



HAL
open science

A new reduction of the old observations of Phoebe and the orbit update

Shanna Li

► **To cite this version:**

Shanna Li. A new reduction of the old observations of Phoebe and the orbit update. Astrophysics [astro-ph]. Université Paris sciences et lettres; Shanghai Astronomical Observatory, 2016. English. NNT : 2016PSLEO008 . tel-01484637

HAL Id: tel-01484637

<https://theses.hal.science/tel-01484637v1>

Submitted on 7 Mar 2017

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

THÈSE DE DOCTORAT

de l'Université de recherche Paris Sciences et Lettres
PSL Research University

Préparée dans le cadre d'une cotutelle entre
IMCCE, l'Observatoire de Paris
et **Shanghai Astronomical Observatory, CAS**

A new reduction of the old observations of Phoebe and the orbit update

**Une nouvelle réduction des anciennes observations de
Phoebe et amélioration de son orbite**

Ecole doctorale n°127

Astronomie et Astrophysique d'Ile-de-France

Spécialité Astronomie et Astrophysique

Soutenue par Shanna LI
le 23 09 2016

Dirigée par
Jean-Eudes ARLOT
et
Zheng-Hong TANG

COMPOSITION DU JURY :

Mme. DUCOURANT Christine
Observatoire de Bordeaux, Rapporteur

M. FU Yan-Ning
Purple Mountain Observatory, CAS,
Rapporteur

M. PENG Qing-Yu
JI-NAN University, Membre du jury

M. SHEN Kai-Xian
National Time Service Center, CAS,
Membre du jury

M. SOUCHAY Jean
Observatoire de Paris, Président du jury

M. ARLOT Jean-Eudes
Observatoire de Paris, Directeur de thèse

M. TANG Zheng-Hong
Shanghai Astronomical Observatory, CAS,
Codirecteur de thèse

M. VIENNE Alain
Université Lille 1, Co-encadrant de thèse



Remerciement

En premier lieu, j'aimerais exprimer mes plus profondes gratitudees à mes deux directeurs de thèse, Jean-Eudes ARLOT et Zheng-Hong TANG qui m'ont proposé cette recherche. Sans leurs encouragements constants et leurs judicieux conseils, cette thèse n'aurait jamais pu aboutir. J'ai beaucoup appris d'eux, tant sur la recherche que sur la vie. Leurs travaux scientifiques et leur enthousiasme dans le recherche continueront sûrement à m'inspirer.

Je suis profondément reconnaissante à mon co-encadrant de thèse Alain VIENNE pour sa gentillesse, ses disponibilités et son aide précieuse.

Je remercie chaleureusement Christine DUCOURANT et Yan-Ning FU qui m'ont fait l'honneur de rapporter ma thèse. Je leur exprime ma respectueuse gratitude et ma reconnaissance notamment pour leurs lectures attentives du manuscrit de ma thèse, leurs commentaires et suggestions enrichissantes.

Je remercie également Qing-Yu PENG, Kai-Xian SHEN, et Jean SOUCHAY qui se sont intéressés à mon travail et m'ont fait le grand honneur de siéger dans le jury.

Mes remerciements s'adressent à mes collaborateurs Josselin DESMARS, Rong-Chuan QIAO, Shu-He WANG, Yong YU, Xiao-Jin XI, Hui-Yan ZHANG, Laurène BEAUVALET. Les travaux avec eux constituent des parties substantielles de ma thèse. Je leur témoigne ici mon profond respect.

Je remercie vivement Daniel HESTROFFER, directeur de l'IMCCE (Institut de Mécanique Céleste et de Calcul des Éphémérides) pour m'avoir accueillie dans le laboratoire qui m'a donné des conditions de travail idéales, et William THUILLOT, responsable du Doctorat de l'Observatoire de Paris pour mon inscription à l'ED127. Merci aussi aux membres de l'IMCCE, particulièrement aux membres de l'équipe PECASE pour son soutien et les discussions enrichissantes qui s'y sont déroulées.

J'aimerais aussi remercier Ming ZHAO, Jian-Hai ZHAO, et l'ensemble des membres de l'Observatoire de Shanghai.

Je tiens à remercier également François COLAS, Hong-Jian PAN, pour leur accueil chaleureux lors de mes observations, et tous les assistants de l'Observatoire du Pic du Midi, de la Station d'observation de SheShan, de XingLong, de LiJiang, et de l'Observatoire de

KunMing.

Cette thèse a été financée partiellement par une bourse du gouvernement chinois: China Scholarship Council, et aussi supportée par le Project 11573029 du National Natural Science Foundation of China. Je leur remercie sincèrement.

Je remercie tous mes amis, Li ZHANG, Ni XIAO, Huan HUANG, Kun-Liang YAO, Jia-Cheng LIU, pour leurs aides et encouragements dans la vie et les études.

En fin, je remercie mon époux Lingmin LIAO et ma famille, sans eux je n'aurais pas pu terminer ma thèse dans de si bonnes conditions.

Contents

1	Introduction	11
1.1	History of Phoebe	12
1.2	Chapter introduction	13
2	Astrometric reductions: Analysis of the catalogues used for the astrometric reduction	15
2.1	Introduction	15
2.2	Reference Systems and Frames, Fundamental Astronomy Coordinates . . .	15
2.2.1	ICRS, ICRF	16
2.2.2	Type of the coordinates	17
2.2.3	Different reference systems where we get observed coordinates and general change to ICRS	20
2.3	Time scales	22
2.4	Catalogs used for reductions	25
2.4.1	Catalogs used for old reductions	25
2.4.2	Modern catalogs	29
2.4.3	Catalog comparison and the statistics on the used catalogues	33
2.5	Other corrections for astrometric reduction	36
3	Search and selection of published observations	39
3.1	Introduction	39
3.2	Old photographic observations	41
3.2.1	Observatory and telescopes	41

3.2.2	Different sets of observations	41
3.3	New observations	45
4	The re-reduction of old observations	55
4.1	Introduction	55
4.2	Introduction to the used method	56
4.3	Identifying the old reference stars in new catalogs	57
4.4	Choosing new catalogs and the case of no information on reference stars . . .	59
4.5	Catalog bias	63
4.6	Result	64
5	A new ephemeris of Phoebe	65
5.1	Introduction	65
5.2	The numerical model	66
5.2.1	Perturbations	66
5.2.2	Equations of motion	69
5.2.3	Observations used to fit the dynamical model	69
5.2.4	Numerical integration	69
5.3	The frequency analysis	71
5.3.1	Definition of the elements	72
5.3.2	Developpement of quasi-periodic series	72
5.3.3	The principle of the fine analysis	73
5.3.4	Data windowing	75
5.3.5	The procedure of the frequencies analysis	75
5.3.6	Identification and synthetic representation	77
5.3.7	Conclusion	82
6	Comparisons and validation of the new ephemeris of Phoebe	83
6.1	Observations Comparison	83
6.1.1	CCD observations calibrated with different catalogs	83

6.1.2	Old published photographic observations and the reduced observations	88
6.2	Ephemeris Comparison	92
7	Conclusion and future work	97

List of Figures

2-1	Gaia catalogue Positional accuracy (F. Mignard, private communication) . . .	32
2-2	The statistics of the difference in angular separations of the stars in old and modern catalogs	34
3-1	The photographic observations of Phoebe	40
3-2	Original table for Date	42
3-3	Coordinates of the reference stars	43
3-4	Original measures	43
3-5	Measuring squares	43
3-6	Positions of Phoebe	44
3-7	Coordinate of reference stars	44
3-8	Positions of Phoebe	44
3-9	O-C for the observations at Pic du midi	47
4-1	Reference stars' <i>J</i> 2000 positions in two catalogs in a plate in the year 1900 and the position differences. The base of the arrow presents the reference stars' position in catalog CPD, while the head of the arrow presents the positions in catalog Tycho2	58
4-2	The positions of the comparison stars and Phoebe. The black stars mean the 10 reference stars mentioned in the paper Pickering [1908], the red marks mean the reference stars found through our method. The circles stand for the seven reduced positions of Phoebe and the green points mean the positions of Phoebe in the JPL ephemeris.	60

4-3	The positions of one reference star in the epoch J2000.0 (lower) and B1900.0 (upper) calculated with the positions and the proper motions in four different catalogs: TYCHO, USNOB1.0, UCAC2, and HIPPARCOS.	63
4-4	Example of result list	64
5-1	Image of the function $\frac{\sin(2\pi x)}{2\pi x}$	74
6-1	The residuals in RA and DEC of the observations reduced with catalog UCAC3 at Pic du midi in 2013.	85
6-2	The residuals in RA and DEC of the observations reduced with catalog UCAC2 at Pic du midi in 2013.	86
6-3	The residuals in RA and DEC of the observations reduced with catalog USNO-B1.0 at Pic du midi in 2013.	87
6-4	The comparison of residuals in RA of the observations before and after our method.	90
6-5	The comparison of residuals in DEC of the observations before and after our method.	91

List of Tables

2.1	Participating observatories and number of stars measured in the context of the Astrographic Catalogue	28
2.2	The principal characteristics of the Tycho-2 catalog	30
2.3	Statistics on the catalogues used to reduce the astrometric positions of Phoebe	35
3.1	Telescope in Arequipa Observatory with which the first plate of Phoebe had been taken	39
3.2	All the studied photographic observations	41
3.3	Positions of Phoebe observed at Pic du midi	48
4.1	Statistics of the position difference between two catalogs	59
4.2	O-C of the Phoebe with different catalogs	60
4.3	O-C of Phoebe position with different methods for choosing reference stars	61
4.4	Statistics of residuals with different methods for choosing reference stars . .	61
5.1	Mean and root mean square (rms) of O-C in arcsec for the different models (Desmars et al. [2013]).	68
5.2	Dynamical constants of Saturn's system (Jacobson et al. [2006]).	68
5.3	Saturnicentric starting state vector at JD 2440600.5 in ICRS (Shen et al. [2011]).	70
5.4	The resulting initial Saturnicentric vectors at JD 2440600.5 for Phoebe after adjustment in ICRS	71
5.5	Fundamental arguments of the Saturn system used for the identification in 52488 days with the Model 1	78
5.6	Linear part in the mean longitudes of Phoebe in 52488 days with Model 1 .	78

5.7	Complex z of Phoebe in 52488 days with Model 1, the series is in complex exponential.	79
5.8	Semimajor axis of Phoebe in 52488 days with Model 1, the series is in cosine.	79
5.9	Fundamental argument of the Saturn system used for the identification in 729000 days with the Model 4	80
5.10	Semimajor axis of Phoebe in 729000 day with the Model 4, the series is in cosine	80
5.11	$\Delta\lambda$ of Phoebe in 729000 days with the Model 4, $\lambda = -1.680167769776 + 4.17034475t + \Delta\lambda$, the series is in cosine	81
5.12	Complex z of Phoebe in 729000 days with the Model 4, the series is in complex exponential	81
5.13	Complex ζ of Phoebe in 729000 days with the Model 4, the series is in complex exponential	82
6.1	Statistics for the observed-minus-computed residuals for the photograph observations grouped by reference	89
6.2	Statistics for the observed-minus-computed residuals for all observations in using different ephemeris	94
6.3	Statistics for the observed-minus-computed residuals for photographic observations in using different ephemeris	95
6.4	Statistics for the observed-minus-computed residuals for CCD observations in using different ephemeris	96

Chapter 1

Introduction

The aim of the present work is to focus on the recalibration of historical observations of Phoebe, the 9th satellite of Saturn, and its orbit improvement based on the recalibrated data.

The accuracy of ephemeris depends on the theoretical model's quality (internal error) and also depends on the accuracy and the quantity of observations used to fit the model (external error). Phoebe is far away from its planet and other major satellites. The theoretical model is not very complicated, so the internal error is well known and can be generally controlled. However, the external error is in general unknown and is the main cause of the global error. Since the brightness of Phoebe is faint ($V_{\text{mag}}=16.5$), not so many observations have been made in the past as other nature satellites. The ephemeris of Phoebe is not as good as those of other major satellites of Saturn.

The coordinates of Phoebe over one hundred years after the astrometric reduction were referred to different catalogues. There are some inconsistencies in the observation data obtained from different authors and different epochs. The observations have been published in different reference frames and in different formats. These observations cannot be used directly to fit the dynamical model. The systematic differences should be corrected, by using the latest astronomical constants and reference frame. The observations should be re-input into a unified data format, with the uniform time scale TT Julian days and the equatorial astrometric coordinates in the same ICRF reference frame.

The earliest observations of Phoebe have bad accuracy and do not fit the existing models very well. At that time, the catalog they used for reduction did not contain enough stars, so

there were not many catalog stars on the plates. Because of the unsatisfactory precision of the old catalog, the positions of the reference stars were not very precise which make the position of Phoebe inaccurate. We will introduce a method by which allows to reduce the positions of Phoebe or other natural satellites from the old published observations using the modern precise astrometric catalogs, such as UCAC or GAIA in the future.

Desmars et al. [2009] have shown that a most adequate model of satellite motion should be constructed and built not only based on high-accurate observations but also on data with long period. The theoretical model of natural satellite motion needs to be improved when a considerable amount of new observations are accumulated over a significant time interval. After the historical observation data are re-calibrated and the new observations reduction have been finished, the new orbital model of Phoebe will be derived, and then the numerical integration process of the orbit can be well improved. The improvement of the astrometry and of the dynamical model may allow quantifying physical perturbations which were usually neglected.

1.1 History of Phoebe

Being the sixth planet from the Sun, Saturn is the second largest planet in the Solar System. It is a gas giant like Jupiter, Uranus and Neptune. With a diameter of about nine times that of the Earth, Saturn has at least 150 moons and moonlets, 53 of which were confirmed and named. The satellites of Saturn have various sizes from very small moons of less than 1 km in diameter across to the enormous Titan which is larger than the planet Mercury. As the second largest moon in the Solar System among the identified satellites, Titan has a diameter about 5000 km. Twenty-four of Saturn's moons are regular satellites which have prograde orbits almost circular and slightly inclined relative to the equatorial plane of the planet. The other 38 moons are irregular satellites whose orbits are much farther from Saturn and strongly inclined to the equatorial plane of the planet. Some of the irregular satellites have the particularity to present a retrograde orbit and are most likely captured, as suggested by their inclinations. All such bodies with retrograde orbits have a radius less than 30 km, with the exception of Phoebe, the ninth satellite of Saturn which has a

magnitude of 16.5 and a size of about 106.5 km as mean radius. Phoebe has the orbit nearly 13 million kilometers from Saturn. Since 2000, the systematic search of satellites using the imagery with a large field helped discovering enough bodies (Gladman et al [2001]) to classify them into 3 groups: Norse, Inuit and Gauls. The satellites in group Norse are the satellites with retrograde orbits, like Phoebe. They may be mentioned: S-9 Phoebe, S-27 Skathi, S-31 Narvi, S-25 Mundilfari, S-23 Suttungr, S-30 Thrymr, S-19 Ymir and other satellites that have not yet been definitively denominated. Phoebe was the first encountered target upon the arrival of the Cassini spacecraft to the Saturn system in 2004. Cassini's trajectory and arrival time to Saturn were specifically chosen to permit this flyby.

The dynamics of the eight main satellites were well studied by several people (Taylor & Shen [1988]; Duriez & Vienne [1991]; Vienne & Duriez [1991], [1992], [1995]; Dourneau [1993]; Harper & Taylor [1993]; Lainey et al. [2012]). We present here the studies of an irregular natural satellite of Saturn which is the first natural satellite discovered using photographic plates: Phoebe.

At the end of the 19th Century, the development in photographic plates and in the long time exposure technique allows the discovery of new satellites. Phoebe, identified in 1899 by W.H Pickering, is the first satellite discovered by analyzing the photographic plates. Since the invention of the charge-coupled devices (CCD) by the Bell Telephone Laboratories in 1969, photographic plates are gradually less in use in professional observatories. The usage of photographic plates has been declined significantly since the early 1980s, replaced by CCD.

1.2 Chapter introduction

We will start by presenting the general principles. We will recall the reference systems, the coordinates, the star catalogs and astrometric reductions that will be used in our work. At the end of this part, we will compare the old catalogs and new catalogs. This section is the first work of this thesis and has been published in Desmars et al. [2013]. We will introduce our work in the following parts. All the contents with no citation are our own work.

In the Chapter 3 we will provide the information of old observations. From the ref-

erences of these old observations, we will look for the details of observatory stations, the characteristics of telescopes, the catalogues used at that epoch and the methods to calibrate the astrometric coordinates.

Then, we will explain the methods to improve the reduction of the old observations. Among the different star catalogs used to reduce the observations, we will determine which catalog will be used to produce the most accurate results. Old publications gave Phoebe's positions by analyzing photographic plates, while we will try to reanalyze the plates and recalculate the positions of Phoebe.

In the next chapter, we will first present the dynamical model of Phoebe and the integration of the equations of motion. We will use all recalculated observations and recent observations to fit dynamical model by the Least Squares Method which then will improve the accuracy of Phoebe's ephemeris. Then we discuss the determination of orbital motion frequency using the quasi-periodic series. Our aim is not only to get a good presentation of the movement over a given interval, but also to give an access to a good knowledge of physical frequencies which characterizes the studies over long period dynamical system.

To validate the feasibility of the developed methods, we will show the comparison of the statistics in the observed-minus-computed residuals for the published observations and the reduced observations, the comparison of the ephemerides accuracy before and after the reduction of the observations, and the comparison of different ephemerides to fit the reduced observations.

Chapter 2

Astrometric reductions: Analysis of the catalogues used for the astrometric reduction

2.1 Introduction

To re-reduce or correct old data for our purpose, it is essential to understand what was done for the reductions and what were the reference frames of the data. This will help us to define the process useful to improve old data. In this chapter, our work consists in finding out the reference system, coordinates, the time scales and catalogs used in old and recent observations, reductions and ephemeris of Phoebe. We will recall the general concept of the astrometric reductions and compare the old catalogs and new catalogs.

2.2 Reference Systems and Frames, Fundamental Astronomy Coordinates

The motion of a natural satellite can be expressed as position, velocity and acceleration with time. We can define a position relative to a reference system. It is necessary to give some definitions to secure the frame within which we will work by using each of the observations

that will be described.

Old data used for the making of Phoebe's ephemeris were used until today as published by their authors. Doing so, different reference frames were mixed making biases in the ephemeris. Our purpose is to make a new reduction of old observations or to make corrections to old data. So that we must understand what was done in the past by the observers and how to make now useful corrections. In some cases, we will be able to make a complete new reduction and otherwise, only partial corrections. In this section we will prepare tools for the making of a new procedure reduction.

2.2.1 ICRS, ICRF

The IAU Working Group on Nomenclature for Fundamental Astronomy has recommended the following definitions for the ICRS and ICRF:

International Celestial Reference System (ICRS): The idealized barycentric coordinate system to which celestial positions are referred. It is kinematically non-rotating with respect to the ensemble of distant extragalactic objects. It has no intrinsic orientation but was aligned close to the mean equator and dynamical equinox of J2000.0 for continuity with previous fundamental reference systems. Its orientation is independent of epoch, ecliptic or equator and is realized by a list of adopted coordinates of extragalactic sources.

International Celestial Reference Frame (ICRF): A set of extragalactic objects whose adopted positions and uncertainties realize the ICRS axes and give the uncertainties of the axes. It is also the name of the radio catalogue whose 212 defining sources are currently the most accurate realization of the ICRS. Note that the orientation of the ICRF catalogue was carried over from earlier IERS radio catalogs and was within the errors of the standard stellar and dynamic frames at the time of adoption. Successive revisions of the ICRF are intended to minimize rotation from its original orientation.

The ICRS is a fundamental celestial reference system for high-precision positional astronomy. It is meant to represent the most appropriate coordinate reference system for expressing reference data on the positions and motions of celestial objects.

A reference frame is the physical realization of a reference system, i.e., the reference

frame is the reported coordinates of datum points. The ICRS is an idealization with defined origin and axis. The ICRF consists of a set of identifiable fiducially points on the sky along with their coordinates, which serves as the practical realization of the ICRF. The ICRF is now the standard reference frame used to define the positions of the planets (including the Earth) and other astronomical objects.

Although the directions of the ICRS coordinate axes are not defined by the kinematics of the Earth, the ICRS axes (as implemented by the ICRF) closely approximate the axes that would be defined by the mean Earth equator and equinox of J2000.0 (to within about 0.02 arcsecond), if the latter is considered to be a barycentric system. Because the ICRS axes are non-rotating, there is no date associated with the ICRS. Furthermore, since the defining radio sources are assumed to be so distant that their angular motions, seen from Earth, are negligible, there is no epoch associated with the ICRF. It is technically incorrect, then, to say that the ICRS is a "J2000.0 system", even though for many current data sources, the directions in space defined by the equator and equinox of J2000.0 and the ICRS axes are the same within the errors of the data.

For other reference frame, Geocentric means that the reference system centre is in the Earth. The Topocentric means the centre is the observer.

The numerical integration of the dynamical model allows the computation of the positions of Phoebe in the same reference system as that of the planetary ephemeris. The planetary ephemeris develops and the reference system changes and recently we use ICRS. we have to transform all the observations to the same reference system ICRS in order to compare computed and observed positions. The classical transformations have been done with the Software Routines from the IAU SOFA Collection [2010]. Copyright at International Astronomical Union Standards of Fundamental Astronomy (<http://www.iausofa.org>).

2.2.2 Type of the coordinates

The coordinates for observation:

1) The "absolute" equatorial coordinates:

Right Ascension and Declination, the notation is (RA, DEC) ou (α, δ) . The coordinates

are usually defined in terrestrial equatorial system. Most of the observations of Phoebe are in absolute coordinates. The absolute coordinates are determined by calculation in using reference stars' coordinates in star catalogs. The center of the reference system may be topocentric or geocentric.

2) Separation (arcsec) and Position angle (degree):

Separation is the apparent angular distance between the selected satellite and the reference object. The satellite position angle refers to the reference object counted from North to East. Many satellites near the primary are within this coordinates.

3) (X, Y) Differential Coordinates:

The difference of the equatorial coordinates between satellite and its reference body onto the celestial parallel and celestial meridian mutually intersecting in the reference body which can be an object or the geometrical center for all of them. The notation is $(\Delta\alpha, \Delta\delta)$ or $(\Delta\alpha \cos \delta, \Delta\delta)$.

4)(X, Y) tangential coordinates:

The coordinates measured on the tangent plane of the celestial sphere at the point of reference body. Usually X is measured to the east, Y is measured to the north. Sometimes we call them standard coordinates.

If we know the equatorial coordinate (α_0, δ_0) of the optical center of the instrument and the equatorial position (α, δ) of a celestial body, we can deduce its tangential coordinates (X,Y) relative to the field center, as follows gnomonic projection:

$$\begin{aligned} X &= \frac{\cos \delta \sin(\alpha - \alpha_0)}{\sin \delta \sin \delta_0 + \cos \delta \cos \delta_0 \cos(\alpha - \alpha_0)} \\ Y &= \frac{\sin \delta \cos \delta_0 - \cos \delta \sin \delta_0 \cos(\alpha - \alpha_0)}{\sin \delta \sin \delta_0 + \cos \delta \cos \delta_0 \cos(\alpha - \alpha_0)}. \end{aligned} \tag{2.1}$$

5) (x, y) measured coordinates :

We note (x, y) for the coordinates measured in the photographic plates or CCD images. Theoretically the measured coordinates should be the same as the tangential coordinates if the measurement is from the center of the image and the reference axis are from west to east for x and from south to north for y. But in reality the optical center is not determined exactly and therefore the position (α_0, δ_0) is an approximation; the inaccuracy of the focal

length of the instrument and the orientation of the plate causes a rotation and uneven scale effect of the reference axis; the distortion of the optic and the effect of the atmosphere. These errors will be corrected by least squares method in the astrometric reduction in using the positions of the reference stars.

The transformation of (x, y) to (X, Y) defined by:

$$\begin{aligned} X &= a_1x + b_1y + c_1 + d_1x^2 + e_1y^2 + f_1xy + T_1(x, y) \\ Y &= a_2x + b_2y + c_2 + d_2x^2 + e_2y^2 + f_2xy + T_2(x, y). \end{aligned} \quad (2.2)$$

The terms $T_1(x, y), T_2(x, y)$ are the terms with the order higher than 3. The constants $a_1, b_1, c_1, d_1, e_1, f_1, a_2, b_2, c_2, d_2, e_2, f_2$, are the characteristic of the plate for each observation called plate constants. If we know these constants we can get objet's tangential coordinates from the measured coordinates of the objet's, then with the equatorial coordinate of the optical center, the equatorial coordinate of the objet can be solved out.

We can get equatorial coordinates of the reference stars from star catalog, and with the equatorial coordinate of the optical center, the tangential coordinates (X, Y) are provided. The measurements give the (x, y) of the reference stars, if we have enough reference stars, with the equations above in using the least squares method through several iterations we can determine the plate constants. We need at least 3 stars to define 6 constants for the first order of the equations, which means we ignore the distortion. At least 6 stars are needed to define 12 constants for the second order and 10 stars are needed to define 20 constants for the third order.

The coordinates for ephemerides:

6) x, y, z, V_x, V_y, V_z vectors:

The rectangular vector coordinates and the velocity of satellite. These coordinates are solved out from the equations of motion and used to calculate the orbital elements. The vectors are converted to absolute equatorial coordinates in order to compare computed and observed positions. The vector can be defined in the planetocentric or barycentric and terrestrial ecliptic or equatorial system. The center of the reference system may be another satellite. The vectors from the ephemeris TASS (Vienne & Duriez [1991], [1992]),

an ephemeris of main satellites of Saturn, are in the planetocentric and terrestrial ecliptic reference system. The epoch epoch of ecliptic and equinox is J2000.0. The ephemeris of Phoebe provides the positions vector in Saturn-centric ICRS reference system.

2.2.3 Different reference systems where we get observed coordinates and general change to ICRS

When we want to compare the observed positions of the satellite with the computed positions which at first are the vectors solved from the dynamical equations, we should transform these vectors from planetocentric equator or ecliptic system to the reference systems of the observations. We need to find out the reference system for every observation and for simplifying the process of the vectors transformation, we try to reduce the observations in the same reference system. The following reference systems were used in observations of Phoebe and the number that indicate the reference system will be present in the figure 4-4. For each reference system we introduce the general method to change the coordinates in this reference system to the coordinates in the ICRS reference system. We will introduce a new reduction in Chapter 4 with which we do not need to change the reference system as usual for some old observations. The main idea of this new reduction was published in Desmars et al. [2013] at the beginning of our thesis work and only some of earliest old observations had been reduced in that article.

1) ICRS:

The reference frame of most of the modern catalogs are considered to be very close to ICRF and the coordinates of the stars are in ICRS reference system, such as Hipparcos Catalogue, TYCHO catalogs, UCAC catalogs and USNO catalogs except USNO A1.0. The observations reduced with these catalogs are in ICRS reference system.

The coordinates relating to the reference system of the planetary ephemeris DE4XX / LE4XX or INPOP are also considered to be very close to the ICRS.

2) J2000:

The reference system center is in the barycentre of the Solar System, The coordinates reported at the Earth's mean equator and equinox in J2000.0. There is a very slight differ-

ence between the ICRS frame and the J2000 frame as mentioned before, we can use the transformation between J2000 and ICRS of the IERS (<http://www.iers.org>). The accuracies of the observation and the ephemeris of Phoebe do not reach to the difference between ICRS and J2000, so we can treat the transformation as an identity in our work. But we still distinguish the notation in the observation table for future work.

3) B1950:

The axes of the reference system are defined by equinox and mean equator of B1950.0 in FK4 catalog. The coordinates are transformed to ICRS with Newcomb precession, including the elliptical aberration. The parameter from Aoki [1983], Kinoshita [1975] and Smith [1989]

4) Apparent:

Case when the coordinates are in true equator and equinox of date. The coordinates are transformed with the precession model including frame bias, and the notation model adopted by IAU in 2006, the SOFA routines are PB06 and MUT06A.

The coordinates can be geocentric or topocentric.

5) B1900:

The axes of the reference system are defined by equinox and mean Equator B1900.0. The coordinates are transformed to ICRS with Newcomb precession.

6) Year: The coordinates reported at the mean equator and equinox at 1 January of the year of observation. The coordinates are transformed to ICRS by using the precession model adopted by IAU in 2006. The SOFA routine is PB06. The correction of the elliptical aberration must be done at first for the observations using the catalogues before 1984.

7) B1875:

Equinox and mean Equator B1875.0. The coordinates are first corrected from elliptical aberration, then transformed to mean equator and equinox of the date using the precession value of Newcomb (Kinoshita [1975]). Finally, the coordinates are transformed to ICRS by using current values of precession provided by IAU 2006.

2.3 Time scales

The time scale used for observations must very carefully verified and converted if necessary. Phoebe is moving about 1.8 km/second so that an error of 10 seconds in the timing of an observation leads to an error of 18 km in space.

The time used for dynamical model should be continuous and uniform. But most of the data to record the old observation time are relative to the local time.

1) Greenwich Mean Time (GMT)

It was originally reckoned from noon to noon. In 1925, some countries shifted GMT by 12 hours so that it would begin at Greenwich midnight. This new definition is used for world time and in the navigational publications of English-speaking countries. The designation Greenwich Mean Astronomical Time (GMAT) is reserved for the reckoning of time from noon (and previously called GMT). Before 1805 the Royal Navy Day started 12 hours before local mean solar time, thus the Royal Navy Day was then 24 hours ahead of GMT.

2) Coordinated Universal Time (UTC)

It was introduced in 1972. Now it is the basis of all civilian time throughout the world. Because most daily life is still organized around the solar day, UTC was defined to closely parallel Universal Time, and UTC is uniform between two leaps while UT1 is based on Earth's rotation, which is gradually slowing. In order to keep the two times within 0.9 seconds of each other, a leap second is added to UTC about once every 12 to 18 months. The GMT presented in the publications before 1925 is 12h after UTC.

GMT and UTC are not a time continuing, and cannot be used as a dynamical time even they are used to date the observations.

3) UT1 - Universal Time

Universal Time (UT1) is a measure of the actual rotation of the earth, independent of observing location. It is the observed rotation of the earth with respect to the mean sun corrected for the observer's longitude with respect to the Greenwich Meridian and for the observer's small shift in longitude due to polar motion.

UT1 is not a uniform time since the rate of the earth's rotation is not constant, and its

difference with atomic time is no predictable. As of December 1995, UT1 was drifting about 0.8 seconds per year with respect to atomic time (TAI or UTC). The difference between UT1 and UTC is never greater than 0.9 since the leap seconds defined for UTC to keep this difference.

$$UT1 = UTC + DUT1$$

DUT1 is published weekly in IERS (International Earth Rotation Service) Bulletin A along with predictions for a number of months into the future.

UT1 is continuous but not uniform, so can't be used as a dynamical time.

4) GMST - Greenwich Mean Sidereal Time

Sidereal time is the measure of the earth's rotation with respect to distant celestial objects. Compare this to UT1, which is the rotation of the earth with respect to the mean position of the sun. One sidereal second is approximately 365.25/366.25 of a UT1 second. In other words, there is one more day in a sidereal year than in a solar year.

By convention, the reference points for Greenwich Sidereal Time are the Greenwich Meridian and the vernal equinox (the intersection of the planes of the earth's equator and the earth's orbit, the ecliptic). The Greenwich sidereal day begins when the vernal equinox is on the Greenwich Meridian. Greenwich Mean Sidereal Time (GMST) is the hour angle of the average position of the vernal equinox, neglecting short term motions of the equinox due to nutation.

In conformance with IAU conventions for the motion of the earth's equator and equinox GMST is linked directly to UT1 through the equation

$$\begin{aligned} &GMST \text{ (in seconds at } UT1 = 0) \\ &= 24110.54841 + 8640184.812866 * T + 0.093104 * T^2 - 0.0000062 * T^3 \end{aligned} \tag{2.3}$$

where T is in Julian centuries from 2000 Jan. 1 12h UT1,

$$\begin{aligned} T &= d/36525 \\ d &= JD - 2451545.0 \end{aligned} \tag{2.4}$$

5) International Atomic Time (Temps Atomique International = TAI)

It is defined as the weighted average of the time kept by about 200 atomic clocks in over 50 national laboratories worldwide. UTC is different from TAI by changing an integral number of seconds.

6) Terrestrial Dynamical Time (TDT, TD)

It was introduced by the IAU in 1979 as the coordinate time scale for an observer on the surface of Earth. It takes into account relativistic effects and is based on TAI. The time TDT is the atomic time used in the theories of motion for bodies in the Solar System. In 1991, the IAU refined the definition of TDT to make it more precise.

7) Barycentric Dynamical Time (TDB)

It is used as a time-scale of ephemerides referred to the barycentre of the Solar System. It is different from TDT by at most a few milliseconds.

8) Terrestrial Time (TT)

It was originally used instead of TDT or TDB when the difference between them did not matter. It was defined in 1991 to be consistent with the SI second and the General Theory of Relativity, replaced TDT in the ephemerides from 2001 and on.

$$TT - TAI = 32.184s$$

$$TDB = TT + 0.001658s * \sin(g) + 0.000014s * \sin(2 * g)$$

$$g = 357.53d + 0.98560028d * (JD - 2451545.0)$$

(higher order terms neglected; g = Earth's mean anomaly)

The FK5 Earth precession model is expressed in terms of GMST. The DEXXX is expressed in terms of TT or TDB, which is more-or-less a fixed offset from Atomic Time (TAI). The clock on the computers used to record the observations time shows UTC. One UTC second is equal to one TAI second. The time scale used in dynamical models is TT. The leap second tables for UTC are maintained by the IERS.

The UT1 might be used for planning a telescope to follow the earth's rotation, the TT for searching an ephemeris of a planet or satellite, and the TDB for interpreting pulsar observations. TAI would be useful for calculating time intervals between observed events, as well as the best way of making time critical applications leap-second-proof. TCG, TCB would have more specialized dynamical uses. The UTC itself would be suitable for record-

ing and for conversion into local time, and essentially nothing else.

For our research, all the observation time are first recorded as UTC in format Julian days (JD). As the accuracy of the ephemeris of Phoebe is around 0.1 second of time, we consider no difference between TT and TDB. We give the difference between UTC and TT in the list of the observations in order to get the TDB Julian day to be easily used in the theories of motion for bodies in Solar System.

2.4 Catalogs used for reductions

We are looking for old observations which had been reduced with different reference star catalogues. We need to identify which catalogs were used to be able to identify which stars were used to the reduction and then to make corrections to the astrometric positions. The following information for catalogs are from ViZieR database (Ochsenbein [2000]) from CDS (Centre de Données astronomiques de Strasbourg), and the mark I/number is the catalog identification code in ViZieR.

2.4.1 Catalogs used for old reductions

1) **BD:(Argelander 1859-1903). Catalog number in CDS: I/122**

The Bonner Durchmusterung (BD, Argelander [1859-62], Kuestner [1903], Becker [1951], Schmidt [1968]) is a visual survey of stars in the declination zones +89 to -01 degrees. Actual magnitude estimates were made and reported to 0.1 mag for all stars down to 9.5 mag, with fainter stars being assigned to 9.5. Positions are given to the nearest 0.1 sec in right ascension and 0.1 arcminute in declination. Positions are for Equator 1855 and no proper motion is provided. It has not been used to identify the stars around Phoebe because of the declination zones but it is a well known catalogue used before 1910. The catalogs published before 1920 usually have the BD number in the catalog.

2) **SD: I/119**

The Southern Durchmusterung (SD, Schoenfeld [1886], Becker [1949], Schmidt [1967]) is a catalog which covers the declination zones from -02 to -23 degrees. It is completed as an extension to BD. The SD magnitude estimates extend to 9.9 mag with all fainter stars

assigned a magnitude of 10. As the same to the BD, the SD contains a rather large number of stars fainter than 10.0 mag and even occasionally as faint as 11 mag. Positions are given to the nearest 0.1 sec in right ascension and 0.1 arcmin in declination as that in the BD. Positions are for Equator 1855 and no proper motion is provided. It has not been used to calibrate Phoebe.

3) CPD: I/108

The "Cape Photographic Durchmusterung" (CPD, Gill and Kapteyn [1895-1900]) is a photographic survey of southern stars in the declination with range -18 degree to -90 degrees, using photographic plates which would provide a permanent record of the sky at the epoch of observation. The summary of the positional uncertainties quoted in the third volume of the published catalog gives $\pm 0.28 \text{sec(R.A.)}$, $\pm 0.044 \text{arcmin (Dec.)}$ for zones -18 to -57 degrees, $\pm 0.157 \text{sec} + 0.0764/\cos(\delta) \text{sec (R.A.)}$, $\pm 0.056 \text{arcmin (Dec.)}$ for zones -58 to -85 degrees, and $\pm 0.157 \text{sec} + 0.0353/\cos(\delta) \text{sec (R.A.)}$, $\pm 0.0127 \text{arcmin (Dec.)}$ for the polar plate where, as explained in the introduction to the third volume, many positions were derived from rectangular coordinates (these are positions reported to 0.1 sec (R.A.) and 0.001 arcmin (Dec.) in the -86 to -89 degree zones in the catalog). Positions are for Equator 1875 and no proper motion is provided. It has been used to identify the reference stars of Phoebe from 1898 to 1902.

4) AGK1 catalog I/310

The Catalogue of 5954 Stars in Declination Zone from -2° to $+1^\circ$. Kortazzi I. [1900]
It has been used to identify the reference stars of Phoebe from 1904 to 1910.

5) AG catalog

Astronomische Gesellschaft Katalog in German (AGK; "Astronomical Society Catalog"), compilation of the positions of all stars brighter than the ninth magnitude, compiled by the Astronomische Gesellschaft of Germany. Friedrich W.A. Argelander, founder of the society, proposed the star catalog in 1867, after completing the Bonner Durchmusterung ("Bonn Survey"). The massive project gave each participating observatory responsibility for mapping a specific zone of declination. Many observatories around the world took part in the work. The first version of the Astronomische Gesellschaft Katalog (AGK1) covered the sky north of 18° south declination and was completely published in 1912. A second

(AGK2), based on photographs rather than direct observations, was begun in 1924 and published in 1951-58. A third catalog (AGK3) I/61B included the stars' proper motions and became available in 1975.

6) Abbadia Catalog between $+5^{\circ}15'$ and $-3^{\circ}15'$ (Hendaye [1914]) I/65

The catalog contains the mean positions of 13532 stars observed at the Abbadia Observatory (near Hendaye, France), from observations made between 1906 and 1912 with the meridian circle of the observatory. The observations were made mainly for the reduction of the Alger zone of the Astrographic Catalog. The accuracy of this catalog is not well known but a similar catalog present latter has been given the accuracy. Abbadia catalog has been used to identify the reference stars of Phoebe during the year 1922.

7) Abbadia Catalog of 14263 Stars, $+16^{\circ}$ to $+24^{\circ}$ (Hendaye [1915]) I/57

This catalog contains meridian circle observations of 14192 reference stars in the Paris Observatory zone of the Astrographic Catalog, $+16deg$ to $+24deg$, made from 1899 to 1906. The positions have been reduced to 1900.0 on the basis of Newcomb's constants. The probable errors for most stars range from $0.0093s$ to $0.0161s$ in right ascension and from $0.096''$ to $0.162''$ in declination, depending on the number of observations.

8) AC2000 Catalog I/247

The Carte du Ciel and the Astrographic Catalogue (or Astrographic Chart) were distinct but connected components of a massive international astronomical project. This project was started over 100 years ago, and the positions that have been derived from the AC data are being used, in combination with modern epoch positions, to determine accurate proper motions.

The United States Naval Observatory has completed the reductions of the Astrographic Catalogue data (AC) to a consistent system. The resulting catalog, called AC 2000, contains 4,621,836 stars covering the entire sky, at an average epoch of 1907. The positions are on the Hipparcos reference frame (J2000.0) at the epochs of observation. Twenty observatories from around the world participated in exposing and measuring more than 22,000 (glass) photographic plates in an enormous observing program extending over several decades. It has been used to calibrate Phoebe from year 1940 to 1969.

9) AGK3R mean positions and proper motions (Corbin, [1978]) I/72

Table 2.1: Participating observatories and number of stars measured in the context of the Astrographic Catalogue

Observatory (Zone)	Declination		Epoch	No. of stars
	From	To		
Greenwich	+90°	+65°	1892-1905	179 000
Vatican	+64°	+55°	1895-1922	256 000
Catania	+54°	+47°	1894-1932	163 000
Helsingfors	+46°	+40°	1892-1910	159 000
Potsdam	+39°	+32°	1893-1900	108 000
Hyderabad north	+39°	+36°	1928-1938	149 000
Uccle	+35°	+34°	1939-1950	117 000
Oxford 2	+33°	+32°	1930-1936	117 000
Oxford 1	+31°	+25°	1892-1910	277 000
Paris	+24°	+18°	1891-1927	253 000
Bordeaux	+17°	+11°	1893-1925	224 000
Toulouse	+10°	+05°	1893-1935	270 000
Algiers	+04°	-02°	1891-1911	200 000
San Fernando	-03°	-09°	1891-1917	225 000
Tacubaya	-10°	-16°	1900-1939	312 000
Hyderabad south	-17°	-23°	1914-1929	293 000
Cordoba	-24°	-31°	1909-1914	309 000
Perth	-32°	-37°	1902-1919	229 000
Perth/Edinburgh	-38°	-40°	1903-1914	139 000
Cape Town	-41°	-51°	1897-1912	540 000
Sydney	-52°	-64°	1892-1948	430 000
Melbourne	-65°	-90°	1892-1940	218 000

The AGK3R and SRS are lists of reference stars containing, respectively, 21,499 stars in the northern hemisphere and 20,500 stars in the southern hemisphere. This paper presents computations of proper motions of these two groups of stars that will permit the use of the observed positions away from the epochs of observation. The Positions and proper motions are in the FK4 reference frame for Equator B1950.0 and for the epochs referred. The accuracy are $1ms$ for right ascension and $10mas$ for declination. It has been used from 1975 to 1981.

10) Yale Zone Catalogues Integrated (Yale Univ. [1939-1983]) I/141

The coordinates on the B1950 system are given for the epoch, Ep. Proper motions must be applied to change the epoch. The accuracy are $1ms$ for right ascension and $10mas$ for declination. It has been used during the year 1952.

2.4.2 Modern catalogs

We know the accuracy of modern catalogues and it will be necessary to link them to the old ones.

1) The Hipparcos and Tycho Catalogues (ESA [1997a], [1997b]) I/239

The Hipparcos catalog serves as primary realization of the ICRF in visible wavelengths, but the orientation of its axes relative to the axes of the ICRF, which is an essential element to enable it to carry out a fundamental reference system, should be regularly measured. To monitor the most accurate proper motions of stars in the Hipparcos catalog, photographic and CCD astrometric instruments are used in the United States, Russia and Europe.

The Hipparcos and Tycho Catalogues contains a large quantity of very high quality astrometric and photometric data. In addition there are associated annexes featuring variability and double/multiple star data, and solar system astrometric and photometric measurements.

Median astrometric standard errors (in position, parallax, and annual proper motion) are in the range $0.7 - 0.9$ milliarcsec (mas) for stars brighter than 9 mag at the catalogue epoch J1991.25. The catalogue is a materialisation of the ICRS reference system, coinciding with its principal axes at the level of $\pm 0.6mas$, and with proper motions consistent with an

inertial system at the level of $\pm 0.25 \text{ mas/yr}$. The 118218 constituent stars provide a mean sky density of 3 stars/deg^2 .

2) The Tycho-2 Catalog (Hog+ [2000]) I/259

The Tycho-2 Catalog is an astrometric reference catalog containing positions and proper motions as well as two-color photometric data for the 2.5 million brightest stars in the sky. The Tycho-2 positions and magnitudes are based on precisely the same observations as the original Tycho Catalog collected by the star mapper of the ESA Hipparcos satellite, but Tycho-2 is much bigger and slightly more precise, owing to a more advanced reduction technique. Components of double stars with separations down to 0.8 arcsec are included. Proper motions with precise to about 2.5 mas/year are given as those derived from a comparison with the Astrographic Catalog and 143 other ground-based astrometric catalogs, all reduced to the Hipparcos celestial coordinate system.

The principal characteristics of the Tycho-2 Catalog are summarized below. By means of proper motions the positions are transformed to the year J2000.0, the epoch of the catalog. The median values of internal standard errors are given.

Table 2.2: The principal characteristics of the Tycho-2 catalog

Mean satellite observation epoch	$\sim J1991.5$
Epoch of the Tycho-2 Catalogue	$J2000.0$
Reference system	ICRS
coincidence with ICRS (1)	$\pm 0.6 \text{ mas}$
deviation from inertial (1)	$\pm 0.25 \text{ mas/yr}$
Number of entries	2539913
Astrometric standard errors (2)	
VT < 9 mag	7 mas
all stars positions	60 mas
all stars proper motions	2.5 mas/yr
Photometric std. errors (3) on VT	
VT < 9 mag	0.013 mag
all stars	0.10 mag
Star density	
b= 0 deg	150 stars/sq.deg.
b= ± 30 deg	50 stars/sq.deg.
b= ± 90 deg	25 stars/sq.deg.
Completeness to 90 per cent	V ~ 11.5 mag
Completeness to 99 per cent	V ~ 11.0 mag
Number of Tycho observations	$\sim 300 \cdot 10^6$

Phoebe is a faint satellite with magnitude about 16.5. Recently the observations are usually done with the telescope whose focus is more than 1 meter, and have a small field of view that don't contain enough reference stars in catalog TYCHO. So we use other catalogs.

3) UCAC Catalog

UCAC2 Catalogue (Zacharias+ [2004]) I/289

The UCAC2 is a high density, highly accurate, astrometric catalog of 48,330,571 stars covering the sky from -90 to $+40$ degrees in declination and going up to $+52$ degrees in some areas. The northern limit is a function of right ascension. Proper motions and photometry are provided for all stars. Positions and proper motions are on the ICRS (International Celestial Reference System) and given at the epoch J2000.0.

UCAC4 (Zacharias+, [2013]) I/322A

UCAC4 is a compiled, all-sky star catalog. It covers mainly the range from 8 to 16 magnitude in a single bandpass between V and R. Positional errors are about 15 to 20 mas for the stars in the range from 10 to 14 mag. Proper motions have been derived for most of the about 113 million stars utilizing about 140 other star catalogs with significant epoch difference to the UCAC CCD observations. All bright stars not observed with the astrophotography have been added to UCAC4 from a set of Hipparcos and Tycho-2 stars.

The proper motions of bright stars are based on about 140 catalogs, including Hipparcos and Tycho, as well as all catalogs used for the Tycho-2 proper motion construction. Proper motions of faint stars are based on re-reductions of early epoch SPM data (-90 to about -20 deg Dec) and NPM (PMM scans of early epoch blue plates) for the remainder of the sky. These early epoch SPM data have also been combined with the late epoch SPM data to arrive at proper motions partly independent from UCAC4. No Schmidt plate data are used in UCAC4. The unpublished plate measure data obtained by StarScan from the AGK2, the Hamburg Zone Astrograph, the USNO Black Birch Astrograph, and the Lick Astrograph have contributed to considerable improvement in proper motions for stars mainly in the range from 10 to 14 mag (down to the UCAC limit for Lick data). However, these data do not cover all sky.

4) GAIA.

The European mission GAIA based on optical interferometric observations of a million star bursts. In the final Gaia catalogue, brighter objects (3-13 magnitude) will have positions measured to a precision of 5 microarcseconds, Figure 2-1 shows the accuracy for the position. The results will be expected in the best case around 2020.

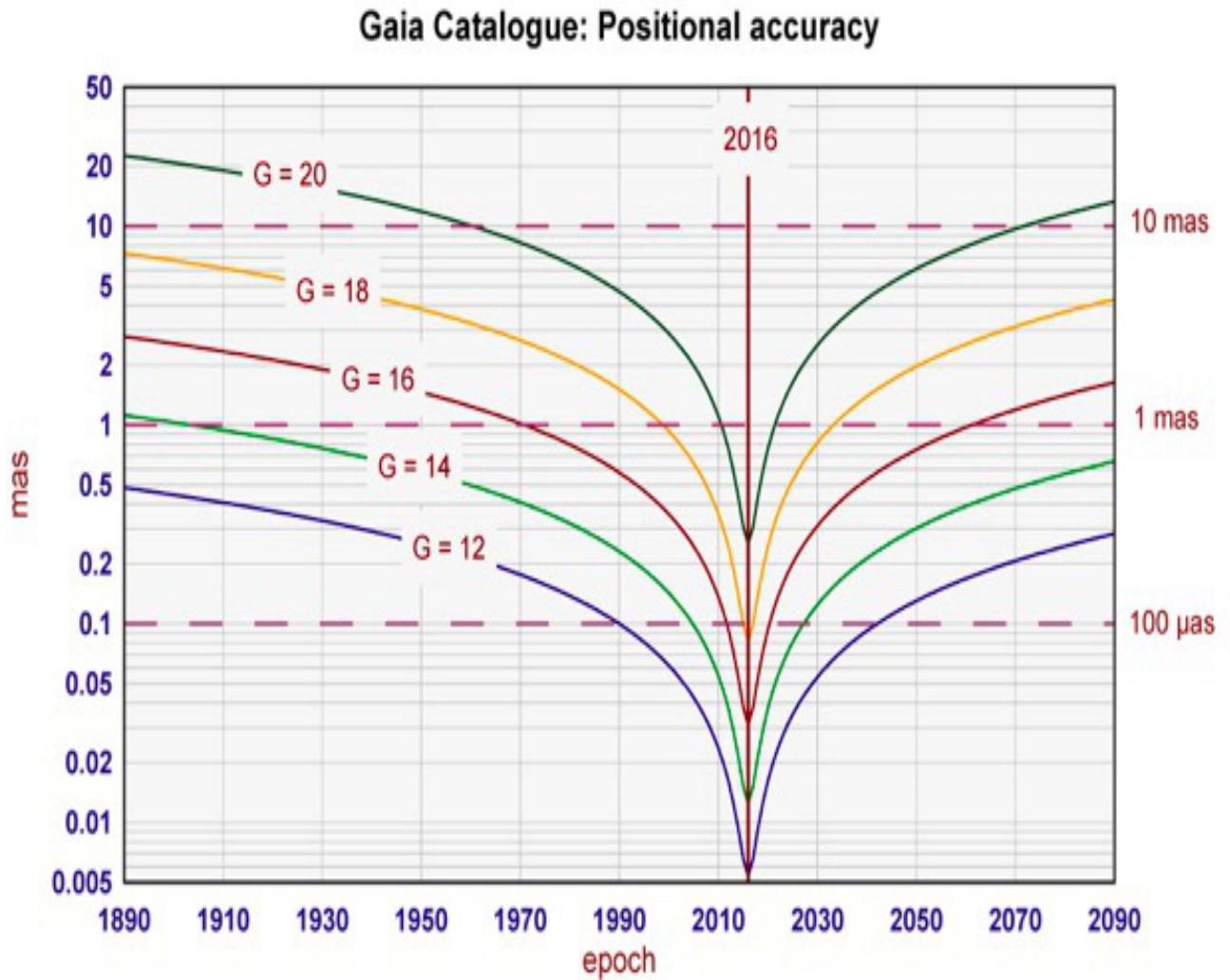


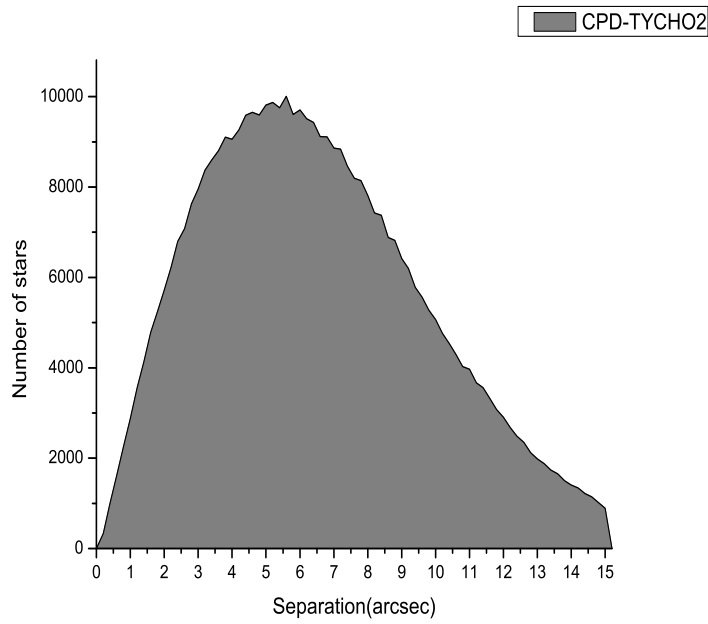
Figure 2-1: Gaia catalogue Positional accuracy (F. Mignard, private communication)

These catalogs provide most accurate proper motions; we will use these catalogs to recalculate the positions of reference stars at the date of the observations.

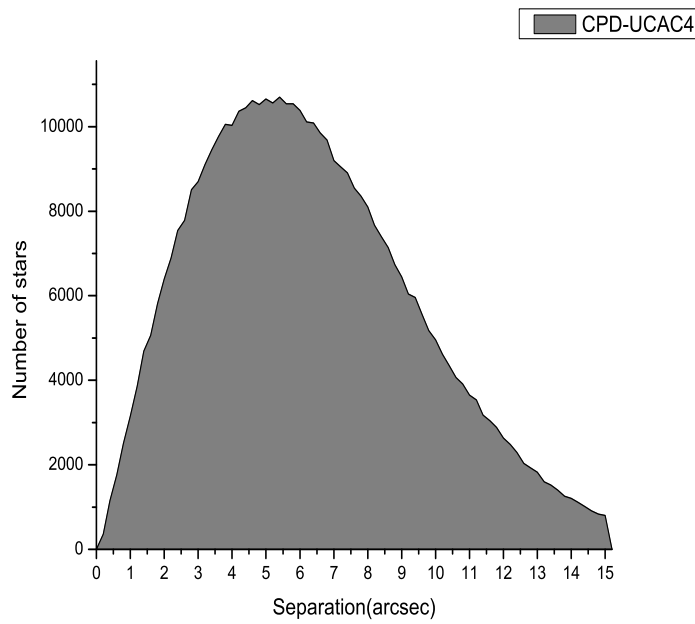
2.4.3 Catalog comparison and the statistics on the used catalogues

The number of reference stars of the old catalogues on photographic plates is small and their positions are inaccurate. As an illustration, figure 2-2 represents the statistics of the difference in angular separation of the stars in old catalogue and modern catalogues used in current reduction (Here angular separation is given by $s = \sqrt{\Delta\alpha^2 \cos^2 \delta + \Delta\delta^2}$ where $\Delta\alpha$ and $\Delta\delta$ are the right ascension and declination difference between older and modern catalogues.). For many stars, the difference in position between the older and recent catalogues is more than 5 arcsec. The positions of the reference stars in the old catalogues represent a source of systematic errors on Phoebe's positions.

Phoebe is far away from its planet, most of the observations are in absolute coordinates. The catalogues used in astrometric reduction cause the systematic error. This error depends on the catalogue used and on the zone on the celestial sphere. We will introduce the correction of the catalog bias in chapter 4. Table 2.3 provides statistics on the catalogues used to reduce the astrometric positions of Phoebe for the observations mentioned in section 5.2.3. The number codes indicate the catalogs used to reduce the photographic observations and the alphabet code are similar to MPC flag that used to reduce the CCD observations.



(a) $CPD - TYCHO2 > 5as$



(b) $CPD - UCAC4 > 5as$

Figure 2-2: The statistics of the difference in angular separations of the stars in old and modern catalogs

Table 2.3: Statistics on the catalogues used to reduce the astrometric positions of Phoebe

Code	Catalogue	Number	Percentage	Time-span
3	CPD I/108	42	1.0 %	1898-1902
4	AGK1 I/310	63	1.5 %	1904-1910
6	Abbadia I/65	5	0.1 %	1922-1922
8	AC2000 I/247	56	1.4 %	1940-1969
5	Yale I/141	7	0.2 %	1952-1952
9	AGK3R I/72	6	0.1 %	1975-1976
a	USNO A1.0	8	0.2 %	2000-2000
b	USNO SA1.0	3	0.1 %	2000-2000
c	USNO A2.0	384	9.3 %	1998-2012
d	USNO SA2.0	12	0.3 %	2001-2003
g	Tycho-2	236	5.7 %	2000-2011
l	ACT	5	0.1 %	2000-2000
o	USNO B1.0	272	6.6 %	2005-2012
r	UCAC2	2392	57.8 %	1996-2015
t	UCAC3-beta	6	0.1 %	2011-2012
u	UCAC3	76	1.8 %	2010-2012
v	NOMAD	95	2.3 %	2008-2009
w	CMC	2	0.0 %	2010-2010
z	GSC(generic)	27	0.7 %	2000-2000
	Unknown	442	10.7 %	1898-2014

This section was published in Desmars, et al. [2013] and we developed here the statistics for old catalogs in details.

2.5 Other corrections for astrometric reduction

Proper motions

The Proper motions (pmRA, pmDE) or (μ_α, μ_δ) are the rates of the change in positions of stars defined in the same reference system as the stars which are highly necessary to measure the coordinates of the reference stars' position at the time of the observation. Because most proper motions are much less than one arcsec per year, most modern catalogs, like UCAC, now express proper motion in terms of milliarcseconds per year (mas/yr). The proper motion of Right Ascension in the catalog AGK3R (Corbin, [1978]) is in terms of microsecond per year (us/yr). The modern catalogs usually give the proper motion in right ascension multiplied by $\cos(\delta)$ instead of the real proper motion in right ascension, noted μ_α^* .

$$\Delta\alpha \cos \delta = \mu_\alpha^* \Delta T$$

$$\Delta\delta = \mu_\delta \Delta T$$

The earliest observations of Phoebe are in the year 1898, about 115 years before. The modern catalog used to calibrate these observations should both have precise positions and precise proper motions.

Aberration

The aberration of light is a phenomenon which produces a motion of light direction caused by a moving observer compared to a stable observer at the same place and the same time. Because there are always the Earth's revolution, rotation and other reasons, the aberration exists always when we observe the celestial objects from the Earth. The change in angle depends on observer's speed and the direction of motion. The correction of aberration for the observer on the Earth is close to v/c where c is the speed of light and v is the velocity of the observer to the target object.

Atmospheric refraction

Atmospheric refraction is the deviation of light or other electromagnetic wave from a straight line as it passes through the atmosphere due to the variation in air density as a function of altitude. It is depend also on the temperature and the atmospheric pressure. Whenever possible, astronomers will schedule their observations around the time of culmination of an object when it is highest in the sky. We will never shoot a star which is not at least 20° or more above the horizon.

Atmospheric refraction of the light from a star is zero in the zenith, less than one arc-minute at 45° apparent altitude, and still only $5.3'$ at 10° altitude; it quickly increases as altitude decreases, reaching $9.9'$ at 5° altitude, $18.4'$ at 2° altitude, and $35.4'$ at the horizon; all values are for 10°C and 101.3 kPa in the visible part of the spectrum (Allen, C.W. [1976]).

For observations with reference stars, we have no need to make the correction of atmospheric refraction except if we have too few stars in the field.

Chapter 3

Search and selection of published observations

3.1 Introduction

Phoebe was discovered by William Henry Pickering on March 17, 1899, on the photographic plates made between 16 and 18 August 1898 by the American astronomer Delisle Stewart at the astronomical observation station Arequipa in Peru, which depended on the Harvard College Observatory. All these plates are conserved in Harvard Plate stacks and to be scanned. Table 3.1 shows the information of the telescope with which the first plate of Phoebe had been taken.

All the observations of Phoebe from 1898 to 1989 are photographic observations, and after 1989 all the observations are CCD observations or space observations. We consider all the published observations included in the Natural Satellite Data Center (NSDC, Arlot & Emelyanov [2009]), the Minor Planet Center (MPC) since 1898 and new CCD observations

Table 3.1: Telescope in Arequipa Observatory with which the first plate of Phoebe had been taken

Observatory Name	Arequipa
Aperture (m)	0.6
Scale (arcsec/mm)	59.57
Telescope	24-inch Bruce Doublet
Field of plates	1 degree

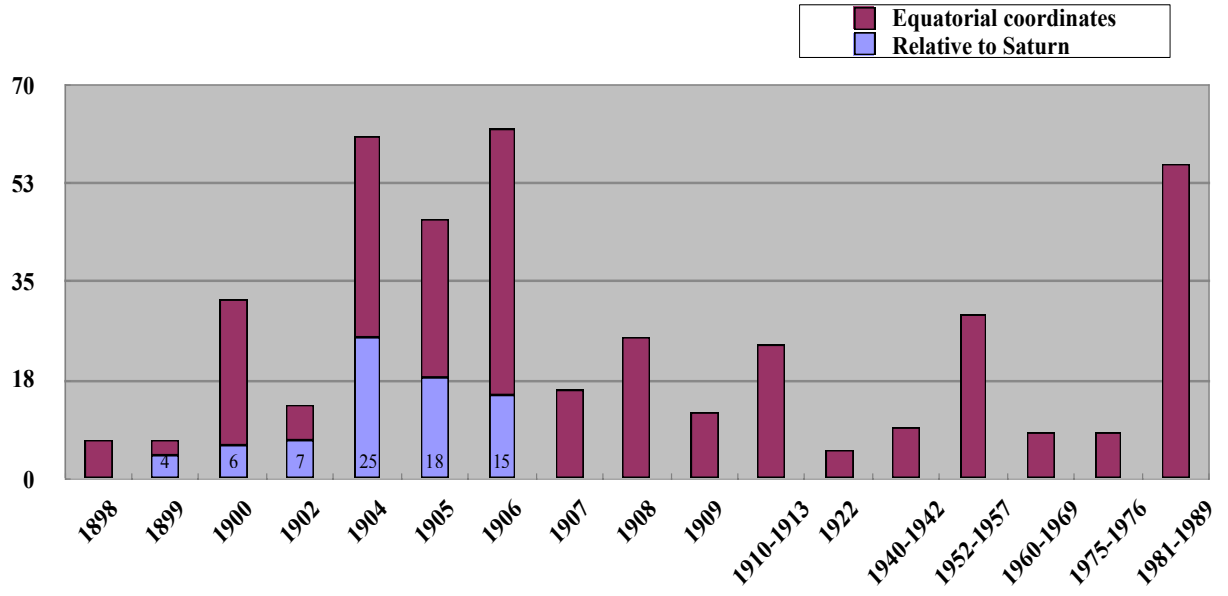


Figure 3-1: The photographic observations of Phoebe

at Pic-du-midi in the year 2013 and 2015 to develop the ephemeris, with 243 photographic observations in 23 articles and other CCD observations. Figure 3-1 shows the numbers of the early photographic observations of Phoebe from 1898 to 1989. Most of the available observations of Phoebe are absolute positions (RA,DEC) derived from a large variety of catalogs, but some of them are relative to Saturn or other satellites. We do not re-reduce these relative observations here but they can be used to improve the orbit of Phoebe. Consequently, the observed absolute positions can be affected by possible significant systematic errors due to the errors of catalogs. We re-reduced these observations with new precise catalogs in next chapter.

In this chapter, our work consists in classifying the old photographic observations of Phoebe to 4 different sets in order to re-reduced them with different methods. In the last section of the chapter we reduce the new CCD observations and give the positions of Phoebe for these observations.

3.2 Old photographic observations

3.2.1 Observatory and telescopes

As presented in Jaccobison [1998], table 3.2 presents all the studied photographic observations of Phoebe in our work, with the year of the observations, the observatory, the telescope used to take the photograph, reference paper, the code of the observatory, the amount of observations during the year of observations and the set number which will be introduced in the following section and indicate the type of the observations in the published papers. The Phoebe's positions of these observations have been published in absolute equatorial coordinates.

Table 3.2: All the studied photographic observations

Year	Observatory	Instrument	Reference	CODE	Obs NO	Set number
1898 -1902	Arequipa	24 in Bruc	Pickering [1908]	800	42	1
1904	Lick	Crossley reflector	Perrine [1904]	662	5/7	2
1905	Lick	Crossley reflector	Albrecht & Smith [1909]	662	11	4
1906-1908	Lick	Crossley reflector	Perrine [1909]	662	10	4
1907	Greenwich	30 in reflector	MNRAS68- [1908]	000	16	1
1908	Greenwich	30 in reflector	MNRAS69- [1909]	000	23	1
1909	Greenwich	30 in reflector	MNRAS70- [1910]	000	12	1
1910	Greenwich	30 in reflector	MNRAS71- [1911]	000	7	1
1912/13	Yerkes	40 in refractor	Barnard [1913]	754	2/12	4
1913	Yerkes	40 in refractor	Barnard [1914]	754	2/5	2
1922	Yerkes	24 in reflector	van Biesbroeck [1922]	754	5	3
1940	Mt. Wilson	100 in reflector	Richmond & Nicholson [1943]	672	1	3
1942	McDonald	82 in reflector	van Biesbroeck [1944]	711	8	3
1952	Cordoba	Normal astrograph	Bobone [1953]	822	7	3
1955	Yerkes	24 in reflector	van Biesbroeck [1957]	754	11	3
1955	McDonald	82 in reflector	van Biesbroeck [1957]	711	3	3
1957	Bloemfontien	ADH telescope	van Biesbroeck [1958]	074	8	3
1960	Flagsta	40 in reflector	Roemer & Lloyd [1966]	689	2	3
1968	Crimean	40 cm astrograph	Chernykh & Chernykh [1971]	094	2	4
1969	Kitt Peak	213 cm reflector	van Biesbroeck et al. [1976]	695	1	3
1969	Catalina	154 cm reflector	van Biesbroeck et al. [1976]	693	3	3
1975/76	McDonald	2.1 m reflector	Mulholland & Shelus [1980]	711	6/8	3
1981	Lowell	0.33 m reflector	Smith&Bowell [1981]	688	8	3
1981	La Silla	40 cm GPO	Debehogne [1981]	809	22	4
1982	La Silla	40 cm GPO	Debehogne [1981]	809	18	4
1989	Bordeaux-Flourac	60 cm reflector	Dourneau G. [1991]	999	5	4

3.2.2 Different sets of observations

1). Known measured coordinates of stars and Phoebe:

The following article Pickering [1908] includes the first observations of Phoebe. It contains the details of the plates and observations. The author of this article reduced the

spherical positions-Right Ascension and Declination of the natural satellite from its relative positions to reference stars on the photographic plates. He first gave the data relating to plates in details which present in figure 3-2 for the first two rows. The table contains at first two columns the series number of the plates with which we can search for it now in Harvard Astronomical Plate Collection; following columns are the observation date and time which is at the middle time of the exposition, the exposition time, the coordinate of the plate center, the number of the measurements, the number of the reference stars in the plate and the factor used to reduce the readings to seconds of arc. The coordinates for assumed centers which used to calculate the standard coordinates are considered to be the same in order to simplify the calculations.

TABLE I.
DATA RELATING TO PLATES.

Plate.		Date.				Exp.	Sid. T.		G.M.T.	J.D. 2410000+	Actual Centre.			Assumed Centre.			No. of Meas.	No. of Stars.	Reduction to Arc.		
A.	D.	y.	m.	d.	m.	h.	m.	h.	m.		R.A. 1875.	Dec. 1875.	R.A. 1875.	Dec. 1875.							
											h.	m.	o	''	h.	m.	o	''			
3227	10811	98	8	16	60	17	11	12	16	4518.511	16	15.8	-20	4	16	15.9	-19	59	16	10	2.98,2.98
3228	10815	98	8	16	120	19	13	14	18	4518.596	16	15.8	-20	4	α	α	α	α	16	10	2.99,2.98

Figure 3-2: Original table for Date

Figure 3-3 presents the coordinates of the reference stars. The author used the assumed center to calculate the computed coordinates of Phoebe. The measured coordinates were introduced in the following. These 10 stars were used as reference stars in 7 plates. There are 42 plates mentioned in the article and these plates are in Harvard Astronomical Plate Collection waiting to being scanned.

Figure 3-4 shows that at that time, Phoebe was measured at the beginning, in the middle, and at the end of each series. Other satellites of Saturn were also measured, such as Titan, Hyperion, Iapetus... We can also get relative positions between Phoebe and other satellites of Saturn from this table. The readings of the left, lower, right, and upper sides of the squares 3-5 are designated as A, B, C and D.

At the end, with 42 plates we have 42 positions of Phoebe. In figure 3-6 we present 7 positions which have been calculated with 10 reference stars mentioned before. The coordinates are in equatorial reference system at epoch 1875.0. The coordinates x and y are the standard coordinates (tangential coordinates) on the plates .

TABLE II.
STANDARD STARS.

Plate A.	Star.	DM.	Magn.	R.A. 1875.			Dec. 1875.		Computed coörd.		Measured coörd.		
				<i>h.</i>	<i>m.</i>	<i>s.</i>	<i>°</i>	<i>'</i>	<i>''</i>	<i>X</i>	<i>Y</i>	<i>x</i>	<i>y</i>
3227	1	-20° 6322	10.0	16	14	32.9	-20	13	18	-1141.1	- 859.0	-1138.9	- 865.3
	2	-19° 5982	10.0	16	14	40.7	-19	57	0	-1033.5	+ 119.0	-1032.5	+ 123.4
	3	-19° 5983	9.6	16	14	43.2	-19	4	12	-1003.8	+ 3287.3	-1002.0	+ 3293.8
	4	-20° 6325	9.3	16	15	22.4	-20	13	24	- 444.8	- 864.0	- 452.6	- 867.8
	5	-20° 6328	9.4	16	16	13.9	-20	6	48	+ 279.8	- 468.0	+ 283.2	- 468.6
	6	-20° 6331	9.4	16	18	3.6	-20	1	36	+1826.5	- 159.0	+1824.9	- 150.4
	7	-19° 5985	8.4	16	18	8.2	-19	32	48	+1897.1	+1569.0	+1897.8	+1568.9
	8	-20° 6332	9.8	16	18	12.4	-20	11	12	+1948.5	- 735.0	+1945.2	- 740.5
	9	-20° 6333	10.0	16	18	46.7	-20	9	48	+2431.4	- 653.0	+2437.2	- 649.0
	10	-19° 5986	9.8	16	21	21.1	-19	1	18	+4639.5	+3444.3	+4637.0	+3437.3
3228	1	-20° 6322	10.0	16	14	32.9	-20	13	18	-1141.1	- 859.0	-1137.7	- 863.4
	2	-19° 5982	10.0	16	14	40.7	-19	57	0	-1033.5	+ 119.0	-1033.2	+ 124.7

Figure 3-3: Coordinates of the reference stars

TABLE III.
ORIGINAL MEASURES.

Plate A	Star.	Estimates.		Scale Readings.				Uncorrected.		Plate A	Star.	Estimates.		Scale Readings.				Uncorrected.	
		<i>x</i>	<i>y</i>	A	B	C	D	<i>x</i>	<i>y</i>			<i>x</i>	<i>y</i>	A	B	C	D	<i>x</i>	<i>y</i>
3227	P	30.7	45.5	2.5	4.9	7.6	5.1	1815.2	2699.4	3228	4	14.9	9.6	0.5	5.6	9.6	4.5	867.2	543.3
	2	2.2	27.6	8.1	6.0	2.0	4.1	101.8	1625.7		P	33.7	44.1	2.8	0.5	7.2	9.5	1992.2	2613.2
	1	0.4	11.0	5.5	10.2	4.5	10.1	5.0	630.3		6	53.1	21.6	3.9	7.0	1.1	3.0	3156.7	1271.9
	3	2.6	80.7	4.2	7.9	5.9	2.1	125.1	4817.3		5	27.3	16.3	7.5	3.0	2.6	7.1	1605.3	947.3
	T	4.8	5.4	1.9	4.0	8.1	6.1	258.5	3053.7		8	55.2	11.8	3.2	8.0	1.9	2.1	3281.2	677.6
	H	10.6	52.3	4.3	2.9	5.9	7.2	604.8	3107.2		9	62.4	13.4	5.9	3.5	4.1	6.5	3774.6	771.1
	I	14.3	51.4	7.1	3.9	2.9	6.2	827.5	3053.1		7	54.3	50.6	6.9	5.5	3.1	4.5	3228.6	3003.0
	P	30.8	45.5	2.4	5.0	7.8	5.1	1816.1	2699.7		10	100.1	82.1	3.5	0.5	1.5	9.6	5979.1	4892.9
	4	11.9	10.9	0.3	9.9	9.7	0.1	688.0	629.2		J	17.3	49.9	6.9	9.4	3.1	0.8	1008.6	2965.6
	5	24.3	17.7	7.1	7.1	3.0	2.9	1427.8	1032.5		P	33.7	44.1	3.0	0.4	7.1	9.7	1992.3	2612.3
3228	6	50.1	23.1	8.5	1.0	1.6	9.1	2979.4	1355.9	3230	P	31.2	46.6	7.9	6.0	2.1	4.2	1842.7	2765.4
	8	52.2	13.2	8.1	2.0	2.0	8.0	3101.8	762.1		1	2.3	28.6	8.0	7.1	2.1	3.0	102.4	1692.2
	7	51.4	51.9	6.9	9.5	3.1	0.5	3048.7	3096.8		2	2.0	81.9	10.1	9.9	10.1	0.2	90.0	4888.9
	9	60.4	14.7	3.5	7.6	4.5	2.5	3597.0	855.2		T	4.6	52.6	3.0	6.8	7.1	3.3	252.3	3130.4
	10	97.2	83.4	8.0	3.9	2.1	6.2	5802.4	4973.1		I	15.1	52.1	9.9	0.5	0.2	9.5	871.0	3093.2
	P	30.7	45.5	2.5	4.9	7.6	5.1	1815.2	2699.4		3	0.6	12.1	3.1	0.9	7.0	9.2	11.7	693.3
	P	33.7	44.1	2.9	0.5	7.1	9.6	1992.6	2612.9		4	12.3	12.3	7.9	1.9	2.1	8.2	702.7	701.2

Figure 3-4: Original measures



Figure 3-5: Measuring squares

TABLE IX.
POSITIONS OF PHOEBE.

Plate A.	Date.			J. D. 24100+	x	y	R. A. 1875.0.			Declination 1875.0.		
	y.	m.	d.				h.	m.	s.	°	'	"
3227	1898	8	16	4518.511	+ 672.1	+1186.6	16	16	41.6	-19	38	12
3228	"	"	"	4518.596	+ 670.8	+1184.8	16	16	41.5	19	39	14
3230	"	"	17	4519.539	- 8027.5	+1349.1	16	16	54.0	19	39	35
3233	"	"	18	4520.550	+ 740.7	+1140.0	16	16	46.5	19	40	0
3304	"	9	15	4548.531	+1550.4	- 22.2	16	20	31.9	19	55	20
3308	"	"	16	4549.528	+1742.2	- 66.3	16	20	45.5	19	56	3
3312	"	"	17	4550.528	+1934.7	- 109.1	16	20	59.2	19	56	38

Figure 3-6: Positions of Phoebe

2). Known reference stars' catalog coordinates

In some older publications such as Perrine [1904] shown in figure 3-7, the authors gave the spherical positions of the natural satellite and the spherical positions of reference stars on the photographic plates shown in figure 3-8.

MEAN PLACES OF COMPARISON STARS FOR 1904.0

*	α	δ	Authority.
1	21 ^h 7 ^m 33.33	-17° 34' 22.4	Washington A.G. Zones
2	8 24.67	-17 57 38.5	" "
3	8 46.97	-17 40 4.7	" "
4	9 23.62	-17 36 57.7	" "
5	9 36.11	-17 58 17.7	" "
6	21 9 44.42	-17 44 33.6	" "

Figure 3-7: Coordinate of reference stars

POSITIONS OF PHOEBE FOR 1904.0

Plate No.	Date 1904	P.S.T.	α		δ		Parallax.	
			α	δ	α	δ		
1345	November 6	7 ^h 26 ^m 5	21 ^h 8 ^m 22.734	-17° 40' 46.86	+0.016	+0.72		
1346	" 7	7 32	8 29.862	-17 40 13.16	.018	.72		
1347	" 8	7 23	8 37.168	-17 39 35.58	.017	.72		
1350	" 9	7 30	8 44.929	-17 38 56.66	.019	.71		
1351	" 10	7 30	21 8 53.161	-17 38 16.00	+0.020	+0.71		
SATURN								
1345	" 6	7 26.5	21 9 50.067	-17 33 32.76	+0.016	+0.72		

Figure 3-8: Positions of Phoebe

3). Unknown reference stars' catalog positions but known the catalog used to reduce the observations .

Most of the articles do not mention the information of the reference stars, but introduced the catalog used at that time.

4). Unknown information about reference stars and catalog.

For some published observations as notices, the author just published the observation site, time and the corresponding Phoebe's positions, no other information about the observations.

3.3 New observations

To generate a precise ephemeris, we need the observations completely disturbed over time. That is why we need both the old and the new observations at present.

We observed Phoebe for seven nights and got 321 observations of Phoebe in April 2013 and 2015 at the Observatory Pic-du-Midi with one-meter telescope. The observers were F. COLAS and me in 2013, F.COLAS, Q.F.ZHANG, A. VIENNE and E. Saquet in 2015. The image of Phoebe was taken every two minutes in 2013 and every one minute in 2015. The author reduced the observations with the software *ASTROMETRICA* [Raab]. At first we reduced the observation in 2013 and we found that most of the differences between the ephemeris and the observed positions reduced with catalog UCAC3 or UCAC4 in Declination are bigger than 0.5 arcsecond. That is because Phoebe was in south sky at that time and the catalog UCAC4 and UCAC3 have a system error in this area. The comparison of the residuals of O-C between catalog UCAC2 and UCAC3 will be shown in Chapter 6 in Figure 6-1 and Figure 6-2. The observations reduced with catalog UCAC2 presents a good adaption with the ephemeris. Even though the CCD images of the observations on the night of April 20, 2013 didn't contain many catalog reference stars (only 4 or 5 catalog stars identified), the re-calibrated positions still have small residuals of O-C. Therefore, we used catalog UCAC2 to reduce the observations in 2015 and all the 321 new positions of Phoebe reduced with UCAC 2 present in Table 3.3. The observation time was UTC, not correct the light time, the coordinates were in ICRS reference system. The value of O-C residuals of these observations is indicated in the figure 3-9 with the computed positions calculated from the PH12 (developed by Desmars [2013]). We had good results for the first two nights, with the residuals about 0.1 arcsecond. The sequence numbers for these two nights are from 1 to 104. From the residuals in right ascension we can see that there is a leap for the third night, with the sequence number from 105 to 156. This means that there

are system errors may be caused by the catalog bias or the bad seeing. We calibrated these observations with another catalog USNO-B1.0, and the comparisons will be presented in chapter 6.

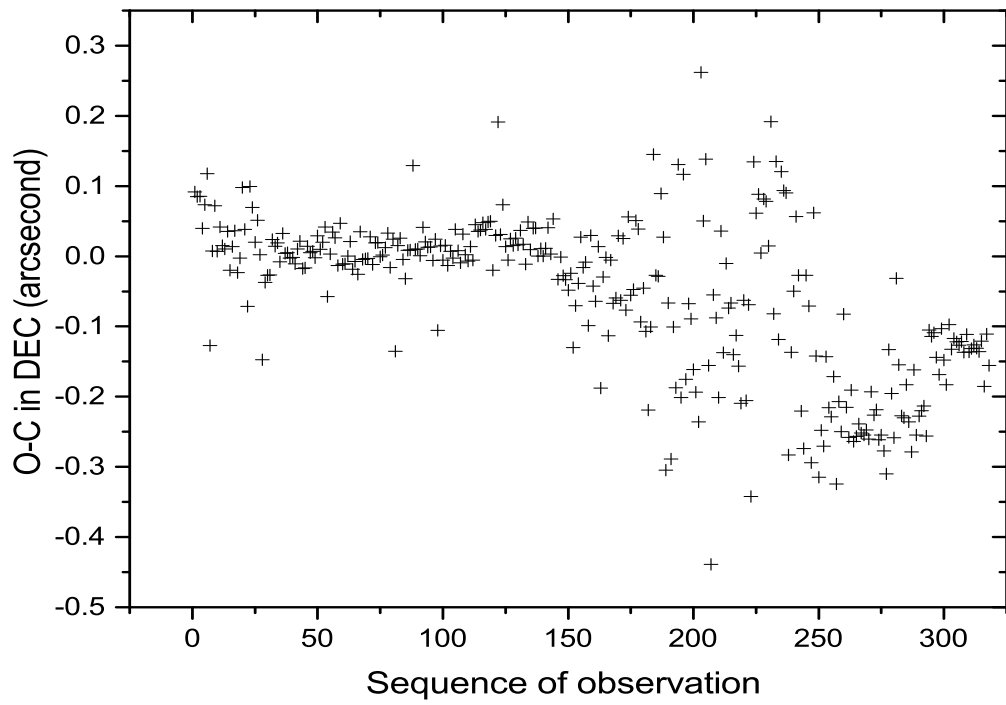
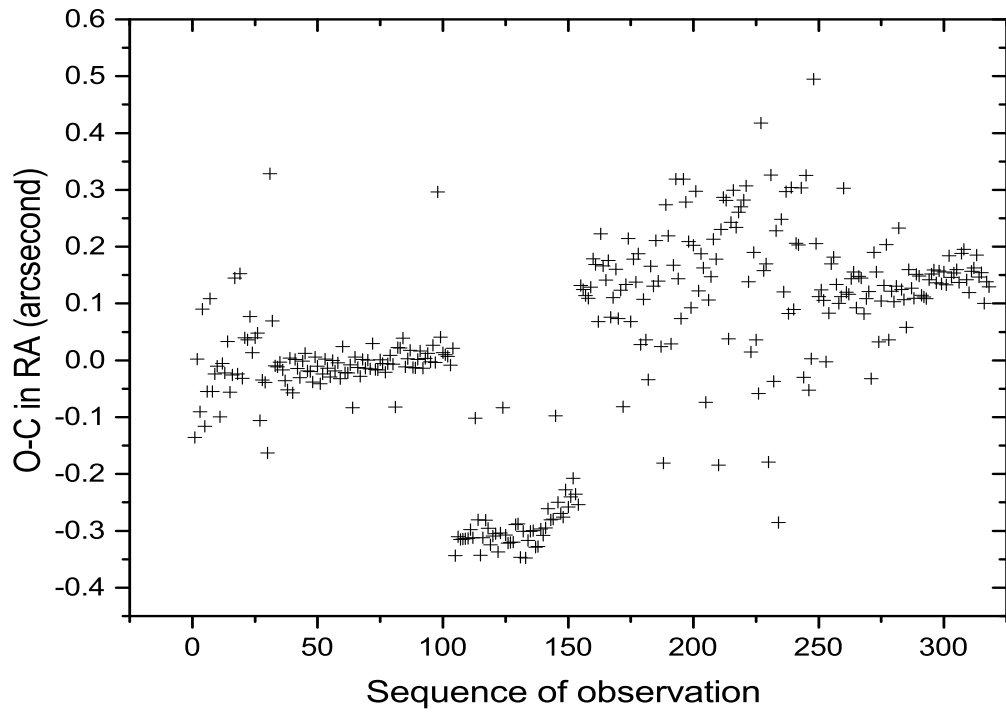


Figure 3-9: O-C for the observations at Pic du midi

Table 3.3: Positions of Phoebe observed at Pic du midi

UTC			RA			DEC			Catalog	No.Observatory
2013	04	21.04918	14	29	47.744	-11	57	39.64	UCAC2	586
2013	04	21.05054	14	29	47.728	-11	57	39.52	UCAC2	586
2013	04	21.05161	14	29	47.702	-11	57	39.42	UCAC2	586
2013	04	21.05253	14	29	47.697	-11	57	39.38	UCAC2	586
2013	04	21.05303	14	29	47.674	-11	57	39.30	UCAC2	586
2013	04	21.05352	14	29	47.669	-11	57	39.21	UCAC2	586
2013	04	21.05400	14	29	47.671	-11	57	39.41	UCAC2	586
2013	04	21.05449	14	29	47.651	-11	57	39.23	UCAC2	586
2013	04	21.05498	14	29	47.644	-11	57	39.12	UCAC2	586
2013	04	21.05546	14	29	47.636	-11	57	39.14	UCAC2	586
2013	04	21.05595	14	29	47.621	-11	57	39.06	UCAC2	586
2013	04	21.05645	14	29	47.618	-11	57	39.04	UCAC2	586
2013	04	21.05693	14	29	47.608	-11	57	39.00	UCAC2	586
2013	04	21.05988	14	29	47.557	-11	57	38.70	UCAC2	586
2013	04	21.06037	14	29	47.542	-11	57	38.71	UCAC2	586
2013	04	21.06226	14	29	47.509	-11	57	38.50	UCAC2	586
2013	04	21.06363	14	29	47.495	-11	57	38.35	UCAC2	586
2013	04	21.06502	14	29	47.458	-11	57	38.28	UCAC2	586
2013	04	21.06641	14	29	47.444	-11	57	38.13	UCAC2	586
2013	04	21.06780	14	29	47.406	-11	57	37.90	UCAC2	586
2013	04	21.06919	14	29	47.385	-11	57	37.83	UCAC2	586
2013	04	21.07058	14	29	47.359	-11	57	37.81	UCAC2	586
2013	04	21.07197	14	29	47.336	-11	57	37.51	UCAC2	586
2013	04	21.07336	14	29	47.306	-11	57	37.41	UCAC2	586
2013	04	21.07475	14	29	47.282	-11	57	37.33	UCAC2	586
2013	04	21.07613	14	29	47.257	-11	57	37.17	UCAC2	586
2013	04	21.07752	14	29	47.221	-11	57	37.09	UCAC2	586
2013	04	21.07891	14	29	47.200	-11	57	37.11	UCAC2	586
2013	04	21.08030	14	29	47.174	-11	57	36.87	UCAC2	586
2013	04	21.08169	14	29	47.140	-11	57	36.73	UCAC2	586
2013	04	21.08308	14	29	47.147	-11	57	36.60	UCAC2	586
2013	04	21.08447	14	29	47.104	-11	57	36.42	UCAC2	586
2013	04	21.08586	14	29	47.073	-11	57	36.30	UCAC2	586
2013	04	22.93351	14	29	13.304	-11	54	44.32	UCAC2	586
2013	04	22.93488	14	29	13.279	-11	54	44.22	UCAC2	586
2013	04	22.93628	14	29	13.252	-11	54	44.05	UCAC2	586
2013	04	22.93766	14	29	13.225	-11	54	43.95	UCAC2	586
2013	04	22.93905	14	29	13.198	-11	54	43.82	UCAC2	586
2013	04	22.94043	14	29	13.176	-11	54	43.70	UCAC2	586
2013	04	22.94182	14	29	13.146	-11	54	43.57	UCAC2	586
2013	04	22.94321	14	29	13.124	-11	54	43.45	UCAC2	586
2013	04	22.94460	14	29	13.097	-11	54	43.30	UCAC2	586
2013	04	22.94599	14	29	13.070	-11	54	43.16	UCAC2	586
2013	04	22.94738	14	29	13.046	-11	54	43.07	UCAC2	586
2013	04	22.94877	14	29	13.021	-11	54	42.94	UCAC2	586
2013	04	22.95016	14	29	12.993	-11	54	42.78	UCAC2	586
2013	04	22.95155	14	29	12.967	-11	54	42.66	UCAC2	586
2013	04	22.95293	14	29	12.940	-11	54	42.53	UCAC2	586
2013	04	22.95432	14	29	12.917	-11	54	42.41	UCAC2	586
2013	04	22.95571	14	29	12.890	-11	54	42.25	UCAC2	586
2013	04	22.95710	14	29	12.862	-11	54	42.14	UCAC2	586
2013	04	22.95850	14	29	12.837	-11	54	42.00	UCAC2	586
2013	04	22.95988	14	29	12.813	-11	54	41.85	UCAC2	586
2013	04	22.96127	14	29	12.786	-11	54	41.82	UCAC2	586
2013	04	22.96266	14	29	12.759	-11	54	41.63	UCAC2	586
2013	04	22.96405	14	29	12.735	-11	54	41.47	UCAC2	586
2013	04	22.96543	14	29	12.708	-11	54	41.35	UCAC2	586

continued on next page

continued from previous page

UTC			RA			DEC			Catalog	No.Observatory
2013	04	22.96682	14	29	12.683	-11	54	41.26	UCAC2	586
2013	04	22.96822	14	29	12.655	-11	54	41.07	UCAC2	586
2013	04	22.96960	14	29	12.633	-11	54	41.00	UCAC2	586
2013	04	22.97099	14	29	12.604	-11	54	40.87	UCAC2	586
2013	04	22.97238	14	29	12.578	-11	54	40.73	UCAC2	586
2013	04	22.97377	14	29	12.553	-11	54	40.58	UCAC2	586
2013	04	22.97516	14	29	12.522	-11	54	40.49	UCAC2	586
2013	04	22.97655	14	29	12.502	-11	54	40.35	UCAC2	586
2013	04	22.97793	14	29	12.475	-11	54	40.24	UCAC2	586
2013	04	22.97932	14	29	12.448	-11	54	40.05	UCAC2	586
2013	04	22.98071	14	29	12.424	-11	54	39.96	UCAC2	586
2013	04	22.98210	14	29	12.397	-11	54	39.83	UCAC2	586
2013	04	22.98349	14	29	12.372	-11	54	39.70	UCAC2	586
2013	04	22.98488	14	29	12.345	-11	54	39.54	UCAC2	586
2013	04	22.98627	14	29	12.322	-11	54	39.45	UCAC2	586
2013	04	22.98766	14	29	12.293	-11	54	39.29	UCAC2	586
2013	04	22.98905	14	29	12.267	-11	54	39.16	UCAC2	586
2013	04	22.99045	14	29	12.242	-11	54	39.05	UCAC2	586
2013	04	22.99182	14	29	12.216	-11	54	38.92	UCAC2	586
2013	04	22.99321	14	29	12.189	-11	54	38.78	UCAC2	586
2013	04	22.99460	14	29	12.164	-11	54	38.63	UCAC2	586
2013	04	22.99599	14	29	12.139	-11	54	38.55	UCAC2	586
2013	04	22.99738	14	29	12.112	-11	54	38.38	UCAC2	586
2013	04	22.99877	14	29	12.081	-11	54	38.41	UCAC2	586
2013	04	23.00016	14	29	12.062	-11	54	38.13	UCAC2	586
2013	04	23.00155	14	29	12.036	-11	54	37.99	UCAC2	586
2013	04	23.00295	14	29	12.011	-11	54	37.89	UCAC2	586
2013	04	23.00432	14	29	11.982	-11	54	37.79	UCAC2	586
2013	04	23.00571	14	29	11.957	-11	54	37.62	UCAC2	586
2013	04	23.00710	14	29	11.932	-11	54	37.49	UCAC2	586
2013	04	23.00849	14	29	11.904	-11	54	37.24	UCAC2	586
2013	04	23.00988	14	29	11.878	-11	54	37.23	UCAC2	586
2013	04	23.01127	14	29	11.853	-11	54	37.10	UCAC2	586
2013	04	23.01266	14	29	11.828	-11	54	36.98	UCAC2	586
2013	04	23.01405	14	29	11.800	-11	54	36.81	UCAC2	586
2013	04	23.01545	14	29	11.775	-11	54	36.70	UCAC2	586
2013	04	23.01682	14	29	11.750	-11	54	36.58	UCAC2	586
2013	04	23.01821	14	29	11.723	-11	54	36.45	UCAC2	586
2013	04	23.01960	14	29	11.699	-11	54	36.34	UCAC2	586
2013	04	23.02099	14	29	11.671	-11	54	36.18	UCAC2	586
2013	04	23.02238	14	29	11.665	-11	54	36.18	UCAC2	586
2013	04	23.02377	14	29	11.622	-11	54	35.93	UCAC2	586
2013	04	23.02517	14	29	11.594	-11	54	35.82	UCAC2	586
2013	04	23.02655	14	29	11.568	-11	54	35.67	UCAC2	586
2013	04	23.02793	14	29	11.542	-11	54	35.57	UCAC2	586
2013	04	23.02932	14	29	11.515	-11	54	35.42	UCAC2	586
2013	04	23.03071	14	29	11.491	-11	54	35.30	UCAC2	586
2013	04	23.95580	14	28	54.437	-11	53	09.17	UCAC2	586
2013	04	23.95718	14	28	54.433	-11	53	08.83	UCAC2	586
2013	04	23.95858	14	28	54.409	-11	53	08.73	UCAC2	586
2013	04	23.95995	14	28	54.383	-11	53	08.62	UCAC2	586
2013	04	23.96134	14	28	54.357	-11	53	08.45	UCAC2	586
2013	04	23.96273	14	28	54.331	-11	53	08.35	UCAC2	586
2013	04	23.96412	14	28	54.305	-11	53	08.23	UCAC2	586
2013	04	23.96551	14	28	54.280	-11	53	08.08	UCAC2	586
2013	04	23.96690	14	28	54.253	-11	53	07.97	UCAC2	586
2013	04	23.96829	14	28	54.241	-11	53	07.79	UCAC2	586
2013	04	23.96968	14	28	54.203	-11	53	07.67	UCAC2	586
2013	04	23.97106	14	28	54.173	-11	53	07.54	UCAC2	586
2013	04	23.97245	14	28	54.149	-11	53	07.40	UCAC2	586

continued on next page

continued from previous page

UTC			RA			DEC			Catalog	No.Observatory
2013	04	23.97384	14	28	54.125	-11	53	07.28	UCAC2	586
2013	04	23.97523	14	28	54.098	-11	53	07.14	UCAC2	586
2013	04	23.97662	14	28	54.070	-11	53	07.01	UCAC2	586
2013	04	23.97801	14	28	54.045	-11	53	06.95	UCAC2	586
2013	04	23.97941	14	28	54.019	-11	53	06.77	UCAC2	586
2013	04	23.98079	14	28	53.991	-11	53	06.48	UCAC2	586
2013	04	23.98219	14	28	53.967	-11	53	06.51	UCAC2	586
2013	04	23.98356	14	28	53.956	-11	53	06.34	UCAC2	586
2013	04	23.98495	14	28	53.915	-11	53	06.27	UCAC2	586
2013	04	23.98634	14	28	53.888	-11	53	06.16	UCAC2	586
2013	04	23.98773	14	28	53.862	-11	53	06.00	UCAC2	586
2013	04	23.98912	14	28	53.836	-11	53	05.88	UCAC2	586
2013	04	23.99051	14	28	53.812	-11	53	05.74	UCAC2	586
2013	04	23.99190	14	28	53.786	-11	53	05.62	UCAC2	586
2013	04	23.99329	14	28	53.756	-11	53	05.47	UCAC2	586
2013	04	23.99468	14	28	53.733	-11	53	05.36	UCAC2	586
2013	04	23.99606	14	28	53.704	-11	53	05.26	UCAC2	586
2013	04	23.99745	14	28	53.680	-11	53	05.07	UCAC2	586
2013	04	23.99884	14	28	53.655	-11	53	04.98	UCAC2	586
2013	04	24.00023	14	28	53.629	-11	53	04.82	UCAC2	586
2013	04	24.00162	14	28	53.601	-11	53	04.69	UCAC2	586
2013	04	24.00301	14	28	53.575	-11	53	04.60	UCAC2	586
2013	04	24.00440	14	28	53.551	-11	53	04.46	UCAC2	586
2013	04	24.00580	14	28	53.524	-11	53	04.34	UCAC2	586
2013	04	24.00718	14	28	53.499	-11	53	04.20	UCAC2	586
2013	04	24.00858	14	28	53.475	-11	53	04.04	UCAC2	586
2013	04	24.00995	14	28	53.448	-11	53	03.95	UCAC2	586
2013	04	24.01134	14	28	53.422	-11	53	03.77	UCAC2	586
2013	04	24.01273	14	28	53.375	-11	53	03.69	UCAC2	586
2013	04	24.01412	14	28	53.382	-11	53	01.64	UCAC2	586
2013	04	24.03336	14	28	53.011	-11	53	01.80	UCAC2	586
2013	04	24.03473	14	28	52.984	-11	53	01.64	UCAC2	586
2013	04	24.03876	14	28	52.908	-11	53	01.29	UCAC2	586
2013	04	24.03925	14	28	52.902	-11	53	01.25	UCAC2	586
2013	04	24.03973	14	28	52.891	-11	53	01.22	UCAC2	586
2013	04	24.04054	14	28	52.877	-11	53	01.12	UCAC2	586
2013	04	24.04103	14	28	52.870	-11	53	01.18	UCAC2	586
2013	04	24.04189	14	28	52.852	-11	53	01.04	UCAC2	586
2013	04	24.04337	14	28	52.823	-11	53	00.87	UCAC2	586
2015	04	04.08254	16	08	59.575	-18	44	40.88	UCAC2	586
2015	04	04.08397	16	08	59.562	-18	44	40.88	UCAC2	586
2015	04	04.08469	16	08	59.555	-18	44	40.85	UCAC2	586
2015	04	04.08602	16	08	59.543	-18	44	40.90	UCAC2	586
2015	04	04.08674	16	08	59.538	-18	44	40.75	UCAC2	586
2015	04	04.08746	16	08	59.535	-18	44	40.80	UCAC2	586
2015	04	04.08818	16	08	59.528	-18	44	40.80	UCAC2	586
2015	04	04.08890	16	08	59.515	-18	44	40.70	UCAC2	586
2015	04	04.08962	16	08	59.519	-18	44	40.88	UCAC2	586
2015	04	04.09033	16	08	59.509	-18	44	40.70	UCAC2	586
2015	04	04.09105	16	08	59.501	-18	44	40.65	UCAC2	586
2015	04	04.09177	16	08	59.497	-18	44	40.74	UCAC2	586
2015	04	04.09249	16	08	59.484	-18	44	40.61	UCAC2	586
2015	04	04.09321	16	08	59.480	-18	44	40.65	UCAC2	586
2015	04	04.09393	16	08	59.477	-18	44	40.62	UCAC2	586
2015	04	04.09464	16	08	59.465	-18	44	40.51	UCAC2	586
2015	04	04.09536	16	08	59.462	-18	44	40.58	UCAC2	586
2015	04	04.09608	16	08	59.442	-18	44	40.47	UCAC2	586
2015	04	04.09680	16	08	59.450	-18	44	40.55	UCAC2	586
2015	04	04.09867	16	08	59.439	-18	44	40.36	UCAC2	586
2015	04	04.09938	16	08	59.423	-18	44	40.45	UCAC2	586

continued on next page

continued from previous page

UTC			RA			DEC			Catalog	No.Observatory
2015	04	04.10010	16	08	59.424	-18	44	40.42	UCAC2	586
2015	04	04.10082	16	08	59.415	-18	44	40.30	UCAC2	586
2015	04	04.10154	16	08	59.412	-18	44	40.29	UCAC2	586
2015	04	04.10226	16	08	59.395	-18	44	40.40	UCAC2	586
2015	04	04.10298	16	08	59.394	-18	44	40.33	UCAC2	586
2015	04	04.10369	16	08	59.383	-18	44	40.37	UCAC2	586
2015	04	04.10441	16	08	59.372	-18	44	40.46	UCAC2	586
2015	04	04.10513	16	08	59.379	-18	44	40.32	UCAC2	586
2015	04	04.10657	16	08	59.364	-18	44	40.03	UCAC2	586
2015	04	04.10729	16	08	59.363	-18	44	40.18	UCAC2	586
2015	04	04.10800	16	08	59.352	-18	44	40.16	UCAC2	586
2015	04	04.10872	16	08	59.338	-18	44	40.02	UCAC2	586
2015	04	04.10944	16	08	59.318	-18	44	40.06	UCAC2	586
2015	04	04.11016	16	08	59.342	-18	44	40.37	UCAC2	586
2015	04	04.11088	16	08	59.332	-18	44	40.11	UCAC2	586
2015	04	04.11160	16	08	59.313	-18	44	40.31	UCAC2	586
2015	04	04.11231	16	08	59.316	-18	44	40.10	UCAC2	586
2015	04	04.11448	16	08	59.307	-18	44	40.12	UCAC2	586
2015	04	04.11520	16	08	59.289	-18	44	39.78	UCAC2	586
2015	04	04.11592	16	08	59.278	-18	44	40.09	UCAC2	586
2015	04	04.11664	16	08	59.288	-18	44	39.75	UCAC2	586
2015	04	04.11736	16	08	59.279	-18	44	40.02	UCAC2	586
2015	04	04.11808	16	08	59.268	-18	44	39.89	UCAC2	586
2015	04	04.11879	16	08	59.254	-18	44	39.89	UCAC2	586
2015	04	04.11951	16	08	59.255	-18	44	39.94	UCAC2	586
2015	04	04.12023	16	08	59.255	-18	44	39.95	UCAC2	586
2015	04	04.12095	16	08	59.237	-18	44	39.97	UCAC2	586
2015	04	04.12167	16	08	59.235	-18	44	39.45	UCAC2	586
2015	04	04.12239	16	08	59.227	-18	44	39.64	UCAC2	586
2015	04	04.12310	16	08	59.205	-18	44	39.53	UCAC2	586
2015	04	05.06310	16	08	51.093	-18	44	10.42	UCAC2	586
2015	04	05.06417	16	08	51.086	-18	44	10.67	UCAC2	586
2015	04	05.06630	16	08	51.071	-18	44	10.22	UCAC2	586
2015	04	05.06736	16	08	51.059	-18	44	10.22	UCAC2	586
2015	04	05.06844	16	08	51.025	-18	44	10.30	UCAC2	586
2015	04	05.06950	16	08	51.043	-18	44	10.03	UCAC2	586
2015	04	05.07057	16	08	51.037	-18	44	10.17	UCAC2	586
2015	04	05.07163	16	08	51.027	-18	44	10.01	UCAC2	586
2015	04	05.07270	16	08	51.001	-18	44	10.04	UCAC2	586
2015	04	05.07376	16	08	51.005	-18	44	10.00	UCAC2	586
2015	04	05.07483	16	08	50.999	-18	44	10.04	UCAC2	586
2015	04	05.07589	16	08	50.985	-18	44	09.98	UCAC2	586
2015	04	05.07696	16	08	50.977	-18	44	09.99	UCAC2	586
2015	04	05.07802	16	08	50.968	-18	44	10.01	UCAC2	586
2015	04	05.07909	16	08	50.959	-18	44	09.83	UCAC2	586
2015	04	05.08015	16	08	50.951	-18	44	09.94	UCAC2	586
2015	04	05.08122	16	08	50.930	-18	44	09.77	UCAC2	586
2015	04	05.08229	16	08	50.912	-18	44	10.01	UCAC2	586
2015	04	05.08335	16	08	50.914	-18	44	09.50	UCAC2	586
2015	04	05.08442	16	08	50.894	-18	44	09.54	UCAC2	586
2015	04	05.08548	16	08	50.878	-18	44	09.48	UCAC2	586
2015	04	05.08655	16	08	50.900	-18	44	09.53	UCAC2	586
2015	04	05.08761	16	08	50.873	-18	44	09.42	UCAC2	586
2015	04	05.08868	16	08	50.864	-18	44	09.39	UCAC2	586
2015	04	05.08975	16	08	50.831	-18	44	09.42	UCAC2	586
2015	04	05.09081	16	08	50.855	-18	44	09.21	UCAC2	586
2015	04	05.09188	16	08	50.821	-18	44	09.45	UCAC2	586
2015	04	05.09294	16	08	50.829	-18	44	09.20	UCAC2	586
2015	04	05.09401	16	08	50.785	-18	44	09.42	UCAC2	586
2015	04	05.09692	16	08	50.794	-18	44	09.09	UCAC2	586

continued on next page

continued from previous page

UTC			RA			DEC			Catalog	No.Observatory
2015	04	05.09799	16	08	50.813	-18	44	09.15	UCAC2	586
2015	04	05.09905	16	08	50.766	-18	44	09.05	UCAC2	586
2015	04	05.10012	16	08	50.768	-18	44	09.02	UCAC2	586
2015	04	05.10118	16	08	50.744	-18	44	09.36	UCAC2	586
2015	04	05.10225	16	08	50.749	-18	44	09.18	UCAC2	586
2015	04	05.10331	16	08	50.725	-18	44	09.06	UCAC2	586
2015	04	05.10438	16	08	50.723	-18	44	08.92	UCAC2	586
2015	04	05.10545	16	08	50.713	-18	44	08.97	UCAC2	586
2015	04	05.10651	16	08	50.710	-18	44	09.13	UCAC2	586
2015	04	05.10758	16	08	50.678	-18	44	09.15	UCAC2	586
2015	04	05.10864	16	08	50.692	-18	44	08.87	UCAC2	586
2015	04	05.10971	16	08	50.657	-18	44	08.88	UCAC2	586
2015	04	05.11077	16	08	50.651	-18	44	09.07	UCAC2	586
2015	04	05.11184	16	08	50.674	-18	44	08.68	UCAC2	586
2015	04	06.03886	16	08	42.325	-18	43	39.14	UCAC2	586
2015	04	06.03958	16	08	42.312	-18	43	39.29	UCAC2	586
2015	04	06.04030	16	08	42.306	-18	43	39.20	UCAC2	586
2015	04	06.04102	16	08	42.298	-18	43	39.20	UCAC2	586
2015	04	06.04174	16	08	42.284	-18	43	39.05	UCAC2	586
2015	04	06.04245	16	08	42.283	-18	43	39.10	UCAC2	586
2015	04	06.04317	16	08	42.282	-18	43	39.09	UCAC2	586
2015	04	06.04389	16	08	42.276	-18	43	39.01	UCAC2	586
2015	04	06.04461	16	08	42.266	-18	43	39.14	UCAC2	586
2015	04	06.04533	16	08	42.257	-18	43	39.00	UCAC2	586
2015	04	06.04605	16	08	42.251	-18	43	39.02	UCAC2	586
2015	04	06.04676	16	08	42.257	-18	43	38.83	UCAC2	586
2015	04	06.04748	16	08	42.238	-18	43	38.94	UCAC2	586
2015	04	06.04820	16	08	42.231	-18	43	38.96	UCAC2	586
2015	04	06.04892	16	08	42.226	-18	43	38.87	UCAC2	586
2015	04	06.04964	16	08	42.220	-18	43	38.92	UCAC2	586
2015	04	06.05036	16	08	42.209	-18	43	38.89	UCAC2	586
2015	04	06.05107	16	08	42.206	-18	43	38.85	UCAC2	586
2015	04	06.05179	16	08	42.199	-18	43	38.84	UCAC2	586
2015	04	06.05251	16	08	42.188	-18	43	38.82	UCAC2	586
2015	04	06.05323	16	08	42.183	-18	43	38.79	UCAC2	586
2015	04	06.05395	16	08	42.177	-18	43	38.78	UCAC2	586
2015	04	06.05467	16	08	42.160	-18	43	38.69	UCAC2	586
2015	04	06.05539	16	08	42.168	-18	43	38.70	UCAC2	586
2015	04	06.05610	16	08	42.159	-18	43	38.67	UCAC2	586
2015	04	06.05682	16	08	42.144	-18	43	38.69	UCAC2	586
2015	04	06.05754	16	08	42.142	-18	43	38.66	UCAC2	586
2015	04	06.05826	16	08	42.137	-18	43	38.66	UCAC2	586
2015	04	06.05898	16	08	42.135	-18	43	38.67	UCAC2	586
2015	04	06.05970	16	08	42.117	-18	43	38.47	UCAC2	586
2015	04	06.06041	16	08	42.116	-18	43	38.51	UCAC2	586
2015	04	06.06113	16	08	42.108	-18	43	38.55	UCAC2	586
2015	04	06.06185	16	08	42.103	-18	43	38.30	UCAC2	586
2015	04	06.06257	16	08	42.103	-18	43	38.40	UCAC2	586
2015	04	06.06329	16	08	42.089	-18	43	38.45	UCAC2	586
2015	04	06.06401	16	08	42.081	-18	43	38.43	UCAC2	586
2015	04	06.06472	16	08	42.071	-18	43	38.36	UCAC2	586
2015	04	06.06544	16	08	42.071	-18	43	38.39	UCAC2	586
2015	04	06.06616	16	08	42.062	-18	43	38.41	UCAC2	586
2015	04	06.06688	16	08	42.054	-18	43	38.27	UCAC2	586
2015	04	06.06760	16	08	42.050	-18	43	38.34	UCAC2	586
2015	04	06.06832	16	08	42.043	-18	43	38.29	UCAC2	586
2015	04	06.06903	16	08	42.034	-18	43	38.26	UCAC2	586
2015	04	06.06975	16	08	42.027	-18	43	38.23	UCAC2	586
2015	04	06.07047	16	08	42.020	-18	43	38.25	UCAC2	586
2015	04	07.05469	16	08	32.822	-18	43	05.78	UCAC2	586

continued on next page

continued from previous page

UTC			RA			DEC			Catalog	No.Observatory
2015	04	07.05619	16	08	32.808	-18	43	05.74	UCAC2	586
2015	04	07.05726	16	08	32.798	-18	43	05.70	UCAC2	586
2015	04	07.05833	16	08	32.786	-18	43	05.70	UCAC2	586
2015	04	07.05939	16	08	32.777	-18	43	05.69	UCAC2	586
2015	04	07.06046	16	08	32.765	-18	43	05.59	UCAC2	586
2015	04	07.06152	16	08	32.756	-18	43	05.60	UCAC2	586
2015	04	07.06259	16	08	32.744	-18	43	05.60	UCAC2	586
2015	04	07.06365	16	08	32.737	-18	43	05.48	UCAC2	586
2015	04	07.06472	16	08	32.724	-18	43	05.48	UCAC2	586
2015	04	07.06579	16	08	32.714	-18	43	05.43	UCAC2	586
2015	04	07.06685	16	08	32.704	-18	43	05.40	UCAC2	586
2015	04	07.06792	16	08	32.692	-18	43	05.37	UCAC2	586
2015	04	07.06898	16	08	32.685	-18	43	05.33	UCAC2	586
2015	04	07.07005	16	08	32.675	-18	43	05.31	UCAC2	586
2015	04	07.07111	16	08	32.661	-18	43	05.25	UCAC2	586
2015	04	07.07218	16	08	32.649	-18	43	05.24	UCAC2	586
2015	04	07.07325	16	08	32.641	-18	43	05.20	UCAC2	586
2015	04	07.07431	16	08	32.631	-18	43	05.16	UCAC2	586
2015	04	07.07538	16	08	32.622	-18	43	05.13	UCAC2	586
2015	04	07.07644	16	08	32.609	-18	43	05.10	UCAC2	586
2015	04	07.07751	16	08	32.599	-18	43	05.05	UCAC2	586
2015	04	07.07857	16	08	32.585	-18	43	05.08	UCAC2	586
2015	04	07.07964	16	08	32.577	-18	43	04.97	UCAC2	586
2015	04	07.08070	16	08	32.566	-18	43	04.98	UCAC2	586

Chapter 4

The re-reduction of old observations

4.1 Introduction

In this chapter, our work consists in the methods to re-reduce the old photographic observations of Phoebe. The positions of Phoebe are deduced from the photographic plates in old publications. We try to reanalyze the plates and recalculate the positions of Phoebe. Since Phoebe is far away from the main planet, contrary to the positions of other satellites measured relative to the planet, most positions of Phoebe were measured on the plates with reference to comparison stars. Thus, the Right Ascensions and Declinations of Phoebe can be deduced directly from the measured coordinates and the catalog positions of the reference stars by solving the plate constants which is mentioned in Section 2.2.2.

All the ancient observations are in different reference systems. In general work, to develop the ephemeris of Phoebe, only the transformation of the observational coordinates in different reference system was done to fit the dynamical model. However, in early times, the position of Phoebe was calculated from the positions of reference stars in ancient catalogs like CPD. The number of reference stars in these catalogs which can be identified in photographic plates is small. Moreover, the positions of such reference stars are inaccurate. As mentioned before, Figure 2-2 in Chapter 2 represents the statistics of the difference in angular separations of the stars in CPD and in modern catalogs used in current reduction. For many stars, the differences of positions between the older and recent catalogs are more than five arcseconds. The positions of the reference stars in the CPD catalog represent a

source of systematic errors on Phoebe's positions. Nowadays, although these plates have not been available yet, it is possible to reduce the positions of Phoebe from data provided by the publications with the modern and precise astrometric catalogs, such as TYCHO2 and UCAC4. This proves to be true especially when the measured coordinates of Phoebe and the stars are given in articles and when the more precise catalogs such as Gaia come out.

4.2 Introduction to the used method

To eliminate the error of the distortion of the telescope, we calculate the constants of the plate by using the measured coordinates of at least four reference stars and the least squares method. Afterwards, with the measured coordinates of Phoebe and the catalog coordinates of the reference stars, we can get the equatorial coordinates of Phoebe.

The method used to reduce the plates has not been changed. The standard coordinates obtained from the celestial coordinates of the plate center and the reference stars are compared with the measured coordinates to get the plates parameters. The formulae used in early times are as follows: (Turner [1894])

$$\xi = \frac{\tan(\alpha - A) \sin q}{\cos(P - q)}, \eta = \tan(P - q), \text{ where } \tan q = \tan p \cos(\alpha - A), \quad (4.1)$$

and

$$k\xi = (1 + \alpha)x + \beta y + \gamma, k\eta = \delta x + (1 + \epsilon)y + \zeta \quad (4.2)$$

Here P , A are Right Ascension and N.P.D (North Polar Distance) of the center of the plate; p , α are the RA and NPD of a star; ξ , η are the standard coordinates and x , y are the measured coordinates, the rest parameters are the plate constants.

Currently, we use DEC in place of NPD with $\text{DEC} = \pi/2 - \text{NPD}$. Then the above formulae are the same with the formulae 2.1 and 2.2 that are used now in the section 2.2.2.

If we know the measured coordinates of reference stars and Phoebe, as the set number 1 mentioned in Section 3.2.2, we will replace the old catalog coordinates of the reference

stars by the new catalog coordinates and then get the reduced Phoebe position in the same reference system as that of the new catalog. We do not need to transform the reference system.

In the case as the set number 2, with the coordinates of the reference stars and Phoebe known, we can get the tangential coordinates of the reference stars and Phoebe by the formula 2.2.2 in Section 2.2.2. Then we consider these tangential coordinates as the measured coordinates and resolve the problem as in the previous case.

For the set number 3, if old papers did not note the comparison stars, but mentioned the telescope for the observations and the old catalog that was used, we can re-calibrate the positions of Phoebe by comparing the positions of the stars in the field of view of the telescope in old catalogs with those of the new accurate catalog. The details will be presented in the following sections.

If nothing but the positions of Phoebe were presented in the papers, we are just doing the transformations of the coordinates in different reference systems.

The most important and difficult procedure of our re-reduction is to identify the old reference stars in new catalogs.

4.3 Identifying the old reference stars in new catalogs

The method of reduction consists of identifying the reference stars on the plates and applying corrections of the proper motion for each star on the day of observation. To identify the stars, the parameters introduced in Chapter 2 are used to change the reference system from the old catalog's reference system to the new one. Since some old catalogs failed to indicate the proper motions of stars, we use the position of the stars and the proper motions in new catalogs to return the stars' positions from the epoch of the new catalogs to that of the old ones. It is very important to choose a catalog with high precision for the proper motions. After calculations, the positions of stars in two catalogs are given in the same reference system and at the same epoch. If one of the two stars is in an old catalog and the other in a new catalog, with a position difference of less than 15" and a magnitudes difference of less than one, it is assumed that they are the same star. Normally, we should

find almost all the old catalog stars in the new catalog, but only half of the stars in the old catalogs such as CPD can be identified in the new catalog.

The difficulty to identify the reference stars is comes from the poor precision of the old catalog and the lack of accuracy of the proper motions of stars in the new catalog. The difference of a star position in two catalogs can reach 11 arcseconds. Figure 4-1 shows the positions of reference stars of two catalogs in a plate in the year 1900 in reference system ICRS, as well as the differences in positions. The base of the arrow presents the reference stars' position in catalog CPD, while the head of the arrow presents the positions in the catalog Tycho2 which have the most precise proper motions. The directions of the arrows are random which shows that there is no absolute system difference between the two catalogs in this area. Table 4.1 gives the statistics of the position difference of these stars between two catalogs. From this table we can see that in the plate the average Declination difference between two catalogs is more than two arcseconds and the maximum difference is more than 11 arcseconds. We should consider how to choose a precise catalog.

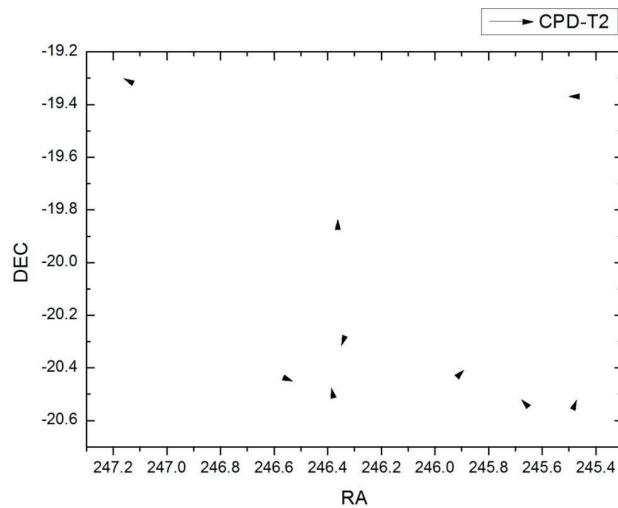


Figure 4-1: Reference stars' *J*2000 positions in two catalogs in a plate in the year 1900 and the position differences. The base of the arrow presents the reference stars' position in catalog CPD, while the head of the arrow presents the positions in catalog Tycho2

Table 4.1: Statistics of the position difference between two catalogs

Unit :as	Mean	Standard Deviation	Sum	Min	Median	Max
RA	-0.749	4.768	-7.486	-7.732	-0.734	7.898
DEC	-2.541	5.098	-25.414	-11.485	-1.855	5.422

4.4 Choosing new catalogs and the case of no information on reference stars

We take the publication of Pickering [1908] as an example to find out which new catalog we should choose. We try to reduce observations from Pickering [1908] by using several stellar catalogs (TYCHO, USNOB, and UCAC4). The Figure 4-2 presents the positions of Phoebe and the reference stars. The black stars mean the 10 reference stars mentioned in the paper Pickering [1908], the red marks mean the reference stars found through our method. We compare the positions of stars in two catalogs and identify the same stars. The two compared catalogs are the CPD catalog that was used at that time and UCAC2 (including U2SUP, which means the UCAC2 Bright Star Supplement) (Urban et al. [2006]). The circles stand for the seven reduced positions of Phoebe and the green points refer to the positions of Phoebe in the JPL ephemeris. We can see that the red marks are identical with the black stars, which means that through our method, the real reference stars can be found. Because of the unsatisfactory precision of the old catalog and the proper motions of the new catalog, sometimes it is difficult to identify all the reference stars.

We also used the TYCHO2 catalog to identify these reference stars. The following tables give some comparisons of the observed positions of Phoebe and the IMCCE ephemeris positions in different cases. Table 4.2 shows the resulting O-C of the positions of Phoebe in Right Ascension and Declination. The first column gives the dates of observations; the second two columns are the results derived from the positions of Phoebe given in the Pickering [1908] by only changing the reference system from the B1875 to ICRS; the next two columns are the results obtained by using TYCHO2 as the new catalog to recalculate the positions of the ten mentioned reference stars in Pickering [1908]; the last two columns have the similar results to the two previous columns, but by using the UCAC2 catalog. This table shows that the results turn out to be better when the new catalog is used. On the

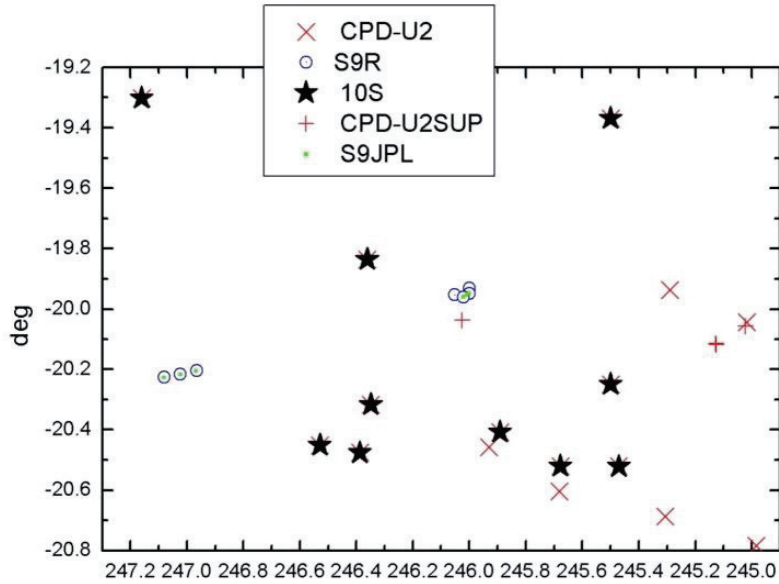


Figure 4-2: The positions of the comparison stars and Phoebe. The black stars mean the 10 reference stars mentioned in the paper Pickering [1908], the red marks mean the reference stars found through our method. The circles stand for the seven reduced positions of Phoebe and the green points mean the positions of Phoebe in the JPL ephemeris.

other hand, the results derived from the catalog UCAC2 are a little better than those using TYCHO2.

We still take the same observations from Pickering [1908] as mentioned in Figure 4-2 as an example to introduce how to get the reduced positions of Phoebe for set number 3. Suppose that we do not know the positions of the reference stars. We search for the stars in distance with 1 degree of field of view around the Phoebe. Then we consider two stars respective in UCAC2 and CPD catalogs as the same star when the position difference is

Table 4.2: O-C of the Phoebe with different catalogs

DATE	1875-ICRS (")		TYCHO2 (")		UCAC2 (")	
	RA	DEC	RA	DEC	RA	DEC
1898 8 17.011	4.05	-4.10	1.25	-2.40	1.64	-1.60
1898 8 17.096	0.30	-3.47	2.23	-1.62	-2.08	-1.18
1898 8 18.039	4.95	-3.14	3.25	-1.54	3.25	-1.49
1898 8 19.050	1.80	2.40	0.60	4.20	0.21	4.72
1898 9 16.031	-7.95	1.10	-8.23	-1.01	-8.27	-0.38
1898 9 17.028	-2.75	0.53	1.65	-1.83	-2.59	-1.11
1898 9 18.028	-1.05	-8.66	3.37	-11.28	-1.24	-10.56

Table 4.3: O-C of Phoebe position with different methods for choosing reference stars

DATE	With stars (")		I (")		II (")	
	RA	DEC	RA	DEC	RA	DEC
1898 8 17.011	1.64	-1.60	3.79	-3.68	3.22	-2.17
1898 8 17.096	-2.08	-1.18	0.31	-3.01	-0.29	-1.51
1898 8 18.039	3.25	1.49	6.01	-2.94	5.50	-1.36
1898 8 19.050	0.21	4.72	3.03	2.75	2.45	4.30
1898 9 16.031	-8.27	-0.38	-6.75	-3.28	-5.87	-0.03
1898 9 17.028	-2.59	-1.11	-1.17	-4.13	-0.19	-0.79
1898 9 18.028	-1.24	-10.56	0.25	-13.60	1.33	-10.17

Table 4.4: Statistics of residuals with different methods for choosing reference stars

	1875- J2000 (S,")		TYCHO2(S,")		UCAC2(S,")		I(S,")		II(S,")	
μ	1.217	-1.113	2.058	-0.700	-0.135	-0.173	1.242	-2.382	1.304	-0.260
σ	2.967	2.776	1.106	2.442	2.270	2.435	2.083	2.553	1.562	2.347

less than 10 arcseconds and the magnitude difference is less than one. This is the Case "I". As a result, we can find ten reference stars. If we choose the same radius around Phoebe and the same magnitude difference, 15 arcseconds for the different distance of the cross identification. This is called Case "II". We can find 16 stars.

Table 4.3 shows the comparison of results adopting different parameters to choose the reference stars with the UCAC2 catalog. The second two columns are the same with the last column of the Table 4.2. Here "with stars" means that we reduce the position of Phoebe with the 10 reference stars mentioned in Pickering [1908]. The other columns for cases I and II are the cases for the set number3 of the observations.

From Table 4.3 we can see that the results obtained through the new reduction method are not better than those directly from the transformation the reference system for the last observations. The reason should be that Phoebe is far away from the center of these reference stars, but we have supposed it as the plate center and there are not enough reference stars to correct this error by least squares method. In these cases, the average positions of the reference stars found by our method are considered as the plate centers or we the average residuals of the reference stars are used to correct the positions of Phoebe.

We reject the observations with O-C more than 5 arcseconds. Table 4.4 gives the statistics of the O-C of Phoebe with different methods. We can see that the differences are not

significant; however, the results are better in Case "II". The increase of the differences (O-C) in Case "I" by comparison with case "With stars" indicates that one or more stars were incorrectly identified by comparing the two catalogs. The high density of stars in the UCAC2 catalog and the errors of the proper motions could be affected by this. In Case "II" the identification was more successful, but the improvement was only for the plates with the optical center not successfully selected before. We will try to find more reference stars by choosing suitable parameters in the following study.

The catalog TYCHO2 has less reference stars but more accurate proper motions; the catalog UCAC2 has more stars but less accurate proper motions. It's better to use TYCHO2 to deal with the early observations and use UCAC2 for the recent ones in order to have more matching reference stars. The last two columns show that we can still find the reference stars to improve the accuracy of the position, even if the reference stars were not mentioned in the old articles.

In fact, the main difficulty of this method is the poor knowledge of the proper motion of the reference stars. The observations were realized more than 100 years ago and consequently inaccurate values of the proper motion led to inaccurate positions of stars at the date of observation. For example, Figure 4-3 (published in Desmars et al. [2013]) represents the position of a reference star given by four different catalogs at J2000.0 and B1900.0 epochs using the star proper motions from these catalogs. For the J2000.0 epoch, the positions of the star are quite similar for the different catalogs. For B1900.0, the difference in the positions of the stars can reach more than one arcsecond, according to the used catalogs. The HIPPARCOS catalog is expected to give accurate values of proper motions but only a small number of stars from this catalog are available on the plate (less than four).

For a publication that does not give a comparison of star positions, we can find the size of the field of view by the information provided by the telescope used to observe. Then we can choose the stars of magnitude nine or ten around Phoebe with the same size of view in the catalog used at that time, as mentioned in the article. We can also find the positions of these comparison stars in a new catalog: TYCHO2 or UCAC, and calculate these positions at that epoch in the reference system ICRS by using the proper motions in the new catalog. With the new precise star positions, the precise positions of Phoebe in

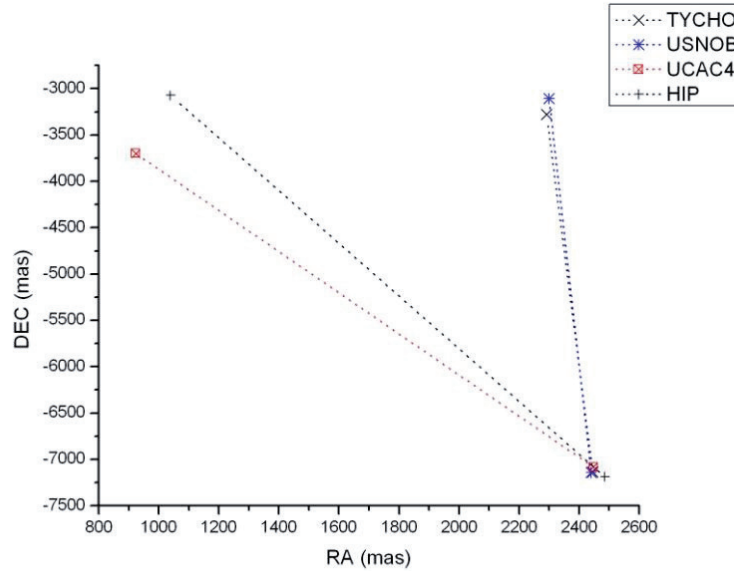


Figure 4-3: The positions of one reference star in the epoch J2000.0 (lower) and B1900.0 (upper) calculated with the positions and the proper motions in four different catalogs: TYCHO, USNOB1.0, UCAC2, and HIPPARCOS.

ICRS reference system are calculated.

Because of the uncertainty of the proper motion, we especially study at recent observations to avoid the errors caused by the long time span. There are more and more observations at recent time. But to reduce the length of the articles, the authors of the articles do not give the information of the reference stars anymore. In this case we must find the used catalogue and add the catalogue bias corrections to the positions of Phoebe.

4.5 Catalog bias

For recent CCD observations of Phoebe, we have much more reference stars since recent catalogs have a bigger star density. Reference stars' positions are not presented in the publication any more, but the catalogs are usually indicated. The catalogs used in astrometric reduction are usually a source of systematic error. This error depends on the used catalogue and on the zone on the celestial sphere. Since Phoebe is far away from Saturn and its main satellites, most of the positions of Phoebe are reduced with reference stars in catalogs. This is not the case for the positions of some other satellites that are reduced with the related

positions. Desmars et al. [2013] applied the treatment of star catalog biases proposed by Chesley et al. [2010] to Phoebe. A bias in star positions of five stellar catalogues (Tycho-2, UCAC2, USNO B1.0, USNO A2.0, USNO A1.0) have been detected compared with 2MASS stellar catalogue. As many observations of Phoebe are observed in the same celestial zone, these observations have the same bias correction. We take the positions of Phoebe with the corrections of the catalog bias, using the same treatment to remove the biases for new CCD observations of Phoebe, as is presented in Desmars et al. [2013]. Table 2.3 in Chapter 2 provides statistics on the catalogues used to reduce the astrometric positions of Phoebe. The stellar catalogue can be identified for 3697 observations. Among these observations, 3292 (89%) use one of the five catalogues studied in Chesley et al. [2010] and are pertained to the treatment. The bias removal can reach about 0.5 as both in Right Ascension and in Declination.

4.6 Result

The table of the reduced photographic observations contains: the number of the plates, the date of the observation, the original coordinate transferred to the reference system ICRS; the original coordinate; the reference system of the original coordinate; the observatory code; the number of reference stars; the catalog; the reduced position in the reference system ICRS; the original O-C; the reduced O-C.

N Plate	/Date	/Original position	/Original reference system/obs code/N	Reference stars/Catalogue code/	Reduced position ICRS	/Original O-C	/Reduced O-C
3227	1898 8 17.0110000	244.1733333 -19.6533333	1875 800 10	1	246.0009082 -19.9477830 999 999	1.242 0.260	
3228	1898 8 17.0960000	244.1729167 -19.6538889	1875 800 10	1	246.0006004 -19.9484928 999 999	-1.917 -0.731	
3230	1898 8 18.0390000	244.2250000 -19.6597222	1875 800 10	1	246.0113689 -19.9537673 999 999	3.679 -1.130	
3233	1898 8 19.0500000	244.1937500 -19.6666667	1875 800 10	1	246.0216976 -19.9593283 999 999	0.482 -0.454	

Figure 4-4: Example of result list

Chapter 5

A new ephemeris of Phoebe

5.1 Introduction

Most of the ephemeris of satellites comes from the numerical integration of equations of motion with all the perceptible effects on movement. We can get very precise positions by adjusting the observations which have a limited time span. The accuracy of the ephemeris depends on the initial conditions, the completeness of dynamical model, the integration method and the accuracy of the observations. However, the numerical method cannot present the informations on the dynamical characteristics of the trajectories.

We also have analytical method to resolve the equations of motion which introduce the details of the dynamical system, such as the theory TASS for major satellites of Saturn. We obtain a trigonometric series of osculating elements which is an analytical expression for the evolution of the system over time. The analytical theory solves the problem directly from the dynamical motions and can provide an ephemeris for a much longer time span. However, it is very difficult to get an explicit solution for a complex dynamics problem and we often need to simplify the problem. But this then makes it difficult to achieve the precision provided by numerical integration.

A synthetic representation exists between the two previous methods. We can determine the frequencies and amplitudes directly from the result of the numerical method as in the case of analytical method, by researching for the integer combinations of the fundamental frequencies of the system for each determined frequency. Then we use the determined fre-

quencies and amplitudes series to provide ephemeris. The information about the dynamical characteristics of the trajectories is hence obtained.

In this chapter, we first introduce the numerical method to provide the ephemeris of Phoebe. Our work consists in the resulting initial vectors for Phoebe after adjustment of the re-reduced observations with introduced models and numerical integration. Then we build a synthetic representation for the movement of Phoebe. At present our version of the synthetic representation gives some information on dynamics although it does not have the same precision with the numerical integration. Some of the contents of this chapter have been published in Desmars et al. [2013].

5.2 The numerical model

5.2.1 Perturbations

The Sun is always considered as the main source for the perturbations, and Rose [1979] includes only the perturbation of the Sun. The planets, the satellites of Saturn and the flatness (J2) of Saturn are also taken into account in dynamical models. Jupiter's perturbation is added by Bykova & Shikhalev [1982]. The effects of the inner planets have been checked too small to be taken into account. As Titan is the most massive body in the Saturnian system, its perturbations on Phoebe were first taken into account by Bec-Borsenberger & Rocher [1982] and then by Jacobson [1998], Shen et al. [2005] and Arlot et al. [2009]. Jacobson [1998] added the perturbation of Titan for pre-1966 observations and the perturbations of the main Saturnian satellites for post-1966 observations. However, Jacobson had shown that the perturbations by Titan were so small that they can be neglected. Shen et al. [2005], [2011] included the perturbations of the Sun, Jupiter and Uranus and the flatness (J2) of Saturn, as well as Emelyanov [2007] took into account the perturbations of the Sun, Jupiter, Uranus, Neptune and Saturn's flatness (J2, J4). In the last two publications, the Saturnian satellites are taken into account by adding their masses to the mass of Saturn and by correcting the J2 and J4 values reported by Emelyanov [2007]. A complete model was provided by Jacobson et al. [2006] determining the motion of all Saturnian satellites includ-

ing Phoebe. Desmars et al. [2013] compared four models including a complete model and a simple model, which will be introduced later, and showed that a simple model is enough. The comparison result presented in Table 5.1. All the observations used to fit these four models were never corrected by adopting our new methods introduced in Chapter 4.

We first use Model 1 to get the computed positions in order to compare the computed positions minus observed positions before and after re-calibration by our methods. The positions of the Saturnian satellites are given by Lainey et al. [2012]. The comparison result will be presented in Table 6.1 of chapter 6. Then we use Model 4 to get the computed positions, in which we take only the perturbations by the Sun and the Jupiter, but not the perturbations by other planets, other satellites of Saturn including Titan, and the perturbation due to the oblateness of Saturn. We added the mass of the eight major satellites to the mass of Saturn as Shen et al. [2005], [2011] and Emelyanov [2007] did. The planet positions are computed from the planetary ephemeris DE421 (Folkner et al. [2009]). To avoid a too simple model, we add the perturbation by Uranus as the Model 5. We compare our recalibrate observations with the computed positions generated with the Model 1 and the Model 5, and the comparison result will be presented in Table 6.3 of chapter 6. The values for the model parameters which are derived from Jacobson et al. [2006] are given in Table 5.2 .

- Model 1 is the most complete version with almost all the perturbations: gravitational perturbations of the Sun, the Moon and all the planets, the perturbations of the eight main satellites of Saturn, and the flatness parameter J2;
- Model 2 includes the perturbations of the Sun, Jupiter, Uranus and Neptune, the three main Saturnian satellites (Titan, Iapetus, Rhea), and the J2 parameter;
- Model 3 includes the perturbations of the Sun, Jupiter and Titan;
- Model 4 includes the perturbations of the Sun and Jupiter;
- Model 5 includes the perturbations of the Sun, Jupiter and Uranus.

Table 5.1: Mean and root mean square (rms) of O-C in arcsec for the different models (Desmars et al. [2013]).

		μ	rms
Model 1	Ra	0.0089	0.4467
	Dec	0.0564	0.4617
Model 2	Ra	0.0088	0.4468
	Dec	0.0564	0.4617
Model 3	Ra	0.0084	0.4514
	Dec	0.0532	0.4612
Model 4	Ra	0.0079	0.4675
	Dec	0.0458	0.4649

Table 5.2: Dynamical constants of Saturn's system (Jacobson et al. [2006]).

Name	Value	Units
Phoebe GM	0.5534	$\text{km}^3 \text{s}^{-2}$
Mimas GM	2.5023	$\text{km}^3 \text{s}^{-2}$
Enceladus GM	7.2096	$\text{km}^3 \text{s}^{-2}$
Tethys GM	41.2097	$\text{km}^3 \text{s}^{-2}$
Dione GM	73.1127	$\text{km}^3 \text{s}^{-2}$
Rhea GM	153.9416	$\text{km}^3 \text{s}^{-2}$
Titan GM	8978.1356	$\text{km}^3 \text{s}^{-2}$
Hyperion GM	0.3727	$\text{km}^3 \text{s}^{-2}$
Iapetus GM	120.5117	$\text{km}^3 \text{s}^{-2}$
Saturn equatorial radius	60330	km
Saturn J2	290.71	$\times 10^6$
Saturn pole α_P	40.5955	degrees
Saturn pole δ_P	83.5381	degrees
Saturn polar rate $\dot{\alpha}_P$	-0.04229	deg century ⁻¹
Saturn polar rate $\dot{\delta}_P$	-0.00444	deg century ⁻¹

5.2.2 Equations of motion

The motion of Phoebe can be described as in the following equation:

$$\frac{d^2\vec{r}}{dt^2} = -K(M + m)\frac{\vec{r}}{r^3} + \sum_{i=1}^n Km_i\left(\frac{\vec{r}_i - \vec{r}}{\|\vec{r}_i - \vec{r}\|^3} - \frac{\vec{r}_i}{r_i^3}\right) + P_{J_2, J_4} \quad (5.1)$$

with the mass of Saturn M , the mass of Phoebe m , the mass of other objects m_i , vector from Saturn center to Phoebe \vec{r} , the vectors from Saturn center to other objects \vec{r}_i and the perturbation of J_2, J_4 of Saturn P_{J_2, J_4} .

5.2.3 Observations used to fit the dynamical model

We re-calibrate all the old observations described in Chapter 3 to get the reduced positions. These reduced observations and the new observations have not been used to fit the model before. All the observations used to calculate our ephemerides are as follows:

- 1). 42 observations from 1898 to 1904 (Pickring [1908]), but only 10 of them are valuable to fit the dynamical model;
- 2). 4139 observations made from 1904 to 2014 available with 224 re-calibrated observations and 451 observations are the newest observations that have not been used in Desmars et al. [2013] from 2011 to 2014;
- 3). 321 new observations of Phoebe made in April 2013 and 2015, observed at Observatory Pic-du-Midi with the one-meter telescope.

5.2.4 Numerical integration

The calculated Phoebe's orbit is generated by a numerical integration of its equations of motion expressed in Saturnicentric rectangular coordinates with reference system ICRS. We start at epoch JD 244 0600.5. The sets of initial positions and velocities before adjustment are given in Table 5.3 from Shen et al. [2011]. The similar integration with Shen [2011] is carried out by using a 12th-order Runge-Kutta-Nystrom formula with the variable-step-size method.

Table 5.3: Saturnicentric starting state vector at JD 2440600.5 in ICRS (Shen et al. [2011]).

Component	Position (au)	Velocity (au/d)
x	-0.8222971225450D-01	0.2851019713340D-03
y	0.3487253432435D-01	0.7987119269534D-03
z	0.2605950144252D-01	0.3433839208685D-03

The computed positions are then obtained by conducting numerical integration of the equation of motion for the models from 1 to 5. Beginning with the initial conditions in Table 5.3, then, by the least-squares method, we correct the initial epoch state vector which minimizes the difference between the observed positions and the computed positions. The observed positions are reduced from the observations mentioned above. The resulting initial epoch state vectors are presented in Table 5.4. With the new initial epoch state vectors and the numerical integration, we can get new ephemeris of Phoebe.

Table 5.4: The resulting initial Saturncentric vectors at JD 2440600.5 for Phoebe after adjustment in ICRS

	Component	Position (au)	Velocity (au/d)
Model 1	x	-0.822332848127D-01	0.2849415511644D-03
	y	0.3486347194433D-01	0.7994264977317D-03
	z	0.2604434454622D-01	0.3433273093274D-03
Model 2	x	-0.8223364309325D-01	0.2849300720644D-03
	y	0.3486240979463D-01	0.7994316228317D-03
	z	0.2604389433852D-01	0.3433308532274D-03
Model 3	x	-0.8223190191195D-01	0.2849300361644D-03
	y	0.3486030546823D-01	0.7994487982317D-03
	z	0.2604690499022D-01	0.3433322073274D-03
Model 4	x	-0.8223008250135D-01	0.2848949774644D-03
	y	0.3485270514373D-01	0.7994932085317D-03
	z	0.2605294420552D-01	0.3433345827274D-03
Model 5	x	-0.8223246745887D-01	0.2850857392821D-03
	y	0.3487044641511D-01	0.7987019336651D-03
	z	0.2605900267052D-01	0.3433774867359D-03

5.3 The frequency analysis

Spectral analysis method is mainly used to detect the main fundamental frequencies from the numerical solution of dynamical systems. We can obtain approximate expressions for the study of the long-term evolution of the system. Spectral analysis method described in this chapter is based on the numerical analysis of the fundamental frequency (Laskar [1992], [1993]). The algorithm we used is very similar to the one used in Vienne & Duriez [1992].

5.3.1 Definition of the elements

We have six classical orbit elements: semi-major axis a , eccentricity e , inclination i , longitude of the ascending node Ω , argument of pericenter ω , mean anomaly M .

We can get the positions and velocities of a satellite from these elements and the vice versa also holds.

However, when the elements e and i are close to zero then it is difficult to obtain the elements from the position and velocity coordinates. Due to the singularity produced by the traditional definition of the elements, it can cause the problems in process. Therefore, we need another representation for elements for the orbit that has a small eccentricity and inclination but no singularity. We change the elements to $a, \lambda, \text{Re}z, \text{Im}z, \text{Re}\zeta, \text{Im}\zeta$ defined as follows:

$$\begin{aligned}
 a & \text{ with } n^2 a^3 = G(M_s + m) \\
 \lambda & = \varpi + M = \Omega + \omega + M \\
 z & = e \cdot \exp(\sqrt{-1}\varpi) \\
 \zeta & = \sin \frac{i}{2} \cdot \exp(\sqrt{-1}\Omega)
 \end{aligned} \tag{5.2}$$

where G is the gaussian constant of gravitation, M_s is the masses of Saturn, m is the masses of the satellite, n is the mean motion and $\varpi = \Omega + \omega$ is the longitude of pericenter.

5.3.2 Developpement of quasi-periodic series

Let $f(t)$ be a function describing our mechanical system. For example, $f(t)$ can be one of the variables described in the equation (5.2). We can write the function $f(t)$ as a Fourier series of θ :

$$f(t) = \sum_{k \in \mathbb{Z}^l} a_k \exp \sqrt{-1}k \cdot \theta(t) \text{ with } a_k \in \mathbb{C} \tag{5.3}$$

where $\theta(t) = \omega t + \theta_0 \in \mathbb{T}^l$ is a vector named the vector of proper argument. The vector $\omega = (\omega_1, \dots, \omega_l)$ is named the vector of proper frequency. The number l is the degree of the

freedom of the system.

By developing the scalar product $k \cdot \theta(t)$, we get a series of the forms:

$$f(t) = \sum_{j \in \mathbb{N}} A_j \exp \sqrt{-1} \nu_j t \text{ with } A_j \in \mathbb{C} \quad (5.4)$$

where the ν_j are the integer combinations of the proper frequencies ω_j .

The solution $f(t)$ of a dynamical system may come from a numerical integration of the equations. The numerically determined series (5.3) are continuous and analytic functions describing the evolution of the system: they can be used for ephemeris or other analytical studies for the same system or related systems.

5.3.3 The principle of the fine analysis

The aim is to reconstruct a quasi-periodic function $f(t)$ with numerical methods from a series of values on the interval $[-T, T]$. It is therefore to determine the different frequencies and the corresponding complex amplitudes.

For any quasi-periodic function with a general form (5.4), the Fourier series of $f(t)$ in $[-T, T]$ is defined by

$$\tilde{F}(t) = \sum_{n=-\infty}^{+\infty} \langle f, u_n \rangle u_n(t) \text{ with } u_n(t) = \exp(\sqrt{-1} n \nu_0 t) \text{ where } \nu_0 = \pi/T.$$

$\tilde{F}(t)$ is the projection of the $f(t)$ in the space generated by the vectors $\{u_n\}_{n \in \mathbb{Z}}$. Here the product $\langle \cdot, \cdot \rangle$ is defined as

$$\langle f, g \rangle = \frac{1}{2T} \int_{-T}^T f(t) \bar{g}(t) dt.$$

The integration is done by adopting the Hardy method in order 6. Note that if the $\{\nu_k\}$ in (5.4) are not integer multiples of ν_0 , then $\tilde{F}(t)$ is a very poor representation of $f(t)$ with the form (5.4), in which the obtained frequency is close to ν_0 .

In fact, we can obtain a better precision in frequency by studying an "amplitude function" defined by:

$$A(\nu) = \langle f(t), \exp(\sqrt{-1} \nu t) \rangle.$$

If $f(t)$ has only one term, we have

$$A_1(\nu) = \langle \exp(\sqrt{-1}\nu_1 t), \exp(\sqrt{-1}\nu t) \rangle = \frac{\sin(\nu_1 - \nu)T}{(\nu_1 - \nu)T}.$$

The image of the function $\frac{\sin(2\pi x)}{2\pi x}$ is given in Figure 5-1. The value of the $A_1(\nu)$ becomes larger when ν approaches ν_1 . The maximum value of the $A_1(\nu)$ is reached when $\nu = \nu_1$. The determination of ν_1 is to search for the maximum of the function $A_1(\nu)$. Fast Fourier Transform (FFT, Cooley-Tukey, 1965) can locate the maximum in a short interval, then a quadratic interpolation algorithm is used for the fine research.

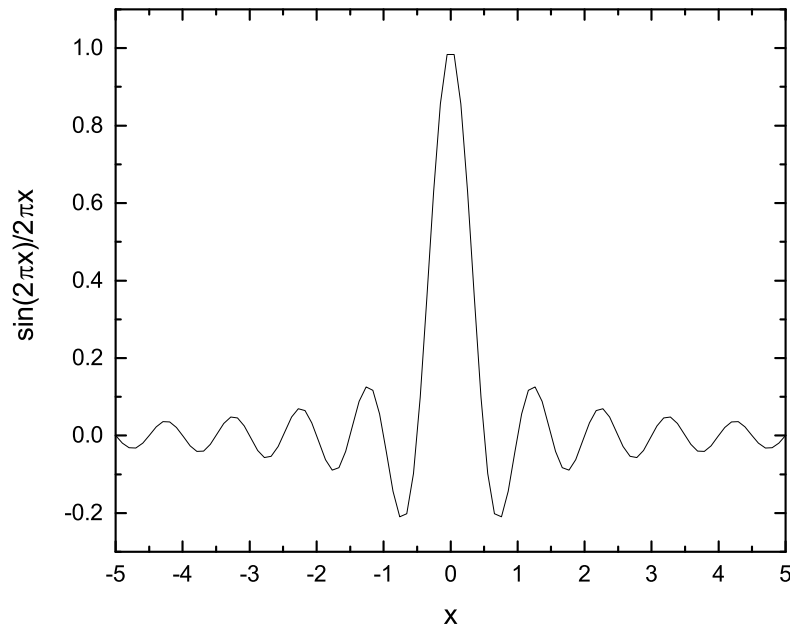


Figure 5-1: Image of the function $\frac{\sin(2\pi x)}{2\pi x}$

In a more general case where $f(t)$ is the sum of several terms, a maximum will be associated at each frequency. Adding a term causes a distortion of the peaks associated with other terms that are no longer of the same height or the exact centers on ν_j . Fortunately, the influence will be minimized for the well-separated frequencies and we can correct this effect in part by adding a weight function to the scalar product.

5.3.4 Data windowing

To minimize mutual interference between different terms, it is essential to reduce the oscillations of $|A(\nu)|$ around its maximum (see Figure 5-1). This can be realized by a redefinition of the scalar product:

$$\langle f, g \rangle = \frac{1}{2T} \int_{-T}^T f(t) \bar{g}(t) \chi(t) dt$$

where $\chi(t)$ is a weight function, that is a positive function of norm equal to 1, and

$$\frac{1}{2T} \int_{-T}^T \chi(t) dt = 1.$$

We use the Hanning filter as the weight function, that is $\chi(t) = 1 + \cos(\nu_0 t)$ with $\nu_0 = \pi/T$.

The function $A_1(\nu) = \langle \exp(\sqrt{-1}\nu_1 t), \exp(\sqrt{-1}\nu t) \rangle$ becomes

$$\frac{\sin(\nu_1 - \nu)T}{(\nu_1 - \nu)T} \cdot \frac{\pi^2}{(\nu_1 - \nu)^2 T^2 - \pi^2}.$$

This new product makes a decrease of the oscillations and reduces the perturbation of other terms. The Hanning window doubles the width of the pick area but it is not a problem if the frequencies are well separated.

The FFT method is an efficient algorithm but the limited precision is up to the fundamental frequency $\nu_0 = \pi/T$. Long integration and many data are needed to get high precision frequencies (3 millions years for the precision of 10^{-6} rad/year). With the Hanning window the method provides $1.24 \sqrt{\epsilon} \nu_0$ times more precise than what a simple Fourier series will do, where ϵ is the epsilon-machine precision of the computer.

5.3.5 The procedure of the frequencies analysis

The procedure of the analysis is as follows.

- 1, Use the method FFT to find out an approximation of the maximum value term of the

$$|A(\nu)| = |\langle f(t), \exp(\sqrt{-1}\nu t) \rangle|$$

and get the precise frequency ν_1 corresponding to the maximum amplitude

$$A_1^{(1)} = |\langle f(t), \exp(\sqrt{-1}\nu_1 t) \rangle|.$$

2, Let $e_1 = \exp(\sqrt{-1}\nu_1 t)$ be the first base of function. Find out the frequency ν_2 corresponding to the maximum of the function

$$A^1(\nu) = |\langle f^1, \exp(\sqrt{-1}\nu t) \rangle|$$

where

$$f^1 = f - A_1^{(1)} \exp(\sqrt{-1}\nu_1 t).$$

3, As the two functions e_1 and $e_2 = \exp(\sqrt{-1}\nu_2 t)$ are not orthogonal for the used scalar product, we use the Gramm-Schmidt method to obtain an orthogonal base. We calculate the projections of the function $f(t)$ to the new orthogonal bases. Then the projection $A_1^{(2)}$ on e_1 , $A_2^{(2)}$ on e_2 can be obtained as amplitudes by the changes of coordinates from the two new orthogonal bases back to (e_1, e_2) . Let

$$f^2 = f - A_1^{(2)} \exp(\sqrt{-1}\nu_1 t) - A_2^{(2)} \exp(\sqrt{-1}\nu_2 t).$$

Then we find out the frequency ν_3 corresponding to the maximum of the function f^2 .

4, This process is then iterated again to detect the amplitudes and to determine the next functions, and then the next frequencies.

5, In fact, it is very difficult to detect and separate two close frequencies. If in one step of the procedure, we find a frequency separated from a previously term by less than $2\nu_0$, we put this term back into the signal to determine it again. Sometimes it is enough, and the procedure continues with the discovery of new frequencies. But, if the problem occurs again for the same term, then we determine all the previous terms again, one by one.

The algorithm is ended either when the last determined amplitude is smaller than a given limit or when the last-determined frequency is close to an obtained frequency with certain limited distance.

5.3.6 Identification and synthetic representation

In this section we will use the frequency analysis to find the analytical expression of the orbit elements of Phoebe.

At first we use Model 1 which includes all the perturbations to get four series for six orbit elements with 0.1 day step during 52488 days, from the year 1876 to 2020. These series are with short time span because of the time limit of the ephemeris of Saturn's other satellites. The fundamental frequency is $\nu_0 = 4.372301161117482E - 002$ rad/year, and the detected frequency is less than 11474.6671672367 rad/year.

Then we use Model 4 in order to get a long time span ephemeris which is about 2000 years and to get four series of orbit elements with 0.1 day step during 729000 days. The fundamental frequency is $\nu_0 = 3.148056836004587E - 003$ rad/year, and the detected frequency is less than 11474.6671672367rad/year.

We try to identify the perturbations that we assumed in the model from these data.

We expect the proper arguments in the numerical representations issued from the frequency analysis process. These arguments are:

L_S : the linear part, in time, in the mean longitude of the Sun.

L_J : the linear part, in time, in the mean longitude of the Jupiter.

L_i : the linear part in the mean longitudes of the satellite i .

ϖ_i : the proper modes in the longitudes of the pericenters of the satellite i .

Ω_i : the proper modes in the longitudes of the nodes of the satellite i .

The values of these arguments are given in Table 5.5 for a short time span about 144 years and in Table 5.9 for a long time span of about 2000 years. We have not given the phase because it has not been used for identification. The solutions of the argument are given in Tables 5.6 to 5.8 for a short time span with Model 1 and in Tables 5.10 to 5.13 for a long time span with Model 4. Each line presents an argument of a trigonometric function (sine, cosine, or complex exponential) with the formula: phase + frequency $\times t$, where $t = 1000 + (DJ - 2451545.0)/365.25$ (that is, the time from 1000 years before J2000, in years). The time scale is TDB. The amplitudes are expressed in AU for semi-major axis and in RAD in other variables. The solutions are expressed in cosine for the semi-

major axis, in sine for the mean longitude, and in complex exponential for the variables z (eccentricity-pericenter) and ζ (inclination-node).

The last terms of these tables display an indication of the precision of the corresponding elements. For example, the last term of Table 5.8 is $7.7 \times 10^{-7} au$ and the RMS is $1.6 \times 10^{-6} au$.

For Model 1, since the data have a short time span, we can only obtain the results for elements a and z , from which the perturbations of the Sun and the Jupiter can be identified. The reason for the absence of the other two elements is that the short time span is only about 144 years. Therefore, the precession rates on the pericenter and the node are too small to be in the analysis. However, among the elements that have been detected, many terms are found with identification. In order to identify the perturbations, we try to express the frequency as the integer combinations of the proper frequencies which signify the perturbations for identification. The differences between the frequency we get and the integer combinations of the fundamental frequencies are about $10^{-3} rad/year$.

Table 5.5: Fundamental arguments of the Saturn system used for the identification in 52488 days with the Model 1

n	Period(year)	Frequency(rad/year)
L_S	29.46	0.21329912
L_J	11.86	0.52967961
L_9	1.51	4.17307754

Table 5.6: Linear part in the mean longitudes of Phoebe in 52488 days with Model 1

n	Frequency/ ν_0	Period(year)	Frequency(rad/year)	Phase(rad)
1	95.44	1.506	4.17307754	-1.187263768479

Table 5.7: Complex z of Phoebe in 52488 days with Model 1, the series is in complex exponential.

n	Frequency/ ν_0	Period (year)	Frequency (rad/year)	Amp (rad)	Phase (rad)	Identification	Error
1	0.17	823.300809	0.00763170	0.16414891	1.26147180		
2	-9.79	-14.671534	-0.42825687	0.01540244	-2.90730226	$2L_S$	0.001658634
3	-105.08	-1.367621	-4.59424358	0.00510523	2.70753334	$2L_S+L_9$	-0.005432195
4	-14.85	-9.677429	-0.64926181	0.00184399	-0.20371919	$3L_S$	0.009364452
5	-4.96	-28.982724	-0.21679071	0.00096750	-1.74505000	L_S	0.003491587
6	-109.96	-1.306935	-4.80757270	0.00088426	-1.39611338	$3L_S+L_9$	-0.0054022

Table 5.8: Semimajor axis of Phoebe in 52488 days with Model 1, the series is in cosine.

n	Frequency/ ν_0	Period (year)	Frequency (rad/year)	Amp (au)	Phase (rad)	Identification	Error
1	0.00		0.00000000	0.08645001	0.00000000		
2	105.25	1.365329	4.60195582	0.00014523	1.69267076	$2L_S+L_9$	0.00228004
3	295.79	0.485832	12.93282960	0.00004989	-0.06072608	$2L_S+3L_9$	-0.013001263
4	205.40	0.699630	8.98072330	0.00005110	0.21252408	$3L_S+2L_9$	-0.005329144
5	110.13	1.304810	4.81540317	0.00002556	-0.48762352	$3L_S+L_9$	0.002428262
6	95.26	1.508550	4.16504907	0.00001804	0.70864681	L_9	-0.008028466
7	300.70	0.477900	13.14749106	0.00001104	-2.35862114	$3L_S$	-0.011638919
8	391.06	0.367475	17.09827055	0.00000825	-2.50939259	$2L_S+4L_9$	-0.020637854
9	210.30	0.683337	9.19486244	0.00000683	-2.04828710	$4L_S+2L_9$	-0.004489115
10	195.64	0.734518	8.55416016	0.00000592	1.42643069	L_S+2L_9	-0.005294033
11	115.00	1.249556	5.02833248	0.00000310	-2.64103590	$4L_S+L_9$	0.002058455
12	100.42	1.431086	4.39050054	0.00000372	0.56943306	L_S+L_9	0.004123885
13	305.88	0.469805	13.37403323	0.00000266	-2.63357777	$4L_S+3L_9$	0.001604133
14	395.98	0.362909	17.31338372	0.00000202	1.43705688	$3L_S+4L_9$	-0.018823799
15	90.37	1.590254	3.95105686	0.00000205	2.87468104	$-L_S+L_9$	-0.008721562
16	190.50	0.754337	8.32941881	0.00000202	1.22802442	$2L_9$	-0.016736267
17	486.33	0.295490	21.26364851	0.00000142	1.33159948	$2L_S+5L_9$	-0.028337432
18	310.69	0.462530	13.58439236	0.00000103	1.77962187	$5L_S+3L_9$	-0.001335865
19	215.20	0.667782	9.40902990	0.00000087	1.89599210	$5L_S+2L_9$	-0.003620785
20	202.93	0.708149	8.87268868	0.00000077	-2.78037055	$2L_S+L_J$	-0.003145998

For Model 4, with long time span ephemeris, the time span is 14 times longer. So we can get the frequency analysis result for all the four elements. Unfortunately, it is not enough to obtain the precession rate. We still cannot find the longitudes of the pericenter and the nodes of the Phoebe whose period is more than two thousand or even three thousand years. That is why we have two extra arguments: E71 and F71 with the period of one thousand years,

without knowing the identifications of these arguments. It is also the reason for which the comparison between a and z with Model 1 is not relevant. The differences between the frequency we get and the integer combinations of the fundamental arguments are smaller than that of the short time span. The smaller differences can reach 10^{-6} rad/year.

Table 5.9: Fundamental argument of the Saturn system used for the identification in 729000 days with the Model 4

n	Period(year)	Frequency(rad/year)
L_S	29.46	0.21329912
L_J	11.86	0.52967961
L_9	1.51	4.17034475
F71	1270.159	0.00494677
E71	-1677.680	-0.00374516

Table 5.10: Semimajor axis of Phoebe in 729000 day with the Model 4, the series is in cosine

n	Frequency/ ν_0	Period (year)	Frequency (rad/year)	Amp (au)	Phase (rad)	Identification	Error
1	0.00	infini	0.00000000	0.08644988	0.00000000		
2	2784.98	0.717	8.76728140	0.00029753	-1.33229674	$2L_S+2L_9$	-0.00000634
3	1461.82	1.365	4.60188541	0.00014399	0.41282812	$2L_S+L_9+F71$	-0.00000435
4	4108.15	0.486	12.93268325	0.00004960	0.05883335	$2L_S+3L_9-F71$	-0.00000247
5	2852.70	0.700	8.98046504	0.00005421	-1.63072741	$3L_S+2L_9$	-0.00012182
6	1529.54	1.305	4.81508246	0.00002641	0.10059505	$3L_S+L_9+F71$	-0.00010642
7	1323.16	1.508	4.16539503	0.00001821	-1.74481818	L_9-F71	-0.00000295
8	4176.00	0.478	13.14628535	0.00000803	-0.65687767	$3L_S+3L_9-F71$	0.00030051
9	5431.31	0.367	17.09808178	0.00000815	1.45330725	$2L_S+4L_9-2F71$	-0.00000192
10	2717.25	0.735	8.55405203	0.00000600	2.15074767	L_S+2L_9	0.00006341
11	2920.41	0.683	9.19363059	0.00000691	-1.91458583	$4L_S+2L_9$	-0.00025539
12	2787.12	0.716	8.77400888	0.00000420	-1.83657161	$2L_S+2L_9-F71-3E71$	0.00043243
13	2782.86	0.717	8.76060844	0.00000391	2.28015660	$2L_S+2L_9+F71+3E71$	-0.00039059
14	4246.84	0.470	13.36930896	0.00000421	-1.06474126	$4L_S+3L_9+F71$	0.00013146
15	1393.97	1.432	4.38829479	0.00000365	-2.02769677	L_S+L_9+F71	-0.00029585
16	1597.26	1.250	5.02827426	0.00000339	-0.20527030	$4L_S+L_9+F71$	-0.00021374
17	1463.94	1.363	4.60857716	0.00000195	-0.08997259	$2L_S+L_9-3E71$	0.00039869

Table 5.11: $\Delta\lambda$ of Phoebe in 729000 days with the Model 4, $\lambda = -1.680167769776 + 4.17034475t + \Delta\lambda$, the series is in cosine

n	Frequency/ ν_0	Period (year)	Frequency (rad/year)	Amp (rad)	Phase (rad)	Identification	Error
1	2.75	725.293	0.00866296	0.41644552	-2.87313358	F71-E71	-0.00002897
2	5.46	365.821	0.01717556	0.03935902	-0.79801376	2F71-2E71	-0.00020830
3	132.76	15.034	0.41794230	0.00849779	-1.19933758	$2L_S$ -F71+E71	0.00003599
4	67.72	29.473	0.21318252	0.00815245	1.27456466	L_S	-0.00011660
5	2784.98	0.717	8.76728303	0.00596069	0.24278322	$2L_S+2L_9$	-0.00000471
6	8.27	241.367	0.02603164	0.00579829	0.69894887	3F71-3E71	-0.00004415
7	1461.82	1.365	4.60188683	0.00517556	1.98840475	$2L_S+L_9$ +F71	-0.00000293
8	138.69	14.391	0.43661192	0.00330363	-2.70562765	$2L_S+2F71$	0.00012014

Table 5.12: Complex z of Phoebe in 729000 days with the Model 4, the series is in complex exponential

n	Frequency/ ν_0	Period (year)	Frequency (rad/year)	Amp (rad)	Phase (rad)	Identification	Error
1	1.57	1270.159	0.00494677	0.15579183	-3.07440912	F71	0.00000000
2	-1.19	-1677.680	-0.00374516	0.03259879	1.40546768	E71	0.00000000
3	4.32	461.946	0.01360156	0.03392359	1.90982057	2F71-E71	-0.00003714
4	-137.08	-14.560	-0.43153946	0.01427815	-2.09426584	$-2L_S$ -F71	0.00000555
5	-3.92	-509.265	-0.01233776	0.00657943	-0.50509617	2E71-F71	0.00009933
6	-1460.25	-1.367	-4.59694149	0.00487278	-0.33955163	$-2L_S-L_9$	0.00000150
7	-139.85	-14.272	-0.44025072	0.00293239	2.39010830	$-2L_S-2F71$ +E71	-0.00001378
8	-134.35	-14.856	-0.42293990	0.00292569	3.02180796	$-2L_S$ -E71	-0.00008682
9	-204.81	-9.745	-0.64476813	0.00187771	-1.72523176	$-3L_S$ -F71	0.00007600
10	-1463.01	-1.364	-4.60565341	0.00099832	-2.13433702	$-2L_S-L_9$ +E71-F71	-0.00001849
11	-1527.98	-1.306	-4.81016057	0.00089111	0.01899360	$-3L_S-L_9$	0.00008154
12	1324.73	1.507	4.17032156	0.00097641	1.51474713	L_9	-0.00002319
13	-1457.49	-1.369	-4.58825783	0.00106695	-1.64516502	$-2L_S-L_9$ -E71+F71	-0.00000677
14	-6.61	-301.731	-0.02082376	0.00103868	-2.87268496	-5F71-E71	0.00016493
15	-131.18	-15.215	-0.41296645	0.00072151	-0.30943218	$-2L_S+2F71$ -E71	-0.00000691

Table 5.13: Complex ζ of Phoebe in 729000 days with the Model 4, the series is in complex exponential

n	Frequency/ ν_0	Period (year)	Frequency (rad/year)	Amp (rad)	Phase (rad)	Identification	Error
1	0.00	2243022.218	0.00000280	0.968273645757	-3.041052293409		
2	-2.75	-725467	-0.00866088	0.108512773563	1.406224497001	E71-F71	0.00003105
3	2.75	725297	0.00866291	0.102459182324	1.941472004089	-E71+F71	-0.00002902
4	-5.47	-364799	-0.01722370	0.015419439182	-0.588436309898	2E71-2F71	0.00016016
5	5.42	368210	0.01706416	0.004330874250	-2.032908827026	-2E71+2F71	-0.00031970
6	-8.25	-242050	-0.02595816	0.002645028327	-2.323527482508	3E71-3F71	0.00011763
7	-132.77	-15033	-0.41795336	0.002184962990	-0.239556350382	-2L _S -E71+F7	-0.00004705
8	132.75	15034	0.41792012	0.002093293576	-2.621902794025	2L _S +E71-F71	0.00001381

5.3.7 Conclusion

We can build a synthetic representation for the movement of Phoebe from the above tables. Although currently the precision of our representation is not good enough, the representation gives the explicit information about the dynamical system: solar perturbations and Jupiter perturbations. Unfortunately, for even over 2000 years, we are not able to find out the precession rates. The series presented here are useful to have compact ephemeris. But more investigations are needed to collect more dynamical information. A possible way could be to extend Model 1 or Model 4 over a long time span. Another way could be to use an analytical study, to constrain the formula (5.3). Thus we can expect a better precision and more dynamical information with a long time span. Our method to build a synthetic representation is not only to generate an ephemeris but also to get special information about the system such as the rotation of the planets and satellites, the tide, the resonance and the stability of the dynamical system.

Chapter 6

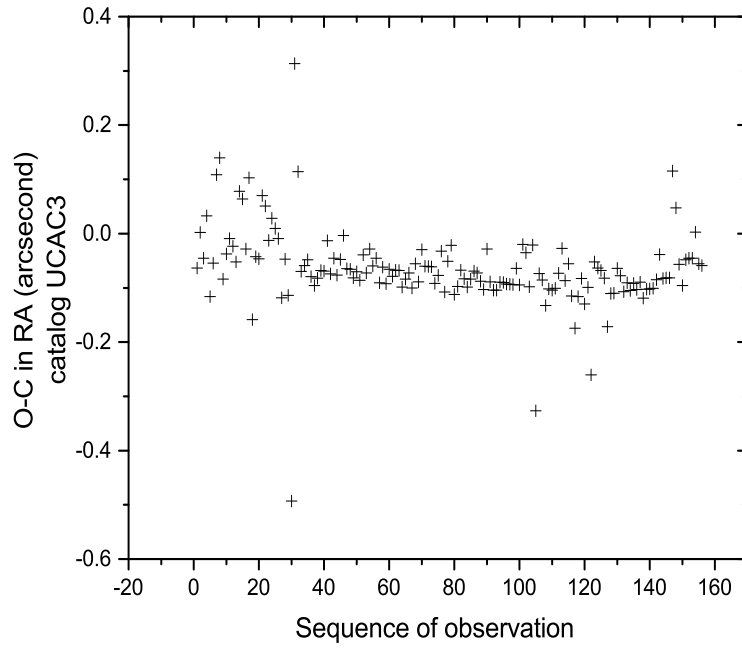
Comparisons and validation of the new ephemeris of Phoebe

6.1 Observations Comparison

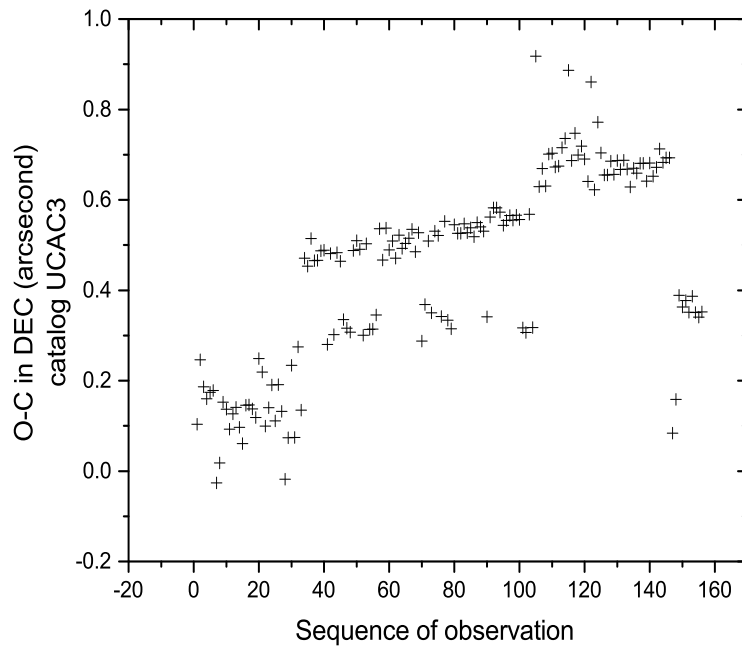
6.1.1 CCD observations calibrated with different catalogs

Phoebe was observed at pic du midi in 2013 over the nights of April 20th, 22nd, and 23rd. The image of Phoebe was taken every two minutes. We reduce the 156 CCD images observed at first with catalogs UCAC3, but the residuals of O-C are not good. Then we reduce some of them with the catalog UCAC4 to check if we can have the better residuals. These two catalogs in the celestial zone of Phoebe provide almost the same reference stars and the positions of them, so the residuals of O-C are not better than before. We reduce these observations with catalog UCAC2 and get the better residuals. For the night of April 23 with the sequence number from 105 to 156, we also used the catalog USNO-B1.0 to reduce the observations. The comparisons are shown from Figure 6-1 to Figure 6-3. We used ephemeris PH12 developed by Desmars et al. [2013] with model 1 to generate the computed position, Figure 6-1(a) and Figure 6-1(b) present the O-C residuals for all 156 observations when we used the catalog UCAC3 to reduced the images. The residuals in Right Ascension are very good which are about 0.1 arcsecond, however, that the ones in Declination are bad for the observations over the last two nights, with sequence numbers

from 34 to 156, and the residuals more than 0.5 arcsecond. These residuals are not suitable to adapt the dynamical model for CCD images. Figure 6-2(a) and Figure 6-2(b) show the residuals when we used the catalog UCAC2. The residuals are very good with an accuracy about 0.1 arcsecond in both RA and DEC for the first two nights and in DEC for the third night. The residuals are not very good in RA for the third night which are between 0.2 and 0.3 arcsecond. The reason might be that at that night we didn't have many UCAC2 catalog stars as the reference stars to reduce the image. We have just four or five reference stars. So we changed the catalog and try to have more reference stars to reduce the observations of this night. We used the catalog USNO-B1.0. The O-C residuals are shown in Figure 6-3(a) and Figure 6-3(b). We can see that the residuals in RA are also 0.2 to 0.3 arcsecond and the residuals in DEC are about 0.05 arcsecond. The result is not better than that of the observations reduced with catalog UCAC2. As the accuracy of the catalog UCAC2 is more precise than that of the catalog USNO-B1.0, we finally used the positions reduced with catalog UCAC2 .

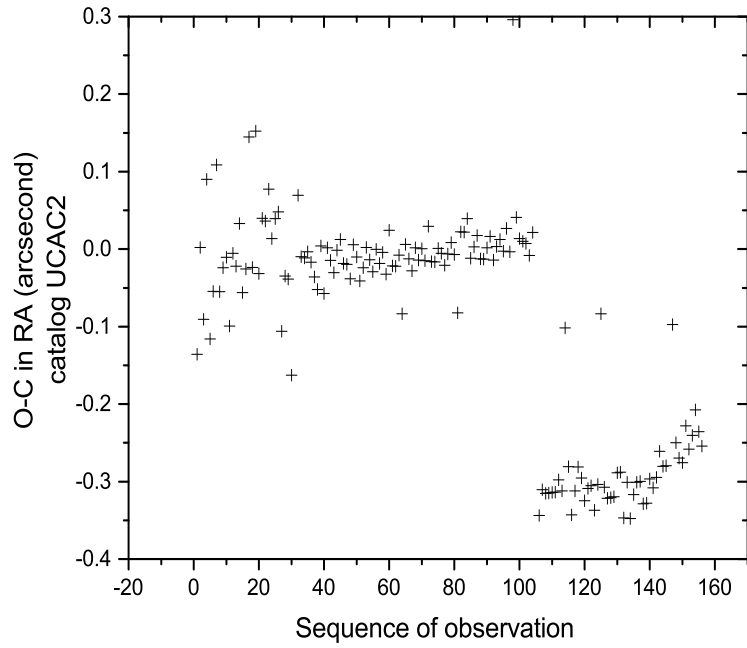


(a) O-C in RA with catalog UCAC3

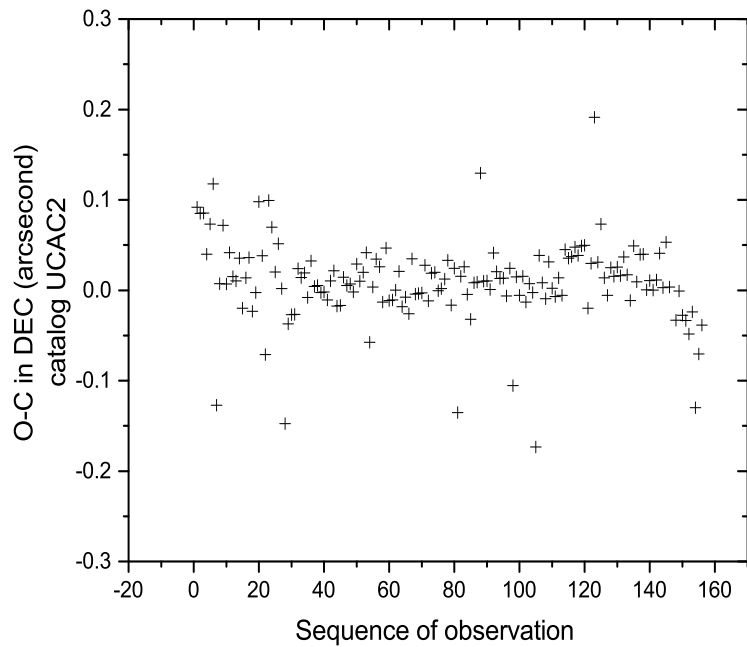


(b) O-C in DEC with catalog UCAC3

Figure 6-1: The residuals in RA and DEC of the observations reduced with catalog UCAC3 at Pic du midi in 2013.

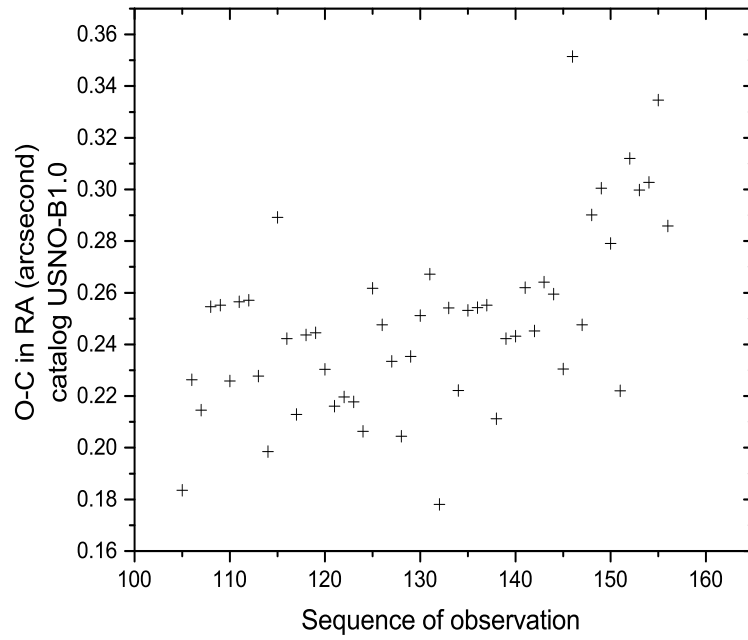


(a) O-C in RA with catalog UCAC2

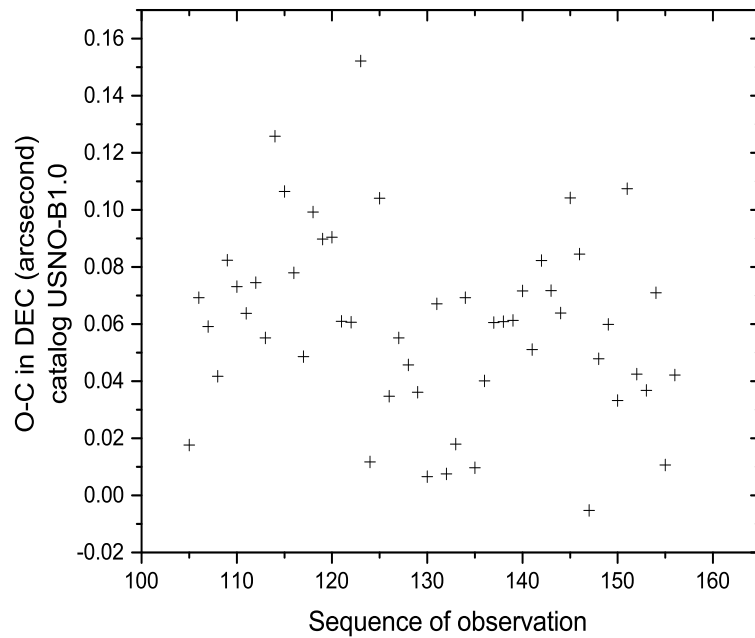


(b) O-C in DEC with catalog UCAC2

Figure 6-2: The residuals in RA and DEC of the observations reduced with catalog UCAC2 at Pic du midi in 2013.



(a) O-C in RA with catalog USNO-B1.0



(b) O-C in DEC with catalog USNO-B1.0

Figure 6-3: The residuals in RA and DEC of the observations reduced with catalog USNO-B1.0 at Pic du midi in 2013.

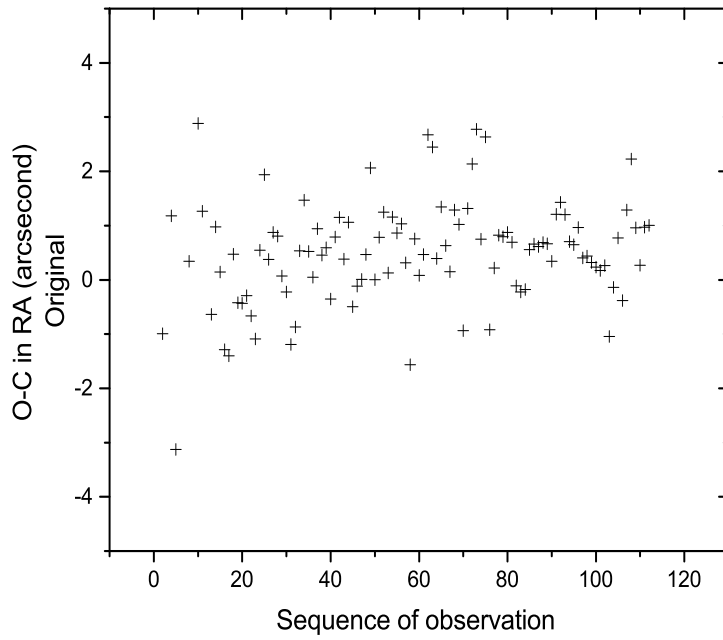
6.1.2 Old published photographic observations and the reduced observations

The following table 6.1 gives the statistics for the observed-minus-computed residuals for the photograph observations grouped by reference. The computed positions come from IMCCE PH12 developed by Desmars et al. [2013]. For each set, the table indicates the type of observations and the number used versus the total number available. The statistics include the sample mean of the residuals and the root mean square before and after the re-calibration.

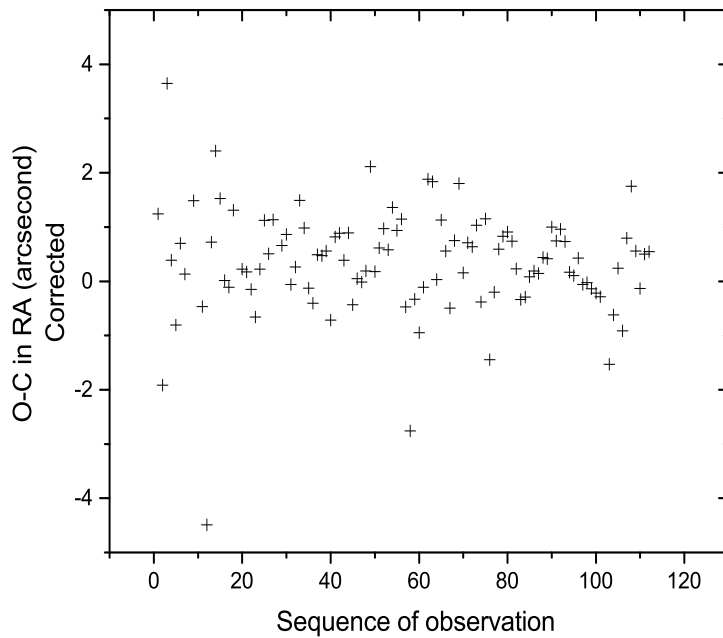
The result is much better for the observations set number 1 which indicates the measured coordinates of reference stars and Phoebe, especially for the first observations of Phoebe, published by Pickering [1908]. In Desmars et al. [2013] we had the first results for re-reducing the observations, with the method for knowing the absolute coordinates of the reference stars, only six observations had been used to fit the dynamical model and the residuals are about three arcseconds, but here we have ten observations available and the residuals are better, which is less than two arcseconds. Figure 6-4 and Figure 6-5 show the residuals of O-C for the observation set number 1, 2, and 3 mentioned in Chapter 3 before and after our re-reduction. The corrections are more obviously in DEC, with most of the residuals smaller than 1.5 arcseconds. Most of the residuals in RA are in the band -1 to 1 arcsecond and close to 0 after correction.

Table 6.1: Statistics for the observed-minus-computed residuals for the photograph observations grouped by reference

Year	Set N.	Reference	n/N	Uncorrected O-Cph12				Corrected O-Cph12			
				MeanRa	MeanDec	RmsRa	RmsDec	REMeanRa	REMeanDec	REERmsRa	REERmsDec
1898 -1902	1	Pickering [1908]	10/42	1.807	-0.201	2.891	3.252	-0.037	0.1	2.082	1.284
1904	2	Perrine [1904]	5/5	-0.441	-0.107	0.999	0.528	0.635	0.459	1.006	0.595
1905	4	Albrecht & Smith [1909]	11	-2.374	1.344	2.523	2.137	-1.233	1.419	0.915	1.788
1906	4	Perrine [1909]	10	-1.029	0.057	1.679	0.795	0.138	0.02	1.319	0.898
1907	1	MINRAS68 [1908]	13/15	0.787	1.325	1.078	1.527	0.229	0.332	0.688	0.505
1908	1	MINRAS69 [1909]	23/23	0.618	0.875	0.867	1.398	0.534	0.488	0.631	0.962
1909	1	MINRAS70 [1910]	12/12	0.749	1.188	1.308	1.532	0.247	0.002	1.212	1.015
1910	1	MINRAS71 [1911]	7/7	1.386	1.476	1.830	1.617	0.751	1.022	0.727	0.604
1912/13	4	Barnard [1913]	2/12	0.463	-0.475	1.415	0.577	1.275	-0.691	1.905	0.484
1913	2	Barnard [1914]	2/5	-0.907	-0.542	1.027	0.546	0.149	-0.525	0.676	0.106
1922	3	van Biesbroeck [1922]	0/5								
1940	3	Richmond & Nicholson [1943]	1/1	-0.553	0.050	0.553	0.050	0.169	-0.452	0.169	0.452
1942	3	van Biesbroeck [1944]	8	1.420	-0.067	1.818	0.359	1.936	-0.508	1.216	0.394
1952	3	Bobone [1953]	0/7								
1955	3	van Biesbroeck [1957]	9/11	0.577	0.467	1.251	0.652	0.116	0.004	1.07	0.449
1955	3	van Biesbroeck [1957]	3/3	1.702	0.581	1.811	0.908	1.382	0.332	0.532	0.886
1957	3	van Biesbroeck [1958]	8/8	-0.696	1.164	0.935	1.335	0.082	0.729	0.711	0.68
1960	3	Roemer & Lloyd [1966]	2	1.011	0.320	1.017	0.341	0.613	-0.133	0.169	0.197
1968	4	Chernykh & Chernykh [1971]	2	0.585	-1.082	1.451	1.141	0.618	-1.119	1.887	0.522
1969	3	van Biesbroeck et al. [1976]	1	-0.394	0.877	-0.394	0.877	0.166	0.338	0.166	0.338
1969	3	van Biesbroeck et al. [1976]	2/3	-1.711	0.662	3.317	0.705	-1.070	0.050	3.943	0.434
1975/76	3	Mulholland & Shelus [1980]	6/8	0.414	-0.097	0.757	0.471	0.440	-0.095	0.605	0.644
1981	3	Smith & Bowell [1981]	8/12	0.336	-1.521	0.824	1.611	0.385	-1.484	0.806	0.569
1981	4	Debehogne [1981]	22	0.524	-0.113	0.678	0.556	0.547	-0.039	0.434	0.581
1982	4	Debehogne [1981]	18	-0.241	-0.612	0.550	0.814	-0.275	-0.644	0.508	0.553
1989	4	Dourneau G. [1991]	5	0.185	-0.215	0.233	0.307	0.170	-0.245	0.155	0.243

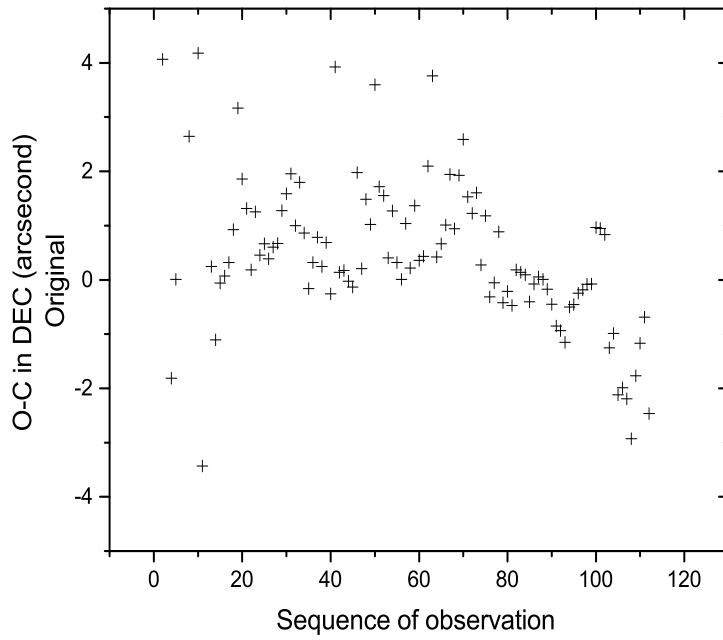


(a) O-C in RA before re-reduction

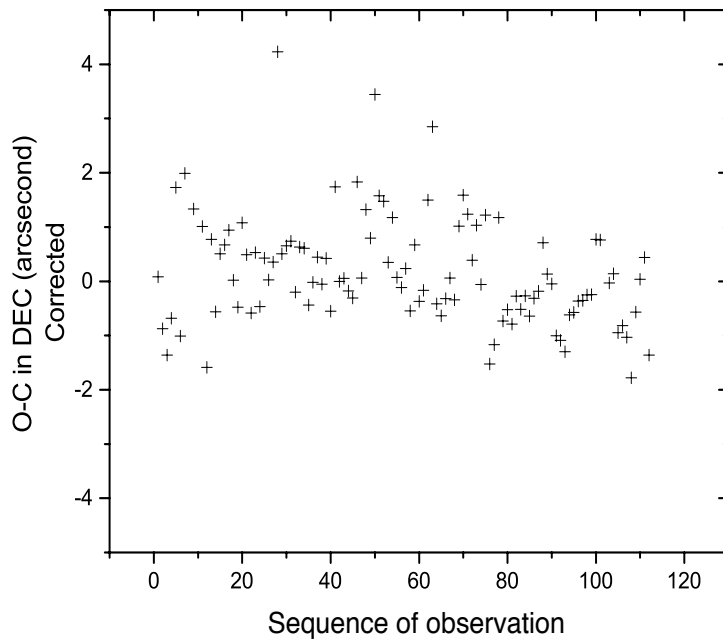


(b) O-C in RA after re-reduction

Figure 6-4: The comparison of residuals in RA of the observations before and after our method.



(a) O-C in DEC before re-reduction



(b) O-C in DEC after re-reduction

Figure 6-5: The comparison of residuals in DEC of the observations before and after our method.

6.2 Ephemeris Comparison

We generate the ephemeris with all the re-calibrated observations and CCD observations as mentioned in Section 5.2.3 with Model 3, 4 and 5. Model 4 is a simple model, with which the routine to generate the ephemeris runs quickly and we can get an ephemeris with a long time span. Model 3 adds the perturbation of Titan so it needs the ephemeris of the Titan which has a limit time span. We compare these two models and try to find out the advantage of each. Table 6.2 shows the Statistics for the observed-minus-computed residuals for all the observations before and after the adjustment of re-reduced observations with Model 3 and simple Model 4. The Photographic observation and the CCD observation provide different precision. We separate them to compare the ephemeris. Model 5 does not consider the perturbation of the satellites of Saturn and adds the perturbation of Uranus. An ephemeris with a long time span can be generated with Model 5, but its precision is expected to be known. We use model 5 and PHE12 to get the computed positions of Phoebe. With the residuals of the O-C of the photographic observations, we compare the two ephemeris, as the statistic of the result shows is shown in table 6.3. Then we display in table 6.4 the statistic results of O-C by using these two ephemeris for the CCD observations.

After adjustment, the residuals of photometric observations and ccd observations get worse in DEC for model 4, but the residuals turn better with other models. It indicates that compared to other models the Model 4 is a little simpler to generate a long interval ephemeris. Model 5 get better result than PH12 for CCD observations but no improvement for photographic observations. That means Model 5 is suitable for recent observations but cannot provide accurate positions for a long time. We should develop other models to generate the ephemeris to conduct frequency analysis. From the tables we can also see that the CCD observations provide much more precise positions than photographic observations. Because of the development of the telescopes, more precise digital measurements and the improvement of the catalogs become possible. However, the residuals are better after using the corrected photographic observations. Although old photographic observations don't have a good precision as CCD observations can provide, they extend and complete the time span and they play an important role to improve the ephemeris. We can expect the

improvement of the ephemeris with the digital photographic plates.

Table 6.2: Statistics for the observed-minus-computed residuals for all observations in using different ephemeris

		O-C before adjustment						O-C after adjustment					
		MeanRA	MeanDEC	RmsRA	RmsDEC	RmsRA	RmsDEC	REMeanRA	REMeanDEC	RERmsRA	RERmsDEC	RERmsRA	RERmsDEC
Model 4	Photographic observation	0.2742	-0.0169	1.1129	1.0050	0.2843	-0.0062	1.0466	1.0070				
	CCD observation	0.0067	0.0322	0.3369	0.3588	0.0110	0.0300	0.3346	0.3521				
	All	0.0217	0.0294	0.4204	0.4223	0.0266	0.0281	0.4090	0.4170				
Model 3	Photographic observation	0.2736	0.0465	1.0569	0.9627	0.2960	0.0567	1.0157	0.9563				
	CCD observation	0.0076	0.0350	0.3361	0.3579	0.0064	0.0351	0.3300	0.3553				
	All	0.0226	0.0357	0.4117	0.4160	0.0227	0.0363	0.4010	0.4130				

Table 6.3: Statistics for the observed-minus-computed residuals for photographic observations in using different ephemeris

Year	Set N.	Reference	Corrected O-Cph12			Corrected O-C5			n/N	REMeanRa	REMeanDec	RERmsRa	RERmsDec	REMeanRa	REMeanDec	RERmsRa	RERmsDec
			n/N	REMeanRa	REMeanDec	RERmsRa	RERmsDec	REMeanRa									
1898 -1902	1	Pickering [1908]	10/42	-0.037	0.1	2.082	1.284	9/42	0.488	0.296	1.583	1.160					
1904	2	Perrine [1904]	5/5	0.635	0.459	1.006	0.595	5/5	0.745	0.481	1.009	0.594					
1905	4	Albrecht & Smith [1909]	11	-1.233	1.419	0.915	1.788	11	-1.257	1.417	0.902	1.806					
1906	4	Perrine [1909]	10	0.138	0.02	1.319	0.898	10	0.040	-0.027	1.350	0.879					
1907	1	MNRAS68 [1908]	13/15	0.229	0.332	0.688	0.505	13/15	0.318	0.370	0.651	0.501					
1908	1	MNRAS69 [1909]	23/23	0.534	0.488	0.631	0.962	23/23	0.582	0.514	0.636	0.965					
1909	1	MNRAS70 [1910]	12/12	0.247	0.002	1.212	1.015	12/12	0.165	-0.039	1.225	1.011					
1910	1	MNRAS71 [1911]	7/7	0.751	1.022	0.727	0.604	7/7	0.671	0.991	0.738	0.617					
1912/13	4	Barnard [1913]	2/12	1.275	-0.691	1.905	0.484	2/12	1.346	-0.677	1.888	0.491					
1913	2	Barnard [1914]	2/5	0.149	-0.525	0.676	0.106	2/5	-0.049	-0.567	0.680	0.106					
1922	3	van Biesbroeck [1922]	0/5					0/5									
1940	3	Richmond & Nicholson [1943]	1/1	0.169	-0.452	0.169	0.452	1/1	0.244	-0.419	0.244	-0.419					
1942	3	van Biesbroeck [1944]	8	1.936	-0.508	1.216	0.394	8	1.856	-0.531	1.214	0.394					
1952	3	Bobone [1953]	0/7					0/7									
1955	3	van Biesbroeck [1957]	9/11	0.116	0.004	1.07	0.449	9/11	0.056	0.028	1.077	0.447					
1955	3	van Biesbroeck [1957]	3/3	1.382	0.332	0.532	0.886	3/3	1.324	0.353	0.532	0.887					
1957	3	van Biesbroeck [1958]	8/8	0.082	0.729	0.711	0.68	8/8	0.117	0.725	0.700	0.683					
1960	3	Roemer & Lloyd [1966]	2	0.613	-0.133	0.169	0.197	2	0.704	-0.127	0.169	0.197					
1968	4	Chernykh & Chernykh [1971]	2	0.618	-1.119	1.887	0.522	2	0.624	-1.115	1.879	0.525					
1969	3	van Biesbroeck et al. [1976]	1	0.166	0.338	0.166	0.338	1	0.116	0.315	0.116	0.315					
1969	3	van Biesbroeck et al. [1976]	2/3	-1.070	0.050	3.943	0.434	2/3	-1.130	0.024	3.949	0.431					
1975/76	3	Mulholland & Shelus [1980]	6/8	0.440	-0.095	0.605	0.644	6/8	0.472	-0.098	0.625	0.645					
1981	3	Smith & Bowell [1981]	8/12	0.385	-1.484	0.806	0.569	8/12	0.343	-1.463	0.806	0.570					
1981	4	Debehogne [1981]	22	0.547	-0.039	0.434	0.581	22	0.511	-0.020	0.434	0.581					
1982	4	Debehogne [1981]	18	-0.275	-0.644	0.508	0.553	18	-0.262	-0.649	0.509	0.553					
1989	4	Dourneau G. [1991]	5	0.170	-0.245	0.155	0.243	5	0.164	-0.247	0.159	0.243					

Table 6.4: Statistics for the observed-minus-computed residuals for CCD observations in using different ephemeris

CCD	n/N	O-Cph12						O-C5		
		MeanRA	MeanDEC	RmsRA	RmsDEC	MeanRA	MeanDEC	RmsRA	RmsDEC	
	3436/3443	0.0044	0.0344	0.3198	0.3568	0.0019	0.0361	0.3080	0.3439	

Chapter 7

Conclusion and future work

The publication that presents the earliest observations of Phoebe has been found. The measurements of old plates are obtained directly from the old publications or reduced by our method. The old observations are re-reduced with the accurate catalogs to extend and develop a new ephemeris of Phoebe.

The analysis of the old and recent catalogs in Chapter 2 shows that the separation between the position of a star in an old catalog and the position of such star in a recent accurate catalog can be more than 15 arcseconds. With a new accurate catalog, we can re-reduce the old observations of Phoebe to improve the precision of the old astrometric positions of Phoebe.

The examples in Chapter 3 indicate that it is possible to find out useful data from old publications, such as the reference system, the observation time, the positions of reference stars, the catalogs and other information needed for the astrometric reduction. The concepts of these information are introduced in Chapter 2.

Then we can re-reduce the old data by the methods presented in Chapter 4. The proper motions in the new catalog were used to calculate the positions of reference stars in the day of the observation. A star in the old catalog and a star in the new catalog are assumed to be the same when their position difference is less than 15 arcseconds and their magnitude difference is less than one. It is better to choose the catalog TYCHO2 that provides accurate proper motions to deal with the early observations, and to use the catalog UCAC2 for the recent ones in order to have more matching reference stars. The observations which

indicate the coordinates of the reference stars or catalogs have been re-reduced. After the re-reduction, the corrections are more obvious in declination, with most of the residuals smaller than 1.5 arcseconds. The residuals in right ascension are almost in the band -1 to 1 arcsecond and close to 0 after the correction. The observations which indicate the measured coordinates of the reference stars and Phoebe present a good result. We can expect the improvement of the position precision after reducing the digital photographic plates.

The most important and difficult step of our method to re-reduced historical observations is to identify the old catalog reference stars in new catalogs. Since the old observations were taken hundreds of years ago, the proper motions of reference stars are much more important than expected. The recent UCAC catalogues do not allow to go back one century to identify all the reference stars in old catalogs with a high accuracy. However, from the comparison results we can see that our new reductions and corrections have improved the final observation data and the ephemeris.

After the adjustment of the re-reduced old observations and the new CCD observation with the computed positions calculated by numerical method, the initial position and velocity vectors can be corrected and a new ephemeris of Phoebe can be built. We consider different perturbations of planets and satellites of Saturn to have five different models. The comparison of the ephemeris in Chapter 6 provides the validation of our new ephemeris of Phoebe. The residues of O-C between different models indicate that the accuracy of the ephemeris generally depends on the precision of the observations. From the state vector of Phoebe calculated by the ephemeris with a step of 0.1 day over hundreds and thousands years, we can also build the synthetic representations for the movement of Phoebe by the frequency analysis method. Unfortunately, for even over 2000 years, we are not able to find out the precession rates. Although currently the precision of our representation is not good enough, the representation gives the explicit information about the dynamical system: solar perturbations and Jupiter perturbations. Our method to build a synthetic representation is not only to generate an ephemeris but also to obtain special information about the system such as the rotation of the planets and satellites, the tide, the resonance and the stability of the dynamical system.

The next step of our work is to use the catalogue Gaia which provides more accurate proper motions to identify more reference stars and get more precise positions of reference stars. Then we are able to re-reduce other historical observations of faint natural satellites. But if the proper motions from Gaia will be globally accurate to one mas one century ago, some stars would not have this accuracy and, since the number of reference stars is small on old plates, the inaccuracy of one reference star can influence the result. Thus one should be careful when re-reducing old plates.

We expect to extend the complete model over a longer time span to find the precession rates with the frequency analysis. With more proper arguments identified by frequency, we shall also identify the phase of the proper arguments. Then we can build a more precise synthetic representation for the movement of Phoebe.

Bibliography

- [1909] Albrecht, S., & Smitil, E. 1909, Lick Observatory Bulletin, 5, 109
- [1976] Allen, C.W., 1976. Astrophysical quantities (3rd ed. 1973, Repr. with corrections 1976. ed.). London: Athelone Press., 125.
- [1859-62] Argelander, F. W. A. 1859-1862, Bonner Sternverzeichniss, erste bis dritte Sektion, Astronomischen Beobachtungen auf der Sternwarte des Koeniglichen Rhein., Friedrich-Wilhelms-Universitat zu Bonn, Bande 3-5.
- [1983] Aoki S., Soma M., Kinoshita H., Inoue K., 1983, A&A 128, 263
- [2009] Arlot, J.-E., & Emelyanov, N. V. 2009, A&A, 503, 631
- [1913] Barnard, E. E. 1913, Astron. Nachr., 194, 333
- [1914] Barnard, E. E. 1914, Astron. Nachr., 198, 223
- [1982] Bec-Borsenberger, A., & Rocher, P. 1982, A&AS, 50, 423
- [1949] Becker, F. 1949, Bonner Durchmusterung, Suedlicher Teil, Deklinations-Zonen -2 bis -22 grade Sternverzeichnis, zweite, berichtigte Auflage (Bonn: Ferd. Duemmlers Verlag)
- [1951] Becker, F. 1951, Bonner Durchmusterung, Noerdlicher Teil, Deklinations-Zonen -1 bis +89 grade Sternverzeichnis, dritte, berichtigte Auflage (Bonn: Ferd. Duemmlers Verlag).
- [1953] Bobone, J. 1953, AJ, 58, 172

- [1982] Bykova, L. E., & Shikhalev, V. V. 1982, *Astronomicheskij Tsirkulyar*, 1237, 6
- [1971] Chernykh, L. I., & Chernykh, N. S. 1971, *Byulleten? Instituta Teoreticheskoy Astronomii (Leningrad)*, 12, 739
- [2010] Chesley, S. R., Baer, J., & Monet, D. G. 2010, *Icarus*, 210, 158
- [1978] Corbin T.E., 1978, *IAU Coll.* 48, 505
- [1981] Debehogne, H. 1981, *IAU Circ.*, 3629, 3
- [2009] Desmars, J., Arlot, S., Arlot, J.-E., Lainey, V., & Vienne, A. 2009, *A&A*, 499, 321
- [2013] Desmars, J.; Li, S. N.; Tajeddine, R.; Peng, Q. Y.; Tang, Z. H. 2013, *A&A*, 553, 10
- [1991] Dourneau, G., Bec-Borsenberger, A., & LeCampion, J.-F. 1991, *Internal Communication of the Bordeaux observatory*
- [1993] Dourneau G., 1993, *A&A*, 267, 292
- [1991] Duriez L., & Vienne A., 1991, *A&A*, 243, 263
- [2007] Emelyanov, N. V. 2007, *A&A*, 473, 343
- [1997a] ESA, 1997, *The Hipparcos Catalogue*, ESA SP-1200
- [1997b] ESA, 1997, *The Tycho Catalogue*, ESA SP-1200
- [2009] Folkner, W., Williams, J., & Boggs, D. 2009, *JPL IPN Progress Reports*, 42-178, [HYPERLINK "http://ipnpr.jpl.nasa.gov/index.cfm"](http://ipnpr.jpl.nasa.gov/index.cfm)
- [1895-1900] Gill, D. and Kapteyn, J. C. 1895-1900, *Cape Photographic Durchmusterung*, *Ann. Cape Obs.* 3 (1895, Part I: zones -18 to -37 degrees); 4 (1897, Part II: zones -38 to -52 degrees); 5 (1900, Part III: zones -53 to -89 degrees)
- [2001] Gladman B., Kavelaars J.J., Holman M. et al., 2001, *Nature*, 412, 163
- [1993] Harper D., & Taylor D. B., 1993, *A&A*, 268, 326

- [1914] Hendaye, 1914, Imprimerie de l'observatoire d'Abbadia
- [1915] Hendaye, 1915, Imprimerie de l'observatoire d'Abbadia
- [2000] Hog E., Fabricius C., Makarov V.V., Urban S., Corbin T., Wycoff G., Bastian U., Schwekendiek P., Wicenc A., 2000, *Astron. Astrophys.* 355, L27
- [2010] IAU SOFA Board. 2010, IAU SOFA Software Collection
- [1998] Jacobson, R. A. 1998, *A&AS*, 128, 7
- [2006] Jacobson, R. A., Antreasian, P. G., Bordi, J. J., et al. 2006, *AJ*, 132, 2520
- [1975] Kinoshita, H. 1975, *SAO Special Report*, 364
- [1900] Kortazzi I., 1900, *Catalog der Astronomische Gesellschaft*, 1-121,
- [1903] Kuestner, F. 1903, *Bonner Durchmusterung des Noerdlichen Himmels, zweite berichtigte Auflage*, Bonn Universitats Sternwarte (Bonn: A. Marcus und E. Weber's Verlag)
- [1999] Ledovskaya, I. V., Jockers, K., Karpov, N. V., & Sergeev, A. V. 1999, *Kinematikai Fizika Nebesnykh Tel*, 15, 483
- [2012] Lainey, V., Karatekin, A., Desmars, J., et al. 2012, *ApJ*, 752, 14
- [1993] Laskar, J. 1993, *PhyD*, 67, 257
- [1992] Laskar, J., Froeschlé, C., & Celletti, A. 1992, *PhyD*, 56, 253
- [1908] MNRAS68-1908. 1908, *MNRAS*, 68, 211
- [1909] MNRAS69-1909. 1909, *MNRAS*, 69, 213
- [1910] MNRAS70-1910. 1910, *MNRAS*, 70, 503
- [1911] MNRAS71-1911. 1911, *MNRAS*, 71, 713
- [1980] Mulholland, J. D., & Shelus, P. J. 1980, *AJ*, 85, 1112

- [2000] Ochsenbein, F., Bauer, P., & Marcout, J. 2000, A&AS, 143, 230
- [1904] Perrine, C. D. 1904, Lick Obs. Bull., 3, 52
- [1909] Perrine, C. D., Albrecht, S., & Smith, E. 1909, Lick Obs. Bull., 5, 106
- [1908] Pickering, E. C. 1908, Annals of Harvard College Observatory, 60, 45
- [Raab] Raab, <http://www.astrometrica.at/>
- [1943] Richmond, M. L., & Nicholson, S. B. 1943, AJ, 50, 163
- [1966] Roemer, E., & Lloyd, R. E. 1966, AJ, 71, 443
- [1979] Rose, L. E. 1979, AJ, 84, 1067
- [1967] Schmidt, H. 1967, Bonner Durchmusterung, Suedlicher Teil, Deklinations- Zonen -2 bis -22 grade Sternverzeichnis, dritte Auflage (Bonn: Ferd. Duemmlers Verlag)
- [1968] Schmidt, H. 1968, Bonner Durchmusterung, Noerdlicher Teil, Deklinations-Zonen -1 bis +89 grade Sternverzeichnis, vierte Auflage (Bonn: Ferd. Duemmlers Verlag)
- [1886] Schoenfeld, E. 1886, Bonner Sternverzeichniss, vierte Sektion, Astronomische Beobachtungen auf der Sternwarte der Koeniglichen Rheinischen Friedrich-Wilhelms-Universitat zu Bonn 8, Part IV (Bonn: Adolph Marcus)
- [2005] Shen, K. X., Harper, D., Qiao, R. C., Dourneau, G., & Liu, J. R. 2005, A&A, 437, 1109
- [2011] Shen, K. X., Li, S. N., Qiao, R. C., et al. 2011, MNRAS, 417, 2387
- [1981] Smith, B. A., Larson, S. M., Fountain, J. W., & Bowell, E. 1981, IAU Circ., 3602, 1
- [1989] Smith C.A., Kaplan G.H., Hughes J.A., Seidelmann P.K., Yallop B.D., Hohenkerk C.Y., 1989, AJ 97, 265
- [1988] Taylor, D. B.,& Shen, K. X., 1988, A&A, 200, 269

- [1894] Turner, 1894, MNRAS, 11, 25
- [2006] Urban et al. 2006 U.S.N.O. 2012, Flagsta Astrometric Scanning Transit Telescope Planet and Planetary Satellite Observations, <http://www.nofs.navy.mil/data/plansat.html>
- [1922] van Biesbroeck, G. 1922, AJ, 34, 167
- [1944] van Biesbroeck, G. 1944, AJ, 50, 184
- [1957] van Biesbroeck, G. 1957, AJ, 62, 136
- [1958] van Biesbroeck, G. 1958, AJ, 63, 147
- [1976] van Biesbroeck, G., Vesely, C. D., Aksnes, K., & Marsden, B. G. 1976, AJ, 81, 122
- [1991] Vienne A., & Duriez L., 1991, A&A, 246, 619
- [1992] Vienne A., & Duriez L., 1992, A&A, 257, 351
- [1995] Vienne A., & Duriez L., 1995, A&A, 297, 588
- [1939-1983] Yale University, 1939-1983, trans. Astron. Obs., Yale Univ. 11-32
- [2004] Zacharias, N., Urban, S. E., Zacharias, M. I., et al. 2004, AJ, 127, 3043
- [2013] Zacharias N., Finch C.T., Girard T.M., Henden A., Bartlett J.L., Monet D.G., Zacharias M.I., 2013, AJ. 145, 44

Résumé

Dans cette thèse, la recherche se concentre sur le recalibrage des observations historiques de Phoebe, le 9ème satellite de Saturne, et l'amélioration de son orbite sur la base des données re-calibrés. La précision de calcul de l'orbite dépend de la qualité du modèle théorique (erreur interne) et de la précision des observations utilisées pour ajuster le modèle (erreur externe). Phoebe est loin de sa planète et des satellites majeurs. Le modèle théorique n'est pas très compliqué et l'erreur interne est bien connue et généralement bonne. Toutefois, l'erreur externe est en général inconnue et est la principale cause de l'erreur globale. Comme la luminosité de Phoebe est faible ($V_{\text{mag}} = 16,5$), il n'y a pas beaucoup d'observations faites dans le temps passé. La précision de son orbite n'est pas aussi bonne que celles des autres grands satellites de Saturne. Un modèle plus adéquat du mouvement du satellite devrait être construit non seulement sur la base des observations de haute-précision, mais aussi sur des données couvrant une période aussi longue que possible. Les observations de Phoebe sur plus de cent ans ont été calibrées avec différents catalogues. Il y a quelques incohérences dans les données d'observation obtenues à partir de différents auteurs à différentes époques. De nombreuses observations ont été publiées dans différents formats; elles ne peuvent pas être utilisées directement pour ajuster le modèle dynamique; elles doivent être converties dans un format de données unifié et être corrigées pour des erreurs systématiques, avec les dernières constantes astronomiques dans le système de référence. Les premières observations de Phoebe ont une mauvaise précision et ne collent pas bien aux modèles. A cette époque, les catalogues utilisés ne contenaient pas assez d'étoiles avec une précision suffisante, donc il n'y avait pas beaucoup de étoiles du catalogue sur la plaque. En raison de la précision insatisfaisante des catalogues à l'époque, les positions des étoiles de référence ne sont pas très précises. Cela influe sur la position de Phoebe. Dans le présent travail, nous introduisons une méthode permettant, même si nous ne disposons pas des vraies plaques, de réduire les positions des satellites naturels avec des catalogues astrométriques modernes, tels que UCAC ou GAIA dans le futur. Après la re-calibration des données d'observation historiques, la réduction des observations a été effectuée et la nouvelle éphéméride de Phoebe on a été dérivée. La représentation synthétique au mouvement orbital de Phoebe à partir des éphémérides a été réalisée.

Mots Clés

Anciennes observations, éphéméride, astrométrie

Abstract

In this thesis, the research focuses on the recalibration of historical observations of Phoebe, the 9th satellite of Saturn, and its orbit improvement based on the re-calibrated data. The accuracy of orbit predictions depends on the theoretical model's quality (internal error) and on the quantity and accuracy of the observations used to fit the model (external error). Phoebe is far away from its planet and other major satellites, so that the theoretical model is not very complicated and the internal error is well known and generally good. However, the external error is generally unknown and is the main cause of the global error. Since the brightness of Phoebe is faint ($V_{\text{mag}}=16.5$), not so many observations have been made in the past time. The accuracy of its ephemeris is not as good as those of other major satellites of Saturn. A most adequate model of satellite motion should be built not only based on high-accurate recent observations but also on data over a period of time as long as possible. The observations of Phoebe over one hundred years have been calibrated with different catalogs. There are some inconsistencies in observational data obtained from different authors at different epochs. Many observations have been published in different formats; they cannot be used directly to fit the dynamical model, should be re-input into unified data format, and be corrected for systematic differences, with the latest astronomical constants and reference frame. The earliest observations of Phoebe have bad accuracy and not fit the models very well. At that time, the catalogues they used did not contain enough stars with a precise accuracy, so there were not many catalogue stars on the plate. Because of the unsatisfactory precision of the old catalogues at the epoch, the positions of the reference stars were not very precise. This brings today inaccuracy on the position of Phoebe. In the present work, we introduce a method, even if we do not have real plates, to reduce the positions of the natural satellites with modern astrometric catalogues, such as UCAC or GAIA in the future. After the recalibration of historical observation data, reduction of observations has been completed and the new ephemeris Phoebe was derived. Then the synthetic representation of Phoebe's orbital motion from the ephemeris was provided.

Keywords

old observations, ephemeris, astrometry