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# Impact of the unrealized gain or loss on stock returns : theory and tests in an alternative utility framework

Shoujun Li

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## THÈSE

Pour obtenir le grade de

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Appliquées à la Gestion (CERAG)**  
dans l'**École Doctorale Sciences de Gestion**

# **Impact des gains ou pertes non réalisés sur les rentabilités des actions : théories et tests dans un cadre alternatif d'utilité**

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# **Impact of the Unrealized Gain or Loss on Stock Returns: Theory and Tests in an Alternative Utility Framework**

**DISSERTATION**

submitted to obtain the degree of

**Doctor of Université Grenoble Alpes**

specialty: Management Science

presented by **Shoujun LI**

directed by **Madame Isabelle GIRERD-POTIN**

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**Centre d'Etudes et de Recherches Appliquées à la Gestion (CERAG)**

**École Doctorale Sciences de Gestion**

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# CONTENTS

|   |           |
|---|-----------|
| <b>Introduction .....</b>                                   | <b>1</b>  |
| Research background.....                                    | 1         |
| Research question .....                                     | 4         |
| Literature summary.....                                     | 5         |
| Methodology and data .....                                  | 7         |
| Dissertation structure.....                                 | 7         |
| <b>Chapter 1 Literature review .....</b>                    | <b>11</b> |
| 1.1 Momentum effect and empirical evidence .....            | 11        |
| 1.1.1 The momentum strategy.....                            | 11        |
| 1.1.2 Empirical evidences of the momentum effect.....       | 13        |
| 1.1.3 The momentum effect and other anomalies .....         | 15        |
| 1.1.4 Momentum effect as a measure of fund performance..... | 16        |
| 1.2 Decomposition of momentum profit .....                  | 18        |
| 1.2.1 Jegadeesh and Titman’s approach.....                  | 18        |
| 1.2.2 Lo and MacKinlay’s approach .....                     | 19        |
| 1.2.3 Lewellen’s analysis .....                             | 21        |
| 1.3 Studies on momentum under risk-return framework.....    | 26        |
| 1.3.1 Potential sources of momentum profit .....            | 26        |

|   |   |           |
|---|---|-----------|
| 1.3.2   | Industry momentum .....   | 35        |
| 1.3.3   | Trading cost.....   | 36        |
| 1.3.4   | Other determinants .....  | 37        |
| 1.4   | Irrational model of momentum.....   | 38        |
| 1.4.1   | Underreaction and overreaction in stock prices .....                      | 38        |
| 1.4.2   | Behavior model based on cognitive biases .....                            | 40        |
| 1.4.3   | Behavior model of momentum based on the disposition effect .....          | 46        |
| 1.5   | The alternative utility theories .....                                    | 54        |
| 1.5.1   | Differences between the classic and the alternative utility theories..... | 54        |
| 1.5.2   | The regret theory .....   | 56        |
| 1.5.3   | Quantitative measurement of regret .....                                  | 61        |
| 1.5.4   | The prospect theory .....   | 69        |
| 1.5.5   | Application of prospect theory .....                                      | 73        |
| 1.6   | Literature summary.....   | 78        |
| 1.6.1   | Existing approach to explain the momentum effect.....                     | 79        |
| 1.6.2   | A direct approach with the alternative utility theories .....             | 80        |
| <b>Chapter 2 Theoretical analyses on unrealized gain/loss in regret aversion and loss aversion models .....</b> |   | <b>81</b> |
| 2.1   | A decision making scenario involving unrealized gain or loss .....        | 82        |
| 2.1.1   | Variable transformation.....  | 83        |
| 2.1.2   | The theoretical significance of the Relative Capital Gain Overhang.....   | 86        |
| 2.2   | The regret theory applies to the scenario .....                           | 89        |
| 2.2.1   | The utility function and rationality under the regret theory .....        | 90        |
| 2.2.2   | Preference function elicitation .....                                     | 91        |
| 2.2.3   | Preference function applies to market scenario.....                       | 96        |
| 2.3   | The prospect theory applied to the scenario .....                         | 98        |
| 2.3.1   | The S-shaped utility function and investor rationality .....              | 98        |

|                  |   |            |
|------------------|---|------------|
| 2.3.2            | Preference function of prospect theory .....                | 99         |
| 2.3.3            | Investor behavior under unrealized gain or loss.....        | 102        |
| 2.4              | Combining loss aversion and regret aversion.....            | 107        |
| 2.4.1            | S-shaped choiceless utility function.....                   | 107        |
| 2.4.2            | The combined utility function applies to the scenario ..... | 108        |
| 2.4.3            | Distinguishing regret aversion and loss aversion.....       | 113        |
| 2.5              | Discussions .....   | 115        |
| 2.5.1            | The demand side of the market in the scenario.....          | 116        |
| 2.5.2            | From preference to risk premium.....                        | 118        |
| 2.5.3            | The case with a positive expected outcome .....             | 120        |
| 2.5.4            | EU function in preference/RCGO plan .....                   | 122        |
| 2.5.5            | Connection with the disposition effect.....                 | 125        |
| 2.6              | Prediction and test design.....                             | 126        |
| 2.6.1            | Result interpretation .....                                 | 126        |
| 2.6.2            | Prediction on stock return .....                            | 127        |
| 2.6.3            | Test design.....  | 128        |
| 2.7              | Summary.....  | 129        |
|                  | Annex 2.1: Zero-sum game preference .....                   | 130        |
|                  | Annex 2.2: Finding $g()$ from $Q()$ .....                   | 131        |
| <b>Chapter 3</b> | <b>Preliminary tests.....</b>                               | <b>133</b> |
| 3.1              | Review of Grinblatt and Han's model and test.....           | 134        |
| 3.1.1            | Test model.....   | 134        |
| 3.1.2            | Estimation of the key variables .....                       | 136        |
| 3.1.3            | Regression method .....                                     | 138        |
| 3.2              | Observations and replication test hypotheses.....           | 139        |
| 3.2.1            | Remarks on variable estimation .....                        | 139        |
| 3.2.2            | Influence of time horizon on predictability.....            | 142        |

|                  |   |            |
|------------------|---|------------|
| 3.2.3            | Hypotheses .....  | 144        |
| 3.3              | Replication tests and results .....                     | 146        |
| 3.3.1            | Data description.....                                   | 146        |
| 3.3.2            | Summary statistics.....                                 | 149        |
| 3.3.3            | Test result .....                                       | 152        |
| 3.4              | Summary of preliminary tests.....                       | 158        |
| 3.4.1            | Market reaction to the disposition effect.....          | 158        |
| 3.4.2            | Link to following tests.....                            | 159        |
| Annex 3.1:       | Comparison of CGO .....                                 | 161        |
| <b>Chapter 4</b> | <b>Test on the unrealized gain/loss model.....</b>      | <b>163</b> |
| 4.1              | Empirical application of regret and loss aversion ..... | 164        |
| 4.1.1            | Estimation of RCGO in an empirical context .....        | 165        |
| 4.1.2            | RCGO influence on stock return .....                    | 169        |
| 4.2              | Data description.....                                   | 171        |
| 4.2.1            | Estimation of variables.....                            | 171        |
| 4.2.2            | Summary statistics of RCGO .....                        | 174        |
| 4.3              | RCGO sorting test .....                                 | 176        |
| 4.3.1            | Average RCGO deciles return.....                        | 177        |
| 4.3.2            | T-test over RCGO deciles return.....                    | 179        |
| 4.3.3            | Average decile return adjusted for risks.....           | 179        |
| 4.4              | Regression based on Grinblatt and Han's model.....      | 181        |
| 4.4.1            | Summary statistics.....                                 | 183        |
| 4.4.2            | Single grouped test .....                               | 184        |
| 4.4.3            | Double grouped test .....                               | 185        |
| 4.4.4            | Quadruple grouped test .....                            | 187        |
| 4.4.5            | Test summary .....                                      | 190        |
| 4.5              | Residual regression test .....                          | 191        |

|                  |  |            |
|------------------|--|------------|
| 4.5.1            | Test model .....                                       | 191        |
| 4.5.2            | Summary statistics.....                                | 193        |
| 4.5.3            | Single grouped test .....                              | 194        |
| 4.5.4            | Double grouped test .....                              | 196        |
| 4.5.5            | Test summary .....                                     | 199        |
| 4.6              | RCGO Extremity minus Middle portfolio return .....     | 200        |
| 4.6.1            | The EMM strategy .....                                 | 200        |
| 4.6.2            | The EMM profit .....                                   | 201        |
| 4.7              | Summary.....   | 203        |
| Annex 4.1        | Forecasting SD of next period: .....                   | 205        |
| <b>Chapter 5</b> | <b>The Unrealized gain/loss model in dynamic .....</b> | <b>207</b> |
| 5.1              | The model in dynamic .....                             | 207        |
| 5.1.1            | Interactive relation between RCGO and price .....      | 208        |
| 5.1.2            | Evolution of stock price and RCGO in dynamic.....      | 211        |
| 5.2              | Implication of the dynamic model.....                  | 215        |
| 5.2.1            | Explaining previous test results.....                  | 215        |
| 5.2.2            | Convergence and divergence of RCGO .....               | 218        |
| 5.3              | Tests of the dynamic model.....                        | 219        |
| 5.3.1            | Prediction of the dynamic model .....                  | 219        |
| 5.3.2            | Test design.....                                       | 221        |
| 5.3.3            | Dynamic model test results .....                       | 222        |
| 5.4              | Summary.....   | 228        |
| <b>Chapter 6</b> | <b>RCGO explaining the momentum effect.....</b>        | <b>231</b> |
| 6.1              | The momentum effect.....                               | 232        |
| 6.1.1            | The momentum strategy .....                            | 232        |
| 6.1.2            | Replication of the momentum strategy .....             | 233        |
| 6.2              | Link between RCGO and the momentum effect .....        | 236        |

|       |  |            |
|-------|--|------------|
| 6.2.1 | Similarities between RCGO and momentum effect..... | 236        |
| 6.2.2 | Sources of the momentum profit.....                | 239        |
| 6.2.3 | Autocorrelation due to unrealized gain/loss.....   | 242        |
| 6.3   | Direct test on momentum profit.....                | 243        |
| 6.3.1 | Test design.....                                   | 244        |
| 6.3.2 | Similarity in portfolio composition.....           | 249        |
| 6.3.3 | Autocorrelation test and result.....               | 254        |
| 6.4   | Summary.....                                       | 258        |
|       | <b>General conclusion .....</b>                    | <b>261</b> |
|       | Main results .....                                 | 261        |
|       | Major contributions .....                          | 265        |
|       | Limitation and future research.....                | 267        |
|       | <b>Bibliography: .....</b>                         | <b>269</b> |
|       | <b>List of tables:.....</b>                        | <b>273</b> |
|       | <b>List of figures:.....</b>                       | <b>274</b> |

# Introduction

This dissertation aims to apply alternative utility theories, namely the prospect theory and the regret theory, to try to explain a well-known market anomaly, the momentum effect. The study performed in this dissertation focuses on the concept of the unrealized gain/loss, which indicates if an investor is currently at a winning or a losing position. Under the framework of the alternative utility theories, and inspired by existing literature, a model is established to capture the influence of the unrealized gain/loss as measured by a new variable named Relative Capital Gain Overhang. The model also shows that the investors are highly reluctant to sell their stocks in a large gain or in a large loss situation. Tests on stock return confirm this conclusion. Moreover, this finding suggests that stocks, especially stocks with large gain or large loss, could show return autocorrelation as a result of the behavior factor introduced by the alternative utility theories. According to existing literature, such an autocorrelation in stock returns is consistent with one source of the momentum effect. Tests show that the unrealized gain/loss could contribute to the momentum effect, but is not the only source. The momentum effect could be a result of a combination of many complex factors.

## Research background

In the studies on financial markets, theories and anomalies have always been a pair of dialectic forces that keep deepening our understanding. On the theoretical side, the Capital Asset Pricing Model (CAPM) laid down the foundation of asset pricing that the value of a financial asset originates from two sources: the time value and the risk premium for bearing market systematic risk. On the anomaly side, empirical studies on the CAPM reveal



inconsistency with the model: some kind of assets could consistently yield higher returns than that expected from the theory.

Fama and French studied these anomalies and expanded the CAPM model to a three-factor model. Besides the risk factor related to the market, the Fama-French Three-Factor model includes two more risk factors related to firm market capitalization and book-to-market ratio. The work of Fama and French deepens our understanding to asset pricing. Risk factors are not only limited to market risk, other attributes of firms could also be related to risks. However, even though the Fama-French Three-Factor model showed a great success in research, anomalies were still documented in the financial market. This dissertation focuses on one of them, the momentum/reversal effect.

The momentum effect was first documented by Jegadeesh and Titman in 1993. They found that the past winner stocks could consistently outperform the past loser stocks over a time horizon of about one year. This anomaly has its own specificity: the performance of a stock is, to some extent, related to its past performance. Unlike other anomalies mentioned above, the momentum effect does not relate to any firm attributes such as the size or book-to-market ratio, but mainly depends on past stock performance. Therefore, it is not possible to interpret the momentum effect a risk factor. On the contrary, the momentum effect can be considered as a market failure. The classic theories based on a risk-return framework have difficulty to explain this anomaly.

In research in the field financial markets, the classic theories based on the risk-return framework are in a dominating position. However, a new branch of theory known as the behavioral finance has emerged since the 1990s. Instead of using a pure mathematical foundation such as the expected utility theory, behavioral finance adopts some findings in the field of psychology. The behavioral finance asserts that after all, the participants of the financial market are people, and psychological findings could provide more accurate descriptions on how people make their decisions, on the factors that could influence the decision making process, and even on what kind of “error” people are likely to commit when making decisions.

The behavior finance provides a new perspective to look at the market anomalies. The classic financial theories are based on the assumption of a rational individual whose behavior is modeled by the expected utility theory. Since this theory is purely mathematic, it could be a simplified, or an idealized model to describe the decision making of individuals. In fact the

decision making could be more complex and to some extent, is different from what the expected utility theory described. As a result, the classic financial theories could capture major factors in the financial market such as the time value and the risk premium, but it could still fail to capture some elements originating from individual behavior. These elements influence the decision making, hence could contribute to asset pricing. This provides an explanation to the market anomalies that puzzle the classic theories.

The development of the behavioral finance suffers from a slow progression because of its own limitations. The first difficulty in this domain is to give directly and quantitatively a measure of the behavior factors. Many studies in the behavioral finance use proxy variables to represent behavior factors, but using such approximations will result in decreasing the test accuracy. The second difficulty is that there is not a theory that could unify the behavioral finance. Some studies in this field could provide insight on some aspects, but the big picture is still missing. Finally, it is also difficult to find synergy between the behavior factors and classic financial theory. However, the alternative utility theories have their advantages that could potentially overcome these difficulties. The prospect theory developed by Kahneman and Tversky in 1979 is so far the most successful alternative utility theory. The regret theory developed separately by Loomes and Sugden (1982), and Bell (1983) is also a well accepted alternative utility theory.

The alternative utility theories investigate the underlying foundation of financial and economical theories: utility. They integrate individual behavior patterns into utility, and derive utility functions that have other properties than the classic utility theory. Under the framework of the alternative utility theory, the individuals are rational since their decision making will still maximize their utility according to their utility function. These two characteristics suggest that the alternative utility theories still follow the basic principles developed by the classic theory. Hence the financial model that is derived from the alternative utility theories could share some elements in common with the classic financial theories.

This dissertation stands at the intersection of the classic financial theories and the behavioral finance. On one hand, there are market anomalies that still puzzle the classic theories. On the other hand, there are alternative utility theories that wait to be applied to the financial markets. Some recent studies attempted to tackle the task of applying the alternative utility theories. The research of Grinblatt and Han (2005) and Barberis and Xiong (2009) show the potential in this domain. This dissertation is inspired by these pioneering works.

## **Research question**

This dissertation aims to provide an answer to this question:

Could we explain a major market anomaly, the momentum effect, by applying the alternative utility theories?

This research question contains three elements: the momentum effect is the application field of this research; the alternative utility theories are the theoretical foundation to study the research subject. And finally, the most crucial element in this study is to find the link between the momentum effect and the behavior factor included in the alternative utility theories.

Regarding the momentum effect, it is manifested by the phenomenon that past winner stocks outperform past loser stocks. Beyond such a basic observation, more detailed and profound understanding on this subject is needed. The same level of work is needed to understand the alternative utility theories. In order to be able to apply these theories, a profound understanding to these theories is required, not only about the general principles, but also about the technical details. In addition, learning how existing work applies the alternative utility theories in practice is also an important part of the study. Review on the existing literature about the momentum effect and the alternative utility theories will fulfill this task.

The most crucial element of this research question is finding the link between the momentum effect and the alternative utility theories. The first step to reach this goal is to find out an attack point to apply the alternative utility theories. The differences between the alternative utility theories and the classic expected utility theory are reasonable attack points. The prospect theory introduces a reference point that distinguishes gains from losses. The regret theory introduces a complementary utility for experiencing regret or rejoice. Application of these theories requires integrating these features into a model that could describe investor decision making in the market. Consequently, such an application should reveal differences in investor behavior compared with classic utility theory, and could even provide prediction n stock performances under the framework of alternative utility theories.

At this level, the research encounters an underlying question: could the behavior factors captured by the alternative utility theories have influence stock returns? This question is a necessary condition for explaining the momentum effect by the alternative utility theories. If the behavior factors represented by the alternative utility theories can explain the momentum

effect, first they must have an influence on stock returns. Moreover, the influence of behavior factors on stock return is in fact a more general and more fundamental issue to understand the way the financial market works. Therefore, this underlying question will be first studied. The research will look into the predictions deduced from the alternative utility theories, and empirical tests will be performed to verify the theoretical model.

Based on the result that behavior factors could influence stock performance, the next step of the research is devoted to apply this result to explain the momentum effect. The impact of the behavior factor could potentially be the link that connects the momentum effect and the alternative utility theories. In an ideal situation, one could expect that the model under the framework of the alternative utility theories may predict a stock performance pattern that is consistent with the momentum effect. For example, investors could prefer buying a winning stock, and selling a losing stock. However, the real connection between these two elements may be much more complex than the ideal case.

### **Literature summary**

As mentioned in the research question, review of existing literature is the first step of the research in this dissertation, and is also the key step for understanding and grasping the important concepts of the research subject. In this dissertation, the literature review covers two aspects: the momentum effect and the alternative utility theories.

The literature review first looks into the momentum effect. The literature includes some fundamental works such as the work of Jegadeesh and Titman (1993 and 2001), Rouwenhorst (1998), Lewellen (2002), etc. This section is organized in three steps. The first step identifies the problem. In this step, the literature review focuses on the formation of the momentum strategy and on empirical evidence of the momentum effect documented all over the world and on various sample periods. The second step analyzes the problem. This subsection covers theoretical studies on the sources of the momentum profit. Three major sources are identified: Exposure to systematic risks, lead-lag effect of factor realization, and autocorrelation in idiosyncratic component. The final step consists of solving the problem. This subsection summarizes studies that aim to explain the momentum effect. These studies are categorized according to the source of the momentum profit they try to explain. On one hand, under the classic risk-return framework, literature finds that neither exposure to risks nor the factor

realization is auto-correlated can explain the momentum profit. On the other hand, by relaxing the assumption of an efficient market, studies show interest in stock price under-reaction or over-reaction. In this approach, studies tend to resort to behavior factors to explain the reason for under-reaction or over-reaction. The behavior models created by Barberis Shleifer and Vishny (1998) and Hong and Stein (1999) could predict the momentum effect in simulation, but there is no empirical test performed because of the difficulty in measuring the behavior factors.

The alternative utility theories compose the second section of the literature review. This section emphasizes on the regret theory founded by Loomes and Sugden (1982), and Bell (1983), and the prospect theory founded by Kahneman and Tversky (1979). The literature review in this section first focuses on the characteristics of the alternative utility theories, especially how these theories incorporate the behavior factors in individual decision and how these behavior factors are formulated in a mathematical form. Next, the literature review focuses on how alternative utility theories could be applied. For the regret theory, the application mainly consists of quantitative measurement of regret and elicitation of its utility function. For the prospect theory, the literature review is devoted to recent studies explaining the disposition effect, since this latter could potentially be related to the momentum effect.

The literature review also tries to find existing studies that establish a link between the momentum effect and the alternative utility theories. However, very few existing works are found on this subject. The most important literature is the work of Grinblatt and Han (2005). They identified the disposition effect as the intermediary between the prospect theory and the momentum effect. They suggest that some investors in the market behave according to the prospect theory, and show disposition effect when facing unrealized gain/loss. As a result, these investors create excess/insufficient supply of stock that results in stock price under-reaction; and such an under-reaction contributes to the momentum effect. However, more recent studies show that the disposition effect cannot link the prospect theory to the momentum effect. On one hand, the prospect theory cannot consistently lead to the formation of the disposition effect. This point is demonstrated by Barberis and Xiong (2009). On the other hand, Birru (2015) finds that the momentum effect is still significant after controlling for the disposition effect. Although the existing works have denied the disposition effect as the link between the momentum and the alternative utility theory, the work of Grinblatt and Han (2005) still provide a great value in terms of theoretical work and empirical test. The research in this dissertation is inspired by their work.

## **Methodology and data**

To answer the research question, the studies in this dissertation involve both theoretical modeling and empirical tests. The research process follows these steps: establishing a theory, testing the theory, improving the theory, and applying the theory. The theory is established by a mathematical development of the existing alternative utility theories. Then, the empirical tests confirm the theory, but also find problem on the periodicity of the model. Solving this problem leads to improving the theory by proposing a dynamic version. This dynamic model provides an approach to apply this theory to the momentum effect.

The empirical tests are essential in this dissertation to check the theoretical models. The tests performed in this dissertation include both non parametrical tests and regressions. The sorting test is used to compare stock performance among groups divided according to certain criteria, and graphics are often used to give an intuitive understanding. The regressions are performed with different methods. For panel data, the Fama-Macbeth method is often applied. In certain cases where the test intends to isolate the behavior factor from the risk factors, a two-step regression is used. The first step is to collect the residuals that are orthogonal to the risk factors; the second step is to perform the regression on the residuals.

The data used in this dissertation includes all common stocks in the NYSE and AMEX from January 1982 to December 2012. Returns are calculated on a weekly basis. The data covers the same market as the study of Grinblatt and Han (2005). In terms of sample periods, our data and the data of Grinblatt and Han (2005) both contain the period from 1982 to 1996. It allows making comparison between the test results. The Fama-French factor portfolio data are directly downloaded from the online database of the Data Library of Kenneth French. Note that the Fama-French factors are calculated from the data of NYSE AMEX and Nasdaq. Because the stocks in Nasdaq yield significant higher returns than the other two markets, the risk adjusted returns calculated in this dissertation tend to have negative values. Unfortunately, limited by our access to database, this dissertation cannot overcome this problem.

## **Dissertation structure**

This dissertation is divided into six chapters. The first chapter is dedicated to the literature review.

The second chapter elaborates a theoretical model under the framework of alternative utility theories. This model emphasizes the influence of the unrealized gain/loss on the investor preference to sell or hold a stock, which is not captured by the classic utility theory. This model introduces the variable Relative Capital Gain Overhang which measures the relative level of unrealized gain/loss with respect to the volatility of the stock. The model suggests that investors have a decreasing propensity to sell the stock when facing either an unrealized gain or an unrealized loss. The RCGO model can only give a prediction on investor preferences, and can suggest that RCGO influence stock performance. The model cannot yet provide a link between the RCGO and the momentum effect. At this stage, the empirical tests of the RCGO model will focus on the influence of RCGO on stock performance.

The third chapter runs a preliminary test. Before performing tests on the RCGO model, this preliminary test mainly consists of a replication of the test performed by Grinblatt and Han (2005). This step is important in the research. The variable RCGO has a close relation to the variable Capital Gain Overhang introduced in their work. Therefore, the preliminary test first serves as a learning process. The replication of the existing test allows understanding how existing work estimates the key variables and how to convert a theoretical prediction into an empirical test. These experiences will be transferred to later tests on the RCGO model. In addition, the preliminary test also includes tests on the immediate and the subsequent reaction of the market. It helps to understand the way by which the impact of the behavior factor is transferred to stock performance. Last, the replication test is performed on a more recent sample period. This allows making comparison between the results of the replication and the original tests.

The fourth chapter performs empirical tests on the RCGO model. In the first section, by applying the experiences learned from the preliminary test, the RCGO model is adapted in order to be empirically tested. Next, the RCGO model is tested through various methods. These methods introduced with an increasing complexity. The first test is a non-parametrical sorting test. It finds that stocks with extreme high or low RCGO yield higher average returns. Then this pattern observed in the sorting test is studied by regressions. One regression model is Grinblatt and Han's test, the other regression model is a single variable regression on the residuals that are orthogonal to the Fama-French model. Finally the test explores the pattern related to RCGO, and forms a zero-cost portfolio that yields a positive profit after controlling for risks. The tests in this chapter focus on verifying if the RCGO influences stock

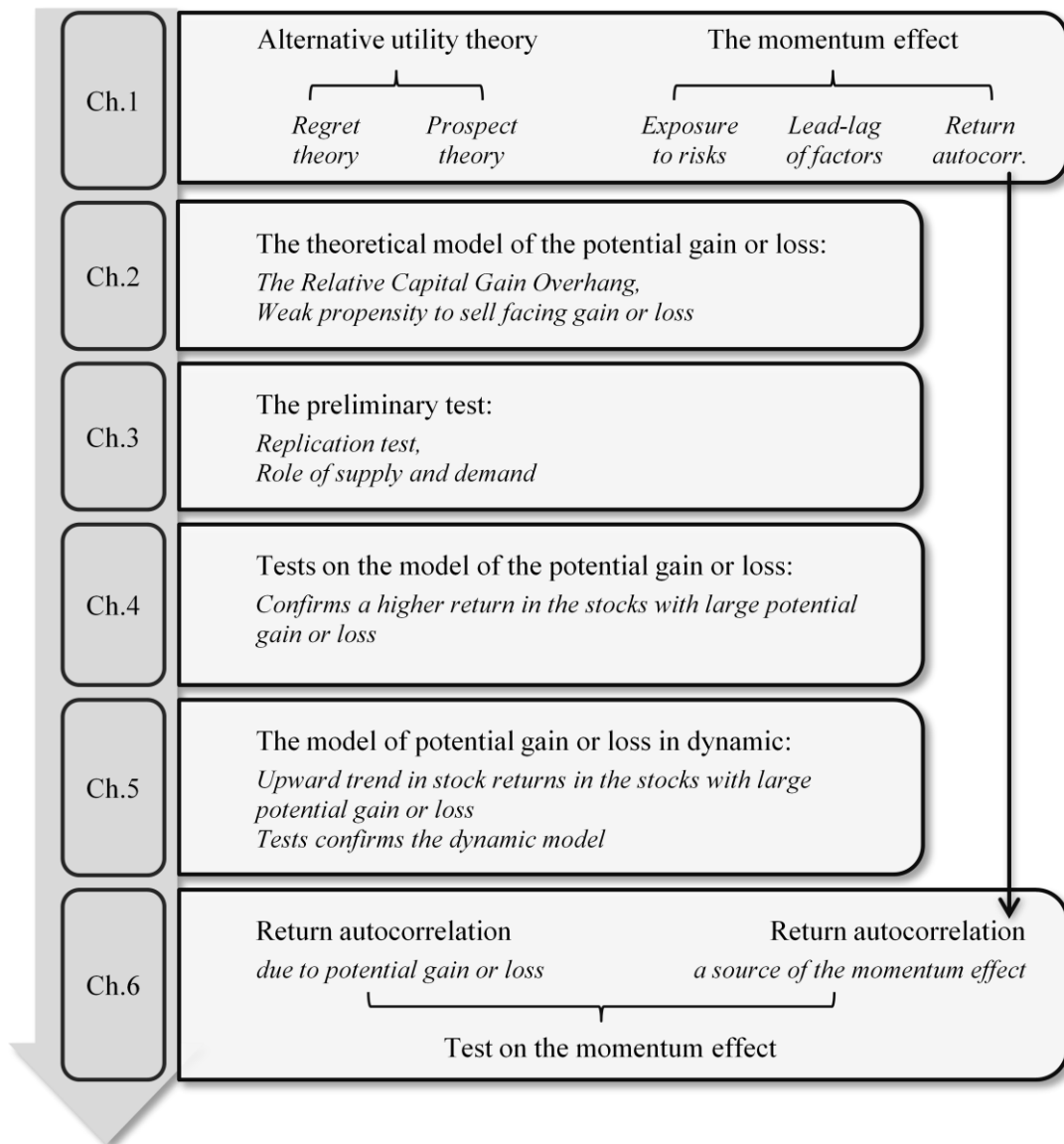
performance as predicted by the RCGO model. In general, the tests confirm the theoretical model. However, the tests also lead to one interesting question: why the influence of RCGO is significant over long term horizon (over 6 months to one year) rather than in the short term?

The fifth chapter goes back to the theoretical study on the RCGO model and aims to provide an answer to the question found in the previous empirical tests. By revising the theoretical model and its adaptation to empirical tests, a dynamic version of the RCGO model is established. The dynamic model finds that by aggregating the reference prices at which investors have purchased their stock, the variable RCGO could contain past information on the unrealized gain/loss over past one year. This suggests that the influence of RCGO could persist over time. Comparing with the static version that describes the cross-sectional relation, the dynamic model emphasizes the time-serial relation between RCGO and stock performance. It suggests that the larger the unrealized gain/loss is, the longer its influence could persist. The persistence of the influence of RCGO could explain the test results in the previous chapter. More importantly, such persistence could also imply a strong positive return autocorrelation among the stocks with extreme unrealized gain or loss. Such a positive return autocorrelation is consistent with one of the three sources that contribute to the momentum profit.

Finally, the sixth chapter tries to link the momentum effect to the RCGO model. Following the studies performed in the previous chapters, a potential link between the alternative utility theories and the momentum effect is identified: a positive return autocorrelation in the stocks with extreme RCGO. The empirical test on this subject examines the correlation between the momentum profit and the profit of a strategy based on RCGO. However, this test does not show the expected result. It suggests that the momentum profit could be a result of complex reasons; more thorough studies are still needed in future research.



Figure 1: Structure of this dissertation



# Chapter 1

## Literature review

This chapter reviews the existing literature on the dissertation subject. The chapter will be organized as follows: first, the momentum effect and the empirical evidence of this effect; second, the existing explanations to the momentum effect in both rational and behavioral framework; third, the alternative utility theories; and last, the existing attempts to link alternative utility theories to the momentum effect.

### 1.1 Momentum effect and empirical evidence

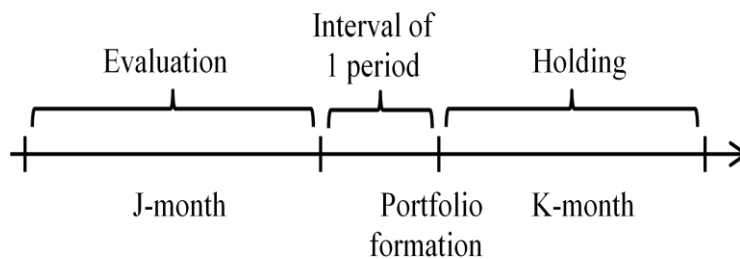
The momentum effect refers to the phenomenon that past winner stocks will outperform past loser stocks in the following six months to one year. Jegadeesh and Titman (1993) documented this effect in the US market using the J-month/K-month strategy (also known as the relative strength strategy, or momentum strategy). Later, the momentum effect has also been documented in other stock markets over the world.

#### 1.1.1 The momentum strategy

Jegadeesh and Titman (1993) refer to this strategy as a J-month/K-month strategy. At any time spot, past J-month returns are used to select stocks in order to form a zero-cost arbitrage portfolio. Then the portfolio is kept for the following K-month to harvest arbitrage profit.

At the beginning of month  $t$ , the stocks are sorted in ascending order based on their  $J$  month past returns. Then, the stocks are divided into ten deciles, the first decile is called the “losers” decile because it contains the least profitable stocks; the last decile is called the “winner” decile for including the most profitable stocks over the past  $J$  months. Then, the strategy suggests buying one share of every stock in the winner group, meanwhile, short selling one share of every stock in the loser group. Therefore, an equal-weighted zero-cost arbitrage portfolio is constructed. Then, the strategy suggests holding this portfolio for the next  $K$  months. Theoretically, if the market is efficient, there should be no arbitrage opportunity. In the long run, such a strategy should yield zero profit. However, the momentum strategy systematically yields positive profits. In actual empirical tests, an interval of one period is taken between  $J$ -month and  $K$ -month in order to avoid some influence linked to market micro structure. The time line of the  $J$ -month/ $K$ -month strategy is shown in the following figure:

Figure 1.1-1: The time line of the  $J$ -month/ $K$ -month strategy



Jegadeesh and Titman (1993) tested this strategy on the monthly data over the period from 1965 to 1989. The stock selection and holding period is on a quarterly basis, which means,  $J$  and  $K$  take the value of 3, 6, 9, and 12. Their result shows that all the combinations of  $J$  and  $K$  generate statistically significant positive return from all the zero-cost portfolios.

A variant of the relative strength strategy is the weighted relative strength strategy (WRSS), this requires constructing a zero-cost portfolio that weights stocks according to the difference between their past returns and the past equally weighted index returns (Lo and MacKinlay 1990). The WRSS yields a profit and is highly correlated to the equally weighted relative strength strategy. Other weights are also used in momentum portfolio formation; most commonly used is the value weighted portfolio (Stivers and Sun 2010, Mclean 2010, Novy-Marx 2012 etc.). It consists in using stock market capitalization as the weight in forming momentum portfolio. In addition to the value-weighting method, Mclean (2010) also uses the stock’s idiosyncratic risk to define stock weights in the momentum portfolio.

Other classification methods can also be applied to the momentum strategy. Lesmond Schill and Zhou (2004) sort stocks into three groups according to past gross return: poor performers, moderate performers, and strong performers. Moskowitz and Grinblatt (1999) use the 30<sup>th</sup> percentile to form industry momentum strategy. Stivers and Sun (2010) also study the “decile-9 minus decile-2” momentum strategy in addition to the “winner minus loser” strategy.

Novy-Marx (2012) studies the term-structure of momentum effect, and finds that the stock’s performance in the past intermediate horizon gives a more important contribution to the momentum effect than the recent past performance. This study compares the momentum portfolio selected using recent past 6-month return ( $r_{-6;-1}$ ) and the portfolio selected using intermediate-horizon past 6-month return ( $r_{-12;-7}$ ). The latter yields significantly higher profits.

Instead of using past raw returns, Grundy and Martin (2001) use stock past cumulative excess returns as the selection criterion to perform the momentum strategy. They also find a significant profit for the momentum strategy.

### **1.1.2 Empirical evidences of the momentum effect**

The momentum effect is documented in many studies all over the world, and in different time periods. In the US market, Jegadeesh and Titman (1993) documented a significant monthly profit of 1.31% ( $t = 4.03$ ) by implementing a 12-month/6-month strategy using NYSE and AMEX data over the period 1965 - 1989.

After the first study on the momentum strategy in the early 1990s, this subject becomes popular in the academic and professional worlds. However, the momentum profit has not seemed to disappear in the US market. Jegadeesh and Titman (2001) reexamined the momentum effect. Applying the same momentum strategy over the period from 1965 to 1998, also using NYSE and AMEX data, they documented a monthly momentum profit of 1.23% ( $t = 6.46$ ). Grundy and Martin (2001), using cumulative monthly excess return as a criterion for selecting stocks, have also documented a profit of 0.44% ( $t = 1.83$ ) over the period from 1926 to 1995.

Chapter 1:  
Literature review

Table 1.1-1: Momentum profit documented by Jegadeesh and Titman (2001)

|         |    | Panel A |        |        |        | Panel B |        |        |        |        |
|---------|----|---------|--------|--------|--------|---------|--------|--------|--------|--------|
|         |    | K-month |        |        |        | K-month |        |        |        |        |
|         |    | 3       | 6      | 9      | 12     | 3       | 6      | 9      | 12     |        |
| J-month | 3  | profit  | 0.0032 | 0.0058 | 0.0061 | 0.0156  | 0.0073 | 0.0078 | 0.0074 | 0.0077 |
|         |    | P-value | 1.10   | 2.29   | 2.69   | 3.53    | 2.61   | 3.16   | 3.36   | 4.00   |
|         | 6  | profit  | 0.0084 | 0.0095 | 0.0102 | 0.0086  | 0.0114 | 0.0110 | 0.0108 | 0.009  |
|         |    | P-value | 2.44   | 3.07   | 3.76   | 3.36    | 3.37   | 3.61   | 4.01   | 3.54   |
|         | 9  | profit  | 0.0109 | 0.0121 | 0.0105 | 0.0082  | 0.0135 | 0.0130 | 0.0109 | 0.0085 |
|         |    | P-value | 3.03   | 3.78   | 3.74   | 2.89    | 3.85   | 4.09   | 3.67   | 3.04   |
|         | 13 | profit  | 0.0131 | 0.0114 | 0.0093 | 0.0068  | 0.0149 | 0.0121 | 0.0096 | 0.0069 |
|         |    | P-value | 3.74   | 3.4    | 2.95   | 2.25    | 4.28   | 3.65   | 3.09   | 2.31   |

Source: Jegadeesh and Titman (2001) Momentum, working paper Table 1. Panel A shows result using J-month/K-month strategy without interval; Panel B shows result with one week interval between J-month and K-month.

More recent studies also documented momentum effect. Mclean (2010) studied the period from 1965 to 2004. The momentum portfolio is constructed using different weighing. The study shows that the equal weighted portfolio yields a profit of 0.74% (t = 3.39) per month; the value weighted portfolio yields a profit of 0.88% (t = 3.95); the portfolios weighted by idiosyncratic risk and by the inverse of the idiosyncratic risk have respectively a return of 0.38% (t = 1.42) and 0.72% (t = 3.38). Novy-Marx (2012) studies the term structure of momentum effect. Based on the data from 1926 to 2010 on the US market, the study shows that the equal weighted momentum portfolio selected according to recent past return yields a profit of 0.67% per month, while the portfolio selected according to intermediate horizon past return yields a profit up to 1.20% per month. The latter generates 0.54% higher profit, and such difference is statistically significant.

The momentum effect is also documented internationally. Rouwenhorst (1998) studied the financial markets in 12 European countries using the same method. Based on the data from 1978 to 1995, the study had found significant profit for momentum strategy in 11 among 12 European countries. The cross-country average momentum profit is 0.93% (t = 5.36). This study also shows that the internationally diversified portfolio experiences the momentum effect too. By mixing the stocks in the 12 European financial markets, the 12-month/3-month momentum strategy yields an average profit of 1.35% (t = 3.97), even slightly higher than that in the US market.

Abbes Boujelbene and Bouri (2006) concentrate their study on the French market. Based on the data from 1995 to 2004 on Paris Stock Exchange, the study shows that a 12-

month/6-month strategy could generate a profit up to 6.73% ( $t=42.29$ ), which is 1.21% per month.

Other studies show that the momentum profit is also found in less developed countries. Bartens and Hassan (2011) study the South Africa stock market from 1987 to 2004. Instead of using the momentum strategy, they incorporate the momentum effect into a model of portfolio selection along with other criteria such as size, book-to-market, and beta. They find that the optimal portfolio could always beat the market in the in-sample period; but in the out-of-sample period, the profit is not stable. Su (2011) studies the momentum effect in the Chinese market. Based on the data from 1994 to 2008 of Shanghai and Shenzhen Stock Exchanges, the study finds that a Weighted Relative Strength Strategy of 1-month/1-month combination could yield a monthly return up to 1.313%.

Besides the equity market, the momentum effect is also found in future market. Pirrong (2005) studied found that future prices show the pattern of momentum and long-term reversal. The momentum effect in the future market is correlated to stock market momentum.

### **1.1.3 The momentum effect and other anomalies**

Other market anomalies can have influences on the momentum effect. The most visible one is the January effect. The momentum strategy yields a different return in January comparing to the other months. Jegadeesh and Titman (2001) show that the momentum profit drops to -1.55% ( $t = -1.87$ ) in January, while in other months, the momentum profit is up to 1.48% ( $t = 7.89$ ). Grundy and Martin (2001) have documented same phenomenon. In January, the average momentum profit is -5.85% ( $t = -4.93$ ), in other months, the profit is 1.01% ( $t = 4.44$ ). Regarding winning rates, Jegadeesh and Titman (1993) show that the momentum strategy earns in average a positive return in 67% of the months, but only in 24% of the months in January months. If the latter are excluded, the momentum strategy earns a positive return in 71% of the months. However, in the study of Grinblatt and Han (2005), the momentum effect is stronger in January compared with the other months in the year. Similarly, Su (2011) shows that in the Chinese market, the momentum strategy generates higher profits in January (1.957%) than in other months (1.254%).

The momentum effect is also related to the size effect. Jegadeesh and Titman (2001) found that the top decile and the bottom decile of the momentum portfolio are composed by small sized firms. Because small firms are more volatile, hence they are more likely to have

extreme returns. This relation is confirmed by further tests. The most convincing one is the test on size based subsamples. The momentum strategy is implemented separately on small size; medium size and large size subsamples. The momentum profit tends to be higher for small firms, but the momentum effect does not disappear in large size subsample.

Novy-Marx (2012) studied the momentum in stock market capitalization five quintiles. Using the recent past return based strategy, the momentum profit is higher in the smallest size quintile (0.53%,  $t = 2.99$ ), and is lower in the biggest size quintile (0.24%,  $t = 1.25$ ). Using the intermediate horizon past return based strategy, the momentum profit is similar in the smallest size quintile (0.83%,  $t = 5.17$ ) and in the biggest size quintile (0.82%,  $t = 4.66$ ).

#### 1.1.4 Momentum effect as a measure of fund performance

Instead of explaining the momentum effect, Carhart (1997) took an opposite perspective on the momentum effect: using the momentum effect as a factor to explain the persistence of certain mutual fund performances. The approach Carhart took is to add the momentum effect as a new factor to the Three-Factor model of Fama and French, and to form a Four-Factor model such as the following equation:

$$r_{it} = \alpha_{iT} + b_{iT}RMRF_t + s_{iT}SMB_t + h_{iT}HML_t + p_{iT}PR1YR_t + e_{iT}$$

Where:

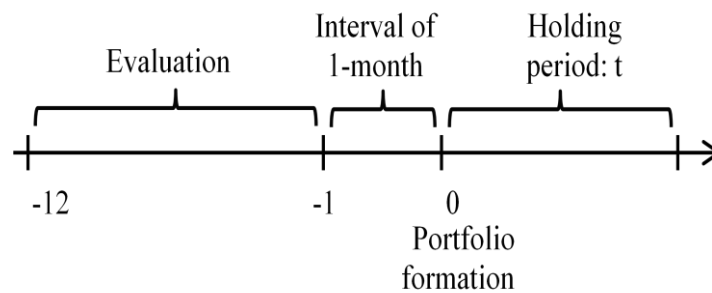
$r_{it}$  : is the return of mutual fund.

RMRF, SMB, HML: are the three risk factors in Fama French model.

PR1YR : is the one year momentum in stock return.

In this model, Carhart constructs the PR1YR factor as follows: the portfolio selection is based on monthly returns. The evaluation period is month -12 to month -1, equivalent to a J-month strategy of 11-month. Month-1 to month 0 is an interval of one month. At month 0 the PR1YR portfolio is formed. The following figure demonstrates the timeline of this portfolio:

Figure 1.1-2: The timeline of Carhart's PR1YR factor



The sample includes all stocks in NYSE, Amex, and Nasdaq. The winner group contains the stocks with the highest 30% past returns; the loser group contains the stocks with the lowest 30% past returns. The PR1YR portfolio is formed by taking the equal weighted average of the stock returns.

Comparing with the formation of the momentum factor, the economical meaning of this four-factor model is the most essential idea to capture. The four-factor model aims to explain the persistence of mutual fund performance, and according to Carhart, the factors in this model are interpreted as four investment strategies: “high versus low beta stocks, large versus small market capitalization stocks, value versus growth stocks, and one-year return momentum versus contrarian stocks.” This interpretation of the model factors suggests first that this model is self-consistent among the factors. The Fama-French factors are simply considered as portfolio outcomes; hence all factors represent one type of investment strategy. A positive coefficient implies that the mutual fund benefits from this type of investment strategy.

Second, the interpretation suggests that this model has a limited application field. This four-factor model is an empirical model concerning investment strategies, not an asset pricing model at a theoretical level. Although this model is developed from the Fama-French three-factor model, the factors have different meanings. As an asset pricing model, the three factors are risk premiums. These factors represent systematic risks related to certain attribute of the firm. However, regarding the momentum effect, it is documented as a market anomaly. The characteristic of the momentum effect is the the past winners outperform the past losers. It suggests that this anomaly could be potentially related to time serial autocorrelation of stock returns. In some sense, the momentum effect is an evidence of stock price over-reaction or under-reaction to information. Therefore, the momentum effect is a violation to classic financial theories, and could hardly be considered as a risk factor.



## 1.2 Decomposition of momentum profit

Jegadeesh and Titman (1993) proposed a decomposition of momentum return based on a factorial model. In the study of the reversal effect, Lo and MacKinlay (1990) proposed an approach based on a portfolio model. Then Lewellen (2002) adapted Lo and MacKinlay's model in the study of momentum effect.

### 1.2.1 Jegadeesh and Titman's approach

Based on a single factor model, Jegadeesh and Titman (1993) decompose the momentum profit into three possible sources: the cross-sectional dispersion in expected returns; the time serial correlation of the factor; and the average serial covariance of the security returns' specific feature. The model and the decomposition are the following:

$$r_{it} = \mu_i + b_i f_t + e_{it}$$

$$\text{With: } E(f_t) = 0$$

$$E(e_{it}) = 0$$

$$\text{Cov}(e_{it}, f_t) = 0$$

$$\text{Cov}(e_{it}, e_{jt-1}) = 0$$

Where:

$r_{it}$  is the return on security  $i$  at time  $t$ ,

$\mu_i$  is the unconditional expected return on security  $i$ ,

$f_t$  is the unconditional unexpected return on a factor-mimicking portfolio,

$e_{it}$  is the firm-specific component of return,

$b_i$  is the factor loading of security  $i$ .

The momentum effect can be expressed as a positive correlation between past returns and present returns:

$$E\{(r_{it} - \bar{r}_t)(r_{it-1} - \bar{r}_{t-1})\} > 0$$

With the one-factor model given above, the momentum profits can be decomposed into three terms:

$$E\{(r_{it} - \bar{r}_t)(r_{it-1} - \bar{r}_{t-1})\} = \sigma_{\mu}^2 + \sigma_b^2 Cov(f_t, f_{t-1}) + \overline{Cov}_l(e_{it}, e_{it-1})$$

Where  $\sigma_{\mu}^2$  and  $\sigma_b^2$  are the cross-sectional variances of expected returns and factor sensitivities.

In this equation, the first item in the right side:  $\sigma_{\mu}^2$  represents the cross-sectional dispersion in expected returns. The second item:  $\sigma_b^2 Cov(f_t, f_{t-1})$  means that the momentum effect is sensitive to the serial correlation of the factor mimic portfolio. And the last item:  $\overline{Cov}_l(e_{it}, e_{it-1})$  represents the average serial covariance of the idiosyncratic components of stock returns.

### 1.2.2 Lo and MacKinlay's approach

Lo and MacKinlay (1990) use a portfolio approach to study the reversal effect. At time  $t$ , a zero cost arbitrage portfolio is formed. In such portfolio, the weight of stock  $i$  is the difference between stock's return and the equal weighted market portfolio's return in past  $k$  period.

$$\omega_{i,t}(k) = -\frac{1}{N}(R_{i,t-k} - R_{m,t-k})$$

Where:

$\omega_i$ : is the weight of stock  $i$  in the arbitrage portfolio.

$k$ : is the evaluation period.

$N$ : is the total number of stocks in the market.

$R_i$ : is the return of stock  $i$  at a given period.

$R_m$ : is the return of the equal weighted market portfolio at a given period.

The above weighting formula represents a reversal strategy. If a stock outperformed the market in the past  $k$  period, its weight in the portfolio is negative, suggesting short selling of this stock. If a stock underperformed the market in the past, its weight will be positive. Then, the profit of this portfolio is the sum of the return times the weight of each stock in the market.

$$\pi_t(k) = \sum_{i=1}^N \omega_{i,t}(k) R_{i,t}$$

Where:

$\pi$ : is the return of the arbitrage portfolio at a given period.

$\omega_i$ : is the weight of stock  $i$  in the arbitrage portfolio.

$k$ : is the evaluation period.

$N$ : is the total number of stocks in the market.

$R_i$ : is the return of stock  $i$  at a given period.

Replacing the weight by its formula and developing the function of portfolio profit, Lo and MacKinlay demonstrate that the expected return of the reversal portfolio is the following:

$$E[\pi_t(k)] = \frac{I' \Gamma_k I}{N^2} - \frac{1}{N} \text{tr}(\Gamma_k) - \frac{1}{N} \sum_{i=1}^N (\mu_i - \mu_m)^2$$

Where:

$\pi$ : is the return of the arbitrage portfolio at a given period.

$k$ : is the evaluation period.

$I$ : is the unit vector of  $(1, 1, \dots, 1)$ .

$\Gamma_k$ : is the auto-covariance matrix of all stocks for the past  $k$  period.

$\text{tr}()$ : is the operator of the trace of a matrix.

$N$ : is the total number of stocks in the market.

$\mu$ : is the expected return of a given asset at a given period.

By rearranging the above expression, the profit of the reversal portfolio can be decomposed into three parts:

$$E[\pi_t(k)] = C_k + O_k - \sigma^2(\mu)$$

Where:

$$C_k = \frac{1}{N^2} [I' \Gamma_k I - tr(\Gamma_k)]$$

$$O_k = - \left( \frac{N-1}{N^2} \right) tr(\Gamma_k)$$

$\sigma^2$ : is the variance operator.

The first term  $C_k$  is the auto correlation matrix minus its trace. This represents a time serial correlation of one stock with another stock. The second term  $O_k$  includes only the trace of the matrix, this term represent the auto correlation of each stock to itself. And the last term represents the cross-sectional dispersion of the expected stock returns in market.

### 1.2.3 Lewellen's analysis

Lewellen (2002) adapted Lo and MacKinlay's method, and applied this approach to study the momentum effect. The momentum portfolio is formed using the weighting of stock's excess return with respect to the equal weighted market portfolio:

$$\omega_{i,t} = \frac{1}{N} (R_{i,t-1} - R_{m,t-1})$$

Where:

$\omega_i$ : is the weight of stock i in the arbitrage portfolio.

N: is the total number of stocks in the market.

$R_i$ : is the return of stock i at a given period.

$R_m$ : is the return of the equal weighted market portfolio at given period.

Such a weighting suggests buying the winning stocks and shorting losing stocks to form a zero cost portfolio, which corresponds to the momentum strategy. Using the same development as Lo and MacKinlay's, the momentum profit can be decomposed into the following form:

$$E[\pi_t] = \left( \frac{N-1}{N^2} \right) tr(\Gamma) - \frac{1}{N^2} [I' \Gamma I - tr(\Gamma)] + \sigma^2(\mu)$$

Where:

$\pi$ : is the return of the arbitrage portfolio at given period.

$I$ : is the unit vector of  $(1,1,\dots,1)$ .

$\Gamma$ : is the auto covariance matrix of all stocks over the past period.

$\text{tr}()$ : is the operator of the trace of a matrix.

$N$ : is the total number of stocks in the market.

$\mu$ : is the expected return of given asset at a given period.

Such decomposition suggests that the momentum profit can originate from a positive autocorrelation of stocks' returns, the first term of the equation; from a negative cross-serial correlation between a stock and other stocks, the second term of the equation; and from the cross sectional dispersion of the expected stock returns, the last term of the equation.

Lewellen (2002) gives further analysis to some factors that might cause stock return autocorrelation: under reaction, overreaction, and time-varying risk premium. Lewellen adapts Summers (1986) and Fama and French (1988)'s representation of the stock price:

$$p_t = q_t + \varepsilon_t$$

Where:

$p$ : is stock's log price

$q$ : follows a random walk

$\varepsilon$ : is a white noise.

In such a configuration, the stock price at time  $t$  is determined by the present value of future dividend  $q$ , and other noise that could influence the stock price. The variation in dividend can be represented as:

$$q_t = \mu + q_{t-1} + \eta_t$$

Where:

$\mu$ : is the expected drift in dividend

$\eta$ : is a white noise in dividend

The stock return can be represented as:

$$r_t = \mu + \eta_t + \Delta\varepsilon_t$$

Where:

$$\Delta\varepsilon_t = \varepsilon_t - \varepsilon_{t-1}$$

In the case of under reaction, the stock price reacts slowly to the dividend information, which means that the non-expected variation of price  $\varepsilon$  partially reacts to the non-expected variation of dividend  $\eta$ , and also partially reacts to past  $\eta$ :

$$\varepsilon_t = -\rho\eta_t - \rho^2\eta_{t-1} - \rho^3\eta_{t-2} - \dots$$

Where :

$\rho$ : is the reaction rate, and  $0 < \rho < 1$

In such a configuration, the autocorrelation of stocks' returns is:

$$cov(r_t, r_{t-1}) = \left(\rho \frac{1 - \rho}{1 + \rho}\right)\Gamma$$

And in the case of under reaction, the momentum profit can be decomposed into the following form:

$$E[\pi_t] = \rho \frac{1 - \rho}{1 + \rho} \left[ \frac{1}{N} tr(\Gamma) - \frac{1}{N^2} I' \Gamma I \right] + \sigma^2(\mu)$$

Where:

$\pi$ : is the return of the arbitrage portfolio at given period.

$\rho$ : is the reaction rate, and  $0 < \rho < 1$

$I$ : is the unit vector of  $(1, 1, \dots, 1)$ .

$\Gamma$ : is the auto covariance matrix of all stocks for the past period.

$tr()$ : is the operator of the trace of a matrix.

$N$ : is the total number of stocks in the market.

$\mu$ : is the expected return of given asset at given period.

Since  $\rho$  is smaller than zero, the momentum profit will be strictly positive when under reaction exists.

To study the case of overreaction, Lewellen assumes that investors can correctly evaluate the information concerning the stock, but they will over estimate the information's influence on other stocks, thus, overreaction appears when valuing other stocks. In the price forming model, such an overreaction is represented as:

$$\varepsilon_t = B\rho\eta_t + B\rho\eta_{t-1} + B\rho^2\eta_{t-2} - \dots$$

Where:

$$\rho: 0 < \rho < 1$$

B: is the overreaction matrix with zero diagonal and positive off-diagonal terms

In such a configuration, the autocorrelation of stocks' returns will be:

$$\text{cov}(r_t, r_{t-1}) = \sigma_\eta^2(\rho - 1) \left[ B + \frac{2}{1 + \rho} BB' \right]$$

With some restrictions on the overreaction matrix, the momentum profit can be written as the following:

$$B = b[ii' - I]$$

Where:

b: is a scalar that  $0 < b < 1$

i: is a vector of ones

I: is a identity matrix

The momentum profit in the case of overreaction can be represented as:

$$E[\pi_t] = \sigma_\eta^2 \frac{b(\rho - 1)(N - 1)}{N} \left[ \frac{b}{1 + \rho} - 1 \right] + \sigma^2(\mu)$$

Where:

$\pi$ : is the return of the arbitrage portfolio at given period.

$\eta$ : is a white noise in dividend

$b$ : is a scalar such that  $0 < b < 1$

$\rho$ : is the reaction rate, and  $0 < \rho < 1$

$N$ : is the total number of stocks in the market.

$\mu$ : is the expected return of given asset at given period.

This equation suggests that the momentum profit is possible when the overreaction among the stocks is not too strong ( $0 < b < 1$ ).

Lewellen also studies the case where a time-varying risk premium causes excess covariance. Such scenario can be represented as:

$$\varepsilon_t = x_t \beta$$

Where:

$x$ : is an auto correlated scalar with mean zero, with  $x_t = \rho x_{t-1} + v_t$

$\beta$ : is the vector of the sensibilities of stock returns to risk premium

The autoregressive process  $x$  suggests that the change in risk premium is auto correlated. In addition, the change in risk premium is positively related to the change in dividend. Therefore, the autocorrelation of stocks can be represented as:

$$cov(r_t, r_{t-1}) = \rho_{\Delta x} \beta \beta' + (\rho - 1) \beta \delta'$$

Where:

$\rho_{\Delta x}$ : is the auto covariance of  $\Delta x$ , and  $\rho_{\Delta x}$  is negative

$\delta$ : is the covariance of the change in dividend and the change in risk premium

In the case of time-varying risk premium, the profit of momentum strategy can be represented as:

$$E[\pi_t] = \rho_{\Delta x} \sigma_\beta^2 + (\rho - 1) \sigma_{\beta, \delta} + \sigma_\mu^2$$

Where:



$\pi$ : is the return of the arbitrage portfolio at a given period.

$\beta$ : is the vector of the sensitivities of stock returns to risk premium

$\rho_{\Delta x}$ : is the auto covariance of  $\Delta x$

$\delta$ : is the covariance of the change in dividend and the change in risk premium

$\mu$ : is the expected return of a given asset at a given period.

In such a decomposition, the first term of the equation suggest that the momentum profit is related to the cross sectional dispersion of risk sensitivity. The sign of this term is determined by the auto correlation of the change in risk premium. The second term of the equation relates momentum profit to the correlation between the changes in risk premium and the changes in dividend. Since  $\rho_{-1}$  is negative, such a correlation reduces the momentum profit. The last term of the equation is, as always, the cross-sectional dispersion of stock returns.

### **1.3 Studies on momentum under risk-return framework**

After the momentum effect has been documented, many studies were conducted in order to explain this phenomenon. Under the risk-return framework, the studies are mainly focused on the following aspects: risk exposure of the momentum portfolio, autocorrelation of stock returns, the industry factor, and the trading cost.

#### **1.3.1 Potential sources of momentum profit**

The above section summaries the theoretical work of decomposition of momentum profit. In general, the existing literatures suggest two types of sources of momentum profit. The first type is related to momentum portfolio's exposure to risks. The zero-cost arbitrage portfolio might take more risks than presumed, hence could yield positive return as compensation. The second type of source is related to return auto correlation. Such an auto correlation could be found either at the market level, at the factor level, or at the idiosyncratic level.

### 1.3.1.1 Cross-sectional dispersion of stock returns

The cross-sectional dispersion in stock expected returns, named  $\sigma_{\mu}^2$ , appears in all the above decompositions of momentum profit. The momentum strategy consists of buying the stocks having highest actual return and selling the ones with the lowest actual return. Since stocks' actual returns and expected returns are closely related, the momentum strategy might pick up automatically the stocks with high expected returns. This means taking systematically more risks, and the momentum profit is the compensation for bearing these risks.

One way to study whether the cross-sectional dispersion contributes to momentum profit is to examine the factor loadings based on an asset pricing model such as the CAPM model or Fama-French Three Factors model. Jegadeesh and Titman (1993) calculated the average CAPM betas for the ten deciles of the stock, and the beta for the portfolio "winner – loser" is -0.08. Also in their study in 2001, they calculated the Fama-French factor sensitivities for "winner – loser" portfolio and they found -0.04 for market; -0.13 for SMB; and -0.22 for HML. All these sensitivities of the momentum portfolio are negative, and very close to 0. Such results suggest that the momentum strategy is not systematically taking more risks, so the momentum profit is not the compensation for bearing extra risks, hence, the cross-sectional dispersion in expected returns can be ruled out as the source of the momentum profit.

In a much more recent study, Novy-Marx (2012) also finds negative factor loadings for momentum profit in both CAPM model (beta = -0.52) and Fama-French model test (MKT = -0.41, SMB = -0.24, HML = -0.44). Mclean (2010) tests the momentum profit with other model, the factor loadings on size and book-to-market ratio are also negative.

However, the study shows that the loser stocks are more sensitive to SMB and HML factor, because during the holding period, the winner stocks raise and the loser stocks fall, this brings changes in stocks' market capitalization and the book-to-market ratio which affects the sensitivities to the factors.

Another way is to study the estimated alphas in the CAPM and Fama-French model regression, for the estimated alphas represent the returns of momentum strategy after adjusting risks for market, size or book-to-market ratio. The following equation shows a test on the risk adjusted return based on the CAPM model (Jegadeesh and Titman 1993):

$$r_{pt} - r_{ft} = \alpha_p + \beta_p(r_{mt} - r_{ft}) + e_{it}$$

Where:

$r_{pt}$ : is the momentum profit

$r_{ft}$ : is risk-free rate

$r_{mt}$ : is market return

$\alpha_p$ : is the risk-adjusted returns that will be estimated

Jegadeesh and Titman (2001) reports a result of 1.24 (t = 6.50) for CAPM alpha and 1.36 (t = -7.04) for Fama-French model alpha. The values of alpha are very close to the momentum profit. This means that the momentum profit can hardly be explained by any of the CAPM or Fama-French model factors, hence, the momentum profit cannot be interpreted as a compensation for taking extra risks. The same result is verified by other studies. Novy-Marx (2012) finds an intercept of 0.99 for CAPM model, and of 1.16 for Fama-French model; Mclean (2010) finds an intercept of 1.61.

Furthermore, Jegadeesh and Titman (1993) implement the 6-month/6-month momentum strategy on three size based subsamples: small medium and large; and on three beta based subsamples: low beta medium beta and high beta. This test is aimed to verify the robustness of the momentum strategy and also to study the potential sources of momentum profit.

Former studies indicate that firm size and beta are related to stock's risk and expected return. By dividing into subsamples, the dispersion of size and beta factors are reduced within subsamples, hence the dispersion of expected return is reduced too. If momentum profit is related to the cross-sectional dispersion of expected return, it is expected to see that the momentum profits are smaller when the strategy is implemented on subsamples. Alternatively, the momentum profit is likely to be related to other causes such as serial correlated firm idiosyncratic components.

The results on the subsample tests support the alternative hypothesis. They find that there is no significant difference in terms of momentum profits when the strategy is implemented on subsamples or on the entire sample. However, the momentum profit seems to

be related to firm size and beta. Small firms and high beta firms tend to earn higher momentum profit and large firms and low beta firms tend to generate lower momentum profit. This evidence indicates that the cross-sectional dispersion of expected returns is not a primary source of momentum profit. It is more likely related to serial correlation of firm idiosyncratic component of return.

Also, Jegadeesh and Titman (1993) studied the risk-adjusted returns of the momentum strategies implemented on different subsamples using a CAPM model. Their results show no significant difference between risk-adjusted returns and raw returns for the subsamples or the entire sample.

However, the study of Novy-Marx (2012) shows that momentum profit varies in size sub samples. Comparing to the full sample profit of 0.49%, the profits of the five increasing size deciles are respectively: 0.53%, 0.73%, 0.52%, 0.53%, and 0.24%. The risk adjusted return of momentum profit also varies in the sub samples in the same way as the raw profit, but the momentum effect is still significant.

In addition, Jegadeesh and Titman find that the after controlling for the market factor, the winner portfolio earns a statistically significant profit, while the profit of the loser portfolio is not significant. This indicates that the momentum profit is due to the buy side of the transaction rather than the sell side.

#### 1.3.1.2 Dynamic exposure to risks

In previous studies concerning cross-sectional dispersion, Jegadeesh and Titman show that the momentum profit cannot be explained by taking systematic risks, because in average, the beta of the momentum portfolio is not significantly different from zero (-0.08 for CAPM model, and -0.04 for Fama French model). Such results suggest that in general, the momentum portfolio has very little or no exposure to the market factor.

However, the momentum strategy is a rolling strategy which requires stock evaluation and adjustment of the portfolio every month, therefore, the average beta of the portfolio that has been calculated above, does not provide enough information about how the momentum portfolio is exposed to the market factor. And it is reasonable to study the portfolio's dynamic exposure to factors.

The momentum strategy involves changing factor exposure. In a single factor model such as CAPM model, in an upward market, the momentum strategy longs high beta stocks, which are the winner stocks, and shorts low beta stocks (the losers), while in a bear market, the strategy longs low beta stocks, which in this scenario, are winner stocks, and shorts high beta stocks (the losers). This suggests that instead of having a very weak factor exposure, as described by the static model, the momentum portfolio may have either positive or negative factor exposure in different period.

The theoretical study of Grundy and Martin (2001) on the dynamic exposure is based on a two-factor model:

$$r_i = \alpha_i + \beta_i r_{EW} + s_i OMT + e_i$$

Where:

$r_i$ : is the cumulative return of stock  $i$  from  $t-7$  to  $t-1$

$r_{EW}$ : is the cumulative excess return of equal weighted market portfolio from  $t-7$  to  $t-1$

$OMT$ : is the sum of the return of size decile one minus decile ten from  $t-7$  to  $t-1$

This model suggests that the cumulative excess return is related to two factors: the market factor  $r_{EW}$ , and the size factor  $OMT$  (One Minus Ten in the market capitalization deciles, with one being the smallest). In addition, the error is assumed to be independently identically and normally distributed:  $e \sim N(0, \sigma^2)$ , the factor loadings are assumed to follow a bivariate normal distribution:

$$\begin{pmatrix} \beta_i \\ s_i \end{pmatrix} \sim N \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\beta^2 & \sigma_{\beta s} \\ \sigma_{\beta s} & \sigma_s^2 \end{pmatrix} \right)$$

Then, the cumulative excess returns will follow a normal distribution:  $r_i \sim N(r_{EW}, \gamma)$  where, the standard deviation of the cumulative excess returns of stock  $i$  :

$$\gamma = \sigma_\beta^2 r_{EW}^2 + \sigma_s^2 OMT^2 + 2\sigma_{\beta s} r_{EW} OMT + \sigma_e^2$$

Grundy and Martin (2001) show that the conditional expected factor loading of the momentum portfolio is related to factor realization:

$$E\{\beta_W|r_{EW}, OMT\} - E\{\beta_L|r_{EW}, OMT\} = 2 \times 1.754 \frac{\sigma_\beta^2 r_{EW} + \sigma_{\beta_S} OMT}{\sqrt{\gamma}}$$

In this equation,  $\sqrt{\gamma}$ ,  $\sigma_\beta^2$ , and  $\sigma_{\beta_S}$  can only have positive values. So considering a scenario of an upward market, where  $r_{EW} > 0$ , and  $OMT = 0$ , the right side of the equation will be positive, which means that the momentum portfolio (winner minus loser) has a positive exposure to market risk. On the contrary, in a down market scenario, the momentum portfolio will have a negative exposure.

The empirical tests confirm this theory. Grundy and Martin (2001) use regressions based both on their two-factor model and on the Fama-French three-factor model with dummy variables that allow distinguishing the UP, FLAT and DOWN stat of factor realization for market factor, size factor and book-to-market factor. Their data are NYSE and AMEX stocks covering the period from 1926 to 1995. The results show that for the two-factor model, the average beta (exposure) of the momentum portfolio is -0.190, however, the dynamic betas of the momentum portfolio are -0.452 for down market, -0.005 for flat market, and 0.409 for up market. Similarly for the Fama-French model, the average market beta is -0.114, and the dynamic betas are -0.766, 0.061, and 0.354 respectively.

This empirical evidence confirms that the estimation of an average beta during the whole sample period cannot represent the real risk exposure of the momentum portfolio, because the positive and negative risk exposures in different market states are canceling each other. This means that the momentum strategy is riskier than originally considered. Naturally, the following question is whether the momentum profit can be explained by the dynamic exposure to market, size and book-to-market risks.

To answer this question, Grundy and Martin (2001) study the risk adjusted return while hedging out the dynamic exposure.

$$r_{W-L,\tau} = \alpha_{W-L} + \beta_{W-L} r_{EW,\tau} + s_{W-L} OMT_\tau + e_{W-L,\tau}, \tau = t, \dots, t + 5$$

This equation regress the momentum profit on the market realization and size factor, but instead of estimating the factor loadings  $\beta_{W-L}$  and  $s_{W-L}$ , they use the estimated values in the portfolio's holding period for each operation. This method is equivalent to hedge the factor exposure in each month, and the  $\alpha_{W-L}$  is the risk adjusted profit of the momentum strategy. If the momentum profit comes from the dynamic exposure to systematic risks, then the risk adjusted profit should be statistically insignificant. However, the test result shows that the

hedged momentum strategy earns even more profit than the non-hedged strategy. Comparing to the non-hedged strategy which generates a profit of 0.44% and 0.78% in the two-factor and three-factor models, the risk adjusted profit of the hedged strategy in the two-factor model is up to 1.34% ( $t=12.11$ ) and for the three-factor model, the profit is 1.48% ( $t=7.83$ ). This result shows that the momentum profit cannot be explained by bearing systematic risks that are related to the dynamic factor exposure.

Grundy and Martin (2001) give mainly two reasons for this increase in momentum profit. The first reason is that the hedging strategy improves greatly the portfolio's performance in the January. This strategy wins in 43 over 69, comparing to 15 wins over 69 for non-hedged strategy, and the average January return is increased from -5.85% to 0.49%. The second reason is that "hedging the strategy's dynamic factor exposure hedges out its implicit bet on momentum in the factors." This allows the strategy to be still profitable in the sub-sample period when the market experiences great losses, for example, the 1926 to 1945 period.

In practice, the hedged momentum strategy seems very appealing. But a perfect hedging is not possible, for the investors cannot have the holding period factor loadings at the beginning of the operation. A feasible way of hedging is to estimate the factor loadings over the formation period or using long-term historical data. Grundy and Martin (2001) show that this feasible hedged momentum strategy can generate a profit of 0.63% ( $t=3.30$ ) for the period from 1929 to 1995.

#### 1.3.1.3 Serial covariance and lead-lag effect

In the decomposition of the momentum profit, studies show that the profit could be caused by autocorrelation either at the market level, at the factor level, or at the idiosyncratic level. In the Jegadeesh and Titman's approach, it appears in the equation as the serial covariance of factor returns:  $Cov(f_t, f_{t-1})$ , and the serial covariance of idiosyncratic term:  $\overline{Cov}_i(e_{it}, e_{it-1})$ . In the studies of Lo and MacKinlay (1990) and Lewellen (2002), the serial covariance appears in the form of the auto covariance matrix:  $\Gamma_k$ . This matrix is then decomposed into its diagonal terms, representing the autocorrelation effect within each stock; and the off-diagonal terms of the matrix, representing the serial covariance between one stock and other stocks.

In the scenario when the factor is serially correlated the factor tends to keep its moving trend. In this case, the momentum strategy picks up automatically the stock with high sensitivity to the factor. In other words, the momentum profit comes from momentum of the fundamentals.

In the study of Jegadeesh and Titman (2001), the covariance of factor returns is transformed in the serial covariance of stocks' actual returns.

$$\sigma_b^2 Cov(f_t, f_{t-1}) = Cov(\bar{r}_t, \bar{r}_{t-1})$$

Intuitively, if the factor is serial correlated, we should find a serial correlation of stock returns in a general way. Jegadeesh and Titman find that a serial correlation of 6-month return equals -0.0028. This finding shows that there exists a very weak but negative correlation in stock returns. This does not support a covariance of factor returns.

Grundy and Martin (2001) performed tests directly on factor realizations using a first order autoregressive model:

$$f_{jt} = \theta_{0j} + \theta_{1j} \sum_{\tau=t-7}^{t-2} f_{j\tau} + u_{jt}$$

Where:

$f_{jt}$ : is the realization of factor j in month t

This equation tests if the realization of factor j in month t is related to the realization in past 5 months, and with 1 month of delay. The results are the following: for market factor,  $\theta_{1j}$  equals -0.0249 (t = -1.93), and for size factor,  $\theta_{1j}$  equals -0.0305 (t = -2.21). This result indicates that the factors do not have momentum effect; hence, the momentum profit cannot be explained by covariance of factor realization.

Delayed reaction to common factor can also cause momentum profits to appear. This has been demonstrated in the work of Lewellen (2002). If the market reacts to information with a delay, investors can anticipate future stock price based on current information. The following model is widely used to study the lead-lag effect:

$$r_{it} = \mu_i + \beta_{0,i}f_t + \beta_{1,i}f_{t-1} + e_{i,t}$$

Where:



$\mu_i$  is the unconditional expected return on security  $i$

$\beta_{0,i}$  is the sensitivity to the current factor realization

$\beta_{1,i}$  is the sensitivity to the previous factor realization

If  $\beta_{1,i} > 0$ , the stock continues to react in the same direction with previous factor realization, then the stock reacts with a delay. If  $\beta_{1,i} < 0$ , the stock continues to react in the opposite direction to previous factor realization, then the stock overreacts. This type of model has been used in Lo and MacKinlay (1990), Jegadeesh and Titman (1995), etc.

Based on the above lead-lag effect model, the momentum profit can be decomposed as:

$$E\{(r_{it} - \bar{r}_t)(r_{it-1} - \bar{r}_{t-1})\} = \sigma_\mu^2 + \delta\sigma_f^2$$

Where:

$$\delta = \frac{1}{N} \sum_{i=1}^N (\beta_{0,i} - \bar{\beta}_0)(\beta_{1,i} - \bar{\beta}_1)$$

$\bar{\beta}_0$  and  $\bar{\beta}_1$ : are the cross-sectional averages of  $\beta_{0,i}$  and  $\beta_{1,i}$ .

To guarantee a momentum profit,  $\delta$  has to be greater than 0, which means that:

$$(\beta_{0,i} - \bar{\beta}_0)(\beta_{1,i} - \bar{\beta}_1) > 0$$

This indicates that the stocks should be more sensitive to factor realization than the average level for both periods. Momentum profits are more likely to appear with high beta stocks, because large factor realization will be followed by a large delayed reaction, and the extent of the momentum profit will be related to the magnitude of the factor realization in the previous period. Therefore, a positive correlation is expected to be found between momentum profit and factor realization.

Empirically, Jegadeesh and Titman use the following model to test the correlation:

$$r_{p,t-6} = a_i + \theta r_{m,t-6}^2 + \mu_{it}$$

Where:

$r_{p,t-6}$ : is the momentum profit

$r_{m,t-6}^2$ : is the return of market portfolio

The return of the market portfolio is calculated by the value-weighted index in the month  $t-6$  through  $t-1$ . Over the sample period of 1965 to 1989, the estimate of  $\theta$  is  $-1.77$  ( $t = -3.56$ ). This result shows that there is not a positive correlation between momentum profit and factor realization, hence rules out any market-wide lead-lag effect as source of the momentum profits.

Other than the serial covariance of the common factors, the autocorrelation of stock return can be linked to serial covariance of idiosyncratic components. This potential source of momentum profit is related to stock's specific risk. In the CAPM paradigm, the stock's specific risk can be completely diversified, and should not be compensated if investor bears this kind of risk. Therefore, if the momentum profit comes from the idiosyncratic components, then the market is not efficient.

Regarding the idiosyncratic components, Grundy and Martin (2001) compare the performances of momentum strategies based on different selection standards. Based on a regression of the two-factor and the three-factor model, they found that the strategy generate a higher profit if the portfolio is selected on the stocks' risk adjusted return (the alpha), compared to a selection on the return related to factors ( $R_m + SMB + HML$ ). This result suggests that the momentum profit is more closely related to idiosyncratic components of stock return, than to systematic risks.

Some studies suggest that the idiosyncratic risk can limit arbitrage practice, and permit the momentum profit to exist. However, McLean (2010)'s empirical test shows no significant difference in momentum profit for high or low idiosyncratic risk groups. Such results suggest that momentum is not related to the idiosyncratic risk level, and rule out this explanation.

### **1.3.2 Industry momentum**

The studies above have ruled out the possibility that any common factor could explain the momentum profit. But these studies are carried out according to a simplified single factor model, and only the market factor and the idiosyncratic component are taken into account. However, in the real world, there could be a middle level between common factor and firm specific factor, for example: industry factors. Momentum profit can be found if some industry factors experience a serial correlation.

Moskowitz and Grinblatt (1999) studied the momentum effect in industry returns. Their method is to form industry portfolios by ranking stocks based on past industry returns, and to compare the returns between this strategy and a “random industry” strategy. They found that the industry portfolios can generate a significant momentum profit, while the “random industry” strategy earns a return close to zero. So they conclude that industry momentum could contribute to the momentum effect.

Grundy and Martin (2001) reexamine the industry momentum using the same method as Moskowitz and Grinblatt (1999). However, they find that the industry momentum profit is very sensitive to the way the portfolio is constructed. If the holding period begins directly after the ranking period, the industry momentum strategy can earn a significant profit of 0.78%. But if the holding period begins one month after the ranking period, the industry profit is close to zero. Considering that the momentum strategy with individual stocks earns consistently a significant profit of 0.79%, it is not so sure that industry momentum is the source of momentum profit.

### **1.3.3 Trading cost**

The transaction cost can have an important influence on momentum profit, because the relative strength strategy requires frequent transactions. Jegadeesh and Titman (1993) affirms that the momentum strategy is still profitable even after including transaction costs in their study over the period from 1965 to 1989. They suggest that the momentum strategy results in a turnover of 84.8% semiannually. Considering a 0.5% of one-way transaction cost, the risk-adjusted return of momentum strategy will still be 9.29% per year.

Lesmond Schill and Zhou (2004) suggest that Jegadeesh and Titman underestimated the transaction cost for implementing the momentum strategy. They tested the profitability of three specific momentum strategies, and used more sophisticated methods to estimate the transaction costs including Bid-Ask spread, commissions, short sale costs etc. They find that the momentum portfolio is composed of stocks with high trading costs, and the momentum return is related to the portfolio’s transaction costs.

McLean (2010) studies stock idiosyncratic risk. This factor is often used as a proxy for arbitrage cost. High idiosyncratic risk implies high holding costs for arbitrage operations, which creates additional trading friction in the market. Such an extra market friction may

cause the momentum profit to exist. However, McLean (2010)'s empirical test shows no significant difference in momentum profit for high or low idiosyncratic risk groups.

### 1.3.4 Other determinants

Studies have shown that the momentum effect is likely to be related to firm specific information, and it is interesting to find out which type of information has greater influence on the momentum effect. Jegadeesh and Titman find in their study that small firms tend to generate greater momentum effect. Lesmond, Schill and Zhou (2004) suggest that the most important cross-sectional predictor is stock price level. One thing in common between firm size and price level is the transaction cost; hence the above results may suggest that the momentum profit is at least partially canceled by transaction costs.

In terms of information, Hong, Lim, and Stein (2000) find that the momentum profit is related to analyst coverage. Even after controlling for size, the firms with low analyst coverage generate a higher momentum profit. Hong and Stein (1999) suggest that slow public information dissemination can increase momentum profits. And Daniel, Hirshleifer and Subrahmanyam (1998) suggest that more private information will also increase momentum profits. These results are highly consistent. Intuitively, firms with less analysts following will have their information revealed slower, and consequently have more private information.

Daniel and Titman (1993) studied the relation between momentum and book-to-market ratio. They find that momentum profits are significantly higher when the strategy is implemented on growth stocks (low book-to-market ratio) rather than value stocks (high book-to-market ratio). This may be due to the fact that growth stocks are harder to evaluate than value stocks, hence have greater volatility than value stocks.

Lee and Swaminathan (2000) documented a correlation between momentum profits and turnover. The momentum strategy earns more profit when implemented on stocks with high turnover. Stivers and Sun (2010) report that the momentum profit is related to the economic cycle, and also related to market cross-sectional dispersion level on stock returns.

## 1.4 Irrational model of momentum

This section reviews some of the major studies on the momentum effect in the field of the behavior finance. This latter took a different perspective with respect to the classic financial theories by dropping the assumptions such as strict rationality and homogeneity in investors' expectation. Behavioral finance adopts findings in psychology suggesting that individual decision making is influenced by factors such as sentiments, heuristics etc..

Emerging and development of behavioral finance is closely related to empirical evidence found in financial market that is against market efficiency. The momentum effect is one of such anomalies. Behavioral finance suggests that the momentum is a failure of market efficiency. Due to certain behavior bias of investors, stock prices fail to incorporate new information instantaneously, hence stock prices could show trend of underreaction or overreaction after a piece of news is published. Such a trend then contributes to the momentum effect.

Theoretical models have been formed to describe behavior factors and were aimed to provide generic explanation to market anomalies. Barberis Shleifer and Vishny (1998) proposed a behavior model based on confirmation bias of investor. Hong and Stein (1999) proposed a model based on heterogeneity of investor. Both behavior models are able to predict overreaction and underreaction of stock prices.

### 1.4.1 Underreaction and overreaction in stock prices

The stock price underreaction or overreaction suggests that stock price incorporates information with a delay. The stock price underreaction describes the phenomenon of short term return autocorrelation after a release of news. For example, after good news is released, instead of instantaneously jumping to the new level, stock price gradually raises to the new level. Therefore, it is possible to observe a positive drift on stock returns after good news is released. In the work of Barberis Shleifer and Vishny (1998), they propose a definition for underreaction:

$$E(r_{t+1}|z_t = G) > E(r_{t+1}|z_t = B)$$

Where:

$z_t = G$  represents good news at time  $t$ ;

$z_t = B$  represents bad news at time  $t$ ;

The expected return of a stock in the period following good news is greater than the expected return in the period following bad news. If the stock price incorporates information instantaneously, no information will be incorporated into stock price at  $t+1$ , therefore, the expected return will be the same. When there is underreaction, the stock return in the period following good news will be higher than normal due to slow information incorporation; while stock return following bad news will be lower than normal.

The stock return overreaction describes the phenomenon that stock return show a reversal over the long term. Stocks with very bad performance over the past period tend to yield high return. The opposite is also true. Stocks with very good performance in the past tend to yield low return. This phenomenon suggests that after a series of successive good news or bad news, investors tend to form expectation upon past experience, and give overly high expectation for past winner stocks and overly low expectation for past loser stocks. However, when news is finally released, the expectations are proven to be incorrect. Then stock returns revert to correct level.

In the work of Barberis Shleifer and Vishny (1998), they gave the definition of stock return overreaction on the same principle:

$$E(r_{t+1} | z_t = G, z_{t-1} = G, \dots, z_{t-j} = G) < E(r_{t+1} | z_t = B, z_{t-1} = B, \dots, z_{t-j} = B)$$

Where:

$z_t = G$  represents good news at time  $t$ ;

$z_t = B$  represents bad news at time  $t$ ;

In this formula, the terms  $z_t z_{t-1} \dots z_{t-j}$  represent a series of past events,  $r_{t+1}$  represents the stock return over the next period. If a stock experiences overreaction due to successive past good news, its price has been already inflated, hence at  $t+1$ , the stock price should return to a correct level, and yields a relatively lower return over  $t+1$ . The same logic applies to the case of past bad news; and in this case, the stock return over  $t+1$  should be relatively higher.

The momentum effect, stock return autocorrelation, and underreaction/overreaction are closely related. Stock underreaction is confirmed by positive autocorrelation of stock return over intermediate time horizon, usually 12 to 18 months. Stock overreaction is confirmed by

negative return autocorrelation over long time horizon, usually 3 years. Zero-cost portfolios are formed to exploit return autocorrelation, and could systematically earn positive profit.

However, there is one point that needs to be clarified about stock return autocorrelation. This term can refer to either a time serial autocorrelation, or a cross sectional autocorrelation. Note that the descriptions of stock underreaction and overreaction are based on time serial autocorrelation. It concerns only a given stock, and in the underreaction case, after good news is released, the stock return will yield a series of returns that are higher than **its own average**. The same is true in the overreaction case. If a stock had successive good news, in the following periods, the stock will yield a series of returns that are lower than its own average return. The definitions of underreaction and overreaction given by Barberis Shleifer and Vishny are also based on such a time serial autocorrelation of stock returns. However, in empirical studies, the anomaly documented corresponds to cross sectional autocorrelation. Take the momentum effect as an example: past winner stocks outperform past loser stocks. It suggests that if a stock has a higher return than the **cross sectional average of all stocks** in the market in the past, this stock will be likely to have a higher return than the cross sectional average in the future.

Lewellen's decomposition of the momentum profit (Lewellen 2002, mentioned in section 1.2.3) established the link between the time serial and cross sectional autocorrelations of stock return. This model shows that by introducing a time serial autoregressive factor that represents the underreaction/overreaction in stock prices in the portfolio formation, this autoregressive factor also appears in the momentum profit. This study suggests that stock underreaction or overreaction could contribute to the momentum effect. However, this model cannot provide an answer to why underreaction or overreaction could first exist. The cause of underreaction or overreaction might be very complex.

#### **1.4.2 Behavior model based on cognitive biases**

A behavior model aims to explain market anomalies such as price underreaction and overreaction by behavior factors, which means relaxing some of the basic hypotheses of the classic theory, including complete rationality of individual or investors homogeneity. Behavior models search supporting evidence from studies in the field of psychology. Research in psychology has shown conclusive evidence that individual decision making suffers from cognitive biases such as: individualism, over-confidence, confirmation bias,

conservatism etc. These studies in psychology questioned the validity of the rational individual hypothesis in classic theory: if individual decision making is biased, the classic theories will not be able to perfectly describe what actually happens in financial markets, and financial models should take behavior bias into consideration. However, implementing such an idea is very difficult. Barberis Shleifer and Vishny (1998), Hong and Stein (1999) separately established behavior models that incorporate different aspects of behavior bias: cognitive heuristics and investor heterogeneity, into stock return. And their works also show that these models could reproduce market anomalies of underreaction and overreaction.

#### 1.4.2.1 Heuristics model

Barberis Shleifer and Vishny (1998) are interested in two important phenomena that had been documented in the field of psychology: conservatism and the representativeness heuristic. The conservatism bias refers to the phenomenon that individuals are reluctant to change their beliefs when facing new evidence. The representativeness heuristic bias suggests that individuals cannot fully understand the complexity of an uncertain event, and then subjectively use simplified observations to represent the uncertain event.

The model of Barberis Shleifer and Vishny captures the above heuristic bias by the following configuration: the earning stream of a firm follows a random walk; the discount rate  $\delta$  is constant; investors in the market are risk-neutral and do not understand the randomness of earning realization, but believe that the earning realization depends on the state of the economy. It could be either a trend state or a reversal state. In a trend state, if present earning realization is positive/negative, the earning in the next period is also likely to be positive/negative. In a reversal state, the earning in the next period is likely to have the opposite sign to present earning realization.

Table 1.4-1: Earning realization in the reversal and the trend states

| Reversal | $Y_{t+1}=y$ | $Y_{t+1}=-y$ | Trend    | $Y_{t+1}=y$ | $Y_{t+1}=-y$ |
|----------|-------------|--------------|----------|-------------|--------------|
| $Y_t=y$  | $\pi_L$     | $1-\pi_L$    | $Y_t=y$  | $\pi_H$     | $1-\pi_H$    |
| $Y_t=-y$ | $1-\pi_L$   | $\pi_L$      | $Y_t=-y$ | $1-\pi_H$   | $\pi_H$      |

Source: Barberis et al./ Journal of Financial Economics 49 (1998) page 321;  $\pi$  is the corresponding probability, with  $\pi_L$  smaller than 0.5,  $\pi_H$  larger than 0.5.

Investors also believe that the reversal state and the trend state can transfer into each other with a given probability:



Chapter 1:  
Literature review

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Table 1.4-2: Transition of the reversal and the trend states

|                    |                      |                      |
|--------------------|----------------------|----------------------|
|                    | S <sub>t+1</sub> = 1 | S <sub>t+1</sub> = 2 |
| S <sub>t</sub> = 1 | 1 - λ <sub>1</sub>   | λ <sub>1</sub>       |
| S <sub>t</sub> = 2 | λ <sub>2</sub>       | 1 - λ <sub>2</sub>   |

Source: Barberis et al./ Journal of Financial Economics 49 (1998) page 322; State 1 is the reversal state, State 2 is the trend state; λ indicates corresponding probability.

In Barberis Shleifer and Vishny's model, the only deviation from the classic hypothesis of investor rationality is that investors do not know the randomness of the market. Other than that, investors make their decisions based on the probability calculated by Bayes law. Investors observe past realization of earnings, and calculate the probability that currently the market is at the reversal state:  $q_t = \Pr(s_t=1|y_t, y_{t-1}, q_{t-1})$ . The decision making process of the investors is rational by the standards of classic rationality, but this rational process is based on investors personal belief rather than the reality.

In the configuration introduced by Barberis Shleifer and Vishny (1998), investors believe in an alternating trend/reversal states of earning realization, while in fact the earning realization is purely random. In fact, this model can be applied to the cases where investors fail to understand the way in which the market works; and the cause of such misunderstanding cannot only be the conservative bias proposed by Barberis Shleifer and Vishny, other factors that obstruct the information transmission or hamper the formation of correct understanding of the market could also be described by this model. Such factors could be behavioral, such as over confidence, optimism/pessimism etc.; the factors could also be pure informational: the true market mechanism is too complicated for investors to understand; or it is too expensive to cover every piece of information.

Despite the fact that investors are under the influence of conservatism and representativeness biases, they still use classic pricing model to evaluate assets where the current price of an asset equals the present value of expectations of future cash flows calculated at a given discount rate:

$$P_t = E_t \left\{ \frac{N_{t+1}}{1 + \delta} + \frac{N_{t+2}}{(1 + \delta)^2} + \dots \right\}$$

Where:

P<sub>t</sub>: is asset price evaluated at time t

N: is earning stream

$\delta$ : is discount rate

However, since investors are influenced by behavior biases, and believe that the earning realization depends on the alternating reversal/trend state of the market rather than a random distribution, the asset price evaluated by such investors will differ from the true value of the asset. Barberis Shleifer and Vishny demonstrate that the asset price formed by the above investors has the following expression:

$$P_t = \frac{N_t}{\delta} + y_t(p_1 - p_2q_t)$$

Where:

$y$ : is the unexpected earning

$q$ : is the probability that the current state is a reversal state based on past information

$p_1 p_2$ : are constants representing the earning distribution probabilities

The asset pricing formula of the investors under behavior biases can be interpreted into two parts: the first part is  $N_t/\delta$ . This term is discounted the earning realization. This part consists of the correct value of the asset. The rest of the formula is composed by the parameters representing investors' subjective belief: the parameters  $p_1 p_2$  include how earnings are distributed in the reversal/trend states ( $\pi_L \pi_H$ ), the probability of the states transfer into each other ( $\lambda_1 \lambda_2$ ). This second part of the formula consists of the deviation of asset price from its true value.

Barberis Shleifer and Vishny also run simulations to demonstrate that this behavior model is able to generate price overreaction or underreaction under certain settings of  $\pi_L \pi_H \lambda_1 \lambda_2$ . The definitions of price overreaction or underreaction have been given by Barberis Shleifer and Vishny at the beginning of their work. The underreaction is found when the expected return following good news is higher than the one following bad news. The overreaction is found when the expected return following a series of good news is lower than the one following a series of bad news. Lam Liu and Wong (2012) provide a recent development of this behavior model, and show that this model is also able to explain other market anomalies such as excess volatility.

The behavior model established by Barberis Shleifer and Vishny (1998) provides a general description on asset price formation when investors do not have correct estimation of

asset fundamentals. However, this model is very difficult apply on empirical tests because it is almost impossible to measure investor behavior biases in an empirical context.

#### 1.4.2.2 Heterogeneity model

Hong and Stein (1999) also established a behavior model. Their model looks at the influence of interacting heterogeneous investors, namely the interaction between news-watchers and momentum traders.

The model assumes that an asset generates a single dividend payoff at the liquidating time T in a random way:

$$D_T = D_0 + \sum_{j=0}^T \varepsilon_j$$

Where:

$\varepsilon_j$ : is a random variable representing update of dividend information at time j,

The news-watchers closely follow the news of dividend diffusion. However the ability for news-watchers acquiring information is limited, news-watcher can only observe partial information in time, but the news-watcher will gradually receive the rest of information in the following periods. This setting suggests that information released at a given time spot will need several time periods to spread out on the market. The news-watchers evaluate asset prices only from fundamentals, but since the information about dividend diffusion occurs over time, Hong and Stein suggests that the evaluation by the news-watchers will be:

$$P_t = D_t + \{(z - 1)\varepsilon_{t+1} + (z - 2)\varepsilon_{t+2} + \dots + \varepsilon_{t+z-1}\}/z - \theta Q$$

Where:

z: is the rate at with information spreads

$\theta$ : is investor's risk aversion level

Q: is the supply of the stock

This formula suggests that the asset price incorporates information gradually over time, hence is consistent with the price underreaction.

On the other hand, momentum traders also exist in the market. The momentum traders are trend chasers, they make forecast on asset prices based on price evolution in the last period ( $P_{t-1} - P_{t-2} = \Delta P_{t-1}$ ), and then they place trading orders to make transaction with news-watchers. Opposite to the news-watchers, the momentum traders have short investment horizon, and do not consider the fundamentals of the asset. Hong and Stein suggest that the activity of the momentum traders will influence asset pricing by introducing variation in asset supply, and the asset price will be:

$$P_t = D_t + \frac{\{(z-1)\varepsilon_{t+1} + (z-2)\varepsilon_{t+2} + \dots + \varepsilon_{t+z-1}\}}{z} - \theta Q + jA + \sum_{i=1}^j \phi \Delta P_{t-i}$$

Where:

A: is a constant related to trading orders

j: is the period for which momentum traders hold their positions

$\phi$ : is the elasticity of parameter

The last terms in this formula is the variation in asset supply due to the momentum traders. Hong and Stein demonstrate that equilibrium can be achieved under certain conditions on the parameters in the above formula. Moreover, this model is also able to generate short term price underreaction and long term price overreaction. The price underreaction is rooted in the configuration of slow spreading information about asset fundamentals. The price overreaction is related to trend chasing transactions made by the momentum traders. Hong and Stein also demonstrate by simulations that the scale of the underreaction/overreaction of asset prices is linked to the trading horizon of momentum traders, the risk tolerance of momentum traders and the information diffusion speed.

In terms of empirical implications on the momentum effect, Hong and Stein's model suggests that the momentum strategy will be more effective on the stocks with slow information diffusion. They also propose two proxy variables to represent information diffusion of an asset: firm size and residual analyst coverage. They also point out that one should be careful when using these proxies, because they also contain other factors besides information diffusion.

### 1.4.3 Behavior model of momentum based on the disposition effect

#### 1.4.3.1 The disposition effect

The disposition effect is commonly known as a behavior bias that investors are likely to sell winning stocks too soon while they are likely to keep losing stocks too long. Shefrin and Statman (1985) provided some empirical evidence from both individual investors and mutual fund data. Their study shows that about 58% of all the transactions were made in a gain situation. Odean (1998) established an empirical measurement of the disposition effect. If the Proportion of Gains Realized (PGR) is statistically higher than the Proportion of Losses Realized (PLR), the disposition effect is observed. The variables PGR and PLR are defined as follow:

$$\frac{\textit{RealizedGains}}{\textit{RealizedGains} + \textit{PaperGains}} = \textit{ProportionofGainsRealized}$$

$$\frac{\textit{RealizedLosses}}{\textit{RealizedLosses} + \textit{PaperLosses}} = \textit{ProportionofLossesRealized}$$

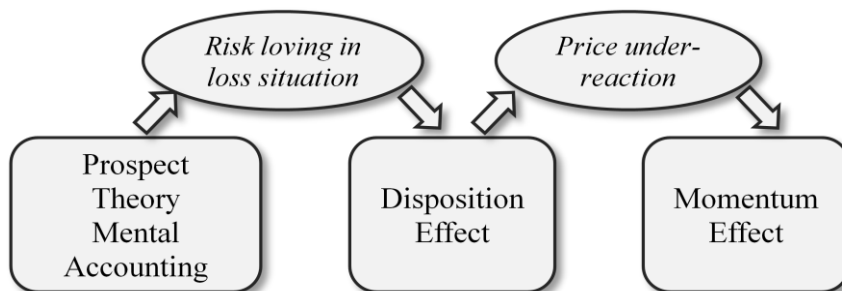
Using such a measurement Odean tested on the data from brokerage a house of randomly selected 10000 customers' trading records. The results show that PGR is statistically higher than PLR by 0.05. The results also show that the disposition effect is reversed in December, and investors are more likely to realize small amount of gains or losses. More interestingly, study on the ex-post stock return shows that the stocks with paper gain will continue to have positive return over 6 months to one year after they are sold. Stocks with paper losses will continue to have negative return. Such an empirical evidence matches the description of selling the winning stocks too soon while holding the losing stocks too long.

Intuitively, such a behavior bias might cause effects contrary to that of the momentum effect. If investors sell winning stocks too soon, as suggested by the disposition effect, future stock returns of the past winning stocks should go down instead of continuing to win. For the past losing stocks, the existence of the disposition effect should prevent their prices from continuing to fall because of investors' reluctance to sell the stocks.

1.4.3.2 A model based on under-reaction caused by the disposition effect

Grinblatt and Han (2005) established a link between the behavior bias of disposition effect and the market anomaly of momentum effect through the dynamic of stock price evolution. If the disposition effect exists on the market a winning stock will suffer from extra selling pressure from the holders and its price will decrease. Since the disposition effect is purely behavioral and has no influence on future cash flow of the stock, a low price means that the stock is under-valued. For a losing stock, its price will be too high because of a lack of selling pressure, suggesting that it is over-valued. In the long run, such a miss-pricing of stocks will eventually disappear as a result of market correction, therefore, for a winning stock that is under-valued, its price will gradually move up to the correct level; while for a losing stock that is over-valued, its price will finally drop to meet its fundamental value. Following this reasoning, the disposition effect creates an extra selling pressure for winning stocks and an insufficient selling pressure for losing stocks, such pressures work as resistance or friction on price evolution for winning or losing stocks, and makes it longer for prices to reach the correct value. Due to the under-reaction induced by the disposition effect, the stock price of a past winning stock will continue to go up and stock price of a past losing stock will continue to drop. Such a price evolution corresponds to the momentum effect. To conclude, the disposition effect could cause stock price under-reaction, and this latter is the direct reason for forming the momentum effect.

Figure 1.4-1: Illustration of the Grinblatt and Han's model



In the empirical studies in Grinblatt and Han's (2005) work, the key variable is the Capital Gain Overhang (CGO here after). This variable is defined as:

$$CGO_t = (P_t - R_t)/P_t$$

Where:

CGO: is the capital gain overhang

P: is stock price

R: is the reference price

t: indicates the time period

The Capital Gain Overhang is an incremental value of a stock with respect to a reference price, at which the investor had bought the stock. This variable measures the unrealized gain or loss that an investor is facing, according to Grinblatt and Han, it is a proxy for the disposition effect.

Grinblatt and Han's tests show evidence regarding the link between the disposition effect and the momentum effect. The double sorting test compares the average return of the 25 groups (5 by 5 groups) divided according to the past one-year return and the Capital Gain Overhang. The test first creates five groups according to past return, and then within each past return group, five CGO sub-groups are created. It is observed that within each past return group, the average return of CGO sub-groups increases with CGO, and the average return difference between the top CGO group and the bottom CGO group is significantly positive. Conversely, when the double sorting is done by first dividing five CGO groups and then five past return sub-groups within each CGO group, the average returns are not significantly different between past winners and past losers. The momentum effect is not observed when controlling for the CGO level.

In addition, Grinblatt and Han tested the predictability of CGO over stock return using Fama-Macbeth regression on the following model:

$$r = a_0 + a_1 r_{-4:-1} + a_2 r_{-52:-5} + a_3 r_{-156:-53} + a_4 \bar{V} + a_5 s + a_6 cgo$$

Where:

$r$  is the weekly stock return;

$r_{-4:-1}$  is the cumulative return from week -4 to week -1;

$r_{-52:-5}$  is the cumulative return from week -52 to week -5;

$r_{-156:-53}$  is the cumulative return from week -156 to week -53;

$\bar{V}$  is the average turnover from week -52 to -1;

$s$  is the market capitalization of week -1;

$cgo$  is the capital gain overhang of week -1;

In this model, the control variable  $r_{-52:-5}$  represents the stock return autocorrelation over one-year horizon, during which the momentum effect is expected to be observed. In the regression without the variable CGO, the coefficient of  $r_{-52:-5}$  is 0.0014 (t-value: 3.57). Such significant positive correlation suggests that current stock returns tend to move in the same direction as past returns over one-year, and corresponds to the momentum effect. However, in the regression including the variable CGO, the coefficient of  $r_{-52:-5}$ , indicator of the momentum effect, becomes insignificant. The results from these two tests suggest that after controlling for the Capital Gain Overhang, a measurement of the disposition effect, the momentum strategy no longer yields significant profits.

Frazzini (2006) developed further the idea of price under-reaction caused by the disposition effect. If investors tend to sell a winning stock as a result of the disposition effect, there should be a stronger under-reaction of stock price when good news is announced, but there should be no under-reaction when bad news is announced. For winning stocks, the disposition effect hampers price rise but not price decreases; and for losing stocks, the disposition effect hinders prices drop but not price rise. There should be a stronger under-reaction of losing stocks when bad news is released, while no under-reaction when good news is released.

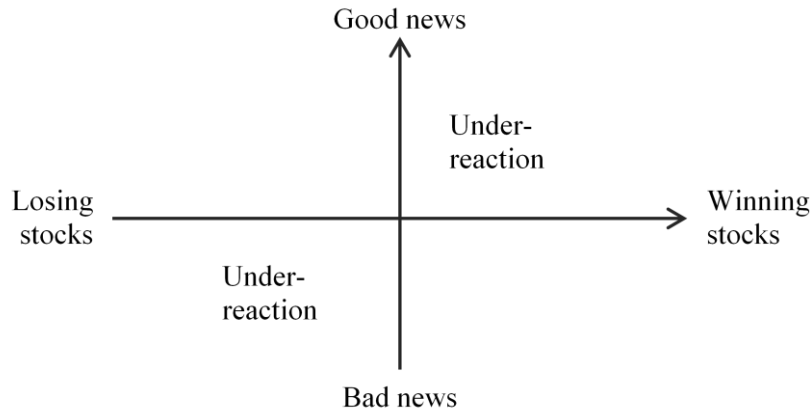
In the empirical test, Frazzini (2006) examines the stock price under-reaction to information release by studying the post-earning announcement drift. If winning stocks under-react to good news, there should be a positive drift after the announcement, and if losing stocks under-react to bad news, there should be negative drift. In order to determine the winning stocks and the losing stocks, Frazzini also uses the variable Capital Gain Overhang but it is calculated using actual mutual fund holding data. Such data provides reliable measurement of the reference price. The test on the post-earning announcement drift consists of forming a zero-cost portfolio by longing the stocks that are expected to have a positive drift and shorting the stocks that are expected to have a negative drift.

The result shows that the portfolio of good news high CGO stocks minus bad news low CGO stocks yields a significant positive profit after adjusting for risks. The control group which is a portfolio of good news low CGO minus bad news high CGO does not yield



significant profit. The study of Frazzini (2006) does involve a direct test on the link between the disposition effect and the momentum effect but is clear evidence on the stock price under-reaction due to the disposition effect. The results of this study help to better understand the crucial intermediate link between the disposition and the momentum effects.

Figure 1.4-2: Illustration of Frazzini's hypothesis



Some studies explore the subject of investor sophistication. Hur Pritamani and Sherma (2010), Bhootra and Hur (2012) suggest that the disposition effect, as a behavioral bias, should be more visible among the individual investors than among institutional investors. Prior study of Grinblatt and Han (2005) has established the link between disposition and momentum effects. Based on this result, the momentum effect that is induced by the disposition effect should be stronger among the stocks with high fraction of individual investors. Hur Pritamani and Sherma (2010) use the fraction of share outstanding owned by individual investors and the fraction of trading volume due to individual investors as measurements of investor sophistication. Bhootra and Hur (2012) use the stock co-integration as another measurement. Both studies find evidences supporting their hypotheses.

Despite a relative success of this model, the empirical studies do not provide sufficiently conclusive evidence to link the momentum effect to the disposition effect and to the prospect theory. The regression test of Grinblatt and Han is performed directly on stock return, but not on the profit of the momentum strategy. This test could explain the middle term return autocorrelation by the Capital Gain Overhang, but the essential idea of the momentum effect is that past winners out-performs past losers. Time serial autocorrelation of stock return cannot capture this characteristic.

Similarly, the work of Frazzini (2006) provides convincing evidence on the under-reaction induced by Capital Gain Overhang, but there is no test performed directly on the

disposition effect and the momentum profit. Moreover, the tests of Frazzini focus on the under-reaction over short term horizons, generally from one week to three months. Such time horizons could not match the momentum effect which is generally documented over longer time horizons, from six months to one year. Hur Pritamani and Sherma (2010), Bhootra and Hur (2012) adopt the results of Grinblatt and Han's study, and assume that the momentum is caused by the disposition effect. Their results directly confirm that the momentum profit is higher for stocks held by individual investors, but they do not provide evidence linking the disposition and momentum effects.

Overall, the behavioral model proposed by Grinblatt and Han is still an interesting approach to explore the link between behavioral factors and the momentum effect. Particularly, the variable Capital Gain Overhang carries valid economical meaning of the unrealized gain or loss that investors are facing. Under the framework of prospect theory, investor's decision making is determined by this factor. Therefore, this dissertation will study the unrealized gain or loss under alternative utility theories, and aims to find the influence of unrealized gain or loss over stock return and momentum effect.

#### 1.4.3.3 Opposing evidence

The empirical study of Birru (2015) suggests that the disposition effect cannot be the only cause of the momentum effect. This work approaches the subject of the disposition and the momentum effects from the angle of stock split. Recall that the disposition effect suggests that investors consider their purchase price as a reference, when the current stock price is higher than the reference the investors tend to sell the stock, and when the current stock price is lower than the reference the investors tend to hold the stock. However, when a stock split takes place, the stock price will be divided by the split ratio. Hence the price at which the investors have purchased the stock will be no more a valid reference for distinguishing gain or loss. If the investors fail to correctly update their reference, they will mistakenly think a winning stock as a losing one, or conversely think a losing stock as a winning one. If it is the case, then after an event of stock split, the disposition effect should vanish or at least, attenuate. If the momentum effect is completely caused by the disposition effect, then after the stock split, when the disposition effect is attenuated, the momentum effect should also diminish.

The work of Birru (2015) first examines if the disposition effect persists after a stock split. The intuition is that when a stock split happens, the reference price of the investor should be adjusted downward. If the investor still use the old reference price which is much higher than it should be, the investor will be more likely to consider a winning stock as a losing one. Hence, instead of selling the stock, the investor will keep the stock. Therefore, for a winning stock, there should be less selling transactions if the stock has recently undergone a split. The empirical test involves performing a logit regression of investor's selling action on the gain, and introduces the split as a dummy variable. The baseline of this regression is the following:

$$Sale_{i,t} = \beta_0 + \beta_1 Gain_{i,t} + control\ variables + \varepsilon_{i,t}$$

The variable  $Sale_{i,t}$  is a binary variable. It takes the value 1 if a sale happened at time  $t$ , 0 if not. The coefficients in this model can be interpreted as the probability that a sale could occur conditionally to a certain factor. This regression is performed on the data at the individual investor level from 1991 to 1996. The regression result shows that the coefficient for Gain is 0.391, while the coefficient for Split×Gain is -0.366. Both coefficients are significant at 1% level. This result suggests first the the disposition effect is confirmed in general cases. As the baseline, the coefficient is up to 0.4. However, when introducing the stock split, this factor reduces the effect of the Gain by -0.366. This result confirms that stock splits could disrupt the disposition effect. When stock split takes place, investors fail to update their reference price, and misjudge the winner or loser stocks. As a result, the disposition effect has been largely attenuated after a stock split.

Next, the author studies the stock performance after the stock split event. Existing theory suggests that the disposition effect could cause extra selling pressure on the winning stocks causing such stocks to be under valued. According to the above test result, after a split, the disposition effect is disrupted, without the extra selling pressure there should be a rebound in the returns on the under valued stocks. Such a return rebound should be greater for stocks that had large gain before the split, because large gain implies a strong disposition effect and hence strong under-valuation of the stock.

To verify the influence on stock performance, the empirical tests performed by Birru focus on the adjusted abnormal return of stocks over two days after the split event. These tests use the data on a market level. The data includes the stocks on NYSE and AMEX from 1967 to 2011. The gain of a stock is estimated using the method introduced by Grinblatt and Han

(2005) for calculating the Capital Gain Overhang. By sorting the winning stocks into five quintiles according to their Capital Gain Overhang, the quintile with the highest gain has significantly higher adjusted abnormal returns than the quintile with the lowest gain. The difference is up to 1.23% on the basis of two days. The tests also include a regression of the adjusted abnormal return on Capital Gain Overhang. The regression shows a positive correlation between the adjusted abnormal returns and the Capital Gain Overhang with a coefficient of 0.025 and with a confident level of 99%.

Finally, Birru performs tests on the momentum effect. The above tests confirm that the disposition effect and its influence on stock performance are indeed weakened by the split event. If the disposition effect is the origin of the momentum, one could expect a decrease of the momentum effect after a stock split.

The empirical test is based on the regression model of Grinblatt and Han (2005). Grinblatt and Han apply this model to study the influence of Capital Gain Overhang on stock return. They also find that when the Capital Gain Overhang is introduced in the regression, the autocorrelation of stock returns over intermediate time horizon becomes no more significant. This find suggests that the Capital Gain Overhang could explain the momentum effect. Birru incorporates the stock split into the regression model:

$$r = a_0 + a_1 r_{-4:-1} + a_2 r_{-52:-5} \times 1_{split} + a_3 r_{-52:-5} \times 1_{nosplit} + a_4 r_{-156:-53} + a_5 \bar{V} + a_6 S \\ + a_7 g \times 1_{split} + a_8 g \times 1_{nosplit} + a_9 1_{split}$$

The regression of this model shows that the coefficient  $a_2$  equals 0.0035 and is significant; while  $a_3$  is not significant. Regarding the Capital Gain Overhang, the coefficient  $a_7$  is not significant;  $a_8$  equals 0.0032 and is significant at 1% level. This suggests that when there is no stock split, the disposition effect is not disrupted. Therefore, the intermediate term return autocorrelation is explained by the disposition effect. However, the intermediate term return autocorrelation is still significant when stock split occurs. This result means that there must be other factors that contribute to the return autocorrelation. The disposition effect cannot be the only explanation.

## 1.5 The alternative utility theories

The alternative utility theories could capture some behaviors and describe them using mathematical language, and could be considered as complements to the Expected Utility Theory, which is the well-established foundation of economic theories. The alternative utility theories offer a potential way to integrate investor behaviors into the framework of financial theories. In this dissertation, the regret theory and the prospect theory are studied.

### 1.5.1 Differences between the classic and the alternative utility theories

The expected utility rule is founded by two mathematicians von Neumann and Morgenstern in their study on Game Theory. They suggest that a rational individual should make decisions following four axioms: Completeness, Transitivity, Independence, and Continuity. As long as the individual's preference follows these axioms, the individual makes decisions by maximizing his expected utility of wealth.

However, these axioms are proposed for the purpose of providing mathematically sound foundation for using the expected utility criterion. They are not obtained from observing people's actual decision makings. Therefore, when studies look at how individuals make their decisions in a risky framework, violations to the axioms of the expected utility theory are found. One of such violations is known as the "Allais' paradox". It was introduced by French economist Allais in 1953. Kahneman and Tversky (1979) also study this violation the certainty effect. They show it in their study with the following experiment:

Table 1.5-1: The certainty effect in decision making

| Problem 1: choose between A and B |       |           |       | Problem 2: choose between C and D |       |           |       |
|-----------------------------------|-------|-----------|-------|-----------------------------------|-------|-----------|-------|
| A: payoff                         | proba | B: payoff | proba | C: payoff                         | proba | D: payoff | proba |
| 2500                              | 0.33  | 2400      | 1     | 2500                              | 0.33  | 2400      | 0.34  |
| 2400                              | 0.66  |           |       | 0                                 | 0.67  | 0         | 0.66  |
| 0                                 | 0.01  |           |       |                                   |       |           |       |
| N=72                              |       | 18%       |       | 82%                               |       | N=72      |       |
|                                   |       |           |       | 83%                               |       | 17%       |       |

Source: Kahneman and Tversky (1979) Prospect theory: an analysis of decision under risk, *Econometrica* vol 47 no.2 page265-266.

The options in problem 1 and 2 are equivalent from the point of view of the expected utility. The axiom of independence suggests that if the option L is preferred to M, then by adding a same lottery to both L and M will not change their preference:

$$\text{If } L \succ M, \text{ then for any } N \text{ and } p \in (0, 1],$$

$$pL + (1 - p)N > pM + (1 - p)N$$

In the problem 1 of the above table, the option B is preferred to option A. But assume that there is an option with  $N=-2400$  and  $p=0.66$ . Then applying the independence axiom allows obtaining the two options C and D in the problem 2. An individual should prefer D to C. However, in the problem 1, 82% of the individuals prefer B to A, while in the problem 2, the preference is reversed. 83% of the individuals prefer C to D. Hence the independence axiom is violated.

The study of Kahneman and Tversky also demonstrates other examples of violation of the expected utility theory such as the probabilistic insurance, the reflection effect, and the isolation effect. The reason for these violations is that the expected utility theory is built on a pure theoretical level, and is a mathematical idealization of individual decision making. This theory sets a model of the way an individual should behave but it is not a description of how an individual actually behaves.

Alternative utility theories take the opposite approach to the expected utility theory. These theories begin with observing the behavior of individual decision making, and then try to build a theory that could describe the behavior pattern of individuals. The regret theory developed by Loomes and Sugden (1982) and Bell (1983), is based on researches in the field of psychology. The feeling of regret is documented as a factor that could influence individual decision making. Then the regret theory aims to use mathematical language to describe the regret as the difference in the individual's satisfaction between what has happened and what could have happened. The individual will show regret-averse attitude, and the decision made by such an individual will try to maximize his/her total satisfaction level while taking the regret into consideration.

A similar approach is also taken by Kahneman and Tversky to establish the prospect theory. They have conducted experiments in a controlled environment to find behavioral patterns in individual decision making. Their experiment allows collecting data by using monetary units and probability to describe choices. From the data collected in experiments, the authors are able to elicit the S-shaped utility function. This utility function suggests that the individual is risk-averse when facing gains, but is risk-loving when facing losses. In addition to the S-shaped utility function, Kahneman and Tversky also find in the experiment that individuals are likely to over-estimate the likelihood of a low probability event, and under-estimate the likelihood of a high probability event. Based on this observation, they

developed the subjective probability function. Under the framework of the prospect theory, one individual's decision making is determined by the S-shaped utility function and the subjective probability function. The individual will maximize his/her expected utility according to the above functions.

The alternative utility theories could capture some behavior patterns of the individuals. Meanwhile, they also imply that the individuals are rational in their own context. In both the prospect theory and the regret theory, individuals always maximize their expected utility according to their utility functions, the same as a rational individual in the expected utility theory. However, in alternative utility theories, the utility functions include the factors of individual's emotion and behavior pattern. In reality, emotions such as regret, fear, and attachment to family or to friends are the factors that influence individual's decision making, and cannot be excluded. In this sense, under the framework of the alternative utility theories, the individuals are not "irrational". The individuals make their decisions under a "relaxed" or "generalized" rationality.

The alternative utility theories are established "bottom-up" by observing the behavior of the individuals, while the expected utility theory is build "top-down" by using mathematical deduction. Such a difference between these two types of utility theory suggests that the alternative utility theories could better describe how individuals actually make decisions, but they are difficult to apply to studies in economics and finance.

## **1.5.2 The regret theory**

The regret is an important feeling that humans could experience in daily life. Studies on this subject begin in the field of psychology at first. Later, the regret theory is formed in the field of economics to capture the effect of regret on the individual utility.

### **1.5.2.1 The regret as a sentiment**

"Regret is a negative, cognitively based emotion that we experience when realizing or imagining that our present situation would have been better, had we decided differently."(Zeelenberg 1999)

How the sentiment of regret can influence the behavior is a subject that has been studied in the field of psychology since the 1970s. These studies clearly showed that the anticipated

regret can affect the decision making in the way that the fear of regret makes the individual to behave more cautiously, and make more “rational” decisions. However, in some other studies, it is documented that an individual can have a riskier behavior when the sentiment of regret is taken into consideration in the decision making process. This means that the regret can cause both risk-avoiding and risk-seeking behaviors.

Zeelenberg (1999) studies in what situation the risk-avoiding and risk-seeking behaviors take place. Based on a sufficient amount of experiments, he found that this behavior depends on which decision is the regret-minimized decision. And one condition for individual to take regret into consideration is that the individual could have the information feedback of the result of the option. The evidence that the regret comes into play is that the decision making of the individual is different when the feedback of an option is available. In the experiment of Zeelenberg (1999), where the access to feedback of the choices is manipulated by the designer, the subjects tend to choose the option of which the feedback is available.

Table 1.5-2: Percentage of participants deciding for the safer or riskier option

|                         | Decision       |              |
|-------------------------|----------------|--------------|
|                         | Riskier option | Safer option |
| Feedback riskier option | 37%            | 63%          |
| No feedback             | 21%            | 79%          |
| Feedback safer option   | 11%            | 89%          |

Source: Zeelenberg 1999 Anticipated regret expected feedback and behavioral decision making Journal of Behavioral Decision Making Vol. 12, Iss. No. 2

The result of this experiment shows that the regret plays an important role in decision making. In the control group, the feedback is not available, only 21% of the subjects choose the riskier option. In this case, the subjects only know the outcome of the option that they choose, while the outcome of the alternative is unknown. Therefore, the subject will not be exposed to the regret sentiment. In the other two cases, where the feedback of one option is available, the subjects know that they will be informed with the outcome of the specified option after the choice is made. For example, in the first scenario, if a subject chooses the safer option at first, he will later receive the outcome of the safer option, moreover, he will be notified of the outcome of the riskier option. The comparison ex-post between the result he got and the result that he could have got if he chose differently, will give rise to the feeling of regret. Thus, if a subject is facing a riskier choice with feedback available, the fear of experiencing the regret will naturally be taken into the decision making process. And in order to avoid the ex-post feeling of regret, the subject is more likely to take the riskier option. In



this case, it is better riskier than regretful. The experiment verifies this theory. More test subjects choose the riskier option when the feedback is available.

This finding in the field of psychology provides a theoretical support for bringing behavioral factors, in this case, the sentiment of regret, into the field of finance. In the financial market, the information feedback on the alternative option is easily accessible. An investor can easily compare the return of his real investment choice to the return he could have if he chose a different investment strategy. The difference between the returns makes the investor experience an ex-post feeling of rejoice or regret. Therefore, at the moment of the decision making, the investor will anticipate the regret feeling, and modify the investment strategy to avoid the regret. If this hypothesis is true, then the financial market will be slightly different from what the traditional theory describes.

#### 1.5.2.2 The theoretical framework of the regret theory

The regret theory, developed separately by Loomes and Sugden (1982), and Bell (1983), models the feeling of regret in the framework of utility theory. The basic idea is to add a function that describes the feeling of regret and rejoice to the traditional utility function. According to Loomes and Sugden (1982) and Bell (1983), the new value function which includes the regret is written:

$$m_{ik}^j = c_{ij} + R(c_{ij} - c_{kj})$$

In this equation,  $m$  is called the Modified Utility, it indicates the total satisfaction level of an individual when he chooses the action  $k$  instead of the alternative action  $i$  in the state of the world  $j$ . The total satisfaction is divided into one part  $c()$ , which is a traditional utility function that measures the satisfaction related to acquisition of wealth, and the other part  $R$ , which is a function that measures the satisfaction related to making a good or a bad decision compared with the observed result of the alternative action, in other words, rejoice or regret.

The  $R()$ , the regret-rejoice function introduces a new dimension in utility theory. Not only the material aspect, but the mental aspect of an outcome will also influence the satisfaction level. Loomes and Sugden (1982) use an example to show the difference between choiceless utility and regret-rejoice: Comparing losing 100 dollars because of tax increase, and losing the same money for gambling in a horse race, one will feel much worse for losing money in horse race, because he/she had choice not to gamble. Inversely, one will feel much

happier for winning 100 dollars in a horse race than receiving the same amount of money for tax reduction.

Within the framework of regret theory, a rational individual will maximize his value function. In a risky situation, using the same principle as in Expected Utility Theory, the rationality will be expressed as maximization of Expected Modified Utility:

$$E(m_i^k) = \sum_{k=1}^n p_k m_{i,j}^k$$

Where:

m: is the modified utility

p<sub>k</sub>: is the probability that stat k happens

The preference order, in the same way as in the Expected Utility Theory, is defined as follows: if an action k is preferred to an action i for one individual, then, the expected modified utility of action k should be greater than the expected modified utility of action i:

$$\sum_{j=1}^n p_j [c_{ij} - c_{kj} + R(c_{ij} - c_{kj}) - R(c_{kj} - c_{ij})] \geq 0$$

Where:

c: is the choiceless utility function

R: is the regret-rejoice function

p: is the probability

### 1.5.2.3 The representation of regret: function R() and function Q()

In the value function of regret theory:  $m_{ik}^j = c_{ij} + R(c_{ij} - c_{kj})$ , which represents the utility of choosing option i, instead of option k. The R() is a “regret-rejoice” function. By introducing this regret-rejoice function, the above function shows that the preference for an option is now different from the one under the expected utility theory, if option i is preferred to option k. By transformation of the above function, we can write it as follow:

$$E(c_{ij}) - E(c_{kj}) + E[R(c_{ij} - c_{kj})] - E[R(c_{kj} - c_{ij})] \geq 0$$

The first part of this formula:  $E(c_{ij}) - E(c_{kj})$  is the difference between the expected choiceless utilities of the two options. The second part:  $E(R(c_{ij}-c_{kj})) - E(R(c_{kj}-c_{ij}))$  is the difference of expected regret-rejoice. Together with the effects of the two parts, this formula can determine the individual's preference. But in this form, it is hard to see the individual's attitude towards regret, therefore, it is convenient to define a function  $Q(y)$  such that for all  $y$ ,

$$Q(y) = y + R(y) - R(-y)$$

Then, that option  $i$  is preferred to option  $k$  is equivalent to:

$$\sum_{j=1}^n p_j [Q(c_{ij} - c_{kj})] \geq 0$$

In this form, individual's attitude towards regret can be interpreted by the looking at shape of function  $Q$ . First, function  $Q$  is increasing and symmetric ( $Q' > 0$ , and  $Q(-y) = -Q(y)$ <sup>1</sup>). Then, a linear function  $Q$  ( $Q'' = 0$ ) means indifference to regret, in such case, the value function will be the same as the traditional utility function. A concave function  $Q$  ( $Q'' < 0$ ) means regret loving; and a convex function  $Q$  ( $Q'' > 0$ ) means regret aversion. According to Loomes and Sugden (1982), there is no priory privilege for any form of the  $Q()$  function, but a concave  $Q()$  function can successfully explain the inconsistencies of the Expected Utility Theory.

Intuitively, in a convex function  $Q$ ,  $Q(c_{ij}-c_{kj}) > (c_{ij}-c_{kj})$  for  $(c_{ij}-c_{kj}) > 0$ . This means that if his actual choice is the better option, the individual will experience more satisfaction than the utility purely related to an increase of outcome, and this rejoice feeling will be even greater if the positive difference is bigger. On the contrary, if the alternative option is better, the individual will experience regret, which will decrease the satisfaction level than a pure decrease of outcome. This means that the individual is adverse to regret.

The above mathematical transformation shows that, the function  $Q$  is a transformation of the function  $R$ , hence it is not a utility function, but rather an indicator for the attitude towards regret. And the function  $R$  is actually the rejoicing - regret function. If  $Q$  is a convex function, we can deduce the property of function  $R$ :  $Q''(y) = R''(y) - R''(-y) > 0 \Rightarrow R''(y) > R''(-y)$ .

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<sup>1</sup> It is easy to prove that:  $Q(-y) = -y + R(-y) - R(y) = -Q(y)$

The regret theory has profound implication in describing individual's preference. This theory successfully explains by a mathematical logic some of the inconsistencies of traditional expected utility theory, such as “the common ratio effect”, “Allais paradox”, “the isolation effect” etc. Among these, one particular inconsistency called “the reflection effect” could explain why people have mixed risk attitude, such as they take both insurances and gambles. The application on the financial market could help to explain the excess risk-loving attitude. Moreover, the regret theory could offer an explanation to one behavioral bias in the market called the disposition effect (Muermann and Volkman 2007).

### **1.5.3 Quantitative measurement of regret**

Since the appearance of the regret theory in the 1980s, many theoretical discussions have been delivered to show this theory better describes the decision making of an individual than the traditional utility theory. However, few empirical works have been done in order to give concrete evidence of this point. Cagno and Hey (1988) have performed a direct test on the regret theory, but their result is not conclusive. Recently, Bleichrodt Cillo and Diecidue (2010) measured quantitatively the regret using a lottery based experimentation. This part will briefly summarize the method of Bleichrodt Cillo and Diecidue (2010) to measure the regret while comparing with Cagno and Hey's method(1988).

In the field of experimentation, the test of regret is usually performed on function  $Q()$ , because it is more straightforward for comparing the preferences, and based on the shape of function  $Q()$  it is easy to determine the attitude towards regret. Both tests by Cagno and Hey (1988) and by Bleichrodt Cillo and Diecidue (2010) are performed on this function.

Both tests use a similar procedure. The first step is to discover the choiceless utility function  $c = u(x)$ , then, the function  $Q(c_i - c_k)$  is elicited based on the result of the first step. Both experiments are conducted with questionnaires on computers. The questions are presented in the form of lotteries. In the experiment of Cagno and Hey (1988), the subjects are asked to give the exact value that satisfies the indifference between two lotteries. And in the experiment of Bleichrodt Cillo and Diecidue (2010), the subjects are asked to choose the preferred case in a binary situation, and the indifference level is derived from the answers.

Despite the similarity, the two tests use different method of elicitation. Cagno and Hey (1988)'s test uses a traditional method of Certainty-equivalent method in the elicitation of  $c$  and  $Q$ , while the test of Bleichrodt Cillo and Diecidue (2010) uses the trade-off method in the

elicitation of choiceless function. The trade-off method allows generating a standard sequence of utility, which can be considered as unit utility in the elicitation of  $Q$ , and may improve the accuracy of the final result. And another difference is that the Cagno and Hey (1988) test covers the negative value of utility.

### 1.5.3.1 The test of Cagno and Hey

As mentioned above, this test is designed in two parts of elicitation of utility and a third part of validity check. The method of elicitation used is the certainty-equivalent method. This method requires the test subjects to compare a certain amount  $(x,1)$  with a lottery  $(g,p;G,1-p)$ ,  $g$  and  $G$  are reference points, the value of  $x$  varies until the individual is indifferent between the two options. The utility of  $x$  can be deduced from:  $u(x) = p u(g) + (1-p) u(G)$ . In practice, it is usually assumed that  $u(g)=0$ ,  $u(G)=1$  and the probability is set to be equals to  $3/4$ ,  $1/2$ ,  $1/4$ . Then test gives the value of  $x$  at the points where the utility is equal to  $0.25$ ,  $0.5$ ,  $0.75$ . In the test, Cagno and Hey use money as stimulant, the reference points are set as  $g = 0$  and  $G = 100$  units of currency. One unit is 1 pound sterling for British subjects, and is 1000 liras for Italian subjects.

Under the framework of regret theory, the indifference of  $(x,1)$  and  $(g,0.5;G,0.5)$  means:

$$0.5Q(c(x) - c(g)) + 0.5Q(c(x) - c(G)) = 0$$

Because  $Q$  is symmetric:

$$0.5Q(c(x) - c(g)) = 0.5Q(c(G) - c(x))$$

Because  $Q$  is strictly increasing:

$$0.5(c(x) - c(g)) = 0.5(c(G) - c(x))$$

Assuming that  $c(g) = 0$  and  $c(G) = 1$ :

$$c(x) = 0.5$$

Then, using the newly discovered  $x$ , the choiceless utility at the point of 0.25 0.75 can be elicited in the same way. Note that in this part, only 50-50 lotteries can be used in the elicitation of choiceless utility, if not, the equality will not hold when canceling  $Q$ .<sup>2</sup>

The second part consists in eliciting the function  $Q$  with the same method but a different setting. In this part, the reference points are set as  $Q(0) = 0$ ,  $Q(1) = 1$ , the subjects are asked to compare between a certainty of nothing: (0,1) and a lottery ( $x,p;Y,1-p$ ). Here,  $Y$  can take two values for  $c(Y) = 1$ , or  $c(Y) = -1$ , hence two ways to elicit the  $Q()$  function. Cagno and Hey (1988) call them separately: the positive way and the negative way. The positive way is that  $Y$  is discovered in the first part as  $c(Y) = -1$ . Then  $x$  varies until two options are considered as indifferent. This indifference means:

$$pQ(c(x) - c(0)) + (1 - p)Q(c(Y) - c(0)) = 0$$

Because  $c(0) = 0$ ,  $c(Y) = -1$

$$Q(c(x)) = \frac{1 - p}{p}$$

Three probabilities are used for  $p$ : 4/7, 2/3, 4/5, to generate the value of  $x$  where  $Q(c(x))$  equals 0.75, 0.5, and 0.25. Then  $c(x)$  can be calculated by interpolation of choiceless utility function.

The negative way is where  $Y$  takes the value for  $c(Y) = 1$ . Using the same method, the elicitation of  $Q()$  function is:

$$Q(c(x)) = \frac{p}{1 - p}$$

To generate the value of  $x$  where  $Q(c(x))$  equals to 0.75, 0.5, 0.25, the values of the probability are set to be 3/7, 1/3, 1/5. Cagno and Hey (1988) include both positive and negative ways in the experiment to elicit the  $Q()$  function, theoretically, these two ways should yield the same result.

The third part of the test provides a consistency check on the regret theory. The questions in this part are designed on the answers of previous questions, but a “correct” answer is expected from the subjects. The following exhibit shows how the questions of the

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<sup>2</sup> for any  $p \neq 0.5$ ,  
 $Q(c(G)-c(x))/Q(c(x)-c(g)) = p/(1-p)$   
 $\Rightarrow (c(G)-c(x))/(c(x)-c(g)) = p/(1-p)$  because  $Q(ax) = aQ(x)$  if and only if  $a = 1$

test are organized. In part three, seven consistency checking questions are asked, but only two questions are showed in the table as examples.

Table 1.5-3: Summary of experiment questions in Cagno and Hey (1988)

| part 1 |           |         |                |                       |       |                 |
|--------|-----------|---------|----------------|-----------------------|-------|-----------------|
| a1     | satisfies | [a1,1]  | indifferent to | [(100,0.5),(0,0.5)]   | hence | c(a1)=0.5       |
| a2     | satisfies | [a2,1]  | indifferent to | [(a1,0.5),(0,0.5)]    | hence | c(a2)=0.25      |
| a3     | satisfies | [a3,1]  | indifferent to | [(100,0.5),(a1,0.5)]  | hence | c(a3)=0.75      |
| a4     | satisfies | [0,1]   | indifferent to | [(100,0.5),(a4,0.5)]  | hence | c(a4)=-1        |
| a5     | satisfies | [a5,1]  | indifferent to | [(a4,0.5),(0,0.5)]    | hence | c(a5)=-0.5      |
| a6     | satisfies | [a6,1]  | indifferent to | [(a5,0.5),(0,0.5)]    | hence | c(a6)=-0.25     |
| a7     | satisfies | [a7,1]  | indifferent to | [(a4,0.5),(a5,0.5)]   | hence | c(a7)=-0.75     |
| part 2 |           |         |                |                       |       |                 |
| a8     | satisfies | [0,1]   | indifferent to | [(a4,4/7),(a8,3/7)]   | hence | Q[c(a8)]=0.75   |
| a9     | satisfies | [0,1]   | indifferent to | [(a4,2/3),(a9,1/3)]   | hence | Q[c(a9)]=0.5    |
| a10    | satisfies | [0,1]   | indifferent to | [(a4,4/5),(a10,1/5)]  | hence | Q[c(a10)]=0.25  |
| a11    | satisfies | [0,1]   | indifferent to | [(100,3/7),(a11,4/7)] | hence | Q[-c(a11)]=0.75 |
| a12    | satisfies | [0,1]   | indifferent to | [(100,1/3),(a12,2/3)] | hence | Q[-c(a12)]=0.5  |
| a13    | satisfies | [0,1]   | indifferent to | [(100,1/4),(a13,3/4)] | hence | Q[-c(a13)]=0.25 |
| part 3 |           |         |                |                       |       |                 |
| a14    | satisfies | [a14,1] | indifferent to | [(a1,0.5),(a5,0.5)]   | hence | a14 should = 0  |
| a15    | satisfies | [a15,1] | indifferent to | [(a3,0.5),(a7,0.5)]   | hence | a15 should = 0  |

Source: Cagno D. and Hey J. (1988) A direct test of the original version of regret theory Journal of behavioral decision making, Vol. 1, 43-56 ; "ai" denotes the answer to question i

It is necessary to set the test in two parts for  $c(x)$  and  $Q(c(x))$ . The goal of the test is to elicit the function  $Q()$ , which, by definition is a function of the difference between the choiceless utility, but in the test, the data is collected as outcome, not utility. Therefore, it is necessary to discover the utility function  $c()$  at first to convert the outcome in utility, in order to show the real form of the function  $Q()$ . It is also notable that the theoretical basis and the method used in the elicitation of  $c()$  and  $Q()$  are very similar, Both of the elicitations are based on the same equation of the indifference between a certainty and a lottery:

$$pQ(c(x) - c(g)) + (1 - p)Q(c(x) - c(G)) = 0$$

But in fact, as this equation is written in a form of a composite function, the two elicitations deal separately the two layers of the equation. In the first part, the reference points are set for the utility function, as:  $c(g) = 0$ ,  $c(G) = 1$ . Because in this part, the designers are interested only in choiceless utility, and the  $Q()$  function is canceled by setting the probability to 0.5. In the second part, the reference points are set for  $Q()$  function as:  $Q(0) = 0$ ,  $Q(1) = 1$ , and with the help of the utility function elicited in the first part, the outcomes and the utilities

can be converted. In this way, the designers can ignore the choiceless utility function, and consider the  $Q()$  as a function whose independent variable is utility.

The experiment is realized at the University of York in Britain, and at L.U.I.S.S in Rome. Three sets, for a total of 64 subjects participated in the experiment. The subjects answer questions on computer, the questions are expressed in words, such as "what is  $x$  which makes you indifferent between nothing for sure and a 50-50 gamble between 100 and  $x$ ?".

However, the results of Cagno and Hey (1988)'s test is not promising. Among the 64 subjects, only 9 of them (14%) have the behavior consistent to with the regret theory. The elicited choiceless function has a concave shape, which suggests a risk avoiding behavior. This result is consistent with traditional utility theory, but opposite to the assumption of a linear utility function in the regret theory. For the function  $Q()$ , the positive way and negative way have elicited different functions, which is against the theoretical prediction.

To sum up, the test of Cagno and Hey (1988) fails to prove the regret theory. The main reason according to the authors is "that the experiment is badly designed". The questions are not designed in a very comprehensive way knowing that many subjects did not fully understand the questions. The result reports that five subjects did not finish answering all the questions, and 15 subjects, according to the authors, "gave some grounds for suspicion that they had not really understood the questions". The two cases combined, there are 20 subjects out of 64 who failed to understand the questions, almost one third of all the subjects. This is highly likely to corrupt the data and damage the result. Despite of the defects, the theoretical foundation and the experimental design of Cagno and Hey (1988)'s test are sound, and the later test of Bleichrodt Cillo and Diecidue (2010) use basically the same structure.

### 1.5.3.2 The test of Bleichrodt Cillo and Diecidue

The test of Bleichrodt Cillo and Diecidue (2010) is also designed in two parts, the first part for eliciting  $c()$ , and the second part for eliciting  $Q()$ . One improvement of this test is to adopt the trade-off method (Wakker and Deneffe 1996) in eliciting the choiceless utility function. This method is developed to elicit utility under expected utility criterion, and it remains valid in dealing with alternative utilities such as prospect theory, rank-dependent utility.



The elicitation of utility by trade-off method is implemented as follows: First, reference points (“gauge outcomes”) are designated as:  $G$  and  $g$  to be any value for  $G > g$ . Then, the subjects are asked to compare two lotteries:  $(x_0, p; G, (1-p))$  and  $(x_1, p; g, (1-p))$ , and  $x_1$  varies until the two lotteries are indifferent. According to regret theory, the indifference implies:

$$pQ(c(x_1) - c(x_0)) + (1 - p)Q(c(g) - c(G)) = 0$$

By transformation:

$$Q(c(x_1) - c(x_0)) = \frac{1 - p}{p} Q(c(G) - c(g))$$

Next, the subject is asked to compare two other lotteries:  $(x_1, p; G, (1-p))$  and  $(x_2, p; g, (1-p))$ . Here,  $x_1$  is the result from the previous question, and  $x_2$  will be changed until the two lotteries are indifferent. Similarly, this indifference implies:

$$Q(c(x_2) - c(x_1)) = \frac{1 - p}{p} Q(c(G) - c(g))$$

From the two equations above, another equivalence is established:

$$Q(c(x_1) - c(x_0)) = Q(c(x_2) - c(x_1))$$

Because  $Q()$  is strictly increasing:

$$c(x_1) - c(x_0) = c(x_2) - c(x_1)$$

By repeating the same procedure  $k$  times, the trade-off method can generate a “standard sequence”:  $\{x_0, x_1, \dots, x_k\}$ , for which:

$$c(x_1) - c(x_0) = c(x_{j+1}) - c(x_j)$$

For  $j = 1, \dots, k-1$

The reference points are set as  $x_0 = 0$ ,  $x_k = 1$ , then,  $c(x_j) = j / k$ .

The trade-off method clearly has some advantages compared with the certainty equivalent method that is used in Cagno and Hey (1988)’s test. First, the trade-off method does not involve a probability distribution, because the probability is canceled. Therefore, misconception or distortion of probability will not affect the test result. The test subjects are not needed to have a good understanding of probability in order to give a reasonable answer

to test questions. Second, the gauge outcomes “G and g” can be set with different intervals in between, this allows the designers to control the precision of their test as they wish. In other words, the standard sequence can be generated for any size of interval. And last, only one value  $c(0) = 0$  is designated arbitrarily, for  $c(x_k) = 1$ ,  $x_k$  is a value elicited in the test.

The second part of Bleichrodt Cillo and Diecidue (2010)’s test uses the same method of elicitation as in Cagno and Hey (1988)’s test. The test subjects are asked to compare two lotteries:  $(x_4, p; x_0, (1-p))$  and  $(x_3, p; z, (1-p))$ ,  $x_3, x_4$  are the results elicited from part 1 of the test,  $z$  will be modified until two lotteries are considered as indifferent. According to regret theory:

$$pQ(c(x_4) - c(x_3)) + (1 - p)Q(c(x_0) - c(z)) = 0$$

In this part of the test, the reference point is set at  $c(x_0) = 0$  and  $Q(1/k) = 1$ . In the previous part of test,  $1/k$  is the interval of elicited standard sequence, therefore,  $c(x_{j+1}) - c(x_j) = 1/k$ , and  $Q(c(x_{j+1}) - c(x_j)) = 1$ . Then:

$$Q(c(z)) = \frac{p}{1 - p}$$

The value of  $p$  are set to  $1/4, 2/5, 3/5, 3/4$ , in order to elicit the value of  $z$  where  $Q(c(z)) = 1/3, 2/3, 1, 3/2, \text{ and } 3$ . After knowing the value of  $z$ ,  $c(z)$  can be inferred by interpolation from choiceless utility function.

Table 1.5-4: The measurement method of Bleichrodt et al. (2010)

|        | Assessed quantities | Indifference                                    | Implication                             | Stimuli  |  |
|--------|---------------------|---|---|--|--|
|        |                     |   |   | Measurement 1  | Measurement 2  |
| Part 1 | $x_1, \dots, x_5$   | $x_{j+1}pG \sim x_jpG, j = 0, \dots, 4$         | $u(x_{j+1}) - u(x_j) = u(x_1) - u(x_0)$ | $p = 1/3$<br>$G = \text{€}16$<br>$g = \text{€}11$<br>$x_0 = \text{€}20$<br>$p_1 = 1/4$ | $p = 1/2$<br>$G = \text{€}17$<br>$g = \text{€}13$<br>$x_0 = \text{€}20$<br>$p_1 = 1/4$ |
| Part 2 | $z_1, \dots, z_4$   | $x_{4p_j}x_0 \sim x_{3p_j}z_j, j = 1, \dots, 4$ | $Q(u(z_j)) = p_j / (1 - p_j)$           | $p_2 = 2/5$<br>$p_3 = 3/5$<br>$p_4 = 3/4$  | $p_2 = 2/5$<br>$p_3 = 3/5$<br>$p_4 = 3/4$  |

Source: Bleichrodt, Cillo, and Diecidue 2010: A Quantitative Measurement of Regret Theory Management Science 56(1), pp. 161–175

Comparing to Cagno and Hey (1988)’s test, one important difference of Bleichrodt Cillo and Diecidue (2010)’s test is the use of “standard sequence” in the elicitation. This standard sequence has the role of a “scale”, In the first part of the test, the elicitation of choiceless utility begins with  $x_0 = 0$ , then goes up one scale at each round of question until

sufficient data are collected. While in Cagno and Hey (1988)'s test, the whole set ( $x = 0$  to  $x = 100$ ) is considered as the "scale", and each round of question divides the scale into halves and quarters. In the second part of the test, the standard sequence has same interval between neighboring point, is naturally used as "scale", the "unit", or the value of "1". Other intervals of utility  $Q(c(z)-c(x_0))$  will be compared to the "scale"  $Q(c(x_{j+1})-c(x_j)) = 1$  to identify its value. In in Cagno and Hey (1988)'s test, the whole set  $Q(c(100)-c(0))$  is considered as "scale", as they did in the first part. Generally speaking, Cagno and Hey (1988)'s test uses a "top down" way, and Bleichrodt Cillo and Diecidue (2010)'s test uses a "bottom up" way. Using the standard sequence as the "scale" in the elicitation may improve the precision of the test, and may explain the difference in the results of the two tests.

The experiment of Bleichrodt Cillo and Diecidue (2010)'s test is conducted in Pompeu Fabra University in Spain. The test subjects include 55 students in different departments. The questions are presented vividly with help of tables and diagrams. The indifference value is not given directly by the subjects, but is determined by the subjects' answers to a series of binary choices. By changing the gauge outcome,  $G$  and  $g$ , the designers performed the test twice in order to check the validity of the test.

The result of Bleichrodt Cillo and Diecidue (2010)'s test is satisfying. At an aggregate level, the choiceless utility function shows a linear shape in both measurements, and the results are very consistent. The authors explain the absence of concavity of utility function that "Under regret theory, part of people's attitude toward risk is captured by the regret function  $Q$ ". At the individual level, more than half of the subjects have a concave utility. The result also shows that in the aggregate level, the preference function  $Q()$  has a convex shape in both measurements as predicted by regret theory. At individual level, some of the subjects are removed from analyses. One subject is removed because the subject does not understand the questions. Some subjects are removed because their answers violate the monotonicity. All remaining subjects (42 in the first measurement, 46 in the second) show the aversion to regret.

To sum up, compared with Cagno and Hey (1988)'s test, Bleichrodt Cillo and Diecidue (2010)'s test used the trade-off method for eliciting the choiceless utility function, that could improve the precision of test. Moreover, in their test, the questions are much more clearly presented, which improves significantly the quality of the answers. Combining these two major reasons, Bleichrodt Cillo and Diecidue (2010)'s test yields a supporting result for regret theory.

### 1.5.4 The prospect theory

Kahneman and Tversky (1979) founded the prospect theory as an alternative to the expected utility theory. The prospect theory aims to offer an explanation to the constantly observed violations of expected utility theory.

The prospect theory defines a prospect as a contract that yields different outcomes  $x_i$  with corresponding probability  $p_i$ :

$$(x_1, p_1; x_2, p_2; \dots; x_n, p_n)$$

The outcomes of a prospect, unlike the expected utility theory, represent the net gain or loss of individual's wealth; and are also sorted in ascending order:  $x_1 < x_2 < \dots < x_n$ . The value of a prospect is defined as the average value of each outcome weighted by the corresponding decision weight.

$$V(x_1, p_1; x_2, p_2; \dots; x_n, p_n) = \sum_{i=1}^n \pi(p_i) v(x_i)$$

Where:

V: is the value of the prospect

v: is value function of an outcome

$\pi$ : is the decision weight depending on probability

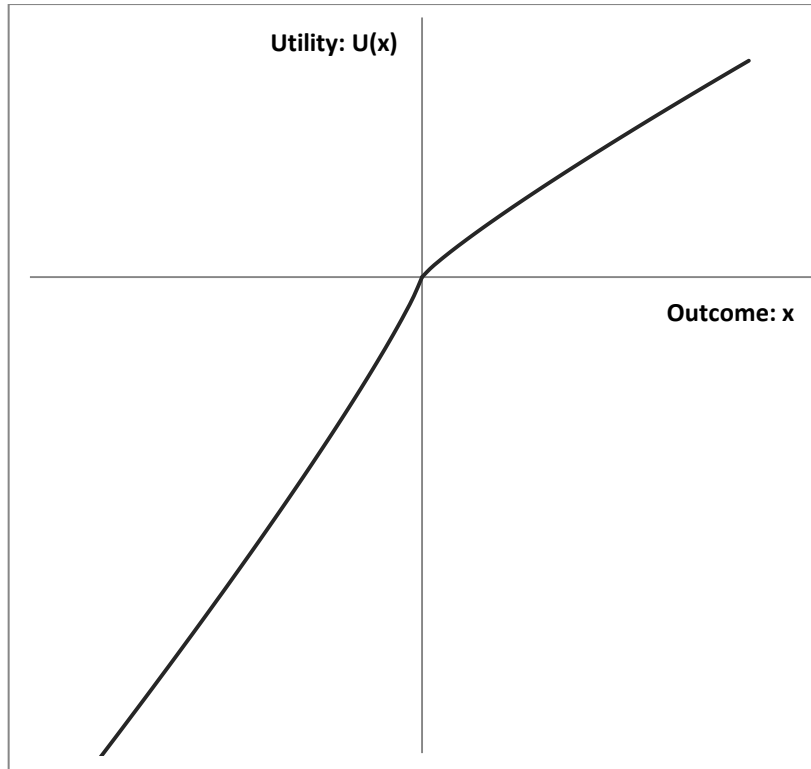
Such a definition has a similar form to the one of the expected utility, however the mathematical terms carry different economical meanings. First, the outcomes of a prospect are in the form of net gain or loss, meaning that  $x_i$  could have negative values. Second, investor's decision making is not only depending on probabilities. Investor's subjective perception influences the decision making. Last, the value function captures how much the investor is sensitive to gain and loss.

#### 1.5.4.1 S-shaped value function

The prospect theory is well-known for its S-shaped value function. The properties of the value function are derived from controlled experimentations that confirmed the violations of the expected utility theory. Kahneman and Tversky (1979) proposed a value function that it is

"defined on deviations from the reference point", is "generally concave for gains and commonly convex for losses", and is "steeper for losses than for gains".

Figure 1.5-1: The S-shaped value function



The horizontal axis represents the wealth in terms of gain or loss, the vertical axis represents corresponding utility.

The S-shaped value function suggests that investors make their decision based on losses and gain instead of considering their total wealth level. Dealing with gains, investors show risk aversion behavior while dealing with losses, investors show risk-loving behavior. In other words, investors prefer a smaller but sure gain than a risky larger gain, but, investors would rather take the risk of possible larger losses than suffer a sure loss. Finally, investors are more sensitive to losses than to gains, but in both situations, investors' sensitivity decreases with the amount of gains and losses. Such a behavior pattern is called "loss aversion".

In the study of Kahneman and Tversky (1992), they proposed a specific expression of the S-shaped value function:

$$v(x) = \begin{cases} x^\alpha, & x \geq 0 \\ -\lambda(-x)^\beta, & x < 0 \end{cases}$$

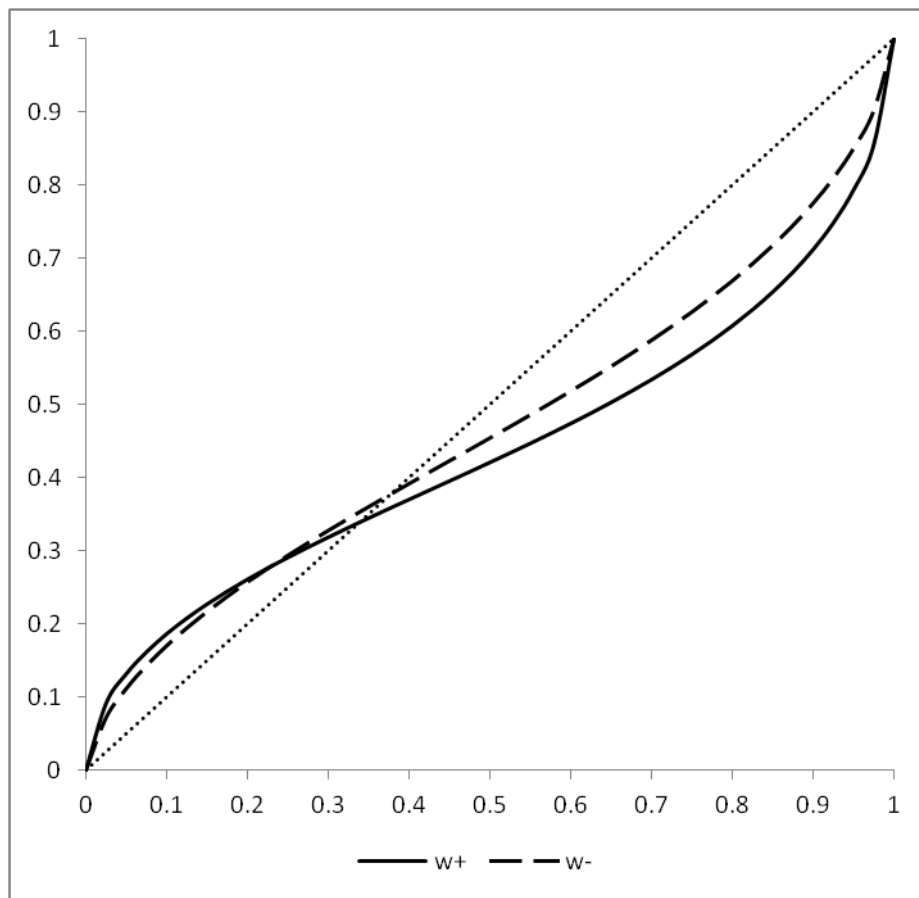
Where  $\alpha=\beta=0.88$ ,  $\lambda=2.25$

The  $\alpha$  and  $\beta$  in the equation give the function concave shape in the positive part of the horizontal axis, and convex shape in the negative part. The coefficient  $\lambda$  makes the curve of the function steeper in the negative part.

#### 1.5.4.2 Weighting function

In the prospect theory, the investor assigns decision weight to each possible outcome. The decision weight is not a probability, but a subjective perception of how likely an outcome will be produced, even though such a perception is based on objective probabilities.

Figure 1.5-2: The decision weighting



The horizontal axis represents the objective probability, the vertical axis represents the subjective probability. The solid line is the curve of  $W^+$ , the dashed line is the curve of  $W^-$ .

In the figure, the straight line represents the objective probability, the curves of  $W^+$  and  $W^-$  represent the decision weighting of individuals when facing winning and losing situations. Following functions are derived from experiments.

$$w^+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$$

$$w^-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{1/\delta}}$$

Where:  $\gamma = 0.61$  and  $\delta = 0.69$

The result shows that individuals are likely to over-estimate the events with very low probabilities and to under-estimate the events with very high probabilities.

#### 1.5.4.3 Shifts of reference

Kahneman and Tversky (1979) pointed out that the reference point that distinguishes gains from losses may alter. In their paper, the gains and losses are defined according to individual's current situation; however they argue that an individual can equally set his or her expectation as his reference point. In addition, a sudden change in individual's current situation may also lead to a shift of the reference point.

One example given by Kahneman and Tversky is that one person who recently lost 2000 and is facing a choice between a sure gain of 1000 and an equal chance of winning 2000 or nothing. If this person could not make peace with the loss that has already happened, instead of thinking the options as (1000, 1) and (2000, 0.5; 0, 0.5), this person could consider the options as (-1000, 1) and (0, 0.5; -2000, 0.5). In this case, a shift of reference point transfers a winning situation into a losing situation. According to previous conclusions about the prospect theory, the individual will take the risky option because of the shift of reference point.

Kahneman and Tversky demonstrated that, for  $x, y, z > 0$ , if an individual is indifferent between a risky prospect  $(x, p; -y, 1-p)$  and doing nothing  $(0, 1)$ , then by subtracting the outcomes by  $z$ , the new prospect  $(x-z, p; -y-z, 1-p)$  will be preferred over  $(-z, 1)$ .

Based on the above conclusion, Kahneman and Tversky suggest that in economic activity, investor's decision making may depend on how the investor perceives the investment options. They show such an example: for a gamble that requires paying 10 and yields 1000 with 1% probability, the prospect could be written either as  $(990, 0.01; -10, 0.99)$  and  $(0, 1)$ ,

or as (1000, 0.01; 0, 0.99) and (-10, 1).<sup>3</sup> In the second case, the individual considers the gamble and its cost separately. The gamble will be purchased if  $\pi(0.01)v(1000) + v(-10) > 0$ .

Kahneman and Tversky also suggest that if an individual makes decision based on his or her final total wealth, as in the expected utility theory, he can be considered as shifting the reference point to the left until there is no negative value. As a result, the convex part of the value function is eliminated, the individual will no longer show risk-seeking behavior.

### 1.5.5 Application of prospect theory

It has been over thirty year since Kahneman and Tversky founded the prospect theory, its application in the field of economics and finance is limited. Barberis (2013) explains the reason in his review over the prospect theory that "it is often unclear how to define precisely what a gain or loss is".

#### 1.5.5.1 Difficulty in defining the reference point

The discussions about the shifts of reference point have already shown such a difficulty. Kahneman and Tversky have pointed out that individual's perception over the lottery may changes the reference point. Barberis (2013) also pointed out that in practice, defining the reference point is even more complicated. On one hand, an investor may set the reference points for each stock that he/she had acquired, or the investor may set one reference points for his/her entire portfolio. On the other hand, many criteria can be adopted as reference point. The most intuitive criterion is the purchase price of the asset. A positive raw return means a gain and a negative raw return means a loss. But other criteria may also be used depending on the investor's objective. If the investor considers the time value of the investment, then he/she could set the reference point at the risk free rate. In the case where the investor considers a certain market index as a benchmark for his/her investment, the return of the index could becomes the reference point of the investor.

In the work of Grinblatt and Han (2005), they present the variable Capital Gain Overhang. This variable measures the hanging gain or loss of a stock, and uses the purchase price as the reference point. The variable Capital Gain Overhang is well accepted, and the

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<sup>3</sup> These two formulations of the same gamble do not match the definition of shift of reference point. The second formulation should be (1000, 0.01; 0, 0.99) and (10, 1). In such formulation, individual considers not playing the gamble as gaining 10.

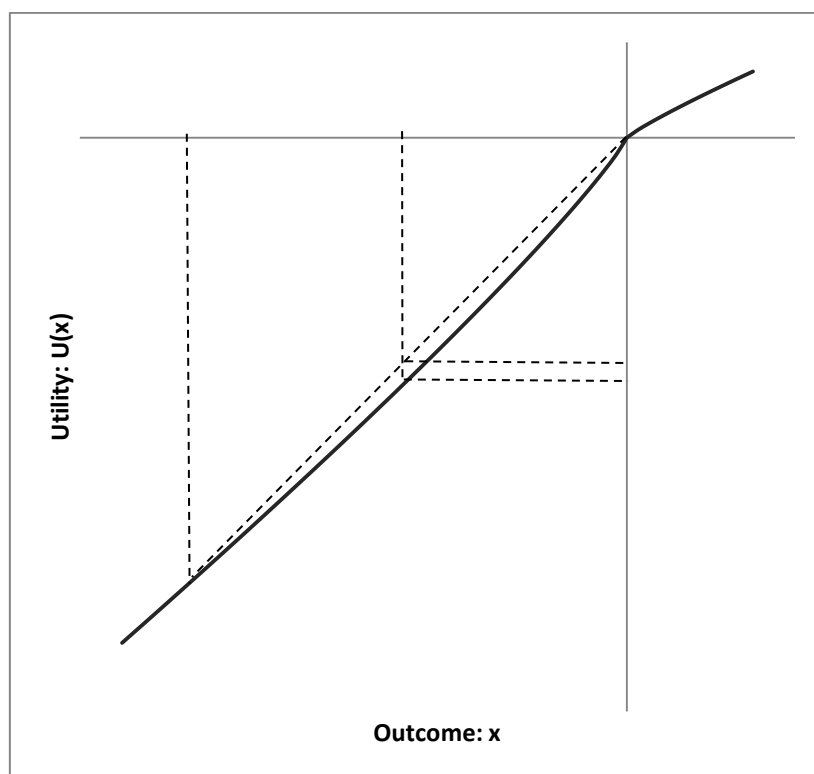


idea of using the purchase price as the reference point is also adopted by other researchers such as Frazzini (2006), Kaustia (2010) etc.

### 1.5.5.2 Prospect theory explaining the disposition effect

One major application of the prospect theory is to explain the disposition effect. The reason is that the loss-averse investors could show risk-loving attitude when facing losses, hence might want to hold the losing stock. However, more comprehensive studies do not provide conclusive evidence supporting this idea.

Figure 1.5-3: Holding the risky asset is preferred in the case of loss



What causes the disposition effect has been an interesting subject of research. The empirical work of Shefrin and Statman (1985) and Odean (1998) have ruled out the tax-reduction and portfolio rebalancing as causes of the disposition effect. In addition, Shefrin and Statman (1985) listed four major possible causes: prospect theory, mental accounting, seeking pride and avoiding regret, and Self-control. Comparing with other possible causes, the prospect theory is a well established theory with clearly defined mathematical models. For this reason, many studies try to link the disposition effect to the prospect theory.

Intuitively, the prospect theory could provide a good explanation to the disposition effect. One feature of the prospect theory is that investors have aversion to loss. The S-shaped

value function proposed by the prospect theory is convex in the negative area, this characteristic implies that when facing losses, investors are willing to make risky decisions and bet on the good chance that could help to get out of the losing situation. If a loss aversion investor is holding a losing stock, the expected value for holding the stock is higher because there is a chance that stock price re-bounces to the level at which it has been purchased by the investor, and the decrease in loss would largely increase investor's utility. Therefore the loss aversion investor is reluctant to sell the losing stock. Such a behavior matches the description of the disposition effect. Such reasoning is adopted by many studies including Grinblatt and Han (2005) and Frazzini (2006) however, more recent studies on this subject show results that challenge the idea of loss aversion causing the disposition effect.

Other studies have adopted more sophisticated mathematical method to explore the link between the disposition effect and the prospect theory or other causes. Kaustia (2010) calculated the incremental utility from selling a risky asset on the basis of the S-shaped utility function from prospect theory. Assuming that a stock return follows a normal distribution with  $N(\mu, \sigma)$ , the prospect value is calculated as:

$$Prospect\ value = \int_{-\infty}^{+\infty} v(x)f(x)dx$$

Where:  $x$  is the unrealized gain,  $f()$  is the probability density function.

The calculation shows that the value of selling the risky asset is always lower than the value of holding the asset. The highest incremental value is reached at the point where unrealized gain or loss equals zero. Once the price deviates from the reference price at which the asset was purchased, the incremental value decreases with both the unrealized gain and unrealized loss. Furthermore, Kausita changes the parameters of return distribution as well as the parameters of the S-shaped utility function:  $\alpha, \beta, \lambda$ .

$$v(x) = \begin{cases} x^\alpha, & x \geq 0 \\ -\lambda(-x)^\beta, & x < 0 \end{cases}$$

The parameters  $\alpha, \beta$  determine the degree of sensitivity to gain and loss and the parameter  $\lambda$  determines the level of the loss aversion. The calculation shows that in order to have the model make the prediction of selling the risky asset to realize a gain, the parameters of the S-shaped utility function should describe a high level of sensitivity. In many cases, there is a need of an exogenous factor to induce a sell.

The empirical study of Kaustia (2010) takes a sample from the Finish stock market. Data contains trading information of all individual investors. Such detailed information helps to determine at a given time, if an investor sells or not his portfolio and also the acquisition cost of the portfolio. The method used is to perform regression of the portfolio position (1 for sale and 0 for holding) on the variable of Capital Gain Overhang, which is calculated using first-in-first-out inventory accounting method. Results show that the likelihood for realizing a sell in winning position is higher, and investors are reluctant to realize large losses. However, the results also show a low propensity for sales in large gain situation.

Gomes (2005), Barberis and Xiong (2009), Hens and Vlcek (2011) and Meng (2014 working paper) adopted the optimum portfolio allocation approach. This method consists of modeling investor's utility under the framework of the prospect theory, usually by including the reference point and a convex utility function for losses, and then the optimum portfolio allocation between the risk-free asset and a risky asset is found by maximizing investor's utility.

Barberis and Xiong (2009) used the original S-shaped utility function and conventional parameter values to formulate the model. The total trading period is set to be one year, during which, there are multiple trading dates. Stock price evolves along a binomial tree with multiple periods. Investors maximize the expected utility at the end of the year, hence at each trading date they adjust their distribution of wealth between the risky asset and the risk-free asset accordingly. Under the standardized setting of the prospect theory, the simulation of Barberis and Xiong shows that the evolution of the optimum portfolio allocation does not match the disposition effect, but rather, in some cases, matches the opposite of the disposition effect. Investors increase the holding of risky asset in winning situation, and in some losing situations, investors decrease the holding of risky asset.

Table 1.5-5: Simulation of Barberis and Xiong (2009)

| Risky Asset Price $P_{t,j}$       |      | State Price Density $q_{t,j}$ |        |
|-----------------------------------|------|-------------------------------|--------|
|                                   | 72.9 |                               | 0.46   |
|                                   | 62.7 |                               | 0.56   |
|                                   | 54.0 | 55.6                          | 0.68   |
|                                   | 46.5 | 47.9                          | 0.83   |
| 40                                | 41.2 | 42.4                          | 1      |
|                                   | 35.5 | 36.5                          | 1.18   |
|                                   | 31.4 | 32.4                          | 1.38   |
|                                   | 27.9 |                               | 1.62   |
|                                   | 24.7 |                               | 1.91   |
| Risky Asset Shares Held $x_{t,j}$ |      | Wealth $W_{t,j}$              |        |
|                                   | -    |                               | 163.39 |
|                                   | 6.8  |                               | 94.70  |
|                                   | 3.5  | -                             | 64.25  |
|                                   | 1.8  | 0.5                           | 50.75  |
| 1.7                               | 0.2  | -                             | 40     |
|                                   | 1.5  | 0.5                           | 32.45  |
|                                   | 2.7  | -                             | 26.26  |
|                                   | 5.2  |                               | 16.51  |
|                                   | -    |                               | 0      |

Source: Barberis, N., & Xiong, W. (2009). What Drives the Disposition Effect? An Analysis of a Long-Standing Preference-Based Explanation. *the Journal of Finance*, 64(2), 751–784. Page 763

Next, Barberis and Xiong (2009) modified the framework of their model by using the realized gain or loss, instead of the annual gain or loss to derive investor's utility. In the model using realized gain or loss, the investor receives utility only when he sells total or a part of his asset. The difference lays in the input of the S-shaped utility function. The annual gain or loss model considers the total portfolio position as the input while the realized gain or loss model only uses the variation of the portfolio as input. The realized gain or loss model is further developed in the work of Barberis and Xiong (2012). Simulations on the realized gain or loss model could produce the disposition effect more reliably, but still cannot rule out the opposite of disposition effect.

Using the same optimum portfolio allocation approach, Gomes (2005) studied the link between loss aversion and disposition effect from a different angle. A different model is proposed with parameters representing the loss aversion. This model is concave in the positive part representing risk aversion in winning situation, and convex in the negative part for risk loving in loss situation. But when the loss is greater than a threshold, the function becomes concave again. The optimum portfolio allocation allows plotting the demand curve for risky asset for loss aversion investors. The demand is generally higher in losing situation

than in winning situation. In addition, the demand for risky asset increases when the current asset price deviates away from the reference price in both winning and losing directions.

Both work of Gomes (2005) and Barberis and Xiong (2009) mentioned one problem. In order to introduce the unrealized gain or loss, the models usually assume that the investor has bought the risky asset at the beginning of the period but at the given parameters for utility function and return distribution, the investor has no interest in acquiring such risky asset at the beginning. The work of Hens and Vlcek (2011) suggests that after adjusting for this problem, the prospect theory could not predict the disposition effect.

To sum up, many studies are devoted to link the disposition effect to the prospect theory. However, the models under the configuration given by the original prospect theory do not give convincing evidence. Modifications of the models are made to explore different possibilities. Gomes (2005), Kausita (2010) change the parameters representing the level of loss aversion and investor's sensibility. Barberis and Xiong (2009, 2012) modify the model by using an unconventional way of measuring the utility based on the realized gain or loss. Meng (2014 working paper) provides a new perspective on the reference point at which the gain and loss are distinguished. Instead of setting the reference point at the acquisition cost of the risky asset, Meng uses the investor's expectation at the beginning of the period as the reference point.

Overall, despite the fact that the risk-seeking behavior predicted by the prospect theory appears to match the disposition effect, rigorous models under the framework of the prospect theory still have difficulty to explain this behavior bias. The true cause of the disposition effect could be more complex and might involve many other factors than the prospect theory. However, existing researches on the prospect theory yield a common prediction on investor behavior: in either large gain or large loss situations, investors have lower propensity to sell the stock. Such a result provides a new perspective to apply the prospect theory to the field of finance.

## **1.6 Literature summary**

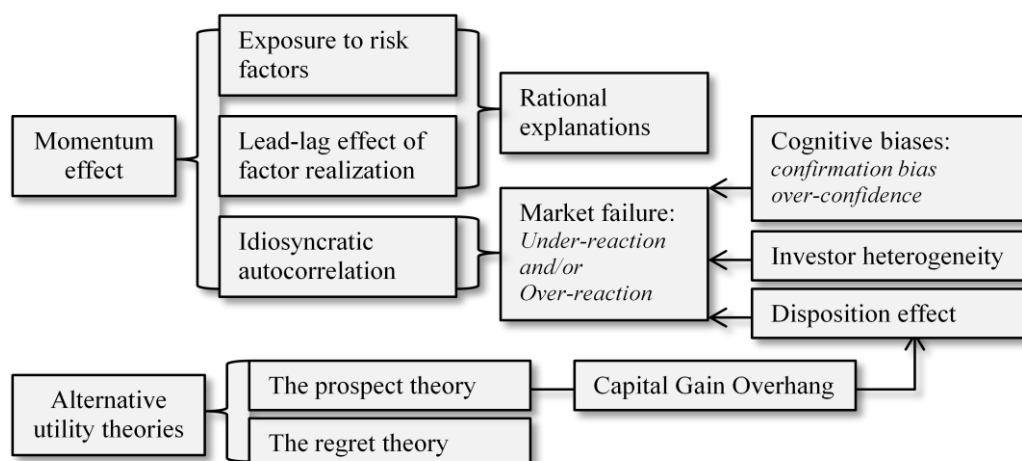
The literature review is organized according to the research question. In order to apply the alternative utility theories to explain the momentum effect, the literature review is dedicated to understanding the momentum effect and the alternative utility theories. The first

section is focused on the momentum effect: the empirical evidence, the theoretical study, and the rational and irrational models that try to explain it. The second section is devoted to the alternative utility theories, namely the prospect theory and the regret theory. However, the existing literature provides little evidence on how to link the alternative utility theories to the momentum effect. The disposition effect distinguishes itself in the literature. Both works involving the momentum effect and the alternative utility theories mentioned the disposition effect. The existing studies do not give conclusive evidence to link the disposition effect to the momentum effect, but the disposition effect could be one clue for the research.

### 1.6.1 Existing approach to explain the momentum effect

The structure of the literature review is explained by the following figure. The first part of the literature review focuses on the momentum effect. This part summarizes both theoretical and empirical works. The most important element in this part is the decomposition of the momentum profit. The decomposition identifies three potential sources of the momentum profit: exposure to risk factors, lead-lag effect of factor realization, and return autocorrelation in idiosyncratic component. Then the literature review looks at how existing works cover these three sources. The first and the second sources are under the framework of classic financial theory. However, these two sources are rejected by empirical tests. The third source implies market inefficiency and investors' irrationality. Stock prices tend to under-react or over-react to information. Behavior models have tried to explain the market failure. The models study the cognitive biases such as the confirmation bias, and the heterogeneity of investors. The advantage is that they are well developed models, but the disadvantage is that these behavior factors are impossible to measure in empirical tests.

Figure 1.6-1: Connection between literature fields



The second part of the literature review covers the well-known alternative utility theories: the regret theory and the prospect theory. The literature review first emphasizes the theoretical framework of these theories. This helps to understand how the theories are founded. Next, the literature review presents their application to real cases. Concerning the regret theory, the focus is on the measurement of regret, because such a measurement allows eliciting a specific utility function based on regret theory. Concerning the prospect theory, the focus is on its application to explain the disposition effect, since this latter has a potential connection to the momentum effect.

### **1.6.2 A direct approach with the alternative utility theories**

Although the disposition effect appears in both the first and the second part of the literature review, more detailed studies could not provide conclusive evidence showing that the disposition effect is the link between the momentum effect and the alternative utility theories. On one hand, there are studies suggesting that the prospect theory could predict the opposite of the disposition effect in certain situations. On the other hand, a recent study shows that when controlling for the disposition effect, the momentum strategy could still yield significant profits. These studies suggest that the disposition effect fails to connect either to the momentum effect or to prospect theory.

Yet, the studies on the disposition effect are inspiring in terms of applying the prospect theory to the practice. These studies, one of which is the work of Grinblatt and Han's (2005), highlight the importance of the investor's reference point where the gain and loss are separated; and also point out the mechanism through which behavior factor could have an impact on stock performances: the investor's preference at an aggregated level could influence the demand or the supply of an asset, hence have an impact on its return. In this sense, the studies on the disposition effect have paved the foundation to apply the alternative utility theories.

This dissertation learns from the existing work in this field. On one hand, this dissertation will follow the two essential points: the influence of the reference point and the transmission of preference to stock performances via the force of demand and supply. On the other hand, this dissertation will avoid the impasse of the disposition effect, and will look at a more direct approach of the alternative utility theories.

## **Chapter 2**

# **Theoretical analyses on unrealized gain/loss in regret aversion and loss aversion models**

The existing studies on the subject of unrealized gain or loss, which is also referred as capital gain overhang in an empirical context, have shown interesting discoveries. In this part, the unrealized gain or loss will be studied under the framework of alternative utility theories, namely the regret theory and the prospect theory.

Like commodity market, the financial market is also driven by the force of supply and demand from the investors. If the traditional utility theory does not capture certain behavior patterns of the investors, these behavior patterns could influence investor's investment decision making, which consists essentially buying or selling certain assets. As a result, the aggregated force of buying or selling under the influence of behavior factors may cause market to deviate from the equilibrium described by the classic theories.

Existing literature that studies the disposition effect follows this line of thought. Both theoretical and empirical works are dedicated to apply the alternative utility theories, especially the prospect theory. Although there is no conclusive evidence showing that the cause of the disposition effect is the prospect theory, all theoretical work suggest that under alternative utility theory, investors' preferences and decision makings are different from the classic theory. For instance, the work of of Gomes (2005), Barberis and Xiong (2009) demonstrate such differences. In terms of empirical evidence, Grinblatt and Han (2005)



studied the influence of prospect theory and mental accounting investor on the market, they suggest that existence of such investors can cause extra selling pressure when stock price is going up, and also cause insufficient selling pressure when stock price is going down.

Instead of concentrating on the disposition effect, this section focuses on the theoretical analyses of the behavior pattern that can be deduced from applying the alternative utility theories to a conventional decision making scenario between a risky choice and a certain choice. In view of existing literature, the theoretical analyses aim to explore the existence of a reference price at which investor had purchased a stock. Because of such reference price, investors' follow-up decision making will be exposed to unrealized gain or loss that was not taken into consideration by classic utility theory. In this section, the value functions of the prospect theory and the regret theory are applied to the above scenario. Investor's preference to one choice is represented by the difference in the expected value of the value functions. Calculation shows that investors have higher propensity to sell stocks in small unrealized gain or loss situations than in large unrealized gain or loss situations. Such result does not perfectly match the disposition effect, and does not contradict to it either; but similar behavior pattern can be observed from existing theoretical works such as Gomes (2005), Kausita (2010), Barberis and Xiong (2009, 2012) and Meng (2014).

Following parts in this section will demonstrate the processes of developing a theoretical model. The section 2.1 introduces the reference point in a decision making scenario. The section 2.2 2.3 and 2.4 apply the alternative utility theories to the decision making scenario. Such applications allow calculating the expected value of the different choices, and compare investor preference. Finally, the section 2.5 discusses the pertinence of this behavior model, and the section 2.6 provides the prediction based on the theoretical model.

## **2.1 A decision making scenario involving unrealized gain or loss**

Assume that the evolution of the stock price follows a binomial tree. At a given time  $t$ , the price is  $p_t$ . For the next period  $t+1$ , the stock price has equal chance for increasing by  $\Delta p$  or decreasing by  $\Delta p$ . Since the stock price cannot be negative, the variation of stock price  $\Delta p$  cannot be greater than  $p$  ( $0 < \Delta p < p$ ). Under this limit, the stock price at  $t+1$  can be represented as:

$$p_{t+1} : (p_t + \Delta p, 0.5; p_t - \Delta p, 0.5)$$

In order to introduce the unrealized gain or loss, the reference point is set to be the acquisition price at which an investor had purchased the stock. The acquisition price is considered as an exogenous parameter that is independent of the stock price evolution. Suppose that investor's acquisition price of the stock is  $p^*$ , hence at time  $t$ ,  $p_t - p^*$  is the increment in that price that represents the gain or loss that the investor could potentially obtain if the investor chooses to sell the stock.  $p_t - p^*$  is also called the Capital Gain Overhang.

At time  $t$ , the investor who is holding the stock, faces two options: sell the stock (option Sell), or hold the stock (option Hold). In the option Sell, the investor cashes in the unrealized gain/loss. Hence the payoff for this option is the Capital Gain Overhang, and the outcome is sure. In the option Hold, investor does not cash in the hanging gain/loss. Since the stock price varies by  $\Delta p$  for the next time period, the outcome of this option will be Capital Gain Overhang +  $\Delta p$  or Capital Gain Overhang -  $\Delta p$ , with 50-50 chance. To sum up, the payoffs of the two options are:

$$\text{Sell: } (p - p^*, 1)$$

$$\text{Hold: } (p - p^* + \Delta p, 0.5; p - p^* - \Delta p, 0.5)$$

In such a configuration, the unrealized gain/loss is included into investor's decision making; meanwhile, the decision making scenario remains a traditional risky versus certain situation. The above configuration contains two independent variables: CGO and  $\Delta p$ . The Capital Gain Overhang represents the unrealized gain or loss from the past, the  $\Delta p$  represents how much the stock price will vary in the future.

### 2.1.1 Variable transformation

The above configuration contains two independent variables: the Capital Gain Overhang:  $p - p^*$  and stock price variation  $\Delta p$ . The Capital Gain Overhang measures the unrealized gain or loss from the past, the  $\Delta p$  measures by how much the stock price will vary in the future. In order to simplify the analyses, the model will limit the number of variables to one.

### 2.1.1.1 Standardization by Capital Gain Overhang

In the above market decision making scenario, where the stock price evolution follows a binominal tree. At a given time  $t$ , the price is  $p_t$ . For the next period  $t+1$ , the stock price has equal chance of increasing by  $\Delta p$  or decreasing by  $\Delta p$  ( $0 < \Delta p < p$ ). Therefore the stock price at  $t+1$  will be:

$$p_{t+1} = (p + \Delta p, 0.5; p - \Delta p, 0.5)$$

At time  $t$ , investor has the options to sell the stock or hold the stock for the next period. Suppose that the investor bought the stock at  $p^*$ , then the payoff for selling the stocks is  $p - p^*$  with certainty. The term  $p - p^*$  is the Capital Gain Overhang, which can be either positive (a unrealized gain) or negative (a unrealized loss). The payoff for holding the stock is  $p + \Delta p - p^*$  with a probability of 50%, or  $p - \Delta p - p^*$  with the same probability.

To standardize the Capital Gain Overhang, all the payoffs will be divided by the absolute value of the Capital Gain Overhang:  $|p - p^*|$ . The payoff for selling option will be:

$$\text{Sell: } (\pm 1, 1)$$

The payoff the holding option will be:

$$\text{Hold: } \left( \pm 1 + \frac{\Delta p}{|p - p^*|}, 0.5; \pm 1 - \frac{\Delta p}{|p - p^*|}, 0.5 \right)$$

In this expression, the term “ $\pm 1$ ” is the standardized scale of unrealized gain or loss. To distinguish a gain from a loss, the term will be written as “1” for unrealized gain, and “-1” for unrealized loss. In the outcome of the Hold option, the new variable term:  $\Delta p / |p - p^*|$  is the price variation divided by Capital Gain Overhang, represents the relative price variation level with respect to the scale of the unrealized gain or loss.

Since both  $\Delta p$  and  $|p - p^*|$  are positive, the term of relative price variation level will always be positive. If  $\Delta p < |p - p^*|$ , which means the price variation level is smaller than the unrealized gain or loss, the whole term of relative price variation level will be smaller than 1. If  $\Delta p > |p - p^*|$ , the price variation level is larger than the unrealized gain or loss, the relative price variation level will be larger than 1.

The above development allows reducing variables to one by standardizing the unrealized gain or loss. However, there are some drawbacks in this development. First, the

unrealized gain or loss is the subject to study, but is transformed to a constant. Second, the standardized unrealized gain or loss is scaled to “ $\pm 1$ ”, meaning that the following analysis will be performed separately in both the situations involving unrealized gain: “1” and unrealized loss: “-1”. It will be a complication in the analysis for comparison and demonstration. Last, the relative price variation level:  $\Delta p/|p-p^*|$  is not mathematically defined at the point where  $|p-p^*|=0$ , but in reality, it is possible that a zero Capital Gain Overhang case happens.

Due to the above disadvantages, the theoretical analyses look for other way that could offer a better solution to this problem.

#### 2.1.1.2 Standardization by price variation level

Using the same configuration of the binomial tree price evolution, stock price can vary for  $\pm \Delta p$  ( $\Delta p > 0$ ) by equal chance. The acquisition price for the investor is  $p^*$ , then the unrealized gain or loss is  $p-p^*$ . The payoff for cashing in the unrealized gain or loss by selling the stock at  $t$  is:

$$\text{Sell: } (p-p^*, 1)$$

The payoff for holding the stock until next period is:

$$\text{Hold: } (p-p^*+\Delta p, 0.5; p-p^*-\Delta p, 0.5)$$

To standardize the payoffs by the price variation level, all the outcomes are divided by the stock price variation:  $\Delta p$ , then the payoffs after standardization for selling and for holding options will be:

$$\text{Sell: } \left(\frac{p-p^*}{\Delta p}, 1\right)$$

$$\text{Hold: } \left(\frac{p-p^*}{\Delta p} + 1, 0.5; \frac{p-p^*}{\Delta p} - 1, 0.5\right)$$

This standardization processes does not influence the sign of the Capital Gain Overhang as did in the previous transformation, hence the unrealized gain or loss remain represented by the sign of new variable. The price variation is standardized to be 1 or -1 with 50-50 chance. The term  $(p-p^*)/\Delta p$  represents the unrealized gain or loss adjusted by the price variation level, and will be denoted as the new variable called Relative Capital Gain Overhang because this variable equals to the Capital Gain Overhang in the work of Grinblatt and Han (2005) divided by stock price variation.

$$RCGO = \frac{p - p^*}{\Delta p}$$

In reality it is very rare that stock price does not vary over time. Hence, in this model,  $\Delta p$  is assumed to be strictly greater than zero. Therefore, the Relative Capital Gain Overhang is mathematically defined for all real numbers, and could better be adapted to the cases that could happen in real market. A unrealized gain or a unrealized loss can be represented by the sign of the variable. Therefore, the gain or loss situation is not required to be studied separately. Furthermore, stock price variation level represents the risk of the stock. Standardization by price variation in fact uniforms the factor of risk, allowing isolating the behavior factor of unrealized gain or loss. In general, relative capital gain overhang is a better variable to be used in future studies than the relative price variation.

### 2.1.2 The theoretical significance of the Relative Capital Gain Overhang

Recall that the Relative Capital Gain Overhang is defined as the capital gain overhang divided by the price variation level. The payoffs of the Sell and Hold options are:

Sell: ( $rcgo$ , 1)

Hold: ( $rcgo + 1$ , 0.5;  $rcgo - 1$ , 0.5)

In the above form, the price variation is regulated to one unit, and the relative capital gain overhang is the number of unrealized gain or loss for each unit of price variation. The feature of this new variable can be demonstrated in two aspects.

#### 2.1.2.1 A shifting reference point

Assuming that the RCGO is a constant, then the lotteries of the Sell and Hold given previously corresponds to a classic decision making situation of certainty versus risk. Imagine a special scenario where RCGO equals to zero, then the payoffs of the Sell and Hold options will be:

Sell: (0, 1)

Hold: (1, 0.5; -1, 0.5)

The Sell option offers a certain but neutral outcome, while the Hold option offers 50% of chance for gaining one unit of wealth or losing one unit of wealth for the same chance.

These two options have equal expected outcome: zero. The outcome of the risky option is symmetric in terms of both the winning or losing quantities and the probabilities. This special scenario is equivalent to a classic case of gambling by throwing a coin, which was commonly used to as an example of risk aversion. The preference of individuals in such a case is well studied under the framework of the expected utility theory. One important implication of it in the financial field is the investor's attitude towards risk.

Introducing the Relative Capital Gain Overhang expands the above lottery in to a more complex circumstance. The risk factor is fixed to one unit. The RCGO exogenously defines a reference point from where the lottery plays. The reference point can vary from negative to positive, and the scale can change from small to large. In the same time, the expected outcome of the two options remains equal. Such configuration allows studying if individuals' preference can be influenced by one's initial winning or losing condition while the risk factor is fixed.

Take the same example of gambling by throwing a coin. Instead of studying one round of game, player can play as many rounds as he/she wishes. After each round of game, scores are kept, but only until the player quits the game, the wager will be settled. The RCGO can be considered as the 'score' kept during the game. In such case, it is natural that current game score could influence the player at least in his/her mental state. If current score is positive, the player may feel confident, relaxed; if current score is negative, the player may feel stressed, anxious or even desperate. Here lies the disagreement between classic utility theory and alternative utility theory. In the classic framework, a rational individual should be immune to the sentiments mentioned above when making decisions. Individual should make identical decision regardless current score being negative positive or zero. Alternative utility theories suggest the opposite. Human behavior will inevitably be influenced by emotions. The prospect theory suggests that individual is reluctant to accept losses; while the regret theory suggests that individual also takes the feeling of regret into consideration when making decisions. In the practice of finance, investing in stocks has similar feature to above example. One investor bought a stock, the evolution of stock price is the 'score' that reminds investor that his/her is winning or losing, but there is no real cash-flow until the moment investor sell the stock he/she holds.

Kahneman and Tversky (1979) discussed the scenario of shift of reference. They suggests that for  $x, y, z > 0$ , if a risky prospect  $(x, p; -y, 1-p)$  is indifferent from doing nothing

(0, 1) for an individual, then by subtracting the outcomes by  $z$ , the new prospect  $(x-z, p; -y-z, 1-p)$  will be preferred over  $(-z, 1)$ . Such proposition is consistent to the decision making scenario proposed in previous section:

Sell:  $(rcgo, 1)$

Hold:  $(rcgo + 1, 0.5; rcgo - 1, 0.5)$

It contains a risky prospect and a certain prospect. The RCGO is equivalent to the value  $z$  that captures the shift of reference. In addition, the Sell/Hold scenario has some developments from Kahneman and Tversky's lottery. First, this model trades the shift of reference as a variable that could take a positive value as well as a negative value. It allows performing a generalized study on the change of individual's preference caused by shift of reference.

Second, the risky prospect is simplified to represent one unit of risk. By this simplification, the influence from the factor of risk is isolated. For the same one unit of risk, investor's preference may change depending on current winning or losing situation. Such development implies that shift of reference point could potentially influence individual's attitude to risk. It suggests that for the same amount of risks, investor demands different compensation, in other word risk premium, in unrealized gain or loss situation.

#### 2.1.2.2 Four gain/loss situations

According to the relation between relative capital gain overhang and the situations of price variation, four situations can be distinguished.

1: Large unrealized loss situation: In this case, Relative Capital Gain Overhang is smaller than -1 ( $RCGO < -1$ ). A negative RCGO means that investor is currently in a loss situation. In addition, the scale of the loss is larger than the scale that stock price could vary. In such situation, even if the stock price goes up, the investor will remain in an unrealized loss situation. In other word, there is no chance that investor could recover from loss situation in the next time period.

2: Small unrealized loss situation: In this case, relative capital gain overhang is bigger than -1 and smaller than 0 ( $-1 \leq RCGO < 0$ ). A negative RCGO still means that investor is currently in a loss situation. However, the scale of the unrealized loss is smaller than the scale

that stock price could vary. In this case, if stock price goes up, investor could recover at least all the unrealized loss; but if stock price goes down, investor will suffer even greater loss.

3: Small unrealized gain situation: In this case, relative capital gain overhang is bigger than 0 and smaller than 1 ( $0 \leq \text{RCGO} < 1$ ). A positive RCGO means that investor is currently in a unrealized gain situation, more precisely, a non-loss situation. However, this winning situation is not secured for the next time period, because the scale of the unrealized gain is smaller than the scale of stock price variation. If stock price goes down in the next time period, investor will lose all the unrealized gain, and sink to an unrealized loss situation.

4: Large unrealized gain situation: In this situation, relative capital gain overhang is bigger than 1 ( $\text{RCGO} \geq 1$ ). A positive RCGO means that investor is currently in a unrealized gain situation; moreover, the winning situation is secured to remain to the next time period. The scale of unrealized gain is larger than the scale of stock price variation, meaning that even the worst case happens, stock price goes down, and investor will still be in an unrealized gain situation.

Other than the conventional definition of risk and certainty, the above division of situations captures the idea that the investor's current winning or losing situation will or will not remain to the next time period. The relative capital gain overhang measures the unrealized gain or loss with respect to the price variation, hence could capture the feature that how secure the investor's current winning or losing situation is. A large positive RCGO does not necessarily mean a large sum of unrealized gain, but means that investor's winning situation is safe for the next time period, no matter stock price may raise or fall the investor will always be in a winning situation.

## **2.2 The regret theory applies to the scenario**

In this section, the utility function of the regret theory will be applied to the market decision scenario. Such application will reveal if investor's preference is influenced by the existence of unrealized gain or loss when the emotion of regret is taken into consideration.



### 2.2.1 The utility function and rationality under the regret theory

The regret theory established by Looms and Sugden (1982) suggests that the value function  $V()$ , also called “modified utility function”, is a combination of a choiceless utility function  $c()$  and a regret-rejoice function  $g()$ :

$$V(x, y) = c(x) + g[c(x) - c(y)]$$

Where:

$V()$ : is the modified utility function

$c()$ : is the choiceless utility function

$g()$ : is the regret-rejoice function

$x$ : is the outcome of the chosen option

$y$ : is the outcome of the alternative option

The value function measures the individual’s total utility obtained by choosing the option  $X$  instead of the option  $Y$ . The choiceless function, according to Looms and Sugden (1982): "is the utility that the individual would derive from the consequence  $x$  if he experienced it without having chosen it." It is in fact the utility function in the traditional utility theory. In addition to the traditional theory, the regret-rejoice function is added. It measures the additional satisfaction that is linked to the rejoicing or regret feeling for choosing  $X$  instead of  $Y$  to be a bad or a good decision in the end. In the theoretical discussion of Looms and Sugden (1982) and Bell (1983), the choiceless utility function is simplified by a linear function:  $c(x) = x$ , which is verified by the work of Bleichrodt et al. (2010), and the regret function should have the properties of  $g(0) = 0$ ,  $g'(\xi) \geq 0$ , and  $g''(\xi) > g''(-\xi)$  for all  $\xi > 0$ , to satisfy the regret aversion.

Looms and Sugden (1982) follows the definition of rationality of individual from traditional utility theory but suggests that a rational individual will maximize the modified utility, i.e. the value function. In risky situation, rational individual will maximize the mathematical expectation of the value function. If a rational individual prefers the option  $x$  to the option  $y$ , it implies that option  $x$  offers more utility than option  $y$ :

$$V(x, y) - V(y, x) \geq 0$$

Using proposed value function:  $V(x, y) = c(x) + g(c(x) - c(y))$  and denote that

$$Q(x, y) = V(x, y) - V(y, x)$$

By transformation:

$$Q(x, y) = (c(x) - c(y)) + g(c(x) - c(y)) - g(-(c(x) - c(y)))$$

The function  $Q(x,y)$ , also called the preference function, represents the extra utility obtained by choosing the option  $x$ , instead of choosing the option  $y$ . If  $Q(x,y)$  is positive, investor obtain more utility for choosing  $x$ , hence the option  $x$  is preferred. In a risky situation, rational investor will examine the mathematical expectation of  $Q(x,y)$ . If  $EQ(x,y)$  is positive, then option  $x$  is preferred.

The preference function  $Q()$  allows directly comparing two options of one lottery, hence is the ideal tool for studying the market decision scenario involving unrealized gain or loss. However, the work of Looms and Sugden does not provide the functions with specific form or parameters. In order to obtain intuitive result; it requires eliciting the preference function.

## 2.2.2 Preference function elicitation

Bell (1983) proposes a regret-rejoice function in his theoretical discussion, with the linear choiceless utility function, it is able to develop the preference function. On the other hand, using experimentation, Bleichrodt Cillo and Diecidue (2010) measure the regret aversion of individual. The result of this experiment allows eliciting a preference function.

### 2.2.2.1 A theoretical proposition

Bell (1983) proposes a regret function  $g(\xi)$  that is consistent with regret aversion:

$$g(\xi) = 1 - e^{-a\xi}$$

Where:  $a > 0.625$

The value of  $a$  will not affect the shape of the value function, hence, it will not change individual's preference predicted by the value function. In order to simplify the calculation,  $a$  is assumed to be  $a = 1$ , and the value function in regret theory will be:

$$V(x, y) = x + 1 - e^{-(x-y)}$$

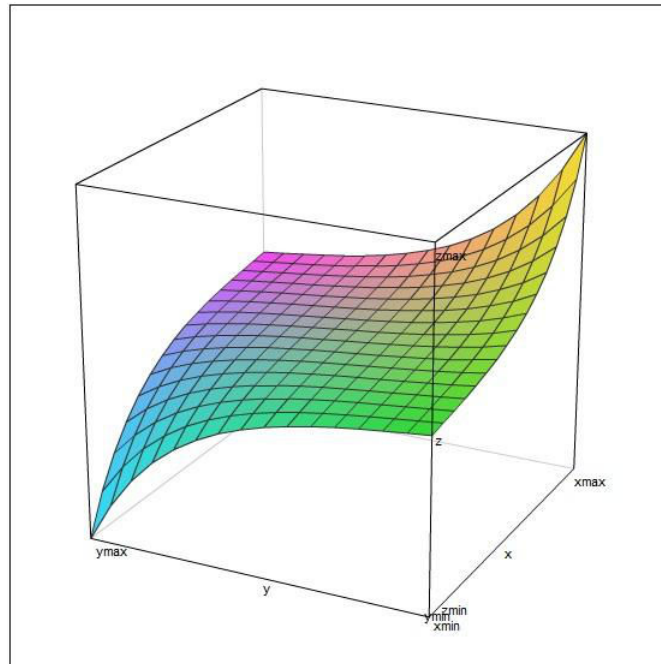
The function is a binary equation with variable  $x$  for the outcome of chosen option and variable  $y$  for the outcome of alternative option. Negative values of  $x$  and  $y$  are allowed, suggesting that individual reasons in terms of gain and loss.

Given the specific form of the choissless function  $c()$  and the regret-rejoice function  $g()$ , the preference function  $Q()$  can be written in the following form:

$$Q(x, y) = x - y - e^{y-x} + e^{x-y}$$

The  $Q$  function gives a clear picture for the investor's preference to one specific option in a decision making situation. If the  $Q$  function is positive, the option  $x$  is preferred to option  $y$ , and vice versa. The following figure shows the shape of the  $Q$  function.

Figure 2.2-1: The graphic of  $Q(x,y)=(x-y)+e^{(x-y)}-e^{-(x-y)}$



The  $x$  axis represents the outcome of the chosen option; the  $y$  axis represents the outcome of the alternative option;  $z$  axis represents investor's preference to the chosen option. Graphed by Online 3D Function Grapher

This preference function corresponds to the requirements described in regret theory. Let  $x-y=\xi$ , then:

$$Q(\xi) = \xi + e^{\xi} - e^{-\xi}$$

It is easy to prove that  $Q(\xi)$  is an increasing function, and is concave over negative values, convex over positive values. The function is also symmetric to the origin point:

$$Q'(\xi) > 0$$

$$Q''(\xi) < 0 \text{ for } \xi < 0, Q''(\xi) > 0 \text{ for } \xi > 0$$

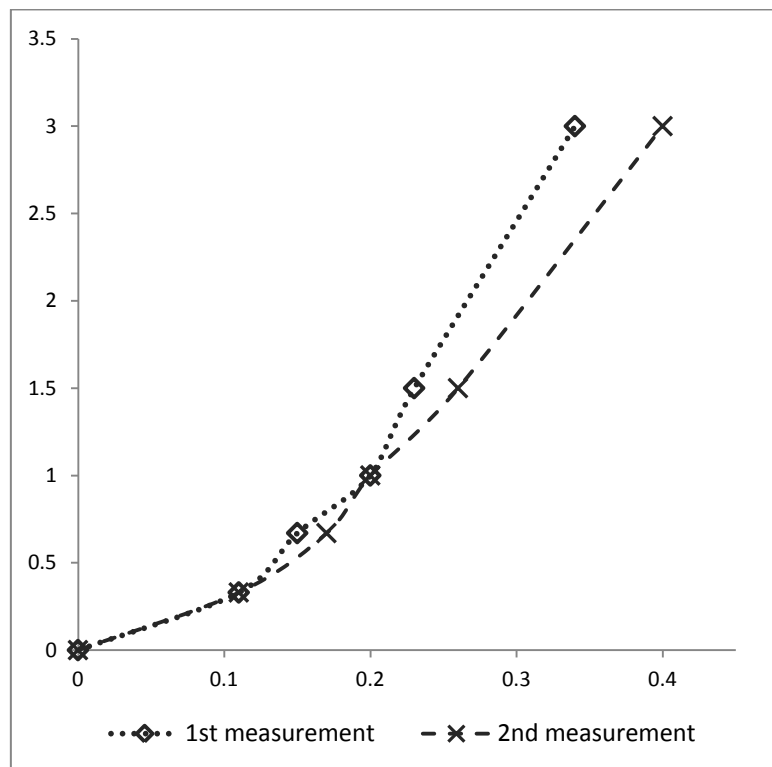
$$Q(-\xi) = -Q(\xi)$$

In fact, by defining  $x-y=\zeta$ , the curve of  $Q(\zeta)$  is the intersection of the surface  $Q(x,y)$  and the surface  $x=-y$ . In the 3-D graph, the area where  $Q$  is greater than zero indicates all possible combination of option  $x$  and  $y$  that  $x$  is preferred.

### 2.2.2.2 An elicitation from experiment

Another formula of the  $Q$  function can be estimated according to Bleichrodt Cillo and Diecidue (2010)'s measurement of regret. Their experiment data shows a convex and increasing shape of  $Q$  function over positive values. The following table and figure show the elicited  $Q$  function with two different methods of measurement. Bleichrodt Cillo and Diecidue (2010)'s measurement supports the assumption of regret aversion individual. Based on this quantitative measurement, it is possible to estimate a specific preference function.

Figure 2.2-2: Graphic of Bleichrodt Cillo and Diecidue (2010)'s measurements



Source: Bleichrodt, H., Cillo, A., Diecidue, E., 2010. A quantitative measurement of regret theory. *Management Science* 56, page 170.

Table 2.2-1: The data from Bleichrodt Cillo and Diecidue (2010)'s measurements

| Q()  | c(x)-c(y)                   |                             |
|------|-----------------------------|-----------------------------|
|      | 1 <sup>st</sup> measurement | 2 <sup>nd</sup> measurement |
| 0.00 | 0.00                        | 0.00                        |
| 0.33 | 0.11                        | 0.11                        |
| 0.67 | 0.15                        | 0.17                        |
| 1.00 | 0.20                        | 0.20                        |
| 1.50 | 0.23                        | 0.26                        |
| 3.00 | 0.34                        | 0.40                        |

Source: Bleichrodt, H., Cillo, A., Diecidue, E., 2010. A quantitative measurement of regret theory. *Management Science* 56, page 170. The data is collected from the graphics.

In order to find an appropriate preference function, it should take into consideration the symmetry shape of the function; it should be within the feasibility of linear regression. And finally the function should be in a simple form. One possible Q function is:

$$Q(\xi) = a\xi^3 + b\xi$$

Where  $\xi$  represents the utility difference of the two options, which is  $\xi = c(x)-c(y)$ .

Using linear Ordinary Least Square regression, the coefficients a and b are estimated:

Table 2.2-2: Estimation of the Q function parameters

| Coefficient        | a      | b      | Adj-R <sup>2</sup> |
|--------------------|--------|--------|--------------------|
| First measurement  |        |        |                    |
| Estimation         | 47.86  | 3.37   | 0.9986             |
| P-value            | <.0001 | <.0001 |                    |
| Second measurement |        |        |                    |
| Estimation         | 23.73  | 3.77   | 0.9986             |
| P-value            | <.0001 | <.0001 |                    |

Taking the average value of a and b in both regressions and round the value at integral numbers for the purpose of simplicity, the Q function can be written as:

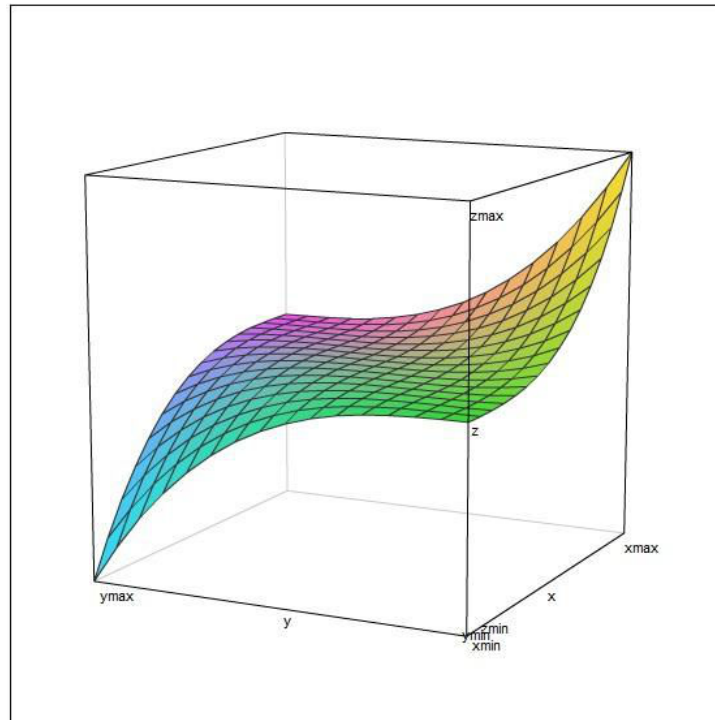
$$Q(\xi) = 36\xi^3 + 3\xi$$

Following the configuration of a linear choiceless utility function,  $\xi$  is replaced by x-y, and then the preference function will be found in such form:

$$Q(x, y) = 36(x - y)^3 + 3(x - y)$$

The following figure shows the shape of this preference function.

Figure 2.2-3: The graphic of  $Q(x,y)=36(x-y)^3+3(x-y)$



The x axis represents the outcome of the chosen option; the y axis represents the outcome of the alternative option; z axis represents investor's preference to the chosen option. Graphic by Online 3D Function Grapher

In this case, the preference function is found through regression on experiment result. Based on the Q function, it is possible to find the utility function of individual. By transformation (the calculation is shown in annex 2.2), the regret-rejoice function  $g()$  can be found as:

$$g(x - y) = \frac{2}{3}(x - y)^3$$

hence the value function will be:

$$V(x, y) = x + \frac{2}{3}(x - y)^3$$

Unlike the exponential  $g()$  function, which is concave for all real numbers, in this case,  $g()$  is increasing and concave for  $(x-y)<0$ , but is convex for  $(x-y)>0$ . The concave part suggests that in a losing situation, individual is more sensible for the fear for making the situation worse, The convex part suggests that in a winning situation, the individual is more motivated for making the situation even better. Despite the difference in the shape of  $g()$  function, the  $Q()$  functions developed from the two types of value function have the same shape and property. Therefore, they will yield consistent result.

A rational regret aversion individual will maximize his utility according to regret aversion value function. In a decision making situation, the preference of such individual will be determined by the preference function  $Q()$ . The above study proposes two types of preference function; the first one is deduced from theoretical framework, the second one is elicited from empirical experiment. Despite some differences in each function's property, these two types of function share similar shape and convexity, and could be consistent to each other. With the help of the preference functions, it is possible to study the decision making problem in financial market.

### 2.2.3 Preference function applies to market scenario

Recall the market scenario involving unrealized gain or loss. The payoff for the Sell option and the Hold option is given by the lottery:

Sell:  $(x, 1)$

Hold:  $(x+1, 0.5; x-1, 0.5)$

The variable  $x$  represents the unrealized gain or loss adjusted by stock price variation level, which is the relative capital gain overhang. Applying the payoff into the preference functions, firstly, the theoretical function:

$$Q(\xi) = \xi + e^{\xi} - e^{-\xi}$$

Where:

$\xi$ : is the difference in outcomes for Sell and Hold

In the case where Sell is the chosen option, if stock price goes up, the difference in outcomes will be -1; if stock price goes down, the difference in outcomes will be 1. Therefore, the expected preference for chose the Sell option is:

$$Q(S, H) = 0.5(-1 + e^{-1} - e^1) + 0.5(1 + e^1 - e^{-1}) = 0$$

This result suggests that investor is indifferent between the Sell option and the Hold option. Applying the lottery into the preference function obtained by experiment:

$$Q(\xi) = 36\xi^3 + 3\xi$$

The function also yields zero as result:

$$Q(S, H) = 0.5(36 + 3) + 0.5(-36 - 3) = 0$$

Both the theoretical and empirical preference functions predict that investor is indifferent either selling the stock or holding the stock. Such a null result is due to the assumptions given to the utility functions and the symmetric payoff of the lottery. Recall the modified utility function of the regret theory:

$$V(x, y) = c(x) + g[c(x) - c(y)]$$

The preference function  $Q()$  is obtained by:

$$Q(x, y) = V(x, y) - V(y, x)$$

By transformation:

$$Q(x, y) = (c(x) - c(y)) + g(c(x) - c(y)) - g(-(c(x) - c(y)))$$

With the assumptions that  $c()$  is a linear function, and  $g()$  has any form. The preference function can be written as:

$$Q(\xi) = \xi + g(\xi) - g(-\xi)$$

Where:

$$\xi = x - y$$

In a lottery with symmetric payoff such as the market scenario, the difference between the outcomes of Sell and Hold is  $\xi$  in the good state, while the difference between the outcomes is  $-\xi$  in the bad state. With equal probabilities, the expected value of  $Q$  will always be zero:

$$EQ = \frac{1}{2}[\xi + g(\xi) - g(-\xi) - \xi + g(-\xi) - g(\xi)] = 0$$

In this situation, the probability distribution and the outcomes of raising and dropping in the Hold action are strictly symmetric. Therefore, the regret for “losing” and the regret for “not winning” cancel each other in such symmetric lottery. (In the annex, the asymmetric probability situation is demonstrated.) In addition, the assumption is given that the choiceless utility function is linear, suggesting that investors are neutral towards risks. For the two reasons, the investor will always be indifferent between Sell and Hold options.



To sum up, under the framework of regret theory, especially for the given condition of regret aversion and risk neutral investor, neither the unrealized gain or loss nor the stock price variation could influence investor's preference. Investor is strictly indifferent between Sell option and Hold option.

### 2.3 The prospect theory applied to the scenario

Prospect theory (Kahneman and Tversky 1979) proposes an S-shaped utility function that captures the loss aversion of individual. This utility function introduces a reference point that distinguishes gain from loss. Such an utility function will be applied to the market decision making scenario involving unrealized gain or loss:

Sell: ( $rcgo$ , 1)

Hold: ( $rcgo + 1$ , 0.5;  $rcgo - 1$ , 0.5)

#### 2.3.1 The S-shaped utility function and investor rationality

The founders of prospect theory have given a very specific utility function:

$$u(x) = \begin{cases} x^\alpha, & x \geq 0 \\ -\lambda(-x)^\beta, & x < 0 \end{cases}$$

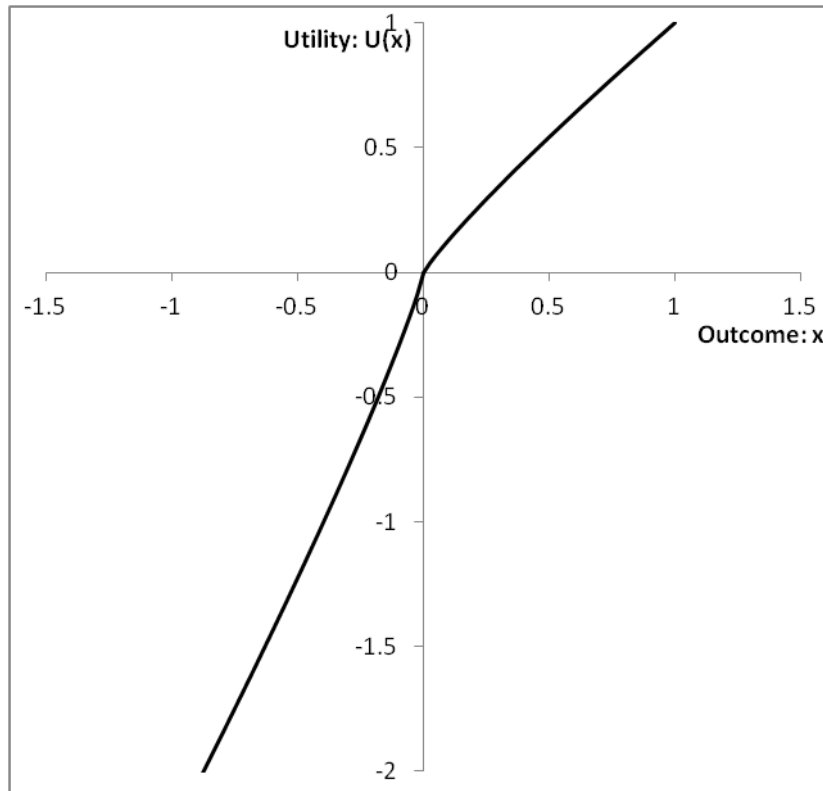
Where:

$$\alpha = \beta = 0.88$$

$$\lambda = 2.25$$

The shape of this function is illustrated by the following graph.

Figure 2.3-1: The S-shaped utility function



The horizontal axis represents the wealth in terms of gain or loss, the vertical axis represents corresponding utility.

The above figure demonstrates the S-shaped utility function. The origin point is a reference point separating gain and loss. The function is concave in the section  $x > 0$  and convex in the section  $x < 0$ . Such propriety describes the loss aversion behavior of investors.

Following the definition of rationality in the expected utility theory, investor maximizes the expected utility, hence prefers the option which offers higher expected utility.

### 2.3.2 Preference function of prospect theory

The preference function in regret theory offers a clear way to compare investor's preference between two options. Adopting the same idea, the preference function of prospect theory will be developed to study the market decision making scenario.

The preference for the option Sell is the difference between the utility of Sell and the expected utility of Hold. In the given market decision making scenario, the outcomes of the Sell and the Hold options are written in terms of the Relative Capital Gain Overhang, therefore, the preference function  $Q()$  is also written in terms of RCGO:

$$Q(x) = u_S(x) - E(u_H(x))$$

Where:

$x$ : represents the Relative Capital Gain Overhang

$u$ : is the prospect theory utility function

In this equation, the prospect value of the Sell option is:

$$u_S(x) = u(x)$$

The expected prospect value of the Hold option is:

$$E(u_H(x)) = \frac{1}{2}(u(x-1)) + \frac{1}{2}(u(x+1))$$

In this way, the preference for selling the stock is described as a function of unrealized gain or loss hence could directly show its influence on the decision making. Recall that the S-shaped utility function has different forms in positive and negative domains; therefore the preference function requires to be discussed in four sections in order to apply correct form of the utility function.

Section  $x < -1$ , corresponds to the large loss situation:

In this section,  $x < 0$ ,  $x+1 < 0$ ,  $x-1 < 0$ . The preference function in this section is:

$$Q(x) = -\lambda(-x)^\beta - \frac{1}{2}\{-\lambda[-(x+1)]^\beta - \lambda[-(x-1)]^\beta\}$$

Section  $-1 \leq x < 0$ , corresponds to the small loss situation. In this section,  $x < 0$ ,  $x+1 > 0$ ,  $x-1 < 0$ . The preference function in this section is:

$$Q(x) = -\lambda(-x)^\beta - \frac{1}{2}\{(x+1)^\alpha - \lambda[-(x-1)]^\beta\}$$

Section  $0 \leq x < 1$ , corresponds to the small gain situation: In this section,  $x > 0$ ,  $x+1 > 0$ ,  $x-1 < 0$ . The preference function in this section is:

$$Q(x) = x^\alpha - \frac{1}{2}\{(x+1)^\alpha - \lambda[-(x-1)]^\beta\}$$

Section  $x > 1$ , corresponds to the large gain situation: In this section,  $x > 0$ ,  $x+1 > 0$ ,  $x-1 > 0$ . The preference function in this section is:

$$Q(x) = x^\alpha - \frac{1}{2}\{(x+1)^\alpha + (x-1)^\alpha\}$$

To sum up, the preference function for the option Sell in the given scenario is:

$$Q(x) = \begin{cases} -\lambda(-x)^\beta - \frac{1}{2}\{-\lambda[-(x+1)]^\beta - \lambda[-(x-1)]^\beta\}, & x < -1 \\ -\lambda(-x)^\beta - \frac{1}{2}\{(x+1)^\alpha - \lambda[-(x-1)]^\beta\}, & -1 \leq x < 0 \\ x^\alpha - \frac{1}{2}\{(x+1)^\alpha - \lambda[-(x-1)]^\beta\}, & 0 \leq x < 1 \\ x^\alpha - \frac{1}{2}\{(x+1)^\alpha + (x-1)^\alpha\}, & x \geq 1 \end{cases}$$

Where:

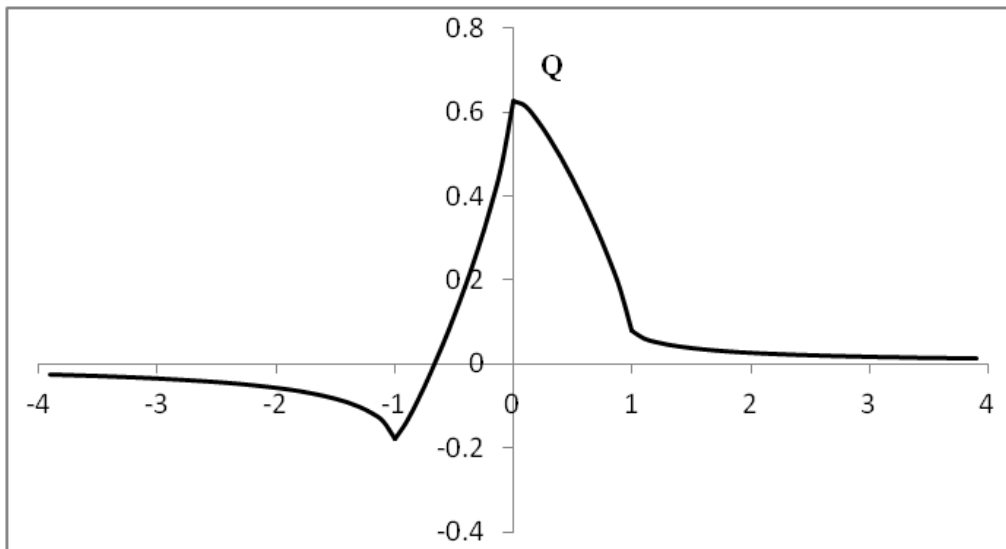
Q: is investor's preference for the option Sell

x: is the relative capital gain overhang

$\alpha, \beta, \lambda$ : are parameters of the S-shaped utility function, with  $\alpha = \beta = 0.88, \lambda = 2.25$

The graphic of the preference function Q provides an intuitive illustration of the influence of unrealized gain or loss on the decision making.

Figure 2.3-2: The preference for Sell depending on unrealized gain or loss



The horizontal axis represents unrealized gain or loss, the vertical axis represents investor's preference for the option Sell.

The above figure shows that the preference function is not a constant. It may be positive or negative depending on relative capital gain overhang. It suggests that according to prospect theory the decision making of a loss aversion investor is influenced by unrealized gain or loss.

### **2.3.3 Investor behavior under unrealized gain or loss**

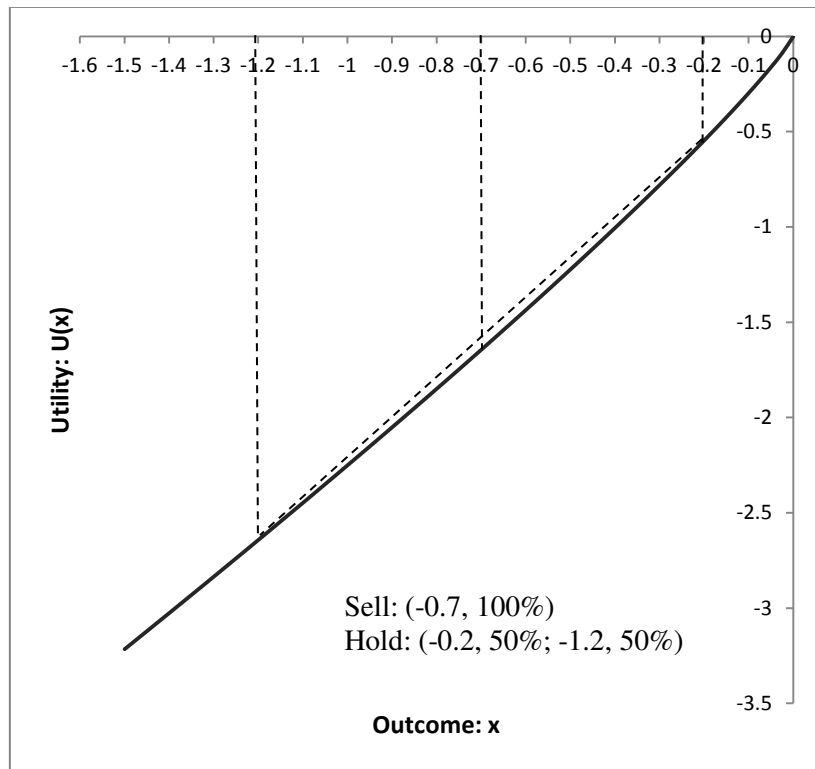
The graphic of the preference function provides an illustrative prediction of a loss aversion investor's decision making facing unrealized gain or loss. Overall, investor has strong preference for sell or holding the stock when the unrealized gain or loss is relatively small; and the two options tends to be indifferent for investor when either the gain or the loss is very large.

The Figure 2.3-2 shows that the curve of the preference function has three inflection points that divide the curve into four sections. These four sections correspond to the potentially large loss, small loss, small gain and large gain situation in the market decision making scenario. All these situations will be explained with the help of graphics.

#### The large loss situation

Once in a large loss situation, the unrealized loss is larger than the stock price could vary. Even if the price goes up in the next period, investor will be still at a unrealized loss. In the following figure, the unrealized loss is down to -0.7, even if the stock price goes up, the investor will still at a loss of -0.2. For a loss aversion investor, when the loss is inevitable, the investor prefers to take a chance and hope the best case happens in order to reduce the unrealized loss.

Figure 2.3-3: Investor's preference in the large loss situation



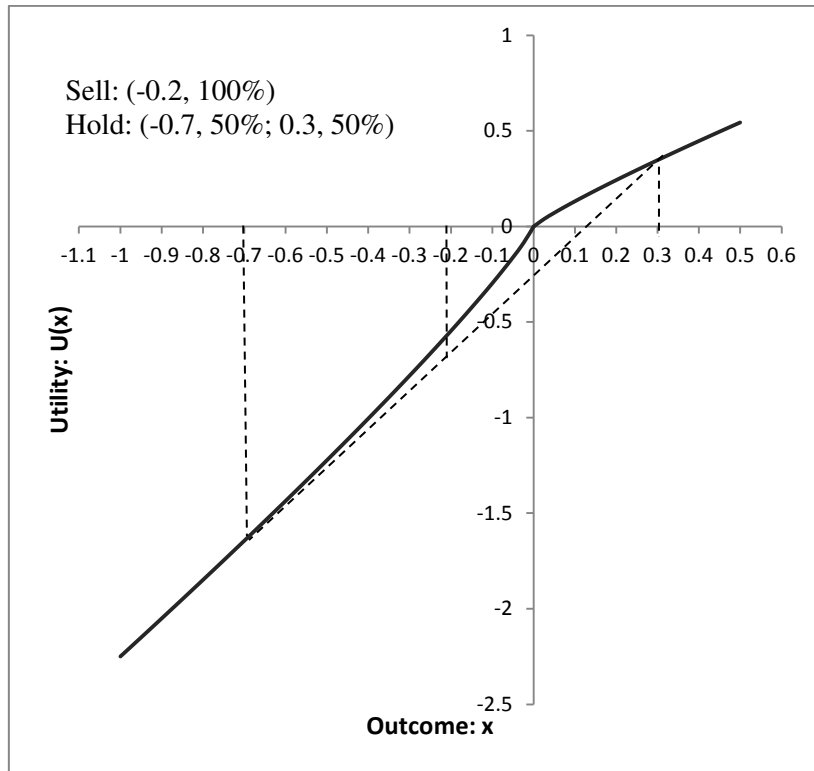
In this situation, due to the convex shape of the utility function, larger the unrealized loss is, comparatively the unit of risk matters less to investor. Therefore, although the investors still favors to hold, their preference for holding slightly decreases with the amount of the unrealized loss, and tends to be indifferent when facing very large unrealized losses.

#### The small loss situation

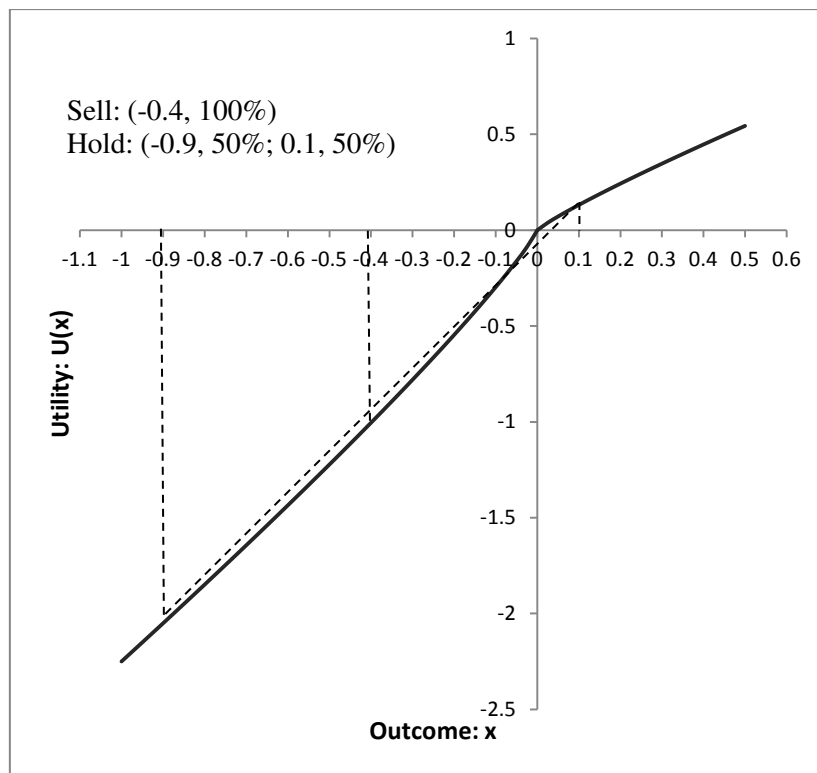
In the small loss situation, the Sell option allows investor to cut loss and accept the loss; the Hold option offers a gamble with the possibility to get out of the losing situation, but with the risk to fall deeper into an even worse situation. For a loss-averse investor, when facing a very small amount of unrealized loss, the investor prefers to cut loss in exchange for avoiding the risk of falling into an even worse situation. The panel A of the following figure illustrates this preference for cutting losses. However, as the initial unrealized loss grows larger, it becomes more difficult for investor to cut loss, the preference for the option Sell becomes weaker; finally when the initial unrealized loss is too large that investor becomes risk loving, and is willing to take the risk and hope for the best case happens that allowing the investor to recover all of the unrealized loss. The panel B of the following figure demonstrates this preference for gambling.

Figure 2.3-4: Investor's preference in the small loss situation

Panel A: Cutting losses is preferred



Panel B: Gambling for a win is preferred

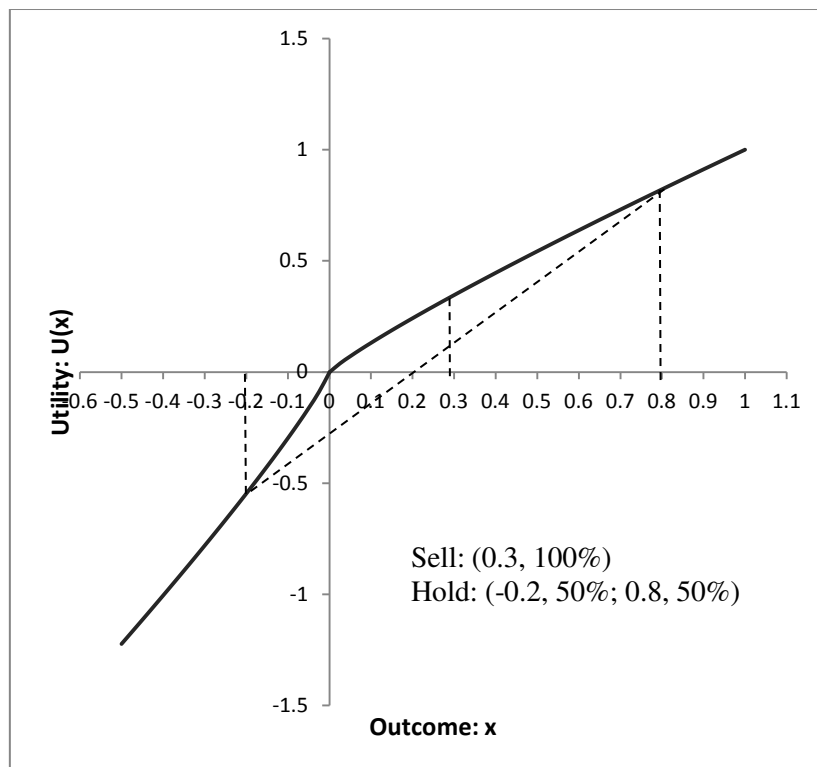


In a small loss situation, the investor could show preference for either the Sell option or for the Hold option. The loss aversion attitude could manifest as “fear” to potentially suffer greater losses when an investor is already facing a small amount of loss, especially when the amount of the loss that the investor is actually facing is much smaller than the amount of the loss that the investor will suffer from if the gamble fails. Therefore, in this case, a preference to the option Sell can still be observed.

#### The small gain situation

In the small gain situation, the Sell option allows investor to cash in the unrealized gain, while the Hold option offers the possibility to increase the gain, but with the risk of falling into a losing situation. A loss aversion investor prefers to sell the stock so that the gain is guaranteed, and risk of losing is avoided. As the following figure illustrates, the investor is currently facing a gain of 0.3. For this investor, accepting this gain is much better than taking a gamble and risking falling into a loss of -0.2, because the dissatisfaction for a loss of -0.2 out-weights the satisfaction for a win of 0.8.

Figure 2.3-5: Investor’s preference in the small gain situation



However, investor's motivation for selling the stock becomes weaker when the initial unrealized gain increases. With a larger the initial unrealized gain, if the investor chooses to hold the stock, he would suffer from a smaller loss when the stock price goes down, and

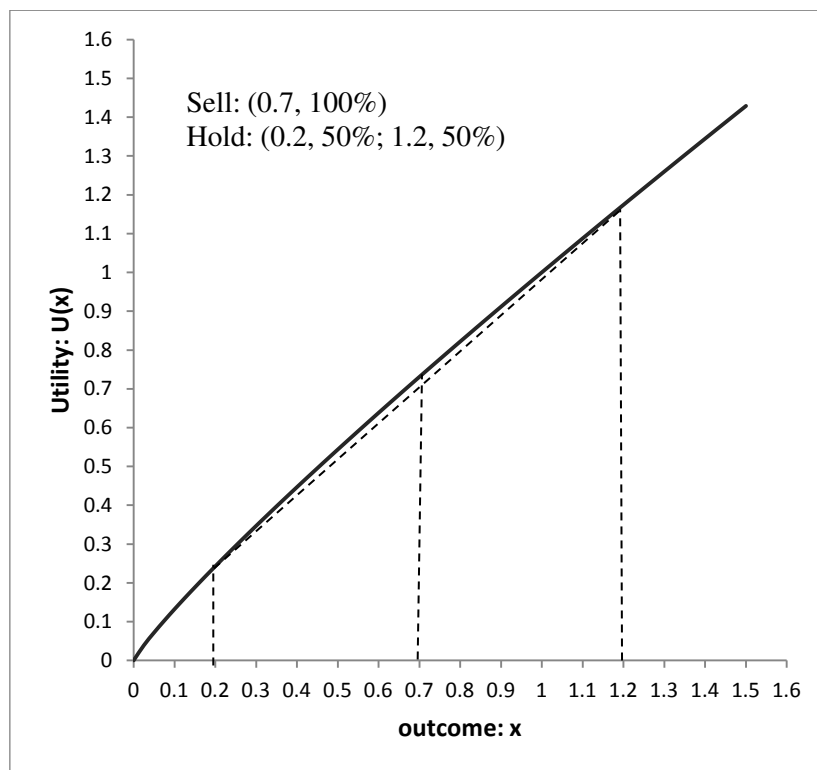


would enjoy a larger gain when the stock price goes up. This means that the option Hold becomes more attractive when the initial unrealized gain increases. Therefore, investor's preference for selling the stock becomes weaker when investor has larger initial unrealized gain.

#### The large gain situation

In the large gain situation, no matter stock price goes up or down, the investor remains in a winning situation, because the initial gain is even larger than the stock price could vary. As the following figure demonstrates, the investor is facing a unrealized gain of 0.7. If the investor keeps on holding the stock, the worst outcome is still a gain of 0.2. In such guaranteed winning situation, the prospect theory suggests that a loss-averse investor should also show risk aversion. Therefore, the option Sell will always be preferred. However, such a preference will decrease when the initial gain becomes even larger. With a greater initial gain, the impact of one unit of risk will be smaller on investor's final wealth level. Hence, the investor tends to become indifferent between option Sell and Hold when the initial unrealized gain is very large comparing to the stock price variation.

Figure 2.3-6: Investor's preference in the large gain situation



To sum up, in a unrealized gain situation, the option for selling the stock is always preferred, but such a preference decreases with a increase of the unrealized gain. In a

unrealized loss situation, the investor's preference to sell also shows a decreasing trend with the increase of the unrealized loss, but when the loss is sufficiently large, the investor will prefer to hold the stock.

## 2.4 Combining loss aversion and regret aversion

Both regret aversion and loss aversion could influence investor's decision making, and in real situation, the influence could be mixed. Therefore, in this part, a model is proposed to combine both effects.

### 2.4.1 S-shaped choiceless utility function

In previous part, applying the regret theory to the unrealized gain or loss situation yields a null result. Investor is strictly indifferent to selling or holding the stock. Individual's indifference in equal probability situation is caused by the assumption of a linear choiceless utility function. Recall the modified utility function of the regret theory:

$$V(x, y) = c(x) + g[c(x) - c(y)]$$

Where:

$c()$  is the choiceless utility function, and  $c(x) = x$

$g()$  is the regret-rejoice function

However, the regret theory does not give any restriction to the form of the choiceless utility function; therefore, it is possible to apply the S-shaped utility function from the prospect theory in the case of the choiceless utility function. The S-shaped utility function from the prospect theory (Kahneman and Tversky 1992):

$$u(x) = \begin{cases} x^\alpha, & x \geq 0 \\ -\lambda(-x)^\beta, & x < 0 \end{cases}$$

Where  $\alpha=\beta=0.88$ ,  $\lambda=2.25$

This new S-shaped choiceless utility function could describe the behavior of loss aversion, while the regret-rejoice function describe the behavior of regret aversion. In such way, the utility function could describe the combined effect of loss aversion and regret

aversion. Therefore, the regret and loss aversion value function for choosing option  $x$  instead of  $y$  is:

$$V(x, y) = c(x) + g[c(x) - c(y)]$$

Where:

$c()$  is the choiceless function in the form of the S-shaped utility function:

$$c(\xi) = \begin{cases} \xi^{0.88}, & \xi \geq 0 \\ -2.25(-\xi)^{0.88}, & \xi < 0 \end{cases}$$

$g()$  is the regret-rejoice function

Recall that in previous part, two forms of regret-rejoice functions are used, the first one is proposed by the theory:

$$g(\xi) = 1 - e^{-\xi}$$

The second one is elicited from experiment:

$$g(\xi) = \frac{2}{3}(\xi)^3$$

Both forms of regret-rejoice function will be used in studying investor's preference with the combined utility function.

#### 2.4.2 The combined utility function applies to the scenario

Recall the market decision making scenario involving unrealized gain or loss, investor faces such lottery:

Sell: (RCGO, 1)

Hold: (RCGO + 1, 0.5; RCGO - 1, 0.5)

The same as in the previous analyses, the study will focus on investor's preference of the option Sell over the option Hold. The preference function developed from the combined utility function will be used in the study.

$$Q(rcgo) = E[V(rcgo, rcgo \pm 1)] - E[V(rcgo \pm 1, rcgo)]$$

Where:

RCGO: is relative capital gain overhang, the input of the function

$V(rcgo, rcgo \pm 1)$ : represents the utility for choosing Sell instead of Hold

$V(rcgo \pm 1, rcgo)$ : represents the utility for choosing Hold instead of Sell

Replacing the utility function  $V()$  with the combined form, the preference function will be:

$$Q(rcgo) = \frac{1}{2} \{c(rcgo) + g[c(rcgo) - c(rcgo - 1)] + c(rcgo) + g[c(rcgo) - c(rcgo + 1)]\} - \frac{1}{2} \{c(rcgo - 1) + g[c(rcgo - 1) - c(rcgo)] + c(rcgo + 1) + g[c(rcgo + 1) - c(rcgo)]\}$$

Where  $c()$  is the S-shaped utility function:

$$c(\xi) = \begin{cases} \xi^{0.88}, & \xi \geq 0 \\ -2.25(-\xi)^{0.88}, & \xi < 0 \end{cases}$$

Applying the two difference form of the regret-rejoice function  $g()$ , two specific preference functions could be found:

The theoretical deduced function, with  $g(\xi) = 1 - e^{-\xi}$ :

$$Q(rcgo) = \frac{1}{2} \{c(rcgo) - c(rcgo - 1) - e^{c(rcgo-1)-c(rcgo)} + e^{c(rcgo)-c(rcgo-1)} + c(rcgo) - c(rcgo + 1) - e^{c(rcgo+1)-c(rcgo)} + e^{c(rcgo)-c(rcgo+1)}\}$$

Where:

$$c(\xi) = \begin{cases} \xi^{0.88}, & \xi \geq 0 \\ -2.25(-\xi)^{0.88}, & \xi < 0 \end{cases}$$

The empirically elicited function, with  $g(\xi) = \frac{2}{3}(\xi)^3$ :

$$Q(rcgo) = \frac{1}{2} [36(c(rcgo) - c(rcgo - 1))^3 + 3(c(rcgo) - c(rcgo - 1)) + 36(c(rcgo) + c(rcgo + 1))^3 + 3(c(rcgo) + c(rcgo + 1))]$$

Where:

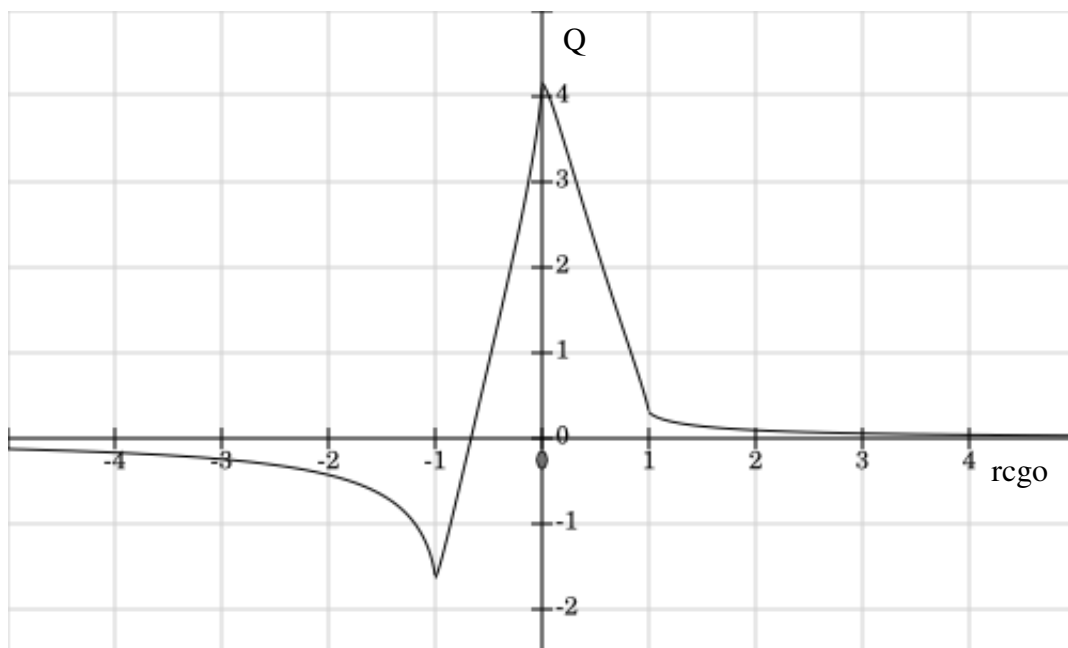
$$c(\xi) = \begin{cases} \xi^{0.88}, & \xi \geq 0 \\ -2.25(-\xi)^{0.88}, & \xi < 0 \end{cases}$$

Due to the propriety of the S-shaped utility function, the preference function will have different forms in the sections of  $RCGO < -1$ ,  $-1 \leq RCGO < 0$ ,  $0 \leq RCGO < 1$ , and  $RCGO \geq 1$ , corresponding to the large unrealized loss situation, small unrealized loss situation, small unrealized gain situation, and large unrealized gain situation.

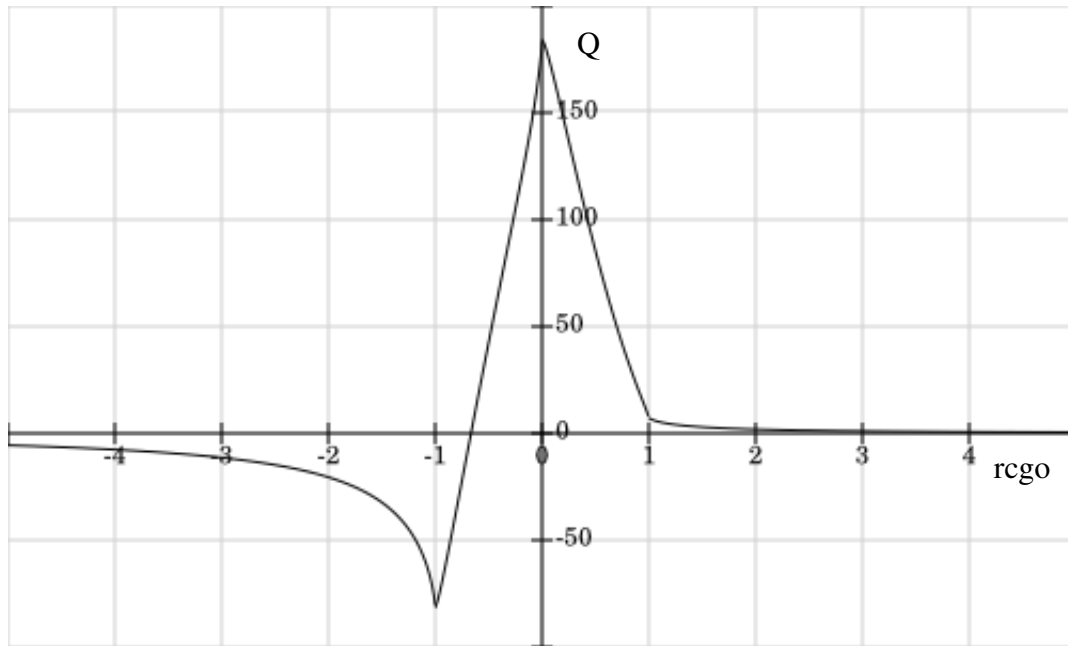
The specific preference function in each section will not be demonstrated because of its lengthy form. Instead, the graphic of the preference function will be an efficient way for illustrating investor's preference in different unrealized gain or loss situations when both loss aversion and regret aversion are taken into consideration. The following graph shows the graphic of both theoretically and experimentally obtained preference functions for all four situations.

Figure 2.4-1: The preference for selling the stock of a loss aversion and regret aversion investor

A: Using theoretical preference function:



B: Using elicited preference function:



In both of the graphs, the vertical axis represents investor's preference for selling the stock; the horizontal axis represents the initial unrealized gain or loss. Graphic by online function grapher

The result suggests that, in large unrealized loss situation, investor shows weak preference for holding the stock, but the preference grows stronger with the decrease of the unrealized loss, and reaches its peak when the scale of unrealized loss equals to the scale of volatility. In small unrealized loss situation, investor prefers to hold the stock when the scale of unrealized loss is slightly smaller than the scale of price volatility. With the decrease of the unrealized loss, the option for selling the stock becomes increasingly interesting, and finally is preferred by the investor. In unrealized gain situation, the option for selling the stock is always preferred, but when the unrealized gain is at a very large scale, the investor become almost indifferent to the selling or holding option. The following table compares the predictions in four situations by the two regret functions.

Table 2.4-1: Comparison between results using different  $g$  functions

|                       | Theoretical function with $g(\xi) = 1 - e^{-\xi}$                          | Empirical function with $g(\xi) = \frac{2}{3}\xi^3$                        |
|-----------------------|--|--|
| Large unrealized loss | Very weak preference for holding, increases with loss approach to -1       | Very weak preference for holding, increases with loss approach to -1       |
| Small unrealized loss | Maximum preference for holding at -1, Maximum preference for selling at 0. | Maximum preference for holding at -1, Maximum preference for selling at 0. |
| Small unrealized gain | Strong preference for selling and decreasing with gain                     | Strong preference for selling and decreasing with gain                     |
| Large unrealized gain | Weak preference for selling, and decreasing with gain                      | Weak preference for selling, and decreasing with gain                      |

The table shows that the predictions given by the different functions are identical.

In terms of consistency, the above graph shows that the curves of the two functions have the same shape, and the only difference is the scale in the vertical axis. This suggests that the two types of the regret-rejoice functions used in the two studies are highly consistent. In addition, comparing to the previous study on the prospect theory, each of the four part yields consistent result to their counterpart in the previous study.

To sum up, at unrealized gain, investor always prefers to sell the stock for certain gain, but the selling intention decreases with the scale of unrealized gain. If the unrealized gain in large enough, investor is almost indifferent between the Sell and Hold options. At unrealized loss, the Sell option is preferred when the unrealized loss is small in order to prevent any further loss. While the unrealized loss grows to match the price variation level, investor favors strongly the Hold option hoping for turning the loss to gain. When the unrealized loss is large, the Hold option become decreasingly interesting because even the price goes up at nest period, the investor will still be in a losing situation.

This result reveals a new behavior pattern of investor: they are sensitive to change in current winning or losing situation. In the graph, the maximum value and the minimum value are found in the sections of small gain and small loss. Within this interval, investor's preference changes drastically with variation of RCGO. On the other hand, in the large gain and large loss section, investor's preference tends to be indifferent between the two options. From a different point of view, this result may suggest that individual may not perceive risks in terms of number of the outcome, but in terms of winning or losing. In a very large gain or loss situation, the variation of the payout is the same as in small gain or loss situation, but such variation is not able to change current winning to losing situation, hence, investor is indifferent to it.

The result of this study suggests also that when the regret aversion and loss aversion are took into consideration; investor's preference will be different from what the traditional theory describes. In this study, the price variation level, considered as the risk, is standardized. According to traditional theory, for a given level of risk, the risk aversion investor demands a certain amount of compensation for bearing the risk. Therefore, in the same scenario as in the above study, the traditional risk aversion investor should exhibit a constant degree for the preference to the selling option. In the graph, the curve representing such investor's preference will be a line that is parallel to the x axis. Suppose that the market equilibrium is defined according to traditional theory, while the investor shows the pattern of regret and loss

aversion, the excess or the lack in the actual preference will introduce extra or insufficient selling pressure in the market, and pushes the price to deviate from equilibrium level.

### 2.4.3 Distinguishing regret aversion and loss aversion

The market decision making scenario is set to study how unrealized gain or loss influences the preference for selling or holding the stock for certain types of investors, namely regret aversion investors and loss aversion investors. According to the scenario, the following lottery is proposed:

Sell: (RCGO, 1)

Hold: (RCGO + 1, 0.5; RCGO - 1, 0.5)

At a given moment, RCGO is the relative capital gain overhang, representing unrealized gain or loss. The RCGO could be positive or negative for representing a unrealized gain or unrealized loss situations.

Regret theory proposes the rationality of regret aversion, and investor maximizes the modified utility function as the following:

$$V(x, y) = c(x) - g[c(x) - c(y)]$$

Where:

$$c(x) = x, \quad g''(\xi) > g''(-\xi) \text{ for } \xi > 0$$

Despite that regret theory has many interesting properties, for this specific lottery, the theory predicts that a regret aversion investor is absolutely indifferent between the Sell and Hold action.

$$EQ(s, h) = E[V(s, h) - V(h, s)] = 0$$

In fact, in this lottery, the probability distribution and the payout of raising and dropping in the Hold action are strictly symmetric. Therefore, the regret for “losing” and the regret for “not winning” cancel each other in such a symmetric lottery, and the investor will always be indifferent between Sell and Hold action.

The prospect theory introduces investor’s aversion to loss. This property is represented by an S-shaped utility function:



$$u(x) = \begin{cases} x^\alpha, & x \geq 0 \\ -\lambda(-x)^\beta, & x < 0 \end{cases}$$

Where:  $\alpha=\beta=0.88, \lambda=2.25$

Applying this utility function on the given lottery, investor's preference can be predicted by:

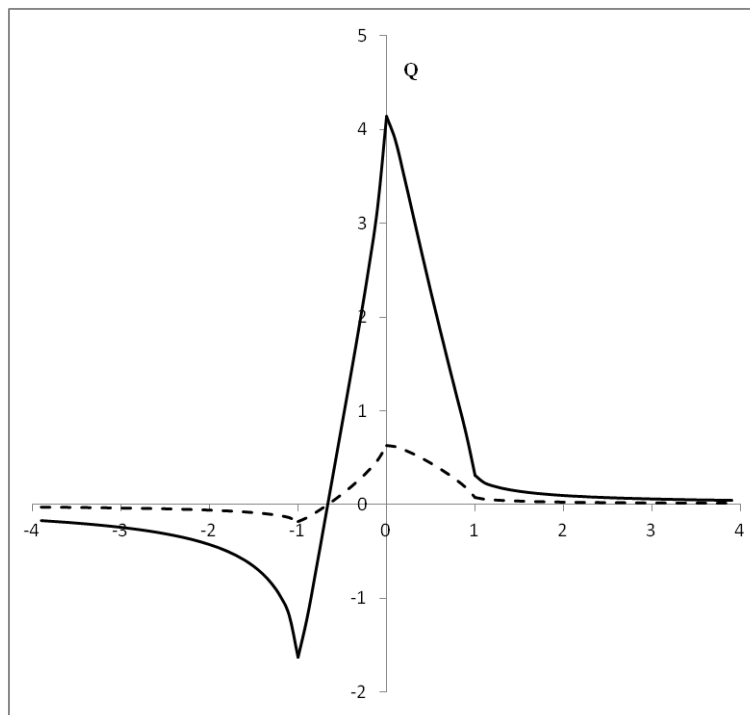
$$Eq(S, H) = E[u(s) - u(h)]$$

The calculation is performed in four sections:  $x < -1, -1 \leq x < 0, 0 \leq x < 1, x \geq 1$ . The result shows that investor's preference for selling the stock varies depending on the level of the unrealized gain or loss.

Using the S-shaped utility function as the choiceless utility function in the regret theory provides one way to combine the behavior of regret aversion and loss aversion. The combined utility function yields the same prediction as the prospect theory.

To compare the results obtained from difference models, the graphs of the preference functions from regret aversion model, loss aversion model, and the combine model are demonstrated in the same plan, as presented in the following graph:

Figure 2.4-2: Comparison between regret and loss aversion model and prospect theory



The horizontal axis represents capital gain overhang; the vertical axis represents investor's preference for Sell action. The dashed line represents the model including only prospect theory; the solid line represents the combined model.

This graph shows that the curve of prospect theory is almost identical to the curve of regret and loss aversion model, but is only in a smaller scale. This suggests that the variation of investor's preference originates from the loss aversion. The regret aversion, gives a null prediction if a linear choicless utility function is applied, but acts as an amplifier of the loss aversion effect in the combined model.

This result is also consistent to the mathematical formula of the preference function  $Q()$ . When only the loss aversion is considered, the preference is the difference between the  $u(s)$  and  $u(h)$ . However, when both loss aversion and regret aversion are combined, the preference is the difference between the  $V(s,h)$  and  $V(h,s)$ , which are also functions of  $u(s)$  and  $u(h)$ , because  $u()$  is considered as the choicless function. As a result, the preference function in a combined framework  $Q(s,h)$  is in fact a function of the preference function in the prospect theory framework  $q(s,h)$ :

$$Q(s, h) = F[q(s, h)]$$

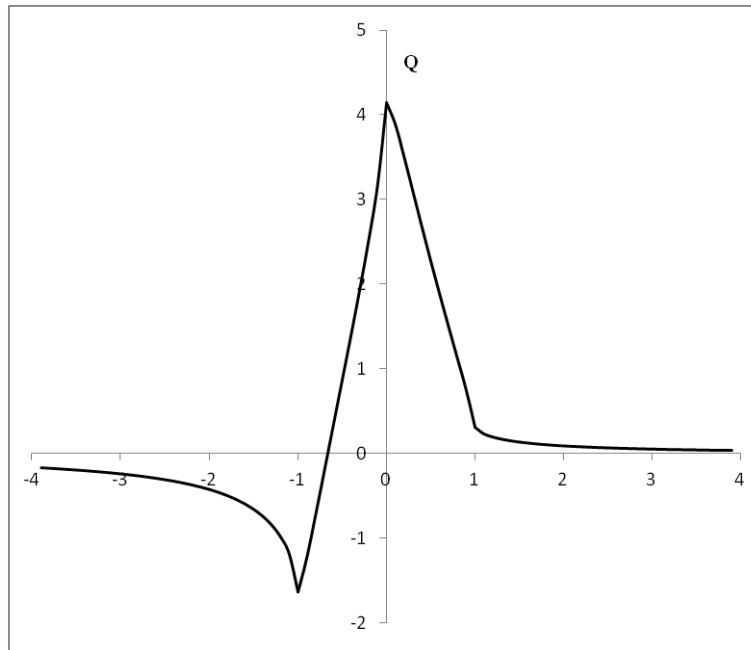
In this formula, the function  $F()$  represents the influence from regret aversion. From this formula, one can deduce that the influence the regret aversion functions as an indirect factor. It deals with the result from the loss aversion, and amplifies the investor's preference.

Overall, the results of the above studies suggest that for regret and loss aversion investor, the decision making processes is influenced by the combined effect of the two factors. The loss aversion plays a major role in determining investor preference to sell or to hold the stock. The regret aversion enhances the effect of the loss aversion.

## 2.5 Discussions

The model combining regret aversion and loss aversion gives the prediction that investor's preference alters depending on unrealized gain or loss. The following figure illustrates the relation between preference and RCGO.

Figure 2.5-1: Investor's preference to Sell stock under combined model



In this section, discussions are brought on some theoretical conditions of the model, and its application to empirical level.

### 2.5.1 The demand side of the market in the scenario

In the above section, it is assumed that investors have the options for either sell the stocks or hold the stocks. Such configuration implies that the analysis focus on the supply side in a financial market. In this section, the same scenario is used to analyze the demand side of the financial market.

Taking the same assumption that stock price follows a binomial distribution. There is equal probability that the price goes up or down by  $\Delta p$ . The stock price at  $t+1$  will hence be:

$$p_{t+1} : (p_t + \Delta p, 0.5; p_t - \Delta p, 0.5)$$

To represent the demand side of the market, it is assumed that investors have the options for either buy or not to buy the given stock. If investor chooses to buy the stock, the outcome of this option will be the stock price at  $t+1$ . If investor chooses not to buy the stock, in other word, to wait, the outcome will be zero. Therefore, the demand side investors will face the following lottery:

$$\text{Wait: } (0, 1)$$

Buy:  $(+ \Delta p, 0.5; - \Delta p, 0.5)$

Since the acquisition cost of buying the stock is the stock price at time  $t$ , which is  $p_t$ , the capital gain overhang for the buyers of the stock equals to zero. In terms of gain or loss, the outcome of the option Buy will be  $\Delta p$  or  $-\Delta p$ , in other word, the variation of stock price.

Applying the same process of standardization by stock price variation as in the analysis on the supply side of the market, all the outcomes in the lottery are divided by the stock price variation. The outcome of the Wait option will still be zero for sure, and the outcome of the Buy option will be:

Buy:  $(\frac{\Delta p}{\Delta p}, 0.5; \frac{-\Delta p}{\Delta p}, 0.5)$

The payoffs of the lottery become:

Wait:  $(0, 1)$

Buy:  $(1, 0.5; -1, 0.5)$

The lottery for the demand side investors in the market is classic and straightforward risk versus certainty situation. The variable of capital gain overhang does not appear in the lottery, suggesting that the unrealized gain or loss is not a concern for the demand side investors. It is very obvious that the acquisition cost does not exist for those who have not yet acquired the stock.

The unrealized gain or loss, as a behavior factor, appears only on the supply side of the market, and does not appear on the demand side of the market. Therefore, if such factor could influence investors' preference, there will be effect that could make the supply curve of the market to move. On the other hand, the demand curve is not influenced by the unrealized gain or loss, hence remains not moved. With the static demand curve, any movement in the supply curve will result to change in the equilibrium price.

At a deeper level, such observation could potentially challenge the hypothesis of homogeneity in the classic financial pricing theory. Even though investors share same expectation on the risk level of an asset, the buyers and sellers of the asset could have disagreement on the compensation for bearing the risk.

### 2.5.2 From preference to risk premium

So far, the study on unrealized gain or loss focuses on the influence on investor's preference, but the study could be extended to the influence on risk premium. Take the market decision making scenario again, investor faces the following lottery:

Sell: (RCGO,1)

Hold: (RCGO+1,0.5; RCGO-1,0.5)

This lottery is basically a case of decision making of certainty versus risk with a shifting reference point. Therefore, the lottery can be renamed as:

Riskless: (RCGO,1)

Risky: (RCGO+1,0.5; RCGO-1,0.5)

According to its definition, risk premium is the compensation required by an investor for the expected utility of the risky option equals to the utility of the riskless option.

$$U(rcgo) = EU(rcgo + \tilde{x} + \pi)$$

Where:

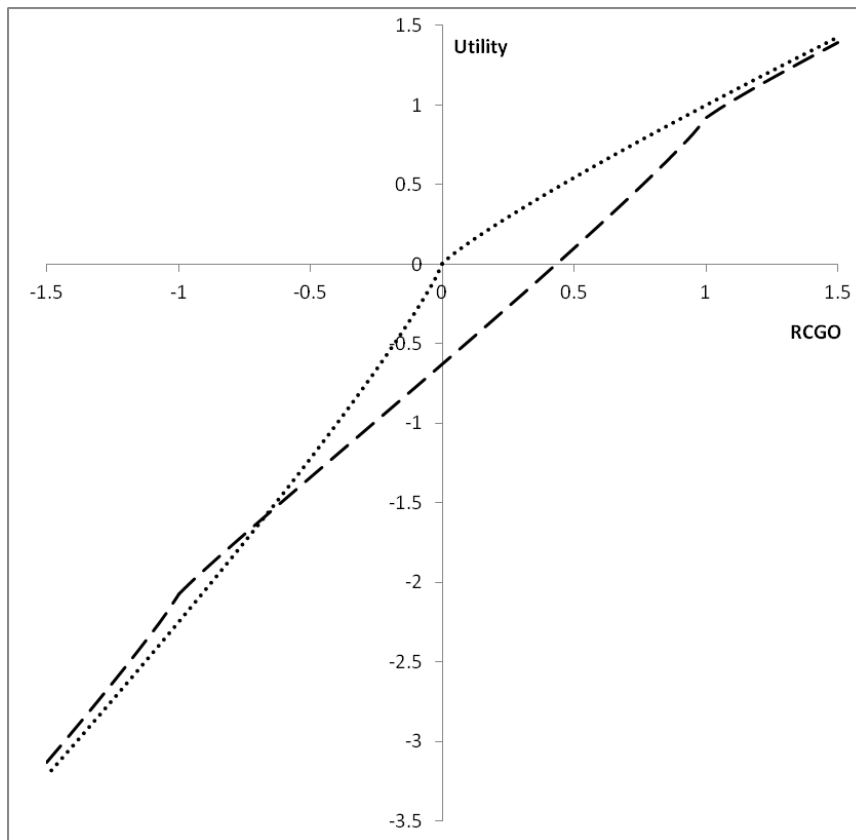
$\tilde{x}$ : is the zero mean risk component,  $\tilde{x} = 1$  or  $-1$ .

$\pi$ : is the risk premium

Studies in previous parts have provided very specific utility functions hence the utility of the riskless option and the expected utility of the risky option can be illustrated in the following figure.

According to the definition, the horizontal distance between the curve of the utility function of the riskless option and the expected utility function of the risky option is the risk premium of the risky option at given unrealized gain or loss level. For example, in a situation where there is no unrealized gain or loss. The utility of the riskless option equals to zero, while the expected utility of the risky option is negative. In order to compensate for bearing risk, it requires the risky option offers a risk premium that equals to the horizontal intercept of the expected utility curve of the risky option so that both options have the same utility level.

Figure 2.5-2: Comparison of utility of riskless and risky options



The dot line is the utility of the riskless option; the dash line is the expected utility of the risky option. The horizontal distance between the lines is the risk premium.

The lottery given by the market decision making scenario takes place in a localized point; it does not have a drastic influence on investor's wealth level. In addition, the risk level is standardized to one unit of risk for any level of unrealized gain or loss. In such condition, the risk premium should be a constant according to classic theory. However, the graph shows that under the assumption of loss aversion investor and unrealized gain and loss, the risk premium of one unit of risk, which is the horizontal distance between the two curves, varies depending investor's potential winning or losing situation.

According to the graph, for one unit of risk, the premium is the highest when investor is facing no unrealized gain or loss. With the increase in either unrealized gain or loss, the investor requires less compensation for bearing the risk, the risk premium diminishes. In the case of large unrealized loss the risk premium even becomes negative. A general trend can be observed: the risk premium decrease with the absolute value of relative capital gain overhang.

Assume that investors are influenced by both risk aversion and loss aversion; the risk premiums caused by these two factors could be summed to form investor's final demand of compensation for bearing one unit of risk.

$$\pi_{total} = \pi_{EU} + \pi_{LA}$$

Where:

$\pi_{EU}$ : represents the risk premium under expected utility theory

$\pi_{LA}$ : represents the risk premium of loss aversion

In this manner, the total risk premium can be considered as the classic risk premium plus a distortion caused by loss aversion in the presence of unrealized gain or loss. The amount of such distortion can be proximately measured by the relative capital gain overhang, more precisely, the absolute value of the relative capital gain overhang.

For a stock in the market, the relative capital gain overhangs of the stock measures on average how much unrealized gain or loss that the holders are experiencing. According to above analyses on the risk premium and on the demand side of the market, a large RCGO in absolute value implies that the risk premium for the holders of this stock has more distortion caused by loss aversion comparing to the investors who are going to invest in the stock. Such discrepancy in risk premium in the sellers and buyers of the stock should result to changes in stock price, and hence leads to changes in stock return. The mechanism of how idiosyncratic distortion of risk premium influences the stock return is unknown, but it is able to predict that the evolution of stock return should be different in stocks with extreme RCGO and stocks with null RCGO.

### **2.5.3 The case with a positive expected outcome**

In previous sections, all analyses are based on the decision making scenario that offers the riskless option and the risky option with equal expected outcome. The payout for the sell option is the unrealized gain or loss, and the expected outcome of the hold option is the same. However, in many existing theoretical work, it usually assumes that the risky option could offer a higher expected outcome in order to attract risk aversion investors. In this section, such case will be considered.

In order to introduce a positive expected outcome into the risky option, there will be a positive constant term added in the risky option of the original lottery.

Sell: (RCGO,1)

Hold: (RCGO+1+c,0.5; RCGO-1+c,0.5)

In this new lottery, the RCGO is still the variable representing the shift of reference point. the lottery remains the basic form as in previous analyses. The constant  $c$  ( $c > 0$ ) makes the difference in expected outcome between the riskless option and the risky option.

With this new lottery, investor's preference will always be represented by the difference between the utility of the riskless option and the expected utility of the risky option. Same method can be used as in previous studies. Investor's preference is represented by the preference function  $Q$  depending on the variable RCGO. Under uncertainty, investor's decision of taking the Sell or the Hold option depends on the expected value of the preference function.

$$EQ_{S,H}^n(RCGO) = E[u_S^n(RCGO) - u_H^n(RCGO)]$$

Where:

The superscript  $n$  indicates the preference or the utility of the new lottery.

The subscript  $S, H$  indicate the Sell option or the Hold option.

The term  $Q_{S,H}$  indicates the preference of choosing Sell option instead of Hold option.

In this equation, the utility of the Sell option is the same as in the original lottery, and since  $c$  is a constant, the expected utility of the Hold option is the expected utility of the original Hold option plus the utility of  $c$ :

$$Eu_H^n(RCGO) = Eu_H^o(RCGO) + u(c)$$

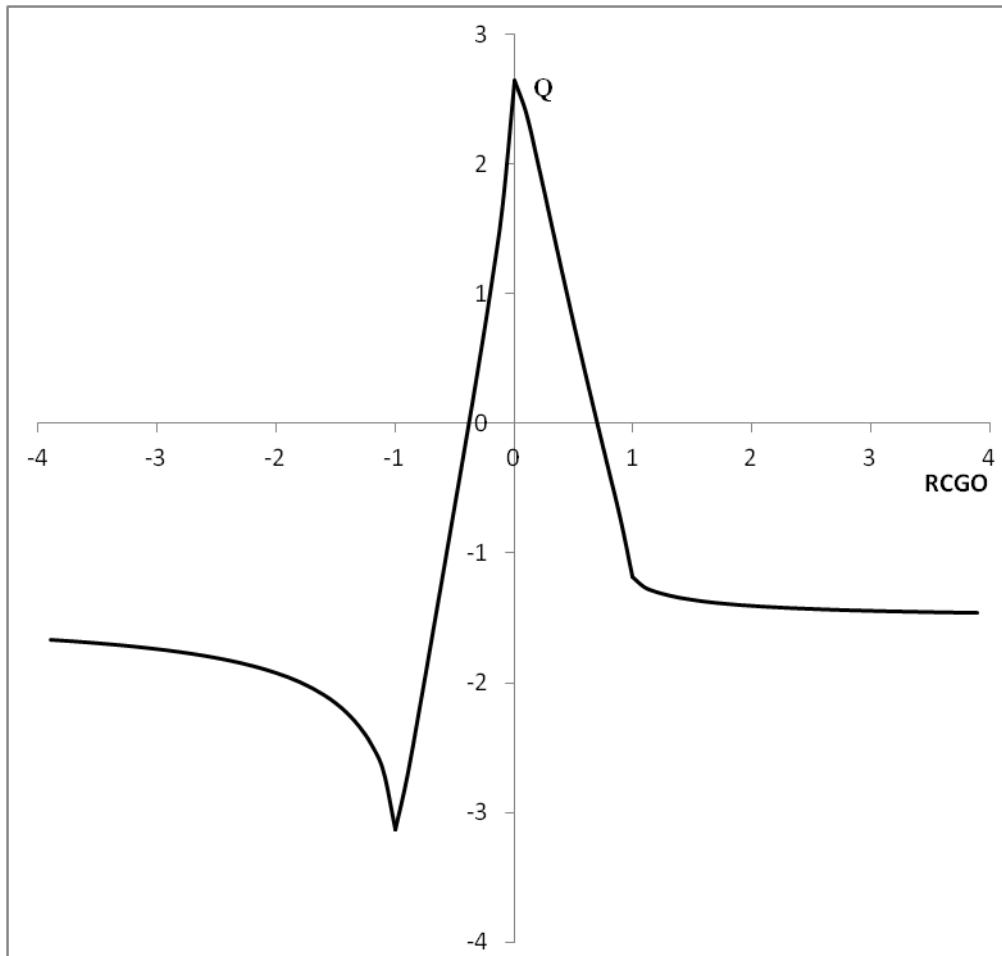
Replacing this term in the preference function, it can obtain that the expected preference of the new lottery is the expected preference of the original lottery minus the constant:

$$EQ_{S,H}^n(RCGO) = EQ_{S,H}^o(RCGO) - u(c)$$

This equation indicates that introducing a positive expected value in the risky option could result to the curve of the preference function moving towards the downside in the vertical axis. However, this shift does not change other property of the curve of the preference function.



Figure 2.5-3: The preference curve for the new lottery



The curve of investor's preference is shifted to the downside in the vertical axis comparing to the original curve.

The investor's preference curve in the new lottery with a positive expected value for the risky option suggests that first, the risky option becomes more favorable. This is a reasonable consequence for the risky option offering higher compensation. Second, the shape of the preference curve does not change. Investor's preference is still a function of the RCGO. Same trend is found in this case: investor has stronger preference to sell the stock when experiencing small unrealized gain or loss, while has stronger preference to hold the stock when experiencing large unrealized gain or loss.

#### 2.5.4 EU function in preference/RCGO plan

One obvious way to compare the prediction from the expected utility theory to the prediction from the combined model is to integrate preference function of expected utility theory into the preference/RCGO plan. However, such approach is not feasible.

The expected utility theory studies the total wealth level of an individual, while the prospect theory studies the variation of wealth around a reference point. The difference in such conditions implies that the expected utility theory cannot apply to the negative wealth situation.

Furthermore, the scale of the utility functions is completely incomparable. In the previous analyses, the exponential function is used to describe the regret aversion behavior. The same type of function is also used as the utility function in the expected utility theory. Though using the same function, in EU theory, the utility function represents the entire wealth level; the function in the combined model represents only a small variation on one localized point in the utility curve. Therefore these two models are not in the same scale.

For these two reasons, it is not appropriate to directly add the preference function of expected utility in the preference/RCGO plan to compare with the combined model. However, from certain property of the expected utility theory, it is able to give an approximation of the preference function of the expected utility theory.

The concave shape of the utility function in EU theory suggests that investors are risk-averse, and the degree of risk-aversion changes with investor's total wealth level. For an identical amount of risk, the higher the wealth level is, the less sensitive investors are. Other factors do not influence investor's attitude towards risk.

Recall the market decision making scenario, investor faces the following lottery:

Sell: (RCGO,1)

Hold: (RCGO+1,0.5; RCGO-1,0.5)

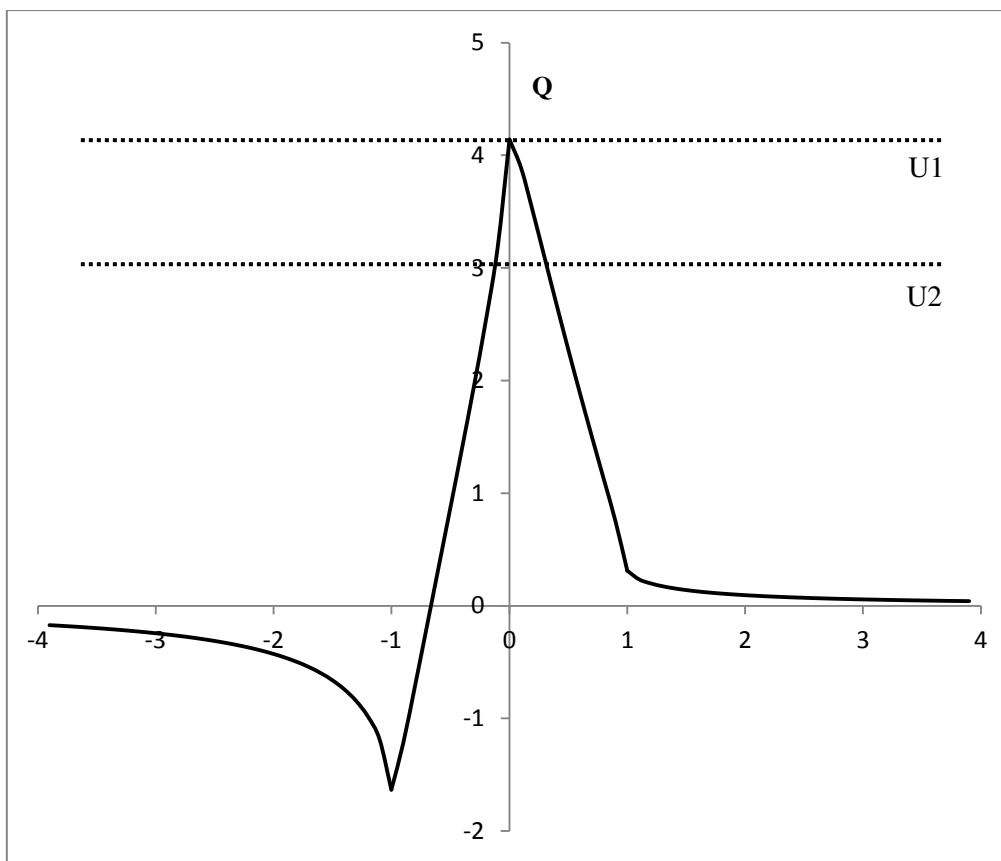
At a given moment, RCGO is the relative capital gain overhang. It could be positive or negative for representing a unrealized gain or unrealized loss. At this moment, the Sell option offers a riskless outcome, which equals to the unrealized gain or loss; while the Hold option offers a risky outcome with 50-50 chance for gaining or losing one more unit of value. The Sell and Hold option have same expected outcome, but the Hold action is a risky one.

For a risk-averse investor, if the expected outcomes of the two options are equal, the riskless option will always be preferred. In addition, the market decision making scenario is assumed to be applied to a localized point in the utility curve representing the total wealth level of an investor. This suggests that the variation of the outcome of the Hold option does

not have a significant influence on the total wealth level. Therefore, it can be assumed that investor's risk-aversion degree does not change depending on the variation of the outcome. This means that, for a risk-averse investor in the EU theory, his/her preference for the Sell option is constant. In the preference/RCGO plan, the curve of the preference of such investor is a straight line above, and parallel to the horizontal axis.

However, the above analyses do not provide enough evidence to define the exact intercept of the curve of the preference function under expected utility theory. Under the assumption that the market decision making scenario applies to a localized point of the total wealth and the variation of the outcome is trivial comparing with the total wealth, it is able to conclude that investor's preference for option Sell will not be too strong. Such a conclusion is valid in the scale of total wealth, but in the scale of localized point, it is not able to determine if the intercept is larger or smaller than the maximum value of the preference function of the combined model.

Figure 2.5-4: Possible integration with expected utility functions



The blue curve represents the investor's preference under the alternative utility model, the dot lines  $U_1$ ,  $U_2$  represents two possibilities of investor's preference under the classic expected utility function.

One possible approach is to look at the combined model where RCGO equals zero, where the unrealized gain or loss play no role in describing investor's preference. But it is not appropriate to consider the value of this point as the value of EU line's intercept. These two models capture different behavior aspects: the combined model captures the aversion to regret and to loss, EU model captures the aversion to risk. There is no clear evidence showing that these two effects are equivalent.

Overall, it is not able to make direct comparison between investor's preference under the expected utility theory and under the regret and loss aversion model. Despite such constrain, the combined model shows different behavior pattern to the conventional model. Investor's preference is influenced by unrealized gain or loss and risky option could be preferred under certain unrealized loss situation.

### **2.5.5 Connection with the disposition effect**

In the previous part, one behavior pattern named the disposition effect is mentioned. Such effect describes the phenomenon that investors are likely to sell the winning stocks too soon, and are likely to hold the losing stocks too long. Such description implies a comparison to a norm, which is the behavior under classic utility theory.

Such description of the disposition effect makes it difficult to link the combined model to the effect, since the direct comparison between classic utility theory and the combined model is not possible. However some similarity and differences can still be found between the disposition effect and the combined model.

It is possible to say that the combined model could match the description of disposition effect in some specific situations. In general, investor's preference could change depending on the potential winning or losing situation. The combined model predicts preference for Sell when investor is at unrealized gain and preference for Hold when investor is at certain level of unrealized loss. Similarly, the disposition effect records the preference changes in the same directions.

The regret loss aversion model suggests that the preference is not evenly distributed for any RCGO, but is concentrated in the small unrealized gain/loss situation. However, this concentration does not mean an excessive preference with respect to traditional theory, since it is not able to directly compare these two models' preferences in one coordinate system.

Consequently, it is not able to conclude that the relatively high selling preference in small unrealized gain situation is actually higher than the ‘normal’ selling preference. In addition, the regret and loss aversion model predicts the opposite disposition effect in small unrealized loss situation. In such situation, instead of holding the losing stocks, as described in disposition effect, the regret and loss aversion investor tends to sell the stock in order to prevent further loss. Therefore, despite the similarity, there is no exact evidence for the unrealized gain/loss model to explain the disposition effect.

## **2.6 Prediction and test design**

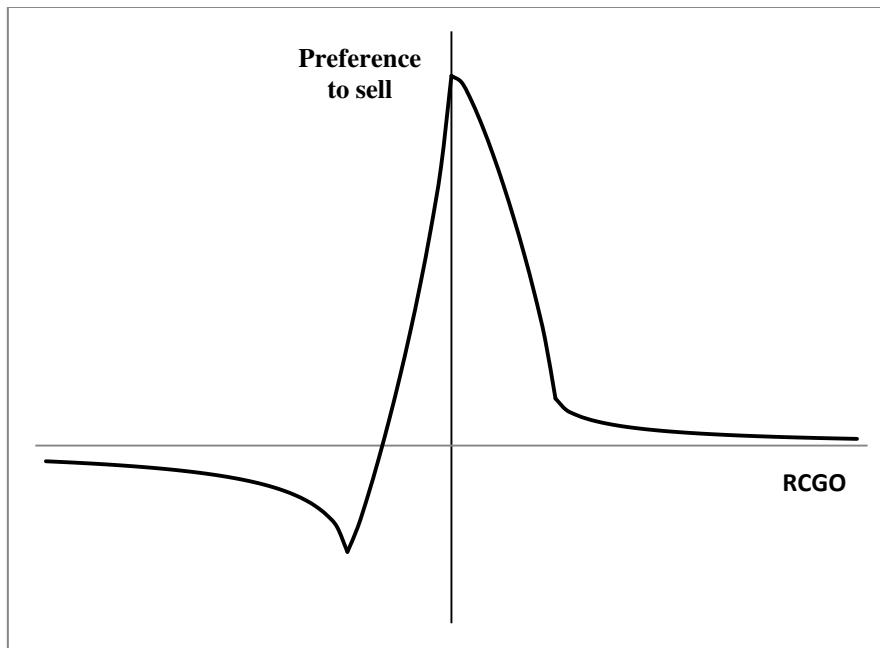
This section summarizes the theoretical analysis in this chapter and proposes tests for verifying the unrealized gain/loss model. The theoretical analyses in this chapter suggest that unrealized gain or loss is another factor that could influence investor’s preference in his/her decision making. The theoretical analyses begin with setting up a lottery that represents a market decision making scenario, and introducing a shifting reference point representing investor’s unrealized gain or loss situation.

### **2.6.1 Result interpretation**

There are two features about this configuration: first, the lottery is a typical riskless option versus risky option choice which is used in classic utility theory. Second, the construction of the variable Relative Capital Gain Overhang allows measuring the unrealized gain or loss level while controlling the risk level that investor is exposed to. These two features allow comparing the change in preference between the classic utility theory and the alternative utility theories, while isolating the influence caused by exposure to risks.

The influence of the unrealized gain or loss on investor’s preference under the framework of alternative utility theories is illustrated by the following graph:

Figure 2.6-1: Influence of RCGO on preference



It should be cautious to interpret this result. The previous sections have discussed the difficulty to directly incorporate the Expected Utility theory into this graph; therefore it is not appropriate to use descriptions referring to a comparison to a benchmark, such as ‘extra preference’ or ‘excessive preference’. As a result, this graph suggests that first investor’s preference is not a constant as suggested in the classic theory. Second, investor’s propensity to sell his/her stock decreases when the investor experiences either an increasing unrealized gain or an increasing unrealized loss. Finally investor is more sensitive to change in winning or losing situation.

### 2.6.2 Prediction on stock return

Until here, the result obtained from the theoretical analyses is limited to the level of decision making. The model suggests a decreasing propensity to sell a stock when either the unrealized gain or the unrealized loss increases. But this model cannot explain how such behavior pattern could transfer to stock price or stock return.

One approach for connecting behavior pattern to stock return is by the market mechanism of supply and demand. In an aggregated level, if in average the holders of a stock are found be in a large unrealized gain or a large unrealized loss situation, they have less propensity to sell the stock. If the demand of this stock is not perfectly elastic, a reduction in supply could cause stock price to rise, hence leads to higher stock return.

However, classic financial theory suggests that stock price should be the actualization of future cash-flows. Deviation from this correct price, such as the over-valuation caused by unrealized gain/loss should be corrected by the market through the mechanism of arbitrage. Therefore, one should expect lower stock return after observing a large unrealized gain or a large unrealized loss. Apparently, these two approaches give opposite prediction on how unrealized gain or loss influences stock return. Moreover, under the framework of alternative utility theory, change in investor's preference can be justified as variation in risk premium depending on unrealized gain or loss. As discussed in previous section, in large unrealized gain or loss situation, investor tends to have lower demand of compensation for bearing a constant level of risk. Consequently, the seller and buyer of the stock do not have homogeneity in risk premium. In this sense, the 'miss-pricing' in the classic framework can be justified as a response to variation in risk premium.

From the perspective of informational efficiency, the Efficient Market Hypothesis (Fama 1965, 1970) suggests that stock price should incorporate all information available on the market. According to the result from previous section, investors have weak propensity to sell the stock when experiencing either large unrealized gain or large unrealized loss. Such behavior pattern could result to less trading to occur, hence cause a slowed price reaction to new information. By such approach, it could suggest that stocks with large unrealized gain or loss suffer from strong under-reaction to information.

To sum up, the overall effect of the unrealized gain or loss cannot be clearly determined. For such reason, the theoretical model could only yield one weak prediction: the evolution of stock return should be different between stocks with large unrealized gain or loss and stocks with small unrealized gain or loss. Consequently, at this stage, the theoretical model could not provide an explanation of the momentum effect.

### **2.6.3 Test design**

In the following sections, empirical tests will be conducted. They aim first at verifying the prediction given by the theoretical model. Moreover, the empirical test expects to find certain pattern in return evolution between stocks with large gain/loss and stocks with small gain/loss. Such empirical results might help to better understand how the behavior pattern could influence stock return, and also might provide new clues that allow linking the alternative utility theories to the momentum effect.

Performing an empirical test is a complex job involving many detailed works. For such reason, the first empirical test in this dissertation will be a replication of an existing study. The replication object is the work of Grinblatt and Han (2005). This study is one of the pioneering researches in this field. Its empirical test provides a very good example to follow. Next, the empirical model of Grinblatt and Han's will be adapted to perform tests on the subject of Relative Capital Gain Overhang.

## **2.7 Summary**

This chapter presents a theoretical model under the framework of prospect theory and regret theory. This model focuses on the influence of the unrealized gain or loss. In order to isolate this factor from risks, the unrealized gain or loss is defined in the form of a ratio of the amount of gain or loss to the variation level of the outcome. As a result, the variable Relative Capital Gain Overhang is proposed. This variable allows distinguishing four situations: large loss, small loss, small gain, and large gain. In a large gain/loss situation, the unrealized gain/loss is so large that the investor will certainly remain in the same gain/loss situation in the next period. In a small gain/loss situation, if the investor chooses the risky option, there is a chance that his gain/loss situation inverts in the next period.

Applying the utility functions of prospect theory and regret theory to RCGO, this model shows that investors' preferences undergo some drastic variations due to RCGO. In an unrealized gain situation, investors prefer to sell the stock. But their preferences decrease with the unrealized gain. In an unrealized loss situation, investors could show preference to sell in order to cut a small unrealized loss. But when the loss is large, investors prefer to hold the stock. In general, investors' preferences to sell the stock decrease when facing either a gain or a loss compared with a null RCGO situation.

This model of unrealized gain or loss could describe investors' preferences, but it cannot provide a direct prediction on how this factor impacts stock returns. Existing literature suggests that the forces of supply and demand on the market to transmit investor preferences to stock prices. When investors show a decreased preference to sell a stock at the aggregated level, there should be insufficient supply of the stock. Therefore, the stock price will go up. The empirical tests in the following chapters will focus on examining the impact of the unrealized gain or loss on stock returns.



### Annex 2.1: Zero-sum game preference

- **Indifference in equal probability game**

Two lotteries:

$$A: (x,1)$$

$$B: (x+y, 0.5; x-y, 0.5)$$

The expected preference for choosing A:

$$\begin{aligned} EQ(A, B) &= \frac{1}{2}Q(x, x + y) + \frac{1}{2}Q(x, x - y) \\ &= \frac{1}{2}[x - x - y + g(x - x - y) - g(x + y - x) + x - x + y + g(x - x + y) - g(x - \\ & \quad y - x)] \\ &= \frac{1}{2}[-y + g(-y) - g(y) + y + g(y) - g(-y)] \\ &= 0 \end{aligned}$$

Therefore, for any function  $g()$ , regret aversion individual is always indifferent for playing or not.

- **Probability dependent preference**

Two lotteries:

$$A: (0,1)$$

$$B: (1, p; -p/(1-p), 1-p). (0 < p < 1)$$

B is the payoff of a zero-sum game  $E(B)=0$ , A stands for not to play the game. Hence the preference is:

$$\begin{aligned} EQ(A, B) &= p[-1 + g(-1) - g(1)] + (1 - p) \left[ \frac{p}{1-p} + g\left(\frac{p}{1-p}\right) - g\left(-\frac{p}{1-p}\right) \right] \\ &= \left[ -\frac{p}{1-p} + \frac{p}{1-p}g(-1) - \frac{p}{1-p}g(1) + \frac{p}{1-p} + g\left(\frac{p}{1-p}\right) - g\left(-\frac{p}{1-p}\right) \right] / (1 - p) \end{aligned}$$

$$= \left\{ \left[ \frac{p}{1-p} g(-1) - g\left(-\frac{p}{1-p}\right) \right] + \left[ g\left(\frac{p}{1-p}\right) - \frac{p}{1-p} g(1) \right] \right\} / (1-p)$$

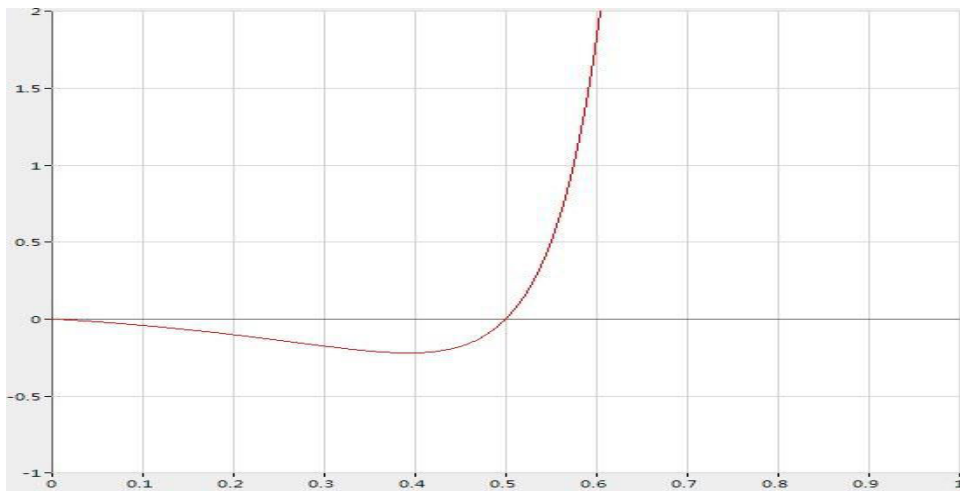
EQ=0 when p=0.5. However, for other probability, EQ can be positive or negative when the regret function is not linear. Assuming that g() is concave as:

$$g(x) = 1 - e^{-x}$$

Then:

$$EQ(A, B) = \left\{ \left[ \frac{p}{1-p} e^{-1} - e^{-\frac{p}{1-p}} \right] + \left[ e^{\frac{p}{1-p}} - \frac{p}{1-p} e \right] \right\} / (1-p)$$

The graphic of this function is the following (as the preference for NOT buying)



Source: Online Function Grapher

EQ is negative when p is smaller than 0.5, and positive when p is greater than 0.5. Individual prefers to play the zero-sum game when the game is more likely to yield a gain. However, this result is obtained under the assumption of a linear choiceless function and a concave regret function.

### Annex 2.2: Finding g() from Q()

$$Q(\xi) = 36\xi^3 + 3\xi$$

Let  $\xi = x/3$ , then:

$$Q = 36\left(\frac{x}{3}\right)^3 + 3\left(\frac{x}{3}\right) = \frac{4}{3}x^3 + x = x + \frac{2}{3}x^3 - \frac{2}{3}(-x)^3$$

The original form of Q is:

$$Q(x) = x + g(x) - g(-x)$$

Hence:

$$g(x) = \frac{2}{3}x^3$$

# Chapter 3

## Preliminary tests

This chapter conducts the first empirical test in this dissertation. The test will follow the example of an existing test performed in the work of Grinblatt and Han (2005). Replicating this test is expected to offer inspiration on many aspects for performing future tests, such as adaptation of theoretical model to empirical test model; definition of test variables; and test methodology.

In the following sections of this chapter, Grinblatt and Han's test will be reviewed in order to understand how the test model is applied, how the test variables are estimated and the method used in the test. Next, understanding the existing test allow reflection being made, and lead to propose hypotheses on the time horizon of the test. Finally, empirical test will be performed to replicating Grinblatt and Han's test, and also to verify the hypotheses.

This chapter does not only consist of a simple replication of Grinblatt and Han's test. First, the sample period in this chapter covers a more recent period of time. More importantly, this chapter investigates the way the variable Capital Gain Overhang is estimation, and suggests that the estimation method of CGO could distinguish the immediate reaction or the subsequent reaction of the market to CGO. The test does not find consistent result with Grinblatt and Han's. It suggests that Grinblatt and Han's test result is sensitive to sample period and to the method used in estimating the CGO.

### 3.1 Review of Grinblatt and Han's model and test

The work of Grinblatt and Han (2005) studies the disposition effect under the framework of prospect theory and mental account, and its influence on stock return. In this study, the disposition effect is explained by the prospect theory. The reasoning is that there are a part of investors in the market who show loss aversion, hence in winning position, they show risk-averse attitude, and tend to sell the stock; in a losing position, they show risk loving attitude, and tend to hold the stock. Therefore, the extra selling pressure will appear to the winning stocks, the stocks will be undervalued, then the market will correct such an undervaluation, positive return is expected. Through the same reasoning, a losing stock will cause insufficient selling pressure, the stock will be overvalued, the market corrects the overvaluation, and negative return is expected.

The reference price is the price at which the investor has bought the stock. It distinguishes the winning or losing stocks. Winning stock will have a positive Capital Gain Overhang; losing stock will have a negative Capital Gain Overhang. Therefore, there should be a positive correlation between stock return and Capital Gain Overhang.

#### 3.1.1 Test model

The theoretical analysis of Grinblatt and Han suggests that the disposition effect, closely linked to the capital gain overhang, will impact the stock price; while the market corrects such an impact, it cause a positive correlation between stock return and capital gain overhang. To test this correlation, they proposed an empirical test model allowing performing regression test.

$$r = a_0 + a_1 r_{-4:-1} + a_2 r_{-52:-5} + a_3 r_{-156:-53} + a_4 \bar{V}_{-52:-1} + a_5 s_{-1} + a_6 cgo_{-1}$$

Where:

$r$  is the weekly stock return;

$r_{-4:-1}$  is the cumulative return from week -4 to week -1;

$r_{-52:-5}$  is the cumulative return from week -52 to week -5;

$r_{-156:-53}$  is the cumulative return from week -156 to week -53;

$\bar{V}$  is the average turnover from week -52 to -1;

$s$  is the market capitalization of week -1;

$cgo$  is the capital gain overhang of week -1;

Grinblatt and Han's empirical model seeks a positive correlation between stock return and Capital Gain Overhang. Other variables are also added in this regression model as control variables, including short intermediate and long term past return, trading volume of the stock in the past 52 weeks, and the market capitalization of the stock. The control variables in this model suggest that stock return could be related to its past performance, stock's liquidity, and the size of the firm.

Notice that this regression model is formulated purely at the empirical level, it does not have a pricing theory to support it. The reason for using such an empirical model instead of using the well established pricing model such as the CAPM or Fama-French Three-Factor model is not explained in their paper. One possible reason might be that the theoretical study of Grinblatt and Han is not performed under the classic risk-return framework, but rather under a supply - demand framework.

The regression is actually performed in two steps. In the first step, one regression is performed on the model without the variable CGO, in the second step, regression is performed on the complete model.

Table 3.1-1: The result of Grinblatt and Han's test (2005)

$$r = a_0 + a_1 r_{-4:-1} + a_2 r_{-52:-5} + a_3 r_{-156:-53} + a_4 \bar{V}_{-52:-1} + a_5 s_{-1} + a_6 cgo_{-1}$$

| Period  | $a_1$                 | $a_2$                | $a_3$                | $a_4$                | $a_5$                | $a_6$                |
|---------|-----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| All     | -0.0425<br>(-35.9364) | -0.0002<br>(-0.6794) | -0.0007<br>(-5.0871) | -0.0188<br>(-0.9364) | -0.0004<br>(-5.2885) | 0.0040<br>(7.7885)   |
| Jan     | -0.0520<br>(-10.9905) | -0.0001<br>(-0.0477) | -0.0025<br>(-3.8964) | -0.0620<br>(-0.9768) | -0.0026<br>(-8.4381) | -0.0117<br>(-4.9519) |
| Feb-Nov | -0.0407<br>(-32.6251) | -0.0000<br>(-0.0768) | -0.0006<br>(-3.6950) | -0.0291<br>(-1.3143) | -0.0002<br>(-2.8816) | 0.0050<br>(9.4191)   |
| Dec     | -0.0498<br>(-10.8151) | -0.0022<br>(-1.8953) | -0.0005<br>(-1.3410) | 0.1238<br>(1.7980)   | 0.0001<br>(0.2702)   | 0.0104<br>(6.2673)   |

Source: Grinblatt, M., Han, B., 2005. Prospect theory, mental accounting, and momentum. *Journal of financial economics* 78, 311–339.

The test of Grinblatt and Han found positive correlation between Capital Gain Overhang and stock returns, which is consistent with the result of their theoretical work. Furthermore, by introducing the Capital Gain Overhang in the model, the coefficient

representing the intermediate horizon past return becomes non-significant, suggesting that the effect of momentum is ruled out by Capital Gain Overhang.

### 3.1.2 Estimation of the key variables

In Grinblatt and Han's model, the Capital Gain Overhang is the variable that represents the disposition effect. In the theoretical study, the Capital Gain Overhang is defined as the difference between current stock price and the reference price, which is the investor's purchase price of the stock. In the empirical test, the stock price can be directly obtained, but the reference price cannot. Therefore it is crucial to make accurate estimation of the reference price. This section reviews how Grinblatt and Han estimate the reference price and the Capital Gain Overhang.

#### 3.1.2.1 The reference price

In the empirical context, Grinblatt and Han define the reference price as the average of all investor's purchase price of the stock. Grinblatt and Han provide one way to estimate this variable: calculating the weighted average between the stock price at date  $t$  and the reference price at date  $t-1$ . The weight is the percentage of the stock that is traded at date  $t$ . The equation for the calculation is the following:

$$R_t = v_t p_t + (1 - v_t) R_{t-1}$$

Where:

$R_t$  is the reference price for a stock at the end of week  $t$ ,

$p_t$  is the stock price at week  $t$ ,

$v_t$  is the trading volume in percentage:  $v_t = \frac{\text{volume}_t}{\text{share outstanding}_t} \times 100\%$

The idea behind this estimation is straightforward. At a given time, if the stock is traded during this time period, the investors bought the stock at current price  $p_t$ . If the stock is not traded, the investor's purchase price remains the same as in the last period. The weight is naturally the trading volume of the stock at this time period.

Grinblatt and Han (2005) also demonstrate an equivalent way to estimate the reference price. This method avoids using the reference price of the previous period, hence involves

only the past stock prices and considers the corresponding trading volume as the probability at which one share is traded in the corresponding time period.

$$R_t = \sum_{n=1}^{\infty} (v_{t-n} \prod_{\tau=1}^{n-1} (1 - v_{t-n+\tau})) p_{t-n}$$

Where:

$R_t$  is the reference price for a stock at the end of week  $t$ ,

$p_t$  is the stock price at week  $t$ ,

$v_t$  is the trading volume represented in percentage.

At a given time  $i$ , the probability for one share is trade at time period  $i$  and is no longer traded until the present period is  $(v_{t-i} \prod_{\tau=1}^{i-1} (1 - v_{t-i+\tau}))$ . Hence, in this formula, the reference price is considered as the expected value of the past stock prices over all the past periods, weighted by the probability of the stock being traded for the last time at the corresponding time period. In their empirical test, Grinblatt and Han use the second formula to estimate the reference price.

### 3.1.2.2 Capital gain overhang

In Grinblatt and Han (2005)'s study, the definitions of the variable Capital Gain Overhang are different in terms of time horizon in the theoretical model and the empirical test. In theoretical model, they define the CGO at period  $t$  as such:

$$CGO_t = (P_t - R_t)/P_t$$

Where:

CGO: is the capital gain overhang

P: is stock price

R: is the reference price

$t$ : indicates the time period

In this definition, CGO, price and reference price are all in the same time period.



The empirical test of Grinblatt and Han (2005), they use the one-period lagged capital gain overhang, calculated on the two-period lagged stock price. The variable is estimated in such form:

$$CGO_{t-1} = (P_{t-2} - R_{t-1})/P_{t-2}$$

According to Grinblatt and Han, the reason for using one-period lagged stock price is to avoid confounding market microstructure effects, such as bid-ask bounce.

### 3.1.3 Regression method

In financial studies, tests often encounter panel data. It contains cross-sectional data over large period of time. Grinblatt and Han adopt Fama-Macbeth regression (Fama and Macbeth 1973) to deal with panel data in this test.

The Fama-Macbeth method consists of performing cross-sectional regressions at each time period, and then using the estimated coefficients to perform a time-serial t-test for statistic significance. The Fama-Macbeth method allows giving a more accurate estimation of the standard error of the coefficients when cross-sectional correlation exists in the data, which is usually the case of financial data. Therefore, the t-test gives a more reliable result. The actual work of Fama and Macbeth (1973) contains three steps: first, estimation of beta; second, cross-sectional regression for factor loadings (coefficients); and finally, t-test on the time-serial data of factor loadings.

Concerning to the regret and loss aversion model test, the key factor to be estimated is the average acquisition price. But unlike the beta, the mathematical property of the average acquisition price is not discussed; therefore, it is not clear whether the method of forming portfolio could be applied on estimation the average acquisition price.

The estimation of average acquisition price requires five years of data; the stocks that have less than five years of records are deleted from the test. Following the Fama-Macbeth procedure, at each week  $t$ , one regression will be performed on the cross-section of all existing stocks at the time. The regressions will generate a serie of coefficients for different period:  $\hat{b}_{1t}, \hat{b}_{2t} \dots \hat{b}_{it}$ . The average of  $\hat{b}_{it}$  will be the final estimation of the coefficient and the standard deviation of  $\hat{b}_{it}$  will be used for calculating the t-statistics:

$$t(b_i) = \frac{m(\hat{b}_i)}{s(\hat{b}_i)/\sqrt{n}}$$

Where:

$m()$ : is the mean of the estimations

$s()$ : is the standard deviation of the estimations

In Grinblatt and Han's test, the cross-sectional regression at each time period is performed on individual stocks instead of on portfolios as was performed in Fama and Macbeth's test. Using individual stocks provides a much larger regression sample and could yield more accurate estimation of coefficients.

## 3.2 Observations and replication test hypotheses

After reviewing the test of Grinblatt and Han, this section brings up some observations and reflections over their study. The reflections mainly concern the change in time horizon in the tests as a result of using one-period lagged stock price to estimate the reference price. The change in time horizon is crucial for understanding how the disposition effect makes influence on stock return. Based on this observation, the replication test proposes additional hypotheses to verify the influence of the disposition effect over immediate time horizon and over subsequent time horizon.

### 3.2.1 Remarks on variable estimation

#### 3.2.1.1 The reference price

Regarding the reference price, Grinblatt and Han demonstrated two formulas to estimate the reference price. One formula adopts the reasoning of weighted average; the other formula adopts the thinking in terms of probability. But mathematically, these two formulas are equivalent.

However, there is one problem if trading volumes are used as the weight or the probability. On one hand, because of the existence of short time speculation activity, some shares can be traded multiple times during one period. Such activity could inflate the number

of shares traded. Hence the trading volume can be greater than one, which does not make sense neither as a weight nor as a probability. On the other hand, there are investors who intend to hold the stock over very long time. Therefore, some shares are not traded even if the trading volume exceeds one.

Because of this problem, directly using the trading volume to estimate the reference price will lead to incorrect value. To deal with this problem, Grinblatt and Han introduce a constant that corrects the sum of the probabilities to one.

$$R_{t-1} = \frac{1}{k} \sum_{n=1}^{260} (v_{t-1-n} \prod_{\tau=1}^{n-1} (1 - v_{t-1-n+\tau})) p_{t-1-n}$$

with  $k$  a constant that makes the weights on past prices sum to one.<sup>4</sup>

Another solution to this problem is to set a maximum value of the trading volume in the calculation of the reference price. Therefore, in actual calculation, a ceiling is arbitrarily set at 0.95 to prevent the probability from going higher. This solution could apply to both formulas used for estimation.

The first formula:

$$R_t = \min(0.95, v_t) p_t + (1 - \min(0.95, v_t)) R_{t-1}$$

The second formula:

$$R_t = \sum_{n=1}^{260} \left\{ \min(0.95, v_{t-n}) \prod_{\tau=1}^{n-1} [1 - \min(0.95, v_{t-n+\tau})] \right\} p_{t-n}$$

where:

$R_t$  is the average acquisition price for a stock at the end of week  $t$ ,

$p_t$  is the stock price at week  $t$ ,

$v_t$  is the trading volume represented in percentage

$\min()$ : is the operation of taking the minimum value between 0.95 and  $v_t$

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<sup>4</sup> Page 321, Grinblatt, Han Journal of Financial Economics 78 (2005)

In either method, the calculation requires using past data. The first method uses the average acquisition price of the previous period; the second method uses a series of past stock prices. Such a calculation will inevitably introduce time-serial auto-correlation in the data, and should be considered when performing the test.

From the formula that estimates the reference price one can remark that the reference price is mainly influenced by short term past stock price. The second formula offers a clear vision on this point. The weight of the price at past n period in the calculation of reference price is the product of a series of probabilities. Since the value of a probability is smaller than one, the weight on the earlier period past prices will geometrically decrease. This suggests that the reference price has larger weight on more recent stock prices.

### 3.2.1.2 Capital gain overhang

The definition of the variable capital gain overhang implies that it is the difference between current stock price and the reference price, which is the average acquisition cost of all investors in the market. In Grinblatt and Han (2005)'s study, the theoretical model defines the CGO at period t in the following way:

$$CGO_t = (P_t - R_t)/P_t$$

Where:

CGO: is the capital gain overhang

P: is stock price

R: is the reference price

t: indicates the time period

In the empirical test of Grinblatt and Han (2005), the Capital Gain Overhang is calculated on a lagged stock price. The variable is estimated by the following formula:

$$CGO_t = (P_{t-1} - R_t)/P_{t-1}$$

However, in other empirical studies, the estimation of capital gain overhang may still adopt the non-lagged stock price, in other word, the definition used in the theoretical studies. Frazzini (2006), Bhootra and Hur (2012) use this formula to calculate CGO in their empirical

tests. They calculate the variable using the stock price and the acquisition cost at the same date. It will be mentioned as the theoretical capital gain overhang ( $CGO^T$ ).

$$CGO_t^T = (P_t - R_t)/P_t$$

This estimation uses one-period lagged stock price to calculate capital gain overhang, and will be mentioned as the empirical capital gain overhang ( $CGO^E$ ).

$$CGO_t^E = (P_{t-1} - R_t)/P_{t-1}$$

By replacing the  $R_{t-1}$  by its definition, the following equation shows that the empirical CGO is in fact related to one-period lagged theoretical CGO (Details will be displayed in annex 3.2-1):

$$CGO_{t-1}^E = v_{t-1}(-r_{t-1}) + (1 - v_{t-1})CGO_{t-2}^T$$

Where:

E: indicates an empirical variable (defined with lagged past stock price)

T: indicates a theoretical variable

v: is stock's turnover

r: is one-period stock return

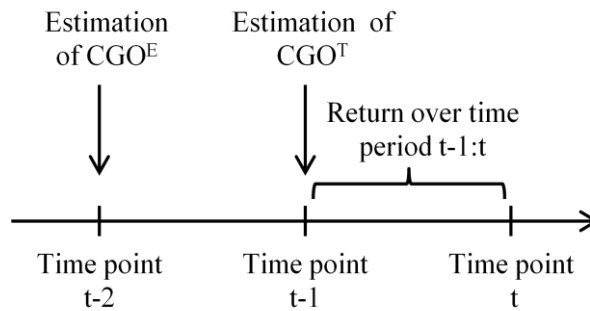
Using lagged or not lagged stock price in estimating capital gain overhang makes difference when studying the predictability of capital gain overhang on stock return in short horizon. The empirical one-period lagged CGO can be considered similar to the theoretical two-period lagged CGO.

### 3.2.2 Influence of time horizon on predictability

In terms of the time horizon, investors evaluate their unrealized gain or loss situation at the end of the current time period, and make their decision of sell or hold the stock in the next period. Hence, the stock price should react instantly to the change in selling pressure on the market. Therefore, stock with high selling pressure is expected to have low return in the following time period.

Assuming that the one-period stock return is measured at the time point of  $t = 0$ , for the period from  $t = -1$  to  $t = 0$ . Using the theoretical CGO implies that the unrealized gain or loss is measured at the time point of  $t = -1$ . Therefore, the one-period stock return will incorporate the immediate effect of existing unrealized gain or loss. The well documented disposition effect suggests that in winning situation, investors are likely to sell the winning stocks too soon. Such phenomenon implies that a positive capital gain overhang should cause a decrease of stock return in immediate horizon.

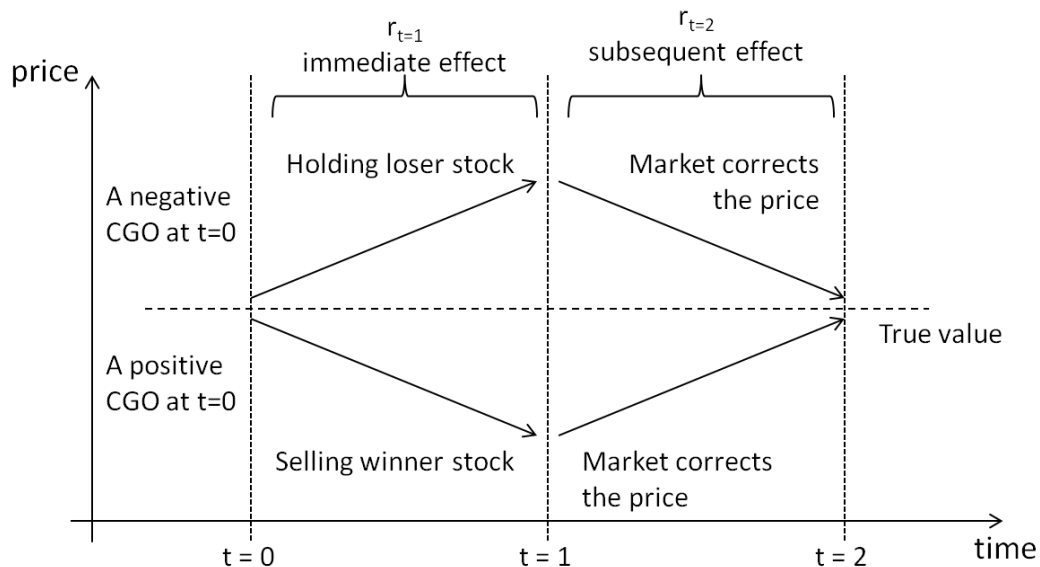
Figure 3.2-1: Time line of estimating CGO



In the other case, given that one-period stock return is measured at the time point of  $t = 0$ , using the empirical CGO implies that the unrealized gain or loss is measured at the time point of  $t = -2$ . There is a one-period interval between the time when CGO is measured and the time when stock return is measured, which means that the immediate effect of CGO is skipped, the stock return captures the subsequent effect of unrealized gain or loss.

In order to clearly demonstrate the immediate effect and the subsequent effect of capital gain overhang, an initial time  $t=0$  is set at the time when the unrealized gain or loss is measured. If the measurement is performed using the theoretical CGO, then the following stock return corresponds to the period of  $t = 0$  to  $t = 1$ . If using the empirical CGO, then the following stock return corresponds to the period of  $t = 1$  to  $t = 2$ . The Figure 3.2-2 demonstrates the case of a positive CGO.

Figure 3.2-2: The immediate and subsequent effect of CGO



In a positive CGO case, the immediate effect of CGO should be a decrease of stock price, while the subsequent effect of CGO should be an increase of stock price

With the existence of the disposition effect and market correction, the immediate effect and the subsequent effect of capital gain overhang act in different directions. The relation between stock return and the empirical CGO or a two-period lagged theoretical CGO, captures the subsequent effect of unrealized gain or loss, hence a positive relation is expected between CGO and stock returns. Such a positive relation is verified by Grinblatt and Han (2005), using the empirical CGO, and Bhootra and Hur (2012) using the two-period lagged theoretical CGO, though the work of Bhootra and Hur (2012) focuses on long horizon future returns for 6 to 12 months beginning at  $t = 2$ .

To sum up, the methods for estimating the capital gain overhang may lead to opposite correlation between CGO and stock return, because it could capture either the immediate effect or the subsequent effect. Both effects will be studied in the empirical tests.

### 3.2.3 Hypotheses

As a starting point of a series of tests, Grinblatt and Han's model is replicated on the data of all stocks in NYSE and AMEX ranging from January 1982 to December 2012. Using the same regression method, the same definition of variables, and the same time horizon, the replication is expected to yield similar result.

In addition, reflection on Grinblatt and Han's tests draws remark on the time horizon over which the variable Capital Gain Overhang is estimated. The replication test will also include test on this matter, and expects to capture the immediate effect and the subsequent effect of Capital Gain Overhang.

This replication first expects to reproduce Grinblatt and Han's results. It includes first finding positive correlation between stock returns and the empirical CGO. Second, they draw attention to the correlation between stock returns and the control variable of intermediate term past returns, because the latter represents the momentum effect. Such a correlation should be insignificant in the model with CGO. The hypothesis 1 and 2 are proposed respectively:

**Hypothesis 1:** The one-period lagged empirical capital gain overhang is positively related to stock return:  $a_6 > 0$  in the model with  $cgO_{-1}^E$ .

**Hypothesis 2:** The effect of momentum ( $a_2$ ) should become insignificant in the model including CGO.

In addition, this replication also tests the immediate effect and the subsequent effect of capital gain overhang. Over immediate time horizon, the disposition effect should cause stock price to deviate from correct value, hence negative correlation is expected. Over subsequent time horizon, the market should correct the stock miss-pricing, hence positive correlation is expected. The hypothesis 3 and 4 are proposed respectively:

**Hypothesis 3:** The one-period lagged theoretical capital gain overhang is negatively related to stock return:  $a_6 < 0$  in the model with  $cgO_{-1}^T$ .

**Hypothesis 4:** The two-period lagged theoretical capital gain overhang is also positively related to stock return:  $a_6 > 0$  in the model with  $cgO_{-2}^T$ .

The Hypotheses 1 and 2 aim to replicate the results of Grinblatt and Han's test. The Hypothesis 3 verifies the immediate effect of capital gain overhang, and Hypothesis 4 verifies if the  $cgO_{-1}^E$  and  $cgO_{-2}^T$  are equivalent, and if these variables could capture the subsequent effect of capital gain overhang.



### 3.3 Replication tests and results

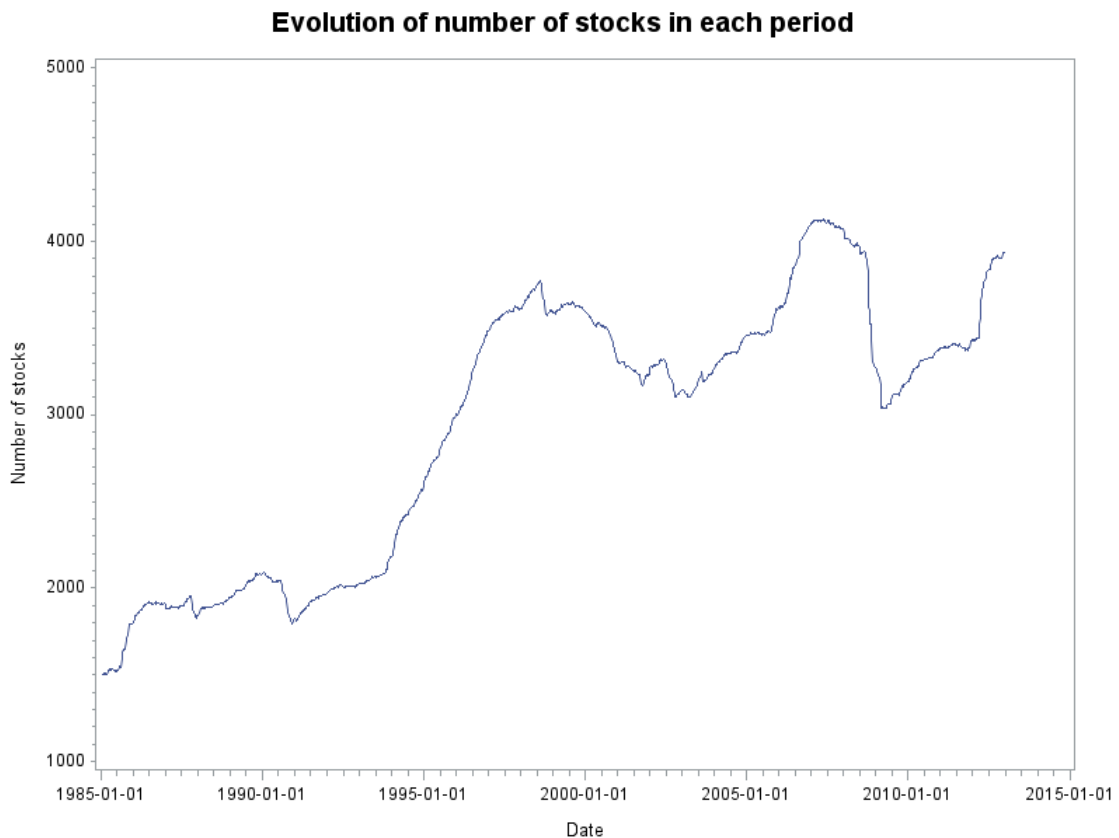
This section reports the replication tests. The following parts will report the procedure of refining data, estimating variables, and the results of the regressions.

#### 3.3.1 Data description

The data includes stocks of NYSE and AMEX from January 1982 to December 2012. The data is converted to weekly data, and is refined to delete the observations having missing data on stock price, share outstanding and trading volume. In order to ensure the estimation of all the variables gives reliable value, a valid observation requires at least 157 continuous records. It means that the observations that have less than 156 continuous previous observations are also deleted.

After the refinement, there are in a total of 7765076 observations included in the test, and for each week, there are between 4000 and 6000 observations. This provides good sample for the cross-sectional regressions at each time point. The evolution of number of stocks in each period is demonstrated by the following figure.

Figure 3.3-1: Evolution of the number of stocks included in the regression



The variables that will be used in the test include stock return, trading volume, market capitalization of stock, and the Capital Gain Overhang. The next section will explain the way in which these variables are calculated.

Concerning stock price, the test uses the close price of a stock at the trading day. If no trade occurred in the trading day, the test uses the average between the bid price and the ask price. The prices are adjusted for splits and distributions according to the cumulative factor for adjusting price that is provided by the database.

Return of individual stocks over different time horizons are required. The test first calculates stock return on a weekly basis. The weekly return is calculated on the stock price adjusted for splits and distributions. The stock return at week  $t$  is calculated according to the following equation:

$$r_t = \frac{p_t}{p_{t-1}} - 1$$

Where:

$r_t$  is the stock weekly return at week  $t$ .

$p_t$  is the stock price at week  $t$ .

Then, stock return over longer period is calculated as cumulative return of multiple weekly returns according to the following formula:

$$r_{t:t+n} = \prod_{i=1}^n (r_{t+i} + 1) - 1$$

Trading volume and share outstanding are directly provided in the unit of numbers of shares. The values of the two variables are first adjusted for splits and distributions according to the cumulative factor for adjusting shares that is provided by the database. Then the trading volume is standardized into ration.

$$v_t = \frac{\text{trading volume}_t}{\text{share outstanding}_t}$$

The market capitalization of stock is calculated by multiplying the stock price at time  $t$  by the share outstanding of the stock at time  $t$ .

$$s_t = price_t \times share\ outstanding_t$$

In this replication test, the reference price will be renamed as the average acquisition price of the stock. It will be noted as  $aqp$ . The reference price and its notation  $R$  is confusing with the notation of stock return  $r$ . The estimation of the average acquisition price will adopt the following formula:

$$aqp_t = \min(0.95, v_t)p_t + (1 - \min(0.95, v_t))R_{t-1}$$

Where:

$aqp_t$  is the average acquisition price for a stock at the end of week  $t$ ,

$p_t$  is the stock price at week  $t$ ,

$v_t$  is the trading volume represented in percentage.

This formula is chosen of its simplicity in calculation. In the actual estimation, the  $aqp$  is calculated based on the time-series data series of individual stocks from the very first observation to the last one. For the initial  $aqp$  where  $t$  equals to 1, the average acquisition price is the stock price at the time. Since it is the first time the stock appears in the market, every investor has bought the stock for the first time.

The Capital Gain Overhang is the variable representing the disposition effect. This variable is defined as the difference between current stock price and the average acquisition cost divided by current stock price. As mentioned in previous sections, in this test three types of capital gain overhang will be used.

One-period lagged theoretical CGO:

$$CGO_{t-1}^T = (P_{t-1} - R_{t-1})/P_{t-1}$$

One-period lagged empirical CGO, as in Grinblatt and Han (2005)'s study:

$$CGO_{t-1}^E = (P_{t-2} - R_{t-1})/P_{t-2}$$

Two-period lagged theoretical CGO:

$$CGO_{t-2}^T = (P_{t-2} - R_{t-2})/P_{t-2}$$

Where:

CGO: is the capital gain overhang

E: indicates an empirical variable (defined with lagged past stock price)

T: indicates a theoretical variable

P: is stock price

R: is the average acquisition cost

t: indicates the time period

The different versions of capital gain overhang correspond to the effect in different horizon. The replication test will be performed separately with the one-period lagged theoretical CGO, which corresponds to the immediate effect; the one-period lagged empirical CGO, and the two-period lagged theoretical CGO, which correspond to the subsequent effect.

### 3.3.2 Summary statistics

The summary statistic of all the variables is demonstrated in table 3.3-1 including both empirical and theoretical capital gain overhang. The sample time period includes the full sample period from 1982 to 2012; and also includes two half sample periods from 1982 to 1996 and from 1997 to 2012.

There are two reasons for dividing the subsamples by the end of year 1996. Grinblatt and Han's data sample ranges from 1967 to 1996; hence the first half sample is also a sub-period of Grinblatt and Han's sample. Therefore, the first half sample is expected to produce the result that is similar to Grinblatt and Han's work. The second reason is that from 1997, the world had experienced some severe crisis, namely the Asian crisis, the "dot com" bubble, and

Chapter 3:  
Preliminary tests

the “subprime” crisis. These anomalies have impact on the data, hence could influence the test result. The second half subsample isolates such impact.

Table 3.3-1: Summary statistic of variables

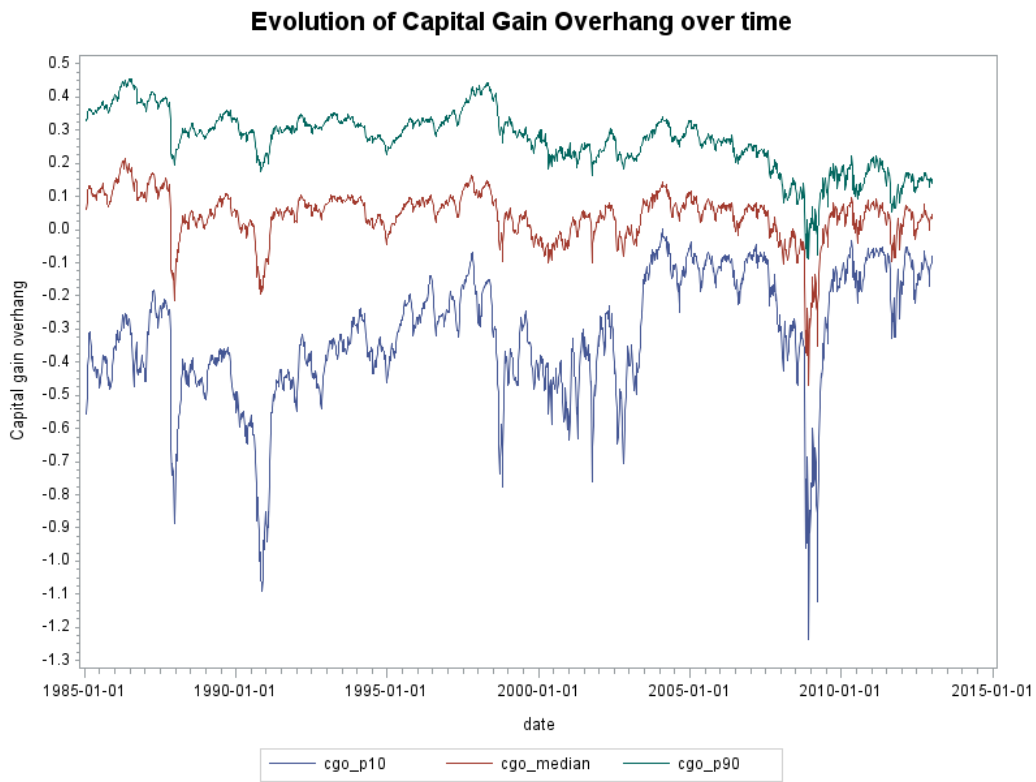
|           |           | $r$     | $r_{-4:-1}$ | $r_{-52:-5}$ | $r_{-156:-53}$ | $V$    | $s$     | $CGO^E$ | $CGO^T$ |
|-----------|-----------|---------|-------------|--------------|----------------|--------|---------|---------|---------|
| 1982-2012 | Mean      | 0.0014  | 0.0057      | 0.0754       | 0.2371         | 0.0322 | 12.9360 | -0.0285 | -0.0306 |
|           | S.D.      | 0.0579  | 0.1113      | 0.4088       | 0.7354         | 0.1147 | 1.9241  | 0.6572  | 0.6783  |
|           | 10th Pctl | -0.0526 | -0.1048     | -0.3226      | -0.3651        | 0.0036 | 10.5761 | -0.3139 | -0.3243 |
|           | Median    | 0.0000  | 0.0030      | 0.0377       | 0.1092         | 0.0149 | 12.8396 | 0.0401  | 0.0412  |
|           | 90th Pctl | 0.0550  | 0.1137      | 0.4720       | 0.9172         | 0.0633 | 15.4604 | 0.2806  | 0.2843  |
| 1982-1996 | Mean      | 0.0016  | 0.0063      | 0.0743       | 0.2179         | 0.0144 | 12.3200 | -0.0509 | -0.0528 |
|           | S.D.      | 0.0546  | 0.1034      | 0.3610       | 0.5921         | 0.0264 | 1.8241  | 0.9196  | 0.9352  |
|           | 10th Pctl | -0.0488 | -0.0957     | -0.2929      | -0.3324        | 0.0026 | 10.0680 | -0.4009 | -0.4092 |
|           | Median    | 0.0000  | 0.0000      | 0.0403       | 0.1154         | 0.0095 | 12.2087 | 0.0623  | 0.0634  |
|           | 90th Pctl | 0.0521  | 0.1087      | 0.4500       | 0.8561         | 0.0294 | 14.7647 | 0.3283  | 0.3313  |
| 1996-2012 | Mean      | 0.0013  | 0.0054      | 0.0759       | 0.2458         | 0.0403 | 13.2174 | -0.0182 | -0.0204 |
|           | S.D.      | 0.0593  | 0.1147      | 0.4289       | 0.7921         | 0.1365 | 1.9029  | 0.4925  | 0.5200  |
|           | 10th Pctl | -0.0548 | -0.1093     | -0.3364      | -0.3799        | 0.0044 | 10.8760 | -0.2776 | -0.2887 |
|           | Median    | 0.0000  | 0.0041      | 0.0364       | 0.1065         | 0.0199 | 13.1204 | 0.0338  | 0.0348  |
|           | 90th Pctl | 0.0563  | 0.1161      | 0.4824       | 0.9469         | 0.0757 | 15.7533 | 0.2498  | 0.2542  |

Recall that  $r$  is the weekly stock return;  $r_{-4:-1}$  is the cumulative return from week -4 to week -1;  $r_{-52:-5}$  is the cumulative return from week -52 to week -5;  $r_{-156:-53}$  is the cumulative return from week -156 to week -53;  $V$  is the average turnover from week -52 to -1;  $s$  is the market capitalization of week -1;  $cgo$  is the capital gain overhang at week -1.

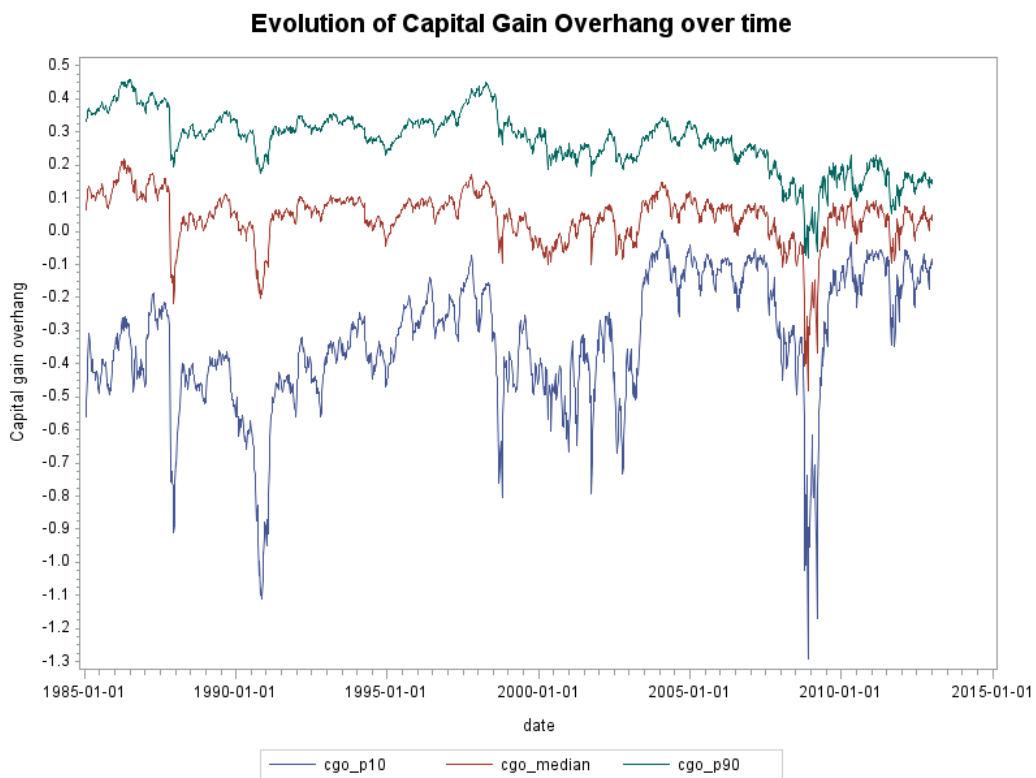
The evolution of capital gain overhang is displayed with a chart, showing separately the 10<sup>th</sup> percentile, 50<sup>th</sup> percentile (median) and 90<sup>th</sup> percentile at each time period. Both the empirical and theoretical capital gain overhang are demonstrated. The graph shows that the 10<sup>th</sup> percentile has slightly larger fluctuation than the median and 90<sup>th</sup> percentile. The evolution the medians in both CGO are very close; however, the theoretical CGO has larger extreme values than the empirical CGO. Such difference is observed mainly in the first half of the sample time period.

Figure 3.3-2: Evolution of different types of Capital Gain Overhang overtime

A: Evolution of the EMPIRICAL capital gain overhang



B: Evolution of the THORETICAL capital gain overhang



Chapter 3:  
Preliminary tests

Before performing the Fama-Macbeth regression, the matrix of correlation is presented. The matrix suggests that the hypothesis of no correlation is rejected for most of the explanatory variables. However, the rejection could be resulted of the enormous sample size. Positive correlation is found between capital gain overhang and cumulative return of different time horizons.

Table 3.3-2: The correlation matrix of the regressors

| Pearson Correlation Coefficients, N = 7765076, Prob >  r  under H0: Rho=0 |                   |                    |                      |          |         |                  |
|---|-------------------|--------------------|----------------------|----------|---------|------------------|
|   | r <sub>4:-1</sub> | r <sub>52:-5</sub> | r <sub>156:-53</sub> | V        | s       | cgo <sup>E</sup> |
| r <sub>4:-1</sub>   | 1                 | -0.03307           | -0.00144             | -0.00927 | 0.00279 | -0.00382         |
|   |                   | <.0001             | 0.0029               | <.0001   | <.0001  | <.0001           |
| r <sub>52:-5</sub>  | -0.03307          | 1                  | -0.0139              | -0.01726 | 0.00376 | 0.04317          |
|   | <.0001            |                    | <.0001               | <.0001   | <.0001  | <.0001           |
| r <sub>156:-53</sub>  | -0.00144          | -0.0139            | 1                    | -0.09156 | 0.00452 | 0.13596          |
|   | 0.0029            | <.0001             |                      | <.0001   | <.0001  | <.0001           |
| V   | -0.00927          | -0.01726           | -0.09156             | 1        | 0.0248  | 0.13423          |
|   | <.0001            | <.0001             | <.0001               |          | <.0001  | <.0001           |
| s   | -0.00279          | -0.00376           | -0.00452             | 0.0248   | 1       | 0.08149          |
|   | <.0001            | <.0001             | <.0001               | <.0001   |         | <.0001           |
| cgo <sup>E</sup>  | -0.00382          | 0.04317            | 0.13596              | 0.13423  | 0.08149 | 1                |
|   | <.0001            | <.0001             | <.0001               | <.0001   | <.0001  |                  |

### 3.3.3 Test result

Following Grinblatt and Han's approach, this test performs Fama Macbeth regression on Grinblatt and Han's model. The first regression will not include the capital gain overhang; while the second regression will be performed on the complete model.

The Fama Macbeth method consists of, at each time period, a cross sectional regression is performed on Grinblatt and Han's model. Then, the time serial average of the coefficients and the t-statistic of the coefficients are calculated. Table 3.3-3 demonstrates the result of the regression without capital gain overhang, Table 3.3-4 demonstrates the result of the regression with different definitions of capital gain overhang.

Chapter 3:  
Preliminary tests

Table 3.3-3: Average coefficients and their t-statistics of the model without CGO

| $r = a_0 + a_1r_{-4:-1} + a_2r_{-52:-5} + a_3r_{-156:-53} + a_4\bar{V}_{-52:-1} + a_5s_{-1}$ |             |        |                |         |          |                |        |        |
|--|-------------|--------|----------------|---------|----------|----------------|--------|--------|
|  | coefficient | $a_0$  | $a_1$          | $a_2$   | $a_3$    | $a_4$          | $a_5$  | $R^2$  |
| Full period<br>1982-2012   | mean        | 0.0008 | <b>-0.0238</b> | 0.0003  | -0.0002  | <b>-0.0092</b> | 0.0000 | 0.0378 |
|  | t-value     | 0.99   | -17.77         | 0.71    | -1.38    | -2.30          | 0.65   |        |
|  | p-value     | 0.3214 | <.0001         | 0.4790  | 0.1655   | 0.0216         | 0.5131 |        |
| Sub period<br>1982-1996  | mean        | 0.0010 | <b>-0.0300</b> | 0.0008* | -0.0004* | <b>-0.0172</b> | 0.0001 | 0.0254 |
|  | t-value     | 0.86   | -20.48         | 1.61    | -1.80    | -2.18          | 0.84   |        |
|  | p-value     | 0.3923 | <.0001         | 0.1073  | 0.0731   | 0.0293         | 0.4038 |        |
| Sub period<br>1997-2012  | mean        | 0.0006 | <b>-0.0192</b> | 0.0000  | -0.0001  | -0.0036        | 0.0000 | 0.0494 |
|  | t-value     | 0.57   | -9.34          | 0.03    | -0.50    | -0.89          | 0.17   |        |
|  | p-value     | 0.5667 | <.0001         | 0.9735  | 0.6174   | 0.3761         | 0.8656 |        |

Coefficients in bold face indicate significant level at 95% or more; \* indicates a marginally significant level at 90%.

The first regression examines the predictability of short, intermediate, and long term past return, trading volume and size. The intermediate and long term past returns represent the momentum effect and the reversal effect.

The coefficients of determination of the regressions vary from 0.025 to 0.049, indicating that the explanative power of the variables is not very satisfactory. Only about 3%-5% of the variation is captured by the independent variables. Since Grinblatt and Han did not report the coefficients of determination of their regressions, comparison cannot be made.

Over the full sample period from 1982 to 2012, these tests only find significant correlation in short term past return, and size. Comparing with the result of Grinblatt and Han's, they found a positive coefficient for the intermediate term past return, which represents the momentum effect, and a negative coefficient for the long term past return, which represents the reversal effect. However, this replicating test does not reproduce their result on the more recent test sample. The regression gives a positive value of  $a_2$ , and a negative value to  $a_3$ . The signs of the coefficients are consistent to Grinblatt and Han's result, but the confidence levels in the replicating test are too low to be accepted. This test does not find significant correlations on the intermediate term and the long term past returns over the full sample period.

The regressions on both sub periods yield consistent results with the full period regression. They only find significant correlation over short term past returns. However, the test on the early half of the sample period from 1982 to 1996, which partially covers the



sample period of Grinblatt and Han's test, is able to find a marginally significant correlation on the intermediate term past return and on the long term past return. The coefficient  $a_2$  is positive with a confidence level of 89%, and the coefficient  $a_3$  is negative with a confidence level of 92%. But in the late half of the sample period from 1997 to 2012, there is no significant correlations on the intermediate term past return, nor on the long term past return.

Although this test does not reproduce the result of Grinblatt and Han's test, it suggests that the difference in sample period could be the main cause. Grinblatt and Han's test covers a much earlier period, and documented significant correlations on intermediate and long term past returns. Over the sample period from 1982 to 1996, which is covered both in the original test and the replication test, the correlations becomes marginally significant. Over the most recent period, the correlation is not significant. Such an observation suggests that the autocorrelation in stock return has been reduced over time. This result also raises the question on the accuracy for using the  $a_2$  and  $a_3$  to represent the momentum and the reversal effects. The replicating test finds no significant correlation, while the momentum effect has still been documented by literature over recent sample period.

Chapter 3:  
Preliminary tests

Table 3.3-4: Average coefficients and their t-statistics of the full model

$$r = a_0 + a_1r_{-4:-1} + a_2r_{-52:-5} + a_3r_{-156:-53} + a_4\bar{V}_{-52:-1} + a_5s_{-1} + a_6cgo_{-1}$$

| period   | coefficient | $a_0$         | $a_1$          | $a_2$         | $a_3$   | $a_4$          | $a_5$         | $a_6$          | $R^2$  |
|--|-------------|---------------|----------------|---------------|---------|----------------|---------------|----------------|--------|
| Panel A: $cgo_{-1}^T$ is defined as: $cgo_{-1} = (P_{-1} - aqp_{-1})/P_{-1}$ |             |               |                |               |         |                |               |                |        |
| Full period  | mean        | -0.0003       | <b>-0.0227</b> | <b>0.0016</b> | 0.0000  | <b>-0.0118</b> | 0.0001        | <b>-0.0015</b> | 0.0439 |
| 1982-2012  | (p-value)   | (.7093)       | (<.001)        | (.0145)       | (.8282) | (.0031)        | (.0755)       | (<.001)        |        |
| Sub period   | mean        | -0.0008       | <b>-0.0283</b> | <b>0.0017</b> | -0.0000 | <b>-0.0213</b> | <b>0.0002</b> | <b>-0.0017</b> | 0.0300 |
| 1982-1996  | (p-value)   | (.4892)       | (<.001)        | (.0003)       | (.9913) | (.0065)        | (.0308)       | (<.001)        |        |
| Sub period   | mean        | 0.0001        | <b>-0.0189</b> | 0.0007        | 0.0001  | -0.0048        | 0.0001        | <b>-0.0014</b> | 0.0510 |
| 1997-2012  | (p-value)   | (.9184)       | (<.001)        | (.3261)       | (.7920) | (.2138)        | (.5862)       | (.0005)        |        |
| Panel B: $cgo_{-1}^E$ is defined as: $cgo_{-1} = (P_{-2} - aqp_{-1})/P_{-2}$ |             |               |                |               |         |                |               |                |        |
| Full period  | mean        | 0.0006        | <b>-0.0242</b> | 0.0005        | -0.0001 | -0.0099        | 0.0001        | -0.0002        | 0.0420 |
| 1982-2012  | (p-value)   | (.4068)       | (<.001)        | (.3092)       | (.3743) | (.0128)        | (.4173)       | (.5361)        |        |
| Sub period   | mean        | 0.0005        | <b>-0.0300</b> | <b>0.0010</b> | -0.0003 | <b>-0.0183</b> | 0.0001        | -0.0004        | 0.0293 |
| 1982-1996  | (p-value)   | (.6610)       | (<.001)        | (.0326)       | (.2214) | (.0194)        | (.2256)       | (.1362)        |        |
| Sub period   | mean        | 0.0007        | <b>-0.0198</b> | 0.0001        | -0.0001 | -0.0037        | 0.0000        | 0.0000         | 0.0515 |
| 1997-2012  | (p-value)   | (.4738)       | (<.001)        | (.8961)       | (.7827) | (.3320)        | (.9467)       | (.9809)        |        |
| Panel C: $cgo_{-2}^T$ is defined as: $cgo_{-2} = (P_{-2} - aqp_{-2})/P_{-2}$ |             |               |                |               |         |                |               |                |        |
| Full period  | mean        | 0.0006        | <b>-0.0242</b> | 0.0005        | -0.0001 | <b>-0.0102</b> | 0.0001        | -0.0002        | 0.0421 |
| 1982-2012  | (p-value)   | (.4311)       | (<.001)        | (.2543)       | (.3816) | (.0111)        | (.4116)       | (.4588)        |        |
| Sub period   | mean        | 0.0005        | <b>-0.0300</b> | <b>0.0011</b> | -0.0002 | -0.0189        | 0.0001        | -0.0004        | 0.0295 |
| 1982-1996  | (p-value)   | (.6661)       | (<.001)        | (.0189)       | (.2341) | (.0164)        | (.2351)       | (.0905)        |        |
| Sub period   | mean        | <b>0.0006</b> | <b>-0.0198</b> | 0.0001        | -0.0001 | -0.0037        | 0.0000        | -0.0000        | 0.0508 |
| 1997-2012  | (p-value)   | (.5052)       | (<.001)        | (.8605)       | (.7792) | (.3318)        | (.9199)       | (.9868)        |        |

Panel A shows the result using the one-period lagged theoretical CGO, and shows negative relation; panel B shows the result using the one-period lagged empirical CGO, panel C shows the result using the two-period lagged theoretical CGO, both panel B and C show positive relation. Coefficients in bold face indicate significant level at 95% or higher.

The second regression introduces the variable of CGO in addition to the first regression. The result of this test is generally consistent with the result of the previous test in which CGO is not included.

First, the result shows that the coefficients of determination of the regressions vary from 0.029 to 0.051, indicating that the explanatory power is at almost the same level as in previous regression. Adding the variable of CGO could only slightly improve the explanatory power of the model, but such improvement is very limited. Second, the replication test finds a significant negative correlation between stock current return and short term past return over all the sample periods and for all the regressions using different forms of CGO. The test also finds a positive correlation on the intermediate term past returns over the early half sample period in all the three regressions using different CGO. Third, for the Capital Gain Overhang, this test only finds a negative correlation between stock returns and the **one-period lagged**

**theoretical** CGO over all three sample periods. But the correlations on other forms of CGO are not significant. Last, for other control variables, the coefficients of the trading volume variable are negative. The significances of these coefficients vary in the regressions using different CGO, and also depend on the sample periods. The coefficient of the size variable is generally non significant.

Comparing with the test of Grinblatt and Han, this replicating test does not reproduce their test result. Their result mainly consists of finding a non significant correlation on the intermediate past return and a positive correlation on CGO. Panel B of the above table reports the result of the regression using the same CGO as in their tests. In this replicating test, the correlation on the intermediate past return is not significant, but is not the result for including the CGO in the regression. The correlation between current stock return and CGO is not significant. This result rejects hypothesis 1 and 2.

Concerning the time horizon on the effect of capital gain overhang, the hypotheses expect to find negative correlation over immediate time horizon, while positive correlation over subsequent time horizon.

In this test, the dependent variable of the regression is weekly return at current period which indicates also the reference time point. The **one-period lagged theoretical** capital gain overhang captures the unrealized gain or loss at the beginning of current period (the end of previous period), hence covers the immediate time horizon. The panel A of the previous table reports the result of the regression using this form of CGO. The result shows that this CGO is negatively related to the stock return of current period at 99% confidence level. It suggests that the CGO has a negative impact on stock returns over the immediate time horizon. A negative relation is consistent to the description of the disposition effect; investors are likely to sell winning stock too soon, and to hold losing stock too long. When a unrealized gain appears, immediately market will show extra selling pressure, and the latter will bring down stock price; when a unrealized loss appears, market shows insufficient selling pressure and it pushes up stock price. The Hypotheses 3 is confirmed.

The **one-period lagged empirical** capital gain overhang and the **two-period lagged theoretical** capital gain overhang are considered as equivalent for representing the subsequent time horizon. Despite some differences in calculations of these two variables, they are both considered to measure the unrealized gain/loss one period earlier. It means that there is an interval of one week between the time when CGO is measured and the time when the current

stock return is measured. The panel B and panel C of the above table reports the regression results using these CGO. There are no significant correlation between current stock returns and CGO. This result rejects the hypotheses 4.

However, the regression results show that the coefficient of the two-period lagged theoretical CGO and the coefficient of the one-period lagged empirical CGO are almost identical in the two regressions. And the overall regression results using these two forms of CGO are also very similar. This observation could suggest that the one-period lagged empirical CGO used by Grinblatt and Han and the two-period lagged theoretical CGO proposed in this replicating test capture the same effect. In this sense, the assumption that these two forms of CGO are equivalent in terms of representing the subsequent effect of the disposition effect can be justified.

This test using three forms of CGO could reveal the evolution of the influence of the disposition effect over the immediate and the subsequent time. The disposition effect impacts the stock return in a negative direction over immediate time horizon. The investor's reluctance to sell the losing stock increases the stock return; the investor's haste to sell the winning stocks decreases the stock return. Over the subsequent time horizon, the impact of the disposition effect fades out.

Regarding measuring the momentum effect, using proxy variable such as the intermediate term autocorrelation is arguable. The coefficient on the intermediate horizon past return is intended to represent the momentum effect; however, this variable could not completely fulfill its task. The first reason is related to the portfolio formation. The momentum effect is the result of a portfolio strategy that past winners outperform the past losers, but it is not necessary that past winner keeps winning. Therefore, a positive relation between current return and intermediate horizon past return cannot capture the essence of the momentum effect. The second reason is related to the time horizon. The momentum effect is observed at intermediate term in the past and future. Jegadeesh and Titman's J-month/K-month strategy suggests that after the strategy portfolio is formed, it is held for at least three months. In Grinblatt and Han's model, the holding period corresponds to the dependent variable which is the current weekly stock return. Comparing to the K-month holding period, the weekly time horizon is too short, and is very likely to be influenced by other factors. Therefore, the weekly stock return may not be able to capture the momentum effect.

For the above two reasons, the insignificant coefficient of this variable such as in Grinblatt and Han's work, cannot rule out the presence of the momentum effect. For the same reasons, the negative relation found in the replication does not necessarily contradict the momentum effect. Concerning the proxy of reversal effect, the same argument can be put forward. The proxy does not match the portfolio formation and time horizon of the reversal effect, hence the negative relation between current stock return and long term past return that is expected, does not necessarily represent the existence of reversal effect.

### **3.4 Summary of preliminary tests**

This chapter replicates the test of Grinblatt and Han (2005), and also tests the time horizon of the influences from the disposition effect on stock returns. The replication test did not reproduce the results in Grinblatt and Han's test. The test on the time horizon of the disposition effect finds an instant negative impact on stock return. This impact then fades out over the subsequent period.

The replication test also allows reflecting on the limitation of existing study. It concerns mainly about the theoretical support of the regression model and the test involving the momentum effect. Overall, replication provides good understanding about the advantages and limitations of Grinblatt and Han's work. Experience obtained from this test will guide the following study on the influence of unrealized gain/ loss over stock performance.

#### **3.4.1 Market reaction to the disposition effect**

Besides verifying the result of Grinblatt and Han's work, the additional tests of the replication allow showing the complete reaction of the market to a presence of the disposition effect. Using the capital gain overhang estimated by different formulas, the replication test finds a negative impact of the disposition effect over immediate time horizon. Such a negative effect can be explained by the market mechanism of supply and demand. The disposition effect implies an insufficient supply of the stock in loss and an excess supply of the stock in gain. The imbalanced supply power then pushes the stock prices away from the former equilibrium level. Such a direct influence from the disposition effect should act over immediate time horizon. Over the subsequent time horizon, this impact dissipates.

Comparing with the theory proposed by Grinblatt and Han, the results of the replicating test suggests that the disposition effect affects the stock prices in a more direct way through the forces of supply and demand. The mechanism of market correction suggested by Grinblatt and Han is not observed in this test using a different sample period.

This finding can be applied to the model of the unrealized gain/loss proposed in the previous chapter. When an unrealized gain or loss appears, the behavior factor will immediately affect investor's decision making for selling or holding the stock. The aggregated selling or holding decision then introduces extra or insufficient selling pressure in the market. Such pressure will influence stock's supply on the market. Meanwhile, the demand side of the market is not affected by the presence of unrealized gain or loss, the demand of the stock remains constant (which has been discussed in part 2.5.1). Driven by the force of supply and demand, the newly formed equilibrium price will deviate from the true value of the stock.

### **3.4.2 Link to following tests**

In chapter 2 of this dissertation, a market decision making scenario is formed to include the unrealized gain or loss, and analyses are performed on such a scenario under the framework of alternative utility theories. The studies attempt to fit the behavior factor related to the unrealized gain/loss into the well established system of utility and rationality. The following test aims to examine the validity of the prediction given by theoretical model of unrealized gain or loss that is developed under the framework of alternative utility theories.

Grinblatt and Han's test provides an example for conducting following tests. The method for estimating the reference price and the Capital Gain Overhang will be adopted in following tests. In fact, the variable in this dissertation that represents the influence of unrealized gain/loss is a ratio of the Capital Gain Overhang to the volatility of the stock. The regression model of Grinblatt and Han's test is a well accepted empirical model that links stock returns to behavior factors. It can be applied as well in the following tests in this dissertation.

This replication test also finds the market reaction to the disposition effect over the immediate and the subsequent time horizon. The interpretation of the test result offers a plausible mechanism by which the disposition effect could affect the stock prices. Understanding such mechanism will be beneficial in the future for studying how behavior effect transmits in financial market.

However, the limitations of Grinblatt and Han's test should be aware of. First, there is no conclusive evidence suggesting that the loss aversion attitude from the prospect theory is the cause of the disposition effect. More rigorous studies suggest that there is not a linear correlation between investor's preference and Capital Gain Overhang. For such reason, it requires more cautious interpretation of the key variables.

The regression model used by Grinblatt and Han is not compatible with the well established risk-return paradigm. In this dissertation, theoretical analyses is founded on alternative utility theories, and considers the behavior factors as alternative rationality which is a complimentary or a modification of classic rationality. Moreover, the key variable Relative Capital Gain Overhang is defined by controlling the risk level of the lottery. Under such framework, testing the influence of behavior factors should be based on ruling out the conventional risk factors within stock return. Therefore, a test model rooted from risk-return paradigm will still be needed.

The initial objective of this dissertation is to study the momentum effect from the perspective of alternative utility theory. Regarding the momentum effect, Grinblatt and Han's test did not provide convincing explanation. First, the momentum effect refers to the observation that past winners out-perform past losers. In this sense, the intermediate term autocorrelation of individual stock return, which is tested in Grinblatt and Han's regression, cannot accurately capture the momentum effect. Second, the profit form momentum strategy makes best variable to represent momentum effect. To fully study this anomaly, direct tests on the momentum profit is indispensable.

To sum up, future tests in this dissertation will be organized in two steps. First, tests will be performed to examine the influence of unrealized gain or loss over individual stock return. This step aims to verify the theoretical analyses in chapter three. Moreover, empirical results from the tests will be helpful to understand whether the unrealized gain or loss could contribute to form the momentum effect. Second, direct test on momentum profit will be performed in order to examine if alternative utility theories could explain the momentum effect.

### Annex 3.1: Comparison of CGO

In theory, the Capital Gain Overhang at the time point of t-1, is defined as:

$$CGO_{t-1}^T = (P_{t-1} - R_{t-1})/P_{t-1}.$$

In the empirical test, the same variable is defined as:

$$CGO_{t-1}^E = (P_{t-2} - R_{t-1})/P_{t-2}.$$

The reference price at the time point of t, is defined as:

$$R_t = V_t P_t + (1 - V_t) R_{t-1},$$

Where:  $V_t$  is the stock's turnover at the end of the period.

According to the definition of  $R_t$ , Replace  $R_{t-1}$  in the expression of empirical CGO:

$$CGO_{t-1}^E = \frac{1}{P_{t-2}} \{P_{t-2} - [V_{t-1} P_{t-1} + (1 - V_{t-1}) R_{t-2}]\}$$

$P_{t-2}$  in the parentheses can be written as:

$$P_{t-2} = V_{t-1} P_{t-2} + (1 - V_{t-1}) P_{t-2}$$

Therefore:

$$CGO_{t-1}^E = \frac{1}{P_{t-2}} \{V_{t-1} P_{t-2} + (1 - V_{t-1}) P_{t-2} - [V_{t-1} P_{t-1} + (1 - V_{t-1}) R_{t-2}]\}$$

Combining similar terms:

$$CGO_{t-1}^E = \frac{1}{P_{t-2}} \{V_{t-1} (P_{t-2} - P_{t-1}) + (1 - V_{t-1}) (P_{t-2} - R_{t-2})\}$$

Switching terms:

$$CGO_{t-1}^E = (1 - V_{t-1}) \frac{P_{t-2} - R_{t-2}}{P_{t-2}} - V_{t-1} \frac{P_{t-1} - P_{t-2}}{P_{t-2}}$$

Then,  $\frac{P_{t-2} - R_{t-2}}{P_{t-2}}$  is the theoretical CGO at t-2,  $\frac{P_{t-1} - P_{t-2}}{P_{t-2}}$  is the stock return at t-1.

$$CGO_{t-1}^E = (1 - V_{t-1}) CGO_{t-2}^T - V_{t-1} r_{t-1}$$





## Chapter 4

### Test on the unrealized gain/loss model

In this chapter, the study focuses on performing empirical tests over the theoretical work done in chapter three. The theoretical work analyses the influence of unrealized gain or loss over investor's decision making under the frame work of prospect theory and regret theory. Despite that the theoretical model could predict a decrease of propensity to sell in situations of large unrealized gain and large unrealized loss, it could not provide better prediction than the existence of a difference between the extreme unrealized gain/loss stocks and the null unrealized gain/loss stocks in terms of stock return.

The empirical study starts with adapting the theoretical model into the empirical context. It involves transforming the behavior factor of the unrealized gain/loss into a test variable of the Relative Capital Gain Overhang that can be estimated accurately, and transforming theoretical prediction into testable hypotheses.

Next, a sorting test will be performed. This test divides all stocks into ten deciles according to the RCGO, and shows the average returns of all deciles over various time horizons. This test will provide a figurative illustration of the dispersion of stock returns in different unrealized gain/loss situations. Based on the result of the sorting test, regressions will be performed to examine the cross-sectional correlation between the RCGO and the stock returns. One regression will use the model of Grinblatt and Han, another regression will test the correlation between RCGO and the residual from Fama-French Three-Factor model.

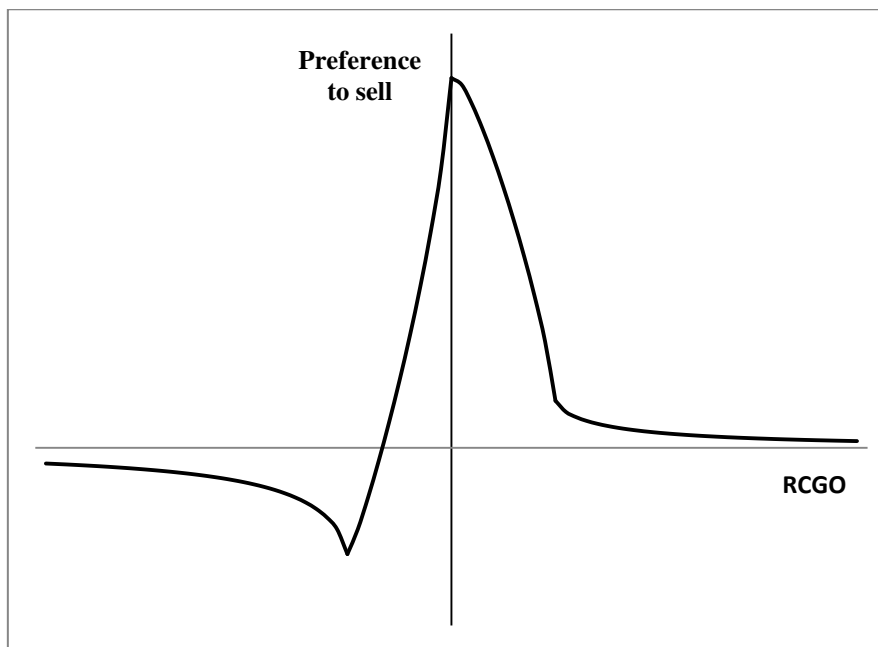
The results from the sorting tests and the regressions suggest that the stocks with extreme RCGO will outperform the stocks with RCGO close to zero over a intermediate to

long term. This result leads to form an arbitrage strategy: the Extremity minus Middle strategy. Finally, test shows that such strategy is significantly profitable.

#### 4.1 Empirical application of regret and loss aversion

The theoretical analyses in chapter three suggest that assuming the presence of investor's regret aversion and loss aversion attitude, the investor's preference for selling a stock depends on the unrealized gain or loss that the investor is experiencing. Relative Capital Gain Overhang is the variable that measures the unrealized gain or loss. The following figure recalls such a relation between preference and relative capital gain overhang.

Figure 4.1-1: Investor's preference for selling with RCGO



The horizontal axis represents relative capital gain overhang; the vertical axis represents preference for selling the stock.

Recall that the Relative Capital Gain Overhang is defined as a ratio that is standardized by the variation of stock price. This definition allows measuring the scale of the unrealized gains or losses with respect to the scale of risks. The variable RCGO indicates if the unrealized gain or the unrealized loss is large enough to make sure that investor's current winning or losing situation remains in next period. Hence, RCGO captures investor's sensibility to change in winning or losing situation. The definition of RCGO also means that the risk factors are controlled for, and the RCGO only covers the influences at the behavior

level. Therefore, in empirical tests, the influence of RCGO on stock returns should be able to be observed after ruling out the risk factors from stock return.

For this reason, the Relative Capital Gain Overhang is the most important factor in both theoretical and empirical study. An accurate estimation will be crucial to perform following tests.

#### 4.1.1 Estimation of RCGO in an empirical context

In this study, the key variables include the Relative Capital Gain Overhang and price variation level. These variables are defined in a pure theoretical context. In order to adapt to empirical test, some modification or use of proxy is necessary.

The unrealized gain/loss model assumes a simplified situation where the stock price distribution is binary. Under such an assumption, the model explains that investor's preference is influenced by the fact that the investor is at a winning situation or at a losing situation. The theoretical model defines the Relative Capital Gain Overhang as a ratio of two values:

$$RCGO = \frac{\text{Incremental price}}{\Delta P} = \frac{P_t - aqp_{t-1}}{|P_{t+1}^* - P_t|}$$

Where:

P: is the stock price at a given period;

\*: indicates a forecast value;

aqp: is the acquisition price of the stock.

This variable is defined in the form of a ratio of the unrealized gain/loss to the price variation. Such a definition allows distinguishing four situations: large loss, small loss, small gain, and large gain. In the large loss situation, the loss is greater than price variation, meaning that even if the price goes up, it cannot cancel the existing unrealized loss. In this case, the investor will be sure to remain in a loss situation. In the small loss situation, however, if stock price goes up, investor could get out of the losing situation. Same reasoning can be applied in the winning cases.

Under the configuration of binary distribution of stock price, the price variation level can be considered as a boundary. If investor's current unrealized gain or loss is greater than the boundary, the winning or the losing situation of the investor will be secured for the next period.

#### 4.1.1.1 Empirical form of RCGO

In the empirical study, the stock price is considered to follow a continuous distribution. Hence there is no clear boundary that limits stock price variation as in the theoretical model. However, the same idea can be expressed using interval of confidence in the case of a continuous distribution. To apply this idea in empirical test, it is necessary to modify the mathematical form of Relative Capital Gain Overhang.

From the above formula, divide both Numerator and denominator by  $P_t$  to transform values into ratios:

$$RCGO = \frac{(P_t - aqp_{t-1})/P_t}{|P_{t+1}^* - P_t|/P_t} = \frac{(P_t - aqp_{t-1})/P_t}{|r_{t+1}^*|}$$

In this form, the denominator: the price variation transforms to return, more precisely, the absolute value of the forecast return for the next period. The numerator: the unrealized gain or loss is represented in the form of a ratio with respect to current stock price. In fact, the numerator has the identical definition to the variable Capital Gain Overhang in the theoretical work of Grinblatt and Han (2005).

$$CGO_t = (P_t - R_t)/P_t$$

Where:

R: is the reference price, which is the acquisition price at the previous time.

Assuming that the stock return follows a normal distribution:  $\tilde{r}_{t+1} \rightarrow \mathcal{N}(0, \sigma)$ , where the expected return is zero,  $\sigma$  is the standard deviation of stock return. According to the property of normal distribution, the probability for the actual stock return of next period  $r_{t+1}$  to be within a  $2\sigma$  range from mean is 95.4%. In other word, there is less than 5% of chance that the actual  $r_{t+1}$  is greater than  $2\sigma$  or smaller than  $-2\sigma$ . Therefore, the stock variation in the initial form of RCGO can be described in terms of a confidence level. For example, giving a

confidence level of 95%, the amplitude of stock return variation can be considered as twice the standard deviation of stock return.

To sum up, in the empirical context, the relative capital gain overhang can be rewritten into the following form without changing its original meaning:

$$RCGO = \frac{cgo_t}{2\sigma_{t+1}^*}$$

Where:

*cgo*: is capital gain overhang defined as a ratio with respect to current price:

$$cgo_t = (P_t - aqp_{t-1})/P_t$$

$\sigma$ : is the standard deviation of stock return, the \* indicates that it is a forecast value.

In this form, the relative capital gain overhang is a ratio between two rates that have same reference  $P_t$ . The CGO represents the amplitude of unrealized gain/loss, the  $2\sigma$  represents the amplitude of stock return variation at 95% confidence level. If CGO is greater than  $2\sigma$ , investor's winning/losing situation is considered as secured, if CGO is smaller than  $2\sigma$ , investor's winning/losing situation is challenged. In addition, this definition of the Relative Capital Gain Overhang can adapt to different decision making horizon depending which time horizon the standard deviation covers.

This adaptation of variable also has its limitation. The confidence level is given arbitrarily. There is no theoretical or empirical support for choosing 95% as the confidence level. The consequence for using such a confidence level is that the four gain/loss groups will be unbalanced. Most of the observations will be included in the small gain/loss groups, only few extreme observations will be included in the large gain/loss groups.

Note that using either a  $2\sigma$  or a  $\sigma$  in the estimation of RCGO does not influence the quality of the estimation. If one stock has higher or lower RCGO than another stock using the first method, it will also have a higher or lower RCGO using the second method. Because this variable is the ratio of the amount of unrealized gain/loss to the amount of risks, factoring a constant on the denominator will not change the order of the results. In fact, this property could apply to the estimation of RCGO over different investment horizons. A longer investment horizon implies that the standard deviation will be amplified by time. But it will not influence the estimation of RCGO.

Therefore, without damaging the estimation quality but has a simpler mathematical form, in the empirical test, the Relative Capital Gain Overhang will be defined as:

$$RCGO = \frac{cgo_t}{\sigma_{t+1}^*}$$

Where:

*cgo*: is Capital Gain Overhang defined as a ratio with respect to current price:

$$cgo_t = (P_t - aqp_{t-1})/P_t$$

$\sigma$ : is the standard deviation of stock return,

\*: indicates that it is a forecast value.

In financial studies, the standard deviation of stock return is commonly accepted as a measurement of risk. The definition using standard deviation could capture the basic idea of isolating the unrealized gain/loss from the risk factors. The small or large gain/lose groups will be divided according to the median of positive/negative RCGO instead of using 1 or -1.

#### 4.1.1.2 Standard deviation of stock return

This test uses the standard deviation of stock return to measure the amplitude of variation. The definition of relative capital gain overhang requires the standard deviation of stock return to be a forecast of the next period. Such forecast standard deviation will be estimated by the Exponentially Weighted Moving Average (EWMA) method (JP Morgan, R., 1996. RiskMetrics Chapter 5). This method gives more weight to recent observations, and can capture the dynamic features of volatility. Moreover, the EWMA method allows giving forecast standard deviation of the next period.

First, estimate the standard deviation of stock return of past T period using EWMA method:

$$\sigma_t = \sqrt{(1 - \lambda) \sum_{t=1}^T \lambda^{T-t} (r_t - \bar{r})^2}$$

Where:

$\lambda$  is the decay factor with  $0 < \lambda < 1$

$\bar{r}$  is the mean return during T period.

Next, based on the estimation, forecast the standard deviation of the next period by the following formula:

$$\sigma_{t+1}^* = \sqrt{\lambda\sigma_t^2 + (1 - \lambda)(r_t - \bar{r})^2}$$

Where:

$\sigma_t$  is the estimated value of standard deviation at period t.

$\bar{r}$  is the mean return.

This formula implies the assumption that the average returns for T period and for T+1 period are equal. Details are demonstrated in Annex 4.1.

To sum up, the transformation of the Relative Capital Gain Overhang keeps the essential concept of measuring the unrealized gain or loss relative to risk level. Estimating the RCGO requires first the estimation of Capital Gain Overhang using the method provided by Grinblatt and Han, and then the estimation of the future standard deviation of stock return using the Exponentially Weighted Moving Average (EWMA) method.

#### **4.1.2 RCGO influence on stock return**

In chapter 3, discussions were made in a theoretical contest about the significance of the Relative Capital Gain Overhang and the appropriate way to interpret RCGO. This variable captures the fact that investors are sensible to change in winning or losing situation. In other word, if the volatility of future outcome is unlikely to alter current winning or losing situation, investor will have less motivation to make risk-aversion decision. Such behavior pattern can be observed from the figure in the previous sub-section.

Under the theoretical context, four gain/loss situations can be distinguished by the value -1 0 and 1 of RCGO: the large loss situation ( $RCGO < -1$ ), the small loss situation ( $-1 < RCGO < 0$ ), the small gain situation ( $0 < RCGO < 1$ ), and the large gain situation ( $RCGO > 1$ ). In large gain or large loss situations, investor's propensity to sell the stock is significantly weaker than in small gain or small loss situations. The empirical adaptation allows

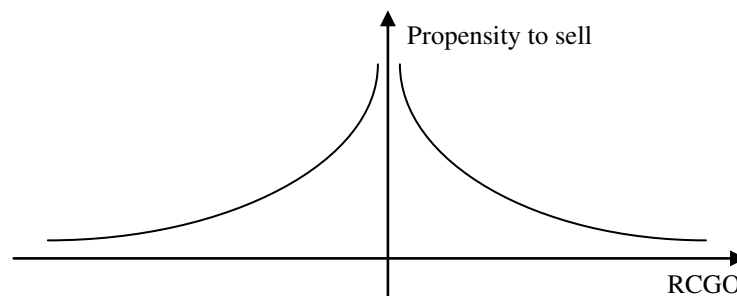


transforming the theoretical notion of unrealized gain or loss into an empirically testable variable. The empirical variable of RCGO remains the form of the scale of gain or loss comparing to the volatility of stock and hence maintains the measurement of investor's sensibility to changes in one's winning or losing situation. In the theoretical model, because the stock variation is binary, it is able to give very clear separation between large or small unrealized gain/loss. The large loss and small loss situations are distinguished by the value of -1, the large gain and small gain situations are distinguished by the value of 1. If RCGO is smaller than -1 or greater than 1, it will be certain that the current winning or losing situation will not change in the next period.

In the empirical model, the stock return is continuously distributed hence there is no clear boundary that distinguishes the large or small unrealized gain/loss such as 1 and -1. Instead the boundary is replaced by the idea of probability. A large RCGO in absolute value suggests that investor's current winning or losing situation is unlikely to change in the next period, while a small RCGO in absolute values means that investor's current situation is not stable.

In the context of empirical study, the influence of RCGO on investor's decision making will be simplified. Without clear distinction between the large or the small unrealized gain/loss, the influence can be considered as a downward trend of investor's propensity to sell when the RCGO deviates from zero. Investors are more reluctant to sell the stock when facing a large unrealized gain or loss than facing a small unrealized gain or loss.

Figure 4.1-2: The influence of RCGO in a simplified form



The discussions in previous part conclude that the unrealized gain/loss model suggests lower propensity to sell in large gain/loss situation. However, the model does not describes the way such behavior pattern impacts stock performance. It is unclear how such a behavior patter will impact stock return. Although in chapter 3, the empirical tests on the disposition

effect suggests that the force of supply and demand could be the media in between, still, it is difficult to give precise prediction of stock return evolution depending on RCGO.

For this reason, the hypothesis for the empirical study will be developed in a weak form. A null hypothesis will be posed that the Relative Capital Gain Overhang does not have influence on stock return. The empirical tests expect to reject this hypothesis.

**Hypotheses 1:** H0: There is no significant relation between stock return and Relative Capital Gain Overhang over different time horizons.

In the following sections, a sorting test will first be performed to offer first impression on stock performance in different RCGO groups and over various time horizons. Then a regression test will be performed using the model derived from Grinblatt and Han's test model. The test will study the influence of Relative Capital Gain Overhang on stock return, while controlling other effects caused by past stock performance size and trading volume.

A second regression test will be performed using a two-step regression. The first step is a time-serial regression on Fama-French model. The second step is a Fama-Macbeth regression of relative capital gain overhang on the residuals from the first regression.

## 4.2 Data description

The test uses weekly data of NYSE and AMEX stocks from January 1982 to December 2012. Observations with missing data or stock price smaller than 5\$ are deleted. Then, observations that have at least prior 158 continuous observations are selected in the tests. The data of Fama-French factors are obtained from the on-line database of the Data Library of Kenneth French.

### 4.2.1 Estimation of variables

In following tests, variables will be estimated using the same method in previous replication tests.

Stock price is the close price of a stock at the trading day. If no trade occurred in the trading day, the test uses the average between the bid prices and the ask price. The prices are

adjusted for splits and distributions according to the cumulative factor for adjusting price that is provided by the database.

Return of individual stocks are calculates stock return on weekly basis. Weekly return is calculated on the stock price adjusted for splits and distributions. The stock return at week t is calculated according to the following equation:

$$r_t = \frac{p_t}{p_{t-1}} - 1$$

Where:

$r_t$  is the stock's weekly return at week t.

$p_t$  is the stock's price at week t.

Then, stock return over longer period is calculated as cumulative return of multiple weekly returns according to the following formula:

$$r_{t:t+n} = \prod_{i=1}^n (r_{t+i} + 1) - 1$$

Trading volume and share outstanding are directly provided in the unit of numbers of shares. The values of the two variables are first adjusted for splits and distributions according to the cumulative factor for adjusting shares that is provided by the database. Then the trading volume is standardized into ration.

$$v_t = \frac{\text{trading volume}_t}{\text{share outstanding}_t}$$

The market capitalization of stock is calculated by multiplying the stock price at time t by the share outstanding of the stock at time t.

$$s_t = \text{price}_t \times \text{share outstanding}_t$$

The estimation of the average acquisition price will adopt the following formula:

$$aqp_t = \min(0.95, v_t)p_t + (1 - \min(0.95, v_t))R_{t-1}$$

Where:

$aqp_t$  is the average acquisition price for a stock at the end of week  $t$ ,

$p_t$  is the stock price at week  $t$ ,

$v_t$  is the trading volume represented in percentage.

The Capital Gain Overhang is defined as the difference between current stock price and the average acquisition cost divided by current stock price.

$$CGO_t = (P_t - aqp_{t-1})/P_t$$

Where:

$P$ : is stock price

$aqp$ : is the average acquisition cost

$t$ : indicates the time period

The forecast the standard deviation of the next period will be estimated using the EWMA method by the following formula:

$$\sigma_{t+1}^* = \sqrt{\lambda\sigma_t^2 + (1 - \lambda)(r_t - \bar{r})^2}$$

Where:

$\sigma_t$  is the estimated value of standard deviation at period  $t$ .

$\bar{r}$  is the mean return.

The variable Relative Capital Gain Overhang will be estimated using the formula discussed in previous section.

$$RCGO = \frac{cgo_t}{\sigma_{t+1}^*}$$

Where:

cgo: is capital gain overhang

$\sigma^*$ : is the forecast standard deviation of stock return

#### 4.2.2 Summary statistics of RCGO

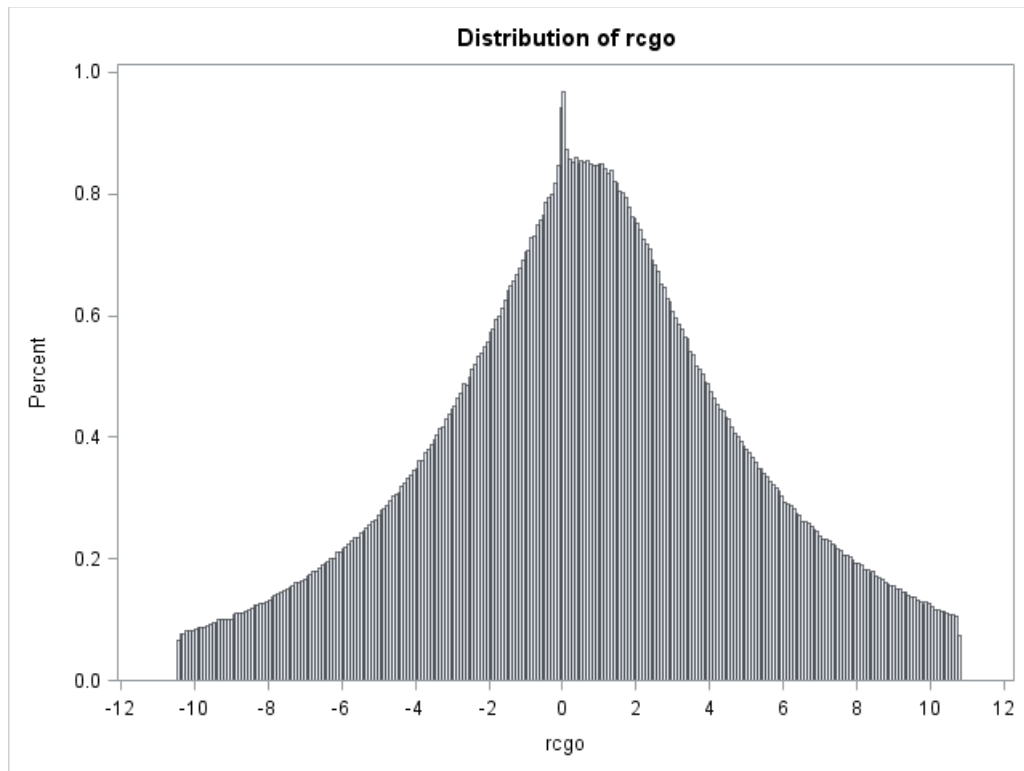
The relative capital gain overhang, defined by  $RCGO = CGO/\sigma$ . The  $\sigma$  is the forecasted standard deviation of stock return, estimated using the EWMA method. This definition of RCGO will not try to label a “safe” or “not safe” unrealized gain/loss. Instead, it gives a value of capital gain overhang adjusted by the stock’s volatility. The descriptive statistics of the data is displayed in the following table.

Next, more detailed statistical information about the Relative Capital Gain Overhang is given in this part, including the moments, quantiles and its distribution.

Table 4.2-1: The moments and quantiles of RCGO

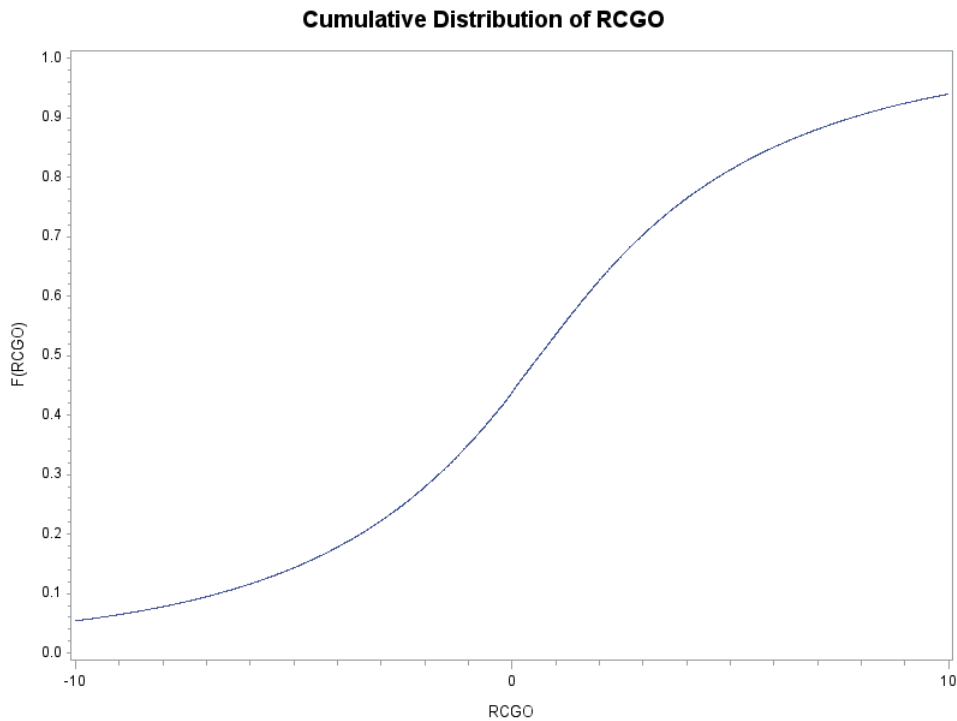
| <b>Moments</b>   |                      |                 |                 |                 |
|------------------|----------------------|-----------------|-----------------|-----------------|
| <b>Mean</b>      | <b>Std Deviation</b> | <b>Variance</b> | <b>Skewness</b> | <b>Kurtosis</b> |
| 0.1806           | 13.3877              | 179.2305        | -60.8074        | 7900.0689       |
| <b>Quantiles</b> |                      |                 |                 |                 |
| <b>5%</b>        | <b>25%</b>           | <b>50%</b>      | <b>75%</b>      | <b>95%</b>      |
| -10.4681         | -2.4847              | 0.6284          | 3.7388          | 10.7791         |

Figure 4.2-1: Distribution of RCGO



This figure reports the density of the distribution of RCGO within the range from 5 percentile to 95 percentile. The spick actually appears at RCGO=0.

Figure 4.2-2: Cumulative distribution of RCGO

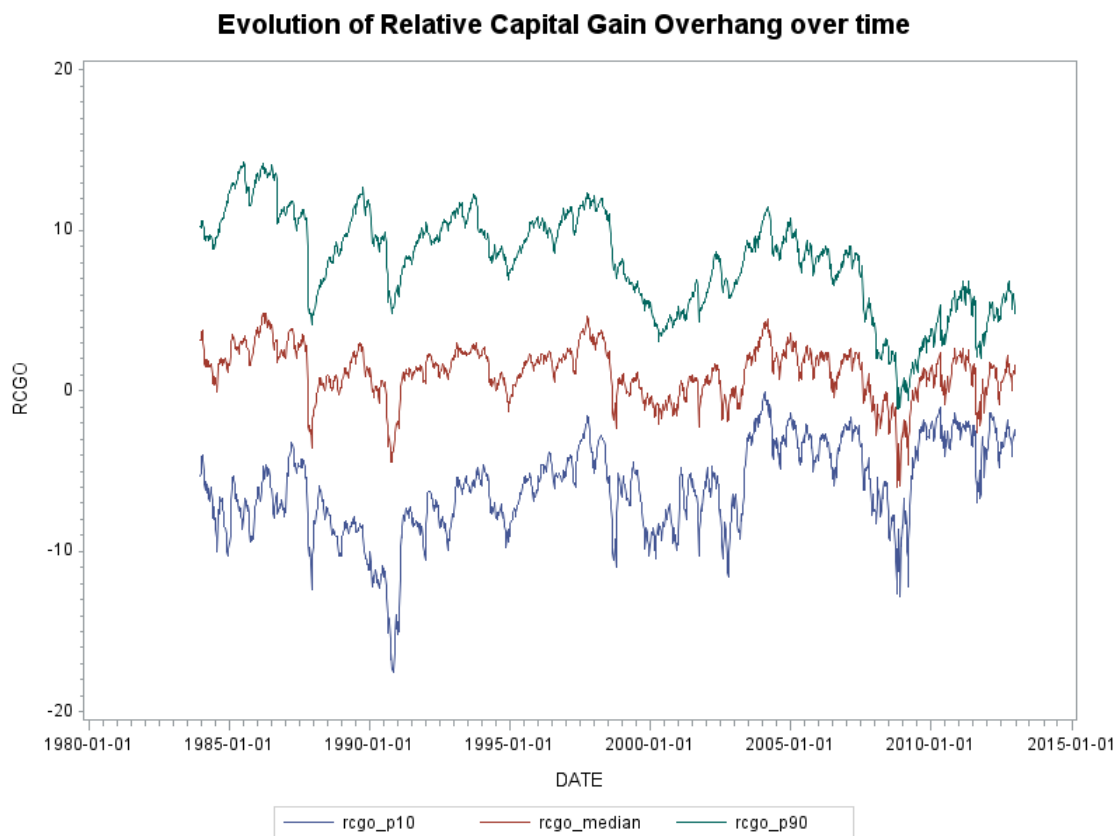


This figure reports the cumulative distribution of RCGO within the range from 5 percentile to 95 percentile

The mean and the median of RCGO are both positive and are close to zero. The distribution of RCGO is slightly skewed to the left. This suggests that there are less

observations with negative value, but the negative observations tend to have extreme values. The statistic also shows that about 80% of the observations are concentrated in the interval of -10 and 10. With such a dispersed distribution, it is impossible to distinguish large unrealized gain/loss from small unrealized gain/loss by the value of -1 and 1 as in the theoretical model. The empirical adaptation of the RCGO shows its advantage in this circumstance.

Figure 4.2-3: Evolution of RCGO, 10<sup>th</sup> percentile, median, and 90<sup>th</sup> percentile



The evolution of RCGO shows that it is more volatile in the early half of the sample period than in the late half of the sample period. Comparing with the evolution of capital gain overhang, RCGO values have a much larger range: 10<sup>th</sup> percentile could surpass the value of -10, and 90<sup>th</sup> percentile could reach the value of 10. The RCGO is more volatile in the first half of sample period, which is consistent with the evolution of the capital gain overhang.

### 4.3 RCGO sorting test

This test first divides all stocks into ten deciles in ascending order based on stock's Relative Capital Gain Overhang, and then compares the average return among the deciles. Such test will provide an illustration of the return level of each RCGO deciles, and allow

intuitive comparison among the deciles. The objective of this test is to verify the null hypothesis that the RCGO has no influence on stock performances.

The division of deciles is performed in two steps: first, all observations are separated into two groups for  $RCGO < 0$  and  $RCGO \geq 0$ ; then, five deciles are divided within the winning or the losing group in ascending order of RCGO. The five deciles in the negative RCGO group are labeled as deciles 1 to 5, and the five deciles in the positive RCGO group are labeled as deciles 6 to 10. The reason for performing such a division is to make sure that the winning stocks and the losing stocks are not mixed in the middle deciles.

This division is performed at the beginning of each week ( $t_0$ ). Therefore, the values that cut the different deciles vary in time. However, the winning stocks and losing stocks are always separated into deciles 5 and 6. The following table reports the cutting values by given the minimum, the median, and the maximum values. For instance, the median value of RCGO separating deciles 1 and 2 is -6.316. The cutting value of deciles 5 and 6 is strictly zero as was intended to.

Table 4.3-1: The cutting values of RCGO deciles

| Deciles | 1/2     | 2/3    | 3/4    | 4/5    | 5/6 | 6/7   | 7/8   | 8/9   | 9/10   |
|---------|---------|--------|--------|--------|-----|-------|-------|-------|--------|
| Min     | -13.733 | -8.735 | -5.285 | -2.983 | 0   | 0.074 | 0.333 | 0.657 | 1.177  |
| Median  | -6.316  | -3.696 | -2.145 | -0.968 | 0   | 1.466 | 2.959 | 4.826 | 7.785  |
| Max     | -2.929  | -1.511 | -0.754 | -0.257 | 0   | 3.009 | 5.841 | 8.738 | 12.689 |

### 4.3.1 Average RCGO deciles return

Next, the equal weighted average return of each decile is calculated for the following time horizon: Immediate 1-week ( $t_0-t_1$ ), subsequent 1-week ( $t_1-t_2$ ), subsequent 3-month ( $t_1-t_{13}$ ), subsequent 6-month ( $t_1-t_{26}$ ), subsequent 9-month ( $t_1-t_{39}$ ), and subsequent 12-month ( $t_1-t_{52}$ ). The following table and graphic demonstrate the average deciles return for the full sample period.



Chapter 4:  
Tests on the unrealized gain/loss model

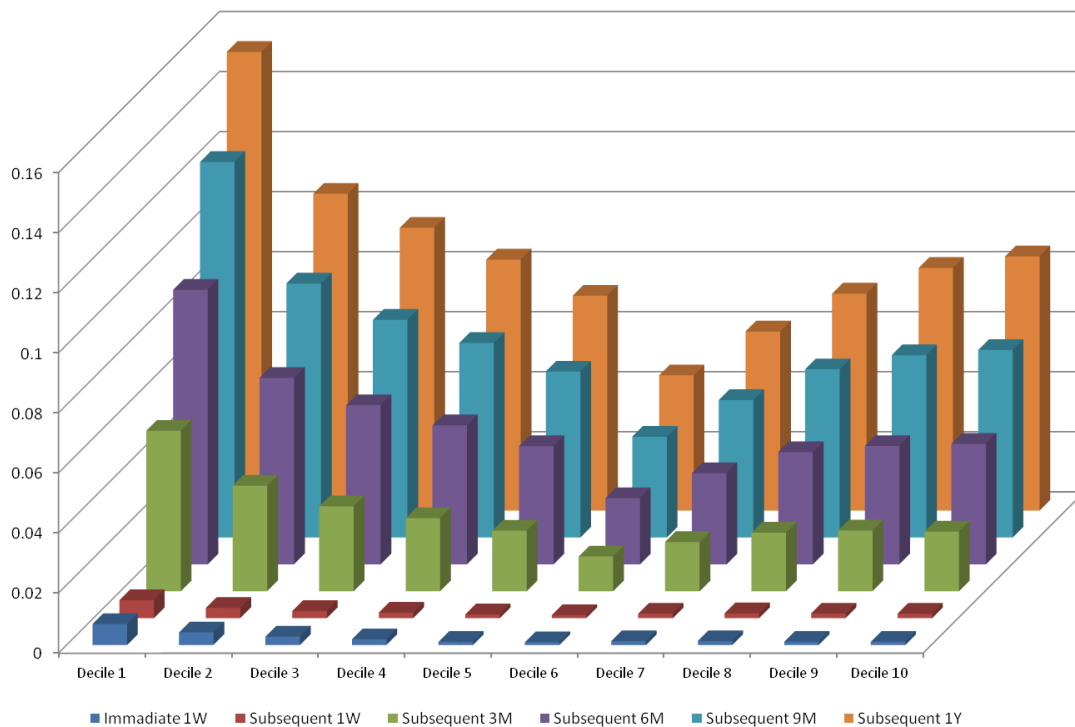
Table 4.3-2: Average stock return on RCGO deciles

| RCGO Deciles | $r_{0:1}$  | $r_{1:2}$  | $r_{1:13}$ | $r_{1:26}$ | $r_{1:39}$ | $r_{1:52}$ |
|--------------|------------|------------|------------|------------|------------|------------|
| (lowest) 1   | .006973545 | .005963343 | 0.053450   | 0.091461   | 0.12507    | 0.15278    |
| 2            | .004187192 | .003418728 | 0.035153   | 0.062096   | 0.08457    | 0.10551    |
| 3            | .002806294 | .002357480 | 0.028278   | 0.053013   | 0.07249    | 0.09420    |
| 4            | .001903460 | .001799785 | 0.024331   | 0.046330   | 0.06480    | 0.08361    |
| 5            | .001056791 | .001237654 | 0.020161   | 0.039382   | 0.05526    | 0.07157    |
| 6            | .000935637 | .001004048 | 0.011563   | 0.022040   | 0.03353    | 0.04503    |
| 7            | .001292319 | .001484106 | 0.016381   | 0.030356   | 0.04575    | 0.05965    |
| 8            | .001280274 | .001598842 | 0.019511   | 0.037369   | 0.05602    | 0.07229    |
| 9            | .001083526 | .001493496 | 0.020224   | 0.039423   | 0.06071    | 0.08083    |
| (highest) 10 | .001062411 | .001475667 | 0.019875   | 0.040131   | 0.06246    | 0.08465    |

All average returns are statistically significant different from zero at 99.9% confidence level

The above table reports the equal weighted average raw returns of all the ten RCGO deciles. All values are statistically significant. For allowing more intuitive comparison, the above data will be illustrated by graph.

Figure 4.3-1: Average raw return on RCGO deciles



The above figure shows that the cross-section of the RCGO decile returns behave differently over short and long time horizons. Over the short time horizon, especially the immediate time horizon, one could observe a negative relation either in the loss situation or in gain situation. And no clear pattern can be seen over subsequent one-week to 13-week time

horizons. However, in longer time horizon, a pattern begins to emerge. The stocks with extreme low RCGO and extreme high RCGO have higher average return than the stocks with RCGO close to zero. This pattern is more visible with the increase of time horizon. This figure also shows that the stocks with losses could have higher returns than the stocks with gains, and the decile 1, which contains the stocks with the largest losses, yield the highest returns over all 6 time horizons.

### 4.3.2 T-test over RCGO deciles return

In order to verify the statistical significance of the results in the sorting, T-tests are performed. The RCGO sorting data allows calculating the difference in performance over 1-week to 1-year time horizons between the extreme deciles and the middle deciles in each week. The difference of stock returns is calculated as such: (decile 1 + decile 10) - (decile 5 + decile 6). Finally, T-tests will be performed on the time-serial data of the differences of returns to examine if they are statistically different from zero.

Table 4.3-3: Difference in average return between the middle deciles and other deciles

|                      |         | Immediate<br>1-week | Subsequent<br>1-week | Subsequent<br>3-months | Subsequent<br>6-months | Subsequent<br>9-months | Subsequent<br>12-months |
|----------------------|---------|---------------------|----------------------|------------------------|------------------------|------------------------|-------------------------|
| (D1+D10)<br>-(D5+D6) | mean    | 0.0060              | 0.0048               | 0.0367                 | 0.0592                 | 0.0821                 | 0.0993                  |
|                      | T-value | 10.33               | 8.80                 | 17.56                  | 20.72                  | 24.13                  | 24.65                   |
|                      | P-value | <.0001              | <.0001               | <.0001                 | <.0001                 | <.0001                 | <.0001                  |

The T-test confirms the observations from the figure. Over all the 6 time horizons, the extreme deciles outperform the middle deciles, and all the differences are significant at 99% confidence level.

### 4.3.3 Average decile return adjusted for risks

The sorting test on stock raw returns shows that the stocks with extremely high RCGO or extremely low RCGO can outperform the stocks with neutral RCGO. However, test on the raw return cannot rule out the possibility that the extreme RCGO deciles might be exposed to systematic risk factors, and result in higher return than other groups. To test this possibility, same sorting test will be performed on the abnormal return adjusted by Fama-French model.

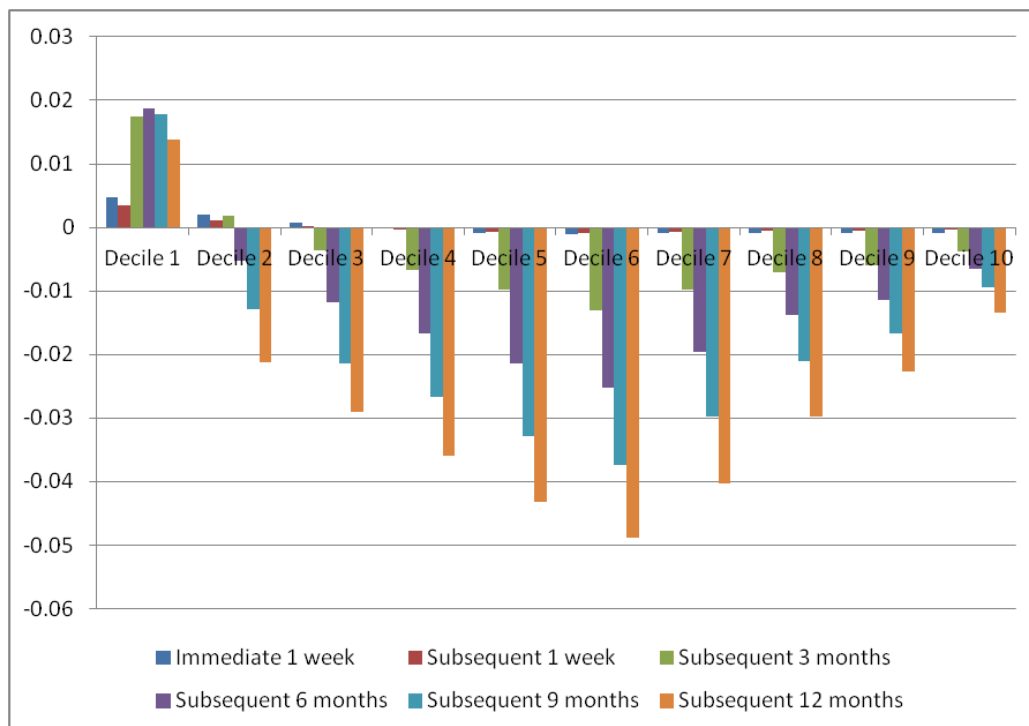
Before the sorting, a time serial regression will be performed on each individual stock over the total sample period on the Fama-French three-factor model:

$$r_i - r_f = \beta_1(r_m - r_f) + \beta_2SMB + \beta_3HML + \varepsilon$$

By this model, stock return that is related to risk factors of the market, size, and book-to-market ratio will be controlled. There is no intercept term in this model therefore all errors that deviate from the model's prediction are captured by the residual term  $\varepsilon$ .

Next, the same sorting procedure will be performed on the residuals of the above regression. Since the residuals are orthogonal to the risk factors, if any pattern is observed within the residuals, it could be due to other factors such as the investor behavior. The sorting results are reported in the following figure.

Figure 4.3-2: Average return residual of RCGO deciles



The sorting on return residuals yields consistent result with the sorting on raw returns. It shows the same pattern that stocks with extreme Relative Capital Gain Overhang outperform the stocks with neutral RCGO over intermediate to long time horizons. Similarly, the stocks with extreme unrealized losses, namely the decile 1, still yield higher performance than the stocks in other deciles.

One critic to this graph is that all risk adjusted returns are all negative in all tested time horizons. The reason for such an anomaly may be the inconsistency of data. Due to limited data source, the test data includes only the stocks in NYSE and AMEX. However, the Fama-French risk factors are directly downloaded from the Data Library of Kenneth French. The

latter are however calculated from stock in NYSE, AMEX and NASDAQ. The latter had a much higher return over the sample period. Hence, this discrepancy of data source may cause that the testing stocks are systematically underperform the Fama-French factors.

To sum up, the RCGO decile sorting test shows that extreme RCGO stocks will outperform neutral RCGO stocks over intermediate to long time horizons. The T-test show significant difference in stock returns between these groups. Such result rejects the null hypothesis that RCGO has no influence over stock performance. However, the test results raise more questions about the transmission of investor behavior to stock return. The major doubt is why the influence of RCGO, developed from a model that describes investor preference in short term, takes effect over long time horizon. Such question will inspire further development of the theoretical model.

In the following section, regression tests will be performed to further verify the cross-sectional relation between RCGO and stock return. The regressions will provide more convincing evidence of the influence of RCGO.

#### **4.4 Regression based on Grinblatt and Han's model**

In this section, Grinblatt and Han's model will be adapted to perform regression on stock return and Relative Capital Gain Overhang. The original model is used to test the influence of the disposition effect over stock returns. The disposition effect is represented by the Capital Gain Overhang, and it assumes that there is linear relation between these two elements. Other factors such as past return size and trading volume are also controlled.

The reason for choosing this model is that it is a well established and well accepted empirical model in the field of behavior finance. Moreover, mathematically speaking, the variable RCGO can be considered as a variant of the CGO, although the economical meanings of these two variables are different. The CGO represents the disposition effect, while the RCGO captures investor's sensitivity to the change from a winning to a losing situation or the change in the opposite direction.

The adaptation of Grinblatt and Han's test model consists of first replacing the variable CGO by RCGO. Next, since the theoretical model about the unrealized gain or loss predicts decreasing propensity to sell stock in gain and in loss situations, the test will be able to

separate the positive RCGO from the negative RCGO. Finally, the theoretical model also suggests that investors should be less sensitive in large unrealized gain/loss situations. Therefore, the model will be able to distinguish all the four large/small gain/loss situations.

Following this adaptation, the regression test base on Grinblatt and Han's test model will be divided into three steps. First, the variable CGO in the original model is replaced by RCGO. Second, one dummy variable is introduced in order to separate the positive RCGO group and the Negative RCGO group. Finally, three dummy variables are introduced in order to distinguish the large loss, the small loss, the small gain, and the large gain situations.

The basic test model used in the following series of regressions is the Grinblatt and Han's test model with replacement of CGO by RCGO

$$r = a_0 + a_1 r_{-4:-1} + a_2 r_{-12:-5} + a_3 r_{-64:-13} + a_4 \bar{V}_{-52:-1} + a_5 S_{-1} + a_6 rcgo_{-1}$$

Where:

$r$ : is the weekly stock return

$r_{-4:-1}$ : is the accumulative return from week -4 to week -1

$r_{-12:-5}$ : is the accumulative return from week -12 to week -5

$r_{-64:-13}$ : is the accumulative return from week -64 to week -13

$\bar{V}$ : is the average stock turnover from week -52 to week -1

$S$ : is the stock's market capitalization

$rcgo$ : is the relative capital gain overhang, defined as:

$$RCGO = \frac{cgo_t}{\sigma_{t+1}^*}$$

Where:

$cgo$ : is Capital Gain Overhang defined as a ratio with respect to current price:

$$cgo_t = (P_t - aqp_{t-1})/P_t$$

$\sigma^*$ : is the forecasting standard deviation of stock return,

In this regression model proposed by Grinblatt and Han, the dependant variable  $r$  is the stock weekly raw return, suggesting that this regression test the correlation over short time horizon. The variable CGO is estimated using non-lagged stock price, indicating that it corresponds to the theoretical CGO mentioned in previous chapter. This suggests that the unrealized gain/loss is measured at the beginning of the week when the stock return is calculated. For this reason, this study will focus on the effect of relative capital gain overhang on stock return over immediate time horizon. According to previous sorting test, over the immediate time horizon, there is generally a negative correlation between RCGO and stock return. The regression is expected to verify such a correlation.

#### 4.4.1 Summary statistics

The data used in this test is the same as in the sorting test. The data consists of all stocks in NYSE and AMEX from January 1982 to December 2012. The variables are also estimated by the same method. The summary statistics of the variables used in this test are reported by following tables.

Table 4.4-1: Descriptive statistics of the variables

| Variable        | Mean    | Std Dev | 10th Pctl | Median  | 90th Pctl |
|-----------------|---------|---------|-----------|---------|-----------|
| $r$             | 0.0014  | 0.0579  | -0.0526   | 0.0000  | 0.0549    |
| $r-4:-1$        | 0.0043  | 0.0967  | -0.0909   | 0.0015  | 0.0976    |
| $r-52:-5$       | 0.0752  | 0.4090  | -0.3232   | 0.0377  | 0.4719    |
| $r-156:-53$     | 0.2349  | 0.7318  | -0.3656   | 0.1083  | 0.9127    |
| $V$             | 0.0322  | 0.1148  | 0.0036    | 0.0149  | 0.0633    |
| $s$             | 12.9366 | 1.9243  | 10.5764   | 12.8402 | 15.4611   |
| CGO             | -0.0305 | 0.6781  | -0.3242   | 0.0412  | 0.2843    |
| Forecasted S.D. | 0.0674  | 0.0508  | 0.02461   | 0.0547  | 0.1236    |
| RCGO            | 0.9014  | 11.5179 | -5.9647   | 1.0410  | 8.5139    |

The above table reports the descriptive statistics of testing variables. It also includes the Capital Gain Overhang and the forecasted standard deviation of stock returns. Based on these two variables, the RCGO is estimated. This table shows that the values of the forecasted standard deviation of stock return are smaller than one. Therefore, the values of RCGO are scaled up by the forecasted S.D. comparing to the values of CGO. Note that the mean the CGO is negative, but the mean of the RCGO is a positive value. This change suggests that the RCGO is not a manipulation of CGO in the form. In fact the difference in their economic meanings could results to difference in the values of the variables in actual tests.

Table 4.4-2: Matrix of correlation of the variables

| Pearson Correlation Coefficients<br>Prob >  r  under H0: Rho=0 |                    |                    |                     |                       |                    |                    |                    |
|--|--------------------|--------------------|---------------------|-----------------------|--------------------|--------------------|--------------------|
|  | r                  | r <sub>-4:-1</sub> | r <sub>-52:-5</sub> | r <sub>-156:-53</sub> | V <sub>-1</sub>    | s <sub>-1</sub>    | RCGO <sub>-1</sub> |
| r  | 1                  | -0.03307<br><.0001 | -0.00144<br>0.0029  | -0.00927<br><.0001    | -0.00279<br><.0001 | -0.00382<br><.0001 | -0.00608<br><.0001 |
| r <sub>-4:-1</sub>   | -0.03307<br><.0001 | 1                  | -0.0139<br><.0001   | -0.01726<br><.0001    | -0.00376<br><.0001 | 0.04317<br><.0001  | 0.12547<br><.0001  |
| r <sub>-52:-5</sub>  | -0.00144<br>0.0029 | -0.0139<br><.0001  | 1                   | -0.09156<br><.0001    | -0.00452<br><.0001 | 0.13596<br><.0001  | 0.23664<br><.0001  |
| r <sub>-156:-53</sub>  | -0.00927<br><.0001 | -0.01726<br><.0001 | -0.09156<br><.0001  | 1                     | 0.0248<br><.0001   | 0.13423<br><.0001  | 0.09724<br><.0001  |
| V <sub>-1</sub>  | -0.00279<br><.0001 | -0.00376<br><.0001 | -0.00452<br><.0001  | 0.0248<br><.0001      | 1                  | 0.08149<br><.0001  | -0.02111<br><.0001 |
| s <sub>-1</sub>  | -0.00382<br><.0001 | 0.04317<br><.0001  | 0.13596<br><.0001   | 0.13423<br><.0001     | 0.08149<br><.0001  | 1                  | 0.15447<br><.0001  |
| RCGO <sub>-1</sub>   | -0.00608<br><.0001 | 0.12547<br><.0001  | 0.23664<br><.0001   | 0.09724<br><.0001     | -0.02111<br><.0001 | 0.15447<br><.0001  | 1                  |

Matrix of correlation is demonstrated by the above table. Similar to the replication test, the non-correlation hypothesis is rejected for any of two variables. This means that the regression may suffer from the problem of co-linearity. Considering that the sample contains a very large number of observations, it could boost the significance of the statistic on the correlations.

#### 4.4.2 Single grouped test

This sub-section performs the single grouped regression test. This regression aims to verify the linear correlation between RCGO and stock returns at a general level. No sub groups will be distinguished based on unrealized gains or losses. The model used in this single grouped test is the model of Grinblatt and Han, with modification that replaces the original CGO with the RCGO:

$$r = a_0 + a_1 r_{-4:-1} + a_2 r_{-52:-5} + a_3 r_{-156:-53} + a_4 \bar{V}_{-52:-1} + a_5 s_{-1} + a_6 rcgo_{-1}$$

The Fama-Macbeth method is applied to perform this regression. At each time, a cross-sectional regression is performed on all the stocks to get the coefficients. Then T-test is performed on all the estimated coefficients over all time periods to produce the statistics. The following table reports the coefficients of this single grouped regression.

Table 4.4-3: Estimation of coefficients of the single group regression

| $r = a_0 + a_1 r_{-4:-1} + a_2 r_{-52:-5} + a_3 r_{-156:-53} + a_4 \bar{V}_{-52:-1} + a_5 s_{-1} + a_6 rcgo_{-1}$ |        |                |        |         |                |        |         |           |
|---|--------|----------------|--------|---------|----------------|--------|---------|-----------|
| Coef.   | $a_0$  | $a_1$          | $a_2$  | $a_3$   | $a_4$          | $a_5$  | $a_6$   | Adj $R^2$ |
| Mean  | 0.0006 | <b>-0.0305</b> | 0.0002 | -0.0002 | <b>-0.0096</b> | 0.0001 | -0.0000 | 0.0409    |
| T-value   | 0.80   | -20.89         | 0.36   | -0.94   | -2.41          | 0.89   | -1.39   |           |
| P-value   | 0.4241 | <.0001         | 0.7189 | 0.3484  | 0.0159         | 0.3748 | 0.1637  |           |

The above table demonstrates the Fama-Macbeth regression result of the single grouped model. The adjusted R-squared is 0.04, suggesting that the test model is able to explain about 4% of the variance of the data. The intercept of this model is positive and statistically significant. For control variables, negative correlation is found for short term past return. The coefficient of the intermediate term past return is positive. The one of the long term past return is negative, but these two coefficients are not significant. A negative correlation is found for the trading volume of stock. The coefficient of the market capitalization is not significant. The result of the control variables is consistent with the replicating test in chapter 3. However, for the variable Relative Capital Gain Overhang, this regression does not find a significant correlation to current stock return.

Comparing with the replication of Grinblatt and Han's test in the previous chapter, these test results have some similarities. The adjusted R-squared are at a 4% to 5% level. The general explanatory power of the regression model remains. In this test, the estimation of the coefficients of the control variables is also very similar to the ones in the replication test. The terms representing the momentum effect and the reversal effect are not significant. The major difference between these two tests is observed on the coefficient  $a_6$ . In the replication test, a negative correlation between stock return and CGO is found, which is consistent with the disposition effect. In this test, no significant correlation is found between stock return and RCGO. This result suggests that first, the RCGO represents a behavior factor other than the disposition effect, and the overall influence of RCGO is not significant over the immediate time horizon.

#### 4.4.3 Double grouped test

In this section, a regression test is performed to verify if the influences of the unrealized gain and the unrealized loss on stock return are different. The reason for performing such a test is that the single grouped test did not find a significant correlation between stock returns



and RCGO. However, the theoretical model predicts a decrease in propensity to sell the stock when facing an unrealized gains or losses. Therefore, in a loss situation, investors are more likely to hold the stock when the loss becomes larger. In other word, the value of RCGO becomes smaller. Hence one can expect an increase in stock return with respect to a decrease in RCGO. It corresponds to a negative correlation between stock return and RCGO in an unrealized loss situation. In a winning situation, investors are less likely to sell the stock with an increase in gains. Hence, stock return will increase with RCGO. One can expect a positive correlation in an unrealized gain situation. To verify such correlations, a test will be able to distinguish the effect of a positive RCGO from the effect of a negative RCGO.

For the above reason, in this test, a dummy variable is introduced in order to separate the observations with negative RCGO from the observations with positive RCGO. The test aims to examine the different correlations. Therefore, the dummy variable will be applied on the variable RCGO in order to capture the difference in the slopes between the positive and negative RCGO groups. The double grouped test model is the following:

$$r = a_0 + a_1r_{-4:-1} + a_2r_{-52:-5} + a_3r_{-156:-53} + a_4\bar{V}_{-52:-1} + a_5s_{-1} + a_6rcgo_{-1} + a_7Nrcgo_{-1}$$

Where:

N = 1 if RCGO < 0, N = 0 if else.

Other variables are defined the same way as in the previous model. The dummy variable N identifies the observations with a negative RCGO. Its counterpart, the positive RCGO group is considered as the base group. The coefficient of the base group  $a_6$  indicates the correlation between RCGO and stock returns in a winning situation, the coefficient  $a_7$  captures the difference of the slopes of the negative RCGO group with respect to the base group. The dummy variable separates a negative RCGO from the positive one, allowing distinguishing the influences of unrealized gain from the influence of unrealized loss on stock returns.

The regression also adopts the Fama-Macbeth method. At each time period, a cross-sectional regression is performed to estimate the regression coefficients. The T-test is performed on the coefficients over all time periods. The following table demonstrates the result of Fama-Macbeth regression of the double grouped model.

Chapter 4:  
Tests on the unrealized gain/loss model

Table 4.4-4: The double grouped test result

Panel A: The coefficients of the regression

$$r = a_0 + a_1r_{-4:-1} + a_2r_{-52:-5} + a_3r_{-156:-53} + a_4\bar{V}_{-52:-1} + a_5s_{-1} + a_6rcgo_{-1} + a_7Nrcgo_{-1}$$

| Coef.   | $a_0$  | $a_1$   | $a_2$  | $a_3$   | $a_4$   | $a_5$  | $a_6$  | $a_7$   | $R^2$  |
|---------|--------|---------|--------|---------|---------|--------|--------|---------|--------|
| Mean    | 0.0001 | -0.0304 | 0.0002 | -0.0002 | -0.0061 | 0.0001 | 0.0000 | -0.0001 | 0.0431 |
| T-value | 0.07   | -21.06  | 0.35   | -1.24   | -1.57   | 1.13   | 0.90   | -3.54   |        |
| P-value | 0.9423 | <.0001  | 0.7263 | 0.2149  | 0.1167  | 0.2603 | 0.3566 | 0.0004  |        |

Panel B: Coefficients of the groups:

| Group | unrealized gain ( $a_6$ ) | unrealized loss ( $a_6+a_7$ ) |
|-------|---------------------------|-------------------------------|
| Coef. | 0                         | -0.0001***                    |

\*\*\* indicates the coefficient is significant at 99% confidence level.

The above table demonstrates the Fama-Macbeth regression result of the double grouped regression test. Panel A reports the estimated coefficients and the statistics of the regression, Panel B reports the correlation of the positive and negative RCGO groups. The control variables show similar result comparing to the previous test. The intercept is not significant. The short term past return is negatively correlated to current stock return. The intermediate term past return and the long term past return are not correlated to current stock return. The adjusted R squared in this regression is also at the same level as in the single grouped regression. Concerning to the unrealized gain or loss, the coefficient of the base group, where the observations has positive RCGO, is  $a_6 = 0$ . The coefficient of the negative RCGO group is  $a_6+a_7 = -0.0001$ . It is significant at 99% confidence level. This result suggests that the impact of Relative Capital Gain Overhang over immediate time horizon is found in the stocks with unrealized losses.

#### 4.4.4 Quadruple grouped test

The model of the Relative Capital Gain Overhang suggests that investors are sensitive to changes in his/her winning or losing situation. If the variation in the outcome is likely to turn a winning situation into a loss (or otherwise), the investors will have a strong preference towards one option. If not, the investors tend to be indifferent. A transition from a gain to a loss situation (or the reverse scenario) is more likely to occur in a small unrealized gain or a small unrealized loss situation. Therefore, one can expect that the investor will show stronger motivation to make a decision, whether the decision concerns selling or holding the stock. This feature suggests that within the unrealized gain or the unrealized loss group, the

correlation between RCGO and stock return should not be linear. The slope should be steeper in the small gain/loss situation than in the large gain/loss situation.

In this section, the test aims to verify such feature. It hence requires separating the sample into four groups: large loss, small loss, small gain, and large gain. This issue has been discussed in previous sub-sections. According to the discussion, the small or large gain/loss groups will be divided by the medians of the loss group or the gain group. Using the medians as the dividing value is an intuitive way to divide these four groups, moreover, it allows forming the subsample of the large loss, small loss, small gain, and large gain groups with approximately equal subsample sizes.

The test model is further derived from previous test models. Dummy variables LL SL SG are introduced to distinguish different winning or losing situation. The base line is set to be the large gain situation; hence no dummy variable is formed to represent the large gain group. These dummy variables will capture the deviation of the coefficients of each group from the base line group.

The quadruple grouped test model:

$$r = a_0 + a_1 r_{-4:-1} + a_2 r_{-52:-5} + a_3 r_{-156:-53} + a_4 \bar{V}_{-52:-1} + a_5 s_{-1} + a_6 rcgo_{-1} + a_7 LLrcgo_{-1} + a_8 SLrcgo_{-1} + a_9 SGrcgo_{-1}$$

Where:

LL = 1 if stock is in the lower half of the losing group, LL = 0 if not;

SL = 1 if stock is in the upper half of the losing group, SL = 0 if not;

SG = 1 if stock is in the lower half of the winning group, SG = 0 if not;

Other variables are defined the same way as in previous model.

The theoretical model predicts that the investors are more sensitive to change in winning/losing situations, suggesting that the investors are more likely to hold stock in the small loss situation than in large loss situation; and are more likely to sell stock in small gain situation than in large gain situation. Therefore, one can expect a stronger negative correlation between stock return and RCGO in small gain and small loss groups, and a weaker negative correlation in large gain and large loss groups.

Chapter 4:  
Tests on the unrealized gain/loss model

The regression also adopts the Fama-Macbeth method. At each time period, a cross-sectional regression is performed to estimate the regression coefficients. The T-test is performed on the coefficients over all time periods to calculate the statistic. The following table reports the regression result of the quadruple grouped test.

Table 4.4-5: The loss and regret aversion mode, four-group test

Panel A: The coefficients of the regression

$$r = a_0 + a_1r_{-4:-1} + a_2r_{-52:-5} + a_3r_{-156:-53} + a_4\bar{V}_{-52:-1} + a_5s_{-1} + a_6rcgo_{-1} + a_7LLrcgo_{-1} + a_8SLrcgo_{-1} + a_9SGrcgo_{-1}$$

| coefficient | mean    | T-value | P-value |
|-------------|---------|---------|---------|
| $a_0$       | 0.0002  | 0.22    | 0.8275  |
| $a_1$       | -0.0308 | -21.39  | <.0001  |
| $a_2$       | 0.0000  | 0.01    | 0.9940  |
| $a_3$       | -0.0002 | -1.37   | 0.1704  |
| $a_4$       | -0.0050 | -1.31   | 0.1916  |
| $a_5$       | 0.0001  | 0.96    | 0.3353  |
| $a_6$       | 0.0000  | 0.90    | 0.3676  |
| $a_7$       | -0.0001 | -3.73   | 0.0002  |
| $a_8$       | 0.0002  | 2.90    | 0.0038  |
| $a_9$       | 0.0002  | 2.06    | 0.0393  |
| $R^2$       | 0.0443  |         |         |

Panel B: The coefficients of the groups

| Group | Large loss ( $a_6+a_7$ ) | Small loss ( $a_6+a_8$ ) | Small gain ( $a_6+a_9$ ) | Large gain ( $a_6$ ) |
|-------|--------------------------|--------------------------|--------------------------|----------------------|
| Coef. | -0.0001***               | 0.0002**                 | 0.0002**                 | 0                    |

\*\* indicates the coefficient is significant at 95% confidence level, \*\*\* indicates the coefficient is significant at 99% confidence level.

The above table reports the regression result of the quadruple grouped regression. Panel A of the table reports the estimation of the coefficients in the regression model. Panel B reports the coefficients of all four testing groups. The result of the control variables remains the same as in the above tests. The short term past return is negatively correlated to current stock return. The other control variables are not significant. Focusing on the four groups, the coefficients in different groups are respectively: in the large loss group:  $a_6+a_7 = -0.0001$ ; in the small loss group:  $a_6+a_8 = 0.0002$ ; in the small gain group:  $a_6+a_9 = 0.0002$ ; and in the large gain group:  $a_6 = 0$ . The coefficients of RCGO in all the groups are significantly negative except for the large gain group.

This result offers more detailed information about the correlation between RCGO and stock return over immediate time horizon. When four groups are divided, this result finds a positive correlation in small gain group and a non-significant correlation in large gain group.

The result on these two groups is consistent to the predicted influence of RCGO. The investors are more likely to sell the stocks when facing a small gain than facing a large gain. However, in the small loss group, the test finds a positive correlation between RCGO and stock returns. This result is difficult to explain.

#### **4.4.5 Test summary**

In this section, tests are performed to study the influence of the unrealized gain or loss over stock returns. The model used in this series of test is based on the empirical model from Grinblatt and Han (2005). In addition to replacing the variable CGO by the RCGO, the tests also developed the double grouped and the quadruple grouped regression following the suggestion provided by the theoretical model.

The single grouped regression examines the correlation at a general level. It does not find significant correlation. The double grouped regression studies the correlations separately for the winning group and the losing group. The result shows negative correlation in the losing group, while a non significant correlation in the winning group. Finally, the quadruple grouped regression examines if the correlation is stronger in the small gain/loss groups than in the large gain/loss groups. The result in the winning groups support this assumption, but the result in the losing groups rejects it. Overall, the results of the series of tests suggest that over the immediate time horizon, the influence of RCGO is not strong. Other factors, for example the disposition effect, could possibly interfere with this influence.

Comparing to previous chapter of replicating the test of Grinblatt and Han, the results are consistent for the control variables. The short term past return is negatively correlated to current stock return, but all other control variables are not significant. The explanatory power of these two tests is at 4% to 5% level. However, the RCGO does not have a negative correlation with stock return as the CGO does. This finding suggests that the CGO and the RCGO represent different factors.

The major limitation of the test model is that it is not compatible with the well established pricing theory under the risk-return framework. This test model is an empirical model applied in the field of behavioral finance. This model suggests that stock returns are influenced by past stock returns, trading volume, company size, and behavior factor. Despite the fact that it is validated by empirical test, it does not have a strong theoretical support. For

such reason, the following section will conduct tests using a model based on Fama-French three-factor model.

## **4.5 Residual regression test**

In this section, tests will be performed using a regression model that is compatible with the risk-return framework. The reason for using an alternative test model is that first, the test model used in previous test does not have a strong theoretical support. Second, the previous test model also have co-linearity problem, because the independent variables are correlated. Finally, using alternative model will improve the robustness of the test result.

This test still focuses on examining the influence of unrealized gain or loss on stock return. In terms of the time horizon, this test will not only examine the influence over immediate time horizon, but also other time horizons including 3-month, 6-month, 9-month, and 1-year. The test will adopt a two staged regressions. The first regression aims to obtain the residuals from a Fama-French model, then the second regression will be performed on the residuals and Relative Capital Gain Overhang. A dummy variable will also be introduced in order to examine separately the gain situation and the loss situation.

The test result suggests that the pattern observed on the sorting test is confirmed. Over intermediate to long time horizons, the stocks with extreme RCGO will outperform the stocks with neutral RCGO.

### **4.5.1 Test model**

In chapter 3, this dissertation proposed a theoretical model about how unrealized gain or loss may influence investor preference under the framework of prospect theory and regret theory. One feature of this theoretical model is that the factor of risk level is controlled when constructing the Relative Capital Gain Overhang. More specifically, this model alters the level of unrealized gain or loss while keeping the risk level constant. This configuration of the model suggests that the investor preference pattern predicted by this model is a pure behavior factor; risk factor does not contribute to such pattern.

Such a feature of the theoretical model on the unrealized gain or loss provides a support for applying risk-return model in empirical test. Since the behavior factor is isolated from risk

factors, it is able to perform test on the returns that are not explained by classic risk factors. The classic financial theory on asset price is established on the risk-return framework. Stock return originates from either the time value of the investment or from exposure to systematic risk. The most commonly used risk-return model in empirical test is the Fama-French three factor model:

$$r_i = r_f + \beta_1(r_m - r_f) + \beta_2SMB + \beta_3HML$$

Stock return can be explained by the time value, and the risk factors linked to the market, the firm size, and the book-to-market ratio of the stock. In this test, all these factors will be eliminated from stock by taking the residuals of a time serial regression on the following model:

$$r_i - r_f = \beta_1(r_m - r_f) + \beta_2SMB + \beta_3HML + \varepsilon$$

In this regression model, there is no regression intercept term. All the errors from the predicted value by the three factors are captured by the residual term of  $\varepsilon$ . Hence, using this model, the systematic risks are ruled out. The residuals are considered containing idiosyncratic component of the stock, and this term is the part of stock return that cannot be explained by risk factors.

In this regression, the residuals of the regression are kept for next test step. The reason for using the residual is to match the variable of RCGO in the next step. The RCGO measures the unrealized gain or loss level of each individual stock at the end of each week. Therefore RCGO is a panel data that contains both time serial and cross sectional information. Using the intercept would not be suitable as it measures the overall level of return that is not explained by the model over the total sample period. It contains only cross sectional information, and is not compatible with the data of RCGO.

In the second step, all the residuals from the Fama-French three-factor model will be used to perform test over Relative Capital Gain Overhang. The regression test will be performed using a very simple model:

$$\varepsilon = a_0 + a_1rcgo$$

This test consists of a two-step regression instead of using one regression model that contains both risk factors and RCGO. Such a two-step test has better consistency with

financial theories. The Fama-French model is strictly oriented to risks; all terms in this model are risk premiums. Moreover, the risk factors are formed by the difference in returns of some specific portfolios. Therefore it is not appropriate to mix with the variable of RCGO, which does not carry the theoretical meaning of risk premium or formed by portfolio return difference. In order to keep theoretical consistency, this test will be performed in two steps. The first step gathers the residuals from the model under the risk-return framework. The second step examines the correlation between the residuals and the behavior factor under the behavioral finance framework.

#### 4.5.2 Summary statistics

The test uses weekly data of NYSE and AMEX stocks from January 1982 to December 2012. The observations with stock price smaller than 5 dollars are eliminated. All valid observations require at least 18 months of continuous records. The data of Fama-French factors are obtained from the on-line database of the Data Library of Kenneth French.

Variables are defined in the same way as in previous tests. Stock returns over different horizons are calculated using weekly cumulative return. The relative Capital Gain Overhang, defined by  $RCGO = CGO/\sigma$ . The  $\sigma$  is the forecasted standard deviation of stock return, estimated using the EWMA method.

Table 4.5-1: Summary statistic of stock returns and RCGO

| Variable           | Mean   | Std Dev | 25th Pctl | Median | 75th Pctl |
|--------------------|--------|---------|-----------|--------|-----------|
| RCGO               | 0.1806 | 13.388  | -2.4847   | 0.6284 | 3.7388    |
| Immediate 1-week   | 0.0020 | 0.0568  | -0.0218   | 0.0000 | 0.0234    |
| Subsequent 1-week  | 0.0020 | 0.0569  | -0.0218   | 0.0000 | 0.0234    |
| Subsequent 13-week | 0.0232 | 0.2023  | -0.0719   | 0.0140 | 0.1052    |
| Subsequent 26-week | 0.0433 | 0.2906  | -0.0993   | 0.0241 | 0.1568    |
| Subsequent 39-week | 0.0627 | 0.3637  | -0.1203   | 0.0324 | 0.2000    |
| Subsequent 52-week | 0.0812 | 0.4327  | -0.1403   | 0.0397 | 0.2405    |

This table reports the summary statistic of the variable of RCGO, and the stock returns over various time horizons that will be examined by the residual regression test. The immediate 1-week time horizon is the week right after the RCGO is measured. The subsequent time horizons introduce an interval of one week after the RCGO is measured, then begins calculating stock returns.



### 4.5.3 Single grouped test

In this section, the single grouped residual regression will be performed. The regression examines the overall correlation between residuals from Fama-French model and RCGO over various time horizons. The winning group and the losing group will not be distinguished in this test.

First, a time serial regression of each stock over the total sample period will be performed on the Fama-French three-factor model:

$$r_i - r_f = \beta_1(r_m - r_f) + \beta_2SMB + \beta_3HML + \varepsilon$$

Then, a Fama-Macbeth regression will be performed on residuals and RCGO by the single grouped model:

$$\varepsilon = a_0 + a_1rcgo$$

Using this model, the systematic risks are ruled out in the first step. The residual are considered containing idiosyncratic component of the stock, and the influence of RCGO will be examined in the second step. The following table reports the results.

Chapter 4:  
Tests on the unrealized gain/loss model

Table 4.5-2: Coefficients of the single grouped residual regression

| $\varepsilon_{i,t} = a_0 + a_1rcgo_{i,t-1}$ |          |                |                |                    |
|---|----------|----------------|----------------|--------------------|
| Horizon                                     | variable | a <sub>0</sub> | a <sub>1</sub> | Adj R <sup>2</sup> |
| Immediate                                   | mean     | 0.00004        | -0.00018       | 0.0040             |
|   | T-value  | 0.43           | -10.20         |                    |
|   | P-value  | 0.6671         | <.0001         |                    |
| Subsequent<br>1 week                        | mean     | -0.00011       | -0.00011       | 0.0037             |
|   | T-value  | -0.14          | -6.69          |                    |
|   | P-value  | 0.8889         | <.0001         |                    |
| Subsequent<br>13 week                       | mean     | -0.00491       | -0.00059       | 0.0075             |
|   | T-value  | -11.96         | -9.60          |                    |
|   | P-value  | <.0001         | <.0001         |                    |
| Subsequent<br>26 week                       | mean     | -0.0130        | -0.00077       | 0.0093             |
|   | T-value  | -21.47         | -8.81          |                    |
|   | P-value  | <.0001         | <.0001         |                    |
| Subsequent<br>39 week                       | mean     | -0.0216        | -0.00077       | 0.0101             |
|   | T-value  | -27.55         | -7.12          |                    |
|   | P-value  | <.0001         | <.0001         |                    |
| Subsequent<br>52 week                       | mean     | -0.0301        | -0.00061       | 0.0104             |
|   | T-value  | -31.94         | -4.83          |                    |
|   | P-value  | <.0001         | <.0001         |                    |

The result of the single grouped residual regression shows that all coefficients are significant except for the intercepts in the regressions over the immediate and the subsequent 1-week time horizons. Negative correlations between RCGO de the return residuals are found over all the time horizons. The intercept terms of all the regressions are negative. The reason for this problem is the same as in the sorting test. The Fama-French factors obtained from French's website are based form NYSE, AMEX and NASDAQ. The latter is not included in the data of individual stocks. The adjusted R-squared is only at 0.1% level. It suggests that the explicating power is not satisfying.

The single grouped test examines the overall influence of RCGO on stock return, but the theoretical model of RCGO has better interest on the influence of gains and losses over stock returns. In the following section, double grouped test will be performed to examine such issue.

#### 4.5.4 Double grouped test

In the previous section, the sorting tests show that stocks with extremely high or low RCGO out-perform the stocks with RCGO close to zero over three months to one year time horizon, and such a difference in stock returns cannot be explained by exposure to risk factors. In this part, a regression test will be performed to examine the cross-sectional correlation between stock returns and RCGO.

In the beginning of this chapter, the sorting test is performed to reject a null hypothesis: Hypothesis 1: There is no significant relation between stock return and Relative Capital Gain Overhang over different time horizons. Such a hypothesis in a weak form is developed because the theoretical model does not predict how RCGO will influence stock return. However, the result of the sorting test shows the pattern of stock return with respect to RCGO. This empirical observation may allow deducing the way the behavior factor transmits to stock performance.

The theoretical model suggests that in an unrealized gain situation, investors show weaker propensity to sell stock when the unrealized gain grows. The empirical tests show higher stock return in the stocks with a large positive RCGO. Therefore, one could expect positive correlation between stock returns and RCGO in winning situation. On the other hand, in an unrealized loss situation, the investor's propensity to sell is lower when the loss becomes greater. Empirical tests confirm higher return in the stocks with very low RCGO. One could expect a negative correlation between stock return and RCGO in losing situation. Based on such a theory, in the following regression tests, the basic hypothesis 1 can be further developed into two subsidiary hypotheses:

**Hypothesis 2:** In losing situation ( $RCGO < 0$ ), there is significant negative correlation between stock return and RCGO.

**Hypothesis 3:** In winning situation ( $RCGO > 0$ ), there is significant positive correlation between stock return and RCGO.

The double grouped residual regression will be performed in two steps. The first step is a time serial regression of each stock over the total sample period on the Fama-French three-factor model:

$$r_i - r_f = \beta_1(r_m - r_f) + \beta_2SMB + \beta_3HML + \varepsilon$$

Using this model, the systematic risks are ruled out. The residuals of each observations are saved in a data set for the next step.

The second step is a Fama-Macbeth regression of the residuals on the one-period lagged relative Capital Gain Overhang. At each week, a cross-sectional regression is performed on the residual from the Fama-French model and RCGO calculated at this week. The coefficients of the regressions form a time-serial data set. T-test over the time-serial data allows determining the significant level of the coefficients. The hypothesis of this test predicts a negative correlation in loss situation and a positive correlation in gain situation. To separate the winning stocks and losing stocks, a dummy variable is introduced.

$$\varepsilon_{i,t} = a_0 + a_1rcgo_{i,t-1} + a_2N \times rcgo_{i,t-1}$$

Where:

$N = 1$  if  $RCGO < 0$ ,  $N = 0$  if else.

In the two grouped test, the dummy variable  $N$  is for negative RCGO. The positive RCGO group is considered as the base group.  $N$  equals to 1 if RCGO is inferior to zero; otherwise,  $N$  equals to zero.

The following table reports the coefficients, their t-value p-value and the R-squared of the double group regression over all five horizons.

Chapter 4:  
Tests on the unrealized gain/loss model

Table 4.5-3: Coefficients of the double grouped residual regression

$$\varepsilon_{i,t} = a_0 + a_1rcgo_{i,t-1} + a_2N \times rcgo_{i,t-1}$$

| Horizon                 | variable | a <sub>0</sub> | a <sub>1</sub> | a <sub>2</sub> | Loss group<br>(a <sub>1</sub> +a <sub>2</sub> ) | Gain group<br>(a <sub>1</sub> ) | Adj R <sup>2</sup> |
|-------------------------|----------|----------------|----------------|----------------|---|---------------------------------|--------------------|
| Immediate<br>1 week     | mean     | -0.00032       | -0.00011       | -0.00017       | -0.00028***                                     | -0.0001***                      | 0.0058             |
|                         | T-value  | -2.83          | -4.89          | -5.41          |   |                                 |                    |
|                         | P-value  | 0.0048         | <.0001         | <.0001         |   |                                 |                    |
| Subsequent<br>1 week    | mean     | -0.0004        | -0.00004       | -0.00015       | -0.00019**                                      | -0.00004***                     | 0.0054             |
|                         | T-value  | -3.26          | -1.92          | 3.13           |   |                                 |                    |
|                         | P-value  | 0.0011         | 0.0550         | 0.0018         |   |                                 |                    |
| Subsequent<br>3 months  | mean     | -0.00830       | 0.00009        | -0.00131       | -0.00094  | 0.00009***                      | 0.0110             |
|                         | T-value  | -18.30         | 1.12           | -13.58         |   |                                 |                    |
|                         | P-value  | <.0001         | 0.2608         | <.0001         |   |                                 |                    |
| Subsequent<br>6 months  | mean     | -0.0192        | 0.000643       | -0.00242       | -0.001777***                                    | 0.000643***                     | 0.0140             |
|                         | T-value  | -28.05         | 5.89           | -17.24         |   |                                 |                    |
|                         | P-value  | <.0001         | <.0001         | <.0001         |   |                                 |                    |
| Subsequent<br>9 months  | mean     | -0.0305        | 0.00129        | -0.00349       | -0.0022***                                      | 0.00129***                      | 0.0156             |
|                         | T-value  | -33.82         | 8.88           | -18.87         |   |                                 |                    |
|                         | P-value  | <.0001         | <.0001         | <.0001         |   |                                 |                    |
| Subsequent<br>12 months | mean     | -0.0409        | 0.00182        | -0.0042        | -0.00238***                                     | 0.00182***                      | 0.016              |
|                         | T-value  | -37.19         | 10.20          | -18.89         |   |                                 |                    |
|                         | P-value  | <.0001         | <.0001         | <.0001         |   |                                 |                    |

\*\*: significant at 95% confidence level; \*\*\*: significant at 99% confidence level

This table reports the result of the double grouped residual regression.  $a_1$  and  $a_2$  are respectively the coefficients of the regression of the base group and the difference in slope in the alternative group comparing to the base group. The majority of the coefficients are statistically significant. The columns of the gain group and the loss group report the final coefficients of RCGO in the winning situation and the losing situation.

Over short terms, namely the immediate 1-week to 3 months, negative correlations are found in both winning situation and losing situation. However, over a period of 6 months to 1 year, there is a significant negative correlation between stock return and RCGO in the unrealized loss situation ( $RCGO < 0$ ), and a significant positive correlation between stock return and RCGO in the unrealized gain situation ( $RCGO \geq 0$ ). Such a regression result is

consistent to the observations made from the sorting test. Over longer terms, the stocks with extreme RCGO will outperform the stocks with neutral RCGO. And the hypotheses 2 and 3 are confirmed over long time horizons.

Over long term, the positive correlation between RCGO and stock return is much stronger in the winning situation than in the losing situation. The coefficients of RCGO is twice as large in the positive group as in the negative group, hence could result to the positive correlation observed in the single-group regression. In terms of the adjusted R-squared, it is at 0.3% level, which is very small. However, its values have increased comparing to the single-group regression.

#### **4.5.5 Test summary**

In this section, tests are performed by using the two-step regression. Using such a test model is in response to the tests performed using Grinblatt and Han's model. The two-step regression allows establishing a link between the classic risk-return model with the theoretical model developed in this dissertation. Essentially, the two-step regression eliminates all risk factors in the first step, and then it examines the influence of RCGO on the stock return residuals that have not been explained by Fama-French model.

The objective of the residual regression is still to examine the influence of Relative Capital Gain Overhang on stock returns. Particularly, this test aims to verify the pattern observed in the sorting test that stocks with extreme RCGO could outperform the stock with neutral RCGO over intermediate to long time horizon.

Both single grouped and double grouped tests are performed. The single grouped test finds negative correlation between RCGO and stock return in short term, and positive correlation in long term. This result is consistent with the assumption that the disposition effect impacts stock return in short term, and market corrects the mispricing over long term. The double grouped test examines the influence of RCGO separately in winning and in losing situations. The test result shows negative correlation between RCGO and stock return in losing situation, and positive correlation in winning situation. Test results confirm the hypothesis that stock with extreme RCGO could outperform stocks with neutral RCGO.

The extremely low explaining power of the regression model is the main critic of this test. The test model can only capture up to 0.3% of the variance of the sample. Considering

that the regression is performed on the residuals of the Fama-French model, the residuals are already the errors that cannot be explained. They contain all potential factors beyond the systematic risks that could influence stock return. Therefore such a low R-squared is not surprising. Despite the low R-squared value, the coefficients of the regressions are significant at 99% level. This may suggest that the correlation is not linear between RCGO and stock returns. By separating the gain and the loss situations, the regression managed to double or triple the R-squared. Therefore, adopting a non-linear regression model could potentially increase the explaining power of the model.

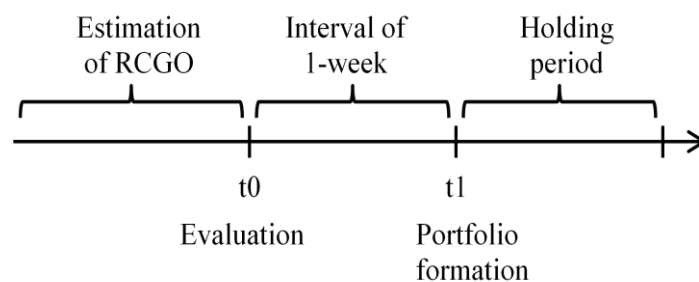
## 4.6 RCGO Extremity minus Middle portfolio return

In previous sections of this chapter, the tests confirm that stocks with extreme RCGO could outperform stocks with neutral RCGO. Based on such result, this part aims to test the profitability of a strategy that exploits the result of the above tests. Since the stocks with extreme RCGO could out-perform the stocks with middle RCGO over 6-month to 1 year time horizon, the strategy consists of longing the stocks with very large unrealized gain or loss while shorting the stocks with very small unrealized gain or loss.

### 4.6.1 The EMM strategy

At each time point  $t_0$ , the stocks are evaluated. All stocks are divided into 10 deciles in ascending order of Relative Capital Gain Overhang. The zero-cost portfolio will be formed by longing the stocks in deciles 1 and 10, while shorting the stocks in deciles 5 and 6. From time point  $t_1$ , the extremity minus middle (EMM) portfolio will be hold for 3, 6, 9, or 12 months, then the positions are cleared. The following figure illustrates the timeline of the EMM strategy.

Figure 4.6-1: Time line of the EMM strategy



According to the previous theoretical model, for the stocks in the extremity portfolio, their holders have lower propensity to sell the stock. Between the holders and buyers of the stock, there is an important difference in the expectation. As a result, the stock return will be influenced by such a strong behavior factor. In the middle portfolio, the effect of the behavior factor is not significant, and the stock return will not be influenced. The EMM portfolio is formed by longing the stocks that are strongly exposed to the behavior factor and shorting the stocks that are not or weakly exposed to the behavior factor. The profit of the EMM portfolio originates from the difference in returns in response to the influence of loss aversion. Hence the EMM could represent such behavior factor.

Risk factors needs to be considered in the portfolio because it is possible that the profit of the EMM portfolio comes from bearing systematic risks. The extremity portfolio is composed by stocks having large unrealized gain or large unrealized loss. The RCGO is calculated using past returns. It suggests that the extremity portfolio could contain stocks that are highly exposed to risks. On the contrary, the middle portfolio could contain stocks with low exposure to risks. If it is the case, the EMM portfolio is a speculative portfolio that is financed by shorting low risky assets and investing in high risk assets.

#### **4.6.2 The EMM profit**

In this section, the EMM strategy will be applied on the data sample to see if such strategy could systematically yield a significant profit. To rule out the systematic risk factors, a time serial regression will be performed on the raw EMM profit using the Fama-French Three-Factor model:

$$EMM_t = \alpha + \beta_1 RmRf_t + \beta_2 SMB_t + \beta_3 HML_t$$

In the test, the EMM portfolio will be formed by both equal weighting and value weighting. The regression result is reported by the following table.



Chapter 4:  
Tests on the unrealized gain/loss model

Figure 4.6-2: The EMM portfolio profit adjusted by Fama-French factors

Panel A: Equal weighted EMM portfolio

|           |         | Equal weighted EMM portfolio |          |          |         |                     |
|-----------|---------|------------------------------|----------|----------|---------|---------------------|
|           |         | Intercept                    | Rm-Rf    | SMB      | HML     | adj. R <sup>2</sup> |
| 3 Months  | coef    | 0.02864                      | -0.22542 | -0.34836 | 0.48474 | 0.3417              |
|           | T-value | 16.69                        | -10.33   | -10.11   | 16.30   |                     |
|           | P-value | <.0001                       | <.0001   | <.0001   | <.0001  |                     |
| 6 Months  | coef    | 0.04994                      | -0.17173 | -0.27470 | 0.46708 | 0.2548              |
|           | T-value | 19.09                        | -7.85    | -7.50    | 16.48   |                     |
|           | P-value | <.0001                       | <.0001   | <.0001   | <.0001  |                     |
| 9 Months  | coef    | 0.06892                      | -0.07744 | -0.11925 | 0.44749 | 0.1728              |
|           | T-value | 20.06                        | -3.46    | -3.22    | 15.84   |                     |
|           | P-value | <.0001                       | 0.0006   | 0.0013   | <.0001  |                     |
| 12 Months | coef    | 0.08062                      | 0.0427   | -0.11456 | 0.44454 | 0.1449              |
|           | T-value | 18.75                        | 1.73     | -2.97    | 15.56   |                     |
|           | P-value | <.0001                       | 0.0842   | 0.003    | <.0001  |                     |

Panel B: Value weighted EMM portfolio

|           |         | Value weighted EMM portfolio |          |          |         |                     |
|-----------|---------|------------------------------|----------|----------|---------|---------------------|
|           |         | Intercept                    | Rm-Rf    | SMB      | HML     | adj. R <sup>2</sup> |
| 3 Months  | coef    | 0.01239                      | -0.16257 | -0.1100  | 0.38454 | 0.1414              |
|           | T-value | 6.10                         | -6.30    | -2.70    | 10.93   |                     |
|           | P-value | <.0001                       | <.0001   | 0.0070   | <.0001  |                     |
| 6 Months  | coef    | 0.02148                      | -0.11237 | -0.00803 | 0.45262 | 0.1518              |
|           | T-value | 7.45                         | -4.66    | -0.20    | 14.46   |                     |
|           | P-value | <.0001                       | <.0001   | 0.8423   | <.0001  |                     |
| 9 Months  | coef    | 0.02768                      | -0.06211 | -0.06656 | 0.53593 | 0.1845              |
|           | T-value | 7.31                         | -2.52    | -0.63    | 17.21   |                     |
|           | P-value | <.0001                       | 0.0119   | 0.1033   | <.0001  |                     |
| 12 Months | coef    | 0.02687                      | 0.00392  | -0.04878 | 0.49181 | 0.1536              |
|           | T-value | 5.79                         | 0.16     | -1.17    | 15.96   |                     |
|           | P-value | <.0001                       | 0.8762   | 0.2410   | <.0001  |                     |

The results show that both equal weighting EMM portfolio and value weighting EMM portfolio yield significant positive profit over all 4 time horizons. The factor loadings are also significant in the majority of the cases. Such results suggest that risk factors could explain part of the profit of the EMM strategy, but the profit is still significant after ruling out the systematic risks.

Comparing the results from equal weighting and the value weighting EMM portfolio, the regression intercepts (the alpha) are much smaller in the value weighting portfolio than in the equal weighting portfolio. This suggests that by reducing the weight of the small capitalization stocks in the composition of portfolio, the profit decreases. The small capitalization stocks have greater contribution to the profit of EMM strategy. This result is consistent with the observation on the coefficient of the size factor SMB. In the panel A, the correlation between EMM profit and the size factor is significant. The coefficient for SMB is up to -0.11 over the one year horizon. In the panel B, the portfolio is formed using value weighting, the size factor is already controlled by the formation of the portfolio. Hence the coefficient of the variable SMB in panel B is not significant any more. One possible explanation to this observation is that small sized stocks are considered less liquid than big sized stocks, their unrealized gain or loss is updated less frequently. Therefore unrealized gain or loss has stronger influence on small sized stocks.

In both regressions, the coefficients of the factor HML are significant over all four time horizons. The coefficients are all positive and goes up to 0.75 in the value weighted portfolio over one year. This indicates that the profit of EMM portfolio have strong positive covariance with the factor of book-to-market ratio. This ratio represents investors' expectation of future performance of a firm. High book-to-market ratio suggests that the firm is conceded as underestimated by the market, and is the firm is expected to create more value in the future. Investors are more interested in firm's future than its present. Hence the investors have even less propensity to sell such stocks.

To sum up, the profit of EMM strategy is correlated to risk factors of market, size, and book-to-market ratio. The behavior factor is not completely isolated from risks. However, EMM strategy could still yield positive profit after ruling out systematical risk factors. Such result could confirm the influence of unrealized gain or loss on stock performance.

## **4.7 Summary**

The object of this chapter is to examine the influence of the Relative Capital Gain Overhang on stock return by empirical tests. The theoretical model on the RCGO predicts that the investors have a weaker propensity to sell when facing unrealized gains or unrealized losses. This prediction only allows developing a hypothesis in weak form: Stock return should

be different in the extreme RCGO stocks and the neutral RCGO stocks. To verify this hypothesis, the sorting test is performed. By dividing all stocks into deciles by RCGO, the sorting test shows that stocks with large unrealized gain or loss tend to have higher return than stocks with small unrealized gain or loss over intermediate and long time horizons.

The result of the sorting test shows in which direction the unrealized gain or loss influences stock returns. Based on this result, regression tests are conducted to verify the cross sectional correlation between RCGO and stock return over various time horizons. Two different regression models are used. The first one is derived from Grinblatt and Han's model. This model is a well accepted empirical model. The test using this model focuses on the immediate time horizon. The result of this test is not conclusive. The second test model aims to establish a link with the classic risk-return framework. It consists of first capturing the residuals of the Fama-French model; then performs a second regression to test the correlation between the residuals and the RCGO. Such a two-step model allows isolating the risk factors in the first step, and then examines the behavior factor in the second step. This test confirms the long term influence of RCGO on stock return. The stocks with extreme RCGO could outperform the stock with neutral RCGO.

The empirical test results that are consistent with the theoretical prediction motivate the dissertation to design an investment strategy that could exploit the above finding. Therefore, the test examines the profitability of an Extremity minus Middle strategy. This strategy consists of longing the stocks in the top and bottom deciles divided according to RCGO; while short selling the stock in the middle deciles. The test shows that holding the EMM portfolio over 6 months to 1 year could yield a profit of 3% after adjusting for risks. According to the theoretical model, the extremity portfolio and the middle portfolio represents different investor behavior facing unrealized gains or losses. The positive profit of the EMM strategy might imply that financial market does react to behavior factors in stock pricing.

However, there is still one unsolved question. The influence of unrealized gain or loss takes effect over long term instead of over short term. The theoretical model is developed on the basis of investor behavior over very short time horizons. Investor should instantly react to change in winning or losing positions, and modifies the propensity to sell the stock accordingly. How could the influence of the unrealized gain or loss transmit to long term? This puzzle motivates a review of the theoretical model, and develops a dynamic version of the theoretical model.

### Annex 4.1 Forecasting SD of next period:

According to the EWMA method:

$$\sigma_t = \sqrt{(1 - \lambda) \sum_{t=1}^T \lambda^{T-t} (r_t - \bar{r})^2}$$

$$\sigma_{t+1}^2 = (1 + \lambda) \{ (r_{t+1} - \bar{r})^2 + \lambda^1 (r_t - \bar{r})^2 + \lambda^2 (r_{t-1} - \bar{r})^2 + \dots \}$$

$$\sigma_{t+1}^2 = (1 + \lambda) (r_{t+1} - \bar{r})^2 + \lambda (1 + \lambda) \{ (r_t - \bar{r})^2 + \lambda^1 (r_{t-1} - \bar{r})^2 + \dots \}$$

$$\sigma_{t+1}^2 = (1 + \lambda) (r_{t+1} - \bar{r})^2 + \lambda \sigma_t^2$$

This transformation implies that the mean returns for T period and T+1 period are equal.



# Chapter 5

## The Unrealized gain/loss model in dynamic

In previous chapter, tests are performed to examine the correlation between the behavior factor RCGO and stock performance over various time horizons, namely from one week to one year. The tests find that stocks are likely to have higher returns if they are in a large loss or in a large gain situation. This pattern is consistent with the theoretical prediction of the RCGO model, but the major problem with the test results is that the periodicity does not match. The theoretical model assumes that investors make decision of either sell or hold their stock at each investment period. Since stock market is a very active market, it is reasonable to assume that investors would revise their investment decision frequently. Therefore, the prediction given by the theoretical model should apply to short time horizon. However, it is over medium to long time horizon that the test results show the predicted pattern. To tackle this problem, this chapter reviews the theoretical model proposed in the previous chapters, and develops is to adapt to the aggregated preference of investors in a dynamic processes.

### 5.1 The model in dynamic

This section aims to develop the previous theoretical model of RCGO into a dynamic process. Such a dynamic process originates from the inter-reactive relation between RCGO and stock price over time. The basic reasoning is the following: For a given RCGO at a given time, it will influence stork price in the following time period. But when trades take place at the newly formed price, the RCGO will be updated according to the new price that was formed under the influence of past RCGO. When such a process repeats itself over time, the

influence of the initial RCGO may last certain period of time. Therefore, in a dynamic process, the RCGO could influence the stock price in the next period, but could also influence RCGO itself in the next period because the update RCGO is based on the newly formed stock price.

### **5.1.1 Interactive relation between RCGO and price**

The static model focuses only on the influence of the unrealized gain or loss on stock price, while the dynamic model includes the feedback of stock price on the formation of new RCGO. The interactive relation between the Relative Capital Gain Overhang and stock price is the key concept that distinguishes the dynamic model from the static model. In this section, both directions of the interactive relation will be examined.

#### **5.1.1.1 Impact of RCGO on stock price**

The impact of unrealized gain or loss on stock price has been so far the core issue in this dissertation. Understanding such an impact involves both the theoretical analyses and the empirical tests on the RCGO model.

The theoretical model studies the investor preference for selling a stock in a winning or in a losing situation under the framework of the prospect theory and regret theory. This model suggests that the investor has higher propensity to sell a stock facing a small gain/loss than facing a large gain/loss. However, this prediction is limited, since it gives prediction only on individual preferences, and is not sufficient to predict how stock price or return will respond to changes in preferences. A common assumption on how individual preferences transmit to stock price is by the market mechanism of supply and demand.

The empirical studies complete the link between investor preference and stock returns. The replication test on the Capital Gain Overhang over very short time horizon shows evidence of the disposition effect and its influence on stock return. In a loss situation, investor tends to hold the stock, creating shortage of stock supply, hence raises the price. Through the same mechanism, in gain situation, investor tends to sell the stock, and stock price will decrease. Such empirical results confirm the assumption that the influence of investor transmits to stock price by supply and demand. The tests on the RCGO find that the stocks with large unrealized gain/loss have higher returns. This result also supports the above assumption.

Combining the theoretical model and the empirical result, it is able to establish the whole process of how RCGO impacts stock price: Investors show attitude of loss aversion and regret aversion. Therefore, the unrealized gain or loss is a factor that influences investor preference. The higher the gain or the loss is, the stronger the intention the investors have in order to conserve their stocks. Such an influence at the individual level can be aggregated at the market level. If on average, the stock' holders are in a large gain or in a large loss situation, then the holders tend to keep the stock. As a result, there is a decrease in supply of the stock, and the stock price will go up in the next period.

#### 5.1.1.2 Feedback of stock price on RCGO

In the theoretical context, the Relative Capital Gain Overhang is defined as the difference between the stock's current price and the reference price at which the investor purchased the stock. The theoretical model studies investor preference at an individual level, hence the reference price is considered as an exogenous factor because the purchase had happened in the past, and the reference price will not evolve over time. In this sense, the reference price is static.

When applying the model to the empirical test, the individual investor preferences are aggregated in order to represent the market. In such a context, the reference price will not be the purchase price of an individual investor, but redefined as the average acquisition price of all holders at a given time. Recall that the average acquisition price is estimated by the following formula:

$$aqp_t = v_t p_t + (1 - v_t) aqp_{t-1}$$

This estimation formula suggests that the stock price at current time contributes to form the reference price. The newly formed reference price is the past reference price that has been partially updated by current stock price. How much past reference price will be replaced depends on the turnover ratio. If a stock is highly liquid, and large quantities of shares are traded on a regular base, the average acquisition price will be very close to the stock price at that time period. Such a definition implies that at the aggregated level, the reference price is no longer a **static** factor that is exogenous to the model, but becomes a **dynamic** process that will be continuously updated by the latest transaction of stocks.

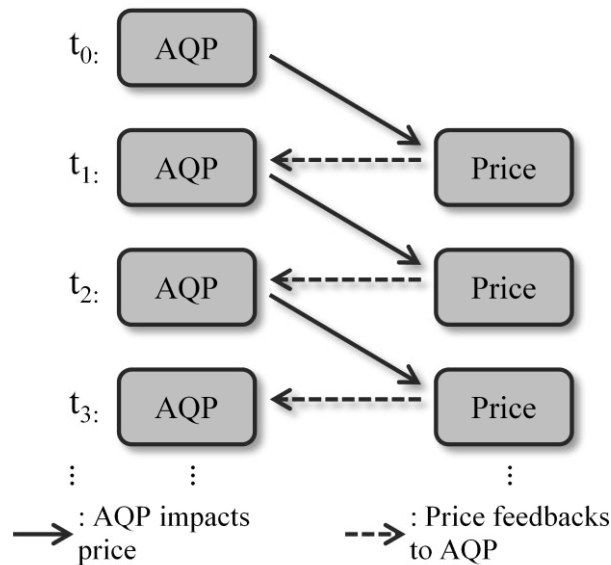


To conclude, the theoretical model studies individual investor preference in a static mode. However, when the individual preferences are aggregated, the definition of the reference price inevitably evolves toward a dynamic process.

### 5.1.1.3 Illustration of the interaction between RCGO and current stock price

The stock price and the average acquisition price interact with each other over time. The price formation at a given time is influenced by the unrealized gain or loss at the last time period, which is determined by the reference price, in other words, the purchase price of the investors. However, at the end of a given time period, the new reference price will be updated by the influenced stock price. In this sense, it is possible that the impact of the initial unrealized gain or loss could persist over time, especially when the turnover ratio is so small that the newly formed reference price inherits the majority of past reference price.

Figure 5.1-1: Interaction between the stock price and the average acquisition price



The above figure illustrates the interaction between the average acquisition price and the stock price. For clarity purposes, the figure shows a simplified case. In fact, the impact on stock price is caused by unrealized gain or loss, the difference between current stock price and current reference price. The formation of the new average acquisition price is determined not only by new stock price, but also by the past AQP.

It should also point out that the interaction between the stock price and the average acquisition price is not the determine factor in stock price formation. Stock price should

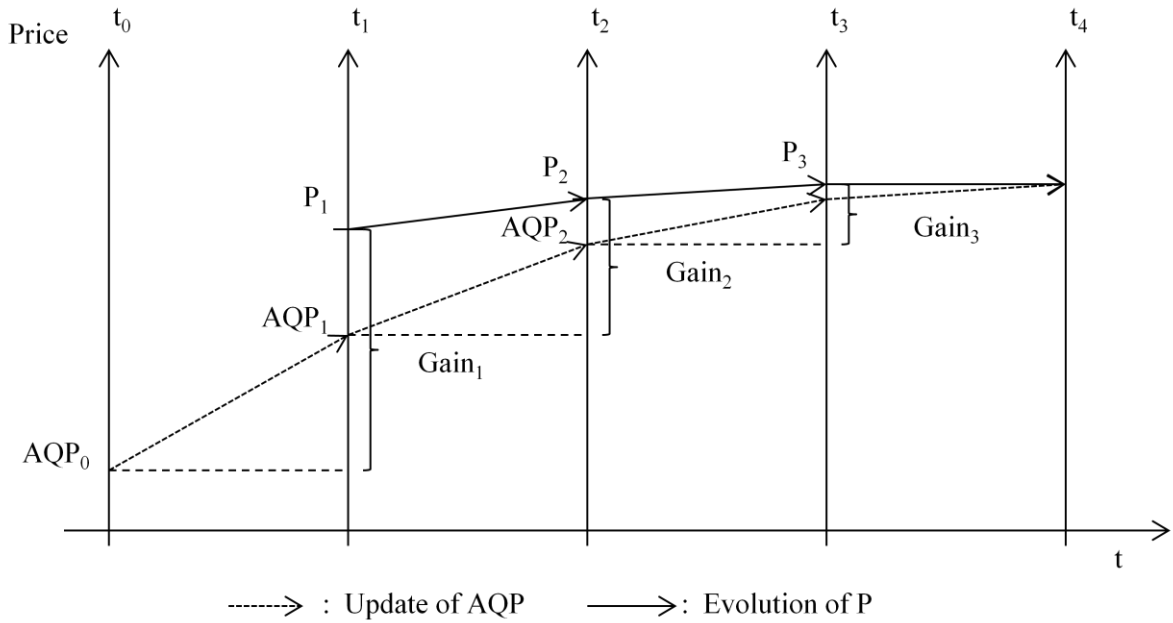
reflect its true value, but behavior factor such as the unrealized gain or loss could influence the supply in the market, and cause stock price to deviate from true value.

### 5.1.2 Evolution of stock price and RCGO in dynamic

Following the idea proposed in the previous section, this part aims to describe the evolution of the impact of the unrealized gain or loss by using reasonable assumptions. The assumptions originate from the static model. The higher the unrealized gain or the unrealized loss is, the more reluctant the investors are to sell the stock. As a result, the stock price in the next period will increase, and the trading volume will decrease.

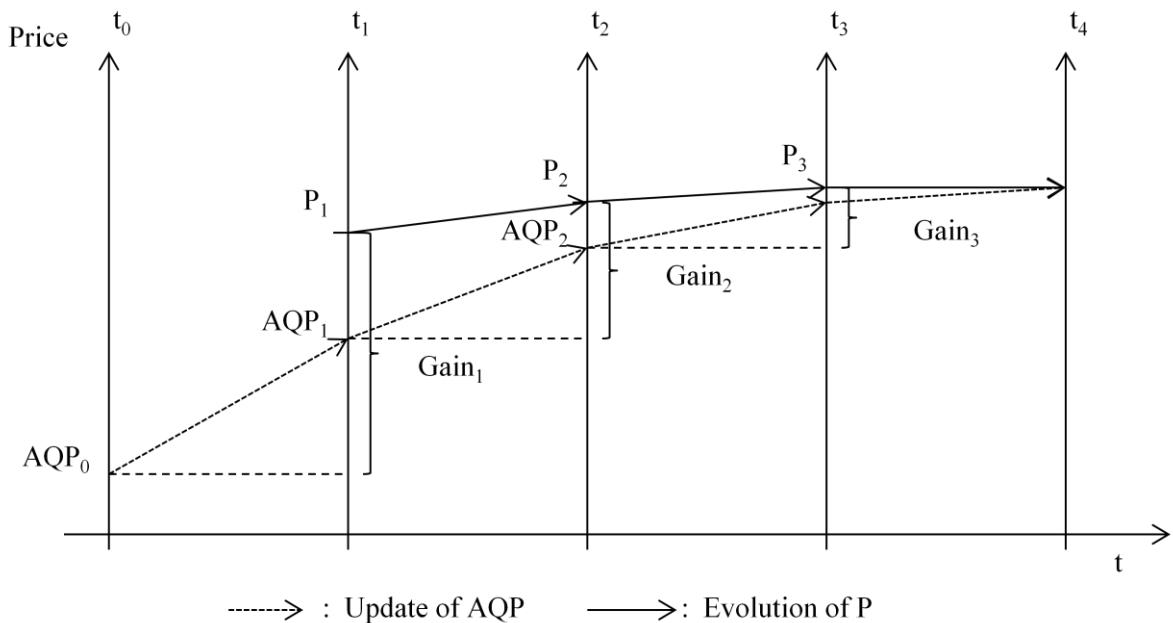
In a small unrealized gain or loss situation, the impact of the unrealized gain/loss on the stock price is small. The investors are willing to sell the stock. Hence one can expect that the trading volume is relatively high. Note that in this situation the trading volume is in fact slightly decreased compared with a situation in which the unrealized gain/loss is strictly zero. But comparing with a large gain/loss situation, in which the investors have very low propensity to sell the stock, the trading volume is relatively higher in a small gain/loss situation. Consequently, a high trading volume implies that a large proportion of past average acquisition price will be updated by current stock price. In the following figure, this effect is represented by a large increase in AQP in the next period (the dashed line), while the increase in price is small (the solid line). As a result, the average acquisition price quickly converges to the current stock price, suggesting that the influence of a small unrealized gain quickly diminishes over time.

Figure 5.1-2: Evolution of stock price and the acquisition price over time in SMALL GAIN situation



Similarly in a small loss situation, the increase in stock price due to unrealized loss is small, but the trading volume is relatively high, therefore the average acquisition price quickly converges to current stock price. The impact of a small loss quickly disappears.

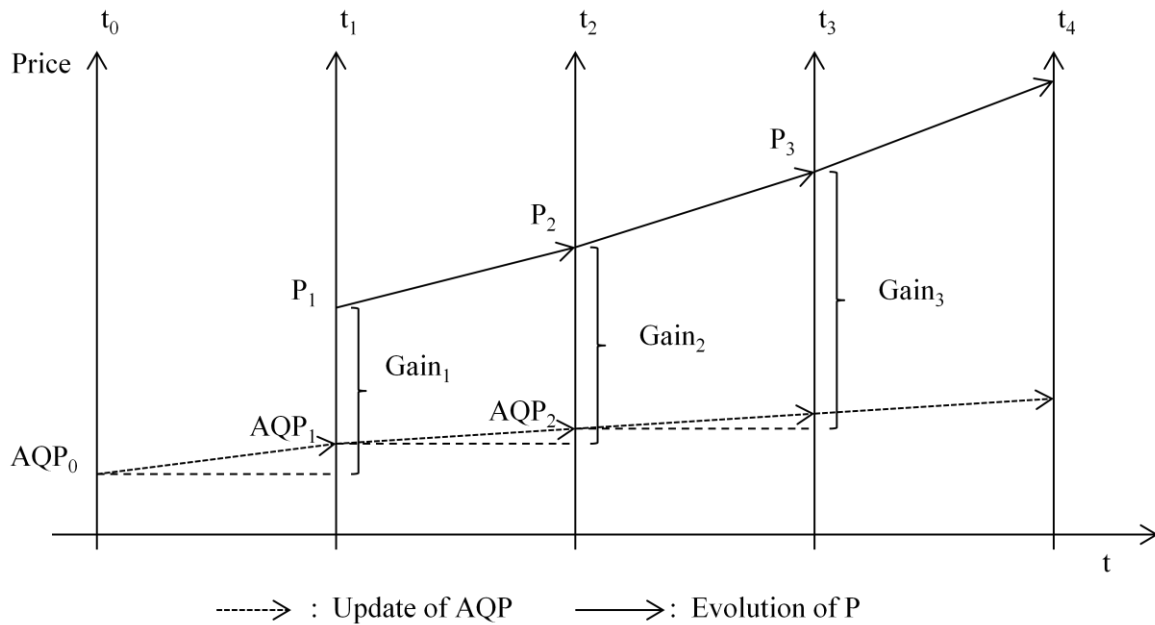
Figure 5.1-3: Evolution of stock price and the acquisition price over time in SMALL LOSS situation



In contrast to the small gain or loss situations, in the large gain or loss situations because investors are strongly reluctant to sell, stock price should greatly increase in the following period. In addition, due to weak trading volume, average acquisition price will only have a small portion that will be updated by current stock price.

In large gain situation, both stock price and average acquisition price move upward, but the increase in stock price is larger than the increase in average acquisition price. As a result, the unrealized gain in the following periods will continue to increase and the stock price also tends to go higher in the following periods. The influence of a large unrealized gain echoes and is amplified by itself over time.

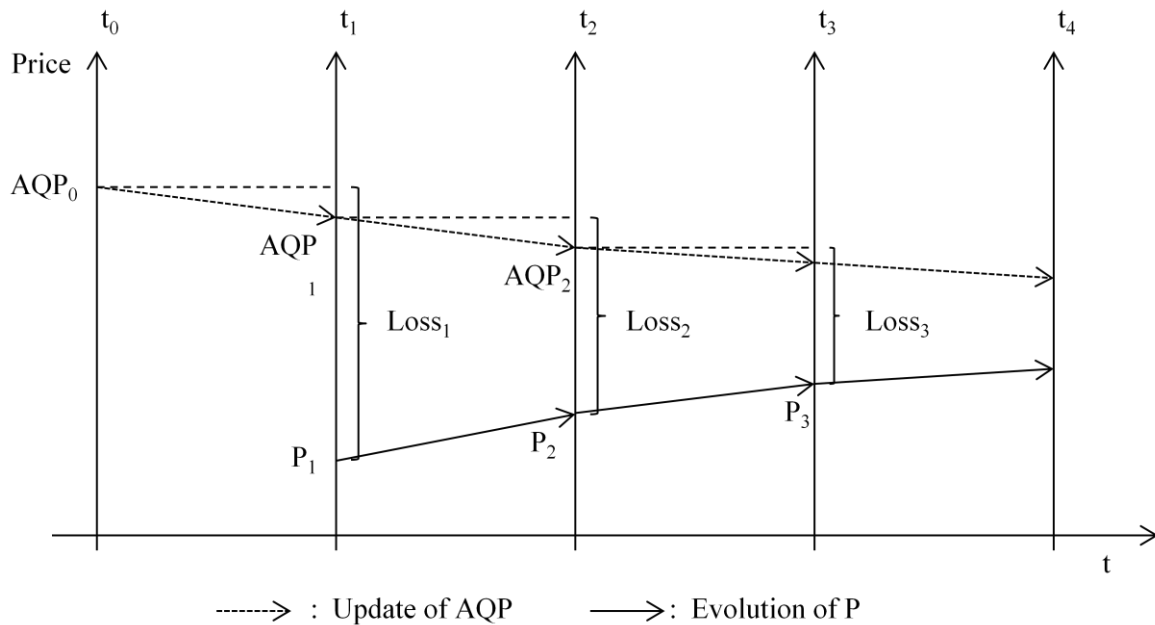
Figure 5.1-4: Evolution of stock price and the acquisition price over time in LARGE GAIN situation



Note that the evolution of stock price and the acquisition price over time in the large gain situation could be consistent with a financial bubble. The stock price deviates from its true value and continuously goes up. This dynamic process could provide one perspective to explain the formation of a financial bubble. If the financial market is in a gain situation, all the investors will be reluctant to sell their stocks. A shortage in stock supply appears, and the demanders in the market will have to offer a higher purchase price if they want to acquire the stocks even though the demander fully acknowledges that the stock price have equal chance to go up or go down. The financial bubble could be another application of the unrealized gain or loss for future research.

In a large loss situation, stock price goes up due to a high reluctance to sell, and the trading volume is small. Despite the fact that stock price goes up in the next period, the average acquisition price is updated slowly by stock price. But the average acquisition price and stock price have a trend to move towards each other, therefore, the unrealized loss will eventually converges to zero with lower speed. In this situation, the influence of large unrealized loss also diminishes over time but with a lower speed.

Figure 5.1-5: Evolution of stock price and the acquisition price over time in LARGE LOSS situation



To sum up, in small unrealized gain/loss situations, the gain or loss is quickly updated by current stock price, and the influence will disappear. In large unrealized gain or loss situations, the impact of gain or loss persists for a long period due to slow update rate of the average acquisition price. Such an impact manifests in a gain situation as a force that continues pushing stock price upward, and in a loss situation as a slow convergence of stock price towards the average acquisition price.

In all four situations, the stock price show upward trend under the influence of unrealized gain or loss. This trend could result to positive auto-correlation in stock returns. Particularly, in large gain or large loss situations, the auto-correlation should be stronger and lasts for longer time. This result could match the empirical result that stock returns are higher in the Extremity RCGO groups than the Middle groups over intermediate to long time horizons.

In addition, the dynamic model suggests that in large unrealized gain situation, the influence amplifies over time and causes stock price to continuously go up without limit. While in the large loss situation, stock price also goes up under the influence of unrealized loss, but stock price will eventually converge to current stock price. This gives one reason that past winner stocks could out-perform past loser stocks and forms the momentum effect.

## **5.2 Implication of the dynamic model**

Development of the dynamic model is a response to the question met in the results of the empirical tests over unrealized gain/loss and stock returns. The original static model that describes investor preference cannot explain that the expected relation between RCGO and stock return is observed over long time horizon rather than over immediate or short time horizon. A revision on the static model suggests that by adapting the theoretical model to the empirical test, the definition of the reference price also change from a static form to a dynamic process. The reference price and the stock price interact with each other over time, allowing the initial influence of RCGO to persist.

Further analyses suggest that the influence of RCGO on stock price described by the static model is more likely to last in both large gain and large loss situations. In such situations, investors have very low propensity to sell stocks, therefore less transaction are made; only a very small portion of past reference price will be updated by the new stock price. Contrarily, in the small gain and small loss situations, the investors have higher propensity to sell, hence the new reference price will be mainly composed by new stock price. In small gain or loss situations, the influence of RCGO will quickly diminish.

### **5.2.1 Explaining previous test results**

The previous empirical tests on the Relative Capital Gain Overhang find that the stocks with either very high RCGO or very low RCGO will have higher returns than stocks with RCGO close to zero over a period from 6 months to one year. This observation has been confirmed by regressions of residual returns on RCGO. The tests find that in the loss situations, RCGO is negatively correlated to stock return over long term; in the gain situations, RCGO is positively correlated to long term stock return.

In a losing situation, the static model suggests that the investor's propensity to sell the stock drops with the increase of unrealized loss. But if the unrealized loss is small, its influence on stock price is weak, this influence will quickly disappear because a large portion of the reference price will be updated by current stock price, hence the reference price will quickly converge to stock price. In the next period, the unrealized loss will approach zero. However, if the unrealized loss is large, on one hand, the impact of stock price is strong, since the investors have strong intention to hold the stock. On the other hand, such a strong intention to hold will reduce the trading volume of the stock; hence prevent the reference price from converging to stock price. In the next period, the unrealized loss will remain a relatively large scale. For this reason, the empirical test finds that stocks with large unrealized loss tend to have higher return than stocks with small unrealized loss over long term.

The same reasoning applies to the winning situation. With an increase in unrealized gain, the investors have decreasing propensity to sell the stock. In a small gain situation, investors have a high propensity to sell. Hence the reference price will be quickly updated, and converges to stock price. In a large gain situation, investors have a very low propensity to sell. Such a tendency will cause the stock price to go up; meanwhile it will also reduce trading volume. Therefore, the newly formed stock price will be higher, but the newly formed reference price will remain at the same level. Consequently, the unrealized gain in the next period will be even larger. For this reason, the stocks with large unrealized gain will increase in price over long term.

The above reasoning suggests that the trading volume is the key aspect that determines the update speed of the reference price. Recall that the estimation of the reference can be considered as a weighted average of the stock prices in past periods. Grinblatt and Han (2005) demonstrate this relation by the following formula:

$$R_t = \sum_{n=1}^{\infty} (v_{t-n} \prod_{\tau=1}^{n-1} (1 - v_{t-n+\tau})) p_{t-n}$$

Where:

$R_t$  is the reference price for a stock at the end of week  $t$ ,

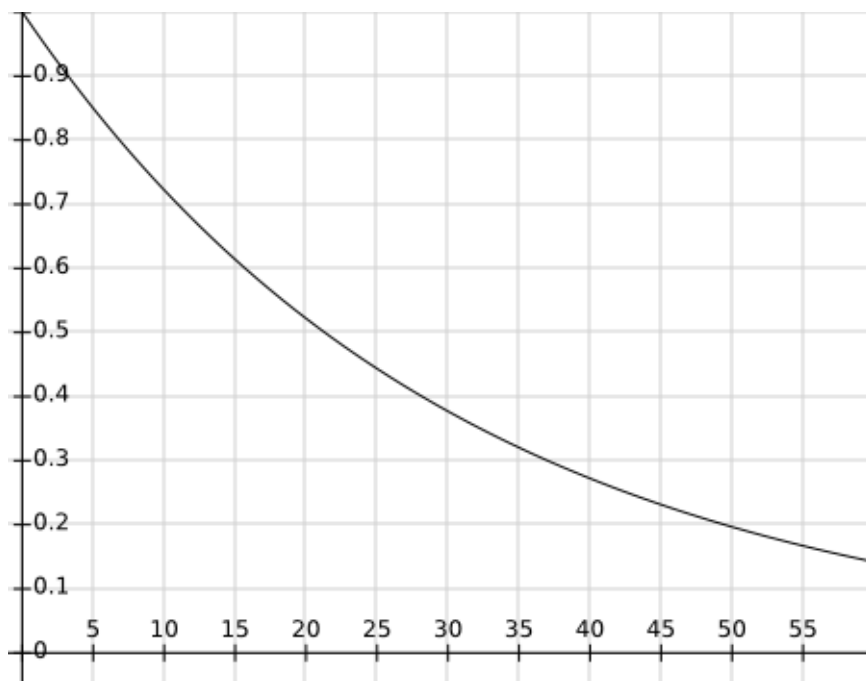
$p_t$  is the stock price at week  $t$ ,

$v_t$  is the trading volume represented in percentage.

According to this formula, at a given time  $t$ , the reference price  $R_t$  is composed by: the most recent trading price  $P_{t-1}$  with the stock turnover ratio as the weight; the trading prices in all the past periods with the weight representing the portion of the stock that have not been traded since the past  $n^{\text{th}}$  period. This latter suggests that for a stock that has been traded for the last time at a certain past period, this trading price was the past purchase cost at is carried on to the current reference price. This formula also shows that the higher the stock turnover is, the less the past purchase cost the current reference price contains.

In previous chapters, the test data shows that the average weekly turnover ratio is 3.2%. Assuming that the weekly turnover ratio is constant and based on the above estimation formula of the reference price, the weight on the purchase cost at the past  $n^{\text{th}}$  period will be the  $n^{\text{th}}$  power of 0.968. This simple simulation suggests that the weight on the past purchase cost decreases exponentially with time. The following figure demonstrates the evolution of the weight over time.

Figure 5.2-1: Weight of past purchase cost in current acquisition price



The vertical axis represents the weight; the horizontal axis represents the number of week prior to current period.

This simulation is not an exact estimation of the reference price. With the weight of the most recent purchase cost being one, the weight in the above figure represents a relative strength of earlier purchase cost comparing to the most recent purchase cost.



This figure shows that with a turnover ratio of 3.2%, the weight of past purchase cost drops to 0.2 when the purchase occurred 50 weeks ago, meaning that the purchase cost from one year ago still have one fifth of the weight comparing with the most recent purchase cost. It suggests that the past purchase cost still maintain a certain contribution to the current reference price, and indicate that the influence of the unrealized gain/loss is able to persist over a period of one year as in the previous empirical tests.

The dynamic model on RCGO provides an explanation to the long term influence observed in the empirical tests. When adapting the theoretical model to the empirical test, the reference price becomes a dynamic concept. In a small gain or loss situation, the influence of RCGO is diluted by the update of the reference price. Only when the gain or loss is sufficiently large, its influence can persist.

### **5.2.2 Convergence and divergence of RCGO**

In the previous section, the dynamic model is applied separately in the four situations: large loss, small loss, small gain and large gain. The influence of the Relative Capital Gain Overhang shows difference patterns in these situations.

The analyses show that in the situations such as large loss, small loss, and small gain, the reference price converges to the stock price. It means that in such situations, the unrealized gain or loss has the tendency to stabilize towards zero. Especially in a small gain situation and a small loss situation, because of a high trading volume, the reference price is being constantly updated; therefore the influence of behavior factor will be quickly eliminated from the market. In a large loss situation, the update of reference price is slow, but the reference price and the stock price move towards each other. In the end, even though the influence of behavior factor lasts for a longer time, it will be eliminated eventually. The convergence of RCGO towards zero suggests that in the large loss, small loss, and small gain situations the market activity is self-stabilizing.

In a large gain situation, the RCGO shows divergence pattern. Facing very large gain, investors have weak propensity to sell the stock. By the market mechanism of stock supply, a very large gain should raise the stock price. For the same reason, for a low propensity to sell, the update ratio of the reference price will also be very small. If the increase in the reference price cannot outrun the increase in stock price, the RCGO will become larger in the next period. Then the same process will repeat, and the RCGO will become even larger in later

periods. This pattern suggests that if an unrealized gain is sufficiently large, it will become self-enhanced over time.

However, from the previous tests in chapter 4, especially the figure 4.3-1 of the RCGO sorting test, it is not able to observe the convergence or the divergence pattern of the influence of RCGO on stock returns. The RCGO sorting test shows that the large loss group has much higher returns than the large gain group in the subsequent time periods, which may suggest a stronger overall influence of unrealized loss, but the sorting test focuses on cross-sectional comparison among RCGO deciles, and cannot fully demonstrate the time-series evolution of the influence of RCGO in the different deciles. Further tests will be necessary to study the convergence and the divergence pattern.

The divergence of RCGO in large gain situation makes the winner stock a special group. The influence of unrealized gain is self-enhancing. Under such an influence, the stock price can be continuously boosted up, hence can outperform other stocks. In this sense, the divergence of RCGO could drive the momentum effect. Moreover, in a scenario where the entire market is in a very large gain situation, the divergence of the RCGO of the market might inflate the market and result in the formation of a financial bubble.

### **5.3 Tests of the dynamic model**

In this section, tests will be performed to examine the dynamic model. The tests will focus on the time series evolution of stock performances in different RCGO quintiles. The stock price and the cumulative abnormal return of RCGO quintiles will be studied over one year time horizon. The tests show that the top quintile stocks (large gain) and the bottom quintile (large loss) stocks have an upward trend in returns comparing with the middle quintile. The test result is consistent with the prediction made by the dynamic model.

#### **5.3.1 Prediction of the dynamic model**

The static model focuses on the cross sectional relation between RCGO and stock performance. At a given time, the static model compares between the stocks with extreme RCGO and the stocks with neutral RCGO. The dynamic model draws the attention to the time series evolution of stock performances under the influence of RCGO. The dynamic model

suggests that if a stock is experiencing large gain or large loss, the influence of RCGO will likely to persist over time, and generates an upward trend in stock performance.

The upward trend of stock performance in the extreme RCGO stocks can be interpreted as return autocorrelation over time. If other conditions are equal, stocks with extreme RCGO could yield returns that are above the average return at given time period. In the following time period, the influence of unrealized gain/loss will persist, and the same stocks will still yield above average return. As a result, one could observe positive autocorrelation in the sample stocks.

Although the dynamic model could predict an autocorrelation in stock returns, it is still difficult to verify the upward trend by a return autocorrelation test. Autocorrelation in stock returns has been already documented by existing literature as an anomaly in the financial market, and various theories have been developed to explain it. Therefore, by performing a test on return autocorrelation over the sample is not able to identify the RCGO as the cause of return autocorrelation.

Testing the return autocorrelation within RCGO quintiles is one way to introduce the factor of RCGO in the test. However, such a test is not consistent with the upward trend predicted by the dynamic model. The common autocorrelation tests involving quintiles often examine a **cross-autocorrelation**. This type of test looks at the position of one stock in its quintile at different times. If the stock is likely to have a higher return than the average of the quintile at time  $t$  and  $t+1$ , it is documented as a positive autocorrelation. As for an upward trend, it looks at the time serial evolution of the stock return. It implies a continuous positive return regardless the position of the stock in its quintile. Therefore, the common autocorrelation tests are not suitable for examining the upward trend caused by RCGO.

For the reason mentioned above, the test on autocorrelation of stock returns will not be used. Instead, the test will simply look into the evolution of stock returns and the cumulative abnormal returns of the stocks in the same quintile, and makes comparisons between the quintiles. The periodicity of this test will be one year following the Relative Capital Gain Overhang is measured. Such a periodicity corresponds to the long time horizon in previous tests.

The dynamic model of RCGO predicts an upward trend in stock performance on the stocks with extreme RCGO. In the empirical test, one could expect the following hypothesis:

**Hypothesis 1:** In the extreme RCGO group, stock return and the cumulative abnormal return will increase over time during a period of one year after RCGO is measured.

Moreover, the test will also examine the divergence pattern of the influence of RCGO in large gain situation. This divergence pattern suggests that the upward trend in the large gain group will be stronger over time; hence over the one year horizon after RCGO is measured, the total return in the large gain group should be higher than in the large loss group. In the empirical test, one could expect such a hypothesis:

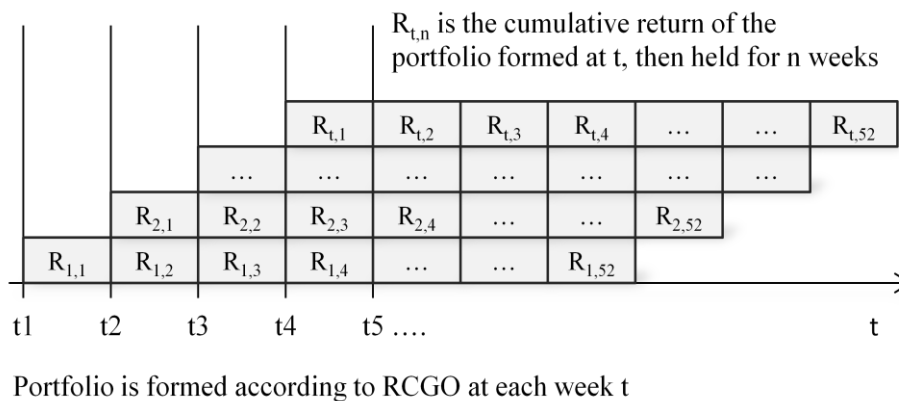
**Hypothesis 2:** The increase in stock price and the cumulative abnormal return will be higher in the top quintile than in the bottom quintile.

### 5.3.2 Test design

The test on dynamic model focuses on the evolution of stock raw returns and the cumulative abnormal returns after the Relative Capital Gain Overhang is measured. The time horizon will be 52 weeks after RCGO is measured, because it covers the same time period as in previous tests. For the purpose of simplifying the calculation, this test will divide stocks into 5 RCGO quintiles instead of 10 RCGO deciles.

At each time  $t$ , all stocks will be sorted into 5 quintiles in ascending order according to their RCGO at time  $t$ . The quintile 1 is the large loss group, the quintile 3 is the neutral group, also called the middle group, and the quintile 5 is the large gain group. The test then calculates the cross sectional average of stock raw returns and the cumulative abnormal returns of the quintiles in each week over the next 52 weeks. Therefore, at each time  $t$ , there is a set of portfolio raw returns and cumulative abnormal returns in week 1, week 2... until week 52 after the measurement of RCGO for each quintile. Finally, the time serial average of the raw returns and the cumulative abnormal returns of each quintile from  $t_1$  to the last time period of the sample will be calculated. The following figure illustrates the timeline of this test.

Figure 5.3-1: Illustration of the dynamic model test timeline



The dynamic model gives a description on the evolution of stock performances. Under the influence of unrealized gain or loss, stock prices should show upward trend in the periods following an unrealized gain or loss, and such an upward trend should be stronger in the large gain stocks than in the large loss stocks. Therefore, the test expect to find an increase of stock raw returns and cumulative abnormal returns over time in the top and bottom quintiles; while in the middle quintile, the above variables remain constant over time. The increase in stock returns will be illustrated by graphs and will also be verified through time serial regressions.

### 5.3.3 Dynamic model test results

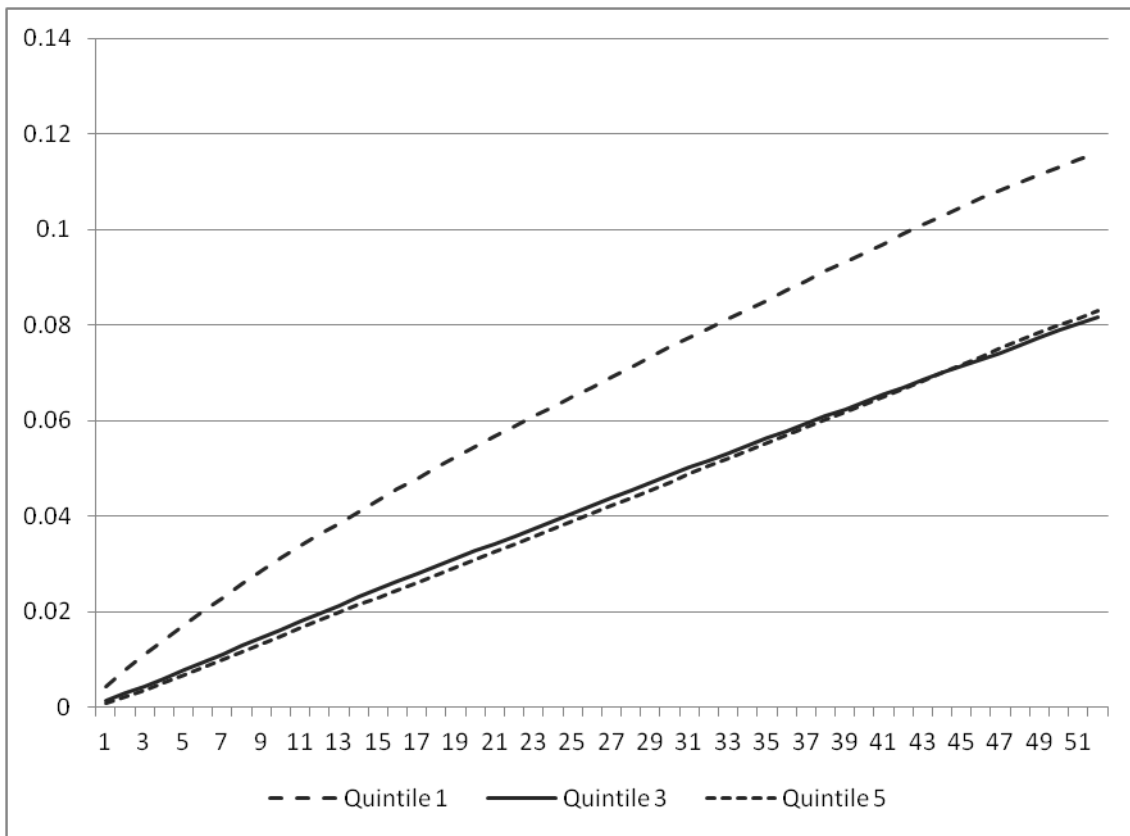
This section reports the test result. For both the stock raw returns and the cumulative abnormal returns, the test finds upward trend in the top and bottom RCGO quintiles over the following one year time horizon after dividing the quintiles. However, the bottom quintile (loss group) shows stronger upward trend than the top quintile (gain group).

#### 5.3.3.1 Test on the stock raw returns

The dynamic model suggests that in large unrealized gain or loss situation, the influence on stock price persists over long term; and this influence could push stock price to go up. Therefore, the most direct way to verify this influence is to look at the evolution of stock returns.

This test first examines the stock raw return. The following figure shows the evolution of average stock raw returns in a cumulative way of the RCGO portfolio of quintile 1, 3, and 5 over one year after the portfolio formation.

Figure 5.3-2: Cumulative return of RCGO quintile 1, 3, and 5 over 52 weeks



The vertical axis is the cumulative raw return; the horizontal axis is the week number after portfolio formation.

This figure shows that there are upward trends of raw return evolution in all the three groups. It is normal to see such upward trends in cumulative raw returns, because the tests in previous chapters show that the stocks yield in average a positive weekly return of 0.15% over the sample period from 1982 to 2012. This figure also shows that the quintile 1 (the loss group) has a much stronger upward trend than the quintile 3 and the quintile 5. However, the quintile 5 (the gain group) has a return evolution almost identical to the quintile 3.

This test result suggests that the influence of RCGO might be stronger in the losing situation than in the winning situation. In a large unrealized loss situation, investors are extremely reluctant to realize losses. Such a behavior pattern is consistent with the loss aversion behavior described in the prospect theory.

### 5.3.3.2 Test on cumulative abnormal returns

This test will be performed on cumulative abnormal returns of RCGO quintiles. Using the abnormal returns allows controlling for the time value and the risk factors in RCGO quintile returns.

This section repeats the test method from previous section, and applies it to the cumulative abnormal return in order to control for risk factors. The test will demonstrate the evolution of the cumulative abnormal return of the top middle and bottom RCGO quintile portfolios over 52 weeks after the portfolios are formed. Finally, time serial regression will be applied to verify the slope.

The cumulative abnormal return is obtained using the same method as in the residual regression in chapter 4. For each individual stock, a time serial regression is performed on the Fama-French three-factor model:

$$r_i - r_f = \beta_1(r_m - r_f) + \beta_2SMB + \beta_3HML + \varepsilon$$

The regression does not contain the intercept term; hence the residual captures all errors that cannot be explained by the model. The residual  $\varepsilon$  of the regression is collected as the abnormal return. Then, for each time period, the cumulative abnormal return of the  $i$ th week is calculated over the horizon of 52 weeks. This means that at each time period, an individual stock will have a series of cumulative abnormal return from the first week to the 52th week after time  $t$ . The cumulative abnormal return will be estimated by the sum of abnormal returns:

$$car_{t,i} = \sum_{n=1}^i ar_n$$

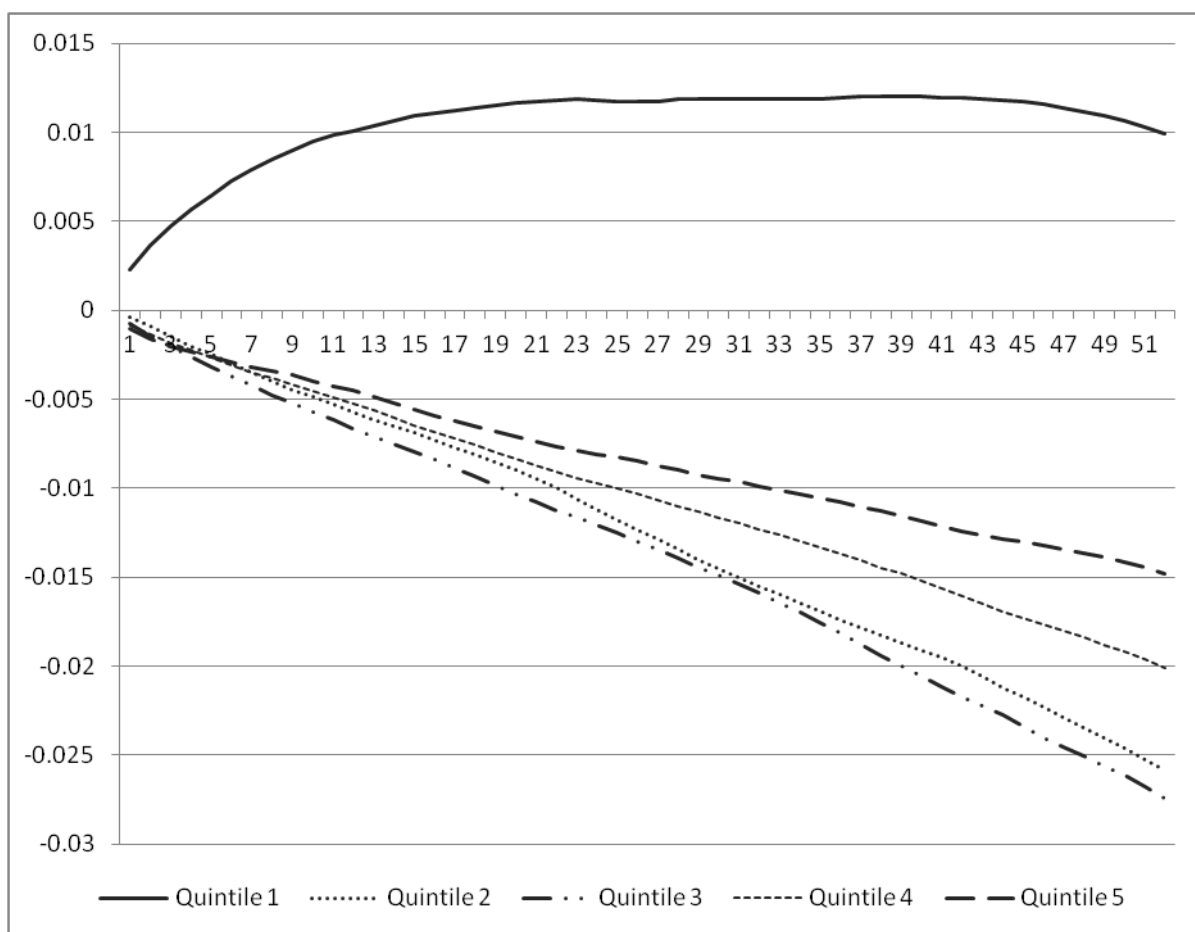
Where:

$car_{t,i}$  : is the cumulative abnormal return of  $i$ th week after time  $t$ .

$ar$  : is the abnormal return.

Next, the RCGO quintile portfolios are formed in the same way as in the previous section. At each time all the stocks are divided to five RCGO quintiles. The bottom quintile (quintile 1) covers the stocks with large loss the top quintile (quintile 5) covers the stocks with large gain. Finally, the test calculates the average cumulative abnormal return of the  $i$ th week in each RCGO quintile portfolio in all time periods. The following figure illustrates the evolution of the cumulative abnormal returns of all the five quintiles over 52 weeks after the portfolio formation. The bottom (quintile 1), middle (quintile 3), and top (quintile 5) groups are highlighted with bold lines.

Figure 5.3-3: Cumulative abnormal return of RCGO quintiles over 52 weeks



The vertical axis is the cumulative abnormal return; the horizontal axis is the week number after portfolio formation.

From the above figure, one could observe that first, in the quintile 3 and the quintile5, the cumulative abnormal returns show decreasing trend over the one year horizon after the portfolio formation. The reason for such a decreasing trend is the same as in previous tests. The Fama-French risk factor performances which are downloaded from the on-line database of the Data Library of Kenneth French, includes the more profitable market of NASDAQ comparing to the test sample that includes only NYSE and AMEX. Therefore, the residuals from the regression are generally negative. Second, over the entire 52-week time horizon, the bottom quintile has the highest cumulative abnormal return while the middle quintile has the lowest cumulative abnormal return. This result is consistent with the cross sectional relation in RCGO deciles found in previous tests. Finally, at the time-serial level, an upward trend can be observed in the bottom and the top quintiles compared with the middle quintile. At the beginning of the 52-week horizon, the weekly returns of all the three quintiles are more or less at the same level. During the 52-week horizon, the bottom and the top quintiles consistently yield higher returns than the middle quintile. As a result, at the end of the 52-



week horizon, the cumulative returns in the bottom and the top quintiles are much higher than the middle quintile. This result is consistent with the hypothesis 1.

The above figure shows higher cumulative abnormal return in the bottom quintile (loss group) than in the top quintile (gain group), which is not consistent with the hypothesis 2. This hypothesis expects higher return in the gain group as a result of the divergence pattern of the influence of RCGO. However, this test result could still contribute to the issue of the convergence/divergence pattern of RCGO. Comparing the evolution of the cumulative abnormal return of the bottom quintile with the top quintile, the curve of the top quintile has a linear shape, while the curve of the bottom quintile has a clear concave shape. A concave curve suggests that the return of the bottom quintile increases at a decelerating speed. Over a very short horizon after a large loss, the influence of RCGO is very strong. However, the influence of RCGO reduces in its power over time, and finally fades out. The influence of RCGO converges to zero over long term. In the large gain group, the cumulative abnormal return has a linear shape. It suggests that the return increases at a constant speed. The influence of RCGO does not appear to reduce in power over time, hence show divergence pattern.

To complete the test on the cumulative abnormal return, a time serial regression will also be performed to verify the trends observed in the above figure. In this test on the cumulative abnormal returns, the decreasing trend is dominant. Therefore, instead of looking for a positive slope in the top and the bottom RCGO quintiles, this test will introduce dummy variables in the regression which will allow identifying the differences in slopes between the middle quintile and the top/bottom quintile. The regression model is:

$$CAR = \alpha_0 + a_1top + a_2bottom + a_3t + a_4top \times t + a_5bottom \times t$$

Where:

CAR : is the cumulative abnormal return

t : is the week number after portfolio formation

top =1 if observation is from quintile 5, else top =0

bottom =1 if observation is from quintile 1, else bottom =0

The variables *top* and *bottom* are dummy variables introduced in the regression, representing the top quintile and the bottom quintile. The middle quintile is considered as the base group to compare with. This regression model will distinguish the difference between the base group and the top/bottom group in terms of intercept and as well as the slope. The interest of this regression is in the difference in slopes, if the coefficients  $a_4$  and  $a_5$  are positive, it could suggest that the top and bottom quintiles show upward trend, or at least, a relative upward trend compared with the middle RCGO quintile.

The regression method used in this test is similar to the Fama-Macbeth regression method: At each time period  $t$ , one regression is performed on the cumulative abnormal returns to obtain the coefficients; then the statistics are calculated from all coefficients estimated by the regressions. The following table reports the regression result.

Table 5.3-1: Time serial regression on cumulative abnormal return over time

| $CAR = \alpha_0 + a_1top + a_2bottom + a_3t + a_4top \times t + a_5bottom \times t$ |         |         |        |         |        |        |
|---|---------|---------|--------|---------|--------|--------|
|   | $a_0$   | $a_1$   | $a_2$  | $a_3$   | $a_4$  | $a_5$  |
| Coef.   | -0.0005 | -0.0010 | 0.0084 | -0.0006 | 0.0003 | 0.0007 |
| T-value   | -1.41   | -2.14   | 13.73  | -20.77  | 11.38  | 17.49  |
| P-value   | 0.1579  | 0.0325  | <.0001 | <.0001  | <.0001 | <.0001 |

The regression shows that the coefficient of the base group is significantly negative, suggesting that in general, a decreasing trend is present in the evolution of the cumulative abnormal return. However, the coefficients of  $a_4$  and  $a_5$  are both positive and are significant at 99% confidence level; suggesting that in the top and bottom quintiles, the decreasing trend is weaker than in the middle quintile. In addition, the coefficient of  $a_4$  is smaller than  $a_5$ , suggesting that the bottom quintile has a stronger overall upward trend than the top quintile.

The result of the regression test is consistent with the observation from the Figure 5.3-3. After ruling out the risk factors, there is an upward trend existing in the large unrealized gain and large unrealized loss situations. This result could confirm the hypothesis 1. The test result also shows a higher return in the bottom quintile than in the top quintile. This result contradicts the hypothesis 2. However, this result also provides a different interpretation about the convergence and the divergence patterns of the RCGO's influence over long term. The high return in the bottom quintile results from a strong influence of RCGO in the short term, but such an influence weakens over time. In the top quintile, the influence of RCGO is not as strong as in the bottom quintile, but such an influence could persist over time.

## 5.4 Summary

This chapter contains a development of the theoretical analyses and tests on the model concerning the unrealized gain or loss under the framework of alternative utility theories. Elaboration of such a dynamic model is a response to the problem about the periodicity in the empirical test on the static model. The major problem in the test results is the expected relation between stock return and the Relative Capital Gain Overhang is not found over the short term, but over the long term, more specifically, over one year time horizon.

Revising the static model and the adaptation of the theoretical model to empirical test shows that by aggregating the investors' preferences on the market, the reference price at which investors purchased the stock, becomes a dynamic factor. It suggests that the reference price could influence the stock price in the next time period, but in the meantime, the reference price also partially influences itself in the next time period because when trade occurs, the newly formed stock price becomes the reference price of the new buyers. In this way, the RCGO can be considered as a recursive process over time, and the initial influence of RCGO could persist over long time horizon.

The dynamic model studies the four situations concerning large loss, small loss, small gain, and large gain. Another key factor in this analysis is the stock turnover, which determines the update rate of the reference price. In small loss and small gain situations, the reference price is updated very quickly. Hence the RCGO will converge to zero very quickly. But in the large loss and large gain situations, since the reference price update rate is very slow, the RCGO converges to zero over long term in the large loss case, and the RCGO even becomes divergent in the large gain case. Therefore, the influence of RCGO is found over long time horizons.

Unlike the previous tests on the static model, which look into the cross-sectional relation between return and RCGO, the test on the dynamic model emphasizes the time serial evolution of stock performances over one year time horizon after the RCGO is measured. The test results show that the stocks with large gains or large losses will show upward trend in their returns over the following one year. Such an upward trend is even stronger in the large loss stocks than in the large gain stocks. The results are generally consistent with the prediction of the dynamic model. However, both the dynamic model and the related tests are

still in a primitive stage. Further development and refinement are still needed in future research.

The static and dynamic models and the tests show how unrealized gain or loss could influence stock returns. It provides evidence for behavior factor playing a role in the financial market. Moreover, such influences of the unrealized gain or loss cannot be considered as a bias or irrationality, because under the alternative utility theory framework, investor maximizes his/her utility function. Such a rational characteristic of the behavior factor might explain why some patterns that are considered as market anomalies by classic financial theory could be consistently documented.

One application of the unrealized gain/loss model is to the momentum effect. The study on the unrealized gain/loss model suggests that there is an upward trend performance in the large gain and large loss stocks. Such trend could result in return auto-correlation of stock for the winner and loser stocks, and might contribute to the momentum effect. In the next chapter, this aspect will be examined.



## Chapter 6

# RCGO explaining the momentum effect

This chapter aims to establish the link between the RCGO model and the momentum effect and to perform tests on this link. On one hand, the unrealized gain/loss has certain similarity to past returns, and could lead to the momentum portfolio containing the stocks with large gain/loss. On the other hand, the dynamic model of unrealized gain/loss developed in the last chapter could predict a positive stock return autocorrelation. For the stocks with a large unrealized gain/loss, investors have much lower propensity to sell the stocks comparing to other stocks, and such behavior pattern tends to last over a long time horizon. Therefore, the prices of stocks with large unrealized gain/loss will have tendency to go up over 6 month to one year. The above description given by the dynamic model corresponds to one source of the momentum effect: the autocorrelation in idiosyncratic component of stock returns. The behavior factor related to unrealized gain/loss could contribute to the momentum effect.

In terms of empirical work, this chapter applies the J-month/K-month strategy on the data from 1982 to 2012, and also documents the momentum effect. The momentum profits are then used in the test that relates the momentum effect to the unrealized gain/loss. The tests focus on using the strategy profits as test variables because the profits are the most important evidence of the momentum effect. Regression is performed on momentum profit and the profits of RCGO Top minus Bottom strategy, and finds a positive correlation. It suggests that the momentum portfolio and the TMB portfolio share similar stock composition. However, the regressions on momentum profit and the profits of the RCGO Extremity minus Middle strategy find negative correlations. The analysis of this result shows that the influence of the unrealized gain/loss could not only result in a positive return autocorrelation in individual

stock, but could also result in a positive cross-correlation among stocks with large gain/loss. According to existing theoretical work, this latter has a negative influence on the momentum profit.

The tests in this chapter suggest that the relation between unrealized gain/loss and the momentum effect is very complex. The unrealized gain/loss could have both positive and negative influence on the momentum effect through either the return autocorrelation or the cross-correlation. More detailed studies are expected in future research.

## **6.1 The momentum effect**

The momentum effect refers to the phenomenon that past winner stocks will outperform past loser stocks over a time horizon of six months to one year. This effect was first documented by Jegadeesh and Titman (1993) in the US stock market. Later, the momentum effect was also documented in European markets and as well as in emerging markets. Rouwenhorst (1998) studied the financial markets in 12 European countries using the data from 1978 to 1995; Bartens and Hassan (2011) study the South Africa stock market from 1987 to 2004; Su (2011) studies the momentum effect in Chinese market using the data of Shanghai and Shenzhen Stock Exchanges from 1994 to 2008. Moreover, recent studies on the US markets, such as Grundy and Martin (2001), Mclean (2010), Novy-Marx (2012), show evidence that the momentum still persists after it has been identified in early 1990s.

### **6.1.1 The momentum strategy**

The momentum effect is defined by a positive profit from implementing the momentum strategy. According to Jegadeesh and Titman, it is also referred as the J-Month/K-Month strategy. At one time spot, past J-month returns are used to evaluate stocks in order to form a zero-cost arbitrage portfolio. Then the portfolio will be kept for the following K-month to harvest arbitrage profit.

In empirical tests, Jegadeesh and Titman tested the combination of 3 6 9 and 12 months both as J-month and K-month. At the end of a given time period, all stocks on the market are divided into 10 deciles in ascending order according to past J-month return. Decile 1 contains stocks with the lowest past returns, and is called the loser group. Decile 10 contains stocks with the highest past returns, and is called the winner group. At the next time period, a zero-

cost arbitrage portfolio is formed by longing the winner group while selling short the loser group. Such a momentum portfolio will be kept over the following K-month. The momentum strategy is a rolling process. It can be repeated on monthly, weekly, or even daily basis throughout the total test period. Some tests may vary the composition of the winner and loser groups by dividing all stocks into three or five groups according to past returns. The momentum portfolio can be formed either by equal weighted average or by value weighted average.

The empirical result of the momentum profit varies among the studies. Jegadeesh and Titman (1993) reports that the combination of 12-month/3-month strategy could yield an average raw return of 1.31% on monthly basis. Rouwenhorst (1998) finds that on average the momentum strategy could yield 0.93% in the European market. Other studies based on different simple period or using different portfolio formation documented the annualized momentum profit ranging from 0.54% to 1.20% on monthly basis.

### **6.1.2 Replication of the momentum strategy**

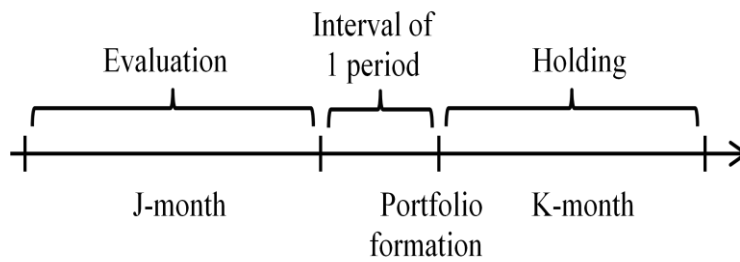
In this dissertation, the momentum strategy will also be implemented on the test sample. In this part, the momentum effect will be tested using Jegadeesh and Titman's J-Month/K-Month strategy.

The test runs a replication of the J-Month/K-Month strategy over the same sample as in previous tests. The sample contains stocks in the NYSE and AMEX within a period from January 1982 to December 2012. The observations with stock price smaller than 5 dollars are eliminated for lack of liquidity. A valid observation also requires having 150 continuous observations before it. In this test, J and K also take the value of 3 6 9 12 months. The J-Month/K-Month strategy is repeated at each week since the sample consists of weekly data.

At the end of each week ( $t_0$ ), all stocks are sorted into ten deciles in ascending order according to their returns over past J-month. The decile 1 consists of the stocks experiencing the heaviest losses in past J-month, hence is called the loser group; the decile 10 consists of the stocks that had the highest returns, hence is called the winner group. At the next week ( $t_1$ ), an equal weighted zero-sum portfolio is formed by longing the winner group while shorting the loser group. Then this portfolio is held for K-month. The following figure illustrates the timeline of the momentum strategy.



Figure 6.1-1: Time line of the J-month/K-month strategy



Performing the J-month/K-month strategy with one combination of J-month/K-month allows collecting a time serial data of the momentum profit at each week over the entire sample period for the specific J/K time horizon. Finally, t-tests over the time serial data allow testing the significance of the momentum profits.

The following table shows the average return of the winner minus loser portfolio for all 16 combinations of J-month/K-month equal to 3, 6, 9, and 12 months, equivalent to 13, 26, 39, and 52 weeks since the data is on a weekly basis. In the following table, the returns over different K-month are converted into annual rate of return.

Table 6.1-1: The profits of J-Month/K-Month strategy

|         |          | K-Month  |          |           |            |            |
|---------|----------|----------|----------|-----------|------------|------------|
|         |          | 13 weeks | 26 weeks | 39 weeks  | 52 weeks   |            |
| J-Month | 13 weeks | Profit   | 0.0086   | 0.012***  | 0.0078***  | 0.0047**   |
|         |          | T-value  | 1.03     | 2.99      | 2.94       | 2.04       |
|         |          | P-value  | (0.3053) | (0.0028)  | (0.0034)   | (0.0416)   |
|         | 26 weeks | Profit   | 0.0111   | 0.0298*** | 0.012***   | -0.0061    |
|         |          | T-value  | 1.33     | 4.55      | 2.74       | -1.79      |
|         |          | P-value  | (0.1844) | (<.0001)  | (0.0062)   | (0.0736)   |
|         | 39 weeks | Profit   | 0.0121   | 0.0077    | -0.0002    | -0.0224*** |
|         |          | T-value  | 1.51     | 1.25      | -0.03      | -5.06      |
|         |          | P-value  | (0.131)  | (0.2106)  | (0.9721)   | (<.0001)   |
|         | 52 weeks | Profit   | 0.0052   | -0.0158** | -0.0366*** | -0.0493*** |
|         |          | T-value  | 0.63     | -2.55     | -6.65      | -9.13      |
|         |          | P-value  | (0.531)  | (0.0107)  | (<.0001)   | (<.0001)   |

The table reports annualized raw return; \*\*: significant at 95% confidence level; \*\*\*: significant at 99% confidence level

From the above table, one can observe that 15 of all the 16 combinations of the J-month/K-month strategy yield results significantly different from zero. The highest profit is

obtained from the 6-month/6-month strategy. It could yield a profit up to 2.98% annually, equivalent to a result of 0.25% per month. The second highest profit is obtained from the 6-month/9-month combination. It yields a profit of 1.2% per year, equals to 0.1% per month. The lowest profit is obtained from the 12-month/12-month combination, which is the longest time horizon. The portfolio yields a loss of -4.93% annually, equivalent to -0.41% per month.

This table also shows a pattern of the profit from J-month/K-month strategy. Considering the sum of J-month and K-month to be the overall time horizon of the strategy, it can be observed that the profit increases firstly with the length of the time horizon. Over a total time horizon less than one year, this strategy generally yields positive outcome. It suggests that past winner stocks outperform past loser stocks. The momentum effect is confirmed. However, over a total time horizon that is longer than one year, the profits of the J-month/ K-month strategy begin to decrease. Finally, over a total time horizon from 21 months to two years, the portfolio yields negative results. This observation is consistent with the reversal effect over long term.

Comparing with the existing studies on the momentum effect, the study of Jegadeesh and Titman documented a momentum profit up to 1.31% per month on the US markets over the period 1965 to 1989. Other studies report the momentum profits ranging from 0.54% to 1.20%. The samples cover the markets all over the world and the sample periods vary from 1970s to 2000s. The momentum strategy tested in this dissertation yields a significant but much lower profit than in the previous studies. This test also contains some J-month/K-month combinations that yield non-significant profit such as the 6-month/3-month combination and the 9-month/9-month combination. The reason could be the fact that this test uses the data from 1982 to 2012, which consists of a more recent sample period. Both of a lower value and a weaker significance of the momentum profit suggest that the momentum effect has been decreasing during recent period.

Recall that in Chapter 3, the tests on the Capital Gain Overhang also draw the attention to the momentum effect. According to Grinblatt and Han's model, the momentum effect is represented by an autocorrelation between stock's current return and its return over one year in the past. The test result in Table 3.3-3 shows that using recent data, this dissertation did not find significant return autocorrelation over intermediate term in the sample period from 1982 to 2012. However, the test on the first half sub-sample shows marginally significant intermediate term returns autocorrelation with a P-value of 0.107. This result is consistent

with the observation of a decreasing momentum effect found by the J-month/K-month strategy. Another implication from the comparison between the test in Chapter 3 and the J-month/K-month strategy is that the return autocorrelation over the intermediate term is not an accurate measurement of the momentum effect. Over the same sample period, the test in Chapter 3 shows no significant return autocorrelation while the J-month/K-month strategy is still able to yield a positive and significant profit. This finding is an evidence for directly using the momentum profit as a test variable in order to perform more accurate tests.

The following tests will be performed on the most profitable time horizon, which is the 6-month/6-month horizon. The tests related to the momentum profit will also adopt the same time horizon.

## **6.2 Link between RCGO and the momentum effect**

To find the link that connects the behavior factor represented by RCGO and the momentum effect, this study first revisits the test results obtained in the previous chapters. These results show some similarities between the RCGO decile portfolio and the momentum portfolio in terms of portfolio composition and time horizon. Revising the existing theoretical decomposition reveals that the return autocorrelation induced by RCGO could be one source of the momentum effect.

### **6.2.1 Similarities between RCGO and momentum effect**

During the study on both of the momentum effect and the RCGO model, some features have been noticed. These remarks suggest that the RCGO deciles portfolios and the momentum decile portfolios share some common points.

First, the unrealized gain or loss seems to be more or less related to past J-month return in terms of their definitions. From the perspective of an individual investor, these two concepts are the same thing. The actual return from an investment is the realization of the unrealized gain or loss. From the aggregated perspective at the market level, the reference point in these two concepts differs. Stock return measures how much stock price has varied in the past J-month, the reference point is the past price J-month ago; while the unrealized gain or loss measures if most of the investors are winning or losing, the reference point is the average acquisition price. Intuitively, if a stock has performed well in the past, it is very likely

that holders of this stock are winning. However, as suggested in the dynamic model, it should take into consideration the update rate of the average acquisition price. When the stock is traded, the latest stock price will participate in the formation of the new average acquisition price. In previous sections of this dissertation, discussions have addressed this issue, and the Figure 5.2-1 also illustrates the influence (or the weight) of past average acquisition price in the latest RCGO. This figure shows a clearly decreasing weight for longer past time. It also shows that the past acquisition prices one year ago could still have a certain influence in forming the latest RCGO. For the above reason, on one hand, the unrealized gain or loss is more strongly influenced by short term past stock price. The data in this dissertation is on weekly basis, hence normally, past prices that longer than one month will have reduced influence on unrealized gain or loss. On the other hand, the evaluation period for the RCGO could be significantly longer than the momentum strategy. For the momentum strategy, the evaluation period of the momentum strategy (the J-month) usually covers past 6 months. Comparing with the RCGO decile portfolio, its evaluation period is not limited to a given length. The previous analyses show that the past acquisition price older than one year could still have an influence.

As a result of the above reasons, the RCGO decile portfolio and J-month/K-month portfolio are likely to be related. Both portfolios could include certain stocks, but the different evaluation criterion also imply different periodicity and weighting in portfolio selection. Both portfolios will not be identical.

Second, the momentum effect and the impact of RCGO share similar time horizon. The empirical studies show that the momentum effect is often documented over intermediate time horizon, usually from 12 to 18 months. Longer than this horizon, the J-month/K-month strategy will yield a negative profit, suggesting a dominant reversal effect. For the influence of unrealized gain or loss, the empirical tests in this dissertation show that the stocks with a large unrealized gains and large unrealized losses outperform the stocks with small or neutral gains/losses. Such a pattern is not observed over short term, but it begins to emerge from 6-month horizon, and becomes increasingly apparent over one year horizon. It should be noticed that the time horizon of the momentum effect refers to the time of J months plus K months. In other word, it includes both the evaluation period and the holding period. The empirical test in previous section shows that the momentum profit tends to reach the highest level at the total time horizon of  $J+K=12$  months. Longer than 12 months, the momentum profit begins to decrease. This pattern is to some extent similar to the influence of RCGO.

Supposing that one stock has a very high RCGO at time  $t$ , then under the influence of a very weak propensity to sell the stock, the stock price will continue to rise in the following 12 month. At time  $t+J$ , as the stock has a good performance in past  $J$ -month, this stock will likely to be selected in the winner group of the  $J$ -month/ $K$ -month strategy. Next, as the price of this stock continues to be pushed up in the following  $K$ -month by the self-enhancing influence of RCGO in a large gain situation, the winner group will yield higher returns. As a result, the momentum effect is observed.

Last, the empirical tests so far also show inconsistent results between RCGO and momentum strategies. In the RCGO sorting test in Chapter 4 and the dynamic model test in Chapter 5, the results shows that the stocks with large unrealized loss have even higher return than the stocks with large unrealized gain over 6-month to one year time horizon. This result is likely caused by a very strong reluctance to realize loses for investors. However, according to this result, the loss stocks should outperform the gain stocks. Therefore there should be an opposite of the momentum effect over the time horizon of 6-month to one year. One explanation could be the compositions of the portfolios are different. The momentum strategy selects stocks based on past 6-month return, while the estimation of RCGO does not have a specific estimation period. According the analyses in Chapter 5, past stock price from one year ago could still have certain contribution to stock's current RCGO. As a result to this difference in stock selection, the RCGO portfolio could give higher weight to long term past performances of the stocks.

The model developed in this dissertation suggests that the unrealized gain/loss could induce stock return autocorrelation in the large gain and large loss stocks. Such an influence is caused by purely behavior factor; hence the stock return autocorrelation is at idiosyncratic level. This pattern corresponds to one of the three components of the momentum profit. Besides such a link at theoretical level, empirical study also show similarities between the momentum effect and the influence of unrealized gain/loss in terms of portfolio composition and the time horizon. To sum up, the unrealized gain/loss could contribute to the momentum effect, and empirical tests will be performed to examine such hypothesis in the following section.

### 6.2.2 Sources of the momentum profit

The momentum effect is considered as an anomaly in stock market. According to classic financial theory, the J-month/K-month strategy should yield zero profit after adjusted for risks because the momentum portfolio is a zero-cost arbitrage portfolio. From the day when the momentum effect was documented, studies have been devoted to identify the cause of the momentum effect. However, there is still no clear answer to such anomaly.

Jegadeesh and Titman (1993) and Lo and MacKinlay (1990) have separately proposed decomposition of the momentum profit from different perspectives. Their work has paved the foundation for theoretical study on the momentum effect. The decomposition of Jegadeesh and Titman (1993) takes the approach of asset pricing. Return of one asset is described by a single-factor model:

$$r_{it} = \mu_i + b_i f_t + e_{it}$$

The momentum effect is represented by such a formula:

$$E\{(r_{it} - \bar{r}_t)(r_{it-1} - \bar{r}_{t-1})\} > 0$$

It suggests that if a stock has a higher return than the average at a given time, the stock should also have a higher return than the average in the following time. If the stock has a lower return than the average, it should also have a lower return in the following time. With such a setup, Jegadeesh and Titman are able to decompose the momentum profit into the following expression:

$$E\{(r_{it} - \bar{r}_t)(r_{it-1} - \bar{r}_{t-1})\} = \sigma_\mu^2 + \sigma_b^2 Cov(f_t, f_{t-1}) + \overline{Cov}_i(e_{it}, e_{it-1})$$

The term  $\sigma_\mu^2$  and  $\sigma_b^2$  are respectively the cross-sectional variances of expected returns and factor sensitivities.  $Cov(f_t, f_{t-1})$  is the time serial covariance in factor mimicking portfolio. Therefore, in this equation, the first item in the right side:  $\sigma_\mu^2$  represents the cross-sectional dispersion in expected returns. The second item:  $\sigma_b^2 Cov(f_t, f_{t-1})$  means that the momentum is caused by exposure to the serial correlation of the factor mimic portfolio. And the last item:  $\overline{Cov}_i(e_{it}, e_{it-1})$  suggests that a positive time-serial autocorrelation in the idiosyncratic components of stock returns could contribute to the momentum effect.

On the other hand, the decomposition of Lo and MacKinlay (1990) and Lewellen (2002) takes an approach of portfolio performance. In fact, Lo and MacKinlay's work intended to

study the reversal effect, which is the opposite of momentum that takes place over long term. Lewellen adopts this approach and modifies it to fit the momentum effect. Lo and MacKinlay's approach is more straightforward. They form a portfolio that contains all stocks in the market, but the weight of each individual stock in this portfolio is how much the stock outperforms the market. The momentum effect is observed when this portfolio yields a positive profit:

$$\pi_t(k) = \sum_{i=1}^N \omega_{i,t}(k) R_{i,t} > 0$$

According to their formula, if the stock  $i$  has a higher return than the market during past  $k$  period, the weight for the stock:  $\omega_i$  will be positive; if the stock has lower return than the market, its weight will be negative. Mathematically, the weight of stock  $i$  is:

$$\omega_{i,t} = \frac{1}{N} (R_{i,t-1} - R_{m,t-1})$$

In this approach, Lewellen also decompose the momentum profit in the following mathematical form:

$$E[\pi_t] = \left( \frac{N-1}{N^2} \right) tr(\Gamma) - \frac{1}{N^2} [I' \Gamma I - tr(\Gamma)] + \sigma^2(\mu)$$

In this formula,  $N$  is the total number of stocks in the market;  $\Gamma$  is the auto-covariance matrix of all the stock;  $tr()$  is the operator of the trace of a matrix. In the above formula, the first term and the second term of the right side contains the auto-covariance matrix. The first term only includes the diagonal of the matrix, therefore it represents the return autocorrelation of individual stocks. The second term includes the rest of the elements in the matrix, hence it represents a cross-correlation between any of two stocks. A positive cross-correlation between stock A and B means that if the stock A has a high return in time  $t$ , then the stock B is likely to have a high return as well in time  $t+1$ . Note that the second term has a negative sign in the formula. It means that a positive cross-correlation could in fact decrease the momentum effect. The last term in this formula is the variance in the expected returns. This term is also present in Jegadeesh and Titman's decomposition.

Despite the different approaches to decompose the momentum profit, Jegadeesh and Titman (1993) and Lo and MacKinlay (1990) have however reached the same conclusion on this matter. The sources of the momentum profit can be classed into two categories: the cross

sectional dispersion of individual stock expected return, and time-serial autocorrelation of stock returns. Both of the decompositions give the same explanation to the cross sectional dispersion of individual stock expected return, but they take different perspectives to explain the return autocorrelation. Jegadeesh and Titman distinguish the autocorrelation in the factor realization from the one in the idiosyncratic component of individual stocks. The lead-lag effect of the factor realization is one source within the classic framework of risk-return, and the autocorrelation in the idiosyncratic component suggests an influence from the factors other than risks. Lewellen's decomposition distinguishes the stock return autocorrelation with respect to itself from a cross-correlation with respect to other stocks.

Among the sources of the momentum profit, the cross sectional dispersion of individual stock expected return and autocorrelation of factor return belong to the classic framework of risk-return paradigm. It suggests that the momentum portfolio, although zero-cost, is exposed to systematic risk factors. The profit of the momentum strategy originates from taking risks. However, empirical tests show that even after controlling for risk factors using the CAPM model or the Fama-French Three-Factor model, the momentum portfolio still yields a significant positive return. In a review of the momentum effect, Jegadeesh and Titman (2001) reports a result of 1.24 ( $t = 6.50$ ) for CAPM alpha and 1.36 ( $t = 7.04$ ) for Fama-French model alpha. Many other studies also confirm that the momentum profit cannot be ruled out by exposure to systematic risks.

The third component of the momentum profit from Jegadeesh and Titman's decomposition is the autocorrelation of the idiosyncratic part of stock return. According to the decomposition model, when the first two components failed to explain the momentum profit, it could only be attributed to the idiosyncratic part of stock return. In this situation, it implies that the market is no longer efficient because in classic financial theory, idiosyncratic component should be completely eliminated by diversification, hence stock return should not compensate such part.

The autocorrelation of the idiosyncratic part of stock return could have various causes. In the field of behavior finance, individual behavior bias such as conservatism, over confidence, may cause stock prices to over-react or to under-react to information. Such behavior biases could cause trouble in dissemination of information in the market, and stock price cannot instantly incorporate new information. As a result, with such a gradual



integration of information, stock price show patterns of under reaction or overreaction and leads to auto correlation in stock returns.

### **6.2.3 Autocorrelation due to unrealized gain/loss**

The explanation of the momentum effect fell on the autocorrelation on idiosyncratic component of stocks. Stock prices over-reaction or under-reaction can be the cause of such an autocorrelation. But an answer at a deeper level is still needed to understand the cause of stock price over-reaction or under-reaction.

In the field of behavior finance, some theories have been established to associate autocorrelation to behavior factors. However, in empirical studies, it is very difficult to find variables that could precisely capture the behavior factors. Regarding the measurement of a behavior factor, the RCGO model has its advantage. As discussed in theoretical part of this dissertation, the variable Relative Capital Gain Overhang is developed from a model that combines the loss aversion attitude from the prospect theory and the regret aversion from the regret theory. Therefore, this variable is able to capture the behavior pattern that investors are more sensitive to change in winning or losing situations than change in the payout value. For this reason, the variable RCGO has very specific economic and behavioral meaning comparing with proxy variables that are commonly used in empirical tests involving behavior factors.

The static model of RCGO suggests that investors will show a weaker propensity to sell the stock when they are either in a large gain situation or in a large loss situation. Such a preference should cause stocks with large gain or large loss to yield higher return than their counterpart. The empirical tests following this model show that such a predicted stock performance pattern is observed over one year time horizon. Next, the dynamic model of RCGO is developed. The dynamic model provides one explanation to the empirical results. Due to a slow update rate of the reference price, in large gain and large loss situation, the initial impact of unrealized gain or loss could persist over time. Moreover, the dynamic model also suggests that, unlike in other situations where the initial impact attenuate with the evaluation of stock price, in large gain situation, the initial impact could enhance itself overtime, hence could continuously push the stock prices upward.

The dynamic model suggests that the behavior factor represented by the Relative Capital Gain Overhang can generate an upward trend in stock performance in the stocks with

large gains or large losses. In the previous chapter, such an upward trend can be observed from the cumulative abnormal returns in the winner and loser group. This trend in stock performance could contribute to a return autocorrelation in the winner stocks and in the loser stocks. In addition, the empirical result on the cumulative abnormal returns also suggests that such an autocorrelation originates from the idiosyncratic component of stock return, and such an autocorrelation could last for over one year after an extreme RCGO. This one year time horizon of the autocorrelation could cover the total time horizon of the momentum strategy. According to the theoretical decomposition of the momentum strategy, positive autocorrelation of stock returns between the evaluation time and the holding period is one source of the momentum profit.

To sum up, the autocorrelation of the idiosyncratic component of stock returns is the link between the momentum effect and the unrealized gain or loss. Existing works have ruled out the exposure to systematic risk as a cause of the momentum profit. According to the decomposition, it leaves the autocorrelation of idiosyncratic component as the only source of momentum effect. Meanwhile, both the static and the dynamic model of unrealized gain/loss, combined with related empirical tests, provide an explanation that unrealized gain or loss could lead to positive autocorrelation of idiosyncratic component. This finding suggests that the unrealized gain/loss might contribute to formation of momentum effect.

### **6.3 Direct test on momentum profit**

This section performs empirical tests on the momentum profit and the Relative Capital Gain Overhang. Tests will focus on the positive profit obtained from the momentum strategy, because a positive profit is the most conclusive evidence of the momentum effect. Previous sections suggest that some similarities are found between momentum effect and the influence of unrealized gain/loss. The RCGO also allows forming profitable arbitrage strategies in a similar way to the momentum strategy. Both strategies could share some common portfolio composition. At a theoretical level, the unrealized gain/loss can be linked to the momentum effect, and the autocorrelation in idiosyncratic component is the key to the link. For such facts, the empirical test will focus on the profits from both momentum and RCGO strategies.

### 6.3.1 Test design

Empirical tests aim to examine the links between the influence of unrealized gain/loss and the momentum effect. The tests will be performed directly on strategy profits. The tests will look into the similarities between the RCGO and the momentum effect, and will also verify if the return autocorrelation caused by RCGO could be related to the momentum profit.

#### 6.3.1.1 A direct approach through strategy profits

This test emphasizes examining the profits obtained from momentum strategy or from the RCGO strategy. The reason for focusing on the strategy profits is that the profits, such as the momentum profit, is the most direct evidence of the momentum effect. Studies over the momentum effect without performing tests on the momentum profit will hardly provide a convincing result.

The second reason for focusing on the momentum profit is related to the study of Grinblatt and Han (2005). The subject of their study is the disposition effect, but their empirical test model also includes test on the momentum effect. The test model of Grinblatt and Han (2005), which has been replicated in previous part of this dissertation, is the following:

$$r = a_0 + a_1r_{-4:-1} + a_2r_{-52:-5} + a_3r_{-156:-53} + a_4\bar{V}_{-52:-1} + a_5s_{-1} + a_6cgo_{-1}$$

This model does not aim to directly test the momentum effect, but it contains the term of  $r_{-52:-5}$ , which is the past return from 52 weeks ago to 5 weeks ago. The coefficient of this term represents the return autocorrelation over intermediate term. In this test, Grinblatt and Han consider a positive coefficient for this term as an evidence of the momentum effect. This consideration makes sense. Because the regression uses the Fama-Macbeth method: at each time, one regression is performed on cross-sectional data. A positive coefficient of  $a_2$  suggests that at a given time, if the past return of one stock is higher(lower) than the cross-sectional average of past returns of all stocks, its current return is also likely to be higher(lower) than the average of all current stock returns. However, using the return autocorrelation over intermediate term is rather an approximation to the momentum effect. Previous tests have shown insignificant intermediate term return autocorrelation, but the momentum strategy is still able to yield a positive profit. Therefore, the return autocorrelation cannot replace the momentum profit as a measurement of this anomaly. Moreover, the time horizon of the return

autocorrelation in Grinblatt and Han's model can be compared to a 1-year/1-week combination, which is not consistent with the time horizon of J-month/K-month in the momentum strategy. For the above reasons, the tests in this section will be performed on the strategy profits.

#### 6.3.1.2 Portfolio Similarities

The previous section has discussed some similarities between the momentum effect and the influence of unrealized gain/loss. It suggests that both phenomena share similar portfolio composition and take effect at similar time horizon. Concerning the time horizons of these two phenomena, it is observed from previous tests that they last over a period of about one year. Regarding the portfolio composition, previous analyses suggest that it is possible that the winner group and loser group in the momentum strategy contain the same stocks as the large gain group and the large loss group in RCGO sorting.

In the first part, the test checks whether the RCGO deciles portfolios and the momentum portfolio have similar stock composition. When the momentum portfolio is formed the winner group could include some stocks whose RCGO is also the highest. If it is the case, these stocks with large unrealized gains could continue to yield high returns in the following periods under the influence of RCGO. Therefore, the RCGO could contribute to the performance of the winner group. However, for the loser group, the influence of the RCGO is against the momentum effect. If the loser group is composed of large loss stocks, these stocks will also yield high returns, and even higher returns than the large gain stocks, in the following periods. Therefore, a reversal effect should be observed. This analysis suggests that it is interesting to look at the RCGO of the momentum portfolio, and find out if the winner/loser group of the momentum strategy is equal to the gain/loss group in the RCGO deciles.

Another way to look into the portfolio compositions is to construct a Relative Capital Gain Overhang Top minus Bottom strategy, and study the profits of the both strategies. The portfolio formation of the RCGO Top minus Bottom strategy, similar to the J-month/K-month strategy, is also a past winner minus past loser strategy. It longs the stocks with the highest RCGO while short selling the stocks with the lowest RCGO. In order to be consistent with the momentum strategies, the holding period of this strategy is also 6 months.

However, despite the similarities to the momentum strategy in terms of the portfolio formation, previous empirical tests may suggest that such a Top minus Bottom strategy will yield a negative profit. The previous tests show that both the top and the bottom RCGO deciles have higher returns than the middle deciles, but the stocks with highest unrealized gains yields lower average return than the bottom RCGO decile stocks. The profit of the RCGO Top minus Bottom strategy originates from the difference in performances of the large gain stocks and the large loss stocks after the portfolio is formed. Such different performances are illustrated in the Figure 5.3-2 and the Figure 5.3-3 that shows the evolution of RCGO quintiles cumulative returns. The bottom quintile (large loss) stocks show higher returns compared with the top quintile stocks. This result is consistent with the loss aversion attitude of the prospect theory. Investors are reluctant to realize losses and prefer to continue holding the losing stocks. Therefore, the RCGO Top minus bottom strategy could to some extent capture the behavior factor of loss aversion.

Comparing the Top minus Bottom strategy and the J-month/K-month strategy, the portfolio composition could be very similar. Both strategies form a zero cost portfolio on a winner minus loser basis, but both strategies use their own definition for winners and losers. As a result, the winner and the loser portfolios in these strategies could share some common but not identical stocks. In the previous subsection 6.2.1, analyses have been made about this point. The analyses suggest that the evaluation by RCGO involves a much longer time horizon. Past performance over one year ago may still be included in estimating the RCGO. Comparing with the momentum strategy whose evaluation horizon is usually 6 months, the total time horizon of the RCGO Top minus Bottom strategy could be largely extended, and become comparable to the long term J-month/K-month strategy. Over such a long time horizon, the reversal effect becomes dominant, and the J-month/K-month strategy yields negative profit instead of a positive profit. This is consistent with the negative profit expected from the Top minus Bottom strategy.

### 6.3.1.3 Autocorrelation and momentum profit

In the second part, the test will examine if the momentum effect is related to the autocorrelation caused by RCGO.

In the previous chapters, the result from the empirical test on stock return and unrealized gain/loss allows developing the RCGO Extremity minus Middle strategy. This

strategy consists of forming the RCGO Extremity minus Middle portfolio and holding the position for 6 to 12 months. The RCGO deciles are divided in ascending order according to the Relative Capital Gain Overhang measured at current time. The EMM portfolio is a zero-cost portfolio formed by longing the stocks in RCGO decile 1 and decile 10 while short selling the stocks in RCGO decile 5 and decile 6. Comparing the EMM strategy with the Top minus Bottom strategies, both strategies are based on the Relative Capital Gain Overhang, which represents behavior factors involving unrealized gain/loss. The TMB strategy can be considered as an approximation to the momentum effect. It emphasizes the similarities in both portfolios. The EMM strategy represents the behavior pattern that investors are reluctant to sell stock in large unrealized gain and loss situations comparing with neutral situation. This behavior pattern will result in return autocorrelation in the stocks with large unrealized gain/loss. According to the decomposition of the momentum profit, stock return autocorrelation is one source of the momentum effect.

The RCGO Extremity minus Middle strategy exploits the behavior pattern that investors are more sensible to return variation when the latter could result in moving from a winning to a losing situation, or in the opposite direction. The static RCGO model provides theoretical support. In this model the variable RCGO is defined to measure the unrealized gain/loss while controlling for the risk factors. Under the framework of the prospect theory, investors will show much lower propensity to sell a stock when the stock is at large gain or at large loss. The aggregated investor preference will result to higher return in stocks with large gain or large loss. The dynamic version of the RCGO model suggests that at an aggregated level, the influence of an extreme RCGO could persist over time, and creates a positive return autocorrelation in the extremity group in the EMM strategy. Therefore, by longing the extreme RCGO stocks while short selling the stocks that have a null unrealized gain/loss, the EMM strategy is able to yield a positive profit. According to this previous analysis, the EMM strategy could capture the behavior factor represented by the RCGO. The positive profit earned by the EMM strategy originates from investors' different reactions or decisions when facing large unrealized gain/loss. The persistence of the influence of such a behavior pattern is the direct source of the EMM strategy profit.

Based on the above analysis, it is able to conclude that the TMB strategy based on RCGO is only an analogy to the momentum effect. This strategy does not carry any economic meaning. The EMM strategy has a solid theoretical support that the profit of the EMM

strategy is a measurement of investor's behavior factor, and also could represent the return autocorrelation caused by extreme RCGO.

#### 6.3.1.4 Test hypotheses

The tests on the link between the momentum effect and the unrealized gain/loss will include two parts: a test on similarity in portfolio composition and a test on return autocorrelation. The test on the similarity in portfolio composition will first compare the RCGO of the momentum portfolios, and then examine the correlation between the momentum profit and the profit of RCGO Top minus Bottom strategy. In this part, the tests expect to verify:

**Hypothesis 1:** The average RCGO of momentum deciles increases with decile ranks.

**Hypothesis 2:** The correlation between the momentum profit and TMB strategy profit is positive.

The test on the return autocorrelation examines if the return autocorrelation caused by the unrealized gain/loss is the major source of the momentum profit. This test will be performed on the profit of the momentum strategy and the profit of the RCGO EMM strategy. The EMM profit is a measurement of the autocorrelation caused by RCGO. If it is the major sources of the momentum effect, then a high EMM profit implies a strong return autocorrelation, and results to a high momentum profit. Hence, the test expects a positive correlation.

In addition to a positive correlation between the strategies profits, the test also expects an insignificant intercept in the regression of the momentum profit on EMM profit. It suggests that the variation in the momentum profit can be explained by the variation of the RCGO profit. After the RCGO profit is taken into account, the momentum portfolio will no longer yield significant profit.

**Hypothesis 3:** The correlation between the momentum profit and the EMM profit is positive.

**Hypothesis 4:** The intercept in the regression of the momentum profit on the EMM profit is not significant.

### 6.3.2 Similarity in portfolio composition

This part studies the portfolio composition of momentum and RCGO deciles. The test first shows that the average RCGO of the momentum deciles is consistent with expectation. The loser groups have negative RCGO, and the winner groups have positive RCGO. The test then runs a time serial regression between the profits of both strategies. The regression shows a positive correlation.

#### 6.3.2.1 RCGO of momentum portfolio

This test is a simple comparison of the average RCGO in all ten deciles divided by past J-month returns. Recall the timeline of the momentum strategy: At time  $t_0$ , all stocks are evaluated according to past J-month return; hence  $t_0$  is called the evaluation time. At time  $t_1$ , the winner and the loser portfolios are formed; hence  $t_1$  is called the formation time. The time interval between  $t_0$  and  $t_1$  is one week. The formula for estimating the RCGO suggests that RCGO can be positively related to past stock return. Therefore, in order to match the time horizon of past J-month return, the RCGO will also be measured at the evaluation time of  $t_0$ .

The same data is used in this test as other tests in this dissertation including the replication of the momentum strategy. The data covers stocks in the NYSE and AMEX within a period if from January 1982 to December 2012. The observations with stock price smaller than 5 dollars are eliminated for lack of liquidity. A valid observation requires having 150 continuous previous observations.

This test adopts the 6-Month/6-Month strategy, because in previous test, the 6-Month/6-Month combination yields the highest momentum profit. All ten deciles divided by past 6-month will be reported in this test. The Relative Capital Gain Overhang of each individual stock is estimated at the evaluation time, which is the end of the first 6-month. The RCGO of each decile portfolio is the equal weighted average of the RCGO of the stocks in the decile. The decile portfolios are also formed by equal weighting. The following table reports the average annualized raw returns and the average RCGO of each momentum decile.



Chapter 6:  
RCGO explaining the momentum effect

Table 6.3-1: 6-Month/6-Month portfolio return and the average RCGO at the evaluation time

|           | <b>Variable</b> | <b>Mean</b> | <b>Std Dev</b> | <b>Variable</b> | <b>Mean</b> | <b>Std Dev</b> |
|-----------|-----------------|-------------|----------------|-----------------|-------------|----------------|
| Decile 1  | <b>Return</b>   | 0.06414     | 0.8094         | <b>RCGO</b>     | -5.9178     | 17.4010        |
| Decile 2  | <b>Return</b>   | 0.07596     | 0.6252         | <b>RCGO</b>     | -2.2852     | 11.1886        |
| Decile 3  | <b>Return</b>   | 0.08091     | 0.5532         | <b>RCGO</b>     | -0.6527     | 13.1536        |
| Decile 4  | <b>Return</b>   | 0.08001     | 0.5044         | <b>RCGO</b>     | 0.58733     | 10.1245        |
| Decile 5  | <b>Return</b>   | 0.08068     | 0.4813         | <b>RCGO</b>     | 1.56406     | 11.2753        |
| Decile 6  | <b>Return</b>   | 0.08656     | 0.4947         | <b>RCGO</b>     | 2.43498     | 11.3623        |
| Decile 7  | <b>Return</b>   | 0.09163     | 0.5129         | <b>RCGO</b>     | 3.14487     | 8.4949         |
| Decile 8  | <b>Return</b>   | 0.09774     | 0.5479         | <b>RCGO</b>     | 3.60953     | 7.9736         |
| Decile 9  | <b>Return</b>   | 0.10470     | 0.6096         | <b>RCGO</b>     | 3.76194     | 7.6763         |
| Decile 10 | <b>Return</b>   | 0.10024     | 0.7908         | <b>RCGO</b>     | 3.20393     | 5.9940         |

The variable "Return" is the annualized K-month return of decile, the variable "RCGO" is the average RCGO of individual stocks in the decile, and is measured at the evaluation time.

The above table reports the yields of all ten decile portfolios over K-month and their Relative Capital Gain Overhang at the evaluation time. The deciles are divided in ascending order according to stock return over past 6 months: Decile 1 contains the stocks with the lowest return in the past 6 months, or the loser group, decile 10 contains the stocks that had the best performance in the past 6 months, or the winner group. The table shows generally an increasing trend in both the K-month return and the average RCGO.

This result confirms the momentum effect and gives more detailed information about the distribution of K-month returns in all ten deciles. The momentum strategy involves longing the decile 10 while short selling the decile 1. Clearly, such a winner minus loser strategy will yield a positive profit. It is also observed that on average, all ten deciles yield positive K-month returns. It suggests that the momentum effect cannot be considered as loser stocks continue to lose, and winner stocks continue to win. In this case, both loser stocks and winner stocks will win in the following months, but the winner stocks win more than the loser stocks.

This result shows that seven of all ten momentum deciles are at unrealized gain at the evaluation time. The average Relative Capital Gain Overhang also shows an increasing trend with the decile ranks. This result suggests that RCGO and past 6-month performance of a portfolio are positively related. Past loser stocks carries larger unrealized loss at the evaluation time; while past winner carries larger unrealized gain. Such a result is consistent

with the assumption that the momentum decile portfolio, especially the winner and the loser group, may contain the same stocks as in the RCGO decile portfolios. In addition, note that the standard deviation of the RCGO in decile 1 is much larger than that in decile 10. In fact, the RCGO of the decile 1 is the most dispersed. It suggests that although the loser group mainly contains the stocks with the lowest RCGO, it could also pick up the stocks with high RCGO. In contrast to the loser group, the winner group has the most concentrated distribution of RCGO. It suggests that the winner group could consistently pick up the stocks with very large RCGO.

However, inconsistency also can be observed. From decile 1 to decile 3, the average RCGOs are smaller than zero but increasing. This pattern is consistent with a decreasing unrealized loss situation in RCGO model. The RCGO model predicts that in unrealized loss situation, the cross sectional stock returns is negatively related to RCGO. This prediction means that the stocks that are in a large loss (decile 1) should yield higher returns in the following period compared with stocks in a small loss (decile 3). Therefore, the K-month returns from decile 1 to decile 3 are expected to decrease with rank of deciles. In the above table, the K-month returns from decile 1 to decile 3 are not decreasing, but are increasing with the decile rank. One explanation attributes this inconsistency to the time horizon of the influence of RCGO. Previous tests shows that the influence of RCGO takes effect from 6-month to one year. While in this momentum strategy, the K-month holding period is set to be 6 months, which is at the minimum range. Over 6-month time horizon, the influence of RCGO might be still too weak to have a significant effect on stock return.

Another inconsistency is observed in the winner deciles. From decile 7 to decile 10, the momentum portfolios are sorted in ascending order of past 6-month returns. However the average RCGOs do not monotonically increase with decile ranks. The highest average RCGO is found in decile 9, not in the decile that had the best performance in the last 6 months. In contrast, the decile 10 has a very low average RCGO of 3.20, which is lower than that of decile 8, and is similar to that of decile 7. This result could suggest that the momentum portfolio and the RCGO portfolio still have different stock composition. The difference in evaluation time horizons between these two portfolios could help to explain this result. The evaluation of RCGO includes much longer past period. If a stock has experienced a very strong increase during a six-month period beginning one year ago, but only has a mild increase in the last 6 months, the stock could have a very large RCGO while is sorted in the 8<sup>th</sup> or 9<sup>th</sup> decile in the momentum strategy. For example, in the decile 10, the high

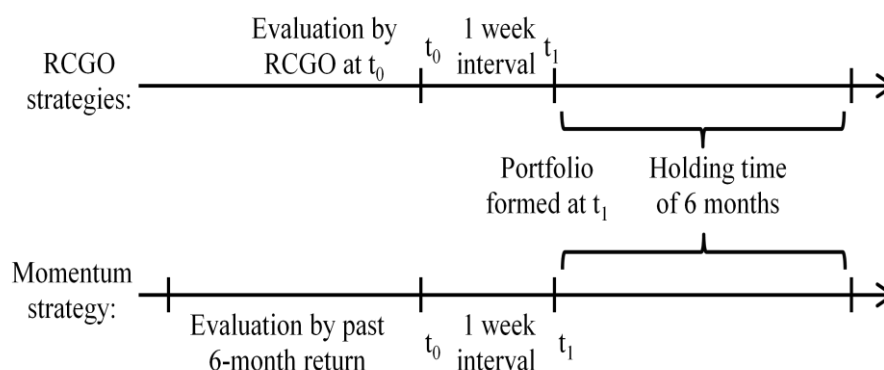
performance in past 6 months is composed mostly by recent high returns; and the decile 8 and decile 9 could pick up the winner stocks of previous investment cycle of the momentum strategy.

Finally, the above table shows that the K-month return is consistent with the average RCGO of the momentum portfolios. From the decile 4 to the decile 9, both the average RCGOs and the K-month returns of the deciles increase with the decile rank. The decile 9 has the highest average RCGO, and also has the highest K-month return. It is consistent with the empirical result in Chapter 4 that in a gain situation, stock performance is positively correlated to its unrealized gain. However, the decile 10, which experiences lower RCGO than the decile 8, has a better performance than the deciles 8. It suggests that there are other factors that could contribute to the K-month return of the decile 10.

### 6.3.2.2 Regression between momentum profit and TMB profit

In this part, the test aims to verify the correlation between the momentum profit and the TMB profit. The momentum profit is obtained by performing the 6-month/6-month strategy. The TMB profit is obtained by performing a 6-month TMB strategy which consists of longing the Top RCGO decile (the gain group) while selling short the Bottom RCGO decile (the loss group), and holding the portfolio for 6 months. The time line of both strategies matches each other. This means that at a given time, the evaluation time, the portfolio formation time, and the holding time of both strategies are identical. The following figure illustrates the time line of the both strategies.

Figure 6.3-1: Time line of the test



The strategies are implemented in a rolling way. At each week, the same procedure is repeated. Finally, a time serial data of the strategy profits can be collected for regression test. The data used in this test is the same as in the previous tests.

The reason for choosing such a regression model is that neither the momentum nor the RCGO could fit in the classic risk return framework. The momentum effect is considered as an anomaly of the market. Numerous studies have documented that the momentum profit cannot be explained by risk factors. Concerning the RCGO profits, the RCGO strategies are rooted from behavior finance model. The RCGO profits represent the behavior factor related to unrealized gain/loss, which is not included in the classic financial theories. In this test, the interest lies in the correlation between momentum profit and RCGO profit, hence the simple regression model is able to accomplish this task.

In order to control for risk factors, this test will also adopt a two-step regression. The first regression will be performed on Fama-French three-factor model so that the residuals of the regression can be collected. Then the second regression is a simple time-serial regression of the residuals of the momentum profit on the residuals of the TMB profit.

$$R_{mo} = \alpha + \beta R_{TMB}$$

In this regression model,  $R_{mo}$  represents the momentum profit residuals;  $R_{TMB}$  represent the residuals of the RCGO Top minus bottom strategy profit. The following tables report the statistic of the regression variables and the regression result.

Table 6.3-2: Descriptive statistics

| Risk adjusted profits (Residuals) |         |         |           |        |           |
|-----------------------------------|---------|---------|-----------|--------|-----------|
| Variable                          | Mean    | Std Dev | 10th Pctl | Median | 90th Pctl |
| $R_{mo}$                          | 0.0333  | 0.2594  | -0.2039   | 0.0692 | 0.2584    |
| $R_{TMB}$                         | -0.0006 | 0.1151  | -0.1439   | 0.0166 | 0.1258    |

Table 6.3-3: Regression on momentum profit and TMB profit

|  | Intercept | Coefficient | Adj-R <sup>2</sup> |
|--|-----------|-------------|--------------------|
| Model: $R_{mo} = \alpha + \beta R_{TMB}$ |           |             |                    |
| Estimation                               | 0.0339    | 1.0688      | 0.2243             |
| P-value                                  | <.0001    | <.0001      |                    |

The regression finds a positive correlation between the momentum profit and the RCGO Top minus Bottom strategy profit. The correlation is significant at 99% confidence level. This result confirms the hypothesis.

This test also shows that the TMB strategy over a 6-month horizon yields a negative result. This finding is consistent with the expectation, since previous tests results show higher returns in the loss group than in the gain group. In contrast, the momentum strategy could yield a positive profit over the same time horizon. In the previous subsection, explanations of such opposite results have been proposed. The stocks in both strategies are not identical. The TMB strategy could have a longer evaluation period due to the way the RCGO is estimated. As a result, the total time horizon of the TMB strategy could be extended to a longer term horizon, over which the reversal effect becomes dominant.

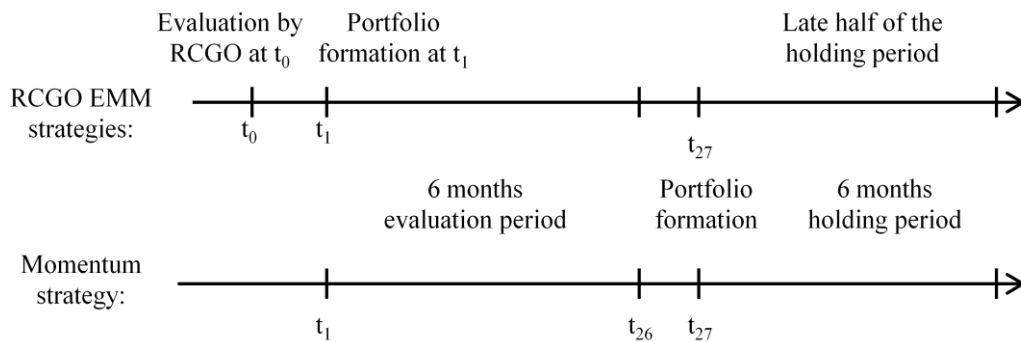
Despite a loss in the TMB strategy, a positive correlation between the strategy profits is still found. Both strategies are formed on the basis of a winner group minus a loser group. The positive correlation suggests that when the winner group in the momentum strategy have a better performance than the loser group, the same is likely to happen in the TMB strategy. The stocks with large gain have better returns compared with the stocks with large loss. This could suggest that the winner groups in the both strategies may include certain stocks in common that contribute to the performance of the winner groups.

### **6.3.3 Autocorrelation test and result**

This part reports the regression test over the momentum profit and the profits from EMM strategies. A high EMM strategy profit represents a strong return autocorrelation, and could contribute to the momentum effect.

In this test, there is a difference in the time horizon of the strategies. The momentum strategy remains a 6-month/6-month strategy, but the EMM strategy will adopt the one year time horizon. Moreover, for a momentum portfolio formed at a given time, its corresponding EMM portfolio is actually formed 6 months earlier. The formation time of the EMM portfolio equals to the beginning of the evaluation time of the momentum portfolio. However, in order to match the periodicity of the momentum strategy, the regression will use the EMM profit over the late half of the EMM holding period, which is from week 27 to week 52. The following figure demonstrates the time line of both the strategies.

Figure 6.3-2: Time line of the momentum test



The reason for such an arrangement of the strategies is to match the momentum strategy and the autocorrelation created by the influence of RCGO. Recall the existing theories on the decomposition of the momentum profit, the autocorrelation in the idiosyncratic component of stock return contributes to the profit. In terms of the time horizon, the theories also suggest that such an autocorrelation occurs through the entire J-month/K-month strategy horizon. More specifically, the return of a stock in the evaluation period is positively correlated to its return in the holding period. Regarding the RCGO model, it predicts a persistent upward trend in return evolution after a very large unrealized gain/loss. As a result, at the evaluation stage of the momentum strategy, since these stocks have performed well in the past J-month because of an extreme RCGO J-month ago, they will be selected in the winning group of the momentum strategy. In the following K months, since the upward trend in these stocks still persists, they will continue to perform well, and contribute to the momentum profit. For this reason, this test matches the 6-month/6-month momentum strategy to the 12-month RCGO EMM strategy, with the time line shown in the above figure.

The strategies are implemented in a rolling way. Each week, the same procedure is repeated. Finally, a time serial data of the strategy profits can be collected for regression test. The regression model in this test will also be a simple one that relates the EMM profit to the momentum profit.

This test will also adopt a two-step regression. The first step consists of a regression of strategy profit on the Fama-French factors. The residuals will be recorded for the second step, which is a single variable regression of residual of the momentum profit on the residual of the RMM profit:

$$R_{mo} = \alpha + \beta R_{EMM}$$

In the models the momentum profit will be denoted as  $R_{mo}$ , the profit from the RCGO Extremity minus Middle strategy will be denoted as  $R_{EMM}$ .

This test uses the same data as in previous tests in this dissertation. The same data refining criterion are applied: The sample contains stocks in the NYSE and AMEX within a period if from January 1982 to December 2012. The observations with stock price smaller than 5 dollars are eliminated for lack of liquidity. Valid observation requires having 150 continuous observations before it. The profit of the momentum effect is obtained from a 6-month/6-month strategy; the profit of the EMM is obtained from a 12-month RCGO Extremity minus Middle strategy. The following table reports the simple statistic and the correlation matrix of the variables.

Table 6.3-4: Descriptive statistics

| Strategy profits adjusted for risks (residuals) |        |         |           |        |           |
|---|--------|---------|-----------|--------|-----------|
| Variable  | Mean   | Std Dev | 10th Pctl | Median | 90th Pctl |
| $R_{mo}$  | 0.0333 | 0.2594  | -0.2039   | 0.0692 | 0.2584    |
| $R_{EMM}$                                       | 0.0370 | 0.0956  | -0.0746   | 0.0312 | 0.1572    |

The descriptive statistics shows that the EMM strategy yields a slightly higher profit than the 6-month/6month momentum strategy. Note that the  $R_{EMM}$  only count the strategy profit during the same holding period of 6 months as the momentum strategy. And the momentum profit has higher volatility. The regression result is reported in the following table.

Table 6.3-5: Regression on momentum profit and EMM profit

|  | Intercept | Coefficient | Adj-R <sup>2</sup> |
|--|-----------|-------------|--------------------|
| Model A: $R_{mo} = \alpha + \beta R_{EMM}$ |           |             |                    |
| Estimation                                 | 0.0458    | -0.3782     | 0.0185             |
| P-value                                    | <.0001    | <.0001      |                    |

Test result shows that there is a negative correlation between the 6-month/6-month momentum profit and the EMM profit. The intercept is positive. Both the coefficient and the intercept are significant at 99% confidence level. This result rejects Hypothesis 3 and 4.

Contrary to the expectation, this test gives opposite results to the hypotheses. The hypothesis expects a positive correlation between the momentum profit and the EMM profit, since the RCGO model predicts positive return autocorrelation in stock with extreme RCGO, and such an autocorrelation contributes to the momentum profit. However, the test finds a

negative correlation. It suggests that the momentum profit is actually lower when the EMM strategy yields a high profit. This result is paradoxical. However, the decomposition of the momentum profit could provide one way to explain this result.

Recall the decomposition of Lewellen (2002). Using the approach founded by Lo and MacKinlay, the momentum profit can be decomposed into the following form:

$$E[\pi_t] = \left(\frac{N-1}{N^2}\right) tr(\Gamma) - \frac{1}{N^2} [I'\Gamma I - tr(\Gamma)] + \sigma^2(\mu)$$

In the above formula, there is a negative term  $-\frac{1}{N^2} [I'\Gamma I - tr(\Gamma)]$ . The terms in the square brackets are the auto-covariance matrix of all stocks minus the trace of the matrix, which is the autocorrelation of each stock. Therefore, this term represents the cross-covariance between any two stocks, and between past time and present time. It means that if one stock performed well/bad in the past period, other stocks will perform well/bad in the next period. Since the entire term is negative, it suggests that a positive cross-covariance among the stocks will reduce the momentum profit.

Regarding the RCGO model, it predicts an upward trend in the stocks with large unrealized gain/loss. Such an upward trend does not only result in return autocorrelation for individual stocks, it also results in cross-correlation among the stocks with extreme RCGO: for one stock with a large gain/loss that performed well in the J-month, other stock with large gain/loss will also perform well in the K-month under the persistent influence of RCGO during the total period. With respect to the RCGO EMM strategy, if this strategy yields a high profit, it means the unrealized gain/loss has a strong influence on stock returns. As a result, there is not only a strong return autocorrelation for individual stocks, but there also is a strong positive cross-covariance among the large gain/loss stocks. The cross-covariance has a negative impact on the momentum profit as suggested by Lewellen's model, therefore the momentum strategy will yield a lower profit.

From a more intuitive perspective, the composition of the momentum portfolio could also help to explain the negative correlation between the momentum profit and the EMM profit. In the previous section, test show that the winner group of the momentum portfolio generally has a large unrealized gain while the loser group of the momentum portfolio has a large unrealized loss. According to the test results in the chapter 4, the stocks with large unrealized loss have in average a higher return than the stocks with large gain. When the



EMM strategy has a high profit, the influence of RCGO is strong. The gain group will perform well but the loss group will perform even better. For this reason, the momentum profit will decrease.

The result of this test suggests that the relation between the momentum effect and the influence of unrealized gain/loss is much more complex than expected. The unrealized gain/loss could on one hand contribute to the momentum effect. The upward trend in the stocks with large gain/loss could contribute to the formation of the winner group of the momentum effect, and the persistence of the influence from RCGO also helps the winner group to continue performing well. On the other hand it could also weaken the momentum effect. The upward trend also results in a positive cross-correlation among the large gain/loss stocks which reduces the momentum profit. Moreover, since the stocks with large loss have higher returns than the stocks with large gain, the total effect of the unrealized gain/loss is negative on the momentum profit.

Concerning the test method, the regression between the strategy profits aims to find the correlation between these two factors: the momentum effect and the influence of unrealized gain/loss. However, the profit of the EMM strategy could only represent the magnitude of the influence of the unrealized gain/loss, but cannot distinguish the return autocorrelation of individual stocks and the cross-correlation among all the stocks with large unrealized gain/loss. Distinguishing these two opposite effect on the momentum profit will be one possible approach for future study in this subject.

## **6.4 Summary**

This chapter focuses on applying the model of unrealized gain/loss on the momentum effect. First, by replicating the momentum strategy on the sample data, the test shows that the momentum effect is still documented over the period from 1982 to 2012.

Then by revising the previous chapters of this dissertation, two possible links between the momentum effect and the unrealized gain/loss are found. At an empirical level, the momentum portfolio could have a similar composition with a RCGO Top minus Bottom portfolio. At a theoretical level, the RCGO model predicts a return autocorrelation in the idiosyncratic component of stock return which is consistent to one source of the momentum profit identified by existing works.

Finally, tests are performed to examine the correlation between the momentum effect and the influence of unrealized gain/loss at both the empirical and theoretical levels. The test finds evidence supporting that the momentum and RCGO TMB portfolios share similar composition. But the test finds negative correlation between momentum profit and RCGO EMM profit. This result suggests that the influence of unrealized gain/loss has in general a negative impact on the momentum profit. More importantly, this test result shows that the relation between the momentum effect and the unrealized gain/loss is very complex. The upward trend in stock returns created by a large unrealized gain/loss could explain the formation and the performance of the winner group in the momentum effect. However, the same upward trend could also introduce a positive cross-correlation among stocks that weakens the momentum effect.

To sum up, the tests on the momentum effect do not show conclusive results. However, these test results proved interesting clues. The return autocorrelation and cross-correlation caused by the unrealized gain/loss could be the starting point for future studies.



# General conclusion

This dissertation aims to apply the alternative utility theories to explain the momentum effect. The theoretical work in this dissertation established a model under the framework of the alternative utility theories suggesting that an unrealized gain/loss could generate an upward trend in stock returns. It means that a large unrealized gain/loss could induce a positive autocorrelation in stock returns that contributes to the momentum effect. The empirical work in this dissertation documented the influence of unrealized gain/loss on stock performance. However, there is no conclusive empirical evidence proving that the autocorrelation induced by the unrealized gain/loss and the momentum effect are connected.

## Main results

This dissertation proposed a theoretical model under the framework of the alternative utility theories, and suggests that the unrealized gain/loss could influence the individual preferences. The application of the alternative utility theories is implemented on a decision making scenario in which the unrealized gain/loss is taken into consideration, while the amount of gain/loss is standardized by the outcome variation level. The advantage of such a configuration is that first, the risk factor is isolated from the unrealized gain/loss, and the risk level is controlled at one unit of risk. Second, this decision making scenario is essentially a classic risky versus riskless situation but with the unrealized gain/loss involved. Hence it is possible to make comparisons between the classic scenario and the scenario involving the unrealized gain/loss.

The application of the alternative utility theories is realized by calculating and comparing the expected utilities of the riskless and the risky options using the utility functions proposed by the prospect theory and the regret theory. The investors' preference for the Sell option, which is the riskless option with respect to the Hold option, which is the risky option, is measured by the difference in the expected utilities between the Sell option and the Hold option. The calculation shows that the investor's preference undergoes drastic changes along with the investors' unrealized gain/loss. When facing either a unrealized gain or a unrealized loss, an individual tends to have a decreasing preference for the riskless option compared with in a zero unrealized gain/loss situation. Recall that in the above decision making scenario, a zero unrealized gain/loss situation is equivalent to the riskless versus risky case in the classic utility theory. Therefore, this result suggests that the unrealized gain/loss does influence investor's decision making; and this factor is not included in the classic utility theory. In fact, when an unrealized gain/loss is present, individual will show less risk-aversion than that described in the classic utility theory.

In the financial market, the stock prices are influenced by the supply and demand of the stock. At the demand side of the stock, the demand consists of the investors who are looking for purchasing the stock. Since they are not actually holding the stock, their unrealized gain/loss is strictly zero. Therefore, their preference for buying or not buying the stock is described by the classic utility model. However, at the supply side of the stock, the supply consists of the willingness for selling the stock of the investors who are actually holding the stock. These investors are exposed to the unrealized gain/loss; hence their preference for selling or holding the stock is described by the model proposed in this dissertation. Therefore, in a situation where the investors are facing large unrealized gain or large unrealized loss, they will have very low propensity to sell the stock. At an aggregated level, if the majority of the investors who are holding the stock are at a large unrealized gain/loss, there should be a low supply of the stock. As a result, the stock will show an increase in stock price and yields a higher return after a large unrealized gain or a large unrealized loss.

Moving from an individual level to an aggregated level, the model of the unrealized gain/loss also moves from a static version to a dynamic version. Since the stocks are traded every day, the average acquisition price, a key variable for estimating the unrealized gain/loss, will be partially updated every day by the trading price of the stock at the current day. In the sample markets, the average weekly turnover is about 3%. It means that with such a low update rate, there is still more than 10% of the unrealized gain/loss measured today that will

not be updated 60 weeks later in the future. This dynamic version of the unrealized gain/loss model suggests that the influence of the unrealized gain/loss could persist over up to one year time horizon. If a large unrealized gain/loss occurs, the influence will continuously push the stock price upward over a period up to one year, and hence will result in a positive return autocorrelation.

The dynamic version of the unrealized gain/loss model also provides a theoretical link between the alternative utility theories and the momentum effect. The decomposition of the momentum profit suggests that a positive return autocorrelation is one source contributing to the momentum effect. Such an autocorrelation could exist at either a systematic level or at an idiosyncratic level. The former has been ruled out by existing research on the lead-lag effect of the factor realization. This dissertation suggests that a positive return autocorrelation could originate from a behavior factor related to the unrealized gain/loss. Since the behavior factor does not belong to the risk-return framework, it corresponds to the idiosyncratic level of stock performance. If a stock is experiencing a large unrealized gain/loss, it will yield a higher return in the following one year. During this period, the stock is likely to be selected in the winner group of the momentum strategy, and hence contribute to the momentum effect.

The above paragraphs summarize the theoretical work in this dissertation. The static model of the unrealized gain/loss is presented in the chapter 2; the dynamic model of the unrealized gain/loss is presented in the first half of the chapter 5; and the link between the momentum effect and the alternative utility theories is presented in the first half of the chapter 6. This dissertation also mentioned other potential applications of the unrealized gain/loss model. This model could be extended to describe investor's risk premium. It also provides a clue for explaining the formation of market bubbles.

The empirical tests in this dissertation are organized to check the theoretical models. Chapter 3 reports a preliminary test that replicates the test performed by Grinblatt and Han (2005). This test aims to examine the correlation between stock returns and the disposition effect, which is represented by the variable Capital Gain Overhang. Comparing with a positive correlation found by Grinblatt and Han's results, this replication test finds negative correlation between CGO and the weekly returns immediately following the CGO estimation. Such a negative correlation is consistent with a direct impact of an unbalanced stock supply caused by the disposition effect. The disposition effect describes the phenomenon that investors tend to sell a winning stock too soon and to hold a losing stock too long. It implies

that there is an excessive selling pressure on the winning stocks and an insufficient selling pressure on the losing stocks. As a result, the winning stocks should have lower returns while the losing stocks should have higher returns. The test result of the replication test shows evidence for the above reasoning. Over the immediate time horizon, the stock prices react to the unbalanced forces of supply and demand.

Chapter 4 reports tests on the static model of unrealized gain/loss. The model predicts that investors have a lower propensity to sell the stocks with either a large unrealized gain or a large unrealized loss. This prediction suggests that there will be insufficient selling pressure in these stocks, and through the market mechanism observed from the test in chapter 3, these stocks should yield higher returns. The tests in chapter 4 include a sorting test, a regression test with two different models, and a strategy simulation. The sorting test finds that by sorting the stocks into 10 deciles according to their Relative Capital Gain Overhang, a variable representing the unrealized gain/loss, the deciles at the two extremities yield higher returns than the deciles in the middle, over a period of 6 months to one year. Such a cross sectional relation between RCGO and stock returns is then verified by regressions. One regression model is the one used by Grinblatt and Han (2005), the other regression model is based on the Fama-French model in order to control for risks. Both regressions find a negative correlation between RCGO and stock returns in the loss situation, and a positive correlation in the gain situation. Finally an investment strategy is proposed based on the test results. This strategy consists of longing the stocks with extreme RCGO while short selling the stocks with neutral RCGO. The result shows that holding such an Extremity minus Middle portfolio could yield a positive profit up to 8% over one year period.

Chapter 5 studies the dynamic version of the unrealized gain/loss model. The empirical tests in this chapter aim to verify the time serial evolution of stock returns under the influence of the unrealized gain/loss. The tests first compare the evolution of the average cumulative abnormal returns among the stocks with large loss, neutral gain/loss, and large gain over the horizon of one year after the estimation of RCGO. The groups with large gain or large loss show a steeper slope in the graphe. This observation is then confirmed by a time serial regression. This regression examines the correlation between the cumulative abnormal returns and time. It also introduces dummy variables to distinguish these three groups. The neutral gain/loss group is set as the baseline, and the regression find positive and significant coefficients for both the large gain group and the large loss group. This result suggests that the large gain group and the large loss group show a stronger upward trend in stock returns

compared with the middle group. This test confirms the prediction made by the dynamic model of unrealized gain/loss that this latter could generate a positive autocorrelation in stock returns.

Chapter 6 links the momentum effect and the alternative utility theories with the positive return autocorrelation related to the unrealized gain/loss. The empirical tests first examine the similarity between the momentum strategy and strategies based on unrealized gain/loss. The result suggests that the winner group in the momentum strategy is likely to consist of stocks with relatively large unrealized gain. The influence of the unrealized gain could contribute to the performance of the winner group. Next, tests examine the correlation between the autocorrelation caused by the unrealized gain/loss and the momentum profit. The test uses the profit from the Extremity minus Middle (EMM) strategy to represent the autocorrelation, and expects a positive correlation between momentum profit and EMM profit. However, the regression finds a negative correlation between the EMM profit and the momentum profit. This unexpected result could be explained by a strong cross-correlation among the stocks that is also caused by the unrealized gain/loss, and the EMM profit cannot isolate the autocorrelation from the cross-correlation. According to Lewellen's decomposition of the momentum profit, such a positive cross-correlation cancels the momentum effect. The overall conclusion extracted from this dissertation is that the behavior factors captured by the unrealized gain/loss could partially contribute to the momentum effect, but the cause of this anomaly could be very complex; more detailed studies are still needed to explain the momentum effect.

## **Major contributions**

This dissertation contributes to the existing literature in the following aspects. Regarding the theoretical aspect, this dissertation proposed one approach to apply the alternative utility theories, in particular the prospect theory and the regret theory, to the studies on financial markets. The model of the unrealized gain/loss proposed in this dissertation is able to provide a different prediction from that of the classic utility theory on investor preference between a riskless option and a risky option. Based on such a difference in the investor preference, this dissertation documents some patterns in stock performances that are consistent with the theoretical model. These results could validate this application of the alternative utility theories to the financial markets; and also suggest that the properties or the



behavior patterns proposed by the prospect theory and the regret theory can be observed on the financial markets.

The model of the unrealized gain/loss, developed from a static version to a dynamic version, provides a complete description on how a behavior factor is integrated in the decision making on the market, then influences stock return over intermediate to long term horizons, and finally contributes to the momentum effect. This model also shows the potential to describe changes in investor's risk premium. It suggests that the application of the alternative utility theories proposed in this dissertation could be on a good track towards more profound and sophisticated studies integrating the behavior factors within the core of the classic financial theories such as the risk premium, and the risk-return framework.

Regarding the empirical aspect, inspired by the work of Grinblatt and Han, this dissertation developed the variable Relative Capital Gain Overhang. This variable is supported by the model of unrealized gain/loss, hence it represents a very specific factor: changes in investor preferences in response to exposure to the unrealized gain/loss due to loss aversion and regret aversion. This characteristic distinguishes the RCGO. One of the major difficulties in studies in the field of behavioral finance is to measure a behavior factor. Since the behavior factors are beyond the financial system, the studies have to use proxy variables that are within the financial or economic system to approximate the behavior factors. This compromise results in downgrades in the accuracy and the quality of the studies. The RCGO overcomes this difficulty. This variable is derived under the framework of the alternative utility theories; moreover, it is estimated using financial data. Therefore, the RCGO could provide an accurate and reliable measure of the behavior factor.

This dissertation also proposed an investment strategy, called the Extremity minus Middle strategy, based on the model of unrealized gain/loss. This strategy consists of longing the stocks with either a very large unrealized gain or a very large unrealized loss while selling short the stocks with a neutral unrealized gain/loss. Empirical test shows that this strategy could yield positive profit. From an empirical point of view, the positive profit documented by applying this strategy can be considered as a market anomaly under the classic framework of risk-return; and can only be explained by behavior factors. It is evidence against an efficient market because this strategy is able to make profit by using only past information, namely the unrealized gain/loss, which is estimated from past stock prices and trading volumes. Moreover, different from other well-known anomalies such as the time effect, the

size effect, and including the momentum effect, which are found by pure empirical discovery, the EMM strategy is found under the guidance of a theoretical model. The profitability of this strategy shows the power of the alternative utility theories.

The EMM strategy also carries managerial value. This strategy could be easily implemented in practice. On one hand, the RCGO can be estimated using only past stock prices and trading volumes. On the other hand, the implementation could consist of simply picking up the stocks with high positive RCGO or with low negative RCGO when selecting one's portfolio. Therefore, the RCGO could be a new element to look at when investing over intermediate to long time horizons.

### **Limitation and future research**

This dissertation accomplished its task in applying the alternative utility theories to explain the momentum effect. However, there are still some limitations that could be improved in future research.

First, the theoretical model of the unrealized gain/loss could only provide predictions on investor preferences. To answer the question of how such preferences impact stock performance, this dissertation relies only on empirical results. The model of the unrealized gain/loss deals with investor's decision making, but does not incorporate any element about asset pricing. This latter point could be one path for future research.

Second, the empirical tests on the momentum effect are not satisfying. They could not confirm the prediction made by the dynamic version of the unrealized gain/loss model. The theoretical model is able to deduce that the return autocorrelation could contribute to the momentum effect, but in the empirical test, the dependent variable of the regression is not able to isolate return autocorrelation from other influences. In future research, empirical tests will continue to look for evidence that supports the theoretical model. Improvements can be made on finding better variables to represent the return autocorrelation caused by behavior factor. More sophisticated methods will also be applied to verify if the behavior factor induces return autocorrelation and its link to the momentum effect.

Third, a limited access to financial data is one important restriction to the empirical studies in this dissertation. One major issue in this regard is that the empirical tests have to

use the Fama-French factor portfolio data downloaded from the Data Library of Kenneth French to control for risk factors. Because this data contains Nasdaq, a more profitable market, the risk adjusted returns estimated from the Fama-French model tend to have negative values. This problem can be observed in the sorting test in chapter 4 and in the test on the cumulative abnormal returns in chapter 5. In future research, an advanced database will solve this problem. Moreover, if detailed data including individual level trading information is available, it could help to give much more accurate estimation on the average acquisition price and the Relative Capital Gain Overhang of a stock.

Following this dissertation, future research could be carried out in two directions. The first one consists in elaborating the model of unrealized gain/loss in depth. As mentioned in the previous paragraphs, future studies on this model are expected to reveal the asset pricing mechanism through investor preference and behavior factors. In the section 2.5.2, this dissertation has shown the possibility to incorporate the unrealized gain/loss with risk premium, which is an underlying concept in asset pricing. The second direction consists of applying the model of unrealized gain/loss to other market anomalies. In this dissertation, this model is applied to explain the momentum effect. This application is still an interesting subject for future research. Moreover, the unrealized gain/loss model could also be applied to study a more generic anomaly known as the under-reaction/over-reaction of stock prices. In the section 5.2.2, this dissertation also mentioned the possibility to apply the dynamic model of unrealized gain/loss to the financial bubbles, because the dynamic model could describe a self-enhancing process of an upward trend in the stocks with large unrealized gains.

To sum up, this dissertation shows one approach to apply the alternative utility theories to the studies in financial markets, and the results are satisfying. There are some limitations to this work, but it also opens up a path for promising future research.

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# List of tables:

|   |     |
|---|-----|
| Table 1.1-1: Momentum profit documented by Jegadeesh and Titman (2001) .....                    | 14  |
| Table 1.4-1: Earning realization in the reversal and the trend states .....                     | 41  |
| Table 1.4-2: Transition of the reversal and the trend states .....                              | 42  |
| Table 1.5-1: The certainty effect in decision making.....                                       | 54  |
| Table 1.5-2: Percentage of participants deciding for the safer or riskier option .....          | 57  |
| Table 1.5-3: Summary of experiment questions in Cagno and Hey (1988).....                       | 64  |
| Table 1.5-4: The measurement method of Bleichrodt et al. (2010) .....                           | 67  |
| Table 1.5-5: Simulation of Barberis and Xiong (2009) .....                                      | 77  |
| Table 2.2-1: The data from Bleichrodt Cillo and Diecidue (2010)'s measurements .....            | 94  |
| Table 2.2-2: Estimation of the Q function parameters .....                                      | 94  |
| Table 2.4-1: Comparison between results using different g functions.....                        | 111 |
| Table 3.1-1: The result of Grinblatt and Han's test (2005) .....                                | 135 |
| Table 3.3-1: Summary statistic of variables .....   | 150 |
| Table 3.3-2: The correlation matrix of the regressors .....                                     | 152 |
| Table 3.3-3: Average coefficients and their t-statistics of the model without CGO .....         | 153 |
| Table 3.3-4: Average coefficients and their t-statistics of the full model .....                | 155 |
| Table 4.2-1: The moments and quantiles of RCGO .....  | 174 |
| Table 4.3-1: The cutting values of RCGO deciles .....   | 177 |
| Table 4.3-2: Average stock return on RCGO deciles.....  | 178 |
| Table 4.3-3: Difference in average return between the middle deciles and other deciles.....     | 179 |
| Table 4.4-1: Descriptive statistics of the variables .....                                      | 183 |
| Table 4.4-2: Matrix of correlation of the variables .....                                       | 184 |
| Table 4.4-3: Estimation of coefficients of the single group regression .....                    | 185 |
| Table 4.4-4: The double grouped test result.....  | 187 |
| Table 4.4-5: The loss and regret aversion mode, four-group test .....                           | 189 |
| Table 4.5-1: Summary statistic of stock returns and RCGO .....                                  | 193 |
| Table 4.5-2: Coefficients of the single grouped residual regression.....                        | 195 |
| Table 4.5-3: Coefficients of the double grouped residual regression .....                       | 198 |
| Table 5.3-1: Time serial regression on cumulative abnormal return over time .....               | 227 |
| Table 6.1-1: The profits of J-Month/K-Month strategy .....                                      | 234 |
| Table 6.3-1: 6-Month/6-Month portfolio return and the average RCGO at the evaluation time ..... | 250 |
| Table 6.3-2: Descriptive statistics .....   | 253 |
| Table 6.3-3: Regression on momentum profit and TMB profit .....                                 | 253 |
| Table 6.3-4: Descriptive statistics .....   | 256 |
| Table 6.3-5: Regression on momentum profit and EMM profit .....                                 | 256 |



# List of figures:

|  |     |
|--|-----|
| Figure 1.1-1: The time line of the J-month/K-month strategy .....  | 12  |
| Figure 1.1-2: The timeline of Carhart's PR1YR factor.....  | 17  |
| Figure 1.4-1: Illustration of the Grinblatt and Han's model .....  | 47  |
| Figure 1.4-2: Illustration of Frazzini's hypothesis .....  | 50  |
| Figure 1.5-1: The S-shaped value function .....  | 70  |
| Figure 1.5-2: The decision weighting .....   | 71  |
| Figure 1.5-3: Holding the risky asset is preferred in the case of loss .....                               | 74  |
| Figure 1.6-1: Connection between literature fields.....  | 79  |
| Figure 2.2-1: The graphic of $Q(x,y)=(x-y)+e^{(x-y)}-e^{-(x-y)}$ .....                                     | 92  |
| Figure 2.2-2: Graphic of Bleichrodt Cillo and Diecidue (2010)'s measurements .....                         | 93  |
| Figure 2.2-3: The graphic of $Q(x,y)=36(x-y)^3+3(x-y)$ .....   | 95  |
| Figure 2.3-1: The S-shaped utility function .....  | 99  |
| Figure 2.3-2: The preference for Sell depending on unrealized gain or loss .....                           | 101 |
| Figure 2.3-3: Investor's preference in the large loss situation .....                                      | 103 |
| Figure 2.3-4: Investor's preference in the small loss situation .....                                      | 104 |
| Figure 2.3-5: Investor's preference in the small gain situation.....                                       | 105 |
| Figure 2.3-6: Investor's preference in the large gain situation .....                                      | 106 |
| Figure 2.4-1: The preference for selling the stock of a loss aversion and regret aversion investor.....    | 110 |
| Figure 2.4-2: Comparison between regret and loss aversion model and prospect theory.....                   | 114 |
| Figure 2.5-1: Investor's preference to Sell stock under combined model .....                               | 116 |
| Figure 2.5-2: Comparison of utility of riskless and risky options.....                                     | 119 |
| Figure 2.5-3: The preference curve for the new lottery .....   | 122 |
| Figure 2.5-4: Possible integration with expected utility functions.....                                    | 124 |
| Figure 2.6-1: Influence of RCGO on preference.....   | 127 |
| Figure 3.2-1: Time line of estimating CGO .....  | 143 |
| Figure 3.2-2: The immediate and subsequent effect of CGO .....   | 144 |
| Figure 3.3-1: Evolution of the number of stocks included in the regression .....                           | 146 |
| Figure 3.3-2: Evolution of different types of Capital Gain Overhang overtime .....                         | 151 |
| Figure 4.1-1: Investor's preference for selling with RCGO.....   | 164 |
| Figure 4.1-2: The influence of RCGO in a simplified form.....  | 170 |
| Figure 4.2-1: Distribution of RCGO .....   | 175 |
| Figure 4.2-2: Cumulative distribution of RCGO .....  | 175 |
| Figure 4.2-3: Evolution of RCGO, 10 <sup>th</sup> percentile, median, and 90 <sup>th</sup> percentile..... | 176 |
| Figure 4.3-1: Average raw return on RCGO deciles.....  | 178 |
| Figure 4.3-2: Average return residual of RCGO deciles.....   | 180 |
| Figure 4.6-1: Time line of the EMM strategy .....  | 200 |

|   |     |
|---|-----|
| Figure 4.6-2: The EMM portfolio profit adjusted by Fama-French factors.....                             | 202 |
| Figure 5.1-1: Interaction between the stock price and the average acquisition price .....               | 210 |
| Figure 5.1-2: Evolution of stock price and the acquisition price over time in SMALL GAIN situation..... | 212 |
| Figure 5.1-3: Evolution of stock price and the acquisition price over time in SMALL LOSS situation..... | 212 |
| Figure 5.1-4: Evolution of stock price and the acquisition price over time in LARGE GAIN situation..... | 213 |
| Figure 5.1-5: Evolution of stock price and the acquisition price over time in LARGE LOSS situation..... | 214 |
| Figure 5.2-1: Weight of past purchase cost in current acquisition price.....                            | 217 |
| Figure 5.3-1: Illustration of the dynamic model test timeline.....                                      | 222 |
| Figure 5.3-2: Cumulative return of RCGO quintile 1, 3, and 5 over 52 weeks .....                        | 223 |
| Figure 5.3-3: Cumulative abnormal return of RCGO quintiles over 52 weeks .....                          | 225 |
| Figure 6.1-1: Time line of the J-month/K-month strategy .....   | 234 |
| Figure 6.3-1: Time line of the test.....  | 252 |
| Figure 6.3-2: Time line of the momentum test.....   | 255 |

## Résumé

Cette thèse applique la prospect theory et la théorie du regret à l'étude de la performance des actions et à explication d'une anomalie de marché bien connue, l'effet momentum. Un modèle théorique est proposé liant les facteurs comportementaux à la performance des actions et à l'effet momentum, puis des vérifications empiriques sont effectuées. Dans le chapitre 2, le modèle est établi sur un concept de gains/pertes potentiels, indiquant si un investisseur se trouve actuellement dans une situation gagnante ou perdante. Ensuite, le modèle montre que les investisseurs sont très réticents à vendre leurs actions dans une situation de grands gains ou de grandes pertes. Les chapitres 3 et 4 présentent des tests empiriques sur le modèle de gains/pertes potentiels. L'échantillon de données comprend toutes les actions du NYSE et de l'AMEX de 1982 à 2012. Les tests sont en mesure de confirmer l'influence des gains/pertes potentiels sur les rentabilités des actions. En outre, une stratégie à coût nul baptisée Extrémité moins Moyen (EMM), consistant à détenir les titres à forts gains ou pertes potentiels et vendre à découvert les titres à faible gain ou pertes potentiels, est mise en œuvre et s'avère rentable après contrôle des risques. Dans le chapitre 5, le modèle des gains/pertes potentiels est développé dans une version dynamique. Il suggère que l'influence des gains/pertes potentiels pourrait persister sur un horizon moyen et long terme, et générer une tendance à la hausse des actions présentant un(e) grand(e) gain/perte potentiel(le). Les tests empiriques dans ce chapitre se concentrent sur l'évolution des séries temporelles des rentabilités. Les tests montrent que les actions présentant un(e) grand(e) gain/perte potentiel(le) ont une plus forte tendance à la hausse. Le chapitre 6 applique les résultats du chapitre précédent pour expliquer l'effet momentum. La tendance à la hausse correspond à une auto-corrélation positive des rentabilités, ce qui est l'une des sources contribuant au profit momentum. Les tests empiriques dans ce chapitre explorent la similitude entre la stratégie momentum et celle fondée sur les gains/pertes potentiels, et examinent également la corrélation entre le profit momentum et le profit de la stratégie EMM. Les tests montrent que les gains/pertes potentiels pourraient contribuer à l'effet momentum, mais ne suffisent pas à l'expliquer. L'effet momentum semble être le résultat d'une combinaison de plusieurs facteurs complexes.

Mots clés: finance comportementale, théorie des prospects, théorie du regret, capital gain overhang, auto-corrélation des rendements, *l'effet momentum*

## Abstract

This dissertation applies the prospect theory and the regret theory to the study of stock performance and tries to explain one well-known market anomaly called the momentum effect. The dissertation proposes a theoretical model that links the behavior factors to stock performance and the momentum effect, and performs empirical tests to check the theoretical model. In chapter 2, the model is established on the concept of the unrealized gain/loss, which indicates if an investor is currently at a winning or a losing position. The model then shows that the investors are highly reluctant to sell their stocks experiencing a large gain or a large loss. The chapters 3 and 4 perform empirical tests on the model of unrealized gain/loss. The test sample includes all stocks in NYSE and AMEX from 1982 to 2012. The tests confirm the influence of the unrealized gain/loss on stock returns. Moreover, a zero-cost Extremity minus Middle (EMM) strategy consisting in holding stocks with high potential gain/loss and short selling stocks with low potential gain/loss is carried out and turns out to be profitable after controlling for risks. In chapter 5, the model of unrealized gain/loss is developed into a dynamic version. It suggests that the influence of a unrealized gain/loss could persist over intermediate to long term horizons, and generates an upward trend in performance for stocks with large unrealized gain/loss. The empirical tests in this chapter focus on the time serial evolution of returns. The tests show that stocks with large unrealized gain/loss have a stronger upward trend. The chapter 6 applies the results from the previous chapter to explain the momentum effect. The upward trend corresponds to a positive return autocorrelation, which is one of the sources that contribute to the momentum profit. The empirical tests in this chapter look into the similarity between the momentum strategy and the one based on unrealized gain/loss, and also examine the correlation between the momentum profit and the profit from the EMM strategy. Tests show that the unrealized gain/loss could contribute to the momentum effect, but is not the only source. The momentum effect could be the result of a combination of many complex factors.

Keywords: behavioral finance, prospect theory, regret theory, capital gain overhang, return autocorrelation, momentum effect