La théorie des jeux pour l’allocation de ressources dans les réseaux à petites cellules
Kenza Hamidouche

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Par

Ms. Kenza Hamidouche

La Théorie des jeux pour l’allocation de ressources dans les réseaux à petites cellules

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Acronyms

BGE Boltzmann-Gibbs equilibrium. 66–70, 72
BMMG backhaul management minority game. 61, 63, 64, 66
BMRL backhaul management reinforcement learning. 70–72
BR best response. 39, 40, 43
CGA centralized greedy algorithm. 70–72
CP content provider. vii, 35–45
DE Debreu equilibrium. 75, 79, 80
FPK Fokker-Planck-Kolmogorov. 25, 30, 32, 33
HetNet heterogeneous network. 35, 36, 41, 42
HJB Hamilton-Jacobi-Bellman. 25, 29–33
LBT listen-before-talk. 75
LTE Long Term Evolution. vii, 75–79, 82
MBS macro base station. 60, 61, 64, 69
MDS maximum distance separable. 26
MFG mean-field game. 25
MG minority game. 59, 61–64, 66
MNO mobile network operator. vii, 35–43, 45
NE Nash equilibrium. 35, 39–43
OCA optimal centralized algorithm. 71, 72
PMNE proper mixed Nash equilibrium. 59, 64–66, 69, 73
PNE pure Nash equilibrium. 63
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<td>SINR</td>
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<td>WU</td>
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Chapter 1

Introduction and Background

The exponential growth of mobile devices such as smartphones and tablets, coupled with proliferation of online social networks have considerably increased the traffic in cellular networks. In contrast to classical cellular traffic that was only based on voice and audio communications, the recent technologies have resulted in bandwidth-intensive services such as video streaming, and video conferencing which represent more than 50% of the traffic exchanged between users [1]. This traffic surge has strained the capacity of existing wireless networks which makes it difficult for the operators to ensure the high quality-of-service (QoS) required by the evolving cellular services. To deal with the limited capacity of existing cellular networks and keep up with the strict QoS requirements, in terms of data rate and delay tolerable application-specific delays, a new generation of wireless networks has emerged. To achieve the requirements of this new generation and provide efficient infrastructure support for this data deluge, several research challenges must be addressed and solved.

This chapter is organized as follows. In Section 1.1, an overview of emerging wireless networks is provided. Moreover, distributed caching methods and LTE over unlicensed bands techniques are introduced as a way of dealing with the challenges driven by the emerging networks. Section 1.2 reviews existing works on distributed caching in small cell networks and LTE over unlicensed channels. The main contributions of this thesis are discussed in Section 1.3.

1.1 Emerging Wireless Heterogeneous Networks

In this section, we overview next-generation cellular networks and discuss briefly the challenges that need to be addressed before the deployment of these systems.


1.1. Emerging Wireless Heterogeneous Networks

1.1.1 Small cell networks

To support the growth of mobile traffic demands, mobile network operators have evolved their networks from classical wireless networks that are based on the sparse deployment of macro base stations (MBSs) to heterogeneous small cell networks (SCNs) [2,3]. In SCNs, a large number of small base stations of different types, such as picocells and femtocells, is densely deployed closer to the end-users. These short range and low power small base stations are used to offload the MBSs or complement their coverage [4]. Based on the concept of frequency reuse in SCNs, a large number of small cells may reuse the same spectrum locally resulting in increased spectral efficiency, improved signal quality, better cellular coverage in rural and isolated areas, and higher capacity at the edge of the macro cells. The main differences and new challenges that arise in SCNs can be given as follows [3,5,6]:

- **Heterogeneity of the backhaul:** In SCNs, the SBSs cannot all be connected to the core network via fiber backhaul due to the cost and geographical deployment challenges in urban environments. The SBSs are thus connected to the core network via capacity-limited wireless backhaul links which makes it difficult for the SBSs to meet the QoS requirements of the users, especially during peak hours. The capacity of conventional wireless networks is inherently limited because of the limited licensed spectrum. Thus, to support all the traffic surge, MNOs have started considering the use of all the available and underused spectrum such as high frequency bands via millimeter wave communication and the exploitation of unlicensed bands. This results in a heterogeneous wireless backhaul with different characteristics making high capacity and reliable communications via the backhaul one of the key challenges of SCNs deployment.

- **Dense deployment of SBSs:** One of the solutions for boosting the capacity of wireless networks, is the extensive spatial reuse through the dense orthogonal deployment of small cells. In fact, the number of SBSs is already larger than the number of deployed MBSs and is envisioned to exceed the number of mobile devices. The dense deployment of small sized cells creates more boundaries resulting in more frequent handovers to support mobile users compared to traditional cellular networks. This comes with the expense of significant signaling overhead, transmission delays and packet losses which affect considerably the QoS experienced by the end-users. Classical handover techniques that are based on the received signal power are less efficient in SCNs as all SBSs with nearly equal strength will result in very frequent handovers between the multiple small cells. Thus, new techniques must allow the SBSs to cooperatively serve the users while accounting for the heterogeneous and capacity limited backhaul that connect the base stations to one another.

- **Energy consumption:** In contrast to conventional cellular networks in which the MBSs are continuously powered, in SCNs, it is not always possible to provide grid power to all SBSs due to their possible locations which could be hard-to-reach in rural zones, or due to the remote deployment in outdoor environments. One way
1.1. Emerging Wireless Heterogeneous Networks

to deal with this problem is through the development of wireless energy harvesting
techniques to power the SBSs either from renewable energy sources or from dedi-
cated energy sources. Moreover, the introduction of millimeter wave backhaul in
small cell networks induces a significant increase of power consumption compared
to classical wireless networks. In fact, the SBSs require more power to perform the
beamforming techniques which are necessary for transmissions over high frequency
bands, especially when the SBSs are equipped with multiple antennas. Thus, in
contrast to classical cellular networks in which the energy efficient techniques ig-
nore the processing power and only account for optimizing the transmit power, new
energy efficient approaches need to be developed for SCNs.

The ever increasing cellular traffic and the dense deployment of SBSs have made of
backhaul the major bottleneck of SCNs due its limited capacity. To cope with this prob-
lem that is essential to fully reap the potential of SCNs, new technologies have emerged
including distributed caching and backhaul management techniques as discussed in the
following sections.

1.1.2 Distributed caching in small cell networks

The idea of distributed caching has emerged based on two observations, the nature of the
traffic popularity and the availability of storage space at very low prices. In fact, only
20% of the content is responsible of more than 80% of the traffic load while the prices of
storage units with a capacity in the order of terabits keep decreasing. These two factors
have led to the concept of distributed caching which consists in equipping the SBSs with
storage units to reduce the amount of redundant requests that are served via the backhaul
[7]. For this, the SBSs predict the requests of users and download the most popular files
ahead of time during peak-off hours, and cache these files in their storage units. Thus,
the SBSs can serve most of the requests locally without using the backhaul allowing to
significantly reduce the backhaul load during peak hours. The idea of distributed caching
was further extended to users’ devices with the introduction of device-to-device (D2D)
communications as part of future cellular networks. Such network architecture allows users
that are endowed with large storage capacities to cache content in their devices and share
the cached content by communicating directly with one another via D2D communications.
An illustration of a caching system is provided in Figure 1.1.

To successfully deploy SCN caching solutions, MNOs require the cooperation of the
CPs that have to share their contents with the MNOs and provide these contents’ global
popularity to the MNOs [8,9]. However, although CPs can improve the QoS of their users
by caching, they might be reluctant to share their content with the MNOs. This can be due
to reasons such as privacy since, once the content cached at the SBSs, the MNOs can get
access to all the CP’s files as well as the traffic dynamics of the users subscribed to the CP
[8]. Thus, the MNOs must provide incentives to the CPs to share their data and cache it
at the SBSs by introducing suitable economic arrangements that can be beneficial for both
the MNO and the CPs. On the other hand, the gain of the users from the caching process
1.1. Emerging Wireless Heterogeneous Networks

depends on the caching policy that is used by the MNO which consists in determining which file to cache at each of the SBSs and when. To develop efficient caching policies, the MNOs need to account for multiple network properties at the SBSs such as the dynamics of the content popularity, the position of the base stations (BSs), their traffic load as well as the limited capacity of their storage units. Moreover, the contents requested by users exhibit significant similarities that can be captured by exploiting their social interactions in online social networks (OSNs). These caching problems, from different perspectives, shall be addressed and analyzed in the first part of this thesis.

1.1.3 LTE over unlicensed bands

All the current mobile communication systems operate on the licensed spectrum, especially the low-frequency bands, which is limited, and is rapidly getting congested with the dense deployment of SBSs and the growing number of mobile users. This has compelled network operators to exploit the readily available unlicensed spectrum by enabling LTE-U BSs to aggregate the licensed LTE spectrum with the unlicensed 5 GHz band that is currently used primarily for WiFi [10]. However, although offloading part of the LTE traffic to the unlicensed bands can considerably increase the performance of cellular networks, the heterogeneity of the devices coupled with the disparity of the medium access protocols that are used by the WiFi access points (WAPs) and the LTE-U BSs, rise new challenges that need to be addressed before these networks can reach their true potential [11]. To ensure a fair usage of the unlicensed bands regulations are imposed forcing the BSs to use the same medium access policy as the WiFi access points (APs). In these networks, the WAPs sense the channel and transmit only if the sensed interference level does not exceed a predefined threshold. In countries such as United States, China and South Korea, new policies need to be developed to ensure a harmonious coexistence of LTE and WiFi over
1.2. Related works

Multiple techniques have been advocated recently by academia and industry to prevent the LTE BSs from harming the WiFi users and can be classified into licensed assisted access (LAA) and LTE-U. In LAA, an listen-before-talk (LBT) like technique is used to access the unlicensed bands with longer sensing periods. Based on the observed medium activities, the BSs can then determine the fraction of time they use the channels. On the other hand, LTE-U is a more open model in which BSs do not have any regulation constraints regarding the fraction of time they use the unlicensed channels or the interference caused to the WiFi users. However, regular users (i.e., the common people using WiFi), are starting to raise serious objections about cellular operators taking over their bands [12]. In fact, without regulatory requirements in LTE-U, LTE BSs can significantly impact the performance of WiFi users as WiFi users cannot access the channels when they sense a high interference level from the continuously transmitting BSs. Thus, LTE-U introduces new challenges that must be addressed to make its impact on WiFi of equal magnitude as that of another WiFi network. The coexistence problem of LTE and WiFi over unlicensed bands is addressed in the second part of this thesis.

Figure 1.2: LTE-U network model.

unlicensed bands. An illustration of an LTE-U cellular system is provided in Figure 1.2.

In this section, we overview the existing literature on both distributed caching and LTE-U coexistence approaches.
1.2. Related works

1.2.1 Distributed caching in cellular networks

Beyond caching in small cells, there has been considerable works on caching in the computer science community. The closest caching models to the considered one in this thesis, is caching in content delivery networks and content centric networks [13, 14]. The idea consists in storing data at the closest proxy servers of the content delivery networks to the end users, known as the network edge. The goal from this approach is to balance the load over the servers, reduce the bandwidth requirements and thus reduce the users service time [15]. Content centric networks rely on the same idea of caching with more intelligent forwarding strategies. Indeed, the content files are identified by name instead of their location, allowing to spread the content all over the Internet network in a smart way [16, 17]. Recently, the idea of caching was introduced in cellular networks to deal with the capacity-limited backhaul in small cell networks [14, 18]. Despite the similarities with caching in the Internet, the network structure of SCNs is significantly different from Internet architecture. Thus, new challenges arise in SCNs such as accounting for channel characteristics and interference, that make the previously proposed approaches for the Internet not applicable, as discussed in [19–23]. This led to the recent emergence of a large literature that aims to address the caching problem while taking into account the specific characteristic of SCNs, as discussed previously. Here, we classify these works based on their similarities and directions.

Caching and content popularity estimation

The optimality of caching policies in SCNs relies on the accuracy of files popularity estimations. Several works have addressed this problem and showed the key role of exploiting contextual information such as users’ content viewing history, social connections that can be made available from D2D interactions and online social networks. In this regard, the authors in [24] proposed a transfer-learning approach in which the SBSs can exploit the social interactions between users, referred as source domain, to estimate the popularity of the files, referred as target domain. It is shown that the proposed model engenders significant backhaul offloading gains and resource savings as compared to random caching, and estimation methods that are based on collaborative filtering. The work in [25] proposed an online learning approach that captures the fast evolution of files popularity and then designed an efficient caching policy for trend caching systems. Moreover, the authors proposed a collaborative algorithm that enables a given SBS to exploit the learning results at the neighboring SBSs with whom it has a high level of similarity to improve the performance of its local learning estimations. The authors in [26] proposed a caching algorithm in cloud access radio networks, in which the caching nodes can learn the distribution of the files’ popularity as well as the mobility of the users using echo state networks. A sub-linear algorithm is then proposed to determine the cache placement policy. In [27], the authors proposed a coded caching framework in which the SBS learn the popularity of the files based on reinforcement learning algorithms. To determine the caching policy based on the popularity estimation, the problem is formulated as a linear program.
relaxation of the problem is then derived and solved. The social interactions between users is exploited in [28] where users have caching capabilities and can share the content via D2D communications. In this system, the SBSs predict the set of most influential users and cache the files at the users and SBSs based on files’ popularity. The impact of the correlation of users’ interest on the caching policy is considered in [29]. The authors in [30] assumed that the SBSs do not have any information about the files popularity and can only observe the instantaneous demand. Using multi-armed bandit theory, three algorithms are proposed to define the caching policies without knowing the popularity of the files. Similarly in [31], the authors address the problem of cache placement when only the instantaneous estimation of the content popularity is known at the SBSs. A central-ized algorithm is proposed to simultaneously estimate files popularity and determine the most efficient caching policy.

Coded caching

Coded caching enables the SBSs to cache fractions of a given a file at multiple SBSs. To this end, the files are coded using different coding schemes such erasure generating codes or maximum-distance separable (MDS) codes. Thus, a user can be served from multiple SBSs and recover the original file after collecting a given number of fractions of the file that it requested. The idea of coded caching was introduced in [32] where the authors showed that in contrast to single-SBS caching systems, caching the most popular files in a SCN composed of multiple SBSs is not the most optimal caching policy. A grouped coded caching scheme is proposed to minimize the expected load of the backhaul links while assuming that the popularity of the files follow a heavy-tailed distribution. An information theoretic formulation of the caching problem is introduced in [33] and a coded caching scheme that exploits both local and global gains of caching is proposed. The fundamental lower bounds on the rate required for serving the files, using the proposed coding scheme, are derived and the rate of the user is shown to increase within a factor 12 compared to uncoded caching schemes. Based on the same idea, in [34], the authors assumed fixed set of files and popularity, and formulated the caching problem as an Index coding problem. Moreover, the authors showed that the further performance of the caching scheme proposed in [33] under the considered assumptions can only be achieved by coded cache placement. In [20], a greedy algorithm is proposed to assign a complete file or a coded chunk of a file to a given SBS while minimizing the total delay. Two hierarchical coded caching scheme with two caching levels are proposed in [35]. The first approach provides coded multicasting opportunities within each level while the second approach provides coded multicasting opportunities across multiple levels. The work in [36] proposed an online caching scheme and proved that the performance of the optimal online scheme is comparable with the performance of the optimal offline scheme, in which the storage units are refreshed based on the entire set of popular files before each new request. The work in [37] proposed a coded caching for multi-level popularity degrees in which the set of files is divided into multiple levels based on the popularity of the files. Moreover, the effect of users connected to each SBS on the caching scheme is analyzed.
In another line of works, coded caching policies were proposed in MIMO systems to exploit the cooperation possibilities. In [38], the joint problem of power and encoded cache control is formulated using an optimization approach. The goal in this model is to create more opportunities for serving users cooperatively by multiple SBSs that are equipped with multiple antennas. The performance of video streaming for similar caching schemes is analyzed in [39]. The authors in [40] proposed a physical layer caching scheme for wireless adhoc networks. Then, the achievable throughput by the users is analyzed and the impact of different system parameters studied.

Caching in D2D-enabled networks

The large available storage space at the users’ devices has motivated the extension of the idea of caching into users devices. Thus, a given user can cache the most popular files and serve the users in its velocity via D2D communications. The authors in [41] used tools from stochastic geometry to derive the probability of successful transmissions in cache enabled networks in the presence of D2D communications. An optimization problem is then formulated to determine the best caching policy that maximizes the probability of successful file transmissions. In [42], the authors analyzed the performance of caching in D2D networks under two different caching policies. In the first approach, the files are cached at a device that is randomly selected from all the devices in the network while in the second approach, the files are cached at the device based on their distance from the requesting devices. An experimental analysis of these systems is conducted in [43] while accounting for the mobility of devices. In another line of works, the performance of caching in SCNs with enabled D2D connections is addressed from an information theoretic point of view [44]. The authors in [45] proposed a decentralized random caching placement in SCNs where a device can be served either directly or via multi-hop D2D communications from the closest device that caches the requested file. The work in [46] investigated the energy efficiency of regenerating codes and derived the popularity thresholds in which redundancy induced by such coding methods is beneficial to cope with nodes failure. The authors in [47] considered a caching system in which the devices cache files that are encoded using an erasure correcting code and can leave or arrive from the cellular network. A repair scheduling is introduced to recover the lost content when a device leaves the network. In [48], the problem of caching placement in two-tier wireless networks is formulated as difference of convex problem. The optimization problem is then relaxed and solved using classical water-filling method. It is shown that in caching SCNs, it is more efficient for the devices and SBSs to cache the most popular files under low cache node density while it is more efficient to evenly cache different files under high cache nodes density.

Caching in cellular networks deployment

Several caching policies have been proposed in the literature and take into account of different wireless parameters such as the geographical position of the SBSs, the mobility of the users, the type of transmission between the SBSs and users as well as the energy
consumption at the SBSs. A hierarchical caching policy that exploits users mobility is proposed in [49]. The mobility of users in [21] is modeled as a discrete-time Markov chain model. Then, the caching problem is formulated as an optimization problem and the optimal caching policy is determined while accounting for the delay constraints of the users. The goal of the proposed caching policy is to minimize the probability of serving fractions of the requested content from the core network. The authors in [22] proposed a caching policy that accounts for the geographical position of the SBSs and users. The goal is to maximize the probability of finding the requested files by the users at the SBSs cache. In [50], a joint encoded caching and routing problem is formulated and then reduced to a tractable facility location problem. Energy efficiency in caching cellular networks is addressed in [51–56]. In [52], the authors proposed an energy efficient power control scheme for a single SBS that is equipped with a storage unit and connected to the core network via a capacity-limited backhaul link. The authors in [53] analyzed the energy efficiency in networks with a macro BS and multiple SBSs equipped with caches. While accounting for content popularity and request arrival density, it is shown that the energy efficient cannot be improved when the SBSs are equipped with large sized storage units. The work in [54] analyzed the impact of caching on the energy consumption in SCN using stochastic geometry. In [55], the authors analyzed the maximal energy efficiency gain brought by caching and showed that caching a number of content that exceeds a given threshold, the SBSs may not achieve higher energy efficiency. The work in [56] proposed a caching policy that optimizes the overall energy consumption of the SBSs while accounting for the multicast opportunities. The authors in [57] exploited the interdependence between caching and the MAC layer coordination. In particular, the caching problem that is considered accounts for the cooperation of multiple SBSs that can serve the same users simultaneously. The multiplexing gain is analyzed and the performance improvement in terms of delivery delay is shown. An analysis of cooperation schemes using stochastic geometry is provided in [58].

1.2.2 Coexistence of LTE-U and WiFi over unlicensed bands

Many works have proposed and analyzed novel mechanisms for the deployment of LTE-U over unlicensed channels. Here, we classify these existing works based on their similarities.

Performance analysis

The performance of LTE-U wireless networks is analyzed in [59–64]. The work in [59] evaluated the performance of coexisting WiFi and time division duplex-LTE in the 5 GHz. The work in [60] analyzed the effect of WiFi and WLAN when coexisting over the unlicensed bands by developing a novel inter-system interference analysis technique. In [61], the authors analyzed the coexistence of scheduled and random access LTE users on the same unlicensed channel. In [62], the authors analyzed the performance dynamics of integrated LTE-U and WiFi when sharing the unlicensed bands. The impact of LTE-U on the WiFi network is analyzed in [63] using stochastic geometry. The impact of two
channel access protocols were compared including continuous transmission of the LTE-U BSs over the unlicensed channels, and LTE with LBT. In [64], the authors showed that the benefit of implementing LBT at the LTE-U BSs using stochastic geometry.

Classical optimization-based coexistence frameworks

The problem of coexistence of unlicensed spectrum was extensively studied using classical optimization tools while taking into account different network parameters and objectives. In [65], the load balancing between licensed and unlicensed bands is formulated as an optimization problem to maximize the overall LTE users’ rate. In [66], an optimization problem is formulated to jointly allocate licensed and unlicensed bands while maximizing the users’ sum rate and ensuring fairness among these users. The work in [67] proposed a distributed traffic offloading scheme for LTE-U scenarios with a single base station. In [68], the authors proposed the notion of hyper-access points (HAP) in which the functions of both an LTE-U SBS and a Wi-Fi AP are combined in the same HAP. The work in [69] proposed an inter-cell interference coordination mechanism as a time-domain multiplexing technique for interference mitigation that allows multiple LTE-U SBSs that are held by different operators to share an unlicensed channel. In [70], the authors proposed a cooperative optimization framework that allows the WUs and SBSs to coordinate dynamically via power control and time division channel. In [71], the authors formulated the spectrum sharing problem between WiFi users (WUs) and LTE-U SBSs as a mixed integer optimization problem while ensuring fairness between the two technologies. The authors in [72] developed a hybrid method for LTE-U SBSs to use the unlicensed bands by combining both spectrum sharing and traffic offloading methods. All these works have addressed the problem of coexistence over unlicensed spectrum based on classical optimization tools. Thus, they do not account for the interactions and the interdependence between the traffic offloaded by the different LTE-U SBSs on one another and on the WUs over the unlicensed bands.

Game-theoretic coexistence frameworks

Several works have investigated the problem of coexistence between LTE-U and WiFi from a game-theoretic point of view [26, 73–76]. The work in [26] formulated the unlicensed spectrum allocation problem with uplink-downlink decoupling as a noncooperative game in which the SBSs are the players that select the unlicensed channels over which they serve their users. The authors in [73] formulated the unlicensed spectrum allocation problem as a student-project matching problem with externalities. A learning approach is conducted in [74], where the authors proposed a cross-system learning algorithm that allows the SBSs to autonomously select the unlicensed channels over which they transmit and determine their transmit power. The economic aspect of LTE-U is addressed in [75, 76] where the authors designed an auction-based mechanism in which a number of WiFi access point owners negotiate with the LTE providers to sell their channel access opportunities.
1.3 Thesis Outline and Contributions

This thesis contains three parts. In Part I, we focus on the problem of distributed caching in SCNs using tools from game theory.

In Chapter 3, the problem of caching in small cell networks is addressed from a game theoretic point of view. In particular, the caching problem is formulated as a many-to-many matching game between small base stations and service providers’ servers. The servers store a set of videos and aim to cache these videos at the small base stations in order to reduce the experienced delay by the end-users. On the other hand, small base stations cache the videos according to their local popularity, so as to reduce the load on the backhaul links. We propose a new matching algorithm for the many-to-many problem and prove that it reaches a pairwise stable outcome. Simulation results show that the number of satisfied requests by the small base stations in the proposed caching algorithm can reach up to three times the satisfaction of a random caching policy. Moreover, the expected download time of all the videos can be reduced significantly.

In Chapter 4, the problem of distributed caching in dense wireless SCNs is studied using mean field games (MFGs). In the considered SCNs, SBSs are equipped with data storage units and cooperate to serve users’ requests either from files cached in the storage or directly from the capacity-limited backhaul. The goal of the SBSs is to define a caching policy that reduces the load on the capacity-limited backhaul links. This cache control problem is formulated as a stochastic differential game (SDG). In this game, each SBS takes into consideration the storage state of the other SBSs to decide on the fraction of content it should cache. To solve this problem, the formulated SDG is reduced to a mean-field game (MFG) by considering an ultra-dense network of SBSs in which the existence and uniqueness of the mean-field equilibrium is shown to be guaranteed. Simulation results show that this framework allows an efficient use of the available storage space at the SBSs while properly tracking the files’ popularity. The results also show that, compared to a baseline model in which SBSs are not aware of the instantaneous system state, the proposed framework increases the number of served files from the SBSs by more than 69%.

In Chapter 5, an incentive proactive cache mechanism in cache-enabled SCNs is proposed, in order to motivate the CPs to participate in the caching procedure. A network composed of a single MNO and multiple CPs is considered. The MNO aims to define the price it charges the CPs to maximize its revenue while the CPs compete to determine the number of files they cache at the MNO’s SBSs to improve the QoS of their users. This problem is formulated as a Stackelberg game where a single MNO is considered as the leader and the multiple CPs willing to cache files are the followers. The followers’ game is modeled as a noncooperative game and both the existence and uniqueness of a Nash equilibrium (NE) are proved. The closed-form expression of the NE which corresponds to the amount of storage each CP requests from the MNO is derived. An optimization problem is formulated at the MNO’ side to determine the optimal price that the MNO should charge the CPs. Simulation results show that at the equilibrium, the MNO and
CPs can all achieve a utility that is up to 50% higher than the cases in which the prices and storage quantities are requested arbitrarily.

In Chapter 6, a novel approach for providing incentives for caching in SCNs is proposed based on the economics framework of contract theory. In this model, a MNO designs contracts that will be offered to a number of CPs to motivate them to cache their content at the MNO’s SBSs. A practical model in which information about the traffic generated by the CPs’ users is not known to the MNO is considered. Under such asymmetric information, the incentive contract between the MNO and each CP is properly designed so as to determine the amount of allocated storage to the CP and the charged price by the MNO. The contracts are derived by the MNO in a way to maximize the global benefit of the CPs and prevent them from using their private information to manipulate the outcome of the caching process. For this interdependent contract model, the closed-form expressions of the price and the allocated storage space to each CP are derived. This proposed mechanism is shown to satisfy the sufficient and necessary conditions for the feasibility of a contract. Moreover, it is shown that the proposed pricing model is budget balanced, enabling the MNO to cover all the caching expenses via the prices charged to the CPs. Simulation results show that none of the CPs will have an incentive to choose a contract designed for CPs with different traffic loads.

In the second part of the thesis, namely in Part II, we address the problem of spectrum sharing in cache-enabled networks as well as the problem of coexistence in LTE-U systems using tools from game theory.

In Chapter 7, the heterogeneous backhaul management problem in cache-enabled SCNs is formulated as a minority game in which each SBS has to define the number of predicted files to download, without affecting the required transmission rate of the current requests. For the formulated game, it is shown that a unique fair proper mixed Nash equilibrium (PMNE) exists. A self-organizing reinforcement learning algorithm is proposed and proved to converge to a unique Boltzmann-Gibbs equilibrium which approximates the desired PMNE. Simulation results show that the performance of the proposed approach is close to that of the ideal optimal algorithm in 85% of the cases while it outperforms a centralized greedy approach by up to 50% in terms of the amount of data that is cached without jeopardizing the quality-of-service of current requests.

In Chapter 8, a new framework for optimizing the coexistence of LTE and WiFi over unlicensed bands is proposed. This coexistence problem is formulated as multi-game composed of two games of different types. In this model, the WiFi users are prioritized when using the unlicensed bands while the SBSs adapt their traffic based on the WiFi traffic load. In particular, two optimization problems are jointly considered. First, the power allocation of the SBSs on each of the unlicensed channels is analyzed. Then, the assignment of the WUs to the unlicensed channels is considered. In such a model, the achievable throughput by a given WU or SBS does not only dependent on the channel over which the WU/SBS transmits. In fact, it also depends on the transmit power of the other SBSs and WUs serving their content over the same unlicensed channel. The competition between the WUs to access the unlicensed channels is formulated as a one-
sided matching game while the power allocation problem of the SBS is formulated as a noncooperative game. For the multi-game, the existence of a unique stable outcome is shown and a decentralized algorithm that converges to this outcome is proposed.

Finally, Part III includes conclusions and future works related to work presented in this thesis. We note that each chapter contains its own mathematical notation.

1.4 Publications

List of publications during the course of this PhD are listed below. The results and details provided in this manuscript are marked with *.

Journal Articles


Conference Papers


[81]* K. Hamidouche, W. Saad, M. Debbah, “Multi-Games for LTE and WiFi Coexistence over Unlicensed Channels”, International conference on Network Games, Control and Optimization, Avignon, France, 2016. (Chapter 8)

1.4. Publications


[84]* K. Hamidouche, W. Saad, M. Debbah, V. H. Poor, “Mean-Field Games for Distributed Caching in Ultra-Dense Small Cell Networks”, American Control Conference, Boston, USA, 2016. (Chapter 4)


Chapter 2

Game Theoretic Frameworks for Resource Allocation in Cellular Networks

Recent advanced technologies discussed in Chapter 1 will potentially provide new opportunities to yield significant performance gains required by the future heterogeneous cellular networks. Nonetheless, a successful integration of these technologies in one unified system needs novel resource management techniques. In particular, some recent major paradigm shifts for resource management can be underlined as follows:

- Unlike conventional cellular networks in which an MBS acts as a controlling entity for the users, next generation of wireless user equipments (UEs) may not be fully controlled by a centralized station. In fact, UEs are able to directly communicate with one another via D2D links, simultaneously connect to multiple BSs, and transmit or receive over different frequency bands [87].

- Dense deployments of SBSs in heterogeneous networks requires fast algorithms for cell association and resource allocation. In addition, vertical handovers between different radio access technologies (RATs) will increase the complexity of resource management [88].

- As discussed in Chapter 1, the development of effective resource allocation mechanisms requires the cooperation of multiple rational actors such as content providers, network operators and users. For instance, to gain from caching solutions, the content providers must provide their content to MNOs so that they can cache the files. Similarly, the MNO can offload part of their traffic over the unlicensed bands only if the users accept such mechanism as LTE-U does not provide any QoS guarantees [86].

Therefore, resource management algorithms have to be robust, user-centric, and consequently self-organizing. In particular, managing resources of a complex and dense cellular
network warrants a fundamental shift from traditional centralized mechanisms toward self-organizing and self-optimizing approaches. This paradigm shift is motivated, given 1) the density of next generation SCNs, 2) the limited capacity of backhaul and fronthaul solutions in SCN, and 3) the need for low latency solutions and cost-effective network management which requires decentralized control. In fact, self-organization allows SBSs and UEs to use some intelligence to make fast resource allocation decisions [87]. To this end, new optimization schemes and game-theoretic solutions are introduced in the literature to address the specific challenges of the future heterogeneous networks. However, optimization problems are mostly suitable for centralized implementations provided with the global information, which may impose significant overhead and complexity. This becomes even more important when dealing with combinatorial integer programming problems such as channel allocation and caching problems.

During the past few years, multiple game-theoretic frameworks are envisioned as promising approaches to solve the resource allocation problems in heterogeneous networks [40, 39, 41]. In this chapter, we present different frameworks each introduced to account for specific characteristics of next-generation cellular networks. The rest of this chapter is organized as follows. In Section 2.1, we review the fundamentals of mechanism design theory. In Section 2.2, we introduce the class of anonymous games. Finally, in Section 2.3, we present hierarchical games and their applications in the context of cellular networks.

2.1 Mechanism Design

Mechanism design is a field of applied mathematics and economics that takes an engineering approach to designing economic mechanisms or incentives towards desired objectives. Mechanism design as a field covers multiple game theoretic tools that enforce agreements between agents and stakeholders. In wireless communications literature, mechanism design has recently become attractive to solve resource allocation problems based on matching theory [89] and motivate agents to take appropriate actions through the introduction of contract theory [90]. Next, we present the fundamentals of both matching theory and contract theory in the context of wireless networks.

2.1.1 Fundamentals of Contract Theory

Contract theory is a mathematical tool that studies the construction of contractual arrangements between two entities with different and interdependent interests. It allows an entity of the system to motivate decision makers to take appropriate actions [90]. It was initially used in political science and economics to model problems such as employment and voting processes in which the actors that correspond to politicians and employers align the interests of voters and workers with their own interests. Recently, contract theory was adopted in wireless networks to ensure a successful deployment of multiple resource allo-
cation solutions. This interest is due to the properties that can be used to account for the involvement of multiple stockholders that must cooperate in next-generation cellular networks.

In wireless networks, contract theory enables a network actor called principal such as MNOs and BSs to motivate agents including users, content providers and MNOs to participate or accept the deployment of optimal resource allocations solutions in the presence of asymmetric information. This information asymmetry captures the inability of one party of the system to observe relevant information that is private to the other party and can be used to manipulate the agreements and improve its own performance in the network. Private information may correspond to the willingness of users to serve other users via D2D communications or the willingness of CPs to cache their content in BSs and users’ devices. Based on the party that exhibits private information, we distinguish two different contract theory frameworks, moral hazard and adverse selection [91].

In moral hazard, the agents can take actions that are not observed by the principal. Such information asymmetry appears after the principal and the agents sign the contracts, known as postcontractual asymmetric information. In adverse selection, the agents have private information that the principal is not aware of. The information asymmetry between the principal and the agents is present before the principal proposes an agreement to the agents, known as precontractual asymmetric information [92].

In wireless networks, the goal of a principal is to incite the agents to use network resources to improve the performance of user (in terms of delay and data rate) in exchange of another service or a monetary asset. Indeed, the MNOs cannot ensure alone the deployment of the novel solutions that are proposed to boost the capacity of cellular networks such as distributed caching and LTE over unlicensed channels. The successful deployment of distributed caching solutions relies on the cooperation of CPs that must accept to cache their content at the BSs. In LTE-U enabled-networks, users must accept to receive their requests via unlicensed bands with best-effort service and without any QoS guarantee.

To incite agents to use network resources, the MNO defines and proposes a set of contracts to the agents while accounting for information asymmetry. Next, we define the key components and terminologies for every contract design problem in adverse selection class in the context of wireless networks.

- **Principal**: The goal of the principal is to motivate the agents to use network resources by designing and offering a set of contracts. We denote $m$ as the principal.

- **Agents**: Every agent might choose one of the contracts proposed by the principal in the goal of improving its performance in the network. If an agent does not benefit from any of the contracts, it can decline all the offers. We denote the set of agents as $N$.

- **Agent’s type**: The agents have private information that models either their willingness to use network resources or their impact on the network resources. Thus,
every agent $n \in \mathcal{N}$ is characterized by a type $\theta_n \in \Theta$, where $\Theta$ is the set of all possible types.

- **Set of contracts**: The principal proposes a contract $y_{\theta_n} = (r_n(\theta_n), \pi_n(\theta_n))$ for every type of agents $\theta_n$, where $r_n$ is the network resource (e.g., frequency band, caching space) that the principal allocates to agent $n$ in exchange of a monetary asset $\pi_n$. We denote the set of contracts $\mathcal{Y} = \{(r_n(\theta_n), \pi_n(\theta_n))\}_{\theta_n \in \Theta}$.

- **Utility function**: Determines the utilities that every agent and principal assigns to each type of contract. We denote the utility of an agent $n$ as $U_n(.) : \mathcal{Y} \times \Theta \rightarrow \mathbb{R}$ and can be given by:

$$U_n(r_n(\theta_n), \pi_n(\theta_n), \theta_n) = u_n(r_n(\theta_n), \theta_n) - \pi_n(\theta_n),$$

where $u_n$ is the valuation function of agent $n$ and $\pi_n$ is the price that the MNO charges the agent. The utility $V_m(.) : \mathcal{Y} \rightarrow \mathbb{R}$ of the MNO for every contract it proposes is defined as follows:

$$V_m(\pi_n) = \sum_{n \in \mathcal{N}} [v_n(r_n(\theta_n)) + \pi_n(\theta_n)],$$

where $v_n$ is the cost of serving agent $n$ by the MNO over the network resource $r_n$. In essence, $U_n(.)$ and $V_m(.)$ are the objective functions of the agents and the principal, respectively. Thus, every agent selects the contract that maximizes its utility and the principal designs a contract for each agent’s type so that it maximizes its utility.

The goal of the MNO is to identify the set of contracts $\mathcal{Y}$ that maximizes its utility $V_m(.)$ subject to being implementable and individually rational. These two conditions known as **individual rationality** and **incentive compatibility** are necessary and sufficient for the feasibility of the contracts and are defined as follows:

- **Individual rationality (IR)**: An allocation $y_{\theta_n} = (r_n, \pi_n)$ is individually rational if,

$$U_n(r_n(\theta_n), \pi_n(\theta_n), \theta_n) \geq 0, \forall \theta_n \in \Theta.$$  

This condition motivates every agent of type $\theta_n$ to participate in the resource allocation process by ensuring a positive utility for the agent under any contract.

- **Incentive compatibility (IC)**: An allocation $y_{\theta_n} = (r_n(\theta_n), \pi_n(\theta_n))$ is incentive compatible (IC) or truthfully implementable if there does not exist another type $\theta_i$ with $i \neq n$ such that:

$$U_n(r_n(\theta_n), \pi_n(\theta_n), \theta_n) \geq U_n(r_n(\theta_i), \pi_n(\theta_i), \theta_n).$$

This condition ensures that every agent of type $\theta_n$ would prefer to select a contract that is design for its type than selecting a contract that is designed for another type.
Thus, the MNO’s optimization problem for designing the most optimal resource allocation policy while inciting the agents to participate can be defined as follows:

$$\max_{\{(r_n, \pi_n)_{\theta_n} \in \Theta\}} \sum_{n \in N} [v_n(r_n(\theta_n), \theta_n) + \pi_n(\theta_n)],$$

subject to

$$U_n(r_n(\theta_n), \pi_n(\theta_n), \theta_n) \geq 0, \quad \forall \theta_n \in \Theta \text{ and } \forall n \in N,$$

$$U_n(r_n(\theta_n), \pi_n(\theta_n), \theta_n) \geq U_n(r_n(\theta_i), \pi_n(\theta_i), \theta_n), \quad \forall n \in N.$$  \hspace{1cm} (2.5)

To solve this problem, classical optimization theory is used to develop algorithms that identify the optimal decisions.

### 2.1.2 Fundamentals of Matching Theory

Matching theory is a mathematical framework that studies the formation of beneficial relationships between unmatched agents [89,93,94]. It was initially used in labor economics to model the assignment problem of jobs to workers in firms, residents to hospitals and also to describe the marriage problem in which men are assigned to women. Recently, matching theory was extended and adopted in wireless communications theory to address multiple resource allocation problems. This interest is due to several properties that can be used to capture the specific characteristics of next-generation cellular networks.

Matching games are used in wireless networks for the allocation of network resources such as caching space, frequency bands, and power, to the demanding agents including content providers, mobile users, and access points. Based on the types of the agents and the network resources, matching games can be classified into three classes:

- **One-sided matching games:** This class of matching problems considers one set of agents for which the goal is to form groups of agents by matching themselves to one another [95,96]. In wireless networks, the goal is to efficiently allocate network resources that are not controlled by any entity such as unlicensed bands to a set of agents such as users, base stations or access points. Each agent has a preference list that represents the agents with which it prefers to share a given network resource. Based on its requirement in terms of network resources, every agent can limit the number of agents to which it accepts to be matched over each item in the set of resources.

- **Two-sided matching games:** In two sided matching games, the assignment is between two disjoint sets of players [97]. This class of games is the most extensively used to model resource allocation problems in cellular networks. It is more suitable for the assignment of users to cells, the allocation of licensed bands that can be controlled directly by the base stations or indirectly by the network operator. In this class of games, the agents of each set have different capacity limits. The matching problem is called one-to-one matching if the players of both sets are matched to only one player of the opposite set. A many-to-one matching game if the players of one set are matched to only one player of the opposite set, while the players of
the second set accept multiple partners. In a many-to-many matching game, the players of both sets are matched to multiple partners from the opposite set [98].

- **Three-sided matching games:** A more complex class of matching games is the three-sided matching games in which the players of three disjoint sets are matched to one another [99,100]. In cellular networks, this class of games enables the network agents to account for two types of resources simultaneously. For instance, the cell association and frequency bands allocation problems can be considered jointly as the performance of the users is affected by both its serving BS and the frequency band over which the users are served. Thus, a three-sided matching with three disjoint set of players that correspond to users, resources blocks and BSs can be used to model such resource allocation problems.

Although different in their structure, the goal in any class of matching game is to optimally allocate the network resources to the demanding entities while accounting for the limitations imposed by the wireless network architecture. For clarity, next, we present the components of *two-sided matching games* in the context of network resource allocation:

- **Sets of players:** In a two-sided matching game, a matching $\mu$ assigns the players in one set to the players of the opposite set. One of the sets corresponds to the demanding agents $D$ and the other set contains the network resources $R$. The resources that are matched to an agent $d \in D$ by matching $\mu$ are denoted $\mu(d) \subseteq R$. Similarly, $\mu(r) \subseteq D$ denotes the set of agents that is assigned to the network resource $r \in R$.

- **Quota:** For each player, the quota determines the maximum number of players from the opposite set to which it can be matched. The quota of each resource and demanding agent are denoted $q_r$ and $q_d$, respectively. Thus, matching $\mu$ must satisfy the two constraints: $|\mu(r)| \leq q_r$ and $|\mu(d)| \leq q_d$.

- **Preference relation:** Each player $i \in R \cup D$ uses a relation $\prec_i$ to rank the players of the other set based on a predefined metric. This metric is captured by a utility function $u_i(j)$ that every user $i \in R \cup D$ assigns to a player $j$ of the other set. Thus, a resource item $r$ can define its preference relation $\prec_r$ as $m_1 \prec_r m_2$ if and only if $u_r(d_1) \leq u_r(d_2)$.

- **Utility function:** Represents the benefit that each player can achieve when matched to every player of the other side. In wireless networks, a utility $u_d(r)$ is an objective function that quantifies the QoS of the agent $d$ for a resource-agent matching. Thus, the goal of every agent is to maximize its utility function $u_d(.) : R \rightarrow \mathbb{R}$.

- **Outcome of the matching problem:** The outcome of the matching problem is a function $\mu: D \cup R \rightarrow R \cup D$ such that: 1) $\forall d \in D, \mu(d) \subseteq R$ 2) $\forall r \in R, \mu(r) \subseteq D$ 3) $|\mu(d)| \leq q_d$ and $|\mu(r)| \leq q_r$, and 4) $r \in \mu(d)$ if and only if $d \in \mu(r)$. An agent-resource pair $(d,r) \in \mu$ if $d$ and $r$ are matched, and $(m,r) \notin \mu$ otherwise.
2.2 Anonymous Games

- **Solution concept:** The goal in each matching problem is to reach a stable and efficient outcome. A matching $\mu$ is said two-sided stable if there does not exist a pair of players $(d, r) \in D \times R$ that blocks the matching $\mu$. A pair $(d, r)$ is a blocking pair if 1) $d \notin \mu(r)$ and $r \notin \mu(d)$, and 2) $r <_d \mu(d)$ and $d <_r \mu(r)$ which corresponds to the case in which both agent $d$ and resource item $r$ prefer one another to their partners under matching $\mu$. On the other hand, a matching $\mu$ is **Pareto efficient** if there is does not exist another matching $\mu'$ under which at least one of the players $i \in D \cup R$ strictly prefers $\mu'(i)$ to $\mu(i)$ while all the others are either indifferent between their partners over the two matchings or prefer their partners under matching $\mu'$ to the ones under matching $\mu$.

Note that Pareto efficiency is a more general solution concept than two-sided stability as it applies to all the classes of matching problems we defined. However, the two solution concepts are equivalent in the class of two-sided matching games. To reach a stable outcome, different algorithms are proposed in the literature for each class of matching problems. For two-sided one-to-one and many-to-one matching games, the deferred acceptance algorithm proposed in [94] by Gale and Shapely, guarantees the convergence to a two-sided stable matching. However, such algorithm does not exist for the many-to-many matching problem due to its complexity as compared with the one-to-one and many-to-one matching problems. An extension of the deferred acceptance algorithm is proposed in [100] to reach a stable outcome in the three-sided matching problems under strong conditions. Such conditions enforce the players to prioritize the players in one of the two other sets. For the one-sided matching, the centralized algorithms **serial dictatorship** and **top-trading cycle** algorithms guarantee the convergence to a Pareto-efficient outcome [95].

In [101], matching problems are classified based on wireless parameters into 1) **canonical matching**, 2) **matching with externalities** [102], 3) **matching with dynamics** [103]. The class of canonical matching represents the basic form of matching problems in which the preference of the players are fixed and do not change over time of the resource allocation (i.e., fixed strategies). The class of matching problems with externalities includes scenarios in which preferences of every player depend on other players’ preferences. Hence, players may change their preferences (i.e. varying strategies) within the time frame of the resource allocation. The class of matching with dynamics represents matching problems in which the strategy of players in the current resource allocation may depend on the strategies in the past resource allocations.

### 2.2 Anonymous Games

We introduce the class of anonymous game theory as a statistical mechanics approach for modeling and describing resource allocation problems in which the interactions between the agents results in a collective phenomena. In such problems, the agents select their resources based on the collective behavior through the distribution law of many individual agent decisions [104, 105]. Unlike conventional approaches, the agents do not have any
2.2. Anonymous Games

information about other agents and their behavior varies over time. Thus, this class of mechanisms is developed to explicitly consider the dynamical approach towards the steady state through the dynamics of agents’ states and the adaptation of their actions over time. The dynamical nature of such resource allocation problems gives rise to new challenges regarding the stability of the system and whether the agents can reach the steady state with a minimum communication overhead. In this section, we overview the class of minority games and mean-field games that were recently adopted in the wireless communication community as powerful tools for studying resource allocation problems in dynamical and dense wireless cellular networks.

2.2.1 Minority Games

The class of minority games was initially introduced to model the evolution of financial markets with a number of adaptive agents. In this model of repeated games, the agents convert the history or the information available at each time stage into expectations about the future and then, take their decisions based on the expected benefit. The original minority game is a mathematical formulation of the “El farol Bar” problem in which a number of individuals have to decide whether to go or not to the “El farol Bar” every Friday [104]. In this problem, the El Farol bar has a limited number of seats and the agents enjoy being in the bar only if the bar is not crowded or empty. The main challenge of this minority problem is driven by the absence of coordination between the agents that can take their decision only based on the decisions of other agents in the previous weeks. This class of games is suitable to model resource allocation problems as it enables users to select their network resources independently without coordination with other users or any centralized entity. Solving a minority game problem consists in the two following steps.

- **The minority mechanism**: In the first step, we model the interactions between the agents and the impact of the history of actions on the action chosen by every agent at the considered time slot.

- **The learning scheme**: Represents the algorithms that determine the allocation of the resources to the agents and the performance achieved by each of the agents.

To introduce the main components of a minority game, we consider a wireless network composed of a set \( \mathcal{N} \) of \( N \) agents that exploit a set \( \mathcal{R} \) of \( R \) network resources with \( R \geq 2 \) during \( T \) time periods.

- **Players**: The set of agents \( \mathcal{N} \) corresponds to the players. In a wireless system, the players could be users, small base stations, content providers, and WiFi access points.

- **Actions**: The set of actions represents the set of choices for all the agents, from which every agent selects a network resource. Thus, every agent \( n \) can choose a resource \( r_n(t) \in \mathcal{R} = \{1,\ldots,R\} \) at time period \( t \). The network resources may correspond to the frequency bands, storage space, and the set of access points.
2.2. Anonymous Games

- **Resources capacity:** The main characteristic of minority games appears in the limited capacity of the resources. Each network resource can support a limited number of agents $q_r$ with $\forall r \in \mathcal{R}$. Thus, the goal of the agents is to select the resources without coordination, while ensuring that each network resource is neither underused or overused. This guarantees a full usage of the network resources while all the agents are served.

- **Utility:** The goal of each player in a minority game is to select a network resource that maximizes a given utility. To ensure a full usage of the network resources, each agent defines its utility based on the capacity of the network resource. For a given resource $r \in \mathcal{R}$ that can only support $q_r$ agents, the capacity represents the limit starting from which the utility of the players begins to decrease. Indeed, if an agent $n$ decides to use the resource $r$ while already $q_r$ agents are being served over the same resource, this will degrade the QoS of all the agents using the same resource. Thus, the utility of every agent depends on the actions chosen by other agents as well as the capacity of every network resource and can be given by:

\[
  u_n(r_n, r_{-n}, t) = \begin{cases} 
  v_n(r_n, s_r, t) & \text{if } s_r(t) > q_r, \\
  -v_n(r_n, s_r, t) & \text{otherwise},
\end{cases}
\]

(2.6)

where $s_r$ is the current number of agents that select network resource $r$ and depends on the actions $r_{-n}$ chosen by all agents except agent $n$. The utility of the agent $n$ is the objective function of the agent that it aims to maximize. In a minority game, the utility of every agent has the form of (2.6) which means that the objective function of agent $n$ reaches the maximum when the total number of agents that select resource $r$ equals the capacity $q_r$ of the resource.

- **Solution concepts:** The goal of the agents in a minority game is to reach a stable situation in which none of the agents has an incentive to deviate from its current behavior. Such a state is known a Nash equilibrium and can formally defined as follows:

**Definition 1.** A strategy profile $[r^*_1, ..., r^*_N]$ is a Nash equilibrium is for all agent $n \in \mathcal{N}$ and $r_n$ we have that,

\[
  u_n(r^*_n, s^*_r, t) \geq u_n(r_n, s^*_r, t),
\]

(2.7)

where $s^*_r$ is the number of agents that choose resource $r$ at the equilibrium and is defined based on the actions of all the agents $r^*_{-n}$.

2.2.2 Mean-Field Games

In the context of wireless networks, mean-field game theory was introduced to account for the ultra-density of next-generation cellular networks in terms of users and SBSs. In fact, mean-field theory is an economic model that was developed to study the strategic decisions
of a large systems of interacting agents with symmetric payoffs [106]. The problem of power allocation in ultra-dense cellular networks has been extensively modeled in the literature as a mean-field problem while accounting for different network parameters. The components of a mean-field game can be defined as follows [].

- **Players**: The set of players in cellular systems corresponds to the set of SBSs in the networks or the set of devices which we denote by \( \mathcal{N} \). In a mean-field game, the number of players is very large which captures one of the key characteristics of 5G networks.

- **Action**: Depending on the considered resource allocation problem, every player \( n \in \mathcal{N} \) has a set of actions \( \mathcal{A}_n \) from which it can choose its strategy. It may correspond to the transmit power of user \( n \), the the fraction of a given file that should be cached by a user \( n \).

- **User’s state**: Every player \( i \) is defined by a state \( X_i(t) \) that may represent the power level of its local battery, the channel state with the served user, or the state of the storage unit.

- **Mean-field**: In every game, the state and the strategies of every user are defined based on the interactions with the other users and their state. Due to the large number of players in the system we consider, instead of determining the action of a generic user \( i \) based on the state of all the other users, user \( i \) only accounts of its own state and the distribution of the other users’ state that can be defined as:

\[
M(t) = \frac{1}{N} \sum_{i=1}^{N} \delta_{X_i(t)}, \tag{2.8}
\]

where \( \delta \) is the Dirac measure and \( N \) is the total number of users.

- **State dynamics**: The dynamics of the generic user’s state is expressed by an Itô formula as follows:

\[
dX(t) = b(m(t), a(t))dt + \sigma d\beta_t, \tag{2.9}
\]

where \( b(m(t), a(t)) \) is the drift function and represents the evolution of the players’ state and \( \beta_t \) is the \( d \)-dimensional Brownian motion that captures the randomness in the evolution of the state.

- **Cost function**: Represents the cost of each player when selecting a control \( a(t) \). We denote the instantaneous cost by \( c(t, m(t), a(t)) \) and the goal of a given player is to minimize its cost over a finite time horizon \([t_0, T]\) given by:

\[
J_T(t, X, a(t), m(t)) = \mathbb{E} \left[ \int_{t_0}^{T} c(t, m(t), a(t)) + g(T)dt \right], \tag{2.10}
\]

where \( g(T) \) is the terminal cost.
2.2. Anonymous Games

- **Solution concept:** In a game with infinite number of players, the goal of the players is to reach a stable point in which none of the players can lower its cost without increasing the cost of another player in the system. More formally the mean-field equilibrium can be defined as follows.

**Definition 2.** A control strategy \( a^*_t \) is a mean-field equilibrium if and only if \( \forall a_t \in \mathcal{A} \) it holds that:

\[
J_T(t, X, a^*(t), m(t)) \geq J_T(t, X, a(t), m(t)).
\] (2.11)

To solve the formulated optimization problem, we define a valuation function \( v(t, x) \) as follows,

\[
v(t, X) = a int J_T(t, X, a(t), m(t)),
\] (2.12)

which solves the Hamilton-Jacobi-Bellman equation,

\[
\begin{cases}
- \frac{\partial v}{\partial t} - \frac{\partial^2}{2} \Delta_a v + H(\nabla_a v) = b(a(t), m), \\
v(T, X) = g(X),
\end{cases}
\] (2.13)

where \( H \) is the Hamiltonian associated to the cost function \( c \) and the feedback control

\[
a(t, X) = -\nabla_a H(\nabla v(t, X))
\] (2.14)

is optimal. Thus, the optimal process

\[
dX^*_t = -\nabla_a H(\nabla v(X^*(t)))dt + \delta d\beta_t
\] (2.15)

has a distribution whose density \( m \) solves the Kolmogorov-Fokker-Plank equation

\[
\begin{cases}
\frac{\partial m}{\partial t} - \frac{\partial^2}{2} \Delta m + div(m \nabla H(\nabla v)) = 0, \\
m(0, x) = m_0(x).
\end{cases}
\] (2.16)

where \( m_0(x) \geq 0 \) and \( \int_{\mathbb{R}^d} m_0(x)dx = 1 \) is the distribution of the initial state of the system.

Thus, at the mean-field equilibrium, the value function and the density of the optimal process are characterized by the two following partial deferential equations (PDEs):

\[
\begin{cases}
\frac{\partial v}{\partial t} - \frac{\partial^2}{2} \Delta_a v + H(\nabla_a v) = b(a(t), m), \\
\frac{\partial m}{\partial t} - \frac{\partial^2}{2} \Delta m + div(m \nabla H(\nabla v)) = 0, \\
v(T, X) = g(X), m(0, X) = m_0(x),
\end{cases}
\] (2.17)

where \( m \) is an equilibrium distribution of the users.
2.3 Hierarchical Games

Hierarchical games theory is an economic framework that was introduced to model the competition between multiple agents with different status with respect to their roles in the system. In two-level hierarchical games, one can distinguish two types of actors with different weights called leaders and followers. The leaders move first knowing that their actions are observed by the followers, and then the followers observe the actions of the leaders and respond to it by determining their actions. By providing the agents different weights, the game can be extended to multiple levels. We can distinguish two classes of hierarchical games *Stackelberg games* that we present next in the context of wireless networks and *multi-games* that we introduce specifically to address the problem of coexistence of LTE and WiFi over unlicensed bands.

2.3.1 Stackelberg Game Framework

A Stackelberg game is a two-stage game that is extensively used in cellular networks for regulating the access to network resources (e.g., frequency bands, transmit power, and storage space) via pricing and modeling resource allocation problems (e.g., spectrum sharing in cognitive systems) in the presence of licensed (incumbent) users and unlicensed (non-incumbent) users. In such resource allocation problems, the licensed users have priority in accessing the network resources. Thus, first, the incumbent users determine the spectrum frequency to use or the access period while predicting the actions of the unlicensed users. Then, the unlicensed users observe the actions of the licensed users and determine when and which network resource to access. To limit the impact of sharing their network resources with the unlicensed users on their own performance in terms of delay and data rate, the incumbent users may impose a price to the unlicensed users. The licensed users determine the price of using every network resource unit based on their global traffic load, the demand and the capacity of the resources. The components of every Stackelberg game can be given as follows.

- **Players:** In a Stackelberg game, two sets of players compete for the same set of network resources. One of the sets corresponds to the incumbent users and the other set contains the non-incumbent users that are known as leaders and followers.
  - **Leaders:** The users in the set of leaders denoted by $\mathcal{L}$ have the ability to anticipate the responses of the followers and take their decisions accordingly.
  - **Followers:** The users in the set of followers denoted by $\mathcal{F}$ adapt their policies based on their observations.

- **Actions:** Each player is associated to a set of actions from which it can choose its strategy. Two sets of actions are defined for the leaders and followers denoted by $\mathcal{A}$ and $\tilde{\mathcal{A}}$, respectively. The set of actions may correspond to the transmit power of the players, the frequency bands, the amount of storage space or the charged price for the non-incumbent users.
2.3. Hierarchical Games

- **Utilities:** The goal of each user in the system is to select the action that maximizes its own benefit in the system. A utility $u_l(a_l, a_{-l}, \tilde{a})$ of a user $l \in L$ is a function of the actions selected by all the other leaders except user $l$ denoted by $a_{-l}$ and the actions selected by the users in $F$ denoted by $\tilde{a}$. Similarly, we define a utility function $\tilde{u}_f(a_f, a, \tilde{a}_{-f})$ for a user $f \in L_f$ that represents the objective function of user $f \in F$. In wireless networks, the utility function captures the physical or economic reward that every user $i \in L \cup F$ can achieve when selecting an action $a_i \in \mathcal{A} \cup \tilde{\mathcal{A}}$ given that all the other users have selected their actions. In such a system, the users can select two types of strategies, pure and mixed strategies. In pure strategies, every user $i$ selects one of the actions $a_i \in \mathcal{A} \cup \tilde{\mathcal{A}}$ with probability one and the other actions with probability zero. When mixed strategies are allowed, a user can select multiple actions with different probabilities different that range from zero to one.

- **Solution concept:** The Stackelberg problem can be solved using the concept of **backward induction** that starts by determining the optimal actions of the followers first, and then move to the leaders problem [107]. That is, by anticipating the best actions of the followers in each situation, it is possible to determine what the leaders will do. Thus, two optimizations problems or games need to be solved depending on the number of leaders and followers, starting by the followers problem first. The goal of the followers is to reach a state in which none of the followers can improve its utility given that the actions of the leaders and the other followers are fixed. Such a state is called the **sub-game Nash equilibrium** and can be defined more formally as follows:

**Definition 3.** A strategy profile $\tilde{a}^*$ is called a sub-game Nash equilibrium iff $\forall f \in F$ and $a_f \in \tilde{\mathcal{A}}$,

$$
\tilde{u}_f(a_f^*, a^*, \tilde{a}_{-f}^*) \geq \tilde{u}_f(a_f, a^*, \tilde{a}_{-f}^*),
$$

(2.18)

where $a^*$ represents the optimal actions of the leaders and $\tilde{a}_{-f}^*$ corresponds to the optimal actions of all the followers except user $f$.

The same solution concept can be defined for the case of mixed strategies where the goal of every user is to maximize its expected utility. Given the solution of the followers problem, the leaders problem is then solved and the complexity of the problem depends on the number of leaders in the system.

Stackelberg games can be classified into two classes based on the number of leaders in the game [108]:

- **Single-leader follower game:** In this class, the game is played by a single leader and a number of followers that compete noncooperatively. Given a strategy of the leader $a$, the followers choose their strategies such that,

$$
\tilde{a}_f^* \in \left\{ \max_{\tilde{a}_f \in \tilde{\mathcal{A}}} u_f(\tilde{a}_f, a_l, \tilde{a}_{-f}) \right\} \quad \text{s.t.} \quad c_f(\tilde{a}_f, a_l, \tilde{a}_{-f}) \geq 0.
$$

(2.19)
where $c_f(a_f) \geq 0$ is a constraint of user $f$. Given that the followers choose their strategies at the equilibrium, the leader optimizes its objective function subject to its own constraint:

$$a_l^* \in \left\{ \begin{array}{l}
\max_{a_l \in A} u_l(a_l, \tilde{a}^*), \\
\text{s.t. } c_l(a_l, \tilde{a}^*) \geq 0
\end{array} \right\} \quad (2.20)$$

- **Multiple-leader follower game:** This class of games arises when two or more leaders with identical followers compete noncooperatively for the same network resources. Such games can be modeled as equilibrium problems with equilibrium constraints as in (2.20). The aim is to find an equilibrium point where no user can improve its utility without degrading the utility of at least one other user. Solving this class of problems is more challenging due to the complexity driven by the number of competing leaders.

### 2.3.2 Multi-Game Framework

Here, we introduce a novel game-theoretic framework namely multi-game, that generalizes existing game-theoretic solutions and enables one to capture all the following characteristics of LTE-U:

- **Disparity in the used access protocols by the LTE BSs and the WAPs on the unlicensed bands,**

- **Heterogeneity of the network devices with the integration of LTE-U BSs and the existing WAPs and BSs,**

- **Different access bands and QoS requirements,** and

- **Need of WiFi-aware algorithms to ensure a fair coexistence on the unlicensed spectrum.**

In this regard, we describe the proposed multi-game model and new solution concepts that are desirable in such networks. In particular, we propose the framework of *multi-game theory* as a new tool suitable to capture the inter-network and intra-network interactions between the users in LTE-WiFi networks. A multi-game $G$ is a game that is composed of multiple interdependent games $G = \{G_1, \ldots, G_N\}$. Each game $G_1$ models a specific resource management problem at the WiFi level, LTE level or coupled level. Multi-games can be defined by three parameters, the players, the actions, and the utility functions. The players can be BSs, WAPs, LTE UEs, or WiFi user equipments (WUEs), each of which is associated a set of actions from which it can choose its strategy. The goal of each player is to select the strategy that maximizes a utility function that corresponds to its goal in the network. For example, BSs aim to maximize the experienced quality by their users over both licensed and unlicensed bands. On the other hand, based on the best-effort service, WiFi users would prefer to deliver a maximum amount of data. The utility is given by
a function \( u_n(a_{-m}, \{ o_i \}_{n=1, \ldots, N\backslash n}) \), which depends on the actions that are selected by all its co-players in the game \( G_n \) as well as the outcome of all the other games denoted \( o_i \) for each game \( G_i \). For instance in LTE-U, the throughput of a WiFi user is limited by the interference from both BSs and WiFi users transmitting over the same channel. The defined multi-game framework for LTE-U have two key properties:

- **Heterogeneity of the games type:** The formulated games to model the interconnected resource allocation problems can be of different type. For instance, the allocation of the unlicensed bands to the WAPs can be formulated as a cooperative game while the allocation of the licensed bands to the BSs as a two-sided matching game. When considering the power allocation problem of the APs, a noncooperative game can be more suitable. The problems can even be formulated as classical non-game theoretic optimization problems.

- **Interdependence between games:** In addition to the utilities of the players that depend on the outcome of the other games, the dependence between the games can appear in two other ways. Each game of the multi-game can be an inter-network or intra-network game, thus, a game can be played exclusively by WiFi users, LTE users, or network operators but it can also be played by a combination of these players. Moreover, the set of actions of the players can be the same for many games such as the APs that can be selected by either WUEs, LTE UEs and UEs having both functionalities.

Hence, in contrast to the popular two-level Stackelberg games in which the dependence between the games and the players appears only through the utility that is a function of the outcome of the other game, in the proposed multi-game model, in addition to the multi-level structure of the game, the dependence between the games can appear in various ways.

In such multi-level games, the incumbent players at the considered network should have priority in accessing the network resources. Thus, when allocating unlicensed bands, the WiFi game is considered as the first-level game giving priority to the WAPs. Then, the players of each game respond to the decisions of the players at the higher-level games while simultaneously, anticipating the decisions of the players at the lower-level games and their possible reactions. The proposed multi-game framework is aligned with the idea of multi-resolution game theory in [109]. However, in contrast to the concepts in [109] which are mainly aligned with problems in security and resilience of control systems, we consider games of different types while focusing on LTE-U resource management.

### Multi-Game Solution Concepts

Having presented the general multi-game model, the goal now is to define suitable solution concepts that can achieve an optimal resource allocation across the LTE and WiFi networks as well as the unlicensed bands. NE and stability are the two solution concepts that can be desirable depending on the formulated games. The NE is suitable for games
2.3. Hierarchical Games

such as noncooperative games while stability is defined in games such as cooperative and matching games in which groups of players are formed/matched. An equilibrium represents the state in which none of the players can improve its utility by any deviation, i.e., by selecting another action given that the actions of the other players are fixed. On the other hand, stability refers to the state in which a player would not have any incentive to leave its group of players given that the other players have selected their partners or group mates.

Applying classical equilibrium and stability notions to LTE-U will not be able to capture the interdependence between the games. Such conventional solutions do not account for the WiFi users’ performance which is affected by the LTE users through the increase of collisions number. Thus, novel LTE-U oriented equilibrium and stability concepts must be defined to protect the WiFi users while maximizing the benefit of the LTE users on the unlicensed bands. To this end, we propose two solution concepts, multi-game equilibrium (MGE) and multi-game stability (MGS) that can be characterized depending on the class of games used to formulate a given LTE-U problem. MGE and MGS represent the states in which all the WiFi users have reached a minimum target utility and all the LTE users have maximized their utility. In this state:

1. None of the WiFi users can reach the target utility by changing their strategies, given the actions of all LTE and other WiFi users fixed,
2. None of the LTE users can improve their utilities by changing their strategies, given the actions of all WiFi and other LTE users fixed.

At the equilibrium, all the WiFi users are satisfied by their utility and none of them can make any further improvements. Next, we present two new classes of algorithms to reach the equilibrium states in the defined multi-game.

Algorithmic Solutions

To allow the players to achieve the MGE and MGS, the classical algorithms that are used in single games cannot be applied anymore. In fact, such algorithms do not capture the interdependences between multiple heterogeneous games, and do not allow the players to exploit the specific properties in multi-games as some players can participate in multiple games at the same time and thus can exploit the feedback from one game to improve their performance on another game. Since the games are of multiple types, multiple algorithms must be combined to reach the equilibria or stable outcomes. More importantly, the algorithms developed must be distributed and provide adaptation to networks’ changes. Following the introduced solution concept, we can define two classes of algorithms.

- **Multi-game stability:** To reach the stable state in wireless networks when the resource allocation problems are formulated as matching games and coalitional formation games, decentralized algorithms such as deferred acceptance [110] and merge-and-split [111] are used. These decentralized algorithmic solutions have been shown
to significantly improve the performance of wireless networks with a low complexity. However, they are confined to single games.

Hence, to reach the MGS outcome defined in the previous section, new cross-system algorithms must be developed. In such algorithms, dual-mode BSs must be able to observe their environment on the WiFi network and exploit this information to define their preferred actions on the LTE network which may correspond to the cells they want to join, or the other BSs with which they prefer to share the bands. Moreover, the multiplicity of the stable states in classical algorithms, introduces new algorithmic challenges that must be accounted for at each of the algorithms that are defined to solve the games. In some cases, the players must be able to determine the worst possible outcome, i.e. stable state, of all the other algorithms and respond to it.

- **Multi-game Nash equilibrium:** Nash equilibrium is the basic solution concept in noncooperative games. To reach this equilibrium, many learning algorithms can be used depending on the characteristics of the considered game [112]. Similar to the stability case, classical learning algorithms are designed for single games and do not account for the target utility requirements of the players and hence cannot be used to reach the introduced MGE.

Consequently, new cross-system learning need to be developed for the multi-level games, to account for the interdependence between the games. In such algorithms, the players can explore the environment, learn their actions and exploit the feedback from the other bands and games. The players can then learn their strategies, estimate and update their utilities iteratively based on their observations.
2.3. Hierarchical Games
Part I

Distributed Caching in Small Cell Networks
Chapter 3

Proactive Social-Caching in Wireless Small Cell Networks

3.1 Overview

The main contribution of this chapter is to develop a novel caching algorithm that aims to reduce the backhaul load and the experienced delay by the end-users when accessing shared videos in OSNs. We leverage the information hosted by OSNs to design an accurate proactive caching strategy, in which the SBSs predict the users’ requests and download ahead of time the related videos. We formulate the cache placement problem using matching theory which is a game theoretic approach that provides solid mathematical tools suitable to study the considered problem [89]. While most of the works in wireless networks rely on the one-to-one and many-to-one matchings [113–118], we formulate the caching problem as a many-to-many matching game which is less understood in the literature. The formulated game is a matching problem between the two sets of Service Provider Servers (SPSs) and SBSs. Each SPS that hosts a set of videos aims to be matched to a group of SBSs where it can cache related videos. On the other hand, the SBSs are matched to a set of SPSs and indirectly to a set of videos that are stored in those SPSs. Subsequently, the SPSs and SBSs rank one another so as to decide on which files will be cached and at which SBSs. To solve the many-to-many game, we propose a novel, distributed algorithm and prove that it can reach a stable outcome, i.e., no player will have the incentive to leave his partners for other players.

The rest of this chapter is organized as follows: Section 3.2 presents the system model. Section 3.3 presents the game formulation and properties. In Section 3.4, we study the proposed stable matching algorithm. Simulation results are presented and analyzed in Section 3.5. Finally, conclusions are drawn in Section 3.6.
3.2 System Model and Game Formulation

Consider two networks, a virtual network and a real network. The virtual network represents an OSN through which \( N \) UEs in the set \( \mathcal{N} = \{u_1, u_2, u_3, \ldots, u_N\} \) are connected to one another via friendship relationships. Thus, these users can interact, communicate and share information with their friends. Suppose that the \( N \) UEs share and watch videos chosen from a library of \( V \) videos in the set \( \mathcal{V} = \{v_1, v_2, \ldots, v_V\} \). The service providers that supply the videos store the provided videos in their SPSs. To ensure a better quality-of-experience (QoE) for the end-users, instead of serving users via capacity-limited backhaul links, service providers prefer to store copies of the videos deeper in the network, i.e., at the SBSs, closer to UEs. The real network consists of the \( N \) UEs, served by \( K \) SPSs in the set \( \mathcal{C} = \{c_1, c_2, \ldots, c_K\} \) and \( M \) SBSs in the set \( \mathcal{S} = \{s_1, s_2, s_3, \ldots, s_M\} \). The real network’s topology is illustrated in Fig. 3.1. Each SPS \( c_i \) is connected to an SBS \( s_j \) via low-rate backhaul link of capacity \( b_{ij} \), using which the SBS downloads videos from that SPS.

The SBSs are equipped with storage units of high but limited storage capacities \( Q = [q_1, q_2, q_3, \ldots, q_M] \), expressed by the number of videos that each SBS can store. Thus, the service providers can cache their videos in the SBSs such that each SBS \( s_i \) can locally serve a UE \( u_j \) via a radio link of capacity \( r_{ij} \).

In this scenario, our goal is to produce a proactive download of video content at the SBSs level. A caching is said proactive if the SBSs can predict the users’ requests and download ahead of time the related videos. Each SBS captures users’ requests for the shared videos based on users’ interests and interactions in the OSN. The most important properties of social content that can be used to design efficient proactive caching strategies are discussed next.
3.2.1 Social Interactions

The videos that a user typically watches depend strongly on the friends who share them. In fact, a user is more likely to request a video, shared by one of his friends, if this user is used to watch the shared videos by that friend [119]. This induced popularity can be given by:

$$I_{social} = \frac{\alpha_{ln}}{\sum_{j=1}^{F_i} \alpha_{jl}}$$

where $\alpha_{ln}$ is the number of videos previously shared by user $u_n$ and viewed by user $u_l$, $F_i$ is the number of $u_i$’s friends.

3.2.2 Sharing Impact

Given that each video file can belong to a distinct category (e.g., news, music, games, etc.), we let $S_{gl}$ the number of videos of category $g$ shared by a user $u_l$. Whenever a user $u_l$’s request for a specific video is predicted and cached in its serving SBS $s_m$, sharing this video with this user’s friends can have an important impact on the traffic load. This sharing impact depends on the number of user $u_l$’s friends that are connected to the same SBS and the probability that user $u_l$ shares the video. More formally, the sharing impact is given by:

$$I_{sharing} = F_i^m \cdot \frac{S_{gl}}{\sum_{i=1}^{G} S_{il}}$$

where $F_i^m$ is the number of $u_l$’s friends that are connected to the SBS $s_m$ and $G$ is the total number of the considered video categories.

3.2.3 Users’ Interests

Whenever a user is interested in a certain topic, it can request a video that belongs to its preferred categories irrespective of the friend who shared it [119],[120]. Based on the categories of the previously watched videos by a user $u_l$, an SBS can predict the user $u_l$’s interests. The impact of this parameter is computed using:

$$I_{Interests} = \frac{V_{gn}}{\sum_{i=1}^{H} V_{il}}$$

with $V_{gn}$ being the number of viewed videos of category $g$ by a user $u_n$, and $H$ being the number of videos in the history of user $u_l$.

Given these factors, our goal is to predict users’ requests and accordingly select and cache a set of videos at each SBS. Hence, we formulate this caching problem as a many-to-many matching game, in which the SPSs aim to cache their videos in the SBSs that offer the smallest download time for the requesting users, while the SBS prefer to cache the videos that can reduce the backhaul load.

3.3 Proactive-Caching as a Many-to-Many Matching Game

3.3.1 Matching Concepts

To model the system as a many-to-many matching game [98], we consider the two sets $C$ of SPSs and $M$ of SBSs as two teams of players. The matching is defined as an assignment
3.3. Proactive-Caching as a Many-to-Many Matching Game

of SPSs in C to SBSs in M. The SPSs acts on behalf of the video files and each of them decides on its own videos. Meanwhile, SBSs store videos depending on their storage capacity. In a matching game, the storage capacity of an SBS s as well as the number of SBSs, in which an SPS c would like to cache a given file v are known as the quotas, q_s and q_{(c,v)}, respectively [89]. Since an SPS decides in which SBS it caches a video v independently from the other owned files, for ease of notation, we use v instead of (c,v) pairs.

**Definition 4.** A many-to-many matching µ is a mapping from the set $\mathcal{M} \cup \mathcal{V}$ into the set of all subsets of $\mathcal{M} \cup \mathcal{V}$ such that for every $v \in \mathcal{V}$ and $s \in \mathcal{M}$ [121]:

1. $\mu(v)$ is contained in $S$ and $\mu(s)$ is contained in $V$;
2. $|\mu(v)| \leq q_v$ for all $v \in \mathcal{V}$;
3. $|\mu(s)| \leq q_s$ for all $s \in \mathcal{S}$;
4. $s$ is in $\mu(v)$ if and only if $v$ is in $\mu(s)$,

with $\mu(v)$ being the set of player v’s partners under the matching $\mu$.

The definition states that a matching is a many-to-many relation in the sense that each stored video in an SPS is matched to a set of SBSs, and vice-versa. In other words, an SPS can decide to cache a video in a number of SBSs and an SBS cache videos originating from different SPSs. Before setting an assignment of videos to SBSs, each player needs to specify its preferences over subsets of the opposite set based on its goal in the network. We use the notation $S \succ_m T$ to imply that SBS $m$ prefers to store the videos in the set $S \subseteq \mathcal{V}$ than to store the ones proposed in $T \subseteq \mathcal{V}$. A similar notation is used for the SPSs to set a preference list for each video. Faced with a set $S$ of possible partners, a player $k$ can determine which subset of $S$ it wishes to match to. We denote this choice set by $C_k(S)$. Let $A(i, \mu)$ be the set of $j \in \mathcal{V} \cup \mathcal{M}$ such that $i \in \mu(j)$ and $j \in \mu(i)$.

To solve the matching game, we are interested to look at a stable solution, in which there are no players that are not matched to one another but they all prefer to be partners. In many-to-many models, many stability concepts can be considered depending on the number of players that can improve their utility by forming new partners among one another. However, the large number of SBSs which is expected to exceed the number of UEs [122], makes it difficult to identify and organize large coalitions than to consider pairs of players and individuals. Hence, in this work we are interested in the notion of pairwise stability, defined as follows [123]:

**Definition 5.** A matching $\mu$ is pairwise stable if there does not exist a pair $(v_i, s_j)$ with $v_i \notin \mu(s_j)$ and $s_j \notin \mu(v_i)$ such that $T \in C_{v_i}(A(v_i, \mu) \cup \{s_j\})$ and $S \in C_{s_j}(A(s_j, \mu) \cup \{v_i\})$ then $T \succ_{v_i} A(v_i, \mu)$ and $S \succ_{s_j} A(s_j, \mu)$

In the studied system, the SBSs and SPSs are always interested in the gain they can get from individuals of the opposite set. For instance, an SBS would always like to first
3.4. Proactive Caching Algorithm

cache the most popular file as long as that file is proposed to it. Thus, even though the set of stable outcomes may be empty [98], SPSs and SBSs have substitutable preferences, defined as follows [124]:

**Definition 6.** Let $T$ be the set of player $i$’s potential partners and $S \subseteq T$. Player $i$’s preferences are called substitutable if for any players $k, k' \in C_i(S)$ then $k \in C_i(S \setminus \{k\})$.

In fact, substitutability is the weakest needed condition for the existence of a pairwise stable matching in a many-to-many matching game [123].

### 3.3.2 Preferences of the Small Base Stations

Based on the social features previously discussed, we define the local popularity of a video $v_i$ at the $m$th SBS as follows:

$$P_{v_i} = \sum_{l=1}^{m} I_{\text{sharing}}(\gamma \cdot I_{\text{social}} + (1 - \gamma) I_{\text{Interests}}),$$  \hspace{1cm} (3.1)

where $\gamma \in [0, 1]$ is a weight that balances the impacts of social interactions and users’ interests on the local popularity of a video.

### 3.3.3 Preferences of the Service Provider Servers

The goal of service providers is to enhance the quality experienced by the end-users. In fact, an SPS $c_i$ would prefer to cache a video $v_k$ at the SBS $s_j$ that offers the smallest download time for the expected requesting UEs. The download time depends on the capacity of the backhaul link $b_{ij}$ and the radio links $r_{jn}$ that connect the SBS $s_j$ to the UE $u_n$. The video file is first downloaded by $s_j$ which then serves the UEs. Thus, in the worst case, downloading a video stored in $c_i$ takes the required time to pass by the link with the poorest capacity. When many UEs are expected to request the same file from $s_j$, the download time is given by:

$$T_D = \frac{1}{\min(b_{ij}, \sum_{n=1}^{N} r_{jn})}.$$  \hspace{1cm} (3.2)

Since each video might be requested by different UEs, an SPS defines its preferences over the SBSs for each owned video file.

### 3.4 Proactive Caching Algorithm

To our knowledge, [125] is the only work that deals with many-to-many matchings in wireless networks. The proposed algorithm deals with responsive preferences which is a
3.4. Proactive Caching Algorithm

Table 3.1: Proposed Proactive Caching Algorithm

<table>
<thead>
<tr>
<th>Phase 1 - Network Discovery:</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Each SPS discovers its neighboring SBSs and collects the required network parameters.</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Phase 2 - Specification of the preferences</th>
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<tbody>
<tr>
<td>- Each SPS and SBS sets its preference list(s).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Phase 3 - Matching algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>- The SPs propose each owned video to the SBSs in $C_{v_i}(\mathcal{M})$ to cache it.</td>
</tr>
<tr>
<td>- Each SBS rejects all but the $q_1$ most preferred videos.</td>
</tr>
</tbody>
</table>

**Repeat**
- The SPs propose each file to the related most preferred set of SBSs that includes all of those SBSs whom it previously proposed to and who have not yet rejected it (**Substitutability**).
- Each SBS rejects all but the $q_s$ most preferred videos.

**Until** convergence to a stable matching

stronger condition than **substitutability**. Thus, the algorithm could not be applied to our case. Under substitutable preferences, a stable matching algorithm has been proposed in [89] for many-to-one games. The pairwise stable matching in the many-to-many problem, has been proved to exist between firms and workers when salaries (money) are explicitly incorporated in the model [123]. Here, we extend and adapt these works to our model in order to propose a new matching algorithm that is proven to reach a pairwise stable.

3.4.1 Proposed Algorithm

After formulating the caching problem as a many-to-many game, we propose an extension of the **deferred acceptance** algorithm [94] to the current model with SPs proposing. The algorithm consists of three phases. During the first phase, SPs and SBSs discover their neighbors and collect the required parameters to define the preferences, such as the backhaul and radio links capacities. This can be done for instance, by exchanging **hello** messages periodically. In the second phase, SPs define a preference list for each owned file over the set of SBSs, while the SBSs define their preferences over the set of videos that would be proposed by the SPs. The last phase consists of two steps. In the first step, every SP proposes an owned video to the most preferred set of SBSs that offer the shortest download time for that video. Afterwards, each SBS $s_j$ rejects all but the $q_j$ most popular videos from the set of alternatives proposed to it. In the second step, the SPs propose an owned video to the most preferred set of SBSs, which includes the SBSs to which they previously proposed that video and have not yet rejected it (**substitutability**). Each SBS rejects all but its choice set from the proposed videos. The second step is repeated until no rejections are issued. The algorithm is summarized in Table 3.1.

3.4.2 Pairwise Stability

Let $\mathcal{P}_{s_j}(k)$ be the set of proposals received by an SBS $s_j$ at step $k$, and $C_{v_i}(\mathcal{M}, k) \subseteq \mathcal{M}$ be the choice set of SBSs to which a video $v_i$ has been proposed at step $k$. By analyzing the algorithm, we state the following propositions.
Proposition 1. Offers remain open: For every video \( v_i \), if an SBS \( s_j \) is contained in \( C_{v_i}(M, (k - 1)) \) at step \( k - 1 \) and did not reject \( v_i \) at this step, then \( s_j \) is contained in \( C_{v_i}(M, k) \).

Proof. Note that \( C_{v_i}(M, k - 1) = C_{v_i}(C_{v_i}(M, k - 1) \cup C_{v_i}(M, k)) \), since \( C_{v_i}(M, k - 1) \) is video \( v_i \)'s choice set from all those sets whose elements have not been rejected prior to step \( k - 1 \), while \( C_{v_i}(M, k) \) is the choice set from the smaller class of sets whose elements have not been rejected prior to step \( k \). Here, substitutability implies that if \( s_j \in C_{v_i}(M, k - 1) \), then \( s_j \in C_{v_i}(C_{v_i}(M, k) \cup \{s_j\}) \). So if \( s_j \in C_{v_i}(M, k - 1) \) is not rejected at step \( k - 1 \), it must be contained in \( C_{v_i}(M, k) \), since otherwise \( C_{v_i}(C_{v_i}(M, k) \cup \{s_j\}) \succ v_i C_{v_i}(M, k) \), violating the requirement that \( C_{v_i}(M, k) \) is the most preferred set whose elements have not been rejected.

\( \Box \)

Proposition 2. Rejections are final: If a video \( v_i \) is rejected by an SBS \( s_j \) at step \( k \) then at any step \( p \geq k \), \( v_i \notin C_{s_j}(P_{s_j}(p) \cup \{v_i\}) \).

Proof. Assume that the proposition is false, and let \( p \geq k \) be the first step at which \( v_i \in C_{s_j}(P_{s_j}(p) \cup \{v_i\}) \). Since \( C_{s_j}(P_{s_j}(p)) \) contains \( C_{s_j}(P_{s_j}(p - 1)) \) by Proposition 1, substitutability implies that \( v_i \in C_{s_j}(C_{s_j}(P_{s_j}(p - 1)) \cup \{v_i\}) \) and thus \( v_i \in C_{s_j}(P_{s_j}(p - 1) \cup \{v_i\}) \), which contradicts the definition of \( p \) and completes the proof of the proposition.

\( \Box \)

Theorem 1. The proposed matching algorithm between SPSs and SBSs is guaranteed to converge to a pairwise stable matching.

Proof. We make the proof by contradiction. Suppose that there exists a video \( v_i \) and an SBS \( s_j \) with \( v_i \notin \mu(s_j) \) and \( s_j \notin \mu(v_i) \) such that \( T \in C_{v_i}(A(v_i, \mu) \cup \{s_j\}) \), \( S \in C_{s_j}(A(s_j, \mu) \cup \{v_i\}) \) and \( T \succ v_i A(v_i, \mu) \) and \( S \succ s_j A(s_j, \mu) \). Then, the video \( v_i \) was proposed to the SBS \( s_j \) which rejected it at some step \( k \). So \( v_i \notin C_{s_j}(A(s_j, \mu) \cup \{v_i\}) \), and thus \( \mu \) cannot be unstable.

\( \Box \)

3.5 Simulation Results and Analysis

For simulations, we consider a network in which we set the number of SPSs, SBSs, UEs and videos to \( K = 80 \), \( M = 150 \), \( N = 400 \) and \( V = 100 \), respectively. We assume that all the SBSs have the same storage capacity, while the backhaul links capacities are lower than radio links capacities which captures the real network characteristics [?]. We set the total backhaul links capacities and radio links capacities to \( B = 80 \) Mbit/time slot and \( R = 180 \) Mbit/time slot, respectively. Due to the unavailability of social dataset that includes the addressed parameters in this chapter (e.g., the accessed video categories and shared videos by users) because of privacy reasons, we generate files popularity and user requests pseudo-randomly. The popularity of files is generated at each SBS with a Zipf distribution which is commonly used to model content popularity in networks [114]. Users’ requests are generated using a uniform distribution.
To evaluate the performance of the proactive caching algorithm, we implement the proposed MA as well as a RA, in which the SBSs are filled randomly with videos from the SPSs to which they are connected to, until reaching their storage capacity limit. We run the two algorithms for different values of a storage ratio $\beta$, which represents the number of files that each SBS has the capacity to store. More formally, $\beta = \frac{q}{V} \forall i \in V$. In the numerical results, we show the evolution of the satisfaction ratio, which corresponds to total number of served requests by the SBSs over the total number of requests, while increasing the number of requests in the network. Moreover, we compare the mean required time for downloading all the videos by the requesting users.

In Fig. 3.2, we show the proportion of served users for $\beta \in \{0.25, 0.75, 1\}$. The satisfaction ratio decreases when the storage capacity of the SBSs decreases. This is evident as less files are cached in the SBSs. Fig. 3.2 shows that, when each SBS has the capacity to store all the proposed videos by the SPSs ($\beta = 1$), the satisfaction ratio remains equal to 1 irrespective of the used caching policy. This result stems from the fact that all the requests are served locally by the SBSs. When the SBSs have the capacity to store 25% and 75% of the proposed videos by the SPSs, i.e., $\beta = 0.25$ and $\beta = 0.75$ respectively, the satisfaction of the MA is up to three times higher than the RA ($\beta = 0.25$). Under the MA, the number of served users by the SBSs decreases by increasing the number of requests. This is due to the fact that SBSs choose to cache first the files with a higher local popularity. Fig. 3.2 shows that, as the number of requests increases, the satisfaction ratio of the RA changes only slightly due to the uniform selection of videos under the same distribution of files popularity.

Fig. 3.3 shows the expected download time of all the files in the network for $\beta \in \{0.25, 1\}$. The download time is lower when the requests are served by the SBSs ($\beta = 1$) compared to the case in which some requests need to be served by the SPSs ($\beta = 0.25$).
This is due to the fact that those videos would require more time to pass through the backhaul links which are of lower capacities compared to the radio links. In Fig. 3.3, we can see that, when $\beta = 1$, the expected download time is identical for both RA and MA. When $\beta = 0.25$, although the download time increases by increasing the number of requests in the network due to the network congestion, the MA outperforms the RA. In fact, the higher satisfaction ratio of the MA compared to RA leads to a smaller expected download time because most of the requests are served via the radio links.

As seen from Figs. 3.2 and 3.3, clearly, the proposed MA allows to efficiently overcome the backhaul bottleneck and improve the performance of video download in small cell networks.

3.6 Closing Remarks

In this chapter, we have proposed a novel caching approach for overcoming the backhaul capacity constraints in wireless small cell networks. We have formulated a many-to-many matching game while considering the limited capacity of backhaul links as well as the impact of the local popularity of each video at the SBSs. To solve the game, we have proposed a new matching algorithm that assigns a set of videos to each SBSs. Simulation results have shown that the proposed matching game enables the SBSs and SPSs to decide strategically on a cache placement that reduces the backhaul links load as well as the experienced delay by the end-users.
3.6. Closing Remarks
Chapter 4

Distributed Caching in Dense Small Cell Networks

4.1 Overview

In the previous chapter, we addressed the caching problem while considering the impact of online social networks on the SBSs decisions. In this chapter, we develop a novel approach to analyze the use of coded caching in a network with a large number of SBSs. For a given SCN, we assume that each SBS has a state vector which is composed of the state of the wireless channel between the SBS and the served UE as well as the state of the storage unit. The dynamics of such a state vector is modeled via an Itô process. Then, we formulate the cache decision problem as a SDG in which the SBSs’ goal is to maximize the number of requests served from the cache while taking into account the state of all the other SBSs. We show that, by considering a dense network of homogeneous SBSs, the SDG can be reduced to a MFG [105, 106, 126], in which the existence and uniqueness of the mean-field equilibrium are guaranteed. In this MFG, the individual state of the SBSs can be replaced by an average overall state, called the mean-field which captures the global system state. Moreover, it is shown that deriving the equilibrium of the formulated MFG amounts to solving a coupled system of Hamilton-Jacobi-Bellman (HJB) and Fokker-Planck-Kolmogorov (FPK) equations. Simulation results show that the proposed framework allows an effective use of the storage space by modeling the dynamics of the storage spaces based on files’ popularities. Moreover, in the proposed framework, the number of served requests from the SBSs can be increased by more than 69% compared to a baseline, static model in which the SBSs are not aware of the instantaneous system state.

The rest of the chapter is organized as follows. Section 4.2 presents the system model. In Section 4.3, we formulate and analyze the stochastic differential game and the mean field game. Simulation results are presented in Section 4.4. Finally, conclusions are drawn in Section 4.5.
4.2 System Model

Consider a wireless SCN composed of a set $\mathcal{N}$ of $N$ SBSs. The SBSs are equipped with data storage units that allow them to serve users’ requests via radio links. Users can request videos from a set $\mathcal{V}$ of $V$ videos. When the requested files are not available in the storage units, users are served from the core network via backhaul links. The aim from equipping the SBSs with storage units is to reduce the traffic load of the capacity-limited backhaul links especially during peak hours. To this end, users’ requests must be predicted before being requested and cached at the network edge to serve users locally via the neighboring SBSs, without using backhaul links. An illustration of the system model is given in Fig. 4.1.

We consider a caching model similar to the one proposed in [38], in which each video is coded using an ideal maximum distance separable (MDS) rateless code, and then cached in the storage units. An MDS code divides the original file $k$ into segments of $q_k$ bits, and each segment is coded into a longer sequence of parity bits, such that any arbitrary $q_k$ bits are sufficient to regenerate the original segment. This caching structure allows each node to control the number of bits to be stored in its storage. Here, a user can be served by more than one node at the same time without having to cache the whole file at all the serving nodes. Moreover, cooperation between SBSs may allow users to receive the requested content within a shorter time duration. The main challenge for each SBS $i$ is to define the fraction of each file $k$ that should be stored while optimizing a given cost. This is done by defining the values of the cache control variables $\mathbf{n}^{(i)} = [n^{(i)}_{1,t}, ..., n^{(i)}_{k,t}, ..., n^{(i)}_{V,t}]$, where $n^{(i)}_{k,t} \in [0, 1], \forall k \in \mathcal{V}$ is the fraction of file $k$ that will be downloaded by node $i$ at time $t$.

The wireless network is assumed to operate using a single frequency band, over which the two sets of SBSs and UE request and exchange video files from the set $\mathcal{V}$ of videos. We
4.2. System Model

assume that the users’ requests follow a Zipf distribution which is a common model used for describing realistic file popularity distributions [18]. Thus, the probability of receiving a request for the $k^{th}$ video at the SBSs is given by the following probability mass function:

$$
\Omega_t(k, V, \beta) = \frac{\frac{1}{k^\beta}}{\sum_{i=1}^{V} \frac{1}{V}},
$$

where the parameter $\beta$ characterizes the steepness of the distribution. Based on this distribution, the nodes need to define a caching policy that allows them to download parts of the requested videos, while taking into account the network conditions such as the channel model as well as the internal state of each node described by the storage capacity [127]. The instantaneous dynamics of these parameters are given next.

### 4.2.1 Channel model

SBSs are configured in order to enhance the coverage of UEs by deploying them closer to the UEs. They can be deployed either by the operators in densely built-up areas or by the UEs inside the buildings. To capture the effects of the channel fading on the transmitted signal in such environments, we use the model in [128], which showed that the instantaneous dynamics of non-stationary channel models can be described by a mean-reverting square process or a mean-reverting Ornstein-Uhlenbeck process. Here, we consider a time-varying additive Gaussian channel model which is given by $|h_{i,t}|^2$. We model the dynamics of the channel by a mean-reverting Ornstein-Uhlenbeck process which is a general model that describes additive Gaussian channels [128]. The dynamics of the channel are given by

$$
dh_{i,t} = \frac{\alpha}{2} (\mu_h - h_{i,t}) \, dt + \frac{1}{2} \sigma_h \, dB_{i,t},
$$

where $\mu_h > 0$, $\sigma_h > 0$ and $B_{i,t}$ is a standard Brownian motion.

### 4.2.2 SINR model

At a given time $t$, the signal received by a user from its serving SBS will experience interference from other SBS links. The signal-to-interference-plus-noise-ratio (SINR) is given by

$$
\gamma_{i,t} = \frac{p_i |h_{i,t}|^2}{N_0 + \frac{1}{N} \sum_{k \neq i, k \in N} p_k |h_{k,t}|^2},
$$

where $p_i$ is the transmit power of SBS $i$, $N_0$ is the background noise level at the served user, and $N$ is the set of all the potential transmitters. Analogous to [129] and [130], the interference from all the other SBSs is normalized to keep it bounded in a dense area of SBSs.
4.3 Problem Formulation

4.2.3 Storage Unit Dynamics

The available storage capacity at the SBSs changes stochastically depending on external control. Indeed, the storage of SBSs can be updated by either the users or the operator. Each UE is served by an SBS \( i \) at an instantaneous rate given by

\[ \kappa_{i,t} = \log(1 + \gamma_{i,t}) \]

We let \( s_{k,i,t}^{(i)} \) be the number of bits from video \( k \) that are stored at SBS \( i \) at time \( t \). To capture the randomness of users’ behavior, we model the dynamics of the storage unit \( s_{k,i,t}^{(i)} \) while serving file \( k \) which was requested at \( l_{k}^{(i)}(x) \), as follows:

\[
d s_{k,i,t}^{(i)} = [n_{k,i,t}^{(i)}q_{k} - \beta(1 - p_{k,t})\bar{\zeta}_{i,t}] \ dt + \sigma_{s} \ dB_{i,t}, \tag{4.3}
\]

where \( n_{k,t} \) is the download rate of video file \( k \) by the SBS and \( q_{k} \) is the size of the file. The second term is the removal rate of file \( k \) at the SBS, where \( \beta > 0 \) is a parameter, \( p_{k,t} \) is the popularity of file \( k \) at time \( t \), and \( \bar{\zeta}_{i} \) is the mean number of bits downloaded by all the users up to time \( t \). In fact, this term models the tradeoff between the popularity of the file and the number of bits that have been downloaded by the users in the time duration \([0,t]\). Here, we assume that there is no broadcasting and only one user can download a content item \( k \) at time \( t \) from a node \( i \). \( \bar{\zeta}_{i} \) is the number of bits of file \( k \) downloaded from node \( i \) by the currently served user and is given by

\[
\bar{\zeta}_{i,t} = \min \left\{ \kappa_{i,t}, s_{k,i,t}^{(i)} - \int_{t_{l}}^{t} \zeta_{i,z} \ dz + \int_{t_{l}}^{t} n_{k,z}^{(i)}q_{k} \ dz \right\}, \tag{4.4}
\]

where \( t_{l} = l_{k}^{(i)}(x) \) is the time at which the SBS \( i \) starts serving the requested file \( k \). The number of downloaded bits is the minimum between the transmission rate from SBS \( i \) to the served UE and the available bits from file \( k \) at node \( i \) which have not been downloaded yet by the served UE .

4.3 Problem Formulation

The goal of an SBS \( i \) is to decide on the values of the cache control variables and find the fraction \( n_{k,i,t}^{(i)} \in [0, 1] \), of video \( k \) that should be downloaded at time \( t \) for serving users. Due to the limited capacity of the backhaul links, SBS \( i \) cannot download more than \( B_{k}^{(i)} \) bits from the core network, resulting in the backhaul download constraint \( \sum_{k \in V} B_{k,i,t}^{(i)} \leq B_{k}^{(i)} \), with \( B_{k}^{(i)} \) the maximum allocated backhaul for downloading the fraction \( n_{k,i,t}^{(i)} \) of file \( k \). Each SBS \( i \) in the system aims at finding the optimal control vector \( \mathbf{n}_{t}^{(i)} = [n_{1,t}^{(i)}, ..., n_{V,t}^{(i)}] \), among the set of all admissible storage allocations \( \mathcal{A}_{i} \), that optimizes a cost function which is defined next.
4.3. Problem Formulation

4.3.1 SBSs’ Cost Function

The goal for each SBS \( i \) is to determine the values of the cache control variables that maximize the amount of cached bits for file \( k \) subject to its storage capacity \( o_i \), in terms of bits. We denote the instantaneous download profile of file \( k \) at all the SBSs by \( n_{k,t} = [n_{k,t}^{(1)}, ..., n_{k,t}^{(N)}] \). The global cost at a given SBS is affected by the following factors.

- **The inter-SBS redundancy cost**: This represents the cost of caching parts of a file \( k \) knowing that this file was already cached by other SBSs. This cost is determined by the function \( c_{k,t}^{(i)} : \mathbb{R}^N \rightarrow \mathbb{R}, n_{k,t} \mapsto c_{k,t}^{(i)}(n_{k,t}) \), which models the dependence between the cache decisions of all the SBSs. Since a UE can be served by many SBSs at the same time, the cache states of the other SBSs need to be considered. This will prevent the SBS \( i \) from caching the same content as the files already cached by the other SBSs. Since the cache state of an SBS is a function of its own cache decision (see (4)), then, the cache decision \( n_{k,t}^{(i)} \) of SBS \( i \) depends implicitly on the cache decisions \( n_{k,t}^{(-i)} = [n_{k,t}^{(1)}, ..., n_{k,t}^{(i-1)}, n_{k,t}^{(i+1)}, ..., n_{k,t}^{(N)}] \) chosen by all the other SBSs. The function \( c_{k,t}^{(i)} \) will be defined later in (13) (see Section 3.C).

- **The in-SBS redundancy cost**: This represents the cost of caching the same bits from the same file at a given SBS. To avoid this, we limit the maximum number of cached bits from file \( k \) at an SBS \( i \) to the size of the file \( q_k \). This constraint can be modeled by \( \nu_i [s_{k,t}^{(i)} - q_k] \), where \( \nu_i \) is a constant.

- **The backhaul cost**: This represents the cost of downloading a fraction \( n_{k,t}^{(i)} \) of file \( k \) in order to be cached. This fraction of file \( k \) is downloaded through the allocated backhaul \( B_{k,t}^{(i)} \) for that file. This cost is given by the following function: \( g_{t}^{(i)}(n_{k,t}^{(i)}) : \mathbb{R}^+ \rightarrow \mathbb{R}, n_{k,t}^{(i)} \mapsto g_{t}^{(i)}(n_{k,t}^{(i)}) \):

\[
g_{t}^{(i)}(n_{k,t}^{(i)}) = \begin{cases} -\log\left(\frac{B_{k,t}^{(i)} - q_k n_{k,t}^{(i)}}{q_k}\right) & \text{if } n_{k,t}^{(i)} < \frac{B_{k,t}^{(i)}}{q_k}, \\ +\infty & \text{if } n_{k,t}^{(i)} \geq \frac{B_{k,t}^{(i)}}{q_k}. \end{cases} \tag{4.5} \]

Note that, the proposed framework can accommodate any other form of the backhaul cost function.

- **The storage cost**: This represents the cost of storage at the SBS and allows modeling the limited storage capacity of the SBS \( i \) which should not exceed \( o_i \) bits. It is given by \( \omega_i \left[\sum_{k=1}^{K} s_{k,t}^{(i)} - o_i\right] \), where \( \omega_i \) is a constant.

Thus, the global cost function can be written as:

\[
J_{k,t}^{(i)}(n_{k,t}^{(i)}, n_{k,t}^{(-i)}) = c_{k,t}^{(i)}(n_{k,t}) + g_{t}^{(i)}(n_{k,t}^{(i)}) + \nu_i \left[s_{k,t}^{(i)} - q_k\right] + \omega_i \left[\sum_{k=1}^{V} s_{k,t}^{(i)} - o_i\right]. \tag{4.6} \]
Next, we formulate the cache control problem as a stochastic differential game.

### 4.3.2 Stochastic Differential Game Formulation

Let $\mathcal{N}$ be the set of players. The state of a player $i$ at time $t$ with respect to a given file $k$ is defined as $y_{k,t}^{(i)} = (h_{i,t}, s_{k,t}^{(i)})$, $\forall i \in \mathcal{N}, k \in \mathcal{V}$. The stochastic differential caching game is defined by $(\mathcal{N}, (\mathcal{Y}_i)_{i \in \mathcal{N}}, A_{i \in \mathcal{N}}, (J_i)_{i \in \mathcal{N}})$ where

- $\mathcal{N}$ is the set of SBSs;
- $\mathcal{Y}_i$ is the set space of SBS $i$ and follows
  \begin{align*}
  dh_{i,t} &= \frac{\alpha}{2} (\mu_h - h_{i,t}) \, dt + \frac{1}{2} \sigma_h \, dB_{i,t}, \\
  ds_{k,t}^{(i)} &= [n_{k,t}^{(i)} - \beta (1 - p_{k,t}) \bar{z}_{i,t}] \, dt + \sigma_s \, dB_{i,t}.
  \end{align*}
  (4.7)

- $A_i$ is the set of admissible caching control policies for node $i$; and
- $J_{k}^{(i)}$ is the cost function of node $i$ defined as follows:
  \begin{align*}
  J_{k}^{(i)} &= E \left[ \int_0^T J_{k,t}^{(i)} (n_{k,t}, n_{k,t}^{(-i)}) \, dt + \psi^{(i)}(\lambda_T) \right],
  \end{align*}

where the function $\psi^{(i)}(\lambda_T) : [0, 1] \to \mathbb{R}, \lambda_T \to \psi^{(i)}(\lambda_T)$ models the cost of having a fraction $\lambda_T$ of free storage space at the end of the period $[0, T]$. This function guarantees that the SBS’s owner will not use all the storage space but will keep a fraction for specific functionalities such as system updates or for the user’s own usage when the SBS is owned by a user.

Assume that, at each time $t \in [0, T]$, a player $i$ can observe the current state $y_{k,t} = (y_{1,t}^{(i)}, ..., y_{k,t}^{(i)}, ..., y_{N,t}^{(i)})$ of the system with respect to file $k$. However, this player has no additional information about the strategy of the other players. In particular, it cannot predict the future actions of the other players. In this case, the solution of the game can be captured via the following equilibrium concept:

**Definition 7.** A control strategy $n_{k,t}^*$ is said to be a feedback Nash equilibrium of the SDG if and only if $\forall i \in \mathcal{N}, \forall k \in \mathcal{V}$, $n_{k,t}^{(i)*}$ is the solution of the control problem

\begin{align*}
  \psi_{k,t}^{(i)}(y_{k,t}) = \inf_{n_{k,t}^{(i)}} J_{k}^{(i)}.
\end{align*}

$\psi_{k,t}^{(i)}(y_{k,t})$ is called the value function.
4.3. Problem Formulation

A condition for the existence of a feedback Nash equilibrium for the SDG is the existence of a solution to the following HJB equations for each SBS $i$ and file $k$ [131]:

$$
\partial_t v^{(i)}_{k,t}(y_k) + \left[ q^{(i)}_{k,t} - \beta (1 - p_{k,t}) \bar{\zeta}_{i,t} \right] \partial_s v^{(i)}_{k,t}(y_k) + \frac{\alpha}{2}(\mu_{k} - h_{k,t}) \partial_h v^{(i)}_{k,t}(y_k) + \frac{\sigma_h^2}{2} \partial^2_{hh} v^{(i)}_{k,t}(y_k) + \frac{\sigma_s^2}{2} \partial^2_{ss} v^{(i)}_{k,t}(y_k) + J^{(i)}_{k,t} (n^{(i)}_{k,t}, n^{(-i)}_{k,t}) = 0.
$$

A sufficient condition for the existence and uniqueness of a solution $v^{(i)}_{k,t}(y_k)$ to the corresponding HJB equation is the smoothness of the drift functions of the dynamic equations and the cost function, i.e. the functions belong to $C^\infty$ [131]. However, even if the choice of the functions in our system guarantees the existence and uniqueness of the solution, solving the $V \times N$ coupled HJB equations can be complex in a dense network of SBSs. Moreover, it is very difficult for a given SBS to observe all the states of the other nodes in a large scale wireless network. Interestingly, the analysis of the system becomes tractable using a mean-field approximation, when the number of players is considered very large. This solution is mainly practical for the emerging SCNs that are expected to include millions of connected devices. Thus, we will study the asymptotic case in the following section.

4.3.3 Mean-Field Game Formulation

We are interested in solving a stochastic optimal control problem when the number of SBSs is large ($N \to \infty$). To study the convergence of the system into the mean field, we assume that the state and download control preserve the exchangeability property which is defined as follows.

**Definition 8.** The states $y^{(1)}_{k,t}, y^{(2)}_{k,t}, \ldots, y^{(N)}_{k,t}$ are said to be exchangeable under the strategy $n^{(i)}_{k,t}$ if they generate a joint law which is invariant by permuting the SBSs’ indices, i.e.,

$$
\mathcal{L}(y^{(1)}_{k,t}, \ldots, y^{(N)}_{k,t} | n^{(i)}_{k,t}) = \mathcal{L}(y^{(\pi(1))}_{k,t}, \ldots, y^{(\pi(N))}_{k,t}),
$$

for any bijection $\pi$ defined over $\{1, \ldots, N\}$.

To guarantee this property, we make the following assumptions:

- Each SBS knows its individual state; and
- Each SBS implements a homogeneous caching policy: $n^{(i)}_{k,t} = f_k(t, y^{(i)}_{k,t})$.

Due to this exchangeability property, all the players become indistinguishable and thus we can focus on a generic SBS whose state is now given by $y_{k,t} = [h_t, s_{k,t}]$. Under the exchangeability property, we can simplify the previous system of coupled HJB equations.
by considering that a given player defines its control policy based only on its state and the mean-field. By considering such a system, a player does not require the knowledge of each player’s state in the system but only the distribution of those players over the states. The convergence of the SDG to a mean field game is provided in the following result.

**Theorem 2.** Define $M^N_{k,t} = \frac{1}{N} \sum_{i=1}^{N} \delta_{y^{(i)}_{k,t}}$ as the occupancy measure of the $N$ SBSs. Suppose that the states $y^{(i)}_{k,t}$ and the caching control $n^{(i)}_{k,t}$ preserve the exchangeability property, then $M^N_{k,t}$ converges in distribution to $m_k$. Moreover, the law $m_k$ is the solution of the following FPK equation:

$$m_{k,0}(y_k) = \rho_0(y_k), \forall y_k$$

$$\partial_t m_{k,t}(y_k) + \left[ n_{k,t} q_k - \beta (1 - p_{k,t}) \bar{\zeta}_t \right] \partial_s m_{k,t}(y_k) + \frac{\alpha}{2} (\mu - h_t) \partial_h m_{k,t}(y_k) - \frac{\sigma^2}{2} \partial^2_{ss} m_{k,t}(y_k)$$

$$- \frac{\sigma^2}{2} \partial^2_{hh} m_{k,t}(y_k) = 0.$$

**Proof.** The proof is given in Appendix A. \qed

Under the exchangeability property and for $N \to \infty$ and $\forall k \in V$ we have, $\gamma_{i,t} \to \gamma_t$, which can be derived as in [130].

Hence, the dynamics of the state for a generic SBS can be defined by the following system of differential equations:

$$\begin{align*}
\frac{dh_t}{dt} &= \frac{\alpha}{2} (\mu - h_t) \ dt + \sigma_h \ dB_t, \\
\frac{ds_{k,t}}{dt} &= \left[ n_{k,t} q_k - \beta (1 - p_{k,t}) \bar{\zeta}_t \right] \ dt + \sigma_s \ dB_t.
\end{align*}$$

(4.10)

The aim for each node is to choose a caching control $n_{k,t}$ for each file $k \in V$ in order to minimize the following cost function:

$$J_k = \mathbb{E} \left[ \int_0^T J_{k,t}(n_{k,t}, m_{k,t}) \ dt \right] + \psi(\lambda_T).$$

(4.11)

Now we can redefine the cost function as a function of the mean field process. In this regard, we define the function $c_{k,t}(m_{k,t})$ which is now a function of the mean field process $m_{k,t}$. Intuitively, the larger is the expected cached fractions of file $k$ in the network, given by $\phi_k = \frac{d}{dt} \int_0^t s_k m_{k,t}(y_k) \ dh$, the lower is the interest of the SBS in caching bits from that file. However, when the mean number of cached bits of file $k$ is low, the cost should be defined in order to encourage the SBS to cache bits of file $k$ until a given threshold. On the other hand, an SBS would aim to cache more bits from file $k$ when the expected requests for that file is high. Thus, the cost depends as well on the distribution of users’ requests. The cost can then be written as

$$c_{k,t}(m_{k,t}) = \exp(-\phi_t \phi_k) + \frac{\theta_2 \phi_k}{\Omega_t(k, V, \beta)},$$

(4.12)
4.3. Problem Formulation

where $\varrho_1$ and $\varrho_2$ are constants.

The value function for a generic SBS is given by

$$v_{k,t}(y_k) = \inf_{n_{k,t}} \{ J_k \}. \quad (4.13)$$

Finding the optimal control of a given SBS and file $k$ amounts to jointly solving the following mean field problem:

$$\partial_t v_{k,t}(y_k) + [n_{k,t}q_k - \beta(1 - p_{k,t})\bar{\zeta}_t] \partial_s v_{k,t}(y_k) + \alpha \left( \mu - h_t \right) \partial_h v_{k,t}(y_k) + \frac{\sigma^2}{2} \partial_{ss} v_{k,t}(y_k) + J_{k,t}(n_{k,t}, m_{k,t}) = 0,$$

$$m_{k,0}(y_k) = \rho_0(y_k), \forall y_k,$$

$$\partial_t m_{k,t}(y_k) + [n_{k,t}q_k - \beta(1 - p_{k,t})\bar{\zeta}_t] \partial_s m_{k,t}(y_k) + \frac{\alpha}{2} \left( \mu - h_t \right) \partial_h m_{k,t}(y_k) - \frac{\sigma^2}{2} \partial_{ss} m_{k,t}(y_k) = 0.$$  

The advantage of the mean field formulation is that SBSs do not need full knowledge of the state or the caching strategy of other SBSs to compute the outcome of the game. Also, in order to find the optimal power control an SBS has to solve only one HJB equation for a given file. To compute the optimal download control, we have to solve the two coupled equations in $v_k$ and $m_k$. From the optimization standpoint, finding the solution of the stochastic optimal control amounts to finding the optimal caching control that minimizes the Hamiltonian. The Hamiltonian function is defined as follows:

$$H(y_k, m_{k,t}, \nabla v_k) = \left\{ \frac{\alpha}{2} \left( \mu - h_t \right) \partial_h v_{k,t}(y_k) + [n_{k,t}q_k - \beta(1 - p_{k,t})\bar{\zeta}_t] \partial_s v_{k,t}(y_k) \right\} + J_{k,t}(n_{k,t}, m_{k,t}) \quad (4.14)$$

with $\nabla v_k$ the gradient.

The optimal value of the number of bits that should be downloaded by each SBS is given in the following result.

**Proposition 3.** The optimal control $n_{k,t}^*$ of the stochastic mean field game is given by

$$n_{k,t}^* = \frac{1}{q_k} \left[ B_{t,k} - \frac{1}{2\alpha \partial_s v_{k,t}(y_k)} \right]. \quad (4.15)$$

**Proof.** The proof is given in Appendix A. \qed
4.4 Numerical Results

To solve the HJB-FPK system of equations, we proceed by solving iteratively the two equations using a simple fixed-point algorithm until convergence. We assume a static channel model and thus the state is only defined by the cache state of the SBSs. The transmit power is set to $p = 1$ W and the noise to $N_0 = -80$ dBm. In Fig. 4.2, we show the evolution of $m^*$ for one file whose popularity increases over 24 hours. For this, the file size is normalized to 1 and the storage capacity of the SBSs is set to $\phi = 2/5$. The initial distribution of the SBSs follows a normal distribution $\mathcal{N}(0, 0.2)$. Fig. 4.2 shows that the cached fraction of the file at the SBSs decreases when the file’s popularity is low, thus making the storage space available for other more popular files. When more requests are expected for that file, all the SBSs cache a higher fraction of the file, in the limit of their storage capacity. This allows all the users to be served by any random subset of SBSs in their proximity, improving the experienced quality in terms of download time.

In Fig. 4.3, we compare the proposed framework with a baseline method in which the SBSs are not aware of the instantaneous global state of the system. To define their optimal download rate and minimize the cost, the SBSs make their decisions based on the information from their previous experience by averaging the storage state over the past time periods. The two methods are compared in terms of the fraction of satisfied requests from the cache of the SBSs without using the backhaul with respect to the inter SBS site distance that models the density of the SBSs in the network. That is, the smaller is the distance between the SBSs, the denser is the network; here, one unit of inter SBS distance is set equal to 25 m. The comparison is performed for two different cases. The first case corresponds to scenarios that exhibit a large variability in the file popularity (LVP), from a given time period to another. In the second case, the popularity is assumed to change slightly (SVP). We can see from Fig. 4.3, that the denser the network, the higher is the number of requests served by the SBSs. In fact, densifying the networks results in a given user being in the coverage area of a large number of SBSs which increases the probability of finding the requested file at one of the SBSs in its vicinity. The performance of the baseline model is closer to the proposed model when the popularity of the files changes slightly.
4.5 Closing Remarks

This is due to the fact that the SBSs in this model update their storage space based on the past information which remains valid when the popularity changes slightly. However, the proposed algorithm outperforms the baseline model by increasing the number of served requests from the SBSs by up to 69%, when the popularity of the files varies largely. In this case, fewer requests are served from the SBSs in the baseline model due to the significant changes in the SBSs' states over time, which does not allow the SBSs to adapt their control variables according to the real system state.

4.5 Closing Remarks

In this chapter, we have studied the problem of distributed caching in ultra-dense small cell networks. We have formulated the cache control problem as a mean-field game in which the SBSs aim to minimize a given cost function while taking into account the cache state of all the other SBSs in the network. We have analyzed and showed the existence and uniqueness of the mean field equilibrium which is the solution of a coupled system of HJB and FPK equations. We have showed through simulations, that the proposed method enables efficient use of the storage space efficiently by adapting the control variable to the popularity of the files and the global state of the storage spaces in the network. Moreover, the proposed approach significantly decreases the load on the backhaul links by serving more requests locally from the SBSs.
4.5. Closing Remarks
Chapter 5

Breaking the Economic Barrier of Caching in Cellular Networks

5.1 Overview

In the two previous chapters, we have proposed new caching mechanisms that enable the SBSs to decide independently which files to cache. In this chapter, we address the economic aspect of caching and propose a cache incentive mechanism between an MNO and multiple CPs. We formulate the cache incentive problem as a Stackelberg game in which the MNO is a leader and the CPs are the followers. The MNO predicts the requests profile of the CPs and define the price that maximizes its revenue. On the other hand, due to limited storage capacity of the SBSs, CPs cannot cache all their files, thus, the CPs compete to maximize the amount of storage space they request given the fixed price by the MNO. The competition between the CPs is formulated as a non-cooperative sub-game in which each CP aims to maximize the satisfaction of its users in terms of transmission rate. Both the existence and uniqueness of the NE are proved. The NE represents a state in which none of the CPs can improve its benefit by requesting a different amount of storage space given the requested amount by the other CPs fixed. We provide closed-form expressions of the storage amount the CPs must request at the NE. Then, given the request profiles of the CPs, an optimization problem is formulated at the MNO to determine the optimal price that should be charged to the CPs to maximize the MNO’s revenue. Simulation results show that the MNO can get a revenue that is up to 50% higher compared to the case in which the prices are selected arbitrarily. Moreover, at the equilibrium, we show that the CPs can achieve a utility that is 20% to 50% higher compared to the two cases in which half and the double of the storage space at the NE are requested by the CPs, respectively.

The rest of the chapter is organized as follows. In Section 5.2, we define the system and the caching models. The Stackelberg game is formulated in Section 5.3, and the analysis is conducted in Section 5.4. The numerical results are presented in Section 5.5. We finally draw conclusions in Section 5.6.
5.2 Preliminaries

5.2.1 System Model

We consider a heterogeneous network (HetNet) which consists of \( N \) cache-enabled SBSs and is controlled by one MNO. There are \( M \) CPs that are willing to cache their files in the HetNet in order to enhance the QoS of their users. Let \( \mathcal{N} = \{1, 2, \ldots, N\} \) denote the set of cache-enabled SBSs, each of which has a limited capacity \( s_n \), and \( \mathcal{M} = \{1, 2, \ldots, M\} \) denote the set of CPs. Each CP requests from the MNO to cache its \( q_m \) most popular files. The \( N \) SBSs are connected to the MNO with a backhaul such as DSL, optical fibers or wireless backhaul. All the UEs communicate with the corresponding SBSs. If the desired files from the UEs are not cached in the SBSs, they are served by the MNO via the capacity-limited backhaul links. The system model is depicted in Fig. 5.1.

Each CP \( m \) has a local content catalog \( \mathcal{F}_m \) with \( F_m \) files. The global files catalog is denoted \( \mathcal{F} = \bigcup \mathcal{F}_m \), and all the files are assumed to have the same unit size. We assume that each file \( f_i \) is cached by the MNO \( p_{fi} \) times at the SBSs based on the popularity of the file. The goal of each CP is to cache its most popular files locally at the SBSs so that the users experience an improved data transmission rate. However, the storage capacity of the SBSs is limited and not all the files can be cached with sufficient copies. The notations used in the rest of this chapter is summarized in Table 5.1.

5.2.2 Cache Incentive Model

We formulate the cache incentive problem as a Stackelberg game in which a non-cooperative game is involved as a sub-game. As shown in Fig. 5.1, the Stackelberg game is played between the MNO and the \( M \) CPs. On the one hand, all the CPs wish to cache as many as possible files at the MNO’s HetNet such that the QoS of their users (for example the delay) is improved. On the other hand, there exists a cost for caching. Therefore, the
5.3. Stackelberg Game Formulation

Table 5.1: List of Notations.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{N} = {1, 2, \ldots, N}$</td>
<td>set of SBSs</td>
</tr>
<tr>
<td>$\mathcal{M} = {1, 2, \ldots, M}$</td>
<td>set of CPs</td>
</tr>
<tr>
<td>$q_m$</td>
<td>The number of files that CP $m$ wants to cache</td>
</tr>
<tr>
<td>$p_{f_i}$</td>
<td>The number of cached copies of file $f_i$</td>
</tr>
<tr>
<td>$s_n$</td>
<td>The cache size of each SBS $n$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>The charged price for caching a given file</td>
</tr>
<tr>
<td>$u_o$</td>
<td>The utility of MNO</td>
</tr>
<tr>
<td>$u_m$</td>
<td>The utility of CPs</td>
</tr>
<tr>
<td>$Q_m$</td>
<td>The set of files requested by each CP $m$</td>
</tr>
<tr>
<td>$p_m$</td>
<td>The mean access probability of CP $m$’s files</td>
</tr>
<tr>
<td>$d(Q_m)$</td>
<td>The total caching copies of requested files of all CPs</td>
</tr>
</tbody>
</table>

Incentive proactive caching mechanism is controlled by a charge price determined by the MNO to optimize its revenue in the proposed Stackelberg game. The sequential game (Stackelberg) leads to a more competitive equilibrium than the simultaneous move game: The MNO plays as the leader providing the caching price $\pi$ to all the CPs, while the CPs play as the followers reacting with their optimal number of files they want to cache based on the given price. Both the MNO and the CPs are rational and optimize their own utilities denoted as $u_o$ and $u_m$, respectively. The strategy of the leader MNO is the caching price $\pi$ and the strategies of the followers CPs are their the number of files they want to cache $q_m(\pi)$. It should be noted that the price defined by the MNO depends on the used caching policy and the storage capacity of all the SBSs.

Due to the limited caching capacity, the $M$ CPs compete for the possible caching quantities. Therefore, a non-cooperative game is played as a sub-game among the $M$ CPs. We assume that the file access probability of each file is perfectly known at all the entities (i.e., CPs and MNO). The strategies of the CPs are the quantity of their caching requests. Under the perfect knowledge of the access probability of each caching file, the CPs will choose to cache the first $q_m^*$ files ordered with the highest access probability, i.e., popularity.

5.3 Stackelberg Game Formulation

In this section, we provide the utilities of the MNO and the CPs, respectively. The proposed Stackelberg game is played as follows. The leader plays first by providing its optimal strategy to all the followers after predicting the strategies of the followers. Then, the followers reply with their best strategies given the strategy of the leader. The result of the Stackelberg game is that both the leader and the followers optimize their utilities. The Stackelberg equilibrium is exactly the point that the leader wishes. The basic idea of
5.3. Stackelberg Game Formulation

the utilities for both the MNO and the CP can be formulated as a general function, such as $Utility = Revenue - Cost$. In the following, we will analyse the utilities of the MNO and the CPs, respectively.

5.3.1 Utility of the MNO

For the MNO, the main cost is the caching storage cost denoted as $C_o$. This caching storage cost is a function of the caching capacity of all the SBSs and the number of times each file is cached. Therefore, the cost of caching all CPs files $C_o$ for the MNO can be defined as the following barrier function:

$$C_o = \begin{cases} \frac{1}{S-d(Q_M)} & \text{if } 0 < d(Q_M) \leq S \\ \infty & \text{otherwise}, \end{cases} \quad (5.1)$$

where $d(Q_M)$ is the amount of all the cached files by the MNO and is given by:

$$d(Q_M) = \sum_{m=1}^{M} \sum_{f_i \in Q_m} p_{f_i}, \quad (5.2)$$

where $p_{f_i}$ is the number of copies of file $f_i$ that are cached at the SBSs and defined by the MNO depending on the popularity of file $f_i$. The caching capacity of the MNO is given by:

$$S = \sum_{n=1}^{N} s_n. \quad (5.3)$$

The revenue of the MNO in the caching problem is the total charge from all the CPs for caching their files. The price of caching one file is denoted by $\pi$ and the number of files that is requested by a CP $m$ to be cached is $q_m$. Thus, the total revenue of the MNO can be given by

$$R_o = \pi \sum_{m=1}^{M} q_m(\pi). \quad (5.4)$$

Now we obtain the utility function of the MNO in our proposed Stackelberg game as a function of the quantity of caching request from all the CPs, $q = [q_1, \ldots, q_M]$.

$$u_o(q(\pi)) = R_o(q(\pi)) - C_o(q(\pi)), \quad (5.5)$$

where $R_o$ and $C_o$ are defined in (5.1) and (5.4), respectively.
5.3.2 Utility of the CPs

Given that the MNO fixes a price $\pi$ for a storage unit, all the CPs reply with their quantity of caching request by optimizing their own utilities. We assume that each CP is independent of the others and makes its best strategy only based on its local information.

The cost of CPs for the caching requests are the charge paid to the MNO for their desired quantity of caching files. For each CP $m$, the cost $C_m$ is defined as

$$C_m = \pi q_m(\pi),$$

(5.6)

where $q_m(\pi)$ is a function of the provided charge price $\pi$. $C_m$ is an increasing function of the caching quantity $q_m(\pi)$.

In fact, the CPs such as Facebook and Youtube are concerned about the satisfaction of their users. This user satisfaction, which can be the delay of downloading a file or the data throughput, is an increasing function of the caching request quantity $q_m$ of CP $m$ and a decreasing function of the caching request quantities $q_{-m}$ of others because of the limited caching capacity. $q_{-m}$ is defined as the caching request quantities of all the other CPs except CP $m$, i.e.,

$$q_{-m} = [q_1, \ldots, q_{m-1}, q_{m+1}, \ldots, q_M].$$

(5.7)

The revenue of CPs can be easily defined as the satisfaction function of each CP. We assume that the $q_m$ files with the highest access probabilities in each CP $m$ are requested. Then the satisfaction function of CP $m$ is as follows, which is an increasing function of the caching request quantity.

$$R_m = \log \left(1 + \frac{q_m}{1 + \frac{1}{\alpha_m} J_m}\right),$$

(5.8)

where $J_m = \sum_{l \neq m} q_l$ is the quantity of caching files requested by all the other CPs except CP $m$ and $\alpha_m$ is the amount of generated requests by the users of CP $m$. The satisfaction $R_m$ of a CP $m$ was chosen as it is an increasing function of the number of files that are cached by a CP $m$ as well as its traffic load $\alpha_m$. Indeed, the more files are cached at the SBSs and the higher the popularity of the files, the higher is the number of requests that can be served locally from the SBSs. Thus, more users of CP $m$ can experience a higher transmission rate. On the other hand, the satisfaction of a CP $m$ decreases as the number of files cached by the other CPs increases. In this case, less storage is available for CP $m$ and a higher number of its requests need to be served through the backhaul. It should be noted that any other function that has the same properties as $R_m$ can be chosen as a satisfaction function.

Given the constraints on the caching capacity of the mobile network, now we obtain the utility function of each CP $m$ as

$$u_m = R_m - C_m,$$

(5.9)
5.4. Stackelberg Game Solution

where the cost $C_m$ and the revenue $R_m$ are defined in (5.6) and (5.8), respectively. It is obvious that $u_m$ is a concave function of the quantity of the caching requests.

Given the charge price from the MNO, all the CPs respond with their quantities of caching requests simultaneously by maximizing their own utilities. Due to the limited caching capacity $S$, all the CPs can be considered as competitive players competing for the caching quantities. Therefore, we can formulate the quantity optimization problem of CPs as a non-cooperative game $G = G(M, Q, U)$ consisting of the following components:

- The set of players in $G$ is the set $M$ of CPs.
- Given the quantity profile $q_{-m} \in Q_m = \prod_{l \neq m} Q_l$ of the opponents of player $m$, the feasible action set of CP in the presence of the caching capacity $d(Q_M) \leq S$ is
  \[ Q_M(q_{-m}) = \{ q_m \in Q_m : q_m \geq 0 \}. \] (5.10)
- The utility $u_m$ of player $m$ is given by (5.9).

In this context, the most adopted solution concept is the NE.

**Definition 9.** A quantity profile $q^*$ is a NE of the caching request quantity game $G$ if

\[ q_m^* \in Q_m(q_{-m}^*), \quad \forall m \in M, \] (5.11)

\[ u_m(q^*) \geq u_m(q_m, q_{-m}^*), \quad \forall q_m \in Q_m(q_{-m}^*). \] (5.12)

Based on the utility functions of the MNO and CPs, we derive the optimal strategies of the leader and the followers in the next section. In the Stackelberg game, the followers’ strategies are predicted before the leader makes its own strategy. Therefore, the optimal quantity of caching requests of the CPs is analyzed first.

### 5.4 Stackelberg Game Solution

In this section, we derive the solution of the proposed Stackelberg game. In the sequential (Stackelberg) game, the leader moves first by predicting the strategies of the followers. The followers reply by optimizing their own utilities given the strategy of the leader. The result of the game is that the followers play exactly what the leader wishes. Therefore, the optimal strategy of the followers should be predicted first. In the following, we obtain the optimal quantities of caching requests of the CPs in the closed form.

#### 5.4.1 Optimal Quantity of Caching Request

The CPs optimize their strategies of the quantity of the caching requests and provide them to the MNO given the charge price $\pi$ announced by the MNO. The optimal $q_m$ is obtained by solving the following problem.

\[ q_m = \arg \max_q u_m \quad \text{subject to} \quad q \geq 0. \] (5.13)
5.4. Stackelberg Game Solution

Given the charge price for the caching files, the CPs compete for the quantity of caching requests, which formulates a non-cooperative game. In the following, we find the solution for this non-cooperative game.

**Proposition 4** (Best Response). *Given the charge price $\pi$ announced by the MNO, the best response (BR) of each CP is the quantity of caching files it requests, which is*

$$q_{m}^{BR} = \left(\frac{1}{\pi} - 1 - \frac{J_{m}}{\alpha_{m}}\right)^{+},$$  \hspace{1cm} (5.14)

*with $\alpha_{m} \geq M$.\)

**Proof.** The BR of the caching request quantity $q_{m}^{BR}$ is obtained by checking the first derivative of $u_{m}$ with respect to $q_{m},$

$$\frac{\partial u_{m}}{\partial q_{m}} = \frac{\alpha_{m}}{\alpha_{m} + \alpha_{m}q_{m} + J_{m}} - \pi = 0. \hspace{1cm} (5.15)$$

The second derivative of $u_{m}$ with respect to $q_{m}$ is

$$\frac{\partial^{2} u_{m}}{\partial q_{m}^{2}} = \frac{-\alpha_{m}^{2}}{(\alpha_{m} + \alpha_{m}q_{m} + J_{m})^{2}} < 0, \hspace{1cm} (5.16)$$

which guarantees a global optimal of $u_{m}$. Since the utility of each CP $u_{m}$ is a convex function of $q_{m}$, (5.14) is proved. The function $()^{+}$ is to ensure the requested caching quantity to be a non-negative value.\)

**Remark 1.** The total quantity of the caching requests of all the other CPs can be learnt or fed back at each CP. The $q_{m}^{BR}$ is based on the local information $\alpha_{m}, q_{m}$ and the feedback $J_{m}$ as a single value of the summation.

Now we prove that the proposed non-cooperative game admits a unique NE.

**Theorem 3** (Nash Equilibrium). *Given the charge price $\pi$ announced by the MNO, the NE of each CP is the quantity of caching files it requests, which is*

$$q_{m}^{NE} = \frac{D_{m}}{D} = \frac{(\frac{1}{\pi} - 1)a_{m}b_{m}}{D},$$  \hspace{1cm} (5.17)

*where $a_{m}$ is*

$$a_{m} = \begin{cases} 1 - \frac{1}{\alpha_{m}} - \frac{1-\alpha_{1}}{\alpha_{1}} \sum_{l=2,l\neq m}^{M} \frac{\alpha_{m}-\alpha_{l}}{(\alpha_{1}-1)\alpha_{l}} & \text{if } m \neq 1 \\ 1 - \frac{1}{\alpha_{1}} - \frac{1-\alpha_{M}}{\alpha_{M}} \sum_{l=2}^{M-1} \frac{\alpha_{M}-\alpha_{l}}{(\alpha_{1}-1)\alpha_{l}} & \text{if } m = 1 \end{cases} \hspace{1cm} (5.18)$$

*and $b_{m}$ is*

$$b_{m} = \begin{cases} \prod_{l=2,l\neq m}^{M}(1 - \frac{1}{\alpha_{l}}) & \text{if } m \neq 1 \\ \prod_{l=2}^{M-1}(1 - \frac{1}{\alpha_{l}}) & \text{if } m = 1 \end{cases} \hspace{1cm} (5.19)$$
and

\[ D = \left( 1 - \frac{1 - \alpha_1}{\alpha_1} \sum_{l=2}^{M} \frac{1}{\alpha_l - 1} \right) \prod_{l=2}^{M} \left( 1 - \frac{1}{\alpha_l} \right). \] (5.20)

If \( M = 2 \), then \( \sum_{l=2}^{M-1} \frac{\alpha_1 - \alpha_l}{(\alpha_l - 1)\alpha_1} = 0 \) and \( \prod_{l=0}^{M-1} 1 - \frac{1}{\alpha_l} = 1 \).

**Proof.** Here, we give only the sketch of the proof. The trick for finding the NE of the proposed non-cooperative game is to jointly solve the \( M \) functions of the BR for all the CPs. The \( M \) functions of the BR can be formulated as a matrix function denoted as \( Dq = C \) where the matrix \( D \) and vectors \( q \) and \( C \) are as follows, respectively.

\[
\begin{pmatrix}
\frac{1}{\alpha_1} & \frac{1}{\alpha_1} & \cdots & \frac{1}{\alpha_M} \\
\frac{1}{\alpha_2} & 1 & \cdots & \frac{1}{\alpha_2} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{1}{\alpha_M} & \frac{1}{\alpha_M} & \cdots & 1
\end{pmatrix}
\begin{pmatrix}
q_1 \\
q_2 \\
\vdots \\
q_M
\end{pmatrix}
= \begin{pmatrix}
\frac{1}{\pi} - 1 \\
\frac{1}{\pi} - 1 \\
\vdots \\
\frac{1}{\pi} - 1
\end{pmatrix}. \] (5.21)

The NE quantity of each CP is solved by applying the Cramer’s rule \( q_m = \frac{\det(D_m)}{\det(D)} \), where \( \det(D) \) is the determinant of matrix \( D \) and \( \det(D_m) \) is the determinant of matrix \( D_m \) which is formed by replacing the \( m \)-th column of \( D \) by the column vector \( C \).

**Remark 2.** In caching, there are two basic modes. One is coded caching in which the SBSs cache any number of bits from the files. In this case, the NE quantity derived in (5.17) is the actual quantity of files requested by CP \( m \). The other one is uncoded caching, in which a given file can only be cached as a whole or equally divided chunks. In this case, the round function \( \lfloor \cdot \rfloor \) is applied to \( q^{NE}_m \) before CP \( m \) replies to the MNO in order to obtain the integer optimal quantity of caching request. In our model, we assume the uncoded caching case so that only integer quantity of caching requests are considered.

### 5.4.2 Optimal Charge Price

After predicting the strategy policies of the CPs, the MNO optimizes the charge price \( \pi \) by solving the following problem.

\[ \pi^* = \arg \max_{\pi} u_o(q(\pi)) \quad \text{subject to} \quad \pi \geq 0. \] (5.22)

We observe from (5.1) and (5.2) that the quantity of caching request from each CP \( m \) exists in the size of the set \( Q_m \). Therefore, we assume that the number of copies of each file \( f_i \in Q_m \) that are cached are defined as the quantized vector given by:

\[ p_{f_i} = [\ldots, p_m - \Delta p_m, p_m, p_m + \Delta p_m, \ldots] \] (5.23)

with size of \( q_m \) for each CP \( m \). \( p_m \) is the mean access probability of files requested by CP \( m \) and \( \Delta p_m \) is the step size.
5.4. Stackelberg Game Solution

Given the number of times $p_{f_i}$ each file $f_i$ is cached as defined in (5.23), we obtain

$$
\sum_{f_i \in Q_m} p_{f_i} = q_m^{NE} f(p_m), \quad (5.24)
$$

where $f(p_m)$ is a function of the mean access probability $p_m$ with the following form

$$
f(p_m) = \begin{cases} 
p_m & \text{when } q_m \text{ is odd}, 
p_m + \frac{\Delta p_m}{2} & \text{when } q_m \text{ is even}. 
\end{cases} \quad (5.25)
$$

Since the MNO can predict the NE quantities of all the CPs, which are functions of the charge price $\pi$, the utility function of the MNO is then

$$
u_o = \pi \sum_{m=1}^{M} q_m(\pi) - \frac{1}{S - \sum_{m=1}^{M} q_m(\pi) f(p_m)}. \quad (5.26)
$$

By predicting the requesting $q_m(\pi)$ from each CP $m$, the MNO can make its own optimal strategy, which is the charge price $\pi^*$. Here we analyse the case where the total caching requests do not exceed the caching capacity of the HetNet.

**Proposition 5 (Optimal Price).** The optimal charge price $\pi^*$ provided by the MNO to maximize its own utility $u_o$ is as follows.

$$
\pi^* = \sqrt{\frac{r}{S + r}}, \quad (5.27)
$$

where $r$ is a function of $\alpha$ and $t$ is a function of both $\alpha$ and $p$ defined as

$$
t = \sum_{m=1}^{M} \frac{a_m b_m}{D}, \quad (5.28)
$$

and

$$
r = \sum_{m=1}^{M} \frac{a_m b_m f(p_m)}{D}. \quad (5.29)
$$

**Proof.** Since $r$ and $t$ are independent of the optimization variable $\pi$, we rewrite the utility function of MNO $u_o$ as

$$
u_o = (1 - \pi) t - \frac{1}{S - (\frac{1}{\pi} - 1)r}. \quad (5.30)
$$

Then the first derivative with respect to $\pi$ is

$$
\frac{\partial u_o}{\partial \pi} = \frac{r}{(S\pi - (1 - \pi)r)^2} - t. \quad (5.31)
$$
The second derivative with respect to $\pi$ is

$$\frac{\partial^2 u_o}{\partial \pi^2} = \frac{-2r(S\pi - (1 - \pi)r)(S + r)}{(S\pi - (1 - \pi)r)^4}.$$  \hfill (5.32)

By observing that the MNO will definitely provide a price in order to ensure $d(Q_m) \leq S$, $S\pi - (1 - \pi)r > 0$ and then $\frac{\partial^2 u_o}{\partial \pi^2} < 0$, which guarantees a global maximum $\pi^*$. By solving $\frac{\partial u_o}{\partial \pi} = 0$ in (5.31), we get the value of $\pi^*$ in (5.27), which completes the proof.

Notice that the charge price should be a positive value $\pi^* > 0$, therefore the result $-\sqrt{r + r} \frac{S + r}{S + r}$ is dropped.

To ensure a positive total charge and also that the total caching requests do not exceed the caching capacity $S$, i.e., the cost $C_o$ is a positive limited value, the charge price given by the MNO should be restricted in the following range.

**Corollary 1.** The caching capacity $S$ of the HetNet can be fully exploited if the charge price given by the MNO follows the range

$$\frac{r}{S + r} < \pi < 1.$$  \hfill (5.33)

**Proof.** $\pi < 1$ is proved by ensuring $\frac{1}{\pi} - 1 > 0$ in (5.30). $\frac{r}{S + r} < \pi$ is proved by ensuring $\frac{1}{S - \left(\frac{r}{S} - 1\right)r} > 0$ in (5.30).

**Remark 3.** The optimal price $\pi^*$ provided in (5.27) always satisfies the feasible price range in (5.33).

### 5.4.3 Example

In this part, we show the 2-CP case as an example to illustrate the incentive Stackelberg game on caching. The protocol of the proposed Stackelberg game played between the MNO and CPs and the non-cooperative game played among the CPs are described as follows.

- The MNO predicts the NE quantities of the caching requests from all the CPs before making its own strategy.

$$q_1^{NE} = \frac{(\frac{1}{2})(\alpha_1 - 1)\alpha_2}{\alpha_1\alpha_2 - 1},$$ \hfill (5.34)

$$q_2^{NE} = \frac{(\frac{1}{2})(\alpha_2 - 1)\alpha_1}{\alpha_1\alpha_2 - 1}.$$ \hfill (5.35)
5.5. Numerical Results

- The optimal charge price $\pi^*$ is the strategy of the MNO by maximizing its utility $u_o$.

$$\pi^* = \frac{r + \sqrt{r}}{r + S},$$

where $r = \sum_{m=1}^{2} (\alpha_{m-1})_{m} \alpha_{m}$, $t = \sum_{m=1}^{2} (\alpha_{m-1})_{m} f(p_m)$ and $f(p_m) = p_m + \Delta p_m$.

- Given the charge price for each file of the caching request, each CP chooses a quantity of caching files as its best response.

$$q_{1{BR}} = \left( \frac{1}{\pi} - 1 \right) - \frac{q_2}{\alpha_1},$$

$$q_{2{BR}} = \left( \frac{1}{\pi} - 1 \right) - \frac{q_1}{\alpha_2}.$$

- The BR quantities of both CPs converge to the NE quantities in (5.34) and (5.35), respectively. The resulting $q_{m{NE}}$ maximizes both the utilities of the MNO and CPs.

5.5 Numerical Results

In order to illustrate the outcome of the proposed game-theoretical cache problem, without loss of generality, we simulate the scenario of one MNO and 2 CPs with different parameters. We assume identical file size as 10GB for high quality videos.

Fig. 5.2 shows the BR dynamic of the non-cooperative game for the 2-CP case. The parameters of the simulations are as follows: $\alpha_1 = 5$, $\alpha_2 = 7$. The charge price provided by the MNO is set as $\pi = 0.3$. The initial value of CP 2 is set to be $q_2^{int} = 0$. We can observe that the proposed non-cooperative game converges very rapidly. The convergence is irrespective of the initial points. The convergence values result in the same quantities as the theoretical NE solutions given in (5.17) in Theorem 3.

Fig. 5.3 shows the utility function of the MNO with respect to the charge price. We can see that the proposed utility of the MNO always admits a global optimum for different sets of parameters. The utilities of the MNO with higher caching capacity are higher. This is because more caching requests can be served and less caching cost is spent for the same amount of caching files. The difference of the starting points of the curves are due to the feasible price region in Corollary 1. The global optimum in Fig. 5.3 result in the same price derived in (5.27). With the optimum price, the utility of the MNO improves even up to 50% than arbitrary chosen prices.

Fig. 5.4 shows the utility of a single CP with respect to the total number of CPs $M$ while the quantity of the caching request $q_m$ changes. The charge price is provided as the optimal price $\pi^*$ for different total number of CPs. The optimal quantity $q_{m*}$ is derived accordingly. The utilities with double quantity $2q_{m*}$ and half quantity $\frac{q_{m*}}{2}$ are provided for
5.5. Numerical Results

Figure 5.2: Convergence of the best response for 2-CP case.

Figure 5.3: Utility function of the MNO with respect to the charge price $\pi$ for 2-CP case.
5.6 Closing Remarks

In this chapter, we have studied a Stackelberg game of proactive edge caching between the leader MNO and the followers CPs. The best response, the resulting NE caching quantities, and the optimum charge price have been derived in closed forms; and the convergence of our proposed incentive mechanism has been validated via numerical studies. These numerical results also showed that both the MNO and CPs can achieve up to 50% higher utilities in the proposed Stackelberg game. This clearly points out the need of incentive caching mechanisms in 5G wireless networks.
5.6. Closing Remarks
Chapter 6

Cache-Incentive Frameworks in the presence of Asymmetric Information

6.1 Overview

In the previous chapter, we addressed the cache incentive problem in SCNs while assuming that all the involved agents are truthful. In this chapter, we introduce a novel incentive mechanism for practical scenarios in the presence of asymmetric information for facilitating the deployment of caching in SCNs. We consider a model in which an MNO proposes agreements to the CPs to incite them to cache their content. In the presence of asymmetric information, the CPs can be of different, private types that are unknown to the MNO. This private information pertains to the level of generated CP traffic which corresponds to the popularity of its content. The proposed approach, based on contract theory [90], allows the MNO to define a contract for each CP in the presence of asymmetric information, by fixing a price for the allocated storage space to the CP. The goal of the MNO is to maximize the global reward of the CPs and cover the expenses of the caching process. Unlike classical contract-theoretic models [132–134], we show that the proposed model exhibits strategic interdependence between the CPs. Consequently, for this interdependent contract model, we derive the closed-form expression of the price and prove that the resulting mechanism satisfies the feasibility constraints of a contract. The designed mechanism is then shown to be budget balanced as it allows the MNO to offload its backhaul while covering the cost of caching the CPs’ content. Simulation results show the performance advantage of the proposed scheme as well as its ability to incite truthfulness on the CPs.

The rest of the chapter is organized as follows. In Section 6.2, we present the system model. In Section 6.3, we formulate the problem as an incentive mechanism problem and propose a new mechanism that satisfy the desirable constraints. Simulation results are presented in Section 6.4 and conclusions are drawn in Section 6.5.
6.2 System Model

Consider a SCN composed of a set $M$ of MBSs and a set $S$ of SBSs deployed by an MNO. The SBSs are connected to the MBSs via capacity-limited backhaul links and serve a set of users $U$ that are subscribed to multiple CPs from a set $C$. The SBSs can cache content to offload the MNO’s backhaul. However, the MNO needs the CPs’ cooperation to cache their content. To motivate the CPs to participate in the caching, the MNO must offer contracts that present significantly improved QoS for the CPs’ users. The contract terms between an MNO and a CP determine the price charged to the CP by the MNOs and the amount of storage space offered.

We consider heterogeneous CPs with different traffic loads and content popularity. Based on this traffic load, the CPs have different incentive levels towards sharing their content with the MNO. Naturally, there is an information asymmetry between the MNO and the CPs. The CP is aware of its users’ traffic as well as its preferences while the MNO may not have that information. Consequently, the CPs may have an incentive to conceal their correct types so as to pay lower prices to the MNO. To overcome this challenge, the MNO must specify a suitable performance-reward bundle contract $(\pi, \rho)$, where $\pi$ is the monetary reward that is paid by the CP, and $\rho$ is the storage space allocated to the CP.

The goal of each CP $k \in C$ is to maximize the performance of its users which depends on the amount of content that other CPs in the set $C \setminus k$ will be willing to cache at the SBSs. In fact, the higher the traffic load of the CPs in $C \setminus k$, more storage space is allocated to cache their content. Thus, less storage space can be made available for the CP $k$. This will negatively impact the data rate of CP $k$’s users as more requests need to be served via the capacity-limited backhaul. Moreover, by introducing caching, the traffic load of each SBS increases when caching highly popular files. Thus, more power is required at the SBS to serve all the requests for a highly popular file. Consequently, other CPs’ users might experience large interference level from the SBSs that cache the files of CPs having a higher traffic load.

6.2.1 Transmission Data Rate

The performance $r_k$ of a CP $k$ that results from caching its contents is measured by the transmission rate that its users experience from the serving MBS or SBS. When an SBS $i$ serves a user $j$, the data rate will be:

$$\alpha_{ij}(\theta_i, \theta) = E_t \left[ w_{ij} \log \left( 1 + \frac{p_{ij}(\rho, \theta_i)|h_{ij}|^2}{\sigma^2 + I(\rho, \theta)} \right) \right],$$

(6.1)

where $I(\rho, \theta) = \sum_{k \in S \setminus i} p_{kj}(\rho, \theta)|h_{kj}|^2$ is the interference experienced by user $j$ from all the other SBSs. $w_{ij}$ is the channel bandwidth, $p_{ij}(\rho, \theta)$ is the transmit power from SBS $i$ to user $j$, $|h_{ij}|^2$ is the channel gain between SBS $i$ and user $j$, and $\sigma^2$ is the variance of the Gaussian noise. The vector $\theta = [\theta_1, ..., \theta_C]$ represents the traffic load of the CPs. Thus, the higher the traffic load of the CPs, the higher is the interference experienced by the
users served from the neighboring SBSs. Since caching is done during off-peak periods, the transmit power is averaged over the considered time period.

If the file is cached at the SBSs, then the users will experience a relatively high data rate as the content is closer to them. However, if the data is not available at the associated SBS to serving a certain user, then the SBS must fetch the user content from the MBS over the capacity-limited and congested backhaul, yielding higher delays. The data rate of a user \( j \) requesting file \( f \) from its associated SBS \( i \) can be given by:

\[
r_{ij}(\theta) = (1 - \beta_{if}(\rho(\theta), \theta)) \min \{\alpha_{ij}(\rho(\theta), \theta), \alpha'_{mi}\} + \beta_{if}(\rho(\theta), \theta)\alpha_{ij}(\rho(\theta), \theta),
\]

where \( \beta \in \{0, 1\}^{S \times F_k} \) is the outcome of the MNO’s storage allocation \( \rho \) and \( F_k \) is the cardinality of the set of files \( \mathcal{F}_k \) provided by a CP \( k \). \( \beta \) depends on the caching policy \( \rho \) and the traffic load of the CPs. For instance, when a CP \( k \) has highly popular files or its willingness level to cache its content is high, it will impact the storage allocation to the other CPs in \( \mathcal{C} \setminus k \). The larger the number of files that CP \( k \) wants to cache, the lower is the storage space that will be allocated to other CPs \( \mathcal{C} \setminus k \) and vice versa. Each entry \( \beta_{if} \) is a binary variable that equals 1 if file \( f \) is cached at SBS \( i \) and 0 otherwise. \( \alpha'_{mi} \) is the data rate from the MBS \( m \) to SBS \( i \) and is given by:

\[
\alpha'_{mi} = E_t \left[ w_m \log \left( 1 + \frac{p_{mi}|h_{mi}|^2}{\sigma^2 + I'} \right) \right],
\]

where \( I' = \sum_{l \in \mathcal{M} \setminus m} p_{li}|h_{li}|^2 \) is the interference experienced by SBS \( i \) from all the other transmitting MBSs.

Thus, the total rate of the users of CP \( k \) can be given by:

\[
r_k(\rho(\theta), \theta_k) = \sum_{i \in S} \sum_{j \in \Omega_{ki}} r_{ij}(\theta),
\]

where \( \Omega_{ki} \subseteq \Omega_k \) is the set of users that request at least one file from CP \( k \) by using SBS \( i \), and \( \Omega_k \) is the set of users requesting files of CP \( k \). Here, we note that, in our model, each CP will have private information that is modeled as a type of CP as discussed next.

### 6.2.2 Content Provider Type

We define the CP’s type to be a representation of its traffic load and content popularity. In fact, when the MNO offers a contract, it must account for the generated traffic by the CPs. For example, by caching the contents of CPs with a high traffic, the MNO can serve more requests locally, thus decreasing its backhaul load considerably. Here, we consider that the number of CP types belongs to a discrete, finite space and grouped as follows:

**Definition 10.** There are \( C \) CPs that generate traffic over an MNO’s network. The CPs’ types are sorted in an ascending order and classified into \( K \) types \( \theta_1, ..., \theta_K \) with \( K \leq C \). Each type includes properties such as the willingness to cache and the global popularity of the CPs files. The types are ordered as follows: \( \theta_1 < ... < \theta_k < ... < \theta_K, \quad k \in \{1, ..., K\} \).
Since the types are not known by the MNO, the CPs can announce wrong information about their types so that they improve the performance of their users. For example, by claiming that its content popularity is higher than it actually is, a CP $k$ can mislead the MNO to allocate more storage space. In such a case, the CP can end up paying lower prices while also lowering the interference experienced by its users. Indeed, the truthful popularity information is necessary for the MNO to define the contracts that optimize the benefit of the CPs and cover the implementation costs of caching. Such cost includes the expenses of deploying storage devices and the required power to download the content and refresh the storage units. Here, our goal is to design contracts that incentivize the CPs to reveal the true values of their types $\theta$ to the MNO. To this end, the contracts will be designed such that no CP can profit by choosing a contract that is designed for other types.

### 6.2.3 Content Provider Model

The utility function of a CP $k$ of type $\theta_k$ that decides to cache a set of files $F_k$ at the operator’s network is:

$$u_k(\theta) = r_k(\rho_k(\theta), \theta_k) - \pi_k(\theta), \quad (6.5)$$

where $r_k(\rho)$ is defined in (6.4) and represents the valuation function regarding the rewards, which is a strictly increasing concave function of $\rho_k$, with $r(0) = 0$ and $r'(\rho_k) > 0$, $r''(\rho) < 0$ for all $\rho_k$. $\pi_k$ represents the price charged by the MNO for a storage allocation $\rho_k$.

### 6.2.4 Mobile Network Operators Model

By caching the content of the CPs, the MNO will be able to reduce the traffic load on its backhaul. This benefit depends on the traffic load of the CPs as dictated by the popularity of their cached traffic. Thus, the MNO will generally prefer to cache the most popular files. By doing so, for the same storage capacity, the load can be reduced more for a CP whose files have a high popularity compared to other CPs. Thus, the cost of storage $c_s$ at the MNO can be given as a function of the traffic load of the considered CP as, $c(\theta) = \log (1 + \theta)$. This storage cost function increases quickly up to a certain threshold and then increases slowly. It is suitable to model the storage cost as the MNO must allocate more storage space to serve a given traffic load, and this cost becomes insignificant when the traffic load increases as some requests become redundant.

A proper utility function for the MNO can be defined as the monetary reward that is charged to the CPs minus the cost of the allocated resources by the MNO, including storage.

$$v_k(\theta) = \pi_k(\theta) - c_k(\theta, \theta_k), \quad (6.6)$$

where $\pi_k$ is the price that the operator charges CPs of type $k$. The total expected utility
of the operator can be given by
\[ v = \sum_{k \in C} v_k(\theta). \] (6.7)

In the considered model, the MNO is assumed to get the CP’s types directly from the CPs. Based on this information, the goal of the MNO is to determine a contract for all possible CP types that maximizes the global benefit of the CPs. At the same time, the MNO ensures that its utility is nonnegative by making the prices charged to the CPs to at least cover its cost. This optimization problem can be defined as follows:

\[ \max_{(\pi_k, \rho_k)} \sum_{k \in C} u_k(\rho_k(\theta), \theta_k), \] subject to \[ v \geq 0. \] (6.8)

In this formulation, we do not make any constraint on the participation of the CPs. Thus, when proposing the contracts resulting from solving (6.8), CPs may prefer not to select any of the contracts or select contracts that are not designed for their types. To analyze this economic incentive problem, next, we propose a solution based on the framework of contract theory for designing feasible contracts [90].

### 6.3 Proposed Incentive Mechanism for Caching

The QoS achieved by the CPs’ users depends on the interference from the other SBSs as shown in (6.1) and (6.3). Moreover, the allocated storage capacity to a given CP depends on the number of CPs that have signed contracts with the same MNO. For instance, the more storage is allocated to a CP \( k \) the lower is the available storage capacity for other CPs. Thus, in the considered model, there is interdependence between the signed contracts by the different agents.

Classical contract theory models that are used to model resource allocation problems in wireless networks such as in [132–134] cannot be applied for the analysis of caching incentive problem between CPs and an MNO defined in (6.8). In fact, these works assume that the contract selected by a CP does not impact the utility of other CPs or focus only on models with one MNO and one CP. Thus, none of the existing works account for the interactions between the CPs. Moreover, the revelation of misleading information by a given CP in a multiple CPs model not only impacts the MNO but also impacts other CPs, which is not considered in [132–134].

To define the most appropriate contract for the formulated problem (6.8), we consider the so-called truthful dominant strategy implementation. Under such contracts, the solution that maximizes the utility function of the CPs will require those CPs to reveal their private information which, in our model, pertains to the real popularity of their content. The goal of the MNO is to maximize the social welfare which effectively captures the global QoS that is experienced by the users of all CPs. Moreover, the MNO ensures that the cost of serving these users is at least covered by the price charged to the CPs. To
6.3. Proposed Incentive Mechanism for Caching

incite the CPs to collaborate with the MNO via caching, the contract that a CP selects must be feasible in that it satisfies the following necessary and sufficient constraints:

Definition 11. Ex-post Individual Rationality (IR): The contract that a CP selects should guarantee that the utility of the CP is nonnegative for any \( \theta_{-k} \) declared by the other CPs,

\[
r_k(\rho_k(\theta_k, \theta_{-k}), \theta_k) - \pi_k \geq 0, \quad \forall k \in \{1, \ldots, K\}.
\]  

(6.9)

Definition 12. Incentive Compatibility (IC): A contract satisfies incentive compatibility constraint if each CP of type \( \theta_k \) prefers to reveal its real type \( \theta_k \) rather than another type \( \hat{\theta}_k \), i.e.,

\[
r_k(\rho_k(\theta_k, \theta_{-k}), \theta_k) - \pi_k \geq r_k(\rho_k(\hat{\theta}_k, \theta_{-k}), \theta_k) - \pi_k.
\]  

(6.10)

6.3.1 Incentive Mechanism Analysis

The goal of the MNO is to determine a pricing policy that motivates the CPs to declare their real type and simultaneously participate in the caching system through a budget balanced mechanism, i.e., the MNO would not experience a negative utility and its effort is covered by the price charged to the CPs. To this end, the CPs need to declare their types for the MNO that in turn optimizes their utility while accounting for the necessary conditions for contracts feasibility. The optimization problem of the MNO can be defined as follows:

\[
\max_{(\pi_k, \rho_k)} \sum_{k \in \mathcal{C}} u_k(\rho_k(\theta_k), \theta_k) \quad \text{subject to (6.9), (6.10), } v \geq 0.
\]  

(6.11)

The solution of this problem consists in the determination of the components of a contract that consist in the allocated storage space and the price charged to each CP. The closed-form of the contract is provided by the following theorem.

Theorem 4. The unique efficient solution of the optimization problem (6.11) can be given by:

\[
\rho_k^* \in \arg \max_{\rho_k} \sum_i \left[ r_i(\rho_i(\hat{\theta}), \hat{\theta}_i) - c_i(\hat{\theta}) \right], \quad \forall k,
\]  

(6.12)

\[
\pi_k(\hat{\theta}) = \left[ \max_{\rho_i} \sum_{i \neq k} r_i(\rho_i(\hat{\theta}_{-k}), \hat{\theta}_i) - c_i(\hat{\theta}_{-k}) \right] - \left[ \sum_{i \neq k} r_i(\rho_i^*(\hat{\theta}), \hat{\theta}_i) - c_i(\hat{\theta}) \right], \quad \text{(a)} \quad \text{and} \quad \text{(b)}
\]  

(6.13)

where (a) represents the maximized social welfare when CP \( k \) is not considered while in (b), CP \( k \) is considered. Moreover, \( \hat{\theta} \) represents the revealed type by the CPs while \( \theta \) is the real type of the CPs.

Proof. The proof is provided in Appendix B. \( \square \)

This result shows that, in order to determine the terms of a contract with a CP \( k \), the MNO first, allocates the storage space to CP \( k \) by solving the optimization problem
(6.12). It is clear that the problem in (6.12) is NP-hard and thus it is challenging to find the optimal storage allocation. To solve (6.12), we use the framework of matching theory to analyze the assignment of storage space between the MNO that acts on behalf of its SBSs and the CPs [135]. Matching theory is a suitable framework to solve NP-hard assignment problems such as in (6.12).

To define the preference relations of the players, we introduce individual utility function for each CP and SBS, using which they can rank one another. In the proposed cell association problem, the preference relations of the SBSs depends on the traffic load $\theta_{k,f}$ generated by each file $f$, while a CP $k$ prefers to cache its file $f$ at multiple SBSs given that the achievable rate by its users and the.

Once the storage space is allocated to the CPs, the price paid by a CP $k$ is found from (6.13), which accounts for the impact of CP $k$ on the utility of other CPs. This price represents the difference between the global utility achieved by all the CPs when CP $k$ participates in the caching process, and the global utility achieved by the CPs when CP $k$ does not participate. Note that a CP $k$ can impact other CPs utilities in two ways. The first one is through the allocated storage space. In fact, when more storage space is allocated to CP $k$, less storage is available for other CPs and thus more requests of these CPs are served via the backhaul. The second is the traffic load of CP $k$ as the transmit power of the SBSs increases by increasing the number of served requests for that CP’s files. Thus, we can deduce that higher traffic load of a CP $k$ and large amounts of allocated storage to CP $k$ will result in an increase in the price charged by the MNO. The dependence of the price on the traffic load of the CPs, i.e., CPs type, is given next.

**Corollary 2.** When $\theta_k \geq \theta_l$ then we have $\pi_k \geq \pi_l$.

**Proof.** This results follows directly from the monotonicity property of the rate function and the structure of (6.13).

### 6.4 Simulation Results

For our simulations, we consider five CPs with different traffic load levels from 1 to 5 with type 5 being the highest load. A type-1 CP is chosen with no traffic and is used as a baseline to compare the performance of our mechanism with the case in which there is no caching. We consider a set of 100 files whose popularity follows a Zipf distribution of parameter $\alpha = 0.2$. The MNO has one MBS that serves all the requests that cannot be served from the SBSs’ cache. The number of SBSs is 10 and the total storage capacity of the SBSs is 1 Gbits. The transmit power of the SBSs is 1 W and the bandwidth capacity to 100 MHz.

In Fig. 8.2, we show that the amount of content that is served via the backhaul for every CPs’ users when a CP selects the contracts designed by the MNO for each type. We account for the fact that the price that can be paid by each CP is limited and the limit increases by increasing the type of the CPs. From Fig. 8.2, we can observe that when high
6.4. Simulation Results

Figure 6.1: Amount of content served via the backhaul with respect to each CP’s type.

Figure 6.2: Utility of the CPs as a function of the CPs’ type and the used storage allocation model.
6.4. Simulation Results

The mean utility of the CPs $\rho$ is equal to $0.5^* \rho^*$ and $\alpha = 2$ based on the proposed mechanism and $\alpha = 0.2$.

Figure 6.3: Mean utility of a CP with respect to the total number of CPs and the popularity distribution of the files.

Type CPs select contracts that are designed for CPs with lower traffic load, the amount of content that is served via the backhaul increases until it reaches the maximum which corresponds to the lowest type contract. Similarly, when low type CPs select the contracts designed for higher types, the amount of traffic served from the backhaul increases. This is due to the high charged price by the MNO for high type CPs and thus CPs of lower type cannot afford that which results in a larger amount of served content from the backhaul. Thus, this result validates the fact the proposed approach for cache incentive compatible as is forces each CP to choose the contract designed for its own type.

In Fig. 8.3, we compare the case in which all the CPs choose the contracts defined for their corresponding types by the MNO and the case in which the storage space is allocated equally by the MNO for all the CPs. Fig. 8.3 show that the model of equal storage allocation outperforms the proposed mechanism for low type CPs. In this case, the allocated storage space for the CPs is higher than the required storage by the CPs which appears through the utilities of type-2 CP and type-3 CP that are only 2% to 10% higher than their utilities when following the proposed mechanism. On the other hand, the proposed mechanism outperforms the equal storage space allocation model for high type CPs. In fact, the utility of high level types is higher when selecting the offered contracts by the MNO for their specific types. The utilities of type-4 and type-5 CPs are 50% to 140% higher than their achievable utilities in the model of equally allocated storage space. The proposed mechanism is more beneficial for the CPs as it allows the MNO to offer to all CPs only the amount of storage space they need. In contrast, when using the equal allocation approach, the MNO allocates to the low type CPs more than their storage requirements and insufficient space for high level type CPs.

In Fig. 8.4, we show the variation of the mean utility of the CPs when increasing the
total number of CPs. We consider two different values for the parameter $\alpha$, 0.2 which corresponds to the case in which the files have comparable popularity, and 2 for the case in which some files are very popular while others have a very low popularity. We can observe that the mean utility for the CPs decreases by increasing the total number of CPs in the model. This is due to the data rate function that depends on the additional interference from the added CPs and the decrease of the available storage space that can be allocated to each CP. The achievable utility by a CP is up to 20% larger compared to the cases in which the CPs request a storage capacity that is larger or lower than the offered one by the MNO. Moreover, we can see that the popularity of the files impacts the mean utility of the CPs. In fact, the CPs can achieve a larger utility when the distribution of the popularity of the files is steep and thus by caching a file, a large amount of the requests can be served from the cache of the SBSs.

6.5 Closing Remarks

In this chapter, we have proposed a new incentive framework to motivate the CPs to cooperate with an MNO and cache their content at the MNO’s SBSs. Based on contract theory, we have designed an incentive mechanism that allows the MNOs to offer a contract for each CP in which it sets the allocated storage for the CP and the charged price by the MNO for the caching service. This model accounts for both asymmetry of information and the interdependence between the different contracts. We have then derived the optimal pricing mechanisms and contracts that motivate the CPs to cache their content and reveal their private information. Simulations have shown the effectiveness of the proposed approach in inciting the participation of CPs in caching.
Part II

Spectrum Management in Small Cell Networks
Chapter 7

Backhaul Management in Cache-enabled Cellular Networks

7.1 Overview

In the previous chapters, we investigated caching problems in small cell networks. In this chapter, we address the problem of backhaul management in a wireless cellular network having caching capabilities and a heterogeneous backhaul. In particular, we propose a novel framework using which the SBSs can determine the number of predicted files to download at each time stage, without affecting the download rate of the current critical requests. We consider a SCN with different coexisting backhauls including wired links, mmW and sub-6 GHz bands that can only support a limited number of files at each time period. The problem is then formulated as a minority game (MG), in which the SBSs are the players that must decide independently, on the number of predicted files to download while taking into account other SBSs’ decisions. We study the properties of the game and prove that there exists a unique fair PMNE in which all SBSs have an equal chance of using the backhaul. Moreover, we propose a self-organizing reinforcement learning (RL) algorithm with incomplete information that allows the SBSs to reach a Boltzmann-Gibbs equilibrium without communicating with one another. Also, we provide a formal proof of the convergence of the RL algorithm to a unique Boltzmann-Gibbs equilibrium which approaches the PMNE in the formulated game. The proposed approach allows the SBSs to take their decisions autonomously and manage the optimization operations locally without coordinating with one another or with a centralized entity. In fact, having such self-organizing SBSs is of high importance in 5G systems due to the high density of SBSs and the capacity-limited backhaul links [136–138]. To our knowledge, this is the first work that jointly considers SCN backhaul management with caching by taking into account the impact of having predicted and current user requests on the backhaul allocation in cache-enabled SCNs. Simulation results show that the amount of cached data can be 50% higher compared to the centralized algorithm due to the reduction of information exchange. Moreover, the performance of the proposed algorithm will match
The performance of the optimal, ideal centralized algorithm in more than 85% of the cases, under properly chosen parameters.

The rest of this chapter is organized as follows. Section 7.2 presents the system model. In section 7.3, we formulate the problem as an MG and study its properties. In Section 7.4, a distributed RL algorithm is proposed and its convergence to a unique Boltzmann-Gibbs equilibrium is proved. Section 7.5 provides the simulation results and Section 7.6 concludes the chapter.

7.2 System Model

Consider a small cell network composed of a set $M$ of MBSs and a set $N$ of SBSs. Each SBS can be connected to the MBSs via one or many backhaul links of different types which can be cable, mmW band or sub-6 GHz band. Such heterogeneous backhauls have been proposed recently as a solution to improve SCN performance as discussed in [136]. An illustration of the system model is given in Fig. 7.1. The wireless link is divided into two sets of backhaul resource blocks denoted by $K_1$ and $K_2$ for mmW band and sub-6 GHz band, respectively. Then, depending on the required rate by each SBS, the backhaul resource blocks are allocated to the SBSs. The wired link of maximum capacity $C_{\text{max}}$ is assumed to be shared by many SBSs. The maximum achievable backhaul rate for a given SBS over the wireless backhaul, is subject to different effects such as interference between the transmitting MBSs when using sub-6 GHz band and atmospheric attenuations when using the mmW band. Indeed, since mmW bands operate at high frequencies, an antenna at a given MBS is able to provide high directional gain and thus the signals do not interfere with one another. However, the transmission rate over the mmW band is limited by rain and atmospheric attenuations as well as the distance between the transmitting MBS and
the receiving SBS. For mmW, the path loss is given by [139]:

\[ L_{mn}^{\text{mmW}} = \beta + \alpha 10\log_{10}(\delta_{mn}) + \chi, \]

where \( \alpha \) is the slope of the fit, \( \delta \) is the distance between the MBS and the served SBS, \( \beta \) is the path loss for 1 meter of distance, and \( \chi \) is the deviation in fitting which is a Gaussian random variable with zero mean and variance \( \zeta^2 \). The signal-to-noise ratio (SNR) at a receiving SBS \( n \) in the mmW band is given by:

\[ \gamma_{mk1n} = \frac{10\log_{10}(P_{mk1n}) - L_{mn}^{\text{mmW}}}{N_1}, \] (7.1)

where \( P_{m1} \) denotes the transmission power of the MBS \( m \) serving SBS \( n \) over backhaul resource block \( k_1 \in K_1 \) and \( N_1 \) is the variance of the receiver’s Gaussian noise. For sub-6 GHz bands, the rate of an SBS is usually limited by the interference experienced from the other transmitting MBSs. The signal-to-noise-interference ratio (SINR) at a receiving SBS \( n \) in the sub-6 GHz band is given by:

\[ \gamma_{mk2n} = \frac{P_{mk2n}|h_{mk2n}|^2}{N_2 + \sum_{i\in M, i\neq m} P_{ik2n}|h_{ik2n}|^2}, \] (7.2)

where \( P_{m2} \) denotes the transmission power of the MBS \( m \) serving SBS \( n \) over backhaul resource block \( k_2 \in K_2 \). In addition, \( h_{mk2j} \) and \( N_2 \) represent, respectively, the channel state of the link between MBS \( m \) and SBS \( j \) over backhaul resource blocks \( k_2 \) and the variance of the receiver’s Gaussian noise. For the wired backhaul, even though the transmission is interference-free, the achievable capacity by a given SBS is limited by the number of SBSs that are served using the same link since all the served SBSs share the wired capacity \( C_{\text{max}} \).

An SBS is assumed to have current and predicted requests to serve. Hence, downloading the files to serve the predicted requests during high traffic times will affect badly the backhaul rate of the SBS for serving the current requests due to congestion in the wired backhaul or interference in the wireless backhaul. Assume that, at a given time period, an SBS needs a rate \( R_n \) to serve all the current requests and rate \( D_n(s_n) \) to download \( s_n \) files to serve the predicted requests. In order to serve the requests, a backhaul allocation algorithm is used to assign each backhaul resource block to a given SBS. Without loss of generality, we assume that an algorithm such as the one proposed in [137] for mmW and sub-6 GHz backhaul resource blocks is used in this context. The algorithm results in an assignment of SBSs to the backhaul resource blocks that aims to satisfy the required rate by each SBS. However, the requested rate by each SBS depends on the number of files that each SBS requests. Thus, the output of the backhaul allocation algorithm is a function of the global rate \( R = \sum_{n\in N} R_n \) that is required for serving the current requests, and the global set of predicted files denoted \( \mathcal{F}_c \), and is given by a matrix \( \eta_k(\mathcal{F}_c, R) \in \{0,1\}^{M\times N} \), for each backhaul resource block \( k \in K \). An entry \( \eta_{mkn}(\mathcal{F}_c, R) \) of the matrix \( \eta_k(\mathcal{F}_c, R) \) equals 1 if MBS \( m \) allocates backhaul resource block \( k \) to SBS \( n \), and equals 0 otherwise. We use \( f_c = \sum_{n\in N} s_n \) to denote the cardinality of the set \( \mathcal{F}_c \), which corresponds to the total number of predicted files that all the SBSs decide to download. Given
7.3 Problem Formulation

the backhaul resource blocks assignment algorithm, the total achievable backhaul rate for SBS \( n \) is given by:

\[
r_n = \sum_{m \in M} \left[ c_{mn}(F_c, R) + \sum_{k \in K} \omega_k \log(1 + \gamma_{mkn}(\eta_{mkn}(F_c, R))) \right],
\]

(7.3)

where \( \omega_k \) is the bandwidth capacity of backhaul resource block \( k \), and \( c_{mn} \) is the wired allocated capacity by MBS \( m \) to SBS \( n \). The wired backhaul link’s capacity is assumed to be shared between all the SBSs based on the remaining traffic load that could no be served through the wireless backhaul. The allocated wired backhaul by MBS \( m \) to SBS \( n \) is given by, \( c_{mn} = \sigma_n c'_m \), where \( c'_m \) is the available wired backhaul capacity at MBS \( m \) and \( \sigma_n \) is the traffic load of SBS \( n \) over the total traffic load of all the SBSs. Since an SBS perceives only the interference from the MBSs transmitting over the same resource blocks, we rewrite the interference as a function of the outcome of the backhaul assignment algorithm \( \eta_{mkn}(F_c, R) \).

Based on the total capacity of the heterogeneous backhaul and the number of urgent requests, each SBS has to decide, without a direct communication with the other SBSs, on the number of predicted files to download without reducing the transmission rate of the current requests in the network. This problem is formulated in the next section, as a minority game.

7.3 Problem Formulation

7.3.1 Backhaul Management Minority Game

We formulate the problem of backhaul management as a one stage MG, in which the SBSs are the players and each of them has to determine the number of predicted files that must be downloaded from the core network at a given time period, without coordinating with the other SBSs. We consider that, regardless of the traffic load, the SBSs must serve the urgent requests whenever they receive them but they have to decide whether to download or not files that can be cached to serve the predicted requests. Depending on the traffic load, assume that the maximum number of predicted files that can be downloaded at a given time period without affecting the service of the current requests is given by \( \phi \in [0, F] \), where \( F \) is the cardinality of the set of files \( \mathcal{F} \) from which users can pick their requests. It should be noted that the value of \( \phi \) is fixed for the considered time period but can vary from a given time stage to another one. Moreover, the statute of a given request can evolve over time, from a predicted request to an urgent request. Since we consider a one stage MG, there is no need to account for the evolution of the requests as the statute’s changes are implicitly considered by defining the static sets of current and predicted requests at each time period. However, we do not make any restriction on the cooperation between the SBSs. Thus, when determining the caching policy, a given file can be divided into small chunks each of which will be cached at a different SBS. Each file chunk will be considered by a SBS as a complete files in our model and will add it to
the set of predicted files. In this model, the storage space is allocated more efficiently and
a user can be served by multiple SBSs at the same time.

In an MG, each SBS \( n \) has to select a strategy \( s_n \) from a set \( S_n = \{0, 1, \ldots, F_n\} \), where \( F_n \) corresponds to the number of files for which the SBS \( n \) predicts requests and these files must be cached at the SBS. Note that even by caching the predicted files, these files may not be requested by the users in the future which can result in a waste of backhaul capacity if the critical urgent requests are not prioritized. Moreover, the files in the set of predicted files can become current requests if the SBSs are not able to cache these files before the users request them. In this case, the files are removed from the set of predicted requests and added to set of current requests to serve them instantaneously. The capacity \( \phi \) represents the limit starting from which the utility of the players will begin to decrease. Indeed, assuming that all the files have the same size, if the SBSs decide to download a large number of predicted files, this will reduce the allocated backhaul rate per SBS and hence degrade the QoS of the requests that are currently being served from the backhaul, as these urgent files will not be served on time. This is equivalent to deciding on the number of backhaul resource blocks that an SBS needs to use at each time period: the higher the number of files an SBS decides to download, the higher the number of backhaul resource blocks that must be assigned to that SBS. Thus, an SBS delays the service of its own current requests if the total number of predicted files that are requested by the SBSs exceeds \( \phi \).

The formulated game is classified as a minority game [140], due the limited number of predicted files that can be supported by the backhaul links, as well as the nature of the SBSs’ utility. Essentially, in an MG, players are always better off when they select the action selected by the minority group. The size of the minority group is determined by the maximum system resources that can be allocated to the players. In our context, an SBS would prefer not to request predicted files if more than \( \phi \) predicted files are requested by the SBSs, in which case the set of SBSs not requesting files will constitute the minority group. Similarly, the SBSs would prefer to request predicted files if less than \( \phi \) files are requested by the other SBSs. The minority group in this case corresponds to the SBSs that choose to request predicted files. The main challenge in this game is that the SBSs do not communicate with one another and if they all think that the backhaul will be congested, none of the SBSs will requests files and the backhaul will be underused. On the other hand, if all the SBSs think that the other SBSs will not request predicted files, the backhaul will be congested and the utility of the SBSs decreases.

The utility of an SBS \( n \) when it decides to download \( s_n \) predicted files, is given by:

\[
\begin{align*}
    u_n(s_n, F_c) &= -R_n - D_n(s_n) + \sum_{m \in M} \left( c_{mn}(F_c, R) + \sum_{k \in K} \omega_k \log(1 + \gamma_{mkn}(F_c, R)) \right), \text{ if } f_c \geq \phi, \\
    u_n(s_n, F_c) &= R_n + D_n(s_n) - \sum_{m \in M} \left( c_{mn}(F_c, R) + \sum_{k \in K} \omega_k \log(1 + \gamma_{mkn}(F_c, R)) \right), \text{ if } f_c \leq \phi,
\end{align*}
\]

where \( f_c \) is the total number of requested files by all the SBSs. This utility represents the difference between the allocated backhaul rate for SBS \( n \) and the rate it requires to
serve all the current requests and the \( f_n \) predicted requests. Note that when the maximum backhaul capacity is reached, i.e. \( f_c \geq \phi \), the higher is the number of requested files by the SBSs, the lower is the number of assigned backhaul resource blocks and wired capacity to the SBSs. Thus, the utility of a given SBS is a decreasing function of the total number of the requested files by the SBSs. Moreover, in order to avoid underusing the backhaul, the utility obtained by an SBS that chooses not to request predicted files when \( f_c \leq \phi \), is defined as an increasing function of the number of requested files until all the backhaul is efficiently allocated, i.e., \( f_c = \phi \).

Having defined the utility functions, the goal is to find a solution for the defined game. For this, we distinguish between the pure strategy and proper mixed strategy cases.

**Pure Strategies**

In the pure strategy game, each SBS selects its strategies deterministically, i.e., with probability 1 or 0. The **pure Nash equilibrium** is defined as follows [141].

**Definition 13.** Let \( s_n \) be the strategy selected by SBS \( n \in \mathcal{N} \) and \( s_{-n} = [s_1, \ldots, s_{n-1}, s_{n+1}, \ldots, s_N] \) the strategy profile of all the other SBSs except SBS \( n \). A strategy profile \( s^* = [s^*_1, \ldots, s^*_N] \) is a pure Nash equilibrium (PNE) if:

\[
\forall n, s_n \in S_n, u_n(s^*_n, s_{-n}^*) \geq u_n(s_n, s_{-n}^*). \tag{7.5}
\]

In MG literature, results on the existence of PNEs were provided when the number of strategies is the same for all the players and equal to two [140, 142]. However, in the formulated backhaul management minority game (BMMG), each SBS has a larger set of strategies which changes from an SBS to another SBS. For the BMMG, we can derive the following result:

**Theorem 5.** There exists a PNE obtained when the total number of predicted files that are requested by the SBSs at the considered time stage, equals \( \phi \).

**Proof.** A PNE is the state in which none of the SBSs can improve its utility by unilaterally changing its strategy. Denoting \( s^*_n \) the strategy chosen by SBS \( n \) in the PNE. When an SBS changes its strategy from \( s^*_n \) to \( s_n \), two cases can be considered: \( s_n > s^*_n \) and \( s_n < s^*_n \). Thus, at the PNE, the two following conditions must be satisfied:

\[
\begin{align*}
    u_n(s^*_n, \phi) &\geq u_n(s_n, \phi + (s_n - s^*_n)) \quad \text{if } s_n > s^*_n, \tag{7.6} \\
    u_n(s^*_n, \phi) &\geq u_n(s_n, \phi - (s^*_n - s_n)) \quad \text{if } s_n < s^*_n. \tag{7.7}
\end{align*}
\]

From (4), we can deduce that if the SBS selects another strategy \( s_n > s^*_n \), then \( u_n(s^*_n, \phi) \geq u_n(s_n, \phi + (s_n - s^*_n)) \). This is because the utility is a decreasing function of the total number of requested files when \( f_c \geq \phi \), which is the case when \( \phi + (s_n - s^*_n) > \phi \).

On the other hand, assuming \( \phi > 0 \) and \( s_n < s^*_n \), we have \( u_n(s_n, \phi) \geq u_n(s_n, \phi - (s^*_n - s_n)) \). This is due to the fact that the utility is increasing by increasing the number of requested files when the total number of requested files does not exceed \( \phi \), which occurs when SBS
$n$ chooses a strategy $s_n < s_n^*$. 
From these two cases, we can conclude that $f_c = \phi$ is a PNE.

In the pure strategy case, we can notice that any combination of strategies that satisfy $\sum_{n=1}^{N} s_n = \phi$ is a PNE, resulting in a large number of equilibria. Thus, in the repeated BMMG, it is difficult to capture the frequency with which each SBS downloads predicted files over a large time horizon. In fact, for a given available backhaul capacity, a subset of SBSs may keep requesting a large number of files with probability 1 at each time period, while other SBSs never download any predicted files. In order to ensure fairness between the SBSs, in terms of backhaul usage over a large time duration, we consider the proper-mixed strategy case in which each SBS $n$ selects one of the strategies $s_i \in S_n^i$ with a given probability $p_i(n) \in (0, 1)$, thus allowing a fairer backhaul use as shown next.

**Proper-Mixed Strategies**

In the mixed strategy game, an SBS $n \in \mathcal{N}$ can play the strategies in $S_n$ with a probability profile $p^{(n)} = [p_1^{(n)}, ..., p_{F_n}^{(n)}]$, where $p_i^{(n)} \in (0, 1)$ [141].

**Definition 14.** A PMNE specifies an optimal mixed strategy $p^{(n)*}$ for each SBS $n \in \mathcal{N}$ such that:

$$\bar{u}_n(p^{(1)*}, ..., p^{(n-1)*}, p^{(n)*}, p^{(n+1)*}, ..., p^{(N)*}) \geq \bar{u}_n(p^{(1)*}, ..., p^{(n-1)*}, p^{(n)*}, p^{(n+1)*}, ..., p^{(N)*}),$$

(7.8)

where $\bar{u}_n$ is the expected utility of SBS $n$ when the used probability profile by all the SBSs is $p = [p^{(1)}, ..., p^{(N)}]$. 

Even though proving the existence and uniqueness of the PMNE is possible when the number of players is two and the number of strategies is also equal to two, it is very challenging to extend this results to the case of multiple players and strategies even for MGs [140,142]. In order to solve the BMMG, we reduce the problem to a minority game with multiple players, each of which has two strategies. We study this simplified game and then map the results to the original backhaul management MG.

### 7.3.2 simplified backhaul management minority game (SBMMG)

To cast the backhaul management problem as a simplified minority game, we introduce an additional set $\mathcal{V} = \bigcup_{n \in \mathcal{N}} \mathcal{V}_n$ of $\mathcal{V}$ virtual SBSs. In fact, for each real SBS $n$ that has a strategy set composed of $F_n$ predicted files, we consider one real SBS $n$ and create a set $\mathcal{V}_n$ of $F_n - 1$ virtual SBSs. In this modified model, each real and virtual SBS $n \in \mathcal{G} = \mathcal{N} \cup \mathcal{V}$ is assigned one predicted file, i.e., $s_n = 1$, and has to decide whether to download or not that predicted file. The strategy set for all the SBSs becomes a binary set $S = \{c, d\}$, in which the strategy $c$ corresponds to requesting and caching the file, and $d$ corresponds to not requesting the file from the MBSs. The set of current requests for the real SBSs is
the same while it is empty for the virtual SBSs and consequently \( R_n = 0 \), \( \forall n \in \mathcal{V} \). The utilities for an SBS \( n \) of choosing a strategy \( c \) or \( d \) when the total number of predicted files that will be requested by all virtual and real SBSs equals \( f_c \), are given by:

\[
\begin{align*}
    u_n(c, f_c) &= -R_n - D_n(s_n) + \sum_{m \in \mathcal{M}} \left( c_{mn}(F_c, R) + \sum_{k \in \mathcal{K}} \omega_k \log \left( 1 + \gamma_{mkn}(\eta_{mkn}(F_c, R)) \right) \right), \\
    u_n(d, f_d) &= -u_n(c, f_c + 1),
\end{align*}
\]

(7.9)

where \( f_d = \sum_{n \in \mathcal{N}} F_n - f_c \) is the number of predicted files that the SBSs decide to not download.

In this SBMMG formulation, the number of requested files corresponds to the number of SBSs using strategy \( c \). In an MG, a player is better off if it chooses the strategy chosen by the minority. In our context, the SBS gets a positive utility, i.e. serves the predicted file without affecting the QoS of the current requests, if the total number of predicted files that is requested by all the SBSs does not exceed \( \phi \). On the other hand, if the SBS chooses not to download the predicted file while the total number of requested files does not exceed \( \phi \), the SBS gets a negative utility. This represents the regret of not downloading the file when it is possible and waisting backhaul.

Remark 4. Note that this problem formulation is equivalent to the BMMG as the SBSs still take their decision independently. Moreover, in the BMMG, the decisions taken by one SBS do not depend on the identity of the SBS itself but on the mean number of requested files. Thus, introducing a set of virtual SBSs that take decisions independently on their real related SBSs keeps the model valid.

Next, we study the proper mixed Nash equilibrium.

Proper-Mixed Strategies

In this section we start by studying the formulated SBMMG and then extend the results for the BMMG. In particular, we are interested in finding the PMNE where all the SBSs take their decision probabilistically and do not have the incentive to deviate from their chosen strategy. In contrast to existing works on minority games [140] which provide conditions for the existence of mixed equilibria when \( \phi = \frac{F - 1}{2} \) with \( F = 2k + 1 \) and \( k \in \mathbb{N}_0 \), here, we need to extend the results for any \( \phi \) and \( F \in \mathbb{N}_0 \). For the SBMMG, we have the following result.

Proposition 6. In the proposed SBMMG, there exists at most two PMNEs.

Proof. The expected utility for an SBS \( n \) of choosing each of the strategies \( c \) and \( d \) are
7.3. Problem Formulation

given respectively, by:
\[ \bar{u}_n(c, p_{\neg n}) = \prod_{i \neq n} (1 - p_i) u_n(c, 1) + \sum_{i \neq n} p_i \prod_{j \neq \{i,n\}} (1 - p_j) u_n(c, 2) + ... + \prod_{l \neq n} p_l u_n(c, G). \]  
(7.10)

\[ \bar{u}_n(d, p_{\neg n}) = \prod_{i \neq n} p_i u_n(d, 1) + \sum_{i \neq n} (1 - p_i) \prod_{j \neq \{i,n\}} p_j u_n(d, 2) + ... + \prod_{l \neq n} (1 - p_l) u_n(d, G), \]  
(7.11)

where \( G \) is the cardinality of the set \( G \), \( p_i \) is the probability that SBS \( i \) downloads its assigned file, and \( p_{\neg n} \) is the probability profile of all SBSs except SBS \( n \).

By using the indifference principle, a PMNE exists when the expected utility of requesting the predicted file is equal to the expected utility of not requesting that file, i.e., \( u_n(c, p_{\neg n}) = u_n(d, p_{\neg n}) \). By substituting \( u_n(d, f_d) \) based on its definition in (9) and equating the utilities (10) and (11), we get:
\[ \prod_{i \neq n} (1 - p_i) u_n(c, 1) + \sum_{i \neq n} p_i \prod_{j \neq \{i,n\}} (1 - p_j) u_n(c, 2) + ... + \prod_{l \neq n} p_l u_n(c, G) = 0. \]  
(7.12)

When solving (7.12) for each SBS, we end up with two analytical expressions for each probability \( p_i, i \in G \), which we do not include due to space limitation. From this, we can conclude that there is at most two PMNEs.

However, a PMNE may not be fair in the sense that some SBSs will request their file with higher probability compared to the other SBSs. In order to allow the SBSs to equally use the available backhaul, we are interested in looking for a fair PMNE.

**Proposition 7.** In the SBMMG, there exists a unique fair PMNE where all the SBSs select strategy \( c \) with the same probability \( p \).

**Proof.** When all the SBSs choose strategy \( c \) with the same probability \( p \), the utilities of selecting one of the strategies \( c \) and \( d \) write as follows:
\[ \bar{u}_n(c, p_{\neg n}) = \sum_{k=0}^{G-1} \binom{G-1}{k} p^k (1-p)^{G-(k+1)} u_n(c, k+1). \]  
(7.13)

\[ \bar{u}_n(d, p_{\neg n}) = \sum_{k=0}^{G-1} \binom{G-1}{k} p^k (1-p)^{G-(k+1)} u_n(d, G-k). \]  
(7.14)

Using the indifference principle and (7.9), there exists a PMNE when:
\[ \sum_{k=0}^{G-1} \binom{G-1}{k} p^k (1-p)^{G-(k+1)} u_n(c, k+1) = 0. \]  
(7.15)

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To prove that \( p_i = p, \forall i \in \mathcal{N} \), is a solution for (7.15), assume the case where all the SBSs except SBS \( n \) deviate from their mixed strategy and choose a pure strategy \( c \) with probability \( \sigma_i = 0, \forall i \neq n \). Then, from (7.13) and (7.14), we have:

\[
\bar{u}_n(c, \sigma_{-n}) = u_n(c, 1) > u_n(d, G) = \bar{u}_n(d, \sigma_{-n}).
\]

(7.16)

This comes from the fact that \( u_n(d, G) = -u_n(c, 1) \) and \( u_n(c, 1) > 0 \). Similarly, assume that all the SBSs except SBS \( n \) select strategy \( c \) with probability \( \pi_i = 1, \forall i \neq n \). From (7.13) and (7.14), we have:

\[
\bar{u}_n(d, \pi_{-n}) = u_n(d, 1) > u_n(c, G) = \bar{u}_n(c, \pi_{-n}).
\]

(7.17)

From (7.16) and (7.17), we have:

\[
\begin{cases}
\bar{u}_n(c, \sigma_{-n}) - \bar{u}_n(d, \sigma_{-n}) > 0, \\
\bar{u}_n(c, \pi_{-n}) - \bar{u}_n(d, \pi_{-n}) < 0.
\end{cases}
\]

(7.18)

By using the intermediate value theorem, we deduce that there exists a probability profile \( p = [p_1, \ldots, p_G] \), with \( p_i = p, \forall i \in \mathcal{G} \), and \( p \in (0,1) \) solving (7.12). Since, the utility of selecting strategy \( c \) in (7.13) is a decreasing function of \( p \) and the utility of selecting strategy \( d \) in (7.14) is an increasing function of \( p \), then, the two utilities meet in only one point which is \( p \).

\( \square \)

From Proposition 3, we can deduce the following:

**Corollary 3.** There exists a unique fair PMNE for the BMMG, where each SBS \( n \in \mathcal{N} \), chooses a strategy profile \( p_n = [B(1, F_n, p), B(2, F_n, p), \ldots, B(i, F_n, p), \ldots, B(F_n, F_n, p)] \). Here, \( B(i, F_n, p) \) is the binomial distribution and the probability \( p \) is the same for all the SBSs in \( \mathcal{N} \).

**Proof.** Since at the fair PMNE in the SBMMG, each virtual/real SBS downloads one file with probability \( p \), then the related real SBS \( n \) in the BMMG, will decide to download each file from the \( F_n \) files with an independent probability \( p \). The probability of selecting \( i \) files is hence given by the binomial distribution \( B(i, F_n, p) \).

\( \square \)

The result in Corollary 3 does not follow directly from existing works on MGs [140] since those works are restricted to games with the same binary set of strategies for all the players, which is not the case in the BMMG. Moreover, the formulated BMMG has the independence characteristic, given in Remark 4, which is the main parameter that allows the introduction of the set of virtual SBSs and thus the derivation of Corollary 3.

While analytically characterizing the uniqueness and properties of the PMNE is possible, the next step is to develop a practical algorithm that enables the SBSs to reach this PMNE or its neighborhood. To this end, we will next develop a reinforcement learning algorithm that converges to a refinement of the fair PMNE which is known as the Boltzmann-Gibbs equilibrium (BGE) [143]. The BGE is a special case of the \( \epsilon \)-Nash equilibrium which is a solution concept in which the players are within \( \epsilon \) of the sought equilibrium. In other words, at an \( \epsilon \)-equilibrium no deviating SBS can improve its expected utility by a small amount \( \epsilon \).
7.4 Self-Organizing Learning Algorithm

To find an approximation of the PMNE, we propose an algorithm based on RL in which the players do not need to know any information about the actions of the other players. At each time period, the SBSs need to only observe an estimation of their utility and select their strategy accordingly. In contrast to works that use RL approaches such as [144], in which the convergence to a BGE and the uniqueness are not ensured, in this work we prove that the RL algorithm converges to a unique BGE that approaches the PMNE of the formulated SBMMG. First, we need to define the notion of a smoothed best response.

**Definition 15.** The smoothed best response function $\beta_n^{(\kappa_n)} : [0, 1]^{(G-1)\times 2} \rightarrow [0, 1]^2$, with parameter $\kappa_n > 0$, is defined as follows:

$$
\beta_n^{(\kappa_n)}(p-n) = \left( \beta_n^{(\kappa_n)}(c, p-n), \beta_n^{(\kappa_n)}(d, p-n) \right),
$$

and $\forall \ a \in \{c, d\}$, $\beta_n^{(\kappa_n)}(a, p-n)$ is given by the Boltzmann-Gibbs distribution:

$$
\beta_n^{(\kappa_n)}(a, p-n) = \frac{\exp \left( \kappa_n \bar{u}_n(a, p-n) \right)}{\exp \left( \kappa_n \bar{u}_n(c, p-n) \right) + \exp \left( \kappa_n \bar{u}_n(d, p-n) \right)}.
$$

Here, we note that depending on the value of the parameter $\kappa_n$, the smoothed best response of SBS $n$ changes. In fact, as $\kappa_n \rightarrow 0$, the smoothed best response of SBS $n$ converges to the uniform probability distribution, i.e., $\beta_n^{(\kappa_n)}(p-n) = (1/2, 1/2)$, irrespective the strategies adopted by all the other players. However, when $\kappa_n \rightarrow \infty$, the smoothed best response is a uniform probability distribution over the pure strategies that are best responses to the strategies adopted by all the other players. The parameter $\kappa_n$ represents the exploitation/exploration rate that enables the small base station to make a decision about whether to just exploit by always selecting the action with the maximum utility in the current stage, or fold in some exploration and try other actions to discover more information about the network that can be used to achieve better long-term rewards. During the exploitation period, the SBSs do not stop the learning process as they will still use the reward received at the reached sub-slot, to adapt its behavior in the future sub-slots. However, the SBSs may be blocked at local minima which prevent them from reaching the highest possible utility. At the exploration phase, the SBSs determines which action to choose so that the SBSs learn perfectly the best actions that will allow them to determine how behave in the future. Eventually, when everything to know is learned by the SBSs, there is no need to continue the exploration, and the SBSs must act optimally according to the best learned and possible policy. The desired algorithm must allow the exploration probability to decrease as the SBSs gather enough information and the network is better known. This can enable for learning the optimal policy by the end of the time period. The value of $\kappa_n$ is commonly chosen to be $\frac{1}{t}$, where $t$ is the current sub-slot.

Now, we define the BGE which is also known as the logit equilibrium as follows:
Definition 16. The strategy profile $p^* = [p^*_1, ..., p^*_c]$ is a BGE with parameter $\kappa_k > 0$ of the game if $\forall$ $n \in G$,

$$p^*_n = \beta_{n}^{\kappa_n}(p^*_{-n}).$$  \hfill (7.21)

In an SCN, the decentralized nature of the operations at the SBSs makes it difficult for each SBS to communicate with all the other SBSs and get access to their selected strategies. Thus, the SBSs cannot compute the exact value of their utility at a given time period. Hence, we assume that the utilities of the SBSs are subject to random errors and an SBS $n$ can only observe an estimation $\tilde{u}_n(a_n(t))$ of its utility function $u_n(a_n(t), a_{-n}(t))$ for selecting action $a$ at time $t$ when all the other SBSs select the actions given in $a_{-n}$. The estimated utility is given as follows:

$$\tilde{u}_n(a_n(t)) = u_n(a_n(t), a_{-n}(t)) + \epsilon_{n,a_n(t)}(t),$$ \hfill (7.22)

where $\forall n \in G$, $a_n(t) \in \{c, d\}$, and $\epsilon_{n,a_n(t)}$ is a random variable that represents the distribution of the observation error on the instantaneous utility when SBS $n$ selects action $a$. Its expected value is assumed to be 0, i.e., $E[\epsilon_{n,a_n(t)}] = 0$. The variable $t$ is used to denote the number of the current sub-slot as a given time period is divided into a multiple sub-slots. To estimate the instantaneous utility, an SBS does not require any communication with the other SBSs nor with a centralized entity. The value of $\tilde{u}_n(a_n(t))$ can be computed based on the received power from the serving macro base station as well as the perceived interference power from all the other macro base stations when serving other SBSs over the same backhaul resource blocks.

Knowing the estimated value of the utility at each time period as well as the selected actions, each SBS can estimate the achievable expected utility for each of the actions in its strategy set. Based on these estimations, the SBS selects the action with the highest expected utility. As a consequence of the Boltzmann-Gibbs distribution, by adapting the parameters $\kappa_n$, the SBSs can be pushed to explore new actions and not always select the ones achieving the highest performance. This allows the SBSs to try all their set of actions looking for any possible improvement of the expected utility. In what follows, we present a decentralized RL algorithm for backhaul management in which the SBSs simultaneously learn both utilities and strategies. First, we denote the estimate of the expected utility of an SBS $n$ by:

$$\hat{u}_n(t) = \left(\hat{u}_n(c, t), \hat{u}_n(d, t)\right),$$ \hfill (7.23)

where $\hat{u}_n(a, t)$ is the estimate of $\tilde{u}_n(a_n, a_{-n}(t))$. The expected utility is updated at each time period based on the instantaneous observations $\tilde{u}_n(t)$ while the probability of selecting each action is a function of the estimated smoothed best response function. Before providing the RL algorithm, we define the estimate best response function $\bar{\beta}_n^{\kappa_n} : \mathbb{R}^2 \rightarrow \mathbb{R}_+$ based on the estimated utility function $\hat{u}_n$ as follows:

$$\bar{\beta}_n^{\kappa_n}(\hat{u}_n(t)) = \left(\bar{\beta}_n^{\kappa_n}(c, \hat{u}_n(t)), \bar{\beta}_n^{\kappa_n}(d, \hat{u}_n(t))\right),$$ \hfill (7.24)

with $\hat{u}_n(t) = [\hat{u}_n(c, t), \hat{u}_n(d, t)]$ and

$$\bar{\beta}_n^{\kappa_n}(a, \hat{u}_n(t)) = \frac{\exp(\kappa_n \hat{u}_n(a, t))}{\exp(\kappa_n \hat{u}_n(c, t)) + \exp(\kappa_n \hat{u}_n(d, t))}.$$ \hfill (7.25)
where \( a \in \{c, d\} \).

As first proposed in [145], any RL algorithm can be defined as follows \((\forall n \in G, \forall a \in \{c, d\})\):

\[
\begin{align*}
\hat{u}_n(a, t) &= \hat{u}(a, t - 1) + \\
&\quad + \alpha(t)1_{\{a_n(t)=a\}}\left(\tilde{u}_n(a(t)) - \hat{u}_n(a, t - 1)\right), \\
p_n(t) &= p_n(t - 1) + \lambda_n(t)\left(\hat{u}_n(t) - p_n(t - 1)\right),
\end{align*}
\]

where, \((\hat{u}_n(0), p_n(0)) \in \mathbb{R}^2 \times [0, 1]^2\), is an arbitrary initialization of player \( n \). For instance, \( \hat{u}_n(0) = (0, 0) \) and \( p_n = (1/2, 1/2) \), can be defined as the initial values. Moreover, the following conditions must be satisfied for all \((j, n) \in G^2\):

\[
\begin{align*}
&\lim_{T \to \infty} T \sum_{t=1}^{T} \alpha_n(t) = +\infty, \quad \lim_{T \to \infty} T \sum_{t=1}^{T} \alpha_n^2(t) < +\infty, \\
&\lim_{T \to \infty} T \sum_{t=1}^{T} \lambda_n(t) = +\infty, \quad \lim_{T \to \infty} T \sum_{t=1}^{T} \lambda_n^2(t) < +\infty, \\
&\lim_{t \to \infty} \lambda_n(t) = 0.
\end{align*}
\]

and either,

\[
\begin{align*}
\forall n \in G, \quad \lambda_n = \lambda, \quad \text{or}, \\
\forall n \in G \setminus \{G\}, \quad \lim_{t \to \infty} \frac{\lambda_n(t)}{\lambda_{n+1}(t)} = 0.
\end{align*}
\]

It should be noted that the proposed RL algorithm is (7.26) does not require any assumption on the dynamics of the channel state as it is able to capture the changes of both channel and users’ demands. In fact, in contrast to other learning algorithms, the SBSs in the proposed algorithm decide whether to download predicted files or not based on the estimation of their utility that is computed using the channel and traffic statistics from a number of previous sub-slots with different channel state and demand profiles. For RL algorithms that follow (7.26), there is no guarantee of convergence to an equilibrium even if the algorithm has a steady point [145,146]. Moreover, most works that are able to prove convergence to an equilibrium such as [144], cannot guarantee the uniqueness of this equilibrium. For the studied SBMMG, we provide the following result on the convergence of the algorithm in (7.26) to a unique BGE.

**Theorem 6.** The algorithm in (7.26) converges to a unique BGE with parameter \( \kappa_n \), \( \forall n \in G \), in the SBMMG and we have:

\[
\begin{align*}
\lim_{t \to \infty} p_n(t) &= \tilde{p}_n^*, \\
\lim_{t \to \infty} \hat{u}_n(a, t) &= \tilde{u}_n(a, \tilde{p}_n^*).
\end{align*}
\]

**Proof.** The proof is given in Appendix C. \( \square \)
7.5. Simulation Results and Analysis

The main challenge is to prove that the proposed algorithm converges to a unique point for the formulated game and show that this point corresponds to the BGE. In contrast to most works that use similar RL algorithms, we do not only prove that the algorithm converges but we also ensure that the reached equilibrium is unique for the formulated game. To this end, we first write (7.26) as an approximated ordinal differential equation and prove that it admits at least one rest point. Then, to show the uniqueness of the rest point, we prove that the defined ordinal differential equation is a contraction which in turn guarantees the uniqueness of equilibrium.

Since a BGE is a special case of $\epsilon$-Nash equilibrium, the BGE is an approximate equilibrium of the fair PMNE that is within $\epsilon$ of the PMNE. Here, we provide a bound for the utility improvement an SBS can obtain by unilaterally deviating from the BGE.

**Proposition 8.** At the BGE, assume the strategy profile of the SBMMG with parameters $\kappa_n > 0$ is $p^*$. Then, $p^*$ is an $\epsilon$-equilibrium with $\epsilon = \frac{1}{\kappa_n} \ln(2)$.

**Proof.** This result follows directly from [112] based on the definition of the best response $\beta_{n}^{(\kappa_n)}(p_{-n}^*)$ which writes as follows:

$$\beta_{n}^{(\kappa_n)}(p_{-n}^*) = \arg \max_{p_n \in [0,1]^2} \left[ \tilde{u}_n(p_n, p_{-n}^*) - H(\beta_{n}^{(\kappa_n)}(p_{-n}^*), p_{-n}^*) \right],$$

where $H(\beta_{n}^{(\kappa_n)}(p_{-n}^*)) = \frac{1}{\kappa_n} \left( p_n^* \log(p_n^*) + (1 - p_n^*) \log(1 - p_n^*) \right)$, and the following property of the entropy function $H(\beta_{n}^{(\kappa_n)}(p_{-n}^*))$:

$$H(\beta_{n}^{(\kappa_n)}(p_{-n}^*)) - H(p_{-n}) \leq H(p_n^0),$$

where, $p_n^0 = (\frac{1}{2}, \frac{1}{2})$ is the initial uniform distribution over the set of strategies. Thus, we can deduce the following:

$$\tilde{u}_n(p_k, p_{-k}^*) - \tilde{u}_n(\beta_{n}^{(\kappa_n)}(p_{-n}^*), p_{-k}^*) \leq \frac{1}{\kappa_n} \left( H(\beta_{n}^{(\kappa_n)}(p_{-n}^*)) - H(p_n) \right),$$

$$\leq - \frac{1}{\kappa_n} \log(\frac{1}{2}),$$

$$\leq \frac{1}{\kappa_n} \log(2).$$

The BGE is equal to the fair PMNE when $\epsilon = 0$. Thus, by putting $\kappa_n \to \infty$ while satisfying the condition given in the Appendix, we can make sure to approach the fair PMNE.

7.5 Simulation Results and Analysis

For our simulations, we consider a 2 km $\times$ 2 km area which is covered by two MBSs and five SBSs. The SBSs are connected to the MBSs via a heterogeneous backhaul having
a total capacity of 1 Gbps unless stated otherwise. In order to ensure fairness between the SBSs, we always set the minimum number of backhaul resource blocks equal to the number of SBSs. Then, without loss of generality, we use a matching algorithm similar to the one proposed in [137], for allocating backhaul capacity. The total number of predicted files is set to 150 randomly distributed over the SBSs. All statistical results are averaged over 100 independent runs.

Once the utilities are defined based on the backhaul allocation algorithm, we use the proposed RL algorithm, abbreviated by backhaul management reinforcement learning (BMRL), with \( \alpha_n(t) = \frac{1}{t} \) and \( \lambda_n(t) = \frac{1}{t^2} \), to reach the BGE. The algorithm is run until convergence for different configurations. To show the performance gain of the proposed framework, we compare the decentralized BMRL with a centralized greedy algorithm (CGA). In CGA, a central entity receives information from all the SBSs regarding the number of their current and predicted requests. It is also aware of the capacity of the backhaul links. At each iteration, the central entity allows some SBSs to download a fixed number of predicted files. The chosen SBSs at each iteration are determined based on a fairness rule, i.e., the SBSs that have the lowest number of downloaded predicted files are selected. We also compare the BMRL with an ideal and optimal centralized algorithm (OCA) which is similar to centralized greedy algorithm (CGA) in which there is no information exchange between the SBSs and the central entity. The CGA is optimal as it never exceeds the capacity of the backhaul and guarantees fairness between the SBSs. Even though, CGA is not realistic or practical, it allows benchmarking the proposed approach against an ideal and optimal scheme.

In Fig. 7.2, we assess the impact of the parameter \( \kappa_n \) on the achievable utility by the SBSs in the BMRL. Fig. 7.2 shows the the variation of the difference between the available backhaul capacity and the required capacity for serving all the predicted files that are requested by the SBSs, while increasing the number of predicted files in the network. Note that the available backhaul capacity is the same for all the configurations while the number of SBSs having predicted files is being increased. By increasing the number of files in the network, the allocated backhaul to each SBS decreases resulting in a decreasing average utility. In Fig. 7.2, we can see that the parameter \( \kappa_n \) has a significant impact on the performance of the proposed algorithm. In fact, when the value of \( \kappa_n \) is high, the SBSs tend to choose more frequently the actions that are best responses to the actions of the other SBSs. Hence, when the backhaul capacity allows to serve all the predicted files, i.e., the total number of files is up to 60 in Fig. 7.2, higher values of \( \kappa_n \) (\( \kappa_n = 0.005 \) and \( \kappa_n = 0.017 \)) allow the BMRL to achieve the same performance as the OCA. In this case, BMRL is also as good as CGA due to the available backhaul that can support the extra exchanged packets in CGA. In contrast, decreasing the value of \( \kappa_n \) will lead the SBSs to play all the actions uniformly. In this case, the performance of the BMRL is close to OCA and much higher than CGA when the predicted files cannot all be served through the backhaul. In fact, by increasing the number of files, more backhaul is allocated for the information exchange resulting in a decreasing performance in CGA. Thus, by properly choosing the values of the parameter \( \kappa_n \), we can achieve optimal performance at a lower signaling overhead compared to CGA.
7.5. Simulation Results and Analysis

In Fig. 7.3, the performance of the BMRL is compared with OCA in three different cases:

- **Case 1**: The available backhaul capacity is higher than the required capacity to serve the current requests, but the extra backhaul capacity can only be used to serve up to 60 predicted files.

- **Case 2**: The backhaul capacity (50 Mbps) is lower than the required capacity to serve the current requests.

- **Case 3**: The backhaul capacity (3 Gbps) is sufficient to serve all the current requests and up to 150 predicted files.

For cases 2 and 3, choosing relatively high values of $\kappa_n$ allows one to achieve exactly the same performance as the OCA. In Case 2, by choosing $\kappa_n = 0.001$, the SBSs download all their predicted files with a high probability approaching $p_n = 1$. On the other hand, in Case 3, by putting $\kappa_n = 1$, none of the SBSs requests a predicted file and the probability of requesting a predicted file approaches $p_n = 0$. Finally, in case 1, when the capacity of the backhaul is not sufficient for serving all the predicted files, $\kappa_n$ should be chosen carefully depending on the backhaul capacity and the approximate total number of files in the network. For this case, $\kappa_n = 0.001$ according to Fig. 7.2.

Fig. 7.4 shows the total amount of predicted data that is requested by the SBSs in BMRL, OCA, CGA and a centralized random algorithm (RFA) in which neither the
Figure 7.3: The difference between the available backhaul capacity and the required capacity for serving the predicted files that are requested by the SBSs with respect to the number of files.

capacity of the backhaul nor the requests profile of the SBSs are known to the central entity. We can observe that the backhaul usage in BMRL is similar to the backhaul usage in OCA in 85% of the cases while it outperforms CGA and RFA by up to 50% in terms of the amount of cached content and the rate with which the current requests are served, respectively. In fact, when the available backhaul capacity is higher than the total number of predicted files in the network (up to 60 predicted files), in CGA, BMRL and OCA all the predicted files are requested. The backhaul usage in the RFA is lower compared to the other algorithms, because the capacity of the backhaul is selected randomly at each iteration and since the backhaul is allocated fairly to the SBSs, each SBS downloads the predicted file with probability $p_n = 0.5$. This results in an inefficient backhaul usage whether the backhaul is available or not. When the capacity of the backhaul is not sufficient to support all the predicted files, the amount of downloaded content in CGA decreases by increasing the number of files in the network which is due to the extra packets that are transmitted over the backhaul for coordination with the central entity. In this same case, both OCA and BMRL allow the SBSs to download files without exceeding the capacity of the backhaul. Note that in Fig. 7.4, the values of $\kappa_n$ were chosen based on the maximum amount of files that can be downloaded without exceeding the capacity of the backhaul.
7.6 Closing Remarks

In this chapter, we have proposed a novel backhaul management approach for cache-enabled small cell networks, while taking into account the heterogeneity of the backhaul links. We divided the requests of each SBS into predicted requested that can be served locally by the SBSs and current requests that must be served instantaneously from the backhaul. We have formulated a minority game in which the SBSs have to define the number of predicted files to download at each time period without deteriorating the QoS of the current requests. We have proved the existence of multiple pure Nash equilibria and the existence of a unique fair mixed Nash equilibrium allowing all the SBSs to use the backhaul evenly. Moreover, we have proposed a self-organizing reinforcement learning algorithm that reaches a unique Boltzmann-Gibbs equilibrium that approximates the PMNE. Simulation results have shown that the proposed algorithm outperforms the centralized greedy algorithm and its performance is exactly the same as the performance of the ideal and optimal algorithm in more than 85% of the cases. The impact of the caching and replacement policies at the SBSs, on the number of predicted files as well as the whole backhaul management approach, is left for future work.

Figure 7.4: Amount of requested predicted data with respect to the backhaul capacity.
Chapter 8

Coexistence of LTE and WiFi over Unlicensed Bands

8.1 Overview

In the previous chapter, we investigated the problem of spectrum management in cache-enabled SCNs. In this chapter, we study the spectrum management problem in LTE-U systems. In particular, we propose a novel unlicensed spectrum allocation approach that allows the LTE-U SBSs to fairly coexist with the WAP over the unlicensed channels. This is done by enabling the LTE-U SBSs to adapt their transmit power on each of the unlicensed bands based on the assigned channels to each of the WAPs as well as their traffic load. We formulate a multi-game framework [12] in which the WUs are considered as leaders and the LTE-U SBSs are the followers. In contrast to classical hierarchical Stackelberg games, two different type of games are formulated to account for the interdependence between the SBSs and the WUs, while prioritizing the WUs when accessing the unlicensed channels. For the followers, the power allocation problem at the SBSs over the unlicensed channels is formulated as a noncooperative game allowing the SBSs to respond to the selected unlicensed channels by the WUs [147]. For the leaders, we formulate the channel allocation problem as a one-sided matching game with externalities [95, 96], in which the WUs predict the transmit power of the SBSs and autonomously select the channel over which they serve their traffic. For the formulated multi-game, we introduce multi-game stability as a new solution concept and prove the existence and uniqueness of this stable outcome for the formulated multi-game. This is done by first proving the existence of a unique Debreu equilibrium (DE) for the power allocation problem. Then, we propose a new distributed matching algorithm for the assignment of the WUs to the unlicensed channels and prove that the proposed algorithm converges to a unique stable matching that is in the core of the game.

The rest of this chapter is organized as follows. In Section 8.2, we present the system model. The problem of unlicensed channel allocation is formulated as a multi-game and analyzed in Section 8.3. Section 8.5 concludes the chapter.
8.2 System Model

Consider a wireless network composed of a set \( \mathcal{N} \) of \( N \) users served by a set \( \mathcal{S} \) of \( S \) LTE-U small cell base stations and a set \( \mathcal{W} \) of \( W \) WiFi users. The WUs transmit their content over a set of unlicensed frequency bands, while the LTE-U SBSs can serve the users by using both licensed and unlicensed frequency bands. We denote the set of \( C \) unlicensed channels by \( C \). Here, each channel can be allocated to multiple WUs at a given time but only one WU can transmit at a time based on the LBT scheduling scheme. These same bands when available, are allocated to the SBSs allowing them to exploit LTE-U transmissions to improve the QoS of the served users. The system model is shown in Figure 8.1.

We consider that the SBSs only serve downlink traffic over unlicensed channels while in the WiFi network, both downlink and uplink transmissions can occur. Thus, the set of WiFi users includes WiFi stations and WiFi access points that exchange information with one another. This set is denoted by \( W \) and is composed of \( W \) WUs. The achievable throughput, \( R_w(\mathcal{W}_c, \mathcal{S}_c) \), of a WiFi user \( w \) in the network depends on the number of active WUs and SBSs that transmit over the same unlicensed channel \( c \in C \), where \( \mathcal{W}_c \) the cardinality of the set \( \mathcal{W}_c \) that represents the set of active WUs over channel \( c \) and \( \mathcal{S}_c \) the cardinality of the set \( \mathcal{S}_c \) of SBSs that transmit over channel \( c \) with a positive transmit power. The fraction of time during which each WU \( w \) uses a channel \( c \), also known as the channel busy time, is given by [148]:

\[
t_{wc}(\mathcal{W}_c, \mathcal{S}_c) = \frac{l_{wc}}{R_w(\mathcal{W}_c, \mathcal{S}_c)} + \gamma_{wc},
\]

where \( l_w \) is the size of transmitted file by the WU \( w \) and \( \gamma_{wc} \) is the channel access overhead and the protocol overhead which depends on the type of the file. The WiFi network throughput \( R_w(\mathcal{W}_c, \mathcal{S}_c) \) is given by:

\[
R_w(\mathcal{W}_c, \mathcal{S}_c) = \frac{P_{ws}^s L}{P_{ws}^s T_s + P_{wc}^c T_c + P_{ws}^c T_c},
\]

where \( P_{ws} = \tau_w \prod_{i \neq w}^{\mathcal{W}_c + \mathcal{S}_c} (1 - \tau_i) \) is the probability of a successful transmission which corresponds to the probability of having only one transmission over the channel and \( \tau_w \) is
the probability of transmission by a given WU \( w \). \( \bar{L} \) is the average size of a packet file, \( T^s \) is the average time needed to transmit a packet of size \( \bar{L} \), \( P^\sigma = \prod_{i=1}^{W} (1 - \tau_i) \) is the probability of the channel being idle, \( T^c \) is the duration of the idle period, \( P_c \) is the probability of collision, and \( T_c \) is the average time spent in the collision. The WUs detect collisions based on the power or interference they sense over the unlicensed channels. In fact, if the sensed interference from all other WUs and SBSs over a channel \( c \) exceeds a given threshold \( I_{th} \), the WUs consider the channel as busy and will backoff. For the SBSs, the achievable physical throughput by a user \( j \) that is served by SBS \( i \) is given by:

\[
R_{ic} = \omega_c \log \left( 1 + \frac{p_{ic}|h_{ic}|^2}{\sigma^2 + \sum_{i \in S \setminus i} p_{ic}|h_{ic}|^2 + \sum_{j \in W} \alpha_{jc} p_{jc}|h_{jc}|^2} \right),
\]

(8.3)

where \( p_{ic} \) is the transmission power of SBS/WU \( i \) over channel \( c \), \( |h_{ic}| \) is the channel gain of SBS/WU \( i \) over channel \( c \), and \( \omega_c \) is the bandwidth of band \( c \). \( \alpha_{jc} \) is a Boolean that is related to the CSMA/CA transmission mode indicating that WU \( j \) is transmitting over channel \( c \) when \( \alpha_{jc} = 1 \), and \( \alpha_{jc} = 0 \) otherwise. Thus, a user \( j \) served over channel \( c \) experiences interference from all the other SBSs and WUs transmitting over the same channel.

In such LTE-U networks, if the SBSs do not control their transmit power and the amount of traffic that they offload to the unlicensed channels, they can monopolize the access to these channels preventing the WUs from transmitting their content. In fact, if the interference experienced by the WUs is higher than a predefined threshold \( I_{th} \), the WUs will completely stop their data transmissions. To ensure a harmonious coexistence between the SBSs and the WUs that are using the same unlicensed channels, the SBSs must adapt their transmit power according to the WiFi traffic load. Moreover, as given in (8.1), the achievable throughput by a given WU does not only depend on the transmit power of the SBSs but also on the WUs that transmit over the same unlicensed channel. Thus, a WU has to strategically select the channel over which it serves its content while accounting for the channels that are selected by the other WUs as well as the transmit power of the SBSs.

To ensure a fair usage of the unlicensed channels, a priority should be introduced and provided to the WUs while the SBSs must adapt their offloaded traffic according to the WiFi load. Such mechanism allows the WUs to access the unlicensed bands without being affected by the LTE-U traffic generated by the SBSs. Next, based on a new class of games that we have introduced in [12], we propose a new coexistence mechanism in which the WUs select first the unlicensed channel over which they serve their traffic. Then, the SBSs control their transmit power on each unlicensed channel according to the WiFi traffic load.
8.3 Game Formulation and Analysis

In an LTE-U system, the data rate that is achieved by an SBS over the unlicensed bands depends on both the transmit power of other SBSs and the WUs that serve their content over the same unlicensed channels. Similarly, the throughput experienced by a WU is related to the interference experienced from the SBSs and WUs that serve their users over the same unlicensed channel. To ensure a harmonious coexistence between the SBSs and the WUs, we formulate a multi-game problem which is a new game-theoretic framework that we introduced in [12]. A multi-game is a hierarchical game in which multiple interconnected sub-games are formulated. Different from classical Stackelberg games, the games can be of different types and the sets of players are not necessarily disjoint. In our context, the WUs are regarded as leaders in the formulated multi-game, since they have a priority for using the unlicensed spectrum. On the other hand, the SBSs are considered as followers that can access the unlicensed spectrum only if they do not affect the transmitting leaders. The goal of the WUs is to serve their traffic within a predefined time period $T_{\text{max}}$ without any guarantee of QoS. The SBSs can transmit over these unlicensed channels only if they do not prevent the WUs from serving their traffic in the time period $T_{\text{max}}$. The formulated multi-game is composed of two games. A high level game which is played by the WUs and a low-level game that is played by the SBSs. The WUs must select the unlicensed channel with the use of which they transmit their content while anticipating the SBSs’ possible reactions. In contrast, the SBSs follow the leaders decisions and respond to it by determining their transmit power over each of the channels.

8.3.1 Low-Level Game

A set $\mathcal{S}$ of SBSs compete over the unlicensed channels $\mathcal{C}$ to transmit their content over the unlicensed bands with a predefined QoS requirement depending on the type of the served data. The goal of each SBS $i$ is to determine its transmit power over each unlicensed channel to serve its request of size $l_i$. The transmit power must allow the SBS to serve its content in a limited time period while accounting for the WiFi traffic on these unlicensed bands which is given by the sets $\mathcal{W}_c, \forall c \in \mathcal{C}$ that represents the set of WUs that are assigned to each unlicensed channel $c$. In this low-level game, we assume that these sets are fixed and we analyze in the next section the assignment problem of the WUs to the unlicensed channels.

We define $\mathbf{p}^\text{min}_i = [p^\text{min}_i, ..., p^\text{min}_{i,C}]$ as the vector of minimum transmit powers over all channels that are required by an SBS $i$ to meet the delay constraint, given the power allocation of other SBSs over the same unlicensed channels. The goal of each SBS is to minimize its transmit power over the unlicensed channels while accounting for the served
8.3. Game Formulation and Analysis

WUs over the same channels. More formally, this problem can be defined as follows:

\[ p_{ic} = \arg\min_{\{p_{ic}\}} \sum_{c=1}^{C} p_{ic}(p_{-i}) \]  
(8.4a)

\[ \text{s.t. } R_{ic} = w_c \log(1 + \gamma(p_{ic}, p_{-ic})) \geq \frac{l_i}{T_{\text{max}} - t_{wc}}, \]
\[ \sum_c p_{ic} \leq p_{ic}^{\text{max}}, \]
\[ p_{ic} \leq p_{ic}^{\text{max}}, \]

where \( p_{-i} \) is the power allocation of all other SBSs except SBS \( i \) over all the channels \( c \in C \), \( l_{ic} \) is the amount of traffic that SBS \( i \) wants to transmit over the unlicensed channel \( c \), \( p_{ic}^{\text{max}} \) is the maximum transmit power that is allowed over the unlicensed channel \( c \), and \( t_{wc} \) is the fraction of time the WUs require to transmit their packets successfully over the selected unlicensed channel \( c \). \( R_{ic} \) is the mean achievable rate of the LTE users served by SBS \( i \) over the unlicensed channel \( c \) as defined in (8.3) and can be rewritten as:

\[ R_{ic} = \omega_c \log \left( 1 + \frac{p_{ic}|h_{ic}|^2}{N + \sum_{l \in S \setminus i} \alpha_{jc} p_{lc}|h_{jc}|^2} \right), \]  
(8.5)

where the noise plus interference term from the WUs \( N = \sigma^2 + \sum_{j \in W} \alpha_{jc} p_{jc}|h_{jc}|^2 \), can be seen as a constant, which is independent of the power allocation of the SBSs. This is because the WUs define their channels in the high level game and the SBSs respond to these actions by determining their power allocation. The power allocation problem (8.4a) is equivalent to the following problem:

\[ p_{ic} = \arg\min_{\{p_{ic}\}} \sum_{c} p_{ic} \]  
(8.6a)

\[ \text{s.t. } w_c \log(1 + \gamma(p_{ic}, p_{-ic})) = \frac{l_i}{T_{\text{max}} - t_{wc}}, \]
\[ \sum_c p_{ic} \leq p_{ic}^{\text{max}}, \]
\[ p_{ic} \leq p_{ic}^{\text{max}}, \forall c, \forall i \]

where equality holds for the rate requirements in the first constraint in (8.6).

This problem can be formulated as a noncooperative game:

\[ \mathcal{G}(t_w) = \{S, \{P_i\}_{i \in S}, \{v_i(p_i, p_{-i})\}_{i \in S}\}, \]  
(8.7)

where the set of SBSs \( S \) corresponds to the set of players, \( v_i(p_i, p_{-i}) \) is the cost function per SBS and given by:

\[ v_i(p_i, p_{-i}) = \sum_{c \in C} p_{ic}, \]  
(8.8)
and $\mathcal{P}_i$ is the strategy set of SBS $i$ given by:

\[
\mathcal{P}_i(p_{-i}) = \left\{ p_{ic} \in \mathbb{R}^+ : \sum_{c \in \mathcal{C}} p_{ic} \leq p_{ic}^{\max}, p_{ic} \leq p_{ic}^{m}, \right\}.
\]

Here, the cost function $v_i(p_i, p_{-i})$ of an SBS $i$ corresponds to the transmitted power as defined by the objective function (8.6). Due to the dependence between the strategy set $\mathcal{P}_i(p_{-i})$ of a given player $i$ on other players’ strategies $p_{-i}$, a suitable solution for this game $\mathcal{G}$ will be the so-called Debreu equilibrium also known as the generalized Nash equilibrium [149], which can be defined as follows:

**Definition 17.** A strategy profile $p^*$ is a DE of the game $\mathcal{G}(t_w)$ if, for all the SBSs $i \in \mathcal{S}$, we have $p^*_i \in \mathcal{P}_i(p_{-i})$ with

\[
v_i(p^*_i, p^*_{-i}) \leq v_i(p_i, p^*_{-i}), \tag{8.10}\]

for all $p_i \in \mathcal{P}(p^*_{-i})$.

When all the SBSs except SBS $i$ select their transmit power strategies $p_{-i}$, the best choice of SBS $i$ consists in a selected strategy from the best-response strategies set $\mathcal{B}_i(p_{-i})$ which is given by:

\[
\mathcal{B}_i(p_{-i}) = \arg\min_{p_i \in \mathcal{P}_i(p_{-i})} v_i(p_i, p_{-i}). \tag{8.11}\]

Thus, the DEs can be derived by solving the fixed point problem (8.11) for all the $S$ SBSs. The resulting set of DEs might be empty or it may contain many DEs. Therefore, to analyze the hierarchical game, we first need to establish the existence and uniqueness of a DE in the formulated generalized noncooperative game. The time constraint bound in this case:

\[
w_c \log(1 + \gamma(p_{ic}, p_{-ic})) = \frac{l_{ic}}{T_{\text{max}} - t_w}, \tag{8.12}\]

Thus, the minimum required transmission power of an SBS $i$ to serve its requests without exceeding the time limit is given by:

\[
p_{ic}^{\text{DE}} = \min \{p_{ic}^{\max}, p_{ic}^{\text{DE}}\}, \tag{8.13}\]

where,

\[
p_{ic}^{\text{DE}} = \arg\min_{p_{ic} \in \mathbb{R}^+} v_i(p_{ic}, p_{-i}). \tag{8.14}\]

The following proposition provides the closed-form solution of the optimal power allocation and shows that this DE is unique.

**Theorem 7.** The unique DE power allocation strategy for SBS $i$ over channel $c$ to its served LTE users is given by:

\[
p_{ic}^{\text{DE}} = \frac{N}{|h_{ic}|^2} \cdot \frac{1 - 2^{-\alpha_{ic}}}{\sum_{j=1}^{S} 2^{-\alpha_{jc}} - S + 1}. \tag{8.15}\]
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**Proof.** The DE power allocation can be solved by finding the fixed point of problem (8.6). We define \( \alpha_{ic} = \frac{h_{ic}}{\tau_{max} - t_{wc}} \cdot \frac{1}{\omega_c} \). The constraint,

\[
\log \left( 1 + \frac{p_{ic}|h_{ic}|^2}{N + \sum_{l \in S \setminus i} p_{lc}|h_{lc}|^2} \right) = \alpha_{ic}, \forall c \in C, \tag{8.16}
\]

can be written as follows:

\[
p_{ic}|h_{ic}|^2 = (2^{-\alpha_{ic}} - 1)N + (2^{-\alpha_{ic}} - 1) \sum_{l \in S \setminus i} p_{lc}|h_{lc}|^2, \forall c \in C,
\]

which is equivalent to:

\[
\frac{1}{2^{-\alpha_{ic}} - 1} p_{ic}|h_{ic}|^2 - \sum_{l \in S \setminus i} p_{lc}|h_{lc}|^2 = N, \forall c \in C.
\]

Therefore, to jointly solve the \( C \) problems, we formulate a linear matrix equation \( Dp = C \) given by:

\[
\begin{bmatrix}
\frac{|h_{1c}|^2}{2^{-\alpha_{1c}} - 1} & -|h_{2c}|^2 & \cdots & -|h_{Sc}|^2 \\
-|h_{1c}|^2 & \frac{|h_{2c}|^2}{2^{-\alpha_{2c}} - 1} & \cdots & -|h_{Sc}|^2 \\
\vdots & \vdots & \ddots & \vdots \\
-|h_{1c}|^2 & -|h_{2c}|^2 & \cdots & \frac{|h_{Sc}|^2}{2^{-\alpha_{Sc}} - 1}
\end{bmatrix}
\begin{bmatrix}
p_{1c} \\
p_{2c} \\
\vdots \\
p_{Sc}
\end{bmatrix}
= \begin{bmatrix}
N \\
N \\
\vdots \\
N
\end{bmatrix}.
\]

The power allocation \( p_{ic} \) can be solved by applying the Cramer’s rule \( p_{ic} = \frac{\det(D_m)}{\det(D)} \) [147], where \( \det(D) \) is the determinant of matrix \( D \) and \( \det(D_m) \) is the determinant of matrix \( D_m \) which is formed by replacing the \( m \)-th column of \( D \) by the column vector \( C \).

The determinant of \( D_m \) and the determinant of \( D \) are as follows, respectively.

\[
\det(D_m) = N \cdot \prod_{j \neq i} \frac{2^{-\alpha_{jc}}}{2^{-\alpha_{jc}} - 1} |h_{jc}|^2, \tag{8.17}
\]

\[
\det(D) = \sum_{j=1}^{S} (2^{-\alpha_{jc}} - S + 1) \prod_{j} |h_{jc}|^2 \cdot \frac{2^{-\alpha_{jc}}}{2^{-\alpha_{jc}} - 1}. \tag{8.18}
\]

Then the power allocation of \( p_{ic}^{DE} = \frac{\det(D_m)}{\det(D)} \) is derived. \( \square \)

Given the single SBS power constraint \( p_{i}^{\text{max}} \) and the power constraint on each unlicensed channel \( c \), the rate requirement is only feasible if the following conditions are met.

**Proposition 9.** The DE power allocation \( p_{ic} \) is feasible under the power constraints if and only if,
8.3. Game Formulation and Analysis

\[ \sum_c N \frac{1 - 2^{-\alpha_{ic}}}{\sum_j S 2^{-\alpha_{jc}} - S + 1} \leq p_{ic}^{\max}, \quad (8.19) \]

and,

\[ \max_{1 \leq j \leq S} \left( \frac{N(1 - 2^{-\alpha_{jc}})}{p_{jc}^{\max} |h_{jc}|^2} \right) + S - 1 < \sum_{j=1}^S 2^{-\alpha_{jc}} < S. \quad (8.20) \]

Proof. Condition (8.19) ensures that the total power allocation on all the channels does not exceed the power constraint of SBS \( i \).

Since the rate requirement \( \alpha_{ic} \) over each channel \( c \) is positive, \( 0 < 2^{-\alpha_{jc}} < 1 \). The rate requirement \( \alpha_{ic} \) is achievable with positive power allocation if and only if \( \sum_{j=1}^S 2^{-\alpha_{jc}} - S + 1 > 0 \), because all the other parts \( \frac{N}{|h_{ic}|^2} \) and \( 1 - 2^{-\alpha_{ic}} \) are positive. Then we obtain

\[ S - 1 < \sum_{j=1}^S 2^{-\alpha_{jc}} < S. \]

In order to solve the constraint \( p_{ic} < p_{ic}^{\max} \), we obtain \( \sum_{j=1}^S \frac{1}{2^{-\alpha_{jc}}} > \frac{1 - \frac{1}{p_{ic}^{\max} |h_{ic}|^2}}{1 - \frac{1}{p_{ic}^{\max} |h_{ic}|^2}} + S - 1, \quad j = 1, \ldots, S. \) Since \( \left( \frac{1 - \frac{1}{p_{ic}^{\max} |h_{ic}|^2}}{1 - \frac{1}{p_{ic}^{\max} |h_{ic}|^2}} \right) \) is always positive, \( S > \max_{1 \leq i \leq S} \left( \frac{1 - \frac{1}{p_{ic}^{\max} |h_{ic}|^2}}{1 - \frac{1}{p_{ic}^{\max} |h_{ic}|^2}} \right) + S - 1 > S - 1. \) Therefore, condition (8.20) is satisfied.

Given the existence and the uniqueness of the DE for the power allocation game on the unlicensed channels, the goal of the WUs is to choose the unlicensed channels over which they serve their traffic. Next, we formulate and solve this problem as the high-level game.

8.3.2 High-Level Game

Given the power allocation strategies of the SBSs, the WUs have to choose the channels over which they transmit their traffic. The set of WUs that select a given channel \( c \in \mathcal{C} \) is \( \mathcal{W}_c \). In contrast to the problem of licensed bands assignment, in the unlicensed spectrum, there is no entity that controls the access to channels such as the operators or the SBSs. Thus, the WUs must independently select the unlicensed channel over which they prefer to serve their traffic while accounting for the other WUs that transmit over each of the channels as well as the transmit power of the SBSs. To account for the specific properties of unlicensed channels, we model the assignment problem of the unlicensed channels to the WUs as a one-sided matching game with externalities that is played by the WUs \([95,96,101]\). In a one-sided matching game, there exists one set of players that want to be matched to one another to form groups of players allowing each player to maximize a given utility. In our context, the goal of each WU is to select the channel which it shares with other WUs, over which it maximizes its achievable throughput. A one-sided matching game is defined by one set of players \( \mathcal{W} \) and a preference relation \( \prec_w \) that allows each
player \( w \in \mathcal{W} \) to rank the unlicensed channels from the most preferred channel to the least preferred one. This matching game is called with externalities due to the dependence of the preference relation of the players.

**Remark 5.** In contrast to classical one-sided matching problems such as the house allocation problem, the matching problem we formulate here for the association of the WUs to the unlicensed bands exhibits externalities. In fact, the preference relation of a given WU over the other WUs changes by changing the considered unlicensed channel. This is due to variations of the transmit power of the WUs and the channel gain between the transmitting and receiving WUs over each of the unlicensed channels.

In fact, a WU prefers to serve its traffic over the less loaded channels while the load of each channel decreases by decreasing the total number of WUs on the same channel. This WUs-channels allocation problem can be formulated as a house allocation problem with existing tenants [95, 96]. The house allocation problem with existing tenants is a one-sided matching problem that is composed of a set of houses and a number of agents that want to rent a house while some of the houses are already occupied and their tenants may choose to participate or not in the assignment process. Similarly, some agents may already have a house while others do not. In our context, the WUs are the agents and the unlicensed channels correspond to the houses. The unlicensed allocation game can be defined as a tuple \( \mathcal{P} = (\mathcal{W}, \mathcal{A}, \{v_{wc}\}_{w \in \mathcal{W}, c \in \mathcal{C}}, \{\prec_w\}_{w \in \mathcal{W}}, \mu) \) with each element defined as follows:

- The set \( \mathcal{W} \) of WUs represents the set of players.
- The set \( \mathcal{A} = \mathcal{C} \cup \{c_0\} \) of unlicensed channels is the set of actions that can be selected by each of the WUs, where \( c_0 \) corresponds to the case in which none of the unlicensed channels in chosen by the considered WU.
- \( v_{wc} \) is the utility of WU \( w \) when serving its traffic over the unlicensed channel \( c \).
- \( \prec_w \) is the preference relation of the WUs. The preference relation \( \prec_w \) is transitive and complete. We use \( c \prec_w c' \) to denote that WU \( w \) prefers to serve its content over channel \( c \) than serving it over channel \( c' \) for \( c \neq c' \).
- \( \mu \) is the result of the actions selected by all the WUs.

The house allocation problem is a one-sided matching problem in which only the WUs have preferences over the channels while the unlicensed channels do not participate in the game by taking strategic decisions. This assumption captures the main characteristic of unlicensed bands which corresponds to the fact that unlicensed spectrum is free and can be accessed by any user and thus, there does not exist any entity that can act on behalf of the unlicensed bands and set their preferences. The outcome of the house allocation problem with existing tenants can be defined as follows.

**Definition 18.** A matching between the WUs and unlicensed bands \( \mu \) is a mapping from the set \( \mathcal{W} \cup \mathcal{C} \) into the set \( \mathcal{W} \cup \mathcal{C} \) such that for every \( w \in \mathcal{W} \) and \( c \in \mathcal{C} \):
• \( \mu^{-1}(w) \) is contained in \( C \) and \( \mu(c) \) is contained in \( W \),
• \(|\mu^{-1}(w)| \leq 1 \) for all \( w \in W \),
• \(|\mu(c)| \leq q_c \) for all \( c \in C \),
• \( c \in \mu^{-1}(w) \) if and only if \( w \in \mu(c) \),

where \( q_c \) is the maximum number of WUs that can be served over channel \( c \).

The value of \( q_c \) is not predefined at the channels and depends on the amount of LTE traffic that each WU decides to serve over that channel as well as the amount of traffic that each WU decides to serve over the unlicensed channels. Definition 18 states that the outcome of the matching game is an assignment of the WUs to the unlicensed channels. More precisely, a WU \( w \) can only select one unlicensed band \( \mu^{-1}(w) \) while an unlicensed band \( c \) can serve multiple WUs \( \mu(c) \), depending on its capacity and the WiFi traffic load. Before setting the assignment of the WUs to the unlicensed channels, each WU needs to specify its preferences over the unlicensed channels based on its utility function. The externalities in the formulated matching problem appear in the throughput of a given WU that depends on the assigned users to each WU. Thus, the preferences of a WU may change by changing the assigned WUs to each of the unlicensed channels as shown in (8.2). In the considered game, only the WUs have preferences over the unlicensed channels and based on this information, the WUs can define the most profitable allocation for all the WUs and unlicensed channels.

The goal of each WU \( w \) is to serve its content within the time duration \( T_{\text{max}} \). The utility of a WU \( w \), when transmitting over channel \( c \), is given by:

\[
v_{wc}(c, p_c, \mu(c)) = T_{\text{max}} - \hat{t}_{ic}(p_c) - t_{wc}(\mu(c), S_c),
\]

where \( p_c = [p_{1c}, ..., p_{Sc}] \) is the transmit power of all the SBSs over channel \( c \), and \( t_{wc}(\mu(c), S_c) \) is given in (1). Assuming all the SBSs transmit at the same time, \( \hat{t}_{ic} \) is the maximal duration during which the SBSs transmit and is given by:

\[
\hat{t}_{ic}(p_c) = \arg \max_{\tau_{ic}} \left\{ \tau_{ic} = \frac{\bar{l}_i}{w_c \log(1 + \gamma_c(p_c))}, \forall i \in S_c \right\},
\]

where \( S_c \) is the set of SBSs that decide to transmit over the unlicensed channel \( c \) and \( \tau_{ic} \) is the fraction of time during which an SBSs \( i \) uses channel \( c \).

From (8.21), we can see that the utility of a WU \( w \) not only depends on the set of WUs \( \mu(c) \) that are assigned to channel \( c \), but also on the interference generated by other SBSs transmitting over the same channel. Based on the defined utility function, each WU can define its preference relation \( \prec_w \) over the set of channels \( C \), such that for any two channels \( c, c' \in C \) with \( c \neq c' \), and two matchings \( \mu, \mu' \in W \times C \), \( s \in \mu(c), w \in \mu'(c') \):

\[
(\mu, c, p) \prec_w (\mu', c', p) \iff v_{wc}(\mu, p) < v_{wc'}(\mu', p),
\]

(8.23)
8.3. Game Formulation and Analysis

To solve the matching problem, we are interested in finding a desirable matching outcome that is Pareto optimal and in the core which are two natural solution concepts for the one-sided matching games and can be defined as follows.

**Definition 19.** A matching $\mu$ is Pareto optimal if there does not exist another matching $\mu'$ under which:

- $\forall w \in W$, $(\mu', \mu'^{-1}(w), p) \preceq_w (\mu, \mu^{-1}(w), p)$,
- $\exists w \in W$, such that $(\mu', \mu'^{-1}(w), p) \prec_w (\mu, \mu^{-1}(w), p)$.

This definition states that a matching $\mu$ is Pareto efficient if there does not exists a matching $\mu'$ such that $\mu' \neq \mu$ in which at least one of the WUs can improve its utility while none of the WUs will degrade its utility compared to matching $\mu$. Next, we define the concept of core for the one-sided matching problem which represents the set of undominated matchings.

**Definition 20.** A matching $\mu$ lies in the core of the one-sided matching $(W, C, \prec, \{u_{wc}\}_{w \in W, c \in C_{\mu}})$, if there is no coalition of WUs, $W' \subseteq W$, and a matching $\mu'$ such that:

- $\mu'^{-1}(W) \in \{c_0\}_{s \in W'}$ for all $s \in W'$,
- $(\mu, \mu^{-1}(w), p) \preceq_w (\mu', \mu'^{-1}(w), p)$ for all $w \in W'$,
- $(\mu, \mu^{-1}(w), p) \prec_w (\mu', \mu'^{-1}(w), p)$ for some $w \in W'$.

Definition 4 states that a matching $\mu$ lies in the core if there does not exists a set $W'$ of WUs that can exchange their assigned channels among one another resulting in a new matching $\mu'$, in which at least one of the WUs prefers its assigned unlicensed channel under matching $\mu'$ to its assigned channel under $\mu$, while all others are indifferent between the two matchings.

In the formulated problem, the channels can serve multiple WUs while in the original house allocation problem only one agent can be assigned to a given house. Thus, the existing algorithms do not account for the externalities that appear in the utility function (8.21) of the WUs as it depends on the WUs that are assigned to each of the channels. Moreover, the existing top trading cycle algorithm [95, 96] for solving such problems is centralized and cannot be applied for the assignment of WUs to the unlicensed channels in which it is preferable to use decentralized algorithms due to the dense deployment of access points and the resulting communication overhead. Next, we propose a new decentralized algorithm to solve the formulated one-sided matching game with externalities.

**Proposed Algorithm**

To match the WUs to the unlicensed channels, we propose a distributed algorithm provided in Algorithm 1. The idea is based on an extension of the swap-matching idea that
8.3. Game Formulation and Analysis

is extensively used to deal with externalities in two-sided matching problems such as in [150], and [151]. In a two-sided matching problem, two sets of players are matched to one another. A swap matching consists in switching the places of two players, i.e., WUs, that can improve their utilities by exchanging their partners, i.e. unlicensed channels, while keeping all other WUs’ assignments the same. In two-sided matching games, the swap can only happen between two players that prefer to exchange their partners. However, this cannot be applied to one-sided matching problems, in which there is only one set of players that are matched to one another. Thus, we propose a new matching algorithm in which we allow a group of WUs to exchange their channels given that they can all improve their utilities by doing so.

In the first step of the algorithm, each of the WUs selects one of the channels randomly. Given the current matching, the WUs update their utilities based on (8.21) and define their lists of preferences over the unlicensed channels. In the second step, to solve the conflict between the WUs that want to access the same channel, we introduce an ordering and rank the WUs randomly. The ordering can also be done based on different parameters such as the amount of traffic of each WU or their traffic type. Following the defined order, if a WU is not associated to its most preferred unlicensed channel that allows it to strictly increase its current utility, the WU $w_0$ sends a request to one of the WUs that is assigned to its most preferred channel called $w_1$. Upon receiving a request, if WU $w_1$ is not matched to its most preferred unlicensed channel, it sends a request to a WU $w_2$ that is assigned to its most preferred channel. It also includes a list that contains the WU $w_1$ from which it received a request and the WU $w_0$ that has sent a request to WU $w_1$, thus, including all the WUs that lead to the initial requesting WU. At the end of the requests process, each WU checks the existence of a cycle in the received list from its requesting WUs. Once all the WUs emit their requests, the WUs that have initiated the requests procedure, check if a cycle is detected in the list they receive. If so, the WU with the highest order accepts the requests of its preceding WU in the list and transmit the list by removing itself from it. Each WU in the cycle does the same thing until the last WU of the list. Then, all the WUs that belong to a cycle leave the matching game and the WUs update their preferences list based on the remaining WUs. Thus, all the WUs that are assigned in the first round of the process get their most preferred channel. It should be noted that there always exists at least one cycle, called also top trading cycle. Moreover, the WUs might prefer not to be matched to any channel. In this case, the WU forms a cycle with itself and it is allocated channel $c_0$. The assignment process is then repeated among the remaining WUs and their associated channels.

**Theorem 8.** The WUs-channels assignment that results from the proposed algorithm is in the core of the game.

**Proof.** Let $\mu$ be the matching that results from the proposed algorithm. Suppose that there is a group of WUs $W' \subseteq W$ that can improve their utility by exchanging their partners and thus their assigned channels, inducing a new matching outcome $\mu'$. Thus, a subset of WUs $W'$ prefer their assigned channels and WUs partners under $\mu'$ to the ones in $\mu$. More formally, this means that $\exists w \in W'$, such that $(\mu, \mu^{-1}(w)) \prec_w (\mu', \mu'^{-1}(w))$, where
Algorithm 1: WUs-Channels Association Algorithm

**Input:** Set of WUs, SBSs and unlicensed bands.

**Output:** Matching $\mu$ of the WUs to the unlicensed bands.

1 **Phase 1 - Specification of the preferences**
   - Each WU computes utility as given in (8.21).
   - Each WU selects a channel randomly and defines the list of its preferences.

2 **Phase 2 - Matching Algorithm**
   repeat
      - Each WU $w$ updates its utility $v_{wc}(\mu, p)$ on every channel $c$ and initializes an empty list $L_w = \emptyset$.
      - Each WU $w$ for which $\exists c \text{ s.t. } v_{wc}(\mu, p) > v_{w\mu(w)}(\mu, p)$, do not participate in the next steps.
      - An ordering $\mathcal{O}$ is defined among all the participating WUs.
      - Following the order $\mathcal{O}$, the first WU $w$ sends a request to the WUs $w' \in \mu(c)$ that are assigned to its most preferred channel $c$ including to its request the list $L_w = L_w \cup \{w\}$.
      - if $\exists w' \in \mu(c)$, s.t. $(\mu, \mu^{-1}(w'), p) \prec_w (\mu, c, p)$ then
         - WU $w'$ sends a request following the same process including the list $L_w' = L_w \cup \{w'\}$, and so on.
      else
         - The WUs rejects the request of the WU $w$, and WU $w$ sends a request to the WUs that are assigned to its next preferred channel.
   end
   for every list $L$ that is detected to be a cycle by a WU do
      - The WU with the highest order in the list $L$ accepts the request of its preceding WU in the cycle.
      - Each WU $w$ in the cycle that receives an acceptance message, accepts the request of its preceding WU and transmits the cycle to that WU.
   end
   until $\exists w \in W \text{ s.t. } (\mu, c, p) \prec_w (\mu, \mu^{-1}(w), p)$;
\( \mu'^{-1}(w) \in W' \) is the resulting matching after WU \( w \) exchanges its assigned unlicensed channel with another WU in \( W' \). To prove that such a set does not exist, we proceed by contradiction. For this, we show that at each round of proposals, such a set does not exist. First, we denote cycle \( i \) that results from the first round proposal by \( c_i \) which is composed of a finite number of WUs \( c_i = \{w_k, w_l, ..., w_k\} \).

Let \( W_1 = \bigcup_i \{c_i\} \) be the union of all the cycles that result from the first proposals round of Phase 2 in the proposed algorithm. Initially, all the WUs \( w \in W_1 \) get their most preferred channel given the current matching \( \mu \). In fact, the WUs in the first round of proposals, send requests to the WUs that are assigned to their most preferred unlicensed channels. Thus, all the WUs \( w \in c_i, \forall i \), are assigned to their most preferred channels, i.e., \( \forall w \in W_1 \), we have that
\[
(\mu, c, p) \prec_w (\mu, \mu^{-1}(w), p), \quad \forall c \in \mathcal{C} \setminus \mu^{-1}(w). \tag{8.24}
\]

Given the outcome of the first proposals round, two cases are possible, the intersection between the set of matched WUs at the first round \( W_1 \) and the set \( W' \) could be either empty or not empty.

- **Case 1:** Empty intersection, \( W_1 \cap W' = \emptyset \): Based on the definition of \( W' \) we have that, \( \exists \ w \in W', \exists \ c_2 \in \mathcal{C} \) such that, \( (\mu, c_1, p) \prec_w (\mu, c_2, p), \ \forall c_1 \in \mathcal{C} \) and \( c_2 = \mu^{-1}(w') \) while \( w' \in W' \). However, WU \( w \) must have sent an offer to WU \( w' \) to exchange its unlicensed channel which did not result in a cycle and \( w' \) rejected the offer as it is assigned to its most preferred channel. Thus, based on (8.24), \( w' \in W_1 \). However, since, \( W_1 \cap W' = \emptyset \), then \( w' \notin W' \) which contradicts the definition of a blocking-core coalition. This means that the most preferred channel of some WU \( w \in W' \) is assigned to a WU that does belong to the coalition \( W' \).

- **Case 2:** Non-empty intersection, \( W_1 \cap W' \neq \emptyset \), then \( W' \) must consist of the union of some cycles that result from the initial proposals round, \( W' \subseteq W_1 = \bigcup_i \{c_i\} \). In this case, the matching \( \mu' \) coincides with the allocation \( \mu \) in \( W_1 \cap W' \).

Repeating the argument, we find that \( W' \cap W_2 \) consists of the possibly empty union of cycles in the second proposals round, and \( \mu' \) coincides with \( \mu \) on \( W' \cap W_2 \), and so on. Thus, at any proposals round \( t \), if the intersection between the coalition \( W' \) and the set of resulting cycles \( W_t \) is not empty, then the matching \( \mu' \) coincides with the matching \( \mu \), and if the intersection is empty, then the most preferred channel of some WU is assigned to a WU that does not belong to the set \( W' \) which contradicts the definition of a blocking coalition \( W' \) of the core.

\[ \square \]

This theorem shows that there does not exist a set of WUs that can exchange their assigned unlicensed channels so that they all reduce the delay for serving their contents. Moreover, we prove that this matching is unique.
Theorem 9. The outcome of the proposed matching algorithm is the unique matching in the core.

Proof. A matching is said in the core if \( \not\exists W' \in W \), a matching \( \mu' \), and a cycle \( c_t = \{w_1, w_2, ..., w_k\} \), where, \( \{w_1, w_2, ..., w_k\} \subseteq W', (\mu, \mu^{-1}(w_1), p) \prec_{w_1} (\mu', w_{i+1}, p) \) and \( (\mu, \mu^{-1}(w_k), p) \prec_{w_k} (\mu', w_1, p) \). We have proved that such a coalition does not exists in the proposed channel allocation algorithm. To prove its uniqueness, we define for each WU \( w \in W \), the \( i \)-th most preferred channel as \( B_i(w) \) and prove the uniqueness by construction. Every proposal round \( t \) is played by the WUs that could not get their \( t-1 \) most preferred choices, and this round results in two new sets:

- A set \( \hat{W} \) with \( \hat{W} = \emptyset \) at the first round of proposals. At the end of each round \( t \), \( \hat{W} \) is updated as follows \( \hat{W} = \hat{W} \cup W_t \), where, \( \forall w \in W_t, \mu^{-1}(w) = B_i(w) \). Thus, all the WUs that are matched at round \( t \) are assigned to their \( t \)-th most preferred channel and leave the game.
- The set of remaining players \( W \setminus \hat{W} \) that were rejected by their \( t \)-th most preferred WUs, participate again in the assignment process of round \( t + 1 \).

Given these two cases and the fact that the preferences of the WUs are strict, we prove by contradiction and construction the uniqueness of the core matching. Assume that there exist another core matching \( \mu^* \neq \mu \), and the first WU to be matched \( w_i \) with \( \mu^{-1} \neq \mu^* \) following the initial order that is defined in the proposed matching algorithm.

At a given round of proposals \( t \), we assume that \( w_1 \in W_t \), and \( \forall w_j \in \hat{W} \) satisfy that \( \mu^{-1}(w_i) = \mu^*^{-1}(w_j) \).

By construction of the proposed matching algorithm, \( (\mu, \mu^{-1}(w_j), p) \succeq_{w_j} (\mu^*, \mu^*^{-1}(w_j), p) \) for all \( w_j \in W_t \) which follows from the definition of the sets \( W_t \) in which \( \forall w \in W_t \) that was assigned to an unlicensed channel at round \( t' \), we have, \( (\mu^*, \mu^*^{-1}(w'), p) \succ_{w} (\mu^*, \mu^*^{-1}(w), p), \forall w \in W \setminus W_{t-1} \). We do not account the WUs in \( W_{t-1} \) because all the most preferred WUs of WU \( w \) are assigned at the \( t-1 \) previous rounds and thus, \( \mu^*^{-1}(w) \) can in the best case be as good as \( \mu^{-1}(w) \).

Furthermore, since we assumed that \( \mu^{-1}(w_i) \neq \mu^*^{-1}(w_i) \) and \( w_i \in \hat{W} \) then, we have that \( (\mu, \mu^{-1}(w_i), p) \succ_{w_i} (\mu^*, \mu^*^{-1}(w_j), p) \).

Since, for any \( w_j \in \hat{W} \), \( \mu^*^{-1}(w_j) \) is the unlicensed channel that is assigned to a WU in \( \hat{W} \).

From the two last factors, we can deduce that \( \hat{W} \) is profitable for WU \( w_j \) and can be achieved by deviating from \( \mu^* \) to \( \mu \).

After proving the uniqueness of the matching outcome that results from the proposed algorithm, we provide a result regarding the efficiency of this matching.

\( \square \)
Theorem 10. The proposed algorithm is Pareto efficient.

Proof. Assume that a matching \( \mu \) is not Pareto efficient and there exists a matching \( \mu' \) that allows a WU \( w \) to improve its utility,

\[
(\mu, \mu^{-1}(w), p) \prec_w (\mu', \mu'^{-1}(w), p).
\] (8.25)

Given that WU \( w \) is matched to its unlicensed band at time period \( t \), then there exists a cycle \( c^i_l \) at time period \( l \) prior \( t \), i.e., \( l < t \), in which WU \( w \) could have been matched to an unlicensed band that would have increased its utility without decreasing the utility of the other assigned WUs. We prove that such a case is not possible by construction of the proposed algorithm.

In the first round of proposals, the WUs that belong to at least one cycle are matched to their best choices which correspond to the unlicensed channels that maximize their utilities. Thus, at the first time period, there does not exists a matching \( \mu' \), that satisfies (8.25). The WUs that are assigned in the first round of the process leave the assignment process. Thus, in the second round, the existence of a matching \( \mu' \) and a WU \( w \) for which (8.25) is satisfied implies that \( \exists w' \in W \) such that \( w, w' \in c^2_i \). However, in this case \( (\mu, \mu'^{-1}(w')) \prec_{w'} (\mu', \mu'^{-1}(w')) \) and as the highest utility of \( w' \) is achievable by its most preferred channel to which it is assigned in round 1. Since it is not possible for a WU \( w \) that is matched in round 1 to improve its utility at an upcoming round, it is also not possible for the WUs in round 2 to improve their utilities at another round without degrading the utility of the WUs that are matched at previous time periods, which completes the proof.

This theorem states that all the WUs achieve the lowest possible delay for serving their data under matching \( \mu \) and their does not exist any other matching different from \( \mu \) under which the WUs can be better off in terms of delay. The result on the complexity of the proposed algorithm to this efficient outcome is given next.

Proposition 10. The proposed algorithm is guaranteed to converge to the unique matching in the core with complexity \( O(W \times C \times (2^W - 1)) \).

Proof. In the proposed algorithm, the WUs can send requests to other WUs only if they can strictly improve their utility. Since the number of WUs and unlicensed channels is limited, the WUs evaluate all the possibilities/cycles and if none of the WUs can improve their utility and no new cycle is detected then, all the WUs are better of by remaining assigned to their current unlicensed channels. This ensures the convergence of the proposed algorithm. The total number of channels is limited and equal to \( C \) and each channel may have \( 2^W \) possible subsets of WUs. Thus, every WU may send \( \sum_{k=1}^{W} \binom{W}{k} = 2^W - 1 \) requests per channel \( c \).

Now, we introduce a new solution concept for the formulated multi-game.
**Definition 21.** A strategy profile \((p^*, \mu)\) is a multi-game stable outcome of the game \(M\) that is composed of the noncooperative game \(G\) and the one-sided matching game \(\mu\) if for all SBS \(i \in S\) and for all WU \(w \in W\), the two following conditions hold:

- \(v_i(p^D_i, p^D_{-i}, \mu) \leq v_i(p_i, p^D_{-i}, \mu)\), and

- \(\not\exists \mu', \text{ and } W' \subseteq W \text{ such that, } \forall w \in W', v_{wc}(p^D, \mu, \mu^{-1}(w)) \geq w_{wc}(p^D, \mu', \mu'^{-1}(w)).\)

**Corollary 4.** The formulated multi-game admits a stable outcome if and only if conditions (19) and (20) are satisfied.

**Proof.** This result follows directly from Theorem 7, Proposition 9 and Theorem 8.

### 8.4 Simulation Results

To show the performance of the proposed multi-game framework, we perform simulations on a networks composed of dual-mode BSs and WUs that contend for 10 unlicensed bands. The network parameters are provided in Table 8.1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of BSs</td>
<td>40</td>
</tr>
<tr>
<td>Mean time of a successful transmission</td>
<td>5 (\mu s)</td>
</tr>
<tr>
<td>Mean time of a collision</td>
<td>1 (\mu s)</td>
</tr>
<tr>
<td>Mean time of idle channel</td>
<td>3 (\mu s)</td>
</tr>
<tr>
<td>Number of channels</td>
<td>10</td>
</tr>
<tr>
<td>RTS</td>
<td>20 bytes</td>
</tr>
<tr>
<td>CTS</td>
<td>14 bytes</td>
</tr>
<tr>
<td>DIFS</td>
<td>34 (\mu s)</td>
</tr>
<tr>
<td>SIFS</td>
<td>16 (\mu s)</td>
</tr>
<tr>
<td>Number of WUs</td>
<td>30</td>
</tr>
<tr>
<td>Transmit power of the WUs</td>
<td>0.5W</td>
</tr>
<tr>
<td>Transmit power of the BSs</td>
<td>1W</td>
</tr>
</tbody>
</table>

- **Multi-game vs. classical single game**

![Multi-game vs. single game in U-bands allocation](image)

Figure 8.2: Multi-game vs. single game in U-bands allocation.
8.4. Simulation Results

Figure 8.2 shows the sum-throughput of all the WUs as a function of the traffic in the network. Two approaches are compared, the single game and multi-game models. In the former, there is no priority and all the BSs and WiFi can access the network similarly. In the latter, the WUs are given the priority by playing the two first games and choosing the actions that maximize their utilities. Then, the BSs respond to their actions by using the unlicensed channels only when they are not used by the WUs. From Figure 8.2, we can see that the network saturates faster in the single-game model compared to the multi-game. This is due to the fact that the BSs can monopolize the network and transmit for an infinite time when the WUs are not given priority. This results in a sum-throughput that is 50% lower for the WUs, in the single game compared to the multi-game case.

- BSs utility

![Figure 8.3: BSs sum-rate with respect to the WiFi traffic load and number of WUs.](image)

Figure 8.3 shows the amount of offloaded traffic to the unlicensed channels based on the number of WUs that are considered in the network. In fact, the ability of adaptation to network changes at the BSs allows us to show how the proposed multi-game framework impacts the performance of the LTE-U network as compared to LBT and single game models. In particular, we show the sum-rate of the BSs when increasing the traffic load of the WUs and the number of WUs is the network. Figure 8.3 shows that as the number of WUs increases, the sum-rate of the BSs decreases. This is due to the limited capacity of the unlicensed bands that saturates when increasing the load. Thus, based on the priority model, the BSs decrease the amount of offloaded traffic over the unlicensed bands as the WiFi traffic increases.

- Multi-game vs. LBT
In Figure 8.4, we show the sum-throughput of the WUs as a function of the WUs traffic load. We compare the proposed multi-game model with LBT that is used in LTE-U and licensed assisted access (LAA) and LTE-WiFi link aggregation (LWA). In the case of LBT, the WiFi network saturates faster compared to the multi-game case and this is due to the BSs that try to maximize the amount of offloaded content to the unlicensed bands. Moreover, since all the BSs and WUs use the same backoff parameters, all the APs have the same priority. In the multi-game mode, the throughput increases by increasing the load in the network until the network saturates. The sum-throughput of the WUs significantly higher in the multi-game compared to LBT.

8.5 Closing Remarks

In this chapter, we have proposed a multi-game framework to model the coexistence problem over unlicensed bands. In particular, we have formulated a multi-game composed of two interdependent sub-games of different types in which the WiFi users are the leaders and the LTE-U SBSs are the followers. For the followers, we have formulated a power allocation problem as a noncooperative game in which the SBSs respond to actions of the WUs [147]. For the leaders, we have formulated the channel allocation problem as a one-sided matching game with externalities [95, 96], in which the WUs predict the transmit power of the SBSs and autonomously select the channel over which they serve their traffic. For the formulated multi-game, we have introduced multi-game stability as a new solution concept and proved the existence and uniqueness of this stable outcome for the formulated multi-game. This is done by first proving the existence of a unique DE for the power allocation problem and then, proposing a new distributed matching algorithm for the assignment of the WUs to the unlicensed channels.
8.5. Closing Remarks
Chapter 9

Conclusions and Outlook

In this thesis, we have developed self-organizing resource management mechanisms to boost the capacity of existing cellular networks and optimize the QoS across the network. Moreover, we have designed novel economic frameworks to ensure the successful deployment of such resource allocation solutions. To do so, different game theoretic frameworks are introduced as suitable approaches to design low-complexity, distributed resource management protocols for wireless systems while accounting for the specific characteristics of next-generation cellular networks. The key research goals that were accomplished include in this thesis the following:

- In the first part of this thesis, we have addressed the problem of distributed caching in small cell networks. In particular, we have proposed a novel matching-based caching mechanism for overcoming the backhaul capacity constraints in wireless small cell networks while accounting for the local popularity of the files at the SBSs. We have then studied the problem of distributed caching in ultra-dense cellular networks and formulated the cache control problem as a mean-field game problem to capture the ultra-density of small cell networks. To ensure the successful deployment of the caching approaches, we have proposed two cache-incentive mechanisms for motivating the content providers to cooperate with a network operator and cache their files at the operators’ SBSs. In the first model, we have assumed that the agents are truthful and formulated the problem as a Stackelberg game between a single MNO and multiple CPs. In the second model, we have considered the presence of asymmetric information that corresponds to the popularity of the files that is private to the CPs and not known by the MNO. To account for such information asymmetry, we have introduced contract theory as a suitable framework based on which we have derived the optimal pricing mechanisms and contracts that motivate the CPs to cache their content and reveal their private information to the MNO.

- In the second part of this thesis, we have studied the problem of spectrum management in two different scenarios, i) the backhaul allocation problem in cache-enabled systems, and ii) the coexistence problem of LTE-U SBSs and WiFi APs over unlicensed channels. In particular, we have proposed a novel distributed backhaul
management approach in a cellular network having caching capabilities and heterogeneous backhaul links. In the proposed framework, the SBSs can determine the number of predicted files to download at each time frame so that they do not affect and deteriorate the download rate of the urgent critical requests. To this end, we formulated the backhaul management problem as a minority game and proposed a self-organizing reinforcement learning algorithm that is proved to reach a unique Boltzman-Gibbs equilibrium for the formulated game. Moreover, we have proposed a multi-game framework to address the new resource allocation problems raised by the introduction of LTE-U into wireless networks. In contrast to classical hierarchical Stackelberg games, two different type of games are formulated to account for the interdependence between the LTE-U SBSs and the WUs, while prioritizing the WUs when accessing the unlicensed channels. For the formulated multi-game, we introduced multi-game stability as a new solution concept and proved the existence and uniqueness of this stable outcome.

Ongoing and future work will revolve around the following key directions.

In Chapter 3, the model can be improved by considering:

- **Impact of contents length:** For ease of analysis, most of existing works on distributed caching have assumed that the files have the same length. In the caching problem we considered, such assumption in the model may impact the cache placement outcome as the notion of quotas that we introduced as the number of files that each small base station can cache, does not hold anymore. Thus, this assumption could be relaxed to consider a more realistic system, design novel cache placement algorithms that account for the heterogeneity in the content length, and analyze the gain and performance of caching in cache-enabled cellular networks.

- **Global popularity:** In the proposed model, we assumed that the small base stations can locally determine the popularity of the files. However, due to the spatial correlation of users’ requests and the ultra-dense deployment of small base stations, the SBSs may end up caching the same files which impacts the global efficiency of the caching system. Thus, the proposed model can be extended by enabling the SBSs to account for both global and local popularity of the files when determining the caching policies. A distributed model that allows the SBSs to account for both types of popularity will reduce the cooperation overhead between the SBSs and improve the global gain of caching in terms of backhaul load.

- **Social network structure:** The gain of distributed caching depends highly on the popularity distribution of the files. In the system model we considered, we assumed that the small base stations have access to information that are private to the users such as the files to which they access, their list of friends in the social network, and the user’s interactions with its friends. However, not all users can be willing to reveal such information which impacts the accuracy of the popularity estimation using the model we proposed. A logical extension of this work is to develop a new learning
model that is only based on the general structure of the online social network and analyze the performance improvement of such framework on the caching gain.

- **Impact of content chunking:** Existing works on coded caching assume that the files are encoded and divided into chunks of the same size. In such systems, a user can only recover the original file it requests after downloading a given number of the file’s chunks. Thus, it is important to account for the distributed nature of caching as it impacts the performance that the users can achieve when the files are divided into multiple chunks. This implies that the small base stations must coordinate when determining the optimal cache placement as a given user may be served by multiple small base stations at the time. In this situation, one can assume identical size of the files and extend the model by focusing on the design of novel cooperative caching policies to account for the impact of content chunking.

- **Presence of asymmetric information:** In practical systems, the small base stations, content providers and network operators have different interests and may not agree on the same caching policies. In such system, each of the actors may have private information that it is not willing to share with the other agents. For instance, the network operator may not want to reveal the real caching policy it deploys in its storage units while the social network keeps private the structure of its network, the social interactions between users and the files popularity. Thus, dropping the assumption on the truthfulness of the agents that we considered in the model will introduce new challenges that need to be addressed for the design of effective and practical caching systems.

- **Operators cooperation:** The small base stations are deployed by multiple network operators that need to coexist in the same areas. The introduction of caching in the small base station forces the network operators to cooperate and thus coordinate to determine the most optimal cache placement for the users that are subscribed to the services of all the operators. Such cooperation framework can enable the operators to offload the traffic on their overloaded small base stations and associate their users to less offloaded small base stations that serve the users with better QoS from their caches. To successfully implement such cache cooperation model between the network operators, a pricing model can be designed to define the terms of the agreements.

- **Density of the SBSs:** The model proposed in this chapter can be extended to account for the ultra-density of cellular networks. Similar to most of matching-based resource allocation frameworks in the literature, the caching algorithm we proposed in this paper is based on the assumption that there is a finite and small number of small base stations. Thus, to account for the presence of a large number of SBSs, one must extend the matching-based caching algorithm to the case of a continuum set of SBSs and analyze the existence and convergence to a stable outcome.

- **Traffic arrival/departure process:** In this chapter, we analyzed the caching problem at a single time slot in which the sets of SBSs and users as well as users are fixed. The
notion of time together with traffic arrival/departure process can lead to interesting insights about the overall performance system and design of more effective caching policies. However, introducing time dynamics in the game framework requires the design of novel solutions, as for instance the interference becomes temporally dependent. The traffic process together with some approximations and mild assumptions might bring fruitful design insights to the network designers.

In Chapter 4, the model can be extended by considering:

- **Content popularity:** The gain of caching policies depends highly on the accuracy of the popularity estimation. In this chapter, we have assumed that the instantaneous popularity of the files is known at the SBSs ahead of time. However, this popularity varies over time and need to be captured instantaneously to design an effective caching policy. Thus, the mechanism we proposed can be extended by introducing a learning based approach to learn and estimate the popularity of every file at each time frame.

- **D2D communications:** In the system model we have considered, only the small base stations can cache files in their storage units. However, exploiting the large storage spaces that are available at the users devices can enable cellular networks to achieve a further gain in terms of bachkaul offloading. An interesting path to extend the caching problem we considered in dense cellular networks is to introduce users devices as a new type of caching units. The main challenge in such systems is modeling the interdependence between the two classes of agents with different characteristics.

- **Mobility of the users:** Introducing mobility into the system model is another important extension of the caching mechanism we proposed. The dynamics of the mobility can be modeled as a Levy walk pattern that emulates human walk patterns in outdoor mobile network environments. However, the incorporation of a mobility model will introduce an additional practical complexity for solving the HJB-PFK system of equations. Moreover, the performance that is achieved by every user as well as the content popularity of the files will be influenced by the movement of the users.

- **Energy efficiency:** Power consumption and energy efficiency is of high interest when designing caching systems. Even though the unit energy consumed by the caches prefetching/delivery is in general assumed to be lower than the unit energy in the backhaul prefetching/delivery, the consumption that results from the caching model in D2D networks is important as it may dissuade users from participating in the caching process. Thus, designing energy efficient caching approaches will result in a more efficient load balancing between the devices by accounting for their local energy level.

The work on cache incentive in Chapter 5 and 6 can be extended and improved by considering:
• **Multiple operators case:** In both cache-incentive models we considered a single mobile network operator. This model can be extended to multiple network operators that compete to motivate content providers to cache their content at the small base stations. The multiplicity of the network operators introduces new challenges as there is a loss of generality in focusing on simple incentive compatible direct mechanisms as done in Chapter 6.

• **Deployment-based incentive:** The contracts defined in the model we proposed depend on the content provider and the popularity of their files. However, the price could also depend on the quality of service that is achievable by the users of each content provider. Thus, a new cache incentive model can be designed in which the price of renting each storage unit is defined based on the quality of service it can offer to the users of each content provider.

• **Hidden actions case:** In Chapter 6, we have assumed the presence of asymmetric information which corresponds to the popularity of each content provider’s files. However, another type of private information may exist in such system which corresponds to the caching policy deployed by the network operator and is not known to the content provider. Such private information is called hidden actions model in contract theory and can be used to model such incentive caching problems.

• **Joint pricing and caching:** In both Chapters 5 and 6, we have only considered the economic aspect of the caching problem and assumed that the caching problem is developed independently on the pricing model. A possible extension of this work is designing a joint caching and pricing model and analyzing the impact of the caching policy on the pricing model and vice versa.

The backhaul management mechanism proposed in Chapter 7 can be improved by considering:

• **User-cell association policies:** A joint user-cell association and backhaul management problem can be formulated to optimize both backhaul and access links allocation while fairly balancing the traffic load among the SBSs. In fact, the associated users to each SBS impact the sets of urgent and non-urgent requests and thus, the backhaul management mechanism should be adapted to approach used for associating users to cells.

• **Joint caching and backhaul management:** In this chapter, we have focused on the backhaul management problem that can be applied on top of any caching model. It would be interesting to extend the proposed model by developing an online caching policy while accounting for the instantaneous variations of the files popularity over time. The impact of the caching approach on the backhaul management mechanism can then be analyzed.

• **Multi-casting opportunities:** The introduction of multi-casting opportunities at the base stations can considerably reduce the traffic load on the backhaul links as multiple SBSs can receive the same files at the same time from the core network. However,
In such model, the backhaul management approach we proposed must be modified by enabling the SBSs to account not only for the non-urgent and urgent requests of the other SBSs but also for the exact files that are requested.

In Chapter 8, the coexistence optimization of LTE and WiFi over unlicensed bands can be improved by considering:

- **LTE-U incentive:** Over the unlicensed channels, the SBSs serve their users based on the best-effort service and do not guarantee any QoS. Thus, the users may be reluctant to accepting being served over the unlicensed channels. To cope with this problem, an economic framework can be proposed to motivate the users to accept offloading their traffic into the unlicensed bands. Such framework must account for the QoS requirements of the users and their traffic type as bandwidth-intensive applications are more sensitive to the achievable data rate by the requesting users. The pricing model must also account for the WiFi traffic that should be impacted by the LTE-U offloaded traffic.

- **Context-aware LTE-U:** In the proposed model, we have ignored the impact of the interdependence of the LTE and WiFi networks. In other words, the outcome of the resource allocation and the possible available information on one of the networks is not exploited at the other network. Such information can be easily made available by the LTE-U SBSs that belong to both WiFi and LTE networks and thus, can be used as a feedback at one of the networks to improve its performance. Hence, the model in this chapter can be extended to account for such context information for a better optimization of unlicensed channels allocation.
Appendix A

Proofs of Results on Distributed Caching in Ultra-Dense Small Cell Networks

A.1 Proof of Theorem 2

To prove the weak convergence of the occupancy measure $M_N^t$ to the mean field process $m$ in the caching control problem, we use Theorem 25.10 in [152] which gives a necessary and sufficient condition for weak convergence. Theorem 25.10 which is a result of Helly’s Theorem, states that tightness is a necessary and sufficient condition for weak convergence. Thus, since $(R^2, B(R^2))$ is a separable complete metric space, then every sequence of probability measure defined on this space is tight, which leads to the weak convergence of $M_N^t$ to the mean field process $m$. Next we derive the FPK equation which describes the evolution of the density of users per state. In what follows, we omit the file index $k$ for ease of notation and the proof is applicable $\forall k \in \mathcal{V}$:

$$m_0(y) = \rho_0(y), \forall y$$
$$\partial_t m_t(y) + \hat{u}_1(y,t) \partial_y m_t(y) + \hat{u}_2(y,t)m_t(y) + \hat{\sigma}_1 \partial^2_{ss} m_t(y) + \hat{\sigma}_2 \partial^2_{hh} m_t(y) = 0,$$

where $\hat{u}_1(y,t) = -\left[n_k,t q_k - \beta(1 - p_k,t)\zeta_t\right]$, $\hat{u}_2(y,t) = \frac{\alpha}{2}(\mu - h_t)$, $\hat{\sigma}_2 = \frac{\sigma_2}{2}$ and $\hat{\sigma}_1 = \frac{\sigma_1}{2}$. Let $\varphi(y,t)$ be a test function that belongs to $C^2(R^2)$ in space and $C^1(R)$ in time. By applying Ito’s lemma we obtain

$$d\varphi(y,t) = \sum_{i=1}^{2} \hat{f}_i \frac{\partial \varphi}{\partial y_i}(y,t) + \sum_{i,j=1}^{2} \hat{\sigma}_{ij} \frac{\partial \varphi}{\partial y_i \partial y_j}(y,t) + \sum_{i=1}^{2} \hat{\sigma}_i \frac{\partial \varphi}{\partial y_i}(y,t) dB_i,$$

where $\hat{f} = (\hat{u}_1(y,t), \hat{u}_2(y,t))$ and $\hat{f}_i$ represents the $i$th entry of the vector $\hat{f}$. $\hat{\sigma} = \frac{1}{2}(\sigma_1, \sigma_2)(\sigma_1, \sigma_2)^T$ and $\hat{\sigma}_{ij}$ represents an entry of the matrix $\hat{\sigma}$. By taking the expecta-
A.2. Proof of Proposition 3

Because Brownian motion is a martingale, thus \( E[\sum_{i=1}^{2} \sigma_i \frac{\partial \varphi}{\partial y_i}(y,t)dB_i] = 0 \). By definition, we have \( E[\varphi(y,t)] = \int_{\mathbb{R}^2} \varphi(x,t)m_{k,t}(x)dx \).

By substituting in (17), we obtain

\[
\int_{\mathbb{R}^2} \varphi(y,t)m_t(y)dy = \int_{\mathbb{R}^2} \left[ \sum_{i=1}^{2} f_i \frac{\partial m_t(y)}{\partial y_i}(y,t) + \sum_{i,j=1}^{2} \hat{\sigma}_{ij} \frac{\partial m_t(y)}{\partial y_i} \frac{\partial \varphi}{\partial y_j}(y,t) \right] m_t(y)dy.
\] (A.3)

A further integration by part of the second term on the right-hand side gives

\[
\int_{\mathbb{R}^2} \varphi(y,t)m_t(y)dy = \int_{\mathbb{R}^2} \left[ \sum_{i=1}^{2} f_i \frac{\partial m_t(y)}{\partial y_i}(y,t) - \sum_{i,j=1}^{2} \hat{\sigma}_{ij} \frac{\partial^2 m_t(y)}{\partial y_i \partial y_j}(y) \right] \varphi(y,t)dy.
\] (A.4)

By substituting in (18), we have

\[
\int_{\mathbb{R}^2} \varphi(y) \left[ m_t(y) - \sum_{i=1}^{2} f_i \frac{\partial m_t(y)}{\partial y_i} - \sum_{i,j=1}^{2} \hat{\sigma}_{ij} \frac{\partial^2 m_t(y)}{\partial y_i \partial y_j} \right] dy = 0.
\] (A.5)

Then using the generalized variational lemma we have that [lemma 7.1.2] [153]

\[
m_t(y) - \sum_{i=1}^{2} f_i \frac{\partial m_t(y)}{\partial y_i} - \sum_{i,j=1}^{2} \hat{\sigma}_{ij} \frac{\partial^2 m_t(y)}{\partial y_i \partial y_j} = 0,
\] (A.6)

which completes the proof.

A.2 Proof of Proposition 3

\[
H(y_k,m_{k,t},\nabla v_k) = \{ [\mu_{k,t}q_k - \beta(1 - p_{k,t})\zeta_t] \partial_s v_{k,t}(y_k) + \frac{\alpha}{2} (\mu_k - h_k) \partial h v_{k,t}(y_k) + J_{k,t}(m_{k,t},m_{k,t}) \}.
\] (A.7)
A.2. Proof of Proposition 3

The derivative of $H(y_k, m_{k,t}, \nabla v_k)$ with respect to $n_{k,t}$ gives

$$\partial_n H(y_k, m_{k,t}, \nabla v_k) = q_k \left[ \partial_s v_{k,t}(y_k) + \frac{1}{B_{k,t} - q_k n_{k,t}} \right]. \tag{A.8}$$

By setting it to 0, we get $n_{k,t}^*$ which can be written as follows:

$$n_{k,t}^* = \frac{1}{q_k} \left[ B_{k,t} + \frac{1}{\partial_s v_{k,t}(y_k)} \right]. \tag{A.9}$$
Appendix B

Proofs of Results on Cache Incentive in Small Cell Networks

We prove Theorem 4 by showing the following Lemmas.

Lemma 11. The proposed mechanism is incentive compatible.

Proof. We show this result by contradiction. Suppose that ρ is an efficient decision rule but (ρ, π) is not dominant strategy incentive compatible. Then, there exists i, θ, and \( \hat{\theta} \) such that:

\[
\begin{align*}
    r_i(\rho_i(\hat{\theta}_i, \theta_{-i}), \theta_i) - \pi_i(\theta_{-i}, \hat{\theta}_i) &> r_i(\rho_i(\theta), \theta_i) - \pi_i(\theta) \\
\end{align*}
\]

From (6.13), this implies that

\[
\begin{align*}
    r_i(\rho_i(\hat{\theta}_i, \theta_{-i}), \theta_i) - (a) &> r_i(\rho_i(\theta), \theta_i) - (b),
\end{align*}
\]

which is equivalent to

\[
\begin{align*}
    r_i(\rho_i(\hat{\theta}_i, \theta_{-i}), \theta_i) - \sum_{j \neq i} r_j(\rho_j(\hat{\theta}_i, \theta_{-i}), \theta_j) - c_i(\theta_i, \theta_{-i}) &> r_i(\rho_i(\theta), \theta_i) - \sum_{j \neq i} r_j(\rho_j(\theta_i, \theta_{-i}), \theta_j) - c_i(\theta_i, \theta_{-i}).
\end{align*}
\]

This contradicts the efficiency of ρ based on (6.12) and thus, the assumption was incorrect.

Lemma 12. Truth telling is a dominant strategy under (6.13).

Proof. Consider the problem of choosing the best type \( \hat{\theta}_i \) by a CP i. A best strategy for CP i solves

\[
\max_{\hat{\theta}_i} r_i(\rho_i(\hat{\theta}), \hat{\theta}_i) - \pi_i(\hat{\theta}).
\]

Substituting the payment function by the proposed mechanism (6.12), we get

\[
\max_{\hat{\theta}_i} \left[ r_i(\rho_i(\hat{\theta}), \hat{\theta}_i) - \sum_{j \neq i} r_j(\rho(\hat{\theta}_{-i}), \theta_j) - c_j(\theta_j, \hat{\theta}_{-i}) + \sum_{j \neq i} r_j(\rho_j(\hat{\theta}), \hat{\theta}_j) \right].
\]

This completes the proof.

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Since (a) does not depend on \( \hat{\theta}_i \), it is sufficient to solve
\[
\max_{\hat{\theta}_i} \left( r_i(\rho_i(\hat{\theta}), \hat{\theta}_i) + \sum_{j \neq i} r_j(\rho_j(\hat{\theta}), \hat{\theta}_j) \right). 
\]
Thus, CP \( i \) would pick a declaration \( \hat{\theta}_i \) that will lead the mechanism to pick a \( \rho \) which solves
\[
\max_{\rho} \left( r_i(\rho_i(\hat{\theta}), \hat{\theta}_i) + \sum_{j \neq i} r_j(\rho_j(\hat{\theta}_{-i}), \hat{\theta}_j) \right). 
\]

(B.1)

Under the proposed mechanism,
\[
\rho^* \in \arg \max_{\rho} \left( r_i(\rho_i(\hat{\theta}), \theta_i) + \sum_{j \neq i} r_j(\rho_j(\hat{\theta}_{-i}), \theta_j) \right). 
\]

The proposed mechanism (6.13) will choose \( \rho \) in a way that solves the maximization problem (B.1) with \( \hat{\theta}_i = \theta_i \). Thus, truth-telling is a dominant strategy for CP \( i \).

\[\square\]

Lemma 13. The proposed mechanism (6.13) is ex-post individually rational.

Proof. At the equilibrium, all the CPs are truthful and declare their real types. Thus, by replacing (6.13) in the utility of a CP \( i \), we have
\[
\begin{align*}
u_i &= \sum_i r_i(\rho_i^*(\theta), \theta_i) - c_i(\theta) - \sum_{j \neq i} r_j(\rho_j^*(\theta_{-i}), \theta_j) - c_j(\theta), 
\end{align*}
\]
where \( \rho^* \) is the outcome that maximizes the social welfare. The CPs could have picked \( \rho_i(\theta_{-i}) \) instead of \( \rho_i(\theta) \) as a solution of the optimization problem, as it is one of the possible strategies. This is possible because the set of strategies, i.e., the total storage capacity of the MNO, is fixed and does not change by changing the set of participating CPs. Thus,
\[
\sum_j r_j(\rho_j^*(\theta), \theta_j) \geq \sum_j r_j(\rho_j^*(\theta_{-i}), \theta_j).
\]

Furthermore, we know that the rate of a participating CP cannot be negative, i.e.,
\[
r_j(\rho_j^*(\theta_{-i}), \theta_j) \geq 0. 
\]

Therefore,
\[
\sum_i r_i(\rho_i^*(\theta), \theta_i) \geq \sum_{j \neq i} r_j(\rho_j^*(\theta_{-i}), \theta_j).
\]

Thus, (B.2) is non-negative and the proposed mechanism is ex-post individual rational.

\[\square\]

Lemma 14. The proposed mechanism (6.13) is weakly budget-balanced.
Proof. Since the CPs are truth-telling at the equilibrium then we have

\[ \sum_i \pi_i(\theta) = \sum_i \left( \left[ \sum_{j \neq i} r_j(\rho_j(\theta_{-i}), \theta_j) - c_j(\theta_{-i}, \theta_j) \right] - \left[ \sum_{j \neq i} r_j(\rho_j(\theta), \theta_j) - c_j(\theta, \theta_j) \right] \right). \]

Moreover, since the utility of a CP is a decreasing function of the number of the set of participating CPs, we have that, \( \forall i, \)

\[ \sum_{j \neq i} r_j(\rho_j(\theta_{-i}), \theta_j) - c_j(\theta_{-i}, \theta_j) \geq \sum_{j \neq i} r_j(\rho_j(\theta), \theta_j) - c_j(\theta, \theta_j). \]

Thus, the proposed mechanism is weakly budget-balanced. \( \square \)

Next, we prove the provided result in Theorem 4.

Proof. Based on Lemma 11, Lemma 13 and Lemma 14, we can deduce that all the conditions of the optimization problem (6.11) are satisfied. Based on Lemma 12, we can deduce that the proposed mechanism (6.13) is the unique efficient solution of the formulated problem (6.11). \( \square \)
Appendix C

Proofs of Results on Backhaul Management in Small Cell Networks

The proof of the convergence of the algorithm in (7.26) to a unique BGE can be decomposed into two parts. We first start by proving that the algorithm converges surely to a BGE and then prove that the algorithm admits a unique steady point which corresponds to the unique BGE.

Since a minority game is a special case of congestion games and based on [154, Theorem 3.1], we can deduce that the formulated SBMMG admits a potential function. On the other hand, based on [145, Theorem 7] and knowing that the SBMMG accepts a finite number of BGEs, then the algorithm in (7.26) admits at least one steady point and converges with probability 1 to a BGE.

In order to prove the uniqueness of the steady point of the algorithm in (7.26) we analyze the Robin-Monro iteration form of (7.26) [155]. The limiting ordinal differential equations (ODE) of the Robin-Monro equations write as follows:

\[
\begin{align*}
\dot{u}_n(a,t) &= E_p[u_n(a(t))] - \hat{u}_n(a,t), \\
\dot{p}_n(t) &= \beta(\kappa_n^*) (\hat{u}_n(t)) - p_n(t - 1),
\end{align*}
\]  
(C.1)

Given the existence of at least one fixed point for the ODE function:

\[
\dot{p}_n = \beta(\kappa_n)(p_n - p_n),
\]  
(C.2)

then, we have:

\[
p_n^* = \beta(\kappa_n)(p_n^*),
\]  
(C.3)

and by replacing with (7.22) and (C.3) in \(\dot{u}_n(a,t)\), we get:

\[
\dot{u}_n(a,t) = E_p(u_n(a(t), p_n^*)) + E_p[\epsilon_{n,a_n(t)}(t)] - \hat{u}_n(a,t),
\]  
(C.4)

which reduces to solving the ODE:

\[
\dot{u}_n(a,t) = u_n(a(t), p_n^*) - \hat{u}_n(a,t),
\]  
(C.5)
Now, we prove the existence of a unique fixed point for the ODE (C.5). Given Banach fixed point theorem which says that a contraction has a unique fixed point, it is sufficient to prove that the ODE in (C.5) is a contraction in order to prove the uniqueness of the fixed point $u_n(a(t), p^-_n)$ given by:

$$
\bar{u}_n(c, p^-_n) = \sum_{k=0}^{G-1} \binom{G-1}{k} p^k (1-p)^{G-(k+1)} u_n(c, k+1),
$$

with $p_n = \beta_n^{(s_n)}(p^-_n)$.

**Definition 22.** (Contraction): A map function $g : X \rightarrow X$ is said to be a $\theta$-contraction if $\exists \ 0 < \theta < 1$ such that:

$$
|g(x_1, x_2)| \leq \theta |x_1 - x_2|.
$$

Consider the difference $|\bar{u}_n(c, p^-_n) - \bar{u}_n(c, p'_n)|$, we have:

$$
|\bar{u}_n(c, p^-_n) - \bar{u}_n(c, p'_n)| = \\
|p_n \sum_{k=0}^{G-1} \binom{G-1}{k} p^k (1-p)^{G-(k+1)} u_n(c, k+1) - \\
p'_n \sum_{k=0}^{G-1} \binom{G-1}{k} (p')^k (1-p')^{G-(k+1)} u_n(c, k+1)| = \\
\left| \sum_{k=0}^{G-1} \left( p_n - p'_n \right) \binom{G-1}{k} \left( (p')^k - (p)^k \right) (1-p)^{G-(k+1)} u_n(c, k+1) \right| \\
\leq \left| \left( p_n - p'_n \right) \sum_{k=0}^{G-1} u_n(c, k+1) \right|, \\
\leq \left| \sum_{k=0}^{G-1} u_n(c, k+1) \right||p_n - p'_n|.
$$

By replacing with the best response functions we get:

$$
|p_n - p'_n| = |\beta_n^{(s_n)}(c, p^-_n) - \beta_n^{(s_n)}(c, p'_n)| = \\
\left| \exp \left( \kappa_n u_n(a, p^-_n) \right) - \exp \left( \kappa_n u_n(a, p'_n) \right) \right| \\
\left| \exp \left( \kappa_n u_n(c, p^-_n) \right) + \exp \left( \kappa_n u_n(d, p^-_n) \right) \right|.
$$

After some numerical computation we get:

$$
|p_n - p'_n| \leq \kappa_n ||p - p'||_{\infty}
$$

by replacing in (37) we have:

$$
|\bar{u}_n(c, p^-_n) - \bar{u}_n(c, p'_n)| \leq \kappa_n \sum_{k=0}^{G-1} u_n(c, k+1)||p - p'||_{\infty}.
$$

Since we have $u_n(c, p^-_n)u_n(d, p^-_n) \leq 0$, we can conclude that $u_n(p)$ is an $\infty$-contraction and admits a unique fixed point if $\kappa_n \leq \left| \sum_{k=0}^{G-1} u_n(c, k+1) \right|$. Following the results from stochastic approximation algorithms and considering the Lyapunov function $V(p) = ||p - p'||_{\infty}$ for ODE (C.2), we deduce that $p'$ is the unique globally asymptotically stable point of (C.2). Thus, the formulated SBMMG admits a unique fixed point which is the BGE of the game.
Appendix D

La théorie des jeux pour l’allocation de ressources dans les réseaux à petites cellules

La nouvelle génération de réseaux cellulaires sans fil sera caractérisée par un mélange de liens de communication hétérogènes, et d’un déploiement dense de petites stations de base. Pour bénéficier de cet écosystème émergent, plusieurs défis techniques doivent être abordés en termes de gestion des interférences, d’allocation des ressources, et de gestion de backhaul. En fait, les bandes fréquence radio qui sont actuellement exploitées sont utilisées à la fois comme liens backhaul et liens d’accès et doivent être partagées par un grand nombre de stations de base, ce qui engendre la congestion de ces liens. Des approches prometteuses ont été récemment proposées pour relever ce défi: a) le stockage distribué et b) LTE sur des canaux non-licenciés (LTE-U).

D’une part, le stockage distribué permet aux stations de base de stocker les fichiers les plus populaires à l’avance afin de servir les requêtes des utilisateurs localement et ainsi réduire la charge sur les liaisons backhaul. D’une autre part, le LTE-U permet aux petites stations de base de transmettre une partie de leur trafic sur les canaux non-licenciés afin de réduire le trafic transmis par les bandes de fréquence licenciées. Cependant, ces deux solutions donnent lieu à de nouveaux défis techniques qui doivent être traités avant que ces réseaux puissent atteindre leur véritable potentiel. Ces défis comprennent la conception de mécanismes de stockage distribué auto-organisés et de nouvelles méthodes de gestion de backhaul qui prennent en compte l’hétérogénéité des types de trafic et les exigences des utilisateurs en termes de qualité de service. Dans les systèmes LTE-U, de nouvelles techniques de gestion de ressources sont nécessaires pour assurer la coexistence harmonieuse des systèmes LTE-U et WiFi. Cette thèse consiste donc à présenter de nouvelles contributions fondamentales pour relever ces défis.

Tout d’abord, des outils de la théorie des jeux sont présentés comme des approches adaptées à la conception de protocoles pour la gestion des ressources distribuées, à faible complexité, tout en tenant compte des caractéristiques spécifiques des réseaux cellulaires.
futurs. Puis, dans la première partie de cette thèse, les défis techniques et économiques pour la mise en œuvre des politiques de stockage distribué sont traitées pour surmonter les contraintes de capacité des liens de transmission dans les réseaux à petites cellules. En particulier, un mécanisme de stockage proactif est proposé dans lequel les stations de base peuvent exploiter les informations extraites à partir des réseaux sociaux en ligne afin d’estimer la popularité locale des fichiers. Une autre approche de stockage optimisé est proposée pour tenir compte de l’ultra-densité des réseaux cellulaires futurs et des variations instantanées de l’état des unités de stockage. Pour faciliter le déploiement de ces solutions de stockage, de nouveaux mécanismes économiques sont conçus pour motiver les fournisseurs de contenu à coopérer avec les opérateurs de réseaux et de stocker leurs fichiers au sein des petites stations de base. Puis, dans la deuxième partie, le problème de la gestion du spectre est étudié dans des réseaux contenant des stations de base avec des capacités de stockage ainsi que dans les systèmes LTE-U. En particulier, une approche de gestion de backhaul de manière distribuée dans un réseau cellulaire ayant des capacités de mise en cache et des liens de backhaul hétérogènes est proposée. Le but de cette approche est de permettre à chaque petite station de base de déterminer le nombre de fichiers non-urgent à télécharger à chaque période de temps de manière à ce que ces les fichiers téléchargés ne détériorent pas la qualité de service des requêtes critiques et urgentes. Un algorithme d’apprentissage auto-organisé est ensuite proposé pour assurer une utilisation efficace des liaisons backhaul. D’autre part, un modèle multi-jeux est présenté comme un nouvel outil de la théorie des jeux pour faire face aux nouveaux problèmes d’allocation de ressources soulevés par l’introduction de la technologie LTE-U dans les réseaux sans fil. À cet égard, un multi-jeu composé de deux sous-jeux de types différents est formulé pour optimiser la coexistence des utilisateurs WiFi et les petite stations de base LTE-U sur les bandes non-licenciées, tout en empêchant les stations de base LTE-U d’affecter la qualité de service des utilisateurs WiFi. Pour le multi-jeu formulé, la stabilité du multi-jeu est présentée comme un nouveau point de stabilité et le multi-jeu formulé est prouvé d’accepter un point stable unique.

Les principales contributions de cette thèse sont donc la conception de nouvelles approches d’auto-organisation pour l’optimisation et la gestion de l’allocation des ressources dans les systèmes émergents hétérogènes et à grande échelle. L’objectif à travers les mécanismes proposés est d’utiliser efficacement toutes les ressources disponibles telles que le stockage et le spectre, tout en minimisant les transmissions de contrôle. En outre, de nouveaux mécanismes économiques sont proposés pour réussir le déploiement des solutions d’allocation des ressources proposées.
Bibliography


[81] ——, “Multi-games for LTE and WiFi coexistence over unlicensed channels,” in *International conference on Network Games, Control and Optimization*, 2016.


Abstract: This thesis consists in developing distributed mechanisms for resource allocation in next-generation cellular networks. In the first part of this thesis, the technical and economic challenges for the implementation of distributed storage policies in small cell networks are addressed. In particular, a proactive storage approach is proposed enabling the small base stations to exploit the information extracted from online social networks to estimate the local popularity of the files. Another optimized storage approach is proposed for ultra-dense cellular networks while accounting for the instantaneous variations of the state of the storage units. To facilitate the deployment of these storage solutions, new economic mechanisms are developed to motivate content providers to cooperate with network operators and store their files within the operators’ small base stations. In the second part of this thesis, the problem of spectrum management is studied in cache-enabled small cell networks as well as LTE-U systems. In particular, a distributed backhaul management approach is proposed for cellular networks with heterogeneous backhaul links. On the other hand, a multi-game framework is proposed as a new game theoretic tool to cope with the new resource allocation problems that emerge with the introduction of LTE-U technology in wireless networks. In this regard, a multi-game composed of two sub-games of different types is formulated to optimize the coexistence of LTE-U base stations and WiFi users over unlicensed bands, while preventing LTE-U base stations from jeopardizing the WiFi users.

Résumé : Cette thèse consiste à développer des mécanismes distribués pour la gestion de ressources dans les réseaux cellulaires futurs. Dans la première partie de cette thèse, les défis techniques et économiques pour la mise en œuvre des politiques de stockage distribuées dans les réseaux à petites cellules sont traités. En particulier, un mécanisme de stockage proactif est proposé permettant aux stations de base d’exploiter les informations extraites des réseaux sociaux afin d’estimer la popularité locale des fichiers avant de les stocker. Une autre approche de stockage optimisé est proposée pour les réseaux cellulaires ultra-denses tout en prenant en compte les variations instantanées de l’état des unités de stockage. Pour faciliter le déploiement de ces solutions de stockage, de nouveaux mécanismes économiques sont développés pour motiver les fournisseurs de contenu à coopérer avec les opérateurs réseau et stocker leurs fichiers au sein des petites stations de base. Dans la deuxième partie de cette thèse, le problème de gestion du spectre est étudié dans des réseaux contenant des stations de base munies de capacités de stockage ainsi que dans les systèmes LTE-U. En particulier, une approche de gestion de backhaul distribuée est proposée pour des réseaux cellulaire ayant des capacités de stockage et des liens de backhaul hétérogènes. D’autre part, un modèle multi-jeux est proposé comme un nouvel outil de la théorie des jeux pour faire face aux nouveaux problèmes d’allocation de ressources qui émergent avec l’introduction de la technologie LTE-U dans les réseaux sans fil. À cet égard, un multi-jeux composé de deux sous-jeux de types différents est formulé pour optimiser la coexistence des stations de base LTE-U et utilisateurs WiFi sur les bandes non-licenciées, tout en empêchant les stations de base LTE-U de dégrader la performance du réseau WiFi.