COA) which returns the center of the area under the curve is also used in

defuzzification process in fuzzy modeling applying fuzzy clustering:

$$\text{Index of UE per year} = \frac{\sum(\mu_i \times m_i^*)}{\sum \mu_i} \quad (5.3)$$

5.2 Fuzzy MIMIC method

It was mentioned earlier, in contrast with traditional logic theory in which the elements of a crisp set definitely do belong to the set, in a fuzzy set, elements of the set have a degree of membership. This membership value can range from 0 (not an element of the set) to 1 (a full member of the set). The fuzzy set theory can also be extended in other mathematical methods such as fuzzy regression model. A solution for Multiple Causes Multiple Indicators (MIMIC) method with fuzzy data is proposed in this work. This solution is further applied to estimate the size of the underground economy. The details of the procedure is discussed next.

5.2.1 Regression methods

In some situations, like when the environment conditions or individual responses affect the results, the observations cannot be described accurately. In such conditions, only an approximate description of results or an interval to enclose them can be provided. The uncertainty of the model in such cases is different from randomness and it is sometimes referred to as vagueness. For example, when the temperature is measured by a thermocouple, the data are shown theoretically continuous but they are discrete in reality. The reason is that the measurement proceedings are not precise [D'Urso and Gastaldi, 2002].

The difference between the observed values and the estimates are assumed to be due to random errors in conventional regression analysis. Thus, statistical techniques are used to do the estimation and inference in these methods. But, the differences are sometimes due to imprecise observations or the ambiguity of the structure of the system. In this case, the uncertainty becomes fuzziness, not randomness. In these conditions, fuzzy linear regression can be used to model vague and imprecise phenomena using fuzzy model parameters. Fuzzy regression is a non statistical method unlike ordinary least squares methods [Kim and Chen, 1997]. In ordinary regression analysis, if the assumptions are violated, the performance and validity of least squares method will be degraded.

It should be noted that only a part of the total uncertainty is considered in both fuzzy regression and ordinary regression. In other words, fuzziness and randomness are two different kinds of uncertainty [Chang and Ayyub, 2001].

5.2.2 Linear regression

The general form of the classical statistical linear regression is:

$$y_i = a_0 + a_1x_{i1} + \dots + a_kx_{ik} + \varepsilon_i \quad (5.4)$$

where y_i is the dependent or response variable, x_{ij} is the independent or explanatory variable and a_j is the coefficient. These variables are crisp and ε_i is a crisp random error term with a normal distribution ($E(\varepsilon_i) = 0$, $Var(\varepsilon_i) = \sigma^2$). The covariance between each pair of error terms is zero ($Cov(\varepsilon_i, \varepsilon_j) = 0$, $\forall i, j \ i \neq j$).

Some complications may affect the results of the classical linear regression, such as:

- Inadequate number of observations or samples (Small data set)[Bárdossy, 1990; Bárdossy et al., 1990, 1992];
- Unclear relationship between response and explanatory variables [Asai, 1982; Tanaka, 1987];
- Difficulties verifying distribution assumptions;
- Ambiguity of events or degree to which they occur;
- Inaccuracy and distortion due to linearity assumption;
- When the human judgments are involved, and;
- If the errors are due to vagueness of the model structure and indefiniteness of human perception of the model [Asai, 1982; Özelkan and Duckstein, 2000; Shapiro, 2005].

These are the very situations fuzzy regression is meant to address. The fuzzy linear regression is a method which is proposed as a reliable method in these situations to prevent inefficient and unreliable results. Linear regression with fuzzy coefficients was firstly introduced by Tanaka and his colleagues in 1982 [Asai, 1982]. The basic idea was to minimize the fuzziness of the model by minimizing the total spread of the fuzzy coefficients, subject to including all the given data [Asai, 1982; Shapiro, 2005]. This suggestion was the start of applying the fuzzy linear regression in different topics and domains.

5.2.3 Triangular Fuzzy Number and "h-certain" Factor

For fuzzy numbers, there are different forms of membership functions the most common types of which are triangular and trapezoidal shapes. In this study, to find the coefficients of a fuzzy regression model, fuzzy components of the equation are assumed as Symmetrical Triangular Fuzzy Numbers (STFN) in order to avoid the complications of modeling. Figure (5.7) depicts

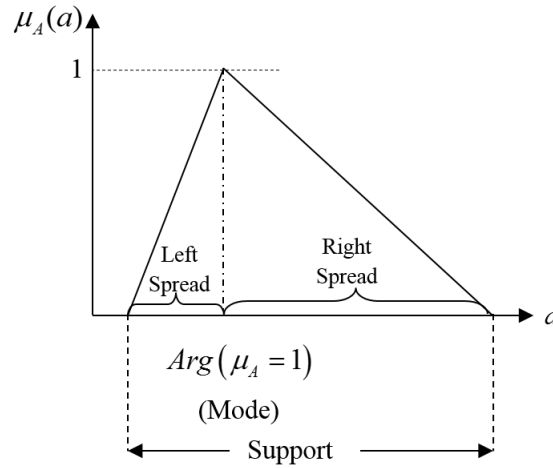


Figure 5.7: Triangular Fuzzy Number(TFN) and its membership function.

the membership function of a Triangular Fuzzy Number(TFN). The main features of the TFN are mode, left and right spread and its support. As it is shown in Figure (5.7), the left and right spreads are not always equal. However, when they are equal, the TFN is known as a Symmetrical Triangular Fuzzy Number (STFN). The membership function of TFN is defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \frac{a-x}{S^L} & a - S^L \leq x \leq a \\ 1 - \frac{x-a}{S^R} & a \leq x \leq a + S^R \end{cases} \quad (5.5)$$

where a , S^R and S^L are respectively mode, right and left spreads. In case of STFN, the right and left spreads, S^R and S^L are equal (Figure (5.8)).

The STFN can be presented as $\tilde{A} = (a, s)_T = (a - s_T, a, a + s_T)$. Its membership function in some references is [Shapiro, 2005]:

$$\mu_{\tilde{A}}(x) = \max \left\{ 1 - \frac{|x - a|}{S}, 0 \right\} \quad (5.6)$$

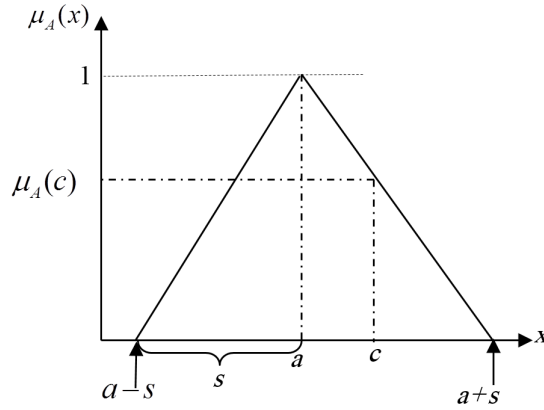


Figure 5.8: Symmetrical Triangular Fuzzy Number(STFN) and its membership function.

When the coefficients are STFN, the fuzzy linear regression can be written as:

$$\tilde{y}_i = \tilde{a}_0 + \tilde{a}_1 x_{i1} + \dots + \tilde{a}_k x_{ik} = \tilde{a}_0 + \sum_{j=1}^k \tilde{a}_j x_{ij} = (a_0, s_0) + \sum_{j=1}^k (a_j, s_j) x_{ij} \quad (5.7)$$

$$\tilde{y}_i = (a_0, s_0) + \sum_{j=1}^k (a_j, s_j) x_{ij} = (a_0 + \sum_{j=1}^k a_j x_{ij}, s_0 + \sum_{j=1}^k s_j x_{ij}) \quad (5.8)$$

This is a very useful and applicable formulation as it indicates the mode and spread of the fuzzy parameters, also the dependent variable. In Figure (5.9), the point "h" on the y-axis is called "h-certain" factor. By controlling the size of the feasible data interval (the mentioned

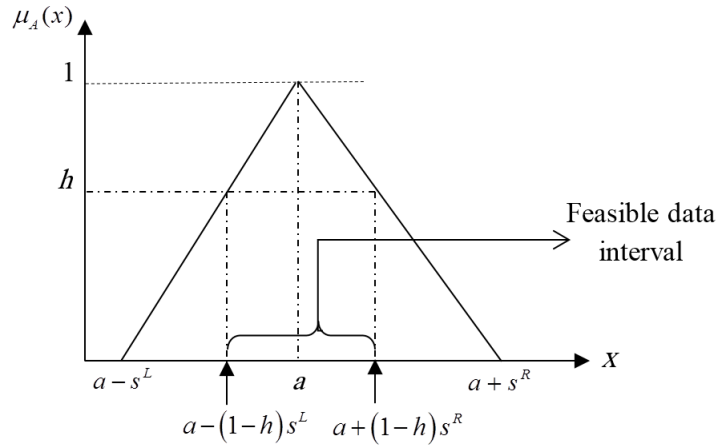


Figure 5.9: h-Certain Factor and feasible data interval.

area in Figure (5.9)) the support of the membership function extends [Shapiro, 2005]. It can be seen that as the magnitude of the h increases and approaches the maximum membership function for a given data set, the feasible data interval and the fuzziness decrease.

5.2.4 Fitting fuzzy regression model

When the observations are defined by fuzzy member functions rather than the classical probability distributions, the associated regression models are called possibility or fuzzy regression model [Kao and Chyu, 2003].

Fuzzy linear regression model was first introduced in 1982 by Tanaka et al. Later on, Diomand developed fuzzy linear least squares models by defining the metric on triangular fuzzy number spaces [Ruoning, 1997]. Several other works on fuzzy regression models have been published after which can be generally divided into two main categories: linear-programming-based methods, also called possibilistic approach, and fuzzy least-squares methods, also known as least squares approach [Chen, 2001].

First approach, the possibilistic model, aims at minimizing the fuzziness of the model by reducing the total spreads of the fuzzy coefficients through setting the data points of each sample within a specified feasible data interval. The objective of the second approach, the least-squares model, is minimizing the error term of the equation or in other words, the distance between the output of the model and the observed output, based on modes and spreads of the output of the model [Arabpour and Tata, 2008; Shapiro, 2005].

In both of these fuzzy regressions, "best fit" represents the optimization of a functional associated with the problem. In particular, in the possibilistic approach, "this functional takes the form of a measure of the spreads of the estimated output, either as a weighted linear sum involving the estimated coefficients in linear regression, or as quadratic form in the case of exponential possibilistic regression". In the least-squares approach "the functional to be minimized is an L_2 distance between the observed and estimated outputs. This reduces to a class of quadratic optimization problems and constrained quadratic optimization" [D'Urso, 2003]. These approaches are discussed by details in the next two sections.

Approach 1. The possibilistic fuzzy regression model

As it was explained, this approach focuses on minimizing the spread of the output of the model or the dependent variables (5.10) by considering the adequate containment of the data,

which implies that:

$$\min \left[s_0 + \sum_{j=1}^k s_j |x_{ij}| \right], \quad s_j \geq 0 \quad (5.9)$$

The mode and the spread of the output of the model is depicted in Figure (5.10).

Now, putting the observed fuzzy output together with the fuzzy output of the model (Figure

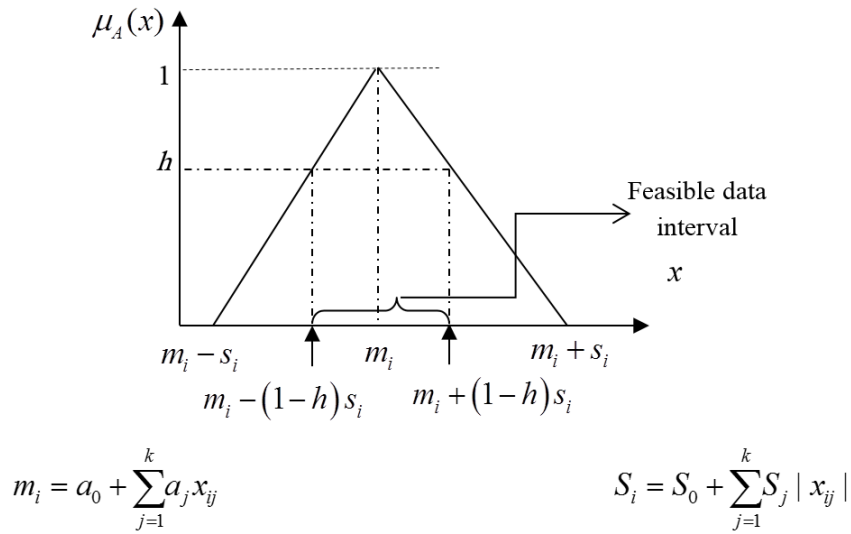


Figure 5.10: Fuzzy output of the model.

(5.11)), it can be seen how the estimated fuzzy output may be fitted to the observed fuzzy data.

The key point of this approach is that the observed fuzzy data, adjusted for the h-certain factor,

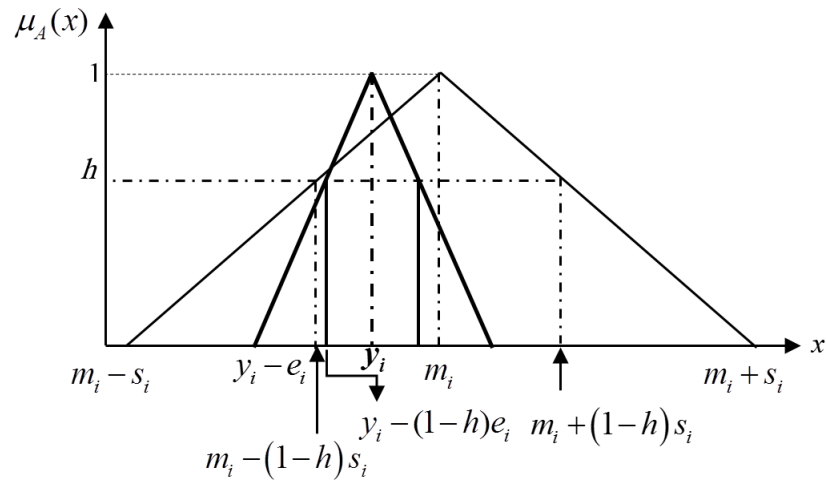


Figure 5.11: Fitting the observed output to the estimated output.

is contained within the estimated fuzzy outputs, adjusted for the h-certain factor; which means that:

$$m_i + (1 - h)s_i > y_i + (1 - h)e_i \quad \text{and} \quad m_i - (1 - h)s_i < y_i - (1 - h)e_i \quad (5.10)$$

or

$$\begin{cases} a_0 + \sum_{j=1}^k a_j x_{ij} + (1 - h) \left[s_0 + \sum_{j=1}^k s_j |x_{ij}| \right] > y_i + (1 - h)e_i \\ a_0 + \sum_{j=1}^k a_j x_{ij} - (1 - h) \left[s_0 + \sum_{j=1}^k s_j |x_{ij}| \right] < y_i - (1 - h)e_i \end{cases} \quad (5.11)$$

$$s_j \geq 0, \quad i = 0, 1, \dots, m, \quad j = 0, 1, \dots, k$$

Chang and Ayyub [2001] showed that by increasing the h-certain factor, the confidence interval expands and consequently, it is more probable that out-of-sample values fall within the model. This is comparable to increasing the confidence in statistical regression by increasing the confidence interval.

Approach 2. The fuzzy least-squares regression model

In classical statistical linear regression equation (5.4), the independent variables and coefficients may be either fuzzy or crisp, based on which, 3 cases are possible. It is considerable that in all cases, the dependent variables are naturally fuzzy. These cases are:

Case 1: Fuzzy independent variables and crisp coefficients

$$\widetilde{y}_i = a_0 + a_1 \widetilde{x}_{i1} + \dots + a_k \widetilde{x}_{ik} + \widetilde{\varepsilon}_i, \quad i = 1, 2, \dots, m \quad (5.12)$$

Case 2: Crisp independent variables and fuzzy coefficients

$$\widetilde{y}_i = \widetilde{a}_0 + \widetilde{a}_1 x_{i1} + \dots + \widetilde{a}_k x_{ik} + \widetilde{\varepsilon}_i, \quad i = 1, 2, \dots, m \quad (5.13)$$

Case 3: Fuzzy independent variables and fuzzy coefficients

$$\widetilde{y}_i = \widetilde{a}_0 + \widetilde{a}_1 \widetilde{x}_{i1} + \dots + \widetilde{a}_k \widetilde{x}_{ik} + \widetilde{\varepsilon}_i, \quad i = 1, 2, \dots, m \quad (5.14)$$

where \widetilde{y}_i 's are fuzzy dependent or response variable, \widetilde{x}_{ij} 's are independent variables and a_i 's are coefficients. In fuzzy least-squares regression model, the purpose is the estimation of the coefficients in the relevant equation which fits closely the data. Rearranging equation (5.12):

$$\widetilde{\varepsilon}_i = \widetilde{y}_i - (a_0 + a_1 \widetilde{x}_{i1} + \dots + a_k \widetilde{x}_{ik}), \quad i = 1, 2, \dots, m \quad (5.15)$$

To find the best equation, the error terms should be minimized. The equation that minimizes the sum of the errors is the one that gives the possible fit. As the error terms are either positive or negative, to account for the signs in minimizing the summation of them, the terms must be squared first:

$$\min\left(\sum_{i=1}^m \widetilde{\varepsilon}_i^2\right) = \min \sum_{i=1}^m \{\widetilde{y}_i - (a_0 + a_1 \widetilde{x}_{i1} + \dots + a_k \widetilde{x}_{ik})\}^2 \quad (5.16)$$

There are different methods to implement Fuzzy Least-Squares Regression (FLSR), but the most common approaches are FLSR using distance measures and FLSR using compatibility measures which are described next [Shapiro, 2005].

Fuzzy least-squares regression using distance measures

FLSR using distance measures was first introduced by Diamond [1988] which is the most commonly used methodology in the FLSR. He explains that for the two triangular fuzzy numbers $\widetilde{A}_1 = (a_{1l}, a_{1m}, a_{1r})$ and $\widetilde{A}_2 = (a_{2l}, a_{2m}, a_{2r})$, the distance will be defined as:

$$d^2(\widetilde{A}_1, \widetilde{A}_2) = (a_{1l} - a_{2l})^2 + (a_{1m} - a_{2m})^2 + (a_{1r} - a_{2r})^2 \quad (5.17)$$

The given distance provides a measure of the distance between two fuzzy numbers based on their modes, left and right spreads. It is obvious that this distance satisfies the usual distance

properties, i.e.:

$$\begin{aligned}
 d(\tilde{A}_1, \tilde{A}_2) &\geq 0 \\
 d(\tilde{A}_1, \tilde{A}_2) = 0 &\Leftrightarrow \tilde{A}_1 = \tilde{A}_2 \text{ or } (a_{1l} = a_{2l}, a_{1m} = a_{2m}, a_{1r} = a_{2r}) \\
 d(\tilde{A}_1, \tilde{A}_2) &\leq d(\tilde{A}_1, \tilde{A}_3) + d(\tilde{A}_3, \tilde{A}_2)
 \end{aligned} \tag{5.18}$$

Also, the distance between two trapezoidal numbers $\tilde{A}_1 = (a_{1l}, a_{1m}, a_{1u}, a_{1r})$ and $\tilde{A}_2 = (a_{2l}, a_{2m}, a_{2u}, a_{2r})$ can be defined as:

$$d^2(\tilde{A}_1, \tilde{A}_2) = (a_{1l} - a_{2l})^2 + 0.5(a_{1m} - a_{2m})^2 + 0.5(a_{1u} - a_{2u})^2 + (a_{1r} - a_{2r})^2 \tag{5.19}$$

If $a_{1m} = a_{1u}$ and $a_{2m} = a_{2u}$, there are two triangular numbers and this distance reduces to triangular distance.

For the two fuzzy numbers \tilde{y}_i (observed fuzzy output) and \tilde{Y}_i (calculated fuzzy output of the model) as:

$$\begin{aligned}
 \tilde{y}_i &= (y_i - s_{y_i}^L, y_i, y_i + s_{y_i}^R), \\
 \tilde{Y}_i &= (a_0 + a_1\tilde{x}_{i1} + \dots + a_k\tilde{x}_{ik}) = (m_i - s_i, m_i, m_i + s_i)
 \end{aligned} \tag{5.20}$$

where $m_i = a_0 + \sum_{j=1}^k a_j x_{ij}$ and $s_i = s_0 + \sum_{j=1}^k s_j |x_{ij}|$. Also, the distance of the numbers can be written as:

$$\min \{d(\tilde{y}_i, \tilde{Y}_i)\} = \min \sum \{(y_i - s_{y_i}^L - m_i + s_i)^2 + (y_i - m_i)^2 + (y_i + s_{y_i}^R - m_i - s_i)^2\} \tag{5.21}$$

If a solution exists, the $k+1$ coefficients satisfy a system of $k+1$ equations arising from derivatives associated with equation (5.21) being set to zero [Arabpour and Tata, 2008; Shapiro, 2005].

Fuzzy least-Squares regression using compatibility measures

Celmiņš [1987] introduced an alternate least-squares approach called the compatibility measure between data and the fitted model. Defining μ_A and μ_B as the membership functions of

the two fuzzy quantities \tilde{A} and \tilde{B} , Celmins suggested compatibility measure between \tilde{A} and \tilde{B} as $\gamma(\tilde{A}, \tilde{B})$. Figure (5.12) indicates the definition of γ for two fuzzy numbers (\tilde{A} and \tilde{B}) [Chang and Ayyub, 2001] in various conditions.

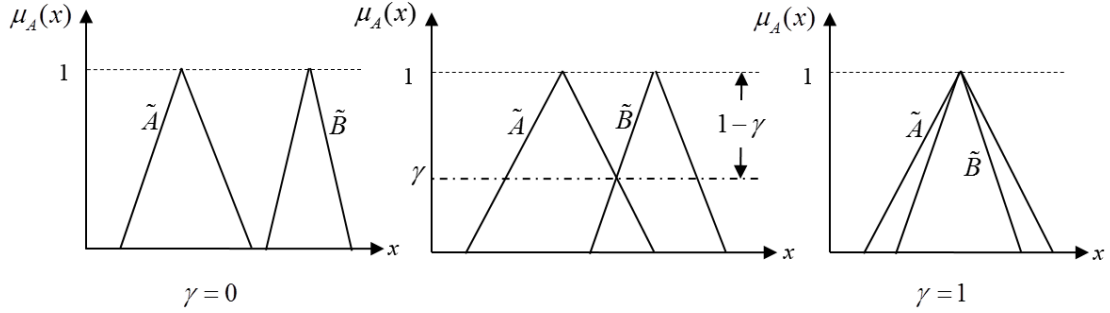


Figure 5.12: Compatibility Measure.

According to Figure (5.12), γ can take values between 0 and 1; the minimum value of γ is when the membership functions are mutually exclusive and the maximum value of γ is gained when the modes of the membership functions coincide.

In Celmins compatibility model which aims at maximizing the compatibility between the data and the fitted model, the objective function for the data which involves minimizing the sum of the squares of the deviations is [Celmiņš, 1987]:

$$\sum_{i=1}^m (1 - \gamma_i)^2 \quad (5.22)$$

As an example, when there is a single crisp independent variable, it can be written as [Chang and Ayyub, 2001]:

$$\tilde{Y} = \tilde{A}_0 + \tilde{A}_1 x = m_0 + m_1 x \pm \sqrt{s_0 + 2s_{01}x + s_1^2 x^2} \quad (5.23)$$

where m_0 and m_1 are determined using weighted least-squares regression and s_0 , s_1 and s_{01} are determined using iteration and desired compatibility measured.

A brief introduction to Fuzzy Linear Regression was presented up to here and the possible solutions to solve equations and gain the coefficients were discussed. In next section, the fuzzy MIMIC method and its structure will be explained.

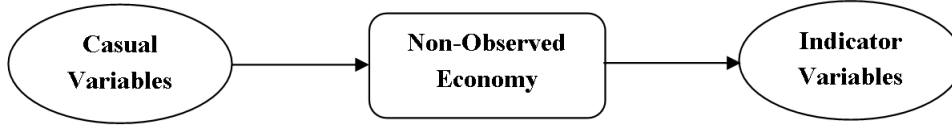


Figure 5.13: Relations among variables in MIMIC Method

5.2.5 Fuzzy MIMIC Structure

It was explained that MIMIC is a common method for the estimation of the underground economy. The Multiple Indicators and Multiple Causes or MIMIC is represented in chapter 6 of Giles and Tedds' book [Giles and Tedds, 2002] as a relation between three sets of variables: first set, Indicator Variables ($Y_{p \times 1}$); second set, Casual Variables ($X_{q \times 1}$) which are exogenous and all of the other variables depend on them directly or indirectly, and last set, a latent variable (η). In Figure (5.13), it is illustrated that the casual and indicator variables are connected by an unobserved or latent variable η which is also scalar:

$$y_t = \lambda \eta_t + \varepsilon_t \quad (5.24)$$

$$\eta_t = \gamma' x_t + \xi_t \quad (5.25)$$

where, $\gamma_{q \times 1}$ and $\lambda_{p \times 1}$ are the vectors of unknown parameters³. It was discussed that this model cannot specify the scale of all of the parameters, so a normalization condition is required and based on Giles convention, it was assumed that the first element of λ is unit, $\lambda_1 = 1$.

It was also mentioned that the error term $\varepsilon_{p \times 1}$ is assumed to have normal distribution with zero mean and variance $\Theta = \text{diag}(\theta_1, \dots, \theta_p)$. The error term ξ (scalar) is also assumed to have normal distribution with zero mean and variance Ψ . It was assumed that these error terms are not correlated with each other over time.

After a brief explanation about MIMIC model, fuzzy MIMIC and its structure will be presented. This fuzzy model is based on structural equation modeling (Giles and Tedds, 2002) which contains a latent variable (the underground economy) and two main sets of variables including: casual variables which affect the latent variable and indicator variables which represent the latent variable variations as depicted in Figure (5.14). This model comprises two sets of equations (equations (5.26) and (5.27)); one between casual variables and latent variable and

³ The vector γ' is transpose of the vector γ and therefore, $\gamma'_{1 \times q}$.

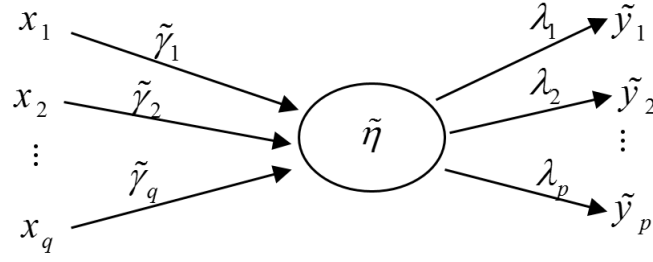


Figure 5.14: Relations among variables in fuzzy MIMIC Method

the other between latent variable and indicator variables:

$$\tilde{y}_t = \lambda \tilde{\eta}_t \quad (5.26)$$

$$\tilde{\eta}_t = \tilde{\gamma}' x_t \quad (5.27)$$

where, $\tilde{y}_{p \times 1}$ is the vector of indicator variables with fuzzy value, $x_{q \times 1}$ is the vector of casual variables or independent variables that affect model without being affected by the model and $\tilde{\eta}$ (scalar) is the latent variable. The latent variable is necessarily a fuzzy variable, so, if x is assumed to be crisp, the coefficients of the first set of equations must be fuzzy ($\tilde{\gamma}_{q \times 1}$). According to the fuzzy nature of the latent variable ($\tilde{\eta}$), \tilde{y} must be fuzzy. Consequently, the coefficients of the second set of equations ($\lambda_{p \times 1}$) can be either fuzzy or crisp which is assumed to be crisp in this study. It is noticeable that the vector $\tilde{\gamma}'_{1 \times q}$ is transpose of the vector $\tilde{\gamma}$.

The model comprising of equations (5.26) and (5.27) cannot specify the scale of all of the parameters, so, a normalization condition is required. To do so, Giles and Tedds [2002] propose the convention of setting the first element of λ to unit, as $\lambda_1 = 1$. The equation (5.28) is written as the reduced form of the two equations (5.26) and (5.27):

$$\tilde{y}_t = \tilde{\Pi} x_t, \quad \tilde{\Pi} = \lambda \tilde{\gamma}' \quad (5.28)$$

Unlike other regression analysis, the uncertainty in the fuzzy regression model is not random, but is fuzziness [Kazemi et al., 2011]. So, the random errors do not appear in fuzzy regressions and the uncertainty of the model emerges in form of spreads or fuzziness of the variables [Bargiela et al., 2007; Donoso et al., 2006; Kazemi et al., 2011].

Based on Giles and Tedds convention, it is assumed that the coefficient λ is a vector ($p \times 1$)

with the first element initialized to one ($\lambda_1 = 1$). So, λ and $\widetilde{\Pi}(= \lambda \widetilde{\gamma}')$ can be written as (equation (5.29)):

$$\lambda = \begin{bmatrix} 1 \\ \lambda_2 \\ \vdots \\ \lambda_{p-1} \\ \lambda_p \end{bmatrix} \Rightarrow \widetilde{\Pi} = \lambda \times \widetilde{\gamma}' = \begin{bmatrix} \widetilde{\gamma}' \\ \lambda_2 \widetilde{\gamma}' \\ \vdots \\ \lambda_{p-1} \widetilde{\gamma}' \\ \lambda_p \widetilde{\gamma}' \end{bmatrix} \quad (5.29)$$

Writing the reduced form equations in full gives:

$$\widetilde{y}_{1t} = \widetilde{\gamma}' x_t \quad (5.30)$$

$$\left\{ \begin{array}{l} \widetilde{y}_{2t} = \lambda_2 \widetilde{\gamma}' x_t \\ \widetilde{y}_{3t} = \lambda_3 \widetilde{\gamma}' x_t \\ \vdots \\ \widetilde{y}_{pt} = \lambda_p \widetilde{\gamma}' x_t \end{array} \right. \quad (5.31)$$

$$\widetilde{y}_{it} = \lambda_i \widetilde{\gamma}' x_t$$

As can be seen, the coefficients of the variable x_t in the equations (5.30) and (5.31) have a constant ratio (This constant ratio is λ_i). So, by substituting equation (5.30) in equation (5.31):

$$\widetilde{y}_{it} = \lambda_i \widetilde{\gamma}' x_t = \lambda_i (\widetilde{\gamma}' x_t) = \lambda_i \widetilde{y}_{1t} \Rightarrow \widetilde{y}_{it} = \lambda_i \widetilde{y}_{1t} \quad (5.32)$$

For more clarification, the case of three indicators ($p = 3$) and six causes ($q = 6$) which is applied to estimate the underground economy in this study will be discussed.

The coefficient λ is a vector (3×1) with the first element set to one ($\lambda_1 = 1$) based on Giles and Tedds' convention. The restricted reduced form equations of the model can be written as

(equation (5.33)):

$$\widetilde{y}_{jt} = \widetilde{\Pi}_j x_t, \quad j = 1, 2 \text{ and } 3 \quad \lambda = \begin{bmatrix} 1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} \Rightarrow \widetilde{\Pi} = \lambda \times \widetilde{\gamma}' = \begin{bmatrix} \widetilde{\gamma}' \\ \lambda_2 \widetilde{\gamma}' \\ \lambda_3 \widetilde{\gamma}' \end{bmatrix} \quad (5.33)$$

Representing the restricted reduced form equation in full gives (as equations (5.34) and (5.35)):

$$\widetilde{y}_{1t} = \widetilde{\gamma}' x_t \quad (5.34)$$

$$\widetilde{y}_{it} = \lambda_i \widetilde{\gamma}' x_t \quad i = 2 \text{ and } 3 \quad (5.35)$$

As it was explained, the resulted equations in the restricted reduced form can be fitted by two different possible methods: the possibilistic regression model and the fuzzy least-squares regression model.

Now, the possibilistic regression model which minimizes the spread of the dependent variable will be used to fit the data in the equations of the fuzzy model. Based on the possibilistic regression model, each equation has an objective function and two constraints. Here, there are three objective functions ($k = 3$) and consequently six constraints for the restricted reduced form equations. To gain the objective function of final non-linear model, the weighted sum method for Multi-Objective optimization [Marler and Arora, 2004] will be used where w_i is the weight of the i^{th} objective function ($F_i(x)$):

$$\text{Final Objective Function } (\underline{FOF}) = \sum_{i=1}^k w_i \times F_i(x) \quad (5.36)$$

There is no priority among these objective functions, thus, the equal weights for objective functions are considered:

$$\underline{FOF} = \sum_{i=1}^k w_i \times F_i(x) \xrightarrow{w_i=m(\text{constant})} \sum_{i=1}^k m \times F_i(x) = m \times \sum_{i=1}^k F_i(x) \quad (5.37)$$

From other side, based on optimization rules in Operation Research (OR), a constant coefficient in objective function of a model has no effect on result and can be ignored. So, the

final objective function can be written as the summation of the objective functions (equation (5.38)). The constraints of the final model are the collection of conditions which were written for each equation.

$$\text{Final Objective Function} = \sum_{i=1}^{k=3} F_i(x) \quad (5.38)$$

Finally, GAMS software ⁴ is used to optimize the resulted model.

The GAMS results are $\lambda_{(3 \times 1)}$ and $\tilde{\gamma}_{(6 \times 1)}$ (or $\tilde{\gamma}'_{(1 \times 6)}$, that each elements of it is a fuzzy number written as $(a_i, s_i) = (a_i - s_i, a_i, a_i + s_i)$, $i = 1, 2, \dots \text{ and } 6$) which are the coefficients in equations (5.26) and (5.27). Different time series of the underground economy are obtained with $\lambda_{(3 \times 1)}$ and $\tilde{\gamma}_{(6 \times 1)}$ based on different *h-certain* factors. These time series can be written as equation (5.27) which indicates the relation between the causal variables (x) and the latent variable variable (η).

⁴ The General Algebraic Modeling System (GAMS) is a high-level modeling system for mathematical programming and optimization. It consists of a language compiler and a stable of integrated high-performance solvers. GAMS is tailored for complex, large scale modeling applications, and allows to build large maintainable models that can be adapted quickly to new situations.

5.3 Summary

In this chapter,:

- Three fuzzy methods have been proposed to estimate the size and trend of the underground economy: fuzzy modeling applying mean and variance, fuzzy modeling applying fuzzy clustering, and structural equation modeling with fuzzy data.
- Fuzzification, fuzzy inference and defuzzification are the three steps of the first two fuzzy methods which have been completely discussed.
 1. The main difference of the first two fuzzy methods is in the fuzzification process which converts the initial crisp data to initial fuzzy data. In other words, two different fuzzifiers were proposed for these methods: one was mean and variance used in the first model and the other one was fuzzy clustering used in the second model.
 2. In fuzzy inference step, the decision making process on initial fuzzy data were done. To do so, the fuzzy rules which are the basis of the fuzzy inference table were used. In other words, impacts of the initial data on the underground economy were considered based on the relationship of the fuzzy terms of the initial data and the underground economy.
 3. The defuzzifier in the defuzzification step converted the linguistic variables of the underground economy including the linguistic terms and their membership function values to the crisp values of the underground economy to make it possible to compare the results, depicting the graphs, etc.
- In the third method, called structural equation modeling with fuzzy data or fuzzy MIMIC (Fuzzy Multiple Indicators Multiple Causes), a solution was proposed to structure equation modeling with fuzzy initial data that involved two main steps: changing the structural equation model to a reduced form, then making a non-linear model from reduced-form equations applying fuzzy linear regression concept and optimizing the resulted non-linear model based on fitting the initial data.

The equations of the MIMIC model for the underground economy are composed of two sets of equations: the equations between the causes and the underground economy as the latent variable, and the equations between the underground economy and the indicators. These two sets of equations were also used in the structural equation modeling of the underground economy in our research and were converted to the reduced form equations during the process of changing the structural equation model to a reduced form. To fit the initial data to the obtained equations in reduced form, a non-linear model was developed applying the fuzzy regression model and its fitting methods. Resulted non-linear model was further optimized for the fuzzy initial data with the GAMS software (a software frequently used in Operation Research).

Chapter 6

Numerical Results and Discussion

Introduction

Three methods including: fuzzy modeling applying mean and variance, fuzzy modeling applying fuzzy clustering, and structural equation approach with fuzzy data were proposed to determine the size and trend of the underground economy. These methods were theoretically described in previous chapter.

It was said that the first two fuzzy modeling methods are composed of three main steps: fuzzification, fuzzy inference and defuzzification. Their main difference is that they apply two different fuzzification methods: one uses fuzzification using mean and variance while the other one applies fuzzification using fuzzy clustering. The two other steps are the same for these fuzzy modeling methods.

A detailed description of the structural equation approach with fuzzy data was also provided in previous chapter. A solution was proposed to convert the structural equation model that consists of two sets of equations (the equations between the causes and the underground economy, and the equations between the indicators and the underground economy) to reduced form equations. The fuzzy linear regression and its fitting methods were then applied to fit the initial data into these reduced form equations which further resulted a non-linear model. The resulted non-linear model was then optimized for the fuzzy initial data with the GAMS software in order to obtain the size of the underground economy.

In this chapter, the proposed methods are applied for five countries; France, Germany, Italy, Canada and the United States. The numerical calculations are illustrated for one of the countries. The same procedure will be applied to other countries. The numerical results obtained for all these mentioned countries via the proposed methods are provided. These results are compared with the findings of MIMIC models (Schneider and Buehn, 2012; Dell’Anno et al., 2007) and a time series estimated by Elgin and Oztunali (2012).

6.1 Fuzzy Modeling Method applying Mean and Variance

6.1.1 Fuzzification

The linguistic terms were defined for the causes and the underground economy. Five linguistic terms of the causes are: *Very Low (VL)*, *Low (L)*, *Normal (N)*, *High (H)* and *Very High (VH)* and the related linguistic terms of underground economy are: *Very Small (VS)*, *Small (S)*, *Average (A)*, *Big (B)* and *Very Big (VB)* same as other methods.

To convert crisp raw data to fuzzy raw data, the average and variance of raw data are required in different periods. Here, the average of five past years up to the desired year and the standard deviation of these years make the structure of this transformation. In Fuzzification step, we deal with pair of causes, i.e. the linguistic terms and membership functions will be determined for each pair of causes.

The fuzzification of initial data for France in 1989 is explained as an example next ¹. The procedure is illustrated for a pair of two causes: unemployment rate (UR) and government regulation (GR). The same procedure will be applied to all other pairs of causes. The two other pairs of causes are : Indirect Tax Rate (ITR) and Inflation Rate (IR) and, Direct Tax Rate (DTR) and Self-employment (SE).

To calculate mean and standard deviation value for this pair of causes (unemployment rate and government regulation) in 1989, the crisp raw data of these causes from 1985 to 1989 are needed. These data and their mean and standard deviation are provided in Table (6.1). From Table (6.1), the value of unemployment rate in 1989 (UR_{1989}) and government regulation in

¹ The sample data used to explain the fuzzy modeling applying mean and variance is related to France.

1989 (GR_{1989}) are respectively 8.34 and 48.93.

Table 6.1: Crisp raw data of two causes: unemployment rate (UR) and government regulation (GR) and their mean and standard deviation.

Year	UR	GR
1985	8.93	51.88
1986	9.08	51.28
1987	9.26	50.71
1988	8.83	50.06
1989	8.34	48.93
Mean (m_N^*)	8.88	50.572
Standard Deviation(SD)	0.346	1.139

Now, the linguistic terms should be assigned to unemployment rate and government regulation in 1989 (UR_{1989} and GR_{1989}). Having mean and standard deviation for each of these causes (Table (6.1)), the value associated with the pick of membership function of each linguistic term is calculated (Table(6.2)). For unemployment rate , these values are: 8.195 for VL ($\mu_{VL}(8.195) = 1.0$), 8.541 for L ($\mu_L(8.541) = 1.0$), 8.888 for N ($\mu_N(8.888) = 1.0$), 9.235 for H ($\mu_H(9.235) = 1.0$) and 9.583 for VH ($\mu_{VH}(9.583) = 1.0$).

According to the value of unemployment rate in 1989 ($UR_{1989} = 8.34$) and the value associated with the pick of membership function of each linguistic term, we can say that UR_{1989} stands somewhere between $Low(L)$ and $Very Low(VL)$ in 1989. Doing the same calculations to determine the value associated with the pick of membership function of each linguistic term for government regulation, we can say that the value of government regulation in 1989 ($GR_{1989} = 48.93$) stands somewhere between $Low(L)$ and $Very Low(VL)$ as it could be seen in Table(6.2).

As Table (6.2), for government regulation and unemployment rate, the actual value of each cause can be associated with one or more than one linguistic terms.

Now, having the linguistic terms for each of the causes, the associated membership functions can be determined using Table (6.3). For unemployment rate with the linguistic term *Very Low (VL)*, we check the conditions given in second column of Table (6.3). The value of unemployment rate ($UR_{1989} = 8.34$) is between $m_N^* - 2 \times SD$ and $m_N^* - SD$, thus, the

Table 6.2: Fuzzification of crisp raw data for unemployment rate and government regulation in 1989.

	VL	L	N	H	VH
	$m_N^* - 2 \times SD$	$m_N^* - SD$	m_N^*	$m_N^* + SD$	$m_N^* + 2 \times SD$
$UR_{1989} = 8.34$	8.195	8.541	8.888	9.235	9.583
	VL	L			
$GR_{1989} = 48.93$	48.294	49.433	50.572	51.711	52.850
	VL	L			

Table 6.3: The linguistic terms and their membership functions. (m_i^* of each linguistic term is the point with maximum membership function $\mu(m_i^*) = 1.0$.)

Linguistic terms	Related Conditions	Membership Function
<i>Very Low (VL)</i>	<i>if</i> $x \leq m_N^* - 2SD$	$\mu_{VL}(x) = 1.0$
	<i>if</i> $m_N^* - 2SD \leq x \leq m_N^* - SD$	$\mu_{VL}(x) = \frac{(m_N^* - SD) - x}{SD}$
$i=(L), (N) \text{ and } (H)$	<i>if</i> $L.B_i \leq x \leq m_i^*$	$\mu_i(x) = \frac{x - (L.B_i)}{SD}$
	<i>if</i> $m_i^* \leq x \leq U.B_i$	$\mu_i(x) = \frac{(U.B_i) - x}{SD}$
<i>Very High (VH)</i>	<i>if</i> $m_N^* + SD \leq x \leq m_N^* + 2SD$	$\mu_{VH}(x) = \frac{x - (m_N^* + SD)}{SD}$
	<i>if</i> $x \geq m_N^* + 2SD$	$\mu_{VH}(x) = 1.0$

membership function is calculated from equation (6.1):

$$\mu_{VL}(x) = \frac{(m_N^* - SD) - x}{SD} = \frac{8.541 - 8.34}{0.346} = 0.58 \quad (6.1)$$

For unemployment rate with the linguistic term *Low(L)*, the value of unemployment rate ($UR_{1989} = 8.34$) is between $L.B_i$ and m_i^* , therefore, the membership function is calculated from the equation (6.2):

$$\mu_L(x) = \frac{x - (L.B_i)}{SD} = \frac{x - (m_N^* - 2 \times SD)}{SD} = \frac{8.34 - (8.88 - 2 \times 0.346)}{0.346} = 0.42 \quad (6.2)$$

For the government regulation and its linguistic terms, the same procedure explained above must be undertaken. Table (6.4) presents the linguistic terms and membership functions of unemployment rate and government regulation in 1989.

Table 6.4: Membership functions and linguistic terms of two causes: unemployment rate and government regulation in 1989.

Causes		Linguistic Terms				
		Very Low	low	Normal	High	Very High
UR	Linguistic Term(s)	VL	L			
	Membership Function	0.58	0.42			
GR	Linguistic Term(s)	VL	L			
	Membership Function	0.44	0.56			

Figure (6.1) depicts the membership function of unemployment rate in 1989.

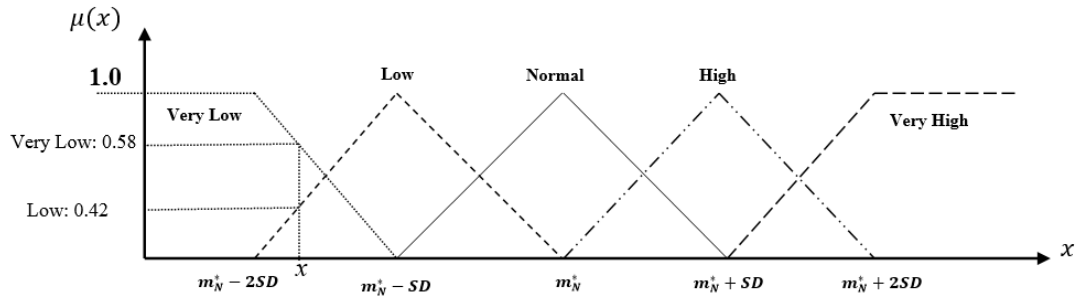


Figure 6.1: The associated membership function of unemployment rate in 1989.

6.1.2 Fuzzy Inference

Now, having the linguistic terms and membership functions for the two causes, the linguistic terms of this pair of causes and its related degree can be determined. We obtained two linguistic terms for each of the causes unemployment rate and government regulation. Thus, at most $2 \times 2 = 4$ decision rules with associated degrees must be applied to determine the linguistic terms and membership functions for the pair of these two causes. Different combinations of the linguistic terms of unemployment rate and government regulation with their related decision rules are presented in Table (6.5).

To determine decision rules, linguistic terms and degrees, we use Table (6.6). To calculate the membership function of the pair of causes, we consider minimum value of the membership

Table 6.5: Fuzzy rules and results for the pair of two causes (unemployment rate and government regulation) using fuzzification with mean and variance method.

Order	UR	GR	Rule	Linguistic term and Degree	Degree \times Min { TR, GR }	Linguistic term and Membership function
1	VL	VL	(5,5)	VS, 1.0	$1.0 \times 0.44 = 0.44$	VS, 0.44
2	VL	L	(5,4)	VS, 1.0	$1.0 \times 0.56 = 0.56$	VS, 0.56
3	L	VL	(4,5)	VS, 1.0	$1.0 \times 0.42 = 0.42$	VS, 0.42
4	L	L	(4,4)	S, 1.0	$1.0 \times 0.42 = 0.42$	S, 0.42

functions of the two causes, thus, min-operator is used (equation (6.3)):

$$\mu_{Cause\ 1\ and\ cause\ 2} = Min\{\mu_{Cause\ 1}, \mu_{Cause\ 2}\} \times d \quad (6.3)$$

where, d is the degree written next to each linguistic term in Table (6.6).

Table 6.6: Table of fuzzy rules and relevant degrees. Rule (i,j) represents the rule in row i and column j.

			Cause 2				
			1	2	3	4	5
			VH	H	N	L	VL
Cause 1	1	VH	VB, 1	VB, 1	B, 1	B, 0.8	A, 0.5
	2	H	VB, 0.95	B, 1	B, 0.8	A, 1	S, 0.8
	3	N	B, 0.95	B, 0.75	A, 1	S, 0.8	S, 1
	4	L	B, 0.75	A, 0.95	S, 0.75	S, 1	VS, 1
	5	VL	A, 0.45	S, 0.75	S, 0.95	VS, 1	VS, 1

In Table (6.5), the rules in rows 1, 2 and 3 result in the same linguistic term for the pair of causes (VS) but with different membership functions. It is clear that only one membership function is acceptable. Based on fuzzy logic, OR-operator (or max-operator) must be used, thus:

$$\text{Rows 1, 2 and 3} = \text{the same linguistic term (VS) and } Max(0.44, 0.56, 0.42) = VS, 0.56$$

Finally, two linguistic terms with their membership functions are obtained for the pair of two causes (unemployment rate and government regulation): VS, 0.56 and S, 0.42 (Table 6.5).

The linguistic terms with their membership functions must be determined for other pairs of causes (totally 3 pairs of causes) repeating the procedure explained above. Then, using these linguistic terms and membership functions, the linguistic term and membership function of the underground economy can be obtained in aggregation step.

6.1.3 Defuzzification

The outputs of fuzzy inference, explained above, including linguistic terms and membership functions of three pairs of causes are not crisp, thus, they must be converted to crisp data for comparing or ranking through defuzzification. But, before defuzzification, the aggregation must be done. In aggregation step, having the linguist terms and membership functions for three pairs of causes, the linguistic terms and membership functions of the underground economy will be determined.

Before going to aggregation step, the final results of fuzzy inference including linguistic terms and membership functions of three pairs of causes (UR and GR, ITR and IR, and DTR and SE) are summarized in Table (6.7):

Table 6.7: Final results of fuzzy inference step using fuzzy method with mean and variance: linguistic terms and membership functions of the pairs of causes

Pair of Causes	Linguistic term and membership function
UR and GR	S, 0.42
	VS, 0.56
ITR and IR	S, 0.53
	VS, 0.05
DTR and SE	S, 0.58
	VS, 0.42

In aggregation step, three possible operations can be performed on fuzzy inference results (Table (6.7)):

- Maximum : pointwise maximum over all fuzzy sets;
- Sum : pointwise sum over all fuzzy sets; and,

- Probabilistic sum or probabilistic OR.

In both fuzzy models proposed in our work, maximum operation is used to aggregate fuzzy inference results. It means that when different values of membership function are assigned to the same linguistic term in one year, the fuzzy *OR-operator* must be used which implies that the *maximum* membership function must be selected. In other words, the membership function with the highest value is assigned to each linguistic term of the underground economy.

So, for the linguistic term *S* and *VS* :

$$\text{Same linguistic terms (S) and Max (0.42, 0.53, 0.58) = S, 0.58}$$

$$\text{Same linguistic terms (VS) and Max (0.56, 0.05, 0.42) = VS, 0.56}$$

The linguistic terms and membership functions of the underground economy in 1989 are presented in Table (6.8).

Table 6.8: Final results of aggregation step (inputs of defuzzification step) in fuzzy method using means and variance.

Linguistic term and membership function	
Underground Economy	VS, 0.56
	S, 0.58
	N, 0.0
	B, 0.0
	VB, 0.0

To convert the fuzzy data of the underground economy obtained in aggregation step to crisp data, Average of First Maxima (equation (6.4)) [Ross, 2004; Yen and Langari, 1998] is used in defuzzification step:

$$\text{Index of UE} = \frac{\sum_{i=1}^n (m_i^* \pm (1 - \mu_i) \times S D_i)}{n} \quad \begin{cases} + & \text{VS Linguistic term} \\ - & \text{Other Linguistic terms} \end{cases} \quad (6.4)$$

where, m_i^* is the value of each linguistic term of the underground economy with maximum membership function $\mu(m_i^*) = 1.0$, μ_i is the associated membership function to the linguistic

term, SD_i is the standard deviation related to each linguistic term and n represents the number of linguistic terms of the underground economy in the year under analysis.

The main question arises here is that what are the related values of linguistic terms of the underground economy or in other words, what are the values of m_i^* in equation (6.4)?

About the values of linguistic terms of the underground economy, Lindstrom explains that people evaluate various concepts differently; as an example, the phrase "high unemployment rate" can be described in many ways depending on the condition. He adds that in Sweden, where the unemployment rate once used to be below 3%, a high unemployment rate means any rate above 5%. However, in another country, the unemployment rate cannot be considered high unless it exceeds 10% [Lindström, 1998].

By this explanation, the maximum estimated value for the underground economy of France is 28.5% of GDP which is estimated by Dell'Anno et al. [2007], while Schneider et al. [2010] believes that the expected interval for the size of the underground economy is between 13% and 16% of GDP. Based on these values, Figure (6.2) is proposed for the linguistic terms of the underground economy in France. For other countries, the linguistic terms and their membership functions are similarly determined.

According to Figure (6.2), if the underground economy is very small then m_1^* will be 0.05;

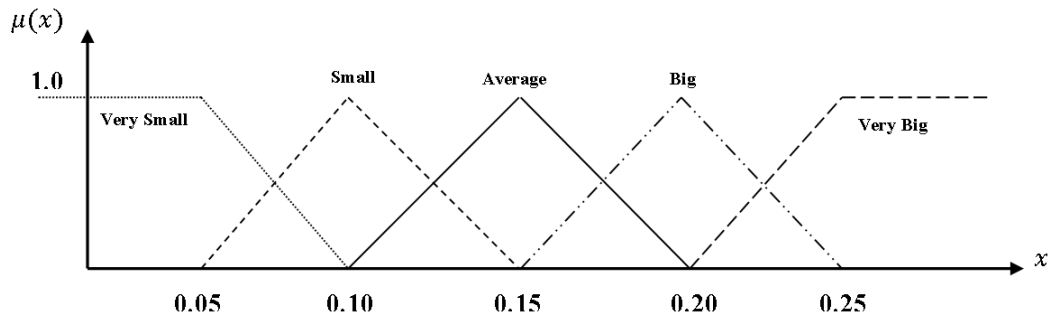


Figure 6.2: The linguistic terms of the underground economy (in France).

if the underground economy is small then m_2^* will be 0.10; if the underground economy is moderate then m_3^* will be 0.15; if the underground economy is big then m_4^* will be 0.20 and if the underground economy is very big then m_5^* will be 0.25.

Now, the results presented in Table (6.8) can be converted to crisp data for comparing or ranking through defuzzification. Using Average of First Maxima, the index of the underground

economy in 1989 will be:

$$\text{Index of UE} = \frac{(0.05 - (1 - 0.56) \times 0.05) + (0.1 - (1 - 0.58) \times 0.05)}{2} = 0.755$$

This result indicates that the underground economy in 1989 is 7.55 percent of GDP. The same calculations presented for 1989 are performed for the next years up to 2010. The final results are presented in result and discussion section.

6.2 Fuzzy Modeling Method applying Fuzzy Clustering

6.2.1 Fuzzification

It was explained already that the causes are transformed to five linguistic terms: *Very Low* (VL), *Low* (L), *Normal* (N), *High* (H) and *Very High* (VH) and the related linguistic terms of underground economy are *Very Small* (VS), *Small* (S), *Average* (A), *Big* (B) and *Very Big* (VB) same as other methods.

Next, fuzzy c-means clustering is used to generate the matrix of membership functions for each cause. The algorithm of fuzzy c-means method is depicted in Figure (6.4). The final

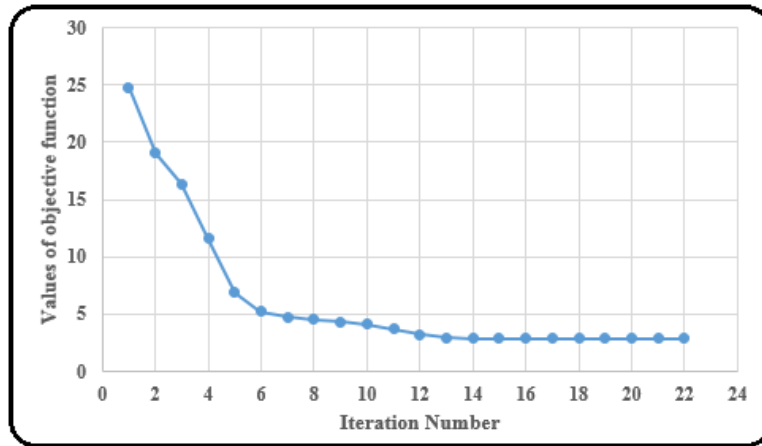


Figure 6.3: The different values of objective function in different iterations (Cause: indirect tax rate of Canada).

matrix of membership function has n rows (number of the years) and c columns (number of clusters). The number of clusters is equal to the number of linguistic terms; five here, which means that each cluster stands for one linguistic term. The termination condition of iteration

in fuzzy c-means clustering is that the difference of two consecutive objective functions in two consecutive iterations (epsilon) must be below 10^{-5} (0.00001).

To clarify how the iterations are done, the diagram of objective function in different iterations for indirect tax rate (ITR) of Canada is depicted in Figure (6.3). As can be seen in Figure (6.3), the algorithm satisfies the termination condition in 22th iteration. The final matrix of membership function (for ITR) in final iteration for recent years (2001-2010) is presented in Table (6.9).

Table 6.9: The membership functions of indirect tax rate (ITR); result of fuzzy c-means algorithm.

	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5		⇐ Clusters	
	31.7885	28.8434	26.2532	25.1866	24.0967		⇐ $V_{(i_{final})}$	
Year	VH	H	N	L	VL		⇐ Linguistic terms	
2001	0.0001	0.0002	0.0020	0.9954	0.0023		⇐ Matrix of membership function $(U_{(i_{final})})$	
2002	0.0008	0.0039	0.9753	0.0156	0.0044			
2003	0.0000	0.0001	0.9997	0.0002	0.0000			
2004	0.0036	0.0137	0.5741	0.3635	0.0452			
2005	0.0004	0.0014	0.0205	0.9659	0.0118			
2006	0.0020	0.0057	0.0348	0.2201	0.7372			
2007	0.0001	0.0002	0.0011	0.0041	0.9944			
2008	0.0050	0.0119	0.0440	0.1145	0.8246			
2009	0.0015	0.0038	0.0158	0.0486	0.9303			
2010	0.0011	0.0031	0.0174	0.0917	0.8867			

Based on vector $V(i)$ which indicates the center of clusters, the linguistic terms are assigned to the clusters. Each element of this matrix indicates the membership function of the related cluster (or linguistic term) in the related year. In other words, each value of membership function indicates the degree of dependency of input value to the related cluster or linguistic term.

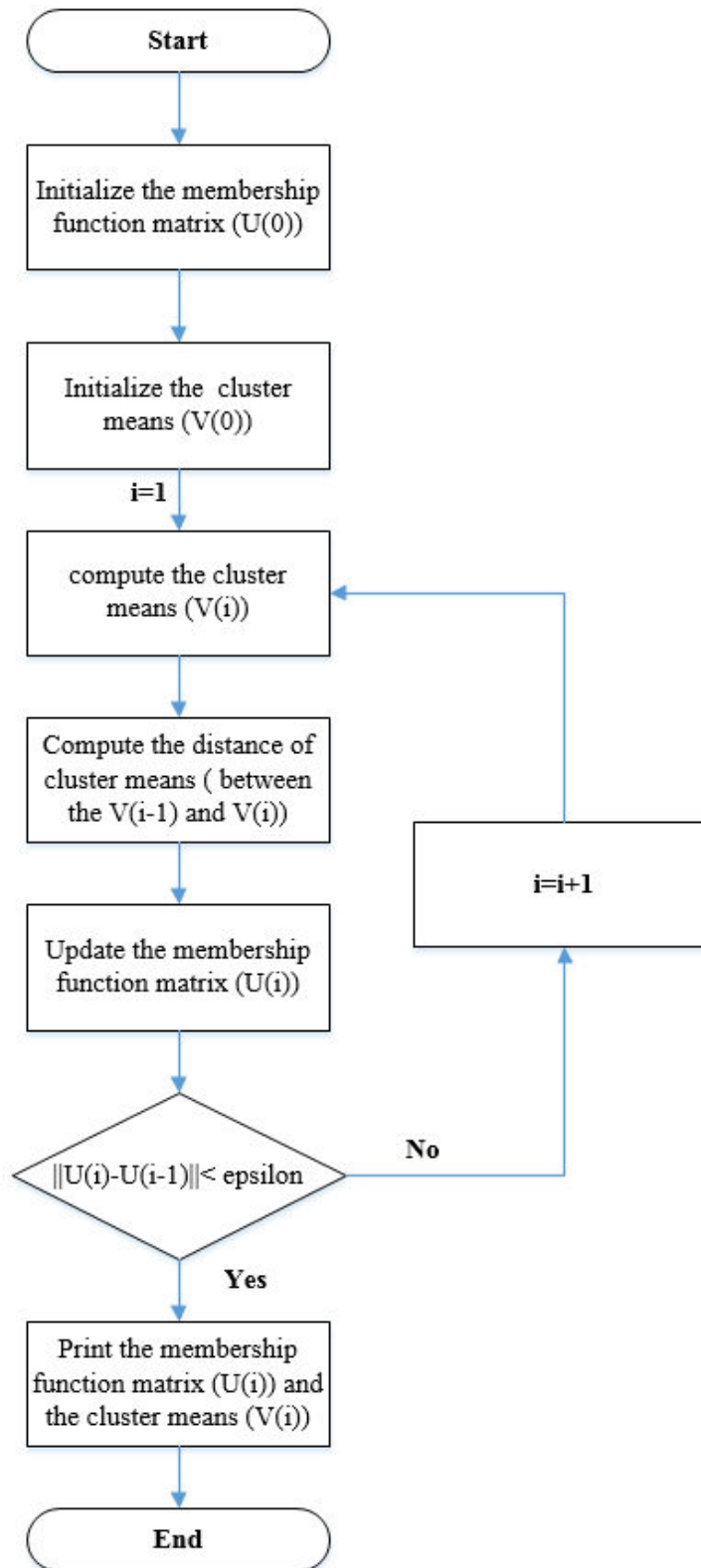


Figure 6.4: Algorithm of the fuzzy c-means method.

6.2.2 Fuzzy Inference

Now, the linguistic terms of the underground economy can be obtained. In most cases, each cause has five different linguistic terms and membership functions based on the number of clusters defined in c-means clustering. Consequently, each pair of causes has twenty five decision rules and degrees which will be used to get the linguistic terms and membership functions of the underground economy. If several membership functions are obtained for a linguistic terms, the maximum membership function will be assigned to that linguistic term, So, *Max-operator* is used. Different combinations of linguistic terms for two causes (Indirect tax rate and inflation rate of Canada) as well as decision rules in 1989 are summarized in Table (6.10).

Table 6.10: Fuzzy rules and results for the pair of two causes (indirect tax rate and inflation rate of Canada) in 1989 using fuzzification clustering method.

Order	ITR	IR	Rule	Linguistic term and Degree	Degree \times Min { UR, IR }	Linguistic term and Membership function
1	VH	VH	(1,1)	VB, 1.0	$1.0 \times (0.0389, 0.9486) = 0.0389$	VB, 0.0389
2	VH	H	(1,2)	VB, 1.0	$1.0 \times (0.0389, 0.0427) = 0.0389$	VB, 0.0389
...						
6	H	VH	(2,1)	VB, 0.95	$0.95 \times (0.5620, 0.9486) = 0.5334$	VB, 0.5334
...						
14	N	L	(3,4)	S, 0.8	$0.8 \times (0.2622, 0.0028) = 0.0023$	S, 0.0023
...						
24	VL	L	(5,4)	VS, 1.0	$1.0 \times (0.0455, 0.0028) = 0.0028$	VS, 0.0028
25	VL	VL	(5,5)	VS, 1.0	$1.0 \times (0.0455, 0.0014) = 0.0014$	VS, 0.0014

The rules in the rows 1, 2 and 6 result in the same linguistic term for the underground economy but with different membership functions. As described, based on fuzzy logic, *OR-operator* must be applied, thus:

Rows 1, 2 and 6 = the same linguistic term (VB) and $Max(0.0389, 0.0389, 0.5334) = VB, 0.5334$

Therefore, 1th and 2nd rows and their results must be ignored and the linguistic term (VB) with

membership function value of 0.5334 go to next step. The same procedure will be applied to other linguistic terms until all linguistic terms and membership functions of all pairs of causes are obtained.

6.2.3 Defuzzification

Table (6.11) summarizes the linguistic terms and membership functions of each pair of causes. These linguistic terms and membership functions are resulted from fuzzy inference step for the causes of Canada in 1989.

Table 6.11: Final results of fuzzy inference step using fuzzy clustering method: linguistic terms and membership functions of the pairs of causes

Pair of Causes	UR and GR	ITR and IR	DTR and SE
Linguistic term and membership function	VB, 0.0011	VB, 0.5334	VB, 0.0001
	B, 0.0101	B, 0.2490	B, 0.0008
	A, 0.0801	A, 0.0411	A, 0.0213
	S, 0.6431	S, 0.0320	S, 0.6674
	VS, 0.0146	VS, 0.0028	VS, 0.0559
UR:Unemployment rate		GR:Government regulation	
ITR:Indirect tax rate		IR:Inflation rate	
DTR:Direct tax rate		SE:Self employment	

Next, the result of the inference step must be aggregated. Based on using the *OR-operator* as aggregation operator, the maximum value of the membership function of each linguistic terms or the maximum value of each row are selected as final membership function, as an example, the final membership function of the linguistic term **VB** is:

$$\text{Row 1} = \text{the same linguistic term (VB) and } \text{Max}(0.0011, 0.5334, 0.0001) = \text{VB}, 0.5334$$

Table (6.12) illustrates the result of aggregation which is initial data of the defuzzification step.

To convert the fuzzy data of the underground economy obtained in aggregation step to crisp

Table 6.12: Final results of aggregation step (inputs of defuzzification step) in fuzzy clustering method.

Underground Economy	Linguistic term and membership function
	VB, 0.5334
	B, 0.2490
	A, 0.0801
	S, 0.6674
	VS, 0.0559

data, Center Of Area (COA)(equation (6.5) [Ross, 2004; Yen and Langari, 1998] is used for defuzzification:

$$\text{Index of underground economy} = \frac{\sum_{i=1}^n (\mu_i \times m_i^*)}{\sum_{i=1}^n \mu_i} \quad (6.5)$$

where, m_i^* is the value of each linguistic term of the underground economy with maximum membership function $\mu(m_i^*) = 1.0$, μ_i is the associated membership function with the linguistic term and n represents the number of linguistic terms of the underground economy in the year under analysis.

To determine the values of linguistic terms of the underground economy or the values of m_i^* in equation (6.5), Lindstrom description is applied. Regarding this description and based on Figure (6.5), the values of m_i^* for Canada are obtained and summarized in Table (6.13).

Table 6.13: The related values (m_i^*) of the linguistic terms of the underground economy in different countries.

Linguistic term	The value of m_i^*				
	Canada	France	Germany	USA	Italy
VB	$m_{VB}^* = 0.20$	$m_{VB}^* = 0.25$	$m_{VB}^* = 0.24$	$m_{VB}^* = 0.16$	$m_{VB}^* = 0.38$
B	$m_B^* = 0.18$	$m_B^* = 0.20$	$m_B^* = 0.20$	$m_B^* = 0.13$	$m_B^* = 0.34$
A	$m_A^* = 0.16$	$m_A^* = 0.15$	$m_A^* = 0.16$	$m_A^* = 0.10$	$m_A^* = 0.30$
S	$m_S^* = 0.14$	$m_S^* = 0.10$	$m_S^* = 0.12$	$m_S^* = 0.07$	$m_S^* = 0.26$
VS	$m_{VS}^* = 0.12$	$m_{VS}^* = 0.05$	$m_{VS}^* = 0.08$	$m_{VS}^* = 0.04$	$m_{VS}^* = 0.22$

Defuzzification of the linguistic terms and membership functions of the underground economy

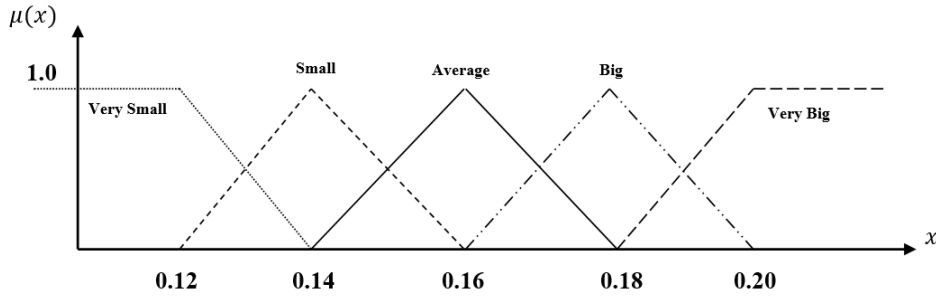


Figure 6.5: Linguistic terms of the underground economy in Canada.

in that particular year is done as follows regarding Tables (6.12) and (6.13):

$$\begin{aligned}
 UE_{1989} &= \frac{\sum_{i=1}^n (\mu_i \times m_i^*)}{\sum_{i=1}^n \mu_i} = \frac{0.20 \times 0.5334 + \dots + 0.12 \times 0.0559}{0.5334 + \dots + 0.0559} \\
 &= 0.1668
 \end{aligned}$$

This result indicates that the underground economy of Canada in 1989 is 16.7 percent of GDP. The calculations for the next years up to 2010 are done as explained. The final results are presented in result and discussion section.

6.3 Result and discussion of fuzzy models

Two different estimation methods were introduced thoroughly until here. These methods were used to estimate the underground economy of different countries. Table (6.14) presents the average estimated values of the underground economy for these fuzzy models ², MIMIC [Dell'Anno et al., 2007; Schneider and Buehn, 2012] and Elgin and Oztunali [2012]'s macro economic model for different countries in the period 1989 to 2010.

It is clear from the Table (6.14) that there is no significant difference between average estimated values of the underground economy for different countries in different methods. There is only a significant difference in estimation of Elgin and Oztunali [2012] for France with other methods which needs to be discussed.

About the estimations for Canada (Figure (6.6)), the trend of both fuzzy models are very similar. On the whole, all models show a decreasing trend for the size of underground economy

² whole of the tables of these models are presented in Appendix.

Table 6.14: Average of results of fuzzy models (clustering and mean and variance), MIMIC and Elgin's macro-economic model.

	Fuzzy Models		MIMIC	Elgin and
	Mean and Variance	Clustering	Schneider (2012) Dell'Anno (2007)	Oztunali (2012)
Canada	15.00	15.10	15.61	16.53
France	12.58	12.86	12.84	15.54
Germany	15.21	15.96	15.68	15.44
Italy	27.68	29.72	28.34	26.92
USA	8.60	9.48	8.75	9.36

while fuzzy models have some oscillations.

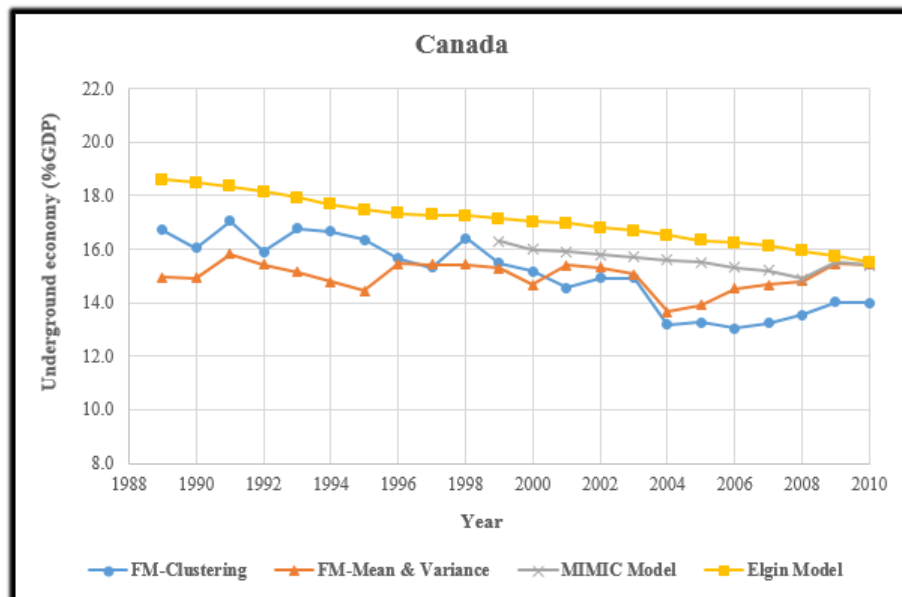


Figure 6.6: Time series of Canada's underground economy estimated by fuzzy models (clustering and mean and variance), Elgin-Oztunali model and MIMIC.

From Figure(6.6), it can be seen that the size of the underground economy has increased significantly after the economic crisis in 2008 based on MIMIC model and fuzzy models whilst Elgin and Oztunali [2012]'s model shows a constant downward trend which is different from what is expected.

For France also, there is a significant difference between the average of Elgin and Oztunali [2012]'s estimation and other models (Figure (6.7)). The MIMIC estimation of Schneider

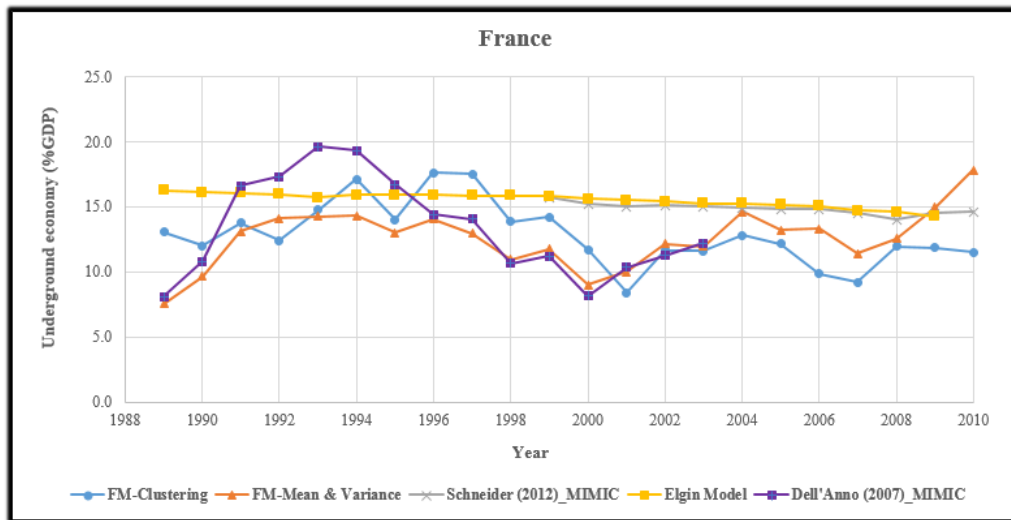


Figure 6.7: Time series of France's underground economy estimated by fuzzy models (clustering and mean and variance), Elgin-Oztunali model and MIMIC.

and Buehn [2012] are very similar to the estimation of Elgin and Oztunali [2012] while this similarity exists between the trend of the fuzzy models and the MIMIC estimation of the Dell'Anno et al. [2007] in first 14 years. It must be also considered that these estimations exist while Frey and Schneider [2000] indicated that expected interval for the size of France's underground economy is between 13% and 16% of GDP.

As illustrated in Figure (6.8), the trend of the underground economy is increasing between 2000-2010 unlike the Elgin-Oztunali result.

The effects of the two causes, unemployment rate and government regulation, on the size

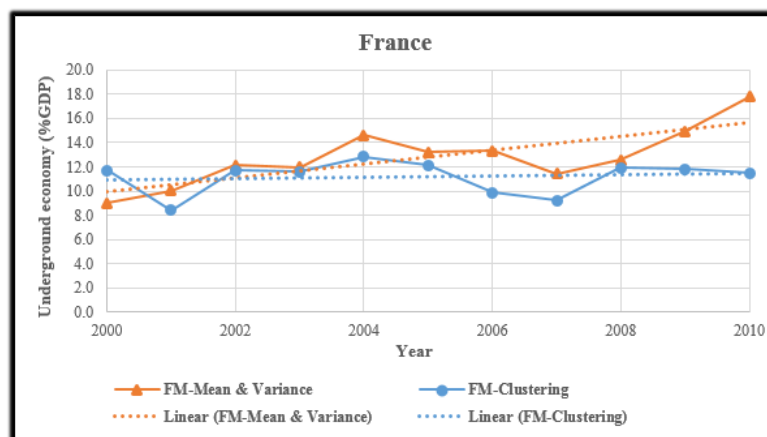


Figure 6.8: The trend of time series of France's underground economy between 2000-2010 estimated by fuzzy models).

of France's underground economy is depicted in Figure (6.9). The magnitude of each cause

is indicated on one axis and different colors of diagram represent the magnitude of the underground economy. As it was expected, increasing the unemployment rate together with government regulations, the size of the underground economy increases. Also, in a condition with a definite government regulations (or unemployment rate), the increase in unemployment rate (or government regulations) will augment the size of the underground economy. It can be seen that it is possible to make a trade-off between the intensity of unemployment rate and government regulation to control the size of the underground economy; as an example, by increasing the unemployment rate from 6 to 8, if the government does not change the regulations, the size of the underground economy will not change for every amount of the government regulation. Also, if the unemployment rate increases up to 10 but the government does not change the regulations, the size of the underground economy will not change. In this case, even if the government regulations increases, the size of the underground does not vary significantly. In another case, if the government regulations are controlled to 52, even if the unemployment rate increases, the size of the underground economy will be still acceptable. Thus, it can be observed that there is a trade-off between the intensity of unemployment rate and government regulation which controls the size of the underground economy.

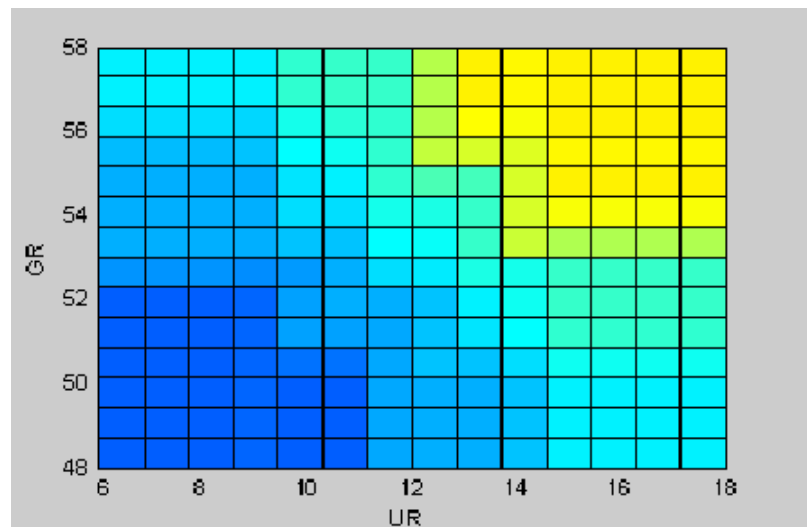


Figure 6.9: Effects of two causes (unemployment rate and government regulation) on the size of France's underground economy.

Considering the effects of government regulation on the size of the underground economy while the magnitudes of other causes are kept constant (Figure (6.10)), it would be possible

to decide about the regulations related to the underground economy such as determining the punishment of entering the underground activities, etc.

As Figure (6.10) depicts, it seems that increasing the amount of government regulations up to 50, the size of the underground economy remains the same. But beyond that limit, the size of the underground economy increases with increasing the amount of government regulations up to 56. Afterwards, the size of the underground economy does not change with further increase in the amount of regulations. Thus, understanding how different causes affect the size of underground economy, the decisions made about the regulations related to the underground economy will be more accurate and promising.

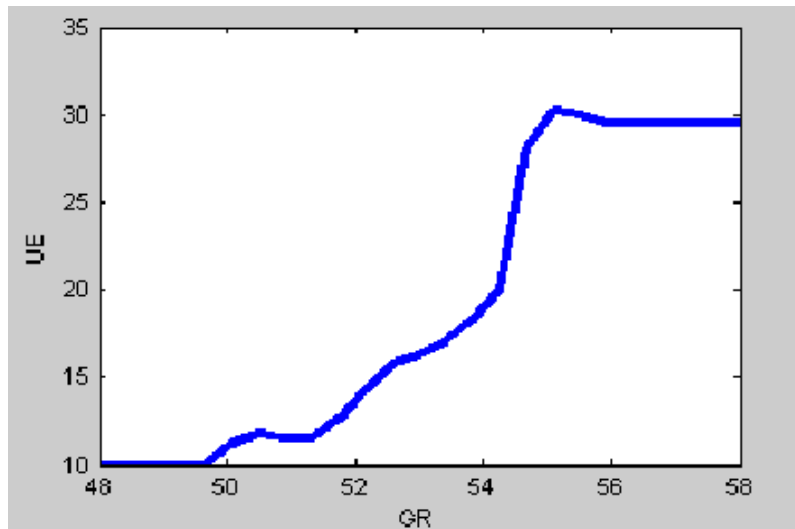


Figure 6.10: The effect of GR on the underground economy (constant values for others) in France.

As it is illustrated in Figure (6.11), the trend of Germany's underground economy is nearly constant. Unlike other estimations, fuzzy modeling applying fuzzy clustering have some oscillations in its trend. The trend and values of MIMIC and fuzzy modeling applying mean and variance are very similar after 2000, also those of Elgin and Oztunali [2012]'s.

Table (6.14) represents also that the average estimated values of all methods are nearly equal. About Germany, it is considerable that after 2008, the size of the underground economy increases. Although, Germany's underground economy has an increasing trend like France based on fuzzy models, but its rising rate is lower than that of France. It must be considered that the increasing trend of underground economy in Germany is also estimated by MIMIC

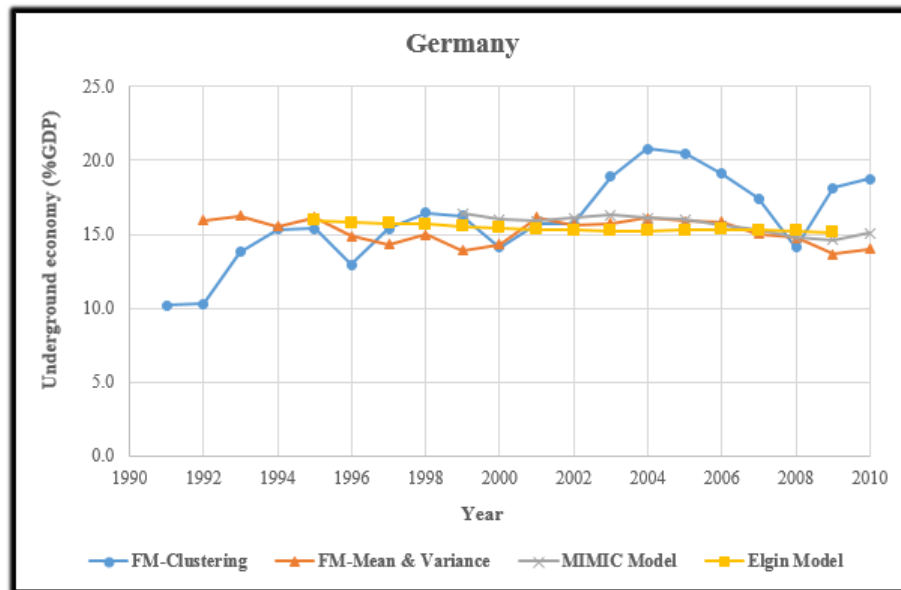


Figure 6.11: Time series of Germany's underground economy estimated by fuzzy models (clustering and mean and variance), Elgin-Oztunali model and MIMIC.

Buehn and Goethel [2010]. Both fuzzy models estimate the value of underground economy in 2004 as maximum in the period 1989 to 2010. About Italy's underground economy, the average values of Elgin and Oztunali [2012]'s estimations are different from other estimations (Figure (6.12)). Fuzzy models represent similar trends for Italy's underground economy with an ascending trend after 2008 just like in MIMIC estimation by Schneider and Buehn [2012]. It is considerable that Italy holds the maximum average estimated value of the underground economy among the countries studied in this thesis.

Figure (6.13) illustrates the trend of the underground economy in the United States. It shows that the underground economy has an increasing trend in the united states after economic crisis in 2008. Also, the similar trend of estimated by fuzzy model applying clustering and MIMIC by Schneider and Buehn [2012] is a highlight point of the united states underground economy from 2006-2010. About the average estimated value, the average of MIMIC and fuzzy modeling applying mean and variance are very close, unlike the results of fuzzy modeling applying fuzzy clustering and Elgin and Oztunali [2012]'s estimation which have close averages.

It can be claimed that the fuzzy modeling applying fuzzy clustering method can be applied to estimate the underground economy in different countries as the results have sufficient robust

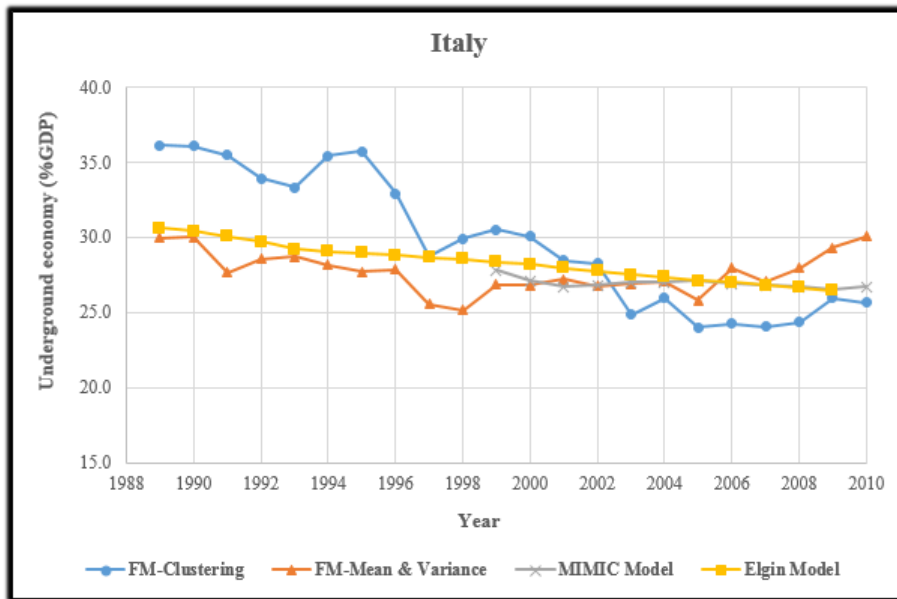


Figure 6.12: Time series of Italy's underground economy estimated by fuzzy models (clustering and mean and variance), Elgin-Oztunali model and MIMIC.

in their trend and size. As a proof, some analyses to examine the robustness of the resulting underground economy series have been carried out and it revealed that the results are not sensitive to the assumptions of the model. For instance, changing the degree used in fuzzy statement, the number of linguistic terms in the fuzzification step or the fuzzification method, or using the median instead of the mean, the trend of the results won't change.

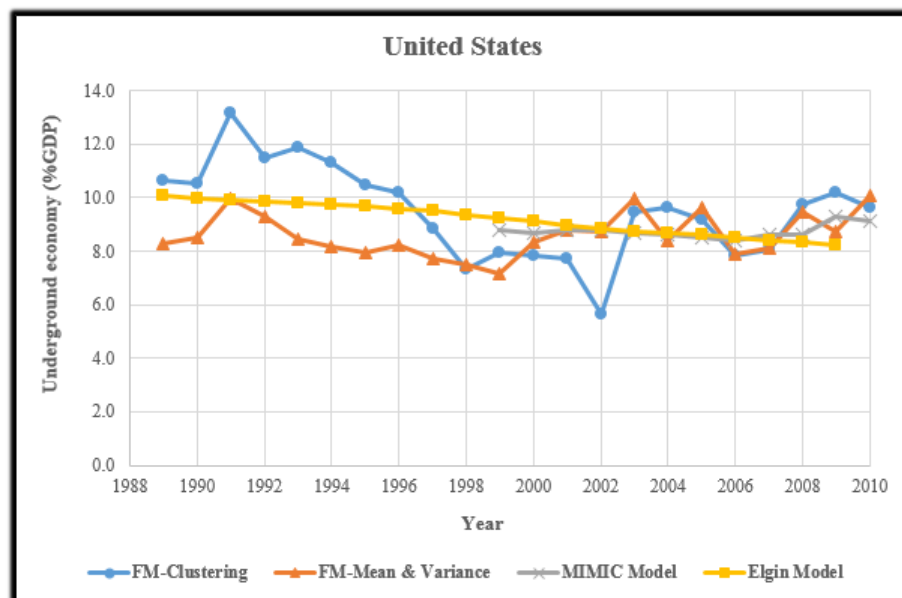


Figure 6.13: Time series of the United States' underground economy estimated by fuzzy models (clustering and mean and variance), Elgin-Oztunali model and MIMIC.

6.4 Fuzzy MIMIC Method

As it was explained in previous chapter, the equations of fuzzy MIMIC are firstly converted to reduced form equations (equations (6.6) and (6.7)):

$$\widetilde{y}_{1t} = \widetilde{\gamma}' x_t \quad (6.6)$$

$$\widetilde{y}_{it} = \lambda_i \widetilde{\gamma}' x_t \quad i = 2 \text{ and } 3 \quad (6.7)$$

The resulted equations in the restricted reduced form are then fitted by possibilistic regression model to determine the coefficients of the fuzzy model. This approach focuses on minimizing the spread of the dependent variable by considering adequate containment of data.

Based on possibilistic regression model, we have three equations in the restricted reduced form and each equation has an objective function and two constraints. There are totally three objective functions ($k = 3$) and consequently six constraints for the restricted reduced form equations.

It was also proved that the final objective function is written as the summation of the objective functions and the constraints of the final model are the collection of conditions written for each equation. So, the final objective function (objective functions of equations (6.6) and (6.7)) is (equation (6.8)):

$$\min \left\{ \sum_{j=1}^3 \lambda_j \times \left\langle \sum_{t=1}^{T_f} \sum_{i=1}^6 S_i |x_{it}| \right\rangle \right\} \quad s_i \geq 0 \quad (6.8)$$

Each objective function has two constraints ((equations (6.9) and (6.10))):

$$\lambda_j \times \sum_{i=1}^6 a_i x_{it} + (1 - h) \times \left\{ \lambda_j \times \sum_{i=1}^6 s_i x_{it} \right\} > y_{jt} \quad , \quad \forall j, j = 1, 2 \text{ and } 3 \quad (6.9)$$

$$\lambda_j \times \sum_{i=1}^6 a_i x_{it} - (1 - h) \times \left\{ \lambda_j \times \sum_{i=1}^6 s_i x_{it} \right\} < y_{jt} \quad , \quad \forall j, j = 1, 2 \text{ and } 3 \quad (6.10)$$

In the resulting model, there 162 main mathematical expressions when optimizing the objective function. Some additional mathematical expressions are also added as constraints of the model; for example, the expressions which are imposed on the model due to the positive and negative

variables in the objective function.

Using GAMS, the coefficients of equations (6.6) and (6.7) are calculated: $\lambda_{(3 \times 1)}$ and $\tilde{\gamma}_{(6 \times 1)}$ (or $\tilde{\gamma}'_{(1 \times 6)}$). It is noteworthy that each element of $\tilde{\gamma}'_{(1 \times 6)}$ is a fuzzy number written as $\tilde{a}_i = (a_i, s_i) = (a_i - s_i, a_i, a_i + s_i)$, $i = 1, 2, 3, \dots \text{ and } 6$. The time series of the underground economy based on various h-certain factors can be written as equation (6.7) which is the relation between the causal variables ($x_{(6 \times 1)}$) and the latent variable ($\tilde{\eta}$).

The fuzzy number of underground economy ($\tilde{\eta}$) in each year (t) (equation (5.27)) can be written as:

$$\begin{aligned} \tilde{\eta}_t = \tilde{\gamma}' x_t = \tilde{a}_1 x_{1t} + \dots + \tilde{a}_6 x_{6t} &= \sum_{i=1}^6 \tilde{a}_i x_{it} = \sum_{i=1}^6 (a_i, s_i) x_{it} = \sum_{i=1}^6 (a_i - s_i, a_i, a_i + s_i) x_{it} \\ &= \left(\sum_{i=1}^6 (a_i - s_i) x_{it}, \sum_{i=1}^6 a_i x_{it}, \sum_{i=1}^6 (a_i + s_i) x_{it} \right) \end{aligned} \quad (6.11)$$

$t=1,2,\dots,T_f$

Based on equation (6.11), the central value (equation (6.12)) and left and right bounds (equations (6.13) and (6.14)) of the underground economy in year (t) are:

$$\text{Central Value in year } t \text{ which makes central time series: } \sum_{i=1}^6 a_i x_{it} \quad (6.12)$$

$t=1,2,\dots,T_f$

$$\text{Left Value in year } t \text{ which makes lower bound time series: } \sum_{i=1}^6 (a_i - s_i) x_{it} \quad (6.13)$$

$t=1,2,\dots,T_f$

$$\text{Right Value in year } t \text{ which makes upper bound time series: } \sum_{i=1}^6 (a_i + s_i) x_{it} \quad (6.14)$$

$t=1,2,\dots,T_f$

The time series of the underground economy obtained by varying h-certain factor are discussed next.

6.5 Results and discussion of Fuzzy MIMIC

There are different criteria for evaluating fuzzy regression models. In this study we use Index of Confidence [Wang and Tsaur, 2000] to evaluate different results obtained by varying

h-factor (equation (6.15)):

$$\begin{aligned}
 IC &= 1 - \frac{SSE}{SST} \\
 SSE &= 2 \sum_{t=1}^{T_f} (Y_{mim,t} - \tilde{Y}_{Fmim,t}^a)^2 \\
 SST &= \sum_{t=1}^{T_f} \langle (Y_{mim,t} - \tilde{Y}_{Fmim,t}^L)^2 + (Y_{mim,t} - \tilde{Y}_{Fmim,t}^R)^2 \rangle
 \end{aligned} \tag{6.15}$$

where, $Y_{mim,t}$ is the underground economy time series obtained from MIMIC model [Dell'Anno et al., 2007; Schneider and Buehn, 2012] and $(\tilde{Y}_{Fmim,t}^L, \tilde{Y}_{Fmim,t}^a, \tilde{Y}_{Fmim,t}^R)$ are fuzzy numbers of the underground economy in each year obtained from fuzzy model.

Average absolute error percentage (*AAEP*) is also applied to select the most suitable h-certain factor yielding the best estimate. *AAEP* is calculated from equation (6.16) [Kazemi et al., 2011]:

$$AAEP = \frac{1}{n} \sum_{t=1}^{T_f} \left| \frac{\tilde{Y}_{Fmim,t}^a - Y_{mim,t}}{Y_{mim,t}} \right| \tag{6.16}$$

where, $\tilde{Y}_{Fmim,t}^a$ is the central estimated value of the underground economy gained from fuzzy model and $Y_{mim,t}$ is the underground economy time series obtained from MIMIC.

There is a trade-off between minimum *IC* and minimum *AAEP*. The best time series will be selected on the basis on these two items (Table (6.15)). Average absolute error percentage (*AAEP*) is less in cases 2 and 5. Between these two cases, case 2 has the lowest index of confidence, thus, the h-certain factor used in case 2 is the best. So, h-certain factor is set to 0.4 for the data set of France. The final time series for France's underground economy, resulting

Table 6.15: Evaluation of model based on different h-certain values (data for France).

Case	h-Certain	SSE	SST	IC	AAEP
1	0.2	0.166155	0.241169	0.311043	0.28854
2	0.4	0.174953	0.404872	0.567882	0.278663
3	0.5	0.173016	0.434624	0.601919	0.285188
4	0.6	0.165269	0.404741	0.591666	0.282714
5	0.8	0.14623	0.436384	0.664906	0.274314

from the model with h-certain factor set to ($h = 0.4$), is scaled in Figure (6.14). Other time series estimated by MIMIC models [Dell’Anno et al., 2007; Schneider and Buehn, 2012] and a time series estimated by Elgin and Oztunali [2012] are also shown to compare the trend of these four results.

It can be seen from Figure (6.14) that the trend and turning points of Dell’Anno results and

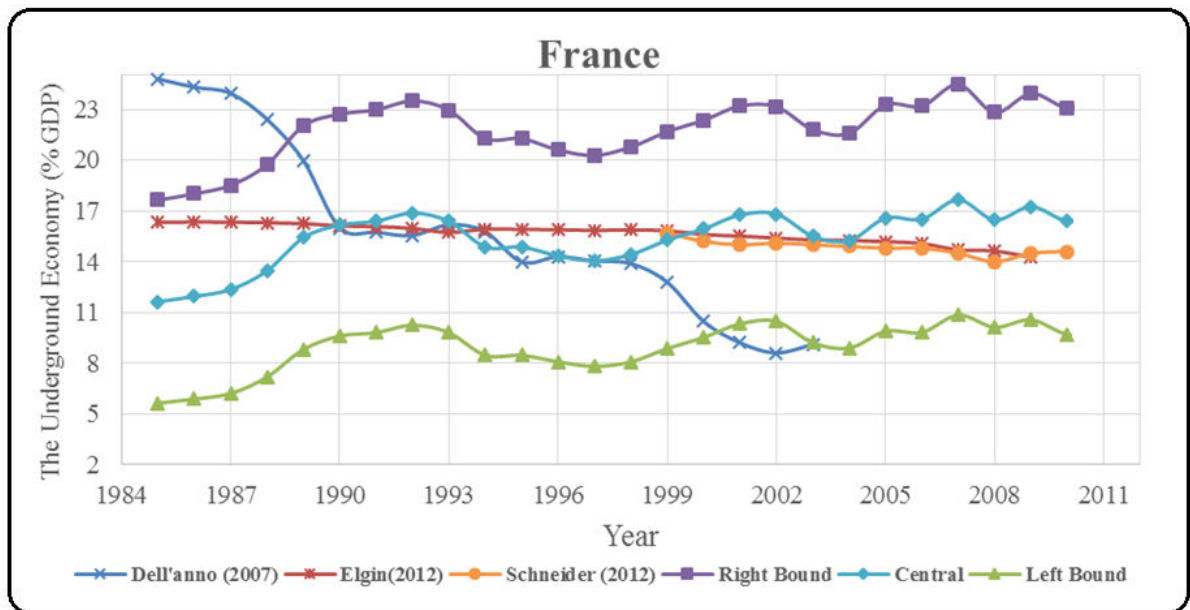


Figure 6.14: Fuzzy model, Elgin-Oztunali and MIMIC time series of France’s underground economy.

fuzzy curves are nearly similar between 1980 and 1990. Also, a similar trend is observed for Schneider-Buehn’s result and fuzzy result from 2001 to 2010 with an increasing trend for the subsequent years. Elgin-Oztunali’s result does not show any significant variation in 1985-99, whereas it reveals a generally downward trend for the following years.

The magnitude of France’s underground economy generally increases after 2004 based on Schneider-Buehn’s and the fuzzy estimates (unlike the Elgin-Oztunali results) which is reasonable. It means that without changing the policies and regulations, the working situation in the underground economy will be favorable and it may have undesirable effects on the official economy.

As demonstrated in Figure (6.14), the presented results for the France’s underground economy differ in magnitude. But all off them including: the results of fuzzy model in the central graph (the more likely value of the underground economy or the mode of the fuzzy number of the

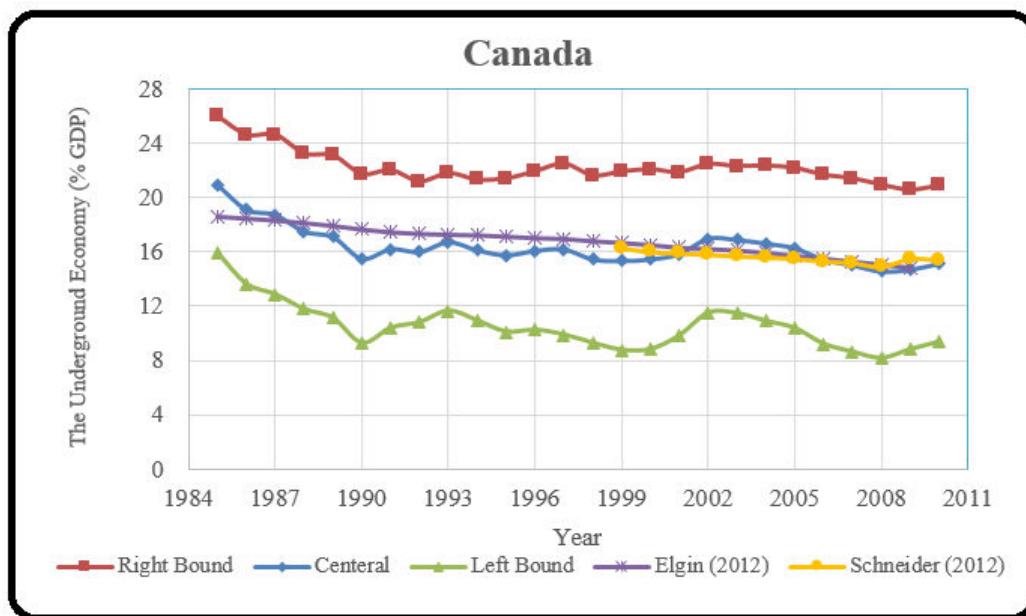


Figure 6.15: Time series of Canada's underground economy estimated by fuzzy, Elgin-Oztunali model and MIMIC.

underground economy for each year), Dell'Anno's (1990 – 1999), Schneider-Buehn's and Elgin-Oztunali's time series are within the limit determined by Frey and Schneider [2000]. As it was mentioned earlier, they imply that the expected interval for the size of France's underground economy is between 13% and 16% of GDP. On the whole, the central time series of the fuzzy method coincides with Dell'Anno's between 1990 and 1998 and with Schneider-Buehn's estimates from 1999 to 2010. However, it should be noted that the true time series of the underground economy is unknown; there is no way of saying which of these methods is more accurate [Dell'Anno et al., 2007; Schneider and Buehn, 2012].

As illustrated in Figures (6.15) and (6.16), for Canada and Italy, all three models estimate a downward trend for the underground economy with average sizes of respectively 16.36% and 28.34% of official GDP in fuzzy model, 16.82% and 28.81% in Elgin-Oztunali model, and 15.61% and 26.93% in Schneider-Buehn estimate.

For the United States as is shown in Figure (6.17), the trend of fuzzy results and those of Schneider-Buehn are nearly similar with an increase in the magnitude of the underground economy after 2007, whereas the Elgin-Oztunali model shows a downward trend.

For Germany (Figure (6.18)), Elgin-Oztunali model shows a nearly constant trend unlike Schneider-Buehn's and fuzzy results which estimate increasing trends after 2007. Also,

MIMIC estimate of Buehn and Goethel [2010] for Germany is very similar in trend and size to the result of fuzzy method unlike MIMIC estimate of Schneider and Buehn [2012].

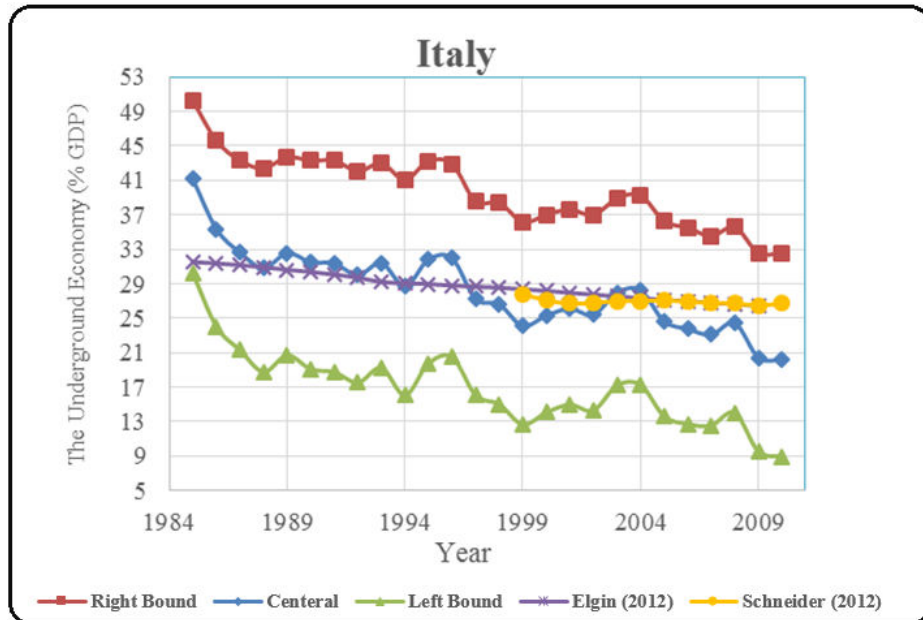


Figure 6.16: Time series of Italy's underground economy estimated by fuzzy, Elgin-Oztunali model and MIMIC.

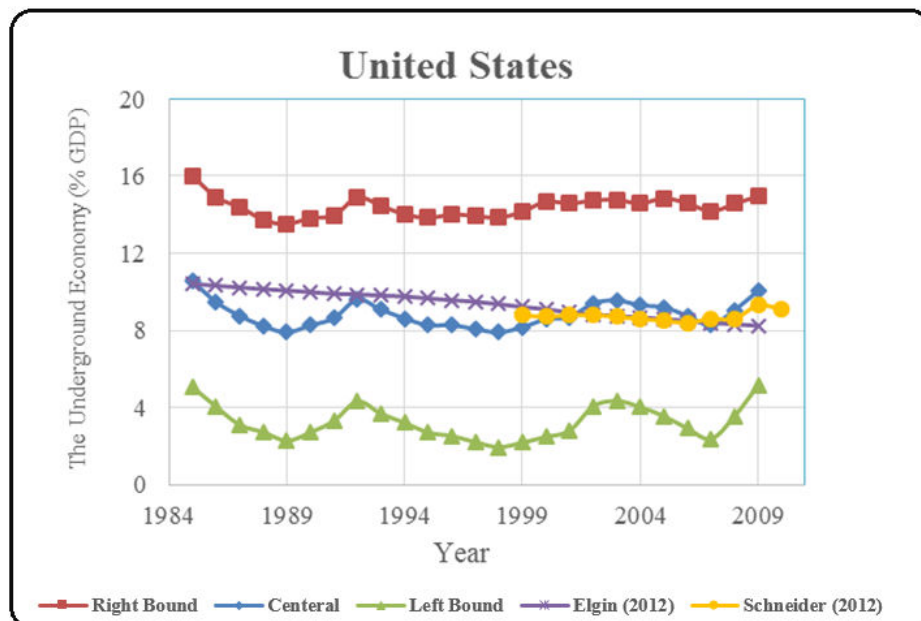


Figure 6.17: Time series of the United States' underground economy estimated by fuzzy, Elgin-Oztunali model and MIMIC.

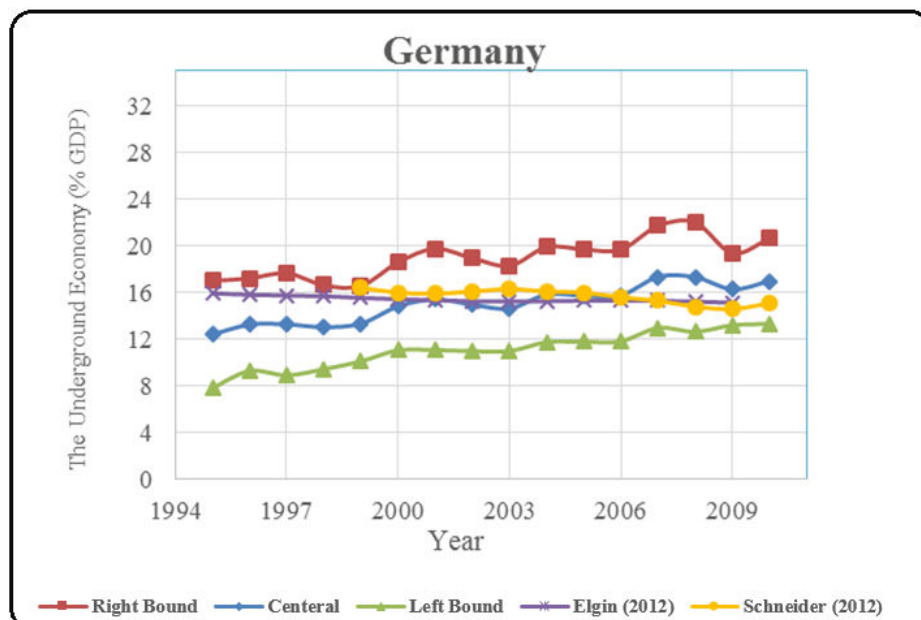


Figure 6.18: Time series of the Germany's underground economy estimated by fuzzy, Elgin-Oztunali model and MIMIC.

6.6 Summary

In this chapter,:

1. The three proposed fuzzy methods and their results were presented. The methods are: fuzzy modeling applying mean and variance, fuzzy modeling applying fuzzy clustering and structural equation modeling with fuzzy data or fuzzy Multiple Indicators Multiple causes.

The first two fuzzy methods consist of three steps: fuzzification or converting crisp data to fuzzy data; fuzzy inference or decision making process on the fuzzy data; and defuzzification or converting the fuzzy results to crisp results. The main difference of the first two fuzzy models is using different fuzzification methods; the first method uses the mean and variance while the second one uses the fuzzy clustering.

About the structural equation modeling with fuzzy data, firstly, the two sets of equations are converted to the reduced form equations and afterwards, a non-linear model is written and optimized by GAMS software based on fitting the fuzzy regression methods.

2. Comparing the fuzzy methods with MIMIC, it seems that all of the methods can be suitable when the raw data are adequate. But, MIMIC method is very sensitive to the size of the raw data and does not give reliable results unless adequate number of observations is provided (the size of data set is important). Various factors including: the difficulty verifying normal distribution of the term error, unclear relationships between variables, ambiguity associated with the event and inappropriate linearity assumption pose problems for the statistical regression underpinning the MIMIC method. In contrast, the proposed fuzzy methods are not based on strong assumptions and can be applied despite difficulties verifying distribution assumptions. Also, the fuzzy methods involve no complicated mathematical modeling and expressions, are less sensitive to the size of the data set and are not sensitive to data specification, dimension, etc. (See Breusch (2005) for more on the weaknesses of MIMIC). Furthermore, the uncertainty of the raw data is included in the time series obtained for the underground economy from structural equation modeling with fuzzy data.

3. Comparing fuzzy methods with Elgin-Oztunali model, we realize that despite some

similarities between these methods, they are completely different. Fuzzy methods focus on macroeconomic aspects of the underground economy whereas the Elgin-Oztunali model focuses on microeconomic aspects (behavior of the actors of the underground economy). It seems that modeling actors' behavior in economic events is not a suitable method if the same model is meant to be applied to all countries. For example in France, welfare services, regulatory sanctions and deterrents are the main factors which prevent people from working in the underground economy. But the Elgin-Oztunali model makes no allowance for these factors and considers the tax rate as the only regulation. Meanwhile in Iran, there are no comprehensive welfare services and regulatory deterrents are weak. It follows that it is not a good idea to use the same model (or similar constants in the model) for both countries. Moreover, the differences in the average size of the underground economy between these two countries in 1990-2008 is only 2.37% of the official GDP which seems unlikely, given that one of these countries (France) is developed and the other (Iran) developing (See Figure (6.19)). Setting aside this difference, the similar trend of the underground economy time series for both countries in 1990-2008 suggests that the model overlooks some predominant causes in each country.

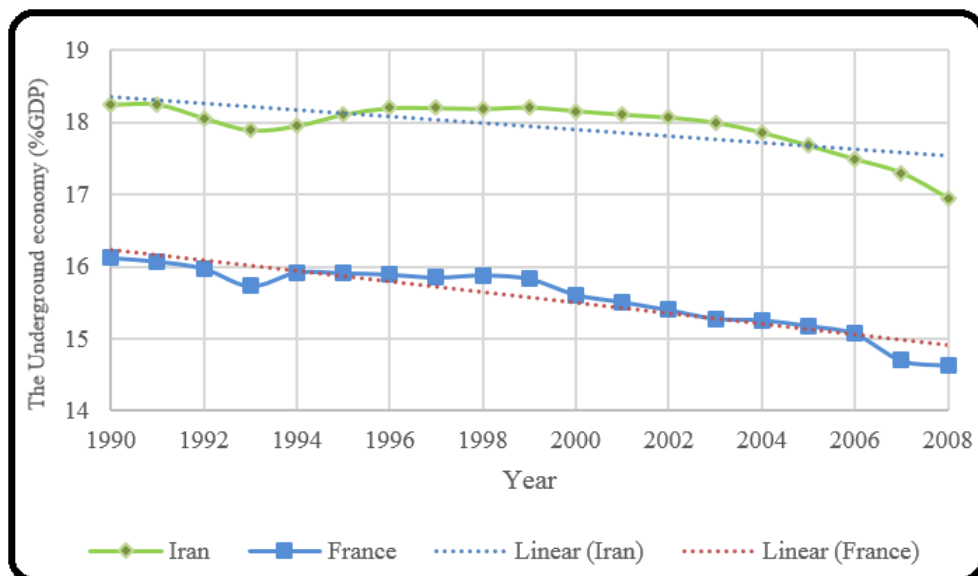


Figure 6.19: The time series of the underground economy of Iran and France estimated by Elgin and Oztunali (2012).

4. The underground economy is unobservable; the causes of the underground economy are

often vague and there is very limited knowledge about the nature of the relationships between these causes or they may be intrinsically non-linear and making the mathematical modeling of the underground economy difficult or even impossible. Among the methods proposed to model the underground economy, fuzzy methods (especially, the first two methods) allow for rapid modeling, even with imprecise and incomplete data, as it is not necessary to know all the details about the relations and distributions before starting. They also let us to model non-linear functions of arbitrary complexity and achieve simplicity and flexibility using fuzzy modeling strategy. From other side, it is not possible to determine the true series for the size of the underground economy or to determine which model is more accurate. But, the application of the fuzzy methods in the estimation of underground economy seems promising due to the mentioned features of the underground economy.

Summary of Part II

The second part of this research was dedicated to introduce the proposed fuzzy methods in this thesis including: the fuzzy modeling applying Mean and Variance; fuzzy modeling applying Fuzzy clustering and Fuzzy MIMIC.

The following Table (6.16) helps to have a complete overview on these methods:

Table 6.16: The proposed estimation methods.

	Steps of the model	Related method or concept of the step	Result of the step
Fuzzy modeling applying Mean and Variance	1. Fuzzification	Mean and Variance method	Crisp data of the causes are converted to Fuzzy data
	2. Fuzzy Inference	Fuzzy Inference Table	Decision making process on the fuzzy data
	3. Defuzzification	The Average of First Maxima	Fuzzy data of the underground economy are converted to crisp data
Fuzzy modeling applying Fuzzy Clustering	1. Fuzzification	Fuzzy Clustering method	Crisp data of the causes are converted to Fuzzy data
	2. Fuzzy Inference	Fuzzy Inference Table	Decision making process on the fuzzy data
	3. Defuzzification	The Center of Area	Fuzzy data of the underground economy are converted to crisp data
Fuzzy MIMIC —	1. Converting the two sets of equations to reduced form equations	Reduced form equations concept	Reduced form equations
	2. Make a model to fit the initial data to the reduced form equations	Fitting Fuzzy regression models	Non- Linear model
	3. Fitting the initial data to the obtained non-linear model	Operation Research concept	Time series of the underground economy

Part III

Summary and Conclusion

Chapter 7

Summary, Conclusion and Discussion

Introduction

In previous chapters, the focus of the research was on the modeling and the required details for this purpose. The necessary definitions, raw data and parameters were thoroughly discussed and the proposed fuzzy methods with their results were presented. The existing methods (either direct, indirect or modeling methods) have some shortcomings and strengths from an economic point of view such as being sensitive to the sample size, data specifications, data dimensions and distribution of the data or being time and cost consuming (especially, direct methods). But, how about the fuzzy methods? what are the strong points and weaknesses of these methods? Are the proposed fuzzy methods reliable enough to be used by the economists and policy makers? These questions need to be discussed.

The fuzzy methods proposed in this research are less sensitive to the data sample size, data specifications and data distributions like any other fuzzy model. They include some assumptions but as it was explained, most of them were examined to see their effects on the obtained results. Changing the degree used in the fuzzy statement, the number of linguistic terms used in the fuzzification step or the fuzzification method and using the median instead of the mean were among the assumptions verified. It revealed that the results had adequate robust and changing these items do not change the trend of the results and the changes in the size was really negligible. These are some advantages which may motivate the policy

makers rely on these fuzzy methods. Besides, the first two proposed fuzzy methods are free of mathematical complications and they do not have complicated mathematical equations. These characteristics motivate to use them. Moreover, fuzzy economy can eliminate the need for the complex calculations of conventional econometric models. Also, the fuzzy rules with the linguistic terms are easy for the human to understand. It means that the ability of fuzzy logic to translate the natural phenomenon into easy to understand linguistic terms using the simple mathematical models which do not need many data, make this method more favorable for policy makers.

About the structural equation modeling with fuzzy data it can be said that it is not only a simple method but also has some strengths which distinguish it from other methods. Some shortcomings of the MIMIC (which can be considered as a kind of the structural equation modeling) such as data distribution assumption, sensitivity to sample size, data quality and dimensions are eliminated in this method. Unlike some other methods, some initial conditions of the underground economy are also considered in this method such as: the uncertainty of the initial data, unclear relationship between the causes and the underground economy and between the underground economy and the indicators.

It must be considered that these models have also some shortcomings which should be taken into account and will be discussed next. But still the strong points of these methods are enough to motivate the policy makers and economists to apply and rely on them.

7.1 Summary and Conclusion

The underground economy briefly consists of transactions, activities and services which can be either legal or illegal. The underground activities are not reported or recorded in spite of government regulations due to unconsciousness, evasion or avoidance of government regulations, or the shortcomings of government agents. This work aimed at proposing the fuzzy methods to estimate the annual size of the underground economy in France, Germany, Canada, Italy and the United States for the period 1989-2010. The outcomes are the following methods:

- fuzzy modeling applying mean and variance;

- fuzzy modeling applying fuzzy clustering; and,
- fuzzy multiple indicators multiple causes (MIMIC) approach or in other words, the multiple indicators multiple causes (MIMIC) approach with fuzzy initial data.

To best of the author's knowledge, these methods were applied for the first time to estimate the size and trend of the underground economy and proved to be promising enough for this purpose.

This dissertation was composed of two main parts: a literature review on the underground economy together with various definitions proposed to describe this economic phenomenon which gave an insight about its actors and their incentives as well; and, a brief presentation of various methods used to estimate the size of the underground economy, the proposition of three fuzzy methods used in this study, their algorithms and results followed by conclusions and remarks.

About the consequences of the underground economy, it was stated that some adverse impacts of the underground sector are reduction in tax revenues, distortion of statistics, inefficiency in resource allocation and disintegration of social norms. However, it is believed that the underground economy has also some positive impacts such as provision of additional employment during recession, bringing competitive power and dynamism to economy, or the fact that a major part of money obtained from underground activities is spent in the official sector. Some researchers argue that it cannot be proven whether it is better to do an activity in the underground economy or not to do it at all.

Various direct, indirect and modeling methods are used to estimate the underground economy among which MIMIC (a special case of structural equation models (SEM)) is the most famous one. Although MIMIC method is the most common method for the estimation of the underground economy, it has some shortcomings like when the data set is too small for efficient statistical regression analysis or the statistical distribution assumption cannot be justified and pose problems for the statistical regression underpinning the MIMIC method. Improper linearity assumption involved in MIMIC method, difficulty verifying the normal distribution of the error term, uncertain relationships among variables and ambiguity of the underground economy are some other complications faced when applying MIMIC method to estimate the underground economy. Applying fuzzy concept in the estimation of the underground economy

has proved however to be advantageous as it allows for rapid modeling, even with imprecise and incomplete data and let model non-linear functions of arbitrary complexity and achieve simplicity and flexibility. The two proposed fuzzy methods and their steps were thoroughly discussed in chapters, including: fuzzification, converting the crisp data to fuzzy data; fuzzy inference, making decision from fuzzy data; and aggregation and defuzzification, aggregating the results of the causes and obtaining the linguistic terms and membership functions of the underground economy and converting them to crisp data to make decision and comparing. As it was already mentioned, the main difference between these two methods is related to the fuzzification and defuzzification steps. In fuzzy modeling applying mean and variance, the mean and variance of the five consecutive years is applied to make the fuzzification step while the fuzzy clustering is used to make fuzzification in the other fuzzy model. Also, two different defuzzification methods are applied to convert the fuzzy results to crisp in defuzzification step of these methods.

The third method, MIMIC method with fuzzy data or fuzzy MIMIC method, was completely different from the two other methods. To solve the MIMIC method with fuzzy data which can be named fuzzy MIMIC, the fuzzy MIMIC equations were converted to reduced form equations and the final non-linear model was written to fit the fuzzy data in model and gain the coefficients of the first equations. The final model was solved by GAMS software.

The causes of the underground economy considered in all these methods were: taxes on incomes and profits as the effective direct tax rate or the share of direct taxation in GDP, taxes on goods and services as the effective indirect tax rate or the share of indirect taxation in GDP, ratio of the long term unemployment to labor force, proxy of government regulation defined as the ratio of government consumption and expenditure to GDP, unemployment rate, ratio of the self-employment to labor force and inflation rate. Real GDP per capita, which is calculated by dividing the real GDP by population in working age (15-64 years old), participation ratio of the labor force which is calculated by dividing the total labor force by population in working age and the household net saving rate were used as the indicators of the underground economy. Detailed explanations about the calculations were provided for each model followed by the obtained results ¹. The resulted time series of the underground economy obtained from three methods proposed in this study as well as the results of a MIMIC method [Dell'Anno et al.,

¹ All calculations and results are available in Appendix.

2007; Schneider and Buehn, 2012] and a macro economic model proposed by Elgin and Oztunali [2012] were provided to better compare the results. Comparing these estimates of the underground economy, it can be seen that nearly all estimates are close for most years and there are some similarities among them in trend and size. But, it is still difficult to say which method is more accurate.

The estimated values of the underground economy in 2010 are shown in Table (7.1) for the developed countries studied in this work. It can be said that the absolute values of the underground economy have overall increased by time in these developed countries (France, Germany, Italy, the United States and Canada). These results manifest the significance of the underground economy even in these countries and demonstrate a need for revising and reformulating some financial and other policies in each case.

The ambiguous character of the underground economy raises many problems in the research

Table 7.1: The size of the underground economy in different countries in 2010

	Fuzzy Modeling		
	Mean and Variance	Fuzzy Clustering	Fuzzy MIMIC
France	17.82	11.49	16.4
Germany	14.01	18.73	16.98
Italy	30.05	25.65	20.14
Canada	15.42	14	15.11
USA	10.09	9.64	9.55

in this area. Since there are a variety of models for estimating the size of the underground economy, it is necessary to know which of them suits perfectly for the development and comparative analyses among countries. It is nearly impossible to say which approach is superior to others in representing the underground economy. Each method has its own econometric techniques and its results. Comparing the results of different approaches, there are always discrepancies which originate from not having a previous reliable estimate (base value) and cause the estimates to be most likely biased. Although, it seems that MIMIC approach with fuzzy data is one step forward as it takes into account various causes and indicators of the underground economy, while fuzzy modeling approach only takes into consideration some causes. In the process of policy making, regarding the purpose of the policy, it can be decided

to consider which measure by policy makers based on the advantages and disadvantages of each model.

If the goal of the policy is revising the tax system and increasing the tax revenues, fuzzy approach (applying mean and variance or applying fuzzy clustering) might be more relevant. The reason is that the fuzzy modeling is a facile method that can be used with two, four, six or more causes which are related to tax system such as direct and indirect tax, tax rate, tax burden, etc.

If the goal of the policy is improving the efficiency of economic policy and the quality of official statistics, then MIMIC approach with fuzzy data might be more appropriate. Fuzzy MIMIC applies some indicators in addition to causes which lets investigate more precisely the economic situation. It is also possible to improve the results by increasing the number of causes and indicators. Being a bit complex is the only weakness of fuzzy MIMIC compared to the fuzzy methods.

7.2 Discussion

7.2.1 Economic crisis, GDP growth and the underground economy

After the worldwide economic crisis in 2008 that affected the economic systems in many countries, a question arose that how economic crisis affects the size of the underground market and the incentives of people for participating in the underground activities. To answer this question, we consider the trend of the underground economy from 2000 until 2010 for the countries studied earlier (Figure (7.1) and Figure (7.2)).

At the first look, it can be seen that the underground economy has generally increased in most

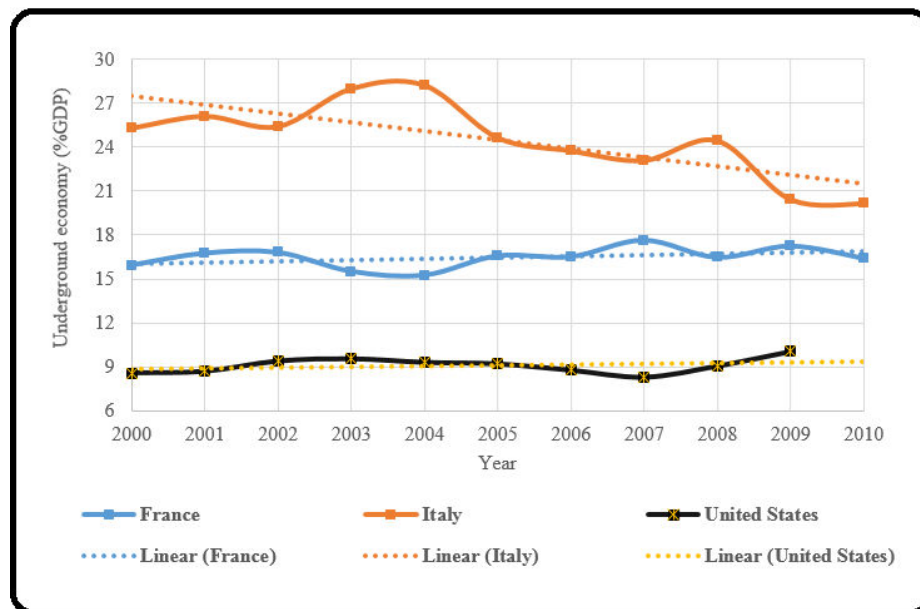


Figure 7.1: The trend of the underground economy in France, Italy and the United States.

of the investigated countries. With economic crisis in 2007-8, the benefit of the official market has significantly decreased. So, they preferred to continue their job in a market that costs less (i.e., a non-taxed market). The underground markets are usually cheaper than the official ones with reduced costs of products and services. From other side, purchasing power of the people has diminished after economic crisis, so, they demanded cheaper goods and services mostly provided by the underground markets. The increase of demands in the underground markets consequently tempts the individuals to participate in these markets.

Hence, it can be claimed that the decrease in purchasing power of people is the main reason that push them to the underground markets in order to decrease the price of goods and services

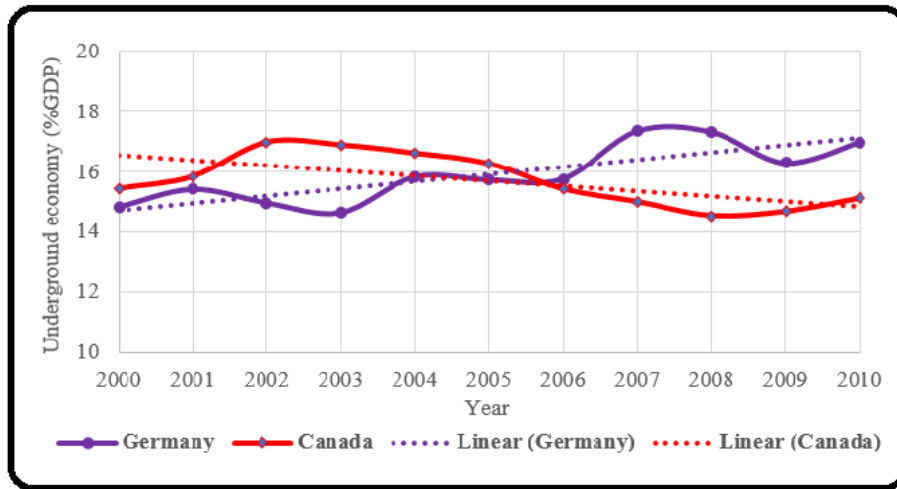


Figure 7.2: The trend of the underground economy in Germany and Canada.

in one hand, and earn more revenues to increase their purchasing power on the other hand. This participation in the underground economy can be entirely or partially; i.e., a small company produces goods completely in the underground market or a service provider might eliminate the tax for some customers.

The entirely or partially movement to the underground economy might happen for different

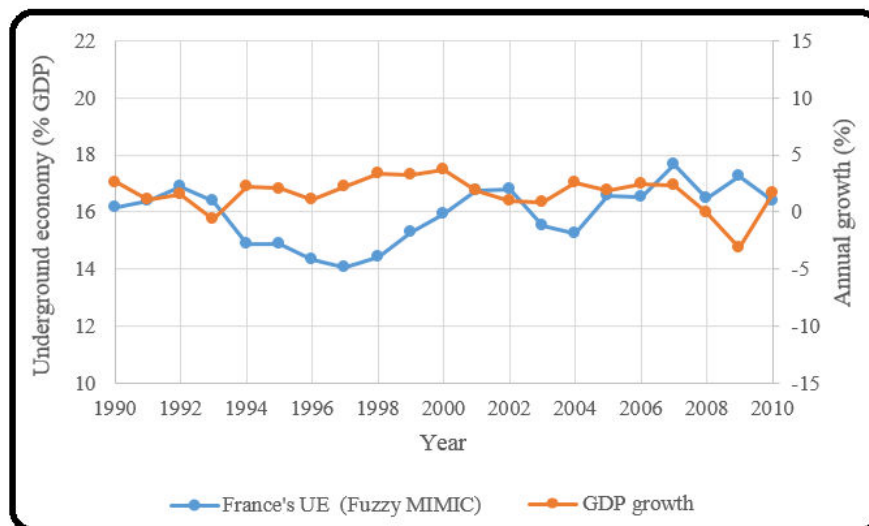


Figure 7.3: The trend of the underground economy and GDP growth in France.

actors of the official economy. This can be recognized from the benefit and purchasing power theories. These theories imply that people try to get more benefits by decreasing expenses or increasing their revenues. During economic crisis, it is not possible to increase revenues by increasing the production or the price of services because the purchasing power of people

has already diminished. So, decreasing the costs and expenses is a reasonable act. It seems, however, that the economic situation of countries before economic crisis is also determinant on the decision of people to participate in the underground market.

The underground economy and GDP growth time series of different countries are depicted in Figures (7.3)-(7.7). It seems that in most countries with GDP growth in 2006-7 (the United States, Canada, Germany and France), the size of the underground economy increased in 2009 as a result of the world financial and economic crisis.

According to Figure (7.3), the trends of the underground economy time series and that of the GDP growth seem to be similar with a delay of one to two years in case of France. It can be claimed that the government intervention in the economy, social security and social aids of the government to the people, integration process of the immigrants, social capital, etc. may affect the decision of people to move into or quit the underground market.

In case of Italy, the GDP growth is shown to be negative in 2007-2008, which means that the

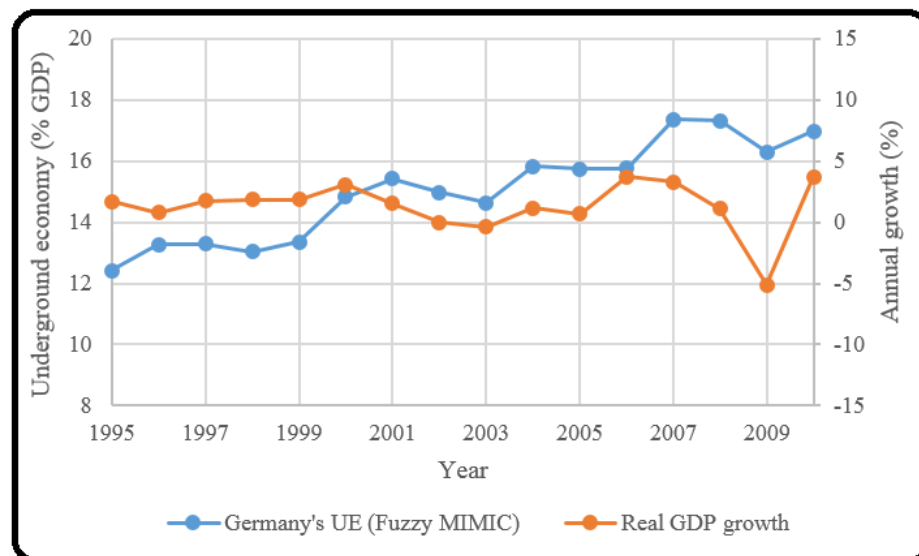


Figure 7.4: The trend of the underground economy and GDP growth in Germany.

economic situation of Italy was not suitable in the mentioned years. This trend has led to a fall down in the trend of the underground economy based on the fuzzy MIMIC approach. Based on GDP growth in 2009-2010, the trend of the underground economy could be expected to be positive in 2010-2011.

Canada and the United states have almost a positive economic growth in different years based on Figure (7.6) and (7.7). The stable economic growth in these countries influences the main

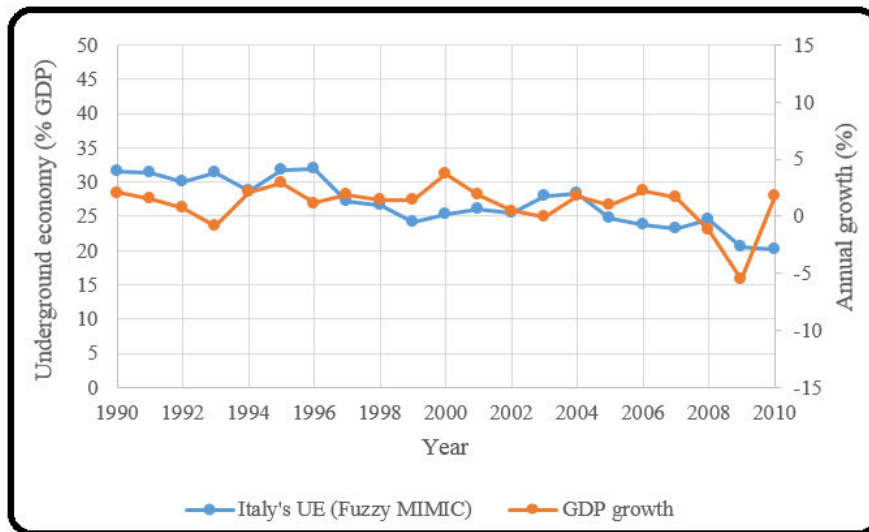


Figure 7.5: The trend of the underground economy and GDP growth in Italy.

causes of the underground economy such as unemployment rate, inflation rate, etc. It seems that the stable trend of the causes of the underground economy may lead to the stable trend in the underground economy without government interventions or changing the government policies. This case is completely clear from the less fluctuation of the trend of the underground economy in these countries.

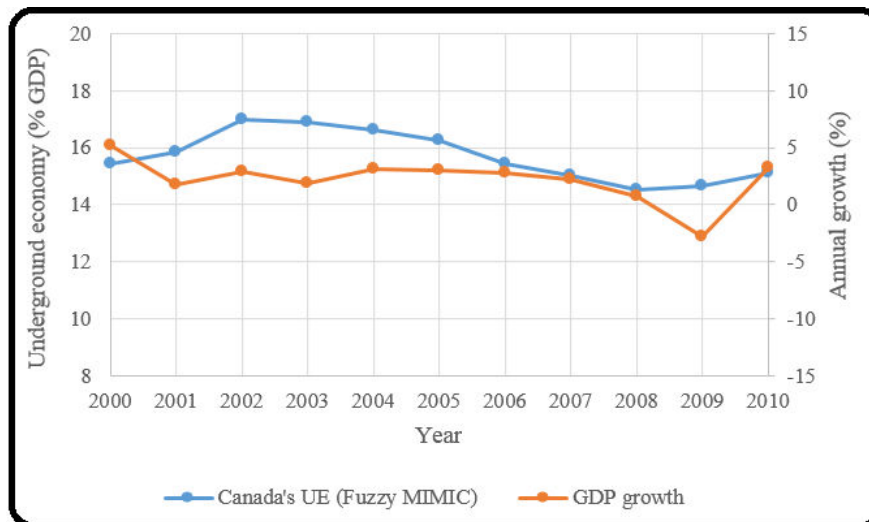


Figure 7.6: The trend of the underground economy and GDP growth in Canada.

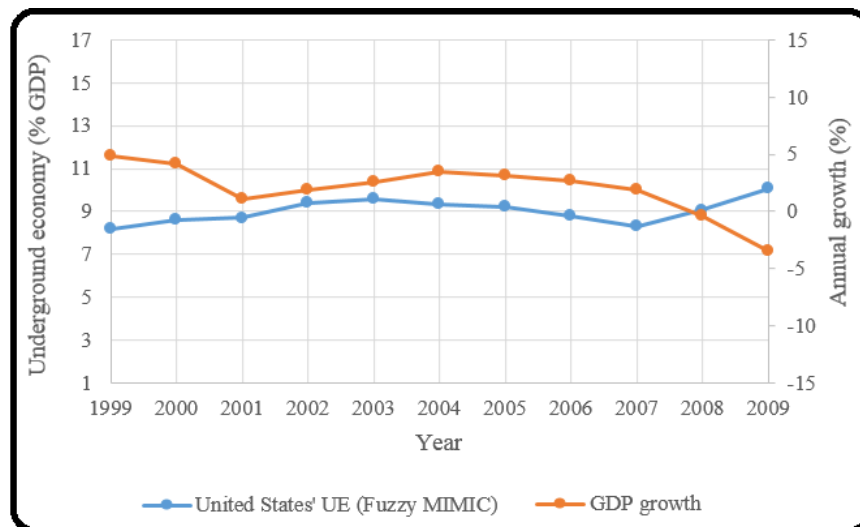


Figure 7.7: The trend of the underground economy and GDP growth in the United States.

7.2.2 Developed countries and the underground economy

The general consequences of the underground economy were already discussed. A discussion on the consequences of the underground economy in developed countries will be informative. A first considerable effect of the underground economy in developed and developing countries is the tax evasion. Being a secret illegal activity, it is difficult to precisely estimate the tax evasion in the economy. Although, some researchers (Tanzi (1983), Giles (1999), Faal (2007), etc.) have provided some estimates for the size of the tax evasion in various countries using the estimates of the underground economy in those countries.

Any decrease in the government tax revenues, especially because of the tax evasion, results in budget deficit due to the gap between the real and the expected tax revenues. This gap makes some crucial issues concerning the economy. The very first impacts of the reduced government revenues will come up in the quality and quantity of public goods and services provided by the government. Besides increasing the tax rate, the government might be obliged to borrow money from domestic or external sources to compensate for tax revenue deficit. The increase in the public debt consequently increases the public debt service (interest and principal) payments in the public budget, which might considerably worsen the public budget deficit.

The impacts of the underground economy on the poverty are also considerable, but, the net effect of the underground economy on poverty is unclear. From one hand, the deficit in the

government revenues caused by the tax evasion of the underground activities has a negative effect on poverty. It means that it reduces the government social programs and allocations that aimed at combating poverty. On the other hand, the income earned from the underground activities has a positive impact on alleviating poverty.

Since people who are engaged in the underground activities prefer cash transactions to evade paying income and sales taxes, the demand for money in circulation might also increase as a result of the increase in the size of the underground economy. Since the expansion in the money supply is specified by the central bank based on the expected growth of the official output (GDP), the supply of money in circulation will be less than the demand which should put upward pressure on interest rates and on the general prices level.

Furthermore, the expansion in government demand for borrowing to cover the loss of tax revenues due to the tax evasion will rival the private sector for banking deposits which will also cause a rise in interest rates and, consequently, an increase in the cost of investment.

The underground markets can also affect the efficiency of the industries and the size of firms. As it is burdensome for a large company to hide its activities from official authorities, a company that works underground tends to stay small in order to remain undetected. On the contrary, being a small company deprives that firm from benefits of the economies of scale and directly influences its technologies, efficiency, etc. Thus, the underground economy also affects negatively the plan of official firms for expansion.

True and real performance and situation of the economy might also be biased due to the existence of the underground activities that makes the official national accounts statistics inaccurate. Particularly, the unreported underground activities underestimate the officially published GDP obtained from the size of the underground economy. All crucial economic indicators that are expressed as a percent of GDP such as the budget deficit, the current account deficit, and the public debt will be consequently overestimated. The GDP per capita, which indicates the standard of living and is used as a measure of comparison among various countries, will be also underestimated if the underground economy is not reported.

High tax rates is an incentive for the laborers to move from the official economy to the underground economy. Most laborers who are involved in the underground activities are not included in the official labor census due to the hidden nature of the underground activities. Thus, the official employment statistics underestimate the unemployment rates. Other eco-

economic policies and indicators associated with unemployment, such as poverty indicators and social welfare allocations and government subsidies, are consequently distorted.

The existence of the underground economy also affects the inflation. The consumer price index indicates the general price level of goods and services in the official market but it does not include the price level in the underground markets. It is however known that the prices of the underground goods and services are lower than their prices in the formal economy due to the lower production cost and the tax evasion in the underground economy. Official inflation rates, hence, overestimate the price level in the economy which biases related economic indicators such as the real economic growth, the real interest rates, and the real effective exchange rate.

Policy making and the underground economy

It was explained that the underground economy is a complex economic phenomenon that exists in all types of economy (developing, transition and highly developed). Various governments have employed different strategies through all the years to combat this phenomenon. To do so, they must frame more effective policy measures which either hinder underground activities or encourage their actors to move into the official markets. But this requires detailed knowledge of the size and development of the underground economy, the individuals engaged in it and their incentives.

People tend to move to the underground economy for many reasons the most important of which can be the government actions, most notably, taxation and regulation. The governments therefore play an important role in extension or restriction of the underground economy. The causes of the underground economy are however often vague and there is very limited knowledge about the nature of the relationships between these causes. But, there is a possibility to assess the effect of each cause on the underground economy using estimation models and methods.

As It was stated, we used fuzzy methods to determine the trend and size of the underground economy. Here, we are going to focus more on the results of fuzzy MIMIC for policy making in different countries. The resulted time series of fuzzy MIMIC provide information about the maximum and minimum possible size of the underground economy as a fuzzy number in each year. The central time series represents the more probable size of the underground

economy and its upper and lower bounds (the spreads of the mode) in each year demonstrate the appropriate and inappropriate conditions and help governments of countries to purify the economic criteria and legislate.

The methods developed in this dissertation are not data-dependent and can be applied to different country. Moreover, the fuzziness or spreads of the results of fuzzy MIMIC (the divergence of the left and right bounds from the central time series) indicates the magnitude of the uncertainty of the raw data for each country. In other words, it tells us how accurately the assumed initial conditions for decision policies represent the real initial conditions. There are also grounds for claiming that fuzziness is inversely related to the achievement of policy goals. So, any increase in fuzziness decreases the achievement of various policy goals and vice versa.

The results (three time series) of the proposed fuzzy MIMIC model may therefore be crucial to decision-makers in each country as they can obtain a size for the underground economy by calculating the weighted average of the three time series. The greater the fuzziness of the results, the more advisable it is to increase the weighting percentage of the right and left bounds.

Moreover, policy-makers need to base their decisions on the information regarding the magnitude, development and the attributes of the underground economy. Without reliable statistics about unemployment, GDP per capita, family savings, etc., the regulations will not be effective. The results of the fuzzy MIMIC model provide an overall view of possible interval for the size of the underground economy which can help policy makers. They can consider the different conditions for the size of the underground economy, based on the central time series as the most likely size of the underground economy and its upper and lower bounds for each year.

It is notable that the positive and negative impacts of the underground economy on the economic and social criteria of a country must be determined according to the type of the country (developing or developed). For instance, in a highly developed country, the government may not be inclined substantially to reduce the underground economy because [Schneider, 2007] :

1. The income earned in the underground economy augments the welfare and standard of living of at least 1/3 of the working population;

2. At least $2/3$ of the income earned in the underground economy is spent in the official economy, then, tax losses may consequently be slight;
3. Between 40 and 50% of the underground activities yield additional added value, increasing overall GDP; and,
4. Workers engaged in the underground activities have less time for other things such as taking part in demonstrations.

On the other hand, the activities of the underground economy must be investigated through field research in order to determine how much revenue these activities provide and for which group of the working population. In a developed country such as France, the government authorizes daily markets which might evade taxation, whereas the ratio of the public welfare expenditure to the tax revenue in this country is about 70%. Thus, if the tax evasion is the main incentive for working in the underground economy, it can seriously affect the welfare system and social services.

In each country either developed or developing, self-employment and unemployment are the two predominant causes which act as an incentive to take part in the underground activities. Unemployment can be controlled by government via economic policies, but the government has less control over the impact of self-employment on the underground economy. However, a government can encourage people to become their own entrepreneurs, as a way of reducing unemployment on one hand, and control welfare contributions in the underground economy, on the other hand. Such actions however need to be backed by stronger institutions and a stricter moral stance on tax, in order to reduce the probability of self-employed people shifting a substantial share of their business into the underground economy, which would undermine government policies supporting self-employment.

Adjustment of the underground economy measures in national accounts

As can be seen in the results of different methods and models, the size of the underground economy increases in different countries, but, the rate of increase is different among various countries.

The underground economy has a distorting effect on the accuracy of a country's national accounts statistics. It was mentioned that the official GDP per capita will be underestimated if

the underground economy is not reported. All crucial economic indicators that are expressed as a percent of GDP will be consequently erroneous. As most laborers working in the underground economy are not registered in the official labor statistics, the official unemployment statistics overestimate the unemployment rates. In addition, the prices of underground goods and services produced are lower than the prices in the official economy, thus, the official inflation rates will overstate the price level in the economy.

Since the underground economy does exist (in value added terms), countries have made necessary adjustments of the measures of the underground economy in their national accounts². Andrews et al. [2011] has detected and reported seven of them:

- A1:** A producer deliberately does not register to avoid tax and social security obligations;
- A2:** A producer deliberately does not register as a legal identity or as an entrepreneur because of being involved in illegal activities;
- A3:** A producer is not required to register because he has no market output;
- A4:** A legal person is not surveyed due to the reasons such as the business register is out of date or the updating procedures are inadequate;
- A5:** Registered entrepreneurs may not be surveyed since the statistical office does not conduct a survey of the registered entrepreneurs;
- A6:** Cross output is underreported and/or intermediate consumption is overstated; and,
- A7:** The data are either not complete or not collected or not directly collectable and/or data are incorrectly handled.

Table (7.2) demonstrates that the countries which make some adjustments, their adjustments are significantly different from other countries. Since a part of the underground economy has been already considered in these countries (starting from Australia and ending with the United States), the measures of the size and development of the underground economy expressed as the percent of official GDP is then biased. This is a further difficulty when comparing the size and development of the underground economies between countries.

² The following text closely follows Andrews et al. [2011] pages 11 and 12, Box 2. The Table (7.2) is also reproduced from this reference.

Table 7.2: Adjustment of the underground economy in National Accounts, around 2000.

	Activities included						A7: Incorrect data
	A1: Non registered producers	A2: Non registered identity of a producers	A3: No requirement to register	A4: Non registration to old state	A5: Not captured by the state office	A6: Under reporting of output	
Australia			X			X	X
Austria	X		X	X	X	X	X
Belgium	X		X			X	X
Canada	X	X	X			X	X
Czech Rep.	X	X	X	X	X	X	X
Estonia	X	X				X	X
Finland	X		X			X	
Germany							
Hungary	X	X	X			X	X
Ireland			X	X		X	X
Italy	X		X	X	X	X	
Mexico		X	X				
Netherlands						X	X
Norway			X	X	X	X	X
Poland	X	X	X			X	X
Russia	X		X	X	X	X	X
Spain	X			X		X	X
Sweden		X				X	X
Turkey	X		X			X	
UK	X		X		X	X	X
US						X	

Policy implication of the underground economy

The negative and positive impacts of the underground economy on the official economy have been already discussed. Regarding the negative consequences, the governments should employ effective strategies to reduce the size of the underground economy and minimize these consequences. This is not however an easy task as it entails implementing some economic, legal, and administrative measures and needs enough time. But some of these negative consequences, particularly those related to the accuracy of the national account statistics, might be controlled more easily. In this section, we analyze the policy implications of the underground economy.

Policy makers use the official statistics for economic policies they design. Thus, the efficiency of the policies depends to a large extent on the accuracy of those statistics. Since the underground economy distort the official statistics due to its hidden nature, the outcome of the economic policies tend to be inefficient. As it was explained before, the simplest solution to this problem is that the policy makers take into consideration the existence of the underground economy when making policies. In other words, it is necessary to reduce the discrepancy between the official statistics and the real statistics.

The proposed fuzzy MIMIC method which estimates the size and development of the underground economy as an interval, provides policy makers with some valuable information. The governments can therefore conduct periodic estimates of the size of the underground economy and include these results in the official statistics. A specialized unit should be based in the department of economy for this purpose and staffed with qualified experts to conduct this mission.

To eliminate the underground economy or alleviate it, government and policy makers should think about some issues:

- Level of tax and social insurance burden;
- License and permission requirements;
- Legislation related to the entry and the exit of businesses;
- Institutional capacity and enforcement of laws;
- Lack of broader public control over the public administration and institutions; and,

- Corruption and red-tapes.

Since the taxes are known to be the main driving force to enter the underground economy, revising the tax system is expected to contribute to the reduction in the size of the underground economy. Fixing appropriate tax rates is an important task which requires a separate research. High taxes do not necessarily provide high tax revenues for the governments since there is always the tax evasion. Governments should consequently concentrate on specifying the tax base in a way to collect sufficient tax revenues instead of raising individual tax rates. Legal reforms should be also associated with tax revisions. The reforms should contribute to the strict implementation of laws against working in the underground economy and evading taxes, including enforcement of strict penalties and financial sanctions.

Enforcing tax incentive policy undermines the incentives to participate in the underground activities. The tax incentives may include adopting tax exemption in some specific periods for individual firms in the sectors of large underground activities. This might encourage these firms to report their economic activities to the official authorities. Moreover, imposing lump-sum tax on small and micro-enterprises are the simple and cost effective techniques that helps the government to capture some underground activities. This requires however to explore the impact of tax incentives and incidence of existing tax rates on the underground economy.

Excessive government intervention in the economy in various forms such as license and permission requirements, legislation related to entry and exit of businesses, institutional capacity and enforcement of laws, etc., is another reason that force people to move to the underground economy. Over-regulation in the official economy affects negatively the market competition, sets up barriers to entering the formal market and increases the costs of working in the formal economy, consequently, people tend to work in the underground economy. To reduce incentives to enter the underground economy, the government should lessen the complexity and rigidity of regulations and reduce unnecessary intervention in the economy. Although, some degree of regulation is necessary.

Briefly, an attractive business environment must be maintained in the formal economy. It can be achieved by enhancing economic freedom, removing market barriers, and simplifying the bureaucratic procedures that are required for working in the formal economy. Governments should also emphasize the rule of law by enforcing regulations in a transparent way.

It necessitates, therefore, that the government and policy makers at all levels of state administration seek for a solution to the underground economy problem through making decisions and taking measures that result in the long-run. The natural political contradiction between the short-term pre-election period and the long-term benefits should be neglected in this regard. Actual decisions should concentrate on efforts to:

- Reduce the tax burden by lower tax rates and rapid VAT refund: since high tax rate is the main incentive to do business in the underground economy, the best policy measure is to reduce the tax burden in order to reduce the underground economy;
- Provide an attractive business environment: fiscal and business regulations should be designed in a way to motivate the businesses to work in the official economy. The number of licenses and permissions should be reduced and if not possible, their enforcement should be removed from the discretion of the ruling bodies;
- Reduce the social and payroll contribution of employees: it is known that if the social contributions and employee benefits paid by the businesses reduces reasonably and the gross salary concept is implemented, the evasion of the contributions will be less and employees are more motivated to operate in the official economy;
- Reduce corruption in all levels: to have a successful tax system, tax authorities should be autonomous and carry out their duties without political influences. It means that the tax authorities should be staffed with adequate experienced personnel with essential expertise. This should be accompanied with regulation reforms in order to make the economy more competitive, reduce the incentives for corruption, and encourage the businesses to move from the underground economy into the official economy. Enhancing the transparency of the rules and the regulations can also diminish the possibility for rent and consequently reduces the corruption;
- Improve the efficiency of the rule of law: this implies that necessary measures should be taken to improve the enforcement actions against the tax evaders and accelerate the persistent slowness of the judicial system; and,
- Improve and increase the regulatory control: improved and more regular control on the

VAT payments, for instance among green markets, property rental contracts, and all handicraft activities, might contribute to the lower underground economy.

The amount of social transfers paid by the government also affects the underground economy. High social transfers bring about unwanted effects on the underground economy as it dissuades the unemployed from operating in the official economy but incentivizes to operate in the underground economy and simultaneously receiving the social aid.

7.2.3 Tax, tax rate and the underground economy

Since tax is a compulsory payment to the government, studies of the tax rates can provide valuable insight into the changes in the size of the underground economy. Taxation lowers the purchasing power of the consumers and the ability to invest, thus, affects significantly the growth of the underground economy. It can be claimed that a rising tax burden provides great incentives to participate in the underground economy. According to Cebula [1997], 1% increase in the tax will lead to 1.4% increase in the underground economy.

The impacts of taxation on the underground economy needs to be examined carefully, not only because the tax burden itself is an important aspect of economic imbalance, but also because it is associated with laborers participation in the official economy. Since the underground economy does not meet certain labor market standards or regulations such as minimum wage law, maximum working hours, etc., there is an increasing incentive to enter the underground activities. From other side, taxation affects the labor-leisure choices and stimulate the participation in the underground activities. Any increase in the marginal tax rate, for example, leads to substitution effects. The decline in marginal utility of productive labor due to an increase in the tax rates, makes taxpayers to choose leisure instead of working in the official economy. This will consequently suppress the labor participation in the official economy.

A great difference between the total cost of labor and the after-tax earnings in the official economy deters the labor from staying in the official economy. This results in the reallocation from more efficient and productive businesses in the official economy to less efficient businesses in the underground economy which, in turn, impairs the efficiency of the economy.

As it was already stated, the increase in the tax rates with the aim of compensating for the shortage in tax revenues caused by tax evasion does not necessarily provide high tax revenues

for the governments. Instead, the result might be the increase in the amount of tax evasion and in the size of the underground economy. On the other hand, if the government borrows money from domestic or external sources to compensate for tax revenue deficit, the increase in the public debt consequently increases the public debt service payments in the public budget, which might considerably worsen the public budget deficit. Although it is clear that tax is one of the main causes of the underground economy, it is important to choose the appropriate tax rate.

Tax rates can be generally classified into average tax rate and marginal tax rate. Average tax rate represents the institution effect and can be reasonably well captured by their ratio of total tax revenue, i.e. the tax ratio [Lee, 2005]. A study by Hill and Kabir [1996] revealed that the early researches applying average tax rates were carried out by Cagan [1958] and Macesich [1962]. They defined average tax rate as the total amount of taxes paid by an individual or business divided by the taxable income. Cagan further explained that this series presumes that the amount of tax evasion depends directly on the rewards and believed that the changes in the taxes affect currency holdings with a delay as it took some time until people start to escape them.

Tanzi [1982b] has explained in his work the role of average tax rates stating that: "if the average tax rate is also high, there could be an income effect that might reinforce the taxpayers' propensity to evade tax." In his researches about the average personal income tax, obtained via dividing personal income taxes by personal income net of transfers, he believe in the removal of transfer income as it is largely non-taxable. He explains that only if all the taxes paid from transfer payment were taken away from the numerator, all transfers should be removed from the denominator.

Marginal tax rates such as: statutory tax rates, aggregate effective marginal rates and tax ratio, represent the incentive effect, but, they cannot be readily measured [Lee, 2005]. Lee also argues for a need to generate a weighted average of various statutory tax rates since there are various taxes in any economy, including personal income tax, corporate income tax, commodity taxes, etc. This process however involves strong assumptions and approximations that might cause the loss of many observations.

Koester and Kormendi [1989] suggested a simple method in this regard to estimate the aggregate effective marginal tax rates. The tax revenue was regressed on GDP for each country and

the estimated coefficient of the tax revenues was used as a measure of the effective aggregate marginal tax rates. Only one aggregate effective marginal tax rate can be estimated, otherwise, it causes loss within country variations in tax rates. Moreover, these effective aggregate marginal tax rates are not noticeably different from the tax ratio, which is an average tax rate that can also work as a proxy for weighted average of numerous marginal tax rates. According to Schneider [2004a] and Schneider and Enste [2000], the higher the marginal tax rate, the larger is the substitution effect and the greater the distortion of the labor-leisure decision.

In order to decide which tax rates must be used, we can use the Laffer curve [Lemieux et al.,

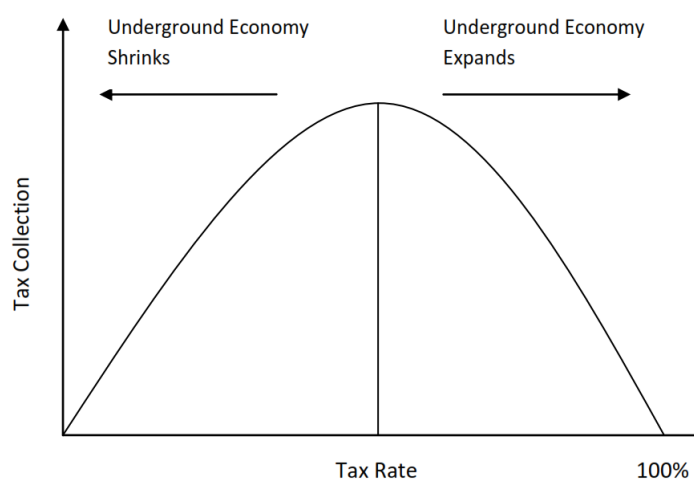


Figure 7.8: Laffer curve and the underground economy [Sharpe, 2007].

1994]. This curve (Figure (7.8)) depicts the relationship between possible rates of taxation and the resulting levels of government revenue. It is used to demonstrate the concept of taxable income elasticity in which taxable income will change in response to the changes in the tax rate. Figure (7.8) shows that a 0% tax rate raises no tax revenue, neither does a 100% tax rate since a rational taxpayer has no incentive to earn income in such a case. Thus, there must be at least one rate between 0% and 100% rate of taxation where the tax revenue would be a non-zero maximum. The Laffer curve suggests that a rise in the marginal tax rate will cause the tax revenue to decrease when the tax rate is high Schneider [2004a]; Schneider and Enste [2000]. It means that the tax revenue of government will increase as the tax rates increase, but increasing the tax rates beyond a certain point will be counter-productive for raising further tax revenue. It is obvious from the Laffer curve that the underground economy shrinks as the tax rates increase from low level to high level while the tax revenue collected

by the government increases.

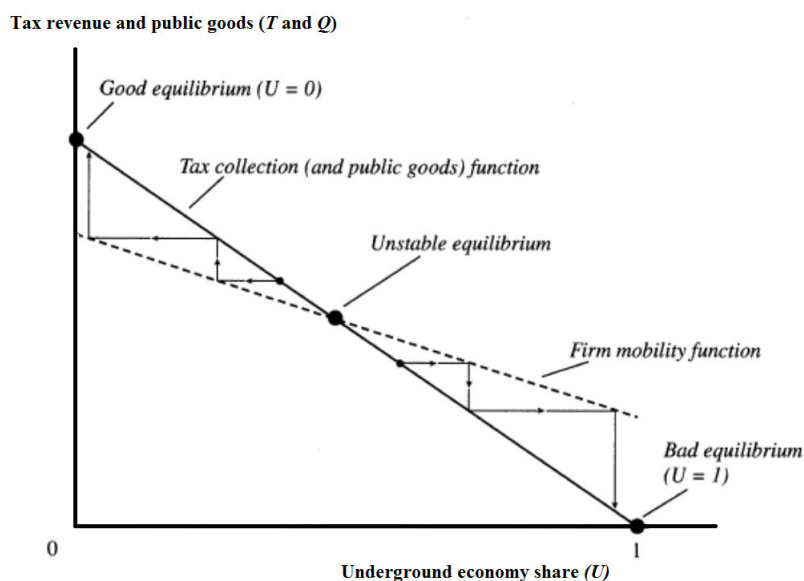
Schneider and Enste [2002] explains that If a growing underground economy is due to a rise in the tax rates beyond the maximum level, it may lead eventually to an erosion of tax revenue and social security base. This, consequently, forces the government to raise the tax rates on the official sector which, in turn, incentivize to participate in the underground economy. A vicious cycle arises therefor if the budget deficits or the tax rates further increase with the consequence of continues growth of the underground economy [Schneider, 2004a; Schneider and Enste, 2000].

If the decrease in the underground economy is due to an increase in the tax rates from low level, tax revenue collected by the government may increase which consequently makes up budget deficit or reduces tax rates. This, in turn, undermine the incentive to move into the underground economy, leading to a virtue cycle.

In this regard, Johnson et al. [1997] proposed a graph (Figure (7.9)) that depicts the equilibrium of the underground economy. From Figure (7.9), if the public good provision increases, more individuals will join the official economy, economies of scale will exist and the returns to official economy will be more than the entries to the underground economy. The tax revenue collected by the government will consequently increase, and the provision of public goods increases further.

It was explained that according to the Laffer curve (Figure (7.10)), if government increases the tax rate beyond the revenue maximizing point (threshold), the tax revenue will begin to drop. In such conditions, the equilibrium point in Figure (7.9) will begin to move down until it finally reaches the bad equilibrium point. Briefly, any increase in the tax rates beyond the threshold will result in a decrease in tax the revenue which forces the whole economy into high tax rate and the underground economy with low tax revenue and public good provision. Giles and Caragata [2001] believed that the effective tax rate affects the underground economy, but they did not indicate the reverse causal effect. It means that there is no bi-causality effect which also implies the absence of a vicious cycle. If we accept that a vicious cycle exist, then reversing the cycle by decreasing the tax rates may contribute to bringing the economy to its desirable equilibrium level.

If the response of the underground economy to an increase in the taxes is different from the response to a decrease in the taxes, then the response of the underground economy is asymmetric.



Source: Johnson, Kaufmann, and Shleifer (1997)

Figure 7.9: The underground economy and the collapse of public finance.

Christopoulos [2003] supported the symmetry hypothesis in a study of symmetrical response of the underground economy to direct and indirect tax for the case of Greece. Although Giles and Caragata [2001] did not find asymmetric effect of the underground economy to upward and downward movement in the effective tax rate, there were marked differences in the point elasticities associated with upward and downward changes in the tax rates.

In another study by Schneider [1998], it was demonstrated that a tax reform, such as the direct tax decrease, does not lead to the decrease in the size of the underground economy. This only helps to stabilize the size of the underground economy but not to reduce it. If the asymmetric hypothesis is true, the government will face difficulties in using tax reformation as a means to improve the underground economy problem. Therefore, the government might have to seek for other solutions.

Selecting the appropriate tax rate is not therefore as easy as it seems. It depends on several criteria in the society and economy some of which were discussed above. Besides imposing reasonable tax rates which is the government duty, government needs to reform the tax system, simplifies the tax procedures and adopts the highest level of transparency and accountability in the tax system in order to reduce the incentives to work in the underground economy. The government should concentrate more on extending the tax base to collect sufficient tax revenues rather than raising the tax rates. About individual firms and micro-enterprises, the

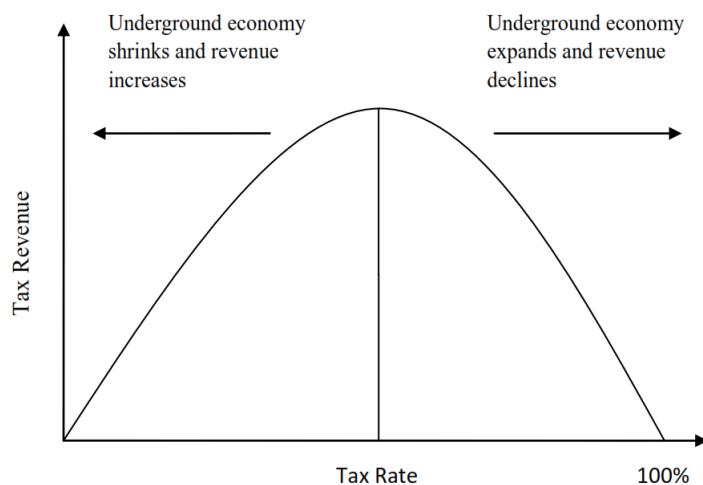


Figure 7.10: Laffer's Curve, the underground economy and tax revenue [Sharpe, 2007].

government should implement tax incentive policies to dissuade them from working in the underground economy. The government should also lessen the complexity and rigidity of the regulations and reduce the unnecessary intervention in the economy in order to provide an attractive business environment in the official economy and to reduce the incentives to work in the underground economy.

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Part IV

Appendices

Appendix A

Initial Data and Calculations

A.1 Initial data

Table A.1: The initial data of France (OECD statistics Data Base)

Year	Causes						Indicators		
	DTR	ITR	UR	GR	SER	IR	GDPC	HSR	PRL
1985	15.985	29.680	8.93	51.88	15.02	5.83	20.428	10.3	65.678
1986	16.525	29.279	9.08	51.28	14.74	2.54	15.545	9.24	65.709
1987	16.674	29.256	9.26	50.71	14.51	3.29	13.737	7.19	65.641
1988	16.082	29.388	8.83	50.06	14.18	2.7	14.213	7.55	65.603
1989	16.086	28.735	8.34	48.93	13.71	3.5	15.851	8.1	65.705
1990	16.048	28.405	8	49.58	13.36	3.23	13.940	9.04	68.251
1991	16.860	27.202	8.46	50.69	12.86	3.22	14.615	10.06	68.286
1992	15.996	26.470	9.32	52	12.33	2.41	13.901	10.84	68.463
1993	15.833	26.204	10.55	55.07	11.89	2.08	14.803	11.98	68.154
1994	16.104	27.056	11.12	54.07	11.44	1.67	14.937	11.33	68.505
1995	16.274	27.675	10.53	54.39	10.97	1.79	13.775	12.47	68.491
1996	16.863	27.954	10.97	54.5	10.61	2	14.319	11.44	68.875
1997	18.192	27.434	11.14	54.18	10.29	1.19	16.899	12.46	69.040
1998	23.120	26.959	10.74	52.76	9.91	0.64	17.731	11.86	69.637
1999	23.937	26.594	10.38	52.6	9.62	0.54	19.161	11.26	70.422
2000	24.932	25.811	9.02	51.69	9.31	1.69	23.347	11.01	70.702
2001	25.363	25.272	8.18	51.66	9.01	1.63	24.832	11.66	70.898
2002	23.920	25.735	8.31	52.87	8.91	1.92	23.987	12.97	71.028
2003	23.189	25.565	8.89	53.4	8.9	2.11	19.356	11.87	71.057
2004	23.365	25.591	9.28	53.26	8.97	2.13	17.868	12.22	70.930
2005	23.537	25.374	9.28	53.57	9.05	1.74	18.387	11.08	70.854
2006	24.240	24.916	9.25	52.98	9.06	1.68	18.971	11.21	70.966
2007	23.884	24.845	8.38	52.61	9.01	1.49	17.846	11.72	70.905
2008	24.155	24.592	7.78	53.28	9.03	2.81	17.051	11.7	70.550
2009	20.727	25.094	9.53	56.77	9.19	0.09	17.507	12.55	70.844
2010	21.885	25.035	9.77	56.55	9.21	1.53	18.634	12.06	70.927

Table A.2: The initial data of Italy (OECD statistics Data Base)

Year	Causes						Indicators		
	DTR	ITR	UR	GR	SER	IR	GDPC	HSR	PRL
1985	36.831	25.386	8.14	49.84	29.7	9.21	21.840	..	61.450
1986	36.175	26.087	8.88	50.45	29.85	5.82	16.626	..	62.169
1987	36.129	26.438	9.63	49.77	29.77	4.75	14.415	..	62.415
1988	35.686	28.041	9.7	50.38	29.41	5.06	14.604	..	62.756
1989	37.028	27.031	9.7	51.59	29.12	6.26	15.545	..	62.583
1990	36.481	28.016	8.87	52.64	28.67	6.46	13.274	21.65	63.050
1991	36.097	28.067	8.52	53.7	28.55	6.25	13.397	21.44	62.931
1992	37.522	27.150	8.79	55.03	28.57	5.27	13.124	20.24	62.070
1993	36.947	25.917	9.73	55.99	28.92	4.63	16.480	19.52	58.702
1994	34.692	28.330	10.65	53.22	29.03	4.05	16.983	18.13	58.483
1995	35.294	27.293	11.18	52.22	29.32	5.24	17.136	16.69	58.634
1996	34.632	25.789	11.17	52.17	29.34	4.01	15.999	17.48	59.095
1997	35.312	25.866	11.22	49.96	29.14	2.04	17.801	14.72	59.586
1998	32.717	27.508	11.3	48.9	29.08	1.96	18.569	10.74	60.433
1999	33.976	27.534	10.87	47.85	28.64	1.66	19.458	9.97	61.072
2000	33.168	27.888	10.05	45.86	28.49	2.54	23.337	7.94	61.763
2001	34.136	26.837	9.01	47.71	28.16	2.79	24.673	9.96	62.428
2002	32.454	26.985	8.48	47.12	27.67	2.47	22.432	10.81	62.966
2003	30.954	25.731	8.43	48.08	27.53	2.67	18.521	10.28	63.065
2004	31.453	26.447	8.03	47.53	28.39	2.21	16.729	10.5	62.961
2005	31.569	26.422	7.7	47.92	27.03	1.99	16.836	10.22	62.935
2006	32.931	26.217	6.8	48.45	26.71	2.09	17.644	9.55	63.321
2007	33.756	25.253	6.1	47.63	26.36	1.83	16.623	8.95	63.144
2008	34.358	24.538	6.73	48.6	25.73	3.35	16.039	8.45	63.668
2009	32.727	24.344	7.8	51.88	25.24	0.77	15.841	6.95	63.066
2010	32.868	25.773	8.43	50.51	25.48	1.53	16.777	5.1	62.855

Table A.3: The initial data of Germany (OECD statistics Data Base)

Year	Causes						Indicators		
	DTR	ITR	UR	GR	SER	IR	GDPC	HSR	PRL
1985	34.784	25.665	2.07	25.283	..	66.557
1986	34.552	25.174	-0.13	18.839	..	67.253
1987	34.142	25.434	0.25	15.926	..	67.800
1988	34.214	25.229	1.27	16.356	..	68.018
1989	35.054	25.614	2.78	18.283	..	68.482
1990	32.385	26.724	2.7	16.305	..	69.539
1991	31.826	27.053	5.53	46.25	9.79	4.01	14.107	..	71.555
1992	32.178	27.086	6.57	47.18	10.05	5.14	13.037	..	71.463
1993	30.878	27.979	7.79	48.15	10.37	4.39	13.355	..	71.336
1994	29.556	28.883	8.45	47.82	10.61	2.76	13.344	..	71.093
1995	30.279	27.969	8.25	54.86	10.71	1.75	11.955	11.21	71.009
1996	28.618	28.071	8.94	49.06	10.76	1.38	12.793	10.76	71.196
1997	28.166	27.917	9.68	48.22	10.88	1.93	15.019	10.32	71.514
1998	29.373	27.320	9.45	48	10.99	1	15.565	10.3	72.109
1999	29.842	27.967	8.63	48.22	10.84	0.55	16.766	9.65	71.235
2000	30.139	28.083	8.01	45.1	10.96	1.42	20.131	9.38	71.317
2001	28.780	28.725	7.86	47.61	11.15	1.94	21.245	9.55	71.749
2002	27.982	29.077	8.68	47.95	11.18	1.48	20.419	10.07	71.773
2003	27.446	29.318	9.81	48.46	11.44	1.04	17.414	10.43	71.754
2004	27.379	29.094	10.5	47.07	12.13	1.65	16.385	10.58	72.935
2005	28.184	28.955	11.28	46.91	12.45	1.52	17.067	10.72	74.240
2006	30.242	28.359	10.28	45.35	12.21	1.6	18.182	10.82	75.520
2007	31.151	29.237	8.66	43.51	12.06	2.26	17.349	11.02	76.243
2008	31.458	29.042	7.54	44.05	11.71	2.6	17.072	11.68	76.622
2009	28.851	29.654	7.76	48.1	11.65	0.38	16.937	11.09	77.073
2010	28.642	29.496	7.08	47.5	11.62	1.12	18.811	11.27	77.333

Table A.4: The initial data of The United States (OECD statistics Data Base)

Year	Causes						Indicators		
	DTR	ITR	UR	GR	SER	IR	GDPC	HSR	PRL
1985	45.366	18.811	7.19	36.85	9.09	3.55	15.206	8.53	69
1986	44.920	18.556	6.99	37.26	8.9	1.9	15.589	7.84	69.68
1987	46.760	17.596	6.19	37.06	8.93	3.66	15.944	6.73	70.74
1988	45.561	17.830	5.49	36.21	8.98	4.08	16.451	7.11	71.65
1989	46.469	17.055	5.27	36.19	8.88	4.83	16.879	6.87	72.5
1990	45.960	17.367	5.62	37.24	8.8	5.4	17.002	6.73	72.19
1991	44.022	18.081	6.82	38.67	9.02	4.23	16.733	7.29	70.97
1992	43.776	18.320	7.51	38.65	8.7	3.03	17.075	7.27	70.81
1993	44.580	18.252	6.9	38.11	8.82	2.95	17.336	6.11	71.21
1994	44.727	18.509	6.08	37.17	8.8	2.61	17.830	5.72	72
1995	46.046	17.937	5.61	37.13	8.52	2.81	18.068	5.72	72.53
1996	47.521	17.316	5.42	36.59	8.42	2.93	18.536	5.12	72.86
1997	48.500	16.888	4.95	35.46	8.25	2.34	19.142	4.75	73.5
1998	49.228	16.581	4.51	34.62	7.94	1.55	19.754	5.44	73.85
1999	49.537	16.398	4.22	34.17	7.66	2.19	20.461	3.25	73.95
2000	50.491	16.028	3.99	33.88	7.42	3.38	21.082	3.03	74.1
2001	48.715	16.297	4.73	34.98	7.36	2.83	21.096	2.79	73.13
2002	44.184	17.735	5.78	35.89	7.24	1.59	21.274	3.65	71.93
2003	43.149	18.296	5.99	36.27	7.57	2.27	21.614	3.76	71.22
2004	43.667	18.260	5.53	36.06	7.57	2.68	22.163	3.54	71.22
2005	46.701	17.656	5.07	36.32	7.47	3.39	22.634	1.65	71.53
2006	48.315	17.133	4.62	36.02	7.4	3.23	23.015	2.7	72
2007	48.780	16.762	4.62	36.85	7.22	2.85	23.223	2.42	71.78
2008	45.596	17.464	5.78	39.09	7.02	3.84	22.925	5.53	70.89
2009	40.547	18.532	9.27	42.66	7.09	-0.36	21.967	5.35	67.62
2010	42.972	17.922	9.62	42.46	7.05	1.64	22.399	5.47	66.69

Table A.5: The initial data of Canada (OECD statistics Data Base)

Year	Causes						Indicators		
	DTR	ITR	UR	GR	SER	IR	GDPC	HSR	PRL
1985	44.113	31.817	10.67	48.3	10.66	3.96	17.248	16.02	65.8
1986	45.816	29.591	9.68	47.58	9.81	4.19	17.652	13.69	67.29
1987	47.255	29.089	8.83	46.06	9.57	4.36	17.034	12.12	68.59
1988	46.718	28.334	7.77	45.36	9.49	4.03	16.222	12.57	69.98
1989	47.689	27.792	7.57	45.84	9.39	4.98	15.595	13.34	70.75
1990	48.565	25.857	8.15	48.8	9.47	4.78	15.245	13.33	70.34
1991	47.031	25.893	10.33	52.26	9.82	5.63	14.543	13.59	68.26
1992	44.760	26.327	11.19	53.34	10.13	1.49	15.429	13.26	66.82
1993	44.291	26.682	11.4	52.23	10.66	1.87	16.812	12.05	66.46
1994	44.766	26.156	10.42	49.74	10.74	0.17	18.568	9.61	67.04
1995	46.444	25.415	9.51	48.48	10.58	2.15	18.955	9.41	67.52
1996	47.157	24.972	9.61	46.59	11.1	1.57	18.971	7.16	67.33
1997	48.843	24.421	9.12	44.28	11.4	1.62	20.011	4.95	67.99
1998	48.365	24.741	8.28	44.8	11.76	1	22.440	4.94	68.93
1999	49.849	24.059	7.59	42.68	11.3	1.73	23.468	4.09	70
2000	50.056	24.214	6.82	41.11	10.61	2.72	23.613	4.78	70.91
2001	47.805	25.136	7.23	41.99	9.9	2.53	24.962	5.26	70.79
2002	45.791	26.408	7.67	41.23	9.83	2.26	25.504	3.53	71.42
2003	45.597	26.267	7.58	41.18	9.82	2.76	23.026	2.7	72.18
2004	46.714	25.781	7.18	39.86	9.54	1.86	21.771	3.24	72.46
2005	47.378	25.322	6.76	39.3	9.47	2.21	20.986	2.18	72.4
2006	48.603	24.482	6.33	39.45	9.17	2	20.104	3.61	72.78
2007	49.196	24.022	6.03	39.35	9.33	2.14	19.188	2.9	73.51
2008	49.194	23.449	6.13	40.03	9.08	2.37	18.780	4.04	73.62
2009	47.329	23.774	8.26	44.37	9.48	0.3	19.776	4.7	71.48
2010	46.882	24.362	8	44.08	9.22	1.78	17.867	4.88	71.54

A.2 Fuzzy modeling applying Mean and Variance

Table A.6: The initial data of France (Only the values of the causes)

Year	Causes of France					
	DTR	ITR	UR	GR	SER	IR
1985	15.985	29.680	8.93	51.88	15.02	5.83
1986	16.525	29.279	9.08	51.28	14.74	2.54
1987	16.674	29.256	9.26	50.71	14.51	3.29
1988	16.082	29.388	8.83	50.06	14.18	2.7
1989	16.086	28.735	8.34	48.93	13.71	3.5
1990	16.048	28.405	8	49.58	13.36	3.23
1991	16.860	27.202	8.46	50.69	12.86	3.22
1992	15.996	26.470	9.32	52	12.33	2.41
1993	15.833	26.204	10.55	55.07	11.89	2.08
1994	16.104	27.056	11.12	54.07	11.44	1.67
1995	16.274	27.675	10.53	54.39	10.97	1.79
1996	16.863	27.954	10.97	54.5	10.61	2
1997	18.192	27.434	11.14	54.18	10.29	1.19
1998	23.120	26.959	10.74	52.76	9.91	0.64
1999	23.937	26.594	10.38	52.6	9.62	0.54
2000	24.932	25.811	9.02	51.69	9.31	1.69
2001	25.363	25.272	8.18	51.66	9.01	1.63
2002	23.920	25.735	8.31	52.87	8.91	1.92
2003	23.189	25.565	8.89	53.4	8.9	2.11
2004	23.365	25.591	9.28	53.26	8.97	2.13
2005	23.537	25.374	9.28	53.57	9.05	1.74
2006	24.240	24.916	9.25	52.98	9.06	1.68
2007	23.884	24.845	8.38	52.61	9.01	1.49
2008	24.155	24.592	7.78	53.28	9.03	2.81
2009	20.727	25.094	9.53	56.77	9.19	0.09
2010	21.885	25.035	9.77	56.55	9.21	1.53

Table A.7: Fuzzification of the Direct Tax Rate (for France in FM applying Mean and Variance)

Year	DTR (Direct Tax Rate)			VL	L	N	H	VH	First LV		Second LV	
	DTR Value	Mean	SD	N-2SD	N-SD	N	N+SD	N+2SD	LT	MF	LT	MF
1989	16.086	16.270	0.308	15.655	15.963	16.270	16.578	16.886	L	0.599	N	0.401
1990	16.048	16.283	0.294	15.694	15.989	16.283	16.577	16.872	L	0.799	N	0.201
1991	16.860	16.350	0.387	15.576	15.963	16.350	16.736	17.123	H	0.682	VH	0.318
1992	15.996	16.214	0.363	15.489	15.852	16.214	16.577	16.939	L	0.601	N	0.399
1993	15.833	16.165	0.400	15.364	15.764	16.165	16.565	16.965	L	0.827	N	0.173
1994	16.104	16.168	0.399	15.369	15.769	16.168	16.568	16.967	L	0.160	N	0.840
1995	16.274	16.214	0.395	15.423	15.818	16.214	16.609	17.004	N	0.847	H	0.153
1996	16.863	16.214	0.397	15.421	15.818	16.214	16.611	17.007	H	0.364	VH	0.636
1997	18.192	16.653	0.939	14.775	15.714	16.653	17.593	18.532	H	0.362	VH	0.638
1998	23.120	18.111	2.918	12.275	15.193	18.111	21.029	23.947	H	0.283	VH	0.717
1999	23.937	19.677	3.595	12.486	16.082	19.677	23.273	26.868	H	0.815	VH	0.185
2000	24.932	21.409	3.631	14.146	17.777	21.409	25.040	28.672	N	0.030	H	0.970
2001	25.363	23.109	2.884	17.341	20.225	23.109	25.993	28.877	N	0.218	H	0.782
2002	23.920	24.255	0.893	22.469	23.362	24.255	25.147	26.040	L	0.375	N	0.625
2003	23.189	24.268	0.871	22.526	23.397	24.268	25.139	26.011	VL	0.239	L	0.761
2004	23.365	24.154	0.959	22.236	23.195	24.154	25.113	26.071	L	0.823	N	0.177
2005	23.537	23.875	0.875	22.125	23.000	23.875	24.750	25.624	L	0.386	N	0.614
2006	24.240	23.650	0.426	22.797	23.224	23.650	24.077	24.503	H	0.618	VH	0.382
2007	23.884	23.643	0.421	22.801	23.222	23.643	24.064	24.485	N	0.428	H	0.572
2008	24.155	23.836	0.380	23.076	23.456	23.836	24.216	24.597	N	0.161	H	0.839
2009	20.727	23.309	1.469	20.371	21.840	23.309	24.778	26.247	VL	0.757	L	0.243
2010	21.885	22.978	1.586	19.807	21.392	22.978	24.564	26.150	L	0.689	N	0.311

Table A.8: Fuzzification of the Indirect Tax Rate (for France in FM applying Mean and Variance)

Year	ITR (Indirect Tax Rate)			VL	L	N	H	VH	First LV		Second LV	
	ITR Value	Mean	SD	N-2SD	N-SD	N	N+SD	N+2SD	LT	MF	LT	MF
1989	28.735	29.267	0.342	28.583	28.925	29.267	29.610	29.952	VL	0.556	L	0.444
1990	28.405	29.012	0.423	28.165	28.589	29.012	29.436	29.859	VL	0.435	L	0.565
1991	27.202	28.597	0.875	26.847	27.722	28.597	29.472	30.347	VL	0.595	L	0.405
1992	26.470	28.040	1.183	25.673	26.857	28.040	29.223	30.407	VL	0.327	L	0.673
1993	26.204	27.403	1.132	25.139	26.271	27.403	28.535	29.667	VL	0.059	L	0.941
1994	27.056	27.067	0.853	25.362	26.214	27.067	27.920	28.773	L	0.013	N	0.987
1995	27.675	26.921	0.588	25.745	26.333	26.921	27.510	28.098	H	0.718	VH	0.282
1996	27.954	27.072	0.751	25.569	26.320	27.072	27.823	28.575	H	0.826	VH	0.174
1997	27.434	27.265	0.679	25.908	26.586	27.265	27.943	28.622	N	0.750	H	0.250
1998	26.959	27.416	0.417	26.582	26.999	27.416	27.833	28.250	VL	0.096	L	0.904
1999	26.594	27.323	0.547	26.229	26.776	27.323	27.871	28.418	VL	0.332	L	0.668
2000	25.811	26.950	0.816	25.317	26.134	26.950	27.767	28.583	VL	0.396	L	0.604
2001	25.272	26.414	0.871	24.671	25.543	26.414	27.285	28.157	VL	0.310	L	0.690
2002	25.735	26.074	0.686	24.703	25.389	26.074	26.760	27.445	L	0.494	N	0.506
2003	25.565	25.796	0.492	24.811	25.303	25.796	26.288	26.780	L	0.468	N	0.532
2004	25.591	25.595	0.207	25.181	25.388	25.595	25.802	26.009	L	0.019	N	0.981
2005	25.374	25.508	0.184	25.140	25.324	25.508	25.692	25.875	L	0.725	N	0.275
2006	24.916	25.436	0.318	24.800	25.118	25.436	25.754	26.073	VL	0.636	L	0.364
2007	24.845	25.258	0.356	24.547	24.903	25.258	25.614	25.970	VL	0.160	L	0.840
2008	24.592	25.064	0.408	24.247	24.655	25.064	25.472	25.880	VL	0.155	L	0.845
2009	25.094	24.964	0.292	24.381	24.673	24.964	25.256	25.547	N	0.554	H	0.446
2010	25.035	24.896	0.196	24.504	24.700	24.896	25.093	25.289	N	0.294	H	0.706

Table A.9: Fuzzification of the Unemployment Rate (for France in FM applying Mean and Variance)

Year	UR (Unemployment Rate)			VL	L	N	H	VH	First LV		Second LV	
	UR Value	Mean	SD	N-2SD	N-SD	N	N+SD	N+2SD	LT	MF	LT	MF
1989	8.340	8.888	0.347	8.195	8.541	8.888	9.235	9.581	VL	0.581	L	0.419
1990	8.000	8.702	0.523	7.656	8.179	8.702	9.225	9.748	VL	0.342	L	0.658
1991	8.460	8.578	0.483	7.612	8.095	8.578	9.061	9.544	L	0.244	N	0.756
1992	9.320	8.590	0.504	7.581	8.086	8.590	9.094	9.599	H	0.553	VH	0.447
1993	10.550	8.934	1.026	6.882	7.908	8.934	9.960	10.986	H	0.425	VH	0.575
1994	11.120	9.490	1.331	6.827	8.159	9.490	10.821	12.153	H	0.776	VH	0.224
1995	10.530	9.996	1.081	7.835	8.915	9.996	11.077	12.157	N	0.506	H	0.494
1996	10.970	10.498	0.707	9.083	9.791	10.498	11.205	11.913	N	0.333	H	0.667
1997	11.140	10.862	0.301	10.259	10.561	10.862	11.163	11.465	N	0.077	H	0.923
1998	10.740	10.900	0.261	10.377	10.639	10.900	11.161	11.423	L	0.612	N	0.388
1999	10.380	10.752	0.311	10.131	10.441	10.752	11.063	11.373	VL	0.198	L	0.802
2000	9.020	10.450	0.849	8.753	9.601	10.450	11.299	12.147	VL	0.685	L	0.315
2001	8.180	9.892	1.246	7.401	8.646	9.892	11.138	12.383	VL	0.374	L	0.626
2002	8.310	9.326	1.178	6.970	8.148	9.326	10.504	11.682	L	0.863	N	0.137
2003	8.890	8.956	0.874	7.208	8.082	8.956	9.830	10.704	L	0.076	N	0.924
2004	9.280	8.736	0.472	7.792	8.264	8.736	9.208	9.680	H	0.847	VH	0.153
2005	9.280	8.788	0.523	7.743	8.265	8.788	9.311	9.833	N	0.059	H	0.941
2006	9.250	9.002	0.421	8.161	8.581	9.002	9.423	9.843	N	0.410	H	0.590
2007	8.380	9.016	0.392	8.232	8.624	9.016	9.408	9.800	VL	0.623	L	0.377
2008	7.780	8.794	0.686	7.423	8.108	8.794	9.480	10.165	VL	0.479	L	0.521
2009	9.530	8.844	0.737	7.370	8.107	8.844	9.581	10.318	N	0.069	H	0.931
2010	9.770	8.942	0.836	7.271	8.106	8.942	9.778	10.613	N	0.009	H	0.991

Table A.10: Fuzzification of the Government Rate (for France in FM applying Mean and Variance)

Year	GR (Government Rate)			VL	L	N	H	VH	First LV		Second LV	
	GR Value	Mean	SD	N-2SD	N-SD	N	N+SD	N+2SD	LT	MF	LT	MF
1989	48.930	50.572	1.139	48.294	49.433	50.572	51.711	52.850	VL	0.442	L	0.558
1990	49.580	50.112	0.923	48.267	49.189	50.112	51.035	51.957	L	0.577	N	0.423
1991	50.690	49.994	0.759	48.476	49.235	49.994	50.753	51.512	N	0.083	H	0.917
1992	52.000	50.252	1.171	47.910	49.081	50.252	51.423	52.594	H	0.507	VH	0.493
1993	55.070	51.254	2.431	46.392	48.823	51.254	53.685	56.116	H	0.430	VH	0.570
1994	54.070	52.282	2.285	47.712	49.997	52.282	54.567	56.852	N	0.218	H	0.782
1995	54.390	53.244	1.830	49.583	51.414	53.244	55.074	56.905	N	0.374	H	0.626
1996	54.500	54.006	1.178	51.650	52.828	54.006	55.184	56.362	N	0.581	H	0.419
1997	54.180	54.442	0.390	53.663	54.052	54.442	54.832	55.221	L	0.672	N	0.328
1998	52.760	53.980	0.703	52.575	53.277	53.980	54.683	55.385	VL	0.736	L	0.264
1999	52.600	53.686	0.927	51.832	52.759	53.686	54.613	55.540	VL	0.171	L	0.829
2000	51.690	53.146	1.169	50.807	51.977	53.146	54.315	55.485	VL	0.245	L	0.755
2001	51.660	52.578	1.029	50.521	51.549	52.578	53.607	54.635	L	0.893	N	0.107
2002	52.870	52.316	0.593	51.130	51.723	52.316	52.909	53.502	N	0.066	H	0.934
2003	53.400	52.444	0.759	50.926	51.685	52.444	53.203	53.962	H	0.740	VH	0.260
2004	53.260	52.576	0.845	50.886	51.731	52.576	53.421	54.266	N	0.191	H	0.809
2005	53.570	52.952	0.767	51.418	52.185	52.952	53.719	54.486	N	0.194	H	0.806
2006	52.980	53.216	0.290	52.636	52.926	53.216	53.506	53.796	L	0.814	N	0.186
2007	52.610	53.164	0.378	52.409	52.786	53.164	53.542	53.919	VL	0.467	L	0.533
2008	53.280	53.140	0.362	52.415	52.778	53.140	53.502	53.865	N	0.614	H	0.386
2009	56.770	53.842	1.675	50.492	52.167	53.842	55.517	57.192	H	0.252	VH	0.748
2010	56.550	54.438	2.044	50.351	52.394	54.438	56.482	58.525	H	0.967	VH	0.033

Table A.11: Fuzzification of the Self-Employment Rate (for France in FM applying Mean and Variance)

Year	SER (Self-Employment Rate)			VL	L	N	H	VH	First LV		Second LV	
	SER Value	Mean	SD	N-2SD	N-SD	N	N+SD	N+2SD	LT	MF	LT	MF
1989	13.710	14.432	0.508	13.416	13.924	14.432	14.940	15.448	VL	0.422	L	0.578
1990	13.360	14.100	0.567	12.967	13.533	14.100	14.667	15.233	VL	0.306	L	0.694
1991	12.860	13.724	0.653	12.418	13.071	13.724	14.377	15.030	VL	0.323	L	0.677
1992	12.330	13.288	0.721	11.846	12.567	13.288	14.009	14.730	VL	0.329	L	0.671
1993	11.890	12.830	0.740	11.350	12.090	12.830	13.570	14.310	VL	0.270	L	0.730
1994	11.440	12.376	0.761	10.854	11.615	12.376	13.137	13.898	VL	0.230	L	0.770
1995	10.970	11.898	0.739	10.420	11.159	11.898	12.637	13.376	VL	0.256	L	0.744
1996	10.610	11.448	0.690	10.068	10.758	11.448	12.138	12.828	VL	0.215	L	0.785
1997	10.290	11.040	0.639	9.761	10.401	11.040	11.679	12.319	VL	0.173	L	0.827
1998	9.910	10.644	0.593	9.459	10.051	10.644	11.237	11.829	VL	0.238	L	0.762
1999	9.620	10.280	0.538	9.204	9.742	10.280	10.818	11.356	VL	0.227	L	0.773
2000	9.310	9.948	0.518	8.913	9.430	9.948	10.466	10.983	VL	0.233	L	0.767
2001	9.010	9.628	0.500	8.628	9.128	9.628	10.128	10.628	VL	0.235	L	0.765
2002	8.910	9.352	0.418	8.517	8.934	9.352	9.770	10.187	VL	0.059	L	0.941
2003	8.900	9.150	0.311	8.529	8.839	9.150	9.461	9.771	L	0.805	N	0.195
2004	8.970	9.020	0.168	8.684	8.852	9.020	9.188	9.356	L	0.297	N	0.703
2005	9.050	8.968	0.064	8.840	8.904	8.968	9.032	9.096	H	0.722	VH	0.278
2006	9.060	8.978	0.075	8.827	8.903	8.978	9.053	9.129	H	0.911	VH	0.089
2007	9.010	8.998	0.065	8.867	8.933	8.998	9.063	9.129	N	0.816	H	0.184
2008	9.030	9.024	0.036	8.952	8.988	9.024	9.060	9.096	N	0.832	H	0.168
2009	9.190	9.068	0.071	8.926	8.997	9.068	9.139	9.210	H	0.278	VH	0.722
2010	9.210	9.100	0.093	8.913	9.007	9.100	9.193	9.287	H	0.821	VH	0.179

Table A.12: Fuzzification of the Inflation Rate (for France in FM applying Mean and Variance)

Year	IR (Inflation Rate)			VL	L	N	H	VH	First LV		Second LV	
	IR Value	Mean	SD	N-2SD	N-SD	N	N+SD	N+2SD	LT	MF	LT	MF
1989	3.500	3.572	1.324	0.925	2.248	3.572	4.896	6.219	L	0.054	N	0.946
1990	3.230	3.052	0.411	2.230	2.641	3.052	3.463	3.874	N	0.567	H	0.433
1991	3.220	3.188	0.295	2.598	2.893	3.188	3.483	3.778	N	0.892	H	0.108
1992	2.410	3.012	0.444	2.124	2.568	3.012	3.456	3.900	VL	0.356	L	0.644
1993	2.080	2.888	0.609	1.670	2.279	2.888	3.497	4.106	VL	0.327	L	0.673
1994	1.670	2.522	0.693	1.136	1.829	2.522	3.215	3.908	VL	0.229	L	0.771
1995	1.790	2.234	0.621	0.992	1.613	2.234	2.855	3.476	L	0.715	N	0.285
1996	2.000	1.990	0.286	1.418	1.704	1.990	2.276	2.562	N	0.965	H	0.035
1997	1.190	1.746	0.351	1.044	1.395	1.746	2.097	2.448	VL	0.584	L	0.416
1998	0.640	1.458	0.545	0.367	0.913	1.458	2.003	2.549	VL	0.500	L	0.500
1999	0.540	1.232	0.658	-0.084	0.574	1.232	1.890	2.548	VL	0.052	L	0.948
2000	1.690	1.212	0.638	-0.064	0.574	1.212	1.850	2.488	N	0.251	H	0.749
2001	1.630	1.138	0.537	0.063	0.601	1.138	1.675	2.213	N	0.084	H	0.916
2002	1.920	1.284	0.644	-0.003	0.640	1.284	1.928	2.571	N	0.012	H	0.988
2003	2.110	1.578	0.611	0.356	0.967	1.578	2.189	2.800	N	0.129	H	0.871
2004	2.130	1.896	0.231	1.433	1.665	1.896	2.127	2.359	H	0.989	VH	0.011
2005	1.740	1.906	0.221	1.464	1.685	1.906	2.127	2.348	L	0.750	N	0.250
2006	1.680	1.916	0.206	1.504	1.710	1.916	2.122	2.328	VL	0.144	L	0.856
2007	1.490	1.830	0.280	1.269	1.550	1.830	2.110	2.391	VL	0.212	L	0.788
2008	2.810	1.970	0.524	0.922	1.446	1.970	2.494	3.018	H	0.397	VH	0.603
2009	0.090	1.562	0.971	-0.381	0.591	1.562	2.533	3.505	VL	0.515	L	0.485
2010	1.530	1.520	0.966	-0.413	0.554	1.520	2.486	3.453	N	0.990	H	0.010

Table A.13: Fuzzy inference result of the Direct Tax Rate and Self-Employment Rate (for France in FM applying Mean and Variance)

Year	DTR					SER					DTR & SER				
	VH	H	N	L	VL	VH	H	N	L	VL	VB	B	A	S	VS
1989	0.000	0.000	0.401	0.599	0.000	0.000	0.000	0.000	0.578	0.422	0.000	0.000	0.000	0.578	0.422
1990	0.000	0.000	0.201	0.799	0.000	0.000	0.000	0.000	0.694	0.306	0.000	0.000	0.000	0.694	0.306
1991	0.318	0.682	0.000	0.000	0.000	0.000	0.000	0.000	0.677	0.323	0.000	0.255	0.677	0.259	0.000
1992	0.000	0.000	0.399	0.601	0.000	0.000	0.000	0.000	0.671	0.329	0.000	0.000	0.000	0.601	0.329
1993	0.000	0.000	0.173	0.827	0.000	0.000	0.000	0.000	0.730	0.270	0.000	0.000	0.000	0.730	0.270
1994	0.000	0.000	0.840	0.160	0.000	0.000	0.000	0.000	0.770	0.230	0.000	0.000	0.000	0.616	0.160
1995	0.000	0.153	0.847	0.000	0.000	0.000	0.000	0.000	0.744	0.256	0.000	0.000	0.153	0.595	0.000
1996	0.636	0.364	0.000	0.000	0.000	0.000	0.000	0.000	0.785	0.215	0.000	0.509	0.364	0.172	0.000
1997	0.638	0.362	0.000	0.000	0.000	0.000	0.000	0.000	0.827	0.173	0.000	0.511	0.362	0.139	0.000
1998	0.717	0.283	0.000	0.000	0.000	0.000	0.000	0.000	0.762	0.238	0.000	0.573	0.283	0.191	0.000
1999	0.185	0.815	0.000	0.000	0.000	0.000	0.000	0.000	0.773	0.227	0.000	0.148	0.773	0.181	0.000
2000	0.000	0.970	0.030	0.000	0.000	0.000	0.000	0.000	0.767	0.233	0.000	0.000	0.767	0.186	0.000
2001	0.000	0.782	0.218	0.000	0.000	0.000	0.000	0.000	0.765	0.235	0.000	0.000	0.765	0.218	0.000
2002	0.000	0.000	0.625	0.375	0.000	0.000	0.000	0.000	0.941	0.059	0.000	0.000	0.000	0.500	0.059
2003	0.000	0.000	0.000	0.761	0.239	0.000	0.000	0.195	0.805	0.000	0.000	0.000	0.000	0.761	0.239
2004	0.000	0.000	0.177	0.823	0.000	0.000	0.000	0.703	0.297	0.000	0.000	0.000	0.177	0.527	0.000
2005	0.000	0.000	0.614	0.386	0.000	0.278	0.722	0.000	0.000	0.000	0.000	0.461	0.366	0.000	0.000
2006	0.382	0.618	0.000	0.000	0.000	0.089	0.911	0.000	0.000	0.000	0.382	0.618	0.000	0.000	0.000
2007	0.000	0.572	0.428	0.000	0.000	0.000	0.184	0.816	0.000	0.000	0.000	0.458	0.428	0.000	0.000
2008	0.000	0.839	0.161	0.000	0.000	0.000	0.168	0.832	0.000	0.000	0.000	0.666	0.161	0.000	0.000
2009	0.000	0.000	0.000	0.243	0.757	0.722	0.278	0.000	0.000	0.000	0.000	0.182	0.325	0.209	0.000
2010	0.000	0.000	0.311	0.689	0.000	0.179	0.821	0.000	0.000	0.000	0.000	0.233	0.655	0.000	0.000

Table A.14: Fuzzy inference result of the Unemployment Rate and Government Regulation (for France in FM applying Mean and Variance)

Year	UR					GR					UR & GR				
	VH	H	N	L	VL	VH	H	N	L	VL	VB	B	A	S	VS
1989	0.000	0.000	0.000	0.419	0.581	0.000	0.000	0.000	0.558	0.442	0.000	0.000	0.000	0.419	0.558
1990	0.000	0.000	0.000	0.658	0.342	0.000	0.000	0.423	0.577	0.000	0.000	0.000	0.000	0.577	0.342
1991	0.000	0.000	0.756	0.244	0.000	0.000	0.917	0.083	0.000	0.000	0.000	0.567	0.232	0.062	0.000
1992	0.447	0.553	0.000	0.000	0.000	0.493	0.507	0.000	0.000	0.000	0.468	0.507	0.000	0.000	0.000
1993	0.575	0.425	0.000	0.000	0.000	0.570	0.430	0.000	0.000	0.000	0.570	0.425	0.000	0.000	0.000
1994	0.224	0.776	0.000	0.000	0.000	0.000	0.782	0.218	0.000	0.000	0.224	0.776	0.000	0.000	0.000
1995	0.000	0.494	0.506	0.000	0.000	0.000	0.626	0.374	0.000	0.000	0.000	0.494	0.374	0.000	0.000
1996	0.000	0.667	0.333	0.000	0.000	0.000	0.419	0.581	0.000	0.000	0.000	0.465	0.333	0.000	0.000
1997	0.000	0.923	0.077	0.000	0.000	0.000	0.000	0.328	0.672	0.000	0.000	0.262	0.672	0.062	0.000
1998	0.000	0.000	0.388	0.612	0.000	0.000	0.000	0.000	0.264	0.736	0.000	0.000	0.000	0.388	0.612
1999	0.000	0.000	0.000	0.802	0.198	0.000	0.000	0.000	0.829	0.171	0.000	0.000	0.000	0.802	0.198
2000	0.000	0.000	0.000	0.315	0.685	0.000	0.000	0.000	0.755	0.245	0.000	0.000	0.000	0.315	0.685
2001	0.000	0.000	0.000	0.626	0.374	0.000	0.000	0.107	0.893	0.000	0.000	0.000	0.000	0.626	0.374
2002	0.000	0.000	0.137	0.863	0.000	0.000	0.934	0.066	0.000	0.000	0.000	0.103	0.819	0.049	0.000
2003	0.000	0.000	0.924	0.076	0.000	0.260	0.740	0.000	0.000	0.000	0.000	0.555	0.072	0.000	0.000
2004	0.153	0.847	0.000	0.000	0.000	0.000	0.809	0.191	0.000	0.000	0.153	0.809	0.000	0.000	0.000
2005	0.000	0.941	0.059	0.000	0.000	0.000	0.806	0.194	0.000	0.000	0.000	0.806	0.059	0.000	0.000
2006	0.000	0.590	0.410	0.000	0.000	0.000	0.000	0.186	0.814	0.000	0.000	0.149	0.590	0.328	0.000
2007	0.000	0.000	0.000	0.377	0.623	0.000	0.000	0.000	0.533	0.467	0.000	0.000	0.000	0.377	0.533
2008	0.000	0.000	0.000	0.521	0.479	0.000	0.386	0.614	0.000	0.000	0.000	0.000	0.367	0.455	0.000
2009	0.000	0.931	0.069	0.000	0.000	0.748	0.252	0.000	0.000	0.000	0.711	0.252	0.000	0.000	0.000
2010	0.000	0.991	0.009	0.000	0.000	0.033	0.967	0.000	0.000	0.000	0.032	0.967	0.000	0.000	0.000

Table A.15: Fuzzy inference result of the Indirect Tax Rate and Inflation Rate (for France in FM applying Mean and Variance)

Year	ITR					IR					ITR & IR				
	VH	H	N	L	VL	VH	H	N	L	VL	VB	B	A	S	VS
1989	0.000	0.000	0.000	0.444	0.556	0.000	0.000	0.946	0.054	0.000	0.000	0.000	0.000	0.528	0.054
1990	0.000	0.000	0.000	0.565	0.435	0.000	0.433	0.567	0.000	0.000	0.000	0.000	0.412	0.424	0.000
1991	0.000	0.000	0.000	0.405	0.595	0.000	0.108	0.892	0.000	0.000	0.000	0.000	0.103	0.565	0.000
1992	0.000	0.000	0.000	0.673	0.327	0.000	0.000	0.000	0.644	0.356	0.000	0.000	0.000	0.644	0.356
1993	0.000	0.000	0.000	0.941	0.059	0.000	0.000	0.000	0.673	0.327	0.000	0.000	0.000	0.673	0.327
1994	0.000	0.000	0.987	0.013	0.000	0.000	0.000	0.000	0.771	0.229	0.000	0.000	0.000	0.617	0.013
1995	0.282	0.718	0.000	0.000	0.000	0.000	0.000	0.285	0.715	0.000	0.000	0.282	0.715	0.000	0.000
1996	0.174	0.826	0.000	0.000	0.000	0.000	0.035	0.965	0.000	0.000	0.035	0.661	0.000	0.000	0.000
1997	0.000	0.250	0.750	0.000	0.000	0.000	0.000	0.000	0.416	0.584	0.000	0.000	0.250	0.584	0.000
1998	0.000	0.000	0.000	0.904	0.096	0.000	0.000	0.000	0.500	0.500	0.000	0.000	0.000	0.500	0.500
1999	0.000	0.000	0.000	0.668	0.332	0.000	0.000	0.000	0.948	0.052	0.000	0.000	0.000	0.668	0.332
2000	0.000	0.000	0.000	0.604	0.396	0.000	0.749	0.251	0.000	0.000	0.000	0.000	0.574	0.297	0.000
2001	0.000	0.000	0.000	0.690	0.310	0.000	0.916	0.084	0.000	0.000	0.000	0.000	0.655	0.233	0.000
2002	0.000	0.000	0.506	0.494	0.000	0.000	0.988	0.012	0.000	0.000	0.000	0.379	0.470	0.009	0.000
2003	0.000	0.000	0.532	0.468	0.000	0.000	0.871	0.129	0.000	0.000	0.000	0.399	0.445	0.097	0.000
2004	0.000	0.000	0.981	0.019	0.000	0.011	0.989	0.000	0.000	0.000	0.000	0.736	0.018	0.000	0.000
2005	0.000	0.000	0.275	0.725	0.000	0.000	0.000	0.250	0.750	0.000	0.000	0.000	0.250	0.725	0.000
2006	0.000	0.000	0.000	0.364	0.636	0.000	0.000	0.000	0.856	0.144	0.000	0.000	0.000	0.364	0.636
2007	0.000	0.000	0.000	0.840	0.160	0.000	0.000	0.000	0.788	0.212	0.000	0.000	0.000	0.788	0.212
2008	0.000	0.000	0.000	0.845	0.155	0.603	0.397	0.000	0.000	0.000	0.000	0.452	0.377	0.116	0.000
2009	0.000	0.446	0.554	0.000	0.000	0.000	0.000	0.000	0.485	0.515	0.000	0.000	0.446	0.515	0.000
2010	0.000	0.706	0.294	0.000	0.000	0.000	0.010	0.990	0.000	0.000	0.000	0.565	0.294	0.000	0.000

Table A.16: Final fuzzy inference results (for France in FM applying Mean and Variance)

Year	UR & GR					ITR & IR					DTR & SER				
	VB	B	A	S	VS	VB	B	A	S	VS	VB	B	A	S	VS
1989	0.000	0.000	0.000	0.419	0.558	0.000	0.000	0.000	0.528	0.054	0.000	0.000	0.000	0.578	0.422
1990	0.000	0.000	0.000	0.577	0.342	0.000	0.000	0.412	0.424	0.000	0.000	0.000	0.000	0.694	0.306
1991	0.000	0.567	0.232	0.062	0.000	0.000	0.000	0.103	0.565	0.000	0.000	0.255	0.677	0.259	0.000
1992	0.468	0.507	0.000	0.000	0.000	0.000	0.000	0.000	0.644	0.356	0.000	0.000	0.000	0.601	0.329
1993	0.570	0.425	0.000	0.000	0.000	0.000	0.000	0.000	0.673	0.327	0.000	0.000	0.000	0.730	0.270
1994	0.224	0.776	0.000	0.000	0.000	0.000	0.000	0.000	0.617	0.013	0.000	0.000	0.000	0.616	0.160
1995	0.000	0.494	0.374	0.000	0.000	0.000	0.282	0.715	0.000	0.000	0.000	0.000	0.153	0.595	0.000
1996	0.000	0.465	0.333	0.000	0.000	0.035	0.661	0.000	0.000	0.000	0.000	0.509	0.364	0.172	0.000
1997	0.000	0.262	0.672	0.062	0.000	0.000	0.000	0.250	0.584	0.000	0.000	0.511	0.362	0.139	0.000
1998	0.000	0.000	0.000	0.388	0.612	0.000	0.000	0.000	0.500	0.500	0.000	0.573	0.283	0.191	0.000
1999	0.000	0.000	0.000	0.802	0.198	0.000	0.000	0.000	0.668	0.332	0.000	0.148	0.773	0.181	0.000
2000	0.000	0.000	0.000	0.315	0.685	0.000	0.000	0.574	0.297	0.000	0.000	0.000	0.767	0.186	0.000
2001	0.000	0.000	0.000	0.626	0.374	0.000	0.000	0.655	0.233	0.000	0.000	0.000	0.765	0.218	0.000
2002	0.000	0.103	0.819	0.049	0.000	0.000	0.379	0.470	0.009	0.000	0.000	0.000	0.000	0.500	0.059
2003	0.000	0.555	0.072	0.000	0.000	0.000	0.399	0.445	0.097	0.000	0.000	0.000	0.000	0.761	0.239
2004	0.153	0.809	0.000	0.000	0.000	0.000	0.736	0.018	0.000	0.000	0.000	0.000	0.177	0.527	0.000
2005	0.000	0.806	0.059	0.000	0.000	0.000	0.000	0.250	0.725	0.000	0.000	0.461	0.366	0.000	0.000
2006	0.000	0.149	0.590	0.328	0.000	0.000	0.000	0.000	0.364	0.636	0.382	0.618	0.000	0.000	0.000
2007	0.000	0.000	0.000	0.377	0.533	0.000	0.000	0.000	0.788	0.212	0.000	0.458	0.428	0.000	0.000
2008	0.000	0.000	0.367	0.455	0.000	0.000	0.452	0.377	0.116	0.000	0.000	0.666	0.161	0.000	0.000
2009	0.711	0.252	0.000	0.000	0.000	0.000	0.000	0.446	0.515	0.000	0.000	0.182	0.325	0.209	0.000
2010	0.032	0.967	0.000	0.000	0.000	0.000	0.565	0.294	0.000	0.000	0.000	0.233	0.655	0.000	0.000

Table A.17: Final defuzzification results and time series of the underground economy (for France in FM applying Mean and Variance)

UE (LT & MF)							
Year	VB	B	A	S	VS	Year	UE
1989	0.000	0.000	0.000	0.578	0.558	1989	7.55
1990	0.000	0.000	0.412	0.694	0.342	1990	9.61
1991	0.000	0.567	0.677	0.565	0.000	1991	13.01
1992	0.468	0.507	0.000	0.644	0.356	1992	14.08
1993	0.570	0.425	0.000	0.730	0.327	1993	14.25
1994	0.224	0.776	0.000	0.617	0.160	1994	14.32
1995	0.000	0.494	0.715	0.595	0.000	1995	13.01
1996	0.035	0.661	0.364	0.172	0.000	1996	14.04
1997	0.000	0.511	0.672	0.584	0.000	1997	12.94
1998	0.000	0.573	0.283	0.500	0.612	1998	10.93
1999	0.000	0.148	0.773	0.802	0.332	1999	11.74
2000	0.000	0.000	0.767	0.315	0.685	2000	8.99
2001	0.000	0.000	0.765	0.626	0.374	2001	10.03
2002	0.000	0.379	0.819	0.500	0.059	2002	12.05
2003	0.000	0.555	0.445	0.761	0.239	2003	11.90
2004	0.153	0.809	0.177	0.527	0.000	2004	14.58
2005	0.000	0.806	0.366	0.725	0.000	2005	13.16
2006	0.382	0.618	0.590	0.364	0.636	2006	13.32
2007	0.000	0.458	0.428	0.788	0.533	2007	11.43
2008	0.000	0.666	0.377	0.455	0.000	2008	12.50
2009	0.711	0.252	0.446	0.515	0.000	2009	14.91
2010	0.032	0.967	0.655	0.000	0.000	2010	17.76

Table A.18: Final defuzzification results and time series of the underground economy (for Canada in FM applying Mean and Variance)

UE (LT & MF)							
Year	VB	B	A	S	VS	Year	UE
1989	0.000	0.525	0.767	0.630	0.024	1989	14.95
1990	0.000	0.454	0.472	0.458	0.000	1990	14.92
1991	0.585	0.470	0.572	0.022	0.000	1991	15.82
1992	0.329	0.671	0.451	0.553	0.447	1992	15.42
1993	0.000	0.558	0.527	0.627	0.000	1993	15.14
1994	0.000	0.315	0.606	0.695	0.080	1994	14.77
1995	0.000	0.491	0.180	0.595	0.405	1995	14.43
1996	0.346	0.654	0.158	0.729	0.271	1996	15.45
1997	0.462	0.538	0.188	0.683	0.317	1997	15.42
1998	0.340	0.660	0.406	0.578	0.422	1998	15.42
1999	0.166	0.584	0.281	0.576	0.424	1999	15.27
2000	0.000	0.475	0.525	0.669	0.331	2000	14.67
2001	0.442	0.558	0.364	0.635	0.479	2001	15.41
2002	0.305	0.592	0.280	0.576	0.491	2002	15.31
2003	0.000	0.850	0.569	0.723	0.043	2003	15.05
2004	0.000	0.000	0.243	0.590	0.340	2004	13.66
2005	0.000	0.000	0.746	0.554	0.446	2005	13.90
2006	0.000	0.362	0.548	0.626	0.497	2006	14.52
2007	0.000	0.353	0.559	0.607	0.193	2007	14.66
2008	0.000	0.257	0.775	0.182	0.000	2008	14.81
2009	0.703	0.227	0.916	0.401	0.599	2009	15.46
2010	0.032	0.966	0.187	0.590	0.233	2010	15.42

Table A.19: Final defuzzification results and time series of the underground economy (for Germany in FM applying Mean and Variance)

UE (LT & MF)						Year	UE
Year	VB	B	A	S	VS		
1992	0.339	0.707	0.673	0.234	0.000	1992	15.95
1993	0.276	0.975	0.967	0.014	0.000	1993	16.23
1994	0.053	0.562	0.482	0.442	0.000	1994	15.54
1995	0.749	0.309	0.614	0.498	0.000	1995	16.17
1996	0.000	0.709	0.749	0.708	0.000	1996	14.89
1997	0.000	0.467	0.815	0.459	0.000	1997	14.32
1998	0.211	0.520	0.534	0.726	0.274	1998	14.97
1999	0.000	0.542	0.392	0.496	0.000	1999	13.91
2000	0.103	0.799	0.331	0.276	0.598	2000	14.33
2001	0.443	0.557	0.452	0.691	0.000	2001	16.14
2002	0.228	0.493	0.562	0.350	0.000	2002	15.63
2003	0.576	0.424	0.580	0.161	0.000	2003	15.74
2004	0.000	0.536	0.514	0.000	0.000	2004	16.10
2005	0.326	0.481	0.796	0.291	0.000	2005	15.89
2006	0.604	0.396	0.333	0.512	0.000	2006	15.84
2007	0.485	0.543	0.000	0.505	0.495	2007	15.04
2008	0.308	0.471	0.503	0.608	0.392	2008	14.80
2009	0.000	0.246	0.692	0.790	0.085	2009	13.64
2010	0.000	0.375	0.882	0.860	0.106	2010	14.01

Table A.20: Final defuzzification results and time series of the underground economy (for USA in FM applying Mean and Variance)

UE (LT & MF)							
Year	VB	B	A	S	VS	Year	UE
1989	0.000	0.111	0.846	0.891	0.109	1989	8.30
1990	0.000	0.462	0.553	0.474	0.000	1990	8.49
1991	0.503	0.553	0.385	0.492	0.000	1991	9.95
1992	0.406	0.594	0.554	0.653	0.347	1992	9.32
1993	0.000	0.405	0.768	0.256	0.000	1993	8.43
1994	0.000	0.108	0.894	0.683	0.090	1994	8.20
1995	0.000	0.483	0.396	0.691	0.309	1995	7.95
1996	0.000	0.392	0.510	0.869	0.131	1996	8.23
1997	0.000	0.235	0.706	0.523	0.518	1997	7.71
1998	0.000	0.086	0.603	0.592	0.634	1998	7.49
1999	0.000	0.000	0.597	0.772	0.228	1999	7.14
2000	0.000	0.229	0.718	0.906	0.094	2000	8.32
2001	0.000	0.569	0.460	0.738	0.000	2001	8.77
2002	0.509	0.564	0.295	0.274	0.726	2002	8.75
2003	0.183	0.817	0.718	0.225	0.000	2003	9.96
2004	0.000	0.644	0.713	0.021	0.000	2004	8.38
2005	0.009	0.601	0.678	0.189	0.000	2005	9.61
2006	0.000	0.474	0.580	0.513	0.405	2006	7.87
2007	0.000	0.517	0.445	0.587	0.076	2007	8.11
2008	0.729	0.479	0.000	0.488	0.449	2008	9.43
2009	0.612	0.264	0.269	0.370	0.650	2009	8.72
2010	0.144	0.856	0.479	0.647	0.000	2010	10.09

Table A.21: Final defuzzification results and time series of the underground economy (for Italy in FM applying Mean and Variance)

Year	UE (LT & MF)					Year	UE
	VB	B	A	S	VS		
1989	0.623	0.377	0.567	0.394	0.000	1989	29.96
1990	0.062	0.938	0.536	0.457	0.000	1990	29.99
1991	0.000	0.626	0.617	0.502	0.090	1991	27.65
1992	0.000	0.325	0.685	0.916	0.000	1992	28.57
1993	0.242	0.758	0.376	0.499	0.501	1993	28.70
1994	0.000	0.424	0.757	0.441	0.000	1994	28.16
1995	0.000	0.296	0.815	0.173	0.000	1995	27.71
1996	0.000	0.090	0.888	0.952	0.048	1996	27.88
1997	0.000	0.000	0.932	0.374	0.626	1997	25.57
1998	0.000	0.000	0.664	0.400	0.705	1998	25.15
1999	0.000	0.126	0.825	0.621	0.699	1999	26.87
2000	0.000	0.716	0.035	0.748	0.688	2000	26.81
2001	0.000	0.510	0.362	0.546	0.227	2001	27.19
2002	0.000	0.122	0.403	0.604	0.396	2002	26.73
2003	0.000	0.094	0.848	0.471	0.529	2003	26.88
2004	0.000	0.189	0.763	0.548	0.419	2004	27.08
2005	0.000	0.000	0.666	0.537	0.354	2005	25.80
2006	0.000	0.257	0.679	0.547	0.000	2006	27.98
2007	0.000	0.313	0.608	0.616	0.482	2007	27.06
2008	0.000	0.450	0.806	0.207	0.000	2008	27.95
2009	0.741	0.259	0.000	0.654	0.346	2009	29.31
2010	0.353	0.631	0.407	0.655	0.000	2010	30.05

A.3 Fuzzy modeling applying Fuzzy C-means Clustering

Table A.22: The fuzzification step of Indirect Tax Rate (for Canada in FM applying Fuzzy Clustering)

Year	ITR Value	Indirect Tax Rate					VOF
		VH	H	N	L	VL	
		31.789	28.843	26.253	25.187	24.097	
		Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5	
1989	27.792	0.039	0.562	0.262	0.091	0.045	24.812
1990	25.857	0.003	0.012	0.703	0.246	0.036	19.030
1991	25.893	0.003	0.011	0.759	0.197	0.030	16.349
1992	26.327	0.000	0.001	0.994	0.004	0.001	11.611
1993	26.682	0.006	0.034	0.865	0.071	0.024	6.870
1994	26.156	0.000	0.001	0.986	0.010	0.002	5.226
1995	25.415	0.001	0.004	0.067	0.901	0.027	4.778
1996	24.972	0.001	0.003	0.026	0.916	0.055	4.555
1997	24.421	0.002	0.004	0.026	0.148	0.820	4.363
1998	24.741	0.003	0.007	0.055	0.633	0.302	4.123
1999	24.059	0.000	0.000	0.000	0.001	0.999	3.727
2000	24.214	0.000	0.001	0.003	0.014	0.981	3.198
2001	25.136	0.000	0.000	0.002	0.995	0.002	2.945
2002	26.408	0.001	0.004	0.975	0.016	0.004	2.887
2003	26.267	0.000	0.000	1.000	0.000	0.000	2.870
2004	25.781	0.004	0.014	0.574	0.363	0.045	2.865
2005	25.322	0.000	0.001	0.020	0.966	0.012	2.864
2006	24.482	0.002	0.006	0.035	0.220	0.737	2.863
2007	24.022	0.000	0.000	0.001	0.004	0.994	2.863
2008	23.449	0.005	0.012	0.044	0.115	0.825	2.863
2009	23.774	0.002	0.004	0.016	0.049	0.930	2.863
2010	24.362	0.001	0.003	0.017	0.092	0.887	2.863

Table A.23: The fuzzification step of Inflation Rate (for Canada in FM applying Fuzzy Clustering)

Year	IR value	Inflation Rate					VOF
		VH	H	N	L	VL	
		5.158	4.141	2.389	1.712	0.306	
		Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5	
1989	4.980	0.949	0.043	0.004	0.003	0.001	12.999
1990	4.780	0.716	0.251	0.018	0.011	0.005	8.733
1991	5.630	0.874	0.088	0.019	0.013	0.007	5.487
1992	1.490	0.003	0.006	0.055	0.903	0.032	3.985
1993	1.870	0.002	0.004	0.083	0.901	0.009	3.385
1994	0.170	0.001	0.001	0.004	0.008	0.987	2.686
1995	2.150	0.005	0.011	0.749	0.223	0.013	2.297
1996	1.570	0.002	0.003	0.029	0.955	0.012	1.960
1997	1.620	0.001	0.001	0.014	0.979	0.005	1.544
1998	1.000	0.012	0.021	0.110	0.417	0.440	1.308
1999	1.730	0.000	0.000	0.001	0.999	0.000	1.219
2000	2.720	0.015	0.045	0.833	0.090	0.016	1.190
2001	2.530	0.003	0.007	0.957	0.029	0.004	1.179
2002	2.260	0.002	0.004	0.938	0.052	0.004	1.174
2003	2.760	0.019	0.058	0.803	0.101	0.018	1.172
2004	1.860	0.002	0.004	0.071	0.915	0.008	1.171
2005	2.210	0.003	0.007	0.870	0.112	0.008	1.171
2006	2.000	0.005	0.011	0.342	0.624	0.018	1.170
2007	2.140	0.005	0.011	0.725	0.245	0.013	1.170
2008	2.370	0.000	0.000	0.999	0.001	0.000	1.170
2009	0.300	0.000	0.000	0.000	0.000	1.000	1.170
2010	1.780	0.000	0.001	0.012	0.985	0.002	1.170

Table A.24: The fuzzification step of Direct Tax Rate (for Canada in FM applying Fuzzy Clustering)

Year	DTR value	Direct Tax Rate					VOF
		VH	H	N	L	VL	
		49.834	48.645	47.146	45.814	44.474	
		Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5	
1989	47.689	0.043	0.215	0.667	0.056	0.019	18.360
1990	48.565	0.004	0.992	0.003	0.001	0.000	11.150
1991	47.031	0.002	0.005	0.983	0.009	0.002	7.388
1992	44.760	0.003	0.005	0.013	0.067	0.912	4.781
1993	44.291	0.001	0.002	0.004	0.014	0.979	3.812
1994	44.766	0.003	0.005	0.014	0.070	0.908	3.162
1995	46.444	0.017	0.040	0.397	0.495	0.051	2.891
1996	47.157	0.000	0.000	1.000	0.000	0.000	2.697
1997	48.843	0.038	0.944	0.013	0.004	0.002	2.460
1998	48.365	0.033	0.904	0.048	0.011	0.005	2.208
1999	49.849	1.000	0.000	0.000	0.000	0.000	2.051
2000	50.056	0.966	0.024	0.006	0.003	0.002	1.987
2001	47.805	0.056	0.329	0.535	0.059	0.021	1.959
2002	45.791	0.000	0.000	0.000	0.999	0.000	1.942
2003	45.597	0.002	0.005	0.018	0.940	0.035	1.931
2004	46.714	0.014	0.037	0.748	0.172	0.028	1.924
2005	47.378	0.008	0.031	0.934	0.021	0.006	1.920
2006	48.603	0.001	0.998	0.001	0.000	0.000	1.917
2007	49.196	0.402	0.538	0.039	0.014	0.007	1.915
2008	49.194	0.397	0.542	0.039	0.014	0.007	1.915
2009	47.329	0.005	0.018	0.959	0.014	0.004	1.914
2010	46.882	0.007	0.020	0.906	0.055	0.011	1.914

Table A.25: The fuzzification step of Unemployment Rate (for Canada in FM applying Fuzzy Clustering)

Year	UR Value	Unemployment Rate					VOF
		VH	H	N	L	VL	
		10.889	9.530	8.219	7.505	6.307	
		Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5	
1989	7.570	0.000	0.001	0.010	0.986	0.003	15.958
1990	8.150	0.001	0.002	0.984	0.011	0.001	10.685
1991	10.330	0.619	0.302	0.043	0.024	0.012	7.913
1992	11.190	0.949	0.031	0.010	0.006	0.004	5.743
1993	11.400	0.887	0.066	0.023	0.015	0.009	3.791
1994	10.420	0.735	0.204	0.033	0.019	0.010	2.371
1995	9.510	0.000	0.999	0.000	0.000	0.000	1.903
1996	9.610	0.004	0.991	0.003	0.001	0.001	1.778
1997	9.120	0.040	0.743	0.154	0.048	0.016	1.741
1998	8.280	0.001	0.002	0.990	0.006	0.001	1.730
1999	7.590	0.001	0.002	0.018	0.975	0.004	1.726
2000	6.820	0.009	0.021	0.077	0.321	0.572	1.725
2001	7.230	0.005	0.012	0.065	0.843	0.075	1.725
2002	7.670	0.002	0.007	0.081	0.896	0.013	1.725
2003	7.580	0.001	0.001	0.014	0.981	0.003	1.725
2004	7.180	0.006	0.015	0.077	0.792	0.110	1.725
2005	6.760	0.008	0.018	0.064	0.246	0.664	1.725
2006	6.330	0.000	0.000	0.000	0.000	0.999	
2007	6.030	0.003	0.006	0.015	0.033	0.943	
2008	6.130	0.001	0.003	0.007	0.016	0.973	
2009	8.260	0.000	0.001	0.995	0.003	0.000	
2010	8.000	0.005	0.017	0.807	0.158	0.013	

Table A.26: The fuzzification step of Government Regulation (for Canada in FM applying Fuzzy Clustering)

Year	GR value	Government Regulation					VOF
		VH	H	N	L	VL	
		52.565	48.447	45.025	41.568	39.600	
		Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5	
1989	45.840	0.013	0.084	0.858	0.031	0.015	88.195
1990	48.800	0.009	0.979	0.009	0.002	0.001	80.372
1991	52.260	0.991	0.006	0.002	0.001	0.001	60.903
1992	53.340	0.960	0.024	0.008	0.004	0.003	36.809
1993	52.230	0.988	0.008	0.002	0.001	0.001	28.585
1994	49.740	0.158	0.754	0.057	0.019	0.012	23.952
1995	48.480	0.000	1.000	0.000	0.000	0.000	19.764
1996	46.590	0.036	0.369	0.519	0.050	0.026	15.511
1997	44.280	0.007	0.028	0.877	0.066	0.022	12.116
1998	44.800	0.001	0.004	0.989	0.005	0.002	10.275
1999	42.680	0.009	0.026	0.160	0.712	0.093	9.508
2000	41.110	0.001	0.004	0.012	0.900	0.083	9.253
2001	41.990	0.002	0.004	0.018	0.947	0.029	9.175
2002	41.230	0.001	0.002	0.008	0.949	0.041	9.151
2003	41.180	0.001	0.003	0.009	0.931	0.056	9.143
2004	39.860	0.000	0.001	0.002	0.023	0.974	9.140
2005	39.300	0.000	0.001	0.003	0.017	0.979	9.139
2006	39.450	0.000	0.000	0.001	0.005	0.994	9.139
2007	39.350	0.000	0.001	0.002	0.012	0.985	9.139
2008	40.030	0.001	0.002	0.007	0.072	0.918	9.138
2009	44.370	0.006	0.023	0.904	0.049	0.017	9.138
2010	44.080	0.010	0.038	0.803	0.114	0.036	9.138

Table A.27: The fuzzification step of Self-Employment Rate (for Canada in FM applying Fuzzy Clustering)

Year	SER value	Self-Employment Rate					VOF
		VH	H	N	L	VL	
		11.752	11.276	10.645	9.846	9.375	
		Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5	
1989	9.390	0.000	0.000	0.000	0.001	0.999	3.804
1990	9.470	0.002	0.003	0.006	0.059	0.931	2.062
1991	9.820	0.000	0.000	0.001	0.995	0.003	1.251
1992	10.130	0.020	0.040	0.197	0.651	0.092	0.758
1993	10.660	0.000	0.001	0.999	0.000	0.000	0.492
1994	10.740	0.008	0.029	0.947	0.011	0.005	0.434
1995	10.580	0.003	0.009	0.978	0.008	0.003	0.421
1996	11.100	0.058	0.798	0.120	0.016	0.008	0.418
1997	11.400	0.107	0.861	0.023	0.005	0.003	0.416
1998	11.760	1.000	0.000	0.000	0.000	0.000	0.411
1999	11.300	0.003	0.995	0.001	0.000	0.000	0.393
2000	10.610	0.001	0.003	0.993	0.002	0.001	0.372
2001	9.900	0.001	0.001	0.005	0.982	0.010	0.366
2002	9.830	0.000	0.000	0.000	0.998	0.001	0.359
2003	9.820	0.000	0.000	0.001	0.995	0.003	0.350
2004	9.540	0.004	0.007	0.017	0.218	0.754	0.339
2005	9.470	0.002	0.003	0.006	0.059	0.931	0.331
2006	9.170	0.006	0.008	0.017	0.082	0.887	0.329
2007	9.330	0.000	0.001	0.001	0.008	0.990	0.328
2008	9.080	0.010	0.015	0.029	0.122	0.823	0.328
2009	9.480	0.002	0.003	0.007	0.075	0.913	0.328
2010	9.220	0.003	0.005	0.011	0.057	0.923	0.328

Table A.28: The fuzzy inference result of ITR (first cause) and IR (second cause)- Part 1 (for Canada in FM applying Fuzzy Clustering)

	VH+VH	VH+H	VH+N	VH+L	VH+VL	H+VH	H+H	H+N	H+L	H+VL	N+VH	N+H	N+N
	VB	VB	B	B	A	VB	B	B	A	S	B	B	A
Year	1	1	1	0.8	0.5	0.95	1	0.8	1	0.8	0.95	0.75	1
1989	0.039	0.039	0.004	0.002	0.001	0.534	0.043	0.004	0.003	0.001	0.249	0.032	0.004
1990	0.003	0.003	0.003	0.003	0.002	0.012	0.012	0.010	0.011	0.004	0.668	0.188	0.018
1991	0.003	0.003	0.003	0.002	0.001	0.011	0.011	0.009	0.011	0.005	0.721	0.066	0.019
1992	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.003	0.005	0.055
1993	0.002	0.004	0.006	0.005	0.003	0.002	0.004	0.027	0.034	0.007	0.002	0.003	0.083
1994	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.004
1995	0.001	0.001	0.001	0.001	0.001	0.004	0.004	0.003	0.004	0.003	0.004	0.008	0.067
1996	0.001	0.001	0.001	0.001	0.000	0.001	0.003	0.002	0.003	0.002	0.001	0.002	0.026
1997	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.004	0.004	0.004	0.001	0.001	0.014
1998	0.003	0.003	0.003	0.002	0.001	0.007	0.007	0.006	0.007	0.006	0.012	0.016	0.055
1999	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.003	0.002	0.003
2001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.002	0.002
2002	0.001	0.001	0.001	0.001	0.000	0.002	0.004	0.003	0.004	0.003	0.002	0.003	0.938
2003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.018	0.044	0.803
2004	0.002	0.004	0.004	0.003	0.002	0.002	0.004	0.011	0.014	0.007	0.002	0.003	0.071
2005	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.003	0.006	0.020
2006	0.002	0.002	0.002	0.002	0.001	0.005	0.006	0.005	0.006	0.005	0.005	0.008	0.035
2007	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001
2008	0.000	0.000	0.005	0.001	0.000	0.000	0.000	0.010	0.001	0.000	0.000	0.000	0.044
2009	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.003	0.000	0.000	0.000
2010	0.000	0.001	0.001	0.001	0.001	0.000	0.001	0.002	0.003	0.002	0.000	0.001	0.012

Table A.29: The fuzzy inference result of ITR (first cause) and IR (second cause)- Part 2 (for Canada in FM applying Fuzzy Clustering)

	N+N	N+L	N+VL	L+VH	L+H	L+N	L+L	L+VL	VL+VH	VL+H	VL+N	VL+L	VL+VL
	A	S	S	B	A	S	S	VS	A	S	S	VS	VS
Year	1	0.8	1	0.75	0.95	0.75	1	1	0.45	0.75	0.95	1	1
1989	0.004	0.002	0.001	0.069	0.041	0.003	0.003	0.001	0.020	0.032	0.004	0.003	0.001
1990	0.018	0.009	0.005	0.184	0.234	0.013	0.011	0.005	0.016	0.027	0.017	0.011	0.005
1991	0.019	0.010	0.007	0.148	0.083	0.014	0.013	0.007	0.014	0.023	0.018	0.013	0.007
1992	0.055	0.722	0.032	0.002	0.004	0.003	0.004	0.004	0.000	0.001	0.001	0.001	0.001
1993	0.083	0.692	0.009	0.002	0.004	0.053	0.071	0.009	0.001	0.003	0.023	0.024	0.009
1994	0.004	0.006	0.986	0.001	0.001	0.003	0.008	0.010	0.000	0.001	0.002	0.002	0.002
1995	0.067	0.053	0.013	0.004	0.010	0.562	0.223	0.013	0.002	0.008	0.026	0.027	0.013
1996	0.026	0.021	0.012	0.001	0.003	0.022	0.916	0.012	0.001	0.002	0.027	0.055	0.012
1997	0.014	0.021	0.005	0.001	0.001	0.011	0.148	0.005	0.000	0.001	0.013	0.820	0.005
1998	0.055	0.044	0.055	0.009	0.020	0.082	0.417	0.440	0.006	0.016	0.104	0.302	0.302
1999	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.000	0.000	0.001	0.999	0.000
2000	0.003	0.003	0.003	0.011	0.014	0.011	0.014	0.014	0.007	0.034	0.792	0.090	0.016
2001	0.002	0.002	0.002	0.002	0.007	0.718	0.029	0.004	0.001	0.002	0.002	0.002	0.002
2002	0.938	0.041	0.004	0.001	0.004	0.012	0.016	0.004	0.001	0.003	0.004	0.004	0.004
2003	0.803	0.081	0.018	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2004	0.071	0.459	0.008	0.001	0.004	0.053	0.363	0.008	0.001	0.003	0.043	0.045	0.008
2005	0.020	0.016	0.008	0.002	0.007	0.652	0.112	0.008	0.001	0.006	0.011	0.012	0.008
2006	0.035	0.028	0.018	0.004	0.011	0.165	0.220	0.018	0.002	0.008	0.324	0.624	0.018
2007	0.001	0.001	0.001	0.003	0.004	0.003	0.004	0.004	0.002	0.008	0.689	0.245	0.013
2008	0.044	0.001	0.000	0.000	0.000	0.086	0.001	0.000	0.000	0.000	0.783	0.001	0.000
2009	0.000	0.000	0.016	0.000	0.000	0.000	0.000	0.049	0.000	0.000	0.000	0.000	0.930
2010	0.012	0.014	0.002	0.000	0.001	0.009	0.092	0.002	0.000	0.001	0.012	0.887	0.002

Table A.30: Final fuzzy inference results (for Canada in FM applying Fuzzy Clustering)

Year	UR & GR					ITR & IR					DTR & SER				
	VB	B	A	S	VS	VB	B	A	S	VS	VB	B	A	S	VS
1989	0.001	0.009	0.080	0.643	0.015	0.534	0.249	0.041	0.032	0.003	0.000	0.001	0.021	0.667	0.056
1990	0.002	0.734	0.011	0.006	0.001	0.012	0.668	0.234	0.027	0.011	0.003	0.005	0.059	0.745	0.001
1991	0.619	0.041	0.006	0.005	0.001	0.011	0.721	0.083	0.023	0.013	0.000	0.001	0.005	0.786	0.003
1992	0.949	0.024	0.008	0.005	0.004	0.001	0.005	0.055	0.722	0.004	0.005	0.015	0.038	0.187	0.651
1993	0.887	0.022	0.007	0.006	0.001	0.004	0.027	0.083	0.692	0.024	0.001	0.001	0.004	0.930	0.000
1994	0.735	0.204	0.033	0.019	0.012	0.001	0.001	0.004	0.986	0.010	0.005	0.010	0.028	0.862	0.011
1995	0.000	0.999	0.000	0.000	0.000	0.004	0.008	0.067	0.562	0.027	0.009	0.032	0.397	0.371	0.008
1996	0.034	0.415	0.050	0.021	0.001	0.001	0.003	0.026	0.916	0.055	0.000	0.599	0.120	0.013	0.000
1997	0.028	0.594	0.154	0.053	0.022	0.001	0.004	0.014	0.148	0.820	0.101	0.861	0.013	0.004	0.003
1998	0.001	0.003	0.989	0.005	0.002	0.007	0.016	0.055	0.417	0.440	0.859	0.045	0.002	0.000	0.000
1999	0.002	0.013	0.025	0.712	0.093	0.000	0.000	0.000	0.001	0.999	0.995	0.001	0.000	0.000	0.000
2000	0.004	0.010	0.021	0.321	0.572	0.001	0.011	0.014	0.792	0.090	0.003	0.966	0.006	0.002	0.002
2001	0.004	0.010	0.018	0.843	0.075	0.000	0.002	0.007	0.718	0.004	0.001	0.045	0.329	0.428	0.021
2002	0.002	0.006	0.008	0.896	0.041	0.002	0.004	0.938	0.041	0.004	0.000	0.000	0.000	0.998	0.001
2003	0.001	0.002	0.009	0.931	0.056	0.000	0.044	0.803	0.081	0.000	0.000	0.002	0.005	0.940	0.035
2004	0.001	0.005	0.015	0.077	0.792	0.004	0.011	0.071	0.459	0.045	0.007	0.014	0.037	0.748	0.172
2005	0.001	0.006	0.017	0.064	0.664	0.001	0.006	0.020	0.652	0.012	0.003	0.007	0.031	0.931	0.021
2006	0.000	0.000	0.000	0.001	0.994	0.005	0.008	0.035	0.324	0.624	0.005	0.014	0.082	0.710	0.000
2007	0.001	0.002	0.006	0.015	0.943	0.000	0.003	0.004	0.689	0.245	0.001	0.006	0.201	0.430	0.014
2008	0.001	0.002	0.007	0.016	0.918	0.000	0.010	0.044	0.783	0.001	0.015	0.098	0.199	0.434	0.014
2009	0.001	0.017	0.904	0.040	0.003	0.000	0.000	0.001	0.016	0.930	0.003	0.006	0.018	0.913	0.014
2010	0.009	0.028	0.803	0.118	0.036	0.001	0.002	0.012	0.092	0.887	0.005	0.009	0.020	0.906	0.055

Table A.31: Final defuzzification results and time series of the underground economy (for Canada in FM applying Fuzzy Clustering)

UE (LT & MF)							
Year	VB	B	A	S	VS	Year	UE
1989	0.534	0.249	0.080	0.667	0.056	1989	16.68
1990	0.012	0.734	0.234	0.745	0.011	1990	15.99
1991	0.619	0.721	0.083	0.786	0.013	1991	17.03
1992	0.949	0.024	0.055	0.722	0.651	1992	15.91
1993	0.887	0.027	0.083	0.930	0.024	1993	16.84
1994	0.735	0.204	0.033	0.986	0.012	1994	16.67
1995	0.009	0.999	0.397	0.562	0.027	1995	16.40
1996	0.034	0.599	0.120	0.916	0.055	1996	15.58
1997	0.101	0.861	0.154	0.148	0.820	1997	15.30
1998	0.859	0.045	0.989	0.417	0.440	1998	16.34
1999	0.995	0.013	0.025	0.712	0.999	1999	15.49
2000	0.004	0.966	0.021	0.792	0.572	2000	15.18
2001	0.004	0.045	0.329	0.843	0.075	2001	14.55
2002	0.002	0.006	0.938	0.998	0.041	2002	14.92
2003	0.001	0.044	0.803	0.940	0.056	2003	14.91
2004	0.007	0.014	0.071	0.748	0.792	2004	13.18
2005	0.003	0.007	0.031	0.931	0.664	2005	13.25
2006	0.005	0.014	0.082	0.710	0.994	2006	13.04
2007	0.001	0.006	0.201	0.689	0.943	2007	13.21
2008	0.015	0.098	0.199	0.783	0.918	2008	13.52
2009	0.003	0.017	0.904	0.913	0.930	2009	14.01
2010	0.009	0.028	0.803	0.906	0.887	2010	14.00

Table A.32: Final defuzzification results and time series of the underground economy (for France in FM applying Fuzzy Clustering)

Year	UE (LT & MF)					Year	UE
	VB	B	A	S	VS		
1989	0.556	0.347	0.055	0.677	0.886	1989	13.03
1990	0.160	0.746	0.019	0.800	0.921	1990	12.02
1991	0.013	0.682	0.711	0.204	0.514	1991	13.76
1992	0.017	0.049	0.634	0.763	0.049	1992	12.43
1993	0.147	0.727	0.279	0.939	0.096	1993	14.75
1994	0.904	0.068	0.016	0.779	0.141	1994	17.13
1995	0.009	0.999	0.812	0.239	0.658	1995	14.01
1996	0.952	0.537	0.289	0.959	0.020	1996	17.61
1997	0.965	0.094	0.414	0.782	0.039	1997	17.53
1998	0.021	0.535	0.225	0.948	0.014	1998	13.86
1999	0.036	0.649	0.327	0.789	0.114	1999	14.23
2000	0.003	0.031	0.446	0.963	0.021	2000	11.70
2001	0.000	0.003	0.413	0.371	0.993	2001	8.39
2002	0.002	0.010	0.491	0.961	0.019	2002	11.68
2003	0.002	0.012	0.386	0.736	0.035	2003	11.62
2004	0.002	0.041	0.917	0.757	0.021	2004	12.83
2005	0.004	0.250	0.605	0.630	0.332	2005	12.15
2006	0.000	0.003	0.926	0.050	0.995	2006	9.84
2007	0.006	0.023	0.498	0.821	0.928	2007	9.19
2008	0.014	0.048	0.497	0.797	0.079	2008	11.94
2009	0.034	0.841	0.025	0.643	0.909	2009	11.83
2010	0.191	0.551	0.022	0.768	0.957	2010	11.49

Table A.33: Final defuzzification results and time series of the underground economy (for Italy in FM applying Fuzzy Clustering)

UE (LT & MF)							
Year	VB	B	A	S	VS	Year	UE
1989	0.999	0.712	0.033	0.011	0.004	1989	36.12
1990	0.993	0.735	0.033	0.010	0.005	1990	36.08
1991	0.896	0.753	0.075	0.060	0.016	1991	35.45
1992	0.026	0.999	0.022	0.008	0.003	1992	33.92
1993	0.953	0.175	0.366	0.438	0.017	1993	33.30
1994	0.746	0.910	0.039	0.026	0.011	1994	35.44
1995	0.990	0.946	0.039	0.030	0.005	1995	35.74
1996	0.982	0.598	0.043	0.783	0.006	1996	32.93
1997	0.078	0.984	0.036	0.168	0.813	1997	28.74
1998	0.042	0.441	0.986	0.516	0.024	1998	29.92
1999	0.019	0.754	0.872	0.477	0.025	1999	30.49
2000	0.001	0.752	0.077	0.697	0.017	2000	30.06
2001	0.010	0.069	0.813	0.610	0.035	2001	28.46
2002	0.010	0.042	0.875	0.481	0.122	2002	28.27
2003	0.006	0.032	0.040	0.990	0.609	2003	24.83
2004	0.004	0.009	0.022	0.978	0.066	2004	25.95
2005	0.002	0.004	0.011	0.902	0.957	2005	24.01
2006	0.006	0.013	0.023	0.926	0.863	2006	24.26
2007	0.020	0.036	0.048	0.483	0.892	2007	24.07
2008	0.003	0.007	0.019	0.958	0.774	2008	24.34
2009	0.004	0.018	0.889	0.062	0.960	2009	25.95
2010	0.007	0.030	0.747	0.161	0.998	2010	25.65

Table A.34: Final defuzzification results and time series of the underground economy (for USA in FM applying Fuzzy Clustering)

Year	UE (LT & MF)					Year	UE
	VB	B	A	S	VS		
1989	0.382	0.630	0.043	0.895	0.044	1989	10.62
1990	0.023	0.933	0.181	0.629	0.020	1990	10.52
1991	0.236	0.876	0.055	0.032	0.017	1991	13.17
1992	0.048	0.868	0.851	0.043	0.008	1992	11.49
1993	0.026	0.696	0.371	0.026	0.012	1993	11.85
1994	0.039	0.671	0.969	0.015	0.003	1994	11.29
1995	0.095	0.785	0.429	0.649	0.011	1995	10.47
1996	0.052	0.541	0.880	0.474	0.023	1996	10.19
1997	0.021	0.875	0.091	0.870	0.475	1997	8.84
1998	0.087	0.414	0.048	0.059	0.956	1998	7.35
1999	0.015	0.693	0.033	0.052	0.927	1999	7.94
2000	0.016	0.621	0.113	0.513	0.820	2000	7.84
2001	0.008	0.551	0.141	0.991	0.703	2001	7.70
2002	0.005	0.031	0.057	0.561	0.793	2002	5.63
2003	0.004	0.649	0.143	0.882	0.038	2003	9.48
2004	0.030	0.709	0.122	0.974	0.017	2004	9.61
2005	0.039	0.404	0.982	0.616	0.258	2005	9.15
2006	0.021	0.614	0.344	0.436	0.966	2006	7.84
2007	0.003	0.079	0.444	0.896	0.082	2007	8.06
2008	0.012	0.769	0.269	0.922	0.018	2008	9.75
2009	0.988	0.191	0.235	0.011	0.992	2009	10.21
2010	0.990	0.017	0.114	0.683	0.814	2010	9.64

Table A.35: Final defuzzification results and time series of the underground economy(for Germany in FM applying Fuzzy Clustering)

UE (LT & MF)						Year	UE
Year	VB	B	A	S	VS		
1991	0.005	0.011	0.028	0.948	0.948	1991	10.18
1992	0.004	0.024	0.070	0.893	0.958	1992	10.30
1993	0.023	0.033	0.988	0.420	0.311	1993	13.83
1994	0.007	0.593	0.932	0.885	0.074	1994	15.32
1995	0.040	0.642	0.008	0.921	0.023	1995	15.40
1996	0.024	0.608	0.161	0.758	0.881	1996	12.93
1997	0.008	0.999	0.513	0.513	0.437	1997	15.40
1998	0.029	0.875	0.127	0.655	0.035	1998	16.48
1999	0.003	0.799	0.302	0.655	0.016	1999	16.27
2000	0.005	0.012	0.879	0.766	0.026	2000	14.11
2001	0.043	0.626	0.359	0.759	0.048	2001	15.69
2002	0.058	0.724	0.084	0.834	0.066	2002	15.72
2003	0.738	0.947	0.077	0.591	0.046	2003	18.90
2004	0.900	0.842	0.432	0.018	0.003	2004	20.77
2005	0.874	0.873	0.437	0.060	0.017	2005	20.47
2006	0.199	0.750	0.412	0.041	0.007	2006	19.10
2007	0.545	0.739	0.100	0.969	0.010	2007	17.42
2008	0.028	0.947	0.035	0.043	0.945	2008	14.14
2009	0.910	0.071	0.989	0.554	0.001	2009	18.12
2010	0.996	0.088	0.186	0.693	0.019	2010	18.73

Appendix B

The GAMS Model and Results of the Fuzzy MIMIC

B.1 GAMS Model

Sets

```
t time periods /1*27/  
i related to x /1*6/  
j related to y /1*3/  
k related to landa /2*3/
```

Scalar h our constant between 0&1 /.4/;

Variables

```
SSgamma(i)  
Gamma(i)  
landa(i)  
Z  
x(i,t)  
y(j,t);
```

Positive variables

```
y  
landa  
SSgamma;
```

Equations

Object the objective variable value

```
constant  
Eq1(t)  
Eq2(t)  
Eq3(t)  
Eq4(t)
```

Eq5(t)

Eq6(t);

Object..

Z=e=landa("1")*sum(t,sum(i,x(i,t)*SSgamma(i))
 +landa("2")*sum(t,sum(i,x(i,t)*SSgamma(i))
 +landa("3")*sum(t,sum(i,x(i,t)*SSgamma(i))));

constant..

landa("1")=e=1;

Eq1(t) ..

y("1",t)=g=landa("1")*sum(i,x(i,t)*Gamma(i))
 -(1-h)*(landa("1")*sum(i,x(i,t)*SSgamma(i)));

Eq2(t) ..

y("1",t)=l=landa("1")*sum(i,x(i,t)*Gamma(i))
 +(1-h)*(landa("1")*sum(i,x(i,t)*SSgamma(i)));

Eq3(t) ..

y("2",t)=g=landa("2")*sum(i,x(i,t)*Gamma(i))
 -(1-h)*(landa("2")*sum(i,x(i,t)*SSgamma(i)));

Eq4(t) ..

y("2",t)=l=landa("2")*sum(i,x(i,t)*Gamma(i))
 +(1-h)*(landa("2")*sum(i,x(i,t)*SSgamma(i)));

Eq5(t) ..

y("3",t)=g=landa("3")*sum(i,x(i,t)*Gamma(i))
 -(1-h)*(landa("3")*sum(i,x(i,t)*SSgamma(i)));

Eq6(t) ..

y("3",t)=l=landa("3")*sum(i,x(i,t)*Gamma(i))
 +(1-h)*(landa("3")*sum(i,x(i,t)*SSgamma(i)));

model FuzzyMIMIC / ALL / ;

FuzzyMIMIC.reslim=3000;

```
* Read Excel File   for x
$CALL GDXXRW.exe dataMos.xlsx par=sa rng=A1:AB7;
parameter sa(i,t);
$GDXIN dataMos.gdx
$Load sa
$GDXIN
x.fx(i,t)=sa(i,t);
***** end of read excel file for X

* Read Excel File   for Y
$CALL GDXXRW.exe dataMos.xlsx par=sa2 rng=A9:AB12;
parameter sa2(j,t);
$GDXIN dataMos.gdx
$Load sa2
$GDXIN
y.fx(j,t)=sa2(j,t);
***** end of read excel file for Y

solve FuzzyMIMIC minimizing Z using nlp;
```

B.2 GAMS Model Description

In this section, different sections of the GAMS codes of Fuzzy MIMIC will be explained.

In this section the related arrays of variables will be defined:

Sets

```
t time periods /1*27/
i related to x /1*6/
j related to y /1*3/
k related to landa /2*3/
```

The value of the h-certain factor is determined in the following line of the program:

```
Scalar h our constant between 0&1 /.4/;
```

Different variables are defined as below:

Variables

```
SSgamma(i)
Gamma(i)
landa(i)
Z
x(i,t)
y(j,t);
```

Some of the variables must be positive and cannot take negative values, they are defined here:

Positive variables

```
y
landa
SSgamma;
```

Equations

The name of the equations are defined in this section with the following codes:

Object the objective variable value

constant

Eq1 (t)

Eq2 (t)

Eq3 (t)

Eq4 (t)

Eq5 (t)

Eq6 (t) ;

The Objective function of the program is defined as below:

Object ..

```
Z=e=landa("1")*sum(t,sum(i,x(i,t)*SSgamma(i)))
+landa("2")*sum(t,sum(i,x(i,t)*SSgamma(i)))
+landa("3")*sum(t,sum(i,x(i,t)*SSgamma(i))) ;
```

The "landa (1)" is initialized as it was explained:

constant ..

landa("1")=e=1;

Different equations related to fitting the initial data to the reduced form equations are defined here:

Eq1 (t) ..

```
y("1",t)=g=landa("1")*sum(i,x(i,t)*Gamma(i))
-(1-h)*(landa("1")*sum(i,x(i,t)*SSgamma(i))) ;
```

Eq2 (t) ..

```
y("1",t)=l=landa("1")*sum(i,x(i,t)*Gamma(i))
+(1-h)*(landa("1")*sum(i,x(i,t)*SSgamma(i))) ;
```

Eq3 (t) ..

```
y("2",t)=g=landa("2")*sum(i,x(i,t)*Gamma(i))
-(1-h)*(landa("2")*sum(i,x(i,t)*SSgamma(i))) ;
```

Eq4 (t) ..

```
y("2",t)=l=landa("2")*sum(i,x(i,t)*Gamma(i))
+(1-h)*(landa("2")*sum(i,x(i,t)*SSgamma(i))) ;
```

Eq5(t) ..

```
y("3",t)=g=landa("3")*sum(i,x(i,t)*Gamma(i))
      -(1-h)*(landa("3")*sum(i,x(i,t)*SSgamma(i)));
```

Eq6(t) ..

```
y("3",t)=l=landa("3")*sum(i,x(i,t)*Gamma(i))
      +(1-h)*(landa("3")*sum(i,x(i,t)*SSgamma(i)));
```

Name of the model is defined and its maximum possible iteration is determined here:

```
model FuzzyMIMIC / ALL / ;
FuzzyMIMIC.reslim=3000;
```

Read the initial data from excel file which is named dataMos.xlsx (for the Causes):

```
* Read Excel File for x
$CALL GDXXRW.exe dataMos.xlsx par=sa rng=A1:AB7;
parameter sa(i,t);
$GDXIN dataMos.gdx
$Load sa
$GDXIN
x.fx(i,t)=sa(i,t);
***** end of read excel file for X
```

Read the initial data from excel file which is named dataMos.xlsx (for the Indicators):

```
* Read Excel File for Y
$CALL GDXXRW.exe dataMos.xlsx par=sa2 rng=A9:AB12;
parameter sa2(j,t);
$GDXIN dataMos.gdx
$Load sa2
$GDXIN
y.fx(j,t)=sa2(j,t);
***** end of read excel file for Y
```

Define the type of the objective function (minimize or maximize) and the method which is used to solve the mode (nlp mean non-linear programming):

```
solve FuzzyMIMIC minimizing Z using nlp;
```

B.3 Fuzzy MIMIC Results

Table B.1: Final Time series of the underground economy for Germany (Fuzzy MIMIC)

Year	Fuzzy MIMIC			Elgin (2012)	Schneider (2012)
	Right Bound	Central	Left Bound		
1995	17.01	12.41	7.81	15.94	
1996	17.23	13.26	9.30	15.82	
1997	17.66	13.29	8.92	15.75	
1998	16.63	13.04	9.45	15.69	
1999	16.58	13.35	10.12	15.55	16.40
2000	18.60	14.83	11.07	15.44	16.00
2001	19.76	15.42	11.08	15.34	15.90
2002	18.95	14.97	10.99	15.27	16.10
2003	18.29	14.64	11.00	15.24	16.30
2004	19.91	15.83	11.76	15.26	16.10
2005	19.69	15.74	11.79	15.31	16.00
2006	19.70	15.76	11.83	15.33	15.60
2007	21.72	17.35	12.98	15.30	15.30
2008	22.01	17.33	12.65	15.23	14.80
2009	19.37	16.29	13.21	15.14	14.60
2010	20.64	16.98	13.32		15.10

Table B.2: Final Time series of the underground economy for France (Fuzzy MIMIC)

Year	Fuzzy MIMIC			Dell'anno (2007)	Elgin (2012)	Schneider (2012)
	Right Bound	Central	Left Bound			
1985	17.64	11.64	5.64	24.80	16.33	
1986	18.01	11.95	5.88	24.32	16.35	
1987	18.52	12.36	6.20	23.95	16.34	
1988	19.73	13.46	7.18	22.42	16.30	
1989	22.03	15.42	8.81	19.97	16.25	
1990	22.72	16.16	9.60	16.01	16.12	
1991	22.98	16.39	9.80	15.74	16.07	
1992	23.50	16.87	10.24	15.55	15.97	
1993	22.96	16.38	9.80	16.19	15.73	
1994	21.28	14.87	8.46	15.77	15.92	
1995	21.28	14.87	8.46	13.97	15.91	
1996	20.61	14.34	8.08	14.30	15.89	
1997	20.28	14.05	7.81	14.04	15.85	
1998	20.78	14.43	8.08	13.89	15.88	
1999	21.68	15.27	8.86	12.79	15.83	15.70
2000	22.36	15.94	9.52	10.46	15.61	15.20
2001	23.20	16.76	10.32	9.20	15.51	15.00
2002	23.16	16.82	10.47	8.60	15.40	15.10
2003	21.81	15.51	9.21	9.12	15.28	15.00
2004	21.59	15.24	8.89		15.26	14.90
2005	23.27	16.58	9.89		15.18	14.80
2006	23.21	16.51	9.82		15.07	14.80
2007	24.47	17.66	10.84		14.70	14.50
2008	22.82	16.47	10.12		14.63	14.00
2009	23.94	17.25	10.56		14.26	14.50
2010	23.10	16.40	9.71			14.60

Table B.3: Final Time series of the underground economy for USA (Fuzzy MIMIC)

Year	Fuzzy MIMIC			Elgin (2012)	Schneider (2012)
	Right Bound	Central	Left Bound		
1985	16.04	10.55	5.06	10.42	
1986	14.91	9.47	4.04	10.32	
1987	14.41	8.75	3.09	10.23	
1988	13.74	8.23	2.72	10.14	
1989	13.52	7.90	2.27	10.07	
1990	13.83	8.27	2.71	9.99	
1991	13.96	8.63	3.31	9.90	
1992	14.90	9.61	4.31	9.87	
1993	14.46	9.07	3.67	9.81	
1994	14.03	8.62	3.21	9.76	
1995	13.84	8.27	2.70	9.66	
1996	14.02	8.27	2.52	9.57	
1997	13.94	8.07	2.20	9.49	
1998	13.86	7.90	1.94	9.36	
1999	14.16	8.17	2.18	9.22	8.80
2000	14.69	8.58	2.47	9.10	8.70
2001	14.59	8.69	2.80	8.94	8.80
2002	14.75	9.40	4.05	8.83	8.80
2003	14.78	9.56	4.34	8.75	8.70
2004	14.59	9.31	4.02	8.67	8.60
2005	14.85	9.20	3.55	8.59	8.50
2006	14.61	8.76	2.92	8.50	8.40
2007	14.18	8.27	2.37	8.40	8.60
2008	14.57	9.05	3.54	8.32	8.60
2009	14.96	10.06	5.15	8.24	9.30
2010					9.10

Table B.4: Final Time series of the underground economy for Canada (Fuzzy MIMIC)

Year	Fuzzy MIMIC			Elgin (2012)	Schneider (2012)
	Right Bound	Central	Left Bound		
1985	25.95	20.95	15.94	18.61	
1986	24.59	19.12	13.65	18.47	
1987	24.60	18.74	12.88	18.33	
1988	23.22	17.50	11.79	18.14	
1989	23.07	17.10	11.12	17.93	
1990	21.71	15.49	9.28	17.66	
1991	22.00	16.20	10.40	17.46	
1992	21.17	15.99	10.81	17.34	
1993	21.77	16.72	11.66	17.28	
1994	21.30	16.12	10.93	17.24	
1995	21.39	15.75	10.11	17.14	
1996	21.91	16.08	10.24	17.03	
1997	22.48	16.18	9.89	16.97	
1998	21.61	15.44	9.28	16.80	
1999	21.89	15.33	8.76	16.68	16.30
2000	22.06	15.44	8.82	16.52	16.00
2001	21.85	15.84	9.83	16.32	15.90
2002	22.44	16.98	11.52	16.23	15.80
2003	22.29	16.88	11.47	16.12	15.70
2004	22.33	16.61	10.90	15.94	15.60
2005	22.16	16.27	10.37	15.74	15.50
2006	21.66	15.43	9.20	15.52	15.30
2007	21.40	15.01	8.62	15.30	15.20
2008	20.93	14.55	8.16	15.08	14.90
2009	20.56	14.68	8.80	14.85	15.50
2010	20.87	15.12	9.36		15.40

Table B.5: Final Time series of the underground economy for Italy (Fuzzy MIMIC)

Year	Fuzzy MIMIC			Elgin (2012)	Schneider (2012)
	Right Bound	Central	Left Bound		
1985	50.22	41.14	30.19	31.54	
1986	45.61	35.35	24.03	31.38	
1987	43.32	32.62	21.29	31.17	
1988	42.39	30.82	18.78	30.93	
1989	43.72	32.57	20.71	30.63	
1990	43.29	31.51	19.10	30.39	
1991	43.33	31.37	18.76	30.07	
1992	42.06	30.00	17.52	29.72	
1993	42.99	31.41	19.25	29.21	
1994	41.08	28.70	16.08	29.08	
1995	43.26	31.82	19.76	28.98	
1996	42.87	31.95	20.46	28.80	
1997	38.57	27.23	16.08	28.67	
1998	38.43	26.61	15.01	28.54	
1999	36.18	24.13	12.69	28.38	27.80
2000	37.01	25.30	14.06	28.19	27.10
2001	37.63	26.12	14.97	27.96	26.70
2002	36.99	25.41	14.30	27.75	26.80
2003	38.91	27.99	17.21	27.53	27.00
2004	39.28	28.24	17.27	27.36	27.00
2005	36.28	24.66	13.64	27.13	27.10
2006	35.52	23.76	12.73	27.00	26.90
2007	34.53	23.10	12.57	26.80	26.80
2008	35.59	24.45	14.03	26.65	26.70
2009	32.58	20.44	9.55	26.43	26.50
2010	32.60	20.14	8.94		26.70

