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Hang Luo

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**THESE DE DOCTORAT DE
L'UNIVERSITE PIERRE ET MARIE CURIE**

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Hang Luo

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Influence in Combinatorial and Collective Decision-making
--by the Example of UN Security Council Voting

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Abstract

Influence study in combinatorial and collective decision-making, is an interdisciplinary research direction combining computer science and social science, especially artificial intelligence and collective decision-making. The influence among each other is one natural or social instinct for people, and with the help of large-scale communication via networks, the interaction and influence among people have become much more convenient and intensive. In the context of combinatorial and collective decision-making, this calls for a study of how influences works among multi-agents and multi-issues, how influences and decision-making are interleaved, and how the structures of influence among agents and issues produce an effect. In the thesis, we mainly performed three aspects of work:

Firstly, build complex models of influence based on preference representation languages and social influence models, proposed a series of new patterns of influence to better describe the complex influences in real-world situation, and discussed a series of theoretical problems of influencing and influenced structure, influence from more than one origins, and influence with abstentions and constraints (which are common in real-world situation while ignored in previous works).

Then test the models of influence from an exemplary perspective for interdisciplinary study, from both social science and computer science paradigms, by both qualitative case studies approach and quantitative matching algorithms approach, to provide an evaluation for the models of influence.

Furthermore, use the models of influence after tested to do simulation experiments, by the example UN Security Council voting. Design the experiments from both social and computer science perspectives, and discuss the interleaved effects between new cases of influence and different SC reform schemes, provide with both theoretical thoughts and practical advices, which is a frontier expansion of artificial intelligence models and complex algorithms design in social sciences, especially the international politics, endowed with new research perspective and methodological system.

This work thus presents a new theory of influence study in combinatorial collective decision-making, and its application at the intersected domains of computer science, artificial intelligence, decision theory and international politics.

Key words: Influence; Combinatorial and Collective Decision-making; Preference Representation Languages; Social Influence Modes; UN Security Council

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Table of Main Symbols

Theory of Influence

CCS	a society of combinatorial and collective decision-making system embedded with structures of influence, $CCS = \langle \mathbb{N}, \mathbb{M}, \text{PR}/\text{WT} \rangle$
$A_{(i)}$	the agent i
\mathbb{N}	the set of all agents
n	the number of agents
$F_{(k)}$	the feature k
\mathbb{M}	the set of all features/issues
m	the number of features/issues
PR	the matrix of priority of influence
$pr_{(i,j)}(k, q)$	the priority of influence from variable $x_{i,k}$ (agent i , feature k) to variable $x_{j,q}$ (agent j , feature q)
WT	the matrix of weight of influence
$wt_{(i,j)}(k, q)$	the weight of influence from variable $x_{i,k}$ (agent i , feature k) to variable $x_{j,q}$ (agent j , feature q)
$S_{(ij)}(kq)$	the structure of influence from variable $x_{i,k}$ (agent i , feature k) to variable $x_{j,q}$ (agent j , feature q)
$\mathcal{S}[\mathbb{N}]$	the structure of all basic/independent influences among agents \mathbb{N}
$\mathcal{S}[\mathcal{S}[\mathbb{N}]]$	the structure of all aggregated/interacted influences among agents \mathbb{N}
$\mathcal{S}[\mathbb{M}]$	the structure of all basic/independent influences among features \mathbb{M}
$\mathcal{S}[\mathcal{S}[\mathbb{M}]]$	the structure of all aggregated/interacted influences among features \mathbb{M}
$\mathcal{S}[\mathbb{N} \times \mathbb{M}]$	the structure of all basic/independent influences in whole system \mathbb{M}
$\mathcal{S}[\mathcal{S}[\mathbb{N} \times \mathbb{M}]]$	the structure of all aggregated/interacted influences in whole system
$P_{(k)}(i)$	the preference of agent i on feature k
$C_{(k)}(i)$	the choice/decision of agent i on feature k
k, l, s, q, t, d	a portion of No. of features
i, j, e, r, g, h	a portion of No. of agents
\mathbb{A}	a subset of agents \mathbb{N}
Pa	the intra-influence function
In	the inter-influence function
$Pa \perp In$	the intra-inter influence function
$In \perp Pa$	the inter-intra influence function
$\alpha, \beta, \sigma, \gamma, \delta$	a portion of categories of influence functions
ϕ	the function for the influential effect of intra-structure
φ	the function for the influential effect of inter-structure
ψ	the function for the influential effect of structure, crossing intra-dimension and inter dimension
χ	the function for the influential effect of a group of agents

Application of influence

$PN_o.MF$	Matching Frequency for Influence Pattern No.
$PN_o.MR$	Matching Ratio for Influence Pattern No.
FV	the factual vote
EV	the expected vote
IFV	the factual vote on the influencing feature or the influencing agent
IEV	the expected vote on the influencing feature or the influencing agent
FVD	the factual vote on the influenced feature or the influenced agent
EVD	the expected vote on the influenced feature or the influenced agent
WT_Y	the accumulated vote weight for affirmative influence
WT_A	the accumulated vote weight for absent influence
WT_N	the accumulated vote weight for opposing influence
WTS_Y	the accumulated structure weight of positive influence
WTS_N	the accumulated structure weight of negative influence
$OOP_{(t)}(i)$	the member i 's original preference for the bill t before influenced
$ONW_{(t)}(i)$	the weight of influence from own for member i on bill t
$ALP_{(t)}(i)$	the preferences of member i 's allies for bill t
$ALW_{(t)}(i)$	the weight of influence from allies for member i on bill t
$OPP_{(t)}(i)$	the preferences of member i 's opponents for bill t
$OPW_{(t)}(i)$	the weight of influence from opponents for member i on bill t
$IOP_{(t)}(i)$	the member i 's eventual preference for for the bill t after influenced
$Vote_{(t)}(i)$	the member i 's vote decision for the bill t , with a ternary domain $\{Y, A, N\}$
$Rule(t)$	the collective voting rule for bill t
$FO(t)$	the factual voting outcome for bill t , with a binary domain $\{Passed, Failed\}$
$EO_{(t)}(i)$	the expected voting outcome of member i for bill t , with a ternary domain $\{Passed, Indifferent, Failed\}$
$OC_{(t)}(i)$	the cumulative frequency of un-match between expected outcome and factual outcome of member i after bill t
$AC_{(t)}(i)$	the comparison of member i 's own match status (between expected outcome and factual outcome) with allies' match status after bill t
$EP_{(t)}(i)$	the expectation psychology status of member i after bill t
$CP_{(t)}(i)$	the comparison psychology status of member i after bill t
Num_Y	the number of members voting affirmative
$PNum_N$	the number of permanent members voting negative
NUM^*	the threshold for accumulative unmatch (between expected outcome and factual outcome) to reverse the expectation psychology
MOI	the Mild-to-Opponent Inclination
OPI	the Own-Centripetal-Inclination
AFI	the Ally-Centrifugal-Inclination

Part I

Basics of Influence

Chapter 1

Introduction

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The core question in this thesis is, in a multi-issue and multi-agent (namely combinatorial and collective) decision-making context, how agents (as decision-makers) influence and are influenced with each other, and how the individual decision-making behavior and the evolving structure among multi-agents and multi-issues is interleaved and interacted through the function of influence, more specifically: are the existing patterns of influence expressive enough to describe the complex influences in real-world situation? How to deal with the influence from more than one origins crossing both different agents and issues, with varied weights and contradictive directions? How to deal with the influential effect from the structures (of influencing relations) among multi-decisions besides the influential effect from independent individual decisions? And how to re-discuss the influence under the ignored cases of abstention and constraint? To solve these questions, we design a new system of patterns of influence (by graphical representation and mathematic modeling), design a series of influence pattern matching algorithms for testing, and do simulation experiments to study the effect of influence models.

We consider a multi-agent setting where a set of agents take decisions in sequence, based on their preferences over the possible alternatives toward each issue, and then aggregate a collective decision (according to a voting rule). While agents have their initial inclinations, they may interact with each other, influence and be influenced by each other, and therefore modify their preferences [Maran et al., 2013]. The influence is achieved by the information exchange before agents actually declaring their final vote. The information agents exchange could be the mere observation of others' vote, agents may revise their vote on the basis of the observed votes of others [Maran et al., 2013]; or according to others' declared and shared preferences (underlying their votes).

Recent work has usually modeled the influence phenomenon in the case of voting over a single issue, while we assume a model to account for preferences over combinatorially structured domains including several issues, and propose a way to model influence when agents express their preferences as CP-nets, referring to the work of [Maran et al., 2013], while we provide a more complex description on the role, pattern and effect of influence.

1.1 Computational Social Choice

To study the collective decision-making from the perspective of computer science, a related concept is Computational Social Choice [Chevalleyre et al., 2007, Brandt et al., 2013], which is an interdisciplinary research area of Computer Science and Social Science, especially the Artificial

Intelligence and the Collective Decision-making [Chevaleyre et al., 2007]. In short, computational social choice aims at studying the computational aspects of collective decision-making, which is typically studied in social choice. As an interdisciplinary area, the computational social choice involve the bi-directions of theoretical interaction and crossed application between computer science and social science, and there are basically two directions or facets of research in computational social choice.

One facet is from computer science to social science [Chevaleyre et al., 2007, Brandt et al., 2013], applies techniques and tools originating from computer science (such as complexity analysis and algorithm design) to study the procedures usually discussed in human societies, such as the collective voting or resource allocation and so on. For instances, with the help of notions borrowed from theoretical computer science, the communication complexity of a resource allocation mechanism can be formally evaluated, as can be the computational complexity of determining the winner for a given collective decision-making procedure. Besides, recent advances in compact representation in game theory permit to conduct massive computer simulations which light on some complex problems (in social sciences/human societies) which can be extremely difficult to deal with in other approaches, especially the traditional methods of social sciences.¹ [Thompson et al., 2013].

On the other facet, it is also very interesting to see how solutions originating in social sciences have found applications [Chevaleyre et al., 2007, Brandt et al., 2013] in seemingly remote contexts (such as computer science, artificial intelligence and so on). One motivation for this trend of research lies on the fact that the deployment of large-scale communication and computation via networks opens new and challenging perspectives. For instance, social choice axioms have been adapted to analyze the behaviour of search engines.² Criteria of fairness³ studied in political science have been used to evaluate how computing resources should be shared in cloud computing [Ghodsai et al., 2011].

These new contexts of application (combining computer science and social sciences) are challenging because they impose new and original constraints, usually by technological qualifications or social complexity (particularly for the human psychologies and behaviors). Especially with the large-scale communication via networks beyond the limitation of space, time and environment [Luo et al., 2010]), the current research should typically involve very large number of agents, as opposed to the restricted case consisting of very few agents only. In this context, not only the psychology and behavior of individual agent should be discussed, but also the interaction, influence or the collective effect among agents.

In this context, the study of Influences in Collective Decision-making has received intensive attentions recently. **Decision**, as the core activity of social procedures, involve nearly every aspects of human societies, on economic fields, such as how to allocate resources, what categories and amounts of goods to produce and exchange, on political fields, choose who as the president, governor from candidates and so on. Particularly in the discipline of management, decision has been endowed with the most important role, deemed as the core activity of whole management procedures [Simon, 1965]. Furthermore, especially for the social nature of humans, collective decision-making (making decisions together by more than one person) is the common form for decision-making, required by the system of democracy, modern enterprise institution, the need to share the risk, the bond of affections and so on.

One reason for studying the **Influence** in collective decision-making is the push of tools allowing massive online decision-making. In such environment, the fact that decisions of agents

¹Such as social experiment, questionnaire, interviewees, statistical analysis and so on.

²For example, the different search engines would have different rankings about the searched content, it also need some kind of social choice rule (or collective decision-making rule) to aggregate a eventual ranking from all these engines to provide for clients.

³Criteria of fairness, such as equality, maximum of minimum, performance-based and so on.

may be influenced by others are in the norm rather than in the exception: for example, agents may observe the made decision of others, or communicate with others about planned decision, they may deliberate, exchange arguments, prior to a vote, etc. In short, such environment typically allow agents to access some more information before they opt for a given decision, the decisions for now are far from the simple aggregation of sperate and divided preferences. Therefore, this calls for a study of how influences works among agents, and how influences and decision-making are interleaved.

Furthermore, even without these communication tools and even before the era of internet, the influence is kind of natural instinct for people, if there is a society of more than one people, then there will always be influence among people, no matter unconsciously or intentionally, people will influence other people, and be influenced by other people. For example, it is human nature to be influenced by families, close friends and so on, there is a natural empathy with the feelings of families and close friends. Also, people have varied reasons and motivations to influence others, you hope to persuade others to believe what you believe, or what you want them to believe, you like to lead or mislead others for better interests, and so on.⁴

Besides, the interactions and influences among decision-makers are very common particularly in political collective decision-making procedures, decision-makers in a same group would positively influence with each other (in the form of supporting, keeping uniform and so on), and decision-makers from confronting groups would negatively interact with each other (in the form of opposing, suppressing and so on). For the example of UN security council voting, which is full of positive influence among allies and negative influence among opponents, and with varied purposes, strengths and directions.

1.1.1 The Framework of Decision-Influence-Structure

To understand the complex influences in collective decision-making, intuitively the core work is to build the **Models of Influence**, but which can not be analyzed separately, there are other two related models have to be discussed surrounding the models of influence, and integrally as a whole system, which are the **Models of Decision** and the **Models of Structure**, to investigate the influence in a more systematic and rounded perspective.

As the key for the influence (models) is allowing the capability (or possibility) for agents to access some (new) information before they make decision, therefore, the influence could be expressed as the transmission or the flows of information among agents. And if the transmissions of information are considered, then so should the acquisition and processing of information be considered, (and in a collective decision-making system), the agents mainly need information for two procedures:

- On one hand, there is no doubt that the core role in a collective decision-making system is to make decision, the decision could be the object of influence your decision would possibly be influenced by other agents' decisions, and the decision could also be the subject of influence, your decision would also possibly in turn affect the decisions of other agents, which means there are influences between decisions and decisions, decisions could both be influencing subject and influenced object. And there are interactions between (the models of) influence and (the models of) decision, which can not be discussed separately.

⁴People try to influence others from varied purposes, such as economic interests, politic claims, religion believes and so on, and on some other circumstances, people would possibly influence others unconsciously and unintentionally, for the most influential people (such as the opinion leaders) in the social networks, even not on purpose, their attitudes, behaviors and so on will influence other agents in the social networks imperceptibly.

- On the other hand, the construction and evolution of structure (such as the social networks)⁵ is also very important in the collective decision-making, the influences flow in the specific structure (for instance in the social networks, the influences have to flow through the links, only linked agents can influence each other), in another word, the influence must have specific structure as its platform or carrier. Though series of work study the influence in social networks [Jackson, 2008, Grabisch and Rusinowska, 2010a, Grabisch and Rusinowska, 2010c, Grabisch and Rusinowska, 2010b, Grabisch and Rusinowska, 2011a, Salehi-Abari and Boutilier, 2014], but all assumed the (social networks) structure, the links among agents, as fixed, which would affect the decision as an environment, but would not be influenced and changed in return during the process. Furthermore, perceive the structure just as the “path” or “channel” for influence, it is the decision through the links of structure influencing, but it is not the structure itself exerting the influence. Therefore, both the subject of influences and the object of influences are only decisions (of individual agent), but not perceive the structure (the influencing relations among agents or among features) itself as a influencing subject or influenced object.

However, the structure could be endowed with a more living and dynamic role, which is not just fixed and objective, but also an autonomous choice for agents, for example (in the social networks), agents could choose the social relations by their own (choose which one to be friends with, which one to oppose to, and more generally, which one to communicate and interact with, and so on), especially in the context of influences, expressed as choosing which one to influence or which one to be influenced by, and so on. Therefore, both the structure and decision, should be endowed with a equal status⁶ in the context of influences, both could be the autonomous choices for agents, just one in micro level and one in macro level, and naturally both could undertake the influencing and influenced role.

Specially discuss about the influencing and influenced role of structure, while constructing the (social network) structure, like choosing the links from oneself to whom or from who to oneself, the agents obtain the information through the influence, and perform the evolution of structure, in which the structure is assumed as the objective of influence (influenced role). Besides, the agents can also transmit the information about decisions through the structure to make the structure acts on other decisions or structures, or the structure itself directly exerts an influential effect on other decisions or structures (which would be discussed in details in chapter 7), in which the structure is assumed as the subject of influence (influencing role). Mark Granovetter [Granovetter, 1985] also think that economic behaviors are embedded in the social structure, and affected by social relationship. Which means that there are also interactions between the (models of) influence and the (models of) structure. Therefore, to better study the influence in the collective decision-making, the model of influence should be combined with the model of decision and model of structure, in an integrated and systematic perspective.⁷

⁵The structure describe the relations among agents or among features, and the structure used most in current works [Jackson, 2008, Grabisch and Rusinowska, 2010a, Grabisch and Rusinowska, 2010c, Grabisch and Rusinowska, 2010b, Grabisch and Rusinowska, 2011a, Salehi-Abari and Boutilier, 2014] are social networks, in which the links among nodes (agents) constitute the structure.

⁶In fact, the decision discussed in literatures mainly refer to the decision about candidate, and the structure discussed in the thesis could be understood as the decision about structure, both are decisions made by agents, and can influence mutually.

⁷Specifically, the agent’s decision (and its related information, such as the results of decision and so on) is the content of the influence, and the agent also acquire and process the information, adjust the decision to adapt the influence, the (social network) structure is not only the platform/carrier of influence, but also the content of influence, and the agent also acquire and process the information, adjust the structure to adapt the influence.

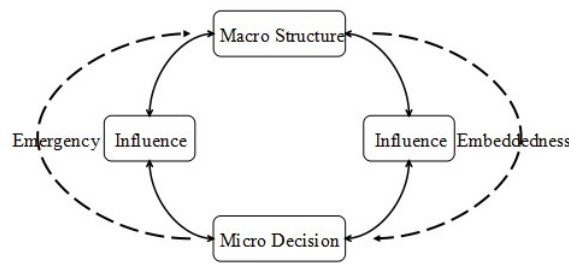


Figure 1.1: The Framework of Decision-Influence-Structure

Furthermore, from the perspective of the complex adaptive system, the models of decision describe the decision-making behavior of the micro level, and the models of structure describe the structure evolution of the macro level, through the influence (information transmission) among agents described by the model of influence, achieve the bidirectional feedback and dynamic interaction between micro decision and macro structure. On one hand, the micro decision-making behavior is embedded to macro (social networks) structure, on the other hand, the macro (social networks) structure emerge from the micro decision-making interaction. Therefore, there might be a relationship among the three models: **Model of Decision** \leftrightarrow **Model of Influence** \leftrightarrow **Model of Structure**, in which, the model of influence is the core of the integrated system, and is the medium to perform the system. The mechanism could be illustrated in figure 1.1.

1.2 What is the Influence?

The study of **Influences in Collective Decision-Making** has received a lot of attentions recently, partly due to the push of massive online communication and decision-making (which benefits from the advancement of ICT technology, and with the help of online platforms such as Facebook, Twitter, Wechat and so on), the influencing relations among people have been much more intensive, people can share the opinions and emotions, communicate information and knowledge, show the behaviors and activities much more convenient and time-saving, beyond the limitations of space, time and environment [Luo et al., 2010, Luo, 2013, Luo et al., 2013, Luo and Meng, 2013b)]⁸, and furthermore, even without these communication tools and before the era of internet, the influence among agents is kind of natural instinct for people, if there is a society of more than one people, then there will be influence among people, no matter unconsciously or intentionally, people will influence other people, and people will be influenced by other people. In the context of influences, agents may observe the behaviors of others, communicate the minds with others, and they may deliberate, exchange arguments, prior to a decision, etc. In short, the models of influence typically allow agents to access some (new) information before they opt for a given decision.

To comprehend the concept and mechanism of the influence, the first step would be to give a definition. There are many work [Jackson, 2008, Gaspers et al., 2013, Grabisch and Rusinowska, 2010a, Grabisch and Rusinowska, 2010c, Grabisch and Rusinowska, 2010b, Grabisch and Rusinowska, 2011a, Maran et al., 2013] understanding the influence from different ways, but not purposely define the definition of influence. Summarize the current work, a very straight-forward definition of influence is proposed as the **“Change Because of Others”**.⁹ Though simplified, this

⁸With such tools, the fact that decisions of agents may be influenced by others is the norm rather than the exception. Before the help of online communication tools, people usually interact just with families, friends who are close in the space distance.

⁹The others could represent other agents, such as people, software entities, artificial lives and all other forms of decision-maker, specifically in the context of multi-agent system or collective decision-making system. As in non multi-agent system, the influence might come from the environment, materials and so on, but not just from other

definition can contain the core elements of the influence as much as possible. Review separately, this definition of influence consists of two elements, one is the “change”, and the other is the “others”. For the “change”, the influence must be reflected/revealed at least in some senses of “change”, either in behaviors, preferences, attitudes or believes and so on¹⁰. And for the “others”, it is also very critical to distinguish the sources/origins of the change, which might be within oneself, or also might be from the effect of others, and usually only the latter could be defined as influence.

The above definition of influence, “change because of others” is proposed naturally due to the intuition, which is that if the change is from own¹¹ but not from other agents, then it should be the original status, but could not be called as “influenced”. However, a broader understand about the influence should be proposed, not only “the change from other agents”, but also “the change from other features”. In a multi-feature/multi-issue decision-making system, usually there are also influences from some features to some other features but within one agent. When reviewing the influence among agents, it is from other agents, and when reviewing the influence among features, the “others” means other features, and it might be still from the same agent, for example, your former choice(s) will “influence” your own latter choice(s). However, this kind of “influence” is usually called *dependency* but not influence in the literature [Boutilier et al., 2004a, Maran et al., 2013], such as CP-nets and the extended influenced CP-nets. Actually, your choice will not only be influenced by other agents, and also will be influenced by your own former choices, it is equivalent by perceiving your current choice be dependent on your former choice(s), or be influenced by your former choice(s). No matter using the term “dependency” or the term “influence”, they express the same thing. Natively, the influence among agents and the influence among features work in the same way, only different in the direction or dimension, for instance in the framework of the influenced CP-nets [Maran et al., 2013], the influences among agents work in the “horizontal” dimension, and the influences or dependencies among features work in the “vertical” dimension.

To fully develop the definition of influence, we introduce a pair of terms from logistics: **Connotation** and **Denotation**. In common saying, connotation is about the implication (intension), pertaining to the inner aspect of the concept, and denotation is about the sphere (extension), pertaining to the external aspect of the concept. In logic and semantics, connotation is roughly synonymous with intension. Connotation is often contrasted with denotation, which is more or less synonymous with extension.

1.2.1 The Connotation of Influence

The connotation of influence could be discussed from two different perspectives, one is the **Inner Process** and the other is the **External Outcome**¹², or the former is about the **Inner Psychological Change** such as preferences, beliefs, attitudes and so on, and the latter is about the **External Behavioral Change** such as votes, decisions, social relations and so on.

For identifying the influence in the sense of (change of) outcome [Jackson, 2008, Grabisch

agents, or more broadly, as the “change because of all *other factors*”, the influence is defined as originating from any external factors (concluding other agents, environment, materials and so on), For example, besides influenced by other agents, we can learn from books and influenced by the new knowledge and experience, also, we can learn from practice and influenced by the interactions between people and environment, etc, compared with the (own) internal factors. However, is the influence only coming from external factor? There are also “influence” among features within the same agent, which can be deemed as the “within” factor. However, it is still about the change of others, either others agent (refer to own agent) or other features (refer to the current feature for decision-making).

¹⁰For example, after influenced, the behaviors, preferences, attitudes or believes of the agent should be different from former state or initial state, at least be different on one aspect, but could be stable or indifferent on all aspects.

¹¹Own preference, attitude, believe or something.

¹²The distinction and comparison between outcome and process has been used a lot in both daily lives and academic work, and are very familiar to all readers.

and Rusinowska, 2010a, Grabisch and Rusinowska, 2010c, Grabisch and Rusinowska, 2010b, Grabisch and Rusinowska, 2011a, Maran et al., 2013], which (mainly) means that the influence must be reflected in the observed or visible outcome. For example, [Grabisch and Rusinowska, 2011a] define the influence as the difference between agents' (initial) inclinations and (eventual) decisions, "Due to the influences, a decision of an agent may differ from his preliminary inclination. Such a transformation between agents' inclinations and their decisions are represented by an influence function." Also in the influenced CP-nets [Maran et al., 2013], due to the newly proposed ci-statement (conditional influence statement), distinguished from the traditional cp-statement (conditional preference statement), the preferences (or inclinations) of influenced variables are changed from initial ones, and consequently changing the decision. Besides, some other work such as [Leskovec et al., 2006], just investigate the evolution of behaviors, focusing on the directly observed changes.¹³

The other perspective is identifying the influence in the sense of (change in) process, which means the influence might not be (always) presented in a directly observed or visible way, for instance as the change of outcome. In another word, the change could be revealed in the inward aspect, be represented in the level of psychology, in the inner process of mind, which are not directly "observable". For example, in the context of influence, there might be a common case that only the preferences of agents are changed by the influence, but the behavior of agents might be still constant.¹⁴ For a preference (the ordering of candidates) of an agents, the gap between top preferred candidate and second preferred candidate might be increased or decreased, but only if the comparison has not be reversed, then the choice would still be the same. For example while you buy a telephone and compare Apple and Samsung, your initial preference is "I prefer Apple much more than Samsung", and the corresponding behavior would be to buy the Apple without any hesitation, but then a salesperson saying that there is a very good mode of Samsung just close to the end of promotion, and with a high cost performance, then your preference might be influenced, and the new preference change to "I prefer the Apple (a little) than the Samsung", however, the corresponding behavior would still be the same, just with more a more difficult choice. Compared with the change of the behaviors, this transformation in the preferences is much more implicit than explicit.¹⁵ Besides, if using the probabilistic distribution (of candidates) to model the preference¹⁶, for example of the multi-agent influence diagrams [Koller and Milch, 2003] based on bayesian networks, dealing with the preference about candidates as a probabilistic distribution over domains, which is a probabilistic approach but not a deterministic approach¹⁷, and implying that the change from preferences to decision¹⁸ is also probabilistic. For one example of the probabilistic preferences, assume agent i have a probabilistic distribution over domain $\{a,b,c\}$ as $\{0.8,0.1,0.1\}$, the choice of agent i is most probably to be a , then agent i is influenced by other agent(s) and its preferences over $\{a,b,c\}$ becomes to $\{0.7,0.2,0.1\}$, however, the choice of agent i

¹³Kind of "ignoring" the underlying preferences, and usually the preferences underly the decisions, and the decisions of agents are determined by the preferences of agents, however, which not means that these work deny the role of preferences behind (under) the behaviors, but dealing the process in some sense of integrating the preferences and behaviors, deeming that the varying of behaviors naturally reflect the varying of preferences.

¹⁴That is, (in a very common saying), it influence the process but could not influence the outcome.

¹⁵Actually, the ordinal approach to represent the preference, such as the ordering of candidates, reveal limited information. For example of a preference $a \succ b \succ c$, we know that a is preferred to b , and b is preferred to c , but we do not know how much a is preferred to b , or how intensive b is preferred than c . However, the ordinal approach does have its advantage, because the candidates for many social issues can only be compared by the sequence, and could not be compared directly by the numbers or scores.

¹⁶Which belongs to the cardinal approach, compared with the classical ordinal approach in the computational social choice.

¹⁷For the deterministic approach, if there is a ordering $a \succ b \succ c$, then the choice would be deterministically a , but not b or c .

¹⁸Wether the changed/new preference would lead to changed/new decision.

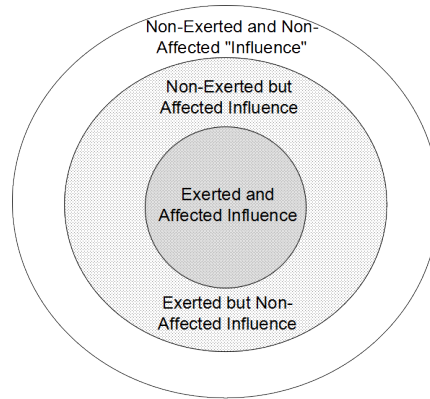


Figure 1.2: The Extension of the Influence

might still be a, being constantly, though the preference has already been changed. This is another example of influencing the preference but not the behavior, or influencing the “inner process” but not the “external outcome”. This perspective to understand the influence inspire that the (expression of) influence might not always be that direct, apparent or easily observed, and more indirect, underlying or invisible elements in the influence should be considered, to gain a more deliberate view about the influence.

1.2.2 The Denotation of Influence

The denotation pertains to the (extensive) field of one concept. For the influence, it is meaningful to accurately draw the “territory” of this concept, such as where is the kernel area (for the “standard” or “pure” influence), where is the bordered area (for the “quasi” or “half” influence), and where is non-included area or “out of bounds” (for the non-influence). We discuss the denotation of influence from an interesting dimension of {affected, non-affected}¹⁹ (in another word, {effective or ineffective}, {successful or failed}), and combining with another dimension of {exerted or non-exerted}²⁰ (or {intentional or unintentional}), it could be perceived as that the former dimensions is about the objective outcome, and the latter dimension is about the subjective willingness. Specifically, with the permutation and combination of two dimensions {Exerted, non-Exerted} \times {Affected, non-Affected}, to define and differentiate four statuses of influence: {Exerted, Affected}, {non-Exerted, Affected}, {Exerted, non-Affected}, {non-Exerted, non-Affected}. Further, in order to discuss more elaborately, the subject of influence and the object of influence should be distinguished, promptly the influencing agent and influenced agent:

- **{Exerted, Affected}**: the subject of influence exerts to influence the object, and affected (changed) the object, which means that the influencing agent has the intention to influence and also achieves as wish. This situation is undoubtedly deemed as the influence, and actually is the typical influence discussed by most of current work [Grabisch and Rusinowska, 2010a, Grabisch and Rusinowska, 2010c, Grabisch and Rusinowska, 2010b, Grabisch and Rusinowska, 2011a, Maran et al., 2013, Maudet et al., 2012a]. This case of influence contain two requirements, one is the change of the object of influence, the other is the exertion (or intention) of the subject of influence.²¹ However, what is interesting is that if deprived of

¹⁹Common speaking, distinguish whether the agent has been changed or not by the influence.

²⁰Which distinguish the agents influence other agents consciously or unconsciously.

²¹In another word, two requirements are the objective change of the influenced agent and the subjective intention of influencing agent.

(or relaxing) one of the two requirements, can the “influence” still be deemed as influence, or quasi influence, half influence or just non-influence?

- **{non-Exerted, Affected}**: the subject of influence does not exert to influence the object, but affected (changed) the object as the consequence, which means that, subjectively, the influencing agent does not have the intention (or intentional actions) to influence other agents, but objectively, affecting other agents, and generating unintentional (or unconscious) outcome. For the formation and evolution of public opinions on internet, there exist some influential users (such as the stars, the so-called “common awareness” and so on) of the microblogs, who have abundant fans, these influential users deliver their preferences, attitudes, claims about happening events and affairs, sometimes intentionally to influence others (directly for their fans, and indirectly for their fans’ fans²²), but sometimes without particular purposes²³, just do their own ways, convey their own views (according to their own preferences), to support or oppose, to express the favor or dislike about something, some events or some people, but objectively their attitudes and behaviors might stimulate a big disturbance, and have tremendous impact on other people.²⁴ These “influential” people are very related to the concept of the opinion leaders [Katz and Lazarsfeld, 1970]. Another similar example is the phenomenon of opinion leaders in the political elections (such as elections for the president, governor, or mayor), the candidates or stakeholders of candidates try their best (by means of medias, advertise, speeches and so on) to influence voter, however might not have a very effective outcome, but for some opinion leaders among the voters, such as the stars, religion leaders, etc, a single vote of a opinion leader would have tremendous impact on other voters (particular for their fans, followers), and stimulate much more vote for the same candidate). It should be noticed that, this kind of unintentional influence has specific meaning for (computational) social choice, because in the context of social choice, the influence is (mostly) related to the bribery, before specifically study the models of influence, the most common form of influence in social choice discussed in literatures [Maran et al., 2013, Mattei et al., 2012, Faliszewski et al., 2009] is the bribing. In which, the bribers pay the voters to influence them, intending to change their votes (as the briber wish) and eventually change the aggregating outcome through this portion of bribed voters. Therefore, the influence in the context of social choice is usually highly purposive, however, the kind of unintentional influence provides another new perspective to view the influence, and which would stimulate new thoughts about the influence, and how it works, what it generates, and so on.
- **{Exerted, non-Affected}**: the subject of influence exerts to influence the object, but did not affect (change) the object, which means that the “influencing” agent tries to influence other agents, spending efforts and costs, but can not achieve as wish. According to previous discussion (referring to the division of inner process and external outcome), the unaffected status can be classified into two levels, one is just unaffected in the observed behavior (decision, vote, etc), but might affected in the invisible psychology (attitude, belief, etc), the other is totally unaffected, even in the aspect of inner psychological process, which means the exertion of “influencing” agent does not generate effect on any level and any aspect

²²Which will propagate from their fans to the fans of their fans, and spread or diffuse to a very broad area in a relatively short time.

²³In this context, they not not mean to influence others, to change others.

²⁴For example in China, there are several popular microblogs, such as Sina Weibo, Sohu Weibo, Tencent Weibo and so on, the VIP users (usually as the stars, so-called common awareness, famous enterprisers and so on) in these microblogs, have a great quantity of fans (even more than ten millions), these “VIP” have enormous influential power on the internet, and their attitudes and behaviors have stimulated some mass incidents (group events), such as the PM2.5, PX program event in China.

of the “influenced” agent. Furthermore, what is interesting and controversial is that, this kind of “influence” should be deemed as influence or not? Especially for the “influence” while the object being unaffected even in the level inner process. Actually, the point is, should the status {exert, unaffected} be indifferent with the status {non-exert, unaffected}, or still has some meaning for the influence. For example, at least the “influencing” agent spend some resources²⁵ to exert the influence, and let the “influenced” agent feel the attitude and efforts, and also there might be a feedback to the influencing agent (that the exertion failed), which both would possibly influence the following psychologies or behaviors of the “influencing” and “influenced” agent. Actually, the status of {exert, unaffected} are very rarely discussed in current work [Grabisch and Rusinowska, 2010a, Grabisch and Rusinowska, 2010c, Grabisch and Rusinowska, 2010b, Grabisch and Rusinowska, 2011a, Maran et al., 2013, Maudet et al., 2012a], the typical influence investigated are {exert, affected} and {non-exert, affected}, discussing about the effective influence with specific purposes (such as the briber to change the outcome) [Maran et al., 2013, Maudet et al., 2012a, Mattei et al., 2012, Faliszewski et al., 2009], or studying the effective influence without (the direct relations with) purposes (such as influenced by the group pressure, peer pressure, represented as the enough ratio of other agents holding different preferences with the influenced ones [Grabisch and Rusinowska, 2010a, Grabisch and Rusinowska, 2010c, Grabisch and Rusinowska, 2010b, Grabisch and Rusinowska, 2011a]).²⁶

- {**non-Exerted, non-Affected**}: of course, this status (of combination) does not belong to the influence in any sense.

1.3 Overview of the Thesis

- In chapter 1, we introduced the core question of the thesis, proposed a framework of decision-influence-structure, and define the connotation and denotation of the concept of influence;
- In chapter 2, we discussed the related works, regarding Decision-Theoretic Agents, Combinatorial Domains, Collective Decision-making and Influence among Agents;
- In chapter 3, we described the Influence (Models) from a 5W1H Framework;
- In chapter 4, we summarized the three ignored questions to be studied: Influencing and Influenced Structure, Influence from More than One Origins and Influence with Abstention and Constraint;
- In chapter 5, we introduced the different patterns of influence that may occur in a multi-agent and multi-issue context, especially with new influences involved with influencing and influenced structure;
- In chapter 6, we discussed how to model influence from more than one origin in that setting, proposed two approaches of prominent influence and collective influence;
- In chapter 7, we discussed the influences with abstention and constraints, and re-discussed the collective decision-making under these special cases;
- In chapter 8, we introduced the example—the United Nations Security Council voting—which we used to test and simulate the models of influence;

²⁵No matter the form of resources is time, money, energy or spirit and so on.

²⁶However, it should be noticed that, these work do not deny the purposes underlying the influences, just not directly studying the purposes, and investigating the change (of decisions) as the outcome of objective environment (such as the peer pressure.)

- In chapter 9, we test the models of influence from a social science paradigm, by qualitative case studies;
- In chapter 10, we test the models of influence from a computer science paradigm, by quantitative algorithm designs;
- In chapter 11, we used the models of influence to do simulation experiments, designed the experiment plans from both social science and computer science perspectives, and discussed the interplay effects between different cases of influence and different SC reform schemes.

Chapter 2

Related Works

Contents of Chapter 2

- 2.1 Decision-Theoretic Agents
- 2.2 Combinatorial Domains
- 2.3 Collective Decision-making
- 2.4 Influence among Agents

2.1 Decision-Theoretic Agents

In a collective decision-making system, the elemental concept is the agent (as decision-maker).

2.1.1 Agent

The meaning of the term agent differs in different natural science or social sciences. In philosophy, it is usually simply an entity which is capable of *action*. A related concept is *agency*, which is simply the capacity of an agent to act in a given environment. Most relevant for us are the definitions in economics and artificial intelligence:

In economics —an agent is an actor or decision-maker in a model, and they may be of different types. For instance, buyers and sellers are two common types of agents in partial equilibrium models of a single market, and households, firms, and governments or banks are the main types of agents in the macroeconomic models. Typically, every agent makes decisions by solving an optimization/choice problem.

In Computer Science and Artificial Intelligence —a typical meaning is that of an Intelligent Agent [Russell and Norvig, 2003], an autonomous entity which can *observe* (through sensors) and acts upon *environment* (through actuators) and directs its activity towards achieving their *goals*, the intelligent agents may also *learn* or use *knowledge*. *Decision-theoretic agents* [Parsons and Wooldridge, 2002] rely on decision-theory to implement their behaviors.

Agent Structure. In [Russell and Norvig, 2003] a simple agent structure is proposed, as illustrated on Figure 2.1. The same behaviour can also be defined as a function [Russell and Norvig, 2003] mapping every possible perceptual inputs (P^*) to a possible action (A) the agent can perform, or to a coefficient, feedback element, function or constant that affects eventual actions: $f : P^* \rightarrow A$.

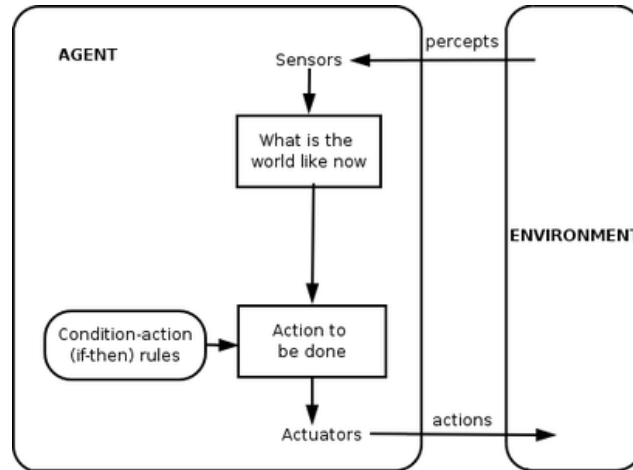


Figure 2.1: The Simple Reflex Agent by [Russell and Norvig, 2003]

At the interface of economics and computer science also lies the paradigm of agent-based simulation, and in particular *agent-based economics* [Page, 2008], whereby economics models are approached, refined and evaluated thanks to agent simulations. We shall rely on such techniques in part of this thesis. It is an interdisciplinary paradigm combining computer science and economics, in which, corresponding agents are “*computational objects* modeled as interacting according to rules” over space and time. Quoting [Page, 2008]: “*The rules are formulated to model behavior and social interactions based on stipulated incentives and information*”. In an economic model, if all agents of a given type (such as all consumers, or all firms) are assumed to be identical, then it is called as a *representative agent model*, while if the differences among agents of a given type are recognized, then it is called as a *heterogeneous agent model*. Representative agent models are often used by economists as traditional economics paradigm, to describe the economy in the simplest form, while on many cases heterogeneous agent models should be used when differences among agents are critical and directly relevant to questions and outcomes [Ríos-Rull, 1995], especially for a complex question interleaved with complexity, then the agent-based computational paradigm would be more appropriate, to avoid the oversimplifying.

Notation 1 (Agent) *An agent is a decision-maker in a collective (multi-agent) decision-making system. We denote by $\mathbb{N} = \{A_{(1)}, A_{(2)}, \dots, A_{(n)}\}$ the set of all agents, where $A_{(i)}$ represents the agent i .*

2.1.2 Preferences

Each agent needs to make decision based on his or her own preferences. These preferences represent how good are different options, alternatives, outcomes, for the agent (for [Lichtenstein and Slovic, 2006], preferences “could be conceived of as an individual’s attitude towards a set of objects”). Several aspects of preferences are studied in different fields.

In psychology —the elements that build and modify preferences are primarily studied, for instance how emotions may impact them [Scherer, 2005], how specific circumstances may play a role, how they may change over time (“A preference is not necessarily stable over

time, which can be notably modified by decision-making processes, such as choices” [Sharot et al., 2009], even in an unconscious way [Coppin et al., 2010]).

In Economics —the relation from preferences to actual choices is studied, and —although only actual choices can eventually be observed—, they are a key element in many theoretical studies ranging from voting, markets, consumer choices, etc. Issues of misrepresentation of preferences (manipulation) have for instance a long tradition of study.

In computer science —preferences have gained popularity with the rise of decision-theoretic agents. One specific issue which is studied in computer science is related to the representation of preferences (how to compactly encode a preference structure which may naively be prohibitive to represent, as is the case in particular in combinatorial domains), and of course to the computation of various reasoning tasks related to preferences (for instance, checking that an option is preferred to another one, or computing an aggregation).

Preferences can be modeled either ordinally or cardinally.

Cardinal Preferences —under a cardinal approach, preferences usually have a quantitative measure. The classical way is to rely on an *utility function*. For one example, when you choose a mobile phone, there are three makers as alternatives $\{Apple^{\text{®}}, Samsung^{\text{®}}, Nokia^{\text{®}}\}$, and your degree of liking are respectively a utility of 10, 3 and -5, showing you are a big fan of $Apple^{\text{®}}$, and could still accept $Samsung^{\text{®}}$, but dislike or even hate $Nokia^{\text{®}}$. Thus, through the cardinal preference, we not only know which alternative is preferred to which other alternative, but also know how much is one alternative preferred than another one. However, note that when dealing with cardinal preferences and aggregating preferences amongst agents the issue of *interpersonal comparisons* of utility or preference comes up: indeed this is usually meaningless because there is no simple way to interpret how different agents value their options.

Ordinal Preferences —under an ordinal approach, preferences are only captured by orders, such as full rankings or orderings. Taking the same example as above, there are three alternatives $\{Apple^{\text{®}}, Samsung^{\text{®}}, Nokia^{\text{®}}\}$ too, and your preference is expressed as an ordering:

$$Apple^{\text{®}} \succ Samsung^{\text{®}} \succ Nokia^{\text{®}}$$

Thus, when an ordinal approach is used, the preference provides an ordering between alternatives of a choice set, but tells nothing about the related strength of preferences. Through above ordering, we know that you prefer $Apple^{\text{®}}$ more to $Samsung^{\text{®}}$ and $Nokia^{\text{®}}$, prefer $Samsung^{\text{®}}$ more to $Nokia^{\text{®}}$ but less to $Apple^{\text{®}}$, and prefer $Nokia^{\text{®}}$ less to both $Apple^{\text{®}}$ and $Samsung^{\text{®}}$, but nothing more. Although it is clear that you prefer $Apple^{\text{®}}$ most and prefer $Nokia^{\text{®}}$ least, but it is unclear to what extent you prefer $Apple^{\text{®}}$ more to $Samsung^{\text{®}}$, and also not clear you like or dislike these alternatives, for example, the ordering might be understood as you hate $Nokia^{\text{®}}$, or you just prefer less to $Nokia^{\text{®}}$ than other two, you still could accept $Nokia^{\text{®}}$; also, the ordering could be understood that you are a big fan of $Apple^{\text{®}}$, but the truth might be you like none of the three makers, but just hate $Apple^{\text{®}}$ not that much. As lacking a respective value for each of alternative but just an ordering between them, some information about the preference would be *missing*. However, as the interpersonal comparisons of cardinal utility are usually deemed as unfeasible, and on many situations, the information about preference are natively limited or incomplete, we might just have the information about the ordering, but unknown about the detailed and delicate magnitudes.

It should be noted that, preference orderings used in this thesis are *transitive*. Transitivity is a key property of both partial order relations and equivalence relations. In mathematics, a binary relation R over a set X is transitive if whenever an element a is related to an element b , and b is in turn related to an element c , then a is also related to c . However, they will be required to be *complete*.

2.1.3 Decision

Under the assumption of rationality, decisions and preferences are related: different preferences lead to corresponding decisions and the decision is made based on the preference. *If the preference is the underlying or internal psychology (determining the decision), then the decision is the superficial or external behavior (reflecting the preference).*

Decision-making can be perceived as a *cognitive process* based on preferences to select an alternative or option among several ones. According to [Simon, 1965], “decision-making is one of the central activities of management and is also a critical part of any process of implementation”. Decisions has been the subject of active research from several different perspectives, that we list here following [Wikipedia, 2015d]:

- *Psychological Decision*—“studies individual decisions in the context of a set of needs, preferences and values the individual has or seeks”.
- *Cognitive Decision*—“means the decision-making process is regarded as a continuous process integrated in the interaction with the *environment*”.
- *Normative Decision*— “concerned with the logic of decision-making and rationality and the choice it leads to” (which is the perspective of decision mainly used in computer science and artificial intelligence) [Kahneman and Tversky, 2000].

2.2 Combinatorial Domains

2.2.1 Feature/Issue

Combinatorial domains (see *e.g.* [Chevaleyre et al., 2009]) occur when the alternatives of the decision-making setting are defined upon different features, criteria or dimensions. In that case, the dimension of the domain of alternatives grows fast, since they are as many options as the Cartesian product of the domains on each feature.

When the features are dependent, then this poses a challenge for the representation of preferences of the agents, since in principle it may be required to enumerate all the feasible alternatives.

To worsen matters, a collective decision may have to be taken on such domains. For example [Grandi et al., 2014] take the combinatorial and collective decision-making context of a family buying a car, usually the car has more than one features to make choices about, such as the *Maker*, *Mode* and *Color*, each of these features should be decided, which make buying a car a multi-features decision-making but not a single feature decision-making. The feature here means the property of one entity to make choices about.

Besides, many decision-making questions are not about making choices on multi-features of an entity, but (directly) making choices on multi-entities, usually namely as multi-issues for decision-making. If we take the example of the United Nations Security Council voting, there are a lot of bills proposed ceaselessly and put on table for collective voting, which is a typical multi-issue decision-making. In some cases, some bills are clearly related to other ones.

Notation 2 (Feature) *Feature is the property of the decision-making object in combinatorial decision-making, one feature is denoted as F , the set of all features $\mathbb{M} = \{F_{(1)}, F_{(2)}, \dots, F_{(m)}\}$, with finite domains $D(F_{(1)}), D(F_{(2)}), \dots, D(F_{(m)})$, $F_{(k)}$ represents the k -th feature.*

Notation 3 (Variable) *Variable is the decision-making on one feature by one agent in combinatorial and collective decision-making, one variable is denoted as V , the set of all variables $\mathbb{N} \times \mathbb{M} = \{V_{(1)}(1), \dots, V_{(m)}(n)\}$, with finite domains $D(V_{(1)}(n)), \dots, D(V_{(m)}(n))$, $V_{(k)}(i)$ represents the decision-making variable of i -th agent on k -th feature.*

Notation 4 (Alternative) *Alternative is the candidate for decision-making in the domain of feature or variable (a domain is constituted by alternatives), alternatives of feature is denoted as O , Alternatives on feature k $D(F_{(k)}) = \{o_1, o_2, \dots, o_t\}$, which could be shorted as $O_{(k)} = \{o_1, o_2, \dots, o_t\}$, and a full alternatives on all features $\mathbb{O} = O_{(1)} \times O_{(2)} \times \dots \times O_{(m)}$, which is the cartesian product of all domains $\{D(F_{(1)}), D(F_{(2)}), \dots, D(F_{(m)})\}$.*

We are now in a position to define the notations for preferences and choices, introduced earlier:

Notation 5 (Preference) *In a collective (multi-agents) decision-making system, preference is the ordering or magnitude of alternatives according to the degree of like or dislike, Preference is denoted as P , the set of all preferences $\mathbb{P} = \{P_{(1)}(1), \dots, P_{(m)}(n)\}$, $P_{(k)}(i)$ represents the preference of agent i on feature k , the preferences of agent i on all features $\mathbb{P}(i) = \{P_{(1)}(i), \dots, P_{(m)}(i)\}$, the preferences on feature k by all agents $\mathbb{P}(k) = \{P_{(k)}(1), \dots, P_{(k)}(n)\}$.*

Notation 6 (Decision/Choice) *In a collective (multi-agents) decision-making system, decision is the selection of an alternative among several alternative possibilities based on the preferences of the decision maker, Decision/Choice is denoted as C , the set of all choices $\mathbb{C} = \{C_{(1)}(1), \dots, C_{(m)}(n)\}$, $C_{(k)}(i)$ represents the decision/choice of agent i on feature k , the choice of agent i on all features $\mathbb{C}(i) = \{C_{(1)}(i), \dots, C_{(m)}(i)\}$, the choices on feature k by all agents $\mathbb{C}(k) = \{C_{(k)}(1), \dots, C_{(k)}(n)\}$.*

As mentioned above, combinatorial domains are a challenge for preference representation. We now present the approach of CP-nets that we shall mostly use in this work.

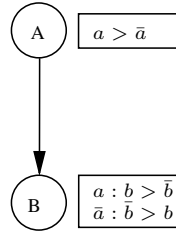


Figure 2.2: A Simple Example for the CP-nets

2.2.2 CP-nets

CP-nets [Boutilier et al., 2004a] (for Conditional Preference networks) are a graphical model for compactly representing conditional and qualitative preference relations. They are sets of ceteris paribus (all other things being equal) preference statements (cp-statements). As explained in [Maran et al., 2013], “the cp-statement “I prefer red wine to white wine if meat is served.” asserts that, given two meals that differ only in the kind of wine served and both containing meat, the meal with red wine is preferable to the meal with white wine.” Technically, a CP-net is defined as a set of features (or issues) $F = \{x_1, \dots, x_n\}$ with finite domains $D(x_1), \dots, D(x_n)$. Then, for each feature x_i , a set of parent features $Pa(x_i)$ is given, that can affect the preferences over the values of x_i . This results in a dependency graph in which each node x_i has $Pa(x_i)$ as its immediate predecessors. However, CP-nets is mainly concerned to describe the dependency among features or issues (of one agent), while not in a multi-agent influence context (until the work of influenced CP-nets [Maran et al., 2013]).

Example 2.1 (CP-nets) For a simple general example for the model of CP-nets, assume there are two features A and B , both are binary, respectively with domain $\{a, \bar{a}\}$, $\{b, \bar{b}\}$. The preference for A is $a \succ \bar{a}$ (a is preferred to \bar{a}), while the preference for feature B is dependent on the choice of feature A , if A is chosen as a , then $b \succ \bar{b}$ (b is preferred to \bar{b}), but if A is chosen as \bar{a} , then $\bar{b} \succ b$ (\bar{b} is preferred to b), which means that there is a dependency between feature A and feature B , the decision of latter one is dependent on the decision of former one, and the dependency relations or the cp-statements could be stated as: $a \succ \bar{a}, a : b \succ \bar{b}, \bar{a} : \bar{b} \succ b$.

Then given a practical example to help understand, for example of buying a car, assumed there are two features *Mode* and *Color* to make choices about, respectively with binary domain as $\{\text{Commercial car}, \text{Sports car}\}$, $\{\text{Red}, \text{Black}\}$. The preference for feature *Mode* is *Commercial car* \succ *Sports car* (*Commercial car* is preferred to *Sports car*), while the preference for the feature *Color* would be dependent on the choice of the feature *Mode*, if *Mode* is chosen as *Commercial car*, then *Black* \succ *Red* (*Black* is preferred to *Red* on the condition that *Commercial car* is determined, as *Black* would be more appropriate than *Red* for a *Commercial car*), but if *Mode* is chosen as *Sports car*, then *Red* \succ *Black* (*Red* is preferred to *Black* on the condition that *Sports car* is determined, as *Red* might seem more energetic than *black* for a *Sports car*). Therefore, there is a very straightway dependency between feature *Mode* and feature *Color*, the decision of *Color* would be dependent on the decision of *Mode*, and the cp-statements to express the dependency relations would be *Commercial car* \succ *Sports car*, *Commercial car* : *Black* \succ *Red*, *Sports car* : *Red* \succ *Black*.

2.3 Collective Decision-making

Collective decision-making is the problem of aggregating the preferences of several agents to single out one winning option, or even sometimes a representative preference for the group. The most classical methods for collective-decision making (beyond deliberation) are voting-based methods. While there are many such methods, we only present the relevant ones for our work. These methods are simple *scoring based* methods, whereby we assign points to certain positions in the preference ordering (assumed to be complete here).

Voting-based Methods.

- *Borda rule*—if there are p candidates, the top candidate gets p points, the second one gets $p - 1$ points, etc.
- *Plurality*—only the top option of each candidate is considered, and the one with the highest score wins, even if it falls short of a majority (lower than 50%).

Consensus Decision-making. This approach requires an option to be approved by a majority but the minority should also agree to go along with that option, which means if the minority disagree or oppose to the option, then the option should be modified or compromised to reduce the objection as possible. We see that this approach is more deliberative since the nature of the option may vary during the process. The veto power in UN security council is a typical negative example for consensus decision-making, ignoring the objections from minority or even majority.

2.3.1 Decision-making in Combinatorial Domains

[Xia et al., 2007] discussed the sequential composition of voting rules in multi-issue domains, as dealing with combinatorial domains leads to the well-known dilemma: either ask the voters to vote separately on each issue (and aggregate the votes on each issue independently), which may lead to the so-called *multiple election paradoxes*; or allow voters to express their full preferences on the set of all combinations of values, which as we mentioned may be practically impossible even for a few issues. [Xia et al., 2007] try to reconcile both views and find a middle way, by relaxing the extremely demanding separability restriction (which would guarantee sequential votes to be well-behaved) into the so-called *o-legality* notion:

Definition 1 *There exists a linear order $X_1 > \dots > X_P$ on the set of issues such that for each voter, every issue X_i is preferentially independent of X_{i+1}, \dots, X_P given X_1, \dots, X_{i-1} .*

This leads to define a family of *sequential voting rules*, defined as the sequential composition of local voting rules. These rules relate to the setting of conditional preference networks (CP-nets [Boutilier et al., 2004a]). They study in detail how these sequential rules inherit, or do not inherit, the properties of their local components.

2.4 Influence among Agents

For a combinatorial and collective decision-making system, another important aspect is the influencing relations among agents. For example of a family buying a car, which is a collective decision collectively by husband, wife and even kids, wife's preference on maker might be influenced by her husband, while husband's preference on model might be influenced by his wife.

2.4.1 Structure of Influence

Influencing relations among agents is close to the concept of *social relation* or *social interaction* in social science. Social relation is any interpersonal relationship among at least two individuals. The prerequisite of social relations is the individual agency¹, and then the gathering of social relations is the *social structure*.

Interpersonal Relationship

Most works about social relations are about personal social relations, namely the *interpersonal relationship*. An interpersonal relationship is a association or acquaintance between two or more people that may vary on both strength and duration. The concept of transitivity in relations is also possessed by interpersonal relationships, for example of the friendships, “my friend’s friend may become my friend”. Actually, the study of interpersonal relationships has received attentions from scientists from several different fields, such as sociology, psychology, artificial intelligence and so on. The scientific study of relationships came to be referred to as “relationship science” [Berscheid, 1999]. Interpersonal ties are also a subject in mathematical sociology [Kelley et al., 1983].

Human beings are innately social and are shaped by their relations and interactions with others. There are multiple perspectives to understand this inherent motivation to interact with others. As summarized in [Wikipedia, 2015h]:

- **Need to belong:** According to Maslow’s hierarchy of needs [McLeod, 2007], humans need to feel accepted in various social groups (like family, peer groups).
- **Social exchange:** Individuals engage in relations that are rewarding, this concept fits into a larger theory of *social exchange*: “The theory is based on the idea that relationships develop as a result of *cost-benefit analyses*. Individuals seek out rewards in interactions with others and are willing to pay a cost for said rewards. In the best-case scenario, rewards will exceed costs, producing a *net gain*. This can lead to “shopping around” or constantly comparing alternatives to maximize the benefits (rewards) while minimizing costs” [Wikipedia, 2015h].
- **Relational self:** “Relationships are also important for their ability to help individuals develop a *sense of self*. The relational self is the part of an individual’s self-concept that consists of the feelings and beliefs that one has regarding oneself that develops based on interactions with others [Andersen and Chen, 2002]. In other words, one’s emotions and behaviors are shaped by prior relationships. Thus, relational self theory posits that prior and existing relationships influence one’s emotions and behaviors in interactions with new individuals, particularly those individuals that remind him of others in his life” [Wikipedia, 2015h]. In short, the prior relationship with others become one part of self, and affecting how interacting with new others [Hinkley and Andersen, 1996].

Interpersonal relationships should be regarded as *dynamic systems* that change continuously during their “life cycle”: they may vary according to circumstances, being strengthen or weakened are people get closer or not. One of the most influential models of relationship development was proposed by [Kelley et al., 1983].

¹“In the social sciences, agency refers to the capacity of individuals to act independently and to make their own free choices. By contrast, structure are those factors of influence (such as social class, religion, gender, ethnicity, customs, etc.) that determine or limit an agent and his or her decisions” [Barker, 2003].

Theory of Interpersonal Relationship

Example 2.2 (Confucianism) “Confucianism is a theory of relationships especially within hierarchies. Social harmony—the central goal of Confucianism—results in part from every individual knowing his or her place in the social order, and playing his or her part well. Particular duties arise from each person’s particular situation in relation to others. The individual stands simultaneously in several different relationships with different people: as a junior in relation to parents and elders, and as a senior in relation to younger siblings, students, and others. Juniors are considered in Confucianism to owe their seniors reverence and seniors have duties of benevolence and concern toward juniors. A focus on mutuality is prevalent in East Asian cultures to this day” [Richey, 2005].

Example 2.3 (Minding relationships) “The mindfulness theory of relationships shows how closeness in relationships may be enhanced. Minding is the ‘reciprocal knowing process involving the nonstop, interrelated thoughts, feelings, and behaviors of persons in a relationship.’ ” [Harvey and Pauwels, 2009].

Social Networks

A social network is a social structure made up of a set of agents (*social actors*) (which can be individual agents but also organizations) and a set of bilateral relations among them (also called *ties*). The discipline of social network has recently emerged has a very successful interdisciplinary area of research.

Interpersonal Ties. In mathematical sociology, interpersonal ties are defined as “information-carrying connections” between people. Interpersonal ties generally come in three varieties: *strong*, *weak*, *absent*, and with two directions: *positive*, *negative* [Granovetter, 1973, Granovetter, 1983, Granovetter, 2005] argued that *weak social ties* are responsible for the majority of the embeddedness and structure of social networks and the transmission of information through these networks, specifically, more new information flows to individuals through weak rather than strong ties, as our families and close friends tend to be in the same circles with us, the information they possess usually overlap with what we already graph. However, [Granovetter, 1983] deemed that “weak ties provide people with access to information and resources beyond those available in their own social circle; but strong ties have greater motivation to be of assistance and are typically more easily available”, therefore there exists the *weak/strong ties paradox*. According to [Granovetter, 1973], “absent ties are those relationships without substantial significance, such as ‘nodding’ relationships between people living on the same street, the fact that two people may know each other by name does not necessarily qualify the existence of a weak tie. If their interaction is negligible the tie may be absent. The ‘strength’ of an interpersonal tie is a linear combination of the amount of time, the emotional intensity, the intimacy (or mutual confiding), and the reciprocal services which characterize each tie.”

The Weak and Strong Tie Hypothesis. Consider three agents A, B, C (as shown in figure 2.3), if A is strongly tied to both B and C , then the hypothesis is that the $B - C$ link is always present, whether weak or strong, given the other links. This results in bridges that can connect different groups in the network.

David Krackhardt proposed a new idea of *the strength of strong ties* [Krackhardt, 1992] contrast to the *strength of weak ties* [Granovetter, 1983], deemed that strong ties are very important in severe changes and uncertainty: “People resist change and are uncomfortable with uncertainty. Strong ties constitute a base of trust that can reduce resistance and provide comfort in the face

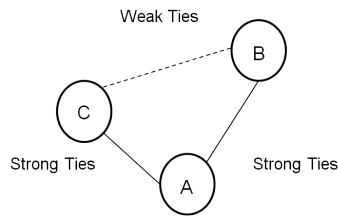


Figure 2.3: Weak Tie Hypothesis

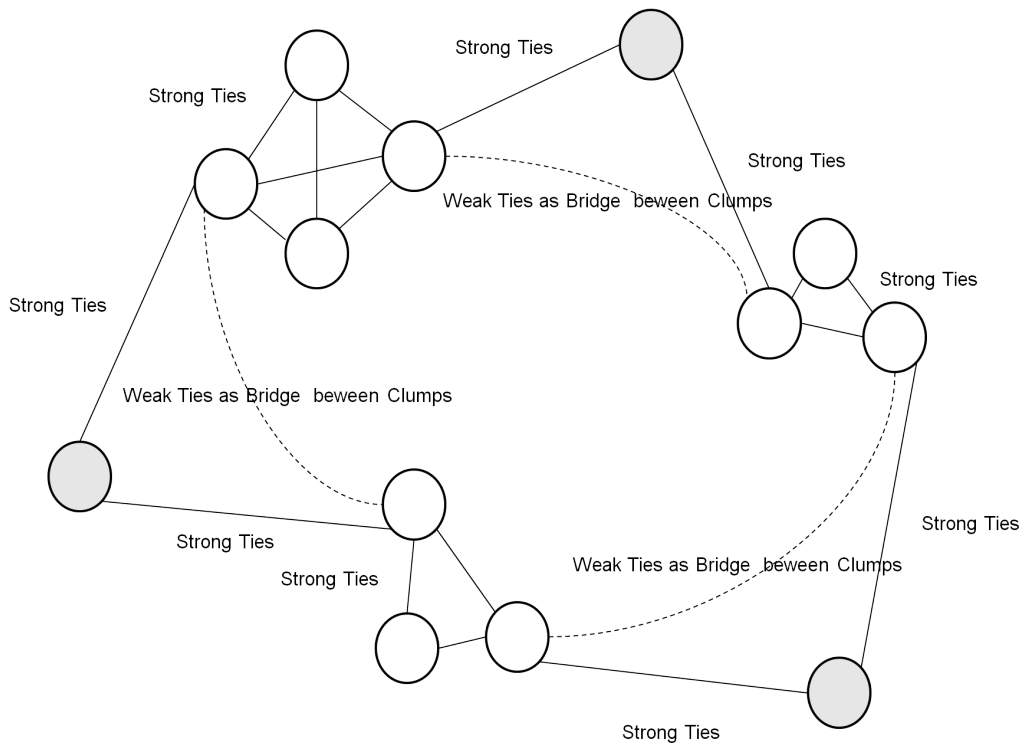


Figure 2.4: Weak Ties as Bridges

of uncertainty. This it will be argued that change is not facilitated by weak ties, but rather by a particular type of strong tie.” In a word, Granovetter finds the value of weak ties introducing the new information from disparate clumps, while ignoring the risk and intrust of the new information by weak ties.

Positive Ties and Negative Ties. While acquaintances are mostly positive ties, it is also possible to conceive negative ties, representing enemies, agents with negative appreciations of each others. This can be modeled thanks to *signed graphs* that represent both positive and negative sentiment relations. According to [Harary et al., 1953], “a signed graph is called *balanced* if the product of the signs of all relations in every cycle (links in every graph cycle) is positive. A signed graph is unbalanced if the product is ever negative. It is proven that if a network of interrelated positive and negative ties is balanced, then it consists of two subnetworks such that each has positive ties among its nodes and negative ties between nodes in distinct subnetworks”, which means a social system that splits into two cliques. “However, one of the two subnetworks may be empty, which might occur in very small networks.” To conclude, note that the ties discussed so far are assumed to be symmetric. This may not be the case in influence relations.

Notation 7 (Structure of Influence) *In the combinatorial and collective decision-making context, structure of influence is the influencing relations among decision-making agents/features/variables, making the decisions of some entities be influenced by or dependent on the decisions of other entities. The structure of Influence is denoted as S , all structures in the whole graph $\mathbb{S}[\mathbb{N} \times \mathbb{M}] = \{S_{(11)}(11), \dots, S_{(mm)}(nn)\}$, $S_{(kq)}(ij) \in \mathbb{S}[\mathbb{N} \times \mathbb{M}]$ represents the influencing relations from decision-making variable $x_{i,k}$ (by agent i on feature k) to decision-making variable $x_{j,q}$ (by agent j on feature q), and for the section graphs, structures within the horizontal dimension feature k as $\mathbb{S}[\mathbb{N}]_{(k)} = \{S_{(k)}(11), S_{(k)}(nn)\}$, and structures within the vertical dimension agent i as $\mathbb{S}[\mathbb{M}]_{(i)} = \{S_{(11)}(i), \dots, S_{(mm)}(i)\}$.*

2.4.2 Social Influence

Social influence is a typical model capturing the influencing relations among agents, in short, when decisions of agents are affected by others.

Kelman’s Varieties. [Kelman, 1958] identified three broad varieties of social influence (compliance, identification, internalization), to help determine the effects of social influence, in particular to separate *public conformity* (when the agent acts to comply with the group without changing his own views) from *private acceptance* (when the private mental state of the agent is truly affected).

Psychological Needs. [Deutsch and Gerard, 1955] described two psychological needs that lead humans to conform to the expectations of others. These include our need to be right, the need for truth (*informational social influence*), and our need to be liked, the need for affection (*normative social influence*).

- **Informational influence** (or **social proof**) as is usually the case in situation where an agent doesn’t know how to act exactly and will rely on the assumption that the other agents must have a better knowledge of the situation.
- **Normative influence** is “an influence to conform to the *positive expectations* of others”, not letting others down. “Normative social influence’s power stems from the human identity as a social creature, with a need for companionship and association [Aronson et al., 2013]. This fact often leads to people exhibiting public compliance—but not necessarily private acceptance—of the group’s social norms in order to be accepted by the group” [Wikipedia, 2015k], which is different from the social proof (informational social influence).

Informational social influence is just about information and knowledge, and normative social influence is about emotion and feeling. In terms of Kelman’s typology, normative influence leads to public compliance, whereas informational influence leads to private acceptance, not necessarily to internalization, the influenced one might accept innerly but deny externally.

Major Types

According to [Wikipedia, 2015m], there are several typical types of social influence (quotations from [Wikipedia, 2015m]):

- “**Conformity** is a type of social influence involving a change in behavior, belief or thinking to align with others or normative standards. Conformity research tends to distinguish between two varieties: informational conformity (also called social proof, or “internalization” in Kelman’s terms) and normative conformity (‘compliance’ in Kelman’s terms) [Aronson et al., 2005].”
- “**Persuasion** is the process of guiding oneself or another toward the adoption of attitudes by rational or symbolic means [Cialdini, 2009]. defined six “weapons of influence”: reciprocity, commitment, social proof, authority, liking, and scarcity. Persuasion can occur through appeals to reason or appeals to emotion.”
- “**Self-fulfilling prophecy** is the *prediction* that directly or indirectly causes itself to become true, due to a positive feedback between belief and behavior. A prophecy declared as truth (when it is actually false) may sufficiently influence people, either through fear or logical confusion, so that their reactions ultimately fulfill the once-false prophecy. The term is proposed by [Merton, 1948].”
- **Obedience** requires the existence of an *authority* figure. It is well documented in the literature: “The Milgram experiment [Milgram, 1963, Blass, 1999], Zimbardo’s Stanford prison experiment, and the Hofling hospital experiment [Hofling et al., 1966] are three well-known experiments on obedience, and they all conclude that humans behave surprisingly obedient in the presence of perceived legitimate authority figures.”
- **Reactance** is in a sense opposite to conformity. “While the results are the opposite of what the influencer intended, this reactive behavior is the result of social pressure [Brehm, 1966]. It is notable that anticonformity does not necessarily mean independence, but possible as a negative influence (go to the opposition of the influencer). In many studies, reactance manifests itself in a deliberate rejection of an influence, even when the influence is clearly correct [Frager, 1970].”
- **Majority influence** takes place when a minority is influenced to accept the beliefs or behaviors of a majority, which is easy to understand, as a majority would form a social or peer pressure for the minority. Majority influence would usually be affected by the ratio of the sizes between majority and minority groups. Majority influence most often operates through normative social influence because the minority would be afraid to be disliked by the majority, besides, majority influence could also be introduced by informational social influence because the minority deems that the majority hold the truth therefore there are more persons with the same beliefs or behaviors in their group.
- “**Minority influence** takes place when a majority is influenced to accept the beliefs or behaviors of a minority. Minority influence can be affected by the sizes of majority and minority groups, the level of consistency of the minority group and situational factors (such as the affluence or social importance of the minority) [Nemeth, 1986]. Minority influence most often operates through informational social influence (as opposed to normative social influence) because the majority may be indifferent to the liking of the minority [Wood et al., 1994]”, but caring about the truth held by the minority.

2.4.3 Convergence to consensus

Recently in economics, the question of the *dynamics* of influence has been intensely studied. For indeed, influence is an iterative process: it may well be the case that agent i influences agent

j , which influences agent k , which in turn influences agent i . In particular, one central question has been whether a consensus state could be reached at the end of the process.

Most of these models assume a simple belief model for the agent (a value), which will be affected by neighbors depending on the social structure. In this line, the model of [DeGroot, 1974] is foundational. More recently, [Jackson, 2008] discussed a social network environment where one agent's utility would be influenced (in the form of learning) by own and all other agents' with a weight unequal to 0, where weights may be positive or negative to represent the polarity of influence. Conditions on the matrix representing influences allow to determine whether consensus will be reached in the end or not.

In a different setting of binary decisions, we also mention the important work of Grabisch and Rusinowska, who did a lot of works about influence, such as influence functions (follower functions, command game) [Grabisch and Rusinowska, 2011a], single step or iterating influence embedded in the social networks [Grabisch and Rusinowska, 2010b, Grabisch and Rusinowska, 2011b]. As mentioned the influence are all assumed in binary decision over a single issue. For example, [Grabisch and Rusinowska, 2011a] study and compare two frameworks: a model of influence and command games. In the influence model, in which players are to make a certain acceptance/rejection binary decision, due to influence of other players, the decision of a player may be different from his inclination. It is also assumed by [Grabisch and Rusinowska, 2010b] that each player has an inclination to say YES or NO which, due to influence of other players, may be different from the decision of the player. And [Grabisch and Rusinowska, 2011b] further generalize a yes-no model of influence in a social network with a single step of mutual influence to a framework with iterated influence. Each agent makes an acceptance-rejection decision and has an inclination to say either "yes" or "no" (but still a binary decision toward a single issue). "Due to influence by others, an agent's decision may be different from his original inclination, such a transformation from the inclinations to the decisions is defined as *influence*, and represented by an *influence function*." They analyze the decision process in which the mutual influence does not stop after one step but iterates, and give a complete description of terminal classes.

Influence Functions. In [Grabisch and Rusinowska, 2010b, Grabisch and Rusinowska, 2010c, Grabisch and Rusinowska, 2011b], an influence function B over n agents is a function that maps every vector of inclinations $I = (I_1, \dots, I_n) \in \{-1, +1\}^n$, where I_i is the inclination of the agent i , into a vector of decisions $B(I) = (B_1(I), \dots, B_n(I)) \in \{-1, +1\}^n$, where $B_i(I)$ denotes the decision made by the agent i . Stable states satisfy $I_i^{(k)} = I_i^{(k+1)}$, for every agent i , starting from a certain k , where k is the number of iterations and $I_i(k)$ denotes the inclination (state) of agent i at the iteration k . A set of agents such that their $B_i(I)$ coincide in a stable state is a consensus group. The influence function can be modelled via a graph where nodes are states and arcs model state transitions via the influence function. Starting from an initial state, via the influence function we may pass from state to state until stability holds (in the graph formulation, we are in a state represented by a node with a loop), or we may also not converge. Here are some examples of influence functions by [Grabisch and Rusinowska, 2011b]:

Example 2.4 (Fol) *"is an influence function between two agents, each always following the inclination of the other one (for instance the agent always thinking the other's choice is better than oneself). It converges to stability when the initial inclination is a consensus between the two agents. Otherwise, influence iteration never stops."*

Example 2.5 (Gur) *"is an influence function where one of the agents is the guru (which is similar to the opinion leader or the Charisma) and all other agents follow him. It has two stable states, which both represent consensus. Given any initial inclination, the iteration will converge to one of the stable states (according to the guru's initial inclination)."*

Example 2.6 (Conf3) “models a community with a king, a man, a woman, and a child, following a Confucian model²: the man follows the king, the woman and child follow the man, and the king is influenced by others only if he has a positive inclination, in which case he will follow such an inclination only if at least one of the other people agrees with him. This influence function always converges to one of two stable states, which both represent consensus, depending on the initial state.”

Example 2.7 (Social Influence) Given a simple example of the social influence model (in the context of Social Networks) as shown in figure 2.5³, assume there are four agents A, B, C, D , and influencing and influenced by each other. The matrix of weight of influence among these agents could be expressed as following (the horizontal axis is the influencing agents, and the vertical axis is the influenced agents). Assumed $wt_{i,j}$ represents the weight of influence from agent i to agent j , and the weight indicates the strength and direction of the influence, $wt_{i,j} > 0$ means a positive influence, $wt_{i,j} < 0$ means a negative influence, and $wt_{i,j} = 0$ means there is no influence between agent i and agent j . Actually, the matrix of weight of influence and the graph of structure of influence could be mutually transformed.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{pmatrix} & A & B & C & D \\ \begin{pmatrix} 1 & 2 & -3 & 0 \\ 1 & 2 & 1 & 1 \\ 2 & 3 & -1 & 3 \\ -1 & 0 & -2 & 0 \end{pmatrix} \end{pmatrix}$$

As observed in above matrix, four agents are full of influences, either positive or negative, except absent from agent D to A , from agent B to D , and from agent D to own. To understand the weight of influence among agents, for the influenced aspect of agent A , he thinks agent B is his friend ($wt_{B,A} = 2$), while agent C is his worst enemy ($wt_{C,A} = -3$), and not care about agent D ($wt_{D,A} = 0$, which means agent D might be “nobody” for agent A , and would not influence agent A ’s preference and decision.) And for the influencing aspect of agent A , he is counted as friend by agent B and C ($wt_{A,B} = 1$ and $wt_{A,C} = 2$), while counted as enemy by agent D ($wt_{A,D} = -1$).

Besides, it is interesting to note that the weight of influence from agent C to own is set as negative ($wt_{C,C} = -1$), although on most cases agent would be positively determined or affected by own preference, while the negative influence is still reasonable in some cases, the agent might be very disappointed and unsatisfied with own former behaviors or current status, lose confidence on oneself, and be eager to make difference, and improve the situation, then one solution would be to refer more to other agents’ preferences and behaviors, especially for those experts, authorities, winners.

Furthermore, it should be noticed that the weight of influence might not be symmetrical between two agents, such as $wt_{A,C} = 2$ while $wt_{C,A} = -3$, which means agent C perceive agent A as friend (with positive influence) while agent A perceive agent C as enemy (with negative influence), this kind of “contradiction” is broadly common in real-world situation, for example sometimes you think someone is your friend, while he doesn’t think similarly, and counts you as a bother.

2.4.4 A Note on Information Cascades

According to [Wikipedia, 2015g], “an information (or informational) cascade occurs when a person observes the actions of others and then—despite possible contradictions in his own private

²In the Confucianism, the state and the family have the same structure, the man listens to king and the woman and children listen to the man.

³Black lines represent the influence to agent A , yellow lines represent the influence to agent B , green lines represent the influence to agent C , and white lines represent the influence to agent D .

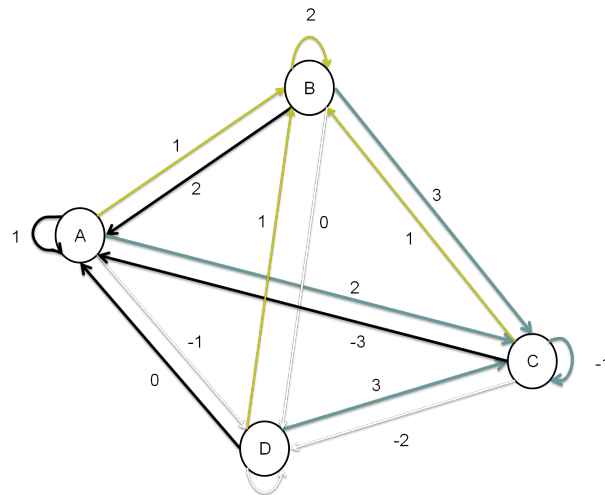


Figure 2.5: One Example of Weight of Influence and Structure of Influence

information signals—engages in the same acts. A cascade develops, when people “abandon their own information in favor of inferences based on earlier people’s actions” [Easley and Kleinberg, 2010]. Information cascades explain why this may occur, under what probabilities, and what are the chances to start a cascade given the conditions [Bikhchandani et al., 1992]. It is sometimes challenged that agents always act rationally. As a result, social perspectives of cascades have emerged, taking into account the fact that agents may act different from optimal when social pressures is present [Shiller, 1995].

Key Conditions

There are four key conditions in an information cascade model [Aronson et al., 2005], cited in [Wikipedia, 2015g]:

- Agents make rational decisions based on the available information (a discussed assumption, as mentioned above);
- “Agents make decisions sequentially”, while for some social influence studies, [Gaspers et al., 2013] also discussed a model where agents in a social network vote sequentially and are influenced by the vote of those of their friends who voted before them;
- “Agents do not have access to the private information of others”, (just could observe the actions of others, contrast to some social influence models, which assumed agents know the preference of others);
- “A limited action space exists” (e.g. an adopt/reject decision).

Comparison with Social Influence

Some competing models of information cascade exist, it is often the problem that the concept of an information cascade is conflated with some similar concept, like social proof, information diffusion [Gruhl et al., 2004], social influence and so on, while which might not match all the key conditions of information cascade discussed above.

One of the most similar and typical model would be social influence, which is also one of the main referring model of the thesis, and would be discussed in details in following. Although

there are lots of similarity between social influence and information cascade, both about agents are affected by others, there are still some differences:

- Social influence centered on the change of one's preference by others, while information cascade centered on the renewal of one's information by others, (but both eventually leading to the transform or update of actions);
- Social influence could make the influenced agent personally changed, usually renewed with a new preference more similar or like the influencing one, which is related to the concept of "assimilation", while information cascade would just give the affected agent more information of others, possessed more information or knowledge for decision-making, but not necessarily personally changed;
- Information cascade is more like a pure rational mode, in which the decision-maker constantly collect, process and use the information of previous participants, update own possessed information, to make own choices more correct (although the individual rationality might lead to the collective incorrectness), while social influence are sometimes emotional rather than rational, for example you are easily to be positive influenced by your friends and families, and negatively influenced by your enemies and foes (be empathetic with someone you like and oppose particularly to someone you hate), although the influences and reactions might abandon yourself to emotions.
- Information cascade is triggered by the observation of the actions of others, but social influence might also be triggered by the acquaintance of the preferences of others, as the unaccessible to the private information of others is just assumed by information cascade but not social influence. And this also explains why information cascade focus on the information, more specifically, (the information) about the actions of others, while social influence focus on the preference, as preference belong to the private information of others, which usually could not be directly observed.

Actually, action and preference are highly interrelated, preference is the underlying determinant of action, while action is the apparent expression of preference. As assumed the inaccessible to the private information of others, thus information cascade could be just measured by the observable and external elements (usually as actions).

In fact, the term information cascade has been wrongly used to refer to such processes of social influence [Sadikov et al., 2011].

Example 2.8 (Information Cascades) *Quote one example from [Wikipedia, 2015g], "Information cascades occur when external information obtained from **previous participants** in an event **overrides** one's own private signal, irrespective of the correctness of the former over the latter. The experiment conducted in [Anderson and Holt, 1997] is a useful example of this process. The experiment consisted of two urns labeled A and B. Urn A contains two balls labeled 'a' and one labeled 'b'. Urn B contains one ball labeled 'a' and two labeled 'b'. The urn from which a ball must be drawn during each run is determined randomly and with equal probabilities (from the throw of a dice). The contents of the chosen urn are emptied into a neutral container. The participants are then asked in random order to draw a marble from this container. This entire process may be termed a 'run', and a number of such runs are performed. Each time a participant picks up a marble, he is to decide which urn it belongs to. His decision is then announced for the benefit of the remaining participants in the room. Thus, the $(n + 1)$ th participant has information about the decisions made by all the n participants preceding him, and also his private signal which is the label on the ball that he draws during his turn. The experimenters observed that an information*

cascade was observed in 41 of 56 such runs. This means, in the runs where the cascade occurred, at least one participant gave precedence to earlier decisions over his own private signal. It is possible for such an occurrence to produce the wrong result. This phenomenon is known as ‘Reverse Cascade’.”

2.4.5 Influence with Ordinal Preferences

Voting and social influence. As mentioned already, one very specific type of influence in voting is bribery, whereby an agent *pays a price* to influence the vote of another agent. This does not exactly fit our context of social influence though. We now list a couple of relevant works:

- The recent work [Gaspers et al., 2013] studies a model where agents in a social network vote sequentially and are influenced by the vote of their friends who voted before them. In their model, agents have candidates that they like (or are acceptable), among which they single out one favorite candidate. The influence model is as follows: agents vote in some order, and can observe the votes of friends who voted before them: in case more than half of them voted for a candidate that x likes, they x would follow them, otherwise he would stick to his preferred candidate.
- The model of empathetic social networks [Salehi-Abari and Boutilier, 2014] also discusses the influence from more than one agents combining with the *decision-making* question, in the form of empathetic social choice on social networks, assumed one agent’s preference would be empathetic with both own all other neighbours’ preferences according to allocated weights (as a weighted form of classical preference aggregation), though assumed a multi-choices but not a single choice, while didn’t set the dependency or combinatorial structures among multi-choices. Empathetic Social Choice [Salehi-Abari and Boutilier, 2014] is a very recent model to capture the influence from more than one agents on a social networks environment. They introduce a model for social choice aiming at consensus decision making on social networks. They define an *empathetic social choice* framework in which agents derive their utility based on both their own intrinsic preferences (before being influenced) and the satisfaction of their neighbors (other linked agents or agents with influencing relations to him). Interestingly, the problem can be mapped into a (weighted) form of preference aggregation (e.g., maximization of social welfare for certain forms of voting).

Influenced CP-nets. However, in multi-issue domains, a crucial new aspect is that an agent may be influenced by another agent on given issue, but by another agent (for instance), on another issue. *Influenced CP-nets* [Maran et al., 2013] is a very recent model of multi-agent influence in such combinatorially structured domains. [Maran et al., 2013] consider multi-agent settings where a set of agents want to take a collective decision, based on their preferences over the possible candidate options. While agents have their initial inclination, they may interact and influence each other, and therefore modify their preferences, until hopefully they reach a stable state and declare their final inclination. At that point, a voting rule is used to aggregate the agents’ preferences and generate the collective decision.

Based on the CP-nets [Boutilier et al., 2004a], they adapt such frameworks to incorporate influences among agents, by allowing influences to be over the same issue or also among different issues. An interesting feature of the influenced CP-nets is that influence is embedded smoothly in the *multi-agent CP-net profile* [Rossi et al., 2004a], and there is a convenient coincidence between the optimal outcomes of certain CP-nets and the stable states of the influence iterative process.

And by combining with the recent works (such as [Grabisch and Rusinowska, 2011a, Grabisch and Rusinowska, 2010b, Grabisch and Rusinowska, 2011b]) modelling the influence phe-

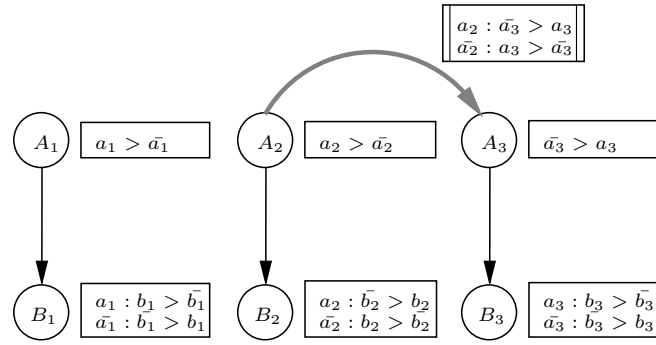


Figure 2.6: An Example of Influenced CP-nets [Maran et al., 2013]

nomenon in the case of voting over a single issue, they generalize the model to account for preferences over combinatorially structured domains including multi-issues that may be dependent on each other, and propose a way to model influence when agents express their preferences as CP-nets [Boutilier et al., 2004a].

Furthermore, they study two procedures for aggregating preferences and finding a collective decision in this context which interleaves voting and influence convergence:

Final aggregation (FA) —performs influence iteration at each level of the CP-nets and it aggregates agents' preferences only at the end,

Local aggregation (LA) —performs influence iteration and preference aggregation at each level.

They then evaluate such procedures in terms of resistance to bribery. Bribery in voting may be regarded as a type of influence, although it does not involve an iterative process: an external agent (the briber) wants to influence the result of the voting process by convincing some agents to change their vote, in order to get a collective result which is more preferred to him; there is usually a limited budget to be spent by the briber to convince agents [Faliszewski et al., 2009]. They show that the presence of inter-agent influence makes bribery computationally difficult, even in a very restrictive setting, both for LA and FA, thus making such procedures resistant to bribery.

Example 2.9 (Influenced CP-nets) *As figure 2.6, in line with the CP-nets graphical notation (using vertical-line to model dependencies), [Maran et al., 2013] use hyper-arcs (named as ci-arcs) to graphically model influences, going from the influencing variables to the influenced variable. In the graph, there are three agents 1,2,3 and thus three CP-nets with two binary issues: A and B. There is an ordering O of decision-making is $A \succ B$. Each variable A_i (resp., B_i), with $i \in \{1, 2, 3\}$, has two values denoted by a_i and \bar{a}_i (resp., b_i and \bar{b}_i). Value a_i for the variables A_i correspond to value a for A , and similarly for B . Variables A_i belong to the first level while variables B_i belong to the second level. Cp-statements (representing dependencies) are denoted by vertical black-line arrows while ci-statements (representing) are denoted by horizontal gray-line arrows. Agent 3 is influenced on issue A by agent 2.*

Although [Maran et al., 2013] consider acyclic CP-nets, while ci-arcs may create loops due to the iterative nature of influences: a self-influencing variable models the fact that the value of the variable in the next state depends on its value in the current state.

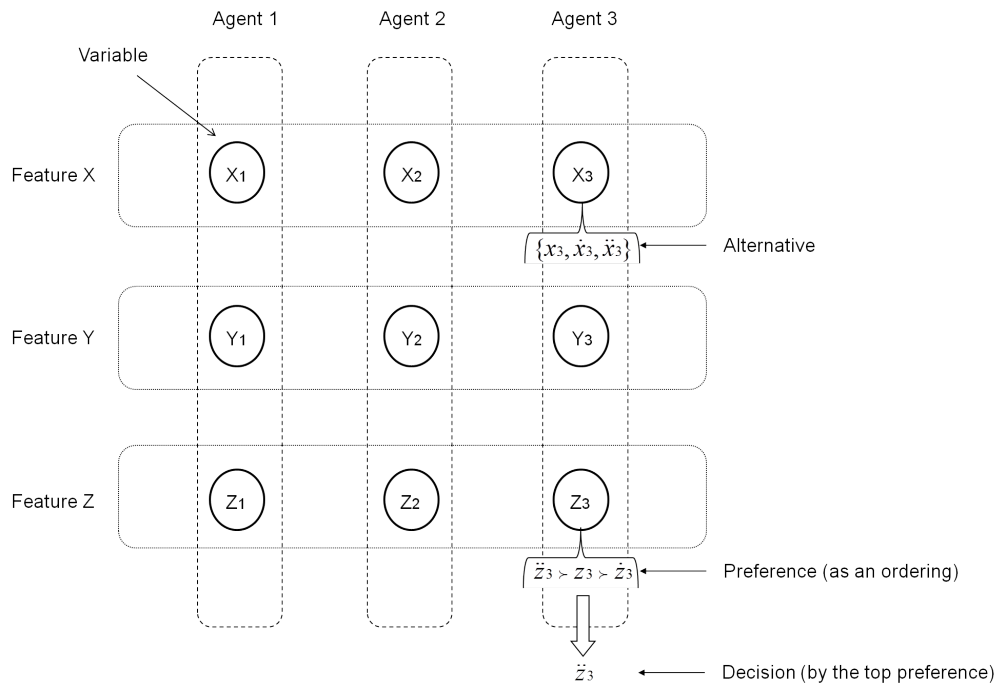


Figure 2.7: A Combinatorial and Collective Decision-making System: Basic Concepts and Their Relations

2.4.6 Summary of our approach

Based on above definitions, a *Combinatorial and Collective Decision-making System* could be built:

Notation 8 (Combinatorial and Collective Decision-making System) *CCS is a society of Combinatorial and Collective decision-making system embedded with Structures of influence, $CCS = \langle N, M, PR/WT \rangle$, N is the set of all agents, M is the set of all features, PR/WT is the matrix of priority/weight of influence, which is the numeric expression of structures of influence, and could be mutual transformed with the graphic expression of structures of influence (as nodes and links).*

Finally, we should summarize some basic assumptions of the model we will deal with in this thesis (unless specified otherwise of course):

- as [Xia et al., 2007] we assume issues can be ranked following an *o-legal* order;
- agents will make decisions sequentially, on these different issues;
- agents will observe decisions of others and possibly be influenced by these decisions;
- influence may come from various agents and from various features;
- in general, we will not be much concerned with problems of convergence, that is, we shall assume that influence is either one shot or stabilize after a few rounds.

Chapter 3

Understanding Influence (Models) from the 5W1H Framework

Contents of Chapter 3

- 3.1 What Influence
- 3.2 Where Influence
- 3.3 When Influence
- 3.4 Who Influence
- 3.5 Why Influence
- 3.6 How Influence

In order to survey the models of influence, discover what is missing in current work and point out the direction for advancements, a systematic framework to organize the existed work [Grabisch and Rusinowska, 2010a, Grabisch and Rusinowska, 2010c, Grabisch and Rusinowska, 2010b, Grabisch and Rusinowska, 2011a, Maran et al., 2013, Maudet et al., 2012a, Salehi-Abari and Boutilier, 2014, Koller and Milch, 2003] should be proposed.

One traditional way is to classify the models of influence according to the disciplines. As discussed before, the models of influence (in computational social choice) pertain to the interdisciplinary fields between social science and computer science¹, on the left wing such as economics, politics, management (more specifically, social choice, decision theory, game theory and so on), on the right wing such as artificial intelligence, multi-agent system, operation research, computational logic and so on [Chevalyere et al., 2007].

As the research about influence has obtained increasing interests from widespread and seemingly distinct disciplines, for instance, (welfare) economics², artificial intelligence, complex networks analysis, (computational) social choice and so on, many researchers from different disciplines all focus on the influence but possibly from different perspectives or with different bias (emphasis), “influenced” by their different disciplinary backgrounds and knowledge systems.

For example, the economics, artificial intelligence and computational social choice focus on the preference a lot, because the preference determined the resource allocation, the voting, which are very important to these disciplines, but comparatively, the complex networks (analysis) focus on the behavior a lot, studying the behavior of agents directly while “ignoring” the preference. There are many reasons, one is that the complex networks science emphasis not on the transfor-

¹Especially combining the collective decision-making and artificial intelligence.

²For example, *Social and Economic Networks* by [Jackson, 2008] investigate the influence in the context of social networks (in which combine the concept of learning and influence), Grabisch and Rusinowska [Grabisch and Rusinowska, 2010a, Grabisch and Rusinowska, 2010c, Grabisch and Rusinowska, 2010b, Grabisch and Rusinowska, 2011a] also discuss different approaches to influence based on social networks, and combining with simple games, and so on.

mation from the preference to the behavior, but on the integral structural property of the networks constituted by the collective behaviors. Therefore, it is one way to organize based on different discipline.

However, in the computational social choice, natively as an interdisciplinary field, the differences among different disciplines have become more and more minimal (it is very common that researchers from different disciplines and with different backgrounds focus on the same topics and could communicate their work barrier-free, transcending the different paradigms of disciplines), actually, the approach to organize work by disciplines might have been slighted outdated³, especially for a cutting-edge interdisciplinary field.

Therefore, a framework ignoring the boundaries of disciplines but emphasis the functions of systems should be proposed to to organize the current work.⁴ For this interdisciplinary area of Social Science and Computer science, it is reasonable to introduce the framework of 5W1H from the management (science) to organize the current work about the models of influence. 5W1H namely consists of What, Where, When, Who, Why (5W) and How (1H), which is an intuitive and systematic framework.

3.1 What Influence

The 1st W “What” is mainly about the content of the influence. For example in the social networks, if the links from agent to agent represents the “path” of influence, then “What” of influence is the “Cargo” transporting on the “Paths”. As discussed before, there are agents as the subject of influence and also the agents as the object of influence, which actually means the influences are directed, (starting) from the influencing agent (and going) to the influenced agent. Therefore, the “What” of influence could be discussed from the perspective of influenced by What and influenced to What.

Influenced by What Influenced by What is discussed in substantial work as varied forms⁵, summary as following, the former three are about (the information of) decision-makers/voters⁶, and the last one is about (the information of) candidates and environments:

- An agent can be directly influenced by the (observed) decision [Grabisch and Rusinowska, 2010a, Grabisch and Rusinowska, 2010c, Grabisch and Rusinowska, 2010b, Grabisch and Rusinowska, 2011a, Maran et al., 2013] of another agent, typically a friend or a opponent (due to intensive relations), or an expert (due to professional knowledge and prospect better payoff) and so on. Which is influenced by the present and eyeable fact of decision.
- An agent may be influenced by the prospective result of a social choice (such as a poll of an election [Reijngoud and Endriss, 2012]). Which is influenced by the future and prospective forecast of decision.
- An agent may be influenced by arguments among agents during a deliberation. Which is influenced by the ongoing process of agents’ communication about (made or planned)

³Many scholars contribute to overturn the borders between different disciplines. A very famous example is the complexity science, which devotes to build theories crossing disciplines.

⁴In fact, what counts should be the nature of what the researchers study, and the way how they think of, but not which discipline they come from.

⁵There might be also another kind of (influenced by) What, the constraints [Rossi et al., 2011, Meseguer et al., 2005, Dechter, 2003, Rossi et al., 2006, Grandi et al., 2014], although in many cases constraints are discussed distinguished from the influence, but these two possessing one common point that, they are all about the former features “influencing” the decision of latter features.

⁶or decisions and votes.

decision. There might be one kind of (directed) argumentation as the recommendation, [Leskovec et al., 2006] discuss the patterns of influence in a recommendation network, in which the purchase behaviors of consumers in a large on-line retailer could be influenced by the recommendations of previous buyers.⁷

- An agent might be influenced by some new information, new cases and new events about the collective decision. Different from above three forms, this is unrelated to the decision-makers, but about the candidates, environments and so on.

Above are several typical types of influenced by “What” mainly discussed in current work [Grabisch and Rusinowska, 2010a, Grabisch and Rusinowska, 2010c, Grabisch and Rusinowska, 2010b, Grabisch and Rusinowska, 2011a, Maran et al., 2013, Reijngoud and Endriss, 2012, Leskovec et al., 2006, Gilboa and Vieille, 2004]. Though diversified and complicated in varied forms, but still there is a common point (or a uniform reason) for all these influenced by “What”, it is the changing of interests underlying the changing of decisions. Based on the assumption of rationality⁸, all choices (from more than one candidates) are based on the comparison of the payoff/utility (accompanied with each candidate), and the agent make choices in order to gain the maximum of the payoff/utility (in a collective decision-making context, either genuinely choose the top preferred candidate, or manipulatively choose not exactly according to own preference, in order to get the more preferred one to be chosen). Therefore, if the agent is influenced and changed, then no matter influenced by “What” (such as the observed vote, prospective result, argumentations, new cases and so on), the “What” would eventually lead to the change of interests over different candidates. For example, if the original most preferred candidate⁹ has been replaced by another candidate, then of course the agent would change the choice to this new candidate (if according to the genuine preference).

Although varied types of influenced by “What” have been discussed, but nearly all the influences coming from the decisions of agents, but what should be noticed that in the context of influence (for example in the social networks of influence), the structure (of influences)¹⁰ among agents are also very important, which can be autonomously determined by the agents (there are choices about candidates, and also choices about structures, for example, you can choose who to communicate with, who to interact with, particular in the context of influence, you can choose who to influence and who to be influenced by, and so on), and same with the decision, the structure could also play a influential effect, and can influence each other and be influenced by each other.¹¹ The influence about the structure is one of the core part in the thesis, and would be discussed in details in the part of Influence Models.

Influence to What After discussed the influence from “What”, according to the direction of influence, then the influence to “What”. In another word, discuss how the agent be influenced, influenced on which aspect, and influenced to what extent, and so on. The influence usually acts on specific aspects (or attributes) of the object (agent), and there are mainly three perspectives to “cut-in”. As in the context of collective decision-making, the most important two attributes

⁷Such as the Amazon, the Dangdang in China and so on, after you have bought a book, you could recommend the book to your friends by e-mail, and once your friend also buy the book, you as the recommend could have some bonus, and of course the your friends usually could buy the book at some discounts by your recommendations.

⁸In economics and many disciplines, especially for the game theory, very apparently, the agents make choices on the strategies in order to gain higher or the highest payoff/utility, etc.

⁹Which is the candidate with the highest payoff compared with other candidates.

¹⁰Which could be expressed as the structure of links among agents in the social networks.

¹¹In the model of the thesis, both the decision and the structure, or the decision about candidates and the decisions about the structure are endowed with a equal status and influencing and influenced role, both can influence and be influenced by, and the decision and structure can influence each other.

are preferences and decisions, which can be combined to three cases: the change from (initial) preference to (eventual) decision [Grabisch and Rusinowska, 2011a] (more specifically, the new preference after influenced leading to the new decision¹²); the change of preference, from original preference to influenced new preference [Salehi-Abari and Boutilier, 2014], not directly discuss the transform from preference to decision; the change of decision (or other behaviors) [Leskovec et al., 2006, Koller and Milch, 2003], just discuss the renewal of (observed) behaviors, while “ignore” the underlying preference.

3.2 Where Influence

The 2nd W “Where” is mainly about the field or platform¹³ of the influence. For example in the social networks, the influence has to work through the links among agents, in which, the “Where” of influence (the links) are like the paths of influence, and the “What” of influence are like the cargos transmitting on the paths. In a word, the (function of) Influence need a concrete field or platform to take place. There are several typical models for the “Where” of influence discussed as following:

- Most of works (currently) [Jackson, 2008, Grabisch and Rusinowska, 2011a, Golub and Jackson, 2010, Salehi-Abari and Boutilier, 2014, Luo et al., 2013, Luo and Meng, 2013b] about influence are investigated in the context of social networks, which are composed of persons (agents) presented as nodes and influence between persons to persons (agents to agents) presented as the links between nodes to nodes, which has been a mainstream approach to model the “where” of influence, especially in the multi-agent system. This kind of “Where” is defined by the social relations, whether the influence works/happens or not is determined by the existence of personal relations and the willingness to interact.
- Another kind of the “Where” of influence is the physical space, one renowned example is the cellular automata.
- Furthermore, it should be noticed that the social networks is not the only networks model for the “Where” of influence, actually, the social networks models devote to describe the social relations among persons (or the personal relations among agents). Actually, there is another kind of networks relatively less discussed compared with the social networks in the context of influence, which is the causal relationship networks among events [Koller and Milch, 2003], in which, related variables are linked not by the (subjective) personal relations but by the (objective) casual relations between events¹⁴, this kind of casual relationship networks are usually represented in the framework of the Bayesian Networks, which means that the status of some events will influence the probability of some other events, or event the decisions of agents.

From one perspective, the causal relationship networks are related to the the **System Dynamics**, in which the sub-systems (or the components) are interconnected with each other (by the causality). Contrast to the casual networks, the social networks are about the **Network Dynamics**, concerning the evolution and interaction of multi-agents. Compared further, the casual networks

¹²In another word, the preference influenced/changed leads to the decision influenced/changed.

¹³Or replaced as the area, place, site, space, ground of the influence and so on.

¹⁴The relations could be replaced by the relevances, the dependencies, and so on. [Koller and Milch, 2003] describe the event relations as the strategic relevance, which can be used to decompose large games into a set of interacting smaller games, and can be solved in sequence and lead to substantial savings in the computational cost of finding Nash equilibria in these games.

could describe the relations of multi-issues within one single agent, and the social networks could describe the relations of multi-agents on one single issue, therefore, casual networks and social networks could be combined to better discuss the multi-issue multi-agent decision-making situation.

One approach to advance the networks models is to improve the social influence networks with adding the probabilistic casual networks, integrating the network dynamics and system dynamics, not only considering the influence among agents, but also the influence among agents, events (issues), and environment.¹⁵

3.3 When Influence

The 3rd W “When” is mainly about the time of influence, which is also an important factor for the influence, different time point of the influence take effect would generate different outcome. As the decision is the core role in collective decision-making, therefore, assume the decision as the reference point (for the time of influence), then there will be kinds of “When” as following:

- Influence **After** the decision, for example be influenced by the observed decision or observed outcome, which means the decision of agent be influenced by other agent(s) already making decisions before them, in fact in many cases, the agents make decisions sequentially (with a specific or random order¹⁶);
- Influence **Before** the decision, such as be influenced by the forecast, future expectation and prospective results of the collective decisions, once agent estimate or obtain the prospective results of the collective decisions or other agents’ decisions, then could possibly purposely reveal the fake preference or make choices not exactly according to own genuine preference, to pursue better payoff for oneself, which is named as the manipulation [Taylor, 2005] in social choice;
- Influence **During** the decision, for the example of the discussion, argumentation, persuasion and so on among agents during the decision-making process. It is about the information communication (during a deliberation) among agents (for instance, agents communicate with other agents of own genuine or fake preference, try to represent own claims, “spy” on other agents’ preference, or attempt to persuade other agents to change their decisions, etc).

3.4 Who Influence

The 4th W “Who” is mainly about the agent of influence (including both the subject of influence and the object of influence). For example in the social networks, the influences flow (through the links) from nodes to nodes, in which the nodes represent the agents. To study the “Who” of influence, the categories or attributes, properties of the influencing and influenced agents should be considered. Organized the categories of influencing agent¹⁷ usually discussed as following:

- Friends (or Families, Relatives and so on), which is the category of influencing agent discussed regularly, all of these influencing agents have a common point, which is the close,

¹⁵For instance, the events happening or not would influence the decisions, and the evolution of the environment would also affect the decisions, containing both subjective agents and objective environment.

¹⁶In some cases, there are rules about the sequence of the votes, and even without the rules, there will still be a sequence of the votes, for example, some agents are eager to observe other agents’ vote (if the vote is not opaque), then vote correspondingly.

¹⁷These categories nearly all apply to the influencing agent, just play different roles, as different ends, in the directed influence.

intimate relations with the influenced agent. Due to the nature of human (ties of blood and friendship, and the trust from these ties¹⁸), agent is naturally more inclined to accept, adopts, and persuaded by the preferences and recommendations from families and friends, or more inclined to be empathetic with the feelings of their families and friends, and be easier to influenced, changed by the close ones;

- Experts (with knowledge), experts are identified as agents having abundant knowledge in specific (professional) area, as we know, knowledge (or information) is regarded as the (most) valued resources in the information society, the chosen choice (and its payoff accompanied) is highly related to the knowledge (or information) obtained and possessed, the quality of decision is highly based on the degree of integrity of information, therefore, it is reasonable to assume agents would be (more possibly and easily) influenced by experts;
- Enemies (or opponents, foes and so on), another kind of influencing agents relatively rarely discussed is the enemies or opponents¹⁹, if assume agent is easier to be influenced by close friends or families (because of trust and empathy), similarly and reversely (similar on the intensity of the relations but reverse on the direction of the relations), it is also reasonable to assume agent is more inclined to be influenced by their opponents or enemies, for instance, agents would be more inclined to purposively choose the opposite choice against its opponent, especially for those decision indifferent for them²⁰, therefore, it would be reasonable to assume that agents would choose against the opponent rather than randomly choose.

Above are just one kind of criteria to classify “Who” influence (mainly from the dimension of attributes of agents). Besides, considering the two positions or roles of agents in the influence, as the subject of influence or the object of influence, the “Who” can also be classified into three kinds as below:

- Who both influence other agents and be influenced by other agents, in networks represented by the nodes with both inward links and outward links, this type of agent is the one mainly discussed in current work [Jackson, 2008, Grabisch and Rusinowska, 2010a, Grabisch and Rusinowska, 2010c, Grabisch and Rusinowska, 2010b, Grabisch and Rusinowska, 2011a, Maran et al., 2013], which usually would not specifically discriminate the agents who can only influence others and the agents who can only be influenced by others, and generally speaking, it is actual and reasonable for persons to both influence and be influenced;
- Who influences other agents but is not influenced by other agents, this type of agents could be deemed as external information providers or external information sources. In fact, the “pure” or “absolute” external influencer does not exist, and is assumed to be exist depending on specific assumptions and situations.²¹

¹⁸People natively trust more on their families and friends.

¹⁹The influences from friends and enemies are usually respectively classified as the positive influence and the negative influence.

²⁰Which means the different values and results of the decisions are relatively indifferent for the agents.

²¹For an example of voting presidents of voters in a small community, the opinions and choices of voters in the community could influence each other, interacting with each other, and also easy to be influenced by the claims and activities of the presidential candidates by varied forms of medias (TV, newspapers, internet), actually, the presidential candidates and their teams emphasis highly on the usage of varied media and its transmission. If we investigating the voting system of the community, it is easy to assume that presidential candidates influence voters substantially, but nearly not be directly influenced by the voters of a small common community, at least not in a totally symmetric ways (considering the degree of these two directions of influence), of course, the candidates would be influenced by the public opinion, or the poll of the whole elections, but only one small community might only constitute very tiny portion, however, for every voter (particularly for those keen on politics) in the community, the candidates’ activities or claims might generate a big influence.

- Who is influenced by other agents but does not influence other agents, this type of agent could be deemed as internal information receiver. In fact, the “pure” or “absolute” influenced agent does not exist either, and is assumed to exist depending on specific assumptions and situations. For example in a group (like a class), usually there are one or few isolated and ignored members (students), nobody in the group (class) like them or want to communicate with them, or to be friends with them, not to mention being influenced by their opinions or behaviors. But conversely, these isolated and ignored members (students) are eager to join and merge into the group, and trying best to imitate from other people (for example, those “unpopular” students also want to be cool guys like other students). Therefore if we discuss specifically on this system of group, to some extent, the isolated and ignored agents could be identified as “Who” only be influenced but not influence.

3.5 Why Influence

The 5th W “Why” is mainly about the reason and motivation of influence. Nearly all work about the influence [Jackson, 2008, Grabisch and Rusinowska, 2010a, Grabisch and Rusinowska, 2010c, Grabisch and Rusinowska, 2010b, Grabisch and Rusinowska, 2011a, Maran et al., 2013] deals with influence as a priori given way, for example, if the decision of one agent is changed from the initial inclination, then it is deemed as the influence, namely identifying the influence from the (external) eyeable phenomenons or results. But the influence is far more than the external phenomenons, what is really meaningful is the internal psychological process or inner mental evolution of the influence. For example why the influenced agent give up the initial inclination and change to the new preference and the corresponding new behavior, in real-world, why a people initially have faith in what he believes or what he prefers (even firmly) but then be shaken, give up and transformed to new believes or new preferences, Furthermore, why this change happens on some agents but not on some other agents, and happens on some circumstances but not on some other circumstances. However, this kind of internal psychological process or evolution, which fundamentally determines the influence, especially for the part of why “changed” are rarely discussed.²²

For studying and modeling the internal psychological evolving process, to explain “why” influence, we propose several framework, one is based on the concept of the threshold (of one value), the other is based on the idea of the different competing forces (expressed by different values):

- For the idea of **Threshold**, assume every agent has an value for change (shorted as VfC.) (of the preference and the corresponding behavior), which measures the degree of the agents prone to change the current preference or decision. While under the influence of other agents, the value for change would be dynamic rather than constant, would be increased or decreases (accumulated or lessened) according to the different situations. For example, for an agent, if its close friend possessing different preference with oneself, the agent would possibly increase the value for change, but if its close friends possessing the same preference with oneself, the agent would possibly decrease the value for change. Conversely, for other agents the agent dislike, such as the enemies or opponents, the agent would possibly increase the value for change while facing the same preference, and decrease the value for change while facing the different preference. And the change is determined by this value, when the evolving value exceeds a specific threshold, then the agent would change its preference

²²What have done is investigating the influence according to the observed outcome (such as the change of behavior as one typical example), and accordingly, modeling the influence in the form of observed change (from the initial inclination to eventual decision), but ignoring the inner psychology of agents (how is the evolution of the minds), and why influenced or not influenced (for certain agent and on certain circumstance).

and (the corresponding) choice, but if the evolving value is dynamic but always within the threshold, then its preference and corresponding choice would keep constant.

- For the idea of **Competing Forces**, assume for a decision there are more than one candidates (choices) in the domain, and each corresponding to a different force underlying the inner reasoning process of the decision-making agent. Just like the different voices in your heart trying to persuade you to make different choices, choose different candidates, these forces are usually contradicting and competing. For an agent, which force hold superiority, then the candidate corresponding to the force would be chosen, or the behavior corresponding to the force be adopted. The forces inner heart representing different candidates are usually dynamic rather than constant, declining or inclining in varied ways, under the influence from other agents. Just like the value of change discussed above, if the initial superior force is then dominated by other force, then the preference and decision of agent would also be transformed corresponding to the new dominated force.

Above all is actually discussing about “Why” the influenced agent be influenced, but the other aspect of “Why” is why the influencing agent want to influence other agent(s), which two together constitute the whole aspects of “why” influence. Usually, exerting influence need the efforts and costs, then the influencing agent according would expect some repays, and exert influence or not is mainly based on the comparison between cost and repay according to a rational assumption. For example of the bribery in social choice, the bribers pay to influence the votes of voters to order to make the voting outcome better satisfy (or match) with their purposes. But bribing who or bribing (pay) to what extent need to be considered under the limited budget, so “why influence” is important for the influencer. Besides, it should be noted that the question of “Why” influence is not meaningful for all situations, as discussed in the *denotation of influence*, some influences are not intentional and the influencing agent do not affect other agents on purpose, therefore the “Why” question (or the weighing between cost and payoff) is not meaningful for this kind of influence.

3.6 How Influence

The one 1 H “How” is mainly about the rule of influence (how the influences works, functions, operates, or takes effect) . According to different rules, the processes and the results of influence would be different even under the total identity of other conditions (such as identical on 5W) .

For example, for the same kind of “What” influence, given influenced by the directly observed decisions of other agent(s), and by different rules, the processes and results of influence would be different, if for the rule of “following your best friend”, one agent would change his choice once observed his closest friend adhering to a choice different from him; but for the rule of “following the majority”, one agent would change his choice if there is a majority of group holding a choice different from him.

Besides, for the same “Where”, given influencing in the social networks, for the rule of “direct influence”, the agents can only influence and be influenced through direct links (only with direct neighbours), which means that in the networks the information flows can not be transited through the mediation (relay) of nodes and can only transmit one step; but for the rule of “indirect influence”, the agent can also influence and be influenced through mediating and relaying of nodes step by step, the information of influence can flow more than one steps, commonly speaking, you can be influenced not only by your friends, but also by your friends of friends.

The model of influence could be constituted by the establishment of systematic rules of influence. There are several models of influence discussed by [Grabisch and Rusinowska, 2011b] as **Fol**, **Gur** and **Conf3** (which is stated in chapter 2). However, all above mainly discuss about

the positive influence (like one follows others), but not the negative influence (like one go against others), three models could be built contradictively corresponding to above three (also assumed a binary choice):

- **Opp** is an influence function between two agents, each always going against (or “following” the opposite of) the inclination of the other one (for instance thinking own choice is always better than the other one)²³. It converges to stability when the initial inclination is a discord between the two agents. Otherwise, influence iteration never stops.
- **Bor** is an influence function where one of the agents is the bore and all other agents “dislike” him, and are afraid to stand by the same side of him. It has two stable states, which both represent consensus (of all other agents) except one. Given any initial inclination, the iteration will converge to one of the stable states.
- **Reb** models a community where the influence rule works as rebel (which is full of confrontation and opposition), Contradictive to Conf3 (a Confucian model), which is about the authority and orderliness, social estate and loyalty. Due to varied opposing structures, The Reb could hardly converges to consensus and stable states. For example, in a group with three agents *A*, *B* and *C*, *A* is an authority or sage (a wise man), and *B* and *C* are two rebels (such as treacherous adolescents), making decision on a binary variable (such as 0 or 1), among them *B* want to be different from *A*, *C* also want to be different from *A*, and *B* and *C* want to be different from each other, if the “unconformable” desire of agent *B* and *C* couldn’t be satisfied, the agents would change the choice in next step, it is easy to image that this group of agents would never reach to stable state.

Above two groups of rules of “how” influence exclusively consider either positive influence or negative influence, it is meaningful to consider about the conflict brought by **more than one origins of influence**, in many cases, the agent would face more than one influencing agent, and could possibly simultaneously receive both positive influence and negative influence. If the influencing agents all take a part and work collectively, for example, the agent be influenced by both friends and opponents, guru or bore, and to different degree (weight). Given a specific example, while computing the next state of an agent (after influenced), for the weight allocation of influence, one 1/3 of portion is determined by the former state of oneself (influenced by own), the other 1/3 of portion is influenced by the state of friends (positively influenced²⁴, this portion of weight could be summed up by “+” in the computation), and another 1/3 of portion is affected by the state of opponents (negatively influenced²⁵, this portion of weight could be summed up by “-” in the computation).

Of course, above are just one dimensions to classify about “how” influence, there are also many other dimensions, such as direct influence or indirect influence, prominent one influence or collective influence and so on, worth to discuss in details.

Besides, above discussion about “how” influence all fall within the field of deterministic model (or determinate theory), nearly all work²⁶ discuss the influence in the deterministic “paradigm” [Jackson, 2008, Grabisch and Rusinowska, 2010a, Grabisch and Rusinowska, 2010c, Grabisch and Rusinowska, 2010b, Grabisch and Rusinowska, 2011a, Maran et al., 2013], which means that (after the influence) the agent either be influenced or not, there is no so-called “uncertainty” or “randomness”, for example of the “why” influence framework of the threshold, if the value for change

²³Or each agent always detests or disdains the other agent.

²⁴For instance the more friends prefer to a choice/candidate, the more the agent would prefer to the same one.

²⁵For instance the more opponents prefer to a choice/candidate, the more the agent “dislike or hate” the same one, want to be different from this one as possible.

²⁶Especially in traditional computational social choice.

exceeds the threshold, then the decision should be changed deterministically, without any probabilistic issues, also for the “why” influence framework of different forces, if the initial superior force which corresponds to the top candidate is surpassed/dominated by other forces during the influence, then the choice will be changed deterministically. But in real-world, the influence might not be solely or absolutely **deterministic**, which could also work in a **probabilistic** way, and can be presented basically in two forms:

- One is the influence itself may be represented in a probabilistic way, but the preference (over decisions) is still deterministic, for example, the domain for agent i is $\{a, b, c\}$, and the preference ordering is $a \succ c \succ b$, if without any influence, the choice of agent i is definitely (deterministically) a (which is the most preferred or top candidate), if agent i being exerted an influence leading to the choice b , under the probabilistic assumption, the influence possibly takes effect, and the agent i change the choice to b , or the influence possibly doesn't work, and agent i (still) insist to a , due to the certain probability of the influence.
- The other is the decision may be presented in a probabilistic way²⁷, for example, the influencing agent j has three values $\{a, b, c\}$ for choice, for the influenced agent i , the initial preference (presented as the probability distribution) over domain $\{a, b, c\}$ is respectively $\{0.6, 0.3, 0.1\}$, then after influenced by agent j , changed to $\{0.7, 0.2, 0.1\}$ if under the choice a of agent j , or changed to $\{0.4, 0.5, 0.1\}$ if under the choice b of agent j , or $\{0.3, 0.3, 0.4\}$ if under the choice c of agent j . Therefore, the influence changes the probability distribution of decisions, but with different influenced preferences, the choice of agent i might still be constant, not deterministically changed, due to a specific probability.²⁸ One relevant research is the multi-agent influence diagrams [Koller and Milch, 2003] based on bayesian networks.

In fact, whether the deterministic assumption or the probabilistic assumption, both depict the “how” influenced to some sense, and both be reasonable in some aspects. For the deterministic assumption, assuming that the agent is purely rational and always choose the most preferred candidate according to own preference ordering. For example of the preference ordering $a \succ b \succ c$, the choice would be certainly a , without any probabilistic or random issues. But for the probabilistic assumption, basically assuming the agent is rational but also combining with irrationality under certain circumstances (for sometimes and somewhere). For example of a probability distribution $\{0.7, 0.2, 0.1\}$ over $\{a, b, c\}$, which also reveals the preference ordering of $a(0.7) > b(0.2) > c(0.1)$, and the most preferred choice a has the highest probability to be chosen, but still not deterministically, the less preferred choice and even the least preferred choice also have the (only relatively low) possibility to be chosen, which can be explained due to varied internal or external issues. In real-world, it has been observed or experienced a lot that people did not choose what they preferred most and regret for what they have done in the past, and can't figure out why they used to choose like that, this kind of irrationality might be named as the “probabilistic rationality”.

²⁷Which means that the preference itself over decision is probabilistic.

²⁸It need to be noted that, though the decision of influenced agent works in probabilistic ways, and the decision of influencing agent also works in probabilistic ways, but the influence mechanism itself is deterministic, according to previous discussion, the decision probability distribution for influenced agent is deterministic under specific choice of influencing agent.

Chapter 4

What is Missing?

Contents of Chapter 4

4.1 Influencing and Influenced Structure

4.2 Influence from More than One Origins

4.3 Influence with Abstention and Constraint

Previous works [Jackson, 2008, Salehi-Abari and Boutilier, 2014, Grabisch and Rusinowska, 2010a, Grabisch and Rusinowska, 2010c, Grabisch and Rusinowska, 2010b, Grabisch and Rusinowska, 2011a] usually discuss about the decision-making and influence among multi-agents for one single (binary) issue or independent multi-issues. On the other side, although the combinatorially structured domains of multi-issues has been discussed a lot in works about preference representation (for example of the CP-nets [Boutilier et al., 2004a]), but usually be considered in an individual decision-making process or independent collective decision-making process. For most cases, previous works either study a multi-agent decision-making for a single issue or independent issues, focusing on the influences among multi-agents but ignoring the dependencies among multi-issues, or study a multi-issue choice by a single agent or independent agents, focusing on the dependencies among multi-issues but ignoring the influences among multi-agents. In the thesis, a multi-issue and multi-agent decision-making context is built, integrating with both the dependencies among multi-issues and the influences among multi-agents.

Actually, both the combinatorial structure among issues and influential structure among agents are common in real-world situation, the objective of a decision may be *multi-dimensional*, the decision-maker has to consider and make choices about more than one issues, for instance, while you choose a car, you need to make choices on the budget, brand/maker, color, a given model, and a number of other options. It is very clear for the existences of dependency between budget and brand, once you just have a lean budget, then many brands (like Ferrari) would be unavailable for you. Furthermore, the influence among agents on different issues would be varied, the agent may be influenced by some agents regarding a specific issue, but this may be different for another issue [Maudet et al., 2012a], for example, you may be influenced by the preferences of your families for the color of the car, but not for the model of the car, which might be influenced by the advices of some friends with abundant experiences, and so on.¹ Under this context, it becomes difficult to understand how these different influences or dependencies will affect the final outcome. Furthermore, the risk of ignoring the combinatorial structure of multi-issues and deal with it simply as independent issues is very serve, for the example of *Multiple elections Paradoxes* [Xia et al., 2007].

¹It is also very common for influential relations among agents, not to mention the voting procedure in the democratic system, the candidates and the medias struggle to influence the voters.

Although the previous works have a lot of meaningful points for reference. There are still following points not being discussed yet, which is necessary to better explain the complex influences in real-world situation:

4.1 Influencing and Influenced Structure

Firstly, previously work only discuss the influence from decision to decision, perceive the structure (typically as the influencing relations among multi-agents or the dependent relations among multi-issues) just as the path or channel of influence, (for example in a social networks environment, the influence from agent's decision to agent's decision can only flow among linked one, through the existing links, and the link among agents construct the structure of networks, namely just perceive the structure among agents as the "path" of influence), it is the decision through the structure influencing, while ignore the structure itself could also exert an influential effect. And the structures are usually assumed as fixed and static, just as an environment functioning, which can affect the decision-making and influencing process among agents, but can not be influenced and changed back during the process. But in fact, both decision and structure can play the role of influencing subject and influenced object. The structure of influence should be endowed with a more living and active role, actually, the structure itself could be perceived as an autonomous choice for agents too (besides the widely recognized choice for candidates), for example, agents can make decisions about which agents to communicate or interact with, and particular in the context of influence, agents could choose which agents to influence or to be influenced by, therefore, different choices would generate different structures of influence. In a word, the influence between decisions and structures should be bi-directional but not one-way.

Naturally, if introduced the influence originating coming from the structure itself, then how to deal with the influence from structure to decision should be discussed, as which are two disparate things, structure is the relations among more than one agents, while decision is the behavior of one individual agent, how to realize the transmission of influence between structure and decision, especially to achieve the transformation between structure and decision in mathematics, there is no work yet to specifically discuss about.

For one example of the influential effect from the structure itself, when I am influenced by two agents A and B , if I deem the two agents make own decisions independently, or I know that agent B just follows agent A but not from own thoughts, the outcome might be different. I would possibly reduce the weight of influence from agent B , which means the influencing relations (B follows A) within two influencing agents A and B would also produce an influential effect besides their own individual influence.

4.2 Influence from More than One Origins

Further, previous works usually discuss the influence from single agent to single agent, or from multi-agent to one agent for a single (binary) issue [Salehi-Abari and Boutilier, 2014], but did not fully discuss the influence from more than one origins (from both different agents and different issues)². Very few work discuss the influence in a multi-agent and multi-issue setting, for example, while making a multi-choices, I may be influenced by agent A on issue X , but be influenced by agent B on issue Y instead, with influence crossing both agents and issues. Especially when a choice is simultaneously influenced by more than one influencing agents or issues with

²A decision would be simultaneously influenced by more than one agents, and a decision would also be simultaneously influenced by more than one issues, then the origin is a more general concept, including both influencing agents and issues.

contradictive influencing directions (positive or negative), and varied influencing strengths (weak or strong), how to produce the result of collective influence would be an interesting question.

The influences from more than one origins are very common in real-world situation, for example from more than one agents, it is too naive to assume that each agent is influenced by only one another agent at a time or in a round, the truth is we could be influenced by more than one friends or families for a same issue (for instance while a family choose a car for purchase, sometimes the choice of the husband would be influenced by both his wife and kids); and we could also be simultaneously influenced by both friends and enemies (for instance the voting of United Nations Security Council, due to the existence of different allies and state interests³, the vote of one member state would be positively influenced by the preference of his allies and negatively influenced by the preference of his enemies, and some other more complex circumstances. It is easy to set the rule of influence from one individual agent to another individual agent, while it would be more complicated to design the rule of influence from more than one agents (especially from more than one origins crossing both agents and issues).

In the thesis, two rules of influence from more than one origins are constructed. As in the multi-agents and multi-issues context, each influencing and influenced entity is a decision-making variable, shortly as *decision*, decision-making variable $x_{i,k}$ means the decision of agent i on issue k . One rule is to assume that a decision would be (just) influenced by the most predominant decision (with “strongest” influencing power) from all influencing decisions, and a concept of the *priority of influence* is proposed to distinguish the predominant influencing decision. Assume there are $n \times m$ decision-making variables (n agents and m issues), there are a priority of influence from each decision variable $x_{i,k}$ to decision variable $x_{j,q}$ ($i, j \in \mathbb{N}$, $k, q \in \mathbb{M}$), a influenced decision would be influenced by the influencing decision with the highest priority compared with all other influencing decision; the other rule is to assume that the decision would be collectively influenced by all the influencing decisions (just to different extents according to their respective weights of influence), the KSB metric [Kemeny and Snell, 1962] could be used to describe the collective influence and to compute the influenced outcome by minimum distance. And a weight matrix is built to describe the weight relations of influence between each decision variable $x_{i,k}$ and decision variable $x_{j,q}$ ($i, j \in \mathbb{N}$, $k, q \in \mathbb{M}$), the preference ordering with the minimal distance compared with all influencing preference orderings would be the eventual influenced results. This rule of collective influence is similar to the empathetic model [Salehi-Abari and Boutilier, 2014] in which the agent is influenced by both other agents and oneself collectively, while from a cardinal approach but not ordinal approach, and just from more than one agents for each single issue but not from more than one origins crossing both agents and issues.⁴

³It is usually assumed that there are currently at least two “political allies” among UN Security Council members (the western camp led by US, and the other camp led by China and Russia). And in the history, before the disintegration of the Soviet Union, the socialist camp led by the Soviet Union and the capitalist camp led by the United States are two contradictive political groups (intensively expressed as the confrontation of two military alliances of NATO and WarPact. The members among the same alliance support each other, but the members between different alliances oppose each other, in fact, SC resolution decision is the game between big country alliances. It should be noticed that the consideration of members’ vote-decision is not only limited to the own (individual) revenue, also need to refer to the standpoint and revenue of their allies and opponents [Luo and Meng, 2013a].

⁴In [Salehi-Abari and Boutilier, 2014], the weighted borda score (of relevant intrinsic preference) is computed for each empathetic preference. It should be noticed that a new empathetic preference is constituted but the intrinsic preference for each agent still remains unchanged. While in the thesis, the preference ordering with minimum distance would be found to replace the original preference, the intrinsic preference for influenced agent would be renewed.

4.3 Influence with Abstention and Constraint

Besides, there is a special but common case⁵ in combinatorial and collective decision-making mostly overlooked in previous work, the *Abstention and Constraint*. For example there are three agents 1,2,3 and three issues X, Y, Z , then there are three decision-making variables for each agent and 3×3 in total, one variable represents one agent's decision on one issue (for example variable X_1 means agent 1's decision on issue X), it should be noticed that for each issue's decision, not all agents will participate, for example the aggregated choice on issue X is collectively determined by $\{Y_1, Y_3\}$ but not $\{Y_1, Y_2, Y_3\}$, as agent 2 didn't participate on issue Y . It is normal for portion of agents being absent or missing on some issues' decision (namely the Abstention), due to different knowledge distributions or different interest distributions and so on (which would be discussed in details in chapter 8). And for each agent's decision on one issue, the domain for choices (the candidates or alternatives) might not be identical and complete for each agent, for example the possible full domain for issue X is $\{a, b, c\}$, for part of agents the domain might be partial, such as $\{a, b\}$ for X_1 , or $\{b, c\}$ for X_2 , but not full for all agents. Partial domains are due to varied reasons like objective qualifications and subjective willingness and so on, a very typical reason is the constraint. Once accompanied with the special cases of abstention and constraint, the procedures of preferences aggregation and collective voting might need to be reconsidered.

In summary, firstly in theoretical modeling, the study of influence in combinatorial and collective decision-making in the thesis is advanced on following aspects:

- Discuss the multi-agent collective decision-making not within one single issue or separate multi-issues, but with combinatorial structures of dependencies among multi-issues, constituting a combinatorial and collective decision-making context;
- not just perceive the structure, the influencing relations among agents and issues, as the path or channel of influence, but consider the influential effect of the structure itself and the feedback of influence from decision to structure, endow the structure with a dynamics, interactive and bidirectional with the decision (the structure has the equal status as the decision, both can be the influencing and influenced entities);
- not only discuss the influence from individual decision to individual decision, but investigate the influence from more than one origins crossing both different agents and issues, and deal with the conflict of influences from more than one origins with varied weights and contradictive directions of influence, through the perspective of prominent one influence or the collective group influence;
- not “naively” assume the procedure of combinatorial collective decision-making with a full and identical variables and domains for every agents, deal with the abstention and constraint, and discuss how these special cases would affect the process, rule and outcome of combinatorial collective decision-making.

Secondly, use a typical example of multi-issue multi-agent decision-making, the voting of United Nations Security Council (UN SC)⁶, with detailed voting data⁷ from the official website of

⁵Special in academic discussion but common in real-world situation.

⁶One reason to choose UN Security Council voting as the example is that the data about every agent's vote are attainable and very detailed, much more importantly, the UN SC voting has thousands of bills' voting with dependencies relations among them (many similar topics happens again and again), and the UN SC voting also has diversified influences among agents, such as the positive influence among allies (especially from the leader of the ally), and the negative influence from the opponents and so on.

⁷Organize the data of SC resolutions into different subject by similar or relevant topics and contents, to build series of dependency graph of different subjects, to discuss the combinatorial structure within resolutions and the effect on collective voting

United Nations⁸, to test the built models, and by the methods and from the paradigms of both computer sciences and social sciences (quantitative matching algorithms and qualitative case studies), especially to test whether the new patterns of influence proposed exist or not in real-world situation.⁹

Lastly, after the model verified and validated, the models and patterns of influence could be used to solve many practical problems, to better understand the interaction and influence process in multi-issue and multi-agent context, with the help of mathematical modeling and computer simulation. For example of the UN Security Council, by setting the varied influencing patterns among SC members and inputting the existing voting data, many key questions of the UN Security Council, for example of the controversial UN Security Council reform, could be discussed, a series of simulation experiments could be designed and ran to compare the collective decision-making process and outcome under different reform plans (such as the “Uniting for Consensus” plan, the “G4” plan and the “Africa Union” plan, etc), and to forecast and obtain insights about the future reform. Similar with many other social procedures or systems, the research of UN Security Council could only be discussed in mathematical modeling and virtual simulation (with existing empirical data), typical methods of social sciences such as empirical survey or authentic experiment¹⁰ are usually not down to earth (realistic and feasible).

⁸<https://www.un.org/en/sc/>

⁹It is not enough to build the models of influence only in mathematic, only by testing whether the patterns of influence exist or not in the real-world situation of collective voting with empirical data, the model proposed can be more convincing and meaningful.

¹⁰it is not possible to experiment and compare different SC reform plans in the real-world, the change of the rule or procedure of SC voting is very deliberate, once one reform plan is adopted, it will be stable in a very long time, since the establishment of United Nations (about 70 years history since 1945), the SC has only been reformed once in 1966 (enlarging the non-permanent members from 6 to 10 seats, and not involving any core issues like the number of permanent members and the veto power), until now, the set of SC has been stable for about half century without any change.

Part II

Theory of Influence

Chapter 5

Influencing and Influenced Structure— the Extended Patterns of Influence

Contents of Chapter 5

- 5.1 A Framework of Combinatorial and Collective Decision-making
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- 5.7 Pattern 13-15 Intra-inter influence of Decision
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- 5.9 Pattern 19-21 Inter-intra influence of Decision
- 5.10 Pattern 22-24 Inter-intra Influence of Structure

To discuss the influence crossing both multi-agents and multi-issues/features, combining the framework of CP-nets [Boutilier et al., 2004a, Maran et al., 2013] (capturing the dependencies among issues) and the social influence model [Jackson, 2008, Maran et al., 2013] (representing the influences among agents), and referring to the collective influence (from more than one agents with allocated weights) [Salehi-Abari and Boutilier, 2014], we propose a model of influence in combinatorial and collective (multi-issue and multi-agent) decision-making context, provide with a systematical discussion of new pattern of influence (by role of structure), and handle with common but special¹ cases such as influence from more than one origins and under abstention and constraint, to better describe the complex influences in real-world situation.

Actually, CP-nets and social influence models are both committed to model the *interacting relations* (either represented as dependencies or influences) among *decision-making variables* (either as issues/features or agents), while CP-nets mainly focus on the dependency relations among multi-issues/features (but within one agent) and social influence model mainly focus on the influence relations among multi-agents (but within one issue/feature), therefore, combining CP-nets with social influence model to mutually complete (each other) would be meaningful.

¹Common because these cases are common in the real-world, special because these cases are ignored in previous works.

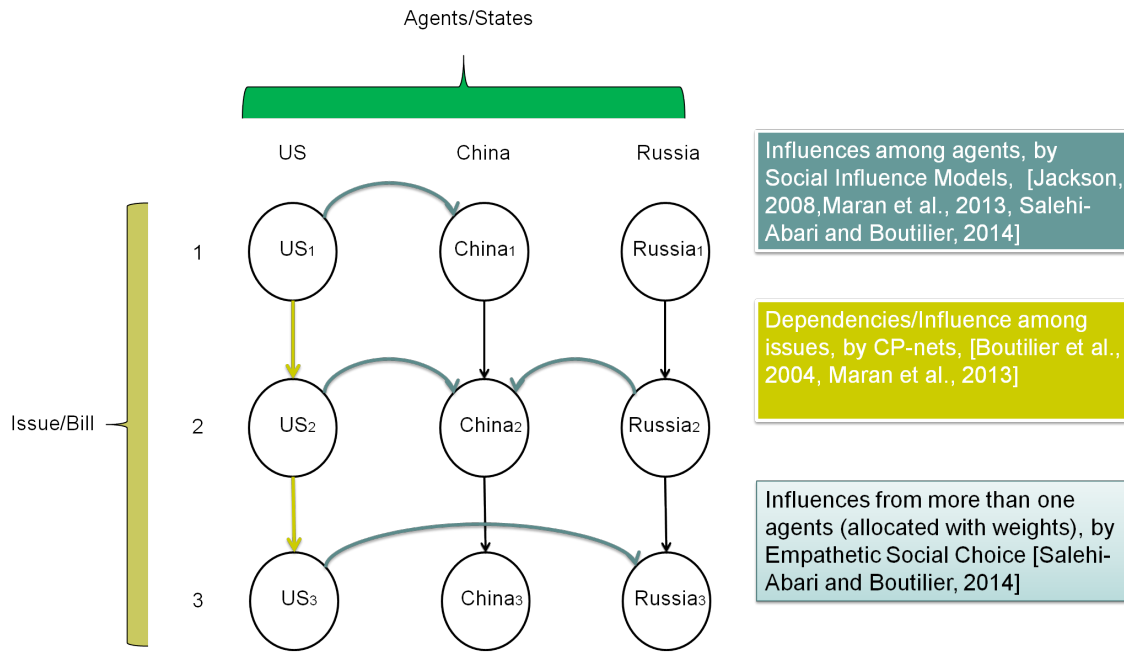


Figure 5.1: UN SC voting in Combinatorial and Collective Decision-making Framework

5.1 A Framework of Combinatorial and Collective Decision-making

A preliminary combined framework for combinatorial and collective decision-making is the *influenced CP-nets* [Maran et al., 2013], which is proposed based the traditional CP-nets [Boutilier et al., 2004a], and combining with Social Influence, not only use cp-statements (conditional preference statements) to express the dependency among issues, but also introduced the new ci-statements (conditional influence statements) to express the influence among agents, thus in the framework of influenced CP-nets, there are both the influences among multi-agents flowing at the “horizontal” dimension, and there are the influences/dependencies among multi-issues flowing at the “vertical” dimension, therefore, the influenced CP-nets has already formed a preliminary model for the multi-issues and multi-agents (combinatorial and collective) decision-making and influence.

However, to fully describe the complex influence in real-world situation, what is still missing for the influenced CP-nets are the living role of structure (influencing and be influenced), and how to deal with the influence from more than one origins (simultaneously crossing both different agents and issues), and how to reconsider the influence under the special context of abstention and constraint.²

A practical example of combinatorial and collective decision-making is given, the different aspects (dependency/influence among multi-issues, influence among multi-agents, and influence from more than one origins) are explained, with corresponding referential works and their relationships.

Example 5.1 (Combinatorial and Collective Decision-making) *Use a simple example to illustrate the related works about influence, for instance of UN security council voting, which is a*

²In the influenced CP-nets [Maran et al., 2013], the structures of dependency among issues and influences among agents are assumed to be fixed, which can not be influenced and changed by decisions or other structures during the process, and only discuss the influence from individual decision to individual decision, without the influence from more than one decisions simultaneously, and haven’t full discussed the influence crossing different agents and issues, flowing in “diagonal” direction.

typical example of multi-issues and multi-agents decision-making with influence among them.

Firstly, it is a multi-issues decision, member states make decisions on thousands of bills since the establishment of UN, and many bills with the same subject (such as Palestine Conflicts, Iraq Wars and so on) happens with high frequencies. And usually the decision of one state on latter bill would be influenced by his own decision on former related bill, which means that there are dependencies (represented as the vertical golden line) among issues.

Secondly, it is a multi-agents decision, a collective decision of members states. For instance, a member state would try to persuade his allies (representing as positively influencing) or oppose to his opponents (representing as being negatively influenced), in order to make his preferred outcome achieved. It's easy to find that the UN security council is full of varied game, interaction and influence among states, which means there are influences (expressed as the horizontal green line) among agents.

Thirdly, it should be noticed that the influence among member states are not simply from individual one to individual one, but usually simultaneously from more than different ones. For instance, the decision of China might be simultaneously positively influenced by Russia and negatively influenced by US, then how to process the influenced outcome? Especially the complex multi-influences with varied influencing directions and diversified influencing weights?

5.1.1 CP-nets with Initial Inclinations

In [Maran et al., 2013], the notion of initial inclination is introduced as a way to capture the default preference of agents regarding variables without any parents in CP-nets. This is important in their setting since this gives the default value, prior to any influence by other agents, and may affect the eventual outcome of the influence process. In this thesis we extend this notion to any variable occurring in an influenced CP-net. The reader may be wondering why this is useful: after all, any variable dependent on other variable should see this default preference overwritten. In fact there are different reasons why this will prove important in our setting:

- since dependencies and influence may be dynamic, we cannot assume that variable will indeed be affected as specified a priori;
- in a context of constraint and partial domains, it is important to cater for situations where the variable we depend upon will not receive any value;
- it is important to distinguish semantically between these different situations, the value is affected or from default preference.

Definition 2 (Initial Inclination) *The initial inclination of an agent towards a given issue is the preference of this agent regarding this issue, disregarding the value on any other issues. It correspond to the decision which would be taken on that issue alone, or more precisely if all parent variables were discarded. Of course in some situations this decision may not be meaningful (if I don't get an engine it does not really make sense to choose the color of seats).*

Example 5.2 (Initial and Influenced Preference) *For example, when a UN SC member state face two sequent bills to sanction another country (one is mild and the other is tough), and this member deems the country just deserve a mild sanction but not tough sanction, therefore, if only votes alone respectively on two bills, this member inclines to vote Y(Yes) on mild sanction bill but N(No) on tough sanction bill, which reveal the initial inclinations. However, it happens that, if the vote of this member on mild sanction is N(No), different from his initial inclination, which might be due to the influence from his allies and under pressures (such as US as the superpower and leader of NATO can exert big influences on other allies, especially his "little brothers"). On this*

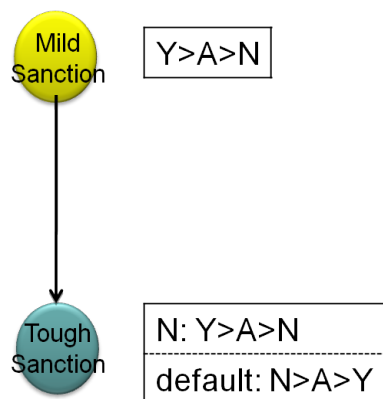


Figure 5.2: UN SC Voting with Initial Inclination

cases, this member might feel unsatisfied and unbalanced, and it is possible for him to vote *Y*(Yes) on the following tough sanction, which is an affected vote but not from a default preference.

As figure 5.2, the set of initial inclination for each issue is useful, as voting in the domain, we could get CP-nets as each agent's profile, and in the statements each case is specially stated, if influenced and affected by former choices, modify initial preference then make choices, otherwise used own initial preference. In the statement boxes, the statements above dotted line are influenced ones, and below are initial ones.

The influence discussed by previous works all come from decisions and go to decisions, but ignore the structure (expressed as the influencing relations) among multi-decisions can exert influencing, and be influenced during the process. Actually, there is a *structure dynamics* worth to discuss, both decision and structure should be equally endowed with an active and living role, and can play both the roles of influencing subject and influenced object. In fact, not only the decision (about the candidates) is an autonomous choice for agents, but also the decision about structure, such as the relations with other agents, is also an autonomous choice for agents, in many cases, you can choose who to interact and communicate with, who to influence or be influenced by, and so on.

5.2 The System of Influence Patterns by the DIS Framework

Based on the Decision-Influence-Structure (DIS) framework, the decision and the structure can both influence and be influenced by each other, we can build a system of patterns of influence with 4 categories and 24 patterns (6 patterns per each categories). As shown in table 5.1, classifying different patterns according to three dimensions:

- the first dimension is the *facet of influence*, the influence might flow among different issues but within one agent (represented as the vertical lines in the CP-nets [Boutilier et al., 2004a]), named as *intra-influence*³, or among different agents but within one issue (represented as horizontal arcs in the influenced CP-net [Maran et al., 2013]), named as *inter-influence*⁴, or crossing both different agents and different issues (which has not been fully discussed previous works, named as *inter-intra* or *intra-inter* influence;
- the second dimension is the *influencing factors*, is the influence coming from or originating from decision or structure;

³As it is the influence among multi-issues but intra/within one agent.

⁴As it is the influence on one issue but inter/among multi-agents.

- the third dimension is the *influenced factors*, is the influence going to or affecting on decision, structure, or both decision and structure. The Cartesian product for patterns of influence are $\{\text{inter-influence, intra-influence, intra-inter influence, inter-intra influence}\} \times \{\text{from decision, from decision and structure, from structure}\} \times \{\text{to decision, to decision and structure, to structure}\}$. Different categories and relevant patterns are discussed in the following.

As in the table, organizing all patterns of influence by horizontal axis (the facet of influence, where the influence flows at) and the vertical axis (the influencing factors and influenced factors). For every pattern of influence, the first rows mark the No of the influence (from Pattern 1 to 24), the second rows indicate the pattern of influence is already existed in previous works or firstly proposed in the thesis (as new patterns), the third row assign the corresponding statement to describe the influence (which are discussed in details in following), and the last rows state the innovation points of extended patterns compared with existed patterns (for example, it is crossing both different agents and different issues, or introducing the active role of the structure). Compared with the exiting patterns (1 and 7), other new patterns either make the structure could both be the subject and object of influence, or the influence could follow simultaneously at vertical and horizontal dimension.

5.2.1 New Influences and New Statements beyond CP-statement and CI-statement

In the traditional CP-nets [Boutilier et al., 2004a], the cp-statement (conditional preference statements) is used to describe the dependencies among multi-issues (within one agent). And in the influenced CP-nets [Maran et al., 2013], an extended statement, ci-statement (conditional influence statements), is proposed to express the influence among multi-agents within one issue. Both cp-statement and ci-statement discuss the influencing or independent relations among preferences/decisions, just in different dimensions (like vertical or horizontal). Actually, the ip-statement (influential preference statement), rather than the ci-statement (conditional influence statement), might be a more appropriate term to capture the influence among multi-agents, and to be compared and symmetric with the cp-statement. As for the two categories of influence and two corresponding statements discussed, the common point is both about the relations among *preferences*, but just different in the *facet* of influence (where the influence flows on, vertically among issues or horizontally among agents, and the influence among issues are usually named as dependent or conditional relations by previous works). Therefore, the ci-statement could be renamed as ip-statement, to both underline the common (relating to preferences) and difference (vertical conditional relations/horizontal influential relations) with cp-statements.

Referring to cp-statement and ip-statement [Boutilier et al., 2004a, Maran et al., 2013], a system of statements correspondingly to patterns of influence should also be built, and named by the different combinations of $c/i, p/s$, the former notation c or i represents conditional or influential (the vertical dependent relations among issues or the horizontal influential relations among agents), and the latter notation p or s represents preferences or structures (which means it is the relations between preferences/decisions, between structures, or between both preferences/decisions and structures⁵). A system of statements would include:

Statement 5.1 (cp-statements) *Conditional preference statements (proposed by [Boutilier et al., 2004a]), express the dependencies among decisions crossing different issues within one agent. The*

⁵Preferences/decisions are used to represent the individual or independent preferences/decisions, and structure are used to represent the influencing/independent relations among multi-preferences/decisions, actually, the preference and decision could be in common use in this context, as once influenced on the preference, then the decision determined by preference would be influenced either, and influenced agent could usually only observes the external decision of influencing agents, but not the inner preference of influencing ones, but the influence would firstly affect on the preference of influenced agent, then leading to the change of decision.

Table 5.1: The System of Patterns of Influence

	Intra-influence	Inter-influence	Intra-inter Influence	Inter-intra Influence
From Decision to Decision	Pattern 1 existed cps-statement [Boufflier et al., 2004a]	Pattern 7 existed ip-statement [Marran et al., 2013]	Pattern 13 extended cip-statement cross features then agents	Pattern 19 extended icp-statement cross agents then features
From Decision to Decision and Structure	Pattern 2 extended cps-statement influenced structure	Pattern 8 extended ips-statement influenced structure	Pattern 14 extended cips-statement influenced structure cross features then agents	Pattern 20 extended icps-statement influenced structure cross agents then features
From Decision to Structure	Pattern 3 extended cps-statement influenced structure	Pattern 9 extended ips-statement influenced structure	Pattern 15 extended cips-statement influenced structure cross features then agents	Pattern 21 extended icps-statement influenced structure cross agents then features
From Structure to Decision	Pattern 4 extended cps-statement influencing structure	Pattern 10 extended ips-statement influencing structure	Pattern 16 extended cips-statement influencing structure cross features then agents	Pattern 22 extended icps-statement influencing structure cross agents then features
From Structure to Decision and Structure	Pattern 5 cps-statement extended influencing and influenced structure	Pattern 11 ips-statement extended influencing and influenced structure	Pattern 17 cips-statement extended influencing and influenced structure cross features then agents	Pattern 23 icps-statement extended influencing and influenced structure cross agents then features
From Structure to Structure	Pattern 6 extended cs-statement influencing and influenced structure	Pattern 12 extended is-statement influencing and influenced structure	Pattern 18 extended cfs-statement influencing and influenced structure cross features then agents	Pattern 24 extended ics-statement influencing and influenced structure cross agents then features

cp-statement is used to describe the intra-influence from decision to decision (Pattern 1).

Statement 5.2 (cps-statements) *Conditional preference-structure statements, express the dependencies among decisions and structures crossing different issues within one agent. The cps-statement is used to describe the intra-influence of decision to decision and structure (Pattern 2), from decision to structure (Pattern 3), from structure to decision (Pattern 4) and from structure to decision and structure (Pattern 5).*

Statement 5.3 (cs-statements) *Conditional structure statements, express the dependencies among structures crossing different issues within one agent. The cs-statement is used to describe the intra-influence of structure to structure (Pattern 6).*

Statement 5.4 (ip-statements) *Influential preference statements, named as conditional influence statements (ci-statements) in [Maran et al., 2013]), express the influences among decisions crossing different agents within one issue. The ip-statement is used to describe the inter-influence of decision to decision (Pattern 7).*

Statement 5.5 (ips-statements) *Influential preference-structure statements, express the influence between decisions and structures crossing different agents within one issue. The ips-statement is used to describe the inter-influence from decision to decision and structure (Pattern 8), from decision to structure (Pattern 9), from structure to decision (Pattern 10) and from structure to decision and structure (Pattern 11).*

Statement 5.6 (is-statements) *Influential structure statements, express the influence among structures crossing different agents within one issue. The is-statement is used to describe the inter-influence from structure to structure (Pattern 12).*

Statement 5.7 (cip-statements) *Conditional-influential preference statements, express the influence among decisions crossing both different issues and agents, and the influence starting from the vertical (intra) dimension and leading to the horizontal (inter) dimension. The cip-statement is used to describe the intra-inter influence of decision to decision (Pattern 13).*

Statement 5.8 (cips-statements) *Conditional-influential preference-structure statements, express the influence among decisions and structures crossing both different issues and agents, and the influence starting from the vertical (intra) dimension and leading to the horizontal (inter) dimension. The cips-statement is used to describe the intra-inter influence from decision to decision and structure (Pattern 14), from decision to structure (Pattern 15), from structure to decision (Pattern 16) and from structure to decision and structure (Pattern 17).*

Statement 5.9 (cis-statements) *Conditional-influential structure statements, express the influence among structures crossing both different issues and agents, and the influence starting from the vertical (intra) dimension and leading to the horizontal (inter) dimension. The cis-statement is used to describe the intra-inter influence of structure to structure (Pattern 18).*

Statement 5.10 (icp-statements) *Influential-conditional preference statements, express the influence among decisions crossing both different agents and issues, and the influence starting from the horizontal (inter) dimension and leading to the vertical (intra) dimension. The icp-statement is used to describe the inter-intra influence of decision to decision (Pattern 19).*

Statement 5.11 (icps-statements) *Influential-conditional preference-structure statements, express the influence among decisions and structures crossing both different agents and issues, and the influence starting from the horizontal (inter) dimension and leading to the vertical (intra) dimension.*

The *icps*-statement is used to describe the inter-intra influence of decision to decision and structure (Pattern 20), decision to structure (Pattern 21), structure to decision (Pattern 22), structure to decision and structure (Pattern 23).

Statement 5.12 (ics-statements) *Influential-conditional structure statements, express the influence among structures crossing both different agents and issues, and the influence starting from the horizontal (inter) dimension and leading to the vertical (intra) dimension. The ics-statement is used to describe the inter-intra influence of structure to structure (Pattern 24).*

5.3 Pattern 1-3 Intra-influence of Decision

Firstly, discuss about the intra-influence from decision, which is the influence among different issues within one same agent. Build CP-nets with issues X, Y, Z, W (as shown in figure 5.3), with binary domains containing s and \bar{s} if S is the name of the issue, and in which a table $x : X \rightarrow Y$ means the specific choice of x on issue X will lead an edge of influence from issue X to issue Y , more specifically, take an edge of influence X to Y with attached restricted domain D'_X , the semantics is that if the values of the variable X belong to the domain D'_X on the edge, then the variable Y will be influenced by X , otherwise it will take the default value. Hence we only need to specify the restricted domain in the tables. Of course, mathematically speaking, the same expressivity can be achieved by relying on the classical CP-net framework, by simply stating the values of Y for the full however this would overlook the fact that the influence is dependent on specific values of the former variable.

It should be noticed that, the intra-influence is usually named as the dependencies among features/issues in former works [Boutilier et al., 2004a, Maran et al., 2013], but not as the influence, only the “interaction” among agents is deemed as the influence, but not the “interaction” among different issues/features within one agent. Actually, your choice will not only be influenced by other agents (such as friends, opponents and so on), and also will be influenced by your former choices (of your own), of course it could be perceived as your current choice would be dependent on your former choice(s), or be influenced by your former choice(s), but no matter using the term “dependency” or the term “influence”, they describe the same thing, the latter choice would be affected or determined by the former choices in some sense, therefore, the dependencies among issues/features could be named as the intra-influence (the influence among issues/features intra the agent), and could be integrated as one category of influence in the system of influence built in the thesis.

Pattern 1 Intra-influence from Decision to Decision

The choice of agent i on latter issue $k + l$ would be influenced by own choice on former issue k . In order to avoid the cycle, it is assumed that the decision must be sequential, and correspondingly the influences are from former issues to latter issues, but can not be reversed. *The detailed mathematical expression of influence pattern 1 and all other patterns (with variations) can be found in appendix B.1.*

Abstract Example 5.1 (Pattern 1) *As shown in sub figure 1 of figure 5.3, agent 1 make choices on two issues X and Y , the choices of agent 1 on issue Y is influenced by (or dependent on) the choices of agent 1 on issue X . If agent 1 choose x_1 on issue X , then will choose y_1 on latter issue Y , while if agent 1 choose \bar{x}_1 on issue X , then the choice on issue Y will be \bar{y}_1 instead. This pattern of influence is the dependency among multi-issues discussed in CP-nets*

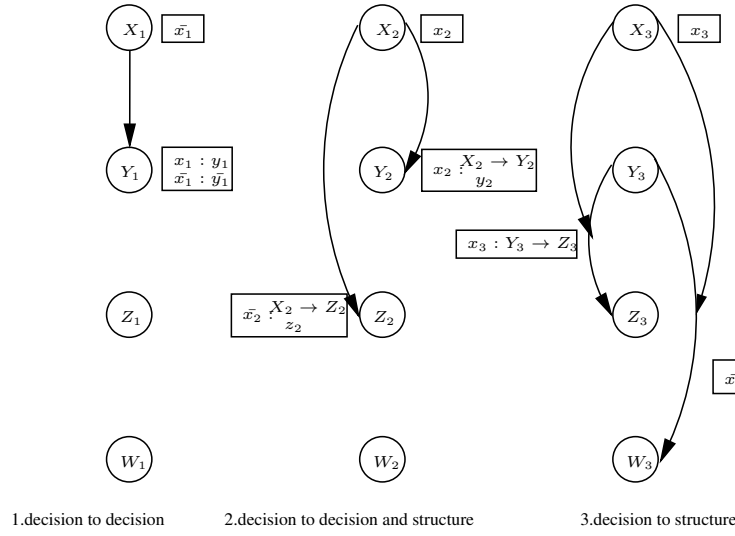


Figure 5.3: Intra-influence of Decision

as *cp*-statements [Boutilier et al., 2004a, Maran et al., 2013], which could be represented as: $\bar{x}_1 \succ x_1, x_1 : y_1 \succ \bar{y}_1, \bar{x}_1 : \bar{y}_1 \succ y_1$.

Pattern 2 Intra-influence from Decision to Decision and Structure

The different choices of agent i on former issue k will influence to own choices on different latter issues, $k + l$ or $k + l + s$. The influence is from decision to both decision and structure, not only the choice of latter issues being affected, but also the directions or the structures of influence being affected by the former choice.

Abstract Example 5.2 (Pattern 2) As shown in sub figure 2 of figure 5.3, agent 2 make choices on three issues X, Y, Z , if agent 2 chooses x_2 , then the decision on issue Y will be influenced as y_2 , while if agent 2 chooses \bar{x}_2 , then the decision on issue Z instead would be influenced as z_2 . The choice on X would affect the structure or direction of influence (from X to Y or from X to Z), then the new *cps*-statements (conditional preference-structure statements) could be used to expressed as: $x_2 \succ \bar{x}_2, x_2 : y_2 \succ \bar{y}_2, \bar{x}_2 : z_2 \succ \bar{z}_2$.

Here intuitively, if the value of X_2 is x_2 then X will affect Y (but otherwise Y will be unaffected by X and keep its initial inclination value). More generally, what this means is that an edge from X to Y is present relatively to given (partial) domain, otherwise initial inclinations are used.

Pattern 3 Intra-influence from Decision to Structure

The different choices on former issue k of an agent i will generate different dependent relations among latter issues, $k + l \rightarrow k + l + s$ or $k + l \rightarrow k + l + s + t$. The influence is from decision to just structure, the dependent relations of latter issues will be affected, but the specific choice for latter issue would not be directly affected by the former choice.

Abstract Example 5.3 (Pattern 3) As shown in sub figure 3 of figure 5.3, agent 3 makes choices on four issues X, Y, Z, W , if agent 3 chooses x on issue X , there will be a dependency on issue Y by issue Z (influence from Y to Z), but if agent 3 choose \bar{x} on issue X , then there will be another dependency on issue Y by issue W (influence from Y to W), the *cps*-statement could be expressed as: $x_3 \succ \bar{x}_3, x_3 : Z_3 = Pa(Y_3), \bar{x}_3 : W_3 = Pa(Y_3)$.

Compared with the former pattern 2 of intra-influence from decision to decision and structure, in which the decision on issue X not only influence the structure of influence (“ $x : X \rightarrow Y$ ” or “ $\bar{x} : X \rightarrow Z$ ”), but also directly influence the decision (“ $x : y$ ” or “ $\bar{x} : z$ ”), which could be expressed as a statement similar to the cp-statement: $x_2 \succ \bar{x}_2, x_2 : y_2 \succ \bar{y}_2, \bar{x}_2 : z_2 \succ \bar{z}_2$, however, when the decision only influences the structure of other issues, but not directly influences the decision of other issues, the statement should be adjusted, using Pa to express the structure of dependency/influence among issues.

5.4 Pattern 4-6 Intra-influence of Structure

Secondly, discuss about the intra-influence from structure, which is the influence from dependent relations among former issues within one same agent.⁶ Build CP-nets as shown in figure (as shown in figure 5.4)⁷. To be honest, the interpretation of this group of patterns is somewhat controversial and rather different from the usual CP-nets, and they are not the typical patterns mainly illustrated in next test and simulation. Therefore, here we keep patterns 4-6 for completeness and symmetry of the system of influence patterns, while the detailed discussions and examples can be found in appendix B.1.

Pattern 4 Intra-influence from Structure to Decision

The different dependent relations among former issues determined by agent i will influence own decision on one latter issue.

Pattern 5 Intra-influence from Structure to Decision and Structure

The different dependent relations among former issues determined by agent i will influence to own decision on different latter issues.

Pattern 6 Intra-influence from Structure to Structure

The different dependent relations among former issues determined by agent i will generate own different dependent relations among latter issues.

5.5 Pattern 7-9 Inter-influence of Decision

Thirdly, discuss about the inter-influence, which is just “contrary” to the intra-influence, the intra-influence is the “influence” among multi-issues/ features but within (intra) one same

⁶Our mind should be reformed from that only the choice (of values) could be determined by agents, actually the structure among issues could also be determined by agents, thus, there are not only decisions about the candidates, but also decisions about the structure. For example, while I make choices on three sequential issues $X > Y > Z$, while choosing on issue Z , if I think issue X is more important than issue Y , I will make choice on Z dependent on X , while if I think issue Y is more important than issue X , I will make choice on Z dependent on Y instead, which means the structures of dependency would be dynamic and could be autonomously determined by decision-maker on cases. The structure among issues are not necessarily objective (which is fixed and can not be affected by the agent’s willingness), but can also be subjective, autonomously determined by agent.

⁷In tables $X \rightarrow Y : z$ means an edge of influence from issue X to issue Y will lead the choice on issue Z to be z , and $X \rightarrow Y : Z \rightarrow W$ means an edge of influence from issue X to issue Y will lead an edge of influence from issue Z to issue W .

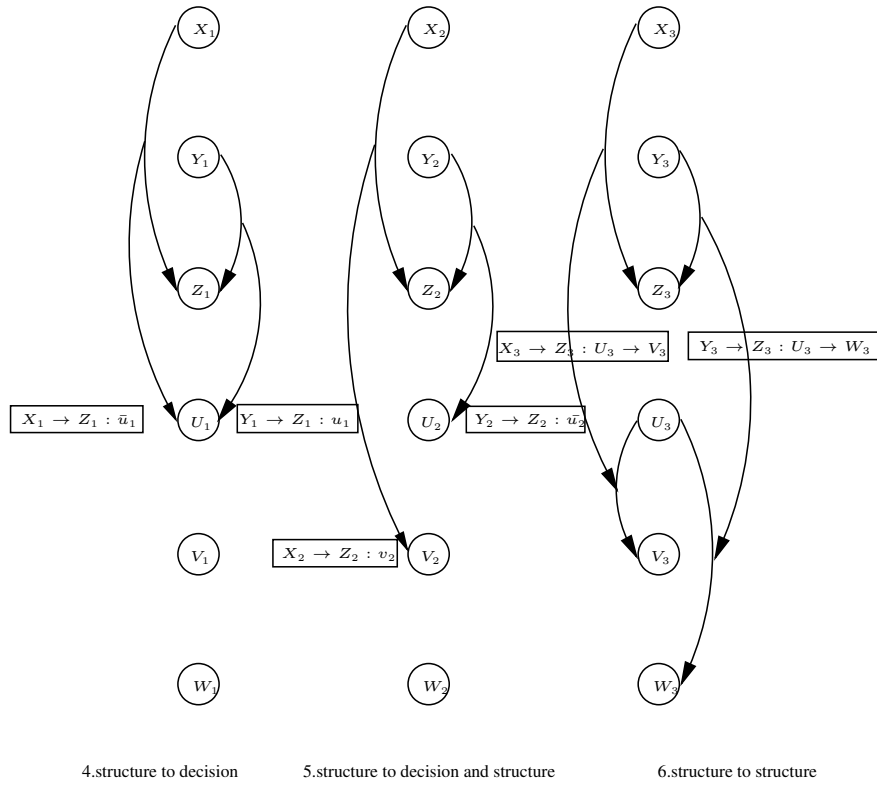


Figure 5.4: Intra-influence of Structure

agent, while the inter-influence is the influence among (inter) multi-agents but within one same issue/feature, in another word, referring to the framework of *influenced CP-nets*, the intra-influence flows in the vertical dimension but the inter-influence flows at the horizontal dimension. Build CP-nets as shown in figure 5.5 to express this group of influences, and in which a table $x_1 : X_1 \rightarrow X_2$ means the specific choice of x_1 on variable X_1 (of agent 1) will lead an edge of influence from agent 1 to agent 2, more specifically, take an edge of influence from 1 to 2 with attached restricted domain D'_{X_1} , the semantics is that if the values of the variable X_1 belong to the domain D'_{X_1} on the edge, then the variable X_2 (of agent 2) will be influenced variable by X_1 (of agent 1), otherwise it will take the default value. Hence we only need to specify the restricted domain in the tables.

Pattern 7 Inter-influence from Decision to Decision

The decision of one agent i will influence the decision of another agent j on one same issue k , which is the most simple and common form of influence [Maran et al., 2013].

Abstract Example 5.4 (Pattern 7) As shown in sub figure 7 of figure 5.5, agent 1 and agent 2 make decisions on one issue X , this is the standard pattern of influence discussed in *influenced CP-nets* [Maran et al., 2013] by *ci*-statements, if agent 1 choose x_1 , then agent 2 would choose x_2 , but if agent 1 choose \bar{x}_1 , then agent 2 would choose \bar{x}_2 instead. The *ip*-statement (influential preference statement) could be expressed as: $\bar{x}_1 \succ x_1, x_1 :: x_2 \succ \bar{x}_2, \bar{x}_1 :: \bar{x}_2 \succ x_2$. It should be noticed that, the *ci*-statement [Maran et al., 2013] is renamed as the *ip*-statement to describe this pattern of influence, to be better embedded in the system of patterns of influence, and also better

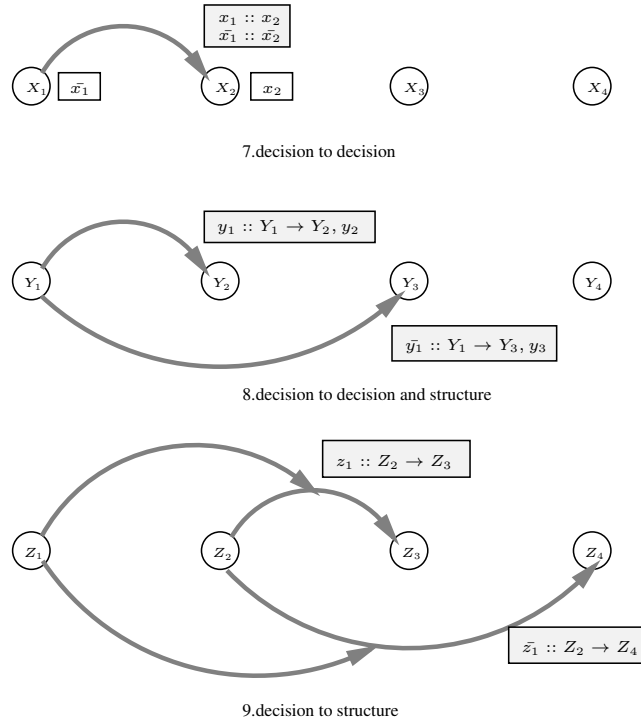


Figure 5.5: Inter-influence of Decision

grasping the common and difference compared with other statements (which has been discussed in details in section 6.2.)

Pattern 8 Inter-influence from Decision to Decision and Structure

The different choices of agent i on issue k will influence to the choices of different other agents j, g on the same issue. This pattern of influence is common in real-world situation, as different peoples have different sensitive nerves.

Abstract Example 5.5 (pattern 8) As shown in sub figure 8 of figure 5.5, agent 1,2,3 make decisions on one issue Y , the different choices of agent 1 would lead the influence to different agents (2 or 3), when the choice of agent 1 is y_1 , the influence leads to agent 2 and makes decision as y_2 , but when the choice of agent 1 is \bar{y}_1 , the influence leads to agent 3 instead and make decision as y_3 , the different choices of agent 1 not only affect the structure of influence ($y_1 :: Y_1 \rightarrow Y_2$ or $\bar{y}_1 :: Y_1 \rightarrow Y_3$), but also affect the choices of other agents ($y_1 : y_2$ or $\bar{y}_1 : y_3$). Integrally, this pattern of influence could be represented in a new statement, the *ips-statement* (influential preference and structure statement), as: $y_1 :: Y_1 \rightarrow Y_2, y_2, \bar{y}_1 :: Y_1 \rightarrow Y_3, y_3$. The statements are composed of two parts, not only including the influenced preferences (just like the traditional *cp-statements* [Boutilier et al., 2004a] and *ci-statements* [Maran et al., 2013]), which is the latter half, but also the influenced structure, which is the former half. This statement could also be simplified or reduced as $y_1 :: y_2, \bar{y}_1 :: y_3$, which is then close to the *ci-statement* [Maran et al., 2013].

Pattern 9 Inter-influence from Decision to Structure

The different choices of agent i on issue k will generate different influencing relations among other agents, $j \rightarrow g$ or $j \rightarrow h$.

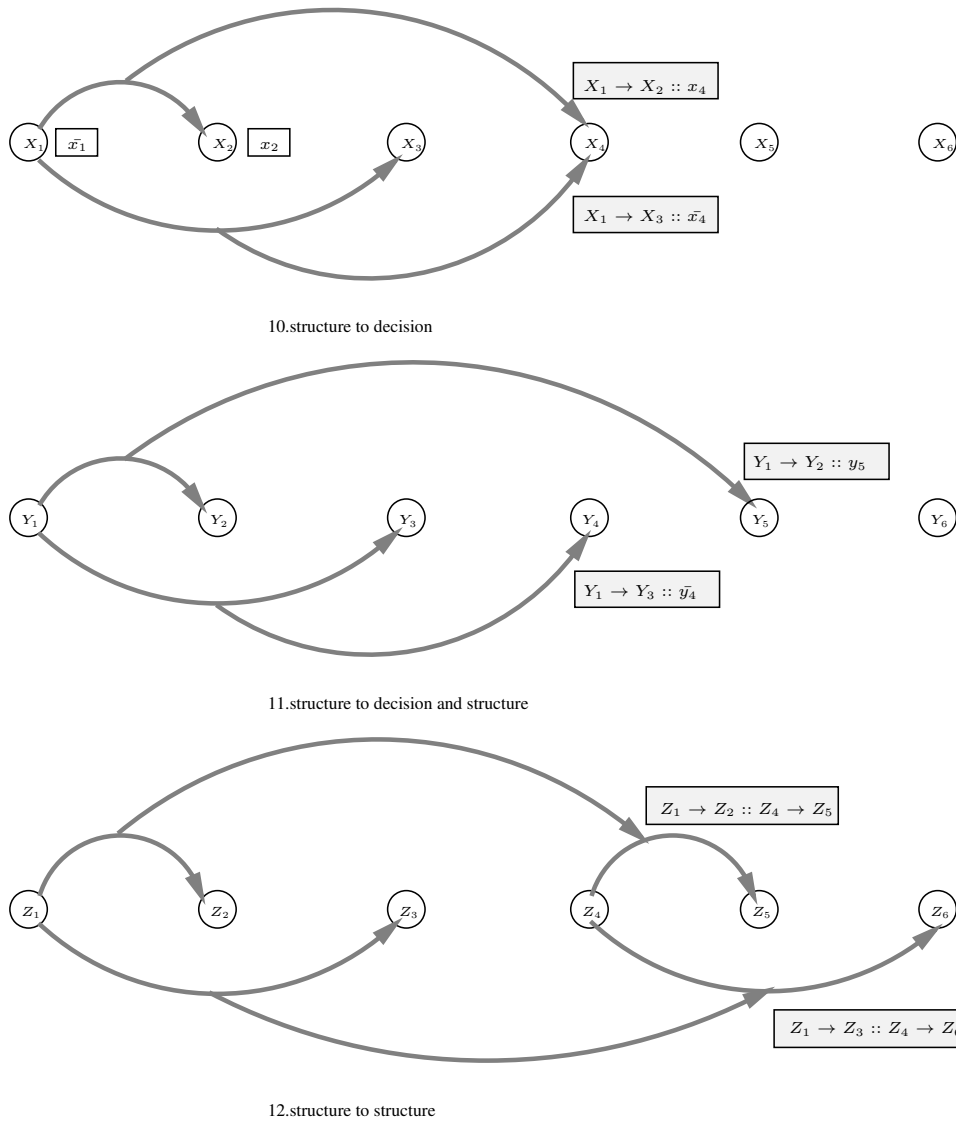


Figure 5.6: Inter-influence of Structure

Abstract Example 5.6 (Pattern 9) As shown in sub figure 9 of figure 5.5, agent 1,2,3,4 make decisions on issue Z, if agent 1 choose z_1 on issue Z, then agent 3 would be influenced by agent 2, but if agent 1 choose \bar{z}_1 on issue Z, then agent 4 would be influenced by agent 2. The ips-statements could be represented as: $z_1 :: Z_3 = In(Z_2)$, $\bar{z}_1 :: Z_4 = In(Z_2)$.

5.6 Pattern 10-12 Inter-influence of Structure

Fourthly, discuss about the inter-influence from structure among agents. Build CP-nets with five agents 1,2,3,4,5 and three issues X, Y, Z (as shown in figure 5.6).

Similarly as discussed in section 6.4 Intra-influence of Structure, it should be noted that not only the choice about candidates could be determined by the agent, but also the influencing rela-

tions (structure of influence) among agents. The influencing agent can choose who to influence, and the influenced agent can also choose who to be influenced by, and so on.

The choices of which agent to influence or which agent to be influenced by are determined by lots of factors, which are very common in real-world situation, for influencing agents, for example of the common *bribery* problem in computational social choice, while briber hope to persuade some agents to change their votes, bound to the limited budget, the briber usually obey two principles to choose the object to bribe, one is the agent with more influential power (like the opinion leader [Katz and Lazarsfeld, 1970]), once (successfully) buying the widely influential agent, there will be much more agents following, and the other principle is the ease degree to be bribed, for the agent with same influential power, the briber is inclined to choose the easy one, to save more budget and to convince other more agents.

Besides, for influenced agents, determining which one to follow or to be influenced by are also very common, for example, agents are more inclined to listen to the advise of experts with professional knowledge, or listen to family members or friends with intimate relations. Therefore, the influencing relations (structure of influence) among agents are not only objectively (which is fixed and not affected by the agent's willingness), but can also be subjectively, would be affected by agents' different preferences, willingness, believes, targets or strategies or so on.

Pattern 10 Inter-influence from Structure to Decision

The different influencing relations determined by agent i ($i \rightarrow g$ or h , g or $h \rightarrow i$ ⁸) on issue k will influence to different choices (k or \bar{k}) of another agent j on the same issue.

Abstract Example 5.7 (Pattern 10) *As shown in sub figure 10 of figure 5.6, agent 1,2,3,4 make decisions on issue X, if agent 1 (chooses to) influence agent 2, then agent 4 will be influenced and choose x_4 , but if agent 1 (chooses to) influence agent 3, then agent 4 will also be influenced but choose \bar{x}_4 instead. The influence can be expressed as ips-statement: $X_2 = In(X_1) :: x_4, X_3 = In(X_1) :: \bar{x}_4$.*

Pattern 11 Inter-influence of Structure from Decision and Structure

The different influencing relations determined by agent i ($i \rightarrow g$ or h , g or $h \rightarrow i$) on issue k will influence to different agents (j or g)' choices on the same issues.

Abstract Example 5.8 (Pattern 11) *As shown in sub figure 11 of figure 5.6, agents 1,2,3,4,5 make decisions on one issue Y, if agent 1 determine to influence agent 2, then agent 5 would be affected and choose y_5 , but if agent 1 determine to influence agent 3, instead agent 4 would be affected and choose \bar{y}_4 . The influence can be expressed as ips-statement: $Y_2 = In(Y_1) :: y_5, Y_3 = In(Y_1) :: \bar{y}_4$.⁹*

⁸Choosing which one to influence or which one to be influenced by.

⁹This kind of influence is very common in real-world situation, for example, there are 5 agents 1,2,3,4,5, agent 1 would like to influence other agents, agent 2 and agent 5 have very intimate relations, and agent 3 and agent 4 have very close friendships. If agent 2 always follows agent 1 (or always be influenced by agent 1), then agent 5, perceiving agent 2 as "best friend", would possibly feel jealous or angry, or some other negative emotions, then agent 5 would be affected by the structure of influence (agent 2 follows agent 1). However, if agent 3 always follows agent 1 (or always be influenced by agent 1), then it will be the agent 4 instead, treating agent 3 as "best friend", might feel bad emotions and be affected and take some actions correspondingly.

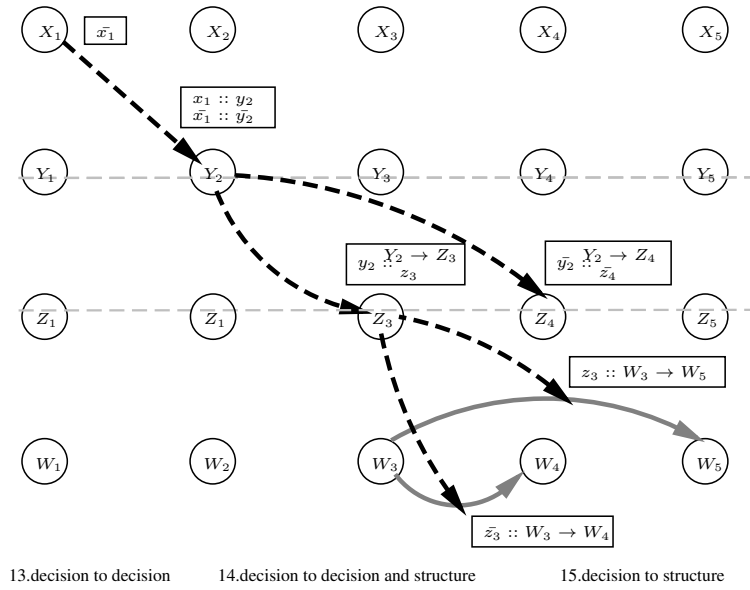


Figure 5.7: Intra-inter Influence of Decision

Pattern 12 Inter-influence from Structure to Structure

The different influencing relations determined by agent i ($i \rightarrow e$ or r, e or $r \rightarrow i$) on issue k will introduce different influencing relations determined by agents j ($j \rightarrow g$ or h, g or $h \rightarrow j$)¹⁰.

Abstract Example 5.9 (Pattern 12) As shown in sub figure 12 of figure 5.6, agents 1,2,3,4,5,6 make decisions on issue Z , if agent 1 (chooses to) influence agent 2, then agent 4 would influence agent 5, but if agent 1 (chooses to) influence agent 3, then agent 4 would influence agent 6. The new is-statements (influential structure statements, without preferences directly involved) could be used to expressed as: $Z_2 = In(Z_1) :: Z_5 = In(Z_4), Z_3 = In(Z_1) :: Z_6 = In(Z_4)$.

5.7 Pattern 13-15 Intra-inter influence of Decision

Fifthly, discuss about the intra-inter influence (from decision), namely the influence not only crossing different issues, but also crossing different agents. As discussed before, the intra-influence is among different issues but within one same agent, flowing on the vertical dimension of influenced CP-nets, and the inter-influence is among different agents but within one same issue, flowing on the horizontal dimension of influenced CP-nets. But the intra-inter influence flows in a diagonal direction, through both vertical distance and horizontal distance.

It is necessary to distinguish the intra-inter influence and inter-intra influence, this two categories of influence both flow at horizontal and vertical dimension, but with different sequence (or with different distance comparison). Generally, intra-inter influence is identified as the influence firstly starting from or flowing among different issues (in the vertical dimension), but then pointing to or affecting on different agents (in the horizontal dimension), comparatively, the inter-intra influence is identified as the influence firstly starting from or flowing among different agents (in the horizontal level), but eventually pointing to or affecting on different issues (in the vertical dimension). Build CP-nets with five agents 1,2,3,4,5 and four issues X, Y, Z, W as shown in 5.7.

¹⁰For example, if agent j observed agent i influenced agent e , then agent j would influence agent g , but if agent j observed agent i influenced agent r , then agent j would influence agent h instead.

Pattern 13 Intra-inter Influence from Decision to Decision

The decision of agent i on former issue k will influence the decision of another agent j on the latter issue $k + l$.

Abstract Example 5.10 (Pattern 13) *As shown in sub figure 13 of figure 5.7, agent 1,2 make decisions on issue X,Y, if agent 1 choose x_1 on issue X, then agent 2 would choose y_2 on issue Y, but if agent 1 choose \bar{x}_1 on issue X, then agent 2 would choose \bar{y}_2 instead on issue Y. This pattern of influence simultaneously involves the relations between different issues ($X \rightarrow Y$) and the relations between different agents ($1 \rightarrow 2$), the former one is usually represented by the cp-statement [Boutilier et al., 2004a], and the latter one is represented by the ci-statement (in the influenced CP-nets [Maran et al., 2013]). And it is not appropriate to simply borrow the existed cp-statement or ci-statement to express this pattern of influence. It is necessary to integrate the cp-statement and ci-statement (the intra aspect expressed by cp-statement and inter aspect expressed by ci-statement), and considering about the properties of intra-inter influence (starting from the vertical dimension to horizontal dimension, or more influencing distance on vertical dimension than horizontal dimension), to propose a new cip-statement (the conditional and influential preference statement), simultaneously describing the dependent and influenced preference. The cip-statement in this example can be expressed as: $x_1 :: y_2, \bar{x}_1 :: \bar{y}_2$.*

Pattern 14 Intra-inter Influence from Decision to Decision and Structure

The different choices (k or \bar{k}) of agent i on former issue k will influence to the decisions of different other agents (j or g) on latter issue $k + l$.

Abstract Example 5.11 (Pattern 14) *As shown in sub figure 14 of figure 5.7, agents 2,3,4 make decisions on two issues Y and Z, if agent 2 choose y_2 on issue Y, then agent 3 would be influenced and choose z_3 on issue Z, but if agent 2 choose \bar{y}_2 , then agent 4 would be influenced instead and choose \bar{z}_4 , as this pattern of influence involves the influencing relations between both decisions \bar{y}_2 or \bar{z}_4 and structures $Y_2 \rightarrow Z_3$ or $Y_2 \rightarrow Z_4$, therefore, the cip-statement (just between decisions and decisions) discussed before would be not capable to express, the new cips-statement (the conditional-influential preference-structure statement) should be used to express as $y_2 :: Y_2 \rightarrow Z_3, z_3, \bar{y}_2 :: Y_2 \rightarrow Z_4, \bar{z}_4$.*

Pattern 15 Intra-inter Influence from Decision to Structure

The different choices (k or \bar{k}) of agent i on former issue k will introduce different influencing relations determined by agent i ($i \rightarrow g$ or h, g or $h \rightarrow i$) on latter issue $k + l$.

Abstract Example 5.12 *As shown in sub figure 15 of figure 5.7, agents 3,4,5 make decisions on issues Z and W, if agent 3 choose z_3 on issue Z, then agent 3 would influence agent 5 on issue W, but if agent 3 choose \bar{z}_3 on issue Z, then agent 3 would influence agent 4 instead on issue W. The cips-statement could be expressed as: $z_3 :: W_5 = In(W_3), \bar{z}_3 :: W_4 = In(W_3)$.*

5.8 Pattern 16-18 Intra-inter influence of Structure

Sixthly, discuss the intra-inter influence from structure, namely the influence originating from the dependent relation(s) among issues within/intra one agent, then crossing among/inter different agents.

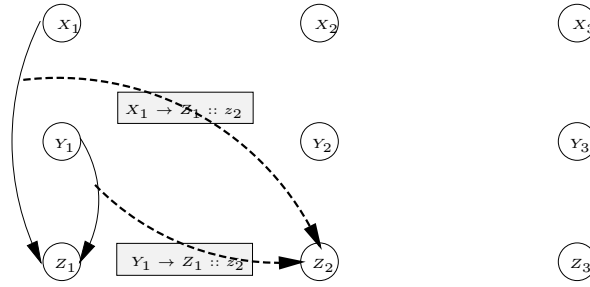


Figure 5.8: 16.Intra-inter Influence of Structure to Decision

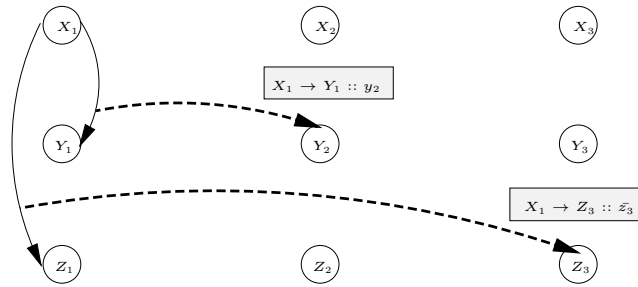


Figure 5.9: 17.Intra-inter Influence of Structure to Decision and Structure

Pattern 16 Intra-inter Influence from Structure to Decision

The different dependent relations among issues (k or $k + l \rightarrow k + l + s$, $k \rightarrow k + l$ or $k + l + s$ ¹¹) determined by agent i will influence to the different choices of another agent j on issue later than $k + l + s$.

Abstract Example 5.13 (Pattern 16) As shown in figure 5.8, build CP-nets with three agents 1,2,3 and three issues X,Y,Z with binary domains, which is one example of one agent making decisions on one latter issue dependent on different former issues. If agent 1 make decisions on issue Z dependent on issue X, then agent 2 would be influenced and choose z_2 on issue Z, but if agent 1 make decisions on issue Z dependent on issue Y, then agent 2 (the same one) would be influenced but choose \bar{z}_2 instead. The cips-statement could be expressed as: $Z_1 = Pa(X_1) :: z_2, Z_1 = Pa(Y_1) :: \bar{z}_2$.

Pattern 17 Intra-inter Influence from Structure to Decision and Structure

The different dependent relations among issues (k or $k + l \rightarrow k + l + s$, $k \rightarrow k + l$ or $k + l + s$) determined by agent i will influence to the decisions of different agents j or q on issue later than $k + l + s$.

Abstract Example 5.14 (Pattern 17) As shown in figure 5.13, build CP-nets with three agents 1,2,3 and three issues X,Y,Z with binary domains, if agent 1 makes decisions on issue Y conditional on issue X, then agent 2 would be influenced on issue Y and choose y_2 , but if agent 1 make decisions on issue Z conditional on issue X, then agent 3 instead would be influenced on issue Z and choose \bar{z}_3 , the cips-statement could be expressed as: $Y_1 = Pa(X_1) :: y_2, Z_1 = Pa(X_1) :: \bar{z}_3$.

¹¹Choosing which one to dependent on or which one to be dependent by.

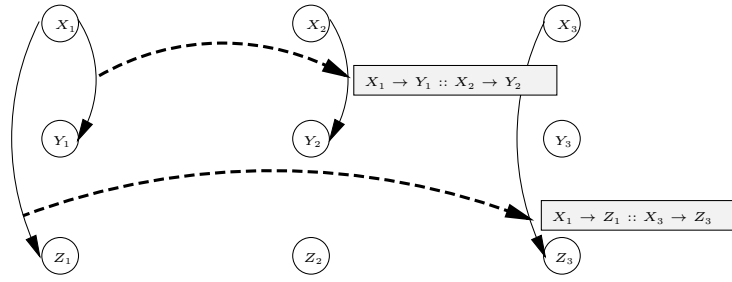


Figure 5.10: 18. Intra-inter Influence of Structure to Structure

Pattern 18 Intra-inter Influence from Structure to Structure

The different dependent relations among issues (k or $k + l \rightarrow k + l + s$, $k \rightarrow k + l$ or $k + l + s$) determined by agent i will introduce different dependent relations among (k or $k + l \rightarrow k + l + s$, $k \rightarrow k + l$ or $k + l + s$) determined by agent j (or g).

Abstract Example 5.15 As shown in figure 5.14, build CP-nets with three agents 1,2,3 and three issues X, Y, Z with binary domains, if agent 1 make decision on issue Y dependent on issue X , then agent 2 would be influenced and also make decision on issue Y conditional on issue X , but if agent 1 make decision on issue Z dependent on issue X , then agent 3 instead would be influenced and also make decision on issue Z conditional on issue X , this pattern of influence does not directly touch the specific choice of value for individual variables, therefore, the new cis-statement (conditional and influential structure statement) could be used to express as: $X_1 = Pa(Y_1) :: X_2 = Pa(Y_2)$, $X_1 = Pa(Z_1) :: X_3 = Pa(Z_3)$.

5.9 Pattern 19-21 Inter-intra influence of Decision

Seventhly, discuss about the inter-intra influence from decision, which also crossing both different agents and cross different issues, but contrast to the intra-inter influence from decision, the inter-intra influence flows on the inter (horizontal) level, and then affects on the intra (vertical) level. Build CP-nets with four agents 1,2,3,4 and five issues X, Y, Z, W, V with binary domains.

Pattern 19 Inter-intra Influence from Decision to Decision

The decision of agent i on former issue k will influence the decision of another agent j on latter issue $k + l$.

Abstract Example 5.16 (Pattern 19) As shown in sub figure 19 of figure 5.11, agents 1,2 make decisions on issues X, Y , if agent 1 choose x_1 on issue X , then agent 2 will choose y_2 on issue Y , but if agent 1 choose \bar{x}_1 on issue X , then agent 2 will choose \bar{y}_2 instead on issue Y , the new icp-statement (influential and conditional preference statement) can be expressed as: $x_1 :: y_2, \bar{x}_1 :: \bar{y}_2$.

Pattern 20 Inter-intra Influence from Decision to Decision and Structure

The different choices (k or \bar{k}) of agent j on former issue k will influence to the decisions of another agent j on different latter issues $k + l$ or $k + l + s$.

Abstract Example 5.17 (Pattern 20) As shown in sub figure 20 of figure 5.11, agents 2,3 make decisions on Y, Z, W , if agent 2 choose y_2 on issue Y , then agent 3 would be affected on issue Z and choose z_3 , but if agent 2 choose \bar{y}_2 , then agent 3 would be affected on issue W instead and

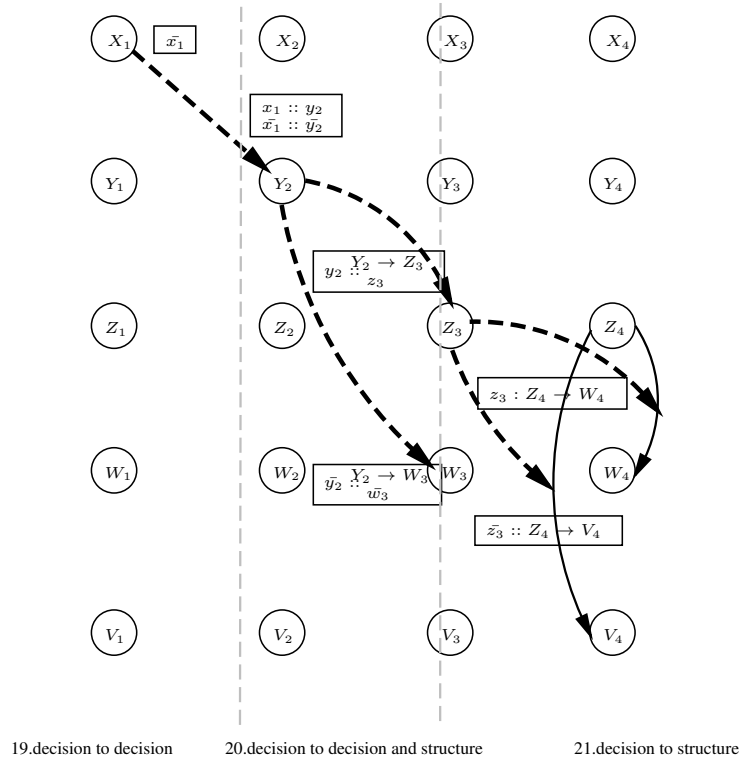


Figure 5.11: Inter-intra influence of Decision

choose \bar{w}_3 , this pattern of influence flows at the horizontal dimension and splits at (affects on) the vertical dimension, then the icps-statement (the influential and conditional preference and structure statement, involving both the influence with decision and structure) could be used to express as $y_2 :: Y_2 \rightarrow Z_3, z_3, \bar{y}_2 :: Y_2 \rightarrow W_3, \bar{w}_3$.

Pattern 21 Inter-intra Influence from Decision to Structure

The different choices (k or \bar{k}) of agent i on former issue k will introduce different dependent relations among latter issues ($k \rightarrow k + l$ or $k + l + s$) of another agent j .

Abstract Example 5.18 (Pattern 21) As shown in sub figure 21 of figure 5.11, agents 3,4 make decisions on three issues Z,W,V, if agent 3 choose z_3 on issue Z, then agent 4 would make decisions on issue W conditional on issue Z, but if agent 3 choose \bar{z}_3 on issue Z, then the agent 4 would make decisions on issue V conditional on issue Z. This pattern of influence is from decision to structure, the icps-statement (influential-conditional preference-structure statement) could be expressed as: $z_3 :: W_4 = Pa(Z_4), \bar{z}_3 :: V_4 = Pa(Z_4)$.

5.10 Pattern 22-24 Inter-intra Influence of Structure

Eighthly, discuss the inter-intra influence originating from structure, namely the influence coming from influencing relation(s) among agents on former issue, then going to decision(s) or influencing relation(s) on latter issue(s). The influence starts from the horizontal/inter dimension and flows at the vertical/intra dimension.

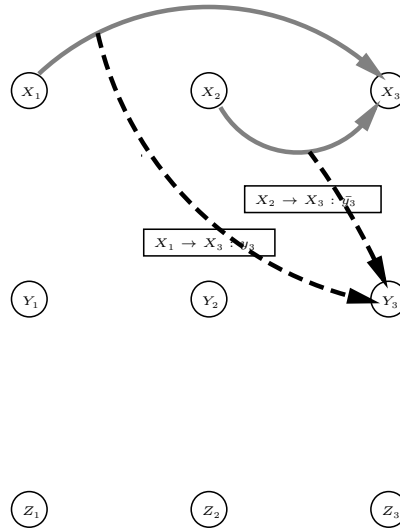


Figure 5.12: 22. Inter-intra Influence of Structure to Decision

Pattern 22 Inter-intra Influence from Structure to Decision

The different influencing relations among agents (i or $j \rightarrow g$, j or $g \rightarrow i$) determined by agent i on former issue k will influence to the decision of agent i or j or g or h on latter issue $k + l$.

Abstract Example 5.19 (Pattern 22) As shown in figure 5.12, build CP-nets with three agents 1,2,3 and three issues X,Y,Z with binary domains, this is an example of agent (choosing to) be influenced by who, if agent 3 be influenced by agent 1 on issue X, then agent 3 would choose y_3 on issue Y, but if agent 3 be influenced by agent 2 on issue X, then agent 3 would choose \bar{y}_3 instead on issue Y. The icps-statement could be expressed as: $X_3 = In(X_1) :: y_3$, $X_3 = In(X_2) :: \bar{y}_3$.

Pattern 23 Inter-intra Influence from Structure to Decision and Structure

The different influencing relations among agents (j or $g \rightarrow i$, j or $g \rightarrow i$) determined by agent i on former issue k will influence to the decisions of different agents i or j or g or h on issue latter than $k + l$.

Abstract Example 5.20 (Pattern 17) As shown in figure 5.13, build CP-nets with three agents 1,2,3 and three issues X,Y,Z with binary domains, if agent 1 (determines to) influence agent 2 on issue X, then agent 2 would choose y_2 on issue Y, but if agent 1 (determines to) influence agent 3 on issue X, then agent 3 would choose \bar{z}_3 on issue Z. The icps-statement could be expressed as: $X_2 = In(X_1) :: y_2$, $X_3 = In(X_1) :: \bar{z}_3$.

Pattern 24 Inter-intra Influence from Structure to Structure

The different influencing relations among agents (j or $g \rightarrow i$, j or $g \rightarrow i$) determined by agent i on former issue k will introduce different influencing relations among agents (j or $g \rightarrow i$, j or $g \rightarrow i$) on latter issue $k + l$ (or $k + l + s$).

Abstract Example 5.21 (Pattern 24) As shown in figure 5.14, build CP-nets with three agents 1,2,3 and three issues X,Y,Z with binary domains, if agent 2 is influenced by agent 1 on issue X, then agent 1 will be influenced (back) by agent 2 on issue Y, but if agent 3 is influenced by agent

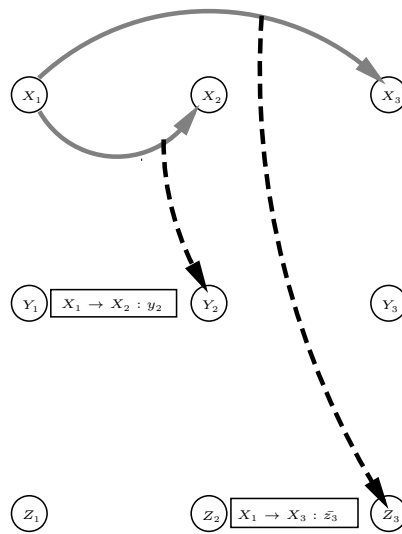


Figure 5.13: 23. Inter-intra Influence of Structure to Decision and Structure

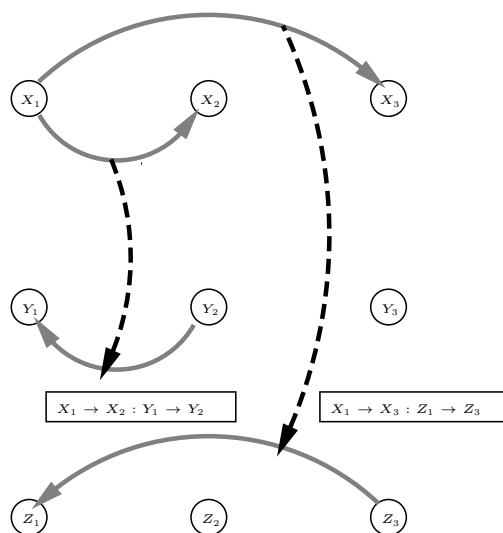


Figure 5.14: 24. Inter-intra Influence of Structure to Structure

I on issue X, then agent 1 would will be influenced (back) by agent 3 on issue Z. The new ics-statement (influential-conditional structure statement, without decisions directly involved) could be used to express as: $X_2 = In(X_1) :: Y_1 = In(Y_2)$ or $X_3 = In(X_1) :: Z_1 = In(Z_3)$.

With more specific meaning, three agents 1,2,3 collectively make decisions on issues X,Y,Z, for issue X which is very important to agent 1, and agent 1 is eager for the support from agent 2 and 3, exerting positive influences to both agent 2 and 3 (such as persuasion, bribery and so on). If agent 2 follows agent 1 (be positively influenced by agent 1), then agent 1 would possibly give a favor back to agent 2 on latter issue Y (which is important to agent 2), follows the vote of agent 2, but if agent 3 follows agent 1 (be positively influenced by agent 1), then agent 1 would likely give a favor back to agent 3 on another latter issue Z (which is important to agent 3), supports the vote of agent 3. This kind of interactions between influential relations could be summarized in a simple word, if you play nice to me, then I will play nice back, if you play hard to me, then I will play hard back.

Chapter 6

Influence from More than One Origins

Contents of Chapter 6

6.1 The Prominent One Influence-by the Priority of Influence

6.2 The Collective Influence-by the Weight of Influence

6.3 How to Compute the Influential Effect from Structure

When one decision (namely a decision-making variable of one agent or on one issue) simultaneously faces several influences, of different strength and even conflicting directions, then how to produce the result of this collective influence is a complex question. To facilitate exposure, we will first make a simplifying assumption: we shall only talk about influence among agents, and ignore the fact influence may cross different issues (note that this just to make things more readable). Now, to deal with such influence from more than one agents, there are many ways. In this chapter we discuss three specific approaches:

- in the prominent influence model, only the variable with the highest “strength” of influence will eventually be considered, even though other influences may exist (Section 6.1);
- in the simple collective influence, all influencing decisions contribute to collective effect together and each to different extents or directions, but *independently*, which can be expressed as the weights of influence (among decisions) which is also determined by a series of factors (Section 6.2);
- finally, in the combinatorial collective influence model, all influencing decisions contribute, but in a way which may be dependent on the structure (Section 6.3).

Before we get deeper in the details of these models, we note that in the influence mechanism, they are two components to start with:

- the first is the own preference of the agent regarding the considered variable,
- the second are the external influences, which, as we may indeed discuss in this chapter, may come from several origins.

Even though we may treat similarly these different factors, they play intuitively a different role. An agent may be more or less open-minded, more or less open to contradicting views, more or less firmly convinced of his own views.

Metrics of Influence 1 (Strength of Willingness) *describe how one agent insist on or be sure about own view or preference, and how is his willpower to influence on other agents. Some agents might have explicit or strict views, but others might be unclear, indifferent or ignored about own*

views. The so-called indifference or ignorance about preference is similar to the partially ordered preference [Pini et al., 2009], for some agents, part of candidates are incomparable or indifferent for them, it is not easy to tell which one is better or which one is more preferable than the other, under this situation, it is easy for them to be influenced by the agents with clear and definite views, or with strong and intense willingness to influence others. Therefore, it is reasonable to assume that agents indifferent or ignored about own preferences are easier to be influenced by the agents with explicit and strict preferences, but not reversely.

6.1 The Prominent Influence-by the Priority of Influence

Intuitively, in this first model, we can think of the agent as considering several influencing decisions, but only taking into account the most convincing one in the end. Of course, stated like this the priority of influence remains a very abstract concept. We can think of different metrics to measure, to distinguish the most prominent influencing agent, explained as following (for example of the priorities of influence among agents):

Metrics of Influence 2 (Strength of Power) *In many contexts it is natural that agents have different status that give them different strength of power to influence others, we shall assume the concept of power is exogenously given in our model. It is natural to assume that commonly the agents with weaker power are easier to be influenced by the agents with stronger power, but not reversely. There are several reasons, one is that usually the agent would face the “pressure” if his view is different from the group, majority or other agents, and the more powerful of another agent with different view, then easier for the agent to fell pressed and be influenced by. We also note that strength may vary considerably depending on the issue considered.*

Example 6.1 (Strength of Power) *For example of a general member state in the NATO, while facing the influence and pressure from different member states, such as US, UK, France and some other allies, it is more likely to be influenced by the most powerful one (US), as the leader of NATO and the most powerful state in “western world”. The other reason is that usually agents would perceive the stronger agents as the more successful ones, and assuming that it is the better choices or wiser preference leading to his success.*

Metrics of Influence 3 (Closeness of Relations) *which could not only be measured in the intensity, but also be measured in the direction (good/bad, friend/foe, etc), as positive influencing relation or negative influencing relation. It is reasonable to assume that the agent are easier to be influenced by other agent with more intense relations. For example while you face more than one friends with different preferences influencing you simultaneously, in most cases you will follow your best friends among them; similarly, while you face more than one enemies with different preferences influencing you simultaneously, in most cases you will oppose to the worst one. However, above just compare the intensity of relations among friends (in which the best friend dominate) or among enemies (in which the worst foe dominate), but how to distinguish the most prominent agent from both friends and foes together, a solution is to just compare the intensity of relations (and ignoring the directions of influences).*

Metrics of Influence 4 (Similarity of Preference) *describes the similarity degree of the preference among agents. It is usually the case that the agent would be more inclined to be influenced by other agents with more similarities compared with oneself. As it is likely to engender the friendliness (or familiarity) between people with more similarities like preferences, views, favors and so on. Besides, the agents with more similarities would provide more referential meaning for the influenced agent’s decision (as thinking we face the same/similar situation, or we are “of the same kind”, and so on).*

Example 6.2 (Similarity of Preference) *Recommender systems provide a good example of such principle: in particular those based on collaborative filtering principle capture preferential information of users. The rationale is that my decision to buy a given item will be primarily influenced by “similar users”. Preference information is a natural way of capturing what it is that an agent wants or intends to do [Wicker and Doyle, 2007, Wicker and Doyle, 2008].*

Amon these metrics, note that the closeness of relations has variations not only on the scale of the value as the others, but also on the polarity (positive/negative) of the value.

In most cases of real-world situations, a lot of factors would simultaneously come into play, therefore, assume all above metrics collectively form the priority of influence. The four metrics could be considered so as to obtain the matrix of priority of influence. Of course, for these numbers to be meaningful, some attention is required: in particular, normalization has to be performed.

Assume a society of combinatorial and collective decision-making system $CCS = \{\mathbb{N}, \mathbb{M}, \mathbb{PR}\}$, $\mathbb{N} = \{A_{(1)}, A_{(2)}, \dots, A_{(n)}\}$, $\mathbb{M} = \{F_{(1)}, F_{(2)}, \dots, F_{(m)}\}$, \mathbb{N} is the set of all agents (n agents), \mathbb{M} is the set of all issues (m issues), and \mathbb{PR} is the matrix of priority of influence among them, which can be determined collectively by several metrics discussed before. Note that this is a very cumbersome matrix, of dimension $(n \times m)^2$. If only influence among agents is considered, then we end up with a simple $n \times n$ matrix. One such example and a specific computing procedure under the context of UN security council can be found in appendix C.2.

Definition 3 (Matrix of Priority of Influence) \mathbb{PR} is the matrix of priorities of influence of the whole decision-making system, in which the value $pr_{j,i(q,k)}$ is the priority of influence from the variable $x_{j,q}$ (the decision of agent j on issue q) to the variable $x_{i,k}$ (the decision of agent i on issue k), if $pr_{j,i(q,k)} = 0$, this means there is no influence from variable $x_{j,q}$ to $x_{i,k}$, but if $pr_{j,i(q,k)} \neq 0$, this means there is an influence from variable $x_{j,q}$ to $x_{i,k}$, and the bigger of the absolute value $|pr_{j,i(q,k)}|$, the higher of the priority of influence.

As assumed, only variable on the former issue could influence variable on the latter issue, and can not be reversed, which satisfying:

$$pr_{j,i(q,k)} = 0 \text{ if } q > k \quad (6.1)$$

The influenced variable $x_{i,k}$ ($i \in \mathbb{N}$, $k \in \mathbb{M}$), would be influenced by the influencing variable $x_{j,q}$ ($j \in \mathbb{M}$, $q \in \mathbb{M}$) with the highest priority of influence ($P_{(k)}(i)$ is the preference of agent i on issue k):

$$P_{(k)}(i) = P_{(q)}(j) \text{ with } q, j \in \text{argmax} pr_{j,i(q,k)} \quad (6.2)$$

Observe that here we simply assume that the initial preferences of the agent are overriden. The Complementary Illustration of Priority of Influence can also be found in appendix B.3

6.2 The Collective Influence-by the Weight of Influence

As is the case with the priority of influence, the weights of influence among variables could be measured by a series of metrics as discussed before. A specific computing procedure under the context of UN security council can be found in appendix C.2.

However this weights do not tell us much so far, because the problem is that being influenced by several variables, we don't know yet how these different influences will be aggregated. There are different ways to approach the problem:

- in a cardinal approach, or even in an ordinal but complete setting, we could for example compute the weighted Borda score of every candidate by all influencing preferences, the

candidate with the highest weighted Borda score will be the most preferred candidate after influenced;

- in a purely ordinal approach, the fact that orders may be partial makes the previous approaches inappropriate. We could instead compute a weighted distance, find the preference ordering with the weighted minimal distances in total compared with all influencing preferences, and return it as the influenced result. It should be noticed that, the weight could be both negative or positive, for the negatively influencing preference, it is finding the maximal distance for this part of weight (want to be as far/different as possible).

In what follows we make use of the KSB distance [Kemeny and Snell, 1962, Bogart, 1973, Wicker and Doyle, 2007, Wicker and Doyle, 2008]. In fact [Bogart, 1973] generalizes the work by [Kemeny and Snell, 1962] that obtains a distance measure on strict partial orderings as the unique metric satisfying several natural axioms [Wicker and Doyle, 2007]. This metric, called as KSB metric, is defined in terms of a *matrix representation* of the orderings.

Define a concept of ordering matrix as OM , $OM = [om_{o,o'}]$, for an ordering $P_{(i)}$, the corresponding ordering matrix is $OM^{P_{(i)}}$, $OM^{P_{(i)}} = [om_{o,o'}^{P_{(i)}}]$, $P_{(i)} \in \mathbb{P}$ (\mathbb{P} is the set of all possible preference ordering according to candidates¹), and in which o, o' ($o, o' \in O_{(i)} = \{o_1, \dots, o_t\}$) are two different candidates in the preference ordering $P_{(i)}$:

$$om_{o,o'}^{P_{(i)}} = \begin{cases} 1 & \text{if } o \text{ is strictly preferred to } o' \text{ by } P_{(i)}, \\ -1 & \text{if } o' \text{ is strictly preferred to } o \text{ by } P_{(i)}, \\ 0 & \text{otherwise (indifference, incomparability, missing)}. \end{cases} \quad (6.3)$$

An ordering matrix is in canonical form if the column and row are ordered lexicographically with the candidates in the ordering, let $om_{o,o'}^{P_{(i)}}$ and $om_{o,o'}^{P_{(j)}}$ be the corresponding ordering matrix entries from the respective preference orderings $P_{(i)}$ and $P_{(j)}$ ($P_{(i)}, P_{(j)} \in \mathbb{P}$), then the KSB distance between $P_{(i)}$ and $P_{(j)}$ are (define Dis as the distance function by KSB metrics)

$$Dis(P_{(i)}, P_{(j)}) = \sum_{o=o_1}^{o_t} \sum_{o'=o_1}^{o_t} |om_{o,o'}^{P_{(i)}} - om_{o,o'}^{P_{(j)}}| \quad (6.4)$$

For an preference $P_{(q)}(j)$ (the preference of agent j on issue q) being collectively influenced, the resulted/influenced outcome would be one of possible preference $P_* \in \mathbb{P}$ which make the sum of KSB distance from all influencing preferences (with *weight* $\neq 0$) be minimal, compared with all other possible preferences.

$$P_{(q)}(j) = Argmin \left[\sum_{i=1}^n \sum_{k=1}^m wt_{i,j(k,q)} Dis(P_*, P_{(k)}(i)), P_* \right] \quad (6.5)$$

Another rule is to find one possible preference $P_* \in \mathbb{P}$ which make the maximum of KSB distance from all influencing preferences (with *weight* $\neq 0$) be minimal, compared with all other possible preferences, thus not asking the minimal of sum distance, but the minimal of maximum of distance.

$$P_{(q)}(j) = Argmin \left[\max_{i=1}^n \max_{k=1}^m wt_{i,j(k,q)} Dis(P_*, P_{(k)}(i)), P_* \right] \quad (6.6)$$

¹For example, if there are three candidates for ranking, then the possible combinations of preference orderings would be $C_3^1 \times C_2^1 \times C_1^1 = 3 \times 2 \times 1 = 6$.

In chapter 6, we define Pa , In , $Pa_{\perp}In$, $In_{\rightarrow}Pa$ respectively as the general functions of intra-influence, inter-influence, intra-inter influence and inter-intra influence, which are just general notations to indicate all influencing preferences, and which can use different specific rules such as Borda score, KSB metric to compute the influenced result.

For example finding the preference with minimal distance as influenced result by KSB metric, for intra-influence function Pa among issues, $P_{(q)}$ is the influenced preference on issue q (m is the number of all issues):

$$P_{(q)} = Pa(\sum_{k=1}^m wt_{k,q}P_{(k)}) = \begin{cases} Argmin[\sum_{k=1}^m wt_{k,q}Dis(P_*, P_{(k)}), P_*] & \text{if Minimal of Sum of Distance} \\ Argmin[\max_{k=1}^m wt_{k,q}Dis(P_*, P_{(k)}), P_*] & \text{if Minimal of Maximum of Distance} \end{cases} \quad (6.7)$$

For inter-influence function In among agents, $P_{(j)}$ is the influenced preference of agent j (n is the number of all agents):

$$P_{(j)} = In(\sum_{i=1}^n wt_{i,j}P_{(i)}) = \begin{cases} Argmin[\sum_{i=1}^n wt_{i,j}Dis(P_*, P_{(i)}), P_*] & \text{if Minimal of Sum of Distance} \\ Argmin[\max_{i=1}^n wt_{i,j}Dis(P_*, P_{(i)}), P_*] & \text{if Minimal of Maximum of Distance} \end{cases} \quad (6.8)$$

For intra-influence function $Pa_{\perp}In$ and inter-intra influence $In_{\rightarrow}Pa$ crossing both agents and issues, $P_{(q)}(j)$ is the influenced preference on issue q of agent j .

$$P_{(q)}(j) = Pa_{\perp}In(\sum_{i=1}^n \sum_{k=1}^m wt_{i,j(k,q)}P_{(k)}(i)) = In_{\rightarrow}Pa(\sum_{i=1}^n \sum_{k=1}^m wt_{i,j(k,q)}P_{(k)}(i)) = \begin{cases} Argmin[\sum_{i=1}^n \sum_{k=1}^m wt_{i,j(k,q)}Dis(P_*, P_{(k)}(i)), P_*] & \text{if Minimal of Sum of Distance} \\ Argmin[\max_{i=1}^n \max_{k=1}^m wt_{i,j(k,q)}Dis(P_*, P_{(k)}(i)), P_*] & \text{if Minimal of Maximum of Distance} \end{cases} \quad (6.9)$$

Example 6.3 (Collective Influence by Weighted Borda Score) As shown in figure 6.1, there are agents F_1 (Friend 1), F_2 (Friend 2), E (Enemy), M (Me) making decisions on one issue with domain $\{a,b,c\}$, agent M is influenced by agent F_1, F_2, E and agent M own, with preferences respectively as $c \succ b \succ a$, $a \succ c \succ b$, $b \succ c \succ a$, $a \succ b \succ c$, with weights of influence respectively as 3,2,-1,2.

If the influence is from all influencing agents' made decisions to agent M 's decision, and decisions are made according to the top preferred candidate, then the decisions would be c for agent F_1 , a for agent F_2 , b for agent E and a for agent M , based on the rule of (weighted) plurality, the score for candidate a is $2+2=4$ (2 from agent F_2 and 2 from agent M), the score for candidate b is -1 (from agent E), and the score for candidate c is 3 (from agent F_1), thus the candidate a with the highest score will be the resulted/influenced decision for agent M .

And if the influence is from all influencing agents' preferences to agent M 's preference, based on the rule of weighted borda score, assuming given 3 score for top preferred candidate, 2 score for second preferred candidate, and 1 score for last preferred candidate, then the weighted borda score for candidate a is $1 \times 3 + 3 \times 2 + 1 \times -1 + 3 \times 2 = 14$ (1×3 means the score of a given by agent F_1 is 1 as a is last preferred by agent F_1 , and the weight of influence from agent F_1 is 3, followed by the scores and weights from agent F_2, E and M), the weighted borda score for candidate b is $2 \times 3 + 1 \times 2 + 3 \times -1 + 2 \times 2 = 9$, and the weighted borda score for candidate c is $3 \times 3 + 2 \times 2 + 2 \times -1 + 1 \times 2 = 13$, thus a has highest borda score, c second and b third, then the resulted/influenced preference for agent M will be $a \succ c \succ b$.

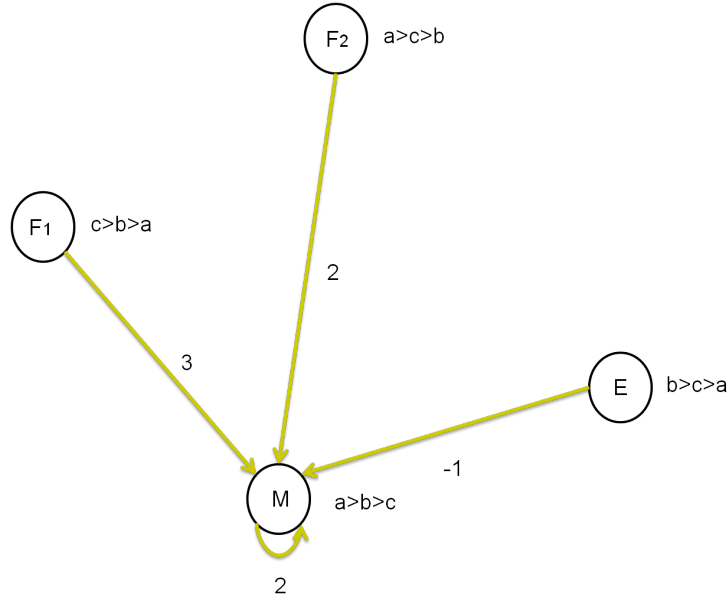


Figure 6.1: An Example if Influence from More than One Decisions/Preferences

Example 6.4 (Collective Influence by Switch Distance) As a further illustration, suppose that the distance of preference i from preference j is defined as the frequencies of switching two candidates next to each other in the preference ordering i in order to make it be identical with the preference ordering j (a related concept is the Kendall tau rank distance, which counts the number of pairwise disagreements between two preference rankings). For two preferences $c \succ b \succ a$ and $a \succ b \succ c$, the latter one need at least 3 times of switches to transform to the former one, $a \succ b \succ c \rightarrow 1st. b \succ a \succ c \rightarrow 2nd. b \succ c \succ a \rightarrow 3rd. c \succ b \succ a$.

In following matrix, the horizontal axis represents all influencing preferences with their weights in (\cdot) , the vertical axis represents all possible influenced preferences (there are $C_3^1 \times C_2^1 \times C_1^1 = 3 \times 2 \times 1 = 6$ combinations of possible preferences), according to below outcomes, the preference $c \succ a \succ b$ with the minimal weighted distances 7 (from all influencing preferences) compared with all other possible preferences, would be the resulted/influenced preference.

	$c \succ b \succ a(3)$	$a \succ c \succ b(2)$	$b \succ c \succ a(-1)$	$a \succ b \succ c(2)$	Weighted Distance
$a \succ b \succ c$	3×3	1×2	2×-1	0×2	9
$a \succ c \succ b$	2×3	0×2	3×-1	1×2	8
$b \succ a \succ c$	2×3	2×2	2×-1	3×2	14
$b \succ c \succ a$	1×3	3×2	0×-1	2×2	14
$c \succ a \succ b$	1×3	1×2	2×-1	2×2	7
$c \succ b \succ a$	0×3	2×2	2×-1	3×2	8

Example 6.5 (Collective Influence by Weighted KSB Distance) Also as shown in figure 6.1, agent M 's preference is influenced by agent F_1, F_2, E and agent M (own)'s preferences, by the rule of KSB metric, then the possible preference with the minimal weighted (sum/maximum of) distances comparing with all influencing preferences will be chosen as the influenced/resulted preference of agent M .

To compare the distance between those preferences, according to KSB metric, firstly the ordering matrix OM for each of (existent and possible) preference ordering should be given, as following ($a \succ b \succ c, a \succ c \succ b, b \succ c \succ a, c \succ b \succ a$ are preferences ordering possessed by

any of agent, and $b \succ a \succ c, c \succ a \succ b$ are preferences ordering not possessed but are possible and available theoretically):

$$OM_{a \succ b \succ c} = OM_{P_{(M)}} = \begin{matrix} & a & b & c \\ a & \left(\begin{array}{ccc} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{array} \right) \\ b & & & \\ c & & & \end{matrix}$$

$$OM_{a \succ c \succ b} = OM_{P_{(F_2)}} = \begin{matrix} & a & b & c \\ a & \left(\begin{array}{ccc} 0 & -1 & -1 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{array} \right) \\ b & & & \\ c & & & \end{matrix}$$

$$OM_{b \succ a \succ c} = \begin{matrix} & a & b & c \\ a & \left(\begin{array}{ccc} 0 & 1 & -1 \\ -1 & 0 & -1 \\ 1 & 1 & 0 \end{array} \right) \\ b & & & \\ c & & & \end{matrix}$$

$$OM_{b \succ c \succ a} = OM_{P_{(E)}} = \begin{matrix} & a & b & c \\ a & \left(\begin{array}{ccc} 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & 1 & 0 \end{array} \right) \\ b & & & \\ c & & & \end{matrix}$$

$$OM_{c \succ a \succ b} = \begin{matrix} & a & b & c \\ a & \left(\begin{array}{ccc} 0 & -1 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & 0 \end{array} \right) \\ b & & & \\ c & & & \end{matrix}$$

$$OM_{c \succ b \succ a} = OM_{P_{(F_1)}} = \begin{matrix} & a & b & c \\ a & \left(\begin{array}{ccc} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{array} \right) \\ b & & & \\ c & & & \end{matrix}$$

If by the rule of minimum of sum of distance, finding one preference P_* ($P_* \in \{a \succ b \succ c, a \succ c \succ b, b \succ a \succ c, b \succ c \succ a, c \succ a \succ b, c \succ b \succ a\}$) with the minimal sum of distance comparing with $P_{(F_1)}, P_{(F_2)}, P_{(E)}, P_{(M)}$:

$$P_M = \text{Argmin}[3\text{Dis}(P_*, P_{(F_1)}) + 2\text{Dis}(P_*, P_{(F_2)}) - \text{Dis}(P_*, P_{(E)}) + 2\text{Dis}(P_*, P_{(M)}), P_*]$$

Omitted the process of computation, the influenced/resulted preference ordering is $c \succ a \succ b$, with sum of distance $2 \times 3 + 2 \times 2 + 4 \times -1 + 4 \times 2 = 14$.

6.3 The role of structure in collective influence

So far we have assumed that the influence of each agent could be assumed independent. In what follows we sketch a more sophisticated model where the structure among agents is considered. The structure here means the influencing relations among agents or the dependent relations among issues. Further, how to deal with the influential effect from structure to decision, as structure and decision are two disparate things, one is about the behavior of individual agent, and the other is about the relations among multi-agents², how to achieve the transformation of influence

²Or one is the behavior on individual feature, and the other is about the relations among multi-features

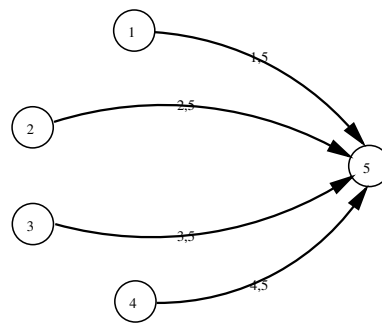


Figure 6.2: Phase 1-Influence from More than one Independent Agents-Marked Influence 1,5 2,5 3,5 4,5

from structure to decision especially in mathematics, and how to build a bridge to make up the gap between decision and structure, is a key question.

6.3.1 Three Levels of Influence: from Independent Agents, Grouped Agents to Influencing Agents

We could look back the former works about the influences from more than one agents, and the dealing with the influence from structure to decision actually identify the place of our work in the system of influence study, is to advance the influence from the first level *Separate Agents* and the second level *Grouped Agents* to the third level *Influencing Agents*, only in the third level, the influence from the structure among multi-agents are not ignored.

- Level 1, Influence from **Independent Agents or Separate Agents**, one agent is influenced by more than one agents, but these influencing agents are independent from each other, individually exerting the influence, and the influence from different agents could be directly and simply summed up, just by corresponding weights. As in the case of figure 6.2, agent 1,2,3 and 4 are influencing agents, all exert own influence separately to agent 5. This is what we assumed so far.
- Level 2, Influence from **Ganged Agents or Grouped Agents**, not only consider the influence from agents individually and independently, but also perceive (portion of) influencing agents as a united or uniform group, mainly for those agents possessing with the same believes and opinions, which under certain circumstances would form (a kind of) group effect. There are often special effects related with *majority* or *unanimity*, which go beyond the mere consideration of distinct influences separately.³ As in the case of figure 6.3, agent 1,2,3 and 4 are influencing agents, and agent 5 is the influenced agent, agent 1,2,3 have the same belief or preference, and agent 4 has different belief or preference, then not only four agents 1,2,3,4 all individually exert an influence, but also the group of agent 1,2,3 as an ensemble (a opinion alliance) would exert a group effect.
- Level 3, Influence from **Interacting Agents or Influencing Agents**, which is not a group of independent or separate agents, but there are influencing and influenced relations among

³For instance, when you submit a paper to a conference, and get three reviews back, if only one or two reviewers give a negative feedback, it might would not severely harm your confidence and feelings, but if three independent reviewers from different disciplines all judge your paper as “rubbish”, then you would likely feel despairing, which is kind of group pressure produced by more than one influencing agents possessing a same view.

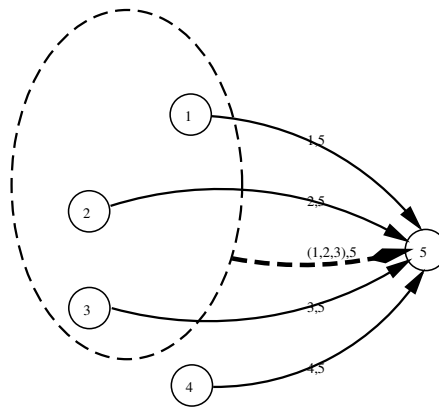


Figure 6.3: Phase 2-Influence from More than one Ganged Agents-Marked Influence: 1,5 2,5 3,5 4,5 (1,2,3),5

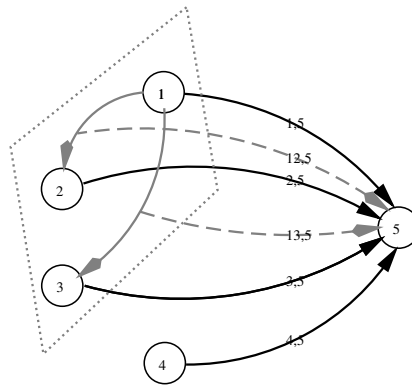


Figure 6.4: Phase 3-Influence from More than one Interacted Agents-Marked Influence: 1,5 2,5 3,5 4,5 12,5 13,5

them, and the structures of influence would also produce an influential effect on the influenced agent (which is outside the group of influencing agents). The reasons for the influential effect from structure could be discussed in details in following. As in the case of figure 6.4, agent 1,2,3 and 4 are influencing agents, among the influencing agents 1,2,3 there are two influencing relations as from agent 1 to 2 and from agent 1 to 3, then not only four agents 1,2,3,4 all individually exert an influence to agent 5, but also the two influence relations, $1 \rightarrow 2$ and $1 \rightarrow 3$ would exert structure influential effects to agent 5.

Three Levels of Influence in Mathematical Formulations

Level 1 To represent the influence from more than one agents but independent from each other, as the figure 6.2, there are four individual influences from 1 to 5 (1,5), from 2 to 5 (2,5), from 3 to 5 (3,5) and from 4 to 5 (4,5). The influence is expressed as an arrow line with mark x, y , in which x is the influencing component, and y is the influenced component. The component could be individual decision, interactive structure or both.

If represented in a cardinal form, preferences are expressed as score-based utility, the influenced preference will be the weighted sum of all influencing preferences, $wt_{x,y}$ is the weight of influence from x to y :

$$\begin{aligned} s.t. & |wt_{1,5}| + |wt_{2,5}| + |wt_{3,5}| + |wt_{4,5}| = 1 \\ P_5 & = wt_{1,5}P_1 + wt_{2,5}P_2 + wt_{3,5}P_3 + wt_{4,5}P_4 \end{aligned} \quad (6.10)$$

If represented in an ordinal form, preferences are expressed as ordering, the influenced outcome will be the preference with the minimal weighted distance compared with all influencing preferences:

$$\begin{aligned} s.t. & |wt_{1,5}| + |wt_{2,5}| + |wt_{3,5}| + |wt_{4,5}| = 1 \\ P_5 & = \text{Argmin}[wt_{1,5}Dis(P_1, P_*) + wt_{2,5}Dis(P_2, P_*) + wt_{3,5}Dis(P_3, P_*) + wt_{4,5}Dis(P_4, P_*)] \end{aligned} \quad (6.11)$$

If represented in the general form, with a general influence function In . The preference of an agent j would be influenced by the preferences all other agents with the weight of influence $\neq 0$.

$$\begin{aligned} s.t. & \sum_{n \in \mathbb{N}} |wt_{n,j(k)}| = 1 \\ P_{(k)}(j) & = In\left(\sum_{n \in \mathbb{N}} wt_{n,j(k)}P_{(k)}(n)\right) \end{aligned} \quad (6.12)$$

Level 2 To represent the influence from more than one agents including group(s) of agents possessing the same preferences. As the figure 6.3, except the four individual influence from 1 to 5 (1,5), from 2 to 5 (2,5), from 3 to 5 (3,5) and from 4 to 5 (4,5), and there is one influential effect from the group of agent 1,2,3 (possessing a same preference) to agent 5 $\{(1,2,3),5\}$. If represented in an specific form as follow, in which $wt_{(1,2,3),5}$ represents the weight of influence from agent (1,2,3) as a uniform group to agent 5, and χ represents the group influential effect function. If the influence from grouped agents is expressed as X, y , then X is the set of influencing agents (x_1, x_2, \dots, x_g) with the same views, and y is the influenced agent.

If represented in a specific cardinal form (weighted utility):

$$\begin{aligned} s.t. & |wt_{1,5}| + |wt_{2,5}| + |wt_{3,5}| + |wt_{4,5}| + |wt_{(1,2,3),5}| = 1 \\ P_5 & = wt_{1,5}P_1 + wt_{2,5}P_2 + wt_{3,5}P_3 + wt_{4,5}P_4 + wt_{(1,2,3),5}\chi[P_1, P_2, P_3] \end{aligned} \quad (6.13)$$

If represented in a specific ordinal form (weighted distance):

$$\begin{aligned} s.t. & |wt_{1,5}| + |wt_{2,5}| + |wt_{3,5}| + |wt_{4,5}| + |wt_{(1,2,3),5}| = 1 \\ P_5 & = \text{Argmin}[wt_{1,5}Dis(P_1, P_*) + wt_{2,5}Dis(P_2, P_*) + wt_{3,5}Dis(P_3, P_*) + wt_{4,5}Dis(P_4, P_*) \\ & \quad + wt_{(1,2,3),5}Dis(\chi[P_1, P_2, P_3], P_*)] \end{aligned} \quad (6.14)$$

If represented in the general form, in which $\mathbb{G}[\mathbb{N}]_{(k)}$ is the set of all groups of agents with same preference on issue k , $\mathfrak{g} \in \mathbb{G}[\mathbb{N}]_{(k)}$.

$$\begin{aligned} s.t. & \sum_{n \in \mathbb{N}} |wt_{n,j(k)}| + \sum_{\mathfrak{g} \in \mathbb{G}[\mathbb{N}]_{(k)}} |wt_{\mathfrak{g},j(k)}| = 1 \\ P_{(k)}(j) & = In\left\{\sum_{n \in \mathbb{N}} wt_{n,j(k)}P_{(k)}(n), \sum_{\mathfrak{g} \in \mathbb{G}[\mathbb{N}]_{(k)}} wt_{\mathfrak{g},j(k)}\chi[\mathfrak{g}]\right\} \end{aligned} \quad (6.15)$$

Level 3 To represent the influence from more than one agents interacting or influencing among each other, as the figure 6.4, besides the four individual influences 1,5, 2,5, 3,5 and 4,5, there are two influential effect from structures (12,5 and 13,5). If represented in a specific form as below, $wt_{12,5}$ and $wt_{13,5}$ respectively represent the weight of influence from structure (from agent 1 \rightarrow 2) and structure (from agent 1 \rightarrow 3) on agent 5, in which φ represents the structure influential effect function. If the influence from structure is expressed as xy, z , then the influencing component is the structure from x to y , and the influencing component is the single agent z .

If represented in a specific cardinal form (weighted utility):

$$\begin{aligned} s.t. & |wt_{1,5}| + |wt_{2,5}| + |wt_{3,5}| + |wt_{4,5}| + |wt_{12,5}| + |wt_{13,5}| = 1 \\ P_5 & = wt_{1,5}P_1 + wt_{2,5}P_2 + wt_{3,5}P_3 + wt_{4,5}P_4 + wt_{12,5}\varphi[P_1, P_2] + wt_{13,5}\varphi[P_1, P_2] \end{aligned} \quad (6.16)$$

If represented in a specific ordinal form (weighted distance):

$$\begin{aligned} s.t. & |wt_{1,5}| + |wt_{2,5}| + |wt_{3,5}| + |wt_{4,5}| + |wt_{12,5}| + |wt_{13,5}| = 1 \\ P_5 & = Argmin[wt_{1,5}Dis(P_1, P_*) + wt_{2,5}Dis(P_2, P_*) + wt_{3,5}Dis(P_3, P_*) + wt_{4,5}Dis(P_4, P_*) \\ & + wt_{12,5}Dis(\varphi[P_1, P_2], P_*) + wt_{13,5}Dis(\varphi[P_1, P_2], P_*)] \end{aligned} \quad (6.17)$$

If represented in the general form, in which $\mathbb{S}[\mathbb{N}]_{(k)}$ is the set of all influencing relations (structures of influence) among agents on issue k , $\mathfrak{s} \in \mathbb{S}[\mathbb{N}]_{(k)}$.

$$\begin{aligned} s.t. & \sum_{n \in \mathbb{N}} |wt_{n,j(k)}| + \sum_{\mathfrak{s} \in \mathbb{S}[\mathbb{N}]_{(k)}} |wt_{\mathfrak{s},j(k)}| = 1 \\ P_{(k)}(j) & = In\left\{ \sum_{n \in \mathbb{N}} wt_{n,j(k)}P_{(k)}(n), \sum_{\mathfrak{s} \in \mathbb{S}[\mathbb{N}]_{(k)}} wt_{\mathfrak{s},j(k)}\varphi[\mathfrak{s}] \right\} \end{aligned} \quad (6.18)$$

6.3.2 The Influential Effect from Structure among Agents (an ordinal approach)

There is a **Preassumption** that people usually know the relations between oneself and others, you have idea about who is your friend or who is your foe, who is your genuine friend and who is your pretend friend, the relations between oneself and others is more kind of a subjective cognition than an objective fact. Even your perception is incorrect, for example you think agent 1 is your friend, but in fact he is just an insincere goody, who doesn't hope you better deep down; or you see agent 2 as your enemy and think agent 1 stand with you against agent 2, but in fact agent 1 cooperate with agent 2 very well privately. Though the cognition about relations between oneself and others, or the relations among others, might be wrong from the reality, but it doesn't matter the fact that the agent's behavior is determined by his own thoughts, whether it is right or wrong to the reality. And as the cognition or perception about the relations among others is a subjective concept but not an objective concept, therefore, usually every agent has a full information about the relations among all agents, and the thoughts about relations are usually varied and even contradictive, as they are from different agents' own perspectives.

To deal with the influence from structure among agents, for the most basic case (as shown in figure 6.5), there are three agents 1,2,3, agent 3 is influenced by two agents 1 and 2 with the structure of influence from agent 1 to agent 2. One specific example of the influence mode is agent 2 just follows agent 1, would make the same choice with agent 1.

If considering the influential effect of the structure among influencing agents, there are different perspectives to understand and deal with, as human minds are natively very complicated, different people have varied personalities and senses of values, and even for one same person his

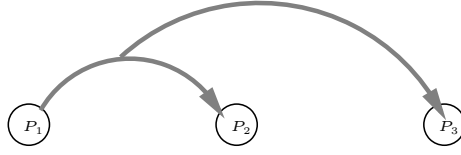


Figure 6.5: One Example of the Effect of Structure among Agents

cognition would be different under changing environments and cases. Just give some simple and common examples of perspectives to deal with the influence from structure.

For one angle of view, the influenced agent 3 would think agent 2 follows agent 1 and does not have independent idea or own mind, therefore, because of the influencing structure (agent 2 follows agent 1), the agent 3 is inclined to “ignore” or decrease the influence from agent 2 (as the follower).

$$\begin{aligned}
P_3 &= \text{Argmin}[wt_{1,3}Dis(P_1, P_*) + wt_{2,3}Dis(P_2, P_*) + wt_{12,3}Dis(\varphi[P_1, P_2], P_*), P_*] \\
&= \text{Argmin}[wt_{1,3}Dis(P_1, P_*) + wt_{2,3}Dis(P_2, P_*) - wt_{12,3}Dis(P_2, P_*), P_*] \\
&\text{if } wt_{12,3} = wt_{2,3} \text{ then} \\
&= \text{Argmin}[wt_{1,3}Dis(P_1, P_*), P_*]
\end{aligned} \tag{6.19}$$

If in the general form, influenced by all agents with the weight of influence $\neq 0$ and the influencing structure among them. $\mathbb{S}[\mathbb{N}]_{(k)}$ is the set of structures of influence among the agents \mathbb{N} on feature k . $\mathfrak{s} \in \mathbb{S}[\mathbb{N}]_{(k)}$, as a most basic structure (one agent influence one agent), \mathfrak{s} is composed by two agents $\hat{n}_{\mathfrak{s}}$ and $\check{n}_{\mathfrak{s}}$, the former is the influencing agent in \mathfrak{s} , and the latter is the influenced agent in \mathfrak{s} , $\hat{n}_{\mathfrak{s}}, \check{n}_{\mathfrak{s}} \in \mathfrak{s}$, $\{\hat{n}_{\mathfrak{s}}, \check{n}_{\mathfrak{s}}, \hat{n}_{\mathfrak{s}} \rightarrow \check{n}_{\mathfrak{s}}\} = \mathfrak{s}$.

$$\begin{aligned}
P_{(k)}(j) &= \text{Argmin}\left[\sum_{n \in \mathbb{N}} wt_{n,j(k)}Dis(P_{(k)}(n), P_*) + \sum_{\mathfrak{s} \in \mathbb{S}[\mathbb{N}]_{(k)}} wt_{\mathfrak{s},j(k)}Dis(\varphi[P_{(k)}(\hat{n}_{\mathfrak{s}}), P_{(k)}(\check{n}_{\mathfrak{s}})], P_*), P_*\right] \\
&= \text{Argmin}\left[\sum_{n \in \mathbb{N}} wt_{n,j(k)}Dis(P_{(k)}(n), P_*) - \sum_{\mathfrak{s} \in \mathbb{S}[\mathbb{N}]_{(k)}} wt_{\mathfrak{s},j(k)}Dis(P_{(k)}(\check{n}_{\mathfrak{s}}), P_*), P_*\right]
\end{aligned} \tag{6.20}$$

For another angle of view, the influenced agent 3 focus on the influencing one (agent 1) but not the influenced one (agent 2), thinks that agent 1 is very influential (like the opinion leader) and maybe it is because agent 1 indeed has the correct or better idea (or have the truth), then due to the influencing structure (agent 1 influenced agent 2), agent 3 is inclined to be more influenced by agent 1 or referring more to agent 1. Compared with not considering the structure, it is kind of “emphasising” (more weight) on the agent 1 (as the influencer).

$$\begin{aligned}
P_3 &= \text{Argmin}[wt_{1,3}Dis(P_1, P_*) + wt_{2,3}Dis(P_2, P_*) + wt_{12,3}Dis(\varphi[P_1, P_2], P_*), P_*] \\
&= \text{Argmin}[wt_{1,3}Dis(P_1, P_*) + wt_{2,3}Dis(P_2, P_*) + wt_{12,3}Dis(P_1, P_*), P_*] \\
&\text{if } wt_{12,3} = wt_{1,3} \text{ then} \\
&= \text{Argmin}[2wt_{1,3}Dis(P_1, P_*) + wt_{2,3}Dis(P_2, P_*), P_*]
\end{aligned} \tag{6.21}$$

If in the general form, from the perspective of appreciating the influence from “influencer”, the influenced preference would be:

$$\begin{aligned}
P_{(k)}(j) &= \text{Argmin}\left[\sum_{n \in \mathbb{N}} wt_{n,j(k)}Dis(P_{(k)}(n), P_*) + \sum_{\mathfrak{s} \in \mathbb{S}[\mathbb{N}]_{(k)}} wt_{\mathfrak{s},j(k)}Dis(\varphi[P_{(k)}(\hat{n}_{\mathfrak{s}}), P_{(k)}(\check{n}_{\mathfrak{s}})], P_*), P_*\right] \\
&= \text{Argmin}\left[\sum_{n \in \mathbb{N}} wt_{n,j(k)}Dis(P_{(k)}(n), P_*) + \sum_{\mathfrak{s} \in \mathbb{S}[\mathbb{N}]_{(k)}} wt_{\mathfrak{s},j(k)}Dis(P_{(k)}(\hat{n}_{\mathfrak{s}}), P_*), P_*\right]
\end{aligned} \tag{6.22}$$

The Interaction among Positive and Negative Influences

However, above just assume all influencing relations as positive, but not consider about the mixing effect among positive influence and negative influence. In real world, it would be much more complicated, the relations among agents are diversified (as strong/weak or good/bad and so on), there are both positive influence and negative influence.

For above example (as figure 6.5), the influential effect of the structure (from agent 1 to agent 2) on agent 3 is actually affected by at least three different relations, between agent 1 and agent 2, between agent 1 and agent 3, and between agent 2 and agent 3 (of course all from the perspective or cognition of the influenced agent 3, it is about how agent 3 perceive the relations between own and agent 1,2, and the relation between agent 1 and agent 2). By considering more varied combinations of positive/negative influencing relations, the influential effect of structure would be more complex. As in the most basic model, there are three relations (between agent 1 and agent 2, between agent 1 and agent 3, and between agent 2 and agent 3), with binary states $\{positive, negative\}$, then there are 8 ($2 \times 2 \times 2$) variations of combinations in total. The specific procedures of 8 variations can be found in appendix B.6

The Results Table for the Influential Effect of Basic Structure among Agents Summary the influential effects from basic structures by all combinations as below, the column is the combinations of bilateral relations among the basic structure, the second column is the result not considering the effect from structure (as the control group), the third column is the influenced result if focusing on the influencing agent in the structure, and the last column is the influenced result if focusing on the influenced agent in the structure.

$$\begin{array}{c}
\begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{pmatrix} 1 & 2 & 3 \\ + & & \\ - & + & \end{pmatrix} \\
\begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{pmatrix} 1 & 2 & 3 \\ + & & \\ + & - & \end{pmatrix} \\
\begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{pmatrix} 1 & 2 & 3 \\ - & & \\ - & + & \end{pmatrix} \\
\begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{pmatrix} 1 & 2 & 3 \\ - & & \\ + & - & \end{pmatrix} \\
\begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{pmatrix} 1 & 2 & 3 \\ + & & \\ + & + & \end{pmatrix} \\
\begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{pmatrix} 1 & 2 & 3 \\ + & & \\ - & - & \end{pmatrix} \\
\begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{pmatrix} 1 & 2 & 3 \\ - & & \\ + & + & \end{pmatrix} \\
\begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{pmatrix} 1 & 2 & 3 \\ - & & \\ - & - & \end{pmatrix}
\end{array}
\left(\begin{array}{ccc}
\textit{control} & \textit{influence} & \textit{influenced} \\
-wt_{1,3}P_1 + wt_{2,3}P_2 & -wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_1 & -wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_2 \\
wt_{1,3}P_1 - wt_{2,3}P_2 & wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_1 & wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_2 \\
-wt_{1,3}P_1 + wt_{2,3}P_2 & -wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_1 & -wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}P_2 \\
wt_{1,3}P_1 - wt_{2,3}P_2 & wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_1 & wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}P_2 \\
wt_{1,3}P_1 + wt_{2,3}P_2 & wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_1 & wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}P_2 \\
-wt_{1,3}P_1 - wt_{1,2}P_2 & -wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}P_1 & -wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_2 \\
wt_{1,3}P_1 + wt_{2,3}P_2 & wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}P_1 & wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_2 \\
-wt_{1,3}P_1 - wt_{2,3}P_2 & -wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{1,3}P_1 & -wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{1,3}P_2
\end{array} \right)$$

Furthermore, there are also the influential effect from structure among issues, which is similar with above discussion, and the detailed reasoning can be found in appendix B.5.

6.3.3 The Interplay of Group and Structure Effect (a cardinal approach)

To better understand the interplay between group and structure effects, we provide the illustration of simple cardinal approach for influence among agents.

As shown in figure 6.6, assume one agent is simultaneously influenced by 7 agents, with 4 agents saying 1 and 3 agents saying 0, and there are two influencing relations among 3 agents saying 1, assume the influencing relations specific as “following” (the influenced one follows the influencing one), and assume all weights of influence identical as 1.

If not considering the structure effect (the influential effect from influencing relations), just consider about 7 agents’ independent individual influences, it is easy to get a linear function for the influenced outcome. Assume \mathcal{P}_1 is the probability of influenced agent saying 0, n_0 is the number of influencing agents saying 0, n_1 is the number of influencing agents saying 1.

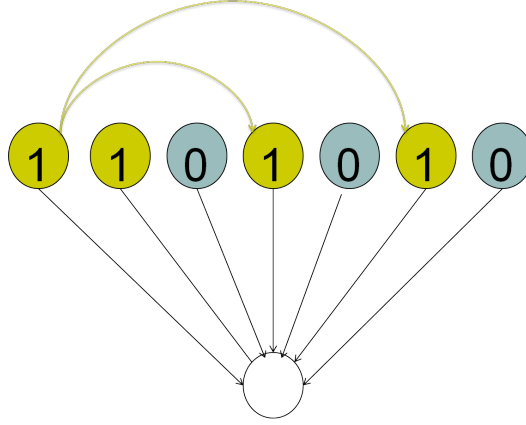


Figure 6.6: An Example of Mixed Group Effect and Structure Effect

$$\mathcal{P}_1 = \frac{n_1}{n_0 + n_1} \quad (6.23)$$

For this specific example:

$$\mathcal{P}_1 = \frac{4}{3 + 4} = 0.57 \quad (6.24)$$

Structure Effect

However, if we consider about the structure effect, specially for one rule of weakening or eliminating the weight of influence from “follower”, then firstly we should distinguish the influencing agents with original preference and the influencing agents with own preferences influenced by other influencing ones, n_1^U is the number of influencing agents saying 1 while Uninfluenced by any other, n_1^I is the number of influencing agents saying 1 while Influenced by other influencing ones, α and β are respectively the structure effect coefficient times by the number of uninfluenced agents and the number of influenced agents. As the structure effect is to weakening the weight from “followers”, then assume $\alpha + \beta = 1$ and $\alpha \geq \beta$, while $\alpha = 0.5, \beta = 0.5$, it equals with not considering the structure effect, while $\alpha = 1, \beta = 0$, it means totally eliminating the weight from “followers”.

$$\mathcal{P}_1 = \frac{\alpha n_1^U + \beta n_1^I}{\alpha n_0^U + \beta n_0^I + \alpha n_1^U + \beta n_1^I} \quad (6.25)$$

For the specific example and assume $\alpha = 0.8, \beta = 0.2$:

$$\mathcal{P}_1 = \frac{2 \times 0.8 + 2 \times 0.2}{2 \times 0.8 + 2 \times 0.2 + 3 \times 0.8} = 0.45 \quad (6.26)$$

And if assume $\alpha = 1, \beta = 0$, which means totally eliminating the influence from “followers” (as figure 6.7):

$$\mathcal{P}_1 = \frac{2 \times 1 + 2 \times 0}{2 \times 1 + 2 \times 0 + 3 \times 1} = 0.4 \quad (6.27)$$

While it is still different from all these eliminated “followers” saying 1 being replaced by agents saying different as 0 (as figure 6.8).

$$\mathcal{P}_1 = \frac{2 \times 1}{2 \times 1 + 5 \times 1} = 0.29 \quad (6.28)$$

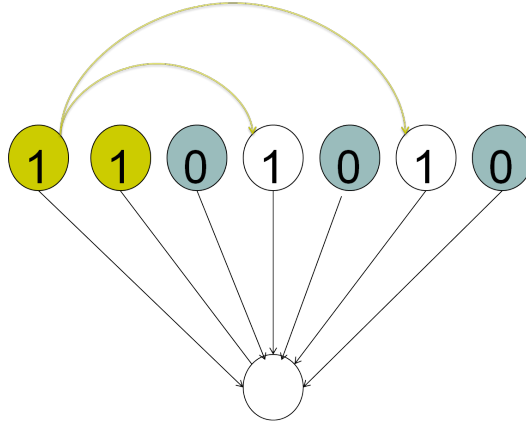


Figure 6.7: Structure Effect by Eliminating the Weights of “Followers”

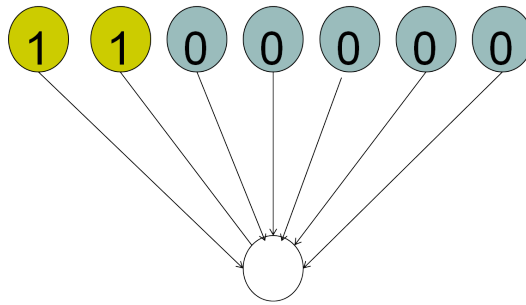


Figure 6.8: Eliminated “Followers” Replaced by Saying Different

Group Effect

If further consider the group effect, assume \mathcal{P}_1 is the probability of saying 1 before group effect, and \mathcal{P}'_1 is the probability of saying 1 after group effect.

If without any group effect (as figure 6.9):

$$\mathcal{P}'_1 = \mathcal{P}_1 \quad (6.29)$$

If the group effect works by majority rule, which means once the influencing agents saying 1 outreach 0.5, then the influenced outcome will be 1 for sure, as figure 6.10.

$$\mathcal{P}'_1 = \begin{cases} 1 & \text{if } \mathcal{P}_1 \geq 0.5 \\ 0 & \text{if } \mathcal{P}_1 < 0.5 \end{cases} \quad (6.30)$$

If the group effect works by majority increasing and minority decreasing rule, which means once the ratio/probability without group effect outreach 0.5, then the probability with group effect accelerate increasing, while if not outreach 0.5, then the probability with group effect accelerate decreasing, as $0 < \mathcal{P} < 1$, ${}^{in}\sqrt{\mathcal{P}}$ make it more close to 1 and \mathcal{P}^{de} make it more close to 0, and *in* and *de* are respectively representing the speed of increasing and decreasing, therefore there would be at least two cases:

$$\mathcal{P}'_1 = \begin{cases} {}^{in}\sqrt{\mathcal{P}_1} & \text{if } \mathcal{P}_1 > 0.5 \\ \mathcal{P}_1 & \text{if } \mathcal{P}_1 = 0.5 \\ 1 - {}^{in}\sqrt{\mathcal{P}_1} & \text{if } \mathcal{P}_1 < 0.5 \end{cases} \quad (6.31)$$

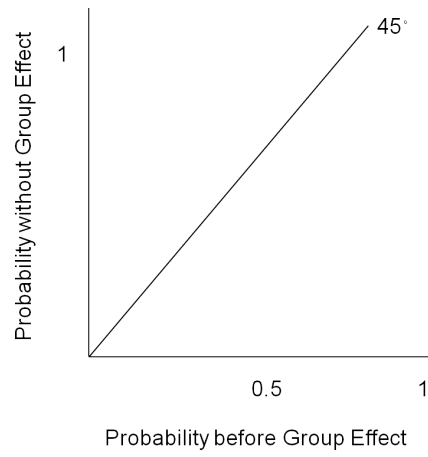


Figure 6.9: Probability Without Group Effect

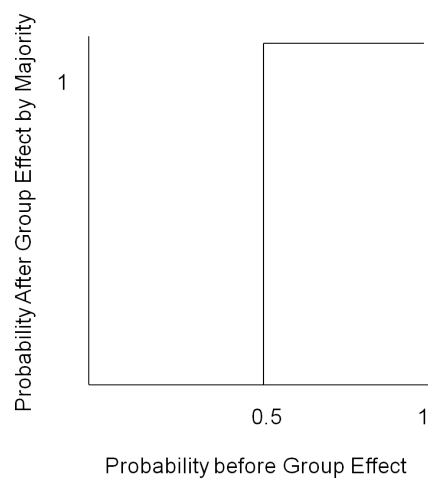


Figure 6.10: Probability after Group Effect by Majority Rule

$$\mathcal{P}'_1 = \begin{cases} 1 - \mathcal{P}_1^{de} & \text{if } \mathcal{P}_1 > 0.5 \\ \mathcal{P}_1 & \text{if } \mathcal{P}_1 = 0.5 \\ \mathcal{P}_1^{de} & \text{if } \mathcal{P}_1 < 0.5 \end{cases} \quad (6.32)$$

Chapter 7

Influence with Abstention and Constraints

Contents of Chapter 7

7.1 Abstention

7.2 Constraints and Partial Domains

7.3 Constrained CP-nets

7.4 Collective Decision-making with Constrained Profiles

7.5 Collective Decision-making with Abstention

7.6 Domains and Influence: perspectives

Decision-making is about choosing a value from the domain of a variable. The most simple decision-making is the individual single-issue decision-making, for example of one agent i making decision about one issue k (if assumed to be binary) with the domain of $\{k_a, k_b\}$. The most simple form of decision-making can be extended in two directions, one is from single-issue to multi-issues (even with combinatorial structures), the other is from the individual decision to the collective decision of more than one agents (even with influencing relations).

7.1 Abstention

Contrary to most works on multi-issue collective decision-making (especially based on the CP-nets), where the variables of the decision-making system are assumed to be completely specified (that is, if there is a system with n agents and m issues (or features), then there would be $n \times m$ variables in total for decision-making), we will now consider that some variables may not be specified by agents. This may be due to the fact that the agent himself is not aware of this variable, or that he may strategically avoid to vote (we discuss this latter). In any case, we shall observe an *abstention* from the agent on the given variable.

In works involving multi-agent CP-nets [Rossi et al., 2004b], it is usually assumed that all agents share the same set of variables. In fact, it is sometimes even assumed that agents share some common structure among the variables, such as the *o-legality* notion defined in [Lang and Xia, 2009]. And this assumption of complete variables, or uniform decision-making issues for all agents and uniform decision-making agents for all issues, are accepted by nearly all works [Chevalerey et al., 2007, Maudet et al., 2012b, Maran et al., 2013] naturally and undoubtedly, but it might be questioned that whether it is always the complete variables, however, in real-world situation, most of decision-making circumstance might not be that “perfect” and “symmetrical”, there might be variables “missing”.

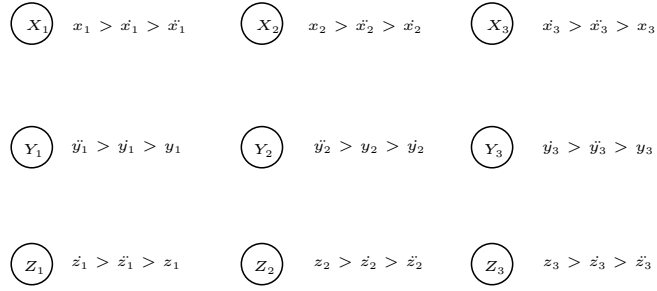


Figure 7.1: One Example without Abstention and Constraint (Partial Domains)

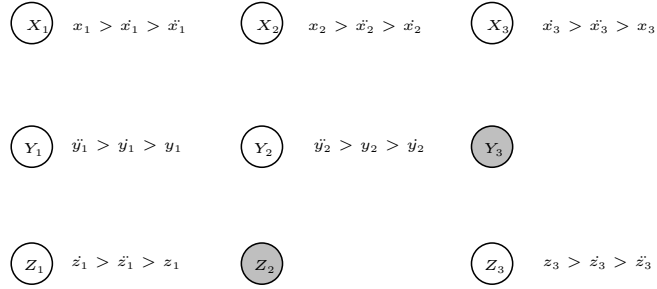


Figure 7.2: One Example with Abstention

Abstract Example 7.1 (Abstention) Contrast with a example without the abstention (as figure 7.1), assume a decision-making circumstance not that “uniform” (as shown in figure 7.2), lacking two variables Y_3 and Z_2 (marked as gray), from the perspective of the agent, the agent 1 participate in full issues $\{X, Y, Z\}$, but the agent 2 just participate in partial issues $\{X, Y\}$, and for the agent 3 just participate in partial issues of $\{X, Z\}$; and from the perspective of the issue, the issue X is determined by the entire agents $\{1,2,3\}$, the issue Y is determined by the partial agents $\{1,2\}$, and the issue Z is determined by the partial agents $\{1,3\}$.

As mentioned, the incomplete (or fragmentary) variables set are due to varied reasons, which is mainly discussed from the perspective of agent, as agent is the independent decision-making unit, but not the feature (or issue)¹, several reasons are proposed, including but not limited to:

- For different agents, there are **Different Knowledge Distributions** over issues. For example, some agents might be the expert for a specific issue (know abundant knowledge in this field), also there might be some other agents knowing nothing or very few about this issue (I would call it “Unawareness”), and it is reasonable to assumed that generally the “expert” is more inclined to participate the decision of the issue than the “ignorant”, and it is likely for the people knowing few to just quit this decision, thinking that “I know nothing about it, it would be more beneficial for me to let the experts make decision”.

The abstention due to knowledge distribution are related to the concept of **Incomparability**, as the agent know nothing or few about certain issue, and does not possess enough information to form a preference ordering over alternatives, or to perform a comparison among alternatives in the issue. In this sense, the alternatives in the domain are incomparable, for a issue X with domain $\{x, \dot{x}, \ddot{x}\}$, the “preference” (which is actually nonexistent to some sense) could be expressed as $x? \dot{x}? \ddot{x}$, based on the figure 7.2 of one example with abstention, if on the case of incomparability, the example could be extended as figure 7.3.

¹The agent is the (active) subject of decision-making, and the issue is just the (passive) object of decision-making.

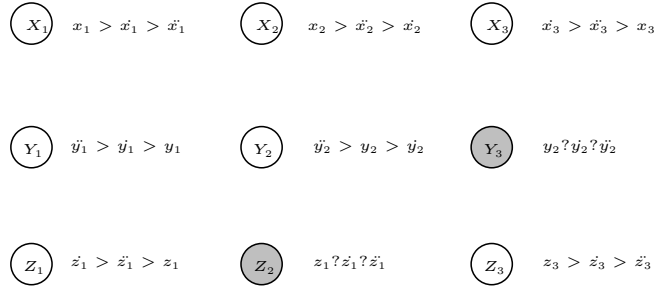


Figure 7.3: Abstention due to Knowledge Distribution (Incomparability)

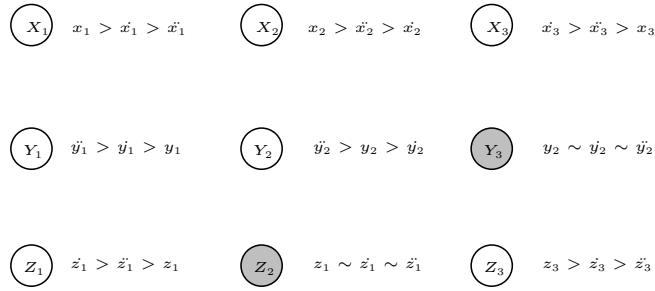


Figure 7.4: Abstention due to Interest Distribution (Indifference)

- For different agents, there are **Different Interest Distributions** over issues, in other words, the *weighted importance* for different issues are different. For the example of figure 7.2 with agents $\{1,2,3\}$, in which the agent 2 just participate partial issues of $\{X, Y\}$, it is possibly to assume that the decision and outcome² of issue X and issue Y is more important for agent 2, but the decision on issue Z is not that important, or comparatively indifferent to agent 2, therefore agent 2 just participate the decision of $\{X, Y\}$, and as we know, participating the decision itself needs certain efforts and costs, you need to collect or search the information³, and do the reasoning and consideration, so it is reasonable to think that why bothering to make decision on a trivial issue (whose outcome would make no difference for oneself).

The abstention due to interest distribution are related to the concept of **Indifference**, as the different alternatives chosen on one issue would not affect or change the agent's interest, and the agent do not care about the outcome on this issue. In this sense, the alternatives in the domain are indifference for the agent, for a issue X with domain $\{x, \hat{x}, \ddot{x}\}$, the indifferent preference could be expressed as $x \sim \hat{x} \sim \ddot{x}$, based on the figure 7.2 of one example with abstention, if on the case of indifference, the example could be extended as figure 7.4.

According to the notation 6, Decision (or Choice) is denoted as \mathbb{C} , for a combinatorial and collective decision-making context (with m features and n agents in total), the set of all possible choices $\mathbb{C} = \{C_{(1)}(1), \dots, C_{(m)}(n)\}$, $C_{(k)}(i)$ represents the decision/choice of agent i on feature k , the choice of agent i on all features $\mathbb{C}(i) = \{C_{(1)}(i), \dots, C_{(m)}(i)\}$, the choices on feature k by all agents $\mathbb{C}(k) = \{C_{(k)}(1), \dots, C_{(k)}(n)\}$.

Definition 4 (Abstention) *In the combinatorial and collective decision-making context, abstention means not all possible choices are made, define the set of all made choices in the decision-*

²Mainly about the payoff brought by the decision, more generally, the value (gained or lost) by the decision.

³As we know, the information is a very important resources, especially in information society, and the abundant and complete degree of information would directly determine the quality of decision.

making system as \mathbf{C} , $\mathbf{C} \subset \mathbf{C} = \{C_{(1)}(1), \dots, C_{(m)}(n)\}$ means abstention. For the decision-making of agent i , define the set of all made choices by agent i as $\mathbf{C}(i)$, $\mathbf{C}(i) \subset \mathbf{C}(i) = \{C_{(1)}(i), \dots, C_{(m)}(i)\}$, which means there is abstention for agent i on $\mathbf{C}(i) \setminus \mathbf{C}(i)$; for the decision-making on feature k , define the set of all agents making choices on feature k as $\mathbf{C}(k)$, $\mathbf{C}(k) \subset \mathbf{C}(k) = \{C_{(k)}(1), \dots, C_{(k)}(n)\}$, which means there is abstention for feature k on $\mathbf{C}(k) \setminus \mathbf{C}(k)$.

It should be noticed that, if the abstention are due to the indifference of alternatives, then only the made choices are incomplete compared with all possible choices, while the preference underlying the choices are complete (as agent quit the decision-making on one variable due to the indifference about its different candidates, which means different outcomes would make no difference, and the preference is like equal or similar among all alternatives). While if the abstention are due to the incomparability of alternatives, then both the made choices and the possessed preferences are incomplete, and the incomplete made choices are determined by the incomplete possessed preferences, as due to the lacked or limited knowledge about certain field, agent might be unable to compare different alternatives, and not hold a clear preference (ordering) over alternatives.

According to the Definition 5, the set of all possible preferences $\mathbb{P} = \{P_{(1)}(1), \dots, P_{(m)}(n)\}$, $P_{(k)}(i)$ represents the preference of agent i on feature k , the preferences of agent i on all features $\mathbb{P}(i) = \{P_{(1)}(i), \dots, P_{(m)}(i)\}$, the preferences on feature k by all agents $\mathbb{P}(k) = \{P_{(k)}(1), \dots, P_{(k)}(n)\}$. On the case of abstention due to the incomparability, defined the set of all possessed preference in the decision-making system as \mathbf{P} , then $\mathbf{P} \subset \mathbb{P} = \{P_{(k)}(1), \dots, P_{(k)}(n)\}$, for the preferences of agent i , $\mathbf{P}(i) \subset \mathbb{P}(i) = \{P_{(1)}(i), \dots, P_{(m)}(i)\}$, and for the preferences on feature k , $\mathbf{P}(k) \subset \mathbb{P}(k) = \{P_{(k)}(1), \dots, P_{(k)}(n)\}$.

7.1.1 Comparison between Value Gained and Cost

In summary, the decision to participate an activity or not is usually determined by the comparison between the value gained and the cost (of the activity), only the value gained outweighs the cost, then agent would take the activity, which is of course applicable to whether or not participate a decision. Therefore, usually if an agent determine not to participate on a issue's decision, it is because the cost is too high (unacceptable) or the value gained is too low (unattractive). In fact, above discussed two reasons can both fall under the value gained-cost framework, for the **Different Interest Distribution**, it is because the value gained for some issues' decisions are too small, and for the **Different Knowledge Distribution**, it is because some agents have too few information on some issues, and to seek enough information and make a proper decision on these issues would cost too much.

Further, to provide an specific example to explain the abstention, considering the decision-making process of cabinet of ministers (of a central government), as we know, the decision-making of important state issues need the overall participation of all cabinet members (mainly the ministers). It is due to the legal procedures that every cabinet members need to be presented and make decisions collectively. However, the real or factual participation situation might be different from the "legal" or "theoretical" one, for different fields of issues, the degree of participation of every minister might be very different, due to the heterogenous distribution of knowledge and interest and so on. For a issue in the area of education (such as the budget allocation planing for universities), the minister of education will definitely play an important and active role in the decision-making process, and the minister of finance will also participate to be responsible of the budget planing, but for other ministers such as minister of defence, minister of justice and so on, might not participate in the decision process (although be presented)⁴, on one side, the minister of

⁴At least not possible to play an important role as the minister of education and minister of finance, due both to the different distribution of knowledge and the interest.

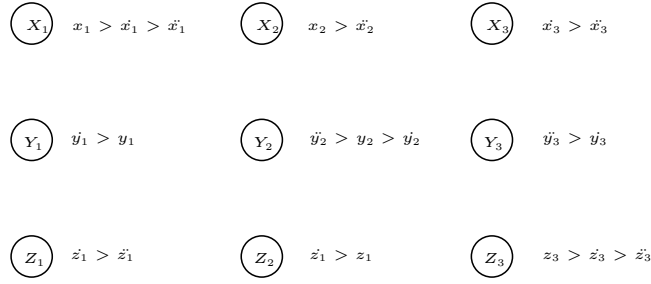


Figure 7.5: One Example with Constraints and Partial Domains

defence and minister of justice might know nothing or very few about the field of budget planning and resources allocation for universities (they are not the expert with professional knowledge on this field), on the other side, this area is not the field of their functions and responsibilities, and not their place of “interest”, a good participation on this irrelevant field would not bring a value gained, and a “omission” on this irrelevant field would also not bring a value loss.

7.2 Constraints and Partial Domains

The domains (value sets) of variables are also a basic element of decision-making. It should be noticed that in all works [Chevaleyre et al., 2007, Maudet et al., 2012b, Maran et al., 2013] of multi-issue collective decision-making, for one (same) issue or feature, its domain (candidate set) for every agent (to make choices) are set to be uniform and symmetric too.

Abstract Example 7.2 (Full Domains) *As shown in figure 7.1 with three issues $\{X,Y,Z\}$ and three agents $\{1,2,3\}$, in which each variable represents one decision of one agent on one issue, such as for variable X_1 means the decision of agent 1 on issue X. It is usually assumed that for one same issue, its domain for all agents are all identical, and all as the full domain, for this example (of ternary choices), the full domains for features X,Y,Z could be respectively expressed as $\{x, \dot{x}, \ddot{x}\}$, $\{y, \dot{y}, \ddot{y}\}$, $\{z, \dot{z}, \ddot{z}\}$, and assume the domain for agent 1,2,3 on issue X (variable X_1, X_2, X_3) are all $\{x, \dot{x}, \ddot{x}\}$, on issue Y (variable Y_1, Y_2, Y_3) are all $\{y, \dot{y}, \ddot{y}\}$, on issue Z (variable Z_1, Z_2, Z_3) are all $\{z, \dot{z}, \ddot{z}\}$.*

Abstract Example 7.3 (Partial Domains) *As shown in figure 7.3, it might be a circumstance with domains not that “uniform” and “complete”, for the issue X, the domain for X_1 (the domain of issue X for agent 1) is full as $\{x_1, \dot{x}_1, \ddot{x}_1\}$, but the domain for X_2 (of issue X for agent 2) is partial as $\{x_2, \ddot{x}_2\}$, the domain for X_3 (of issue X for agent 3) is partial as $\{\dot{x}_3, \ddot{x}_3\}$, and it is the same for the issue Y and issue Z, there also exists partial domains (for Y_1 and Y_3 , Z_1 and Z_2).*

The nonuniform and incomplete of domains are due to varied reasons, including but not limited to, which is also from the perspective of the agent:⁵

- **Unacceptance** (the constraints of the agent), for some issues, part of values (or candidates) in the domain might be totally unacceptable for (or strongly disgusted by) some agents. For example, in a ordering of full domain of X_i as $x_i > \dot{x}_i > \ddot{x}_i$, it means that x_i and \dot{x}_i are both preferred by agent i to \ddot{x}_i , but we can not distinguish or tell that \ddot{x}_i is just less preferred (but still “liked”) by agent i , or agent i “dislikes” or “disgusts” \ddot{x}_i (even totally

⁵For one issue, the domain should be identical, originally and objectively, but become (partially) different for different agents, subsequently and subjectively, due to agents’ different personalities or qualifications and so on.

can not accept \ddot{x}_i), if it is the latter reason, through directly eliminating \ddot{x}_i from the domain, a partial domain ordering as $x_i > \dot{x}_i$ is much more suitable than the full domain ordering $x_i > \dot{x}_i > \ddot{x}_i$, to express the factual preference or real mind of agent i , therefore, it is more reasonable to assume the domain for agent i as partial. By distinguishing the full domain (ordering) and the partial domain (ordering) it can reveal much more information about the real preference of agent. Therefore the framework of (different) partial domains can better capture the psychology or mind of agent in an practical and delicate way.

Specific Example 1 (Constraints by Unacceptance) *For one example of a man buying a car, there are multi-features to make decisions about, such as maker, given model and so on, in which the color of the car is also a very important feature. If assume there is a domain of color as $\{\text{red, black, pink}\}$, and for this man, the preference ordering could be $\text{black} > \text{red} > \text{pink}$, however, a more appropriate ordering is $\text{black} > \text{red}$, because the pink is totally unacceptable for him.*

- **Nonexistence** (the constraints of the domain), as some specific combinations are not feasible, and these *constraints on domains* are uniform for all agents, but not varied for different agents like the *constraints of agents*.

Specific Example 2 (Constraints by Unacceptance) *For the same example of a man buying a car, assume there are two features as the maker (brand) and the given model to make choices about, the domain of decision-maker is $\{\text{Ferrari}^{\text{®}}, \text{Benz}^{\text{®}}, \text{BMW}^{\text{®}}\}$, and the domain of model is $\{\text{racing, roadster, SUV}\}$, for instance if you choose the maker as $\text{Ferrari}^{\text{®}}$, then the domain would become partial as $\{\text{racing, roadster}\}$, because $\text{Ferrari}^{\text{®}}$ does not produce SUV, and SUV is an unavailable or non-existed value after chosen maker as $\text{Ferrari}^{\text{®}}$.*

The nonexistence by constraints (especially by other features within the set) have been studied intensively [Dechter, 2003, Rossi et al., 2006, Bessiere, 2005, Dechter, 2005], such as the hard constraints and extensive soft constraints [Pozza et al., 2011, Meseguer et al., 2005], and the common ground of all works is dealing with the constraints in a uniform (or universal) way, for example of a hard constraint on issue X and Y as $X \neq Y$, it means that for all agents, $X_i \neq Y_i$.

There are some similarities between the partial domains due to the unacceptance and the nonexistence, both about eliminating part of values from the domain, but the unacceptance is mainly based on the **Subjective Willingness**, such as the agent can't accept the "disgusted" choice, but the nonexistence is mainly due to the **Objective Rules**, as some candidates in the domain do not exist in the real-world, specially under some certain circumstances or constraints.

According to the Notation 4, Alternative is denoted as O , and for a combinatorial and collective decision-making context (with m features and n agents in total), the set of all possible local alternatives on one feature k is $O_{(k)} = \{o_1, o_2, \dots, o_t\}$, the set of all possible global alternatives $\mathbb{O} = O_{(1)} \times O_{(2)} \times \dots \times O_{(m)}$. Above assumed that alternatives (or a domain) on one feature for all agents are uniform, otherwise, alternatives on feature k for agent i is denoted as $O_{(k)}(i)$, and the set of all possible global alternatives of agent i is $\mathbb{O}(i) = O_{(1)}(i) \times O_{(2)}(i) \times \dots \times O_{(m)}(i)$.

Definition 5 (Partial Global Domains) *A combinatorial and collective decision-making context has partial domain when the set of feasible global alternatives:*

$$\mathcal{O} \subset \mathbb{O} = O_{(1)} \times O_{(2)} \times \dots \times O_{(m)}$$

We also denote the set of feasible alternatives for agent i as $\mathcal{O}(i)$

Definition 6 (Partial Domains for Features) For a given feature k and a given agent i , the domain $O_{(k)}(i)$ is partial when $O_{(k)}(i) \subset O_{(k)} = \{o_1, o_2, \dots, o_t\}$ where $O_{(k)} = \{o_1, o_2, \dots, o_t\}$ is the full domain for feature k .

Similarly, the **Unacceptance** and **Nonexistence** could also be explained in the framework of (comparison between) value gained and cost, if a candidate is eliminated from the domain, it is either due to the value gained too low, or the cost too high. For the **Unacceptance**, it is possibly for some candidates, their value gained are too trivial, or even are value loss, then naturally the agent would get rid of these candidates from the domain. And for the **Nonexistence**, the fact that some candidates do not exist might be due to the two high cost (or outrageous cost), for example compared with the unacceptance of the Ferrari colored in pink, Ferrari might not produce the pink car, but it is easy to paint a Ferrari with pink, it is feasible and doable for a pink Ferrari, but for the nonexistence of the Ferrari SUV, it is not possible to get a Ferrari SUV without a systematic production line, of course, theoretically it is possible for the Ferrari to research and develop a SUV production line, but it would obey the orientation and strategy of Ferrari, and the cost would be outrageous. Therefore, from a objective and factual perspective, under certain circumstances and constraints, the Ferrari SUV is nonexistent due to outrageous cost.

7.3 Constrained CP-nets

A constrained CP-net is just a CP-net N with the addition of a set of constraints C over the same variables⁶. These constraints could thus specify unfeasible outcomes, and thus partial global domains as explained above. Given this, we say that an outcome is feasible if it satisfies all constraints in C . Then, extending the classical definition, an optimal outcome for a constrained CP-net (N, C) is a feasible outcome which is not dominated by any other feasible outcome in the CP-net preference ordering.

While for acyclic CP-nets, we recall that finding an optimal outcome is computationally easy, for acyclic constrained CP-nets it is as difficult as solving (possibly several times) the constraint set C . In [Boutilier et al., 2004b] an algorithm (Search-CP) is defined to find an optimal outcome in a constrained CP-net. Therefore, when the constraint set is tractable, for example it has a tree structure, then this problem is computationally easy. Here we consider constrained CP-nets where the CP-net is acyclic.

Example. Consider three agents, each expressing their preferences over candidates defined by 3 binary features. So we have 3 CP-nets N_1 , N_2 , and N_3 , with features A , B , and C , where each feature X has values x and \bar{x} . N_1 contains the preferential statements $a \succ \bar{a}$, $b \succ \bar{b}$, $(a \wedge b) \vee (\bar{a} \wedge \bar{b}) : c \succ \bar{c}$, $(a \wedge \bar{b}) \vee (\bar{a} \wedge b) : \bar{c} \succ c$. We recall that $a \succ \bar{a}$ represents the unconditional preference for $A = a$ over $A = \bar{a}$, while $(a \wedge \bar{b}) \vee (\bar{a} \wedge b) : \bar{c} \succ c$ states that $C = c$ is preferred to $C = \bar{c}$, when $A = a$ and $B = \bar{b}$ and also when $A = \bar{a}$ and $B = b$. Thus, in N_1 , A and B are independent variables, while C depends on both A and B . N_2 contains instead the following preferential statements: $a \succ \bar{a}$, $a : b \succ \bar{b}$, $\bar{a} : \bar{b} \succ b$, $b : c \succ \bar{c}$, $\bar{b} : \bar{c} \succ c$. Thus, in N_2 , A is an independent variable, while B depends on A and C depends on B . N_3 is defined by: $a \succ \bar{a}$, $b \succ \bar{b}$, $c \succ \bar{c}$. Thus, in N_3 , all variables are independent. Figure 7.6 shows this profile.

Consider now ordering $O = \langle A, B, C \rangle$, and let us apply the sequential voting procedure with voting rule Majority for all three variables. For variable A , we have the preferences $a \succ \bar{a}$ from all agents, thus we select $A = a$. Then, given this choice, we pass on to variable B , getting preferences $b \succ \bar{b}$ from all agents. Thus we choose $B = b$. Notice that, while feature B is independent from A in agent 1 and 3, in agent 2 it depends on A . Thus the preferences on the

⁶The content of this section and the following has been published in [Grandi et al., 2014]

values of B in such an agent are those corresponding to the value of A chosen in the previous step. Passing on to C , we get preferences $c \succ \bar{c}$ from all agents, thus we choose $C = c$. Thus the sequential procedure chooses the variable assignment $\langle A = a, B = b, C = c \rangle$.

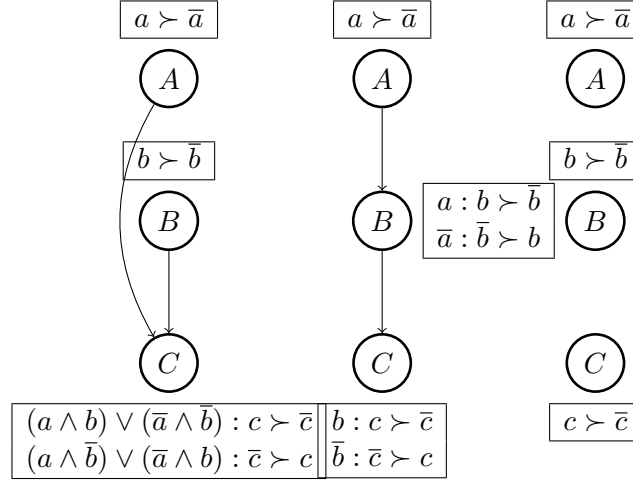


Figure 7.6: A profile of CP-nets.

So far the literature had focussed on finding optimal outcomes of constrained CP-nets, it is also an interesting question to know whether preferences expressed by the CP-net comply (and to what extent) with the constraints. In fact, there are several possible ways to define such a compliance.

7.3.1 Consistency notions

Given a constrained CP-net (N, C) , here are some notions of consistency between the preference structure expressed by N and the set of constraints C . The reason we are interested in these consistency notions is that in some cases they make help for the aggregation phase, when preferences are expressed by a collection of constrained CP-nets, as we will see later.

The first notion of consistency relates the optimal outcome of the CP-net to the constraints.

Definition 7 A constrained CP-net (N, C) is top-consistent if the optimal outcome of N satisfies the constraints in C .

For example, the CP-net of agent 1 in Figure 7.6 is top-consistent with the set of constraints $\{A = B\}$.

The next notion of consistency acts at the variable level and makes sure that feasibility is maintained when passing from the parents of the variable to its most preferred value.

Definition 8 A constrained CP-net (N, C) is locally-consistent if there is no line in the CP-tables of N of the form $o : b > \bar{b}$ such that o is feasible but ob is not.

Since o and ob can be partial outcomes, that is, assigning values to only some of the variables, we recall that a partial outcome is feasible if it can be extended to a solution.

The third notion of consistency we define is a structural property, that related the dependency graph of the CP-net to the path-closure graph of the constraints.

Definition 9 A constrained CP-net (N, C) is dependency-consistent if the path-closure graph of C is a subset of the undirected version of the dependency graph of N .

Dependency consistency can be natural in several settings. For example, if constraints are known in advance, the process of specifying a CP-net will exploit preferential dependencies among variables connected by a constraint to express qualitative preferences over the partial outcomes over such variables.

Theorem 1 *If a constrained CP-net is both locally and dependency-consistent, then it is also top-consistent.*

Proof 1 *If we have both local and dependency consistency, the optimal outcome is feasible (that is, we have top consistency). In fact, let us compute the optimal outcome by instantiating one variable at a time, in an order which is compatible with the dependency graph of the CP-net (that is, parents come before their children). We start from the independent variables and we give them their most preferred value. This is a feasible partial assignment since, by dependency consistency, there are no constraints among independent variables. At any step, we instantiate a new variable to its most preferred value given the chosen instantiation of its parents. If the partial assignment before this step was feasible, also the new partial assignment is feasible because of local and dependency consistency. Thus all partial assignments built during the procedure, included the last one which is the optimal outcome, are all feasible. A feasible complete assignment is, by definition, a solution. \square*

It is easy to see that neither local nor dependency consistency alone imply top consistency, and viceversa.

Example. *Consider the CP-nets in Figure 7.6 and the constraints $c_{AB} = \{(A = a, B = b), (A = \bar{a}, B = \bar{b})\}$ and $c_{BC} = \{B = b, C = \bar{c}\}, (B = \bar{b}, C = c)\}$. None of the CP-nets are top consistent. Moreover, N_1 is not locally consistent because of the CP-table for feature C : ab is a partially feasible assignment but abc is not. N_2 is not locally-consistent either, again because of the CP-table for C : b is feasible but bc is not. On the other hand, n_3 is locally consistent. Only N_2 is dependency consistent.*

7.3.2 Checking the consistency notions

We now study the computational complexity of checking the above three notions of consistency in a constrained CP-net. In what follows we assume CP-nets to be acyclic.

Theorem 2 *Given a constrained CP-net (N, C) , it is polynomial to check whether it is top-consistent or dependency consistent.*

Proof 2 *For top consistency, it is sufficient to find the optimal outcome of N and check whether it satisfies the constraints in C . Since N is acyclic, this is computationally easy.*

For dependency consistency, we just need to compare the dependency graph and the path-closure graph of the constraints. Once we have the two graphs, this is linear in their size. The path-closure graph can be obtained by achieving 3-consistency on the constraints, which is polynomial. \square

Theorem 3 *Given a constrained CP-net (N, C) with Boolean variables, with C a set of binary constraints, it is polynomial to check whether it is locally consistent.*

Proof 3 *We need to check that ob is feasible, for each row in the CP-tables of the form $o : b > \bar{b}$ such that o is feasible. Since constraints are binary, for each row in a CP-table, this can be done in polynomial time. In fact, checking that a partial outcome is feasible with a set of binary constraints*

is computationally easy if variables are Boolean (it amounts to solving a 2SAT problem). The number of rows in a CP-net may be exponential in the number of issues, but not in the size of the CP-net which is given in the input. Thus the overall complexity is polynomial. \square

Observe that local consistency in general cannot be checked in polynomial time if constraints are not binary, even if variables are Boolean, since it would require solving a SAT problem, while in the binary case it is 2-SAT.

7.3.3 Achieving top and local consistency in constrained CP-nets

Assume now that (N, C) is a constrained CP-net which is *not* top-consistent or *not* locally-consistent. This can happen in scenarios in which we have our own preferences over the outcomes expressed via a CP-net, and somebody gives us the constraints describing the feasible outcomes, and the two things together do not have the desired notion of consistency. We would like to modify our CP-net as little as possible in order to obtain either top consistency or local consistency.

Top consistency. To achieve top consistency, we may adopt the following procedure. Let us start from any independent variable (there must be one since N is acyclic) and have one step for each variable, in an order which is compatible with the dependency graph of the CP-net (parents come before their children), computing the optimal outcome. If at any step j , the partial outcome o obtained so far is not feasible, then we modify the row of the CP-table of variable x_j corresponding to the parents' assignment in o doing a switching of the ordering. This assures that the new partial outcome is feasible. This algorithm will produce in polynomial time a CP-net which is top-consistent if the constraints are binary. However, it does not assure that the resulting CP-net is minimally distant from the given one, if the distance is the number of different orderings in the CP-tables. However we conjecture it would be computationally difficult to find the one which is minimally distant.

Local consistency. Instead, to obtain a CP-net which is locally-consistent, it is sufficient to check each row in the CP-tables for the condition of local consistency, again following an order of the variables which is compatible with the dependency graph. If one of the rows fails the consistency check, then the preference expressed in this row needs to be inverted. Since we are moving forward following the dependency structure of N , we are guaranteed that one of the two possible orders in a row of the CP-table must be consistent. Notice that this algorithm is different from the previous one since we need to check all rows of the CP-tables and not just those involved in the computation of the optimal outcome. Again, the assumption of binary constraints is crucial for this algorithm to be polynomial. Unlike the previous one, this algorithm guarantees that the resulting CP-net is minimally distant from the given one, if the distance is the number of different orderings in the CP-tables.

A constrained profile models the scenario in which we have several individuals who express their preferences over a common set of outcomes by using CP-nets, and the constraints model the set of feasible outcomes. Only those outcomes that satisfy all constraints can be returned as the result of the aggregation of the preferences of the individuals.

Formally, a *constrained profile* is a collection of CP-nets $\{N_1, \dots, N_n\}$ plus a set of constraints C . This can also be seen as a collection of constrained CP-nets $\{(N_1, C), \dots, (N_n, C)\}$, all having the same constraints.

Notice that all CP-nets share the same set of feasible (and thus unfeasible) candidates, which are those defined by C . Moreover, the CP-nets of all agents share also the variables and the variable domains. So, what can be different in two agents is the dependency graph of their CP-nets, as well as CP-tables of the CP-nets.

Recall that we restrict our attention to constrained profiles which are O-legal, and notice that O-legality implies that all CP-nets in the constrained profile are acyclic.

Example. *As an example of a constrained profile, let us consider the CP-nets in Figure 7.6, with the addition of the set of constraints $c_{AB} = \{(A = a, B = b), (A = \bar{a}, B = \bar{b})\}$, $C_{BC} = \{B = b, C = \bar{c}\}, (B = \bar{b}, C = c)\}$. It is easy to see that this profile is O-legal: there is an ordering O of the variables which is compatible with all dependency links, namely $O = (A, B, C)$. Observe that the top outcome is abc for all three agents, and this would be the result of sequential majority over the three CP-nets. However, this outcome is not feasible (only $ab\bar{c}$ and $\bar{a}\bar{b}c$ are).*

7.4 Collective decision-making with Constrained Profiles

Now our goal is to take a constrained profile and return a feasible outcome, which should satisfy the preferences of the individual CP-nets as much as possible. As we know, when we have no constraints on the feasible outcomes, sequential voting is used to perform such an aggregation. We will now see that sometimes sequential voting is all we need also in presence of constraints. In general, however, we need to take constraints into account. This can be done by adapting the sequential voting procedure, while maintaining a polynomial time complexity if constraints are tractable.

7.4.1 Top, local, and dependency consistency

Under assumptions the consistency notions introduced in Section 7.3.1, sequential aggregation using the majority rule outputs a feasible outcome. The first result applies when we have top consistency, but requires CP-nets to be separable, that is, to have no dependency structure.

Theorem 4 *If $\langle (N_1, \dots, N_n), C \rangle$ is a constrained profile such that all N_i are top-consistent and separable, and C is a set of binary constraints, then the winner determined by sequential voting with the majority rule is feasible.*

Proof 4 *Since all N_i are separable (and variables are binary), then the order followed by sequential majority is irrelevant, and the problem is equivalent to binary aggregation in which all individuals submit their top outcome and issue-by-issue majority voting is used. We can therefore use the following result from the binary aggregation literature: issue-by-issue majority outputs a feasible outcome given feasible input (that is, it is collectively rational) if and only if the constraints are equivalent to a conjunction of disjunctions of size 2 [Grandi and Endriss, 2013, List and Puppe, 2009]. First we observe that, by top consistency, each individual top outcome satisfies the constraints. Second, since we assume constraints in C to be binary, each constraint can be written as a conjunction of disjunctions of size 2, thus the whole set of constraints can also be written in this way. Therefore this result applies here. If the constraints are not binary, it is possible to find examples in which the outcome of sequential majority is not feasible. \square*

When the CP-nets have a non-empty dependency structure, we can still apply standard sequential voting to get a feasible outcome if they are both locally and dependency consistent (and thus also top consistent). So we need a stronger property on the CP-nets when we have preferential dependencies.

Theorem 5 *If $\langle (N_1, \dots, N_n), C \rangle$ is a constrained profile such that all N_i are locally-consistent and dependency-consistent, and C is a set of binary constraints, then the winner determined by sequential voting with the majority rule is feasible.*

Proof 5 We will prove by induction on the number of variables that, at each step i between 1 and m , the partial assignment generated until step i is feasible. For step 1, it is trivially true since the CP-nets are locally consistent, so the most preferred value in an independent variable must be feasible. This means that all CP-nets vote for a feasible value for the first variable, and thus majority chooses a feasible value.

Let us assume that the statement is true until step i , and let us consider step $i + 1$. We have a feasible partial assignment $\langle v_1, \dots, v_i \rangle$ obtained so far. For variable $i + 1$, assume that there is a majority in favor of b , i.e., at least a majority of the individual CP-nets prefer b to \bar{b} given the partial assignment obtained so far. This means that if individual j is part of this majority, then N_j contains the row $o_j : b > \bar{b}$, where o_j is the assignment of the parent variables of variable i in CP-net N_j which occurs in the current feasible assignment. By dependency consistency we know that the parent variables of variable i in each individual CP-net include all variables k that are related with i by a constraint. By local consistency, we also know that $o_j b$ is feasible for each j between 1 and n . Thus also $\langle v_1, \dots, v_i, b \rangle$ is feasible.

Therefore all partial assignments generated during the sequential voting procedure are feasible, including the last one, which is a complete assignment and thus a solution of all the constraints. \square

7.4.2 Aggregation in non-consistent profiles

When none of the sufficient conditions mentioned above hold, we can obtain a feasible outcome by modifying the sequential voting procedure to take the constraints into account. Starting from the LA procedure already defined in the literature to aggregate CP-nets, we define the procedure CLA, for Constrained LA procedure. CLA is very similar to LA, except that it will work on possibly reduced variable domains, because of the constraints. As each step, the constraints will tell us what domain values to consider, in order to get a feasible outcome.

Algorithm 1 CLA

Input: A constrained profile $\langle (N_1, \dots, N_m), C \rangle$, n voting rules r_1, \dots, r_n , an ordering $O = \langle x_1, \dots, x_n \rangle$

Output: a variable assignment $\langle x_1 = v_1, \dots, x_n = v_n \rangle$

for $i = 1$ to n **do**

$T_i =$ the constraint graph of C (a tree), rooted at x_i

$C' = \text{DAC}(T_i)$

$D_i =$ the domain of x_i in C'

if $D_i = \emptyset$ **then**

Return No feasible candidate

end if

for $j = 1$ to m **do**

$o_j =$ the ordering over D_i given by the CP-table in N_j for

$x_1 = v_1, \dots, x_{i-1} = v_{i-1}$

$o'_j = o_j$ restricted to D'_i

end for

$v_i = r_i(o'_1, \dots, o'_n)$

 Add the constraint $x_i = v_i$ to C

end for

Return $\langle x_1 = v_1, \dots, x_n = v_n \rangle$

The first thing we need to do is to preprocess the constraints in C so to bring to the variable domains the information about the feasible candidates. In fact, since LA is a sequential voting

procedure which considers one variable at a time, it is important to leave in the domain of each variable only the values that belong to feasible candidates.

As in the classical sequential voting procedure, we have a collection of m voting rules $\langle r_1, \dots, r_m \rangle$ that will be used in the m steps of the procedure, one step for each variable. If variables are Boolean, of course all r_i will be the majority voting rule. Assume for now that the constraint set has a bounded tree-width, so it belongs to a tractable class. For sake of easiness of presentation, let us consider a tree-like shape. However, the CLA procedure works also for bounded tree-width constraint sets.

Since the constraints have a tree shape, it is indeed possible to leave in the domain of each variable only those values that appear in some feasible candidate. We just need to consider the variable ordering O , take the first variable x_1 , use it as the root of the tree, and achieve directional arc-consistency to this tree. At the end, the new domain of x_1 , say D'_1 , will contain only those values that appear in some feasible candidate. We can now apply the voting rule r_1 to the profile over variable x_1 , where however the domain of x_1 has been reduced to D'_1 . This will choose a value for x_1 , say v_1 , which is feasible (that is, it can be extended to a solution).

Let us now pass to the second variable x_2 . Given the value v_1 chosen for x_1 , we set $x_1 = v_1$ in C and in all the CP-nets and we apply again DAC bottom-up, now by using x_2 as the root of the tree. This will generate a new domain for x_2 , say D'_2 , which will contain only those values that appear in some feasible candidate. We can now apply the voting rule r_2 to the profile over variable x_2 (given $x_1 = v_1$), where however the domain of x_2 has been reduced to D'_2 . This will choose a value for x_2 , say v_2 . Now we have the partial assignment $\langle x_1 = v_1, x_2 = v_2 \rangle$, which is feasible.

We then continue like this until all variables have been assigned. The winning candidate is then $\{x_1 = v_1, \dots, x_n = v_n\}$.

Since C is a tree, the first application of DAC will tell us if there are feasible candidates. If no variable domain is empty after the first DAC, then we know there is at least one feasible candidate, and the later applications of the DAC procedure will never generate any empty variable domain.

Example. *As an example of the application of the CLA algorithm to a constrained profile, consider again the constrained profile in Figure 7.6, with ordering $O = \{A, B, C\}$ and constraints $c_{AB} = \{(A = a, B = b), (A = \bar{a}, B = \bar{b})\}$, $c_{BC} = \{B = b, C = \bar{c}\}, (B = \bar{b}, C = c)\}$. Since we do not have top consistency, nor local and dependency consistency, for all CP-nets, we cannot use classical sequential voting to select a feasible outcome. We will thus use the CLA procedure. Since the variables are binary, we use majority voting at each step. CLA first achieves DAC to the constraint set, which is a tree, rooted at A . This removes the value \bar{b} from the domain of B , because of the constraint c_{BC} , and it also removes the value \bar{a} from the domain of A , because of constraint c_{AB} . We now apply majority voting to the profile related to variable A , getting $A = a$. We then add the constraint $A = a$ to the initial set of constraints and we pass on to the second variable, B . We achieve DAC to the tree rooted at B , which does not remove anything from any domain. We apply majority to the profile for B , getting $B = b$, and we add this as a new constraint. Finally, we achieve DAC on the tree rooted at C , leaving only \bar{c} in the domain of C , and, by majority voting, we get $C = \bar{c}$. Thus the result of CLA is $(A = a, B = b, C = \bar{c})$. This variable assignment satisfies all constraints. Notice that the outcome of a sequential voting procedure over the same profiles, without the constraints, would be $(A = a, B = b, C = c)$, which does not satisfy the constraints.*

On the other hand, if C is not tree-shaped, achieving DAC could leave in the variable domains also values that do not appear in any feasible candidate. Thus, once a value for a variable is chosen, it could be that there is no value for the next variable which is compatible with it. This means that the CLA procedure should backtrack its previous choices (for example the last one made) and

replace it with another value. It could also be the case that no feasible candidate exists, and this will result in backtracking over the choices until no more alternative choice is available. Thus the CLA procedure needs to perform search if achieving DAC (or adaptive consistency) does not leave the domains *minimal*, that is, containing only the values that participate in at least a solution.

7.4.3 Properties of CLA

The most important property to prove is that, in the setting we are considering, CLA always returns feasible outcomes, in time polynomial in the size of the input. We recall that our setting assumes that we have a constrained profile $\langle (N_1, \dots, N_n), C \rangle$, where C has a tree-like constraint graph, m voting rules r_1, \dots, r_m , and an ordering $O = \langle x_1, \dots, x_m \rangle$ which makes the profile O-legal.

Theorem 6 *The variable assignment $\langle x_1 = v_1, \dots, x_m = v_m \rangle$ returned by CLA satisfies all constraints in C .*

Proof 6 *Consider the output of CLA, say $\langle v_1, \dots, v_m \rangle$. Take any constraint in C , say c , between variables x_i and x_j . We need to prove that $\langle x_i = v_i, x_j = v_j \rangle$ satisfies c . At step i , CLA applied DAC to the tree rooted at x_i , restricting the domain of x_i . So, by definition of DAC, v_i is a value for x_i such that there is a value in x_j (and in any other variable) which satisfies c . After doing that, CLA has added the constraint $x_i = v_i$ to C . Then, at step j , CLA applied DAC again, to the tree rooted at x_j , thereby reducing the domain of x_j to only those values that have support in the domains of all variables, thus also in the domain of x_i , which is now containing just the value v_i . Since v_j is in the domain of x_j , this means that $\langle x_i = v_i, x_j = v_j \rangle$ satisfies c . \square*

Theorem 7 *CLA works in time $O(n \times (md^2 + n + t))$, where m is the number of variables, d is the size of the largest domain among D_1, \dots, D_m , n is the number of agents, and $t = f(n, d)$ is the time complexity for winner determination in the most computational expensive of the voting rules r_1, \dots, r_m .*

Proof 7 *CLA performs at most m steps. At each step, it achieves DAC on a tree with m variables with domain size at most d . This takes $O(md^2)$ time. It then reduces the n orderings over the current variable domain to the new domains computed by DAC. Finally, it applies the voting rule for that step to such orderings. \square*

It is worth noting that, in the case of just one voter, we have a single constrained CP-net, and CLA returns a feasible outcome which is undominated in the CP-net preference ordering. This is equivalent to what is done in [Boutilier et al., 2004b]. However, since we consider tree-shaped constraint sets, we can get this outcome in polynomial time. We therefore get this useful result out of our aggregation procedure. Observe moreover that if we start from acyclic CP-nets, an order O that makes the profile O-legal (or the conclusion that there is no such order) can be found in time polynomial in the number of variables. It is indeed sufficient to take the union of all dependency graphs and take any linearisation of it, if there is one. Any such ordering would give the same result of sequential majority, since preferential dependencies are all taken care of in the union graph.

Theorem 8 *Given a constrained CP-net $\langle N, C \rangle$, where N is acyclic and the constraint graph of C is a tree, finding an undominated feasible outcome is in \mathcal{P} .*

Proof 8 By Theorem 6, the variable assignment returned by CLA is feasible, that is, it satisfies all constraints in C . We have just one CP-net N and a set of constraints C . Thus, every time we use a voting rule r_i , this voting rule acts as the identity, thus returning the top choice in the preference ordering it gets in input. We will prove by induction on i that, after step i , $\langle x_1 = v_1, \dots, x_i = v_i \rangle$ is undominated by feasible outcome (in the outcome ordering of N restricted to x_1, \dots, x_i) and it satisfies all constraints in C . It is trivial for step 1, since v_1 is the top choice in the restricted domain of x_1 , obtained after applying DAC to the tree C rooted at x_1 . Assume the statement is true at step i and let us now consider step $i + 1$. CLA returns v_{i+1} , which is the top element of the preference ordering in the restricted domain of x_{i+1} . Since $\langle x_1 = v_1, \dots, x_i = v_i \rangle$ is an undominated outcome over variables x_1, \dots, x_i , and since x_{i+1} is the top element in the feasible domain of x_{i+1} , there is no other extension of $\langle x_1 = v_1, \dots, x_i = v_i \rangle$ to variable x_{i+1} which can be more preferred to $\langle x_1 = v_1, \dots, x_i = v_i, x_{i+1} = v_{i+1} \rangle$. \square

Sequential voting with CP-nets has been studied also from the point of view of the properties which can, or cannot, be transferred from the “local” voting rules r_1, \dots, r_n to the “global” sequential voting rule LA [Lang and Xia, 2009]. It is easy to see that, if a property transfers from local to global in the LA procedure, then it also transfers for CLA. In fact, CLA is just LA but on possibly smaller variable domains. So we are performing a domain restriction on the set of possible profiles. If a property is true of a sequential voting procedure when considering a larger set of profiles, it will remain true when we consider a smaller set. This is true for properties like anonymity, consistency, strong monotonicity, and monotonicity (of r_m), as shown in [Lang and Xia, 2009].

If instead a property does not transfer from local to global, then by passing from LA to CLA the same examples showing this still hold. Examples of properties that do not transfer from local to global LA are neutrality, efficiency, and participation (see again [Lang and Xia, 2009]). However, it could be that the domain restriction imposed by the constraints removes those profiles which are problematic for that property. We plan to study specific constraint classes that could allow some properties to transfer from local to global for CLA, even though they do not do so for LA.

Theorem 9 *If all voting rules r_1, \dots, r_m are anonymous (resp., consistent, strong monotone), then so is CLA. If r_m is monotone, then so is CLA. If r_1, \dots, r_m satisfy neutrality (resp., efficiency, participation), it could be that CLA does not satisfy it.*

Proof 9 *It follows directly from the analogous results in [Lang and Xia, 2009] for LA.* \square

7.5 Collective Decision-making with Abstention

Suppose an agent abstains on a variable x_i : how shall we interpret this? Perhaps the agent actually *knew* the issue and abstained strategically: in that case the other decisions may have been taken following the value you would have voted. Thus, the vote is not expressed, but the final outcome remains intuitively “close” from the actual outcome. But another interpretation is that the agent *didn't know the issue*, in that case this may deeply affect the preferred outcome of the agent.

Let us see the implications it may have on the aggregation methods:

- For the LA procedure, the outcome can be computed easily.

Abstract Example 7.4 (LA Procedure of Voting with Abstention) *For the example of figure 7.2, with three issues $\{X, Y, Z\}$ and three agents $\{1, 2, 3\}$, for issue X is determined by*

$\{X_1, X_2, X_3\}$ (agent 1,2,3), for issue Y is determined by $\{Y_1, Y_2\}$ (lacking agent 3), and for issue Z is determined by $\{Z_1, Z_3\}$ (lacking agent 2). One natural and neutral way to handle the missing information is to compute the outcome with respect to the expressed vote, just ignoring the absent agents and compute the vote of the (remaining) participated agents by the common method. If using the plurality rule in the case of figure 7.2, for the issue X , there are three votes as x, x, \hat{x} , then x is selected, and for the issue Y , there are two votes as \hat{y}, \hat{y} , then \hat{y} is chosen, and for the issue Z , there are two different votes as \hat{z}, z , then Z is, say z by tie-breaking. Finally, the chosen candidate (overall winner) is $(X = x, Y = \hat{y}, Z = z)$.

- For the FA procedure, this turns out to be more difficult to handle.

Abstract Example 7.5 (FA Procedure of Voting with Abstention) For the case of figure 7.2, the top candidate for agent 1 is $(X = x, Y = \hat{y}, Z = \hat{z})$, shorted as $x\hat{y}\hat{z}$, for agent 2 is $(X = x, Y = \hat{y})$, shorted as $x\hat{y}$, for agent 3 is $(X = \hat{x}, Z = z)$, shorted as $\hat{x}z$, then it is not straightforward to see how to compute the collective outcome based on these partial votes?

Dealing with partial votes. In what follows we shall assume that the sought final outcome is a complete outcome, and that the plurality voting rule is used. As we mentioned, there are two categories of votes: *full votes* and *partial vote*. The non-ambiguous situation is when full votes alone suffice to determine the outcome, in other words, there is a necessary winner [Lang and Xia, 2009], even disregarding the *partial votes*.

Definition 10 A partial vote v is compatible with a full vote v' if v can be completed with the abstained variable so as to give v' .

If this is not the case, we have to consider the partial votes, and we may have different strategies for doing so. Here we discuss a couple of them:

- under the *blind completion*, full votes receive additional point for any compatible partial vote expressed.
- under the *majoritarian completion*, all partial votes are completed by taking the majoritarian value for the abstained values.

Example 7.1 Suppose there are 5 votes: $x_1y_1z_1, x_1y_2, x_1y_2z_2, x_2y_2z_2, y_1z_1$. Here we have x_1y_2 compatible with $x_1y_2z_2$, and y_1z_1 compatible $x_1y_1z_1$, and y_1z_2 is not compatible with any full votes. Under the blind completion, $x_1y_1z_1$ and $x_1y_2z_2$ are on tie with 2 points. Under the majoritarian completion, the abstention on X is completed as x_1 while the abstention on Z is completed as z_1 (assuming z_1 wins by tie-breaking). In that case, $x_1y_1z_1$ is the only plurality winner.

7.6 Domains and Influence: perspectives

If the variable sets and value sets could be varied and heterogeneous for different agents, then it is more reasonable to assume the variable sets and value sets of agents would be dynamic in the process, being influenced and evolving: it may be expanding, contracting during the process.

Influence on the variables. For the influencing and evolving of variables set, as previously discussed, the variables set of the agent might be partial due to varied reasons like the knowledge and interest distribution, the agent might know nothing or very few about the field of certain variable then decide to abstain on the variable and let other “experts” make decision, but it is possible that there are other agent(s) who are close (in relationships) to the agent and deemed as “expert” by the agent, trying to persuade the agent with all kinds of reasons to participate the decision, in order to have more votes for their preferred candidate. For the agent initially knowing few, it is very “vulnerable” for him to be persuaded by “experts” with close relations. Therefore, the variables set for this agent will be expanded adding the new participated decision. Besides, once the new variables added to the set, it might also reform the combinatorial structure of the variables, changing or adding the dependencies among variables.

Influence on the domains. For the influencing and evolving of values set (domain), as previously discussed, the values set of agent might be partial due to varied reasons like the unacceptance and constraints, one is about the (subject) attitude and the other is about the (objective) capacity, but which are not absolutely constant or totally fixed, other agents will possibly try to persuade the agent to change his minds or provide the required resources for the agent to overcome the constraints restricting the agent’s domain, to let the agent stand by them enlarging the consensus votes. Besides, it is also possible that the attitude and capacity of agents autonomously be dynamic, (changing with time, environment and so on, or through the learning, observing, feeling and so on), the attitude might change and the capacity might improve. Therefore, the value sets (domains) would be transformed, agents would add new choice or eliminate original choice. Furthermore, once new choice added to the domain, the comparison of other choices (the preferences of other choices) might be reformed (which is contradictive to the independence of irrelevant alternatives, but this desiderata itself might be unreasonable and contradictive to real-world or real-psychology), very related to the reason-based model [Shafir et al., 2004]. Likewise, the eliminating of some choice would also change the comparison.

Domains as a heuristic for the similarity metrics. By comparing the domains of preference between the influencing variable and the influenced variable, the higher of the similarity, the higher of the weight of influence between them. For the aspect of domain, the weight of influence could be affected by:

$$sim_{i,j} = \frac{\sum_{k \in F} |O_{(k)}(i) \cap O_{(k)}(j)|}{\sum_{k \in F} |O_k|} \quad (7.1)$$

For example, for agent 1 the domain of preference is {a,b,c}, there are two influencing agents 2 and 3 with the domain respectively as {a,b,c} and {b,c}, considering all other conditions identical, but just considering the similarity of domain of preference, it is naturally to assume that agent 1 is more inclined to be influenced by agent 2, as thinking them have the same domain and face the same circumstance, it is more meaningful to refer from agent 2.

Part III

Application of Influence

We use a typical example of multi-issue and multi-agent decision-making—UN Security Council voting, to test the models of influence, by both the *Qualitative* approach (Cases Studies) and the *Quantitative* approach (Algorithm Designs)⁷, then we use the models to do simulation experiments, to simulate the voting and influencing process of UN Security Council, and discuss the interplay effect between new cases of influence and SC reform schemes. Before doing that, we need to discuss about the voting system of UN Security Council, and review the relevant works about UN Security Council voting, point out our innovations.

Voting System of UN SC

The Security Council is one of the United Nation's six principal organs. According to the Charter of United Nations, Security Council takes the responsibility of maintaining international peace and security, and is the only UN body with the authority to take enforcement actions. Its powers include the establishment of peacekeeping operations, the establishment of international sanctions, and the authorization of military action through Security Council resolutions.

Security Council is constituted of 5 permanent members and 10 non-permanent members, 5 permanent members are China, United States, Russia, France and United Kingdom, non-permanent members are constituted by the election of General Assembly of the United Nations, their tenures are 2 years and replaced 5 of them per year by election, the last term of non-permanent members can not be re-elected for the next term.

The UN SC voting is a very standard collective decision-making, for the decision-making rule of SC, each member state has one vote, for the non-procedural issues or called substantive issues, it needs at least 9 affirmative votes to pass, and the permanent member has veto right, which means only one negative vote from permanent member will fail the bill (which is called the "*Principle of Unanimity*" of great powers). Security Council holds two regular meetings every year, and can convene conference anytime if the chairman of SC affirm it is necessary, or through the request of the General Assembly, Secretary-General, or any SC member [Luo and Meng, 2013a].

Background of UN SC Reform

Since the establishment of United Nation, accompanied with the dynamic evolution of the international situation, the appeals for UN reform have been ceaseless, in which the appeals for Security Council reform are most intensive. As the core of power of the United Nation, the reform of SC affects most widely and touches most deeply on the critical state interests (especially for the five permanent members, which are the biggest vested interests for the current system), therefore, it is the most crucial, complicated, controversial and difficult part of UN reforms [Luo and Meng, 2013a, Hosli et al., 2011, Weiss and Young, 2005]. With the sequential independence of former colonial countries (peaked by 1960, the independence year of Africa), the scale of members of the whole United Nations have enlarged profoundly; furthermore, after the end of the Cold War (the disintegration of the Soviet Union), the international situation has been rebuilt thoroughly, which make the member composition, power allocation, and working procedure of current Security Council system confront gross challenges. More specifically explained as following:

- On one hand, the number of UN members has increased tremendously, especially after the developing countries have occupied the majority in UN, while the scale of SC has not enlarged correspondingly and proportionately, especially for the seats of developing countries (particularly the African countries) in SC, which make the wideness of representativeness and the democracy of decision-making of SC being severely doubted and challenged.

⁷The former is from a *Social Sciences Paradigm*, and the latter is from a *Computer Science Paradigm*.

- On the other hand, the international structure and situation nowadays is quite different from the period when UN just established (the end of World War II) and also from the era of Cold War, international community becomes more and more unsatisfied for the current power allocation and decision-making process of SC. For instance, the Germany, Japan, Brazil, India and so on are not satisfied for their current status, and unsatisfied for the current position of those declining “great powers”, and hope to become permanent members either.

In fact, although nearly all UN members hold the affirmative attitude for the SC reform, no one dare to apparently oppose to the reform⁸, but there are many disputes and controversies among members on many specific issues about SC reforms, such as the category and number of SC member enlargement (add how many new members? and add permanent members or non-permanent members?), the qualification of SC member (how to set the threshold for the new members?), the manner of election and the term of SC member, and so on [Luo and Meng, 2013a]. Nearly all countries have their own and varied claims, and hardly to compromise and reach consensus. Until now, although series of reform plans came on stage, there is no one which can obtain the support of above 2/3 members of UN assembly, making the process of SC hard to progress [Blum, 2005].

Previous Works and Our Advancements

There are a series of work about UN SC reform, mainly about the comparison among varied reform schemes, the reasoning and motivation of reform pushers and obstructors, the inference and forecast about the progress of SC reform, and so on [Luo and Meng, 2013a, Hosli et al., 2011, Weiss and Young, 2005, Crossette, 2000].

Common Focus: Efficiency and Fairness

Although the focus of different work are varied, but the orientation of reform target and the criteria of scheme evaluation mainly all focus on two aspects of *Efficiency* and *Fairness*, commonly deeming that any reform of SC should contribute to the efficiency value and fairness value:

- On one side, the SC is usually inefficient, hardly to reach consensus on many critical issues involving the world peace and security, and paralysed on the crucial or urgent moment, resulting in many (should-pass) constructive bills failed (or not implemented in time), which is one point that the SC be intensively criticized at. China⁹ and United States [Crossette, 2000] as the permanent members of SC even jointly deem that any change in SC should be based on the premise of making SC more efficient [Luo and Meng, 2013a];
- On the other side, with the continuous expansion of UN members and the constant scale of SC members, the democracy, representativeness and legality of SC have been questioned and challenged heavily. As the majority of United Nations members, small and medium countries deem their discourse rights and state interests can not be effectively guaranteed, and the SC is doubted as merely the arena of great powers [Luo and Meng, 2013a].

Room for Advancements: Micro-Macro Analysis and Complex System Design

Previous works have many reference values, but still embodied two potential improvements:

⁸Even for the vested interests the permanent members, they do not want to stand at the opposite side of so-called fairness and justices. And they are hardly named as enthusiast in the process of reform.

⁹http://news.xinhuanet.com/zhengfu/2005-06/08/content_3058885.htm

- Firstly, as SC reform gained the interests mainly from social scientists (particularly political scientist), but lacked the focus of scientists from other disciplines such as computer science, artificial intelligence, decision science and so on.

The social scientist are usually interested on the macro-level questions, like what is the critical differences among reform plans, how to design a feasible and satisfying reform plan, and what is the effectiveness of reform plans, but might ignore the micro-level questions and inner-process details. For example, under the background of SC reform, how is the gaming process and interacting pattern among SC members. Although these micro setups are not directly related to the SC reform, but they are the underlying and inner determinant elements for the whole reform. Even if we know the contents and targets of reform plans, if blind and ignored about the interacting mechanism and influencing pattern among individual member states, it is still impossible for us to discuss and forecast the effectiveness of different reform plans. *If there is no micro support, then there would be no macro analysis.* Actually, in a complex system, the law in the macro level usually emerge from the interaction in the micro level.

- Secondly, the previous work mainly stay in the stage of qualitative analysis, or descriptive, static and linear quantitative analysis¹⁰, until now, there is hardly research based on ABMS (Agent-based Modeling and Simulation) approach, which is the most suitable approach to study the complex systems. As in which, the macro laws emerge through the micro interaction among individual agents, and would possibly display series of complexity characteristics like the nonlinear, uncertainty and polymorphic equilibrium, which are hard to study by qualitative approaches or static, linear quantitative approaches.

In fact, the UN SC voting could be perceived as a typical complex system, which is composed of multi-agents with independent voting right and respective interests and preferences, they are multi-period interacting, heterochromous composed, self-organizing and self-adapting. Confronting the sequential bills affecting the world situation and involving (own) state interest, on the basis of information exchanges and communications, all members interact constantly, game repeatedly, influence mutually, and gradually achieve the adaption among member states, among alliances and opponents, and between the external environment and the collective objective. In the thesis, the research of UN SC reform based on ABMS provides an example for interdisciplinary study, endow the international politics with *new research perspective and methodological system.*

Analysis Framework: CP-nets and Social Influence Model

As discussed above, we discuss the UN SC voting based on an integrated framework combining CP-nets with social influence models. CP-nets represent the dependencies among features (or issues), social influence models represent the influences among agents, therefore, the two models should be integrated to fully describe the diverse influencing patterns in UN SC voting. We discussed the characteristics of these two models and their applications on UN SC voting:

- **CP-nets:** SC voting is a typical multi-issue (combinatorial) decision-making system, there are sequential bills for voting, and with dependency relations among these bills. Therefore, The CP-nets are very matched to model the SC voting process, in which, each feature/issue represents a SC bill, and the cp-statements could be used to describe the dependency relations among different bills. For example, there are a lot of bills with the same topics happening again and again, such as the high-frequency issues of Palestine, Iraq, Cyprus,

¹⁰Which might be partially due to the limited mathematic and science foundations of social scientists.

and so on. For those correlated bills, naturally, the member's vote on the current bill will be influenced by his own vote on the former bill, especially for a great power or super power, who should be persistent on the opinion of the same topic.

However, the traditional CP-nets are not designed to solve the collective voting and influence among multi-agents, the *influenced CP-nets* [Maran et al., 2013] is an advancement (although just discussed the influence from one single agent to another single agent), in which the new ci-statements (conditional influence statements) are proposed to describe the influences among agents.

Furthermore, the cp-statements and ci-statements are just the most basic form to describe the very limited patterns of influence, the influences indeed are varied and complex, a series of new patterns with corresponding statements (beyond the cp-statement and ci-statement) are proposed in part II, by involving an active role of structure, or crossing different agents and issues, to better capture the complex influences in UN SC.

- **Social Influence Model:** SC voting is also a typical multi-agent (collective) decision-making system, there are 15 member states collectively voting, and full of influencing relations among these member states, varying as negative or positive, strong or weak, etc.

On one side, the member state has to consider the preferences of his allies (especially the alliance leader); on the other side, the alliance leader should also consider about his allies or followers.

Example 7.2 (Positive Influence among Member States) *UK often follows the same vote of US even though his preferences are not completely uniform with US; on the other side, US as the leader of NATO and broadly the "western world", dominate the decisions of his allies on many international affairs, but still can not be totally unrestrained or do anything as please, US also has to consider about his allies's attitudes, claims and states preferences, to stand by their sides on some cases.*

On the other side, it is also common for member states to be influenced by their opponents or enemies, but reversely in a negative way. It can not be denied that there are always confrontations or conflicts among states or alliances in SC, no matter in an apparent way or an invisible way.

Example 7.3 (Negative Influence among Member States) *A typical example is the confrontations between the United States and the former Soviet Union, and between NATO and Warsaw Pact before the disintegration of Soviet Union. For now, there are still competitions or confrontations between US and Russia, between US and China, between China and Japan and many other cases. For own state preference, every state is inclined to go against his enemies or opponents¹¹.*

Furthermore, in real cases of SC voting, the influence are usually coming from more than one member states, and possibly from both allies and opponents at the same time.

Example 7.4 (Influences from More than One Member States) *The vote of US might be a synthesis process, a weighted sum of (referring positively to) own preference, allies' preferences (such as UK, Germany, Japan, etc), and (referring negatively to) opponents' preferences (such as Russia, China, etc). This synthesis process is universe for nearly all states.*

¹¹Which means that the more his enemies oppose some bills, then the state would support more on these bills, or in other words, the bills are more adverse to the preference of his enemies, then the state would possibly favor more.

As same with the CP-nets, the social influence models are also not enough to model the integral SC voting process, the discussion about dependencies among bills is lacked in the traditional social influence models, which would discuss a multi-period or multi-issue decision-making, but assumed issues are independent from each other, and have not dealt with the combinatorial domains. Therefore, the two models of CP-nets and social influence models need to be combined to describe the complex influence in UN SC voting. For a member state's voting, besides positively influenced by allies, friends, negatively influenced by enemies, opponents, and also positively influenced by successful/satisfied votes and negatively influenced by failed/unsatisfied votes on the former issues.

UN SC as a Combinatorial and Collective Decision-Making System

We express the UN SC voting system in the framework of CP-nets combining with social influence models, each agent represents one member state, and each issue represents one bill, and each variable represents a decision of one member state on one bill. There are dependencies among different bills, and also influences among different member states. And for the domain of each bill, according to the voting rule of UN SC, the full domain of choices are *affirmative*, *absent*, *negative/veto*. For the permanent members with veto power, the negative vote means veto.

Besides, we should also consider about the abstentions and partial domains in the circumstance of UN SC voting, as these two “special” phenomena are also very common in UN SC situation.

Abstentions in UN SC Voting

It happens that part of members would possibly choose to be absent of the meeting or be present but not vote for some bills. The abstentions are due to many reasons (including but not limited to):

- Firstly, assume facing two groups with conflicting claims for one bill (one side support strongly and the other side oppose strongly), and the state does not want to offend any side, then a wise choice may be to be absent.
- Secondly, assume from own state interest, the member state would be inclined to vote negative (or affirmative) on one bill, but which would make him stand by the opposition of the named justice and fairness and face the pressures from international community, then the member state would possibly choose a promised plan, to just vote absent, especially for the permanent member with veto power, directly vote negative to veto the bill as own wish would face too much pressure from opponents and international community.

Example 7.5 (US's Abstentions due to International Pressure) *There are many bills “against” Israel proposed in UN SC, though the US usually stand by the side of Israel but just vote absent but not vote veto.*

Example 7.6 (USSR's Abstentions due to International Pressure) *For the bill about the UN command (against the North Korea in 1953), the representative of former Soviet Union was late for the voting, which might be possibly on purpose, on the following, Soviet Union secretly provide military assistance to China and North Korea against the UN command. What is more interesting, the former Soviet Union used to let its union republic Ukraine (whose vote is totally controlled by Soviet Union) to vote negative for one bill, but Soviet Union itself just vote absent, to both reveal the opposing attitude, and also not goes severely against the majority (totally veto the bill).*

- Thirdly, some bills and their outcomes are unimportant or indifferent for some member states, which are unrelated to their core state interest, and it is also possible for them to be absent of the meeting or not vote.

Partial Domains in UN SC Voting

There is one peculiarity between the abstentions and partial domains for the UN SC voting system that one choice “absent” in the domain equals to being absent of the meeting, at least mathematically in the computation. For the partial domains in UN SC situation, it is mainly due to **Unacceptance** but not **Nonexistence**, as theoretically, every member state can vote any affirmative, negative and absent on every bill according to their own willingness, the choice domain would not be partial due to constraints. Actually, what lead to the partial domains are subjective factors but not objective factors, there are many reasons (including but not limited to):

- Firstly, assume one bill which is totally “disgusted” by the member state, and who would never accept this bill passed, therefore, the choice domain for him would be parted as $\{absent, negative\}$, the member would never even consider about voting affirmative. On the contrary if it is totally keened by the member state, then the choice domain would be parted as $\{affirmative, absent\}$, which means the member state can not accept this bill failed. This unacceptance is from the own preference.
- Secondly, assume one bill which is strongly “detested” by the member state’s allies or even alliance leader, and who would never accept the bill passed, therefore, to consider the preference his allies, or compelled by the pressure from his alliance leader (like US in the NATO, and former Soviet Union in the Warsaw Pact), then the choice domain for him would be parted to $\{absent, negative\}$, the member could not consider about affirmative vote, even thought inner heart he favor the bill very strong, but could only vote absent at most. On the contrary if it is totally keened by his allies or alliance leader, even from own preference, the member state dislike the bill, but the choice domain has to be parted to $\{affirmative, absent\}$, and could only vote absent at most. Above case is applied specially for the small or medium countries. And this unacceptance is from others’ preference, and similar to the concept of peer pressure, obedience, conformity and so on.
- Thirdly, assume one bill which is strongly preferred or disliked by the member state’s enemies or arch rival, the choice domain might also be partial, just in the contrary way compared with above.

Chapter 8

Testing the Models of Influence with UN SC Gaming Example—by the Approach of Qualitative Case Studies

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- 8.1 Great Powers Worship the Reputation—An Example of Pattern 1 Intra-influence of Decision to Decision
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- 8.4 Be Close to Your Friends When Your Enemies be Close to Theirs—An Example of Pattern 12 Inter-influence of Structure to Structure
- 8.5 How to Deal with Contradictory Multipartite Relations: Balance Strategy and Revenge Strategy—An Example of Pattern 15 Intra-inter Influence from Decision to Structure
- 8.6 How to Maintain a Relationships: Unilateral Continuity or Mutual Interaction—An Example of Pattern 24 Inter-intra Influence of Structure to Structure

After we build the system of patterns of influence, and before we use these influence models to do simulation experiments, a prerequisite question would be: do these (new) patterns of influence exist in real-world and could be used to solve real-world problems, and describe the real-world process?¹ The answer would be Yes, the influence models could be used in many social disciplines or social fields, for example of the UN Security Council voting, which is a very critical question of international politics (as the Security Council is the hardcore of the United Nation, and the core field of great powers' game), and is also a very typical multi-issue and multi-agent (combinatorial and collective) decision-making example. The patterns of influence could be employed to describe the influencing relations among member states, and the influencing (independent) relations among bills of the UN Security Council.

In order to prove that, we can adopt qualitative or quantitative approach. A most common qualitative approach is the case studies, which is to propose factual examples and do thick descriptions, in this situation is about how member states interact and influence with each other. In the thesis, a series of factual examples in real-world international politics are found to provide evidences for each pattern of influence.

¹The theoretical and practical meaning are both important for the models of influence, in particular for an interdisciplinary study between computer science and social sciences.

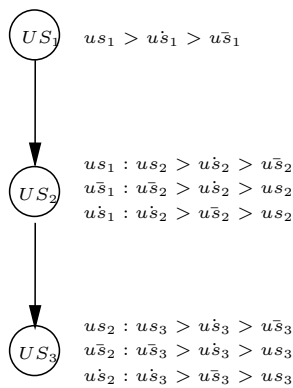


Figure 8.1: The Example of Influence Pattern 1 Intra-influence of Decision to Decision

8.1 “Great Powers Worship the Reputation”— An Example of Pattern 1 Intra-influence of Decision to Decision

The intra-influence from decision to decision is the dependencies among issues (within one agent) originally discussed in CP-nets by cp-statement . In UN Security Council voting, it is easy to find that member state’s current votes would be influenced by their own former votes on correlated bills. One common form of this influence is the latter choice “follow” the former choice. For a series of bills pertaining to the same topic, the member state is inclined to be persistent on his attitude or opinion, especially for the super powers and great powers, who are more independent, not easily be influenced by others, and mainly make choices on one’s own standpoint, and furthermore, who worship more of own reputation, would not change easily and frequently, and hope to give no matter allies or opponents with stable expectations and reliable reputations. In some cases, even when their interests and preferences are already different (or changed) from before, they would still try to be persistent on the claim and keep the same opinion, only if it would not severely count against their current interests and preferences.

Example 8.1 (US Worships his Reputation?) *For one example as shown in figure 8.1, assume US faces three correlated issues 1,2,3 (with the same topic), as a super power and the leader of “western world”, it is reasonable for him to be persistent on the votes, if vote affirmative on former bill(s), then would be inclined to vote the same on latter bill(s). Assume if S is the vote of one member state, s represents the affirmative vote, \dot{s} represents the absent (vote), and \bar{s} represents the negative vote.*

US has been advocating the “Human Rights Above Sovereignty” all over the world for a long time, and use this argument to intervene the domestic affairs of many other countries (especially for those countries disliked by US), and even eager to intervene some “potential replacer” like China who also chosen different developing road and political system from US; besides, US also has already started wars to violently overthrow a few countries’ governments such as Iraq, Afghanistan.

There is a negative example of bad result of US “betraying” the sustained claims, in the Ukraine (Crimina) Crisis, due to the long-time dispute between pro-western and pro-Russia² camp, a severe conflict broke out since January 2014, the president Yanukovich of Ukraine who is pro-Russia had to fleet away from Ukraine to Russia, and the pro-western camp control the central government of Ukraine. Then the people of Crimea hope to join Russia, as nearly 60% of

²More specifically, there is a harsh dispute on the relations of Ukraine with the European Union, whether go closer to the European Union or not.

citizens of Crimea are Russian ethnic, the Crimea used to belong to Russia Federation and was given to Ukraine as a gift (to celebrate the alliance between Russia and Ukraine) by the leader of former Soviet Union (Khrushchev) in 1955, furthermore, the living standard in Russia is much higher than in Ukraine, once the Crimea is integrated into Russia, the pension of people in Crimea would be compatible with the Russian standard, and which would be 3 times than before. Due to the historical bond and the realistic interest, most of citizens in Crimea hope to be independent from Ukraine or directly join in Russia. And of course, the central government of Ukraine can not tolerate the disintegration, and US and Europe Union also are hard to accept the Crimea (which has a very important strategic status) leave from Ukraine and back to Russia.

When the parliament of Crimea passed the resolution of the merge to Russia, and the referendum has also passed by the people of Crimea. US is very unsatisfied and oppose to the referendum of Crimea, claiming that the referendum of Crimea has to be agreed by the central government of Ukraine. However, this attitude is totally contradictive to US's former claim “Human Rights Above Sovereignty”, but has totally changed to “Sovereignty Above Human Rights”, which means that the citizens of Crimea can not determine their destinies by their own. Therefore, the US's reputation has been severely doubted on this event, US would play double standard totally according to own interest. People from other countries would think that, if US want to intervene some countries, then claiming “Human Rights Above Sovereignty”, the war to Iraq and Afghanistan and so on are not invasion, just to liberate their people, but if US does not want some other countries to be intervened (such as Crimea, most of the people are even Russian Ethnic, but the US will loudly claim “Sovereignty Above Human Rights”, and condemning Russia's invasion. The “double standard” (or the inconsistent or unconfordable attitudes) for the same topic would severely harm a state's reputation, and a sensible leader would not easily reverse his long-term advocacy. Above is a very typical negative example that unstable attitude would generate a big cost, therefore, for most of cases, the member state, especially a super power or a great power, are inclined to insist on their former attitude on the same topic, in another word, to positively refer to or be influenced by the former choices³.

8.2 “Side with Allies and Go against Enemies”— Example of Pattern 7 Inter-influence from Decision to Decision

The inter-influence from decision to decision is the influences among agents (within one issue) discussed in influenced CP-nets [Maran et al., 2013] by ci-statement. In the UN Security Council voting, it is very common for the influence from one member state to another member state, no matter it is the positive influence from allies, or the negative influence from opponents. In order to maximum own state interest, the member states would try to unite with some states (with the same ideology and common interest), further forge an alliance, to oppose to some other states, or an opponent alliance (with contradictive ideologies or conflicting interests), just like the confrontations between Entente Countries and the Ally Countries during the World War I, between Fascist Alliance and Anti-Fascist Alliance during the World War II, and between NATO and Warsaw Pact during the Cold War, and many other examples. Proposed three typical examples of influence from one member state to another member state in the UN SC as following:

Example 8.2 (Positive Influence from US to UK) *US and UK have close historical, cultural, and blood bonds, and share the same langue as English, whom natively have the familiarity or*

³It should be noticed that, sometimes it is not the change of attitude on the same topic means agent be influenced, sometimes it is the same attitude means agent be influenced, for example, the interest of the state for the similar affairs has changed, but in order to be persistent (to protect the reputation), the agent would still keep the same attitude (obeying to former choice and influenced by former choices).

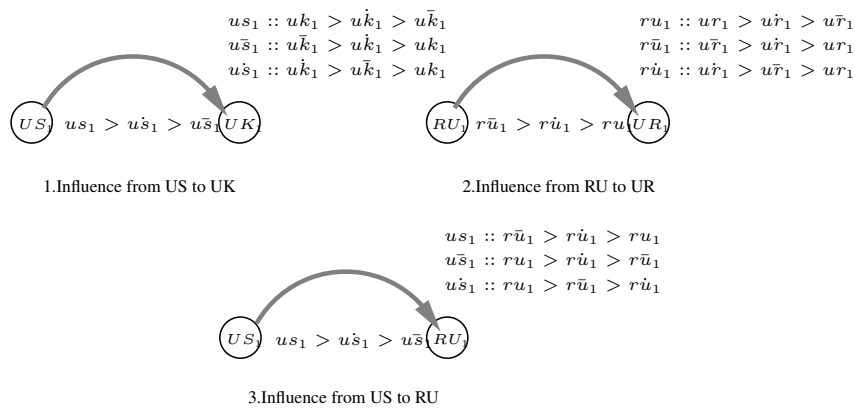


Figure 8.2: The Example of Influence Pattern 7 Inter-influence from Decision to Decision

friendliness between each other. Furthermore, due to the political reality, US and UK belong to the same military alliance NATO, and US is the leader of the alliance, even the leader of the whole capitalist bloc or the western bloc⁴. It is highly reasonable to assume that there is an influencing relation between US and UK, and considering the status as leader and the national power of US (compared with UK), it is also rational to assume that it is more possible for the influence from US to UK than from UK to US, namely, it is more possible that UK's decision is influenced by US's than the US's decision is influenced by UK's. This typical influence also could be proved in many voting records, such as the vote of UK follows closely with the vote of US on Iraq issue, especially compared with other US's allies, who acted contrary to US, like France, Germany, etc. As shown in sub figure 1 of figure 8.2, if US vote affirmative, then UK would be inclined to vote affirmative as well, but if US vote negative, UK would be inclined to vote negative either, which is a very typical positive influence from one agent to another.

Example 8.3 (Positive Influence from the Soviet Union to Ukraine) Ukraine used to be an union republic of Soviet Union. As initially establishing of United Nations, in order to appease the deeply imbalance felt by Soviet Union (because the majority of UN members, especially the UN SC members⁵, belongs to the capitalist bloc or the western bloc), a compromised arrangement was agreed that Soviet Union obtained three votes (one for the Union, one for Ukraine, and one for Belarus). What is interesting is, both Ukraine and Belarus were the union republics and the constituent part of Soviet Union, but not independent sovereign states. Therefore, the votes of UN representatives of Ukraine and Belarus are absolutely controlled by the central government of Soviet Union, and in another word, the preferences/votes of the two union republics are completely influenced by the preference/vote of the Soviet Union. This absolute influence could be approved by the voting records, when the Ukraine and Belarus were elected as UN SC member, their votes are highly congruent with the Soviet Union's vote, there was never a conflicting attitudes between Ukraine, Belarus and Soviet Union, and what is more interesting is, sometimes, the Soviet Union let the two "little brothers" took some "special tasks", for example, the Soviet Union used to vote absent while let the Ukraine vote negative, which might be because the Soviet Union not only wanted to show the opposing attitude, but also not wanted to directly veto the majority, to

⁴"The Western Bloc or Capitalist Bloc during the Cold War refers to the countries allied with the United States and NATO against the Soviet Union and its allies. The latter were referred to as the Eastern Bloc, a more common term in English than Western Bloc. The governments and press of the Western Bloc were more inclined to refer to themselves as the 'Free World' or 'Western World' " [Luke, 1999].

⁵For the five permanent members, there are four US, UK, France and China (before 1971) all belongs to the capitalist bloc.

8.3. “DIFFERENT INFLUENCING RELATIONS TOUCH DIFFERENT SENSITIVE NERVES”¹²³

undertake too much pressure (as one negative vote from Soviet Union would totally fail the bill).

Example 8.4 (Negative Influence from US to Soviet Union) *Due to the conflicts in state interests, economic and political systems, ideologies and so on, US and Soviet Union had been confronting with each other during the Cold War. It is naturally for one state to oppose to what his opponents prefer, and it is likely your preference has been negatively influenced by your opponents. The nonconformity between US's votes and Soviet Union's votes are very common in SC voting records. To specifically describe the psychologies in negatively influence, assume US prefer to vote affirmative on a bill, if Soviet Union initially prefers to vote negative, which means the bill is beneficial to the interest of US but harmful to the interest of Soviet Union, and Soviet Union knows the attitude of US or observes the vote of US, then of course Soviet Union would be more inclined to vote negative; and if Soviet Union initially prefer to vote absent, which means the bill passed or not affects little on the interest of Soviet Union, but due to the opposing relationships between two states, it is also possible for Soviet Union to adjust his preference and vote negative opposing to what US; furthermore, if Soviet Union also initially prefers to vote affirmative, the affirmative vote are both the most preferred choice for US and Soviet Union independently, but the preferred degree or interest gained are still different, for instance the interest of bill passed for US is 100 units, of bill failed is 0, but for Soviet Union is only 10 unites, of bill failed is also 0, therefore affirmative vote would be the most preferred choice for both states independently. While on some severe confronting situations, if Soviet Union acquires the information of prospective enormous interest gained by US, considers the interest gap and the opposing relation, it is highly possible for Soviet Union to vote negative even opposing to his initial preference.*

8.3 “Different Influencing Relations Touch Different Sensitive Nerves” —An Example of Pattern 11 Inter-influence from Structure to Decision and Structure

The inter-influence from structure to decision and structure is a newly proposed pattern of influence by endowing the structure with an influencing role. UN Security Council voting as a very typical international political gaming, it is full of interactions, influences, aligning and confronting among states. And these states' strategies of following, opposing, alliance formation and so on would profoundly affect own or other state interests, and possibly inducing the change of their decisions. Therefore, the structures of influence (expressed as the influencing relations) among member states would affect the decisions, and usually different structures of influence would touch the sensitive nerves of different member states and leading to different reactions.

Example 8.5 (Influencing Relations between US and South Korea Touch North Korea?) *For the influencing relations among US, Japan, South Korea, North Korea and China (respectively abbreviated as US, JP, KR, KP, CN), as shown in the figure 8.3.*

Firstly for the influencing component (the influencing relations among US, Japan and South Korea), US as a super power and the leader of “Western World”, has extensive and enormous influences on many other states, especially his allies or “sidekicks”. Two typical “sidekicks” in US's Asia-Pacific strategy are Japan and South Korea, who both respectively signed bilateral military alliance treaty with US. But for these “little brothers”, they do not exactly follow US from time to time. Sometimes they behave better and sometimes they behave worse. For instance of Japan, when his relationships with neighbour countries are worse, such as the disputes and conflicts with China and South Korea⁶ becoming serve, then he would feel unsafe and would possibly

⁶Japan usually has bad relations with his neighbours, such as South Korea, China and North Korea, a very important

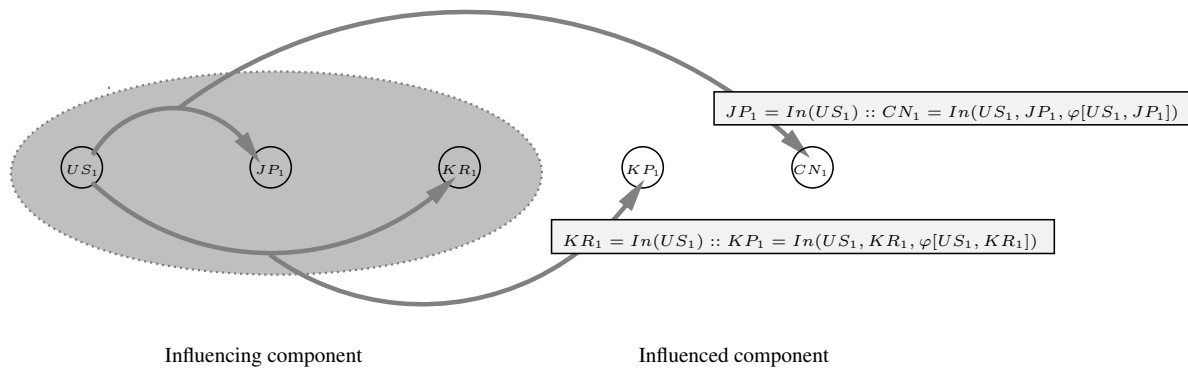


Figure 8.3: The Example of Influence Pattern 11 Inter-influence from Structure to Decision and Structure

behave better and follows more closely with US. However, when the periphery international environment is more calm and peaceful, or the economy and strength of Japan has been going well or even stronger⁷, or the inconformity of states interests between Japan and US have been intensified⁸, under these circumstances, Japan would behave in own ways and not follow US that closely. Therefore, even for the states belong to the same alliance, the influencing relations among allies are sometimes stronger, but sometimes weaker, and it is similar for the influencing relation from US to South Korea (which has going closer and closer to China and makes US very “headache”).

Secondly, for the influenced component (leading to different decisions of China or North Korea), as usually the aligning relations built by US and other states have a (focusing) target, for example NATO confronting former Soviet Union and nowadays Russia, and the “Asian version” NATO (the military alliances between US and Japan, between US and South Korea) aiming at China, North Korea, and Russia. Therefore, the structure of influence between US and his allies (expressed as influences from who to who, and how is the intensities and frequencies of influence and so on) would affect other states. For instance, when South Korea follows US not that closely and even has “rebels” with US, the “sworn opponent” of South Korea, the North Korea would feel ease and the sensitive nerve would not be “touched”, however, if South Korea follows US very closely, and plays exactly correspondingly to US’s strategy, and the relations between South Korea and US has become very intimate, then North Korea would feel nervous and would likely adopt many counter-measures, such as threatening to do nuclear tests, and to launch missiles and so on. Likewise, when Japan follows US very closely and play a good teamwork (for example US support Japan firmly on the Diaoyu Islands dispute with China, and states that the Diaoyu Islands apply to the US-Japan Security Treaty), then other related countries, such as China, would be “touched” and then do reactions.

8.4 “Be Close to Your Friends When Your Enemies be Close to Theirs” —An Example of Pattern 12 Inter-influence from Structure to Structure

The inter-influence from structure to structure is a newly proposed pattern of influence by endowing the structure with both influencing and influenced roles. In international politics, in order

reason is that Japan denies on the historical reality of World War II and Japan glorifies his invasion in the World War II, and also about the territorial disputes on Diaoyu Islands with China and on Lone Islands with South Korea, and so on.

⁷For example of the gold era of Japan in 80s of last century.

⁸For example Japan hope to strengthen military force and even be eager for the nuclear weapons.

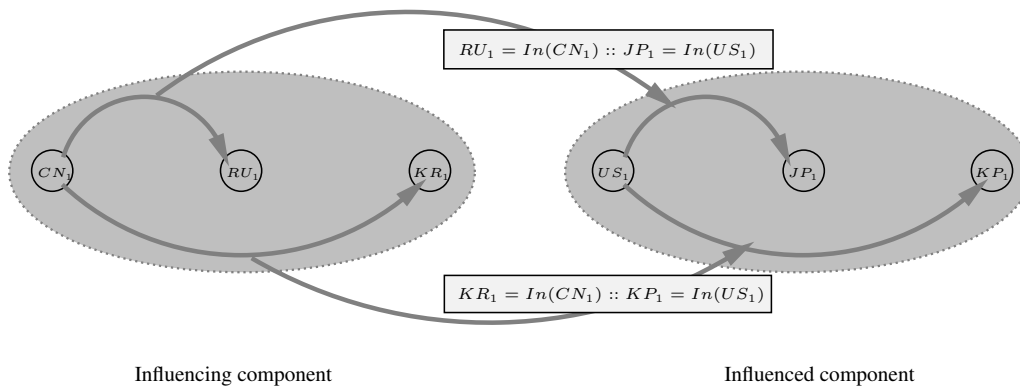


Figure 8.4: The Example of Influence Pattern 12 Inter-influence from Structure to Structure

to maximum own state interests and suppress their opponents, the states would form alliances (or friendships) to unite and strengthen force, and to confront the challenge from their opponents and alliances united by them. Only if the states exist, then the cooperations and alignments, and the conflicts and confrontations among states (based on the similarities or differences on state interests) would never stop. Furthermore, every state will vary his strategy ceaselessly, and adjusting his relations with other states constantly, to cope with the changing situation and pursue the maximum of state interest. One common strategy is, when your enemies go closer to their friends, you would better also go closer to yours either.

Example 8.6 (China and Russia Come Closer, then US and Japan Come Closer?) *For the influencing relations among China, Russia, South Korea, US, Japan, North Korea (abbreviated as CN, RU, KR, US, JP, KP), as shown in figure 8.4.*

Firstly for the influencing component (the influencing relations among China, Russia and South Korea), China as the rising super power who is widely recognized as the potential rival to replace US⁹, no matter what strategy he adopts, it always easily touches the sensitive nerve of US. Actually, US takes many counter-strategies hoping to suppress the rise of China, or at least set obstructions and lag the rise of China, for example of the Asia-Pacific Rebalance strategy performed by US and his allies to re-balance the power and influence of China (especially in Asia-Pacific), and the subtext for the re-balance strategy is that “China is too much strong and needed to be balanced”¹⁰ In fact, no matter it is an important state decision of China, or his relations with other states (with who becoming better or worse), would all easily touch the sensitive nerve and produce an influencing effect on US.

Secondly for the influenced component (leading to the influencing relations from US to Japan, or US to North Korea), due to the historical bond¹¹ and realistic demand¹², China and Russia become closer and closer, and play a very good teamwork toward many international affairs (such

⁹There is a concept as “Chinese Century”, which suggests the possibility that the 21st century will be dominated by the People’s Republic of China, similarly to how “the American Century” refers to the 20th century and “the British Century” refers to the 19th century [Rees-Mogg, 2005]. The phrase is used particularly in the assertion that the economy of China will overtake the economy of the United States as the largest national economy in the world [Elliott, 2007], a position it held from around 1000 AD-1700 AD or 221 BC-1830 AD, depending on source [Dahlman and Aubert, 2001].

¹⁰Though US not admitting publicly it is targeting at China, but it is widely known that only China is worth the efforts of US, to put in more military forces and perform dollar diplomacy in Asia-Pacific area even though facing the poor economics.

¹¹The history of Russia as used to be the former Soviet Union and his help to China’s industrial construction, and so on, which did affect several generations of China and Russia.

¹²Both facing the same opponent as US, and both facing uneasy pressure and challenge from US.

as the Syria issues, Crimea issues, etc), and form a very special “quasi-alliance” relationship¹³. This “sweet” relations between China and Russia would of course make US and Japan feel unhappy. Specifically in Northeast Asian region, the “alliance” between China and Russia would enormously reshape the geopolitical pattern, and make Japan feel very nervous and unsafe. Therefore, facing the constantly bettering relations between China and Russia, it is reasonable for US and Japan to become closer and keep in step with each, together against the challenge. On the other, with the reviving of Chinese economy and culture, and China and South Korea sharing more tacit understanding, South Korea has gone closer and closer to China¹⁴, then North Korea would be nervous and angry, and take revenge trying to seek for the reconciliation with US, and hope to improve the terrible relation with US. For instance, Jeong eun Kim invited Dennis Rodman to visit North Korea, in order to release a good will, however, obtained no respond from US officials.

8.5 How to Deal with Contradictory Multipartite Relations: Balance Strategy and Revenge Strategy—An Example of Pattern 15 Intra-inter Influence from Decision to Structure

In international politics, it is common for some small countries who survives difficultly in the crack between two or more super (great) powers, playing the balance diplomatic strategy, trying best to offend neither side and please all sides, and would not totally lean to any side. However, on cases a state has to express an explicit attitude, to offend one side and stand by the other side, and take the “revenge” from the offended side. As an inexplicit attitude might offend both sides.

8.5.1 Balance Strategy: Offend Neither Side, or Offend One Side then Please the Same Side Later

Example 8.7 (Triangular Relations among Ukraine, US and Russia) *The pro-western force in Ukraine claim to join the European Union and even NATO, but the pro-Russia force emphasis on the the hardly snipped economic¹⁵ and blood tie¹⁶ between Ukraine and Russia. With the alternative rules of two forces, the Ukraine has been playing the balance strategy between western world (European Union, US) and Russia since the independence, keeps intimate relation with US and European Union for a period of time, and also go close with Russia for a period of time, but hardly totally leans to any side.*

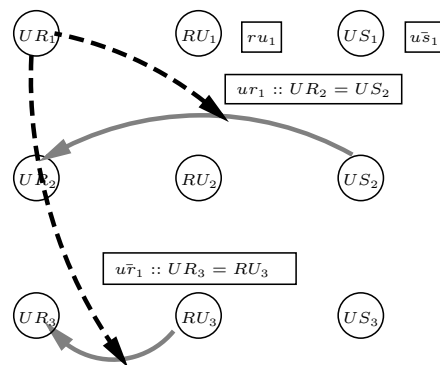
As shown in sub figure 1 of figure 8.5, a collective voting and influencing relations among Ukraine, Russia and United States (as the representation of Western world) is displayed (in which UR is the abbreviation for Ukraine), assume there are three sequential bills 1,2,3 for voting, bill 1 is a very tricky and sensitive issue, and Ukraine is irresolute to make decision on it, because if Ukraine votes negative, it will offend Russia (as whose preference is topped as affirmative and hope the bill passed), on the contrary, if Ukraine votes affirmative, it will then offend US (because whose preference is topped as negative and expect the bill failed), therefore, no matter vote affirmative or

¹³“Diplomatic relations between People’s Republic of China and the Russian Federation dramatically improved after the dissolution of the Soviet Union and the establishment of the Russian Federation in 1991. The two countries share a long land border which was demarcated in 1991, and they signed a Treaty of Good-Neighborliness and Friendly Cooperation in 2001. On the eve of a 2013 state visit to Moscow by Chinese President Xi Jinping, Russian President Vladimir Putin remarked that the two nations were forging a special relationship” [Wikipedia, 2015].

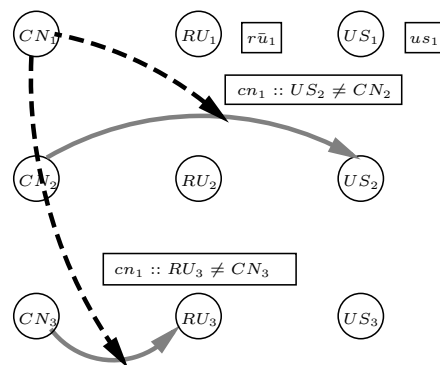
¹⁴Korea used to be the vassal state of China for a long time, the good relations between South Korea and China would easily make US think too much.

¹⁵Ukraine used to be one part of the whole Soviet Union planned economic system, the standard of many productions of Ukraine can only be integrated with Russia.

¹⁶Part of Ukrainian belongs to the Russian Ethnic, especially in the eastern region.



1. Influence from decision (of UR)
to structure (from US to UR)
or to structure (from RU to UR)



2. Influence from decision (of CN)
to structure (from CN to US)
or to structure (from CN to RU)

Figure 8.5: The Example of Influence Pattern 15 Intra-inter influence from Decision to Structure

negative, Ukraine has to displease either one, however, to maximum own interest, Ukraine would try best to keep a good relationship with both sides.

One answer is that Ukraine just be absent on bill 1, and hope it would not offend any side, which is the strategy of “**Offend Neither Side**”, however, it should be noticed that sometimes this kind of strategy “Offend Neither Side” would fail and factually offend both sides, so it does not work on all cases.

Therefore, Ukraine might have to choose his side, and express an explicit attitude (either affirmative or negative, support which side and against which side). However, even though, the state can still find ways to fix the “wounded” relationship with the offended side. As shown in sub figure 1 of figure 8.5, if Ukraine vote affirmative on bill 1 and displease US, then Ukraine would find chance to “flatter” to US, and for the following bill 2 which is beneficial to US (and also not very crucial to Russia), the Ukraine would likely vote affirmative correspondingly on bill 2, which can be perceived as a positive influence from US to Ukraine (one specific form is $ur_1 :: UR_2 = US_2$, the Ukraine vote just the same as US). On the other side, if Ukraine vote negative on bill 1 and displease Russia, then Ukraine would try to “play up to” Russia, for the following bill 3 which is beneficial to Russia (and also not very critical to US), Ukraine would be more inclined to vote affirmative on it purposively, which also can be perceived as a positive influence from Russia to Ukraine (one specific form is $ur_1 :: UR_3 = RU_3$, the Ukraine vote just identical with Russia). Above is then the strategy of “**Offend One Side then Please the Same Side Later**”.

We build a general table to discuss this pattern of influence and relevant strategies as following, assume there are three agents as A, B, C , and agent A, B are two conflicting agents influencing on agent C , and agent C hope balancing between two agents A or B .

	C's vote on Former Issue	C's Vote on Latter Issue
1. Offend Neither Side	<i>Absent</i>	P_C
2. Offend and then Please one Side	P_A	P_B
	P_B	P_A

8.5.2 Revenge Strategy: Offend Neither Side, or Offend One Side then Wait for the Revenge from the Same Side

Example 8.8 (Triangular Relations among US, China and Russia) *The triangular relations among US, China and Russia is very subtle, as shown in figure 8.6, China and Russia are more like friends (who become closer and closer under the common pressure from US¹⁷, and form a quasi-alliance relation); Russia and US are more like opponents or enemies (the bilateral relation between them has become more and more deteriorated due to the conflicts in Syria issue, Crimea issue and so on), and US and China are kind of like “Frenemies” (they have a very intimate economic bond but so different on politic systems, cultures and ideologies, and they are also the competitors for the future supremacy). Though US and Russia have been offending each other openly, and China and Russia never cover up their intimate friendships, but the relations between US and China are bewildering, nearly never publicly opposing to each other (since the reform and openness of China), and also hardly saying as true friends. Therefore, the current triangular relations among China, US and Russia (which might be the most important triangular relations in international politics¹⁸) is likely as shown in sub figure 1 of figure 8.6, in which the relations between US and China is*

¹⁷Such as the NATO against Russia and the Asian “NATO” (US, Japan, South Korea) against China.

¹⁸What is interesting is, Chinese scholars think there is only one super power as US, US scholars think there are two super powers as US and China (the G2 system), and Russia scholars think there are three super powers as US, China and Russia.

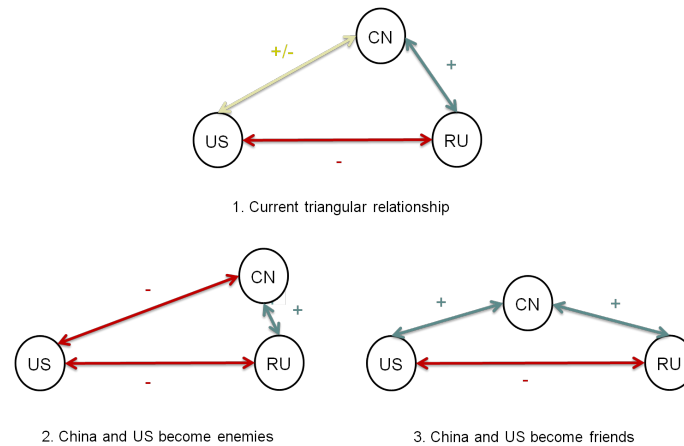


Figure 8.6: The Triangular Relationships among US, China and Russia and the Possible Evolutions

a *X-factor*, which would possibly become better, or possibly go worse in the foreseeable future, and two possible dynamics and evolutions of the triangular relations are shown in sub figure 2 of figure 8.6 (in which China and US become enemies and US would have to face the alliance of China and Russia) and 3 (in which China and US become friends and China would become the mediator of US-Russia relation and might be flattered by both two countries).

As shown in sub figure 2 of figure 8.5, an example of collective voting and influencing relations among China, US and Russia is displayed, assume voting on three sequential bills 1,2,3, for instance bill 1 is about preventing the disintegration of Ukraine and banning the intervene from Russia (on Crimea, and other Ukraine regions eager to be separate from Ukraine and integrate into Russia), which is very sensitive to US and Russia, and the two states have severely conflicting claims (US threatened to sanction Russia and indeed took a series of measures). However, the bill is not that sensitive to China, and the settlement of Ukraine issue would not affect the state interest of China directly and enormously like Russia and US. However, the vote-decision of China (as one of the most influential and powerful countries), would still touch the sensitive nerves of other states (especially US and Russia). Specifically, as China is the “Ally” of Russia, if China vote affirmative, it will definitely offend Russia, as Russia hopes heavily China as his “best friend” can also vote veto together with him, or at least vote absent, to share and relieve the pressure of Russia from other states¹⁹, but if China vote affirmative, Russia would highly possibly feel disappointed or even betrayed. However, if China vote veto together with Russia, it might offend and enrage US and other western countries, claiming that China has gone near and near to Russia and stand at the “wrong” or even “evil” side. Therefore, no matter China vote affirmative or negative, he would always displease one side, and possibly have to undertake the “revenge” in return.

To prevent the worsening of relations, one answer is to just be absent, and hope it would not offend any side, but it does not work all times.

On many case, the state has to choose his side, and offend one side and support the other side, to express a clear attitude, just as the old saying “Of two evils choose the lesser”. Then if China vote affirmative (it might mean China think offending US is worse than offending Russia, at least on this bill), then it is possibly for Russia to find chances to “revenge” at China, such as on bill 3 (which is critical to China but not Russia), then Russia would vote against China on purpose (one specific form is $cn_1 :: RU_3 \neq CN_3$; but if China vote affirmative, it is possibly for US to find chance to “punish” at China, such as on bill 2 (which is crucial to China but not to US) , then US

¹⁹As if only Russia votes veto but all other SC members vote affirmative, the pressure of Russia would be very heavy and fall into the absolute minority (hindering the willingness of majority), and even be accused as nonmoral and injustice, therefore, Russia would expect a lot China would vote veto together.

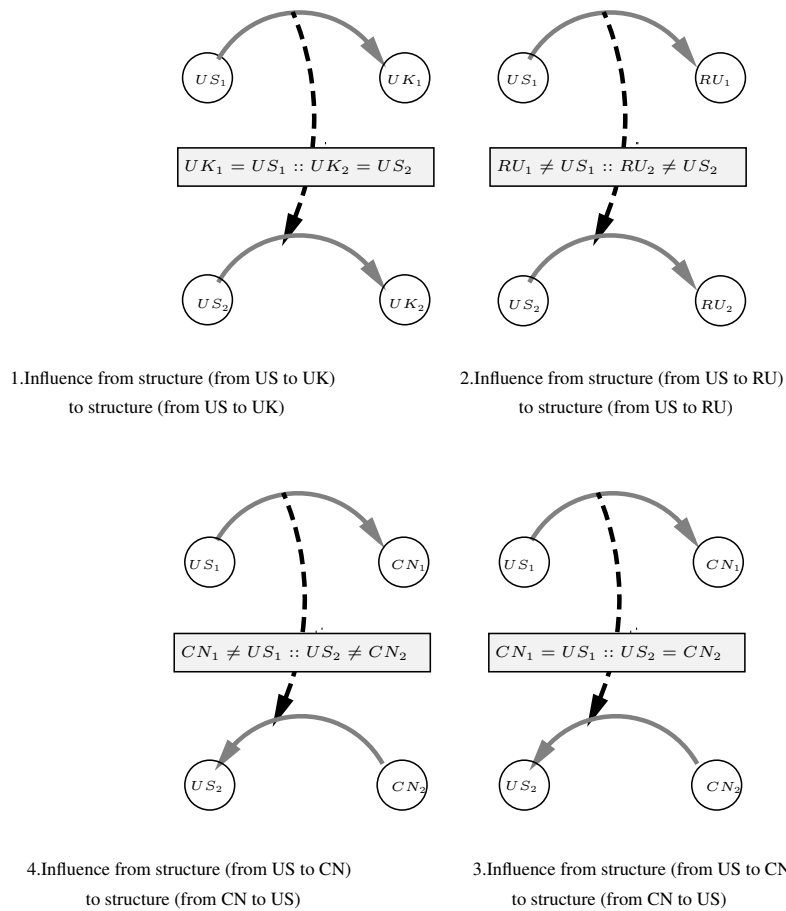


Figure 8.7: The Example of Influence Pattern 24 Inter-intra influence from Structure to Structure

would vote against China purposively (one specific form is $cn_1 :: US_2 \neq CN_2$).

Similarly, we build a general table to discuss this pattern of influence and relevant strategies as following.

	C's Former Vote	A's Latter Vote	B's Latter Vote
1. Offend Neither Side	<i>Absent</i>	P_A	P_B
2. Offend and Wait for	$\neq P_A$	$\neq P_C$	P_B
Revenge from one Side	$\neq P_B$	P_A	$\neq P_C$

8.6 How to Maintain Stable Relationships: Unilateral Continuity or Mutual Interaction—An Example of Pattern 24 Inter-intra Influence from Structure to Structure

8.6.1 Unilateral Loyalty or Bear Grudge: Once I Follow You Then I Always Follow You, Once I Oppose to You Then I Always Oppose to You

In international politics, it is common to observe that the relations between states possess continuity, no matter it is the alliance relation (between allies) or it is the hostile relation (between enemies). Especially for the super powers and great powers who value a lot about reputations,

their relations with other states are very stable and foreseeable²⁰, which are usually determined by their traditions in the history, and it is hard to find them suddenly “changing faces” (unexpectedly turning hostile from friendly, or turning friendly from hostile).

Example 8.9 (Continuity of Followership: UK and US) *The alliance between US and UK is a typical stable alliance relation since 1940²¹, the two countries are bound together by shared history, an overlap in religion and a common language and legal system, and kinship ties that reach back hundreds of years. In current British foreign policy, the relationship with the United States represents the “most important bilateral partnership”; and the American foreign policy also affirms its relationship with Britain as its most important bilateral relationship. Though UK used to be the suzerain of US before its independence in 1776, but US became the largest economy in late 19th century and became the absolute leader of the western world after World War II. Therefore, for an intimate relationship between US and UK, it is reasonable to assume that on most cases it is UK following US but not US following UK, and more precisely, it is more usually that UK is influenced by US and supports US to achieve his strategic objective state interest, but not reversely. And after reviewing the history, UK does play a very well role of partner and “sidekick” of US, UK stands firmly and closely by the side of US, even on many controversial issues (such as the Iraq war), US was opposed by nearly all other states, even including the NATO allies such as France, Germany, but UK not only supported US verbally, but also directly sent army to aid the US military action.*

It should be noticed that the follow of UK with US are not all always beneficial to the state interest of UK on every individual issue, on some cases there are conflicts between the interest of US and own, but UK would still support US and “sacrifice” on own interest to a certain extent. Therefore, it is not only the common interest on every individual issue determining the following behavior of UK, but it is also the alliance and friendship in the history, even as traditions, determining the following behavior of UK. Though UK might sacrifice own interest to support US on some individual issues, but in the whole, and for the long-time, the special relationships with US is beneficial to the UK’s state interest. This is also very common in real-world daily life, if one person always behave totally according to own interest, and never concern about friend, he would have no friend, and eventually will not get what he wants. Only the person valuing the friendships and help friends evening sacrificing own when needed, then friends will help him to achieve what he wants.

As shown in sub figure 1 of figure 8.7, if UK follows with US on bill 1, then UK would also follow with US on latter relevant bill 2.

We build a general table to describe this pattern of influence from structure to structure (or from influencing relations to influencing relations) as following, assume there is one agent and his ally, and the preference for his ally is P_A , if on former issue the agent follows his ally, then on latter issue the following would continue.

	Ally’s Vote	Own Vote
On Former Issue	P_{1A}	P_{1A}
On Latter Issue	P_{2A}	P_{2A}

Example 8.10 (Continuity of Opponency: Soviet Union and US) *Besides the continuity of loyalty discussed above, it is also common for the continuity of confrontation. The confrontation between US and Soviet Union is a very typical and stable opposing relations. Since the establishment*

²⁰ As small countries will follow the super/great powers with good reputations, not changing easily and frequently.

²¹ “Anglo-American relations, encompass many complex relations ranging from two early wars to competition for world markets. Since 1940 they have been close military allies enjoying the Special Relationship built as wartime allies, and NATO partners” [Wikipedia, 2011].

of Soviet regime in Russia, then a mutual hostility between Russia and US began. During World War II, the two countries were briefly allies. But at the end of war, the signs of post-war mistrust and hostility began to appear, and escalating into the Cold War. US and Soviet Union opposed to each other on many issues, in Europe, in Cuba, in Afghanistan and many others, which is essentially due to the conflicts on ideologies, state interests and the rival for world supremacy. US and Soviet Union not only threatened each other in military force, but also oppose to each other in UN Security Council. US vetoed the bills proposed by Soviet Union or favor to Soviet Union, and Soviet Union also vetoed the bills proposed by US or favor to US, the UN SC representative of Soviet Union, Vyacheslav Mikhailovich Molotov, was even named as “Mr. Veto” by the western diplomats.

It should be noticed that the opponency of Soviet Union to US are not always due to the conflicting preferences on every individual issue. On some issues which are beneficial to US but might be non-harmful or irrelevant to Soviet Union, Soviet Union would still likely oppose to US and vote against US. Therefore, it is not only the conflicting interests on every individual issue determining the opposing behavior, but it is the hostile relation in the history, the long-term confrontations and mutual counteracts, and the deeply remembered antagonism determining the opposing behavior. As Soviet Union knew, I used to oppose to US many times and had long ago offended US, US would do the same facing the same situation, oppose to what I prefer.

As shown in sub figure 2 of figure 8.7, if Soviet Union opposes to US on bill 1 (a specific form as voting different from US), then Soviet Union would also oppose to US on latter relevant bill 2.

We build a general table to describe this pattern of influence, assume there is one agent and his opponent, and the vote for the opponent is P_O , if on former issue the agent opposed to his opponent, on latter issue the opposing would repeat.

	Opponent's Vote	Own Vote
On Former Issue	P_{1O}	$\neq P_{1O}$
On Latter Issue	P_{2O}	$\neq P_{2O}$

8.6.2 Mutual Favor or Mutual Harm: If You Play Nice to Me Then I Play Nice Back, If You Play Hard to Me Then I Play Hard Back

However, it is usual that a state would not be forever persistent on own former opinions and behaviors unilaterally and unconditionally. For the decision-makers of every state, they are people with feelings, with rationalities, and will weigh the gains and loss. It is hard to maintain a loyalty to your friend or your leader, if they never concern about your interest and even harm you repeatedly; and it is also hard to maintain a hostility to your enemies in mind, if they “requite evil with good”, not oppose to you like you oppose to them, and even do favor to you. Therefore, on many cases, the influencing relations among states are affected by their multi-phase mutual interactions, and how you treat others is determined by how others treat you, which is especially suitable to the delicate bilateral relationships with two who are neither purely friendship or purely hostility, and without a long-term traditions between them. Therefore, the developing trend of the relationship would be highly determined by the interaction between each other.

Example 8.11 (US and China: Mutual Favor?) A typical example is the “frenemy” relation between US and China, which has ups and downs, and is determined by both side’s actions, and how each side treats the other side.

As shown in sub figure 3 of figure 8.7, assume bill 1 is critical and beneficial to US, and US try to persuade China to vote the same with US (namely US expect a positive influence from US to China), and if China does vote the same with US, then for a following bill 2 which is critical and beneficial to China, then it is likely for US to play nice back, and also vote the same with China.

We build a general table to describe this pattern of influence (positive influence and then positive influence back) as following, assume there are agents A and B , firstly agent B supported (be positively influenced by) agent A on former issue 1 ($P_{1B} = P_{1A}$), then agent A would support (be positively influenced by) agent B on latter issue 2 ($P_{2A} = P_{2B}$).

$$\begin{array}{l} \text{On Former Issue} \\ \text{On Latter Issue} \end{array} \begin{array}{cc} \text{A's Vote} & \text{B's Vote} \\ \left(\begin{array}{cc} P_{1A} & P_{1A} \\ P_{2B} & P_{2B} \end{array} \right) \end{array}$$

Example 8.12 (US and China: Mutual Harm?) *As shown in sub figure 4 of figure 8.7, similarly assume bill 1 is critical and beneficial to US, and US try to persuade China to vote the same (namely US expect a positive influence from US to China), but if China vote against US (namely the influence come true but negatively), then for a following bill 2 which is critical and beneficial to China, then it is likely for US to take “revenge”, play tough back, and also votes the opposition expected by China.*

We build a general table to describe this pattern of influence (negative influence and then negative influence back) as following, firstly agent B opposes to (be negatively influenced by) agent A on former issue 1 ($P_{1B} \neq P_{1A}$), then agent A would oppose to (be negatively influenced by) agent B on latter issue 2 ($P_{1A} \neq P_{2B}$).

$$\begin{array}{l} \text{On Former Issue} \\ \text{On Latter Issue} \end{array} \begin{array}{cc} \text{A's Vote} & \text{B's Vote} \\ \left(\begin{array}{cc} P_{1A} & \neq P_{1A} \\ \neq P_{2B} & P_{2B} \end{array} \right) \end{array}$$

The Layout of Patterns of Influence with Case Studies

As discussed above, six patterns of influence are discussed by cases studies (with “thick description” on real-world examples), and these patterns are distributed in the patterns system as following (in which \mathcal{D} represents the decision, and \mathcal{S} represents the structure).

Table 8.1: Patterns of Influence Provided with Case Studies

	Intra-influence	Inter-influence	Intra-inter Influence	Inter-intra Influence
From \mathcal{D} to \mathcal{D}	Pattern 1	Pattern 7		
From \mathcal{D} to \mathcal{D} and \mathcal{S}				
From \mathcal{D} to \mathcal{S}			Pattern 15	
From \mathcal{S} to \mathcal{D}				
From \mathcal{S} to \mathcal{D} and \mathcal{S}		Pattern 11		
From \mathcal{S} to \mathcal{S}		Pattern 12		Pattern 24

Limited on space, although only 6 patterns of influence are provided with case studies, but they are distributed relatively dispersedly. On each of four categories of 1.intra-influence, 2.inter-influence, 3.intra-inter influence and 4.inter-intra influence, there are at least one patterns of influence being thickly described, therefore, the 6 discussed patterns possess a relatively good representativeness and typical significance, as patterns 11,12,15,24 are all newly proposed patterns with influencing or influenced structure, and patterns 15,24 represents the complex influences crossing both multi-agents and multi-issues.

Chapter 9

Testing the Models of Influence with UN SC Voting Data—by the Approach of Quantitative Matching Algorithm

Contents of Chapter 9

9.1 Test Sample: Passed Resolutions with at least One Different Voices

9.2 Test Method: Influence Pattern Matching Algorithm Design

9.3 Test Outcome: Influence Patterns Matching and Influencing Relations Ranking

Before we use the influence models to do simulation experiments, also test all these patterns of influence with quantitative empirical data under the context of UN Security Council voting¹ (which can be obtained from the official web database of United Nations):

<http://www.un.org/en/sc/documents/resolutions/>.

9.1 Test Sample: Passed Resolutions with at least One Different Voices

The data sample chosen to test the patterns of influence is all passed resolutions except the “*Assent and Consent*” (which means the resolutions voted affirmative by all SC members, or unanimously approved by all SC members) ones. The reason to choose passed resolutions is that each of passed resolution has complete voting record (the vote choice of each SC member) on the UN web database, but for those failed bills, the complete voting record are available only after 1994, there are no official record for the failed bills before 1994 (as shown in table D.1 in Appendix D.1, both passed and failed bills’ record are available only since 1994s). However, the scope of voting data chosen for test is the whole voting process since the establishment of United Nations (in 1946), to better investigate the influencing and interacting process of SC member states with

¹As the influence is represented as the change between initial preference and eventual behavior. For the data obtained from the official website of United Nations, one deficiency in testing influence is that, there are only eventual (or factual) vote data for each SC member, but lacking the initial preference data. However, this is basically inevitable, because it is nearly impossible to obtain all the initial preference data by formal or empirical methods. It is not feasible to acquire the intrinsic and genuine thoughts of every SC representative by questionnaire or investigation or some other approaches, and Furthermore, usually the representatives of states are not willing to reveal the true attitude, which are mostly pertain to national interest and national secrets. Therefore, we have to use the only vote data (the factual vote) to test the models of influence. Actually, after checked and reviewed a lot of collective voting empirical data, only having the factual vote, but lacking the initial inner preference is the common fault, compared with all other available data, the UN SC voting data obtained from the official website of United Nations is the most complete and detailed, not only including each member’s vote for each of more than thousands of bills, but also with the detailed record of the content of the bill, and relevant voters’ claims.

the integral historical background. Therefore, to guarantee the uniformity of chosen data at any year and any phase, only the passed resolutions are chosen to constitute the test sample.

Furthermore, the reason to eliminate the “*Assent and Consent*” (unanimously approved) resolutions is that, testing the “conflicting” resolutions (with at least one different “voices”, namely one absent or negative vote but not all affirmative votes from member states) would be more appropriate to reveal the influence, interaction, aligning and opposing among member states. Because many of unanimously approved resolutions² are principally constructive to the world development and interest as a whole (such as protecting women and children, etc), or urgent for helping some “innocent” small countries which are during unrest, wars or natural disasters (such as relieving Haiti in the earthquake and tsunami, etc). For those kinds of bills, all SC members are willing to vote affirmative, due to purposes such as reputation, humanitarianism, and furthermore, these bills are also not touching or harming the crucial interests of these member states. Then of course all of the SC members would be inclined to vote affirmative by their own (from the state reputation and international morality), without hesitation and ambivalence. However, there are hardly apparent interest conflicts among member states in these bills, therefore the space for “influence” (for example the member try to persuade his allies or oppose to his enemies, in order to manipulate the outcome and make it better beneficial to own interest), is also very limited. Because there are no motivation for the intention and behavior of influence, all members can obtain the expected outcome, and are not worried about the big loss as these bills do not touch their crucial state interests. Thus, only in the bills with intensive interest conflicts, diversified preference distributions, and confronting state objectives (among different member states) , and they have different appeals and expectations for the voting results, then naturally, there would be more quantity and intensity of “influences”, therefore, the resolutions with at least one different “voice” would be more appropriate to investigate and test the models of influence than the unanimously approved ones.

To organize the test sample data, considering the major reform of UN SC (since 1966, the number of non-permanent members enlarged from 6 to 10) and the grand event re-structuring international politics in the history (the disintegration of Soviet Union at the end of 1991), the sample data (of all passed resolutions except assent and consent ones since the establishment of United Nations) could be divided as three phases or parts, the part 1 is since 1992 (after the disintegration of Soviet Union) , which could be found in table D.2 in Appendix D.2, the part 2 is from 1966 to 1993 (before the disintegration of Soviet Union, and the number of non-permanent members has enlarged from 6 to 10 since 1966), which could be found in table D.3, and the part 3 is from 1946 to 1965 (with only 6 non-permanent members)³, which could be found in table D.4.

9.1.1 Classified as Different Subjects with Dependencies among Resolutions

In the process of collective voting of UN SC, it is undoubtedly that there are varied influences among different member states, like the positive influences among allies (such as UK follows US, Poland followed Soviet Union) and the negative influences among opponents (such as Soviet Union opposed to US) and so on, therefore, it is reasonable to make assumptions and do

²There are a lot of unanimously approved resolutions, which are relatively less meaningful for the test of influence, especially for the latter phases of UN SC. However, in the early phase of UN SC (after UN just established), the veto is very dense, many bills are about the admission of new membership to the United Nations, at that period (with confrontation between two “camps”, one is the so-called “Capitalism” and the other is the so-called “Communism”), it is usually inevitable that the new membership candidate state has its “camp” belonging, then it is highly possibly that the member states within the same camp would vote affirmative, correspondingly, the opponent member states would also not spare of the negative vote or veto. And also there is a series of bills about military movements, like the former Soviet Union’s military operation on Iran, the power/interest allocation between superpowers after war, and so on, which all would invoke the different voices (absent and negative votes). However, for nowadays many resolution is about the humanities rescue and so on, which would be more easy to obtain 15 affirmative votes.

³All three phases with 5 permanent members as the same, the number of permanent members has never changed.

test about the influence among member states. However, are there influences (or dependencies) among different resolutions? In order to make assumptions for testing about the influences among resolutions, the current sample data (ordered just by the temporal sequence) should be reclassified, according to different subjects (as many resolutions with the same subjects happen again and again, and there are a series of high-frequency subjects, such as the Iraq issue, the Palestine issue, etc), therefore, it would be much more reasonable and logical to assume there are dependencies among those resolutions with the correlated contents or belonging to the same subject⁴, rather than assume the dependencies among irrelevant resolutions or random resolutions just with temporal sequence. Therefore, an important step before the test is to classify the whole sample data into sub-samples by different subjects.

Based on the sample data, we choose out correlated resolutions to constitute a series of subjects, particularly those with high frequency, as following (the details data for each subject could be found in Appendix D.3):

Table 9.1: The List of Subjects Chosen from UN SC Sample Data

No.	Key Word	Content	Frequency	Duration
1	Membership	the Admission of New Memberships	15	1946-2000
2	Iraq	the Iraqi Invasion to Kuwait and the Sanctions and Wars against Iraq	21	1990-2010
3	Palestine	the Israeli-Palestinian Conflicts	31	1948-2009
4	Former Yugoslavia	the Conflicts between and within the Constituent Republics of the Former Yugoslavia, usually named as the Yugoslav War	28	1992-2012
5	India and Pakistan	the conflicts and wars between India and Pakistan	13	1948-1971
6	Portuguese Colony	the Decolonization of Territories under Portuguese Administration and Portugal Military Operations on African countries	11	1963-1972
7	South Africa	the Invasion on African Countries and the Apartheid Policy of South Africa	27	1960-1988
8	Southern Rhodesia	the Minority Regime and the Invasion on other African Countries by Souther Rhodesia (Zimbabwe)	20	1965-1980

9.2 Test Method: Influence Pattern Matching Algorithm Design

9.2.1 Making Assumptions about Influences

Based on the official voting data (classified into 8 subjects), we firstly make assumptions about the influencing relations for test, usually based on following methods:

- **Common Sense;**

Example 9.1 (Common Sense) *As we know UK has an intimate relation with US, and UK follows closely with US on many international issues, therefore there would be a positive influence between US and UK; and considering the national power comparison between US and UK, while two countries vote the same, it is more reasonable to assume that US influenced UK's decision than UK influenced US's decision, at least for most of cases. And we can also assume that there might be negative influences between US and Soviet Union, as US and Soviet Union with conflicting state interests and ideologies have been confronting and opposing to each other during the whole Cold War. There are many other assumptions about influences due to common sense likewise.*

⁴Usually, those subjects happening again and again are also much more important, or as controversial issues, compared with other one-off/one-time subjects, the former are more worthy for us to study about.

- checking the **Detailed Historical Materials**, there are a lot of sources about the opinions and intentions of member states' voting in the history, like the official statements (by the president, the foreign minister, etc), diplomatic news conference and so on;

Example 9.2 (Historical Materials) *As we know, before the outbreak of Iraq War, UK stood firmly by the side of US, and the letter from the British former prime minister Tony Blair to the US former president George Bush was recently exposed that Blair stated clearly to support Bush to wage the war against Iraq, and declare UK will join the possible military operations led by US; but on the contrast, France had estrangements with US on the Iraq issue, the French former president Jacques Chirac even threatened to veto US's bill proposed (for the military operation against Iraq) to SC, and the relations between France and US was much worse than between UK and US, therefore we could assume that, compared with UK, the attitude or belief of France leader is much less persuaded or influenced by US, and mostly insisted on own (original) attitude or belief. And There are many other assumptions about influences could be made due to known historical records.*

- **Observation of Official Voting Data.**

Example 9.3 (Official Voting Data) *According to the official voting data of UN Security Council, it could be found that China chose to be absent on most of bills about Yugoslavia War and Cyprus Dispute, while US chose to be absent or even veto (as the absolute minority) on many bills against Israel. And many interesting voting patterns of other member states can be found by observing voting data .*

9.2.2 Influence Pattern Matching Algorithms

Based on the assumptions about influences made by above methods, and further design the Influence Pattern Matching Algorithms, to test the assumed influencing relations by the official voting data⁵.

The basic thoughts and steps are:

1. according to assumptions of influencing relations as different patterns, choose corresponding Influence Pattern Matching Algorithm to run the voting data, to compute the *Expected Vote* if after influenced as assumed on *each bill*;
2. and then compare the *Expected Vote* (shorted as *EV*), which is trough computation of voting data by algorithm, with the *Factual Vote* (shorted as *FV*), which can be directly observed from the existing official data, if the *Expected Vote* is identical with the *Factual Vote*, then it is deemed as a match for the assumed influence pattern, otherwise, it is deemed as a mismatch for the assumed influence pattern;
3. for a data sample (by subjects) with a series of bills, on the more bills (namely more times) the computed *Expected Vote* and the observed *Factual Vote* are matched, the better significant of the assumed influence pattern, represented by Pattern *n*-th Matching Frequency (shorted as *PnMF*) and Pattern *n*-th Matching Ratio (shorted as *PnMR*)⁶.

We propose the matching algorithms for 6 influence patterns (1-th, 7-th, 13-th, 18-th and 24-th) as following. For example of Algorithm 2 for influence pattern 1 (which is just from decision to decision), we assume the agent's vote on current issue will be influenced by own votes on

⁵<http://www.un.org/en/sc/documents/resolutions/>

⁶*n*-th is the number of the pattern of influence, from 1 to 24.

all former issues, in which i is the number of agent, q, k is the number of issue; $opposeVotes(x)$ ($x \in \{Y, A, N\}$) means returning $\{Y, A, N\} \setminus \{x\}$; $\mathcal{WT}_{(k,q)}(i)[FV_{(k)}(i)]$ means the accumulative weight of influence for voting $[FV_{(k)}(i)]$ ($\in \{Y, A, N\}$), which is from issue $1, 2, \dots, k$ to issue q on agent i , $wt_{(k,q)}(i)[FV_{(k)}(i)]$ means the current weight of influence for voting $[FV_{(k)}(i)]$, from issue k to issue q on agent i .

For another example of Algorithm 6 for influence pattern 18 (which involves the influence from structure), we assume the agent's vote on current issue will be influenced by own votes on former issues in the same way (positive/negative) like other agents (how are their current votes influenced by their former votes), in which $\mathcal{WT}_{\mathcal{S}(q,q)}(\mathbb{N}, j)[Po/Ne]$ means the accumulative weight of influencing structure as positive/negative, from all agents \mathbb{N} to agent j on issue q .

It should be noticed that weights can be both positive and negative as discussed before.

The extended expression, detailed explanations of procedures and corresponding examples to display these matching algorithms can be found in appendix C.1.

Algorithm 2 Testing Algorithm for Influence Pattern 1

Input: Given A society $\mathcal{ST}(\mathbb{N}, \mathbb{M}, \mathcal{WT})$, $\mathbb{N} = \{1, \dots, n\}$, $\mathbb{M} = \{1, \dots, m\}$

Output: Pattern 1 Matching Frequency (P1MF) and Pattern 1 Matching Ratio (P1MR) of $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$

```

for  $i = 1$  to  $n$  do
  for  $q = 1$  to  $m$  do
    for  $k = 1$  to  $q - 1$  do
       $\mathcal{WT}_{(k,q)}(i)[FV_{(k)}(i)] \leftarrow \mathcal{WT}_{(k-1,q)}(i)[FV_{(k)}(i)] + wt_{(k,q)}(i)[FV_{(k)}(i)]$ 
    end for
     $maxVote \leftarrow \max(|\mathcal{WT}_{(q-1,q)}(i)[Y]|, |\mathcal{WT}_{(q-1,q)}(i)[A]|, |\mathcal{WT}_{(q-1,q)}(i)[N]|)$ 
    if  $|\mathcal{WT}_{(q-1,q)}(i)[maxVote]| > 0$  then
       $EV_{(q)}(i) \leftarrow maxVote$ 
    else
       $EV_{(q)}(i) \leftarrow opposeVote(maxVote)$ 
    end if
    if  $FV_{(q)}(i) \subseteq EV_{(q)}(i)$  then
       $count\ P1MF_{(q)}(i) = P1MF_{(q-1)}(i) + 1$ 
    else
       $count\ P1MF_{(q)}(i) = P1MF_{(q-1)}(i)$ 
    end if
     $P1MR_{(q)}(i) = \frac{P1MF_{(q)}(i)}{q}$ 
  end for
end for
Return P1MF and P1MR of  $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$ 

```

9.3 Test Outcome: Influence Patterns Matching and Influencing Relations Ranking

In order to test the patterns of influence quantitatively by empirical data, we collected the testing data sample and designed the matching algorithms for different influence patterns, further, we need to display and analyze the testing outcome. To run the testing procedure of algorithms, a crucial information needs to be collected is the allocation of weight of influence among different agents (or issues). In Appendix C.2, we preliminarily proposed an example to discuss the weight of influence among 5 permanent members mainly referring to the data by Ray Cline [Cline, 1980] about the national power comparison in 1978. However, the data is about only one time point, while the UN Security Council has been operating since 1946, and the sample data collected for testing is from 1946 to 2013, which is a continuous period for about 70 years. However, nearly all works about the national power has their specific year point, but not last ceaselessly for at least more than 10 years. Furthermore, works about national power usually just evaluate the major countries in the world, which are at least great powers (such as China, UK, France) or

Algorithm 3 Testing Algorithm for Influence Pattern 7**Input:** Given A society $\mathcal{ST}\langle \mathbb{N}, \mathbb{M}, \mathbb{WT} \rangle$, $\mathbb{N} = \{1, \dots, n\}$, $\mathbb{M} = \{1, \dots, m\}$ **Output:** Pattern 7 Matching Frequency (P7MF) and Pattern 7 Matching Ratio (P7MR) of $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$

```

for  $k = 1$  to  $m$  do
  for  $j = 1$  to  $n$  do
    for  $i = 1$  to  $n$ ,  $i \neq j$  do
       $\mathcal{WT}_{(k)}(\mathbb{N}, j)[FV_{(k)}(i)] \leftarrow \mathcal{WT}_{(k)}(\mathbb{N}, j)[FV_{(k)}(i)] + wt_{(k)}(i, j)[FV_{(k)}(i)]$ 
    end for
     $maxVote \leftarrow \max(|\mathcal{WT}_{(k)}(\mathbb{N}, j)[Y]|, |\mathcal{WT}_{(k)}(\mathbb{N}, j)[A]|, |\mathcal{WT}_{(k)}(\mathbb{N}, j)[N]|)$ 
    if  $\mathcal{WT}_{(k)}(\mathbb{N}, j)[maxVote] > 0$  then
       $EV_{(k)}(j) \leftarrow maxVote$ 
    else
       $EV_{(k)}(j) \leftarrow opposeVote(maxVote)$ 
    end if
    if  $FV_{(k)}(j) \subseteq EV_{(k)}(j)$  then
       $P7MF_{(k)}(j) = P7MF_{(k)}(j) + 1$ 
    else
       $P7MF_{(k)}(j) = P7MF_{(k)}(j)$ 
    end if
     $P7MR_{(k)}(j) = \frac{P7MF_{(k)}(j)}{k}$ 
  end for
end for
Return P7MF and P7MR of  $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$ 

```

Algorithm 4 Testing Algorithm for Influence Pattern 13**Input:** A society $\mathcal{ST}\langle \mathbb{N}, \mathbb{M}, \mathbb{WT} \rangle$, $\mathbb{N} = \{1, \dots, n\}$, $\mathbb{M} = \{1, \dots, m\}$ **Output:** Pattern 13 Matching Frequency (P13MF) and Pattern 13 Matching Ratio (P13MR) of $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$

```

for  $j = 1$  to  $n$  do
  for  $q = k$  to  $m$  do
    for  $k = 1$  to  $q - 1$  do
       $\mathcal{WT}_{(k, q)}(i, j)[FV_{(k)}(i)] \leftarrow \mathcal{WT}_{(k-1, q)}(i, j)[FV_{(k)}(i)] + wt_{(k, q)}(i, j)[FV_{(k)}(i)]$ 
    end for
     $maxVote \leftarrow \max(|\mathcal{WT}_{(q-1, q)}(i, j)[Y]|, |\mathcal{WT}_{(q-1, q)}(i, j)[A]|, |\mathcal{WT}_{(q-1, q)}(i, j)[N]|)$ 
    if  $\mathcal{WT}_{(q-1, q)}(i, j)[maxVote] > 0$  then
       $EV_{(q)}(j) \leftarrow maxVote$ 
    else
       $EV_{(q)}(j) \leftarrow opposeVote(maxVote)$ 
    end if
    if  $FV_{(q)}(j) \subseteq EV_{(q)}(j)$  then
       $count\ P13MF_{(q)}(j) = P13MF_{(q-1)}(j) + 1$ 
    else
       $count\ P13MF_{(q)}(j) = P13MF_{(q-1)}(j)$ 
    end if
     $P13MR_{(q)}(j) = \frac{P13MF_{(q)}(j)}{q}$ 
  end for
end for
Return P13MF and P13MR of  $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$ 

```

Algorithm 5 Testing Algorithm for Influence Pattern 19**Input:** A society $\mathcal{ST}(\mathbb{N}, \mathbb{M}, \mathcal{WT})$, $\mathbb{N} = \{1, \dots, n\}$, $\mathbb{M} = \{1, \dots, m\}$ **Output:** Pattern 19 Matching Frequency (P19MF) and Pattern 19 Matching Ratio (P19MR) of $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$

```

for  $q = 1$  to  $m$  do
  for  $j = 1$  to  $n$  do
    for  $i = 1$  to  $n$   $i \neq j$  do
       $\mathcal{WT}_{(k,q)}(\mathbb{N}, j)[FV_{(k)}(i)] \leftarrow \mathcal{WT}_{(k,q)}(\mathbb{N}, j)[FV_{(k)}(i)] + \mathcal{WT}_{(k,q)}(i, j)[FV_{(k)}(i)]$ 
    end for
     $maxVote \leftarrow \max(|\mathcal{WT}_{(k,q)}(\mathbb{N}, j)[Y]|, |\mathcal{WT}_{(k,q)}(\mathbb{N}, j)[A]|, |\mathcal{WT}_{(k,q)}(\mathbb{N}, j)[N]|)$ 
    if  $|\mathcal{WT}_{(k,q)}(\mathbb{N}, j)[maxVote]| > 0$  then
       $EV_{(q)}(j) \leftarrow maxVote$ 
    else
       $EV_{(q)}(j) \leftarrow opposeVote(maxVote)$ 
    end if
    if  $FV_{(q)}(j) \subseteq EV_{(q)}(j)$  then
       $P19MF_{(q)}(j) = P19MF_{(q)}(j) + 1$ 
    else
       $P19MF_{(q)}(j) = P19MF_{(q)}(j)$ 
    end if
     $P19MR_{(q)}(j) = \frac{P19MF_{(q)}(j)}{q}$ 
  end for
end for
Return P19MF and P19MR of  $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$ 

```

regional power (such as India, Brazil, Australia)⁷, but would not evaluate the small countries which constitute the majority of United Nations. Actually, many small and medium countries would also participate the UN SC voting and interact with other members, influence each other and be influenced by each other, while the information about their national power or weight of influence (affected by national power) is lacked, hardly received attentions and studies. Considering the limited data and the complexity of measuring the influencing power among different states, we might choose a straight and easy approach, but not a tangled and complicated way, to roughly allocate the weight of influence among states. Based on the power classification widely recognized in international politics, we can directly distinguish the categories of national powers, such as super power, great power, and other states. More specifically, [Chapnick, 1999] classify the categories of powers as super power, great power, regional power, middle power and small power, as shown in figure 9.1, which is mainly about the situation of international relations after the Cold War, in which United States is the only super power, and the great powers include Russia, China, United Kingdom, France, Germany and Japan.

The basic assumption about weight of influence is that with other conditions identical, the bigger of the national power, the bigger of the weight of influence, we can define a concept as Power Index (shortly as *Pindex*), and set the *Pindex* for each categories of power, at least satisfying: $Pindex_{super} > Pindex_{great} > Pindex_{regional} > Pindex_{middle} > Pindex_{small}$, for example, we can assume $Pindex_{super} = 5$, $Pindex_{great} = 4$, $Pindex_{regional} = 3$, $Pindex_{middle} = 2$, $Pindex_{small} = 1$. For an influence $x \rightarrow y$, set $Pindex(x)$ as the power index of influencing state x , and $Pindex(y)$ as the power index of influenced state y . Then the weight of influence from x to y would be: $+/- \frac{Pindex(x)}{Pindex(y)}$, the polarity (+/-) is usually determined by the alliance relations (positive among allies and negative among opponents), and the strength $\frac{Pindex(x)}{Pindex(y)}$ is determined by the power comparison between influencing and influenced states. Of course this is not the only approach to allocate the weight of influence, which is just a straightforward and facilitating example, and more detailed and delicate setups could be discussed in future work.

After the preliminary weight allocation, we could further use the influence pattern matching algorithms to test the proposed patterns of influence by the official voting data. And the testing

⁷For example, Cline [Cline, 1980] just evaluate 9 states like US, Soviet Union, UK, France, West Germany, China, Japan, Brazil and Australia.

Algorithm 6 Testing Algorithm for Influence Pattern 18**Input:** A society $\mathcal{ST}(\mathbb{N}, \mathbb{M}, \mathcal{WT})$, $\mathbb{N} = \{1, \dots, n\}$, $\mathbb{M} = \{1, \dots, m\}$ **Output:** Pattern 18 Matching Frequency (P18MF) and Pattern 18 Matching Ratio (P18MR) of $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$

```

for  $j = 1$  to  $n$  do
  for  $q = k$  to  $m$  do
    for  $i = 1$  to  $n$ ,  $i \neq j$  do
      for  $k = 1$  to  $q - 1$  do
         $\mathcal{WT}_{(k,q)}(i, i)[FV_{(k)}(i)] \leftarrow \mathcal{WT}_{(k-1,q)}(i, i)[FV_{(k)}(i)] + wt_{(k,q)}(i, i)[FV_{(k)}(i)]$ 
      end for
       $maxVote \leftarrow \max(|\mathcal{WT}_{(q-1,q)}(i, i)[Y]|, |\mathcal{WT}_{(q-1,q)}(i, i)[A]|, |\mathcal{WT}_{(q-1,q)}(i, i)[N]|)$ 
      if  $\mathcal{WT}_{(q-1,q)}(i, i)[maxVote] > 0$  then
         $EV_{(q)}(i) \leftarrow maxVote$ 
      else
         $EV_{(q)}(i) \leftarrow opposeVote(maxVote)$ 
      end if
      if  $EV_{(q)}(i) \subseteq FV_{(q)}(i)$  then
         $\mathcal{WTS}_{(q,q)}(\mathbb{N}, j)[Po] \leftarrow \mathcal{WTS}_{(q,q)}(\mathbb{N}, j)[Po] + wts_{(q,q)}(i, j)[Po]$ 
      else
         $\mathcal{WTS}_{(q,q)}(\mathbb{N}, j)[Ne] \leftarrow \mathcal{WTS}_{(q,q)}(\mathbb{N}, j)[Ne] + wts_{(q,q)}(i, j)[Ne]$ 
      end if
    end for
    for  $k = 1$  to  $q - 1$  do
       $\mathcal{WT}_{(k,q)}(j, j)[FV_{(k)}(j)] \leftarrow \mathcal{WT}_{(k-1,q)}(j, j)[FV_{(k)}(j)] + wt_{(k,q)}(j, j)[FV_{(k)}(j)]$ 
    end for
     $maxVote \leftarrow \max(|\mathcal{WT}_{(q-1,q)}(j, j)[Y]|, |\mathcal{WT}_{(q-1,q)}(j, j)[A]|, |\mathcal{WT}_{(q-1,q)}(j, j)[N]|)$ 
    if  $\mathcal{WTS}_{(q,q)}(\mathbb{N}, j)[Po] \geq \mathcal{WTS}_{(q,q)}(\mathbb{N}, j)[Ne]$  then
      if  $\mathcal{WT}_{(q-1,q)}(j, j)[maxVote] > 0$  then
         $EV_{(q)}(j) \leftarrow maxVote$ 
      else
         $EV_{(q)}(j) \leftarrow opposeVote(maxVote)$ 
      end if
    else
      if  $\mathcal{WT}_{(q-1,q)}(j, j)[maxVote] > 0$  then
         $EV_{(q)}(j) \leftarrow opposeVote(maxVote)$ 
      else
         $EV_{(q)}(j) \leftarrow maxVote$ 
      end if
    end if
    if  $EV_{(q)}(j) \subseteq FA_{(q)}(j)$  then
       $P18MF_{(q)}(j) = P18MF_{(q-1)}(j) + 1$ 
    else
       $P18MF_{(q)}(j) = P18MF_{(q-1)}(j)$ 
    end if
     $P18MR_{(q)}(j) = \frac{P18MF_{(q)}(j)}{q}$ 
  end for
end for
Return P18MF and P18MR of  $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$ 

```

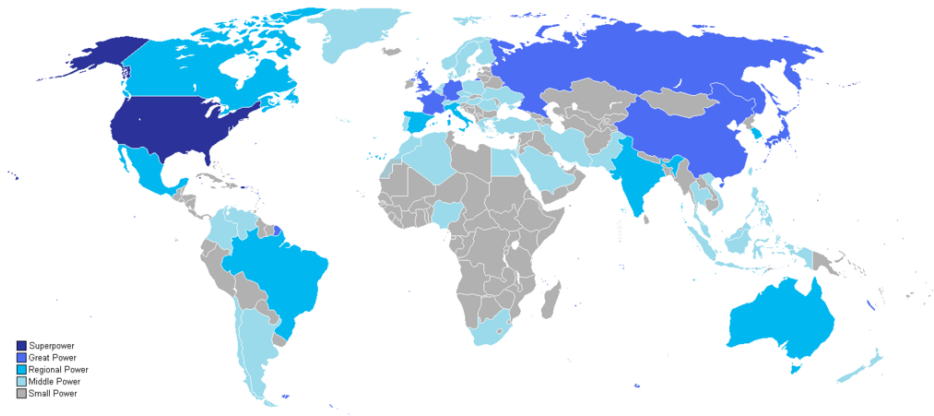


Figure 9.1: Map Reflecting the Categories of Power in International Relations [Chapnick, 1999] after Cold War

Algorithm 7 Testing Algorithm for Influence Pattern 24**Input:** A society $\text{ST}(\mathbb{N}, \mathbb{M}, \text{WT})$, $\mathbb{N} = \{1, \dots, n\}$, $\mathbb{M} = \{1, \dots, m\}$ **Output:** Pattern 24 Matching Frequency (P24MF) and Pattern 24 Matching Ratio (P24MR) of $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$

```

for  $q = k$  to  $m$  do
  for  $j = 1$  to  $n$  do
    for  $k = 1$  to  $q - 1$  do
      for  $i = 1$  to  $n$ ,  $i \neq j$  do
         $\text{WT}_{(k,k)}(\mathbb{N}, j)[FV_{(k)}(i)] \leftarrow \text{WT}_{(k,k)}(\mathbb{N}, j)[FV_{(k)}(i)] + \text{wt}_{(k,k)}(i, j)[FV_{(k)}(i)]$ 
      end for
       $\text{maxVote} \leftarrow \max(|\text{WT}_{(k,k)}(\mathbb{N}, j)[Y]|, |\text{WT}_{(k,k)}(\mathbb{N}, j)[A]|, |\text{WT}_{(k,k)}(\mathbb{N}, j)[N]|)$ 
      if  $\text{WT}_{(k,k)}(\mathbb{N}, j)[\text{maxVote}] > 0$  then
         $\text{EV}_{(k)}(j) \leftarrow \text{maxVote}$ 
      else
         $\text{EV}_{(k)}(j) \leftarrow \text{opposeVote}(\text{maxVote})$ 
      end if
      if  $\text{EV}_{(k)}(j) \subseteq FV_{(k)}(j)$  then
         $\text{WTS}_{(k,q)}(\mathbb{N}, j)[Po] \leftarrow \text{WTS}_{(k-1,q)}(\mathbb{N}, j)[Po] + \text{wts}_{(k,q)}(\mathbb{N}, j)[Po]$ 
      else
         $\text{WTS}_{(k,q)}(\mathbb{N}, j)[Ne] \leftarrow \text{WTS}_{(k-1,q)}(\mathbb{N}, j)[Ne] + \text{wts}_{(k,q)}(\mathbb{N}, j)[Ne]$ 
      end if
    end for
    for  $i = 1$  to  $n$ ,  $i \neq j$  do
       $\text{WT}_{(q,q)}(\mathbb{N}, j)[FV_{(q)}(i)] = \text{WT}_{(q,q)}(\mathbb{N}, j)[FV_{(q)}(i)] + \text{wt}_{(q,q)}(i, j)[FV_{(q)}(i)]$ 
    end for
     $\text{maxVote} \leftarrow \max(|\text{WT}_{(q,q)}(\mathbb{N}, j)[Y]|, |\text{WT}_{(q,q)}(\mathbb{N}, j)[A]|, |\text{WT}_{(q,q)}(\mathbb{N}, j)[N]|)$ 
    if  $\text{WTS}_{(q-1,q)}(\mathbb{N}, j)[Po] \geq \text{WTS}_{(q-1,q)}(\mathbb{N}, j)[Ne]$  then
      if  $\text{WT}_{(q,q)}(\mathbb{N}, j)[\text{maxVote}] > 0$  then
         $\text{EV}_{(q)}(j) \leftarrow \text{maxVote}$ 
      else
         $\text{EV}_{(q)}(j) \leftarrow \text{opposeVote}(\text{maxVote})$ 
      end if
    else
      if  $\text{WT}_{(q,q)}(\mathbb{N}, j)[\text{maxVote}] > 0$  then
         $\text{EV}_{(q)}(j) \leftarrow \text{opposeVote}(\text{maxVote})$ 
      else
         $\text{EV}_{(q)}(j) \leftarrow \text{maxVote}$ 
      end if
    end if
    if  $\text{EV}_{(q)}(j) = FA_{(q)}(j)$  then
       $P24MF_{(q)}(j) = P24MF_{(q-1)}(j) + 1$ 
    else
       $P24MF_{(q)}(j) = P24MF_{(q-1)}(j)$ 
    end if
    if
       $P24MR_{(q)}(j) = \frac{P24MF_{(q)}(j)}{q}$ 
    end if
  end for
end for
Return P24MF and P24MR of  $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$ 

```

procedure is carried on 8 subject samples (such as Iraq issue, Palestine issue, etc) separately. For each subject sample, we give the specific voting data and the content on every bill, to describe how the situation of this event or issue happens and develops with the time, and how it is going on gradually. The detailed sample data (with voting record and bill description) can be found in Appendix D.3.

Before we move on to exposing our results, three remarks are in place to appreciate the scope and limit of our findings:

Distinguishing initial inclination and influence— a very difficult, well-known, issue is to distinguish causality in the available data; that is, is the case that the observed votes are indeed a consequence of some other votes. Here we make some assumptions on the existing patterns, as explained earlier. However our result are certainly optimistic in the sense that they count any matching occurrence of the tested pattern as positive evidence. As such, they constitute some upper limit for the occurrence of the tested patterns. More refined approaches is left for future work.

Voting on the same issues— another assumption open to discussion is the fact that when counting votes on the same issues, it is assumed that bills are put forward in a coherent manner, in the sense that a country is deemed constant when voting similarly. It could be that the nature of the bills could actually explain that two different votes are in fact *coherent*, but we make this simplifying assumption here.


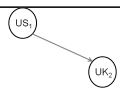
Interpreting the results— of course, there are many potential explanations to the observed outcomes. In the following section, we propose some possible interpretations, keeping in mind that this is by no means exhaustive nor open to critic and alternative explanations.

9.3.1 Subject 1-Admission of New Memberships

Subject 1 **Membership** (Frequency: 15, Range: 1946-2000) is about the Admission of New Memberships to United Nations. At its founding, the UN had only 51 member states, but there are now 193, which is enlarged step by step. While deciding the admission of new member of United Nations, as the member state candidate usually belongs to different blocs, especially before the disintegration of Soviet Union and the end of Cold War, there are candidates from the Capitalist bloc (Western bloc), and also candidates from the Socialist bloc (Eastern bloc), usually the member candidates would get the support from their allies, but not from their opponents. And for current members in United Nations, once the candidate from the same bloc vetoed or opposed by rival bloc, it is possible for them to take revenge in the next time (deciding on the membership of another candidate from the rival bloc), and this kind of mutual negative-vote or mutual non-support happens. For instance, US used to vote absent to Albania, Hungary, Mongolia (which all belong to Socialist Countries at that time).

We ran the influence pattern matching algorithms based on the sample data of subject 1 in table D.5 (in Appendix D.3). The texting outcome is illustrated in table 9.2, in which *IPMF* is a shortcut for the influence pattern matching frequency, and *IPMR* is short for the influence pattern matching ratio.

Table 9.2: Test Outcome for Patterns of Influence with Subject 1-Admission of New Memberships

No	Mode	Rank	<i>IPMF</i>	<i>IPMR</i>	No	Mode	Rank	<i>IPMF</i>	<i>IPMR</i>
PT 1		4	12/14	0.86	PT 13		1	13/14	0.93

PT 1		1	13/14	0.93	PT 13		4	12/14	0.86
PT 1		4	12/14	0.86	PT 13		10-	1/14	0.07
PT 1		8	8/10	0.80	PT 13		10-	1/14	0.07
PT 1		1	13/14	0.93	PT 13		10-	2/14	0.14
PT 7		8	12/15	0.80	PT 24		10	11/14	0.79
PT 7		10-	11/15	0.73	PT 24		10-	10/14	0.71
PT 7		10-	3/15	0.20	PT 24		10	11/14	0.79
PT 7		10-	2/15	0.13	PT 24		4	12/14	0.86
PT 7		10-	3/15	0.20	PT 24		10	11/14	0.79

- **Pattern 1:** Make assumptions about the influencing relations of one state's latter decisions influenced by own former decisions, particularly for permanent members, as super powers and great powers are usually more inclined to be influenced by own, be persistent on own former preferences and behaviors, worship own reputations, but not easily be manipulated and affected by others. The five permanent members except "China"⁸ are all super powers or great powers. And the (Influence Pattern) Matching Ratio of five permanent members are all very high, and in which the four super powers and great powers are higher than "China", the testing outcome is matched to our inference.
- **Pattern 7:** Make assumptions about the influencing relations from one state's current decision to another state's current decision, such as the positive influence from US to his two allies UK and France, and the negative influence from Soviet Union to his opponents, US, UK and France and so on. The Matching Ratio of influence from US to UK is higher than from US to France, which is matched to the common sense, UK is more close to US and follows closely with US on many crucial issues; however, France though also belonging to Western World and NATO, the relations with US is not that close, and sometimes even went against US's claim, such as on Iraq issue. However, the testing outcome for three negative influences are not ideal, in fact, the two blocs (especially US and Soviet Union) indeed vetoed each other side's candidates, while in our testing sample there are only passed resolutions, therefore the conflicting and confrontation between two blocs might seem not that intensive.
- **Pattern 13:** Make assumptions about the influencing relations from one state's former decisions to another state's latter decision, such as the positive influence from US's former decisions to his two allies UK and France's latter decisions, and the negative influence from

⁸Before 1971, the permanent member seat is taken by "Republic of China" (Taiwan).

US's to Soviet Union's, and from Soviet Union's to UK and France's and so on. The Matching Ratio of US's former decisions to UK and France's current decisions are both very high, and in which the influence to UK's is higher than to France's, which is matched to the common sense, UK concerns comparatively more about US's long-term preference and interest, based on the best alliance relationship and shared blood and culture. However, the testing outcome for these negative influences are not ideal, which is similar with the testing of pattern 7, as only passed resolutions are included.

- **Pattern 24:** Make assumptions about the stability of long-term relations between states, particularly permanent members, the Matching Ratio are nearly all ideal, which is matched to the reality, the relations between super powers and great powers are usually more stable, no matter it is alliance or opponency relations. Powers would not change their attitudes so easily and shortly, but would be persistent in a long run and worship their reputations, to provide a stable expectations for both allies and opponents.

Table 9.3: Top 10 Influencing Relations Tested by Subject 1-Admission of New Memberships

Rank	Pattern	Subject-Object	Content
1	PT 1	UK influenced by own former decisions	As a great power, United Kingdoms keep attitude persistent on this subject, voted affirmative on all resolutions except the admission of Israel, which might be due to the special relations
1	PT 1	Soviet Union influenced by own former decisions	As a super power, Soviet Union keep attitude persistent on this subject, voted affirmative on all resolutions except the admission of Mauritania
1	PT 13	UK influenced by US's former decisions	The intimate relations between UK and US, UK follows with US closely
4	PT 1	US influenced by own former decisions	As a super power, US keep attitude persistent on this subject, voted affirmative on all resolutions except two, involving the admission of Albania, Jordan, Hungary and Mongolia, which are either socialist countries or has special relations with Soviet Union
4	PT 1	France influenced by own former decisions	As a great power, France keep attitude persistent on this subject, voted affirmative on all resolutions except two, involving the admission of Guinea and Comoros, which both used to be the colonies of France
4	PT 13	France influenced by US's former decisions	The ally relations between France and US, though not that close like UK and US, France still concerns about US's opinions
4	PT 24	Relations between Soviet Union and UK influenced by former relations between them	The interacting relations between this super power and great power as opponents is very stable
8	PT 1	"Republic of China " influenced by own former decisions	"Republic of China" keep attitude basically persistent on this subject, voted affirmative on all resolutions except the admission of Indonesia and Hungary
8	PT 7	UK influenced by US's current decision	UK follows closely with US on each of decision-making issues
10	PT 24	Relations between US and UK influenced by former relations between them	The interacting relations between these two allies (with special intimacy) is very stable
10	PT 24	Relations between US and Soviet Union influenced by former relations between them	The interacting pattern between these two confronting super powers is stable
10	PT 24	Relations between Soviet Union and France influenced by former relations between them	The interacting relations between this superpower and great power as opponents is stable

9.3.2 Subject 2-the Iraqi Invasion of Kuwait and the Sanctions against Iraq

Subject 2 **Iraq** (Frequency: 21, Range: 1990-2010) is about the Iraqi Invasion to Kuwait and the Sanctions and Wars against Iraq. According to the Wikipedia entry on the issue [Wikipedia, 2015i]:



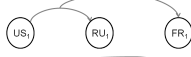
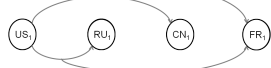
“The Invasion of Kuwait, also known as the Iraq-Kuwait War, was a major conflict between the Ba’athist Iraq and the State of Kuwait, which resulted in the seven-month long Iraqi occupation of Kuwait, and subsequently led to direct military intervention by U.S.-led forces in the Gulf War.”

Besides, during and after the invasion of Kuwait by Iraq, there was a series of bills to sanction and restrict Iraq proposed in the UN Security Council, and there are not a less of different voices among permanent members for two decades. For example, US claimed clearly to sanction and even start war against Iraq in 2003, but had difficulty to get the support from his allies such as France, Germany, while UK stood firmly by the side of US.

We ran the influence pattern matching algorithms based on the sample data of subject 2 in table D.6 (in Appendix D.3). The text outcome is illustrated in table 9.4.

Table 9.4: Test for Patterns of Influence with Subject 2-the Iraqi Invasion of Kuwait and the Sanctions against Iraq

No	Mode	Rank	IPMF	IPMR	No	Mode	Rank	IPMF	IPMR
PT 1		1	20/20	1.00	PT 13		1	20/20	1.00
PT 1		1	20/20	1.00	PT 13		8	15/20	0.75
PT 1		8	15/20	0.75	PT 13		10-	14/20	0.70
PT 1		10-	14/20	0.70	PT 13		10-	14/20	0.70
PT 1		10-	14/20	0.70	PT 13		10-	7/10	0.70
PT 7		1	21/21	1.00	PT 24		1	20/20	1.00
PT 7		7	16/21	0.76	PT 24		8	15/20	0.75
PT 7		10-	6/21	0.29	PT 24		10-	14/20	0.70
PT 7		6	8/10	0.80	PT 24		10-	14/20	0.70
PT 7		10-	7/11	0.64	PT 24		10-	6/10	0.60
PT 10,11		10-	3/6	0.50					

PT 10,11		10-	3/6	0.50
PT 10,11		10-	3/5	0.60
PT 10,11		10-	3/5	0.60
PT 10,11		10-	3/5	0.60

- Pattern 1:** Make assumptions about the influencing relations of one state's latter decisions influenced by own former decisions, particularly for permanent members. The Matching Ratio on US and UK are much higher than on France, China and Russia, which is matched to the real situation. US and UK kept a very firm attitude on the bills against Iraq, while France kept different voices, and even went against US threatening to veto US's proposal to start a war, and for China and Russia, their attitudes are relatively ambivalent on Iraq issues.
- Pattern 7:** Make assumptions about the influencing relations from one state's current decision to another state's current decision. In which there are three influencing relations highly matched, which are the positive influence from US to UK and from US to France, and the negative influence from US to Cuba, which are all the typical influence between allies or opponents. For example, Cuba voted against US on 8 of 10 collectively voting times.
- Pattern 10,11:** Make assumptions about the influence from the influencing relations among some states to the decision of another state, for example, when China voted different from US, would Russia stand by the side of China together against US? And when France as the ally of US hold different voices against US, would France (be dare to) vote different alone or need to be shared pressure with China and Russia, namely just voted different while China or Russia also voted different with US at the same time. According to the testing outcome, the Matching Ratio of this patterns of influence are not very prominent.
- Pattern 13:** Make assumptions about the influencing relations from one state's former decisions to another state's latter decision, such as the positive influence from US to his two allies UK and France, the positive influence from China to Russia, and the negative influence from US to China and to Russia. The Matching Ratio of US's former decisions on UK and France's current decisions are higher, especially for the influence from US to UK, which is matched to the common sense, UK concerns comparatively more about US's long-term preference and interest, based on the best alliance relationship and own state interest.
- Pattern 24:** Make assumptions about the stability of long-term relations between states, particularly between permanent members, the relations between US and UK, and between US and France are more stable (with a higher Matching Ratio), especially for the relations between US and UK, which is matched to the reality. The relations among US, UK and France are typical alliance (all belong to NATO and US is the leader of NATO), China and Russia though keep a very good relation after the disintegration of Soviet Union, still never become allies and have no intentions to form an alliance.

Table 9.6: Top 10 Influencing Relations Tested by Subject 2-the Iraqi Invasion of Kuwait and the Sanctions against Iraq

Rank	Pattern	Subject-Object	Content
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1	PT 1	US influenced by own former decisions	As a super power, US claimed very firmly to sanction Iraq, voted affirmative on all resolutions against Iraq
1	PT 1	UK influenced by own former decisions	As a great power, United Kingdoms kept constant on Iraq issue, voted affirmative on all resolutions
1	PT 7	UK influenced by US 's current decision	UK follows closely with US on all Iraq bills, stand closely by US's side
1	PT 13	UK influenced by US 's former decisions	UK not only consider the US 's current decisions, but would also consider US 's long-term preference on the whole Iraq issue
1	PT 24	Relations between US and UK influenced by former relations between them	The interacting pattern between these two allies (with special intimacy) is very stable
6	PT 7	Cuba (negatively) influenced by US 's current decision	Cuba voted different with US on nearly all collectively made voting, there is tradition of confrontation with US for Cuba
7	PT 7	France influenced by US 's current decision	The ally relations between France and US, though not that close like UK and US, is still stable
8	PT 1	France influenced by own former decisions	As a great power, France keep attitude basically constant on Iraq issue, but still voted five different voices, didn't have a very firm attitude like US and UK against Iraq
8	PT 13	France influenced by US 's former decisions	France not only consider US 's current decisions, but would also consider US 's long-term preference on the whole Iraq issue, though was not that obedient like UK
8	PT 24	Relations between US and France influenced by former relations between them	The interacting relations between these two allies (one as super power and one as great power) is stable

9.3.3 Subject 3-Israeli and Palestinian Conflicts

Subject 3 **Palestine** (Frequency: 31, Range: 1948-2009) is about the ongoing Israeli-Palestinian Conflicts which began in the mid-20th century.⁹ The Israeli-Palestinian conflict has formed the core part of the wider Arab-Israeli conflict. It has widely been referred to as the world's "most intractable conflict" [Fortna, 2004, Falk, 2005]. According the wikipedia entry on the topic [Wikipedia, 2015j]

"Despite a long-term peace process and the general reconciliation of Israel with Egypt and Jordan, Israelis and Palestinians have failed to reach a final peace agreement. The remaining key issues include mutual recognition, borders, security, water rights, control of Jerusalem, Israeli settlements, and so on. The violence of the conflict, in a region rich in sites of historic, cultural and religious interest worldwide, has been the object of numerous international conferences dealing with historic rights, security issues and human rights".

In UN security council, there are a lot of bills proposed in order to solve or relieve the Israeli-Palestinian Conflicts (mainly to restrict or demand on Israel), for those bills, US voted different voices many times, even as the absolute minority (all other 14 members vote affirmative, but only US vote absent or even veto, to support and stand by the side of Israel). For many historical reasons, US has a good relationship (or intimacy) with Israel. Thus on bills supporting palestinian demands, there are many cases where all other states except US vote affirmative. Besides, US could vote different or even veto alone, with no veto partner like China and Russia, who usually veto together. Though there is only one veto from US, without other support from his allies (such as UK, France, Germany and some other western countries), US still clearly express own different

⁹The conflict is wide-ranging, and the term is sometimes also used in reference to the earlier sectarian conflict in Mandatory Palestine, between the Zionist yishuv and the Arab population under British rule.

attitude, and not fears heavy pressure from other allies and the international community full of the “demeanour” of super power.

We ran the influence pattern matching algorithms based on the sample data of subject 3 in table D.7 (in Appendix D.3), the texting outcome is illustrated in table 9.7.

Table 9.7: Test for Patterns of Influence with Subject 3-Israeli and Palestinian Conflicts

No	Mode	Rank	<i>IPMF</i>	<i>IPMR</i>	No	Mode	Rank	<i>IPMF</i>	<i>IPMR</i>
PT 1		10-	17/30	0.57	PT 13		1	30/30	1.00
PT 1		4	29/30	0.97	PT 13		1	30/30	1.00
PT 1		1	30/30	1.00	PT 13		10-	9/30	0.30
PT 1		5	16/17	0.94	PT 13		10-	7/8	0.88
PT 1		10-	21/30	0.70	PT 13		10-	7/8	0.88
PT 7		10-	17/31	0.55	PT 24		10-	16/30	0.53
PT 7		10-	18/31	0.58	PT 24		10-	17/30	0.57
PT 7		10-	20/31	0.65	PT 24		10-	9/12, 9/17	0.75, 0.53
PT 7		9	8/9	0.89	PT 24		10-	19/30	0.63
PT 7		9	8/9	0.89	PT 24		10-	7/8	0.88
PT 10,11		8	10/12	0.91					
PT 10,11		10-	11/20	0.55					
PT 10,11		6	13/14	0.93					
PT 10,11		10-	12/14	0.86					
PT 10,11		6	13/14	0.93					

- Pattern 1:** Makes assumptions about the influencing relations of one state’s latter decisions influenced by own former decisions, particularly for permanent members. The Matching Ratio on UK, France and China are higher than on US and Soviet Union (Russia), which means UK, France and China kept a more firm attitude on Israeli-Palestinian conflicts than US and Soviet Union (Russia), which is actually matched to the real situation. The diplo-

matic policy of US usually backs Israel, but can also consider other interests. Similarly, Soviet Union (Russia) also has special relations with Middle East Arab Countries, supported them in the Middle East Wars, and showed a relatively ambivalent attitude on the Israeli-Palestine conflicts. Comparatively, UK, France and China do not have that intimate relations with either side, and show a more firm attitude on relevant bills.

- **Pattern 7:** Makes assumptions about the influencing relations from one state's current decision to another state's current decision. In which the Matching Ratio of positive influence from US to UK and from US to France are not very ideal, especially for the influence from US to UK, as for most of cases UK follows closely with US, confirmedly stands by the side of US. While in the Israel-Palestinian issue, UK just voted half the same with US, which might because there is no special relation between UK and Israel, and UK just voted affirmative together with nearly all others UN SC members "against" Israel (for many bills against Israel, US voted absent alone while all others voted affirmative). Comparatively, the attitude of China and Russia are highly uniform.
- **Pattern 10,11:** Makes assumptions about the influence from the influencing relations among some states to the decision of another state, for example, when China voted different from US, would Russia stood by the side of China together against US? And when UK as the ally and "sidekick" of US hold different voices against US, would UK (dare to) vote different alone or need to be shared pressure with other allies like France or even non-allies like China and Russia, namely just voted different while other members also voted different with US at the same time. According to the testing outcome, the Matching Ratio of this patterns of influence is overall high. For most cases when China voted different with US, Russia would support China and voted different with US the same. And for most cases when UK dared to speak different with US, there are always some other countries voting different the same with UK.
- **Pattern 13:** Make assumptions about the influencing relations from one state's former decisions to another state's latter decision. The testing outcome is better than corresponding pattern 7 (with the identical influencing and influenced states), especially for the positive influence from US to UK and from US to France, which means though UK and France didn't follow with US exactly on each bill, but on the long-term, UK and France would still consider about US's preferences and interests, would not offend US overmuch and risk own state interest.
- **Pattern 24:** Makes assumptions about the stability of long-term relations between states, particularly between permanent members. As observed, the testing outcome is not very ideal. Only the Matching Ratio of influencing relations between China and Russia is relatively high, after the disintegration of Soviet Union, the relations between China and Russia become very close soon, under the common threaten and pressure from US, the friendships between China and Russia is very stable.

Table 9.9: Top 10 Influencing Relations Tested by Subject 3-Israeli and Palestinian Conflicts

Rank	Pattern	Subject-Object	Content
1	PT 1	France influenced by own former decisions	As a great power, France kept attitude constant on Palestine issue, voted affirmative on all resolutions, without any dilemma like US
1	PT 13	UK influenced by US's former decisions	Though UK did not follow with US's decision at each individual step (compared with Pattern 7 US to UK), but would consider US's long-term decisions as a whole

1	PT 13	France influenced by US's former decisions	Though France did not followed with US's decision at each individual step (compared with Pattern 7 US to France), but would consider US's long-term decisions as a whole
4	PT 1	UK influenced by own former decisions	As a great power, United Kingdoms keep a constant attitude on Palestine issues, voted affirmative on all resolutions except one, not with an ambivalent attitude like US
5	PT 1	China influenced by own former decisions	As a great power, China keep a constant attitude on Palestine issues, voted affirmative on all resolutions except one
6	PT 10,11	UK influenced by the influencing relations between US and France	UK would be afraid to voted different with US alone, on 14 times when UK voted different from US, there 13 times UK and France voted the same and together against US
6	PT 10,11	UK influenced by the influencing relations between US and Soviet Union	UK would be afraid to voted different with US alone, on 14 times when UK voted different from US, there 13 times Soviet Union also voted different from US
8	PT 10,11	Soviet Union influenced by the influencing relations between US and China	Russia (Soviet Union) would stand by the side of China together against US, on 12 times when China voted different from US, there are 10 times Russia (Soviet Union) voted the same with China together against US
9	PT 7	Russia influenced by China's current decision	After the disintegration of Soviet Union, the relation between Russia and China become closer and closer, Russia stays in step with China on many international issues
9	PT 7	Ukraine influenced by Soviet Union's current decision	This is a special influence, as Ukraine was the constitute republic of Soviet Union, which is an absolute influence, and Ukraine voted all the same with Soviet Union, except one, Soviet Union voted absent, but let Ukraine voted negative, as kind of strategy

9.3.4 Subject 4-Yugoslav Wars

Subject 4 **Former Yugoslavia** (Frequency: 28, Range: 1992-2012) is about the Conflicts between and within the Constituent Republics of the Former Yugoslavia, usually named as the Yugoslav Wars. According to the wikipedia entry on the subject [Wikipedia, 2015p]:

“The Yugoslav Wars were ethnic conflicts fought from 1991 to 1999 on the territory of former Yugoslavia. The wars followed the breakup of the country, where its constituent republics declared independence, but the issues of ethnic minorities in the new countries, chiefly Serbs in central parts and Albanians in the southeast, were left unsolved. The wars are generally considered to be a series of largely separate but related military conflicts occurring and affecting most of the former Yugoslav republics.”

And the United Nations proposed and passed a series of resolutions about sanctions, humanitarian assistance, send of UN force toward these involved states. One interesting point on the voting record is about China, for 28 passed resolutions (with different voices) about the former Yugoslavia, China voted 18 absent, besides, Russia also be absent for 13 times, the frequency of absent for China and Russia is much higher than other member states like US, UK and France (the frequency of absent respectively as 1,0,0). The high ratio of absent of China and Russia might be due to the special relations between them and former Yugoslavia countries. Besides, China and Russia opposed frequently to the intervene from western countries (especially the military operations) on former Yugoslavia.

We ran the influence pattern matching algorithms based on the sample data of subject 4 in table D.8 (in Appendix D.3), the text outcome is illustrated in table 9.10.

Table 9.10: Test for Patterns of Influence with Subject 4-Yugoslav Wars

No	Mode	Rank	IPMF	IPMR	No	Mode	Rank	IPMF	IPMR
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PT 1		5	26/27	0.96	PT 24		5	26/27	0.96
PT 1		1	27/27	1.00	PT 24		5	26/27	0.96
PT 1		1	27/27	1.00	PT 24		10-	15/27	0.56
PT 1		10-	17/27	0.63	PT 24		10-	19/27	0.70
PT 1		10-	16/27	0.59	PT 24		10-	16/27	0.59
PT 7		5	27/28	0.96	PT 10,11		10-	6/19	0.32
PT 7		5	27/28	0.96	PT 10,11		10-	7/13	0.54
PT 7		10-	19/28	0.68	PT 10,11		10	10/11	0.91
PT 7		10-	14/28	0.50	PT 10,11		1	27/27	1.00
PT 7		10-	9/28	0.32	PT 10,11		1	27/27	1.00
PT 12		10-	2/10	0.20					
PT 12		10-	7/18	0.39					
PT 12		10-	7/18	0.39					
PT 12		10-	7/18	0.39					
PT 12		10-	6/13	0.46					

- Pattern 1:** Makes assumptions about the influencing relations of one state's latter decisions influenced by own former decisions, particularly for permanent members. US, UK and France kept a very persistent attitude on Yugoslav Wars, with a very high Matching Ratio. However, the attitude of China and Russia toward Yugoslav Wars are very ambiguous compared with the firm attitude from Western bloc, especially on those bills against the Federal Republic of Yugoslavia, which might be due to the special relations of China and Russia with Yugoslavia (which used to be socialist state and kept a good friendships with them).
- Pattern 7:** Makes assumptions about the influencing relations from one state's current decision to another state's current decision. In which the Matching Ratio of positive influence from US to UK and from US to France are much higher, which might due to the alliance relations among them, and the common interest against Federal Republic of Yugoslavia.
- Pattern 10,11:** Makes assumptions about the influence from the influencing relations among some states to the decision of another state, for example, when China voted different from

US, would Russia stand by the side of China together against US? And when UK and US become closer and closer, would France be touched or nervous and then also behave better? According to the testing outcome, there are a series of influencing relations with high Matching Ratio. For the two allies or “sidekicks” of US, UK and France, while each one of them goes closely with US, the other one might be touched and also behaves better.

- **Pattern 12:** Makes assumptions about the influence from influencing relations among some states to influenced relations among some other states on one same decision, for example, when the relations among US, UK, France and Japan as alliance become closer, would the relations between China and Russia as a confronting “alliance” also become closer, to respond to possible threats and challenges? As observed, in this sample data about Yugoslav Wars, the testing outcome is not very ideal.
- **Pattern 24:** Makes assumptions about the stability of long-term relations between states, particularly between permanent members. As observed, the Matching Ratio of influencing relations between US and UK, and between US and France are very high, due to the ally relations and common interests.

Table 9.12: Top 10 Influencing Relations Tested by Subject 4-Yugoslav War

Rank	Pattern	Subject-Object	Content
1	PT 1	UK influenced by own former decisions	As a great power, UK keep a constant attitude on Yugoslav War issues, voted affirmative on all resolutions, without dilemma like China and Russia
1	PT 1	France influenced by own former decisions	As a great power, France keep a constant attitude on Yugoslav War issues, voted affirmative on all resolutions, without dilemma like China and Russia
1	PT 10,11	UK influenced by the influencing relations between US and France	When France voted the same with US, UK would also vote the same with US, afraid to lag behind
1	PT 10,11	France influenced by the influencing relations between US and UK	When UK voted the same with US, France would also vote the same with US, afraid to be isolated
5	PT 1	US influenced by own former decisions	As a super power, United States keep a constant attitude on Yugoslav War issues, voted affirmative on all resolutions expect one, which is to terminate measures in the situation in Former Yugoslavia
5	PT 7	UK influenced by US 's current decision	UK followed closely with US on Yugoslav War issues, stood closely by US side, together against China and Russia
5	PT 7	France influenced by US 's current decision	France followed closely with US on Yugoslav War issues, cooperated closely with US, together against China and Russia
5	PT 24	Relations between US and UK influenced by former relations between them	The interacting pattern between these two allies (with special intimacy) is very stable, the loyalty is solid
5	PT 24	Relations between US and France influenced by former relations between them	The interacting relations between these two allies is very stable, the alliance relations is solid
10	PT 10,11	China influenced by the influencing relations between US and Japan	When Japan follows closely with US, China would be touched and voted different from them, on 11 times when Japan voted the same with US, China voted different from them on 10 times

9.3.5 Subject 5-the Conflicts between India and Pakistan

Subject 5 **India and Pakistan** (Frequency: 13, Range: 1948-1971) is about the wars and conflicts between India and Pakistan, usually named as the Indo-Pakistani Wars and Conflicts. According to the Wikipedia entry on the topic [Wikipedia, 2015f]:

“Since the partition of British India in 1947 and the creation of modern republics of India and Pakistan, the two South Asian countries have been involved in four wars, including one undeclared war, as well as many border skirmishes and military stand-offs. The dispute for Kashmir has been the cause, whether direct or indirect of all major conflicts between the two countries with the exception of the Indo-Pakistani

War of 1971, where conflict originated due to turmoil in erstwhile East Pakistan (now Bangladesh).”

The conflicts and wars were mediated and intervened by the UN SC and international society, a series of bills are proposed to relieve the conflicts. One interesting point on the voting record is about Soviet Union, who vote absent on most of resolutions, for 13 resolutions (with different voices) pertaining to the Indo-Pakistani conflicts, Soviet Union voted absent for 11 of them, such an ambiguous attitude might be due to the special relations of Soviet Union with India and Pakistan.

Run the influence pattern matching algorithms based on the sample data of subject 5 in table D.9 (in Appendix D.3), the testing outcome is illustrated in table 9.13.

Table 9.13: Test for the Patterns of Influence with Subject 5-the Conflicts between India and Pakistan

No	Mode	Rank	IPMF	IPMR	No	Mode	Rank	IPMF	IPMR
PT 1		1	12/12	1.00	PT 13		10-	10/12	0.83
PT 1		10-	10/12	0.83	PT 13		3	11/12	0.92
PT 1		3	11/12	0.92	PT 13		3	11/12	0.92
PT 1		1	8/8	1.00	PT 13		10-	10/12	0.83
PT 1		10-	10/12	0.83	PT 13		3	11/12	0.92
PT 7		9	11/13	0.85	PT 24		10-	10/12	0.83
PT 7		3	12/13	0.92	PT 24		3	11/12	0.92
PT 7		9	11/13	0.85	PT 24		10-	10/12	0.83
PT 7		9	11/13	0.85	PT 24		10-	10/12	0.83
PT 7		10-	10/13	0.77	PT 24		10-	9/12	0.75

- **Pattern 1:** Makes assumptions about the influencing relations of one state’s latter decisions influenced by own former decisions, particularly for permanent members. The testing outcome is overall ideal, nearly all states kept a very constant attitude on India-Pakistan issues, be persistent on own former preferences and decisions, and would not change own claim and attitude shortly and easily, which is matched to the common sense, a state specially a super/great power worship own reputation in the international society.
- **Pattern 7:** Makes assumptions about the influencing relations from one state’s current de-

cision to another state's current decision, such as the positive influence from US to UK and from US to France, the negative influence from US to Soviet Union, from Soviet Union to UK and from Soviet Union to France. The Matching Ratio and ranking of all tested influencing relations are very high, just as we know, during the Cold War, the negative interaction (conflicts and confrontations) between opponents such as US and Soviet Union (more broadly the Western bloc and Eastern bloc) are very intensive, and the positive interaction (cooperations and aids) among allies such as US, UK, France and so on are also very intensive, to unite forces, respond to challenges, and defend state interests.

- **Pattern 13:** Makes assumptions about the influencing relations from one state's former decisions to another state's latter decision, which is similar to the influence pattern 7, just influence from the former decisions but not from the current decisions. The testing outcome is also very ideal, with a high Matching Ratio. In the international politics and gaming, a state would consider about his allies' long-term preferences and interests, then interact positively correspondingly, and a state would also think about his opponents' long-term preferences and interests, but interact negatively correspondingly.
- **Pattern 24:** Makes assumptions about the stability of long-term relations between states, particularly between permanent members, such as the relations between US and UK, between US and France, between US and Russia and so on. As observed, the Matching Ratio are very high, due to the common interests or conflicting interests, and due to the reputation and stable expectation, the relations between super powers and great powers are stable, not changing shortly and easily.

Table 9.14: Top 10 Influencing Relations Tested by Subject 5-the Conflicts between India and Pakistan

Rank	Pattern	Subject-Object	Content
1	PT 1	US influenced by own former decisions	As a super power, United States keep a constant attitude on India-Pakistan issues, voted affirmative on all resolutions
1	PT 1	China influenced by own former decisions	"Republic of China" (after lost the control of Mainland) still kept a constant attitude on India-Pakistan issues
3	PT 1	France influenced by own former decisions	As a great power, France keep a constant attitude on India-Pakistan issues, voted affirmative on all resolutions except one
3	PT 7	France influenced by US's current decision	France cooperated closely with US on India-Pakistan issues, together "against" Russia
3	PT 13	France influenced by US's former decisions	France also considered about the long-term preferences and decisions of US on India-Pakistan Issues
3	PT 13	Soviet Union (negatively) influenced by US's former decisions	As opponents, Soviet Union negatively considered about the long-term preferences and decisions of US, and voted different correspondingly
3	PT 13	France (negatively) influenced by Soviet Union's former decisions	As from confronting blocs, France negatively considered about the long-term preferences and decisions of Soviet Union
9	PT 7	UK influenced by US's current decision	UK followed closely with US, stood closely by US side
9	PT 7	Soviet Union (negatively) influenced by US's current decision	Soviet Union went against US frequently, voted different from US on 11/13 times
9	PT 7	UK (negatively) influenced by Soviet Union's current decision	UK went against Soviet Union frequently, voted different from Soviet Union on 11/13 times






9.3.6 Subject 6-the Decolonization of Territories and Military Operations of Portugal

Subject 6 **Portuguese Colony** (Frequency: 11, Range: 1963-1972) is about the decolonization of territories under Portuguese administration, and Portugal military operations on African countries. The Portugal, as an old empire of colonies that has seen his power weakened, was judged by the current superpowers and great powers (US, Soviet Union, UK, France, etc), a series of bills were proposed against Portugal. But the Portugal still obtained the inapparent support and sympathy from allies of western bloc, and the similar old empires of colonies, such as UK, France, Spain and so on. These states though supported or be sympathetic with the old empire, but could just be absent, still would not vote negative, which would bring too much pressure and lost the morality. However, other countries facing colonization or communist countries, like China, Soviet Union, Zambia, Nepal and so on, voted affirmative very firmly on the bills against the old empire. For example, for 11 resolutions (with different voices) against Portuguese, US voted 10 absent but just 1 affirmative, UK voted 9 absent but just 2 affirmative, France voted 7 absent and 4 affirmative, comparatively, China voted just 1 absent and 10 affirmative, and Soviet Union voted none absent but all affirmative, the contrast is very sharp.

We ran the influence pattern matching algorithms based on the sample data of subject 6 in table D.10 (in Appendix D.3), the texting outcome is illustrated in table 9.15.

Table 9.15: Test for Patterns of Influence with Subject 6-the Decolonization of Territories and Military Operations of Portugal

No	Mode	Rank	<i>IPMF</i>	<i>IPMR</i>	No	Mode	Rank	<i>IPMF</i>	<i>IPMR</i>
PT 1		3	9/10	0.90	PT 13		10-	7/10	0.70
PT 1		10-	7/10	0.70	PT 13		10-	6/10	0.60
PT 1		10-	6/10	0.60	PT 13		3	9/10	0.90
PT 1		6	6/7	0.86	PT 13		10-	7/10	0.70
PT 1		1	10/10	1.00	PT 13		10-	6/10	0.60
PT 7		7	9/11	0.82	PT 24		8	8/10	0.80
PT 7		10-	6/11	0.55	PT 24		10-	5/10	0.50
PT 7		10-	1/8	0.13	PT 24		3	9/10	0.90
PT 7		10	8/11	0.73	PT 24		10-	7/10	0.70
PT 7		10-	7/11	0.64	PT 24		10-	6/10	0.60

PT 10,11		1	6/6	1.00
PT 10,11		10-	6/9	0.67
PT 10,11		8	8/10	0.80
PT 10,11		10-	6/10	0.60
PT 10,11		10-	1/7	0.14

- **Pattern 1:** Make assumptions about the influencing relations of one state's latter decisions influenced by own former decisions, particularly for permanent members. The Matching Ratio on Soviet Union, US and China are much higher than on UK and France. Facing the bills against the former colonial empire Portugal, Soviet Union, US and China kept a very firm attitude, either against or pro Portugal; while UK and France themselves as former colonial empires, their attitudes are relatively ambiguous, interleaved with the empathy with Portugal and own reputation of justice. Further, compare the voting between US and China, Soviet Union, though all kept a firm attitude, China and Soviet Union voted affirmative on most cases, while US voted absent on most cases, which might be due to the partiality of US as the leader of Western bloc.
- **Pattern 7:** Makes assumptions about the influencing relations from one state's current decision to another state's current decision. In which the Matching Ratio of positive influence from US to UK is higher, which is conforming to the real situation, on most cases UK follows closely with US, confirmedly stand by the side of US. An interesting point is the influence from US to "Republic of China", which was "under protectorate" of US and listened to US on most cases, while on the Portugal issues, "Republic of China" voted different with US on most cases and firmly against Portugal, which might be due to the "bitter" history of being invaded.
- **Pattern 10,11:** Makes assumptions about the influences from the influencing relations among some states to the decision of another state, there are two influencing relations tested highly matched. When France goes closely with US (votes the same with US), UK would also behave well and keep pace with US. Besides, when Soviet Union voted against US, mostly UK would vote the same with US, firmly stand by the side of US. Above two influencing relations is both about UK, influenced by the relations involved US, showing UK emphasis a lot on his relation with US.
- **Pattern 13:** Makes assumptions about the influencing relations from one state's former decisions to another state's latter decision. The most outstanding matched influencing relations is the negative influence from US to Soviet Union, as two opponents during the Cold War, each side would consider about the other side's long-term preference and interest, and correspondingly took actions, supported what the opponent disliked, and opposed to what the opponent likes.
- **Pattern 24:** Makes assumptions about the stability of long-term relations between states, particularly between permanent members. As observed, the testing outcome of the stability of relations between US and UK, and between US and Soviet Union are prominent, which is in accord with the real situation, the relation between US and UK is the best ally relationship, and the relation between US and Soviet Union is the most intensive opponent relationship, therefore, the two relationships are quite stable, for example, you would never easily betray your best friend, and you would never easily become reconciled with your worst enemy.

Table 9.17: Top 10 Influencing Relations Tested by Subject 6-the Decolonization of Territories and Military Operations of Portugal

Rank	Pattern	Subject-Object	Content
1	PT 1	Soviet Union influenced by own former decisions	As a super power, Soviet Union kept a firm attitude on resolutions against Portugal, voted affirmative on all, not with an ambiguous attitude like UK and France (both were colonial empires similar with Portugal)
1	PT 10,11	UK influenced by the influencing relations between US and France	When France followed closely with US, UK would be touched and also voted the same with US, afraid to lag behind
3	PT 1	US influenced by own former decisions	As a super power, United States keep a constantly “neutral” attitude on resolutions against Portugal, voted absent on all resolutions except one, not with an ambiguous attitude like UK and France
3	PT 13	Soviet Union (negatively) influenced by US 's former decisions	As opponents, Soviet Union negatively considered about the long-term preferences and decisions of US on Portugal issues, and voted different from US correspondingly
3	PT 24	Relations between US and Soviet Union influenced by former relations between them	The interacting pattern between these two opponents (both as super powers) is stable, for most of times, Soviet Union would be negatively influenced by or opposed to US
6	PT 1	China influenced by own former decisions	“Republic of China” (Taiwan), tough as a small “state”, still kept a constant attitude on resolutions against Portugal, voted affirmative on all except one, which might be due to the history of being invaded by colonial empires like UK, France, Portugal and so on
7	PT 7	UK influenced by US 's current decision	UK followed closely with US, stood by US side (voted the same) on most cases
8	PT 24	Relations between US and UK influenced by former relations between them	The interacting pattern between these two allies (with special intimacy) is very stable, the loyalty is solid
8	PT 10,11	UK influenced by the influencing relations between US and Soviet Union	UK would stand by the side of US when he is opposed by Soviet Union, for 10 times when Soviet Union voted different from US, there are 8 times UK voted the same together with US
10	PT 7	UK (negatively) influenced by Soviet Union 's current decision	UK went against Soviet Union frequently, voted different from Soviet Union on 8 out of 11 times

9.3.7 Subject 7-the Apartheid Policy and the Invasion by South Africa

Subject 7 **South Africa** (Frequency: 27, Range: 1960-1988) is about South Africa's Invasion on African Countries and Apartheid Policy. According to the Wikipedia entry [Wikipedia, 2015a] on the issue:

“Apartheid was a system of racial segregation in South Africa enforced through legislation by the National Party (NP) governments, the ruling party from 1948 to 1994, under which the rights, associations, and movements of the majority black inhabitants were curtailed and Afrikaner minority rule was maintained. Apartheid was developed after World War II by the Afrikaner-dominated National Party and Broederbond organisations¹⁰ and was practised also in South West Africa, which was administered by South Africa under a League of Nations mandate (revoked in 1966 via United Nations Resolution 2145), until it gained independence as Namibia in 1990. [...] Apartheid sparked significant internal resistance and violence, and a long arms and trade embargo against South Africa [Lodge, 1983]. Since the 1950s, a series of popular uprisings and protests was met with the banning of opposition and imprisoning of

¹⁰The Afrikaner Broederbond (AB) (meaning Afrikaner Brotherhood) or Broederbond was a secret, exclusively male and Afrikaner Calvinist organisation in South Africa dedicated to the advancement of Afrikaner interests.

anti-apartheid leaders. Along with the sanctions placed on South Africa by the international community, this made it increasingly difficult for the government to maintain the regime.”




The UN Security Council also passed a series of resolutions to denounce and sanction the regime of South Africa. However, the attitudes of the western great powers (which used to be colonial empires) toward the bills against South Africa remained quite ambiguous, for 27 resolutions (with different voices) against South Africa, UK voted 17 absent, France voted 14 absent, US voted 14 absent. However, the attitudes of communist countries or former colonial countries are firmly clear, such as Soviet Union just voted 3 absent but 24 affirmative, and China voted all affirmative except one absent, and for other non-permanent members (mainly the small and poor countries used to be colonies), nearly all voted affirmative but none absent, the contrast between two categories of countries is very sharp.

We ran the influence pattern matching algorithms based on the sample data of subject 7 in table D.11 (in Appendix D.3), the texting outcome is illustrated in table 9.18.

Table 9.18: Test for Patterns of Influence with Subject 7-the Apartheid Policy and the Invasion by South Africa

No	Mode	Rank	IPMF	IPMR	No	Mode	Rank	IPMF	IPMR
PT 1					PT 13				
		10-	14/26	0.54			10-	10/26	0.38
PT 1		10-	16/26	0.62	PT 13		10-	9/26	0.35
PT 1		10-	14/26	0.54	PT 13		10	18/26	0.69
PT 1		1,2	8/8,16/17	1,0.94	PT 13		10-	10/26	0.38
PT 1		4	23/26	0.88	PT 13		10-	13/26	0.50
PT 7		10-	10/27	0.37	PT 24				
							10-	17/26	0.65
PT 7		10-	9/27	0.33	PT 24				
							10-	17/26	0.65
PT 7		7,9	7/9, 13/18	0.78, 0.72	PT 24				
							10-	15/26	0.58
PT 7		10-	15/27	0.56	PT 24				
							10-	17/26	0.65
PT 7		3	16/18	0.89	PT 24				
							10-	14/26	0.54

PT 10,11		6	14/17	0.82
PT 10,11		7	14/18	0.78

PT 10,11		10-	7/17	0.41
PT 10,11		10-	8/18	0.44
PT 10,11		5	11/13	0.85

- Pattern 1:** Makes assumptions about the influencing relations of one state's latter decisions influenced by own former decisions, particularly for permanent members. The Matching Ratio on Soviet Union and China are much higher than on US, UK and France. Facing the bills against the "unjust" white regime of South Africa, Soviet Union and China (no matter it is the "Republic of China" or the People's Republic of China) kept a very firm attitude against its apartheid policy and invasion. On the other hand, the attitudes of US, UK and France were not as constant, basically voted half affirmative but half absent. .
- Pattern 7:** Makes assumptions about influencing relations from one state's current decision to another state's current decision. There are several outstanding matched influencing relations, one is the influence from US to China, what is interesting is, as we assumed two phases of influence, one is the positive influence from US to "Republic of China", the other is the negative influence from US to People's Republic of China, and the testing outcome are both ideal, which conforms exactly to the real-world situation. Another prominent example is the positive influence from Soviet Union to People's Republic of China, on many cases, China voted the same with Soviet Union, together against US or the whole Western bloc.
- Pattern 10,11:** Makes assumptions about the influence from the influencing relations among some states to the decision of another state(s), there are several influencing relations highly matched. As observed, on South Africa issues, the opinions and votes among US, UK and France were not very uniform, there were conflicts within Western bloc, and according to the testing outcome, the two "little brothers" UK and FR, under the pressure of superpower US, did not vote against US alone, but usually voted opposing to US together with each other, among the 17 times when UK voted different with US, there are 14 times France voted different the same with UK, and among the 18 times when France voted different with US, there are also 14 times UK voted different the same with France. Besides, the influencing relations among US, Soviet Union and China are also very prominent, China stood by the side of Soviet Union when Soviet Union voted against US for 11 out of 13 times.
- Pattern 13:** Makes assumptions about the influencing relations from one state's former decisions to another state's latter decision. The most prominently matched influencing relations is the negative influence from US to Soviet Union, as two opponents during the Cold War, each side would consider about the other side's long-term preference and interest, and correspondingly took counter-actions, supported what the opponent disliked, and opposed to what the opponent liked.
- Pattern 24:** Makes assumptions about the stability of long-term relations between states, particularly between permanent members. As observed, the testing outcome are not very ideal, especially for the relations between US and UK, between US and France, which are usually very stable as the ally relations. This instability is partially due to the nonuniform opinions within the Western bloc about the Apartheid Policy and Invasion by South Africa, nearly all of them having an ambiguous and dilemma attitudes.

Table 9.20: Top 10 Influencing Relations Tested by Subject 7-the Apartheid Policy and the Invasion by South Africa

Rank	Pattern	Subject-Object	Content
1	PT 1	“Republic of China ” influenced by own former decisions	“Republic of China” (Taiwan), kept a very firm attitude against South Africa’s Apartheid Policy and Invasion, voted affirmative on all resolutions People’s Republic of China, also keep a very firm attitude against South Africa’s Apartheid Policy and Invasion, voted affirmative on all resolutions except one. Whether it is the “Republic of China” or the People’s Republic of China, their attitudes against the injustice in South Africa are the same, unlike US, UK and France
2	PT 1	People’s Republic of China influenced by own former decisions	China cooperated with Soviet Union (voting the same) on most cases, together against other western countries
3	PT 7	China influenced by Soviet Union ’s current decision	As a super power, Soviet Union keep a firm attitude on resolutions against South Africa’s Apartheid Policy and Invasion, voted affirmative on 23 out of 26 times
4	PT 1	Soviet Union influenced by own former decisions	China stood by the side of Soviet Union together against US, for 13 times Soviet Union voted against US, there are 11 times China voted the same with Soviet Union
5	PT 10,11	China influenced by the interacting relations between US and Soviet Union	When France followed closely with US and voted the same with US, UK would be touched and also voted the same with US, afraid to lag behind
6	PT 10,11	UK influenced by the interacting relations between US and France	When UK followed closely with US and voted the same with US, France would also possibly be touched and voted the same with US, afraid to be isolated
7	PT 10,11	France influenced by the interacting relations between US and UK	As the “state under protectorate” of US, “Republic of China” followed closely with US, stood by US’s side on most cases
7	PT 7	“Republic of China ” positively influenced by US ’s current decision	Different from “Republic of China”, there are a lot of ideological difference and interest conflicts between People’s Republic of China and US, and China voted different with US on most cases about South Africa issues
9	PT 7	People’s Republic of China negatively influenced by US ’s current decision	As opponents, Soviet Union negatively considered about the long-term preferences and decisions of US on South Africa Issues, and opposed to US correspondingly
10	PT 13	Soviet Union (negatively) influenced by US ’s former decisions	

9.3.8 Subject 8-the Minority Regime and the Invasion by Southern Rhodesia

Subject 8 **Southern Rhodesia (Zimbabwe)** (Frequency: 20, Range: 1965-1980) is about Southern Rhodesia’s Minority Regime and Invasions on other African Countries. According to the Wikipedia entry [Wikipedia, 2015n] on the topic:

“Initially, the territory was referred to as “South Zambezia”, a reference to the River Zambezi, until the name “Rhodesia” came into use in 1895. This was in honor of Cecil Rhodes, a British empire-builder and key figure during the British expansion into southern Africa. [...] Southern Rhodesia was the self-governing British colony north of the Limpopo River and the Union of South Africa. After its unilateral declaration of independence by a minority white regime in 1965 (which is not recognized by the United Kingdom and the international society), it was known as Rhodesia until 1979.”

The UN security council passed a series of resolutions to sanction Southern Rhodesia, in order to lead the Southern Rhodesia to build a government under the majority rule. While for the 20 resolutions (with different voices) targeting at the minority regime of Southern Rhodesia, the attitudes of different members are diversified, UK voted 9 absent, France voted 8 absent, US voted 11 absent,

the ratios of absent for those western super/great powers is comparatively high, while China voted just 1 absent, and there is nearly no absent from other non-permanent members (mainly small and former colonial countries), all voting as affirmative.

We ran the influence pattern matching algorithms based on the sample data of subject 8 in table D.12 (in Appendix D.3), the text outcome is illustrated in table 9.21.

Table 9.21: Test for Patterns of Influence with Subject 8-the Minority Regime and the Invasion by Southern Rhodesia

No	Mode	Rank	IPMF	IPMR	No	Mode	Rank	IPMR	IPMF
PT 1		10-	10/19	0.53	PT 24		7	17/19	0.89
PT 1		10-	10/19	0.53	PT 24		10-	11/19	0.58
PT 1		10-	11/19	0.58	PT 24		10-	14/19	0.74
PT 1		1,5	5/5,12/13	1.00,0.92	PT 24		10-	12/19	0.63
PT 1		10-	14/19	0.74	PT 24		10-	12/19	0.63
PT 7		6	18/20	0.90	PT 10,11		8	12/14	0.86
PT 7		10-	9/20	0.45	PT 10,11		1	11/11	1.00
PT 7		10,10-	5/6, 11/14	0.83, 0.79	PT 10,11		1	7/7	1.00
PT 7		10-	15/20	0.75	PT 10,11		1	7/7	1.00
PT 7		10-,8	3/6, 12/14	0.50, 0.86	PT 10,11		10-	2/7	0.29

- **Pattern 1:** Makes assumptions about the influencing relations of one state’s latter decisions influenced by own former decisions, particularly for permanent members. The testing outcome is very similar with the above subject 7-the Apartheid Policy and Invasion by South Africa, the Matching Ratio on China and Soviet Union (especially China) are much higher than on US, UK and France. Facing the bills against the “unjust” white minority regime of Southern Rhodesia, Soviet Union¹¹ and China (no matter it is the “Republic of China” or the People’s Republic of China) kept a very firm attitude to vote affirmative; while the attitudes of US, UK and France are not as constant, basically voted half affirmative but half absent, which might be interleaved with the empathy (as old empires of colony) with the white regime and the maintaining of reputation for justice. Actually, Southern Rhodesia used to be the colony of UK, and the minority government is constituted mainly by British people.
- **Pattern 7:** Makes assumptions about the influencing relations from one state’s current decision to another state’s current decision. There are several prominently matched influencing

¹¹The attitude for Soviet Union is more mixed than China, though as communist but used to be imperialist, therefore with a relatively paradoxical attitude.

relations, such as the positive influence from US to UK, the positive influence from US to “Republic of China” and the positive influence from Soviet Union to People’s Republic of China, which is in accord with the real international relations and situations. UK is the best ally of US and Taiwan is under the protection of US, while P.R. China stood by the side of Soviet Union on many cases.

- **Pattern 10,11:** Makes assumptions about the influences from the influencing relations among some states to the decision of another state, the Matching Ratio of most tested influencing relations are all very high. For example China stood by the side of Soviet Union on most bills when Soviet Union voted different with US, and Soviet Union also stood by the side of China on all bills when China voted against US. Besides, we can find that tough the votes between US and France were very incongruous (as the Match Ratio of influence pattern 7 from US to France is just 0.45), but France usually need accompanies to vote different against US, especially the “supports” from super/great powers like Soviet Union and China. For 7 times when France voted different with US, Soviet Union and China all voted different together against US; what is interesting is, as the best ally of US, UK just supported 2 times while France voted different with US, which might show the intimate relationship between US and UK (UK follows closely with US).
- **Pattern 24:** Makes assumptions about the stability of long-term relations between states, particularly between permanent members. As observed, the testing outcome overall are not very ideal, while the Matching Ratio of influencing relation between US and UK is quite high, which is matched to the common sense. As the best ally of US, UK is loyal to US on many crucial and controversial issues, even when US is opposed by France, Germany and some other allies. The relationship between UK and US is highly stable, each side would not easily betray the other side, not only for the friendships, but also for own interest and reputation, especially for super/great powers.

9.3.9 Specific Influencing Relations Ranking

We aggregate the testing outcome of specific influencing relations¹² respectively in 8 subjects into a composite testing outcome, organize a ranking list of the top 30 matched specific influencing relations by the $IPMR_{Average}$ (which is the Weighted Average of Influence Pattern Matching Ratio of every specific influencing relations in 8 subject samples¹³) as shown in table 9.22. In which, $IPMR_{STDEV}$ is the Standard Deviation of $IPMR$, $IPMR_{MAX}$ and $IPMR_{MIN}$ are respectively the Maximum and Minimum of $IPMR$, and $IPMR_{Pattern}$ is the Weighted Average of $IPMR$ of the general influence pattern by all specific influencing relations of the same pattern.

- For 5 influencing relations of pattern 1 ranked top 30 in the list, all 5 are ranked top 10: 2, 3, 7, 8, 10. The most prominent influencing relation of pattern 1 (with the highest $IPMR_{Average} = 0.849$) is $CN_1 \downarrow CN_2$ ¹⁴: the decision of China on latter issue would be influenced by own decisions on former issues. As a growing great power and potential super power, China worships own reputation, and does not easily turn claims and opinions toward a same subject. For example, China firmly advocates the principle of “non-interference in domestic affairs” of other countries, specifically on many controversial issues such as

¹²In a specific influencing relation, the influencing subject and influenced object are both specific states or relations among specific states.

¹³The weight of one influencing relation in each subject sample is determined by the number of voting record of bills computed to obtain the Matching Ratio, the more bills used, the more weight will be given.

¹⁴ \downarrow means the influence from former issue to latter issue (in the vertical dimension.)

Table 9.22: Text Outcome: Specific Influencing Relations Ranking

Rank	Influencing Relations	Pattern	$IPMR_{Average}$	$IPMR_{STDEV}$	$IPMR_{MAX}$	$IPMR_{MIN}$	$IPMR_{Pattern}$
1	$\{US_1 \rightarrow FR_1\} \sim UK_1$	10/11	0.938	0.085	1.00	0.82	0.707
2	$CN_1 \downarrow CN_2$	1	0.849	0.129	1.00	0.63	0.809
3	$UK_1 \downarrow UK_2$	1	0.823	0.184	1.00	0.53	0.809
4	$\{US_1 \rightarrow UK_1\} \downarrow \{US_2 \rightarrow UK_2\}$	24	0.818	0.166	1.00	0.53	0.751
5	$US_1 \leftrightarrow UK_2$	13	0.807	0.238	1.00	0.38	0.662
6	$\{US_1 \rightarrow CN_1\} \sim FR_1$	10/11	0.800	0.283	1.00	0.60	0.707
7	$RU_1 \downarrow RU_2$	1	0.796	0.137	1.00	0.59	0.809
8	$US_1 \downarrow US_2$	1	0.795	0.211	1.00	0.53	0.809
9	$CN_1 \leftrightarrow RU_2$	13	0.790	0.127	0.88	0.70	0.662
10	$FR_1 \downarrow FR_2$	1	0.781	0.190	1.00	0.54	0.809
10	$US_1 \rightarrow UK_1$	7	0.781	0.215	1.00	0.37	0.647
12	$\{RU_1 \rightarrow UK_1\} \downarrow \{RU_2 \rightarrow UK_2\}$	24	0.773	0.085	0.86	0.79	0.751
13	$\{US_1 \rightarrow RU_1\} \downarrow \{US_2 \rightarrow RU_2\}$	24	0.750	0.146	0.92	0.56	0.751
14	$US_1 \leftrightarrow FR_2$	13	0.747	0.239	1.00	0.35	0.662
15	$\{CN_1 \rightarrow RU_1\} \downarrow \{CN_2 \rightarrow RU_2\}$	24	0.740	0.198	0.88	0.60	0.751
16	$\{US_1 \rightarrow FR_1\} \downarrow \{US_2 \rightarrow FR_2\}$	24	0.735	0.183	0.96	0.50	0.751
17	$\{RU_1 \rightarrow FR_1\} \downarrow \{RU_2 \rightarrow FR_2\}$	24	0.725	0.159	0.92	0.59	0.751
18	$CN_1 \rightarrow RU_1$	7	0.720	0.247	0.89	0.32	0.647
19	$\{US_1 \rightarrow UK_1\} \sim FR_1$	10/11	0.685	0.297	1.00	0.29	0.707
19	$\{US_1 \rightarrow RU_1\} \sim UK_1$	10/11	0.685	0.228	0.93	0.41	0.707
21	$\{US_1 \rightarrow CN_1\} \sim RU_1$	10/11	0.682	0.325	1.00	0.32	0.707
22	$\{US_1 \rightarrow RU_1\} \sim FR_1$	10/11	0.680	0.288	1.00	0.44	0.707
23	$US_1 \rightarrow FR_1$	7	0.660	0.222	0.96	0.33	0.647
24	$\{US_1 \rightarrow CN_1\} \downarrow \{US_2 \rightarrow CN_2\}$	24	0.615	0.120	0.70	0.53	0.751
25	$US_1 \leftrightarrow RU_2$	13	0.597	0.341	0.92	0.07	0.662
26	$US_1 \rightarrow CN_1$	7	0.580	0.303	0.79	0.13	0.647
27	$\{US_1 \rightarrow RU_1\} \sim CN_1$	10/11	0.573	0.266	0.86	0.14	0.707
28	$RU_1 \rightarrow UK_1$	7	0.570	0.386	0.85	0.13	0.647
29	$US_1 \rightarrow RU_1$	7	0.543	0.235	0.85	0.20	0.647
30	$RU_1 \leftrightarrow FR_2$	13	0.540	0.321	0.92	0.14	0.662

Cyprus, Yugoslav War and so on, China voted absent steadily, disagreed with the interference by NATO.

- For 6 influencing relations of pattern 7 ranked top 30 in the list, mainly spreading in the middle and lower reaches: 10, 18, 23, 26, 28, 29. The most prominent influencing relation of pattern 7 (with the highest $IPMR_{Average} = 0.781$) is $US_1 \rightarrow UK_1$ ¹⁵: the decision of UK would be influenced by the decision of US on the same issue. UK as the most ardent supporter and firm ally of US, follows closely with US on many international affairs.
- For 7 influencing relations of pattern 10,11 ranked top 30 in the list, the ranking distribution is scattering: 1, 6, 19, 19, 21, 22, 27. The most prominent influencing relation of this pattern (with the highest $IPMR_{Average} = 0.938$) is $\{US_1 \rightarrow FR_1\} \curvearrowright UK_1$ ¹⁶: the decision of UK would be influenced by the influencing relation between US and France on the same issue. The triangular relations among US, France and UK are very subtle, for example, when France follows with US, UK would never lag behind; and UK usually does not dare to vote against US alone, but with the support from France, the two would vote different together.
- For 5 influencing relations of pattern 13 ranked top 30 in the list, the ranking distribution is also scattering: 5, 9, 14, 25, 30. The most prominent influencing relation of pattern 13 (with the highest $IPMR_{Average} = 0.807$) is $US_1 \leftrightarrow UK_2$ ¹⁷: the decision of UK on latter issue would be influenced by the decision of US on former issue, which is also a typical form of bilateral influence between US and UK. The following and positive reference by UK on US is not performed independently on every single issue, but would be represented as the consideration on long-term preferences and decisions of US by UK, only by this a secular and intimate ally relation could be retained.
- For 7 influencing relations of pattern 24 ranked top 30 in the list, mostly distributing in the upper and middle reaches: 4, 12, 13, 15, 16, 17, 24. The most prominent influencing relation of this pattern 24 (with the highest $IPMR_{Average} = 0.818$) is $\{US_1 \rightarrow UK_1\} \downarrow \{US_2 \rightarrow UK_2\}$: the influencing relation between US and UK on the latter issue would be influenced by the influencing relation between themselves on the former issues, which is another form of bilateral influence between US and UK. The cooperation and positively interacting relation between them would not easily change or shake, even confronting the present contradiction of interests, but would still be steadily determined by the long-term intimate relations, represented as the mutual trust and aid between each other.

For the top 10 specific influencing relations, we specifically display a detailed discussion about them, as shown in table 9.23 and 9.24.

9.3.10 General Influence Pattern Comparison

Aggregate the Matching Ratio of specific influencing relations of the same pattern into the Matching Ratio of the general influence pattern, as shown in table 9.26, which means not identifying the specific states any more, but just represented as general influencing and influenced agent. We assumed there are agents A, B, C, D voting on sequential issues X, Y . $IPMR_{Compos}$ is the Weighted Average of $IPMR_{Average}$ of specific influencing relations of the same general pattern, and $Rank_{Mean}$ is the average ranking by $IPMR_{Average}$ of specific influencing relations of the same pattern.

¹⁵ \rightarrow means the influence from one agent to another agent (in the horizontal dimension.)

¹⁶ \curvearrowright means the influence from an influencing relation between agents to another agent on a same issue.

¹⁷ \leftrightarrow means the influence from an agent on former issue to another agent on latter issue.

Table 9.23: Top 10 Influencing Relations Ranking (1-5)

Rank	Composition	Pattern	Content
1	$\{US_1 \rightarrow FR_1\} \rightsquigarrow UK_1$	10/11	The relations among three allies US, UK and France is an important triangular relations in international politics. The intimate degrees among them are not uniform, UK would not easily turn against US, even with severe interest conflicts with US, UK usually would not vote different from US alone, but with the support and companion from France (while France voted the same with UK against US); and when France voted the same with US, UK would not easily lag behind, would vote the same with US as well, to display the image of best ally of US. The $IPMR_{Average}$ of UK's reaction toward the relation between US and France is high as 0.938, showing how much UK attaches importance to his relation with US.
2	$CN_1 \downarrow CN_2$	1	As a responsible great power, China worship his reputation, and never easily turn his claims and opinions toward a same subject, with the $IPMR_{Average}$ as 0.849 of insisting on his former votes, for example, on the Cyprus, Yugoslav War issues and so on, China voted absent steadily.
3	$UK_1 \downarrow UK_2$	1	As a reliable great power, the vote of UK is very constant on the same subject, would not easily change his claims and opinions, with the $IPMR_{Average}$ as 0.823 of being persistent on his former votes, for example, on Iraq, Palestine issues and so on, UK voted affirmative firmly.
4	$\{US_1 \rightarrow UK_1\} \downarrow \{US_2 \rightarrow UK_2\}$	24	The alliance and cooperation relation between US and UK is very steady and reliable, would not easily change or shake, even confronting the present contradiction of state interests, but would be steadily determined by the long-term influencing relations, represented as the mutual trust and aid between US and UK, the $IPMR_{Average}$ is high as 0.818.
5	$US_1 \leftrightarrow UK_2$	13	To maintain the intimate relation with US, the follow and cooperation by UK is not performed independently on every single issue, but would be represented as the consideration of long-term preferences and behaviors of US (UK would not just cooperated or followed with US while needed and temporarily), the $IPMR_{Average}$ of UK voting in accordance with US's former votes is high as 0.807, only by this a secular and stable alliance relation could be preserved.

Table 9.24: Top 10 Influencing Relations Ranking (6-10)

6	$\{US_1 \rightarrow CN_1\} \rightsquigarrow FR_1$	10/11	The relation between US and France is delicate, though belonging to the same side (Western Bloc) and the same alliance (NATO), France would occasionally go against US, perform an independent path from US, different from the following relation between US and UK, however, US is still a super power and the leader of western world, sometimes France would need the supporters while going against US, to share the pressure, just as China as a influential great power and also not totally leaning to either Western Bloc or Soviet Bloc, when China also went against US, the France would much possibly, with the $IPMR_{Average}$ 0.800, "dare" to oppose to US either.
7	$RU_1 \downarrow RU_2$	1	As one of the only two super powers and the leader of Soviet Bloc and Warsaw Pact, Soviet Union worship his reputation, and would not easily turn his claims and opinions toward a same subject, with the $IPMR_{Average}$ as 0.796 of insisting on his former votes, for example, on the India & Pakistan conflicts issue, Soviet Union voted absent steadily, not willingness to interfere the conflicts between them, and on the issue about the colony of Portugal, Soviet Union voted affirmative firmly; to oppose to Portugal.
8	$US_1 \downarrow US_2$	1	Similarly, as one of the only two super powers and the leader of Western Bloc and NATO, US also worship his reputation a lot, to play as a role model for his allies and "little brothers", and would not easily turn his claims and opinions toward a same subject, with the $IPMR_{Average}$ as 0.795 of insisting on his former votes, for example, on the bills about the Iraqi invasion of Kuwait, US voted affirmative unswervingly to sanction Iraq and even start the war against Iraq at last, and on the issue about the Yugoslav wars, US also voted affirmative unwaveringly, to interfere the "civil" wars and sanction the Federal Republic of Yugoslavia.
9	$CN_1 \leftrightarrow RU_2$	13	After the disintegration of Soviet Union, the relation between Russia and China improved soon (the relation between Soviet Union and China has worsen since 1960s), jointly confront the pressure from Western bloc under the new international situation, in order to maintain the mutual trust relationship, the cooperation and coordination with the partner is not performed independently on single issue (not just cooperated or followed the partner while needed and temporarily), but would be represented as the consideration of long-term preferences and behaviors of the partner: the $IPMR_{Average}$ of Russia voting in accordance with China's former votes is high as 0.790, only by this a secular and stable alliance relation could be preserved.
10	$FR_1 \downarrow FR_2$	1	As an unconformable great power, the vote of France is very unalterable on the same subject, would not easily be affected by others and change his claims and opinions, with the $IPMR_{Average}$ as 0.781 of being persistent on his former votes. For example, on the issue of Israel & Palestine conflicts, France voted affirmative, even on the bills against Israel supported by US, also on the issue of Yugoslav Wars, France voted affirmative unswervingly to interfere the conflicts and oppose to the Federal Republic of Yugoslav.
10	$US_1 \rightarrow UK_1$	7	UK as the most ardent supporter and the most firm ally of US, follows closely with US on many international affairs, the follows and cooperations of UK with US are quite common in real-world situations.

Table 9.25: Influential States Extracted by the Top 10 Influencing Relations







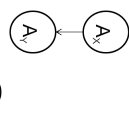
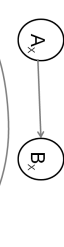
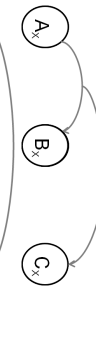

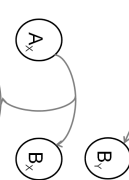
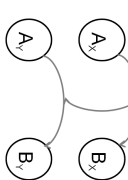
Rank	State	SUM	Influencing	Influenced	Influencing in Influencing	Influenced in Influencing	Influencing in Influenced	Influenced in Influenced
1	US	8						
2	UK	7	1	4	0	1	1	0
3	FR	4	1	2	0	1	0	0
4	CN	4	2	1	0	1	0	0
5	RU	3	1	2	0	0	0	0

Table 9.26: Text Outcome: General Influence Pattern Comparison

Composition	Pattern	$IPMR_{Compos}$	$IPMR_{Median}$	$IPMR_{VAR}$	$IPMR_{STDEV}$	$IPMR_{MAX}$	$IPMR_{MIN}$	TOP 10	Rank Mean
	PT 1	0.809	0.860	0.028	0.166	1.00	0.53	5	6.00
	PT 7	0.647	0.725	0.063	0.250	1.00	0.13	1	22.33
	PT 10/11	0.707	0.780	0.062	0.249	1.00	0.14	2	16.43
	PT 12	0.366	0.390	0.010	0.098	0.46	0.20	0	/
	PT 13	0.662	0.700	0.079	0.282	1.00	0.07	2	16.60
	PT 24	0.751	0.770	0.022	0.148	1.00	0.50	1	14.43

- Influence Pattern 1 Intra-influence from Decision to Decision: The decision of agent A on latter issue Y would be influenced by the decision of own on former issue $X(s)$.
 $IPMR_{Compos}$ and $IPMR_{Median}$ of Influence Pattern 1 are respectively 0.809, 0.860, which are both highest compared with other influence patterns, and in the top 10 specific influencing relations, the specific influencing relations of pattern 1 occupies 5 places.
- Influence Pattern 7 Inter-influence from Decision to Decision: The decision of agent B on issue X would be influenced by the decision of agent A on the same issue X .
 $IPMR_{Compos}$ and $IPMR_{Median}$ of Influence Pattern 7 are respectively 0.647 and 0.725, which are not very prominent compared with other influence patterns.
- Influence Pattern 10/11 Inter-influence from Decision to Decision (and Structure): The decision of agent C on issue X would be influenced by the influencing relations between agent A and agent B on the same issue X .
 $IPMR_{Compos}$ and $IPMR_{Median}$ of Influence Pattern 10/11 are respectively 0.707 and 0.780, which are at the middle level compared with other influence patterns.
- Influence Pattern 12 Inter-influence from Structure to Structure: The influencing relations between agent C and agent D on issue X would be influenced by the influencing relations between agent A and agent B on the same issue X .
 $IPMR_{Compos}$ and $IPMR_{Median}$ of Influence Pattern 12 are respectively 0.366 and 0.390, which are quite low compared with other influence patterns, the overall matching outcome are unideal, and none of specific influencing relation of pattern 12 enters the top 10 and even top 30 .
- Influence Pattern 13 Intra-inter Influence from Decision to Decision: The decision of agent B on latter issue Y would be influenced by the decision of agent A on former issue $X(s)$.
 $IPMR_{Compos}$ and $IPMR_{Median}$ of Influence Pattern 13 are respectively 0.662 and 0.700, which are at the middle level compared with other influence patterns, and $IPMR_{STDEV}$ of pattern 13 is highest as 0.282 compared with all other influence patterns, which means the matching outcome for different influencing relations of pattern 13 is quite scattering, as we can observed, $IPMR_{MAX}$ is highest as 1.00, but $IPMR_{MIN}$ is lowest as 0.07.
- Influence Pattern 24 Inter-intra Influence from Structure to Structure: The influencing relations between agent A and agent B on latter issue Y would be influenced by the influencing relations between agent A and agent B on former issue $X(s)$.
 $IPMR_{Compos}$ and $IPMR_{Median}$ of Influence Pattern 24 are respectively 0.751 and 0.770, which are relatively high compared with other patterns, and the $Rank_{Mean}$ of specific influencing relations of pattern 24 is just second to pattern 1, the matching outcome of this pattern is relatively prominent.

The Layout of Patterns of Influence Tested with Matching Algorithms

The patterns of influence tested by matching algorithms (with voting data from UN official database) are distributed in the patterns system as shown in table 9.27 (in which \mathcal{D} represents the decision, and \mathcal{S} represents the structure).

Table 9.27: Patterns of Influence Provided with Matching Test

Intra-influence	Inter-influence	Intra-inter Influence	Inter-intra Influence
-----------------	-----------------	-----------------------	-----------------------

From \mathcal{D} to \mathcal{D}	Pattern 1	Pattern 7	Pattern 13
From \mathcal{D} to \mathcal{D} and \mathcal{S}			
From \mathcal{D} to \mathcal{S}			
From \mathcal{S} to \mathcal{D}		Pattern 10	
From \mathcal{S} to \mathcal{D} and \mathcal{S}		Pattern 11	
From \mathcal{S} to \mathcal{S}		Pattern 12	Pattern 24

On each of four categories of 1.intra-influence, 2.inter-influence, 3.intra-inter influence and 4.inter-intra influence, there are at least one patterns of influence being tested empirically, therefore providing a relatively good representativeness, also with typical innovative significance (as patterns 10,11,12,13,24 are all newly proposed patterns, pattern 10,11,12,24 introduce the new role of influencing or influenced structure, and patterns 13,24 also represent the complex influence flowing crossing both vertical dimension and horizontal dimension), actually, the influence involved influencing and influenced structure and the influence crossing both different issues and agents are exactly two main innovations of the thesis.

Chapter 10

Modeling and Simulation of the Influence Models in UN SC Voting

Contents of Chapter 10

10.1 Conceptual Model: Reasoning Chart design

10.2 Mathematical Model: Variables Definition and Rules Design

10.3 Computer Model: Multi-agent System Modeling and Simulation

10.4 Simulation Experiments and Analysis

After the models of influence tested, then be used to do simulation experiments by the example of UN Security Council voting, through an integral modeling process of conceptual model, mathematical model and computer model.

10.1 Conceptual Model: Reasoning Chart design

The model is build based on the conceptual model of UN Security Council collective voting designed by [Luo and Meng, 2013a], which combines the multi-agent system and system dynamics, and is improved on many aspects, as shown in figure 10.1, to be more suitable to describe the complicated gaming and influencing process in UN Security Council, and better fit in with the new models of influence proposed.

There are a series of thoughts and setup in the original model [Luo and Meng, 2013a] which are meaningful to be referred to describe the interactions and influences among UN SC member states, as following:

- **The Influence from More than One Agents with Positive/Negative Weight.** The vote of one agent could be influenced by more than one agents simultaneously, and which is positively influenced by own, allies/friends, and negatively influenced by opponents/enemies, accordingly to different weights, namely, there is a weight allocation among own influence, allies' influences and opponents' influence.
- **The Comparison between Expected and Factual Outcome and the Feedback Loop of Expectation Psychologic Status to Weight Allocation of Influence.** The factual voting outcome is determined by the collective votes based on the voting rule, and the expected voting outcome is determined by the agent's own preference, and it happens for the unmatched between expected outcome and factual outcome, and the accumulative frequency of unmatched between what is expected and what is factual would affect the agent's psychological status (defined as Expectation Psychology), which would in return influence the alloca-

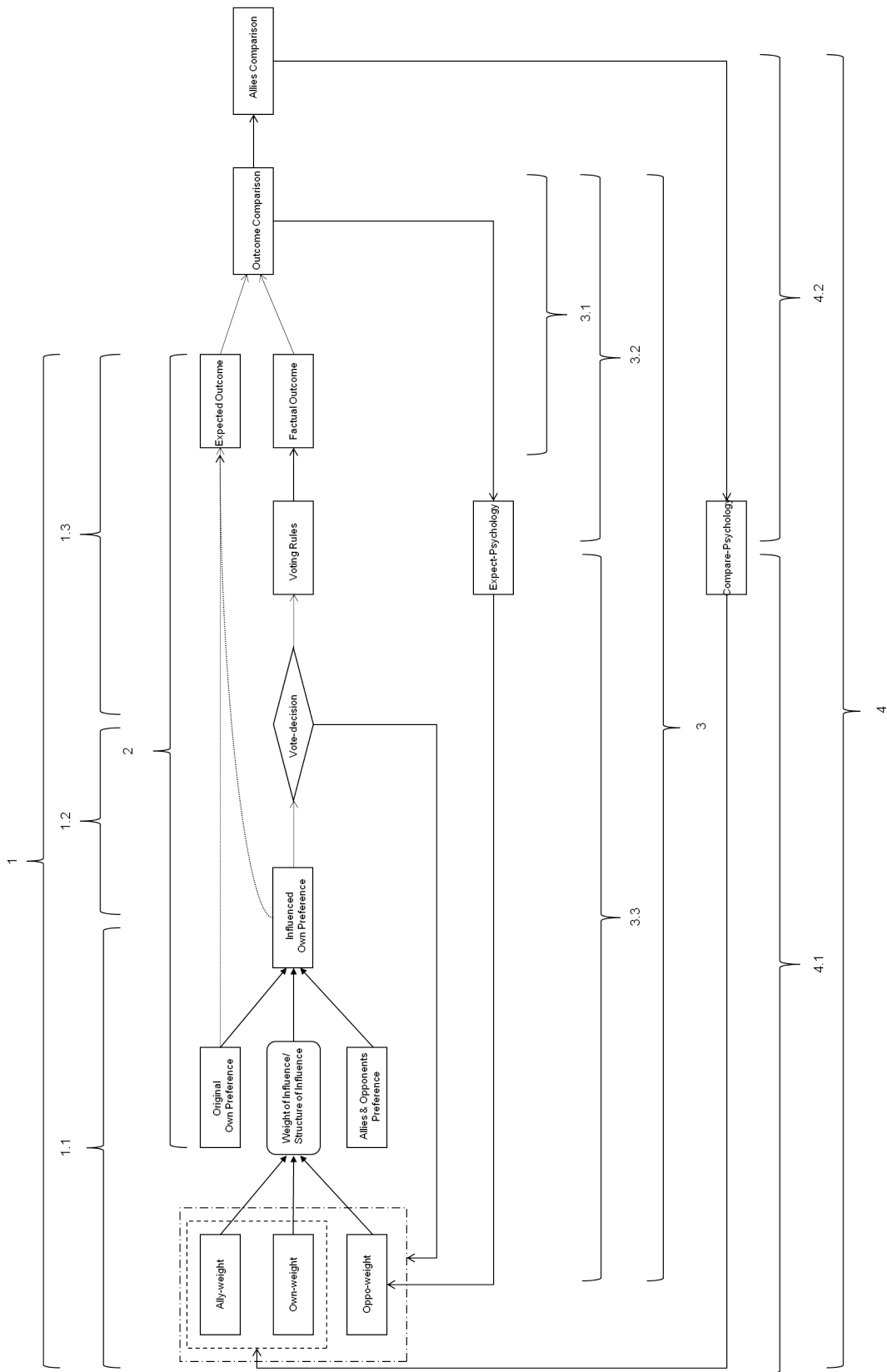


Figure 10.1: Reasoning Chart of SC Voting-Steps Marked for Mathematic Model

tion of weight of influence, while in [Luo and Meng, 2013a], only the weight of influence of opponents/enemies being affected, and simply varying as eliminating as 0 or recovering to the status quo, without continuously changing.

Furthermore, from the original model to the improved model, there are several crucial adjustments and advancements, as following:

- **From Original Own Preference to Influenced Own Preference.** build a new variable as the Influenced Own Preference, which is determined collectively by original own preference and allies' and opponents' preferences. It is the own preference after being influenced, eventually determining the vote-decision. While in the original model, the vote-decision is directly determined by own , allies' and opponents' preference, without the necessary intermediate variable to describe clearly the process of being influenced.
- **The Expected Outcome Determined by Original Own Preference or Influenced Own Preference?** It is undoubted that your own behavior and own expectation are determined by your own preference, though influenced by others, but eventually the vote is independently on your own influenced or uninfluenced preference (only if it is a democracy system).¹ However, it is the original own preference (the *initial dream*) determining your outcome expectation, or the influenced own preference (the *eventual hope*) determining your outcome expectation? Both two assumptions make sense to some extents, while in the original model, only the original own preference determining the expectation was considered.
- **The Comparison between Own Expectation Realizing Situation with Allies' and the Feedback Loop of Comparison Psychological Status to the Weight Allocation of Influence.** Expectation realized or not is usually different for different agents, even in the same alliance, if your expectation realizing situation is apparently lower than your allies, for instance your allies always get what they want, always achieve what they expect, while your expected outcome rarely comes true, it would very likely affect your psychological status (defined as Comparison Psychology), while you feel imbalanced comparing with your allies, it is possible to adjust the weight allocation between own and allies, and more dependent on own, or less influenced by your allies.

Therefore, the feedback loops not only include the expectation psychology [Luo and Meng, 2013a], but also added with the comparison psychology, and the influence not only flowing to the weight of opponents, but also flowing to the weight of own and allies.² As the weight matrix of influence is the numerical expression of the network structure of influence, once the weight of influence being reallocated, then the structure of influencing relations being changed.

10.1.1 Key Concepts and Mechanisms

Further explain the critical concept and mechanism in the improved model, the UN security council voting is a typical multi-agent and multi-phase evolving system, which is composed of independent sovereign state (rational agents) constantly interacting, mutually adapting and collectively evolving. Therefore, the model needs to be integrated with both multi-agent system and system dynamics, use the multi-agent system to describe the gaming and interacting among member states, and use the feedback loops in system dynamics to express the evolution and dynamics

¹For example of the election in a democratic country, although there are enormous influences from medias, from advertises, but the vote is still on the voter.

²Applied with corresponding inclination coefficients to capture the continuous change.

of weight/structure of influence over time. And each member's vote-decision mainly has following aspects of consideration:

- **Original Own Preference and Influenced Own Preference**, the former is the intrinsic preference before influenced, and the latter is the eventual preference after influenced. The original own preference determines the expected outcome directly, or indirectly through the influenced own preference, and eventually acts on the factual outcome through the individual vote.

For determining the expected outcome, if the original/influenced own preference is topped as affirmative vote, then the member state naturally hopes the bill passed; if the original/influenced own preference is topped as negative vote, then the member state naturally hopes the bill failed; and if the original/influenced own preference is topped as absent, then the member state would be indifferent for the outcome of voting.

For determining the factual outcome, as the original own preference as just one portion, together with allies' and opponents' preferences, collectively constituting the influenced own preference, therefore if the original own preference is topped as affirmative, then the member state is more inclined to vote affirmative, but still could be influenced to vote negative or absent; and if the original own preference is topped as negative, then the member state is more probative to vote negative, but still could be influenced to vote affirmative or absent, as the affection of original own preference on the vote-decision has to be intermediated by the influenced own preference.

- **Allies' Preferences and Opponents' Preferences**, it should be noticed that the member state's vote-decision is not only determined by his own original preference, but also need to refer to or consider about the preferences of his allies and opponents, in another word, would be influenced by other allies and opponents, usually it is the influenced own preference but not the original own preference eventually determining own vote-decision. For the allies and friends, the influences are usually positive (expressed as the positive weight of influence), but for enemies and opponents, the influences are usually negative (expressed as the negative weight of influence). Especially for the international politics, which is full of game, strategy, competition and confrontation, and therefore is filled with varied allies and opponents.

No matter admitting or not, there are always at least two confronting alliances in international politics (such as the apparent confrontation between NATO and Warsaw Pact³ during the Cold War, and the nowadays inapparent confrontation between US, US's allies and China, Russia, etc), the states within the same alliance support each other, but the states between different alliances oppose to each other, in fact, the UN security council voting is essentially the game between super/great powers and their allies.

From the original own preference to the influenced own preference, referring to or being influenced by allies' and opponents' preferences, which can be described in specific ways as, for one member state, the more of his allies prefer one choice (as Y or A or N)⁴, the more of this choice be preferred by this member state, but the more of his opponents prefer one choice (as Y or A or N), the less of this choice be preferred by this member state, and for the influence from own original preference, usually it is a positive influence, but also could be negative on some special cases (while the agent is very disappointed and unsatisfied with own status, and unconfident and unsure about own choices).

³“The Warsaw Pact (formally, the Treaty of Friendship, Co-operation, and Mutual Assistance, sometimes, informally WarPac, akin in format to NATO) was a collective defense treaty among eight communist states of Central and Eastern Europe in existence during the Cold War.” [Wikipedia, 2015o]

⁴Y(Yes) shorted for affirmative vote, A(Absent) shorted for absent, and N(No) shorted for negative vote.

	<i>Weight</i>	<i>Influencing Preference</i>	<i>Influenced Preference</i>
<i>Ally</i>	2	$A \succ Y \succ N$	$A \succ Y \succ N$
<i>Opponent</i>	-1	$Y \succ A \succ N$	$N \succ A \succ Y$
<i>Own</i>	3	$Y \succ A \succ N$	$Y \succ A \succ N$

For one example above, one member state is influenced by own intrinsic preference ($A \succ Y \succ N$), one ally's preference ($Y \succ A \succ N$) and one opponent's preferences ($Y \succ A \succ N$) with respective weights 2,-1,3, firstly display the preference outcome of this state respectively and separately influenced by his ally', his opponent' and own intrinsic preference (as shown in column of influenced preference). Secondly discuss about the preference outcome when simultaneously influenced by ally, opponent and own collectively. There are many approaches to deal with this form of influence from more than one origins, like the Weighted Minimal Distance, the Borda Score, and so on. For the Borda rule, based on the three influencing preferences and corresponding weights, the Borda Score for Y is $2 \times 1 - 1 \times 2 + 3 \times 2 = 2 - 2 + 6 = 6$, the Borda Score for A is $2 \times 2 - 1 \times 1 + 3 \times 1 = 4 - 1 + 3 = 6$, and the Borda Score for N is $2 \times 0 - 1 \times 0 + 3 \times 0 = 0$. Then the collectively influenced preference would be $Y \sim A \succ N$, in which Y and A are both preferred to N , but equal preferred to each other (as with identical Borda Scores).

- Weight of Influence/Structure of Influence**, the weight of influence is the numerical expression of the structure of influence, and the structure of influence is the graphical representation of the weight of influence, which could be mutually transformed and substituted, as each element value in the weight matrix represents the *existence or nonexistence, positive or negative, and the strong or weak* of the influence from one agent (member state) to another agent (member state), if the weight of influence between agent i and agent j ($wt_{(i,j)}$) is 0, which means there is no influence from agent i to agent j , but if the weight is higher than 0, which means there is an influence from agent i to j and the influence is positive (agent i is the ally or friend of agent j), and if the weight is lower than 0, which means there is an influence from agent i to j either but the influence is negative (agent i is the opponent or enemy of agent j). And the higher of the absolute value of the weight ($|wt_{(i,j)}|$), the stronger of the influence, although the direction for negative influence and positive influence are contrast, but we can still compare the strength regardless.
- Expectation Psychology Status**, the vote-decision of member states are not a simply weighted sum of own, allies' and opponents' preferences, the weight of influence is not fixed but dynamic during the interacting process. In fact, the vote-decision is also delicately influenced by a lot of psychological factors, such as the expectations realizing situation, more specially, considering the real-time feedbacks of the comparison between expected voting outcome and factual voting outcome. The match status (match or unmatched, conformity or inconformity) between expected outcome and factual outcome would affect the member state's psychological status and then affect the weight allocation of influence.

To explain why the expectation psychology can affect on the adjustment of weight allocation, we need to trace back to the reasons why the expected outcome can not come true. For example, before the disintegration of the Soviet Union and the end of the Cold War, although the Socialist camp (or the Eastern bloc) led by the Soviet Union and the Capitalist camp (or the Western bloc) led by the United States are two contradictive political groups (intensively expressed as the confrontation between two military alliances of NATO and Warsaw Pact), however, the two groups are not in thorough and throat-cutting conflicts all the time, usually have both fight and peace, both confrontation and compromise at the

same time or alternately. In the situation of SC national game, the veto of permanent member makes the confronting states and alliances vulnerable to the vicious circle of mutual vetoes, causing any side's expected bills hardly to pass, and making the UN SC fall into the dilemmas of paralysis and "lose-lose". Therefore, the long expectation contradiction (the inconformity between expected outcome and factual outcome) will force the member to reconsider the weight allocation (particular for opponents), decreasing the confrontation between opponent alliances and weakening the negative weight from opponents, pursuing the "win-win" in some sense.

- **Comparison Psychology Status**, it is naturally to assume that the member state would compare with other allies, about the situation of expected outcome achieved or not, if the member state is hardly to achieve what he wants but his allies achieve a lot what they want, this state might be disappointed, unsatisfied and feel unbalanced, and would possibly reconsider his relations with allies, and specifically representing as the weight (of influence) allocation adjustment from own and allies. Normally, if the average unmatched frequency (which is the number of times when the factual outcome is different with the expected outcome) of agent i is much higher than his allies, the agent would possibly "feel disappointed" of allies, and more dependent on own preference or less influenced by allies' preferences in the following interaction; conversely, if the average unmatched frequency of agent i is much lower than his allies, the agent might "feel satisfied", and a rational response is to maintain the status quo, not changing the current weight allocation.

For example of the unbalanced psychological status, the NATO is built and led by US, in NATO US usually get what he want, while France and some other continental European countries might feel kind of sacrificed for the interest of US, would possibly feel unbalanced and unsatisfied, and be more dependent on own and less closely follow with US.

10.2 Mathematical Model: Variables Definition and Rules Design

10.2.1 Define Variables

(1) $OriginalOwnPreference_{(t)}(i)$ (abbreviated as $OO P_{(t)}(i)$) represents the member i 's intrinsic preference for the bill t before influenced, the domain $Dom[OO P_{(t)}(i)] = \{Y, A, N\}$, the preference is the ordering among $Y(YES)$, $A(Absent)$, and $N(NO)$. $Own-Weight$ (Abbreviated as ONW) represents the weight of referring to original own preference when the member state being influenced to compute the influenced own preference;

(2) $AllyPreference_{(t)}(i)$ (abbreviated as $ALP_{(t)}(i)$), represents the preferences of the member i 's allies for the bill t , which would positively affect member i 's influenced own preference for bill t or after bill t ⁵, $Ally-Weight$ (Abbreviated as ALW) represents the weight of referring to allies' preferences, and the $Ally-weight$ is usually bigger than 0. In the simulation, we preliminarily and conservatively divide two alliances among UN SC members to discuss the current international relations and gaming situations, one is the NATO (more broadly Western bloc) led by US, with members including UK, France, Germany and so on, and NATO is a typical military alliance, the other is the "alliance" led by China and Russia, and the competitions and confrontations between the two groups are obvious and common, especially after the disputes of Syria, Crimea and so on between Russia and Western bloc, Russia has been pushed more closely to China under the restrictions and pressures;

⁵Namely, the more allies prefer the $Y(YES)$ as top, the more possible for the influenced own preference be topped as $Y(YES)$, and then vote affirmative, similarly for $A(Absent)$ and $N(NO)$.

(3) *OpponentPreference*_(t)(*i*) (abbreviated as *OPP*_(t)(*i*)), represents the preferences of the member *i*'s opponents for the bill *t*, which would negatively affect the member *i*'s influenced own preference for the bill *t*⁶, *Oppo-Weight* (Abbreviated as *OPW*) represents the weight of referring to opponents' preferences, and the *Oppo-weight* is usually lower than 0;

(4) *InfluencedOwnPreference*_(t)(*i*) (abbreviated as *IOP*_(t)(*i*)) represents the member *i*'s eventual preference for the bill *t* after influenced, with the domain $Dom[OOP_{(t)}(i)] = \{Y, A, N\}$, which is positively influenced by own and allies' preferences, and negatively influenced by opponents' preferences, according to different weights *OWW*, *ALW*, *OPW*.

(5) *Vote*_(t)(*i*) (abbreviated as *V*_(t)(*i*)) represents the member *i*'s vote-decision for the bill *t*, $Dom[V_{(t)}(i)] = \{Y, A, N\}$, *Y*(*YES*) indicates affirmative vote, *A*(*Absent*) indicates absent, *N*(*NO*) indicates negative vote, *Vote*_(i)(*t*) is determined by the influenced own preference, if it is topped as *Y*(*YES*), then $V_{(t)}(i) = Y$, if it is topped as *A*(*Absent*), then $V_{(t)}(i) = A$, and if it is topped as *N*(*NO*), then $V_{(t)}(i) = N$;

(6) *Rule*(*t*), represents the collective voting rule for the bill No.*t*, for the current rule of UN SC, which need at least 9 affirmative votes from 15 members and none of permanent member voting negative to pass a bill, the most prominent characteristic of the current voting rule is veto right, one negative vote from the permanent members would veto the bill, (namely "the Principle of Unanimity"), which is also a focus of dispute about the Security Council reform. The voting rule can be set up as a variable in simulation experiments, to investigate the effect of different reform plans and voting rules on SC's gaming process and voting outcome.

(7) *FactualOutcome*(*t*) (abbreviated as *FO*(*t*)), represents the factual voting outcome for the bill No.*t*, with domain $Dom[FO(t)] = \{Passed, Failed\}$, which is the outcome of collective voting according to the voting rule;

(8) *ExpectedOutcome*_(t)(*i*) (abbreviated *EO*_(t)(*i*)), represents the expected voting outcome of the member *i* for the bill No.*t*, with $Dom[EO_{(t)}(i)] = \{Passed, Indifferent, Failed\}$, which is determined by the influenced own preference or original own influence, if it is topped as *Y*(*YES*), then $EO_{(t)}(i) = Passed$, if it is topped as *A*(*Absent*), then $EO_{(t)}(i) = Indifferent$, and if it is topped as *N*(*NO*), then $EO_{(t)}(i) = Failed$;

(9) *OutcomeComparison*_(t)(*i*) (abbreviated as *OC*_(t)(*i*)), represents the cumulative number of inconformity or unmatch between expected outcome and factual outcome of the member *i* after the bill No.*t*, the unmatch between what is expected and what is factual would influence the expectation psychology status;

(10) *AlliesComparison*_(t)(*i*) (abbreviated as *AC*_(t)(*i*)), represents the comparison of the member *i*'s own match status (between expected and factual outcome) with his allies' match status after the bill No.*t*, the imbalance between own match status and allies' match status would affect the comparison psychology status;

(11) *ExpectationPsychology*_(t)(*i*) (abbreviated as *EP*_(t)(*i*)), represents the expectation psychological status of the member *i* after the bill No.*t*, with the domain $Dom[EP_{(t)}(i)] = \{Confrontation, NonConfrontation\}$, which is determined by *OutcomeComparison*_(t)(*i*), and would influence back to the weight of opponents. If $EP_{(t)}(i) = C(Confrontation)$, the member *i* would keep on opposing to his opponents, expressed as *Oppo-weight* unchanged; while if $EP_{(t)}(i) = N(NonConfrontation)$, then the member *i* would relatively reduce the opposing intensity, expressed as *Oppo-weight* decreased.

(12) *ComparisonPsychology*_(t)(*i*) (abbreviated as *CP*_(t)(*i*)), represents the comparison psychological status of the member *i* after the bill No.*t*, with the domain $Dom[CP_{(t)}(i)] = \{Balanced, Imbalanced\}$, this psychology is about the psychologic equilibrium compared with other allies, which is determined by the *AlliesComparison*_(t)(*i*) and would influence back to

⁶Namely, the more opponents prefer the *Y*(*YES*) as top, the more possibly for the influenced own preference be topped as *N*(*NO*) or *A*(*Absent*), and then vote negative or absent, just on the contrary of the opponent

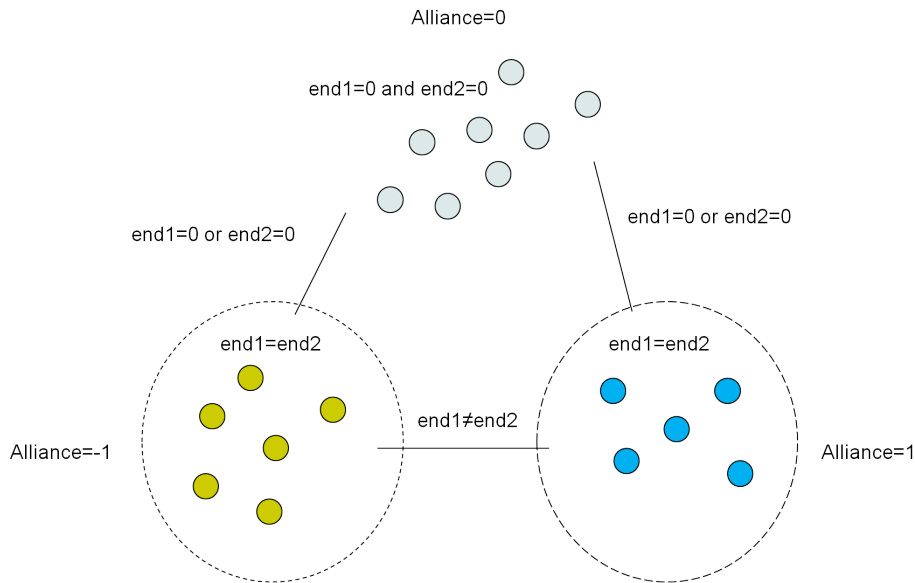


Figure 10.2: Weight of Influence Determined by the Alliance Comparison

the weight allocation between own and allies. If $CP_{(t)}(i) = I(Imbalanced)$, which means the member i feel unbalanced and unsatisfied, he would possibly consider more for himself and less for his allies, be more dependent on own and less influenced by allies in the following interactions, represented as increasing the *Own-Weight* and decreasing the *Ally-Weight*; but if $CP_{(t)}(i) = B(Balanced)$, which means the member i feel balanced and satisfied, then a rational reaction is to maintain the status quo, keep weight allocation unchanged.

10.2.2 Design Rules

Design mathematic rules according to each step in the conceptual model, as shown in figure 10.1. The complementary illustrations can be found in Appendix C.5.

Initial Setup: Weight Allocation Rule The generation of influencing relations is the prerequisite of collective influence and voting. Influence is a directed relation, but not symmetric. It is composed of:

- **Existence:** $\{exist, inexist\}$, whether influence exists or not;
- **Polarity:** $\{positive, negative, neutralz\}$, which is determined by the alliance belongingness of influencing and influenced states;
- **Strength:** $weak \rightarrow strong$, which is determined by the power comparison between influencing and influenced states.

Determining Existence: which can be adjusted by a probability $[0,1]$ to control the existence and integral intensity of influence patterns.

Deciding Polarity: This is determined by drawing probabilities, biased by a priori alliances. The set of Alliances are $\{AL1, AL2, Rest\}$, AL1 represents NATO (more broadly as the western world, such as the Austria), AL2 represents the “opponents” of NATO (typical as Russia and China), Rest represents other states without clear alliance belongingness.

For the polarity of $x \rightarrow y$ (x as influencing state, y as influenced state). As the alliance belongingness of x and y both can be varied as $\{AL1, AL2, Rest\}$, so there are $3 \times 3 = 9$ cases.

If x and y belong to the same alliance, i.e. $x, y \in AL1$, or $x, y \in AL2$, the probabilities should be biased to positive influence, e.g. set as: $Sp^0 = 0.5$, $Sp^+ = 0.4$, $Sp^- = 0.1$, respectively represents the probabilities of neutral/positive/negative influence under same alliance belongingness;

If x and y belong to different alliances, i.e. $x \in AL1, y \in AL2$, or $x \in AL2, y \in AL1$, then the probabilities should be biased to negative influence, e.g. set as: $Dp^0 = 0.5$, $Dp^- = 0.4$, $Dp^+ = 0.1$, respectively represents the probabilities of neutral/positive/negative influence under different alliance belongingness;

If one belongs to one alliance while the other belong to none, i.e. $x \in Rest, y \in AL1$, or $x \in Rest, y \in AL2$, or $y \in Rest, x \in AL1$, or $y \in Rest, x \in AL2$, the influence would be not that intensive as above two, the probabilities would be more biased to neutral, e.g. set as $R1p^0 = 0.7$, $R1p^+ = 0.2$, $R1p^- = 0.1$, respectively represents the probabilities of neutral/positive/negative influence under one belonging to an alliance but one belonging to none situation. The probabilities are more biased to positive compared with negative, as usually trying to persuade from the view of the alliance, or trying to persuade one country from alliance.

If neither of two states belongs to any alliance, i.e. $x, y \in Rest$, the influence would be more light, e.g. $R2p^0 = 0.9$, $R2p^+ = R2p^- = 0.05$, respectively represents the probabilities of neutral/positive/negative influence under two non-alliance states.

Allocating Weights :

We classified all member states into five category: super power, great power, regional power, middle power, small power [Chapnick, 1999]. Respectively, set the power index (shortly as *Pindex*) for five categories of power, at least satisfying:

$$Pindex_{super} > Pindex_{great} > Pindex_{regional} > Pindex_{middle} > Pindex_{small}$$

, e.g.

$$Pindex_{super} = 5, Pindex_{great} = 4, Pindex_{regional} = 3, Pindex_{middle} = 2, Pindex_{small} = 1$$

For influence $x \rightarrow y$, set $Pindex(x)$ as the power index of influencing state x , and $Pindex(y)$ as the power index of influenced state y .

The weight of influence from x to y would be: $+/- \frac{Pindex(x)}{Pindex(y)}$, the polarity (+/-) is determined by the probability biased by priori alliance discussed above; and the strength $\frac{Pindex(x)}{Pindex(y)}$ is determined by the power comparison between two states. It is easy to understand that the weight of influence from a super power to a small power would be different compared with from the small power to the super power.

Initial Setup: Preference Allocation Rule Preference allocation is another prerequisite for collective influence and voting, if from a cardinal approach, which could be expressed by voting probabilities. $PV_{(i)(t)}^N, PV_{(i)(t)}^A, PV_{(i)(t)}^Y$ are respectively the probabilities of member i voting negative, absent or affirmative on bill t .

Full Domains If the voting domain is full, namely as $Dom[Vote_{(t)}(i)] = \{N, A, Y\}$, which means the member i can choose all negative, absent and affirmative on the bill t .

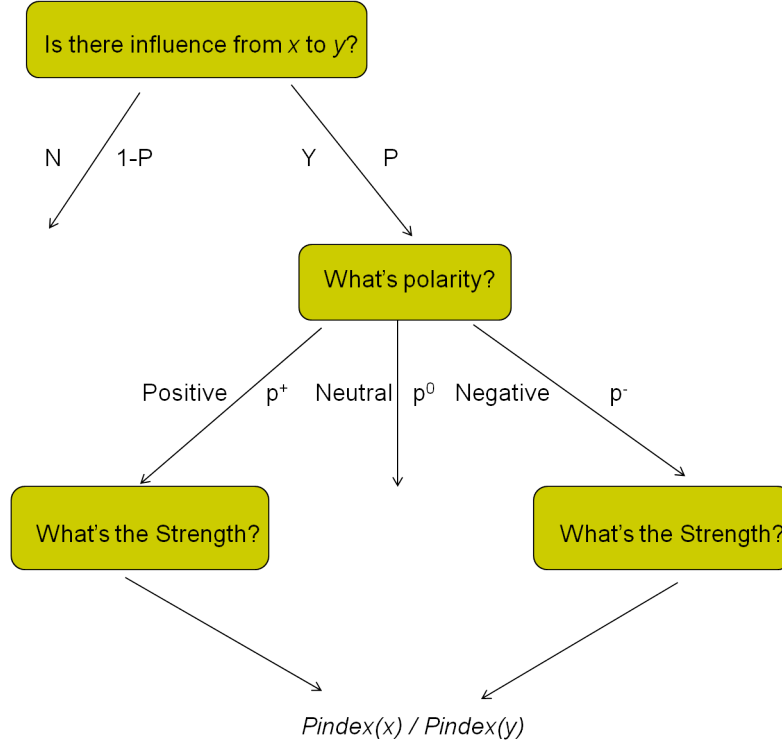


Figure 10.3: Procedures of Determining Polarity and Strength of Influence

$$PV_{(i)(t)}^N = \text{random-float}(1), PV_{(i)(t)}^A = \text{random-float}(3), PV_{(i)(t)}^Y = \text{random-float}(10) \quad (10.1)$$

As the probabilities of voting negative, absent and affirmative are not equal, member state are much more easy to vote affirmative, and very cautious to vote negative, especially for the veto from permanent members, then respectively allocate the probability distribution as $[0,1]$, $[0,3]$ and $[0,10]$.

Partial Domains If voting domain is partial, there will be 6 cases as, $Dom[Vote_{(t)}(i)] = \{N\}, \{A\}, \{Y\}, \{N, A\}, \{A, Y\}$ or $\{N, Y\}$. We only consider the partial domains due to unacceptance, when a member state voting, his domain of choices might be partial but not full under his preference or the pressures from others, then some choices would be unacceptable or impossible for him.

$$\begin{aligned}
 & \text{if } Dom[Vote_{(t)}(i)] = \{N\}, \text{ then } PV_{(i)(t)}^N = 1, PV_{(i)(t)}^A = 0, PV_{(i)(t)}^Y = 0 \\
 & \text{if } Dom[Vote_{(t)}(i)] = \{A\}, \text{ then } PV_{(i)(t)}^N = 0, PV_{(i)(t)}^A = 1, PV_{(i)(t)}^Y = 0 \\
 & \text{if } Dom[Vote_{(t)}(i)] = \{Y\}, \text{ then } PV_{(i)(t)}^N = 0, PV_{(i)(t)}^A = 0, PV_{(i)(t)}^Y = 1 \\
 & \text{if } Dom[Vote_{(t)}(i)] = \{N, A\}, \text{ then } PV_{(i)(t)}^N = \text{random-float}(1), PV_{(i)(t)}^A = \text{random-float}(3), PV_{(i)(t)}^Y = 0 \\
 & \text{if } Dom[Vote_{(t)}(i)] = \{A, Y\}, \text{ then } PV_{(i)(t)}^N = 0, PV_{(i)(t)}^A = \text{random-float}(3), PV_{(i)(t)}^Y = \text{random-float}(10) \\
 & \text{if } Dom[Vote_{(t)}(i)] = \{N, Y\}, \text{ then } PV_{(i)(t)}^N = \text{random-float}(1), PV_{(i)(t)}^A = 0, PV_{(i)(t)}^Y = \text{random-float}(10)
 \end{aligned} \quad (10.2)$$

To be normalized:

$$\begin{aligned}
PV_{(i)(t)}^N &= \frac{PV_{(i)(t)}^N}{PV_{(i)(t)}^N + PV_{(i)(t)}^A + PV_{(i)(t)}^Y} \\
PV_{(i)(t)}^A &= \frac{PV_{(i)(t)}^A}{PV_{(i)(t)}^N + PV_{(i)(t)}^A + PV_{(i)(t)}^Y} \\
PV_{(i)(t)}^Y &= \frac{PV_{(i)(t)}^Y}{PV_{(i)(t)}^N + PV_{(i)(t)}^A + PV_{(i)(t)}^Y}
\end{aligned} \tag{10.3}$$

Step 1 Factual Outcome Rules Computing the bill No t 's voting outcome, which is collectively determined by entire members' votes according to the voting rule, and each member's vote is determined by influenced own preference, which is collectively affected by original own preference, allies' preferences and opponents' preferences, according to their respective weights:

Step1.1 Influenced Own Preference

$$IOP_{(t)}(i) = In\left\{ \begin{array}{l} ONW_{(t)}(i) \times OOP_{(t)}(i), \\ ALW_{(t)}(i) \times ALP_{(t)}(i), \\ OPW_{(t)}(i) \times OPP_{(t)}(i) \end{array} \right\} \left| \begin{array}{l} \text{the Influence from Own} \\ \text{the Influence from Allies} \\ \text{the Influence from Opponents} \end{array} \right. \tag{10.4}$$

As defined and discussed in the Part II and Appendix B.1, In is a general influence function, which can use weighted KSB metric, Borda score for instance as specific rules. If from the cardinal approach, we can use the voting probability to represent the preference and describe the process of influence:

(1) **Prominent One Influence:** If the general influence function In works by the means of prominent one influence, which means for the voting preference of agent i on bill t , it will only be influenced by another voting preference which possess the highest weight/priority of influence, assume it is the voting preference of agent j on bill d , satisfying $|wt_{j,i(d,t)}| = \max_{n \in \mathbb{N}}^{m \in \mathbb{M}} |wt_{n,i(m,t)}|$, then:

$$\begin{aligned}
&\text{if } vote_{(j)}(d) = N \text{ and } wt_{j,i(d,t)} > 0, \text{ then } PV_{(i)(t)}^N = 1, PV_{(i)(t)}^A = 0, PV_{(i)(t)}^Y = 0 \\
&\text{if } vote_{(j)}(d) = N \text{ and } wt_{j,i(d,t)} < 0, \text{ then } PV_{(i)(t)}^N = 0, PV_{(i)(t)}^A = 0, PV_{(i)(t)}^Y = 1 \\
&\text{if } vote_{(j)}(d) = Y \text{ and } wt_{j,i(d,t)} > 0, \text{ then } PV_{(i)(t)}^N = 0, PV_{(i)(t)}^A = 0, PV_{(i)(t)}^Y = 1 \\
&\text{if } vote_{(j)}(d) = Y \text{ and } wt_{j,i(d,t)} < 0, \text{ then } PV_{(i)(t)}^N = 1, PV_{(i)(t)}^A = 0, PV_{(i)(t)}^Y = 0 \\
&\text{if } vote_{(j)}(d) = A \text{ and } wt_{j,i(d,t)} > 0, \text{ then } PV_{(i)(t)}^N = 0, PV_{(i)(t)}^A = 1, PV_{(i)(t)}^Y = 0 \\
&\text{if } vote_{(j)}(d) = A \text{ and } wt_{j,i(d,t)} < 0, \text{ then } PV_{(i)(t)}^N = 0.5, PV_{(i)(t)}^A = 0, PV_{(i)(t)}^Y = 0.5
\end{aligned} \tag{10.5}$$

(2) **Collective Influence:** If the general influence function In works by the means of collective influence, which means all influencing preferences take effect, just according to different weights, assume $WT_{(i)(t)} = \sum_{n \in \mathbb{N}}^{m \in \mathbb{M}} |wt_{n,i(m,t)}|$, which means $WT_{(i)(t)}$ is the sum of absolute values of all influencing weights on member i on bill t , then:

$$\begin{aligned}
PV_{(i)(t)}^N &= \frac{WT_{(i)(t)}^{N,+} + \frac{1}{2}WT_{(i)(t)}^{A,-} + WT_{(i)(t)}^{Y,-}}{WT_{(i)(t)}} \\
PV_{(i)(t)}^A &= \frac{WT_{(i)(t)}^{A,+} + \frac{1}{2}WT_{(i)(t)}^{N,-} + \frac{1}{2}WT_{(i)(t)}^{Y,-}}{WT_{(i)(t)}} \\
PV_{(i)(t)}^Y &= \frac{WT_{(i)(t)}^{N,+} + \frac{1}{2}WT_{(i)(t)}^{A,-} + WT_{(i)(t)}^{Y,-}}{WT_{(i)(t)}}
\end{aligned} \tag{10.6}$$

As shown above, for voting probability of *No*, $PV_{(i)(t)}^N$, which is constituted by three parts, $WT_{(i)(t)}^{N,+}$ is the sum of absolute value of all influencing weights which are positive and voting as $N(No)$, $WT_{(i)(t)}^{A,-}$ is the sum of absolute value of all influencing weights which are negative and voting as $A(absent)$, and $WT_{(i)(t)}^{Y,-}$ is the sum of absolute value of all influencing weights which are negative and voting as $Y(Yes)$.

(3) **Collective Influence with Structure Effect:** On many cases, the collective influence is interleaved with the structure effect, for example while you are influenced by two agents 1 and 2 with influencing relation between them (like agent 2 follows agent 1), then you might think agent 2 is just a follower without independent idea or agent 1 is influential as an opinion leader who might be more possible to hold the truth, then you would possibly increase the weight from agent 1 or decrease the weight from agent 2, which namely means the influencing relation/structure between agents 1 and 2 also takes an effect on the influenced outcome. To express the ebb and flow of weights, we use two coefficient α and β to respectively times on the weights of agents which are *Uninfluenced* in the influencing relations (marked as *U* on the superscript of *WT*), and the weights of agents which are *Influenced* in the influencing relations (marked as *I* on the superscript of *WT*), usually, it should be assumed that $\alpha < \beta$.

$$\begin{aligned} PV_{(i)(t)}^N &= \frac{\alpha WT_{(i)(t)}^{N,+U} + \beta WT_{(i)(t)}^{N,+I} + \alpha \frac{1}{2} WT_{(i)(t)}^{A,-U} + \beta \frac{1}{2} WT_{(i)(t)}^{A,-I} + \alpha WT_{(i)(t)}^{Y,-U} + \beta WT_{(i)(t)}^{Y,-I}}{\alpha WT_{(i)(t)}^U + \beta WT_{(i)(t)}^I} \\ PV_{(i)(t)}^A &= \frac{\alpha WT_{(i)(t)}^{A,+U} + \beta WT_{(i)(t)}^{A,+I} + \alpha \frac{1}{2} WT_{(i)(t)}^{N,-U} + \beta \frac{1}{2} WT_{(i)(t)}^{N,-I} + \alpha \frac{1}{2} WT_{(i)(t)}^{Y,-U} + \beta \frac{1}{2} WT_{(i)(t)}^{Y,-I}}{\alpha WT_{(i)(t)}^U + \beta WT_{(i)(t)}^I} \quad (10.7) \\ PV_{(i)(t)}^Y &= \frac{\alpha WT_{(i)(t)}^{Y,+U} + \beta WT_{(i)(t)}^{Y,+I} + \alpha \frac{1}{2} WT_{(i)(t)}^{A,-U} + \beta \frac{1}{2} WT_{(i)(t)}^{A,-I} + \alpha WT_{(i)(t)}^{N,-U} + \beta WT_{(i)(t)}^{N,-I}}{\alpha WT_{(i)(t)}^U + \beta WT_{(i)(t)}^I} \end{aligned}$$

(4) **Collective Influence with Group Effect:** On many cases, the collective influence is also interleaved with the group (pressure) effect, if it works by Majority Rule, then:

$$\begin{aligned} \text{if } PV_{(i)(t)}^N &= \max(PV_{(i)(t)}^N, \text{ then } PV_{(i)(t)}^A, PV_{(i)(t)}^Y), PV_{(i)(t)}^N = 1, PV_{(i)(t)}^A = 0, PV_{(i)(t)}^Y = 0 \\ \text{if } PV_{(i)(t)}^A &= \max(PV_{(i)(t)}^N, \text{ then } PV_{(i)(t)}^A, PV_{(i)(t)}^Y), PV_{(i)(t)}^N = 0, PV_{(i)(t)}^A = 1, PV_{(i)(t)}^Y = 0 \quad (10.8) \\ \text{if } PV_{(i)(t)}^Y &= \max(PV_{(i)(t)}^N, \text{ then } PV_{(i)(t)}^A, PV_{(i)(t)}^Y), PV_{(i)(t)}^N = 0, PV_{(i)(t)}^A = 0, PV_{(i)(t)}^Y = 1 \end{aligned}$$

Step1.2 (Individual) Vote-decision For the member *i*'s vote on the bill *t*, after obtains the Influenced Own Preference, then the vote-decision can be computed, which is determined by the top preferred vote if from a ordinal approach (expressed as preference ordering), in following, \succ_{IOP} represents the ordering of influenced preference:

$$Vote_{(t)(i)} = \begin{cases} Y & \text{if } Y \succ_{IOP} A \succ_{IOP} N \text{ or } Y \succ_{IOP} N \succ_{IOP} A \\ A & \text{if } A \succ_{IOP} Y \succ_{IOP} N \text{ or } A \succ_{IOP} N \succ_{IOP} Y \\ N & \text{if } N \succ_{IOP} Y \succ_{IOP} A \text{ or } N \succ_{IOP} A \succ_{IOP} Y \end{cases} \quad (10.9)$$

While if the individual vote is expressed by the cardinal approach of voting probability, then:

$$Vote_{(i)(t)} = \begin{cases} N & \text{if } 0 < \text{random-floate}(1) \leq PV_{(i)(t)}^N \\ A & \text{if } PV_{(i)(t)}^N < \text{random-floate}(1) \leq PV_{(i)(t)}^N + PV_{(i)(t)}^A \\ Y & \text{if } PV_{(i)(t)}^N + PV_{(i)(t)}^A < \text{random-floate}(1) \leq 1 \end{cases} \quad (10.10)$$

Step1.3 Factual Voting Outcome Further, computing the factual voting outcome based on every member' votes and according to the collective voting rule, for the existing rule, the bill passes only when no permanent member vote negative and at least 9 members vote affirmative in total:

$$FO_{(t)}(i) = \begin{cases} Passed & \text{if } Num_Y \geq 9 \cap PNum_N = 0 \\ Failed & \text{if } Num_Y < 9 \cup PNum_N > 0 \end{cases} \quad (10.11)$$

In which, the Num_Y represents the number of entire members voting affirmative, and $PNum_N$ represents the number of permanent members voting negative.

Step 2 Expected Outcome Rule Computing the member i 's expected voting outcome for the bill No. t , which is directly determined by original own preference or influenced own preference:

If determined by original own preference:

$$EO_{(t)}(i) = \begin{cases} Passed & \text{if } Y_{OPP} \succ A_{OPP} \succ N \text{ or } Y_{OPP} \succ N_{OPP} \succ A_{OPP} \\ Indifferent & \text{if } A_{OPP} \succ Y_{OPP} \succ N \text{ or } A_{OPP} \succ N_{OPP} \succ Y_{OPP} \\ Failed & \text{if } N_{OPP} \succ Y_{OPP} \succ A_{OPP} \text{ or } N_{OPP} \succ A_{OPP} \succ Y_{OPP} \end{cases} \quad (10.12)$$

If determined by influenced own preference:

$$EO_{(t)}(i) = \begin{cases} Passed & \text{if } Y_{IOP} \succ A_{IOP} \succ N \text{ or } Y_{IOP} \succ N_{IOP} \succ A_{IOP} \\ Indifferent & \text{if } A_{IOP} \succ Y_{IOP} \succ N \text{ or } A_{IOP} \succ N_{IOP} \succ Y_{IOP} \\ Failed & \text{if } N_{IOP} \succ Y_{IOP} \succ A_{IOP} \text{ or } N_{IOP} \succ A_{IOP} \succ Y_{IOP} \end{cases} \quad (10.13)$$

Step 3 Expectation Psychology Rule The member i would compare the factual outcome and expected outcome to get the Outcome Comparison (namely the frequency of Unmatch), then to obtain the Expectation Psychology Status, which would further affect the weight of influence from opponents.

Step 3.1 Outcome Comparison Every time when the expected outcome of member i is different from the factual outcome, then the value of outcome comparison (un-match) would be plus 1.

$$OC_{(t)}(i) = OC_{(t-1)}(i) + 1 \quad \text{if } EO_{(t)}(i) \neq FO(t) \text{ and } EO_{(t)}(i) \neq Indifferent \quad (10.14)$$

Step 3.2 Expectation Psychology When the Outcome Comparison (the frequency of Unmatch between factual outcome and expected outcome) reaches NUM^* or happens NUM^* times accumulatively, the member i will adjust its Expectation Psychology, reversing its EP value, and resetting the outcome comparison, set the frequency of Unmatch as 0, recounting:

$$\begin{aligned} & \text{If } OC_{(t)}(i) = NUM^* \text{ Then} \\ EP_{(t)}(i) &= \begin{cases} Confrontation & \text{if } EP_{(t-1)}(i) = NonConfrontation \\ NonConfrontation & \text{if } EP_{(t-1)}(i) = Confrontation \end{cases} \\ & \text{And Then } OC_{(t+1)}(i) = 0 \end{aligned} \quad (10.15)$$

Step 3.3 Opp-weight When the $ExpectationPsychology_{(t)}(i) = Non-confrontation$, the member i would decrease the weight of influence from opponents (the *Oppo-weight*), to less oppose to his opponents. And a concept “*Mild-to-Opponent-Inclination*” (abbreviated as MOI) is defined to describe the scale of reduction for the *Oppo-weight*.

$$OPW_{(t+1)}(i) = \begin{cases} OPW_{(t)}(i) - OPW_{(t)}(i) \times MOI_{(t)}(i) & \text{If } EP_{(t)}(i) = NonConfrontation \\ OPW_{(t)}(i) & \text{If } EP_{(t)}(i) = Confrontation \end{cases} \quad (10.16)$$

Step 4 Comparison Psychology Rule The member i would compare the own match status (between factual outcome and expected outcome) with his allies’ match status to get the allies’ comparison, then to obtain the Comparison Psychology Status, which would further affect the weight of influence from own and allies.

Step 4.1 Comparison Psychology Define two variables to capture the Allies Comparison, both computed by the Outcome Comparison data, one is the frequency of Own-Historical-Unmatch (abbreviated as $OHU_{(t)}(i)$), and the other is the frequency of Allies-Average-Historical-Unmatch (abbreviated as $AHU_{(t)}(i)$), the comparison between OHU and AHU would affect the Comparison Psychology Status. If the frequency of unmatched (what expected can not be unrealized) of the member i is higher than other allies, the member i would feel imbalanced, and if just on the contrary, the member i would feel balanced.

$$CP_{(t)}(i) = \begin{cases} Balanced & \text{If } OHU_{(t)}(i) \leq AHU_{(t)}(i) \\ Imbalanced & \text{If } OHU_{(t)}(i) > AHU_{(t)}(i) \end{cases} \quad (10.17)$$

Step 4.2 Own-weight and Ally-weight When the member i feel imbalanced (own expectations realized is less than allies), he would feel disappointed and unsatisfied for his allies, and it is reasonable for member i to be more independent on own preference in the next bill, represented as increasing the own-weight, or it is also possible for member i to be less influenced by allies’ preference in the next bill, represented as decreasing the ally-weight; and if member i feel balanced (own expectation realized is not less than allies), he would feel satisfied, and it is rational for him not to casually change the status quo. Two concepts “*Own-CentriPetal-Inclination*” (abbreviated as OPI), “*Allies-CentriFugal-Inclination*” (abbreviated as AFT) is defined to describe the scale of change for the *Own-Weight* and *Ally-Weight*.

$$\begin{aligned} ONW_{(t+1)}(i) &= \begin{cases} ONW_{(t)}(i) & \text{If } CP_{(t)}(i) = Balanced \\ ONW_{(t)}(i) + ONW_{(t)}(i) \times OPI_{(t)}(i) & \text{If } CP_{(t)}(i) = Imbalanced \end{cases} \\ ALW_{(t+1)}(i) &= \begin{cases} ALW_{(t)}(i) & \text{If } CP_{(t)}(i) = Balanced \\ ALW_{(t)}(i) - ALW_{(t)}(i) \times AFT_{(t)}(i) & \text{If } CP_{(t)}(i) = Imbalanced \end{cases} \end{aligned} \quad (10.18)$$

10.3 Computer Model: Multi-agent System Modeling and Simulation

The computer model is constructed based on the conceptual model and mathematical model. There are several platforms such as Swarm, Repast, Ascape, AnyLogic, Matlab for multi-agent system modeling and simulation, in which, Netlogo is one of the advanced simulation softwares

with ease of use and friendly interface, and is chosen as the implementation instrument of this SC multi-agent simulation system [Luo, 2013, Luo et al., 2013, Luo and Meng, 2013b, Luo and Meng, 2013a].

The interface of the simulation system is illustrated as shown in figure 10.4. The left portion is the multi-agents' evolving interaction view, we create $15 \times 100 = 1500$ agents to represent 15 SC member states voting on 100 bills, use every agent to represent a member state voting on one bill, due to the possibility of abstention (being absent of meeting or being present but not voting), we ask all agents, with a probability, "die" (be eliminated). Besides, use friendly shapes to distinguish permanent member and non-permanent member, the shape "person with necktie and briefcase" (set size 0.5) representing permanent member, and the shape "general person" (set size 0.3) representing non-permanent member, and use colors to distinguish every member's vote-decision on each bill, green indicated affirmative vote, red indicated negative vote, and gray indicated absent, and use the directed links among agents to represent the influencing relations among member states, green links as positive influence, red links as negative influence, and gray links as neutral influence.

The right portion is the parameter setup zone and data monitoring windows, on the top, we can adjust the probability of abstention and partial domains (determining the ratio of agents whom will be absent or whose voting domains will be constrained), then is the timely evolving plots and data monitoring windows, we use the plot "vote composition" to monitor the number of negative (red curve), absent (gray curve) and affirmative votes (green curve), and use the plot "efficiency" to monitor the passed frequency of 100 bills (by yellow curve) and the average achieve (namely the match between expected outcome and factual outcome) per 100 agents (by blue curve), which mainly measure the efficiency (how many bills can get passed and how many members can achieve what they want), and use the plot "fairness" to monitor the standard deviation (orange curve) and the minimum (red point) among all member state's achieve average, which mainly measure the fairness (how is the gap among member states' achieved average and how "rich" for the "poorest" one). Below the plots, we can adjust the probability of voting No, absent or voting YES for the initial setup of voting preference, and use the slider PP1 to PP24 to adjust the intensities of all 24 influences patterns⁷, we ask all turtles or links, by a the generation probability, to create corresponding influence patterns (represented as directed links) to other turtles or links. The rule of influence (prominent one influence or weighted collective influence) can be chosen, and the structure effect and group effect can be switched or chosen. Besides, to determine the polarity of influencing relations (positive, neutral or negative), the probability for different polarity would be different according to the relations between the end1 (influencing state) and end2 (influenced state), as we set the alliance of member states as $\{AL1, REST, AL2\}$, *AL1* and *AL2* means two confronting alliances, and *REST* means without clear alliance belongingness, naturally to understand that, if the two states belongs to the same alliance, the influence will be positive with higher probability but negative with lower probability, while if the two states separately belongs to the two confronting alliances, the influence will be negative with higher probability but positive with lower probability. While if one state does not belong to either side or both states do not belong to either side, then the influence would be not that intense and more possible to be neutral compared with other situations.

Improvement Compared with the Original Model [Luo and Meng, 2013a]

- [Luo and Meng, 2013a] distinguish the permanent members and non-permanent members just according to the critical difference of whether possessing veto right or not, but hadn't

⁷Setting the generation probabilities for all influence patterns, from *PP1* to *PP24*, representing the probabilities of influence from pattern 1 to 24.

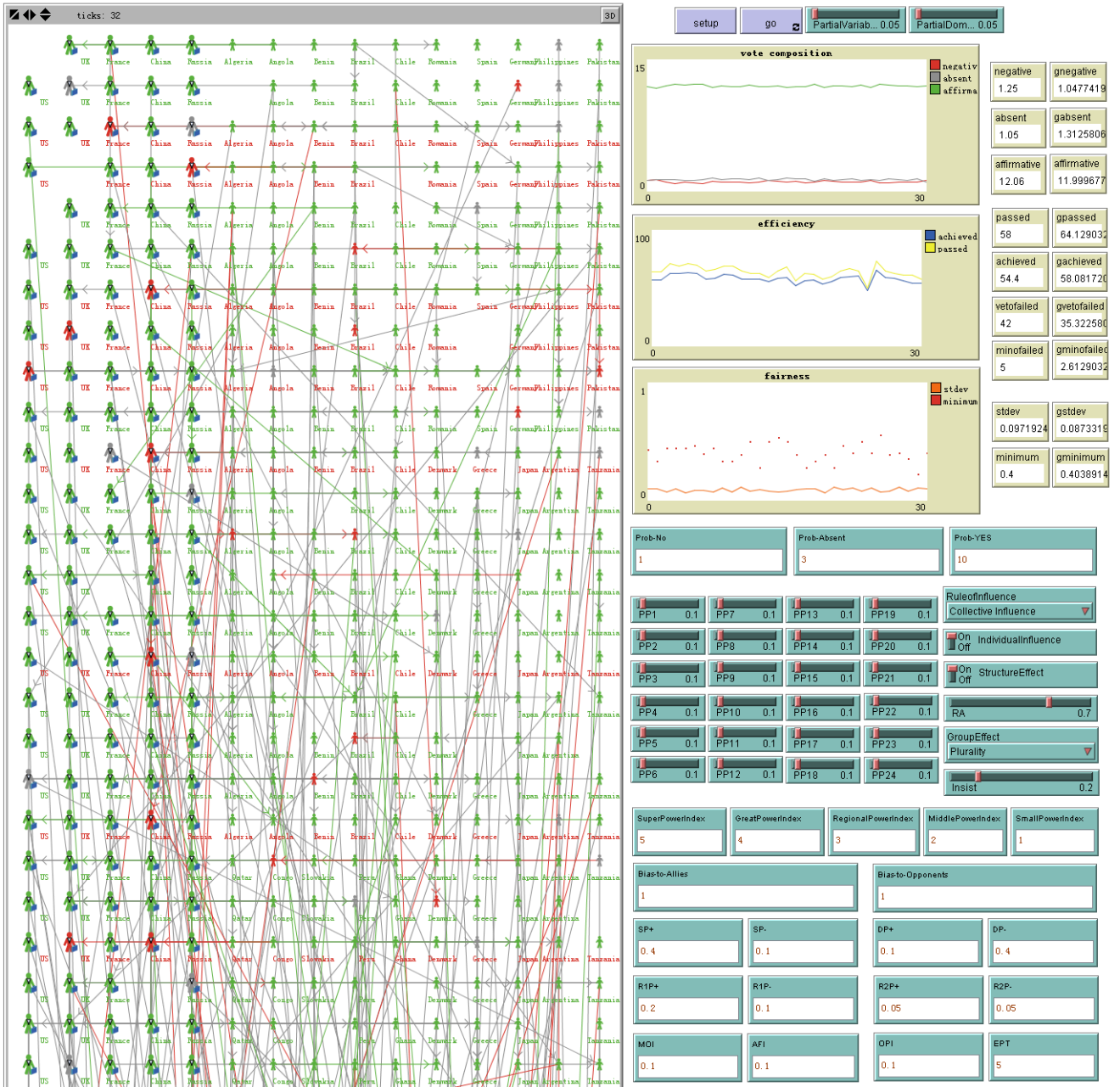


Figure 10.4: Simulation System Interface of UN SC Multinational Game

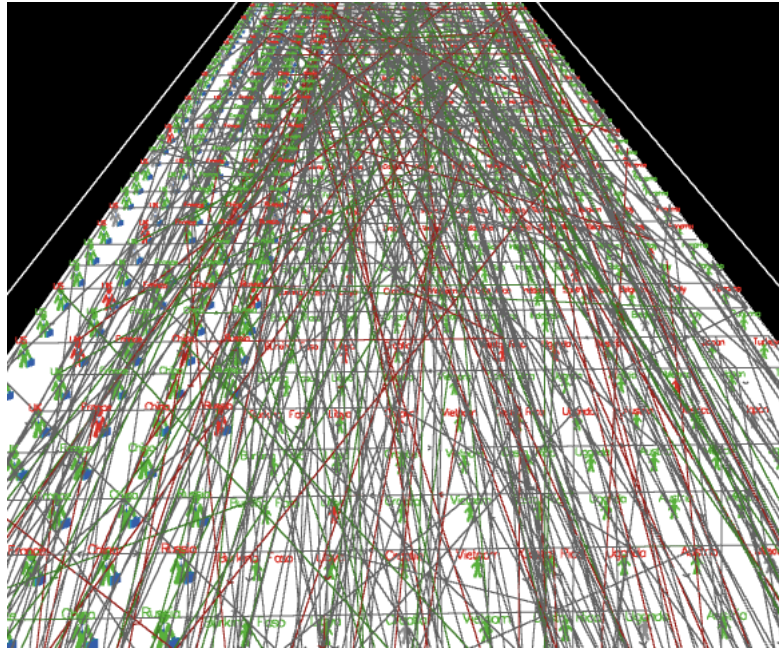


Figure 10.5: 3D view of Simulation System

distinguish the difference on tenures, didn't consider about the alteration of non-permanent members in 100 ticks of simulation (only based on the current SC members composition).

However, in the thesis, by collecting the data about the composition of non-permanent members over the past 10 years, the dynamic alteration of non-permanent members are considered and simulated.

- [Luo and Meng, 2013a] assume every member state's alliance belongingness unchanged, a member state belongs to one same alliance throughout the entire simulation process, although the alliance belongingness of states are indeed very stable in real-world politics, still it is not possible to totally eliminate the frequent alteration in short term; besides, [Luo and Meng, 2013a] didn't distinguish the variation of weights from different allies and opponents, but allocated with an identical weight (e.g. = 1), however, the influences from a superpower and from a small power should be assumed different⁸; Furthermore, [Luo and Meng, 2013a] assumed the weight of influence be stable during the simulations, but not dynamic and evolving.

However, in the thesis, we design the weight allocation as diversified and dynamic, different allies and opponents would have different weights of influence, and the weight would also be influenced and evolved due to some psychological concepts (the Expectation Psychology and Comparison Psychology), for example the positive weight of influence from alliances would possibly decrease into a negative one, and the negative weight of influence from opponents would also possibly increase into a positive one, which means there is an alliance relations dynamics, but not fixed and static.⁹

⁸For instance in the alliance NATO, the US surely has much more weight of influence on his allies than any other member; and while considering the negative influence from opponents, surely US would consider more about the former Soviet Union, than any other communist country, and many other examples.

⁹As common sense, the degree of influence from your friends or opponents would not be all the same (from beginning to end), sometimes you would trust your friend more deep, but other times you might be suspicious about your friend, which would be influenced by a series of objective environment change or objective psychological evolution.

		Intra-Influence	Inter-Influence	Intra-inter Influence	Inter-intra Influence
	To Decision	Pattern 1	Pattern 7	Pattern 13	Pattern 19
From Decision	To Decision and Structure	Pattern 2	Pattern 8	Pattern 14	Pattern 20
	To Structure	Pattern 3	Pattern 9	Pattern 15	Pattern 21
From Structure	To Decision	Pattern 4	Pattern 10	Pattern 16	Pattern 22
	To Decision and Structure	Pattern 5	Pattern 11	Pattern 17	Pattern 23
	To Structure	Pattern 6	Pattern 12	Pattern 18	Pattern 24

Figure 10.6: Matrix of Combinations of Influence Patterns

10.4 Simulation Experiments and Analysis

10.4.1 Experiments Design from Computer Science Paradigm

Design 4 experiments for 4 theoretical innovations, corresponding to each chapter on Part II Theory of influence.

CSP Experiment 1 Combinations of Influence Patterns

Corresponding to Chapter 6 Influencing and Influenced Structures, the Extended Patterns of Influence, the logic of experiment plan designing is, firstly investigated the current influence, then firstly added one new category of influences—crossing both different issues and agents, secondly added the other new category of influences—involved with influencing and influenced structure¹⁰, lastly investigated all influences in the system, as shown in figure 10.6.

- Plan 1: **Pattern** {1,7}, only the existed patterns, namely the influence from individual decision to individual decision and within single agent or single issue;
- Plan 2: **Patterns** {1,7,13,19}, added with influences crossing agents and issues;
- Plan 3: **Patterns** {1-12}, added with influences influencing on or influenced by structure;
- Plan 4: **Patterns** {1-24}, all influence patterns in the system.

CSP Experiment 2 Rules of Influence from More than One Origins

Corresponding to Chapter 7 Influence from More than One Origins¹¹, we proposed two main approaches to deal with the influence from more than one origins, one is the *Prominent One Influence*, in which the influencing preference or decision with the highest *Priority* dominate any

¹⁰As influence involved with structure and influence crossing both different agents and issues are two main innovations for new proposed patterns.

¹¹Section 7.1 The Prominent One Influence-by the Priority of Influence and section 7.2 The Collective Influence-by the Weight of Influence.

others; and the other is the *Collective Influence*, in which all influencing preferences and decisions take part in the influence but just to different extents and directions, according to their respective weights, varying as weak/strong, negative/positive.

- Plan 1: Prominent One Influence by Priority of Influence;
- Plan 2: Collective Influence by Weight of Influence.

CSP Experiment 3 Combinations of Levels of Influence

Corresponding to Chapter 7 Influence from More than One Origins¹², we design four plans to investigate the separate and mixed effect of three levels of influence, as following, level 1 means the influence from independent agents (independent influence) , level two means the influence from grouped agents (grouped influence) , and level 3 means the influence from interacted/influencing agents (structured influence).

- Plan 1: {Level 1}
- Plan 2: {Level 1, Level 2}
- Plan 3: {Level 1, Level 3}
- Plan 4: {Level 1, Level 2, Level 3}

CSP Experiment 4 Influence under Abstention and Constraints

Corresponding to Chapter 8 Influence with Abstention and Constraints, we design four plans to investigate the separate and mixed effect of Abstention and Constraints, as shown in figure 10.7, a set ratio of abstention make the ratio of 15×100 turtles “die” from the beginning in the simulation.

- Plan 1: {Non-abstention, Full domains}
- Plan 2: {Non-abstention, Partial domains}
- Plan 3: {Abstention, Full domains}
- Plan 4: {Abstention, Partial domains}

10.4.2 Experiments Design from Social Sciences Paradigms

Social science and computer science usually have different research interest and focus point, based on the realization of multi-agent simulation system of UN SC voting, a series of experiments are designed compare the effect of different SC reform schemes on the multilateral gaming process and collective voting outcome¹³. We design four sets of experiments, respectively investigate the effect of weight allocation between allies and opponents (as Experiment 1), the reform of veto of current permanent members (as Experiment 2), the enlarged scale of new added members (as Experiment 3), and the veto reform combined with member enlargement (as Experiment 4), which are all critical and controversial issues of UN SC reform, to provide specific thoughts and targeting advices for the prospect and direction of UN SC reform.

¹²Section 7.3 How to Compute the Influential Effect from Structure.

¹³Such as the bills passed or failed, and the state interest achieved or not, and so on.

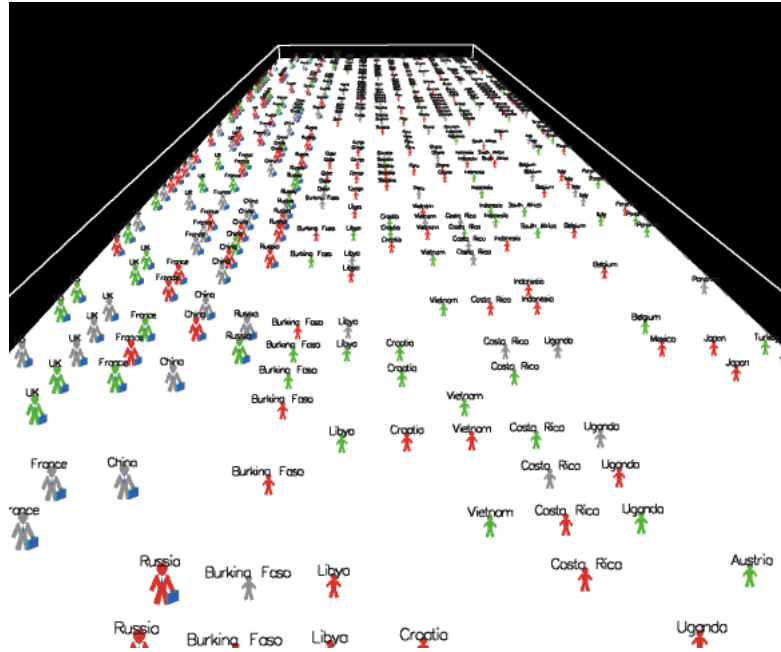


Figure 10.7: Simulation View with Abstentions

To compare and evaluate different reform schemes, the first step is to determine the measurement standard. Referred to previous works about SC reform [Luo and Meng, 2013a, Hosli et al., 2011, Weiss and Young, 2005], although with varied focuses and perspectives, nearly all of them deem that any SC reform should be beneficial to the two core value of *Efficiency* and *Fairness*, which are also the measurements used in the thesis to compare (the effectiveness of) different reform plans and deploy theoretical analysis. As illustrated in table 10.1, the measurement of *Efficiency* include two aspects of *Quantitative Efficiency* and *Qualitative Efficiency*, the former is reflected as the passed frequency of bills, and the latter is reflected as the overall expectation achievements of all members after the voting outcomes; the measurement of *Fairness* include three aspects of *Democracy*, *Representativeness* and *Equality*, the first is reflected as the constraint or weakening on the veto “privilege” of permanent members and improve the equity of vote effectiveness between different members, and the second is reflected as the addition of SC member seats, to improve the representativeness of SC for developing countries, especially the African countries, and the third is about the factual equality of interest satisfying due to the voting outcomes.

Table 10.1: The System of New Measurement Standard based on [Luo and Meng, 2013a]

Value	Sub-value	Explanation
Efficiency	Quantitative Efficiency	Improve the passed frequencies of bills, make the SC plays a more constructive role.
	Qualitative Efficiency	Distinguish the “Good” and “Evil” bills, make more bills beneficial to overall interest passed.
Fairness	Democracy (Right Fairness)	Reduce and weak the difference of vote effectiveness between permanent members and non-permanent members, and constrain or abolish the veto “privilege” of permanent members. This fairness is usually deemed as crucial by those UN members already in the Security Council (as non-permanent members.)

Representativeness (Opportunity Fairness)	Increase the amount of SC members and its representativeness, add more seats for developing countries, especially the African countries. This fairness is usually deemed as crucial by those UN members not yet or hardly be chosen into the Security Council.
Equality (Outcome Fairness)	To prevent the oversize gap between the interest gainer and interest loser (expectation achiever and failure), for instance the interest of super power and great power are fully satisfied, but the interest of small countries are ignored or trampled, there are series of measurement for the equality, such as the Maximum of Minimum [Rawls, 1999], the Lorenz Curve [Lorenz, 1905] and so on.

SSP Experiment 1 Weight Allocation between Allies and Opponents

In the interacting process of national game and specifically UN Security Council, the weight allocation between allies and opponents when members decide the vote could engender important effects on interacting process and voting outcome. The confrontations between opposite alliances have enormously influenced the evolving progress of world pattern in the history, such as the “fight” between the NATO led by the United States and the Warsaw Treaty Organization led by the former Soviet Union. Although the Warsaw Treaty Organization and the Soviet Union have become history, only if the inconformity and conformity between state interests exist, the confrontation between state alliances will exist, no matter the alliance is formal or informal, nominal or virtual, no matter the confrontation is apparent or latent, direct or indirect. For example, in SC’s voting of draft proposal regarding Syria issue in 4th October 2011, China and Russia jointly voted veto together against “Western bloc”. Therefore, the weight referring to allies and opponents is an important variable in SC national game, four plans of different weight allocations are designed (we defined α as the weight multiplier for allies and β as the weight multiplier for opponents):

- Plan 1: Heavily Biased to Allies, $\alpha = 3\beta$;
- Plan 2: Slightly Biased to Allies, $\alpha = 2\beta$;
- Plan 3: Balanced, $\alpha = \beta$;
- Plan 4: Slightly Biased to Opponents, $2\alpha = \beta$;
- Plan 5: Heavily Biased to Opponents, $3\alpha = \beta$.

SSP Experiment 2 Veto Reform

One of the most focused and controversial issue about SC reform is the veto “privilege” of current permanent members. The debate of abolishment and restriction of veto has been existing continuously, small and medium countries usually deem that the veto right violate the principle of democracy and the principle of sovereign equality, and the veto is the big powers’ privilege to impede the majority as the minority, which violates the value of fairness seriously; however, as the “vested interest holder”, the permanent members deem the veto right let them be responsible for the security and peace of the world, preventing from the dangerous consequence brought by the discord among big powers. To scientifically investigate the long-term dynamic effect of current permanent members’ veto reform on SC’s efficiency and fairness value, set three experiment plans:

- Plan 1: control group, current veto right unchanged;

- Plan 2: experimental group, simulate the conservative reform plan reserving part of veto, for example, reform the one-negative veto to two-negative veto, which means only at least two permanent members vote negative then the resolution can be vetoed;
- Plan 3: experimental group, simulate the radical reform plan totally abolishing the veto, make the vote effectiveness of permanent and non-permanent members identical.

SSP Experiment 3 Members Expansion

The member expansion is also an important issue of SC reform, the advocators of reform conformably deem only by adding the number of members can improve the representativeness and democracy of SC, some groups advocate adding only non-permanent members, while some groups claim adding permanent members either, but adding new permanent member inevitably confront the sensitive issue of veto, those “striving to be permanent members” states (such as the “G4”¹⁴, Japan, Germany, India and Brazil) usually have to be realistic, declare they will not consider to seek the veto temporally, in order to deduct the obstruction and join the permanent members as early as possible. Therefore, it is necessary to investigate the effect of SC expansion not involving veto on the efficiency and fairness value, set three experimental plans of different members scale:

- Plan 1: control group, current 15 members;
- Plan 2: experimental group, adding 5 new members, there will be $15 + 5 = 20$ members in total;
- Plan 3: experimental group, adding 10 new members, there will be $15 + 10 = 25$ members in total.

According to the current voting rule of SC, if 9 of 15 members vote affirmative (and none of permanent members vote negative) then the bill get passed, which obeys the absolute majority rule of above 3/5 affirmative, therefore, when the number of members expands to 20 the bill needs 12 affirmative to get passed, and when the number expands to 25 the bill needs 15 affirmative to get passed.

SSP Experiment 4 Members Expansion Involving Veto Reform

As the most critical issues of SC reform, member expansion and veto reform are usually intertwined, which is an important reason why SC reform is so difficult and controversial. The most influential, representative and controversial reform plans include “Uniting for Consensus” (UfC) plan, “G4” plan and “African Union” plan and so on. The G4 (Japan, Germany, Brazil and India) advocate adding both permanent members and non-permanent members, and strive to join the permanent members themselves, the 6 added permanent members include the four states and two African states, and the 4 seats of non-permanent members are respectively distributed to Asia, Africa, Latin America and eastern Europe states, besides, the 6 new permanent members won’t apply veto in 15 years after SC expansion and then the veto issue will be discussed.

The supporters of “Uniting for Consensus” mainly include Italy, Canada, Pakistan, and South Korea and so on, they advocate not adding permanent members but adding roughly 10 non-permanent members, including the new category of long-term and re-elected “quasi-permanent members”. Many of the core assertions of “Uniting for Consensus” and “G4 nations” are contradictory, on one hand, it is very hard for the countries of UfC themselves to join the permanent

¹⁴The G4 nations comprising Brazil, Germany, India, and Japan are four countries which support each others bids for permanent seats on the United Nations Security Council.

members because of limited national power and financial contribution, on the other hand, they are neighborhoods and the competitors of geopolitics of “G4”. For example, Italy is afraid that once Germany’s dream comes true, which will make Italy the only European big power which is not the SC permanent members; Pakistan naturally wouldn’t support its old enemy India; Mexico and Argentina go against Brazil because Brazil is a Portuguese-speaking “exception” in Spanish-dominated South America; South Korea keeps alert of Japan because of historical problems.

Considering the design of two added SC permanent seats for African countries in “G4” plan, the African Union originally supports this plan. However, with the frequent frustration confronted by the “G4” plan and in order to highlight their interest, the Africa Union proposed own plan, to specifically emphasis the representative of African countries in SC, demanding the permanent members distributed to Africa should not be less than two seats and the non-permanent members not less than 5 seats. Besides, the “African Union” plan is very radical in veto reform, asserting to gradually abolish the veto of permanent members and during the existence of veto the new permanent members should also poss equal veto. To investigate the continuous effect of varied reform plans on SC efficiency and fairness value, set three experimental plans corresponding to three reform plans:

- Plan 1: the “Uniting for Consensus” plan, set 25 members in total (including 10 new non-permanent members), with veto right unchanged, which means only 5 original permanent members poss veto;
- Plan 2: the “G4” plan, set 25 members in total (including 6 new permanent members and 4 non-permanent members), considering the periodicity of veto right of new permanent-members (the G4 states declare not using veto in 15 years and then the veto issue will be discussed), assume in 100 ticks of every simulation the new permanent members do not enjoy veto in former 50 ticks and enjoy veto in latter 50 ticks;
- Plan 3: the “Africa Union” plan, set 25 members in total, new permanent members have veto right equally with original permanent members.

10.4.3 Simulation Analysis and Discussion

Combining the Computer Science Paradigm and the Social Science Paradigms, we discuss the simulation outcomes of interleaved effects of both CSP experiments and SSP experiments. And according to different attributes, we arrange four combinations.

Interleaved Effect 1 Simulation Outcome of Weight Allocation (between Allies and Opponents) Combined with Different Rules of Influence

For the first interleaved effect simulation, we discuss the weight allocation combined with the rules of influence, as the weight allocation is a critical factor affecting the process of influence, different weights will determine which influencing agent has the highest priority and takes effect singly, or what extent is the influence from different influencing agents according to their respective weights. The simulation outcome is illustrated in table 10.2 (control all other variables identical, simulate every plan 30 times and run each simulation 100 ticks), the “Pass” represents the accumulative passed frequency in 100 bills’ voting, which can directly measure the SC’s quantitative efficiency (improve the passing probability of bills, making SC play a more active and constructive role), the “Achieve” represents the expectation achieved average for all member states, which can comprehensively measure the SC’s qualitative efficiency (not only simply improve the bill passed frequency, but also prevent the “evil” bills from being passed to harm the members’ interest and make more “good” bills benefiting to entire interest being passed), “Stdev”

Table 10.2: Simulation Outcome of Weight Allocation between Allies and Opponents under Different Rules of Influence

Plan	1. Heavily Bias to Allies Prominent One Influence				2. Slightly Bias to Allies Prominent One Influence				3. Balanced Prominent One Influence				4. Slightly Bias to Opponents Prominent One Influence				5. Heavily Bias to Opponents Prominent One Influence			
	Efficiency		Fairness		Efficiency		Fairness		Efficiency		Fairness		Efficiency		Fairness		Efficiency		Fairness	
	Pass	Achieve	Sidev	Min	Pass	Achieve	Sidev	Min	Pass	Achieve	Sidev	Min	Pass	Achieve	Sidev	Min	Pass	Achieve	Sidev	Min
1	96	80.93	0.055	0.74	93	80.00	0.042	0.73	90	78.80	0.047	0.70	89	77.47	0.057	0.68	89	75.67	0.044	0.69
2	86	75.00	0.051	0.61	90	77.93	0.062	0.64	90	77.13	0.053	0.65	90	77.73	0.064	0.67	90	76.07	0.056	0.73
3	90	75.93	0.031	0.72	94	79.00	0.047	0.67	89	77.07	0.042	0.72	90	75.60	0.051	0.64	89	75.80	0.063	0.55
4	94	80.07	0.041	0.72	90	77.07	0.046	0.70	87	74.33	0.036	0.70	88	78.33	0.056	0.72	87	74.07	0.042	0.70
5	95	80.87	0.041	0.77	94	80.67	0.037	0.77	91	78.73	0.051	0.72	92	74.60	0.052	0.70	92	79.07	0.058	0.66
...																				
26	93	79.00	0.046	0.73	89	76.87	0.046	0.70	84	72.53	0.065	0.66	87	77.47	0.039	0.72	86	74.20	0.044	0.69
27	92	78.07	0.053	0.63	91	76.73	0.054	0.71	94	81.40	0.044	0.77	89	77.20	0.057	0.61	92	77.47	0.055	0.70
28	93	78.80	0.054	0.73	91	77.93	0.037	0.77	92	78.53	0.054	0.63	89	78.73	0.035	0.72	93	79.07	0.054	0.72
29	92	79.13	0.043	0.77	93	78.93	0.053	0.65	94	78.93	0.038	0.76	91	80.00	0.028	0.80	88	75.60	0.037	0.72
30	92	78.40	0.046	0.74	94	80.47	0.037	0.78	89	76.00	0.041	0.69	93	77.00	0.044	0.67	90	77.67	0.031	0.73
Mean	92.3	78.62	0.046	0.72	91.9	78.56	0.046	0.71	90	77.35	0.047	0.70	89.8	77.41	0.049	0.69	89.6	76.47	0.048	0.69
Plan	6. Heavily Bias to Allies Collective Influence				7. Slightly Bias to Allies Collective Influence				8. Balanced Collective Influence				9. Slightly Bias to Opponents Collective Influence				10. Heavily Bias to Opponents Collective Influence			
	Efficiency		Fairness		Efficiency		Fairness		Efficiency		Fairness		Efficiency		Fairness		Efficiency		Fairness	
	Pass	Achieve	Sidev	Min	Pass	Achieve	Sidev	Min	Pass	Achieve	Sidev	Min	Pass	Achieve	Sidev	Min	Pass	Achieve	Sidev	Min
1	76	62.00	0.068	0.40	67	57.00	0.070	0.43	61	53.47	0.085	0.36	62	56.87	0.064	0.43	56	52.07	0.106	0.11
2	61	54.80	0.067	0.45	65	57.53	0.033	0.56	61	55.73	0.067	0.43	55	52.93	0.067	0.41	62	54.73	0.110	0.21
3	76	65.33	0.056	0.59	71	61.53	0.060	0.56	66	56.80	0.069	0.37	61	54.87	0.106	0.36	47	45.07	0.072	0.36
4	74	62.87	0.083	0.50	70	59.33	0.075	0.33	69	60.93	0.079	0.51	67	61.13	0.096	0.46	62	56.47	0.088	0.31
5	72	63.33	0.061	0.62	66	58.67	0.079	0.33	58	54.20	0.074	0.43	58	52.67	0.078	0.38	62	56.07	0.072	0.50
...																				
26	66	58.00	0.094	0.35	70	62.93	0.070	0.48	52	50.27	0.071	0.33	58	54.00	0.094	0.26	46	43.33	0.096	0.28
27	72	62.53	0.073	0.50	60	54.80	0.049	0.50	65	58.67	0.077	0.47	58	54.53	0.067	0.36	57	52.40	0.052	0.49
28	66	59.47	0.046	0.55	69	60.13	0.084	0.39	60	55.40	0.068	0.43	59	55.67	0.092	0.37	54	50.53	0.087	0.26
29	68	60.47	0.086	0.41	70	61.40	0.084	0.44	64	56.47	0.101	0.39	56	52.40	0.093	0.32	58	53.27	0.064	0.43
30	66	59.40	0.064	0.47	62	54.13	0.060	0.44	65	57.20	0.101	0.38	52	49.87	0.094	0.26	60	53.80	0.067	0.37
Mean	69.7	60.82	0.070	0.48	67	58.75	0.066	0.45	62.1	55.91	0.079	0.41	58.6	54.29	0.085	0.36	56.4	51.77	0.085	0.33

represents the standard deviation among all members' achieved average, and "Min" represents the (Maximum of) Minimum of all member's achieved average, both can measure the SC's outcome fairness, as the bigger of the standard deviation, the bigger of the gap among members' interest satisfaction, and the bigger of the minimum, the better fulfilled of the justice principle (by [Rawls, 1999]).

Firstly discussed about the impact of weight allocations (bias to allies or opponents) under different rules of influence, as shown by the statistic data in table 10.3, as observed, from bias to allies to bias to opponents, on the whole, the passed frequency, achieved average and minimum have been decreasing and the standard deviation has been increasing, which means bias to allies is more favor to the efficiency and fairness than bias to opponents, as the majority of initial preference is affirmative, so the bigger of positive weight, the higher possibility to retain the affirmative preference (affirmative \rightarrow $^+$ affirmative), while the bigger of negative weight, the higher possibility to change from affirmative to negative preference (affirmative \rightarrow $^-$ negative). And we can find that the change due to biased weight is much significant on the rule of collective influence than prominent one influence, as under the rule of collective influence, the weight multiply will always reflect its effect on the influencing process, just to different extents according to the multiply and compared with other influencing ones' weights, while under the rule of prominent one influence, the weight multiply will become futile while with few influencing ones, especially with only one influencing agent, therefore the biased weighted represented by the multiply will become totally useless.

Secondly discuss about the impact of rules of influence under different biased weight allocation, as shown by the statistic data in the table 10.4, from prominent one influence to collective influence, both efficiency (measured by passed frequency and achieved average) and fairness (measured by standard deviation and maximum of minimum) have become worsen significantly (according to T test), which means the prominent one influence rule (such as one state loyally follows one other state) is much favor to the efficiency and fairness than the collective influence rule (such as one state comprehensively considering about all other states). Analyzing the reason, as most of initial preference of member states is set as Affirmative according to empirical data, for one member state i 's decision, the ratio of influencing states with affirmative, absent and negative preference is about [0.7,0.16,0.09] according to a set probabilistic distribution, if the prominent one influence works, it is highly possibly for the state i to be influenced by an state with affirmative preference, then posses a voting probability $PV_{(i)}^1 = 1, PV_{(i)}^0 = 0, PV_{(i)}^{-1} = 0$ (according to the rule), then voting affirmative for sure, while if the collective influence works, the state i will have a voting probability $PV_{(i)}^1 = 0.7, PV_{(i)}^0 = 0.16, PV_{(i)}^{-1} = 0.09$ (average speaking and assuming weights identical), though with the highest possibility to vote affirmative, but still with small chances to vote negative or absent, which means the collective influence would likely decrease the ratio of affirmative vote and make less bills got passed, and do harm to efficiency and outcome fairness, therefore, the structure of one follows one or one influences one might be more favor to stability, efficiency and fairness than the structure of multi-influence in international community or general multi-agent community.

Interleaved Effect 2 Simulation Outcome of Veto Reform Combined with Group Effect & Structure Effect

For the second interleaved effect simulation, we discuss about veto reform combined with structure & group effect, as the impact generated by constraining or abolishing veto right might be counteracted with the new discussed structure & group effect. The simulation outcome is illustrated as table 10.5 and 10.6, firstly discuss about the impact of veto reform under different combinations of group & structure effect, as shown by the statistic data of table 10.7, from reserve veto to partial veto, the Passed Frequency, Achieved Average (measuring the efficiency) and

Table 10.3: Statistics of Weight → Allocation between Allies and Opponents under Different Rules of Influence

Pass	Weight Allocation: Heavily, Slightly Bias to Allies, Balanced, Slightly, Heavily Bias to Opponents: AA→A→B→O→OO											
	Average Compare			F test	T1 test (A-A)		T2 test (A-B)		T3 test (B-O)		T4 test (O-O)	
Rule of Influence	Value	Sig.	Value	Value	Sig.	Value	Sig.	Value	Sig.	Value	Sig.	
Prominent One	92.3→91.9→90.0→89.8→89.6	2.78	0.038**	0.37	0.713	1.67	0.113	0.18	0.861	0.22	0.830	
Collective	69.7→67.0→62.1→58.6→56.4	13.61	0.00***	1.36	0.190	2.54	0.021**	1.75	0.098*	0.97	0.346	

Achieve	Weight Allocation: Heavily, Slightly Bias to Allies, Balanced, Slightly, Heavily Bias to Opponents: AA→A→B→O→OO											
	Average Compare			F test	T1 test (A-A)		T2 test (A-B)		T3 test (B-O)		T4 test (O-O)	
Rule of Influence	Value	Sig.	Value	Value	Sig.	Value	Sig.	Value	Sig.	Value	Sig.	
Prominent One	78.6→78.5→77.3→77.4→76.4	2.34	0.079*	0.01	0.995	1.32	0.205	0.01	0.990	1.25	0.228	
Collective	60.8→58.7→55.9→54.2→51.7	11.32	0.00***	1.45	0.151	2.15	0.045**	1.11	0.284	1.54	0.141	

Sidev	Weight Allocation: Heavily, Slightly Bias to Allies, Balanced, Slightly, Heavily Bias to Opponents: AA→A→B→O→OO											
	Average Compare			F test	T1 test (A-A)		T2 test (A-B)		T3 test (B-O)		T4 test (O-O)	
Rule of Influence	Value	Sig.	Value	Value	Sig.	Value	Sig.	Value	Sig.	Value	Sig.	
Prominent One	.046→.046→.047→.049→.048	0.15	0.964	—	0.625	-1.96	0.066*	-0.96	0.352	0.485	0.634	
Collective	.070→.066→.079→.085→.085	2.60	0.049**	0.50	0.625	-1.96	0.066*	-0.96	0.352	0.485	0.634	

Min	Weight Allocation: Heavily, Slightly Bias to Allies, Balanced, Slightly, Heavily Bias to Opponents: AA→A→B→O→OO											
	Average Compare			F test	T1 test (A-A)		T2 test (A-B)		T3 test (B-O)		T4 test (O-O)	
Rule of Influence	Value	Sig.	Value	Value	Sig.	Value	Sig.	Value	Sig.	Value	Sig.	
Prominent One	0.72→0.71→0.70→0.69→0.69	0.53	0.717	—	0.325	1.16	0.262	1.81	0.087*	0.654	0.521	
Collective	0.48→0.45→0.41→0.36→0.33	5.17	0.002***	1.01	0.325	1.16	0.262	1.81	0.087*	0.654	0.521	

Table 10.4: Statistics of Rules of Influence under Different Weight Allocations

Pass	Prominent One Influence	Collective Influence	T test
under Situation	Average Compare	Value	Sig.
Biased Weight Allocation			
AA	92.3→69.7	12.42	0.000****
A	91.9→67.0	19.08	0.000****
B	90.0→62.1	15.04	0.000****
O	89.8→58.6	21.94	0.000****
OO	89.6→56.4	15.57	0.000****

Achieve	Prominent One Influence	Collective Influence	T test
under Situation	Average Compare	Value	Sig.
Biased Weight Allocation			
AA	78.6→60.8	14.93	0.000****
A	78.5→58.7	19.21	0.000****
B	77.3→55.9	17.20	0.000****
O	77.4→54.2	22.98	0.000****
OO	76.4→51.7	16.45	0.000****

Stdev	Prominent One Influence	Collective Influence	T test
under Situation	Average Compare	Value	Sig.
Biased Weight Allocation			
AA	0.046→0.070	-4.56	0.000****
A	0.046→0.066	-3.47	0.003****
B	0.047→0.079	-6.56	0.000****
O	0.049→0.085	-5.81	0.000****
OO	0.048→0.085	-6.22	0.000****

Min	Prominent One Influence	Collective Influence	T test
under Situation	Average Compare	Value	Sig.
Biased Weight Allocation			
AA	0.72→0.48	7.24	0.000****
A	0.71→0.45	8.75	0.000****
B	0.70→0.41	12.97	0.000****
O	0.69→0.36	12.53	0.000****
OO	0.69→0.33	8.38	0.000****

Table 10.5: Simulation Outcome of Veto Reform under Group/Structure Effect-Part I

Plan	1.Current Reserve Veto Group Effect× Structure Effect×						2.Conservative Partial Veto Group Effect× Structure Effect×						3.Radical Abolish Veto Group Effect× Structure Effect×						
	Situation	Vote Composition	Pass	Efficiency	Achieve	Fairness	Situation	Vote Composition	Pass	Efficiency	Achieve	Fairness	Situation	Vote Composition	Pass	Efficiency	Achieve	Fairness	Min
1	[1.60 2.68 9.94]	38	36.60	0.094	0.094	0.21	[1.44 2.71 10.11]	76	58.60	0.061	0.48	[1.45 2.71 10.12]	82	61.00	0.082	0.32			
2	[1.48 2.85 9.89]	46	39.80	0.094	0.094	0.25	[1.80 2.73 9.73]	69	54.07	0.087	0.40	[1.45 2.78 10.05]	82	61.73	0.061	0.58			
3	[1.46 2.81 9.95]	48	42.40	0.070	0.31	0.31	[1.41 2.64 10.21]	75	58.73	0.034	0.55	[1.25 2.84 10.19]	84	62.00	0.069	0.59			
4	[1.57 2.91 9.74]	46	41.13	0.073	0.25	0.25	[1.50 3.04 9.72]	64	51.53	0.046	0.38	[1.48 2.82 9.98]	77	58.53	0.058	0.51			
5	[1.34 2.70 10.18]	52	45.07	0.076	0.26	0.26	[1.28 2.99 9.99]	74	56.60	0.086	0.51	[1.38 2.87 10.03]	85	60.87	0.044	0.54			
...																			
26	[1.54 2.84 9.84]	48	43.07	0.051	0.34	0.34	[1.31 2.66 10.29]	75	58.33	0.082	0.45	[1.49 2.86 9.93]	78	59.93	0.046	0.59			
27	[1.26 2.73 10.23]	55	45.67	0.108	0.29	0.29	[1.81 2.82 9.63]	72	55.47	0.062	0.34	[1.41 2.75 10.12]	84	62.40	0.095	0.46			
28	[1.56 2.67 9.99]	51	44.40	0.079	0.30	0.30	[1.42 2.69 10.15]	75	58.80	0.060	0.45	[1.35 2.95 9.98]	85	61.07	0.057	0.56			
29	[1.51 2.86 9.85]	46	41.47	0.077	0.32	0.32	[1.54 2.51 10.21]	78	60.53	0.046	0.52	[1.35 2.95 9.98]	76	58.73	0.064	0.46			
30	[1.41 2.92 9.89]	44	38.87	0.098	0.25	0.25	[1.46 2.98 9.82]	72	55.73	0.037	0.50	[1.62 2.94 9.72]	75	56.53	0.072	0.48			
Mean	[1.47 2.80 9.95]	47.4	41.85	0.082	0.28	0.28	[1.50 2.78 9.97]	73.0	56.84	0.060	0.46	[1.42 2.85 10.01]	80.8	60.28	0.065	0.51			

Plan	4.Current Reserve Veto Group Effect× Structure Effect×						5.Conservative Partial Veto Group Effect× Structure Effect×						6.Radical Abolish Veto Group Effect× Structure Effect×						
	Situation	Vote Composition	Pass	Efficiency	Achieve	Fairness	Situation	Vote Composition	Pass	Efficiency	Achieve	Fairness	Situation	Vote Composition	Pass	Efficiency	Achieve	Fairness	Min
1	[1.12 2.63 10.57]	61	50.67	0.074	0.37	0.37	[1.45 2.75 10.12]	74	57.47	0.052	0.49	[1.50 2.81 10.07]	80	60.47	0.066	0.53			
2	[1.26 2.53 10.53]	53	45.47	0.074	0.32	0.32	[1.44 2.88 10.00]	76	57.13	0.056	0.46	[1.42 2.85 10.11]	82	60.80	0.052	0.48			
3	[1.20 2.83 10.29]	64	52.13	0.053	0.40	0.40	[1.44 2.74 10.14]	76	59.47	0.068	0.44	[1.43 2.68 10.27]	86	63.53	0.052	0.57			
4	[1.29 2.72 10.31]	56	46.07	0.044	0.37	0.37	[1.40 2.71 10.21]	81	61.40	0.047	0.57	[1.42 2.80 10.16]	79	60.53	0.038	0.50			
5	[1.58 2.74 10.00]	46	40.07	0.064	0.29	0.29	[1.25 2.73 10.34]	77	59.93	0.076	0.51	[1.21 2.67 10.50]	85	64.87	0.070	0.57			
...																			
26	[1.40 2.51 10.41]	44	40.33	0.056	0.33	0.33	[1.32 2.75 10.25]	78	59.53	0.078	0.43	[1.47 2.85 10.06]	83	61.73	0.046	0.56			
27	[1.59 2.79 9.94]	53	44.93	0.092	0.34	0.34	[1.32 2.80 10.20]	76	59.00	0.051	0.54	[1.13 2.88 10.37]	85	64.27	0.076	0.38			
28	[1.35 2.85 10.12]	52	43.67	0.099	0.20	0.20	[1.32 2.88 10.12]	75	57.13	0.055	0.45	[1.54 2.77 10.07]	79	60.80	0.058	0.40			
29	[1.27 2.82 10.23]	52	44.40	0.062	0.33	0.33	[1.52 2.84 9.96]	71	55.13	0.055	0.49	[1.45 2.62 10.31]	88	64.93	0.031	0.64			
30	[1.26 3.11 9.95]	54	44.80	0.049	0.41	0.41	[1.33 2.57 10.42]	78	60.67	0.045	0.56	[1.26 2.63 10.49]	86	65.20	0.073	0.44			
Mean	[1.33 2.75 10.24]	53.5	45.25	0.067	0.34	0.34	[1.37 2.77 10.18]	76.2	58.67	0.058	0.49	[1.38 2.76 10.24]	83.3	62.71	0.056	0.51			

Standard Deviation, Maximum of Minimum (measuring the fairness) have improved significantly (according to the pairwise T1 test), as the restriction on veto can effectively suppress individual or minor permanent members' impediment on the pass of bills; while from partial veto to abolished veto, the improves on Passed Frequency keep significant still, while the improve on Achieved Average, Standard Deviation and Maximum of Minimum become small and stable, insignificant on most cases (according to T2 test), which means from weakening to abolishing of veto, the factual interest (Achieved Average) of members have not improved any more, and the fairness among members (Standard Deviation and Maximum of Minimum) have not advanced either, while the veto reform itself (especially abolishing veto right) would face huge pressure from current permanent members, US, UK, France, China and Russia, the "vested interests". Furthermore, totally abolishing the veto might make against the great powers taking more responsibility (the great powers would lose the ability to prevent "evil" bills from passed and abducted by the "majority", for example, in early period of UN, the Soviet Union was the only Socialism Country in 5 SC permanent members and belonged to the absolute minority, many bills heavily lean to capitalism camp would have passed in UN if without the veto, which would intensify the confrontation between two camps and make the world suffer to the bad consequence of the discord of great powers), therefore, pure and absolute democracy might not be the best form of international politics and could not achieve the best results. In all, the conservative plan restricting the veto achieve a good balance between efficiency and fairness, not only improve the quantitative efficiency and qualitative efficiency of SC (improve the passed frequency and achieved average), but also enhance the fairness and democracy of SC, restricting the "privilege" of permanent members to a degree. Therefore, the conservative reform, weakening veto from one negative veto to two negative combined veto might be a better choice, not only facing less pressure, and improving the efficiency and fairness value significantly.

Secondly discuss about the combinations of group & structure effect under different situations of veto reforms, as shown by the statistic data of table 10.8, overall, both the structure effect and group effect exert evident impacts (according to F test), especially on Passed Frequency, Achieved Average and Maximum of Minimum. As most of initial preference of member states is set as Affirmative according to empirical data, the ratio of affirmative, absent and negative vote is about [0.7,0.16,0.09] according to a probabilistic distribution, there after collective influence, for a state i , on most cases $PV_{(i)}^1 > PV_{(i)}^0$ and $PV_{(i)}^1 > PV_{(i)}^{-1}$, for example $PV_{(i)}^1, PV_{(i)}^0, PV_{(i)}^{-1}$ is 0.7,0.2,0.1, if without group effect, state i mostly possibly vote affirmative, while still has small chances to vote negative or absent, but if the group effect (like majority rule) functions, the state i will vote affirmative for sure, which means the group effect would increase the ratio of affirmative vote and make more bills got passed. Structure effect is also beneficial to the pass of bills, as structure effect would make states refer more to uninfluenced member states (allocating with more weights) and refer less to influenced member states (allocating with less weights), as it is easy for influenced members to be changed on preference (as the initial one is mostly affirmative, so most changes is from affirmative to negative or absent), therefore, the structure effect will make members with less possibility to be influenced by non-affirmative vote, and be favor to the quantitative efficiency. According to table 10.8, we can also find that group effect takes more significant impact than structure effect, comparing the results of T1 and T2 (the impact of structure and group effect singly), and the results of T3 and T4 (the impact of group and structure effect combined with the other).

Interleaved Effect 3 Simulation Outcome of Member Expansion Combined with Abstention & Partial Domains

For the third interleaved effect simulation, we discuss about the member expansion combined with abstentions and partial domains, as the abstention mean part of member states quit and less

Table 10.7: Statistics of Veto Reform under Different Combinations of Group & Structure Effect

Pass		Veto Reform: Current Reserve Veto→Conservative Partial Veto→Radical Abolished Veto							
under Situation		Average Compare		F test		T1 test (Reserve-Partial)		T2 test (Partial-Abolish)	
Group Effect & Structure Effect		Value	Sig.	Value	Sig.	Value	Sig.	Value	Sig.
××		47.4→73.0	0.00***	175.66	0.00***	-13.26	0.00***	-4.39	0.00***
×√		53.5→76.2	0.00***	136.23	0.00***	-10.93	0.00***	-5.40	0.00***
√×		62.1→89.9	0.00***	256.00	0.00***	-15.77	0.00***	-4.84	0.00***
√√		65.1→91.4	0.00***	321.74	0.00***	-16.34	0.00***	-7.55	0.00***
Achieve		Veto Reform: Current Reserve Veto→Conservative Partial Veto→Radical Abolished Veto							
under Situation		Average Compare		F test		T1 test (Reserve-Partial)		T2 test (Partial-Abolish)	
Group Effect & Structure Effect		Value	Sig.	Value	Sig.	Value	Sig.	Value	Sig.
××		41.9→56.8	0.00***	83.77	0.00***	-8.89	0.00***	-1.60	0.127
×√		45.3→58.7	0.00***	136.23	0.00***	-10.14	0.00***	-3.58	0.002***
√×		55.9→73.8	0.00***	245.56	0.00***	-16.94	0.00***	-1.84	0.082*
√√		57.7→76.9	0.00***	290.39	0.00***	-16.37	0.00***	-1.41	0.184
Sidev		Veto Reform: Current Reserve Veto→Conservative Partial Veto→Radical Abolished Veto							
under Situation		Average Compare		F test		T1 test (Reserve-Partial)		T2 test (Partial-Abolish)	
Group Effect & Structure Effect		Value	Sig.	Value	Sig.	Value	Sig.	Value	Sig.
××		.082→.060	0.022**	4.40	0.022**	2.69	0.015**	-0.59	0.561
×√		.067→.058	0.280	1.34	0.280	1.23	0.233	0.347	0.732
√×		.079→.054	0.001***	9.98	0.001***	4.03	0.001***	-0.102	0.920
√√		.090→.052	0.00***	25.99	0.00***	6.56	0.00***	-1.79	0.091*
Min		Veto Reform: Current Reserve Veto→Conservative Partial Veto→Radical Abolished Veto							
under Situation		Average Compare		F test		T1 test (Reserve-Partial)		T2 test (Partial-Abolish)	
Structure Effect & Group Effect		Value	Sig.	Value	Sig.	Value	Sig.	Value	Sig.
××		0.28→0.46	0.00***	33.58	0.00***	-7.26	0.00***	-1.50	0.151
×√		0.34→0.49	0.00***	20.86	0.00***	-6.36	0.00***	-0.42	0.676
√×		0.41→0.63	0.00***	44.50	0.00***	-7.63	0.00***	-1.06	0.302
√√		0.42→0.67	0.00***	63.63	0.00***	-9.10	0.00***	-1.01	0.324

Table 10.8: Statistics of Group & Structure Effect under Different Veto Reforms

Pass		Structure Effect & Group Effect: $\times \times \rightarrow \vee \times \rightarrow \times \vee \times \vee \rightarrow \vee \vee$									
Situation	Average Compare	F test	T1 test ($\times \times - \vee \times$)	T2 test ($\times \times - \times \vee$)	T3 test ($\vee \times - \vee \vee$)	T4 test ($\times \vee - \vee \vee$)	Value	Sig.			
Veto Reform		Value	Value	Value	Value	Value	Value	Value			
Reserve	47.4 \rightarrow 53.5 \rightarrow 62.1 \rightarrow 65.1	27.56	0.00***	-2.90	0.01**	-7.22	0.00***	-4.85	0.00***	-1.42	0.172
Partial	73.0 \rightarrow 76.2 \rightarrow 89.9 \rightarrow 91.4	98.05	0.00***	-2.10	0.05*	-10.89	0.00***	-14.01	0.00***	-1.34	0.198
Abolish	80.8 \rightarrow 83.3 \rightarrow 95.1 \rightarrow 97.4	89.85	0.00***	-1.57	0.135	-10.39	0.00***	-12.96	0.00***	-3.17	0.005***
Achieve		Structure Effect & Group Effect: $\times \times \rightarrow \vee \times \rightarrow \times \vee \times \vee \rightarrow \vee \vee$									
Situation	Average Compare	F test	T1 test ($\times \times - \vee \times$)	T2 test ($\times \times - \times \vee$)	T3 test ($\vee \times - \vee \vee$)	T4 test ($\times \vee - \vee \vee$)	Value	Sig.			
Veto Reform		Value	Value	Value	Value	Value	Value	Value			
Reserve	41.9 \rightarrow 45.3 \rightarrow 55.9 \rightarrow 57.7	57.60	0.00***	-2.21	0.041**	-10.82	0.00***	-7.79	0.00***	-1.27	0.220
Partial	56.8 \rightarrow 58.7 \rightarrow 73.8 \rightarrow 76.9	122.59	0.00***	-0.86	0.401	-9.99	0.00***	-21.39	0.00***	-4.54	0.00***
Abolish	60.3 \rightarrow 62.7 \rightarrow 75.2 \rightarrow 78.6	286.81	0.00***	-2.25	0.037**	-17.90	0.00***	-23.83	0.00***	-5.24	0.00***
Sidev		Structure Effect & Group Effect: $\times \times \rightarrow \vee \times \rightarrow \times \vee \times \vee \rightarrow \vee \vee$									
Situation	Average Compare	F test	T1 test ($\times \times - \vee \times$)	T2 test ($\times \times - \times \vee$)	T3 test ($\vee \times - \vee \vee$)	T4 test ($\times \vee - \vee \vee$)	Value	Sig.			
Veto Reform		Value	Value	Value	Value	Value	Value	Value			
Reserve	.082 \rightarrow .067 \rightarrow .079 \rightarrow .090	3.70	0.02**	1.97	0.064*	-0.42	0.677	-3.06	0.007***	-1.69	0.108
Partial	.060 \rightarrow .058 \rightarrow .054 \rightarrow .052	0.74	0.534	0.25	0.806	0.81	0.429	1.45	0.165	0.37	0.713
Abolish	.065 \rightarrow .056 \rightarrow .054 \rightarrow .060	1.00	0.403	1.25	0.227	1.51	0.149	-0.60	0.557	-0.893	0.384
Min		Structure Effect & Group Effect: $\times \times \rightarrow \vee \times \rightarrow \times \vee \times \vee \rightarrow \vee \vee$									
Situation	Average Compare	F test	T1 test ($\times \times - \vee \times$)	T2 test ($\times \times - \times \vee$)	T3 test ($\vee \times - \vee \vee$)	T4 test ($\times \vee - \vee \vee$)	Value	Sig.			
Veto Reform		Value	Value	Value	Value	Value	Value	Value			
Reserve	0.28 \rightarrow 0.34 \rightarrow 0.41 \rightarrow 0.42	16.81	0.00***	-2.53	0.021**	-6.17	0.00***	-3.46	0.003***	-0.55	0.591
Partial	0.46 \rightarrow 0.49 \rightarrow 0.63 \rightarrow 0.67	24.31	0.00***	-1.17	0.192	-5.46	0.00***	-6.50	0.003***	-1.13	0.273
Abolish	0.51 \rightarrow 0.51 \rightarrow 0.66 \rightarrow 0.69	18.34	0.00***	0.05	0.958	-4.62	0.00***	-5.80	0.00***	-1.16	0.260

Table 10.9: Simulation Outcome of Member Expansion under Abstentions and Partial Domains

Plan	1. Current 15 Members										2. Enlarge 15 + 5 Members										3. Enlarge 15 + 10 Members									
	Abstentions-0.00,Partial Domains-0.00					Abstention-0.00,Partial Domains-0.00					Abstention-0.00,Partial Domains-0.00					Abstention-0.00,Partial Domains-0.00					Abstention-0.00,Partial Domains-0.00									
	Situation	Vote Composition	Passed	Efficiency	Achieved	Fairness	Stdev	Min	Max	Situation	Vote Composition	Passed	Efficiency	Achieved	Fairness	Stdev	Min	Max	Situation	Vote Composition	Passed	Efficiency	Achieved	Fairness	Stdev	Min	Max			
1	[0.05 0.08 0.86]	64	62.00	0.093	0.40	0.093	0.40	0.40	[0.06 0.07 0.87]	52	50.80	0.130	0.10	0.10	0.130	0.10	0.10	[0.06 0.08 0.86]	53	52.16	0.102	0.10	0.102	0.10	0.102	0.40				
2	[0.06 0.08 0.86]	70	65.73	0.094	0.40	0.094	0.40	0.40	[0.05 0.08 0.87]	58	56.65	0.098	0.35	0.35	0.098	0.35	0.35	[0.05 0.08 0.86]	51	49.48	0.090	0.103	0.090	0.103	0.35					
3	[0.06 0.08 0.85]	66	62.73	0.081	0.53	0.081	0.53	0.53	[0.06 0.09 0.86]	61	56.60	0.104	0.40	0.40	0.104	0.40	0.40	[0.05 0.08 0.87]	47	45.40	0.103	0.23	0.103	0.23	0.23					
...																														
28	[0.08 0.08 0.84]	57	54.87	0.091	0.40	0.091	0.40	0.40	[0.06 0.09 0.85]	56	53.15	0.098	0.40	0.40	0.098	0.40	0.40	[0.05 0.09 0.87]	56	53.36	0.120	0.30	0.120	0.30	0.30					
29	[0.06 0.08 0.86]	72	67.53	0.075	0.60	0.075	0.60	0.60	[0.06 0.09 0.86]	62	57.30	0.081	0.40	0.40	0.081	0.40	0.40	[0.05 0.08 0.87]	62	58.44	0.102	0.40	0.102	0.40	0.40					
30	[0.06 0.09 0.85]	60	57.93	0.083	0.45	0.083	0.45	0.45	[0.06 0.08 0.86]	59	57.00	0.099	0.40	0.40	0.099	0.40	0.40	[0.05 0.09 0.86]	54	50.96	0.125	0.35	0.125	0.35	0.35					
Mean	[0.06 0.08 0.85]	64.8	61.8	0.086	0.46	0.086	0.46	0.46	[0.06 0.08 0.86]	58.0	55.25	0.102	0.34	0.34	0.102	0.34	0.34	[0.05 0.08 0.87]	53.8	51.63	0.107	0.34	0.107	0.34	0.34					

Plan	4. Current 15 Members										5. Enlarge 15 + 5 Members										6. Enlarge 15 + 10 Members									
	Abstentions-0.05,Partial Domains-0.00					Abstention-0.05,Partial Domains-0.00					Abstention-0.05,Partial Domains-0.00					Abstention-0.05,Partial Domains-0.00					Abstention-0.05,Partial Domains-0.00									
	Situation	Vote Composition	Passed	Efficiency	Achieved	Fairness	Stdev	Min	Max	Situation	Vote Composition	Passed	Efficiency	Achieved	Fairness	Stdev	Min	Max	Situation	Vote Composition	Passed	Efficiency	Achieved	Fairness	Stdev	Min	Max			
1	[0.06 0.08 0.82]	64	58.93	0.082	0.44	0.082	0.44	0.44	[0.06 0.07 0.82]	55	51.60	0.110	0.34	0.34	0.110	0.34	0.34	[0.05 0.08 0.82]	48	44.08	0.063	0.45	0.063	0.45	0.45					
2	[0.06 0.08 0.80]	61	55.33	0.092	0.39	0.092	0.39	0.39	[0.06 0.08 0.81]	54	49.45	0.082	0.37	0.37	0.082	0.37	0.37	[0.05 0.07 0.82]	48	45.32	0.099	0.28	0.099	0.28	0.28					
3	[0.07 0.07 0.82]	62	57.13	0.096	0.46	0.096	0.46	0.46	[0.07 0.07 0.81]	62	56.25	0.091	0.47	0.47	0.091	0.47	0.47	[0.05 0.08 0.82]	57	52.20	0.089	0.40	0.089	0.40	0.40					
...																														
28	[0.05 0.08 0.82]	76	67.40	0.079	0.50	0.079	0.50	0.50	[0.06 0.08 0.80]	60	53.05	0.099	0.32	0.32	0.099	0.32	0.32	[0.05 0.08 0.82]	53	48.56	0.103	0.35	0.103	0.35	0.35					
29	[0.05 0.08 0.82]	68	60.73	0.052	0.53	0.052	0.53	0.53	[0.06 0.09 0.80]	61	53.40	0.098	0.45	0.45	0.098	0.45	0.45	[0.05 0.07 0.82]	51	47.08	0.092	0.37	0.092	0.37	0.37					
30	[0.07 0.08 0.80]	52	49.80	0.112	0.33	0.112	0.33	0.33	[0.06 0.08 0.81]	52	48.05	0.121	0.21	0.21	0.121	0.21	0.21	[0.05 0.07 0.83]	57	51.56	0.103	0.35	0.103	0.35	0.35					
Mean	[0.06 0.08 0.81]	63.8	58.22	0.086	0.44	0.086	0.44	0.44	[0.06 0.08 0.81]	57.3	51.97	0.100	0.36	0.36	0.100	0.36	0.36	[0.05 0.08 0.82]	52.3	48.13	0.091	0.37	0.091	0.37	0.37					

Plan	7. Current 15 Members										8. Enlarge 15 + 5 Members										9. Enlarge 15 + 10 Members									
	Abstentions-0.00,Partial Domains-0.05					Abstention-0.00,Partial Domains-0.05					Abstention-0.00,Partial Domains-0.05					Abstention-0.00,Partial Domains-0.05					Abstention-0.00,Partial Domains-0.05									
	Situation	Vote Composition	Passed	Efficiency	Achieved	Fairness	Stdev	Min	Max	Situation	Vote Composition	Passed	Efficiency	Achieved	Fairness	Stdev	Min	Max	Situation	Vote Composition	Passed	Efficiency	Achieved	Fairness	Stdev	Min	Max			
1	[0.07 0.09 0.84]	59	56.40	0.092	0.43	0.092	0.43	0.43	[0.06 0.08 0.85]	50	48.75	0.094	0.40	0.40	0.094	0.40	0.40	[0.06 0.09 0.84]	52	50.48	0.095	0.30	0.095	0.30	0.30					
2	[0.07 0.10 0.83]	65	59.47	0.114	0.23	0.114	0.23	0.23	[0.07 0.09 0.84]	57	53.85	0.087	0.33=3	0.33=3	0.087	0.33=3	0.33=3	[0.07 0.09 0.84]	47	45.52	0.109	0.33	0.109	0.33	0.33					
3	[0.07 0.10 0.83]	60	55.40	0.062	0.48	0.062	0.48	0.48	[0.06 0.10 0.84]	55	53.50	0.075	0.40	0.40	0.075	0.40	0.40	[0.06 0.09 0.85]	52	49.36	0.115	0.25	0.115	0.25	0.25					
...																														
28	[0.06 0.09 0.85]	67	62.87	0.076	0.53	0.076	0.53	0.53	[0.06 0.09 0.85]	59	55.15	0.110	0.30	0.30	0.110	0.30	0.30	[0.07 0.08 0.85]	47	46.44	0.090	0.30	0.090	0.30	0.30					
29	[0.07 0.10 0.83]	55	52.53	0.082	0.45	0.082	0.45	0.45	[0.06 0.09 0.85]	67	62.15	0.096	0.37	0.37	0.096	0.37	0.37	[0.06 0.09 0.85]	58	54.24	0.099	0.40	0.099	0.40	0.40					
30	[0.07 0.10 0.83]	56	54.20	0.094	0.38	0.094	0.38	0.38	[0.07 0.10 0.82]	52	49.50	0.109	0.30	0.30	0.109	0.30	0.30	[0.06 0.10 0.84]	56	52.80	0.127	0.30	0.127	0.30	0.30					
Mean	[0.07 0.10 0.84]	60.4	56.81	0.087	0.42	0.087	0.42	0.42	[0.07 0.09 0.84]	56.7	53.82	0.095	0.36	0.36	0.095	0.36	0.36	[0.06 0.09 0.84]	52.0	49.81	0.106	0.31	0.106	0.31	0.31					

Plan	10. Current 15 Members										11. Enlarge 15 + 5 Members										12. Enlarge 15 + 10 Members									
	Abstentions-0.05,Partial Domains-0.05					Abstention-0.05,Partial Domains-0.05					Abstention-0.05,Partial Domains-0.05					Abstention-0.05,Partial Domains-0.05					Abstention-0.05,Partial Domains-0.05									
	Situation	Vote Composition	Passed	Efficiency	Achieved	Fairness	Stdev	Min	Max	Situation	Vote Composition	Passed	Efficiency	Achieved	Fairness	Stdev	Min	Max	Situation	Vote Composition	Passed	Efficiency	Achieved	Fairness	Stdev	Min	Max			
1	[0.08 0.11 0.75]	61	53.47	0.085	0.36	0.085	0.36	0.36	[0.06 0.08 0.81]	66	57.70	0.103	0.41	0.41	0.103	0.41	0.41	[0.06 0.09 0.80]	50	45.88	0.109	0.30	0.109	0.30	0.30					
2	[0.10 0.08 0.76]	61	55.73	0.067	0.43	0.067	0.43	0.43	[0.07 0.09 0.79]	55	49.85	0.103	0.33	0.33	0.103	0.33	0.33	[0.06 0.09 0.80]	50	44.88	0.099	0.40	0.099	0.40	0.40					
3	[0.09 0.09 0.76]	66	56.80	0.069	0.37	0.069	0.37	0.37	[0.08 0.09 0.78]	56	50.55	0.106	0.22	0.22	0.106	0.22	0.22	[0.06 0.08 0.80]	47	43.36	0.098	0.34	0.098	0.34	0.34					
...																														
28	[0.10 0.08 0.76]	60	55.40	0.068	0.43	0.068	0.43	0.43	[0.06 0.09 0.80]	51	46.95	0.108	0.28	0.28	0.108	0.28	0.28	[0.06 0.08 0.80]	56	50.72	0.081	0.41	0.081	0.41	0.41					
29	[0.09 0.09 0.76]	64	56.47	0.101	0.39	0.101	0.39	0.39	[0.07 0.09 0.80]	60	53.60	0.101	0.43	0.43	0.101	0.43	0.43	[0.06 0.09 0.80]	57	50.36	0.110	0.25	0.110	0.25	0.25					
30	[0.09 0.09 0.78]	65	57.20	0.101	0.38	0.101	0.38	0.38	[0.07 0.08 0.80]	51	47.95	0.110	0.22	0.22	0.110	0.22	0.22	[0.06 0.09 0.80]	53	47.12	0.089	0.39	0.089	0.39	0.39					
Mean	[0.10 0.08 0.76]	62.1	55.91	0.079	0.41	0.079	0.41	0.41	[0.07 0.09 0.80]	56.5	51.10	0.105	0.32	0.32	0.105	0.32	0.32	[0.06 0.09 0.80]	52.2	47.05	0.098	0.35	0.098	0.35	0.35					

member states participate in the decision-making process, therefore, the effect generated by abstention would possibly be interacted with the effect generated by member expansion—new part of members added into the decision-making process. The simulation outcome is illustrate in table 10.9, firstly discuss about the impact of member expansion under different combinations of abstention & partial domains, as shown in the statistic data of table 10.10, from 15 to 15 + 5 and to 15 + 10, with the expansion of SC members, on the whole, the Passed Frequency, Achieved Average and Maximum of Minimum decrease, and the Standard Deviation increase, which means the member expansion is unfavor to the efficiency and fairness (mainly the outcome fairness), analyzing the reason, as with the SC expansion, the interest of member states will become more diversified, making the consensus harder to reach, and the game dominating power of permanent 5 be weaken, making the game situation of SC transferring from big power dominated to “tangled warfare”. For example, when the permanent 5 reach consensus, it only needs 4 of 10 (40%) affirmative vote from non-permanent members to pass under the current plan, needs 7 of 15 (46.67%) affirmative vote from non-permanent members to pass under the 15+5 plan, and needs 10 of 20 (50%) affirmative vote from other members to pass under 15+10 plan. Further observed, the reduction on efficiency and fairness is more significant on the expansion from 15 to 15 + 5 than from 15 + 10 (comparing the pairwise T1 test and pairwise T2 test), which means the worsening of member expansion on efficiency and outcome fairness is degressive, reaching stable. Furthermore, it should be noticed that the member expansion would directly increase the opportunity fairness of UN SC, making more UN members participate the Security Council, which will make Security Council more representative, and make more small and middle countries’ voices to be heard.

Secondly discuss about the impact of Abstention and partial domains under different scales of member expansions, as shown by the statistic data of table 10.11, we can observe that the passed frequency and achieved average decrease with the new cases of abstention and partial domains on the whole, as the abstention in UN SC represent part of members not participate the vote on part of bills and naturally decrease the ratio of affirmative and increase the hardness to get passed, and partial domains also generate the possibilities of members’ voting domains constraining to {absent, negative}, {absent}, {negative}, without affirmative include. However, it is also obvious that the change on quantitative efficiency (measured by passed frequency) and qualitative efficiency (measured by achieved average) by different combinations of abstention and partial domains is quite insignificant, which means the abstention and domains can not produce an evident impact on the efficiency of UN SC.¹⁵ However, it might due to the small ratio of abstention and domains (both set to 0.05 as top), however, in real-world situation, the non-participation of members and the non-full of voting domain is quite rare (as member states will actively participate the meet to strive for state interest, obey the organization rule and avoid the criticism, and as all members have the right to choose any affirmative, absent or negative vote according to their willingness, at least theoretically, totally eliminating part of choices from the voting domain is not common), therefore, factually it is unreasonable to set a big ratio for those two partialities, then it is not easy for the two new cases produce a significant impact.

Table 10.10: Statistics of Member Expansion under Different Combinations of abstention & Partial Domains

Pass		Member Expansion: 15→15+5→15+10					
under Situation of Abstention & Partial Domains	Average Compare	F test		T1 test (15 and 15+5)		T2 test (15+5 and 15+10)	
		Value	Sig.	Value	Sig.	Value	Sig.
0.00,0.00	64.8→58.0→53.8	7.76	0.005***	2.46	0.033**	1.64	0.131
0.05,0.00	63.8→57.3→52.3	6.14	0.011**	1.77	0.107	2.10	0.062*
0.00,0.05	60.4→56.7→52.0	3.94	0.042**	1.17	0.271	1.52	0.160
0.05,0.05	62.1→56.5→52.2	9.55	0.002***	2.48	0.033**	1.53	0.157

Achieve		Member Expansion: 15→15+5→15+10					
under Situation of Abstention & Partial Domains	Average Compare	F test		T1 test (15 and 15+5)		T2 test (15+5 and 15+10)	
		Value	Sig.	Value	Sig.	Value	Sig.
0.00,0.00	61.8→55.3→51.6	9.12	0.003***	2.76	0.02**	1.74	0.113
0.05,0.00	58.2→52.0→48.1	8.20	0.004***	2.22	0.049**	2.16	0.056*
0.00,0.05	56.8→53.8→49.8	4.44	0.031**	1.19	0.262	1.61	0.138
0.05,0.05	55.9→51.1→47.1	14.78	0.00***	3.05	0.012**	2.02	0.071*

Sidev		Member Expansion: 15→15+5→15+10					
under Situation of Abstention & Partial Domains	Average Compare	F test		T1 test (15 and 15+5)		T2 test (15+5 and 15+10)	
		Value	Sig.	Value	Sig.	Value	Sig.
0.00,0.00	.086→.102→.107	4.37	0.032***	-2.15	0.057*	-0.64	0.540
0.05,0.00	.086→.100→.091	1.19	0.332	-1.47	0.172	1.04	0.324
0.00,0.05	.087→.095→.106	2.45	0.117	-0.94	0.370	-1.39	0.195
0.05,0.05	.079→.105→.098	6.34	0.010**	-3.44	0.016**	1.56	0.150

Min		Member Expansion: 15→15+5→15+10					
under Situation of Abstention & Partial Domains	Average Compare	F test		T1 test (15 and 15+5)		T2 test (15+5 and 15+10)	
		Value	Sig.	Value	Sig.	Value	Sig.
0.00,0.00	0.46→0.34→0.34	3.55	0.055*	2.03	0.069*	0.06	0.953
0.05,0.00	0.44→0.36→0.37	2.11	0.155	1.67	0.125	-0.15	0.885
0.00,0.05	0.42→0.36→0.31	3.19	0.070*	1.43	0.183	1.32	0.216
0.05,0.05	0.41→0.32→0.35	2.01	0.158	1.99	0.074*	-0.73	0.480

Table 10.1: Statistics of abstention and Domains under Different Member Expansions

Pass under Situation Member Expansion	Average Compare	Abstention & Partial Domain: 0.00,0.00→+0.05,0.00→+0.00,0.05→+0.05,0.05									
		F test Value	Sig.	T1 test Value	Sig.	T2 test Value	Sig.	T3 test Value	Sig.	T4 test Value	Sig.
15	64.8→63.8→60.4→62.1	0.71	0.556	-0.25	0.808	1.47	0.172	0.29	0.775	-1.13	0.284
15+5	58.0→57.3→56.7→56.5	0.11	0.952	0.30	0.774	0.46	0.652	0.29	0.780	0.05	0.962
15+10	53.8→52.3→52.0→52.2	0.22	0.881	0.57	0.583	0.66	0.522	0.07	0.944	-0.07	0.947
Achieve under Situation Member Expansion	Average Compare	Abstention & Partial Domain: 0.00,0.00→+0.05,0.00→+0.00,0.05→+0.05,0.05									
		F test Value	Sig.	T1 test Value	Sig.	T2 test Value	Sig.	T3 test Value	Sig.	T4 test Value	Sig.
15	61.8→58.2→56.8→55.9	1.94	0.155	1.12	0.290	1.95	0.08*	0.80	0.441	0.42	0.690
15+5	55.3→52.0→53.8→51.1	1.46	0.254	1.91	0.085*	0.64	0.536	0.52	0.617	1.05	0.320
15+10	51.6→48.1→49.8→47.1	1.75	0.188	1.51	0.161	0.81	0.437	0.58	0.576	1.38	0.197

Table 10.12: Simulation Outcomes of Member Expansion Involving Veto Reform under Combinations of Influence Patterns

Plan	1. Uniting for Consensus Current Influences						2. G4 Nations Current Influences						3. African Union Current Influences						
	Situation		Efficiency		Fairness		Situation		Efficiency		Fairness		Situation		Efficiency		Fairness		
	Vote Composition	Passed	Achieved	Stdev	Min	Max	Vote Composition	Passed	Achieved	Stdev	Min	Max	Vote Composition	Passed	Achieved	Stdev	Min	Max	
1	[0.07 0.09 0.79]	63	54.2	0.038	0.55	0.99	[0.07 0.09 0.79]	47	43.76	0.06	0.33	0.78	[0.08 0.09 0.78]	40	39.4	0.057	0.24	0.78	
2	[0.07 0.09 0.79]	61	54.28	0.058	0.5	0.99	[0.07 0.08 0.79]	46	43.28	0.046	0.41	0.78	[0.08 0.09 0.78]	32	32.84	0.041	0.26	0.78	
3	[0.08 0.09 0.77]	53	47.24	0.105	0.34	0.79	[0.08 0.08 0.79]	47	43.52	0.063	0.36	0.78	[0.09 0.08 0.78]	40	39.8	0.058	0.33	0.78	
...								...											
28	[0.08 0.09 0.77]	58	50.56	0.092	0.39	0.78	[0.08 0.09 0.78]	50	45.92	0.047	0.41	0.79	[0.08 0.08 0.79]	36	35.08	0.056	0.22	0.79	
29	[0.07 0.08 0.79]	61	52.48	0.068	0.44	0.81	[0.07 0.08 0.81]	41	39.04	0.057	0.36	0.78	[0.08 0.09 0.78]	32	32.96	0.04	0.3	0.78	
30	[0.08 0.08 0.79]	62	53.8	0.088	0.41	0.78	[0.08 0.1 0.78]	52	45.12	0.073	0.23	0.79	[0.08 0.09 0.79]	38	37.32	0.03	0.33	0.79	
Mean	[0.08 0.09 0.78]	59.7	52.09	0.075	0.44	0.79	[0.08 0.09 0.79]	47.2	43.44	0.058	0.35	0.78	[0.08 0.09 0.78]	36.3	36.23	0.048	0.28	0.78	

Plan	4. Uniting for Consensus Added Crossing Influences						5. G4 Nations Added Crossing Influences						6. African Union Added Crossing Influences						
	Situation		Efficiency		Fairness		Situation		Efficiency		Fairness		Situation		Efficiency		Fairness		
	Vote Composition	Passed	Achieved	Stdev	Min	Max	Vote Composition	Passed	Achieved	Stdev	Min	Max	Vote Composition	Passed	Achieved	Stdev	Min	Max	
1	[0.09 0.08 0.78]	49	44.88	0.11	0.37	0.81	[0.08 0.08 0.81]	46	42.76	0.065	0.28	0.79	[0.09 0.08 0.79]	34	34.68	0.056	0.24	0.79	
2	[0.08 0.1 0.77]	51	46.04	0.067	0.42	0.79	[0.08 0.08 0.79]	43	40.64	0.04	0.32	0.77	[0.09 0.1 0.77]	34	34.68	0.055	0.26	0.77	
3	[0.09 0.09 0.78]	54	48.2	0.067	0.44	0.78	[0.09 0.09 0.78]	39	37.68	0.074	0.29	0.77	[0.1 0.09 0.77]	35	37.04	0.052	0.28	0.77	
...								...											
28	[0.08 0.08 0.79]	62	54.2	0.074	0.47	0.81	[0.08 0.08 0.81]	48	45.12	0.061	0.34	0.77	[0.09 0.09 0.77]	38	38.16	0.043	0.28	0.77	
29	[0.09 0.09 0.77]	51	44.56	0.054	0.43	0.79	[0.08 0.09 0.79]	42	39.56	0.045	0.39	0.79	[0.08 0.08 0.79]	32	33.12	0.044	0.29	0.79	
30	[0.09 0.09 0.78]	51	45.96	0.053	0.42	0.79	[0.08 0.08 0.79]	40	39.12	0.055	0.36	0.77	[0.1 0.09 0.77]	33	34.2	0.069	0.15	0.77	
Mean	[0.09 0.09 0.78]	53	47.31	0.071	0.41	0.79	[0.08 0.08 0.79]	43	40.81	0.057	0.33	0.78	[0.09 0.09 0.78]	34.3	35.31	0.053	0.25	0.78	

Plan	7. Uniting for Consensus Added Influences Involved Structure						8. G4 Nations Added Influences Involved Structure						9. African Union Added Influences Involved Structure						
	Situation		Efficiency		Fairness		Situation		Efficiency		Fairness		Situation		Efficiency		Fairness		
	Vote Composition	Passed	Achieved	Stdev	Min	Max	Vote Composition	Passed	Achieved	Stdev	Min	Max	Vote Composition	Passed	Achieved	Stdev	Min	Max	
1	[0.07 0.09 0.81]	60	52.64	0.065	0.43	0.79	[0.07 0.09 0.79]	52	47.52	0.048	0.42	0.76	[0.09 0.1 0.76]	33	32.84	0.055	0.27	0.76	
2	[0.07 0.08 0.81]	63	56.04	0.092	0.48	0.81	[0.06 0.09 0.81]	56	49.76	0.072	0.45	0.79	[0.08 0.09 0.79]	40	38.88	0.038	0.35	0.79	
3	[0.08 0.09 0.79]	62	47	0.082	0.46	0.78	[0.07 0.09 0.78]	49	44.56	0.073	0.38	0.76	[0.08 0.1 0.76]	36	35.12	0.06	0.19	0.76	
...								...											
28	[0.07 0.1 0.79]	59	51.44	0.045	0.44	0.79	[0.08 0.08 0.79]	43	41.32	0.061	0.38	0.81	[0.07 0.09 0.81]	38	36.52	0.054	0.33	0.81	
29	[0.07 0.09 0.81]	60	45.88	0.078	0.42	0.79	[0.07 0.09 0.79]	52	47.68	0.055	0.39	0.79	[0.07 0.08 0.79]	35	34.76	0.053	0.26	0.79	
30	[0.08 0.09 0.81]	65	49.64	0.097	0.49	0.79	[0.07 0.09 0.79]	53	47.2	0.057	0.31	0.79	[0.08 0.08 0.79]	39	39.24	0.054	0.2	0.79	
Mean	[0.07 0.09 0.81]	61.5	53.2	0.077	0.44	0.79	[0.07 0.09 0.79]	50.8	46.34	0.061	0.39	0.78	[0.08 0.09 0.78]	36.8	36.23	0.052	0.28	0.78	

Plan	10. Uniting for Consensus All Influences						11. G4 All Influences						12. African Union All Influences						
	Situation		Efficiency		Fairness		Situation		Efficiency		Fairness		Situation		Efficiency		Fairness		
	Vote Composition	Passed	Achieved	Stdev	Min	Max	Vote Composition	Passed	Achieved	Stdev	Min	Max	Vote Composition	Passed	Achieved	Stdev	Min	Max	
1	[0.08 0.08 0.79]	65	55.84	0.086	0.37	0.79	[0.07 0.09 0.79]	46	43.76	0.054	0.32	0.77	[0.1 0.09 0.77]	25	29.4	0.052	0.2	0.77	
2	[0.08 0.08 0.79]	60	52.68	0.07	0.43	0.81	[0.07 0.08 0.81]	51	46.88	0.047	0.43	0.78	[0.09 0.08 0.78]	32	34.28	0.054	0.2	0.78	
3	[0.08 0.09 0.79]	58	50.2	0.053	0.42	0.79	[0.07 0.09 0.79]	48	44	0.056	0.31	0.78	[0.09 0.09 0.78]	38	37.8	0.037	0.34	0.78	
...								...											
28	[0.08 0.08 0.79]	63	54.68	0.084	0.4	0.79	[0.08 0.09 0.79]	49	44.72	0.057	0.39	0.79	[0.08 0.09 0.79]	46	43.76	0.074	0.28	0.79	
29	[0.09 0.09 0.78]	57	50.12	0.075	0.42	0.79	[0.08 0.08 0.79]	49	44.92	0.045	0.35	0.78	[0.08 0.09 0.78]	35	35.12	0.054	0.2	0.78	
30	[0.07 0.09 0.81]	65	56.08	0.065	0.46	0.79	[0.08 0.09 0.79]	46	42.8	0.062	0.38	0.78	[0.09 0.08 0.78]	37	37.16	0.058	0.25	0.78	
Mean	[0.08 0.09 0.79]	61.3	53.27	0.072	0.42	0.79	[0.08 0.09 0.79]	48.2	44.51	0.054	0.35	0.78	[0.09 0.09 0.78]	35.5	36.25	0.055	0.25	0.78	

Interleaved Effect 4 Simulation Outcome of Popular Reform Scheme Combined with Different Combinations of Influence Patterns

For the fourth interleaved effect simulation, we discuss about popular reform schemes (which simultaneously involve veto and expansion issues) combined with different influence patterns, as the system of new influence patterns is the main innovation in the work, then used to explain the complex and mixed reform issues, and the different combinations of influence patterns (existed, crossing and structured influences) are favor for us to detailed discuss the interacting process and gaming outcome based on different popular reform schemes under varied and complex influencing environment. The simulation outcomes is illustrated in table 10.12, firstly discuss about the impact of popular reform schemes under the situation of different influence patterns, as shown by the statistic data of table 10.13, the quantitative efficiency (passed frequency), qualitative efficiency (achieved average) and outcome fairness (maximum of minimum) of Uniting for Consensus plan are much better significantly compared with both two other plans of G4 nations and African Union (according to two pairwise T1 and T2 test). For the UfC plan, the numbers of permanent members is still 5 and the their proportion in SC has declined substantially compared to the current plan, from 33.3% (5/15) to 20% (5/25), make the “privileged group” more minority, besides, when the permanent 5 reach consensus they need higher proportion of non-permanent members’ support to make the bill get passed, from 40% (4/10) to 50% (10/20), comparatively need more the support from small and medium countries and to reflect their interests, although the “privilege” of permanent 5 haven’t been changed nominally, but have been discounted substantially, which improves the democracy (right fairness) of SC; Furthermore, the planned 10 new members are all non-permanent members, which make many small and medium countries also have opportunities to join SC and is conduct to the advancement of representativeness (opportunity fairness) of SC. However, in other two plans, about half of 10 new seats are distributed to permanent members, which are the small and medium countries occupying the majority of the world can not desire, and their added opportunities are just 5 seats for non-permanent members. Therefore, the “Uniting for Consensus” plan advance the efficiency and fairness value in many senses. For the “Africa Union” plan, all new permanent members enjoy veto and make the proportion of permanent members with veto (the “privileged group”) rise enormously, from 33.3 (5/15) before the reform to 40% (10/25) after the reform, which inevitably make the probability of bill passed declines, not only harming the efficiency, but also violating the fairness (not constrain the veto but intensify it). Comparatively, the G4 nations is comprised on the veto issue and promise they won’t use veto in 15 year, however, after the “promise” duration, there will be 11 permanent members enjoying the veto, which will further blemish the efficiency and fairness value.

Further, both “G4 nations” and “Africa Group” plans will highly possibly intensify the inner confrontation of UN SC, because the new added permanent members usually have clear alliance belongingness, for “G4” plans, new permanent members Germany and Japan are undoubtedly the allies of US in SC, Germany is the member of NATO and Japan, since the World War II, is still secured and influenced by US; although the standpoint of India and Brazil is relatively neutral, but India has increasingly become an important chess piece of US in Asia-Pacific strategy to “re-balance China”, and also have a good relationship with Russia, especially the “honey moon” tradition in the era of former Soviet Union, Brazil as US’s backyard country in traditional sense has been coming close with the rising China (such as the BRICS group), will possibly be competitively strived for by both “alliances”; besides, added two Africa permanent members will highly probably

¹⁵Even for the standard deviation and maximum of minimum, the changes is also quite insignificant in statistics, for the standard deviation, from member expansion 15 to 15 + 5 to 15 + 10, the F-test are respectively 0.11, 0.65 and 1.82, with p-value respectively as 0.953 > 0.1, 0.590 > 0.1, 0.177 > 0.1; and for the maximum of minimum, from member expansion 15 to 15 + 5 to 15 + 10, the F-test are respectively 0.91, 0.26 and 0.85, with p-value respectively as 0.452 > 0.1, 0.851 > 0.1, 0.483 > 0.1.

become the allies of “Sino-Russia” alliance, confronting the alliance led by US, thus, the “G4” plan will undoubtedly intensify the confrontation of SC inner alliances, especially the SC core circle (permanent members), which make SC harder to make consensus and pass constructive bills. For “African Union” plan, although mainly emphasizing the representativeness of African countries, it is nearly impossible for Germany, Japan and other “should have been permanent members” countries to be voted out while expanding the scale of permanent members, the eventual SC member composition won’t be much different from “G4” plan. But for UfC plan, although the belongingness of new non-permanent members are also unavoidable, after all the non-permanent members are mobile and without veto right, which won’t prominently intensify the confrontation of SC members, especially the core circle of permanent members.

Secondly discuss about the impact of combinations of influence patterns under different reform schemes, corresponding to our innovations, we set four combinations—current influences, added with crossing influences, added with influences involved with structure, and all influence patterns, as shown by the statistic data in table 10.14, the change on passed frequency and achieved average due to influence patterns is significantly (according to F test), we can observe that the added structured influence would increase the efficiency, as we discussed before, the structure effect would make members less possibly be influenced by influenced ones with non-affirmative preference, and we can also observe that the added crossing influence would decrease the efficiency, which might be due to that the crossing environment will make member be more vulnerable to non-affirmative votes. Furthermore, we can find that the impact produced by the crossing influence is more significant while singly working than combined with the structured influence (comparing pairwise T1 and T4 test), but the impact produced by the structured influence is more significant while combined with the crossing influence than singly working (comparing pairwise T3 and T2 test), as the unfavor effect of crossing influence will be counteracted to some extent by the favor effect of structured influence, but the structure effect accompanied with the structured influence would play a more evident advancing impact while under more varied influencing relations (non-crossing and crossing ones). Besides, it should be noted that the change on efficiency due to new influence patterns is significant on situation of UfC plans and G4 plan, but not on African Union plan, as in which more than 10 members possess the veto right and the bills are highly vulnerable to be vetoed, therefore the improvement produced by the new influences are quite easily to be futile.

Table 10.13: Statistics of SC Reform Schemes under Different Influence Patterns

Pass		Reform Schemes: Uniting for Consensus→G4 Nations→African Union					
under Situation of Influence Patterns	Average Compare	F test		T1 test (UfC-G4)		T2 test (UfC-AU)	
		Value	Sig.	Value	Sig.	Value	Sig.
Current Influences	59.7→47.2→36.3	59.71	0.00***	5.83	0.00***	11.01	0.00***
Added Crossing Influences	53.0→43.0→34.3	41.05	0.00***	4.20	0.002***	8.92	0.00***
Added Structured Influences	61.5→50.8→36.8	31.71	0.00***	2.07	0.065*	8.51	0.00***
All Influences	61.3→48.2→35.5	46.69	0.00***	8.06	0.00***	8.13	0.00***
Achieve		Reform Schemes: Uniting for Consensus→G4 Nations→African Union					
under Situation of Influence Patterns	Average Compare	F test		T1 test (UfC-G4)		T2 test (UfC-AU)	
		Value	Sig.	Value	Sig.	Value	Sig.
Current Influences	52.1→43.4→36.2	51.24	0.00***	6.06	0.00***	9.31	0.00***
Added Crossing Influences	47.3→40.8→35.3	29.02	0.00***	3.86	0.003***	7.58	0.00***
Added Structured Influences	53.2→46.3→36.2	42.24	0.00***	3.55	0.005***	8.60	0.00***
All Influences	53.3→44.5→36.3	49.99	0.00***	8.78	0.00***	8.41	0.00***
Min		Reform Schemes: Uniting for Consensus→G4 Nations→African Union					
under Situation of Influence Patterns	Average Compare	F test		T1 test (UfC-G4)		T2 test (UfC-AU)	
		Value	Sig.	Value	Sig.	Value	Sig.
Current Influences	0.44→0.35→0.28	9.10	0.003***	2.14	0.058*	4.33	0.001***
Added Crossing Influences	0.41→0.33→0.25	25.40	0.00***	4.45	0.001***	7.03	0.00***
Added Structured Influences	0.45→0.39→0.28	22.95	0.00***	3.04	0.012**	6.57	0.00***
All Influences	0.42→0.35→0.25	18.52	0.00***	2.72	0.022**	6.51	0.00***

Table 10.14: Statistics of Influence Patterns under Different SC Reform Schemes

Pass under Situation of Reform Schemes	Influence Patterns (wether with Crossing and Structured influence): $Non \rightarrow C \rightarrow S \rightarrow CS$										
	Average Compare	F test Value	Sig.	T1 test (Non-C) Value	Sig.	T2 test (Non-S) Value	Sig.	T3 test (C-CS) Value	Sig.	T4 test (S-CS) Value	Sig.
Uniting for Consensus	59.7→53.0→61.5→61.3	7.28	0.002***	2.74	0.021**	-1.04	0.322	-3.49	0.006***	0.10	0.924
G4 Nations	47.2→43.0→50.8→48.2	5.11	0.009***	2.07	0.065*	-1.47	0.172	-3.19	0.010**	1.35	0.208
African Union	36.3→34.3→36.8→35.5	0.39	0.760	1.16	0.272	-0.27	0.792	-0.39	0.702	0.44	0.670

Achieve under Situation of Reform Schemes	Influence Patterns (wether with Crossing and Structured influence): $Non \rightarrow C \rightarrow S \rightarrow CS$										
	Average Compare	F test Value	Sig.	T1 test (Non-C) Value	Sig.	T2 test (Non-S) Value	Sig.	T3 test (C-CS) Value	Sig.	T4 test (S-CS) Value	Sig.
Uniting for Consensus	52.1→47.3→53.2→53.3	4.55	0.014**	2.41	0.036**	-0.80	0.443	-3.53	0.005***	-0.08	0.935
G4 Nations	43.4→40.8→46.3→44.5	5.52	0.006***	1.84	0.096*	-1.93	0.083	-2.76	0.02**	1.55	0.151
African Union	36.2→35.3→36.2→36.3	0.08	0.971	0.43	0.675	0.00	0.990	-0.41	0.692	-0.08	0.940

Min under Situation of Reform Schemes	Influence Patterns (wether with Crossing and Structured influence): $Non \rightarrow C \rightarrow S \rightarrow CS$										
	Average Compare	F test Value	Sig.	T1 test (Non-C) Value	Sig.	T2 test (Non-S) Value	Sig.	T3 test (C-CS) Value	Sig.	T4 test (S-CS) Value	Sig.
Uniting for Consensus	0.44→0.41→0.45→0.42	0.72	0.552	0.39	0.702	-0.45	0.661	-0.46	0.656	2.18	0.054*
G4 Nations	0.35→0.33→0.39→0.35	1.27	0.311	0.62	0.548	-1.15	0.277	-0.59	0.570	1.40	0.191
African Union	0.28→0.25→0.28→0.25	0.49	0.691	1.05	0.319	0.41	0.693	0.16	0.877	0.61	0.555

Conclusion

In the thesis, we mainly perform three aspects of work, firstly, built the complex models of influence based on an ordinal preference representation language and social influence models, proposed a series of new patterns of influence to better describe the complex influences in real-world situation, under the new context of influencing and influenced structure, influence from more than one origins, and influence under context of abstention and constraint (which are common in real-world situation while ignored in former works). Then we test the models of influence from an exemplary perspective for interdisciplinary study, from both social science and computer science paradigms, by both qualitative case studies and quantitative algorithms design, to provide evidence for the models of influence. Finally, we used the models of influence to do simulation experiments, through a systematic modeling procedure of concept model, mathematic model and computer model, to simulate the UN Security Council voting and influencing process, and compare the effectiveness of different SC reform schemes. This is an original application of artificial intelligence in social science (especially international politics), endowing with new research perspective and methodological system.

Influence Modeling

Main Contributions

In the thesis, there are mainly three contributions for the influence study, to further advance the explanatory power of models of influence in the real-world decision-making system.

- Previous works ignore the influence from structure (more specifically the influencing relations among agents or dependent relations among issues), only discuss the influence from (individual) decision to (individual) decision, perceive the structure just as the environment, channel or path of influence, but ignore the structure itself could also exert an influential effect, and assume the structures as fixed but not dynamic, which can affect the decision as the environment but can not be influenced and changed in return during the process.

In the thesis, we build the models of influence based on CP-nets (representing the dependencies among issues) and borrow the ideas from social influence (capturing the influences among agents).

- Previous works mainly discuss the influence from one single agent to one another single agent, but in general do not deal with the influence from more than one agents. In practice, the influenced agent would usually simultaneously face more than one influencing agents, especially with contradictive influencing directions (such as positive influences from friends and negative influences from enemies), and varied influencing strengths (such as strong influences from leaders and weak influences from “nobodies”), under such a complex collective influence context, how to produce the influenced result would be very meaningful to discuss. Besides, although some works discuss the influence from more than one agents,

while mostly over one single issue or independent multi-issues, but not yet fully discuss the influence simultaneously from different agents and different issues.

To deal with the influence from more than one origins (including both agents and issues), we proposed two different approaches, one is the prominent one influence, in which the priority of influence for different influencing decisions are set and the decision with the highest priority of influence dominates any other influencing decisions. The other is the collective influence, in which all the influencing decisions would participate in the influence just to different extents or directions, according to their weights, the matrix of weight of influence among different decisions are set, to synthesize the collective influence from more than one origins.

Furthermore important, once we introduce the influencing and influenced role of structure, than the origins of influence would include the structure itself among agents/issues, to deal with the influential effect from structure to decision (as which are two disparate things, one is the relations among multi decisions, and the other is one individual decision), the gap between them need a bridge to make up, a main contribution in the thesis is to achieve the transformation of influence from structure to decision in mathematics, considering varied combinations of bilateral influencing relations.

- Previous works discuss the influence in a complete context, without abstention and without constraint (partial domains), which means all agents participate all issues' decisions, and the domains of one issues for all agents are complete and uniform.

In the thesis, we discussed different reasons for abstentions (such as different knowledge distributions and different interest distributions) and constraints/partial domains (such as nonexistence and nonacceptance). And we discuss the issues of aggregation under the cases of abstention and constraints, and discuss how the influence with abstention and constraints could not only lead to the preference ordering, but also could lead to the variable sets and value set, which provides a new form of influence. The dynamic and evolving variable sets and value sets under influence are firstly discussed in the thesis, expanding the field of influence study.

Future Work

- In the thesis, the models of influence is mainly based on the CP-nets, which is an ordinal approach, and is a typical approach used in computational social choice to describe the preference representation and voting rule, as the mathematics in theoretical computer science mainly fall under the umbrella of discrete mathematics. However, the Bayesian Networks, which is a cardinal approach but not an ordinal approach, has received attentions in studying the multi-agent interaction and influence, compared with the ordinal approach, one advantage of the ordinal approach is the sophisticated representing ability. Therefore, in the future work, the cardinal approach (typical like Bayesian Networks) could be discussed, to provide another perspective to model the influence in combinatorial and collective decision-making, especially with networks structure.
- For the influence in the multi-agents and multi-issues system, a crucial problem is the cycle of influence, once the influence become cyclic, the computation for the decision-making and influenced outcome would be subject to hard problems. In the thesis, we make an easy and straight assumption that, only the former issue can influence the latter issue, but not reversely back, to make the influence linear and avoid the cycle. This restriction is very effective, but would loss the flexibility of decision-making sequence and the influencing process, in future work, more delicate setup to defend the cyclic could be discussed.

Influence Testing

Having set up this model of influence, we test the models of influence and their relevance in social sciences and social questions. We use the example of UN security council voting to test the models of influence, and proposed an exemplary double paradigms for the interdisciplinary study. One is from the paradigm of social sciences, we test the models of influence with multinational gaming examples in the real-world international affairs, to perform a series of qualitative case studies to discuss the typical patterns of influence; the other is from the paradigm of computer science, we test the models of influence with UN SC official voting data by the quantitative algorithms, organize the test sample by different subjects, designs a series of algorithms for corresponding influence patterns, and then analyze the testing outcome combining with international political theories.

Main Findings

A series of specific examples in real-world international affairs are discussed to prove the proposed patterns of influence whether existed and whether could be applied to explain the real-world situation, we find that the typical patterns of influence could all easily find a corresponding example, and by doing the case study for each pattern of influence, a lot of rules or laws could be found; on the other hand, a series of matching algorithms are designed to give evidence of existence and significance of corresponding patterns of influence. Under our optimistic assumptions, we find that the overall testing outcomes are very good, there are a series of general influence pattern or specific influence relations highly matched by test.

- For influence pattern 1 Intra-influence from Decision to Decision (Compos-*IPMR*: 0.809, Median-*IPMR*: 0.860), “Great Powers Worship the Reputation”, usually the super powers and great powers are more inclined to be influenced by own former preferences and decisions, be persistent on own claims and attitudes, and not easily influenced and manipulated by other states. For example, US (*IPMR*: 0.795, Rank: 8), former Soviet Union/Russia (*IPMR*: 0.796, Rank: 7), UK (*IPMR*: 0.823, Rank: 3), France (*IPMR*: 0.781, Rank: 10) and China (*IPMR*: 0.849, Rank: 2) are more possible to be influenced by own former decisions, as the 5 UN SC permanent members and super powers/great powers, these states worship their reputations in the international societies, would not shortly and easily change their claims and attitudes, but would be persistent and constant, to provide a stable expectation no matter for allies or for enemies.
- For influence pattern 7 Inter-influence from Decision to Decision (Compos-*IPMR*: 0.647, Median-*IPMR*: 0.725), “Side with Allies and against Enemies”, in the international relations, due to the common or conflicting interests, some states would form friendships or even alliance, to united forces and oppose together to the enemies and other alliances formed by enemies. Among the allies, there will be positive influences, and among the enemies (or among confronting alliances), there will be naturally negative influences. Some prominent positive influence among friends or allies are UK influenced by US (*IPMR*: 0.781) and Russia influenced by China (*IPMR*: 0.720).
- For influence pattern 11 Inter-influence from Structure to Decision and Structure (Compos-*IPMR*: 0.707, Median-*IPMR*: 0.780), “Different Influencing Relations Touch Different Sensitive Nerves”, the influencing relations among different states would possibly influence to the decisions of different other states, for example, when Japan goes very closely with US, China would be touched and take actions correspondingly, while when South Korea

goes very closely with US, then North Korea instead would be touched and take actions correspondingly. Besides, there are a series of prominent trilateral relations like UK influenced by the bilateral relations between US and France (*IPMR*: 0.938, Rank: 1), and France influenced by the bilateral relations between US and China (*IPMR*: 0.800, Rank: 6).

- For influence pattern 12 Inter-influence from Structure to Structure (Compos-*IPMR*: 0.366, Median-*IPMR*: 0.390), “Be Close to Your Friends When Your Enemies be Close to Theirs”, the influencing relations among some states would affect the influencing relations among some other states, there is an *alliance relations dynamic* in international relations, for example, when Japan goes closely with US, China might be touched and also goes closely with Russia.
- For influence pattern 15 Intra-inter Influence from Decision to Structure, “How to Deal with Contradictory Multipartite Relations: Balance Strategy or Revenge Strategy”, while there are two states with contradictive preference both trying to influence your decision to their side, and your final decision would offend either of them, and sometimes you have to be clear about your attitude but can not offending neither side, one solution is the “Balance Strategy”, if you offend one side on the current decision, then you will “please” the same side on the latter decision(s), like making the same decision, another solutions is the “Revenge Strategy”, if you offend one side on the current decision, then you will wait for the revenge from the same side on the latter decisions(s), like being opposed by, in a word, “of two evils, the least should be taken”.
- For influence pattern 24 Inter-intra Influence from Structure to Structure (Compos-*IPMR*: 0.751, Median-*IPMR*: 0.770), “How to Maintain a (Lasting) Relationships: Unilateral Continuity or Mutual Interaction”, some stable positive relationships are maintained by “Unilateral Loyalty”, “Once I Follow You Then I Always Follow You”, for example, the relationship between US and UK are very intimate, UK follows closely with US on many international affairs, tough the interests of US and UK are not uniform on every single issue, but the loyalty of UK to US on the long-term is beneficial to the state interest of UK, therefore, even there is some disagrees between US and UK on certain issues, UK will not easily go against US, but still follow with US. And some stable negative relationships are maintained by “Bear Grudge”, “Once I Oppose You Then I Always Oppose You”, for example, during the whole Cold War, Soviet Union and US had kept a long-term confronting relations, even when there is no interest conflicting on certain issues, Soviet Union or US would still oppose to the other, as there has already formed a tradition of confrontation between them, and each side knows, if the other side also has the opportunity, he would oppose to me the same. However, on many cases, the relationships can not be maintained by only one side, they need the interaction between the two sides. There are also two forms, “Mutual Favor”, “If You Play Nice to Me Then I Play Nice to You Later” or “Mutual Harm”, “If You Play Hard to Me Then I Play Hard to You Later”, which means a positive influencing relation would produce a positive influencing relation on latter times, and a negative influencing relations would generate a negative influencing relation on latter times. Some prominent influence between former influencing relations and latter influencing relations are the continuation of bilateral relations between US and UK (*IPMR*: 0.818, Rank: 4), between Russia and UK (*IPMR*: 0.773, Rank: 12), and between US and Russian (*IPMR*: 0.750, Rank: 13) and so on.

Future Work

- In the influence pattern matching algorithms, the weight of influence from different agents or from different issues should be initially set, and the weight of influence would also be evolving but not static during the process, the dynamic weight is the typical characteristic for *Artificial Neural Nets*. In the thesis, the weight of influence as the numeric representation of influencing relations could also be autonomously determined by decisions or affected by other influencing relations during the interacting process. For future work, the influence pattern matching algorithms could be combined with the Artificial Neural Nets, to better test the patterns of influence embedded in a dynamic networks structure.
- In the influence pattern matching algorithms, we have preset the assumed matching pattern before the beginning of testing procedure, to compare the factual decisions and the expected decisions according to the matching algorithms and the voting data, to identify the patterns of influence matched or not. However, the referential matching pattern could be not preset, but explorative and evolving during the process of interaction and constantly inputting new information (which is related to the concept of rule learning). For example, inspiration can be taken from the pattern discovery techniques (such as association rule mining with statistical assessment), which “*explores large search spaces of potential patterns to find those that satisfy some user-specified constraints*” [Webb, 2007]. In future work, our influence pattern matching algorithms could be combined with machine learning (especially rule learning), to be more explorative and intelligent, constantly dealing with the new information and self-adapting to the new pattern during the testing process.

Simulation Experiments

After the models of influence tested, then we used the new influence model to do simulation experiments. We use the example of UN SC voting under background of reform, based on ABMS (agent-based modeling and simulation technology), through a complete modeling process including conceptual model (reasoning chart), mathematical model (rule design) and computer model (Netlogo programming), to simulate and analyze the interacting process and voting outcome among SC member states under simple reform setups and composite reform schemes. The experiment plans are designed combining both social science and computer science perspectives, to study the complicated interleaved effects, and the simulation outcomes are evaluated by a measurement system orienting on the efficiency and the fairness (as the core value of UN security council), to provide both theoretical and practical thoughts.

Main Findings

- The weight (of influence) allocations bias to allies or bias to opponents are critical to UN SC’s interacting process and voting outcome, especially under the rule of collective influence. Weights allocation bias to allies would increase the efficiency and fairness value compared with bias to opponents, a positive orientation to allies rather a negative orientations to opponents should be encouraged to weaken the confrontation between different alliances and increase the efficiency of UN SC, making more constructive bills widely accepted. And about the rules of influence, prominent one influence is significantly superior to collective influence, compared with comprehensively considering about all other states, one state loyally follows one ally or tightly opposes to one enemy might be a better structure (of influence) for the stability, efficiency and fairness of an international community, or more generally a multi-agent community.

- Regarding the veto power possessed by current permanent members, a compromised plan (such as 2-3 vote combined veto) might be a better choice, not only facing less pressure from current permanent members, but also the results of our experiments do not show a significant improvement on efficiency and fairness by “radical abolish veto” plan compared with “conservative partial veto” plan. And about the group and structure effect, both can exert a positive impact evidently, the group pressure (or peer pressure) is a critical factor to achieve the consensus and improve the efficiency and outcome fairness, and the structure effect will make agents be less influenced by “wobbler”, and more influenced by independent ideas, while the positive effect produced by structure effect is not significant as group effect.
- The expansion of UN SC members will advance the representativeness (improve the opportunity fairness value of SC), but also add the difficulty for UN SC to reach consensus and reduce the passed frequency, achieved average and achieved minimum (harm the efficiency and outcome fairness), fortunately, we found that with the further expansion, the worsening of member enlargement on efficiency and outcome fairness is degressive, gradually reaching stable, for example, the worsening from 15 + 5 plan to 15 + 10 is less significant compared with from 15 to 15 + 10 plan, therefore, if a member expansion is considered and available, a big expansion (above 10 new member) rather a small expansion might be a better choice, not only further improving the opportunity fairness, but also with a lower average cost on efficiency and outcome fairness. About the abstention and constraints, which are basically against the passed frequency and achieved average, while the negative effect is quite insignificant, furthermore, both abstention and constraints are rare cases in real-situation, which can not constitute a critical factor to affect the interacting process and outcome.
- Comprehensively compared all popular plans based on the simulation outcomes, the “Uniting for Consensus” plan is significantly superior to the “G4 Nations” plan and the “African Union” plan, not only substantially weaken the veto power of current permanent members and enhance the representativeness of small and medium countries (improve the right fairness and opportunity fairness), but also better avoid the intense confrontation within UN SC members, especially within the core circle—permanent members, reduce the probability of mutual veto between (conflicting) alliances (guarantee the efficiency). About the combinations of influence patterns, the new crossing influences count against the efficiency and outcome fairness, particularly significantly while singly working, while the new structured influences favor the efficiency and outcome fairness, especially significantly while working combined with crossed influences, however, both two influences can not exert an evident effect under the “African Union” plan, while the bills are highly vulnerable to be vetoed by current and new permanent members both with veto power, then counteracted to some extent.

Future Work

- In the thesis, we run four combined experiments to discuss the interleaved effect: weight allocation combined with rules of influence, veto reform combined with group & structure effect, member expansion combined with abstentions and partial domains, and composite reform schemes combined with influence combinations of patterns, in fact, there are more combinations for experiments respectively from social science and computer science paradigms, in our design, there are 4×4 combinations as top, the simulation and analysis of mixed effects of other combined experiments could be performed in future.
- While computing the achieved average (after voting bills passed or failed), to measure the qualitative efficiency, one method is to assume the weight of all members identical, the other

is to allocate the weight according to the population, GDP or some other criteria. Both methods have reasonableness and controversy to some sense, for instance, the former reflects the principle of democracy and sovereign equality (all countries, big or small, should be equal), but ignore the substantial difference between big countries and small countries, for the big countries with much more population, should not they be endowed with higher weight? and it is just contrary for the latter methods, which emphasis on the objective difference between big and small countries, but not the absolute equality. For the two methods, which represents the real fairness might never have a final conclusion. In the thesis, we set the weight of different members all identical, for future work, the differentiated and diversified setup could be discussed.

- Mainly investigate the influence by the (political) alliance status, to distinguish the positives influence (from allies) and the negative influences (from opponents), for a typical example of two confronting alliances, the Capitalist bloc (or NATO) led by United States and the Socialist bloc (or Warsaw Treaty Organization) led by the former Soviet Union before the end of Cold War, and the current competition between the western bloc led by US on one side, and China and Russia on the other side. However, we have to admit that there are more than one conflicting forces shaping the international relations besides (global) political alliances, in fact, the field of real-world international politics is interweaved with varied conflicting forces, it is a very complicated synthetic process. For instance, besides the most prominent global confrontation between two groups discussed above (mainly due to the ideological and cultural difference, the “inertia” of confrontation in the history¹⁶, the challenging from rising super power and so on¹⁷), there is also one ignored but relatively important local confrontation due to the located region and the geopolitics, which is also an influential factors for states’ behaviors and interactions, for instance the relations between Pakistan and India, between Japan and South Korea, and so on. The neighbour states in the same region may have interest conformity (whom form a regional alliance, like the European Union, the Association of Southeast Asian Nations and so on, to form a united force against challenge and for common development), or might have interest conflicts (become geopolitics competitors like Pakistan and India, Japan and Korea, Japan and China).

For many cases, these different conflicting forces take effect simultaneously, for example, in Europe, US built the NATO to confront the former Soviet Union and the nowadays Russia, and in Asia-Pacific, there is also a Asia version of NATO, which mainly means to confront China (there is a “global conflict” between existed superpower US and potential superpower China), constituted by two independent bilateral alliance treaty, one is the alliance between US and Japan, and the other is the alliance between US and Korea, however, there is no alliance relations between Japan and Korea, and they might never form an alliance relations (due to the very complicated historical and realistic reasons), therefore, compared with the uniformity of NATO in the Europe, the Asian “NATO” is much more complicated, US has to ceaselessly deal with the frequently problematical relations between Japan and Korea, and also has to deal with the “centrifugal force” of South Korea, which has gone more and more close to China.

Therefore, in real-world international politics, the conflicting forces among states in the field are more complex, diversified and compositive, the future work could have more delicate considerations and discussions.

¹⁶Mainly about the conflictions between US and Soviet Union in the cold war, which deeply influenced several generations of people in two countries, and the Cold War mentality still exists in many politicians.

¹⁷Mainly referring to China, which is widely deemed to compete with US in the future.

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Appendix

Appendix A

Basics of Influence

A.1 Background: Computational Politics

A broad term *Computational Social Science*¹, as an interdisciplinary field between computer science and social sciences, has got increasing attentions in social sciences, however, it is enormously unbalanced among different social disciplines or social fields.

From the perspective of discipline system and classification, the research of computational social science in *Economics*, *Management* (particularly the Management Science and Engineering), *Sociology* and so on are abundant, such as (Agent-based) Computational Economics², Computational Organization Theory³ in Management, and Artificial Society⁴ in Sociology and so on, however, in other social disciplines (such as *Public Administration*, *International Politics*, *Education and so on*), the relevant research are relatively limited. A special case is Politics, in which there is a lot of works of computational social choices on *Domestic Politics*, such as political elections, voting candidates (for president, governor, mayor and other winners), and discussing the accompanied problems in voting like manipulation, bribery and so on; comparatively, the research of computational social choice on *International Politics* are relatively deficient, however, in fact there are a lot of typical multi-agent and multi-issue decision-making questions in international politics, such as the voting of UN assembly, UN security council and European parliament. And for these multi-nation and multi-bill voting, their processes, rules and consequent outcomes would have great effects on state interests, international relations and situations. Furthermore, the topics in computational social choice such as dependencies among multi-issues, influences among multi-agents exist commonly in these decision-making systems, for example of UN security coun-

¹Computational social science concerned with computational approaches to the social sciences. Fields include computational economics, computational sociology, automated information extraction systems, social network analysis, social geographic information systems (GIS), complexity modeling, and social simulation models.

²“Agent-based computational economics (ACE) is the area of computational economics that studies economic processes, including whole economies, as dynamic systems of interacting agents. As such, it falls in the paradigm of complex adaptive systems” [Arthur, 1994, Tesfatsion, 2003], cited in [Wikipedia, 2014]

³“Computer simulation is a prominent method in organizational studies and strategic management [Harrison et al., 2007]. Most academics in the fields of strategic management and organizational studies have used computer simulation to understand how organizations or firms operate. More recently, however, researchers have also started to apply computer simulation to understand organizational behaviour at a more micro-level, focusing on individual and interpersonal cognition and behavior [Hughes et al., 2012] such as team working [Crowder et al., 2012]” [Wikipedia, 2015c].

⁴“The artificial society has been widely accepted by recent sociology as a promising method characterized by the extensive use of computer programs and computer simulations which include evolutionary algorithms (EA), genetic algorithms (GA), genetic programming (GP), memetic programming (MP), agent based models, and cellular automata (CA). The principle problem is that of classical sociology, the issue of macro-micro linkage: as first articulated by French Sociologist Emile Durkheim, the question of how individuals within a social system influence and are influenced by the macro-social level”, [Wikipedia, 2015b]

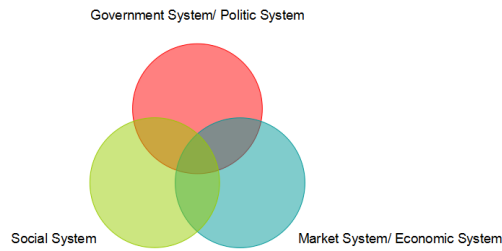


Figure A.1: Market-Government-Society

cial voting, member states would vote for a series of interrelated bills, they would influence and be influenced by each other, convincing others' votes in order to make preferred plan passed and maximum each own state interest. Therefore, the researches of computational social choice in international politics are very meaningful and promising (but relatively ignored in previous works).

From the perspective of application field (of computer science on social problems), according to the (classical) classification of three-dimensional system of *Market-Government-Society*, the relevant research of computational social science are relatively more mature in the market/economic system and the social system, such as the computation and simulation research of market exchange, enterprise competition, industry cluster, innovation diffusion (in the field of market system), and social networks, population system, collaborative cooperation (in the field of social system) and so on, in fact, above research mainly belongs to the discipline of Economics, Management⁵ and Sociology, which also corroborated the former perspective of classification about disciplines. However, the research of computational social science in government system⁶ (which mainly belongs to the disciplinary field of Politics and Public Administration) falls far behind the above two fields (of market and society)⁷, particularly for international government (international politics) system, the relevant works are relatively scarce, and have not yet formed uniform research paradigm or mature theoretical framework, there is a series of reasons, not only due to the challenges of the particular complexity⁸ of international politics system on quantification and computation, and also due to the limitations of mathematics and computer foundations of traditional disciplinary background of politics and public administration scholars. Therefore, it is meaningful to further advance the research of computational social science (especially the computational social choice) in politics and administration system, particularly the international politics system.

⁵Mainly about the private organization (enterprise) management

⁶Which include both the domestic government system and international government system, the former is about elections, government organizations and so on, and the latter is about interactions and games among multi-nations and so on.

⁷Although there are some research samples of public policy simulation, mainly investigate the effect of public policy as an environment on the evolution of economic system and social system, but not investigate the political and administrative systems themselves, in some sense which are just the extensive applications of economics or sociology research, importing public policy as an environment/controlling variable into models of economics and models of sociology.

⁸For example, the game among multi-nations in international politics is very complex, correlated with the interweaving of power, state interest, state strategy, alliance, international situation, and so on, and the result of international game is a complicated process of mixing of varied forces.

Appendix B

Theory of Influence

B.1 Mathematical Expression of Influence Patterns

Pattern 1 Intra-influence from Decision to Decision

To describe this pattern of influence in mathematics, “from decision to decision” just describe the subject and the object of influence, which are both decisions, but how is the subject of influence affects on the object of influence specifically, and the process or rule of influence is not explained. For example of two specific and common forms for this pattern of influence, one is that the object of influence exactly follows the subject of influence, which means that the influence function could be represented as “=” (the influenced preference would be equal to the influencing preference). Given $k + l < m$, $\{k, k + l\} \subseteq \mathbb{M}$, $i \in \mathbb{N}$, \mathbb{M} is the set of all issues, m is the number or sequence of last issue, and \mathbb{N} is the set of all agents. The formula for this form would be (in which $P_{(k)}(i)$ means the preference of agent i on issue k):

$$P_{(k+l)}(i) = P_{(k)}(i) \quad (\text{B.1})$$

The other is just opposite, the object of influence would go against (or oppose to) the subject of influence, and the influence function could be represented as “ \neq ” (the influenced preference should be different or opposite from the influencing preference). This two forms of influence could be deemed as the most specific example for the *positive influence and negative influence*. The formula would be:

$$P_{(k+l)}(i) \neq P_{(k)}(i) \quad (\text{B.2})$$

Besides, above two forms both assume that no matter what is the choice (on the influencing issue), the influence always works in the same way, however, although the direction of influence (from issue k to issue $k+l$) is fixed, but “how” influence could be different due to different choices. For example of the binary choice, for one choice c_k , the form of influence is “=”, but for the other choice \bar{c}_k , the form of influence is “ \neq ”. The formula would be (in which $C_{(k)}(i)$ means the choice of agent i on issue k , which is a binary choice with two values c_k and \bar{c}_k):

$$P_{(k+l)}(i) \begin{cases} = P_{(k)}(i) & \text{if } C_{(k)}(i) = c_k \\ \neq P_{(k)}(i) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \quad (\text{B.3})$$

However, above all are just one specific form for this pattern of influence, a general function Pa could be constructed to describe the general form of the intra-influence, for example, $P_1 =$

$Pa(P_2)$ means that P_1 is intra-influenced by P_2 , indicates that there is an influence from P_1 to P_2 , but “how” the influence works could be expanded in varied forms. The general formula would be:

$$P_{(k+l)}(i) = Pa(P_{(k)}(i)) \quad (\text{B.4})$$

And if the general form of this influence also works in different ways due to different choices. The general formula would be (in which, Pa_α , Pa_β means two different forms of Pa functions, which works by different rules):

$$P_{(k+l)}(i) \begin{cases} = Pa_\alpha(P_{(k)}(i)) & \text{if } C_{(k)}(i) = c_k \\ = Pa_\beta(P_{(k)}(i)) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \quad (\text{B.5})$$

Further, above all formulas assume the preference on the influenced issues is just influenced by other influencing issues, however, in fact it is usual that the eventual preference on one issue is not only influenced by other issues, but also influenced by own (the initial or original preference on this issue), the influence is kind of a collective effect of influencing preference and own preference. Which might explain why equally under the influence, sometimes the object be influenced and changed, but sometimes still insist on the initial own inclination. The general formula would be:

$$P_{(k+l)}(i) \begin{cases} = Pa_\alpha(P_{(k)}(i), P_{(k+l)}(i)) & \text{if } C_{(k)}(i) = c_k \\ = Pa_\beta(P_{(k)}(i), P_{(k+l)}(i)) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \quad (\text{B.6})$$

Pattern 2 Intra-influence from Decision to Decision and Structure

To describe this pattern of influence in general formula, given $k + l + s < m$, $\{k, k + l, k + l + s\} \subseteq \mathbb{M}$, $i \in \mathbb{N}$.

$$\begin{aligned} P_{(k+l)}(i) &= Pa_{(\alpha)}(P_{(k)}(i)) & \text{if } C_{(k)}(i) = c_k \\ P_{(k+l+s)}(i) &= Pa_{(\beta)}(P_{(k)}(i)) & \text{if } C_{(k)}(i) = \bar{c}_k \end{aligned} \quad (\text{B.7})$$

However, there exist a defect in above formula, it describes under certain cases (for issue $k + l$ is the c_k , and for issue $k + l + s$ is the \bar{c}_k), the preferences would be influenced, but not indicate under other cases (for issue $k + l$ is the \bar{c}_k , and for issue $k + l + s$ is the c_k), what the preference would be? In fact, if not influenced by other issue, it should be (determined by) the initial or original preference, therefore, the formula could be complemented as:

$$\begin{aligned} P_{(k+l)}(i) &\begin{cases} = Pa_{(\alpha)}(P_{(k)}(i)) & \text{if } C_{(k)}(i) = c_k \\ = P_{(k+l)}(i) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \\ P_{(k+l+s)}(i) &\begin{cases} = Pa_{(\beta)}(P_{(k)}(i)) & \text{if } C_{(k)}(i) = \bar{c}_k \\ = P_{(k+l+s)}(i) & \text{if } C_{(k)}(i) = c_k \end{cases} \end{aligned} \quad (\text{B.8})$$

Specific Example 3 (Pattern 2) For a daily-life example, when we choose to buy a laptop, assuming there are three important features $\{X, Y, Z\}$ (all with binary values) to make choice about, in which X represents the size of the laptop, which means we can choose a thin laptop (x) or a thick laptop (\bar{x}); Y represents the heat dissipation capacity of the laptop, as we know, if the heat dissipation of the laptop is bad, the laptop would be too hot to run in a proper speed, and would be so “slow” and even “crashed”, then also set two values as good heat dissipation (y) and bad heat

dissipation (\bar{y}); variable Z represents the (computing) performance of the laptop, we could choose a laptop with higher performance (z) and a laptop with relatively lower performance (\bar{z}). As the pattern of influence discussed above, the decision on one feature would influence the decision on other features, and according to the different choices of the influencing feature, the direction of the influence (or the structure of influence), would also be accordingly different. For example, when we choose about the size (X) of a laptop, if we determine to choose a thin one (x), then we usually would focus a lot on the heat dissipation capacity (Y) of the laptop (as we known, when the laptop is very thin, heat dissipation might be a very common headache), which means that, if we choose the value “thin” (x) for feature “size” (X), the feature “heat dissipation” would be influenced and be chosen as “good heat dissipation” (y), and form a cps-statements (conditional preference structure statements)¹ as “ $x : X \rightarrow Y, y$ ”; on the other side, if we choose a “thick” laptop (\bar{x}), then we usually would focus more or expect a lot on the performance (Z) of the laptop, because usually we want the laptop to be very portable and easy to carry, we hope the laptop to be as small and thin as possible, but sometimes in order to have a better performance (for example, to have a bigger screen, to have a CD-ROM and more interfaces, or any other functions, usually the laptop could not be that thin, as we know, nowadays the laptop have become more and more thin, but inevitable we also sacrifice many functions, for instance, most of ultrathin laptops do not have the CD-ROM anymore and the interface are not diversified and abundant enough, we have to prepare many peripheral equipments), therefore, if we choose a relatively thick laptop but not a thin one, usually we would expect more on the performance of the laptop, because we would think that if the laptop is low-performance, why bother to choose to be thick and cumbersome, which means that if we choose the value “thick” (\bar{x}) on feature “size”, then the decision of variable “performance” (Z) would be influenced and be chosen as “higher performance” (z), and form a cps-statement as “ $\bar{x} : X \rightarrow Z, z$ ”. According to this common example, it is easy to understand that the structure of influences would also be influenced by the decision², for the different choice of X , either influencing to Y or Z , which is different from the pattern of influence “decision to decision” discussed before, no matter what the value is chosen, the influence always points from X to Y , which means the decision only influences the decision, but the structure is fixed (uninfluenced).

Pattern 3 Intra-influence from Decision to Structure

The intra-influence of decision to structure could be divided into two components, one is the *influencing component*, which is one decision-making issue, and the other is the *influenced component*, which is the structure of influence/dependency among latter issues. Due to the different structure of the influenced component, “*influence different issues*” or “*influenced by different issues*”, as shown in figure B.1, this pattern of influence could be further classified as two modes.

Influence Which of Influenced Component

In this mode, in the influenced component, the influencing issue is one same, but the influenced issue is more than one. Given $k + t + l + s < m$, $\{k, k + t, k + t + l, k + t + l + s\} \subseteq \mathbb{M}$, $i \in \mathbb{N}$, the general formula would be:

¹Because different from the cp-statements [Boutilier et al., 2004a](in which only the decisions are influenced), both the decision and the structure are influenced.

²For one feature, if we choose a different value for it, the structure of influences among features might also be changed.

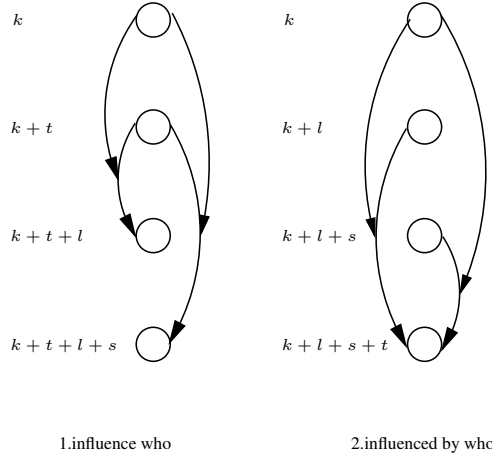


Figure B.1: Two modes of 3. Intra-influence of Decision to Structure

$$\begin{aligned}
 P_{(k+t+l)}(i) & \begin{cases} = Pa_{(\alpha)}(P_{(k+t)}(i)) & \text{if } C_{(k)}(i) = c_k \\ = P_{(k+t+l)}(i) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \\
 P_{(k+t+l+s)}(i) & \begin{cases} = Pa_{(\beta)}(P_{(k+t)}(i)) & \text{if } C_{(k)}(i) = \bar{c}_k \\ = P_{(k+t+l+s)}(i) & \text{if } C_{(k)}(i) = c_k \end{cases}
 \end{aligned} \tag{B.9}$$

Influenced by Which of Influenced Component

In this mode, in the influenced component, the influenced issue is one same, but the influencing issue is more than one. Given $k+l+s+t < m$, $\{k, k+l, k+l+s, k+l+s+t\} \subseteq \mathbb{M}$, $i \in \mathbb{N}$, the general formula would be:

$$P_{(k+l+s+t)}(i) \begin{cases} = Pa_{(\alpha)}(P_{(k+l)}(i)) & \text{if } C_{(k)}(i) = c_k \\ = Pa_{(\beta)}(P_{(k+l+s)}(i)) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \tag{B.10}$$

Specific Example 4 (Pattern 3) Also for the example of buying a laptop, this time we define four variables to make decisions, variable X represents the price, variable Y represents the heat dissipation, variable Z represents the size, variable W represents the (running) performance, every variables has two values, for the price (X): x represents low price, and \bar{x} means the expensive price; for the heat dissipation (Y), y means good heat dissipation, and \bar{y} means bad heat dissipation; for the size (Z), z means thin, and \bar{z} means thick; for the performance (W), w means good performance and \bar{w} means bad performance.

The decision of one variable might influence the structure of influence of other variables, for example, when we firstly determine the value of variable price, if we choose to buy a relatively “cheap” laptop (x), there would be a relationships between the heat dissipation capacity (Y) and size(Z), to be more specifically, there might be a reluctant but inevitable choice between the size and the heat dissipation, actually, both thin (z) and good heat dissipation (y) are wanted by us, but usually when we have already determined to buy a laptop with low price, we have to do trade-offs between these two good attributes, which means that, for the variable size, if we choose to buy a thin one, we may have to put up with a bad heat dissipation under the limitation to buy a low price laptop, on the other side, if we choose to buy a thick one, we may then expect more on the heat

dissipation capacity. However, for the variable price, if we choose expensive but not cheap one from the beginning, there would be no absolute (constraints) relations between the size and heat dissipation capacity, it is highly possible to get a both ultrathin and good heat dissipation laptop under the high price (high manufacture cost). Therefore, it is easy to understand that the decision of one variable might influence how the other variables influence each other.

Furthermore, on the opposite side, if we choose to buy a expensive laptop from the beginning, then there might be (influencing) relationships between different variables. For instance, when we then determine to buy a thin one, there would be a common psychology that “I want both ultrathin and abundant functions because I cost a lot of money!” As we know, for most cases, the thinner the laptop, the less the functions (for example the ultrathin usually sacrifice at abandoning some functions, like losing the CD-ROM, and abating on the graphic performance due to using integrated graphics card but independent graphic card), however, for the consumer who paid a lot of money, would possibly have high expectation on both two good attributes at the same, (think the trade-offs things are just for penny pincher), and specifically emphasis on the functions when buy a thin laptop, therefore, there would be a (influencing) relationships between the variable size and functions. According to different values of some variables (in our example the variable X), the structure of influences among other variables are influenced, based on this example, we could form the cs-statements “ $x : Y \rightarrow Z$ ” and “ $\bar{x} : Z \rightarrow W$ ”.

Pattern 4 Intra-influence from Structure to Decision

Abstract Example B.1 (Pattern 4) As shown in sub figure 4 of figure 5.4, agent 1 make decisions on four issues X,Y,Z,U, if agent 1 make decision on issue Z dependent on issue X, then agent 1 will choose \bar{u}_1 on issue U, but if agent 1 make decision on issue Z dependent on issue Y, then agent 1 will choose u_1 instead on issue U. The cps-statement could be expressed as: $Z_1 = Pa(X_1) : \bar{v}_1, Z_1 = Pa(Y_1) : v_1$.

The influence is from structure to decision, no matter what different dependencies are chosen or determined by the agent $X \rightarrow Z$ or $Y \rightarrow Z$, the influences always point to the choice on the same issue U, only different values to be chosen, u_1 or \bar{u}_1 .

For this pattern of influence, the influencing component is the structure among issues, and the influenced component is one single issue. According to different forms of structure in the influencing component, “influence different issues” or “influenced by different issues”, this pattern of influence could be further classified into two modes.

Influence Which of Influencing Component

Given $k + l + s + t < m$, $\{k, k + l, k + l + s, k + l + s + t\} \subseteq \mathbb{M}$, $i \in \mathbb{N}$, and define $\phi[]$ as the function of influential effect of structure among issues (how it works will be discussed in details in chapter 7), use $\phi[P_1, P_2]$ to express the influential effect from the structure of dependency on P_2 by P_1 , the general formula for this mode would be (combining both the individual influence of issues in the structure and the influential effect of the structure itself, and adding the influence from own issue):

$$P_{(k+l+s+t)}(i) \left\{ \begin{array}{l}
= Pa_{(\alpha)}\{P_{(k)}(i), P_{(k+l)}(i), \phi[P_{(k)}(i), P_{(k+l)}(i)], P_{(k+l+s+t)}(i)\}, \\
\text{if } P_{(k+l)}(i) = Pa_{(\gamma)}(P_{(k)}(i)) \cap [P_{(k+l+s)}(i) \neq Pa_{(\delta)}(P_{(k)}(i))] \\
= Pa_{(\beta)}\{P_{(k)}(i), P_{(k+l+s)}(i), \phi[P_{(k)}(i), P_{(k+l+s)}(i)], P_{(k+l+s+t)}(i)\}, \\
\text{if } P_{(k+l+s)}(i) = Pa_{(\delta)}(P_{(k)}(i)) \cap [P_{(k+l)}(i) \neq Pa_{(\gamma)}(P_{(k)}(i))] \\
= Pa_{(\sigma)}\{P_{(k)}(i), P_{(k+l)}(i), P_{(k+l+s)}(i), \\
\phi[P_{(k)}(i), P_{(k+l)}(i)], \phi[P_{(k)}(i), P_{(k+l+s)}(i)], P_{(k+l+s+t)}(i)\} \\
\text{if } [P_{(k+l)}(i) = Pa_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k+l+s)}(i) = Pa_{(\delta)}(P_{(k)}(i))] \\
= P_{(k+l+s+t)}(i), \\
\text{if } [P_{(k+l)}(i) \neq Pa_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k+l+s)}(i) \neq Pa_{(\delta)}(P_{(k)}(i))]
\end{array} \right. \quad (\text{B.11})$$

Influenced by Which of Influencing Component

Given $l + s + k + t < m$, $\{l, l + s, l + s + k, l + s + k + t\} \subseteq \mathbb{M}$, $i \in \mathbb{N}$, the general formula would be:

$$P_{(l+s+k+t)}(i) \left\{ \begin{array}{l}
= Pa_{(\alpha)}\{P_{(l)}(i), P_{(l+s+k)}(i), \phi[P_{(l)}(i), P_{(l+s+k)}(i)], P_{(l+s+k+t)}(i)\}, \\
\text{if } [P_{(l+s+k)}(i) = Pa_{(\gamma)}(P_{(l)}(i))] \cap [P_{(l+s+k)}(i) \neq Pa_{(\delta)}(P_{(l+s)}(i))] \\
= Pa_{(\beta)}\{P_{(l+s)}(i), P_{(l+s+k)}(i), \phi[P_{(l+s)}(i), P_{(l+s+k)}(i)], P_{(l+s+k+t)}(i)\}, \\
\text{if } [P_{(l+s+k)}(i) = Pa_{(\delta)}(P_{(l+s)}(i))] \cap [P_{(l+s+k)}(i) \neq Pa_{(\gamma)}(P_{(l)}(i))] \\
= Pa_{(\sigma)}\{P_{(l)}(i), P_{(l+s)}(i), P_{(l+s+k)}(i), \\
\phi[P_{(l)}(i), P_{(l+s+k)}(i)], \phi[P_{(l+s)}(i), P_{(l+s+k)}(i)], P_{(l+s+k+t)}(i)\} \\
\text{if } [P_{(l+s+k)}(i) = Pa_{(\gamma)}(P_{(l)}(i))] \cap [P_{(l+s+k)}(i) = Pa_{(\delta)}(P_{(l+s)}(i))] \\
= P_{(l+s+k+t)}(i), \\
\text{if } [P_{(l+s+k)}(i) \neq Pa_{(\gamma)}(P_{(l)}(i))] \cap [P_{(l+s+k)}(i) \neq Pa_{(\delta)}(P_{(l+s)}(i))]
\end{array} \right. \quad (\text{B.12})$$

Pattern 5 Intra-influence from Structure to Decision and Structure

Abstract Example B.2 (Pattern 5) As shown in sub figure 5 of figure 5.4, agent 2 make decision on five issues X, Y, Z, U, V , if agent 2 make decision on issue Z dependent on issue X , the decision of agent 2 on issue U will be influenced and chosen as \bar{v}_2 , but if agent 2 make decision on issue Z dependent on issue Y , then the decision of agent 2 on issue V will be influenced and chosen as v_2 .

And the influence is from structure to both decision and structure, the specific decision on latter issues will be affected \bar{v}_2 or v_2 , also the directions or structures of influence are changed $\rightarrow U$ or $\rightarrow V$, the influence could be represented by cps-statements as: $Z_2 = Pa(X_2) : \bar{v}_2$, $Z_2 = Pa(Y_2) : v_2$.

This pattern of influence could also be classified into two modes, according to the structure of influencing component.

Influence Which

Given $k + l + t < m$, $\{k, k + l, k + l + t\} \subseteq \mathbb{M}$, $k + l + s + d < m$, $\{k, k + l, k + l + s, k + l + s + d\} \subseteq \mathbb{M}$, $k + l + t \neq k + l + s + d$, $i \in \mathbb{N}$, combining the individual influences in the structure, the influential effect of the structure itself and the own influence, the general formula would be:

$$\begin{aligned}
P_{(k+l+t)}(i) & \begin{cases} = Pa_{(\alpha)}\{P_{(k)}(i), P_{(k+l)}(i), \phi[P_{(k)}(i), P_{(k+l)}(i)], P_{(k+l+t)}(i)\}, \\ \text{if } P_{(k+l)}(i) = Pa_{(\gamma)}(P_{(k)}(i)) \\ = P_{(k+l+t)}(i), \\ \text{if } P_{(k+l)}(i) \neq Pa_{(\gamma)}(P_{(k)}(i)) \end{cases} \\
P_{(k+l+s+d)}(i) & \begin{cases} = Pa_{(\beta)}\{P_{(k)}(i), P_{(k+l+s)}(i), \phi[P_{(k)}(i), P_{(k+l+s)}(i)], P_{(k+l+s+d)}(i)\}, \\ \text{if } P_{(k+l+s)}(i) = Pa_{(\delta)}(P_{(k)}(i)) \\ = P_{(k+l+s+d)}(i), \\ \text{if } P_{(k+l+s)}(i) \neq Pa_{(\delta)}(P_{(k)}(i)) \end{cases}
\end{aligned} \tag{B.13}$$

Influenced by Which

Given $l + s + k + t < m$, $\{l, l + s, l + s + k, l + s + k + t\} \subseteq \mathbf{M}$, $l + s + k + d < m$, $\{l, l + s, l + s + k, l + s + k + d\} \subseteq \mathbf{M}$, $l + s + k + t \neq l + s + k + d$, $i \in \mathbf{N}$, the general formula would be:

$$\begin{aligned}
P_{(l+s+k+t)}(i) & \begin{cases} = Pa_{(\alpha)}\{P_{(l)}(i), P_{(l+s+k)}(i), \phi[P_{(l)}(i), P_{(l+s+k)}(i)], P_{(l+s+k+t)}(i)\}, \\ \text{if } P_{(l+s+k)}(i) = Pa_{(\gamma)}(P_{(l)}(i)) \\ = P_{(l+s+k+t)}(i), \\ \text{if } P_{(l+s+k)}(i) \neq Pa_{(\gamma)}(P_{(l)}(i)) \end{cases} \\
P_{(l+s+k+d)}(i) & \begin{cases} = Pa_{(\beta)}\{P_{(l+s)}(i), P_{(l+s+k)}(i), \phi[P_{(l+s)}(i), P_{(l+s+k)}(i)], P_{(l+s+k+d)}(i)\}, \\ \text{if } P_{(l+s+k)}(i) = Pa_{(\delta)}(P_{(l+s)}(i)) \\ = P_{(l+s+k+d)}(i), \\ \text{if } P_{(l+s+k)}(i) \neq Pa_{(\delta)}(P_{(l+s)}(i)) \end{cases}
\end{aligned} \tag{B.14}$$

Pattern 6 Intra-influence from Structure to Structure

Abstract Example B.3 (Pattern 6) As shown in sub figure 6 of figure 5.4, agent 3 make decision on six issues X, Y, Z, U, V, W , if agent 3 make decision on issue Z dependent on issue X , then will make decision on issue V dependent on on issue U , but if agent 3 make decision on issue Z dependent on issue Y , then will make decision on issue W dependent on issue U . The new cs-statement (conditional structure statement, without preference directly involved) could be expressed as: $Z_3 = Pa(X_3) : V_3 = Pa(U_3), Z_3 = Pa(Y_3) : W_3 = Pa(U_3)$.

The influence is from structure to just structure $\{X \rightarrow Z\} \rightarrow \{U \rightarrow V\}$ or $\{Y \rightarrow Z\} \rightarrow \{U \rightarrow W\}$, the structure among latter issues will be affected, but the specific choice on latter issues would not be directly affected by former structures (bur determined by the preferences and dependencies of latter issues own).

As the influencing component and the influenced component are both structures among multi-issues, then both could be divided as two forms, “influence which” and “influenced by which”, and the combination of these two varied structures could constitute four modes, as show in figure B.2.

Influence Which to Influence Which

In this mode, the influencing structure and the influenced structure are both “influence which”. Given $k + l + s + q + t < m$, $k + l + s + q + d < m$, $\{k, k + l, k + l + s, k + l + s + q, k + l +$

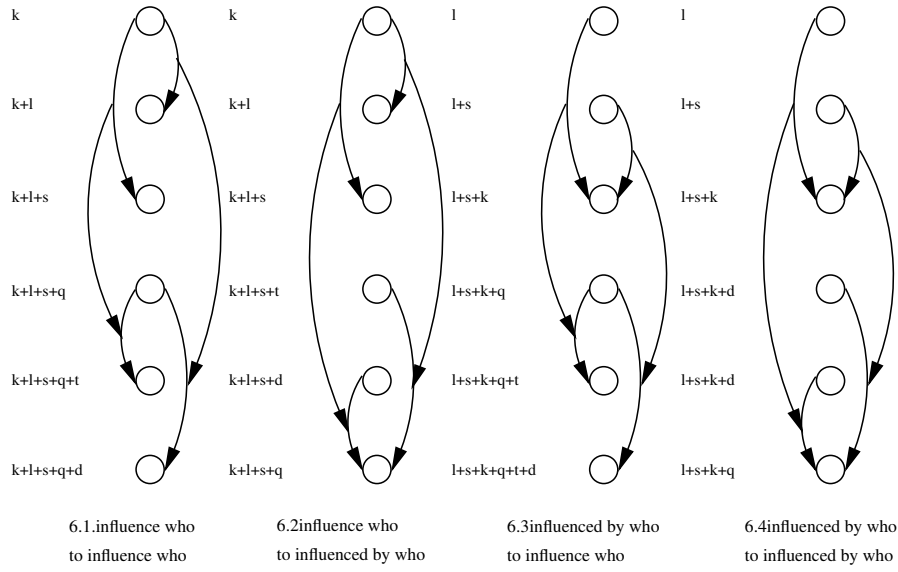


Figure B.2: Four Modes of 6.Intra-influence of Structure to Structure

$s + q + t, k + l + s + q + d\} \subseteq \mathbb{M}, i \in \mathbb{N}$, (if combining the own influence), the general formula would be:

$$\begin{aligned}
 P_{(k+l+s+q+t)}(i) & \begin{cases} = Pa_{(\alpha)}(P_{(k+l+s+q)}(i), P_{(k+l+s+q+t)}(i)) & \text{if } P_{(k+l)}(i) = Pa_{(\gamma)}(P_{(k)}(i)) \\ = P_{(k+l+s+q+t)}(i) & \text{if } P_{(k+l)}(i) \neq Pa_{(\gamma)}(P_{(k)}(i)) \end{cases} \\
 P_{(k+l+s+q+d)}(i) & \begin{cases} = Pa_{(\beta)}(P_{(k+l+s+q)}(i), P_{(k+l+s+q+d)}(i)) & \text{if } P_{(k+l+s)}(i) = Pa_{(\delta)}(P_{(k)}(i)) \\ = P_{(k+l+s+q+d)}(i) & \text{if } P_{(k+l+s)}(i) \neq Pa_{(\delta)}(P_{(k)}(i)) \end{cases}
 \end{aligned} \tag{B.15}$$

Pattern 7 Inter-influence from Decision to Decision

There are many different forms for this influence, we can assume that the influencing agent i influences the influenced agent j positively, for one specific example, agent j follows agent i , and choose exactly the same as agent i . This form of influence is very common in real-world situation, people are inclined to follow the families, relatives, friends, wise man, leaders and so on. Given $k \in \mathbb{M}, \{i, j\} \subseteq \mathbb{N}$, if the specific form of “follow” represented as “=”, the formula would be:

$$P_{(k)}(j) = P_{(k)}(i) \tag{B.16}$$

On the other side, we can also assume that the agent i influences agent j negatively, for one specific example agent j goes against or opposes to agent i . This form of influence is also very common in real-world situation, people are inclined to go against or opposite to their enemies, opponents, and some persons disliked, jealous at and so on. And if the specific form of “go against” or “oppose to” is represented as “ \neq ”, the formula would be:

$$P_{(k)}(j) \neq P_{(k)}(i) \tag{B.17}$$

If the influence works in different ways (but not an uniform way) due to different choices, for example combining the “follow” and “oppose” under different circumstances (such as half follow and half oppose), the formula would be:

$$P_{(k)}(j) \begin{cases} = P_{(k)}(i) & \text{if } C_{(k)}(i) = c_k \\ \neq P_{(k)}(i) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \quad (\text{B.18})$$

Above are just specific forms for this pattern of influence, to represent in the general form, a function In (which is the short of term ‘‘Influence’’) is defined to describe the general form of inter-influence, the function In is similar and symmetrical to the function Pa , both represent the relations of influence, just works in different dimensions, referring to the influenced CP-nets, the function Pa works in vertical dimension, among different issues but within one same agent; on the contrary, the function In works in horizontal dimension, among different agents but within one same issue. If represented in the In function, and combining the influence from the influencing one’s preference and own (initial preference), the formula would be ($P_1 = In(P_2)$) means the P_2 influences P_1 , in another word, there is a structure of influence from P_2 to P_1 :

$$P_{(k)}(j) = In(P_{(k)}(i), P_{(k)}(j)) \quad (\text{B.19})$$

And if the influence works in different ways due to the choices, the formula would be (in which In_α and In_β means two different In function, works by different rules):

$$P_{(k)}(j) \begin{cases} = In_\alpha(P_{(k)}(i), P_{(k)}(j)) & \text{if } C_{(k)}(i) = c_k \\ = In_\beta(P_{(k)}(i), P_{(k)}(j)) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \quad (\text{B.20})$$

Influence Which to Influenced by Which

In this mode, the influencing structure is ‘‘influence which’’, and the influenced structure is ‘‘influenced by which’’. Given $k + l + s + t < m$, $k + l + s + d < m$, $k + l + s + q < m$, $\{k, k + l, k + l + s, k + l + s + t, k + l + s + d, k + l + s + t + q\} \subseteq \mathbb{M}$, $q > t$, $q > d$, $i \in \mathbb{N}$, the general formula would be:

$$P_{(k+l+s+q)}(i) \begin{cases} = Pa_{(\alpha)}(P_{(k+l+s+t)}(i), P_{(k+l+s+q)}(i)) \\ \quad \text{if } [P_{(k+l)}(i) = Pa_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k+l+s)}(i) \neq Pa_{(\delta)}(P_{(k)}(i))] \\ = Pa_{(\beta)}(P_{(k+l+s+d)}(i), P_{(k+l+s+q)}(i)) \\ \quad \text{if } [P_{(k+l+s)}(i) = Pa_{(\delta)}(P_{(k)}(i))] \cap [P_{(k+l)}(i) \neq Pa_{(\gamma)}(P_{(k)}(i))] \\ = Pa_{(\sigma)}(P_{(k+l+s+t)}(i), P_{(k+l+s+d)}(i), P_{(k+l+s+q)}(i)) \\ \quad \text{if } [P_{(k+l)}(i) = Pa_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k+l+s)}(i) = Pa_{(\delta)}(P_{(k)}(i))] \\ = P_{(k+l+s+q)}(i) \\ \quad \text{if } [P_{(k+l)}(i) \neq Pa_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k+l+s)}(i) \neq Pa_{(\delta)}(P_{(k)}(i))] \end{cases} \quad (\text{B.21})$$

Influenced by Which to Influence Which

In this mode, the influencing structure is ‘‘influenced by who’’, and the influenced structure is ‘‘influence who’’. Given $l + s + k + q + t < m$, $l + s + k + q + d < m$, $\{l, l + s, l + s + k, l + s + k + q, l + s + k + q + t, l + s + k + q + d\} \subseteq \mathbb{M}$, $i \in \mathbb{N}$, the general formula would be:

$$\begin{aligned} P_{(l+s+k+q+t)}(i) & \begin{cases} = Pa_{(\alpha)}(P_{(l+s+k+q)}(i), P_{(l+s+k+q+t)}(i)) & \text{if } P_{(l+s+k)}(i) = Pa_{(\gamma)}(P_{(l)}(i)) \\ = P_{(l+s+k+q+t)}(i) & \text{if } P_{(l+s+k)}(i) \neq Pa_{(\gamma)}(P_{(l)}(i)) \end{cases} \\ P_{(l+s+k+q+d)}(i) & \begin{cases} = Pa_{(\beta)}(P_{(l+s+k+q)}(i), P_{(l+s+k+q+d)}(i)) & \text{if } P_{(l+s+k)}(i) = Pa_{(\delta)}(P_{(l+s)}(i)) \\ = P_{(l+s+k+q+d)}(i) & \text{if } P_{(l+s+k)}(i) \neq Pa_{(\delta)}(P_{(l+s)}(i)) \end{cases} \end{aligned} \quad (\text{B.22})$$

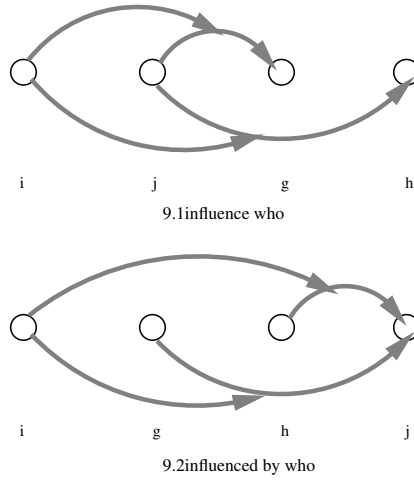


Figure B.3: Two Modes of 9. Inter-influence of Decision to Structure

Influenced by Which to Influence by Which

In this mode, the influencing structure and the influenced structure are both “influenced by which”. Given $l + s + k + t < m, l + s + k + d < m, l + s + k + q < m \{l, l + s, l + s + k, l + s + k + t, l + s + k + d, l + s + k + q\} \subseteq \mathbb{M}, q > t, q > d, i \in \mathbb{N}$, the general formula would be:

$$P_{(l+s+k+q)}(i) \begin{cases} = Pa_{(\alpha)}(P_{(l+s+k+t)}(i), P_{(l+s+k+q)}(i)) \\ \text{if } [P_{(l+s+k)}(i) = Pa_{(\gamma)}(P_{(l)}(i))] \cap [P_{(l+s+k)}(i) = Pa_{(\delta)}(P_{(l+s)}(i))] \\ = Pa_{(\beta)}(P_{(l+s+k+d)}(i), P_{(l+s+k+q)}(i)) \\ \text{if } [P_{(l+s+k)}(i) = Pa_{(\delta)}(P_{(l+s)}(i))] \cap [P_{(l+s+k)}(i) \neq Pa_{(\gamma)}(P_{(l)}(i))] \\ = Pa_{(\sigma)}(P_{(l+s+k+t)}(i), P_{(l+s+k+d)}(i), P_{(l+s+k+q)}(i)) \\ \text{if } [P_{(l+s+k)}(i) = Pa_{(\gamma)}(P_{(l)}(i))] \cap [P_{(l+s+k)}(i) = Pa_{(\delta)}(P_{(l+s)}(i))] \\ = P_{(l+s+k+q)}(i) \\ \text{if } [P_{(l+s+k)}(i) \neq Pa_{(\gamma)}(P_{(l)}(i))] \cap [P_{(l+s+k)}(i) \neq Pa_{(\delta)}(P_{(l+s)}(i))] \end{cases} \quad (\text{B.23})$$

Pattern 8 Inter-influence from Decision to Decision and Structure

Given $\{i, j, g\} \subseteq \mathbb{N}, k \in \mathbb{M}$, i is the influencing agent, and j or g are the two different influenced agent. The formula would be:

$$P_{(k)}(j) \begin{cases} = In_{(\alpha)}(P_{(k)}(i), P_{(k)}(j)) & \text{if } C_{(k)}(i) = c_k \\ = P_{(k)}(j) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \quad (\text{B.24})$$

$$P_{(k)}(g) \begin{cases} = In_{(\beta)}(P_{(k)}(i), P_{(k)}(g)) & \text{if } C_{(k)}(i) = \bar{c}_k \\ = P_{(k)}(g) & \text{if } C_{(k)}(i) = c_k \end{cases}$$

Pattern 9 Inter-influence from Decision to Structure

This pattern of influence could be classified as two modes, considering the structure of the influenced component, as “influence who” or “influenced by who”, as shown in figure B.3.

Influence Who of Influenced Component

Given $\{i, j, g, h\} \subseteq \mathbb{N}$ (i is the influencing agent in the whole structure of influence, j is the influencing agent in the influenced component, and g and h are two different influenced agents in the influenced component, as shown in sub figure 9.1 of figure B.3), $k \in \mathbb{M}$. The general formula would be:

$$\begin{aligned} P_{(k)}(g) & \begin{cases} = In_{(\alpha)}(P_{(k)}(j), P_{(k)}(g)) & \text{if } C_{(k)}(i) = c_k \\ = P_{(k)}(g) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \\ P_{(k)}(h) & \begin{cases} = In_{(\beta)}(P_{(k)}(j), P_{(k)}(h)) & \text{if } C_{(k)}(i) = \bar{c}_k \\ = P_{(k)}(h) & \text{if } C_{(k)}(i) = c_k \end{cases} \end{aligned} \quad (\text{B.25})$$

Influenced by Who of Influenced Component

Given $\{i, g, h, j\} \subseteq \mathbb{N}$ (i is the influencing agent in the whole structure of influence, g and h are the two different influencing agents in the influenced component, and j is the influenced agent in the influenced component, as shown in sub figure 9.2 of figure B.3), $k \in \mathbb{M}$. The formula would be:

$$P_{(k)}(j) \begin{cases} = In_{(\alpha)}(P_{(k)}(g), P_{(k)}(j)) & \text{if } C_{(k)}(i) = c_k \\ = In_{(\beta)}(P_{(k)}(h), P_{(k)}(j)) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \quad (\text{B.26})$$

Pattern 10 Inter-influence from Structure to Decision

This pattern of influence could be classified as two modes, based on the structure of the influencing component, as “*influence who*” and “*influenced by who*”.

Influence Who of Influencing Component

Given $\{i, g, h, j\} \subseteq \mathbb{N}$, $k \in \mathbb{M}$, and define $\varphi[\]$ as the function of influential effect of structure among agents (how it works will be discussed in details in chapter 7), use $\varphi[P_1, P_2]$ to express the influential effect from the structure of influence from P_1 to P_2 , the formula for this mode would be:

$$P_{(k)}(j) \begin{cases} = In_{(\alpha)}\{P_{(k)}(i), P_{(k)}(g), \varphi[P_{(k)}(i), P_{(k)}(g)], P_{(k)}(j)\}, \\ \quad \text{if } [P_{(k)}(g) = In_{(\gamma)}(P_{(k)}(i))] \cap P_{(k)}(h) \neq In_{(\delta)}(P_{(k)}(i)) \\ = In_{(\beta)}\{P_{(k)}(i), P_{(k)}(h), \varphi[P_{(k)}(i), P_{(k)}(h)], P_{(k)}(j)\}, \\ \quad \text{if } [P_{(k)}(h) = In_{(\delta)}(P_{(k)}(i))] \cap [P_{(k)}(g) \neq In_{(\gamma)}(P_{(k)}(i))] \\ = In_{(\sigma)}\{P_{(k)}(i), P_{(k)}(g), P_{(k)}(h), \\ \quad \varphi[P_{(k)}(i), P_{(k)}(g)], \varphi[P_{(k)}(i), P_{(k)}(h)], P_{(k)}(j)\} \\ \quad \text{if } [P_{(k)}(g) = In_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k)}(h) = In_{(\delta)}(P_{(k)}(i))] \\ = P_{(k)}(j), \\ \quad \text{if } [P_{(k)}(g) \neq In_{(\gamma)}(P_{(k)}(i))] \cap P_{(k)}(h) \neq In_{(\delta)}(P_{(k)}(i)) \end{cases} \quad (\text{B.27})$$

Influenced by Who of Influencing Component

Given $\{g, h, i, j\} \subseteq \mathbb{N}$, $k \in \mathbb{M}$, the formula would be:

$$P_{(k)}(j) \begin{cases} = In_{(\alpha)}\{P_{(k)}(g), P_{(k)}(i), \varphi[P_{(k)}(g), P_{(k)}(i)], P_{(k)}(j)\}, \\ \text{if } [P_{(k)}(i) = In_{(\gamma)}(P_{(k)}(g))] \cap [P_{(k)}(i) \neq In_{(\delta)}(P_{(k)}(h))] \\ = In_{(\beta)}\{P_{(k)}(h), P_{(k)}(i), \varphi[P_{(k)}(h), P_{(k)}(i)], P_{(k)}(j)\}, \\ \text{if } [P_{(k)}(i) = In_{(\delta)}(P_{(k)}(h))] \cap [P_{(k)}(i) \neq In_{(\gamma)}(P_{(k)}(g))] \\ = In_{(\sigma)}\{P_{(k)}(g), P_{(k)}(h), P_{(k)}(i), \\ \varphi[P_{(k)}(g), P_{(k)}(i)], \varphi[P_{(k)}(h), P_{(k)}(i)], P_{(k)}(j)\} \\ \text{if } [P_{(k)}(i) = In_{(\gamma)}(P_{(k)}(g))] \cap [P_{(k)}(i) = In_{(\delta)}(P_{(k)}(h))] \\ = P_{(k)}(j), \\ \text{if } [P_{(k)}(i) \neq In_{(\gamma)}(P_{(k)}(g))] \cap [P_{(k)}(i) \neq In_{(\delta)}(P_{(k)}(h))] \end{cases} \quad (B.28)$$

Pattern 11 Inter-influence from Structure to Decision and Structure

This pattern of influence could be further classified into two submodes, based on the structure of the influencing component, as “*influence who*” or “*influenced by who*”.

Influence Who

Given $\{i, h, r, j, q\} \in \mathbb{N}$, $k \in \mathbb{M}$, the formula would be:

$$P_{(k)}(j) \begin{cases} = In_{(\alpha)}\{P_{(k)}(i), P_{(k)}(h), \varphi[P_{(k)}(i), P_{(k)}(h)], P_{(k)}(j)\}, \\ \text{if } P_{(k)}(h) = In_{(\gamma)}(P_{(k)}(i)) \\ = P_{(k)}(j), \\ \text{if } P_{(k)}(h) \neq In_{(\gamma)}(P_{(k)}(i)) \end{cases} \quad (B.29)$$

$$P_{(k)}(g) \begin{cases} = In_{(\beta)}\{P_{(k)}(i), P_{(k)}(r), \phi[P_{(k)}(i), P_{(k)}(r)], P_{(k)}(g)\}, \\ \text{if } P_{(k)}(r) = In_{(\delta)}(P_{(k)}(i)) \\ = P_{(k)}(g), \\ \text{if } P_{(k)}(r) \neq In_{(\delta)}(P_{(k)}(i)) \end{cases}$$

Influenced by Who

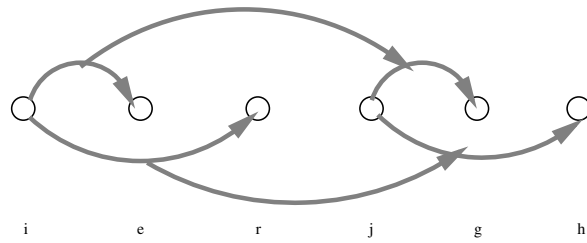
Given $\{h, r, i, j, q\} \in \mathbb{N}$, $k \in \mathbb{M}$, the formula would be:

$$P_{(k)}(j) \begin{cases} = In_{(\alpha)}\{P_{(k)}(h), P_{(k)}(i), \varphi[P_{(k)}(h), P_{(k)}(i)], P_{(k)}(j)\}, \\ \text{if } P_{(k)}(i) = In_{(\gamma)}(P_{(k)}(h)) \\ = P_{(k)}(j), \\ \text{if } P_{(k)}(i) \neq In_{(\gamma)}(P_{(k)}(h)) \end{cases} \quad (B.30)$$

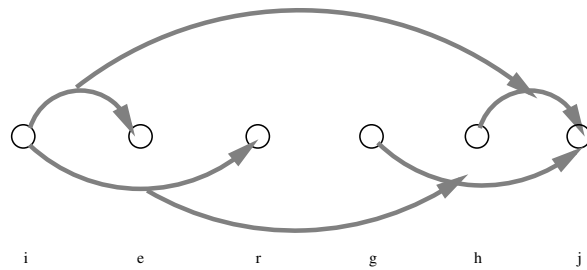
$$P_{(k)}(g) \begin{cases} = In_{(\beta)}\{P_{(k)}(r), P_{(k)}(i), \phi[P_{(k)}(r), P_{(k)}(i)], P_{(k)}(g)\}, \\ \text{if } P_{(k)}(i) = In_{(\delta)}(P_{(k)}(r)) \\ = P_{(k)}(g), \\ \text{if } P_{(k)}(i) \neq In_{(\delta)}(P_{(k)}(r)) \end{cases}$$

Pattern 12 Inter-influence from Structure to Structure

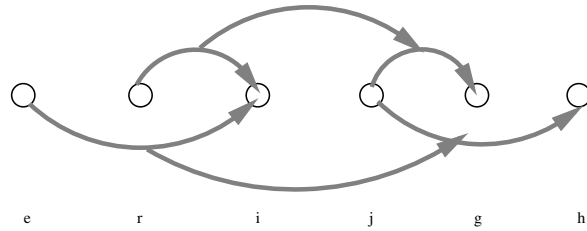
Considering the variations on the structure of influencing component and influenced component (both could be the “*influence who*” or “*influenced by who*”), then the combination would generate four different modes for this pattern of influence, as shown in figure B.4.



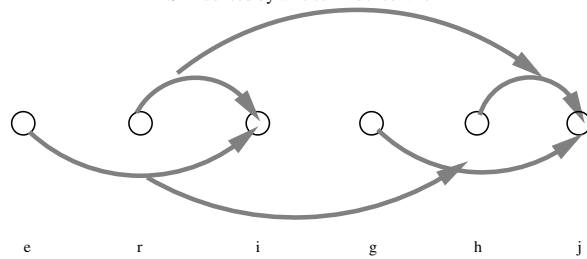
12.1influence who to influence who



12.2influence who to influenced by who



12.3influenced by who to influence who



12.4influenced by who to influenced by who

Figure B.4: Four Modes of 12.Inter-influence of Structure to Structure

Influence Who to Influence Who

This mode is the example discussed in details above, given $\{i, e, r, j, g, h\} \in \mathbb{N}$, $k \in \mathbb{M}$, in which i is the only influencing agent in the influencing component, and j is the only influencing agent in the influenced component, then the formula would be:

$$\begin{aligned}
 P_{(k)}(g) & \begin{cases} = In_{(\alpha)}(P_{(k)}(j), P_{(k)}(g)) & \text{if } P_{(k)}(e) = In_{(\gamma)}(P_{(k)}(i)) \\ = P_{(k)}(g) & \text{if } P_{(k)}(e) \neq In_{(\gamma)}(P_{(k)}(i)) \end{cases} \\
 P_{(k)}(h) & \begin{cases} = In_{(\beta)}(P_{(k)}(j), P_{(k)}(h)) & \text{if } P_{(k)}(r) = In_{(\delta)}(P_{(k)}(i)) \\ = P_{(k)}(h) & \text{if } P_{(k)}(r) \neq In_{(\delta)}(P_{(k)}(i)) \end{cases}
 \end{aligned} \tag{B.31}$$

Influence Who to Influenced by Who

Given $\{i, e, r, g, h, j\} \in \mathbb{N}$, $k \in \mathbb{M}$, in which i is the sole influencing agent in the influencing component, and j is the sole influenced agent in the influenced component, then the formula for this mode would be:

$$P_{(k)}(j) \begin{cases} = In_{(\alpha)}(P_{(k)}(g), P_{(k)}(j)) \\ \text{if } [P_{(k)}(e) = In_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k)}(r) \neq In_{(\delta)}(P_{(k)}(i))] \\ = In_{(\beta)}(P_{(k)}(h), P_{(k)}(j)) \\ \text{if } [P_{(k)}(r) = In_{(\delta)}(P_{(k)}(i))] \cap [P_{(k)}(e) \neq In_{(\gamma)}(P_{(k)}(i))] \\ = In_{(\sigma)}(P_{(k)}(g), P_{(k)}(h), P_{(k)}(j)) \\ \text{if } [P_{(k)}(e) = In_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k)}(r) = In_{(\delta)}(P_{(k)}(i))] \\ = P_{(k)}(j) \\ \text{if } [P_{(k)}(e) \neq In_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k)}(r) \neq In_{(\delta)}(P_{(k)}(i))] \end{cases} \tag{B.32}$$

Influenced by Who to Influence Who

Given $\{e, r, i, j, g, h\} \in \mathbb{N}$, $k \in \mathbb{M}$, in which i is the sole influenced agent in the influencing component, and j is the sole influencing agent in the influenced component, then the formula would be:

$$\begin{aligned}
 P_{(k)}(g) & \begin{cases} = In_{(\alpha)}(P_{(k)}(j), P_{(k)}(g)) & \text{if } P_{(k)}(i) = In_{(\gamma)}(P_{(k)}(e)) \\ = P_{(k)}(g) & \text{if } P_{(k)}(i) \neq In_{(\gamma)}(P_{(k)}(e)) \end{cases} \\
 P_{(k)}(h) & \begin{cases} = In_{(\beta)}(P_{(k)}(j), P_{(k)}(h)) & \text{if } P_{(k)}(i) = In_{(\delta)}(P_{(k)}(r)) \\ = P_{(k)}(h) & \text{if } P_{(k)}(i) \neq In_{(\delta)}(P_{(k)}(r)) \end{cases}
 \end{aligned} \tag{B.33}$$

Influenced by Who to Influenced by Who

Given $\{e, r, i, g, h, j\} \in \mathbb{N}$, $k \in \mathbb{M}$, in which i is the sole influenced agent in the influencing component, and j is the sole influenced agent in the influenced component, then the formula would be:

$$P_{(k)}(j) \begin{cases} = In_{(\alpha)}(P_{(k)}(g), P_{(k)}(j)) \\ \text{if } [P_{(k)}(i) = In_{(\gamma)}(P_{(k)}(e))] \cap [P_{(k)}(i) \neq In_{(\delta)}(P_{(k)}(r))] \\ = In_{(\beta)}(P_{(k)}(h), P_{(k)}(j)) \\ \text{if } [P_{(k)}(i) = In_{(\delta)}(P_{(k)}(r))] \cap [P_{(k)}(i) \neq In_{(\gamma)}(P_{(k)}(e))] \\ = In_{(\alpha)}(P_{(k)}(g), P_{(k)}(h), P_{(k)}(j)) \\ \text{if } [P_{(k)}(i) = In_{(\gamma)}(P_{(k)}(e))] \cap [P_{(k)}(i) = In_{(\delta)}(P_{(k)}(r))] \\ = P_{(k)}(j) \\ \text{if } [P_{(k)}(i) \neq In_{(\gamma)}(P_{(k)}(e))] \cap [P_{(k)}(i) \neq In_{(\delta)}(P_{(k)}(r))] \end{cases} \quad (\text{B.34})$$

Pattern 13 Intra-inter Influence from Decision

Given $\{k, l\} \subseteq \mathbb{M}$, $k + l < m$, $\{i, j\} \subseteq \mathbb{N}$, $l \geq |j - i|$, which means the influence runs longer distance in the vertical dimension than in the horizontal dimension. To represent this pattern of influence in the general form, a function $Pa_{\perp}In$ should be defined, which integrate the function Pa and the function In . As the intra-inter influence firstly starts from or flows at the vertical dimension and then points to or affects on the horizontal dimension (by different sequences), or the intra-inter influence runs longer distance in the vertical direction than in the horizontal dimension (by different distances), therefore in the new defined function, Pa is put before In , and the symbol “ \perp ” is very vivid to describe this influence, the influence firstly flows at the vertical level (represented by the vertical line), and then affects at the horizontal level (represented by the transverse line).

$$P_{(k+l)}(j) = Pa_{\perp}In(P_{(k)}(i), P_{(k+l)}(j)) \quad (\text{B.35})$$

If the influence works in different ways due to the different choices, the formula would be:

$$P_{(k+l)}(j) \begin{cases} = Pa_{\perp}In_{\alpha}(P_{(k)}(i), P_{(k+l)}(j)) & \text{if } C_{(k)}(i) = c_k \\ = Pa_{\perp}In_{\beta}(P_{(k)}(i), P_{(k+l)}(j)) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \quad (\text{B.36})$$

Pattern 14 Intra-inter Influence from Decision and Structure

Same Issue

For the standard form of intra-inter influence from decision to decision and structure, the influence flows at the vertical dimension, and then affects within the horizontal dimension, namely influencing to different agents but within the same issue, just as the vivid symbol “ \perp ” expressed. Given $\{k, k + l\} \subseteq \mathbb{M}$, $k + l < m$, $\{i, j, g\} \subseteq \mathbb{N}$, $\mathbb{A}_1 = \{i, j\}$, $\mathbb{A}_2 = \{i, g\}$, $\mathbb{A}_1 \neq \mathbb{A}_2$, \mathbb{A}_1 and \mathbb{A}_2 are the sets for two different influenced agents, which could be same with the influencing agent i (but on different issues).

$$P_{(k+l)}(\mathbb{A}_1) \begin{cases} = Pa_{\perp}In_{(\alpha)}(P_{(k)}(i), P_{(k+l)}(\mathbb{A}_1)) & \text{if } C_{(k)}(i) = c_k \\ = P_{(k+l)}(\mathbb{A}_1) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \quad (\text{B.37})$$

$$P_{(k+l)}(\mathbb{A}_2) \begin{cases} = Pa_{\perp}In_{(\beta)}(P_{(k)}(i), P_{(k+l)}(\mathbb{A}_2)) & \text{if } C_{(k)}(i) = \bar{c}_k \\ = P_{(k+l)}(\mathbb{A}_2) & \text{if } C_{(k)}(i) = c_k \end{cases}$$

Different Issues

However, for other cases, the influence would affect different agents not within one same issue, but scattering on different issues, then the point would be: how to distinguish the intra-inter influence and intra-inter influence (as the influenced variables are not within one same issue, on the horizontal dimension)? If the structure of the influence could not be the criteria to classify the pattern of influence, then the distance of influence could provide a substitute. One feasible way is to compare the distance of influence flowing on the vertical direction and the horizontal direction, for the intra-inter influence, the figure of the influence should be wider than longer, namely the influence mainly flowing at the vertical dimension. Given $\{k, k + (l), k + (l) + s\}^3$, $k + (l) + s < m$, $\{k, k + (l), k + (l) + s\} \subseteq \mathbb{M}$, $\{i, j, g\} \subseteq \mathbb{N}$, $\mathbb{A}_1 = \{i, j\}$, $\mathbb{A}_2 = \{i, g\}$, $\mathbb{A}_1 \neq \mathbb{A}_2$, $(l) + s \geq |g - j|$ (if $\mathbb{A}_1 = j, \mathbb{A}_2 = g$), $(l) + s \geq |j - i|$ (if $\mathbb{A}_1 = i, \mathbb{A}_2 = j$), $(l) + s \geq |g - i|^4$ (if $\mathbb{A}_1 = i, \mathbb{A}_2 = g$), and if $k + (l) = k$, then $\mathbb{A}_1 \neq i$.⁵

$$\begin{aligned}
 P_{(k+(l))}(\mathbb{A}_1) & \begin{cases} = Pa_{\perp} In_{(\alpha)}(P_{(k)}(i), P_{(k+(l))}(\mathbb{A}_1)) & \text{if } C_{(k)}(i) = c_k \\ = P_{(k+(l))}(\mathbb{A}_1) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \\
 P_{(k+(l)+s)}(\mathbb{A}_2) & \begin{cases} = Pa_{\perp} In_{(\beta)}(P_{(k)}(i), P_{(k+(l)+s)}(\mathbb{A}_2)) & \text{if } C_{(k)}(i) = \bar{c}_k \\ = P_{(k+(l)+s)}(\mathbb{A}_2) & \text{if } C_{(k)}(i) = c_k \end{cases}
 \end{aligned} \tag{B.38}$$

Pattern 15 Intra-inter Influence from Structure

Considered the structure of this pattern of influence, as the influencing component is just one variable (without the variation of structure), but the influenced component is the structures of influence (which has three basic nodes varying as “*influence who*” and “*influenced by who*”), therefore, this pattern of influence could be further classified as two modes:

Influence Who of Influenced Component

In this mode, the structure of influenced component is “one agent influences to two different agents”, and could be further classified into two submodes: influenced component (1) within one same issue or (2) distributed on different issues. If on the same issue, all influenced agents in the influenced component belongs within one same issue (as shown in sub figure 15.1 of figure B.5); and if on different issues, influenced agents in the influenced component are scattered among different issues, for example, half of the influenced structure belongs to one issue, and the other half of the influenced structure belongs to the other issue (as shown in sub figure 15.2 of figure B.5).

Same Issue Given $\{k, k + l\} \subseteq \mathbb{M}$, $k + l < m$, $\{i, j, g, h\} \subseteq \mathbb{N}$, $\mathbb{A}_1 = \{i, j\}$, $\mathbb{A}_2 = \{i, g\}$, $\mathbb{A}_3 = \{i, h\}$, $\mathbb{A}_1 \neq \mathbb{A}_2 \neq \mathbb{A}_3$, \mathbb{A}_1 is the influencing agent in the influenced component, \mathbb{A}_2 and \mathbb{A}_3 are two different influenced agents in the influenced component.

³The $k+(l)$ could be equaled to k , which means for one of the influenced agent, could belongs to the same issue with the influencing agent.

⁴Which means the vertical distance should be longer than the horizontal distance.

⁵Which means that if one influenced agent belongs to the same issue with the influencing agent, then should not also belongs to the same agent, to avoid the overlap.

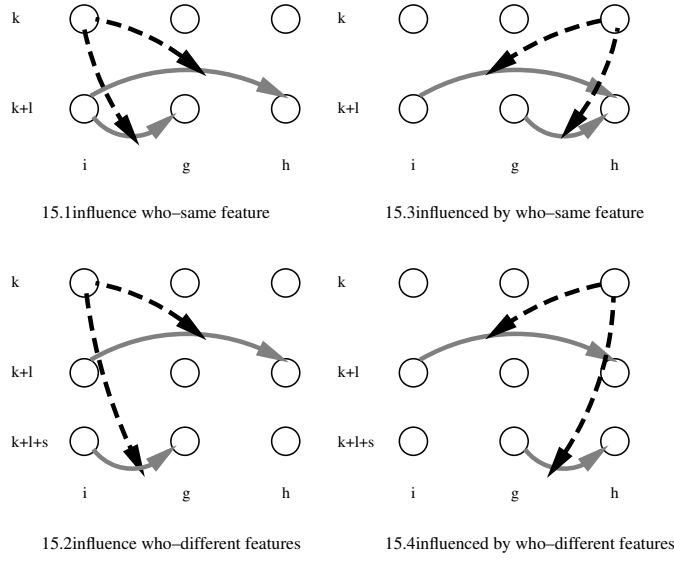


Figure B.5: Four Submodes of 15. Intra-inter Influence from Decision to Structure

$$\begin{aligned}
 P_{(k+l)}(\mathbb{A}_2) & \begin{cases} = Pa_{\perp} In_{(\alpha)}(P_{(k+l)}(\mathbb{A}_1), P_{(k+l)}(\mathbb{A}_2)) & \text{if } C_{(k)}(i) = c_k \\ = P_{(k+l)}(\mathbb{A}_2) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \\
 P_{(k+l)}(\mathbb{A}_3) & \begin{cases} = Pa_{\perp} In_{(\beta)}(P_{(k+l)}(\mathbb{A}_1), P_{(k+l)}(\mathbb{A}_3)) & \text{if } C_{(k)}(i) = \bar{c}_k \\ = P_{(k+l)}(\mathbb{A}_3) & \text{if } C_{(k)}(i) = c_k \end{cases}
 \end{aligned} \tag{B.39}$$

Different Issues Given $\{k, k + (l), k + (l) + s\} \subseteq \mathbb{M}^6$, $k + (l) + s < m$, $\{i, j, g, h\} \subseteq \mathbb{N}$, $\mathbb{A}_1 = \{i, j\}$, $\mathbb{A}_2 = \{i, g\}$, $\mathbb{A}_3 = \{i, h\}$, $\mathbb{A}_1 \neq \mathbb{A}_2 \neq \mathbb{A}_3$, if $k + (l) = k$, then $\mathbb{A}_1 = j$, $\mathbb{A}_2 = g$.⁷

$$\begin{aligned}
 P_{(k+(l))}(\mathbb{A}_2) & \begin{cases} = Pa_{\perp} In_{(\alpha)}(P_{(k+(l))}(\mathbb{A}_1), P_{(k+(l))}(\mathbb{A}_2)) & \text{if } C_{(k)}(i) = c_k \\ = P_{(k+(l))}(\mathbb{A}_2) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \\
 P_{(k+(l)+s)}(\mathbb{A}_3) & \begin{cases} = Pa_{\perp} In_{(\beta)}(P_{(k+(l)+s)}(\mathbb{A}_1), P_{(k+(l)+s)}(\mathbb{A}_3)) & \text{if } C_{(k)}(i) = \bar{c}_k \\ = P_{(k+(l)+s)}(\mathbb{A}_3) & \text{if } C_{(k)}(i) = c_k \end{cases}
 \end{aligned} \tag{B.40}$$

Influenced by Who of Influenced Component

In this mode, the structure of influenced component is “one agent be influenced by two different agents”, and could also be further classified into two submodes: influenced component (1) within one same issue (as shown in the figure 15.3 of figure B.5) or (2) distributed on different issues (as shown in sub figure 15.4 of figure B.5).

Same Issue Given $\{k, k + l\} \subseteq \mathbb{M}$, $k + l < m$, $\{i, g, h, j\} \subseteq \mathbb{N}$, $\mathbb{A}_1 = \{i, j\}$, $\mathbb{A}_2 = \{i, g\}$, $\mathbb{A}_3 = \{i, h\}$, $\mathbb{A}_1 \neq \mathbb{A}_2 \neq \mathbb{A}_3$, \mathbb{A}_1 is the one influenced agent in the influenced component, and \mathbb{A}_2 and \mathbb{A}_3 are two different influencing agents in the influenced component.

⁶The influencing variable lies at issue k , the half of the influenced structure lies at issue $k + (l)$, and the other half of the influenced structure lies at issue $k + (l) + s$, and (l) could equal to 0, then it is possible that $k = k + (l)$, which means half of the influenced structure lies at the same issue with the influencing variable.

⁷Which means that if half of the influenced structure lies at the same issue with the influencing variable, but could not be overlapped with the influencing variable, at least on different agents $\neq i$.

$$P_{(k+l)}(\mathbb{A}_1) \begin{cases} = Pa_{\perp} In_{(\alpha)}(P_{(k+l)}(\mathbb{A}_2), P_{(k+l)}(\mathbb{A}_1)) & \text{if } C_{(k)}(i) = c_k \\ = Pa_{\perp} In_{(\beta)}(P_{(k+l)}(\mathbb{A}_3), P_{(k+l)}(\mathbb{A}_1)) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \quad (\text{B.41})$$

Different Issues Given $\{k, k + (l), k + (l) + s\} \subseteq \mathbb{M}$, $k + (l) + s < m$, $\{i, g, h, j\} \subseteq \mathbb{N}$, $\mathbb{A}_1 = \{i, j\}$, $\mathbb{A}_2 = \{i, g\}$, $\mathbb{A}_3 = \{i, h\}$, $\mathbb{A}_1 \neq \mathbb{A}_2 \neq \mathbb{A}_3$, if $k + (l) = k$, then $\mathbb{A}_1 = j$, $\mathbb{A}_2 = g$. Though the influenced structure is one agent influenced by two different agents, but the decisions of one influenced agent are scattered among two different issues, then the decisions influenced are related to two variables, needed to be represented by two respective formulas, different from the one same variable influenced as before.

$$P_{(k+(l))}(\mathbb{A}_1) \begin{cases} = Pa_{\perp} In_{(\alpha)}(P_{(k+(l))}(\mathbb{A}_2), P_{(k+(l))}(\mathbb{A}_1)) & \text{if } C_{(k)}(i) = c_k \\ = P_{(k+(l))}(\mathbb{A}_1) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \\ P_{(k+(l)+s)}(\mathbb{A}_1) \begin{cases} = Pa_{\perp} In_{(\beta)}(P_{(k+(l)+s)}(\mathbb{A}_3), P_{(k+(l)+s)}(\mathbb{A}_1)) & \text{if } C_{(k)}(i) = \bar{c}_k \\ = P_{(k+(l)+s)}(\mathbb{A}_1) & \text{if } C_{(k)}(i) = c_k \end{cases} \quad (\text{B.42})$$

Pattern 16 Intra-inter Influence from Structure to Decision

Based on the structure of influencing component, this pattern of influence could be classified as two modes, “influence which/dependent by which” and “influenced by which/dependent on which”.

Influence Which/Dependent by Which of Influencing Component

Given $\{k, k + l, k + l + s, k + l + s + (t)\} \subseteq \mathbb{M}^8$, $k + l + s + (t) < m$, $\{i, j\} \subseteq \mathbb{N}$.

$$P_{(k+l+s+(t))}(j) \begin{cases} = Pa_{\perp} In_{(\alpha)}\{P_{(k)}(i), P_{(k+l)}(i), \phi[P_{(k)}(i), P_{(k+l)}(i)], P_{(k+l+s+(t))}(j)\}, \\ \text{if } [P_{(k+l)}(i) = Pa_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k+l+s)}(i) \neq Pa_{(\delta)}(P_{(k)}(i))] \\ = Pa_{\perp} In_{(\beta)}\{P_{(k)}(i), P_{(k+l+s)}(i), \phi[P_{(k)}(i), P_{(k+l+s)}(i)], P_{(k+l+s+(t))}(j)\}, \\ \text{if } [P_{(k+l+s)}(i) = Pa_{(\delta)}(P_{(k)}(i))] \cap [P_{(k+l)}(i) \neq Pa_{(\gamma)}(P_{(k)}(i))] \\ = Pa_{\perp} In_{(\sigma)}\{P_{(k)}(i), P_{(k+l)}(i), P_{(k+l+s)}(i), \\ \phi[P_{(k)}(i), P_{(k+l)}(i)], \phi[P_{(k)}(i), P_{(k+l+s)}(i)], P_{(k+l+s+(t))}(j)\} \\ \text{if } [P_{(k+l)}(i) = Pa_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k+l+s)}(i) = Pa_{(\delta)}(P_{(k)}(i))] \\ = P_{(k+l+s+(t))}(j), \\ \text{if } [P_{(k+l)}(i) \neq Pa_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k+l+s)}(i) \neq Pa_{(\delta)}(P_{(k)}(i))] \end{cases} \quad (\text{B.43})$$

Influenced by Which/Dependent on Which of Influencing Component

Given $\{l, l + s, l + s + k, l + s + k + (t)\} \subseteq \mathbb{M}$, $l + s + k + (t) < m$, $\{i, j\} \subseteq \mathbb{N}$.

⁸ $k + l + s + (t)$ is the issue No. of influenced variable, $(t) \geq 0$, which means the influenced variable could be at the same (last) issue of the influencing structure, as $k + l + s$ while $(t) = 0$, and could also be at the latter feature, as $k + l + s + (t)$ while $(t) \neq 0$.

$$P_{(l+s+k+(t))}(j) \left\{ \begin{array}{l} = Pa_{\perp} In_{(\alpha)} \{P_{(l)}(i), P_{(l+s+k)}(i), \phi[P_{(l)}(i), P_{(l+s+k)}(i)], P_{(l+s+k+(t))}(j)\}, \\ \text{if } [P_{(l+s+k)}(i) = Pa_{(\gamma)}(P_{(l)}(i))] \cap [P_{(l+s+k)}(i) \neq Pa_{(\delta)}(P_{(l+s)}(i))] \\ = Pa_{\perp} In_{(\beta)} \{P_{(l+s)}(i), P_{(l+s+k)}(i), \phi[P_{(l+s)}(i), P_{(l+s+k)}(i)], P_{(l+s+k+(t))}(j)\}, \\ \text{if } [P_{(l+s+k)}(i) = Pa_{(\delta)}(P_{(l+s)}(i))] \cap [P_{(l+s+k)}(i) \neq Pa_{(\gamma)}(P_{(l)}(i))] \\ = Pa_{\perp} In_{(\sigma)} \{P_{(l)}(i), P_{(l+s)}(i), P_{(l+s+k)}(i), \\ \phi[P_{(l)}(i), P_{(l+s+k)}(i)], \phi[P_{(l+s)}(i), P_{(l+s+k)}(i)], P_{(k+l+s+(t))}(j)\} \\ \text{if } [P_{(l+s+k)}(i) = Pa_{(\gamma)}(P_{(l)}(i))] \cap [P_{(l+s+k)}(i) = Pa_{(\delta)}(P_{(l+s)}(i))] \\ = P_{(k+l+s+(t))}(j), \\ \text{if } [P_{(l+s+k)}(i) \neq Pa_{(\gamma)}(P_{(l)}(i))] \cap [P_{(l+s+k)}(i) \neq Pa_{(\delta)}(P_{(l+s)}(i))] \end{array} \right. \quad (B.44)$$

Pattern 17 Intra-inter Influence from Structure to Decision and Structure

Influence Which/Dependent by Which of Influencing Component

In this mode, the structure of influencing component is “one issue conditional by two different issues”, and for the influenced component, which are two different variables, which should at least be different on one dimension, either distributed on different agents, or on different issues, otherwise it would be the same with influence pattern 16, influence leading to a single variable.

Given $k + l + s < m$, $k + l + (t) < m$, $k + l + s + (d) < m$, $\{k, k + l, k + l + s, k + l + (t), k + l + s + (d)\} \subseteq \mathbb{M}$, $\{i, j, g\} \subseteq \mathbb{N}$, $\mathbb{A}_1 = \{i, j\}$, $\mathbb{A}_2 = \{i, g\}$, if $k + l + (t) = k + l$, then $\mathbb{A}_1 = j$, if $k + l + s + (d) = k + l + s$ then $\mathbb{A}_2 = g$, if $\mathbb{A}_1 = i$ then $\mathbb{A}_2 = g$, if $\mathbb{A}_2 = i$ then $\mathbb{A}_1 = j$, if $k + l + (t) = k + l + s + (d)$ then $\mathbb{A}_1 \neq \mathbb{A}_2$, if $\mathbb{A}_1 = \mathbb{A}_2$ then $k + l + (t) \neq k + l + s + (d)$.⁹

$$P_{(k+l+(t))}(\mathbb{A}_1) \left\{ \begin{array}{l} = Pa_{\perp} In_{(\alpha)} \{P_{(k)}(i), P_{(k+l)}(i), \phi[P_{(k)}(i), P_{(k+l)}(i)], P_{(k+l+(t))}(\mathbb{A}_1)\}, \\ \text{if } P_{(k+l)}(i) = Pa_{(\gamma)}(P_{(k)}(i)) \\ = P_{(k+l+(t))}(\mathbb{A}_1), \\ \text{if } P_{(k+l)}(i) \neq Pa_{(\gamma)}(P_{(k)}(i)) \end{array} \right. \quad (B.45)$$

$$P_{(k+l+s+(d))}(\mathbb{A}_2) \left\{ \begin{array}{l} = Pa_{\perp} In_{(\beta)} \{P_{(k)}(i), P_{(k+l+s)}(i), \phi[P_{(k)}(i), P_{(k+l+s)}(i)], P_{(k+l+s+(d))}(\mathbb{A}_2)\}, \\ \text{if } P_{(k+l+s)}(i) = Pa_{(\delta)}(P_{(k)}(i)) \\ = P_{(k+l+s+(d))}(\mathbb{A}_2), \\ \text{if } P_{(k+l+s)}(i) \neq Pa_{(\delta)}(P_{(k)}(i)) \end{array} \right.$$

Influenced by Which/Dependent on Which of Influencing Component

In this mode, the structure of influencing component is “one issue conditional on two different issues”.

Given $l + s + k < m$, $l + s + k + (t) < m$, $l + s + k + (d) < m$, $\{l, l + s, l + s + k, l + s + k + (t), l + s + k + (d)\} \subseteq \mathbb{M}$, $\{i, j, g\} \subseteq \mathbb{N}$, $\mathbb{A}_1 = \{i, j\}$, $\mathbb{A}_2 = \{i, g\}$, if $k + l + s + (t) = k + l + s$, then $\mathbb{A}_1 = j$, if $k + l + s + (d) = k + l + s$, then $\mathbb{A}_2 = g$, if $\mathbb{A}_1 = i$ then $\mathbb{A}_2 = g$, if $\mathbb{A}_2 = i$ then $\mathbb{A}_1 = j$, if $l + s + k + (t) = l + s + k + (d)$ then $\mathbb{A}_1 \neq \mathbb{A}_2$, if $\mathbb{A}_1 = \mathbb{A}_2$ then $l + s + k + (t) \neq l + s + k + (d)$.

⁹Which means the two influenced variables could lie at one same agent or one same issue, but can not be identical on both agent dimension and issue dimension, otherwise would become one single issue.

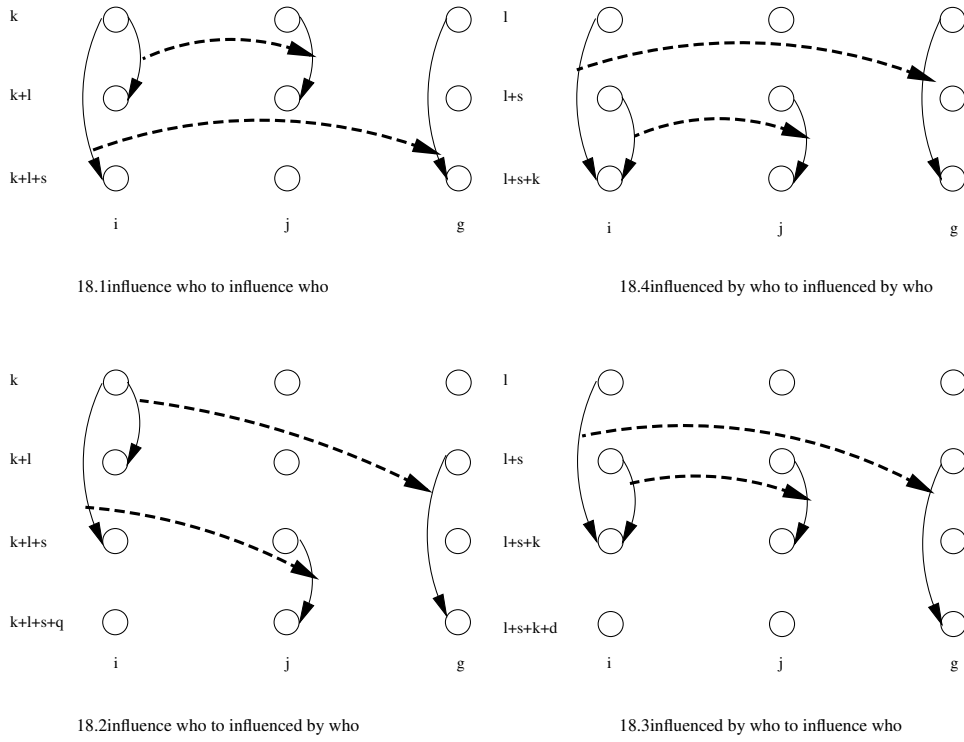


Figure B.6: Four Modes of 18. Intra-inter Influence from Structure to Structure

$$\begin{aligned}
 P_{(l+s+k+(t))}(\mathbb{A}_1) & \begin{cases} = Pa_{\perp} In_{(\alpha)} \{P_{(l)}(i), P_{(l+s+k)}(i), \phi[P_{(l)}(i), P_{(l+s+k)}(i)], P_{(l+s+k+(t))}(\mathbb{A}_1)\}, \\ \text{if } P_{(l+s+k)}(i) = Pa_{(\gamma)}(P_{(l)}(i)) \\ = P_{(l+s+k+(t))}(\mathbb{A}_1), \\ \text{if } P_{(l+s+k)}(i) \neq Pa_{(\gamma)}(P_{(l)}(i)) \end{cases} \\
 P_{(l+s+k+(d))}(\mathbb{A}_2) & \begin{cases} = Pa_{\perp} In_{(\beta)} \{P_{(l+s)}(i), P_{(l+s+k)}(i), \phi[P_{(l+s)}(i), P_{(l+s+k)}(i)], P_{(l+s+k+(d))}(\mathbb{A}_1)\}, \\ \text{if } P_{(l+s+k)}(i) = Pa_{(\delta)}(P_{(l+s)}(i)) \\ = P_{(l+s+k+(d))}(\mathbb{A}_2), \\ \text{if } P_{(l+s+k)}(i) \neq Pa_{(\delta)}(P_{(l+s)}(i)) \end{cases}
 \end{aligned} \tag{B.46}$$

Pattern 18 Inter-intra Influence from Decision to Decision

Further, there are four specific modes for this pattern of influence, separately considering the structures of influencing component and the influenced component, both can be represented in two forms of “one influence which/one dependent by which” or “one be influenced by which/one dependent on which”, therefore, four modes of different combinations could be built, as shown in figure B.6.

Influence Which to Influence Which/Dependent by Which to Dependent by Which

As shown in sub figure 18.1 of figure B.6, given $k+l+s < m$, $k+(q) < m$, $k+l+(t) < m$, $k+l+s+(d) < m$, $k+(q) < k+l+(t)$, $k+(q) < k+l+s+(d)$, $k+l+(t) \neq k+l+s+(d)$,

$\{k, k+l, k+l+s, k+(q), k+l+(t), k+l+s+(d)\} \subseteq \mathbb{M}$, $\{i, j, g\} \subseteq \mathbb{N}$, $\mathbb{A}_1 = \{i, j, g\}$, $\mathbb{A}_2 = \{i, j, g\}$. If $\mathbb{A}_1 = i$ then $\mathbb{A}_2 \neq i$, $k+(q) > k$, $k+l+(t) > k+l$, if $\mathbb{A}_2 = i$ then $\mathbb{A}_1 \neq i$, $k+(q) > k$, $k+l+s+(d) > k+l+s$.

$$\begin{aligned}
P_{(k+l+(t))}(\mathbb{A}_1) & \begin{cases} = Pa_{\perp} In_{(\alpha)}(P_{(k+(q))}(\mathbb{A}_1), P_{(k+l+(t))}(\mathbb{A}_1)) & \text{if } P_{(k+l)}(i) = Pa_{(\gamma)}(P_{(k)}(i)) \\ = P_{(k+l+(t))}(\mathbb{A}_1) & \text{if } P_{(k+l)}(i) \neq Pa_{(\gamma)}(P_{(k)}(i)) \end{cases} \\
P_{(k+l+s+(d))}(\mathbb{A}_2) & \begin{cases} = Pa_{\perp} In_{(\beta)}(P_{(k+(q))}(\mathbb{A}_2), P_{(k+l+s+(d))}(\mathbb{A}_2)) & \text{if } P_{(k+l+s)}(i) = Pa_{(\delta)}(P_{(k)}(i)) \\ = P_{(k+l+s+(d))}(\mathbb{A}_2) & \text{if } P_{(k+l+s)}(i) \neq Pa_{(\delta)}(P_{(k)}(i)) \end{cases}
\end{aligned} \tag{B.47}$$

Influence Which to Influenced by Which/Dependent by Which to Dependent on Which

This mode could be classified into two submodes, influenced structures distributed within one same agent or on different agents, which would affect the composition of the formula.

Same Agent Given $k+l+s < m$, $k+(t) < m$, $k+l+(d) < m$, $k+l+s+(q) < m$, $k+l+(t) \neq k+l+s+(d)$, $k+l+s+(q) > k+l+(t)$, $k+l+s+(q) > k+l+s+(d)$, $\{k, k+l, k+l+s, k+(t), k+l+(d), k+l+s+(q)\} \subseteq \mathbb{M}$, $\{i, j\} \subseteq \mathbb{N}$.

$$P_{(k+l+s+(q))}(j) \begin{cases} = Pa_{\perp} In_{(\alpha)}(P_{(k+(t))}(j), P_{(k+l+s+(q))}(j)) \\ \quad \text{if } [P_{(k+l)}(i) = Pa_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k+l+s)}(i) \neq Pa_{(\delta)}(P_{(k)}(i))] \\ = Pa_{\perp} In_{(\beta)}(P_{(k+l+(d))}(j), P_{(k+l+s+(q))}(j)) \\ \quad \text{if } [P_{(k+l+s)}(i) = Pa_{(\delta)}(P_{(k)}(i))] \cap [P_{(k+l)}(i) \neq Pa_{(\gamma)}(P_{(k)}(i))] \\ = Pa_{\perp} In_{(\sigma)}(P_{(k+(t))}(j), P_{(k+l+(d))}(j), P_{(k+l+s+(q))}(j)) \\ \quad \text{if } [P_{(k+l)}(i) = Pa_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k+l+s)}(i) = Pa_{(\delta)}(P_{(k)}(i))] \\ = P_{(k+l+s+(q))}(j) \\ \quad \text{if } [P_{(k+l)}(i) \neq Pa_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k+l+s)}(i) \neq Pa_{(\delta)}(P_{(k)}(i))] \end{cases} \tag{B.48}$$

Different Agents As shown in sub figure 18.2 of figure B.6, Given $k+l+s < m$, $k+(t) < m$, $k+l+(d) < m$, $k+l+s+(q) < m$, $k+(t) \neq k+l+(d)$, $k+l+s+(q) > k+l+(t)$, $k+l+s+(q) > k+l+s+(d)$, $\{k, k+l, k+l+s, k+(t), k+l+(d), k+l+s+(q)\} \subseteq \mathbb{M}$, $\{i, j, g\} \subseteq \mathbb{N}$, $\mathbb{A}_1 = \{i, j\}$, $\mathbb{A}_2 = \{i, g\}$, $\mathbb{A}_1 \neq \mathbb{A}_2$, if $\mathbb{A}_1 = i$ then $k+(t) > k$, $k+l+s+(q) > k+l+s$, if $\mathbb{A}_2 = i$ then $k+l+(d) > k+l$, $k+l+s+(q) > k+l+s$.

$$\begin{aligned}
P_{(k+l+s+(q))}(\mathbb{A}_1) & \begin{cases} = Pa_{\perp} In_{(\alpha)}(P_{(k+(t))}(\mathbb{A}_1), P_{(k+l+s+(q))}(\mathbb{A}_1)) & \text{if } P_{(k+l)}(i) = Pa_{(\gamma)}(P_{(k)}(i)) \\ = P_{(k+l+s+(q))}(\mathbb{A}_1) & \text{if } P_{(k+l)}(i) \neq Pa_{(\gamma)}(P_{(k)}(i)) \end{cases} \\
P_{(k+l+s+(q))}(\mathbb{A}_2) & \begin{cases} = Pa_{\perp} In_{(\beta)}(P_{(k+l+(d))}(\mathbb{A}_2), P_{(k+l+s+(q))}(\mathbb{A}_2)) & \text{if } P_{(k+l+s)}(i) = Pa_{(\delta)}(P_{(k)}(i)) \\ = P_{(k+l+s+(q))}(\mathbb{A}_2) & \text{if } P_{(k+l+s)}(i) \neq Pa_{(\delta)}(P_{(k)}(i)) \end{cases}
\end{aligned} \tag{B.49}$$

Influenced by Who to Influence Who/Dependent on Which to Dependent by Which

As shown in sub figure 18.3 of figure B.6, given $l+s+k < m$, $l+(q) < m$, $l+s+k+(t) < m$, $l+s+k+(t)+d < m$, $l+(q) < l+s+k+(t)$, $l+(q) < l+s+k+(t)+d$, $\{l, l+s, l+s+k, l+(q), l+s+k+(t), l+s+k+(t)+d\} \subseteq \mathbb{M}$, $\{i, j, g\} \subseteq \mathbb{N}$, $\mathbb{A}_1 = \{i, j\}$, $\mathbb{A}_2 = \{i, g\}$, if $\mathbb{A}_1 = i$ then $\mathbb{A}_2 = g$, $l+(q) > q$, $l+s+k+(t) > l+s+k$, if $\mathbb{A}_2 = i$, then $\mathbb{A}_1 = j$, $l+(q) > q$.

$$\begin{aligned}
P_{(l+s+k+(t))}(\mathbb{A}_1) & \begin{cases} = Pa_{\perp} In_{(\alpha)}(P_{(l+(q))}(\mathbb{A}_1), P_{(l+s+k+(t))}(\mathbb{A}_1)) & \text{if } P_{(l+s+k)}(i) = Pa_{(\gamma)}(P_{(l)}(i)) \\ = P_{(l+s+k+(t))}(\mathbb{A}_1) & \text{if } P_{(l+s+k)}(i) \neq Pa_{(\gamma)}(P_{(l)}(i)) \end{cases} \\
P_{(l+s+k+(t)+d)}(\mathbb{A}_2) & \begin{cases} = Pa_{\perp} In_{(\beta)}(P_{(l+(q))}(\mathbb{A}_2), P_{(l+s+k+(t)+d)}(\mathbb{A}_2)) & \text{if } P_{(l+s+k)}(i) = Pa_{(\delta)}(P_{(l+s)}(i)) \\ = P_{(l+s+k+(t)+d)}(\mathbb{A}_2) & \text{if } P_{(l+s+k)}(i) \neq Pa_{(\delta)}(P_{(l+s)}(i)) \end{cases}
\end{aligned} \tag{B.50}$$

Influenced by Who to Influenced by Who/Dependnet on Which to Dependent on Which

This mode could be classified into two submodes, influenced structures distributed within one same agent or on different agents, which would affect the composition of the formula.

Same Agent Given $l + s + k < m, l + (t) < m, l + s + (d) < m, l + s + k + (q), l + (t) < l + s + k + (q), l + s + (d) < l + s + k + (q), l + (t) \neq l + s + (d), \{l, l + s, l + s + k, l + (t), l + s + (d), l + s + k + (q)\} \subseteq \mathbb{M}, \{i, j\} \subseteq \mathbb{N}$.

$$\begin{aligned}
P_{(l+s+k+(q))}(j) & \begin{cases} = Pa_{\perp} In_{(\alpha)}(P_{(l+(t))}(j), P_{(l+s+k+(q))}(j)) \\ \quad \text{if } [P_{(l+s+k)}(i) = Pa_{(\gamma)}(P_{(l)}(i))] \cap [P_{(l+s+k)}(i) = Pa_{(\delta)}(P_{(l+s)}(i))] \\ = Pa_{\perp} In_{(\beta)}(P_{(l+s+(d))}(j), P_{(l+s+k+(q))}(j)) \\ \quad \text{if } [P_{(l+s+k)}(i) = Pa_{(\delta)}(P_{(l+s)}(i))] \cap [P_{(l+s+k)}(i) \neq Pa_{(\gamma)}(P_{(l)}(i))] \\ = Pa_{\perp} In_{(\sigma)}(P_{(l+(t))}(j), P_{(l+s+(d))}(j), P_{(l+s+k+(q))}(j)) \\ \quad \text{if } [P_{(l+s+k)}(i) = Pa_{(\gamma)}(P_{(l)}(i))] \cap [P_{(l+s+k)}(i) = Pa_{(\delta)}(P_{(l+s)}(i))] \\ = P_{(l+s+k+(q))}(j) \\ \quad \text{if } [P_{(l+s+k)}(i) \neq Pa_{(\gamma)}(P_{(l)}(i))] \cap [P_{(l+s+k)}(i) \neq Pa_{(\delta)}(P_{(l+s)}(i))] \end{cases}
\end{aligned} \tag{B.51}$$

Different Agents As shown in sub figure 18.4 of figure B.6, given $l + s + k < m, l + (t) < m, l + s + (d) < m, l + s + k + (q), l + (t) < l + s + k + (q), l + s + (d) < l + s + k + (q), l + (t) \neq l + s + (d), \{l, l + s, l + s + k, l + (t), l + s + (d), l + s + k + (q)\} \subseteq \mathbb{M}, \{i, j, g\} \subseteq \mathbb{N}, \mathbb{A}_1 = \{i, j\}, \mathbb{A}_2 = \{i, g\}, \mathbb{A}_1 \neq \mathbb{A}_2$, if $\mathbb{A}_1 = i$, then $l + (t) > l, l + s + (d) > l + s$, if $\mathbb{A}_2 = i$, then $l + (t) > l, l + s + k + (q) > l$.

$$\begin{aligned}
P_{(l+s+k+(q))}(\mathbb{A}_1) & \begin{cases} = Pa_{\perp} In_{(\alpha)}(P_{(l+(t))}(\mathbb{A}_1), P_{(l+s+k+(q))}(\mathbb{A}_1)) & \text{if } P_{(l+s+k)}(i) = Pa_{(\gamma)}(P_{(l)}(i)) \\ = P_{(l+s+k+(q))}(\mathbb{A}_1) & \text{if } P_{(l+s+k)}(i) \neq Pa_{(\gamma)}(P_{(l)}(i)) \end{cases} \\
P_{(l+s+k+(q))}(\mathbb{A}_2) & \begin{cases} = Pa_{\perp} In_{(\beta)}(P_{(l+s+(d))}(\mathbb{A}_2), P_{(l+s+k+(q))}(\mathbb{A}_2)) & \text{if } P_{(l+s+k)}(i) = Pa_{(\delta)}(P_{(l+s)}(i)) \\ = P_{(l+s+k+(q))}(\mathbb{A}_2) & \text{if } P_{(l+s+k)}(i) \neq Pa_{(\delta)}(P_{(l+s)}(i)) \end{cases}
\end{aligned} \tag{B.52}$$

Specific Example 5 (Pattern 18) *When a group of friends have a diner at restaurant, for instance they need make choices about three issues: main dish, wine and spirit/alcohol, assume there is an expert (gourmand) who is very good at the combination of delicacies, such as knowing which wine match which dish the best, and the expert's friends trust the his taste very much. Therefore, if the expert choose to match wine with main dish, then other people who like to drink wine would be influenced and follow the match between wine and dish provided by the expert, but, if the expert choose to match alcohol with main dish, then another group of people who like to drink alcohol would then be affected and follow the match between alcohol and dish provided by the expert. The different match (structures of dependency) between issues would influence to different other agents.*

It should be noticed that, it is the dependency relations (the match between main dish and wine or alcohol) of the expert influencing the dependency relations of other agents, not the specific choice of value on individual issues (like choose red wine or white wine, or choose fish or meat.) For example, if the expert choose to match wine with dish, and the people like to match wine and dish will follow the combination, no matter expert match red wine or white wine, the influence always point to those people like to have dish with wine, would not lead to those people like to have dish with alcohol, and the specific choice of value would not affect the structure of dependency of other agents, only the structure of dependency itself.

Pattern 19 Inter-intra Influence from Decision to Decision and Structure

It should be noticed that the construction of pattern 19 (inter-intra influence from decision to decision) is similar to the pattern 13 (intra-inter influence from decision to decision) as shown in sub figure 13 of figure 5.7, the two influence cross both the horizontal dimension and the vertical dimension. In fact, if not comparing the *distance of influence*, the inter-intra influence from decision to decision would be exactly identical with the intra-inter influence from decision to decision, as when the influence comes from sole variable and also leads to sole variable, there would be no difference on the variation of structures (for comparison), only if there are at least two different agents or different issues being influenced, we can distinguish the influence affecting on the intra dimension or inter dimension, for example if two agents within one same issue being influenced, then the influence affects on the inter dimension, and if two issue within one agent being influenced, then the influence affects on the intra dimension. It also applies to the requirements to distinguish the influence coming from intra or inter dimension. At least there are more than one agents or issues on either the “from” side or the “to” side of influence, we can distinguish the inter or intra property of the influence. While the construction of influence could not be compared (like pattern 13 and pattern 19 both from sole variable to sole variable), then the distance of influence could be used to substitute, to compare the distance of influence flowing on intra (vertical) dimension and inter (horizontal) dimension, find which is the principal influencing path.

Given $k + l < m$, $\{k, k + l\} \subseteq \mathbb{M}$, $\{i, j\} \subseteq \mathbb{N}$, and $l \leq |j - i|$, which means the distance between influencing and influenced issues is shorter than the distance between influencing and influenced agents for the inter-intra influence. Compared with $Pa \perp In$, a new function $In \dashv Pa$ is defined, In is put before Pa ¹⁰, and the symbol “ \dashv ” is very vivid to describe this pattern of influence, the influence firstly flows at the inter/horizontal level (represented by the transverse line), and then affects on the intra/vertical level (represented by the vertical line). The general formula would be:

$$P_{(k+l)}(j) = In \dashv Pa(P_{(k)}(i)) \quad (B.53)$$

If the influence works in different ways according to different choices, the formula would be:

$$P_{(k+l)}(j) \begin{cases} = In \dashv Pa_{\alpha}(P_{(k)}(i)) & \text{if } C_{(k)}(i) = c_k \\ = In \dashv Pa_{\beta}(P_{(k)}(i)) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \quad (B.54)$$

¹⁰As the inter-intra influence firstly starts from or flows at the horizontal dimension and then points to or affects on the vertical dimension, or the inter-intra influence runs longer distance in the horizontal direction than in the vertical dimension.

Pattern 20 Inter-intra Influence from Decision to Structure

In fact, this pattern of influence could be classified into two modes, one is the standard one, the influenced component is more than one issues but within one single agent (totally within the intra level), and the other is the extended one, the influenced component could be distributed on both different agents and different issues, then the *distance of Influence* would substitute the *Construction of Influence* to help compare and distinguish between different patterns (like 14 and 20).

Same Agent

Given $k + (l) + s < m$, $\{k, k + (l), k + (l) + s\} \subseteq \mathbb{M}$, $\{i, j\} \subseteq \mathbb{N}$.

$$\begin{aligned} P_{(k+(l))}(j) & \begin{cases} = In_{\rightarrow}Pa_{(\alpha)}(P_{(k)}(i)) & \text{if } C_{(k)}(i) = c_k \\ = P_{(k+(l))}(j) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \\ P_{(k+(l)+s)}(j) & \begin{cases} = In_{\rightarrow}Pa_{(\beta)}(P_{(k)}(i)) & \text{if } C_{(k)}(i) = \bar{c}_k \\ = P_{(k+(l)+s)}(j) & \text{if } C_{(k)}(i) = c_k \end{cases} \end{aligned} \quad (\text{B.55})$$

Different Agents

If influenced variables distributed on different agents, then for the inter-intra influence, the distance of influence splitting on the inter level should be longer than the splitting on the intra level. Given $k + (l) + s < m$, $\{k, k + (l), k + (l) + s\} \subseteq \mathbb{M}$, $\{i, j, g\} \subseteq \mathbb{N}$, $\mathbb{A}_1 = \{i, j\}$, $\mathbb{A}_2 = \{i, g\}$, $\mathbb{A}_1 \neq \mathbb{A}_2$, $s \leq |g - j|$ (if $\mathbb{A}_1 = j$, $\mathbb{A}_2 = g$), $s \leq |j - i|$ (if $\mathbb{A}_1 = i$, $\mathbb{A}_2 = j$), $s \leq |g - i|$ (if $\mathbb{A}_1 = i$, $\mathbb{A}_2 = g$), and if $k + (l) = k$, then $\mathbb{A}_1 = j$, if $\mathbb{A}_1 = i$, then $k + (l) > k$.

$$\begin{aligned} P_{(k+(l))}(\mathbb{A}_1) & \begin{cases} = In_{\rightarrow}Pa_{(\alpha)}(P_{(k)}(i)) & \text{if } C_{(k)}(i) = c_k \\ = P_{(k+(l))}(\mathbb{A}_1) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \\ P_{(k+(l)+s)}(\mathbb{A}_2) & \begin{cases} = In_{\rightarrow}Pa_{(\beta)}(P_{(k)}(i)) & \text{if } C_{(k)}(i) = \bar{c}_k \\ = P_{(k+(l)+s)}(\mathbb{A}_2) & \text{if } C_{(k)}(i) = c_k \end{cases} \end{aligned} \quad (\text{B.56})$$

Pattern 21 Inter-intra Influence from Structure to Decision

Influence Which/Dependent by Which of Influenced Component

In this mode, the structure of influenced component is “one issue influence two different issues” or “one issue be dependent by different issues”. This mode could be further classified into two submodes, based on the range of influenced component on the agent (inter) dimension, within one agent (as shown in sub figure 21.1 of figure B.7) or spread over different agents (as shown in sub figure 21.2 of figure B.7). And the two submodes can be represented by one uniform formula with moderate adjustments (wether $\mathbb{A}_1 = \mathbb{A}_2$ or $\mathbb{A}_1 \neq \mathbb{A}_2$).

Given $k + (t) + l + s < m$, $\{k, k + (t), k + (t) + l, k + (t) + l + s\} \subseteq \mathbb{M}$, $\{i, j, g\} \subseteq \mathbb{N}$, $\mathbb{A}_1 = \{i, j, g\}$, $\mathbb{A}_2 = \{i, j, g\}$, if $k + (t) = k$, then $\mathbb{A}_1 \neq i$ and $\mathbb{A}_2 \neq i$, if $\mathbb{A}_1 = i$ then $\mathbb{A}_2 \neq i$, if $\mathbb{A}_2 = i$ then $\mathbb{A}_1 \neq i$.

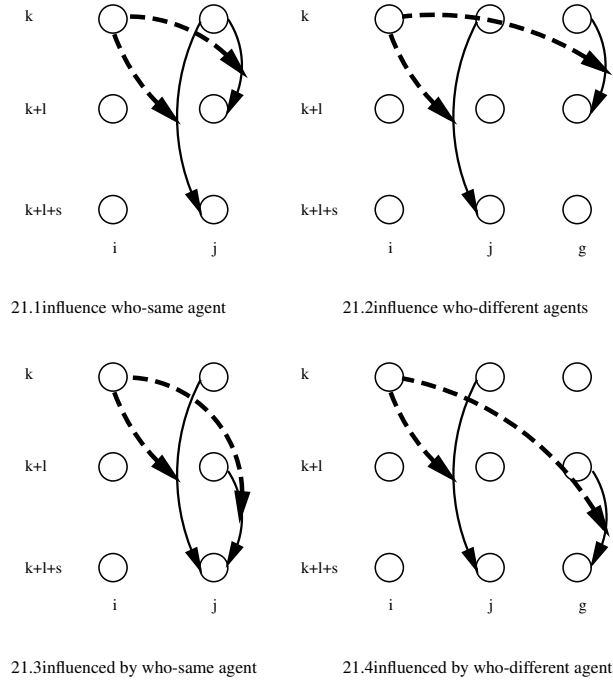


Figure B.7: Four Modes of Inter-intra Influence of Decision to Structure

$$\begin{aligned}
 P_{(k+(t)+l)}(\mathbb{A}_1) & \begin{cases} = In_{\neg}Pa_{(\alpha)}(P_{(k+(t))}(\mathbb{A}_1), P_{(k+(t)+l)}(\mathbb{A}_1)) & \text{if } C_{(k)}(i) = c_k \\ = P_{(k+(t)+l)}(\mathbb{A}_1) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \\
 P_{(k+(t)+l+s)}(\mathbb{A}_2) & \begin{cases} = In_{\neg}Pa_{(\beta)}(P_{(k+(t))}(\mathbb{A}_2), P_{(k+(t)+l+s)}(\mathbb{A}_2)) & \text{if } C_{(k)}(i) = \bar{c}_k \\ = P_{(k+(t)+l+s)}(\mathbb{A}_2) & \text{if } C_{(k)}(i) = c_k \end{cases}
 \end{aligned} \tag{B.57}$$

Influenced by Which/Dependent on Which of Influenced Component

In this mode, the structure of influenced component is “one issue be influenced by two different different issues” or “one issues be dependent on two different issues”. This mode could also be classified into two submodes, based on the range of influenced component on the agent dimension, within one agent (as shown in sub figure 21.3 of figure B.7) or spread over more than one agents (as shown in sub figure 21.4 of figure B.7). And the two submodes can be represented by one uniform formula with moderate adjustments (wether $\mathbb{A}_1 = \mathbb{A}_2$ or $\mathbb{A}_1 \neq \mathbb{A}_2$).

Given $k + (t) + l + s < m$, $\{k, k + (t), k + (t) + l, k + (t) + l + s\} \subseteq \mathbb{M}$, $\{i, j, g\} \subseteq \mathbb{N}$, $\mathbb{A}_1 = \{i, j, g\}$, $\mathbb{A}_2 = \{i, j, g\}$, if $k + (t) = k$, then $\mathbb{A}_1 \neq i$, if $\mathbb{A}_1 = i$ then $\mathbb{A}_2 \neq i$, if $\mathbb{A}_2 = i$ then $\mathbb{A}_1 \neq i$.

$$\begin{aligned}
 P_{(k+(t)+l+s)}(\mathbb{A}_1) & \begin{cases} = In_{\neg}Pa_{(\alpha)}(P_{(k+(t))}(\mathbb{A}_1), P_{(k+(t)+l+s)}(\mathbb{A}_1)) & \text{if } C_{(k)}(i) = c_k \\ = P_{(k+(t)+l+s)}(\mathbb{A}_1) & \text{if } C_{(k)}(i) = \bar{c}_k \end{cases} \\
 P_{(k+(t)+l+s)}(\mathbb{A}_2) & \begin{cases} = In_{\neg}Pa_{(\beta)}(P_{(k+(t)+l)}(\mathbb{A}_2), P_{(k+(t)+l+s)}(\mathbb{A}_2)) & \text{if } C_{(k)}(i) = \bar{c}_k \\ = P_{(k+(t)+l+s)}(\mathbb{A}_2) & \text{if } C_{(k)}(i) = c_k \end{cases}
 \end{aligned} \text{ or } \tag{B.58}$$

Pattern 22 Inter-intra Influence from Structure to Decision

There are two modes for this pattern of influence, considering the variations of structure of influencing component.

Influence Who of Influencing Component

Given $\{i, g, h, j\} \subseteq \mathbb{N}$, $k + l < m$, $\{k, k + l\} \subseteq \mathbb{M}$, the influenced agent is $\mathbb{A} = \{j, i, g, h\}$, which means that the influenced agent could be one of the agents in the influencing component, and also could be another different agent.

$$P_{(k+l)}(\mathbb{A}) \begin{cases} = In_{\neg}Pa_{(\alpha)}\{P_{(k)}(i), P_{(k)}(g), \varphi[P_{(k)}(i), P_{(k)}(g)], P_{(k+l)}(\mathbb{A})\}, \\ \text{if } [P_{(k)}(g) = In_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k)}(h) \neq In_{(\delta)}(P_{(k)}(i))] \\ = In_{\neg}Pa_{(\beta)}\{P_{(k)}(i), P_{(k)}(h), \varphi[P_{(k)}(i), P_{(k)}(h)], P_{(k+l)}(\mathbb{A})\}, \\ \text{if } [P_{(k)}(h) = In_{(\delta)}(P_{(k)}(i))] \cap [P_{(k)}(g) \neq In_{(\gamma)}(P_{(k)}(i))] \\ = In_{\neg}Pa_{(\sigma)}\{P_{(k)}(i), P_{(k)}(g), P_{(k)}(h), \\ \varphi[P_{(k)}(i), P_{(k)}(g)], \varphi[P_{(k)}(i), P_{(k)}(h)], P_{(k+l)}(\mathbb{A})\} \\ \text{if } [P_{(k)}(g) = In_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k)}(h) = In_{(\delta)}(P_{(k)}(i))] \\ = P_{(k+l)}(\mathbb{A}), \\ \text{if } [P_{(k)}(g) \neq In_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k)}(h) \neq In_{(\delta)}(P_{(k)}(i))] \end{cases} \quad (\text{B.59})$$

Influenced by Who of Influencing Component

Given $\{g, h, i, j\} \subseteq \mathbb{N}$, $k + l < m$, $\{k, k + l\} \subseteq \mathbb{M}$, the influenced agent is $\mathbb{A} = \{j, i, g, h\}$.

$$P_{(k+l)}(\mathbb{A}) \begin{cases} = In_{\neg}Pa_{(\alpha)}\{P_{(k)}(g), P_{(k)}(i), \varphi[P_{(k)}(g), P_{(k)}(i)], P_{(k+l)}(\mathbb{A})\}, \\ \text{if } [P_{(k)}(i) = In_{(\gamma)}(P_{(k)}(g))] \cap [P_{(k)}(i) \neq In_{(\delta)}(P_{(k)}(h))] \\ = In_{\neg}Pa_{(\beta)}\{P_{(k)}(h), P_{(k)}(i), \varphi[P_{(k)}(h), P_{(k)}(i)], P_{(k+l)}(\mathbb{A})\}, \\ \text{if } [P_{(k)}(i) = In_{(\delta)}(P_{(k)}(h))] \cap [P_{(k)}(i) \neq In_{(\gamma)}(P_{(k)}(g))] \\ = In_{\neg}Pa_{(\sigma)}\{P_{(k)}(g), P_{(k)}(h), P_{(k)}(i), \\ \varphi[P_{(k)}(g), P_{(k)}(i)], \varphi[P_{(k)}(h), P_{(k)}(i)], P_{(k+l)}(\mathbb{A})\} \\ \text{if } [P_{(k)}(i) = In_{(\gamma)}(P_{(k)}(g))] \cap [P_{(k)}(i) = In_{(\delta)}(P_{(k)}(h))] \\ = P_{(k+l)}(\mathbb{A}), \\ \text{if } [P_{(k)}(i) \neq In_{(\gamma)}(P_{(k)}(g))] \cap [P_{(k)}(i) \neq In_{(\delta)}(P_{(k)}(h))] \end{cases} \quad (\text{B.60})$$

Pattern 23 Inter-intra Influence from Structure to Decision and Structure

Influence Who of Influencing Component

The influenced variables should be different at least on either agent dimension or issue dimension, otherwise the influence would be to sole decision (identical with Pattern 22), but not to both decision and structure. Given $\{i, h, r, j, q\} \subseteq \mathbb{N}$, $k + (l) + (s) < m$, $\{k, k + (l), k + (l) + (s)\} \subseteq \mathbb{M}$, $\mathbb{A}_1 = \{i, h, r, j, g\}$, $\mathbb{A}_2 = \{i, h, r, j, g\}$, if $k + (l) = k$, then $\mathbb{A}_1 = j \cup g$, if $k + (l) + (s) = k$, then $\mathbb{A}_1 = j \cup g$, if $\mathbb{A}_1 = \mathbb{A}_2$ then $k + (l) \neq k + (l) + (s)$.

$$\begin{aligned}
P_{(k+l)}(\mathbb{A}_1) & \begin{cases} = In_{\neg}Pa_{(\alpha)}\{P_{(k)}(i), P_{(k)}(h), \varphi[P_{(k)}(i), P_{(k)}(h)], P_{(k+l)}(\mathbb{A}_1)\}, \\ \text{if } P_{(k)}(h) = In_{(\gamma)}(P_{(k)}(i)) \\ = P_{(k+l)}(\mathbb{A}_1), \\ \text{if } \neg P_{(k)}(h) = In_{(\gamma)}(P_{(k)}(i)) \end{cases} \\
P_{(k+l+s)}(\mathbb{A}_2) & \begin{cases} = In_{\neg}Pa_{(\beta)}\{P_{(k)}(i), P_{(k)}(r), \varphi[P_{(k)}(i), P_{(k)}(r)], P_{(k+l+s)}(\mathbb{A}_2)\}, \\ \text{if } P_{(k)}(r) = In_{(\delta)}(P_{(k)}(i)) \\ = P_{(k+l+s)}(\mathbb{A}_2), \\ \text{if } \neg P_{(k)}(r) = In_{(\delta)}(P_{(k)}(i)) \end{cases}
\end{aligned} \tag{B.61}$$

Influenced by Who of Influencing Component

Given $\{h, r, i, j, q\} \subseteq \mathbb{N}$, $k+(l)+(s) < m$, $\{k, k+(l), k+(l)+(s)\} \in \mathbb{M}$, $\mathbb{A}_1 = \{h, r, i, j, g\}$, $\mathbb{A}_2 = \{h, r, i, j, g\}$, if $k+(l) = k$, then $\mathbb{A}_1 = j \cup g$, if $k+(l)+(s) = k$, then $\mathbb{A}_1 = j \cup g$, if $\mathbb{A}_1 = \mathbb{A}_2$ then $k+(l) \neq k+(l)+(s)$.

$$\begin{aligned}
P_{(k+l)}(\mathbb{A}_1) & \begin{cases} = In_{\neg}Pa_{(\alpha)}\{P_{(k)}(h), P_{(k)}(i), \varphi[P_{(k)}(h), P_{(k)}(i)], P_{(k+l)}(\mathbb{A}_1)\}, \\ \text{if } P_{(k)}(i) = In_{(\gamma)}(P_{(k)}(h)) \\ = P_{(k+l)}(\mathbb{A}_1), \\ \text{if } \neg P_{(k)}(i) = In_{(\gamma)}(P_{(k)}(h)) \end{cases} \\
P_{(k+l+s)}(\mathbb{A}_2) & \begin{cases} = In_{\neg}Pa_{(\beta)}\{P_{(k)}(r), P_{(k)}(i), \varphi[P_{(k)}(r), P_{(k)}(i)], P_{(k+l+s)}(\mathbb{A}_2)\}, \\ \text{if } P_{(k)}(i) = In_{(\delta)}(P_{(k)}(r)) \\ = P_{(k+l+s)}(\mathbb{A}_2), \\ \text{if } \neg P_{(k)}(i) = In_{(\delta)}(P_{(k)}(r)) \end{cases}
\end{aligned} \tag{B.62}$$

Pattern 24 Inter-intra Influence from Structure to Structure

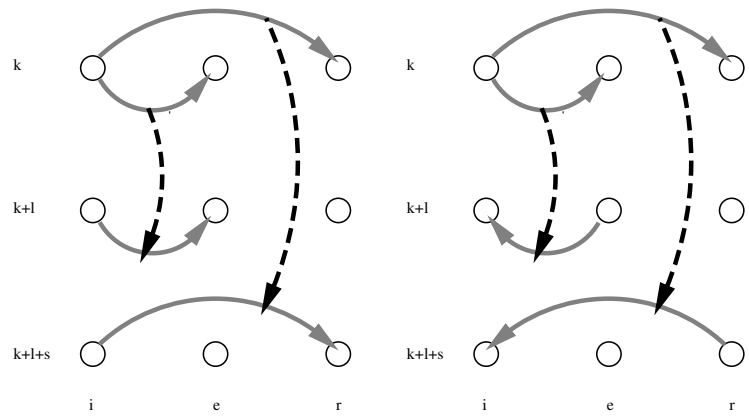
There are four specific modes for this pattern of influence, separately considering the structure of influencing component and influenced component, both can be represented in two forms of “one agent influence different agents” and “one agent be influenced by different agents”, as shown in figure B.8.

Influence Who to Influence Who

Given $\{i, e, r, j, g, h\} \subseteq \mathbb{N}$, $k+(l)+(s) < m$, $\{k, k+(l), k+(l)+(s)\} \subseteq \mathbb{M}$, $\mathbb{A}_1 = \{j, i, e, r\}$, $\mathbb{A}_2 = \{g, i, e, r\}$, $\mathbb{A}_3 = \{h, i, e, r\}$, $\mathbb{A}_1 \neq \mathbb{A}_2 \neq \mathbb{A}_3$, and if $k+(l) = k$, then $\mathbb{A}_1 = j$, $\mathbb{A}_1 = g$, $k+(l)+(s) > k$.¹¹

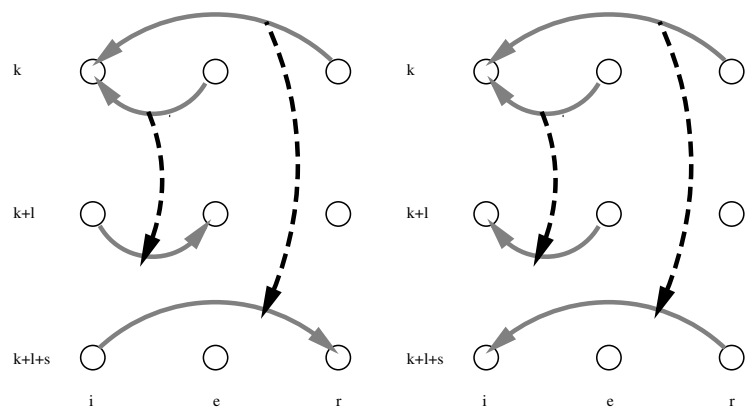
$$\begin{aligned}
P_{(k+l)}(\mathbb{A}_2) & \begin{cases} = In_{\neg}Pa_{(\alpha)}(P_{(k+l)}(\mathbb{A}_1), P_{(k+l)}(\mathbb{A}_2)) & \text{if } P_{(k)}(e) = In_{(\gamma)}(P_{(k)}(i)) \\ = P_{(k+l)}(\mathbb{A}_2) & \text{if } P_{(k)}(e) \neq In_{(\gamma)}(P_{(k)}(i)) \end{cases} \\
P_{(k+l+s)}(\mathbb{A}_3) & \begin{cases} = In_{\neg}Pa_{(\beta)}(P_{(k+l+s)}(\mathbb{A}_1), P_{(k+l+s)}(\mathbb{A}_3)) & \text{if } P_{(k)}(r) = In_{(\delta)}(P_{(k)}(i)) \\ = P_{(k+l+s)}(\mathbb{A}_3) & \text{if } P_{(k)}(r) \neq In_{(\delta)}(P_{(k)}(i)) \end{cases}
\end{aligned} \tag{B.63}$$

¹¹Which means if the issue No. of one of influenced structure is the same with the influencing structure, then the agents could not be $\{i, e, r\}$, otherwise the influenced structure would be lapped with the influencing structure.



24.1influence who to influence who

24.2influence who to influenced by who



24.3influenced by who to influence who

24.4influenced by who to influenced by who

Figure B.8: Four Modes of Inter-intra Influence of Structure to Structure

Influence Who to Influenced by Who

Same Issue Given $\{i, e, r, j, g, h\} \subseteq \mathbb{N}$, $k + l < m$, $\{k, k + l\} \subseteq \mathbb{M}$, $\mathbb{A}_1 = \{j, i, e, r\}$, $\mathbb{A}_2 = \{g, i, e, r\}$, $\mathbb{A}_3 = \{h, i, e, r\}$, $\mathbb{A}_1 \neq \mathbb{A}_2 \neq \mathbb{A}_3$.

$$P_{(k+l)}(\mathbb{A}_1) \begin{cases} = In_{\neg}Pa_{(\alpha)}(P_{(k+l)}(\mathbb{A}_2), P_{(k+l)}(\mathbb{A}_1)) \\ \quad \text{if } [P_{(k)}(e) = In_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k)}(r) \neq In_{(\delta)}(P_{(k)}(i))] \\ = In_{\neg}Pa_{(\beta)}(P_{(k+l)}(\mathbb{A}_3), P_{(k+l)}(\mathbb{A}_1)) \\ \quad \text{if } [P_{(k)}(r) = In_{(\delta)}(P_{(k)}(i))] \cap [P_{(k)}(e) \neq In_{(\gamma)}(P_{(k)}(i))] \\ = In_{\neg}Pa_{(\sigma)}(P_{(k+l)}(\mathbb{A}_2), P_{(k+l)}(\mathbb{A}_3), P_{(k+l)}(\mathbb{A}_1)) \\ \quad \text{if } [P_{(k)}(e) = In_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k)}(r) = In_{(\delta)}(P_{(k)}(i))] \\ = P_{(k+l)}(\mathbb{A}_1) \\ \quad \text{if } [P_{(k)}(e) \neq In_{(\gamma)}(P_{(k)}(i))] \cap [P_{(k)}(r) \neq In_{(\delta)}(P_{(k)}(i))] \end{cases} \quad (\text{B.64})$$

Different Issues Given $\{i, e, r, j, g, h\} \subseteq \mathbb{N}$, $k + (l) + s < m$, $\{k, k + (l), k + (l) + s\} \subseteq \mathbb{M}$, $\mathbb{A}_1 = \{j, i, e, r\}$, $\mathbb{A}_2 = \{g, i, e, r\}$, $\mathbb{A}_3 = \{h, i, e, r\}$, $\mathbb{A}_1 \neq \mathbb{A}_2 \neq \mathbb{A}_3$, and if $k + (l) = k$, then $\mathbb{A}_1 = j$, $\mathbb{A}_2 = g$.

$$P_{(k+(l))}(\mathbb{A}_1) \begin{cases} = In_{\neg}Pa_{(\alpha)}(P_{(k+(l))}(\mathbb{A}_2), P_{(k+(l))}(\mathbb{A}_1)) & \text{if } P_{(k)}(e) = In_{(\gamma)}(P_{(k)}(i)) \\ = P_{(k+(l))}(\mathbb{A}_1) & \text{if } P_{(k)}(e) \neq In_{(\gamma)}(P_{(k)}(i)) \end{cases} \quad (\text{B.65})$$

$$P_{(k+(l)+s)}(\mathbb{A}_1) \begin{cases} = In_{\neg}Pa_{(\beta)}(P_{(k+(l)+s)}(\mathbb{A}_3), P_{(k+(l)+s)}(\mathbb{A}_1)) & \text{if } P_{(k)}(r) = In_{(\delta)}(P_{(k)}(i)) \\ = P_{(k+(l)+s)}(\mathbb{A}_1) & \text{if } P_{(k)}(r) \neq In_{(\delta)}(P_{(k)}(i)) \end{cases}$$

Influenced by Who to Influence Who

Given $\{i, e, r, g, h, j\} \subseteq \mathbb{N}$, $k + (l) + (s) < m$, $\{k, k + (l), k + (l) + (s)\} \subseteq \mathbb{M}$, $\mathbb{A}_1 = \{j, i, e, r\}$, $\mathbb{A}_2 = \{g, i, e, r\}$, $\mathbb{A}_3 = \{h, i, e, r\}$, $\mathbb{A}_1 \neq \mathbb{A}_2 \neq \mathbb{A}_3$, and if $k + (l) = k$, then $\mathbb{A}_1 = j$, $\mathbb{A}_1 = g$, $k + (l) + (s) > k$.

$$P_{(k+(l))}(\mathbb{A}_2) \begin{cases} = In_{\neg}Pa_{(\alpha)}(P_{(k+(l))}(\mathbb{A}_1), P_{(k+(l))}(\mathbb{A}_2)) & \text{if } P_{(k)}(i) = In_{(\gamma)}(P_{(k)}(e)) \\ = P_{(k+(l))}(\mathbb{A}_2) & \text{if } P_{(k)}(i) \neq In_{(\gamma)}(P_{(k)}(e)) \end{cases} \quad (\text{B.66})$$

$$P_{(k+(l)+(s))}(\mathbb{A}_3) \begin{cases} = In_{\neg}Pa_{(\beta)}(P_{(k+(l)+(s))}(\mathbb{A}_1), P_{(k+(l)+(s))}(\mathbb{A}_3)) & \text{if } P_{(k)}(i) = In_{(\delta)}(P_{(k)}(r)) \\ = P_{(k+(l)+(s))}(\mathbb{A}_3) & \text{if } P_{(k)}(i) \neq In_{(\delta)}(P_{(k)}(r)) \end{cases}$$

Influenced by Who to Influenced by Who

Same Issue Given $\{i, e, r, g, h, j\} \subseteq \mathbb{N}$, $k + l < m$, $\{k, k + l\} \subseteq \mathbb{M}$, $\mathbb{A}_1 = \{j, i, e, r\}$, $\mathbb{A}_2 = \{g, i, e, r\}$, $\mathbb{A}_3 = \{h, i, e, r\}$, $\mathbb{A}_1 \neq \mathbb{A}_2 \neq \mathbb{A}_3$.

$$P_{(k+l)}(\mathbb{A}_1) \begin{cases} = In_{\neg}Pa_{(\alpha)}(P_{(k+l)}(\mathbb{A}_2), P_{(k+l)}(\mathbb{A}_1)) \\ \quad \text{if } [P_{(k)}(i) = In_{(\gamma)}(P_{(k)}(e))] \cap [P_{(k)}(i) \neq In_{(\delta)}(P_{(k)}(r))] \\ = In_{\neg}Pa_{(\beta)}(P_{(k+l)}(\mathbb{A}_3), P_{(k+l)}(\mathbb{A}_1)) \\ \quad \text{if } [P_{(k)}(i) = In_{(\delta)}(P_{(k)}(r))] \cap [P_{(k)}(i) \neq In_{(\gamma)}(P_{(k)}(e))] \\ = In_{\neg}Pa_{(\alpha)}(P_{(k+l)}(\mathbb{A}_3), P_{(k+l)}(\mathbb{A}_2), P_{(k+l)}(\mathbb{A}_1)) \\ \quad \text{if } [P_{(k)}(i) = In_{(\gamma)}(P_{(k)}(e))] \cap [P_{(k)}(i) = In_{(\delta)}(P_{(k)}(r))] \\ = P_{(k+l)}(\mathbb{A}_1) \\ \quad \text{if } [P_{(k)}(i) \neq In_{(\gamma)}(P_{(k)}(e))] \cap [P_{(k)}(i) \neq In_{(\delta)}(P_{(k)}(r))] \end{cases} \quad (\text{B.67})$$

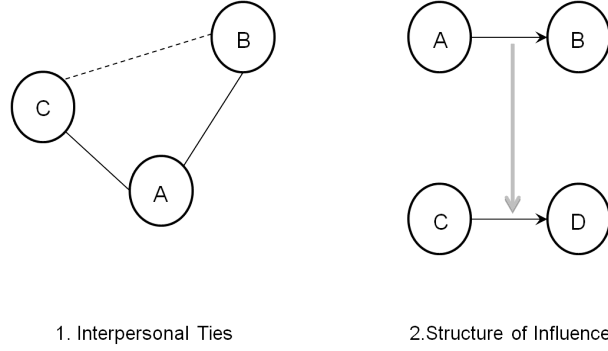


Figure B.9: Interpersonal Ties vs Structure of Influence

Different Issues Given $\{i, e, r, g, h, j\} \subseteq \mathbb{N}$, $k + (l) + s < m$, $\{k, k + (l), k + (l) + s\} \subseteq \mathbb{M}$, $\mathbb{A}_1 = \{j, i, e, r\}$, $\mathbb{A}_2 = \{g, i, e, r\}$, $\mathbb{A}_3 = \{h, i, e, r\}$, $\mathbb{A}_1 \neq \mathbb{A}_2 \neq \mathbb{A}_3$, and if $k + (l) = k$, then $\mathbb{A}_1 = j$, $\mathbb{A}_2 = g$.

$$\begin{aligned}
 P_{(k+(l))}(\mathbb{A}_1) & \begin{cases} = In_{\neg}Pa_{(\alpha)}(P_{(k+(l))}(\mathbb{A}_2), P_{(k+(l))}(\mathbb{A}_1)) & \text{if } P_{(k)}(i) = In_{(\gamma)}(P_{(k)}(e)) \\ = P_{(k+(l))}(\mathbb{A}_1) & \text{if } P_{(k)}(i) \neq In_{(\gamma)}(P_{(k)}(e)) \end{cases} \\
 P_{(k+(l)+s)}(\mathbb{A}_1) & \begin{cases} = In_{\neg}Pa_{(\beta)}(P_{(k+(l)+s)}(\mathbb{A}_3), P_{(k+(l)+s)}(\mathbb{A}_1)) & \text{if } P_{(k)}(i) = In_{(\delta)}(P_{(k)}(r)) \\ = P_{(k+(l)+s)}(\mathbb{A}_1) & \text{if } P_{(k)}(i) \neq In_{(\delta)}(P_{(k)}(r)) \end{cases}
 \end{aligned} \tag{B.68}$$

B.2 Comparison between Strong Ties-Weak Ties Hypothesis and New Influence from Structure to Structure

The generation of weak ties from string ties discussed before could be understood as one influence from interpersonal relations to interpersonal relations, from strong ties to weak ties, and which is also one kind of *influence from structure to structure* (the structure A linked with B and C generate the structure B linked with C), which is the crucial and novel content discussed in the thesis. However, the Weak Tie Hypothesis is still different with the theory of ‘‘Influencing and Influenced Structure’’:

- The interpersonal ties are usually without a specific direction, for example (in sub-figure 1 of figure B.9), A and B as close friends are linked with a tie between them without an arrow, expressed as $A - B$, while in the structure of influence, it should be clear that it is A influencing B or B influencing A , representing the structure as $A \rightarrow B$ or $B \rightarrow A$, it is necessary to identify the influencing one and influenced one, namely the subject or the object of influence.
- Besides, strong ties to weak tie is still within one same clump of agents, from relations $A - B$ and $A - C$ (among a clump $\{A, B, C\}$) to relation $B - C$ (still within the clump of $\{A, B, C\}$), while the influence from structure among one clump can lead to the structure among another clump, for example (in sub-figure 2 of figure B.9) an influence represented as $\{A \rightarrow B\} \rightarrow \{C \rightarrow D\}$, from a clump $\{A, B\}$ to a different clump $\{C, D\}$.

The crucial difference is, whether the agents in the influencing clump are overlapped with the agents in the influenced clump. In fact, in the weak tie hypothesis, it is agent B and C draw support from agent A to form the tie between them, it is like agent A as a mediator contributing to the relationship between B and C , it is still the influence through agent to

agent, while not directly discussing the influence from relationship to relationship, from structure to structure. As shown in figure B.9, in the framework of interpersonal ties, the links is still just from node (agent) to (node) agent, while in the framework of influencing and influenced structure, the links could directly originate from relationships and leads to relationships, like directly perceiving the structure as entity of influence.

- Furthermore, the interpersonal ties are not usually related to the decision, while in influencing and influenced structure, it is assumed that different decisions of agents would lead to different structure of influence/influencing relationships among (other) agents, it is like the structure of influence could be subjectively and determined by agents, and also belongs to one kind of decision-making for agents. But in the weak tie hypothesis, the formation of weak tie is objective and definite (given strong ties between A and B, between A and C, then the tie between B and C is always present [Granovetter, 1983]), not necessarily involving the subjective willingness and autonomous decisions of related agents.

B.3 Complementary Illustration of Priority of Influence

The Priority of Influence among Agents within one Issue

Firstly, assume the matrix of priority of influence in the horizontal dimension, namely among different agents but on one issue k :

$$\mathbb{P}\mathbb{R}_{(k)} = \mathbb{R}\mathbb{E}_{(k)} \times \mathbb{S}\mathbb{W}_{(k)} \times \mathbb{S}\mathbb{P}_{(k)} \times \mathbb{P}\mathbb{F}_{(k)} \quad (\text{B.69})$$

If expressed in the general form, $\mathcal{M}\mathcal{T}_{(k)}$ is the set of matrix of all the contributing metrics to the priority of influence among agents on issue k , $\mathbb{M}\mathbb{T}_{(k)} \in \mathcal{M}\mathcal{T}_{(k)}$.

$$\mathbb{P}\mathbb{R}_{(k)} = \prod_{\mathbb{M}\mathbb{T}_{(k)} \in \mathcal{M}\mathcal{T}_{(k)}} \mathbb{M}\mathbb{T}_{(k)} \quad (\text{B.70})$$

The $\mathbb{P}\mathbb{R}_{(k)}$ is the matrix of priorities of influence among agents on the issue k , in which the value $pr_{i,j(k)}$ is the priority of influence from agent i to agent j for the issue k , if $pr_{i,j(k)} = 0$, which means there is no influence from agent i to agent j , but if $pr_{i,j(k)} \neq 0$, which means there is influence from agent i to agent j , and the bigger of the absolute value $|pr_{i,j(k)}|$, the higher of the priority of influence.¹² The matrix of priority of influence among agents within issue k could be expressed as following:

$$\mathbb{P}\mathbb{R}_{(k)} = \begin{bmatrix} pr_{1,1(k)} & pr_{1,2(k)} & \cdots & pr_{1,n(k)} \\ pr_{2,1(k)} & \cdots & \cdots & pr_{2,n(k)} \\ \cdots & \cdots & \cdots & \cdots \\ pr_{n,1(k)} & pr_{n,2(k)} & \cdots & pr_{n,n(k)} \end{bmatrix} \quad (\text{B.71})$$

For the influenced agent i on the issue k ($k \in \mathbb{M}$), would be influenced by the influencing agent j ($i, j \in \mathbb{N}$) with the highest priority of influence:

$$P_{(k)}(i) = \text{argmax}[pr_{j,i(k)}, P_{(k)}(j)] \quad (\text{B.72})$$

It should be noticed that *the matrix of priority of influence among agents on different issues would be different*, for example, while you making choice about buying a car, for choosing the

¹²However, above just discussed about the priority comparison between individual agent and individual agent, when comparing with the structure composed of (more than one) agents, the priority of influence would become more complex.

color of car, your family members would have higher priority of influence on you, however for choosing the model of car, the experts or friends with abundant experiences might have higher priority of influence on you. The properties of different issues would interact with the metrics, and generating different matrix of priority of influence among agents.

The Priority of Influence among Issues within one Agent

Secondly, assume the matrix of priority of influence in the vertical level, namely among different issues but within one agent i :

$$\mathbb{PR}_{(i)} = \mathbb{RE}_{(i)} \times \mathbb{SW}_{(i)} \times \mathbb{SP}_{(i)} \times \mathbb{PF}_{(i)} \quad (\text{B.73})$$

If expressed in the general form, $\mathcal{MT}_{(i)}$ is the set of matrix of all the contributing metrics to the priority of influence among issues within agent i , $\mathbb{MT}_{(i)} \in \mathcal{MT}_{(i)}$.

$$\mathbb{PR}_{(i)} = \prod_{\mathbb{MT}_{(i)} \in \mathcal{MT}_{(i)}} \mathbb{MT}_{(i)} \quad (\text{B.74})$$

The $\mathbb{PR}_{(i)}$ is the matrix of priorities of influence among issues within the agent i , in which the value $pr_{q,k(i)}$ is the priority of influence from issue q to issue k (in another word, the priority of dependency of issue k on issue q) for the agent i , if $pr_{q,k(i)} = 0$, which means there is no influence from issue q to issue k , but if $pr_{q,k(i)} \neq 0$, which means there is influence from issue q to issue k , and the bigger of the absolute value $|pr_{q,k(i)}|$, the higher of the priority of influence. The matrix of priority of influence among issues within agent i could be expressed as following:

$$\mathbb{PR}_{(i)} = \begin{bmatrix} pr_{1,1(i)} & pr_{1,2(i)} & \dots & pr_{1,m(i)} \\ pr_{2,1(i)} & \dots & \dots & pr_{2,m(i)} \\ \dots & \dots & \dots & \dots \\ pr_{m,1(i)} & pr_{m,2(i)} & \dots & pr_{m,m(i)} \end{bmatrix} \quad (\text{B.75})$$

As assumed, only the former issue could influence the latter issue, and can not be reversed.

$$pr_{q,k(i)} = 0 \text{ if } q > k \quad (\text{B.76})$$

On the influenced issue k for the agent i ($i \in \mathbb{N}$), would be influenced by the influencing issue q ($q, k \in \mathbb{M}$) with the highest priority of influence:

$$P_{(k)}(i) = \text{argmax}[pr_{q,k(i)}, P_{(q)}(i)] \quad (\text{B.77})$$

The Priority of Influence in the Whole Picture

At last, assume the matrix of priority of influence in the whole picture, crossing all agents and issues:

$$\mathbb{PR} = \mathbb{RE} \times \mathbb{SW} \times \mathbb{SP} \times \mathbb{PF} \quad (\text{B.78})$$

And if in general form, \mathcal{MT} is the set of matrix of all the contributing metrics to the priority of influence in the whole picture, $\mathbb{MT} \in \mathcal{MT}$.

$$\mathbb{PR} = \prod_{\mathbb{MT} \in \mathcal{MT}} \mathbb{MT} \quad (\text{B.79})$$

The matrix of priority of influence in the whole picture could be expressed as following:



Figure B.10: One Example of Intra-influence from More than one Decisions

$$\mathbb{PR} = \begin{bmatrix} pr_{1,1(1,1)} & \cdots & pr_{1,1(m,m)} \\ \cdots & \cdots & \cdots \\ pr_{n,n(1,1)} & \cdots & pr_{n,n(m,m)} \end{bmatrix} \quad (\text{B.80})$$

As assumed, only variable on the former issue could influence variable on the latter issue, and can not be reversed.

$$pr_{j,i(q,k)} = 0 \text{ if } q > k \quad (\text{B.81})$$

For the influenced variable $x_{i,k}$ ($i \in \mathbb{N}$, $k \in \mathbb{M}$), would be influenced by the influencing variable $x_{j,q}$ ($j \in \mathbb{M}$, $q \in \mathbb{M}$) with the highest priority of influence:

$$P_{(k)}(i) = \text{argmax}[pr_{j,i(q,k)}, P_{(q)}(j)] \quad (\text{B.82})$$

B.4 Complementary Illustration of Weight of Influence

Intra-influence from More than one Origins

Intra-influence from More than one Decisions

For one specific example of intra-influence from more than one (individual) decisions/preferences, among issues q, d, k of agent i (as shown in the figure B.10), the decision/preference of agent i on issue k is simultaneously influenced by/dependent on his two former issues q and d , and combining with the influence from issue k itself. $\{k, q, d\} \subseteq \mathbb{M}$, $i \in \mathbb{N}$.

$$\begin{aligned} s.t. & |wt_{q,k(i)}| + |wt_{d,k(i)}| + |wt_{k,k(i)}| = 1 \\ P_{(k)}(i) & = Pa(wt_{q,k(i)}P_{(q)}(i), wt_{d,k(i)}P_{(d)}(i), wt_{k,k(i)}P_{(k)}(i)) \end{aligned} \quad (\text{B.83})$$

If expressed in the general form, the decision/preference on issue k of agent i would be influenced by all his former issues $m \in \mathbb{M}$ with the weight of influence on issue k $wt_{m,k(i)} \neq 0$.

$$\begin{aligned} s.t. & \sum_{m \in \mathbb{M}} |wt_{m,k(i)}| = 1 \\ P_{(k)}(i) & = Pa\left(\sum_{m \in \mathbb{M}} wt_{m,k(i)}P_{(m)}(i)\right) \end{aligned} \quad (\text{B.84})$$

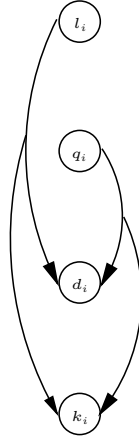


Figure B.11: One Example of Intra-influence from More than one Structures

Intra-influence from More than one Structures

For one specific example of intra-influence from more than one structures, among issues l, q, d of agent i (as shown in the figure B.11), the decision/preference of agent i on issue k is simultaneously influenced by two structures of influence/dependency among his former decisions/preferences (on issue d dependent on issue l and on issue d dependent on issue q , shortly as $l \rightarrow d$ and $q \rightarrow d$). Both consider the individual influence from issues (l, q, d and k itself) and the influential effect of structures among issues ($l \rightarrow d, q \rightarrow d$), the formula would be (in which ϕ represents the function of influential effect of structure among issues, and $wt_{xy,z}$ represents the weight of influence from structure $x \rightarrow y$ to z):

$$\begin{aligned}
 s.t. wt_{l,k(i)} + wt_{q,k(i)} + wt_{d,k(i)} + wt_{ld,k(i)} + wt_{qd,k(i)} + wt_{k,k(i)} &= 1 \\
 P_{(k)}(i) &= In\{wt_{l,k(i)}P_{(l)}(i), wt_{q,k(i)}P_{(q)}(i), wt_{d,k(i)}P_{(d)}(i), \\
 &wt_{ld,k(i)}\phi[P_{(l)}(i), P_{(d)}(i)], wt_{qd,k(i)}\phi[P_{(q)}(i), P_{(d)}(i)], \\
 &wt_{k,k(i)}P_{(k)}(i)\}
 \end{aligned} \tag{B.85}$$

If expressed in the general form, $\mathbb{S}[\mathbb{M}]_{(i)}$ is the set of structures of influence/dependency among issues \mathbb{M} within agent i , $\mathfrak{s} \in \mathbb{S}[\mathbb{M}]_{(i)}$, $\hat{m}_{\mathfrak{s}}$ and $\check{m}_{\mathfrak{s}}$ are respectively the influencing/dependent by issue and influenced/dependent issue in a structure of influence/dependency \mathfrak{s} . $\hat{m}_{\mathfrak{s}} \in \mathfrak{s}, \check{m}_{\mathfrak{s}} \in \mathfrak{s}, \{\hat{m}_{\mathfrak{s}}, \check{m}_{\mathfrak{s}}, \hat{m}_{\mathfrak{s}} \rightarrow \check{m}_{\mathfrak{s}}\} = \mathfrak{s}$.

$$\begin{aligned}
 s.t.s.t. \sum_{m \in \mathbb{M}} |wt_{m,k(i)}| + \sum_{\mathfrak{s} \in \mathbb{S}[\mathbb{M}]_{(i)}} |wt_{\mathfrak{s},k(i)}| &= 1 \\
 P_{(k)}(i) &= In\left\{\sum_{m \in \mathbb{M}} wt_{m,k(i)}P_{(m)}(i), \sum_{\mathfrak{s} \in \mathbb{S}[\mathbb{M}]_{(i)}} wt_{\mathfrak{s},k(i)}\phi[P_{(\hat{m}_{\mathfrak{s}})}(i), P_{(\check{m}_{\mathfrak{s}})}(i)]\right\} \text{ or} \\
 &In\left\{\sum_{m \in \mathbb{M}} wt_{m,k(i)}P_{(m)}(i), \sum_{\mathfrak{s} \in \mathbb{S}[\mathbb{M}]_{(i)}} wt_{\mathfrak{s},k(i)}\phi[\mathfrak{s}]\right\}
 \end{aligned} \tag{B.86}$$

Inter-influence from More than one Origins

Inter-influence from More than one Decisions

For one specific example of inter-influence from more than one (individual) decisions/preferences, among agent g, h, j on issue k (as shown in the figure B.12), the decision/preference of agent j on

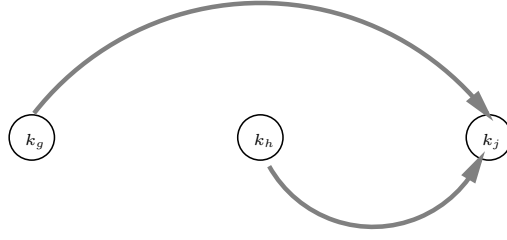


Figure B.12: One Example of Inter-influence from More than one Decisions

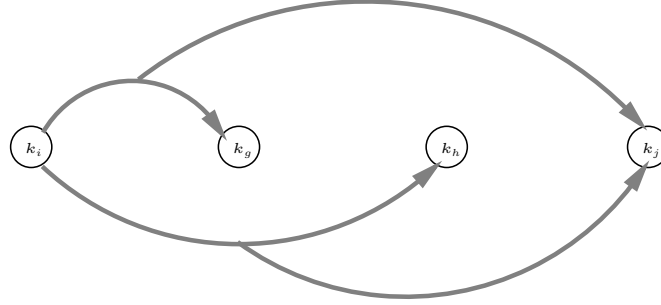


Figure B.13: One Example of Inter-influence from More than one Structures

issue k is simultaneously influenced by two agents g and h , and combining with influence from agent j own. $\{g, h, j\} \subseteq \mathbb{N}, k \in \mathbb{M}$.

$$s.t. |wt_{g,j(k)}| + |wt_{h,j(k)}| + |wt_{j,j(k)}| = 1 \quad (B.87)$$

$$P_{(k)}(j) = In(wt_{g,j(k)}P_{(k)}(g), wt_{h,j(k)}P_{(k)}(h), wt_{j,j(k)}P_{(k)}(j))$$

If expressed in the general form, the decision/preference of agent j on issue k , would be influenced by all influencing agents $n \in \mathbb{N}$ with the weight of influence on agent j $wt_{n,j(k)} \neq 0$.

$$s.t. \sum_{n \in \mathbb{N}} |wt_{n,j(k)}| = 1 \quad (B.88)$$

$$P_{(k)}(j) = In\left(\sum_{n \in \mathbb{N}} wt_{n,j(k)}P_{(k)}(n)\right)$$

Inter-influence from More than one Structures

For one specific example of inter-influence from more than one structures, among agents i, g, h, j on issue k (as shown in figure B.13), the decision/preference of agent j on issue k is simultaneously influenced by two structures of influence among other agents' decisions/preferences (agent i influenced agent g and agent i influenced agent h , shortly as $i \rightarrow g$ and $i \rightarrow h$). Both consider the individual influence from agents i, g, h and j own and the influential effect of structures among agents ($i \rightarrow g, i \rightarrow h$), the formula would be (in which φ represents the function of influential effect of structure among agents):

$$s.t. wt_{i,j(k)} + wt_{g,j(k)} + wt_{h,j(k)} + wt_{ig,j(k)} + wt_{ih,j(k)} + wt_{j,j(k)} = 1$$

$$P_{(k)}(j) = In\{wt_{i,j(k)}P_{(k)}(i), wt_{g,j(k)}P_{(k)}(g), wt_{h,j(k)}P_{(k)}(h),$$

$$wt_{ig,j(k)}\varphi[P_{(k)}(i), P_{(k)}(g)], wt_{ih,j(k)}\varphi[P_{(k)}(i), P_{(k)}(h)],$$

$$wt_{j,j(k)}, P_{(k)}(j)\} \quad (B.89)$$

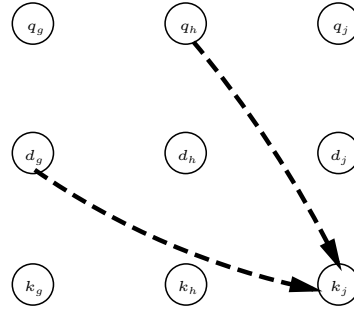


Figure B.14: One Example of Influence from More than one Decisions

If expressed in the general form, $\mathbb{S}[\mathbb{N}]_{(k)}$ is the set of structures of influence among agents \mathbb{N} within issue k , $\mathfrak{s} \in \mathbb{S}[\mathbb{N}]_{(k)}$, $\hat{n}_{\mathfrak{s}}$ and $\check{n}_{\mathfrak{s}}$ are respectively the influencing agent and influenced agent in a structure of influence \mathfrak{s} . $\hat{n}_{\mathfrak{s}} \in \mathfrak{s}, \check{n}_{\mathfrak{s}} \in \mathfrak{s}, \{\hat{n}_{\mathfrak{s}}, \check{n}_{\mathfrak{s}}, \hat{n}_{\mathfrak{s}} \rightarrow \check{n}_{\mathfrak{s}}\} = \mathfrak{s}$.

$$\begin{aligned}
 & s.t. \sum_{n \in \mathbb{N}} |wt_{n,j(k)}| + \sum_{\mathfrak{s} \in \mathbb{S}[\mathbb{N}]_{(k)}} |wt_{\mathfrak{s},j(k)}| = 1 \\
 P_{(k)}(j) &= In\left\{ \sum_{n \in \mathbb{N}} wt_{n,j(k)} P_{(k)}(n), \sum_{\mathfrak{s} \in \mathbb{S}[\mathbb{N}]_{(k)}} wt_{\mathfrak{s},j(k)} \varphi[P_{(k)}(\hat{n}_{\mathfrak{s}}), P_{(k)}(\check{n}_{\mathfrak{s}})] \right\} \text{ or} \\
 & In\left\{ \sum_{n \in \mathbb{N}} wt_{n,j(k)} P_{(k)}(n), \sum_{\mathfrak{s} \in \mathbb{S}[\mathbb{N}]_{(k)}} wt_{\mathfrak{s},j(k)} \varphi[\mathfrak{s}] \right\}
 \end{aligned} \tag{B.90}$$

Influence from More than one Origins

Influence from More than one Decisions

For one specific example of influence from more than one (individual) decisions/preferences, among issue q, d, k and among agents g, h, j (as shown in figure B.14), the variable k_j (decision/preference of agent j on issue k) is simultaneously influenced by two variables d_g and q_h (decisions/preferences of agent g on issue d and of agent h on issue q), and combining with influence from variable k_j itself. $\{q, d, k\} \subseteq \mathbb{M}$, $\{g, h, j\} \subseteq \mathbb{N}$, and PI is the common function for influence simultaneously crossing agents and issues (including $Pa_{\perp}In$ and $In_{\rightarrow}Pa$).

$$\begin{aligned}
 & s.t. |wt_{g,j(d,k)}| + |wt_{h,j(q,k)}| + |wt_{j,j(k,k)}| = 1 \\
 P_{(k)}(j) &= PI(wt_{g,j(d,k)} P_{(d)}(g), wt_{h,j(q,k)} P_{(q)}(h), wt_{j,j(k,k)} P_{(k)}(j))
 \end{aligned} \tag{B.91}$$

If expressed in the general form, the variable k_j would be influenced by all influencing variables m_n ($m \in \mathbb{M}, n \in \mathbb{N}$) with the weight of influence on variable k_j $wt_{n,j(m,k)} \neq 0$.

$$\begin{aligned}
 & s.t. \sum_{n \in \mathbb{N}} \sum_{m \in \mathbb{M}} |wt_{n,j(m,k)}| = 1 \\
 P_{(k)}(j) &= PI\left(\sum_{n \in \mathbb{N}} \sum_{m \in \mathbb{M}} wt_{n,j(m,k)} P_{(m)}(n) \right)
 \end{aligned} \tag{B.92}$$

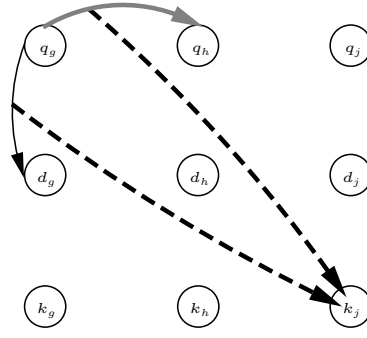


Figure B.15: One Example of Influence from More than one Structures

Influence from More than one Structures

For one specific example of influence from more than one structures, among issues q, d, k and among agents g, h, j (as shown in figure B.15), the variable k_j (decision/preference of agent j on issue k) is simultaneously influenced by two structures of influence among variables (variable q_g influenced d_g and variable q_g influenced q_h , shortly as $q_g \rightarrow d_g$ and variable $q_g \rightarrow q_h$). The formula would be (in which ψ represents the function of influential effect from structure crossing both issues and agents):

$$\begin{aligned}
 s.t. & wt_{g,j(q,k)} + wt_{g,j(d,k)} + wt_{h,j(q,k)} + wt_{gg,j(qd,k)} + wt_{gh,j(qq,k)} + wt_{j,j(k,k)} = 1 \\
 P_{(k)}(j) &= PI\{wt_{g,j(q,k)}P_{(q)}(g), wt_{g,j(d,k)}P_{(d)}(g), wt_{h,j(q,k)}P_{(q)}(h), \\
 & wt_{gg,j(qd,k)}\phi[P_{(q)}(g), P_{(d)}(g)], wt_{gh,j(qq,k)}\psi[P_{(q)}(g), P_{(q)}(h)], \\
 & wt_{j,j(k,k)}, P_{(k)}(j)\}
 \end{aligned} \tag{B.93}$$

If expressed in the general form, $\mathbb{S}[\mathbb{N} \times \mathbb{M}]$ is the set of structures of influence in the whole picture, $\mathfrak{s} \in \mathbb{S}[\mathbb{N} \times \mathbb{M}]$; $\mathbb{S}[\mathbb{N}]$ is the agent/inter dimension of set of structures of influence (among agents \mathbb{N}), \mathfrak{s}_n is the agent dimension of the structure \mathfrak{s} , $\mathfrak{s}_n \in \mathbb{S}[\mathbb{N}]$; $\mathbb{S}[\mathbb{M}]$ is the issue/intra dimension of set of structures of influence (among issues \mathbb{M}), \mathfrak{s}_m is the issue dimension of the structure \mathfrak{s} , $\mathfrak{s}_m \in \mathbb{S}[\mathbb{M}]$. $\hat{n}_{\mathfrak{s}_n}$ and $\check{n}_{\mathfrak{s}_n}$ are respectively the agent coordinate of the influencing and influenced variables in the structure \mathfrak{s} , and $\hat{m}_{\mathfrak{s}_m}$ and $\check{m}_{\mathfrak{s}_m}$ are respectively the issue coordinate of the influencing and influenced variables in the structure \mathfrak{s} . $\hat{n}_{\mathfrak{s}_n}, \check{n}_{\mathfrak{s}_n} \in \mathfrak{s}_n$, $\{\hat{n}_{\mathfrak{s}_n}, \check{n}_{\mathfrak{s}_n}, \hat{n}_{\mathfrak{s}_n} \rightarrow \check{n}_{\mathfrak{s}_n}\} = \mathfrak{s}_n$, $\hat{m}_{\mathfrak{s}_m}, \check{m}_{\mathfrak{s}_m} \in \mathfrak{s}_m$, $\{\hat{m}_{\mathfrak{s}_m}, \check{m}_{\mathfrak{s}_m}, \hat{m}_{\mathfrak{s}_m} \rightarrow \check{m}_{\mathfrak{s}_m}\} = \mathfrak{s}_m$, $\{\mathfrak{s}_n, \mathfrak{s}_m\} = \mathfrak{s}$ or $\{\hat{n}_{\mathfrak{s}_n}, \hat{m}_{\mathfrak{s}_m}, \check{n}_{\mathfrak{s}_n}, \check{m}_{\mathfrak{s}_m}, \{\hat{n}_{\mathfrak{s}_n}, \hat{m}_{\mathfrak{s}_m}\} \rightarrow \{\check{n}_{\mathfrak{s}_n}, \check{m}_{\mathfrak{s}_m}\}\} = \mathfrak{s}$.

$$\begin{aligned}
 s.t. & \sum_{n \in \mathbb{N}}^{m \in \mathbb{M}} |wt_{n,j(m,k)}| + \sum_{\mathfrak{s}_n \in \mathbb{S}[\mathbb{N}]}^{\mathfrak{s}_m \in \mathbb{S}[\mathbb{M}]} |wt_{\mathfrak{s}_n,j(\mathfrak{s}_m,k)}| = 1 \\
 P_{(k)}(j) &= PI\left\{\sum_{n \in \mathbb{N}}^{m \in \mathbb{M}} wt_{n,j(m,k)}P_{(m)}(n), \sum_{\mathfrak{s}_n \in \mathbb{S}[\mathbb{N}]}^{\mathfrak{s}_m \in \mathbb{S}[\mathbb{M}]} wt_{\mathfrak{s}_n,j(\mathfrak{s}_m,k)}\psi[P_{(\hat{m}_{\mathfrak{s}_m})}(\hat{n}_{\mathfrak{s}_n}), P_{(\check{m}_{\mathfrak{s}_m})}(\check{n}_{\mathfrak{s}_n})]\right\} \text{ or} \\
 & PI\left\{\sum_{n \in \mathbb{N}}^{m \in \mathbb{M}} wt_{n,j(m,k)}P_{(m)}(n), \sum_{\mathfrak{s}_n \in \mathbb{S}[\mathbb{N}]}^{\mathfrak{s}_m \in \mathbb{S}[\mathbb{M}]} wt_{\mathfrak{s}_n,j(\mathfrak{s}_m,k)}\psi[\mathfrak{s}]\right\}
 \end{aligned} \tag{B.94}$$

As above, the structure could be represented in two forms, one is displaying the specific details (indicating the influencing variable $\{agent, issue\}$ and influenced variable $\{agent, issue\}$ in the structure, $[P_{(\hat{m}_{\mathfrak{s}_m})}(\hat{n}_{\mathfrak{s}_n}), P_{(\check{m}_{\mathfrak{s}_m})}(\check{n}_{\mathfrak{s}_n})]$), and the other is just abbreviated expressed as \mathfrak{s} .

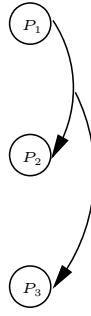


Figure B.16: One Example of the Effect of Structure among Features

B.5 The Influential Effect from Structure among Issues

There is a **Preassumption** that people usually have idea about own former decisions, sometimes you might feel regretful or sad about your past decisions (as it brings bad outcome or undesired payoff), and perceive them as bad example or negative reference for latter decisions, but sometimes you might feel grateful or glad about your past decisions (as it brings good outcome or desired payoff), and perceive them as good example or positive reference for latter decisions. Actually, your idea about own former choices is also more a subjective cognition than an objective fact. Sometimes your judgement is self-righteous or shortsighted, the wise choice from your perspective would be a very bad choice in result (especially in the long term). Though the perspective about about former decisions might be deviated the reality, or from own genuine interest, but it doesn't matter the fact that the agent's behavior is determined by his own thoughts, whether it is right or wrong to the reality. As the cognition about the former decision is a subjective concept but not an objective concept, therefore, usually every agent has the full information about it.

To deal with the influence from structure among issues, for the most basic case (as shown in figure 7.12), there are three issues (or features) 1,2,3, and issue 3 is dependent on (or influenced by) the two issues 1 and 2 with the influencing structure from issue 1 to issue 2¹³. One specific example is the agent make decisions on issue 2 based on issue 1.

If considering the structure among the influencing features, there are different perspectives or different thoughts to understand and deal with the influential effect from structure, as human minds are natively very complicated, different people have varied personalities and sense of values, and even for one same person the cognition would be different under evolving environments and situations.

For one angle of view, the decision-maker would think that the issue 1 is the parent-issue for other issues (influenced other issues or be dependent by other issues) and indeed very important, then would be inclined to make decisions on latter issue 3 referring more to issue 1, and also the agent hope to be consistent on the decision-making, for the former issue 2 is made based on issue 1, then the latter issue 3 should be made 5 to be persistently on issue 1. Then the decision-maker would put more weight on the issue 1 be dependent by. Expressed in one simple and specific way, in which ϕ means the effect of structure on the influenced preference.

$$\begin{aligned}
 P_3 &= wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}\phi[P_1, P_2] \\
 &= wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}P_1 \\
 &\text{if } wt_{12,3} = wt_{1,3} \text{ then} \\
 &= 2wt_{1,3}P_1 + wt_{2,3}P_2
 \end{aligned}
 \tag{B.95}$$

¹³The issue 2 is dependent on (or influenced by) issue 1.

If in the general form, influenced by all issues with the weight of influence $\neq 0$ and the influencing structure among them. $\mathbb{S}[\mathbb{M}]_{(i)}$ is the set of structures of influence/dependencies among the issues \mathbb{M} within agent i . $\mathfrak{s} \in \mathbb{S}[\mathbb{M}]_{(i)}$, as a most basic structure (one issue be influenced by/be dependent on one issue), \mathfrak{s} is composed by two issues $\hat{m}_{\mathfrak{s}}$ and $\check{m}_{\mathfrak{s}}$, the former is the influencing issue in the structure \mathfrak{s} , and the latter is the influenced issue in the structure \mathfrak{s} , $\hat{m}_{\mathfrak{s}} \in \mathfrak{s}$, $\check{m}_{\mathfrak{s}} \in \mathfrak{s}$, $\{\hat{m}_{\mathfrak{s}}, \check{m}_{\mathfrak{s}}\} = \mathfrak{s}$.

$$\begin{aligned} P_{(k)}(i) &= In\left\{\sum_{m \in \mathbb{M}} wt_{m,k(i)} P_{(m)}(i), \sum_{\mathfrak{s} \in \mathbb{S}[\mathbb{M}]_{(i)}} wt_{\mathfrak{s},k(i)} \phi[P_{(\hat{m}_{\mathfrak{s}})}(i), P_{(\check{m}_{\mathfrak{s}})}(i)]\right\} \\ &= In\left\{\sum_{m \in \mathbb{N}} wt_{m,k(i)} P_{(m)}(i), \sum_{\mathfrak{s} \in \mathbb{S}[\mathbb{M}]_{(i)}} wt_{\mathfrak{s},k(i)} P_{(\hat{m}_{\mathfrak{s}})}(i)\right\} \end{aligned} \quad (\text{B.96})$$

For another angle of view, the decision-maker notice on the influenced issue but not the influencing issue, deems that issue 2 is inessential because it is determined by other issues (but not on own), then it is possible to “depreciate” the influence from issue 2, making decisions referring less to issue 2.

$$\begin{aligned} P_3 &= wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}\phi[P_1, P_2] \\ &= wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_2 \\ &\text{if } wt_{12,3} = wt_{2,3} \text{ then} \\ &= wt_{1,3}P_1 \end{aligned} \quad (\text{B.97})$$

If in the general form, from the perspective of depreciating the influence from “issue dependent on others”, the influenced preference would be:

$$\begin{aligned} P_{(k)}(i) &= In\left\{\sum_{m \in \mathbb{M}} wt_{m,k(i)} P_{(m)}(i), \sum_{\mathfrak{s} \in \mathbb{S}[\mathbb{M}]_{(i)}} wt_{\mathfrak{s},k(i)} \phi[P_{(\hat{m}_{\mathfrak{s}})}(i), P_{(\check{m}_{\mathfrak{s}})}(i)]\right\} \\ &= In\left\{\sum_{m \in \mathbb{N}} wt_{m,k(i)} P_{(m)}(i), - \sum_{\mathfrak{s} \in \mathbb{S}[\mathbb{M}]_{(i)}} wt_{\mathfrak{s},k(i)} P_{(\check{m}_{\mathfrak{s}})}(i)\right\} \end{aligned} \quad (\text{B.98})$$

The Interaction among Positive and Negative Dependencies

However, above assume the influencing (or dependent) relations just as positive, but not consider about the mixing effect among positive influence and negative influence. In real world, it would be much more complicated, the influence or dependency among issues might be strong or weak, positive or negative. For some issues, the decisions are “successful” (as expected) and bring good outcome or desired payoff, but for some other issues, the decisions are “failed” (not as expected) and bring bad outcome or undesired payoff. And usually those successful issues would provides good examples and positive references for latter issues’ decision, but those failed issues would provide bad examples and negative references for latter issues’ decision. By considering the interaction among positive dependencies and negative dependencies, the effect of structure would be more complex. For above example, the influential effect of the structure (from issue 1 to issue 2) on issue 3 is actually affected by at least three different bilateral relations, between issue 1 and issue 2, between issue 1 and issue 3, and between issue 2 and issue 3 (all from the perspective of the decision on issue 3), it is about how the decision-maker perceive the reference from issue 1 and issue 2 to issue 3. Considering more varied combinations of influencing relations, as in the most basic model, there are three relations, with binary states (positive and negative), then there are 8 ($2 \times 2 \times 2$) variations of combinations in total. The specific procedures of 8 variations can be found in appendix B.6.

The Results Table for the Influential Effect of Basic Structure among Issues Summary the influential effects from basic structures by all combinations as below, the first column is the combinations of bilateral relations among the basic structure, the second column is the influenced result if not considering the influential effect from structure (as the control group), the third column is the influenced result if perceiving the effect from structure as focusing on the influencing agent, and the last column is the influenced result if perceiving the effect from structure as focusing on the influenced agent.

	<i>control</i>	<i>influence</i>	<i>influenced</i>
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} \\ \\ \end{matrix} \right) \\ 2 & \left(\begin{matrix} + \\ \\ \end{matrix} \right) \\ 3 & \left(\begin{matrix} - & + \\ \\ \end{matrix} \right) \end{matrix}$	$-wt_{1,3}P_1 + wt_{2,3}P_2$	$-wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}P_1$	$-wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_2$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} \\ \\ \end{matrix} \right) \\ 2 & \left(\begin{matrix} + \\ \\ \end{matrix} \right) \\ 3 & \left(\begin{matrix} + & - \\ \\ \end{matrix} \right) \end{matrix}$	$wt_{1,3}P_1 - wt_{2,3}P_2$	$wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}P_1$	$wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_2$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} \\ \\ \end{matrix} \right) \\ 2 & \left(\begin{matrix} - \\ \\ \end{matrix} \right) \\ 3 & \left(\begin{matrix} - & + \\ \\ \end{matrix} \right) \end{matrix}$	$-wt_{1,3}P_1 + wt_{2,3}P_2$	$-wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{1,3}P_1$	$-wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}P_2$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} \\ \\ \end{matrix} \right) \\ 2 & \left(\begin{matrix} - \\ \\ \end{matrix} \right) \\ 3 & \left(\begin{matrix} + & - \\ \\ \end{matrix} \right) \end{matrix}$	$wt_{1,3}P_1 - wt_{2,3}P_2$	$wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_1$	$wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}P_2$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} \\ \\ \end{matrix} \right) \\ 2 & \left(\begin{matrix} + \\ \\ \end{matrix} \right) \\ 3 & \left(\begin{matrix} + & + \\ \\ \end{matrix} \right) \end{matrix}$	$wt_{1,3}P_1 + wt_{2,3}P_2$	$wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}P_1$	$wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_2$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} \\ \\ \end{matrix} \right) \\ 2 & \left(\begin{matrix} + \\ \\ \end{matrix} \right) \\ 3 & \left(\begin{matrix} - & - \\ \\ \end{matrix} \right) \end{matrix}$	$-wt_{1,3}P_1 - wt_{2,3}P_2$	$-wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}P_1$	$-wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_2$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} \\ \\ \end{matrix} \right) \\ 2 & \left(\begin{matrix} - \\ \\ \end{matrix} \right) \\ 3 & \left(\begin{matrix} + & + \\ \\ \end{matrix} \right) \end{matrix}$	$wt_{1,3}P_1 + wt_{2,3}P_2$	$wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_1$	$wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_2$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} \\ \\ \end{matrix} \right) \\ 2 & \left(\begin{matrix} - \\ \\ \end{matrix} \right) \\ 3 & \left(\begin{matrix} - & - \\ \\ \end{matrix} \right) \end{matrix}$	$-wt_{1,3}P_1 - wt_{2,3}P_2$	$-wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_1$	$-wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_2$

B.6 The Structure Effect by Combinations of Positive/Negative Bilateral Relations

The Influential Effect from Structure among Agents

Combination 1 Friend Follows Foe For one example, if from the perspective of agent 3, agent 2 is the friend of agent 3, while agent 1 is the foe of agent 3, and agent 2 follows agent 1 (agent 2

be positively influenced by agent 1), which means that agent 3 observes that his friend follows his foe, this would bring bad “mood”. For example, when you observe your best friend goes with your enemy, it would likely be deemed as a betray by you, and highly possible to take revenge. The combination of relations among agents 1,2,3 could be expressed by the matrix below, in which the horizontal dimension is the influencing agents, and the vertical dimension is the influenced agents.

$$\begin{array}{c} \begin{array}{ccc} & 1 & 2 & 3 \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \left(\begin{array}{ccc} & & \\ + & & \\ - & + & \end{array} \right) \end{array} \end{array} \quad (\text{B.99})$$

If not considering the structure, the influence from agent 1 (as foe) to agent 3 is negative, and the influence from agent 2 (as friend) to agent 3 is positive.

$$P_3 = -wt_{1,3}P_1 + wt_{2,3}P_2 \quad (\text{B.100})$$

If in the general form, to indicate the different influences from friends or foes (positive influence or negative influence), \mathbb{N}^+ is the set of all influencing agents with the weight >0 , $n^+ \in \mathbb{N}^+$, and \mathbb{N}^- is the set of all influencing agents with the weight <0 , $n^- \in \mathbb{N}^-$.

$$P_{(k)}(j) = In\left\{ \sum_{n^+ \in \mathbb{N}^+} wt_{n^+,j(k)}P_{(k)}(n^+), - \sum_{n^- \in \mathbb{N}^-} wt_{n^-,j(k)}P_{(k)}(n^-) \right\} \quad (\text{B.101})$$

But if considering the influential effect from structure (which is negative as friend follows foe will bring a bad feelings), the influenced agent would take actions against the “bad” situation, then how to take revenge, or take revenge on who, then there would be different angles of view, different people would have different thoughts and focus, then leading to different behaviors and choices.

For one angle of view of the influenced agent, it is to take revenge on his foe (who influenced his friend), the agent would hate the foe more deep, and oppose more to him, in the sense of preference, it is to be more far (or more different) from the preference of the foe. ($wt_{x,y}$ is defined as the absolute value for the weight of influence from agent x to agent y):

$$\begin{aligned} P_3 &= -wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}\varphi[-P_1, P_2] \\ &= -wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_1 \\ &\text{if } wt_{12,3} = wt_{1,3} \text{ then} \\ &= -2wt_{1,3}P_1 + wt_{2,3}P_2 \end{aligned} \quad (\text{B.102})$$

If represented in the general form, $\mathbb{S}[\mathbb{N}]^+_{(k)}$ is the set of structures of positive influence on feature k , $\mathfrak{s}^+ \in \mathbb{S}[\mathbb{N}]^+_{(k)}$, $\hat{n}_{\mathfrak{s}^+}^-$ means the influencing agent in the positive structure \mathfrak{s}^+ is the foe of influenced agent, and $\check{n}_{\mathfrak{s}^+}^+$ means the influenced agent in the positive structure \mathfrak{s}^+ is the friend of influenced agent, $\{\hat{n}_{\mathfrak{s}^+}^-, \check{n}_{\mathfrak{s}^+}^+, \hat{n}_{\mathfrak{s}^+}^- \rightarrow \check{n}_{\mathfrak{s}^+}^+\} = \mathfrak{s}^+$. The influential effect from structure (friend follows foe) is expressed as the intensifying the negative influence from the foe.

$$\begin{aligned} P_{(k)}(j) &= In\left\{ \sum_{n^+ \in \mathbb{N}^+} wt_{n^+,j(k)}P_{(k)}(n^+), - \sum_{n^- \in \mathbb{N}^-} wt_{n^-,j(k)}P_{(k)}(n^-), \sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{N}]^+_{(k)}} wt_{\mathfrak{s}^+,j(k)}\varphi[-P_{(k)}(\hat{n}_{\mathfrak{s}^+}^-), P_{(k)}(\check{n}_{\mathfrak{s}^+}^+)] \right\} \\ &= In\left\{ \sum_{n^+ \in \mathbb{N}^+} wt_{n^+,j(k)}P_{(k)}(n^+), - \sum_{n^- \in \mathbb{N}^-} wt_{n^-,j(k)}P_{(k)}(n^-), \sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{N}]^+_{(k)}} wt_{\mathfrak{s}^+,j(k)}P_{(k)}(\hat{n}_{\mathfrak{s}^+}^-) \right\} \end{aligned} \quad (\text{B.103})$$

For another angle of view, it is to take revenge on his friend (who becomes the “betrayer”), the influenced agent would be disappointed and be inclined to depreciate or remove the positive influence from his friend, referring less to him.

$$\begin{aligned}
P_3 &= -wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}\varphi[-P_1, P_2] \\
&= -wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_2 \\
&\text{if } wt_{12,3} = wt_{2,3} \text{ then} \\
&= -wt_{1,3}P_1
\end{aligned} \tag{B.104}$$

If in the general form, the influential effect from structure (friend follows foe) is expressed as reducing the positive influence from the friend.

$$\begin{aligned}
P_{(k)}(j) &= In\left\{ \sum_{n^+ \in \mathbb{N}^+} wt_{n^+,j(k)}P_{(k)}(n^+), - \sum_{n^- \in \mathbb{N}^-} wt_{n^-,j(k)}P_{(k)}(n^-), \sum_{s^+ \in \mathbb{S}[\mathbb{N}]^+_{(k)}} wt_{s^+,j(k)}\varphi[-P_{(k)}(\hat{n}_{s^+}^-), P_{(k)}(\hat{n}_{s^+}^+)] \right\} \\
&= In\left\{ \sum_{n^+ \in \mathbb{N}^+} wt_{n^+,j(k)}P_{(k)}(n^+), - \sum_{n^- \in \mathbb{N}^-} wt_{n^-,j(k)}P_{(k)}(n^-), \sum_{s^+ \in \mathbb{S}[\mathbb{N}]^+_{(k)}} wt_{s^+,j(k)}P_{(k)}(\check{n}_{s^+}^+) \right\}
\end{aligned} \tag{B.105}$$

Combination 2 Foe Follows Friend But for another example (contrast to above) , if agent 2 is the foe of agent 3, and agent 1 instead is the friend of agent 3, and it is also agent 2 follows agent 1, which means that agent 3 observes that his foe follows his friend, this possibly bring good “mood” (especially in the context of collective decision-making, to order to win, and to get the preferred candidate be chosen, the agent would hope other agents’ preferences be more close to own or own friends). For example, when you observe your foe “cleanse the heart from sin” and supports your friend, it is possible to give reward or encourage. And the influence from structure (foe follows friend) would be positive.

$$\begin{array}{ccc}
& 1 & 2 & 3 \\
\begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \left(\begin{array}{cc} & \\ + & \\ + & - \end{array} \right) & &
\end{array} \tag{B.106}$$

But how to give reward, or reward on who, then there would be also different angles of view. For one angle of view, it is to reward on the influencing one (the friend) for the “reclaim”. Besides, it is also likely that my friend indeed has the better preference or the truth, even my foe follows him.

$$\begin{aligned}
P_3 &= wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}\varphi[P_1, -P_2] \\
&= wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_1 \\
&\text{if } wt_{12,3} = wt_{1,3} \text{ then} \\
&= 2wt_{1,3}P_1 - wt_{2,3}P_2
\end{aligned} \tag{B.107}$$

If in the general form, $s^+ \in \mathbb{S}[\mathbb{N}]^+_{(k)}$, $\hat{n}_{s^+}^+$ means the influencing agent in the positive structure s^+ is the friend of influenced agent, and $\check{n}_{s^+}^-$ means the influenced agent in the positive structure s^+ is the foe of influenced agent, $\{\hat{n}_{s^+}^+, \check{n}_{s^+}^-, \hat{n}_{s^+}^+ \rightarrow \check{n}_{s^+}^-\} = s^+$. The influential effect from structure (foe follows friend) is expressed as intensifying the positive influence from the friend.

$$\sum_{s^+ \in \mathbb{S}[\mathbb{N}]^+_{(k)}} wt_{s^+,j(k)}\varphi[P_{(k)}(\hat{n}_{s^+}^+), -P_{(k)}(\check{n}_{s^+}^-)] = \sum_{s^+ \in \mathbb{S}[\mathbb{N}]^+_{(k)}} wt_{s^+,j(k)}P_{(k)}(\hat{n}_{s^+}^+) \tag{B.108}$$

But for another angle, it is to reward on his foe (who be influenced by his friend), for “reform or return to the right way”.

$$\begin{aligned}
P_3 &= wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}\varphi[P_1, -P_2] \\
&= wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_2 \\
&\text{if } wt_{12,3} = wt_{2,3} \text{ then} \\
&= wt_{1,3}P_1
\end{aligned} \tag{B.109}$$

If in the general form, the influential effect from structure (foe follows friend) is expressed as mitigating the negative influence from the foe.

$$\sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{N}]^+_{(k)}} wt_{\mathfrak{s}^+,j(k)}\varphi[P_{(k)}(\hat{n}_{\mathfrak{s}^+}^+), -P_{(k)}(\check{n}_{\mathfrak{s}^+}^-)] = \sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{N}]^+_{(k)}} wt_{\mathfrak{s}^+,j(k)}P_{(k)}(\check{n}_{\mathfrak{s}^+}^-) \tag{B.110}$$

Combination 3 Friend Opposes to Foe However, it should be noticed that all above assume the influencing structure (between agent 1 and 2) is one agent positively influence the other agent (for a specific form, agent 2 follow agent 1), but how is the influencing structure of one agent negatively influence another agent, how would it affect the influential effect from structure?

For one example, if agent 1 negatively influence agent 2 (in a specific form, agent 2 oppose to agent 1 or vote against agent 1), and agent 2 is the friend of agent 3 and agent 1 is the foe of agent 3, which means that the influenced agent 3 observes that his friend oppose to his foe, this would possibly bring agent 3 good “mood” or encouragement, and the influential effect from the structure would be positive. For example, when you observe your friend stand by your side and together against your foe, it is highly possible for you to be motivated to also take actions.

$$\begin{array}{ccc}
& 1 & 2 & 3 \\
1 & \left(\begin{array}{cc} & \\ - & \end{array} \right) \\
2 & & & \\
3 & - & + &
\end{array} \tag{B.111}$$

But how to take actions and take actions on who, there would be different angles of views. For one angle, it is to further support the “good” friend, and refer positively more to the friend, represented as putting more weight on the influence from the friend.

$$\begin{aligned}
P_3 &= -wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}\varphi[-P_1, P_2] \\
&= -wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}P_2 \\
&\text{if } wt_{12,3} = wt_{2,3} \text{ then} \\
&= -wt_{1,3}P_1 + 2wt_{2,3}P_2
\end{aligned} \tag{B.112}$$

If in the general form, $\mathbb{S}[\mathbb{N}]^-_{(k)}$ is the set of structures of negative influence on feature k , $\mathfrak{s}^- \in \mathbb{S}[\mathbb{N}]^-_{(k)}$, $\hat{n}_{\mathfrak{s}^-}$ means the influencing agent in the negative structure \mathfrak{s}^- is the foe of influenced agent, and $\check{n}_{\mathfrak{s}^-}^+$ means the influenced agent in the negative structure \mathfrak{s}^- is the friend of influenced agent, $\{\hat{n}_{\mathfrak{s}^-}, \check{n}_{\mathfrak{s}^-}^+, \hat{n}_{\mathfrak{s}^-} \rightarrow \check{n}_{\mathfrak{s}^-}^+\} = \mathfrak{s}^-$. The influential effect from the structure (friends oppose to foe) could be expressed as intensifying the positive influence from the friend.

$$\sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{N}]^-_{(k)}} wt_{\mathfrak{s}^-,j(k)}\varphi[-P_{(k)}(\hat{n}_{\mathfrak{s}^-}), P_{(k)}(\check{n}_{\mathfrak{s}^-}^+)] = \sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{N}]^-_{(k)}} wt_{\mathfrak{s}^-,j(k)}P_{(k)}(\check{n}_{\mathfrak{s}^-}^+) \tag{B.113}$$

For another angle, it is to further oppose to the foe, and negatively refer more to the foe, to respond to his friend’s action, and stand closer by his friend, which could be represented as putting more negative weight on the influence from the foe.

$$\begin{aligned}
P_3 &= -wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}\varphi[-P_1, P_2] \\
&= -wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_1 \\
&\text{if } wt_{12,3} = wt_{1,3} \text{ then} \\
&= -2wt_{1,3}P_1 + wt_{2,3}P_2
\end{aligned} \tag{B.114}$$

If in the general form, the influential effect from the structure (friends oppose to foe) could also be expressed as intensifying the negative influence from the foe.

$$\sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{N}]^-(k)} wt_{\mathfrak{s}^-,j(k)}\varphi[-P_{(k)}(\hat{n}_{\mathfrak{s}^-}^-), P_{(k)}(\hat{n}_{\mathfrak{s}^-}^+)] = - \sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{N}]^-(k)} wt_{\mathfrak{s}^-,j(k)}P_{(k)}(\check{n}_{\mathfrak{s}^-}^-) \tag{B.115}$$

Combination 4 Foe Opposes to Friend For another example, agent 1 identically negatively influence agent 2, but on the contrary compared with above example, agent 2 is the foe of agent 3 and agent 1 is the friend of agent 3, which means that the influenced agent 3 observes that his foe oppose to his friend, this would possibly bring up the reaction of agent 3, to react this “risk” or “challenge”. And the influential effect from this structure would be negative.

$$\begin{matrix} & 1 & 2 & 3 \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{pmatrix} - & & \\ + & - & \end{pmatrix} & & \end{matrix} \tag{B.116}$$

There would be different angles of views, for one angle, it is to further support the good friend, and refer positively more to the friend, represented as putting more weight on the influence from the friend.

$$\begin{aligned}
P_3 &= wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}\varphi[P_1, -P_2] \\
&= wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_1 \\
&\text{if } wt_{12,3} = wt_{1,3} \text{ then} \\
&= 2wt_{1,3}P_1 - wt_{2,3}P_2
\end{aligned} \tag{B.117}$$

If in the general form, $\mathfrak{s}^- \in \mathbb{S}[\mathbb{N}]^-(k)$, $\hat{n}_{\mathfrak{s}^-}^+$ means the influencing agent in the negative structure \mathfrak{s}^- is the friend of influenced agent, and $\check{n}_{\mathfrak{s}^-}^-$ means the influenced agent in the negative structure \mathfrak{s}^- is the foe of influenced agent, $\{\hat{n}_{\mathfrak{s}^-}^+, \check{n}_{\mathfrak{s}^-}^-, \hat{n}_{\mathfrak{s}^-}^+ \rightarrow \check{n}_{\mathfrak{s}^-}^-\} = \mathfrak{s}^-$. The influential effect from the structure (foe oppose to friend) could be expressed as intensifying the positive influence from the friend.

$$\sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{N}]^-(k)} wt_{\mathfrak{s}^-,j(k)}\varphi[P_{(k)}(\hat{n}_{\mathfrak{s}^-}^+), -P_{(k)}(\check{n}_{\mathfrak{s}^-}^-)] = - \sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{N}]^-(k)} wt_{\mathfrak{s}^-,j(k)}P_{(k)}(\hat{n}_{\mathfrak{s}^-}^+) \tag{B.118}$$

For another different angle, it is to further oppose to the enemy, to take “penalty” on the foe, and negatively refer more to the foe.

$$\begin{aligned}
P_3 &= wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}\varphi[P_1, -P_2] \\
&= wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}P_2 \\
&\text{if } wt_{12,3} = wt_{2,3} \text{ then} \\
&= wt_{1,3}P_1 - 2wt_{2,3}P_2
\end{aligned} \tag{B.119}$$

If in the general form, the influential effect from the structure (foe oppose to friend) could be expressed as intensifying the negative influence from the foe.

$$\sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{N}]^-(k)} wt_{\mathfrak{s}^-, j(k)} \varphi[P_{(k)}(\hat{n}_{\mathfrak{s}^-}^+), -P_{(k)}(\check{n}_{\mathfrak{s}^-}^-)] = \sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{N}]^-(k)} wt_{\mathfrak{s}^-, j(k)} P_{(k)}(\check{n}_{\mathfrak{s}^-}^-) \quad (\text{B.120})$$

Combination 5 Friend Follows Friend Just four cases are discussed above, all different (or unmatch) in the relations between agent 1 and agent 3, and between agent 2 and agent 3, especially, either assume agent 1 is agent 3's friend, agent 2 is agent 3's foe, or agent 1 is agent 3's foe and agent 2 is agent 3's friend. Further, to discuss other four case with the identical relations for agent 1,3 and agent 2,3, which means that from the perspective of agent 3, either agent 1 and 2 are both friends, or both foes.

For one example, agent 2 is positively influenced by agent 1, and agent 1 and agent 2 are both friends of agent 3, which means that the agent 3 observes that his friend follows his friend, friends uniting for a common objective. And the influential effect from this structure would be positive.

$$\begin{array}{c} 1 \quad 2 \quad 3 \\ 1 \left(\begin{array}{cc} & \\ + & \\ + & + \end{array} \right) \end{array} \quad (\text{B.121})$$

One perspective is to reduce or ignore the influence from the “follower”, thinking one friend (agent 2) just follows another friend (agent 3), but without own mind.

$$\begin{aligned} P_3 &= wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}\varphi[P_1, P_2] \\ &= wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_2 \\ &\text{if } wt_{12,3} = wt_{2,3} \text{ then} \\ &= wt_{1,3}P_1 \end{aligned} \quad (\text{B.122})$$

If in the general form, $\mathfrak{s}^+ \in \mathbb{S}[\mathbb{N}]^+(k)$, $\hat{n}_{\mathfrak{s}^+}^+$ means the influencing agent in the positive structure \mathfrak{s}^+ is the friend of influenced agent, and $\check{n}_{\mathfrak{s}^+}^+$ means the influenced agent in the positive structure \mathfrak{s}^+ is the friend of influenced agent either, $\{\hat{n}_{\mathfrak{s}^+}^+, \check{n}_{\mathfrak{s}^+}^+, \hat{n}_{\mathfrak{s}^+}^+ \rightarrow \check{n}_{\mathfrak{s}^+}^+\} = \mathfrak{s}^+$. The influential effect from the structure (friend follows friend) could be expressed as weakening the positive influence from the follower friend.

$$\sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{N}]^+(k)} wt_{\mathfrak{s}^+, j(k)} \varphi[P_{(k)}(\hat{n}_{\mathfrak{s}^+}^+), P_{(k)}(\check{n}_{\mathfrak{s}^+}^+)] = - \sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{N}]^+(k)} wt_{\mathfrak{s}^+, j(k)} P_{(k)}(\check{n}_{\mathfrak{s}^+}^+) \quad (\text{B.123})$$

Another perspective is to increase or intensify the influence from “influencer”, thinking one friend (agent 1) has influential effect on the other friend (agent 2), because agent 1 might have wiser choice.

$$\begin{aligned} P_3 &= wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}\varphi[P_1, P_2] \\ &= wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}P_1 \\ &\text{if } wt_{12,3} = wt_{1,3} \text{ then} \\ &= 2wt_{1,3}P_1 + wt_{2,3}P_2 \end{aligned} \quad (\text{B.124})$$

If in the general form, the influential effect from the structure (friend follows friend) could be expressed as intensifying the positive influence from the influencer friend.

$$\sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{N}]^+_{(k)}} wt_{\mathfrak{s}^+, j(k)} \varphi[P_{(k)}(\hat{n}_{\mathfrak{s}^+}^+), P_{(k)}(\check{n}_{\mathfrak{s}^+}^+)] = \sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{N}]^+_{(k)}} wt_{\mathfrak{s}^+, j(k)} P_{(k)}(\hat{n}_{\mathfrak{s}^+}^+) \quad (\text{B.125})$$

Combination 6 Foe Follows Foe In this example, agent 2 is positively influenced by agent 1 (or specifically, agent 2 follows agent 1), but agent 1 and agent 2 are both foes of agent 3, which means that the agent 3 observes that his foe follows his foe, both foes are “in tackling”, uniting against agent 3. Then it is likely for agent 3 to feel pressed, and would then take actions. The influential effect from this structure (foe follows foe) would be negative.

$$\begin{array}{c} 1 \quad 2 \quad 3 \\ 1 \left(\begin{array}{cc} + & \\ - & - \end{array} \right) \end{array} \quad (\text{B.126})$$

One perspective is to oppose less to the follower, although facing two foes at the same time, but one foe does not have own mind and just follow the other, it is likely for the influenced agent thinking that it is not very necessary to oppose the follower foe purposively.

$$\begin{aligned} P_3 &= -wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}\varphi[-P_1, -P_2] \\ &= -wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}P_2 \\ &\text{if } wt_{12,3} = wt_{2,3} \text{ then} \\ &= -wt_{1,3}P_1 - 2wt_{2,3}P_2 \end{aligned} \quad (\text{B.127})$$

If in the general form, $\mathfrak{s}^+ \in \mathbb{S}[\mathbb{N}]^+_{(k)}$, $\hat{n}_{\mathfrak{s}^+}^-$ means the influencing agent in the positive structure \mathfrak{s}^+ is the foe of influenced agent, and $\check{n}_{\mathfrak{s}^+}^-$ means the influenced agent in the positive structure \mathfrak{s}^+ is the foe of influenced agent either, $\{\hat{n}_{\mathfrak{s}^+}^-, \check{n}_{\mathfrak{s}^+}^-, \hat{n}_{\mathfrak{s}^+}^- \rightarrow \check{n}_{\mathfrak{s}^+}^-\} = \mathfrak{s}^+$. The influential effect from the structure (foe follows foe) could be expressed as weakening the negative influence from the follower foe.

$$\sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{N}]^+_{(k)}} wt_{\mathfrak{s}^+, j(k)} \varphi[-P_{(k)}(\hat{n}_{\mathfrak{s}^+}^-), -P_{(k)}(\check{n}_{\mathfrak{s}^+}^-)] = \sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{N}]^+_{(k)}} wt_{\mathfrak{s}^+, j(k)} P_{(k)}(\check{n}_{\mathfrak{s}^+}^-) \quad (\text{B.128})$$

Another perspective is to focus on the “influencer”, which is kind of like the “opinion leader” in foes, and could be deemed as the “marked man” (or No.1 foe), then would oppose further to this foe.

$$\begin{aligned} P_3 &= -wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}\varphi[-P_1, -P_2] \\ &= -wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}P_1 \\ &\text{if } wt_{12,3} = wt_{1,3} \text{ then} \\ &= -2wt_{1,3}P_1 + wt_{2,3}P_2 \end{aligned} \quad (\text{B.129})$$

If in the general form, the influential effect from the structure (foe follows foe) could be expressed as intensifying the negative influence from the influencer foe.

$$\sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{N}]^+_{(k)}} wt_{\mathfrak{s}^+, j(k)} \varphi[-P_{(k)}(\hat{n}_{\mathfrak{s}^+}^-), -P_{(k)}(\check{n}_{\mathfrak{s}^+}^-)] = \sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{N}]^+_{(k)}} wt_{\mathfrak{s}^+, j(k)} P_{(k)}(\hat{n}_{\mathfrak{s}^+}^-) \quad (\text{B.130})$$

Combination 7 Friend Opposes to Friend In this example, agent 2 opposes to agent 1, but agent 1 and agent 2 are both friends of agent 3, which means that the agent 3 observes that there are estrangement between his two friends, and one of friend oppose to the other friend. The perception or feeling about the effect from this structure would be negative. Then it is likely for agent 3 to take some measures.

$$\begin{array}{c} 1 \quad 2 \quad 3 \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \left(\begin{array}{ccc} & & \\ - & & \\ + & + & \end{array} \right) \end{array} \quad (\text{B.131})$$

Usually when you face two friends oppose to each other, one response is to stand by the side of the friend be opposed, thinking that this friend is the “victim” and the other friend is the “betrayor”, , then would further support this friend by putting more weight on him.

$$\begin{aligned} P_3 &= wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}\varphi[P_1, P_2] \\ &= wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}P_1 \\ &\text{if } wt_{12,3} = wt_{1,3} \text{ then} \\ &= 2wt_{1,3}P_1 + wt_{2,3}P_2 \end{aligned} \quad (\text{B.132})$$

If in the general form, $s^- \in \mathbb{S}[\mathbb{N}]^-(k)$, $\hat{n}_{s^-}^+$ means the influencing agent in the negative structure s^- is the friend of influenced agent, and $\check{n}_{s^-}^+$ means the influenced agent in the negative structure s^- is the friend of influenced agent either, $\{\hat{n}_{s^-}^+, \check{n}_{s^-}^+, \hat{n}_{s^-}^+ \rightarrow \check{n}_{s^-}^+\} = s^-$. The influential effect from the structure (friend opposes to friend) could be expressed as reinforcing the positive influence from the friend be opposed.

$$\sum_{s^- \in \mathbb{S}[\mathbb{N}]^-(k)} wt_{s^-,j(k)}\varphi[P_{(k)}(\hat{n}_{s^-}^+), P_{(k)}(\check{n}_{s^-}^+)] = - \sum_{s^- \in \mathbb{S}[\mathbb{N}]^-(k)} wt_{s^-,j(k)}P_{(k)}(\hat{n}_{s^-}^+) \quad (\text{B.133})$$

Another respond is to punish the friend opposing to another friend, thinking that this friend is “betrayor”, who couldn’t stand by the side of friend, and then the influenced agent would possibly depreciate the weight of influence from him.

$$\begin{aligned} P_3 &= wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}\varphi[P_1, P_2] \\ &= wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_2 \\ &\text{if } wt_{12,3} = wt_{2,3} \text{ then} \\ &= wt_{1,3}P_1 \end{aligned} \quad (\text{B.134})$$

If in the general form, the influential effect from the structure (friend opposes to friend) could be expressed as reducing the positive influence from the friend opposing.

$$\sum_{s^- \in \mathbb{S}[\mathbb{N}]^-(k)} wt_{s^-,j(k)}\varphi[P_{(k)}(\hat{n}_{s^-}^+), P_{(k)}(\check{n}_{s^-}^+)] = \sum_{s^- \in \mathbb{S}[\mathbb{N}]^-(k)} wt_{s^-,j(k)}P_{(k)}(\check{n}_{s^-}^+) \quad (\text{B.135})$$

In summary, for how to deal with the “conflicts” among friends, usually agent would stand by one side, or punish one side, which might be according to the intimacy (which friend is more close to own), or due to the reason (which friend has the truth or justice) and so on.

Combination 8 Foe Opposes to Foe In this example, agent 2 opposes to agent 1, and agent 1 and agent 2 are both foes of agent 3, which means that the agent 3 observes that there are estrangement

between his two foes, this structure would possibly bring good feelings and the effect be perceived as positive by influenced agent 3. And it is possible for agent 3 to take some responding actions, to make use of the estrangement between foes.

$$\begin{array}{c} 1 \quad 2 \quad 3 \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \left(\begin{array}{ccc} & & \\ - & & \\ - & - & \end{array} \right) \end{array} \quad (\text{B.136})$$

One view is to oppose further to the foe be opposed by the other foe, thinking of “adding insult to injury” foe, so would oppose further to agent 1.

$$\begin{aligned} P_3 &= -wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}\varphi[-P_1, -P_2] \\ &= -wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}P_1 \\ &\text{if } wt_{12,3} = wt_{1,3} \text{ then} \\ &= -2wt_{1,3}P_1 - wt_{2,3}P_2 \end{aligned} \quad (\text{B.137})$$

If in the general form, $\mathfrak{s}^- \in \mathbb{S}[\mathbb{N}]^-(k)$, $\hat{n}_{\mathfrak{s}^-}$ means the influencing agent in the negative structure \mathfrak{s}^- is the foe of influenced agent, and $\check{n}_{\mathfrak{s}^-}$ means the influenced agent in the negative structure \mathfrak{s}^- is the foe of influenced agent either, $\{\hat{n}_{\mathfrak{s}^-}, \check{n}_{\mathfrak{s}^-}, \hat{n}_{\mathfrak{s}^-} \rightarrow \check{n}_{\mathfrak{s}^-}\} = \mathfrak{s}^-$. The influential effect from the structure (foe opposes to foe) could be expressed as intensifying the negative influence from the foe be opposed.

$$\sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{N}]^-(k)} wt_{\mathfrak{s}^-,j(k)}\varphi[-P_{(k)}(\hat{n}_{\mathfrak{s}^-}), -P_{(k)}(\check{n}_{\mathfrak{s}^-})] = - \sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{N}]^-(k)} wt_{\mathfrak{s}^-,j(k)}P_{(k)}(\hat{n}_{\mathfrak{s}^-}) \quad (\text{B.138})$$

But sometimes when you observe one foe oppose to the other foe, a different response would be to help the opposer, by relieving the negative influence from the foe opposing to the other foe, thinking that the my enemy’s enemy is my friend, as agent 2 opposing to my foe agent 1, then would oppose less to agent 2, treating him less as a foe or more as a friend.

$$\begin{aligned} P_3 &= -wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}\varphi[-P_1, -P_2] \\ &= -wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_2 \\ &\text{if } wt_{12,3} = wt_{2,3} \text{ then} \\ &= -wt_{1,3}P_1 \end{aligned} \quad (\text{B.139})$$

If in the general form, the influential effect from the structure (foe opposes to foe) could be expressed as relieving the negative influence from the foe opposing.

$$\sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{N}]^-(k)} wt_{\mathfrak{s}^-,j(k)}\varphi[-P_{(k)}(\hat{n}_{\mathfrak{s}^-}), -P_{(k)}(\check{n}_{\mathfrak{s}^-})] = \sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{N}]^-(k)} wt_{\mathfrak{s}^-,j(k)}P_{(k)}(\check{n}_{\mathfrak{s}^-}) \quad (\text{B.140})$$

Summary of How to Perceive the Influential Effect from Basic Structure among Agents by Different Combinations of Bilateral Relations

$$\begin{array}{l} \text{Friend Follows} \\ \text{Friend Opposes to} \\ \text{Foe Follows} \\ \text{Foe Opposes to} \end{array} \begin{array}{c} \text{Friend} \quad \text{Foe} \\ \left(\begin{array}{cc} + & - \\ - & + \\ + & - \\ - & + \end{array} \right) \end{array} \quad (\text{B.141})$$

More specifically, in the main text, we discussed the influenced results by the approach of **Collective Influence**, namely all influencing agents participate in the influence, just to different extents or directions. If transferred into the approach of **Prominent One Influence**, then the influenced results would be as below (through the comparison, the influencing agent with the highest absolute value of the weight of influence takes effects).

	<i>control</i>	<i>influence</i>	<i>influenced</i>
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} + \\ - \end{matrix} \right) \\ 2 & \left(\begin{matrix} + \\ - \end{matrix} \right) \\ 3 & \left(\begin{matrix} - \\ + \end{matrix} \right) \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} \geq wt_{2,3} \\ P_2 & \text{if } wt_{1,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} + wt_{12,3} \geq wt_{2,3} \\ P_2 & \text{if } wt_{1,3} + wt_{12,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} \geq wt_{2,3} - wt_{12,3} \\ P_2 & \text{if } wt_{1,3} \leq wt_{2,3} - wt_{12,3} \end{matrix}$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} + \\ + \end{matrix} \right) \\ 2 & \left(\begin{matrix} + \\ - \end{matrix} \right) \\ 3 & \left(\begin{matrix} + \\ - \end{matrix} \right) \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} \geq wt_{2,3} \\ -P_2 & \text{if } wt_{1,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} + wt_{12,3} \geq wt_{2,3} \\ -P_2 & \text{if } wt_{1,3} + wt_{12,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} \geq wt_{2,3} - wt_{12,3} \\ -P_2 & \text{if } wt_{1,3} \leq wt_{2,3} - wt_{12,3} \end{matrix}$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} - \\ - \end{matrix} \right) \\ 2 & \left(\begin{matrix} - \\ + \end{matrix} \right) \\ 3 & \left(\begin{matrix} - \\ + \end{matrix} \right) \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} \geq wt_{2,3} \\ P_2 & \text{if } wt_{1,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} + wt_{12,3} \geq wt_{2,3} \\ P_2 & \text{if } wt_{1,3} + wt_{12,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} \geq wt_{2,3} + wt_{12,3} \\ P_2 & \text{if } wt_{1,3} \leq wt_{2,3} + wt_{12,3} \end{matrix}$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} - \\ - \end{matrix} \right) \\ 2 & \left(\begin{matrix} - \\ + \end{matrix} \right) \\ 3 & \left(\begin{matrix} + \\ - \end{matrix} \right) \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} \geq wt_{2,3} \\ -P_2 & \text{if } wt_{1,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} + wt_{12,3} \geq wt_{2,3} \\ -P_2 & \text{if } wt_{1,3} + wt_{12,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} \geq wt_{2,3} + wt_{12,3} \\ -P_2 & \text{if } wt_{1,3} \leq wt_{2,3} + wt_{12,3} \end{matrix}$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} + \\ + \end{matrix} \right) \\ 2 & \left(\begin{matrix} + \\ + \end{matrix} \right) \\ 3 & \left(\begin{matrix} + \\ + \end{matrix} \right) \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} \geq wt_{2,3} \\ P_2 & \text{if } wt_{1,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} + wt_{12,3} \geq wt_{2,3} \\ P_2 & \text{if } wt_{1,3} + wt_{12,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} \geq wt_{2,3} + wt_{12,3} \\ P_2 & \text{if } wt_{1,3} \leq wt_{2,3} + wt_{12,3} \end{matrix}$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} + \\ - \end{matrix} \right) \\ 2 & \left(\begin{matrix} + \\ - \end{matrix} \right) \\ 3 & \left(\begin{matrix} - \\ - \end{matrix} \right) \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} \geq wt_{2,3} \\ -P_2 & \text{if } wt_{1,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} + wt_{12,3} \geq wt_{2,3} \\ -P_2 & \text{if } wt_{1,3} + wt_{12,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} \geq wt_{2,3} - wt_{12,3} \\ -P_2 & \text{if } wt_{1,3} \leq wt_{2,3} - wt_{12,3} \end{matrix}$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} - \\ + \end{matrix} \right) \\ 2 & \left(\begin{matrix} - \\ + \end{matrix} \right) \\ 3 & \left(\begin{matrix} + \\ + \end{matrix} \right) \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} \geq wt_{2,3} \\ P_2 & \text{if } wt_{1,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} + wt_{12,3} \geq wt_{2,3} \\ P_2 & \text{if } wt_{1,3} + wt_{12,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} \geq wt_{2,3} - wt_{12,3} \\ P_2 & \text{if } wt_{1,3} \leq wt_{2,3} - wt_{12,3} \end{matrix}$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} - \\ - \end{matrix} \right) \\ 2 & \left(\begin{matrix} - \\ - \end{matrix} \right) \\ 3 & \left(\begin{matrix} - \\ - \end{matrix} \right) \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} \geq wt_{2,3} \\ -P_2 & \text{if } wt_{1,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} + wt_{12,3} \geq wt_{2,3} \\ -P_2 & \text{if } wt_{1,3} + wt_{12,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} \geq wt_{2,3} - wt_{12,3} \\ -P_2 & \text{if } wt_{1,3} \leq wt_{2,3} - wt_{12,3} \end{matrix}$

The Influential Effect from Aggregated Structure among Agents Above just discuss about the influential effect from the most basic structure (with one agent influences another agent), but how is the influential effect from the aggregated structure (the aggregation or collection of more than one basic structures)? Actually, for most cases, the effect from aggregated structure could be decomposed step by step into the basic structures, and dealt just by the influential results of basic structures already discussed.

For one example (as shown in figure 7.11), agent 2 is influenced by agent 1, and agent 3 is influenced by agent 4, the structure (from agent 3 to agent 4) is influenced by the structure (from agent 1 to agent 2), and the agent 5 is influenced by the whole (aggregated) structure. To

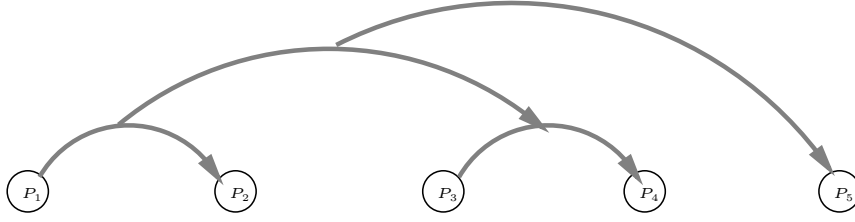


Figure B.17: One Example of the Effect of Aggregated Structure among Agents

compute the influential effect from this aggregated structure, firstly we should know all bilateral relations among these all agents, just assume one combination, composed of all bilateral relations as positive.

$$\begin{array}{c}
 1 \quad 2 \quad 3 \quad 4 \quad 5 \\
 1 \left(\begin{array}{ccccc}
 & & & & \\
 + & & & & \\
 + & + & & & \\
 + & + & + & & \\
 + & + & + & + &
 \end{array} \right)
 \end{array} \quad (\text{B.142})$$

If not considering the structure, expressed in one most simple and specific form (all individual influence as positive).

$$\begin{aligned}
 & s.t. |wt_{1,5}| + |wt_{2,5}| + |wt_{3,5}| + |wt_{4,5}| = 1 \\
 P_5 & = wt_{1,5}P_1 + wt_{2,5}P_2 + wt_{3,5}P_3 + wt_{4,5}P_4
 \end{aligned} \quad (\text{B.143})$$

And if in the general form, as all influencing relations are positive, $n^+ \in \mathbb{N}^+$.

$$P_{(k)}(j) = In\left\{ \sum_{n^+ \in \mathbb{N}^+} wt_{n^+,j(k)} P_{(k)}(n^+) \right\} \quad (\text{B.144})$$

But if considering the effect from structure, based on the rule of the basic structure (friend follows friend), one view is to “ignore” or decrease the influence from the follower, applying the same rule for both two basic structures and the aggregated structure.

$$\begin{aligned}
 & s.t. |wt_{1,5}| + |wt_{2,5}| + |wt_{3,5}| + |wt_{4,5}| + |wt_{12,5}| + |wt_{34,5}| + |wt_{12/34,5}| = 1 \\
 P_5 & = wt_{1,5}P_1 + wt_{2,5}P_2 + wt_{3,5}P_3 + wt_{4,5}P_4 + wt_{12,5}\varphi[P_1, P_2] + wt_{34,5}\varphi[P_3, P_4] + wt_{12/34,5}\varphi\{\varphi[P_1, P_2], \varphi[P_3, P_4]\} \\
 & = wt_{1,5}P_1 + wt_{2,5}P_2 + wt_{3,5}P_3 + wt_{4,5}P_4 - wt_{12,5}P_2 - wt_{34,5}P_4 + wt_{12/34,5}\varphi[-P_2, -P_4] \\
 & = wt_{1,5}P_1 + wt_{2,5}P_2 + wt_{3,5}P_3 + wt_{4,5}P_4 - wt_{12,5}P_2 - wt_{34,5}P_4 + wt_{12/34,5}P_4 \\
 & \text{If } wt_{2,5} = wt_{12,5}, wt_{4,5} = wt_{34,5}, wt_{4,5} = wt_{12/34,5} \text{ then} \\
 & = wt_{1,5}P_1 + wt_{3,5}P_3 + wt_{4,5}P_4
 \end{aligned} \quad (\text{B.145})$$

If in the general form, $\mathbb{S}[\mathbb{N}]^+_{(k)}$ is the set of basic structures of positive influence (from one to one) on issue k , $\mathfrak{s}^+, \hat{\mathfrak{s}}^+, \check{\mathfrak{s}}^+ \in \mathbb{S}[\mathbb{N}]^+_{(k)}$, $\mathcal{S}[\mathbb{S}[\mathbb{N}]^+_{(k)}]$ is the set of aggregated structures composed by basic structures of positive influence $\mathbb{S}[\mathbb{N}]^+_{(k)}$ on issue k , $\mathfrak{s}^+ \in \mathcal{S}[\mathbb{S}[\mathbb{N}]^+_{(k)}]$. \mathfrak{s}^+ is the aggregated structure of basic structure $\hat{\mathfrak{s}}^+$ influencing to basic structure $\check{\mathfrak{s}}^+$, $\{\hat{\mathfrak{s}}^+, \check{\mathfrak{s}}^+, \hat{\mathfrak{s}}^+ \rightarrow \check{\mathfrak{s}}^+\} = \mathfrak{s}^+$. As $\{\hat{n}_{\hat{\mathfrak{s}}^+, \mathfrak{s}^+}^+, \check{n}_{\hat{\mathfrak{s}}^+, \mathfrak{s}^+}^+, \hat{n}_{\hat{\mathfrak{s}}^+, \mathfrak{s}^+}^+ \rightarrow \check{n}_{\hat{\mathfrak{s}}^+, \mathfrak{s}^+}^+\} = \hat{\mathfrak{s}}^+$, $\{\hat{n}_{\check{\mathfrak{s}}^+, \mathfrak{s}^+}^+, \check{n}_{\check{\mathfrak{s}}^+, \mathfrak{s}^+}^+, \hat{n}_{\check{\mathfrak{s}}^+, \mathfrak{s}^+}^+ \rightarrow \check{n}_{\check{\mathfrak{s}}^+, \mathfrak{s}^+}^+\} = \check{\mathfrak{s}}^+$, then more specifically, $\{\hat{n}_{\hat{\mathfrak{s}}^+, \mathfrak{s}^+}^+, \check{n}_{\hat{\mathfrak{s}}^+, \mathfrak{s}^+}^+, \hat{n}_{\hat{\mathfrak{s}}^+, \mathfrak{s}^+}^+ \rightarrow \check{n}_{\hat{\mathfrak{s}}^+, \mathfrak{s}^+}^+\} = \hat{\mathfrak{s}}^+$, $\{\hat{n}_{\check{\mathfrak{s}}^+, \mathfrak{s}^+}^+, \check{n}_{\check{\mathfrak{s}}^+, \mathfrak{s}^+}^+, \hat{n}_{\check{\mathfrak{s}}^+, \mathfrak{s}^+}^+ \rightarrow \check{n}_{\check{\mathfrak{s}}^+, \mathfrak{s}^+}^+\} = \check{\mathfrak{s}}^+$. The influential effect from a basic structure (friend follows friend) could be dealt as weakening the positive influence from the follower friend.

$$\begin{aligned}
P_{(k)}(j) &= In\left\{ \sum_{n^+ \in \mathbb{N}^+} wt_{n^+,j(k)} P_{(k)}(n^+), \sum_{s^+ \in \mathcal{S}[\mathbb{N}]^+(k)} wt_{s^+,j(k)} \varphi[P_{(k)}(\hat{n}_{s^+}^+), P_{(k)}(\check{n}_{s^+}^+)], \right. \\
&\quad \left. \sum_{s^+ \in \mathcal{S}[\mathbb{S}[\mathbb{N}]^+(k)]} wt_{s^+,j(k)} \varphi\{\varphi[P_{(k)}(\hat{n}_{s^+,s^+}^+), P_{(k)}(\check{n}_{s^+,s^+}^+)], \varphi[P_{(k)}(\hat{n}_{s^+,s^+}^+), P_{(k)}(\check{n}_{s^+,s^+}^+)]\} \right\} \\
&= In\left\{ \sum_{n^+ \in \mathbb{N}^+} wt_{n^+,j(k)} P_{(k)}(n^+), - \sum_{s^+ \in \mathcal{S}[\mathbb{N}]^+(k)} wt_{s^+,j(k)} P_{(k)}(\hat{n}_{s^+}^+) \right. \\
&\quad \left. \sum_{s^+ \in \mathcal{S}[\mathbb{S}[\mathbb{N}]^+(k)]} wt_{s^+,j(k)} \varphi[-P_{(k)}(\check{n}_{s^+,s^+}^+), -P_{(k)}(\check{n}_{s^+,s^+}^+)] \right\} \\
&= In\left\{ \sum_{n^+ \in \mathbb{N}^+} wt_{n^+,j(k)} P_{(k)}(n^+), - \sum_{s^+ \in \mathcal{S}[\mathbb{N}]^+(k)} wt_{s^+,j(k)} P_{(k)}(\hat{n}_{s^+}^+), \sum_{s^+ \in \mathcal{S}[\mathbb{S}[\mathbb{N}]^+(k)]} wt_{s^+,j(k)} P_{(k)}(\check{n}_{s^+,s^+}^+) \right\}
\end{aligned} \tag{B.146}$$

The other vies is to “emphasis” or refer more to the influencer (the “opinion leader”), putting more weight on him, also applying the same rule for both two basic structures and the aggregated structure.

$$\begin{aligned}
& s.t. |wt_{1,5}| + |wt_{2,5}| + |wt_{3,5}| + |wt_{4,5}| + |wt_{12,5}| + |wt_{34,5}| + |wt_{12/34,5}| = 1 \\
P_5 &= wt_{1,5}P_1 + wt_{2,5}P_2 + wt_{3,5}P_3 + wt_{4,5}P_4 + wt_{12,5}\varphi[P_1, P_2] + wt_{34,5}\varphi[P_3, P_4] + wt_{12/34,5}\varphi\{\varphi[P_1, P_2], \varphi[P_3, P_4]\} \\
&= wt_{1,5}P_1 + wt_{2,5}P_2 + wt_{3,5}P_3 + wt_{4,5}P_4 + wt_{12,5}P_1 + wt_{34,5}P_3 + wt_{12/34,5}\varphi[P_1, P_3] \\
&= wt_{1,5}P_1 + wt_{2,5}P_2 + wt_{3,5}P_3 + wt_{4,5}P_4 + wt_{12,5}P_1 + wt_{34,5}P_3 + wt_{12/34,5}P_1 \\
& \text{If } wt_{1,5} = wt_{12,5}, wt_{3,5} = wt_{34,5}, wt_{1,5} = wt_{12/34,5} \text{ then} \\
&= 3wt_{1,5}P_1 + wt_{2,5}P_2 + 2wt_{3,5}P_3 + wt_{4,5}P_4
\end{aligned} \tag{B.147}$$

And if in the general form, the influential effect from a basic structure (friend follows friend) could be dealt as intensifying the positive influence from the influencer friend.

$$\begin{aligned}
P_{(k)}(j) &= In\left\{ \sum_{n^+ \in \mathbb{N}^+} wt_{n^+,j(k)} P_{(k)}(n^+), \sum_{s^+ \in \mathcal{S}[\mathbb{N}]^+(k)} wt_{s^+,j(k)} \varphi[P_{(k)}(\hat{n}_{s^+}^+), P_{(k)}(\check{n}_{s^+}^+)], \right. \\
&\quad \left. \sum_{s^+ \in \mathcal{S}[\mathbb{S}[\mathbb{N}]^+(k)]} wt_{s^+,j(k)} \varphi\{\varphi[P_{(k)}(\hat{n}_{s^+,s^+}^+), P_{(k)}(\check{n}_{s^+,s^+}^+)], \varphi[P_{(k)}(\hat{n}_{s^+,s^+}^+), P_{(k)}(\check{n}_{s^+,s^+}^+)]\} \right\} \\
&= In\left\{ \sum_{n^+ \in \mathbb{N}^+} wt_{n^+,j(k)} P_{(k)}(n^+), \sum_{s^+ \in \mathcal{S}[\mathbb{N}]^+(k)} wt_{s^+,j(k)} P_{(k)}(\hat{n}_{s^+}^+) \right. \\
&\quad \left. \sum_{s^+ \in \mathcal{S}[\mathbb{S}[\mathbb{N}]^+(k)]} wt_{s^+,j(k)} \varphi[P_{(k)}(\hat{n}_{s^+,s^+}^+), P_{(k)}(\hat{n}_{s^+,s^+}^+)] \right\} \\
&= In\left\{ \sum_{n^+ \in \mathbb{N}^+} wt_{n^+,j(k)} P_{(k)}(n^+), \sum_{s^+ \in \mathcal{S}[\mathbb{N}]^+(k)} wt_{s^+,j(k)} P_{(k)}(\hat{n}_{s^+}^+), \sum_{s^+ \in \mathcal{S}[\mathbb{S}[\mathbb{N}]^+(k)]} wt_{s^+,j(k)} P_{(k)}(\hat{n}_{s^+,s^+}^+) \right\}
\end{aligned} \tag{B.148}$$

For the influential effect from aggregated structure¹⁴ basically with 5 agents, there are ten (C_5^2) (point to point) relations with binary states (positive or negative), then there are $2^{10}=1024$ variations of combinations in total. Above just one case (all bilateral relations as positive) is discussed, for the other remaining cases, all could be discussed through above method, and based on a results table for the influential effect of basic structure (with 3 agents). Actually, most complex structures could be decomposed step by step and eventually resolved in the most basic structure.

¹⁴From an aggregated structure of two basic structures to one decision, which is the simplest form of aggregated structure or group of structure.

The Influential Effect from Structure among Issues

Combination 1 Good Decision Positively Refer to Bad Decision For one example, if the decision on issue 2 has a desired outcome and the decision on issue 1 has a undesired outcome, namely, issue 2 provides a positive example and issue 1 provides a negative example for issue 3's decision, Furthermore, there is a structure between issue 1 and issue 2 (issue 2 is positively dependent on issue 1), which means the a successful decision is based on a failed decision, and which possibly implies problem or risk remaining under cover. Therefore, it is possible for the decision to take actions on latter issue's decision, to prevent or diminish the bad effect of the "problematic" structure (good decision based on bad decision). The composition of relations among issues 1,2,3 could be expressed by the matrix below.

$$\begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{pmatrix} & & \\ + & & \\ - & + & \end{pmatrix} \end{matrix} \quad (\text{B.149})$$

If not considering the structure, the influence from issue 1 (as a failed decision) on issue 3 is negative, and the influence from issue 2 (as a successful decision) on issue 3 is positive.

$$P_3 = -wt_{1,3}P_1 + wt_{2,3}P_2 \quad (\text{B.150})$$

If in the general form, affected by both positive and negative influences, to indicate the different influences from good decisions or bad decisions, \mathbb{M}^+ is the set of all influencing issues with the weight >0 , $m^+ \in \mathbb{M}^+$, and \mathbb{M}^- is the set of all influencing issues with the weight <0 , $m^- \in \mathbb{M}^-$.

$$P_{(k)}(i) = In\left\{ \sum_{m^+ \in \mathbb{M}^+} wt_{m^+,k(i)} P_{(m^+)}(i), - \sum_{m^- \in \mathbb{M}^-} wt_{m^-,k(i)} P_{(m^-)}(i) \right\} \quad (\text{B.151})$$

If considering the influential effect from the structure (which would negative, as positively referring to a bad decision, but still getting a good one, which is abnormal or illogical), and the decision-maker takes countermeasures against the "potential danger", but how to take measures, or take measures on which issue, then there would be different angles (of view), different people would have different thoughts and focus.

For one angle, it is to take measures on the successful decision (on the "influenced" issue), although the decision on this issue brings desired outcome, but it is based on a failed decision, therefore, in the long-term, it might lurk some risks, then the decision-maker could diminish the influence from this "successful" decision (on issue 2). ($wt_{x,y}$ is the absolute value for the weight of influence from agent x to agent y):

$$\begin{aligned} P_3 &= -wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}\phi[-P_1, P_2] \\ &= -wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_2 \\ &\text{if } wt_{12,3} = wt_{2,3} \text{ then} \\ &= -wt_{1,3}P_1 \end{aligned} \quad (\text{B.152})$$

If represented in the general form, $\mathbb{S}[\mathbb{M}]^+(i)$ is the set of structures of positive influence/dependency on agent i , $\mathfrak{s}^+ \in \mathbb{S}[\mathbb{M}]^+(i)$, $\hat{m}_{\mathfrak{s}^+}^-$ means the influencing issue in the positive structure \mathfrak{s}^+ provides a negative example for the influenced decision, and $\check{m}_{\mathfrak{s}^+}^+$ means the influenced issue in the positive structure \mathfrak{s}^+ provides a positive example for the influenced decision, $\{\hat{m}_{\mathfrak{s}^+}^-, \check{m}_{\mathfrak{s}^+}^+, \hat{m}_{\mathfrak{s}^+}^- \rightarrow$

$\tilde{m}_{s^+}^+$ } = s^+ . The influential effect from structure (Good Decision Positively Refer to Bad Decision) is expressed as weakening the positive influence from the good decision with “potential risk”.

$$\begin{aligned}
P_{(k)}(i) &= In\left\{ \sum_{m^+ \in \mathbb{M}^+} wt_{m^+,k(i)} P_{(m^+)}(i), - \sum_{m^- \in \mathbb{M}^-} wt_{m^-,k(i)} P_{(m^-)}(i), \sum_{s^+ \in \mathbb{S}[\mathbb{M}]^+(i)} wt_{s^+,k(i)} \phi[-P_{(\tilde{m}_{s^+}^-)}(i), P_{(\tilde{m}_{s^+}^+)}(i)] \right\} \\
&= In\left\{ \sum_{m^+ \in \mathbb{M}^+} wt_{m^+,k(i)} P_{(m^+)}(i), - \sum_{m^- \in \mathbb{M}^-} wt_{m^-,k(i)} P_{(m^-)}(i), \sum_{s^+ \in \mathbb{S}[\mathbb{M}]^+(i)} wt_{s^+,k(i)} P_{(\tilde{m}_{s^+}^+)}(i) \right\}
\end{aligned} \tag{B.153}$$

For another angle of view, it is to take measures on the failed decision (on the “influencing” issue), different from the pessimistic perspective as above, the decision-maker might be optimistic, deems that although the first issue’s decision does not bring desired outcome or payoff (for now), but the following decision on latter issue dependent on it are successful, it might make the decision-maker reconsider his perception of the influencing issue, and might adjust (or reduce) the negative influence from this feature.

$$\begin{aligned}
P_3 &= -wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}\phi[-P_1, P_2] \\
&= -wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}P_1 \\
&\text{if } wt_{12,3} = wt_{1,3} \text{ then} \\
&= wt_{2,3}P_2
\end{aligned} \tag{B.154}$$

If in the general form, the influential effect from structure (Good Decision Positively Refer to Bad Decision) is expressed as reducing the negative influence from the bad decision.

$$\begin{aligned}
&= In\left\{ \sum_{m^+ \in \mathbb{M}^+} wt_{m^+,k(i)} P_{(m^+)}(i), - \sum_{m^- \in \mathbb{M}^-} wt_{m^-,k(i)} P_{(m^-)}(i), \sum_{s^+ \in \mathbb{S}[\mathbb{M}]^+(i)} wt_{s^+,k(i)} \phi[-P_{(\tilde{m}_{s^+}^-)}(i), P_{(\tilde{m}_{s^+}^+)}(i)] \right\} \\
&= In\left\{ \sum_{m^+ \in \mathbb{M}^+} wt_{m^+,k(i)} P_{(m^+)}(i), - \sum_{m^- \in \mathbb{M}^-} wt_{m^-,k(i)} P_{(m^-)}(i), - \sum_{s^+ \in \mathbb{S}[\mathbb{M}]^+(i)} wt_{s^+,k(i)} P_{(\tilde{m}_{s^+}^-)}(i) \right\}
\end{aligned} \tag{B.155}$$

Combination 2 Bad Decision Positively Refer to Good Decision For one different example, just contrast to the former one, when issue 2 has a undesired outcome and issue 1 has a desired outcome, namely, issue 2 provides a negative example and issue 1 provides a positive example, and it is the same that issue 2 is dependent on issue 1, which means a latter decision positively referring to a successful former decision but still got failed, which as a whole structure might provide a negative effect (abnormal or illogical) and make the agent to reflect the former decisions, and to take measures on following decisions (issue 3), to diminish or prevent the bad effect of the structure (bad decision coming from good decisions).

$$\begin{matrix} & 1 & 2 & 3 \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \left(\begin{matrix} & & \\ + & & \\ + & - & \end{matrix} \right) & & \end{matrix} \tag{B.156}$$

But how to take reactions, or take reactions on which issue, then there would also be different angles (of view). For one angle, it is to take reactions on the successful decision but causing the failed outcome, therefore, the agent might need to reconsider this issue’s decision, wether existing the “misjudgement”, and wether need to prevent more negative effect brought by this so-called

“successful” decision in the future, and then would possibly diminish the positive influence from this issue (issue 1).

$$\begin{aligned}
P_3 &= wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}\phi[P_1, -P_2] \\
&= wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}P_1 \\
&\text{if } wt_{12,3} = wt_{1,3} \text{ then} \\
&= -wt_{2,3}P_2
\end{aligned} \tag{B.157}$$

If in the general form, $\mathfrak{s}^+ \in \mathbb{S}[\mathbb{M}]^+_{(i)}$, $\hat{m}_{\mathfrak{s}^+}^+$ means the influencing issue in the positive structure \mathfrak{s}^+ provides a positive example for the influenced decision, and $\check{m}_{\mathfrak{s}^+}^-$ means the influenced issue in the positive structure \mathfrak{s}^+ provides a negative example for the influenced decision, $\{\hat{m}_{\mathfrak{s}^+}^+, \check{m}_{\mathfrak{s}^+}^-, \hat{m}_{\mathfrak{s}^+}^+ \rightarrow \check{m}_{\mathfrak{s}^+}^-\} = \mathfrak{s}^+$. The influential effect from structure (Bad Decision Positively Refer to Good Decision) is expressed as weakening the positive influence from the “successful” decision.

$$\sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{M}]^+_{(i)}} wt_{\mathfrak{s}^+,k(i)}\phi[P_{(\hat{m}_{\mathfrak{s}^+}^+)}(i), -P_{(\check{m}_{\mathfrak{s}^+}^-)}(i)] = - \sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{M}]^+_{(i)}} wt_{\mathfrak{s}^+,k(i)}P_{(\hat{m}_{\mathfrak{s}^+}^+)}(i) \tag{B.158}$$

For another angle, it is to take actions on the failed decision coming from the successful decision, different from the pessimistic perspective as above, the agent might be optimistic, deems that although the latter issue’s decision leads to undesired outcome, but it is not the due to the successful former decision, namely it is not because of the making-decision conditional on the former issue leading to the unsatisfied outcome, but it is the decision on the latter issue itself having problems or drawbacks, therefore, there is no need to reflect on the successful decision, and it is possible for the agent to “give another chance” to this influencing issue and “ignore” the negative effect from this issue (which is the failed decision on issue 2 but based on issue 1), reducing the negative influence from issue 2.

$$\begin{aligned}
P_3 &= wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}\phi[P_1, -P_2] \\
&= wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_2 \\
&\text{if } wt_{12,3} = wt_{2,3} \text{ then} \\
&= wt_{1,3}P_1
\end{aligned} \tag{B.159}$$

If in the general form, the influential effect from structure (Bad Decision Positively Refer to Good Decision) is expressed as mitigating the negative influence from the bad decision.

$$\sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{M}]^+_{(i)}} wt_{\mathfrak{s}^+,k(i)}\phi[P_{(\hat{m}_{\mathfrak{s}^+}^+)}(i), -P_{(\check{m}_{\mathfrak{s}^+}^-)}(i)] = - \sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{M}]^+_{(i)}} wt_{\mathfrak{s}^+,k(i)}P_{(\check{m}_{\mathfrak{s}^+}^-)}(i) \tag{B.160}$$

Combination 3 Good Decision Negatively Refer to Bad Decision However, it should be noticed that above (two cases) both assume the structure of influence (between issue 1 and 2) is that one issue is determined positively dependent on the other feature, but how is the structure of influence that one issue is determined negatively dependent on the another issue, how would it affect the influential effect from structure?

For example, if issue 2 is determined negatively dependent on issue 1 (namely the decision on issue 1 is a bad example or negative reference for issue 2), and issue 2 is a good example (as a successful decision) for issue 3, but issue 1 is a bad example (as a failed decision) for issue 3, which means the decision-maker 3 finds out that making decisions on latter issue, by negatively

referring to a bad example (on former issue), then obtain a good outcome on latter one, which would be deemed as a “good experience” (with a positive effect as matching the logical thinking and expectation).

$$\begin{array}{c} 1 \quad 2 \quad 3 \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \left(\begin{array}{ccc} & & \\ - & & \\ - & + & \end{array} \right) \end{array} \quad (\text{B.161})$$

One respond is to further negatively refer to the bad example (issue 1), for other decision (on issue 2) negatively referring to issue 1 obtains a good outcome, then putting more negative weight on issue 1.

$$\begin{aligned} P_3 &= -wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}\phi[-P_1, P_2] \\ &= -wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_1 \\ &\text{if } wt_{12,3} = wt_{1,3} \text{ then} \\ &= -2wt_{1,3}P_1 + wt_{2,3}P_2 \end{aligned} \quad (\text{B.162})$$

If in the general form, $\mathbb{S}[\mathbb{M}]^-(i)$ is the set of structures of negative influence/dependency on agent i , $\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)$, $\hat{m}_{\mathfrak{s}^-}$ means the influencing issue in the negative structure \mathfrak{s}^- provides a negative example for the influenced decision, and $\check{m}_{\mathfrak{s}^-}^+$ means the influenced issue in the negative structure \mathfrak{s}^- provides a positive example for the influenced decision, $\{\hat{m}_{\mathfrak{s}^-}, \check{m}_{\mathfrak{s}^-}^+, \hat{m}_{\mathfrak{s}^-} \rightarrow \check{m}_{\mathfrak{s}^-}^+\} = \mathfrak{s}^-$. The influential effect from the structure (Good Decision Negatively Refer to Bad Decision) could be expressed as intensifying the negative influence from the bad decision.

$$\sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)} wt_{\mathfrak{s}^-,k(i)}\phi[-P_{(\hat{m}_{\mathfrak{s}^-})}(i), P_{(\check{m}_{\mathfrak{s}^-}^+)}(i)] = - \sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)} wt_{\mathfrak{s}^-,k(i)}P_{(\hat{m}_{\mathfrak{s}^-})}(i) \quad (\text{B.163})$$

Another different respond is to further positively refer to the good decision, as a good example, hope would obtain a better outcome on following issues.

$$\begin{aligned} P_3 &= -wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}\phi[-P_1, P_2] \\ &= -wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}P_2 \\ &\text{if } wt_{12,3} = wt_{2,3} \text{ then} \\ &= -wt_{1,3}P_1 + 2wt_{2,3}P_2 \end{aligned} \quad (\text{B.164})$$

If in the general form, the influential effect from the structure (Good Decision Negatively Refer to Bad Decision) could also be expressed as intensifying the positive influence from the good decision.

$$\sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)} wt_{\mathfrak{s}^-,k(i)}\phi[-P_{(\hat{m}_{\mathfrak{s}^-})}(i), P_{(\check{m}_{\mathfrak{s}^-}^+)}(i)] = \sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)} wt_{\mathfrak{s}^-,k(i)}P_{(\check{m}_{\mathfrak{s}^-}^+)}(i) \quad (\text{B.165})$$

Combination 4 Bad Decision Negatively Refer to Good Decision For one example, if issue 2 is still negatively dependent on issue 1 (namely the decision on issue 1 is a bad example or negative reference for issue 2), but contrary to the former example, the issue 2 is a bad decision for issue 3 but issue 1 is a good decision for feature 3, which means the decision-maker finds out that making decision on latter issue by negatively referring to a good example (on former issue) then obtain a bad outcome on latter one, which is kind of lesson, the decision-maker should positively

refer to a good example, but not negatively refer to a good example, in another word, make a good example as a good example, and a bad example as a bad example. However, the structure (negatively referring to a good decision and getting a bad one) itself conforms to logical thinking and common sense, therefore the influential effect is perceived as positive.

$$\begin{array}{c} 1 \quad 2 \quad 3 \\ 1 \left(\begin{array}{cc} & \\ - & \\ + & - \end{array} \right) \end{array} \quad (\text{B.166})$$

One perspective to learn a lesson is to further negatively refer to the bad decision (on issue 2), it is to kind of “punish” the bad decision.

$$\begin{aligned} P_3 &= wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}\phi[P_1, -P_2] \\ &= wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}P_2 \\ &\text{if } wt_{12,3} = wt_{2,3} \text{ then} \\ &= wt_{1,3}P_1 - 2wt_{2,3}P_2 \end{aligned} \quad (\text{B.167})$$

If in the general form, $\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)$, $\hat{m}_{\mathfrak{s}^-}^+$ means the influencing issue in the negative structure \mathfrak{s}^- provides a positive example for the influenced decision, and $\check{m}_{\mathfrak{s}^-}^-$ means the influenced issue in the negative structure \mathfrak{s}^- provides a negative example for the influenced decision, $\{\hat{m}_{\mathfrak{s}^-}^+, \check{m}_{\mathfrak{s}^-}^-, \hat{m}_{\mathfrak{s}^-}^+ \rightarrow \check{m}_{\mathfrak{s}^-}^-\} = \mathfrak{s}^-$. The influential effect from the structure (Bad Decision Negatively Refer to Good Decision) could be expressed as intensifying the negative influence from the bad decision.

$$\sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)} wt_{\mathfrak{s}^-, k(i)} \phi[P_{(\hat{m}_{\mathfrak{s}^-}^+)}(i), -P_{(\check{m}_{\mathfrak{s}^-}^-)}(i)] = - \sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)} wt_{\mathfrak{s}^-, k(i)} P_{(\check{m}_{\mathfrak{s}^-}^-)}(i) \quad (\text{B.168})$$

Another perspective to learn a lesson is to further positively refer to the good decision (on issue 1), it is to kind of compensate the good decision, making a good example indeed exerts a positive influence.

$$\begin{aligned} P_3 &= wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}\phi[P_1, -P_2] \\ &= wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_1 \\ &\text{if } wt_{12,3} = wt_{1,3} \text{ then} \\ &= 2wt_{1,3}P_1 - wt_{2,3}P_2 \end{aligned} \quad (\text{B.169})$$

If in the general form, the influential effect from the structure (Bad Decision Negatively Refer to Good Decision) could be expressed as reinforcing the positive influence from the good decision.

$$\sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)} wt_{\mathfrak{s}^-, k(i)} \phi[P_{(\hat{m}_{\mathfrak{s}^-}^+)}(i), -P_{(\check{m}_{\mathfrak{s}^-}^-)}(i)] = \sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)} wt_{\mathfrak{s}^-, k(i)} P_{(\hat{m}_{\mathfrak{s}^-}^+)}(i) \quad (\text{B.170})$$

Combination 5 Good Decision Positively Refer to Good Decision As the (most) basic model¹⁵, there are three bilateral relations with binary states $\{positive, negative\}$, and there are 8 ($2 \times 2 \times 2$) combinations in total. Above just four cases are discussed, and all are different or unmatched in the two relations, between issue 1 and issue 3, between issue 2 and issue 3, specifically, either assume

¹⁵For the influence from one structure (from one decision to the other decision) to one another decision, there is at least 3 nodes or 3 issues in the graph.

issue 1 as a good decision and issue 2 as a bad decision for issue 3, or assume issue 1 as a bad decision and issue 2 as a good decision for issue 3. Further, to discuss other four cases with the two identical or match relations between issue 1 and issue 3, and between issue 2 and issue 3, which means that from the perspective of the decision on issue 3, either issue 1 and 2 are both good examples, or both bad examples.

For one example, the decision on issue 2 is positively dependent on issue 1, and the decisions on issue 1 and issue 2 are both good examples for issue 3, which means that the decision-maker (while making decision on issue 3) observes that a latter decision positively referring to a good former example also brings out a good outcome, this structure of course would be bring a positive influential effect.

$$\begin{array}{c} 1 \quad 2 \quad 3 \\ 1 \left(\begin{array}{ccc} & & \\ + & & \\ + & + & \end{array} \right) \end{array} \quad (\text{B.171})$$

One reaction is to refer more to the “essential” issue (which determines the decisions on other issues).

$$\begin{aligned} P_3 &= wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}\phi[P_1, P_2] \\ &= wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}P_1 \\ &\text{if } wt_{12,3} = wt_{1,3} \text{ then} \\ &= 2wt_{1,3}P_1 + wt_{2,3}P_2 \end{aligned} \quad (\text{B.172})$$

If in the general form, $\mathfrak{s}^+ \in \mathbb{S}[\mathbb{M}]^+_{(i)}$, $\hat{m}_{\mathfrak{s}^+}^+$ means the influencing issue in the positive structure \mathfrak{s}^+ provides a positive example for the influenced decision, and $\check{m}_{\mathfrak{s}^+}^+$ means the influenced issue in the positive structure \mathfrak{s}^+ provides a positive example for the influenced decision either, $\{\hat{m}_{\mathfrak{s}^+}^+, \check{m}_{\mathfrak{s}^+}^+, \hat{m}_{\mathfrak{s}^+}^+ \rightarrow \check{m}_{\mathfrak{s}^+}^+\} = \mathfrak{s}^+$. The influential effect from the structure (Good Decision Positively Refer to Good Decision) could be expressed as intensifying the positive influence from the “essential” issue.

$$\sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{M}]^+_{(i)}} wt_{\mathfrak{s}^+, k(i)} \phi[P_{(\hat{m}_{\mathfrak{s}^+}^+)}(i), P_{(\check{m}_{\mathfrak{s}^+}^+)}(i)] = \sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{M}]^+_{(i)}} wt_{\mathfrak{s}^+, k(i)} P_{(\hat{m}_{\mathfrak{s}^+}^+)}(i) \quad (\text{B.173})$$

Another reaction is to refer less to the “inessential” issue (which is dependent on other issues but not on own).

$$\begin{aligned} P_3 &= wt_{1,3}P_1 + wt_{2,3}P_2 + wt_{12,3}\phi[P_1, P_2] \\ &= wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_2 \\ &\text{if } wt_{12,3} = wt_{2,3} \text{ then} \\ &= wt_{1,3}P_1 \end{aligned} \quad (\text{B.174})$$

If in the general form, the influential effect from the structure (Good Decision Positively Refer to Good Decision) could be expressed as weakening the positive influence from the “inessential” issue.

$$\sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{M}]^+_{(i)}} wt_{\mathfrak{s}^+, k(i)} \phi[P_{(\hat{m}_{\mathfrak{s}^+}^+)}(i), P_{(\check{m}_{\mathfrak{s}^+}^+)}(i)] = - \sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{M}]^+_{(i)}} wt_{\mathfrak{s}^+, k(i)} P_{(\check{m}_{\mathfrak{s}^+}^+)}(i) \quad (\text{B.175})$$

Combination 6 Bad Decision Positively Refer to Bad Decision For one example, the decision on issue 2 is positively dependent on issue 1, but the decisions on issue 1 and issue 2 are both bad examples for issue 3, which means that the decision-maker (while making decision on issue 3) observes that a latter decision positively referring to a bad former example also brings out a bad outcome, which is also a lesson for the following decisions, a bad example should be negatively referred to. However, the structure (positively referring to a bad decision and getting a bad one) itself is in accord with logics and common sense, and be perceived as a positive influential effect.

$$\begin{array}{c} 1 \quad 2 \quad 3 \\ 1 \left(\begin{array}{ccc} & & \\ + & & \\ - & - & \end{array} \right) \end{array} \quad (\text{B.176})$$

One response is to further negatively refer to the “harmer”, it is kind of punishment for the bad example (as the bad decision on issue 1 makes issue 2 also a bad outcome), and it is also a correction, making the bad example a bad example, and indeed be negatively referred by.

$$\begin{aligned} P_3 &= -wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}\phi[-P_1, -P_2] \\ &= -wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}P_1 \\ &\text{if } wt_{12,3} = wt_{1,3} \text{ then} \\ &= -2wt_{1,3}P_1 - wt_{2,3}P_2 \end{aligned} \quad (\text{B.177})$$

If in the general form, $\mathfrak{s}^+ \in \mathbb{S}[\mathbb{M}]^+(i)$, $\hat{m}_{\mathfrak{s}^+}^-$ means the influencing issue in the positive structure \mathfrak{s}^+ provides a negative example for the influenced decision, and $\check{m}_{\mathfrak{s}^+}^-$ means the influenced issue in the positive structure \mathfrak{s}^+ provides a negative example for the influenced decision either, $\{\hat{m}_{\mathfrak{s}^+}^-, \check{m}_{\mathfrak{s}^+}^-, \hat{m}_{\mathfrak{s}^+}^- \rightarrow \check{m}_{\mathfrak{s}^+}^-\} = \mathfrak{s}^+$. The influential effect from the structure (Bad Decision Positively Refer to Bad Decision) could be expressed as reinforcing the negative influence from the bad example positively referred by.

$$\sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{M}]^+(i)} wt_{\mathfrak{s}^+, k(i)} \phi[-P_{(\hat{m}_{\mathfrak{s}^+}^-)}(i), -P_{(\check{m}_{\mathfrak{s}^+}^-)}(i)] = - \sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{M}]^+(i)} wt_{\mathfrak{s}^+, k(i)} P_{(\hat{m}_{\mathfrak{s}^+}^-)}(i) \quad (\text{B.178})$$

Another response is to “forgive” the mistake of decision on issue 2, thinking that the bad outcome is just due to following a bad example, there might be not very necessary to negatively refer to this issue purposively.

$$\begin{aligned} P_3 &= -wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}\phi[-P_1, -P_2] \\ &= -wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_2 \\ &\text{if } wt_{12,3} = wt_{2,3} \text{ then} \\ &= -wt_{1,3}P_1 \end{aligned} \quad (\text{B.179})$$

If in the general form, the influential effect from the structure (Bad Decision Positively Refer to Bad Decision) could be expressed as weakening the negative influence from the bad decision (which is just based on another bad decision).

$$\sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{M}]^+(i)} wt_{\mathfrak{s}^+, k(i)} \phi[-P_{(\hat{m}_{\mathfrak{s}^+}^-)}(i), -P_{(\check{m}_{\mathfrak{s}^+}^-)}(i)] = \sum_{\mathfrak{s}^+ \in \mathbb{S}[\mathbb{M}]^+(i)} wt_{\mathfrak{s}^+, k(i)} P_{(\hat{m}_{\mathfrak{s}^+}^-)}(i) \quad (\text{B.180})$$

Combination 7 Good Decision Negatively Refer to Good Decision In this example, issue 2 is negatively dependent on issue 1, and the decisions on issue 1 and issue 2 are both good examples for issue 3, which means that the decision-maker finds that a latter decision is made negatively referring to a former good decision, but still obtain a good outcome, which is irreconcilable with common sense, and there might lurk some problems under cover, then would be perceived as a negative influential effect.

$$\begin{array}{ccc} & 1 & 2 & 3 \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \left(\begin{array}{ccc} & & \\ - & & \\ + & + & \end{array} \right) & \end{array} \quad (\text{B.181})$$

To take reactions against the “potential risk”, one view is to refer less to the good decision be negatively referred by, which might be deemed as “suspectable”, because other decisions negatively referring to it still obtain a good outcome.

$$\begin{aligned} P_3 &= wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}\phi[P_1, P_2] \\ &= wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_1 \\ &\text{if } wt_{12,3} = wt_{1,3} \text{ then} \\ &= wt_{2,3}P_2 \end{aligned} \quad (\text{B.182})$$

If in the general form, $\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)$, $\hat{m}_{\mathfrak{s}^-}^+$ means the influencing issue in the negative structure \mathfrak{s}^- provides a positive example for the influenced decision, and $\check{m}_{\mathfrak{s}^-}^+$ means the influenced issue in the negative structure \mathfrak{s}^- provides a positive example for the influenced decision either, $\{\hat{m}_{\mathfrak{s}^-}^+, \check{m}_{\mathfrak{s}^-}^+, \hat{m}_{\mathfrak{s}^-}^+ \rightarrow \check{m}_{\mathfrak{s}^-}^+\} = \mathfrak{s}^-$. The influential effect from the structure (Good Decision Negatively Refer to Good Decision) could be expressed as reducing the positive influence from the good decision (be negatively referred by).

$$\sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)(k)} wt_{\mathfrak{s}^-,k(i)}\phi[P_{(\hat{m}_{\mathfrak{s}^-}^+)}(i), P_{(\check{m}_{\mathfrak{s}^-}^+)}(i)] = \sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)(k)} wt_{\mathfrak{s}^-,k(i)}P_{(\hat{m}_{\mathfrak{s}^-}^+)}(i) \quad (\text{B.183})$$

Another view is to refer less to the good decision negatively referring to other good decision, which might be also deemed as “suspectable”, as there might be some potential problems, this decision negatively referring to a good example but still obtain a good outcome.

$$\begin{aligned} P_3 &= wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}\phi[P_1, P_2] \\ &= wt_{1,3}P_1 + wt_{2,3}P_2 - wt_{12,3}P_2 \\ &\text{if } wt_{12,3} = wt_{2,3} \text{ then} \\ &= wt_{1,3}P_1 \end{aligned} \quad (\text{B.184})$$

If in the general form, the influential effect from the structure (Good Decision Negatively Refer to Good Decision) could be expressed as reducing the positive influence from the good decision (negatively referring to other good decision).

$$\sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)} wt_{\mathfrak{s}^-,k(i)}\phi[P_{(\hat{m}_{\mathfrak{s}^-}^+)}(i), P_{(\check{m}_{\mathfrak{s}^-}^+)}(i)] = \sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)} wt_{\mathfrak{s}^-,k(i)}P_{(\check{m}_{\mathfrak{s}^-}^+)}(i) \quad (\text{B.185})$$

Combination 8 Bad Decision Negatively Refer to Bad Decision In this example, issue 2 is negatively dependent on issue 1, and the decisions on issue 1 and issue 2 are both bad examples

for issue 3, which means that the decision-maker finds that a latter decision is made negatively referring to a former bad decision, but still obtain a bad outcome, which is irreconcilable with common sense (as usually when we refer negatively to a bad example, we would expect a good outcome), and there might lurk some problems under cover, therefore, the influential effect from this structure would also be negative.

$$\begin{matrix} & 1 & 2 & 3 \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{pmatrix} & & \\ - & & \\ - & - & \end{pmatrix} & & \end{matrix} \quad (\text{B.186})$$

To take actions to prevent potential problems (remaining in the future), one view is to refer less to the bad decision (negatively referred by other decision but still leads to a bad outcome), this bad decision might be “really bad”, then would better refer less to it, even negatively.

$$\begin{aligned} P_3 &= -wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}\phi[-P_1, -P_2] \\ &= -wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_1 \\ &\text{if } wt_{12,3} = wt_{1,3} \text{ then} \\ &= -wt_{2,3}P_2 \end{aligned} \quad (\text{B.187})$$

If in the general form, $\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)$, $\hat{m}_{\mathfrak{s}^-}$ means the influencing issue in the negative structure \mathfrak{s}^- provides a negative example for the influenced decision, and $\check{m}_{\mathfrak{s}^-}$ means the influenced issue in the negative structure \mathfrak{s}^- provides a negative example for the influenced decision either, $\{\hat{m}_{\mathfrak{s}^-}, \check{m}_{\mathfrak{s}^-}, \hat{m}_{\mathfrak{s}^-} \rightarrow \check{m}_{\mathfrak{s}^-}\} = \mathfrak{s}^-$. The influential effect from the structure (Bad Decision Negatively Refer to Bad Decision) could be expressed as weakening the negative influence from the bad decision be negatively referred by.

$$\sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)} wt_{\mathfrak{s}^-, k(i)} \phi[-P_{(\hat{m}_{\mathfrak{s}^-})}(i), -P_{(\check{m}_{\mathfrak{s}^-})}(i)] = - \sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)} wt_{\mathfrak{s}^-, k(i)} P_{(\hat{m}_{\mathfrak{s}^-})}(i) \quad (\text{B.188})$$

Another view is to refer less to the bad decision (negatively referring to other decision but still gets to a bad outcome), this bad decision might be deemed as a “victim”, which did the right thing negatively referring to the bad example, then the decision-maker would not refer negatively to this issue that purposively, at least weakening the negative weight of influence from this issue.

$$\begin{aligned} P_3 &= -wt_{1,3}P_1 - wt_{2,3}P_2 - wt_{12,3}\phi[-P_1, -P_2] \\ &= -wt_{1,3}P_1 - wt_{2,3}P_2 + wt_{12,3}P_2 \\ &\text{if } wt_{12,3} = wt_{2,3} \text{ then} \\ &= -wt_{1,3}P_1 \end{aligned} \quad (\text{B.189})$$

If in the general form, the influential effect from the structure (Bad Decision Negatively Refer to Bad Decision) could be expressed as weakening the negative influence from the bad decision negatively referring to other bad decision.

$$\sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)} wt_{\mathfrak{s}^-, k(i)} \phi[-P_{(\hat{m}_{\mathfrak{s}^-})}(i), -P_{(\check{m}_{\mathfrak{s}^-})}(i)] = - \sum_{\mathfrak{s}^- \in \mathbb{S}[\mathbb{M}]^-(i)} wt_{\mathfrak{s}^-, k(i)} P_{(\check{m}_{\mathfrak{s}^-})}(i) \quad (\text{B.190})$$

Summary of How to Perceive the Influential Effect from Basic Structure among Issues by Different Combinations of Bilateral Relations

	<i>Good Decision</i>	<i>Bad Decision</i>
<i>Positively Refer to Good Decision</i>	+	-
<i>Positively Refer to Bad Decision</i>	-	+
<i>Negatively Refer to Good Decision</i>	-	+
<i>Negatively Refer to Bad Decision</i>	+	-

$$\left(\begin{matrix} & & \\ & & \\ & & \\ & & \end{matrix} \right) \quad (\text{B.191})$$

Specifically, in the main text we discussed the influenced results by the approach of **Collective Influence**, namely all influencing issues participate in the influence, just to different extents or directions. If transferred into the approach of **Prominent One Influence**, then the influenced results would be as below (through the comparison, the influencing issue with the highest absolute value of the weight of influence takes effects).

	<i>control</i>	<i>influence</i>	<i>influenced</i>
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} \\ + \\ - \end{matrix} \right) \\ 2 & \left(\begin{matrix} \\ + \\ - \end{matrix} \right) \\ 3 & \left(\begin{matrix} \\ + \\ - \end{matrix} \right) \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} \geq wt_{2,3} \\ P_2 & \text{if } wt_{1,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} - wt_{12,3} \geq wt_{2,3} \\ P_2 & \text{if } wt_{1,3} - wt_{12,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} \geq wt_{2,3} - wt_{12,3} \\ P_2 & \text{if } wt_{1,3} \leq wt_{2,3} - wt_{12,3} \end{matrix}$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} \\ + \\ + \end{matrix} \right) \\ 2 & \left(\begin{matrix} \\ + \\ - \end{matrix} \right) \\ 3 & \left(\begin{matrix} \\ + \\ - \end{matrix} \right) \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} \geq wt_{2,3} \\ -P_2 & \text{if } wt_{1,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} - wt_{12,3} \geq wt_{2,3} \\ -P_2 & \text{if } wt_{1,3} - wt_{12,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} \geq wt_{2,3} - wt_{12,3} \\ -P_2 & \text{if } wt_{1,3} \leq wt_{2,3} - wt_{12,3} \end{matrix}$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} \\ - \\ - \end{matrix} \right) \\ 2 & \left(\begin{matrix} \\ - \\ + \end{matrix} \right) \\ 3 & \left(\begin{matrix} \\ - \\ + \end{matrix} \right) \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} \geq wt_{2,3} \\ P_2 & \text{if } wt_{1,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} + wt_{12,3} \geq wt_{2,3} \\ P_2 & \text{if } wt_{1,3} + wt_{12,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} \geq wt_{2,3} + wt_{12,3} \\ P_2 & \text{if } wt_{1,3} \leq wt_{2,3} + wt_{12,3} \end{matrix}$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} \\ - \\ + \end{matrix} \right) \\ 2 & \left(\begin{matrix} \\ - \\ - \end{matrix} \right) \\ 3 & \left(\begin{matrix} \\ + \\ - \end{matrix} \right) \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} \geq wt_{2,3} \\ -P_2 & \text{if } wt_{1,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} + wt_{12,3} \geq wt_{2,3} \\ -P_2 & \text{if } wt_{1,3} + wt_{12,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} \geq wt_{2,3} + wt_{12,3} \\ -P_2 & \text{if } wt_{1,3} \leq wt_{2,3} + wt_{12,3} \end{matrix}$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} \\ + \\ + \end{matrix} \right) \\ 2 & \left(\begin{matrix} \\ + \\ + \end{matrix} \right) \\ 3 & \left(\begin{matrix} \\ + \\ + \end{matrix} \right) \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} \geq wt_{2,3} \\ P_2 & \text{if } wt_{1,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} + wt_{12,3} \geq wt_{2,3} \\ P_2 & \text{if } wt_{1,3} + wt_{12,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} \geq wt_{2,3} - wt_{12,3} \\ P_2 & \text{if } wt_{1,3} \leq wt_{2,3} - wt_{12,3} \end{matrix}$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} \\ + \\ - \end{matrix} \right) \\ 2 & \left(\begin{matrix} \\ + \\ - \end{matrix} \right) \\ 3 & \left(\begin{matrix} \\ - \\ - \end{matrix} \right) \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} \geq wt_{2,3} \\ -P_2 & \text{if } wt_{1,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} + wt_{12,3} \geq wt_{2,3} \\ -P_2 & \text{if } wt_{1,3} + wt_{12,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} \geq wt_{2,3} - wt_{12,3} \\ -P_2 & \text{if } wt_{1,3} \leq wt_{2,3} - wt_{12,3} \end{matrix}$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} \\ - \\ + \end{matrix} \right) \\ 2 & \left(\begin{matrix} \\ - \\ + \end{matrix} \right) \\ 3 & \left(\begin{matrix} \\ + \\ + \end{matrix} \right) \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} \geq wt_{2,3} \\ P_2 & \text{if } wt_{1,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} - wt_{12,3} \geq wt_{2,3} \\ P_2 & \text{if } wt_{1,3} - wt_{12,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} P_1 & \text{if } wt_{1,3} \geq wt_{2,3} - wt_{12,3} \\ P_2 & \text{if } wt_{1,3} \leq wt_{2,3} - wt_{12,3} \end{matrix}$
$\begin{matrix} 1 & 2 & 3 \\ 1 & \left(\begin{matrix} \\ - \\ - \end{matrix} \right) \\ 2 & \left(\begin{matrix} \\ - \\ - \end{matrix} \right) \\ 3 & \left(\begin{matrix} \\ - \\ - \end{matrix} \right) \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} \geq wt_{2,3} \\ -P_2 & \text{if } wt_{1,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} - wt_{12,3} \geq wt_{2,3} \\ -P_2 & \text{if } wt_{1,3} - wt_{12,3} \leq wt_{2,3} \end{matrix}$	$\begin{matrix} -P_1 & \text{if } wt_{1,3} \geq wt_{2,3} - wt_{12,3} \\ -P_2 & \text{if } wt_{1,3} \leq wt_{2,3} - wt_{12,3} \end{matrix}$

The Influential Effect from Aggregated Structure among Features Above just discuss about the influential effect from the most basic structure (one issue influences another issue), but how is the influential effect from the aggregated structure (the aggregation or collection of more than one basic structures)? Actually, for most cases, the effect from aggregated structure could be decomposed step by step into the basic structures, and dealt just by referring to the influential results of basic structures which is already discussed.

For one example (as shown in figure 7.13), starting from the simplest, which is the gather of two interacting basic structures, it should be noticed that the basic structures in the aggregated

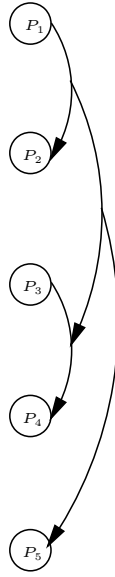


Figure B.18: One Example of the Effect of Aggregated Structure among Features

structure should be related to each other (there is influencing and influenced relations among them), if there are more than one structures but independent from each other, then could directly weighted-sum the influential effects from all these basic structures, but not need to specifically discuss the effect from aggregated structure (complex structure). As in the example, issue 2 is influenced by issue 1, and issue 3 is influenced by issue 4, and the structure of influence (from issue 3 to issue 4) is influenced by the structure of influence (from issue 1 to issue 2), and the issue 5 is influenced by this whole (aggregated) structure. To compute the influential effect from this aggregated structure, firstly we should know all bilateral relations among these all issues, in this example, just assume one combination, composed of all bilateral relations as negative.

$$\begin{array}{c}
 \begin{array}{ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
 1 & & & & & \\
 2 & - & & & & \\
 3 & - & - & & & \\
 4 & - & - & - & & \\
 5 & - & - & - & - &
 \end{array} \\
 \end{array} \quad (B.192)$$

If not considering the structure, expressed in one most simple and specific form, all individual influences assumed as negative.

$$\begin{aligned}
 & s.t. |wt_{1,5}| + |wt_{2,5}| + |wt_{3,5}| + |wt_{4,5}| = 1 \\
 & P_5 = -wt_{1,5}P_1 - wt_{2,5}P_2 - wt_{3,5}P_3 - wt_{4,5}P_4
 \end{aligned} \quad (B.193)$$

And if in the general form, as all influencing relations are assumed as negative, $m^- \in \mathbb{M}^-$.

$$P_{(k)}(i) = In\left\{ \sum_{m^- \in \mathbb{M}^-} wt_{m^-,k(i)} P_{(m^-)}(i) \right\} \quad (B.194)$$

But if considering the influential effect from structure, based on the rule of the basic structure (bad decision negatively refers to bad decision), one perspective is referring less negatively to the “really” bad example, reducing the negative weight of influence from this issue, and applying the same rule for both two basic structures and the aggregated structure.

$$\begin{aligned}
& s.t. |wt_{1,5}| + |wt_{2,5}| + |wt_{3,5}| + |wt_{4,5}| + |wt_{12,5}| + |wt_{34,5}| + |wt_{12/34,5}| = 1 \\
P_5 &= -wt_{1,5}P_1 - wt_{2,5}P_2 - wt_{3,5}P_3 - wt_{4,5}P_4 - wt_{12,5}\phi[-P_1, -P_2] - wt_{34,5}\phi[-P_3, -P_4] \\
& \quad - wt_{12/34,5}\phi\{\phi[-P_1, -P_2], \phi[-P_3, -P_4]\} \\
&= -wt_{1,5}P_1 - wt_{2,5}P_2 - wt_{3,5}P_3 - wt_{4,5}P_4 + wt_{12,5}P_1 + wt_{34,5}P_3 - wt_{12/34,5}\phi[-P_1, -P_3] \quad (B.195) \\
&= -wt_{1,5}P_1 - wt_{2,5}P_2 - wt_{3,5}P_3 - wt_{4,5}P_4 + wt_{12,5}P_1 + wt_{34,5}P_3 + wt_{12/34,5}P_1 \\
& \text{If } wt_{1,5} = wt_{12,5}, wt_{3,5} = wt_{34,5}, wt_{1,5} = wt_{12/34,5} \text{ then} \\
&= wt_{1,5}P_1 - wt_{3,5}P_2 - wt_{4,5}P_4
\end{aligned}$$

If in the general form, $\mathbb{S}[\mathbb{M}]^-(i)$ is the set of basic structures of negative influence/dependency (from one to one/on one by one) within agent i , $\mathfrak{s}^-, \dot{\mathfrak{s}}^-, \ddot{\mathfrak{s}}^- \in \mathbb{S}[\mathbb{M}]^-(i)$, $\mathcal{S}[\mathbb{S}[\mathbb{M}]^-(i)]$ is the set of aggregated structures composed by basic structures of positive influence $\mathbb{S}[\mathbb{M}]^-(i)$ within agent i , $s^- \in \mathcal{S}[\mathbb{S}[\mathbb{M}]^-(i)]$. s^- is the aggregated structure of basic structure $\dot{\mathfrak{s}}^-$ influencing to basic structure $\ddot{\mathfrak{s}}^-$, $\{\dot{\mathfrak{s}}^-, \ddot{\mathfrak{s}}^-, \dot{\mathfrak{s}}^- \rightarrow \ddot{\mathfrak{s}}^-\} = s^-$. As $\{\hat{m}_{\dot{\mathfrak{s}}^-, s^-}, \check{m}_{\dot{\mathfrak{s}}^-, s^-}, \hat{m}_{\dot{\mathfrak{s}}^-, s^-} \rightarrow \check{m}_{\dot{\mathfrak{s}}^-, s^-}\} = \dot{\mathfrak{s}}^-$, $\{\hat{m}_{\ddot{\mathfrak{s}}^-, s^-}, \check{m}_{\ddot{\mathfrak{s}}^-, s^-}, \hat{m}_{\ddot{\mathfrak{s}}^-, s^-} \rightarrow \check{m}_{\ddot{\mathfrak{s}}^-, s^-}\} = \ddot{\mathfrak{s}}^-$, then more specifically, $\{\hat{m}_{\dot{\mathfrak{s}}^-, s^-}, \check{m}_{\dot{\mathfrak{s}}^-, s^-}, \hat{m}_{\ddot{\mathfrak{s}}^-, s^-}, \check{m}_{\ddot{\mathfrak{s}}^-, s^-}, \hat{m}_{\dot{\mathfrak{s}}^-, s^-} \rightarrow \check{m}_{\ddot{\mathfrak{s}}^-, s^-}, \hat{m}_{\ddot{\mathfrak{s}}^-, s^-} \rightarrow \check{m}_{\dot{\mathfrak{s}}^-, s^-}\} = s^-$. The influential effect from the structure (Bad Decision Negatively Refer to Bad Decision) could be expressed as weakening the negative influence from the bad decision be negatively referred by.

$$\begin{aligned}
P_{(k)}(i) &= In\{ \sum_{m^- \in \mathbb{M}^-} wt_{m^-, k(i)} P_{(m^-)}(i), \sum_{s^- \in \mathbb{S}[\mathbb{M}]^-(i)} wt_{s^-, k(i)} \phi[-P_{(\hat{m}_{s^-}^-)}(i), -P_{(\check{m}_{s^-}^-)}(i)], \\
& \quad \sum_{s^- \in \mathcal{S}[\mathbb{S}[\mathbb{M}]^-(i)]} wt_{s^-, k(i)} \phi\{\phi[-P_{(\hat{m}_{\dot{\mathfrak{s}}^-, s^-}^-)}(i), -P_{(\check{m}_{\dot{\mathfrak{s}}^-, s^-}^-)}(i)], \phi[-P_{(\hat{m}_{\ddot{\mathfrak{s}}^-, s^-}^-)}(i), -P_{(\check{m}_{\ddot{\mathfrak{s}}^-, s^-}^-)}(i)]\} \} \\
&= In\{ \sum_{m^- \in \mathbb{M}^-} wt_{m^-, k(i)} P_{(m^-)}(i), - \sum_{s^- \in \mathbb{S}[\mathbb{M}]^-(i)} wt_{s^-, k(i)} P_{(\hat{m}_{s^-}^-)}(i), \\
& \quad \sum_{s^- \in \mathcal{S}[\mathbb{S}[\mathbb{M}]^-(i)]} wt_{s^-, k(i)} \phi[-P_{(\hat{m}_{\dot{\mathfrak{s}}^-, s^-}^-)}(i), -P_{(\check{m}_{\dot{\mathfrak{s}}^-, s^-}^-)}(i)] \} \\
&= In\{ \sum_{m^- \in \mathbb{M}^-} wt_{m^-, k(i)} P_{(m^-)}(i), - \sum_{s^- \in \mathbb{S}[\mathbb{M}]^-(i)} wt_{s^-, k(i)} P_{(\hat{m}_{s^-}^-)}(i), - \sum_{s^- \in \mathcal{S}[\mathbb{S}[\mathbb{M}]^-(i)]} wt_{s^-, k(i)} P_{(\hat{m}_{\dot{\mathfrak{s}}^-, s^-}^-)}(i) \} \quad (B.196)
\end{aligned}$$

Another perspective is to “ignore” or weaken the negative influence from the bad decision (negative referring to another bad decision), to deem the bad decision as kind of “victim”, and not refer negatively to this issue that purposively.

$$\begin{aligned}
& s.t. |wt_{1,5}| + |wt_{2,5}| + |wt_{3,5}| + |wt_{4,5}| + |wt_{12,5}| + |wt_{34,5}| + |wt_{12/34,5}| = 1 \\
P_5 &= -wt_{1,5}P_1 - wt_{2,5}P_2 - wt_{3,5}P_3 - wt_{4,5}P_4 - wt_{12,5}\phi[-P_1, -P_2] - wt_{34,5}\phi[-P_3, -P_4] \\
& \quad - wt_{12/34,5}\phi\{\phi[-P_1, -P_2], \phi[-P_3, -P_4]\} \\
&= -wt_{1,5}P_1 - wt_{2,5}P_2 - wt_{3,5}P_3 - wt_{4,5}P_4 + wt_{12,5}P_2 + wt_{34,5}P_4 - wt_{12/34,5}\phi[-P_2, -P_4] \quad (B.197) \\
&= -wt_{1,5}P_1 - wt_{2,5}P_2 - wt_{3,5}P_3 - wt_{4,5}P_4 + wt_{12,5}P_2 + wt_{34,5}P_4 + wt_{12/34,5}P_4 \\
& \text{If } wt_{2,5} = wt_{12,5}, wt_{4,5} = wt_{34,5}, wt_{2,5} = wt_{12/34,5} \text{ then} \\
&= -wt_{1,5}P_1 - wt_{3,5}P_3 + wt_{2,5}P_4
\end{aligned}$$

And if in the general form, the influential effect from the structure (Bad Decision Negatively Refer to Bad Decision) could be expressed as weakening the negative influence from the bad decision negatively referring to other bad decision.

$$\begin{aligned}
P_{(k)}(i) &= In\left\{ \sum_{m^- \in M^-} wt_{m^-,k(i)} P_{(m^-)}(i), \sum_{s^- \in S[M^-]-(i)} wt_{s^-,k(i)} \phi[-P_{(\tilde{m}_{s^-}^-)}(i), -P_{(\tilde{m}_{s^-}^-)}(i)], \right. \\
&\quad \left. \sum_{s^- \in S[S[M^-]-(i)]} wt_{s^-,k(i)} \phi\{\phi[-P_{(\tilde{m}_{s^-}^-)}(i), -P_{(\tilde{m}_{s^-}^-)}(i)], \phi[-P_{(\tilde{m}_{s^-}^-)}(i), -P_{(\tilde{m}_{s^-}^-)}(i)]\} \right\} \\
&= In\left\{ \sum_{m^- \in M^-} wt_{m^-,k(i)} P_{(m^-)}(i), - \sum_{s^- \in S[M^-]-(i)} wt_{s^-,k(i)} P_{(\tilde{m}_{s^-}^-)}(i), \right. \\
&\quad \left. \sum_{s^- \in S[S[M^-]-(i)]} wt_{s^-,k(i)} \phi[-P_{(\tilde{m}_{s^-}^-)}(i), -P_{(\tilde{m}_{s^-}^-)}(i)] \right\} \\
&= In\left\{ \sum_{m^- \in M^-} wt_{m^-,k(i)} P_{(m^-)}(i), - \sum_{s^- \in S[M^-]-(i)} wt_{s^-,k(i)} P_{(\tilde{m}_{s^-}^-)}(i), - \sum_{s^- \in S[S[M^-]-(i)]} wt_{s^-,k(i)} P_{(\tilde{m}_{s^-}^-)}(i) \right\}
\end{aligned} \tag{B.198}$$

For the influential effect from aggregated structure¹⁶ with minimum 5 issues, there are ten (C_5^2) bilateral relations among them with binary states (positive or negative), then there are $2^{10}=1024$ variations of combinations in total. Above just one case (all bilateral relations as negative) is discussed, for the other remaining cases, all could be discussed through above method, and based on a results table for the influential effect of basic structure (with 3 issues). Actually, most complex structures could be decomposed step by step and eventually resolved in the most basic structure.

¹⁶From an aggregated structure of two basic structures to one decision, which is the simplest form of aggregated structure or group of structure.

Appendix C

Application of Influence

C.1 Matching Algorithms Explanations and Display Examples

Influence Pattern 1 Matching Algorithm

Explanation

For influence pattern 1, which is the influence among different issues but within one agent. The decision of agent i on issue q would be influenced by the decision of agent i own on former issue(s), for example if influenced by all former issues, it would be issues $1, 2, \dots, q - 1$. Design the matching algorithm for influence pattern 1 as Algorithm 8, i is the No. of agent, q is the No. of influenced issues, and k is the No. of influencing issues (therefore $k < q - 1$, only the latter issue could be influenced by the former issue, but not reversely). At the first step, for influencing issue k from 1 to $q - 1$, at each step (of issue k), if $FV_{(k)}(i) = Y$, which means the factual vote (FV) of agent i on issue k is Yes (affirmative), then $\mathcal{WT}_{Y(k,q)}(i) = \mathcal{WT}_{Y(k-1,q)}(i) + wt_{Y(k,q)}(i)$, which means the accumulative weight of influence of affirmative vote would be plus with the weight of influence of the current vote (of issue k), which work similarly if $FV_{(k)}(i) = A$ and $FV_{(k)}(i) = N$. At the second step, for the influenced issue q , assume it would be influenced by all former issues from 1 to $q-1$, if $|\mathcal{WT}_{Y(q-1,q)}(i)| = \text{MAX}(|\mathcal{WT}_{Y(q-1,q)}(i)|, |\mathcal{WT}_{A(q-1,q)}(i)|, |\mathcal{WT}_{N(q-1,q)}(i)|)$, which means the accumulative weight of influence of affirmative vote of all former influencing issues before issue q is the maximum of all accumulative weight of influence (of affirmative vote, absent vote and negative vote), and also $\mathcal{WT}_{Y(q-1,q)}(i) > 0$ (the influence is positive), then $EV_{(q)}(i) = Y$, which means the expected vote (EV) of agent i on influenced issue q computed according to the algorithm would be the same as Yes (affirmative), otherwise if the influence is negative, then $EV_{(q)}(i) = A \text{ or } N$, which means the expected vote would be contrast to or different from Yes (affirmative), which work similarly if $|\mathcal{WT}_{A(q-1,q)}(i)| = \text{MAX}(|\mathcal{WT}_{Y(q-1,q)}(i)|, |\mathcal{WT}_{A(q-1,q)}(i)|, |\mathcal{WT}_{N(q-1,q)}(i)|)$ and $|\mathcal{WT}_{N(q-1,q)}(i)| = \text{MAX}(|\mathcal{WT}_{Y(q-1,q)}(i)|, |\mathcal{WT}_{A(q-1,q)}(i)|, |\mathcal{WT}_{N(q-1,q)}(i)|)$. At the third step, for influenced issue q from k to m , compared the factual vote (according to the official voting data) and the expected vote (by the matching algorithm), at each step (for issue q), if $EV_{(q)}(i) = FV_{(q)}(i)$ (the expected vote is identical with the factual vote), then $P1MF_{(q)}(i) = P1MF_{(q-1)}(i) + 1$ (the pattern 1 matching frequency would be plus 1), and the pattern 1 matching ratio would be obtained by dividing the No. of issue q .

Example

Given a simple example to display the procedure of testing algorithm step by step. Assume there is one agent i , making decisions on 10 issues. Assume the agent i 's current decision is in-

Algorithm 8 Testing Algorithm for Influence Pattern 1

Input: Given A society $\mathcal{ST}(\mathbb{N}, \mathbb{M}, \mathbb{WT})$, $\mathbb{N} = \{1, \dots, n\}$, $\mathbb{M} = \{1, \dots, m\}$, there are n agents and m features.

Output: Pattern 1 Matching Frequency (P1MF) and Pattern 1 Matching Ratio (P1MR) of $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$

```
for  $i = 1$  to  $n$  do
  for  $q = k$  to  $m$  do
    for  $k = 1$  to  $q - 1$  do
      if  $FV_{(k)}(i) = Y$  then
         $\mathcal{WT}_{Y(k,q)}(i) = \mathcal{WT}_{Y(k-1,q)}(i) + wt_{Y(k,q)}(i)$ 
      else
         $\mathcal{WT}_{Y(k,q)}(i) = \mathcal{WT}_{Y(k-1,q)}(i)$ 
      end if
      if  $FV_{(k)}(i) = A$  then
         $\mathcal{WT}_{A(k,q)}(i) = \mathcal{WT}_{A(k-1,q)}(i) + wt_{A(k,q)}(i)$ 
      else
         $\mathcal{WT}_{A(k,q)}(i) = \mathcal{WT}_{A(k-1,q)}(i)$ 
      end if
      if  $FV_{(k)}(i) = N$  then
         $\mathcal{WT}_{N(k,q)}(i) = \mathcal{WT}_{N(k-1,q)}(i) + wt_{N(k,q)}(i)$ 
      else
         $\mathcal{WT}_{N(k,q)}(i) = \mathcal{WT}_{N(k-1,q)}(i)$ 
      end if
    end for
    if  $|\mathcal{WT}_{Y(q-1,q)}(i)| = \text{MAX}(|\mathcal{WT}_{Y(q-1,q)}(i)|, |\mathcal{WT}_{A(q-1,q)}(i)|, |\mathcal{WT}_{N(q-1,q)}(i)|)$  then
      if  $\mathcal{WT}_{Y(q-1,q)}(i) > 0$  then
         $EV_{(q)}(i) = Y$ 
      else
         $EV_{(q)}(i) = A \text{ or } N$ 
      end if
    end if
    if  $|\mathcal{WT}_{A(q-1,q)}(i)| = \text{MAX}(|\mathcal{WT}_{Y(q-1,q)}(i)|, |\mathcal{WT}_{A(q-1,q)}(i)|, |\mathcal{WT}_{N(q-1,q)}(i)|)$  then
      if  $\mathcal{WT}_{A(q-1,q)}(i) > 0$  then
         $EV_{(q)}(i) = A$ 
      else
         $EV_{(q)}(i) = Y \text{ or } N$ 
      end if
    end if
    if  $|\mathcal{WT}_{N(q-1,q)}(i)| = \text{MAX}(|\mathcal{WT}_{Y(q-1,q)}(i)|, |\mathcal{WT}_{A(q-1,q)}(i)|, |\mathcal{WT}_{N(q-1,q)}(i)|)$  then
      if  $\mathcal{WT}_{N(q-1,q)}(i) > 0$  then
         $EV_{(q)}(i) = N$ 
      else
         $EV_{(q)}(i) = A \text{ or } Y$ 
      end if
    end if
    if  $EV_{(q)}(i) = FV_{(q)}(i)$  then
      count  $P1MF_{(q)}(i) = P1MF_{(q-1)}(i) + 1$ 
    else
      count  $P1MF_{(q)}(i) = P1MF_{(q-1)}(i)$ 
    end if
     $P1MR_{(q)}(i) = \frac{P1MF_{(q)}(i)}{q}$ 
  end for
end for
Return P1MF and P1MR of  $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$ 
```

fluenced by all the former decisions, and assume the weight of influence of all former decisions identical as 1 (and as all positive influence). $\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively the accumulative weight of influence of affirmative vote, absent vote and negative vote from former issues.

Table C.1: One Example to Display the Pattern 1 Matching Algorithm

	i
	Y
1	Y
2	A
3	A
4	A
5	Y
6	Y
7	N
8	Y
9	Y
10	Y

The procedure of all steps is displayed as following, just explained the first three steps. For step 1, the former decision is Y, then $\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 1,0,0, the maximum is \mathcal{WT}_Y , therefore the expected vote is Y, and the factual vote on step 1 is also Y, so it is matched, and the matching frequency is 1 and matching ratio is 1.00=1/1; for the second step, the former decisions are Y,Y, then $\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 2,0,0, the maximum is \mathcal{WT}_Y , therefore the expected vote is Y, and the factual vote on step 2 is A, so it is unmatched, and the matching frequency is 1 and matching ratio is 0.50=1/2; for the third step, the former decisions are Y,Y,A, then $\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 2,1,0, the maximum is still \mathcal{WT}_Y , therefore the expected vote is Y, and the factual vote on step 3 is A, so it is unmatched, and the matching frequency is 1 and matching ratio is 0.33=1/3, the following steps work in the same way.

Table C.2: Testing Procedure Step-by-step by the Pattern 1 Matching Algorithm

Step	\mathcal{WT}_Y	\mathcal{WT}_A	\mathcal{WT}_N	MAX	EV	FV	Match	PIMF	PIMR
1	1	0	0	\mathcal{WT}_Y	Y	Y	YES	1	1.00
2	2	0	0	\mathcal{WT}_Y	Y	A	NO	1	0.50
3	2	1	0	\mathcal{WT}_Y	Y	A	NO	1	0.33
4	2	2	0	$\mathcal{WT}_Y/\mathcal{WT}_A$	Y/A	A	YES	2	0.50
5	2	3	0	\mathcal{WT}_A	A	Y	NO	2	0.40
6	3	3	0	$\mathcal{WT}_Y/\mathcal{WT}_A$	Y/A	Y	YES	3	0.50
7	4	3	0	\mathcal{WT}_Y	Y	N	NO	3	0.43
8	4	3	1	\mathcal{WT}_Y	Y	Y	YES	4	0.50
9	5	3	1	\mathcal{WT}_Y	Y	Y	YES	5	0.56
10	6	3	1	\mathcal{WT}_Y	Y	Y	YES	6	0.60

Above displayed the testing outcome if the current decision is influenced just by former decisions, but if combining with the influence from current preference, which means the decision on current issue is the collective effect of the decisions on former issues and the preference of current issue, then the testing outcome would be slightly different.

Table C.3: Testing Procedure Step-by-step by the Pattern 1 Matching Algorithm (with the influence from current preference)

Step	\mathcal{WT}_Y	\mathcal{WT}_A	\mathcal{WT}_N	MAX	EV	FV	Match	PIMF	PIMR
1	2	0	0	\mathcal{WT}_Y	Y	Y	YES	1	1.00
2	2	1	0	\mathcal{WT}_Y	Y	A	NO	1	0.50
3	2	2	0	$\mathcal{WT}_Y/\mathcal{WT}_A$	Y/A	A	YES	2	0.66
4	2	3	0	\mathcal{WT}_A	A	A	YES	3	0.75

5	3	3	0	$\mathcal{WT}_Y/\mathcal{WT}_A$	Y/A	Y	YES	4	0.80
6	4	3	0	\mathcal{WT}_Y	Y	Y	YES	5	0.83
7	4	3	1	\mathcal{WT}_Y	Y	N	NO	5	0.71
8	5	3	1	\mathcal{WT}_Y	Y	Y	YES	6	0.75
9	6	3	1	\mathcal{WT}_Y	Y	Y	YES	7	0.78
10	7	3	1	\mathcal{WT}_Y	Y	Y	YES	8	0.80

Above we just assume the weight of influence from all former decisions all identical (as positive 1), while in real-world, the weight of influence would be varied, weak or strong, negative or positive. For example, we could assume that the more recent of the former decision, the higher weight of influence to current decision, as the fresh memory has a stronger effect; or we could also assume that the more remote of the former decision, the higher weight of influence to current decision, as the oldest memory has a stronger effect. There are many other factors affecting the weight of influence, and it also depends on many situations.

Influence Pattern 7 Matching Algorithm

Explanation

For influence pattern 7, which is the influence among different agents but within one issue. The decision of agent j on issue k would be influenced by the decision of other agent(s) on the same issue k , for example if influenced by all others, it would be the set of all agents \mathbb{N} , agent 1, 2, ..., n except i . Design the matching algorithm for influence pattern 7 as Algorithm 9, k is the No. of issue, j is the No. of influenced agents, and i is the No. of influencing agents ($i \neq j$). At the first step, for influencing agent i from 1 to n , for each agent i , if $FV_{(k)}(i) = Y$, which means the factual vote (FV) of agent i on issue k is Yes (affirmative), then $\mathcal{WT}_{Y(k)}(\mathbb{N}, j) = \mathcal{WT}_{Y(k)}(\mathbb{N}, j) + wt_{Y(k)}(i, j)$, which means the accumulative weight of influence of affirmative vote would be plus with the weight of influence of this agent, which work similarly if $FV_{(k)}(i) = A$ and $FV_{(k)}(i) = N$. At the second step, for the influenced agent j , assume it would be influenced by all agents \mathbb{N} , if $|\mathcal{WT}_{Y(k)}(\mathbb{N}, j)| = \text{MAX}(|\mathcal{WT}_{Y(k)}(\mathbb{N}, j)|, |\mathcal{WT}_{A(k)}(\mathbb{N}, j)|, |\mathcal{WT}_{N(k)}(\mathbb{N}, j)|)$, which means the accumulative weight of influence of affirmative vote of all influencing agents is the maximum of all accumulative weight of influence (of affirmative vote, absent vote and negative vote), and if also $\mathcal{WT}_{Y(k)}(\mathbb{N}, j) > 0$ (the influence is positive), then $EV_{(k)}(j) = Y$, which means the expected vote (EV) of influenced agent j on issue k computed according to the algorithm would be the same as Yes (affirmative), otherwise if the influence is negative, then $EV_{(k)}(j) = A \text{ or } N$, which means the expected vote would be contrast to or different from Yes (affirmative), which work similarly if $|\mathcal{WT}_{A(k)}(\mathbb{N}, j)| = \text{MAX}(|\mathcal{WT}_{Y(k)}(\mathbb{N}, j)|, |\mathcal{WT}_{A(k)}(\mathbb{N}, j)|, |\mathcal{WT}_{N(k)}(\mathbb{N}, j)|)$ and $|\mathcal{WT}_{N(k)}(\mathbb{N}, j)| = \text{MAX}(|\mathcal{WT}_{Y(k)}(\mathbb{N}, j)|, |\mathcal{WT}_{A(k)}(\mathbb{N}, j)|, |\mathcal{WT}_{N(k)}(\mathbb{N}, j)|)$. At the third step, for influenced agent j from 1 to n , compared the factual vote (according to the official voting data) and the expected vote (by the matching algorithm), for each agent j , if $EV_{(k)}(j) = FV_{(k)}(j)$ (the expected vote is identical with the factual vote), then $P7MF_{(k)}(j) = P7MF_{(k)}(j) + 1$ (the pattern 7 matching frequency would be plus 1), and the pattern 7 matching ratio would be obtained by dividing the No. of issue k .

Example

Given a simple example to display the procedure of testing algorithm step by step. Assume there is one agent j influenced by three agents i_1, i_2, i_3 , making decisions on 10 issues. Assume the agent j 's decision on current issue is influenced by all other agents' decisions on the same issue, and assume the weight of influence of all agents identical as 1 (and as all positive influence). \mathcal{WT}_Y , \mathcal{WT}_A , \mathcal{WT}_N are respectively the accumulative weight of influence of affirmative vote, absent vote and negative vote from all influencing agents.

Algorithm 9 Testing Algorithm for Influence Pattern 7

Input: Given A society $\mathcal{ST}(\mathbb{N}, \mathbb{M}, \mathbb{WT})$, $\mathbb{N} = \{1, \dots, n\}$, $\mathbb{M} = \{1, \dots, m\}$
there are n agents and m features.

Output: Pattern 7 Matching Frequency (P7MF) and Pattern 7 Matching Ratio (P7MR) of $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$

```
for k = 1 to m do
  for j = 1 to n do
    for i = 1 to n i ≠ j do
      if  $FV_{(k)}(i) = Y$  then
         $\mathcal{WT}_{Y^{(k)}}(\mathbb{N}, j) = \mathcal{WT}_{Y^{(k)}}(\mathbb{N}, j) + wt_{Y^{(k)}}(i, j)$ 
      end if
      if  $FV_{(k)}(i) = A$  then
         $\mathcal{WT}_{A^{(k)}}(\mathbb{N}, j) = \mathcal{WT}_{A^{(k)}}(\mathbb{N}, j) + wt_{A^{(k)}}(i, j)$ 
      end if
      if  $FV_{(k)}(i) = N$  then
         $\mathcal{WT}_{N^{(k)}}(\mathbb{N}, j) = \mathcal{WT}_{N^{(k)}}(\mathbb{N}, j) + wt_{N^{(k)}}(i, j)$ 
      end if
    end for
    if  $|\mathcal{WT}_{Y^{(k)}}(\mathbb{N}, j)| = \text{MAX}(|\mathcal{WT}_{Y^{(k)}}(\mathbb{N}, j)|, |\mathcal{WT}_{A^{(k)}}(\mathbb{N}, j)|, |\mathcal{WT}_{N^{(k)}}(\mathbb{N}, j)|)$  then
      if  $\mathcal{WT}_{Y^{(k)}}(\mathbb{N}, j) > 0$  then
         $EV_{(k)}(j) = Y$ 
      else
         $EV_{(k)}(j) = A \text{ or } N$ 
      end if
    end if
    if  $|\mathcal{WT}_{A^{(k)}}(\mathbb{N}, j)| = \text{MAX}(|\mathcal{WT}_{Y^{(k)}}(\mathbb{N}, j)|, |\mathcal{WT}_{A^{(k)}}(\mathbb{N}, j)|, |\mathcal{WT}_{N^{(k)}}(\mathbb{N}, j)|)$  then
      if  $\mathcal{WT}_{A^{(k)}}(\mathbb{N}, j) > 0$  then
         $EV_{(k)}(j) = A$ 
      else
         $EV_{(k)}(j) = Y \text{ or } N$ 
      end if
    end if
    if  $|\mathcal{WT}_{N^{(k)}}(\mathbb{N}, j)| = \text{MAX}(|\mathcal{WT}_{Y^{(k)}}(\mathbb{N}, j)|, |\mathcal{WT}_{A^{(k)}}(\mathbb{N}, j)|, |\mathcal{WT}_{N^{(k)}}(\mathbb{N}, j)|)$  then
      if  $\mathcal{WT}_{N^{(k)}}(\mathbb{N}, j) > 0$  then
         $EV_{(k)}(j) = N$ 
      else
         $EV_{(k)}(j) = A \text{ or } Y$ 
      end if
    end if
    if  $EV_{(k)}(j) = FV_{(k)}(j)$  then
       $P7MF_{(k)}(j) = P7MF_{(k)}(j) + 1$ 
    else
       $P7MF_{(k)}(j) = P7MF_{(k)}(j)$ 
    end if
     $P7MR_{(k)}(j) = \frac{P7MF_{(k)}(j)}{k}$ 
  end for
end for
Return P7MF and P7MR of  $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$ 
```

Table C.4: One Example to Display the Pattern 7 Matching Algorithm

	i_1	i_2	i_3	j
1	Y	Y	A	A
2	Y	A	Y	Y
3	Y	A	A	A
4	A	A	Y	N
5	A	Y	A	A
6	A	A	N	A
7	Y	Y	Y	Y
8	A	N	N	A
9	Y	A	Y	Y
10	A	Y	Y	A

The procedure of all steps is displayed as following, just explained the first three steps. For step 1, the decisions of influencing agents are Y,Y,A, then $\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 2,1,0, the maximum is \mathcal{WT}_Y , therefore the expected vote is Y, and the factual vote on step 1 is A, so it is unmatched, and the matching frequency is 0 and matching ratio is $0.00=0/1$; for the second step, the decisions of influencing agents are Y,A,Y, then $\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 2,1,0, the maximum is \mathcal{WT}_Y , therefore the expected vote is Y, and the factual vote on step 2 is also Y, so it is matched, and the matching frequency is 2 and matching ratio is $0.50=1/2$; for the third step, the former decisions are Y,A,A, then $\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 1,2,0, the maximum is still \mathcal{WT}_A , therefore the expected vote is A, and the factual vote on step 3 is also A, so it is matched, and the matching frequency is 2 and matching ratio is $0.67=2/3$, the following steps work in the same way.

Table C.5: Testing Procedure Step-by-step by the Pattern 7 Matching Algorithm

Step	\mathcal{WT}_Y	\mathcal{WT}_A	\mathcal{WT}_N	MAX	EV	FV	Match	P7MF	P7MR
1	2	1	0	\mathcal{WT}_Y	Y	A	NO	0	0.00
2	2	1	0	\mathcal{WT}_Y	Y	Y	YES	1	0.50
3	1	2	0	\mathcal{WT}_A	A	A	YES	2	0.67
4	1	2	0	\mathcal{WT}_A	A	N	NO	2	0.50
5	1	2	0	\mathcal{WT}_A	A	A	YES	3	0.60
6	0	2	1	\mathcal{WT}_A	A	N	NO	3	0.53
7	3	0	0	\mathcal{WT}_Y	Y	Y	YES	4	0.57
8	0	1	2	\mathcal{WT}_N	N	A	NO	4	0.50
9	2	1	0	\mathcal{WT}_Y	Y	Y	YES	5	0.56
10	2	1	0	\mathcal{WT}_Y	Y	A	NO	5	0.50

Above displayed the testing outcome if the agent j 's decision is influenced just by other agents' decisions, but if combining with the influence from own preference, which means the decision on current issue is the collective effect of the decisions of other agents and the preference of own, then the testing outcome would be slightly different.

Table C.6: Testing Procedure Step-by-step by the Pattern 7 Matching Algorithm (with the influence from own preference)

Step	\mathcal{WT}_Y	\mathcal{WT}_A	\mathcal{WT}_N	MAX	EV	FV	Match	P7MF	P7MR
1	2	2	0	$\mathcal{WT}_Y/\mathcal{WT}_A$	Y/A	A	YES	1	1.00
2	3	1	0	\mathcal{WT}_Y	Y	Y	YES	2	1.00
3	1	3	0	\mathcal{WT}_A	A	Y	No	2	0.66
4	1	2	1	\mathcal{WT}_A	A	N	NO	2	0.50
5	1	3	0	\mathcal{WT}_A	A	A	YES	3	0.60
6	0	3	1	\mathcal{WT}_A	A	N	No	3	0.50
7	4	0	0	\mathcal{WT}_Y	Y	Y	YES	4	0.57
8	0	2	2	$\mathcal{WT}_A\mathcal{WT}_N$	A/N	A	YES	5	0.63

9	3	1	0	\mathcal{WT}_Y	Y	Y	YES	6	0.66
10	2	2	0	$\mathcal{WT}_Y/\mathcal{WT}_A$	Y/A	A	YES	7	0.70

Above we just assume the weight of influence from all influencing agents identical (as positive 1), while in real-world, the weight of influence from different agents would be varied, weak or strong, negative or positive. For example, the relations between the influencing agent and influenced agent would affect the weight of influence, the influence from your friend to you would be deemed as positive, and the influence from your enemy to you would be deemed as negative. Besides, the more powerful of the influencing agent, the more similar of the influencing agent with you, the higher would be the weight of influence to you. There are many other factors affecting the weight of influence, and it also depends on many situations.

Influence Pattern 13 Matching Algorithm

Explanation

For influence pattern 13, which is the influence crossing both different agents and different issues. The decision of agent j on issue q would be influenced by the decision of agent i on former issue(s), for example if influenced by all former issues, it would be the decisions of agent i on issues 1, 2, ..., $q - 1$. Design the matching algorithm for influence pattern 13 as Algorithm 10, j is the No. of influenced agent, i is the No. of influencing agent ($i \neq j$), q is the No. of influenced issues, and k is the No. of influencing issues (therefore $k < q - 1$, only the latter issue could be influenced by the former issue, but not reversely). At the first step (about the influencing component), for the influencing agent i 's decisions on issue k from 1 to $q - 1$, at each step (of issue k), if $FV_{(k)}(i) = Y$, which means the factual vote (FV) of agent i on issue k is Yes (affirmative), then $\mathcal{WT}_{Y(k,q)}(i,j) = \mathcal{WT}_{Y(k-1,q)}(i,j) + wt_{Y(k,q)}(i,j)$, which means the accumulative weight of influence of affirmative vote on influenced variable q_j (the decision of agent j on issue q) would be plus with the weight of influence of the current vote (of agent i on issue k), which work similarly if $FV_{(k)}(i) = A$ and $FV_{(k)}(i) = N$. At the second step, for the influenced variable q_j (the decision of agent j on issue q), assume it would be influenced by all former issues from 1 to $q-1$ of agent i , if $|\mathcal{WT}_{Y(q-1,q)}(i,j)| = \text{MAX}_{V \in \{Y,A,N\}}(|\mathcal{WT}_{V(q-1,q)}(i,j)|)$, which means the accumulative weight of influence of affirmative vote of all former influencing issues before issue q of agent i is the maximum of all accumulative weight of influence (of affirmative vote, absent vote and negative vote), and also $\mathcal{WT}_{Y(q-1,q)}(i,j) > 0$ (the influence is positive), then $EV_{(q)}(j) = Y$, which means the expected vote (EV) of influenced agent j on issue q computed according to the algorithm would be the same as Yes (affirmative), otherwise if the influence is negative, then $EV_{(q)}(j) = A \text{ or } N$, which means the expected vote would be contrast to or different from Yes (affirmative), which work similarly if $|\mathcal{WT}_{A(q-1,q)}(i,j)| = \text{MAX}_{V \in \{Y,A,N\}}(|\mathcal{WT}_{V(q-1,q)}(i,j)|)$ and $|\mathcal{WT}_{N(q-1,q)}(i,j)| = \text{MAX}_{V \in \{Y,A,N\}}(|\mathcal{WT}_{V(q-1,q)}(i,j)|)$. At the third step, for influenced issue q from k to m of agent j , compared the factual vote (according to the official voting data) and the expected vote (by the matching algorithm), at each step (for agent j on issue q), if $EV_{(q)}(j) = FV_{(q)}(j)$ (the expected vote is identical with the factual vote), then $P13MF_{(q)}(j) = P13MF_{(q-j)}(i) + 1$ (the pattern 13 matching frequency would be plus 1), and the pattern 13 matching ratio would be obtained by dividing the No. of issue q .

Example

Given a simple example to display the procedure of testing algorithm step by step. Assume there is one influencing agent i and one influenced agent j , making decisions on 10 issues. Assume the agent j 's current decision is influenced by all the former decisions of agent j , and assume the weight of influence of all former decisions identical as 1 (and as all positive influence). \mathcal{WT}_Y ,

Algorithm 10 Testing Algorithm for Influence Pattern 13

Input: A society $\text{ST}(\mathbb{N}, \mathbb{M}, \text{WT})$, $\mathbb{N} = \{1, \dots, n\}$, $\mathbb{M} = \{1, \dots, m\}$,
there are n agents and m features.

Output: Pattern 13 Matching Frequency (P13MF) and Pattern 13 Matching Ratio (P13MR) of $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$

```
for  $j = 1$  to  $n$  do
  for  $q = k$  to  $m$  do
    for  $k = 1$  to  $q - 1$  do
      if  $FV_{(k)}(i) = Y$  then
         $\text{WT}_{Y(k,q)}(i, j) = \text{WT}_{Y(k-1,q)}(i, j) + \text{wt}_{Y(k,q)}(i, j)$ 
      else
         $\text{WT}_{Y(k,q)}(i, j) = \text{WT}_{Y(k-1,q)}(i, j)$ 
      end if
      if  $FV_{(k)}(i) = A$  then
         $\text{WT}_{A(k,q)}(i, j) = \text{WT}_{A(k-1,q)}(i, j) + \text{wt}_{A(k,q)}(i, j)$ 
      else
         $\text{WT}_{A(k,q)}(i, j) = \text{WT}_{A(k-1,q)}(i, j)$ 
      end if
      if  $FV_{(k)}(i) = N$  then
         $\text{WT}_{N(k,q)}(i, j) = \text{WT}_{N(k-1,q)}(i, j) + \text{wt}_{N(k,q)}(i, j)$ 
      else
         $\text{WT}_{N(k,q)}(i, j) = \text{WT}_{N(k-1,q)}(i, j)$ 
      end if
    end for
    if  $|\text{WT}_{Y(q-1,q)}(i, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\text{WT}_{V(q-1,q)}(i, j)|)$  then
      if  $\text{WT}_{Y(q-1,q)}(i, j) > 0$  then
         $EV_{(q)}(j) = Y$ 
      else
         $EV_{(q)}(j) = \text{Aor}N$ 
      end if
    end if
    if  $|\text{WT}_{A(q-1,q)}(i, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\text{WT}_{V(q-1,q)}(i, j)|)$  then
      if  $\text{WT}_{A(q-1,q)}(i, j) > 0$  then
         $EV_{(q)}(j) = A$ 
      else
         $EV_{(q)}(j) = \text{Yor}N$ 
      end if
    end if
    if  $|\text{WT}_{N(q-1,q)}(i, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\text{WT}_{V(q-1,q)}(i, j)|)$  then
      if  $\text{WT}_{N(q-1,q)}(i, j) > 0$  then
         $EV_{(q)}(j) = N$ 
      else
         $EV_{(q)}(j) = \text{Aor}Y$ 
      end if
    end if
    if  $EV_{(q)}(j) = FV_{(q)}(j)$  then
      count  $P13MF_{(q)}(j) = P13MF_{(q-1)}(j) + 1$ 
    else
      count  $P13MF_{(q)}(j) = P13MF_{(q-1)}(j)$ 
    end if
     $P13MR_{(q)}(j) = \frac{P13MF_{(q)}(j)}{q}$ 
  end for
end for
Return P13MF and P13MR of  $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$ 
```

WT_A , WT_N are respectively the accumulative weight of influence of affirmative vote, absent vote and negative vote from the decisions of influencing agent on former issues.

Table C.7: One Example to Display the Pattern 18 Matching Algorithm

	i	j
	Y	Y
1	Y	Y
2	A	Y
3	Y	N
4	Y	Y
5	N	A
6	A	Y
7	A	A
8	A	Y
9	N	Y
10	Y	Y

The procedure of all steps is displayed as following, just explained the first three steps. For step 1, the former decision of agent i is Y, then WT_Y, WT_A, WT_N are respectively 1,0,0, the maximum is WT_Y , therefore the expected vote for agent j on issue 1 is Y, and the factual vote of agent j on step 1 is also Y, so it is matched, and the matching frequency is 1 and matching ratio is $1.00=1/1$; for the second step, the former decisions of agent i are Y,Y, then WT_Y, WT_A, WT_N are respectively 2,0,0, the maximum is WT_Y , therefore the expected vote for agent j on issue 2 is Y, and the factual vote of agent j on step 2 is also Y, so it is matched, and the matching frequency is 2 and matching ratio is $1.00=2/2$; for the third step, the former decisions of agent i are Y,Y,A, then WT_Y, WT_A, WT_N are respectively 2,1,0, the maximum is still WT_Y , therefore the expected vote for agent j on issue 3 is Y, and the factual vote of agent j on step 3 is but N, so it is unmatched, and the matching frequency is 2 and matching ratio is $0.67=2/3$, the following steps work in the same way.

Table C.8: Testing Procedure Step-by-step by the Pattern 13 Matching Algorithm

Step	WT_Y	WT_A	WT_N	MAX	EV	FV	Match	PIMF	PIMR
1	1	0	0	WT_Y	Y	Y	YES	1	1.00
2	2	0	0	WT_Y	Y	Y	YES	2	1.00
3	2	1	0	WT_Y	Y	N	NO	2	0.67
4	3	1	0	WT_Y	Y	Y	YES	3	0.75
5	4	1	0	WT_Y	Y	A	NO	3	0.60
6	4	1	1	WT_Y	Y	Y	YES	4	0.67
7	4	2	1	WT_Y	Y	A	NO	4	0.57
8	4	3	1	WT_Y	Y	Y	YES	5	0.63
9	4	4	1	WT_Y/WT_A	Y/A	Y	YES	6	0.67
10	4	4	2	WT_Y/WT_A	Y/A	Y	YES	7	0.70

Above displayed the testing outcome if the current decision of influenced agent is influenced just by the former decisions (before the current decision) of other influencing agent, but if combining with the influence from the current preference of influencing agent, which means the decision on current issue is the collective effect of the decisions on former issues and the preference of current issue of influencing agent, as below, the testing outcome is the same with just considering the influence from former decisions. Therefore, while the assumptions about rules of influence slightly adjusted, the outcome might be different, and also might be the same.

Table C.9: Testing Procedure Step-by-step by the Pattern 13 Matching Algorithm (with the influence from current preference)

Step	WT_Y	WT_A	WT_N	MAX	EV	FV	Match	PIMF	PIMR
1	2	0	0	WT_Y	Y	Y	YES	1	1.00
2	2	1	0	WT_Y	Y	Y	YES	2	1.00
3	3	1	0	WT_Y	Y	N	NO	2	0.67
4	4	1	0	WT_Y	Y	Y	YES	3	0.75
5	4	1	1	WT_Y	Y	A	NO	3	0.60
6	4	2	1	WT_Y	Y	Y	YES	4	0.67
7	4	3	1	WT_Y	Y	A	NO	4	0.57
8	4	4	1	WT_Y/WT_A	Y/A	Y	YES	5	0.63
9	4	4	2	WT_Y/WT_A	Y/A	Y	YES	6	0.67
10	5	4	2	WT_Y	Y	Y	YES	7	0.70

Influence Pattern 18 Matching Algorithm

Explanation

For influence pattern 18, which is the structure among issues of one agent would be influenced by the structure among issues of other agent(s). The structure of influence among issues of agent j would be affected by the structure of influence among issues of other agent(s), for example if affected by all other agents, it would be agents from 1 to n (except j). Design the matching algorithm for influence pattern 18 as Algorithm 11-Part 1 and Algorithm 12-Part 2), j is the No. of influenced agent, i is the No. of influencing agent ($i \neq j$), q is the No. of influenced issues, and k is the No. of influencing issues (therefore $k < q - 1$, only the latter issue could be influenced by the former issue, but not reversely).

The part 1 is about the structure among issues of influencing agent i . At the first step, for agent i 's decisions on issue k from 1 to $q-1$, at each step (of issue k), if $FV_{(k)}(i) = Y$, which means the factual vote (FV) of agent i on issue k is Yes (affirmative), then $WT_{Y(k,q)}(i, i) = WT_{Y(k-1,q)}(i, i) + wt_{Y(k,q)}(i, i)$, which means the accumulative weight of influence of affirmative vote on variable q_i (the decision of agent i on issue q) would be plus with the weight of influence of the current vote (of agent i on issue k), which work similarly if $FV_{(k)}(i) = A$ and $FV_{(k)}(i) = N$.

At the second step, for the influenced variable q_i (the decision of agent i on issue q), assume it would be influenced by all former issues from 1 to $q-1$ of agent i , if $|WT_{Y(q-1,q)}(i, i)| = MAX_{V \in \{Y, A, N\}}(|WT_{V(q-1,q)}(i, i)|)$, which means the accumulative weight of influence of affirmative vote of all former issues before issue q of agent i is the maximum of all accumulative weight of influence (of affirmative vote, absent vote and negative vote), and also $WT_{Y(q-1,q)}(i, i) > 0$ (the influence is positive), then $EV_{(q)}(i) = Y$, which means the expected vote (EV) of influencing agent i on issue q computed according to the algorithm would be the same as Yes (affirmative), otherwise if the influence is negative, then $EV_{(q)}(i) = A \text{ or } N$, which means the expected vote would be contrast to or different from Yes (affirmative), which work similarly if $|WT_{A(q-1,q)}(i, i)|$ or $|WT_{N(q-1,q)}(i, i)| = MAX_{V \in \{Y, A, N\}}(|WT_{V(q-1,q)}(i, i)|)$.

At the third step, for influenced issue q from k to m of agent i , compared the factual vote (according to the official voting data) and the expected vote (by the matching algorithm), at each step (for agent i on issue q), if $EV_{(q)}(i) = FV_{(q)}(i)$ (the expected vote is the same with the factual vote), then $WT_{S_{P(q,q)}}(\mathbb{N}, j) = WT_{S_{P(q,q)}}(\mathbb{N}, j) + wts_{P(q,q)}(i, j)$, which means the accumulative weight of structure of positive influence would be plus with the weight of current structure.

The part 2 is about the structure among issues of influenced agent j . At the fourth step, for agent j 's decisions on issue k from 1 to $q-1$, at each step (of issue k), if $FV_{(k)}(j) = Y$, which means the factual vote (FV) of agent j on issue k is Yes (affirmative), then $WT_{Y(k,q)}(j, j) = WT_{Y(k-1,q)}(j, j) + wt_{Y(k,q)}(j, j)$, which means the accumulative weight of influence of affir-

Algorithm 11 Testing Algorithm for Influence Pattern 18-Part 1

Input: A society $\mathcal{ST}(\mathbb{N}, \mathbb{M}, \mathcal{WT})$, $\mathbb{N} = \{1, \dots, n\}$, $\mathbb{M} = \{1, \dots, m\}$,
there are n agents and m features.

Output: Pattern 18 Matching Frequency (P18MF) and Pattern 18 Matching Ratio (P18MR) of $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$

```
for  $j = 1$  to  $n$  do
  for  $q = k$  to  $m$  do
    for  $i = 1$  to  $n$ ,  $i \neq j$  do
      for  $k = 1$  to  $q - 1$  do
        if  $FV_{(k)}(i) = Y$  then
           $\mathcal{WT}_{Y(k,q)}(i, i) = \mathcal{WT}_{Y(k-1,q)}(i, i) + wt_{Y(k,q)}(i, i)$ 
        else
           $\mathcal{WT}_{Y(k,q)}(i, i) = \mathcal{WT}_{Y(k-1,q)}(i, i)$ 
        end if
        if  $FV_{(k)}(i) = A$  then
           $\mathcal{WT}_{A(k,q)}(i, i) = \mathcal{WT}_{A(k-1,q)}(i, i) + wt_{A(k,q)}(i, i)$ 
        else
           $\mathcal{WT}_{A(k,q)}(i, i) = \mathcal{WT}_{A(k-1,q)}(i, i)$ 
        end if
        if  $FV_{(k)}(i) = N$  then
           $\mathcal{WT}_{N(k,q)}(i, i) = \mathcal{WT}_{N(k-1,q)}(i, i) + wt_{N(k,q)}(i, i)$ 
        else
           $\mathcal{WT}_{N(k,q)}(i, i) = \mathcal{WT}_{N(k-1,q)}(i, i)$ 
        end if
      end for
      if  $|\mathcal{WT}_{Y(q-1,q)}(i, i)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q-1,q)}(i, i)|)$  then
        if  $\mathcal{WT}_{Y(q-1,q)}(i, i) > 0$  then
           $EV_{(q)}(i) = Y$ 
        else
           $EV_{(q)}(i) = A \text{ or } N$ 
        end if
      end if
      if  $|\mathcal{WT}_{A(q-1,q)}(i, i)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q-1,q)}(i, i)|)$  then
        if  $\mathcal{WT}_{A(q-1,q)}(i, i) > 0$  then
           $EV_{(q)}(i) = A$ 
        else
           $EV_{(q)}(i) = Y \text{ or } N$ 
        end if
      end if
      if  $|\mathcal{WT}_{N(q-1,q)}(i, i)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q-1,q)}(i, i)|)$  then
        if  $\mathcal{WT}_{N(q-1,q)}(i, i) > 0$  then
           $EV_{(q)}(i) = N$ 
        else
           $EV_{(q)}(i) = A \text{ or } Y$ 
        end if
      end if
      if  $EV_{(q)}(i) = FV_{(q)}(i)$  then
         $\mathcal{WTS}_{P(q,q)}(\mathbb{N}, j) = \mathcal{WTS}_{P(q,q)}(\mathbb{N}, j) + wts_{P(q,q)}(i, j)$ 
      else if  $EV_{(q)}(i) \neq FV_{(q)}(i)$  then
         $\mathcal{WTS}_{N(q,q)}(\mathbb{N}, j) = \mathcal{WTS}_{N(q,q)}(\mathbb{N}, j) + wts_{N(q,q)}(i, j)$ 
      end if
    end for
  end for
end for
```

Algorithm 12 Testing Algorithm for Influence Pattern 18-Part 2

```

for  $k = 1$  to  $q - 1$  do
  if  $FV_{(k)}(j) = Y$  then
     $\mathcal{WT}_{Y(k,q)}(j, j) = \mathcal{WT}_{Y(k-1,q)}(j, j) + wt_{Y(k,q)}(j, j)$ 
  else
     $\mathcal{WT}_{Y(k,q)}(j, j) = \mathcal{WT}_{Y(k-1,q)}(j, j)$ 
  end if
  if  $FV_{(k)}(j) = A$  then
     $\mathcal{WT}_{A(k,q)}(j, j) = \mathcal{WT}_{A(k-1,q)}(j, j) + wt_{A(k,q)}(j, j)$ 
  else
     $\mathcal{WT}_{A(k,q)}(j, j) = \mathcal{WT}_{A(k-1,q)}(j, j)$ 
  end if
  if  $FV_{(k)}(j) = N$  then
     $\mathcal{WT}_{N(k,q)}(j, j) = \mathcal{WT}_{N(k-1,q)}(j, j) + wt_{N(k,q)}(j, j)$ 
  else
     $\mathcal{WT}_{N(k,q)}(j, j) = \mathcal{WT}_{N(k-1,q)}(j, j)$ 
  end if
end for
if  $\mathcal{WTS}_{P(q,q)}(\mathbb{N}, j) \geq \mathcal{WTS}_{N(q,q)}(\mathbb{N}, j)$  then
  if  $|\mathcal{WT}_{Y(q-1,q)}(j, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q-1,q)}(j, j)|)$  then
    if  $\mathcal{WT}_{Y(q-1,q)}(j, j) > 0$  then
       $EV_{(q)}(j) = Y$ 
    else
       $EV_{(q)}(j) = A \text{ or } N$ 
    end if
  end if
  if  $|\mathcal{WT}_{A(q-1,q)}(j, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q-1,q)}(j, j)|)$  then
    if  $\mathcal{WT}_{A(q-1,q)}(j, j) > 0$  then
       $EV_{(q)}(j) = A$ 
    else
       $EV_{(q)}(j) = Y \text{ or } N$ 
    end if
  end if
  if  $|\mathcal{WT}_{N(q-1,q)}(j, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q-1,q)}(j, j)|)$  then
    if  $\mathcal{WT}_{N(q-1,q)}(j, j) > 0$  then
       $EV_{(q)}(j) = N$ 
    else
       $EV_{(q)}(j) = A \text{ or } Y$ 
    end if
  end if
else
  if  $|\mathcal{WT}_{Y(q-1,q)}(j, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q-1,q)}(j, j)|)$  then
    if  $\mathcal{WT}_{Y(q-1,q)}(j, j) > 0$  then
       $EV_{(q)}(j) = A \text{ or } N$ 
    else
       $EV_{(q)}(j) = Y$ 
    end if
  end if
  if  $|\mathcal{WT}_{A(q-1,q)}(j, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q-1,q)}(j, j)|)$  then
    if  $\mathcal{WT}_{A(q-1,q)}(j, j) > 0$  then
       $EV_{(q)}(j) = Y \text{ or } N$ 
    else
       $EV_{(q)}(j) = A$ 
    end if
  end if
  if  $|\mathcal{WT}_{N(q-1,q)}(j, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q-1,q)}(j, j)|)$  then
    if  $\mathcal{WT}_{N(q-1,q)}(j, j) > 0$  then
       $EV_{(q)}(j) = A \text{ or } Y$ 
    else
       $EV_{(q)}(j) = N$ 
    end if
  end if
end if
if  $EV_{(q)}(j) = FA_{(q)}(j)$  then
   $P18MF_{(q)}(j) = P18MF_{(q-1)}(j) + 1$ 
else
   $P18MF_{(q)}(j) = P18MF_{(q-1)}(j)$ 
end if
   $P18MR_{(q)}(j) = \frac{P18MF_{(q)}(j)}{q}$ 
end for
end for
Return P18MF and P18MR of  $(FV_{(1)}(1), \dots, FV_{(m)}(n))$ 

```

mative vote on variable q_j (the decision of agent j on issue q) would be plus with the weight of influence of the current vote (of agent j on issue k), which work similarly if $FV_{(k)}(j) = A$ and $FV_{(k)}(j) = N$.

At the fifth step, for the influenced variable q_j (the decision of agent j on issue q), assume it would be influenced by all former issues from 1 to $q-1$ of agent j according to the structure of influence of other (influencing) agents \mathbb{N} . If $\mathcal{WTS}_{P(q,q)}(\mathbb{N}, j) \geq \mathcal{WTS}_{N(q,q)}(\mathbb{N}, j)$, which means the dominating structure of influence affecting on agent j is positive, further, if $|\mathcal{WT}_{Y(q-1,q)}(j, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q-1,q)}(j, j)|)$ (the accumulative weight of influence of affirmative vote of all former issues before issue q of agent j is the maximum of all accumulative weight of influence of affirmative vote, absent vote and negative vote), and also $\mathcal{WT}_{Y(q-1,q)}(j, j) > 0$ (the influence is positive), then $EV_{(q)}(j) = Y$, which means the expected vote (EV) of influencing agent j on issue q computed according to the algorithm would be the same as Yes (affirmative), otherwise if the influence is negative, then $EV_{(q)}(j) = A \text{ or } N$, which means the expected vote would be contrast to or different from Yes (affirmative), which work similarly if $|\mathcal{WT}_{A(q-1,q)}(j, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q-1,q)}(j, j)|)$ and $|\mathcal{WT}_{N(q-1,q)}(j, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q-1,q)}(j, j)|)$. But if back to the first if of this step, if $\mathcal{WTS}_{P(q,q)}(\mathbb{N}, j) < \mathcal{WTS}_{N(q,q)}(\mathbb{N}, j)$, which means the dominating structure of influence is negative, then the whole procedure works contrast to above.

At the sixth step, for issue q from k to m of agent j , compared the factual vote (according to the official voting data) and the expected vote (by the matching algorithm), at each step (for agent j on issue q), if $EV_{(q)}(j) = FV_{(q)}(j)$ (the expected vote is the same with the factual vote), then $P18MF_{(q)}(j) = P18MF_{(q-1)}(j) + 1$, which means the pattern 18 matching frequency is plus with 1, and the pattern 18 matching ratio would be obtained by dividing the No. of issue q .

Example

Further, given a simple example to display the procedure of testing algorithm step by step. Assume just influenced by the structure among issues of one agent, and there is one influencing agent i and one influenced agent j , making decisions on 10 issues. And assume the weight of all structures identical as 1.

Table C.10: One Example to Display the Pattern 18 Matching Algorithm

	i	j
	Y	Y
1	Y	Y
2	A	Y
3	Y	N
4	Y	Y
5	N	A
6	A	Y
7	A	A
8	A	Y
9	N	Y
10	Y	Y

As in the table of testing procedure of this algorithm (table 11.15 and table 11.16), \mathcal{WT}_Y , \mathcal{WT}_A , \mathcal{WT}_N are respectively the accumulative weight of influence of affirmative vote, absent vote and negative vote from former issues' decisions. IEV is the expected vote of Influencing agent, IFV is the factual vote of Influencing agent, IMt is the matching situation between IEV and IFV. \mathcal{WTS}_P is the accumulated weight of structure of positive influence, and \mathcal{WTS}_N is the accumulated weight of structure of negative influence. EVD is the expected vote of influenceD agent, FVD is the factual vote of influenceD agent, and MtD is the matching situation between

EVD and FVD.

The procedure of all steps is displayed as following (table 11.15), just explained the first three steps. For the first step, the former decision of agent i is Y, then $\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 1,0,0, the maximum is \mathcal{WT}_Y , therefore the expected vote for agent j (IEV) on issue 1 is Y, and the factual vote of agent i (IFV) on step 1 is also Y, so IMt is matched, and $\mathcal{WTS}_{P(q,q)}(\mathbb{N}, j)$ and $\mathcal{WTS}_{N(q,q)}(\mathbb{N}, j)$ are respectively 1,0, so the dominating influencing structure is positive, and for influenced agent j , the former decision is Y ($\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 1,0,0), and according to the positive influencing structure, the expected vote of agent j (EVD) is Y, and the factual vote of agent j (FVD) is also Y, so MtD is matched, the matching frequency is 1 and matching ratio is $1.00=1/1$; for the second step, the former decision of agent i is Y,Y, then $\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 2,0,0, the maximum is \mathcal{WT}_Y , therefore the expected vote for agent j (IEV) on issue 2 is Y, and the factual vote of agent i (IFV) on step 2 is A, so IMt is unmatched, and $\mathcal{WTS}_{P(q,q)}(\mathbb{N}, j)$ and $\mathcal{WTS}_{N(q,q)}(\mathbb{N}, j)$ are respectively 1,1, so there is no dominating influencing structure either as positive or negative, and for influenced agent j , the former decision is Y,Y ($\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 2,0,0), according to the unclear influencing structure, the expected vote of agent j (EVD) could be any vote Y/A/N, and the factual vote of agent j (FVD) is Y, so MtD is matched, the matching frequency is 2 and matching ratio is $1.00=2/2$; for the third step, the former decision of agent i is Y,Y,A then $\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 2,1,0, the maximum is \mathcal{WT}_Y , therefore the expected vote for agent j (IEV) on issue 3 is Y, and the factual vote of agent i (IFV) on step 3 is also Y, so IMt is matched, and $\mathcal{WTS}_{P(q,q)}(\mathbb{N}, j)$ and $\mathcal{WTS}_{N(q,q)}(\mathbb{N}, j)$ are respectively 2,1, so the dominating influencing structure is positive, and for influenced agent j , the former decision is Y,Y,Y ($\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 3,0,0), according to the positive influencing structure, the expected vote of agent j (EVD) should be Y, and the factual vote of agent j (FVD) is but N, so MtD is unmatched, the matching frequency is 2 and matching ratio is $0.67=2/3$, the following steps work in the same way.

Above displayed the testing outcome by just comparing the decisions of former issues and the preference of current issue to identify the attribute (positive or negative) of influencing structure, but if also considering the influence from current preference own, comparing the collective effect (combining former decisions and current preference together) with current preference, judge the expected and factual vote matched or not, the testing outcome would be different as below (table 11.22), affected by the adjustment of rule.

Influence Pattern 19 Matching Algorithm

Explanation

For influence pattern 19, which is the influence crossing both different agents and different issues. The decision of agent j on issue q would be influenced by the decision of other agents on former issue k , for example if influenced by all other agents, it would be the decisions of agents set \mathbb{N} except agent j on issue k . Design the matching algorithm for influence pattern 19 as Algorithm 13, q is the No. of influenced issue (k is the No. of influencing issues corresponding to issue q), j is the No. of influenced agent, and i is the No. of influencing agent ($i \neq j$). At the first step, for the influencing agent i from 1 to n on issue k , at each step (of agent i), if $FV_{(k)}(i) = Y$, which means the factual vote (FV) of agent i on issue k is Yes (affirmative), then $\mathcal{WT}_{Y(k,q)}(\mathbb{N}, j) = \mathcal{WT}_{Y(k,q)}(\mathbb{N}, j) + wt_{Y(k,q)}(i, j)$, which means the accumulative weight of influence of affirmative vote on influenced variable q_j (the decision of agent j on issue q) would be plus with the weight of influence of the current vote (of agent i on issue k), which work similarly if $FV_{(k)}(i) = A$ and $FV_{(k)}(i) = N$. At the second step, for the influenced variable q_j (the decision of agent j on issue q), assume it would be influenced by all other agents from 1 to m (except j) on issue k , if $|\mathcal{WT}_{Y(k,q)}(\mathbb{N}, j)| = MAX_{V \in \{Y,A,N\}}(|\mathcal{WT}_{V(k,q)}(i)|)$, which means the

Table C.11: Testing Procedure Step-by-step by the Pattern 18 Matching Algorithm

Step	WT _Y	WT _A	WT _N	IEV	IFV	IMt	WTS _P	WTS _N	WT _Y	WT _A	WT _N	EVD	FVD	MtD	P18MF	P18MR
1	1	0	0	Y	Y	YES	1	0	1	0	0	Y	Y	YES	1	1.00
2	2	0	0	Y	A	NO	1	1	3	0	0	Y/A/N	Y	YES	2	1.00
3	2	1	0	Y	Y	YES	2	1	3	0	0	Y	N	NO	2	0.67
4	3	1	0	Y	Y	YES	3	1	3	0	1	Y	Y	YES	3	0.75
5	4	1	0	Y	N	NO	3	2	4	0	1	Y	A	NO	3	0.60
6	4	1	1	Y	A	NO	3	3	4	1	1	Y/A/N	Y	YES	4	0.67
7	4	2	1	Y	A	NO	3	4	5	1	1	A/N	A	YES	5	0.71
8	4	3	1	Y	A	NO	3	5	2	2	1	A/N	Y	NO	5	0.63
9	4	4	1	Y/A	N	NO	3	6	2	2	1	A/N	Y	NO	5	0.56
10	4	4	2	Y/A	Y	YES	3	7	7	2	1	A/N	Y	NO	6	0.60

Table C.12: Testing Procedure Step-by-step by the Pattern 18 Matching Algorithm (with the influence from own structure)

Step	WT_Y	WT_A	WT_N	IEV	IFV	IMt	WT_{S_P}	WT_{S_N}	WT_Y	WT_A	WT_N	EVD	FVD	MtD	P18MF	P18MR
1	2	0	0	Y	Y	YES	1	0	0	0	0	Y	Y	YES	1	1.00
2	2	1	0	Y	A	NO	1	1	0	0	0	Y/A/N	Y	YES	2	1.00
3	3	1	0	Y	Y	YES	2	1	0	1	1	Y	N	NO	2	0.67
4	4	1	0	Y	Y	YES	3	1	0	1	1	Y	Y	YES	3	0.75
5	4	1	1	Y	N	NO	3	2	1	1	1	Y	A	NO	3	0.60
6	4	2	1	Y	A	NO	3	3	1	1	1	Y/A/N	Y	YES	4	0.67
7	4	3	1	Y	A	NO	3	4	2	1	1	A/N	A	YES	5	0.71
8	4	4	1	Y/A	A	YES	4	4	2	1	1	Y/A/N	Y	YES	6	0.75
9	4	4	2	Y/A	N	NO	4	5	2	1	1	A/N	Y	NO	6	0.67
10	5	4	2	Y	Y	YES	3	7	2	1	1	A/N	Y	NO	7	0.70

Algorithm 13 Testing Algorithm for Influence Pattern 19

Input: A society $\text{ST}(\mathbb{N}, \mathbb{M}, \text{WT})$, $\mathbb{N} = \{1, \dots, n\}$, $\mathbb{M} = \{1, \dots, m\}$,
there are n agents and m features.

Output: Pattern 19 Matching Frequency (P19MF) and Pattern 19 Matching Ratio (P19MR) of $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$

```
for  $q = 1$  to  $m$  do
  for  $j = 1$  to  $n$  do
    for  $i = 1$  to  $n$   $i \neq j$  do
      if  $FV_{(k)}(i) = Y$  then
         $\text{WT}_{Y(k,q)}(\mathbb{N}, j) = \text{WT}_{Y(k,q)}(\mathbb{N}, j) + \text{wt}_{Y(k,q)}(i, j)$ 
      end if
      if  $FV_{(k)}(i) = A$  then
         $\text{WT}_{A(k,q)}(\mathbb{N}, j) = \text{WT}_{A(k,q)}(\mathbb{N}, j) + \text{wt}_{A(k,q)}(i, j)$ 
      end if
      if  $FV_{(k)}(i) = N$  then
         $\text{WT}_{N(k,q)}(\mathbb{N}, j) = \text{WT}_{N(k,q)}(\mathbb{N}, j) + \text{wt}_{N(k,q)}(i, j)$ 
      end if
    end for
    if  $|\text{WT}_{Y(k,q)}(\mathbb{N}, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\text{WT}_{V(k,q)}(i)|)$  then,
      if  $\text{WT}_{Y(k,q)}(\mathbb{N}, j) > 0$  then
         $EV_{(q)}(j) = Y$ 
      else
         $EV_{(q)}(j) = \text{AorN}$ 
      end if
    end if
    if  $|\text{WT}_{A(k,q)}(\mathbb{N}, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\text{WT}_{V(k,q)}(i)|)$  then,
      if  $\text{WT}_{A(k,q)}(\mathbb{N}, j) > 0$  then
         $EV_{(q)}(j) = A$ 
      else
         $EV_{(q)}(j) = \text{YorN}$ 
      end if
    end if
    if  $|\text{WT}_{N(k,q)}(\mathbb{N}, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\text{WT}_{V(k,q)}(i)|)$  then,
      if  $\text{WT}_{N(k,q)}(\mathbb{N}, j) > 0$  then
         $EV_{(q)}(j) = N$ 
      else
         $EV_{(q)}(j) = \text{AorY}$ 
      end if
    end if
    if  $EV_{(q)}(j) = FV_{(q)}(j)$  then
       $P19MF_{(q)}(j) = P19MF_{(q)}(j) + 1$ 
    else
       $P19MF_{(q)}(j) = P19MF_{(q)}(j)$ 
    end if
    if
       $P19MR_{(q)}(j) = \frac{P19MF_{(q)}(j)}{q}$ 
    end if
  end for
end for
Return P19MF and P19MR of  $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$ 
```

accumulative weight of influence of affirmative vote of all other influencing agents on issue k is the maximum of all accumulative weight of influence (of affirmative vote, absent vote and negative vote), and also $\mathcal{WT}_{Y^{(k,q)}}(\mathbb{N}, j) > 0$ (the influence is positive), then $EV_{(q)}(j) = Y$, which means the expected vote (EV) of influenced agent j on issue q computed according to the algorithm would be the same as Yes (affirmative), otherwise if the influence is negative, then $EV_{(q)}(j) = AorN$, which means the expected vote would be contrast to or different from Yes (affirmative), which work similarly if $|\mathcal{WT}_{A^{(k,q)}}(\mathbb{N}, j)| = MAX_{V \in \{Y,A,N\}}(|\mathcal{WT}_{V^{(k,q)}}(i)|)$ and $|\mathcal{WT}_{N^{(k,q)}}(\mathbb{N}, j)| = MAX_{V \in \{Y,A,N\}}(|\mathcal{WT}_{V^{(k,q)}}(i)|)$. At the third step, for influenced issue q from 1 to m of agent j , compared the factual vote (according to the official voting data) and the expected vote (by the matching algorithm), at each step (for agent j on issue q), if $EV_{(q)}(j) = FV_{(q)}(j)$ (the expected vote is identical with the factual vote), then $P19MF_{(q)}(j) = P19MF_{(q-j)}(i) + 1$ (the pattern 19 matching frequency would be plus 1), and the pattern 19 matching ratio would be obtained by dividing the No. of issue q .

Example

Given a simple example to display the procedure of testing algorithm step by step. Assume there is one influenced agent j and three influencing agent i_1, i_2, i_3 , making decisions on 10 issues. Assume the agent j 's current decision is influenced by all other agents' decisions on former issue k (in this example assume $k=q-1$, which means the current decision is influenced by the most recent former decisions), and assume the weight of influence of all other agents identical as 1 (and as all positive influence). $\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively the accumulative weight of influence of affirmative vote, absent vote and negative vote from the decisions of influencing agents on former issue.

Table C.13: One Example to Display the Pattern 19 Matching Algorithm

	i_1	i_2	i_3	j
	A	Y	A	Y
1	Y	Y	A	A
2	Y	A	Y	Y
3	Y	A	A	Y
4	A	A	Y	N
5	A	Y	A	A
6	A	A	N	A
7	Y	Y	Y	Y
8	A	N	N	N
9	Y	A	Y	Y
10	A	Y	Y	Y

The procedure of all steps is displayed as following, just explained the first three steps. For step 1, the former decisions of influencing agents i_1, i_2, i_3 are A,Y,A, then $\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 1,2,0, the maximum is \mathcal{WT}_A , therefore the expected vote for agent j on current issue is A, and the factual vote for agent j on step 1 is A, so it is matched, and the matching frequency is 1 and matching ratio is $1.00=1/1$; for the second step, the former decisions of influencing agents i_1, i_2, i_3 are Y,Y,A, then $\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 2,1,0, the maximum is \mathcal{WT}_Y , therefore the expected vote for agent j on current issue is Y, and the factual vote for agent j on step 2 is also Y, so it is matched, and the matching frequency is 2 and matching ratio is $1.00=2/2$; for the third step, the former decisions of agents i_1, i_2, i_3 are Y,A,Y, then $\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 2,1,0, the maximum is still \mathcal{WT}_Y , therefore the expected vote for agent j on current issue is Y, and the factual vote for agent j on step 3 is also Y, so it is matched, and the matching frequency is 3 and matching ratio is $1.00=3/3$, the following steps work in the same way.

Table C.14: Testing Procedure Step-by-step by the Pattern 19 Matching Algorithm

Step	\mathcal{WT}_Y	\mathcal{WT}_A	\mathcal{WT}_N	MAX	EV	FV	Match	P19MF	P19MR
1	1	2	0	\mathcal{WT}_A	A	A	YES	1	1.00
2	2	1	0	\mathcal{WT}_Y	Y	Y	YES	2	1.00
3	2	1	0	\mathcal{WT}_Y	Y	Y	YES	3	1.00
4	1	2	0	\mathcal{WT}_A	A	N	NO	3	0.75
5	1	2	0	\mathcal{WT}_A	A	A	YES	4	0.80
6	1	2	0	\mathcal{WT}_A	A	A	YES	5	0.83
7	0	2	1	\mathcal{WT}_A	A	Y	NO	5	0.71
8	3	0	0	\mathcal{WT}_Y	Y	N	NO	5	0.63
9	0	1	2	\mathcal{WT}_N	N	Y	NO	5	0.56
10	2	1	0	\mathcal{WT}_Y	Y	Y	YES	6	0.60

Above displayed the testing outcome if the current decision is influenced just by the former decisions of other agents, but if combining with the influence from the former decision of own, which means the decision on current issue is the collective effect of the decisions on former issues of other agents and own together, as below, the testing outcome is the same with just considering the influence from former decisions of other agents. Therefore, while the assumptions about rules of influence slightly adjusted, the outcome might be different, and also might be the same, it depends (on different setup of preferences).

Table C.15: Testing Procedure Step-by-step by the Pattern 19 Matching Algorithm (with the influence from own decision)

Step	\mathcal{WT}_Y	\mathcal{WT}_A	\mathcal{WT}_N	MAX	EV	FV	Match	P1MF	P1MR
1	2	2	0	$\mathcal{WT}_Y/\mathcal{WT}_A$	Y/A	A	YES	1	1.00
2	2	2	0	$\mathcal{WT}_Y/\mathcal{WT}_A$	Y/A	Y	YES	2	1.00
3	3	1	0	\mathcal{WT}_Y	Y	Y	YES	3	1.00
4	2	2	0	$\mathcal{WT}_Y/\mathcal{WT}_A$	Y/A	N	NO	3	0.75
5	1	2	1	\mathcal{WT}_A	A	A	YES	4	0.80
6	1	3	0	\mathcal{WT}_A	A	A	YES	5	0.83
7	0	3	1	\mathcal{WT}_A	A	Y	NO	5	0.71
8	4	0	0	\mathcal{WT}_Y	Y	N	NO	5	0.63
9	0	1	3	\mathcal{WT}_N	N	Y	NO	5	0.56
10	3	1	0	\mathcal{WT}_Y	Y	Y	YES	6	0.60

Influence Pattern 24 Matching Algorithm

Explanation

For influence pattern 24, which is the structure among agents of current issue would be influenced by the structure among agents of former issue(s). The structure of influence among agents on issue q would be affected by the structure of influence among agents of former issues, for example if affected by all former issues, it would be issues 1,2,...,q-1. Design the matching algorithm for influence pattern 24 as Algorithm 14-Part 1 and Algorithm 15-Part 2, q is the No. of influenced issues, and k is the No. of influencing issues ($k < q - 1$), j is the No. of influenced agent, i is the No. of influencing agent ($i \neq j$).

The part 1 is about the structure among agents of influencing issue k . At the first step, for the decisions on issue k of agent i from 1 to $q-1$, for each i , if $FV_{(k)}(i) = Y$, which means the factual vote (FV) of agent i on issue k is Yes (affirmative), then $\mathcal{WT}_{Y(k,k)}(\mathbb{N}, j) = \mathcal{WT}_{Y(k,k)}(\mathbb{N}, j) + wt_{Y(k,k)}(i, j)$, which means the accumulative weight of influence of affirmative vote from all influencing agents on variable k_j (the decision of agent j on issue k) would be plus with the weight of influence of the current vote (of agent i on issue k), which work similarly if $FV_{(k)}(i) = A$ and $FV_{(k)}(i) = N$.

Algorithm 14 Testing Algorithm for Influence Pattern 24-Part 1

Input: A society $ST(\mathbb{N}, \mathbb{M}, \mathbb{WT})$, $\mathbb{N} = \{1, \dots, n\}$, $\mathbb{M} = \{1, \dots, m\}$,
there are n agents and m features.

Output: Pattern 24 Matching Frequency (P24MF) and Pattern 24 Matching Ratio (P24MR) of $\langle FV_{(1)}(1), \dots, FV_{(m)}(n) \rangle$

```
for  $q = k$  to  $m$  do
  for  $j = 1$  to  $n$  do
    for  $k = 1$  to  $q - 1$  do
      for  $i = 1$  to  $n$ ,  $i \neq j$  do
        if  $FV_{(k)}(i) = Y$  then
           $\mathcal{WT}_{Y^{(k,k)}}(\mathbb{N}, j) = \mathcal{WT}_{Y^{(k,k)}}(\mathbb{N}, j) + wt_{Y^{(k,k)}}(i, j)$ 
        end if
        if  $FV_{(k)}(i) = A$  then
           $\mathcal{WT}_{A^{(k,k)}}(\mathbb{N}, j) = \mathcal{WT}_{A^{(k,k)}}(\mathbb{N}, j) + wt_{A^{(k,k)}}(i, j)$ 
        end if
        if  $FV_{(k)}(i) = N$  then
           $\mathcal{WT}_{N^{(k,k)}}(\mathbb{N}, j) = \mathcal{WT}_{N^{(k,k)}}(\mathbb{N}, j) + wt_{N^{(k,k)}}(i, j)$ 
        end if
      end for
      if  $|\mathcal{WT}_{Y^{(k,k)}}(\mathbb{N}, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V^{(k,k)}}(\mathbb{N}, j)|)$  then
        if  $\mathcal{WT}_{Y^{(k,k)}}(\mathbb{N}, j) > 0$  then
           $EV_{(k)}(j) = Y$ 
        else
           $EV_{(k)}(j) = A \text{ or } N$ 
        end if
      end if
      if  $|\mathcal{WT}_{A^{(k,k)}}(\mathbb{N}, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V^{(k,k)}}(\mathbb{N}, j)|)$  then
        if  $\mathcal{WT}_{A^{(k,k)}}(\mathbb{N}, j) > 0$  then
           $EV_{(k)}(j) = A$ 
        else
           $EV_{(k)}(j) = Y \text{ or } N$ 
        end if
      end if
      if  $|\mathcal{WT}_{N^{(k,k)}}(\mathbb{N}, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V^{(k,k)}}(\mathbb{N}, j)|)$  then
        if  $\mathcal{WT}_{N^{(k,k)}}(\mathbb{N}, j) > 0$  then
           $EV_{(k)}(j) = N$ 
        else
           $EV_{(k)}(j) = A \text{ or } Y$ 
        end if
      end if
      if  $EV_{(k)}(j) = FV_{(k)}(j)$  then
         $\mathcal{WTS}_{P^{(k,q)}}(\mathbb{N}, j) = \mathcal{WTS}_{P^{(k-1,q)}}(\mathbb{N}, j) + wts_{P^{(k,q)}}(\mathbb{N}, j)$ 
      else if  $EV_{(k)}(j) \neq FV_{(k)}(j)$  then
         $\mathcal{WTS}_{N^{(k,q)}}(\mathbb{N}, j) = \mathcal{WTS}_{N^{(k-1,q)}}(\mathbb{N}, j) + wts_{N^{(k,q)}}(\mathbb{N}, j)$ 
      end if
    end for
  end for
end for
```

Algorithm 15 Testing Algorithm for Influence Pattern 24-Part 2

```

for  $i = 1$  to  $n$   $i \neq j$  do
  if  $FV_{(q)}(i) = Y$  then
     $\mathcal{WT}_{Y(q,q)}(\mathbb{N}, j) = \mathcal{WT}_{Y(q,q)}(\mathbb{N}, j) + wt_{Y(q,q)}(i, j)$ 
  else
     $\mathcal{WT}_{Y(q,q)}(\mathbb{N}, j) = \mathcal{WT}_{Y(q,q)}(\mathbb{N}, j)$ 
  end if
  if  $FV_{(q)}(i) = A$  then
     $\mathcal{WT}_{A(q,q)}(\mathbb{N}, j) = \mathcal{WT}_{A(q,q)}(\mathbb{N}, j) + wt_{A(q,q)}(i, j)$ 
  else
     $\mathcal{WT}_{A(q,q)}(\mathbb{N}, j) = \mathcal{WT}_{A(q,q)}(\mathbb{N}, j)$ 
  end if
  if  $FV_{(q)}(i) = N$  then
     $\mathcal{WT}_{N(q,q)}(\mathbb{N}, j) = \mathcal{WT}_{N(q,q)}(\mathbb{N}, j) + wt_{N(q,q)}(i, j)$ 
  else
     $\mathcal{WT}_{N(q,q)}(\mathbb{N}, j) = \mathcal{WT}_{N(q,q)}(\mathbb{N}, j)$ 
  end if
end for
if  $\mathcal{WTS}_{P(q-1,q)}(\mathbb{N}, j) \geq \mathcal{WTS}_{N(q-1,q)}(\mathbb{N}, j)$  then
  if  $|\mathcal{WT}_{Y(q,q)}(\mathbb{N}, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q,q)}(\mathbb{N}, j)|)$  then
    if  $\mathcal{WT}_{Y(q,q)}(\mathbb{N}, j) > 0$  then
       $EV_{(q)}(j) = Y$ 
    else
       $EV_{(q)}(j) = A \text{ or } N$ 
    end if
  end if
  if  $|\mathcal{WT}_{A(q,q)}(\mathbb{N}, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q,q)}(\mathbb{N}, j)|)$  then
    if  $\mathcal{WT}_{A(q,q)}(\mathbb{N}, j) > 0$  then
       $EV_{(q)}(j) = A$ 
    else
       $EV_{(q)}(j) = Y \text{ or } N$ 
    end if
  end if
  if  $|\mathcal{WT}_{N(q,q)}(\mathbb{N}, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q,q)}(\mathbb{N}, j)|)$  then
    if  $\mathcal{WT}_{N(q,q)}(\mathbb{N}, j) > 0$  then
       $EV_{(q)}(j) = N$ 
    else
       $EV_{(q)}(j) = A \text{ or } Y$ 
    end if
  end if
else
  if  $|\mathcal{WT}_{Y(q,q)}(\mathbb{N}, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q,q)}(\mathbb{N}, j)|)$  then
    if  $\mathcal{WT}_{Y(q,q)}(\mathbb{N}, j) > 0$  then
       $EV_{(q)}(j) = A \text{ or } N$ 
    else
       $EV_{(q)}(j) = Y$ 
    end if
  end if
  if  $|\mathcal{WT}_{A(q,q)}(\mathbb{N}, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q,q)}(\mathbb{N}, j)|)$  then
    if  $\mathcal{WT}_{A(q,q)}(\mathbb{N}, j) > 0$  then
       $EV_{(q)}(j) = Y \text{ or } N$ 
    else
       $EV_{(q)}(j) = A$ 
    end if
  end if
  if  $|\mathcal{WT}_{N(q,q)}(\mathbb{N}, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q,q)}(\mathbb{N}, j)|)$  then
    if  $\mathcal{WT}_{N(q,q)}(\mathbb{N}, j) > 0$  then
       $EV_{(q)}(j) = A \text{ or } Y$ 
    else
       $EV_{(q)}(j) = N$ 
    end if
  end if
end if
if  $EV_{(q)}(j) = FA_{(q)}(j)$  then
   $P24MF_{(q)}(j) = P24MF_{(q-1)}(j) + 1$ 
else
   $P24MF_{(q)}(j) = P24MF_{(q-1)}(j)$ 
end if
 $P24MR_{(q)}(j) = \frac{P24MF_{(q)}(j)}{q}$ 
end for
end for
Return P24MF and P24MR of  $(FV_{(1)}(1), \dots, FV_{(m)}(n))$ 

```

At the second step, for the influenced variable k_j (the decision of agent j on issue k), assume it would be influenced by all other agents from 1 to m (except agent j) on issue k , if $|\mathcal{WT}_{Y(k,k)}(\mathbb{N}, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(k,k)}(\mathbb{N}, j)|)$, which means the accumulative weight of influence of affirmative vote of all other agents on issue k is the maximum of all accumulative weight of influence (of affirmative vote, absent vote and negative vote), and also $\mathcal{WT}_{Y(k,k)}(\mathbb{N}, j) > 0$ (the influence is positive), then $EV_{(k)}(j) = Y$, which means the expected vote (EV) of influenced agent j on issue k computed according to the algorithm would be the same as Yes (affirmative), otherwise if the influence is negative, then $EV_{(k)}(j) = A \text{ or } N$, which means the expected vote would be contrast to or different from Yes (affirmative), which work similarly if $|\mathcal{WT}_{A(k,k)}(\mathbb{N}, j)|$ or $|\mathcal{WT}_{N(k,k)}(\mathbb{N}, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(k,k)}(\mathbb{N}, j)|)$.

At the third step, for influencing issue k from 1 to $q-1$ of agent j , compared the factual vote (according to the official voting data) and the expected vote (by the matching algorithm), at each step (for agent j on issue k), if $EV_{(k)}(j) = FV_{(k)}(j)$ (the expected vote is the same with the factual vote), then $\mathcal{WTS}_{P(k,q)}(\mathbb{N}, j) = \mathcal{WTS}_{P(k-1,q)}(\mathbb{N}, j) + \text{wts}_{P(k,q)}(\mathbb{N}, j)$, which means the accumulative weight of structure of positive influence would be plus with the weight of current structure, otherwise, the accumulative weight of structure of negative influence would be plus.

The part 2 is about the structure among agents on influenced issue q . At the fourth step, for the decision on issue q of agent i from 1 to n , for each agent i , if $FV_{(q)}(i) = Y$, which means the factual vote (FV) of agent i on issue q is Yes (affirmative), then $\mathcal{WT}_{Y(q,q)}(\mathbb{N}, j) = \mathcal{WT}_{Y(q,q)}(\mathbb{N}, j) + \text{wt}_{Y(q,q)}(i, j)$, which means the accumulative weight of influence of affirmative vote on variable q_j (the decision of agent j on issue q) would be plus with the weight of influence of the current vote (of agent i on issue q), which work similarly if $FV_{(q)}(i) = A$ and $FV_{(q)}(i) = N$.

At the fifth step, for the influenced variable q_j , assume it would be influenced by all other agents from 1 to n on issue q according to the structure of influence of former issues. If $\mathcal{WTS}_{P(q-1,q)}(\mathbb{N}, j) \geq \mathcal{WTS}_{N(q-1,q)}(\mathbb{N}, j)$, which means the dominating structure of influence affecting on agent j on issue q is positive, further, if $|\mathcal{WT}_{Y(q,q)}(\mathbb{N}, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q,q)}(\mathbb{N}, j)|)$ (the accumulative weight of influence of affirmative vote of all other agents except agent j on issue q is the maximum of all accumulative weight of influence of affirmative vote, absent vote and negative vote), and also $\mathcal{WT}_{Y(q,q)}(\mathbb{N}, j) > 0$ (the influence is positive), then $EV_{(q)}(j) = Y$, which means the expected vote (EV) of influencing agent j on issue q computed according to the algorithm would be the same as Yes (affirmative), otherwise if the influence is negative, then $EV_{(q)}(j) = A \text{ or } N$, which means the expected vote would be contrast to or different from Yes (affirmative), which work similarly if $|\mathcal{WT}_{A(q,q)}(\mathbb{N}, j)|$ or $|\mathcal{WT}_{N(q,q)}(\mathbb{N}, j)| = \text{MAX}_{V \in \{Y, A, N\}}(|\mathcal{WT}_{V(q,q)}(\mathbb{N}, j)|)$. But if back to the first if of this step, if $\mathcal{WTS}_{P(q-1,q)}(\mathbb{N}, j) < \mathcal{WTS}_{N(q-1,q)}(\mathbb{N}, j)$, which means the dominating structure of influence is negative, then the whole procedure works contrast to above.

At the sixth step, for issue q from k to m of agent j , compared the factual vote (according to the official voting data) and the expected vote (by the matching algorithm), at each step (for agent j on issue q), if $EV_{(q)}(j) = FV_{(q)}(j)$ (the expected vote is the same with the factual vote), then $P24MF_{(q)}(j) = P24MF_{(q-1)}(j) + 1$, which means the pattern 24 matching frequency is plus with 1, and the pattern 24 matching ratio would be obtained by dividing the No. of issue q , $P24MR_{(q)}(j) = \frac{P24MF_{(q)}(j)}{q}$.

Example

Given a simple example to display the procedure of testing algorithm step by step. Assume just there is one agent j being influenced by three agents i_1, i_2, i_3 at each issue's decision (together making decisions on 10 issues), and the structure of influence among agents at latter issue would be determined by the structure of influence among agents of former issue, in this example, assume just

influenced by the most recent structure, which means the structure on issue q would be influenced by structure on issue $q-1$. And assume the weight of influence of all agents identical as 1 (as all positive influence).

Table C.16: One Example to Display the Pattern 24 Matching Algorithm

	i_1	i_2	i_3	j
	A	Y	A	Y
1	Y	Y	A	A
2	Y	A	Y	Y
3	Y	A	A	Y
4	A	A	Y	N
5	A	Y	A	A
6	A	A	N	A
7	Y	Y	Y	Y
8	A	N	N	N
9	Y	A	Y	Y
10	A	Y	Y	Y

As in the table of testing procedure of this algorithm (table 11.21 and table 11.22), \mathcal{WT}_Y , \mathcal{WT}_A , \mathcal{WT}_N are respectively the accumulative weight of influence of affirmative vote, absent vote and negative vote from other agents' decisions. IEV is the expected vote of influenced agent on Influencing issue, IFV is the factual vote of influenced agent on Influencing issue, IMt is the matching situation between IEV and IFV. \mathcal{WTS}_P is the accumulated weight of structure of positive influence, and \mathcal{WTS}_N is the accumulated weight of structure of negative influence. EVD is the expected vote of influenced agent on influencedD issue, FVD is the factual vote of influenced agent on influencedD issue, and MtD is the matching situation between EVD and FVD.

The procedure of all steps is displayed as following (table 11.21), just explained the first three steps. For the first step (the decision on issue 1), the last decisions of agents i_1, i_2, i_3 are A,Y,A, then $\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 1,2,0, the maximum is \mathcal{WT}_A , therefore the expected vote for agent j (IEV) is A, and the factual vote of agent j (IFV) is Y, so IMt is unmatched (deemed as a negative influence), and $\mathcal{WTS}_{P(q,q)}(\mathbb{N}, j)$ and $\mathcal{WTS}_{N(q,q)}(\mathbb{N}, j)$ are respectively 0,1, the dominating influencing structure is negative, and for the influenced issue-current issue 1, the decisions of agents i_1, i_2, i_3 are Y,Y,A, ($\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 2,1,0), and according to the negative influencing structure, the expected vote of agent j (EVD) on issue 1 should be A or N, and the factual vote of agent j (FVD) on issue 1 is A, so MtD is matched, the matching frequency is 1 and matching ratio is 1.00=1/1; for the second step, the last decisions of agents i_1, i_2, i_3 is Y,Y,A, then $\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 2,1,0, the maximum is \mathcal{WT}_Y , therefore the expected vote for agent j (IEV) on last issue 1 is Y, and the factual vote of agent j (IFV) on last issue 1 is A, so IMt is unmatched (deemed as a negative influence), and $\mathcal{WTS}_{P(q,q)}(\mathbb{N}, j)$ and $\mathcal{WTS}_{N(q,q)}(\mathbb{N}, j)$ are respectively 0,2, so the dominating influencing structure is still negative, and for the influenced issue-current issue 2, the decisions of agents i_1, i_2, i_3 are Y,A,Y, ($\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 2,1,0), according to the negative influencing structure, the expected vote of agent j (EVD) on issue 2 should be A or N, and the factual vote of agent j (FVD) on issue 2 is Y, so MtD is unmatched, the matching frequency is 1 and matching ratio is 0.50=1/2; for the third step, the last decisions of agents i_1, i_2, i_3 are Y,A,Y, then $\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 2,1,0, the maximum is \mathcal{WT}_Y , therefore the expected vote for agent j (IEV) on issue 3 is Y, and the factual vote of agent i (IFV) on issue 3 is also Y, so IMt is matched (deemed as a positive influence), and $\mathcal{WTS}_{P(q,q)}(\mathbb{N}, j)$ and $\mathcal{WTS}_{N(q,q)}(\mathbb{N}, j)$ are respectively 1,2, so the dominating influencing structure is still negative, and for influenced issue 3, the decisions of agents i_1, i_2, i_3 are Y,A,A ($\mathcal{WT}_Y, \mathcal{WT}_A, \mathcal{WT}_N$ are respectively 1,2,0), according to the negative influencing structure, the expected vote of agent j (EVD) on issue 3 should be Y or N, and the factual vote of

agent j (FVD) on issue 3 is Y, so MtD is matched, the matching frequency is 2 and matching ratio is $0.67=2/3$, the following steps work in the same way.

Above displayed the testing outcome by just comparing the decisions of other agents and the decision (or preference) of own to identify the attribute (positive or negative) of influencing structure, but if also considering the influence from own decision, comparing the collective effect (combining other agents' and own decisions together) with own, judge the expected and factual vote matched or not, the testing outcome would be different as below (table 11.22), affected by the adjustment of rule.

C.2 Weight Allocation of Influence among UN SC member states

Above we discuss the mathematic procedures of a series of matching algorithms to test patterns of influence, and basically assume the weight of influence from different agents, issues or structures identical as 1, while in real-world situation, the weight would be much more complicated, varying weaker or stronger, positive or negative, and so on. And in factual applications, to test the patterns of influence or based on the patterns of influence to do experiments, the allocation of weight of influence among different origins would be the first crucial step. For example, while an agent being influenced by more than one different agents, only knowing about the weight of influence among them (which could be represented by a matrix), the influenced outcome could be computed and inferred. But how to obtain the matrix capturing all influencing relations, which is a tricky question, as the complete information is usually not easy to collect in real-world situations.

For example of game and influence among member states of UN security council, to allocate the weight of influence among different member states, as the weight of influence is an abstract term, which needs (a series of) related metrics to measure, we choose three metrics *the strength of power, strength of willingness and closeness of relations* (which have been discussed in chapter 7). As it is reasonable to assume that, in the international politics and relations, it is more easily for one state to be influenced by other states with stronger national strengths, with firmer willingness to influence other states and to pursue own strategic objectives, or with more close and intimate relations with the influenced ones. But how to allocate the weight of influence by strength of power, strength of willingness and closeness of relations among different states, some data and method would be needed for the computation.

A very famous work about the comparison of national power is the formula by Ray Cline. In international politics, Cline devised a widely recognized indexes of national power in 1975. This nonlinear, multi-variable index attempted to integrate both capabilities and commitment (material power and mental power) to create a formula that would rank the perceived power of states. Cline's formula was:

$$P_p = (C + E + M) \times (S + W) \quad (C.1)$$

Where C is critical mass (including population and territory), E is economic capacity (including income plus energy plus non-fuel minerals plus manufacturing plus food plus trade), M is military capacity (including the strategic balance plus combat capabilities plus a bonus for effort), S is the national strategy coefficient, and W is national will (including the level of national integration, the strength of leadership, and the relevance of strategy to the national interest). The formula won a wide readership both in academia and within the defense community, and some variants of it were used to develop the U.S. Army's estimates of long-range trends in the international system [Tellis, 2001].

In a word, C is Critical Mass, E is Economic Capability, M is Military Capability, S is Strategic Purpose, W is Will to Pursue National Strategy, and the former three ($C + E + M$)

Table C.17: Testing Procedure Step-by-step by the Pattern 24 Matching Algorithm

Step	WT _Y	WT _A	WT _N	IEV	IFV	IMt	WTS _P	WTS _N	WT _Y	WT _A	WT _N	EVD	FVD	MtD	P24MF	P24MR
1	1	2	0	A	Y	NO	0	1	2	1	0	A/N	A	YES	1	1.00
2	2	1	0	Y	A	NO	0	2	2	1	0	A/N	Y	NO	1	0.50
3	2	1	0	Y	Y	YES	1	2	1	2	0	Y/N	Y	YES	2	0.67
4	1	2	0	A	Y	NO	1	3	1	2	0	Y/N	N	YES	3	0.75
5	1	2	0	A	N	NO	1	4	1	2	0	Y/N	A	NO	3	0.60
6	1	2	0	A	A	YES	2	4	0	2	1	Y/N	A	NO	3	0.50
7	0	2	1	A	A	YES	3	4	3	0	0	A/N	Y	NO	3	0.43
8	3	0	0	Y	Y	YES	4	4	0	1	2	Y/A/N	N	YES	4	0.50
9	0	1	2	N	N	YES	5	4	2	1	0	Y	Y	YES	5	0.56
10	2	1	0	Y	Y	YES	6	4	2	1	0	Y	Y	YES	6	0.60

Table C.18: Testing Procedure Step-by-step by the Pattern 24 Matching Algorithm (with the influence from own)

Step	WT_Y	WT_A	WT_N	IEV	IFV	IMt	WT_{SP}	WT_{SN}	WT_Y	WT_A	WT_N	EVD	FVD	MtD	P24MF	P24MR
1	2	2	0	Y/A	Y	YES	1	0	2	2	0	Y	A	NO	0	0.00
2	2	2	0	Y/A	A	YES	2	0	3	1	0	Y	Y	YES	1	0.50
3	3	1	0	Y	Y	YES	3	0	2	2	0	A	Y	NO	1	0.33
4	2	2	0	Y/A	Y	YES	4	0	1	2	1	A	N	NO	1	0.25
5	1	2	1	A	N	NO	4	1	1	3	0	A	A	YES	2	0.40
6	1	3	0	A	A	YES	5	1	0	3	1	A	A	YES	3	0.50
7	0	3	1	A	A	YES	6	1	4	0	0	Y	Y	YES	4	0.57
8	4	0	0	Y	Y	YES	7	1	0	1	3	N	N	YES	5	0.63
9	0	1	3	N	N	YES	8	1	3	1	0	Y	Y	YES	6	0.67
10	3	1	0	Y	Y	YES	9	1	3	1	0	Y	Y	YES	7	0.70

constitute the material power, and the latter two ($S + W$) constitute the mental power, in another word, ($C + E + M$) is the resource or capacity one state possesses, and ($S + W$) is the way how one state utilizes his resource and capacity. And the national power is obtained by the combination of material power and mental power: $(C + E + M) \times (S + W)$.

Cline used to give the comparison of national power of several great powers in 1978 as following [Cline, 1980], in which Soviet Union and US respectively ranked No.1 and No.2, basically reflected the situation of two superpowers at that time, and more specifically, though the comprehensive national power of US (304) was behind Soviet Union (458), while actually the material power of US (434) is ahead of Soviet Union (382), but was weaker than Soviet Union on the mental power aspect (0.7 behind 1.2), which might be due to the mistake on strategic purpose and the feeble national will at that period (US had been immersed in the Vietnam war and there was serve disunity in the domestic politics), which weaken the national power of US while competing with Soviet Union, although the material capacity (typical as the economic capacity) of US is ahead of Soviet Union.¹ Another interesting example is China, who ranked only No.7 in the comprehensive national power (83), while the material power ranked high to No.3 in the material aspect $C + E + M$ (139), but pulled by the mental aspect $S + W$ (0.6), which ranked at the last place. Although China has huge population, vast territory and large army, China has just passed the “Great Cultural Revolution” which closed in 1976, and before the era of Reform and Openness (firmly center on the economic construction as national strategic purpose), while during this transition period, the state didn’t have a clear strategy and ambiguous on the future path.

Table C.19: The Comprehensive Nation Power in 1978 by Ray S.Cline

No.	State	$C + E + M$	S	W	$S + W$	National Power
1	Soviet Union	382	0.7	0.5	1.2	458
2	US	434	0.3	0.4	0.7	304
3	Brazil	98	0.6	0.8	1.4	137
4	Germany ²	77	0.7	0.8	1.5	116
5	Japan	77	0.6	0.8	1.4	108
6	Australia	73	0.5	0.7	1.2	88
7	China	139	0.4	0.2	0.6	83
8	France	82	0.4	0.5	0.9	74
9	UK	68	0.5	0.5	1	68

Referring to the data about comprehensive national power (which combine both material aspect and mental aspect), the weight of influence among different states could be initially measured. As we discussed before, the abstract weight of influence could be measured by several related metrics, like the strength of power and the strength of willingness (the state with stronger power and firmer willingness would be more influential), actually, the two metrics could be suitably matched respectively with the material aspect data and mental aspect data of comprehensive national power.

Strength of Power

Which is about the material power (MP) aspect of the comprehensive national power, which could be measured by $C + E + M$ (*criticalmass + economiccapacity + militarycapacity*), and with all other conditions identical, the bigger of the material power, the bigger of the influence power, for example, it is more easier for a small state influenced by a great power, but not a great power influenced by a small state.

¹For instance, the GDP of Soviet Union had never even reached close to US until his disintegration.

²Which is the West Germany. West Germany is the common English name for the Federal Republic of Germany in the period between its creation in May 1949 to German reunification on 3 October 1990. This period is often referred to as the Bonn Republic by academic historians [Nicholls and Nicholls, 1997].

$$MP_{(i)} = C_{(i)} + E_{(i)} + M_{(i)} \quad (C.2)$$

View Oneself and Others Objectively As we assumed, the material power is proportional to the influence power, for one state j being influenced by n other states, in which for one influencing state i , the bigger of the material power, the higher of the weight of influence, and for the influenced state j , the total weight from all other influencing state is assumed as 1, therefore, the weight of influence (determined by strength of power) from state i to state j (defined as $sp_{\alpha[i,j]}$) is:

$$sp_{\alpha[i,j]} = \frac{MP(i)}{\sum_{k=1}^n MP(k)}$$

Above assume the influenced state view the material power of oneself and other influencing states uniformly, would neither overrate or underrate own or others' power, namely with a objective perspective (totally according to the factual material power). Based on the data about material power (in table 11.23) and the function of weight of influence above, the influencing relations among 5 UN SC permanent members are computed as following matrix \mathbb{SP}_{α} (the horizontal axis is the influencing agent, and the vertical axis the influenced agent):

$$\mathbb{SP}_{\alpha} = \begin{matrix} & \begin{matrix} US & UK & FR & CN & RU \end{matrix} \\ \begin{matrix} US \\ UK \\ FR \\ CN \\ RU \end{matrix} & \begin{pmatrix} 0.393 & 0.062 & 0.074 & 0.125 & 0.346 \\ 0.393 & 0.062 & 0.074 & 0.125 & 0.346 \\ 0.393 & 0.062 & 0.074 & 0.125 & 0.346 \\ 0.393 & 0.062 & 0.074 & 0.125 & 0.346 \\ 0.393 & 0.062 & 0.074 & 0.125 & 0.346 \end{pmatrix} \end{matrix} \quad (C.3)$$

The top 5 influencing relations could be ranked as following, the weight of influence from US and Soviet Union ranked No.1 and No.2 as the two super powers have much bigger material power than other three states (usually deemed as great powers).

Table C.20: Top 5 Influencing Relations by Strength of Power–View Oneself and Others Objectively

No	Influence	Weight
1	United States to all	0.393
2	Soviet Union to all	0.346
3	China to all	0.125
4	France to all	0.074
5	United Kingdom to all	0.062

View Oneself More Times than Others While on many cases one state would not view the material power of oneself and others in a very objective way, but would overrate or underrate own or others' power, a very common case is the state would unconsciously overrate own power, and would even not admit overrated own power, therefore, the weight of influence (determined by strength of power) from state i to state j (defined as $sp_{\beta[i,j]}$) would be (in which the influenced agent j would view own weight of influence more ϵ times):

$$sp_{\beta[i,j]} = \begin{cases} \frac{\epsilon MP(i)}{\epsilon MP(j) + \sum_{k \neq j} MP(k)} & i = j \\ \frac{MP(i)}{\epsilon MP(j) + \sum_{k \neq j} MP(k)} & i \neq j \end{cases}$$

If the influenced agent views own material power 3 times bigger, then the weight of influence among 5 UN SC permanent members would be expressed as following matrix \mathbb{SP}_{β} :

if $\epsilon = 3$ then

$$\mathbb{S}\mathbb{P}_\beta = \begin{matrix} & US & UK & FR & CN & RU \\ \begin{matrix} US \\ UK \\ FR \\ CN \\ RU \end{matrix} & \begin{pmatrix} 0.660 & 0.034 & 0.042 & 0.070 & 0.194 \\ 0.350 & 0.164 & 0.066 & 0.112 & 0.308 \\ 0.342 & 0.054 & 0.194 & 0.110 & 0.301 \\ 0.314 & 0.049 & 0.059 & 0.302 & 0.276 \\ 0.232 & 0.036 & 0.044 & 0.074 & 0.613 \end{pmatrix} \end{matrix} \quad (\text{C.4})$$

Further, to the rank the weight of influence of 5×5 groups of influencing relations, because the higher of the weight of influence, the more reasonable to make assumptions about the influence between the two influencing and influenced states. The top 5 influencing relations are ranked as following, all coming from two super powers (US and Soviet Union), and the top 2 are the influence from US to own and the influence from Soviet Union to own, which is matched to the common sense, compared with small states, the super power would be more determined by own preference and interest, but not be influenced and affected by other states.

Table C.21: Top 5 Influencing Relations by Strength of Power–View Oneself More Times (3)

No	Influence	Weight
1	United States to United States	0.660
2	Soviet Union to Soviet Union	0.613
3	United States to United Kingdom	0.350
4	United States to France	0.342
5	United States to China	0.314

If just rank the weight of influence from others as following, it is the same all top influences coming from two super powers, and 3 of 5 from US and 2 of 5 from Soviet Union, as the material power of US (434) is relatively higher than Soviet Union (382).

Table C.22: Top 5 Influencing Relations by Strength of Power–View Oneself More Times (3)(except the own influence)

No	Influence	Weight
1	United States to United Kingdom	0.350
2	United States to France	0.342
3	United States to China	0.314
4	Soviet Union to United Kingdom	0.308
5	Soviet Union to France	0.301

Allocate the Ratio Between Oneself and Others and View Others Objectively While in some cases, the influenced agent would not view the influence from own and the influence from others (or allocate the weight of influence from own and others) in a same criterion, but in a respectively different criterions, for example, the influenced agent would give the weight of own influence with a fixed ratio (defined as λ), and let all other influencing agents share the remaining ratio $(1 - \lambda)$, objectively according to their material powers. Therefore, the weight of influence from state i to state j (defined as $sp_\gamma[i,j]$) would be:

$$sp_\gamma[i,j] = \begin{cases} \lambda & i = j \\ \frac{MP(i)}{\sum_{k \neq j} MP(k)} \times (1 - \lambda) & i \neq j \end{cases}$$

If the influenced agent views own influence with static weight 0.5, and let all other influencing agents shared the other 0.5, then the matrix of weight of influence among 5 UN SC permanent

members would be expressed as following matrix \mathbb{SP}_γ :

if $\lambda = 0.5$ then

$$\mathbb{SP}_\gamma = \begin{matrix} & \begin{matrix} US & UK & FR & CN & RU \end{matrix} \\ \begin{matrix} US \\ UK \\ FR \\ CN \\ RU \end{matrix} & \begin{pmatrix} 0.5 & 0.051 & 0.061 & 0.104 & 0.285 \\ 0.209 & 0.5 & 0.040 & 0.067 & 0.184 \\ 0.212 & 0.033 & 0.5 & 0.068 & 0.187 \\ 0.225 & 0.035 & 0.042 & 0.5 & 0.198 \\ 0.300 & 0.047 & 0.057 & 0.096 & 0.5 \end{pmatrix} \end{matrix} \quad (C.5)$$

The top 5 influencing relations could be ranked as following (just the influence from others, except the influence from own, as all the weights from own influence are set as 0.5, therefore, the comparisons are meaningless). All top influences are coming from two super powers, and US occupied four places due to the advantage in material power. And the mutual influence from US to Soviet Union and from Soviet Union to US ranked as top 2, which is matched to the situation at that period, the confrontation and interaction between two super powers are very intensive, both two deeply influenced and affected the other's behaviors and strategies. While one super power face more than one influencing states, in which, the other super power would share the biggest portion, and has the biggest weight of influence among them.

Table C.23: Top 5 Influencing Relations by Strength of Power—Allocate the Ratio Between Oneself (0.5) and Others (0.5) and View Other Uniformly (except the influences from own)

No	Influence	Weight
1	United States to Soviet Union	0.300
2	Soviet Union to United States	0.285
3	United States to China	0.225
4	United States to France	0.212
5	United States to United Kingdom	0.209

Strength of Power \times Strength of Willingness

The influences among member states are not only affected by the material power, but also affected by the mental power, the influence is the collective effect of both material and mental aspect. The mental power (TP) aspect of the comprehensive national power could be measured by $S + W$ (*strategic purpose + national power*, as shown in table 11.24). And with all other conditions identical, the bigger of the mental power, the bigger of the influence power, for example, it is more easier for a state to be influenced by another state with clear strategic plan and firm willingness to pursue the strategic objective.

$$TP_{(i)} = S_{(i)} + W_{(i)} \quad (C.6)$$

Actually, the weight of influence determined by the strength of willingness could also be computed by three different functions, as $sw_{\alpha[i,j]}$, $sw_{\beta[i,j]}$, $sw_{\gamma[i,j]}$, based on different assumptions, the influenced agent view oneself and others uniformly, or view oneself more times heavier, or view oneself with a static weight and view others uniformly. As the material power and mental power collectively constitute the influence power (material power is like the resource and capacity possessed by one state, and the mental power is about how the state utilize his resource and capacity, and how is the willingness to influence others and to achieve the state objective), therefore, the weight of influence determined by the comprehensive strength (cs) could be computed as following (sp multiply sw):

$$\begin{aligned}
cs_{[i,j]}^* &= sp_{[i,j]} \times sw_{[i,j]} \\
cs_{[i,j]} &= \frac{cs_{[i,j]}^*}{\sum_{k=1}^n cs_{[k,j]}^*}
\end{aligned} \tag{C.7}$$

View Oneself and Others Objectively As we assumed, the mental power is proportional to the influence power, for one state j being influenced by n other states, in which of one influencing state i , the bigger of the mental power, the higher of the weight of influence, and for the influenced state j , the total weight from all other influencing state is assumed as 1, therefore, the weight of influence (determined by strength of willingness) from state i to state j (defined as $sw_{\alpha[i,j]}$) is:

$$sw_{\alpha[i,j]} = \frac{TP(i)}{\sum_{k=1}^n TP(k)}$$

Above assume the influenced state view the mental power of oneself and other influencing states uniformly, would neither overrate or underrate, and with a objective perspective. As the weight of influence is determined by the comprehensive strength combining both material power and mental power, then $cs_{\alpha[i,j]} = sp_{\alpha[i,j]} \times sw_{\alpha[i,j]}$, then the influencing relations determined by both strength of power and strength of willingness could be computed as following matrix CS_{α} :

$$CS_{\alpha} = \begin{matrix} & \begin{matrix} US & UK & FR & CN & RU \end{matrix} \\ \begin{matrix} US \\ UK \\ FR \\ CN \\ RU \end{matrix} & \begin{pmatrix} 0.308 & 0.069 & 0.075 & 0.084 & 0.465 \\ 0.308 & 0.069 & 0.075 & 0.084 & 0.465 \\ 0.308 & 0.069 & 0.075 & 0.084 & 0.465 \\ 0.308 & 0.069 & 0.075 & 0.084 & 0.465 \\ 0.308 & 0.069 & 0.075 & 0.084 & 0.465 \end{pmatrix} \end{matrix} \tag{C.8}$$

The top 5 influencing relations are ranked as following, which is totally matched to the ranking of comprehensive national power by Cline [Cline, 1980], the bigger of the comprehensive strength, the bigger of the influence power. While combining the material power ($C + E + S$) with mental power ($S + W$), Soviet Union replaced US as No.1, as tough behind on the material power, the comprehensive national power of Soviet Union (458) was ahead of US (308) by the data.

Table C.24: Top 5 Influencing Relations by Strength of Power \times Strength of Willingness–View Oneself and Others Objectively

No	Influence	Weight
1	Soviet Union to all	0.465
2	United States to all	0.308
3	China to all	0.084
4	France to all	0.075
5	United Kingdom to all	0.069

View Oneself More Times than Others While if one state would not view the mental power of oneself and others objectively, but would overrate or underrate own or others' mental power, a very common case is the state would unconsciously overrate or overconfident about own mental power (flatter oneself that own strategy is very wise and own will is very firm, and so on), therefore, the weight of influence (determined by strength of willingness) from state i to state j (defined as $sw_{\beta[i,j]}$) would be (in which the influenced agent j would view own weight of influence more ϵ times):

$$sw_{\beta[i,j]} = \begin{cases} \frac{\epsilon TP(i)}{\epsilon TP(j) + \sum_{k \neq j} TP(k)} & i = j \\ \frac{TP(i)}{\epsilon TP(j) + \sum_{k \neq j} TP(k)} & i \neq j \end{cases}$$

As the weight of influence is determined by the comprehensive strength combining both material power and mental power, then $cs_{\beta[i,j]} = sp_{\beta[i,j]} \times sw_{\beta[i,j]}$, then the influencing relations determined by both strength of power and strength of willingness could be computed as following matrix \mathbb{CS}_{β} (assume if the influenced agent views own mental power 3 times bigger):

if $\varepsilon = 3$ then

$$\mathbb{CS}_{\beta} = \begin{matrix} & \begin{matrix} US & UK & FR & CN & RU \end{matrix} \\ \begin{matrix} US \\ UK \\ FR \\ CN \\ RU \end{matrix} & \begin{pmatrix} 0.800 & 0.020 & 0.022 & 0.024 & 0.134 \\ 0.193 & 0.401 & 0.049 & 0.055 & 0.302 \\ 0.192 & 0.043 & 0.420 & 0.053 & 0.291 \\ 0.184 & 0.041 & 0.044 & 0.454 & 0.277 \\ 0.065 & 0.014 & 0.016 & 0.018 & 0.887 \end{pmatrix} \end{matrix} \quad (C.9)$$

The top 5 influencing relations are ranked as following, it is interesting that all top influences are from own to own, as we assume in above functions, every state overrate own material power 3 times and own mental power 3 times, then for every influenced state, the influence from own dominate any influence from other state. And the weight of top 2 influences from US to own and from Soviet Union to own are both way ahead of other influences (the top two are bigger than 0.8 while other three are less than 0.5), which is matched to the common sense, compared with small states, the super power would be more determined by own preference and interest, but not be influenced and affected by other states, and for the two super powers, their comprehensive national power were indeed way ahead of other states.

Table C.25: Top 5 Influencing Relations by Strength of Power \times Strength of Willingness–View Oneself More Times (3)

No	Influence	Weight
1	Soviet Union to Soviet Union	0.887
2	United States to United States	0.800
3	China to China	0.454
4	France to France	0.420
5	United Kingdom to United Kingdom	0.401

As the influence from own are based on the overrated own power, therefore, just rank the weight of influence from others (except the influence from own) would also be meaningful, as following, all top influences come from the two super powers.

Table C.26: Top 5 Influencing Relations by Strength of Power \times Strength of Willingness–View Oneself More Times (3)(except the own influence)

No	Influence	Weight
1	Soviet Union to United Kingdom	0.302
2	Soviet Union to France	0.291
3	Soviet Union to China	0.277
4	United States to United Kingdom	0.193
5	United States to France	0.192

Allocate the Ratio between Oneself and Others and View Others Objectively While in some cases, the influenced agent would not view the influence from own and the influence from others (or allocate the weight of influence from own and others) in a same criterion, but in a respectively different criterions, for example, the influenced agent would give the weight of own influence with

a fixed ratio (defined as μ), and let all other influencing agents share the remaining ratio ($1 - \mu$), objectively according to their mental powers. Therefore, the weight of influence from state i to state j (defined as $sw_{\gamma[i,j]}$) would be:

$$sw_{\gamma[i,j]} = \begin{cases} \mu & i = j \\ \frac{TP(i)}{\sum_{k \neq j} TP(k)} \times (1 - \mu) & i \neq j \end{cases}$$

As the weight of influence is collectively determined by both material power and mental power, then $cs_{\gamma[i,j]} = sp_{\gamma[i,j]} \times sw_{\gamma[i,j]}$, then the influencing relations among 5 UN SC permanent members could be computed as following matrix \mathbb{CS}_{γ} (assume if the influenced agent j give $sp_{\gamma[j,j]}$ as 0.5 and give $sw_{\gamma[j,j]}$ also as 0.5):

if $\mu = 0.5$ then

$$\mathbb{CS}_{\gamma} = \begin{matrix} & \begin{matrix} US & UK & FR & CN & RU \end{matrix} \\ \begin{matrix} US \\ UK \\ FR \\ CN \\ RU \end{matrix} & \begin{pmatrix} 0.784 & 0.022 & 0.023 & 0.026 & 0.145 \\ 0.068 & 0.793 & 0.017 & 0.019 & 0.103 \\ 0.068 & 0.015 & 0.797 & 0.019 & 0.102 \\ 0.066 & 0.015 & 0.016 & 0.802 & 0.100 \\ 0.106 & 0.024 & 0.026 & 0.029 & 0.814 \end{pmatrix} \end{matrix} \quad (C.10)$$

The top 5 influencing relations could be ranked as following (just including the influence from others while removing the influence from own, as the weights from own influence determined by material and mental aspect are both set as 0.5, therefore, the comparisons are meaningless). All top influences are coming from two super powers, and Soviet Union occupied four places due to the advantage in comprehensive national power. And the mutual influence from Soviet Union to US and from US to Soviet Union ranked as top 2, which is matched to the situation at that period, the confrontation and game between two super powers are very intensive, both two deeply influenced and affected the other's behaviors and strategies. While one super power face more than one influencing states, in which, the other super power would share the biggest portion, and has the biggest weight of influence among them.

Table C.27: Top 5 Influencing Relations by Strength of Power \times Strength of Willingness—Allocate the Ratio between Oneself (0.5) and Others (0.5) and View Others Objectively (except the own influence)

No	Influence	Weight
1	Soviet Union to United States	0.145
2	United States to Soviet Union	0.106
3	Soviet Union to United Kingdom	0.103
4	Soviet Union to France	0.102
5	Soviet Union to China	0.100

Strength of Power \times Strength of Willingness \times Degree of Relations

Except affected by the comprehensive national power (material power and mental power), the relations among different states would also produce an effect on the influence. In international politics and affairs, the relations among states could be strong or weak, good or bad, for example there are friends or allies among states, and there are also enemies or opponents among states. And with all other conditions identical, the more intensive of the relations, the bigger of the influence power, for example, while an influenced agent faces more than one friends influencing, the best friend with the most intimate relation would share the largest portion of weight of influence, and while an influenced agent faces more than one enemies influencing, the worst enemy with the

most hostile relation would share the largest portion of weight of influence. Furthermore, due to the introduction of metric of relations, the weight of influence among different states would become negative or positive, but not only varying on the scale.

As discussed above, the material power, mental power and degree of relations would collectively constitute the influence power, therefore, the weight of influence determined by the “related comprehensive strength” (rCS) could be computed as following (sp multiply sw multiply re):

$$rCS_{[i,j]}^* = re_{[i,j]} \times sp_{[i,j]} \times sw_{[i,j]}$$

$$rCS_{[i,j]} = \frac{rCS_{[i,j]}^*}{\sum_{k=1}^n |rCS_{[k,j]}^*|} \quad (C.11)$$

But how to set the degree of relations (re) between different states and constitute the matrix \mathbb{RE} , as there is no existed data like the material power and mental power by Cline’s work [Cline, 1980], in fact, the relation among states is a very subjective concept, and not easy to measure quantitatively and objectively, for now there is no relevant mature work to precisely measure the degree of relations among countries. However, the relation indeed is an important metric to affect the weight of influence, therefore, we preliminarily set the degree of relations among 5 UN SC permanent members mainly according to the common sense, and also assume a principle that the relations between two agents are symmetrical, which means the degree of relation from agent i to agent j (how agent j thinks of his relation with agent i) equals the degree of relation from agent j to agent i (how agent i thinks of his relation with agent j), namely $re_{[i,j]} = re_{[j,i]}$. Of course, the controversy about the setup of degree of relations and the assumption of principles of symmetry are inevitable, the work mainly contributes to provide a thought and approach to study the influence from more than one agents, especially how to measure and allocate the weights of influence from different states in international politics, the handling of specific data could be discussed more in the future work.

The matrix of relations \mathbb{RE}^* is designed as following, we assume the degree of relations varies from -1 to 1 (with a domain [-1,1]). As we can see, the degree of relations in the 45 angle line are all 1 (assume yourself is your own best “friend”, you will always trust yourself will concern your own interest). The relations between US and Uk are set as 1, there is hardly doubt about the intimate friendships between United States and United Kingdoms. The relations between US and France, between UK and France are set as 0.7, tough France belongs to the western world and the alliance NATO, but the relations between France and the other two states are not as thick as thieves like the relations between US and UK, France doesn’t share the common language and ties of blood with them, and France didn’t go closely with US like UK on many crucial international affairs, such as opposing to US on Iraq issue. For China, the relations between China and all other states are not that intensive, China has been always pursued a policy of nonalignment, there is neither intimate allies relations nor hostile enemies relations similar to the countries standing in either of two sides of the Cold War. At that period, the relations between China and US was kind of good (set as 0.5), the US former president Nixon visited China in 1972, improved the relations between US and China, and following there was a honey moon between US and China since 1979 to 1989, together against the threatens from Soviet Union. As US is the leader of whole western world, once US had improved the relations with China, many other countries such as UK, France, Japan followed soon, we set the relations between China and France (0.6) slightly better than the relations between China and UK (0.4), as France used to pursue the independent foreign policy (from US), and become the first western great power to make a diplomatic relation with the people’s republic of China in 1964. At last, for Soviet Union, the relations with all other states had been negative at that time, not only about the confronting and hostile relations with capitalist bloc and NATO member states, but also the relations with China becoming worsen (due to varied disputes and differences, Soviet Union deployed mass troops on the northern border

between Soviet Union and China), making China going closer with US under the severe threatens.

$$\mathbb{RE}^* = \begin{matrix} & US & UK & FR & CN & RU \\ \begin{matrix} US \\ UK \\ FR \\ CN \\ RU \end{matrix} & \begin{pmatrix} 1.000 & 1.000 & 0.700 & 0.500 & -1.000 \\ 1.000 & 1.000 & 0.700 & 0.400 & -1.000 \\ 0.700 & 0.700 & 1.000 & 0.600 & -0.700 \\ 0.500 & 0.400 & 0.600 & 1.000 & -0.500 \\ -1.000 & -1.000 & -0.700 & -0.500 & 1.000 \end{pmatrix} \end{matrix} \quad (C.12)$$

According to above data, we could compare the total intensity of relations from one state to all other states, by summing the absolute value of degree of relations vertically based on above matrix \mathbb{RE}^* (as the vertical axis is the influenced agent and the horizontal axis is the influencing agent). The ranking outcome below is matched to the real-world situations, United States and Soviet Union both ranked top 1, respectively as the leaders of two confronting blocs (western bloc and eastern bloc, or capitalist bloc and socialist bloc), both of them need to unite closely with allies and oppose intensively to opponents, therefore, the relations of them with other states would be more intensive than other general states. The intensity of relations of China ranked at last, which is due to the policy of nonalignment pursued by China, which is also in accord with the real-world politics.

Table C.28: Rank of Intensity of Relations of Influencing States

No	State	Total Intensity	Positive Intensity	Negative Intensity
1	United States	4.2	3.2	-1.0
1	Soviet Union	4.2	1	-3.2
3	United Kingdoms	4.1	3.1	-1.0
4	France	3.7	3.0	-0.7
5	China	3	2.5	-0.5

If to make the sum of absolute value of degree of relations of one influenced state from all other states fixed as 1, then the matrix \mathbb{RE} would be:

$$re_{[i,j]} = \frac{re_{[i,j]}^*}{\sum_{k=1}^n |re_{[k,j]}^*|}$$

$$\mathbb{RE} = \begin{matrix} & US & UK & FR & CN & RU \\ \begin{matrix} US \\ UK \\ FR \\ CN \\ RU \end{matrix} & \begin{pmatrix} 0.238 & 0.238 & 0.167 & 0.119 & -0.238 \\ 0.244 & 0.244 & 0.171 & 0.098 & -0.244 \\ 0.189 & 0.189 & 0.270 & 0.162 & -0.189 \\ 0.167 & 0.133 & 0.200 & 0.333 & -0.167 \\ -0.238 & -0.238 & -0.167 & -0.119 & 0.238 \end{pmatrix} \end{matrix} \quad (C.13)$$

View Oneself and Others Objectively If we assume the influenced state view the material power and mental power of oneself and other influencing states objectively, without any overrating or underrating. And as the weight of influence is determined collectively by material power, mental power and degree of relations (all three metrics are proportional to the influence power), then defined the weight of influence from the perspective of “related comprehensive strength” as $rcs_{\alpha[i,j]} = sp_{\alpha[i,j]} \times sw_{\alpha[i,j]} \times re_{[i,j]}$, then the matrix of weight of influence \mathbb{RCS}_{α} would be:

$$\mathbb{RCS}_{\alpha} = \begin{matrix} & US & UK & FR & CN & RU \\ \begin{matrix} US \\ UK \\ FR \\ CN \\ RU \end{matrix} & \begin{pmatrix} 0.329 & 0.074 & 0.056 & 0.045 & -0.498 \\ 0.333 & 0.074 & 0.057 & 0.036 & -0.500 \\ 0.302 & 0.068 & 0.105 & 0.071 & -0.456 \\ 0.283 & 0.050 & 0.082 & 0.154 & -0.429 \\ -0.329 & -0.074 & -0.056 & -0.045 & 0.496 \end{pmatrix} \end{matrix} \quad (C.14)$$

The top 5 influencing relations are ranked as following, all top influences come from Soviet Union, due to the predominant comprehensive nation power of Soviet Union and the intensive negative relations between Soviet Union and other states. The top two influences are from Soviet Union to UK (-0.500) and from Soviet Union to US (-0.498), which is in accord with the real-world situation, Soviet Union as the leader of eastern bloc, and US as the leader of western bloc and UK as the best ally of US, the confrontation and negative interaction between Soviet Union and US,UK would be more intensive than other relations.

Table C.29: Top 5 Influencing Relations by Strength of Power \times Strength of Willingness \times Closeness of Relations–View Oneself and Others Objectively

No	Influence	Weight
1	Soviet Union to United Kingdom	-0.500
2	Soviet Union to United States	-0.498
3	Soviet Union to Soviet Union	0.496
4	Soviet Union to France	-0.456
5	Soviet Union to China	-0.429

View Oneself More Times than Others While if the influenced state would not view the material power and mental power of oneself and others in a very objective way, but would overrate or underrate, a common case is overrating or overconfident about own power. And as the weight of influence is determined collectively by material power, mental power and degree of relations, then $r_{cs\beta[i,j]} = sp_{\beta[i,j]} \times sw_{\beta[i,j]} \times re_{[i,j]}$, then the matrix of weight of influence \mathbb{RCS}_{β} would be (assume if the influenced agent would overrate own material and mental power both 3 times):

$$\mathbb{RCS}_{\beta} = \begin{matrix} & US & UK & FR & CN & RU \\ \begin{matrix} US \\ UK \\ FR \\ CN \\ RU \end{matrix} & \begin{pmatrix} 0.815 & 0.020 & 0.016 & 0.012 & -0.137 \\ 0.203 & 0.421 & 0.036 & 0.023 & -0.317 \\ 0.164 & 0.037 & 0.512 & 0.039 & -0.249 \\ 0.126 & 0.023 & 0.036 & 0.624 & -0.191 \\ -0.066 & -0.014 & -0.011 & -0.009 & 0.900 \end{pmatrix} \end{matrix} \quad (C.15)$$

The top 5 influencing relations are ranked as following, it is interesting that all top influences are from own to own, as we assume in above functions, every state overrate own material power 3 times and own mental power 3 times, then for every influenced state, the influence from own dominate any influence from other state. And the weight of top 2 influences from US to own and from Soviet Union to own are both way ahead of other influences (the top two are both bigger than 0.8), which is matched to the common sense, compared with small states, the super power would be more determined by own preference and interest, with a more firm persistence on claims and strategies, but not be influenced and affected easily by other states, and for the two super powers, their comprehensive national power were indeed way ahead of other states. Besides, the weight of own influence of China (0.624) is bigger than France (0.512) and UK (0.421), it is not only due to the slight advantage of China in the comprehensive national power, but also due to the policy of nonalignment pursued by China, as in the table 11.32, the total intensity of relations of China with other states are much lower than UK and France, compared with them, China are less influenced by others and more dependent on own interest and strategy.

Table C.30: Top 5 Influencing Relations by Strength of Power \times Strength of Willingness \times Closeness of Relations–View Oneself More Times (3)

No	Influence	Weight
1	Soviet Union to Soviet Union	0.900
2	United States to United States	0.815

3	China to China	0.624
4	France to France	0.512
5	United Kingdom to United Kingdom	0.421

As the influence from own are based on the overrated own power, therefore, just rank the weight of influence from others (except the influence from own) would also be meaningful, as following, all top influences come from the two super powers, due to the predominant national power and intensive positive/negative relations.

Table C.31: Top 5 Influencing Relations by Strength of Power \times Strength of Willingness \times Closeness of Relations–View Oneself More Times (3)(except the own influence)

No	Influence	Weight
1	Soviet Union to United Kingdom	-0.317
2	Soviet Union to France	-0.249
3	United States to United Kingdom	0.203
4	Soviet Union to China	-0.191
5	United States to France	0.164

Allocate the Ratio between Oneself and Others and View Other Agents Objectively While if the influenced agent not views the influence from own and the influence from others (or allocate the weight of influence from own and others) in a same criterion, but in a respectively different criterions, for example given the weight of own influence with a fixed ratio, and let all other influencing agents share the remaining ratio, objectively according to their (material and mental) powers. And as the weight of influence is determined collectively by material power, mental power and degree of relations, then $r_{cs_{\gamma}[i,j]} = sp_{\gamma}[i,j] \times sw_{\gamma}[i,j] \times re_{[i,j]}$, then the matrix of weight of influence \mathbb{RCS}_{γ} would be (assume if the influenced agent gives the weight for own material and the weight for own mental power both as 0.5):

$$\mathbb{RCS}_{\gamma} = \begin{matrix} & \begin{matrix} US & UK & FR & CN & RU \end{matrix} \\ \begin{matrix} US \\ UK \\ FR \\ CN \\ RU \end{matrix} & \begin{pmatrix} 0.800 & 0.022 & 0.016 & 0.013 & -0.148 \\ 0.069 & 0.806 & 0.012 & 0.008 & -0.105 \\ 0.051 & 0.011 & 0.850 & 0.012 & -0.076 \\ 0.037 & 0.007 & 0.011 & 0.890 & -0.055 \\ -0.108 & -0.025 & -0.019 & -0.015 & 0.833 \end{pmatrix} \end{matrix} \quad (C.16)$$

The top 5 influencing relations could be ranked as following (just including the influence from others while removing the influence from own, as the weights from own influence determined by material and mental power, $sp_{[j,j]}$ and $sw_{[j,j]}$, are both set as 0.5, therefore, the comparisons are meaningless). As below, all top influences are coming from two super powers, 3 places of 5 for Soviet Union and 2 places of 5 for United States. And the mutual influence from Soviet Union to US and from US to Soviet Union ranked as top 2, which is matched to the situation at that period, the confrontation and antagonism between the two super powers are very intensive, both two deeply influenced and affected the other's behaviors and strategies. For the computing outcome, on one side, it is due to the most negative relations (set as -1) between US and Soviet Union, on the other side, it is because both sides have huge comprehensive national power. While one super power face more than one influencing states, in which, the other super power would share the biggest portion, and has the biggest weight of influence among them. The negative influence from Soviet Union to UK and France ranked No.3 and No.4, due to the intensive negative relations between the two confronting sides, and the huge national power of Soviet Union. Besides, the positive influence from US to UK also ranked at top 5, due to the intimate relations between them and the advantage national power of US, which is matched to the real-world practice.

Table C.32: Top 5 Influencing Relations by Strength of Power \times Strength of Willingness \times Closeness of Relations—Allocate the Ratio between Oneself (0.5) and Others (0.5) and View Others Objectively (except the own influence)

No	Influence	Weight
1	Soviet Union to United States	-0.148
2	United States to Soviet Union	-0.108
3	Soviet Union to United Kingdom	-0.105
4	Soviet Union to France	-0.076
5	United States to United Kingdom	0.069

C.3 Comparison of Powers of UN SC 5 Permanent Members since 1946

Table C.33: Comparison of Powers of Five UN SC Permanent Members since 1946

Level	State	Period
Super Power	United States	1946-present
	Soviet Union	1946-1991
Great Power	United Kingdom	1946-present
	France	1946-present
	Republic of China (Mainland)	1946-1949
	People's Republic of China	1971-present
	Russia Federation	1991-present
Middle Power	"Republic of China" (Taiwan)	1949-1971

As the UN security council has been operating since 1946, the post-cold war period is just one portion, to organize and display the situation of powers among 5 UN SC permanent members as shown in table C.33, referring to the the classification by [Chapnick, 1999] and the common sense widely accepted, and considering several crucial historical times (or events) reshaping the international politics. As we can see, US has always been the super power since the establishment of United Nations, Soviet Union was widely deemed as a super power before the disintegration in 1991. China is a very special case, before 1971, the permanent member seat of China in UN SC is held not by the People's Republic of China, but by the "Republic of China"³, at the beginning of UN establishment, Republic of China (led by Kuomintang) is the legitimate representative of China, while Kuomintang lost in the civil war with Communist Party of China, and had to flee to Taiwan Island in 1949, losing the control of the mainland of China, and the Communist Party of China build the "New China", the People's Republic of China, since 1 October 1949, however, "Republic of China" on Taiwan Island still held the permanent seat of China in UN security council for more than 20 years, until 25 October 1971, the People's Republic of China replaced Taiwan, to become the only legitimate representative of China in United Nations. The Republic of China after World War II and before 1949 (on the Mainland), and the People's Republic of China are deemed as great power, but the "Republic of China" since 1949 to 1971 (on Taiwan Island) is just a small "state".

³The Republic of China was an era of Chinese history that began in 1912 and ended in 1949. It was preceded by the Qing dynasty and followed by the People's Republic of China. At the end of the Chinese Civil War, the communists gained control of the mainland, while the Kuomintang (nationalists) withdrew to Island of Taiwan and some minor outlying islands where the ROC government still retains power [Dillon, 2008].

C.4 Ranking of GDP, Population, Power Categories of UN SC 5 Permanent members and 7 Main Non-permanent Members

Table C.34: 2014 Ranking of GDP based on PPP of 5 Permanent Members and 7 Main Non-permanent Members

<i>Rank_{of 12}</i>	<i>Rank_{in the world}</i>	State	<i>GDP_{PPP}</i> (Billions Dollars)
1	1	China	17,632
2	2	United States	17,416
3	3	India	7,277
4	4	Japan	4,788
5	5	Germany	3,621
6	6	Russia	3,559
7	7	Brazil	3,073
8	8	France	2,587
9	10	United Kingdoms	2,435
10	25	Argentina	927
11	26	Pakistan	884
12	30	South Africa	683

Table C.35: 2014 Ranking of Population of 5 Permanent Members and 7 Main Non-permanent Members

<i>Rank_{of 12}</i>	<i>Rank_{in the world}</i>	State	Population (Thousands)
1	1	China	1,393,783
2	2	India	1,267,401
3	3	United States	322,583
4	5	Brazil	202,033
5	6	Pakistan	185,132
6	9	Russia	142,467
7	10	Japan	126,999
8	16	Germany	82,652
9	21	France	64,641
10	22	United Kingdoms	63,489
11	25	South Africa	49,512
12	32	Argentina	41,803

Table C.36: Power Comparison of 5 Permanent Members and 7 Main Non-permanent Members

State	Power Categories	SC Position	<i>Rank_{GDP}</i>	<i>Rank_{Population}</i>
United States	Super Power	Permanent	2	3
China	Great Power	Permanent	1	1
Russia	Great Power	Permanent	6	9
France	Great Power	Permanent	8	21
United Kingdom	Great Power	Permanent	10	22
Japan	Great Power	Non-Permanent	4	10
Germany	Great Power	Non-Permanent	5	16
India	Regional Power	Non-Permanent	3	2
Brazil	Regional Power	Non-Permanent	7	5
Argentina	Middle Power	Non-Permanent	25	32
Pakistan	Middle Power	Non-Permanent	26	6
South Africa	Middle Power	Non-Permanent	30	25

For 7 non-permanent members, except India, all other 6 have been elected at least twice over last 10 years.

C.5 Complementary Explanations of Mathematical and Computer Modeling Steps

We describe the steps of programming using natural language in details:

Step 0 Initial Setup We assume 15 SC member states voting on 100 bills, so create $15 \times 100 = 1500$ turtles with order (in netlogo order is named as “who”) from 0 to 1499.

As it may exist the cases of abstentions, we ask turtles, by a probability of *PartialVariables*, “die”.

And we arrange the turtles according to their order, and 15 turtles every row (representing 15 SC member states voting on one bill) and 100 turtles every column (representing one SC member state voting on sequential 100 bills).

For each row, the left 5 turtles represent 5 permanent members which are fixed, with shape set as bigger person with cases, while the right 10 turtles represent 10 non-permanent members which are dynamic, with shape set as smaller person.

According to UN Security Council rule, the term of 10 non-permanent members are 2 years, 5 replaced each year, and can not be reelected in next term. To model the dynamics of non-permanent members, we collect the list of non-permanent members over past 10 years, the constitution of non-permanent members each year running for 10 ticks.

An important attribute for each member state is the alliance belongingness, which will heavily affected the influencing relations. Preliminarily we set two alliances, one is NATO (more broadly western world) led by US, with typical allies like UK, France and so on, and the other is led by China and Russia, with typical allies like Pakistan, African countries and so on, and also there are some member states belonging to neither side.

Assume the *alliance* = $\{-1, 0, 1\}$, -1 and 1 means two contradictive alliances and 0 means no alliance belongingness, firstly for 5 permanent members, set the alliance of US, UK and France as -1, and China and Russian as 1.

Then for non-permanent members, for term of 2004, set Romania (NATO member), Spain (NATO member), Germany (NATO member), Philippines (“Sidekick” and former colony of US) as -1, Pakistan as 1 (“best friend” of China), and Algeria, Angola, Benin, Brazil and Chile as 0; for term of 2005, five new added member states, Denmark (NATO member), Greece (NATO member), Japan (military ally of US) as -1, Argentina as 0, and Tanzania (old “buddy” of China) as 1; for term of 2006, Slovakia (NATO member) as -1, Qatar, Congo, Peru and Ghana as 0; for term of 2007, Belgium (NATO member), Italy (NATO member) as -1, and Indonesia, South Africa and Panama as 0; for term of 2008, Croatia (NATO member) as -1, Burkina Faso, Libya, Vietnam and Costa Rica as 0; for term of 2009, set Austria (EU member), Mexico (special relation with US), Japan (military ally of US) and Turkey (NATO member) as -1, Uganda as 0; for term of 2010, Nigeria, Gabon, Lebanon, Bosnia & Herzegovina and Brazil as 0; for term of 2011, set Portugal (NATO member), Germany (NATO member) as -1, Columbia and India as 0; for term of 2012, set Guatemala, Morocco, Togo and Azerbaijan as 0, and Pakistan as 1 (“follower” of China); for term of 2013, Australia (member of “Western world”), South Korea (military ally of US), Luxembourg (NATO member) as -1, Rwanda and Argentina as 0.

Set the initial preference of turtles, if from a cardinal approach, assume initial preference $\in \{-1, 0, 1\}$, -1 means negative vote, 0 means absent, and 1 means positive vote, and let turtles get a random value from $\{-1, 0, 1\}$.

However, there might exist the cases of partial domain, then the domain of preference could be partial as $\{-1\}$, $\{0\}$, $\{1\}$, $\{-1, 0\}$, $\{-1, 1\}$, $\{0, 1\}$.

Step 1 Generation of Influence Patterns Set the generation probabilities for all influence patterns, from *PP1* to *PP24*, representing the probabilities of influence from pattern 1 to 24. Assume

two variables of turtles as state and ycor (y-coordinate, which indicate the sequence of bills), and two global variables as owns and yc, set owns as the state of turtle be asked, and set yc as the ycor of turtle be asked.

(1) For influence pattern 1,7,13,19, which are influences from individual decision to individual decision.

For influence pattern 1, ask each of turtles, set owns as state and yc as ycor, by a probability $PP1$, to make link to random one of other turtles within the same state and on latter bill (with $state = owns$ and $ycor < yc$).

For influence pattern 7, ask each of turtles, set yc as ycor, by a probability $PP7$, to make link to random one of other turtles within the same bill (with $ycor = yc$).

For influence pattern 13/19, ask each of turtles, set owns as state and yc as ycor, by a probability $PP13/19$, to make link to random one of other turtles on different state and on latter bill (with $state \neq owns$ and $ycor < yc$).

(2) For influence pattern 2,8,14,20, assumed as a binary choice, which are influences from one decision, according to different choices, leading to two different decisions being influenced.

For influence pattern 2, ask each of turtles, set owns as state and yc as ycor, by a probability $PP2$, if $pref/vote=1$, then make link to random one of other turtles on the same state and on latter bill (with $state = owns$ and $ycor < yc$), set who of this turtle as theT, while if $pref/vote=-1$, then make link to different random one of other turtles on the same state and on latter bill (with $who \neq theT$ and $state = owns$ and $ycor < yc$).

For influence pattern 8, ask each of turtles, set yc as ycor, by a probability $PP8$, if $pref/vote=1$, then make link to random one of other turtles on the same bill (with $ycor = yc$), set who of this turtle as theT, while if $pref/vote=-1$, then make link to different random one of other turtles on the same bill (with $who \neq theT$ and $ycor = yc$).

For influence pattern 14/20, ask each of turtles, set owns as state and yc as ycor, by a probability $PP14/20$, if $pref/vote=1$, then make link to random one of other turtles on different state and on latter bill (with $state \neq owns$ and $ycor < yc$), set who of this turtle as theT, while if $pref/vote=-1$, then make link to different random one of other turtles on different state and on latter bill (with $who \neq theT$ and $state \neq owns$ and $ycor < yc$).

(3) For influence pattern 3,9,15,21, which are influences from individual decision to structures among decisions, different choices leading to different structures being influenced/generated.

For influence pattern 3, ask each of turtles, set owns as state and yc as ycor, by a probability $PP3$, if $pref/vote=1$, then ask one of other turtles with $state = owns$ and $ycor < yc$, set yc as ycor of the new turtle being asked, to create links to one of other turtles with $state = owns$ and $ycor < yc$, while if $pref/vote=-1$, then again ask one of other turtles with $state = owns$ and $ycor < yc$, set yc as ycor of the new turtle being asked, to create links to one of other turtles with $state = owns$ and $ycor < yc$.

For influence pattern 9, ask each of turtles, set yc as ycor, by a probability $PP9$, if $pref/vote=1$, then ask one of other turtles with $ycor = yc$, to create links to one of other turtles with $ycor = yc$, while if $pref/vote=-1$, then again ask one of other turtles with $ycor = yc$, to create links to one of other turtles with $ycor = yc$.

For influence pattern 15/21, ask each of turtles, set owns as state and yc as ycor, by a probability $PP15/21$, if $pref/vote=1$, then ask one of other turtles with $state \neq owns$ and $ycor < yc$, set yc as ycor of the new turtle being asked, to create links to one of other turtles with $state \neq owns$ and $ycor < yc$, while if $pref/vote=-1$, then again ask one of other turtles with $state \neq owns$ and $ycor < yc$, set yc as ycor of the new turtle being asked, to create links to one of other turtles with $state \neq owns$ and $ycor < yc$.

(4) For influence pattern 4,10,16,22, which are collective influences, from more than individ-

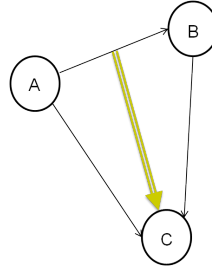


Figure C.1: Individual Influence before the Structure Influence

ual decisions with influencing relations among them to one individual decision ⁴, and for influence pattern 5,11,17,24, which are also collective influences, from more than individual decisions with influencing relations among them but leading to different individual decisions, due to different influencing relations being chosen/generated (in the beginning). The prerequisite for this category of influences coming from structure (influencing relations) to decision is, there are already more than one individual decisions influencing, then the influencing relations among/between them would then take an influential effect. For example, for the most basic form, one turtle C is influenced by two turtles A and B with influencing relations between them, firstly there are two individual influences from A to C and from B to C, then the influencing relations between A and B make effect on C (as shown in C.1).

(5) For influence pattern 6,12,18,24, which are influences from influencing relations to influencing relations, different influencing relations would generate corresponding different influencing relations.

For influence pattern 6, ask each of links within one state (with state of end1 = state of end2), set owns as state of end1 or end2 (as equal), and set yc as $ycor$ of end2 (as latter than end1), by a probability of $PP6$, further ask one of other turtles on the same state and on latter bill (with $state = owns$ and $ycor < yc$), set yc as $ycor$ of the new turtle being asked, to create link to one-of other turtles on the same state and on latter bill (with $state = owns$ and $ycor < yc$).

For influence pattern 12, ask each of links within one bill (with $ycor$ of end1 = $ycor$ of end2), set yc as $ycor$ of end2 or end1 (as equal), by a probability of $PP12$, further ask one of other turtles on the same bill (with $ycor = yc$), to create link to one-of other turtles on the same bill (with $ycor = yc$).

For influence pattern 18/24, ask each of links either crossing different states and crossing different bills (with state of end1 \neq state of end2 and $ycor$ of end1 \neq $ycor$ of end2), set yc as $ycor$ of end2 (as latter than end1), by a probability of $PP12$, further ask one of any other turtles only not on former bill (with $ycor \leq yc$, which means the state and $ycor$ of turtle being asked can be both identical or different from the influencing ones), set yc as $ycor$ of the new turtle being asked, to create link to one-of other turtles not on former bill (with $ycor \leq yc$).

The Possible Maximum and Minimum Number of Influence Patterns Assume there is a decision-making system with n agents and m features, $\mathcal{NP}_{No.}$ is the number of Influence Pattern $No. \in \{1, \dots, 24\}$.

Influence Pattern 1: only former can influence latter, but can not be reversed:

⁴For the most basic form, which are the influences from two decisions with influencing relations between them to one individual decision.

$$\begin{aligned}
\mathcal{NP}_1^{Max} &= n(m-1 + m-2 + \dots + 1) \\
&= n \frac{(m-1)m}{2} \\
\mathcal{NP}_1^{Min} &= 0 \\
\mathcal{NP}_1 &\in [0, \frac{nm(m-1)}{2}]
\end{aligned} \tag{C.17}$$

Influence Pattern 2: only former can influence latter, not reversed; and two influenced decisions must be distributed on two different issues.

$$\begin{aligned}
\mathcal{NP}_2^{Max} &= n[(m-1)(m-2) + (m-2)(m-3) + \dots + 2 \times 1] \\
&= n[(m-1)^2 - (m-1) + (m-2)^2 - (m-2) + \dots + 2^2 - 2] \\
&= n[\frac{(m-1)m(2(m-1)+1)}{6} - 1 - \frac{(m-1+2)(m-2)}{2}] \\
&= n \frac{m^3 - 3m^2 + 2m}{3} \\
\mathcal{NP}_2^{Min} &= 0 \\
\mathcal{NP}_2 &\in [0, n \frac{m^3 - 3m^2 + 2m}{3}]
\end{aligned} \tag{C.18}$$

Influence Pattern 3: only former can influence latter, not reversed; choosing three issues in the latter as influenced structure, in which with one influencing issue and two influenced issue, and three decisions must be distributed on two different issues.

$$\begin{aligned}
\mathcal{NP}_2^{Max} &= n[(m-1)(m-2)(m-3) + (m-2)(m-3)(m-4) + \dots + 3 \times 2 \times 1] \\
&= n[(m-1)^3 - 3(m-1)^2 + 2(m-1) + (m-2)^3 - 3(m-2)^2 + 2(m-2) + \dots + 3^3 - 3 \times 3^2 + 2 \times 3] \\
&= n[[(m-1)(m-1+1)/2]^2 - 2^3 - 1^3 - 3[\frac{(m-1)m(2(m-1)+1)}{6} - 2^2 - 1^2] + 2\frac{(m-1+3)(m-3)}{2}] \\
&= n \frac{m^4 - 6m^3 + 11m^2 - 6m - 24}{4} \\
\mathcal{NP}_2^{Min} &= 0 \\
\mathcal{NP}_2 &\in [0, n \frac{m^4 - 6m^3 + 11m^2 - 6m - 24}{4}]
\end{aligned} \tag{C.19}$$

Influence Pattern 7:

$$\begin{aligned}
\mathcal{NP}_7^{Max} &= m \underbrace{(n-1 + \dots + n-1)}_n \\
&= mn(n-1) \\
\mathcal{NP}_7^{Min} &= 0 \\
\mathcal{NP}_7 &\in [0, mn(n-1)]
\end{aligned} \tag{C.20}$$

Influence Pattern 8:

$$\begin{aligned}
\mathcal{NP}_8^{Max} &= m \underbrace{[(n-1)(n-2) + \dots + (n-1)(n-2)]}_n \\
&= mn(n-1)(n-2) \\
\mathcal{NP}_8^{Min} &= 0 \\
\mathcal{NP}_8 &\in [0, mn(n-1)(n-2)]
\end{aligned} \tag{C.21}$$

Influence Pattern 9:

$$\begin{aligned}
\mathcal{NP}_2^{Max} &= m[\underbrace{(n-1)(n-2)(n-3) + \dots + (n-1)(n-2)(n-3)}_n] \\
&= mn(n-1)(n-2)(n-3) \\
\mathcal{NP}_2^{Min} &= 0 \\
\mathcal{NP}_2 &\in [0, mn(n-1)(n-2)(n-3)]
\end{aligned} \tag{C.22}$$

Influence Pattern 13 and Pattern 19: not distinguish influence pattern 13 and 19, only if the influence crossing both agents and issues from decision to decision.

$$\begin{aligned}
\mathcal{NP}_{13/19}^{Max} &= n^2(m-1) + n^2(m-2) + \dots + n^2 \\
&= \frac{n^2(m-1+1)(m-1)}{2} \\
&= \frac{n^2m(m-1)}{2} \\
\mathcal{NP}_{13/19}^{Min} &= 0 \\
\mathcal{NP}_{13/19} &\in [0, \frac{n^2m(m-1)}{2}]
\end{aligned} \tag{C.23}$$

Influence Pattern 14 and Pattern 20:

$$\begin{aligned}
\mathcal{NP}_{14/20}^{Max} &= n\{n(m-1) \times [n(m-1) - 1]\} + n\{n(m-2) \times [n(m-2) - 1]\} + \dots + n \times \{n - 1\} \\
&= n\{n^2(m-1)^2 - n(m-1) + n^2(m-2)^2 - n(m-2) + \dots + n^2 - n\} \\
&= n\{n^2 \frac{(m-1)(m-1+1)[2(m-1)+1]}{6} - n \frac{(m-1+1)(m-1)}{2}\} \\
&= \frac{n^2m(m-1)(2mn - n + 3)}{6} \\
\mathcal{NP}_{14/20}^{Min} &= 0 \\
\mathcal{NP}_{14/20} &\in [0, \frac{n^2m(m-1)(2mn - n + 3)}{6}]
\end{aligned} \tag{C.24}$$

Step 2 Weight Allocation of Influence Use links to represent the influences, and set the weight of influence based on the alliance situation of two ends (end1 and end2, end1 is the influencing one in the influencing relation, and end2 is the influenced one in the influencing relation).

Assume the alliance of ends = $\{-1, 0, 1\}$, if $end1 = 0$ or $end2 = 0$, set the weight of influence as random value $[-0.3, 0.3]$; if $end1 = 1$ and $end2 = 1$, set the weight of influence as random value $[-0.3, 1]$, bias to positive; if $end1 = -1$ and $end2 = 1$, set the weight of influence as random value $[-1, 0.3]$, bias to negative; if $end1 = 1$ and $end2 = -1$, set the weight of influence as random value $[-1, 0.3]$, bias to negative; if $end1 = -1$ and $end2 = -1$, set the weight of influence as random value $[-0.3, 1]$, bias to positive.

As we can not ask turtles create links to their own in netlogo, we directly ask turtles set a weight as weight of influence from own with random value $[1, 2]$.

Step 3 Preference after Influenced There two two rules while being influenced by more than one turtles, one is the prominent one influence, and the other is the collective influence.

If influenced by the prominent one, ask turtles be influenced by the preference of one of other turtles with the highest weight.

If collectively influenced, there all further three levels, level 1 is just to consider about the influence from individual ones, level 2 adds with the influential effect from group of individuals with same attributes, and level 3 adds the influential effect from structure (influencing relations) within influencing individuals.

If by level 1, let the preference of influenced turtles be the weighted average of all influencing turtles' and own original preference.

If by level 2, let the preference of influenced turtles be the weighted average of all influencing turtles' and own original preference, plus the influential effect from groups possessing the same preference, set a coefficient as GE (Group Effect), and set the function of influential group effect as $(1 + GE)^{\text{count influencing turtles with preference}=\{-1,1\}-1}$.

If by level 3, let the preference of influenced turtles be the weighted average of all influencing turtles' and own original preference, added with the influential effect from influencing relations within influencing groups. For the most basic form, one turtle is influenced by the influencing relations between two turtles, then the influential structure effect would be determined by three bilateral links, assume the turtle in the influencing relation as influencing one as who1, the turtle in the influencing relation as influenced one as who2, and the turtle being influenced by the relation as who3, and respectively with preference1, preference2, preference3, and the weight1, weight2 and weight3, the influential structure effect would be affected by the cartesian product of the signs of three bilateral links (link from who1 to who2, link from who1 to who3 and link from who2 to who3) as $\{+, -\} \times \{+, -\} \times \{+, -\}$. According to the different combinations of signs of bilateral links, the influential structure effect would be given with corresponding influencing preferences as $\{preference1, preference2, -preference1, -preference2\}$, times with corresponding weights as $\{SF \times weight1, SF \times weight2\}$, in which SF is the structure effect with a value from 0 to 1.

Step 4 Vote based on Preference As we first deal with the influence from a cardinal approach, like the weighted average, therefore the influenced preference might not be an integer as $\{-1,0,1\}$ anymore, while the vote domain corresponding to preference domain is $\{-1,0,1\}$, -1 as negative vote (NO), 0 as absent, and 1 as affirmative vote (YES), therefore, the transformation from a non-integer influenced preference to an integer vote need to be built.

We define three distance as $dis - 1$, $dis0$ and $dis1$, which are respectively the distance of influenced preference from point -1, 0 and 1.

If the $dis - 1$ is the minimum compared with other 2 distances, which means the preference of the influenced turtle is most bias to negative vote, then vote as -1 (negative), and set color red; if $dis0$ is the minimum, then vote as 0 (absent), and set color gray; besides, if $dis1$ is the minimum, then vote as 1 (affirmative), and set color green.

Step 5 Factual Result of Collective Vote According to the current voting rules, for 15 UN SC member, only if there are more than 9 members voting affirmative and without any permanent members voting negative (due to the veto of permanent members), then the bill gets passed. Set fo (factual outcome) = $\{1, 0\}$, 1 mean passed, and 0 means failed.

And the voting rule could be deemed as an adjusting variable, to experiment different designed voting rules compared with the current one, liking cancelling or weakening the veto power and so on.

Step 6 Expectation Psychology Rule For one member state, his expected outcome is determined by his preference, if his expected outcome is not indifferent ($\neq 0$) but clearly as affirmative ($= 1$) or negative ($= -1$), and also not equal to factual outcome ($\neq fo$ on one bill, then count the unmatched (between expected outcome and factual outcome) +1, assume the ep (Expectation

Psychology status) = $\{1, 0\}$, which means two different status, 1 means becoming more mild to opponents, and 0 means maintaining the status quo, if unmatched reaches a set threshold value, then the ep would be reserved (from 1 to 0 or from 0 to 1), and reset the unmatched to zero.

While $ep = 1$, the negative weight of influence from opponents would be weakened with a ratio MOI (Mild-to-Opponent-Inclination).

Step 7 Comparison Psychology Rule For one member state, he would possibly compare his own unmatched status with his allies' unmatched status, computes the average of unmatched status of other states with the same alliance belongingness and until the same bill as *allyunmatch*.

If $ownunmatch > allyunmatch$, the member state would feel unbalanced and disappointed, and there are different responds, for example one is to be more dependent on own preference on latter choices, making weight of own increase with a ratio of OPI (*Own-CentriPetal-Inclination*), and another is to be less influenced by his allies on latter choices, making weight from ally decrease with a ratio of AFI (*Ally-Centrifugal-Inclination*).

While if $ownunmatch \leq allyunmatch$, the member state would feel balanced and satisfied, and maintain the status quo about the weight allocation.

Appendix D

Dataset

D.1 General Outcome of UN SC Votes since 1994

Table D.1: General Outcome of UN SC Vote since 1994

¹

Record	Year	Date	Content	Y	N(V)	A	Outcome
S/PV.7088	2013	1218	International Tribunal-Yugoslavia	14	0	1	Passed
S/PV.7060	2013	1115	Peace and security in Africa	7	0	8	Failed
S/PV.7036	2013	0926	Small arms	14	0	1	Passed
S/PV.7014	2013	0730	Cyprus	13	0	2	Passed
S/PV.6908	2013	0124	Cyprus	14	0	1	Passed
S/PV.6889	2012	1217	International Tribunal-Yugoslavia	14	0	1	Passed
S/PV.6838	2012	0919	Children and armed conflict	11	0	4	Passed
S/PV.6819	2012	0731	on the Sudan	14	0	1	Passed
S/PV.6810	2012	0719	Middle East	11	2(2)	2	Failed
S/PV.6809	2012	0719	Cyprus	13	0	2	Passed
S/PV.6711	2012	0204	Middle East situation-Syria	13	2(2)	0	Failed
S/PV.6674	2011	1205	Peace and security in Africa	13	0	2	Passed
S/PV.6627	2011	1004	Middle East situation	9	2(2)	4	Failed
S/PV.6552	2011	0609	Non-proliferation	14	0	1	Passed
S/PV.6498	2011	0317	The situation in Libya	10	0	5	Passed
S/PV.6484	2011	0218	Middle East situation, Palestinian	14	1(1)	0	Failed
S/PV.6463	2010	1222	International Tribunal-Rw & Yu	14	0	1	Passed
S/PV.6450	2010	1215	Iraq	14	0	1	Passed
S/PV.6445	2010	1214	Cyprus	14	1(0)	0	Passed
S/PV.6401	2010	1014	Sudan	14	0	1	Passed
S/PV.6339	2010	0615	Cyprus	14	1(0)	0	Passed
S/PV.6335	2010	0609	Non-proliferation	12	2(0)	1	Passed
S/PV.6254	2009	1223	Peace and security in Africa	13	1(0)	1	Passed
S/PV.6239	2009	1214	Cyprus	14	1(0)	0	Passed
S/PV.6143	2009	0615	Georgia	10	1(1)	4	Failed
S/PV.6132	2009	0529	Cyprus	14	1(0)	0	Passed
S/PV.6063	2009	0108	Middle East situation, Palestinian	14	0	1	Passed
S/PV.6045	2008	1216	Middle East situation, Palestinian	14	0	1	Passed
S/PV.5947	2008	0731	Sudan	14	0	1	Passed
S/PV.5933	2008	0711	Peace and security, Zimbabwe	9	5(2)	1	Failed
S/PV.5848	2008	0303	Non-proliferation Iran	14	0	1	Passed
S/PV.5744	2007	0919	Afghanistan	14	0	1	Passed
S/PV.5742	2007	0914	International Tribunal-Yugoslavia	14	0	1	Passed
S/PV.5710	2007	0629	Iraq	14	0	1	Passed
S/PV.5685	2007	0330	Middle East situation	10	0	5	Passed
S/PV.5619	2007	0112	Myanmar	9	3(2)	3	Failed

¹Y is Yes (affirmative vote), N is No (Negative Vote), V is the number of Veto in all negative votes, and A is Absent.

S/PV.5565	2006	1111	Middle East situation, Palestinian	10	1(1)	4	Failed
S/PV.5519	2006	0831	Sudan	12	0	3	Passed
S/PV.5500	2006	0731	Non-proliferation	14	1(0)	0	Passed
S/PV.5488	2006	0713	Middle East situation, Palestinian	10	1(1)	4	Failed
S/PV.5440	2006	0517	Middle East situation	13	0	2	Passed
S/PV.5423	2006	0425	Sudan	12	0	3	Passed
S/PV.5335	2005	1220	Post-conflict peace building	13	0	2	Passed
S/PV.5158	2005	0331	Sudan	11	0	4	Passed
S/PV.5153	2005	0329	Sudan	12	0	3	Passed
S/PV.5051	2004	1005	Middle East situation, Palestinian	11	1(1)	3	Failed
S/PV.5040	2004	0918	Sudan	11	0	4	Passed
S/PV.5028	2004	0902	Middle East, Lebanon	9	0	6	Passed
S/PV.5015	2004	0730	Sudan	13	0	2	Passed
S/PV.4972	2004	0519	Middle East situation, Palestinian	14	0	1	Passed
S/PV.4947	2004	0421	Cyprus	14	1(1)	0	Failed
S/PV.4934	2004	0325	Middle East situation, Palestinian	11	1(1)	3	Failed
S/PV.4842	2003	1014	Middle East situation, Palestinian	10	1(1)	4	Failed
S/PV.4828	2003	0916	Middle East situation, Palestinian	11	1(1)	3	Failed
S/PV.4820	2003	0912	Libyan Arab Jamahiriya	13	0	2	Passed
S/PV.4808	2003	0814	Iraq-Kuwait	14	0	1	Passed
S/PV.4803	2003	0801	Liberia	12	0	3	Passed
S/PV.4772	2003	0612	United Nations peacekeeping	12	0	3	Passed
S/PV.4761	2003	0522	Iraq-Kuwait	14	0	1	Passed
S/PV.4683	2002	1230	Iraq-Kuwait	13	0	2	Passed
S/PV.4681	2002	1220	Middle East situation, Palestinian	12	1(1)	2	Failed
S/PV.4667	2002	1213	Peace and security-terrorist acts	14	1(0)	0	Passed
S/PV.4614	2002	0924	Middle East situation, Palestinian	14	0	1	Passed
S/PV.4563	2002	0630	Bosnia and Herzegovina	13	1(1)	1	Failed
S/PV.4503	2002	0330	Middle East situation, Palestinian	14	0	1	Passed
S/PV.4489	2002	0312	Middle East situation, Palestinian	14	0	1	Passed
S/PV.4438	2001	1214	Middle East situation, Palestinian	12	1(1)	2	Failed
S/PV.4384	2001	0928	Sudan	14	0	1	Passed
S/PV.4305	2001	0327	Middle East situation, Palestinian	9	1(1)	4	Failed
S/PV.4251	2000	1219	Afghanistan	13	0	2	Passed
S/PV.4248	2000	1218	Middle East situation, Palestinian	8	0	7	Passed
S/PV.4205	2000	1007	Middle East situation, Palestinian	14	0	1	Passed
S/PV.4168	2000	0705	Sierra Leone	14	0	1	Passed
S/PV.4162	2000	0621	Bosnia and Herzegovina	14	0	1	Passed
S/PV.4149	2000	0331	Western Sahara	12	1(0)	2	Passed
S/PV.4103	2000	0217	Admission of new Members-Tuvalu	14	0	1	Passed
S/PV.4094	2000	0131	Georgia	14	0	1	Passed
S/PV.4084	1999	1217	Iraq-Kuwait	11	0	4	Passed
S/PV.4080	1999	1214	Western Sahara	14	0	1	Passed
S/PV.4077	1999	1203	Iraq-Kuwait	11	0	4	Passed
S/PV.4074	1999	1130	Haiti	14	0	1	Passed
S/PV.4017	1999	0625	Admission of new Members-Nauru	14	0	1	Passed
S/PV.4011	1999	0610	Kosovo, Yugoslavia	14	0	1	Passed
S/PV.4003	1999	0514	Kosovo, Yugoslavia	13	0	2	Passed
S/PV.3989	1999	0326	Kosovo, Yugoslavia	3	12	0	Failed
S/PV.3982	1999	0225	Macedonia, Yugoslavia	13	1(1)	1	Failed
S/PV.3949	1998	1125	Haiti	13	0	2	Passed
S/PV.3944	1998	1117	International Tribunal-Yugoslavia	14	0	1	Passed
S/PV.3937	1998	1024	Kosovo, Yugoslavia	13	0	2	Passed
S/PV.3930	1998	0923	Kosovo, Yugoslavia	14	0	1	Passed
S/PV.3868	1998	0331	Kosovo, Yugoslavia	14	0	1	Passed
S/PV.3826	1997	1023	Iraq-Kuwait	10	0	5	Passed
S/PV.3817	1997	0912	Iraq-Kuwait	14	0	1	Passed
S/PV.3791	1997	0619	Albania	14	0	1	Passed
S/PV.3758	1997	0328	Albania	14	0	1	Passed
S/PV.3756	1997	0321	occupied Arab territories	13	1(1)	1	Passed
S/PV.3747	1997	0307	occupied Arab territories	14	1(1)	0	Passed

S/PV.3730	1997	0110	Guatemala	14	1(1)	0	Failed
S/PV.3716	1996	1127	Macedonia	14	0	1	Passed
S/PV.3707	1996	1022	Georgia	14	0	1	Passed
S/PV.3698	1996	0928	occupied Arab territories	14	0	1	Passed
S/PV.3690	1996	0816	Ethiopia-Sudan	13	0	2	Passed
S/PV.3683	1996	0726	Cuba-USA	13	0	2	Passed
S/PV.3670	1996	0530	Macedonia	14	0	1	Passed
S/PV.3660	1996	0426	Ethiopia-Sudan	13	0	2	Passed
S/PV.3595	1995	1122	Serbia and Montenegro-Sanctions	14	0	1	Passed
S/PV.3551	1995	0705	Serbia and Montenegro-Sanctions	14	0	1	Passed
S/PV.3543	1995	0616	United Nations Protection Force	13	0	2	Passed
S/PV.3538	1995	0517	occupied Arab territories	14	0	1	Passed
S/PV.3522	1995	0421	Serbia and Montenegro-Sanctions	13	0	2	Passed
S/PV.3496	1995	0130	Haiti	14	0	1	Passed
S/PV.3487	1995	0112	Serbia and Montenegro-Sanctions	14	0	1	Passed
S/PV.3475	1994	1202	situation in the former Yugoslavia	13	1(1)	1	Failed
S/PV.3470	1994	1129	Haiti	13	0	2	Passed
S/PV.3453	1994	1108	International Tribunal-Rwanda	13	1(0)	1	Passed
S/PV.3437	1994	1015	Haiti	14	0	1	Passed
S/PV.3432	1994	0930	Somalia	14	0	1	Passed
S/PV.3430	1994	0929	Haiti	13	0	2	Passed
S/PV.3428	1994	0923	situation in the former Yugoslavia	11	2(0)	2	Passed
S/PV.3428	1994	0923	situation in the former Yugoslavia	14	0	1	Passed
S/PV.3413	1994	0731	Haiti	12	0	3	Passed
S/PV.3412	1994	0729	Cyprus	14	0	1	Passed
S/PV.3409	1994	0728	Middle East-UNIFIL	14	0	1	Passed
S/PV.3407	1994	0721	Georgia	14	0	1	Passed
S/PV.3392	1994	0622	Rwanda	10	0	5	Passed

D.2 Specific Composition of UN SC Votes since 1946

As following, all member states are shorted as abbreviation (to save the space and fully represent the integral vote composition), for the five permanent members, “US”, “UK”, “FR”, “CN”, “RU” are respectively the abbreviations of United States, United Kingdoms, France, China and Russian, for the non-permanent members, the full names for abbreviations are recorded in Appendix D.4.

Table D.2: Specific Composition of UN SC Vote since 1992

Date	Subject	US	UK	FR	CN	RU	KR	AR	RW	LU	AU	AZ	GT	MA	PK	TG
20131115	Africa	A	A	A	Y	Y	A	A	Y	A	A	Y	A	Y	Y	Y
20130730	Cyprus	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	AZ	GT	MA	PK	TG	CO	DE	IN	PT	ZA
20121212	Tribunal,YU	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
20120919	Children,Arms	Y	Y	Y	A	A	A	Y	Y	Y	Y	Y	A	Y	Y	Y
20120731	Sudan	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	CO	DE	IN	PT	ZA	BH	BR	GA	LB	NG
20110609	Nucleus,Iran	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y
20110317	Libya	Y	Y	Y	A	A	Y	Y	A	Y	A	Y	A	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	BH	BR	GA	LB	NG	AT	JP	MX	TR	UG
20101222	Criminal Tribunal	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
20101215	Iraq,oil-food	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
20101214	Cyprus	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	Y
20101014	Sudan	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
20100615	Cyprus	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	Y
20100609	Nucleus,Iran	Y	Y	Y	Y	Y	N	Y	A	Y	Y	Y	Y	Y	N	Y
Date	Subject	US	UK	FR	CN	RU	AT	JP	MX	TR	UG	BF	CR	HR	LY	VN
20091223	Eritrea,embargo	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	Y
20091214	Cyprus	Y	Y	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y
20090529	Cyprus	Y	Y	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y
20090108	Palestine	A	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	BF	CR	HR	LY	VN	BE	ID	IT	PA	ZA

20081216	Palestine	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
20080731	Sudan	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
20080303	Nuclear,Iran	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y
Date	Subject	US	UK	FR	CN	RU	BE	ID	IT	PA	ZA	CG	GH	PE	QA	SK
20070919	Afghanistan	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
20070914	Tribunal,YU	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
20070629	UNMOVIC,Iraq	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
20070530	Tribunal,Lebanon	Y	Y	Y	A	A	Y	A	Y	Y	A	Y	Y	Y	A	Y
Date	Subject	US	UK	FR	CN	RU	CG	GH	PE	QA	SK	AR	DK	GR	TZ	JP
20060831	Sudan	Y	Y	Y	A	A	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
20060731	Nuclear,Iran	Y	Y	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y
20060517	Lebanon	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
20060425	Sudan	Y	Y	Y	A	A	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	AR	DK	GR	TZ	JP	DZ	BJ	BR	PH	RO
20051220	Peacebuilding	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	A	Y	Y
20050331	Sudan	A	Y	Y	A	Y	Y	Y	Y	Y	Y	A	Y	A	Y	Y
20050329	Sudan	Y	Y	Y	A	A	Y	Y	Y	Y	Y	A	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	DZ	BJ	BR	PH	RO	AO	CL	PK	ES	DE
20040917	Sudan	Y	Y	Y	A	A	A	Y	Y	Y	Y	Y	Y	A	Y	Y
20040730	Sudan	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
20040519	Palestine	A	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y
Date	Subject	US	UK	FR	CN	RU	AO	CL	PK	ES	DE	BG	CM	GN	MX	SY
20030912	Libya	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A
20030814	Iraq,assistance	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A
20030801	Libya	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
20030612	Criminal Court	Y	Y	A	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	A
20030522	Iraq,sanction	Y	Y	A	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	A	A
Date	Subject	US	UK	FR	CN	RU	BG	CM	GN	MX	SY	CO	IE	MU	NO	SG
20021230	Iraq,humanitarian	Y	Y	Y	Y	A	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
20021213	Terrorist,Kenya	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	Y	Y
20020924	Palestine	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
20020330	Palestine	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
20020312	Palestine	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	CO	IE	MU	NO	SG	BD	JM	ML	TN	UA
20010928	Sudan	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	BD	JM	ML	TN	UA	AR	CA	MY	NA	NL
20001219	Afghanistan	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y
20001007	Palestine	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
20000705	Sierra Leone	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y
20000621	B&H	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
20000531	Western Sahara	Y	Y	Y	Y	Y	Y	A	A	Y	Y	Y	Y	Y	N	Y
20000217	Member-Tuvalu	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
20000131	Georgia	Y	Y	Y	A	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	AR	CA	MY	NA	NL	BH	BR	GA	GM	SL
19991217	UNMOVIC,Iraq	Y	Y	A	A	A	Y	Y	A	Y	Y	Y	Y	Y	Y	Y
19991214	Western Sahara	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19991203	Iraq,humanitarian	Y	Y	A	A	A	Y	Y	A	Y	Y	Y	Y	Y	Y	Y
19991130	Haiti	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19990625	Nauru	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19990610	Kosovo	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19990514	Kosovo	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	BH	BR	GA	GM	SL	CR	JP	KE	PT	SE
19981125	Haiti	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19981117	Yugoslavia	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19981030	Western Sahara	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19981024	Kosovo	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19980923	Kosovo	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	CR	JP	KE	PT	SE	CL	EG	GW	PL	KR
19971023	Iraq,refuse	Y	Y	A	A	A	Y	Y	A	Y	Y	Y	A	Y	Y	Y
19970912	Iraq,import petro	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19970619	Albania	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19970328	Albania	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	CL	EG	GW	PL	KR	BW	DE	HN	ID	IT
19961022	Georgia	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19961001	B&H	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19960928	Palestine	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19960816	Ethiopia-Sudan	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19960726	Cuba-USA	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

19960530	UNPREDEP	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19960426	Ethiopia-Sudan	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	BW	DE	HN	ID	IT	AR	CS	NG	OM	RW
19951122	Yugoslavia	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19951122		Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19950705	Yugoslavia	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19950616	UN PROFOR	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19950421	Yugoslavia	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19950130	Haiti	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	y
19950112	YU&BH	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	AR	CS	NG	OM	RW	ES	PK	NZ	DJ	BR
19941129	Haiti	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	A
19941108	Rwanda	Y	Y	Y	A	Y	Y	Y	Y	Y	N	Y	Y	Y	Y	Y
19941015	Haiti	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A
19940929	Haiti	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19940929	Haiti	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	A
19940923	B&H	Y	Y	Y	Y	Y	Y	Y	A	Y	A	Y	N	Y	N	Y
19940923	B&H	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19940731	Haiti	Y	Y	Y	A	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	A
19940729	Cyprus	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
19940728	UNIFIL,Lebanon	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
19940721	Georgia	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
19940622	Rwanda	Y	Y	Y	A	Y	Y	Y	A	Y	Y	Y	A	A	Y	A
Date	Subject	US	UK	FR	CN	RU	ES	PK	NZ	DJ	BR	CV	HU	JP	MA	VE
19931111	Libya	Y	Y	Y	A	Y	Y	A	Y	A	Y	Y	Y	Y	A	Y
19930809	Yugoslavia	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19930604	UNPROFOR	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	A
19930527	Cyprus	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y
19930511	KP,withdraw	Y	Y	Y	A	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y
19930428	Yugoslavia	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19930418	B&H,peace plan	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19930331	B&H,airspace	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	CV	HU	JP	MA	VE	AT	BE	EC	IN	ZW
19921218	Palestine	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19921130	Cambodia	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19921116	B&H	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A
19921009	B&H	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19921002	Iraq,sale petro	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19920919	Member-Yugoslavia	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y
19920914	UNPROFPOR	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y
19920826	Iraq&Kuwait	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y
19920813	B&H	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y
19920530	Yugoslavia	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19920331	Libya	Y	Y	Y	A	Y	A	Y	Y	A	Y	Y	Y	Y	A	Y

Specific Composition of UN SC Votes since 1992 Preliminarily observed the data table (of the phase 1992-2013), there are some interesting (special) points which could be initially discussed about:

- For the non-permanent members, the frequencies to be elected as SC non-permanent members for different UN members (except the five permanent members) are quite unequal, some countries are the “frequent visitors”. For example, Germany, Brazil, Argentina and Japan showed up a lot, Germany has been non-permanent members for 3 times (6 years), and all the latter three countries have been the non-permanent members for 4 times (8 years) during the phase 1992-2013 (22 years), which is a very high proportion. Compared with these UN security council “frequent visitors”, there are many small countries hardly to be elected, even for now there are still more than 60 countries² (of 192 UN members) never becoming the non-permanent members yet, such as Afghanistan, Haiti, Cambodia and so on, which mainly are very small or poor countries. In fact, the international politics run similarly with the domestic politics (like the money politics, in which the powerful and rich people have much more opportunity to be elected for political positions than powerless and poor guys),

²<http://www.un.org/en/sc/members/>

and in international politics, the big and rich countries are much more easier to be elected. For above example, Germany, Japan, Brazil and Argentina all belongs to the powerful countries in the remaining countries except the five permanent members, they are not only very motivated and progressive to become the non-permanent member, but also have enough resources to convince other member states to achieve the objective. By becoming the UN SC non-permanent members and participating the international affairs, these countries can further improve the state interests, influence powers and international reputations. Actually, the G4 nations³ (including Germany, Japan, Brazil and India) not only try to be elected as non-permanent, but also strive to push the reform of SC, to achieve their ambitious goal, eventually making themselves become the permanent members (like US, China, etc).

Table D.3: Specific Composition of UN SC Vote from 1966 to 1991

Date	Subject	US	UK	FR	CN	RU	AT	BE	EC	IN	ZW	KT	CU	RO	YE	ZR
19910919	Iraq,account	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	Y	A	Y
19910815	Iraq,import petro	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	Y	A	Y
19910520	Compensation	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y
19910405	Iraq,repression	Y	Y	Y	A	Y	Y	Y	Y	A	Y	Y	N	Y	N	Y
19910403	Iraq&Kuwait	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	N	Y	A	Y
19910302	Gulf Region	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	KT	CU	RO	YE	ZR	CA	CO	ET	FI	MY
19901222	Trusteeship	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y	Y	Y
19901214	Cyprus	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19901029	Iraq&Kuwait	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y
19900925	Iraq,Air Embargo	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y	Y	Y
19900913	Iraq&Kuwait	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y	Y	Y
19900825	Shipping	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y
19900806	Iraq,Sanction	Y	Y	Y	Y	Y	Y	A	Y	A	Y	Y	Y	Y	Y	Y
19900802	Iraq&Kuwait	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	CA	CO	ET	FI	MY	DZ	BR	NP	SN	VE
19890830	Palestine	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19890706	Palestine	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	DZ	BR	NP	SN	VE	AR	DE	IT	JP	ZW
19881123	South Africa	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y
19880425	Israel&Tunisia	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19880316	South Africa	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19880114	Palestine	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	AR	DE	IT	JP	ZW	BG	CG	GH	AE	VE
19871222	Palestine	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19871030	South Africa	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	BG	CG	GH	AE	VE	AU	DK	MG	TH	TT
19861208	Palestine	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
19860923	UNIF,Lebanon	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19860213	South Africa	Y	A	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	AU	DK	MG	TH	TT	UA	PE	IN	EG	BF
19851017	UNIF,Lebanon	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	A	Y	Y	Y	Y
19851004	Israel&Tunis	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19850726	South Africa	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19850619	South Africa	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19850417	UNIF,Lebanon	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	A	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	UA	PE	IN	EG	BF	MT	NL	NI	PK	ZW
19841023	South Africa	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19841012	UNIF,Lebanon	Y	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y
19840817	South Africa	A	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

³The G4 nations are four countries which support each other's bids for permanent seats on the United Nations Security Council. Unlike the G8, where the common denominator is the economy and long-term political motives, the G4's primary aim is the permanent member seats on the Security Council. Each of these four countries have figured among the elected non-permanent members of the council since the UN's establishment. Their economic and political influence has grown significantly in the last decades, reaching a scope comparable to the permanent members (P5). However, the G4's bids are often opposed by certain countries, particularly their economic competitors or political rivals.

19840601	Gulf region	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	A
19840511	Cyprus	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	Y
19840419	UNIF,Lebanon	Y	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y
19840106	South Africa	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	MT	NL	NI	PK	ZW	GY	JO	PL	TG	CG
19831220	South Africa	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19831118	Cyprus	Y	Y	Y	Y	Y	Y	Y	Y	N	Y	Y	A	Y	Y	Y
19831031	Iran and Iraq	Y	Y	Y	Y	Y	A	Y	A	A	Y	Y	Y	Y	Y	Y
19831028	South Africa	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19831018	UNIF,Lebanon	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	A	Y	Y
19830718	UNIF,Lebanon	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	A	Y	Y
19830118	UNIF,Lebanon	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	A	Y	Y
Date	Subject	US	UK	FR	CN	RU	GY	JO	PL	TG	CG	IE	JP	PA	ES	UG
19821018	UNIF,Lebanon	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y	Y	Y	Y	Y
19820817	UNIF,Lebanon	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y	Y	Y	Y	Y
19820804	Israel&Lebanon	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19820729	Israel&Lebanon	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19820618	UNIF,Lebanon	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y	Y	Y	Y	Y
19820403	Falkland Islands	Y	Y	Y	A	A	Y	Y	A	Y	Y	Y	Y	N	A	Y
19820225	UNIF,Lebanon	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y	Y	Y	Y	Y
19820128	Israel,occupied	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	IE	JP	PA	ES	UG	DDR	MX	NE	PH	TN
19811218	UNDOF	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	A	Y	Y	Y	Y
19811123	UNDOF	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19810619	UNIF,Lebanon	Y	Y	Y	A	A	Y	Y	Y	Y	Y	A	Y	Y	Y	Y
19810604	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19810522	UNDOF	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	DDR	MX	NE	PH	TN	BD	JM	NO	PT	ZM
19801217	UNIF,Lebanon	Y	Y	Y	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y
19801211	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19801126	UNDOF	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19800820	Palestine	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19800630	Palestine	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19800627	South Africa	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19800617	UNIF,Lebanon	Y	Y	Y	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y
19800613	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19800605	Assassination	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19800530	UNDOF	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19800520	Palestine	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19800508	Palestine	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19800202	Southern Rhodesia	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19800109	Afghanistan	Y	Y	Y	Y	N	N	Y	Y	Y	Y	Y	Y	Y	Y	A
Date	Subject	US	UK	FR	CN	RU	BD	JM	NO	PT	ZM	BD	CS	GA	KW	NG
19791231	US&Iran	Y	Y	Y	Y	A	A	Y	Y	Y	Y	Y	A	Y	A	Y
19791221	Southern Rhodesia	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	A	Y	Y	Y
19791219	UNIF,Lebanon	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	A	Y	Y	Y
19791214	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19791130	UNDOF	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19791102	South Africa	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19790720	Israel,occupied	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19790615	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19790614	UNIF,Lebanon	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	A	Y	Y	Y
19790430	UNDOF	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19790328	Southern Rhodesia	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19790328	South Africa	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19790322	Israel&Arab	A	A	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y
19790308	Southern Rhodesia	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19790119	UNIF,Lebanon	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	A	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	BD	CS	GA	KW	NG	MU	IN	DE	CA	VE
19781214	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19781130	UNDOF	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19781113	Elections,Namibia	A	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y
19781023	UNEF	Y	Y	Y	A	A	Y	A	Y	Y	Y	Y	Y	Y	Y	Y
19781010	Southern Rhodesia	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	A	Y
19780929	Group UN,Namibia	Y	Y	Y	A	A	Y	A	Y	Y	Y	Y	Y	Y	Y	Y
19780918	UNIF,Lebanon	Y	Y	Y	A	A	Y	A	Y	Y	Y	Y	Y	Y	Y	Y
19780727	Representative,NM	Y	Y	Y	Y	A	Y	A	Y	Y	Y	Y	Y	Y	Y	Y
19780616	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19780531	UNDOF	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

19780503	UNIF,Lebanon	Y	Y	Y	A	A	Y	A	Y	Y	Y	Y	Y	Y	Y	Y
19780319	UNIF,Lebanon	Y	Y	Y	A	A	Y	A	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	MU	IN	DE	CA	VE	RO	PA	PK	LY	BJ
19771215	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19771130	UNDOF	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	A
19771021	UNEF	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y
19770929	Southern Rhodesia	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19770615	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19770526	UNDOF	Y	Y	Y	A	Y	Y	Y	Y	A	A	Y	Y	Y	Y	Y
19770114	Southern Rhodesia	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	RO	PA	PK	LY	BJ	GY	IT	JP	SE	TZ
19761207	Secretary-General	Y	Y	Y	A	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
19761130	UNDOF	Y	Y	Y	A	Y	Y	Y	Y	A	A	Y	Y	Y	Y	Y
19761122	Member-Angola	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19761022	UNEF	Y	Y	Y	A	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19760730	South Africa&ZM	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19760615	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
19760528	UNDOF	Y	Y	Y	A	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19760422	East Timor	A	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	A	Y	Y
19760331	South Africa&AO	Y	A	A	A	Y	Y	Y	Y	Y	Y	Y	A	A	Y	Y
Date	Subject	US	UK	FR	CN	RU	GY	IT	JP	SE	TZ	CM	MR	IQ	CR	BY
19751213	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19751130	UNDOF	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y
19751023	UNEF	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y
19751017	Member-Comoros	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19750724	UNEF	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y
19750613	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19750528	UNDOF	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y
19750417	UNEF	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y
Date	Subject	US	UK	FR	CN	RU	CM	MR	IQ	CR	BY	AU	AT	ID	KE	PE
19741213	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19741129	UNDOF	Y	Y	Y	A	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y
19741023	UNEF	Y	Y	Y	A	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y
19740816	Cyprus	Y	Y	Y	A	A	Y	Y	A	Y	A	Y	Y	Y	Y	Y
19740815	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19740801	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19740531	IL&SY,UNDOF	Y	Y	Y	A	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y
19740529	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19740528	Iran&Iraq	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19740424	Israel&Lebanon	Y	Y	Y	A	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y
19740408	UNEF	Y	Y	Y	A	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	AU	AT	ID	KE	PE	YU	SD	PA	IN	GN
19731215	Middle East	A	A	A	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19731214	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19731027	UNEF	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19731025	Middle East	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19731023	Israel&Egypt	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19731022	Middle East	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19730615	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19730522	Southern Rhodesia	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19730421	Israel&Lebanon	A	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	A
19730321	Latin America	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19730310	Southern Rhodesia	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19730202	Southern Rhodesia	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19730202	Southern Rhodesia	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	YU	SD	PA	IN	GN	AR	BE	IT	JP	SO
19721212	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19721206	"Homeland"	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19721023	Portugal&Senegal	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y
19720929	Southern Rhodesia	A	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y
19720801	Namibia	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19720728	Southern Rhodesia	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19720721	Israel&SY,LB	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19720626	Israel&Lebanon	A	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y
19720615	Cyprus	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19720228	Southern Rhodesia	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19720204	Portugal	A	A	A	Y	Y	Y	Y	Y	Y	Y	A	A	A	Y	Y
19720204	South Africa	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19720204	South Africa&NA	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

19720204	Group SC,Namibia	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	AR	BE	IT	JP	SO	SY	SL	PL	NI	BI
19711221	India&Pakistan	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	A	Y	Y
19711206	India&Pakistan	Y	A	A	Y	A	Y	Y	Y	Y	Y	Y	Y	A	Y	Y
19711124	Portugal&Senegal	A	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19711020	South Africa	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19710925	Palestine	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y
19710715	Portugal&Senegal	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	SY	SL	PL	NI	BI	CO	FI	NP	ES	ZM
19701208	Portugal&Guinea	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19700905	Israel&Lebanon	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19700729	South Africa	Y	A	Y	Y	A	Y	Y	A	Y	Y	Y	Y	Y	Y	Y
19700729	South Africa	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19700723	South Africa	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19700519	Israel&Lebanon	A	Y	Y	Y	Y	Y	A	Y	A	Y	A	Y	Y	Y	Y
19700318	Southern Rhodesia	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y
19700130	Subcommittee,NM	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	CO	FI	NP	ES	ZM	SN	PY	PK	HU	DZ
19691222	Portugal&Guinea	A	A	A	A	Y	A	Y	Y	A	Y	Y	Y	Y	Y	Y
19691219	Portugal&Senegal	A	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19690915	Palestine	A	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	A	Y	Y	Y
19690728	Portugal&Zambia	A	A	A	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19690401	Israel&Jordan	A	A	Y	Y	Y	A	Y	Y	Y	Y	Y	A	Y	Y	Y
19690320	South Africa	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	SN	PY	PK	HU	DZ	BR	CA	DK	ET	IN
19680927	Israel&Arab	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	A	Y	Y
19680918	Middle East	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
19680619	Nuclear	Y	Y	A	Y	Y	Y	Y	A	Y	A	A	Y	Y	Y	A
19680521	Palestine	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	NG	ML	JP	BG	AR	UG	NZ	JO	NL	UY
19661216	Southern Rhodesia	Y	Y	A	Y	A	Y	A	Y	A	Y	Y	Y	Y	Y	Y
19661125	Israel&Jordan	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y
19660409	Southern Rhodesia	Y	Y	A	Y	A	Y	A	Y	A	Y	A	Y	Y	Y	Y

Specific Composition of UN SC Votes from 1966 to 1991 Preliminarily observed the data table (of the phase 1966-1991), there are some interesting (special) points which could be initially discussed about:

- Due to the principle of equality of United Nation, usually (there is) one vote for one member state (no matter it is a super power or a tiny country), but there are some special cases, for one state, but with more than one votes. The most prominent example is the former USSR with three votes, including the votes of representatives of Soviet Union, Belarus and Ukraine, the latter two are both the union republics of Soviet Union. It is hard to image the non-independent constituent republic can also have a vote, but why the states in the United States of American do not have votes? Actually, it is due to the historical background when the UN established, the members from the socialist camp are much less than the capitalist camp, the Soviet Union felt threatened and unbalanced, in order to better balance between two camps and placate the Soviet Union, the US promise to let Soviet Union has three votes (and considering the Ukraine and Belarus sacrificed huge in the Word War II). What is mentionable is that the Soviet Union preliminarily hope all the union republics join the UN and all has one vote, of course this ambitious plan would never be agreed by the US, and the one state three votes arrangement about Soviet Union is actually the compromised outcome between two superpowers.

Besides, the Soviet Union is not the only example for one state with more than one votes, while in 1945 (the UN just established), the UN member India still belongs to UK (independent since 1947), and Philippines still belongs to US (independent since 1946), before the independence of these states, their votes are controlled by the suzerain, therefore, which also belongs to the special case of one state with more than one votes. And if we consider about

the influence among member states, this kind of influence from suzerain state to colony or union republic is a very strong influence, or absolute influence.

- During the phase 1966-1991, a very important time point (or historical event) is 1971, which is the watershed of the seat of China in United Nations and permanent seat of China in Security Council taken between the “Republic of China” (Taiwan) and the People’s Republic of China. Since 1971, the People’s Republic of China officially take the SC permanent seat from the “Republic of China” (Taiwan). Compared the “Republic of China” and the People’s Republic of China, the former usually follows United States, and the latter is an independent great power, the state interests and strategic objectives, the preference and behavior patterns between these two are totally different. For example, the People’s Republic of China can vote absent or negative individually, even against the United States and Soviet Union, while for the “Republic of China”, usually follows closely with the US, and could only vote as the minority together with US. Therefore, when dealing with the data to test the patterns of influence, it is very important to distinguish the vote of China before 1971 and after 1971.
- The People’s Republic of China voted absent a lot (at least compared with other permanent members or great powers) during this phase, which could be vividly named as the “Mr. Absent” (associated with the “Mr. Veto” named for the representative of the former Soviet Union by western bloc), for many subjects of bills, such as the conflicts in East Middle, Cyprus and so on, China has the different claims with other states, especially with the western bloc (usually China claims to protect the interest of small countries, and firmly advocate the principle of non-interference in the internal affairs of other countries), China hope to declare its position, but is not willing to vote negative against other states, because as the permanent member of SC, the one negative vote of China means veto, totally vetoing the bills, which of course would bring too much cost and pressure for China. Therefore, the best choice for China while facing an disagreed bill, is to be absent (of the meeting) or not vote (at the meeting). Compared with the “Mr. Veto” of Soviet Union, who had the super power accompanied to boldly express its attitude, or to directly veto the majority of other states. While for China in this phase, though has pursued the independent foreign diplomacy, considering the comprehensive strength and the international situation⁴, China has to vote absent, as the most wise choice.
- Latin American countries usually want to be different, there are a lot of different voices or opposing voices coming from Cuba, Venezuela, Argentina, Columbia and some other Latin American countries. The most prominent example is Cuba, Cuba used to be elected as the non-permanent member for 3 times (1949-1950, 1956-1957, 1990-1991), during the term 1990-1991, Cuba participated 14 resolutions (with at least one different voices), in which Cuba voted 7 negative, 4 absent, and just 3 affirmative, which is a very high proportion for the different voices (negative and absent), particularly, for all bills about Iraq issue (the sanction and war against Iraq promoted and leaded by US), Cuba never voted affirmative, and always voted against US. Actually, Cuba and the leader of Cuba Fidel Castro are famous for the image of fighter (anti-US), never concealing the anti-attitude against US, and giving criticism on the system and procedure of United Nations (Security Council). Besides Cuba, the anti-American sentiment⁵ is also popular in some other Latin American countries, such as Argentina⁶, Venezuela (famous for the former president Hugo Chavez) and so on.

⁴Particularly, the relations between China and the former Soviet Union had worsened at this phase.

⁵In fact, the anti-American sentiment might not always conform to the state interest, might be emotional catharsis.

⁶Relations have been tense since the 1940s, when the U.S. feared Argentina was too close to Nazi Germany. Polls compiled by the Chicago Council on Global Affairs showed in 2006 Argentine public opinion was quite negative regarding America’s role in the world.

Table D.4: Specific Composition of UN SC Vote from 1946 to 1965

Date	Subject	US	UK	FR	CN	RU	JO	NL	UY	MY	BO	KT
19651123	Portuguese	A	A	A	Y	Y	Y	A	Y	Y	Y	Y
19651120	Southern Rhodesia	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y
19651112	Southern Rhodesia	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y
19651105	India&Pakistan	Y	A	Y	Y	Y	A	Y	Y	Y	Y	Y
19650920	India&Pakistan	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
19650506	Southern Rhodesia	A	A	A	Y	A	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	BO	KT	CS	BR	MA	NO
19640809	Cyprus	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y
19640618	South Africa	Y	Y	A	Y	A	Y	Y	A	Y	Y	Y
19640609	South Africa	A	A	A	Y	Y	Y	Y	Y	A	Y	Y
19640409	Yemen&UK	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	BR	MA	NO	PH	VE	GH
19631211	Portuguese	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y
19630807	South Africa	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y
19630731	Portuguese	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y
19630611	Reports,Yemen	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	VE	GH	RO	IE	CL	UAR
19621004	Member-DZ	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y
19620409	Israel&Syria	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	CL	UAR	TR	LR	EC	LK
19611124	Congo	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y
19611025	Member-MR	Y	Y	Y	Y	A	Y	N	Y	Y	Y	Y
19611025	Member-MN	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19610722	Tunisia	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19610609	Subcommittee,AO	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y
19610411	Israel&Jordan	Y	Y	Y	Y	A	Y	A	Y	Y	Y	A
19610221	Congo	Y	Y	A	Y	A	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	EC	LK	PL	AR	TN	IT
19600917	Congo	Y	Y	A	Y	N	Y	Y	N	Y	Y	Y
19600909	Dominican	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y
19600809	Congo	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	A
19600719	Cuba&US	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y
19600623	Adolf Eichmann	Y	Y	Y	Y	A	Y	Y	A	NV	Y	Y
19600527	Relations-Powers	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y
19600401	South Africa	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	AR	TN	IT	PA	CA	JP
19590907	Subcommittee,LA	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	PA	CA	JP	CO	IQ	SE
19581209	Member-GN	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y
19580611	Lebanon&Egypt	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	CO	IQ	SE	PH	CU	AU
19571202	India&Pakistan	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19570221	India&Pakistan	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19570124	India&Pakistan	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	CU	AU	YU	PE	IR	BE
19561104	Hungary	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19561031	Suez Canal,EG	Y	N	N	Y	N	Y	A	Y	Y	Y	A
Date	Subject	US	UK	FR	CN	RU	PE	IR	BE	BR	NZ	TR
19551216	UN Charter	Y	Y	A	Y	N	Y	Y	Y	Y	Y	Y
19551214	Member-AL,JO,etc	A	Y	Y	A	Y	Y	Y	A	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	CO	LB	DK	CL	PK	GR
19531203	ICJ-San Marino	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19531203	ICJ-Japan	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19531124	Israel&Jordan	Y	Y	Y	Y	A	Y	A	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	CL	PK	GR	BR	TR	NL
19521223	India&Pakistan	Y	Y	Y	Y	A	Y	A	Y	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	BR	TR	NL	EC	IN	YU
19511110	India&Pakistan	Y	Y	Y	Y	A	Y	Y	Y	Y	A	Y
19510901	Suez Canal,EG	Y	Y	Y	A	A	Y	Y	Y	Y	A	Y
19510518	Israel&Syria	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19510508	Israel&Syria	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19510330	India&Pakistan	Y	Y	Y	Y	A	Y	Y	Y	Y	A	A
Date	Subject	US	UK	FR	CN	RU	EC	IN	YU	CU	EG	NO
19501117	Israel&JO,EG	Y	Y	Y	Y	A	Y	Y	Y	Y	A	Y
19501108	Korea War	Y	Y	Y	N	Y	Y	Y	Y	N	A	Y

19500929	Taiwan	N	Y	Y	N	Y	Y	Y	Y	N	A	Y
19500926	Member-ID	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y
19500731	Korea War	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y
19500707	Korea War	Y	Y	Y	Y	A	Y	A	A	Y	A	Y
19500627	Korea War	Y	Y	Y	Y	A	Y	A	N	Y	A	Y
19500625	Korea War	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y
19500524	SC Procedure	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19500314	India&Pakistan	Y	Y	Y	Y	A	Y	A	A	Y	Y	Y
19500117	Armaments	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	CU	EG	NO	AR	CA	UA
19491018	Armaments	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	A
19491011	Armaments	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	A
19491005	Indonesia	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	N
19490927	SC Procedure	Y	Y	Y	Y	A	A	A	Y	Y	Y	N
19490916	Nuclear	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	A
19490811	Palestine	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	A
19490727	ICT-Liechtenstein	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	A
19490307	Trusteeship	Y	Y	Y	Y	A	Y	A	Y	Y	Y	A
19490304	Member-IL	Y	A	Y	Y	Y	Y	N	Y	Y	Y	Y
19490210	Armaments	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	A
Date	Subject	US	UK	FR	CN	RU	AR	CA	UA	CO	SY	BE
19481229	Palestine	A	Y	Y	Y	A	Y	Y	A	Y	Y	Y
19481228	ID and NL	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y
19481228	ID and NL	Y	A	A	Y	Y	Y	Y	Y	Y	Y	A
19481224	ID and NL	Y	Y	A	Y	A	Y	Y	A	Y	Y	A
19481104	Palestine	Y	Y	Y	Y	A	Y	Y	N	Y	Y	Y
19480729	ID and NL	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y
19480715	Palestine	Y	Y	Y	Y	A	A	Y	A	Y	N	Y
19480707	Palestine	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y
19480622	Nuclear	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y
19480522	Palestine	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y
19480423	Palestine	Y	Y	Y	Y	A	Y	Y	A	A	Y	Y
19480417	Palestine	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y
19480410	Member-MM	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
19480401	Palestine	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y
19480305	Palestine	Y	A	Y	Y	Y	A	Y	Y	Y	A	Y
19480228	ID and NL	Y	Y	Y	Y	A	Y	Y	A	A	A	Y
19480228	Indonesia	Y	Y	Y	Y	A	A	Y	A	Y	Y	Y
19480120	India&Pakistan	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y
19480117	India&Pakistan	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y
Date	Subject	US	UK	FR	CN	RU	CO	SY	BE	BR	AU	PL
19471101	ID and NL	Y	Y	Y	Y	A	A	A	Y	Y	Y	N
19471003	ID and NL	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	A
19470915	Greece	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	N
19470827	SC Procedure	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y
19470826	ID and NL	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y
19470825	ID and NL	Y	Y	Y	Y	A	Y	A	Y	Y	Y	A
19470825	ID and NL	Y	A	Y	Y	A	A	Y	Y	Y	Y	A
19470806	Greece	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	A
19470522	Member-HU	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y
19470430	Member-HU	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	Y
19470418	Greece	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	A
19470409	Greece	Y	A	Y	Y	A	Y	Y	Y	Y	Y	A
19470227	Greece	Y	Y	Y	Y	A	Y	A	Y	Y	Y	A
19470213	Arms Reduction	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19470210	Greece	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19470110	Italy,Trieste	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y
Date	Subject	US	UK	FR	CN	RU	BR	AU	PL	MX	EG	NL
19461216	SC Procedure	A	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19460829	Member-AF,IS,SW	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y
19460508	USSR&Iran	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19460429	Spanish Question	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
19460404	USSR&Iran	Y	Y	Y	Y	N	Y	A	Y	Y	Y	Y

Specific Composition of UN SC Votes from 1946 to 1965 Preliminarily observed the data table (of the phase 1966-1993), there are some interesting (special) points which could be initially discussed about:

- After the SC reform in 1966 and the non-permanent member enlarged to 10, the term for non-permanent members are regular, 2 years for every elected non-permanent members, however, before 1966, the term for the non-permanent members are irregular, for part of them the term is 1 year, and for other part for them the term is 2 years. For example, the first group of non-permanent members are Brazil, Australia, Poland, Mexico, Egypt, Netherlands, in which the term for Brazil, Australia, Poland is 2 two years (1946-1947), while the term for Mexico, Egypt, Netherlands is just 1 year (1946). Other examples for the 1 year term (during this phase 1946-1966) include Yugoslavia (1956), Philippines (1957), Poland (1960), Turkey (1961), Liberia (1961), Romania (1962), Ireland (1962), Philippines (1963), Czechoslovakia (1964), Malaysia (1965), Uganda (1966), New Zealand (1966). In a word, compared with the regular election and term of non-permanent members after 1966, in which 5 new non-permanent members are elected every year, to replace 5 current non-permanent members whose 2 years term have already finished, namely half of 10 non-permanent members replaced every year. While before 1966, not only the terms for non-permanent members are not uniform, but also the number of new elected members, more quota in some years, and less quota in some other years.
- The countries from Eastern bloc (or Soviet bloc) like Soviet Union, Ukraine, Poland, Czechoslovakia and so on vote negative or absent much more frequently than countries from Western bloc (or Capitalist bloc). That's because the member states from the latter bloc constitute the majority of United Nations, the number of socialist countries is the absolute minority compared with capitalist countries, particularly for the permanent members of Security Council of United Nations (which is the core of the core for the power of UN), while in the early phase, the ratio between capitalist bloc and Soviet bloc is 4:1, except Soviet Union, the other four United States, United Kingdoms, France and China (before 1971) are all at the opposite side. As the Western bloc occupied the majority in UN assembly and UN SC, most of bills are proposed by states from Western bloc, and they have a much more discourse power than Eastern bloc. Therefore, in order to express own anti attitude and protect own state interest (under the adverse environment), the countries from Eastern bloc would naturally be inclined to vote negative or absent toward those "unfriendly" bills.
- During the phase 1946-1965, the frequency of resolutions with different voices in the early years is much higher than in the latter years. The total frequency of "conflicting" resolutions of early 5 years (1946-1950) is even higher than following 14 years (1951-1965). That's to say, in the first 5 years while just United Nations just established, the confrontation and competition among different member states, especially among different blocs are very intensive and fierce, which is matching to the common sense, at the initial phase of a new mechanism (about international order and power allocation), usually there would be more frictions and conflicts among contradictive states and alliances, each side would firstly emphasis and holdout on own interest, and try to push the envelope of the opposite side, only after a necessary interaction and co-adaptation among each other, each side would understand the other side's bottom line and also provide a foreseeable estimation about own bottom line for other side, then the conflict and confrontation among them would be relieved, and staying in a relatively low degree (compared with the initial phase). As we observe the voting data in early 5 years, a series of severely conflicting issues happened many times, such as the Greece Civil War⁷, Israeli-Palestinian conflict and Korea War, for these issues, United

⁷"The Greek Civil War was fought from 1946-49 between the Greek government army backed by Great Britain and the United States and the Democratic Army of Greece (DSE), the military branch of the Greek Communist Party (KKE), backed by Yugoslavia and Albania as well as the Soviet Union and Bulgaria. The result was defeat of the Communist insurgents by the government forces [Marantzidis and Antoniou, 2004]" [Wikipedia, 2015e].

States and Soviet Union (more broadly Western bloc and Eastern bloc) have totally contradictory claims and standpoints, for example, United States supported the Greek government army, while Soviet Union supported the Greek Communist Party, United States supported Israel, while Soviet Unions supported Arab countries, United States supported South Korea, while Soviet Union supported North Korea, the conflict and confrontation between them were very apparent and intensive.

D.3 Sample Data of Subject 1-8 for Testing

Table D.5: Sample Data of Subject 1-Admission of New Memberships

Date	Content	US	UK	FR	CN	RU	BR	AU	PL	MX	EG	NL					
19460829	Afghanistan,Iceland and Sweden	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y					
										CO	SY	BE					
19470430	Admission of new member, Hungary	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	Y					
19470522	Admission of new member, Hungary	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y					
								AR	CA	UA							
19480410	Admission of new member, Burma Myanmar	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y					
										CU	EG	NO					
19490304	Admission of new member, Israel	Y	A	Y	Y	Y	Y	Y	Y	Y	N	Y					
								EC	IN	YU							
19500926	Admission of new member, Indonesia	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y					
								PE	IR	BE	BR	NZ	TR				
19551214	Albania, Jordan, Portugal, Hungary,etc	A	Y	Y	A	Y	Y	Y	A	Y	Y	Y					
								PA	CA	JP	CO	IQ	SE				
19581209	Admission of new member, Guinea	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y					
								CL	UAR	TR	LR	EC	LK				
19611025	Admission of new member, Mongolia	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y					
19611025	Admission of new member, Mauritania	Y	Y	Y	Y	A	Y	N	Y	Y	Y	Y					
										GH	RO	IE					
19621004	Admission of new member, Algeria	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y					
								GY	IT	JP	SE	TZ	CM	MR	IQ	CR	BY
19751017	Admission of new member, Comoros	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
													RO	PA	PK	LY	BJ
19761122	Admission of new member, Angola	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
								CV	HU	JP	MA	VE	AT	BE	EC	IN	ZW
19920919	Admission of new member, Yugoslavia	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y
								BD	JM	ML	TN	UA	AR	CA	MY	NA	NL
20000217	Admission of new member, Tuvalu	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

Table D.6: Sample Data of Subject 2-the Iraqi Invasion of Kuwait and the Sanctions against Iraq

Date	Content	US	UK	FR	CN	RU	KT	CU	RO	YE	ZR	CA	CO	ET	FI	MY
19900802	on Iraqi invasion of Kuwait	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19900806	on the Sanctions against Iraq	Y	Y	Y	Y	Y	Y	A	Y	A	Y	Y	Y	Y	Y	Y

19900913	on the Situation regarding foodstuffs in Iraq and Kuwait	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y	Y	Y					
19900925	on Air embargo against Iraq	Y	Y	Y	Y	Y	Y	N	Y	Y	Y	Y	Y	Y	Y	Y					
19901029	Protection of third-State nationals in Iraq and Kuwait	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y					
												AT	BE	EC	IN	ZW					
19910302	the end of hostilities in the Gulf region	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y					
19910403	Restore sovereignty independence and territorial integrity of Kuwait	Y	Y	Y	Y	Y	Y	N	Y	A	Y	Y	Y	A	Y	Y					
19910405	on Repression of the Iraqi civilian population including Kurds	Y	Y	Y	A	Y	Y	N	Y	N	Y	Y	Y	Y	A	Y					
19910815	Permit the import of petroleum and products in Iraq	Y	Y	Y	Y	Y	Y	N	Y	A	Y	Y	Y	Y	Y	Y					
19910919	Release the escrow account for humanitarian assistance to Iraq	Y	Y	Y	Y	Y	Y	N	Y	A	Y	Y	Y	Y	Y	Y					
												CV	HU	JP	MA	VE					
19920826	on demarcation of the boundary between Iraq and Kuwait	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y					
19921002	on the proceeds of sales of Iraqi petroleum and petroleum products	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y					
												CR	JP	KE	PT	SE	CL	EG	GW	PL	KR
19970912	Extension of import temporarily petroleum and products in Iraq	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y					
19971023	Iraq's continued refusal to allow access by the UN Special Commission	Y	Y	A	A	A	Y	Y	A	Y	Y	Y	A	Y	Y	Y					
												AR	CA	MY	NA	NL	BH	BR	GA	GM	SL
19991203	Extension of measures for the humanitarian needs of the Iraqi people	Y	Y	A	A	A	Y	Y	A	Y	Y	Y	Y	Y	Y	Y					
19991217	on establishment of the UNMOVIC ⁸ , Iraq	Y	Y	A	A	A	Y	Y	A	Y	Y	Y	Y	Y	Y	Y					
												BG	CM	GN	MX	SY	CO	IE	MU	NO	SG
20021230	Adjust the Goods review List of the humanitarian programme	Y	Y	Y	Y	A	Y	Y	Y	Y	A	Y	Y	Y	Y	Y					
												AO	CL	PK	ES	DE					
20030522	Lifting economic sanctions on Iraq imposed by resolution	Y	Y	A	Y	Y	Y	Y	Y	A	A	Y	Y	Y	Y	A					
20030814	Establishment of the UN assistance mission for Iraq	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y					
												BE	ID	IT	PA	ZA	CG	GH	PE	QA	SK
20070629	on termination of the mandates of the UNMOVIC, Iraq	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y					
												BA	BR	GA	LB	NG	AT	JP	MX	TR	UG
20101215	Termination of all residual activities under the Oil-for-Food Program	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y					

Table D.7: Sample Data of Subject 3-Israeli and Palestinian Conflicts

Date	Subject	US	UK	FR	CN	RU	AR	CA	UA	CO	SY	BE
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⁸Which is the United Nations Monitoring Verification and Inspection Commission.

19480305	on Implement of General Assembly resolution II on Palestine	Y	A	Y	Y	Y	A	Y	Y	Y	A	Y										
19480401	Request a special session of the GA to discuss the future government	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y										
19480417	Calling all parties to take immediate measures for a truce in Palestine	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y										
19480423	Establishment of the UN Truce Commission for Palestine	Y	Y	Y	Y	A	Y	Y	A	A	Y	Y										
19480522	Call for a cease-fire and maintenance of the truce in Jerusalem	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y										
19480707	Appeal to all parties for prolongation of the truce in Palestine	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y										
19480715	Call all governments to cooperate with the Mediator to maintain peace	Y	Y	Y	Y	A	A	Y	A	Y	N	Y										
19481104	Call to withdraw forces beyond positions and establish truce lines	Y	Y	Y	Y	A	Y	Y	N	Y	Y	Y										
19481229	Call for an immediate cease-fire and the supervision of the truce	A	Y	Y	Y	A	Y	Y	A	Y	Y	Y										
													CU	EG	NO							
19490811	on the armistice agreements and the Palestine question	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y										
													SN	PY	PK	HU	DZ	BR	CA	DK	ET	IN
19680521	on the status of Jerusalem	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	A	Y	Y	Y	Y
																		CO	FI	NP	ES	ZM
19690915	on destruction and profanation of the Holy Places in Jerusalem	A	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y
													AR	BE	IT	JP	SO	SY	SL	PL	NI	BI
19710925	on the status of Jerusalem	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y
													DDR	MX	NE	PH	TN	BD	JM	NO	PT	ZM
19800508	on deportation of Palestinian leaders from territories occupied by Israel	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19800520	on deportation of Palestinian leaders from territories occupied by Israel	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19800630	on the status of Jerusalem	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19800820	on the status of Jerusalem	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
													BG	CG	GH	AE	VE	AU	DK	MG	TH	TT
19861208	on Israeli policy toward 3rds Palestinians in the occupied territories	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19871222	on protection of Palestinian civilians under Israel occupation	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19880114	on the deportation of Palestinian civilians by Israel	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19890706	on the deportation of Palestinian civilians by Israel	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

19890830	on the deportation of Palestinian civilians by Israel	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
CV HU JP MA VE AT BE EC IN ZW																
19921218	on the deportation of Palestinian civilians by Israel	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
CL EG GW PL KR BW DE HN ID IT																
19960928	Situation in Jerusalem, Nablus, Ramallah, Bethlehem and Gaza	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
BD JM ML TN UA AR CA MY NA NL																
20001007	on events throughout the territories occupied by Israel	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
BG CM GN MX SY CO IE MU NO SG																
20020312	Political settlement in the middle East including the Palestine	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
20020330	Cessation of all acts of violence in the territories occupied by Israel	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
20020924	Cessation of all acts of violence and withdrawal of Israeli forces	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
DZ BJ BR PH RO AO CL PK ES DE																
20040519	Humanitarian situation of Palestinians and Israeli obligation	A	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y
BF CR HR LY VN BE ID IT PA ZA																
20081216	Implementation of the Road Map ⁹ to a Permanent Two-State Solution	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y
AT JP MX TR UG																
20090108	a fully respected cease-fire and full withdrawal of Israel from Gaza	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	Y

Table D.8: Sample Data of Subject 4-Yugoslav Wars

Date	Content	US	UK	FR	CN	RU	CV	HU	JP	MA	VE	AT	BE	EC	IN	ZW
19920530	Sanctions against Yugoslavia	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19920813	Humanitarian assistance to B&H	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y
19920914	Enlargement the mandate of UNPROFOPOR	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y
19920919	United Nations Membership of Yugoslavia	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y
19921009	Ban on military flights in the airspace of B&H	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19921116	Interference from outside B&H cease immediately	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A
ES PK NZ DJ BR																
19930331	Extension of the ban on military flights of B&H	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19930418	Peace plan for B&H and measures in Former YU	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19930428	Non-participation of YU in ECOSOC ¹⁰	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19930604	Extending the mandate of UNPROFOR	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	A	Y	Y	Y
19930809	Refusal of YU to allow the CSCE missions	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

⁹Which is the Quartet Performance-based Road Map.

¹⁰Which is the United Nations Economic and Social Council.

		AR	CS	NG	OM	RW															
19940923	Violation of international humanitarian law	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				
19940923	Reinforcement of measures imposed by SC in B&H	Y	Y	Y	Y	Y	Y	Y	A	Y	A	Y	N	Y	N	Y					
												BW	DE	HN	ID	IT					
19950112	Closure of the international border YU&BH	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				
19950421	Extension of the partial suspension of sanctions against YU	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				
19950616	Establish rapid-reaction force, UNPROFOR	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				
19950705	Extension of the partial suspension of sanctions against YU	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				
19951122	Suspension of measures in the situation in Former YU	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				
												CL	EG	GW	PL	KR					
19960530	Extension the mandate of UNPREDEP, Macedonia	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				
19961001	Termination of measures in the situation in Former YU	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				
												BH	BR	GA	GM	SL	CR	JP	KE	PT	SE
19980923	Situation in Kosovo Yugoslavia	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				
19981024	Verification of compliance with the resolution on Kosovo Yugoslavia	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				
19981117	Failure of Yugoslavia to execute arrest warrants	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				
												AR	CA	MY	NA	NL					
19990514	Assistance to Kosovo refugees and internally displaced persons	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				
19990610	Deployment of international civil and security presences in Kosovo	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				
												BD	JM	ML	TN	UA					
20000621	Continuation of the multinational stabilization forces in B&H	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				
												BE	ID	IT	PA	ZA	CG	GH	PE	QA	SK
20070914	Extension of the appointment of the Prosecutor of the ICTY	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				
												AZ	GT	MA	PK	TG	CO	DE	IN	PT	ZA
20121212	Extension of the terms of office of judges at the ICTY	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y				

Table D.9: Sample Data of Subject 5-the Conflicts between India and Pakistan

Date	Content	US	UK	FR	CN	RU	AR	CA	UA	CO	SY	BE					
19480117	Request IN&PK to improve Kashmir Situation	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y					
19480120	Establishing a Commission on IN&PK question	Y	Y	Y	Y	A	Y	Y	A	Y	Y	Y					
												EC	IN	YU	CU	EG	NO
19500314	Appoint a representative and end commission	Y	Y	Y	Y	A	Y	A	A	Y	Y	Y					
												BR	TR	NL			
19510330	Appoint a new UN representative	Y	Y	Y	Y	A	Y	A	A	Y	Y	Y					

19511110	Plan for Demilitarization of Kashmir	Y	Y	Y	Y	A	Y	A	Y	Y	Y	Y				
		CL PK GR														
19521223	Negotiations for Demilitarization of Kashmir	Y	Y	Y	Y	A	Y	A	Y	Y	Y	Y				
		CO IQ SE PH CU AU														
19570124	Final disposition of Jammu and Kashmir	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y				
19570221	Negotiations on the IN&PK question	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y				
19571202	Negotiations on the IN&PK question	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y				
		JO NL UY MY BO KT														
19650920	Immediate cease-fire and Withdrawal of troop	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y				
19651105	Calling IN&PK to schedule troop withdrawal	Y	A	Y	Y	Y	A	Y	Y	Y	Y	Y				
		AR BE IT JP SO SY SL PL NI BI														
19711206	Refer the question to General Assembly	Y	A	A	Y	A	Y	Y	Y	Y	Y	Y	Y	A	Y	Y
19711221	Demand a durable cease-fire be observed	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	A	Y	Y

Table D.10: Sample Data of Subject 6-the Decolonization of Territories and Military Operations of Portugal

Date	Content	US	UK	FR	CN	RU	BR	MA	NO	PH	VE	GH			
19630731	Territories under Portuguese Administration	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y			
19631211	Territories under Portuguese Administration	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y			
		JO NL UY MY BO KT													
19651123	Territories under Portuguese Administration	A	A	A	Y	Y	Y	A	Y	Y	Y	Y			
		CO FI NP ES ZM SN PY PK HU DZ													
19690728	Portugal military operations against Zambia	A	A	A	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
19691219	Portugal military operations against Senegal	A	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
19691222	Portugal military operations against Guinea	A	A	A	A	Y	A	Y	Y	A	Y	Y	Y	Y	Y
		SY SL PL NI BI													
19701208	Portugal military operations against Guinea	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
		AR BE IT JP SO													
19710715	Portugal military operations against Senegal	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19711124	Portugal military operations against Senegal	A	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y
		YU SD PA IN GN													
19720204	Territories under Portuguese Administration	A	A	A	Y	Y	A	A	A	Y	Y	Y	Y	Y	Y
19721023	Portugal military operations against Senegal	A	A	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y

Table D.11: Sample Data of Subject 7-the Apartheid Policy and the Invasion by South Africa

Date	Content	US	UK	FR	CN	RU	EC	LK	PL	AR	TN	IT
19600401	Killings of unarmed and peaceful demonstrators	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y
		BR MA NO PH VE GH										
19630807	Cease the sale and shipment of arms to ZA ¹¹	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y
		BO KT CS										

¹¹Which is the abbreviation of South Africa.

19640609	End the trial against leaders of anti-apartheid	A	A	A	Y	Y	A	Y	Y	Y	Y	Y						
19640618	Persons sentenced for opposition to apartheid	Y	Y	A	Y	A	Y	Y	Y	Y	Y	Y	A					
							CO	FI	NP	ES	ZM	SN	PY	PK	HU	DZ		
19690320	South African administration of Namibia	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
													SY	SL	PL	NI	BI	
19700723	Arms embargo against South Africa	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
19700729	Refrain from any relations with South Africa	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
19700729	Requesting an advisory opinion of the ICT	Y	A	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	A	Y	Y		
							AR	BE	IT	JP	SO							
19711020	Policies of South Africa toward 3rds Namibia	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
													YU	SD	PA	IN	GN	
19720204	South Africa labour policies in Namibia	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
19720204	Policies of apartheid in South Africa	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
							RO	PA	PK	LY	BJ	GY	IT	JP	SE	TZ		
19760331	South Africa's military activities against Angola	Y	A	A	A	Y	Y	Y	Y	Y	Y	Y	A	A	Y	Y		
19760730	South Africa's military activities against Zambia	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
							BD	JM	NO	PT	ZM	BO	CS	GA	KW	NG		
19790328	Policies toward 3rds Angola, front-line states	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
19791102	Policies toward 3rds Angola, front-line states	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
													DDR	MX	NE	PH	TN	
19800627	Policies toward 3rds Angola	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
							MT	NL	NI	PK	ZW	GY	JO	PL	TG	CG		
19831028	Rejecting ZA's insistence about Namibia	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
19831220	South Africa's occupation of Angolan territory	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
													UA	PE	IN	EG	BF	
19840106	South Africa's military attacks on Angola	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
19840817	"New Constitution" of South Africa	A	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
19841023	Demanding immediate eradication of apartheid	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
							AU	DK	MG	TH	TT							
19850619	Install of a interim government in Windhoek	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
19850726	Sanctions against South Africa	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
													BG	CG	GH	AE	VE	
19860213	Threats against States in Southern Africa	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	
							AR	DE	IT	JP	ZW							
19871030	Authorize the SG to arrange a cease-fire	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
													DZ	BR	NP	SN	VE	
19880316	Death sentences imposed on South Africans	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		
19881123	Death sentences imposed on anti-apartheid	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y	Y		

Table D.12: Sample Data of Subject 8-the Minority Regime and the Invasion by Southern Rhodesia

Date	Content	US	UK	FR	CN	RU	JO	NL	UY	MY	BO	KT										
19650506	Request UK to prevent a unilateral independence	A	A	A	Y	A	Y	Y	Y	Y	Y	Y										
19651112	Calling not to recognize the minority regime	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y										
19651120	Independence by the minority regime	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y										
													NG	ML	JP	BG	AR	UG	NZ			
19660409	Oil embargo against Southern Rhodesia	Y	Y	A	Y	A	Y	Y	Y	Y	A	Y	A	Y	A	Y	A	Y				
19661216	Sanctions on commodities to SR ¹²	Y	Y	A	Y	A	Y	Y	Y	Y	A	Y	A	Y	Y	Y	Y	Y				
													SY	SL	PL	NI	BI	CO	FI	NP	ES	ZM
19700318	Expansion of sanctions against the regime	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	A	Y		
													YU	SD	PA	IN	GN	AR	BE	IT	JP	SO
19720228	Urging all states to implement sanction	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19720728	Calling to end economic and other relations	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19720929	Refusal of ZA ¹³ and PT to sanction	A	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
													AU	AT	ID	KE	PE					
19730202	Provocation by SR against Zambia	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19730202	Zambia's decision to sanction against SR	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19730310	Urge UK to convene a constitutional conference	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19730522	Penalties for the evasion or breach of sanctions	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
													MU	IN	DE	CA	VE	RO	PA	PK	LY	BJ
19770114	SR's provocation against Botswana	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19770929	Appointment a representative of the SG	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
													BO	CS	GA	KW	NG					
19781010	Observation of sanctions against SR	A	A	Y	Y	Y	Y	Y	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
													BD	JM	NO	PT	ZM					
19790308	Condemn SR's armed invasions of AO, MZ and ZM	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19790328	Reaffirm the elections to be null and void	A	A	A	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
19791221	Termination of sanctions against SR	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	A	Y	Y	Y	Y	Y	Y	Y	Y
													DDR	MX	NE	PH	TN					
19800202	Call UK to create conditions in SR for free and fair elections	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

D.4 Abbreviations for Non-permanent Members

After 1966 10 non-permanent members, and are elected regularly, two years term for every member.

2013-2014: “KR” for REPUBLIC OF KOREA, “AU” for AUSTRALIA, “RW” for RWANDA, “AR” for ARGENTINA, “LU” for LUXEMBOURG.

2012-2013: “AZ” for AZERBAIJAN, “GT” for GUATEMALA, “MA” for MOROCCO, “PK” for PAKISTAN, “TG” for TOGO.

2011-2012: “CO” for COLOMBIA, “DE” for GERMANY, “IN” for INDIA, “PT” for POR-

¹²Which is the abbreviation of Southern Rhodesia.

¹³Which is the abbreviation of Zimbabwe, the current name for Southern Rhodesia.

TUGAL, “ZA” for SOUTH AFRICA.

2010-2011: “BA” for BOSNIA AND HERZEGOVINA, “BR” for BRAZIL, “GA” for GABON, “LB” for LEBANON, “NG” for NIGERIA.

2009-2010: “AT” for AUSTRIA, “JP” for JAPAN, “MX” for MEXICO, “TR” for TURKEY, “UG” for UGANDA.

2008-2009: “BF” for BURKINA FASO, “CR” for COSTA RICA, “HR” for CROATIA, “LY” for LIBYAN ARAB JAMAHIRIYA, “VN” for VIET NAM.

2007-2008: “BE” for BELGIUM, “ID” for INDONESIA, “IT” for ITALY, “PA” for PANAMA, “ZA” for South Africa.

2006-2007: “CG” for CONGO, “GH” for GHANA, “PE” for PERU, “QA” for QATAR, “SK” for SLOVAKIA.

2005-2006, “AR” for ARGENTINA, “DK” for DENMARK, “GR” for GREECE, “TZ” for UNITED REPUBLIC OF TANZANIA, “JP” for JAPAN.

2004-2005: “DZ” for ALGERIA, “BJ” for BENIN, “BR” for BRAZIL, “PH” for PHILIPPINES, “RO” for ROMANIA.

2003-2004: “AO” for ANGOLA, “CL” for CHILE, “PK” for PAKISTAN, “ES” for SPAIN, “DE” for GERMANY.

2002-2003: “BG” for BULGARIA, “CM” for CAMEROON, “GN” for GUINEA, “MX” for Mexico, “SY” for SYRIAN ARAB REPUBLIC.

2001-2002: “CO” for COLOMBIA, “IE” for IRELAND, “MU” for MAURITIUS, “NO” for NORWAY, “SG” for SINGAPORE.

2000-2001: “BD” for BANGLADESH, “JM” for JAMAICA, “ML” for MALI, “TN” for TUNISIA, “UA” for UKRAINE.

1999-2000: “AR” for ARGENTINA, “CA” for CANADA, “MY” for MALAYSIA, “NA” for NAMIBIA, “NL” for NETHERLANDS.

1998-1999: “BH” for BAHRAIN, “BR” for BRAZIL, “GA” for GABON, “GM” for GAMBIA, “SL” for SLOVENIA.

1997-1998: “CR” for COSTA RICA, “JP” for JAPAN, “KE” for KENYA, “PT” for PORTUGAL, “SE” for SWEDEN.

1996-1997: “CL” for CHILE, “EG” for EGYPT, “GW” for GUINEA-BISSAU, “PL” for POLAND, “KR” for REPUBLIC OF KOREA.

1995-1996: “BW” for BOTSWANA, “DE” for GERMANY, “HN” for HONDURAS, “ID” for INDONESIA, “IT” for ITALY.

1994-1995: “AR” for ARGENTINA, “CS” for CZECH REPUBLIC, “NG” for NIGERIA, “OM” for OMAN, “RW” for RWANDA.

1993-1994: “BR” for BRAZIL, “DJ” for DJIBOUTI, “NZ” for NEW ZEALAND, “PK” for PAKISTAN, “ES” for SPAIN.

1992-1993: “CV” for CAPE VERDE, “HU” for HUNGARY, “JP” for JAPAN, “MA” for MOROCCO, “VE” for VENEZUELA.

1991-1992: “AT” for AUSTRIA, “BE” for BELGIUM, “EC” for ECUADOR, “IN” for INDIA, “ZW” for ZIMBABWE.

1990-1991: “KT” for Republic of Ivory Coast (COTE D’IVOIRE), “CU” for CUBA, “RO” for ROMANIA, “YE” for YEMEN, “ZR” for Zaire. Former USSR.

1989-1990: “CA” for CANADA, “CO” for COLOMBIA, “ET” for ETHIOPIA, “FI” for FINLAND, “MY” for MALAYSIA.

1988-1989: “DZ” for ALGERIA, “BR” for BRAZIL, “NP” for NEPAL, “SN” for SENEGAL, “VE” for VENEZUELA.

1987-1988: “AR” for ARGENTINA, “DE” for FEDERAL REPUBLIC OF GERMANY (before the reunification of Germany), “IT” for ITALY, “JP” for JAPAN, “ZW” for ZIMBABWE.

1986-1987: “BG” for BULGARIA, “CG” for CONGO, “GH” for GHANA, “AE” for United Arab Emirates, “VE” for VENEZUELA.

1985-1986: “AU” for AUSTRALIA, “DK” for DENMARK, “MG” for MADAGASCAR, “TH” for THAILAND, “TT” for TRINIDAD AND TOBAGO.

1984-1985: “UA” for UKRAINE, “PE” for PERU, “IN” for INDIA, “EG” for EGYPT, “BF” for BURKINA FASO.

1983-1984: “MT” for MALTA, “NL” for NETHERLANDS, “NI” for NICARAGUA, “PK” for PAKISTAN, “ZW” for ZIMBABWE.

1982-1983: “GY” for GUYANA, “JO” for JORDAN, “PL” for POLAND, “TG” for TOGO, “CG” for DEMOCRATIC REPUBLIC OF THE CONGO.

1981-1982: “IE” for IRELAND, “JP” for JAPAN, “PA” for PANAMA, “ES” for SPAIN, “UG” for UGANDA.

1980-1981: “DDR” for GERMAN DEMOCRATIC REPUBLIC, “MX” for MEXICO, “NE” for NIGER, “PH” for PHILIPPINES, “TN” for TUNISIA.

1979-1980: “BD” for BANGLADESH, “JM” for JAMAICA, “NO” for NORWAY, “PT” for PORTUGAL, “ZM” for ZAMBIA.

1978-1979: “BO” for BOLIVIA, “CS” for CZECHOSLOVAKIA, “GA” for GABON, “KW” for KUWAIT, “NG” for NIGERIA.

1977-1978: “MU” for MAURITIUS, “IN” for INDIA, “DE” for FEDERAL REPUBLIC OF GERMANY, “CA” for CANADA, “VE” for BOLIVARIAN REPUBLIC OF VENEZUELA.

1976-1977: “RO” for ROMANIA, “PA” for PANAMA, “PK” for PAKISTAN, “LY” for LIBYAN ARAB JAMAHIRIYA, “BJ” for BENIN.

1975-1976: “GY” for GUYANA, “IT” for ITALY, “JP” for JAPAN, “SE” for SWEDEN, “TZ” for UNITED REPUBLIC OF TANZANIA.

1974-1975: “CM” for CAMEROON, “MR” for MAURITANIA, “IQ” for IRAQ, “CR” for COSTA RICA, “BY” for BELARUS.

1973-1974: “AU” for AUSTRALIA, “AT” for AUSTRIA, “ID” for INDONESIA, “KE” for KENYA, “PE” for PERU.

1972-1973: “YU” for YUGOSLAVIA, “SD” for SUDAN, “PA” for PANAMA, “IN” for INDIA, “GN” for GUINEA.

1971-1972: “AR” for ARGENTINA, “BE” for BELGIUM, “IT” for ITALY, “JP” for JAPAN, “SO” for SOMALIA.

1970-1971: “SY” for SYRIAN ARAB REPUBLIC, “SL” for SIERRA LEONE, “PL” for POLAND, “NI” for NICARAGUA, “BI” for BURUNDI.

1969-1970: “CO” for COLOMBIA, “FI” for FINLAND, “NP” for NEPAL, “ES” for SPAIN, “ZM” for ZAMBIA.

1968-1969: “SN” for SENEGAL, “PY” for PARAGUAY, “PK” for PAKISTAN, “HU” for HUNGARY, “DZ” for ALGERIA.

1967-1968: “BR” for BRAZIL, “CA” for CANADA, “DK” for DENMARK, “ET” for ETHIOPIA, “IN” for INDIA.

1966-1967: “NG” for NIGERIA, “ML” for MALI, “JP” for JAPAN, “BG” for BULGARIA, “AR” for ARGENTINA.

Before 1966 6 non-permanent members, elected irregularly, part of non-permanent members for 1 year term, and part of non-permanent members for 2 years term.

1966: “UG” for Uganda, “NZ” for New Zealand.

1965-1966: “JO” for Jordan, “NL” for Netherlands, “UY” for Uruguay.

1965: “MY” for Malaysia.

1964-1965: “BO” for Bolivia, “KT” for Republic of Ivory Coast.

1964: "CS" for Czechoslovakia (before the disintegration).
 1963-1964: "BR" for Brazil, "MA" for Morocco, "NO" for Norway.
 1963: "PH" for Philippines.
 1962-1963: "VE" for Venezuela, "GH" for Ghana.
 1962: "RO" for Romania, "IE" for Ireland.
 1961-1962: "CL" for Chile, questioned "UAR" for United States of Arabia/United Arab Republic (Egypt, Syria, Yemen).
 1961: "TR" for Turkey, "LR" for Liberia.
 1960-1961: "EC" for Ecuador, "LK" for Sri Lanka.
 1960: "PL" for Poland.
 1959-1960: "AR" for Argentina, "TN" for Tunisia, "IT" for Italy.
 1958-1959: "PA" for Panama, "CA" for Canada, "JP" for Japan.
 1957-1958: "CO" for Columbia, "IQ" for Iraq, "SE" for Sweden.
 1957: "PH" for Philippines.
 1956-1957: "CU" for Cuba, "AU" for Australia.
 1956: "YU" for Yugoslavia.
 1955-1956: "PE" for Peru, "IR" for Iran, "BE" for Belgium.
 1954-1955: "BR" for Brazil, "NZ" for New Zealand, "TR" for Turkey.
 1953-1954: "CO" for Columbia, "LB" for Lebanon, "DK" for Denmark.
 1952-1953: "CL" for Chile, "PK" for Pakistan, "GR" for Greece.
 1951-1952: "BR" for Brazil, "TR" for Turkey, "NL" for Netherlands.
 1950-1951: "EC" for Ecuador, "In" for India, "YU" for Yugoslavia.
 1949-1950: "CU" for Cuba, "EG" for Egypt, "NO" for Norway.
 1948-1949: "AR" for Argentina, "CA" for Canada, "UK" for Ukraine (USSR).
 1947-1948: "CO" for Columbia, "SY" for Syria, "BE" for Belgium.
 1946-1947: "BR" for Brazil, "AU" for Australia, "PL" for Poland.
 1946: "MX" for Mexico, "EG" for Egypt, "NL" for Netherlands.

CV

LUO Hang, born on 16/08/1989, in Enshi, Hubei, China. Obtained B.Sc and B.A from Huazhong University of Science and Technology in June 2011, then recommended by Huazhong University of Science and Technology as the No.1 student in the major to Tsinghua University to directly pursue the Doctor of Philosophy in Management, and obtained the government-sponsored scholarship for overseas study, registered in Universite Pierre et Marie Curie, PARIS VI since academic year 2012-2013, to pursue the Doctor of Philosophy in Computer Science.

Education Background

2012.09-, Ph.D Candidate in Computer Science (Artificial Intelligence) at Laboratoire d'Informatique de Paris 6, Universite Pierre et Marie Curie, PARIS VI, advised by Prof. MAUDET Nicolas

2011.09-, Ph.D Candidate in Management (Public Management) at School of Public Management and Policy, Tsinghua University, advised by Prof. MENG Qingguo

2009.02-2011.06, B.A in Literature (English Language and Literature) from School of Foreign Languages, Huazhong University of Science and Technology, as Double Major

2007.09-2011.06, B.SC in Management (Public Management) from School of Public Administration, Huazhong University of Science and Technology, advised by Prof. ZHANG Yi

Published Papers (during the Ph.D)

During his study of Ph.D., LUO Hang has published more than 10 papers, in multi-disciplines of management, politics, computer science and education and so on, a portion of representative published papers as following:

[1] Grandi, U. Luo, H. Maudet, N. and Rossi, F. (2014) Aggregating CP-nets with unfeasible outcomes. In Proceedings of the 20th international conference on principles and practice of constraint programming (indexed by CPCI), Lecture Notes in Computer Science Volume 8656, 2014, PP 366-381. (indexed by EI)

[2] Luo, H. and Meng, Q. (2013). Multi-agent simulation of security council reform and great power games. World Economics and Politics (indexed by CSSCI), 6:136-155. in Chinese.

[3] Luo, H. ZHANG, Y, MENG Q. (2015). Modeling and simulation of multi-cities' policy coordination based on MAS. Chinese Journal of Management Science (indexed by CSSCI), 1:89-98. in Chinese

[4] Luo, H. (2013). Simulation experiment and policy research of urban agglomeration integration and multi-governments interaction. Journal of Dalian University of Technology (indexed by CSSCI), 34(2):46-52. in Chinese.

[5] Luo, H. and Guo, Z. (2014). 2012 Efficiency evaluation of China's project 985 universities—the teaching & research efficiency research and Structure & environment contributing factor anal-

ysis based on the DEA-Tobit model. *Journal of Higher Education* (indexed by CSSCI), 35(12): 35-45. in Chinese.

[6] Luo, H. (2013). 2011 Efficiency evaluation of China's project 985 universities—contributing factors and slack variables. *Tsinghua Journal of Education* (indexed by CSSCI), 34(2):87-95. in Chinese.

[7] Luo, H. and Guo, Z. (2011). (2010) Efficiency evaluation of China's project 985 universities based on SE-DEA model—an improved model integrating china universities evaluation and netbig university ranking. *Journal of Graduate Education* (indexed by CSSCI), (6);48-55. in Chinese.

Honors and Awards (during the Ph.D)

2014.11, *Tsinghua Special Grade Award* (the highest honor of Tsinghua students), awarded by Tsinghua University

2012.09, *Tsinghua First Grade Award*, awarded by Tsinghua University

2011.09, *Entrance Scholarship of Doctoral Student* (Future Scholar Scholarship), awarded by Tsinghua University

2013.09, *National Scholarship of Doctoral Student*, awarded by the Ministry of Education, P.R. China

2012.12, *Promising Young Scholar Award*, awarded by the Ministry of Education, P.R. China

2012.12, *China Development Research Scholarship*, awarded by the Development Research Center of the State Council, P.R. China

2013.05, *Government Sponsorship for Overseas Study*, awarded by the China Scholarship Council

2014.05, *Scholarship of Outstanding Chinese Student in France*, awarded by France Alumni Association, Zhejiang University