



Design and characterization of a MEMS-based rotation sensor for seismic exploration

Maxime Projetti

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MAXIME PROJETTI

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**Design and Characterization of a MEMS-based Rotation Sensor for
Seismic Exploration**

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Chapter 1

Introduction

In seismic exploration, most of the signal acquired by point-receiver geophones is dominated by surface waves or ground rolls. Because they propagate in the near surface, ground rolls do not contain any information on deeper targets. Thus, short spacing between receivers is required so that this noise component can be accurately characterized and removed by digital filtering. However, considering the cost of seismic exploration ventures, new acquisition techniques using fewer point receivers and larger spacing have to be developed. Such a technique is briefly introduced in this dissertation, requiring accurate measurements of ground rotations at the free surface with minimum cost, weight and power consumption.

Therefore, the aim of this dissertation is the design and the characterization of a micro-electromechanical (MEM) angular accelerometer that interfaces with an optimized control electronics to implement compact, high resolution and high sensitivity rotation sensing micro-systems. An angular accelerometer is a sensor used to measure acceleration of rotation unlike well-known gyroscopes which measure velocity of rotation.

In terms of market demands, MEMS angular accelerometers are used primarily to compensate for angular shocks and vibrations in Hard-Disk-Drive assemblies. These devices are very similar to linear accelerometers but they do not meet the same success since few developments have been proposed so far for high-performance applications. The strategy of this dissertation is then to contribute to the development of a high-performance MEMS angular accelerometer by reusing a control electronics developed for MEMS linear accelerometers deployed in seismic exploration. As a result, the work is focused on understanding the behavior of a micro-structure, sensitive to angular accelerations, and how it interacts with the given electronics. In order to define the best structure for the sensitive element,

different sensing principles are studied throughout the dissertation. Then, the prototyping of the best designs allows the characterization in terms of resolution and sensitivity to conclude on the possibility to measure surface waves during seismic exploration.

The thesis is structured as follows:

Chapter 2 deals with the context of the study with a particular focus on the need to use rotation sensors in seismic exploration. A state of the art on technologies used to measure rotations is then presented. To address seismic exploration requirements, the new rotation sensor must be compact, relatively inexpensive with high-resolution and high-sensitivity. MEMS capacitive solutions are promising candidates to meet these requirements.

Chapter 3 provides the working principle of a high-performance capacitive angular accelerometer. Two ideal sensing principles are presented to give design rules: one using lateral capacitors; and the other using gap-closing capacitors. First, the motion equation and the study of electrostatic forces for the two sensing principles are studied. Then, as the sensing and control electronics of the sensor is already set, brief reviews on capacitive sensing and digital control feedback are proposed to understand the different elements of the sensor loop. The interactions between the two sensing principles and the electronics are then studied to fully understand the behavior of the MEMS.

Chapter 4 gives the detailed design for complex topologies inspired by the ideal cases presented in Chapter 3. Each design has to meet technical specifications imposed by the electronics for proper packaging and characterization. An optimization step is performed to produce prototypes with the best performances and at last, simulations are made to foresee the sensor response in its control electronics. An intermediate conclusion on the best topologies is finally given.

Chapter 5 discusses the characterization and the experimental measurements of the sensor designs selected in Chapter 4. Comparisons with models given in Chapter 4 are proposed and the performances are then evaluated and analyzed. Comparative analyses with other technologies are also performed.

At the end, the conclusion on the feasibility of using MEMS angular accelerometers for seismic exploration is given. In addition, different ideas are suggested to improve the prototype performances that have been reached in the scope of this PhD.

Chapter 2

New acquisition system to deal with surface waves during land seismic data acquisition

2.1	Context and principles of seismic exploration	4
2.2	Acquisition of point receiver measurements in seismic exploration	5
2.3	Rotation sensors to deal with coherent noise in land seismic	5
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This chapter presents the context of the study by introducing the difficulty met by geophysicists to deal with surface waves during seismic surveys. An innovative solution using rotation sensors is proposed followed by a technology review about rotation sensing techniques in seismology. At the end, a comparison between the different technologies is done to select a concept suited to meet the specifications of seismic exploration.

2.1 Context and principles of seismic exploration

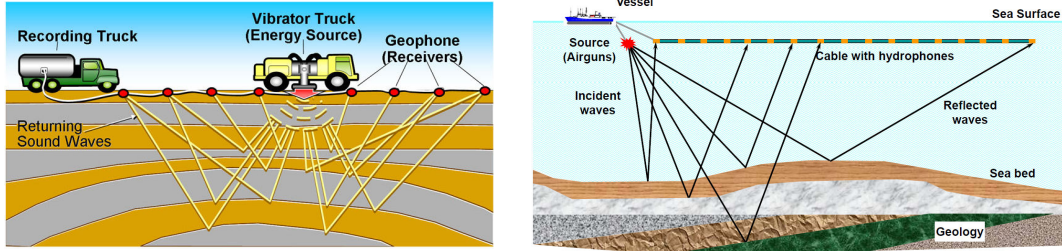


Figure 2.1: Left: Land seismic principle. Right: Marine seismic principle

Exploration seismology is the study of the subsurface by the use of artificially generated seismic waves. This study provides data that, when correlated with other geophysical aspects, can give information on rock properties and mineral resources locations (oil, hydrocarbons, water, etc.). This discipline is encompassed in strong seismology. Seismic waves or earthquakes are generated when a fault breaks open. These waves travel through long distances and can be recorded with seismographs, giving information about the nature of rocks met by the earthquake. Exploration seismology is quite similar except that seismic sources are controlled and the distance between sources and receivers are relatively short.

The measured data during seismic exploration is the travel-time needed for waves to travel from the source to a series of sensors, usually arranged along a straight line directed toward the source. From the travel-time, it is possible to reconstruct the path of the seismic waves. There are two types of seismic surveys: land and marine. Each survey needs particular apparatus to generate and to record seismic waves. For instance, specialized sources are used to generate acoustic waves into the ground such as vibrators or explosive on land and compressed air at sea. These waves are then reflected at the boundary between the different geological layers in the subsurface and recorded by arrays of highly sensitive sensors laid out on the surface: geophones for land and hydrophones for marine (figure 2.1).

2.2 Acquisition of point receiver measurements in seismic exploration

The evolution of high performance computing still follows Moore's law, which enables to solve more and more complex problems and provides new ways for seismic data acquisition. For instance, the advances in hardware, wiring and computer storage have contributed to the acquisition of dense array of single-point receivers in order to preserve as much signal as possible compared to groups of geophones [2]. However, records by single point-receiver are highly dominated by noise. To be able to recover signal, all this noise has to be filtered during data processing. A major component of noise that geophysicists have to deal with is the coherent noise or surface waves.

For land data, most of the source energy is converted into surface waves (Rayleigh waves) [3] which are commonly named as ground-rolls. Because they propagate in the near subsurface, they do not contain information of deeper targets, and due to their strong energy, they dominate seismic records, obscuring reflections from the deep subsurface. A widely used technique to suppress coherent noise is the use of filtering techniques in the wavenumber-frequency ($\kappa - f$) domain [4]. However, for a good attenuation of coherent noise, the point receiver spacing or the spatial resolution must be decreased at least down to the Nyquist distance necessary to keep ground-rolls unaliased. For instance, ground-roll wavelengths can be less than 20 m, resulting in a spacing of 10 m between receivers. This short spacing in seismic surveys means tens of thousand of channels, which has an impact on the team productivity to prepare the set-up, and on the power consumption. Considering costs associated with manpower and logistics, new acquisition techniques have to be developed to keep the price for a seismic survey competitive, leading the way to the exploration of new and complex reservoirs.

2.3 Rotation sensors to deal with coherent noise in land seismic

Another technique presented in [5], introduces rotation sensors in land seismic to measure the gradient of the vertical wavefield measured by conventional geophones. Indeed, rotation is the curl of the wavefield and thus, is related to the derivatives of the displacement wavefield with respect to space coordinates. The

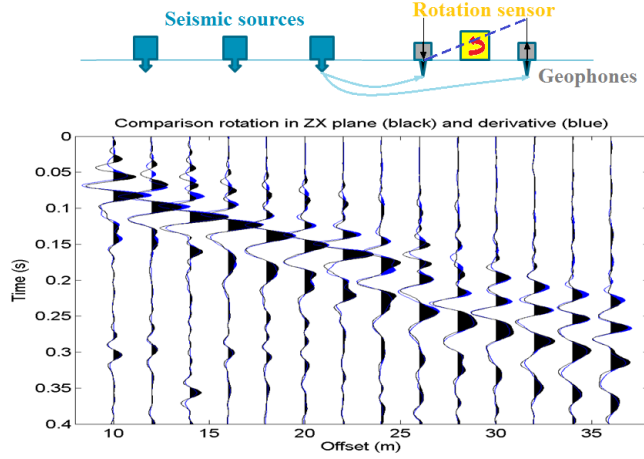


Figure 2.2: Alternative technique to deal with surface waves using rotation sensors

experiment uses coupled geophones on both sides of a pure rotation sensor. An illustration of this set-up is presented in figure 2.2. With different seismic sources, the gradient of the vertical wavefield is calculated from the geophones acquisition and then compared to measurements from the rotation sensor. A good fitting is observed leading to the conclusion that, at the free surface, rotations about a horizontal axis are proportional to the gradient of the wavefield vertical component (also shown in figure 2.2). This result is of main interest since rotation sensors can measure the gradient of any signal acquired by geophones during a seismic survey. However, the seismic wavefield acquired during operations by sensor nodes have different components depending on the type of seismic waves arriving from the subsurface. The question asked here is: which wavefield component is likely to be measured by a rotation sensor?

It is possible to sort seismic waves depending on their apparent wavelengths at the free surface (figure 2.3). The apparent wavelength is a function of the incident angle θ of a seismic wave. For instance, surface waves, e.g. ground rolls in land seismic surveys, have an apparent wavelength close to their true wavelength since they are propagating with a large emergent angle. On the contrary, seismic reflections have a small emergent angle, hence large apparent wavelengths.

Due to their stronger energy and their small apparent wavelength at the free surface, surface waves have higher amplitudes; hence, measuring their spatial gradients using rotation sensors becomes possible whereas it seems more complex to measure the wavefield spatial gradient of seismic reflections. As a consequence, rotation sensors are well suited to measure the spatial gradients of noise components at the free surface. This result is interesting since it could be possible to measure,

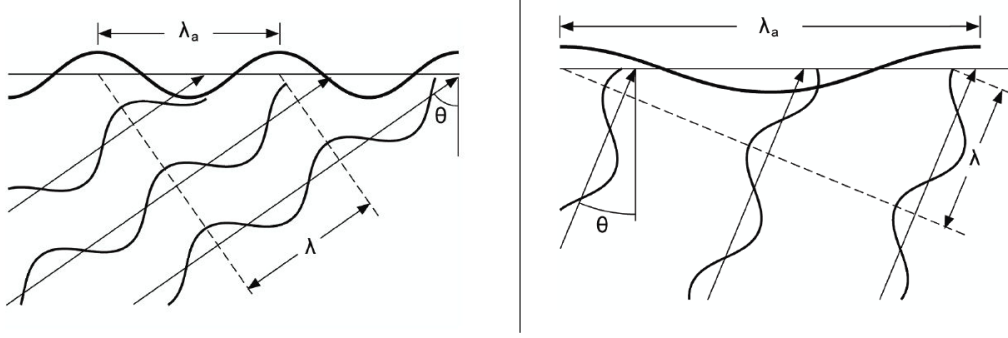


Figure 2.3: Comparison between surface waves and seismic reflections. Left: surface wave profile characterized by a short apparent wavelength at the free surface. Right: typical seismic reflection from deep subsurface characterized by a large apparent wavelength.

at the same node positioned at the free surface, the wavefield vertical component and the gradients of operational noises during a seismic survey.

This possibility can modify strongly the sampling technique used in seismic exploration since it could be possible to move from a standard Nyquist sampling, requiring two measurements for the shortest wavelength of a signal to an improved sampling technique: the Papoulis sampling [6]. Papoulis sampling requires co-located measurements of the vertical wavefield and its gradient at each cycle for the shortest signal wavelength (figure 2.4). As a result, it is possible to interpolate noise components between sensor nodes, allowing sparser spatial sampling.

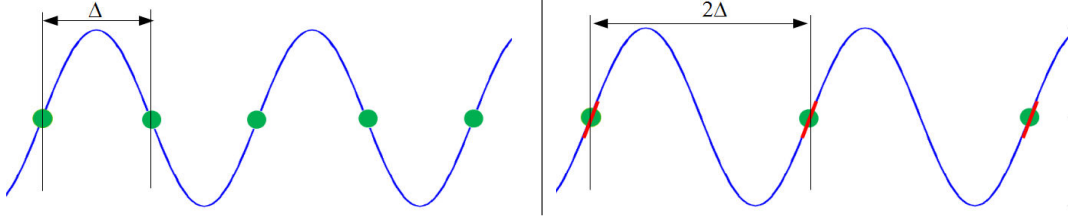


Figure 2.4: Advantage of using rotation sensors for sparser spatial sampling. Left: conventional sensor nodes for Nyquist sampling. Right: nodes using conventional and rotation sensors based on Papoulis sampling.

2.4 Technology review of rotation sensing systems used in seismology

The specifications for rotation sensing systems operable in seismic exploration are confidential. However, it can be said that geophysicists need devices with poorer resolution than for rotational seismology by few orders of magnitude. Specifically, the Root Mean Square (RMS) value of the resolution must be less than 5 mrad.s^{-2} or $30 \text{ } \mu\text{rad.s}^{-1}$ over 100 Hz band. In addition, the frequency band of interest in rotational seismology is below 10 Hz whereas the working bandwidth addressed in this dissertation is between 20 Hz and 200 Hz. Otherwise, the maximum size of the sensor is: $3 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm}$, and the power consumption must be below 50 mW.

Finding a concept to measure ground angular accelerations during seismic surveys starts by reviewing the sensors used in strong seismology. Even though their resolutions are far better than needed for near-field seismic, it allows to extract technical solutions that can be shaped for our application. As a consequence, this section gives a review of different systems used to measure ground rotations divided in families: systems using a pendulum, optical gyroscopes, electro-chemical sensors and inertial sensors. Then, each family is evaluated with respect to different requirements: cost, complexity, power consumption, size and resolution. A summary table (2.1) can be found at the end of this section.

2.4.1 Systems using a pendulum

The first earthquake detector made with a pendulum was the *sismoscope* from the Chinese seismologist Chan Hen in 132 B.C. The principle is well understood, thus, it is not a surprise to find sophisticated instruments using this type of technology.

2.4.1.1 TAPS system

This measuring technique consists in Two Antiparallel Pendulum Seismometers (TAPS) situated at a common vertical axis. The transduction method is electromagnetic with a magnet moving inside a coil. This system has been tested for strong motion seismic. The maximal theoretical resolution of the device, expressed in angular velocity, is 10 nrad.s^{-1} [7]. Figure 2.5 illustrates TAPS device. Considering that this system is used in rotational seismology, its resolution fulfills

our requirements. However, the apparent complexity of this system makes it a bad candidate for large scale deployment in seismic prospecting.

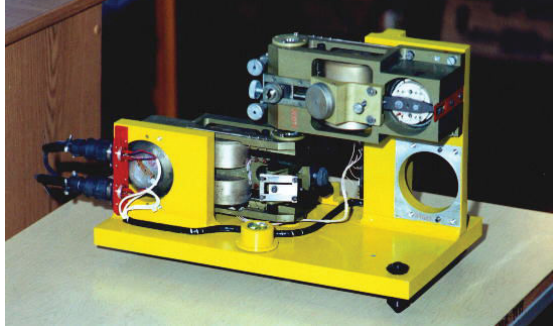


Figure 2.5: Picture of the TAPS system

2.4.1.2 Torsion balance

A torsion balance for near-field seismic and engineering applications is proposed in [8]. The instrument consists in a torsion balance having a natural frequency significantly smaller than the rotational seismic motion to record. The angular position is acquired at regular intervals by means of a high-resolution optical lever of large dynamic range. The principle can be observed in figure 2.6: plate A is used to suspend the balance bob (mirror and extension arms); plate B holds the 200 mm focal length lens, and plate C holds the light source and the Position Sensing Detector (PSD). The noise floor is estimated at $3 \mu rad.s^{-1}$ RMS (10 Hz band). Same comments can be made as TAPS system about the feasibility to use this device in seismic exploration.

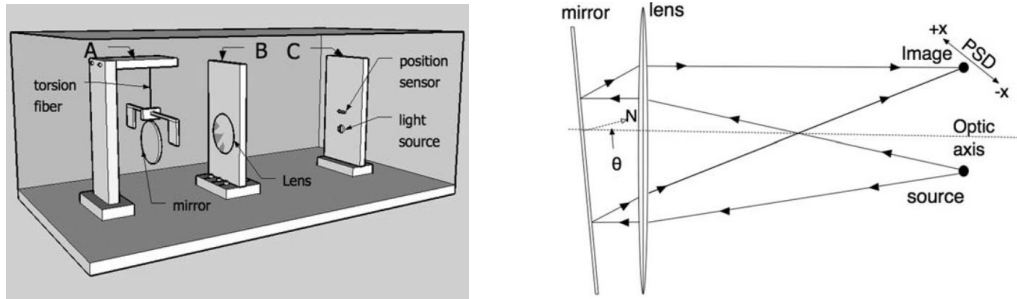


Figure 2.6: Torsional oscillator principle. Left: Illustration of the system; Right: schematic diagram of the optics.

2.4.2 Optical gyroscopes

Optical interferometry has assumed an important role in high-performance applications and especially for strong rotational seismology.

2.4.2.1 Ring Laser Gyroscope

A Ring Laser Gyroscope (RLG), in the original design, contains a ring-shaped cavity filled with a mixed gas, with circulating light beams generated by a laser. It measures the Sagnac beat frequency of two counter-propagating beams. This beat frequency δf is directly proportional to the rotation rate Ω around the normal vector to the laser beam plane $\mathbf{n}$, and given by Sagnac equation :

$$\delta f = \frac{4A}{\lambda P} \mathbf{n} \cdot \Omega \quad (2.1)$$

where Ω is the full rotational velocity, including the Earth rotation rate and local ground rotation, P is the perimeter of the ring, A is the area, and λ is the laser wavelength. It is straightforward, from the equation 2.1, that the RLG is not sensitive to translational motion, but sensitive to deformations since A , P are influenced by them. For seismological application, the scale factor $\frac{4A}{\lambda L}$ must be made much larger than commercial ring lasers since a great sensitivity and a high resolution are needed.

Since a RLG is an active interferometer in which the sensitivity arises from the dependency of the laser frequency from the cavity length, the important requirement for this device is the mechanical stability. In order to operate in the monomode lasing regime, variations of the cavity length must not exceed one wavelength, which sets very high constraints for the rigidity of the laser beam perimeter. As pressure and temperature fluctuations affect the cavity length, the environmental conditions at the location of the RLG must be kept within tight limits (temperature variations less than $0.5^\circ C$ and pressure change less than 10 hPa).

Schreiber reports in [9] the design and first results from the GEOsensor (figure 2.7), which has been specifically built for studies in rotational seismology. The sensor is operated at the Piñon Flat Seismological Observatory in Southern California. The instrument has a length on a side of 1.6 m which provides a total area of 2.56 m^2 . The sensor resolution is estimated at 0.3 nrad.s^{-1} RMS (20 Hz band) under stable operating conditions.

RLG is a complex system which is notably very expensive and power consuming. In spite of an extreme resolution, this system is not suited for seismic exploration.

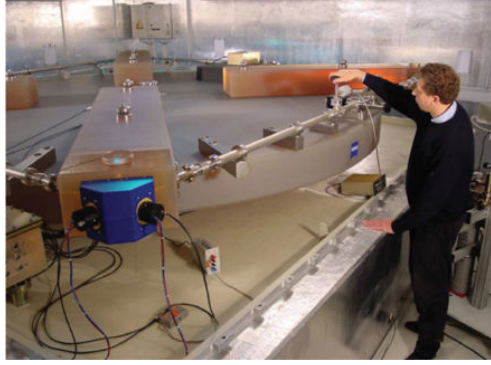


Figure 2.7: Ring Laser Gyroscope for Rotational Seismology (GEOsensor)

2.4.2.2 Fiber-Optic Gyroscopes

The RLG requires sophisticated facilities and complicated operations, which is not compatible with large-scale networks or mobile observation. In comparison, a fiber optic gyroscope (FOG) is small and robust. The principle is quite simple; a narrow-spectral width light beam is generated by a source and passed on to an equal intensity beam splitter. The resultant two light beams are guided around a monomode fiber coil in opposite directions. After passing through the fiber, both beams are superimposed again by the same beam splitter and steered onto a photodetector. If the entire device is at complete rest, each beam travels the same distance and there is no phase difference between them. However, if the FOG is rotating around the normal vector of the fiber coil, the two beams do not travel the same distance and a small phase shift is observed. Because the signal travels at the speed of light, the obtained phase shift remains very small. Therefore, a modulation technique, pulsed operation and $\frac{\pi}{2}$ phase shifting for one sense of propagation are used to maximize the sensitivity of the instrument. Furthermore, the device operates in a closed loop configuration to ensure a wide dynamic range. The observed phase difference is given by the following expression:

$$\delta\psi = \frac{8\pi A}{\lambda c} \mathbf{n} \cdot \boldsymbol{\Omega} \quad (2.2)$$

where c is the speed of light. The design characteristics of a FOG gyroscope for rotational seismology are described in [10] and summarized below:

- Single mode fiber 11 130 *m* long in a 0.63 *m* diameter sensor loop.
- High optical power source.
- Optical loss of 21 *dB*.

Under these conditions, the estimated sensitivity of the FOG system is 42.7 nrad.s^{-1} RMS over the bandwidth 0.1 Hz - 20 Hz. Despite smaller size, same comments can be made as for RLGs on power consumption and cost for the use of this technology in seismic exploration.

2.4.3 Electrochemical sensors

The electrochemical transducer R-1 from Eentec, described in [11], can be used as a sensitive element for rotational sensors (figure 2.8). This transducer replaces the seismic mass of a mechanical pendulum by a liquid electrolyte. The motion of this liquid generates an electrical output signal which is function of the ground motion. True rotational seismometer with a very good resolution of $1 \text{ } \mu\text{rad.s}^{-1}$ RMS over the bandwidth of 0.05 Hz - 20 Hz and 110 *dB* dynamic range are commercially available. In addition, it was found that the R-1 is very sensitive to temperature variations.

The sensor dimensions are: 12 cm×12 cm×10 cm for a weight of 1 kg and a power consumption of 300 mW approximately. In addition, the cost of this sensor is really prohibitive: 5000 USD. Despite its high resolution, it is not suited for seismic exploration.

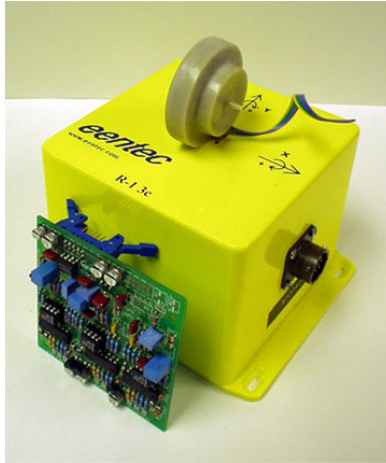


Figure 2.8: Eentec Rotation sensor

2.4.4 Inertial sensors

Inertial sensors are widely used in seismic exploration with the introduction of geophones and, more lately, MEMS accelerometers as point receivers. Very high sensitivity is achievable at reasonable costs. Therefore, one can think of reusing these instruments to measure angular accelerations. Other sensors are available such as vibratory gyroscopes which meet an important success in the inertial navigation industry or angular accelerometers used for vibration monitoring.

2.4.4.1 Translational geophones

Johana Brokešová and Jiří Mělek propose a rotational sensor system called *Rotaphone*, which is based on differential motion measurements of paired sensors (geophones) [12]. The rotational component of ground motions is obtained by calculating the space derivative of the z-velocity about x- and y-axis. According to the figure 2.9, the curl components expressed at the free surface are:

$$\begin{cases} \Omega_x &= \frac{\partial u_z}{\partial y} \\ \Omega_y &= -\frac{\partial u_z}{\partial x} \end{cases} \quad (2.3)$$

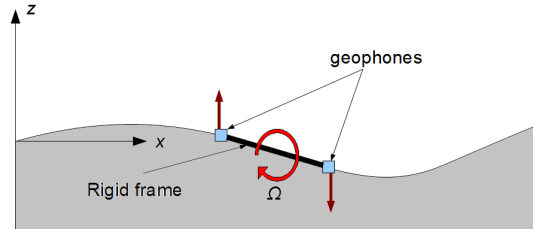
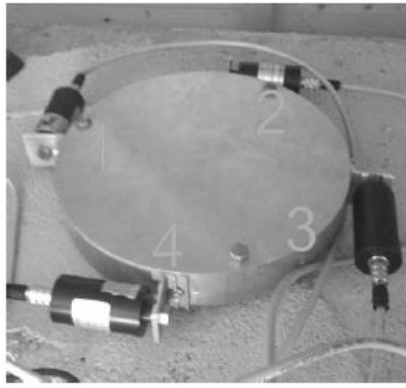
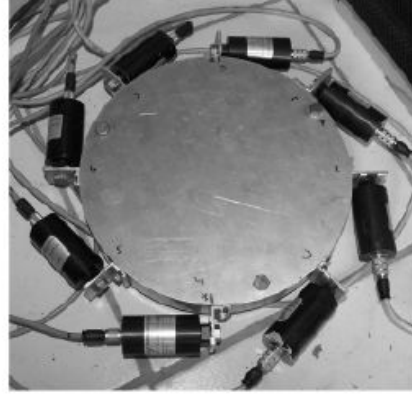


Figure 2.9: Rotaphone principle

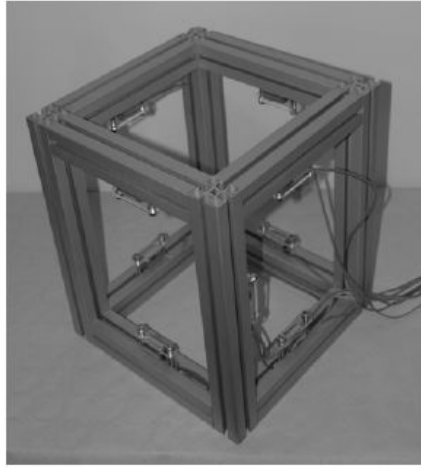
So far, there are three different prototypes shown in figure 2.10. For laboratory prototypes, several high-performance geophones are used. A theoretical resolution of 10 nrad.s^{-1} RMS in the frequency band 1Hz -200 Hz can be achieved. However the diameter of the rigid disk where the geophones are mounted is 25 cm which is not compliant with our specifications.



(a) 4 horizontal geophones in 2 pairs



(b) 8 horizontal geophones in 4 pairs



(c) 8 horizontal geophones in 4 pairs +
1 vertical geophone

Figure 2.10: Rotaphone configurations

The basic features of the *Rotaphone* are: use of highly sensitive geophones connected to a common recorder; the geophones are mounted in diametrical pairs to a rigid frame coupled with the ground; the distance separating the geophones in a given pair is much smaller than the wavelength of the signal to measure.

Since rotations are measured through the derivatives of the displacement wave-field with respect to space coordinates, some issues can be assessed: first, the results are extremely sensitive to errors caused by differences in instrument responses and amplitudes either from instrument noise or site effects. Second, the estimation of the horizontal components of rotations depends on assumptions on subsurface properties.

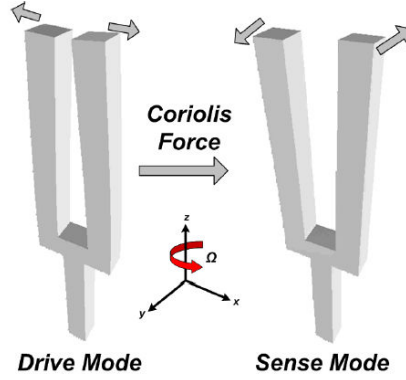


Figure 2.11: Illustration of Coriolis force (from [1])

2.4.4.2 Vibrating gyroscopes

Vibrating gyroscopes are based on *Coriolis effect*. The fundamental principle is simple: a proof mass is excited along a given axis at a frequency ω_1 (drive mode); when the system undergoes a rotation rate Ω , small displacements can be observed in a perpendicular direction in relation to the driving mode due to Coriolis effect with a frequency ω_2 (sense mode). The amplitude of Coriolis force is proportional to the rotation rate Ω through the expression:

$$F_{Coriolis} = -2m\vec{\Omega} \wedge \vec{V}_{drive} \quad (2.4)$$

Where m is the proof mass weight and $\vec{V}_{drive}$ is the proof mass velocity along the driving axis. An illustration of Coriolis effect is shown in the figure 2.11. Quartz tuning fork gyroscopes (Systron Donner “Gyrochip” model QRS11) presented in figure 2.12, were tested to measure the rotational components of strong ground motions [13], [14]). The sensor resolution is 1 mrad.s^{-1} RMS (60 Hz band) for a weight of 60 grams and a power consumption of 400 mW.

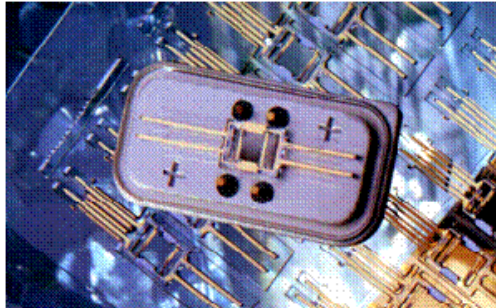


Figure 2.12: Systron Donner quartz rate sensor (model QRS11)

2.4.4.3 Angular accelerometers

Angular accelerometers are used to measure angular shocks or vibrations. These devices are designed with zero pendulosity (in other words, the center of gravity is located at the centroid of the supporting springs). In addition, they completely ignore linear acceleration and measure only rotational acceleration.

Angular acceleration, with associated angular velocity and displacement are very important measurements to be made in the analysis of structural behavior in wind or the process of pointing camera platforms, gun sights, and laser mounts. Delphi and ST Microelectronics, manufacturers of angular accelerometers, use capacitive MEMS sensors and custom Application Specific Integrated Circuits (ASICs) based on the Complementary Metal Oxide Semiconductor (CMOS) technology. One can find in figure 2.13 the angular accelerometer developed by ST Microelectronics (LIS1R02). A resolution of 2.5 rad.s^{-2} over the frequency band 30 Hz - 800 Hz is achieved for a power consumption of 30 mW ([15]). Furthermore, the dimensions of this sensor are approximately: 15 mm×7 mm×5 mm which make it a valuable candidate for seismic exploration except for resolution.

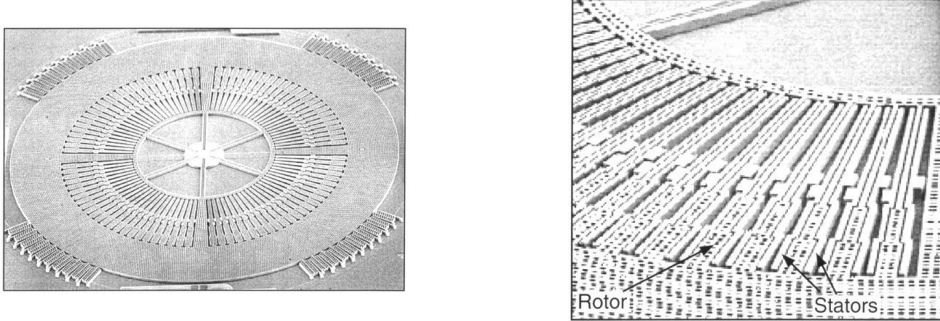


Figure 2.13: Microphotographs of the angular accelerometer from ST Microelectronics; Left: Entire view of the device. Right: Detailed view

This technology has several advantages: simplicity, small size and low cost. Moreover, it is very similar to linear accelerometers in terms of fabrication and readout. However, the main shortcoming is the low resolution compared to other devices stated along this section. Despite all, this technology has not been operated to its maximum potential compared to MEMS accelerometers or vibrating gyroscopes.

2.4.5 Summary table

TECHNOLOGY TYPE		COST	RESOLUTION	SIZE	POWER
PENDULUM SYSTEMS	TAPS system [7]	> 300 USD	10 $nrad.s^{-1}$ RMS (10 Hz band)	- -	-
	Torsion balance [8]	> 300 USD	3 $\mu rad.s^{-1}$ RMS (10 Hz band)		
OPTICAL GYROSCOPES	RLG [9]	> 1000 USD	0.3 $nrad.s^{-1}$ RMS (20 Hz band)	- - -	- - -
	FOG [10]	> 1000 USD	42.7 $nrad.s^{-1}$ RMS (20 Hz band)	- -	- - -
ELECTROCHEMICAL SENSORS	R-1 Eentec [11]	$\approx$ 5000 USD	1 $\mu rad.s^{-1}$ RMS (20 Hz band)	+	-
INERTIAL SENSORS	Rotaphone [12]	> 200 USD	10 $nrad.s^{-1}$ RMS (200 Hz band)	-	-
	Vibrating gyroscopes [14]	$\approx$ 100 USD	1 $mrاد.s^{-1}$ RMS (60 Hz band)	+++	+
	Angular accelerometers [15]	$\approx$ 10 USD	2.5 $rad.s^{-2}$ RMS (800 Hz band)	+++	++
Specifications for seismic exploration		<50 USD	5 $mrاد.s^{-2}$ (100 Hz band) 30 $\mu rad.s^{-1}$	9 cm^3	< 50 mW

Table 2.1: Summary of the technology review

2.5 Chapter summary and conclusions

In seismic exploration, most of the signal acquired by point-receiver geophones is dominated by surface waves or ground rolls. Because they propagate in the near surface, ground rolls do not contain any information on deeper targets. Thus, short spacing between receivers is required so that this noise component can be accurately characterized and removed by digital filtering. However, considering the cost of seismic exploration ventures, new acquisition techniques using fewer point receivers and larger spacing have to be developed. Such a technique has been proposed, but it needs accurate measurements of ground rotations at the free surface with minimum costs.

Rotations can be measured by a broad range of sensors, but none of the solutions developed so far is fully compliant with seismic exploration requirements. Indeed, different instruments have been developed in order to measure ground rotations in earthquake seismology. Nevertheless, seismic data are measured using either sophisticated and thus expensive ring laser technology or cumbersome seismic array techniques including some restrictive assumptions about the wavefield. In land seismic acquisition, where the seismic source and receivers are relatively close compared to strong seismology, the need is to develop low-priced sensors with enough sensitivity to measure the ground-roll horizontal gradient. That explains why geophysicists look at the MEMS industry to extend technology limits met during seismic exploration.

Indeed, the emergence of silicon microfabrication techniques has enabled new market possibilities, cost reduction and enhanced performances in some applications and especially in sensing rotation motions. On one hand, MEMS-based gyroscopes were developed for consumer application (i.e. smartphones) but also for high resolution applications such as inertial navigation. Nonetheless, few tests were performed to validate the use of MEMS gyroscopes in land seismic surveys but resolutions demonstrated by these devices are close to the specifications needed. On the other hand, MEMS angular accelerometers, developed for vibration monitoring, represent an interesting solution since they measure directly the angular vibrations in the same way as MEMS linear accelerometers measure directly the vertical wavefield during seismic surveys.

We have finally selected two potential solutions: vibrating gyroscopes and angular accelerometers. But in the frame of the PhD thesis, the focus has to be put on only one solution. The choice was made to capitalize on a provided ASIC [16] used to control MEMS linear accelerometer. Thus, the MEMS angular accelerometer was adopted. As angular accelerometers and linear accelerometers are based on the same operating principles, a unique control electronics can be used to interface different MEMS devices to measure translation and angular vibrations in a single sensor node during seismic surveys. Consequently, one can imagine the integration of two rotation sensors with one linear accelerometer, all working with the same control electronics as illustrated in figure 2.14 [17].

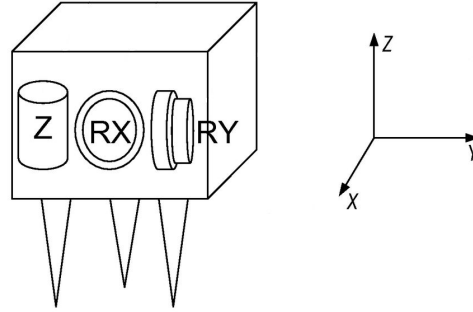


Figure 2.14: Integration of one vertical accelerometer (Z) with two rotation sensors (RX and RY) in one receiver node

In this dissertation, a high performance ASIC operating with differential capacitance accelerometers is reused to interface with an angular accelerometer working on the same principle. To optimize the behavior of this rotation sensor, one has to understand the key features of a rotating proof mass with differential capacitive read-out and the feedback control techniques associated. This is the aim of the next chapter.

Chapter 3

Working principles of a high performance capacitive micromachined rotation sensor

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This previous chapter selected MEMS angular accelerometers as rotation sensing systems for seismic exploration. To develop an optimized device with suitable performances, a high-performance ASIC developed for MEMS accelerometers using differential capacitance measurement is used. Moreover, a digital force-feedback control loop is implemented to improve the system dynamic behavior. The combination of a differential capacitance detection and a digital control loop is widely used for high performance applications ([18], [19]). The objective of this chapter is to give design rules to obtain a micro-structure sensitive to angular accelerations, and able to operate with the provided electronics.

Throughout this chapter, the focus is put on sensor resolution (or noise floor). The sensor noise floor is the total contribution of noise sources that limit the overall resolution. Thus, mechanical and electronic noises have to be carefully studied in order to tailor the appropriate noise distribution in the frequency range needed for seismic applications. The chapter is divided in four sections:

- First, the mechanical equations of the rotation sensor are given. The motion equation of the structure is established to understand how the rotations of a proof mass are related to the angular accelerations of its supporting frame. In a second step, the derivation of the mechanical noise is given.
- Second, the electrostatic behavior of the system is studied with respect to two ideal sensing principles: one with lateral capacitors and the other with gap-closing capacitors. The electrostatic moments are calculated and their influence on the mechanical equilibrium of the system is analyzed.
- Third, the front-end detector of the ASIC provided is studied. The conversion of the mechanical information of interest into an electrical voltage is explained with regards to the two sensing principles discussed before. Then, a review on detection noise is presented followed by a noise modeling of the front-end detector.
- Fourth, the digital force-feedback architecture of the ASIC is described. A brief review on $\Sigma\Delta$ -modulation is provided followed by its implementation to control the MEMS rotation sensor. A noise modeling step of the feedback generator is also given.

This chapter is concluded by two summary tables gathering all the design rules developed in this chapter.

3.1 Mechanical equations of the rotation sensor

This section deals with the mechanical equations of the rotation sensor subjected to angular accelerations of its reference frame. The mechanical sensitivity or gain is defined as well as the mechanical noise. Several assumptions are considered:

- Only small rotations around the vertical axis of the sensors frame (z-axis) are considered.
- Inertia of bending springs is supposed negligible.

3.1.1 Motion equation

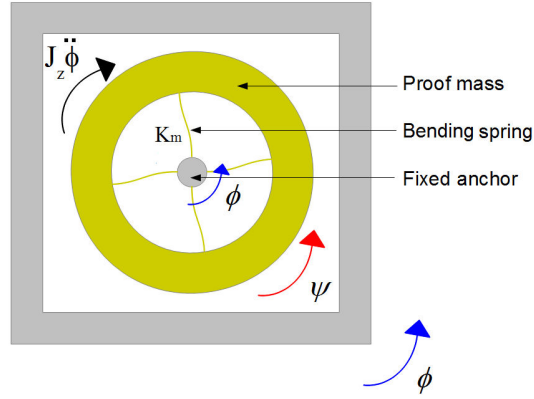


Figure 3.1: Working principle of the angular accerometer

The working principle of an angular accelerometer is shown in the figure 3.1. Due to an angular acceleration of the sensor frame $\ddot{\phi}$, which is the information to measure, an inertial moment acts on the proof mass which rotates by an angular displacement ψ . Supporting beams, acting like springs of constant K_m , are deformed by this inertia effect. Also, let us assume a damping mechanism acting on the system with a damping coefficient D proportional to the relative velocity $\psi - \phi$. The equilibrium equation is given by:

$$J_z \frac{d^2 \psi}{dt^2} + D \left(\frac{d\psi}{dt} - \frac{d\phi}{dt} \right) + K_m (\psi - \phi) = 0 \quad (3.1)$$

where J_z the moment of inertia of the ring. By introducing $\theta = \psi - \phi$, the equation (3.1) becomes:

$$J_z \frac{d^2 \theta}{dt^2} + D \frac{d\theta}{dt} + K_m \theta = -J_z \frac{d^2 \phi}{dt^2} \quad (3.2)$$

It can be noticed from equation (3.2) that the external angular acceleration acts like an inertial moment on the proof mass. As a consequence, measuring the quantity θ can be used to recover the information of interest $\frac{d^2\phi}{dt^2}$.

Equation (3.2) can be rewritten in a canonical form by introducing the natural frequency: $\omega_0 = \sqrt{K_m/J_z}$ and the quality factor: $Q = J_z\omega_0/D$.

Assuming a harmonic excitation of the frame: $\frac{d^2\phi}{dt^2} = -\Phi e^{j\omega t}$, the proof mass response is harmonic and can be solved as follows:

$$\frac{\Theta}{\Phi} = \frac{\frac{1}{\omega_0^2}}{-\frac{\omega^2}{\omega_0^2} + j\frac{\omega}{\omega_0}\frac{1}{Q} + 1} \quad (3.3)$$

where Θ and Φ are the amplitudes of the proof mass rotation and the amplitude of the frame angular acceleration respectively. The mechanical gain can be obtained by taking the magnitude of equation (3.3):

$$G_{mech} = \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{\omega\omega_0}{Q}\right)^2}} \quad (3.4)$$

When operated at frequencies far below from the natural frequency, i.e., when $\omega \ll \omega_0$, the equation (3.4) becomes:

$$G_{mech} = \frac{J_z}{K_m} = \frac{1}{\omega_0^2} \quad (3.5)$$

By observing the equation (3.5), it is obvious that the natural frequency must be as low as possible to obtain a high mechanical sensitivity. In other words, soft spring and great inertia are important to maximize the conversion of frame angular accelerations into proof mass angular displacements. This effect can be observed in the figure 3.3.

Also in the figure 3.3, one can notice the effect of the Q factor on the frequency response of the angular accelerometer. The Q factor is related to damping effects applied on the proof mass. The larger the value of Q, the sharper the resonance peak. If operated in open-loop mode, the Q factor must be at the critical damping value to maximize the bandwidth and the sensitivity of the system [20]. However, damping effects are related to mechanical noise, which will be discussed later, and high values of Q factor are needed to obtain lower noise. This is why

high resolution sensors are operated in closed-loop mode, which allows to work in under-damped conditions with very high values of Q factor.

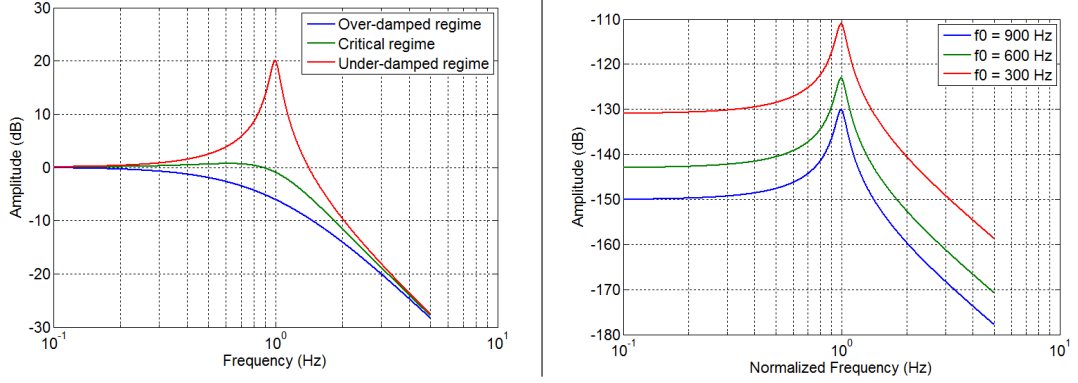


Figure 3.2: Influence of Q (left) and ω_0 (right) on the frequency response of the angular accelerometer

3.1.2 Mechanical noise

Since the introduction of micromachining process, scaling down dimensions brought new sensing possibilities at the expense of an important sensitivity to mechanical noise resulting from molecular motion. Fundamentally, systems that dissipate energy are at the same time a source of noise and vice-versa. For instance, the energy loss of an oscillator caused by damping mechanisms is converted into thermal agitation of the outside environment; this path works both way, which means that thermal agitation causes a motion of the oscillator. This is the fundamental principle of the equipartition theorem which states that energy is shared equally among all energetically accessible degrees of freedom of a system as developed in [21].

Let us consider the harmonic oscillator which has inertia J_z , damping coefficient D and stiffness coefficient K_m [figure 3.3]. Equipartition theorem applies to potential energy and kinetic energy since the system is fully described by two variables: rotation rate Ω and angular position θ . Through the damping mechanism, a force noise M_n is produced for which an expression is developed on the following development.

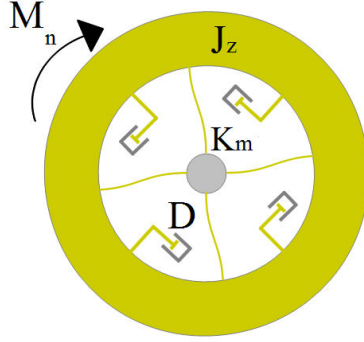


Figure 3.3: model to derive the mechanical noise

The motion equation of the system is:

$$J_z \frac{d^2 \theta_n}{dt^2} + D \frac{d\theta_n}{dt} + K_m \theta_n = M_n \quad (3.6)$$

where the subscript n describes noise functions or random processes. The equation is rewritten in terms of rotation rate $\Omega_n = \frac{d\theta_n}{dt}$ in the frequency domain:

$$j\omega J_z \hat{\Omega}_n + D \hat{\Omega}_n + \frac{K_m}{j\omega} \hat{\Omega}_n = \hat{M}_n \quad (3.7)$$

where $\hat{\Omega}_n$ and $\hat{M}_n$ are the Fourier transforms of Ω_n and M_n respectively. This notation and its applications are detailed in appendix A. The mean square rotation rate due to thermal noise is:

$$\overline{\Omega_n^2} = \frac{\overline{M_n^2}}{D^2 + (\omega J_z - K_m/\omega)^2} \quad (3.8)$$

One can rewrite the equation 3.8 in terms of angular frequency ω_0 and quality factor Q as follows:

$$\overline{\Omega_n^2} = \frac{1}{D^2} \frac{\overline{M_n^2}}{1 + Q^2(\omega/\omega_0 - \omega_0/\omega)^2} \quad (3.9)$$

The kinetic energy stored in a frequency interval is $dK = \frac{1}{2} J_z \overline{\Omega_n^2} d\omega$. Thus, the kinetic energy is given by (assuming $x = \omega/\omega_0$):

$$\langle K \rangle = \frac{1}{2} J_z \int_0^\infty \overline{\Omega_n^2} d\omega = \frac{1}{4\pi D} \int_0^\infty \frac{\overline{M_n^2} Q}{1 + Q^2(x - 1/x)^2} dx \quad (3.10)$$

In order to evaluate the integral in equation (3.10), one can assume that the quality factor is high enough so that all the system energy is confined near the resonance [22]. As a consequence, within a small frequency range, the noise spectral density is approximately constant and the integral can be solved as follows:

$$\langle K \rangle = \frac{\overline{M_n^2}}{8D} \quad (3.11)$$

Equating the kinetic energy expression with the thermal energy $\langle K \rangle = 1/2k_B T$, one can find:

$$\overline{M_n^2} = 4k_B T D \quad (3.12)$$

According to equation (3.12), one should notice that mechanical noise, or Brownian noise, is similar to Johnson-Nyquist noise in resistors [23].

Once the moment spectral density known, one can derive the power spectral density of the angular position noise θ_n :

$$\overline{\theta_n^2} = \frac{\overline{M_n^2}}{D^2 (Q^2 (\omega^2 - \omega_0^2) (\omega^2 + \omega_0^2) + (\omega\omega_0)^2)} \quad (3.13)$$

Considering: $\omega \ll \omega_0$ and according to equation (3.5), the angular acceleration noise density is given by:

$$\Phi_{mn} = \sqrt{\frac{4k_B T D}{J_z}} = \sqrt{\frac{4k_B T \omega_0}{Q J_z}} \quad \left[\frac{\text{rad}}{s^2 \sqrt{\text{Hz}}} \right] \quad (3.14)$$

Preliminary comments on the mechanical equations reveal that maximizing the second moment of inertia of the system J_z along with decreasing the natural frequency ω_0 , increases the mechanical sensitivity and reduces the thermo-mechanical noise. However, this can be obtained only for large chip sizes or heavy proof mass, which can make the process very challenging.

On the other hand, the mechanical noise is linearly dependent on the damping coefficient D . For moving structures with small sizes, the air damping is predominant upon other body forces and this effect becomes larger as micromachined structures decrease in size [24]. Therefore, estimating the damping of the system is one of the most important steps in the design process of micromechanical devices. Air damping mechanisms have different natures, depending on the geometry of moving parts, and must be carefully derived when designing rotation sensor prototypes. This will be studied in section 4.3.

3.2 Electrostatic actuators

Miniaturization of mechanical structures allows the use of electrostatic forces to drive or control moving parts. Depending on the configuration of electrodes, different effects may occur that can trigger non linearity or instability problems. However, an appropriate design allows to shape these effects and to take benefits from them in order to address high resolution applications. Thus, two capacitance sensing principles are studied utilizing a change in area or gap respectively to get a corresponding change in capacitance

Regardless of design issues, a fundamental principle of electrostatic actuators is the generation of only attractive forces [25]. Hence, a double-sided (or differential) capacitor is required to create forces in two directions. In the following development, two configurations using differential capacitor principle are presented: lateral capacitor and gap-closing capacitor. For simplicity, the fringing fields [26] will not be considered for the derivation of design formula.

3.2.1 Lateral-capacitor configuration

A lateral capacitor is composed of two plate electrodes moving laterally with respect to each other. This topology is studied because lateral capacitors have demonstrated linear behavior and high resolution as shown in [27]. In this section, the ideal structure presented in 3.4 is discussed with the calculation of the nominal capacitance and the electrostatic moments. At the end, the influence on the system equilibrium is analyzed.

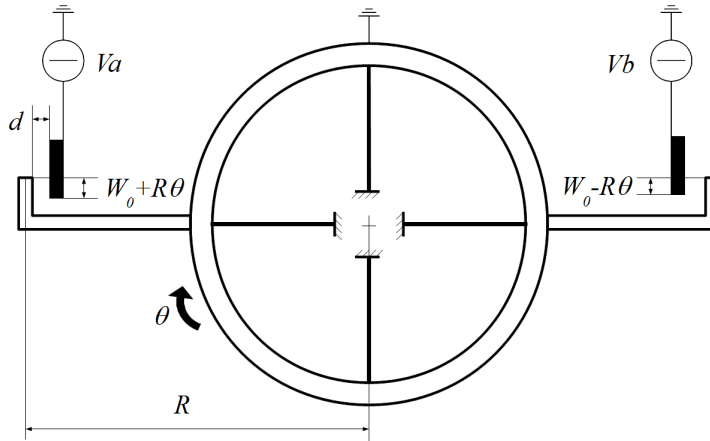


Figure 3.4: Schematic of a sliding capacitor

3.2.1.1 Capacitance calculation and configuration sensitivity

Let us consider a proof mass suspended by four springs and having two legs on both sides which play the role of moving electrodes A and B respectively. At the edge of each leg (at the distance R from the rotation center), a stator electrode is placed at a lateral gap distance d and with a nominal overlap W_0 . Each stator is connected to a supply voltage V_a and V_b respectively. The out-of-plane thickness of this system is T . A schematic of this double-sided capacitor was shown in the figure 3.4.

Assuming the parallel-plate capacitor approximation [25], the capacitances A and B under a small clockwise angular displacement θ of the proof mass are given by:

$$\begin{cases} C_a = \varepsilon_0 \frac{T(W_0 + R\theta)}{d} \\ C_b = \varepsilon_0 \frac{T(W_0 - R\theta)}{d} \end{cases} \quad (3.15)$$

Where ε_0 is the permittivity of air. The capacitance difference is then:

$$\Delta C = C_a - C_b = \varepsilon_0 \frac{2TR}{d} \theta \quad (3.16)$$

Introducing the nominal capacitance value $C_0 = \varepsilon_0 TW_0/d$, the capacitance difference can be rewritten as:

$$\Delta C = C_0 \frac{2R}{W_0} \theta \quad (3.17)$$

The displacement-to-capacitance sensitivity is finally obtained with:

$$K_{\theta-C} = \frac{\partial \Delta C}{\partial \theta} = C_0 \frac{2R}{W_0} \quad (3.18)$$

In this configuration, the capacitance difference is a linear function of the proof mass angular position θ . In addition, the displacement-to-capacitance sensitivity is a pure constant depending linearly on the nominal capacitance C_0 and the moving electrode distance from the rotation center R ; and it is inversely proportional to the overlap W_0 . Thus, to maximize the sensitivity, one has to put electrodes as far as possible from the rotation center with an important nominal capacitance value.

3.2.1.2 Electrostatic moment calculation

The electrostatic moments applied on the proof mass can be derived from the electrostatic energy according to [28]:

$$\begin{cases} M_a = \frac{1}{2} \frac{\partial C_a}{\partial \theta} V_a^2 \\ M_b = \frac{1}{2} \frac{\partial C_b}{\partial \theta} V_b^2 \end{cases} \quad (3.19)$$

substituting the capacitance expressions from equation (3.15), one can find:

$$\begin{cases} M_a = \frac{1}{2} \frac{C_0 R}{W_0} V_a^2 \\ M_b = -\frac{1}{2} \frac{C_0 R}{W_0} V_b^2 \end{cases} \quad (3.20)$$

The resulting electrostatic moment is then given by the following expression:

$$M_{el} = M_a + M_b = \frac{1}{2} \frac{C_0 R}{W_0} (V_a^2 - V_b^2) \quad (3.21)$$

This electrostatic moment can be used for a force-feedback control of the proof mass. However, one can notice from equation (3.21) that the electrostatic moment is a non linear function of applied voltages V_a and V_b . A strategy to linearize the moment uses a Direct Current (DC) bias V_{DC} applied to the two static combs A and B so that the net force on the proof mass is zero. Then, an Alternating Current (AC) feedback voltage V_{fb} is added to the dc bias on one electrode and subtracted from the other electrode([25], [29]). Further details on this feedback voltage V_{fb} will be discussed in section 3.4.2.1. Accordingly, the voltages V_a and V_b can be decomposed as follows:

$$\begin{cases} V_a = V_{DC} + V_{fb} \\ V_b = V_{DC} - V_{fb} \end{cases} \quad (3.22)$$

And consequently,

$$M_{el} = \frac{2C_0 R}{W_0} V_{DC} V_{fb} = K_{\theta-C} V_{DC} V_{fb} \quad (3.23)$$

It can be seen from equation (3.23) that the feedback moment is proportional to the voltage V_{fb} . It is further seen from equation (3.22) that the maximum value of the feedback voltage V_{fb} is the bias voltage V_{DC} . This is because the voltage V_b becomes zero at this voltage. This determines the operating range of the angular accelerometer. In other words, the maximum force that the differential capacitor can bring is given by:

$$M_{el_{max}} = K_{\theta-C} V_{DC}^2 \quad (3.24)$$

3.2.1.3 Influence of the lateral capacitor configuration on the proof mass mechanical behavior

One can notice, according to equation (3.17), the linearity of the capacitance difference with respect to angular position θ . It has been said that the electrical sensitivity can be maximized by increasing the nominal capacitance value along with putting the capacitor far from the centroid. A simple way to increase the nominal capacitance value is to use comb electrodes that allow the addition of several small sliding-plate capacitors.

Rewriting the equilibrium equation of the angular accelerometer from equation (3.1) by adding the electrostatic moment from equation (3.23), one can find:

$$\underbrace{J_z \ddot{\theta} + D \dot{\theta} + K_m \theta}_{\text{proof mass mechanical behavior}} = \underbrace{-J_z \ddot{\phi}}_{\text{external acceleration}} + \underbrace{K_{\theta-C} V_{DC} V_{fb}}_{\text{Electrostatic moment}} \quad (3.25)$$

This section provided an electrostatic study of an ideal structure using lateral capacitors. It was shown that the capacitance difference ΔC is a linear function of the angular position θ . With the particular voltage supply given in equation (3.22), an electrostatic moment was introduced in the system equilibrium that does not depend on θ (see equation (3.25)). This electrostatic moment will be used to implement a force feedback control through the feedback voltage V_{fb} .

3.2.2 Gap-closing capacitor configuration

A sensor using gap-closing capacitors utilizes a change of gap to get a corresponding change in capacitance. Contrary to lateral capacitors studied in the last section, gap changes can create a relatively large displacement-to capacitance sensitivity at the cost of a non-linear response. The electrostatic properties of the structure shown in figure 3.5 are discussed with a particular focus on the influence of the electrostatic moments on the system equilibrium.

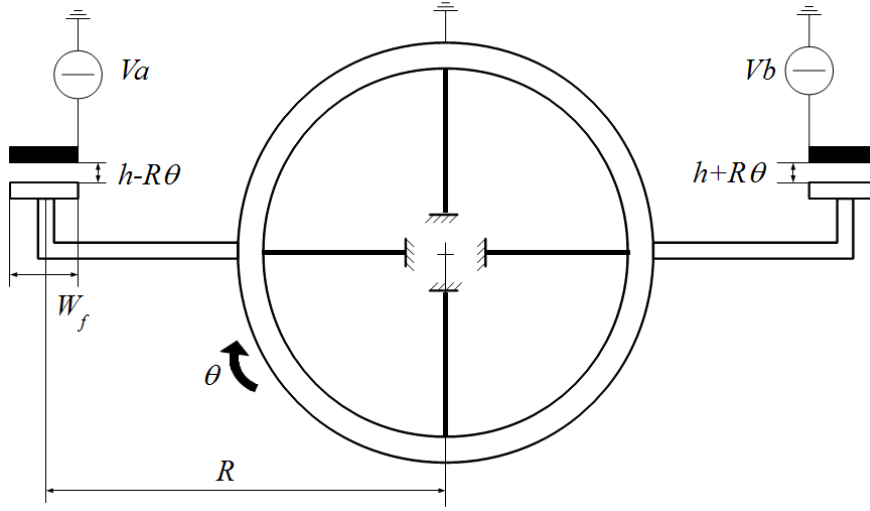


Figure 3.5: Schematic of a normal capacitor

3.2.2.1 Capacitance calculation and configuration sensitivity

Let us consider the same system as the previous section except that, at the edge of each leg, a stator electrode is placed at a normal gap distance h . Here the movable electrode width is W_f . Each stator is connected to a DC supply voltage V_a and V_b respectively. The out-of-plane thickness is T . A schematic of this double-sided capacitor was shown in the figure 3.5.

The capacitances A and B under a small clockwise angular displacement θ of the proof mass are given by:

$$\begin{cases} C_a = \varepsilon_0 \frac{W_f T}{(h - R\theta)} \\ C_b = \varepsilon_0 \frac{W_f T}{(h + R\theta)} \end{cases} \quad (3.26)$$

Introducing the nominal capacitance value $C_0 = \varepsilon_0 TW_f/h$, the capacitance difference can be given by:

$$\Delta C = C_0 \frac{2R\theta/h}{1 - (R\theta/h)^2} \quad (3.27)$$

One can notice that, in gap-closing configuration, the capacitance difference is a nonlinear function of the angular displacement θ , and thus, the displacement-to-capacitance sensitivity is nonlinear too. However, assuming small values of θ so that $R\theta/h \ll 1$, one can introduce the following constant:

$$K_{\theta-C} \approx C_0 \frac{2R}{h} \quad (3.28)$$

From equation (3.28), a simple way to maximize the capacitance in this configuration is to minimize the normal gap dimension h and to maximize the distance from the centroid R . One should notice that the linearized displacement-to-capacitance expression is similar to the one obtained for sliding-plates in equation (3.18) by replacing W_0 by h .

3.2.2.2 Electrostatic moment calculation

Again, the electrostatic moments applied on the proof mass can be derived from the electrostatic energy. Moreover, in order to linearize the resulting electrostatic moment expression with respect to a control voltage, one can assume that the voltages V_a and V_b can be decomposed in two terms, a DC bias V_{DC} and a feedback voltage V_{fb} as defined in equation 3.22. The resulting moment is then given by the expression:

$$M_{el} = M_a + M_b = \frac{2C_0 \frac{R}{h} \left(V_{DC} + V_{fb} \frac{R}{h} \theta \right) \left(V_{DC} \frac{R}{h} \theta + V_{fb} \right)}{\left(-1 + \frac{R}{h} \theta \right)^2 \left(1 + \frac{R}{h} \theta \right)^2} \quad (3.29)$$

Assuming small values of θ so that $R\theta/h \ll 1$, one can write the second order approximation of the moment expression:

$$M_{el} \approx \underbrace{K_{\theta-C} V_{DC} V_{fb}}_{M_{el0}} + \underbrace{2C_0 \left(\frac{R}{h} \right)^2 (V_{DC}^2 + V_{fb}^2) \theta}_{K_{el}} \quad (3.30)$$

In the equation (3.30), one can notice that the resulting moment that can be applied has a constant component: M_{el0} , and a spring component: $M_{el1} = K_{el}\theta$ varying with the angular position θ . Assuming the maximum feedback voltage $V_{fb} = V_{DC}$, the maximum restoring force and the maximum electrostatic spring coefficient are given respectively by:

$$\begin{cases} M_{el_{max}} &= K_{\theta-C} V_{DC}^2 \\ K_{el} &= 4C_0 \left(\frac{R}{h}\right)^2 V_{DC}^2 \end{cases} \quad (3.31)$$

Few comments have to be made on the electrostatic moments calculated in equation (3.30). One can notice the presence of a component M_{el0} which is similar to the one calculated for the lateral configuration (equation 3.23). However, the non-linearity of the gap-closing configuration introduces an electrostatic spring term which increases as the normal gap h decreases in size.

3.2.2.3 Influence of the gap-closing capacitor configuration on the proof mass mechanical behavior

This case brings more complexity due to the nonlinear dependence of the capacitance difference with the angular position θ in (3.27). However, a first order approximation gives a displacement-to-capacitance coefficient very similar to the one obtained in the sliding-plates configuration. Indeed, the comparison between equation (3.28) and (3.18) shows that the overlap parameter W_0 and the normal gap h have the same influence for their respective configuration. Moreover, the gap distance is usually much smaller than the overlap between movable electrodes and fixed electrodes. For equivalent capacitance between the two configurations, it can be written:

$$\frac{K_{\theta-C_{lateral}}}{K_{\theta-C_{normal}}} = \frac{h}{W_0} < 1 \quad (3.32)$$

From the equation (3.32), the linearized gap-closing configuration is much more sensitive to the detection of rotation of a rotating proof mass.

Nevertheless, best sensitivity can only be obtained by using long plate electrode, i.e. an important distance from the centroid R . In this case, the first order approximation is no more valid and one has to look carefully at the non-linearity introduced here.

To this end, let us consider that the proof mass is actuated by a single voltage V at electrode A (i.e. $Vb = 0$). The balance position of the proof mass is given by:

$$M_{net} = \frac{1}{2}C_0V^2 \frac{\frac{R}{h}}{\left(1 - \frac{R}{h}\theta\right)^2} - K_m\theta = 0 \quad (3.33)$$

Now, let us assume small perturbations of the gap $\theta + \delta\theta$. One can write:

$$\delta M_{net} = \frac{\partial M_{net}}{\partial \theta} \delta\theta \quad (3.34)$$

If $\frac{\partial M_{net}}{\partial \theta} > 0$, a small increase $\delta\theta$ creates a moment which tends to pull the moving plate to contact with the fixed plate. For a stable equilibrium, it is necessary to keep $\frac{\partial M_{net}}{\partial \theta} < 0$. The equilibrium condition is then given by:

$$K_m > C_0V^2 \frac{\left(\frac{R}{h}\right)^2}{\left(1 - \frac{R}{h}\theta\right)^3} \quad (3.35)$$

Substituting equation (3.35) in (3.33), the stability region is defined as follows:

$$\theta < \frac{h}{3R} \quad (3.36)$$

Since the equilibrium angular position decreases with increasing voltages, there is a specific voltage from which the stability vanished: it is the so-called pull-in effect occurring at the voltage V_{pi} [28]. The pull-in condition satisfies the condition $M_{net} = 0$, which requires that:

$$\theta_{pi} = \frac{h}{3R} \quad (3.37)$$

Hence, the pull-in voltage is given by:

$$V_{pi} = \frac{h}{R} \sqrt{\frac{8}{27} \frac{K_m}{C_0}} \quad (3.38)$$

One should note that maximizing the distance from the centroid R brings more sensitivity but it reduces also the stability range with a smaller pull-in angle θ_{pi} according to equation (3.37).

So far, in this non-linearity development of the gap-closing capacitor configuration, we have only considered the static equilibrium of the rotation sensor with only one electrode polarized. In what follows, the behavior of the system under the influence of the two electrodes A and B is studied. The voltages applied at the electrodes A and B are functions of the components V_{DC} and V_{fb} , so that we can add the moment expression of equation (3.29) in the sensor dynamic equation:

$$J_z \ddot{\theta} + D \dot{\theta} + K_m \theta = -J_z \ddot{\phi} + \frac{2C_0 \frac{R}{h} \left(V_{DC} + V_{fb} \frac{R}{h} \theta \right) \left(V_{DC} \frac{R}{h} \theta + V_{fb} \right)}{\left(-1 + \frac{R}{h} \theta \right)^2 \left(1 + \frac{R}{h} \theta \right)^2} \quad (3.39)$$

In order to simplify the previous equation and to understand the behavior of a double-actuated capacitor, let us introduce the second order approximation of the moment function from equation (3.30). It gives:

$$J_z \ddot{\theta} + D \dot{\theta} + (K_m - K_{el}) \theta = M_{el0} \quad (3.40)$$

One can notice in equation (3.40) that the electrical spring counteracts the mechanical one. This characteristic is important since the resonance frequency of the rotation sensor is decreased when the electrodes are polarized. Furthermore, a reduced resonance frequency value results in a larger mechanical sensitivity as defined in equation (3.5). Otherwise, the presence of the electrostatic term M_{el0} can be used to implement a force feedback control of the proof mass position.

3.3 Differential capacitive sensing

The proof mass angular displacements must be converted into an electrical signal. The electronics circuitry has to be sensitive enough to detect such change in capacitance. Usually, a detection circuit using a charge amplifier is a standard way to obtain high gain, and thus, high sensitivity [30]. However, such a technique introduces noises which decide the minimum signal detectable or, in other words, the resolution of the position detection [31]. These noise sources come from the operational amplifier used for the detection as it contains input offset, thermal noise at resistors, Flicker noise ($1/f$ noise) when white noise is integrated by the feedback capacitor, etc. . Hence, the design of a front-end electronics is critical to obtain enough sensitivity while keeping noise level low.

In the following development, the front-end detector is studied with a particular attention paid to the two sensing principles seen in the previous section: lateral capacitor and gap-closing capacitor.

3.3.1 Front-end detection using a charge amplifier

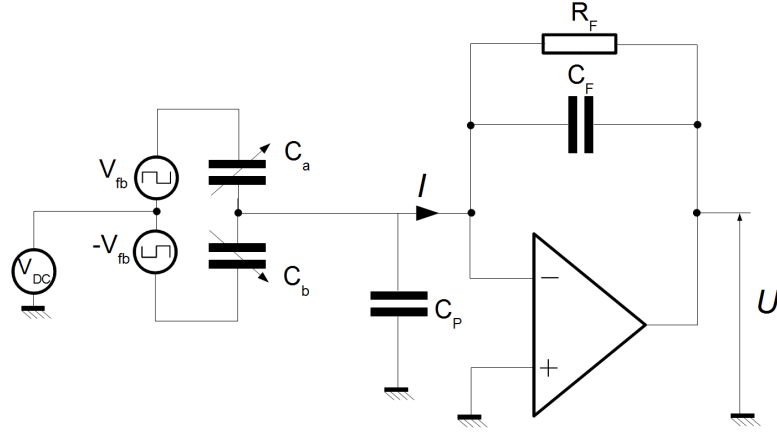


Figure 3.6: Position amplifier principle

A position amplifier is used to detect the capacitance changes in the differential sensor element as shown in figure 3.6. Let us consider that a DC voltage V_{DC} is applied at each electrode and that an AC voltage V_{fb} is added to one electrode and subtracted from the other. The sampling frequency of the detector provided by the ASIC is 1 MHz . The parasitic capacitance C_p does not affect the transduction, but it is included because it has an influence on noise performance. This will be studied further in this section.

The sensor is driven by a high-frequency signal V_{fb} , hence the feedback impedance is dominated by the capacitor C_f . As a result, the voltage output U of the front-end circuit, in function of the charge current I is given by:

$$U = - \int \frac{I}{C_f} dt \quad (3.41)$$

The charge current I is the sum of the the currents passing through the varying capacitors. Since the capacitance variations are of the order of magnitude of the system dynamics ($> 1 \text{ ms}$), and since the voltage variations are of the order of magnitude of the detector sampling frequency ($< 1 \text{ }\mu\text{s}$), one can assume that the capacitances operate in the quasi-static regime. Thus, the equation giving the current I is:

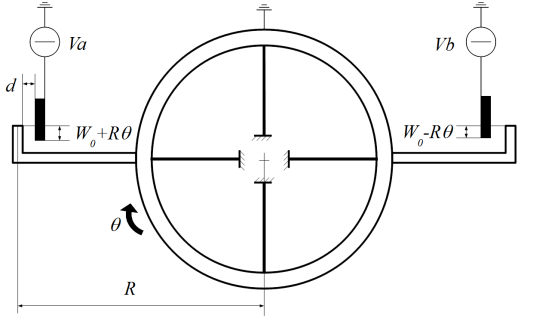
$$I = \frac{dV_{fb}}{dt} (C_a - C_b) \quad (3.42)$$

Replacing the current expression from equation (3.41), the relation between the capacitance difference and the output voltage of the front-end circuit is:

$$U = \frac{\Delta V_{fb}}{C_f} \Delta C = \frac{2V_{DC}}{C_f} \Delta C = G_{amp} \Delta C \quad (3.43)$$

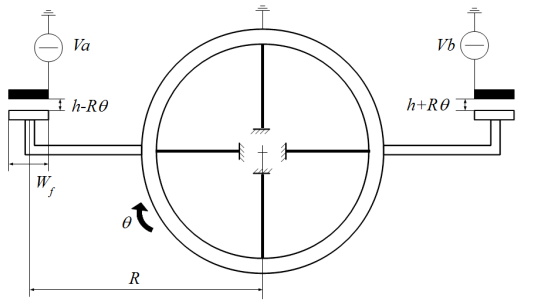
where it is assumed that V_{fb} is a square signal varying between two voltage levels $-V_{DC}$ and $+V_{DC}$, therefore, $\Delta V_{fb} = 2V_{DC}$. According to equation (3.43), one can replace the expressions of ΔC calculated for the two configurations studied in the last section (see figures 3.7 and 3.8).

Consequently, the capacitance variations of the rotation sensor are measured through the voltage U at the output of the charge amplifier. On one hand, it can be noticed that the detection in the lateral capacitor configuration is purely linear with a proportional relation between U and θ (equation (3.44)). On the other hand, the detection is nonlinear in the gap-closing configuration as shown in equation (3.45). However, under small rotation condition, the detection can be linearized according to equation (3.46). Finally, one can notice that the overlap W_0 and the normal gap h play the same role for their respective configuration.



$$U = \frac{2G_{amp}C_0R}{W_0}\theta \quad (3.44)$$

Figure 3.7: Output voltage of the front-end circuit for sliding-plate capacitors



$$U = \frac{4C_0V_{DC}}{C_f} \frac{R\theta/h}{1 - (R\theta/h)^2} \quad (3.45)$$

$$\text{For } \frac{R\theta}{h} \ll 1, \quad U \approx \frac{2G_{amp}C_0R}{h}\theta \quad (3.46)$$

Figure 3.8: Output voltage of the front-end circuit for gap-closing capacitors

3.3.2 Modeling of amplifier noise

High sensitivity is useless if the noise floor of the device exceeds the minimum signal level to detect. This fundamental resolution is induced by noise sources affecting the different stages of the sensor loop. Essentially, noise mechanisms are divided in two groups: mechanical noise and electrical noise. The first mechanism was introduced in the mechanical section whereas the second is studied in the following development. To be more specific about electrical noise, some of the noise sources are dominant in open-loop mode whereas others are dominant in closed-loop mode [32]. In what follows, a brief review on electronic noise of charge amplifier detector is given. In a second step, the modeling of the detector noise provided by the ASIC is presented.

Several techniques have been used in inertial grade accelerometers to reach a good sensitivity while keeping the noise level low. For instance, switched capacitance is a relatively easy technique intended for discrete, sampling based systems [33]. However, the charge amplifier has a bias resistor that generates white current noise. When integrated by the feedback capacitor, this produces a 1/f-

shaped noise spectrum that must be suppressed by the position detector. At high operating frequencies, the $1/f$ -noise decreases below the thermal noise (or white noise) level at the so-called corner frequency. But this white noise is not equal to the operational amplifier thermal noise floor, since high-frequency components are folded back to the baseband due to the sampling action. In other words, the higher the bandwidth of the amplifier, the larger the sampled noise at the output.

This is why we use correlated double sampling (CDS) as an extension of switched-capacitor technique [34]. Indeed, CDS allows to reduce dramatically the $1/f$ noise and enhances the effective gain of the op-amp. Nevertheless, the CDS cannot suppress the wideband noise at the amplifier output, and this noise is aliased to baseband frequencies. Further development and techniques can be found in [35].

Let us consider an equivalent noise model for the charge amplifier presented in figure 3.9. In particular, let us assume that the contribution of amplification stages can be represented as an equivalent noise source at the amplifier input $\overline{V_n^2}$. Then, let us assume that the noise contributions at the amplifier output is modeled by a lumped parameter R_{det} .

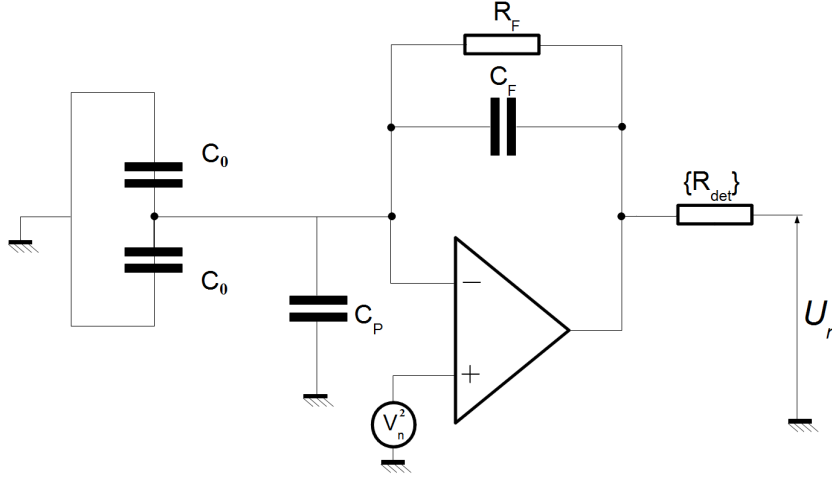


Figure 3.9: equivalent amplifier noise model

Consequently, the amplifier noise is given by:

$$\overline{V_{amp}^2} = G_n^2 \overline{V_n^2} \quad (3.47)$$

where $G_n = \frac{2C_0 + C_p + C_f}{C_f}$ is the noise gain of the amplifier.

Then, the white noise spectrum created at the output of the detector is:

$$\overline{V_{det}^2} = 4k_B T R_{det} \quad (3.48)$$

As a result the equivalent noise of the position detector provided by the ASIC is:

$$\sqrt{\overline{V_{eq}^2}} = \sqrt{\overline{V_{amp}^2} + \overline{V_{det}^2}} \quad (3.49)$$

By replacing equations (3.47) and (3.48) in expression (3.49), the equivalent electrical noise is given finally by:

$$\sqrt{\overline{V_{eq}^2}} = \sqrt{\left(\frac{2C_0 + C_p + C_f}{C_f}\right)^2 \overline{V_n^2} + 4k_B T R_{det}} \quad (3.50)$$

Referring to electrical sensitivity obtained at equation (3.43) along with the electrical noise expression, one can obtain the signal-to-noise ratio (SNR) of the interface circuit of the open-loop mode as follows:

$$SNR = \frac{2V_{DC}\Delta C}{C_f} \frac{1}{\sqrt{G_n^2 \overline{V_n^2} + 4k_B T R_{det}}} \quad (3.51)$$

It is interesting to express the electrical noise with units: $\frac{rad}{s^2} \frac{1}{\sqrt{Hz}}$. This can be made by transporting the noise density through the electrical and mechanical amplification. Finally, the detector noise is:

$$\Phi_{en} = \sqrt{\left(G_n^2 \overline{V_n^2} + 4k_B T R_{det}\right)} \frac{1}{G_{mech} G_{det}} \left[\frac{rad}{s^2} \frac{1}{\sqrt{Hz}} \right] \quad (3.52)$$

Where G_{mech} is the mechanical gain defined in (3.4) and G_{det} is the detector gain defined as:

$$G_{det} = \frac{2V_{DC}}{C_f} K_{\theta-C} = G_{amp} K_{\theta-C} \quad (3.53)$$

Observing equation (3.52), the best resolution is obtained for high mechanical gain G_{mech} and high detector gain G_{det} . Consequently, one has to prefer structures with low resonance frequency and high nominal capacitance values.

3.4 Digital force feedback control loop

High-resolution capacitive sensors are usually operated in the closed-loop mode to increase their bandwidth and performance. In this mode, the output voltage of the capacitance detection circuit is sent back to the proof-mass, generating a restoring electrostatic force, which maintains the proof mass close to its zero position. Moreover, the small amplitudes obtained through this control improve the linearity of the system, especially when gap-closing capacitors are used. The main issue when designing a control loop for capacitive sensor is that the electrostatic force generated is a nonlinear function of the voltage. As a consequence, linearization techniques are needed. In the provided ASIC, a digital feedback using a $\Sigma\Delta$ modulator is implemented.

This section gives a brief review on control loop using $\Sigma\Delta$ modulation followed by a study of the interactions of such a control with the MEMS rotation sensor.

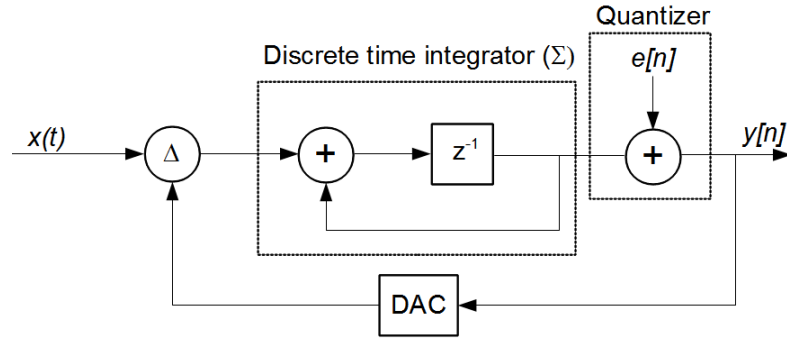
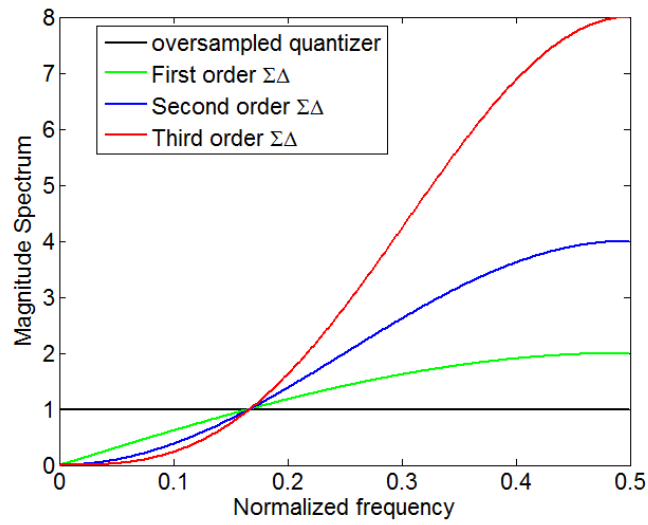
3.4.1 $\Sigma\Delta$ modulation

$\Sigma\Delta$ -modulation [36] is a method for encoding analog signals into digital signals or, in other words, higher-resolution digital signals into lower-resolution digital signals. A $\Sigma\Delta$ analog-to-digital converter (ADC) is performed using error feedback, where the difference between the two signals is measured and used to improve the conversion. The efficiency of the system relies on the use of a lower-resolution high-frequency signal, i.e. oversampling.

According to figure 3.10, the input to the circuit($x(t)$) feeds to the quantizer via an integrator (Σ), and the quantized output ($y[n]$) is fed back to be subtracted from the input signal (Δ). This feedback forces the average value of the quantized signal to track the average input. Any persistent difference between them accumulates in the integrator and eventually corrects itself [37]. If the Digital-to-Analog converter (DAC) is ideal, it is replaced by a unity gain transfer function so that the corresponding time domain expression of the modulator output is:

$$y[n] = x[n-1] + e[n] - e[n-1] \quad (3.54)$$

Where $e[n] - e[n-1]$ is the first order difference of $e[n]$, which is the quantization error.

Figure 3.10: Principle of $\Sigma\Delta$ modulationFigure 3.11: Noise spectrum of different ADC, from an oversampled quantizer to a 4th-order $\Sigma\Delta$ -modulator

$\Sigma\Delta$ -modulation can shape the quantization noise resulting in a attenuated noise in the band of interest. The larger the integrator order, the lower the noise in the band of interest as illustrated in figure 3.11. First order modulators are unconditionally stable, unlike modulators with higher integrator orders. Consequently, one has to pay attention to stability for higher order modulators. Finally, the noise at higher frequencies can be eliminated using digital filters and decimation, resulting in a digital signal with low quantization noise.

3.4.2 Implementation of the digital feedback with the rotation sensor

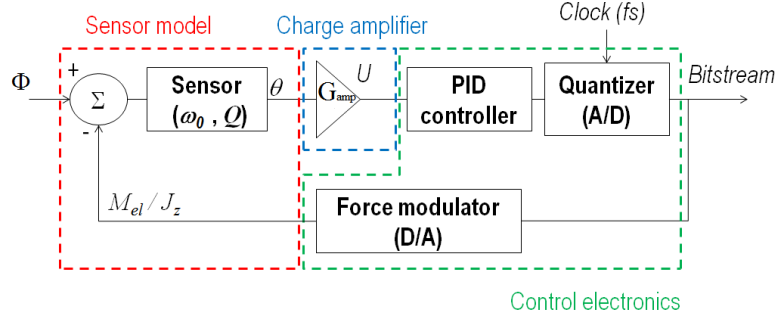


Figure 3.12: Structure of a digital MEMS inertial sensor

The sigma-delta modulator controller, provided by the ASIC, can be used to design a digital, closed-loop rotation sensor. Figure 3.12 shows the structure of a digital MEMS rotation sensor. The signal M_{el} works as the feedback of the system, opposing to the signal Φ . Since the sensor has the dynamics of a second order system, a controller is used to enforce the stability of the whole system, as well as to reduce the noise in the band of interest (20 Hz - 200 Hz).

This structure presents numerous advantages over the analog closed loop implementation as studied in [38]. Some are cited here:

- The output signal is directly digital, no need for an external Digital-to-Analog Converter;
- Trade-off between accuracy and bandwidth is possible;
- Superior stability if compared to analog implementation of closed loop;
- Elimination of the non-linearity between voltage and electrostatic force.

In section 3.2.1, an electrostatic M_{el} moment was identified for the implementation of a force feedback control. This electrostatic moment expression was given in equations (3.23) and (3.30):

$$M_{el} = K_{\theta-C} V_{DC} V_{fb} \quad (3.55)$$

In order to close the loop, one has to understand the generation of the feedback signal V_{fb} by the $\Sigma\Delta$ controller and how it is used to linearize the control moment M_{el} . This is the aim of the next section.

3.4.2.1 Bitstream conversion for linearized feedback

The bitstream output from the $\Sigma\Delta$ -converter represents the analog signal encoded in a pulse density modulation (PDM) signal. In figure 3.13, one can find the bitstream representation of a sine signal. In this illustration, one can notice that the sampling frequency is 50 times higher than the signal frequency. Real implementations can reach an oversampling rate (OSR) of 256 [39]. As explained before, this bitstream can be low-pass filtered (or averaged) to recover the information encoded.

In addition, the bitstream feeds also a force modulator which control the feedback voltage V_{fb} . This feedback voltage can take two levels depending on the bit value: V_{DC} if bit = +1 or $-V_{DC}$ if bit = -1. As a result, one can write the feedback voltage at the DAC output:

$$V_{fb} = D_c \text{sign}(bit)V_{DC} \quad (3.56)$$

Where D_c is a duty-cycle parameter controlling the alternation between actuation and sensing. The average feedback voltage is given by the integration of V_{fb} over a period of the sensing element $T_0 \gg T_s$ where T_s is the sampling period, which can be written as follows:

$$\overline{V_{fb}} = D_c V_{DC} \int_0^{T_0} \text{sign}(bit) dt = D_c V_{DC} \nu \quad (3.57)$$

Through equation (3.57), one can notice the linearization of the control with respect to the control parameter $\nu \in [-1; 1]$. In addition, one should notice that the alternation of V_{fb} between V_{DC} and $-V_{DC}$ does not influence its squared value which remains at V_{DC}^2 .

It is then possible to calculate the average feedback force of the digital controller using the generic electrostatic moment expressions obtained in equations (3.23) and (3.30):

$$\overline{M_{el}} = K_{\theta-C} V_{DC} \overline{V_{fb}} = K_{\theta-C} D_c V_{DC}^2 \nu \quad (3.58)$$

As a result, one can define the feedback gain as follows:

$$K_{fb} = \frac{\overline{M_{el}}}{J_z \nu} = \frac{K_{\theta-C} V_{DC}}{J_z} \overline{V_{fb}} = \frac{K_{\theta-C} D_c V_{DC}^2}{J_z} \quad (3.59)$$

To conclude this development, a linearized electrostatic moment was calculated with respect to a control parameter ν . This control parameter results from the decimation (or the averaging) of the bitstream output given by the $\Sigma\Delta$ modulator. Finally a feedback gain K_{fb} was obtained which represents the full-scale of the rotation sensor or, in other words, the maximum angular acceleration that can be controlled by the system.

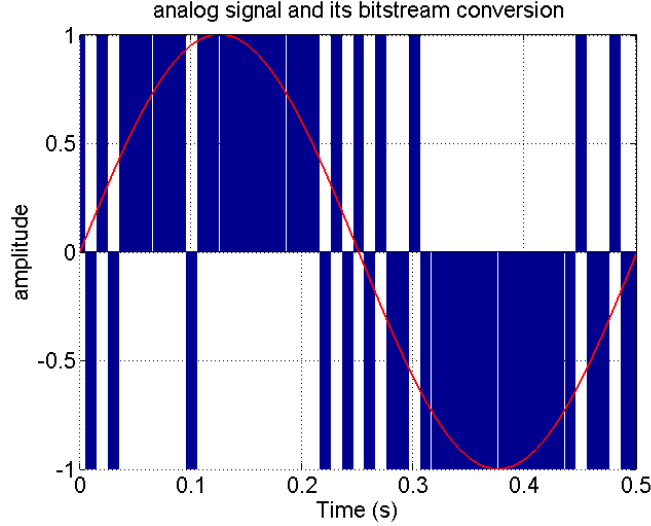


Figure 3.13: Analog signal and its bit-stream conversion

3.4.2.2 Modeling of noise in the closed-loop mode

Additional noise sources occur when the sensor operates in a closed-loop mode. Quantization noise is one of these noise sources but it is not predominant since the $\Sigma\Delta$ strategy is employed [40]. Another source is the proof mass residual motion resulting from the electrostatic feedback [34]. When this feedback is applied through a digital pulse train, it results in a periodic motion of the proof mass around its equilibrium position, even under zero external excitation. This noise is inversely proportional to the square of the sampling frequency, therefore, it could become dominant for low sampling frequency. Since a high sampling frequency of 1 MHz is applied by the ASIC, this source is not predominant in the closed-loop mode of the rotation sensor.

An important noise contribution comes from the DAC when the bitstream output is converted into an analog square signal of amplitude V_{DC} . Indeed, small voltage variations (or noise) are observed which are injected in the sensor loop. Consequently, the noise component in the closed-loop mode of the provided ASIC

can be modeled by a lumped parameter R_{dac} at the Digital-to-Analog Converter generating a white noise spectrum of: $\sqrt{4k_B T R_{dac}}$. This electronic noise results in slight changes in the feedback voltage applied to control electrodes, therefore, this noise is injected in the sensor loop during the force-rebalancing of the proof mass. The DAC noise can be expressed in units: $\frac{rad}{s^2} \frac{1}{\sqrt{Hz}}$ by transporting the noise spectrum through the feedback coefficient such as:

$$\Phi_{fn} = \sqrt{4k_B T R_{dac}} K_{fb} \left[\frac{rad}{s^2} \frac{1}{\sqrt{Hz}} \right] \quad (3.60)$$

this noise depends on the feedback gain K_{fb} which means that the system full-scale needs to be limited for better performances. However, the same electrodes are used for both detection and actuation. Thus, a smaller full-scale will result in a lower detection gain since they depend both on the constant $K_{\theta-C}$.

3.5 Chapter summary and conclusions

This chapter reviewed the design principles to develop high-resolution, high-sensitivity, MEMS-based rotation sensor. The mechanical amplification of an angular-accelerometer structure was given and two capacitive sensing principles, using lateral capacitors and gap-closing plate capacitors respectively, were studied with a particular focus on detection and control. Specifically, it was found that lateral capacitors are less sensitive to proof mass angular displacements than gap-closing capacitors. However, the latter configuration can show unstable behavior for important angular displacements, thus, operating the rotation sensor in the closed-loop mode was found preferable. Indeed, feedback force control keeps the proof mass close to its zero position and, as a consequence, it increases the linearity and the stability of the system. At last, one had to pay attention to noise mechanisms since they are related to the sensor noise floor. This noise floor is the minimum signal that the rotation sensor can detect, hence, it has to be as low as possible.

The two configurations were represented with the control electronics in figures 3.14 and 3.15 as well as their respective design formulas summarized in tables 3.1 and 3.2.

Once the design parameters defined, it is possible to propose different prototypes meeting the requirements of seismic exploration. Designing these prototypes will follow the same logic as this chapter since trade-offs between sensitivity, resolution and stability will be a major consideration.

The next chapter will present and compare two rotation sensor designs : one working with sliding-comb electrodes and another working with pure gap-closing electrodes.

3.5.1 Closed-loop design formula of sliding-plate capacitors

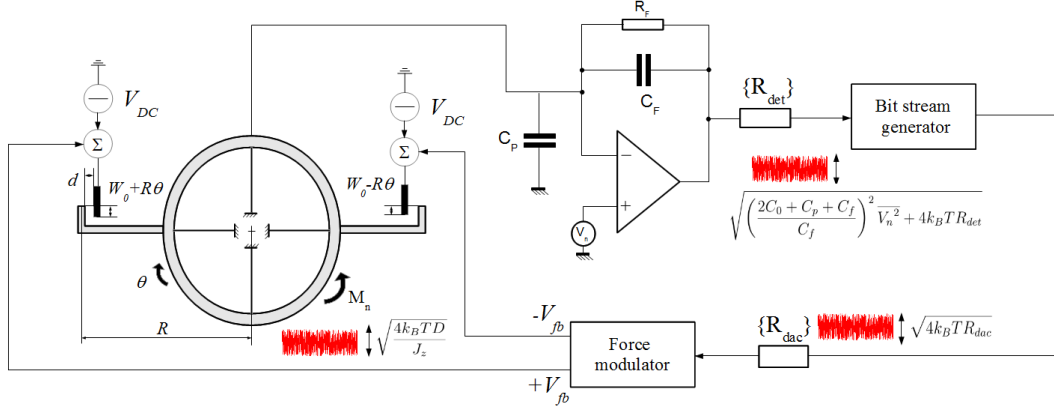


Figure 3.14: Schematic of the sensor loop with sliding-plate capacitors

Sensor parameters		
Mechanical gain (3.4)	$G_{mech} = \frac{1}{\omega_0^2}$	$\left[\frac{rad}{rad/s^2} \right]$
Capacitive Gain (3.18)	$K_{\theta-C} = \frac{2C_0 R}{W_0}$	$\left[\frac{F}{rad} \right]$
Electrical Gain (3.53)	$G_{det} = \frac{2V_{DC} K_{\theta-C}}{C_f}$	$\left[\frac{V}{rad} \right]$
Feedback gain (3.59)	$K_{fb} = \frac{K_{\theta-C} D_c V_{DC}^2}{J_z}$	$\left[\frac{rad}{s^2} \frac{1}{V} \right]$
Noise parameters		
Noise gain (3.47)	$G_n = \frac{2C_0 + C_p + C_f}{C_f}$	
Mechanical noise (3.14)	$\Phi_{mn} = \sqrt{\frac{4k_B T \omega_0}{J_z Q}}$	$\left[\frac{rad}{s^2} \frac{1}{\sqrt{Hz}} \right]$
Detection noise (3.52)	$\Phi_{en} = \sqrt{\left(G_n^2 \bar{V}_n^2 + 4k_B T R_{det} \right)} \frac{1}{G_{mech} G_{det}}$	$\left[\frac{rad}{s^2} \frac{1}{\sqrt{Hz}} \right]$
DAC noise (3.60)	$\Phi_{fn} = \sqrt{4k_B T R_{dac}} K_{fb}$	$\left[\frac{rad}{s^2} \frac{1}{\sqrt{Hz}} \right]$

Table 3.1: Calculated parameters and equivalent noise sources for a rotation sensor with sliding-plate capacitors

3.5.2 Closed-loop design formula of parallel-plate capacitors

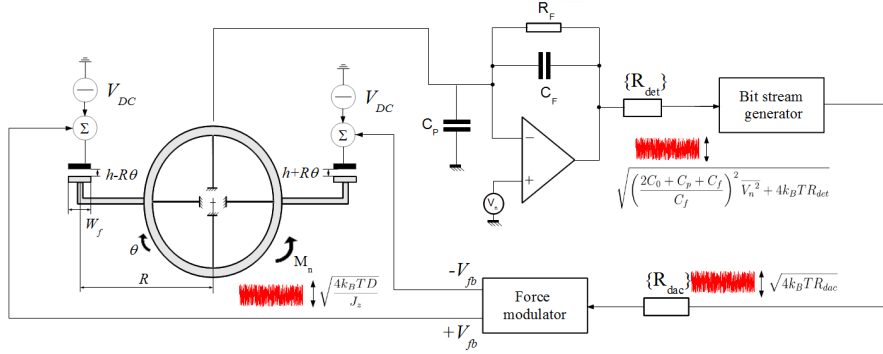


Figure 3.15: Schematic of the sensor loop with parallel-plate capacitors

Sensor parameters		
Capacitive Gain (3.28)	$K_{\theta-C} = \frac{2C_0 R}{h}$	$\left[\frac{F}{rad} \right]$
Electrical Gain (3.53)	$G_{det} = \frac{2V_{DC} K_{\theta-C}}{C_f}$	$\left[\frac{V}{rad} \right]$
Feedback gain (3.59)	$K_{fb} = \frac{K_{\theta-C} D_c V_{DC}^2}{J_z}$	$\left[\frac{rad}{s^2} \frac{1}{V} \right]$
Electrical spring (3.31)	$K_{el} = 4C_0 V_{DC}^2 \left(\frac{R}{h} \right)^2$	$\left[\frac{N.m}{rad} \right]$
modified frequency (3.40)	$\omega_{0m} = \sqrt{\omega_0^2 - \frac{K_{el}}{J_z}}$	$\left[\frac{rad}{s} \right]$
Mechanical gain (3.4)	$G_{mech} = \frac{1}{\omega_{0m}^2}$	$\left[\frac{rad}{rad/s^2} \right]$
Noise parameters		
Noise gain (3.47)	$G_n = \frac{2C_0 + C_p + C_f}{C_f}$	
Mechanical noise (3.14)	$\Phi_{mn} = \sqrt{\frac{4k_B T \omega_0}{J_z Q}}$	$\left[\frac{rad}{s^2} \frac{1}{\sqrt{Hz}} \right]$
Detection noise (3.52)	$\Phi_{en} = \sqrt{\left(G_n^2 V_n^2 + 4k_B T R_{det} \right) \frac{1}{G_{mech} G_{det}}}$	$\left[\frac{rad}{s^2} \frac{1}{\sqrt{Hz}} \right]$
DAC noise (3.60)	$\Phi_{fn} = \sqrt{4k_B T R_{dac}} K_{fb}$	$\left[\frac{rad}{s^2} \frac{1}{\sqrt{Hz}} \right]$

Table 3.2: Calculated parameters and equivalent noise sources for a rotation sensor with parallel-plate capacitors

Chapter 4

Design of rotation sensor prototypes

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This chapter develops two designs for a new MEMS rotation sensor intended for seismic exploration based on the ideal sensing principles presented in Chapter 3. The key features highlighted are used to design two structures based on two capacitor configurations: one using sliding-comb capacitors and the other one using pure gap-closing capacitors. Before giving the detailed design, the technical constraints imposed by the ASIC, the manufacturing process and the packaging are provided. Then, three main analysis are performed:

- First, the mechanical study with the calculation of spring stiffness coefficients and first mode frequencies.
- Second, a damping analysis in order to evaluate the quality factor of the configurations and to define the pressure level needed to limit the mechanical noise.
- Third, the electrical study with the calculation of nominal capacitances and electrostatic moments, which are important to evaluate the detector and the feedback gains of the system.

Then, a non-linear study of the rotation sensor is proposed to predict its behavior when operating with large amplitudes of the seismic mass. Finally, an optimization study is done to extract the design parameters leading to best performances. The designs selected are simulated to foresee their dynamic behaviors as well as their noise distributions in the frequency domain.

4.1 Technical constraints

Before beginning the modeling phase, one should list the different technical constraints in order to define a topology for each design at study.

The first specifications on the design are related to the dimensions of the MEMS to be processed. The rotation sensor will be manufactured according to a specific process developed so far for MEMS accelerometers. More specifically, the fabrication uses optimized Deep Reactive Ion Etching (DRIE) process of Silicon-On-Insulator (SOI) wafers with a thickness of $80\ \mu m$. Consequently, the smallest gap that can be processed is $2\ \mu m$ and the smallest spring width achievable is set at $5\ \mu m$. The releasing process needs the presence of holes in all mobile parts leading to a reduced material density. Thus, one has to consider a new silicon density in the models, which is reduced by a ratio of 20%, giving $\rho = 1864\ kg/m^3$.

Since the silicon is an anisotropic material, one should be aware of the orientation of beams on the wafer. The SOI wafers are oriented such as presented in figure 4.1. The beams are oriented along the $[010]$ crystallographic axis so that the silicon Young modulus is the smallest possible for higher mechanical sensitivity. The resulting Young modulus E_y has a value of $131\ GPa$.

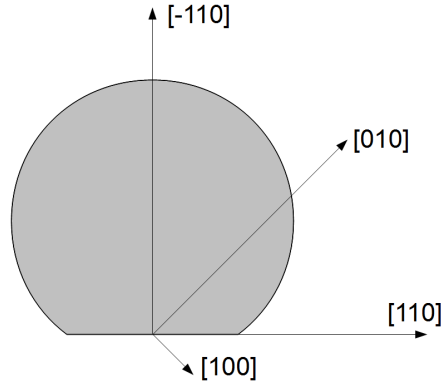


Figure 4.1: Crystallographic axis of SOI wafers used for the rotation sensor process

One should be aware of potential over-etching of the sensitive elements during the process. Indeed, an over-etching can modify the critical parameters of the design such as beam width or air gap between electrodes. Dimensional analyses will be shown in the next chapter to quantify this over-etching. For further details, the manufacturing process is presented in 5.1.

To be able to package the chips and to test them with the provided ASIC, the sensitive element size is fixed at: $3.2\text{ mm} \times 3.2\text{ mm}$. Moreover, the central supporting anchor R_a has a fixed size to ensure the electrical bonding, and it is set at $300\text{ }\mu\text{m}$.

Beyond these considerations issued from the process, one can add some requirements related to the conditions of use of the future rotation sensor. The first one deals with the resonance frequency which has to be below 1 kHz to limit the mechanical noise as developed in equation (3.14). Also, the electronics needs a minimum nominal capacitance to work properly above its intrinsic noise. Therefore, the nominal capacitance value has to be larger than 5 pF .

These requirements (summarized in table 4.1) are at the core of the design since the topologies used to develop our prototypes inherit from them. Indeed, within a surface area of $3.2 \times 3.2\text{ mm}^2$, one has to design a structure which has a relatively small resonance frequency value (e.g. with soft springs and heavy proof mass) while fixing the nominal capacitance at a specific value (e.g. by increasing the number of electrodes used). However, minimizing the resonance frequency while maximizing the capacitance are opposite mechanisms and a trade-off must be accepted at some point.

The two design principles can be seen in figures 4.2 and 4.3. Since the flexible elements and the proof mass are the same for the two configurations, one can derive the stiffness coefficient for both structures. However, separate calculations will occur for the determination of the damping coefficient and the electrical properties.

Specifications	Parameter name	Value
Sensitive element size	C_{size}	$< 3.2 \text{ mm}$
Supporting anchor radius	R_a	$300 \text{ }\mu m$
Wafer thickness	T	$80 \text{ }\mu m$
Critical gap dimension	l_c	$> 2 \text{ }\mu m$
Spring width	W_b	$> 5 \text{ }\mu m$
Structure resonance frequency	f_0	$< 1 \text{ kHz}$
Nominal capacitance	C_0	$> 5 \text{ pF}$

Table 4.1: Technical specifications

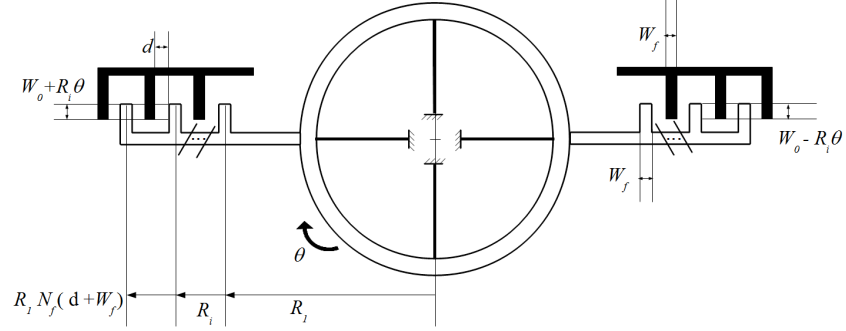


Figure 4.2: Design with sliding-comb capacitors

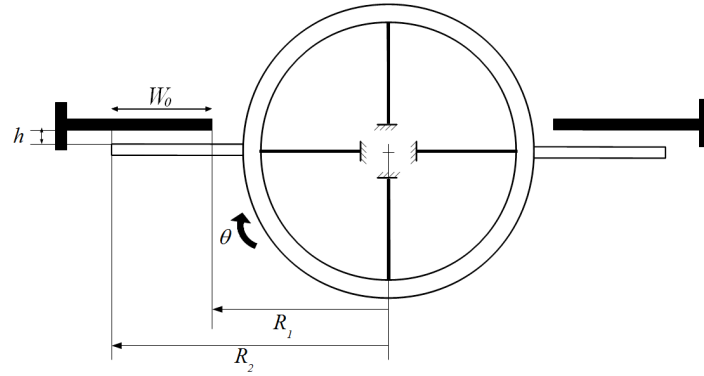


Figure 4.3: Design with gap-closing capacitors

4.2 Mechanical study of the rotation MEMS

This section deals with the mechanical study of the rotation sensor. In a first step, the stiffness coefficients of the first deformation modes are given: rotation in the x-y plane, z-axis translation and rotation in the y-z plane. Then, these stiffness coefficients, coupled with the inertia parameters of the seismic ring, provide the frequency values of the first modes. The calculation of the first mode frequencies is essential for the evaluation of the quality factor and the mechanical noise.

4.2.1 Static model

In the static study, the beam equation under different boundary conditions is solved to determine the stiffness coefficients of the supporting springs. In addition, the beam equation is derived considering several hypothesis:

- The material is assumed to be isotropic with no consideration of Poisson effect.
- The supporting beams operate in the small-strain, small-deflection regime (when specified).
- The beam inertia and shearing effects are supposed negligible.
- Finally, the seismic ring is assumed to be perfectly rigid.

For the calculations, different parameters are introduced in table 4.2:

Beam length	L_b
Beam width	W_b
Structure thickness	T
Central anchor radius	R_a
Ring inner radius	R_i
Ring external radius	R_e
Beam deflection in the x-y plane	$v(x)$
Beam deflection in the z-x plane	$w(x)$

Table 4.2: Parameters used in the derivation of the stiffness coefficients

4.2.1.1 Rotation in the x-y plane

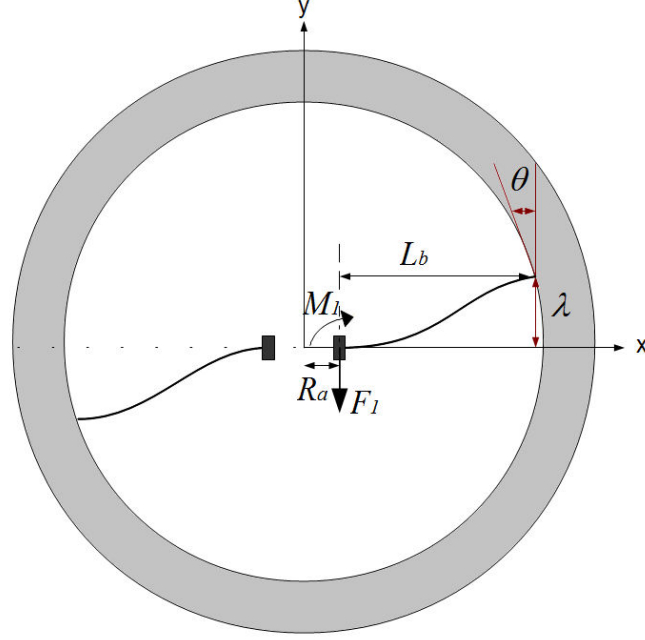


Figure 4.4: model of an isolated beam with its boundary conditions for the rotation in x-y plane

The rotation in the x-y plane is the main motion since the structure has to be sensitive to angular accelerations around the z axis. In order to calculate the stiffness coefficient, one has to isolate one spring from the structure and solve the beam deflection equation with the appropriate boundary conditions (figure 4.4). As the four springs are mounted in parallel, the total stiffness of the structure is obtained by summing all the contributions.

For an isolated beam, the kinematic condition at the free extremity is given by:

$$v(L_b) = \lambda = (L_b + R_a)\theta \quad (4.1)$$

Given the assumptions presented at the beginning, the beam deflection is related to the reactions forces at the fixed point through the following Ordinary Differential Equation (ODE) [41]:

$$\frac{d^2v}{dx^2} = \frac{M_l - F_l x}{E_y I_z} \quad (4.2)$$

Where M_1 is the bending reaction moment and F_1 is the bending reaction force at the fixed extremity, E_y is the Young modulus of the material and I_z is the quadratic moment about z-axis given by:

$$I_z = \frac{TW_b^3}{12} \quad (4.3)$$

The boundary conditions for this particular motion are:

$$\left\{ \begin{array}{lcl} v(0) & = & 0 \\ \frac{dv}{dx} \Big|_{x=0} & = & 0 \\ v(L_b) & = & (L_b + R_a)\theta \\ \frac{dv}{dx} \Big|_{x=L_b} & = & \theta \end{array} \right. \quad (4.4)$$

Using the boundary conditions at the fixed extremity, the deflection function is given by the relation:

$$v(x) = \frac{x^2}{E_y I_z} \left(\frac{M_1}{2} - \frac{F_1}{6} x \right) \quad (4.5)$$

Using the boundary conditions at the free extremity, the reaction efforts are:

$$\begin{aligned} M_1 &= \frac{2E_y I_z (2L_b + 3R_a)}{L_b^2} \theta \\ F_1 &= \frac{6E_y I_z (L_b + 3R_a)}{L_b^3} \theta \end{aligned} \quad (4.6)$$

In order to find the spring constant for this particular motion, one has to transport the reaction forces and moments to the centroid. Considering that the sensor is composed of four springs, the bending moment expressed at the centroid is:

$$M_{center} = 4(M_1 + F_1 R_a) = \frac{4}{3} \frac{E_y T W_b^3 (L_b^2 + 3L_b R_a + 3R_a^2)}{L_b^3} \theta \quad (4.7)$$

One can extract the spring constant for this particular motion:

$$K_m = \frac{4}{3} \frac{E_y T W_b^3 (L_b^2 + 3L_b R_a + 3R_a^2)}{L_b^3} \quad (4.8)$$

4.2.1.2 Influence of large deflections

The deflection of a beam under bending efforts leads to a small elongation. In the case where the deflection is large, the traction force generated by this elongation is no more negligible and another bending contribution has to be considered. The ODE is modified by the addition of a new term T_1 (traction force):

$$\frac{d^2v}{dx^2} - \frac{T_1}{E_y I_z} = \frac{M_1 - F_1 x}{E_y I_z} \quad (4.9)$$

where the boundary conditions are the same as in the previous section. One can introduce the variable k given by the expression:

$$\sqrt{\frac{T_1}{E_y I_z}} \quad (4.10)$$

Using the boundary conditions at the fixed extremity, the deflection equation is given by the following expression:

$$v(x) = \frac{k M_1 (\cosh(kx) - 1) + R_1 x (k - \sinh(kx))}{k^3 E I_z} \quad (4.11)$$

The reaction forces expressions are obtained by using the boundary conditions at the free extremity:

$$\begin{aligned} M_1 &= k E_y I_z \frac{\cosh(k L_b) k (L_b + R_a) - \sinh(k L_b) - k R_a}{2(1 - \cosh(k L_b)) + k L_b \sinh(k L_b)} \theta \\ F_1 &= k^2 E_y I_z \frac{\sinh(k L_b) k (L_b + R_a) - \cosh(k L_b) + 1}{2(1 - \cosh(k L_b)) + k L_b \sinh(k L_b)} \theta \end{aligned} \quad (4.12)$$

It can be noticed that the reaction forces are nonlinear since they depend on the variable k which contains the tensile force parameter. By taking the limit when k is close to 0, the reaction forces expressions are:

$$\begin{aligned} \lim_{k \rightarrow 0} M_1 &= \frac{2 E_y I_z (2 L_b + 3 R_a)}{L_b^2} \theta \\ \lim_{k \rightarrow 0} F_1 &= \frac{6 E_y I_z (L_b + 3 R_a)}{L_b^3} \theta \end{aligned} \quad (4.13)$$

These limits correspond to the expressions found in equation (4.6). Concerning the tensile force expression, one has to calculate the total beam elongation since:

$$T_1 = S\sigma_x = ES\epsilon_x = \frac{E_y S}{L_b} \Delta L_b \quad (4.14)$$

where S is the beam section, σ_x is the stress along x-axis.

Let us consider an infinitesimal length dx which bends with a quantity dv . Consequently, the length of this element dx increases and become $dx + \Delta L_b$. Simple geometric considerations lead to the equivalence:

$$\frac{d\Delta L_b}{dx} \approx \frac{1}{2} \left(\frac{dv}{dx} \right)^2 \quad (4.15)$$

Going back to the tensile force expression:

$$T_1 = \frac{E_y S}{2L_b} \int_0^{L_b} \left(\frac{dv}{dx} \right)^2 dx \quad (4.16)$$

This relation is strongly nonlinear and cannot be solved analytically. But it is possible to find a solution using Newton-Raphson numerical method. Nevertheless, it is possible to find an approximation of the tensile force T_1 for small values of k :

$$T_1 = \frac{1}{30} \frac{E_y T W_b (17L_b^2 + 18R_a^2 + 33L_b R_a)}{L_b^2} \theta^2 \quad (4.17)$$

In figure 4.5, a comparison of the bending moment at the structure centroid (M_{center}) is shown for the small deflection and the large deflection theories.

One can notice that the tensile force T_1 has a large influence on the reaction moment. For instance, assuming a beam with a length of $550 \mu\text{m}$, a width of $5 \mu\text{m}$ and a thickness of $80 \mu\text{m}$, the reaction moment is three times larger than it would be expected from the small deflection theory for $\theta = 15 \text{ mrad}$.

Using series expansion of the deflection function for k close to zero, it is possible to define the validity domain of the small deflection assumption. This validity domain is given by the expression:

$$k^2 \ll \frac{30(L_b^2 + 3R_a L_b + 3R_a^2)}{L_b^2(L_b^2 + 9R_a L_b + 9R_a^2)} \quad (4.18)$$

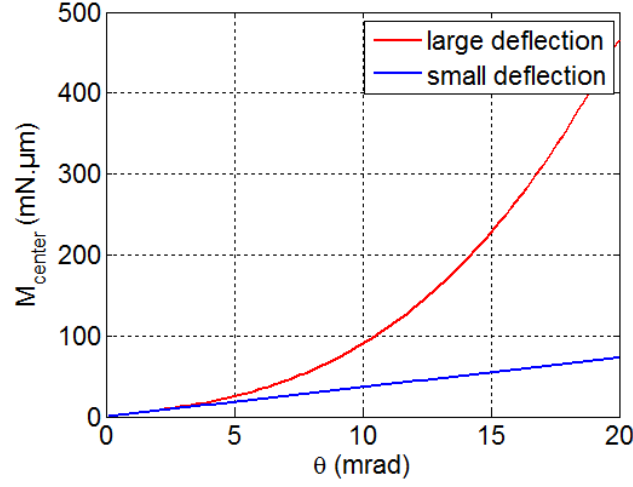


Figure 4.5: Large deflection vs small deflection assumption

Substituting the tensile force expression to equation (4.18) gives:

$$\frac{1}{75} \left(\frac{\theta}{W_b} \right)^2 \frac{(17L_b^2 + 33R_aL_b + 18R_a^2)(L_b^2 + 9R_aL_b + 9R_a^2)}{(L_b^2 + 3R_aL_b + 3R_a^2)} \ll 1 \quad (4.19)$$

If equation 4.19 is not verified, one has to consider higher order terms in the spring force expression. In order to find this spring force expression, one has to transport the reaction efforts to the centroid such as:

$$M_{center} = 4(M_1 + F_1R_a) \quad (4.20)$$

Substituting equation (4.12) to equation (4.20) and taking the Taylor expansion to the third order, the spring force can be approximated by:

$$\begin{aligned} F_{NonLinear} = & \frac{4}{3} \frac{E_y T W_b^3 (L_b^2 + 3L_bR_a + 3R_a^2)}{L_b^3} \theta \\ & + \frac{4}{225} \frac{E_y T W_b (17L_b^2 + 33L_bR_a + 18R_a^2)(L_b^2 + 9L_bR_a + 9R_a^2)}{L_b^3} \theta^3 \end{aligned} \quad (4.21)$$

As explained above, the nonlinear effect makes the spring stiffer for large deflections and will affect dynamic properties. As a conclusion, operating in the linear regime needs a large spring width which is at odds with a low frequency value. This explains why a feedback control is essential to limit the amplitude of rotation θ , thus, ensuring a mechanical linear behavior.

4.2.1.3 Z-axis translation

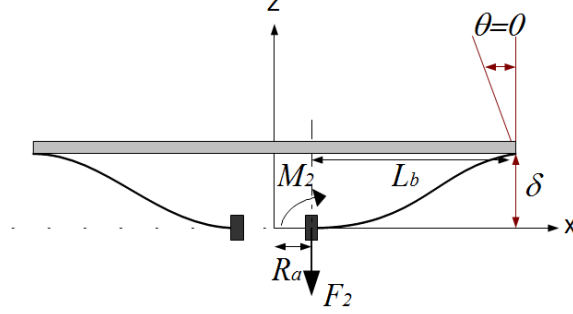


Figure 4.6: model of an isolated beam with its boundary conditions for the translation about z-axis

A parasitic motion of the seismic mass is the translation motion along the z-axis (figure 4.6). In the following development, a small deflection assumption is made. One has to pay attention to the non-rotation kinematic condition at the beam free end. The equation describing the out of plane deflection is thus given by:

$$\frac{dw^2}{dx^2} = \frac{M_2 - F_2 x}{E_y I_y} \quad (4.22)$$

Considering the following boundary conditions:

$$\left\{ \begin{array}{lcl} w(0) & = & 0 \\ \frac{dw}{dx} \Big|_{x=0} & = & 0 \\ w(L_b) & = & \delta \\ \frac{dw}{dx} \Big|_{x=L_b} & = & 0 \end{array} \right. \quad (4.23)$$

Where M_2 is the bending reaction moment and F_2 is the bending reaction force at the fixed extremity, E_y is the Young modulus of the material and I_y is the quadratic moment about y-axis given by:

$$I_y = \frac{W_b T^3}{12} \quad (4.24)$$

The deflection function w can be found by integration:

$$w(x) = \frac{x^2}{E_y I_y} \left(\frac{M_2}{2} - \frac{F_2}{6} x \right) \quad (4.25)$$

M_2 and F_2 are determined by the boundary conditions at the free extremity:

$$\begin{aligned} M_2 &= \frac{6E_y I_y}{L_b^2} \delta \\ F_2 &= \frac{12E_y I_y}{L_b^3} \delta \end{aligned} \quad (4.26)$$

Finally, the stiffness coefficient for this particular motion is:

$$K_{mZ} = \frac{4E_y T^3 W_b}{L_b^3} \quad (4.27)$$

The cubic term in equation (4.27) is the structure thickness T whereas the cubic term in the stiffness coefficient for rotation in the x-y plane is W_b (equation (4.8)). Since T is larger than W_b by a ratio of 16 (see table 4.1), the out-of-plane deformation mode is at a frequency much larger than the in-plane deformation mode. As a result, the structure of the rotation sensor is sensitive to angular accelerations in its reference plane whereas it filters parasitic motions such as translations normal to its reference plane.

4.2.1.4 Rotation in the y-z plane

Another parasitic motion of the seismic mass is the rotation motion in the y-z plane (or in the x-z plane). In the following development, a small deflection assumption is made. It should be noticed that two beams bend about z-axis and two beams twist around x axis (figure 4.7). As a consequence, the study is divided into two parts, one for the bending deformation and another one for the torsion deformation.

The bending constant is obtained in the same way as the rotation in the x-y plane. Therefore, the contribution of the two bending beams is given by the following expression:

$$K_{\varphi_1} = \frac{2}{3} \frac{E_y T^3 W_b (L_b^2 + 3L_b R_a + 3R_a^2)}{L_b^3} \quad (4.28)$$

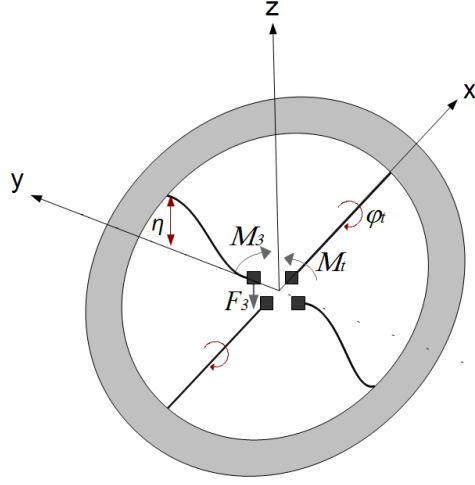


Figure 4.7: model of an isolated beam with its boundary conditions for the rotation in y-z plane

The analysis of the torsion constant of a rectangular bar is quite complicated. According to [42], the theoretical expression for the torsion constant is:

$$\begin{cases} K_{\varphi_2} = \frac{2GW_b^3T}{L_b}\beta\left(\frac{T}{Wb}\right) \\ \beta(\eta) = \frac{1}{3}\left(1 - \frac{192}{\pi^5}\frac{1}{\eta}\sum_{n=1,3,5}\frac{1}{n^5}\tanh\left(\frac{n\pi\eta}{2}\right)\right) \end{cases} \quad (4.29)$$

where G is the material shear modulus. Hence, the combined spring constant for this particular motion is:

$$K_{m_t} = K_{\varphi_1} + K_{\varphi_2} = \frac{2}{3}\frac{E_yTW_b((TL_b)^2 + 3T^2L_bR_a + 3(TR_a)^2) + 3GW_b^2\beta\left(\frac{T}{Wb}\right)L_b^2}{L_b^3} \quad (4.30)$$

Since the cubic term is the structure thickness T in equation (4.28), the frequency value of this deformation mode is larger than the one for rotation in the x-y plane. Consequently, the structure of the rotation sensor filters this motion type.

4.2.2 First mode frequencies

In order to evaluate the frequencies of the first deformation modes, one can consider only the inertia of the seismic ring. As a result, the inertia parameters of the seismic ring (moments around the z-axis and the x-axis as well as the mass) are defined by:

$$\begin{aligned} M &= \rho\pi T (R_e^2 - R_i^2) \\ J_z &= \frac{M}{2} (R_e^2 + R_i^2) \\ J_x &= \frac{J_z}{2} \end{aligned} \quad (4.31)$$

where ρ is the material density. Given stiffness constants calculated in equations (4.8), (4.27) and (4.30), it is possible to calculate the first mode frequencies of the rotation sensor:

$$\left\{ \begin{aligned} f_\theta &= \frac{1}{2\pi} \sqrt{\frac{K_m}{J_z}} = \frac{1}{2\pi} \sqrt{\frac{4 E_y T W_b^3 (L_b^2 + 3L_b R_a + 3R_a^2)}{3 J_z L_b^3}} \\ f_\phi &= \frac{1}{2\pi} \sqrt{\frac{K_{m_t}}{J_x}} = \frac{1}{2\pi} \sqrt{\frac{2 E_y T W_b ((T L_b)^2 + 3T^2 L_b R_a + 3(T R_a)^2 + 3G W_b^2 \beta (\frac{T}{W_b}) L_b^2)}{3 J_x L_b^3}} \\ f_z &= \frac{1}{2\pi} \sqrt{\frac{K_{m_z}}{M}} = \frac{1}{2\pi} \sqrt{\frac{4 E_y T^3 W_b}{M L_b^3}} \end{aligned} \right. \quad (4.32)$$

The frequency expressions presented in (4.32) have to be evaluated and compared with Finite-Element-Method (FEM) simulations to conclude on the model validity. Let us assume a seismic ring suspended by four springs, with parameter values defined in the figure 4.8. A 3D numerical model is made with *COMSOL Multiphysics*® and comparative results are shown in table 4.3. It can be seen that the fit is remarkably good. Furthermore, a comforting observation is that the parasitic motions have resonant frequencies much higher than the one of the motion of interest. For instance, the out-of-plane rotation mode has a resonant frequency which is sixteen times larger than the in-plane rotation mode; and the out-of-plane translation mode has a resonant frequency which is more than twenty-five times larger than the in-plane rotation mode. Therefore, the rotation sensor structure is able to filter parasitic motions while being sensitive to angular accelerations of its reference plane.

As a final remark, one can notice that the resonant frequencies depend strongly on the beam width W_b , which is influenced by over-etching during processing. In conclusion, one will have to expect smaller beam width of rotation sensor prototypes, thus, lower resonant frequencies of the first modes.

Parameter	Value	Unit
L_b	550	μm
W_b	5	μm
T	80	μm
R_a	300	μm
$R_i = R_a + L_b$	850	μm
R_e	1150	μm
J_z	8×10^{-14}	kgm^2
M	66	ng

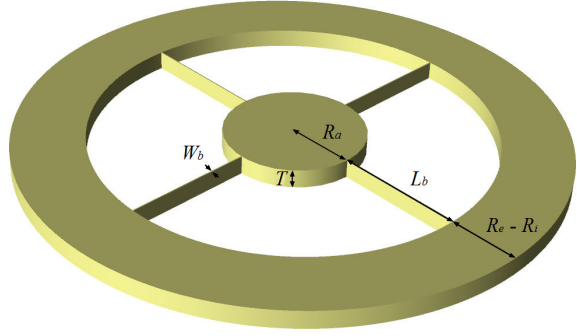


Figure 4.8: model for the validation of first mode frequencies

mechanical frequency	Analytical result (Hz)	COMSOL result (Hz)	$\frac{f_{\text{COMSOL}}}{f_{\text{analytical}}}$
Rotation (x-y plane)	1010	1009	0.99
Rotation (y-z plane)	16154	15698	0.97
Translation (z-axis)	27384	26381	0.96

Table 4.3: Numerical analysis results

4.3 Fluid damping analysis

For moving structures with small sizes, the surface forces (viscous force) become non-negligible with respect to the volume forces (gravity) and this effect becomes greater as micro-machined structures decrease in size. In particular, capacitive micromachined structures use parallel-plate capacitors where a movable electrode interacts with the gas encapsulated between moving and fixed electrodes. At this level, two mechanisms can occur: the viscous lateral damping (or slide-film damping) and the squeeze-film damping.

Slide-film damping occurs when two parallel plates move tangentially with respect to each other. Due to viscosity effects at the gas-structure interface, the fluid exerts a force on the moving plate that opposes the relative motion. This damping force is a function of the velocity gradient in the vicinity of the moving plate, the viscosity of the gas and the overlap area between moving and fixed plate. This damping mechanism is a dominant source in laterally-driven structures like comb-driven systems such as one of our rotation MEMS design: the sliding-comb configuration.

On the other hand, when a gas film between two parallel plates is squeezed by the relative motion of these surfaces, a gas flow is produced, creating a pressure elevation in the cavity. This is the so-called squeeze-film damping. A squeeze film damping is characterized by physical dimensions like width, length and thickness of the gas film. Furthermore, other parameters such as the ambient pressure and the viscosity of the gas play an important role in this type of damping. This mechanism is dominant for driven structures using gap-closing electrodes.

The following section gives a review on damping mechanisms, followed by the calculation of the quality factor for the two configurations studied: sliding-comb capacitors and gap-closing capacitors. The quality factor expressions are then used to determine the pressure level which limits the mechanical noise of the rotation sensor designs.

4.3.1 Damping mechanisms for the sliding-comb configuration of the rotation sensor

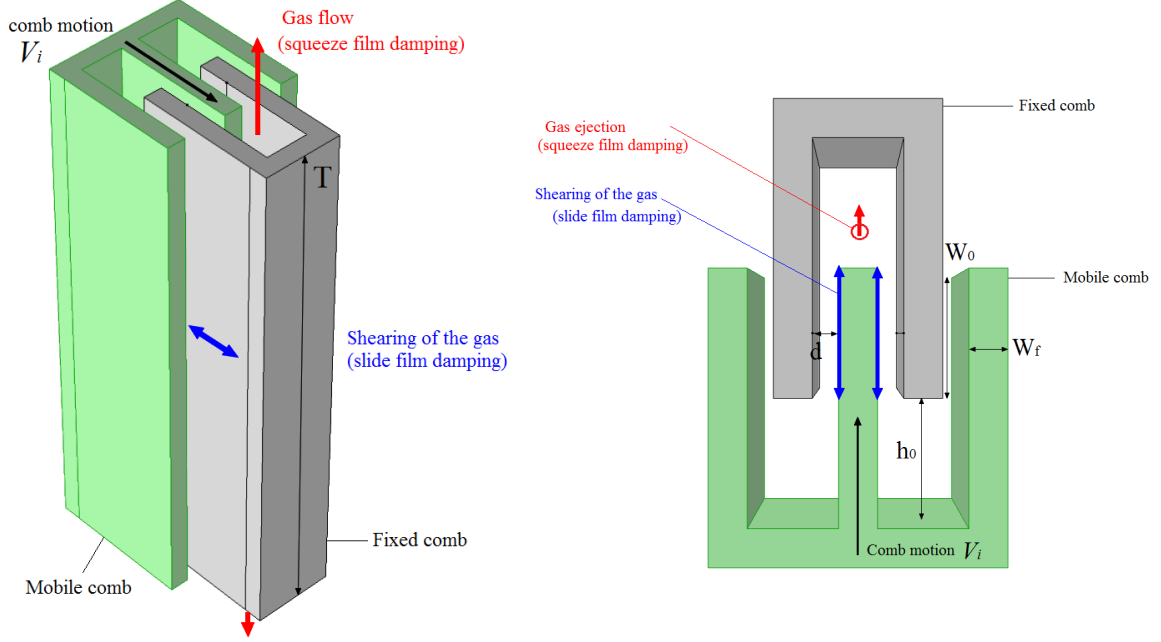


Figure 4.9: Damping mechanisms in a sliding-plate rotation sensor. Left: isometric view; Right: top view

The sliding-capacitor design invokes the two damping mechanisms explained above. Indeed, one can notice on figure 4.9 that slide-film occurs at the lateral boundaries whereas squeeze-film occurs at the extremity boundary for a given finger electrode. The geometrical parameters of the damper are: T , the structure thickness; W_f , the finger width; W_0 the electrode overlap; d the lateral gap; h_0 the normal gap and the v_i the velocity of the i -th electrode. We assume that finger electrodes are strip plates since: $T \gg W_f$.

On one hand, there is a confinement of the gas film between fixed fingers, creating a gas flow along the out-of-plane direction. As a result, one should consider the parameter T and h_0 as the characteristic dimensions of the squeeze damper. On the other hand, the shearing of the gas occurs along the overlap between moving and fixed fingers. The characteristic dimensions of this slide damper are the parameters W_0 and d .

In the following development, each damping mode is derived separately with a particular focus on the influence of gas rarefaction effects.

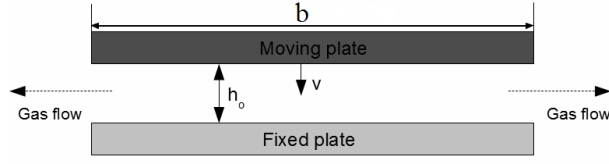


Figure 4.10: Model of a squeeze-film damper

4.3.1.1 Squeeze-film model

In the squeeze-film damper shown in figure 4.10, the inertia effect is negligible due to very small geometries and, under isothermal conditions, the behavior of the squeeze film is governed by the well-known Reynolds equation given by [43]:

$$\nabla \left(\frac{ph^3}{\eta} \nabla p \right) = 12 \frac{\partial(hp)}{\partial t} \quad (4.33)$$

where η is the dynamic viscosity of the gas, p the pressure inside the cavity, ∇ is the gradient operator, and finally h the film gap.

Under small variations of pressure p and gap dimension h , a linearized form of equation (4.33) is proposed in [44] which leads to the damping force expression for a strip plate such as:

$$F_d = \frac{96\eta ab^3}{\pi^4 h_0^3} v \quad (4.34)$$

where a is the damper length, b is the damper width, h_0 is the nominal film gap and v the moving plate velocity amplitude.

Squeeze-film damping coefficient of the rotation sensor design

In the rotation sensor case, squeeze-film damping occurs at the tip of each electrode finger, creating a damping moment about the centroid. In addition, one can notice from figure 4.9 that the gas film is confined between fixed comb fingers, creating a gas flow along the out-of-plane direction.

Let us consider the damping action created at the i -th electrode in figure 4.11. The finger width is small enough so that one can express the finger velocity as follows:

$$v_i = R_i \Omega \quad (4.35)$$

where R_i is the distance from the centroid and Ω is the rotation rate of the proof mass.

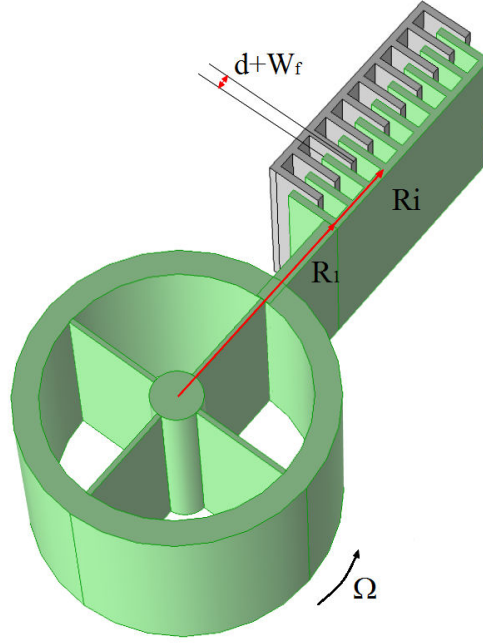


Figure 4.11: Summation of damping actions over a comb

As a result, the damping moment at the i -th finger is given by:

$$M_i = \frac{96\eta W_f T^3 R_i^2}{\pi^4 h_0^3} \delta\Omega \quad (4.36)$$

To calculate the damping moment over a comb, one has to sum all contributions from moving fingers. Assuming N_f fingers on a comb, the expression of the damping force is given by:

$$D_{squeeze} = \sum_{i=1}^{2N_f-1} R_i^2 \frac{96\eta T^3 W_f}{\pi^4 h_0^3} \quad (4.37)$$

where $R_i = R_1 + i(d + W_f) = R_1 + iL_p$. The summation gives at last:

$$D_{squeeze} = R_0 \frac{96\eta T^3 W_f}{\pi^4 h_0^3} \quad (4.38)$$

with the introduction of the constants R_0 such as:

$$R_0 = \frac{8}{3} N_f \left(\left(\frac{1}{8} + N_f^2 - \frac{3}{4} N_f \right) L_p^2 + \frac{3}{2} \left(N_f - \frac{1}{2} \right) R_1 L_p + \frac{3}{4} R_1^2 \right) \quad (4.39)$$

Gas rarefaction effect

In order to minimize the damping effect, one should reduce the gas pressure in the structure cavity. However, such pressure diminution makes the continuum behavior of the gas film no more valid, and thus, a gas rarefaction correction must be taken into account in the model. As specified in [45] and [46], the gas rarefaction effect can be introduced through the Knudsen number defined as follows:

$$K_n = \frac{\lambda_0}{h_0} \quad (4.40)$$

where λ_0 is the mean free path given in function of the gas constant R , the molecular mass M_m and the temperature T_0 by:

$$\lambda_0 = \frac{\eta}{P_a} \sqrt{\frac{2RT_0}{M_m}} \quad (4.41)$$

With this vacuum parameter K_n , it is possible to write a modified viscosity model:

$$\eta_{modified} = \frac{\eta}{Q_{pr}} \quad (4.42)$$

where Q_{pr} is a flow rate coefficient, function of the Knudsen number K_n . For a continuum flow, its value is 1 whereas it increases significantly for higher Knudsen number (or higher vacuum). The flow rate coefficient evaluation requires the solution of the Boltzmann equation, which is complicated even for simple topologies. The table (4.3.1.1) reviews the different flow regime and the associated approximation of Q_{pr} . Substituting Q_{pr} in equation (4.38), one can obtain the squeeze-film damping coefficient of the rotation sensor with gas rarefaction effect:

$$D_{squeeze} = R_0 \frac{96\eta T^3 W_f}{Q_{pr} \pi^4 h_0^3} \quad (4.43)$$

Flow regime	Continuum flow	slip flow	transitional flow	molecular flow
conditions on K_n	$K_n < 0.001$	$0.001 < K_n < 0.1$	$0.1 < K_n < 10$	$K_n > 10$
Q_{pr} function	1	$1 + 6Kn$	$1 + 9.638K_n^{1.159}$	

End effects in squeeze-film damper

The damping coefficient in (4.43) is valid under trivial boundary conditions at the finger electrode boundaries. This condition is a good approximation only if the oscillating finger dimensions are much larger than the film gap: $T, W_f \gg h_0$. However, in practice, the aspect ratio is small enough so that border effects can play an important role in the damping effect. An acoustic border condition is then used to correct this effect as explained in [47]. This model extension needs the calculation of effective plate dimensions which can be defined as follows:

$$\begin{aligned} T' &= T \frac{\sqrt{1+3A_T} (1+4A_{W_f})^{3/8}}{\sqrt{1+3A_{W_f}} (1+4A_T)^{1/8}} \\ W_f' &= W_f \frac{\sqrt{1+3A_{W_f}} (1+4A_T)^{3/8}}{\sqrt{1+3A_T} (1+4A_{W_f})^{1/8}} \end{aligned} \quad (4.44)$$

where the coefficients A_T and A_{W_f} are:

$$\begin{aligned} A_T &= \frac{8}{3\pi} \frac{1 + 2.676K_n^{0.659}}{1 + 0.531K_n^{0.5}(h_0/T)^{0.238}} \frac{h_0}{T} \\ A_{W_f} &= \frac{8}{3\pi} \frac{1 + 2.676K_n^{0.659}}{1 + 0.531K_n^{0.5}(h_0/W_f)^{0.238}} \frac{h_0}{W_f} \end{aligned} \quad (4.45)$$

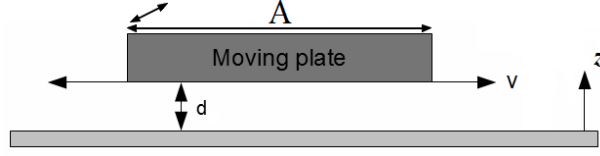


Figure 4.12: Model of a slide-film damper

4.3.1.2 Slide-film damping model

At relatively slow velocities, the gas is excited by the shear force exerted by moving plates (figure 4.12). For low frequency oscillations and small pressure variations across the surface of the damper, the gas flow can be modeled by the diffusion equation [48]:

$$\frac{\partial v}{\partial t} = \eta \Delta u(z) \quad (4.46)$$

where u is the the gas velocity distribution and Δ the Laplace operator. For a fully established flow, the gas flow can be considered a Couette flow with a linear velocity distribution. At the moving plate borders, the damping force is expressed by [49]:

$$F_{shear} = \frac{\eta A}{d} v \quad (4.47)$$

where A is the area of the moving plate and d the thickness of the gas film.

Slide-film damping coefficient of the rotation sensor design

Considering the i -th moving finger border (figure 4.13) and given equation (4.47), the slide-film damping coefficient is given by:

$$\mathbf{D}_{R_i} = \frac{\eta R_i^2 W_0 T}{d} \quad (4.48)$$

Where d is the lateral film thickness, W_0 the shearing length or the overlap between plates and R_i the distance of the damper from the centroid. Summing all the contributions over a comb, the total damping coefficient of the slide film mechanism is given by:

$$\mathbf{D}_{slide} = R_0 \frac{\eta W_0 T}{d} \quad (4.49)$$

where R_0 the constant defined in equation (4.39).

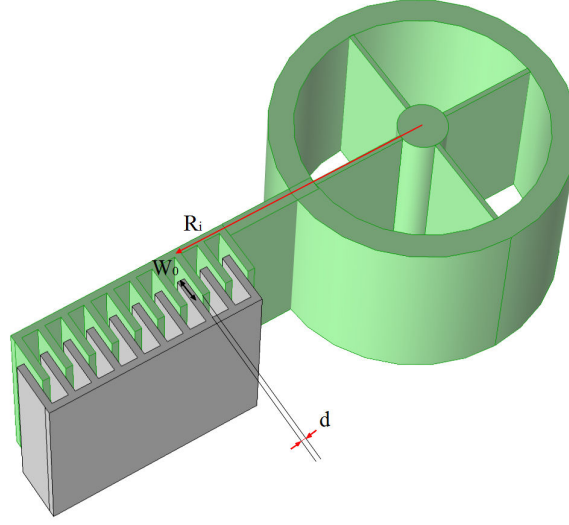


Figure 4.13: slide-film damping coefficient of the rotation sensor

Approximation	Q_{pr2}	Accuracy
Slip flow ($K_{n2} < 1$)	$1 + 2K_{n2}$	-3%
Any flow	$1 + 2K_{n2} + 0.2K_{n2}^{0.788}e^{-K_{n2}/10}$	$\pm 1\%$

Table 4.4: Approximations for Q_{pr2} in slide-film damping

Gas rarefaction effect

The gas rarefaction has an impact on the flow profile but the coefficient Q_{pr} used for the squeeze-film damper is not valid for this present case. Nevertheless, another coefficient Q_{pr2} can be introduced [50], which is a function of the Knudsen number of the slide-film problem K_{n2} and defined as:

$$K_{n2} = \frac{\lambda_0}{d} \quad (4.50)$$

One can find in table 4.4 approximations of Q_{pr2} . Thus, the damping coefficient from equation (4.49) can be rewritten such as:

$$\mathbf{D}_{slide} = R_0 \frac{\eta W_0 T}{Q_{pr2} d} \quad (4.51)$$

End effects in slide dampers

Few references study the end effects of slide dampers. Nevertheless, a simple elongation model is presented in [50] where a modified gap is introduced by the following expression:

$$d' = \frac{d}{1 + 8.5 \frac{d}{a}} \quad (4.52)$$

where a is the characteristic length of the damper, which is, in our case, defined by W_0 .

4.3.1.3 Total damping coefficient of the sliding-comb configuration

One should notice that the squeeze-film damping coefficient is proportional to $1/h_0^3$ whereas the slide-film damping coefficient is proportional to $1/d$. Consequently, the squeeze-film mechanism is dominant over the slide-film one. Given equations (4.43) and (4.49), one can calculate the total damping coefficient given as follows:

$$D = R_0 \left(\frac{96\eta T'^3 W_f'}{Q_{pr}\pi^4 h_0^3} + \frac{\eta W_0 T}{Q_{pr2} d'} \right) \quad (4.53)$$

Considering N_c combs or dampers, one can finally write the expression for the quality factor:

$$Q = \frac{J_z \omega_0}{D} = \frac{J_z \omega_0}{N_c R_0 \eta \left(\frac{96 T'^3 W_f'}{Q_{pr}\pi^4 h_0^3} + \frac{W_0 T}{Q_{pr2} d'} \right)} \quad (4.54)$$

In the sliding-comb configuration, the quality factor expression arises from the summation of two damping mechanisms: slide-film damping and squeeze-film damping. Considering that the quality factor depends on the cube of the normal gap dimension h_0 , structures with large values of h_0 imply higher quality factor levels.

4.3.2 Damping mechanisms for the gap-closing configuration of the rotation sensor

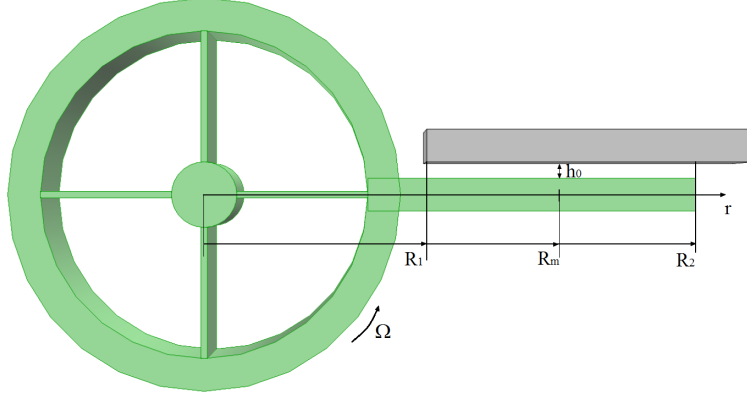


Figure 4.14: Damping mechanisms in a gap-closing rotation sensor

The gap-closing design considered now is more simple since the damping is dominated by squeeze-film damping. According to figure 4.14, a long plate moves with a velocity distribution v over the moving plate. To simplify the model, one can consider that v is constant over the plate and can be approximated by:

$$v \approx R_m \Omega \quad (4.55)$$

Where R_m is the position of the plate center and Ω is the proof mass rotation rate. Detailed calculations of the damping force for a rectangular plate can be found in [51], but here, we will use a practical approximation of the damping force with gas rarefaction effects given in [52]:

$$F_d = \frac{(ab)^2 \eta}{Q_{pr} h_0^3} \left(\frac{1}{f\left(\frac{b}{a}\right)} + \frac{1}{f\left(\frac{a}{b}\right)} \right)^{-1} v \quad (4.56)$$

$$\text{with, } f(\xi) = \xi - 0.63094\xi^2 + 0.47456\xi^{8.138}$$

where a and b the plate dimensions. Considering the approximation given in equation (4.55) and assuming that the damping action is modeled by a single force at the distance R_m from the centroid, the damping coefficient for the rotation

motion is given by:

$$D = \frac{T^2(R_2 - R_1)^2 R_m^2 \eta}{Q_{pr} h_0^3} \left(\frac{1}{f\left(\frac{(R_2 - R_1)}{T}\right)} + \frac{1}{f\left(\frac{T}{(R_2 - R_1)}\right)} \right)^{-1} \quad (4.57)$$

where R_1 and R_2 the positions of the damper extremities, T the plate thickness and Q_{pr} the relative flow rate coefficient depending on the Knudsen number K_n . Considering N_c dampers, one can finally write the quality factor of this design case such as:

$$Q = \frac{J_z \omega_0}{D} = \frac{J_z \omega_0 Q_{pr} h_0^3}{N_c T^2 (R_2 - R_1)^2 R_m^2 \eta} \left(\frac{1}{f\left(\frac{(R_2 - R_1)}{T}\right)} + \frac{1}{f\left(\frac{T}{(R_2 - R_1)}\right)} \right) \quad (4.58)$$

In the gap-closing configuration, only one damping mechanism is involved which is the squeeze-film damping. Increasing the quality factor can be obtained by employing small plates as well as large normal gap dimension. However, this can have a cost on the electrical properties which will be discussed further.

4.3.3 Summary on the damping analysis

In this development, the complete damping models for the two configurations studied were given to calculate the quality factor. The quality factor is an important design parameter since it is related to the mechanical noise in the sensor loop as shown in the previous chapter. Several simplifying assumptions as well as semi-analytical models were used to provide analytical expressions. Indeed, the damping coefficients in equations (4.53) and (4.57) were obtained through continuum flow hypothesis, which is no more valid in molecular regime. That explains why we introduced the relative flow rate coefficients for each damping mechanisms to extend the validity domain of analytical formula. Consequently, the quality factor expressions will be used only to give an order of magnitude of the mechanical noise for each design. For best performances, the mechanical noise has to be as low as possible, thus, very high quality factor values are needed. Maximizing the quality factor by optimizing the geometrical characteristics of the dampers is a good initial strategy but one should consider to reduce the ambient pressure in the MEMS cavity for better results.

4.3.4 Definition of the pressure level needed for low mechanical noise

4.3.4.1 Quality factor evaluation

In what follows, the variation of the quality factor as a function of the ambient pressure P_a is discussed. Let us consider the geometrical parameters of the two design configurations shown in appendices B.1 and B.2. The gas considered is dry air with a free mean path λ_0 of 70 nm at atmospheric pressure. According to equation (4.54) and (4.58), one can observe, in figure 4.15, the quality factor for the two designs and its evolution at low pressure. The sliding plate design has better dynamic properties with a higher Q factor for a given pressure. Furthermore, one can notice that the Q factor increases linearly in logarithmic scale for decreasing pressure levels. And, for a pressure level below 10 Pa, Q values larger than 1000 can be found. As a result, the pressure range of interest will be below 10 Pa. For further evaluations, let us set the pressure value at 1 Pa. In addition to providing high quality factor, this pressure level is chosen because it is the minimum that can be reached in the laboratory experiments detailed in the next chapter.

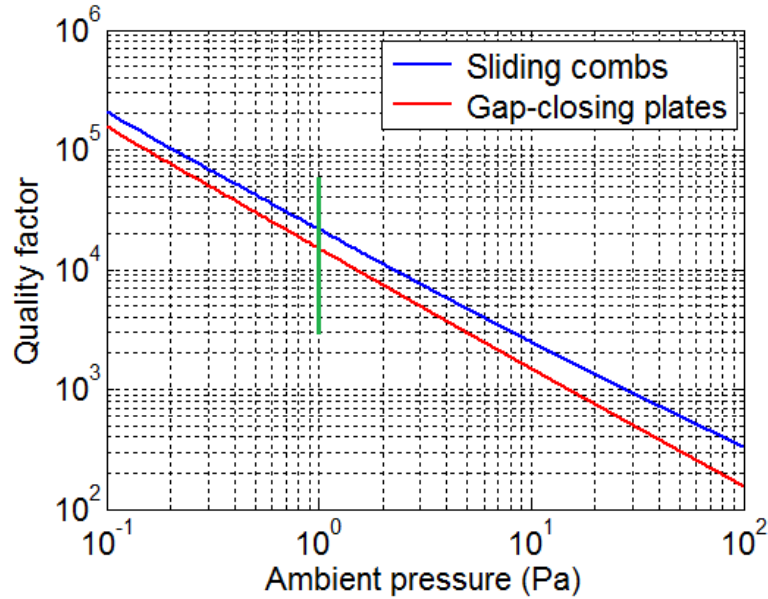


Figure 4.15: Q factor comparison for the two designs studied

4.3.4.2 Mechanical noise evaluation

Once the pressure upper limit given, one can now evaluate the mechanical noise for the two designs considered. One can recall the mechanical noise expression defined in the former chapter (equation (3.14)):

$$\Phi_{mn} = \sqrt{\frac{8\pi k_B T_o f_0}{J_z Q}} \quad (4.59)$$

Using the Q factor values obtained with the parameters defined in appendices B.1 and B.2, the mechanical noise can be calculated and it is shown in figure 4.16. The sliding plate design has a lower mechanical noise level and should be preferable in terms of resolution with a noise floor at $80 \mu\text{rad/s}^2/\sqrt{\text{Hz}}$ against $230 \mu\text{rad/s}^2/\sqrt{\text{Hz}}$ for the gap-closing design. However, one has to consider the overall sensor loop behavior before choosing the better configuration.

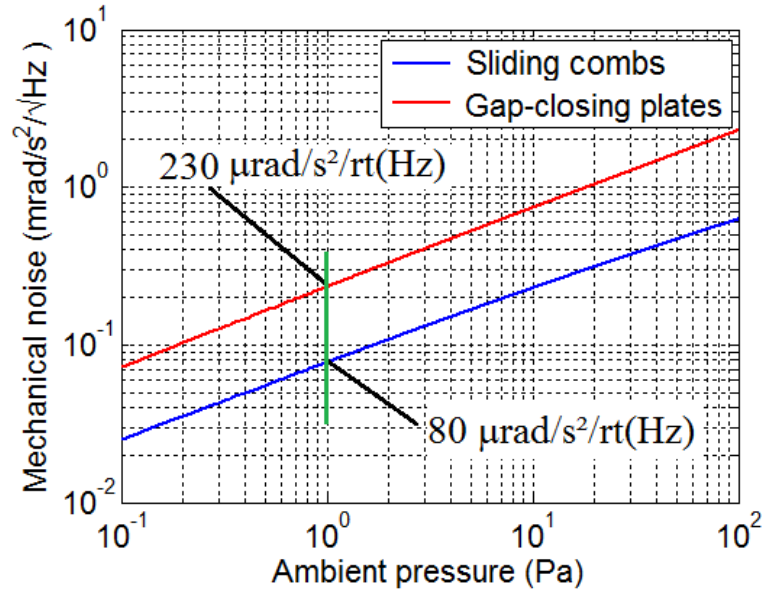


Figure 4.16: Mechanical noise comparison for the two designs studied

4.4 Electrical model of rotation sensor designs

In this section, the electrical properties of the rotation sensor designs are established. First, the study of the sliding-comb configuration is presented which highlight the presence of lateral capacitors as well as gap-closing capacitors. Then, the electrical parameters of the design using gap-closing plates are provided.

4.4.1 Sliding-comb capacitors

In order to evaluate the electrical characteristics of the sliding-comb design, one can consider the model presented in figure 4.17. The geometrical parameters used for calculations are listed in table 4.5.

Vacuum permittivity	ε_0
Structure thickness	T
Electrode overlap	W_0
Finger width	W_f
Lateral gap	d
Normal gap	h

Table 4.5: Geometrical parameters characterizing the electrical properties of the sliding-comb design

Considering the i -th finger electrode, different capacitor configurations can be considered. For instance, there are two sliding components $C_{i,1}$ and $C_{i,2}$ and two gap-closing components: $C_{fr,i}$ and $C_{fs,i}$. Using the parallel-plate approximation, the capacitance expressions are given by:

$$\begin{aligned}
 C_{i,1} &= C_{i,2} = \frac{\varepsilon_0 T (W_0 + R_i \theta)}{d} \\
 C_{fr,i} &= C_{fs,i} = \frac{\varepsilon_0 T W_f}{h - R_i \theta}
 \end{aligned} \tag{4.60}$$

where θ is the angular position of the proof mass and R_i the distance of the i -th electrode from the centroid. Considering N_f finger on a comb electrode and N_c

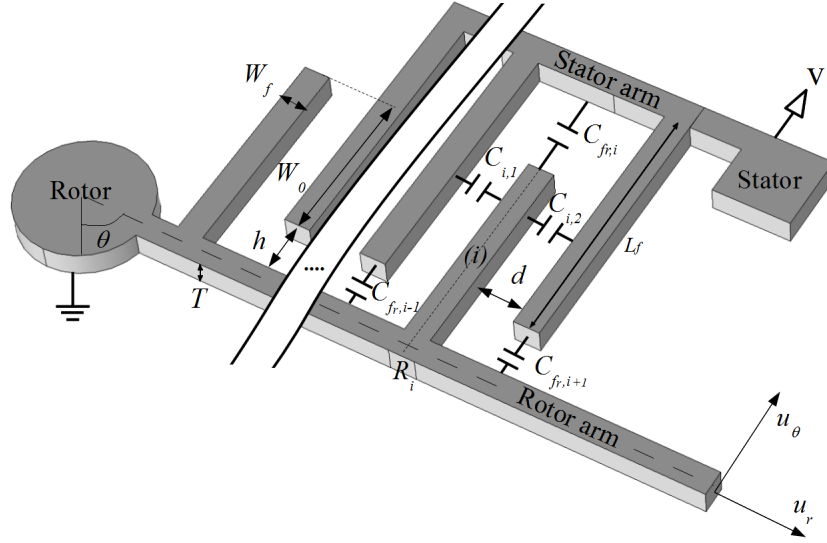


Figure 4.17: Electrical model for sliding-comb capacitors

capacitors, the total capacitance is given by the following expression:

$$C_A = \sum_{i=0}^{2N_f-1} N_c \frac{\varepsilon_0 T (W_0 + (R_1 + iL_p)\theta)}{d} + \sum_{i=0}^{2N_f-1} N_c \frac{\varepsilon_0 T W_f}{h - (R_1 + iL_p)\theta} \quad (4.61)$$

where $L_p = d + W_f$. The Taylor expansion of equation (4.61) gives the following approximation for the capacitance C_A :

$$C_A \approx C_{01} (1 + a_{11}\theta) + C_{02} (1 + a_{21}\theta + a_{22}\theta^2) \quad (4.62)$$

where the different constants introduced in equation (4.62) are given by:

$$\begin{aligned} C_{01} &= N_c \frac{(2N_f - 1) \varepsilon_0 T W_0}{d} \\ C_{02} &= N_c \frac{(2N_f - 1) \varepsilon_0 T W_f}{h} \\ a_{11} &= \frac{R_1 + N_f L_p}{W_0} \\ a_{21} &= \frac{R_1 + N_f L_p}{h} \\ a_{22} &= \frac{4 L_p^2 N_f^2 + (6 R_1 - L_p) L_p N_f + 3 R_1^2}{3 h^2} \end{aligned} \quad (4.63)$$

Since a double sided capacitor configuration is used, one can deduce the capacitance of the opposite comb electrode C_b :

$$C_B \approx C_{01} (1 - a_{11}\theta) + C_{02} (1 - a_{21}\theta + a_{22}\theta^2) \quad (4.64)$$

Finally, the position-to-capacitance constant for this particular design is:

$$K_{\theta-C} = 2 (a_{11}C_{01} + a_{21}C_{02}) \quad (4.65)$$

The position-to-capacitance constant arises from the contribution of two capacitance modes:

- The first one utilizes a change in the lateral area of finger electrodes. This mode is purely linear and it is characterized by the nominal capacitance C_{01} and the parameter a_{11} .
- The second one utilizes a change in gap at the tip of finger electrodes. This mode is non-linear and it is characterized by the nominal capacitance C_{02} and the parameters a_{21} .

Moreover, one can calculate the electrostatic moment applied to the structure using the voltage distribution shown in equation (3.22):

$$M_{el} = K_{\theta-C} (V_{DC}V_{fb}) + 2a_{22}C_{02} (V_{DC}^2 + V_{fb}^2) \theta \quad (4.66)$$

Considering the expressions from equations (4.65) and (4.66) as well as the feedback voltage V_{fb} developed in equation (3.57), one can calculate the amplification gain of the capacitive detection and the feedback gain of the control loop. Therefore, the parameters of the sliding-plate capacitor design are given by:

$$\begin{aligned} G_{det} &= G_{amp}K_{\theta-C} \\ K_{fb} &= \frac{K_{\theta-C}D_cV_{DC}^2}{J_z} \end{aligned} \quad (4.67)$$

where G_{amp} is the amplifier gain, D_c the duty cycle parameter, V_{DC} the reference voltage applied to the system and J_z the moment of inertia of the structure. The second term in equation (4.66) is equivalent to the electrical spring coefficient

introduced in (3.30) and it is given by the following expression:

$$K_{el} = 4a_{22}C_{02}V_{DC}^2 \quad (4.68)$$

It is interesting to notice that, even though we are designing a sliding-capacitor configuration working principally with a change in the electrode overlap W_0 , a gap-closing effect is introduced through the finger tips. This component can be minimized by increasing the normal gap h . In this case, the rotation sensor works with pure sliding capacitors. Otherwise, one can minimize the normal gap h , thus creating a hybrid design mixing sliding capacitors and gap closing capacitors.

4.4.2 Gap-closing capacitors

The same study is performed to evaluate the electrical parameters of a gap-closing capacitor design presented in figure 4.18. The geometrical parameters characterizing this design in terms of electrical properties are summarized in table 4.6.

Vacuum permittivity	ε_0
Structure thickness	T
Electrode surface	W_0
Normal gap	h

Table 4.6: Geometrical parameters characterizing the electrical properties of the gap-closing design

The capacitance expression resulting from N_c long-plate electrodes is given by:

$$C_A = N_c \int_{R_1}^{R_2} \frac{\varepsilon_0 T}{h - r\theta} dr = \frac{N_c \varepsilon_0 T}{\theta} (\ln(R_2\theta + h) - \ln(R_1\theta + h)) \quad (4.69)$$

where $R_2 = R_1 + W_0$ with W_0 the overlap between the moving and the fixed electrode. A Taylor expansion of equation (4.69) gives:

$$C_A \approx N_c \frac{\varepsilon_0 T W_0}{h} \left(1 - \frac{(2R_1 + W_0)\theta}{2h} + \frac{(3R_1^2 + 3R_1 W_0 + W_0^2)\theta^2}{3h^2} \right) \quad (4.70)$$

Through the approximation given in equation (4.70) and using the same method-

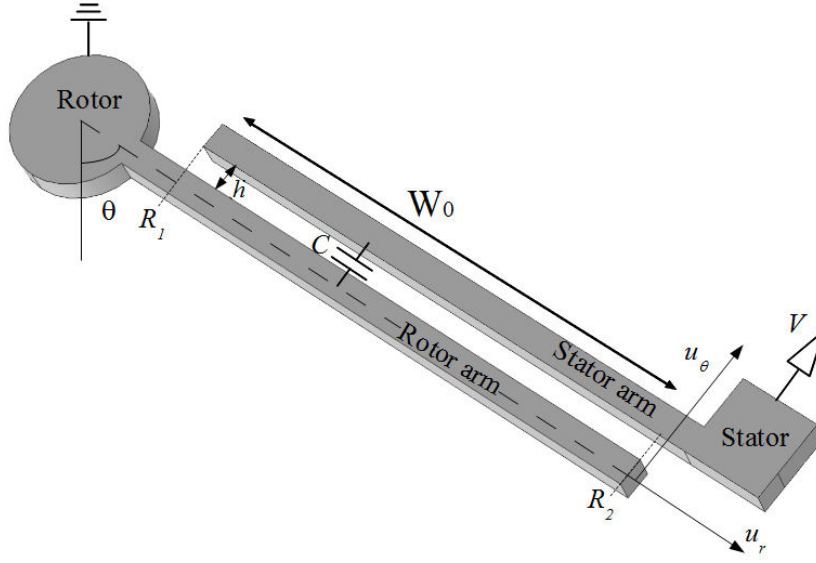


Figure 4.18: Electrical model for gap-closing capacitors

ology as for the sliding-comb design in the previous section, one can calculate the electrical parameters of the gap-closing capacitor design:

$$\begin{aligned}
 C_0 &= N_c \frac{\varepsilon_0 T W_0}{h} \\
 K_{\theta-C} &= C_0 \left(\frac{(2R_1 + W_0)}{h} \right) \\
 G_{det} &= G_{amp} K_{\theta-C} \\
 K_{fb} &= \frac{K_{\theta-C} D_c V_{DC}^2}{J_z} \\
 K_{el} &= 4C_0 V_{DC}^2 \frac{(3R_1^2 + 3R_1 W_0 + W_0^2)}{3h^2}
 \end{aligned} \tag{4.71}$$

This design configuration is based on pure gap-closing electrodes and the capacitance value is set by the electrode dimensions. As a result, one has to prefer long electrodes to obtain increased capacitance values.

4.4.3 Simulation of the nominal capacitance

The parallel-plate approximation was used to calculate the electrical properties of the two rotation sensor configurations. In this section, 3D FEM simulations are done using *COMSOL Multiphysics*[®] to validate this assumption by comparing the simulation of the nominal capacitance with the analytical equations from equations (4.63) and (4.71). The values of the geometrical parameters used for the evaluations are presented in table 4.7.

Sliding-comb design	Value	Unit
T	80	$[\mu m]$
W_0	10	$[\mu m]$
W_f	3	$[\mu m]$
d	2	$[\mu m]$
h	10	$[\mu m]$
N_f	41	
N_c	1	

Gap-closing design	Value	Unit
T	80	$[\mu m]$
W_0	300	$[\mu m]$
h	2	$[\mu m]$
N_c	1	

Table 4.7: Values of the geometrical parameters used for evaluations

The nominal capacitance values are summarized in table 4.8 and show that the gap-closing model fit the simulated value whereas there is an important difference for the sliding-comb model. This can be explained by the influence of the fringing fields that create an accumulation of electrical charges at electrode extremities. Indeed, in the gap-closing case, the capacitor has a large area, hence, the electrostatic energy is determined by the amount of electrical charges accumulated between the electrodes. On the contrary, in the sliding-comb case, the electrodes have small dimensions and the fringing fields have larger influence according to figure 4.19. One can observe in this plot that an important electrostatic energy is accumulated at finger ends which is not negligible compared to the effective electrostatic energy between fingers.

As a final remark on nominal capacitance, the gap parameters d and h are sensitive to over-etching during processing. Thus, to operate safely with the ASIC, one has to design prototypes with a larger nominal capacitance than the one given in the technical constraints from table 4.1.

Design case	C_0 [F] (Analytical)	C_0 [F] (COMSOL)	Relative difference
Sliding-comb (4.4.1)	3×10^{-13}	4.2×10^{-13}	29%
Gap-closing (4.4.2)	1.05×10^{-13}	1.15×10^{-13}	9%

Table 4.8: Comparative table of the nominal capacitance evaluated with COMSOL and with analytical formulas

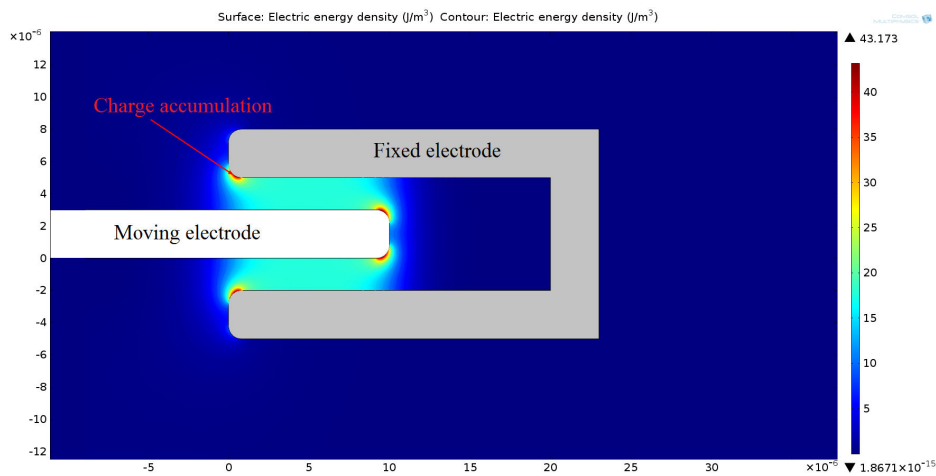


Figure 4.19: Simulation of the electrostatic energy for a comb structure

4.5 Influence of nonlinearities on the MEMS response

Throughout the detailed design of the rotation sensor, different approximations have been made considering small amplitudes θ of the seismic mass. For instance, this allowed us to establish simple expressions of the capacitance as shown in equations (4.62) and (4.70), resulting in first order expressions of the electrostatic moments. Since the system is operated in a closed-loop mode, the approximations made are reasonable. However, open-loop operations of the rotation sensor may be influenced by non-linear terms as it will be noticed in section 5.3.

Experimentally, a nonlinear behavior of the rotation sensor may occur with a resonance peak distorted in lower or higher frequency for strong amplitudes of the proof mass in open-loop mode. Indeed, it is possible to observe the influence of the nonlinear spring coefficient developed in equation (4.21) as well as higher order terms of θ in the electrostatic moment expressions. In order to understand the nonlinear behavior of the rotation sensor, one should expand the nonlinear terms to higher orders and study the consequences when the mechanical nonlinear coefficients are stronger than the electrical nonlinear ones and vice-versa.

In what follows, a nonlinear model of the rotation sensor is derived by considering generic nonlinear terms. Then, in a second step, the generic terms are replaced by the calculated constants of rotation sensor designs to predict the dominant nonlinear mechanism for each configuration (sliding-comb or gap-closing electrodes).

4.5.1 Nonlinear model of the rotation sensor

Let us consider that generic nonlinear terms modify the motion equation as follows:

$$J_z \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt} + K\theta + K_1\theta^2 + K_2\theta^3 = M_{el0} \quad (4.72)$$

Assuming undamped and unforced vibrations regime by setting $D = 0$ and $M_{el0} = 0$, the equilibrium equation (4.72) can be expressed in a canonical form:

$$\frac{d^2\theta}{dt^2} + \omega_0^2\theta + \alpha\varepsilon\theta^2 + \beta\varepsilon^2\theta^3 = 0, \quad \varepsilon > 0 \quad (4.73)$$

In the following development, we use a Lindstedt's method [53] which is a perturbation analysis around linear oscillations at the angular frequency ω_0 . The strategy is to build a period-amplitude relationship into the approximate solution

of the nonlinear equation. First, one has to make a change of variables as follows:

$$\tau = \omega t, \quad \text{where } \omega = \omega_0 + \varepsilon \omega_1 + \varepsilon^2 \omega_2 + \mathcal{O}(\varepsilon^3) \quad (4.74)$$

Next, we expand θ in a power series of ε :

$$\theta(\tau) = \theta_0(\tau) + \varepsilon \theta_1(\tau) + \varepsilon^2 \theta_2(\tau) + \mathcal{O}(\varepsilon^3) \quad (4.75)$$

Substituting equations (4.74) and (4.75) to (4.73), one can find the following equation, sorted with respect to the powers of ε :

$$\begin{aligned} & \left[\omega_0^2 \left(\theta_0 + \frac{d^2 \theta_0}{d\tau^2} \right) \right] + \\ & \varepsilon \left[\omega_0^2 \left(\theta_1 + \frac{d^2 \theta_1}{d\tau^2} \right) + 2\omega_0 \omega_1 \frac{d^2 \theta_0}{d\tau^2} + \alpha \theta_0^2 \right] + \\ & \varepsilon^2 \left[\omega_0^2 \left(\theta_2 + \frac{d^2 \theta_2}{d\tau^2} \right) + 2\omega_0 \omega_1 \frac{d^2 \theta_1}{d\tau^2} + \frac{d^2 \theta_0}{d\tau^2} (\omega_1^2 + 2\omega_0 \omega_2) + 2\alpha \theta_0 \theta_1 + \beta \theta_0^3 \right] \\ & + \mathcal{O}(\varepsilon^3) = 0 \end{aligned} \quad (4.76)$$

To be satisfied for any $\varepsilon \neq 0$, the terms inside brackets in equation (4.76) have to vanish. This results in:

$$\theta_0 = A \cos(\tau) \quad (4.77)$$

Where A is an integration constant representing oscillation amplitude. Substituting equation (4.77) to (4.76) gives:

$$\omega_0^2 \left(\theta_1 + \frac{d^2 \theta_1}{d\tau^2} \right) = 2\omega_0 \omega_1 A \cos(\tau) - \frac{\alpha}{2} A^2 (1 + \cos(2\tau)) \quad (4.78)$$

The participation of the term proportional to $\cos(\tau)$, in the right hand side of equation (4.78), generates a term in $\tau \sin(\tau)$ in the particular solution of θ_1 . this is a so-called *secular term* and it needs to vanish in order to keep the solution periodic. As a consequence, one should remove the secular term by setting $\omega_1 = 0$. Finally, equation (4.78) can be solved as follows:

$$\theta_1(\tau) = \frac{\alpha A^2}{3\omega_0^2} \cos(\tau) + \frac{\alpha A^2}{6\omega_0^2} (\cos(2\tau) - 3) \quad (4.79)$$

From equation (4.79), a first order approximation of θ shows the apparition of two additional frequency components: a DC-term and a higher harmonic term at twice the oscillation frequency.

The same process can be repeated by substituting equations (4.77) and (4.79) to (4.76). The removal of secular terms gives the expression of ω_2 such as:

$$\omega_2 = \frac{A^2}{24\omega_0^3} (9\beta\omega_0^2 + 10\alpha^2) \quad (4.80)$$

Finally, the modified angular frequency function due to nonlinear terms is given by:

$$\omega = \omega_0 + \varepsilon^2 \omega_2 = \omega_0 + \gamma A^2 \quad (4.81)$$

where the coefficient γ depends on the coefficients K , K_1 and K_2 defined in equation (4.72), and expressed by:

$$\gamma = \frac{3}{8} \frac{K_2}{K} \omega_0 - \frac{5}{12} \frac{K_1^2}{K^2} \omega_0 \quad (4.82)$$

Interesting comments can be made from equations (4.81) and (4.82). Indeed, depending on the sign of γ , the angular frequency of the system can shift to a lower or higher frequency for high vibration amplitude as shown in figure 4.20. It could be interesting to evaluate the nonlinear terms of the rotation sensor (both electrical and mechanical) and calculate the constant γ to determine in which direction the resonance peak is distorted and then, concluding on the dominant non-linearity source. This is performed in the next section.

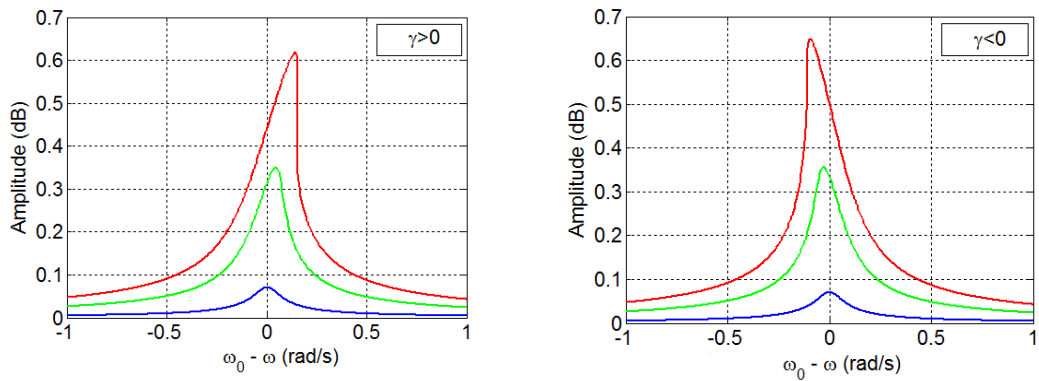


Figure 4.20: Illustration of the resonance peak shift due to nonlinear mechanisms introduced through the coefficient γ ; Left: $\gamma > 0$; Right: $\gamma < 0$. Blue, green and red curves are obtained for small, medium, and strong vibration amplitude A respectively

4.5.2 Prediction of the dominant nonlinear mechanism of the rotation sensor configurations

In this section, we use the electrical formula calculated in sections 4.4.1 and 4.4.2, but with a series expansion to the third order in θ . Considering designs using comb electrodes, the first order coefficient Kec_1 as well as the second order and the third order terms (Kec_2 and Kec_3 respectively) are given as follows:

$$\left\{ \begin{array}{l} Kec_1 = \frac{C_{02}V_{DC}^2}{3} \frac{16 L_p^2 N_f^2 - 4 L_p (L_p - 6 R_1) N_f + 12 R_1^2}{h^2} \\ Kec_2 = 12 C_{02} V_{DC}^2 \frac{(L_p N_f + R_1) \left((N_f^2 - \frac{1}{2} N_f) L_p^2 + L_p N_f R_1 + \frac{1}{2} R_1^2 \right)}{h^3} \\ Kec_3 = \frac{C_{02}V_{DC}^2}{15h^4} \left[(384 N_f^4 - 288 N_f^3 + 16 N_f^2 + 8 N_f) L_p^4 \right. \\ \quad + 960 \left(N_f - \frac{1}{2} \right) N_f^2 R_1 L_p^3 + 960 N_f \left(N_f - \frac{1}{4} \right) R_1^2 L_p^2 \\ \quad \left. + 480 L_p N_f R_1^3 + 120 R_1^4 \right] \end{array} \right. \quad (4.83)$$

Considering designs using gap-closing electrodes, the first order coefficient Keg_1 as well as the second order and the third order terms (Keg_2 and Keg_3 respectively) are given as follows:

$$\left\{ \begin{array}{l} Keg_1 = \frac{4C_0V_{DC}^2}{3} \frac{(3 R_1^2 + 3 R_1 W_0 + W_0^2)}{h^2} \\ Keg_2 = \frac{3C_0V_{DC}^2}{2} \frac{(2 R_1^2 + 2 R_1 W_0 + W_0^2) (W_0 + 2 R_1)}{h^3} \\ Keg_3 = \frac{8C_0V_{DC}^2}{5} \frac{(5 R_1^4 + 10 R_1^3 W_0 + 10 R_1^2 W_0^2 + 5 R_1 W_0^3 + W_0^4)}{h^4} \end{array} \right. \quad (4.84)$$

Given the mechanical spring terms determined in section 4.2.1.2, one can write the first order and the third order terms (Km and Km_3 respectively) as follows:

$$\left\{ \begin{array}{l} Km = \frac{4 E_y T W_b^3 (L_b^2 + 3 L_b R_a + 3 R_a^2)}{3 L_b^3} \\ Km_3 = \frac{4 E_y T W_b (17 L_b^2 + 33 L_b R_a + 18 R_a^2) (L_b^2 + 9 L_b R_a + 9 R_a^2)}{225 L_b^3} \end{array} \right. \quad (4.85)$$

Finally, one can evaluate the constants K , K_1 and K_2 used to calculate γ in equation 4.82:

$$\begin{cases} K &= Km - Ke_1 \\ K_1 &= -Ke_2 \\ K_2 &= Km_3 - Ke_3 \end{cases} \quad (4.86)$$

where $Ke_1 = K_{ec_1}$ or K_{eg_1} ; $Ke_2 = K_{ec_2}$ or K_{eg_2} ; $Ke_3 = K_{ec_3}$ or K_{eg_3} depending on the design case considered.

In what follows, the two configurations of the rotation sensor (sliding-comb capacitor and gap-closing capacitor) are analyzed:

Sliding-comb capacitors

In case of large normal gap h at the finger tips, the non-linear behavior is only induced by the mechanical term K_{m3} since $K_{ec_1} = K_{ec_2} = K_{ec_3} = 0$ in equation 4.83. As a consequence, the resonance peak should shift to the upper frequencies. However, if one considers the geometrical parameters shown in appendix B.4, where the normal gap h has the same value as the lateral gap d , the non-linear electrical terms are no more negligible. Moreover, the coefficient γ of equation (4.82) becomes negative implying a shift of the resonance peak to the lower frequencies.

Gap-closing capacitors

In the pure gap-closing case, the non-linear electrical terms are dominant over the mechanical one. For instance, considering the geometrical parameters shown in appendix B.5, the coefficient γ is negative, thus, it results in a shift of the resonance peak to the lower frequencies.

These predicted behaviors will be confirmed in section 5.3.3.

4.6 Optimization of MEMS rotation sensor designs

In the following section, the design formula obtained through the different analysis are combined to find an optimal structure with the best performances while meeting the technical constraints detailed in table 4.1. In order to give the theoretical resolution and sensitivity of rotation sensors prototypes, one should consider operational parameters measured on the electronics that will be used to control the MEMS sensitive element. Those parameters are defined in tables 3.1 and 3.2 and their values can be found in table 4.9.

For the evaluation of the rotation MEMS performances, we will focus on four output parameters: the noise floor (NF), the mechanical sensitivity (G_{mech}), the detector sensitivity (G_{det}) and the nominal capacitance (C_0), respectively defined by the following expressions:

$$\begin{aligned}
 NF &= \sqrt{\frac{(G_n^2 \overline{V_n^2} + 4k_B T R_{det})}{(G_{mech} G_{det})^2} + \frac{4k_B T \omega_0}{J_z Q} + 4k_B T R_{dac} K_{fb}^2} \\
 G_{mech} &= \frac{1}{\omega_0^2 - \frac{K_{el}}{J_z}} \\
 G_{det} &= \frac{2V_{DC} K_{\theta-C}}{C_f} \\
 C_0 &= \underbrace{N_c (2N_f - 1) \left[\frac{\varepsilon_0 T W_0}{d} + \frac{\varepsilon_0 T W_f}{h} \right]}_{\text{equation (4.63)}} \text{ or } \underbrace{N_c \frac{\varepsilon_0 T W_0}{h}}_{\text{equation (4.71)}}
 \end{aligned} \tag{4.87}$$

Specifications	Parameter name	Value
Electronics supply voltage	V_{DC}	1.5 [V]
Duty-cycle parameter	D_c	0.75
Amplifier noise input	$\overline{V_n}$	25 nV/ $\sqrt{Hz}$
Amplifier noise resistor	R_{det}	10 M Ω
DAC noise resistor	R_{dac}	6 k Ω
Parasitic capacitance of the MEMS	C_p	45 pF

Table 4.9: Measured parameters for operations with the electronics

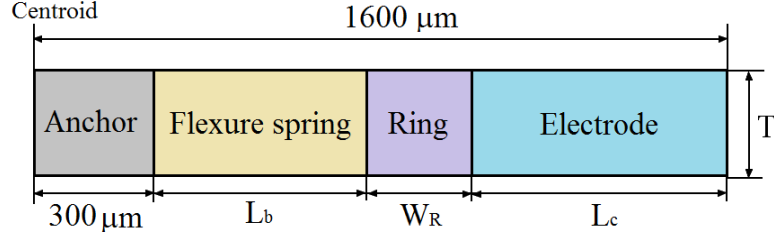


Figure 4.21: Half view of the rotation sensor sensitive element; the anchor radius and the sensitive element size are fixed by the process

The difficulty in the optimization is the counteracting roles of the different design elements (mechanical and electrical) with the chip size fixed at 3.2 mm and considering a fixed anchor radius at $300\text{ }\mu\text{m}$ for electrical bonding (see figure 4.21). for instance, by observing equation (4.87), one should decrease the stiffness coefficient and increase the structure inertia, with the help of a longer spring length L_b and a bigger ring width W_R , so that the noise floor is minimized while the mechanical sensitivity is maximized. However, this will reduce the electrical detection since $K_{\theta-C}$ depends on the electrode length L_c .

The optimization process will be performed as follows: the ring width can be maintained at specific values between $150\text{ }\mu\text{m}$ and $450\text{ }\mu\text{m}$, then the varying parameter will be the beam length, ranging between $350\text{ }\mu\text{m}$ and $800\text{ }\mu\text{m}$. The electrode length L_c is automatically updated so that the total size is fixed at $1600\text{ }\mu\text{m}$.

4.6.1 Sliding-comb designs

This section deals with the optimization of sliding-comb designs. In a first step, a pure sliding-capacitor is studied by taking a large normal gap at the finger tip to avoid non-linear effects. Then, in a second step, a hybrid design is studied mixing sliding capacitors and gap closing capacitors.

4.6.1.1 Pure sliding-plate capacitors

This section deals with the optimization of a design with pure sliding-plate capacitors. In other words, the finger tips are positioned far from the fixed comb so that electrostatic effects are introduced only through lateral edges of finger electrodes. According to the parameters and the constants defined in section 4.4.1, let us consider that the normal gap h is set at 10 microns whereas the lateral gap d is set at 2 microns. The ring width is fixed at two values ($300\text{ }\mu\text{m}$ and $450\text{ }\mu\text{m}$) whereas the spring length varies between $350\text{ }\mu\text{m}$ and $800\text{ }\mu\text{m}$. To ensure the

constraint on the chip size, the number of fingers N_f in a comb is updated at each step.

The variations of the output parameters with respect to the spring length are shown in figure 4.22. It is not a surprise to observe opposite variations of the mechanical sensitivity and the electrical sensitivity with respect to L_b . Moreover, one can notice that the noise floor has lower levels for wider ring along with steeper slopes for capacitance and electrical sensitivity. To operate with the control electronics, we choose two designs represented by the lines *A* and *B* in the figures. The performance values from these two designs can be seen in table 4.10.

In this design analysis, we have chosen structures working in a pure sliding mode. In other words, the electrical parameters are perfectly linear since the coefficient $K_{\theta-C}$ is constant (see section 3.2.1). However, it could be interesting to study the influence of finger tips when they get closer to the opposite fixed comb electrode. This point is studied in the next section.

Parameter name	Design A	Design B
C_0 [pF]	8	7.8
G_{mech} [$\mu\text{rad}/(\text{mrad}/\text{s}^2)$]	0.54×10^{-4}	0.94×10^{-4}
G_{det} [mV/ μrad]	7	5.8
NF [$\text{mrad}/\text{s}^2/\sqrt{(\text{Hz})}$]	5.2	3.7

Table 4.10: Performance of pure sliding combs

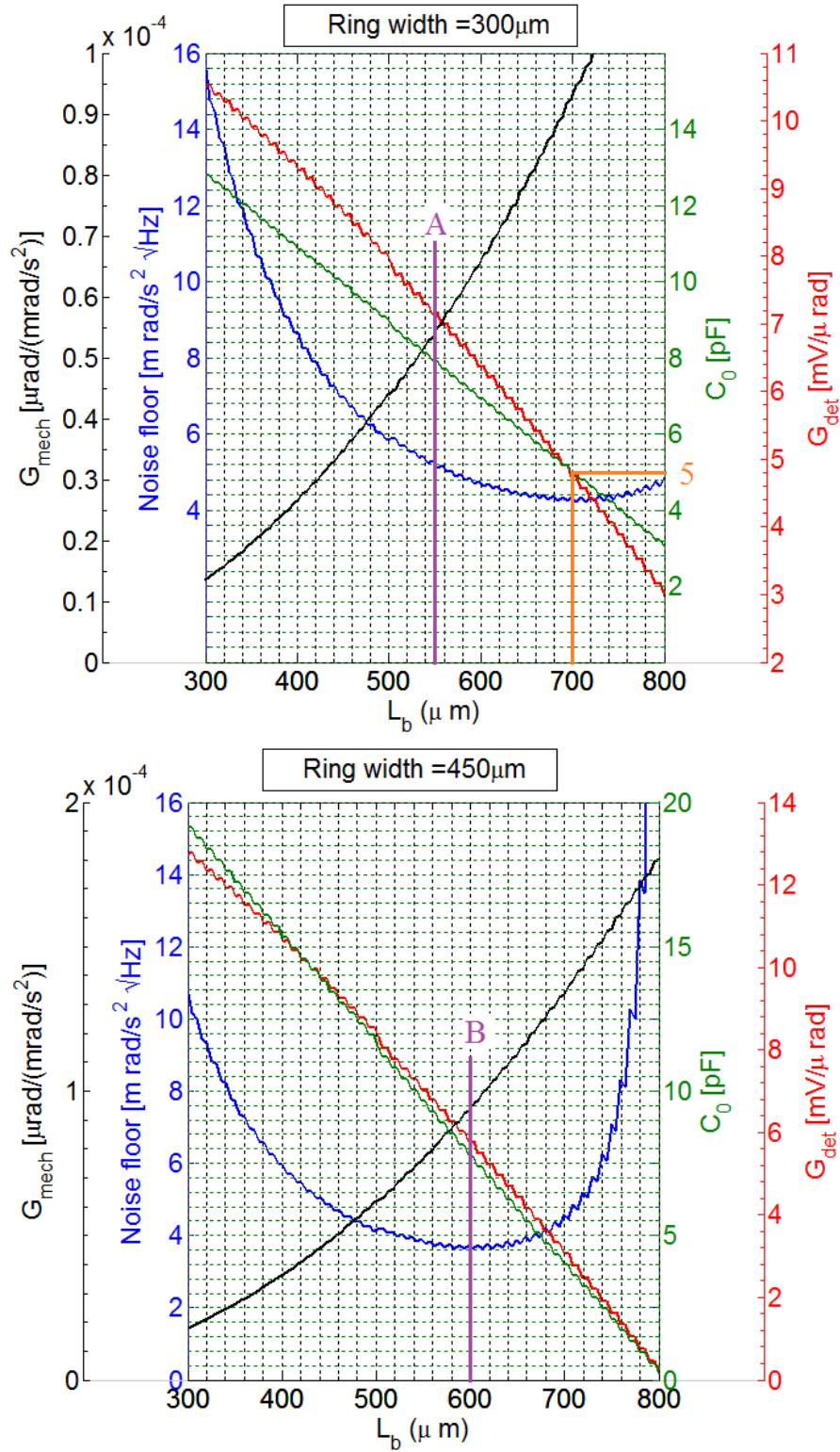


Figure 4.22: Analysis of pure sliding combs

4.6.1.2 Hybrid capacitors

For the study of the hybrid case, we modify the pure sliding designs by setting the normal gap dimension h at 2 microns. The resulting curve is shown in figure 4.23. One should notice that at the optimum point, obtained for $L_b = 550 \mu m$, the overall performances are significantly increased as presented in table 4.11. Indeed, the noise floor is reduced by a factor of 2 while the mechanical and the electrical sensitivity are increased. This can be explained by the influence of the electrostatic spring on the dynamic behavior of the structure. Indeed, one should remember that gap-closing capacitors create a negative electrostatic spring which counteracts the mechanical spring and resulting in larger mechanical sensitivity, thus lower noise floor (see section 3.2.2.3 and table 3.2). However, due to the small dimensions of finger electrodes, this electrostatic spring is too weak to cancel completely the mechanical spring for even more performances.

In spite of linear behavior, pure sliding-capacitor designs show lower performances than hybrid designs in terms of sensitivity and resolution. This is explained by the influence of the electrostatic spring effect introduced by gap-closing electrodes. However, this negative spring turns out to be too weak for a perfect matching with the mechanical spring coefficient. Since non-linear effects are stronger in the pure gap-closing configuration, one should expect better performances for this case which is studied in the next section.

Parameter name	Design C
C_0 [pF]	9.2
G_{mech} [$\mu rad/(mrad/s^2)$]	4×10^{-4}
G_{det} [mV/ μrad]	16.8
NF [$mrad/s^2/\sqrt{(Hz)}$]	0.35

Table 4.11: Performance of hybrid sliding combs

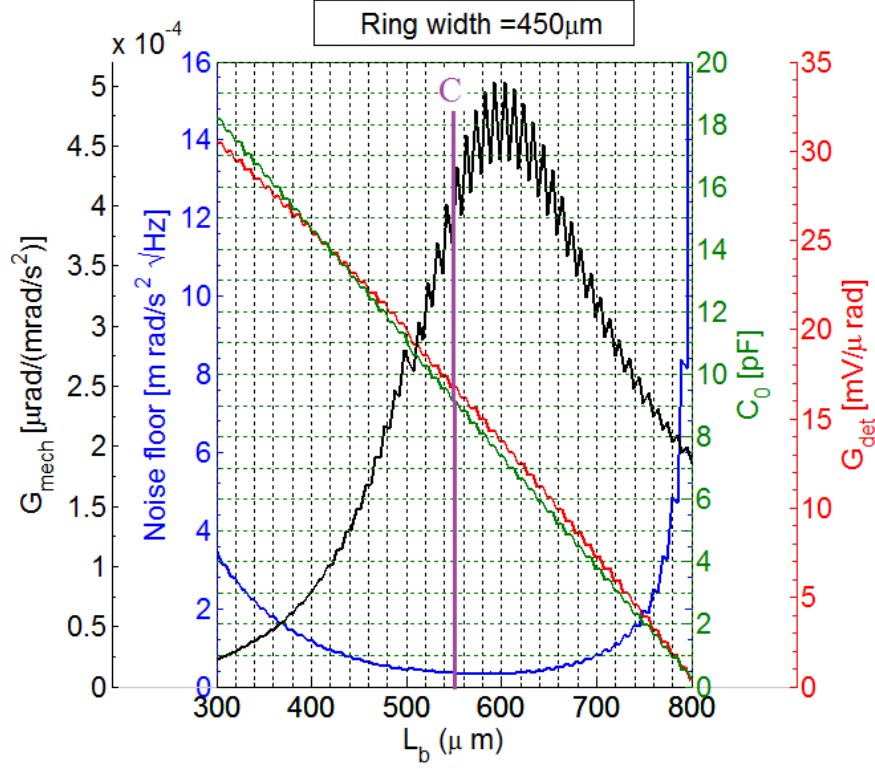


Figure 4.23: Analysis of hybrid sliding combs

4.6.2 Gap-closing designs

This section deals with the optimization of a design with gap-closing capacitors. According to the parameters and the constants used in the section 4.4.2, let us consider that the gap between moving and fixed electrodes is set at 2 microns. Here, the ring width is fixed at two specific values: 150 μm and 200 μm while the beam length varies between 350 μm and 800 μm .

The variations of the output parameters of interest with respect to L_b are shown in figure 4.24. One can notice an interesting behavior of the noise floor which has a local minimum at the same spot as a sharp peak for the mechanical sensitivity. This spot corresponds to the effect mentioned in the sliding capacitor case. Here, the matching between the electrostatic spring and the mechanical spring is perfect, leading to an infinite mechanical gain (see G_{mech} expression in table 3.2). This peak value of the mechanical sensitivity leads to a minimum level of the noise floor according to equation 4.87 since the detection noise of the charge amplifier is significantly reduced. Finally, one can notice that a change of the ring width makes the spot shift. New optimum designs are identified and their performances

are presented in table 4.12.

Parameter name	Design D	Design E
C_0 [pF]	8	6.6
G_{mech} [$\mu\text{rad}/(\text{mrad}/\text{s}^2)$]	2.1×10^{-3}	6.8×10^{-3}
G_{det} [mV/ μrad]	30	25
NF [$\text{mrad}/\text{s}^2/\sqrt{(\text{Hz})}$]	0.28	0.29

Table 4.12: Performance of gap-closing capacitors

As expected, the matching of the electrostatic spring with the mechanical one provides the best scenario to design high-sensitivity, high-resolution rotation sensors. Indeed, the noise floor is ten times better than for pure sliding-capacitor designs and the sensitivity (mechanical and electrical) is significantly improved.

Considering the geometrical parameters of the different designs selected, one has to make sure that the resulting systems are operable with the provided ASIC. This is why a global simulation of the sensor loop has to be performed to check if the rotation sensor designs are well controlled by the electronics. This is the aim of the next section.

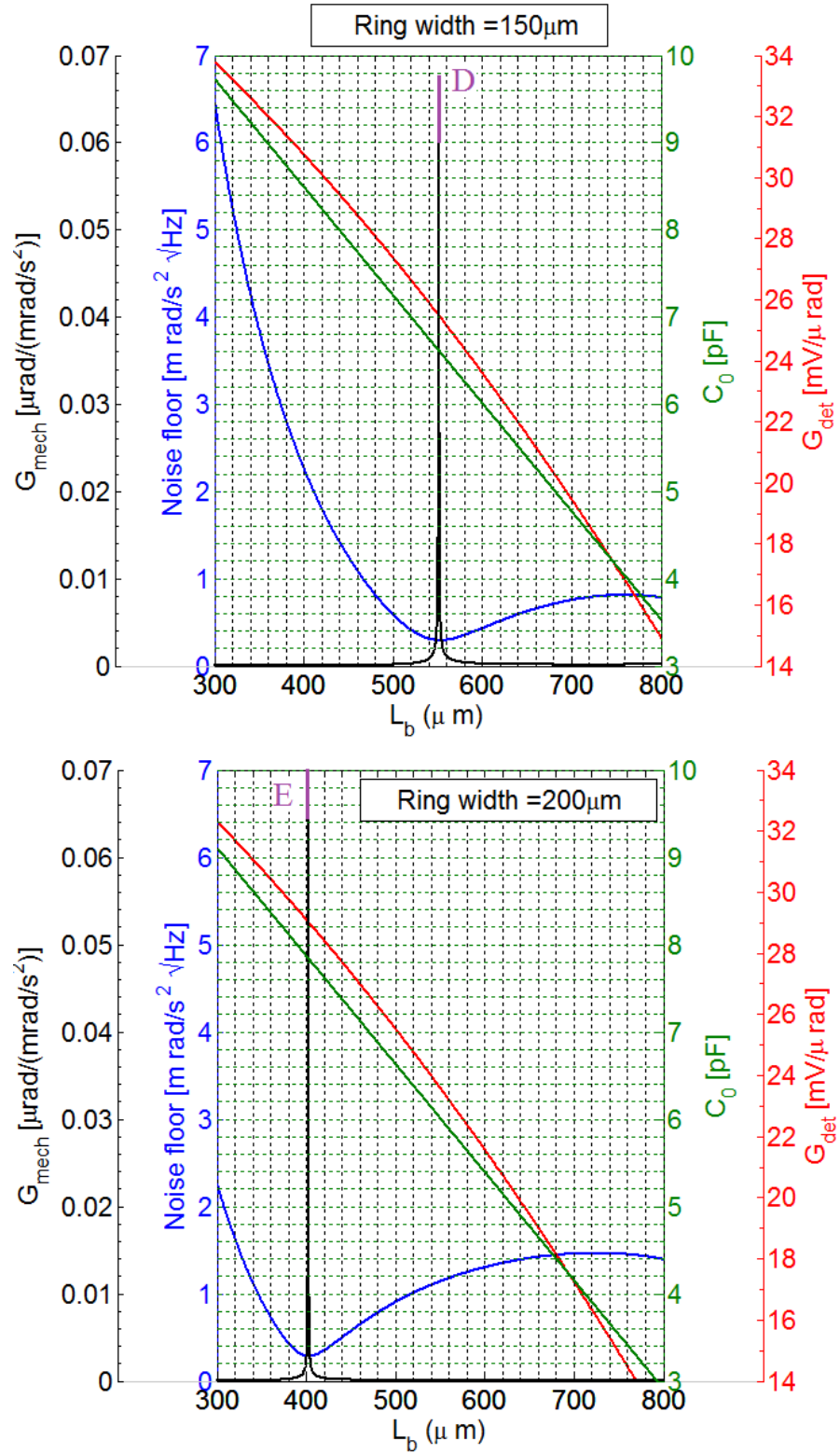


Figure 4.24: Analysis of gap-closing capacitors

4.6.3 Closed-loop simulation of the optimum designs identified

A Simulink interface has been developed to simulate the rotation sensor with its control loop. Three designs presented in the sections 4.6.1 and 4.6.2 are injected into the simulator. More precisely, we take a pure sliding design (B), the hybrid sliding design (C) and a gap-closing design (E). The output bit-stream generated by the simulated electronics is then processed by taking its one-sided Power Spectral Density (PSD). The response is shown in figure 4.25 with no external excitation applied (the amplitude in the graph is expressed in dB of $V/\sqrt{Hz}$).

One can notice a flat and low noise in a low frequency bandwidth between 10 Hz and 800 Hz and higher noise level rising after 1 kHz . This higher noise level after 1 kHz is the result of the $\Sigma\Delta$ modulation (see section 3.4.2). The noise level in the low frequency bandwidth is the noise floor for a given design. One can read a noise level at -124 dB for the design B, -144 dB for the design C and -147 dB for the design E. An interesting observation is the presence of a notch in the sliding designs revealing the structure resonance spot. In the gap-closing design, this notch does not appear in the PSD plot meaning that the control loop cancel the system resonance as well as offering the best resolution.

To conclude this chapter, one can say that designs with gap-closing capacitors are more suited for a high-performance, high-sensitivity rotation sensor. The matching between mechanical spring and electrostatic spring leads to the appearance of an optimization spot where the mechanical sensitivity is maximized while the noise floor is minimized. To validate this statement, the designs selected in this chapter will be processed and measured with our control electronics. For sake of clarity, the designs selected are renamed with the following abbreviations:

- B: Design SC for sliding combs
- C: Design HC for hybrid combs
- E: Design GC for gap-closing electrodes

The geometrical parameters of the three designs can be found in appendices B.3, B.4 and B.5.

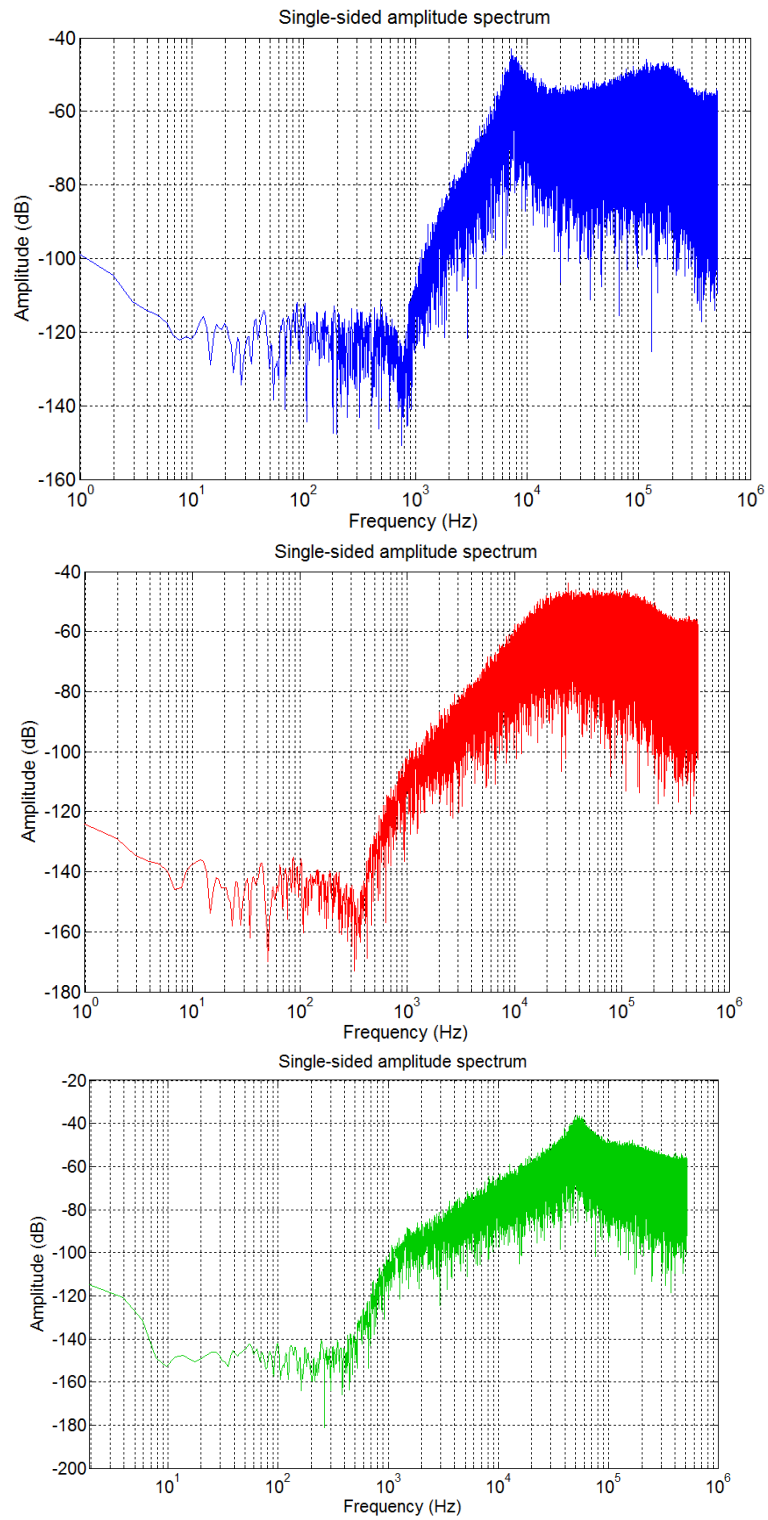


Figure 4.25: Simulated PSD of the three designs selected; Top: design SC; Middle: design HC; Bottom: design GC

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Chapter 5

Measurements and characterization of MEMS-rotation sensor prototypes

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The previous chapter led us to optimum structures that need to be processed for experimental characterization. Three structures were identified:

- Pure sliding-comb design (SC)
- Hybrid-comb design (HC)
- Pure gap-closing design (GC)

In this chapter, designs SC, HC and GC are characterized in details and experimental measurements are compared to the analytical studies in Chapter 4. Before presenting the measurements, the process utilized to manufacture the rotation sensor prototypes is described and the quantification of the over-etching of critical parameters is done using microscopic imaging. Afterwards, the characterization is organized in several steps:

First, the nominal capacitance as well as the parasitic capacitance are measured; and, the variation of the capacitance with respect to the voltage supply is given.

In a second step, an open-loop characterization is performed in order to determine the resonance frequency and the quality factor of our designs. Observations of nonlinear behaviors will be shortly presented.

Finally, the closed-loop characterization of the rotation sensor is performed and the experimental noise floor and sensitivity are determined. The performances of the rotation sensor prototypes are also compared to other rotation sensing systems, measured in the same laboratory conditions.

5.1 Processing of rotation sensor prototypes

A specific process developed so far for MEMS accelerometers is used. More specifically, the fabrication uses optimized Deep Reactive Ion Etching (DRIE) process of Silicon-On-Insulator (SOI) wafers with a thickness of $80\text{ }\mu\text{m}$. The resulting chip size is $7 \times 4.5 \times 1\text{ mm}^3$. The process flow is illustrated in figure 5.1.

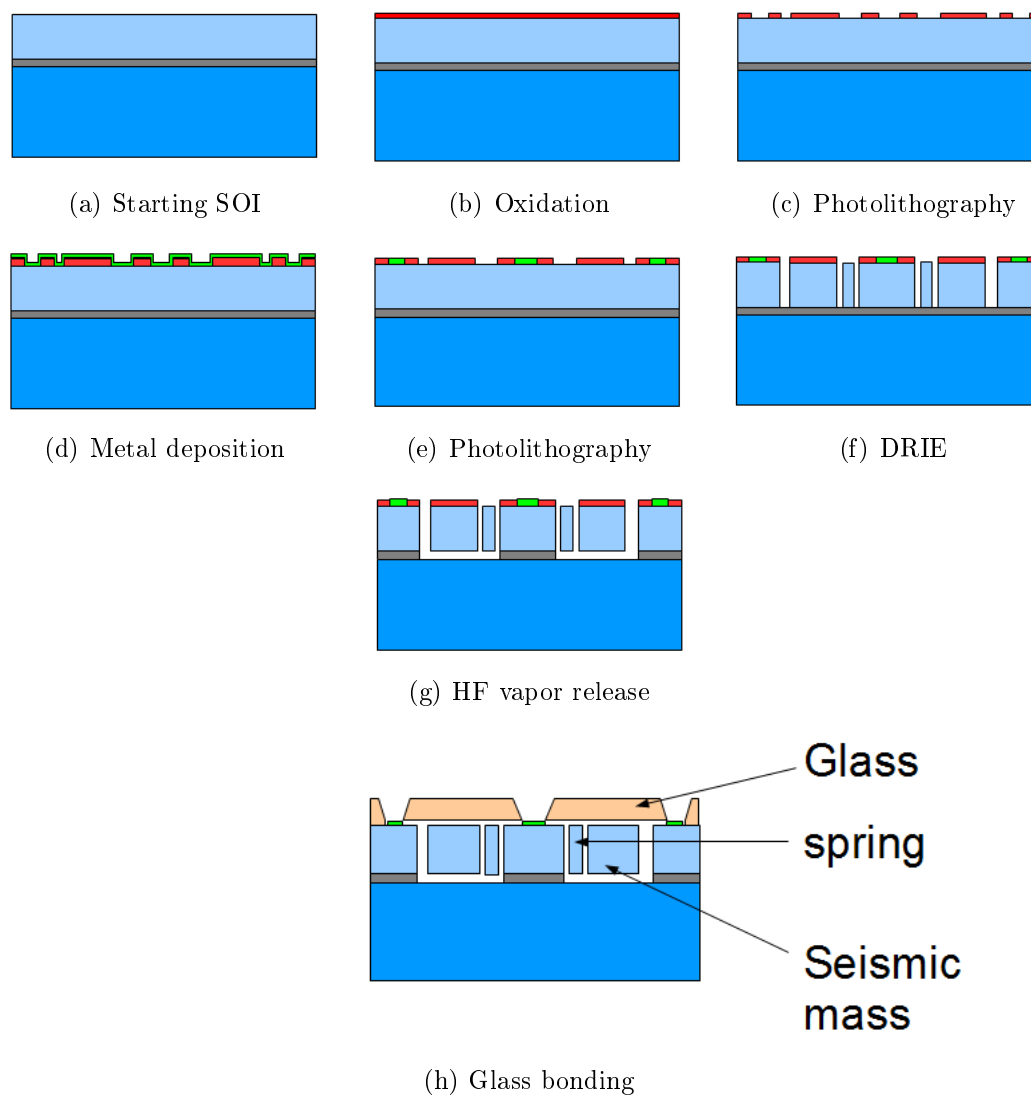


Figure 5.1: MEMS rotation sensor process

The three prototypes to characterize are manufactured based on this process. Observations by Scanning Electron Microscopy (SEM) of the three structures can be found in figures 5.2, 5.3 and 5.4.

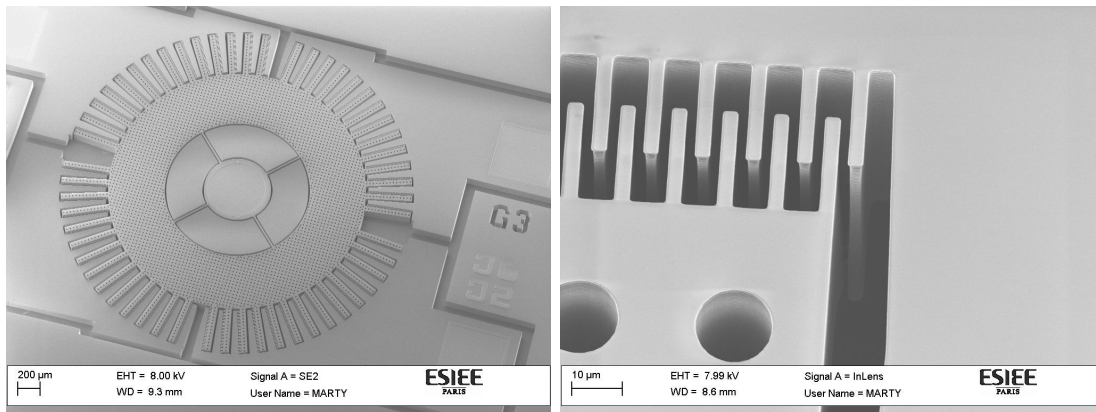


Figure 5.2: Design SC: pure sliding combs

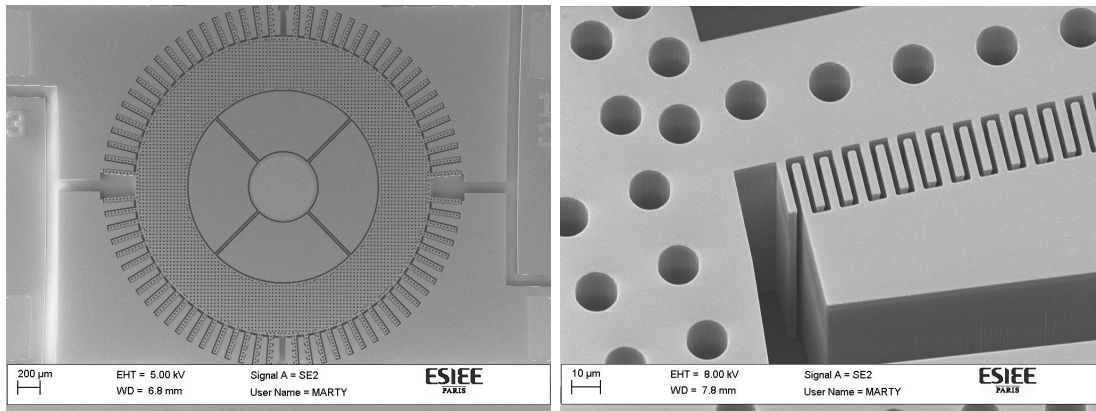


Figure 5.3: Design HC: hybrid combs

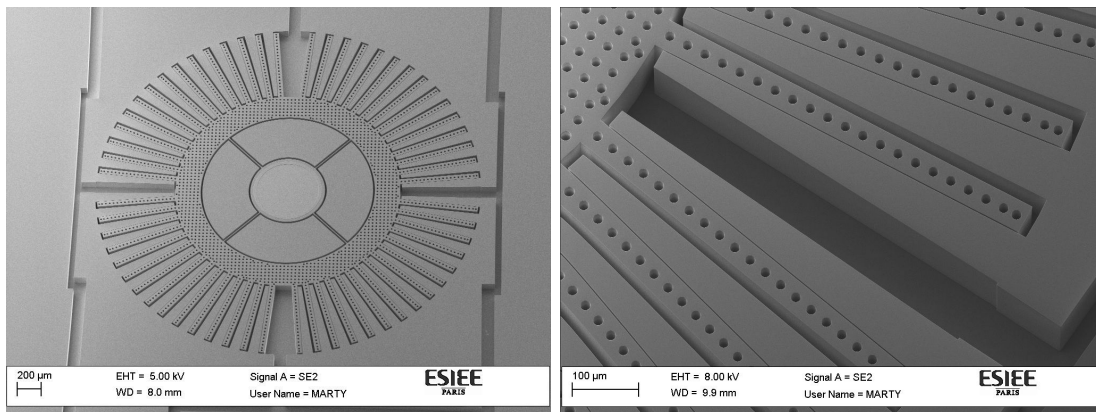


Figure 5.4: Design GC: gap-closing electrodes

One should be aware of potential over-etching of the geometry during the process. Indeed, an over-etching can modify the critical parameters of the design such as the beam width W_b or the air gaps d and h between electrodes. Dimensional analyses of rotation sensor prototypes can be seen in figure 5.5. One can notice an over-etching of $0.5\ \mu\text{m}$ on both sides of finger electrodes. As a result, the finger size is reduced by $1\ \mu\text{m}$ whereas the lateral gap is increased by $1\ \mu\text{m}$. A similar analysis can be performed on the spring width which has been reduced by $1.2\ \mu\text{m}$.

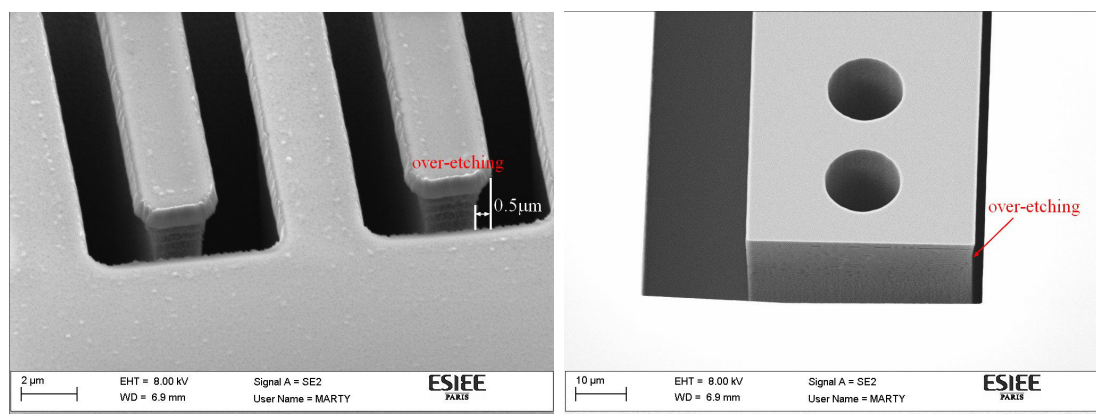


Figure 5.5: Over-etching of electrodes

The conclusion that can be drawn is that the sensitive elements of the rotation sensor will have smaller resonance frequencies than expected by analytical calculations along with modified electrical properties: lower nominal capacitance and lower electrostatic moments. From these dimensional analyses, it is possible to update our models with modified geometrical properties for comparisons with experimental measurements.

5.2 Capacitance vs Voltage (CV) characterization

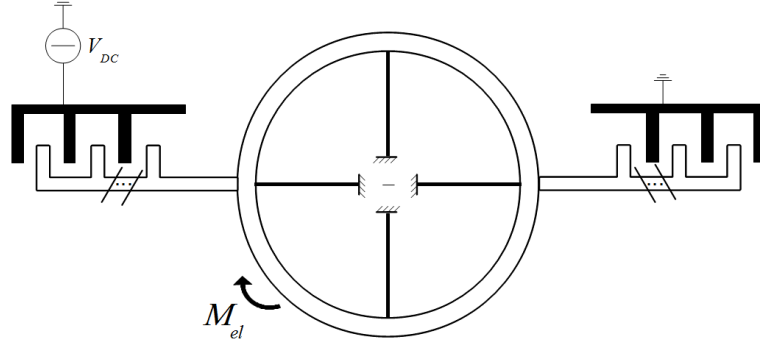


Figure 5.6: Illustration of CV characterization on a sliding case

The capacitance-voltage relationship is based on the static response of the MEMS with respect to a DC voltage input. If we polarize only one set of electrodes (figure 5.6), the seismic mass rotates due to the resulting electrostatic moments. Since the rotation amplitude θ can be important, one has to consider the non-linear terms (mechanical and electrical) established in section 4.5.2. The static equilibrium of the rotation sensor is written as:

$$\left(K_m - \frac{K_{e1}}{4}\right)\theta - \frac{K_{e2}}{4}\theta^2 + \left(K_{m3} - \frac{K_{e3}}{4}\right)\theta^3 = \frac{K_{\theta-C}}{4}V_{DC}^2 \quad (5.1)$$

The nominal capacitance is measured for each prototype and compared with theoretical results from tables 4.10, 4.11 and 4.12. The results are presented in table 5.1. One can notice that despite the over-etching of electrodes, the measured nominal capacitances have an acceptable level considering the technical constraint of the ASIC which imposes a value larger than 5 pF.

On the other hand, the CV profile of each design is shown in figure 5.7. A piecewise representation of the capacitance depending on increasing or decreasing voltage variations is made. Moreover, simulations are performed with updated parameters, which take into account the modified geometry due to over-etching. One can notice a good matching between measurements and simulations after updating the models. As expected, the design SC experiences only linear electrical behavior, thus it does not show pull-in effect unlike designs HC and GC, which have their pull-in voltages at 1.3 V and 1.7 V respectively. Another comment on designs HC and GC is that the releasing of the MEMS after pull-in occurs at different voltages. Finally, design HC presents different CV profiles for positive

and negative voltages explained by an asymmetry of the sensitive element due to etching process.

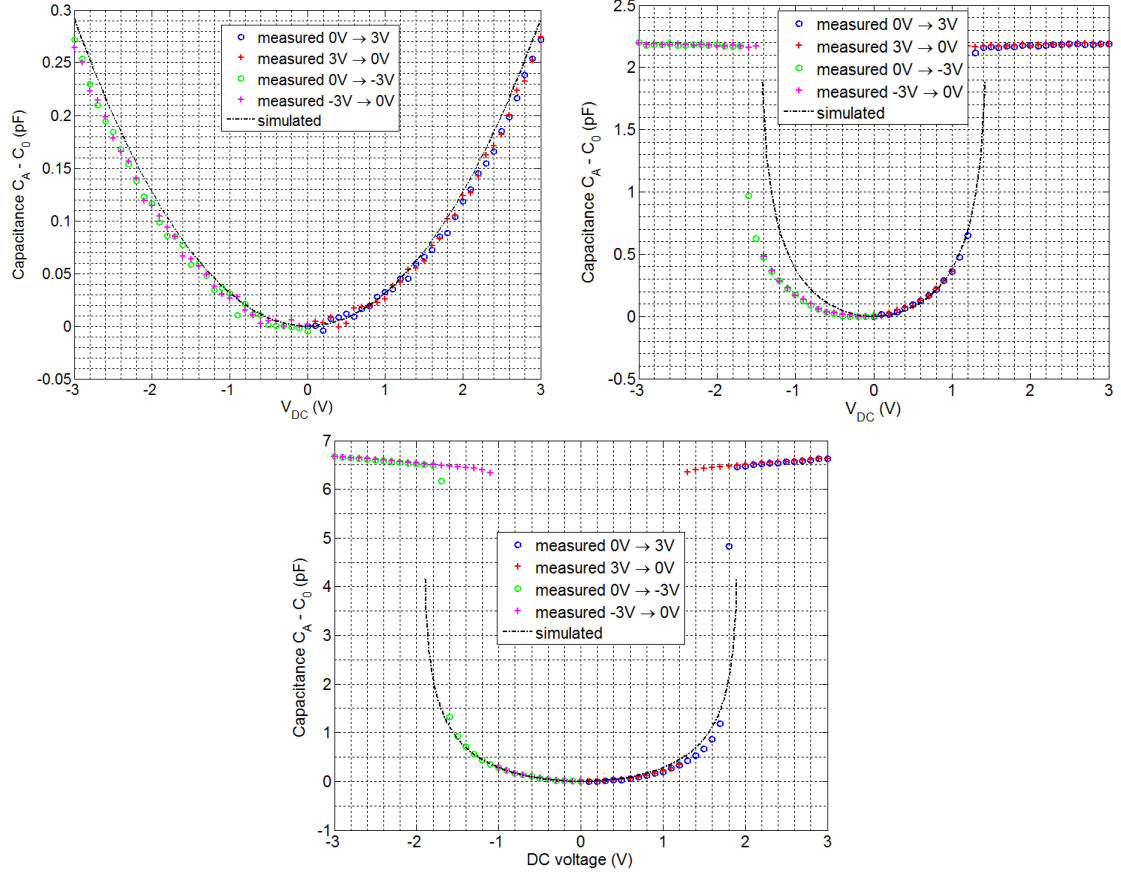


Figure 5.7: Comparison between CV measurements and simulations. Top left: design SC; Top right: design HC; Bottom: design GC

Capacitances	C_0 (measured) [pF]	C_0 (calculated) [pF]	Difference	C_p [pF]
Design SC	6.6	7.8	18%	35
Design HC	8.4	9.2	10%	39
Design GC	6	6.6	10%	44

Table 5.1: Nominal capacitance and parasitic capacitance measurements

In conclusion, the thorough modeling of the rotation sensor performed in Chapter 4 has resulting in a good fitting between measurements and simulations of the MEMS static response. The next step is the characterization of the MEMS dynamic parameters which is the focus of the next section.

5.3 Open-loop characterization of the rotation sensor

In this section, the open-loop characterization of the MEMS rotation sensor is performed. After presenting the set-up, the electrostatic transfer function of each design (SC, HC and GC) is measured to extract its dynamic parameters: resonance frequency and quality factor.

5.3.1 Set-up for the open-loop characterization

This characterization is based on the dynamic response of the MEMS through an electrostatic actuation illustrated in figure 5.8. The two electrodes A and B are polarized with a DC voltage V_{DC} , and with the addition of an AC voltage $V_{AC} \sin(\omega t)$ at one electrode and the subtraction of $V_{AC} \sin(\omega t)$ at the other.

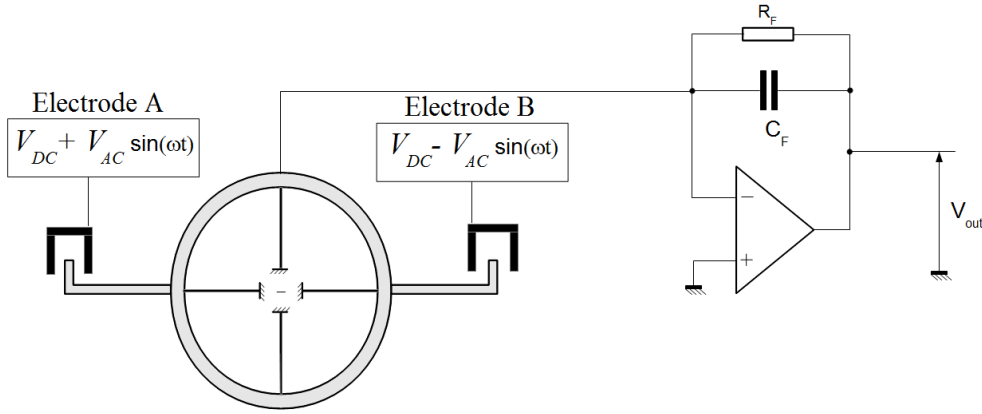


Figure 5.8: Illustration of the open-loop characterization

The voltage V_{DC} is set by the test electronics at 0.6 V. To operate the MEMS in its linear regime, a small voltage V_{AC} is applied. Finally, this experiment can be described by the following equation:

$$\ddot{\theta} + \frac{\omega_0}{Q} \dot{\theta} + \left(\omega_0^2 - \frac{K_{el}}{J_z} \right) \theta = K_{\theta-C} V_{DC} V_{AC} \sin(\omega t) \quad (5.2)$$

where K_{el} and $K_{\theta-C}$ are the electrical parameters established in sections 4.4.1 and 4.4.2.

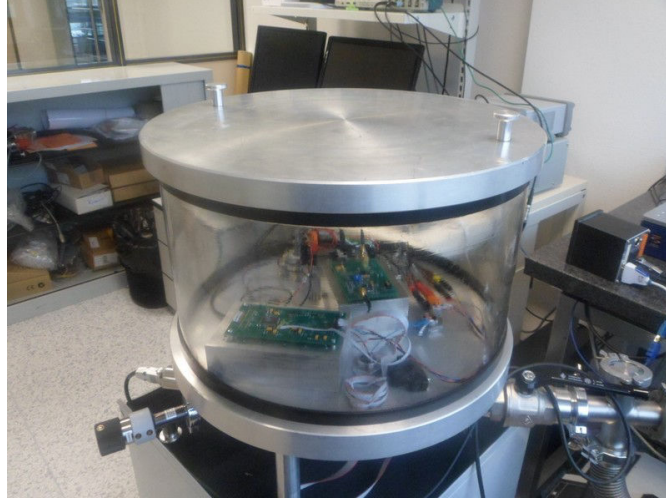


Figure 5.9: Picture of the vacuum chamber used for open-loop characterization

From section 4.3.4.1, the ambient pressure inside the MEMS cavity has a great influence on the quality factor. This is why the set-up is put inside a vacuum chamber as presented in figure 5.9 with a pressure monitored at 20 Pa or 150 mTorr . According to equation 5.2, the negative spring constant can play an important role for designs with gap-closing effect and should reduce the true mechanical resonance frequency. This electrostatic constant will be characterized further but we will use instead the analytical formulas given in equations (4.68) and (4.71). The theoretical electrostatic spring is then subtracted from the mechanical spring term, given in equation (4.8), to calculate the electro-mechanical resonance frequency. This theoretical expression is evaluated with updated geometrical parameters from process and then compared with measurements.

5.3.2 Resonance frequency and Quality factor measurement

One can observe in table 5.2 the comparison between measured and simulated resonance frequency values. With the updating of the rotation sensor models with new geometrical parameters, it can be observed a relatively good agreement between measured and simulated resonance frequency f_{res} . However, the simulated values of the Q factor are significantly different from measurements revealing the lack of reliability of damping models developed in 4.3.4.1. Nevertheless, it can be seen that the damping effects are more important in designs using gap-closing effects which agree the analysis in section 4.3.4.1.

	f_{res} [Hz] (measured)	f_{res} [Hz] (theoretical)	Relative Difference	Q (measured)	Q (theoretical)	Relative Difference
Design SC	338	348	3%	470	1200	150%
Design HC	373	363	3%	240	342	43%
Design GC	635	668	5%	80	273	240%

Table 5.2: Comparison of measured electro-mechanical resonance frequency and quality factor for the three designs considered

In conclusion, one can validate the rotation sensor models developed in Chapter 4 since they corroborate open-loop measurements in terms of electro-mechanical resonance frequency. However, the modeling of the quality factor is found to be inaccurate. Actually, this will not create any issues since a low pressure will be used in closed-loop characterizations. Indeed, one needs the quality factor to be as large as possible to make the mechanical noise negligible during measurements. Consequently, a pressure level of 1 *Pa* should be enough to consider the mechanical noise smaller than electronic noise.

5.3.3 Non-linear behavior in open-loop mode

From section 4.5.2, predictions were made on the dominant non-linear mechanism (mechanical or electrical) of the different configurations studied. It was said that pure-sliding capacitors are dominated by mechanical non-linearity whereas structures using gap-closing electrodes are dominated by electrical non-linearity. These predictions are verified in this section with an observation of the non-linear behavior of rotation sensor prototypes.

Strong voltage input V_{AC} as well as lower pressure levels are necessary to force the non-linear regime of the rotation sensor. The non-linear behavior observed are presented in figure 5.10. On one hand, the design SC has a resonance peak which is distorted toward upper frequencies validating the dominance of mechanical non-linearity. On the other hand, the resonance frequencies of design HC and GC are distorted to lower frequencies. Indeed, since these structures use gap-closing electrodes, the non-linear behavior is induced by electrical non-linearity. The non-linear behavior of the rotation sensor is given as a general observation since the system is planned to operate in a closed-loop mode.

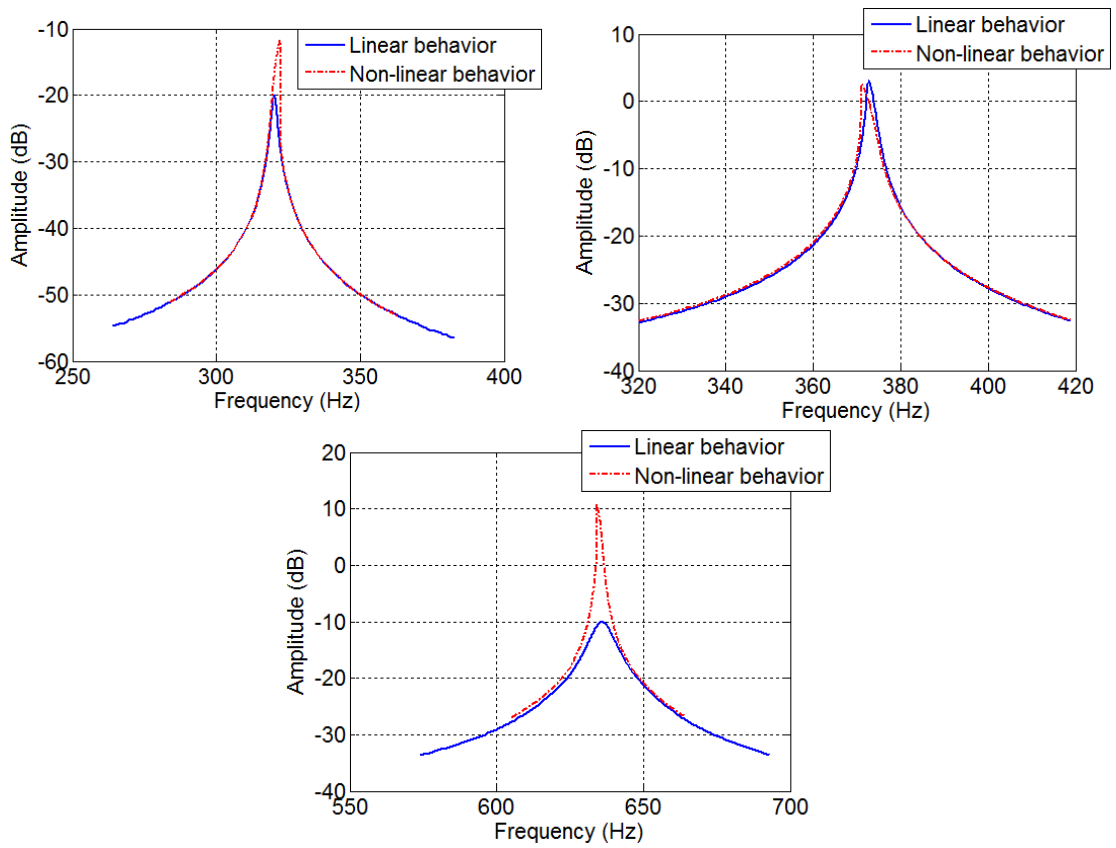


Figure 5.10: Measurements of non-linear transfer functions. Top left: Design SC; Top right: Design HC; Bottom: Design GC

5.4 Closed-loop characterization of the rotation sensor

In this section, the resolution and the sensitivity of the rotation sensor is studied by means of measurements of the MEMS in its control loop. This control loop keeps the proof mass close to its zero position so that the MEMS element works in the linear regime (no influence of the nonlinear spring effect as well as no influence of the higher order electrostatic terms). In this experiment, an important parameter is the reference DC voltage V_{DC} applied to the MEMS element since it influences significantly the negative electrostatic spring of designs HC and GC. Thus a characterization of this effect must be done before evaluating the performances. After the characterization of the electrostatic spring, it will be possible to evaluate the real performances of rotation sensor prototypes.

Throughout this section, the PSD of the MEMS output will be analyzed for characterizations. A typical PSD response of rotation sensor prototypes is presented in figure 5.11.

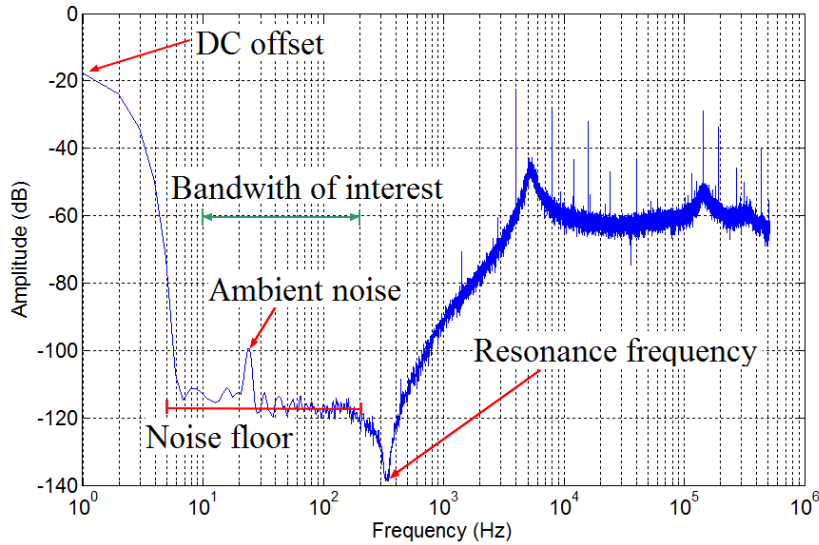


Figure 5.11: Typical PSD response of rotation sensor prototypes

One should notice the good agreement with the simulations shown in figure 4.25 except a rising spectrum at frequencies below 10 Hz. This feature is caused by the sensor DC offset since the structure is not perfectly symmetric due to non-homogeneous etching during the process. After the DC regime, the PSD decreases to reach the noise floor level. Longer time acquisitions allow to minimize this

measurement artifact. Furthermore, one can notice a peak at 24 Hz representing deterministic noise in the laboratory. Finally, the notch in the PSD plot indicates the electro-mechanical resonance frequency of the device. In order to plot the noise density of the sensors studied, we consider a frequency band of interest which is between 10 Hz and 200 Hz .

5.4.1 Electrostatic spring characterization

The resonance frequency spot is a function of the effective stiffness of the device tested. This spot can be used to characterize the electrostatic spring since it depends on the DC voltage V_{DC} . Indeed, variations of V_{DC} make the resonance frequency shift to lower or higher frequencies and measuring this shift allows to evaluate electrostatic parameters.

To this end, let us consider two voltage values E_1 and E_2 . The angular frequencies ω_1 and ω_2 of the electro-mechanical resonance are given by (see 3.2):

$$\begin{aligned}\omega_1 &= \sqrt{\frac{(K_m - K_{el1})}{J_z}} \\ \omega_2 &= \sqrt{\frac{(K_m - K_{el2})}{J_z}}\end{aligned}\tag{5.3}$$

where K_m is the mechanical stiffness, J_z the moment inertia of the structure, K_{el1} the electrostatic spring corresponding to the DC voltage V_{DC1} and K_{el2} the electrostatic spring corresponding to the DC voltage V_{DC2} . According to the definitions of the electrostatic spring in 4.68 and 4.71, one can write:

$$K_{el} = A_{el} V_{DC}^2\tag{5.4}$$

where A_{el} is a constant depending on electrode architecture. Coming back to equation 5.3, one can write:

$$|\omega_2^2 - \omega_1^2| = \frac{A_{el}}{J_z} |V_{DC1}^2 - V_{DC2}^2|\tag{5.5}$$

As a result, in order to characterize the constant A_{el} , one needs to measure the resonance frequency for two DC voltage values and to evaluate the following equation:

$$A_{el} = J_z \frac{|\omega_2^2 - \omega_1^2|}{|V_{DC1}^2 - V_{DC2}^2|}\tag{5.6}$$

The PSD spectrum of the three designs studied are presented in figure 5.12. The first comment is that the resonance peak stays at the same spot for the design SC, which is a comforting behavior since it is based on pure-lateral capacitors, with no electrical spring effect. Next, one can observe a moving resonance peak with respect to the DC voltage V_{DC} for the designs HC and GC. Using the equation 5.6, one can evaluate the electrostatic spring for each design. The results are shown in table 5.3 for a DC voltage V_{DC} of 1.5 V. One can notice a good agreement between our models and measurements.

More generally, one can notice the noise floor diminution with respect to increasing DC voltages. The best resolution can be obtained when the notch vanishes completely from measurements. In other words, the best resolution is obtained when the electrostatic spring matches the mechanical spring. This statement corroborates the design analysis performed in section 4.6.

V_{DC} 1.5 V	Design HC	Design GC
Measured K_{el} [N.m]	5×10^{-6}	7.5×10^{-6}
Theoretical K_{el} [N.m]	6×10^{-6}	8.6×10^{-6}

Table 5.3: Comparison between measured and simulated values of the electrostatic spring

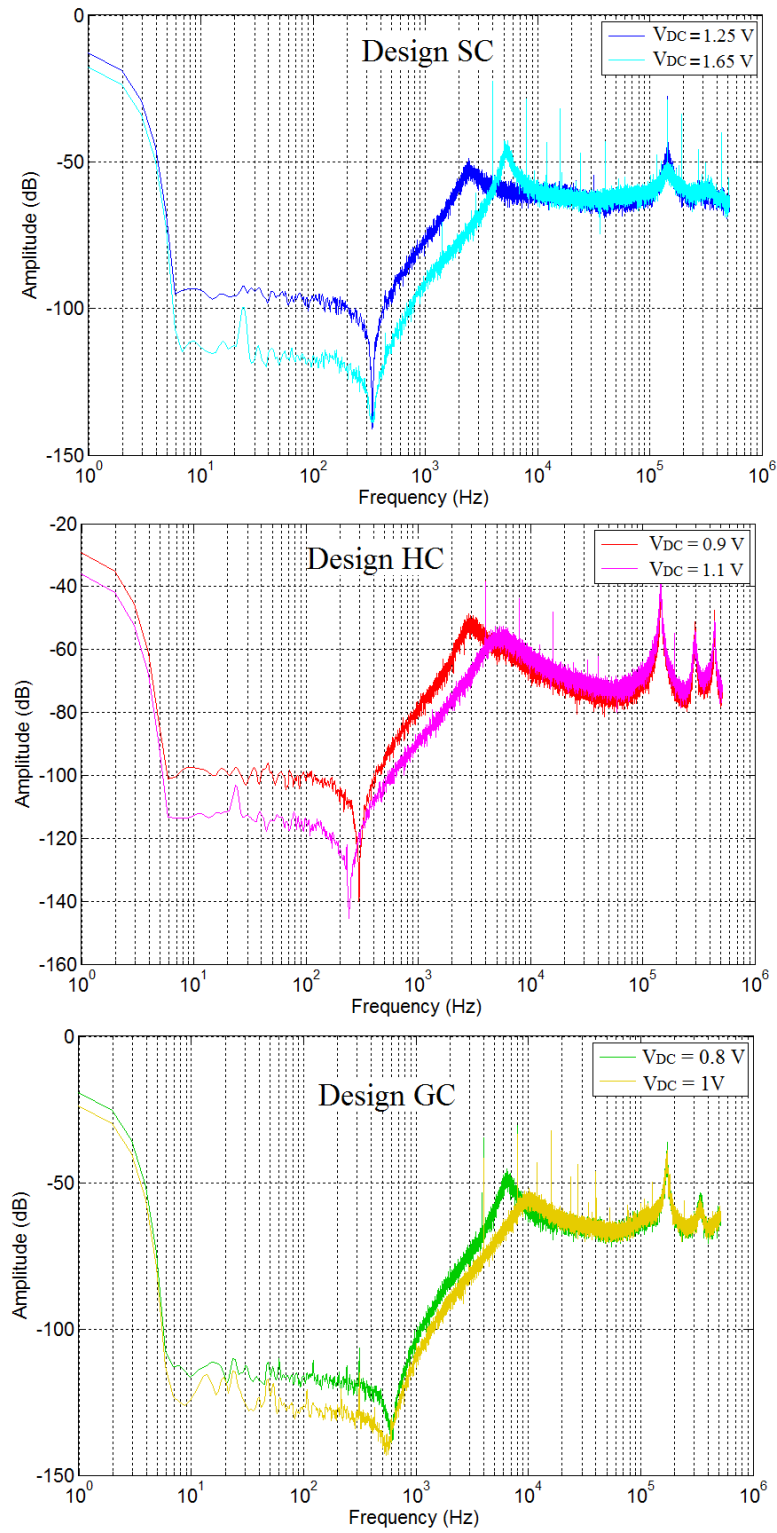


Figure 5.12: Influence of the DC voltage on the PSD response. Top: Design SC; Middle: Design HC; Bottom: Design GC

5.4.2 Experimental set-up for performance characterization

In this section, the DC voltage V_{DC} is set at 1.5 V to ensure the matching of the electrostatic spring with the mechanical one, thus allowing better resolutions. In order to determine the performance of the rotation sensor designs, we use a high resolution rate table capable of generating low-frequency, low-amplitude angular acceleration signals. This rate table as well as the electronic board are shown in figure 5.13.

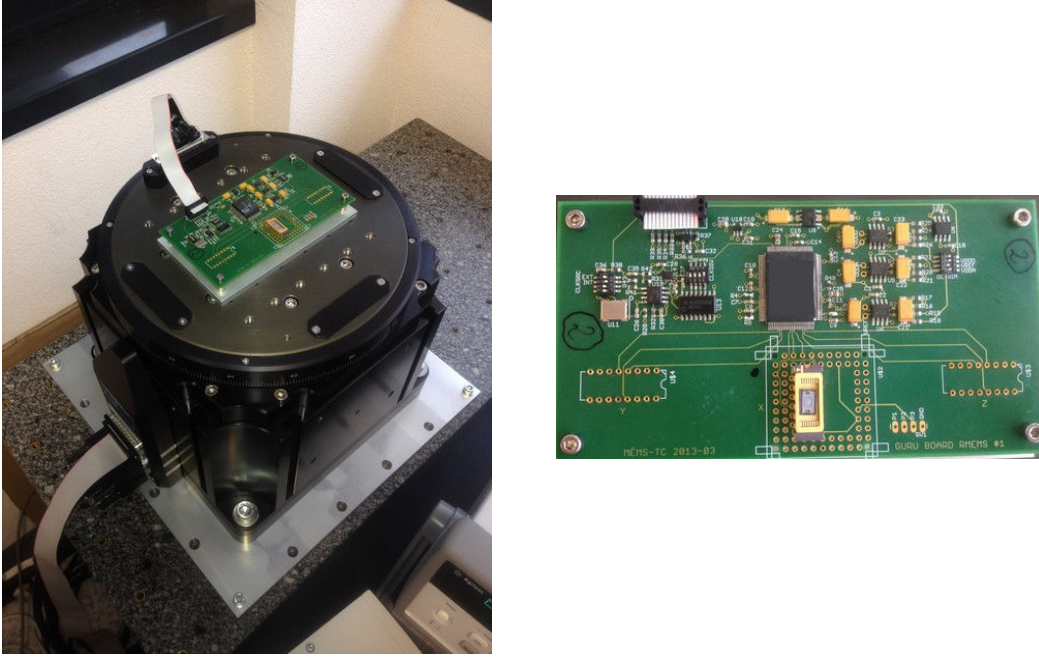


Figure 5.13: Set-up used for the closed-loop characterization of the rotation sensor. Left: the rotation table; Right: the electronic board with the MEMS element

The performance parameters evaluated are the full-scale and the noise floor:

- The full-scale is the maximum external excitation that can be theoretically controlled by the MEMS.
- The noise floor is the minimum angular acceleration that can be measured.

The experimental design developed to characterize the prototypes is organized as follows:

1. A pure sine input from the rate table is applied to the MEMS rotation sensor. The amplitude of angular accelerations is set at 10 rad/s^2 at a frequency of 40 Hz .

2. The RMS amplitude of the resulting peak in the PSD spectrum is measured (figure 5.14). The ratio of this measured value with the acceleration amplitude gives the scale-factor of the prototype.
3. This scale factor is then used to evaluate the full-scale which is obtained for the maximum amplitude of the bitstream output at -3 dB.
4. Finally, the PSD spectrum is multiplied by the full-scale to express it in mechanical units: $mrad.s^{-2}.Hz^{-1/2}$.

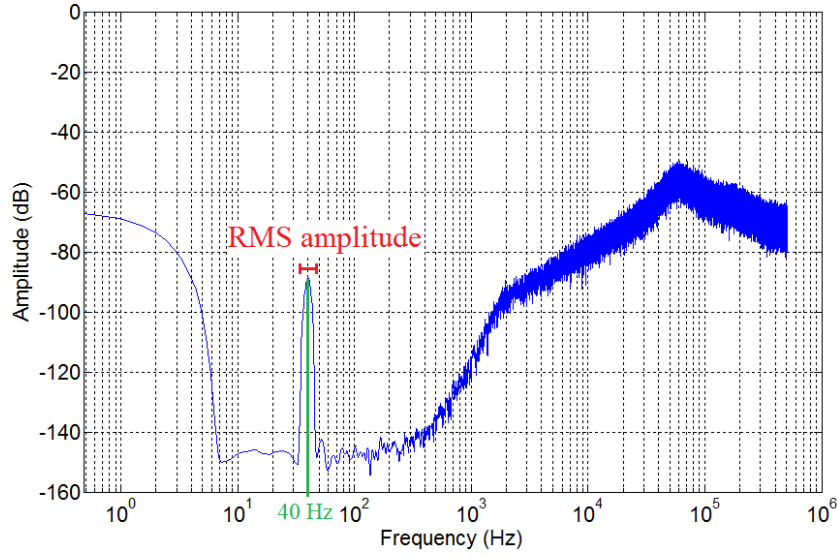


Figure 5.14: Experimental design for performance evaluation

As a remark on the experimental design, the expression relating the Noise Density (ND) expressed in $mrad.s^{-2}.Hz^{-1/2}$ and the noise density expressed in $V.Hz^{-1/2}$ is:

$$\text{Noise Density} \left[mrad.s^{-2}.Hz^{-1/2} \right] = \text{FS} \times \underbrace{\text{Noise Density} \left[V.Hz^{-1/2} \right]}_{\text{Measured PSD}} \quad (5.7)$$

where FS is the full-scale of the MEMS prototype.

In conclusion, high resolution rotation sensor needs a low noise floor but also a low full-scale to measure the smallest angular acceleration levels.

5.4.3 Experimental performance of rotation sensor prototypes

The measured PSD, with no external rotation input, are shown in figure 5.16 for the three rotation sensor prototypes. Following the experimental design detailed in the previous section, the performance of prototypes is finally established. The measured performances are presented in table 5.4 and the noise density plot is presented in figure 5.15.

Performance parameters	Design SC	Design HC	Design GC
Noise floor level $\left[dB(V.Hz^{-1/2}) \right]$	-110	-142	-140
Full-scale $\left[\frac{rad}{s^2} \right]$	1660	6600	12170
Resolution $\left[\frac{mrad}{s^2\sqrt{Hz}} \right]$	4	0.4	1

Table 5.4: Measured performances of the three designs studied

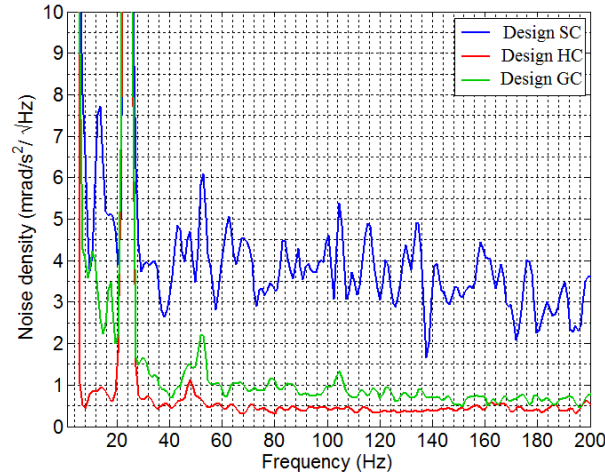


Figure 5.15: noise density of the three MEMS designs

On one hand, the pure sliding case (design SC) has a relatively low full-scale but, as it does not take benefit from the electrostatic spring effect, the mechanical resonance is not canceled and the noise floor is higher with a value of $4 \text{ mrad/s}^2/\sqrt{Hz}$. On the other hand, the gap-closing case (design GC) has an important electrostatic spring along with a large full-scale. In this case, the resonance frequency is canceled and the resolution achieved is $1 \text{ mrad/s}^2/\sqrt{Hz}$. The

best result comes from the hybrid case (design HC) which demonstrates a noise floor at the same level as design GC. However, the full-scale of design C is twice as low as the full-scale of design GC, resulting in a better noise density at $400 \mu\text{rad}/s^2/\sqrt{\text{Hz}}$.

To maximize the performance of the rotation sensor, one has to cancel the resonance frequency by the use of the electrostatic spring introduced by gap-closing electrodes. However, one has to limit also the full-scale so that the resolution is improved as shown in equation (5.7). As a result, the comb architecture developed in design HC is better for high resolution purpose since it cancels the mechanical resonance while offering a lower full-scale.

In conclusion, the RMS resolution over the frequency band 60 Hz 200 Hz is presented in table 5.5 and compared with the specifications given in section 2.4. The challenge proposed in this PhD dissertation has been overcome by achieving the design of a high-performance angular accelerometer based on MEMS technology.

	Design SC	Design HC	Design GC	Specifications
Resolution $\left[\frac{\text{mrad}}{\text{s}^2}\right]$ RMS	34	3	6	< 5
Power mW	35			< 50

Table 5.5: RMS resolution of the three designs studied

To complete the characterization of MEMS rotation sensors, a comparative study is performed with the characterization of other rotation sensing systems in the same laboratory conditions. This is the objective of the next sections.

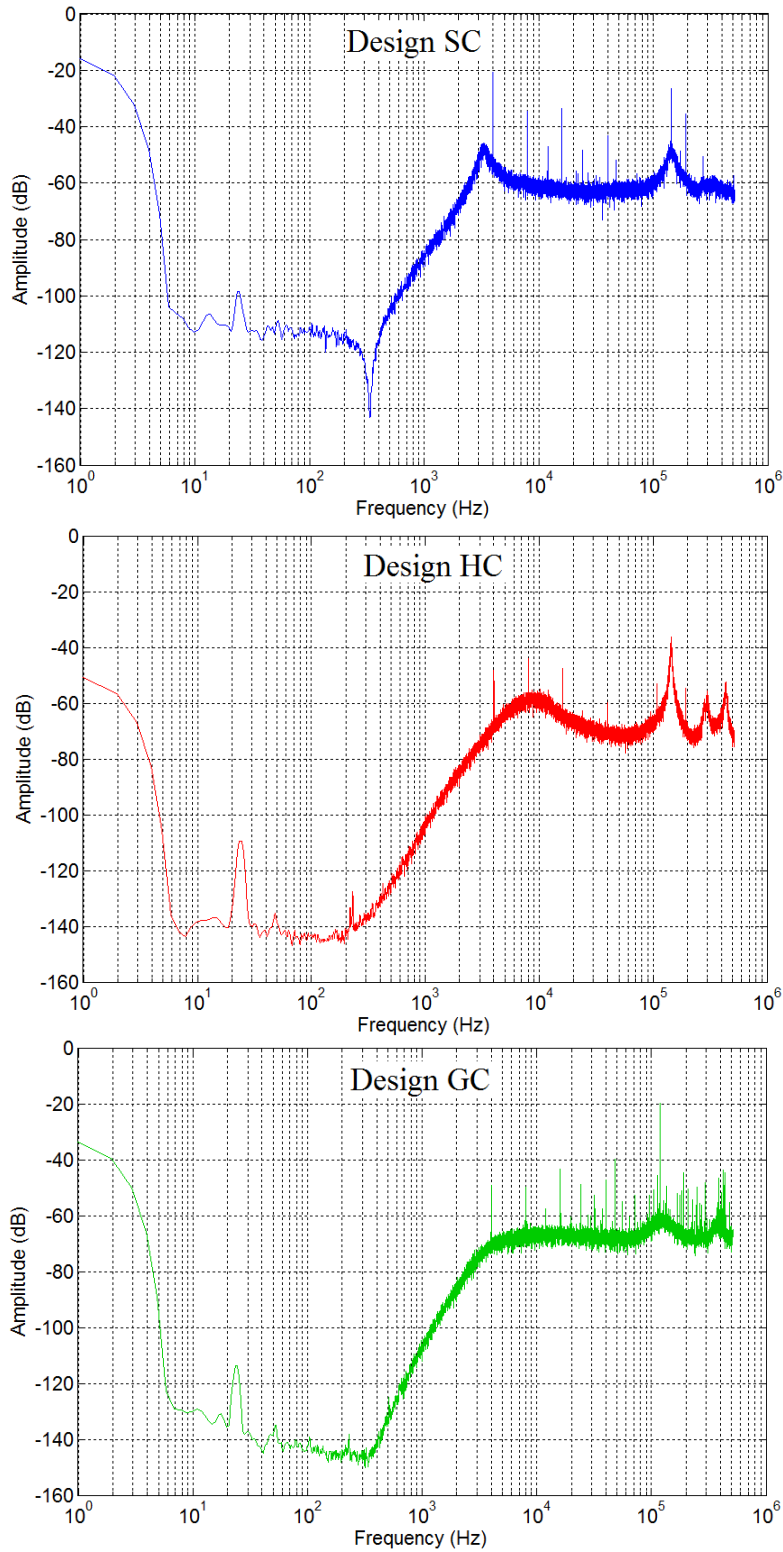


Figure 5.16: Optimized PSD responses with a DC voltage V_{DC} set at 1.5 V; Top: Design SC; Middle: Design HC; Bottom: Design GC

5.5 Comparison with other rotation sensors

The purpose of this section is to compare the MEMS rotation sensors developed in this research with other technologies. The comparison will be conducted in two steps: first, we will compare the rotation MEMS with measurements from coupled MEMS accelerometers in order to recreate in lab the setup proposed in section 2.3. Then, in a second step, we will compare the MEMS rotation sensor with the electro-chemical sensor from Eentec presented in 2.4.3.

5.5.1 Coupled-MEMS accelerometers for angular measurements

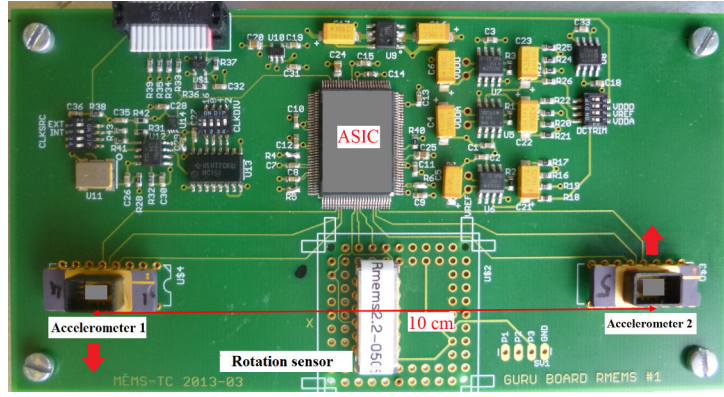


Figure 5.17: Electronic board with two MEMS accelerometers on both sides of a MEMS rotation sensor

The rotation sensor works with an ASIC which can be used also to control differential capacitive accelerometers as shown in figure 5.17. The two MEMS accelerometers are separated by a distance of 10 cm, with a rotation sensor in the middle. The information provided by these three sensors is constrained by the following equation:

$$\ddot{\phi} = \frac{a_2 - a_1}{L} \quad (5.8)$$

where $\ddot{\phi}$ is the angular acceleration measured by the rotation sensor, a_1 and a_2 are the horizontal accelerations measured by the accelerometers 1 and 2 respectively and L is the distance between the two accelerometers. According to equation 5.8, the resolution of the accelerometers and the distance L determines the resolution that can be obtained in the angular measurements. The PSD response of coupled MEMS accelerometers is presented in figure 5.18. It is interesting to observe that

this angular measurement is more sensitive to ambient noise with the presence of a lot of peaks in the frequency band: 10 Hz - 800 Hz compared to the rotation sensor.

Moreover, the accelerometers are calibrated so that the noise density of this type of measurement can be compared with the ones evaluated in the previous section. The plot is shown in figure 5.18. One can notice that angular measurements with coupled accelerometers challenge the best resolution achievable by rotation sensor prototypes. However, the sensitivity of accelerometers to translational noise represents the main limit of this type of measurements whereas the rotation sensor is designed to attenuate out-of-plane motions of the sensitive element thanks to high quadratic moment of the suspension beams. Unfortunately, cross-sensitivity analyses of the rotation sensor prototypes have not been performed during the thesis and it would represent an important measurement for future works on this subject.

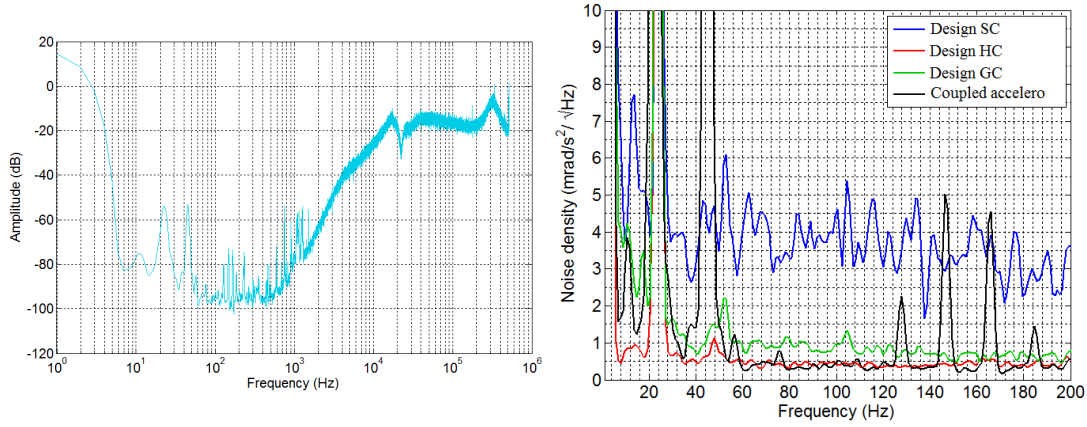


Figure 5.18: Analysis of coupled MEMS accelerometers for angular measurements; Left: PSD plot; Right: noise density plot

5.5.2 Electro-chemical sensor: R-1 Eentec

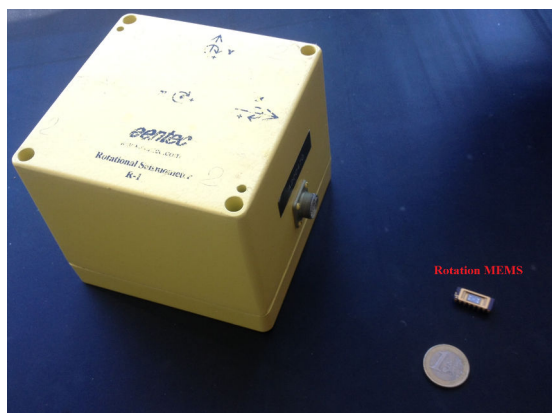


Figure 5.19: Presentation of the R-1 Eentec

The final test is to compare the MEMS rotation sensor prototypes with the reference sensor in rotational seismology: the rotational seismometer R-1 Eentec shown in figure 5.19. The calibration of the sensor is performed at 10 Hz and its noise density is computed in the same lab environment for comparison analysis with MEMS rotation sensor prototypes. The noise density plot is shown in figure 5.20. One can notice that the Eentec noise floor is far better than all other sensors with a value of $50 \mu rad/s^2/\sqrt{Hz}$. However, as explained in section 2.4.3, this solution is not suited for seismic exploration.

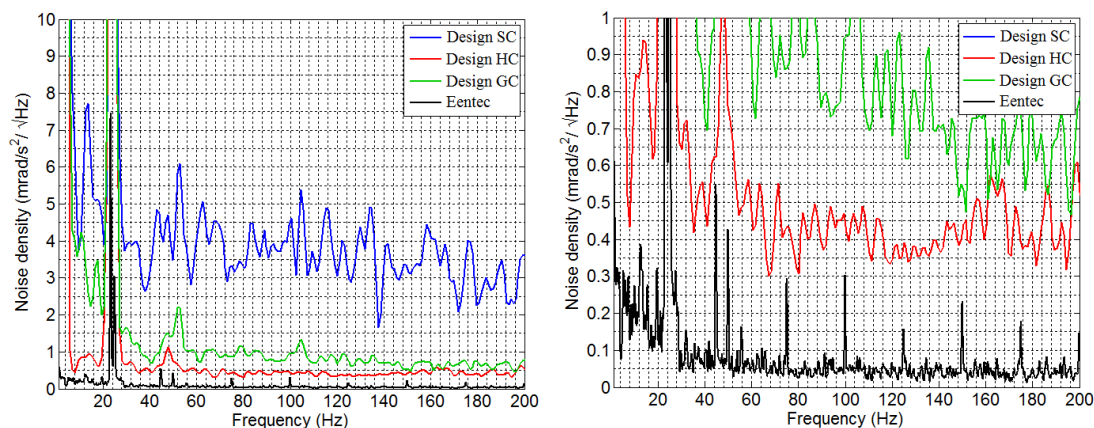


Figure 5.20: Comparison of the noise density of the Eentec with the ones found for the MEMS rotation sensor prototypes; Left: global plot; Right: Zoom on noise density below $1 mrad/s^2/\sqrt{Hz}$

5.5.3 Summary table

The RMS resolution over the frequency band 80 Hz 120 Hz is presented in table 5.6 and compared with the best design of MEMS rotation sensors.

	Dual MEMS accelerometers	R-1 Eentec	MEMS Rotation sensor
Resolution $\left[\frac{mrad}{s^2}\right]$ RMS	2	0.36	3

Table 5.6: RMS resolution of other rotation sensing systems

Given very tight specifications on power consumption, volume size and resolution, it has been achieved a new MEMS capacitive rotation sensor that can challenge the performances of other rotation sensing systems.

5.6 Conclusion on experiments

A detailed characterization of the rotation sensor was presented in this chapter. After updating the geometrical parameters modified by the etching process, it was possible to compare the electrical properties with the models presented in the previous chapter. It resulted in a good fitting of simulated capacitance variations with the measurements performed in lab. This confirmed important properties of the sensor such as the nominal capacitance and the mechanical stiffness. In a next step, frequency sweep analyses were performed to validate the electro-mechanical resonance frequency and the quality factor. It was seen that the analytical models overestimated the quality factor. Finally, after the characterization of the electrostatic spring needed to optimize the sensor response, the behavior of the MEMS in its control loop was studied and the performances were evaluated. The noise density plots revealed that the hybrid design using comb electrodes is the best architecture for high performance sensor. At last, the rotation sensor performances were compared to other technologies such as coupled accelerometers for angular measurements and an electro-chemical solution. The conclusion was that the rotation sensor developed in this dissertation is a good trade-off between performance and size.

Conclusions and perspectives

Summary of the work

In seismic exploration, a new sampling technique of the seismic wavefield was proposed requiring the deployment of rotation sensing systems at the free surface. In terms of requirements, these systems needed to be affordable, small-sized, highly sensitive and with high resolution. To tackle this challenge, a high-performance MEMS angular accelerometer was realized, reusing the electronic interface and the manufacturing process of high resolution MEMS linear accelerometers.

To understand the key features of designing high-performance MEMS rotation sensor, the capacitive detection and the force-feedback controller provided by the ASIC were studied with respect to two ideal sensing principles: the first one utilizing a change in electrode area (lateral capacitor) and the second one utilizing a change in electrode gap (gap-closing capacitor). The separation of the work with respect to the two capacitive sensing principles was chosen to analyze their respective advantages and shortcomings. At last, several design parameters characterizing the sensor behavior were identified for the two configurations studied. Furthermore, the modeling of mechanical and electrical noise sources allowed us to determine the influence of design parameters on the sensor resolution.

Considering the technical constraints imposed by the manufacturing process, the ASIC and the system packaging, two structures were designed based on the two sensing principles discussed. The thorough design of each structure started by a complete calculation of mechanical stiffness coefficients of supporting beams. Then, damping models developed in the literature were used to approximate the quality factor, which is related to mechanical noise. The last step of the design was the determination of the electrical parameters. Finally, an optimization of the sensor structures was performed to identify the best candidates for prototyping.

This optimization revealed that non-linear capacitors introduced an electrostatic spring which balanced the mechanical spring. The matching of this electrostatic spring with the mechanical one allowed us to obtain better performance than for linear capacitors. At last, three designs were selected: a pure sliding capacitor; a hybrid configuration mixing lateral capacitors and gap-closing capacitors; and a pure gap-closing capacitor.

The MEMS rotation sensor prototypes were micro-fabricated using an optimized deep reactive ion etching (DRIE) process of SOI wafers. Several characterization set-ups were employed to validate the model. The good fitting of measured capacitance-voltage characteristics with models was a rewarding observation. Similarly, the good agreement of the electro-mechanical resonance frequency and of the electrostatic spring coefficients proved the reliability of the analytical models. Thus, the measured PSD spectra of the rotation MEMS prototypes operating in closed-loop corresponded to the ones simulated during the modeling phase, with similar noise floor levels. The best performances in terms of noise floor level were obtained with the hybrid prototype. Indeed, the mechanical resonance cancellation by the electrostatic spring along with a lower full-scale than the pure gap-closing design resulted in a resolution of 3 mrad.s^{-2} RMS over the frequency band $60 \text{ Hz} - 200 \text{ Hz}$. In addition to meeting the requirements of seismic exploration, this resolution challenged other rotation sensing systems used in seismology.

Looking ahead

The challenging aspect of this thesis was to achieve the required resolution in spite of the several technical constraints summarized in table 4.1. For instance, the fixed volume size available to design the sensitive element limited the seismic mass dimensions, thus limiting the sensitivity and resolution achievable. Indeed according to equations in tables 3.1 and 3.2, large values of the structure inertia J_z result in lower mechanical noise Φ_{mn} and lower feedback gain K_{fb} , hence, lower full-scale. Consequently, an interesting improvement to implement will be to increase the seismic mass inertia.

Since the electrodes were used for the detection and the actuation, the maximization of the displacement-to-capacitance gain $K_{\theta-C}$ was needed for better capacitive detection. However, large values of $K_{\theta-C}$ resulted in large feedback gain K_{fb} , hence, important full-scale. Thus, one should separate the electrodes used for the detection and the ones used for the actuation. The first set would be designed to maximize the detection while the second set would be used to balance the seismic mass rotation with a limited feedback. However, this strategy would need a more complex ASIC.

Finally, the ultimate step will be intensive field testing of the MEMS rotation sensor to conclude on its capability of measuring the gradient of surface waves during seismic exploration.

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Appendix

Appendix A

Mathematical notions for frequency domain analysis

This appendix is a brief summary of mathematical notions for frequency domain analysis of vibrating mechanical system. These notions are used to derive mechanical noise in section [3.1.2](#).

Let us assume a time dependent function f integrable in the domain $L^1(\mathbb{R})$. For all real number ω , one can define the Fourier transform of f :

$$\widehat{F}(f)(\omega) = \int_{-\infty}^{\infty} e^{-j\omega t} f(t) dt \quad (\text{A.1})$$

It is an important notion in signal processing. A remarkable identity is given by the Parseval's theorem:

$$\int_{\mathbb{R}} |f|^2 dt = \frac{1}{2\pi} \int_{\mathbb{R}} |\widehat{F}|^2 dw \quad (\text{A.2})$$

assuming f a function of the domain $L^1(\mathbb{R}) \cap L^2(\mathbb{R})$.

Given these mathematical notions, one can establish a useful way to quantify noise process by using the average spectral density (or mean square distribution) of the noise in angular frequency space. For instance, let us consider the the noise function $V_n(t)$. The average spectral density of this noise function is:

$$\overline{V_n^2} = \left\langle |\widehat{V}_n|^2 \right\rangle \quad (\text{A.3})$$

where the angle brackets $\langle \dots \rangle$ define the statistical average.

Appendix B

Geometrical parameter values

B.1 Damping analysis of the sliding-comb design

Geometrical parameters used for the evaluation of the Q factor and the mechanical noise of the sliding-comb design in section 4.3.4..

Parameter	Value	Unit
Ring moment of inertia J_z	8×10^{-13}	$kg.m^2$
Mechanical natural frequency f_0	1	kHz
Air dynamic viscosity η	1.85×10^{-5}	$Pa.s$
Wafer thickness T	80	μm
Electrode width W_f	3	μm
Electrode overlap W_0	10	μm
Normal gap h_0	10	μm
Lateral gap d	2	μm
First electrode position R_1	1160	μm
Number of fingers N_f	41	
Number of capacitors N_c	52	

B.2 Damping analysis of the gap-closing design

Geometrical parameters used for the evaluation of the Q factor and the mechanical noise of the gap-closing design in section 4.3.4.

Parameter	Value	Unit
Ring moment of inertia J_z	8×10^{-13}	$kg.m^2$
Mechanical natural frequency f_0	1	kHz
Air dynamic viscosity η	1.85×10^{-5}	$Pa.s$
Wafer thickness T	80	μm
First extremity position R_1	1160	μm
Second extremity position R_2	1660	μm
Normal gap h_0	2	μm
Number of capacitors N_c	52	

B.3 Parameters of design SC

Geometrical parameters found in the optimization study in section 4.6 for a pure-sliding capacitor design.

Parameter	Value	Unit
Wafer thickness T	80	μm
Central anchor radius R_a	300	μm
Beam length L_b	600	μm
Beam width W_b	5	μm
Ring external radius R_e	1350	μm
Electrode length L_f	20	μm
Electrode width W_f	3	μm
Electrode overlap W_0	15	μm
Normal gap h	10	μm
Lateral gap d	2	μm
Number of fingers N_f	41	
Number of capacitors N_c	33	

B.4 Parameters of design HC

Geometrical parameters found in the optimization study in section 4.6 for a hybrid capacitor design.

Parameter	Value	Unit
Wafer thickness T	80	μm
Central anchor radius R_a	300	μm
Beam length L_b	550	μm
Beam width W_b	5	μm
Ring external radius R_e	1300	μm
Electrode length L_f	12	μm
Electrode width W_f	3	μm
Electrode overlap W_0	10	μm
Normal gap h	2	μm
Lateral gap d	2	μm
Number of fingers N_f	26	
Number of capacitors N_c	39	

B.5 Parameters of design GC

Geometrical parameters found in the optimization study in section 4.6 for a pure gap-closing capacitor design.

Parameter	Value	Unit
Wafer thickness T	80	μm
Central anchor radius R_a	300	μm
Beam length L_b	600	μm
Beam width W_b	5	μm
Ring external radius R_e	1350	μm
Normal gap h	2	μm
Number of capacitors N_c	30	

References

- [1] A. Sharma. *CMOS systems and circuits for sub-degree per hour MEMS gyroscopes*. PhD thesis, Georgia Institute of Technology, 2007.
- [2] G.J.M. Baeten, V. Belougne, L. Combee, E. Kragh, A. Laake, and J. Martin. Aquisition And Processing of Point Receiver Measurements in Land Seismic. In *62nd EAGE Conference & Exhibition*, May 2000.
- [3] K. Aki and P.G. Richards. *Quantitative seismology*. University Science Books, 2002.
- [4] R.E. Sheriff and L.P. Geldart. *Exploration Seismology Vol 1: History, Theory, and Data Acqistion*. Cambridge University Press, 1982.
- [5] E. Muyzert, A. Kashubin, E. Kragh, and P. Edme. Land Seismic Data Acquisition Using Rotation Sensors. In *74th EAGE Conference & Exhibition*, June 2012.
- [6] A. Papoulis. Generalized sampling expansion. *IEEE Transactions on Circuits and Systems*, 24(11):652–654, November 1977.
- [7] R. Teisseyre, J. Suchcicki, K. P. Teisseyre, J. Wiszniowski, and P. Palangio. Seismic rotation waves: basic elements of theory and recording. *Annals of Geophysics*, 46(4).
- [8] R. Cowsik, T. Madziwa-Nussinov, K. Wagoner, D. Wiens, and M. Wyssession. Performance Characteristics of a Rotational Seismometer for Near-Field and Engineering Applications. *Bulletin of the Seismological Society of America*, 99(2B):1181–1189, May 2009.
- [9] K.U. Schreiber, J.N. Hautmann, A. Velikoseltsev, J. Wassermann, H. Igel, J. Otero, F. Vernon, and J. P. R. Wells. Ring laser measurements of ground

- rotations for seismology. *Bulletin of the Seismological Society of America*, 99(2B):1190, 2009.
- [10] L. R. Jaroszewicz and Z. Krajewski. Application of fibre-optic rotational seismometer in investigation of seismic rotational waves. *Opto-Electronics Review*, 16(3):314–320, June 2008.
- [11] R. L. Nigbor, J. R. Evans, and C. R. Hutt. Laboratory and Field Testing of Commercial Rotational Seismometers. *Bulletin of the Seismological Society of America*, 99(2B):1215–1227, May 2009.
- [12] J. Brokesová and J. Málek. New portable sensor system for rotational seismic motion measurements. *The Review of scientific instruments*, 81(8):084501, August 2010.
- [13] M. Takeo. Ground rotational motions recorded in near source region of earthquakes. *Geophysical Research Letters*, 25(6):789, 1998.
- [14] R. L. Nigbor. Six-degree-of-freedom ground-motion measurement. *Bulletin Of The Seismological Society Of America*, 84(5):1665, 1994.
- [15] A. Gola, E. Chiesa, E. Lasalandra, F. Pasolini, M. Tronconi, T. Ungaretti, and A. Baschiroto. Interface for MEMS-based rotational accelerometer for HDD applications with 2.5 rad/s^2 resolution and digital output. *IEEE Sensors Journal*, 3(4):383–392, August 2003.
- [16] H. Paulson. Mems-based capacitive sensor, 2012. US Patent 8,104,346.
- [17] E. Muyzert. Seismic acquisition system and technique, 2010. US Patent App. 12/366,449.
- [18] J. Bernstein, R. Miller, W. Kelley, and P. Ward. Low-noise MEMS vibration sensor for geophysical applications. *Journal Of Microelectromechanical Systems*, 8(4):433–438, 1999.
- [19] M. Lemkin and B.E. Boser. A micromachined fully differential lateral accelerometer. In *Proceedings of Custom Integrated Circuits Conference*, pages 315–318. IEEE, 1996.

- [20] R.P. van Kampen and R.F. Wolffenbuttel. A critically-damped integrated capacitive accelerometer using feedback of position and velocity. In *[Proceedings] 1992 IEEE International Symposium on Circuits and Systems*, volume 4, pages 2005–2008. IEEE, 1992.
- [21] T.B. Gabrielson. Mechanical-thermal noise in micromachined acoustic and vibration sensors. *IEEE Transactions on Electron Devices*, 40(5), 1993.
- [22] A.N. Cleland and M.L. Roukes. Noise processes in nanomechanical resonators. *Journal of Applied Physics*, 92(5):2758, 2002.
- [23] H. Nyquist. Thermal Agitation of Electric Charge in Conductors. *Physical Review*, 32(1):110–113, 1928.
- [24] M. Bao. Air Damping. In *Analysis and Design Principles of MEMS Devices*, pages 115–174. Elsevier, 2005.
- [25] M. Bao. Electrostatic Actuation. In *Analysis and Design Principles of MEMS Devices*, pages 175–212. Elsevier, 2005.
- [26] H. Hammer. Analytical Model for Comb-Capacitance Fringe Fields. *Journal of Microelectromechanical Systems*, 19(1):175–182, February 2010.
- [27] B. Homeijer, D. Lazaroff, D. Milligan, R. Alley, J. Wu, M. Szepesi, B. Bicknell, Z. Zhang, R.G. Walmsley, and P.G. Hartwell. Hewlett packard’s seismic grade MEMS accelerometer. In *2011 IEEE 24th International Conference on Micro Electro Mechanical Systems*, pages 585–588. IEEE, January 2011.
- [28] S.D. Senturia. *Microsystem Design*. Kluwer Academic Publishers, 2000.
- [29] R.P. van Kampen, M.J. Vellekoop, P.M. Sarro, and R.F. Wolffenbuttel. Application of electrostatic feedback to critical damping of an integrated silicon capacitive accelerometer. *Sensors and Actuators A: Physical*, 43(1):100–106, 1994.
- [30] M. Lemkin and B.E. Boser. A three-axis micromachined accelerometer with a CMOS position-sense interface and digital offset-trim electronics. *IEEE Journal of Solid-State Circuits*, 34(4):456–468, April 1999.
- [31] L. Yuntao, L. Xiaowei, C. Weiping, and W. Qun. Design and noise analysis of a sigma-delta capacitive micromachined accelerometer. *Journal of Semiconductors*, 31(5):055006, May 2010.

- [32] H. Kulah, J. Chae, N. Yazdi, and K. Najafi. Noise Analysis and Characterization of a Sigma-Delta Capacitive Microaccelerometer. *IEEE Journal of Solid-State Circuits*, 41(2):352–361, February 2006.
- [33] X. Jiang, F. Wang, M. Kraft, and B.E. Boser. An Integrated Surface Micromachined Capacitive Lateral Accelerometer with $2\text{ }\mu\text{g}/\sqrt{\text{Hz}}$ resolution.
- [34] B.E. Boser and B.T. Howe. Surface micromachined accelerometers. In *Proceedings of the IEEE 1995 Custom Integrated Circuits Conference*, pages 337–344. IEEE, 1995.
- [35] D. Zhao. *A low-noise CMOS interface for capacitive microaccelerometers*. PhD thesis, Georgia Institute of Technology, 2009.
- [36] P.M. Aziz, H.V. Sorensen, and J. van der Spiegel. An overview of sigma-delta converters. *IEEE Signal Processing Magazine*, 13(1):61–84, 1996.
- [37] S. Park. *Principles of Sigma-Delta Modulation for Analog-to-Digital Converters*. Motorola.
- [38] M. Kraft, C.P. Lewis, and T.G. Hesketh. Closed-loop silicon accelerometers. *IEE Proceedings - Circuits, Devices and Systems*, 145(5):325, 1998.
- [39] B.E. Boser and B.A. Wooley. The design of sigma-delta modulation analog-to-digital converters. *IEEE Journal of Solid-State Circuits*, 23(6):1298–1308, 1988.
- [40] L. Yin, X. Liu, W. Chen, and Z. Zhou. High resolution interface circuit for closed-loop accelerometer. *Journal of Semiconductors*, 32(4):045005, April 2011.
- [41] S. Timoshenko and J.M. Gere. *Mechanics of materials*. CBS Publisher, second edition, 1972.
- [42] S. Timoshenko and J.N. Goodier. *Theory of Elasticity*, volume 49 of *Course of Theoretical Physics*. Butterworth-Heinemann, 1986.
- [43] W.E. Langlois. Isothermal squeeze films. *Quarterly of Applied Mathematics*, XX(2):131–150, 1962.
- [44] J.J. Blech. On Isothermal Squeeze Films. *Journal of Lubrication Technology*, 105(4):615, October 1983.

- [45] T. Veijola. Compact models for squeezed-film dampers with inertial and rarefied gas effects. *Journal of Micromechanics and Microengineering*, 14(7):1109–1118, 2004.
- [46] F. Sharipov and V. Seleznev. Data on internal rarefied gas flows. *Journal of Physical and Chemical Reference Data*, 27(3):657–706, 1998.
- [47] T. Veijola. End effects of rare gas flow in short channels and in squeezed-film dampers. *Technical Proceedings of the International Conference on Modeling and Simulation of Microsystems*, pages 104–107, 2002.
- [48] S. Vemuri. *Behavioral Modeling in Viscous Damping in MEMS*. PhD thesis, Carnegie Mellon, 2000.
- [49] Y. Cho, A.P. Pisano, and R.T. Howe. Viscous damping model laterally oscillating microstructures. *Journal Of Microelectromechanical Systems*, 3(2):81–87, 1994.
- [50] T. Veijola and M. Turowski. Compact damping models for laterally moving microstructures with gas-rarefaction effects. *Journal of Microelectromechanical Systems*, 10(2):263–273, June 2001.
- [51] M. Bao and H. Yang. Squeeze film air damping in MEMS. *Sensors and actuators. A, Physical*, 136(1):3–27, 2007.
- [52] T. Veijola. Simple but Accurate Models for Squeeze-film Dampers. In *2007 IEEE Sensors*, pages 83–86. IEEE, 2007.
- [53] V. Marinca and N. Herisanu. *Nonlinear Dynamical Systems in Engineering*. Springer Berlin Heidelberg, Berlin, Heidelberg, 2011.

Abstract - Résumé

Abstract

In seismic exploration, most of the signal acquired by point-receiver geophones is dominated by surface waves or ground rolls. Because they propagate in the near surface, ground rolls do not contain any information on deeper targets. Thus, short spacing between receivers is required so that this noise component can be accurately characterized and removed by digital filtering. However, considering the cost of seismic exploration ventures, new acquisition techniques using fewer point receivers and larger spacing have to be developed. Such a technique is briefly introduced in this dissertation, requiring accurate measurements of ground rotations at the free surface with minimum cost, weight and power consumption.

To address this need, the thesis proposes a high-performance rotation sensor based on MEMS technology. Unlike vibrating gyroscopes, sensitive to rotation rates through Coriolis effect, the solution developed is an angular accelerometer designed for differential capacitance measurements. A feedback controller is also implemented utilizing an oversampled $\Sigma\Delta$ -modulator to increase dynamic performances of the system. Thorough analytical designs along with simulations are challenged by fabricated prototypes measurements to achieve a high-sensitivity, high-resolution device. An experimental resolution of 3 mrad.s^{-2} RMS in the frequency band $60 \text{ Hz} - 200 \text{ Hz}$ is then obtained, which is far better than other micro-machined angular accelerometers from literature. Moreover, comparison analyses are performed with specific instruments used for rotational seismology to conclude on the feasibility of a MEMS-based rotation sensor for seismic exploration.

Keywords:

Seismic exploration, wavefield acquisition, MEMS technology, angular accelerometer, high performance, differential capacitor, force-feedback control, $\Sigma\Delta$ -modulation.

Résumé

Lors de la prospection sismique, un réseau de capteurs, utilisant principalement des géophones, est déployé à la surface libre afin d'enregistrer les ondes sismiques provenant du sous-sol. Cependant, l'énergie captée par ces géophones est largement dominée par les ondes de surface ou ondes de Rayleigh produites par la source. Étant donné leur nature, ces ondes de surface ne contiennent aucune information sur la composition des couches géologiques profondes. De ce fait, il est nécessaire d'employer un réseau très fin de capteurs dans le but de caractériser précisément ces composantes puis de les filtrer par des techniques de traitement du signal. Toutefois, les coûts engendrés nécessitent de nouvelles méthodes d'acquisition des ondes sismiques, employant moins de capteurs et permettant d'élargir le pas du réseau. Une telle technique a été mise en évidence, moyennant une mesure précise des rotations de la surface libre.

La piste explorée dans ce manuscrit est l'utilisation d'un capteur MEMS haute performance pour mesurer les rotations de la surface libre, avec un coût, un poids et une consommation électrique minimaux. Plus particulièrement, le choix s'est porté sur la réalisation d'un accéléromètre angulaire, mesurant la rotation d'entraînement de son référentiel. La conception du capteur MEMS proposé utilise une technique de mesure différentielle de capacités et un contrôle en boucle fermée reposant sur la modulation $\Sigma\Delta$. Un important travail de modélisation et de simulation a permis la fabrication de plusieurs prototypes qui ont ensuite été caractérisés. Une résolution fondamentale de 3 mrad.s^{-2} RMS dans une bande de fréquences comprises entre 60 Hz et 200 Hz a ainsi été obtenue. Les performances mesurées surpassent de loin celles d'autres accéléromètres angulaires de la littérature. Finalement, des analyses comparatives avec d'autres instruments de mesure ont permis de conclure sur la faisabilité de notre solution pour la prospection sismique.

Mots-clés:

Prospection sismique, mesure d'ondes, technologie MEMS, accéléromètre angulaire, haute performance, mesure différentielle de capacités, contrôle en boucle fermée, modulation $\Sigma\Delta$.